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Design and Implementation of a Driver for In-Wheel Brushless Motors for Unmanned Vehicles

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To my family and friends:

Thanks for supporting me while I'm learning to fly

— Arturo

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ACRONYMS

BEMF	Back Electromotive Force
PMAC	Permanent Magnet Alternating Current
PMSM	Permanent Magnet Synchronous Motor
BLDC	Brushless Direct Current
PCB	Printed Circuit Board
FOC	Field Oriented Control
DC	Direct Current
PP	Pole Pairs
PWM	Pulse Width Modulation
MOSFET	Metal–Oxide Semiconductor Field-Effect Transistor
IGBT	Insulated-Gate Bipolar Transistor
ADC	Analog-to-Digital Converter

ABSTRACT

The purpose of this thesis is to study the architecture of a commercial brushless motor driving circuit proposed by Texas Instruments and implemented as an electronic speed board as an "open source" project available online, analysing the advantages and disadvantages of such design regarding the implementation of trapezoidal and sinusoidal motor driving and speed and current control techniques for an unmanned vehicle designed for robotic agriculture. After this, the implementation of such driving and control techniques was physically carried out and tested.

This thesis explains in details the most important parts regarding the physical implementation of a motor driving system in such a way that it can be fully replicated. Chapter 1 contains some initial information regarding the electric motor history and the motivation to realize this work. In Chapter 2, we explain more in detail the different reasons why this work was developed and the focus points that were stressed out. In Chapter 3, a simple but sufficient explanation about the theory behind the electric motor is given, explaining also the different existing technologies and their particular driving methods. In Chapter 4 we explain the study case of ROBI', a prototype mobile manipulator for agricultural applications, which uses the in-wheel motor for which the motor driver of this work was developed. In Chapter 5 we explain in deep detail all the work developed around the implementation of the motor driver, both in the software and the hardware fields. In Chapter 6 we explain the final results of the work, showing and comparing graphs and waveforms of the behaviour of the in-wheel motor driven by the system developed. Finally, on Chapter 7, we discuss the results of the work realized and propose a new system, based on the implementation of the researched architecture and the problems encountered while working on the project.

Lo scopo di questa tesi è quello di studiare l'architettura di un circuito di guida del motore brushless commerciale proposto da Texas Instruments e implementato come un tabellone elettronico come un progetto "open source" disponibile online, analizzando i vantaggi e gli svantaggi di tale progetto per quanto riguarda l'implementazione di guida a motore trapezoidale e sinusoidale e tecniche di controllo di velocità e corrente per un veicolo senza pilota progettato per l'agricoltura robotica. Successivamente, l'implementazione di tali tecniche di guida e controllo è stata effettuata e testata fisicamente.

Questa tesi spiega in dettaglio le parti più importanti riguardanti l'implementazione fisica di un sistema di guida del motore in modo tale da poter essere completamente replicato. Chapter 1 contiene alcune informazioni iniziali sulla storia del motore elettrico e la motivazione per realizzare questo lavoro. Nel capitolo 2, spieghiamo più in dettaglio i diversi motivi per cui questo lavoro è stato sviluppato e gli aspetti principali che sono stati evidenziati. In Chapter 3 viene fornita una spiegazione semplice ma sufficiente sulla teoria alla base del motore elettrico, che spiega anche le diverse tecnologie esistenti e i loro particolari metodi di guida. Nel capitolo 4 spieghiamo il caso di studio di ROBI', un prototipo di manipolatore mobile per applicazioni agricole, che utilizza il motore a ruote motrici per il quale è stato sviluppato il driver motorio di questo lavoro. In Chapter 5 spieghiamo dettagliatamente tutto il lavoro sviluppato attorno all'implementazione del driver del motore, sia nel campo del software che in quello dell'hardware. Nel capitolo 6 spieghiamo i risultati finali del lavoro, mostrando e confrontando grafici e forme d'onda del comportamento del motore a ruote motrici guidato dal sistema sviluppato. Infine, in Chapter 7, discutiamo i risultati del lavoro realizzato e proponiamo un nuovo sistema, basato sull'implementazione dell'architettura ricercata e sui problemi incontrati durante il lavoro sul progetto.

PREFACE

*Knowledge is only part of understanding.
Genuine understanding comes from hands-on experience.*

— Seymour Papert *Constructionism* 1991

Motor control is a topic that must be experienced personally to be understood. This is a characteristic of many other engineering topics: they need to be experienced by the engineer or by the student to be fully understood. All the theory behind the movement of the shaft of the electric motor, which explains all the different phenomena interacting to create mechanical motion from electrical energy, should be experienced to fully understand everything that is involved. This is the reason why the practical implementation of theoretical topics is always interesting. Practical implementation makes us realize that there are always challenges that might not be taken into account while they are being studied from books, and they represent new opportunities to drive research and development for improvement.

This text was written with the idea of becoming a guide on the development of a motor controller for further projects, including the explanation of the basic physical phenomena that acts on electric motors and the important parameters to consider for the prediction of its motion, to the implementation of the hardware and software of a driving circuit and a detailed explanation on the most important factors to bring a motor controller alive and to successfully drive a permanent magnet motor. Even if the development of the work was intended for a specific project, all the information related to the development of the motor control systems can be applied to different projects, which is why this work can be useful also as a reference for projects not related to the robotic agriculture.

— Arturo Montúfar Arreola

INTRODUCTION

Electrical science, too, by its fascination, by its promises of immense realizations, of wonderful possibilities chiefly in humanitarian respects, has attracted the attention and enlisted the energies of the artist; for where is there a field in which his God-given powers would be of a greater benefit to his fellow-men than this unexplored, almost virgin, region, where, like in a silent forest, a thousand voices respond to every call?

— Tesla *On Electricity* 1897

The electric motors made their first appearances in the middle of the XIX century right after the invention of the battery by the Italian physicist, chemist and inventor Alessandro Volta in 1800, the discovery of the generation of a magnetic field from an electric current by the Danish physicist and chemist Hans Christian Ørsted in 1820 and the invention of the electromagnet by the English physicist William Sturgeon in 1825 (Doppelbauer, 2012). After these foundations were laid, the development of a machine that generates mechanical power from electrical power has been improving day by day, and, along with that improvement, also its utility has been increasing.

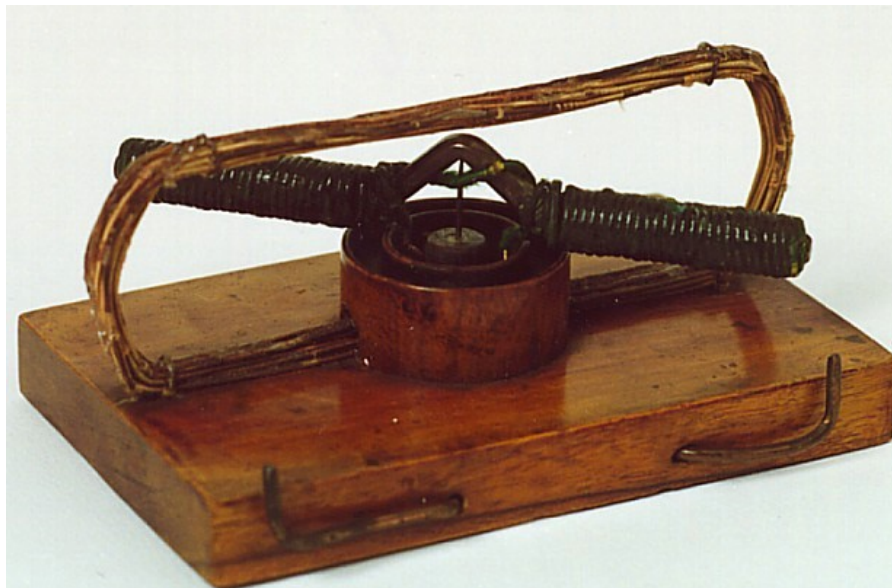


Figure 1.1: Jedlik's "Electromagnetic Self-Rotor". The historic motor created by the Hungarian physicist Ányos Jedlik still works perfectly today in the Museum of Applied Arts in Budapest.

Due to the reduction of the prices of metals and the improvement and automation of manufacture processes, electric motors became

available for a large range of applications, and not only as a research topic, up to the point that nowadays we interact with them in our daily life, sometimes without even noticing it. We have electric motors in all types of devices, from small applications like home appliances and hand-held gadgets, to large applications like robotics, cars and spaceships. As the complexity of the application increases, also the need for accuracy and efficiency does, leading to the development of more advanced electric motor technologies which lead to complex motion control techniques.

One of the most complex applications for motor control is robotics. The motion in a robotic system is part of its definition of automatic movement, therefore, a robotic system needs a predictable driving system to fulfil its purpose, which implies that most of the parameters of the motors inside the robot are known and that they can be controlled in a correct and precise way.

A challenging application regarding motors in robotics is the wheel driving, since it needs to be precise and powerful at the same time to transport the robotic system around large surfaces in an unmanned way, which means that there is no person on board and controlling the robot. For example, in robotic agriculture, which is one of the main reasons why this work was developed, a robot becomes an unmanned vehicle, which must transport itself around in farms, where the road represents harsh conditions for transportation, introducing the need for a precise drive to deal with small crops, a high torque to transport the robot in uneven grounds and a good range of speeds to displace itself in large field areas in the fastest way possible.

With aims to propose a solution for the land transportation in robotic agriculture projects, different types of motor technologies were studied a priori, finding out that in-wheel permanent-magnet brushless motors are one of the most suitable and currently used solutions to develop electric vehicles, since they don't need an external system for power transmission from the motor to the wheels, taking advantage of the high stall torque property of the electric motors and the reduction of the space and weight that having a motor inside a wheel represents (Bascetta, Baur, and Gruosso, 2017).

To correctly drive permanent-magnet brushless motors, it is necessary to apply a proper driving technique, which becomes a complex task when the goal is to achieve the desired characteristics mentioned previously: a good precision, a high torque and a large range of speed. Given such a challenge, the development of electric motor drives becomes one of the topics that draws the interest of many engineers and scientists, also due to the multidisciplinary approach needed to reach the speed, torque and efficiency required to drive the development that the inventions of tomorrow require. The approach to improve the motor drive in the electronics engineering field is directly related to the development of the driving circuit topology and to the imple-

mentation of complex control algorithms in embedded software that improve the performance of the different motor technologies available.

Since the brushless motor technology is quite recent in comparison with the rest, there is still a lot of development going on regarding its driving, with aims to reach the highest efficiency possible at the lowest cost. The objective of this work was to develop an electronic driver to control brushless motors, taking as a study case the in-wheel motors that transport a skid-steering robot designed for robotic agriculture. We studied the available electronic architectures to design a motor driver, the different approaches to drive brushless motors, like the trapezoidal and the sinusoidal drive and the peripherals needed to make these driving happen. After studying these topics, the motor driver was physically implemented and tested, and from this, a new architecture was proposed from the results and conclusions obtained from the implementation and testing of the circuit.



Figure 1.2: Rimac Automobili's electric supercar Concept S. Electric cars are becoming a trend and one of the main drivers for the development of technology related to electric motors.

This thesis explains in detail the most important information regarding the physical implementation of a motor driving system in such a way that it can be fully replicated. In Chapter 2, we explain more in detail the different reasons why this work was developed and the focus points that were stressed out. In Chapter 3, a simple but sufficient explanation about the theory behind the electric motor is given, explaining also the different existing technologies and their particular driving methods. In Chapter 4 we explain the study case of ROBI', a prototype mobile manipulator for agricultural applications, which uses the in-wheel motor for which the motor driver of this work was developed. In Chapter 5 we explain in deep detail all the work developed around the implementation of the motor driver, both

in the software and the hardware fields. In Chapter 6 we explain the final results of the work, showing and comparing graphs and waveforms of the behaviour of the in-wheel motor driven by the system developed. Finally, on Chapter 7, we discuss the results of the work realized and propose a new system, based on the implementation of the researched architecture and the problems encountered while working on the project.

PROBLEM STATEMENT

As motion applications complexity increases, like in the robotics field, the need for a more complex and robust driving system arises, and with it, the need to make decisions regarding the technologies to be implemented for them. For example, when a robotics project begins, with the objective to implement a solution that hasn't been implemented before, one of the main concerns is the motion system. The selection of the motor that is going to be used in a project is not an easy task, since many details must be considered, like the speed and torque required by the application, which lead to the selection of the motor technology, which might be limited by the available power source, the computing power and the economic budget of the project. These aspects have a strong dependency between each other and choosing each one of them represents a trade-off that must be studied in detail to reach an optimal result for the desired application.

2.1 TECHNOLOGICAL TRADE-OFFS

One of the main goals in the technology research and development has always been to reduce the trade-offs between the most important parameters in different engineering applications, using the most suitable technologies available, depending on the application requirements. One of the fields of engineering that has a big focus on the reduction of trade-offs is the motor drive research, since their development is crucial to drive innovation in many engineering fields.

A complicated trade-off to deal with is the cost of a motor drive, which is one of the most limiting characteristics project developments, both for research activities and for industrial and consumer applications. If a project is not expected to generate an income after its development or it's not backed up by a research institute with resources, or if the product is expected to be kept at a low price for many different reasons (like accessibility for low income communities or the possibility of large expansion), the price of a motor drive might be the weak side in a trade-off analysis and the development of the project might not be optimal, since it might require more work to reach an acceptable result.

2.2 LEARNING CURVE

The cost of a project leads to another important trade-off on engineering research and development: the learning curve. In some areas like robotics, a professional engineering project requires thousands of hours of man-work to reach a desirable result. An important amount of the time available for these projects is used in learning theory regarding the technology related to the project and how the available tools for such technology work. This time consumed in learning all the information needed regarding a project is called “learning curve”. The amount of work necessary to reach a good result makes most of the engineering projects expensive and, in many cases, a big part of that time is consumed by the learning curve, time that could be used for developing another part of the project, increasing the speed of the development and, therefore, reducing its cost.

2.3 MOTOR DRIVER DEVELOPMENT

With the aim to solve these two important trade-off factors in motor driving systems, we decided to study the possibility to reduce the cost of a motor drive by using a driver based on a cheap architecture which can be easily modified and allows to understand in a fast way how the motor driver works, making the development around it faster and easier for future projects.

To set up the design specifications for the motor driver to be studied, we based the requirements in the drive characteristics of a robot designed for robotic agriculture designed by the engineering group of the AIRLab of the Politecnico di Milano called ROBI' (Bascetta, Baur, and Gruosso, 2017). This robot is driven by four in-wheel brushless motors that will be described in detail in Chapter 4.

2.4 IMPLEMENTATION OF FIELD ORIENTED CONTROL

Having the objective defined, we looked for an already available platform to develop a motor driver with aims to look for points to improve and define a platform that could be used for different projects involving Permanent Magnet Alternating Current (PMAC) motors, both Brushless Direct Current (BLDC) and Permanent Magnet Synchronous Motor (PMSM). With this aim, we found a board called VESC Board, developed by the Swedish engineer Benjamin Vedder, who made available all the files for its production, including the schematic circuits, the bill of materials, the Printed Circuit Board (PCB) layout files and the source code. Vedder's circuit was very appealing since it had many ports available and it was based on a technology that is easy to

study since it's based in a design proposed by Texas Instrument for three-phase motors.

Since this motor driver was needed to be used in robotics applications, we had the necessity to modify the source code of the microcontroller to apply motor control methods that would be different to the ones available for the VESC board since the code available was designed to drive an electric skateboard. For this aim, we decided to develop our own source code, this way we would have control over everything developed by us and the expansion of the code would follow up without the need of doing reverse engineering over the code that was already available.

Since the motors of the wheels of the robot are [PMSM](#), they have a sinusoidal configuration, and therefore, a sinusoidal waveform was needed to be applied into the motor to reach an optimal efficiency and to control the speed even in low speeds. This specification pointed us towards the implementation of the Field Oriented Control ([FOC](#)) method, which improves the torque capabilities of the motor at controlled low speeds as it will be explained in [Chapter 3](#).

MOTOR CONTROL THEORY

In this chapter, we will review some theoretical principles that concern us regarding motor control. First, we will review the physical principles that rule over the electric motors and we will explain the different motor technologies and its configurations, focusing in the Permanent Magnet Alternating Current (PMAC) motors. Later, we will review the motor drives, the configuration of a driver and the different driving techniques for the PMAC motors. Finally, we will explain the control methods that can be applied to them.

Most of the theory in this chapter was taken from the book *Tecnologie dei sistemi di controllo*, by Magnani, Ferretti, and Rocco, 2007 and from the compilation *Performance and Design of Permanent Magnet AC Motor Drives*, by Bose et al., 1989.

3.1 ELECTRIC MOTOR

An electric motor is an electric machine that transforms electrical power (product between voltage and current):

$$P_{electrical} = V \cdot I \quad (3.1)$$

into mechanical power (product between torque and angular speed):

$$P_{mechanical} = T \cdot \omega \quad (3.2)$$

by means of the electromagnetic phenomena that takes place inside the motor, which is explained by the physical principles mentioned in this chapter.

3.1.1 Physical Principles

The generation of torque in electric motors is based in the interaction of two magnetic fields, one generated by magnets or windings placed in the stator and the other one generated by magnets or windings placed in the rotor as seen in Figure 3.1.

The physical laws that rule over the torque generation in the electric motor are mainly four: The Lorentz's Law, which helps us define the torque generated by an electric charge moving inside a magnetic field; Faraday's Law of Electromagnetic Inductance and the Lenz's Law, which explain the generation of the Back Electromotive Force (BEMF)

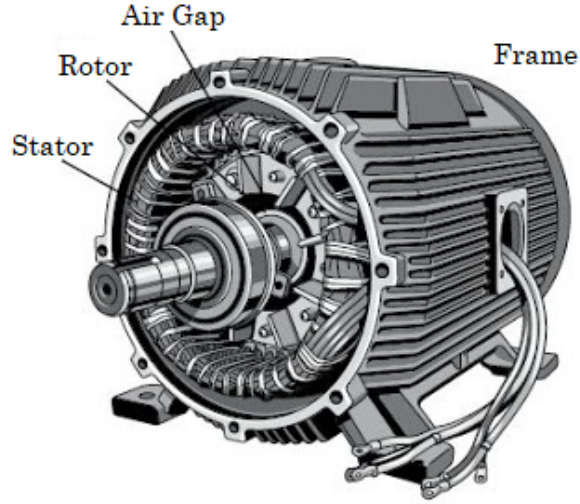


Figure 3.1: Partially Assembled Motor. An electric motor consists mainly in three parts: a rotor, which moves due to the electromagnetic interaction; a stator, the body of the motor; and a frame, which holds the rotor and the stator together. The air gap is the space between the rotor and the stator, where the electromagnetic interaction takes place (*Electrical Motors Basic Components*, 2012).

in the motor coils depending on the speed of the rotor and the influence of the magnetic field generated due to this BEMF respectively; and the Ampere-Laplace Law, which allows us to calculate the magnetic field of a current loop and the mechanical interaction between two magnetic fields.

LORENTZ'S LAW defines a force F , which acts over an electric charge q moving with a speed v inside a magnetic field with intensity B :

$$F = qv \times B \quad (3.3)$$

as seen in Figure 3.2. By defining a current I passing through a conductor with length l we can transform Equation 3.3 into Equation 3.4:

$$F = lI \times B \quad (3.4)$$

Considering the current I flowing through a conductive loop as the one in Figure 3.2 with sides lengths l and h we can see that there is a force F generated in the direction of the cross product of the current I and the magnetic field B . The maximum force F is generated in the sides of the loop where the direction of the current I is perpendicular to the direction of the magnetic field

B (ab and cd), while on the other two sides (ad and bc) the forces generated are cancelled with each other due to the direction of the current respect to the magnetic field.

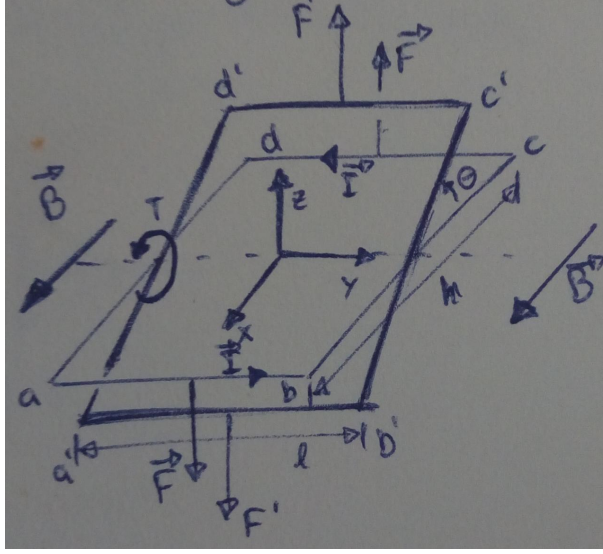


Figure 3.2: Visual explanation of the interaction of the current I and the magnetic field B generating a force F and, consequently, a torque T around the y axis (Magnani, Ferretti, and Rocco, 2007).

Since the forces F generated on the sides ab and cd have the same magnitude but different direction, they create a torque T around the y axis defined by the magnetic field, the current and by the length of the sides of the loop as:

$$T = IhlB \cos \theta \quad (3.5)$$

We can see in Figure 3.2 and in Equation 3.5 that when the angle θ between the sides ad or bc and the direction of the magnetic field B is $\pi/2$ radians, the torque T is zero and when the angle θ is zero or π radians, the torque reaches its maximum possible value.

The dependency of the torque created by the interaction of the current and the magnetic field on the angle between these two physical quantities introduces the need of changing the direction of the magnetic field or the direction of the current, hence, the polarity of the loop, to maintain the loop spinning and a non-zero torque.

FARADAY'S LAW OF ELECTROMAGNETIC INDUCTION states that

In every circuit under the effect of a magnetic field, an electromotive force is induced equal to the derivative respect to the time of the magnetic flux passing through the circuit, with negative sign (Jordan, 1968).

therefore, by indicating with E the electromotive force and with ϕ_m the magnetic flux, we have:

$$E = -\frac{d\phi_m}{dt} \quad (3.6)$$

If we consider a case like the one in Figure 3.2 we can calculate the magnetic flux passing through the loop as:

$$\phi_m = B \cdot u_N S = B \cdot u_N h l = h l B \sin \theta \quad (3.7)$$

where S is the surface of the loop and u_n is the direction normal to the plane of the loop. Therefore, we get an induced electromotive force of:

$$E = -\omega h l B \cos \theta \quad (3.8)$$

where ω is the angular speed of the loop. We can see, comparing Equation 3.8 and Equation 3.5, that the induced electromotive force depends on the angular speed in the same way than the acting torque depends on the current.

LENZ'S LAW can be explained after the explanation of the induced electromotive force. It states that the induced current in a loop has the direction that creates a magnetic field that opposes the change in magnetic flux through the area enclosed by the loop, therefore, the induced current tends to keep the magnetic flux ϕ_m from changing in the circuit.

If the rotation of the loop is generated by the circulation of a current inside a magnetic field, the induced electromotive force will try to oppose to the pass of the current, that's why it's normally referred to as Back Electromotive Force (**BEMF**).

AMPERE-LAPLACE LAW is the last piece to understand the transformation of electrical energy into mechanical energy. It allows us to calculate the magnetic field generated by a closed loop conducting current in a point defined by a vector p as:

$$B(p) = \frac{\mu_0}{4\pi} \oint I \frac{u_t \times u_r}{r^2} dl \quad (3.9)$$

where μ_0 is the vacuum magnetic permeability constant, I is the current circulating through the loop, u_t is the versor with direction of the current in the infinitesimal element dl and u_r

and r are versor and module that define the point p respect to the infinitesimal element of the loop.

Given that the magnetic fields can be generated both by permanent magnets and by current circulation, the electromechanical conversion is obtained due to the interaction of two magnetic fields according to the alignment principle, which states that

In a region of space which hosts two magnetic fields, there is a mechanical action that tends to align both fields. (Magnani, Ferretti, and Rocco, 2007)

If we consider the loop from Figure 3.2 and Equation 3.9, we can see that there is a magnetic field generated around the loop as seen in Figure 3.3, and due to the alignment principle, we will get the strongest coupling torque when the magnetic fields are perpendicular to each other.

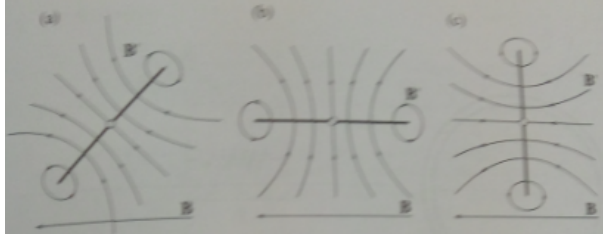


Figure 3.3: Visual explanation of the alignment principle. The alignment torque is the largest in the configuration of figure B and nule in the configuration of figure C.

In the case of electrical motors, there are two different magnetic fields generated in the airgap due to the permanent magnets or to the windings placed in the stator and the rotor which can be considered in radial direction, described by two magnetic fields $B_r(\theta, t)$ and $B_s(\theta, t)$, from which interaction we get the electromechanical conversion, since we have the generation of a torque which tends to align the two fields angles where they have the largest intensity. The alignment torque will be an expression of the type:

$$T_m = k B_r B_s \sin \delta \quad (3.10)$$

where δ is the de-phasing angle between the two fields and the maximum torque will be when $\delta = \pi/2$. In conclusion, by feeding the windings in the right way, we look forward to having a constant 90° de-phase between the two magnetic fields in aims to obtain the maximum torque generation.

In the case of the Direct Current (DC) motor, the perpendicularity condition between the magnetic fields is maintained by a polarity

commutating structure attached to the rotor which is connected to the windings where the current flows and generates the rotor magnetic field that tries to align itself to the magnetic field of the permanent magnets attached to the stator. This commutating system is connected to the power supply by metallic brushes that energise the motor until it reaches a certain angular position and the commutating lead changes to the next one, changing the polarity of the windings. Therefore, the torque obtained is independent from the position of the rotor and it's proportional to the amplitude of the power source.

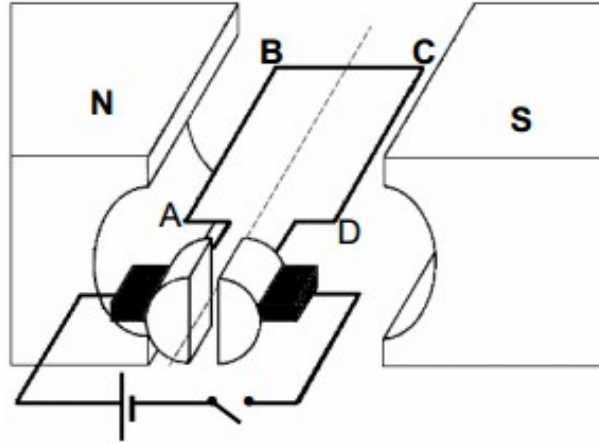


Figure 3.4: Basic schematic representation of the DC motor using the same loop represented in Figure 3.2

Brushless motors, which will be explained later in this chapter, the perpendicularity condition is maintained by feeding in the right time the windings in function of the angular position of the rotor θ , which is one of the main goals to be achieved and explained in this work.

Finally, after all the electromechanical phenomena that interacts inside the motor has been explained, we can obtain the electromechanical conversion between $P_{electrical}$ and $P_{mechanical}$.

If we clear from Equation 3.5, the torque T and the current I , grouping the rest of the variables into one constant, we can write down the following equation:

$$T = K_T I \quad (3.11)$$

where K_T is called Torque Constant and is defined by the geometry of the motor and the magnetic field in the airgap of the motor, which depend on the motor configuration and its construction technology. K_T units are Nm/A .

We can do the same with Equation 3.8, clearing the BEMF and the angular speed ω , getting the following equation:

$$BEMF = K_E \omega \quad (3.12)$$

where K_E is called Voltage Constant and it's defined also by the type of motor we are using. K_E units are $V/(rad/s)$.

If we consider the loop of Figure 3.2 as an equivalent circuit like the one in Figure 3.5, where the inductance L is generated due to the windings of the coil into the motor, and R is the resistance of the conductive material, we can write the Kirchhoff's Voltage Law on its terminals as:

$$v(t) = Ri(t) + L \frac{di(t)}{dt} + bmf(t) \quad (3.13)$$

which at steady state becomes:

$$V = RI + BEMF \quad (3.14)$$

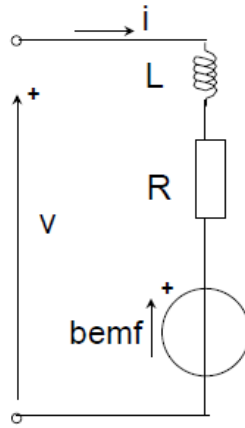


Figure 3.5: Equivalent circuit representation of a motor winding.

We can write down that the net power input into the system $VI - RI^2$ is equal to the electrical power absorbed by the BEMF:

$$P_{electrical} = BEMF \cdot I = K_E \omega I \quad (3.15)$$

and we can also see that the mechanical power can be written using these new constants:

$$P_{mechanical} = T\omega = K_T I \omega \quad (3.16)$$

The two constants, K_E and K_T , synthesize the most important parameters used in the electromechanical power conversion in an electric motor. If we impose a voltage difference between the terminals of the loop of Figure 3.5, we create a current I that depends on the loop impedance. From Equation 3.11, we can see that such current

will generate a proportional torque T . In the same way, from Equation 3.12, we obtain that the BEMF generated by the movement of the coil will define a proportional angular speed ω .

If we rearrange the terms from Equation 3.14 using Equations 3.11 and 3.12, we can see the interrelation between the rotating speed of the motor and the torque being generated:

$$\omega = \frac{V}{K_E} - \frac{R}{K_T K_E} T \quad (3.17)$$

which defines, by imposing a voltage difference between the terminals of the loop, the torque T vs angular speed ω curve that characterizes the mechanical behaviour of an electrical motor.

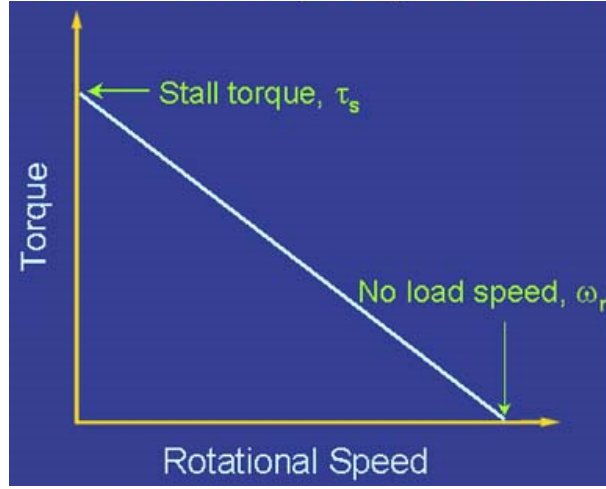


Figure 3.6: Basic representation of the Torque vs Angular Speed characteristic curve of an electric motor

If the speed of the motor ω is equal to zero, then we can rearrange the terms and see that the torque T is defined by $K_T V / R$, which is the maximum torque that the electric motor can provide by a given voltage, also named Stall Torque, T_0 . We can also define a no-load speed ω_0 , for the case when there is no load influencing over the movement of the rotor, as V / K_E .

The relationship between torque and angular speed established in Equation 3.17 will help us select the voltage that we need to regulate in order to control the dynamics of the motor.

It is important to mention that these equations also work if there is an imposed angular speed ω . If we impose an angular speed to the rotor, the motor works as a voltage generator:

$$V = K_E \omega + R \frac{T}{K_T} \quad (3.18)$$

therefore, if we connect the motor to a load, it generates a voltage proportional to the driving speed and provides a current proportional

to the torque driving the rotor. This behaviour is used in the so-called regenerative braking, since we can obtain energy when we want to stop a motor which is moving due to an inertial torque.

3.2 PMAC MOTORS

The Permanent Magnet Alternating Current (PMAC) motor is a kind of electrical motor that doesn't need the mechanical commutators mentioned in 3.1.1 to be driven as in the case of the DC motor, but its windings need to be energised in a specific way to function correctly. Since it doesn't need the mechanical commutators, it also doesn't need the brushes that energise the windings, so it can be said that it's a brushless motor. As seen in Figure 3.7, there are permanent magnets attached to its rotor and the coils are wound into its stator in a three-phase configuration.

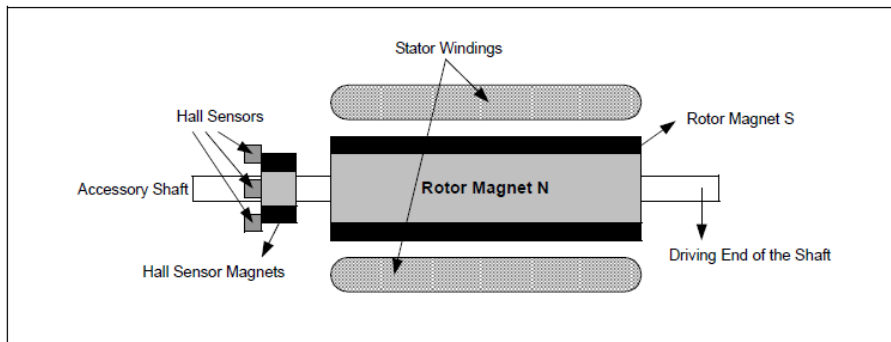


Figure 3.7: Brushless motor transverse section (microchip microchip).

These three phases are feed alternatively in such a way that the magnetic field, generated by the relative currents passing through the coils, should always be orthogonal and synchronous to the magnetic field generated by the rotor's permanent magnets. The characteristics mentioned above give the name to this kind of motors.

To maintain the synchronization, it's necessary to commute, by means of an inverter, the currents in the windings of the stator, taking as a reference the angular position of the rotor, which must be obtained by a sensor.

The number of commutations needed to generate one revolution of the rotor is determined by the number of times that each phase coil is wound in the stator. For example, if each phase coil of the three phase system is wound only once, one commutation would be enough to generate one revolution, but if each phase coil is wound 6 times, we would need to commute the power supply of the coils six times to generate one revolution. This ratio is called Pole Pairs (PP). Normally, the PMAC motors have many PP (six or more) in order to have a lower torque ripple since the alignment of the magnetic fields would be every $2\pi / (PP \times \Phi_N)$ radians of the rotor, where Φ_N is the number

of phases in the motors. For the electromechanical conversion, the angular position of the rotor is substituted by an electrical angular position, which is controlled by the commutator and is defined by:

$$\theta_{electrical} = PP \cdot \theta_{mechanical} \quad (3.19)$$

which also represents a relationship between the mechanical speed of the rotor and the electrical speed of commutation, which is defined by the commutation frequency:

$$\omega_{electrical} = f_{commutation} = PP \cdot \omega_{mechanical} \quad (3.20)$$

so, for example, if we have a motor with 6 PP and we want to drive it at 1kHz, we must commutate the polarity of the inverter at

$$f_{commutation} = PP \cdot \omega_{mechanical} = (6)(1kHz) = 6kHz \quad (3.21)$$

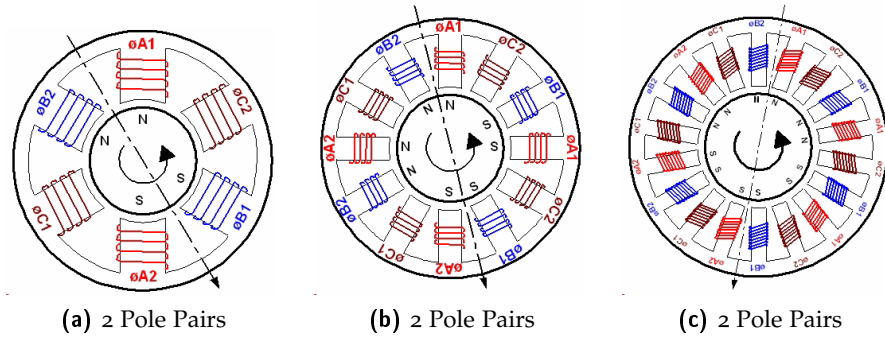


Figure 3.8: Different pole pair configurations

The main characteristics that make the brushless motor a better option for some applications than the DC motor are the following:

- better weight-to-power ratio,
- a more linear acceleration,
- a low inertia,
- a higher reliability,
- smaller dimension,
- reduced need for maintenance,
- high rotation speed,
- ideal for working in hostile environments

There are two disadvantages to this motor technologies: the first one is the need of a rotation sensor; the second one is the need of a complex logic to commutate the currents flowing through the coils. Both of these disadvantages are mainly reflected in a higher price respect to the [DC](#) motor.

It is possible to identify mainly two types of brushless motors. The first one is the Brushless Direct Current ([BLDC](#)) motor, which has a rotor position feedback that is not continuous, since the position of the rotor is given every 60 electrical degrees and its feed in blocks of 120 electrical degrees by simply alternating the voltage in the inverter, and due to these alimentation in blocks, the driving is rectangular, so the ideal [BEMF](#) is trapezoidal. The second type of brushless motors is called Permanent Magnet Synchronous Motor ([PMSM](#)), and it needs a continuous rotor position feedback to feed the motor with sinusoidal current, obtained by Pulse Width Modulation ([PWM](#)) of the [DC](#) bus, therefore the ideal [BEMF](#) is sinusoidal, which generates a lower torque ripple than the trapezoidal one, but needs a more complex control method.

3.2.1 *BLDC Motors*

The stator winding for each one of the three phases of the [BLDC](#) motor consists of a uniform distribution of turns over $N = PP$ sectors of a width equal to 60° . The magnets attached to the stator cover an arc of 180° , and at any instant, each magnet interacts, for 120° , with an arc of stator conductor carrying current.

Due to this "discrete" interaction every 120° , the three-phase switching between the currents of the stator should happen when the edge of the magnet attached to the rotor reaches the boundary between windings every 60° , therefore, a trapezoidal [BEMF](#) is generated in its coils.

The boundary of the rotor magnets with respect to the windings position is detected by three sensors, one every 120° , which send a signal to the driving circuit to change the polarity of the coils depending on the actual value of the three sensors and on the desired direction. The sensors to detect the angular position will be explained in [3.3](#) and the driving sequences will be explained in [3.4](#).

3.2.2 *PMSM Motors*

The stator of the [PMAC](#) motor is fitted with three-phase windings with $N = PP$ turns of each phase distributed sinusoidally around the periphery. If the stator windings are feed by sinusoidal currents, there is a linear current density around the stator periphery and a sinusoidal [BEMF](#) is generated in its coils.

Since the sinusoidal feeding of the current into the coils depends continuously on the magnetic field of the rotor permanent magnets, it is necessary to use an angular position sensor attached to the rotor. The sensors to detect the angular position will be explained in 3.3 and the driving algorithm will be explained in 3.4.

3.3 ELECTRIC DRIVES

An electric drive can be defined as an electromechanical device that converts electrical energy into mechanical energy to impart motion to different machines and mechanisms for various kinds of process control (Ghioni *Introduction to Electrical Drives*).

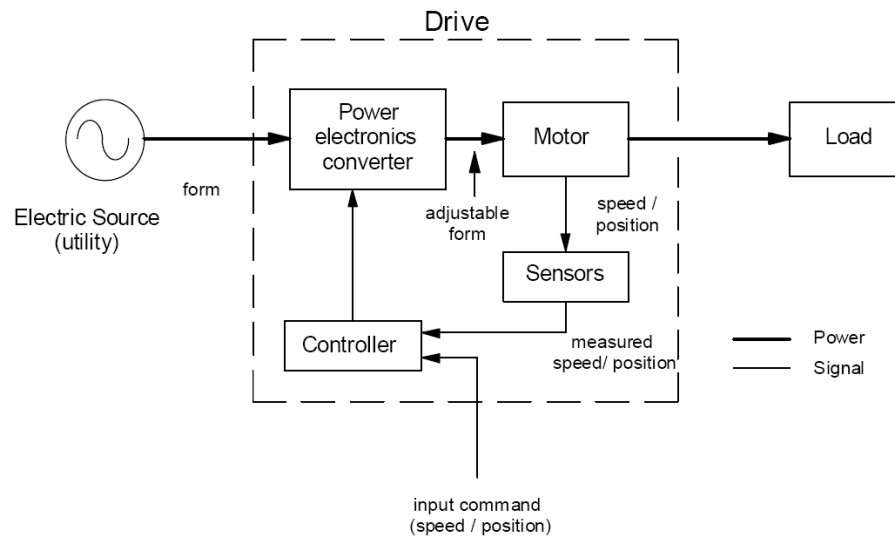


Figure 3.9: Electric Drive Scheme

The brushless motor drive, therefore, consists not only on the electric motor and the inverter, but it also includes the position, speed and current feedback systems, these being external sensors, embedded circuits or just algorithms, the vector, current, speed and position controllers and the DC power supply which can be a battery for mobile applications or a rectified power source for industrial applications.

3.3.1 Inverter

The commutation of the polarities in the windings of the PMAC motors is done by means of a three-phase inverter. The inverter used in PMAC motors avoids the need of the mechanical commutation used in the DC motors, which creates sparks between the commutator and

the brushes due to the discharge of the electromagnetic energy stored in the windings of the rotor.

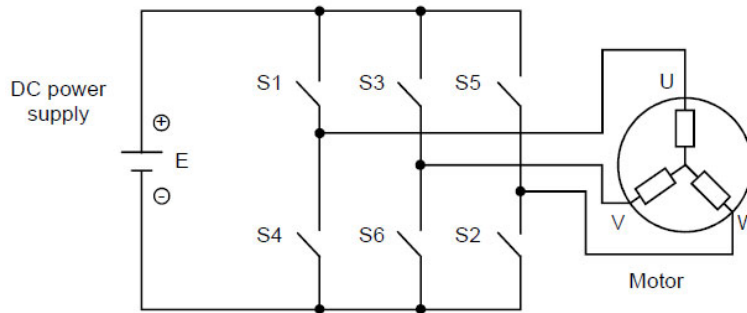


Figure 3.10: Schematic representation of a three-phase inverter

This three-phase inverter configuration consists on 3 branches with 2 switches each one. The middle point between the two switches in each branch is connected to each one of the phases of the motor. To feed the coils of the motor, we need to activate the switches in such a way that the current comes into one of the phases and comes out through another phase. For example, if we want to feed a current through phase *U* and we need it to come out through phase *V* to generate a magnetic field in a certain angle, we would enable switches *S1* and *S6*, having a phase-to-phase voltage in the *U* and *V* coils equal to V_{DC} . The correct feeding sequence should be synchronous with the rotor position to generate an angular rotation of the rotor with the lowest possible torque ripple.

The switching in the three-phase inverter is made by transistors as seen in Figure 3.11. The transistors are chosen accordingly to the power requirements of the application on which the inverter is being used. If the DC voltage is lower than 1000V and the current is lower than 100A, it is recommended to use the Power Metal–Oxide Semiconductor Field-Effect Transistor (MOSFET) technology. If the power consumption of the motor is larger, different technologies should be considered, like the Insulated-Gate Bipolar Transistor (IGBT).

The use of recirculation diodes is necessary to avoid damage in the transistors due to the overvoltage generated by the current transients in the windings LdI/dt that takes place between the switches of a same branch of the inverter while switching from one state to another. For example, when a transistor is suddenly turned off, the current flowing through the coils doesn't instantly disappear, but instead it recirculates through the diodes until it vanishes.

Since Power MOSFETs have large parasitic capacitances, they can't be driven by typical CMOS (0 to 3V) or TTL (0 to 5V) logic signals, which typically have a low current driving capability. Instead, to drive a power MOSFET is necessary to use a more complex driving circuit to rapidly charge the capacitance of the gate and reach a value of V_{GS} large enough to switch the device. Some integrated circuits provide

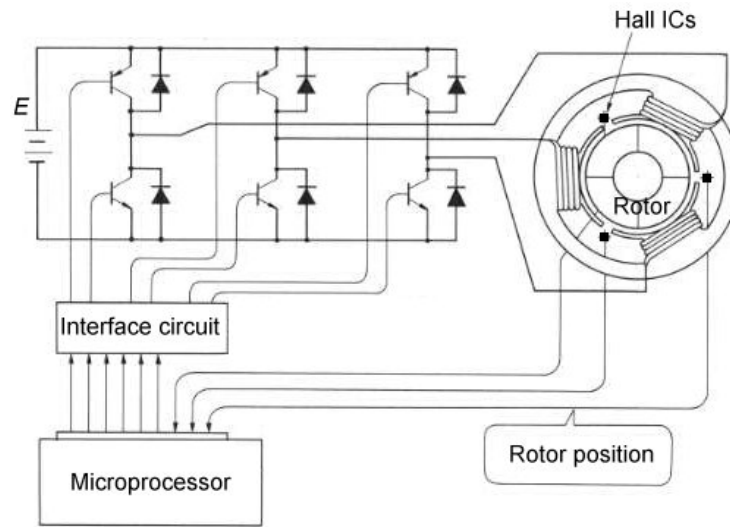


Figure 3.11: Schematic representation of a three-phase inverter built with transistors

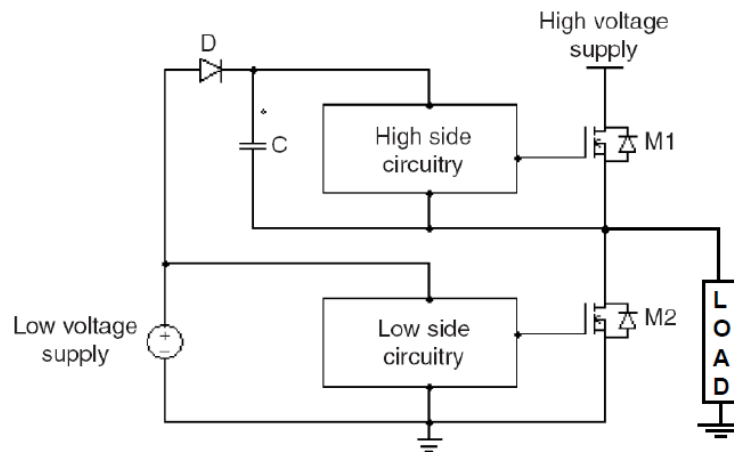


Figure 3.12: Power MOSFET devices require a dedicated circuit to provide enough voltage and current to drive their gates

solutions to solve this problem. These integrated circuits can be considered as part of the inverter, because without them, the inverter won't be driven correctly.

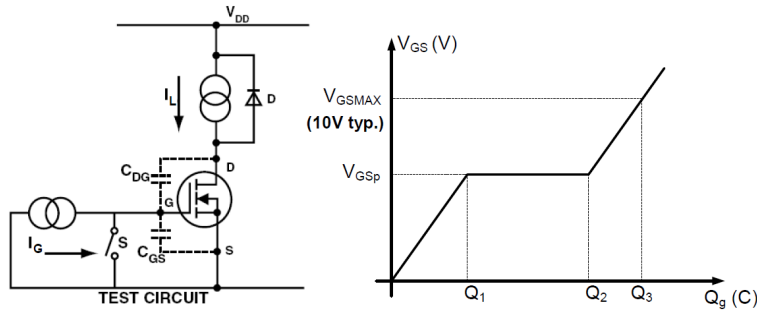


Figure 3.13: Schematic representation of the components influencing on the driving of a Power MOSFET

3.3.2 Angular Position Sensors

It is strictly necessary to obtain the angular position of the rotor in order to energise the coils of a **PMAC** motor in a synchronous way. The angular position detection can be made in different ways, but the solutions are mainly separated into two groups: sensed and sensorless.

The sensed angular detection is made by an external device, which creates a signal that allows us to know the angular position of the rotor. Typically two different approaches are used for the sensed angular detection. The first approach, and the simplest one, is the use of Hall effect sensors. Hall effect sensors are transducers that vary an output voltage signal in response to a magnetic field acting over them, and depending on their configuration the output can be digital (high or low signals) or analog (the output voltage varies proportionally to the detected magnetic field strength). In the case of the brushless motors, since the rotor has permanent magnets attached to it, three Hall effect sensors are mounted inside the motor frame in order to read the magnetic field of those permanent magnets and create a digital output that helps us determine the angular position of the rotor. The Hall effect sensors are mounted every 120 electrical degrees, so they can be mounted in different positions around the rotor, but they will change the output signal every 60 electrical degrees, depending on the pole orientation of the rotor. This approach is mainly used in **BLDC** motors, since they are wound in a trapezoidal distribution with aims to use only these simple sensors and an inverter.

The second approach to the sensed angular detection is the use of absolute rotary encoders. These encoders are attached to the shaft of the motor and, therefore, provide the angle of the rotor continuously. There are different technologies used for this approach. The simplest

one is the resistive encoder, which is practically a potentiometer attached to the rotor's shaft, so it is prone to mechanical disturbances and noise, but is very cheap. The next approach is the optical encoder, which identifies the absolute angular position by means of attaching a disk with holes or with a pattern and optical sensors, like infrared LEDs and infrared detectors. The optical encoder is a very precise approach, but is very prone to mechanical disturbances, so it is mostly used in applications where the conditions to the system don't represent a problem to the encoder. The approach that is being used in this project to detect the absolute angular position is the magnetic encoder, which also uses Hall effect sensors for the absolute angular position determination. The difference between these two approaches is that, in this case, the Hall effect sensors of this encoder provide an analog signal proportional to the angular position of the magnetic field of an external permanent magnet, which is attached to the shaft. Another interesting technology that is being used recently is the capacitive encoder, which detects the capacitance changes, proportional to the angular position, using a high frequency reference signal. The absolute angular position detection is used mainly in the [PMSM](#) technology, since it is necessary to know the angular position of the rotor continuously to create a sinusoidal signal, to drive the sinusoidally distributed coil windings, synchronously to the magnetic field of the permanent magnets attached to the rotor.

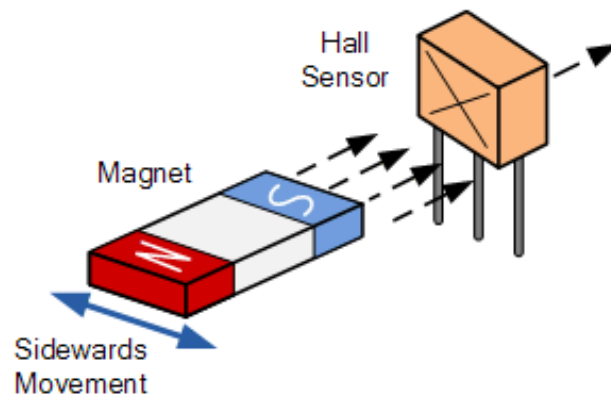


Figure 3.14: Hall Effect Sensor

One important thing that has to be considered when using absolute rotary encoders is that, even if these encoders provide the absolute angle of the rotor, they don't necessarily provide the angular position of the magnetic field of the rotor, introducing an angular slip $\Delta\theta$ which is equal to the difference between the value of the absolute rotary encoder angle and the angular position of the magnetic field. For this reason, the three Hall effect sensors used in the first sensed approach can be found also in [PMSM](#) motors and not only in [BLDC](#) motors, since they can provide the exact position zero of the magnetic field of the permanent magnets.

The sensorless approach to obtain the angular position of the rotor consists in reading the BEMF on an undriven motor terminal during one of the drive phases. This is done by means of connecting a voltage divider to the middle point of each inverter branch, in order to read the analog voltage signal proportional to the BEMF and calculate the angular position of the rotor. This solution can be used for both BLDC and PMSM technologies. This approach reduces the size and the cost of the motor drive, and it's also useful in applications where the rotor runs immersed in fluid. Even if this method is more complex than just reading the angular position, the main disadvantage is that the motor must be moving at a minimum rate to generate a BEMF that can be detected by the controller, therefore, this approach is not convenient where a motor must run at low speeds.

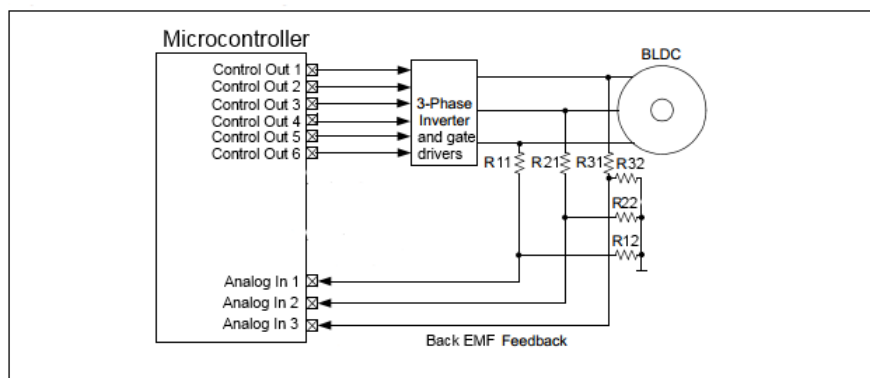


Figure 3.15: Sensorless Angular Detection Circuit

3.3.3 Current Sensors

Knowing the amount of current that is circulating through the motor windings can help us apply different control strategies to control the torque as defined in Equation 3.11. Also, reading the current can help us avoid problems of overcurrent if something goes wrong with the motor drive system.

To read the current we must install an external sensor which can be as complex as the application requires and the budget allows. The simplest approach to measure current, which is also the approach used in this project, is the use of a shunt resistor. The shunt resistor is an electrical resistor with a low (but well controlled) resistance value. This shunt resistor is placed in the path of the current flow of the motor's coils, in such a way that when the current flows through the

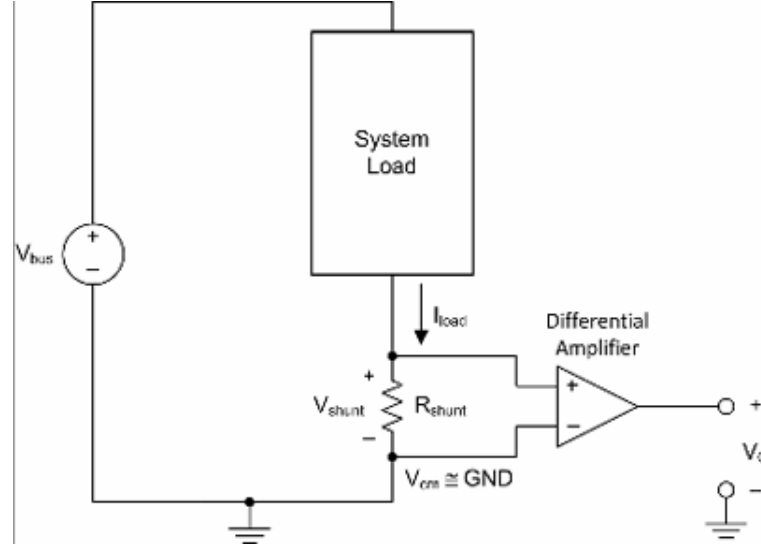


Figure 3.16: Measurement of the current flowing through a load using a shunt resistance

resistor, we have a proportional small voltage drop across it following Ohm's Law:

$$I_{\text{phase-to-phase}} = \frac{V_{\text{shunt}}}{R_{\text{shunt}}} \quad (3.22)$$

Since we need to take care of the power dissipated in the shunt resistor $P_{\text{shunt}} = I^2 R$, small resistance values R with large power dissipation capability are selected, leading to the problem that this generates a proportional voltage signal that is very small and that is prone to noise disturbances. For example, if we have a current of 100mA flowing through a shunt resistance of 0.001Ω , we would get a voltage drop across the shunt resistance equal to 0.1mV , which is a very small value to be read by a typical Analog-to-Digital Converter (ADC). To solve this problem, we need to include an amplifier that allows us to read such a small voltage with a normal ADC, therefore, we can calculate the equivalent current as:

$$I_{\text{phase-to-phase}} = \frac{V_{\text{amplifier}}}{R_{\text{shunt}} G} \quad (3.23)$$

where G is the gain of the voltage amplifier.

3.3.4 Controller

The last piece on the motor drive is the controller, which executes all the logic behind the commutation of the inverter, in order to generate a synchronous driving. Previously, motor controllers were made

by analog electronic circuits. Nowadays, these controllers are implemented using digital circuits that can be programmed, like microcontrollers.

The microcontroller has the task to run the logic that generates a driving signal for the inverter which will depend in the desired dynamic conditions and the actual conditions of the motor, which are sensed and stored by the microcontroller.

For example, if we need to drive a motor at a desired reference speed, we need to apply voltage into its coils. To generate a voltage that is synchronous to the angular position of the magnetic field, we will need to start by defining the absolute angular position of the shaft and, after this, we can apply a voltage into the coils by switching on the respective transistors of the inverter. This voltage will generate a current that will generate a torque and a magnetic field that will interact with the rotor, starting to spin it and, with this, generating a [BEMF](#). As the rotor spins, we must keep reading its angular position, to determine the correct driving voltage needed to keep it spinning. As the motor keeps moving, everytime faster, we need to calculate the speed at which the rotor is spinning, so as the rotor approaches the desired angular speed, we must increase or reduce the commutating frequency of the inverter. All these tasks must be made by the microcontroller in a matter of microseconds.

The microcontroller handles also the interface between other devices, allowing us to modify control parameters, like the desired speed or torque and to read different physical variables from the motor drive that are detected by the system.

3.4 PMAC MOTORS DRIVING METHODS

3.4.1 *Trapezoidal Drive*

3.4.2 *Sinusoidal Drive*

3.5 CONTROL METHODS

3.5.1 *Speed Control*

3.5.2 *Torque Control*

3.5.3 *Field Oriented Control*

3.5.4 *Speed Control with Field Oriented Control*

STUDY CASES

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4.2 RC VEHICLE

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PROJECT DEVELOPMENT

(...)

5.1 HYPOTHESIS

(...)

5.2 VESC BOARD

The electric drive subject of this study is based on an open source project developed by the Swedish engineer Benjamin Vedder called VESC - Open Source ESC, which consists in the hardware design of a Printed Circuit Board (PCB) and the source code used to drive a BLDC motor.

The VESC board was conceived and designed to be used in electric skateboards with the intention to create one of the best electronic speed controllers available. Since it was planned to be used in a skateboard, it has a small form-factor and it can be used for different applications with similar power demand. The hardware can also be modified by changing specific components to drive motors with higher power demand or by modifying the PCB following the schematic design.

The board consists of the following blocks, which will be explained in detail in this chapter:

1. Power supply input
2. Power MOSFETs three-phase inverter
3. MOSFET driver
4. Microcontroller unit
5. Peripherals
 - a) Temperature sensors
 - i. Motor temperature sensor
 - ii. Board temperature sensor
 - b) Hall effect sensors
 - c) RC Servomotor Output

d) Debug LEDs

6. Communication interfaces

- a) USB
- b) CAN
- c) USART
- d) SPI
- e) I2C

5.2.1 *Power Supply Input*

IMAGE

5.2.1.1 *Power Supply Range*

5.2.1.2 *Bulk Electrolytic Capacitor*

1. The largest amount of current required by the motor
2. The capacitance of the power supply and its ability to source current
3. The parasitic inductance between the power supply and the motor
4. The acceptable voltage ripple
5. The type of motor

5.2.1.3 *Power Supply Wiring*

FORMULA

5.2.2 *Power MOSFET Three-Phase Inverter*

5.2.2.1 *Voltage and Current Limits*

5.2.2.2 *Switching Times*

5.2.3 *MOSFET Driver*

5.2.4 *Microcontroller Unit*

High-performance MCUs with DSP and FPU instructions ARM Cortex-M4-based Up to 180 MHz operating frequency Up to 2 Mbytes Flash Up to 384 kB RAM Ethernet IEEE 1588 SDIO 16 and 32-bit timers 2 CAN ports Camera SDRAM SPI 12-bit DAC and ADC USB 2.0 OTG

5.2.5 PCB Layout

(...)

Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Donec odio elit, dictum in, hendrerit sit amet, egestas sed, leo. Praesent feugiat sapien aliquet odio. Integer vitae justo. Aliquam vestibulum fringilla lorem. Sed neque lectus, consectetur at, consectetur sed, eleifend ac, lectus. Nulla facilisi. Pellentesque eget lectus. Proin eu metus. Sed porttitor. In hac habitasse platea dictumst. Suspendisse eu lectus. Ut mi mi, lacinia sit amet, placerat et, mollis vitae, dui. Sed ante tellus, tristique ut, iaculis eu, malesuada ac, dui. Mauris nibh leo, facilisis non, adipiscing quis, ultrices a, dui.

5.3 MEASUREMENT BOARD

(...)

Morbi luctus, wisi viverra faucibus pretium, nibh est placerat odio, nec commodo wisi enim eget quam. Quisque libero justo, consectetur a, feugiat vitae, porttitor eu, libero. Suspendisse sed mauris vitae elit sollicitudin malesuada. Maecenas ultricies eros sit amet ante. Ut venenatis velit. Maecenas sed mi eget dui varius euismod. Phasellus aliquet volutpat odio. Vestibulum ante ipsum primis in faucibus orci luctus et ultrices posuere cubilia Curae; Pellentesque sit amet pede ac sem eleifend consectetur. Nullam elementum, urna vel imperdiet sodales, elit ipsum pharetra ligula, ac pretium ante justo a nulla. Curabitur tristique arcu eu metus. Vestibulum lectus. Proin mauris. Proin eu nunc eu urna hendrerit faucibus. Aliquam auctor, pede consequat laoreet varius, eros tellus scelerisque quam, pellentesque hendrerit ipsum dolor sed augue. Nulla nec lacus.

5.3.1 Voltage Measurement

(...)

Suspendisse vitae elit. Aliquam arcu neque, ornare in, ullamcorper quis, commodo eu, libero. Fusce sagittis erat at erat tristique mollis. Maecenas sapien libero, molestie et, lobortis in, sodales eget, dui. Morbi ultrices rutrum lorem. Nam elementum ullamcorper leo. Morbi dui. Aliquam sagittis. Nunc placerat. Pellentesque tristique sodales est. Maecenas imperdiet lacinia velit. Cras non urna. Morbi eros pede, suscipit ac, varius vel, egestas non, eros. Praesent malesuada, diam id pretium elementum, eros sem dictum tortor, vel consectetur odio sem sed wisi.

5.3.2 *Current Measurement*

(...)

Sed feugiat. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Ut pellentesque augue sed urna. Vestibulum diam eros, fringilla et, consectetur eu, nonummy id, sapien. Nullam at lectus. In sagittis ultrices mauris. Curabitur malesuada erat sit amet massa. Fusce blandit. Aliquam erat volutpat. Aliquam euismod. Aenean vel lectus. Nunc imperdiet justo nec dolor.

5.3.3 *Data acquisition*

(...)

Etiam euismod. Fusce facilisis lacinia dui. Suspendisse potenti. In mi erat, cursus id, nonummy sed, ullamcorper eget, sapien. Praesent pretium, magna in eleifend egestas, pede pede pretium lorem, quis consectetur tortor sapien facilisis magna. Mauris quis magna varius nulla scelerisque imperdiet. Aliquam non quam. Aliquam porttitor quam a lacus. Praesent vel arcu ut tortor cursus volutpat. In vitae pede quis diam bibendum placerat. Fusce elementum convallis neque. Sed dolor orci, scelerisque ac, dapibus nec, ultricies ut, mi. Duis nec dui quis leo sagittis commodo.

5.4 ANGULAR POSITION SENSOR

(...)

Aliquam lectus. Vivamus leo. Quisque ornare tellus ullamcorper nulla. Mauris porttitor pharetra tortor. Sed fringilla justo sed mauris. Mauris tellus. Sed non leo. Nullam elementum, magna in cursus sodales, augue est scelerisque sapien, venenatis congue nulla arcu et pede. Ut suscipit enim vel sapien. Donec congue. Maecenas urna mi, suscipit in, placerat ut, vestibulum ut, massa. Fusce ultrices nulla et nisl.

5.5 MIOSIX

(...)

Etiam ac leo a risus tristique nonummy. Donec dignissim tincidunt nulla. Vestibulum rhoncus molestie odio. Sed lobortis, justo et pretium lobortis, mauris turpis condimentum augue, nec ultricies nibh arcu pretium enim. Nunc purus neque, placerat id, imperdiet sed, pellentesque nec, nisl. Vestibulum imperdiet neque non sem accumsan

laoreet. In hac habitasse platea dictumst. Etiam condimentum facilis libero. Suspendisse in elit quis nisl aliquam dapibus. Pellentesque auctor sapien. Sed egestas sapien nec lectus. Pellentesque vel dui vel neque bibendum viverra. Aliquam porttitor nisl nec pede. Proin mattis libero vel turpis. Donec rutrum mauris et libero. Proin euismod porta felis. Nam lobortis, metus quis elementum commodo, nunc lectus elementum mauris, eget vulputate ligula tellus eu neque. Vivamus eu dolor.

5.6 TEST SETUP

(...)

Nulla in ipsum. Praesent eros nulla, congue vitae, euismod ut, commodo a, wisi. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Aenean nonummy magna non leo. Sed felis erat, ullamcorper in, dictum non, ultricies ut, lectus. Proin vel arcu a odio lobortis euismod. Vestibulum ante ipsum primis in faucibus orci luctus et ultrices posuere cubilia Curae; Proin ut est. Aliquam odio. Pellentesque massa turpis, cursus eu, euismod nec, tempor congue, nulla. Duis viverra gravida mauris. Cras tincidunt. Curabitur eros ligula, varius ut, pulvinar in, cursus faucibus, augue.

5.6.1 Robi Setup

(...)

Nulla mattis luctus nulla. Duis commodo velit at leo. Aliquam vulputate magna et leo. Nam vestibulum ullamcorper leo. Vestibulum condimentum rutrum mauris. Donec id mauris. Morbi molestie justo et pede. Vivamus eget turpis sed nisl cursus tempor. Curabitur mollis sapien condimentum nunc. In wisi nisl, malesuada at, dignissim sit amet, lobortis in, odio. Aenean consequat arcu a ante. Pellentesque porta elit sit amet orci. Etiam at turpis nec elit ultricies imperdiet. Nulla facilisi. In hac habitasse platea dictumst. Suspendisse viverra aliquam risus. Nullam pede justo, molestie nonummy, scelerisque eu, facilisis vel, arcu.

5.6.1.1 Marelli Generator as Load

(...)

Curabitur tellus magna, porttitor a, commodo a, commodo in, tortor. Donec interdum. Praesent scelerisque. Maecenas posuere sodales odio. Vivamus metus lacus, varius quis, imperdiet quis, rhoncus a, turpis. Etiam ligula arcu, elementum a, venenatis quis, sollicitudin

sed, metus. Donec nunc pede, tincidunt in, venenatis vitae, faucibus vel, nibh. Pellentesque wisi. Nullam malesuada. Morbi ut tellus ut pede tincidunt porta. Lorem ipsum dolor sit amet, consectetur adipiscing elit. Etiam congue neque id dolor.

5.6.2 RC Vehicle Setup

(...)

Donec et nisl at wisi luctus bibendum. Nam interdum tellus ac libero. Sed sem justo, laoreet vitae, fringilla at, adipiscing ut, nibh. Maecenas non sem quis tortor eleifend fermentum. Etiam id tortor ac mauris porta vulputate. Integer porta neque vitae massa. Maecenas tempus libero a libero posuere dictum. Vestibulum ante ipsum primis in faucibus orci luctus et ultrices posuere cubilia Curae; Aenean quis mauris sed elit commodo placerat. Class aptent taciti sociosqu ad litora torquent per conubia nostra, per inceptos hymenaeos. Vivamus rhoncus tincidunt libero. Etiam elementum pretium justo. Vivamus est. Morbi a tellus eget pede tristique commodo. Nulla nisl. Vestibulum sed nisl eu sapien cursus rutrum.

5.7 ALGORITHMS IMPLEMENTATION

(...)

Nulla non mauris vitae wisi posuere convallis. Sed eu nulla nec eros scelerisque pharetra. Nullam varius. Etiam dignissim elementum metus. Vestibulum faucibus, metus sit amet mattis rhoncus, sapien dui laoreet odio, nec ultricies nibh augue a enim. Fusce in ligula. Quisque at magna et nulla commodo consequat. Proin accumsan imperdiet sem. Nunc porta. Donec feugiat mi at justo. Phasellus facilisis ipsum quis ante. In ac elit eget ipsum pharetra faucibus. Maecenas viverra nulla in massa.

5.7.1 6-Step Drive Implementation

(...)

Nulla ac nisl. Nullam urna nulla, ullamcorper in, interdum sit amet, gravida ut, risus. Aenean ac enim. In luctus. Phasellus eu quam vitae turpis viverra pellentesque. Duis feugiat felis ut enim. Phasellus pharetra, sem id porttitor sodales, magna nunc aliquet nibh, nec blandit nisl mauris at pede. Suspendisse risus risus, lobortis eget, semper at, imperdiet sit amet, quam. Quisque scelerisque dapibus nibh. Nam enim. Lorem ipsum dolor sit amet, consectetur adipiscing elit. Nunc

ut metus. Ut metus justo, auctor at, ultrices eu, sagittis ut, purus. Aliquam aliquam.

5.7.2 *Field Oriented Control Implementation*

(...)

Etiam pede massa, dapibus vitae, rhoncus in, placerat posuere, odio. Vestibulum luctus commodo lacus. Morbi lacus dui, tempor sed, euismod eget, condimentum at, tortor. Phasellus aliquet odio ac lacus tempor faucibus. Praesent sed sem. Praesent iaculis. Cras rhoncus tellus sed justo ullamcorper sagittis. Donec quis orci. Sed ut tortor quis tellus euismod tincidunt. Suspendisse congue nisl eu elit. Aliquam tortor diam, tempus id, tristique eget, sodales vel, nulla. Praesent tellus mi, condimentum sed, viverra at, consectetur quis, lectus. In auctor vehicula orci. Sed pede sapien, euismod in, suscipit in, pharetra placerat, metus. Vivamus commodo dui non odio. Donec et felis.

5.8 SIMULATION?

(...)

Etiam suscipit aliquam arcu. Aliquam sit amet est ac purus bibendum congue. Sed in eros. Morbi non orci. Pellentesque mattis lacinia elit. Fusce molestie velit in ligula. Nullam et orci vitae nibh vulputate auctor. Aliquam eget purus. Nulla auctor wisi sed ipsum. Morbi porttitor tellus ac enim. Fusce ornare. Proin ipsum enim, tincidunt in, ornare venenatis, molestie a, augue. Donec vel pede in lacus sagittis porta. Sed hendrerit ipsum quis nisl. Suspendisse quis massa ac nibh pretium cursus. Sed sodales. Nam eu neque quis pede dignissim ornare. Maecenas eu purus ac urna tincidunt congue.

RESULTS

(...)

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetur id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

6.1 BLDC

(...)

Nam dui ligula, fringilla a, euismod sodales, sollicitudin vel, wisi. Morbi auctor lorem non justo. Nam lacus libero, pretium at, lobortis vitae, ultricies et, tellus. Donec aliquet, tortor sed accumsan bibendum, erat ligula aliquet magna, vitae ornare odio metus a mi. Morbi ac orci et nisl hendrerit mollis. Suspendisse ut massa. Cras nec ante. Pellentesque a nulla. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Aliquam tincidunt urna. Nulla ullamcorper vestibulum turpis. Pellentesque cursus luctus mauris.

6.1.1 Voltage Waveforms

(...)

Nulla malesuada porttitor diam. Donec felis erat, congue non, volutpat at, tincidunt tristique, libero. Vivamus viverra fermentum felis. Donec nonummy pellentesque ante. Phasellus adipiscing semper elit. Proin fermentum massa ac quam. Sed diam turpis, molestie vitae, placerat a, molestie nec, leo. Maecenas lacinia. Nam ipsum ligula, eleifend at, accumsan nec, suscipit a, ipsum. Morbi blandit ligula

feugiat magna. Nunc eleifend consequat lorem. Sed lacinia nulla vitae enim. Pellentesque tincidunt purus vel magna. Integer non enim. Praesent euismod nunc eu purus. Donec bibendum quam in tellus. Nullam cursus pulvinar lectus. Donec et mi. Nam vulputate metus eu enim. Vestibulum pellentesque felis eu massa.

6.1.2 *Current Waveforms*

(...)

Quisque ullamcorper placerat ipsum. Cras nibh. Morbi vel justo vitae lacus tincidunt ultrices. Lorem ipsum dolor sit amet, consectetur adipiscing elit. In hac habitasse platea dictumst. Integer tempus convallis augue. Etiam facilisis. Nunc elementum fermentum wisi. Aenean placerat. Ut imperdiet, enim sed gravida sollicitudin, felis odio placerat quam, ac pulvinar elit purus eget enim. Nunc vitae tortor. Proin tempus nibh sit amet nisl. Vivamus quis tortor vitae risus porta vehicula.

6.1.3 *Voltage Control Plots*

(...)

Fusce mauris. Vestibulum luctus nibh at lectus. Sed bibendum, nulla a faucibus semper, leo velit ultricies tellus, ac venenatis arcu wisi vel nisl. Vestibulum diam. Aliquam pellentesque, augue quis sagittis posuere, turpis lacus congue quam, in hendrerit risus eros eget felis. Maecenas eget erat in sapien mattis porttitor. Vestibulum porttitor. Nulla facilisi. Sed a turpis eu lacus commodo facilisis. Morbi fringilla, wisi in dignissim interdum, justo lectus sagittis dui, et vehicula libero dui cursus dui. Mauris tempor ligula sed lacus. Duis cursus enim ut augue. Cras ac magna. Cras nulla. Nulla egestas. Curabitur a leo. Quisque egestas wisi eget nunc. Nam feugiat lacus vel est. Curabitur consectetur.

6.1.4 *Current Control Plots*

(...)

Suspendisse vel felis. Ut lorem lorem, interdum eu, tincidunt sit amet, laoreet vitae, arcu. Aenean faucibus pede eu ante. Praesent enim elit, rutrum at, molestie non, nonummy vel, nisl. Ut lectus eros, malesuada sit amet, fermentum eu, sodales cursus, magna. Donec eu purus. Quisque vehicula, urna sed ultricies auctor, pede lorem egestas dui, et convallis elit erat sed nulla. Donec luctus. Curabitur et nunc. Aliquam dolor odio, commodo pretium, ultricies non, pharetra

in, velit. Integer arcu est, nonummy in, fermentum faucibus, egestas vel, odio.

6.2 PMSM

(...)

Sed commodo posuere pede. Mauris ut est. Ut quis purus. Sed ac odio. Sed vehicula hendrerit sem. Duis non odio. Morbi ut dui. Sed accumsan risus eget odio. In hac habitasse platea dictumst. Pellentesque non elit. Fusce sed justo eu urna porta tincidunt. Mauris felis odio, sollicitudin sed, volutpat a, ornare ac, erat. Morbi quis dolor. Donec pellentesque, erat ac sagittis semper, nunc dui lobortis purus, quis congue purus metus ultricies tellus. Proin et quam. Class aptent taciti sociosqu ad litora torquent per conubia nostra, per inceptos hymenaeos. Praesent sapien turpis, fermentum vel, eleifend faucibus, vehicula eu, lacus.

6.2.1 Voltage Waveforms

(...)

Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Donec odio elit, dictum in, hendrerit sit amet, egestas sed, leo. Praesent feugiat sapien aliquet odio. Integer vitae justo. Aliquam vestibulum fringilla lorem. Sed neque lectus, consectetur at, consectetur sed, eleifend ac, lectus. Nulla facilisi. Pellentesque eget lectus. Proin eu metus. Sed porttitor. In hac habitasse platea dictumst. Suspendisse eu lectus. Ut mi mi, lacinia sit amet, placerat et, mollis vitae, dui. Sed ante tellus, tristique ut, iaculis eu, malesuada ac, dui. Mauris nibh leo, facilisis non, adipiscing quis, ultrices a, dui.

6.2.2 Current Waveforms

(...)

Morbi luctus, wisi viverra faucibus pretium, nibh est placerat odio, nec commodo wisi enim eget quam. Quisque libero justo, consectetur a, feugiat vitae, porttitor eu, libero. Suspendisse sed mauris vitae elit sollicitudin malesuada. Maecenas ultricies eros sit amet ante. Ut venenatis velit. Maecenas sed mi eget dui varius euismod. Phasellus aliquet volutpat odio. Vestibulum ante ipsum primis in faucibus orci luctus et ultrices posuere cubilia Curae; Pellentesque sit amet pede ac sem eleifend consectetur. Nullam elementum, urna vel imperdiet sodales, elit ipsum pharetra ligula, ac pretium ante justo a nulla. Cur-

abitur tristique arcu eu metus. Vestibulum lectus. Proin mauris. Proin eu nunc eu urna hendrerit faucibus. Aliquam auctor, pede consequat laoreet varius, eros tellus scelerisque quam, pellentesque hendrerit ipsum dolor sed augue. Nulla nec lacus.

6.2.3 *Voltage Control Plots*

(...)

Suspendisse vitae elit. Aliquam arcu neque, ornare in, ullamcorper quis, commodo eu, libero. Fusce sagittis erat at erat tristique mollis. Maecenas sapien libero, molestie et, lobortis in, sodales eget, dui. Morbi ultrices rutrum lorem. Nam elementum ullamcorper leo. Morbi dui. Aliquam sagittis. Nunc placerat. Pellentesque tristique sodales est. Maecenas imperdiet lacinia velit. Cras non urna. Morbi eros pede, suscipit ac, varius vel, egestas non, eros. Praesent malesuada, diam id pretium elementum, eros sem dictum tortor, vel consectetur odio sem sed wisi.

6.2.4 *Current Control Plots*

(...)

CONCLUSION

(...)

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetur id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

7.1 BLA BLA BLA

(...)

Nam dui ligula, fringilla a, euismod sodales, sollicitudin vel, wisi. Morbi auctor lorem non justo. Nam lacus libero, pretium at, lobortis vitae, ultricies et, tellus. Donec aliquet, tortor sed accumsan bibendum, erat ligula aliquet magna, vitae ornare odio metus a mi. Morbi ac orci et nisl hendrerit mollis. Suspendisse ut massa. Cras nec ante. Pellentesque a nulla. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Aliquam tincidunt urna. Nulla ullamcorper vestibulum turpis. Pellentesque cursus luctus mauris.

7.2 FUTURE WORK

(...)

Nulla malesuada porttitor diam. Donec felis erat, congue non, volutpat at, tincidunt tristique, libero. Vivamus viverra fermentum felis. Donec nonummy pellentesque ante. Phasellus adipiscing semper elit. Proin fermentum massa ac quam. Sed diam turpis, molestie vitae, placerat a, molestie nec, leo. Maecenas lacinia. Nam ipsum ligula, eleifend at, accumsan nec, suscipit a, ipsum. Morbi blandit ligula

feugiat magna. Nunc eleifend consequat lorem. Sed lacinia nulla vitae enim. Pellentesque tincidunt purus vel magna. Integer non enim. Praesent euismod nunc eu purus. Donec bibendum quam in tellus. Nullam cursus pulvinar lectus. Donec et mi. Nam vulputate metus eu enim. Vestibulum pellentesque felis eu massa.

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APPENDIX EXAMPLE: CODE LISTINGS

*We have seen that computer programming is an art,
because it applies accumulated knowledge to the world,
because it requires skill and ingenuity, and especially
because it produces objects of beauty.*

— Knuth, “Computer Programming as an Art,” 1974

A.1 THE `listings` PACKAGE TO INCLUDE SOURCE CODE

Source code is usually not part of the text of a thesis, but if it is an original contribution it makes sense to let the code speak by itself instead of describing it. The package `listings` provide the proper layout tools. Refer to its manual if you need to use it, an example is given in listing [A.1](#).

Listing A.1: Code snippet with the recursive function to evaluate the pdf of the sum Z_N of N random variables equal to X .

```

1 std::vector<int> values_of_x(number_of_values_of_x,
   min_value_of_x);
3 for (unsigned int i = 1; i < number_of_values_of_x; i++) {
   values_of_x[i] = values_of_x[i - 1] + 1;
5 }
   prob_x = 1.0 / number_of_values_of_x;
7 std::vector<std::vector<double>> > p_z;
   for (unsigned int idx = 0; idx < p_z.size(); idx++) {
9     p_z[idx] = std::vector<double>(
       (max_value_of_x * (idx + 1) - min_value_of_x
11        * (idx + 1)) + 1, INIT_VALUE);
   }
13
   double prob(int Z, int value_of_z) {
15     if (value_of_z < min_value_of_x * Z ||
       value_of_z > max_value_of_x * Z) {
17         return 0.0;
   }
19     if (value_of_z < min_value_of_z ||
       value_of_z > max_value_of_z) {
21         return 0.0;
   }
23     int idx_value_of_z = -(min_value_of_z - value_of_z);
     int idx_N = Z - 1;
25     if (p_z[idx_N][idx_value_of_z] == -2.0) {
         if (Z > 1) {
27             double pp = 0.0;
             for (unsigned int i = 0; i < number_of_values_of_x; i++) {
29                 pp += prob(Z - 1, value_of_z - values_of_x[i], p);
             }
31             p_z[idx_N][idx_value_of_z] = prob_x * pp;
         } else {
33             if (Z == 1) {
                 for (unsigned int j = 0; j < number_of_values_of_x; j++)
                     {
35                         if (value_of_z == values_of_x[j]) {
                             p_z[idx_N][idx_value_of_z] = prob_x;
37                             break;
                         }
                     }
39             }
         }
41         if (p_z[idx_N][idx_value_of_z] == INIT_VALUE) {
             p_z[idx_N][idx_value_of_z] = 0.0;
43         }
     }
45 }
47 }

```