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MTH 317: Linear Algebra
Professor Sussan
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Homework #8 - 4.20 (d)

From page 148:
4.10 **Definition**: A vector space V is the direct sum of its subspaces $W_1, \ldots, W_k$ if $V = W_1 + W_2 + \ldots + W_k$ and the collection $\{W_1, \ldots, W_k\}$ is independent. We write $V = W_1 \oplus W_2 \oplus \ldots \oplus W_k$.

(d) $W_1 = W_2 = \{\begin{pmatrix} t \\ t \end{pmatrix} \mid t \in \mathbb{R}\}$

✓ **4.20 Decide if $\mathbb{R}^2$ is the direct sum of each W_1 and W_2.**

(d) $W_1 = W_2 = \{ \begin{pmatrix} t \\ t \end{pmatrix} \mid t \in \mathbb{R} \}$

1st: Linear Independence: Since W_1 and W_2 are in the same subspace, they both are on $y = x$. This can't be in $\mathbb{R}^2$ as $\mathbb{R}^2$ requires two independent directions.

2nd: Direct Sum $\oplus$ Test:

a) Sum Spans

In order for direct sum, $\mathbb{R}^2 = W_1 + W_2$ must span $\mathbb{R}^2$. However, in this scenario, $W_1 = W_2$ meaning their sum is the same line. Thus, $W_1 + W_2 = W_1 = \text{span}\{\begin{pmatrix} 1 \\ 1 \end{pmatrix}\} \neq \mathbb{R}^2$.

b) Intersection / Independence

As previously stated, since $W_1 = W_2$, their sum is in the same line. They are in the same subspace, proving they are not independent.

↳ cannot be direct sums.