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MTH 317: Linear Algebra  
Professor Svssan  
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Homework #4 - 2.22

2.22 Is the vector in the span of the set?

$$\begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix} \left\{ \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \right\}$$

$$\begin{aligned} & \left[ \begin{array}{ccc|c} 2 & 1 & 1 & 1 \\ 1 & -1 & 0 & 0 \\ -1 & 1 & 3 & 3 \end{array} \right] \xrightarrow{\frac{1}{2}R_1 \rightarrow R_1} \left[ \begin{array}{ccc|c} 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 1 & -1 & 0 & 0 \\ -1 & 1 & 3 & 3 \end{array} \right] \xrightarrow{\substack{R_2 - R_1 \rightarrow R_2 \\ R_3 + R_1 \rightarrow R_3}} \left[ \begin{array}{ccc|c} 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & -\frac{3}{2} & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{3}{2} & \frac{7}{2} & \frac{7}{2} \end{array} \right] \\ & \xrightarrow{-\frac{2}{3}R_2 \rightarrow R_2} \left[ \begin{array}{ccc|c} 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & 1 & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{3}{2} & \frac{7}{2} & \frac{7}{2} \end{array} \right] \xrightarrow{R_1 - \frac{1}{2}R_2 \rightarrow R_1} \left[ \begin{array}{ccc|c} 1 & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & 1 & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{3}{2} & \frac{7}{2} & \frac{7}{2} \end{array} \right] \xrightarrow{R_3 - \frac{3}{2}R_2 \rightarrow R_3} \left[ \begin{array}{ccc|c} 1 & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & 1 & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 3 & 3 \end{array} \right] \\ & \quad \downarrow \\ & \text{INCONSISTENT} \end{aligned}$$

Since the 3rd row is inconsistent,  
the target vector  $\begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix}$  is not in the span of the set.