Utsha Saha
MTH 317: Linear Algebra
Professor Sussan
March 29th, 2025
Homework #5 - 1.21(a)

✓ 1.21 Decide whether each subset of $\mathbb{R}^3$ is linearly dependent or linearly independent.

(a) $\left\{\begin{pmatrix}1\\-3\\5\end{pmatrix}, \begin{pmatrix}2\\2\\4\end{pmatrix}, \begin{pmatrix}4\\-4\\14\end{pmatrix}\right\}$

$$\left[\begin{array}{ccc|c} 1 & 2 & 4 & 0 \\ -3 & 2 & -4 & 0 \\ 5 & 4 & 14 & 0 \end{array}\right] \xrightarrow[R_3-5R_1\to R_3]{R_2+3R_1\to R_2} \left[\begin{array}{ccc|c} 1 & 2 & 4 & 0 \\ 0 & 8 & 8 & 0 \\ 0 & -6 & -6 & 0 \end{array}\right] \xrightarrow{\frac{1}{8}R_2\to R_2} \left[\begin{array}{ccc|c} 1 & 2 & 4 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & -6 & -6 & 0 \end{array}\right]$$

$$\xrightarrow[R_3+6R_2\to R_3]{R_1-2R_2\to R_1} \left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

← Cannot pivot any further, thus, → Linearly Dependent