Types of DE

Homogenous form

$$\frac{dy}{dx} = \frac{f_1(x,y)}{f_2(x,y)}$$

Here both f_1 and f_2 are Homogenous fucntion of same order \ degree. Then, let $\frac{y}{x} = v \implies \frac{dy}{dx} = v + x\frac{dv}{dx}$. Therefore above equation transforms to

$$\frac{dy}{dx} = f\left(\frac{y}{x}\right) \implies v + x\frac{dv}{dx} = f(v) \implies \frac{dv}{-v + f(v)} = \frac{dx}{x}$$

Linear DE of first order

Standard Form

$$\frac{dy}{dx} + P(x)y = Q(x)$$
$$I.F = e^{\int P \cdot dx}$$

Solution: $y(I.F) = \int Q \cdot (I.F)dx + c.$

Other Form

$$\frac{dx}{dy} + P(y)x = Q(y)$$
$$I.F = e^{\int P \cdot dy}$$

Bernoullie's Linear DE

$$\frac{dy}{dx} + P(x)y = Q(x)y^n$$

Substitution: $y^{1-n} = v$ then $(1-n)y^{-n}\frac{dy}{dx} = \frac{dv}{dx}$. Therefore,

$$\begin{aligned} &\frac{dv}{dx} + (1-n)Pv = (1-n)Q \\ \implies &\frac{dv}{dx} + P'v = Q' \\ &I.F = e^{\int P'dx} \end{aligned}$$

and the rest continues as usual.

Exact DE

$$Mdx + Ndy = 0$$

If

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

then the above de is in Exact form.

Solution :

$$\int_{\text{y as constant}} Mdx + \int_{\text{free from x terms}} Ndy = C$$

Reducible to exact DE and Integrating factor(I.F)

If DE is not in *exact form* then multiplying a I.F will turn the equation to a *exact DE*. Following the ways to calculate I.F :

- By Inspection:
    1. $xdy + ydx = d(xy)$
    2. $xdy - ydx \implies I.F = \frac{1}{x^2}$
    3. $xdy - ydx \implies I.F = \frac{1}{x^2}$
- If $M(x,y)$ and $N(x,y)$ are homogenous then $I.F = \frac{1}{Mx+Ny}$ such that $Mx + Ny \neq 0$.
- If the DE is in form

$$f_1(xy)dx + f_2(xy)dy = 0$$

  and $Mx - Ny \neq 0$ then $I.F = \frac{1}{Mx-Ny}$
- If

$$\frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = f(x)$$

then $I.F = e^{\int f(x)dx}$

- If

$$\frac{1}{M}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = -f(y)$$

then $I.F = e^{\int -f(y)dy}$