CHAPTER-9
AREAS OF PARALLELOGRAMS AND TRIANGLES

1 Exercise 9.2

Q1. In the figure given below, $ABCD$ is a parallelogram, $AE \perp DC$ and $CF \perp AD$.If $AB = 16cm$, $AE = 8cm$ and $CF = 10cm$, find AD.
Construction

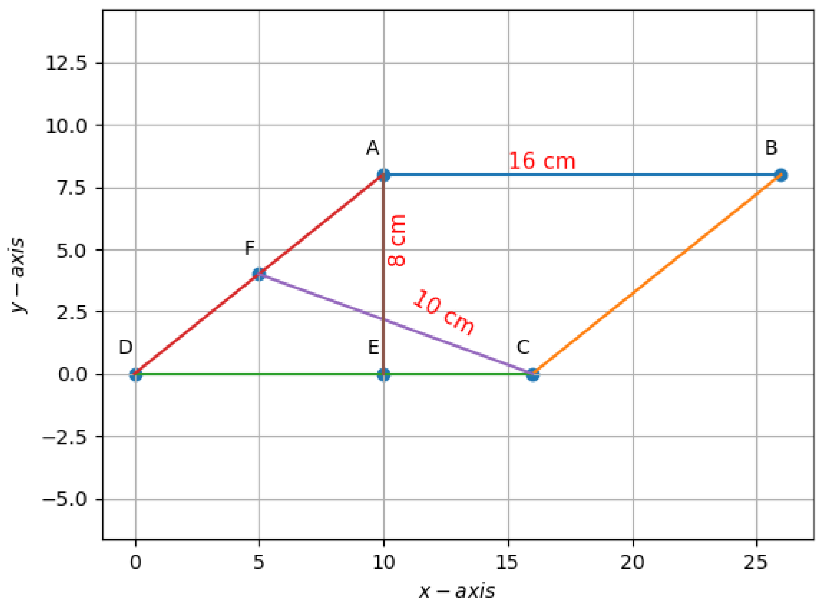

Figure 1: Parallelogram ABCD

The input parameters for the above construction are shown in the table below :

Symbol	Value	Description
AB	16cm	$\parallel \mathbf{B} - \mathbf{A} \parallel$
CD	16cm	$\parallel \mathbf{D} - \mathbf{C} \parallel = \parallel \mathbf{B} - \mathbf{A} \parallel$
AE	8cm	$\parallel \mathbf{E} - \mathbf{A} \parallel$
CF	10cm	$\parallel \mathbf{F} - \mathbf{C} \parallel$
$\angle CFD$	90°	$CF \perp AD$
$\angle AED$	90°	$AE \perp CD$

Table 1: Parameters

The input co-ordinates of the above parallelogram are given in this table :

Point	Co-ordinates	Description
D	(0,0)	Origin(Assumption)
C	(16,0)	(because $\mathbf{CD} = 16cm$)

Table 2: Co-ordinates

The rest of the co-ordinates were obtained in the following way :

1. **To find out the co-ordinates of F:**
   Let equation of **AD** be

$$\mathbf{n}^\top \mathbf{x} = c \tag{1}$$
$$\tag{2}$$

Substituting point D in the above equation in place of x, we get c = 0.Now substituting point A in the above equation, we get :

$$\mathbf{n}^\top \mathbf{A} = 0 \tag{3}$$
$$\tag{4}$$

As **CF** is normal to **AD**, $\mathbf{n} = \mathbf{CF}$ and as point F lies on **AB**, the above equation can be written as

$$(\mathbf{C} - \mathbf{F})^\top \mathbf{F} = 0 \tag{5}$$
$$\tag{6}$$

As it is given that $\|\mathbf{C} - \mathbf{F}\| = 10cm$.So, it can be written as

$$
\begin{aligned}
(\mathbf{C}-\mathbf{F})^\top(\mathbf{C}-\mathbf{F}) &= 100 && (7)\\
\implies (\mathbf{C}-\mathbf{F})^\top\mathbf{C} &= 100 (because (\mathbf{C}-\mathbf{F})^\top\mathbf{C} = 0) && (8)\\
(\mathbf{C}-\mathbf{F})^\top\mathbf{C} &= \|\mathbf{C}\|^2 - \mathbf{F}^\top\mathbf{C} = 100 && (9)\\
& && (10)
\end{aligned}
$$

So, $\mathbf{F}^\top\ \mathbf{C} = 156$, which is

$$
\begin{aligned}
\mathbf{F}^\top \begin{pmatrix} 16 \\ 0 \end{pmatrix} &= 156 && (11)\\
\mathbf{F}e_1 &= \frac{156}{16} && (12)\\
F_x &= 9.75 && (13)\\
& && (14)
\end{aligned}
$$

Based on above equations, $\mathbf{F}^\top\mathbf{F} = \mathbf{F}^\top\mathbf{C}$.So, it can be written as :

$$
\begin{aligned}
\mathbf{F}^\top\mathbf{F} &= 156 && (15)\\
\implies (\mathbf{F}^\top e_1)^2 + (\mathbf{F}^\top e_2)^2 &= 156 && (16)\\
\implies F_x^2 + F_y^2 &= 156 && (17)\\
F_y^2 &= 156 - (9.75)^2 && (18)\\
\implies F_y &= 7.806 && (19)\\
& && (20)
\end{aligned}
$$

The co-ordinates of F are (9.75,7.806).

2. **To find out the co-ordinates of A :**
   The length of AE gives the y co-ordinate of A,which is 8.By substituting in the below equation we can get the x co-ordinate of A :

$$
\begin{aligned}
(\mathbf{C}-\mathbf{F})^\top\mathbf{A} &= 0 && (21)\\
\begin{pmatrix} 6.25 & -7.806 \end{pmatrix} \begin{pmatrix} x \\ 8 \end{pmatrix} &= 0 && (22)\\
(6.25)x &= 62.5 && (23)\\
\implies x &= 10. && (24)
\end{aligned}
$$

   The co-ordinates of A are (10,8).

3. **To find out the co-ordinates of B :**
   As B lies on the same horizontal line as that of A,the y co-ordinate of B

is 8.In order to find out the x co-ordinate of B:

$$As\,\|\mathbf{B}-\mathbf{A}\| = 16 \tag{25}$$
$$B_x = A_x + 16 \tag{26}$$
$$B_x = 10 + 16 = 26 \tag{27}$$
$$\tag{28}$$

So, the co-ordinates of B are (26,8).

Solution
It is given that the length of $AB = 16cm$.So,

$$\|\mathbf{B}-\mathbf{A}\| = 16cm \tag{29}$$
$$\tag{30}$$

As the figure above is a parallelogram,

$$\|\mathbf{B}-\mathbf{A}\| = \|\mathbf{C}-\mathbf{D}\| = 16cm \tag{31}$$
$$\tag{32}$$

In order to find ∠DCF,

$$\text{Let } \theta_2 = \angle FCD \tag{33}$$
$$\mathbf{n_1} = \mathbf{C}-\mathbf{F} = \begin{pmatrix} 6.25 \\ -7.806 \end{pmatrix}, \mathbf{n_2} = \mathbf{C}-\mathbf{D} = \begin{pmatrix} 16 \\ 0 \end{pmatrix} \tag{34}$$
$$\theta_2 = \cos^{-1}\frac{\mathbf{n_1}^\top \mathbf{n_2}}{\|\mathbf{n_1}\|\,\|\mathbf{n_2}\|} \tag{35}$$
$$\implies \theta_2 = \cos^{-1}\frac{\begin{pmatrix} 6.25 & -7.806 \end{pmatrix}\begin{pmatrix} 16 \\ 0 \end{pmatrix}}{(10)(16)} = 51.32^\circ \tag{36}$$

As the sum of interal angles of a triangle are 180°,from $\triangle DFC$

$$\angle CFD + \angle FCD + \angle D = 180^\circ \tag{37}$$
$$\angle D = 180^\circ - (\angle CFD + \angle FCD) \tag{38}$$
$$\angle D = 180^\circ - 141.32^\circ \tag{39}$$
$$\angle D = 38.68^\circ \tag{40}$$
$$\tag{41}$$

The area of parallelogram is calculated using the below formula

$$Area = (base) * (height) \tag{42}$$
$$\tag{43}$$

For the above problem this can be written as,

$$Area = \|\mathbf{A} - \mathbf{E}\| \, \|\mathbf{B} - \mathbf{A}\| \tag{44}$$
$$\implies (8)(16) = 128 \tag{45}$$
$$\tag{46}$$

The area of parallelogram can also be found out using cross product of two vectors,for the above problem it can be written as;

$$Area of parallelogram ABCD = \mathbf{DC} \times \mathbf{AD} \tag{47}$$
$$\mathbf{DC} \times \mathbf{AD} = \|\mathbf{D} - \mathbf{C}\| \, \|\mathbf{D} - \mathbf{A}\| \sin D \tag{48}$$
$$\tag{49}$$

From 32 and 49

$$\|\mathbf{D} - \mathbf{A}\| \, (16) \sin 38.68 = 128 \tag{50}$$
$$\|\mathbf{D} - \mathbf{A}\| \, (16)(\frac{5}{8}) = 128 \tag{51}$$
$$\|\mathbf{D} - \mathbf{A}\| = \frac{64}{5} \tag{52}$$
$$\|\mathbf{D} - \mathbf{A}\| = 12.8cm \tag{53}$$
$$\tag{54}$$

Therefore,$\left| \overrightarrow{\mathbf{AD}} \right| = \mathbf{12.8\ cm}$