CHAPTER-9
AREAS OF PARALLELOGRAMS AND TRIANGLES

# 1 Exercise 9.2

Q1. In the figure given below, $ABCD$ is a parallelogram, $AE \perp DC$ and $CF \perp AD$.If $AB = 16cm$, $AE = 8cm$ and $CF = 10cm$, find $AD$.
**Construction**

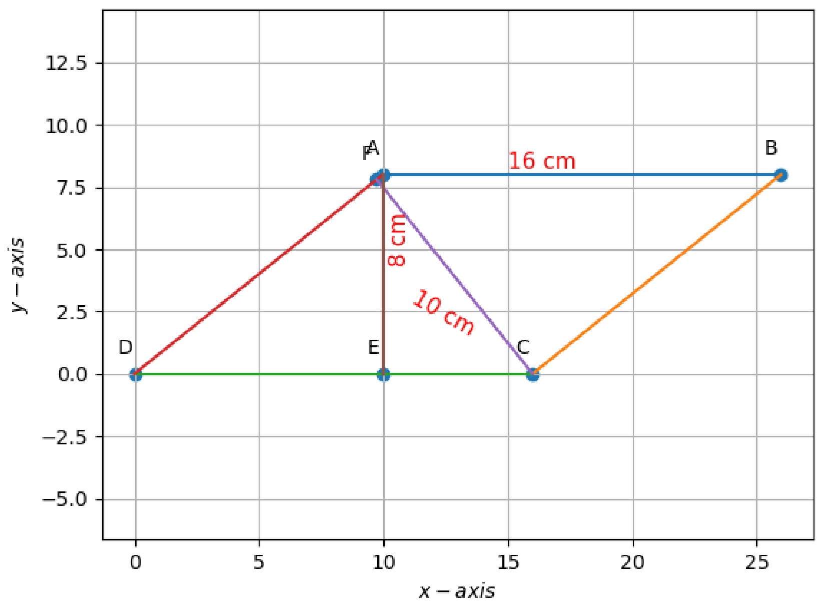


Figure 1: Parallelogram ABCD

The following table consists of given input parameters of the above parallelogram ABCD :

| Symbol | Value | Description |
|---|---|---|
| $x$ | 16cm | **AB** |
| $a$ | 10cm | **CF** |
| $b$ | 8cm | **AE** |
| $\angle CFD$ | 90° | $CF \perp AD$ |
| $\angle AED$ | 90° | $AE \perp CD$ |

Table 1: Parameters

Table below has the given input co-ordinates of the parallelogram :

| Point | Co-ordinates |
|---|---|
| **D** | (0,0) |

Table 2: Co-ordinates

Following table are the unknown lengths and angles and their symbols:

| Symbol | Description |
|---|---|
| c | **DC** |
| r | **AD** |
| d | **DE** |
| b | **AE** |
| $\theta$ | $\angle$**D** |

Table 3: Symbols and Corresponding Vectors

The point co-ordinates are derived in the following way :

1. **To derive the co-ordinates of C :**
   As mentioned in the 3, $\mathbf{DC} = c$.In the above parallelogram it is given that $\mathbf{AB} = 16cm$.According to the properties of a parallelogram the parallel sides are equal in length.So, it can be said that :

$$\mathbf{AB} = \mathbf{DC} = c \tag{1}$$
$$\tag{2}$$

   So, point **C** can be expressed in the following way :

$$\mathbf{C} = c\mathbf{e_1} \tag{3}$$

$$\mathbf{C} = c\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}c\\0\end{pmatrix} \tag{4}$$

$$\tag{5}$$

2. **To derive the co-ordinates of A :**
   A can be expressed in the form of $r\begin{pmatrix}\cos\theta\\ \sin\theta\end{pmatrix}$.In order to obtain r and $\theta$, the following can be done :

   (a) **To find out $\theta$ :**
   To find out $\theta$,let us assume that $\mathbf{CF} = a$, from $\triangle CFD$ :

   $$\sin\theta = \frac{a}{c} \tag{6}$$

   $$\implies \theta = \sin^{-1}\frac{a}{c} \tag{7}$$

   $$\tag{8}$$

   (b) **To find out r :**
   As mentioned in 3, $\mathbf{AD} = r$ and $\mathbf{AE} = b$.In order to find out $r$, from $\triangle ADE$:

   $$\sin\theta = \frac{b}{r} \tag{9}$$

   $$r = \frac{b}{\sin\theta} \tag{10}$$

   $$\tag{11}$$

   So, the co-ordinates of **A** can be written as :

   $$\mathbf{A} = \frac{b}{\sin\theta}\begin{pmatrix}\cos\theta\\ \sin\theta\end{pmatrix} \tag{12}$$

   $$\mathbf{A} = \begin{pmatrix}b\cot\theta\\ b\end{pmatrix} \tag{13}$$

   $$\tag{14}$$

3. **To derive the co-ordinates of B :**
   From parallelogram law of vectors, **B** can be expressed as the sum of **A** and **C**.So, it can be written as,

   $$\mathbf{B} = \mathbf{A} + \mathbf{C} \tag{15}$$

   $$\tag{16}$$

4. **To derive the co-ordinates of E :**
   As mentioned in the 3, $\mathbf{DE} = d$.As, $\mathbf{E}$ lies on x-axis it can be written in the form of $de_1$.So, the co-ordinates can be found out in the following way, from $\triangle DAE$ :

$$\cos\theta = \frac{d}{r} \tag{17}$$

$$d = r\cos\theta \tag{18}$$

$$\tag{19}$$

$$\mathbf{E} = d\mathbf{e_1} = \mathrm{r}\cos\theta \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} r\cos\theta \\ 0 \end{pmatrix}.$$

5. **To derive the co-ordinates of F :**
   As point $\mathbf{F}$ divides $\mathbf{AD}$ in the ratio k : 1.The co-ordinates of $\mathbf{F}$ can be found out in the following way :

$$\mathbf{F} = \frac{k\mathbf{A} + \mathbf{D}}{k+1} \tag{20}$$

$$\tag{21}$$

The following table displays the unknown lengths and angles which were derived from the given quantities :

| Symbol | value | Description |
|---|---|---|
| c | x | **DC** |
| r | $\frac{b}{\sin\theta}$ | **AD** |
| d | $\mathrm{r}\cos\theta$ | **DE** |
| $\theta$ | $\sin^{-1}\frac{a}{c}$ | $\angle D$ |

Table 4: Co-ordinates in terms of given and derived lengths and angles

The following table displays the point co-ordinates in terms of known and derived quantities:

| Point | Co-ordinates |
|---|---|
| **A** | $\begin{pmatrix} b\cot\theta \\ b \end{pmatrix}$ |
| **B** | $\mathbf{A}+\mathbf{C}$ |
| **C** | $\begin{pmatrix} c \\ 0 \end{pmatrix}$ |
| **E** | $\begin{pmatrix} r\cos\theta \\ 0 \end{pmatrix}$ |
| **F** | $\frac{k\mathbf{A}+\mathbf{D}}{k+1}$ |

Table 5: Co-ordinates in terms of known and derived quantities

**Finding the co-ordinates of the above parallelogram:**

1. **Co-ordinates of C:**
   As c = **DC** = 16, the co-ordinates of **C** are:

$$\mathbf{C} = \begin{pmatrix} c \\ 0 \end{pmatrix} = \begin{pmatrix} 16 \\ 0 \end{pmatrix} \tag{22}$$

$$\tag{23}$$

   So, the co-ordinates of **C** are $\begin{pmatrix} 16 \\ 0 \end{pmatrix}$.

2. **Co-ordinates of A:**

   (a) **Finding $\theta$:**

$$\theta = \sin^{-1}\frac{a}{c} \tag{24}$$

$$\theta = \sin^{-1}\frac{10}{16} \tag{25}$$

$$\theta = 38.68^\circ \tag{26}$$

$$\tag{27}$$

   From the above derivations, we got that:

$$\mathbf{A} = \begin{pmatrix} b\cot\theta \\ b \end{pmatrix} \tag{28}$$

$$\implies \begin{pmatrix} 8\cot 38.68 \\ 8 \end{pmatrix} \tag{29}$$

$$\mathbf{A} = \begin{pmatrix} 10 \\ 8 \end{pmatrix} \tag{30}$$

$$\tag{31}$$

so, the co-ordinates of **A** are $\begin{pmatrix} 10 \\ 8 \end{pmatrix}$.

3. **Co-ordinates of B:**
   From above derivation, we got that $\mathbf{B} = \mathbf{A} + \mathbf{C}$.Then, **C** is:

$$\mathbf{B} = \begin{pmatrix} 10 \\ 8 \end{pmatrix} + \begin{pmatrix} 16 \\ 0 \end{pmatrix} \tag{32}$$

$$\mathbf{B} = \begin{pmatrix} 26 \\ 8 \end{pmatrix} \tag{33}$$

$$\tag{34}$$

   So, the co-ordinates of **B** are $\begin{pmatrix} 26 \\ 8 \end{pmatrix}$.

4. **Co-ordinates of E:**

   (a) **Finding r:**
   From above derivation, r can be found out in the following way:

$$r = \frac{b}{\sin\theta} \tag{35}$$

$$r = \frac{8}{\sin 38.68} \tag{36}$$

$$r = 12.8cm \tag{37}$$

$$\tag{38}$$

   The co-ordinates of **E** are:

$$\mathbf{E} = \begin{pmatrix} r\cos\theta \\ 0 \end{pmatrix} \tag{39}$$

$$\implies \begin{pmatrix} (12.8)\cos 38.68 \\ 0 \end{pmatrix} \tag{40}$$

$$\mathbf{E} = \begin{pmatrix} 10 \\ 0 \end{pmatrix} \tag{41}$$

$$\tag{42}$$

So, the co-ordinates for **E** are $\begin{pmatrix} 10 \\ 0 \end{pmatrix}$.

5. **Co-ordinates for F:**
   **F** divides **AD** in the ratio 39 : 1.So, the co-ordinates of **F** are:

$$\mathbf{F} = \frac{k\mathbf{A} + \mathbf{D}}{k+1} \tag{43}$$

$$\implies \frac{(39)\begin{pmatrix} 10 \\ 8 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix}}{39+1} \tag{44}$$

$$\implies \frac{\begin{pmatrix} 390 \\ 312 \end{pmatrix}}{40} \tag{45}$$

$$\mathbf{F} = \begin{pmatrix} 9.75 \\ 7.8 \end{pmatrix} \tag{46}$$

   So, the co-ordinates of **F** are $\begin{pmatrix} 9.75 \\ 7.8 \end{pmatrix}$.

The following table displays the final co-ordinates of the vertices of the parallelogram :

| Point | Co-ordinates |
|---|---|
| **A** | $\begin{pmatrix} 10 \\ 8 \end{pmatrix}$ |
| **B** | $\begin{pmatrix} 26 \\ 8 \end{pmatrix}$ |
| **C** | $\begin{pmatrix} 16 \\ 0 \end{pmatrix}$ |
| **D** | $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ |
| **E** | $\begin{pmatrix} 10 \\ 0 \end{pmatrix}$ |
| **F** | $\begin{pmatrix} 9.75 \\ 7.8 \end{pmatrix}$ |

Table 6: Final co-ordinates of the parallelogram

The length of **AD** was found out in the above process and it is r = 12.8cm.