CHAPTER-9
AREAS OF PARALLELOGRAMS AND TRIANGLES

1 Exercise 9.2

Q1. In the figure given below, $ABCD$ is a parallelogram, $AE \perp DC$ and $CF \perp AD$.If $AB = 16cm$, $AE = 8cm$ and $CF = 10cm$, find AD.
Construction

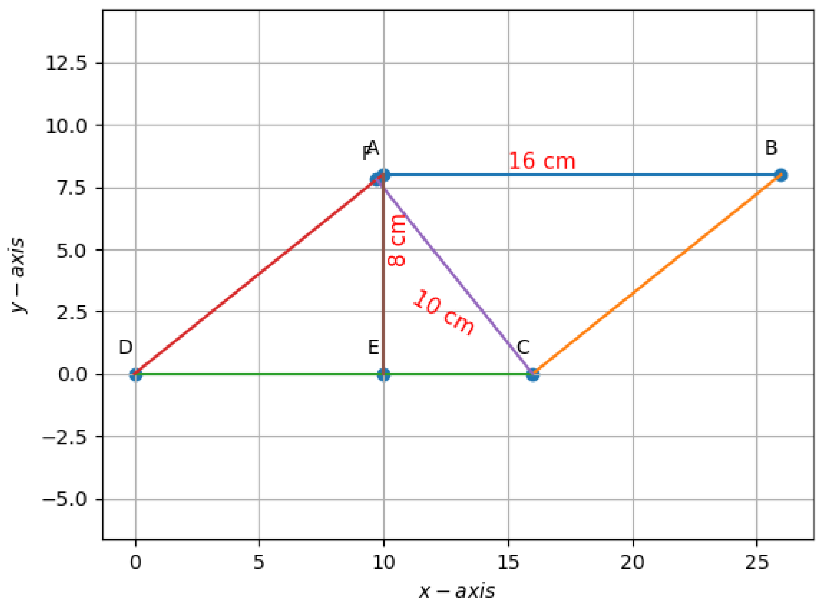

Figure 1: Parallelogram ABCD

The following table consists of given input parameters of the above parallelogram ABCD :

Symbol	Value	Description
AB	16cm	$\parallel \mathbf{B} - \mathbf{A} \parallel$
CD	16cm	$\parallel \mathbf{D} - \mathbf{C} \parallel = \parallel \mathbf{B} - \mathbf{A} \parallel$
AE	8cm	$\parallel \mathbf{E} - \mathbf{A} \parallel$
CF	10cm	$\parallel \mathbf{F} - \mathbf{C} \parallel$
$\angle CFD$	90°	$CF \perp AD$
$\angle AED$	90°	$AE \perp CD$

Table 1: Parameters

Table below has the given input co-ordinates of the parallelogram :

Point	Co-ordinates
D	(0,0)

Table 2: Co-ordinates

Rest of the point co-ordinates are derived in the following way :

1. **To derive the co-ordinates of C :**
   Let us assume that $\|\mathbf{D} - \mathbf{C}\| = \mathbf{c}$.As it is given that $\|\mathbf{D} - \mathbf{C}\| = 16$ and as point C lies on x axis, it can be expressed in the following way :

$$\mathbf{C} = \mathbf{c}e_1 \tag{1}$$

$$\implies (16)\begin{pmatrix}1\\0\end{pmatrix} \tag{2}$$

$$\mathbf{C} = \begin{pmatrix}16\\0\end{pmatrix} \tag{3}$$

$$\tag{4}$$

2. **To derive the co-ordinates of A :**
   A can be expressed in the form of $r\begin{pmatrix}\cos\theta\\\sin\theta\end{pmatrix}$.In order to obtain r and θ, the following can be done :

To find out θ,let us assume that $\|\mathbf{C}-\mathbf{F}\| = \mathbf{a}$

$$from \triangle CFD, \tag{5}$$
$$\sin\theta = \frac{\mathbf{a}}{\mathbf{c}} \tag{6}$$
$$\implies \sin\theta = \frac{10}{16} \tag{7}$$
$$\implies \sin^{-1}\frac{10}{16} = 38.68^\circ \tag{8}$$
$$\tag{9}$$

Let us consider $\|\mathbf{D}-\mathbf{A}\| = \mathbf{r}$ and $\|\mathbf{E}-\mathbf{A}\| = \mathbf{b}$.In order to find out $\mathbf{r}$,

$$from \triangle ADE, \tag{10}$$
$$\sin\theta = \frac{\mathbf{b}}{\mathbf{r}} \tag{11}$$
$$\mathbf{r} = \frac{\mathbf{b}}{\sin\theta} \tag{12}$$
$$\mathbf{r} = \frac{8}{\sin\theta} \tag{13}$$
$$\implies \frac{8}{\frac{5}{8}} \tag{14}$$
$$\mathbf{r} = 12.8 \tag{15}$$
$$\tag{16}$$

So, the co-ordinates of $\mathbf{A}$ can be written as :

$$\mathbf{A} = r\begin{pmatrix}\cos\theta\\ \sin\theta\end{pmatrix} = 12.8\begin{pmatrix}\cos 38.68\\ \sin 38.68\end{pmatrix} \tag{17}$$
$$\implies \mathbf{A} = \begin{pmatrix}10\\ 8\end{pmatrix} \tag{18}$$
$$\tag{19}$$

3. **To find out the co-ordinates of B :**
   From parallelogram law of vectors, $\mathbf{B}$ can be expressed as the sum of $\mathbf{A}$ and $\mathbf{C}$.So, it can be written as,

$$\mathbf{B} = \mathbf{A} + \mathbf{C} \tag{20}$$
$$\mathbf{B} = \begin{pmatrix}10\\ 8\end{pmatrix} + \begin{pmatrix}16\\ 0\end{pmatrix} \tag{21}$$
$$\mathbf{B} = \begin{pmatrix}26\\ 8\end{pmatrix} \tag{22}$$
$$\tag{23}$$

4. **To find out the co-ordinates of E :**
   Let us assume that the $\|\mathbf{D}-\mathbf{E}\| = \mathbf{d}$.As, $\mathbf{E}$ lies on x-axis it can be written in the form of de_1.So, the co-ordinates can be found out in the following way :

$$from \triangle DAE, \tag{24}$$
$$\cos\theta = \frac{\mathbf{d}}{\mathbf{r}} \tag{25}$$
$$\mathbf{d} = \mathbf{r}\cos\theta \tag{26}$$
$$\implies (12.8)\cos 38.68 \tag{27}$$
$$\implies \mathbf{d} = 10 \tag{28}$$
$$\tag{29}$$

$\mathbf{E} = de_1 = (10)\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}10\\0\end{pmatrix}$.

5. **To find out the co-ordinates of F :**
   As point $\mathbf{F}$ divides $\mathbf{AD}$ in the ratio $39 : 1$.The co-ordinates of $\mathbf{F}$ can be found out in the following way :

$$\mathbf{F} = \frac{k\mathbf{A}+\mathbf{D}}{k+1} \tag{30}$$
$$\mathbf{F} = \frac{(39)\begin{pmatrix}10\\8\end{pmatrix} + \begin{pmatrix}0\\0\end{pmatrix}}{40} \tag{31}$$
$$\mathbf{F} = \frac{1}{40}\begin{pmatrix}390\\312\end{pmatrix} \tag{32}$$
$$\mathbf{F} = \begin{pmatrix}9.75\\7.8\end{pmatrix} \tag{33}$$
$$\tag{34}$$

The length of $\mathbf{D}-\mathbf{A}$ was found out in the above process and it is $\|\mathbf{AD}\| = \mathbf{r} = 12.8cm$.