CHAPTER-9
AREAS OF PARALLELOGRAMS AND TRIANGLES

1 Exercise 9.2

Q1. In the figure given below, $ABCD$ is a parallelogram, $AE \perp DC$ and $CF \perp AD$.If $AB = 16cm$, $AE = 8cm$ and $CF = 10cm$, find AD.

Construction

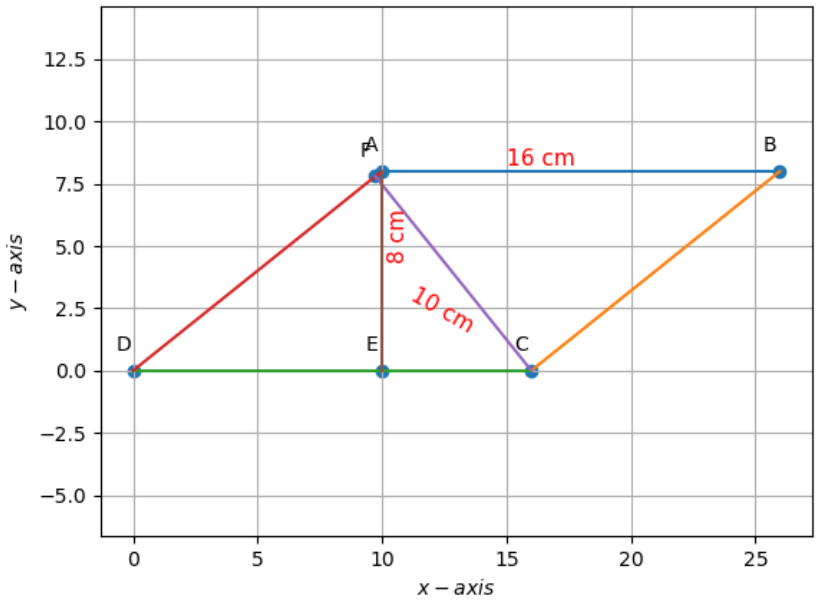

Figure 1: Parallelogram ABCD

The following table consists of given input parameters of the above parallelo-

gram ABCD :

Symbol	Value	Description
AB	16cm	$\| \mathbf{B} - \mathbf{A} \|$
AE	8cm	$\| \mathbf{E} - \mathbf{A} \|$
CF	10cm	$\| \mathbf{F} - \mathbf{C} \|$
$\angle CFD$	90°	$CF \perp AD$
$\angle AED$	90°	$AE \perp CD$

Table 1: Parameters

Table below has the given input co-ordinates of the parallelogram :

Point	Co-ordinates
D	(0,0)

Table 2: Co-ordinates

Following table shows the symbols and it's corresponding descriptions :

Symbol	Description
c	$\|\mathbf{CD}\|$
r	$\|\mathbf{AD}\|$
d	$\|\mathbf{DE}\|$
b	$\|\mathbf{AE}\|$
θ	$\angle\mathbf{D}$

Table 3: Symbols and Corresponding Vectors

Rest of the point co-ordinates are derived in the following way :

$$\mathbf{C} = ce_1, \mathbf{A} = r\begin{pmatrix}\cos\theta\\ \sin\theta\end{pmatrix}, \mathbf{B} = \mathbf{A} + \mathbf{C}, \mathbf{E} = de_1, \mathbf{F} = \frac{k\mathbf{A} + \mathbf{D}}{k+1} \tag{1}$$

1. **To derive the co-ordinates of C :**
   As mentioned in the table3, $\|\mathbf{D} - \mathbf{C}\| = c$.In the above parallelogram it is given that $\|\mathbf{B} - \mathbf{A}\| = 16cm$.According to the properties of a parallelogram the parallel sides are equal in length.So, it can be said that :

$$\|\mathbf{B} - \mathbf{A}\| = \|\mathbf{D} - \mathbf{C}\| = 16 = c \tag{2}$$

$$\tag{3}$$

As point **C** lies on x axis, it can be expressed in the following way :

$$\mathbf{C} = ce_1 \tag{4}$$
$$\implies (16)\begin{pmatrix}1\\0\end{pmatrix} \tag{5}$$
$$\mathbf{C} = \begin{pmatrix}16\\0\end{pmatrix} \tag{6}$$
$$\tag{7}$$

2. **To derive the co-ordinates of A :**
   A can be expressed in the form of $r\begin{pmatrix}\cos\theta\\\sin\theta\end{pmatrix}$.In order to obtain r and θ, the following can be done :

   (a) **To find out θ :**
   To find out θ,let us assume that $\|\mathbf{C}-\mathbf{F}\| = a$

$$from \triangle CFD, \tag{8}$$
$$\sin\theta = \frac{a}{c} \tag{9}$$
$$\implies \sin\theta = \frac{10}{16} \tag{10}$$
$$\implies \sin^{-1}\frac{10}{16} = 38.68^\circ \tag{11}$$
$$\tag{12}$$

   (b) **To find out r :**
   As mentioned in table3, $\|\mathbf{D}-\mathbf{A}\| = r$ and $\|\mathbf{E}-\mathbf{A}\| = b$.In order to find out r,

$$from \triangle ADE, \tag{13}$$
$$\sin\theta = \frac{b}{r} \tag{14}$$
$$r = \frac{b}{\sin\theta} \tag{15}$$
$$r = \frac{8}{\sin\theta} \tag{16}$$
$$\implies \frac{8}{\frac{5}{8}} \tag{17}$$
$$r = 12.8 \tag{18}$$
$$\tag{19}$$

So, the co-ordinates of **A** can be written as :

$$\mathbf{A} = r\begin{pmatrix}\cos\theta\\ \sin\theta\end{pmatrix} = 12.8\begin{pmatrix}\cos 38.68\\ \sin 38.68\end{pmatrix} \tag{20}$$

$$\implies \mathbf{A} = \begin{pmatrix}10\\ 8\end{pmatrix} \tag{21}$$

$$\tag{22}$$

3. **To find out the co-ordinates of B :**
From parallelogram law of vectors, **B** can be expressed as the sum of **A** and **C**.So, it can be written as,

$$\mathbf{B} = \mathbf{A} + \mathbf{C} \tag{23}$$

$$\mathbf{B} = \begin{pmatrix}10\\ 8\end{pmatrix} + \begin{pmatrix}16\\ 0\end{pmatrix} \tag{24}$$

$$\mathbf{B} = \begin{pmatrix}26\\ 8\end{pmatrix} \tag{25}$$

$$\tag{26}$$

4. **To find out the co-ordinates of E :**
As mentioned in the table3, $\|\mathbf{D} - \mathbf{E}\| = d$.As, **E** lies on x-axis it can be written in the form of de_1.So, the co-ordinates can be found out in the following way :

$$from \triangle DAE, \tag{27}$$

$$\cos\theta = \frac{d}{r} \tag{28}$$

$$d = r\cos\theta \tag{29}$$

$$\implies (12.8)\cos 38.68 \tag{30}$$

$$\implies d = 10 \tag{31}$$

$$\tag{32}$$

$\mathbf{E} = de_1 = (10)\begin{pmatrix}1\\ 0\end{pmatrix} = \begin{pmatrix}10\\ 0\end{pmatrix}$.

5. **To find out the co-ordinates of F :**
As point **F** divides **AD** in the ratio 39 : 1.The co-ordinates of **F** can be found out in the following way :

$$\mathbf{F} = \frac{k\mathbf{A} + \mathbf{D}}{k+1} \tag{33}$$

$$\mathbf{F} = \frac{(39)\begin{pmatrix}10\\8\end{pmatrix} + \begin{pmatrix}0\\0\end{pmatrix}}{40} \tag{34}$$

$$\mathbf{F} = \frac{1}{40}\begin{pmatrix}390\\312\end{pmatrix} \tag{35}$$

$$\mathbf{F} = \begin{pmatrix}9.75\\7.8\end{pmatrix} \tag{36}$$

$$\tag{37}$$

The length of $\mathbf{D} - \mathbf{A}$ was found out in the above process and it is $\|\mathbf{AD}\| = \mathbf{r} = 12.8cm$.