CHAPTER-9
AREAS OF PARALLELOGRAMS AND TRIANGLES

1 Exercise 9.2

Q1. In the figure given below, $ABCD$ is a parallelogram, $AE \perp DC$ and $CF \perp AD$.If $AB = 16cm$, $AE = 8cm$ and $CF = 10cm$, find AD.

Construction

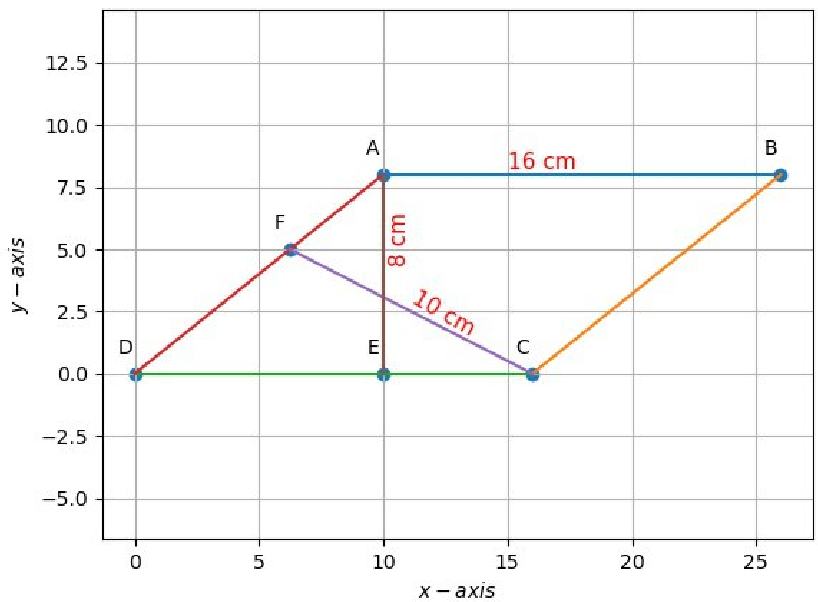

Figure 1: Parallelogram ABCD

The input parameters for the above construction are shown in the table below :

Symbol	Value	Description
D	Origin (0,0)	Vertex D
AB	16cm	$\parallel \mathbf{B}-\mathbf{A} \parallel$
CD	16cm	$\parallel \mathbf{D}-\mathbf{C} \parallel = \parallel \mathbf{B}-\mathbf{A} \parallel$
C	(16,0)	Vertex C (because CD = 16cm)
AE	8cm	$\parallel \mathbf{E}-\mathbf{A} \parallel$
CF	10cm	$\parallel \mathbf{F}-\mathbf{C} \parallel$
A	(10,8)	Ay = Dy + $\lvert\overrightarrow{AE}\rvert$ $Ax = \sqrt{\frac{CF^2}{AB^2-CF^2}}$
B	(26,8)	By = Dy + $\lvert\overrightarrow{AE}\rvert$ $Bx = Ax + AB$
F	(9.75,7.806:)	$Fy = 0.8\ X\ Fx$
E	(10,0)	Ex = Ax, Ey=0

Table 1: Parameters

Solution

It is given that the length of $AB = 16cm$.So,

$$\|\mathbf{B}-\mathbf{A}\| = 16cm \tag{1}$$
$$\tag{2}$$

As the figure above is a parallelogram,

$$\|\mathbf{B}-\mathbf{A}\| = \|\mathbf{C}-\mathbf{D}\| = 16cm \tag{3}$$
$$\tag{4}$$

In order to find ∠DCF,

$$\text{Let } \theta_2 = \angle FCD \tag{5}$$
$$\mathbf{n_1} = \mathbf{C}-\mathbf{F} = \begin{pmatrix} 6.25 \\ -7.806 \end{pmatrix}, \mathbf{n_2} = \mathbf{C}-\mathbf{D} = \begin{pmatrix} 16 \\ 0 \end{pmatrix} \tag{6}$$
$$\theta_2 = \cos^{-1}\frac{\mathbf{n_1}^\top \mathbf{n_2}}{\|\mathbf{n_1}\|\,\|\mathbf{n_2}\|} \tag{7}$$
$$\implies \theta_2 = \cos^{-1}\frac{\begin{pmatrix} 6.25 & -7.806 \end{pmatrix}\begin{pmatrix} 16 \\ 0 \end{pmatrix}}{(10)(16)} = 51.32° \tag{8}$$

As the sum of interal angles of a triangle are 180°,from $\triangle DFC$

$$\angle CFD + \angle FCD + \angle D = 180^\circ \tag{9}$$
$$\angle D = 180^\circ - (\angle CFD + \angle FCD) \tag{10}$$
$$\angle D = 180^\circ - 141.32^\circ \tag{11}$$
$$\angle D = 38.68^\circ \tag{12}$$
$$\tag{13}$$

The area of parallelogram is calculated using the below formula

$$Area = (base) * (height) \tag{14}$$
$$\tag{15}$$

For the above problem this can be written as,

$$Area = \|\mathbf{A} - \mathbf{E}\| \, \|\mathbf{B} - \mathbf{A}\| \tag{16}$$
$$\implies (8)(16) = 128 \tag{17}$$
$$\tag{18}$$

The area of parallelogram can also be found out using cross product of two vectors,for the above problem it can be written as;

$$Area of parallelogram ABCD = \mathbf{DC} \times \mathbf{AD} \tag{19}$$
$$\mathbf{DC} \times \mathbf{AD} = \|\mathbf{D} - \mathbf{C}\| \, \|\mathbf{D} - \mathbf{A}\| \sin D \tag{20}$$
$$\tag{21}$$

From 3 and 20

$$\|\mathbf{D} - \mathbf{A}\| \, (16) \sin 38.68 = 128 \tag{22}$$
$$\|\mathbf{D} - \mathbf{A}\| \, (16)(\frac{5}{8}) = 128 \tag{23}$$
$$\|\mathbf{D} - \mathbf{A}\| = \frac{64}{5} \tag{24}$$
$$\|\mathbf{D} - \mathbf{A}\| = 12.8cm \tag{25}$$
$$\tag{26}$$

Therefore,$|\overrightarrow{\mathbf{AD}}| = \mathbf{12.8\ cm}$