CHAPTER-9
AREAS OF PARALLELOGRAMS AND TRIANGLES

1 Exercise 9.2

Q1. In the figure given below, $ABCD$ is a parallelogram, $AE \perp DC$ and $CF \perp AD$.If $AB = 16cm$, $AE = 8cm$ and $CF = 10cm$, find AD.
Construction

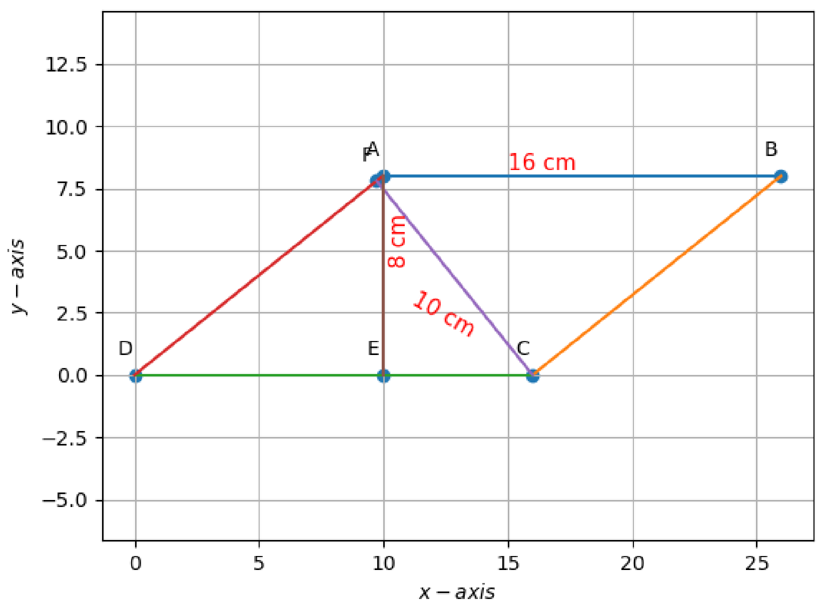

Figure 1: Parallelogram ABCD

The following table displays the given input parameters :

Symbol	Value	Description
a	10cm	CF
b	8cm	AE
l	16cm	AB
$\angle AED$	90°	AE $\perp$ DC
$\angle DFC$	90°	CF $\perp$ AD

Table 1: Input Parameters

The lengths and angles which are to be found out are displayed in the table below along with their symbols :

Symbol	Description
c	CD
d	DE
r	AD
f	DF
θ	$\angle D$

Table 2: Unknown Parameters

The input co-ordinates of the above parallelogram is **D** which is at the origin.The rest of the point co-ordinates can be derived based on this assumption in the following way which is shown in the table below :

Point	Co-ordinates
A	$r\begin{pmatrix}\cos\theta \\ \sin\theta\end{pmatrix}$
B	$\mathbf{A}+\mathbf{C}$
C	$c\mathbf{e_1}$
E	$d\mathbf{e_1}$
F	$f\begin{pmatrix}\cos\theta \\ \sin\theta\end{pmatrix}$

Table 3: Unknown Co-ordinates

Deriving the Unknown lengths and angles in terms of known and derived parameters :

1. **Deriving c:** From Figure1, c is parallel to l(AB paralle to CD).So,
$$c = l \tag{1}$$
2. **Deriving d:** From $\triangle ADE$,
$$\cos\theta = \frac{DE}{AD} = \frac{d}{r} \implies d = r\cos\theta \tag{2}$$
3. **Deriving r:** From $\triangle ADE$,
$$\sin\theta = \frac{AE}{AD} = \frac{b}{r} \implies r = \frac{b}{\sin\theta} \tag{3}$$
4. **Deriving f:** From $\triangle DFC$,
$$\cos\theta = \frac{DF}{DC} = \frac{f}{c} \implies f = c\cos\theta \tag{4}$$

5. **Finding θ:** From $\triangle DFC$,

$$\sin\theta = \frac{CF}{CD} = \frac{a}{c} \implies \theta = \sin^{-1}\frac{a}{c} \tag{5}$$

From eq1,eq2,eq3,eq4 and eq5, table2 can be modified as :

Symbol	value	Description
c	l	DC
r	$\frac{b}{\sin\theta}$	AD
d	$r\cos\theta$	DE
θ	$\sin^{-1}\frac{a}{c}$	$\angle D$
f	$c\cos\theta$	DF

Table 4: Unknown parameters in terms of known and derived parameters

Finding out unknown lengths and angles :

1. **Finding θ:** From eq5,

$$\theta = \sin^{-1}\frac{10}{16} = 38.68^\circ \tag{6}$$

2. **Finding c:** From eq1,

$$c = 16cm \tag{7}$$

3. **Finding r:** From 3, the value of r is :

$$r = \frac{8}{\sin 38.68} = 12.8cm \tag{8}$$

4. **Finding d:** From eq2,

$$d = (12.8)\cos 38.68 = 10cm \tag{9}$$

5. **Finding f:** From eq4,

$$f = (16)\cos 38.68 = 12.48cm \tag{10}$$

Deriving co-ordinates in terms of known and derived parameters:
Based on table4, table3 can be modified as follows :

Based on eq6,eq7,eq8,eq9 and eq10 and table5.The final co-ordinates of the parallelogram are displayed in the table below:

From eq8, we got the length of AD = r = 12.8cm.

Point	Co-ordinates
A	$\frac{b}{\sin\theta}\begin{pmatrix}\cos\theta\\ \sin\theta\end{pmatrix}$
B	$\mathbf{A}+\mathbf{C}$
C	$c\mathbf{e_1}$
E	$r\cos\theta\mathbf{e_1}$
F	$c\cos\theta\begin{pmatrix}\cos\theta\\ \sin\theta\end{pmatrix}$

Table 5: Co-ordinates in terms of known and derived co-ordinates

Point	Co-ordinates
A	$\begin{pmatrix}10\\ 8\end{pmatrix}$
B	$\begin{pmatrix}26\\ 8\end{pmatrix}$
C	$\begin{pmatrix}16\\ 0\end{pmatrix}$
D	$\begin{pmatrix}0\\ 0\end{pmatrix}$
E	$\begin{pmatrix}10\\ 0\end{pmatrix}$
F	$\begin{pmatrix}9.75\\ 7.8\end{pmatrix}$

Table 6: Final Co-ordinates