CHAPTER-9
AREAS OF PARALLELOGRAMS AND TRIANGLES

1 Exercise 9.2

Q1. In the figure given below, $ABCD$ is a parallelogram, $AE \perp DC$ and $CF \perp AD$.If $AB = 16cm$, $AE = 8cm$ and $CF = 10cm$, find AD.

Construction

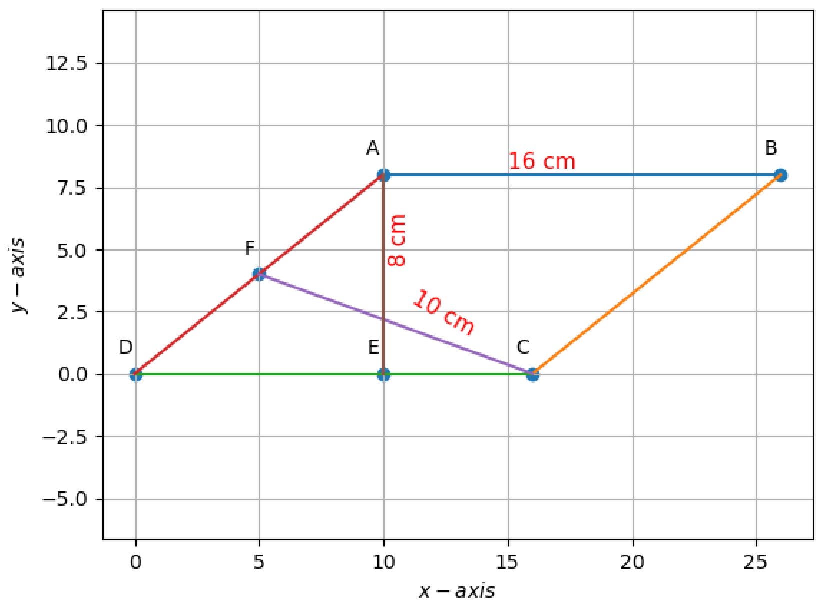

Figure 1: Parallelogram ABCD

The input parameters for the above construction are shown in the table below :

Symbol	Value	Description
AB	16cm	$\parallel \mathbf{B} - \mathbf{A} \parallel$
CD	16cm	$\parallel \mathbf{D} - \mathbf{C} \parallel = \parallel \mathbf{B} - \mathbf{A} \parallel$
AE	8cm	$\parallel \mathbf{E} - \mathbf{A} \parallel$
CF	10cm	$\parallel \mathbf{F} - \mathbf{C} \parallel$
$\angle CFD$	90°	$CF \perp AD$
$\angle AED$	90°	$AE \perp CD$

Table 1: Parameters

The input co-ordinates of the above parallelogram are given in this table :

Point	Co-ordinates	Description
D	(0,0)	Origin(Assumption)
C	(16,0)	(because $\mathbf{CD} = 16cm$)

Table 2: Co-ordinates

The rest of the co-ordinates were obtained in the following way :

1. Deriving the co-ordinates of A :
   A can be expressed in terms of (r$\cos\theta$,r$\sin\theta$).
   To find out the value of r:
   From $\triangle ADE$, r is AD and θ is D.

$$\sin\theta = \frac{AE}{AD} \quad (1)$$

$$\implies \sin\theta = \frac{AE}{r} \quad (2)$$

$$r = \frac{AE}{\sin\theta} \quad (3)$$

$$r = \frac{8}{\sin 38.68} \quad (4)$$

$$r = 12.8cm \quad (5)$$

$$(6)$$

The x co-ordinate is expressed as r$\cos\theta$.

$$A_x = r * \cos\theta \quad (7)$$

$$\implies (12.8) * (\cos 38.68) = 10cm \quad (8)$$

$$(9)$$

The y co-ordinate can be expressed as rsin θ.

$$A_y = r * \sin\theta \tag{10}$$
$$\implies (12.8) * (\sin 38.68) = 8cm \tag{11}$$
$$\tag{12}$$

So, the co-ordinates of A are (10,8).

2. Deriving the co-ordinates for E :
   The co-ordinates of E can be derived using ae_1, where a is the length of DE.To find out the length of DE we have to do the following steps :

$$DE = \cos D * (AD)(from \triangle ADE) \tag{13}$$
$$DE = \cos 38.68 * (12.8) \tag{14}$$
$$DE = 10cm = a \tag{15}$$
$$\tag{16}$$

   Now E can be expresed as a*e_1 which is equal to

$$10\begin{pmatrix}1\\0\end{pmatrix}$$
$$\implies \begin{pmatrix}10\\0\end{pmatrix}$$

   So, the co-ordinates of E are (10,0).

3. Deriving the co-ordinates of B :
   The y co-ordinate of B is same as that of A because both the points are on the same horizontal line which is 8.
   In order to obtain the x co-ordinate :

$$B_x = A_x + 16(because \,\|\mathbf{AB}\| = 16cm) \tag{17}$$
$$B_x = 10 + 16 = 26. \tag{18}$$
$$\tag{19}$$

   The co-ordinates of B are (26,8).

4. Deriving the co-ordinates of F :
   The co-ordinates of F can be derived by applying section formula :
   The value of m = 12.48 and n = 0.32 and m : n = 39 : 1.
   The co-ordinates of point D are cosidered to be (x_1,y_1) and co-ordinates of point A are considered to be (x_2,y_2).

The x co-ordinate is obtained using,

$$F_x = \frac{m(x_2) + n(x_1)}{m+n} \tag{20}$$

$$F_x = \frac{39(10) + 1(0)}{39+1} \tag{21}$$

$$\implies \frac{390}{40} = 9.75 \tag{22}$$

$$\tag{23}$$

The y co-ordinate can be found out using,

$$F_y = \frac{m(y_2) + n(y_1)}{m+n} \tag{24}$$

$$F_y = \frac{39(8) + 1(0)}{39+1} \tag{25}$$

$$\implies \frac{312}{40} = 7.8 \tag{26}$$

$$\tag{27}$$

So, the co-ordinates of F are (9.75,7.8).

Solution

It is given that the length of $AB = 16cm$.So,

$$\|\mathbf{B} - \mathbf{A}\| = 16cm \tag{28}$$

$$\tag{29}$$

As the figure above is a parallelogram,

$$\|\mathbf{B} - \mathbf{A}\| = \|\mathbf{C} - \mathbf{D}\| = 16cm \tag{30}$$

$$\tag{31}$$

In order to find ∠DCF,

$$\text{Let } \theta_2 = \angle FCD \tag{32}$$

$$\mathbf{n_1} = \mathbf{C} - \mathbf{F} = \begin{pmatrix} 6.25 \\ -7.806 \end{pmatrix}, \mathbf{n_2} = \mathbf{C} - \mathbf{D} = \begin{pmatrix} 16 \\ 0 \end{pmatrix} \tag{33}$$

$$\theta_2 = \cos^{-1} \frac{\mathbf{n_1}^\top \mathbf{n_2}}{\|\mathbf{n_1}\| \, \|\mathbf{n_2}\|} \tag{34}$$

$$\implies \theta_2 = \cos^{-1} \frac{\begin{pmatrix} 6.25 & -7.806 \end{pmatrix} \begin{pmatrix} 16 \\ 0 \end{pmatrix}}{(10)(16)} = 51.32° \tag{35}$$

As the sum of interal angles of a triangle are $180°$,from $\triangle DFC$

$$\angle CFD + \angle FCD + \angle D = 180° \tag{36}$$
$$\angle D = 180° - (\angle CFD + \angle FCD) \tag{37}$$
$$\angle D = 180° - 141.32° \tag{38}$$
$$\angle D = 38.68° \tag{39}$$
$$\tag{40}$$

The area of parallelogram is calculated using the below formula

$$Area = (base) * (height) \tag{41}$$
$$\tag{42}$$

For the above problem this can be written as,

$$Area = \|\mathbf{A} - \mathbf{E}\| \|\mathbf{B} - \mathbf{A}\| \tag{43}$$
$$\implies (8)(16) = 128 \tag{44}$$
$$\tag{45}$$

The area of parallelogram can also be found out using cross product of two vectors,for the above problem it can be written as;

$$Areaofparallelogram ABCD = \mathbf{DC} \times \mathbf{AD} \tag{46}$$
$$\mathbf{DC} \times \mathbf{AD} = \|\mathbf{D} - \mathbf{C}\| \|\mathbf{D} - \mathbf{A}\| \sin D \tag{47}$$
$$\tag{48}$$

From **??** and **??**

$$\|\mathbf{D} - \mathbf{A}\| (16) \sin 38.68 = 128 \tag{49}$$
$$\|\mathbf{D} - \mathbf{A}\| (16)(\frac{5}{8}) = 128 \tag{50}$$
$$\|\mathbf{D} - \mathbf{A}\| = \frac{64}{5} \tag{51}$$
$$\|\mathbf{D} - \mathbf{A}\| = 12.8cm \tag{52}$$
$$\tag{53}$$

Therefore,$| \overrightarrow{\mathbf{AD}} | = \mathbf{12.8\ cm}$