

CHAPTER-9
AREAS OF PARALLELOGRAMS AND TRIANGLES

1 Exercise 9.2

Q1. In the figure given below, $ABCD$ is a parallelogram, $AE \perp DC$ and $CF \perp AD$. If $AB = 16\text{cm}$, $AE = 8\text{cm}$ and $CF = 10\text{cm}$, find AD .

Construction

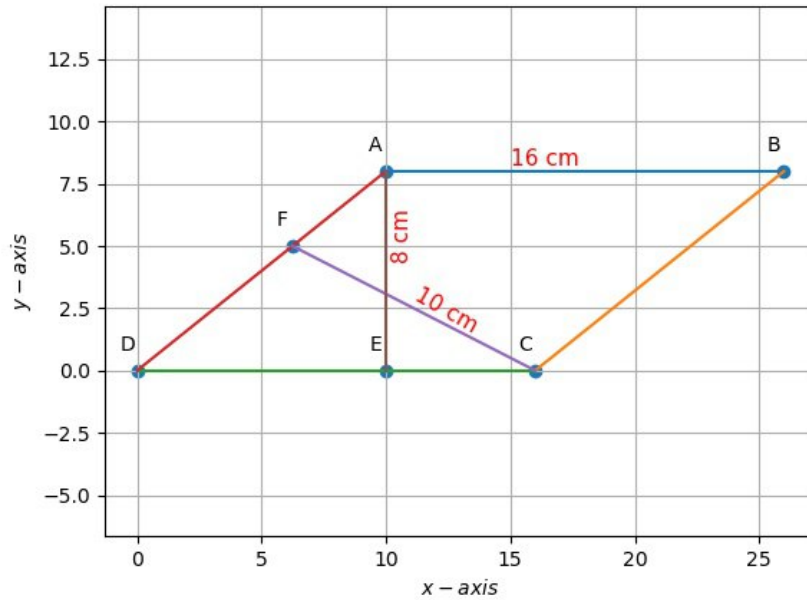


Figure 1: Parallelogram ABCD

The input parameters for the above construction are shown in the table below :

Symbol	Value	Description
AB	16cm	$\ \mathbf{B} - \mathbf{A}\ $
CD	16cm	$\ \mathbf{D} - \mathbf{C}\ = \ \mathbf{B} - \mathbf{A}\ $
AE	8cm	$\ \mathbf{E} - \mathbf{A}\ $
CF	10cm	$\ \mathbf{F} - \mathbf{C}\ $
$\angle CFD$	90°	$CF \perp AD$
$\angle AED$	90°	$AE \perp CD$

Table 1: Parameters

Solution

It is given that the length of $AB = 16cm$. So,

$$\|\mathbf{B} - \mathbf{A}\| = 16cm \quad (1)$$

(2)

As the figure above is a parallelogram,

$$\|\mathbf{B} - \mathbf{A}\| = \|\mathbf{C} - \mathbf{D}\| = 16cm \quad (3)$$

(4)

In order to find $\angle DCF$,

$$\text{Let } \theta_2 = \angle FCD \quad (5)$$

$$\mathbf{n}_1 = \mathbf{C} - \mathbf{F} = \begin{pmatrix} 6.25 \\ -7.806 \end{pmatrix}, \mathbf{n}_2 = \mathbf{C} - \mathbf{D} = \begin{pmatrix} 16 \\ 0 \end{pmatrix} \quad (6)$$

$$\theta_2 = \cos^{-1} \frac{\mathbf{n}_1^\top \mathbf{n}_2}{\|\mathbf{n}_1\| \|\mathbf{n}_2\|} \quad (7)$$

$$\Rightarrow \theta_2 = \cos^{-1} \frac{(6.25 \quad -7.806) \begin{pmatrix} 16 \\ 0 \end{pmatrix}}{(10)(16)} = 51.32^\circ \quad (8)$$

As the sum of internal angles of a triangle are 180° , from $\triangle DFC$

$$\angle CFD + \angle FCD + \angle D = 180^\circ \quad (9)$$

$$\angle D = 180^\circ - (\angle CFD + \angle FCD) \quad (10)$$

$$\angle D = 180^\circ - 141.32^\circ \quad (11)$$

$$\angle D = 38.68^\circ \quad (12)$$

(13)

The area of parallelogram is calculated using the below formula

$$Area = (base) * (height) \quad (14)$$

$$(15)$$

For the above problem this can be written as,

$$Area = \|\mathbf{A} - \mathbf{E}\| \|\mathbf{B} - \mathbf{A}\| \quad (16)$$

$$\implies (8)(16) = 128 \quad (17)$$

$$(18)$$

The area of parallelogram can also be found out using cross product of two vectors, for the above problem it can be written as;

$$Area of parallelogram ABCD = \mathbf{DC} \times \mathbf{AD} \quad (19)$$

$$\mathbf{DC} \times \mathbf{AD} = \|\mathbf{D} - \mathbf{C}\| \|\mathbf{D} - \mathbf{A}\| \sin D \quad (20)$$

$$(21)$$

From ?? and ??

$$\|\mathbf{D} - \mathbf{A}\| (16) \sin 38.68 = 128 \quad (22)$$

$$\|\mathbf{D} - \mathbf{A}\| (16) \left(\frac{5}{8}\right) = 128 \quad (23)$$

$$\|\mathbf{D} - \mathbf{A}\| = \frac{64}{5} \quad (24)$$

$$\|\mathbf{D} - \mathbf{A}\| = 12.8 \text{ cm} \quad (25)$$

$$(26)$$

Therefore, $|\overrightarrow{\mathbf{AD}}| = 12.8 \text{ cm}$