CHAPTER-9
AREAS OF PARALLELOGRAMS AND TRIANGLES

1 Exercise 9.2

Q1. In the figure given below, $ABCD$ is a parallelogram, $AE \perp DC$ and $CF \perp AD$.If $AB = 16cm$, $AE = 8cm$ and $CF = 10cm$, find AD.

Construction

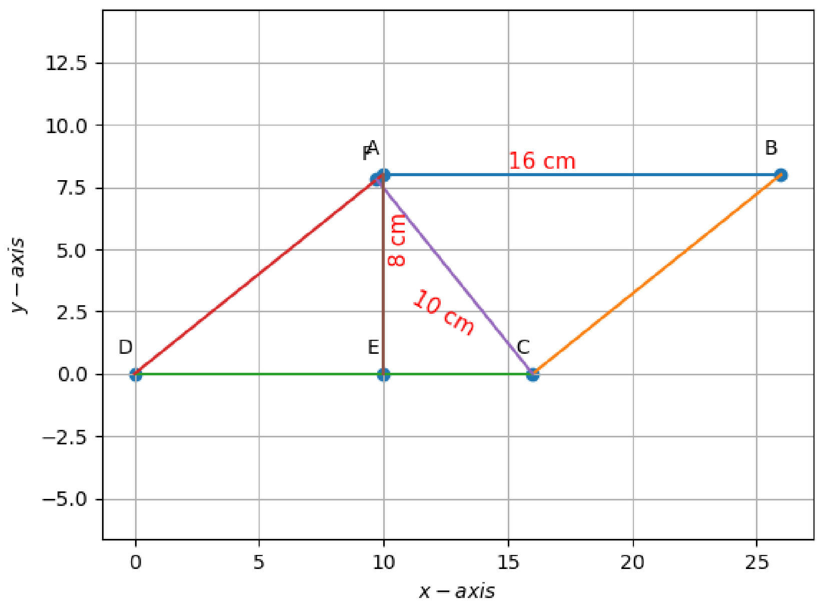

Figure 1: Parallelogram ABCD

The following table displays the given input parameters :

Symbol	Value	Description
a	10cm	CF
b	8cm	AE
l	16cm	AB
$\angle AED$	90°	AE $\perp$ DC
$\angle DFC$	90°	CF $\perp$ AD

Table 1: Input Parameters

The lengths and angles which are to be found out are displayed in the table below along with their symbols :

Symbol	Description
c	CD
d	DE
r	AD
f	DF
θ	$\angle D$

Table 2: Unknown Parameters

The input co-ordinates of the above parallelogram is **D** which is at the origin.The rest of the point co-ordinates can be derived based on this assumption in the following way which is shown in the table below :

Point	Co-ordinates
A	$r\begin{pmatrix}\cos\theta\\ \sin\theta\end{pmatrix}$
B	$\mathbf{A}+\mathbf{C}$
C	$c\mathbf{e_1}$
E	$d\mathbf{e_1}$
F	$f\begin{pmatrix}\cos\theta\\ \sin\theta\end{pmatrix}$

Table 3: Unknown Co-ordinates

Deriving the Unknown lengths and angles in terms of known and derived parameters :

1. **Deriving c:**

From Figure1, c is parallel to l(AB paralle to CD).So,

$$c = l \tag{1}$$
$$\tag{2}$$

2. **Deriving d:**
From $\triangle ADE$,

$$\cos\theta = \frac{DE}{AD} = \frac{d}{r} \tag{3}$$
$$\implies d = r\cos\theta \tag{4}$$
$$\tag{5}$$
$$\tag{6}$$

3. **Deriving r:**
From $\triangle ADE$,

$$\sin\theta = \frac{AE}{AD} = \frac{b}{r} \tag{7}$$
$$\implies r = \frac{b}{\sin\theta} \tag{8}$$
$$\tag{9}$$
$$\tag{10}$$

4. **Deriving f:**
From $\triangle DFC$,

$$\cos\theta = \frac{DF}{DC} = \frac{f}{c} \tag{11}$$
$$\implies f = c\cos\theta \tag{12}$$
$$\tag{13}$$
$$\tag{14}$$

5. **Finding θ:**
From $\triangle DFC$,

$$\sin\theta = \frac{CF}{CD} = \frac{a}{c} \tag{15}$$
$$\tag{16}$$
$$\implies \theta = \sin^{-1}\frac{a}{c} \tag{17}$$
$$\tag{18}$$
$$\tag{19}$$

From 2,5,9,13 and 18, table2 can be modified as :

Symbol	value	Description
c	l	DC
r	$\frac{b}{\sin\theta}$	AD
d	$r\cos\theta$	DE
θ	$\sin^{-1}\frac{a}{c}$	$\angle D$
f	$c\cos\theta$	DF

Table 4: Unknown parameters in terms of known and derived parameters

Deriving co-ordinates in terms of known and derived parameters:
Based on table4, table3 can be modified as follows :

Point	Co-ordinates
A	$\frac{b}{\sin\theta}\begin{pmatrix}\cos\theta\\ \sin\theta\end{pmatrix}$
B	$\mathbf{A}+\mathbf{C}$
C	$c\mathbf{e_1}$
E	$r\cos\theta\mathbf{e_1}$
F	$c\cos\theta\begin{pmatrix}\cos\theta\\ \sin\theta\end{pmatrix}$

Table 5: Co-ordinates in terms of known and derived co-ordinates

Finding Co-ordinates:

1. **Co-ordinates of A:**
   From 18,the value of θ is :

$$\theta = \sin^{-1}\frac{a}{c} = \sin^{-1}\frac{10}{16} \tag{20}$$
$$\implies \theta = 38.68^\circ \tag{21}$$
$$\tag{22}$$

So, co-ordinates of **A** can be derived from table5 and they are :

$$\mathbf{A} = \frac{8}{\sin 38.68}\begin{pmatrix}\cos 38.68\\ \sin 38.68\end{pmatrix} \tag{23}$$
$$\implies \mathbf{A} = \begin{pmatrix}10\\ 8\end{pmatrix} \tag{24}$$
$$\tag{25}$$

2. **Co-ordinates of B:**
   From table5,co-ordinates of **B** are :

$$\mathbf{B} = \begin{pmatrix} 10 \\ 8 \end{pmatrix} + \begin{pmatrix} 16 \\ 0 \end{pmatrix} \tag{26}$$

$$\mathbf{B} = \begin{pmatrix} 26 \\ 8 \end{pmatrix} \tag{27}$$

$$\tag{28}$$

3. **Co-ordinates of C are :**
   From table5,co-ordinates of **C** are :

$$\mathbf{C} = ce_1 = \begin{pmatrix} c \\ 0 \end{pmatrix} \tag{29}$$

$$\implies \mathbf{C} = \begin{pmatrix} 16 \\ 0 \end{pmatrix} \tag{30}$$

$$\tag{31}$$

4. **Co-ordinates of E :**
   From 9, the value of r is :

$$r = \frac{8}{\sin 38.68} = 12.8cm \tag{32}$$

$$\tag{33}$$

   From table5,co-ordinates of **E** are :

$$\mathbf{E} = (12.8) \cos 38.68 \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{34}$$

$$\implies \mathbf{E} = \begin{pmatrix} 10 \\ 0 \end{pmatrix} \tag{35}$$

$$\tag{36}$$

5. **co-ordinates of F:**
   From table5,co-ordinates of **F** are :

$$\mathbf{F} = (16) \cos 38.68 \begin{pmatrix} \cos 38.68 \\ \sin 38.68 \end{pmatrix} \tag{37}$$

$$\implies \mathbf{F} = \begin{pmatrix} 9.75 \\ 7.8 \end{pmatrix} \tag{38}$$

$$\tag{39}$$

Point	Co-ordinates
A	$\begin{pmatrix} 10 \\ 8 \end{pmatrix}$
B	$\begin{pmatrix} 26 \\ 8 \end{pmatrix}$
C	$\begin{pmatrix} 16 \\ 0 \end{pmatrix}$
D	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$
E	$\begin{pmatrix} 10 \\ 0 \end{pmatrix}$
F	$\begin{pmatrix} 9.75 \\ 7.8 \end{pmatrix}$

Table 6: Final Co-ordinates

So, the final co-ordinates of the parallelogram are displayed in the table below:

From 33, we got the length of AD = r = 12.8cm.