CHAPTER-9
AREAS OF PARALLELOGRAMS AND TRIANGLES

1 Exercise 9.2

Q1. In the figure given below, $ABCD$ is a parallelogram, $AE \perp DC$ and $CF \perp AD$.If $AB = 16cm$, $AE = 8cm$ and $CF = 10cm$, find AD.
Construction

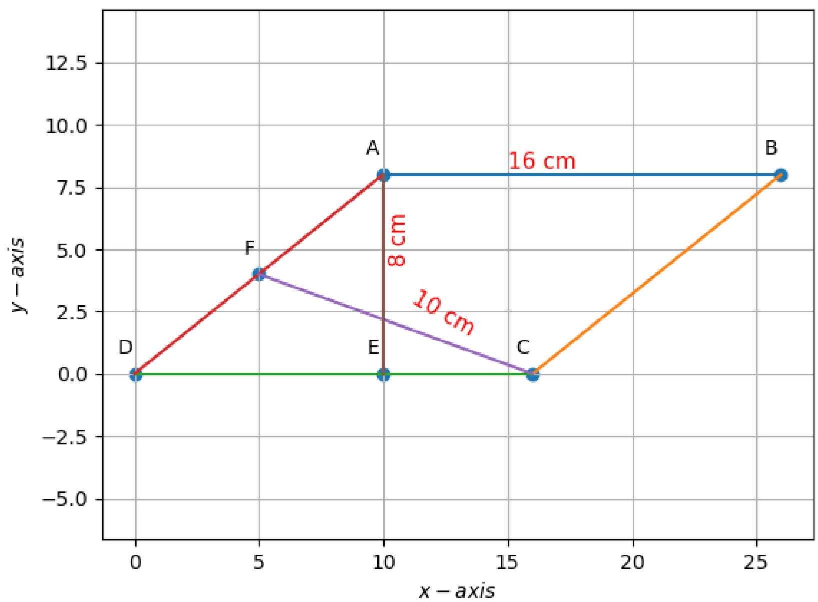

Figure 1: Parallelogram ABCD

The input parameters for the above construction are shown in the table below :

Symbol	Value	Description
AB	16cm	$\parallel \mathbf{B} - \mathbf{A} \parallel$
CD	16cm	$\parallel \mathbf{D} - \mathbf{C} \parallel = \parallel \mathbf{B} - \mathbf{A} \parallel$
AE	8cm	$\parallel \mathbf{E} - \mathbf{A} \parallel$
CF	10cm	$\parallel \mathbf{F} - \mathbf{C} \parallel$
$\angle CFD$	90°	$CF \perp AD$
$\angle AED$	90°	$AE \perp CD$

Table 1: Parameters

The input co-ordinates of the above parallelogram are given in this table :

Point	Co-ordinates	Description
D	(0,0)	Origin(Assumption)
C	(16,0)	(because $\mathbf{CD} = 16cm$)

Table 2: Co-ordinates

The rest of the co-ordinates were obtained in the following way :

1. Deriving the co-ordinates for E :
   The length of DE gives the x co-ordinate of E and it is obtained in the following way :

$$\tan D = \frac{AE}{DE} \tag{1}$$
$$DE = \cot D * AE \tag{2}$$
$$DE = \cot 38.68 * 8cm \tag{3}$$
$$(\frac{10}{8}) * 8cm = 10cm \tag{4}$$
$$\tag{5}$$

   Therefore, the x-coordinate is 10.As, the point E is on the same line as that of D the y co-ordinate is 0.
   So, the co-ordinates of E are (10,0).

2. Deriving the co-ordinates of A :
   As A lies exactly above E the x co-ordinate of A is same as that of E which is 10.
   The length of AE gives the y co-ordinate of A which is 8.So, the co-ordinates of A are (10,8).

3. Deriving the co-ordinates of B :
   The y co-ordinate of B is same as that of A because both the points are on the same horizontal line which is 8.
   In order to obtain the x co-ordinate :

$$B_x = A_x + 16 (because AB = 16cm) \tag{6}$$
$$B_x = 10 + 16 = 26. \tag{7}$$
$$\tag{8}$$

   The co-ordinates of B are (26,8).

4. Deriving the co-ordinates of F :
   Let us initially assume that the co-ordinates of F are (x,y).The co-ordinates of F are drived using distance formula for DF and CF and solving these equations,we get the co-ordinates of F.
   To find out length of DF :

$$\cos D = \frac{DF}{DC} (from \triangle DFC) \tag{9}$$
$$DF = \cos D * DC \tag{10}$$
$$DF = \cos 38.68 * 16 \tag{11}$$
$$DF = 12.48cm \tag{12}$$
$$\tag{13}$$

   By applying distance formula between point D and point F, we get the following equation :

$$x^2 + y^2 = 156 \tag{14}$$
$$\tag{15}$$

   By applying the distance formula between point F and point C, we get te following equation :

$$(x - 16)^2 + y^2 = 100 \tag{16}$$
$$\tag{17}$$

   By solving the above two equations, we get the co-ordinates of F which are (9.75,7.806).

Solution

It is given that the length of $AB = 16cm$.So,

$$\|\mathbf{B} - \mathbf{A}\| = 16cm \tag{18}$$
$$\tag{19}$$

As the figure above is a parallelogram,

$$\|\mathbf{B}-\mathbf{A}\| = \|\mathbf{C}-\mathbf{D}\| = 16cm \tag{20}$$
$$\tag{21}$$

In order to find ∠DCF,

$$\text{Let } \theta_2 = \angle FCD \tag{22}$$
$$\mathbf{n_1} = \mathbf{C}-\mathbf{F} = \begin{pmatrix} 6.25 \\ -7.806 \end{pmatrix}, \mathbf{n_2} = \mathbf{C}-\mathbf{D} = \begin{pmatrix} 16 \\ 0 \end{pmatrix} \tag{23}$$
$$\theta_2 = \cos^{-1} \frac{\mathbf{n_1}^\top \mathbf{n_2}}{\|\mathbf{n_1}\| \|\mathbf{n_2}\|} \tag{24}$$
$$\implies \theta_2 = \cos^{-1} \frac{\begin{pmatrix} 6.25 & -7.806 \end{pmatrix} \begin{pmatrix} 16 \\ 0 \end{pmatrix}}{(10)(16)} = 51.32^\circ \tag{25}$$

As the sum of interal angles of a triangle are 180°,from $\triangle DFC$

$$\angle CFD + \angle FCD + \angle D = 180^\circ \tag{26}$$
$$\angle D = 180^\circ - (\angle CFD + \angle FCD) \tag{27}$$
$$\angle D = 180^\circ - 141.32^\circ \tag{28}$$
$$\angle D = 38.68^\circ \tag{29}$$
$$\tag{30}$$

The area of parallelogram is calculated using the below formula

$$Area = (base) * (height) \tag{31}$$
$$\tag{32}$$

For the above problem this can be written as,

$$Area = \|\mathbf{A}-\mathbf{E}\| \|\mathbf{B}-\mathbf{A}\| \tag{33}$$
$$\implies (8)(16) = 128 \tag{34}$$
$$\tag{35}$$

The area of parallelogram can also be found out using cross product of two vectors,for the above problem it can be written as;

$$Areaof parallelogram ABCD = \mathbf{DC} \times \mathbf{AD} \tag{36}$$
$$\mathbf{DC} \times \mathbf{AD} = \|\mathbf{D}-\mathbf{C}\| \|\mathbf{D}-\mathbf{A}\| \sin D \tag{37}$$
$$\tag{38}$$

From 20 and 37

$$\left\|\mathbf{D}-\mathbf{A}\right\|(16)\sin 38.68 = 128 \tag{39}$$

$$\left\|\mathbf{D}-\mathbf{A}\right\|(16)(\frac{5}{8}) = 128 \tag{40}$$

$$\left\|\mathbf{D}-\mathbf{A}\right\| = \frac{64}{5} \tag{41}$$

$$\left\|\mathbf{D}-\mathbf{A}\right\| = 12.8cm \tag{42}$$

$$\tag{43}$$

Therefore,| $\overrightarrow{\mathbf{AD}}$ | **= 12.8 cm**