CHAPTER-9
AREAS OF PARALLELOGRAMS AND TRIANGLES

1 Exercise 9.2

Q1. In the figure given below, $ABCD$ is a parallelogram, $AE \perp DC$ and $CF \perp AD$.If $AB = 16cm$, $AE = 8cm$ and $CF = 10cm$, find AD.

Construction

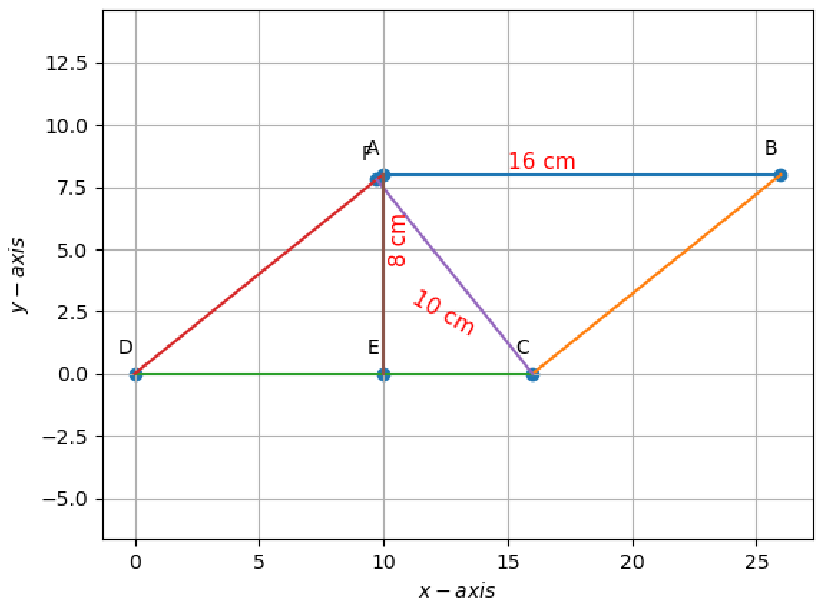

Figure 1: Parallelogram ABCD

The following table consists of given input parameters of the above parallelogram ABCD :

Symbol	Value	Description
x	16cm	$\parallel \mathbf{B} - \mathbf{A} \parallel$
a	10cm	$\parallel \mathbf{F} - \mathbf{C} \parallel$
b	8cm	$\parallel \mathbf{E} - \mathbf{A} \parallel$
$\angle CFD$	90°	$CF \perp AD$
$\angle AED$	90°	$AE \perp CD$

Table 1: Parameters

Table below has the given input co-ordinates of the parallelogram :

Point	Co-ordinates
D	(0,0)

Table 2: Co-ordinates

Following table are the unknown lengths and angles and their symbols:

Symbol	Description
c	$\|\mathbf{D} - \mathbf{C}\|$
r	$\|\mathbf{A} - \mathbf{D}\|$
d	$\|\mathbf{D} - \mathbf{E}\|$
b	$\|\mathbf{A} - \mathbf{E}\|$
θ	$\angle\mathbf{D}$

Table 3: Symbols and Corresponding Vectors

The point co-ordinates are derived in the following way :

1. **To derive the co-ordinates of C :**
   As mentioned in the 3, $\|\mathbf{D} - \mathbf{C}\| = c$.In the above parallelogram it is given that $\|\mathbf{B} - \mathbf{A}\| = 16cm$.According to the properties of a parallelogram the parallel sides are equal in length.So, it can be said that :

$$\|\mathbf{B} - \mathbf{A}\| = \|\mathbf{D} - \mathbf{C}\| = c \tag{1}$$

$$\tag{2}$$

   As point **C** lies on x axis, it can be expressed in the following way :

$$\mathbf{C} = c\mathbf{e_1} \tag{3}$$

$$\mathbf{C} = c\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}c\\0\end{pmatrix} \tag{4}$$

$$\tag{5}$$

2. **To derive the co-ordinates of A :**
   A can be expressed in the form of $r\begin{pmatrix}\cos\theta\\ \sin\theta\end{pmatrix}$.In order to obtain r and θ, the following can be done :

   (a) **To find out θ :**
   To find out θ,let us assume that $\|\mathbf{C}-\mathbf{F}\| = a$

   $$from\triangle CFD, \tag{6}$$

   $$\sin\theta = \frac{a}{c} \tag{7}$$

   $$\implies \theta = \sin^{-1}\frac{a}{c} \tag{8}$$

   $$\tag{9}$$

   (b) **To find out r :**
   As mentioned in 3, $\|\mathbf{D}-\mathbf{A}\| = r$ and $\|\mathbf{E}-\mathbf{A}\| = b$.In order to find out r,

   $$from\triangle ADE, \tag{10}$$

   $$\sin\theta = \frac{b}{r} \tag{11}$$

   $$r = \frac{b}{\sin\theta} \tag{12}$$

   $$\tag{13}$$

   So, the co-ordinates of **A** can be written as :

   $$\mathbf{A} = \frac{b}{\sin\theta}\begin{pmatrix}\cos\theta\\ \sin\theta\end{pmatrix} \tag{14}$$

   $$\mathbf{A} = \begin{pmatrix}b\cot\theta\\ b\end{pmatrix} \tag{15}$$

   $$\tag{16}$$

3. **To derive the co-ordinates of B :**
   From parallelogram law of vectors, **B** can be expressed as the sum of **A**

and $\mathbf{C}$.So, it can be written as,

$$\mathbf{B} = \mathbf{A} + \mathbf{C} \tag{17}$$

$$\tag{18}$$

4. **To derive the co-ordinates of E :**
   As mentioned in the table3, $\|\mathbf{D} - \mathbf{E}\| = d$.As, $\mathbf{E}$ lies on x-axis it can be written in the form of de_1.So, the co-ordinates can be found out in the following way :

$$from \triangle DAE, \tag{19}$$

$$\cos\theta = \frac{d}{r} \tag{20}$$

$$d = r\cos\theta \tag{21}$$

$$\tag{22}$$

$$\mathbf{E} = d\mathbf{e_1} = \text{rcos}\,\theta \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} r\cos\theta \\ 0 \end{pmatrix}.$$

5. **To derive the co-ordinates of F :**
   As point $\mathbf{F}$ divides $\mathbf{AD}$ in the ratio k : 1.The co-ordinates of $\mathbf{F}$ can be found out in the following way :

$$\mathbf{F} = \frac{k\mathbf{A} + \mathbf{D}}{k+1} \tag{23}$$

$$\tag{24}$$

The following table displays the unknown lengths and angles which were derived from the given quantities :

Symbol	value	Description
c	x	$\|\mathbf{D} - \mathbf{C}\|$
r	$\frac{b}{\sin\theta}$	$\|\mathbf{D} - \mathbf{A}\|$
d	rcos θ	$\|\mathbf{D} - \mathbf{E}\|$
θ	$\sin^{-1}\frac{a}{c}$	$\angle D$

Table 4: Co-ordinates in terms of given and derived lengths and angles

The following table displays the point co-ordinates in terms of known and derived quantities:

Point	Co-ordinates
A	$\begin{pmatrix} b\cot\theta \\ b \end{pmatrix}$
B	$\mathbf{A}+\mathbf{C}$
C	$\begin{pmatrix} c \\ 0 \end{pmatrix}$
E	$\begin{pmatrix} r\cos\theta \\ 0 \end{pmatrix}$
F	$\frac{k\mathbf{A}+\mathbf{D}}{k+1}$

Table 5: Co-ordinates in terms of known and derived quantities

Finding the co-ordinates of the above parallelogram:

1. **Co-ordinates of C:**
   As c = $\|\mathbf{D}-\mathbf{C}\| = 16$, the co-ordinates of **C** are:

$$\mathbf{C} = \begin{pmatrix} c \\ 0 \end{pmatrix} = \begin{pmatrix} 16 \\ 0 \end{pmatrix} \tag{25}$$

$$\tag{26}$$

   So, the co-ordinates of **C** are $\begin{pmatrix} 16 \\ 0 \end{pmatrix}$.

2. **Co-ordinates of A:**

   (a) **Finding θ:**

$$\theta = \sin^{-1}\frac{a}{c} \tag{27}$$

$$\theta = \sin^{-1}\frac{10}{16} \tag{28}$$

$$\theta = 38.68^\circ \tag{29}$$

$$\tag{30}$$

   From the above derivations, we got that:

$$\mathbf{A} = \begin{pmatrix} b\cot\theta \\ b \end{pmatrix} \tag{31}$$

$$\implies \begin{pmatrix} 8\cot 38.68 \\ 8 \end{pmatrix} \tag{32}$$

$$\mathbf{A} = \begin{pmatrix} 10 \\ 8 \end{pmatrix} \tag{33}$$

$$\tag{34}$$

so, the co-ordinates of **A** are $\begin{pmatrix} 10 \\ 8 \end{pmatrix}$.

3. **Co-ordinates of B:**
   From above derivation, we got that $\mathbf{B} = \mathbf{A} + \mathbf{C}$.Then, **C** is:

$$\mathbf{B} = \begin{pmatrix} 10 \\ 8 \end{pmatrix} + \begin{pmatrix} 16 \\ 0 \end{pmatrix} \tag{35}$$

$$\mathbf{B} = \begin{pmatrix} 26 \\ 8 \end{pmatrix} \tag{36}$$

$$\tag{37}$$

So, the co-ordinates of **B** are $\begin{pmatrix} 26 \\ 8 \end{pmatrix}$.

4. **Co-ordinates of E:**

   (a) **Finding r:**
   From above derivation, r can be found out in the following way:

$$r = \frac{b}{\sin\theta} \tag{38}$$

$$r = \frac{8}{\sin 38.68} \tag{39}$$

$$r = 12.8cm \tag{40}$$

$$\tag{41}$$

The co-ordinates of **E** are:

$$\mathbf{E} = \begin{pmatrix} r\cos\theta \\ 0 \end{pmatrix} \tag{42}$$

$$\implies \begin{pmatrix} (12.8)\cos 38.68 \\ 0 \end{pmatrix} \tag{43}$$

$$\mathbf{E} = \begin{pmatrix} 10 \\ 0 \end{pmatrix} \tag{44}$$

$$\tag{45}$$

So, the co-ordinates for **E** are $\begin{pmatrix} 10 \\ 0 \end{pmatrix}$.

5. **Co-ordinates for F:**
   F divides **AD** in the ratio 39 : 1.So, the co-ordinates of **F** are:

$$\mathbf{F} = \frac{k\mathbf{A} + \mathbf{D}}{k+1} \tag{46}$$

$$\implies \frac{(39)\begin{pmatrix} 10 \\ 8 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix}}{39+1} \tag{47}$$

$$\implies \frac{\begin{pmatrix} 390 \\ 312 \end{pmatrix}}{40} \tag{48}$$

$$\mathbf{F} = \begin{pmatrix} 9.75 \\ 7.8 \end{pmatrix} \tag{49}$$

   So, the co-ordinates of **F** are $\begin{pmatrix} 9.75 \\ 7.8 \end{pmatrix}$.

The following table displays the final co-ordinates of the vertices of the parallelogram :

Point	Co-ordinates
A	$\begin{pmatrix} 10 \\ 8 \end{pmatrix}$
B	$\begin{pmatrix} 26 \\ 8 \end{pmatrix}$
C	$\begin{pmatrix} 16 \\ 0 \end{pmatrix}$
D	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$
E	$\begin{pmatrix} 10 \\ 0 \end{pmatrix}$
F	$\begin{pmatrix} 9.75 \\ 7.8 \end{pmatrix}$

Table 6: Final co-ordinates of the parallelogram

The length of $\|\mathbf{D} - \mathbf{A}\|$ was found out in the above process and it is r = 12.8cm.