Class 11

Chapter 10 - STRAIGHT LINES

The following problem is question 11 from exercise 10.4

1. Find the equation of the lines through the point (3, 2) which make an angle of 45°with the line x − 2y = 3.

Solution:

The given line parameters are

$$\mathbf{n} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}, c = -5 \tag{1}$$

$$\mathbf{P} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \tag{2}$$

yielding

$$\mathbf{m}_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \tag{3}$$

$$\mathbf{m_2} = \begin{pmatrix} 1 \\ m \end{pmatrix} \tag{4}$$

where m is defined to be the slope of the line. If the angle between the lines be θ,

$$\cos\theta = \frac{\mathbf{m}_1^\top \mathbf{m}_2}{\|\mathbf{m}_1\| \, \|\mathbf{m}_2\|} \tag{5}$$

$$\text{given, } \theta = 45^\circ \tag{6}$$

$$\implies \cos 45^\circ = \frac{\mathbf{m}_1^\top \mathbf{m}_2}{\|\mathbf{m}_1\| \, \|\mathbf{m}_2\|} \tag{7}$$

$$\implies \frac{1}{\sqrt{2}} = \frac{\begin{pmatrix} 2 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ m \end{pmatrix}}{\left\| \begin{pmatrix} 2 \\ 1 \end{pmatrix} \right\| \left\| \begin{pmatrix} 1 \\ m \end{pmatrix} \right\|} \tag{8}$$

$$\implies \frac{1}{\sqrt{2}} = \frac{2+m}{\sqrt{2^2+1}\sqrt{m^2+1}} \tag{9}$$

$$\implies \frac{1}{2} = \frac{m^2+4m+4}{5m^2+5} \tag{10}$$

$$\text{or, } 3m^2 - 8m - 3 = 0 \tag{11}$$

yielding

$$m = -\frac{1}{3}, 3 \tag{12}$$

when m=3,the equation of line passing through $\mathbf{P}$ is then obtained as

$$\mathbf{n}^\top(\mathbf{x} - \mathbf{P}) = 0 \tag{13}$$

$$\text{where,}\mathbf{n} = \begin{pmatrix} m \\ -1 \end{pmatrix} \tag{14}$$

$$\mathbf{n} = \begin{pmatrix} 3 \\ -1 \end{pmatrix} \tag{15}$$

$$\implies \begin{pmatrix} 3 & -1 \end{pmatrix} \left\{ \mathbf{x} - \begin{pmatrix} 3 \\ 2 \end{pmatrix} \right\} = 0 \tag{16}$$

$$= 7 \tag{17}$$

$$\implies \begin{pmatrix} 3 & -1 \end{pmatrix} \mathbf{x} = 7 \tag{18}$$

And, when $m = -\frac{1}{3}$,the equation of the line passing through $\mathbf{P}$ and having a slope of $-\frac{1}{3}$is

$$\mathbf{n}^\top(\mathbf{x} - \mathbf{P}) = 0 \tag{19}$$

$$\mathbf{n} = \begin{pmatrix} -\frac{1}{3} \\ -1 \end{pmatrix} \tag{20}$$

$$\implies \mathbf{n} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \tag{21}$$

$$\implies \begin{pmatrix} 1 & 3 \end{pmatrix} \left\{ \mathbf{x} - \begin{pmatrix} 3 \\ 2 \end{pmatrix} \right\} = 0 \tag{22}$$

$$= 9 \tag{23}$$

$$\implies \begin{pmatrix} 1 & 3 \end{pmatrix} \mathbf{x} = 9 \tag{24}$$

Therefore,the equations of the lines are

$$\begin{pmatrix} 3 & -1 \end{pmatrix} \mathbf{x} = 7 \text{ and } \begin{pmatrix} 1 & 3 \end{pmatrix} \mathbf{x} = 9. \tag{25}$$

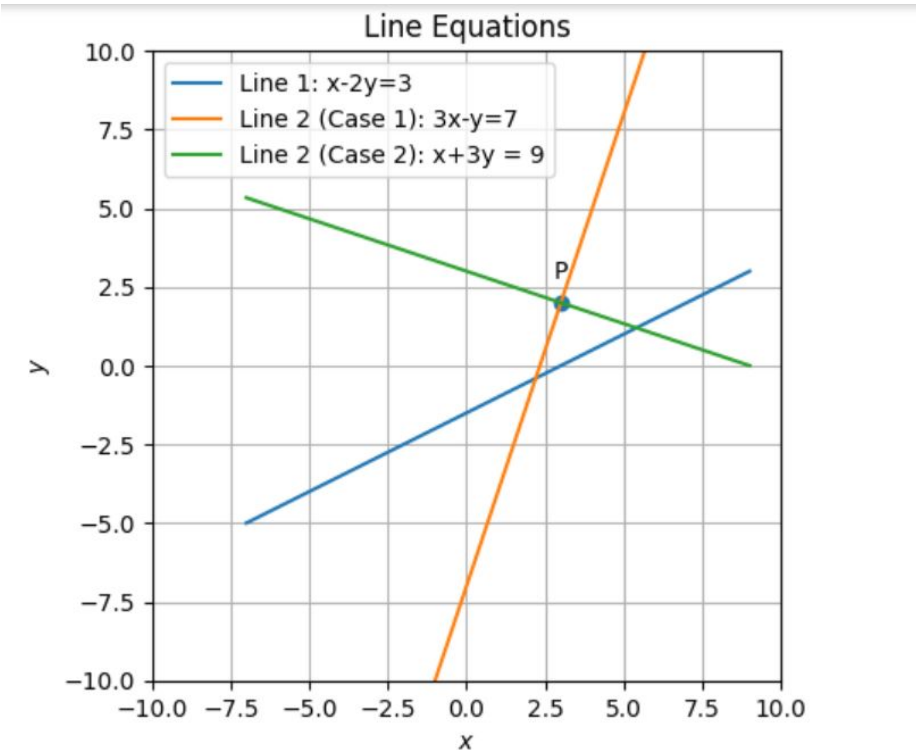

Figure 1: STRAIGHT LINES