

ASSIGNMENT 2

Question : Two lines passing through the point(2,3) intersect each other at an angle of 60° . If slope of one line is 2, find equation of the other line.

Solution :

Symbol	Description	Value
m_1	Slope of one line	2
m_2	Slope of another line	m
P	Intersecting point	$\begin{pmatrix} 2 \\ 3 \end{pmatrix}$
θ	Angle between two lines	60°

Table 1: Table of input parameters

Symbol	Description	Value
\mathbf{m}_1	Direction vector of one line	$\begin{pmatrix} 1 \\ 2 \end{pmatrix}$
\mathbf{n}_1	Normal vector of one line	$\begin{pmatrix} 2 \\ -1 \end{pmatrix}$
\mathbf{m}_2	Direction vector of another line	$\begin{pmatrix} 1 \\ m \end{pmatrix}$
\mathbf{n}_2	Normal vector of another line	$\begin{pmatrix} m \\ -1 \end{pmatrix}$

Table 2: Table of output parameters

So,

$$\cos \theta = \frac{(\mathbf{m}_1^T)\mathbf{m}_2}{\|\mathbf{m}_1\|\|\mathbf{m}_2\|} \quad (1)$$

$$or, \frac{1}{2} = \frac{(1 \ 2) \begin{pmatrix} 1 \\ m \end{pmatrix}}{\sqrt{5}\sqrt{m^2+1}} \quad (2)$$

$$or, \frac{1}{2} = \frac{1+2m}{\sqrt{5m^2+5}} \quad (3)$$

$$or, 11m^2 + 16m - 1 = 0 \quad (4)$$

$$or, m = \frac{-8 \pm 5\sqrt{3}}{11} \quad (5)$$

Therefore, the direction vector is, $\mathbf{m}_2 = \begin{pmatrix} 1 \\ \frac{-8+5\sqrt{3}}{11} \end{pmatrix}$ or, $\begin{pmatrix} 1 \\ \frac{-8-5\sqrt{3}}{11} \end{pmatrix}$

The normal vector is, $\mathbf{n}_2 = \begin{pmatrix} \frac{-8+5\sqrt{3}}{11} \\ -1 \end{pmatrix}$ or, $\begin{pmatrix} \frac{-8-5\sqrt{3}}{11} \\ -1 \end{pmatrix}$

So, the equation of the line is

$$\mathbf{n}^T \mathbf{x} = c \quad (6)$$

$$\begin{pmatrix} \frac{-8+5\sqrt{3}}{11} & -1 \end{pmatrix} \mathbf{x} = c \quad (7)$$

Passes through the point $\mathbf{P} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

$$\begin{pmatrix} \frac{-8+5\sqrt{3}}{11} & -1 \end{pmatrix} \mathbf{P} = c \quad (8)$$

$$or, c = \frac{-49 \pm 16\sqrt{3}}{11} \quad (9)$$

Figure :

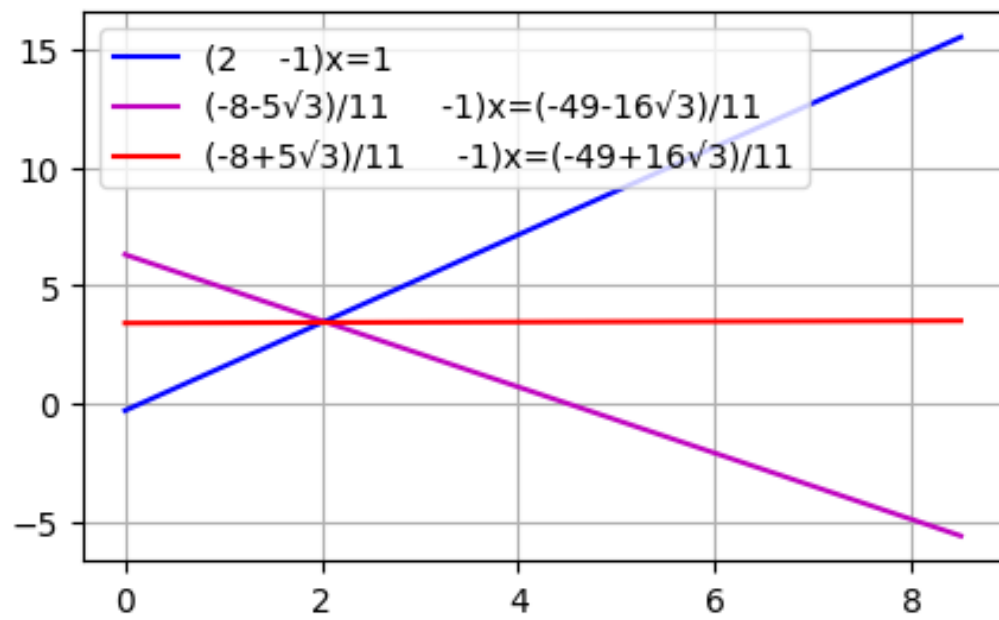


Figure 1: