

$$E = \sum_{i=1}^{\infty} [y_i - f(x_i)]^2$$

Here  $f(x_i) = ax_i^2 + bx_i + c = 0$

$$\therefore E = \sum_{i=1}^{\infty} [y_i - (ax_i^2 + bx_i + c)]^2$$

or  $E = \sum_{i=1}^{\infty} [y_i - ax_i^2 - bx_i - c]^2$

Now,  $\frac{\partial E}{\partial a} = 2 \sum [y_i - ax_i^2 - bx_i - c](-x_i^2) = 0$

$$\Rightarrow \sum y_i (-x_i)^2 + \sum a(x_i)^4 + b \sum (x_i)^3 + c \sum (x_i)^2 = 0$$

$$\Rightarrow \boxed{a \sum (x_i)^4 + b \sum (x_i)^3 + c \sum (x_i)^2 = + \sum y_i (x_i)^2}$$

And  $\frac{\partial E}{\partial b} = 2 \sum [y_i - ax_i^2 - bx_i - c](-x_i) = 0$

$$\Rightarrow -\sum y_i (x_i) + a \sum (x_i)^3 + b \sum (x_i)^2 + c \sum x_i = 0$$

$$\Rightarrow \boxed{a \sum (x_i)^3 + b \sum (x_i)^2 + c \sum x_i = \sum y_i x_i}$$

And  $\frac{\partial E}{\partial c} = 2 \sum [y_i - ax_i^2 - bx_i - c](-1) = 0$

$$\Rightarrow -\sum y_i + a \sum x_i^2 + b \sum x_i + c \sum 1 = 0$$

$$\Rightarrow \boxed{a \sum (x_i)^2 + b \sum x_i + cN = \sum y_i}$$

The matrix is as follows -

$$\begin{pmatrix} \sum (x_i)^4 & \sum (x_i)^3 & \sum (x_i)^2 \\ \sum (x_i)^3 & \sum (x_i)^2 & \sum x_i \\ \sum (x_i)^2 & \sum x_i & \sum 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} \sum y_i (x_i)^2 \\ \sum y_i x_i \\ \sum y_i \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} \sum x_i^4 & \sum x_i^3 & \sum x_i^2 \\ \sum x_i^3 & \sum x_i^2 & \sum x_i \\ \sum x_i^2 & \sum x_i & N \end{pmatrix}^{-1} \begin{pmatrix} \sum y_i (x_i)^2 \\ \sum (y_i x_i) \\ \sum y_i \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} x_a & x_b & x_c \\ x_b & x_c & x \\ x_c & x & N \end{pmatrix}^{-1} \begin{pmatrix} y_a \\ y_b \\ y \end{pmatrix}$$