

⑨ $\overset{\text{rank}}{\text{column space}}(A) + \text{null space}(A) = n = \overset{\text{number of}}{\text{column space}} \text{ columns}$

$$\text{rank}(A) = \dim(C(A))$$

$$\Rightarrow \dim(C(A)) + \dim(N(A)) = n \rightarrow \text{odd}$$

$$\text{but if } \dim C(A) = \dim N(A) = x$$

$$\Rightarrow 2x = n$$

$$x = n/2 \Rightarrow \text{not possible when } n \text{ is odd.}$$

Hence Column-space and Null-space cannot be equal in case of a $n \times n$ matrix where n is odd.