(8) (a) Eigen Values and Eigen Vectors of $A = \begin{bmatrix} 5 & -4 & 0 \\ 1 & 0 & 2 \\ 0 & 2 & 5 \end{bmatrix}$

$\det(A - \lambda I) = 0$

$$\Rightarrow \det\begin{bmatrix} 5-\lambda & -4 & 0 \\ 1 & -\lambda & 2 \\ 0 & 2 & 5-\lambda \end{bmatrix} = 0$$

$$= (5-\lambda)(-\lambda(5-\lambda)-4) + 4(5-\lambda) = 0$$

$$= (5-\lambda)[-5\lambda + \lambda^2 - \cancel{4} + \cancel{4}] = 0$$

$$(5-\lambda)(\lambda^2 - 5\lambda) = 0$$

$$\Rightarrow (\lambda-5)(\lambda-5)(\lambda) = 0$$

$$\lambda = 0, 5, 5$$

$$\lambda_1 = 0 \quad \lambda_2 = 5 \quad \lambda_3 = 5$$

(i) Eigen vectors corresponding to $\lambda_1 = 0$

$$\Rightarrow (A - \lambda_1 I) X = 0$$

$$\begin{bmatrix} 5 & -4 & 0 \\ 1 & 0 & 2 \\ 0 & 2 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\text{rref}\begin{bmatrix} 5 & -4 & 0 \\ 1 & 0 & 2 \\ 0 & 2 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2.5 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2.5 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_1 + 2x_3 = 0$$

$$x_2 + \frac{5}{2}x_3 = 0$$

$$x_3 = 2 \quad \Rightarrow \quad x_2 = -5$$

$$x_1 = -4$$

$$X = \begin{bmatrix} -4 \\ -5 \\ 2 \end{bmatrix}$$

(ii) Eigen vectors corresponding to $\lambda_2 = 5$

$(A - \lambda_2 I)X = 0$

$$\Rightarrow \begin{bmatrix} 5-5 & -4 & 0 \\ 1 & -5 & 2 \\ 0 & 2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & -4 & 0 \\ 1 & -5 & 2 \\ 0 & 2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\text{rref} \begin{bmatrix} 0 & -4 & 0 \\ 1 & -5 & 2 \\ 0 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow \quad x_1 + 2x_3 = 0$

$x_2 = 0 \qquad \Rightarrow x_1 = -2$

$x_3 = 1$

$$\therefore \quad X = \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$$

||^{ly} for $\lambda_3 = 5$

$$\text{Eigen vector}: \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_1 + 2x_3 = 0$

$x_2 = 0 \qquad x_1 = +4$

$x_3 = -2 \Rightarrow \quad X = \begin{bmatrix} +4 \\ 0 \\ -2 \end{bmatrix}$ deficient matrix

Thus eigen vectors for $\lambda = 1, 5, 5$ are

$$\begin{bmatrix} -4 \\ -5 \\ 2 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 \\ 0 \\ -2 \end{bmatrix}$$

(b) It is not possible to diagonalize the matrix A.

Sol) Since the matrix A has 'n-1' independent eigen values (unique), this means the matrix S will have rank < 3 = 2

$$A = S \Lambda S^{-1}$$

rank(S) = 2

as one column is dependent

which mean ~~...~~ the determinant is 0. and hence the S^{-1} does not exist.

⇒ Thus it is not possible to diagonalize the ~~...~~ matrix A