8th September 2021

EAS 501

HW 1

① a) unit vector u_1 in the direction of $u(6,8)$

and v_1 in direction of $v(2,4,4)$

Sol.) $u_1 = \frac{u}{\|u\|_2} = \frac{(6,8)}{\|(6,8)\|_2} = \frac{6}{\sqrt{6^2+8^2}}, \frac{8}{\sqrt{6^2+8^2}}$

$u_1 = (\frac{6}{10}, \frac{8}{10})$

$\boxed{u_1 = (\frac{3}{5}, \frac{4}{5})}$ → unit vector in direction of $u(6,8)$

$v_1 = \frac{v}{\|v\|_2} = \frac{(2,4,4)}{\sqrt{2^2+4^2+4^2}} = \frac{(2,4,4)}{\sqrt{36}} = (\frac{2}{6}, \frac{4}{6}, \frac{4}{6}$

$\boxed{v_1 = (\frac{1}{3}, \frac{2}{3}, \frac{2}{3})}$ → unit vector in direction of $v(2,4,4)$.

b.) unit vector u_2 $\perp^{lr}$ to u_1 and v_2 $\perp^{lr}$ to v_1

let $u_2 : (x, y)$

since $u_2 \perp u_1 \Rightarrow u_2 \cdot u_1 = 0$

$\Rightarrow (x,y) \cdot (3/5, 4/5) = 0$

$\frac{3x}{5} + \frac{4y}{5} = 0$

$\Rightarrow 3x + 4y = 0$; Since u_2 is unit vector $\|u_2\|_2 = 0$

$\Rightarrow x = -\frac{4y}{3}$ $\Rightarrow (x^2+y^2)^{1/2} = 1$

$= x^2 + y^2 = 1$

$\Rightarrow \frac{16y^2}{9} + y^2 = 1$

$\Rightarrow y = \pm 3/5$ $\therefore x = \mp 4/5$

$\boxed{u_2 = (\pm 3/5, \mp 4/5)}$

unit vector v_2 perpendicular to v_1 $(1/3, 2/3, 2/3)$

let $v_3 = [x, y, z]$

Since $v_3 \perp v_1$

$v_3 \cdot v_1 = 0$

$\Rightarrow [x \; y \; z] (1/3, 2/3, 2/3)$

$\Rightarrow \frac{x}{3} + \frac{2y}{3} + \frac{2z}{3} = 0$

$x + 2y + 2z = 0$

choose $x = 2, y = 1$ we get

$2 + 2(1) + 2z = 0$

$z = -2$

Thus $x, y, z = [2, 1, -2] = v_3$

unit vector $= v_2 = \frac{v_3}{||v_3||_2} = \frac{2, 1, -2}{\sqrt{2^2+1^2+(-2)^2}}$

$$\boxed{v_2 = \left(\frac{2}{3}, \frac{1}{3}, -\frac{2}{3}\right)}$$

unit vector $\perp^{r}$ to v_1.