⑥ $P = \begin{bmatrix} 0.6 & 0.2 \\ 0.4 & 0.8 \end{bmatrix}$

Computing eigen values

$\det(P - \lambda I) = 0$

$$\det \left| \begin{bmatrix} 0.6-\lambda & 0.2 \\ 0.4 & 0.8-\lambda \end{bmatrix} \right| = 0$$

$= (0.6-\lambda)(0.8-\lambda) - (0.2)(0.4) = 0$

$\Rightarrow 0.48 + \lambda^2 - 1.4\lambda - 0.08 = 0$

$\lambda^2 - 1.4\lambda + 0.4 = 0$

$10\lambda^2 - 14\lambda + 4 = 0$

$10\lambda^2 - 10\lambda - 4\lambda + 4 = 0$

$10\lambda(\lambda - 1) - 4(\lambda - 1) = 0$

$\Rightarrow \lambda_1 = 1$ and $\lambda_2 = 0.4$

Eigen vectors corresponding to $\lambda_1 = 1$

$\Rightarrow (A - \lambda_1 I)x = 0$

$$\Rightarrow \begin{bmatrix} 0.6-1 & 0.2 \\ 0.4 & 0.8-1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -0.4 & 0.2 \\ 0.4 & -0.2 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \begin{bmatrix} -0.4 & 0.2 \\ 0 & 0 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$\Rightarrow x_2 = 2 \Rightarrow -0.4x_1 + 0.2(2) = 0$

$x_1 = 1$

Eigen vector : $\begin{pmatrix} 1 \\ 2 \end{pmatrix} \Rightarrow$ ~~Recca~~ Equilibrium Probability

$P^n \cdot x = x$ since probabilities must equal 1

Eigen vector $x = \begin{pmatrix} 1/3 \\ 2/3 \end{pmatrix} = \begin{pmatrix} 0.33 \\ 0.67 \end{pmatrix}$

Thus the Equilibrium Probability is by taking eigen vectors of $\lambda = 1 \Rightarrow$ 33% → A

and 67% → B