

11. $A = \begin{bmatrix} 1 & 2 & 3 & 0 & -2 \\ 0 & 0 & c & 2 & 2 \\ 0 & 0 & 0 & d & -2 \end{bmatrix}$ $B = \begin{bmatrix} d & c \\ c & d \end{bmatrix}$

for A to be rank 2:

$c = 0$ and $d = -2$, which makes ~~columns~~

Rows 2 and 3 equal

if $c = 0$ and $d = -2$, (verification of rank 2)

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 & -2 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & -2 & -2 \end{bmatrix}$$

$$\text{rref}(A) = \begin{bmatrix} \textcircled{1} & 2 & 3 & 0 & -2 \\ 0 & 0 & 0 & \textcircled{1} & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

2 pivots, Rank = 2

$B = \begin{bmatrix} d & c \\ c & d \end{bmatrix}$ for rank 2, $\det(B) \neq 0$
(inverse(B) exists)

$$d^2 - c^2 \neq 0$$

$$\Rightarrow c \neq d, c \neq -d$$

$$\Rightarrow c \neq \pm d$$

Since both matrices A and B must have same values of c and d , we get

$$\boxed{\begin{matrix} c = 0 \\ d = -2 \end{matrix}}$$

Verification for B

$B = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$ $\text{rref}(B) = \begin{bmatrix} \textcircled{1} & 0 \\ 0 & \textcircled{1} \end{bmatrix} \Rightarrow 2 \text{ pivots}$
Rank = 2