12.)

(a) $2x_0 + 4x_1 + x_2 + 6x_3 = 7$

$-2x_0 + 2x_2 + x_3 = 12$

$-2x_0 + 6x_1 + 2x_2 + x_3 = 0$

$-8x_0 + -2x_1 + x_2 + x_3 = -11$

Augmented matrix: $[A|b]$

$$\begin{bmatrix} 2 & 4 & 1 & 6 & 7 \\ -2 & 0 & 2 & 1 & 12 \\ -2 & 6 & 2 & 1 & 0 \\ -8 & -2 & 1 & 1 & -11 \end{bmatrix}$$

rref using matlab (rref(Ab))

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 & -2 \\ 0 & 0 & 1 & 0 & 9 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 9 \\ 0 \end{bmatrix}$$

Unique solution.

(c)

$2x_0 + 4x_1 + x_2 + 6x_3 = 7$

$-2x_0 + 2x_2 + x_3 = 12$

$-2x_0 + 6x_1 + 2x_2 + x_3 = 0$

$-4x_0 - 14x_1 - 2x_2 - 12x_3 = -2$

Augmented matrix:

$$\begin{bmatrix} 2 & 4 & 1 & 6 & 7 \\ -2 & 0 & 2 & 1 & 12 \\ -2 & 6 & 2 & 1 & 0 \\ -4 & -14 & -2 & -12 & -2 \end{bmatrix}$$

RREF using matlab

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 & 1.8333 & 3 \\ 0 & 1 & 0 & 0 & -2 \\ 0 & 0 & 1 & 2.3333 & 9 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

# of pivots = 3, consistent

Hence infinite number of solutions.

$\Rightarrow x_0 + 1.8333 x_3 = 3$

$x_1 = -2$

$x_2 + 2.3333 x_3 = 9$

$x_3 \rightarrow$ free variable

$x_0, x_1, x_2 \rightarrow$ fixed variables

assume $x_3 = 6$

we get

$x_0 + (1.8333)(6) = 3$

$x_0 = -7.9998 \approx -8$

$x_2 + (2.3333)(6) = 9$

$x_2 = -5$

one Solution: $\begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -8 \\ -2 \\ -5 \\ 6 \end{bmatrix}$

(b) $2x_0 + 4x_1 + x_2 + 6x_3 = 7$

$-2x_0 + 2x_2 + x_3 = 12$

$-2x_0 + 6x_1 + 2x_2 + x_3 = 0$

$-4x_0 - 14x_1 - 2x_2 - 12x_3 = 0$

Augmented Matrix:

$$\begin{bmatrix} 2 & 4 & 1 & 6 & 7 \\ -2 & 0 & 2 & 1 & 12 \\ -2 & 6 & 2 & 1 & 0 \\ -4 & -14 & -2 & -12 & 0 \end{bmatrix} \xrightarrow{RREF} \begin{bmatrix} 1 & 0 & 0 & 1.8333 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2.333 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

pivots = 3

but the last row is inconsistent

as $0(x_0) + 0(x_1) + 0(x_2) + 0(x_3) = 1$

Not possible

Hence it has no-solutions.