

4.)

a) Dimensions of the matrices (rows \times columns)

$$A = 3 \times 2$$

$$E = 3 \times 3$$

$$B = 3 \times 3$$

$$F = 2 \times 3$$

$$C = 3 \times 1$$

$$G = 1 \times 3$$

$$D = 2 \times 4$$

b) Square matrices are :- B, E

Column matrix :- C

Row matrix :- G

c) $\det(B) \begin{bmatrix} 4 & 3 & 7 \\ 1 & 2 & 7 \\ 2 & 0 & 4 \end{bmatrix}$

$$\det(B) = 4(4(2) - 7(0)) - 3[4(1) - 2(7)] + 7(0(1) - 2(2))$$

$$= 4[8] - 3[4 - 14] + 7(-4)$$

$$= 32 + 30 - 28$$

$$= \underline{\underline{34}}$$

$$\boxed{\det(B) = 34}$$

d) Trace of E

$$\text{Tr}(E) = e_{11} + e_{22} + e_{33}$$

$$\begin{bmatrix} 1 & 5 & 8 \\ 7 & 2 & 3 \\ 4 & 0 & 6 \end{bmatrix}$$

$$= 1 + 2 + 6$$

$$= \underline{\underline{9}}$$

e) $a_{12} = 7$ $b_{23} = 7$ $d_{32} = \text{undefined}$

$e_{22} = 2$ $f_{12} = 0$ $g_{12} = 6$

f) (i) $E + B$

$$\begin{bmatrix} 1 & 5 & 8 \\ 7 & 2 & 3 \\ 4 & 0 & 6 \end{bmatrix} + \begin{bmatrix} 4 & 3 & 7 \\ 1 & 2 & 7 \\ 2 & 0 & 4 \end{bmatrix} = \begin{bmatrix} 5 & 8 & 15 \\ 8 & 4 & 10 \\ 6 & 0 & 10 \end{bmatrix}$$

(ii) $A + F$

$$\begin{bmatrix} 4 & 7 \\ 1 & 2 \\ 5 & 6 \end{bmatrix} + \begin{bmatrix} 3 & 0 & 1 \\ 1 & 7 & 3 \end{bmatrix} \Rightarrow \text{undefined due to different matrix sizes}$$

→ addition requires same dimensions

(iii) AB

$$\begin{bmatrix} 4 & 7 \\ 1 & 2 \\ 5 & 6 \end{bmatrix}_{3 \times 2} \begin{bmatrix} 4 & 3 & 7 \\ 1 & 2 & 7 \\ 2 & 0 & 4 \end{bmatrix}_{3 \times 3}$$

multiplication is not possible due to incompatible dimensions.

$$(3 \times 2) \quad (3 \times 3)$$

↑ ↑
not equal

(iv) BA

$$\begin{bmatrix} 4 & 3 & 7 \\ 1 & 2 & 7 \\ 2 & 0 & 4 \end{bmatrix} \begin{bmatrix} 4 & 7 \\ 1 & 2 \\ 5 & 6 \end{bmatrix} = \begin{bmatrix} 4(4) + 3(1) + 7(5) & 4(7) + 3(2) + 7(6) \\ 1(4) + 2(1) + 7(5) & 1(7) + 2(2) + 7(6) \\ 2(4) + 0(1) + 4(5) & 2(7) + 0(2) + 4(6) \end{bmatrix}$$

$$BA = \begin{bmatrix} 54 & 76 \\ 41 & 25 \\ 28 & 38 \end{bmatrix}$$

(v) D^T

$$D = \begin{bmatrix} 9 & 4 & 3 & -6 \\ 2 & -1 & 7 & 5 \end{bmatrix}$$

$$D^T = \begin{bmatrix} 9 & 2 \\ 4 & -1 \\ 3 & 7 \\ -6 & 5 \end{bmatrix}$$

(vi) C^T

$$C = \begin{bmatrix} 3 \\ 6 \\ 1 \end{bmatrix}$$

$$C^T = \begin{bmatrix} 3 & 6 & 1 \end{bmatrix}$$

(vii) AC

$$\begin{bmatrix} 4 & 7 \\ 1 & 2 \\ 5 & 6 \end{bmatrix} \begin{bmatrix} 3 \\ 6 \\ 1 \end{bmatrix}$$

$3 \times 2 \qquad 3 \times 1$

multiplication not possible due to incompatible dimensions

$(3 \times 2) (3 \times 1)$
not equal sizes

(viii) IB

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 3 & 7 \\ 1 & 2 & 7 \\ 2 & 0 & 4 \end{bmatrix} = \begin{bmatrix} 4 & 3 & 7 \\ 1 & 2 & 7 \\ 2 & 0 & 4 \end{bmatrix}$$

properties $\Rightarrow I \times B = B$

\downarrow
Identity matrix

(ix) $C^T C$

$$\begin{bmatrix} 3 & 6 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 6 \\ 1 \end{bmatrix} = \begin{bmatrix} 3(3) + 6(6) + 1 \end{bmatrix}$$

$1 \times 3 \qquad 3 \times 1$

$$= \begin{bmatrix} 46 \end{bmatrix}$$