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(a) $\begin{bmatrix} 1 & 2 \\ 2 & 5 \\ -2 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$

$[A \vdots b] = \begin{bmatrix} 1 & 2 & b_1 \\ 2 & 5 & b_2 \\ -2 & -4 & b_3 \end{bmatrix}$

$\text{rref}([A \vdots b]) = \begin{bmatrix} 1 & 2 & b_1 \\ 2 & 5 & b_2 \\ -2 & -4 & b_3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 5b_1 - 2b_2 \\ 0 & 1 & b_2 - 2b_1 \\ 0 & 0 & b_3 + 2b_1 \end{bmatrix}$

for a solution R_3 should be consistent

$\Rightarrow$ $b_3 + 2b_1 = 0$

$\Rightarrow$ $\boxed{b_1 = -\left(\frac{b_3}{2}\right)}$ $\boxed{b_1 = \frac{-b_3}{2}}$

(b) $\begin{bmatrix} 1 & -1 & -2 \\ -2 & 2 & 4 \\ -1 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$

$[A \vdots b] = \left[\begin{array}{ccc|c} 1 & -1 & -2 & b_1 \\ -2 & 2 & 4 & b_2 \\ -1 & 1 & 2 & b_3 \end{array}\right]$

$\text{rref} = \begin{bmatrix} 1 & -1 & -2 & b_1 \\ 0 & 0 & 0 & b_2 + 2b_1 \\ 0 & 0 & 0 & b_3 + b_1 \end{bmatrix}$

For a solution R_2 and R_3 should be consistent

$\Rightarrow$ $b_2 + 2b_1 = 0$ $\Rightarrow$ $b_1 = -b_2/2$

$b_3 + b_1 = 0$ $\Rightarrow$ $b_1 = -b_3$

$\Rightarrow$ $\boxed{b_1 = -\frac{b_2}{2} = -b_3}$