(6) given data and trials:

Trial	Data A	Data B
1	0	1
2	1	0
3	0	1
4	1	1

Mean of data A : $\mu_A = \frac{0+1+0+1}{4} = 1/2$

Mean of data B : $\mu_B = \frac{1+0+1+1}{4} = 3/4$

(a) Covariance Matrix = $\begin{bmatrix} Cov(A,A) & Cov(A,B) \\ Cov(A,B) & Cov(B,B) \end{bmatrix}$

$$Cov(A,A) = \frac{1}{N-1} \sum_{i=1}^{N} (x_{A_i} - \bar{x})(\bar{x}_{A_i} - \bar{x}_A)$$

$$= \frac{1}{3}\left[(0-\tfrac{1}{2})(\tfrac{-1}{2}) + (1-\tfrac{1}{2})(1-\tfrac{1}{2}) + (\tfrac{-1}{2})^2 + (\tfrac{1}{2})^2\right]$$

$$= \frac{1}{3}$$

$$Cov(A,B) = \frac{1}{N-1} \sum_{i=1}^{N} (x_{A_i} - \bar{x})(y_{B_i} - \bar{y})$$

$$= \frac{1}{3}\left[(\tfrac{-1}{2})(\tfrac{1}{4}) + (\tfrac{1}{2})(\tfrac{-3}{4}) + (\tfrac{-1}{2})(\tfrac{1}{4}) + \tfrac{1}{2}(\tfrac{1}{4})\right]$$

$$= \frac{-1}{6} = Cov(B,A)$$

$$Cov(B,B) = \frac{1}{3}\left[(\tfrac{1}{4})^2 + (\tfrac{-3}{4})^2 + (\tfrac{1}{4})^2 + (\tfrac{1}{4})^2\right] = \frac{1}{4}$$

Covariance Matrix = $\begin{bmatrix} 1/3 & \frac{-1}{6} \\ \frac{-1}{6} & \frac{1}{4} \end{bmatrix}$

(b) Eigen values of $\begin{bmatrix} 1/3 & -\frac{1}{6} \\ -\frac{1}{6} & \frac{1}{4} \end{bmatrix}$

$\lambda_1 = 0.4635$; $\lambda_2 = 0.1199$

Principal directions (using 'eigs' function)

$\underline{v}_1 = \begin{bmatrix} -0.7882 \\ 0.6154 \end{bmatrix}$ $\underline{v}_2 = \begin{bmatrix} -0.6154 \\ -0.7882 \end{bmatrix}$

(c) PCA scores for the first Principal Direction

PCA scores DB:
(mean shift)

D	A	B		A	B
	$0-\frac{1}{2}$	$1-\frac{3}{4}$		$-\frac{1}{2}$	$\frac{1}{4}$
	$1-\frac{1}{2}$	$0-3/4$	$\Rightarrow$	$\frac{1}{2}$	$\frac{-3}{4}$
	$0-\frac{1}{2}$	$1-3/4$		$\frac{-1}{2}$	$\frac{1}{4}$
	$1-\frac{1}{2}$	$1-3/4$		$\frac{1}{2}$	$\frac{1}{4}$

Scores = DA

$$A = [\underline{v}_1 \ \underline{v}_2] = \begin{bmatrix} -0.7882 & -0.6154 \\ 0.6154 & -0.7882 \end{bmatrix}$$

DA =

$$\begin{bmatrix} -\frac{1}{2} & \frac{1}{4} \\ \frac{1}{2} & -\frac{3}{4} \\ -\frac{1}{2} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{4} \end{bmatrix} \begin{bmatrix} -0.7882 & -0.6154 \\ 0.6154 & -0.7882 \end{bmatrix} = \begin{bmatrix} 0.5479 & 0.1106 \\ -0.855 & 0.2335 \\ 0.5479 & 0.1106 \\ -0.2403 & -0.5048 \end{bmatrix}$$

PCA-1 score ↓ PCA-2 score ↓

PCA scores for 1st principal direction :-

$$\begin{bmatrix} 0.5479 \\ -0.855 \\ 0.5479 \\ -0.2403 \end{bmatrix}$$