3.) a) combination

$$x_1 w_1 + x_2 w_2 + x_3 w_3 = 0$$

$$\Rightarrow x_1 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + x_2 \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} + x_3 \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix} = 0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow x_1 + 4x_2 + 7x_3 = 0 \quad - (1)$$

$$2x_1 + 5x_2 + 8x_3 = 0 \quad - (2)$$

$$3x_1 + 6x_2 + 9x_3 = 0 \quad - (3)$$

Solving $2 \times (1) - (2)$

$$\Rightarrow 2x_1 + 8x_2 + 14x_3 = 0$$

$$-2x_1 - 5x_2 - 8x_3 = 0$$

$$3x_2 + 6x_3 = 0$$

$$x_2 = -2x_3 \rightarrow \text{putting this in } (1)$$

$$\Rightarrow x_1 + -8x_3 + 7x_3 = 0$$

$$\boxed{x_1 = x_3}$$

Thus the above combination has infinite solutions.

$\Rightarrow$ assume $x_1 = 1$

$$x_3 = x_1 = 1$$

$$x_2 = -2(1) = -2$$

$$\boxed{(x_1, x_2, x_3) = (1, -2, 1)}$$

(b.) $W = [w_1 \ w_2 \ w_3]$ $\qquad x = (x_1, x_2, x_3)^T$

$$= \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix} \qquad = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$WX = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$\Rightarrow \quad x_1 + 4x_2 + 7x_3 \quad — (4)$

$2x_1 + 5x_2 + 8x_3 \quad — (5)$

$3x_1 + 6x_2 + 9x_3 \quad — (6)$

These equations are the same as the above (a)

Substituting values of $x_1 = x_3$; ~~[illegible]~~
$x_2 = -2x_3$

we get

Equations (4) $\Rightarrow \quad x_3 + 4(-2x_3) + 7x_3 = 0$

(5) $\Rightarrow \quad 2x_3 + 5(-2x_3) + 8x_3 = 0$

(6) $\Rightarrow \quad 3x_3 + 6(-2x_3) + 9x_3 = 0$

Hence $WX = 0$

The matrix-vector multiplication results in a zero vector $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$.