

$$7) \quad a\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 4x-y \\ xy \end{bmatrix}$$

$$b\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x+y \\ 0 \end{bmatrix}$$

$$a \circ b = a(b\left(\begin{bmatrix} x \\ y \end{bmatrix}\right)) = a\left(\begin{bmatrix} x+y \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 4(x+y)-0 \\ 0 \end{bmatrix}$$

$$\boxed{a \circ b\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) \Rightarrow \begin{bmatrix} 4x+4y \\ 0 \end{bmatrix}}$$

checking if  $a \circ b$  is a linear transformation:

$$\begin{aligned} (i) \quad a \circ b\left(\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} c \\ d \end{bmatrix}\right) &= a \circ \left(\begin{bmatrix} x+y \\ 0 \end{bmatrix} + \begin{bmatrix} c+d \\ 0 \end{bmatrix}\right) \\ &= a \left(\begin{bmatrix} 4(x+y+c+d) \\ 0 \end{bmatrix}\right) \\ &= \begin{bmatrix} 4(x+y) + 4(c+d) \\ 0 \end{bmatrix} \\ &= a \circ b\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) + a \circ b\left(\begin{bmatrix} c \\ d \end{bmatrix}\right) \end{aligned}$$

$$\begin{aligned} (ii) \quad a \circ b\left(\begin{bmatrix} a_1 x \\ a_2 y \end{bmatrix}\right) &= a \circ \begin{bmatrix} a_1(x+y) \\ 0 \end{bmatrix} = \begin{bmatrix} 4a_1(x+y) \\ 0 \end{bmatrix} \\ &= a_1 a \circ b\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) \end{aligned}$$

Hence  $a \circ b$  is a linear transformation.