

(2) $\|v\| = 5$ and $\|w\| = 3$.

(a) Find smallest and largest values of $\|v-w\|$.

$$\|v-w\|^2 = \|v\|^2 + \|w\|^2 - 2\|v\|\|w\|\cos\theta$$

$$= 5^2 + 3^2 - 2(5)(3)\cos\theta$$

$$= 25 + 9 - 30\cos\theta$$

$$\|v-w\|^2 \Rightarrow 34 - 30\cos\theta$$

-1 1

value of $\cos\theta \Rightarrow (-1 \text{ to } 1)$

when $\cos\theta = -1 \Rightarrow 34 - 30(-1) = 64$

when $\cos\theta = 1$

$$\Rightarrow 34 - 30 = 4$$

$$\|v-w\|^2 = 64 \quad ; \quad \|v-w\|^2 = 4$$

$$\|v-w\| = 8$$

$$\|v-w\|^2 = 2$$

$$\|v-w\| = 2$$

Smallest and largest values of $\|v-w\|$ are 2 & 8.

(b) Smallest and largest values of $v \cdot w$

$$v \cdot w = \|v\|\|w\|\cos\theta$$

$$= (5)(3)\cos\theta$$

$$= 15\cos\theta$$

$$-1 = -15$$

$$1 = 15$$

Smallest and largest values of $v \cdot w = (-15, 15)$

Max = 15

Min = -15