15/11/2021

(3) $a = \begin{bmatrix} +1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$ $b = \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix}$ $c = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$ $d = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$

(i) $t_1 = a$ $\quad v_1 = \frac{t_1}{\|t_1\|} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \\ 0 \\ 0 \end{bmatrix}$

(ii) $t_2 = b - \frac{v_1^T b}{v_1^T v_1} v_1 = \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix} - \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \\ 0 \\ 0 \end{bmatrix}$

$t_2 = \begin{bmatrix} +1/2 \\ 1/2 \\ -1 \\ 0 \end{bmatrix}$ $\quad v_2 = \frac{t_2}{\|t_2\|} = \sqrt{\frac{2}{3}} \begin{bmatrix} +1/2 \\ 1/2 \\ -1 \\ 0 \end{bmatrix}$

(iii) $t_3 = c - (v_1^T c) v_1 - (v_2^T c) v_2$

$\begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \\ 0 \\ 0 \end{bmatrix} - \sqrt{\frac{2}{3}} \begin{bmatrix} +\frac{1}{2} & \frac{1}{2} & -1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$

$\times \sqrt{\frac{2}{3}} \begin{bmatrix} +1/2 \\ 1/2 \\ -1 \\ 0 \end{bmatrix}$

$= \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} - \sqrt{\frac{2}{3}} \cdot \sqrt{\frac{2}{3}} \begin{bmatrix} +1/2 \\ 1/2 \\ -1 \\ 0 \end{bmatrix}$

$= \begin{bmatrix} +1/3 \\ 1/3 \\ 1/3 \\ 1 \end{bmatrix}$

$v_3 = \frac{t_3}{\|t_3\|} = \frac{\sqrt{3}}{2} \begin{bmatrix} +1/3 \\ 1/3 \\ 1/3 \\ 1 \end{bmatrix}$

(iv) $t_4 = d - (v_1^T d)\, v_1 - (v_2^T d)\, v_2 - (v_3^T d)\, v_3$

$$\Rightarrow \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} - \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \\ 0 \\ 0 \end{bmatrix} - \sqrt{\frac{2}{3}} \begin{bmatrix} +\frac{1}{2} & \frac{1}{2} & -1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

$$* \sqrt{\frac{2}{3}} \begin{bmatrix} +1/2 \\ 1/2 \\ -1 \\ 0 \end{bmatrix}$$

$$- \frac{\sqrt{3}}{2} \begin{bmatrix} +\frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} \frac{\sqrt{3}}{2} \begin{bmatrix} +1/3 \\ 1/3 \\ 1/3 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$v_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ $\Rightarrow$ which means it is dependent on v_1, v_2 and v_3

Orthonormal basis = $\{v_1, v_2, v_3\}$

$$= \left\{ \begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} +1/\sqrt{6} \\ 1/\sqrt{6} \\ -\sqrt{2/3} \\ 0 \end{bmatrix}, \begin{bmatrix} \frac{-1}{2\sqrt{3}} \\ \frac{1}{2\sqrt{3}} \\ \frac{1}{2\sqrt{3}} \\ \sqrt{3}/2 \end{bmatrix} \right\}$$