9.)

(a) $M = \begin{bmatrix} \frac{1}{4} & \frac{1}{3} \\ \frac{3}{4} & \frac{2}{3} \end{bmatrix}$ $P_0 = \begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \end{bmatrix}$

This matrix M can represent a transition matrix

and P_0 sum is '1' and hence represents probability vector

$$P_1 = MP_0 = \begin{bmatrix} \frac{1}{4} & \frac{1}{3} \\ \frac{3}{4} & \frac{2}{3} \end{bmatrix} \begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \end{bmatrix} = \begin{bmatrix} \frac{2}{12} + \frac{1}{9} \\ \frac{1}{2} + \frac{2}{9} \end{bmatrix} = \begin{bmatrix} \frac{5}{18} \\ \frac{13}{18} \end{bmatrix}$$

$$P_2 = MP_1 = \begin{bmatrix} \frac{1}{4} & \frac{1}{3} \\ \frac{3}{4} & \frac{2}{3} \end{bmatrix} \begin{bmatrix} 5/18 \\ 13/18 \end{bmatrix} = \begin{bmatrix} \frac{5}{72} + \frac{13}{54} \\ \frac{5}{24} + \frac{13}{27} \end{bmatrix} = \begin{bmatrix} \frac{67}{216} \\ \frac{149}{216} \end{bmatrix}$$

(b) $M = \begin{bmatrix} \frac{1}{2} & \frac{1}{3} & 0 \\ 0 & \frac{2}{3} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix}$; $P_0 = \begin{bmatrix} \frac{1}{6} \\ \frac{1}{6} \\ \frac{1}{2} \end{bmatrix}$

Here the pairs the improper due to the reason that

P_0 sum is not equal to 1. ($\because \frac{1}{6} + \frac{1}{6} + \frac{1}{2} = \frac{5}{6}$ is not equal to 1)

Hence Markov chain is not possible.

(c) $M = \begin{bmatrix} \frac{1}{4} & \frac{1}{3} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{6} \\ \frac{1}{4} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}$; $P_0 = \begin{bmatrix} \frac{1}{4} \\ \frac{1}{2} \\ \frac{1}{4} \end{bmatrix}$

The above pair of M and P_0 represent a transition matrix M and initial probability vector P_0. for Markov chain.

$$P_1 = MP_0 = \begin{bmatrix} \frac{1}{4} & \frac{1}{3} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{6} \\ \frac{1}{4} & \frac{1}{3} & \frac{1}{3} \end{bmatrix} \begin{bmatrix} \frac{1}{4} \\ \frac{1}{2} \\ \frac{1}{4} \end{bmatrix} = \begin{bmatrix} \frac{1}{16} + \frac{1}{6} + \frac{1}{8} \\ \frac{1}{8} + \frac{1}{6} + \frac{1}{24} \\ \frac{1}{16} + \frac{1}{6} + \frac{1}{12} \end{bmatrix} = \begin{bmatrix} \frac{17}{48} \\ \frac{1}{3} \\ \frac{5}{16} \end{bmatrix}$$

$$P_2 = MP_1 = \begin{bmatrix} \frac{1}{4} & \frac{1}{3} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{6} \\ \frac{1}{4} & \frac{1}{3} & \frac{1}{3} \end{bmatrix} \begin{bmatrix} 17/48 \\ 1/3 \\ \frac{5}{16} \end{bmatrix} = \begin{bmatrix} \frac{17}{4 \times 48} + \frac{1}{9} + \frac{5}{32} \\ \frac{17}{96} + \frac{1}{9} + \frac{5}{96} \\ \frac{17}{4 \times 48} + \frac{1}{9} + \frac{5}{48} \end{bmatrix} = \begin{bmatrix} \frac{205}{576} \\ \frac{49}{144} \\ \frac{175}{576} \end{bmatrix}$$

(d) $M = \begin{bmatrix} \frac{1}{4} & \frac{1}{3} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{6} \\ \frac{1}{4} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}$ $\quad P_0 = \begin{bmatrix} \frac{1}{4} \\ \frac{1}{2} \\ 1/6 \end{bmatrix}$

Sum $\quad 1 \quad 7/6 \quad 1$

Sum $P_0 \neq 1$

Hence the above ~~matrix~~ ~~transition~~ matrix and probability vector can not form markov chain.

Matrix M cannot form a transition matrix as the column sums exceed 1 (column 2 = $7/6$ = not possible)