③ $A \in \mathbb{R}^{n \times n}$

(a) Storage needed to store original matrix:

$\Rightarrow n \times n = n^2$ values

(b) Storage needed to store original matrix as full rank approximation

$\Rightarrow \underset{\substack{\downarrow \\ rank}}{n} (\underset{\substack{\downarrow \\ u}}{n} + \underset{\substack{\downarrow \\ v}}{n} + \underset{\substack{\downarrow \\ \sigma}}{1}) = \underline{\underline{2n^2 + n}}$

↳ more than original matrix storage

(c) $A_p = p - $rank approximation.

$\Rightarrow$ storage for p-rank approximation

$\Rightarrow p(n+n+1) \Rightarrow p(2n+1)$

for A_p storage to be less than original matrix storage

$\Rightarrow \quad p(2n+1) < n^2$

$$\boxed{p < \frac{n^2}{2n+1}}$$

$$\boxed{P_{max} = \frac{n^2}{2n+1}} \rightarrow \text{cutoff value of rank}$$