13.) $A = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$

(a) Inverse of A using closed form expression:

$$A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{\underbrace{ad-bc}_{\substack{1(1)-(2)(-2) \\ = 1+4 = 5}}} \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{5} \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1/5 & -2/5 \\ 2/5 & 1/5 \end{bmatrix} = \begin{bmatrix} 0.2 & -0.4 \\ 0.4 & 0.2 \end{bmatrix}$$

(b) $[A \ I] = \begin{bmatrix} 1 & 2 & 1 & 0 \\ -2 & 1 & 0 & 1 \end{bmatrix}$

rref using matlab

$$\begin{bmatrix} 1 & 0 & 0.2 & -0.4 \\ 0 & 1 & 0.4 & 0.2 \end{bmatrix}$$

$\underbrace{\quad\quad}$ inverse of A is contained where $I^{2\times2}$ was

(c) To show that results from (a) & (b) are truly the inverse of A, we use the following property

① $A^{-1} A = I$

② $A A^{-1} = I$

(i) $A^{-1} A = \begin{bmatrix} 0.2 & -0.4 \\ 0.4 & 0.2 \end{bmatrix} \begin{bmatrix} 1 & +2 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 0.2+0.8 & 0.4-0.4 \\ 0.4-0.4 & 0.8+0.2 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

(ii) $A A^{-1} = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 0.2 & -0.4 \\ 0.4 & 0.2 \end{bmatrix} = \begin{bmatrix} 0.2+0.8 & -0.4+0.4 \\ -0.4+0.4 & 0.8+0.2 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

Hence results of (a) & (b) are truly inverse of (A).

(d) The reason we get the inverse of a matrix by the augmented matrix may be due to:

$$[A \vdots I] \Rightarrow [I \vdots A^{-1}]$$

Here we are converting any leading number in A to 1 by diving the given number by itself.

i.e we are inversing the number and multiplying

⇒ Hence, the total effect of all row operations being done may be such that we are multiplying the matrix by A^{-1} (inverse of the matrix)

Hence A becomes I (as $A^{-1}A = I$)

and I becomes A^{-1} (as $A^{-1}I = A^{-1}$)

This is possible when the inverse exists.