(3) a) $A = \left\{ \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ -8 \\ 0 \end{bmatrix}, \begin{bmatrix} 8 \\ 4 \\ 6 \end{bmatrix} \right\}$

$\text{rref}(A) = \begin{bmatrix} \textcircled{1} & 0 & 2 & 2 \\ 0 & \textcircled{1} & -1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ 2 pivots

dimension = 2 (# of pivots)

basis set = $\{v_1, v_2\} = \left\{ \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \\ 2 \end{bmatrix} \right\}$

(b) $B = \left\{ \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \\ 2 \end{bmatrix}, \begin{bmatrix} 6 \\ 2 \\ 0 \end{bmatrix} \right\}$

$\text{rref}(B) = \begin{bmatrix} \textcircled{1} & 0 & 0 \\ 0 & \textcircled{1} & 0 \\ 0 & 0 & \textcircled{1} \end{bmatrix}$ 3 pivots

rank = 3

dimension of basis set = 3

basis set = $\{v_1, v_2, v_3\} = \left\{ \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \\ 2 \end{bmatrix}, \begin{bmatrix} 6 \\ 2 \\ 0 \end{bmatrix} \right\}$

(c) $C = \left\{ \begin{bmatrix} 1 \\ -2 \\ 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ -8 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 7 \\ 4 \\ 6 \\ 2 \end{bmatrix} \right\}$

$\text{rref}(C) = \begin{bmatrix} \textcircled{1} & 0 & 0 & 0 \\ 0 & \textcircled{1} & 0 & 0 \\ 0 & 0 & \textcircled{1} & 0 \\ 0 & 0 & 0 & \textcircled{1} \end{bmatrix}$ 4 pivots

rank = 4

dimension = 4

basis set = $\{v_1, v_2, v_3, v_4\}$

basis set = $\left\{ \begin{bmatrix} 1 \\ -2 \\ 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ -8 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 7 \\ 4 \\ 6 \\ 2 \end{bmatrix} \right\}$

(d)

$D = \left\{ \begin{bmatrix} 1 \\ -2 \\ 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ -8 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 8 \\ 4 \\ 6 \\ 2 \end{bmatrix} \right\}$

$\text{rref}(D) = \begin{bmatrix} \textcircled{1} & 0 & 0 & -0.67 \\ 0 & \textcircled{1} & 0 & 3.3 \\ 0 & 0 & \textcircled{1} & 1.3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ 3 pivots

dimension of basis set = 3

basis set = $\{ v_1, v_2, v_3 \}$

$= \left\{ \begin{bmatrix} 1 \\ -2 \\ 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ -8 \\ 0 \\ 1 \end{bmatrix} \right\}$