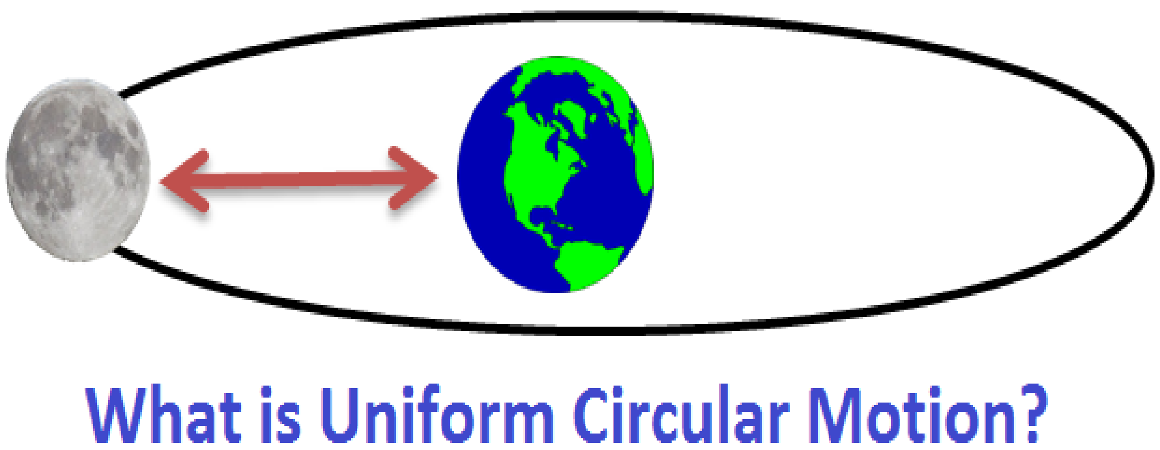
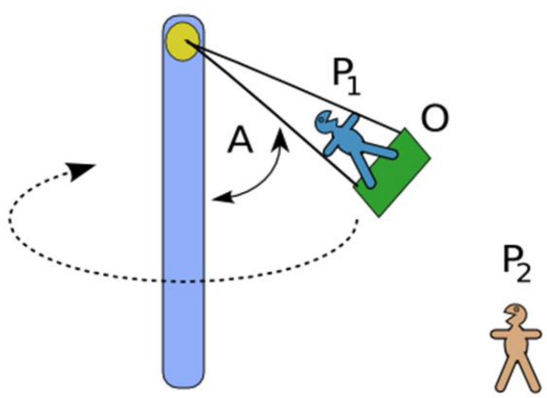
2020

UNIFORM CIRCULAR MOTION





TEACHERS OF PHYSICS

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7/15/2020

1. Define angular displacement.
    - ✓ ***It is the angle subtended by an arc at the center of a circular path.***
2. Define a radian.
    - ✓ ***It is an angle subtended at the center of a circle by an arc length equal to the radius of the circle.***
3. Define centripetal force.
    - ✓ ***It is a force that is required to keep a body moving in a circular path and is directed towards the center of the circular path.***
4. **Define** angular velocity and state its **SI** unit.
    - ✓ ***It is the rate of change of angular displacement. SI unit is radian per second.***
5. Distinguish between *linear velocity* and *angular velocity*.

Linear Velocity	**Angular Velocity**
It is the measure of the rate of change of displacement with respect to time when an object moves along a straight path.	***It is the rate of change of the angular displacement.***
SI unit is meter per second	***SI unit is radian per second***

6. State what is meant by centripetal acceleration.
    - ✓ ***It is the acceleration of a body towards the center of the circle.***
7. Define critical speed.
    - ✓ ***It is the minimum speed that an object moving in a circular path must maintain in order to avoid skidding off the track.***
8. Explain why bodies in circular motion undergo acceleration even when their speed is constant.
    - ✓ ***A body moving in a circular path is said to accelerate even if its speed is constant because its velocity keeps on changing with time as the direction also changes the rate of change of velocity. Thus resulting in acceleration.***
9. Explain why the moon is said to be accelerating when revolving around the earth at constant speed.
    - ✓ ***The moon is said to be accelerating when revolving around the earth at a constant velocity because its velocity keeps on changing as the direction changes with time. This rate of change of velocity results in the moon's acceleration.***
10. State **two** ways in which the centripetal force on the ball in its circular motion, would be reduced.
    - ✓ ***Increasing the radius of the path.***
    - ✓ ***Reducing the angular velocity of the ball.***
11. What is meant by the term "Banking" in roads?
    - ✓ ***It is the act of making the road such that the outer path is slightly raised above the inner side so that the track is slopping towards the center of the curve.***

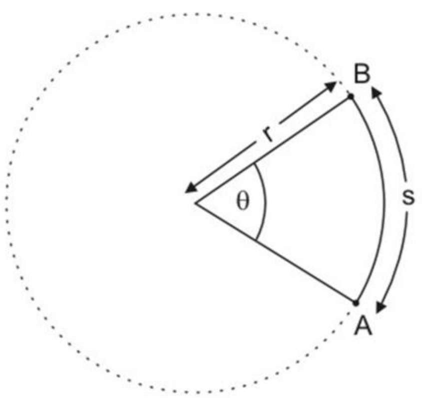
12. State **two** factors which determine the maximum speed at which a car can successfully negotiate a corner on a banked road.
   - ✓ ***Radius of the bend should be bigger.***
   - ✓ ***Angle of banking should be bigger.***
   - ✓ ***Surface of the road should be rough.***

13. State the condition necessary for a car traveling on a banked road not to skid.
   - ✓ ***The critical speed should not be exceeded.***

14. Explain why a pail of water can be swung in a vertical circle without the water pouring out.
   - ✓ ***This is because the centripetal force acting on the water in the vertical position is greater than the weight of the water.***

15. State the reason why banking of a circular part of road is necessary.
   - ✓ ***Banking provides sufficient centripetal friction that keeps the body on a circular path without skidding off.***

16. A body of mass **m** kg moves uniformly around a circle of radius **r** m with a linear velocity v m/s. Derive an expression for the relationship between its linear velocity v and angular velocity $\boldsymbol{\omega}$.



$$Angular\ displacement = \frac{arc\ lenth\ (s)}{radius(r)}$$

$$\theta = \frac{s}{r}$$

From definition;

$$Angular\ velocity, \omega, = \frac{change\ in\ disp.}{change\ in\ time}$$

$$i.e\ \omega = \frac{\Delta\theta}{\Delta t}, but\ \ \theta = \frac{s}{r} \rightarrow \frac{\theta}{t} = \frac{s}{rt}$$

$$Thus\ \frac{\Delta\theta}{\Delta t} = \frac{\nabla s}{r\Delta t}; but\ \omega = \frac{\Delta s}{\Delta t}\ and\ \frac{\Delta s}{r\Delta t} = \frac{v}{r}$$

$$Hence\ \omega = \frac{v}{r} \rightarrow v = \omega r$$

17. A cyclist negotiating a corner at a high speed leans inwards in order to successfully pass. Explain how this action enables him to negotiate.

- ✓ ***Leaning inwards produces the turning effect in the clockwise and anti-clockwise because of the normal and frictional forces thus providing sufficient centripetal force.***

18. A sports car negotiates a corner on a banked road at very high velocity.

    i. What is meant by the term banked road?

    - ✓ ***This is a road that has the outer part slightly raised above the inner side so that the track is slopping towards the center of the curve.***

    ii. Name two factors which determine the magnitude of its velocity.

    - ✓ ***Radius of the corner.***
    - ✓ ***Banking angle.***
    - ✓ ***Nature of the road surface.***

    iii. What provides the centripetal force on the car?

    - ✓ ***Frictional force between the car tyres and the surface of the road OR Horizontal component of reaction force.***

19. What provides for the centripetal force the following cases of circular motion?

    a) The moon moving around the earth.

    - ✓ ***Force of attraction between the earth and the moon.***

    b) A cyclist negotiating a curve.

    - ✓ ***Frictional force between the tyre and the surface of the curve.***

20. When is a satellite said to be in a "parking orbit"?

    - ✓ ***If its periodic time is equal to that of the earth.***

21. State the factors affecting the centripetal force and Explain how each of the factors affect the centripetal force.

    a. Mass of the body

    $\rightarrow \boldsymbol{The\ bigger\ the\ mass, the\ higher\ the\ centripetal\ force\ required.}$

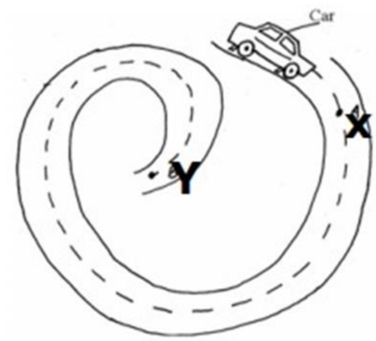
    b. Angular velocity of the object

    $\rightarrow \boldsymbol{The\ greater\ or\ higher\ the\ angular\ velocity, the\ higher\ the\ centripetal\ force}$

    c. Radius of the path

    $\rightarrow \boldsymbol{A\ decrease\ in\ the\ radius\ results\ to\ an\ increase\ in\ centripetal\ force}$

22. The figure below shows a car of mass (m) moving along a curved part of the road with a constant speed.



I. Explain why the car is more likely to skid at **Y** than at **X**.

✓ *From $F_c = \frac{mv^2}{r}$, at point Y the radius of the curve is small. Hence the centripetal force F_c required is higher. The car may slide off since mass and velocity are constant.*

II. If the radius of the path at B is 250m and the car has a mass of 6000kg, determine the maximum speed the car can be driven while at B without skidding. The coefficient of friction between the road and the tyre is 0.3.

✓

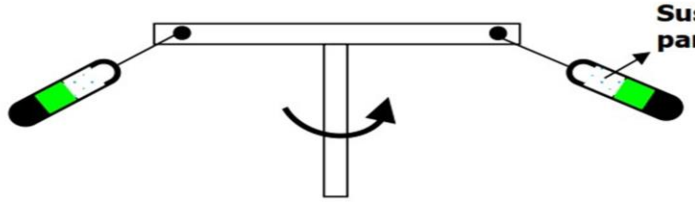
$$F_c = \mu mg \rightarrow 0.3 \times 6000 \times 10 = 18000$$

$$F_c = \frac{mv^2}{r} \rightarrow v^2 = \frac{F_c \times r}{m}$$

$$v^2 = \frac{18000 \times 250}{6000} = 750$$

$$v = \sqrt{750} \rightarrow 27.39m/s$$

23. Fig shows a centrifuge being used in separating particles suspended in a liquid.



Insolvent particles of different masses M_1, and M_2, are suspended in a liquid and system then rotated at high speed as shown.

i. Explain why the particles of different mass will be at different distances from the bottom of the tubes.

- ✓ ***Centripetal force is created as a result of pressure difference at a distance, r, from the center of rotation. Solid particles with greater mass (the more dense) will require greater centripetal force hence will be further away from the center. For lighter particles, the centripetal force would be too too great,($F_c = mr\omega^2$) and r would become smaller. Hence they will be suspended on top of the insolvent.***

ii. If M_1 is greater than M_2, which particle will be further from the base of the tube?

- ✓ ***M_2***

iii. **Explain** how the high speed rotation causes the separation of mud from water.

- ✓ ***Since mud particles are denser than water particles, the mud will require greater centripetal force. Hence will be further away from the center (settles at the bottom) while water floats on mud.***

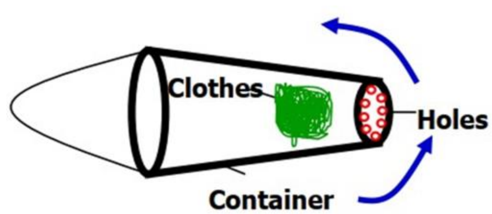
iv. Would the particles separate if the tubes remained vertical during the rotation? Explain.

- ✓ ***No. This is because the centripetal force acting on particles in the vertical position is greater than their weight. Hence they cling together.***

24. A wet umbrella gets dried faster when its handle is rotated at high speed. Explain.

- ✓ ***The force of adhesion of water with the cloth provides the centripetal force necessary for circular motion. At high speed, this force 'gives up' and water molecules fly off the umbrella which gets dried faster.***

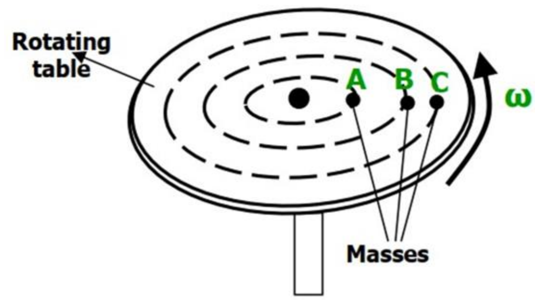
25. The figure below shows a container with small holes at the bottom in which wet clothes have been put.



When the container is whirled in air at high speed, it is observed that the clothes dry faster. Explain how the rotation of the container causes the clothes to dry.

- ✓ ***The force holding the water in the fabric is not strong to hold the water in the same circular path as the clothes when the container rotates. The water therefore breaks from the clothes and flies out through the holes in the container.***

26. Figure shows masses **A**, **B** and **C** placed at different points on a rotating table. The angular velocity of the table can be varied.



a. State two factors that determine whether a particular mass slides off the table or not

- ✓ ***Nature of the surface (rough or smooth)***
- ✓ ***Radius of the path***
- ✓ ***Angular velocity***

b. it is found that masses slide off at angular velocities A, B and C is of decreasing order.

$\boldsymbol{\omega_A > \omega_B > \omega_C}$

27. Three masses are placed in a rotating table at distances 6cm, 9cm and 12cm respectively from the centre of rotation. When the frequency of rotation is varied it is noted that each mass slides off at a different frequency of rotation of the table. Table 1 shows the frequency at which each mass slides off.

Radius, r (cm)	**12**	**9**	**6**
Sliding off frequency rev/s	**0.68**	**0.78**	**1.0**

i. State two factors that determine the frequency at which each mass slides off.

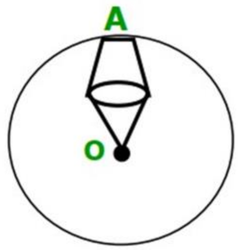
- ✓ ***Mass***
- ✓ ***Radius***
- ✓ ***Friction force***

ii. Oil is now poured on the table before placing the masses. Explain the effect of this on the frequency at which the masses slides off.

- ✓ ***The oil will reduce friction and since friction provides centripetal force, the frequency for sliding off is reduced.***

CALCULATIONS

1. The figure below shows a bucket filled with water of mass **5kg** tied to a string 3.6m long being rotated in a vertical circle with a constant speed of **Vm/s**



i. Write an expression for the minimum force experienced.

$$F_{min} = \frac{mv^2}{r} - mg$$

ii. Use your expression to determine the value of **V**.

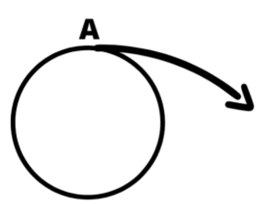
$$\frac{5 \times v^2}{3.6} = 5 \times 10$$

$$v^2 = 36 \rightarrow v = 6m/s$$

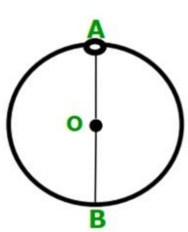
iii. Calculate the minimum speed the bucket takes to rotate in position A so that the water remains in the bucket. (Take g = 10m/s^2).

$$v_{min} = \sqrt{rg} \rightarrow \sqrt{3.6 \times 10}$$
$$= 6m/s$$

iv. Sketch on the figure the path followed by the ball if the strings cuts when the ball is at position **A**.



2. A stone of mass 450g is rotated in a vertical circle at 3 revolutions per second.



If the string has a length of 1.5m, determine:

i. The linear velocity.

$$1rev = 2\pi^c$$

$$3revs = \frac{3 \times 2\pi^c}{1s} \rightarrow 6\pi^c rads^{-1}$$

$$\omega = 6\pi^c rads^{-1}$$

$$But\ v = r\omega \ \rightarrow v = 1.5 \times 6\pi = 9\pi m/s$$

$$= 28.27m/s$$

ii. The tension of the string at positions A, and B.

$$T_A = \frac{0.45 \times (9\pi)^2}{1.5} - (0.45 \times 10)$$

$$(239.83 - 4.5)$$

$$T_A = 235.33N$$

$$T_B = 239.83 + 4.5$$

$$= 244.33N$$

3. A light inextensible string of length L is fixed at its upper end and supports a mass m at the other end. m is rotated at horizontal plane or radius r as shown. The maximum tension the string can withstand without breaking is 2N. Assuming the string breaks when the radius is maximum, calculate the velocity of the mass when the string breaks, given that L 1.25m, and m= 0.1kg.

- ✓ $T_{max} = \frac{mv^2}{r} + mg$
- ✓ $2 = \frac{0.1v^2}{1.25} + (0.1 \times 10)$
- ✓ $0.08v^2 = 1 \Rightarrow v^2 = \frac{1}{0.08}$
- ✓ $\sqrt{v^2} = \sqrt{12.5}; v = 3.54m/s$

1. A particle revolves at 4 HZ in a circle of radius 7cm. calculate its.

i) Linear speed. 2mks

- ✓ $\omega = 2\pi f = 2\pi \times 4 = 8\pi\ rads/s$
- ✓ $v = r\omega = \frac{8\pi \times 7}{100} = 1.76\ m/s$

ii) Angular velocity. (Take) 2mks

- ✓ $\omega = 2\pi f = 2\pi \times 4 = 8\pi\ rad/s$

2. A 150g mass tied to a string is being whirled in a vertical circle of radius 30cm with uniform speed. At the lowest position, the tension in the string is 9.5N. calculate,

i) The speed of the mass. 3mks

- ✓ $9.5 = \frac{0.15v^2}{0.3} + (0.15 \times 10)$
- ✓ $0.5v^2 = 9.5 - 1.5 = 8$
- ✓ $v^2 = \frac{8}{0.5} = 16$
- ✓ $v = \sqrt{16} = 4m/s$

ii) The tension in the string when the mass is at the uppermost position of the circular path. (take g = $10m/s^2$) 3mks

✓ $T_{\text{uppermost}} = \frac{mv^2}{r} - mg$

✓ $\frac{0.15 \times 0.4^2}{0.3} - (0.15 \times 10)$

✓ $8 - 1.5 = 6.5N$

3. A stone 0.5 Kg, was tied to a string that was **120cm** long and whirled in a horizontal circle at a rate of 150 rev/min. Determine:-

i) The periodic time, **T**, of the motion. (2mk)

✓ $f = \frac{150rev}{60s} = 2.5\ \mathcal{HZ}$

✓ $Periodic\ time, T = \frac{1}{2.5} = 0.4s$

ii) The angular velocity, **ω**, of the motion. (1mk)

✓ $\omega = \frac{2\pi}{T} = \frac{2\pi}{0.4} = 5\pi\ rad/s$

4. A lead shot of mass 40g is tied to a string of length 70cm. It is swung vertically at 5 revolutions per second. Determine;

(i) Periodic time. (2mks)

✓ $f = 5rev/s = 5\ \mathcal{HZ}$

✓ $T = \frac{1}{f} = \frac{1}{5} = 0.2\ sec$

(ii) Angular velocity (2mks)

✓ $\omega = \frac{2\pi}{T} = \frac{2\pi}{0.2} = 10\ rad/s$

(iii) Linear velocity (2mks)

✓ $v = r\omega = 0.7 \times 10\pi = 7\pi\ m/s$

✓ $= 21.99 \approx 22\ m/s$

(iv) Maximum tension in the string. (3mks)

✓ $T_{max} = \frac{mv^2}{r} + mg$

✓ $= \frac{0.0422^2}{0.7} + (0.04 \times 10)$

✓ $27.66 + 0.4 = 28.06\ N$

5. A stone is tied to a light string of length 0.5m. If the stone has a mass of 20g and is swung in a vertical circle with a uniform angular velocity of 6 revolutions per second, determine.

(a) The period T. (2 mks)

✓ $T = \frac{1}{f} = \frac{1}{6}s$

(b) The tension of the string when the stone is at

(i) the bottom of the swing. (3 mks)

✓ $v = \frac{2\pi r}{T} = \frac{2\pi \times 0.5}{\frac{1}{6}} = 6\pi\ \frac{m}{s}$

✓ $= 18.85\ m/s$

✓ $T_{bot} = \frac{0.02 \times 18.85^2}{0.5} + (0.02 \times 10)$

✓ $= 14.21 + 0.2 = 14.41\ N$

(ii) the top of the swing. (2 mks)

✓ $14.21 + 0.2 = 14.01\ N$

(iii) the linear velocity. (3 mks)

✓ $V = r\omega = \frac{2\pi r}{T} = \frac{2\pi \times 0.5}{\frac{1}{6}}$

$= 18.85\ m/s$

6. A ball of mass 100g tied to a light to a light string is being whirled in a vertical circle of radius 0.5m with uniform speed. At the lowest position the tension in the string is 2.8N. Calculate

i) The angular velocity of the ball. (3mks)

✓ $F = m\omega^2 r^2 + mg$

✓ $2.8 = (0.1 \times 0.5^2 \times \omega^2) + (0.1 \times 10)$

✓ $0.025\omega^2 = 1.8$

✓ $\sqrt{\omega^2} = \sqrt{72} \Rightarrow \omega = 8.485\ rad/s$

(ii) The tension in the string when the ball is at the upper most position of the circular path. (3mks)

✓ $T = 0.1 \times 0.5^2 \times 72 - (0.1 \times 10)$

✓ $= 1.8 - 1 = 0.8$ N

7. A stone is whirled with uniform speed in horizontal circle having radius of 10cm. it takes the stone 10 seconds to describe an arc of length 4cm.

(i) The angular velocity ω. (3mks

✓ $\boldsymbol{\theta = \frac{arc\ length}{radius} = \frac{0.04}{0.1} = 0.4\ rad}$

✓ $\boldsymbol{\omega = \frac{\Delta\theta}{\Delta t} = \frac{0.4\ rad}{10s} = 0.04\ rad/s}$

(ii) The period time T. (3mks)

✓ $T = \frac{2\pi}{\omega} = \frac{2\pi}{0.04} = 50\pi\ s$

✓ $= 157.08\ s$

11. A car of mass **1,200kg** is moving with a velocity of 25m/s around a flat bend of radius 150m. Determine the minimum frictional force between the tyres and the road that will prevent the car from sliding off.

$$\checkmark \; F_r = \frac{mv^2}{r} = \frac{1200 \times 25 \times 25}{150}$$
$$\checkmark \; = 500\ N$$

12. A marked point on a rim of a wheel has a linear velocity of 11.2m/s. if the rim has a radius of 0.8m. Calculate;

i)The angular velocity of the point (3 marks)

$$\checkmark \; V = r\omega$$
$$\checkmark \; \omega = \frac{11.2\ m/s}{0.8\ m}$$
$$\checkmark \; = 14\ rad/s$$

ii)The centripetal acceleration . (2 marks)

$$\checkmark \; a = \frac{v^2}{r} = \frac{11.2 \times 11.2}{0.8}$$
$$\checkmark \; = 156.8\ m/s$$

13. An object of mass 8 kg is whirled in a vertical circle of radius 2.0m with a constant speed of 6 m/s. Calculate the maximum and minimum tension in the string.(3mk

$$\checkmark \; T_{max} = \frac{8 \times 6^2}{2} + (8 \times 10)$$
$$\checkmark \; = 144 + 80$$

$$\checkmark \; = 224\ \mathrm{N}$$

$$\checkmark \; T_{min} = 144 - 80$$

$$\checkmark \; = 64\ N$$

14. A small object moving in a horizontal circle of radius 0.2m makes 8 revolutions per second. Determine its centripetal acceleration.

$$\checkmark \; f = 8\ rev/s\ = 8\ \mathrm{H}\mathbb{Z}$$
$$\checkmark \; \omega = 2\pi f = 2\pi \times 8 = 16\pi\ rad/s$$
$$\checkmark \; a = r\omega^2 = 0.2 \times (16\pi)^2$$
$$\checkmark \; = 505.32\ m/s^{-2}$$

15. A body moving with uniform angular velocity found to have covered an angular distance 170 radians in t seconds. Thirteen seconds later it is found to have covered a total angular distance of 300 radians. Determine **t** (3mk)

✓ $\omega = \frac{\Delta\theta}{\Delta t}$

✓ $\omega = \frac{300-170}{13} = 10\ rad/s$

✓ $10t = 170$

✓ $\Rightarrow t = 17s$

✓ Or $\frac{t}{13} = \frac{170}{130} \Rightarrow t = \frac{17\times13}{13}$

✓ $t = 17\ s$

16. A wheel rotates at 45 revolutions per minute. What is the angular velocity in rad/s? (3mk)

✓ $1rev = 2\pi\ rads$
✓ $45\ rev/min = \frac{45\times2\pi}{60}$
✓ $1.5\pi\ rd/s$
✓ $\omega = 1.5\ rad/s$
✓ $= 4.71\ rad/s$

17. A wheel has a radius of o.4m and makes 16 revolutions per second. Determine its centripetal acceleration. (3mk)

✓ $\omega = 16 \times 2\pi = 32\pi\ rad/s$
✓ $a = r\omega^2 = 0.4 \times (32\pi)^2$
✓ $= 4042.59\ m/s^2$

18. Determine the angular speed of the wheels of a car of diameter 60cm when the car is moving at $72kmhr^{-1}$.

✓ $v = 72Km/h$ =20 m/s
✓ $r = 0.3\ m$
✓ $\omega = \frac{v}{r} = \frac{20}{0.3}$
✓ $= 66.67\ rad/s$

19. A bucket full of water is whirled in a vertical circular path of radius 1.6M. Determine the minimum speed required to keep the water intact.($g = 10Nkg^{-1}$)

(3mks)

✓ $v_{min} = \sqrt{rg}$
✓ $\sqrt{1.6 \times 10}$
✓ $= 4\ m/s$

20. A turntable of radius 10cm is rotating at 43 revolutions per second. Determine the linear speed of a point on the circumference of the turntable (3mk)

✓ $\omega = \frac{43\times2\pi}{1} = 86\pi\ rad/s$

✓ $v = r\omega = 0.1 \times 86\pi$
✓ $= 27.02\ m/s$

21. A ball of mass 200g is attached to one end of a string and whirled round a vertical circle of radius **1m** once every **second**. Calculate the maximum tension on the string. (3mks)

✓ $\theta = \frac{2\pi r}{r} = \frac{2\pi \times 1}{1} = 2\pi\ rads$
✓ $\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{1} = 2\pi\ rads$
✓ $T_{max} = m\omega^2 r^2 + mg$
✓ $0.2 \times 1^2 \times (2\pi)^2 + (2 \times 10)$
✓ $= 7.9 + 2$
✓ $= \mathbf{9.9\ N}$

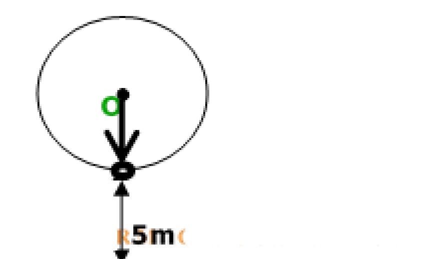
22. A wheel makes 40 revolutions per minute. What is its angular velocity? (3mk)

✓ $\omega = \frac{40\ rev \times 2\pi^c}{1\ rev \times 60\ s}$
✓ $= 1\frac{1}{3}\pi\ rad/s$
✓ $= \mathbf{4.19\ rad/s}$

23. A block of wood of mass 50g is placed 8cm from the centre of a disk which makes 2 revolutions in one second.

(i) Calculate the centripetal acceleration. (1mk)

✓ $\omega = \frac{2\pi \times 2}{1} = 4\pi\ rad/s$
✓ $a = r\omega^2 = 0.08 \times (4\pi)^2$
✓ $= \mathbf{12.63\ m/s^2}$



(ii) Calculate the frictional force between the block and the disc. (3mk)

✓ $F_r = m\omega^2 r$
✓ $= 0.05 \times 0.08^2 \times (4\pi)^2$
✓ $= \mathbf{0.051\ N}$

24. A mass of 100g is attached to a spring of spring constant 50 N/m. The length of the spring together with the mass is 0.4 m. Determine the extension of the spring when it is rotated in a horizontal circle at a speed of $4ms^{-1}$. (3mk)

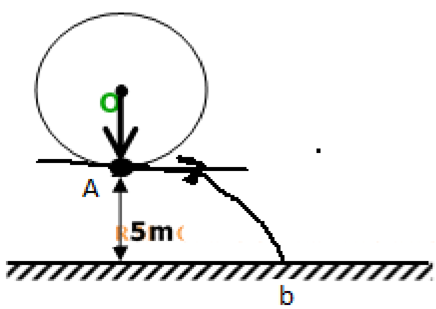
✓ $F_r = \frac{mv^2}{r} = \frac{0.1 \times 4^2}{0.4}$

✓ $= \mathbf{4N}$

✓ $\boldsymbol{F = ke \Rightarrow e = \frac{4N}{50\ N/m}}$

✓ $\mathbf{= 0.08\ m\ or\ 8\ cm}$

25. The diagram below shows a mass m, which is rotated in a vertical circle. The speed of the mass is gradually increased until the string breaks. The string breaks when the mass is at its lowest position A and at a speed of $30ms^{-1}$. Point **a** is 5m above the ground.



a) Show on the diagram.

i) The initial direction of the mass at the point the string breaks.

ii) The path of the mass from A until it strikes the ground at a point **b**.

b) Calculate;

i) The time the mass takes to reach the ground after breaking off.

✓ $H = \frac{1}{2}gt^2$

✓ $t^2 = \sqrt{\frac{5 \times 2}{10}}$

✓ $= \mathbf{1s}$

ii) The horizontal distance the mass travels before it strikes the ground.

✓ $Horizontal\ distance = ut = 30 \times 1$

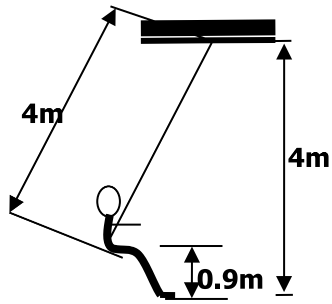
✓ $= \mathbf{30\ m}$

iii) The vertical velocity with which the mass strikes the ground.

✓ $v^2 = u^2 \Rightarrow v = \sqrt{u^2} = \sqrt{30^2}$

✓ $= 30\ m/s$

26. A child of mass 20kg sits on a swing of length 4m and swings through a vertical height of 0.9m as shown in the figure below.



Determine the:

i) Speed of the child when passing through the lowest point.

$$Loss\ in\ P.E = Gain\ in\ K.E$$

✓ $mgh = \frac{1}{2}mv^2$

✓ $v^2 = 2gh$

✓ $v = \sqrt{2 \times 10 \times 0.9}$

✓ $= \mathbf{4.24\ m/s^2}$

ii) Force exerted on the child by the seat of the swing when passing through the lowest point.

✓ $F_e = \frac{mv^2}{r} = \frac{20 \times 4.24^2}{4}$

✓ $= \mathbf{90\ N}$

27. A block of mass 200g is placed on a frictionless rotating table while fixed to the centre of the table by a thin thread. The distance from the centre of the table to the block is 15cm. If the maximum tension the thread can withstand is 5.6N,
determine the maximum angular velocity the table can attain before the thread

cuts. (4mks)

✓ $T = F_r = m\omega^2 r$

✓ $5.6 = 0.2 \times \omega^2 \times 0.15$

✓ $\omega = \sqrt{\frac{5.6}{0.03}}$

✓ $= \mathbf{13.66\ rad/s}$

28. A string of negligible mass has a bucket tied at the end. The string is 60cm long and the bucket has a mass of 45g. The bucket is swung horizontally making 6 revolutions per second. Calculate

(i) The angular velocity (1mk)

✓ $\omega = \frac{2\pi \times 6\ rev}{1 rev \times 1s} = \mathbf{12\ rad/s}$

(ii) The angular acceleration (2mks)

✓ $a = r\omega^2 = 0.6 \times (12\pi)^2$

✓ $= \mathbf{852.73\ m/s^2}$

(iii) The tension on the string (2mks)

✓ $T = m\omega^2\, r$

✓ $= 0.045 \times 0.6 \times (12\pi)^2$

✓ $= \mathbf{38.37\ N}$

(iv) The linear velocity (1mk)

✓ $V = r\omega$

✓ $=0.6 \times 12\pi$

✓ $= \mathbf{22.62\ m/s^2}$

29. Find the maximum speed with which a car of mass 100kg can take a corner of radius 20m if the coefficient of friction between the road and the tyres is 0.5.

✓ $F_C = \mu mg = 0.5 \times 100 \times 10$

✓ $= 500\ N$

✓ $F_C = \frac{MV^2}{r}$

✓ $\sqrt{V^2} = \sqrt{\frac{500\times20}{100}}$

✓ $= \mathbf{10\ m/s}$

30. An object of mass 0.5kg is rotated in a horizontal circle by a string 1m long. The maximum tension in the string before it breaks is 50N. Calculate the greatest number of revolutions per second the object can make.

✓ $F = m\omega^2\, r$

✓ $50 = 0.5 \times 1 \times \omega^2$

✓ $\Rightarrow \sqrt{\omega^2} = \sqrt{\frac{50}{0.5}}$

✓ $\omega = 10\ rad/s$

✓ $No\ of\ revolutions\ per\ sec = \frac{10\ rad/s}{2\pi\ rad}$

✓ $=\mathbf{1.6\ revolutions\ per\ second}$

31. An astronaut is trained in a centrifuge that has an arm length of 6m. If the astronaut can stand the acceleration of 9g. What is the maximum number of revolutions per second that the centrifuge can make?

✓ $a = r\omega^2$

✓ $(90 \times 10) = 6\omega^2$

✓ $\omega^2 = \frac{90}{6} = 15$

✓ $\omega = \sqrt{15} = 3.873\ rad/s^2$

✓ $No\ of\ revolutions\ per\ second = \frac{3.873}{2\pi}$

✓ $= \mathbf{0.62\ rad/s}$

32. A car of mass 1500kg negotiates a bend of radius 45m on a horizontal road. If the frictional force between the road and the tyres is 7200N, Calculate the maximum of speed at which the car be driven, at the bend without going off the road (3mk)

✓ $7200 = \frac{1500 \times V_{max}^2}{45}$

✓ $\sqrt{V_{max}^2} = \sqrt{\frac{7200 \times 45}{1500}}$

✓ $\boldsymbol{v_{max} = 14.7 m/s}$

33. A small body of 200g revolves uniformly on a horizontal frictionless surface attached by a cord 20cm long to a pin set on the surface. If the body makes two revolutions per second. Find the tension of the cord.

✓ $\omega = \frac{2 \times 2\pi}{1} = 4\pi \frac{rad}{s}$

✓ $T = m\omega^2 r = 0.2 \times 0.2 \times (4\pi)^2$

✓ $= \boldsymbol{6.32\ N}$

34. A circular highway curve on a level ground makes a turn 90^0. The highway carries traffic at 120km/h. Knowing that the centripetal force on the vehicle is not to exceed $^1/_{10}$ of its weight, calculate the length of the curve.

✓ $F_C = \frac{1}{10} \times W$, $But\ W = mg$

✓ $F_C = \frac{1}{10} \times m \times 10 = m\ N$

✓ $m = \frac{m \times (33\frac{1}{3})^2}{r}$

✓ $\boldsymbol{r = 1111.1\ m}$

35. A turntable of record player makes 33 revolutions per minute. What is the linear velocity of a point 0.12m from the center?

✓ $\omega = \frac{33 \times 2\pi}{60} = 1.1\pi\ rad/s$

✓ $v = r\omega = 0.12 \times 1.1\pi$

✓ $= \boldsymbol{0.41\ m/s}$

36. An object 0.5kg on the end of a string is whirled around in a vertical circle of radius 2m, with a speed of 10m/s. What is the maximum tension in the string?

✓ $T_{max} = \frac{mv^2}{r} + mg$

✓ $= \frac{0.5 \times 10^2}{2} + (0.5 \times 10)$

✓ $25 + 5$

✓ $\boldsymbol{30\ N}$

37. A 150g mass tied on a string is being whirled in a vertical circle of radius 30cm with a uniform speed. At the lowest position the tension is 9.5N. Calculate

(i) Speed of the mass. (4mk)

✓ $T_{lowest} = \frac{mv^2}{r} + mg$

✓ $9.5 = \frac{0.15v^2}{0.3} + (0.15 \times 10)$

✓ $0.5v^2 = 9.5 - 1.5$

✓ $\frac{0.5v^2}{0.5} = \frac{8}{0.5}$

✓ $v = \sqrt{16} = \mathbf{4\ m/s}$

(ii) The tension of the string when it is at the uppermost point. (3mk)

✓ $T_{uppermost} = \frac{0.15\times4^2}{0.3} - 0.15 \times 10$

✓ $= 8 - 1.5$

✓ $= \mathbf{6.5\ N}$

38. A fun fair ride of diameter 12m makes 0.5 revolutions per second .determine

(i) Its angular velocity (2mk)

✓ $\omega = \frac{0.5\times2\pi}{1} = \boldsymbol{\pi\ rad/s}$

(ii) The linear speed of a child ride in it (3mk)

✓ $v = r\omega = 6 \times \pi = \mathbf{18.85\ m/s}$

(iii) The centripetal acceleration

✓ $v = r\omega^2 = 6 \times (\pi)^2$

✓ $= \mathbf{59.22 m/s^2}$

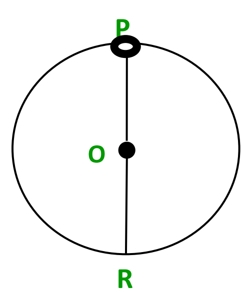
(iv) The centripetal force experienced by the child if his mass was 40kg (3mk)

✓ $F = m\omega^2\, r$

✓ $= 40 \times 59.22$

✓ $2368.8\ N$

39. Fig below shows an object of mass 0.2kg whirled in a vertical circle of radius 0.7m at a uniform speed of 50m/s



Determine the tension on the string at.

i. Position P. (2mk)

✓ $T_P = \frac{0.2\times50^2}{0.7} - 0.2 \times 10$

✓ $= 714.29 - 2$

✓ $= \mathbf{712.29\ N}$

ii. Position R. (2mk)

✓ $T_R = 714.29 + 2$

✓ $\mathbf{716.29\ N}$

iii. At what point is the string likely to cut? Explain. (2mk)

✓ *Point R. Because the string experiences maximum tension.*

40. A particle moving along a circular path of radius 5cm describes an arc length of 2cm every second. Determine:-

i. Its angular velocity. (2mk)

✓ $\theta = \frac{l}{r}$

✓ $= \frac{0.02}{0.05} = 0.4 rads$

✓ $\omega = \frac{\Delta\theta}{\Delta t} = \frac{0.4 rads}{1 sec} = 0.4 rads/s$

ii. Its periodic time. (2mk)

✓ $T = \frac{2\pi}{\omega} = \frac{2\pi}{0.4}$

✓ $= \mathbf{15.71\ sec}$

41. A stone of mass 40g is tied to the end of a string 50cm long and whirled in a vertical circle at 2 revolutions per second. Calculate the maximum tension in the string. (2mk)

✓ $\omega = \frac{2rev \times 2\pi}{1rev \times 1sec} = 4\pi\ rad/s$

✓ $T_{Max} = m\omega^2 r + mg$

✓ $= 0.04 \times 0.5 \times (4\pi^2) + (0.04 \times 10)$

✓ $= 3.158 + 0.4$

✓ $= \mathbf{3.558N}$

42. A car of mass1200kg is moving with a velocity of 25ms^{-1} round a flat bend of radius 150m. Determine:

(i) The minimum frictional force between the tyres and the road that will prevent the car from sliding off. (2mks)

✓ $F_R = \frac{mv^2}{r} = \frac{120 \times 25^2}{150}$

✓ $= \mathbf{500N}$

(ii) Coefficient of limiting static friction between the tyres and the surface(2mks)

✓ $F = \mu mg$

✓ $\mu = \frac{500N}{1200 \times 10N}$

✓ $= \mathbf{0.42}$

43. A stone of mass 0.2 kg tied to a string is whirled in a horizontal circle of radius 1.0 m at a constant speed of 3.0 ms^{-1}.

Determine:

(i) The angular velocity of the stone. (2mks)

✓ $\omega = v/r = 3/1 = \mathbf{3\ rad/s}$

(ii) The tension in the string. (2mks)

✓ $T = r\omega^2 = 0.2 \times 1 \times 3^2$

✓ $= \mathbf{1.8\ N}$

(iii) The number of revolutions the stone makes in one minute. (2mks)

✓ $No\ of\ complete\ rev/min = \frac{3}{2\pi} \times 60$

✓ $= \mathbf{28.65\ rev/min}$

44. A string of negligible mass has a metal ball tied at the end of the string 100cm long and the ball has a mass of 0.04kg. The ball is swinging horizontally, making 4 revolutions per second. Determine;

a) the angular velocity in radian/second (3mk)

✓ $\omega = \frac{2\pi \times 4}{1} = \mathbf{8\pi\ rad/s}$

b) the angular acceleration (2mk)

✓ $a = r\omega^2 = 1 \times (8\pi)^2 = \mathbf{631.65\ m/s^2}$

c) The tension on the string (2mk)

✓ $T = mr\omega^2$

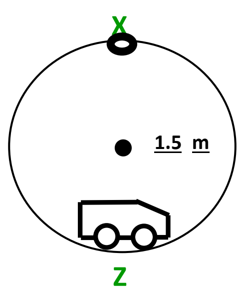
✓ $= 0.04 \times 1 \times (8\pi)^2$

✓ $= \mathbf{25.27N}$

d) The linear velocity (2mk)

✓ $V = \omega r = 1 \times 8\pi = \mathbf{25.13\ m/s}$

45. Figure below shows a trolley moving on a circular rail in a vertical plane. Given that the mass of the trolley is 250g and the radius of the rail is 1.5m



(i) Determine the minimum velocity at which trolley passes point X. (3mks)

✓ $At\ x, \frac{mv^2}{r} = mg$

✓ $v_{min} = \sqrt{rg} = \sqrt{1.5 \times 10}$

✓ $= \mathbf{3.87}\ \boldsymbol{m/s}$

(ii) If the trolley moves with a velocity of 4m/s as it passes point Z, Find;

(I) angular velocity at this point. (3mks)

✓ $\omega = \frac{v}{r} = \frac{4\ m/s}{1.5\ m} = \mathbf{2.67\ rad/s}$

(II) The force exerted on the rails at this point. (2mks)

✓ $F = ma$

✓ $= \frac{0.25 \times 4^2}{1.5}$

✓ $= \mathbf{2.67\ N}$

THE END