B.Tech - Odd Sem : End Semester Exam
Academic Year:2023-2024

22MT2005 - PROBABILITY, STATISTICS &QUEUEING THEORY

Set No: 4

Time: **Max.Marks: 100**

S.NO	Answer All Questions	Choice Options	Marks	CO	CO BTL	COI BTL
1.	Answer all of the following		16Marks	CO4	4	2
1.A.	Interpret the error in the following statement, The probabilities that a printer will make 0, 1, 2, 3, 4 or more mistakes in setting a document are respectively 0.19, 0.34, -0.25, 0.43 and 0.29.		2Marks	CO1	3	2
1.B.	Outline the concept of random variable and give an example.		2Marks	CO1	3	2
1.C.	Summarize the applications of the exponential distribution.		2Marks	CO2	4	2
1.D.	Infer the range of the following sets i) 12,6,7,3,15,10,18,5 ii) 9,3,8,8,9,8,9,18		2Marks	CO2	4	2
1.E.	Show the difference between population and sample.		2Marks	CO3	3	2
1.F.	What is the purpose of the p-value in the context of a paired t-test?		2Marks	CO3	3	2
1.G.	Demonstrate Kendall's Notation in Queuing theory.		2Marks	CO4	4	2
1.H.	Classify the states of the following Markov chain $\begin{pmatrix} 0.5 & 0.25 & 0.25 & 0 \\ 0 & 0 & 1 & 0 \\ 1/3 & 0 & 1/3 & 1/3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$		2Marks	CO4	4	2
2.	Answer all of the following		24Marks	CO4	4	3
2.A.	Amy commutes to work by two different routes A and B. If she comes home by route A, then she will be home no later than 6 P. M. with probability 0.8, but if she comes home by route B, then she will be home no later than 6 P. M. with probability 0.7. In the past, the proportion of times that Amy chose route A is 0.4. If Amy is home after 6 P. M. today, determine the probability that she took route B.		6Marks	CO1	3	3
2.B.	Given the following Aptitude and I.Q. scores for a group of students. Compute rank correlation coefficient between Aptitude scores and I.Q. Scores of the students.		6Marks	CO2	4	3
2.C.	The following table gives the number of accidents that work place in an industry surveying varies days of the week. Test if the accidents are uniformly distributed over the week.		6Marks	CO3	3	3

Aptitude Score	57	58	59	65	60	61	66	64
I.Q. Score	97	108	95	106	120	126	113	110

Days:	Mon	Tue	Wed	Thu	Fri	Sat
No. of accidents	14	18	12	11	15	14

2.D.	Consider a certain community in well-defined area with three types of grocery stores; for simplicity we shall call them I, II and III. Within this community (we assume that the population is fixed) there always exists a shift of customer from one grocery store to another. A study was made on January 1 and it was found that ¼ shopped at store I, 1/3 at store II and 5/12 at store III. Each month store I retains 90% of its customers and loses 10% of them to store II. store II retains 90% of its customers and loses 5% each to store I store III. Store III retains 40% of its customers and loses 50% of them to store I and 10% to store II. i) Estimate the proportion of customers will each store retain by February 1; March 1? ii) Assuming the same pattern continues, what will be the long run distribution of customers among the three stores?		6Marks	CO4	4	3
3.	Answer all of the following	choice Q-4	15Marks	CO1	3	3
3.A.	A store carries flash drives with 1 GB, 2GB, 4 GB, 8GB, and 16GB of memory. The accompanying table gives the distribution of Y=the amount of memory in a purchased drive		7Marks	CO1	3	3

y	1	2	4	8	16
p(y)	0.05	0.10	0.35	0.40	0.10

i) Verify the properties of probability mass function for the given y.

ii) Find the cumulative distribution of y.

3.B.	In a given city, 6% of all drivers get at least one parking ticket per year. Use the Binomial distribution to determine the probabilities that among 80 drivers (randomly chosen in this city): i) 4 will get at least one parking ticket in any given year; ii) atmost 3 will get at least one parking ticket in any given year iii) Anywhere from 3 to 6, inclusive, will get at least one parking ticket in any given year.	8Marks	CO1	3	3
4.	Answer all of the following	15Marks	CO1	3	3
4.A.	The table below, adapted from a snapshot in India today, shows the probability distribution for X, the number of daily coffee breaks taken per day by coffee drinkers	7Marks	CO1	3	3

X	0	1	2	3	4	5
p(x)	0.28	0.37	0.17	0.12	0.05	0.01

i) Find the probability that a randomly selected coffee drinker would take no coffee breaks during the day

ii) Find the probability that a randomly selected coffee drinker would take more than two coffee breaks during the day

iii) Calculate the mean for the random variable X.

4.B. Explain the concept of the Poisson distribution and elaborate on its fundamental characteristics and properties — 8Marks CO1 3 — 3

5. Answer all of the following — choice Q-6 — 15Marks CO2 4 — 4

5.A. In a test on 2000 electric bulbs, it was found that the life of a particular make was normally distributed with an average life of 2040 hours and S.D. of 60 hours. Estimate the number of bulbs likely to burn for (i) more than 2150 hours (ii) less than 1950 hours (iii) more than 1920 hours and but less than 2160 hours. — 7Marks CO2 4 — 4

5.B. During the manufacture of hard disks, the height between the disk and the head, the head lift, must be controlled. One manufacturer recorded: 0.239, 0.246, 0.245, 0.243, 0.239, 0.241, 0.248, 0.246, 0.243, 0.242, 0.251, 0.246. Compute measures of central tendencies. — 8Marks CO2 4 — 3

6. Answer all of the following — 15Marks CO2 4 — 4

6.A. Suppose that a study of a certain computer system reveals that the response time, in seconds, has an exponential distribution with a mean of 3 seconds. Determine i) the probability that response time exceeds 5 seconds ii) the probability that response time is less than 10 seconds iii) the mean and variance of response time. — 7Marks CO2 4 — 3

The presented data on compressive strength x and intrinsic permeability y of various concrete mixes and cures. Summary quantities are

n=14, $\sum_{i=1}^{n} y_i = 572, \sum_{i=1}^{n} y_i^2$=23,530, $\sum_{i=1}^{n} x_i$ =43, $\sum_{i=1}^{n} x_i^2$=157.42, and $\sum_{i=1}^{n} x_i y_i$=1697.80.

6.B. Assume that the two variables are related according to the simple linear regression model: — 8Marks CO2 4 — 4

i) Calculate the least squares estimate of the slope and intercept.

ii) Use the equation of the fitted line to predict what permeability would be observed when the compressive strength is x=4.3.

7. Answer all of the following — choice Q-8 — 15Marks CO3 3 — 3

7.A. Assume that the helium porosity of coal samples taken from any particular seam is normally distributed with true standard deviation 0.75. i) Compute a 95% Confidence interval for the true average porosity of a certain seam if the average porosity for 20 specimens from the seam was 4.85. ii) Compute a 99% confidence interval for the true average porosity of another seam based on 16 specimens with a sample average porosity of 4.56. — 7Marks CO3 3 — 3

7.B.	A random sample of 10 boys had the following I. Q's: 70, 120, 110, 101, 88,83, 95,98,107,100. Do these data support the assumption of a population mean I. Q. of 100? Find a reasonable range in which most of the mean I. Q. values of samples of 10 boys lie.					8Marks	CO3	3	3
8.	Answer all of the following					15Marks	CO3	3	3
	Choose the two causes of variation that arise in testing of hypothesis and also complete the following table for the analysis of variance								

8.A.	Sources of variation	Degrees of freedom	Sum of squares	Mean sum of squares	Variance ration	
					F-calculated value	F-tabulated value
	Blocks	4	26.8	-	-	-
	Treatments	3	-	-	-	-
	Error	-	-	2.5		
	Total	-	85.3	-	-	-

8.A.			7Marks	CO3	3
8.B.	An investigation of two kinds of photocopying equipment showed that 71 failures of the first kind of equipment took on the average 83.2 minutes to repair with a standard deviation of 19.3 minutes, while 75 failures of the second kind of equipment took on the average 90.8 minutes to repair with a standard deviation of 21.4 minutes. Test the null hypothesis $\mu 1 - \mu 2 = 0$ (the hypothesis that on the average it takes an equal amount of time o repair either kind of equipment) against the alternative hypothesis $\mu 1 - \mu 2 \neq 0$ at the level of significance of a = 0.05.		8Marks	CO3	3
9.	Answer all of the following	choice Q-10	15Marks	CO4	4
9.A.	A foreign bank is considering opening a drive-in window for customer service. Management estimates that customers will arrive for service at the rate of 12 per hour. The teller whom it is considering to staff the window can serve customers at the rate of one every three minutes. Assuming Poisson arrivals and Exponential service, Obtain (i) Utilization of teller (ii) Average number in the system (iii) Average waiting time in the line, and (iv) Average waiting time in the system.		7Marks	CO4	4
9.B.	In a railway marshalling yard, goods trains arrive at a rate of 30 trains per day. Assuming that the service time distribution is also follow exponential with an average 30 minutes. Evaluate the following assuming that the capacity of the yard is to admit 9 trains a) Probability that the yard is empty. b) Average no. of trains waiting in the queue and in service. c) Find the effective arrival rate. d) Expected waiting time of a train until it leaves the yard. e) Obtain the probability that a newly arriving train finding the yard full.		8Marks	CO4	4
10.	Answer all of the following		15Marks	CO4	4
10.A.	Categorize the behaviour of customers in waiting line models and also mention the applications of queuing theory.		7Marks	CO4	4
10.B.	A computer centre is equipped with four identical mainframe computers. The number of users at any time is 25. Each user is capable of submitting a job through a terminal every 15 minutes. On the average, but the actual time between submissions is exponential. Arriving jobs will automatically go to the first available computer. The execution time per submission is exponential with mean 2 minutes. Compute the following: i) Average number of jobs waiting for execution. ii) Average number of idle computers.		8Marks	CO4	4