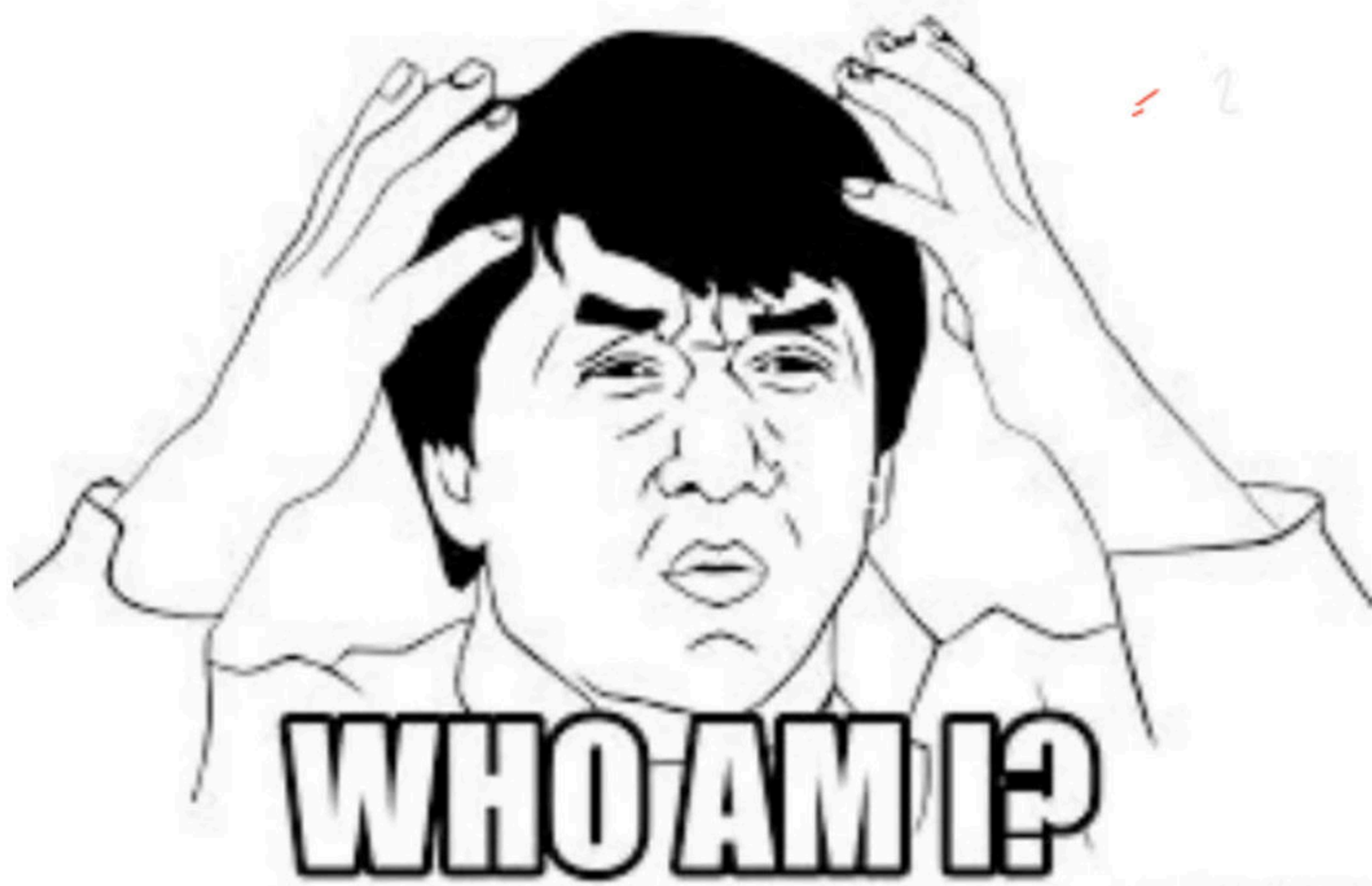




Solving, Coding of Game Theory problems in CP

Special class



ACM ICPC World Finalist (2014, 2015)

2014, 15
2



- Work Experience

- @Google London (2015-2017) //
- @Google MTV (2017-2020) //
- Self Employed (2020-?) //

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- Education

- B.Tech in ECE from IIT Roorkee

- Teaching Experience

- Weekly lectures to my juniors
 - Programming camps
- 
- A hand-drawn red bracket on the right side of the 'Teaching Experience' list, grouping the two items. A red diagonal slash is drawn under the 'Programming camps' item.

Exclusive Batch Starting from 14 December- EVEREST: Complete Course on Competitive Programming



Structured learning for complete beginners to become expert level coders

Instructors: Highly competent technical minds with **ICPC world finals, IOI medals**, IOI team training experience and **Codeforces Grandmasters** as accolades

Get placed in FAANG companies or create your own tech company or crack international coding contests

Industry accepted **Codechef Certification** upon successful course completion

Expenditure of Rs 90/ day empowering you to get placement packages upwards of 30 lacs



Question

What are we doing?

Objective

4 Ann

1. Class 1
 - a. Basics of game theory
 - b. Sample puzzles to develop understanding
2. Class 2
 - a. Minimax
 - b. Nim
3. Class 3
 - a. Nimbers
 - b. Sprague grundy theorem
4. **Class 4**
 - a. Competitive programming problems



Username

evil_maddy

ship_mayni

siya_mona_123

Heenakhan123

rjeshcoder

Rameettimwa

Kaushikjain52



<https://forms.gle/WvUiYxtSPodYuGNx8>

Please enter your correct codechef id before this class finishes!

IMPORTANT!

Codechef laddus : (www.codechef.com/laddu)

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Before we begin the class, let's play a game!

IMPORTANT!

Codechef laddus : (www.codechef.com/laddu)

Before we begin the class, let's play a game!

Free laddus for you, if you win! Chance to win more than 1500 laddus today!

Game!



- Fill the following form within next 5 mins. Only first **50** responses in the next 5 mins to be considered.
- Game is simple, just pick either **500 laddus** or **10 laddus** and enter your *codechef id*.
- If more than two persons of you pick **500 laddus**, then no one gets any laddus. Else you get what you pick.

<https://forms.gle/1ZCrUPKzLTQNrBhB9>

Link will also be available in chat. No cheating, please! Only one response per person!

Game!

- Fill the following form within **next 5 mins**. Only first **50** responses in the **next 5 mins** to be considered.
- Game is simple, just pick either **1000 laddus or 10 laddus** and enter your *codechef id*.
- If **more than one person** of you picks **1000 laddus**, then **no one** gets any laddus. Else you get what you pick.

<https://forms.gle/aecc5UcyVi6iGMC68>

Link will also be available in chat. No cheating, please! Only one response per person!

Games!!

// 5.10

Games!!



x

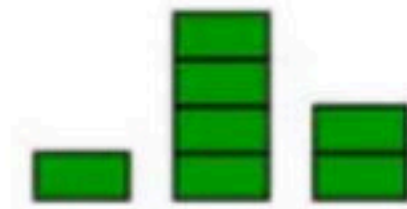
✓



1 2 3
A takes 2 from 1



1 2 3
B takes 3 from 3



1 2 3
A takes 1 from 2



1 2 3
B takes 1 from 2



1 2 3
A takes heap 1



1 2 3
B takes 1 from 2



1 2 3
A takes 1 from 3



1 2 3
B takes heap 2




1 2 3
A takes last coin and wins

Games!!

- **One/Two Person**


Games!!

- One/Two Person
- **Perfect information** 

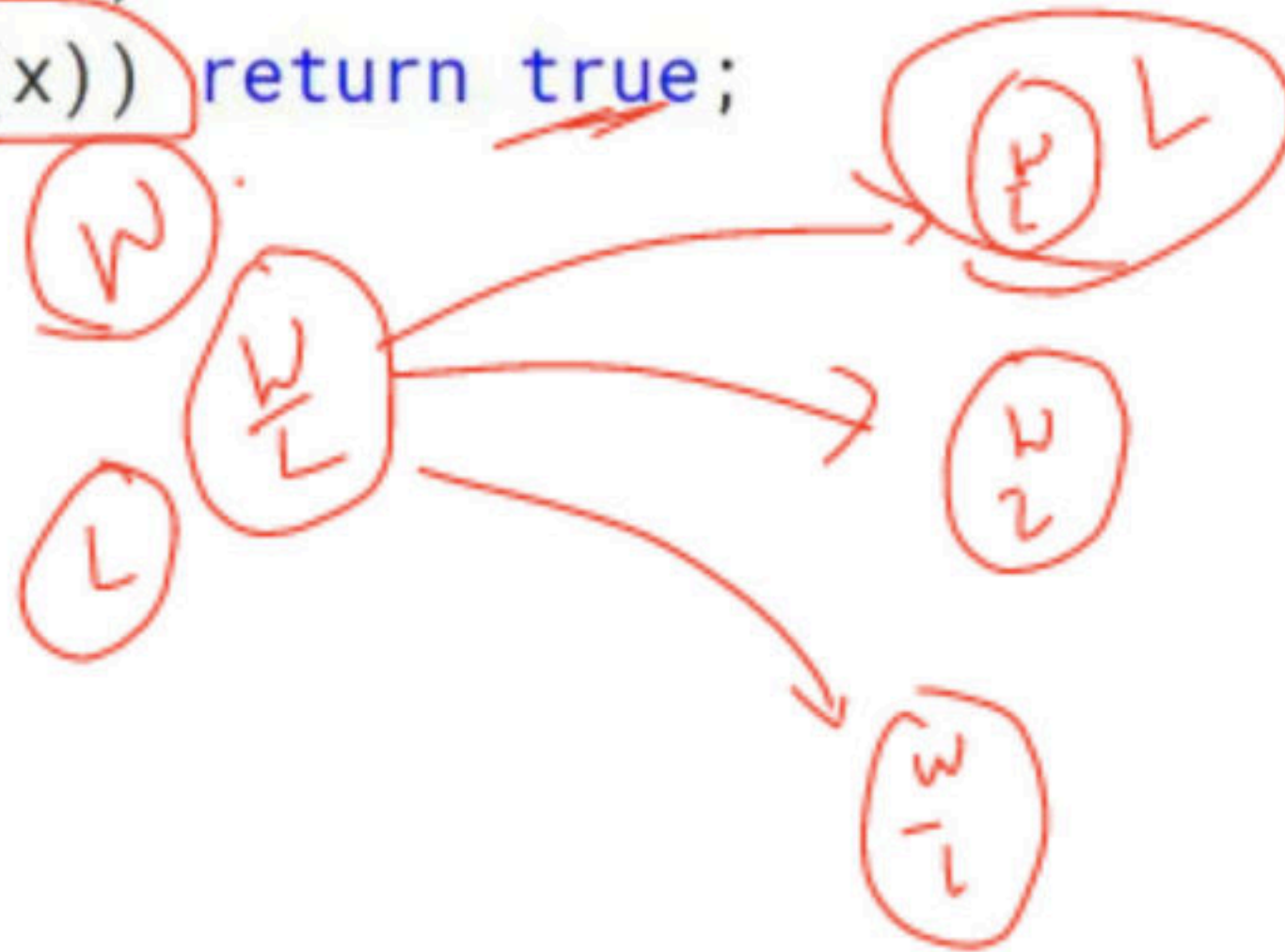
Games!!

- One/Two Person ✓
- Perfect information ✓
- **No chance moves** ✓

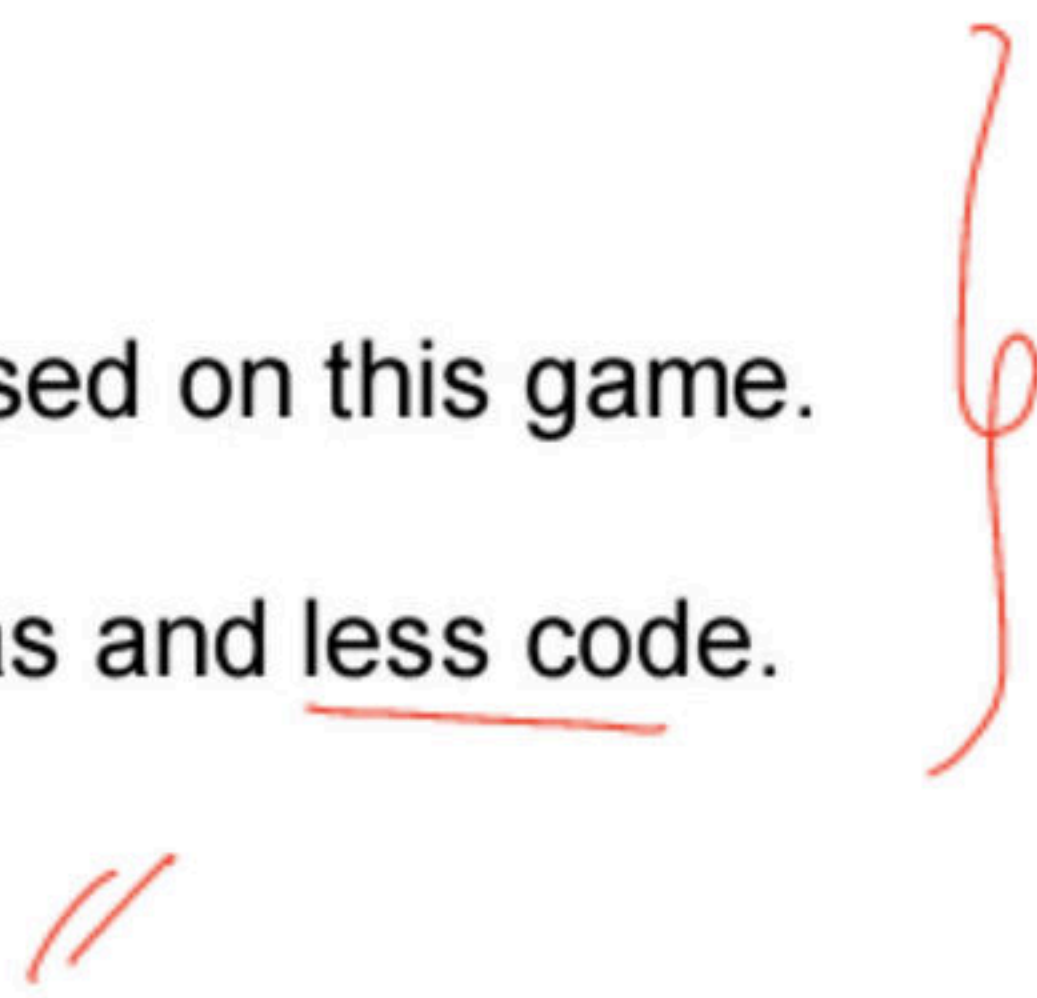
Games!!

- One/Two Person
- Perfect information
- No chance moves
- **Win or lose or draw outcome** 

```
1 boolean isWinning(position pos) {  
2   moves[] = possible positions to which I can move from the  
3   position pos;  
4   for (all x in moves)  
5     if (!isWinning(x)) return true;  
6  
7   return false;  
8 }
```



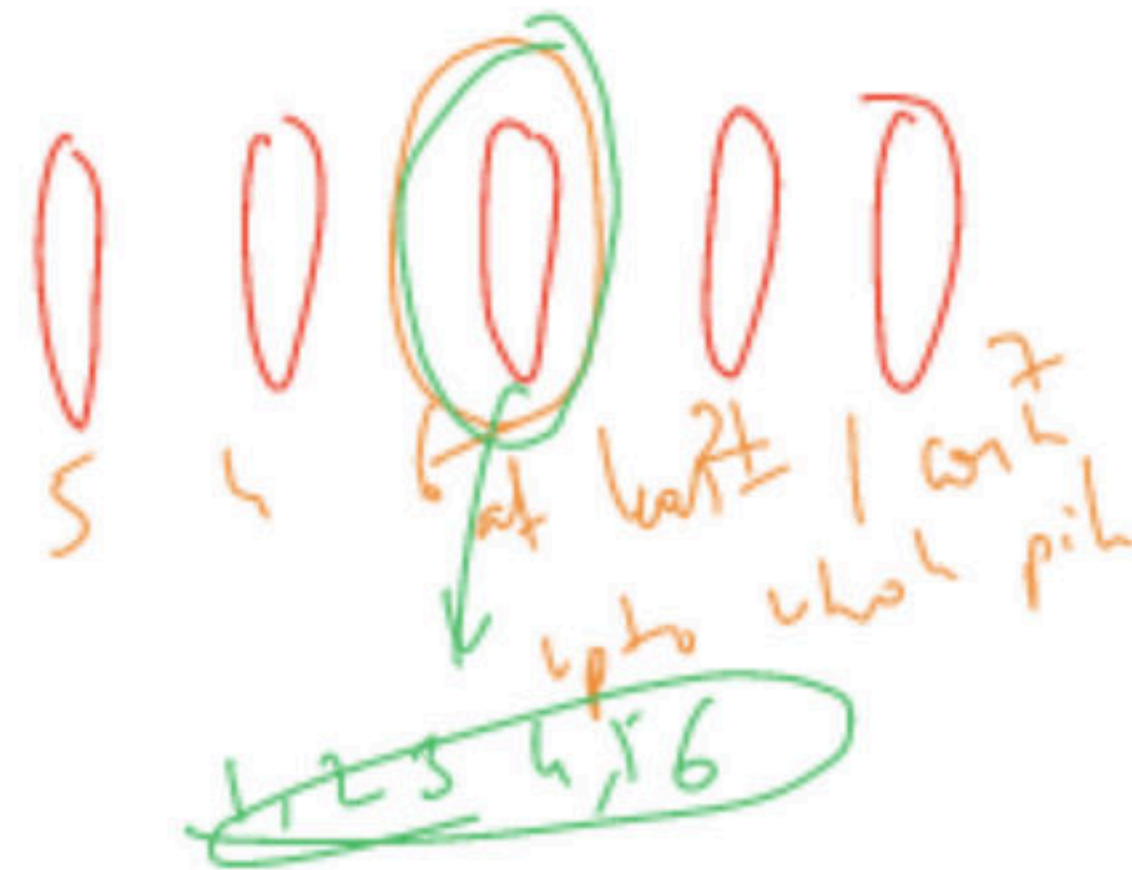
The Game of Nim

- Very famous.
 - Lots of problems based on this game.
 - Requires clever ideas and less code.
- 

The Game of Nim - Statement

- There are n piles of coins.
- When it is a player's turn he chooses one pile and takes at least one coin from it.
- The one who removes the last coin is the winner ✓

(^) $> 0 \rightarrow W$
 $= 0 \rightarrow L$



Minimax Algorithm

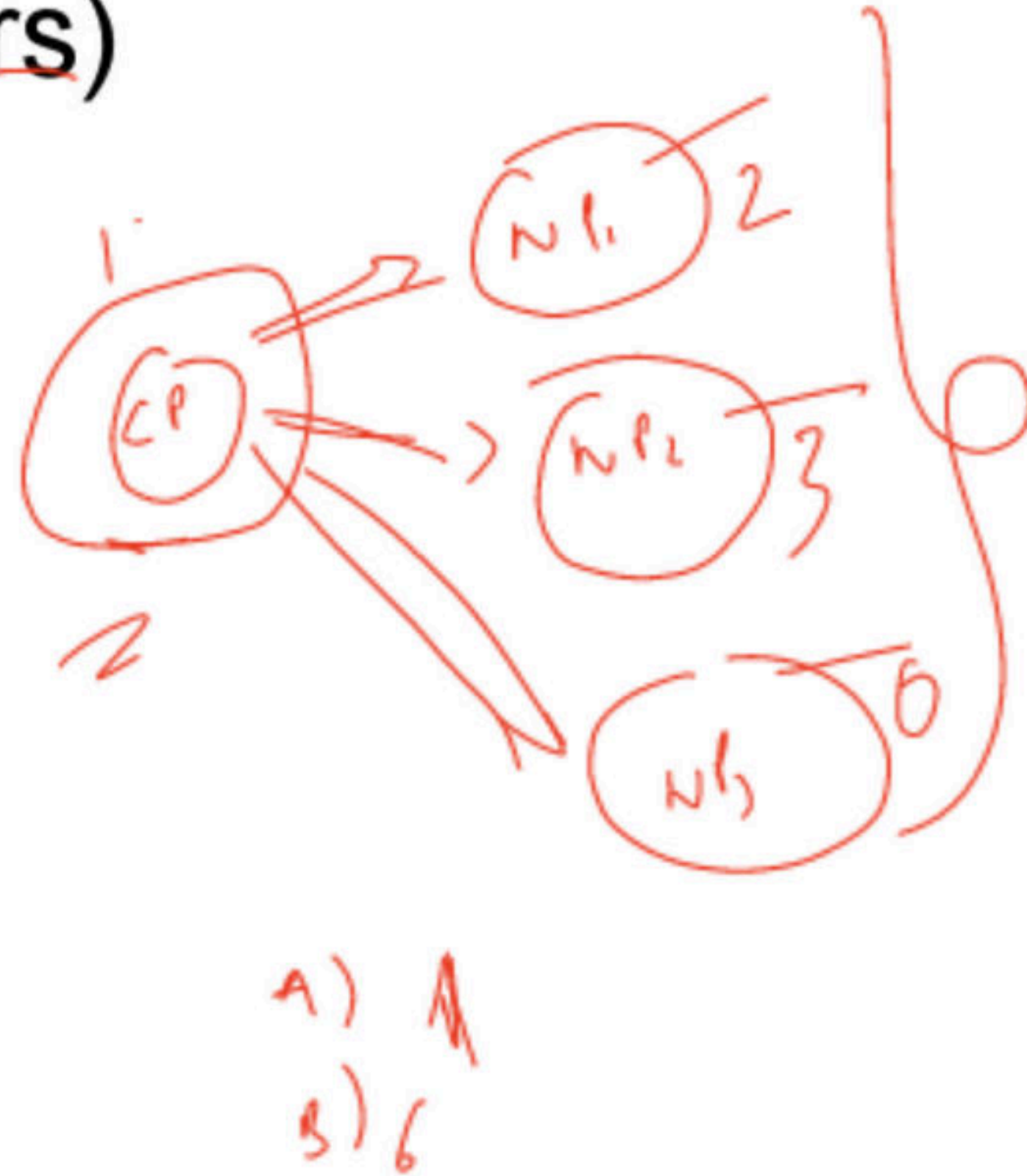
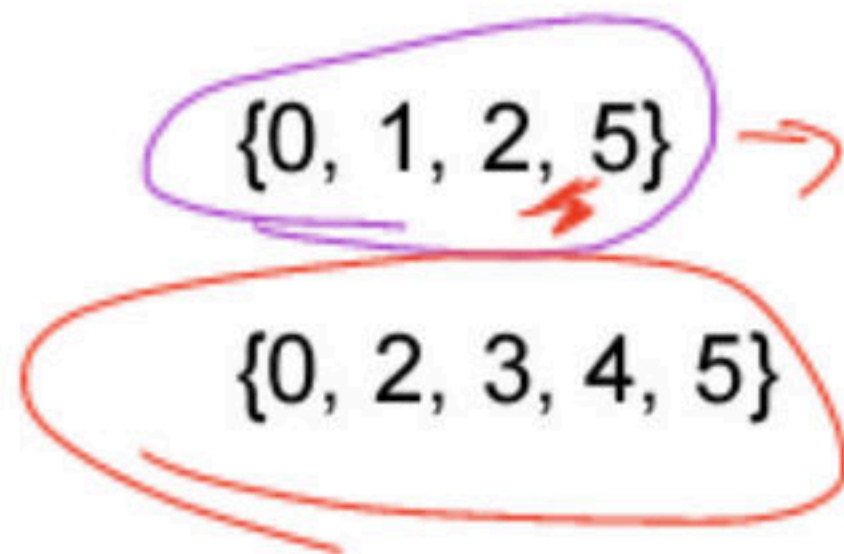
- In Minimax the two players are called maximizer and minimizer.
- The **maximizer** tries to get the highest score possible while the **minimizer** tries to do the opposite and get the lowest score possible
- Backtracking!

Answer g \rightarrow

Grundy Numbers (Nimbers)

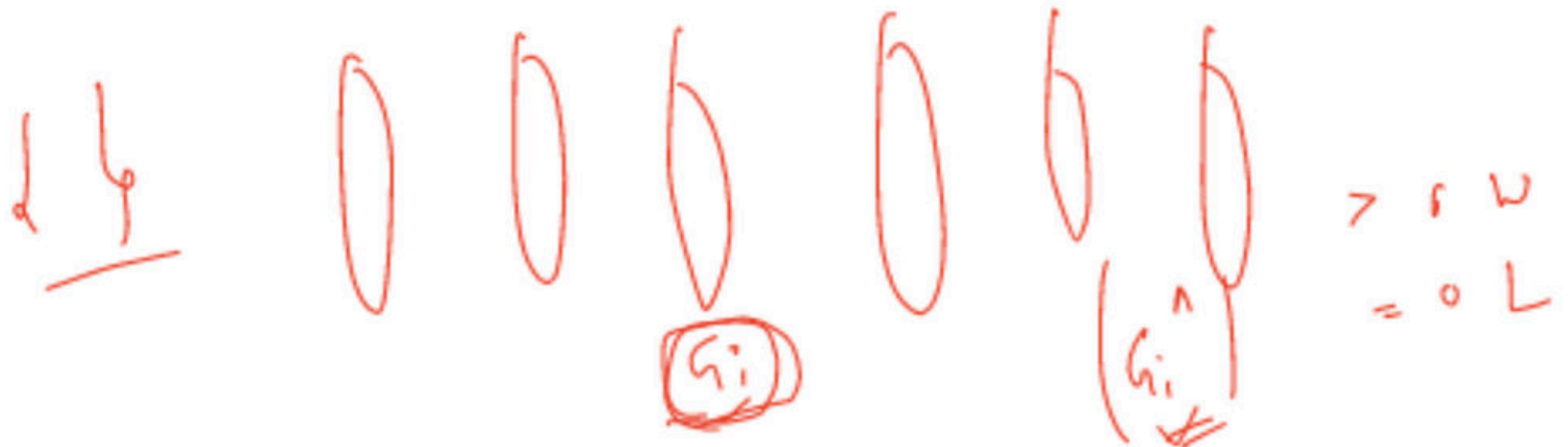
- What are they?

Minimum excludant / **MEX**



Composite Games

- Composite games are combination of multiple simple games.
- Example:
 - Say there are n piles of coins.
 - When it is a player's turn they choose one pile and can take away 1, 3 or 4 coins.
 - The one who removes the last coin is the winner.



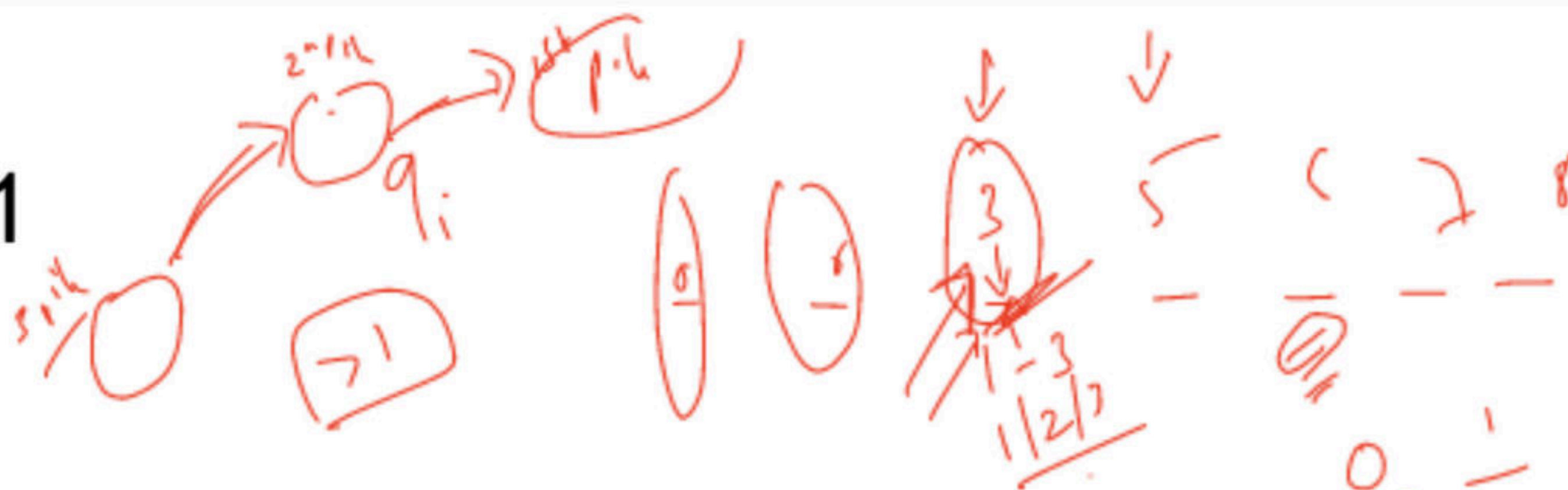
Sprague Grundy Theorem

Sprague-Grundy Theorem says that if both A and B play optimally (i.e., they don't make any mistakes), then the player starting first is guaranteed to win if the XOR of the Grundy numbers of position in each sub-game at the beginning of the game is non-zero.



Questions?

Problem 1



There are n piles of stones, where the i -th pile has a_i stones. Two people play a game, where they take alternating turns removing stones.

In a move, a player may remove a positive number of stones from the **first non-empty pile** (the pile with the minimal index, that has at least one stone). The first player who cannot make a move (because all piles are empty) loses the game. If both players play optimally, determine the winner of the game.

Input

The first line contains a single integer t ($1 \leq t \leq 1000$) — the number of test cases. Next $2t$ lines contain descriptions of test cases.

The first line of each test case contains a single integer n ($1 \leq n \leq 10^5$) — the number of piles.



Problem 2

Mahmoud and Ehab play a game called the even-odd game. Ehab chooses his favorite integer n and then they take turns, starting from Mahmoud. In each player's turn, he has to choose an integer a and subtract it from n such that:

- $1 \leq a \leq n$.
- If it's Mahmoud's turn, a has to be even, but if it's Ehab's turn, a has to be odd.

If the current player can't choose any number satisfying the conditions, he loses. Can you determine the winner if they both play optimally?

Input

The only line contains an integer n ($1 \leq n \leq 10^9$), the number at the beginning of the game.

Handwritten notes:

$n = 100$

$n = 0$

$n = 1$

$n = 2$

A) n

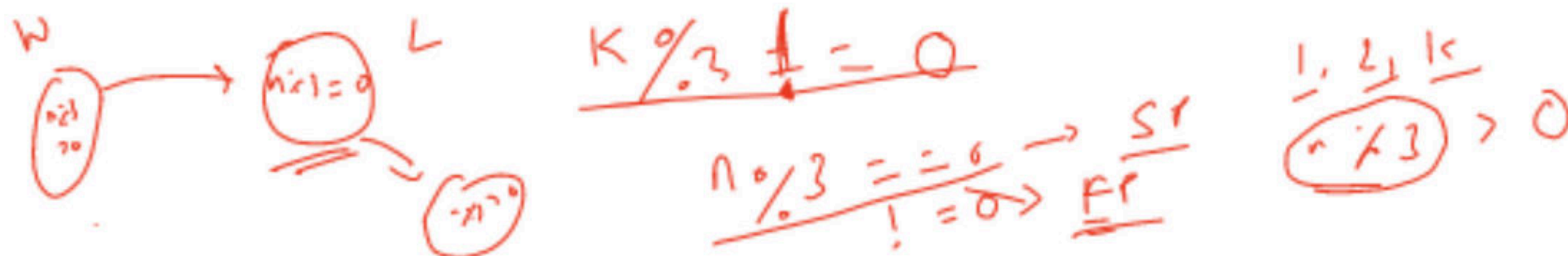
B) n

$n = \text{even}$

$n = \text{odd}$

$n - a$ odd

Problem 3



Alice and Bob play a game. There is a paper strip which is divided into $n + 1$ cells numbered from left to right starting from 0. There is a chip placed in the n -th cell (the last one).

Players take turns, Alice is first. Each player during his or her turn has to move the chip 1, 2 or k cells to the left (so, if the chip is currently in the cell i , the player can move it into cell $i - 1$, $i - 2$ or $i - k$). The chip should not leave the borders of the paper strip: it is impossible, for example, to move it k cells to the left if the current cell has number $i < k$. The player who can't make a move loses the game.

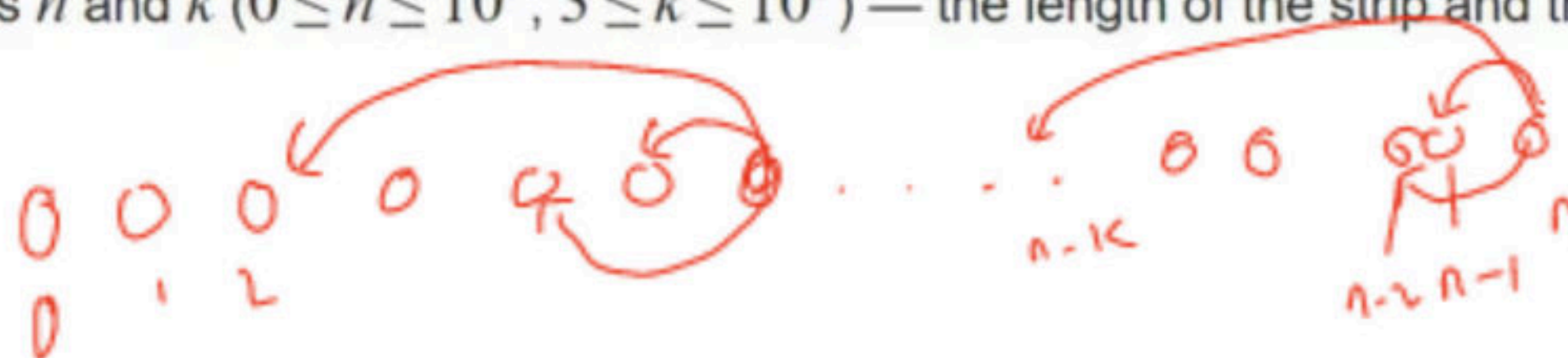
Who wins if both participants play optimally?

Alice and Bob would like to play several games, so you should determine the winner in each game.

Input

The first line contains the single integer T ($1 \leq T \leq 100$) — the number of games. Next T lines contain one game per line. All games are independent.

Each of the next T lines contains two integers n and k ($0 \leq n \leq 10^9$, $3 \leq k \leq 10^9$) — the length of the strip and the constant denoting the third move, respectively.



Problem 4

Ashishgup and FastestFinger play a game.

They start with a number n and play in turns. In each turn, a player can make **any one** of the following moves:

- Divide n by any of its odd divisors greater than 1.
- Subtract 1 from n if n is greater than 1.

Divisors of a number include the number itself.

The player who is **unable to make a move** loses the game.

Ashishgup moves first. Determine the winner of the game if both of them play optimally.

Input

The first line contains a single integer t ($1 \leq t \leq 100$) — the number of test cases. The description of the test cases follows.

The only line of each test case contains a single integer — n ($1 \leq n \leq 10^9$).

Problem 5

Alice and Bob are playing a game with n piles of stones. It is guaranteed that n is an even number. The i -th pile has a_i stones.

Alice and Bob will play a game alternating turns with Alice going first.

On a player's turn, they must choose **exactly** $\frac{n}{2}$ nonempty piles and independently remove a positive number of stones from each of the chosen piles. They **can** remove a **different** number of stones from the piles in a single turn. The first player unable to make a move loses (when there are less than $\frac{n}{2}$ nonempty piles).

Given the starting configuration, determine who will win the game.

Input

The first line contains one integer n ($2 \leq n \leq 50$) — the number of piles. It is guaranteed that n is an even number.

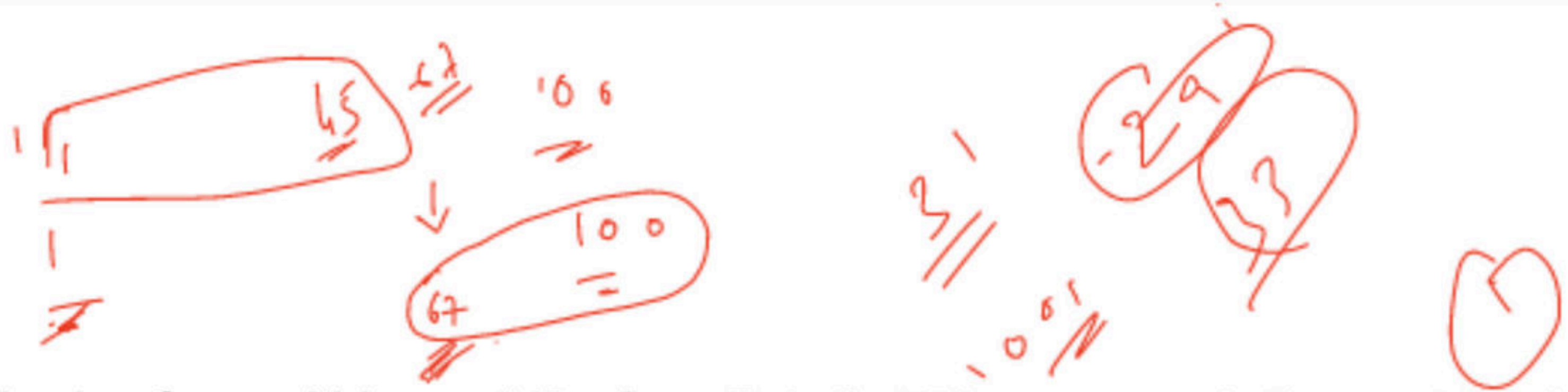
The second line contains n integers a_1, a_2, \dots, a_n ($1 \leq a_i \leq 50$) — the number of stones in the piles.

Summary

1. Basics of game theory, simple games
2. Nim game
3. Composite games - Grundy numbers (Nimbers)
4. Sprague grundy theorem

One last game!?

Game!



- Fill the following form within **next 5 mins**. Only first **50** responses in the **next 5 mins** to be considered.
- Game is simple, select an integer from 1 to 100. and enter your *codechef id*.
- The person who is closest to $\text{int}((\text{average} * 2) / 3)$ will get 100 laddus. If there is more than one person who is closest, all of them will get 100 each.

<https://forms.gle/oe22bcz5yn7Ppntp7>

Link will also be available in chat. No cheating, please! Only one response per person!

