FFT and combinatorics

Polynomial operations and Multipoint evaluation

Special class

Nishchay Manwani • Jan 13, 2021

Prerequisites

→ Polynomial multiplication

→ $p_1(x)$ & $p_2(x)$

→ $p = p_1(x) \times p_2(x)$

$p_1(x) = a_0 x^0 + a_1 x^1 - - - - a_n x^n$

$p_2(x) = b_0 x^0 + b_1 x^1 - - - - b_n x^n$

$p(x) = c_0 x^0 + c_1 x^1 \cdot - - c_{2n} x^n$

$$c_i = \sum_{j=0}^{i} a_j \times b_{i-j} \quad O(n^2)$$

FFT (Fast fourier Transform)

$e^{i\pi \frac{k}{2n}}$

NTT (number Theoretic Transform)

$\% m$

$T(n) = 2T(n) + O(n)$

$T(n) = O(n \log n) \leftarrow 100$

Iterative FFT

$\{x, f(x) = a_0 x^0 + a_1 x^1 + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5 - 10x^6 + 0x^7$

$5 \to 6$

2^k

$f(w_n^0), f(w_n^1), \dots f(w_n^{n-1})$

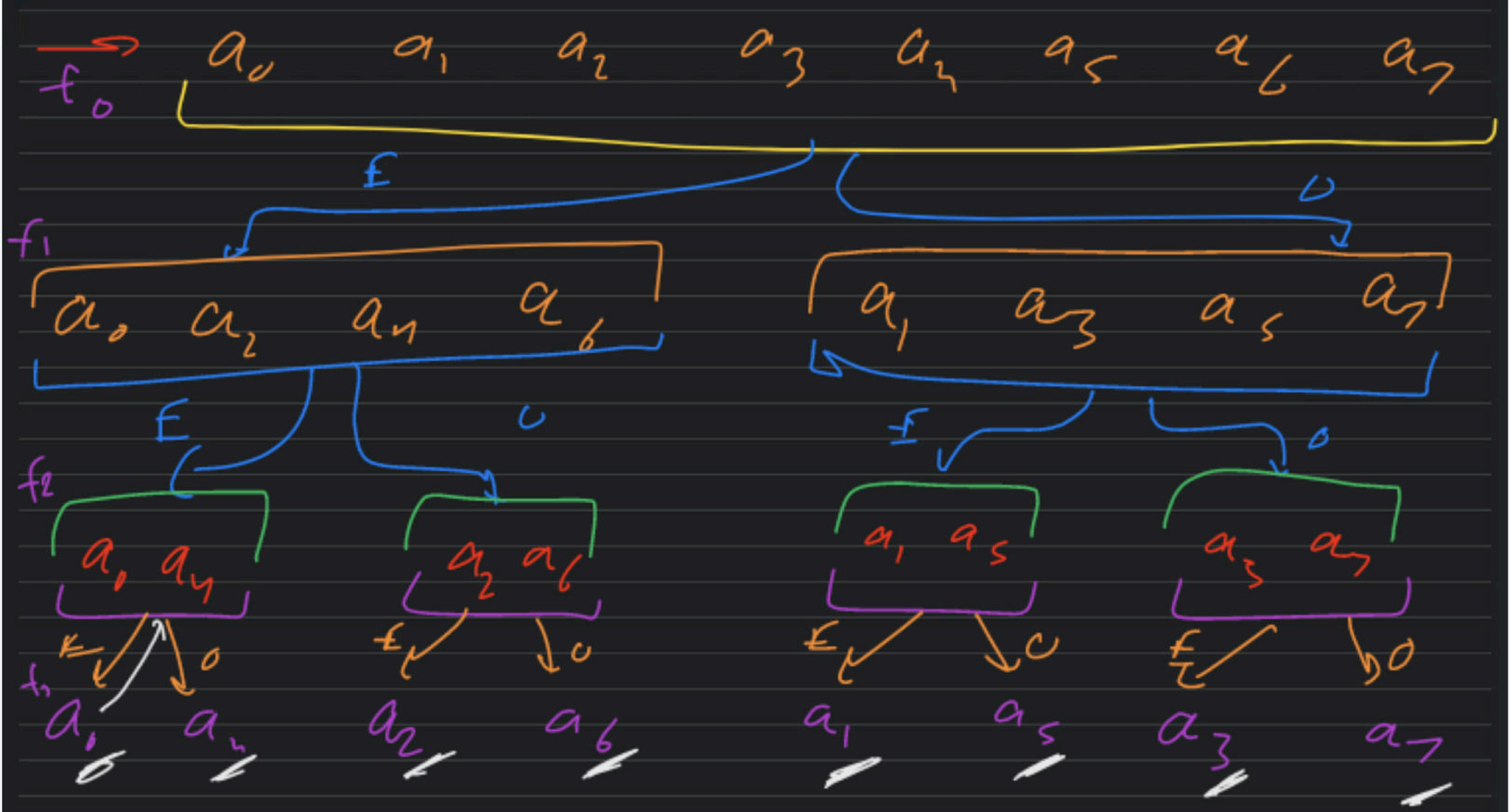
$\underbrace{f(w_8^0) \; \dots \; f(w_8^7)}$

$\hat{w_n} \; g^{\frac{2(n-1)}{n}}$

$\left(\frac{M-1}{n}\right)$

$\underbrace{g^{n-1} \% m = 1}$

$W_n \Rightarrow \underline{g}$



f0
a0 a1 a2 a3 a4 a5 a6 a7
E
O
f1
a0 a2 a4 a6
a1 a3 a5 a7
E
O
E
O
f2
a0 a4
a2 a6
a1 a5
a3 a7
E O
E O
E O
E O
f3
a0 a4 a2 a6 a1 a5 a3 a7

What is special about

↪ 0 2 1 3 $n=4$

↪ 0 4 2 6 1 5 3 7 $n=8$

↪ 0, 8, 4, 12, 2, 10, 6, 14, $n=16$

1, 9, 5, 13, 3, 11, 7, 15,

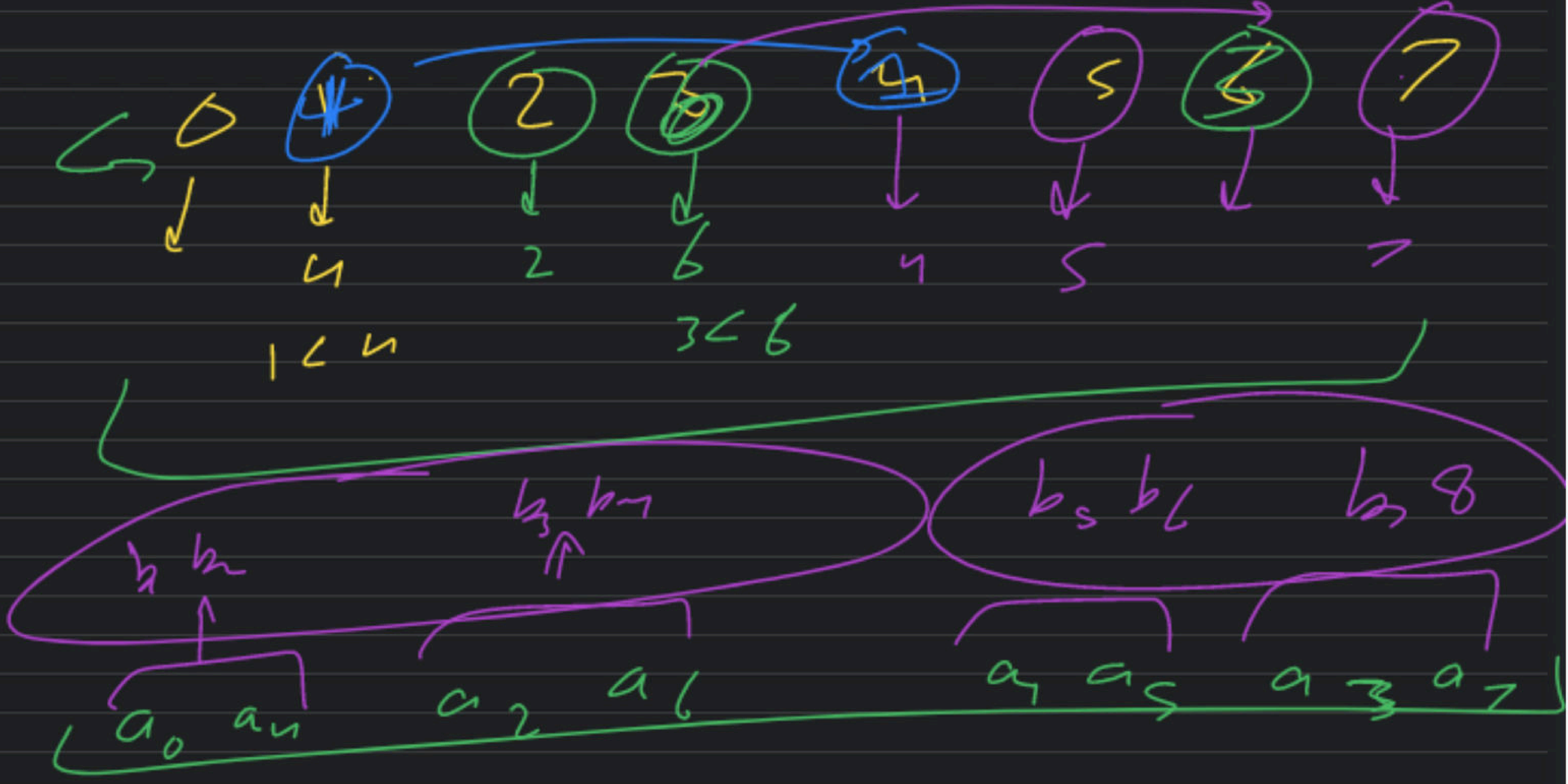
0 4 2 6 1 5 3 7

100 010 110 001 101 011 111

↪ 000 100 010 110 001 101 011 111

↪ 000 001 010 011 100 101 110 111

↪ 0 1 2 3 4 5 6 7


0 4 2 6 4 5 3 7
4 2 6 4 5 7
1 < 4
3 < 6
b_1 b_2
b_3 b_4
b_5 b_6
b_7 8
a_0 a_4
a_2 a_6
a_1 a_5
a_3 a_7

polynomial inverse

division

evaluation

Nishchay Manwani

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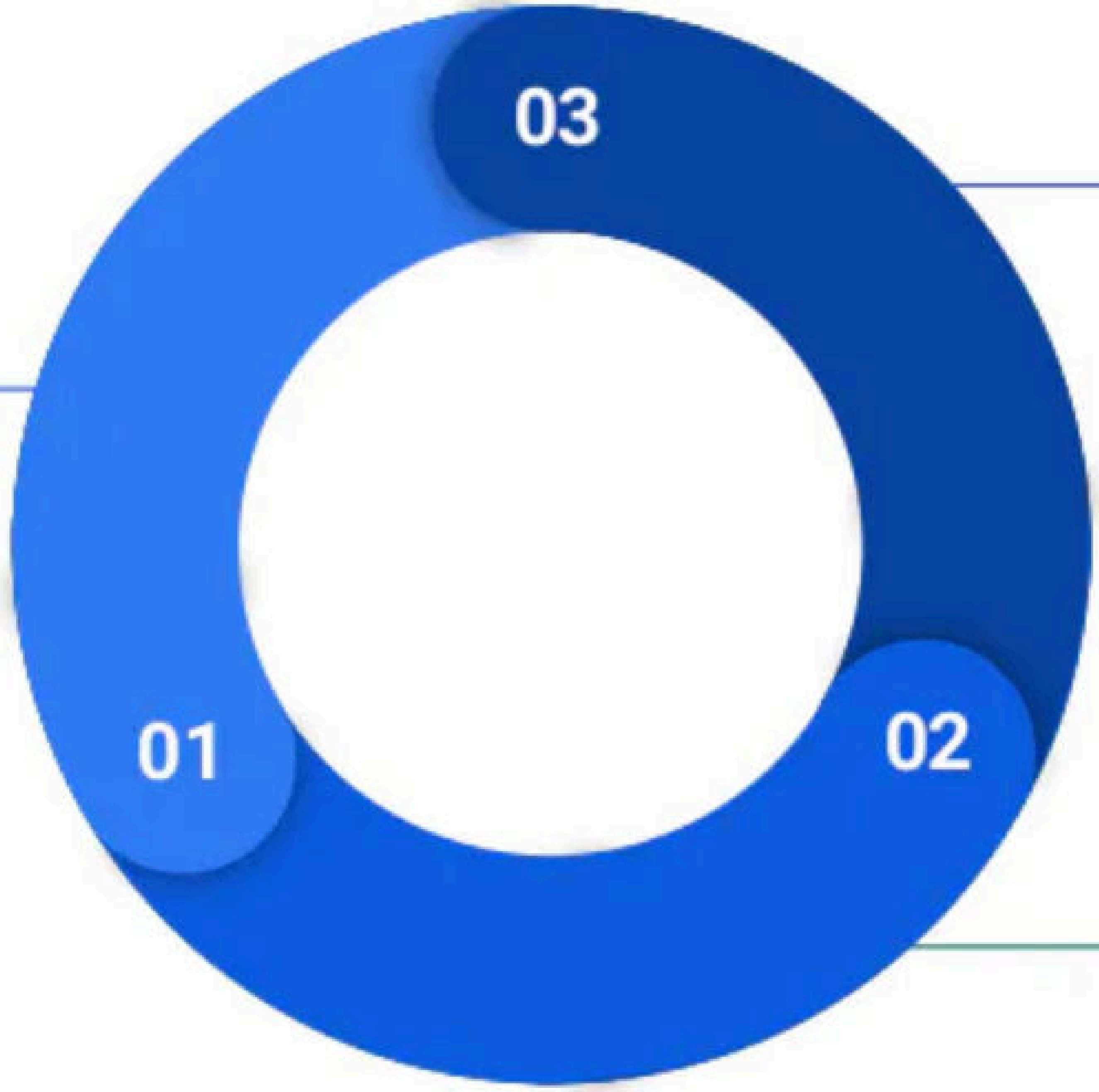

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Week 15-17	Data Structures 2 - Square Root Decomposition and Advanced Problems
Week 18-19	Number Theory and Interview Questions
Week 20-22	Recursion and DP Concepts and Handpicked Problem Solving
Week 23-27	Discrete Mathematics in C++- Concept to Problems
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Week 33-35	Segment Trees
Week 36-38	Advanced Dynamic Programming
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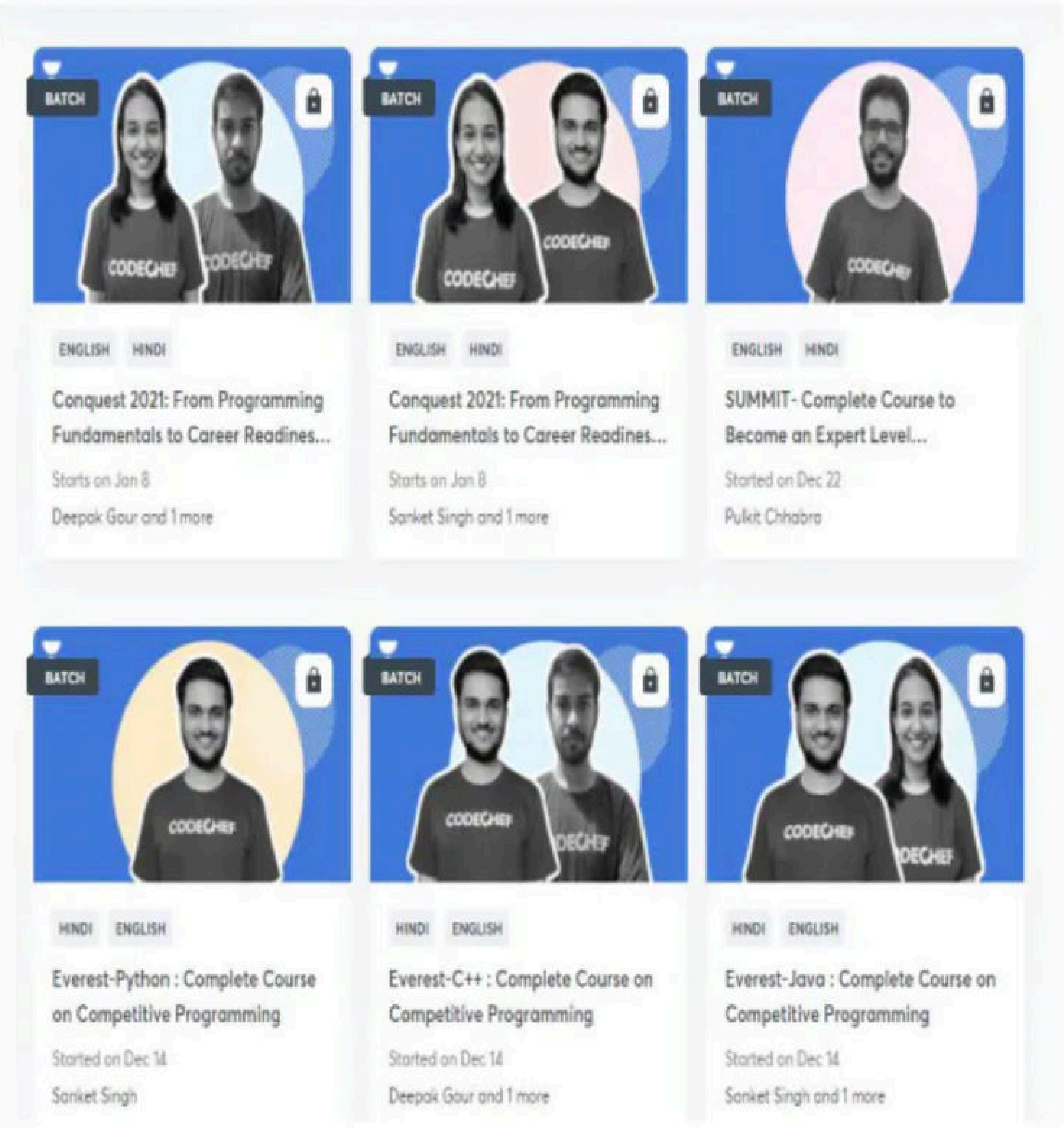


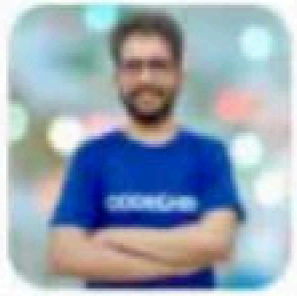
Educators

Tanuj Khattar

ACM ICPC World Finalist - 2017, 2018. Indian IOI Team Trainer 2016-2018. Worked @ Google, Facebook, HFT. Quantum Computing Enthusiast.

Sanket Singh

Software Development Engineer @ LinkedIn | Former SDE @ Interviewbit | Google Summer of Code 2019 @ Harvard University | Former Intern @ISRO



Pulkit Chhabra

Codeforces: 2246 | Codechef: 2416 | Former SDE Intern @CodeNation | Former Intern @HackerRank

Riya Bansal

Software Engineer at Flipkart | Former SDE and Instructor @ InterviewBit | Google Women TechMakers Scholar 2018

Triveni Mahatha

Qualified ICPC 2016 World Final. Won multiple Codechef Long Challenges (India). ICPC Onsite Regionals' Problem setter and Judge. IIT Kanpur.

Deepak Gour

ICPC World Finalist 2020 | Former Instructor @InterviewBit | Software Engineer at AppDynamics

Educators

Himanshu Singh

World Finalist ICPC 2020, Winner Techgig Code Gladiators 2020, Winner TCC '19, 2020 CSE Graduate from IIT BHU, Works at Nutanix

Murugappan S

Software engineer at Google. Have won many programming contests. Max Rating of 2192 in codeforces and 2201 in codechef.

Nishchay Manwani

Hey I am Nishchay Manwani from CSE, IIT Guwahati and I'm a Seven star on Codechef and International Grandmaster on Codeforces.

Vivek Chauhan

Codechef: 7 stars (2612) India Rank 6, Codeforces: MASTER (2279), Won Codechef Long Challenges(India), TCO20 Southern Asia Runner up

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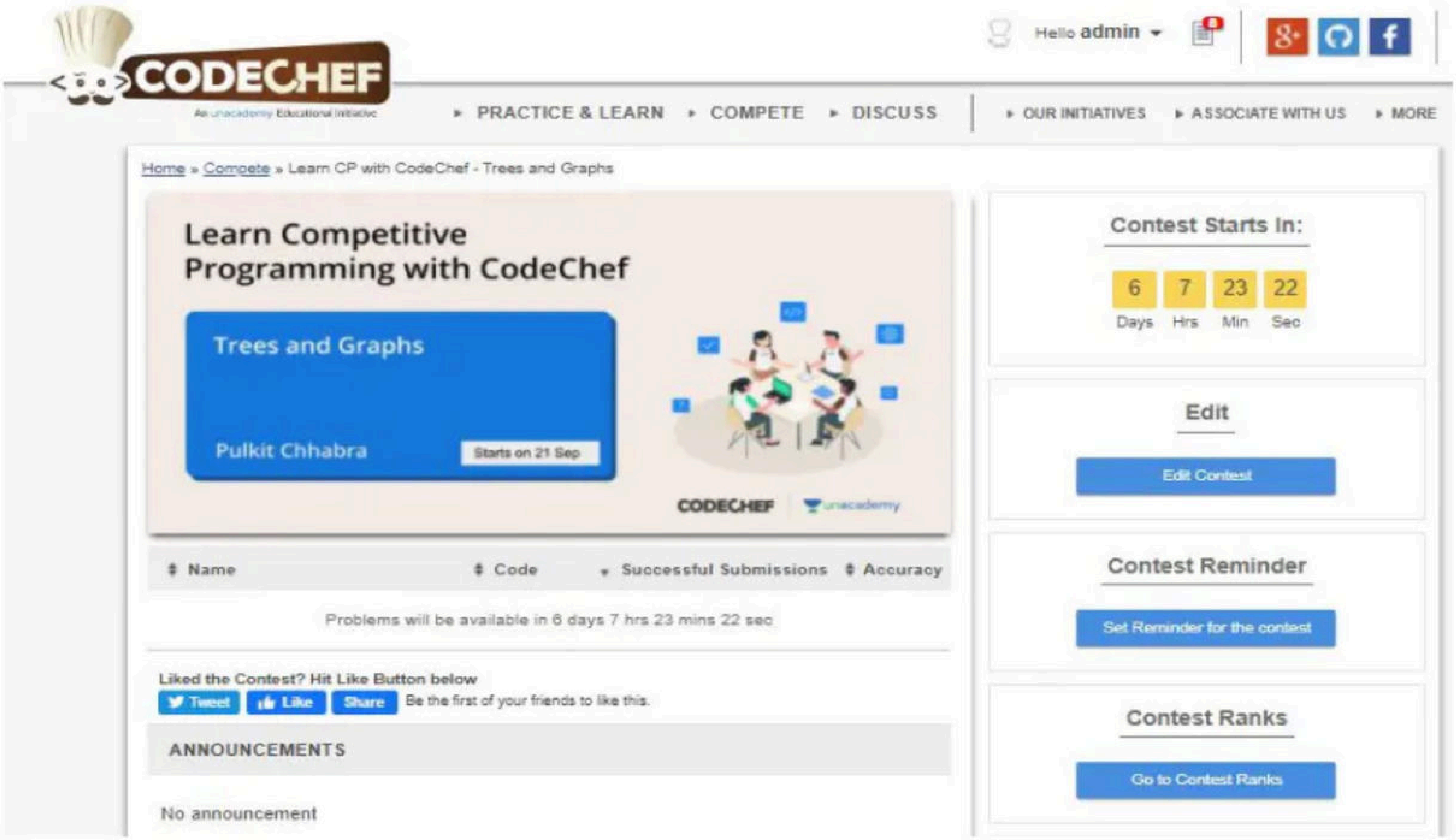
Course-wise Practice Problems



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Polynomial mutiplication

↳ Poly Inverse

$A(x) \rightarrow a_0 x^0 + a_1 x^1 \cdot - - \quad a_{n-1} x^{n-1}$

$A^{-1}(x) \rightarrow b_0 x^0 + b_1 x^1 + - - - - \quad b_n x^n + b_{n+1} x^{n+1}$

$- - b_{\infty} x^{\infty}$

$\hookrightarrow A(x) \times A^{-1}(x) \rightarrow 1 x^0 + 0 x^1 + 0 x^2 - -$

$- - - \quad 0 x^n$

$- - - \infty$

$$\boxed{\underbrace{A(x) \times A^{-1}(x)} \rightarrow 1}$$

$$c_0 = a_0 b_0 = 1$$

$$c_0 = a_0 b_0 = 1$$

$$b_0 = \frac{1}{a_0}$$

$$0 = c_1 = a_0 b_1 + a_1 b_0 \Rightarrow -a_0 b_1 = \frac{a_1}{a_0}$$

$$\Rightarrow b_1 = \frac{-a_1}{a_0^2}$$

$\Rightarrow C_2 = 0 \Rightarrow a_0 b_2 + a_1 b_1 + a_2 b_2$

$b_2 \rightarrow$ -

$A(x) \rightarrow a_0 x^0 + a_1 x^1 - - - a_{n-1} x^{n-1}$

$A^{-1}(x) \rightarrow b_0 x^0 + b_1 x^1 - - - b_{a-1} x^{a-1}$

(first a coefficients only)

$A(x) \times A^{-1}(x) \Rightarrow 1x^0 + 0x^1 + 0x^2 - - - 0x^{a-1}$

$+ (\text{rand}_1 x^a + \text{rand}_2 x^{a+1} + \text{rand}_3 x^{a+2} - - \text{rand } x^{n+a-2})$

$$\boxed{A(x) \times A^{-1}(x) \rightarrow 1 + x^a \times C(x)}$$

$\hookrightarrow$ $\boxed{A^{-1}(x) \Rightarrow B_k(x) \quad \text{(the first } 2^k \text{ coeff)}}$

$B_0(x) \Rightarrow A^{-1}(x) \Rightarrow b_0 x^0$

(only the first coeff)

$b_0 = \dfrac{1}{a_0}$

$$B_0(x) \to B_1(x) \to B_2(x)$$

$$\cdots$$

$$B_{k-1}(x) \to B_k(x) \quad \cdots \to B_k(x)$$

$$\hookrightarrow 2^k > a$$

$\hookrightarrow \quad A(x) \times B_k(x) = 1 + x^{(2^k)} C(x) \quad \text{--- } 1$

$\hookrightarrow \quad \underbrace{B_{k+1}(x) = \underbrace{B_k(x) + x^{(2^k)}} D(x)} \quad \text{--- } 2$

$\hookrightarrow \quad A(x) \times \underbrace{B_{k+1}(x)} = 1 + x^{(2^{k+1})} C_2(x)$

$\hookrightarrow$

$A(x) \times (B_k(x) + x^{(2^k)} D(x)) = 1 + x^{(2^{k+1})} C_2(x)$

$\hookrightarrow \underbrace{A(x) \times B_k(x)}_{\Delta} + \underline{A}(x)\, x^{(2^k)} D(x)$

$\Rightarrow 1 + x^{(2^{k+1})} C_2(x)$

$\hookrightarrow 1 + x^{(2^k)} C(x) + A(x)\, x^{(2^k)} D(x)$

$= 1 + x^{(2^{k+1})} C_2(x)$

look at First 2^{k+1} coeff

$\hookrightarrow \cancel{1} + \cancel{x^{(2^k)}} C(x) + A(x)\, \cancel{x^{(2^k)}} D(x) = \cancel{1}$

$\hookrightarrow \underbrace{C(x)} = -\underbrace{A(x)\, D(x)}$

$\hookrightarrow$ first 2^k coeff

$\hookrightarrow \underbrace{C(x)\, B_k(x) =} -D(x)$

$\hookrightarrow B_{k+1}(x) = B_k(x) + x^{2^k} D(x)$

$\hookrightarrow B_{k+1}(x) = B_k(x) + x^{(2^k)} \times (-C(x)\, B_k(x))$

$$\boxed{B_{k+1} = B_k \times (2 - A \times B_k)}$$

2^{k+1} $\quad 2^k \quad \frac{n}{2}^k \quad \frac{n}{2}^k$

$$\boxed{T(n) = T\left(\frac{n}{2}\right) + O(n \log n)}$$

$$\boxed{T(n) = O(n \log n)}$$

Polynomial Division

$A(x) \quad B(x)$

$$A(x) \div B(x) \to \begin{cases} Q(x) \\ R(x) \end{cases}$$

$$\underbrace{A(x) = B(x) \times Q(x) + R(x)}$$

$$\deg(R(x)) < \deg(B(x))$$

$$A(x) \rightarrow a_0 x^0 + a_1 x^1 - - - \quad a_{n-1} x^{n-1}$$

$$A_R(x) \rightarrow a_{n-1} x^0 + a_{n-2} x^1 + - - - \quad a_0 x^{n-1}$$

$$B(x) \rightarrow b_0 x^0 + b_1 x^1 - - - \quad b_{m-1} x^{m-1}$$

$$B_R(x) \rightarrow b_{m-1} x^0 + b_{m-2} x^1 - - - - b_0 x^{m-1}$$

$$\underbrace{\deg(R)}_{m-2} < \deg(B)$$

$$\underbrace{R(x)}_{m-2} + \underbrace{Q(x)}_{n-m} \cdot \underbrace{B(x)}_{m-1} \longrightarrow \underbrace{A(x)}_{n-1}$$

$n-m + m-1$

$A(x) = a_0 x^0 + a_1 x^1 \; . \; - \; - \qquad a_{n-1} x^{n-1}$

$\underbrace{\qquad\qquad}_{m-2} \qquad \underbrace{\qquad\qquad}_{n-m+1}$

$B(x) = b_0 x^0 + b_1 x^1 \; - \; - \; - \qquad b_{m-1} x^{m-1}$

$Q(x) = q_0 x^0 + q_1 x^1 - - - - - \qquad q_{n-1} x^{n-m}$

$R(x) = r_0 x^0 + \; - \; - \; - \qquad r_{m-2} x^{m-2}$

$n - m - 2$

~~The first $(m-2)$ coefficients of $A(x)$ are not going to affect $Q(x)$~~

(They are going to be affecting $R(x)$)

↳ The last $(n-m+1)$ coefficients of A do not affect $R(x)$

$$A_R(x) \rightarrow a_{n-1}x^0 + a_{n-2}x^1 \cdots - - - \; a_0 x^{n-1}$$

$$\hookrightarrow \left(\underbrace{A_R(x)}\; = \underbrace{B_R(x)} \times \underbrace{Q_R(x)}\right)$$

$\bmod(x^{n-m+1})$

$$\hookrightarrow \left(\underbrace{A_R(x)}_{n-m+1} \times \underbrace{\left(B_R(x)\right)^{-1}}_{n-m+1}\right) = Q_R(x)\;)$$

$\bmod x^{n-m+1}$

Multipoint Evaluation

$$\underbrace{P(x) \rightarrow a_0 x^0 + a_1 x^1 - - - - \quad a_{n-1} x^{n-1}}$$

$b_1, b_2, - - - , b_n$

$P(b_1), P(b_2), P(b_3) - - - , P(b_n)$

$$\underbrace{GP(c)} = \underline{P(x)} \text{ modulo } \underline{(x-c)}$$

$$P(x) \div (x-c) \begin{cases} \Rightarrow Q \\ \Rightarrow R(x) \end{cases}$$

$$\underset{\substack{\downarrow \\ 0}}{\deg}\big(R(x)\big) < \underset{1}{\deg(x-c)}$$

To calculate

$P(b_1) \rightarrow n\log n$

$P(b_2) \rightarrow n\log n$

$n^2\log n$

$$\underbrace{P(x) \ \% \ (x-b_1)(x-b_2)(x-b_3)\ldots(x-b_n)}_{A(x)}$$

$n\log n$ $\qquad$ $n\log n$

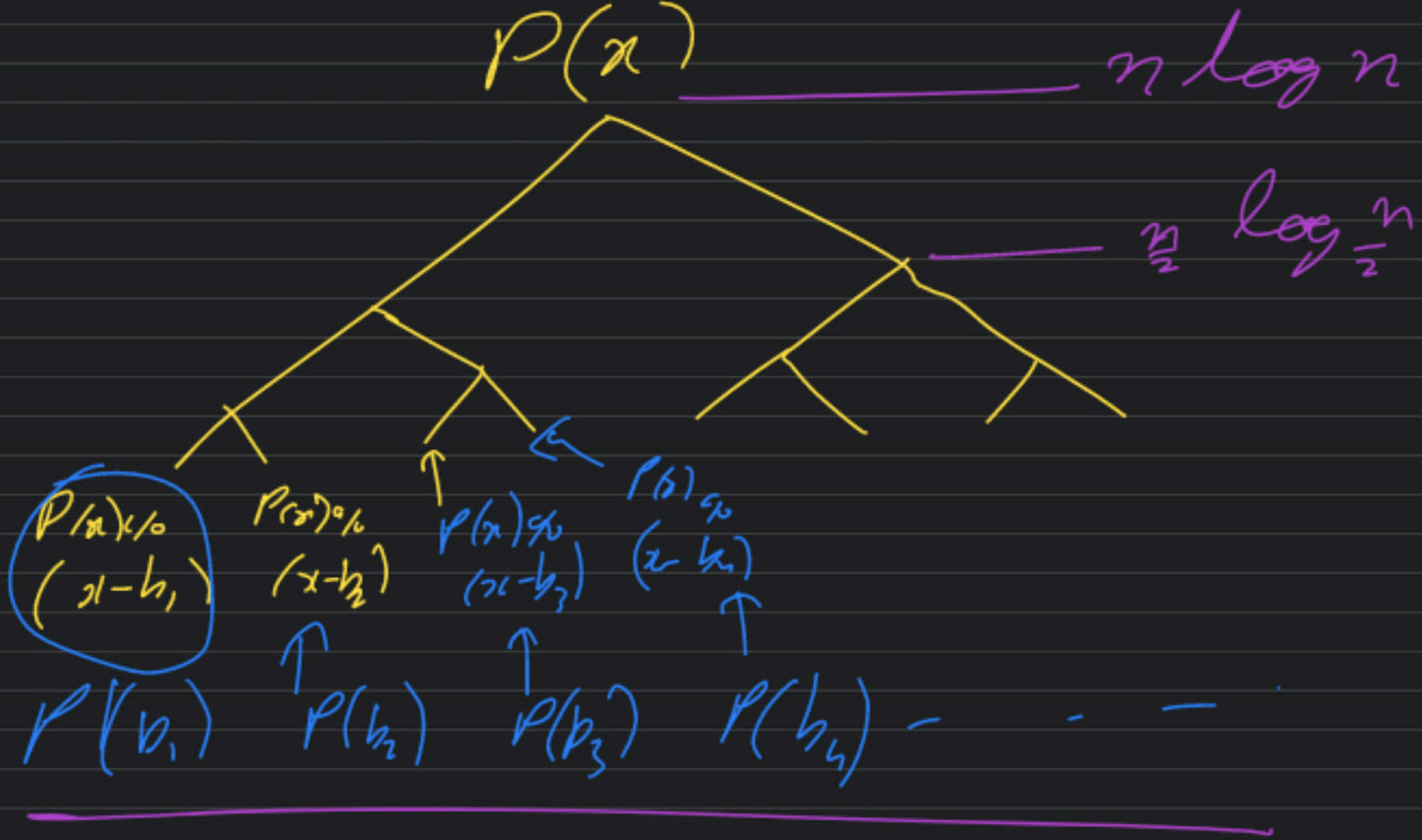
$\frac{n}{2}$ $\qquad$ $\frac{n}{2}$

$$P(x) \ \% \ (x-b_1)(x-b_2)\ldots(x-b_{\frac{n}{2}})$$

$$P(x) \ \% \ (x-b_{\frac{n}{2}+1})(x-b_{\frac{n}{2}+2})\ldots(x-b_n)$$

$\frac{n}{2}\log\frac{n}{2}$

$$A(x) \ \% \ (x-b_1)(x-b_2)\ldots(x-b_{\frac{n}{2}})$$



$P(x)$
$n \log n$
$\frac{n}{2} \log \frac{n}{2}$
$P(x) \% (x-h_1)$
$P(x) \% (x-h_2)$
$P(x) \% (x-h_3)$
$P(x) \% (x-h_4)$
$P(h_1)$ $P(h_2)$ $P(h_3)$ $P(h_4)$ - - -

$$T(n) = 2 \times T\left(\frac{n}{2}\right) + O(n \log n)$$

$$\boxed{T(n) = O(n \log^2 n)}$$

$$\boxed{T(n) = O(\max(r, m)(\log^2 m + \log n))}$$

Exponentiation

log

↪ lagrange interpolation

↪ $P(b_1), P(b_2) \dots P(b_n)$

$Poly \rightarrow P(x) \rightarrow O(n \log^2 n)$

$$\hookrightarrow P(x) = Q(x) \cancel{\times} (x-c) + R(x)$$

$$\hookrightarrow P(c) = Q(c) \times 0 + R(c)$$

$$\hookrightarrow P(c) = \underline{R}$$

Stirling number of second kind