

# Meet In The Middle - II

Special class



# Vivek Chauhan

< Educator Image >

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-

2021 : The Year To QUIT PROCRASTINATING And LEARN CODING

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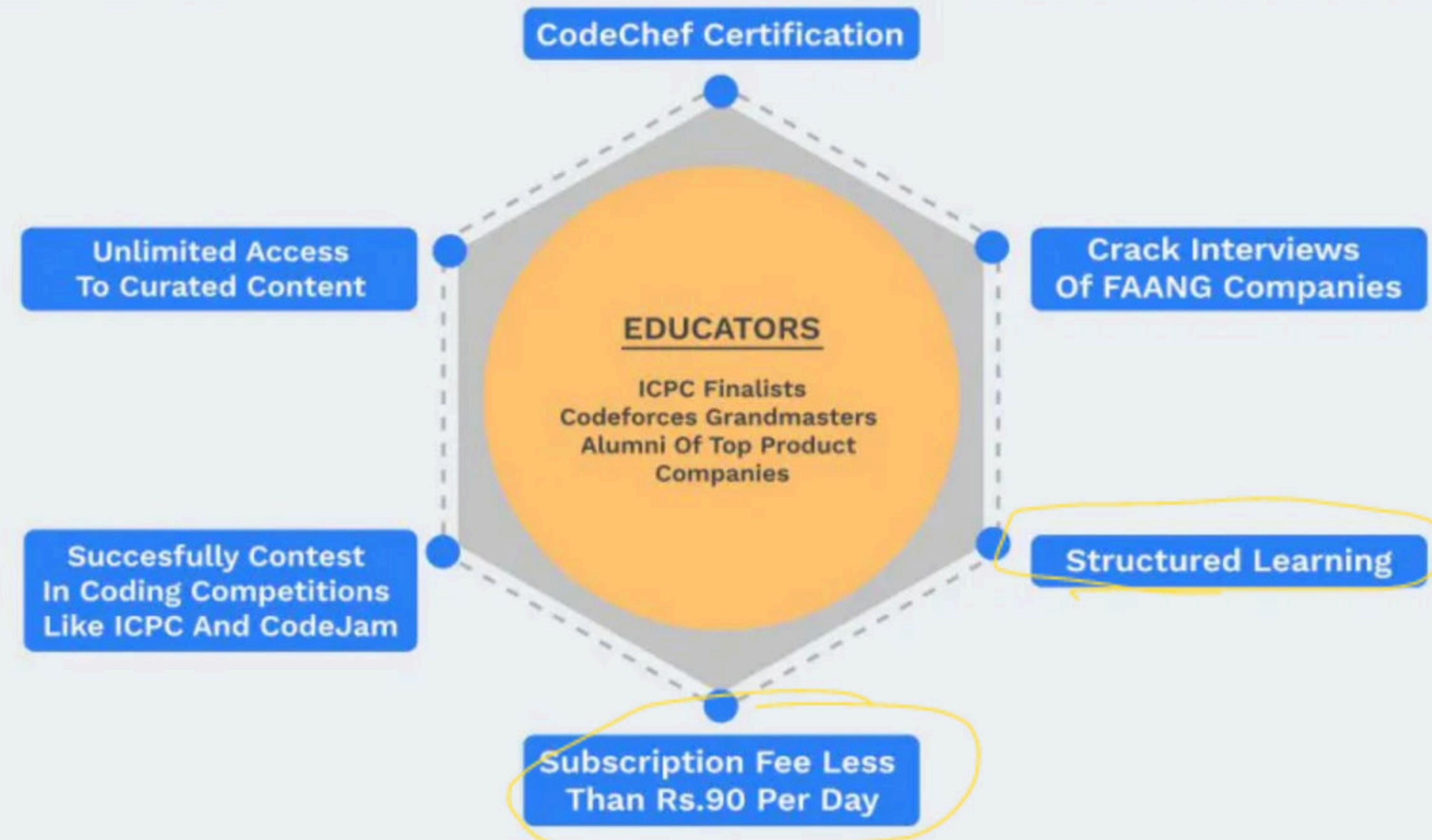
**Pinnacle: Comprehensive and Concise Track to Become an Expert** **GOING LIVE ON 18TH JAN 2021**

- Conquest 2021: Year Long Journey for Intermediate Coders to Become Experts (C++) - Live on 8th Jan 2021
- C++: Conquest 2021: From Programming Fundamentals to Career Readiness - Live on 8th Jan 2021
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# Exclusive Batch Starting On 18th Jan 2021

## PINNACLE - Batch For Intermediates And Beginners

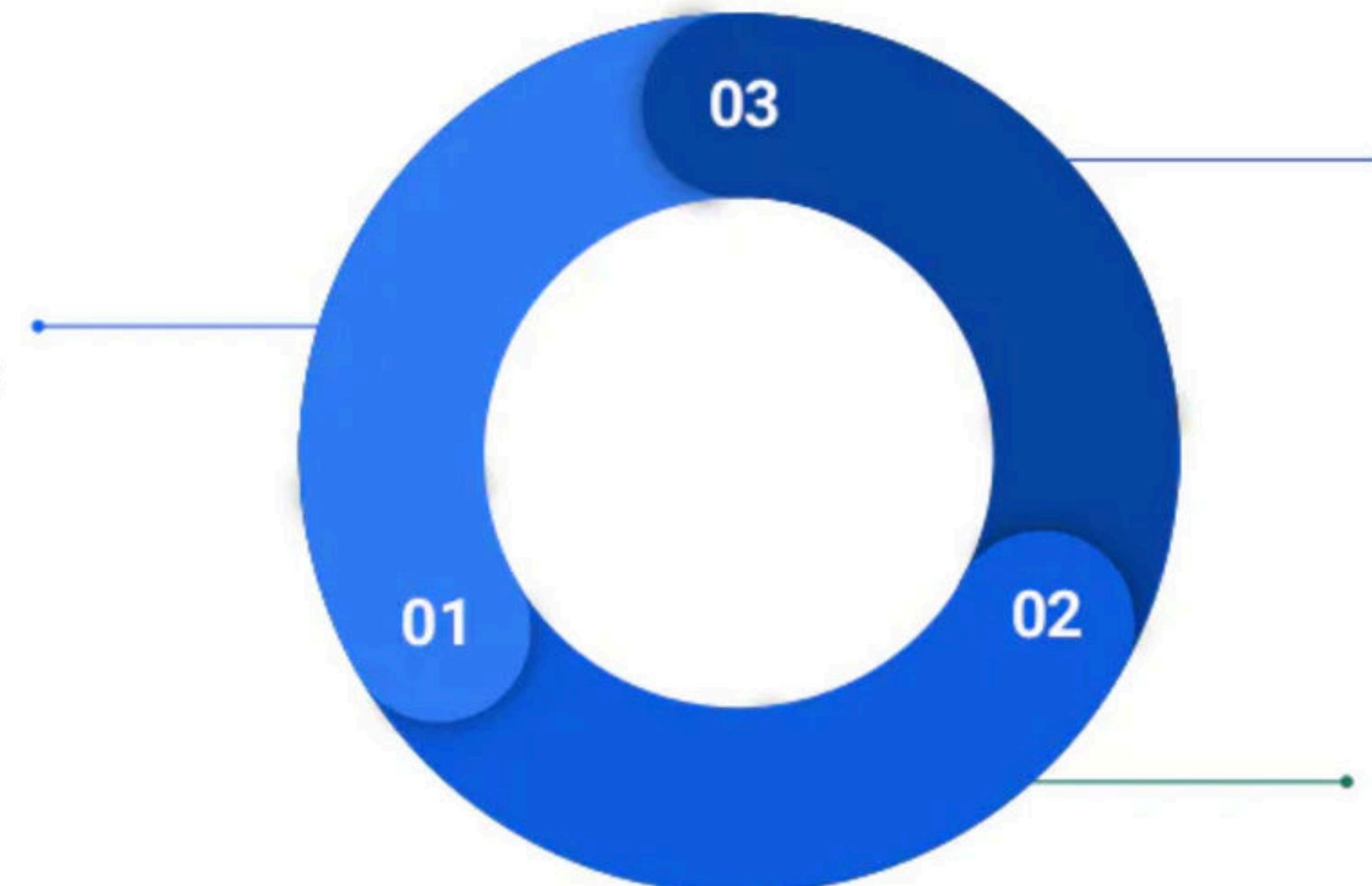




# What you will get

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## Doubt Support

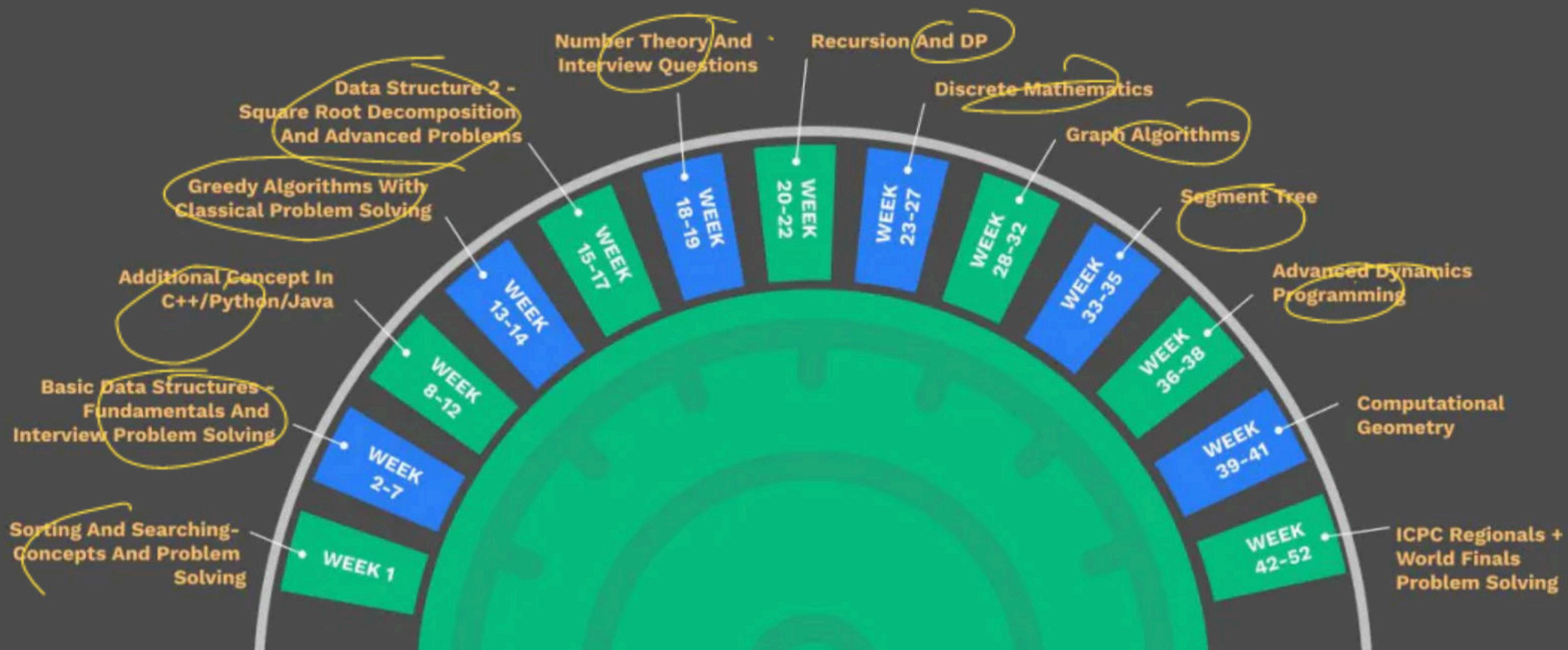
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# Topic-wise Batch Structure

## PINNACLE





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**Batch Getting Live on 18th January 2021:**

**PINNACLE: Comprehensive and Concise Track to Become an Expert (C++)**

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- Conquest 2021: From Programming Fundamentals to Career Readiness (**Java**)
- Conquest 2021: From Programming Fundamentals to Career Readiness (**Python**)
- Conquest 2021: Year Long Journey for Intermediate Coders to Become Experts (**C++**)

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Conquest 2021: From Programming Fundamentals to Career Readiness...  
Starts on Jan 8  
Deepak Gour and 1 more

ENGLISH HINDI

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Starts on Jan 8  
Sanket Singh and 1 more

ENGLISH HINDI

SUMMIT- Complete Course to Become an Expert Level...  
Started on Dec 22  
Pulkit Chhabra



HINDI ENGLISH

Everest-Python : Complete Course on Competitive Programming  
Started on Dec 14  
Sanket Singh

HINDI ENGLISH

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Started on Dec 14  
Deepak Gour and 1 more

HINDI ENGLISH

Everest-Java : Complete Course on Competitive Programming  
Started on Dec 14  
Sanket Singh and 1 more



# Educators

**Tanuj Khattar**

ACM ICPC World Finalist - 2017, 2018. Indian IOI Team Trainer 2016-2018. Worked @ Google, Facebook, HFT. Quantum Computing Enthusiast.

**Sanket Singh**

Software Development Engineer @ LinkedIn | Former SDE @ Interviewbit | Google Summer of Code 2019 @ Harvard University | Former Intern @ISRO

**Pulkit Chhabra**

Codeforces: 2246 | Codechef: 2416 | Former SDE Intern @CodeNation | Former Intern @HackerRank

**Riya Bansal**

Software Engineer at Flipkart | Former SDE and Instructor @ InterviewBit | Google Women TechMakers Scholar 2018

**Triveni Mahatha**

Qualified ICPC 2016 World Final. Won multiple Codechef Long Challenges (India). ICPC Onsite Regionals' Problem setter and Judge. IIT Kanpur.

**Deepak Gour**

ICPC World Finalist 2020 | Former Instructor @InterviewBit | Software Engineer at AppDynamics



# Educators

**Himanshu Singh**

World Finalist ICPC 2020, Winner Techgig Code Gladiators 2020, Winner TCC '19, 2020 CSE Graduate from IIT BHU, Works at Nutanix

**Murugappan S**

Software engineer at Google. Have won many programming contests. Max Rating of 2192 in codeforces and 2201 in codechef.

**Nishchay Manwani**

Hey I am Nishchay Manwani from CSE, IIT Guwahati and I'm a Seven star on Codechef and International Grandmaster on Codeforces.

**Vivek Chauhan**

Codechef: 7 stars (2612) India Rank 6, Codeforces: MASTER (2279), Won Codechef Long Challenges(India), TCO20 Southern Asia Runner up

and many more joining soon...



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# Course-wise Practice Problems

Hello admin

CODECHEF

An Unacademy Educational Initiative

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OUR INITIATIVES ASSOCIATE WITH US MORE

Home » Compete » Learn CP with CodeChef - Trees and Graphs

## Learn Competitive Programming with CodeChef

### Trees and Graphs

Pulkit Chhabra Starts on 21 Sep

CODECHEF unacademy

# Name	# Code	* Successful Submissions	* Accuracy
--------	--------	--------------------------	------------

Problems will be available in 6 days 7 hrs 23 mins 22 sec

Liked the Contest? Hit Like Button below

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#### ANNOUNCEMENTS

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Contest Starts In:

6 Days 7 Hrs 23 Min 22 Sec

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Edit Contest

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Contest Ranks

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No cost EMI available on 6 months & above subscription plans

1 month

₹5,400  
per month

₹5,400  
Total (Incl. of all taxes)

3 months

11% OFF

₹4,800  
per month

₹14,400  
Total (Incl. of all taxes)

6 months

25% OFF

₹4,050  
per month

₹24,300  
Total (Incl. of all taxes)

12 months

54% OFF

₹2,475  
per month

₹29,700  
Total (Incl. of all taxes)



vivek\_1998299



Awesome! You got 10% off!

Proceed to pay



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Proceed to pay





Meet in the Middle

Vivek Chauhan  
(vivek\_1998299)

# Prerequisites

1. Basics of graph theory
2. Basics of bit manipulation

1) Easy - medium  
2) Hard

You are given two strings **S** and **T** of length **N**.

*exactly*

Figure out if we can get string **T** starting from string **S** and applying 4 substring reversal operations.

on S

**Constraints:**

$1 \leq N \leq 300$  20

**S** and **T** consists of lowercase English alphabets only

$S = \underline{c} \underline{b} \underline{e} \underline{d} \underline{f} \underline{g} \underline{h}$

$S = \underline{c} \underline{b} \underline{e} \underline{d} \underline{f} \underline{g} \underline{h}$

$\underline{\underline{I}} \underline{\underline{S}} \underline{\underline{T}} \underline{\underline{R}} = \underline{a} \underline{c} \underline{b} \underline{d}$

$\underline{\underline{20}}$   $\sim b < a \emptyset$

$\underline{\underline{3}}$   $\sim$

$\underline{\underline{m}}$   $\sim d < bc$

$N = 4$

$S = aaba$

$T = aaab$

Yes :

1st

2nd ✓

3rd ✓

4th

$S = \boxed{a} \boxed{a} \boxed{b} \boxed{a}$

a a b a

b a a a

a a a b

a a a b

$T = a a a b$

$$N = 4$$

$$S = \underline{abcd}$$

$$T = \underline{cdba}$$

(-6)

abcd + f g h i  
+ i j k l f e

$$4n = , d b a$$

1st ✓

2nd ✓

3rd ✓

$S = a \cup c \cup d$

$\subseteq b \cup a \cup d$

$\subseteq d \cup a$

$\cup A b a$

$\bar{T} = \underline{c d b a}$

$N = 4$   
 $A = abcd$   
 $B = dcab$

$S =$   
 $\downarrow \downarrow \downarrow$   
 $a b c d$   
 $d c b a$

What will be the answer?

1st       $a c a b$   
2nd       $a c a b$   
3rd       $d c a b$   
4th       $d c a b$   
5th       $-$

N = 4

$$S = abcd$$

$$T = cdःba$$

L2 R2

67

$N = 4$

$S = abcd$

$T = cdba$

$O(n^8) \times O(W)$

$O(N^4)$

$(2o)^4 = \underbrace{1010}_{\{ \} \{ \} \{ \}}$

$\rightarrow$  return No;

string  $\rightarrow$  emp $= \sum_{i=1}^n o(w)$

reverse (temp, l, R<sub>1</sub>);  $O(N)$

reverse (+emp, l<sub>2</sub>, R<sub>2</sub>)  $O(n)$

reverse (temp, l<sub>3</sub>, R<sub>3</sub>);

reverse (+emp, l<sub>4</sub>, R<sub>4</sub>);

if (+emp == T)  $O(N)$

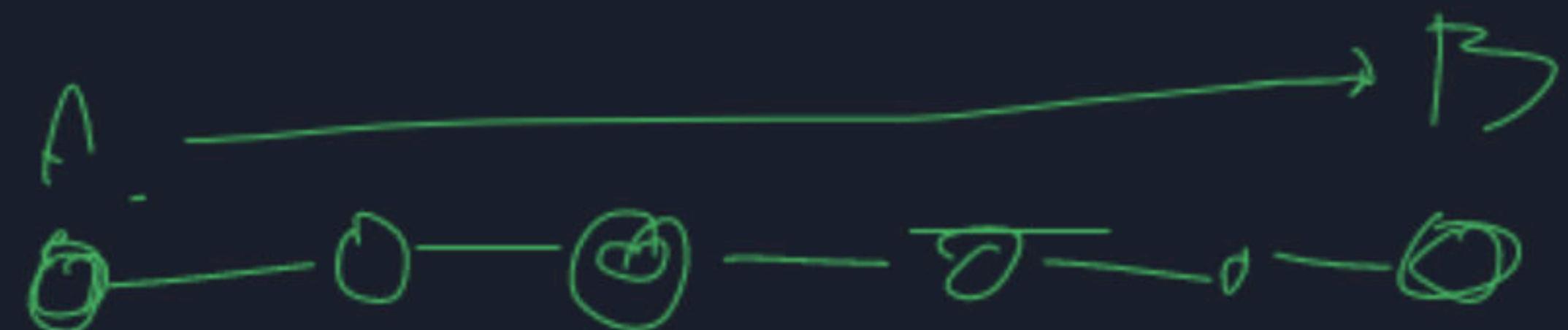
return Yes;

$$N = 4$$

$\begin{matrix} 0 \\ 1 \\ 2 \\ 3 \end{matrix}$

$$S = abcd$$

$$T = \cancel{cdba}$$

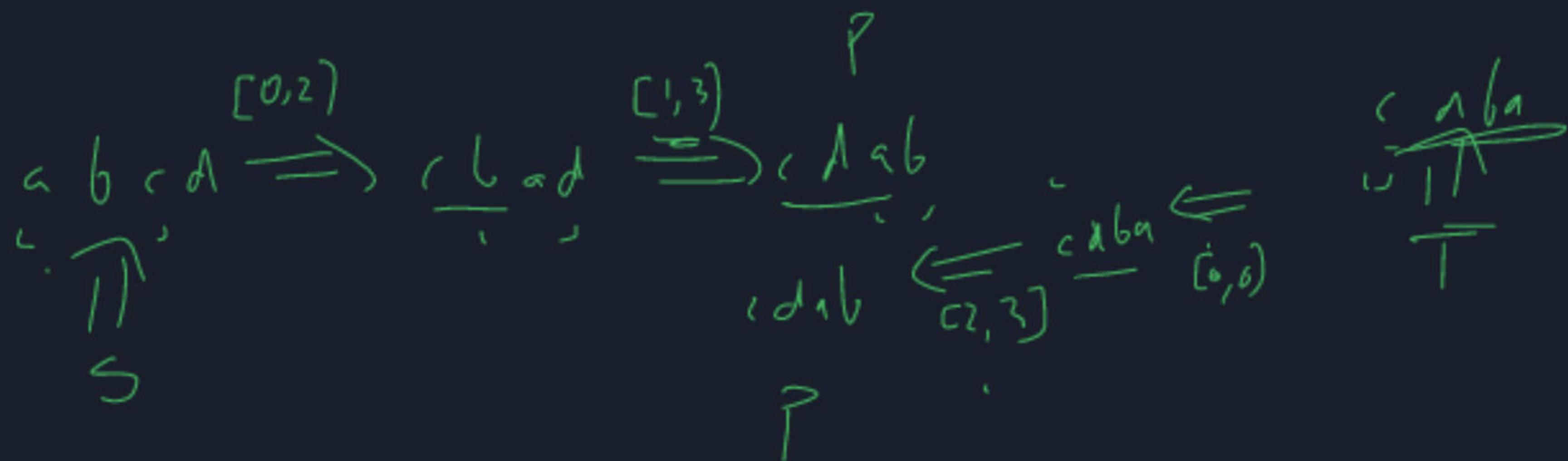


S1

S2

I2

T1



$N = 4$

$S = abcd$

$T = cdba$

### Main Problem

$S \rightarrow T$  by  
Substitution  
Reverse of U.P.

$c \rightarrow b$   $\Rightarrow$   $1 \rightarrow 6$   
 $\Rightarrow$   $(c \rightarrow b)$

$S = \underbrace{ab}_{1} \underbrace{cd}_{2} \Rightarrow \underbrace{b}_{3} \underbrace{(a \rightarrow d)}_{4}$

$a \rightarrow b, d \rightarrow c$   $\Rightarrow$   $\underbrace{bad}_{5} \Rightarrow \underbrace{cda}_{6} \underbrace{b}_{7}$

Find all possible  
strings you can  
generate from  
after 2 subs

- - -  
from T

~~b~~

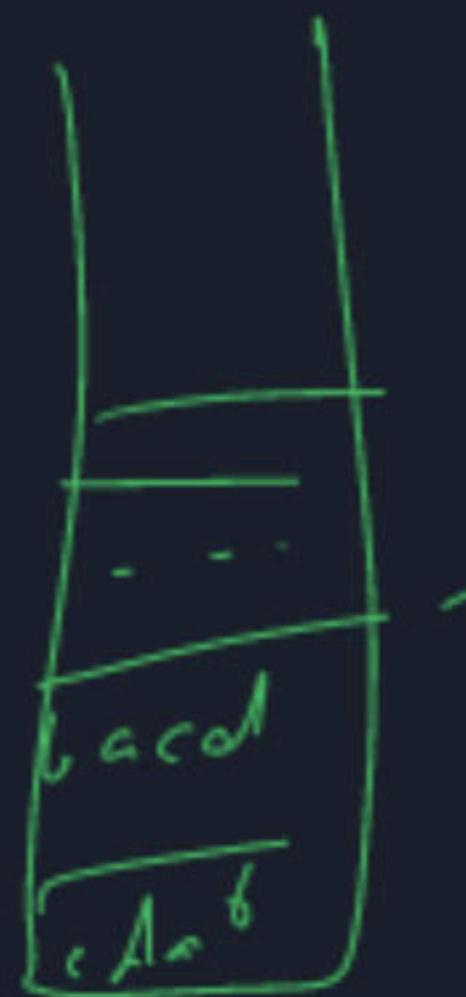
$N = 4$

$S = abcd$

$T = cdba$

$\overline{T} \Rightarrow$

(c d b a)



$S \oplus T$



N = 4

S = abcd

T = cdba



N = 4

S = abcd

T = cdba



N = 4

S = abcd

T = cdba



N = 4

S = abcd

T = cdba



N = 4

S = abcd

T = cdba



N = 4

S = abcd

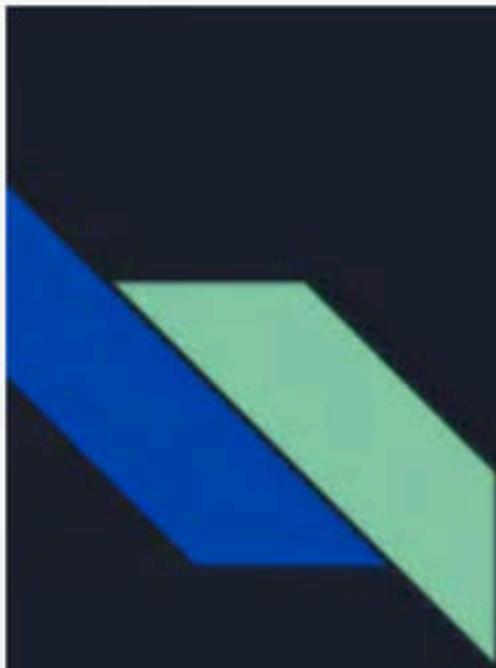
T = cdba



N = 4

S = abcd

T = cdba



N = 4

S = abcd

T = cdba

```
bool solve(string S, string T, int N) {
    set<string> mySet;
    for (int L1 = 0; L1 < N; L1++) {
        for (int R1 = L1; R1 < N; R1++) {
            for (int L2 = 0; L2 < N; L2++) {
                for (int R2 = L2; R2 < N; R2++) {
                    string tempS = S;
                    reverse(tempS.begin() + L1, tempS.begin() + R1 + 1);
                    reverse(tempS.begin() + L2, tempS.begin() + R2 + 1);
                    mySet.insert(tempS);
                }
            }
        }
    }

    for (int L3 = 0; L3 < N; L3++) {
        for (int R3 = L3; R3 < N; R3++) {
            for (int L4 = 0; L4 < N; L4++) {
                for (int R4 = L4; R4 < N; R4++) {
                    string tempT = T;
                    reverse(tempT.begin() + L3, tempT.begin() + R3 + 1);
                    reverse(tempT.begin() + L4, tempT.begin() + R4 + 1);
                    if (mySet.find(tempT) != mySet.end()) {
                        return true;
                    }
                }
            }
        }
    }
    return false;
}
```

| < f +

right

```
bool solve(string S, string T, int N) {
    set<string> mySet;
    for (int L1 = 0; L1 < N; L1++) {
        for (int R1 = L1; R1 < N; R1++) {
            for (int L2 = 0; L2 < N; L2++) {
                for (int R2 = L2; R2 < N; R2++) {
                    string tempS = S;
                    reverse(tempS.begin() + L1, tempS.begin() + R1 + 1);
                    reverse(tempS.begin() + L2, tempS.begin() + R2 + 1);
                    mySet.insert(tempS);
                }
            }
        }
    }

    for (int L3 = 0; L3 < N; L3++) {
        for (int R3 = L3; R3 < N; R3++) {
            for (int L4 = 0; L4 < N; L4++) {
                for (int R4 = L4; R4 < N; R4++) {
                    string tempT = T;
                    reverse(tempT.begin() + L3, tempT.begin() + R3 + 1);
                    reverse(tempT.begin() + L4, tempT.begin() + R4 + 1);
                    if (mySet.find(tempT) != mySet.end()) {
                        return true;
                    }
                }
            }
        }
    }
}

return false;
}
```

```
bool solve(string S, string T, int N) {
    set<string> mySet;
    for (int L1 = 0; L1 < N; L1++) {
        for (int R1 = L1; R1 < N; R1++) {
            for (int L2 = 0; L2 < N; L2++) {
                for (int R2 = L2; R2 < N; R2++) {
                    string tempS = S;
                    reverse(tempS.begin() + L1, tempS.begin() + R1 + 1);
                    reverse(tempS.begin() + L2, tempS.begin() + R2 + 1);
                    mySet.insert(tempS);
                }
            }
        }
    }

    for (int L3 = 0; L3 < N; L3++) {
        for (int R3 = L3; R3 < N; R3++) {
            for (int L4 = 0; L4 < N; L4++) {
                for (int R4 = L4; R4 < N; R4++) {
                    string tempT = T;
                    reverse(tempT.begin() + L3, tempT.begin() + R3 + 1);
                    reverse(tempT.begin() + L4, tempT.begin() + R4 + 1);
                    if (mySet.find(tempT) != mySet.end()) {
                        return true;
                    }
                }
            }
        }
    }
    return false;
}
```

5

L1 R1

L2 R2

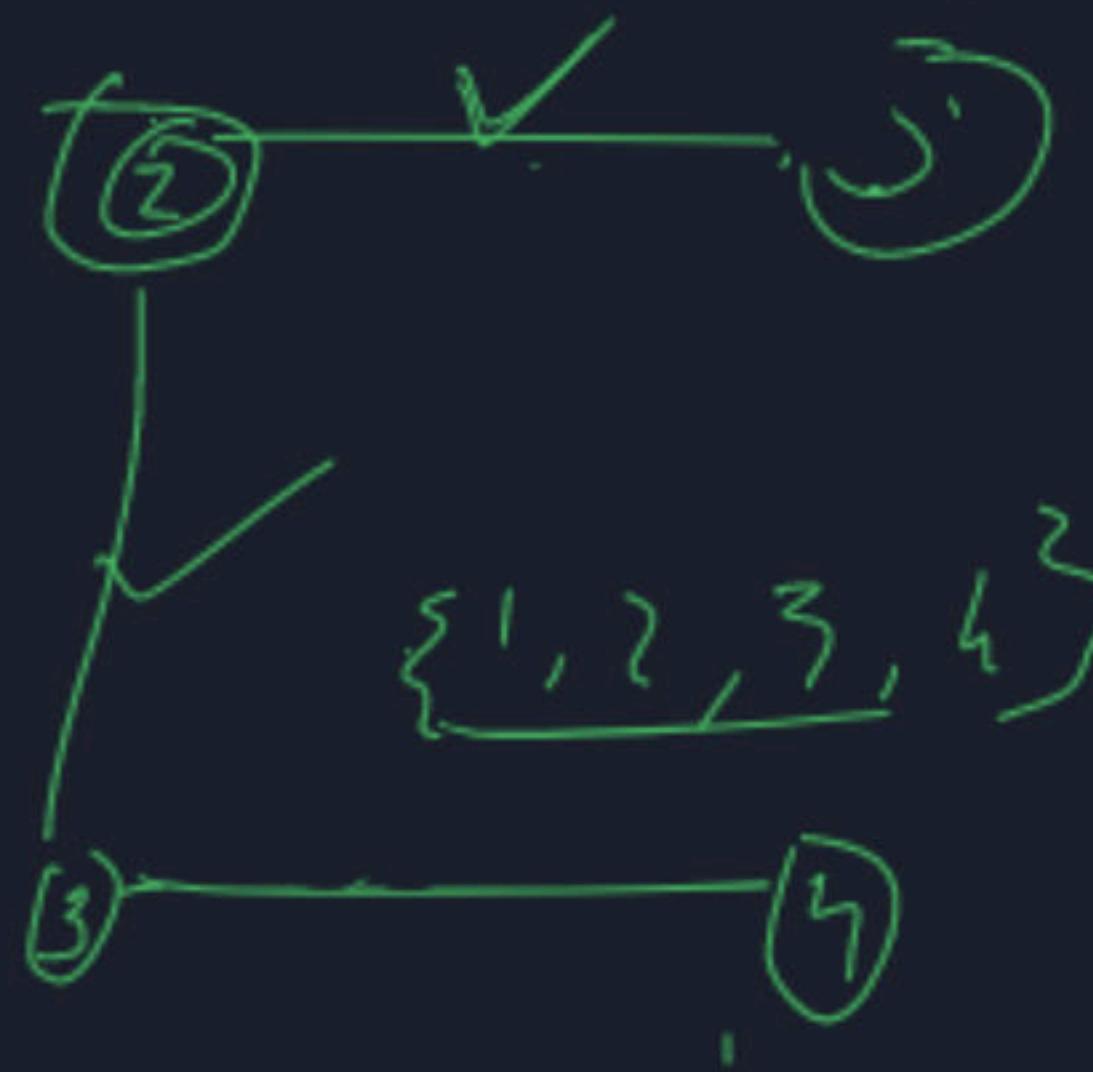
20<sup>5</sup>  
32  
→ YUV<sup>b</sup> VS  
1(N<sup>5</sup>) logN W X  
Assume the time-complexity of  
set.insert and set.find are  
 $O(N^* \log N)$ , what will the be the  
time complexity of solution?

- A.  $O(N^4)$
- B.  $O(N^4 * \log N)$
- C.  $O(N^5)$
- D.  $O(N^5 * \log N)$

```
bool solve(string S, string T, int N) {  
    set<string> mySet;  
    for (int L1 = 0; L1 < N; L1++) { O(N)  
        for (int R1 = L1; R1 < N; R1++) { O(N)  
            for (int L2 = 0; L2 < N; L2++) { O(N)  
                for (int R2 = L2; R2 < N; R2++) { O(N)  
                    string tempS = S;  
                    reverse(tempS.begin() + L1, tempS.begin() + R1 + 1);  
                    reverse(tempS.begin() + L2, tempS.begin() + R2 + 1);  
                    mySet.insert(tempS);  
                } O(N log N)  
            }  
        }  
    } O(N^4)  
    for (int L3 = 0; L3 < N; L3++) {  
        for (int R3 = L3; R3 < N; R3++) {  
            for (int L4 = 0; L4 < N; L4++) {  
                for (int R4 = L4; R4 < N; R4++) {  
                    string tempT = T;  
                    reverse(tempT.begin() + L3, tempT.begin() + R3 + 1);  
                    reverse(tempT.begin() + L4, tempT.begin() + R4 + 1);  
                    if (mySet.find(tempT) != mySet.end()) {  
                        return true;  
                    }  
                }  
            }  
        }  
    }  
    return false;  
}
```

$O(N^4)$

$\{1, 2\}$  Minimum vertex cover  $\Rightarrow$   $\{\underline{2, 3}\} \Leftrightarrow 2$   
vertex cover of size 2



A set of vertices is called  
as a vertex cover if and  
only if for each edge  
at least one endpoint  
lies in the set

$\{\underline{2, 3}\} \Rightarrow$  vertex cover  $\Rightarrow 2$









You are given an undirected graph with **N** vertices and **M** edges.  
Find the size of the minimum vertex cover of the graph.

**Constraints:**

$$3 \leq N \leq 30$$

$$1 \leq M \leq 50$$

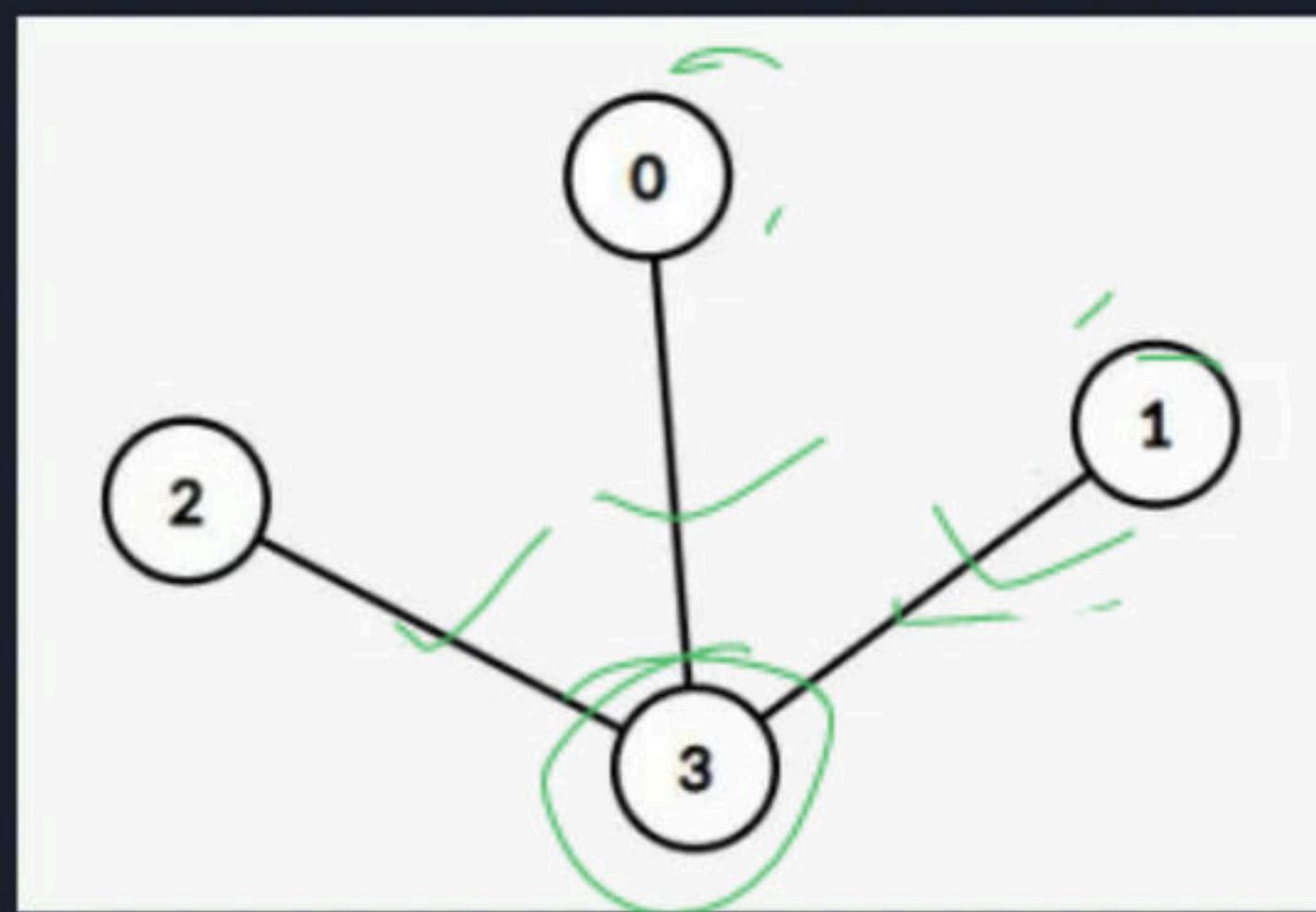
↑  
Vertex cover set of vertices is called as  
vertex cover if and only if for each edge  
at least one endpoint lies in the set  
Min vertex cover vertex cover of minimum size

$$N = \underline{4}$$
$$M = \underline{3}$$

$\{0, 1, 2\} \Rightarrow$  vertex cover

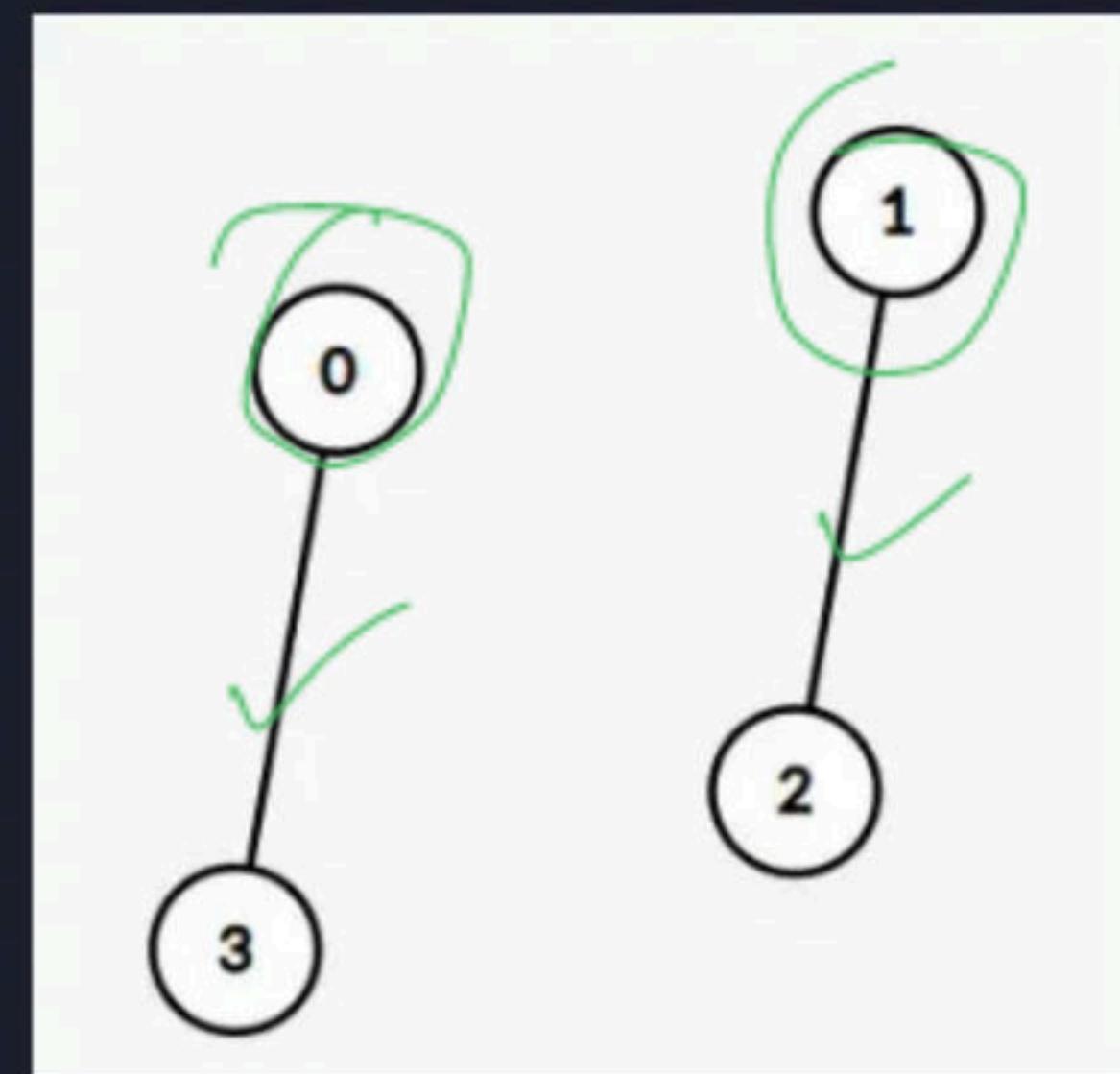
$\Rightarrow 3$

$\overrightarrow{\{2\}} \rightarrow \{3\} \Rightarrow 1$



$N = 4$   
 $M = 2$

$\{0, 1\} \Rightarrow \{2\}$



N = 6  
M = 6

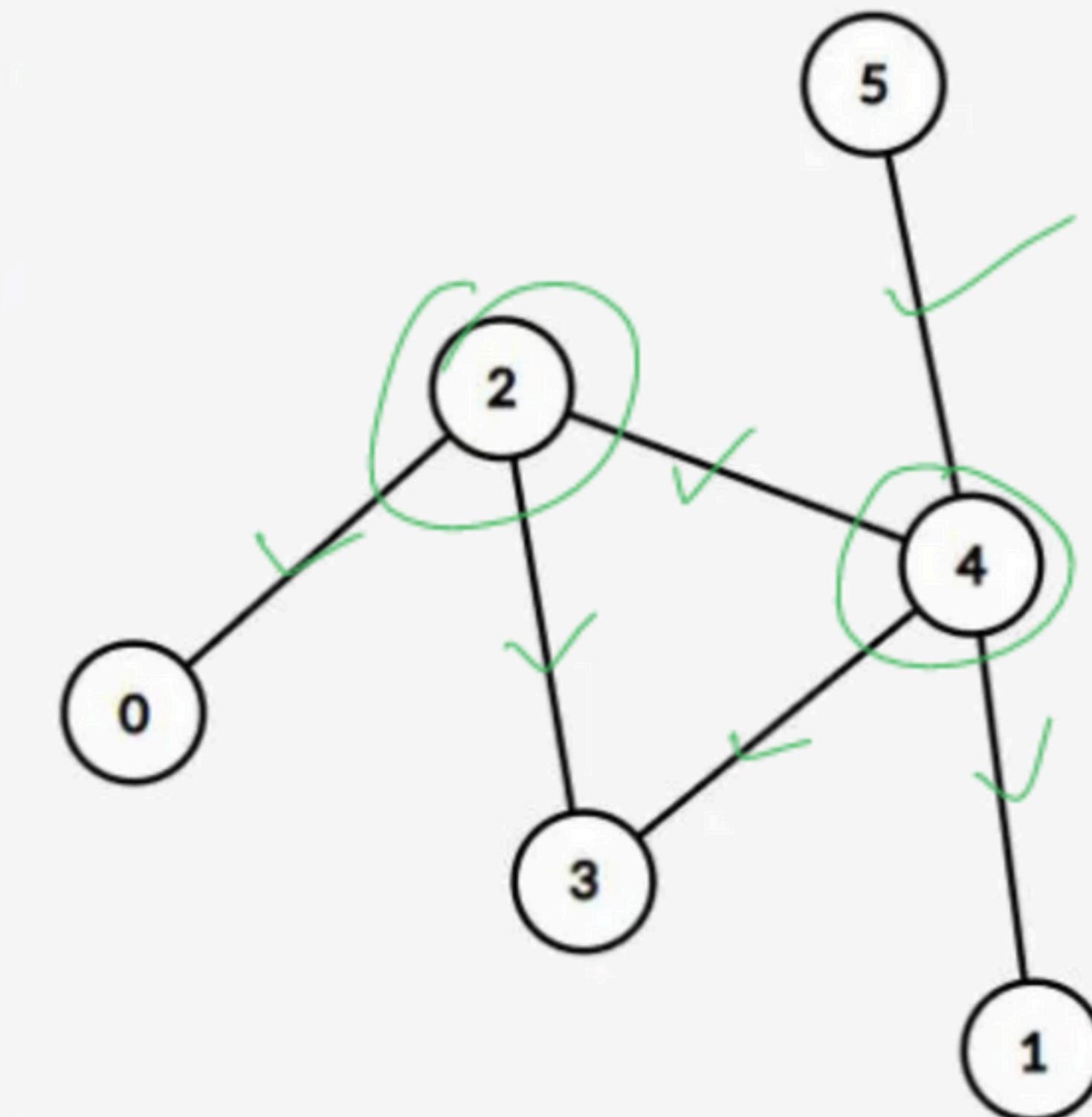
What will be the answer?

A. 1

B. 2

C. 3

D. 4



$N = 6$

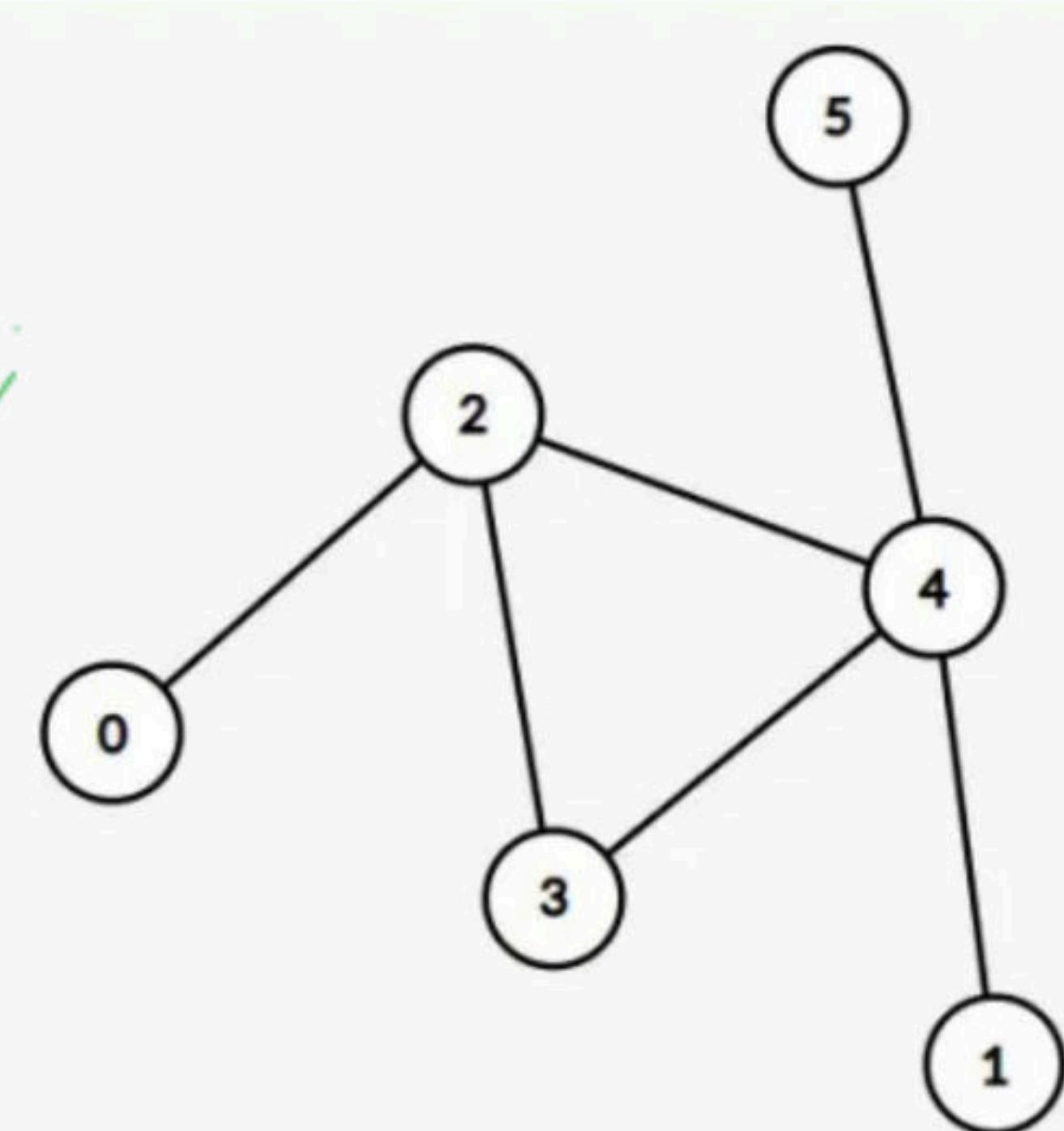
$M = 6$

$\text{res} = \text{inL}$

for (each vertex cover  $V_c$  of graph) {

$\text{res} = \min(\text{res}, |V_c|)$

}



$$N = 6$$
$$M = 6$$

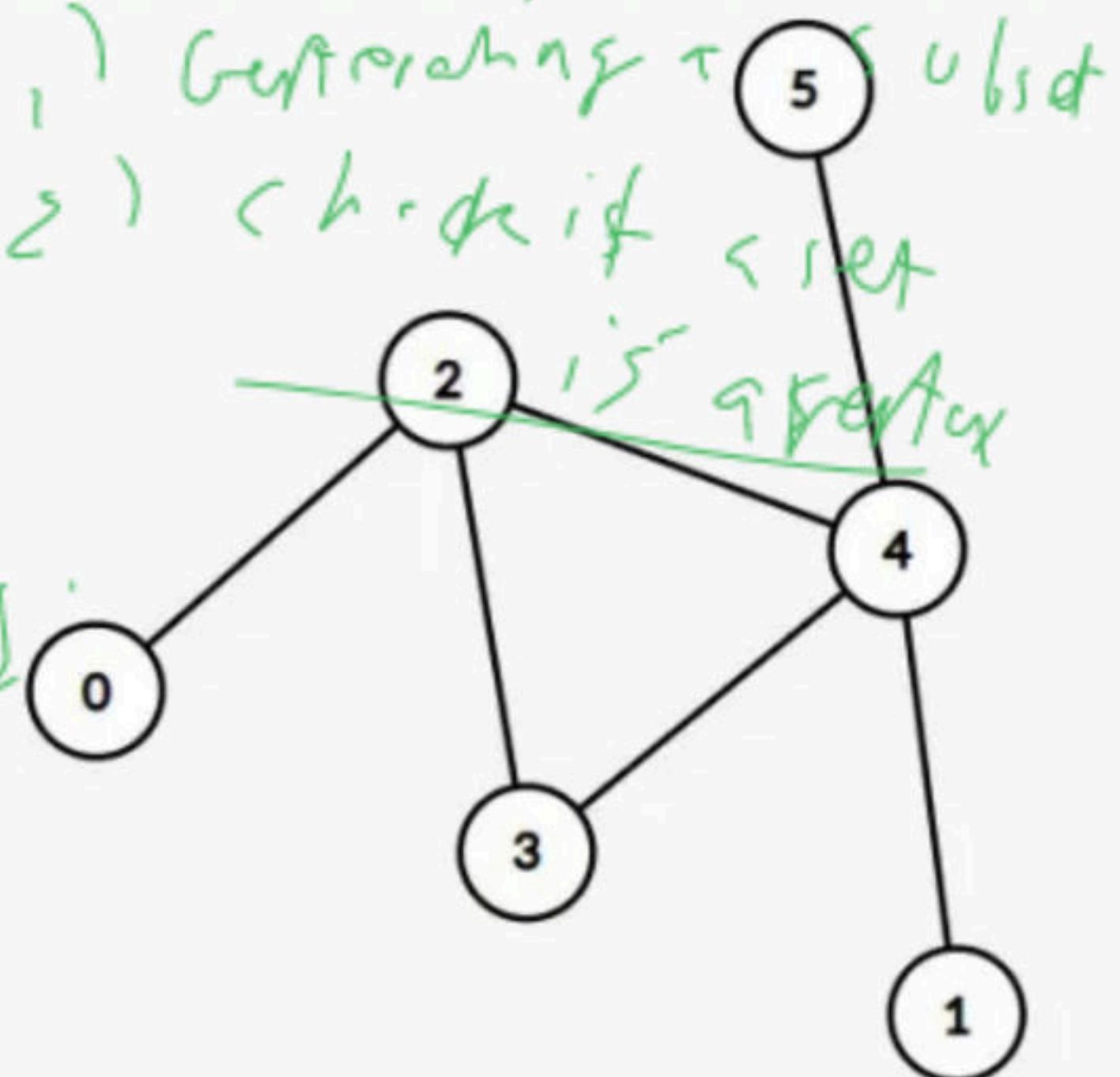
$S = \{0, 1, 2, \dots, N-1\}$

$\{0, 1, 4, 5\}$

$\{0, 2\}$

```
res = inf
for (each subset S, v ∉ S) {
    if (S1 - i is a vertex cover) {
        res = min(res, S1.size)
    }
}
```

- 1) Generating  $\tau^S$  subset
- 2) check if  $S$  set is a vertex



$$N = 6$$

$$M = 6$$

$$S = \{2, 1, 0\} \xrightarrow{\text{underline}}$$

{}

{0}

{1}

{1, 0}

{2}

{2, 0}

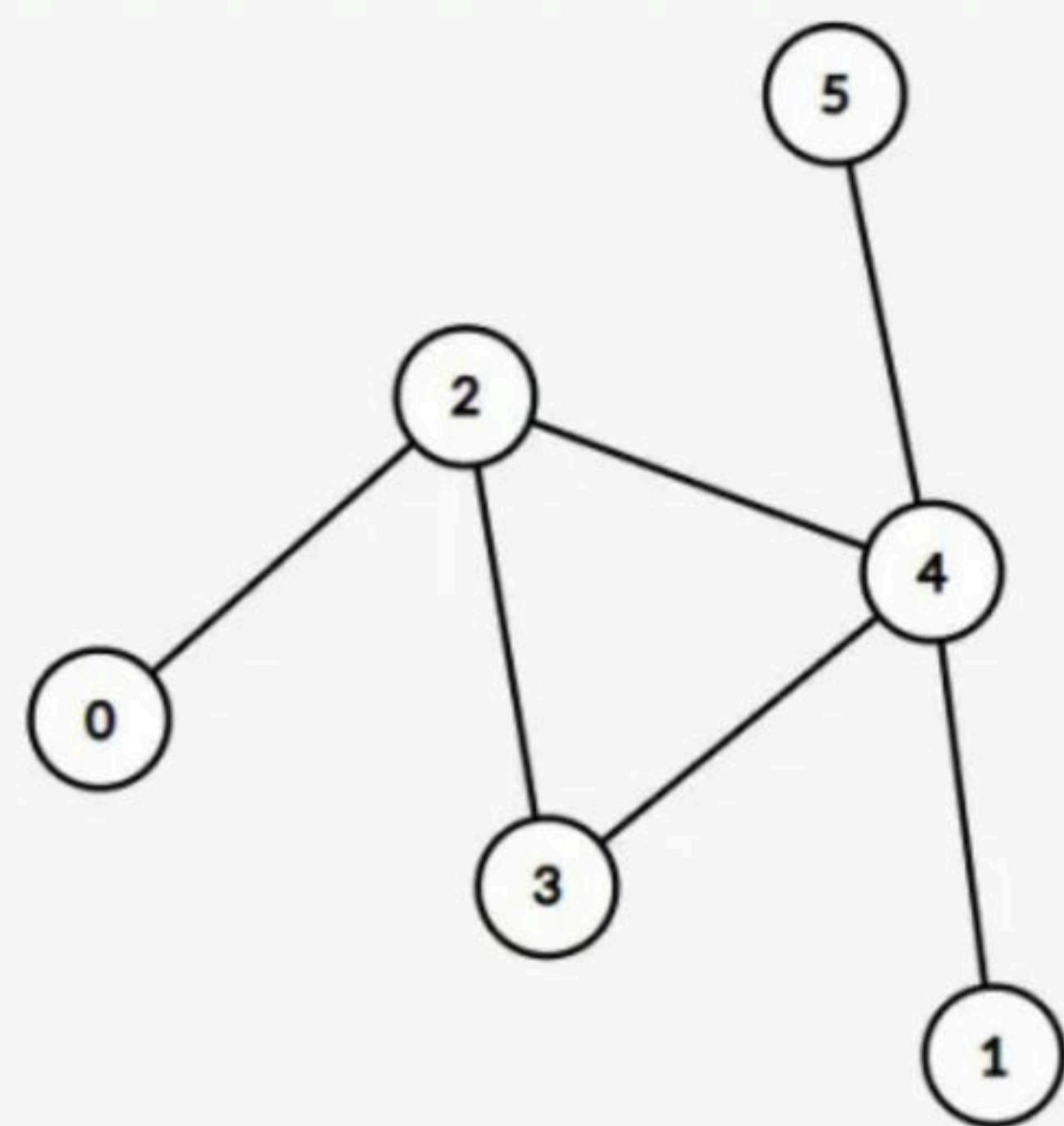
{2, 1, 0}

$$S = \left\{ \begin{array}{c} 0, \\ \diagup \\ 1 \\ \diagdown \\ 2 \end{array}, \begin{array}{c} 1, \\ \diagup \\ 2 \\ \diagdown \\ 2 \end{array}, \begin{array}{c} 3 \\ \diagup \\ 2 \\ \diagdown \\ 2 \end{array}, \begin{array}{c} 3, \\ \diagup \\ 2 \\ \diagdown \\ 2 \end{array}, \begin{array}{c} 4 \\ \diagup \\ 2 \\ \diagdown \\ 2 \end{array} \end{array} \right\}$$

(2)

10

{2, 13}



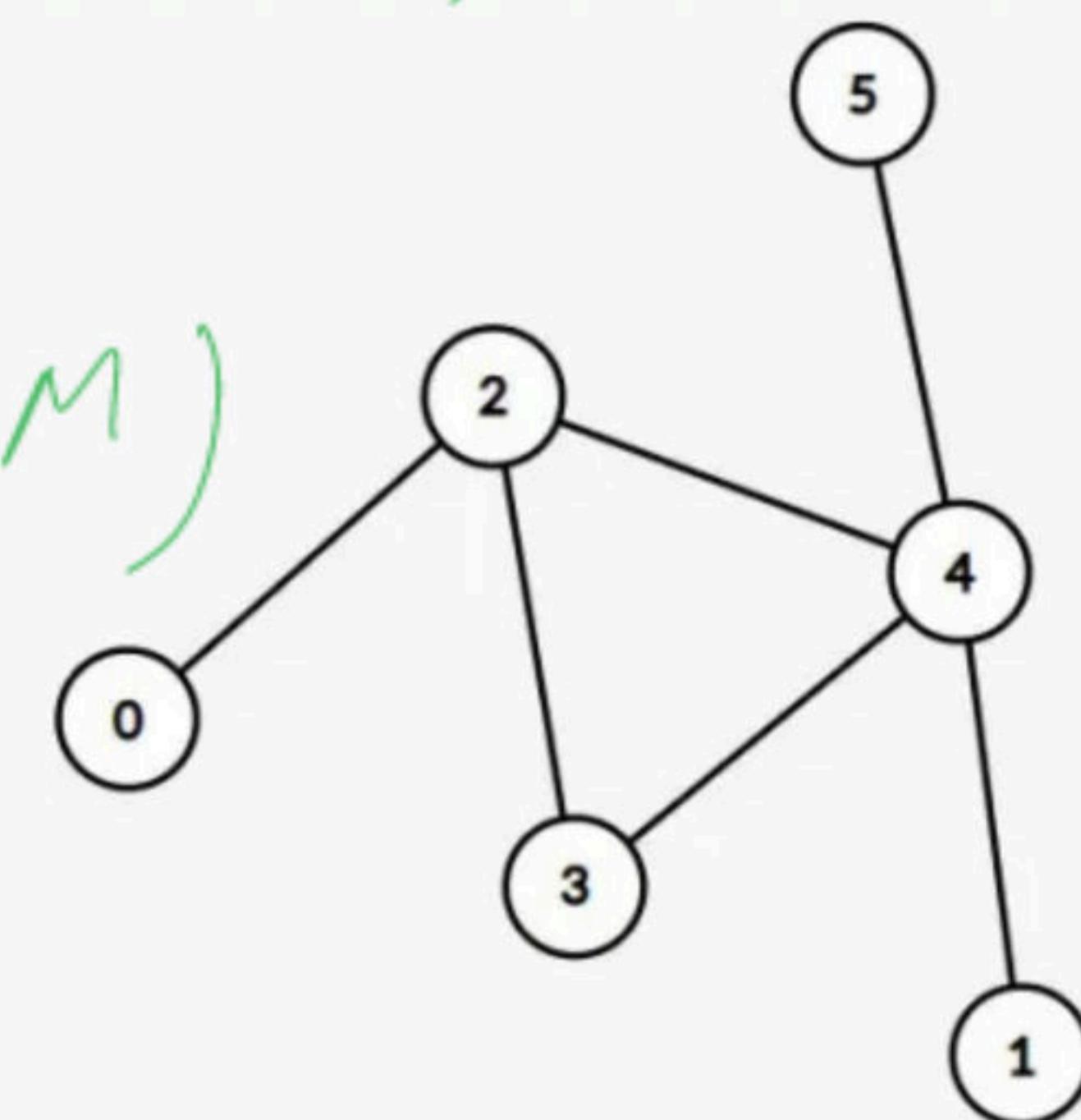
$$N = 6$$
$$M = 6$$

$$\mathcal{O}(2^h \times M)$$

for (mask=0; mask <  $2^h$ ; mask++) {

$S_1 = \{\text{set bits in mask}\}$   
    if ( $S_1$  is a vertex or cover)  $\rightarrow \mathcal{O}(M)$   
    res = min(res,  $S_1 \cdot S_1.size\}$ )

+  
Total  
 $O(2^h)$



$$N = 6$$
$$M = 6$$

$$S = \{0, 1, 4, 5\}$$

$$O(M)$$

$$, (M \times \log N)$$

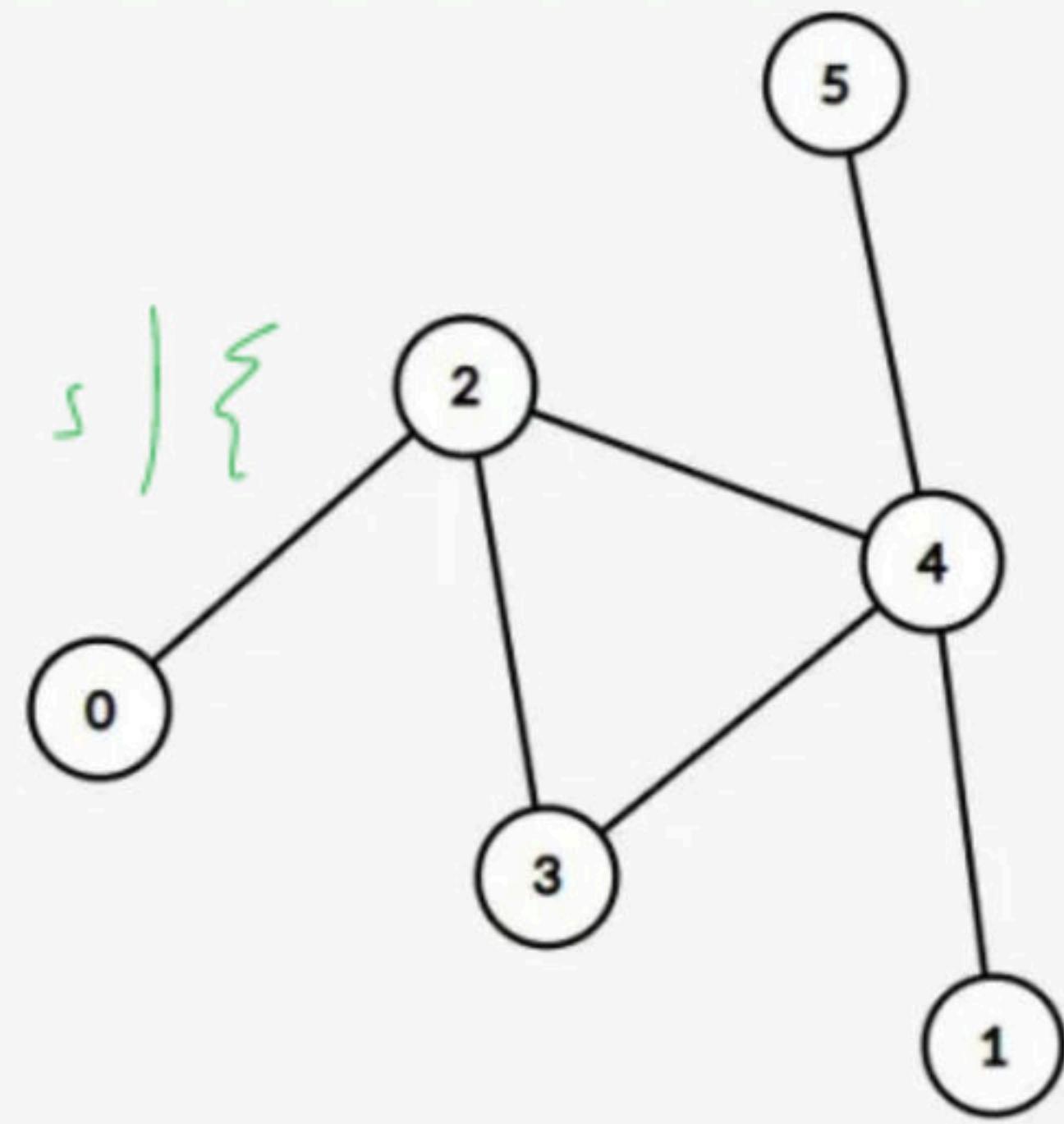
```
for (i=1; i <= M; i++) {
```

$\frac{6}{\text{if } c \text{ is not in } S \text{ & } c \text{ is not in } S \setminus \{}$

return false;

```
}
```

```
return true
```



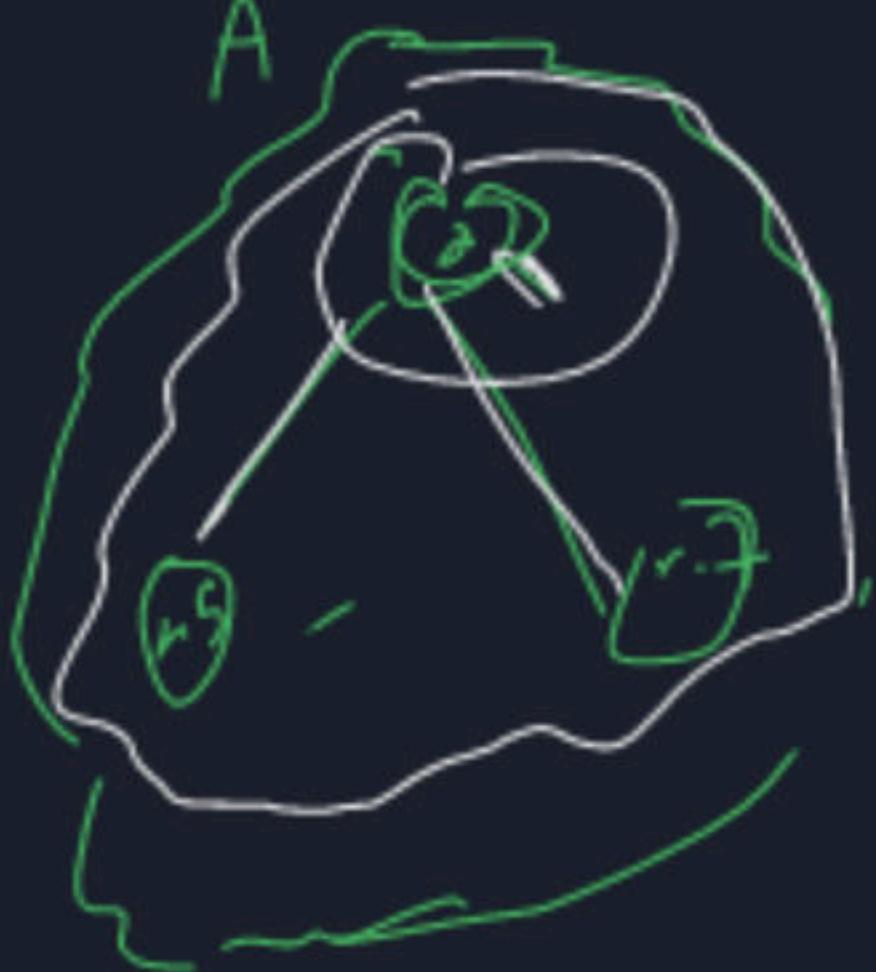
$$N = 6$$

$$M = 6$$

$$S_{Left} = \{1, 2\} \cup \{0\}$$

$$S_{Left} = \{1, 2, 0\}$$

$$\left\{0, 1, 2\right\} < \frac{N}{2}$$



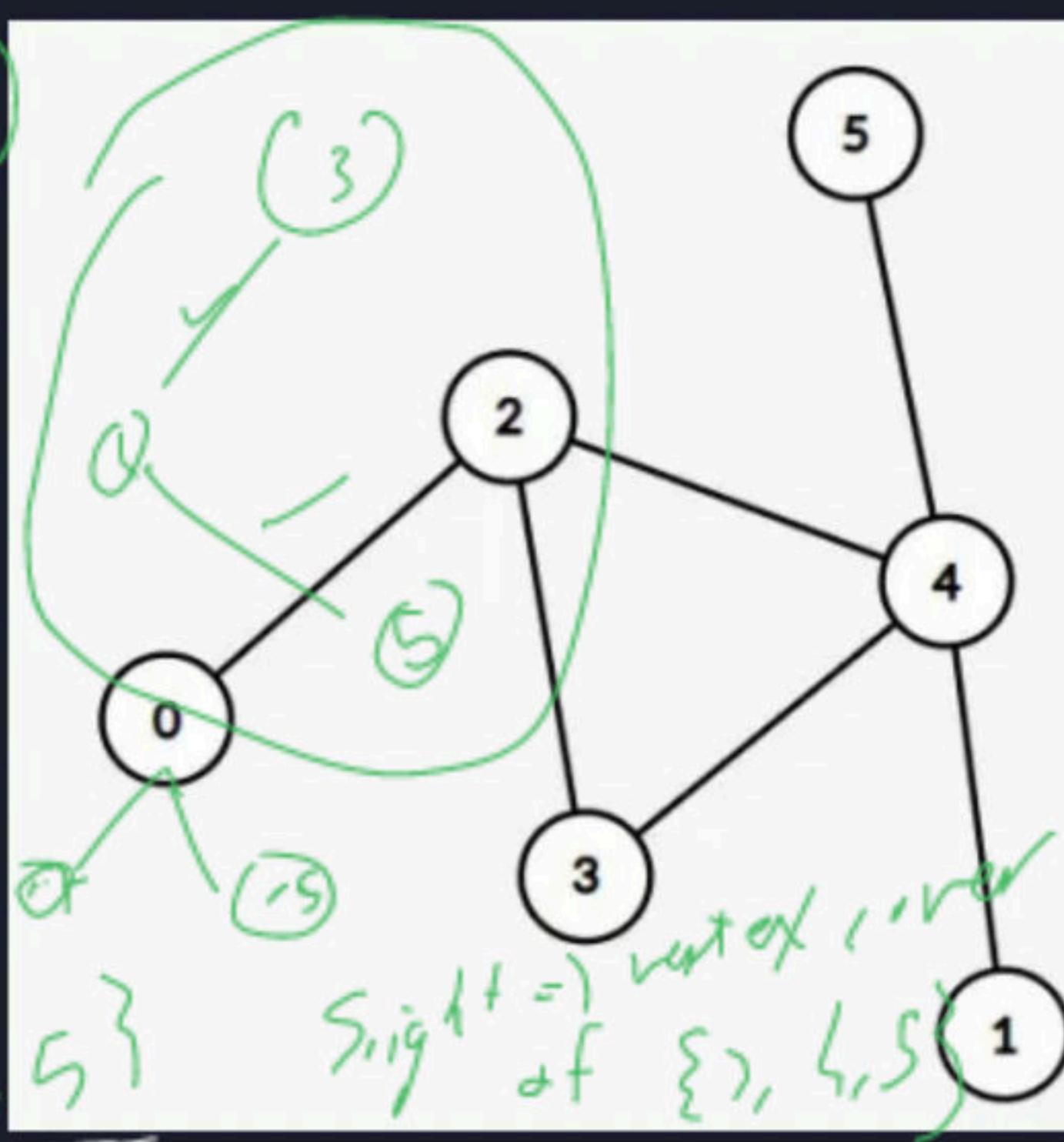
$S_{Left}$

$\{3, 4, 5\}$

$S_{Right} = \{3, 5\}$

$\{3, 4, 5\}$

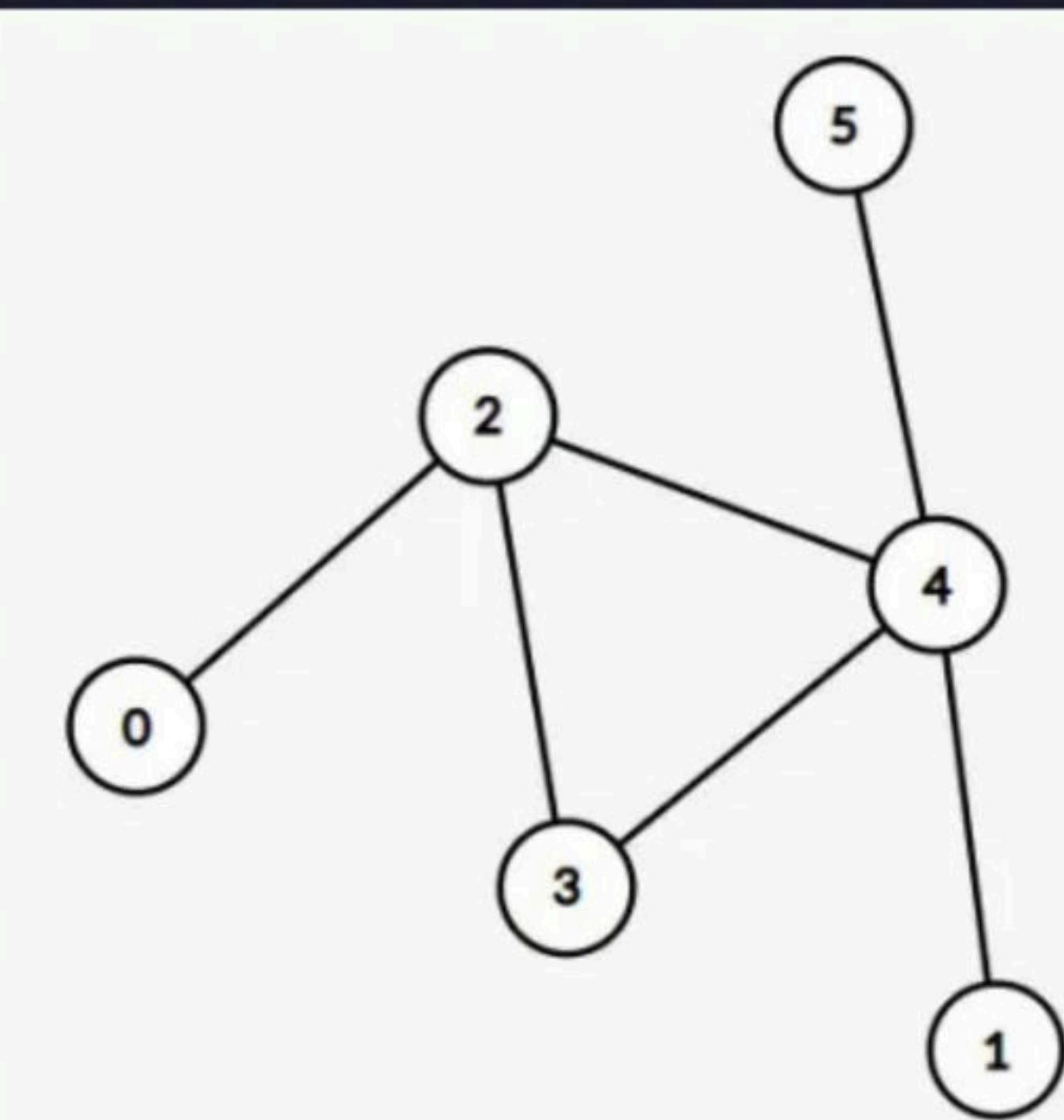
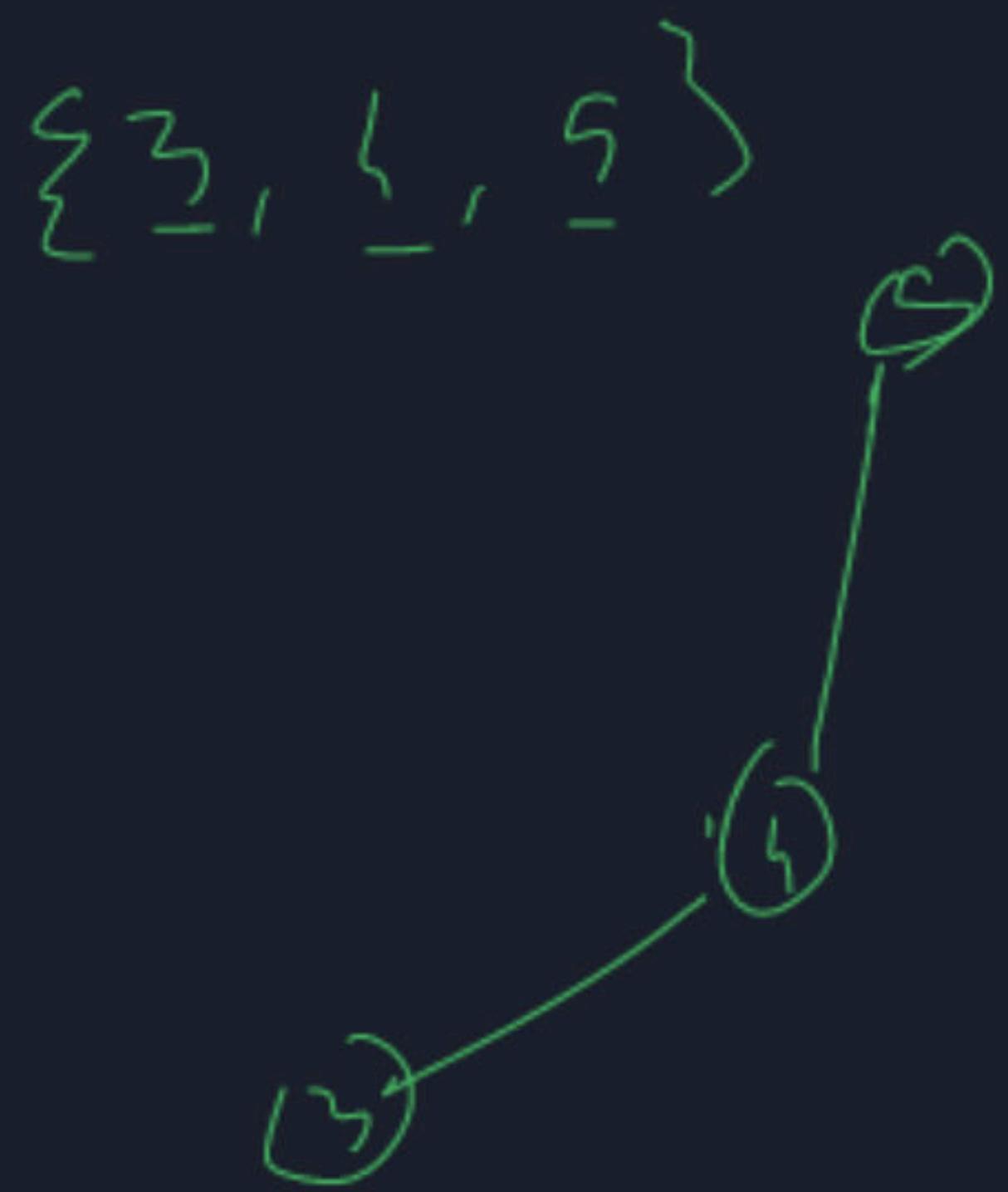
$B$



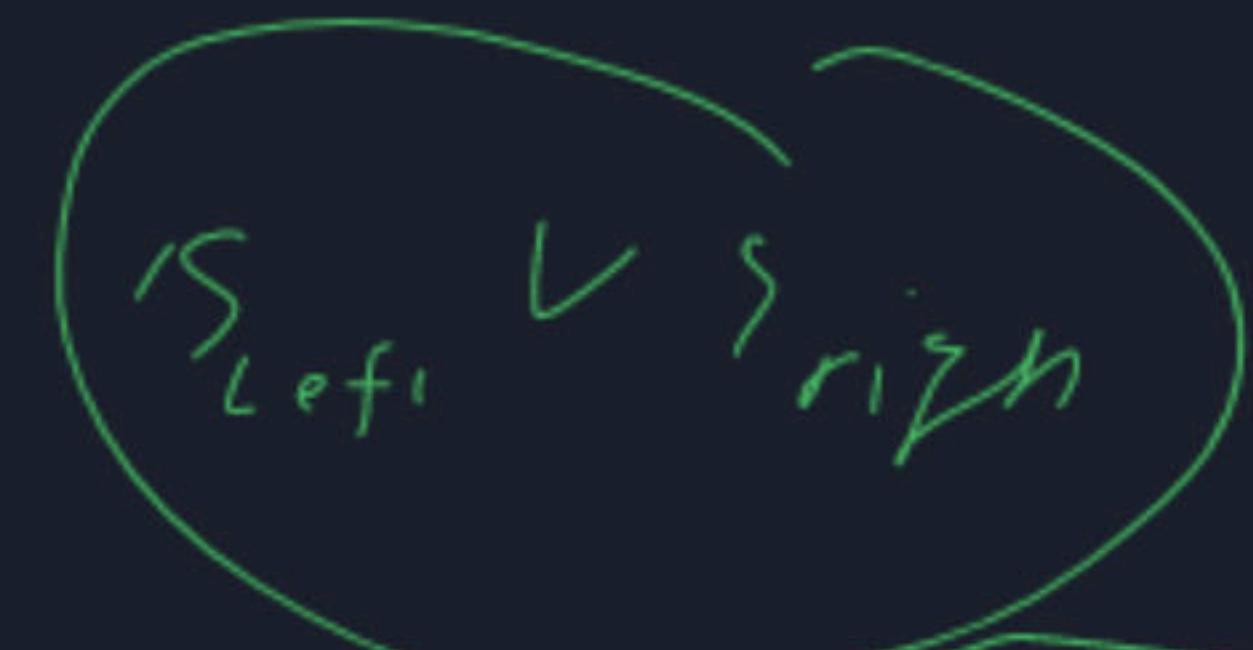
$S_{Right} = \{1, 4, 5\}$

- 1) Edges inside A
- ✓ 2) Edges inside B
- ✓ 3) Edges between A and B

$N = 6$   
 $M = 6$



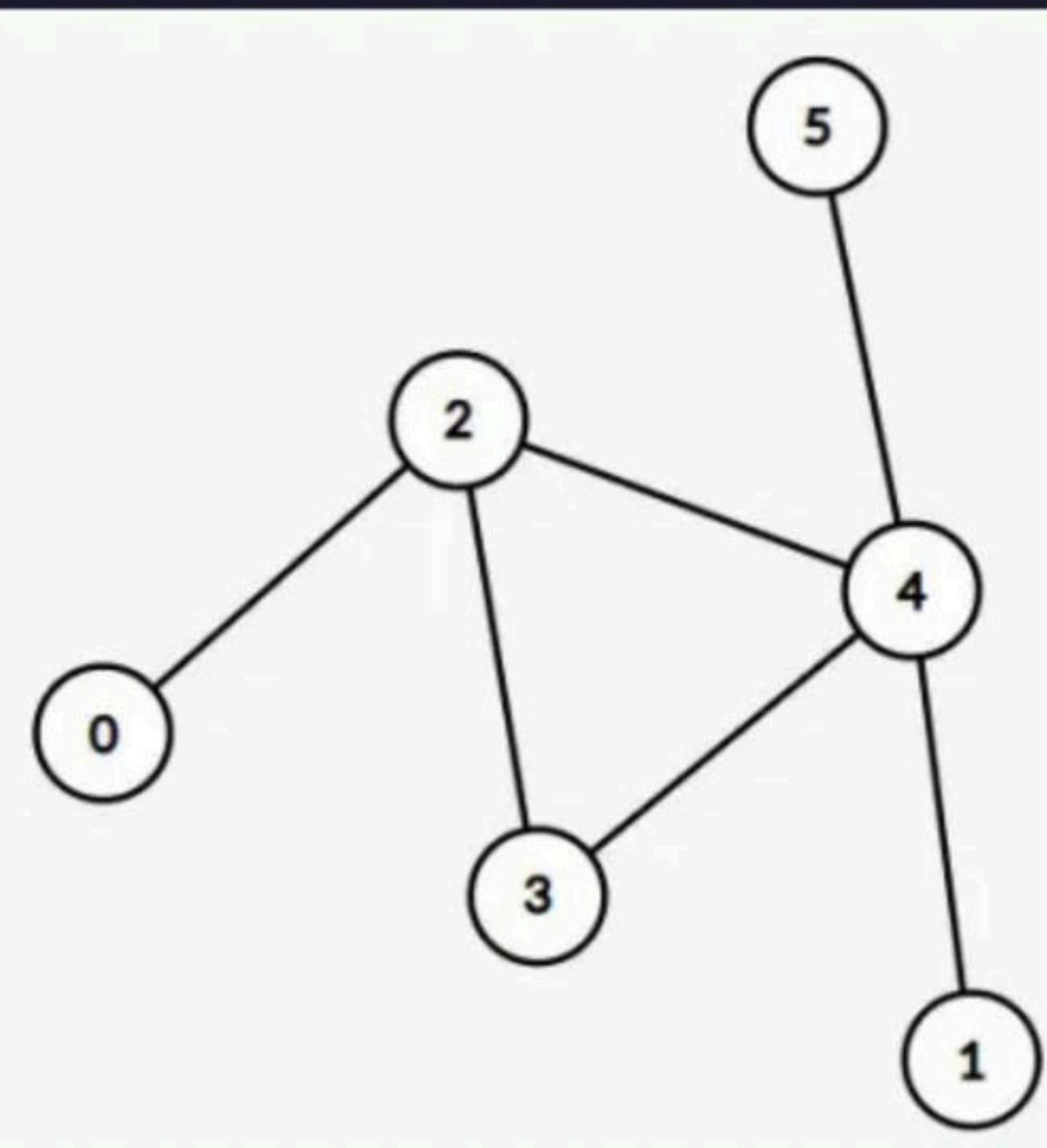
$$N = 6$$
$$M = 6$$



$$S_{left} = \{4, 5\}$$

$$S_{right} = \{5, 6\}$$

$$\{(4, 1), (5, 5)\}$$



$O(3^{\frac{N}{2}} \times m)$

$$N = 6$$

$$M = 6$$



$$V_{13} = \{4\}$$

$$\text{rem}_{13} \{3, 5\}$$

$O(2^{n_{rm}})$

i) Go through all vertex covers  $V_B$  of  $B$

a)  $S_{right} = \{V_3\}$   $\cancel{6X}$

b)  $r_{cM_B} = \cancel{V_B - V_3}$

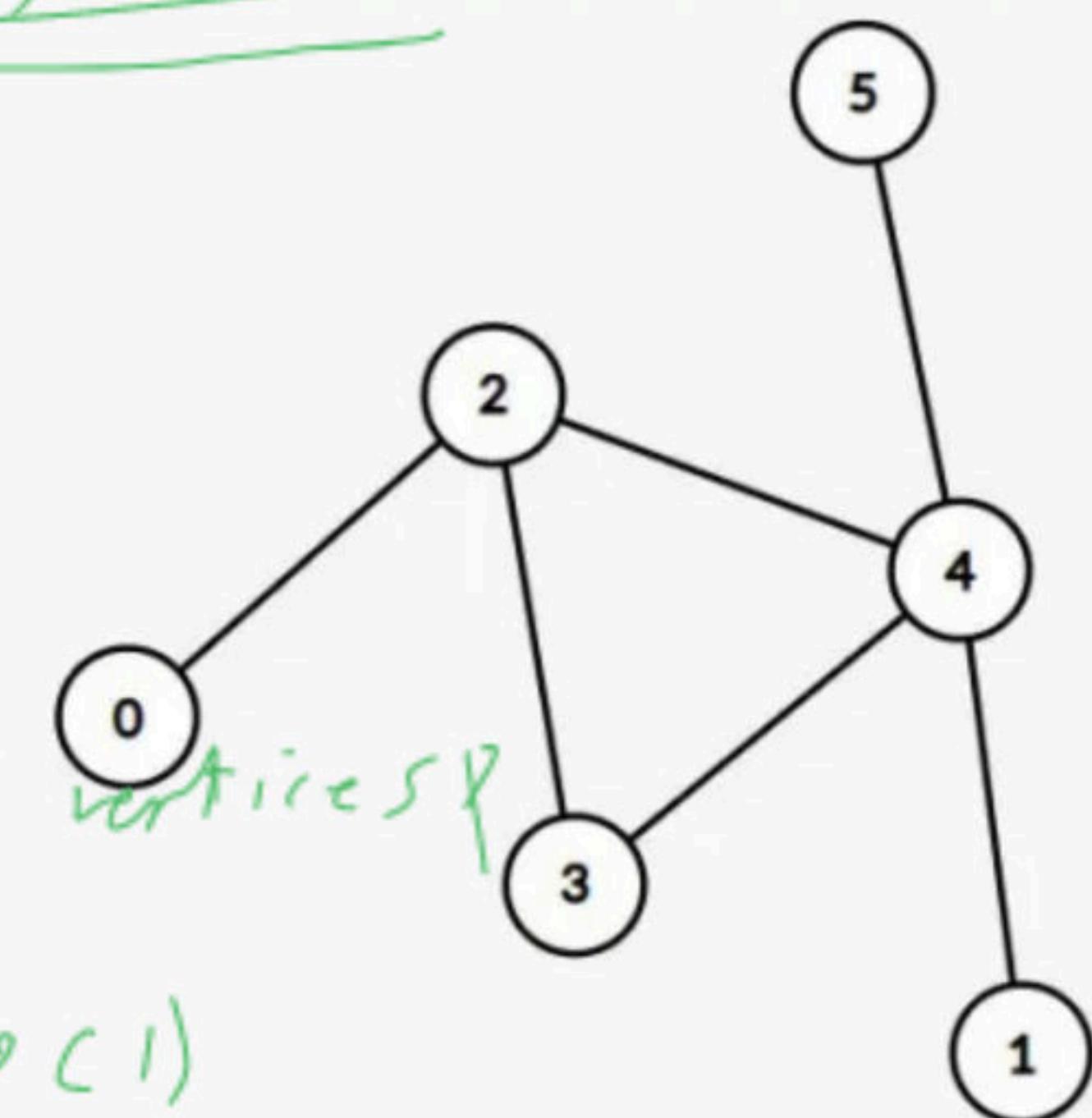
c)  $\text{adj}_A = \cancel{\text{adj}(r_{cM_B})}$

d)  $S_{left} = \{\text{adj}_A\}$   $\pm$  compulsory vertices  $\cancel{S_P}$

e)  $r_{cM_A} = \cancel{A - S_{left}}$

f)  $V_A = \min \text{vertex cover} + \cancel{r_{cM_A}}$

g)  $S_{left} = \{\text{adj}_A\} \cup \min \text{vertex cover}$



$O(1)$

$$N = 6$$

$$M = 6$$

Main problem

Min vertex cover  
but grayed

A

Find min vertex cover  
for all 5 subgraphs  
of A

B

Find all  
vertex covers  
of B

$$\mathcal{O}(3^{N/2} \times M)$$

$$\mathcal{O}(3^{n/2} \times M + 2^{n/2} \times M)$$

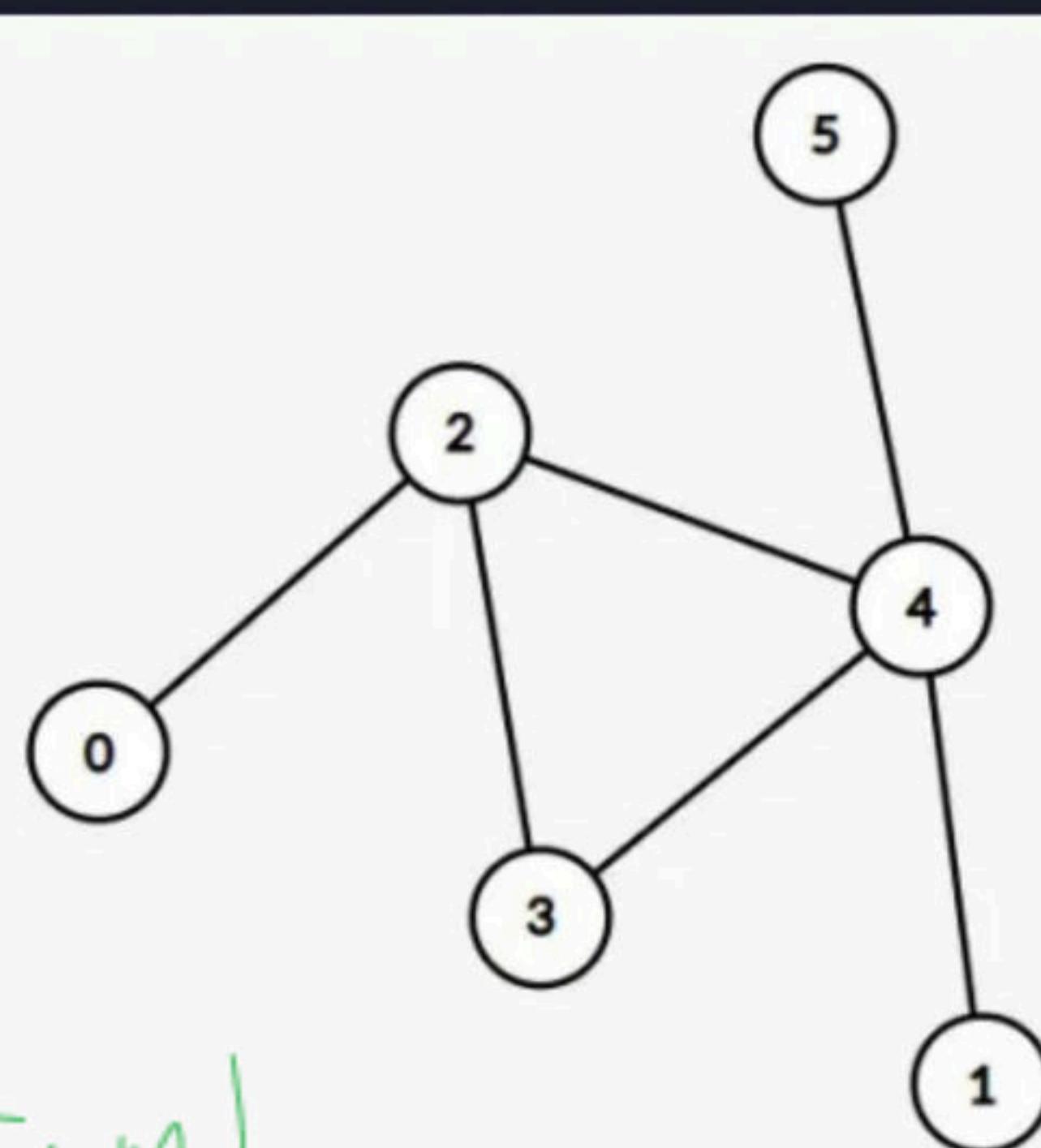
$$\mathcal{O}(2^{n/2} \times M)$$

$$\mathcal{O}(3^{N/2} \times M)$$

$$\{0, 1\}$$

$$\{0, 2\}$$

$$\{1, 2\}$$



$$N = 6$$
$$M = 6$$

B

$$2^{n \times m}$$

$$2^{n/2 \times m}$$

$V_{13} \cup R$ )

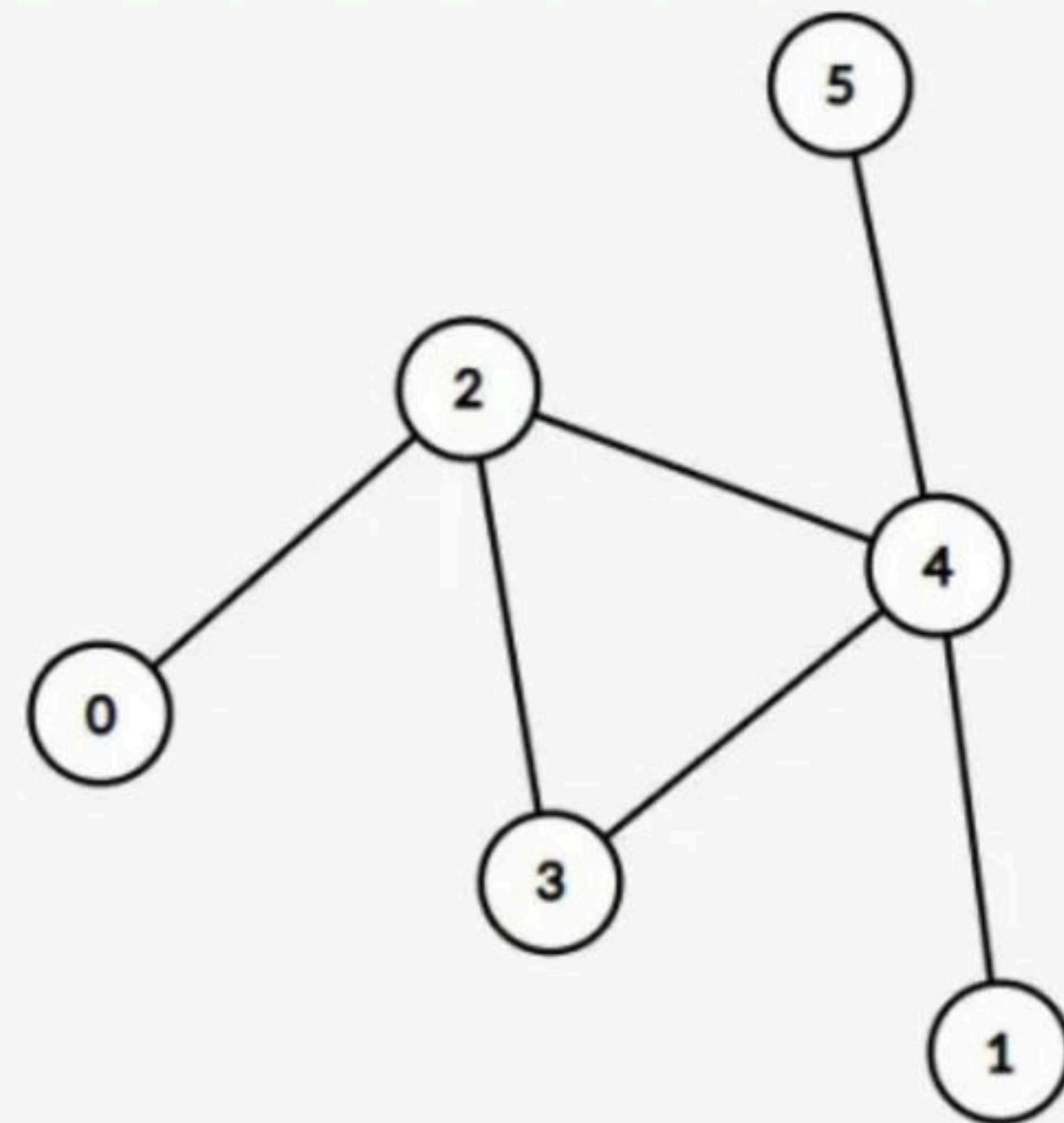
{

for each vertex cover  $V_{13}$  of R

res.append( $(V_{13})$ )

}

for m <



$$2^N + 2^M + 2^M + 2^M + 2^M$$

$$N=6$$

$$M=6$$

$$2^M + 2^M + 2^M$$

$$2^M \times M$$

$$\cancel{2^M \times M}$$

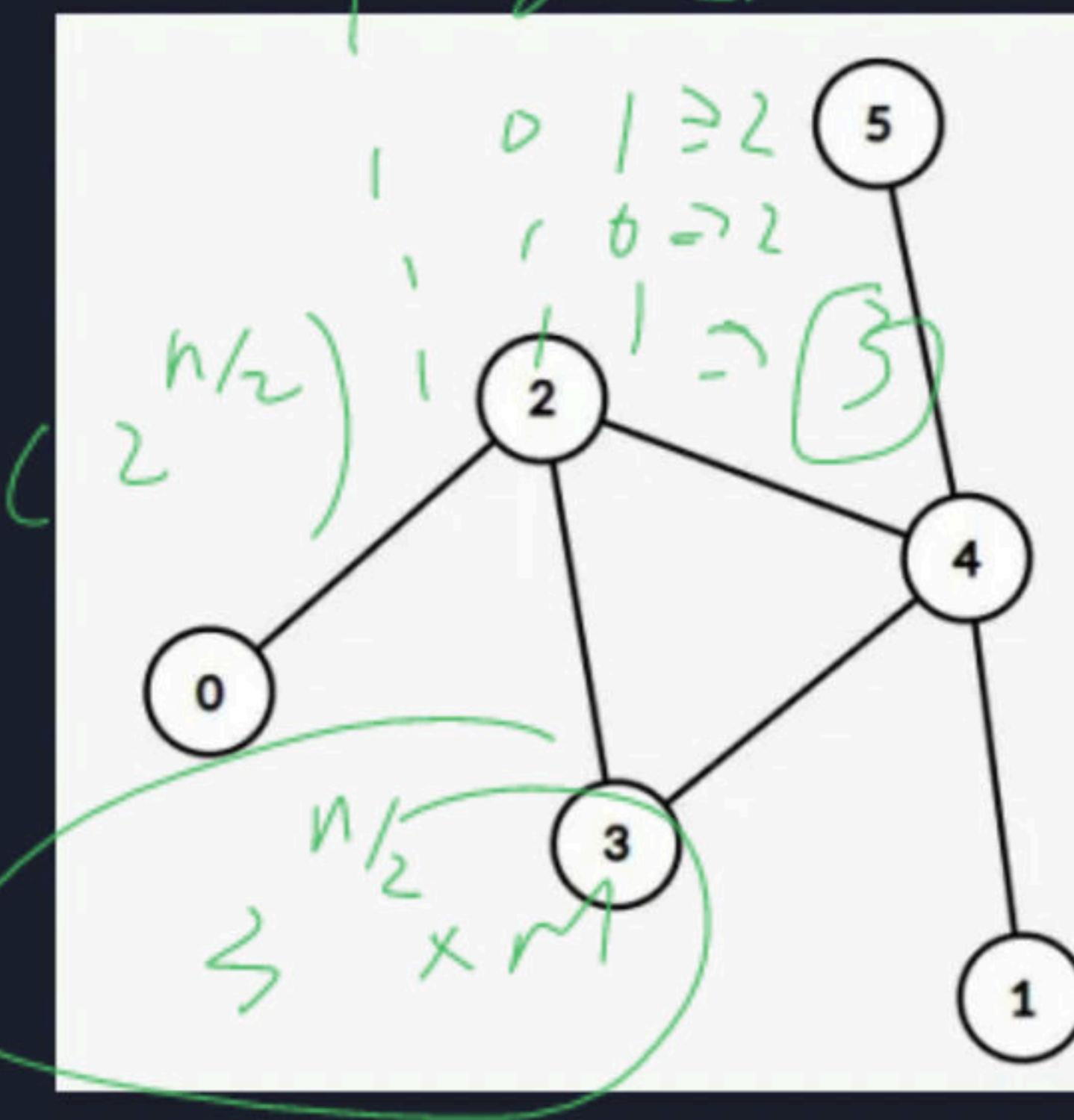
$G$  with  $n$  vertex cover  $\times 2^M$

for (each subgraph  $A'$  of  $A$ ) {  $O(2^{n/2})$

$dp[A'] = \min \text{vertex cover } A'$

}

$000 \Rightarrow 0$   
 $001 \Rightarrow 1$   
 $010 \Rightarrow 1$   
 $011 \Rightarrow 2$   
 $100 \Rightarrow 1$



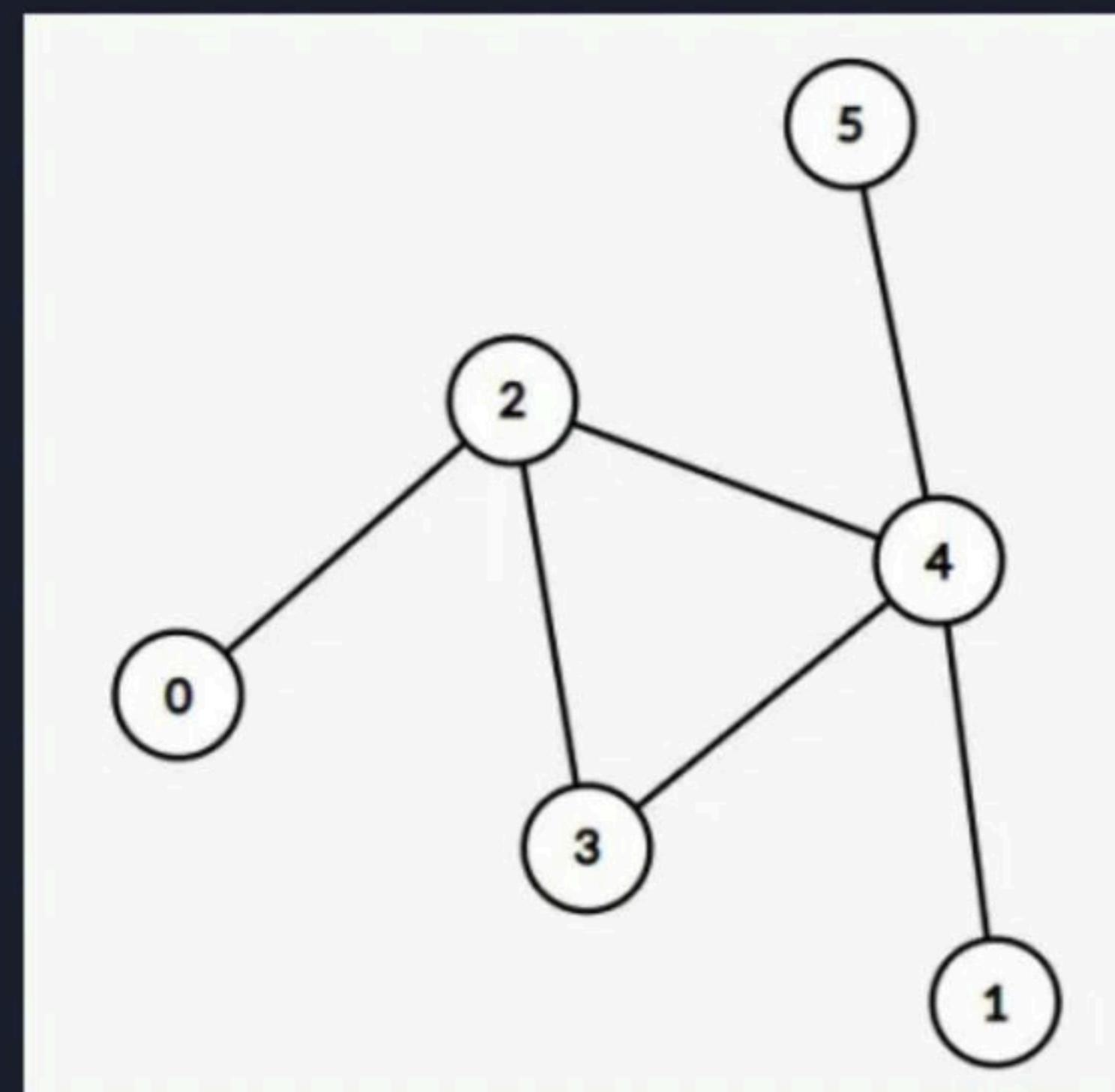
$$N = 6$$
$$M = 6$$

$$n = 3^D$$

$$\left( \sum_{i=0}^{2^k-1} 2^{^{\text{#satellites in } i}} \right) \times M$$

Diagram illustrating a set of 6 nodes (0, 1, 2, 3, 4, 5) connected in a network. Node 0 is connected to node 2. Node 2 is connected to nodes 0, 3, and 4. Node 4 is connected to nodes 2, 3, and 5. Node 5 is connected to node 4. Node 1 is isolated.

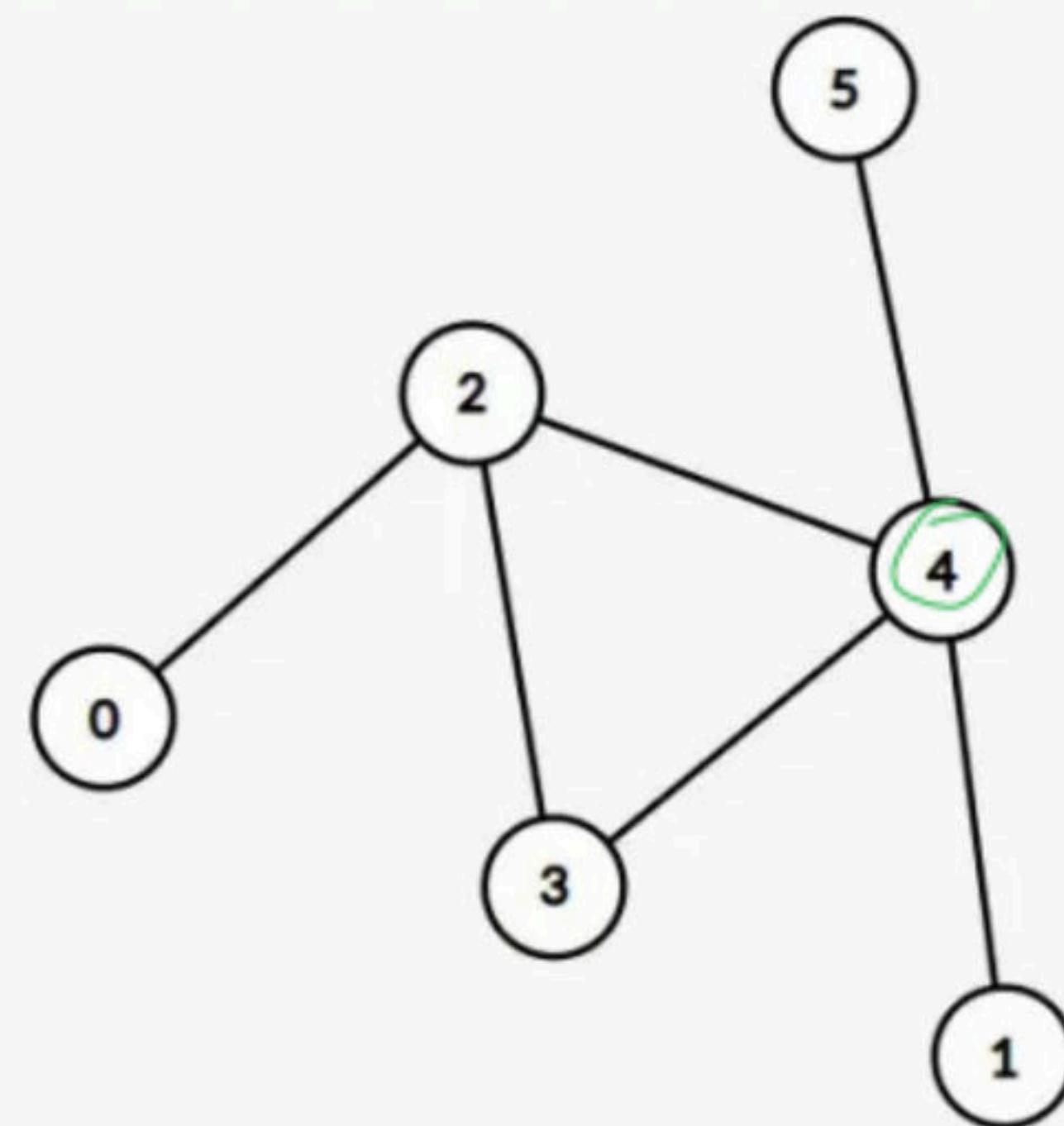
```
graph TD; 0((0)) --- 2((2)); 2 --- 0; 2 --- 3((3)); 2 --- 4((4)); 4 --- 2; 4 --- 3; 4 --- 5((5)); 5 --- 4;
```



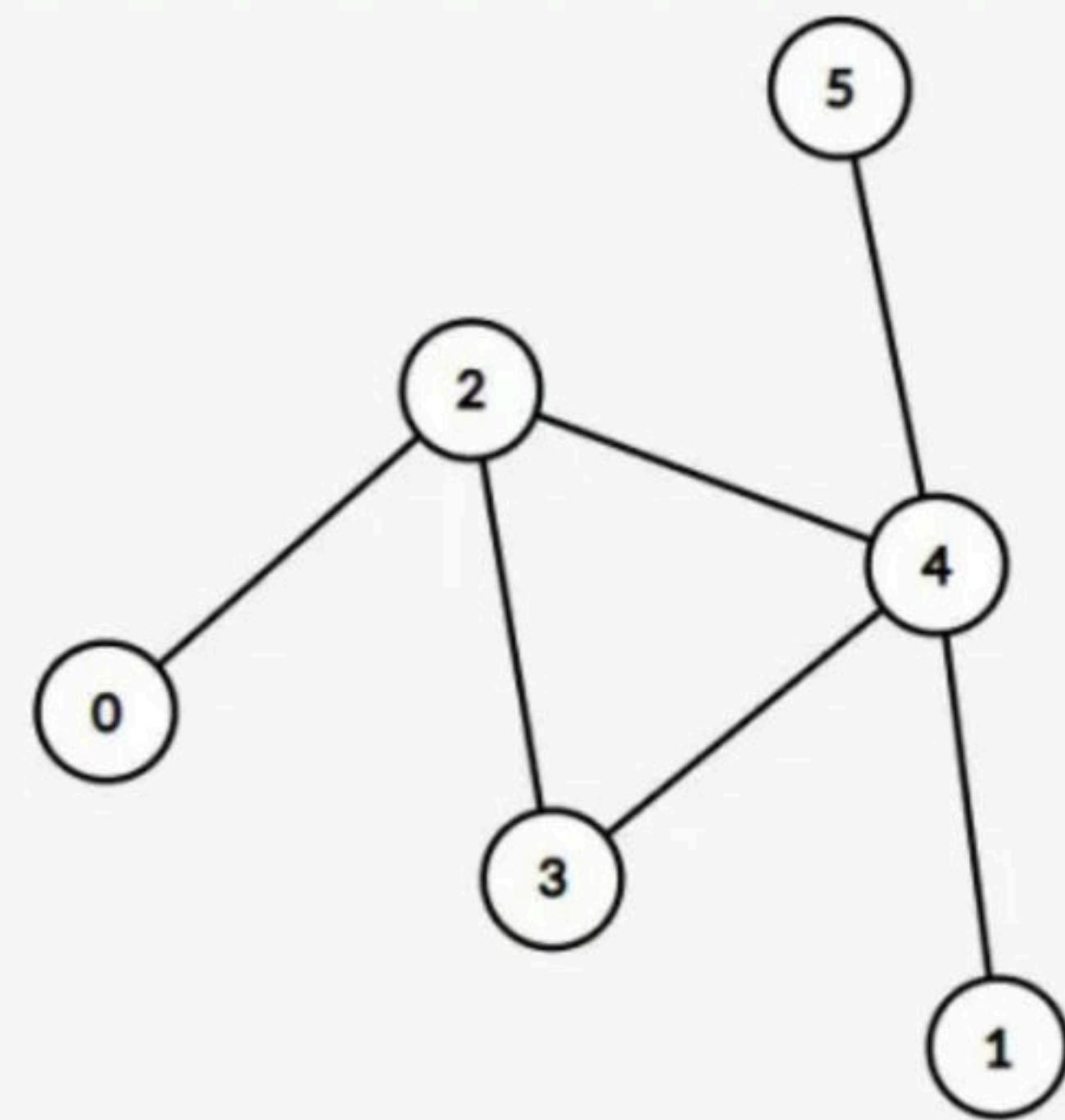
$$\begin{matrix} N = 6 \\ M = 6 \end{matrix}$$

{ 0, 1, 2, 3 }

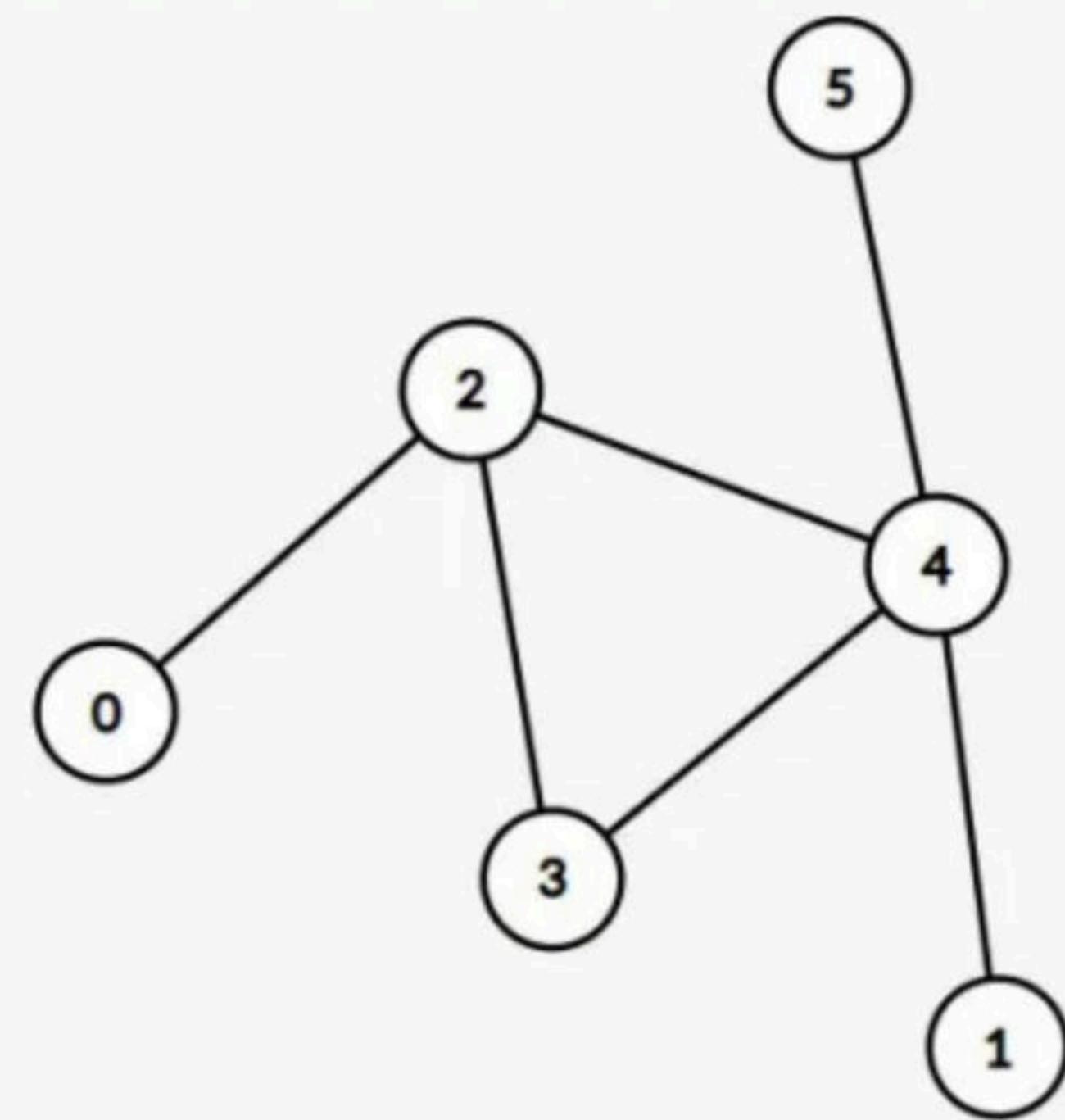
5 1 2 3 4  
| | | |  
0 0 0 0 0  
0 (1)



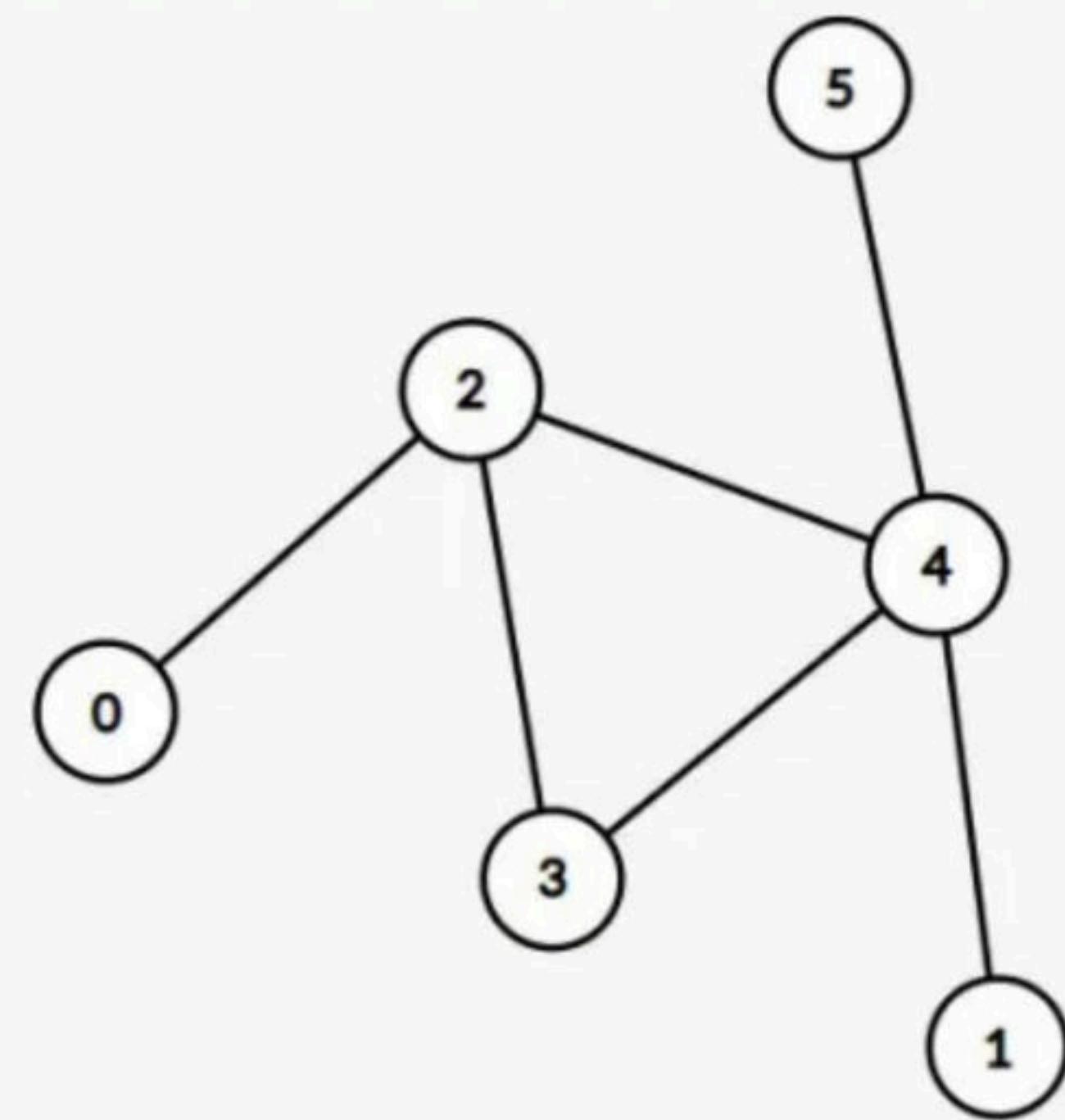
$N = 6$   
 $M = 6$



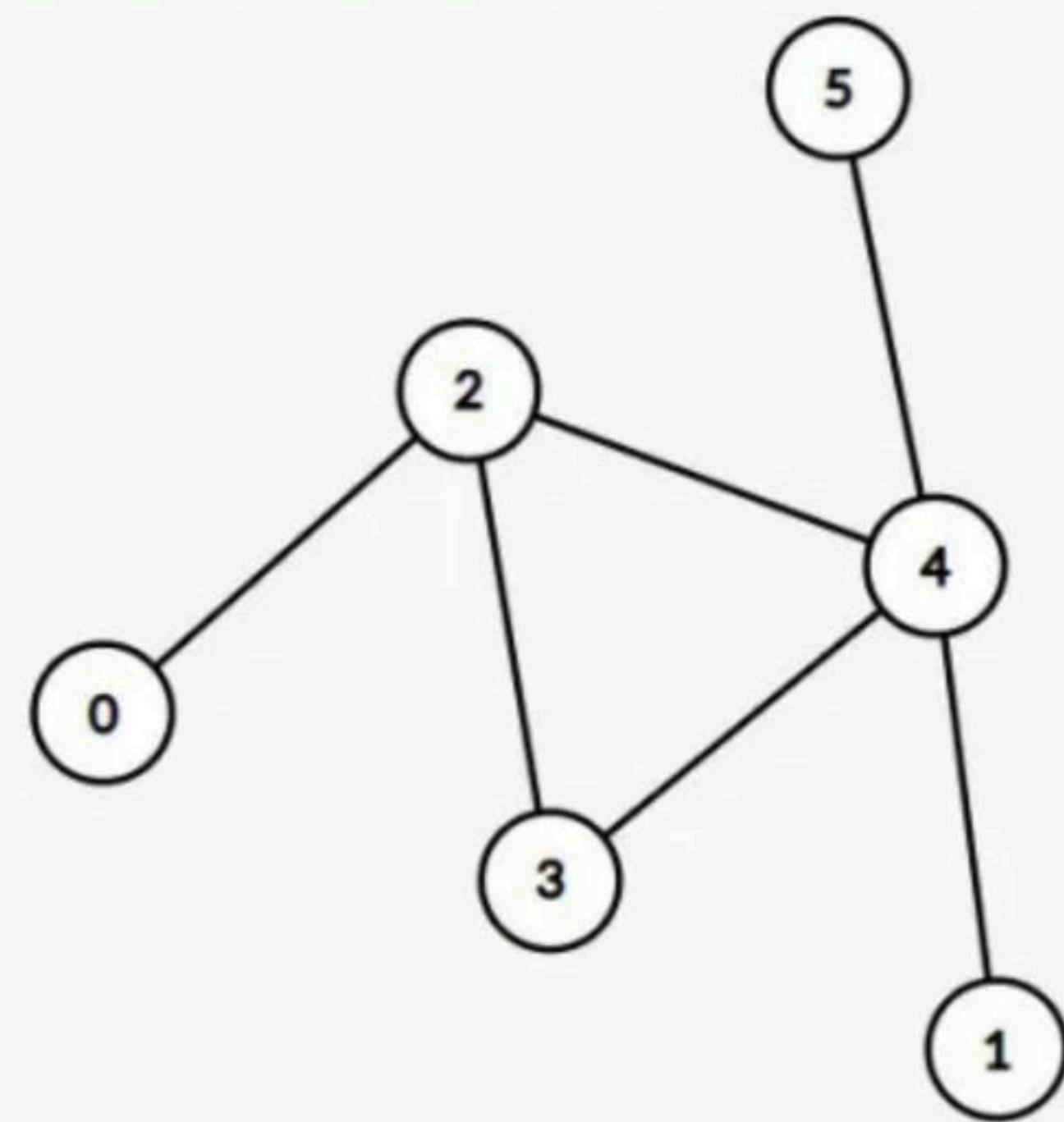
$N = 6$   
 $M = 6$



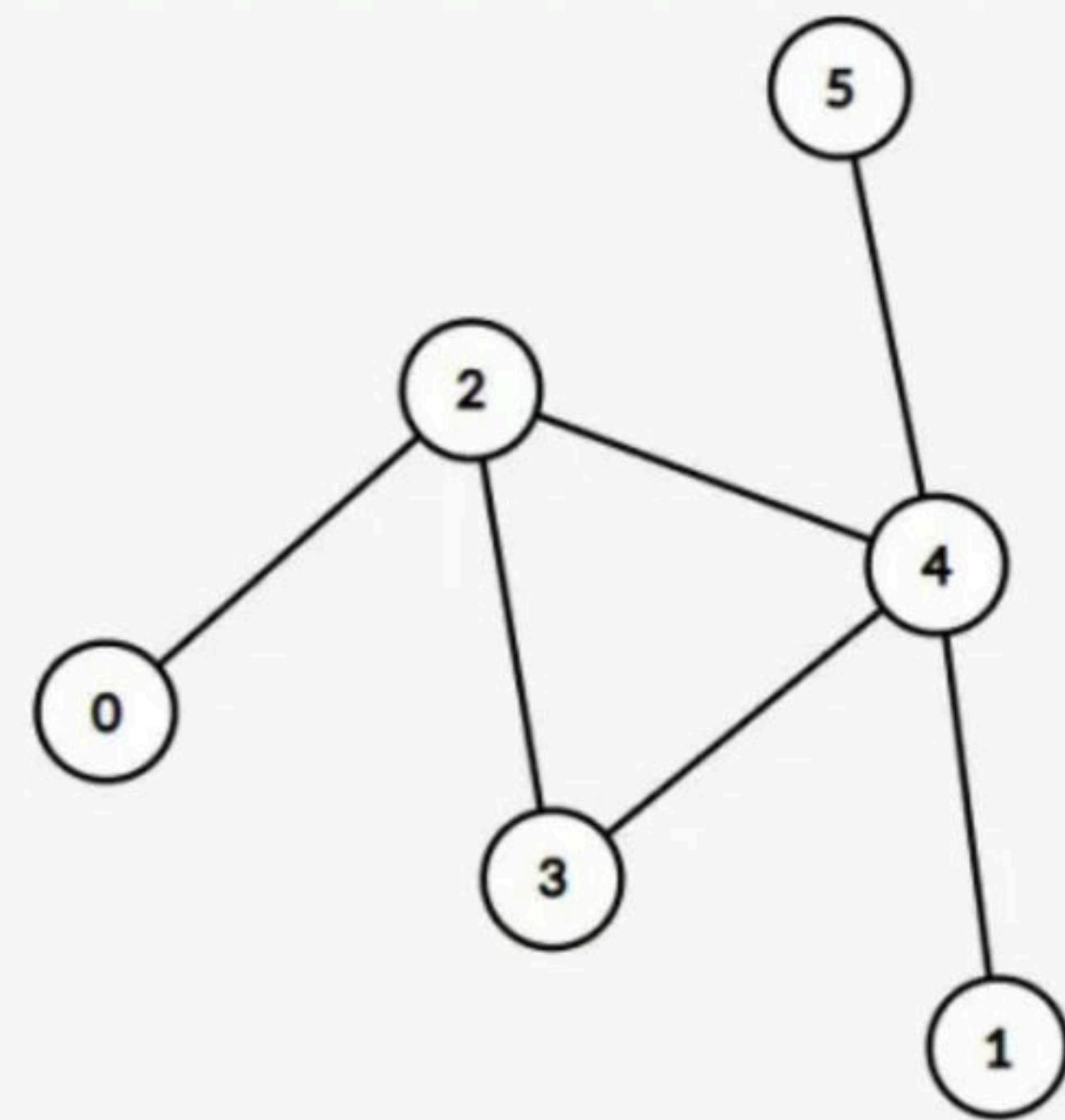
$N = 6$   
 $M = 6$



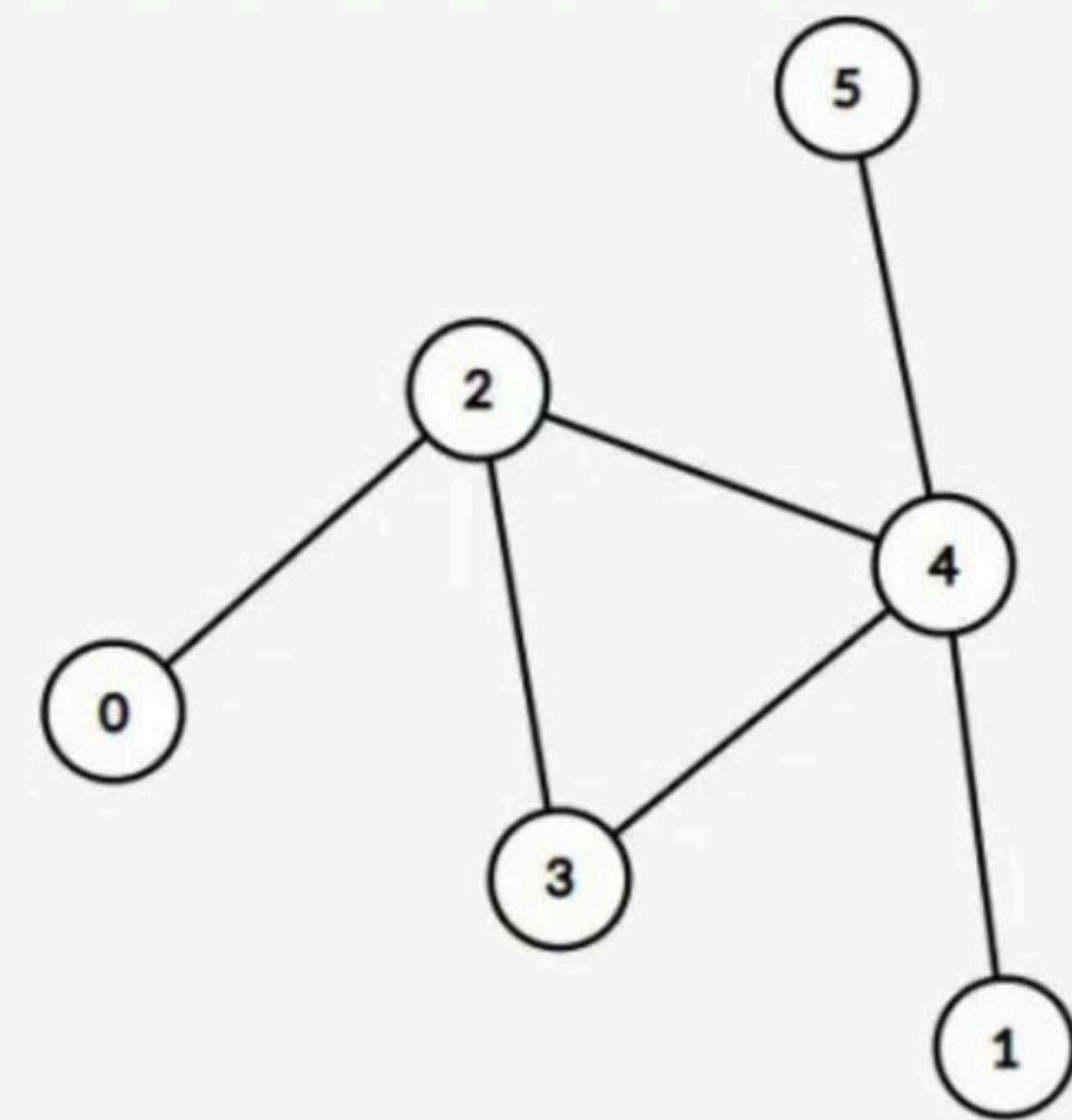
$N = 6$   
 $M = 6$



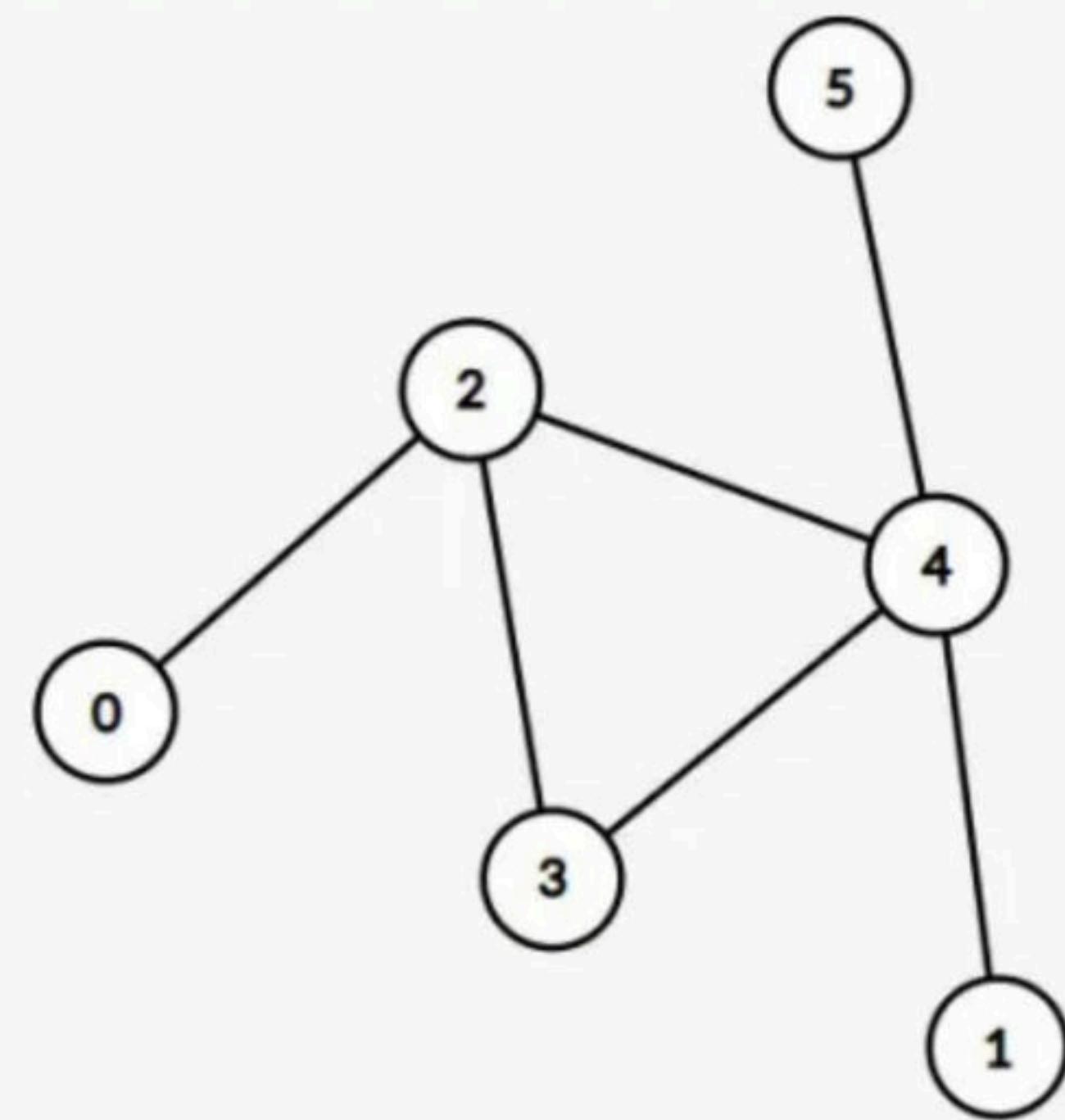
$N = 6$   
 $M = 6$



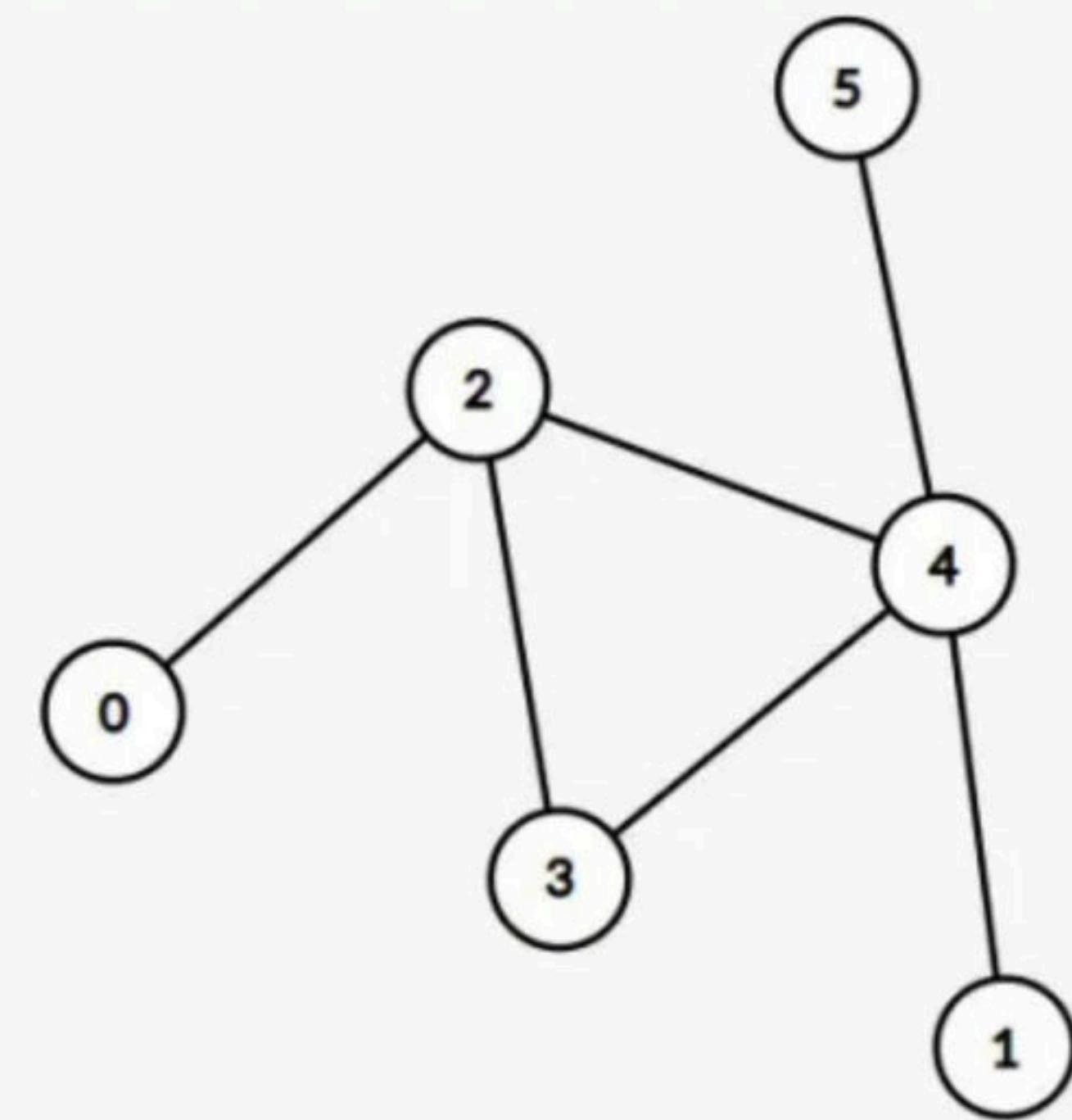
$N = 6$   
 $M = 6$



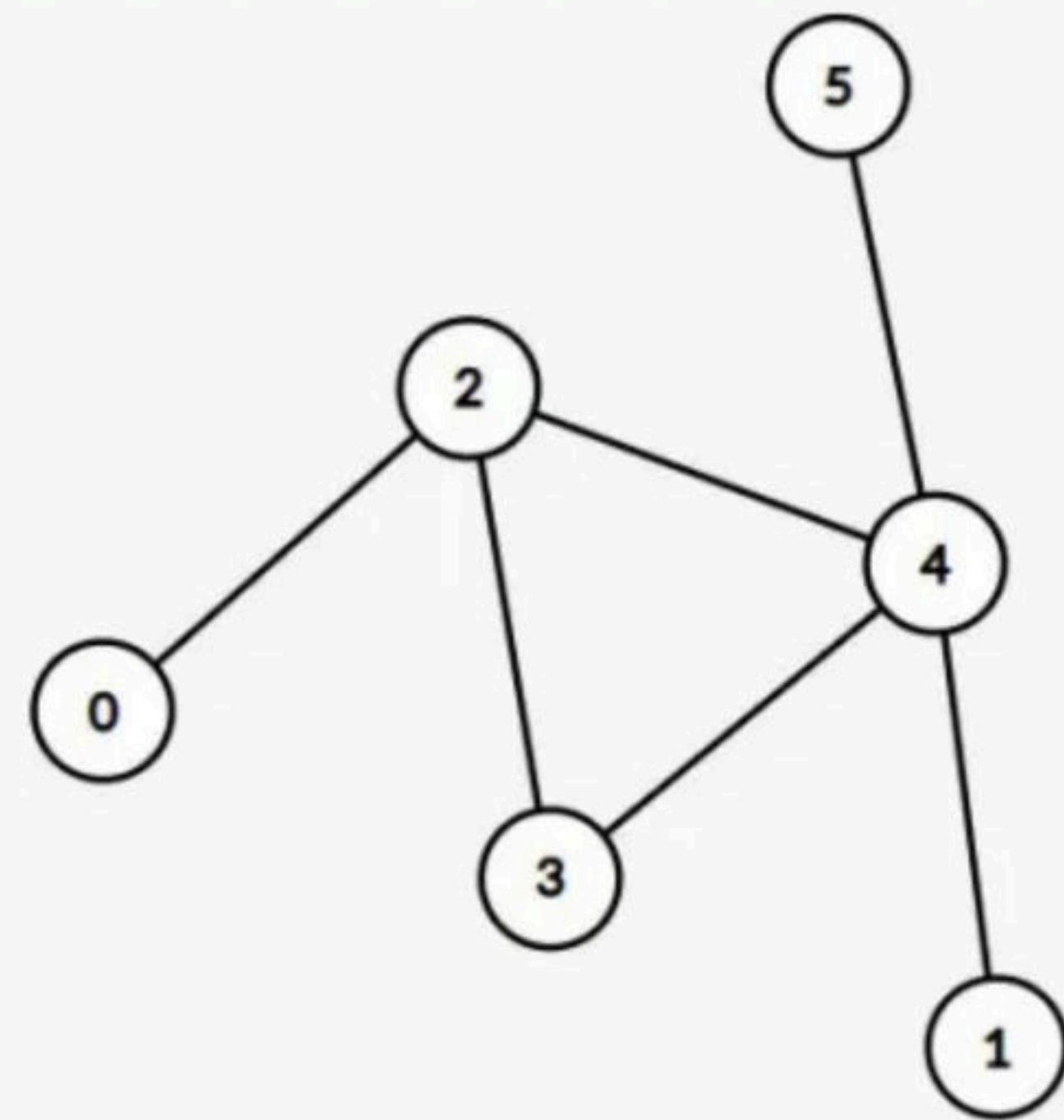
$N = 6$   
 $M = 6$



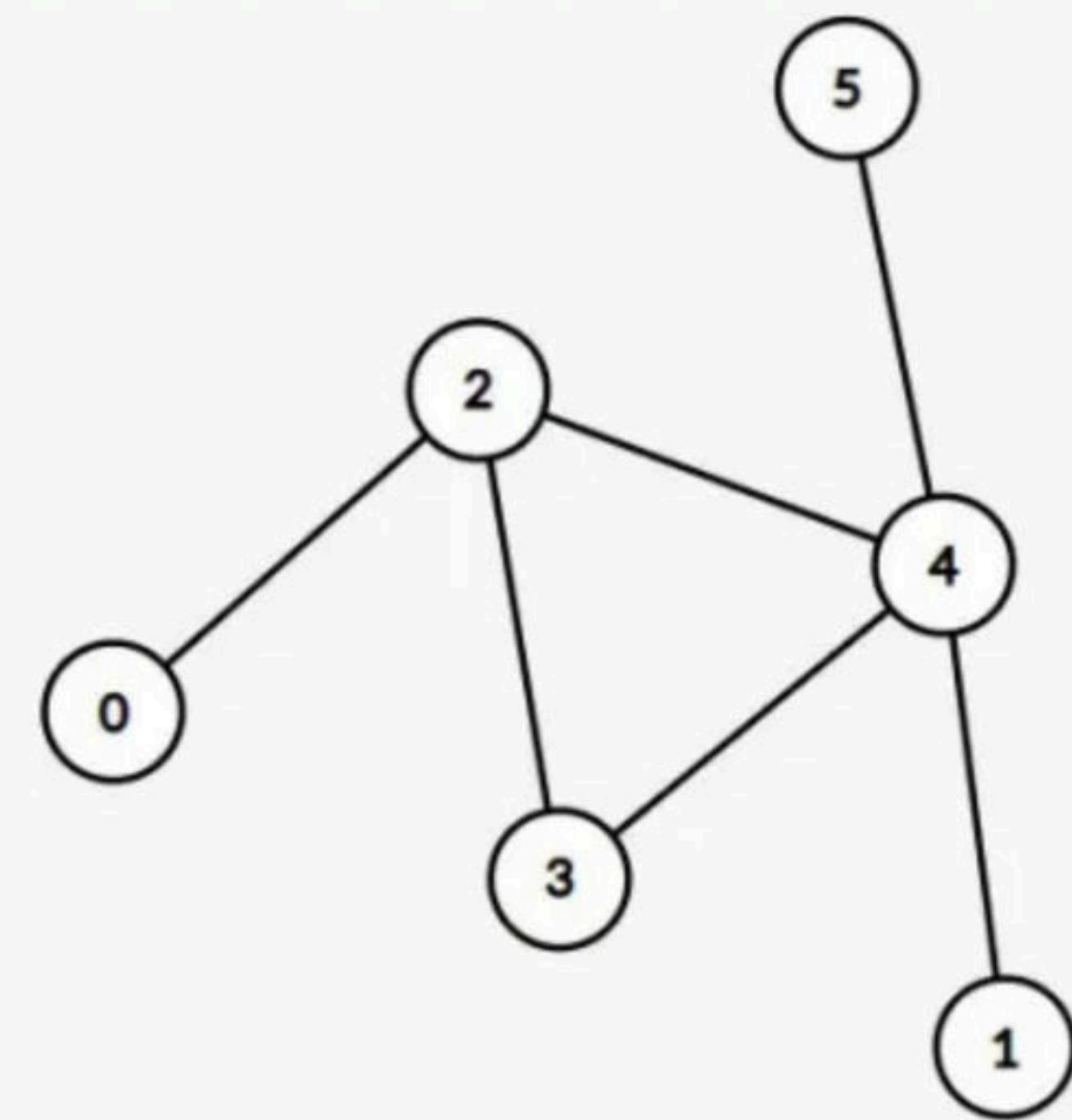
$N = 6$   
 $M = 6$



$N = 6$   
 $M = 6$

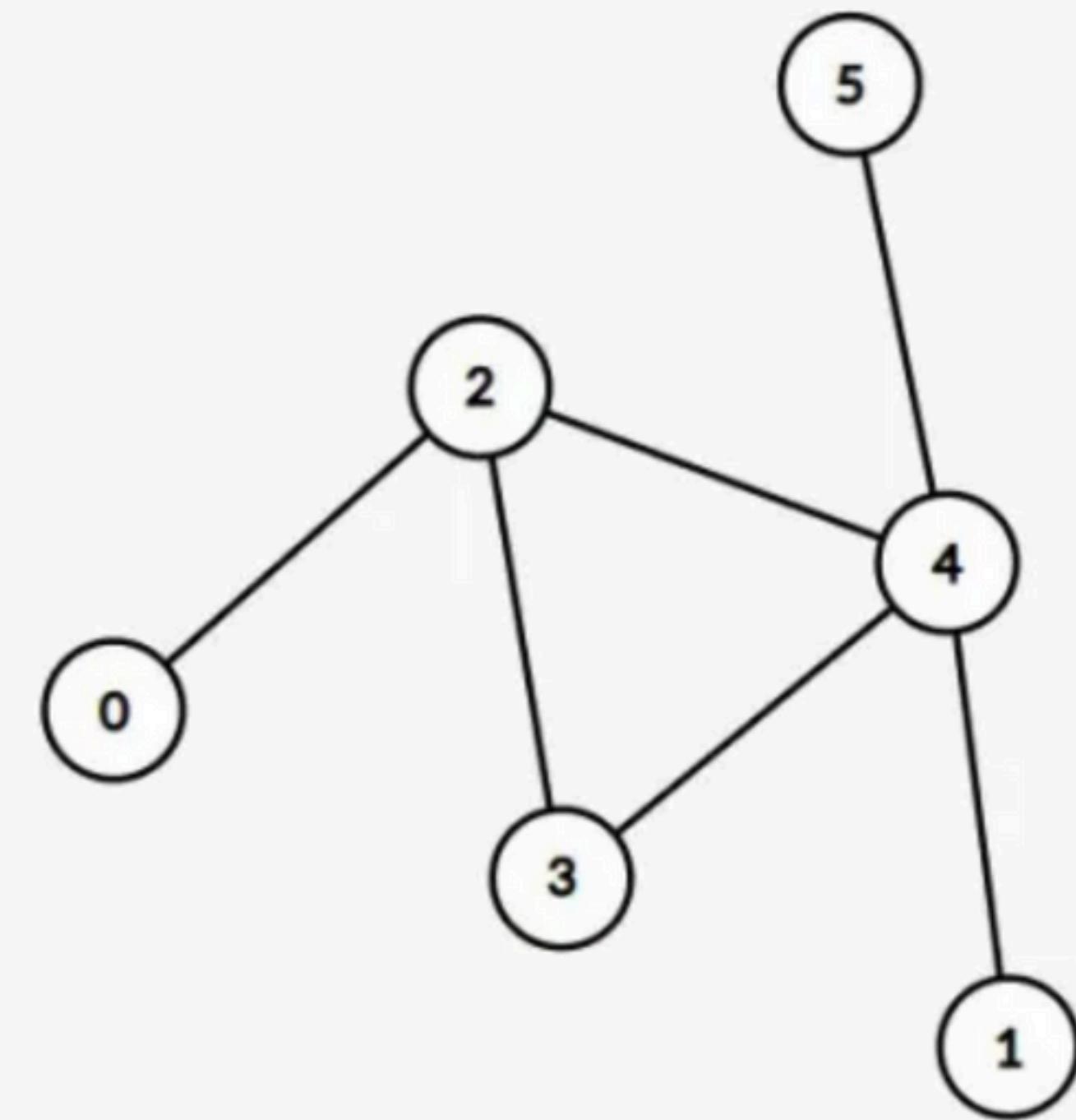


$N = 6$   
 $M = 6$



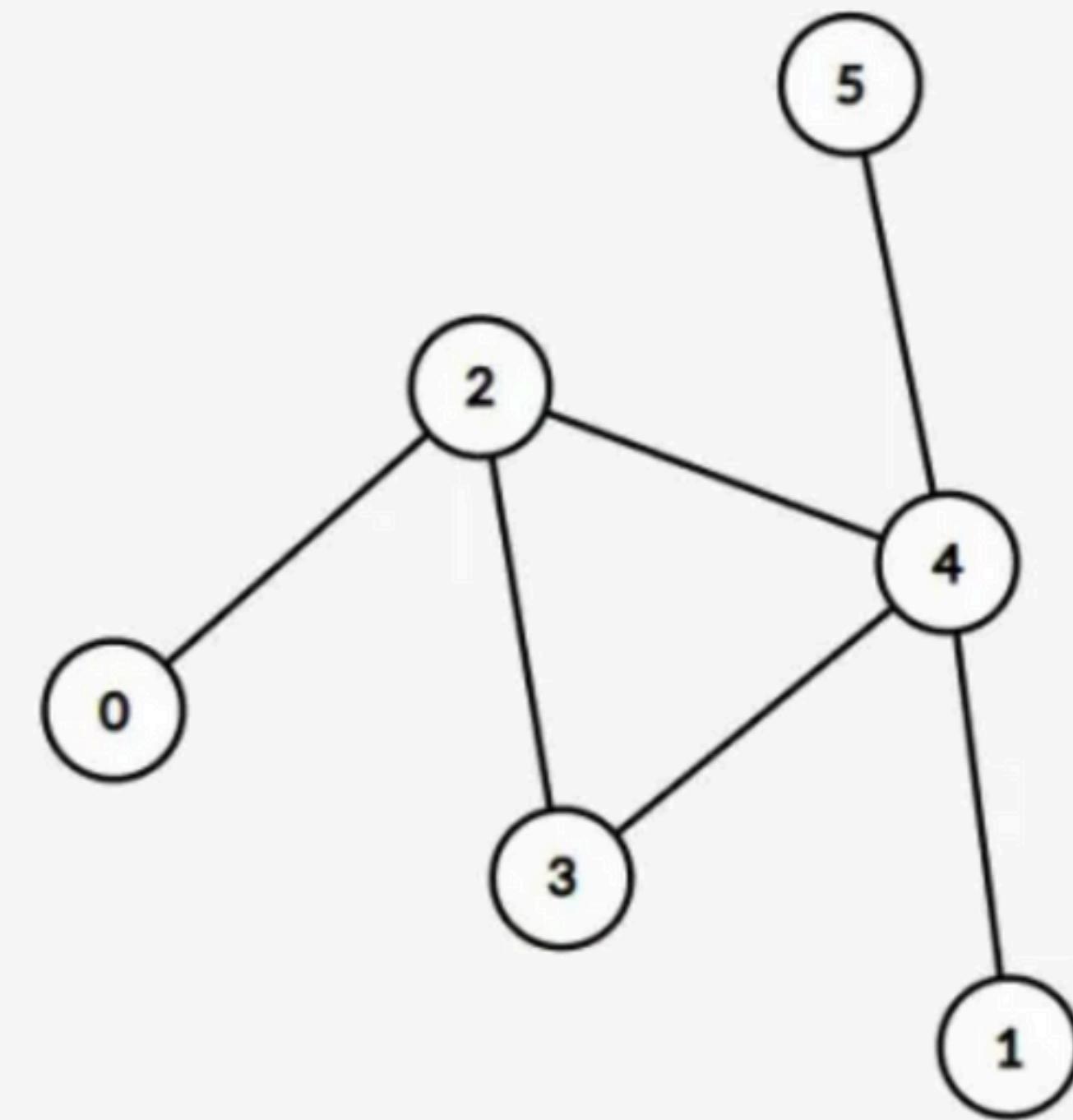
N = 6  
M = 6

```
int res = A[0];
for (int i = mask; i > 0; i = (i - 1) & mask) {
    res += A[i];
}
```



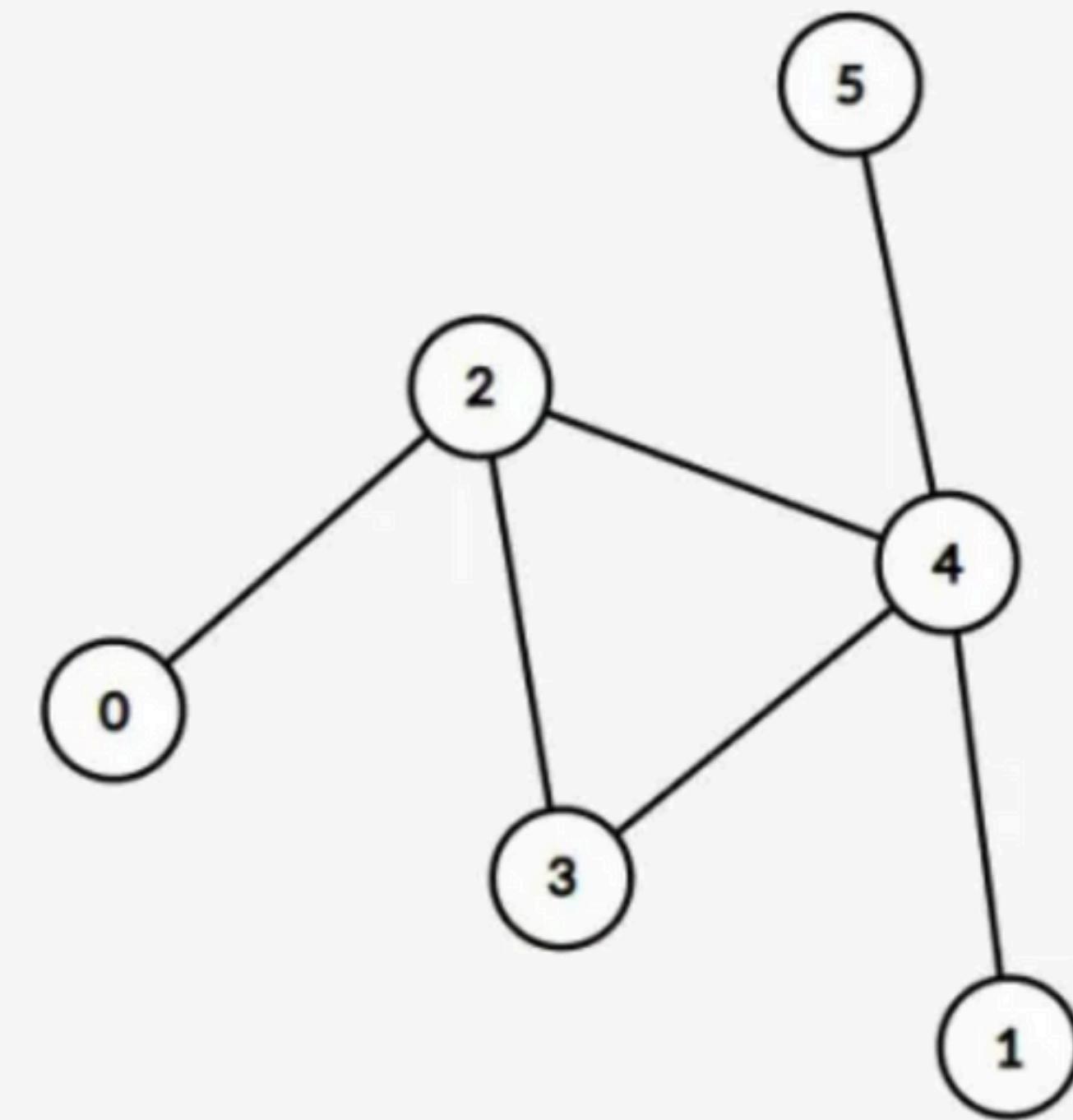
N = 6  
M = 6

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```

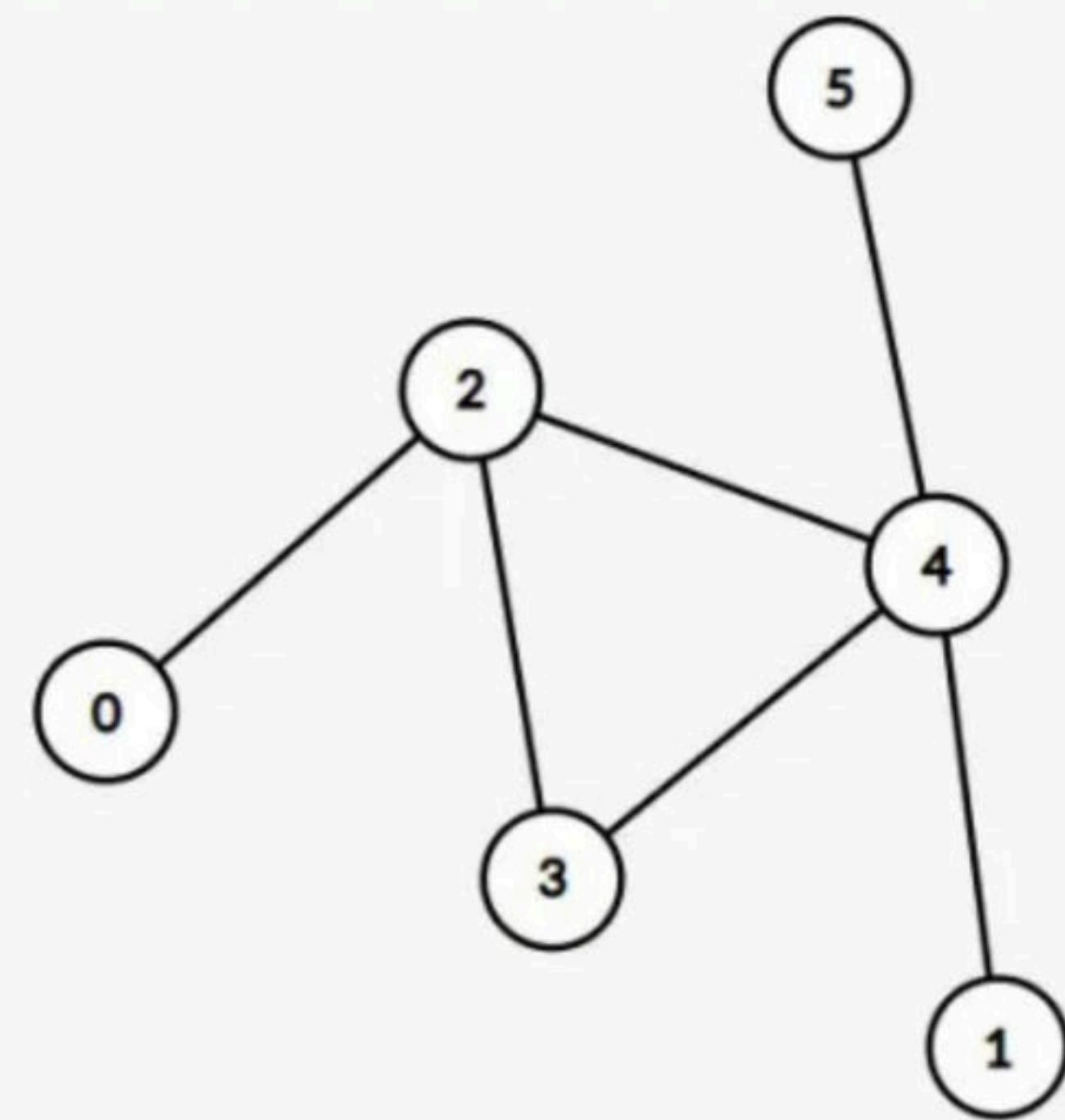


N = 6  
M = 6

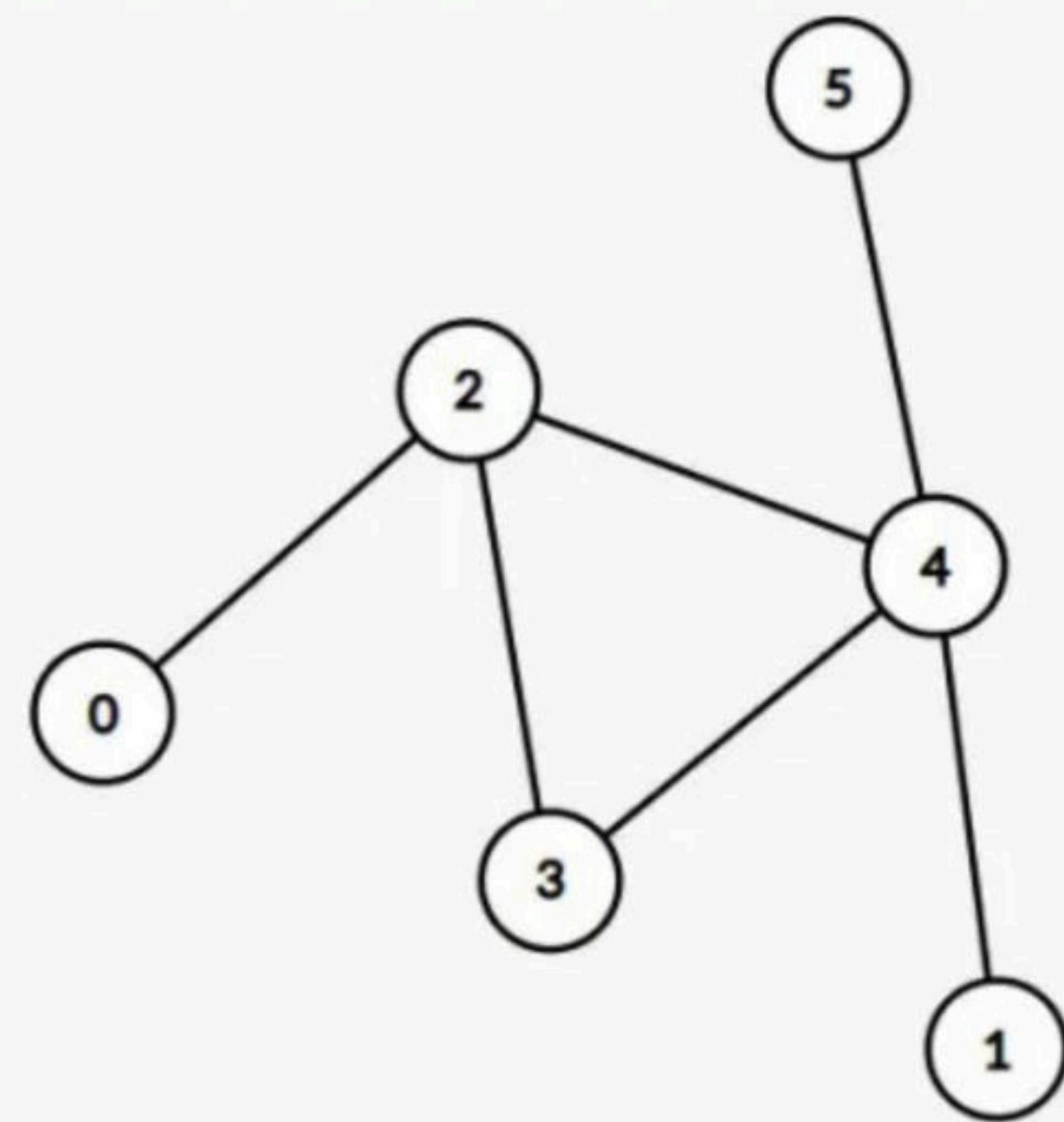
```
int res = A[0];
for (int i = mask; i > 0; i = (i - 1) & mask) {
    res += A[i];
}
```



$N = 6$   
 $M = 6$

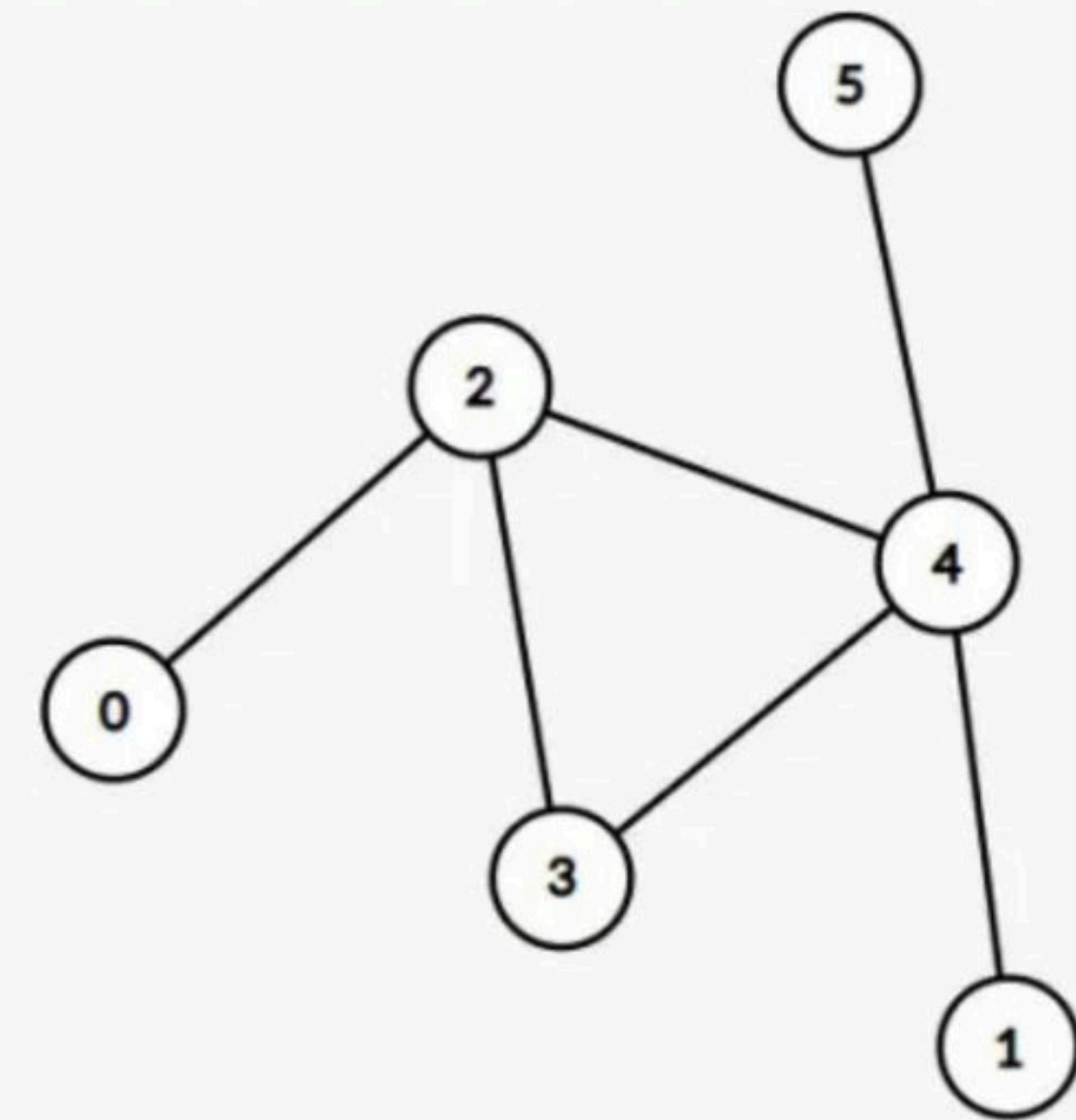


$N = 6$   
 $M = 6$



$N = 6$   
 $M = 6$

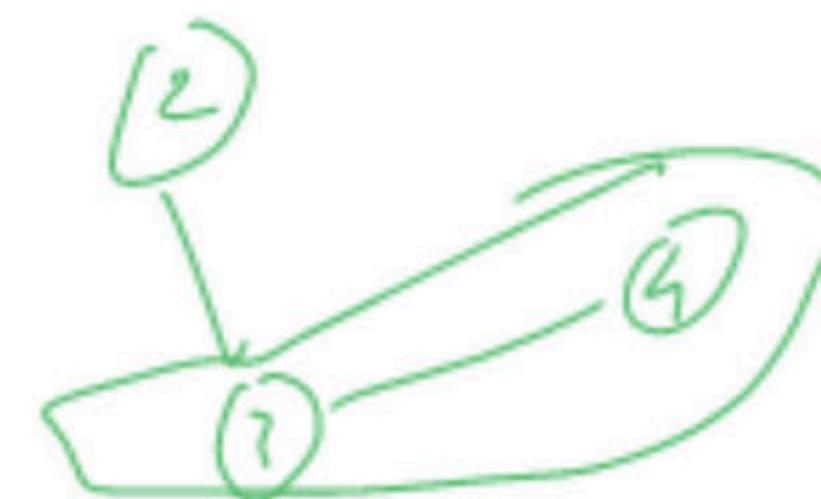
{2, 3}



```

// this function checks whether <vertexCover> is the vertex cover of graph
// <graph> 1 1 2 0 0 0 0 1 1 0
bool check(int graph, int vertexCover, int edges[][2], int M) {
    for (int i = 1; i <= M; i++) {
        int a = edges[i][0];
        int b = edges[i][1];
        if (((graph >> a) & 1) == 0 || ((graph >> b) & 1) == 0) { ✓ O(1)
            // this edge doesn't lie in graph
            continue;
        }
        if (((vertexCover >> a) & 1) == 0 && ((vertexCover >> b) & 1) == 0) { O(1)
            // no edge vertex lies in vertex cover
            return false;
        }
    }
    return true;
}

```



```
// this function checks whether <vertexCover> is the vertex cover of graph
// <graph>
bool check(int graph, int vertexCover, int edges[][2], int M) {
    for (int i = 1; i <= M; i++) {
        int a = edges[i][0];
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        if (((graph >> a) & 1) == 0 || ((graph >> b) & 1) == 0) {
            // this edge doesn't lie in graph
            continue;
        }
        if (((vertexCover >> a) & 1) == 0 && ((vertexCover >> b) & 1) == 0) {
            // no edge vertex lies in vertex cover
            return false;
        }
    }
    return true;
}
```

```

int solve(int edges[][2], int N, int M) {
    const int inf = 1e9;
    int leftSize = N / 2;
    int rightSize = N - leftSize;

    vector<int> adj[N + 5];
    for (int i = 1; i <= M; i++) {
        adj[edges[i][0]].push_back(edges[i][1]);
        adj[edges[i][1]].push_back(edges[i][0]);
    }

    int dp[(1 << leftSize) + 5];
    for (int graph = 0; graph < (1 << leftSize); graph++) {
        int minVertexCover = inf;
        if (check(graph, 0, edges, M))
            minVertexCover = 0;
        for (int vertexCover = graph; vertexCover > 0;
             vertexCover = (vertexCover - 1) & graph) {
            if (check(graph, vertexCover, edges, M)) {
                minVertexCover = min(minVertexCover, countSetBits(vertexCover));
            }
        }
        dp[graph] = minVertexCover;
    }
}

```

{ 2, 3 }

DP[12]

A

k

```

int res = inf;
for (int vertexCover = 0; vertexCover < (1 << rightSize); vertexCover++) {
    int graph = (1 << rightSize) - 1;
    if (!check(graph << leftSize, vertexCover, edges, M))
        continue;
    int rightVertexCover = countSetBits(vertexCover);
    int remGraph = graph ^ vertexCover;

    int leftVerticesToTake = 0;
    for (int remVertex = 0; remVertex < rightSize; remVertex++) {
        if ((remGraph >> remVertex) & 1) {
            int actualVertex = remVertex + leftSize;
            for (int &leftVertex : adj[actualVertex]) {
                if (leftVertex < leftSize)
                    leftVerticesToTake |= (1 << leftVertex);
            }
        }
    }
    int leftVertexCover = countSetBits(leftVerticesToTake) +
        dp[((1 << leftSize) - 1) ^ leftVerticesToTake];
    res = min(res, leftVertexCover + rightVertexCover);
}
return res;
}

```

right

```

int solve(int edges[][2], int N, int M) {
    const int inf = 1e9;
    int leftSize = N / 2;
    int rightSize = N - leftSize;

    vector<int> adj[N + 5];
    for (int i = 1; i <= M; i++) {
        adj[edges[i][0]].push_back(edges[i][1]);
        adj[edges[i][1]].push_back(edges[i][0]);
    }

    int dp[(1 << leftSize) + 5];
    for (int graph = 0; graph < (1 << leftSize); graph++) {
        int minVertexConver = inf;
        if (check(graph, 0, edges, M))
            minVertexConver = 0;
        for (int vertexCover = graph; vertexCover > 0;
             vertexCover = (vertexCover - 1) & graph) {
            if (check(graph, vertexCover, edges, M)) {
                minVertexConver = min(minVertexConver, countSetBits(vertexCover));
            }
        }
        dp[graph] = minVertexConver;
    }

    int res = inf;
    for (int vertexCover = 0; vertexCover < (1 << rightSize); vertexCover++) {
        int graph = (1 << rightSize) - 1;
        if (!check(graph << leftSize, vertexCover, edges, M))
            continue;
        int rightVertexCover = countSetBits(vertexCover);
        int remGraph = graph ^ vertexCover;
        int leftVerticesToTake = 0;
        for (int remVertex = 0; remVertex < rightSize; remVertex++) {
            if ((remGraph >> remVertex) & 1) {
                int actualVertex = remVertex + leftSize;
                for (int &leftVertex : adj[actualVertex]) {
                    if (leftVertex < leftSize)
                        leftVerticesToTake |= (1 << leftVertex);
                }
            }
        }
        int leftVertexCover = countSetBits(leftVerticesToTake) +
            dp[((1 << leftSize) - 1) ^ leftVerticesToTake];
        res = min(res, leftVertexCover + rightVertexCover);
    }
    return res;
}

```

# Summary

