CS331
Haskell Tutorial 03

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Outline

- List Comprehensions : Zip
- Fold : family of higher order functions that process a data structure in some order and build a return value
- Lazy Evaluation
  - Lazyness example
  - Benefits

The Zip Function

A useful library function is <u>zip</u>, which maps two lists to a list of pairs of their corresponding elements.

```
zip :: [a] → [b] → [(a,b)]
```

For example:

```
> zip ['a','b','c'] [1,2,3,4]

[('a',1),('b',2),('c',3)]
```

The Zip Function

Using zip we can define a function returns the list of all pairs of adjacent elements from a list:

```
pairs    :: [a] → [(a,a)]
pairs xs = zip xs (tail xs)
```

For example:

```
> pairs [1,2,3,4]

[(1,2),(2,3),(3,4)]
```

The Zip Function

Using pairs we can define a function that decides if the elements in a list are sorted:

```
sorted    :: Ord a ⇒ [a] → Bool
sorted xs =
   and [x ≤ y | (x,y) ← pairs xs]
```

For example:

```
> sorted [1,2,3,4]
True

> sorted [1,3,2,4]
False
```

The Zip Function

Using zip we can define a function that returns the list of all positions of a value in a list:

```
positions :: Eq a ⇒ a → [a] → [Int]
positions x xs =
   [i | (x’,i) ← zip xs [0..], x == x’]
```

For example:

```
> positions 0 [1,0,0,1,0,1,1,0]
[1,2,4,7]
```

String Comprehensions

A string is a sequence of characters enclosed in double quotes. Internally, however, strings are represented as lists of characters.

```
"abc" :: String
```

Means ['a', 'b', 'c'] :: [Char].

String Comprehensions

Because strings are just special kinds of lists, any <u>polymorphic</u> function that operates on lists can also be applied to strings. For example:

```
> length "abcde"
5

> take 3 "abcde"
"abc"

> zip "abc" [1,2,3,4]
[('a',1),('b',2),('c',3)]
```

String Comprehensions

Similarly, list comprehensions can also be used to define functions on strings, such counting how many times a character occurs in a string:

```
count      :: Char → String → Int
count x xs =
   length [x’ | x’ ← xs, x == x’]
```

For example:

```
> count ’s’ "Mississippi"
4
```

Is the list sorted?

- Using pairs we can define a function that decides if the elements in a list are sorted:

```
sorted        ::  Ord a => [a] -> Bool
sorted xs   =
            and [x<= y | (x,y) <- pairs xs]
```

```
sorted [1,2,3,4]
```

⇨ True

Positions

- Using zip we can define a function that returns the list of all positions of a value in a list:

```
positions          ::  Eq a => a -> [a] -> [Int]
positions x xs   =
            [ i | (x',i) <-  zip xs  [0..n],  x == x']
            where n = length xs - 1

positions  0  [1,0,0,1,0,1,1,0]
```

⇒ [1,2,4,7]

The Foldr function

- A number of functions on lists can be defined using the following simple pattern of recursion:

```
f [ ]      = v
f (x:xs) = x op f xs
```

- **f** maps the empty list to some value **v** and any non-empty list to some function **op** applied to its head and **f** of its tail

Right Folding

```
foldr f u [x1,x2,..,xn]

⇒ f x1 (foldr f u [x2 ..xn])
⇒ f x1 (f x2 (fold f u [x3..xn]))

⇒ f x1 (f x2 (… (fold f u [xn]) …))
⇒ f x1 (f x2 (… (f xn u) …)))
```

associate to right

The Foldr function

- For example

```
sum [ ]            = 0
sum (x:xs) = x + sum xs

product [ ]        = 1
product (x:xs)   = x * product xs

and [ ]            = True
and (x:xs)   = x && and xs
```

The Foldr function

- The higher order function **foldr** (fold right) encapsulates this simple pattern of recursion, with the function **op** and the value **v** as arguments

```
sum            = foldr (+) 0

product        = foldr (*) 1

or             = foldr (||) False

and            = foldr (&&) True
```

The Foldr function

```
foldr :: (a -> b -> b) -> b -> [a] -> b
foldr f  v  [ ]          = v
foldr f  v  (x:xs)   = f x (foldr f v xs)
```

- However it is best to think of **foldr** non-recursively as simultaneously replacing each (:) in a list by a given function and [ ] by a give value

The Foldr function

```
sum [1,2,3]
= foldr (+) 0 [1,2,3]
= foldr (+) 0 (1: (2: (3: [ ])))
= 1 + (2 + (3 + 0)) = 6
```

Replace each (:) by (+) and [ ] by 0

```
product [1,2,3]
= foldr (*) 1 [1,2,3]
= foldr (*) 1 (1: (2: (3: [ ])))
= 1 * (2 * (3 * 1)) = 6
```

Replace each (:) by (*) and [ ] by 1

Left Folding – Tail Recursion

- Accumulate result in a parameter:

```
foldl f u ls =
    case ls of
      []   -> u
      x:xs -> foldl f (f u x) xs
```

based on accumulation

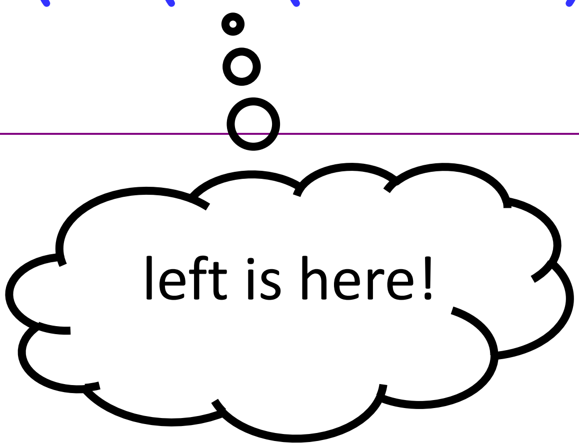
- What is the type of `foldl`?
- Can we compute `factorial` using it?

Left Folding

```
foldl f u [x1,x2,..,xn]

⇒ foldl f (f u x1) [x2 ..xn]
⇒ foldl f (f (f u x1) x2) [x3..xn]))

⇒ foldl f (f … (f (f u x1) x2)… xn) []
⇒ f (… (f (f u x1) x2) …) xn
```



Instance of Left Folding

- Summing a list by accumulation.

```
sumT acc ls =
    case ls of
      []    -> 0
      x:xs -> sumT (x+acc) xs
```

⇩

```
sumList ls = sumT 0 ls

sumT acc ls = foldl (+) acc ls
```

Iterative List Reversal

- Concatenate first element to last position.

```
revT w []          = w
revT w (x:xs)      = revT (x:w) xs
```

⇩

Same as:

```
revT w xs = foldl (\ w x -> x:w) w xs
```

What is the time complexity?

Fusion Transformation

- Consider:

```
    …sum (double xs)…
sum []          = 0
sum x:xs        = x+(sum xs)
double []       = []
double x:xs     = 2*x : (double xs)
```

- Computation reuses smaller functions to build larger ones but may result in unnecessary intermediate structures. They can cause space overheads.

- Solution : Fuse the code together!

Fusion Transformation

- Define:

```
sumdb xs = sum (double xs)
```

- Instantiate: `xs=[]`

```
sumdb []      = sum (double [])
              = sum []
              = 0
```

- Instantiate: `xs=x:xs`

```
sumdb x:xs    = sum (double x:xs)
              = sum (2*x : double xs)
              = 2*x + sum(double xs)
              = 2*x + (sumdb xs)
```

Iteration Transformation

- Define:

```
sumdbT a xs = a+(sumdb xs)
```

- Instantiate: `xs=[]`

```
sumdbT a []  = a+(sumdb [])
             = a
```

- Instantiate: `xs=x:xs`

```
sumdbT a (x:xs)   = a+(sumdb x:xs)
                  = a+(2*x + sumdb xs)
                  = (a+2*x) + (sumdb xs)
                  = sumdbT (a+2*x) xs
```

The Composition function

- The library function (**.**) returns the composition of two functions as a single function

```
(.) :: (b -> c) -> (a -> b) -> (a -> c)
f . g = \x -> f (g x)
```

```
odd :: Int -> Bool
odd = not . even
```

Laziness in Haskell

- When the first print statement ("Hello") happens,
  - The expression myTuple is actually **completely unevaluated**,
  - even though it is defined before the print statement.
- It is represented in memory by what is called a "**thunk**".
  - Of course, our program knows how to evaluate this thunk when the time comes. But for now, no value is there.
- When it prints the first element of the tuple, it still doesn't completely evaluate myTuple.
-

```
main = do
    let myTuple = ("first", map (*2) [1,2,3,4])
    print "Hello"
    print  (fst myTuple)
    print  (head (snd myTuple))
    print (length (snd myTuple))
    print  (snd myTuple)
```

Laziness in Haskell

- When the compiler sees us call the fst function on myTuple, it knows myTuple must be a tuple.
- Instead of seeing a single thunk at this point, the compiler sees myTuple as a **tuple containing two unevaluated thunks**.
- Next, we print the first element of myTuple. Printing an expression forces it to be **completely evaluated**. So after this, the compiler sees myTuple as a tuple containing a string in its first position and an unevaluated thunk in its second position.

```
main = do
    let myTuple = ("first", map (*2) [1,2,3,4])
    print "Hello"
    print  (fst myTuple)
    print  (head (snd myTuple))
    print (length (snd myTuple))
    print  (snd myTuple)
```

Laziness in Haskell

- At the next step, we print (head snd myTuple).
- This tells the compiler this second element must be a non-empty list. If it were empty, our program would actually **crash** here.
- This forces the evaluation of this first element (2). However, the remainder of the list **remains an unevaluated thunk**.

```
main = do
    let myTuple = (“first”, map (*2) [1,2,3,4])
    print “Hello”
    print  (fst myTuple)
    print  (head (snd myTuple))
    print (length (snd myTuple))
    print  (snd myTuple)
```

Laziness in Haskell

- Next, we print the length of the list.
- Haskell will do enough work here to determine how many elements are in the list, but it will not actually evaluate any more items.
- The list is now an **evaluated first element**, and **3 unevaluated thunks**.

```
main = do
    let myTuple = ("first", map (*2) [1,2,3,4])
    print "Hello"
    print  (fst myTuple)
    print  (head (snd myTuple))
    print (length (snd myTuple))
    print  (snd myTuple)
```

Laziness in Haskell

- Finally, we print the full list.
- This evaluates the list in its **entirety**.
  - If we did not do this last step, the final 3 elements would **never be evaluated**.

```
main = do
    let myTuple = (“first”, map (*2) [1,2,3,4])
    print “Hello”
    print  (fst myTuple)
    print  (head (snd myTuple))
    print (length (snd myTuple))
    print  (snd myTuple)
```

Laziness make it faster

- Comparable code in C++ or Java would need to make all three expensive calls to reallyLongFunction before calling F2 with the results.
- But in Haskell, the program **will not call** reallyLongFunction until it absolutely needs to.

```
F1 = F2 exp1 exp2 exp3  where
            exp1 = reallyLongFunction 1234
            exp2 = reallyLongFunction 3151
            exp3 = reallyLongFunction 8571

F2 exp1 exp2 exp3 = if exp1 < 1000
      then exp2
      else if exp1 < 2000
            then exp3
            else exp1
```

Laziness make it faster

- So in this example, we will always evaluate exp1 in order to determine result of if statement in F2.
- if we happen to have exp1 >= 2000, then we'll **never evaluate** exp2 or exp3! We don't need them!
- We'll save ourselves from having to make the expensive calls to reallyLongFunction. As a result, our program will run faster.

```
F1 = F2 exp1 exp2 exp3  where
            exp1 = reallyLongFunction 1234
            exp2 = reallyLongFunction 3151
            exp3 = reallyLongFunction 8571

F2 exp1 exp2 exp3 = if exp1 < 1000
      then exp2
      else if exp1 < 2000
            then exp3
            else exp1
```

Demerit of laziness

- When using recursion or recursive data structures, **unevaluated thunks can build up** in heap memory.
- If they consume all your memory, your program will crash.
- In particular, the foldl function suffers from this problem. The following usage will probably fail due to a memory leak.

foldl (+) 0 [1..10^7]

Demerit of laziness

- Laziness can also make it harder to reason about our code.
  - Just because a number of expressions are defined in the same where clause does **not mean** that they will be **evaluated at the same time**.
  - This can easily confuse beginners.

Thanks