EXCEL 21-127
23/24 March 2020

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1. Go to breakout rooms. First construct the relations satisfying the properties by yourself, then share your answer with the other person. Check each other's answers. You may show handwritten notes via webcam, or directly type into the chat.

a. A relation on [6] that is reflexive, transitive, <u>not</u> symmetric.
b. A relation on [6] that is reflexive, anti-symmetric, <u>not</u> transitive.

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2. Suppose R is a partial order relation on set S. We define the strict preference relation P such that $\forall a, b \in S$,

$$a \sim_P b \Leftrightarrow a \sim_R b \wedge \neg b \sim_R a$$

and define the indifference relation I such that $\forall a, b \in S$,

$$a \sim_I b \Leftrightarrow a \sim_R b \wedge b \sim_R a$$

Prove the following statements about the relations.

a. I is a transitive relation.
b. P is a transitive relation.
c. $\forall a, b, c \in S$. If $a \sim_P b$ and $b \sim_I c$, then $a \sim_P c$.

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3. Recall that we can write a binary relation on set S as a set of ordered pairs/tuples, i.e. a subset of S^2. Consider a function $f: S \to S$. We write the function also as a set of ordered pairs (x, y) such that $(x, y) \in f$ if and only if $f(x) = y$ for all $x, y \in S$.

Compare the set of every binary relations on S and the set of every functions from S to S. Argue that one is a proper subset of another.

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4. Consider the function $f: \mathbb{N}^2 \to \mathbb{N}$ via $f(a,b) = ab + a$. Find $Im_f(\mathbb{N}^2)$.

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5. Consider a function $f: A \to B$. Let $S, T \subseteq A$. Show that

$$Im_f(S \cap T) \subseteq Im_f(S) \cap Im_f(T)$$

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6. Consider a function $f: A \to B$. Let $S \subseteq B$. Show that

$$Im_f\left(PreIm_f(S)\right) \subseteq S$$

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7. Give an example when equality does not hold in the previous question.

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8. Using strong induction, prove that the Fibonacci sequence

$$F_0 = 1$$
$$F_1 = 1$$
$$F_{n+1} = F_n + F_{n-1} \forall n \geq 1$$

satisfies for $n \geq 1$

$$F_n \geq \left(\frac{3}{2}\right)^{n-2}$$

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9. Using strong induction, prove that every positive integer can be written as a sum of distinct powers of 2.
