

Session 7 Group Problem Set

Version A

EXCEL 127 – 1 April 2019

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- 2.1 Cardinality of the set  $\{1, 2, 3, 5\}$
- 2.2 Cardinality of  $[6]$
- 2.3 For an even natural number  $k$  the cardinality of  $\{n \in \mathbb{N} : n \text{ is even} \wedge 0 < n \leq k\}$
- 2.4 Compare the cardinality of  $\mathbb{N}$  and the cardinality of  $\{n \in \mathbb{N} : n \text{ is even}\}$
- 2.5\* Show that  $|\mathbb{N} \times \mathbb{N}| \leq |\mathbb{N}|$ . (Hint: Construct injection. Consider prime factorization.)
- 2.6 Show that  $S = \{n \in \mathbb{N} : n \text{ is prime} \wedge n < 20\}$  is finite.
- 2.7 Construct  $S \subseteq \mathbb{N}$  such that
- a)  $S$  is finite.
  - b)  $S$  is countably infinite.
  - c)  $S$  is countably infinite and  $S^c$  is finite.
  - d)  $S$  is countably infinite and  $S^c$  is countably infinite.
- 2.8 Prove that if  $S \subseteq \mathbb{N}$  is finite,  $\mathbb{N} - S$  is infinite.
- 2.9\* Construct  $S, T \subseteq \mathbb{R}$  such that  $S, T$  are both uncountable and  $S \cap T$  is countable.
- 2.10 Let  $S, T$  be finite sets.  $S \cap T = \emptyset$ . Show that  $S \cup T$  is finite.