EXCEL 151 Final Exam Review Practice
2 December 2019
Sam Yong (Fall 2019)

1. Prove that if $S \subseteq \mathbb{N}$ is finite, $\mathbb{N} - S$ is infinite.
2. Construct $S, T \in \mathbb{R}$ such that S, T are both uncountable, and $S \cap T$ is countable.
3. Compare the cardinality of $\mathbb{N}$ and the cardinality of even natural numbers.
4. Compare the cardinality of $\mathbb{N}$ and the cardinality of $\mathbb{Z}$.
5. Show that the set of all functions $f: [n] \to \mathbb{N}$ is countable.
6. A finite union of countable sets is _____.
7. A finite intersection of countable sets is _____.
8. A finite Cartesian product of countable sets is _____.
9. A countable union of countable sets is _____.
10. A countable intersection of countable sets is _____.
11. A countable Cartesian product of countable sets is _____.
12. Prove that $2^n + 1$ is divisible by 3 for all odd integers n.
13. Prove that $f_n \perp f_{n+1}$ for all $n \geq 1$.
14. Let n be a positive integer. Show that the product of n consecutive integers is divisible by $n!$.
15. Show that $\mathcal{P}(\mathbb{N})$ is uncountable.
16. Show that $\{(a, b, c): a \in \mathbb{N}, b = 2a, c \in \mathbb{Z} \wedge c \leq a\}$ is countably infinite.
17. Find an explicit bijection between $[3, 7]$ and $[0, 1)$.
18. Solve for x where $5x \equiv 22 \bmod 13$.
19. Show that $\left|\{f: \mathbb{N} \to \{0, 1\}\}\right| = |\mathcal{P}(\mathbb{N})|$.
20. Let $A \subseteq [100]$ and has 52 elements. Prove that there always exists $x, y \in A$ such that $x + y = 100$.
21. Three coins are flipped. What is the expected number of heads?
22. 100 dice are rolled, and the 100 results are added together. What is the expected value of the sum?
23. Prove the special number 10 for 3-color version of Pascal's triangle.