

Session 7 Group Problem Set

Version C

EXCEL 127 – 1 April 2019

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- 2.1 Cardinality of the empty set
- 2.2 Cardinality of $[42]$
- 2.3 For a natural number k the cardinality of $\{n \in \mathbb{N} : n \text{ is divisible by } 3 \wedge 0 < n \leq k\}$
- 2.4 Compare the cardinality of \mathbb{Z} and the cardinality of \mathbb{N}
- 2.5* Show that $|\mathbb{N} \times \mathbb{N}| \leq |\mathbb{N}|$. (Hint: Construct injection. Consider prime factorization.)
- 2.6 Show that $S = \{n \in \mathbb{N} : n \text{ is prime} \wedge n < 30\}$ is finite.
- 2.7 Construct $S \subseteq \mathbb{N}$ such that
 - a) S is finite.
 - b) S is countably infinite.
 - c) S is countably infinite and S^c is finite.
 - d) S is countably infinite and S^c is countably infinite.
- 2.8 Prove that if $S \subseteq \mathbb{N}$ is finite, $\mathbb{N} - S$ is infinite.
- 2.9* Construct $S, T \subseteq \mathbb{R}$ such that S, T are both uncountable and $S \cap T$ is countable.
- 2.10 Let S, T be finite sets. $S \cap T = \emptyset$. Show that $S \cup T$ is finite.