Review for Exam 3

EXCEL 151 Fall 19

1. Consider the experiment of flipping a fair coin 10 times.

    a. What is the probability of getting exactly 7 heads?

    b. What is the probability of getting at most 7 heads?

    c. What is the probability that the first head occurs on the second flip?

2. How many 5-digit <u>odd</u> numbers can be made from the digits $\{0, 1, 2, 3, 4, 5\}$ with no repeating digit?

3. How many 5-digit numbers can be made from the digits $\{0, 1, 2, 3, 4, 5\}$ with no repeating digits and such that the number is <u>divisible by 3</u>?

4. How many ways are there to arrange 7 people (A to G) in a line such that A and B are next to each other?

5. How many ways are there to arrange 7 people (A to G) in a line such that C is anywhere in between A and B? (e.g. <u>A</u>DE<u>C</u>BGF is valid)

6. How many ways can we put 8 <u>numbered</u> marbles into 3 different boxes?

7. How many ways can we put 8 <u>identical</u> marbles into 3 different boxes?

8. How many ways can we put 8 identical marbles into 3 different boxes such that
    a. each box has at least 1 marble?
    b. each box has at least 2 marbles?
    c. the first box has at least 2 marbles?

9. Given a 100×100 grid, a lattice path from $(0, 0)$ to $(100, 100)$ is a path that either moves up or right one step at a time.
    a. How many lattice paths are there from $(0, 0)$ to $(100, 100)$?
    b. How many lattice paths do <u>not</u> pass through $(10, 10)$?
    c. How many lattice paths do <u>not</u> pass through $(20, 50)$?

10. Prove the following using counting in two ways.

$$\binom{3n}{3} = n^3 + 6n\binom{n}{2} + 3\binom{n}{3}$$

11. Prove the following using counting in two ways.

$$n^3 = 6\binom{n}{3} + 6\binom{n}{2} + n$$

12. Let x, y, z be non-negative real numbers. Prove

$$xy + xz \geq x\sqrt{yz}$$

13. Let x, y be real numbers. Prove
$$|x| - |y| \leq |x - y|$$

14. Let x, y, z be non-negative real numbers such that $y + z \geq 2$. Prove
$$(x + y + z)^2 \geq 4yz + 4x$$

15. Let x be a positive real number. Prove
$$x + x^{-1} \geq 2$$

16. What is the maximum area of a rectangle with fixed perimeter p? Explain why.

17. Let a, b, c be integers. Find all integers x such that $x \equiv a \bmod 4$, $x \equiv b \bmod 5$, and $x \equiv c \bmod 9$.