15-151 Math Foundations CS – EXCEL

Topic: **Sets & Functions**
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Session Date: **Thu Sep 20**
Academic Development
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Services available: Supplemental Instruction (SI), Academic Counseling in Study Skills, Individual & Walk-in Tutoring

0. Topic Rundown
   - ✓ De Morgan's laws for logical operators
   - ✓ Law of excluded middle
   - ✓ Converse, inverse, contrapositive
   - ✓ Tautology
   - ✓ Universal quantifier, existential quantifier, de Morgan's laws for quantifiers
   - ✓ Quantifier alternation
   - ✓ Set builder notation
   - ✓ Subset and superset, double containment
   - ✓ Cardinality
   - ✓ Intersection, union, relative complement, complement
   - ✓ De Morgan's laws for sets
   - ✓ Cartesian product
   - ✓ Uniqueness
   - ✓ Domain and codomain
   - ✓ Proper notation of function building
   - ✓ Well-definedness (totality, existence, uniqueness)
   - ✓ Identity function and empty function
   - ✓ Function composition
   - ✓ Image and preimage
   - ✓ Injection, surjection, bijection

I. Warm Up

II. Practice Problems

III. Have Fun

IV. Brain Teasers

- $\sum_{i=1}^{n}(i)(i+1)(i+2)$. Derive a formula. Prove it with and without induction.
- Are there more rational numbers than there are integers? Why or why not?
- How do you land a space shuttle from space?
- Let $R = \{x | x \notin x\}$. Prove or disprove: $R \in R$.

V. Weekly Treat

In each of the next 100 days, you will have one date with a different person. After each date, you can decide whether or not to marry the person. It is guaranteed that person will accept your proposal. But once you decide to marry the person, you will not consider anyone else, and you do not know whether you will find a better partner among the rest of your dates.

Suppose there is a unique "best person" among the 100 candidates.

a. How do you optimize your chance to marry the best person?
b. With your strategy, what is the probability that you eventually marry the best person?

There is an optimal solution to this question. The optimal probability is roughly 37%.

Hint: You can achieve optimal 50% chance when you are expecting 3 dates.