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## 21-127 Concepts of Mathematics – EXCEL

Topic: **Counting**

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Session Date: **Mon Apr 29**

Academic Development

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Services available: Supplemental Instruction (SI), Academic Counseling in Study Skills, Individual & Walk-in Tutoring

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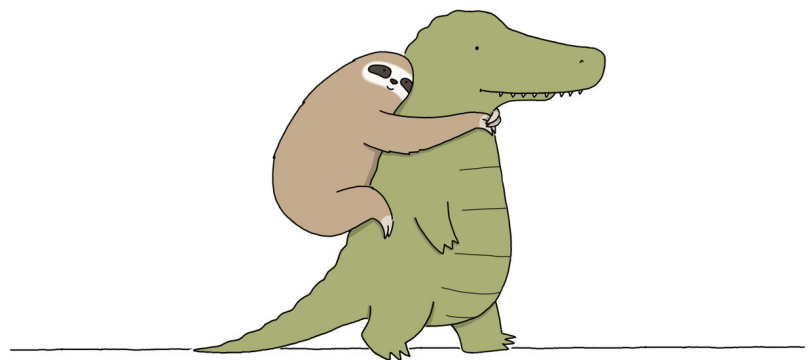
### I. Counting Fest (Timed Quiz!) – 10 minutes

Recall from class: a standard deck consists of 52 cards. Each card is designated by one of the 4 possible suits and one of the 13 possible ranks. A poker hand is a 5-selection from a standard deck.

- a. *One pair* is a hand that contains two cards of the same rank and three cards of three other ranks. How many distinct *one pair* are there?
  
  
  
  
  
  
  
  
  
  
- b. *Two pair* is a hand that contains two cards of one rank, two cards of another rank, and one card of a third rank. How many distinct *two pair* are there?
  
  
  
  
  
  
  
  
  
  
- c. *Three of a kind* is a hand that contains three cards of the same rank and two cards of two other ranks. How many distinct *three of a kind* are there?
  
  
  
  
  
  
  
  
  
  
- d. *Full house* is a hand that contains three cards of one rank and two cards of another rank. How many distinct *full house* are there?
  
  
  
  
  
  
  
  
  
  
- e. How many distinct hands are there?

II. Gotta Count ‘Em All – 20 minutes

- A. How many ways can you choose 4 distinct groups of 4 people from 16 people?
- B. How many ways can you pair up 8 boys and 8 girls?
- C. How many permutations are there of the word “repetition”?
- D. How many 7-digit telephone numbers can be formed if the first digit cannot be 0 or 1?
- E. How many ways can you put 10 indistinguishable desks into 3 offices?
- F. How many numbers can be expressed as a sum of four distinct members of the set  $\{17, 21, 25, 29, 33, 37\}$ ?



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### III. Gotta Count 'Em Twice – counting in two ways

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}, n \in \mathbb{N}, k \leq n$$

$$\sum_{i=1}^n i = \binom{n+1}{2}, n \in \mathbb{N}$$

$$\binom{3n}{3} = 3 \binom{n}{3} + 6n \binom{n}{2} + n^3, n \geq 3$$

$$\sum_{k=0}^n \binom{n}{k} = 2^n, n \in \mathbb{N}$$

$$\sum_{k=0}^n 2^k \binom{n}{k} = 3^n, n \in \mathbb{N}$$

