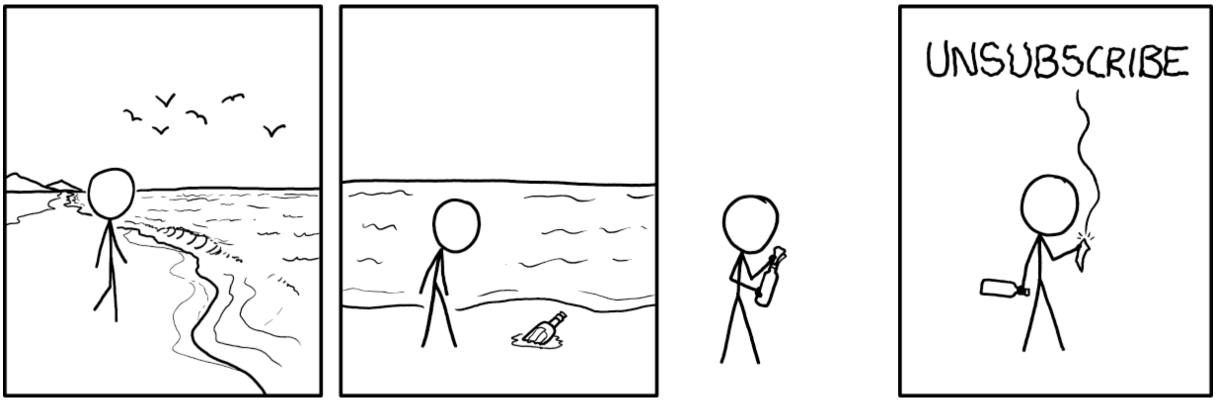
Mathematical Foundations for Computer Science – EXCEL

Relation

Sun 6 / Mon 7 Oct 2019

- Complete mid-semester survey form ~5 mins
- Review induction via a strong induction problem ~10 mins
- Conceptually review relation and equivalence relation ~15 mins
- Relation practice questions ~30 mins

Mid-semester survey form - **https://forms.gle/VenfRvoubSKy5aRc9**



A strong induction problem

The Lucas numbers are defined as $L_1 = 1$, $L_2 = 3$, and $L_n = L_{n-1} + L_{n-2}$ for $n \geq 3$. Show

$$L_n < \left(\frac{7}{4}\right)^n . n \geq 1$$

Relation

Let X, Y be sets. A (binary) **relation** from X to Y is a logical formula $R(x, y)$ with two free variables $x \in$ ______ and $y \in$ ______. We call X ____________ of R and Y ____________ of R.

A relation R is **homogenous** if it has the same ______________________________, in which case we say that R is a relation **on** X.

Given $x \in X$ and $y \in Y$, if $R(x, y)$ then we say ______________________________ and write ____________.

The **graph** of relation $>$ on $[3]$ is ____.

A relation R on X is **reflexive** if ______________________________.

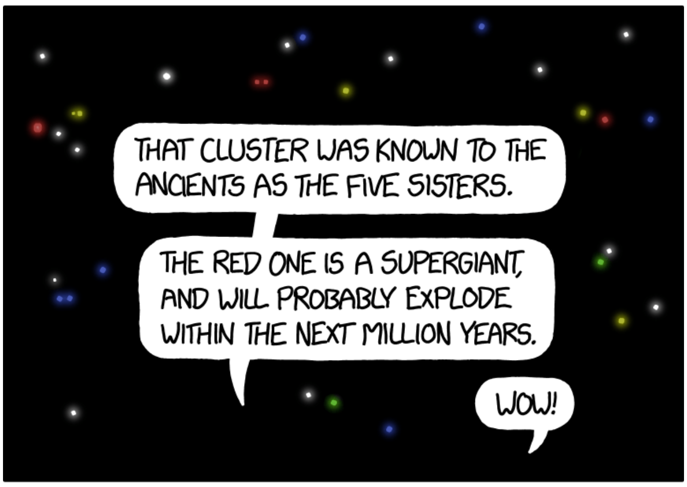
A relation R on X is **symmetric** if ____.

A relation R on X is **antisymmetric** if ____.

A relation R on X is **transitive** if ____.

A relation R on X is an **equivalence relation** if it is ____________________________________.

Let R be an equivalence relation on X. The R-equivalence class of an element $a \in X$ is the set $[a]_R =$ ______________________________.



THERE ARE TOO MANY STATUS LEDs IN MY ROOM.

1. Prove that $\geq$ is a reflexive relation on $\mathbb{Z}$.

2. Prove that $\geq$ is an antisymmetric relation on $\mathbb{Z}$.

3. Prove that $\geq$ is a transitive relation on $\mathbb{Z}$.

4. Prove that "x divides y" is a transitive relation on $\mathbb{Z}$.

5. * The following purports to prove that the reflexivity condition is unnecessary, that is, it can be derived from symmetry and transitivity.
   - Suppose $a\ R\ b$. Then by symmetry, $b\ R\ a$. Since $a\ R\ b$ and $b\ R\ a$, by transitivity, $a\ R\ a$.
   - Therefore, R is reflexive.
   - What is wrong with the argument?

6. * Suppose R is a reflexive relation on X and has the property that for all $a, b, c \in X$, if $a\ R\ b$ and $a\ R\ c$, then $b\ R\ c$. Show that R is an equivalence relation.

7. Let R be a relation on $\mathbb{N}^2$ and $(x_1, y_1)\ R\ (x_2, y_2)$ if and only if $x_1 y_2 = x_2 y_1$. Is R reflexive? Symmetric? Antisymmetric? Transitive?

8. Show the following relation is reflexive, symmetric, and **not** transitive.
$$\{(a, b): a, b \in \mathbb{R}, |a - b| \leq 1\}$$

(what-you-do-**not**-need-to-know-for-the-course-but-I-think-it-is-fun)
The **Golomb sequence** is a non-decreasing integer sequence where a_n is the number of times n occurs in the entire sequence, starting with $a_1 = 1$. In fact, the entire sequence is unique. Find it.

Some miscellaneous review

1. Prove that for every positive integer n
$$\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\cdots+\frac{1}{n^2}\leq 2-\frac{1}{n}$$

2. Prove that function $f: \mathbb{R}^2 \to \mathbb{R}^2$ via $f(x, y) = (x + y, x - y)$ is a bijection.

3. Suppose g is an injective function and f is a surjective function. Is $g\ o\ f$ injective?

4. Prove or disprove: $\mathcal{P}(A \cap B) = \mathcal{P}(A) \cap \mathcal{P}(B)$