Mathematical Foundations for Computer Science – EXCEL

Probability Games
Nov. 24/25

In a TV game show, a contestant selects one of three doors; behind one of the doors there is a price, and behind the other two there are no prizes. After the contestant selects a door, the show host opens one of the remaining doors, revealing that there is no prize behind it. The host then asks the contestant if they want to SWITCH their choice to the other unopened door, or STICK with their original choice. Is it probabilistically advantageous for the contestant to SWITCH, or STICK? Or is the probability of winning the prize the same no matter they SWITCH or STICK?

Three mathematicians enter a room and a red or blue hat is placed on each one's head. The color of each hat is determined (independently) by a fair coin flip. No communication of any sort is allowed, except for an initial strategy discussion before the guessing begins. They can look at the other two's hats color, but not their own's. Then, the mathematicians must simultaneously guess the color of their own hats, or say pass. The goal is to find a strategy such that at least one person guesses correctly and no one guesses incorrectly, with max probability of success.

Ten playing cards numbered from 1 through 10 are randomly shuffled and placed into ten drawers facing down so that each drawer has exactly one card. Ten prisoners are numbered 1 through 10. Now, each prisoner comes to the drawer and picks five drawers (thus five cards) to inspect. Cards are put back into place after each inspection. Communication of any sort is not allowed among the prisoners. If all ten prisoners succeed in finding their own number, they are all granted freedom. However, if any of them fails, they will stay in math prison forever. The goal is to find a strategy that maximizes the probability of all prisoners getting freedom.