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## 15-151 EXCEL

Topic: **Review sets and functions, Integers, Induction basics**  
EXCEL Leader: Sam Yong  
Email: [myong@andrew.cmu.edu](mailto:myong@andrew.cmu.edu)

Session Date: **Sun 22 Sep 19**  
Academic Development  
Cyert Hall B5 | 412-268-6878

Services available: Supplemental Instruction (SI), Academic Counseling in Study Skills, Individual & Walk-in Tutoring

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### 0. Agenda

- 1) Reviewing sets and functions
  - Set equality proofs
  - Well-definedness
  - In-/sur-/bi-jectivity proofs
  - Image/preimage
- 2) Properties of integers
  - Parity and divisions
  - Naturals
  - The Fibonacci numbers
- 3) The concept of induction
  - The principles
  - An example

“The really unusual day would be one where nothing unusual happens.”

– Persi Diaconis

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### 1. Definitions review

[Instruction] Turn to your partner and complete the following together.

- A function  $f: X \rightarrow Y$  is **injective** if \_\_\_\_\_.
- A function  $f: X \rightarrow Y$  is **surjective** if \_\_\_\_\_.
- A function is **bijective** if \_\_\_\_\_.
- Let  $f: X \rightarrow Y$  be a function. A **left inverse** for  $f$  is a function  $g: Y \rightarrow X$  such that  
\_\_\_\_\_.
- Let  $f: X \rightarrow Y$  be a function. A **right inverse** for  $f$  is a function  $g: Y \rightarrow X$  such that  
\_\_\_\_\_.
- Let  $f: X \rightarrow Y$  be a function. An **inverse** for  $f$  is \_\_\_\_\_.
- A function  $f: X \rightarrow Y$  is **injective** if and only if it has a \_\_\_\_\_.
- A function  $f: X \rightarrow Y$  is **surjective** if and only if it has a \_\_\_\_\_.

Let  $f: X \rightarrow Y$  be a function.  $X = \{1, 2, 3\}$ ,  $Y = \{1, 2, 3\}$ .  $f(1) = 1$ ,  $f(2) = 2$ ,  $f(3) = 2$ .

- $f[X] =$  \_\_\_\_\_
- $f[\{2, 3\}] =$  \_\_\_\_\_
- $f^{-1}[Y] =$  \_\_\_\_\_
- $f^{-1}[\{2, 3\}] =$  \_\_\_\_\_
- $f^{-1}[\{3\}] =$  \_\_\_\_\_

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## 2. Some practice problems

[Instruction] Again in your pair, go to a board space for the following practice problems.

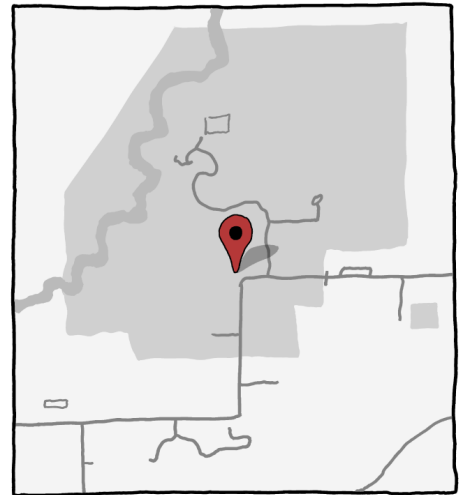
1) Let  $f: X \rightarrow Y$  be a function. Prove  $X \subseteq f^{-1}[f[X]]$ .

2) Let  $f: X \rightarrow Y$  be a function. Prove  $f[f^{-1}[Y]] \subseteq Y$ .

3) Consider function  $f: X \rightarrow X$ . Show that if  $f \circ f$  is injective, then  $f$  is injective.

4) Let  $A, B, C, D$  be sets. Show that if  $A \subseteq B \wedge C \subseteq D$ , then  $A \cap C \subseteq B \cap D$ .

5) Let  $A = \{n: n = 2k + 1, k \in \mathbb{Z}\}$ ,  $B = \{n: n = 2k - 1, k \in \mathbb{Z}\}$ . Prove  $A = B$ .



## CANYON RIVER NUCLEAR LAUNCH FACILITY

What's new about this place?  
It's not.

### REVIEWS (22)

- ★★★★★ GREATEST COUNTRY ON EARTH
- ★★★★☆ LOOKS COOL BUT YOU CAN'T GET IN
- ☆☆☆☆☆ WHAT IS THIS STORE
- ★★★★★ MY COUSIN WORKED HERE
- ★★★★☆ WAITSTAFF HEAVILY ARMED AND VERY RUDE
- ☆☆☆☆☆ STOP DOING CHEMTRAILS
- ☆☆☆☆☆ THIS PLACE IS A SYMPTOM OF THE MILITARY-INDUSTRIAL COMPLEX STRANGLING OUR DEMOCRACY AND... (READ FULL REVIEW-1184 WORDS)
- ★★★★★ ANYONE ELSE NOTICE THE HOLE IN THE WEST FENCE?
- ★★★★★ WHOA, MISSILES!
- ★★★★☆ GOOD IDEA BUT CONFUSING WEB SITE. HOW DO I PREORDER?
- ☆☆☆☆☆ PLEASE DON'T LAUNCH THESE

I LOVE FINDING REVIEWS OF PLACES THAT REALLY DON'T NEED TO HAVE REVIEWS.

### 3. Principle of mathematical induction (weak)

Let  $p(n)$  be logical formula with free variable  $n \in \mathbb{N}$ , and let  $n_0 \in \mathbb{N}$ . If

- i)  $p(n_0)$  is true; and
- ii) For all  $n \geq n_0$ , if  $p(n)$  is true, then  $p(n + 1)$  is true;

then  $p(n)$  is true for all  $n \geq n_0$ .

[Instruction] Identify the following concepts in the principle.

**Base case**

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**Induction step**

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**Induction hypothesis**

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**Induction goal**

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Prove the following formula on  $n \in \mathbb{N}, n \geq 1$ .

$$\sum_{i=1}^n i^2 = \frac{1}{6}n(n+1)(2n+1)$$