15-151 Math Foundations CS – EXCEL

Topic: **Counting Techniques, Principles, Infinite Sets**
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Session Date: **Thu 1 Nov 18**
Academic Development
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"Poirot," I said, "I have been thinking."

"An admirable exercise my friend. Continue it."

– **Agatha Christie**, *Peril at End House*

I. Gotta Count 'Em All (15 minutes)

1. How many ways can you choose 4 distinct groups of 4 people from 16 people?

2. How many ways can you pair up 8 boys and 8 girls?

3. How many permutations are there of the word "repetition"?

4. How many ways can you put 10 indistinguishable desks into 3 offices?

5. How many 7-digit telephone numbers can be formed if the first digit cannot be 0 or 1?

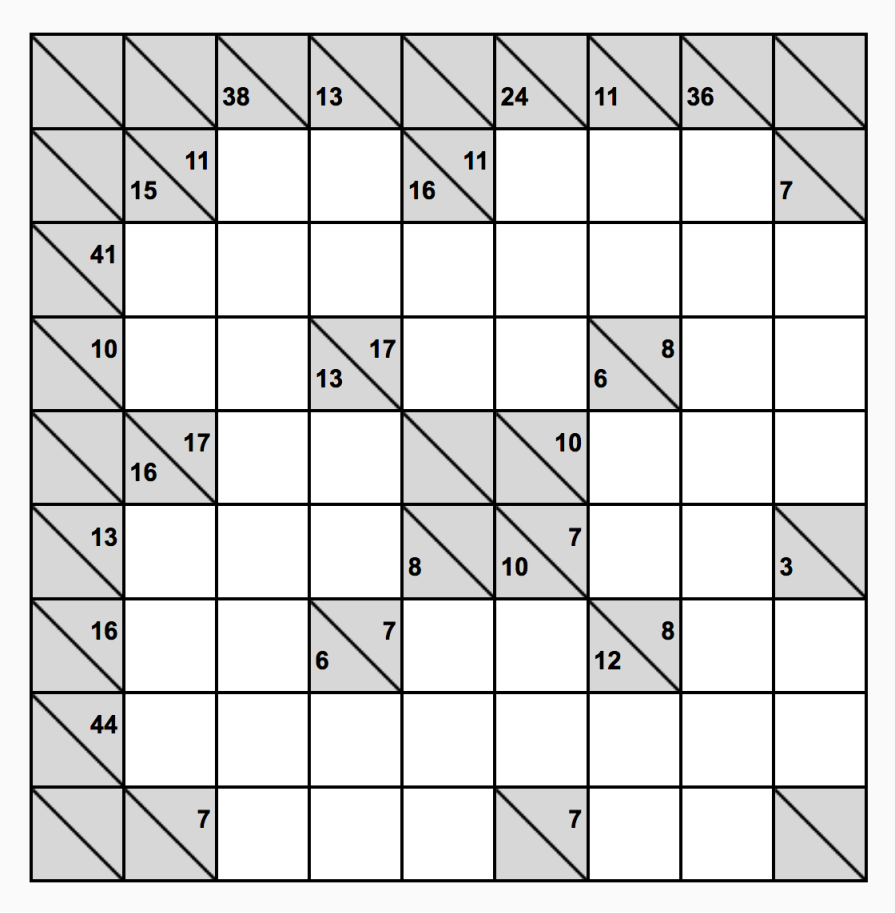
6. * How many numbers can be expressed as a sum of four distinct members of the set $\{17, 21, 25, 29, 33, 37, 41\}$?

II. Infinite Concepts (15 minutes)

1. Prove $\mathbb{N}$ is countably infinite.

2. Prove $\mathbb{Z}$ is countably infinite.

3. * Find a bijection $f: (0,1) \to (0,1]$.



Kakuro

III. An Elegant Proof (10 minutes)

Suppose we have many beads of 3 colors, red, green, and blue. We would like to make a necklace of 5 beads. Investigate how many different necklaces we can make.

We consider necklaces that can be rotated to obtain each other the same, for example, RGGBB is same as GBBRG and only count as one. The only restriction is our necklaces all contain beads of at least 2 distinct colors. We do not make single-color necklaces.

What about making necklaces with 7 beads, from beads of 2 colors?

What about making necklaces with p beads, from beads of a colors?

Use your findings from above, prove a well-known theorem in number theory.

IV. Gotta Count 'Em All, Twice (15 minutes)

1. Prove Pascal's identity:

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

2. * Prove Vandermonde's identity:

$$\sum_{k=0}^{p} \binom{m}{k}\binom{n}{p-k} = \binom{m+n}{p}$$

Reference

- ✓ What does injection, surjection, or bijection imply about the cardinalities of two sets?
- ✓ What is a finite set? An infinite set? A countably infinite set? A countable set?
- ✓ What is an indexed union? An indexed intersection? An indexed product?
- ✓ What is an order-n tuple?
- ✓ What are factorials?
- ✓ What are binomial coefficients?
- ✓ What is a permutation?
- ✓ What is the multiplication principle?
- ✓ What is the addition principle?
- ✓ What is pairwise disjoint?
- ✓ What is a partition?
- ✓ What is the inclusion-exclusion principle?