15-151 EXCEL

Topic: **Review sets and functions, Integers, Induction basics**
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Session Date: **Sun 22 Sep 19**
Academic Development
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Services available: Supplemental Instruction (SI), Academic Counseling in Study Skills, Individual & Walk-in Tutoring

0. Agenda
   1) Reviewing sets and functions
      - Set equality proofs
      - Well-definedness
      - In-/sur-/bi-jectivity proofs
      - Image/preimage
   2) Properties of integers
      - Parity and divisions
      - Naturals
      - The Fibonacci numbers
   3) The concept of induction
      - The principles
      - An example

"The really unusual day would be one where nothing unusual happens."

– Persi Diaconis

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1. Definitions review

[Instruction] Turn to your partner and complete the following together.

- A function $f: X \to Y$ is **injective** if ____.
- A function $f: X \to Y$ is **surjective** if ____.
- A function is **bijective** if ______________________________.
- Let $f: X \to Y$ be a function. A **left inverse** for f is a function $g: Y \to X$ such that ____.
- Let $f: X \to Y$ be a function. A **right inverse** for f is a function $g: Y \to X$ such that ____.
- Let $f: X \to Y$ be a function. An **inverse** for f is ___________________________________.
- A function $f: X \to Y$ is **injective** if and only if it has a _______________.
- A function $f: X \to Y$ is **surjective** if and only if it has a _______________.

Let $f: X \to Y$ be a function. $X = \{1, 2, 3\}, Y = \{1, 2, 3\}$. $f(1) = 1, f(2) = 2, f(3) = 2$.

- $f[X] =$ _______________
- $f[\{2, 3\}] =$ _______________
- $f^{-1}[Y] =$ _______________
- $f^{-1}[\{2, 3\}] =$ _______________
- $f^{-1}[\{3\}] =$ _______________

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2. Some practice problems

[Instruction] Again in your pair, go to a board space for the following practice problems.

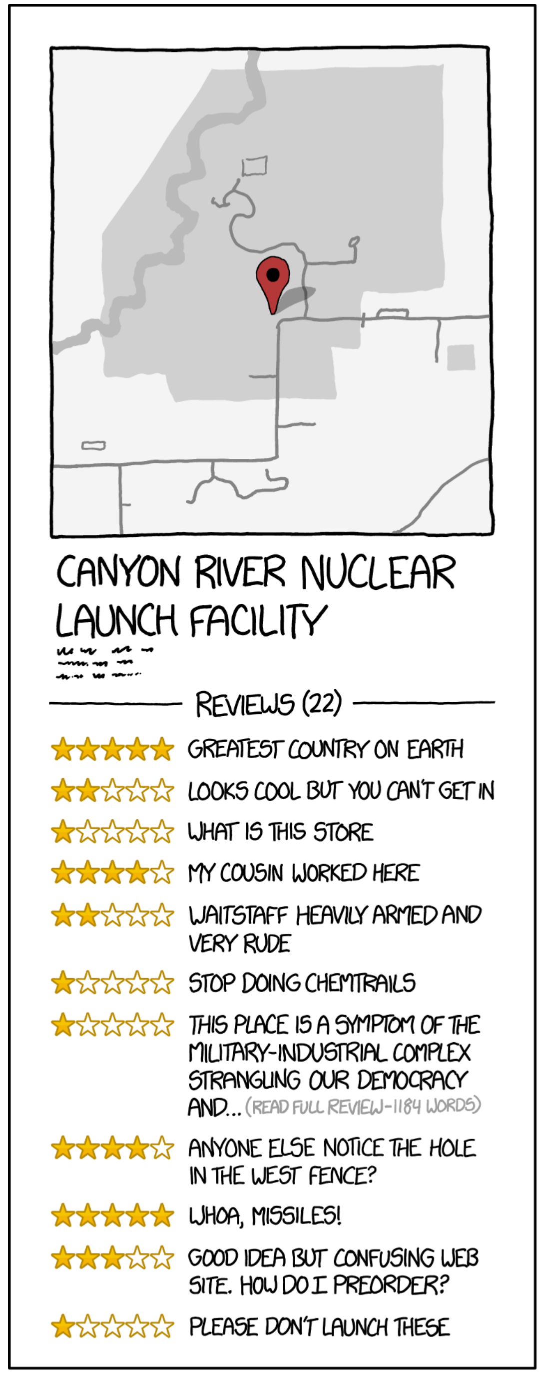
1) Let $f: X \to Y$ be a function. Prove $X \subseteq f^{-1}[f[X]]$.

2) Let $f: X \to Y$ be a function. Prove $f[f^{-1}[Y]] \subseteq Y$.

3) Consider function $f: X \to X$. Show that if $f \circ f$ is injective, then f is injective.

4) Let A, B, C, D be sets. Show that if $A \subseteq B \land C \subseteq D$, then $A \cap C \subseteq B \cap D$.

5) Let $A = \{n: n = 2k + 1, k \in \mathbb{Z}\}, B = \{n: n = 2k - 1, k \in \mathbb{Z}\}$. Prove $A = B$.



I LOVE FINDING REVIEWS OF PLACES THAT REALLY DON'T NEED TO HAVE REVIEWS.

[https://xkcd.com/1803/]

3. Principle of mathematical induction (weak)

Let $p(n)$ be logical formula with free variable $n \in \mathbb{N}$, and let $n_0 \in \mathbb{N}$. If

i) $p(n_0)$ is true; and

ii) For all $n \geq n_0$, if $p(n)$ is true, then $p(n+1)$ is true;

then $p(n)$ is true for all $n \geq n_0$.

[Instruction] Identify the following concepts in the principle.

Base case ______________________________

Induction step ______________________________

Induction hypothesis ______________________________

Induction goal ______________________________

Prove the following formula on $n \in \mathbb{N}, n \geq 1$.

$$\sum_{i=1}^{n} i^2 = \frac{1}{6} n(n+1)(2n+1)$$