Session 7 Group Problem Set

Version B

EXCEL 127 – 1 April 2019

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2.1   Cardinality of the set $\{1, 3, 4, 7, 11\}$

2.2   Cardinality of $[37]$

2.3   For an odd natural number k the cardinality of $\{n \in \mathbb{N}: n \text{ is odd} \wedge 0 < n \leq k\}$

2.4   Compare the cardinality of $\mathbb{N}$ and the cardinality of $\{n \in \mathbb{N}: n \text{ is odd}\}$

2.5*   Show that $|\mathbb{N} \times \mathbb{N}| \leq |\mathbb{N}|$. (Hint: Construct injection. Consider prime factorization.)

2.6   Show that $S = \{n \in \mathbb{N}: n \text{ is prime} \wedge n < 25\}$ is finite.

2.7   Construct $S \subseteq \mathbb{N}$ such that
   a) S is finite.
   b) S is countably infinite.
   c) S is countably infinite and S^C is finite.
   d) S is countably infinite and S^C is countably infinite.

2.8   Prove that if $S \subseteq \mathbb{N}$ is finite, $\mathbb{N} - S$ is infinite.

2.9*   Construct $S, T \in \mathbb{R}$ such that S, T are both uncountable and $S \cap T$ is countable.

2.10   Let S, T be finite sets. $S \cap T = \emptyset$. Show that $S \cup T$ is finite.