21-127 Concepts of Mathematics – EXCEL

Topic: **Math, Sets, Logic**
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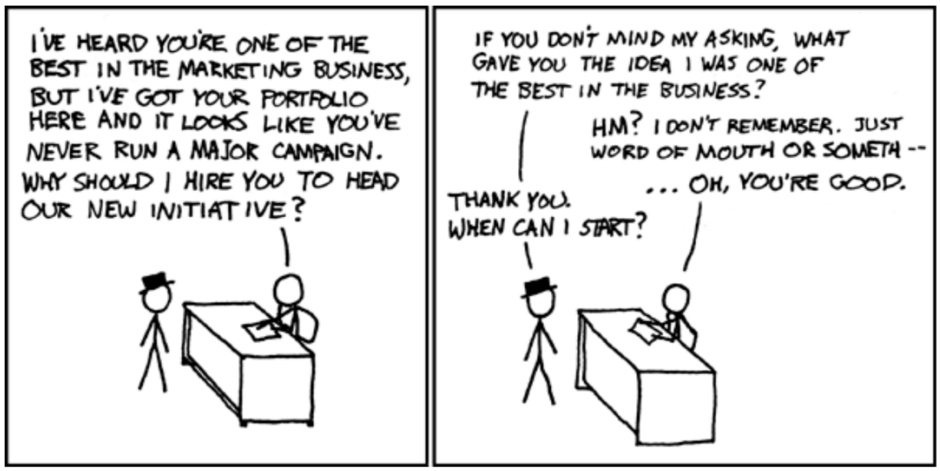

"There are a lot of books on marketing out there. I wonder if you're safest just buying the most popular one."

- A **set** is a collection of all objects that have a common, well-defined property. The objects contained in a set are called _____ of the set. The mathematical symbol _____ represents the phrase "is an element of", and the symbol _____ represents "is not an element of".
- The **empty set** is the set which has no elements. It is denoted by the symbol _____.
- The set of all natural numbers is denoted by _____=_____.
- The notation $[n]$ represents the set _____.
- The set of all integers is denoted by _____=_____.
- The set of all real numbers is denoted by _____, and the set of all complex numbers _____.
- Given two sets A and B, if every element of A is also an element of B, then we say _____ and the mathematical notation is _____. If we want to indicate that, in addition, A is not equal to B, we would write the notation _____ and say that A is a _____.
- Given a set A, the **power set** of A is _____. It is denoted by _____.

- We say two sets A and B are equal if and only if _____.
- Let A and B be any sets. The **intersection** of A and B is the set of elements that _____. It is denoted by _____. Formally, _____=_____. A and B are said to be **disjoint** if _____.
- Let A and B be any sets. The **union** of A and B is the set of elements that _____. It is denoted by _____. Formally, _____=_____.
- The **difference** between sets A and B, denoted by _____, is the set _____. Formally, _____=_____.
- The **complement** of set A is the set _____, denoted by _____. Formally, _____=_____.
- The union of a collection of sets A_i indexed by the set I is _____.
- The intersection of a collection of sets A_i indexed by the set I is _____.
- Given two sets A and B, the **Cartesian product** of A and B denoted by _____ is defined as _____.

- The **universal quantifier** is the symbol _____, meaning the phrase _____.
- The **existential quantifier** is the symbol _____, meaning the phrase _____.
- Let P and Q be mathematical statements. Consider the original claim $P \Rightarrow Q$. The **converse** of the original claim is _____. The **contrapositive** is _____.
- Let P and Q be mathematical statements. We write _____ to indicate they are **logically equivalent**. This is also called _____.
- Let P and Q be mathematical statements. Suppose $P \Rightarrow Q$. We say _____ is a **sufficient** condition for _____, and _____ is a **necessary** condition for _____.

1. Show the following statements are logically equivalent. p, q, r are logical variables.

   $p \wedge (q \vee r)$

   $(p \wedge q) \vee (p \wedge r)$

2. Show the following statements are logically equivalent. p, q, r are logical variables.

   $\neg(p \wedge q)$

   $\neg p \vee \neg q$

3. Negate the following mathematical statements.

   - $\forall x \in \mathbb{N}. x^2$ is even $\Rightarrow x$ is even.

   - $\exists x \in \mathbb{Z}. \forall y \in \mathbb{Z}. x + y = y.$

4. Prove the following statements.

- $(A \cup B)^C = A^C \cap B^C$

- $(A \cup B) \cap A^C = B \backslash A$

5. Let S be a non-empty set of people in a bar. Express the following statement symbolically.

There is a person in the bar such that, if that person is drinking, then everyone else in the bar is drinking.

Prove that it is true.

6. Let $A = \{1, 2, 3\}, B = \{4, 5\}$. Write the following sets to show all the elements.

- $A \cup B =$
- $A - B =$
- $A \times B =$

Space below is for your discussion on the mathematical games. You are in either group 1 or 2.

_____ Group 1: the dollar game

_____ Group 2: the flipping card game