

# Mathematical Foundations for Computer Science – EXCEL

## Counting Part 2

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### Agenda

- Counting poker hands
- Counting in two ways
- Infinite sets counting



A standard deck consists of 52 cards. Each card is designated by one of the 4 possible suits and one of the 13 possible ranks. A poker hand is a 5-selection from a standard deck. (~10min)

1. **One pair** is a hand that contains two cards of the same rank and three cards of three other ranks. How many distinct **one pairs** are there?
2. **Two pair** is a hand that contains two cards of one rank, two cards of another rank, and one card of a third rank. How many distinct **two pairs** are there?
3. **Three of a kind** is a hand that contains three cards of the same rank and two cards of two other ranks. How many distinct **threes of a kind** are there?
4. **Full house** is a hand that contains three cards of one rank and two cards of another rank. How many distinct **full houses** are there?

(~5min)

5.  $n \in \mathbb{N}^+$

$$\sum_{k=0}^n \binom{n}{k} = 2^n$$

(~7min)

6.  $n \in \mathbb{N}^+$

$$n2^{n-1} = \sum_{k=1}^n \binom{n}{k} k$$

Puzzle 1.

Hard

6	2	7	3	5				
		3		6				
	9						1	
	7		9	4	6			
3						7	4	
		8			3	1		
4								8
				1		4		9
							6	

Puzzle 2. Prove  $3 < \pi < 4$  visually.

7. Prove  $|\mathbb{N}| = |\mathbb{N} \times \mathbb{N}|$
  8. Prove  $|(0, 1)| = |(0, 1]|$
  9. Show that the set of all functions  $f: [n] \rightarrow \mathbb{N}$  is countable.
  10. Show that the set of all functions  $f: \mathbb{N} \rightarrow \mathbb{N}$  is uncountable.
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