21-127 EXCEL
10 February 2020

0. Let A, B, C, D be sets. Show that if $A \subseteq B \wedge C \subseteq D$, then $A \cap B \subseteq B \cap D$.

1. Let $A = \{n: n = 2k + 1, k \in \mathbb{Z}\}, B = \{n: n = 2k - 1, k \in \mathbb{Z}\}$. Show that $A = B$.

2. Let X, Y be sets. Prove that $X \subseteq Y$ if and only if $X \cap Y = X$.

3. * Let $R = \{x: x \notin x\}$. Prove or disprove $R \in R$.

4. Negate the following statement: $\forall x \in \mathbb{N}. x^2$ is even $\Rightarrow x$ is even.

5. Let there be a non-empty set of people in the bar. Prove the following statement: *There is a person in the bar such that, if that person is drinking, then everyone else in the bar is drinking.*

6. * The Golomb sequence is a non-decreasing integer sequence where a_n is the number of times that n occurs in the sequence, starting with $a_1 = 1$. Find a_1 to a_{20}.

7. Show that $\cup_{n\in\mathbb{N}} \{1, \dots, n\} = \mathbb{N}$.

8. Find the size of the set $\cap_{n\in\mathbb{N}} \{1, \dots, n\}$.

9. A set S is a rectangle if S can be written as $S = (a, b)\times(c, d)$ where $a, b, c, d \in \mathbb{R}$. Prove that the intersection of two rectangles is a rectangle.

10. * Suppose the circumference of a unit circle is 2π and the area is π. Use geometric construction to show that $3 < \pi < 4$.

11. Symbolically express the following statement: *A natural number's largest divisor is itself.* (hint: to say a is a divisor of b we can write $a|b$)

12. * Let S be a finite set of at least two points in the plane. Assume that no three points in S are collinear. By a <u>windmill</u> we mean a process as follows. Start with a line ℓ going through a point $P \in S$. Rotate ℓ clockwise around the <u>pivot</u> P until the line contains another point $Q \in S$. Now, the point Q takes over as the new pivot. This process continues indefinitely, with the pivot always being a point from S. Show that for a suitable $P \in S$ and a suitable starting line containing P, the resulting windmill will visit each point of S as a pivot infinitely often.

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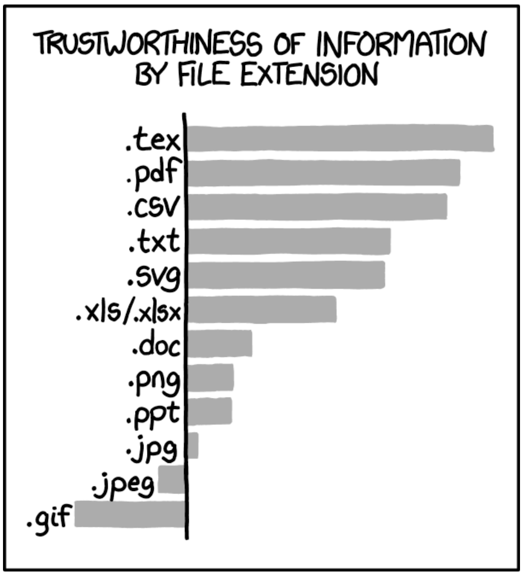

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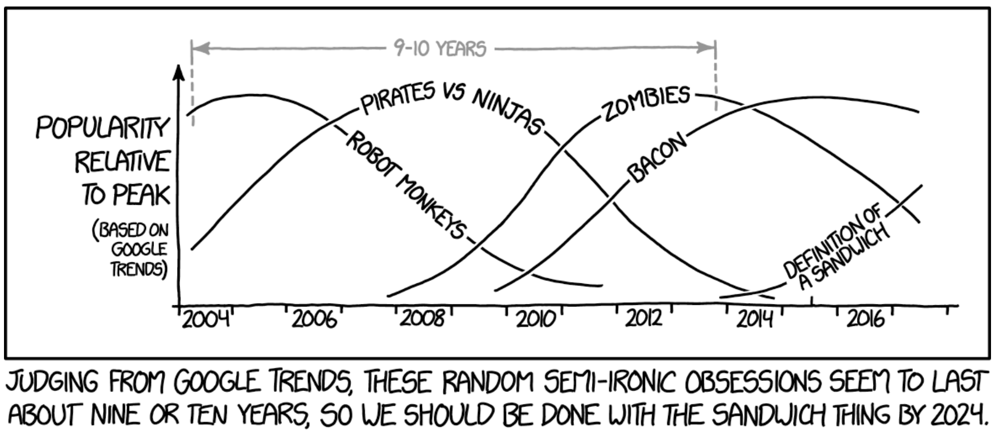

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