
15-151 Math Foundations CS – EXCEL

Topic: **Number Sets, Proof Techniques, Induction, Logic, Sets**
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Session Date: **Thu Sep 13**
Academic Development
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Services available: Supplemental Instruction (SI), Academic Counseling in Study Skills, Individual & Walk-in Tutoring

I. Warm Up [20]

1. Define the following terms in mathematical context. [3]

- a. Proposition
- b. Theorem
- c. Lemma
- d. Corollary

2. Number sets. [3]

- a. A set is a _____ of objects. The objects in the set are called _____ of the set. If X is a set and x is an object, then we write _____ to denote the assertion that x is an – of X .
- b. Describe the following sets.
 - \mathbb{N}
 - \mathbb{Z}
 - \mathbb{Q}
 - \mathbb{R}

3. Explain these proof techniques. [3]

- a. Proof by Contradiction
- b. Proof by Cases

4. Induction. [5]

- a. What is weak induction on natural numbers?
- b. What is strong induction on natural numbers?
- c. What is “strong” about strong induction?

5. Logic and sets. [8]

- a. Define the following terms.
 - Conjunction
 - Disjunction
 - Negation
 - Implication
 - Bi-implication
 - Exclusive or
 - Contrapositive of a proposition
 - Converse of a proposition
 - Tautology
- b. What is the universal quantifier?
- c. What is the existential quantifier?
- d. The empty set, denoted _____, is the set with _____ elements.
- e. What is the set-builder notation?
- f. Let X and Y be sets. We say X is a subset of Y if _____. We abbreviate this proposition by writing X _____ Y , and we write _____ for its negation.
- g. What is the power set of $\{2, 3\}$?
- h. What is the power set of $\{\emptyset, \{\emptyset\}\}$?

II. Practice [35]

1. (Induction) Prove the given formula on natural numbers. [7]

$$\sum_{i=1}^n i^2 = \frac{1}{6}n(n+1)(2n+1)$$

2. (Sets) Let X and Y be sets. Prove that $X \subseteq Y$ if and only if $X \cap Y = X$. [5]
3. (Sets) Let X and Y be sets. Prove that $X \subseteq Y$ if and only if $X \cup Y = Y$.
4. (Sets) Let X be a set. Prove that $X \times \emptyset = \emptyset$.

5. (Number sets) Prove $\sqrt{3}$ is irrational. [[5]]

6. (Number sets) Is natural numbers closed under addition? Subtraction? Multiplication? Division?

7. (Number sets) Is integers closed under addition? Subtraction? Multiplication? Division?

8. (Integers) (*Hard) Find all integers x such that the following is an integer. [7]

$$\frac{x^6 - 3}{x^2 + 2}$$

9. (Integers) Prove that if for some integers a, b, c we have $9|a^3 + b^3 + c^3$, then at least one of the numbers a, b, c is divisible by 3. [5]

10. (Integers) (Take home) Prove that if for integers a, b we have $7|a^2 + b^2$, then $7|a$ and $7|b$.

11. (Integers) Show that for all positive integers n , the number $4^n - 3n + 8$ is divisible by 9. [5]