Topic: **Relation, Probability**
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Just keep swimming,
Just keep swimming,
Just keep swimming, swimming, swimming.
What do we do,
We swim, swim, swim.

– Finding Nemo

1. Distance can be measured. _______________ cannot.

Noise in a communications channel can cause errors in the transmission of binary digits.

Transmit:	*1*	*1*	*0*	*0*	*1*	*0*	*1*
Receive:	*1*	*<u>0</u>*	*0*	*0*	*1*	*<u>1</u>*	*1*

For some types of information, errors can be detected and corrected (below) but not in others (above).

Transmit: *Come to my house at…*

Receive: *Come <u>tc</u> my <u>houzx</u> at…*

In binary error correcting codes, only certain binary sequences (called *code words*) are transmitted. This is similar to having a *dictionary* of allowable words (like above). After transmission over a noisy channel, we can check and see if the received binary sequence is in the dictionary of code words and if not, choose the code word most similar to what we received.

Nature's error correcting code is a mapping of RNA sequences to proteins. Several similar RNA sequences often map to the same amino acid, for example, both UAU, UAC map to Tyrosine.

Here is a simple example of error correcting code on a single binary digit.

Suppose the transmission channel is very noisy. Every bit transmitted has a 20% probability of being flipped. Suppose information is transmitted such that the transmissions of different bits are independent.

The probability that a single bit 0 encoding 0 is correctly received is _______________.

The probability that a single bit 1 encoding 1 is correctly received is _______________.

Now, employing error correcting code…

The probability that a code word 000 encoding 0 is correctly received is _______________.

The probability that a code word 111 encoding 1 is correctly received is _______________.

This looks awful, but the full story is…

The *Hamming distance* between two strings of equal length is the number of positions at which the corresponding symbols are different. In other words, it measures the minimum number of *substitutions* required to change one string into the other.

The Hamming distance between code word 000 and invalid word 001 is __________.

The Hamming distance between code word 111 and invalid word 001 is __________.

The probability that 001 results from errors in the transmission of 000 is _______________.

The probability that 001 results from errors in the transmission of 111 is _______________.

The invalid word 001 most likely results from errors in the transmission of __________.

Similarly, the invalid words 100, 010 most likely result from errors in the transmission of _____.

The invalid words 110, 101, 011 most likely result from errors in the transmission of _____.

Now, suppose we presume the original information is 0 when we receive 000, 001, 100, 010, and presume the original information is 1 when we receive 111, 110, 101, 011.

- What is the probability that the original information 0 (encoded as 000) is incorrectly received and recognized as 1?

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2. A map, it is said, organizes _______________.
   - In pairs, draw a concept map for Clive Notes chapter 5 on Relation, on one of the whiteboards.
   - Be sure to identify key definitions and useful examples.

The *Golomb sequence* is a non-decreasing integer sequence where a_n is the number of times n occurs in the sequence, starting with $a_1 = 1$. In fact, a_n is uniquely defined for all $n > 1$. Find the sequence.

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3. There is little correlation between _______________ and _______________.

3.1. Suppose R is an equivalence relation on set A. Then for all $a, b \in A$, show that the following statements are equivalent.

a) $a\ R\ b$

b) $[a] \cap [b] \neq \emptyset$

c) $[a] = [b]$

3.2. The following purports to prove that the reflexivity condition is unnecessary, that is, it can be derived from symmetry and transitivity.

- Suppose $a \sim b$. By symmetry, $b \sim a$. Since $a \sim b$ and $b \sim a$, by transitivity, $a \sim a$.
- Therefore, $\sim$ is reflexive.
- What's wrong with this argument?

3.3. Suppose R is a relation on A that is reflexive and has the property that for all a, b, c, if $a\ R\ b$ and $a\ R\ c$, then $b\ R\ c$. Show that R is an equivalence relation.

3.4. Let R be a relation defined on $\mathbb{Z}$ via $a\ R\ b$ if $a \neq b$. Is R symmetric? Transitive?

3.5. Show that $[2^7] = [2]$ in $\mathbb{Z}_7$.

3.6. Let $\sim$ be a relation on $\mathbb{Z}^+$ via $a \sim b$ if a divides b. Show that $\sim$ is a partial ordering.

4. The best way to predict the future is to ________________________.

4.1. A fair six-sided dice is rolled three times. What is the probability that the sum of the dice rolls is less than or equal to 12, given that each dice roll shows a power of 2?

4.2. You go to see the doctor about an ingrowing toenail. The doctor selects you at random to have a blood test for swine flu, which for the purpose of this exercise we will say is currently suspected to affect 1 in 10,000 people in Australia. The test is 99% accurate, in the sense that the probability of a false positive is 1%. The probability of a false negative is zero. You test positive. What is the new probability that you have swine flu?

4.3. Now imagine that you went to a friend's wedding in Mexico recently, and (for the purpose of this exercise) it is known that 1 in 200 people who visited Mexico recently come back with swine flu. Given the same test result as above, what should your revised estimate be for the probability you have the disease?

4.4. You toss a fair coin three times. Given that you observe at least one head, what is the probability that you observe at least two heads.

5. The biggest risk a person can take is to ________________________.

Three mathematicians enter a room and a red or blue hat is placed on each person's head. The color of each hat is determined by (an independent) coin toss. No communication of any sort is allowed, except for an initial strategy session before the game begins. Once they have had a chance to look at the other hats (but not their own), the mathematicians must simultaneously guess the color of their own hats or pass. The puzzle is to find a group strategy that maximizes the probability that at least one person guesses correctly and no one guesses incorrectly.

The naïve strategy would be for the group to agree that one person should guess and the others pass. This would have probability 1/2 of success. Find a strategy with a greater chance for success.

(https://www.nytimes.com/2001/04/10/science/why-mathematicians-now-care-about-their-hat-color.html)

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