

# EX 753 Digital Signal Processing

## Lab 6 : Design of FIR Digital Filters

### Background:

In previous experiment, we discussed the most commonly used techniques for design of IIR filters based on transformations of continuous-time IIR systems into discrete time IIR systems. The major difficulty lies in the implementation of the non-iterative direct design method for IIR filters. However FIR filters are almost entirely restricted to discrete-time implementations. Thus the design techniques for FIR filters are based on directly approximating the desired frequency response of the discrete time system. Furthermore, most techniques for approximating the magnitude response of the FIR system assume a linear phase constraint; thereby avoiding the problem of spectrum factorization that complicates the direct design of IIR filters.

The simplest method of FIR design is called the window method. This method generally begins with an ideal desired frequency response  $H_d(e^{j\omega})$ . The impulse response  $h_d[n]$  of the filter exhibiting this desired frequency response can be obtained from inverse Fourier transform of  $H_d(e^{j\omega})$ . However this impulse response exists for  $n = -\infty$  to  $+\infty$  and hence truncation is needed to make the finite duration impulse response. The truncation is similar to the multiplication of the  $h_d[n]$  with the window function  $W[n]$ . The multiplication in discrete time domain is equivalent to the convolution of the two in frequency domain, which actually gives the frequency response of the truncated FIR filter. The design of FIR filters by windowing method assumes that the rectangular window provides the best mean square approximation to a desired frequency response for a given value of  $M$ . That is

$$h[n] = \begin{cases} h_d[n] & 0 \leq n \leq M \\ 0 & \text{otherwise} \end{cases},$$

minimizes the expression

$$\epsilon^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} |H_d(e^{j\omega}) - H(e^{j\omega})|^2 d\omega.$$

The rectangular window exhibits the most abrupt changes while approaching to zero at each end. For the specific value of length of the window, the rectangular window exhibits main lobe with the greatest width and the lowest side lobe attenuation, than the other windows like Bartlett, Hanning, Hamming, Blackman etc that have different tapering of the window to zero at each end. Depending upon the tapering of window, the nature of windows differs. There are also adjustable windows like Kaiser windows whose windowing function  $W[n]$  can be adjusted by changing the value of parameter  $\beta$  according to the stop band attenuation  $A$  as given below:

$$\beta = \begin{cases} 0.1102(A - 8.7), & A > 50, \\ 0.5842(A - 21)^{0.4} + 0.07886(A - 21) & 21 \leq A \leq 50, \\ 0, & A < 21, \end{cases} \quad \text{where } A = -20 \log_{10} \delta.$$

The length of the Kaiser window is given by:

$$M = ((\delta - 7.95)/(14.36 * \Delta w)) + 1, \text{ where } \Delta w = (w_s - w_p)/2.$$

The mean square approximation criterion to a desired frequency response in FIR leads to adverse behavior at discontinuities of  $H_d(e^{j\omega})$ . The window method doesn't permit individual control over the approximation errors in different bands. For many applications, better filters result from minimization of maximum error or a frequency weighted error criterion. Such design can be achieved using algorithmic techniques. One of

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such algorithmic techniques is Remez Exchange Algorithm.

MATLAB Signal Processing Toolbox provides built-in functions to determine and plot the different types of windowing functions and their frequency responses.

For generating the windows, functions like `bartlett()`, `hamming()`, `hanning()`, `blackman()`, `kaiser()` etc. are available.

The functions `fir1()` and `fir2()` can be used for Window-based FIR filter design.

For Remez Exchange Algorithm, MATLAB provides the built in function called `remez()` / `firpm()` to determine filter coefficients.

The frequency response of filter can be obtained from `freqz()`.

Filters and Windows can be analyzed using `fvtool()` and `wvtool()` respectively.

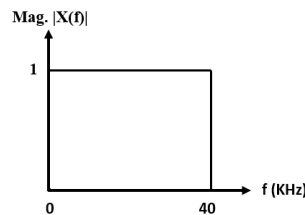
Use MATLAB 'help' for further information of the functions.

### Problems:

1. Design an FIR linear phase digital filter approximating the ideal frequency response

$$H(e^{j\omega}) = \begin{cases} 1 & \text{for } \omega \leq \frac{\pi}{6} \\ 0 & \text{for } \frac{\pi}{6} < \omega \leq \pi \end{cases}.$$

- (a) Plot the window function for Hamming and its frequency response for length of  $M = 31$ .
  - (b) Using the Hamming window plot the frequency response of the truncated FIR filter.
  - (c) Repeat parts (a) and (b) for the Hanning, Blackman and Bartlett windows.
  - (d) Repeat parts (a), (b) and (c) for filter length of  $M = 61$ .
2. Discuss the effects of different types of the windowing functions on the frequency response of the FIR filter. Carry out the comparative study on the basis of the peak side-lobe level, the approximate transition width of the main lobe etc. Also discuss on the effects of increasing value of  $M$ .
  3. An analog signal  $x(t)$  consists of the sum of two components  $x_1(t)$  and  $x_2(t)$ . The spectral characteristics of  $x(t)$  is shown in figure below. The signal  $x(t)$  is band limited to  $40\text{kHz}$  and is sampled at the rate of  $100\text{kHz}$  to yield the sequence  $x[n]$ .

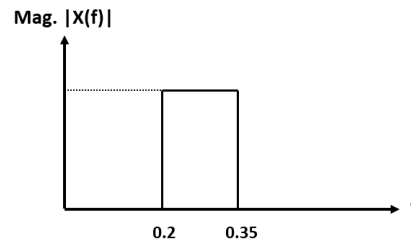


It is desired to suppress the signal  $x_2(t)$  by passing the sequence  $x[n]$  through a digital low pass filter. The allowable distortion on  $X_1(f)$  is  $\pm 2\%$  ( $\delta_1 = 0.02$ ) over the range  $0 \leq f \leq 15\text{ kHz}$ . Above  $20\text{ kHz}$ , the filter must have an attenuation of at least  $40\text{ dB}$  ( $\delta_2 = 0.01$ ).

- (a) For obtaining the filter with above specifications, use the Kaiser window and determine the length of the required window. Plot the frequency and impulse response of the filter.
- (b) If the same filter is to be designed using Hamming window what would be the length of the required window. Plot the frequency and impulse response of the filter.

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4. A band pass filter of length  $M = 32$  is desired to exhibit the magnitude response as shown in figure below:



- (a) Design the filter using the Linear Phase FIR design technique given by Remez Exchange Algorithm. Plot the magnitude response of the filter and determine the attenuation in the stop bands.
  - (b) Observe the response using the same algorithm for the stop band magnitudes of 0.1.
  - (c) Design the above filter using the windowing method. Use Boxcar, hamming, Hanning and Bartlet windows. Compare the responses of each type to that of the Remez Algorithm. Comment on the result.
  - (d) Observe the response of the filter in all the cases for  $M = 64$ .
5. Try above problems using MATLAB's Filter Design and Analysis Tool (FDATool). Observe the results.