EX 753 Digital Signal Processing

Lab 5 : Design of IIR Digital Filters

Background:

There are several methods that can be used to design digital filters having an infinite duration unit sample response. One of the popular methods is based on converting an analog filter into a digital filter. In this method we begin the design of digital filter in the analog domain and then convert the design into the digital domain. For this purpose, depending on the specifications of the required digital filter the various approximations like *butterworth*, *chebyshev*, *chebyshev2,* and *elliptic* filters are used.
Among the different approaches used in the design of digital IIR filters this lab session deals with the,

1. Impulse Invariance method.
2. Bi-Linear Transformation.

In Impulse Invariance method, the objective is to design an IIR filter having an unit sample response $\mathbf{h}[\mathbf{n}]$ that is the sampled version of the impulse response of the analog filter.That is

$$\mathbf{h}[\mathbf{n}] = \mathbf{T_d}\mathbf{h}(\mathbf{nT_d}) \text{ where } \mathbf{n} = 0, 1, 2, \ldots\ldots, \text{and } \mathbf{T_d} \text{ is the sampling interval.}$$

In Bi-Linear transformation a conformal mapping from $\mathbf{s}$ plane to $\mathbf{z}$ plane is carried out with the relation

$$\mathbf{s} = \frac{\mathbf{2}}{\mathbf{T}}\left(\frac{\mathbf{1}-\mathbf{z}^{-1}}{\mathbf{1}+\mathbf{z}^{-1}}\right).$$

For the design of IIR digital filters, there are built-in functions in MATLAB such as `impinvar()`, `bilinear()`, `butter()`, `cheby1()`, `cheby2()`, `ellip()`, `buttord()`, `cheb1ord()`, `cheb2ord()`, `ellipord()`.
Given, the coefficients of the numerator and denominator, the frequency response of analog and digital filters can be obtained from functions `freqs()` and `freqz()` respectively.
For plotting the impulse responses for analog and digital filters, refer to the functions `impulse()` and `dimpulse()`. `fvtool()` can be also be used to analyze digital filter.
For the details of these functions use MATLAB help.

Problems:

1. The analog filter is given as :

$$H_a(s) = \frac{s+0.1}{(s+0.1)^2+9}.$$

   (a) Convert this analog filter into a digital IIR filter by means of the *impulse invariance* method.
   (b) Plot the frequency response (magnitude) of the designed filter taking sampling interval (T) of 0.1 and 0.5 seconds. Compare the response of the filter designed to that of the analog one. Comment on the effect of T on the response.
   (c) Compare the unit sample response of the designed digital IIR filter with the impulse response of analog filter for $T = 0.1$ and 0.5.
   (d) Convert the above analog filter in to a digital IIR filter by means of *bilinear transformation*.
   (e) Repeat 1.(b) and 1.(c) for this designed filter.

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2. An IIR digital low pass filter is required to meet the following specifications:

   (a) Pass band ripple (or peak to peak ripple): $\leq$ 0.5 dB,

   (b) Passband edge: 1.2 kHz,

   (c) Stopband attenuation:$\geq$ 40 dB,

   (d) Stopband edge: 2.0 kHz, and

   (e) Sample rate: 8.0 kHz.

   Use the MATLAB Signal Processing Toolbox functions to determine

   (a) The required filter order,

   (b) The cutoff frequency,

   (c) The numerator and the denominator coefficients

   for the *digital Butterwoth*, *digital Chebyshev* and *digital Elliptic* filters. Also plot their frequency responses. Describe the nature of each response.

3. MATLAB comes with the Filter Design and Analysis Tool (FDATool). It is a Graphical User Interface (GUI) that allows us to design or import, and analyze digital IIR and FIR filters. FDATOOL in matlab can be launched from command window by command `fdatool` (or in newer version of matlab by `filterDesign`). For details see MATLAB help.

   Now design the digital filter for the specifications as given in (2) using this toolbox. Determine the required filter order, the cutoff frequency, and the numerator and the denominator coefficients. Observe the frequency responses and compare then with that in (2).