Sanuj Shakya
IoE, Pulchowk Campus

Lab Module 2

Initial report must include derivations of Fourier series coefficients and MATLAB programs for each of the problems given below. The graphical representation of each signal must be included in the final report along with necessary discussions. Use *xlabel ()*, *ylabel ()* and *title ()* commands to label the axes and add title to each graph.

Section A

Continuous Time Fourier Series Representation

According to Fourier series representation, a continuous time periodic signal *x(t)*, with fundamental period *T* and fundamental frequency $\omega_o = 2\pi/T$, can be represented as a linear combination of harmonically related complex exponential signals with common period *T*. The Fourier series pair for a CT periodic signal *x(t)* with fundamental period *T* and Fourier series coefficients a_k is given by:

$$x(t) \xleftrightarrow{F.S.} a_k$$

Synthesis equation:

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_o t}$$

Analysis equation:

$$a_k = \frac{1}{T}\int_T x(t) e^{-jk\omega_o t} dt$$

Problems

1. A CT periodic signal *x(t)* with fundamental period *T = 4* has Fourier series coefficients a_k given as,

$$a_k = \begin{cases} 0 & for\ k = 0 \\ -j\dfrac{e^{jk\left(\frac{\pi}{2}\right)} \sin\left(k\dfrac{\pi}{2}\right)}{k^2\left(\dfrac{2\pi}{T}\right)^2} & for\ k \neq 0 \end{cases}$$

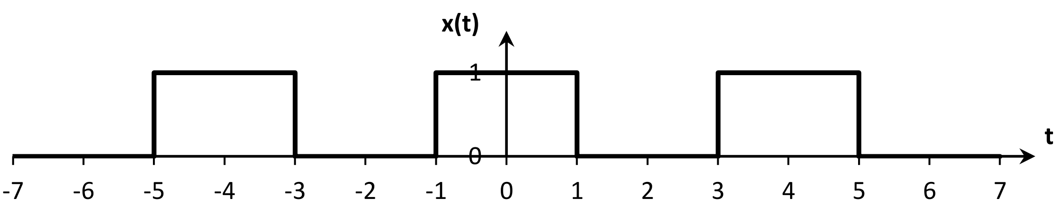
   The following MATLAB function generated the above F.S. coefficients for up to *N* harmonics and also generates the time domain signal $x_N(t)$ using truncated Fourier series.

```
%truncated Fourier series using first N harmonics
function [x,t] = frep(N)

%for freq. domain signal a_k or a[k]
k = -N:N;
a = zeros(size(k));
for i = 1:length(k)
     if k(i) == 0
        a(i) = 0;
     else
        a(i) = -4j*exp(j*k(i)*pi/2)*sin(k(i)*pi/2)/(k(i)*pi)^2;
     end
end
subplot(2,1,1),stem(k,a);

%for time domain signal x(t) using F.S. coefficients
t = -10:0.01:10;
x = zeros(size(t));
for i = 1:length(t)
     x(i) = sum(a.*exp(j*k*2*pi/4*t(i)));
end
subplot(2,1,2),plot(t,x);
```

a. Use above function to generate the Fourier series coefficients for up to 5th harmonics (N = 5) and observe the time domain signal generated by the linear combination of those five harmonics. Predict the nature of the original time domain signal based on the observation.

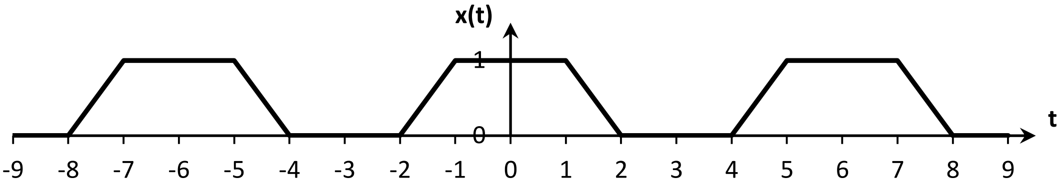
b. Repeat the task in **a.** for N = 50 and observe the generated time domain signal. Compare the observations in **a.** and **b.** and comment upon the results.

2. Find out the Fourier series coefficients for the following CT periodic square wave signal using the Fourier series analysis equation.



Modify the MATLAB function provided in **prob. 1** for the Fourier series coefficients of above signal. Use the modified function to generate the Fourier series coefficients and the time domain representation using truncated Fourier series with N = 5, 25, 50 and 500. Can you observe the Gibb's phenomenon? Comment on the results.

3. Find out the Fourier series coefficients for the following CT periodic trapezoidal signal.



Use truncated Fourier series representation to plot its time domain signal. Observe the signal for N = 1, 5, 9, 13, 17, etc. Comment on your observations.

Section B

Discrete Time Fourier Series Representation

According to Fourier series representation, a discrete time periodic signal *x[n]*, with fundamental period *N* and fundamental frequency $\omega_o = 2\pi/N$, can be represented as a linear combination of harmonically related complex exponential signals with common period *N*. The Fourier series pair for a DT periodic signal *x[n]* with fundamental period *N* and Fourier series coefficients a_k is given by:

$$x[n] \xleftrightarrow{F.S.} a_k$$

Synthesis equation:

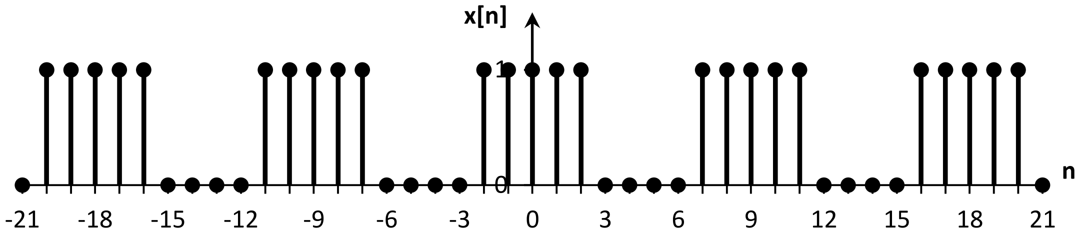
$$x[n] = \sum_{k=<N>} a_k e^{jk\omega_o n}$$

Analysis equation:

$$a_k = \frac{1}{N} \sum_{n=<N>} x[n] e^{-jk\omega_o n}$$

<u>Problems</u>

1. Find out the Fourier series coefficients for the following DT periodic square wave signal using the Fourier series analysis equation.



a. Modify the above program to display the Fourier series coefficients of the given signal and to generate and display the time domain signal using the Fourier series coefficients.

b. Use the modified program to generate and display the time domain signal $x_M[n] = \sum_{k=-M}^{M} a_k e^{jk\omega_o n}$ with M = 1, 2, 3, 4 and 9. Observe and comment upon the result.

Note that only *N* discrete time harmonic complex exponentials are defined, N being the fundamental period of the signal.