EX 753 Digital Signal Processing

Lab 3 : LTI System

Background:

$$x[n] \longrightarrow \boxed{H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_n z^{-n}}{1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_n z^{-n}}} \longrightarrow y[n]$$

A **Linear Time Invariant (LTI)** system is characterized by the transfer function $H(z) = Y(z)/X(z)$, where $X(z)$ and Y(z) are the Z-transforms of the sequences $x(n)$ and $y(n)$ respectively. When the inverse Z-transform of the transfer function $H(z)$ is taken, the resulting difference equation can be written as:

$$y[n] = b_0 x[n] + b_1 x[n-1] + \cdots + b_N x[n-N] - a_1 y[n-1] - a_2 y[n-2] - \cdots - a_N y[n-N] \qquad (1).$$

The digital signal processing basically deals with the methods of implementation of the above difference equation. The equation can be solved using both hardware and software. It can be implemented using microprocessor based designs in which the assembly language programming plays the vital role. For the fast processing of the signals, considering the improved system performance the digital signal processing chips are preferred. The DSP chips are based on the Harvard architecture rather than the Von-Neumann's architecture, usually found in most of the personal computers.

If the transfer function $H(z)$ is known for the given LTI system the MATLAB signal processing toolbox functions can be used to plot the frequency response of the system. For this there is a function; `[H,W]=freqz(b,a,w),` which gives the complex values in amplitude H and angle W radians versus w points frequency. Here 'b' and 'a' are the vector sequences representing the numerator and denominator coefficients of $H(z)$.

The system response in the discrete time domain i.e. $h[n]$ can be observed by taking inverse Fast Fourier transform of the vector $'H'$ (in matlab by function *ifft()*). Now if the input digital signal $x[n]$ is specified, the output $y[n]$ of the system can be obtained from the convolution of $x[n]$ and $h[n]$. Using *fft( )* the input discrete time signal $x[n]$ and the output of tile system $y[n]$ both can be observed in frequency domain. Then the effect of the response of LTI system on the input signal can be visualized.

The difference equation (1) can be solved in MATLAB using the function, `y = filter (b, a, x)`, where 'b' and 'a' are the vectors representing the coefficients of the difference equation and x is the input sequence. The frequency components of the output signal can be determined by performing Fast Fourier Transform (FFT) of the output sequence $y[n]$. The discrete time signal $x[n]$ can be folded using MATLAB function $fliplr()$ and the cross correlation between the two sequences can be determined using the function, *xcorr()*.

Linear Systems Transformation:

In discrete time systems the transfer function in the Z domain plays the key role in determining the nature of the system. The nature of the system is determined from the number and locations of the poles and zeros in the Z-plane. In lower order systems the locations of the poles and zeros can be easily determined from the transfer function of the system. However the higher order systems possess transfer functions with numerator and denominator polynomials of greater degree. As a result the process of determining the poles and zeros of the system becomes complex and tedious. Besides this in most of the higher order discrete time systems, the transfer function is specified in the form of second order sections. If the transfer function of the system consists of a large number of such second order sections either in cascade or parallel form, the determination of the poles and zeros of the system becomes even harder.

MATLAB signal processing toolbox provides a number of functions for transforming the discrete time linear systems from one form to another. The name of the functions and their purpose are listed as follows:

`sos2zp()` : transforms second order sections into zeros and poles.
`sos2tf()` : performs second order sections to transfer function conversion.
`tf2zp()` : transfer function to pole zero conversion.
`zp2sos()` : zero-poles to second order sections.
`zplane()` : plots the pole-zero diagram in Z-plane.
`freqz()` : determines the magnitude and phase of the transfer function.
Also see `fvtool()`, `parallel()`, `series()`, `round()`.

For further information on the above functions, please refer to the MATLAB 'help'.

Statements of the Problem:

1. In the given LTI system of fig above, if the coefficients 'b' and 'a' are specified as
   $b_0 = 0.0663$, $b_1 = 0.1989$, $b_2 = 0.1989$, $b3 = 0.0663$
   $a_0 = 1$, $a_1 = -0.9349$, $a_2 = 0.5668$, $a_3 = -0.1015$,
   then the order of the system is 3 i.e. $N = 3$.
   (a) Plot the frequency response of the system.
   (b) From the magnitude response of the system, find out the cut-off frequency.
   (c) Identify the nature of the system analyzing its frequency response.
   (d) Plot the impulse response of $h[n]$ of the system and for $x[n] = [1\ 2\ 3\ 1\ 0\ 0\ 0\ 1\ 2\ 3\ 1]$ determine the output $y[n]$ of the system. Also observe the plots of $X(w)$ and $Y(w)$ to be familiar with the frequency components present in the input and output signals. Correlate the plots of $X(w)$ and $Y(w)$ with the nature of the system.
   (e) Use MATLAB's '$filter$' function to directly solve the difference equation $y[n]$ for the specified coefficients 'b' and 'a' of the system, taking the same $x[n]$ as the input as in (d). Plot the outout discrete signal $y[n]$ and analyze its frequency components. Compare this plot with the plot from (d).
   (f) Taking same $x[n]$ as in (d), fold the vector sequence $x[n]$. Compute the cross correlation of the folded and unfolded sequences. Use the resulting sequence as input to the same LTI system, repeat procedure (e). Compare output of the system from the output obtained from (e).

2. The transfer function of the fourth-order discrete time system is given as:

$$H(z) = \frac{0.0018 + 0.0073z^{-1} + 0.011z^{-2} + 0.007z^{-3} + 0.008z^{-4}}{1 - 3.0544z^{-1} + 3.8291z^{-2} - 2.2925z^{-3} + 0.55072z^{-4}}.$$

   Using MATLAB
   (a) Find out the poles and zeros of the system and plot them in the z-plane.

(b) Use them to determine the second order sections in the cascaded form.

(c) Plot the frequency response of the system and comment on the nature of the system.

(d) After knowing the numerator and denominator coefficients of each second order section, draw the signal flow graph to represent the cascaded structure.

3. Let a discrete time system be implemented by cascading of the following three second order sections:

Section 1: $$H(z) = \frac{0.0007378(1+2z^{-1}+z^{-2})}{(1-1.2686z^{-1}+0.7051z^{-2})}$$

Section 2: $$H(z) = \frac{1+2z^{-1}+z^{-2}}{(1-1.0106z^{-1}+0.3583z^{-2})}$$

Section 3: $$H(z) = \frac{1+2z^{-1}+z^{-2}}{(1-0.9044z^{-1}+0.2155z^{-2})}$$

(a) Using above three second order sections in cascaded form determine the poles and zeros of the system and plot them in z-plane.

(b) Determine the transfer function of the system, formed by cascading of the above three sections.

(c) Determine the poles and zeros from this transfer function and plot them in z-plane. Your result should match with that from 3(a).

4. Observe the frequency response of the seventh order elliptic filter having the transfer function given by the cascade of the three second order systems and a first order system as follows

$$H(z) = \frac{0.104948+0.104948z^{-1}}{1-0.790103z^{-1}} \times \frac{0.102450-0.007817z^{-1}+0.102232z^{-2}}{1-1.51723z^{-1}+0.714088z^{-2}} \times \frac{0.420100-0.399842z^{-1}+0.419864z^{-2}}{1-1.0421773z^{-1}+0.861895z^{-2}} \times \frac{0.714929-0.8267432z^{-1}+0.71484z^{-2}}{1-1.387447z^{-1}+0.962242z^{-2}}.$$

(a) Now quantize the filter coefficients in three significant digits and observe the response. What is the difference between the response in these two cases?

(b) Plot the poles and zeros of the filter transfer function in unquantized as well as in quantized cases. What is the difference between the pole zero plots in two cases?

5. Plot the response of the filter having following transfer function

$$H(z) = \frac{0.287-0.447z^{-1}}{1-1.297z^{-1}+0.695z^{-2}} + \frac{-2.143+1.145z^{-1}}{1-1.069z^{-1}+0.370z^{-2}} + \frac{1.856-0.630z^{-1}}{1-0.997z^{-1}+0.257z^{-2}}.$$

(a) Now quantize the filter coefficients to two significant digits and plot the response. What is the difference between the responses in these two cases?

(b) Plot the poles and zeros of the transfer function given in 4 and that obtained after coefficient quantization. Observe the difference between the two plots.