Sanuj Shakya
IoE, Pulchowk Campus

Lab Module 4

Initial report must include computation of convolution of given signals as well as MATLAB programs for each of the problems. The properly labeled graphical representation of each signal must be included in the final report along with necessary discussions.

Section A

Convolution of CT and DT Signals

The convolution integral of two CT signals *x(t)* and *h(t)* is given by,

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

Similarly for DT signals *x[n]* and *h[n]*, their convolution sum is given by,

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

It should be noted that the length (no. of samples) of the convolution sum is equal to the sum of the lengths of the signals being convolved minus one i.e.

Length of convolution sum (y[n]) = length of x[n] + length of h[n] – 1

Similarly,

Left extreme point of convolution sum = sum of left extreme points of signals being convolved
Right extreme point of convolution sum = sum of right extreme points of signals being convolved

The convolution sum/integral of two signals can be computed using a MATLAB program that implements the above formula (flip and shift approach). However, MATLAB has an inbuilt function called *conv* to compute the convolution of two signals. Use *help conv* to learn more about this function.

The following MATLAB function can be used to compute the convolution of CT or DT signals.

```
function [y,t] = convolve (x1,t1,x2,t2,del)
%x1 and x2 are the signals to be convolved with time ranges t1
%and t2 respectively.

%del is the step size; del = 1 for DT signals

%y is the resulting signal with time range t

y = conv(x1,x2)*del;
t = min(t1)+min(t2):del:max(t1)+max(t2);
```

Problems

1. Use graphical method to convolve the signals $x_1(t) = \begin{cases} 1 \ ; \ 0 \le t \le 5 \\ 0 \ ; \ otherwise \end{cases}$ and $x_2(t) = \begin{cases} 1 \ ; \ 0 \le t \le 10 \\ 0 \ ; \ otherwise \end{cases}$. Also use a MATLAB program for the convolution of above signals and plot all the signals. Comment on the result.

2. An LTI system has an impulse response $h(t) = e^{0.5t}u(-t)$. If a signal $x(t) = \begin{cases} 1 \ ; \ 2 \le t \le 8 \\ 0 \ ; \ otherwise \end{cases}$ is input to the system, find the corresponding output signal. Use both the graphical method and MATLAB program. Comment on the result.

3. Given the signals $x[n] = \begin{cases} 1 \ ; \ 0 \le n \le 4 \\ 0 \ ; otherwise \end{cases}$ and $y[n] = \begin{cases} 2^n \ ; 0 \le n \le 6 \\ 0 \ \ ; otherwise \end{cases}$. Use overlap and add method as well as MATLAB program to compute the convolution of above signals. Observe and comment on the result.

Section B

First and Second order CT/DT Systems

The first order CT LTI system is described by the differential equation $\tau \frac{dy(t)}{dt} + y(t) = x(t)$ and its frequency response is given by $H(j\omega) = \frac{1}{1+j\omega\tau}$. Here, τ represents the time constant of the system.

The function below plots the frequency response, impulse response and step response of first order CT system.

```
function [] = firstCT (tau)
%system is defined by its frequency response
%H(jw) = (1/tau)/(jw+1/tau)
num = [1/tau];     %coefficient vector in numerator of H(jw)
den = [1 1/tau];   %coefficient vector in denominator of H(jw)

freqs(num,den);    %plots frequency response
figure,impulse(num,den);      %plots impulse response
figure,step(num,den);         %plots step response
```

Similarly, the second order CT LTI system is described by the second order differential equation $\frac{d^2y(t)}{dt^2} + 2\zeta\omega_n\frac{dy(t)}{dt} + \omega_n^2 y(t) = \omega_n^2 x(t)$. Here, ζ is the damping ratio and ω_n is the natural frequency of the system. Its frequency response is given by $H(j\omega) = \frac{\omega_n^2}{(j\omega)^2+2\zeta\omega_n j\omega+\omega_n^2}$. The frequency response, impulse response and step response of this system can be plotted using the above function by modifying the numerator and denominator coefficient vectors as follows.

```
num = [wn*wn];
den = [1 2*zeta*wn wn*wn];
```

The first order DT system is described by the difference equation $y[n] - ay[n-1] = x[n], |a| < 1$ and its frequency response is given by $H(e^{j\omega}) = \frac{1}{1-ae^{j\omega}}$. Here, |a| determines the rate at which the system responds. The following function plots the frequency response, impulse response and step response of first order DT systems.

```
function [] = firstDT (a)
%system is defined by its frequency response
%H(jw) = 1/(1-ae^-jw)
num = [1];         %coefficient vector in numerator of H(e^jw)
den = [1 -a];      %coefficient vector in denominator of H(e^jw)

freqz(num,den);    %plots frequency response
figure,dimpulse(num,den);     %plots impulse response
figure,dstep(num,den);        %plots step response
```

Similarly, the second order DT LTI system is described by the second order difference equation $y[n] - 2rcos\theta y[n-1] + r^2 y[n-2] = x[n], 0 < r < 1, 0 \leq \theta \leq \pi$. Here, r determines the rate of decay and θ determines the frequency of oscillation of the system. The frequency response of this system is given by $H(e^{j\omega}) = \frac{1}{1-2r\,cos\theta\,e^{-j\omega}+r^2\,e^{-j2\omega}}$. We can use the above function to plot the frequency response, impulse response and step response of the second order system by modifying the denominator coefficient vector as follows,

```
den = [1 -2*r*cos(theta) r*r];
```

Problems

1. Plot the frequency, impulse and step responses of the first order continuous time system for given parameters.
   a. $\tau = 0.1$
   b. $\tau = 0.5$
   c. $\tau = 0.9$

   Observe the plots and comment on the response of first order CT systems. Also analyze and comment on the effect of time constant τ.

2. Plot the responses of second order CT system for following parameter values.
   a. $\zeta = 1.5$, $\omega_n = 20$
   b. $\zeta = 1$, $\omega_n = 20$
   c. $\zeta = 0.7$, $\omega_n = 20$
   d. $\zeta = 0.3$, $\omega_n = 20$
   e. $\zeta = 0.1$, $\omega_n = 20$

   Observe and comment on the dependence of time and frequency responses of the system on damping ratio ζ.

3. Plot the time responses and frequency response of first order DT system with given parameters.
   a. a = 0.25, a = -0.25
   b. a = 0.25, a = -0.25
   c. a = 0.25, a = -0.25

   Observe the responses and comment on their dependence on a.

4. Plot the responses of second order DT system for given parameter values.
   a. $\theta = \pi/4$, r = 0.25, r = 0.5, r = 0.75

b. $\theta = \pi/2$, $r = 0.25$, $r = 0.5$, $r = 0.75$

c. $\theta = 3\pi/4$, $r = 0.25$, $r = 0.5$, $r = 0.75$

Discuss about your observation on dependence of time and frequency responses of the system on r and θ.