Exercise Sheet # 0

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a)

$$Ax = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n \end{bmatrix}$$

b)

An eigenvector v is an eigenvector of Matrix A if the length of the vector but not its direction is changed when its multiplied by A.

$$A \cdot v = \lambda \cdot v$$

λ is a scalar known as the eigenvalue associated with the eigenvector v.

c)

the determinant is a value that can be computed from the elements of a square matrix. The determinant of a matrix A is denoted det(A), det A or |A|.

for 2x2: $|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

for 3x3: $|A| = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a \cdot \begin{vmatrix} e & f \\ h & i \end{vmatrix} - b \cdot \begin{vmatrix} d & f \\ g & i \end{vmatrix} + c \cdot \begin{vmatrix} d & e \\ g & h \end{vmatrix}$

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d)

the trace of a $n \times n$ matrix is defined to be the sum of elements on the main diagonal of A.

$$\mathrm{trace}(A) = \sum_{j=1}^{n} a_{jj} = a_{11} + a_{22} + a_{33} + \dots + a_{nn}$$

if $\mathrm{trace}(A) = 0$ the matrix A is called tracefree.

(2)

$$A^{-1} = \begin{bmatrix} -\frac{1}{5} & \frac{4}{5} & -\frac{1}{5} \\ \frac{1}{2} & -\frac{1}{2} & 0 \\ -\frac{1}{10} & -\frac{1}{10} & \frac{2}{5} \end{bmatrix}$$

eigenvalues = $\{-1; 2; 5\}$

eigenvectors:

for -1: $\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$ for 2: $\begin{pmatrix} -2 \\ -1 \\ 3 \end{pmatrix}$ for 5: $\begin{pmatrix} 7 \\ 5 \\ 6 \end{pmatrix}$

eigendecomposition of A = $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

(3)

$$f(x) = \frac{1}{2}x^2 + \frac{1}{24}\sin^3(4x^2) - \cos(e^{2x}) - \log(\sqrt{x}) + \frac{x^2}{5 + e^{-x}}$$

$$f'(x) = x + x \cdot \sin^2(4x^2) \cdot \cos(4x^2) + 2e^{2x}\sin(e^{2x}) + \frac{1}{2} \cdot \frac{1}{x \cdot \ln(10)} + \frac{e^x x (10e^x + x + 2)}{(5e^x + 1)^2}$$

(4)

single variable:

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

$$= f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \dots$$

$$g(x) = \sum_{n=0}^{\infty} \frac{g^{(n)}(x_0)}{n!}(x-x_0)^n$$

$$= g(x_0) + g'(x_0)(x-x_0) + \frac{g''(x_0)}{2!}(x-x_0)^2 + \frac{g'''(x_0)}{3!}(x-x_0)^3 + \dots$$

multi variable:

$$g(x,y) = g(x_0,y_0) + g_x(x_0,y_0)(x-x_0) + g_y(x_0,y_0)(y-y_0) + \frac{g_{xx}(x_0,y_0)}{2!}(x-x_0)^2 + g_{xy}(x_0,y_0)(x-x_0)(y-y_0) + \frac{g_{yy}}{2!}(y-y_0)^2$$

4 $\qquad\qquad\qquad\qquad\qquad\qquad p(B|A) \geq p(B)$

a) true by assumption

b) $p(B|\neg A) \leq p(B)$

$$p(B) \overset{\text{sum}}{=} p(B|A) \cdot p(A) + p(B|\neg A) \cdot p(\neg A) \quad \text{sum rule}$$

$$p(B) \overset{\text{assum}}{\geq} p(B) \cdot p(A) + p(B|\neg A) \cdot p(\neg A)$$

$$p(B) \cdot \overbrace{(1 - p(\neg A))}^{p(A)} \geq p(B|\neg A) \cdot p(\neg A)$$

$$p(B) \geq p(B|\neg A) \quad \square$$

c)

$$p(A|B) \geq p(A)$$

$$p(A|B)\, p(B) = p(B|A) \cdot p(A) \quad \text{Product rule}$$

$$p(A|B)\, p(B|A) \geq p(B|A) \cdot p(A)$$

$$p(A|B) \geq p(A) \quad \square$$

d) $p(A|\neg B) \leq p(A)$

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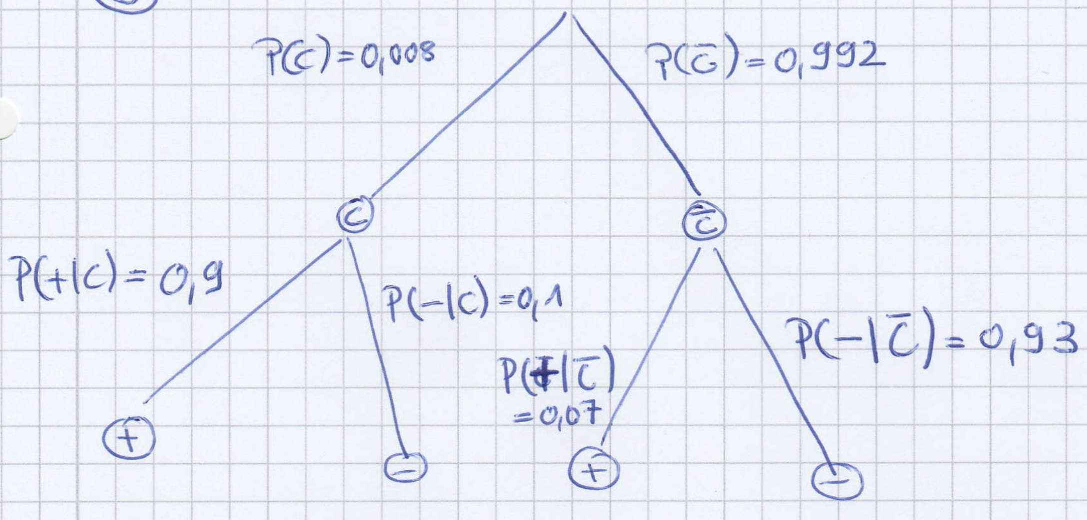

$$P(C|+) = \frac{P(+|C) \cdot P(C)}{P(+)}$$

$$= \frac{P(+|C) \cdot P(C)}{P(+|C) \cdot P(C) + P(+|\bar{C}) \cdot P(\bar{C})}$$

$$= \frac{0{,}9 \cdot 0{,}008}{0{,}9 \cdot 0{,}008 + 0{,}07 \cdot 0{,}992} = \frac{0{,}0072}{0{,}0072 + 0{,}06944}$$

$$\approx 0{,}0939 \mathrel{\hat{=}} \underline{\underline{9{,}39\,\%}}$$