The below questions give a rough idea of the kind of questions to be expected in the written exam (which will take place on 13. February 2018 in HS25 of the Kupferbau). The exam will likely last 110 minutes (+10 minutes set-up and hand-in), and have an overall size corresponding to 100 points (the below examples represent 30 points).

Please note: This example is just provided to help you gauge the level of complexity of the exam. Obviously, the actual exam will contain *different* questions. Some of the questions in the exam may be harder or easier for you than these examples. They may cover different content of the lecture. Do not use absence of certain topics on this sheet as an indication that they will not be in the exam, nor their presence as an indication that they will be. Regardless of what is printed below, you should prepare for the exam by revising the content of *all* lectures in the course.

[Multiple-choice questions can have several correct answers.]

1. *Axioms of Probability* Let $p(x, y)$ be a joint probability distribution over two discrete variables $x, y \in \{1; \ldots; K\}$. Write down an expression for the *marginal* distribution $p(x)$ in terms of the joint. *5 points*

2. *Conjugate priors* Assume you are given observations $\mathbf{x} := [x_1, \ldots, x_n] \in \mathbb{R}_+$ that are drawn independently and identically from the Gamma distribution with unknown rate $\beta \in \mathbb{R}_+$

$$p(x \mid \beta) = \prod_{i=1}^{n} \beta^2 x_i e^{-\beta x_i}$$

   (a) What is the *most likely (i.e. maximum likelihood)* value of β? *10 points*

   (b) Consider the prior (Γ is Euler's Gamma function)

$$p(\beta \mid a, b) = \frac{b^a}{\Gamma(a)} \beta^{a-1} e^{-b\beta}.$$

   What is the posterior distribution $p(\beta \mid x)$? *5 points*

3. *Eigenfunctions.* Which of the following expressions imply that ϕ is an Eigenfunction of the kernel function k (for the nit-picky: wrt. to the Lebesgue measure)?

   (i) $\int k(x, x')\phi(x')dx = \lambda\phi(x)$ (ii) $\int k(x, x')\phi(x)dx = \lambda\phi(x')$
   (iii) $\int k(x, \lambda)\phi(x)dx = \phi(\lambda)$ (iv) $\int k(x, x')\phi(x)dx = \lambda\phi(x)$ *5 points*

4. *Graphical Models* Write down the undirected graphical model (Markov Random Field) corresponding to the joint distribution *5 points*

$$p(a, b, c, d, e) = p(a \mid c, d) \cdot p(c \mid d, e) \cdot p(b \mid c) \cdot p(d) \cdot p(e)$$