SESSION

1

ARITHMETIC

Integers

Integers are whole numbers. We can think of them as the numbers we count with, including negatives and zero. Every integer can be further defined by the terms listed here.

- Positive: Integers greater than zero. Ex: 1, 2, 3, 4...
- Negative: Integers less than zero. Ex: -1, -2, -3, -4...
- Even: Integers divisible by 2. Ex: -4, -2, 0, 2, 4...
- Odd: Integers NOT divisible by 2. Ex: -3, -1, 1, 3, 5...
- Prime: An integer divisible only by 1 and itself. Ex: 2, 3, 5, 7, 11...
- Zero: The only non-negative, non-positive integer.

Example:

Given the set of numbers $\{3, -26, \sqrt{2}, \frac{1}{4}, 0, \text{ and } \pi\}$, which of the numbers are integers?

Positive?

Odd?

Prime?

Exercises

(Answers on page 64.)

Answer the following questions:

1. Which of the digits in the number 27,536 are even?

2. Which of the digits in the number 162,479 are odd?

3. Arrange the following numbers in numerical order:
   14, −23, 0, −30, 6.

4. What are the first five prime numbers greater than 50?

5. What are the first six consecutive positive, even integers?

6. What is the smallest prime number?

Digits are the math alphabet. Just as letters can be grouped and ordered to form different words (*tea*, *ate*, *eat*), digits can also be grouped and ordered to form different numbers (128, 281, 812).

Test Question

7. If $3x + 10$ represents an odd integer, which of the following represents the next largest odd integer?

   (A) $x + 4$
   (B) $3x + 8$
   (C) $3(x + 4)$
   (D) $3(x + 6)$
   (E) $4(x + 1)$

Factors, Multiples, and Remainders

Factor: A positive integer that divides evenly into another given number.

- Factors are the "small" numbers that go into a "bigger" number evenly.
  - **Greatest Common Factor:** The largest factor shared by a set of numbers.

Multiple: An integer that is the result of multiplying an integer by another integer.

- Multiples are the "big" numbers generated by multiplying "smaller" numbers.
  - **Least Common Multiple:** The smallest multiple shared by a set of numbers.

Remainder: The value that is left over in a division problem.

- A remainder is always smaller than the number we are dividing by.

Example:

Determine the greatest common factor of 12 and 18.

List the factors of 12:

List the factors of 18:

Determine the GCF:

Example:

Calculate the remainder when 34 is divided by 3.

Determine the quotient:

Determine the remainder:

Exercises

(Answers on page 65.)

Find the factors of the following numbers:

1. 36
2. 45
3. 121
4. 154

List the first five multiples of the following numbers:

5. 6
6. 13
7. 25
8. 110

Calculate the remainders:

9. 68 divided by 5
10. 93 divided by 4
11. 164 divided by 3
12. 225 divided by 7

Test Question

13. Which of the following is NOT a factor of 168?

    (A) 21
    (B) 24
    (C) 28
    (D) 32
    (E) 42

Prime Factorization: A number expressed as the product of its prime factors. Ex., the prime factorization of 20 is $2 \times 2 \times 5$.

Divisibility Rules

A number is divisible...

- ...by 2 if the number is even
- ...by 3 if the sum of the digits is divisible by 3
- ...by 4 if the 2-digit number formed by the last two digits is divisible by 4
- ...by 5 if the last digit is 0 or 5
- ...by 6 if the number is divisible by both 2 and 3
- ...by 9 if the sum of the digits is divisible by 9
- ...by 10 if the last digit is zero

Example:

Determine whether 216 is divisible...

by 2:

by 3:

by 4:

by 5:

by 6:

by 7:

by 8:

by 9:

by 10:

Exercises

(Answers on page 68.)

Solve the following questions:

1. Which of the following numbers is divisible by 3: 241, 1,662, 4,915, 3,131?

2. Which of the following numbers is divisible by 4: 126, 324, 442, 598?

3. Which of the following numbers is divisible by 6: 124, 252, 412, 633?

4. Which of the following numbers is a multiple of 5: 28, 127, 310, 522?

5. 9 is a factor of which of the following numbers: 487, 315, 1,093, 3,154?

Test Question

6. Which of the following is a multiple of all three integers 2, 3, and 5?

   (A) 525
   (B) 560
   (C) 615
   (D) 620
   (E) 660

Fractions

Fractions signify division. Ex., the fraction ½ represents the result of dividing 1 by 2.

- To reduce a fraction, divide the numerator and denominator by the same number.
- To convert a fraction into
  - an equivalent fraction, multiply or divide the numerator and denominator by the same number.
  - a decimal, divide the numerator by the denominator using long division.
- To convert a decimal into a fraction,
  - move the decimal to the right until the number is an integer to calculate the numerator;
  - count the number of places that the decimal was moved to the right, and then put the same number of 0s after a 1 to calculate the denominator.

Example:

Convert the fraction $\frac{1}{2}$ into an equivalent fraction with a denominator of 6.

What number must we multiply by?

Calculate the equivalent fraction:

Example:

Reduce the fraction $\frac{15}{35}$ to lowest terms.

What number must we divide by?

Calculate the reduced fraction:

Exercises

(Answers on page 70.)

Convert the following fractions to decimals:

1. $\frac{1}{3} =$

2. $\frac{2}{5} =$

3. $\frac{4}{25} =$

4. $\frac{3}{7} =$

5. $\frac{17}{44} =$

Convert the following decimals into fractions and reduce the fractions to lowest terms:

6. $0.45 =$

7. $0.667 =$

8. $2.25 =$

9. $1.35 =$

10. $-0.125 =$

Test Question

11. $6\frac{3}{4} - 6.32 =$

    (A) 0.33
    (B) 0.39
    (C) 0.43
    (D) 0.57
    (E) 0.68

SESSION

1

To convert a mixed fraction:

Step 1: Multiply whole number by denominator

Step 2: Add numerator

Step 3: Place total over denominator

Ex., $1\frac{7}{8}$

Step 1: $1 \times 8 = 8$

Step 2: $8 + 7 = 15$

Step 3: $\frac{15}{8}$

Fraction Operations

- Addition and Subtraction: All fractions must have a common denominator. When possible, determine the Least Common Multiple (LCM) of the denominators.
- Multiplication: Multiply numerators and denominators separately.
- Division: Multiply the top fraction by the reciprocal of the bottom fraction $\left(\text{ex., } \frac{1}{2} \div \frac{3}{4} \text{ becomes } \frac{1}{2} \times \frac{4}{3} = \frac{4}{6} = \frac{2}{3}\right)$.

Example:

Determine the sum of the following expression: $\frac{3}{5} + \frac{2}{3} =$

Find the LCM:

Calculate the equivalent fractions:

Add numerators:

Example:

Determine the quotient: $\frac{4}{3} \div \frac{4}{9} =$

Rewrite as a product:

Multiply numerators and denominators separately:

Reduce to lowest terms:

Exercises

(Answers on page 72.)

Evaluate the following expressions:

1. $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} =$

2. $\frac{12}{25} + \frac{13}{5} =$

3. $\frac{6}{12} + \frac{7}{3} =$

4. $\frac{1}{16} - \frac{3}{4} + \frac{17}{8} =$

5. $4\left(\frac{1}{3} + \frac{1}{12}\right) =$

6. $\frac{1}{2}\left(\frac{1}{3} + \frac{1}{4}\right) =$

7. $\frac{1}{24}(36 + 60) =$

8. $0.21 + 0.946 + 1.324 =$

9. $\left(\frac{12}{16} - \frac{3}{6}\right)^2 =$

10. $1.69 \times 0.002 =$

Test Question

11. $\frac{7}{5} \times \left(\frac{3}{7} - \frac{2}{5}\right) =$

    (A) $\frac{1}{165}$

    (B) $\frac{1}{35}$

    (C) $\frac{1}{25}$

    (D) $\frac{9}{15}$

    (E) 1

Ratios

- A ratio represents the proportional relationship between quantities or numbers.
  - If the ratio of apples to oranges is 2 to 3, you could actually have 4 apples and 6 oranges.
- If you are given the number of each item, you can determine the ratio.
- Ratios may be written with a fraction bar $\left(\frac{x}{y}\right)$ or with a colon (x:y) or in English (ratio of x to y).

Example:

A classroom has a ratio of 4 girls to 5 boys. Determine the ratio of girls to students.

What is the smallest possible number of girls in the class?

What is the smallest possible number of students in the class?

There are 36 students in the class. Determine the number of girls in the class.

Ratio Formula:
Quantity of A =
(Ratio of A to total)(Total)

Determine the ratio of girls to students.

Apply the Ratio Formula:

Exercises

(Answers on page 74.)

Reduce each of the following ratios to lowest terms:

1. 8:128
2. 72:20
3. 14.2:71

Convert the following ratios:

4. A bag contains red and blue balls. If there are 10 red and 12 blue balls, what is the

   a) ratio of red to blue balls?
   b) ratio of red to total balls?
   c) ratio of blue to total balls?

5. A store sells 3 types of shoes: sneakers, sandals, and high heels. The ratio of sneakers to sandals to high heels is 3:5:7. What is the

   a) ratio of sneakers to high heels?
   b) ratio of sandals to shoes?
   c) number of sandals, if the store has 210 pairs of shoes?

Test Question

6. A laboratory has 55 rabbits, some white and the rest brown. Which of the following could be the ratio of white rabbits to brown rabbits in the lab?

   (A) 1:3
   (B) 3:8
   (C) 5:11
   (D) 3:4
   (E) 5:1

Percents

As with any ratio, a percent can be written as a fraction or a decimal.

Percents are a special type of ratio in which the total is always denoted as 100%.

- $x\% = \frac{x}{100}$
- $x\%$ of $y = \left(\frac{x}{100}\right) y$
- Percent $= \frac{\text{Part}}{\text{Whole}}(100\%)$
- Final as percent of original: $\frac{\text{Final}}{\text{Original}}(100\%)$
- Percent Change $= \frac{\text{Difference}}{\text{Original}}(100\%)$

Example:

What is 25% of 36?

Rewrite the percent as a fraction:

Reduce the fraction:

Set up a proportion:

Cross-multiply and solve for x:

Example:

18 is what percent of 3?

Translate the question into an algebraic equation:

Identify the part, whole, and percent:

Apply the Percent Formula:

Exercises

(Answers on page 76.)

Convert to a percent:

1. $0.002 =$
2. $\frac{7}{25} =$
3. $1.31 =$
4. $\frac{12}{5} =$
5. $\frac{1}{400} =$

Convert to a fraction:

6. $16\% =$
7. $24\% =$
8. $0.036\% =$
9. $125\% =$
10. $\frac{1}{4}\% =$

Evaluate:

11. 50% of 12 =
12. 75% of 16 =
13. 150% of 4 =
14. 24% of 2 =
15. 10% of 50 =

Test Question

16. An item is priced at 20% more than its wholesale cost. If the wholesale cost was \$800, what is the price of the item?

    (A) \$ 900
    (B) \$ 960
    (C) \$1,000
    (D) \$1,040
    (E) \$1,200

Averages and Statistics

- Average $= \dfrac{\text{Sum of values}}{\text{Number of values}}$
- Think of the average as the "balanced" value of a set of numbers.
- Median: The middle number in a set of numbers that is arranged from smallest to largest.
- Mode: The number in a set that occurs most frequently. There can be more than one mode for a set of numbers.
- Range: The positive difference between the largest and smallest terms in a set of numbers.
- Standard Deviation: A measure of how spread out a set of numbers is. The larger the standard deviation, the more spread apart the numbers in the set.

Example:

The average of 3, 4, 5, and x is 5. What is x?

Apply the Average Formula:

Solve for x:

Alternate approach:

What is the difference between 3 and 5? 4 and 5? 5 and 5?

What must x equal to balance the other side of the average?

Exercises

(Answers on page 78.)

Find the average:

1. 10, 12, 16, 17, 20

2. 0, 3, 6, 9

3. 12, 24, 36, 48, 60

Find the value of *x*:

4. If the average of -4, 0, 6, and x is 8, then $x = ?$

5. If the average of 8, 3, 12, 11, and x is 0, then $x = ?$

Test Question

6. The average (arithmetic mean) of six numbers is 6. If 3 is subtracted from each of four of the numbers, what is the new average?

   (A) $1\frac{1}{2}$

   (B) 2

   (C) 3

   (D) 4

   (E) $4\frac{1}{2}$

Exponents

- Exponents count the number of times something is multiplied by itself.
- $a^b \times a^c = a^{(b + c)}$
- $\frac{a^b}{a^c} = a^{(b - c)}$
- $(a^b)^c = a^{(b \times c)}$
- $a^{-b} = \frac{1}{a^b}$
- Raising a positive fraction less than 1 to a positive exponent greater than 1 results in a smaller value. The higher the exponent, the smaller the result.
- When a negative number is raised to an even exponent, the result is positive. When a negative number is raised to an odd exponent, the result is negative.
- Any number raised to the "0" power equals 1.

Example:

Simplify the expression $\frac{x^{-2}y^3x^4}{x^6y^2}$.

Combine like bases in the numerator and denominator:

Combine like bases across the fraction bar:

Rewrite as a fraction:

Exercises

(Answers on page 79.)

Solve the following problems:

1. $5^4 =$

2. $2^5 =$

3. $4 \times 3^3 =$

4. $(4 \times 3)^2 =$

5. $(4 + 3)^2 =$

6. $1.016 \times 10^2 =$

7. $(2^2)^4 =$

8. $(3^{-2})^2 =$

9. $5^3 \times 5^{-4} =$

10. $\frac{2^7}{2^3} =$

Test Question

11. If $x = 2$, then $3^x + (x^3)^2 =$

(A) 18
(B) 42
(C) 45
(D) 70
(E) 73

Radicals

- Radicals follow the same rules as exponents.
- $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
- $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$
- $\sqrt{a + b} \neq \sqrt{a} + \sqrt{b}$
- $\sqrt{a - b} \neq \sqrt{a} - \sqrt{b}$
- $(\sqrt{a})^2 = a$
- $\sqrt{a} = a^{\frac{1}{2}}$

Example:

$$\sqrt{\frac{4 \times 9 + 9 \times 16}{25}} =$$

Evaluate the numerator inside the radical:

Factor out any perfect squares from the numerator and denominator separately:

Example:

$$(\sqrt{12})^4 =$$

Rewrite the radical as an exponent:

Simplify the exponent:

Exercises

(Answers on page 80.)

Evaluate:

1. $(\sqrt{2})(\sqrt{8}) =$

2. $(\sqrt{6})(\sqrt{21}) =$

3. $\dfrac{\sqrt{48}}{\sqrt{3}} =$

4. $\sqrt{5} + \sqrt{125} =$

5. $\dfrac{\sqrt{147}}{\sqrt{3}} =$

6. $\sqrt{49 - 36} =$

7. $\sqrt{49} - \sqrt{36} =$

8. $(\sqrt{16})^2 =$

9. $(\sqrt{100})^4 =$

10. $5^3 \times \sqrt{5} =$

Test Question

11. If $x > 0$ and $y > 0$, which of the following is equal to $\dfrac{y}{x}\sqrt{\dfrac{x^3}{y}}$?

(A) $\dfrac{x}{y}$

(B) $\sqrt{x}$

(C) x

(D) $\sqrt{xy}$

(E) $\sqrt{\dfrac{y}{x}}$

Order of Operations

PEMDAS = **P**lease **E**xcuse **M**y **D**ear **A**unt **S**ally. This mnemonic will help you remember the order of operations.

If an expression has nested parentheses, work from the inner-most out.

P = Parentheses
E = Exponents
M = Multiplication
D = Division
A = Addition
S = Subtraction

Multiplication and division come before addition and subtraction. The exception to this rule is when addition or subtraction appears within parentheses.

Example:

$30 - 5 \times 4 + (7 - 3)^2 \div 8 =$

P:

E:

M:

D:

A:

S:

Exercises

(Answers on page 81.)

Evaluate:

1. $7 + 5 \times \left(\frac{1}{4}\right)^2 - 6 \div (2 - 3) =$

2. $4(1.24 - (0.8)^2) + 6 \times \frac{1}{3} =$

3. $\dfrac{\frac{5}{6} + \frac{3}{2} + 2}{\frac{1}{3} + \frac{4}{9} + 4} =$

4. $\dfrac{0.25 \times (0.1)^2}{0.5 \times 40} =$

Test Question

5. $\left(\frac{3}{4}\right)^2 + \frac{5}{24} \div \left(\frac{155}{84} - \frac{5}{12}\right) =$

   (A) $\frac{9}{40}$

   (B) $\frac{34}{75}$

   (C) $\frac{17}{24}$

   (D) $\frac{289}{336}$

   (E) $\frac{17}{16}$