SESSION

2

ALGEBRA

Variables

- Variables are placeholders for numbers that are unknown and which may have no fixed value.
- All math operations that can be performed on numbers can also be performed on variables.

Example:

Evaluate $3x^2 - 4x$ when $x = 2$.

Substitute 2 for each instance of x:

Apply PEMDAS to solve:

Example:

Express $\frac{a}{b - a}$ in terms of x and y if $a = 2x$ and $b = 3y$.

Substitute $2x$ for a:

Substitute $3y$ for b:

Simplify expression:

Exercises

(Answers on page 82.)

Simplify the following expressions:

1. $2x + 4y + 7x - 6y =$
2. $4x\left(3 + \frac{3}{2}\right) =$
3. $(3a + 6b) - (7a + 4b) =$
4. $\frac{x}{4} + \frac{4y}{5} - \frac{3y}{4} + \frac{2x}{5} =$
5. $x^2 + y - 3x^2 + 4y =$

For the following expressions, evaluate for $x = 2$ and $y = 5$:

6. $(y^2 + 1)(x^2 + 1) =$
7. $\frac{x + y}{x - y} =$
8. $9x - 4y + \frac{x}{2} =$
9. $(2x - y)^2 =$
10. $x^2 + 7x + 10 =$

Test Question:

11. What is the value of the expression $x^2 + xy + y^2$ when $x = -2$ and $y = 2$?

    (A) -24
    (B) -4
    (C) 2
    (D) 4
    (E) 6

Linear Equations—Isolating a Variable

The steps for isolating a variable are:

1. Eliminate any **fractions** by multiplying both sides.
2. Put all terms with the variable you're solving for on one **side** by adding or subtracting on both sides.
3. **Combine** like terms.
4. **Factor** out the desired variable.
5. **Divide** to leave the desired variable by itself.

Example:

If $4x - 7 = 2x + 5$, what is x?

Step 1:

Step 2:

Step 3:

Step 4:

Step 5:

Example:

Solve the equation $\frac{x-2}{3} + \frac{x-4}{10} = \frac{x}{2}$.

Step 1:

Step 2:

Step 3:

Step 4:

Step 5:

Exercises

(Answers on page 84.)

Solve for the variable in each of the following equations:

1. $2x + 5 = 10$
2. $3(a - 2) = 6a$
3. $14 - z + 24 = 5z - 3$
4. $(4)(15y)(3) = 2y$
5. $8s + 6 = 12s + 7$
6. $\frac{3}{2}x - \frac{1}{2}x = 6$
7. $\frac{12 + b}{3} = \frac{b + 10}{6}$
8. $9(3 + y) = \frac{18}{5}$
9. $4x - 8 = 12(4 + 3x)$
10. $\frac{3a - 2}{7} - \frac{3 + 2a}{35} = 6$

Test Question

11. If $5 - 2x = 15$, then $5x =$

(A) -25
(B) -10
(C) 1
(D) 5
(E) 10

Systems of Linear Equations

- To solve for all of the variables in a system of equations, we must have at least as many distinct linear equations n as we have distinct variables n.
- There are two ways to solve linear equations:
  - Substitution: Solve one equation for one of the variables, and substitute that variable into the other equation.
  - Combination: Add or subtract one equation from another to cancel out one of the variables.

Example:

Find the values of m and n if $m = 4n + 2$ and $3m + 2n = 16$.

Substitute:

Solve for n:

Solve for m:

Example:

Solve for x and y if $10x - 3y = 10$ and $5x + 6y = 15$.

What can we multiply the first equation by to eliminate y?

Combine the equations:

Solve for x:

Solve for y:

Exercises

(Answers on page 86.)

Solve for each variable:

1. $x + y = 2$

   $x - y = 4$

2. $2x + y = 3$

   $2x + 3y = 6$

3. $2x + 3y = 0$

   $22x + 3y = 6$

4. $21x + 7y = 3$

   $21x + 10y = 3$

5. $x + 2y = 9$

   $2x - 3y = 4$

Test Question

6. If $2x + y = -8$ and $-4x + 2y = 16$, what is the value of y?

   (A) -4
   (B) -2
   (C) 0
   (D) 2
   (E) 4

Quadratic Equations

A quadratic equation has a squared variable (Ex., x^2). Quadratic equations appear in two forms:

Expanded: $a^2 + 5a + 6 = 0$
Factored: $(a + 2)(a + 3) = 0$

To convert from a factored form to the expanded form:

FOIL—First, Outer, Inner, Last

To solve a quadratic equation:

Step 1: Move all terms to one side of the equation, leaving zero on the other side.
Step 2: Factor the expanded equation.
Step 3: Set each expression equal to 0 and solve for the possible values of the variable.

Example:

Expand the expression $(2x + 1)(x - 8)$.

First:

Outer:

Inner:

Last:

Combine like terms:

Example:

If $x^2 - 3x + 5 = 3$, what are the possible values of x?

Step 1:

Step 2:

Step 3:

Exercises

(Answers on page 88.)

Expand each of the binomials:

1. $(x + 2)(x + 5) =$
2. $(a + 4)(a - 2) =$
3. $(2y + 7)(y + 2) =$
4. $(b - 8)(3b + 2) =$
5. $x(x + 1) =$

Solve for the possible values of the variable:

6. $x^2 + 7x + 12 = 0$
7. $b^2 + 3b - 10 = 0$
8. $2y(y - 4) = 0$
9. $2a^2 + 7a + 3 = 0$
10. $z^2 - 11z + 45 = 15$

Test Question

11. What is the set of all values of x for which $x^2 - 3x - 18 = 0$?

(A) $\{-6\}$
(B) $\{-3\}$
(C) $\{-3, 6\}$
(D) $\{3, 6\}$
(E) $\{2, 6\}$

Inequalities

Greater than: $>$
Less than: $<$
Greater than or equal: $\geq$
Less than or equal: $\leq$

Inequalities should be treated exactly like equations, with two exceptions:

1. When we multiply or divide an inequality by a negative number, we must reverse the direction of the inequality sign.
2. Single-variable equations are usually solved for a specific value, whereas inequalities can only be solved for a range of values.

Example:

If $3 - \frac{x}{4} \geq 2$, solve for x.

Eliminate fractions:

Isolate x on one side of the inequality:

Exercises

(Answers on page 89.)

Solve for *x*:

1. $3x + 4 > 64$
2. $2x + 1 < 21$
3. $-x + 1 \leq 63 + x$
4. $21x - 42 \leq 14x$
5. $6 > x + 4 > 4$
6. $2x > x + 10 > -x$
7. $4x + 3 < 24 - 6x$
8. $35 - 7x + 12 > 4(x - 2)$
9. $3x(12) \geq 24$
10. $9 - 12x \leq \frac{1}{3}x$

Test Question

11. The inequality $3x - 16 > 4x + 12$ is true if and only if which of the following is true?

    (A) $x < -28$
    (B) $x < -7$
    (C) $x > -7$
    (D) $x > -16$
    (E) $x > -28$

Symbolism

- Symbolism questions give test takers a definition of a symbol and then ask test takers to apply the definition.
- The definitions given in symbolism questions apply only to the particular question at hand.

Example:

Let x^* be defined by the equation $x^* = \frac{x^2}{1 - x^2}$. Evaluate $\left(\frac{1}{2}\right)^*$.

Plug in $\frac{1}{2}$ anywhere you see x:

Solve the expression:

Exercises

(Answers on page 90.)

1. For all x, the operation # is defined by $\#x = 3x + 4$. Evaluate #7.
2. For all positive x, the operation Δ is defined by $\Delta x = \frac{x}{x + 1}$. Evaluate $\Delta\left(\frac{7}{16}\right)$.
3. For all positive x, the operation $\uparrow$ is defined by $\uparrow x = \frac{5x + 6}{4x + 27}$. Evaluate $\uparrow\left(\frac{7}{4}\right)$.
4. For all x and y, the operation λ is defined by $x \lambda y = 5x - 7y$. Evaluate $8 \lambda 14$.
5. For all x and y, the operation $\square$ is defined by $x \square y = x^y + x + y$. Evaluate $5 \square 3$.
6. The operation $\Leftrightarrow$ is defined for all numbers x, y, and z by the equation $x \Leftrightarrow y \Leftrightarrow z = xy + xz + yz$. Evaluate $\frac{1}{3} \Leftrightarrow \frac{1}{4} \Leftrightarrow \frac{4}{7}$.
7. For all x, the operation Δ is defined by $\Delta x = 7x + 5$. For what value of y is $\Delta y = 173$?
8. For all x and y, the operation $\downarrow$ is defined by $x \downarrow y = xy + 5x + y$. Write an expression for $(d + 4) \downarrow d$ in terms of d and express in simplest form.
9. For all x and y, the operation Φ is defined by $x \Phi y = 12x - 8y$. If $(c + 4) \Phi (2c + 5) = 196$, then what is the value of c?
10. For all x and y, the operation Ω is defined for all x and y by $x \Omega y = x^2 - xy + 25$. If $a \Omega 8 = 10$, then what is the sum of the squares of all the possible values of a?

Test Question

11. If $m \blacktriangle n$ is defined by the equation $m \blacktriangle n = \frac{m^2 - n + 1}{mn}$ for all nonzero m and n, then $3 \blacktriangle 1 =$

(A) $\frac{9}{4}$

(B) 3

(C) $\frac{11}{3}$

(D) 6

(E) 9