Math Refresher

ANSWERS AND EXPLANATIONS

SESSION 1

KAPLAN

TEST PREP AND ADMISSIONS

Arithmetic

Integers (p. 3)

1. 2, 6
2. 1, 7, 9
3. −30, −23, 0, 6, 14
4. 53, 59, 61, 67, & 71
5. 2, 4, 6, 8, 10, 12
6. 2
7. C

1. 2, 6

An integer is even if and only if it is divisible by 2. The digits that are even in the number 27,536 are 2 and 6.

2. 1, 7, 9

An integer is odd if and only if it is not divisible by 2. The digits that are odd in the number 162,479 are 1, 7, and 9.

3. −30, −23, 0, 6, 14

Arranging the numbers 14, −23, 0, −30, 6 in increasing order, we have −30, −23, 0, 6, 14.

4. 53, 59, 61, 67, 71

A prime number is a positive integer greater than 1 whose only positive integer factors are 1 and the integer itself. The first 4 prime numbers are 2, 3, 5, and 7. Another example of a prime number is 17. The first five prime numbers greater than 50 are 53, 59, 61, 67, and 71.

5. 2, 4, 6, 8, 10, 12

An integer is even if and only if it is divisible by 2. The first six consecutive positive, even integers are 2, 4, 6, 8, 10, and 12.

6. 2

A prime number is a positive integer greater than 1 whose only positive integer factors are 1 and the integer itself. The smallest prime number is 2.

Let's note that 2 is the only even prime number.

7. (C)

The odd integers are the integers . . . −7, −5, −3, −1, 1, 3, 5, 7,. . . .

Each odd integer is 2 more than the previous odd integer. The next greater odd integer right after $3x + 10$ is $(3x + 10) + 2$. We have $(3x + 10) + 2 = 3x + 10 + 2 = 3x + 12$. None of the answer choices is written as $3x + 12$. Let's factor 3 out of $3x + 12$. We have $3x + 12 = 3(x + 4)$. Choice (C) is correct.

Factors, Multiples, and Remainders (p. 5)

1. 1, 2, 3, 4, 6, 9, 12, 18, 36
2. 1, 3, 5, 9, 15, 45
3. 1, 11, 121
4. 1, 2, 7, 11, 14, 22, 77, 154
5. 6, 12, 18, 24, 30
6. 13, 26, 39, 52, 65
7. 25, 50, 75, 100, 125
8. 110, 220, 330, 440, 550
9. 3
10. 1
11. 2
12. 1
13. D

1. 1, 2, 3, 4, 6, 9, 12, 18, 36

Let's find pairs of two positive integers whose product is 36. We will systematically list these positive integer factors so that the first positive integer factor in the products will be increasing. Let's start with the factor 1 of 36. We have $36 = 1 \times 36$. After 1, the next positive integer factor of 36 is 2. We have $36 = 2 \times 18$. After 2, the next positive integer factor of 36 is 3. We have $36 = 3 \times 12$. After 3, the next positive integer factor of 36 is 4: $36 = 4 \times 9$. After 4, the next positive integer factor of 36 is 6: $36 = 6 \times 6$. Notice that 6 is the positive square root of 36. We only have to test in this manner up to the positive square root of 36. We know that we have found all the pairs by starting with the first factor being 1 and then, each time, finding the next larger first factor. To be sure, after 6, the next greater positive integer factor of 36 is 9, and $36 = 9 \times 4$. However, we have already listed $36 = 4 \times 9$. Here are the different pairs of positive integers whose product is 36.

$$36 = 1 \times 36$$
$$36 = 2 \times 18$$
$$36 = 3 \times 12$$
$$36 = 4 \times 9$$
$$36 = 6 \times 6$$

All the positive integer factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18, 36.

2. 1, 3, 5, 9, 15, 45

Let's find pairs of two positive integers whose product is 36. We will systematically list these positive integer factors so that the first positive integer factor in the products will be increasing. Let's start with the factor 1 of 45. We have $45 = 1 \times 45$. After 1, the next greater positive integer factor of 45 is 3: $45 = 3 \times 15$. After 3, the next greater positive integer factor of 45 is 5: $45 = 5 \times 9$. We only have to test in this manner up to the (integer closest to the) positive square root of 45; in this case, we only have to test up to 6, since 7 is too large. We know that we have found all the pairs by starting with the first factor being 1 and then, each time, finding the next larger first factor. To be sure, after 45, the next larger positive integer factor of 45 is 9: $45 = 9 \times 5$. However, we have already listed $45 = 5 \times 9$. Here are the different pairs of positive integers whose product is 45.

$$45 = 1 \times 45$$
$$45 = 3 \times 15$$
$$45 = 5 \times 9$$

We can list all the positive integer factors of 45: 1, 3, 5, 9, 15, 45.

3. 1, 11, 121

Let's find pairs of two positive integers whose product is 121. We will systematically list these positive integer factors so that the first positive integer factor in the products will be increasing. Let's start with the factor 1 of 121. We have $121 = 1 \times 121$. After 1, the next greater positive integer factor of 121 is 11: $121 = 11 \times 11$. Notice that 11 is the positive square root of 121. We only have to test in this manner up to the positive square root of 121. We know that we have found all the pairs by starting with the first factor being 1 and then, each time, finding the next larger first factor. To be sure, after 11, the next greater positive integer factor of 121 is 121: $121 = 121 \times 1$. However, we have already listed $121 = 1 \times 121$. Here are the different pairs of positive integers whose product is 121.

$$121 = 1 \times 121$$
$$121 = 11 \times 11$$

We can list all the positive integer factors of 121: 1, 11, 121.

4. 1, 2, 7, 11, 14, 22, 77, 154

Let's find pairs of two positive integers whose product is 154. We will systematically list these positive integer factors so that the first positive integer factor in the products will be increasing. Let's start with the factor 1 of 154. We have $154 = 1 \times 154$. After 1, the next greater positive integer factor of 154 is 2: $154 = 2 \times 77$. After 2, the next greater positive integer factor of 154 is 7: $154 = 7 \times 22$. After 7, the next greater positive integer factor of 154 is 11: $154 = 11 \times 14$. We only have to test in this manner up to the positive square root of 154; in this case, we only have to test up to 12, since 13 is too large, and

12 is not a factor of 154. We know that we have found all the pairs by starting with the first factor being 1 and then, each time, finding the next larger first factor. To be sure, after 11, the next greater positive integer factor of 154 is 14: $154 = 14 \times 11$. However, we have already listed $154 = 11 \times 14$. Here are the different pairs of positive integers whose product is 154.

$$154 = 1 \times 154$$

$$154 = 2 \times 77$$

$$154 = 7 \times 22$$

$$154 = 11 \times 14$$

We can list all the positive integer factors of 154: 1, 2, 7, 11, 14, 22, 77, and 154.

5. 6, 12, 18, 24, 30

The first 5 multiples of 6 are 1×6, 2×6, 3×6, 4×6, and 5×6.

Now $1 \times 6 = 6$, $2 \times 6 = 12$, $3 \times 6 = 18$, $4 \times 6 = 24$, and $5 \times 6 = 30$. Thus, the first 5 multiples of 6 are 6, 12, 18, 24, and 30.

6. 13, 26, 39, 52, 65

The first 5 multiples of 13 are 1×13, 2×13, 3×13, 4×13, and 5×13.

Now $1 \times 13 = 13$, $2 \times 13 = 26$, $3 \times 13 = 39$, $4 \times 13 = 52$, and $5 \times 13 = 65$. Thus, the first 5 multiples of 13 are 13, 26, 39, 52, and 65.

7. 25, 50, 75, 100, 125

The first 5 multiples of 25 are 1×25, 2×25, 3×25, 4×25, and 5×25.

Now $1 \times 25 = 25$, $2 \times 25 = 50$, $3 \times 25 = 75$, $4 \times 25 = 100$, and $5 \times 25 = 125$. Thus, the first 5 multiples of 25 are 25, 50, 75, 100, and 125.

8. 110, 220, 330, 440, 550

The first 5 multiples of 110 are 1×110, 2×110, 3×110, 4×110, and 5×110.

Now $1 \times 110 = 110$, $2 \times 110 = 220$, $3 \times 110 = 330$, $4 \times 110 = 440$, and $5 \times 110 = 550$. Thus, the first 5 multiples of 110 are 110, 220, 330, 440, and 550.

9. 3

Let's divide 68 by 5 using long division.

$$\begin{array}{r} 13 \\ 5\overline{)68} \\ -5 \\ \hline 18 \\ -15 \\ \hline 3 \end{array}$$

We see that when 68 is divided by 5, the quotient is 13 and the remainder is 3.

We can check that our long division and answer are correct by verifying that $68 = 13 \times 5 + 3$.

10. 1

Let's divide 93 by 4 using long division.

$$\begin{array}{r} 23 \\ 4\overline{)93} \\ -8 \\ \hline 13 \\ -12 \\ \hline 1 \end{array}$$

We see that when 93 is divided by 4, the quotient is 23 and the remainder is 1.

We can check that our long division and answer are correct by verifying that $93 = 23 \times 4 + 1$.

11. 2

Let's divide 164 by 3 using long division.

$$\begin{array}{r} 54 \\ 3\overline{)164} \\ -15 \\ \hline 14 \\ -12 \\ \hline 2 \end{array}$$

We see that when 164 is divided by 3, the quotient is 54 and the remainder is 2.

We can check that our long division and answer are correct by verifying that $164 = 54 \times 3 + 2$.

12. 1

Let's divide 225 by 7 using long division.

$$\begin{array}{r} 32 \\ 7\overline{)225} \\ -21 \\ \hline 15 \\ -14 \\ \hline 1 \end{array}$$

We see that when 225 is divided by 7, the quotient is 32 and the remainder is 1.

We can check that our long division and answer are correct by verifying that $225 = 32 \times 7 + 1$.

13. (D)

Let's test each answer choice to see whether or not it is a factor of 168. The correct answer choice will be the choice that is NOT a factor of 168.

Choice (A) is 21. Because $168 \div 21$ is 8 with no remainder, 21 is a factor of 168. Discard choice (A).

Choice (B) is 24. Because 168 ÷ 24 is 7 with no remainder, 24 is a factor of 168. Discard choice (B).

Choice (C) is 28. Because 168 ÷ 28 is 6 with no remainder, 28 is a factor of 168. Discard choice (C).

Choice (D) is 32. Because 168 ÷ 32 is 5 with a remainder of 8, 32 is NOT a factor of 168. Choice (D) is correct.

At this point, we can move on to the next question. For the sake of the discussion, though, let's look at choice (E).

Choice (E) is 42. Because 168 ÷ 42 is 4 with no remainder, 42 is a factor of 168. Discard choice (E).

Divisibility Rules (p. 7)

1. 1,662
2. 324
3. 252
4. 310
5. 315
6. E

1. 1,662

An integer is divisible by 3 if and only if the sum of its digits is divisible by 3. Let's look at each number.

Let's consider 241. The sum of the digits of 241 is $2 + 4 + 1 = 7$. Now 7 divided by 3 is 2 with a remainder of 1. Because the remainder is not 0, 7 is not divisible by 3, and 241 is not divisible by 3.

Let's consider 1,662. The sum of the digits of 1,662 is $1 + 6 + 6 + 2 = 15$. Now 15 divided by 3 is 5 with a remainder of 0. Therefore, 15 is divisible by 3, and 1,662 is divisible by 3.

Let's consider 4,915. The sum of the digits of 4,915 is $4 + 9 + 1 + 5 = 19$. Now 19 divided by 3 is 6 with a remainder of 1. Since the remainder is not 0, 19 is not divisible by 3 Therefore, 4,915 is not divisible by 3.

Let's consider 3,131. The sum of the digits of 3,131 is $3 + 1 + 3 + 1 = 8$. Now 8 divided by 3 is 2 with a remainder of 2. Because the remainder is not 0, 8 is not divisible by 3, and 3,131 is not divisible by 3.

2. 324

An integer is divisible by 4 if and only if the two-digit number formed by the tens digit and the units digit is divisible by 4. Let's look at each number.

Let's consider 126. Now 26 divided by 4 is 6 with a remainder of 2. Since the remainder is not 0, 26 is not divisible by 4. Thus, 126 is not divisible by 4.

Let's consider 324. Now 24 divided by 4 is 6 with a remainder of 0. Therefore, 24 is divisible by 4, and 324 is divisible by 4.

Let's consider 442. Now 42 divided by 4 is 10 with a remainder of 2. Since the remainder is not 0, 42 is not divisible by 4. Thus, 442 is not divisible by 4.

Let's consider 598. Now 98 divided by 4 is 24 with a remainder of 2. Since the remainder is not 0, 98 is not divisible by 4. Thus, 598 is not divisible by 4.

3. 252

An integer is divisible by 6 if and only if the integer is divisible by 2 and 3.

An integer is even if and only if the units digit of the integer is even.

An integer is divisible by 3 if and only if the sum of its digits is divisible by 3.

Now let's look at each number.

Let's consider 124. The units digit of 124 is even. Therefore, 124 is divisible by 2. The sum of the digits of 124 is $1 + 2 + 4 = 7$. Now 7 divided by 3 is 2 with a remainder of 1. Since the remainder is not 0, 7 is not divisible by 3. Thus, 124 is not divisible by 3. Since 124 is not divisible by 3, 124 is not divisible by 6.

Let's consider 252. The units digit of 252 is 2, which is even. Therefore, 252 is divisible by 2. The sum of the digits of 252 is $2 + 5 + 2 = 9$. Now 9 divided by 3 is 3 with a remainder of 0. Thus, 252 is divisible by 3. Since 252 is divisible by 2 and divisible by 3, 252 is divisible by 6.

Let's consider 412. The units digit of 412 is 2, which is even. Therefore, 412 is divisible by 2. The sum of the digits of 412 is $4 + 1 + 2 = 7$. Now 7 divided by 3 is 2 with a remainder of 1. Since the remainder is not 0, 7 is not divisible by 3. Thus, 412 is not divisible by 3 and is not divisible by 6.

Let's consider 633. The units digit of 633 is 3, which is odd. Therefore, 633 it is not divisible by 2. Since 633 is not divisible by 2, 633 is not divisible by 6.

4. 310

An integer is a multiple of 5 if and only if the integer is divisible by 5.

An integer is divisible by 5 if and only if its units digit is a 0 or a 5.

Let's consider 28. The units digit of 28 is 8, which is not 0 or 5. Therefore, 28 is not a multiple of 5.

Let's consider 127. The units digit of 127 is 7, which is not 0 or 5. Therefore, 127 is not a multiple of 5.

Let's consider 310. The units digit of 310 is 0, which is one of 0 and 5. Therefore, 310 is a multiple of 5.

Let's consider 522. The units digit of 522 is 2, which is not 0 or 5. Therefore, 522 is not a multiple of 5.

5. 315

The number 9 is a factor of an integer if and only if the integer is divisible by 9.

An integer is divisible by 9 if and only if the sum of the digits is divisible by 9.

Now let's look at each number.

Let's consider 487. The sum of the digits of 487 is $4 + 8 + 7 = 19$. Now 19 divided by 9 is 2 with a remainder of 1.

Since the remainder is not 0, 19 is not divisible by 9. Thus, 487 is not divisible by 9, and 9 is not a factor of 487.

Let's consider 315. The sum of the digits of 315 is $3 + 1 + 5 = 9$. Now 9 divided by 9 is 1 with a remainder of 0. Thus, 315 is divisible by 9, and 9 is a factor of 315.

Let's consider 1,093. The sum of the digits of 1,093 is $1 + 0 + 9 + 3 = 13$. Now 13 divided by 9 is 1 with a remainder of 4. Since the remainder is not 0, 13 is not divisible by 9. Thus, 1,093 is not divisible by 9, and 9 is not a factor of 1,093.

Let's consider 3,154. The sum of the digits of 3,154 is $3 + 1 + 5 + 4= 13$. Now 13 divided by 9 is 1 with a remainder of 4. Since the remainder is not 0, 13 is not divisible by 9. Thus, 3,154 is not divisible by 9, and 9 is not a factor of 3,154.

6. (E)

Let's test the answer choices until we find one that is a multiple of each of 2, 3, and 5. Here are the rules of divisibility that we will use:

An integer is divisible by 2 if and only if it is even.

An integer is divisible by 3 if and only if the sum of its digits is divisible by 3.

An integer is divisible by 5 if and only if the units digit is a 0 or a 5.

Choice (A) is 525. An integer is even if and only if its units digit is even. The units digit of 525 is 5, which is odd. Discard choice (A).

Choice (B) is 560. An integer is even if and only if its units digit is even. The units digit of 560 is 0, which is even. Moving on, an integer is divisible by 3 if and only if the sum of its digits is divisible by 3. The sum of the digits of 560 is $5 + 6 + 0 = 11$. Now 11 divided by 3 is 3 with a remainder of 2. Therefore, 560 is not a multiple of 3. Discard choice (B).

Choice (C) is 615. An integer is even if and only if its units digit is even. The units digit of 615 is 5, which is odd. Discard choice (C).

Choice (D) is 620. An integer is even if and only if its units digit is even. The units digit of 620 is 0, which is even. Moving on, an integer is divisible by 3 if and only if the sum of its digits is divisible by 3. The sum of the digits of 620 is $6 + 2 + 0 = 8$. Next, 8 divided by 3 is 2 with a remainder of 2. Thus, 620 is not a multiple of 3. Discard choice (D).

Now that all four incorrect answer choices have been eliminated, we know that choice (E) must be correct. For the sake of the discussion, let's look at (E).

Choice (E) is 660. An integer is divisible by 2 if and only if it is even. The units digit of 660 is 0, which is even. Moving on, an integer is divisible by 3 if and only if the sum of its digits is divisible by 3. The sum of the digits of 660 is $6 + 6 + 0 = 12$. Now 12 divided by 3 is 4 with no remainder. Moving on, an integer is divisible by 5 if and only if its units digit is 0 or 5. Since the units digit of 660 is 0, 660 is divisible by 5. Thus, 660 is a multiple of 2, 3, and 5.

Fractions (p. 9)

1. $0.\overline{3}$
2. 0.4
3. 0.16
4. $0.\overline{428571}$
5. $0.38\overline{63}$
6. $\frac{9}{20}$
7. $\frac{667}{1{,}000}$
8. $\frac{9}{4} = 2\frac{1}{4}$
9. $\frac{27}{20} = 1\frac{7}{20}$
10. $-\frac{1}{8}$
11. C

1. $0.\overline{3}$

Let's convert $\frac{1}{3}$ to a decimal using long division.

$$\begin{array}{r} 0.333 \\ 3\overline{)1.000} \\ \underline{-9} \\ 10 \\ \underline{-9} \\ 10 \\ \underline{-9} \\ 1 \end{array}$$

We see that to the right of the decimal, 3 will repeat infinitely. Thus, $\frac{1}{3} = 0.33333333\ldots$. To describe the pattern of repeating 3s to the right of the decimal point, we write $0.\overline{3}$.

2. 0.4

Let's look for a power with a base of 10 and a positive exponent that is a multiple of the denominator 5 of $\frac{2}{5}$. That is, we will look to see if one of 10, 100, 1,000, 10,000,... is a multiple of 5. We see that 10 is a multiple of 5: $10 = 2 \times 5$. Then $\frac{2}{5} = \frac{2}{5} \times \frac{2}{2} = \frac{2 \times 2}{5 \times 2} = \frac{4}{10} = 0.4$.

Alternatively, here is the conversion of $\frac{2}{5}$ to a decimal using long division.

$$\begin{array}{r} 0.4 \\ 5\overline{)2.000} \\ \underline{-20} \\ 0 \end{array}$$

3. 0.16

Let's begin by looking for a power with a base of 10 that is a multiple of the denominator 25 of $\frac{4}{25}$. That is, we will see if one of 10, 100, 1,000, 10,000, . . . is a multiple of 25. While 10 is not a multiple of 25, 100 is a multiple of 25: $100 = 4 \times 25$. Then $\frac{4}{25} = \frac{4}{25} \times \frac{4}{4} = \frac{4 \times 4}{25 \times 4} = \frac{16}{100} = 0.16$.

Alternatively, here is the conversion of $\frac{4}{25}$ to a decimal using long division.

$$\begin{array}{r} 0.16 \\ 25\overline{)4.000} \\ \underline{-25} \\ 150 \\ \underline{150} \\ 0 \end{array}$$

4. $0.\overline{428571}$

Let's convert $\frac{3}{7}$ to a decimal using long division.

$$\begin{array}{r} 0.4285714 \\ 7\overline{)3.0000000} \\ \underline{-28} \\ 20 \\ \underline{-14} \\ 60 \\ \underline{-56} \\ 40 \\ \underline{-35} \\ 50 \\ \underline{-49} \\ 10 \\ \underline{-7} \\ 30 \\ \underline{-28} \\ 2 \end{array}$$

Notice that when we obtained the first digit to the right of the decimal, 30 was divided by 7, and when we obtained the seventh digit to the right of the decimal point, 30 was divided by 7. We have that $\frac{3}{7} = 0.428571428571428571\ldots$.

To indicate that 142857 repeats, we write that $\frac{3}{7} = 0.\overline{428571}$. Notice that we place a line above only those digits that repeat.

5. $0.38\overline{63}$

Let's convert $\frac{17}{44}$ to a decimal using long division.

$$\begin{array}{r}
0.386363\\
44\overline{)17.000000}\\
\underline{-132}\\
380\\
\underline{-352}\\
280\\
\underline{-264}\\
160\\
\underline{-132}\\
280\\
\underline{-264}\\
160\\
\underline{-132}\\
28
\end{array}$$

We see that the repeating pattern begins with the third digit to the right of the decimal point. Beginning with the third digit to the right of the decimal point, we have the repeating pattern of 63636363. . . .

Thus, $\frac{17}{44} = 0.3863636363....$ To indicate that the 63 repeats, we write that $\frac{17}{44} = 0.38\overline{63}$. Notice that we place a line above only those digits that repeat.

6. $\mathbf{\frac{9}{20}}$

To convert a decimal to a fraction, we divide the integer formed by the digits to the right of the decimal point by 10^n, where n is the number of digits to the right of the decimal point.

There are two digits to the right of the decimal point in 0.45. Thus, we have $0.45 = \frac{45}{10^2} = \frac{45}{100} = \frac{45 \div 5}{100 \div 5} = \frac{9}{20}$.

7. $\mathbf{\frac{667}{1{,}000}}$

To convert decimal to a fraction, we divide the integer formed by the digits to the right of the decimal point by 10^n, where n is the number of digits to the right of the decimal point.

There are three digits to the right of the decimal point in 0.667. Thus, we have $0.667 = \frac{667}{10^3} = \frac{667}{1{,}000}$.

8. $\mathbf{\frac{9}{4} = 2\frac{1}{4}}$

We start by dividing the integer formed by the digits to the right of the decimal point by 10^n, where n is the number of digits to the right of the decimal point.

There are two digits to the right of the decimal point in 2.25. Converting 0.25 to a fraction, we have $0.25 = \frac{25}{10^2}$ $\frac{25}{100} = \frac{25 \div 5}{100 \div 5} = \frac{5}{20} = \frac{5 \div 5}{20 \div 5} = \frac{1}{4}$. Since the integer to the left of the decimal point is 2, $2.25 = 2 + \frac{1}{4} = 2\frac{1}{4}$. We can also express that $2\frac{1}{4} = \frac{2 \times 4 + 1}{4} = \frac{9}{4}$.

9. $\mathbf{\frac{27}{20} = 1\frac{7}{20}}$

We start by dividing the integer formed by the digits to the right of the decimal point by 10^n, where n is the number of digits to the right of the decimal point.

There are two digits to the right of the decimal point in 1.35. Thus, we have $0.35 = \frac{35}{10^2} = \frac{35}{100} = \frac{35 \div 5}{100 \div 5} = \frac{7}{20}$. Since the integer to the left of the decimal point is 1, $1.35 = 1 + \frac{7}{20} = 1\frac{7}{20}$.

Also, $1\frac{7}{20} = \frac{1 \times 20 + 7}{20} = \frac{20 + 7}{20} = \frac{27}{20}$

10. $\mathbf{-\frac{1}{8}}$

To convert a decimal to a fraction, we divide the integer formed by the digits to the right of the decimal point by 10^n, where n is the number of digits to the right of the decimal point.

The number -0.125 is a negative number. Let's convert 0.125 to a fraction, then include the negative sign. There are three digits to the right of the decimal point in 0.125. Thus, we have $0.125 = \frac{125}{10^3} = \frac{125}{1{,}000} = \frac{125 \div 5}{1{,}000 \div 5} = \frac{25}{200} = \frac{25 \div 5}{200 \div 5} = \frac{5}{40} = \frac{5 \div 5}{40 \div 5} = \frac{1}{8}$. Since $0.125 = \frac{1}{8}$, $-0.125 = -\frac{1}{8}$.

11. (C)

We want the choice that equals $6\frac{3}{4} - 6.32$. Since all the answer choices are decimals, let's convert $6\frac{3}{4}$ to a decimal. The decimal equivalent of $\frac{3}{4}$ is 0.75. Thus, $6\frac{3}{4} = 6.75$. Then $6\frac{3}{4} - 6.32 = 6.75 - 6.32 = 0.43$. Choice (C) is correct.

Fraction Operations (p. 11)

1. $\frac{13}{12} = 1\frac{1}{12}$
2. $\frac{77}{25} = 3\frac{2}{25}$
3. $\frac{17}{6} = 2\frac{5}{6}$
4. $\frac{23}{16} = 1\frac{7}{16}$
5. $\frac{5}{3} = 1\frac{2}{3}$
6. $\frac{7}{24}$
7. 4
8. 2.48
9. $\frac{1}{16}$
10. 0.00338
11. C

1. $\frac{13}{12} = 1\frac{1}{12}$

To add the fractions, we need a common denominator. The lowest common denominator of the fractions $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ is the least common multiple of the denominators 2, 3, and 4. Since 4 is a multiple of 2. ($4 = 2 \times 2$), the least common multiple of 2, 3, and 4 is the least common multiple of 3 and 4. Since 3 and 4 have no common factors greater than 1, the least common multiple of 3 and 4 is the product of 3 and 4, which is $3 \times 4 = 12$. Thus, the least common multiple of 2, 3, and 4 is 12.

Then $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{1 \times 6}{2 \times 6} + \frac{1 \times 4}{3 \times 4} + \frac{1 \times 3}{4 \times 3} = \frac{6}{12} + \frac{4}{12} + \frac{3}{12} = \frac{6+4+3}{12} = \frac{13}{12}$.

Thus, $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{13}{12}$, or $1\frac{1}{12}$.

2. $\frac{77}{25} = 3\frac{2}{25}$

To add the fractions, we need a common denominator. The lowest common denominator of the fractions $\frac{12}{25}$ and $\frac{13}{5}$ is the least common multiple of the denominators 25 and 5. Since 25 is a multiple of 5. ($25 = 5 \times 5$), the least common multiple of 25 and 5 is 25.

Then $\frac{12}{25} + \frac{13}{5} = \frac{12}{25} + \frac{13 \times 5}{5 \times 5} = \frac{12}{25} + \frac{65}{25} = \frac{12 + 65}{25} = \frac{77}{25}$, or $3\frac{2}{25}$.

3. $\frac{17}{6} = 2\frac{5}{6}$

We want to find $\frac{6}{12} + \frac{7}{3}$. We see that $\frac{6}{12}$ can be reduced: $\frac{6}{12} = \frac{6 \div 6}{12 \div 6} = \frac{1}{2}$. Thus, $\frac{6}{12} + \frac{7}{3} = \frac{1}{2} + \frac{7}{3}$. To add the fractions, we need a common denominator. The lowest common denominator of the fractions $\frac{1}{2}$ and $\frac{7}{3}$ is the least common multiple of the denominators 2 and 3. Since 2 and 3 have no common factor greater than 1, the least common multiple of 2 and 3 is $2 \times 3 = 6$. Therefore, the lowest common denominator of the fractions $\frac{1}{2} + \frac{7}{3}$ is 6.

Then $\frac{1}{2} + \frac{7}{3} = \frac{1 \times 3}{2 \times 3} + \frac{7 \times 2}{3 \times 2} = \frac{3}{6} + \frac{14}{6} = \frac{3 + 14}{6} = \frac{17}{6}$, or $2\frac{5}{6}$.

4. $\frac{23}{16} = 1\frac{7}{16}$

To add or subtract fractions, we need a common denominator. The lowest common denominator of the fractions $\frac{1}{16}$, $\frac{3}{4}$ and $\frac{7}{8}$ is the least common multiple of the denominators 16, 4, and 8. Since 16 is a multiple of 4. ($16 = 4 \times 4$) and 16 is a multiple of 8 ($16 = 2 \times 8$), the least common multiple of 16, 4, and 8 is 16. Therefore, the lowest common denominator of the fractions $\frac{1}{16}$, $\frac{3}{4}$ and $\frac{7}{8}$ is 16.

Then $\frac{1}{16} - \frac{3}{4} + \frac{17}{8} = \frac{1}{16} - \frac{3 \times 4}{4 \times 4} + \frac{17 \times 2}{8 \times 2} = \frac{1}{16} - \frac{12}{16} + \frac{34}{16} = \frac{1 - 12 + 34}{16} = \frac{23}{16}$, or $1\frac{7}{16}$.

5. $\frac{5}{3} = 1\frac{2}{3}$

We want to find the value of $4\left(\frac{1}{3} + \frac{1}{12}\right)$. Let's begin by working with what's inside the parentheses first, which is $\frac{1}{3} + \frac{1}{12}$. To add fractions, we need a common denominator. The lowest common denominator of the fractions $\frac{1}{3}$ and $\frac{1}{12}$ is the least common multiple of the denominators 3 and 12. Since 12 is a multiple of 3 ($12 = 4 \times 3$), the least common multiple of 3 and 12 is 12.

Then $\frac{1}{3} + \frac{1}{12} = \frac{1 \times 4}{3 \times 4} + \frac{1}{12} = \frac{4+1}{12} = \frac{5}{12}$. Thus, $\frac{1}{3} + \frac{1}{12} = \frac{5}{12}$. Therefore, $4\left(\frac{1}{3} + \frac{1}{12}\right) = 4\left(\frac{5}{12}\right)$. Now let's work with $4\left(\frac{5}{12}\right)$, which is $\frac{4 \times 5}{12} = \frac{20}{12} = \frac{5}{3}$, or $1\frac{2}{3}$.

6. $\frac{7}{24}$

We want to find the value of $\frac{1}{2}\left(\frac{1}{3} + \frac{1}{4}\right)$. Let's begin by working with what's inside the parentheses first, which is $\frac{1}{3} + \frac{1}{4}$. To add fractions, we need a common denominator. The lowest common denominator of the fractions $\frac{1}{3}$ and $\frac{1}{4}$ is the least common multiple of the denominators 3 and 4. Since 3 and 4 have no common factors greater than 1, the least common multiple of 3 and 4 is $3 \times 4 = 12$.

Then $\frac{1}{3} + \frac{1}{4} = \frac{1 \times 4}{3 \times 4} + \frac{1 \times 3}{4 \times 3} = \frac{4}{12} + \frac{3}{12} = \frac{4 + 3}{12} = \frac{7}{12}$.

Therefore, $\frac{1}{2}\left(\frac{1}{3} + \frac{1}{4}\right) = \frac{1}{2}\left(\frac{7}{12}\right)$.

Now $\frac{1}{2}\left(\frac{7}{12}\right) = \frac{1 \times 7}{2 \times 12} = \frac{7}{24}$.

7. 4

We want the value of $\frac{1}{24}(36 + 60)$. Let's work with what's inside the parentheses first: $36 + 60 = 96$. Then $\frac{1}{24}(36 + 60) = \frac{1}{24}(96)$. Now $\frac{1}{24}(96) = \frac{96}{24} = 4$.

8. 2.48

To find the value of $0.21 + 0.946 + 1.324$, let's align 0.21, 0.946 and 1.324 in a column where the decimal points are lined up.

$$\begin{array}{r} 0.21 \\ 0.946 \\ 1.324 \\ \hline 2.480 \end{array}$$

Thus, $0.21 + 0.946 + 1.324 = 2.480 = 2.48$.

9. $\frac{1}{16}$

We want the value of $\left(\frac{12}{16} - \frac{3}{6}\right)^2$. The first thing we must do is simplify $\frac{12}{16} - \frac{3}{6}$ inside the parentheses. Notice that both $\frac{12}{16}$ and $\frac{3}{6}$ can be reduced: $\frac{12}{16} = \frac{12 \div 4}{16 \div 4} = \frac{3}{4}$, and $\frac{3}{6} = \frac{3 \div 3}{6 \div 3} = \frac{1}{2}$. Let's replace $\frac{12}{16}$ with $\frac{3}{4}$ and let's replace $\frac{3}{6}$ with $\frac{1}{2}$ so that $\left(\frac{12}{16} - \frac{3}{6}\right)^2 = \left(\frac{3}{4} - \frac{1}{2}\right)^2$.

Now let's work with $\frac{3}{4} - \frac{1}{2}$. To subtract fractions, we need a common denominator. The lowest common denominator of the fractions $\frac{3}{4}$ and $\frac{1}{2}$ is the least common multiple of the denominators 4 and 2. Since 4 is a multiple of 2, the least common multiple of 4 and 2 is 4. Then $\frac{3}{4} - \frac{1}{2} = \frac{3}{4} - \frac{1 \times 2}{2 \times 2} = \frac{3}{4} - \frac{2}{4} = \frac{1}{4}$. Then $\left(\frac{3}{4} - \frac{1}{2}\right)^2 = \left(\frac{1}{4}\right)^2 = \frac{1}{4} \times \frac{1}{4} = \frac{1 \times 1}{4 \times 4} = \frac{1}{16}$.
Thus, $\left(\frac{12}{16} - \frac{3}{6}\right)^2 = \frac{1}{16}$.

10. 0.00338

To multiply two decimals, first multiply the numbers as if there were no decimal points. Then find the total number of digits to the right of the decimal point in the two numbers. Move the decimal point to the left that number of places.

Here, we want to multiply 1.69 and 0.002. Let's first multiply 169 and 2: $169 \times 2 = 338$. There are 2 decimal places in 1.69 and 3 decimal places in 0.002. The total number of places is $2 + 3 = 5$. We move the decimal point in 338 5 places to the left, and we have 0.00338. Thus, $1.69 \times 0.002 = 0.00338$.

11. (C)

We want to work with $\frac{7}{5} \times \left(\frac{3}{7} - \frac{2}{5}\right)$. Let's begin by working inside the parentheses first: $\frac{3}{7} - \frac{2}{5} = \frac{3}{7} \times \frac{5}{5} - \frac{2}{5} \times \frac{7}{7} = \frac{15}{35} - \frac{14}{35} = \frac{1}{35}$.
Then $\frac{7}{5} \times \left(\frac{3}{7} - \frac{2}{5}\right) = \frac{7}{5} \times \frac{1}{35}$.
We can cancel a factor of 7 from the numerator of $\frac{7}{5}$ and the denominator of $\frac{1}{35}$:
$\frac{7}{5} \times \frac{1}{35} = \frac{1}{5} \times \frac{1}{5}$.
And $\frac{1}{5} \times \frac{1}{5} = \frac{1 \times 1}{5 \times 5} = \frac{1}{25}$.
Thus, $\frac{7}{5} \times \left(\frac{3}{7} - \frac{2}{5}\right) = \frac{1}{25}$. Choice (C) is correct.

Ratios (p. 13)

1. 1:16
2. 18:5
3. 1:5

4a. 5:6
4b. 5:11
4c. 6:11
5a. 3:7
5b. 1:3
5c. 70
6. B

1. 1:16

Let's divide both members of the ratio 8:128 by 8. Then we have the ratio 1:16. The ratio 1:16 cannot be reduced any further and thus is the original ratio 8:128 reduced to lowest terms.

2. 18:5

Let's divide both members of the ratio 72:20 by 4. Then we have the ratio 18:5. The ratio 18:5 cannot be reduced any further and thus is the original ratio 72:20 reduced to lowest terms.

3. 1:5

Let's begin by multiplying both members of the ratio 14.2:71 by 10 in order to eliminate the decimal point in 14.2. We have 10(14.2):10(71) and then 142:710. Let's divide both members of the ratio 142:710 by 2. Then we have the ratio 71:355. Now let's divide both members of the ratio 71:355 by 71. Then we have the ratio 1:5. The ratio 1:5 cannot be reduced any further. Thus, 1:5 is the original ratio 14.2:71 reduced to lowest terms.

4a. 5:6

There are 10 red balls and 12 blue balls. The ratio of red balls to blue balls is 10:12. Let's divide both members of the ratio 10:12 by 2. Then we have the ratio 5:6. The ratio 5:6 cannot be reduced any further. Thus, we can say that the ratio of red balls to blue balls is 5:6.

4b. 5:11

We want the ratio of the number of red balls to the total number of balls. There are 10 red balls. Since there are 10 red balls and 12 blue balls, the total number of balls is $10 + 12 = 22$. The ratio of the number of red balls to the total number of balls is 10:22. Let's divide both members of the ratio 10:22 by 2. Then we have the ratio 5:11. The ratio 5:11 cannot be reduced any further. Thus, we can say that the ratio of red balls to the total number of balls is 5:11.

4c. 6:11

We want the ratio of the number of blue balls to the total number of balls. There are 12 blue balls. Since there are 10 red balls and 12 blue balls, the total number of balls is $10 + 12 = 22$. The ratio of the number of blue balls to the total number of balls is 12:22. Let's divide both members of the ratio 12:22 by 2. Then we have the ratio 6:11. The ratio 6:11 cannot be reduced any further. Thus, we can say that the ratio of blue balls to the total number of balls is 6:11.

5a. 3:7

The ratio of sneakers to sandals to high heels is 3:5:7. In this ratio, sneakers are represented by 3 while high heels are represented by 7. So the ratio of sneakers to high heels is 3:7.

5b. 1:3

In this question, one of the keys is to understand that *shoes* means the total number of sneakers, sandals, and high heels.

The ratio of sneakers to sandals to high heels is 3:5:7. In this ratio, sandals are represented by 5. Shoes are represented by the sum of all the numbers in the ratio. Therefore, shoes is represented by $3 + 5 + 7 = 15$. Since sandals are represented by 5 and shoes are represented by 15, the ratio of sandals to shoes is 5:15. Let's divide both members of the ratio 5:15 by 5. Then we have the ratio 1:3. The ratio 1:3 cannot be reduced any further. Thus, the ratio of sandals to shoes is 1:3.

5c. 70

The ratio of the number of pairs of sneakers to the number of pairs of sandals to the number of pairs of high heels is 3:5:7. Let's say that the number of sandals is $3x$, the number of sneakers is $5x$, and the number of high heels is $7x$, where x is a positive integer. The total number of pairs of shoes is $3x + 5x + 7x = 15x$. Now the total number of shoes is 210. So we have the equation $15x = 210$. Dividing both sides by 15, we get $x = 14$. The number of sandals is $5x$. Thus, the number of sandals is $5x = 5(14) = 70$.

6. (B)

To have a possible ratio of white rabbits to brown rabbits, the sum of the parts of the ratio must be a factor of 55. That's because "the number of each type of rabbit" must be an integer. The prime factorization of 55 is 5×11. All the factors of 55 are 1, 5, 11, and 55. The sum of the parts of the two-part ratio must be 1, 5, 11, or 55. Since the answer choices all have ratios of two positive integers, we will not have a case where the sum of the two integers is 1.

Let's look at the answer choices to see which one has the sum of its two integer parts equal to 5, 11, or 55.

Choice (A) is 1:3. We have $1 + 3 = 4$. This is not 5, 11, or 55. Discard choice (A).

Choice (B) is 3:8. We have $3 + 8 = 11$, which is one of 5, 11, or 55. Choice (B) is correct.

At this point, we can move on to the next question. For the sake of the discussion, though, let's look at choices (C), (D), and (E).

Choice (C) is 5:11. We have $5 + 11 = 16$. This is not 5, 11, or 55. Discard choice (C).

Choice (D) is 3:4. We have $3 + 4 = 7$. This is not 5, 11, or 55. Discard choice (D).

Choice (E) is 5:1. We have $5 + 1 = 6$. This is not 5, 11, or 55. Discard choice (E).

Percents (p. 15)

1. 0.2%
2. 28%
3. 131%
4. 240%
5. $\frac{1}{4}\% = 0.25\%$
6. $\frac{4}{25}$
7. $\frac{6}{25}$
8. $\frac{9}{25{,}000}$
9. $1\frac{1}{4} = \frac{5}{4}$
10. $\frac{1}{400}$
11. 6
12. 12
13. 6
14. 0.48
15. 5
16. B

1. 0.2%

To convert a decimal to a percent, multiply it by 100%. Thus, $0.002 = 0.002 \times 100\% = 0.2\%$.

2. 28%

To convert a fraction to a percent, multiply it by 100%. Thus, $\frac{7}{25} = \frac{7}{25} \times 100\% = 7 \times 4\% = 28\%$.

3. 131%

To convert a decimal to a percent, multiply it by 100%. Thus, $1.31 = 1.31 \times 100\% = 131\%$.

4. 240%

To convert a fraction to a percent, multiply it by 100%. Thus, $\frac{12}{5} = \frac{12}{5} \times 100\% = 12 \times 20\% = 240\%$.

5. $\frac{1}{4}\% = 0.25\%$

To convert a fraction to a percent, multiply it by 100%. Thus, $\frac{1}{400} = \frac{1}{400} \times 100\% = \frac{1}{4}\% = 0.25\%$.

6. $\frac{4}{25}$

To convert a percent to a fraction, divide the percent by 100%.

$16\% = \frac{16}{100} = \frac{16 \div 4}{100 \div 4} = \frac{4}{25}$.

Thus, $16\% = \frac{4}{25}$.

7. $\frac{6}{25}$

To convert a percent to a fraction, divide the percent by 100%.

$24\% = \frac{24}{100} = \frac{24 \div 4}{100 \div 4} = \frac{6}{25}$.

Thus, $24\% = \frac{6}{25}$.

8. $\frac{9}{25{,}000}$

To convert a percent to a fraction, divide the percent by 100%.

$0.036\% = \frac{0.036}{100} = \frac{0.036 \times 1{,}000}{100 \times 1{,}000} = \frac{36}{100{,}000} = \frac{36 \div 4}{100{,}000 \div 4} = \frac{9}{25{,}000}$. Thus, $0.036\% = \frac{9}{25{,}000}$.

9. $\frac{5}{4} = 1\frac{1}{4}$

To convert a percent to a fraction, divide the percent by 100%.

$125\% = \frac{125}{100} = \frac{125 \div 5}{100 \div 5} = \frac{25}{20} = \frac{25 \div 5}{20 \div 5} = \frac{5}{4}$. Thus, $125\% = \frac{5}{4}$, or $1\frac{1}{4}$.

10. $\frac{1}{400}$

To convert a percent to a fraction, divide the percent by 100%.

$\frac{1}{4}\% = \frac{\left(\frac{1}{4}\right)}{100} = \frac{1}{4} \times \frac{1}{100} = \frac{1 \times 1}{4 \times 100} = \frac{1}{400}$. Thus, $\frac{1}{4}\% = \frac{1}{400}$.

11. 6

The decimal equivalent of 50% is 0.5. Then 50% of 12 is $0.5 \times 12 = 6$.

If we did not remember the decimal equivalent of 50%, we could have converted 50% to a decimal. To convert a percent to a decimal, divide the percent by 100%. So $50\% = \frac{50\%}{100\%} = \frac{50}{100} = 0.50 = 0.5$.

12. 12

The decimal equivalent of 75% is 0.75. Then 75% of 16 is $0.75 \times 16 = 12$.

13. 6

The decimal equivalent of 150% is 1.5. Then 150% of 4 is $1.5 \times 4 = 6$.

14. 0.48

The decimal equivalent of 24% is 0.24. Then 24% of 2 is $0.24 \times 2 = 0.48$.

15. 5

The decimal equivalent of 10% is 0.1. Then 10% of 50 is $0.1 \times 50 = 5$.

16. (B)

Since the price of the item is 20% more than the wholesale cost, the price of the item is 100% + 20% = 120% of the wholesale cost. To convert a percent to a decimal (or fraction), we divide the percent by 100%. So 120% $= \frac{120\%}{100\%} = \frac{120}{100} = \frac{120 \div 10}{100 \div 10} = \frac{12}{10} = \frac{12 \div 2}{10 \div 2} = \frac{6}{5} = 1.2$. Thus, the decimal equivalent of 120% is 1.2. So the price of the item is (\$800)(1.2) = \$960. Choice (B) is correct.

We could also find the price of the item by finding the increase over the wholesale cost and then adding that increase to the wholesale cost. The price of the item is 20% greater than the wholesale cost. The decimal equivalent of 20% is $\frac{20\%}{100\%} = \frac{20}{100} = \frac{1}{5} = 0.2$. The amount that the price of the item is greater than the wholesale cost is 0.2(\$800) = \$160. Then the price of the item is \$800 + \$160 = \$960. Again, choice (B) is correct.

Averages and Statistics (p. 17)

1. 15
2. $4\frac{1}{2}$
3. 36
4. 30
5. -34
6. D

1. 15

The average formula is Average $= \frac{\text{Sum of the terms}}{\text{Number of terms}}$.
The average of 10, 12, 16, 17, and 20 is $\frac{10 + 12 + 16 + 17 + 20}{5} = \frac{75}{5} = 15$.

2. $4\frac{1}{2}$

The average formula is Average $= \frac{\text{Sum of the terms}}{\text{Number of terms}}$.
The average of 0, 3, 6, and 9 is $\frac{0 + 3 + 6 + 9}{4} = \frac{18}{4} = \frac{18 \div 2}{4 \div 2} = \frac{9}{2} = 4\frac{1}{2}$.

3. 36

The average formula is Average $= \frac{\text{Sum of the terms}}{\text{Number of terms}}$.
The average of 12, 24, 36, 48, and 60 is $\frac{12 + 24 + 36 + 48 + 60}{5} = \frac{180}{5} = 36$.

4. 30

The average formula is Average $= \frac{\text{Sum of the terms}}{\text{Number of terms}}$.
Since the average of -4, 0, 6, and x is 8, we can write the equation $\frac{-4 + 0 + 6 + x}{4} = 8$. Simplifying the numerator of the algebraic fraction on the left side, $\frac{2 \div x}{4} = 8$. Multiplying both sides by 4, we have $2 + x = 32$. Subtracting 2 from both sides, we have $x = 30$.

5. -34

The average formula is Average $= \frac{\text{Sum of the terms}}{\text{Number of terms}}$.
Since the average of 8, 3, 12, 11, and x is 0, we can write the equation $\frac{8 + 3 + 12 + 11 + x}{5} = 0$. Simplifying the numerator of the algebraic fraction on the left side, $\frac{34 + x}{5} = 0$. Multiplying both sides by 5, we have $34 + x = 0$. Subtracting 34 from both sides, we have $x = -34$.

6. (D)

The average formula is Average $= \frac{\text{Sum of the terms}}{\text{Number of terms}}$. We can use this formula in the rearranged form Sum of the terms $=$ Average $\times$ Number of terms. Since the average of six numbers is 6, the sum of these six numbers is Average $\times$ Number of terms $= 6 \times 6 = 36$. If 3 is subtracted from each of four numbers, the sum of the six numbers is reduced by $3 \times 4 = 12$. The resulting new sum of the numbers is $36 - 12 = 24$. The number of numbers is still 6. So the new average is $\frac{24}{6} = 4$. Choice (D) is correct.

Exponents (p. 19)

1. 625
2. 32
3. 108
4. 144
5. 49
6. 101.6
7. 256
8. $\frac{1}{81}$
9. $5^{-1} = \frac{1}{5}$
10. 16
11. E

1. 625

$5^4 = 5 \times 5 \times 5 \times 5 = 25 \times 5 \times 5 = 125 \times 5 = 625$.

2. 32

$2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 4 \times 2 \times 2 \times 2 = 8 \times 2 \times 2 = 16 \times 2 = 32$.

3. 108

$4 \times 3^3 = 4 \times (3 \times 3 \times 3) = 4 \times 3 \times 3 \times 3 = 12 \times 3 \times 3 = 36 \times 3 = 108$.

4. 144

$(4 \times 3)^2 = 12^2 = 12 \times 12 = 144$

5. 49

$(4 + 3)^2 = 7^2 = 7 \times 7 = 49$

6. 101.6

$1.016 \times 10^2 = 1.016 \times (10 \times 10) = 1.016 \times 100 = 101.6$.

7. 256

$(2^2)^4 = (2 \times 2)^4 = 4^4 = 4 \times 4 \times 4 \times 4 = 16 \times 4 \times 4 = 64 \times 4 = 256$

8. $\frac{1}{81}$

$(3^{-2})^2 = \left(\frac{1}{3^2}\right)^2 = \left(\frac{1}{9}\right)^2 = \frac{1}{9} \times \frac{1}{9} = \frac{1 \times 1}{9 \times 9} = \frac{1}{81}$

9. $\frac{1}{5}$

$5^3 \times 5^{-4} = 5^3 \times \frac{1}{5^4} = \frac{5^3}{5^4} = 5^{3-4} = 5^{-1} = \frac{1}{5}$

10. 16

$\frac{2^7}{2^3} = 2^{7-3} = 2^4 = 2 \times 2 \times 2 \times 2 = 4 \times 2 \times 2 = 8 \times 2 = 16$

11. (E)

Let's substitute 2 in for x in the expression $3^x + (x^3)^2$: $3^2 + (2^3)^2 = 3 \times 3 + (2 \times 2 \times 2)^2 = 9 + 8^2 = 9 + 64 = 73$. Thus, when $x = 2$, then the expression $3^x + (x^3)^2$ is equal to 73. Choice (E) is correct.

Radicals (p. 21)

1. 4
2. $3\sqrt{14}$
3. 4
4. $6\sqrt{5}$
5. 7
6. $\sqrt{13}$
7. 1
8. 16
9. 10,000
10. $125\sqrt{5}$
11. D

1. 4

$(\sqrt{2})(\sqrt{8}) = \sqrt{2 \times 8} = \sqrt{16} = 4$

2. $3\sqrt{14}$

$(\sqrt{6})(\sqrt{21}) = \sqrt{6 \times 21} = \sqrt{126} = \sqrt{9 \times 14} = \sqrt{9} \times \sqrt{14} = 3\sqrt{14}$

3. 4

$\frac{\sqrt{48}}{\sqrt{3}} = \sqrt{\frac{48}{3}} = \sqrt{16} = 4$

4. $6\sqrt{5}$

We want to simplify $\sqrt{5} + \sqrt{125}$. Let's work with $\sqrt{125}$ first: $\sqrt{125} = \sqrt{25} \times \sqrt{5} = 5 \times \sqrt{5} = 5\sqrt{5}$.

Then $\sqrt{5} + \sqrt{125} = \sqrt{5} + 5\sqrt{5} = 6\sqrt{5}$.

5. 7

$\frac{\sqrt{147}}{\sqrt{3}} = \sqrt{\frac{147}{3}} = \sqrt{49} = 7$

6. $\sqrt{13}$

Simplifying what is under the radical, $\sqrt{49 - 36} = \sqrt{13}$. There is no more simplifying to be done.

7. 1

$\sqrt{49} - \sqrt{36} = 7 - 6 = 1$

8. 16

$(\sqrt{16})^2 = \sqrt{16} \times \sqrt{16} = 16$

9. 10,000

$(\sqrt{100})^4 = (10)^4 = 10 \times 10 \times 10 \times 10 = 100 \times 10 \times 10 = 1{,}000 \times 10 = 10{,}000$

10. $125\sqrt{5}$

$5^3 \times \sqrt{5} = 5 \times 5 \times 5 \times \sqrt{5} = 25 \times 5 \times \sqrt{5} = 125 \times \sqrt{5} = 125\sqrt{5}$

11. (D)

To solve this question, we will use one law of radicals and two laws of exponents.

We will use the law of radicals that says that if $a \geq 0$ and $b \geq 0$, then $\sqrt{ab} = \sqrt{a}\,\sqrt{b}$.

We will use these two laws of exponents.

(i) If $b \neq 0$, then $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$.

(ii) If $b \neq 0$, $\frac{b^c}{b^d} = b^{c-d}$.

To use the law of radicals we will rewrite $\frac{y}{x}$. Since $x > 0$ and $y > 0$, $\frac{y}{x} = \sqrt{\left(\frac{y}{x}\right)^2}$.

Thus, $\frac{y}{x}\sqrt{\frac{x^3}{y}} = \sqrt{\left(\frac{y}{x}\right)^2}\,\sqrt{\frac{x^3}{y}}$. Using the law of radicals, we get $\sqrt{\left(\frac{y}{x}\right)^2}\,\sqrt{\frac{x^3}{y}} = \sqrt{\left(\frac{y}{x}\right)^2\left(\frac{x^3}{y}\right)}$. Now using the law of exponents that says that if $b \neq 0$, $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$, we get $\left(\frac{y}{x}\right)^2 = \frac{y^2}{x^2}$. Then $\sqrt{\left(\frac{y}{x}\right)^2\left(\frac{x^3}{y}\right)} = \sqrt{\frac{y^2}{x^2}\left(\frac{x^3}{y}\right)} = \sqrt{\frac{x^3(y^2)}{x^2(y)}} = \sqrt{\left(\frac{x^3}{x^2}\right)\left(\frac{y^2}{y}\right)}$. Using the law of exponents that says that if b $\neq 0$, then $\frac{b^c}{b^d} = b^{c-d}$, we will rewrite both $\frac{x^3}{x^2}$ and $\frac{y2}{y}$: $\sqrt{\left(\frac{x^3}{x^2}\right)\left(\frac{y^2}{y}\right)} = \sqrt{(x^{3-2})(y^{2-1})} = \sqrt{(x^1)(y^1)} = \sqrt{xy}$. Choice (D) is correct.

Order of Operations (p. 23)

1. $13\frac{5}{16}$
2. 4.4
3. $\frac{39}{43}$
4. 0.000125
5. C

1. $13\frac{5}{16}$

To work out the value of $7 + 5 \times \left(\frac{1}{4}\right)^2 - 6 \div (2 - 3)$, we must use the order of operations rules (PEMDAS).

We first do what's inside the parentheses. Then we do the exponents. Then we do multiplication and division, which rank equally, from left to right. Then we do addition and subtraction, which rank equally, from left to right.

$$\begin{aligned} & 7 + 5 \times \left(\tfrac{1}{4}\right)^2 - 6 \div (2 - 3) \\ &= 7 + 5 \times \left(\tfrac{1}{4}\right)^2 - 6 \div (-1) \\ &= 7 + 5 \times \left(\tfrac{1}{4} \times \tfrac{1}{4}\right) - 6 \div (-1) \\ &= 7 + 5 \times \tfrac{1}{16} - 6 \div (-1) \\ &= 7 + \tfrac{5}{16} - (-6) \\ &= 13 + \tfrac{5}{16} \\ &= 13\tfrac{5}{16} \end{aligned}$$

2. 4.4

To work out the value of $4(1.24 - (0.8)^2) + 6 \times \frac{1}{3}$, we must use the order of operations rules (PEMDAS).

We first do what's inside the parentheses. Then we do the exponents. Then we do multiplication and division, which rank equally, from left to right. Then we do addition and subtraction, which rank equally, from left to right.

$$\begin{aligned} & 4(1.24 - (0.8)^2) + 6 \times \tfrac{1}{3} \\ &= 4(1.24 - (0.8)(0.8)) + 6 \times \tfrac{1}{3} \\ &= 4(1.24 - 0.64) + 6 \times \tfrac{1}{3} \\ &= 4(0.6) + 6 \times \tfrac{1}{3} \\ &= 2.4 + 6 \times \tfrac{1}{3} \\ &= 2.4 + 2 \\ &= 4.4 \end{aligned}$$

3. $\frac{39}{43}$

To work with the complex fraction $\frac{\frac{5}{6} + \frac{3}{2} + 2}{\frac{1}{3} + \frac{4}{9} + 4}$, let's multiply the numerator and denominator by the same nonzero number that will eliminate the fractions in both. A nonzero number that we can multiply both the numerator and denominator of $\frac{\frac{5}{6} + \frac{3}{2} + 2}{\frac{1}{3} + \frac{4}{9} + 4}$ in order to eliminate the fractions is the least common denominator of the fractions $\frac{5}{6}$, $\frac{3}{2}$, $\frac{1}{3}$, and $\frac{4}{9}$.

The least common denominator of these fractions is the least common multiple of 6, 2, 3, and 9. Since 6 is a multiple of 2 ($6 = 3 \times 2$) and 9 is a multiple of 3 ($9 = 3 \times 3$), the least common multiple of 6, 2, 3, and 9 is the least common multiple of 6 and 9. Now $1 \times 9 = 9$ is not a multiple of 6, while $2 \times 9 = 18$ is a multiple of 6 ($18 = 3 \times 6$). Thus, we will multiply the numerator and denominator by 18:

$$\frac{\frac{5}{6} + \frac{3}{2} + 2}{\frac{1}{3} + \frac{4}{9} + 4} = \frac{18 \times \left(\frac{5}{6} + \frac{3}{2} + 2\right)}{18 \times \left(\frac{1}{3} + \frac{4}{9} + 4\right)} = \frac{18\left(\frac{5}{6}\right) + 18\left(\frac{3}{2}\right) + 18(2)}{18\left(\frac{1}{3}\right) + 18\left(\frac{4}{9}\right) + 18(4)} =$$

$$\frac{3(5) + 9(3) + 36}{6 + 2(4) + 72} = \frac{15 + 27 + 36}{6 + 8 + 72} = \frac{78}{86} = \frac{78 \div 2}{86 \div 2} = \frac{39}{43}$$

Thus, $\frac{\frac{5}{6} + \frac{3}{2} + 2}{\frac{1}{3} + \frac{4}{9} + 4} = \frac{39}{43}$.

4. 0.000125

Let's work with $\frac{0.25 \times (0.1)^2}{0.5 \times 40}$:

$$\frac{0.25 \times (0.1)^2}{0.5 \times 40} = \frac{0.25 \times (0.1 \times 0.1)}{20} = \frac{0.25 \times (0.01)}{20} = \frac{0.0025}{20} = \frac{0.0025}{2(10)}$$

$$= \frac{0.0025}{2}\left(\frac{1}{10}\right) = 0.00125\left(\frac{1}{10}\right) = 0.000125$$

5. (C)

To work out the value of $\left(\frac{3}{4}\right)^2 + \frac{5}{24} \div \left(\frac{155}{84} - \frac{5}{12}\right)$, we must use the order of operations rules (PEMDAS).

We first do what's inside the parentheses. Then we do the exponents. Then we do multiplication and division, which rank equally, from left to right. Then we do addition and subtraction, which rank equally, from left to right.

Let's begin by working with what's inside the parentheses. The denominator 84 of $\frac{155}{84}$ is a multiple of the denominator 12 of $\frac{5}{12}$: $84 = 7 \times 12$.

Then $\frac{155}{84} - \frac{5}{12} = \frac{155}{84} - \frac{5}{12} \times \frac{7}{7} = \frac{155}{84} - \frac{5 \times 7}{12 \times 7} = \frac{155}{84} - \frac{35}{84} = \frac{155 - 35}{84} = \frac{120}{84} = \frac{120 \div 12}{84 \div 12} = \frac{10}{7}$.

Thus, $\left(\frac{3}{4}\right)^2 + \frac{5}{24} \div \left(\frac{155}{84} - \frac{5}{12}\right) = \left(\frac{3}{4}\right)^2 + \frac{5}{24} \div \frac{10}{7}$.

Now let's work with the exponent: $\left(\frac{3}{4}\right)^2 = \frac{3}{4} \times \frac{3}{4} = \frac{3 \times 3}{4 \times 4} = \frac{9}{16}$. Thus, $\left(\frac{3}{4}\right)^2 + \frac{5}{24} \div \frac{10}{7}. = \frac{9}{16} + \frac{5}{24} \div \frac{10}{7}$.

Now let's do the division: $\frac{5}{24} \div \frac{10}{7} = \frac{5}{24} \times \frac{7}{10} = \frac{1}{24} \times \frac{7}{2} = \frac{1 \times 7}{24 \times 2} = \frac{7}{48}$. Thus, $\frac{9}{16} + \frac{5}{24} \div \frac{10}{7} = \frac{9}{16} + \frac{7}{48}$.

Now let's do the addition $\frac{9}{16} + \frac{7}{48}$. To add fractions, we need a common denominator. The denominator 48 in $\frac{7}{48}$ is a multiple of the denominator 16 of $\frac{9}{16}$: $48 = 3 \times 16$.

Then $\frac{9}{16} + \frac{7}{48} = \frac{9}{16} \times \frac{3}{3} + \frac{7}{48} = \frac{9 \times 3}{16 \times 3} + \frac{7}{48} = \frac{27}{48} + \frac{7}{48} = \frac{27 + 7}{48} = \frac{34}{48} = \frac{34 \div 2}{48 \div 2} = \frac{17}{24}$.

We have that $\left(\frac{3}{4}\right)^2 + \frac{5}{24} \div \left(\frac{155}{84} - \frac{5}{12}\right) = \frac{17}{24}$. Choice (C) is correct.