Math Refresher

ANSWERS AND EXPLANATIONS

SESSION 2

KAPLAN
TEST PREP AND ADMISSIONS

Algebra

Variables (p. 27)

1. $9x - 2y$
2. $18x$
3. $-4a + 2b = 2(b - 2a)$
4. $\dfrac{13x + y}{20}$
5. $-2x^2 + 5y$
6. 130
7. $-\dfrac{7}{3}$
8. -1
9. 1
10. 28
11. D

1. $9x - 2y$

Let's simplify $2x + 4y + 7x - 6y$ by combining like terms:

$$\begin{aligned} 2x + 4y + 7x - 6y &= 2x + 7x + 4y - 6y \\ &= 9x - 2y \end{aligned}$$

2. $18x$

We can simplify the expression $4x\left(3 + \frac{3}{2}\right)$ by simplifying $3 + \frac{3}{2}$ inside the parentheses first:

$$4x\left(3 + \tfrac{3}{2}\right) = 4x\left(\tfrac{6}{2} + \tfrac{3}{2}\right) = 4x\left(\tfrac{6+3}{2}\right) = 4x\left(\tfrac{9}{2}\right) = 2x(9) = 18x.$$

We can also simplify the expression $4x\left(3 + \frac{3}{2}\right)$ by distributing $4x$ over each of the numbers inside the parentheses:

$$\begin{aligned} 4x\left(3 + \tfrac{3}{2}\right) &= 4x(3) + 4x\left(\tfrac{3}{2}\right) = 12x + 2x(3) \\ &= 12x + 6x = 18x \end{aligned}$$

3. $-4a + 2b$

First, we will remove the parentheses around $3a + 6b$. Next, we will distribute the negative sign over the terms $7a$ and $4b$ inside the parentheses. Then we will collect like terms.

$$\begin{aligned} (3a + 6b) - (7a + 4b) &= 3a + 6b - (7a + 4b) \\ &= 3a + 6b - 7a - 4b \\ &= -4a + 2b \end{aligned}$$

4. $\frac{13x + y}{20}$

Let's begin by combining like terms. One pair of like terms is $\frac{x}{4}$ and $\frac{2x}{5}$. (If we say that $\frac{x}{4}$ is $\frac{1}{4}x$ and we say that $\frac{2x}{5}$ is $\frac{2}{5}x$, then we readily see that we are combining like terms. Similarly, if we say that $\frac{4y}{5}$ is $\frac{4}{5}y$ and we say that $\frac{3y}{4}$ is $\frac{3}{4}y$, then we see that we are combining like terms.)

We have

$$\frac{x}{4} + \frac{4y}{5} - \frac{3y}{4} + \frac{2x}{5} = \left(\frac{x}{4} + \frac{2x}{5}\right) + \left(\frac{4y}{5} - \frac{3y}{4}\right)$$

Now let's simplify $\frac{x}{4} + \frac{2x}{5}$ and $\frac{4y}{5} - \frac{3y}{4}$. Let's simplify $\frac{x}{4} + \frac{2x}{5}$. To add fractions, we need a common denominator. Let's find the lowest common denominator of the fractions $\frac{x}{4} + \frac{2x}{5}$. Since 4 and 5 have no common factors greater than 1, the lowest common denominator is the least common multiple of 4 and 5: $4 \times 5 = 20$. Similarly, looking at $\frac{4y}{5}$ and $\frac{3y}{4}$, 5 and 4. Since 5 and 4 have no common factors greater than 1. The least common multiple of 5 and 4 is $4 \times 5 = 20$.

Now

$$\frac{x}{4} + \frac{2x}{5} = \frac{x(5)}{4(5)} + \frac{2x(4)}{5(4)} = \frac{5x}{20} + \frac{8x}{20} = \frac{5x + 8x}{20} = \frac{13x}{20}$$

and

$$\frac{4y}{5} - \frac{3y}{4} = \frac{4y(4)}{5(4)} - \frac{3y(5)}{4(5)} = \frac{16y}{20} - \frac{15y}{20} = \frac{16y - 15y}{20} = \frac{y}{20}.$$

Then

$\left(\frac{x}{4} + \frac{2x}{5}\right) + \left(\frac{4y}{5} - \frac{3y}{4}\right) = \frac{13x}{20} + \frac{y}{20}$. The fractions $\frac{13x}{20}$ and $\frac{y}{20}$ have the same denominator, so $\frac{13x}{20} + \frac{y}{20} = \frac{13x + y}{20}$.

Thus, $\frac{x}{4} + \frac{4y}{5} - \frac{3y}{4} + \frac{2x}{5} = \frac{13x + y}{20}$.

5. $-2x^2 + 5y$

Let's combine like terms. One pair of like terms is x^2 and $-3x^2$. The other pair of like terms is y and $4y$. We have $x^2 + y - 3x^2 + 4y = x^2 - 3x^2 + y + 4y = -2x^2 + 5y$.

6. 130

Let's substitute 2 for x and 5 for y into the expression $(y^2 + 1)(x^2 + 1)$. It is important to pay attention to the variables and correctly substitute for each correct variable.

Substituting, we have $(y^2 + 1)(x^2 + 1) = (5^2 + 1)(2^2 + 1) = (25 + 1)(4 + 1) = 26(5) = 130$.

7. $-\frac{7}{3}$

Let's substitute 2 for x and 5 for and 5 for y into the expression $\frac{x + y}{x - y}$. It is important to pay attention to the variables and correctly substitute for each variable.

Substituting, we have $\frac{x + y}{x - y} = \frac{2 + 5}{2 - 5} = \frac{7}{-3} = -\frac{7}{3}$.

8. -1

Let's substitute 2 for x and 5 for y into the expression $9x - 4y + \frac{x}{2}$. It is important to pay attention to the variables and correctly substitute for each variable.

Substituting, we have $9x - 4y + \frac{x}{2} = 9(2) - 4(5) + \frac{2}{2} = 18 - 20 + 1 = -2 + 1 = -1$.

9. 1

Let's substitute 2 for x and 5 for and 5 for y into the expression $(2x - y)^2$. It is important to pay attention to the variables and correctly substitute for each variable.

Substituting, we have $(2x - y)^2 = [2(2) - 5]^2 = (4 - 5)^2 = (-1)^2 = (-1)(-1) = 1$.

10. 28

Let's substitute 2 for x into the expression $x^2 + 7x + 10$. Since y does not appear in the expression $x^2 + 7x + 10$, we will not have to substitute 5 for y.

Substituting, we have $x^2 + 7x + 10 = 2^2 + 7(2) + 10 = 4 + 14 + 10 = 28$.

Let's note that it is correct to say that when $x = 2$ and $y = 5$, $x^2 + 7x + 10 = 28$. It is just that y does not appear in the expression.

11. (D)

Let's substitute 2 for x and -2 for y in the expression $x^2 + xy + y^2$. Then $x^2 + xy + y^2 = 2^2 + (2)(-2) + (-2)^2 = 4 - 4 + 4 = 0 + 4 = 4$. Thus, when $x = 2$ and $y = -2$, $x^2 + xy + y^2 = 4$. Choice (D) is correct.

Linear Equations—Isolating a Variable (p. 29)

1. $x = \dfrac{5}{2}$
2. $a = -2$
3. $z = \dfrac{41}{6}$
4. $y = 0$
5. $s = -\dfrac{1}{4}$
6. $x = 6$
7. $b = -14$
8. $y = -\dfrac{13}{5}$
9. $x = -\dfrac{7}{4}$
10. $a = \dfrac{223}{13}$
11. A

1. $x = \frac{5}{2}$

Let's solve the equation $2x + 5 = 10$ for x. Subtracting 5 from both sides of the equation, we have $2x = 5$. Dividing both sides by 2, we have $x = \frac{5}{2}$.

2. $a = -2$

Let's solve the equation $3(a - 2) = 6a$ for a. Distributing the 3 over the terms inside the parentheses, we have $3a - 6 = 6a$. Subtracting $6a$ from both sides, we have $-3a - 6 = 0$. Adding 6 to both sides, we have $-3a = 6$. Dividing both sides by -3, we have $a = -2$.

3. $z = \frac{41}{6}$

Let's solve the equation $14 - z + 24 = 5z - 3$ for z. Adding the numbers on the left side, we have $38 - z = 5z - 3$. Adding 3 to both sides, we have $41 - z = 5z$. Adding z to both sides, we have $41 = 6z$. Dividing both sides by 6, we have $\frac{41}{6} = z$.

4. $y = 0$

Let's solve the equation $(4)(15y)(3) = 2y$ for y. Simplifying the left side, we have $60y(3) = 2y$ and then $180y = 2y$. Subtracting $2y$ from both sides, we have $178y = 0$. Dividing both sides by 178, we have $y = 0$.

5. $s = -\frac{1}{4}$

Let's solve the equation $8s + 6 = 12s + 7$ for s. Subtracting $8s$ from both sides, we have $6 = 4s + 7$. Subtracting 7 from both sides, we have $-1 = 4s$. Dividing both sides by 4, we have $-\frac{1}{4} = s$.

6. $x = 6$

Let's solve the equation $\frac{3}{2}x - \frac{1}{2}x = 6$. Rewriting the left side of the equation, we have $\frac{3x}{2} - \frac{x}{2} = 6$. These fractions have the same denominator, 2. Subtracting the fractions on the left side, we have $\frac{3x - x}{2} = 6$. Then $\frac{2x}{2} = 6$, and $x = 6$.

7. $b = -14$

Let's solve the equation $\frac{12 + b}{3} = \frac{b + 10}{6}$ for b. Cross multiplying, we have $6(12 + b) = 3(b + 10)$. Distributing the 6 over the terms 12 and b on the left side, and distributing the 3 over the terms b and 10 on the right side, we have $72 + 6b = 3b + 30$. Subtracting $3b$ from both sides, we have $72 + 3b = 30$. Subtracting 72 from both sides, we have $3b = -42$. Dividing both sides by 3, we have $b = -14$.

8. $y = -\frac{13}{5}$

Let's solve the equation $9(3 + y) = \frac{18}{5}$ for y. Multiplying both sides by 5, we have $5[9(3 + y)] = 18$ and then $45(3 + y) = 18$. Distributing the 45 over the terms inside the parentheses, we have $135 + 45y = 18$. Subtracting 135 from both sides, we have $45y = -117$. Dividing both sides by 45, we have $y = \frac{-117}{45} = -\left(\frac{117 \div 9}{45 \div 9}\right) = -\frac{13}{5}$. Thus, $y = -\frac{13}{5}$.

Another way to solve this exercise is to notice that the numerator 18 on the right side of the equation $9(3 + y) = \frac{18}{5}$ is a multiple of the factor 9 on the left side of the equation $(18 = 2 \times 9)$. Let's begin by dividing both sides of the equation by 9. Then $3 + y = \frac{\left(\frac{18}{5}\right)}{9} = \frac{18}{5}\left(\frac{1}{9}\right) = \frac{2}{5}$. Thus, $3 + y = \frac{2}{5}$. Subtracting 3 from both sides, we have $y = \frac{2}{5} - 3 = \frac{2}{5} - \frac{15}{5} = \frac{2 - 15}{5} = \frac{-13}{5}$.

9. $x = -\frac{7}{4}$

Let's solve the equation $4x - 8 = 12(4 + 3x)$ for x. Distributing the 12 over the terms inside the parentheses, we have $4x - 8 = 48 + 36x$. Subtracting $36x$ from both sides, we have $-32x - 8 = 48$. Adding 8 to both sides, we have $-32x = 56$. Dividing both sides by -32, we have $x = \frac{56}{-32}$. Now $\frac{56}{-32} = -\frac{56}{32} = -\frac{56 \div 8}{32 \div 8} = -\frac{7}{4}$. Thus, $x = -\frac{7}{4}$.

10. $a = \frac{223}{13}$

Let's solve the equation $\frac{3a - 2}{7} - \frac{3 + 2a}{35} = 6$ for a. We would like to multiply both sides of this equation by a nonzero number that will lead to an equation without any fractions. We see that the denominators are 7 and 35. Since 35 is a multiple of 7 $(35 = 5 \times 7)$, if we multiply both sides of the equation by 35, we will obtain an equation with no fractions:

$$35\left(\frac{3a - 2}{7} - \frac{3 + 2a}{35}\right) = 35(6).$$

Let's multiply out both sides:

$$35\left(\frac{3a-2}{7}\right) - 35\left(\frac{3+2a}{35}\right) = 210.$$

Simplifying the left side, we have $5(3a - 2) - (3 + 2a) = 210$. Distributing the 5 over the terms $3a$ and -2, and distributing the minus sign over the terms 3 and $2a$, we have $15a - 10 - 3 - 2a = 210$. Combining the a-terms and the numbers on the left side, we have $13a - 13 = 210$. Adding 13 to both sides, we have $13a = 223$. Dividing both sides by 13, we have $a = \frac{223}{13}$.

11. (A)

This question requires the value of $5x$, not the value of x. Let's begin by solving the equation $5 - 2x = 15$ for x. Subtracting 5 from both sides of the equation, we have $-2x = 10$. Dividing both sides by -2, we have $x = -5$. Now we can find the value of $5x$. Let's substitute -5 for x in the expression $5x$: $5x = 5(-5) = -25$. Thus, $5x = -25$. Choice (A) is correct.

Systems of Linear Equations (p. 31)

1. $x = 3, y = -1$
2. $x = \frac{3}{4}, y = \frac{3}{2}$
3. $x = \frac{3}{10}, y = -\frac{1}{5}$
4. $x = \frac{1}{7}, y = 0$
5. $x = 5, y = 2$
6. C

1. $x = 3, y = -1$

If we add the equations $x + y = 2$ and $x - y = 4$, the y-terms will cancel, and we will be left with an equation with the one variable x. We can solve this equation for x. Once we have the value of x, we can use either of the original equations, which are $x + y = 2$ and $x - y = 4$, to solve for the value of y.

Adding the corresponding sides of the equations $x + y = 2$ and $x - y = 4$, we have

$$\begin{array}{r} x + y = 2 \\ \underline{+(x - y = 4)} \\ 2x \quad\;\; = 6 \end{array}$$

Thus, $2x = 6$. Dividing both sides by 2, we have $x = 3$. Now let's use the original equation $x + y = 2$ to find the value of y. Substituting 3 for x, we have $3 + y = 2$. Subtracting 3 from both sides, we have $y = -1$.

Thus, $x = 3$ and $y = -1$.

We can also solve the equations $x + y = 2$ and $x - y = 4$ by substitution. Subtracting x from both sides of the equation $x + y = 2$, we have $y = 2 - x$. Now let's substitute $2 - x$ for y in the equation $x - y = 4$. We then have $x - (2 - x) = 4$. Then $x - 2 + x = 4$, $2x - 2 = 4$, $2x = 6$, and $x = 3$. Now let's substitute 3 for x in the equation $x + y = 2$. We have $3 + y = 2$. Subtracting 3 from both sides, $y = -1$.

Again, $x = 3$ and $y = -1$.

2. $x = \frac{3}{4}, y = \frac{3}{2}$

If we subtract the equation $2x + y = 3$ from the equation $2x + 3y = 6$, the x-terms will cancel, and we will be left with an equation with the one variable y. We can solve this equation for y. Once we have the value of y, we can use either of the original equations to solve for the value of x.

Subtracting the equation $2x + y = 3$ from $2x + 3y = 6$, we have

$$\begin{array}{r} 2x + 3y = 6 \\ \underline{-(2x + y = 3)} \\ 2y = 3 \end{array}$$

Thus, $2y = 3$. Dividing both sides by 2, we have $y = \frac{3}{2}$. Now let's use the equation $2x + y = 3$ to find the value of x. Substituting $\frac{3}{2}$ for y, we have $2x + \frac{3}{2} = 3$. Subtracting $\frac{3}{2}$ from both sides, we have $2x = 3 - \frac{3}{2}$, $2x = \frac{6}{2} - \frac{3}{2}$, $2x = \frac{6-3}{2}$, and $2x = \frac{3}{2}$. Dividing both sides by 2, we have $x = \frac{\left(\frac{3}{2}\right)}{2} = \frac{3}{2}\left(\frac{1}{2}\right) = \frac{3 \times 1}{2 \times 2} = \frac{3}{4}$.

Thus, $x = \frac{3}{4}$ and $y = \frac{3}{2}$.

We can also solve the equations by substitution. Subtracting $2x$ from both sides of the equation $2x + y = 3$, we have $y = 3 - 2x$. Now let's substitute $3 - 2x$ for y in the equation $2x + 3y = 6$. We then have $2x + 3(3 - 2x) = 6$. Then $2x + 9 - 6x = 6$, $-4x + 9 = 6$, $-4x = -3$, $x = \frac{-3}{-4} = \frac{3}{4}$. Thus, $x = \frac{3}{4}$. Now let's substitute $\frac{3}{4}$ for x in the equation $2x + y = 3$. Then $2\left(\frac{3}{4}\right) + y = 3$, $\frac{3}{2} + y = 3$, and $y = 3 - \frac{3}{2} = \frac{6}{2} - \frac{3}{2} = \frac{6-3}{2} = \frac{3}{2}$.

Again, $x = \frac{3}{4}$ and $y = \frac{3}{2}$.

3. $x = \frac{3}{10}, y = -\frac{1}{5}$

If we subtract the equation $2x + 3y = 0$ from $22x + 3y = 6$, the y-terms will cancel, and we will be left with an equation with the one variable x. We can solve this equation for x. Once we have the value of x, we can use either of the original equations to solve for the value of y.

Subtracting the equation $2x + 3y = 0$ from the equation $22x + 3y = 6$, we have

$$\begin{array}{r} 22x + 3y = 6 \\ \underline{-(2x + 3y = 0)} \\ 20x \quad\;\; = 6 \end{array}$$

Thus, $20x = 6$. Dividing both sides by 20, we have $x = \frac{6}{20} = \frac{3}{10}$. Now let's substitute $\frac{3}{10}$ for x in the equation $2x + 3y = 0$. We then have $2\left(\frac{3}{10}\right) + 3y = 0$, $\frac{3}{5} + 3y = 0$, and $3y = -\frac{3}{5}$. Dividing both sides of this equation, $y = \frac{\left(-\frac{3}{5}\right)}{3} = -\frac{3}{5}\left(\frac{1}{3}\right) = -\frac{1}{5}$.

Thus, $x = \frac{3}{10}$ and $y = -\frac{1}{5}$.

4. $x = \frac{1}{7}, y = 0$

If we subtract the equation $21x + 7y = 3$ from the equation $21x + 10y = 3$, the x-terms will cancel, and we will be left with an equation with the one variable y. We can solve this equation for y. Once we have the value of y, we can use either of the original equations to solve for the value of x.

Subtracting $21x + 7y = 3$ from $21x + 10y = 3$, we have

$$\begin{array}{r} 21x + 10y = 3 \\ \underline{-(21x + 7y = 3)} \\ 3y = 0 \end{array}$$

Thus, $3y = 0$. Dividing both sides by 3, we have $y = 0$. Now let's substitute 0 for y in the equation $21x + 7y = 3$. Then $21x + 7(0) = 3$, $21x + 0 = 3$, and $21x = 3$. Dividing both sides by 21, we have $x = \frac{3}{21} = \frac{1}{7}$.

Thus, $x = \frac{1}{7}$ and $y = 0$.

5. $x = 5$, $y = 2$

We have the equations $x + 2y = 9$ and $2x - 3y = 4$. If we multiply both sides of the equation $x + 2y = 9$ by 2, we get $2x + 4y = 18$. If we then subtract the equation $2x - 3y = 4$ from the equation $2x + 4y = 18$, the x-terms will cancel, and we will be left with an equation with the one variable y. We can then solve this equation for the value of y. Once we have the value of y, we can use either of the original equations to solve for the value of x.

Let's subtract the equation $2x - 3y = 4$ from $2x + 4y = 18$.

$$\begin{array}{r} 2x + 4y = 18 \\ \underline{-(2x - 3y = 4)} \\ 7y = 14 \end{array}$$

Thus, $7y = 14$. Dividing both sides by 7, we have $y = 2$. Now let's substitute 2 for y in the equation $x + 2y = 9$. Then $x + 2(2) = 9$, $x + 4 = 9$, and $x = 5$.

Thus, $x = 5$ and $y = 2$.

6. (C)

We see that the coefficient of x in the equation $2x + y = -8$ is 2 and that the coefficient of x in the equation $-4x + 2y = 16$ is -4. If we multiply both sides of the equation $2x + y = -8$ by 2, we get $4x + 2y = -16$. If we then add the equation $4x + 2y = -16$ and the equation $-4x + 2y = 16$, the x-terms will cancel, and we will be left with an equation with the one variable y, which we can solve for y.

Let's add the equations $4x + 2y = -16$ and $-4x + 2y = 16$.

$$\begin{array}{r} 4x + 2y = -16 \\ \underline{+(-4x + 2y = 16)} \\ 4y = 0 \end{array}$$

Thus, $4y = 0$. Dividing both sides by 4, we have that $y = 0$. Choice (C) is correct.

This question can also be answered by solving for one variable in terms of the other in one equation and then substituting for that variable in the other equation. Looking at the equation $2x + y = -8$, we see that if we subtract $2x$ from both sides, we will have solved for y in terms of x. We can then substitute for y in the equation $-4x + 2y = 16$. This will lead to an equation with the one variable x. We can then solve for x. It is important that we keep in mind that this question requires the value of y. Once we have the value of x, we will use one of the original equations to find the value of y.

Subtracting $2x$ from both sides of the equation $2x + y = -8$, we have $y = -2x - 8$. Now let's substitute $-2x - 8$ for y in the equation $-4x + 2y = 16$: $-4x + 2(-2x - 8) = 16$. Then $-4x - 4x - 16 = 16$, $-8x - 16 = 16$, $-8x = 32$, and $x = \frac{32}{-8} = -4$. Thus, $x = -4$. Now let's substitute -4 for x in the equation $2x + y = -8$: $2(-4) + y = -8$, $-8 + y = -8$, and $y = 0$. Again, choice (C) is correct.

Note that the value of x is -4, and -4 is choice (A). Incorrect choice (A) is a trap for those who do not read the question carefully.

Quadratic Equations (p. 33)

1. $x^2 + 7x + 10$
2. $a^2 + 2a - 8$
3. $2y^2 + 11y + 14$
4. $3b^2 - 22b - 16$
5. $x^2 + x$
6. $x = -3, x = -4$
7. $b = 2, b = -5$
8. $y = 0, y = 4$
9. $a = -3, a = -\frac{1}{2}$
10. $z = 5, z = 6$
11. C

1. $x^2 + 7x + 10$

Let's multiply out $(x + 2)(x + 5)$ using FOIL:

$(x + 2)(x + 5) = x^2 + 5x + 2x + 10 = x^2 + 7x + 10.$

2. $a^2 + 2a - 8$

Let's multiply out $(a + 4)(a - 2)$ using FOIL:

$(a + 4)(a - 2) = a^2 - 2a + 4a - 8 = a^2 + 2a - 8.$

3. $2y^2 + 11y + 14$

Let's multiply out $(2y + 7)(y + 2)$ using FOIL:

$(2y + 7)(y + 2) = 2y^2 + 4y + 7y + 14 = 2y^2 + 11y + 14.$

4. $3b^2 - 22b - 16$

Let's multiply out $(b - 8)(3b + 2)$ using FOIL:

$(b - 8)(3b + 2) = 3b^2 + 2b - 24b - 16 = 3b^2 - 22b - 16.$

5. $x^2 + x$

Let's multiply out $x(x + 1)$ by distributing the x that is outside the parentheses over the terms x and 1 inside the parentheses. We get

$x(x + 1) = x(x) + x(1) = x^2 + x.$

6. $x = -3, x = -4$

Let's try to use reverse FOIL to factor $x^2 + 7x + 12$. Testing some values, we find that $x^2 + 7x + 12 = (x + 3)(x + 4)$. So $(x + 3)(x + 4) = 0$. When the product of a group of numbers is 0, at least one of the numbers must be 0. If $(x + 3)(x + 4) = 0$, then $x + 3 = 0$ or $x + 4 = 0$. If $x + 3 = 0$, then $x = -3$. If $x + 4 = 0$, then $x = -4$. All the possible values of x are -3 and -4.

7. $b = -5, b = 2$

Let's try to use reverse FOIL to factor $b^2 + 3b - 10$. Testing some values, we find that $b^2 + 3b - 10 = (b + 5)(b - 2)$. Then $(b + 5)(b - 2) = 0$. When the product of a group of numbers is 0, at least one of the numbers must be 0. If $(b + 5)(b - 2) = 0$, then $b + 5 = 0$ or $b - 2 = 0$. If $b + 5 = 0$, then $b = -5$. If $b - 2 = 0$, then $b = 2$. All the possible values of b are -5 and 2.

8. $y = 0, y = 4$

We have the equation $2y(y - 4) = 0$. When the product of a group of numbers is 0, at least one of the numbers must be 0. In the equation $2y(y - 4) = 0$, the factor 2 is not 0. So we must have $y = 0$ or $y - 4 = 0$. If $y - 4 = 0$, then $y = 4$. All the possible values of y are 0 and 4.

9. $a = -3, a = -\frac{1}{2}$

Let's try to use reverse FOIL to factor $2a^2 + 7a + 3$. Testing some values, we find that $2a^2 + 7a + 3 = (a + 3)(2a + 1)$. Then $(a + 3)(2a + 1) = 0$. When the product of a group of numbers is 0, at least one of the numbers must be 0. If $(a + 3)(2a + 1) = 0$, then $a + 3 = 0$ or $2a + 1 = 0$. If $a + 3 = 0$, then $a = -3$. If $2a + 1 = 0$, then $2a = -1$, and $a = \frac{-1}{2} = -\frac{1}{2}$. All the possible values of a are -3 and $-\frac{1}{2}$.

10. $z = 5, z = 6$

We have the equation $z^2 - 11z + 45 = 15$. Let's begin by subtracting 15 from both sides. Then we have $z^2 - 11z + 30 = 0$. Now let's try to factor $z^2 - 11z + 30$. Testing some values, we find that $z^2 - 11z + 30 = (z - 5)(z - 6)$. So $(z - 5)(z - 6) = 0$. When the product of a group of numbers is 0, at least one of the numbers is 0. If $(z - 5)(z - 6) = 0$, then $z - 5 = 0$ or $z - 6 = 0$. If $z - 5 = 0$, then $z = 5$. If $z - 6 = 0$, then $z = 6$. All the possible values of z are 5 and 6.

11. (C)

Let's try to factor $x^2 - 3x - 18$. Testing some values, we find that $x^2 - 3x - 18 = (x + 3)(x - 6)$. So $(x + 3)(x - 6) = 0$. When the product of a group of numbers is 0, at least one of the numbers must be 0. If $(x + 3)(x - 6) = 0$, then $x + 3 = 0$ or $x - 6 = 0$. If $x + 3 = 0$, then $x = -3$. If $x - 6 = 0$, then $x = 6$. All the possible values of x are -3 and 6. Choice (C) is correct.

Inequalities (p. 35)

1. $x > 20$
2. $x < 10$
3. $x \geq -31$
4. $x \leq 6$
5. $0 < x < 2$
6. $x > 10$
7. $x < \frac{21}{10}$
8. $x < 5$
9. $x \geq \frac{2}{3}$
10. $x \geq \frac{27}{37}$
11. A

1. $x > 20$

We have the inequality $3x + 4 > 64$. Subtracting 4 from both sides, we have $3x > 60$. Dividing both sides by 3, we have $x > 20$.

2. $x < 10$

We have the inequality $2x + 1 < 21$. Subtracting 1 from both sides, we have $2x < 20$. Dividing both sides by 2, we have $x < 10$.

3. $x \geq -31$

We have the inequality $-x + 1 \leq 63 + x$. Adding x to both sides, we have $1 \leq 63 + 2x$. Subtracting 63 from both sides, we have $-62 \leq 2x$. Dividing both sides by 2, we have $-31 \leq x$. Thus, $x \geq -31$.

4. $x \leq 6$

We have the inequality $21x - 42 \leq 14x$. Subtracting $14x$ from both sides, we have $7x - 42 \leq 0$. Adding 42 to both sides, we have $7x \leq 42$. Dividing both sides by 7, we have $x \leq 6$.

5. $0 < x < 2$

We have the multiple inequality statement $6 > x + 4 > 4$.

Let's start with the inequality $6 > x + 4$. Subtracting 4 from both sides of this inequality, we have $2 > x$.

Now let's work with the inequality $x + 4 > 4$. Subtracting 4 from both sides of this inequality, we have $x > 0$.

Since $2 > x$ and $x > 0$, we can say that $2 > x > 0$. This can also be written $0 < x < 2$.

6. $x > 10$

We have the multiple inequality statement $2x > x + 10 > -x$.

Let's start with the inequality $2x > x + 10$. Subtracting x from both sides of this inequality, we have $x > 10$.

Now let's work with the inequality $x + 10 > -x$. Adding x to both sides, we have $2x + 10 > 0$. Subtracting 10 from both sides, we get $2x > -10$. Dividing both sides by 2, we have $x > -5$.

Here is what we now know:

The inequality $2x > x + 10$ requires that $x > 10$.

The inequality $x + 10 > -x$ requires that $x > -5$.

Requiring that $x > 10$ and $x > -5$ is equivalent to requiring that $x > 10$, because if $x > 10$, then it must already be greater than -5. Thus, the solution to $2x > x + 10 > -x$ is $x > 10$.

7. $x < \frac{21}{10}$

We have the inequality $4x + 3 < 24 - 6x$. Adding $6x$ to both sides, we have $10x + 3 < 24$. Subtracting 3 from both sides, we have $10x < 21$. Dividing both sides by 10, we have $x < \frac{21}{10}$.

8. $x < 5$

We have the inequality $35 - 7x + 12 > 4(x - 2)$. Adding the numbers on the left side, we have $47 - 7x > 4(x - 2)$. Multiplying out the right side, we have $47 - 7x > 4x - 8$. Adding $7x$ to both sides, we have $47 > 11x - 8$. Adding 8 to both sides, we have $55 > 11x$. Dividing both sides by 11, we have $5 > x$. Thus, $x < 5$.

9. $x \geq \frac{2}{3}$

We have the inequality $3x(12) \geq 24$. Dividing both sides by 12, we have $3x \geq 2$. Dividing both sides by 3, we have $x \geq \frac{2}{3}$.

10. $x \geq \frac{27}{37}$

We have the inequality $9 - 12x \leq \frac{1}{3}x$. To get rid of the fraction, let's multiply both sides of this inequality by 3. We then have $3(9 - 12x) \leq 3\left(\frac{1}{3}x\right)$. Simplifying both sides, we have $27 - 36x \leq x$. Adding $36x$ to both sides, we have $27 \leq 37x$. Dividing both sides by 37, we have $\frac{27}{37} \leq x$. This can be written $x \geq \frac{27}{37}$.

11. (A)

Let's solve the inequality $3x - 16 > 4x + 12$ for x. Subtracting 12 from both sides, we have $3x - 28 > 4x$. Subtracting $3x$ from both sides, we have $-28 > x$. This is equivalent to $x < -28$, choice (A). Choice (A) is correct.

Symbolism (p. 37)

1. 25
2. $\frac{7}{23}$
3. $\frac{59}{136}$
4. -58
5. 133
6. $\frac{5}{12}$
7. 24
8. $d^2 + 10d + 20$
9. -47
10. 34
11. B

1. 25

Let's substitute 7 for x in the defining equation $\#x = 3x + 4$. Then $\#7 = 3(7) + 4 = 21 + 4 = 25$. Thus, $\#7 = 25$.

2. $\frac{7}{23}$

Let's substitute $\frac{7}{16}$ for x in the defining equation $\triangle x = \frac{x}{x+1}$. Then $\triangle\left(\frac{7}{16}\right) = \frac{\frac{7}{16}}{\frac{7}{16}+1} = \frac{\frac{7}{16}}{\frac{7}{16}+\frac{16}{16}} = \frac{\left(\frac{7}{16}\right)}{\left(\frac{7+16}{16}\right)} = \frac{\left(\frac{7}{16}\right)}{\left(\frac{23}{16}\right)} = \frac{7}{16}\left(\frac{16}{23}\right) = \frac{7}{23}$. Thus, $\triangle\left(\frac{7}{16}\right) = \frac{7}{23}$.

3. $\frac{59}{136}$

Let's substitute $\frac{7}{4}$ for x in the defining equation $\uparrow x = \frac{5x+6}{4x+27}$. Then $\uparrow\left(\frac{7}{4}\right) = \frac{5\left(\frac{7}{4}\right)+6}{4\left(\frac{7}{4}\right)+27} = \frac{\frac{35}{4}+6}{7+27} = \frac{\frac{35}{4}+\frac{24}{4}}{34} = \frac{\left(\frac{35+24}{4}\right)}{34} = \frac{\left(\frac{59}{4}\right)}{34} = \frac{59}{4}\left(\frac{1}{34}\right) = \frac{59(1)}{4(34)} = \frac{59}{136}$. Thus, $\uparrow\left(\frac{7}{4}\right) = \frac{59}{136}$.

4. −58

Let's substitute 8 for x and 14 for y in the defining equation $x \lambda y = 5x - 7y$. Then $8 \lambda 14 = 5(8) - 7(14) = 40 - 98 = -58$. Thus, $8 \lambda 14 = -58$.

5. 133

Let's substitute 5 for x and 3 for y in the defining equation $x \square y = x^y + x + y$. Then $5 \square 3 = 5^3 + 5 + 3 = (5)(5)(5) + 8 = 25(5) + 8 = 125 + 8 = 133$. Thus, $5 \square 3 = 133$.

6. $\frac{5}{12}$

We have the defining equation $x \Leftrightarrow y \Leftrightarrow z = xy + xz + yz$. Let's substitute $\frac{1}{3}$ for x, let's substitute $\frac{1}{4}$ for y, and let's substitute $\frac{4}{7}$ for z. Then $\frac{1}{3} \Leftrightarrow \frac{1}{4} \Leftrightarrow \frac{4}{7} = \left(\frac{1}{3}\right)\left(\frac{1}{4}\right) + \left(\frac{1}{3}\right)\left(\frac{4}{7}\right) + \left(\frac{1}{4}\right)\left(\frac{4}{7}\right) = \frac{1(1)}{3(4)} + \frac{1(4)}{3(7)} + \frac{1(4)}{4(7)} = \frac{1}{12} + \frac{4}{21} + \frac{1}{7}$. To add the fractions $\frac{1}{12}$, $\frac{4}{21}$, and $\frac{1}{7}$, we need a common denominator. Let's try to find the lowest common denominator of the fractions $\frac{1}{12}$, $\frac{4}{21}$, and $\frac{1}{7}$, which is the least common multiple of 12, 21, and 7. Now 21 is a multiple of 7 ($21 = 3(7)$), so the least common multiple of 12, 21, and 7 is the least common multiple of 12 and 21. Now $4(21) = 84$ is a multiple of $12(84 = 7(12))$. Then $\frac{1}{12} + \frac{4}{21} + \frac{1}{7} = \frac{1(7)}{12(7)} + \frac{4(4)}{21(4)} + \frac{1(12)}{7(12)} = \frac{7}{84} + \frac{16}{84} + \frac{12}{84} = \frac{7+16+12}{84} = \frac{35}{84} = \frac{35 \div 7}{84 \div 7} = \frac{5}{12}$. Thus, $\frac{1}{3} \Leftrightarrow \frac{1}{4} \Leftrightarrow \frac{4}{7} = \frac{5}{12}$.

7. 24

We have the defining equation $\triangle x = 7x + 5$. Let's replace x with y in the defining equation. Then $\triangle y = 7y + 5$. Since $\triangle y = 173$, we can write the equation $7y + 5 = 173$. Let's solve for y. Subtracting 5 from both sides, we have $7y = 168$. Dividing both sides by 7, we have $y = 24$.

8. $d^2 + 10d + 20$

We have the defining equation $x \downarrow y = xy + 5x + y$. Let's replace x with $d + 4$, and let's replace y with d. Then $(d + 4) \downarrow d = (d + 4)d + 5(d + 4) + d$. Now let's simplify. We have $(d + 4)d + 5(d + 4) + d = d^2 + 4d + 5d + 20 + d = d^2 + 10d + 20$. Thus, $(d + 4) \downarrow d = d^2 + 10d + 20$.

9. −47

We have the defining equation $x \Phi y = 12x - 8y$. We want to know for what value of c we have $(c + 4) \Phi (2c + 5) = 196$. Let's replace x with $c + 4$ and replace y with $2c + 5$ in the defining equation. Then $(c + 4) \Phi (2c + 5) = 12(c + 4) - 8(2c + 5)$. We're told this equals 196. Let's solve this equation for c.

Multiplying out $12(c + 4) - 8(2c + 5) = 196$, we have $12c + 48 - 16c - 40 = 196$. Gathering the c-terms and the numbers on the left side, we have $-4c + 8 = 196$. Subtracting 8 from both sides, we have $-4c = 188$. Dividing both sides by 4, we have $c = \frac{188}{-4} = -47$. Thus, $c = -47$.

10. 34

We have the defining equation $x \Omega y = x^2 - xy + 25$. We want to find all the possible values of a such that $a \Omega 8 = 10$. Then we will be able to find the sum of the squares of all the possible values of a. Let's replace x with a and let's replace y with 8 in the defining equation. Then $a \Omega 8 = a^2 - (a)(8) + 25 = a^2 - 8a + 25$. Since $a \Omega 8 = 10$, we have the equation $a^2 - 8a + 25 = 10$. Subtracting 10 from both sides, we have $a^2 - 8a + 15 = 0$.

Now let's try to factor the left side of the equation $a^2 - 8a + 15 = 0$. Testing some values, we find that $a^2 - 8a + 15 = (a - 3)(a - 5)$. So the equation $a^2 - 8a + 15 = 0$ can be rewritten $(a - 3)(a - 5) = 0$. When the product of a group of numbers is 0, at least one of the numbers is 0. Here either $a - 3 = 0$ or $a - 5 = 0$. If $a - 3 = 0$, then $a = 3$. If $a - 5 = 0$, then $a = 5$. All the possible values of a are 3 and 5. The sum of the squares of all the possible values of a is $3^2 + 5^2 = 9 + 25 = 34$.

11. (B)

Let's use the defining equation $m \blacktriangle n = \frac{m^2 - n + 1}{mn}$ to find the value of $3 \blacktriangle 1$. Let's replace m with 3 and let's replace n with 1. Then $3 \blacktriangle 1 = \frac{3^2 - 1 + 1}{3(1)} = \frac{9 - 1 + 1}{3(1)} = \frac{9}{3} = 3$. Thus, $3 \blacktriangle 1 = 3$. Choice (B) is correct.