GEOMETRY

3

Lines and Angles

- The sum of angles around a point is 360°.
- A straight line is equivalent to an angle of 180°.
- Perpendicular lines make an angle of 90°.
- Parallel lines will never intersect. When two parallel lines are cut by a transversal, corresponding angles are equal.
- Vertical angles are pairs of equal and opposite angles formed by the intersection of two straight lines.

Example:
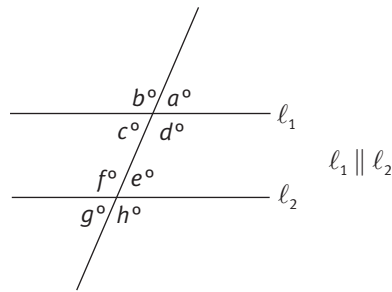
What is the sum of a and b?

What is the sum of $a + b + c + d$?
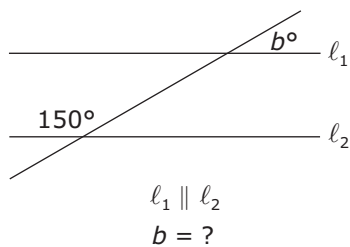
Find the pairs of vertical angles:
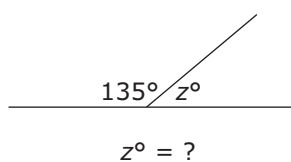
If $a = 45°$, what is the value of c? Of d?

KAPLAN

Exercises

(Answers on page 92.)

Find the indicated values:

1.

$\ell_1 \parallel \ell_2$
$b = ?$

2.

$\ell_1 \parallel \ell_2$
$s = ?$

3.

$\ell_1 \parallel \ell_2$
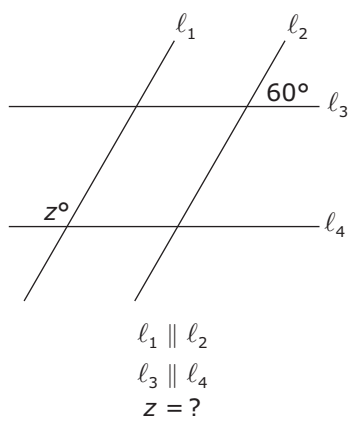$\ell_3 \parallel \ell_4$
$z = ?$

4.

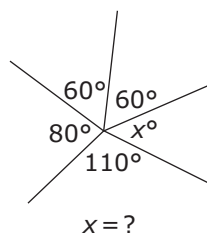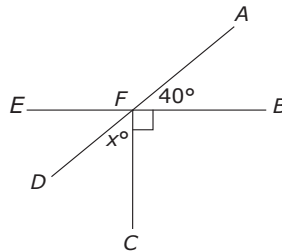$z° = ?$

5.

$y = ?$

6.

$x = ?$
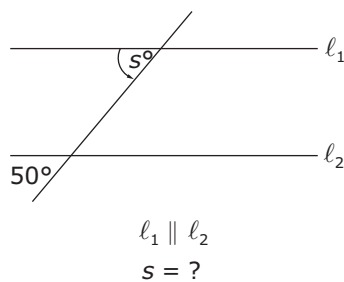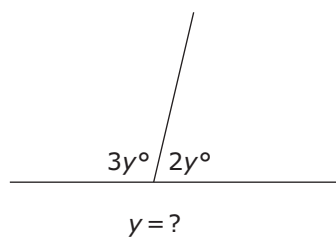
Lines and Angles

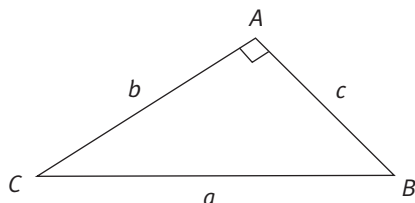**Test Question**

7. In the figure above, *EB* is perpendicular to *FC*, and *AD* and *EB* intersect at point *F*. What is the value of *x*?

    (A)   30

    (B)   40

    (C)   50

    (D)   60

    (E) 130

Triangles

- The sum of all interior angles in a triangle is 180°.
- Perimeter = sum of all sides
- Area = $\frac{1}{2}$(Base) × (Height), where the base and height of the triangle must form a right angle.
- Every side of a triangle must be longer than the difference of the lengths of the other two sides, and shorter than the sum of the lengths of the other two sides.
- Isosceles Triangle: Two sides are equal and two angles are equal.
- Equilateral Triangle: Three sides are equal and all angles equal 60°.

Example:

What is the perimeter?

Identify the base and height of the triangle.

If side a = 5, side b = 4, and side c = 3, how are $\angle CAB$, $\angle ABC$, and $\angle ACB$ related?
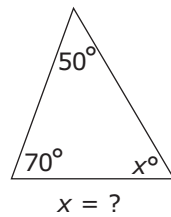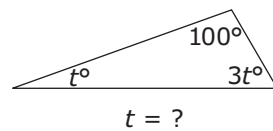
Triangles

Exercises
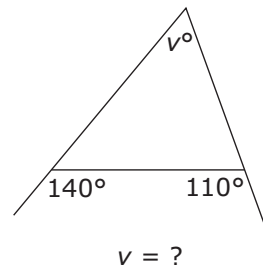
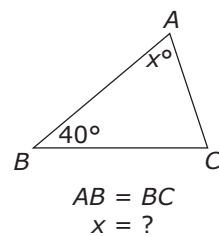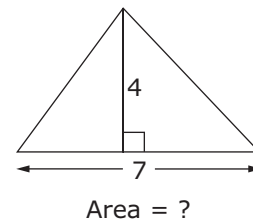**(Answers on page 93.)**

Find the indicated values:

1.

50°

70°   x°

x = ?

2.

100°

t°   3t°

t = ?

3.

v°

140°   110°

v = ?

4.

A

x°

40°

B   C

AB = BC
x = ?

5.

4

7

Area = ?

6.

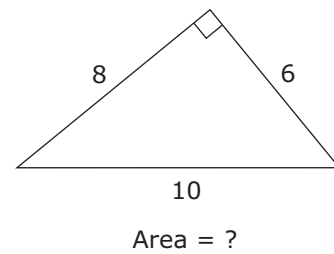8   6

10

Area = ?

7.

A

4

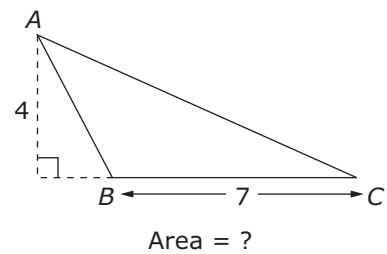B ◄— 7 —► C
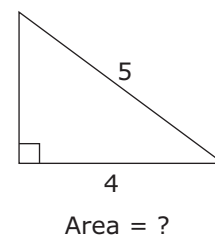
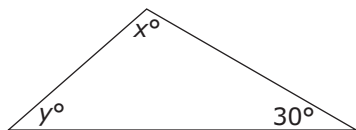Area = ?

8.

5

4

Area = ?

Test Question

9. In the figure above, what is x in terms of y?

    (A) $150 - y$

    (B) $150 + y$

    (C) $80 + y$

    (D) $30 + y$

    (E) $30 - y$

Right Triangles

- A right triangle has one 90° angle.
- The longest side is called the hypotenuse.
- In a right triangle, if the sides are a, b, and the hypotenuse is c, then $a^2 + b^2 = c^2$ (Pythagorean theorem).
- Pythagorean triplets:
  - 3:4:5
  - 5:12:13
- Special triangles:
  - Side lengths: x: x: $x\sqrt{2}$ Angle measures: _____
  - Side lengths: x: $x\sqrt{3}$: $2x$ Angle measures: _____

Example:

What is the length of the hypotenuse of a right triangle with legs of length 9 and 10?

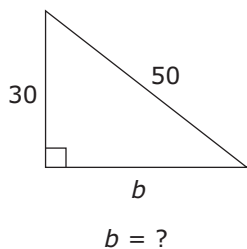Draw figure:

Set up the Pythagorean theorem:
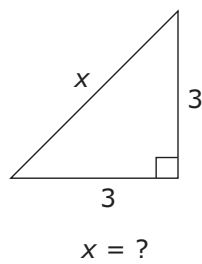
Solve for the hypotenuse:

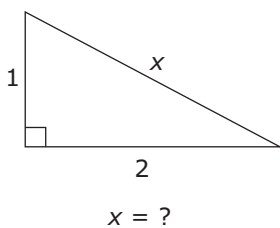# Exercises

(Answers on page 94.)
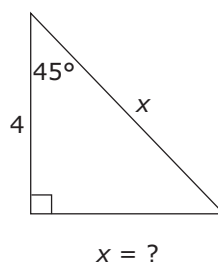
Find the indicated values:

1.

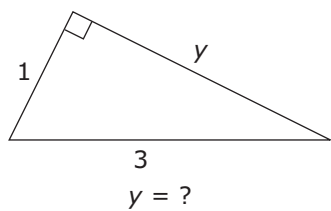
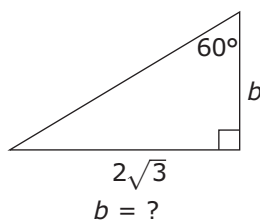
$b = ?$

2.

$x = ?$

3.

$y = ?$

4.

$z = ?$

5.

$x = ?$

6.

$x = ?$

7.

$b = ?$

8.

$a = ?$
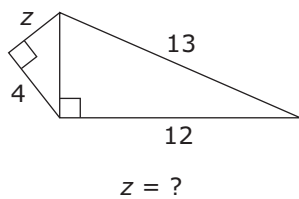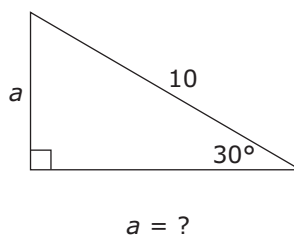
# Right Triangles

Test Question

9. What is the length of the hypotenuse of an isosceles right triangle of area 32?

    (A)   4

    (B) $4\sqrt{2}$

    (C)   8

    (D) $8\sqrt{2}$

    (E)   16

Circles

- Area $= \pi r^2$
- Circumference $= 2\pi r = \pi d$
- The length of an arc as a fraction of a circle's circumference is equal to the degree measure of the corresponding central angle as a fraction of 360.
- The area of a sector as a fraction of a circle's area is equal to the degree measure of the corresponding central angle as a fraction of 360.
- A line that has exactly one point in common with a circle is tangent to the circle, and it is perpendicular to the radius.
- If a triangle is inscribed in a circle so that one of its sides is a diameter of the circle, then the triangle is a right triangle.

Example:
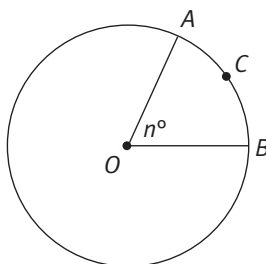
If $OA = 3$, what is the circumference? The area?

If $n = 60°$, what is the length of the arc ACB?

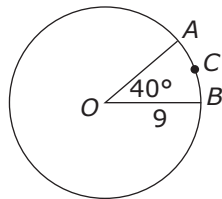If $n = 60°$, what is the area of sector $OACB$?

Circles

Exercises

(Answers on page 96.)

Solve the following:

1. What is the circumference of a circle with the diameter 8?

2. What is the circumference of a circle with the diameter $\frac{3}{4}\pi$?

3. What is the radius of a circle with circumference $\frac{7}{2}\pi$?

4. What is the diameter of a circle with circumference $\frac{\pi}{2}$?

5. What is the area of a circle with radius 8?

6. What is the area of a circle with diameter 12?

7. What is the area of a circle with circumference 8π?

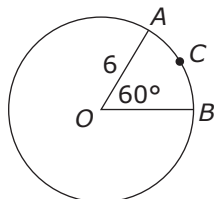8. What is the diameter of a circle with area 49π?
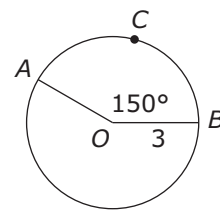
9.

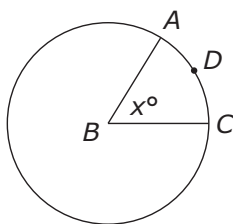Arc *ACB* = ?

10.
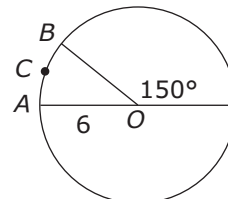
Arc *ACB* = ?

11.

Area of sector *OACB* = ?

12.

Area of sector *OACB* = ?

Test Question

13. In the figure above, the ratio of the circumference of circle B to the length of arc ADC is 8:1. What is the value of x?

(A)   30
(B)   45
(C)   60
(D)   75
(E)   90

Polygons

- A polygon is a closed plane figure formed by three or more line segments.
  - Perimeter = sum of all sides
  - The sum of the interior angle measures of a polygon with n sides is equal to $(n - 2)180°$.

- Special case: Parallelograms (quadrilaterals whose opposite sides are parallel)
  - Parallelograms include rectangles and squares.
  - Opposite sides are equal.
  - Opposite angles are equal.
  - Area = Base $\times$ Height
  - The diagonals bisect each other.

- Special Case: Trapezoid
  - Area $= \frac{1}{2}$ (the sum of the bases) (height)

Example:
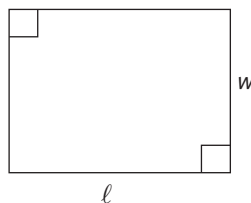
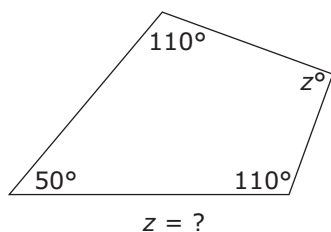If $\ell = 9$ and $w = 12$, what is the perimeter?

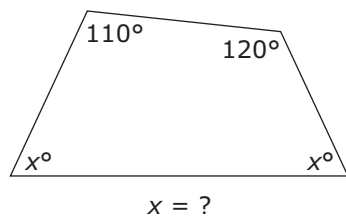If $\ell = 6$ and $w = 6$, what is the area?

Exercises

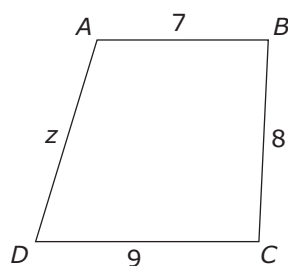**(Answers on page 98.)**

Find the indicated values:

1.

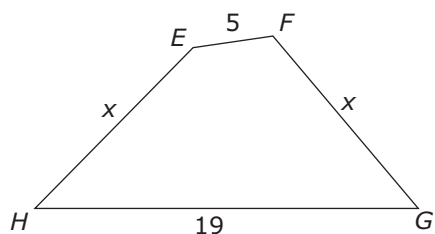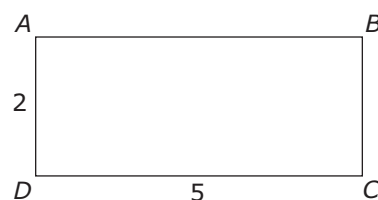$$z = ?$$

2.

$$x = ?$$

3.

The perimeter of *ABCD* is 34.
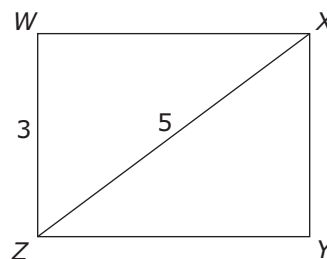$$z = ?$$

4.

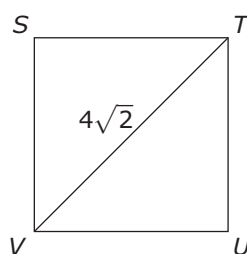The perimeter of *EFGH* is 48.
$$x = ?$$
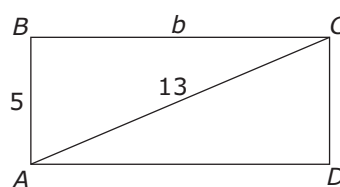
5.

Rectangle *ABCD*
Area = ?

6.

Rectangle *WXYZ*
Area = ?

7.

Square *STUV*
ST = ?

8.

Rectangle *ABCD*
b = ?

Polygons

Test Question

9. If the length of rectangle *A* is one-half the length of rectangle *B*, and the width of rectangle *A* is one-half the width of rectangle *B*, what is the ratio of the area of rectangle *A* to the area of rectangle *B*?

   (A) $\frac{1}{4}$

   (B) $\frac{1}{2}$

   (C) $\frac{1}{1}$

   (D) $\frac{2}{1}$

   (E) $\frac{4}{1}$

Multiple Figures

- Difficult geometry questions often contain complex figures. The key to answering these questions is to break the figures down into simpler figures.
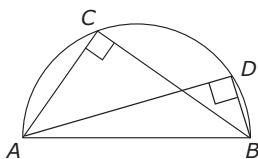
Example:

What side/line/angle is common to both figures?

Right triangle inscribed in a half-circle:

Circle inscribed in a square:

Square inscribed in a circle:

Example:

If the diameter of the semicircle is 15 and $BD = 9$, what is the length of AD?
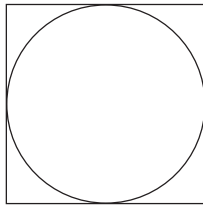
If $AC = BC = 5$, what is the length of the diameter AB?
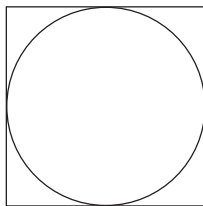
Multiple Figures

Exercises

(Answers on page 99.)

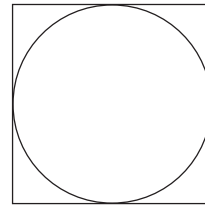**Find the indicated values:**

1.

Area of circle $= 25\pi$
Perimeter of square $= ?$

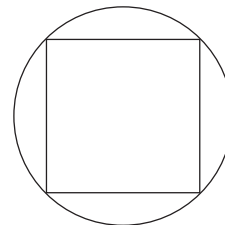2.

Area of square $= 16$
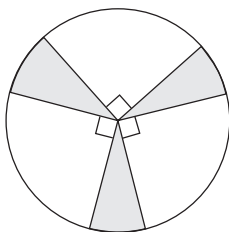Area of circle $= ?$

3.

Circumference of circle $= 6\pi$
Area of square $= ?$

4.

Area of square $= 4$
Area of circle $= ?$

Test Question

5. In the circle above, three right angles have vertices at the center of the circle. If the radius of the circle is 8, what is the combined area of the shaded regions?

(A) 8π

(B) 9π

(C) 12π

(D) 13π

(E) 16π

Solids

- Almost all solids on Test Day are uniform solids (solids in which the measure of each dimension is constant through the entire object).
- To determine the volume of a uniform solid, we multiply the area of the base by the height.
  - Volume of Cylinder $= \pi r^2 h$
  - Volume of Rectangular Solid $= l \times w \times h$

Example:
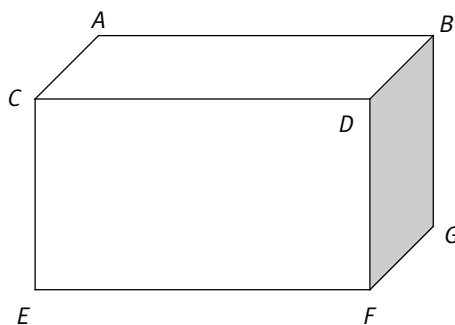
$AB = 10$, $AC = 5$, and $CE = 6$.

What is the volume?

What is the surface area?

Exercises

(Answers on page 100.)

Find the volume and surface area of each of the following solids:

A rectangular solid with dimensions 4, 6, and 8

A rectangular solid with dimensions 3, 4, and 12

A cube with edge 6

A cube with edge $\sqrt{2}$

A cylinder with height 12 and radius 6

Test Question

6. What is the radius of the largest sphere that can be placed inside a cube of volume 64?

   (A) $6\sqrt{2}$
   (B) 8
   (C) 4
   (D) $2\sqrt{2}$
   (E) 2

Coordinate Geometry

- The slope of a line $= \dfrac{\Delta y}{\Delta x} = \dfrac{\text{rise}}{\text{run}}$
- The equation of a straight line in the two-dimensional coordinate plane is typically expressed in the form $y = mx + b$, where m is the slope and b is the point at which the line intercepts the y-axis.
- One way to find a distance in the coordinate plane is to use the Pythagorean theorem.

Example:

Plot the points (3, 2) and (6, 4):
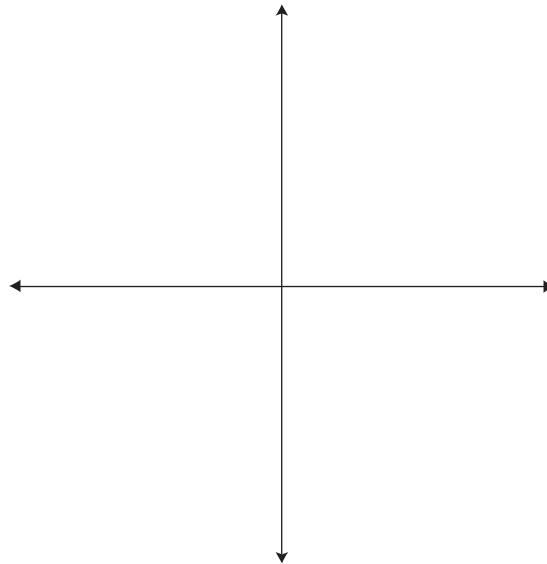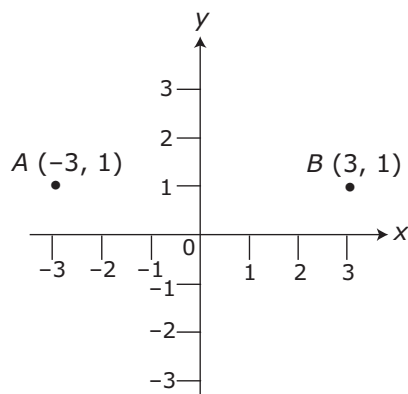
Calculate the slope of the line:

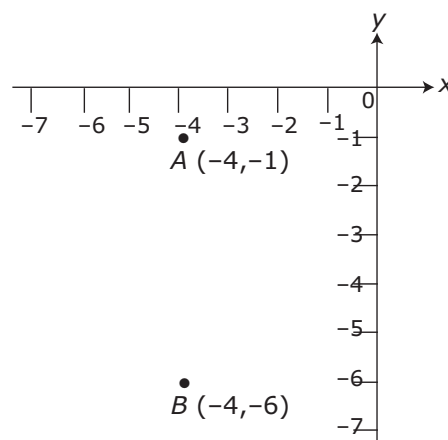Determine the equation of the line:

Exercises

(Answers on page 101.)

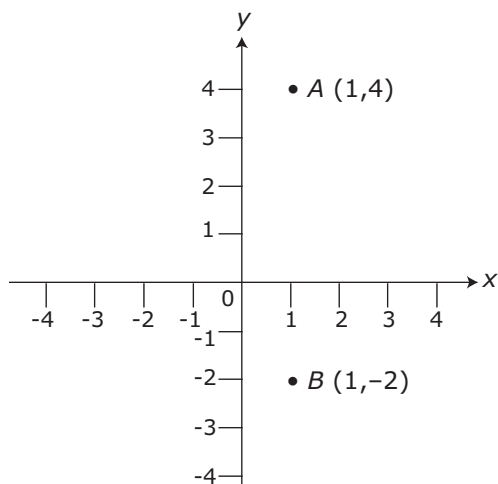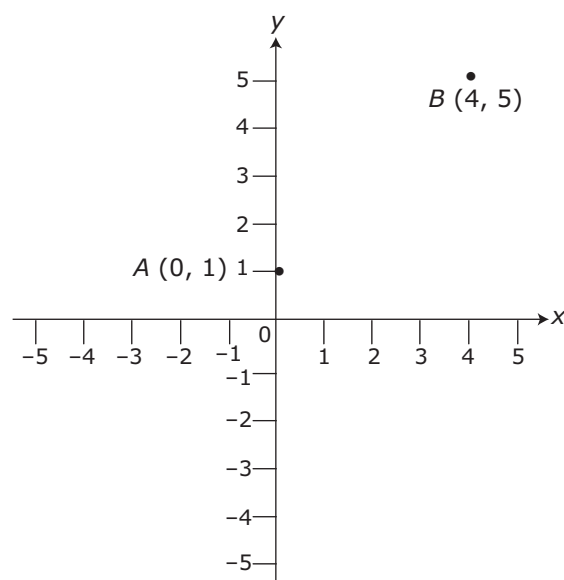**For each of the questions below, calculate the slope and equation of the line for the points:**

1.

A (−3, 1)   *B* (3, 1)

4.

A (−4,−1)

B (−4,−6)

2.

A (1,4)

B (1,−2)

5.

B (4, 5)

A (0, 1)
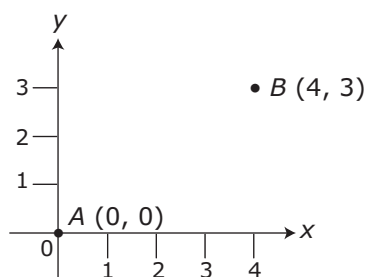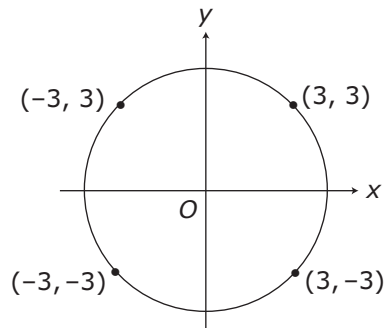
3.

B (4, 3)

A (0, 0)

Coordinate Geometry

Test Question

6. What is the area of the circle above with its center at the origin?

    (A)   9

    (B)   18

    (C) 9π

    (D) 18π

    (E) 36π