Math Refresher

ANSWERS AND EXPLANATIONS

SESSION 3

KAPLAN

TEST PREP AND ADMISSIONS

Geometry

Lines and Angles (p. 41–42)

1. 30
2. 50
3. 120
4. 45
5. 36
6. 50
7. C

1. 30

When parallel lines are crossed by a transversal, and the transversal is not perpendicular to either line it crosses, all the acute angles are equal, all the obtuse angles are equal, and the sum of an acute angle and an obtuse angle is 180 degrees.

In this question, one obtuse angle has a measure of 150 degrees, and one acute angle has a measure of b degrees. We can write the equation $b + 150 = 180$. Subtracting 150 from both sides, we have $b = 30$.

2. 50

When parallel lines are crossed by a transversal, corresponding angles are equal. The angles marked $s°$ and $50°$ are corresponding angles. Therefore, $s = 50$.

3. 120

Let's begin by considering lines ℓ_2, ℓ_3, and ℓ_4. Here, line ℓ_2 is a transversal that crosses lines ℓ_3 and ℓ_4. When parallel lines are crossed by a transversal, corresponding angles are equal. The angle formed by lines ℓ_2 and ℓ_4 is to the right of ℓ_2 and above ℓ_4 and the angle marked $60°$ are corresponding angles. Therefore, these angles are equal. Now let's consider lines ℓ_1, ℓ_2, and ℓ_4. Line ℓ_4 is a transversal that crosses lines ℓ_1 and ℓ_2. When parallel lines are crossed by a transversal, and the transversal is not perpendicular to either line it crosses, all the acute angles are equal, all the obtuse angles are equal, and the sum of an acute angle and an obtuse angle is 180 degrees. The angle marked $z°$ is an obtuse angle, and the angle formed by lines ℓ_2 and ℓ_4 that is to the right of ℓ_2 and above ℓ_4 is an acute angle. Therefore, since the angle formed by lines ℓ_2 and ℓ_4 that is to the right of ℓ_2 and above ℓ_4 has a measure of $60°$, $z + 60 = 180$. Subtracting 60 from both sides, we have $z = 120$.

4. 45

The sum of the angles making up a straight line is $180°$. Here, $135 + z = 180$. Subtracting 135 from both sides, $z = 45$.

5. 36

The sum of the angles making up a straight line is $180°$. Here, $3y + 2y = 180$. Let's solve this equation for y. Combining the y-terms on the left side, we have $5y = 180$. Dividing both sides by 5, we have $y = 36$.

6. 50

The sum of the measures of the angles around a point is $360°$. Here, $110 + 80 + 60 + 60 + x = 360$. Adding the numbers on the left side, we have $310 + x = 360$. Subtracting 310 from both sides, we have $x = 50$.

7. (C)

The sum of the angles making up a straight line is 180 degrees. Here, the sum of the measures of angles *AFB*, *BFC*, and *CFD* is 180 degrees. The measure of angle *AFB* is $40°$, the measure of angle *BFC* is $90°$, and the measure of angle *CFD* is $x°$. We can write the equation $40 + 90 + x = 180$. Let's solve for x. Adding the numbers of the left side, we have $130 + x = 180$. Subtracting 130 from both sides, we have $x = 50$. Choice (C) is correct.

Triangles (p. 44–45)

1. 60
2. 20
3. 70
4. 70
5. 14
6. 24
7. 14
8. 6
9. A

1. 60

The sum of the three interior angles of any triangle is 180 degrees. In this triangle, $70 + 50 + x = 180$. Adding the numbers on the left side, $120 + x = 180$. Subtracting 120 from both sides, $x = 60$.

2. 20

The sum of the three interior angles of any triangle is 180 degrees. In this triangle, $t + 3t + 100 = 180$. Adding the t-terms on the left side, $4t + 100 = 180$. Subtracting 100 from both sides, $4t = 80$. Dividing both sides by 4, $t = 20$.

3. 70

The sum of the three interior angles of any triangle is 180 degrees. In the figure, the angle marked 140° and the interior angle of the triangle that is adjacent to this angle are supplementary angles. If we say this interior angle has a measure of $x°$, then $140 + x = 180$, and $x = 40$. Also, the angle marked 110° and the interior angle of the triangle that is adjacent to this angle are supplementary angles. If we say this interior angle has a measure of $y°$, then $110 + y = 180$, and $y = 70$. The measures of the three interior angles of the triangle are 40°, 70°, and $v°$. The sum of the three interior angles of any triangle is 180 degrees. Here, $40 + 70 + v = 180$. Adding the numbers on the left side, $110 + v = 180$. Subtracting 110 from both sides, $v = 70$.

4. 70

In a triangle, the angles opposite sides of equal lengths are equal. Since $AB = BC$, the angle opposite side AB, which is angle ACB, is equal to the angle opposite side BC, which is angle CAB. Since the measure of angle CAB is $x°$, the measure of angle ACB is $x°$. Thus, the measures of the three interior angles of this triangle are 40°, $x°$, and $x°$. The sum of the three interior angles of any triangle is 180 degrees. So $40 + x + x = 180$. Combining the x-terms on the left side, $40 + 2x = 180$. Subtracting 40 from both sides, $2x = 140$. Dividing both sides by 2, $x = 70$.

5. 14

The area of any triangle is $\frac{1}{2}$ times base times height. One base of this triangle is 7. The height drawn to this base is 4. Therefore, the area of this triangle is $\frac{1}{2} \times 7 \times 4 = 2 \times 7 = 14$.

6. 24

The area of any triangle is $\frac{1}{2}$ times base times height. Here we have a right triangle. The area of a right triangle is $\frac{1}{2} \times \text{leg}_1 \times \text{leg}_2$ because one leg can be considered to be the base and the other leg can be considered to be the height. Here we can say that the leg of length 8 is the base and the leg of length 6 is the height drawn to that base. Thus, the area of the triangle is $\frac{1}{2} \times 8 \times 6 = 4 \times 6 = 24$.

7. 14

The area of any triangle is $\frac{1}{2}$ times base times height. One base of this triangle is CB, which is 7. The height drawn to this base is 4. Even though the height drawn to this base meets the base at a point that is on the extension of the base, this is still a height of the triangle. Therefore, the area of this triangle is $\frac{1}{2} \times 7 \times 4 = 2 \times 7 = 14$.

8. 6

The area of any triangle is $\frac{1}{2}$ times base times height. Here we have a right triangle. The area of a right triangle is $\frac{1}{2} \times \text{leg}_1 \times \text{leg}_2$ because one leg can be considered to be the base and the other leg can be considered to be the height. We know that one leg of this right triangle is 4. This triangle is a special 3-4-5 right triangle. Thus, the other leg of the right triangle is 3. The area of this right triangle is $\frac{1}{2} \times 4 \times 3 = 2 \times 3 = 6$.

9. (A)

The sum of the measures of the three interior angles of any triangle is 180°. The measures of the three interior angles of the triangle in this question are $y°$, $x°$, and 30°. We can write the equation $y + x + 30 = 180$. Let's solve this equation for x in terms of y. Subtracting 30 from both sides, we have $y + x = 150$. Subtracting y from both sides, we have $x = 150 - y$. Choice (A) is correct.

Right Triangles (p. 47–48)

1. 40
2. $\sqrt{5}$
3. $2\sqrt{2}$
4. 3
5. $3\sqrt{2}$
6. $4\sqrt{2}$
7. 2
8. 5
9. D

1. 40

We could use the Pythagorean theorem to fond the value of b. However, there is a quicker way to solve this question. This triangle is a multiple of the special 3-4-5 right triangle. For the hypotenuse of length 50, we have that $50 = 10 \times 5$. For the leg of length 30, we have that $30 = 10 \times 3$. Thus, $b = 10 \times 4 = 40$.

2. $\sqrt{5}$

Let's use the Pythagorean theorem to find x, which is the length of the hypotenuse of a right triangle. We have $x^2 = 1^2 + 2^2$, $x^2 = 1 + 4$, and $x^2 = 5$. If $x^2 = 5$, then $x = \sqrt{5}$ or $x = -\sqrt{5}$. Since lengths cannot be negative, $x = \sqrt{5}$.

3. $2\sqrt{2}$

Let's use the Pythagorean theorem to find y, which is the length of a leg of right triangle. We have that $1^2 + y^2 = 3^2$, $1 + y^2 = 9$, and $y^2 = 8$. If $y^2 = 8$, then $y = \sqrt{8}$ or $y = -\sqrt{8}$. Since lengths cannot be negative, $y = \sqrt{8}$.

We can rewrite $\sqrt{8}$. Let's look for a factor of 8 that is a perfect square. Now 4 is a factor of 8 ($8 = 2 \times 4$) that is a perfect square ($4 = 2^2$). We have the law of radicals that says that if $a \geq 0$ and $b \geq 0$, then $\sqrt{ab} = \sqrt{a}\sqrt{b}$. Then $\sqrt{8} = \sqrt{4 \times 2} = \sqrt{4} \times \sqrt{2} = 2\sqrt{2}$. Thus, we can say that $y = 2\sqrt{2}$.

4. 3

We want the value of z, which is a leg of the right triangle whose other leg has a length of 4. If we knew the hypotenuse of this right triangle, we could find the value of z. Now the hypotenuse of the right triangle whose legs have lengths of z and 4 is also a leg of the right triangle with one leg of length 12 and a hypotenuse of length 13. We could use the Pythagorean theorem to find the length of the other leg of the larger right triangle. However, there is a quicker way to find the length of this leg. The larger right triangle is a special 5-12-13 right triangle. Thus, the length of this leg is 5. Thus, the smaller right triangle has a hypotenuse of length 5.

Now we could use the Pythagorean theorem to find the value of z. However, there is a quicker way to find z. The right triangle with legs of lengths z and 4 and a hypotenuse of length 5 is a special 3-4-5 right triangle. Thus, $z = 3$.

5. $3\sqrt{2}$

We could use the Pythagorean theorem to find x, which is the length of the hypotenuse of a right triangle. However, there is a quicker way to find x. This right triangle whose legs both have lengths of 3 is an isosceles right triangle. The leg to leg to hypotenuse ratio in an isosceles right triangle is x to x to $x\sqrt{2}$. In this question, each leg of the right triangle has a length of 3. So the length of the hypotenuse is $3\sqrt{2}$. Thus, $x = 3\sqrt{2}$.

6. $4\sqrt{2}$

One interior angle of this triangle is a right angle of 90°. Another interior angle of this triangle has a measure of 45°. The sum of the three interior angles of any triangle is 180°. So the third interior angle of this triangle has a measure of $180° - 90° - 45° = 45°$. We have a 45°-45°-90° right triangle. That is, we have an isosceles right triangle. So the length of the unmarked leg of this right triangle is 4.

We could use the Pythagorean theorem to find x, which is the length of the hypotenuse of the right triangle. However, there is a quicker way to find x. The leg to leg to hypotenuse ratio in an isosceles right triangle is x to x to $x\sqrt{2}$. In this question, each leg of the right triangle has a length of 4. So the length of the hypotenuse is $4\sqrt{2}$. Thus, $x = 4\sqrt{2}$.

7. 2

One interior angle of this triangle is a right angle of 90°. Another interior angle of this triangle has a measure of 60°. The sum of the three interior angles of any triangle is 180°. So the third interior angle of this triangle has a measure of $180° - 90° - 60° = 30°$. We have a 30°-60°-90° right triangle.

We could use the Pythagorean theorem to find b, which is the length of the one of the legs of the right triangle. However, there is a quicker way to find b. The leg to leg to hypotenuse ratio in a 30°-60°-90° right triangle x to $x\sqrt{3}$ to $2x$. In this question, the leg opposite the angle of measure 60° has a length of $2\sqrt{3}$. So the length of the leg opposite the angle of measure 30° is $\frac{2\sqrt{3}}{\sqrt{3}} = 2$.

8. 5

One interior angle of this triangle is a right angle of 90°. Another interior angle of this triangle has a measure of 30°.

The sum of the three interior angles of any triangle is 180°. So the third interior angle of this triangle has a measure of $180° - 90° - 30° = 60°$. We have a 30°-60°-90° right triangle.

We could use the Pythagorean theorem to find a, which is the length of the one of the legs of the right triangle. However, there is a quicker way to find a. The leg to leg to hypotenuse ratio in a 30°-60°-90° right triangle is x to $x\sqrt{3}$ to $2x$. In this question, the hypotenuse has a length of 10. Therefore, the leg opposite the angle of measure 30° has a length of $\frac{10}{2} = 5$.

9. (D)

The area of any triangle is $\frac{1}{2}$ times base times height. The area of a right triangle is $\frac{1}{2} \times \text{leg}_1 \times \text{leg}_2$ because one leg can be considered to be the base and the other leg can be considered to be the height. In an isosceles right triangle, the two legs are equal. If the length of a leg of an isosceles right triangle is v, then the area of the isosceles right triangle is $\frac{1}{2} \times v \times v = \frac{v^2}{2}$. Let's say that the length of a leg of the isosceles right triangle in this question is v. Since its area is 32, we have the equation $\frac{v^2}{2} = 32$. Multiplying both sides by 2, we have $v^2 = 64$. Thus, if $v = 8$ or $v = -8$. Since lengths cannot be negative, $v = 8$. Thus, the length of each leg of the isosceles right triangle is 8.

In an isosceles right triangle, the leg to leg to hypotenuse ratio is x to x to $x\sqrt{2}$. Thus, the length of the hypotenuse of this isosceles right triangle is $8 \times \sqrt{2} = 8\sqrt{2}$. Choice (D) is correct.

Circles (p. 50–51)

1. 8π
2. $\frac{3}{4}$
3. $\frac{7}{4}$
4. $\frac{1}{2}$
5. 64π
6. 36π
7. 16π
8. 14
9. 2π
10. π
11. 6π
12. $\frac{15\pi}{4}$
13. B

1. 8π

The circumference C of a circle is related to its diameter d by $C = \pi d$. Here $d = 8$. Then $C = \pi d = 8(\pi) = 8\pi$.

2. $\frac{3}{4}$

The circumference C of a circle is related to its diameter d by $C = \pi d$. Here $d = \frac{3}{4\pi}$. Then $C = \pi d = \pi\left(\frac{3}{4\pi}\right) = \frac{3}{4}$.

3. $\frac{7}{4}$

The circumference C of a circle is related to its radius r by $C = 2\pi r$. Here $C = \frac{7}{2}\pi$. Then $\frac{7}{2}\pi = 2\pi r$. Dividing both sides of this equation by π, we get $\frac{7}{2} = 2r$. Dividing both sides of this equation by 2, we get $\frac{\left(\frac{7}{2}\right)}{2} = r$, $\frac{7}{2}\left(\frac{1}{2}\right) = r$, $\frac{7 \times 1}{2 \times 2} = r$, and $\frac{7}{4} = r$. Thus, $r = \frac{7}{4}$.

4. $\frac{1}{2}$

The circumference C of a circle is related to its diameter d by $C = \pi d$. Here $C = \frac{\pi}{2}$. Then $\frac{\pi}{2} = \pi d$. Dividing both sides by π, we have $\frac{1}{2} = d$. Thus, $d = \frac{1}{2}$.

5. 64π

The area of a circle with a radius r is πr^2. The radius of this circle is 8. Therefore, the area of this circle is $\pi(8^2) = \pi(64) = 64\pi$.

6. 36π

The area of a circle with a radius r is πr^2. The diameter of this circle is 12. The diameter of a circle is always twice the radius. Therefore, the radius of this circle is $\frac{12}{2} = 6$. The area of this circle is $\pi(6^2) = \pi(36) = 36\pi$.

7. 16π

The area A of a circle with a radius r is given by the formula $A = \pi r^2$. The circumference of this circle is 8π. The circumference C of a circle is related to its radius by the formula $C = 2\pi r$. Since the circumference of this circle is 8π, we can say that $8\pi = 2\pi r$. Then $r = \frac{8\pi}{2\pi} = 4$. Since $r = 4$, the area of this circle is $\pi(4^2) = \pi(16) = 16\pi$.

8. 14

The diameter of a circle is twice the radius. Let's find the radius of the circle. Then we will be able to find the diameter. The area of a circle with a radius r is πr^2. Since the area of this circle is 49π, $\pi r^2 = 49\pi$. Dividing both sides of this equation by π, $r^2 = 49$. If $r^2 = 49$, then $r = 7$ or $r = -7$. Since a radius cannot be negative, the radius of the circle is 7. The diameter of a circle is twice the radius. So the diameter of this circle is $2(7) = 14$.

9. 2π

The length of an arc of a circle with a radius r that is intercepted by a central angle whose measure is n degrees is Circumference $\times \left(\frac{n}{360}\right) = 2\pi r \times \left(\frac{n}{360}\right)$. The radius of this circle is 9 and the central angle that intercepts arc AB has a measure of 40 degrees. So the length of arc AB is $2\pi r \times \left(\frac{n}{360}\right) = 2\pi\,(9) \times \left(\frac{40}{360}\right) = 18\pi \times \frac{1}{9} = 2\pi$.

10. π

Angle AOB and the central angle marked 150° make up a straight line. The sum of the angles making up a straight line is 180°. If we say that the measure of angle AOB is x degrees, then $x + 150 = 180$. So $x = 180 - 150 = 30$. Thus, the measure of central angle AOB is 30°.

The length of an arc of a circle with a radius r that is intercepted by a central angle whose measure is n degrees is Circumference $\times \left(\frac{n}{360}\right) = 2\pi r \times \left(\frac{n}{360}\right)$. The radius r of this circle is 6, and the central angle that intercepts arc AB has a measure of $n = 30$ degrees. So the length of arc AB is $2\pi r \times \left(\frac{n}{360}\right) = 2\pi(6) \times \left(\frac{30}{360}\right) = 12\pi \times \frac{1}{12} = \pi$.

11. 6π

The area of a sector of a circle with a radius r that has a central angle whose measure is n degrees is (Area of circle) $\times \left(\frac{n}{360}\right) = \pi r^2 \times \left(\frac{n}{360}\right)$. The radius of this circle is 6, and the central angle of sector OAB has a measure of $n = 60$ degrees. So the area of sector OAB is $\pi r^2 \times \left(\frac{n}{360}\right) = \pi(6^2) \times \left(\frac{60}{360}\right) = 36\pi \times \left(\frac{1}{6}\right) = 6\pi$.

12. $\frac{15\pi}{4}$

The area of a sector of a circle with a radius r that has a central angle whose measure is n degrees is (Area of circle) $\times \left(\frac{n}{360}\right) = \pi r^2 \times \left(\frac{n}{360}\right)$. The radius of this circle is 6 and the central angle of sector OAB has a measure of $n =$ 150 degrees. So the area of sector OAB is $\pi r^2 \times \left(\frac{n}{360}\right) = \pi(3^2) \times \left(\frac{150}{360}\right) = \pi(9) \times \left(\frac{150 \div 30}{360 \div 30}\right) = 9\pi \times \left(\frac{5}{12}\right) = \frac{45\pi}{12} = \frac{15\pi}{4}$.

13. (B)

The length of an arc of a circle with a radius r that is intercepted by a central angle whose measure is n degrees is Circumference $\times \left(\frac{n}{360}\right) = 2\pi r \times \left(\frac{n}{360}\right)$. Then $\frac{\text{Length of arc}}{\text{Circumference}} = \left(\frac{n}{360}\right)$. In this question, $\frac{\text{Length of arc}}{\text{Circumference}} = \frac{1}{8}$, and the central angle that intercepts arc ADC has a measure of $x°$, so we can write the equation $\frac{1}{8} = \left(\frac{x}{360}\right)$. Multiplying both sides of this equation by 360, we have $\frac{1}{8}(360) = x$ and then $45 = x$. Thus, $x = 45$. Choice (B) is correct.

Polygons (p. 53–54)

1. 90
2. 65
3. 10
4. 12
5. 10
6. 12
7. 4
8. 12
9. A

1. 90

The sum of the interior angles of any quadrilateral is 360 degrees. In this question, $50 + 110 + 110 + z = 360$. Adding the numbers on the left side, $270 + z = 360$. Subtracting 270 from both sides, $z = 90$.

2. 65

The sum of the interior angles of any quadrilateral is 360 degrees. In this question, $110 + 120 + x + x = 360$. Adding the numbers on the left side, $230 + 2x = 360$. Subtracting 230 from both sides, $2x = 130$. Dividing both sides by 2, $x = 65$.

3. 10

The perimeter of any polygon is the sum of the lengths of its sides. In this question, the perimeter of quadrilateral *ABCD* is $z + 7 + 8 + 9$. Since the perimeter of quadrilateral *ABCD* is 34, $z + 7 + 8 + 9 = 34$. Adding the numbers on the left side, $z + 24 = 34$. Subtracting 24 from both sides, $z = 10$.

4. 12

The perimeter of any polygon is the sum of the lengths of its sides. In this question, the perimeter of quadrilateral *EFGH* is $x + 5 + x + 19$. Simplifying $x + 5 + x + 19$ by combining the *x*-terms and adding the numbers, $x + 5 + x + 19 = 2x + 24$. Thus, the perimeter of quadrilateral *EFGH* is $2x + 24$. Since the perimeter of this quadrilateral is 48, $2x + 24 = 48$. Subtracting 24 from both sides of this equation, $2x = 24$. Dividing both sides by 2, $x = 12$.

5. 10

The area of a rectangle is length times width. The area of the rectangle in this question is $5 \times 2 = 10$.

6. 12

The area of a rectangle is length times width. The area of rectangle *WXYZ* is $(WX)(WZ)$. We know that $WZ = 3$. We need to find the length of *WX*. Let's look at triangle *WXZ*. Each interior angle of a rectangle is a right angle. Therefore, triangle *WXZ* is a right triangle. In right triangle *WXZ*, we know that $WZ = 3$ and $XZ = 5$. Now we could use the Pythagorean theorem to find the length of *WX*. However, there is a quicker way. Since $WZ = 3$ and $XZ = 5$, this triangle is a 3-4-5 right triangle. Thus, $WX = 4$. We now know that $WX = 4$ and $WZ = 3$. The area of rectangle *WXYZ* is $(WX)(WZ) = (4)(3) = 12$.

7. 4

Each interior angle of a square has a measure of 90 degrees, and the four sides of a square are equal. Therefore, a diagonal of a square divides the square into two identical isosceles right triangles. In this square, we want the length of *ST*. So let's consider isosceles right triangle *STV*, which has side *ST* for one of its legs. The leg to leg to hypotenuse ratio in an isosceles right triangle is x to x to $x\sqrt{2}$. Since the length of hypotenuse *TV* of isosceles right triangle *STV* is $4\sqrt{2}$, the length of leg *ST* of isosceles right triangle *STV* is $\frac{4\sqrt{2}}{\sqrt{2}} = 4$. Thus, $ST = 4$.

8. 12

Each interior angle of a rectangle has a measure of 90 degrees. Therefore, in this question, triangle *ABC* is a right triangle. We could use the Pythagorean theorem to find the value of *b*. which is the length of leg *BC* of this right triangle. However, there is a quicker way to solve this question. Triangle *ABC* is a special 5-12-13 right triangle. Therefore, the length of side *AC* is 12. That is, $b = 12$.

9. (A)

Since the length of rectangle *A* is one-half of the length of rectangle *B* and the width of rectangle *A* is one-half of the width of rectangle *B*, let's say that the length of rectangle *A* is *x* and the width of rectangle *A* is *y*. Then the length of rectangle *B* is $2x$ and the width of rectangle *B* is $2y$. The area of a rectangle is length times width. The area of rectangle *A* is xy. The area of rectangle *B* is $(2x)(2y) = 4xy$.

Here is what we now have.

The area of rectangle *A* is xy.

The area of rectangle *B* is $4xy$.

The ratio of the area of rectangle *A* to the area of rectangle *B* is $\frac{xy}{4xy} = \frac{1}{4}$. Choice (A) is correct.

Multiple Figures (p. 56–57)

1. 40
2. 4π
3. 36
4. 2π
5. E

1. 40

The perimeter of a square is 4 times the length of a side. We need the length of a side of the square in order to find its perimeter. A side of the square is a diameter of the circle. Since the diameter of a circle is twice its radius, if we can find the radius of the circle, we can find its diameter. The area of a circle with a radius r is πr^2. The area of this circle is 25π. So we can say that $\pi r^2 = 25\pi$. Dividing both sides by π, we have that $r^2 = 25$. If $r^2 = 25$, then $r = 5$ or $r = -5$. Since a radius cannot be negative, $r = 5$. The diameter of a circle is twice its radius. The radius of this circle is 5, so the diameter of this circle is 10. Therefore, a side of the square is 10. Then the perimeter of the square is $4 \times 10 = 40$.

2. 4π

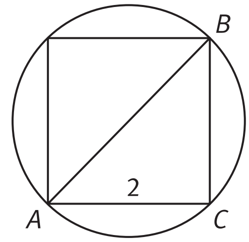
The area of a square is its side squared. The area of this square is 16, so $s^2 = 16$. Then $s = 4$ or $s = -4$. Since lengths cannot be negative, $s = 4$. Thus, the length of each side of the square is 4. A side of the square is equal in length to the diameter of the circle. Therefore, the diameter of the circle is 4. The diameter of a circle is twice the radius. Therefore, the radius of the circle is $\frac{4}{2} = 2$. The area of a circle with a radius r is πr^2. The area of this circle is $\pi(2^2) = \pi(4) = 4\pi$.

3. 36

The circumference C of a circle is related to its diameter d by the formula $C = \pi d$. The circumference of this circle is 6π. Thus, $\pi d = 6\pi$. Dividing both sides by π, we have that $d = 6$. Thus, the diameter of this circle is 6. The diameter of the circle is equal to a side of the square. Therefore, a side of the square is 6. The area of a square is its side squared. The area of this square is $6^2 = 36$.

4. 2π

The area of a square is its side squared. Here $s^2 = 4$. So $s = 2$ or $s = -2$. Since lengths cannot be negative, $s = 2$. The length of a side of the square is 2. Let's redraw the picture for this exercise, indicating that a side of the square is 2 and adding a diagonal of the square.



This diagonal of the square divides it into two identical isosceles right triangles. Let's refer to isosceles right triangle ABC. The leg to leg to hypotenuse ratio in an isosceles right triangle is x to x to $x\sqrt{2}$. Since $AC = 2$, $AB = 2\sqrt{2}$. Now AB is the diameter of the circle. Thus, the diameter of the circle is $2\sqrt{2}$. The diameter of a circle is twice the radius. Therefore, the radius of the circle is $\frac{2\sqrt{2}}{2} = \sqrt{2}$. The area of a circle with a radius r is πr^2. The area of this circle is $\pi(\sqrt{2})^2 = \pi \times \sqrt{2} \times \sqrt{2} = 2\pi$.

5. (E)

The area of a sector of a circle with a radius r that has a central angle whose measure is n degrees is (Area of circle) $\times \left(\frac{n}{360}\right) = \pi r^2 \times \left(\frac{n}{360}\right)$.

If we say that the degree measures of the central angles of the shaded regions are x, y, and z, respectively, then the sum of the areas of the shaded regions is $\pi r^2 \times \left(\frac{x}{360}\right) + \pi r^2 \times \left(\frac{y}{360}\right) + \pi r^2 \times \left(\frac{z}{360}\right) = \pi r^2 \times \left(\frac{x}{360} + \frac{y}{360} + \frac{z}{360}\right) = \pi r^2 \times \left(\frac{x+y+z}{360}\right)$. We know that $r = 8$. Let's try to find the value of $x + y + z$. The sum of the measures of the angles around a point is 360 degrees. The sum of the 3 central angles of the shaded regions and the 3 central angles that are right angles is 360 degrees. Thus, $x + 90 + y + 90 + z + 90 = 360$. Then $x + y + z + 270 = 360$, and $x + y + z = 90$.

Now we know that $r = 8$ and $x + y + z = 90$. The sum of the areas of the shaded regions is $\pi r^2 \times \left(\frac{x+y+z}{360}\right) = \pi(8^2) \times \left(\frac{90}{360}\right) = 64\pi \times \frac{1}{4} = 16\pi$. Choice (E) is correct.

Solids (p. 59)

1. $V = 192$, $SA = 208$
2. $V = 144$, $SA = 192$
3. $V = 216$, $SA = 216$
4. $V = 2\sqrt{2}$, $SA = 12$
5. $V = 432\pi$, $SA = 216\pi$
6. E

1. $V = 192$, $SA = 208$

The volume V of a rectangular solid with a length ℓ, a width w, and a height h is given by the formula $V = \ell wh$. The volume of a rectangular solid with dimensions 4, 6, and 8 is $4 \times 6 \times 8 = 24 \times 8 = 192$.

The total surface area SA is given by the formula $SA = 2(\ell w + \ell h + wh)$. The total surface area of a rectangular solid with dimensions 4, 6, and 8 is $2(4 \times 6 + 4 \times 8 + 6 \times 8) = 2(24 + 32 + 48) = 2(104) = 208$.

2. $V = 144$, $SA = 192$

The volume V of a rectangular solid with a length ℓ, a width w, and a height h is given by the formula $V = \ell wh$. The volume of a rectangular solid with dimensions 3, 4, and 12 is $3 \times 4 \times 12 = 12 \times 12 = 144$.

The total surface area SA is given by the formula $SA = 2(\ell w + \ell h + wh)$. The total surface area of a rectangular solid with dimensions 3, 4, and 12 is $2(3 \times 4 + 3 \times 12 + 4 \times 12) = 2(12 + 36 + 48) = 2(96) = 192$.

3. $V = 216$, $SA = 216$

The volume V of a cube with an edge of length e is given by the formula $V = e^3$. The volume of a cube with an edge of length 6 is $6^3 = 6 \times 6 \times 6 = 36 \times 6 = 216$.

The total surface area SA of a cube with an edge of length e is given by the formula $SA = 6e^2$. The total surface area of a cube with an edge of length 6 is $6(6^2) = 6(6 \times 6) = 6(36) = 216$.

Let's note that the volume of the cube is 216 cubic units, while the total surface area of the cube is 216 square units.

4. $V = 2\sqrt{2}$, $SA = 12$

The volume V of a cube with an edge of length e is given by the formula $V = e^3$. The volume of a cube with an edge of length 6 is $(\sqrt{2})^3 = \sqrt{2} \times \sqrt{2} \times \sqrt{2} = 2 \times \sqrt{2} = 2\sqrt{2}$.

The total surface area SA of a cube with an edge of length e is given by the formula $SA = 6e^2$. The total surface area of a cube with an edge of length $\sqrt{2}$ is $6(\sqrt{2})^2 = 6 \times \sqrt{2} \times \sqrt{2} = 6 \times 2 = 12$.

5. $V = 432\pi$, $SA = 216\pi$

The volume V of a right circular cylinder with a radius r and a height h is given by the formula $V = \pi r^2 h$. The volume of a right circular cylinder with a height h of 12 and a radius r of 6 is $\pi(6^2)(12) = \pi(36)(12) = \pi(432) = 432\pi$. Note that in this exercise, the height is mentioned before the radius. It is important to read questions carefully.

The total surface area SA of a right circular cylinder is given by the formula $SA = 2\pi r^2 + 2\pi rh$. The total surface area of a right circular cylinder with a height h of 12 and a radius r of 6 is $2\pi(6^2) + 2\pi(6)(12) = 2\pi(36) + 2\pi(72) = 72\pi + 144\pi = 216\pi$.

6. (E)

The diameter of the largest sphere that can be placed inside a cube is equal to an edge of the cube. The volume of a cube with an edge of length e is e^3. Since the volume of this cube is 64, $e^3 = 64$. So $e = 4$. Thus, the edge of the cube is 4. Therefore the diameter of the largest sphere that can be placed inside the cube is 4. The diameter of a sphere is twice the radius. So the radius of the largest sphere that can be placed inside the cube is $\frac{4}{2} = 2$. Choice (E) is correct.

Coordinate Geometry (p. 61–62)

1. $m = 0, y = 1$
2. m is undefined, $x = 1$
3. $m = \frac{3}{4}, y = \frac{3}{4}x$
4. m is undefined, $x = -4$
5. $m = 1, y = x + 1$
6. D

1. $m = 0, y = 1$

The slope m of a line going through two points is given by the formula $m = \frac{\Delta y}{\Delta x}$, where Δy means the change in y and Δx means the change in x. The slope of the line going through the points (x_1, y_1) and (x_2, y_2) is given by $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

The slope of the line going through the points $A(-3, 1)$ and $B(3, 1)$ is $m = \frac{1 - 1}{3 - (-3)} = \frac{0}{3 + 3} = \frac{0}{6} = 0$.

Since both of the points $A(-3, 1)$ and $B(3, 1)$ have a y-coordinate of 1, every point on the line must have a y-coordinate of 1. Therefore, the equation of the line going through the points $A(-3, 1)$ and $B(3, 1)$ is $y = 1$.

2. m is undefined, $x = 1$

The slope m of a line going through two points is given by the formula $m = \frac{\Delta y}{\Delta x}$, where Δy means the change in y and Δx means the change in x. The slope of the line going through the points (x_1, y_1) and (x_2, y_2) is given by $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$.

The slope of the line going through the points $A(1, 4)$ and $B(1, -2)$ is $m = \frac{4 - (-2)}{1 - 1} = \frac{4 + 2}{0} = \frac{6}{0}$. The slope is undefined. (Whenever a line is parallel to the y-axis, the slope is undefined.)

Since both of the points $A(1, 4)$ and $B(1, -2)$ have an x-coordinate of 1, every point on the line must have a x-coordinate of 1. Therefore, the equation of the line going through the points $A(1, 4)$ and $B(1, -2)$ is $x = 1$.

3. $m = \frac{3}{4}, y = \frac{3}{4}x$

The slope m of a line going through two points is given by the formula $m = \frac{\Delta y}{\Delta x}$, where Δy means the change in y and Δx means the change in x. The slope of the line going through the points (x_1, y_1) and (x_2, y_2) is given by $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$.

The slope of the line going through the points $A(0, 0)$ and $B(4, 3)$ is $m = \frac{3 - 0}{4 - 0} = \frac{3}{4}$.

The slope-intercept form of an equation is $y = mx + b$, where m is the slope and b is the y-intercept. Since we have found that the slope of the line is $m = \frac{3}{4}$, the equation of the line is $y = \frac{3}{4}x + b$. To find the value of b, we can look at the figure and see that the line intersects the y-axis where $y = 0$. Thus, $b = 0$. The equation of the line is $y = \frac{3}{4}x$.

4. m is undefined, $x = -4$

The slope m of a line going through two points is given by the formula $m = \frac{\Delta y}{\Delta x}$, where Δy means the change in y and Δx means the change in x. The slope of the line going through the points (x_1, y_1) and (x_2, y_2) is given by $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$.

The slope of the line going through the points $A(-4, -1)$ and $B(-4, -6)$ is $m = \frac{-6 - (-1)}{-4 - (-4)} = \frac{-6 + 1}{-4 + 4} = \frac{-5}{0}$. The slope is undefined. (Whenever a line is parallel to the y-axis, the slope is undefined.)

Since both of the points $A(-4, -1)$ and $B(-4, -6)$ have an x-coordinate of -4, every point on the line must have an x-coordinate of -4. Therefore, the equation of the line going through the points $A(-4, -1)$ and $B(-4, -6)$ is $x = -4$.

5. $m = 1, y = x + 1$

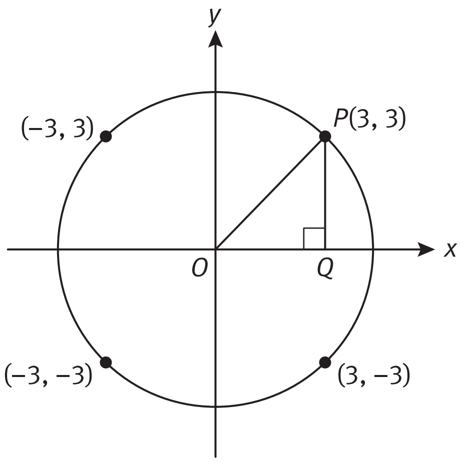
The slope m of a line going through two points is given by the formula $m = \frac{\Delta y}{\Delta x}$, where Δy means the change in y and Δx means the change in x. The slope of the line going through the points (x_1, y_1) and (x_2, y_2) is given by $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$.

The slope of the line going through the points $A(0, 1)$ and $B(4, 5)$ is $m = \frac{5 - 1}{4 - 0} = \frac{4}{4} = 1$.

The slope-intercept form of an equation is $y = mx + b$, where m is the slope and b is the y-intercept. Since we have found that the slope of the line is $m = 1$, the equation of the line is $y = x + b$. To find the value of b, we can use either of the points (0, 1) or (4, 5). It appears that the coordinates (0, 1) will be easier to work with. Let's substitute 0 for x and 1 for y in the equation $y = x + b$. Then $1 = 0 + b$, and $1 = b$. Thus, $b = 1$. The equation of the line is $y = x + 1$.

6. (D)

The area of a circle with a radius r is πr^2. To find the radius of the circle, let's find the distance from the point (3, 3) on the circle to the point (0, 0), which is the center of the circle. Let's call the point with coordinates (3, 3) point P. Let's drop a perpendicular from point $P(3, 3)$ to the x-axis. This perpendicular will meet the x-axis at the point Q.



Since *PQ* is parallel to the *y*-axis, point *Q* has the same *x*-coordinate that point *P* has. Thus, the *x*-coordinate of point *Q* is 3. Since point *Q* is on the *x*-axis, the *y*-coordinate of point *Q* is 0. Thus, the coordinates of point *Q* are (3, 0). Since points *P* and *Q* have the same *x*-coordinate of 3, the length of *PQ* is the positive difference of the *y*-coordinates of points *P* and *Q*. Thus, the length of *PQ* is $3 - 0 = 3$. Since points *O* and *Q* have the same *y*-coordinate of 0, the length of *OQ* is the positive difference of the *x*-coordinates of points *O* and *Q*. Thus, the length of *OQ* is $3 - 0 = 3$. Thus, right triangle *OPQ* is an isosceles right triangle because $PQ = OQ$. In an isosceles right triangle, the leg to leg to hypotenuse ratio is 1 to 1 to $\sqrt{2}$.Since $PQ = 3$, $OP = \sqrt{2} \times 3 = 3\sqrt{2}$. Now *OP* is a radius of the circle. Thus, the radius of the circle is $3\sqrt{2}$. The area of a circle with a radius *r* is πr^2. The area of the circle in this question is $\pi(3\sqrt{2})^2 = \pi(3\sqrt{2})(3\sqrt{2}) = \pi(3)(3)(\sqrt{2})(\sqrt{2})p =$ $\pi(9)(2) = 18\pi$. Choice (D) is correct.