7 CAMS

Introduction

A cam is a mechanical member used to impart desired motion to a follower by direct contact. The cam may be rotating or reciprocating whereas the follower may be rotating, reciprocating or oscillating. Complicated output motions which are otherwise difficult to achieve can easily be produced with the help of cams. Cams are widely used in automatic machines, internal combustion engines, machine tools, printing control mechanisms, and so on. They are manufactured usually by die-casting, milling or by punch-presses.

A cam and the follower combination belong to the category of higher pairs. Necessary elements of a cam mechanism are

- A driver member known as the *cam*
- A driven member called the *follower*
- A *frame* which supports the cam and guides the follower

The chapter outlines the methods of drawing the cam profiles as well as analysis to determine the velocity and acceleration at various positions of the follower. This information is useful in determining the smooth operation of the cam.

7.1 TYPES OF CAMS

Cams are classified according to
1. shape,
2. follower movement, and
3. manner of constraint of the follower.

According to Shape

1. *Wedge and Flat Cams* A wedge cam has a wedge *W* which, in general, has a translational motion [Figs 7.1(a) and (b). The follower *F* can either translate [Fig. 7.1(a)] or oscillate [Fig. 7.1(b)]. A spring is, usually, used to maintain the contact between the cam and
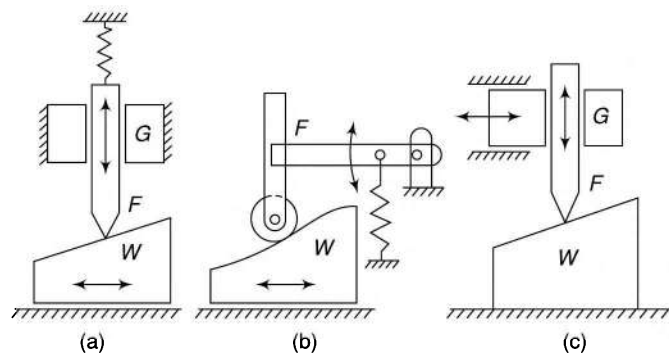
Fig. 7.1

the follower. In Fig. 7.1(c), the cam is stationary and the follower constraint or guide *G* causes the relative motion of the cam and the follower.

In stead of using a wedge, a flat plate with a groove can also be used. In the groove the follower is held as shown in Fig. 7.2. Thus, a positive drive is achieved without the use of a spring.
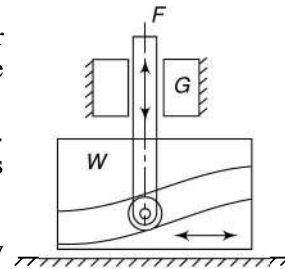

Fig. 7.2

2. ***Radial or Disc Cams*** A cam in which the follower moves radially from the centre of rotation of the cam is known as a radial or a disc cam [Fig. 7.3(a) and (b)]. Radial cams are very popular due to their simplicity and compactness.
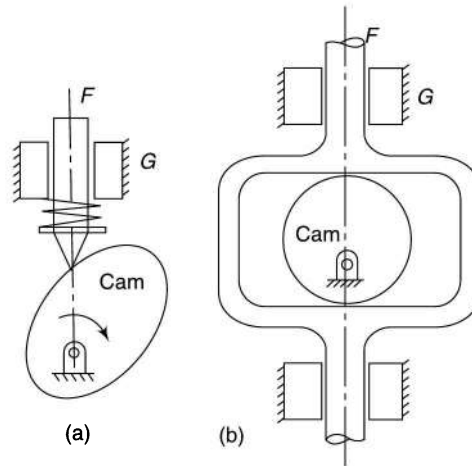

(a)   (b)

Fig. 7.3

3. ***Spiral Cams*** A spiral cam is a face cam in which a groove is cut in the form of a spiral as shown in Fig. 7.4. The spiral groove consists of teeth which mesh with a pin gear follower. The velocity of the follower is proportional to the radial distance of the groove from the axis of the cam.

   The use of such a cam is limited as the cam has to reverse the direction to reset the position of the follower. It finds its use in computers.

4. ***Cylindrical Cams*** In a cylindrical cam, a cylinder which has a circumferential contour cut in the surface, rotates about its axis. The follower motion can be of two types as follows:

   In the first type, a groove is cut on the surface of the cam and a roller follower has a constrained (or positive) oscillating
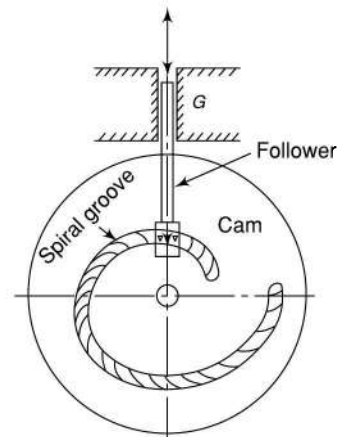

Fig. 7.4

motion [Fig. 7.5(a)]. Another type is an end cam in which the end of the cylinder is the working surface (7.5b). A spring-loaded follower translates along or parallel to the axis of the rotating cylinder.
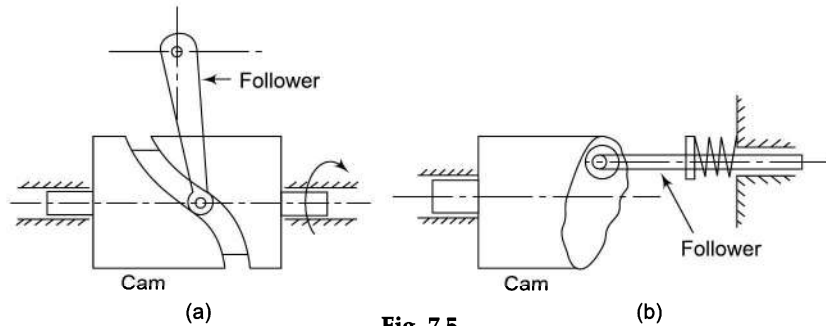

Fig. 7.5

Cylindrical cams are also known as *barrel* or *drum* cams.

5. *Conjugate Cams* A conjugate cam is a double-disc cam, the two discs being keyed together and are in constant touch with the two rollers of a follower (Fig. 7.6). Thus, the follower has a positive constraint. Such a type of cam is preferred when the requirements are low wear, low noise, better control of the follower, high speed, high dynamic loads, etc.

6. *Globoidal Cams* A globoidal cam can have two types of surfaces, convex or concave. A circumferential contour is cut on the surface of rotation of the cam to impart motion to the follower which has an oscillatory motion (Fig. 7.7). The application of such cams is limited to moderate speeds and where the angle of oscillation of the follower is large.
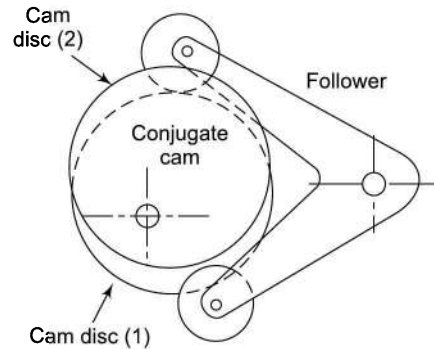

Fig. 7.6

7. *Spherical Cams* In a spherical cam, the follower oscillates about an axis perpendicular to the axis of rotation of the cam. Note that in a disc cam, the follower oscillates about an axis parallel to the axis of rotation of the cam.
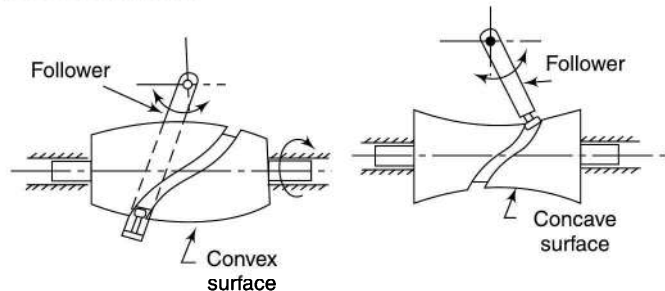
A spherical cam is in the form of a spherical surface which transmits motion to the follower (Fig. 7.8).
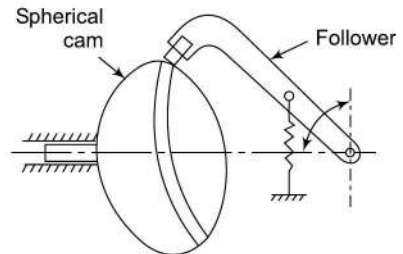

Fig. 7.7

According to Follower Movement

The motions of the followers are distinguished from each other by the dwells they have. A dwell is the zero displacement or the absence of motion of the follower during the motion of the cam.

Cams are classified according to the motions of the followers in the following ways:

Fig. 7.8

1. *Rise-Return-Rise* **(R-R-R)** In this, there is alternate rise and return of the follower with no periods of dwells (Fig. 7.9a). Its use is very limited in the industry. The follower has a linear or an angular displacement.

2. *Dwell-Rise-Return-Dwell (D-R-R-D)* In such a type of cam, there is rise and return of the follower after a dwell [Fig. 7.9(b)]. This type is used more frequently than the R-R-R type of cam.

3. *Dwell-Rise-Dwell-Return-Dwell* **(D-R-D-R-D)** It is the most widely used type of cam. The dwelling of the cam is followed by rise and dwell and subsequently by return and dwell as shown in Fig. 7.9(c). In case the return of the follower is by a fall [Fig. 7.9(d)], the motion may be known as Dwell-Rise-Dwell (D-R-D).
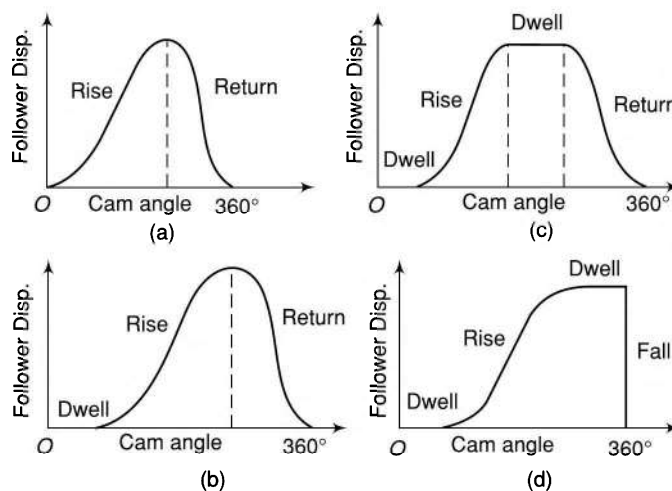
Fig. 7.9

According to Manner of Constraint of the Follower

To reproduce exactly the motion transmitted by the cam to the follower, it is necessary that the two remain in touch at all speeds and at all times. The cams can be classified according to the manner in which this is achieved.

1. *Pre-loaded Spring Cam* A pre-loaded compression spring is used for the purpose of keeping the contact between the cam and the follower [Figs 7.1(a) and (b), 7.3(a), 7.5(b) and 7.8].

2. *Positive-drive Cam* In this type, constant touch between the cam and the follower is maintained by a roller follower operating in the groove of a cam [Figs 7.2, 7.3(b). 7.4, 7.5(a) and 7.7]. The follower cannot go out of this groove under the normal working operations. A constrained or positive drive is also obtained by the use of a conjugate cam (Fig. 7.6).

3. *Gravity Cam* If the rise of the cam is achieved by the rising surface of the cam and the return by the force of gravity or due to the weight of the cam, the cam is known as a gravity cam. Figure 7.2(c) shows such a cam. However, these cams are not preferred due to their uncertain behaviour.

7.2 TYPES OF FOLLOWERS

Cam followers are classified according to the
1. shape,
2. movement, and
3. location of line of movement.

According to Shape

1. *Knife-edge Follower* It is quite simple in construction. Figure 7.1(a) shows such a follower. However, its use is limited as it produces a great wear of the surface at the point of contact.

2. *Roller Follower* It is a widely used cam follower and has a cylindrical roller free to rotate about a pin joint [Figs 7.1(b), 7.2, 7.5, 7.8]. At low speeds, the follower has a pure rolling action, but at high speeds, some sliding also occurs.

In case of steep rise, a roller follower jams the cam and, therefore, is not preferred.

3. *Mushroom Follower* A mushroom follower (Fig. 7.10) has the advantage that it does not pose the problem of jamming the cam. However, high surface stresses and wear are quite high due to deflection and misalignment if a flat-faced follower is used [Fig. 7.10(a)]. These disadvantages are reduced if a spherical-faced follower [Fig. 7.10(b)] is used instead of a flat-faced follower.
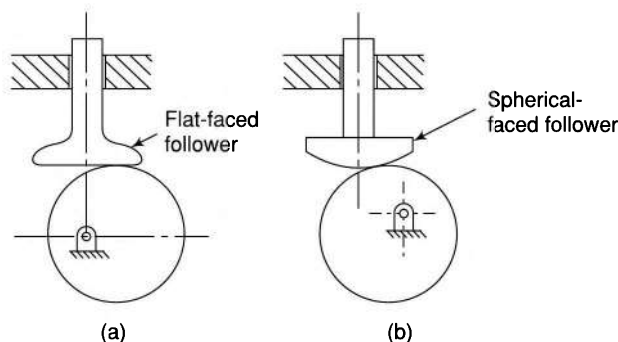

Fig. 7.10

According to Movement

1. *Reciprocating Follower* In this type, as the cam rotates, the follower reciprocates or translates in the guides [Fig. 7.1(a)].
2. *Oscillating Follower* The follower is pivoted at a suitable point on the frame and oscillates as the cam makes the rotary motion [Fig. 7.1(b)].

According to Location of Line of Movement

1. *Radial Follower* The follower is known as a radial follower if the line of movement of the follower passes through the centre of rotation of the cam [Figs 7.3(a) and (b)].
2. *Offset Follower* If the line of movement of the roller follower is offset from the centre of rotation of the cam, the follower is known as an offset follower [Fig. 7.10(b)].

A cam with two oscillating followers. This can operates the inlet as well as the exhaust valves of the engine

7.3 DEFINITIONS

With reference to Fig. 7.11, some definitions are given below:

Base Circle It is the smallest circle tangent to the cam profile (contour) drawn from the centre of rotation of a radial cam.

Trace point It is a reference point on the follower to trace the cam profile such as the knife-edge of a knife-edged follower and centre of the roller of a roller follower.

Pitch Curve It is the curve drawn by the trace point assuming that the cam is fixed, and the trace point of the follower rotates around the cam.

Pressure Angle The pressure angle, representing the steepness of the cam profile, is the angle between the normal to the pitch curve at
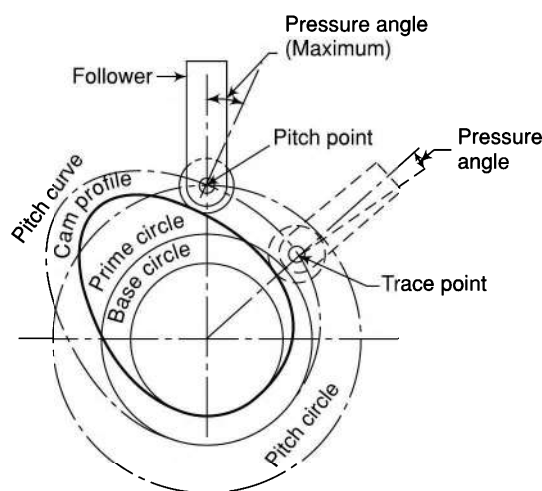

Fig. 7.11

a point and the direction of the follower motion. It varies in magnitude at all instants of the follower motion. A high value of the maximum pressure angle is not desired as it might jam the follower in the bearings.

Pitch Point   It is the point on the pitch curve at which the pressure angle is maximum.

Pitch Circle   It is the circle passing through the pitch point and concentric with the base circle.

Prime Circle   The smallest circle drawn tangent to the pitch curve is known as the prime circle.

7.4 FOLLOWER DISPLACEMENT PROGRAMMING

As a cam rotates about the axis, it imparts a specific motion to the follower which is repeated with each revolution of the cam. Thus, it is enough to know the motion of the follower for only one revolution. During rotation of the cam through one revolution, the follower is made to execute a series of events such as rises, dwells and returns. The motion of the cam can be represented on a graph, the *x*-axis of which may represent the angular displacement of the cam and *y*-axis, the angular or the linear displacement of the follower. The follower displacement is measured from its lowest position and is plotted with the same scale as is to be used in the layout of the cam profile. The following terms are used with reference to the angular motion of the cam:

Fig. 7.12

- *Angle of Ascent* (ϕ_a) It is the angle through which the cam turns during the time the follower rises.
- *Angle of Dwell* (δ) The angle of dwell is the angle through which the cam turns while the follower remains stationary at the highest or the lowest position.
- *Angle of Descent* (ϕ_d) It is the angle through which the cam turns during the time the follower returns to the initial position.
- *Angle of Action* The angle of action is the total angle moved by the cam during the time, between the beginning of rise and the end of the return of the follower.

To satisfy the given requirements of the follower displacement, a programme can be made keeping in view the following points:

1. In a given specific interval of time, due consideration to the velocity and the acceleration must be given, the effects of which are manifested as inertia loads. The dynamic effects of acceleration, usually limit the speed of the cams. Moreover, effects of jerks (rate of change of acceleration) in case of high-speed mechanisms produce vibrations of the system, which is undesirable for a follower motion. Though it is very difficult to completely eliminate jerk, efforts are to be made to keep it within tolerable limits.

Fig. 7.13

2. The force exerted by a cam on the follower is always normal to the surface of the cam at the point of contact. The vertical component ($F \cos\alpha$) lifts the follower whereas the horizontal component ($F \sin\alpha$) exerts lateral pressure on the bearing as shown in Fig. 7.12.

In order to reduce the lateral pressure or $F \sin \alpha$, α has to be decreased which means making the surface more convex and longer (dotted lines). This results in reduced velocity of the follower and
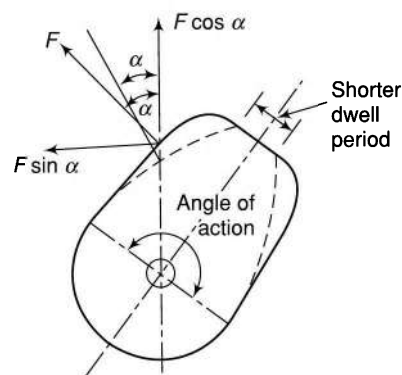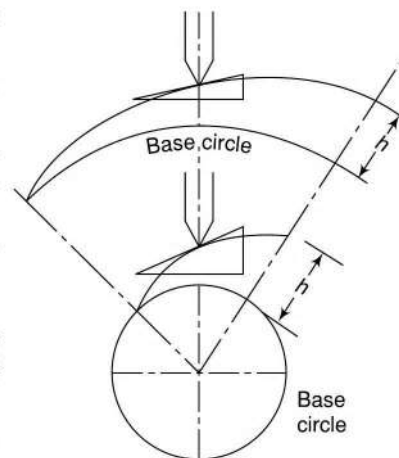
more time for the same rise. This also reduces the dwell period for a fixed angle of action. In internal combustion engines, a shorter dwell period means a smaller period of valve-opening, resulting in less fuel per cycle and lesser power production. Thus, the minimum value of α cannot be reduced from a certain value.

3. The size of the base circle controls the pressure angle. As shown in Fig. 7.13, the increase in the base circle diameter increases the length of the arc of the circle upon which the wedge (the raised portion) is to be made. A short wedge for a given rise requires a steep rise or a higher pressure angle, thus increasing the lateral force.

7.5   DERIVATIVES OF FOLLOWER MOTION

The derivatives of follower motion can be *kinematic* (with respect to θ) which relate to the geometry of the cam system or *physical* (with respect to time) which relate to the motion of the follower of the cam system.

Kinematic Derivatives

A displacement diagram of the follower motion is plotted with the cam angle θ as the abscissa and the follower linear or angular motion as the ordinate. Thus, it is a graph that relates the input and the output of the cam system. Mathematically, if s is the displacement of the follower. Then

$$s = s\,(\theta)$$

Differentiating it with respect to θ provides the first derivative,

$$\dot{s}(\theta) = \frac{ds}{d\theta}$$

It represents the slope or the steepness of the displacement curve at each position of the cam angle. A higher value of this means a steep rise or fall which hampers the smooth running of the cam.

The second derivative is represented by

$$\ddot{s}(\theta) = \frac{d^2s}{d\theta^2}$$

This derivative is related to the radius of curvature of the cam at different points along its profile and is in inverse proportion. Thus, with an increase in its value, the radius of curvature decreases. If its value becomes infinity, the cam profile becomes pointed at that position which is undesirable from the point of view of stresses between the cam and follower surfaces.

The next derivative can also be taken if desired:

$$\dddot{s}(\theta) = \frac{d^3s}{d\theta^3}$$

It is not easy to describe it geometrically. However, it should also be controlled as far as possible while choosing the shape of the displacement diagram for smooth working of the cam.

Physical Derivatives

We have, $s = s(\theta)$ and $\theta = \theta(t)$

Taking the first derivative with respect to time,

$$\dot{s} = \frac{ds}{dt} = \frac{ds}{d\theta}\frac{d\theta}{dt} = \omega\frac{ds}{d\theta}$$

which represents the velocity of the follower.

The second derivative is

$$\ddot{s} = \frac{d^2s}{dt^2} = \omega^2 \frac{d^2s}{d\theta^2}$$

It represents the *acceleration* of the follower. A higher value of acceleration means a higher inertia force. A third derivative is known as the *jerk*.

$$\dddot{s} = \frac{d^3s}{dt^3} = \omega^3 \frac{d^3s}{d\theta^3}$$

For smooth movement of the follower, even the high values of the jerk are undesirable in case of high-speed cams.

7.6   HIGH-SPEED CAMS

A real follower always has some mass and when multiplied by acceleration, inertia force of the follower is obtained. This force is always felt at the contact point of the follower with the cam surface and at the bearings. An acceleration curve with abrupt changes exerts abrupt stresses on the cam surfaces and at the bearings accompanied by detrimental effects such as surface wear and noise. All this may lead to an early failure of the cam system. Thus, it is very important to give due consideration to velocity and acceleration curves while choosing a displacement diagram. They should not have any step changes.

In low-speed applications, cams with discontinuous acceleration characteristics may not show any undesirable characteristic, but at higher speeds such cams are certainly bound to show the same. The higher the speed, the higher is the need for smooth curves. At very high speeds, even the jerk (related to rate of change of acceleration or force) is made continuous as well. For most of the applications, however, this may not be needed. In Section 7.8, standard cam motions have been discussed from which some comparison can easily be made for suitable selection.

7.7   UNDERCUTTING

Sometimes, it may happen that the prime circle of a cam is proportioned to provide a satisfactory pressure angle; still the follower may not be completing the desired motion. This can happen if the curvature of the pitch curve is too sharp.
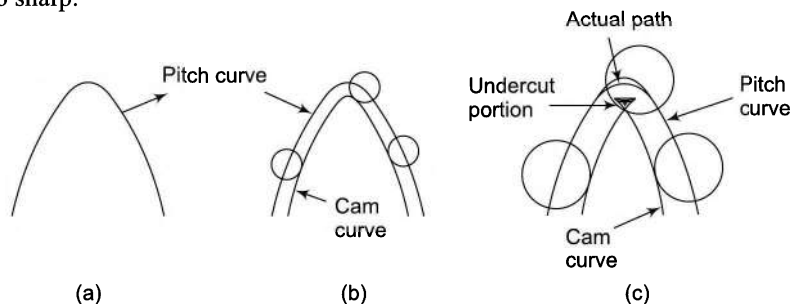

Fig. 7.14

Figure 7.14(a) shows the pitch curve of a cam. In Fig. 7.14(b), a roller follower is shown generating this curve. In Fig. 7.14(c), a larger roller is shown trying to generate this curve. It can easily be observed that the

cam curve loops over itself in order to realize the profile of the pitch curve. As it is impossible to produce such a cam profile, the result is that the cam will be undercut and become a pointed cam. Now when the roller follower will be made to move over this cam, it will not be producing the desired motion.

It may be observed that the cam will be pointed if the radius of the roller is equal to the radius of curvature of the pitch curve. Thus, to have a minimum radius of curvature of the cam profile, the radius of curvature of the prime circle must always be greater than that of the radius of the roller.

7.8 MOTIONS OF THE FOLLOWER

Though the follower can be made to have any type of desired motion, knowledge of the existing motion programmes saves time and labour while designing the cams.

Following are some basic displacement programmes:

1. Simple Harmonic Motion (SHM)

This is a popular follower motion and is easy to lay out.

Let s = follower displacement (instantaneous)

h = maximum follower displacement

v = velocity of the follower

f = acceleration of the follower

θ = cam rotation angle (instantaneous)

φ = cam rotation angle for the maximum follower displacement

β = angle on the harmonic circle.

Construction   The follower rises through a distance h while the cam turns through an angle φ. Construct the follower displacement curve as follows (Fig. 7.15).
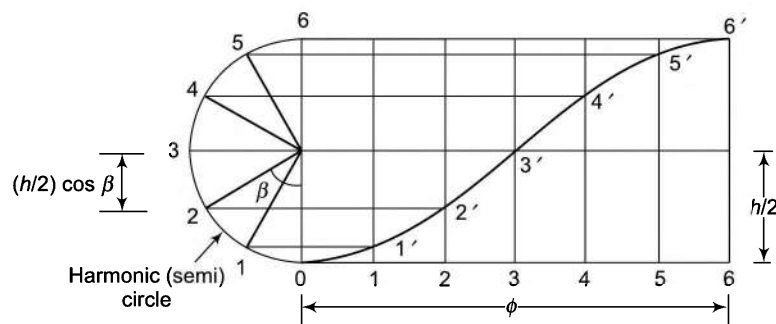
Fig. 7.15

(i) Draw a semicircle with cam rise (or fall) as the diameter. This is, usually known as the harmonic (semi) circle. Divide this semicircle into n equal arcs (*n* even).

(ii) Divide the cam displacement interval into n equal divisions.

(iii) Project the intercepts of the harmonic semicircle to the corresponding divisions of the cam displacement interval.

(iv) Join the points with a smooth curve to obtain the required harmonic curve.

Displacement   At any instant, displacement of the follower is given by,

$$s = \frac{h}{2} - \frac{h}{2}\cos\beta$$

$$= \frac{h}{2}(1 - \cos\beta) \tag{i}$$

For the rise (or fall) h of the follower displacement, the cam is rotated through an angle φ whereas a point on the harmonic semicircle traverses an angle π. Thus, the cam rotation is proportional to the angle turned by the point on the harmonic semicircle, i.e.

$$\beta = \pi\frac{\theta}{\varphi}$$

Thus β can be replaced by θ and φ in Eq. (i) above,

$$s = \frac{h}{2}\left(1 - \cos\frac{\pi\theta}{\varphi}\right) \tag{7.1}$$

The expression is also valid for β more than 90°. In that case, $\cos\beta$ or $\cos\pi\theta/\varphi$ becomes negative so that s is again positive and more than $h/2$.

Let ω = Angular velocity of the cam

$$\therefore \qquad\qquad \theta = \omega t$$

and

$$s = \frac{h}{2}\left(1 - \cos\frac{\pi\omega t}{\varphi}\right)$$

$$v = \frac{ds}{dt} = \frac{h}{2}\frac{\pi\omega}{\varphi}\sin\frac{\pi\omega t}{\varphi}$$

$$= \frac{h}{2}\frac{\pi\omega}{\varphi}\sin\frac{\pi\theta}{\varphi} \tag{7.2}$$

$$v_{max} = \frac{h}{2}\frac{\pi\omega}{\varphi} \text{ at } \theta = \frac{\varphi}{2} \tag{7.3}$$

$$f = \frac{dv}{dt} = \frac{h}{2}\left(\frac{\pi\omega}{\varphi}\right)^2\cos\frac{\pi\omega t}{\varphi}$$

$$= \frac{h}{2}\left(\frac{\pi\omega}{\varphi}\right)^2\cos\frac{\pi\theta}{\varphi} \tag{7.4}$$

$$f_{max} = \frac{h}{2}\left(\frac{\pi\omega}{\varphi}\right)^2 \text{ at } \theta = 0 \tag{7.5}$$

Let $\quad\varphi_a$ = angle of ascent

$\quad\varphi_d$ = angle of descent

$\quad\delta_{1,2}$ = angles of dwells

It can be seen from the plots of Fig. 7.16 that there is an abrupt change of acceleration from zero to maximum at the beginning of the follower motion and also from maximum (negative) to zero at the end of the follower motion when the follower rises. Similar abruption would also be there at the start and end

of the return motion. As these abrupt changes result in infinite jerk, vibration and noise, the programme should be adopted only for low or moderate cam speeds.

2. Constant Acceleration and Deceleration (Parabolic)

In such a follower programme, there is acceleration in the first half of the follower motion whereas it is deceleration during the later half. The displacement curve is found to be parabolic in this case. The magnitude of the acceleration and the deceleration is the same and constant in the two halves.

Construction   Refer Fig. 7.17(a).
   (i) Divide each half of the cam displacement interval into n equal divisions.
  (ii) Divide half the follower rise into n^2 equal divisions.
 (iii) Project 1^2 displacement interval to the first ordinate of the cam displacement, 2^2 to the second ordinate, 3^2 to the third, and so on.
  (iv) The second half of the curve is similar to the first half.

   Alternatively, divide half of the follower rise at the central ordinate of the cam displacement into n equal divisions [Fig. 17.17(b)]. Joining the zero point with the first division gives 1^2 on the first ordinate, with the second division 2^2 on the second ordinate, and so on.

   The equation for the linear motion with constant acceleration f (during the first half of the follower motion) is found as follows:

$$s = v_0 t + \frac{1}{2} ft^2$$

where v_o is the initial velocity at the start of the motion (rise or fall) and is zero in this case.
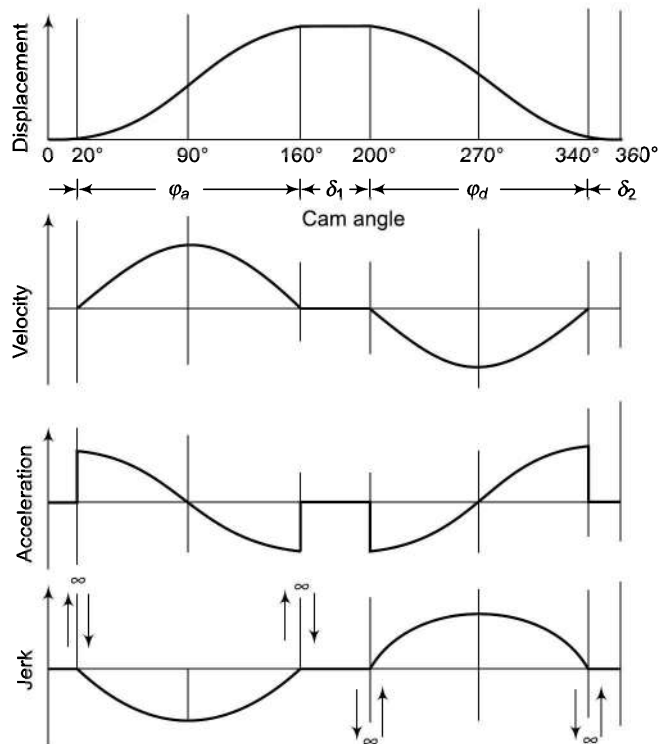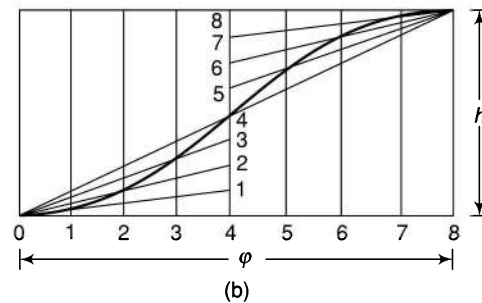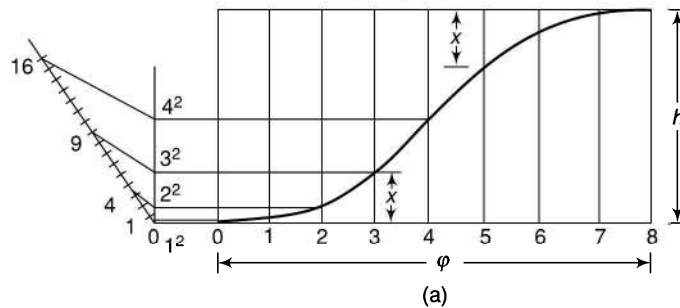
$$\therefore \qquad S = \frac{1}{2} ft^2$$

Fig. 7.16

(a)

(b)

Fig. 7.17

or $\qquad\qquad f = \dfrac{2s}{t^2} = \text{constant}$ $\qquad\qquad\qquad\qquad\qquad\qquad$ (7.6)

As f is constant during the accelerating period, considering the follower at the midway,

$$s = \frac{h}{2} \quad \text{and} \quad t = \frac{\varphi/2}{\omega}$$

$\therefore \qquad\qquad f = \dfrac{2h/2}{\varphi^2/4\omega^2} = \dfrac{4h\omega^2}{\varphi^2}$ $\qquad\qquad\qquad\qquad\qquad$ (7.6a)

The velocity is linear during the period and is given by

$$v = \frac{ds}{dt} = \frac{1}{2} \times 2ft = ft \qquad\qquad\qquad\qquad (7.7)$$

$$= \frac{4h\omega^2}{\varphi^2} \frac{\theta}{\omega} \qquad\qquad\qquad (\theta = \omega t)$$

$$= \frac{4h\omega}{\varphi^2} \theta \qquad\qquad\qquad\qquad\qquad\qquad (7.7a)$$

The velocity is maximum when θ is maximum or the follower is at the midway, i.e., when $\theta = \varphi/2$.

$$v_{\max} = \frac{4h\omega}{\varphi^2} \frac{\varphi}{2} = \frac{2h\omega}{\varphi} \qquad\qquad\qquad (7.8)$$

During the second half of the follower motion, the follower is decelerated at constant rate so that the velocity reduces to zero at the end.

It can be observed from the plots shown in Fig. 7.18 that there are abrupt changes in the acceleration at the beginning, midway and the end of the follower motion. At midway, an infinite jerk is produced. Thus, this programme of the follower is adopted only up to moderate speeds.

3. Constant Velocity

Constant velocity of the follower implies that the displacement of the follower is proportional to the cam displacement and the slope of the displacement curve is constant (Fig. 7.19).
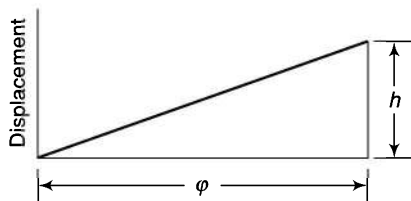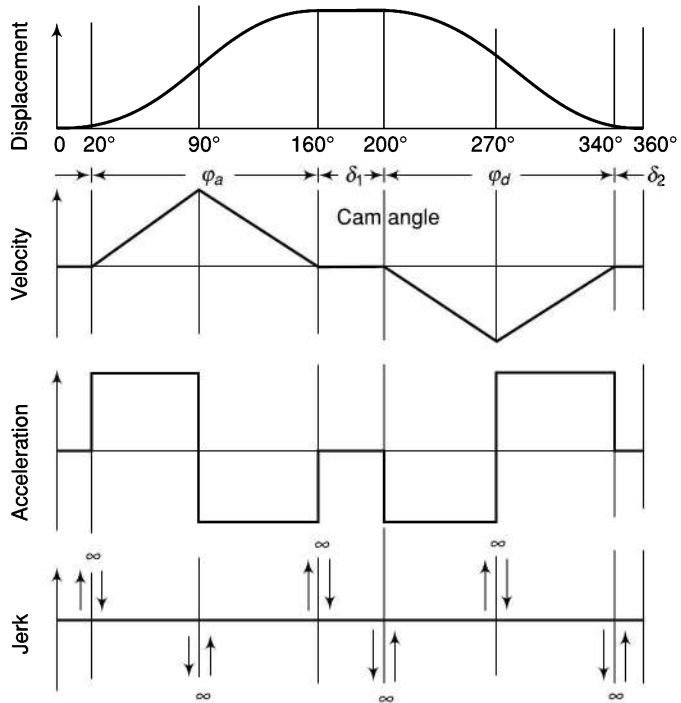
Displacement of the follower for the

Fig. 7.19

Fig. 7.18

angular displacement θ of the cam is given by

$$s = h\,\frac{\theta}{\varphi} = h\,\frac{\omega t}{\varphi} \tag{7.9}$$

$$v = \frac{ds}{dt} = \frac{h\omega}{\varphi}\ \text{constant} \tag{7.10}$$

$$f = \frac{dv}{dt} = 0 \tag{7.11}$$

As seen in the plots of Fig. 7.20(a), though acceleration is zero during the rise or the fall of the follower, it is infinite at the beginning and end of the motion as there are abrupt changes in velocity at these points. This results in infinite inertia forces and thus is not suitable from the practical point of view.

A modified programme for the follower motion can be evolved in which the accelerations are reduced to finite values. This can be done by rounding the sharp corners of the displacement curve so that the velocity changes are gradual at the beginning and end of the follower motion. During these periods, the acceleration may be assumed to be constant and of finite values. A modified constant velocity programme is shown in Fig. 7.20(b).

(a)   (b)

Fig. 7.20

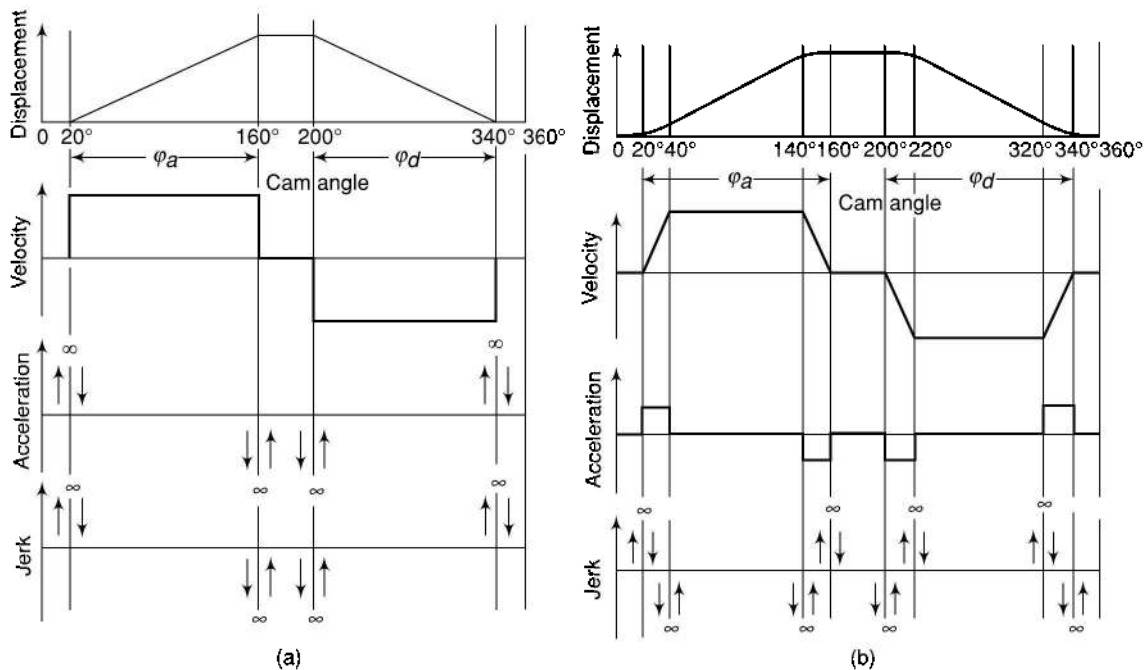## 4. Cycloidal

A cycloidal is the locus of a point on a circle rolling on a straight line.

Construction   (Refer Fig. 7.21)
   (i) Divide the cam displacement interval into n equal parts (n even).
   (ii) Draw the diagonal of the diagram and extend it below.
   (iii) Draw a circle with the centre anywhere on the lower portion of the diagonal such that its circumference

is equal to the follower displacement, i.e., $2\pi r = h$ or $r = h/2\pi$.

(iv) Divide the circle into n equal arcs and number them as shown in the diagram.

(v) Project the circle points to its vertical diameter and then in a direction parallel to the diagonal of the diagram to the corresponding ordinates.

Joining the points with a curve gives the required cycloidal. Mathematically, a cycloidal is expressed by

Fig. 7.21

$$s = \frac{h}{\pi}\left(\frac{\pi\theta}{\varphi} - \frac{1}{2}\sin\frac{2\pi\theta}{\varphi}\right) \qquad (7.12)$$

$$v = \frac{ds}{dt} = \frac{ds}{d\theta}\frac{d\theta}{dt}$$

$$= \left[\frac{h}{\varphi} - \frac{h}{2\pi}\frac{2\pi}{\varphi}\cos\frac{2\pi\theta}{\varphi}\right]\omega$$

$$= \frac{h\omega}{\varphi} - \frac{h\omega}{\varphi}\cos\frac{2\pi\theta}{\varphi} \qquad (7.13)$$

$$= \frac{h\omega}{\varphi}\left(1 - \cos\frac{2\pi\theta}{\varphi}\right)$$

$$v_{max} = \frac{2h\omega}{\varphi} \text{ at } \theta = \frac{\varphi}{2} \qquad (7.14)$$

$$f = \frac{dv}{dt} = \frac{dv}{d\theta}\frac{d\theta}{dt}$$

$$= \left[\frac{h\omega}{\varphi}\frac{2\pi}{\varphi}\sin\frac{2\pi\theta}{\varphi}\right]\omega$$

$$= \frac{2h\pi\omega^2}{\varphi^2}\sin\frac{2\pi\theta}{\varphi} \qquad (7.15)$$

$$f_{max} = \frac{2h\pi\omega^2}{\varphi^2} \text{ at } \theta = \frac{\varphi}{4} \qquad (7.16)$$

From the plots of Fig. 7.22, it is observed that there are no abrupt changes in the velocity and the acceleration at any stage of the motion. Thus, it is the most ideal programme for high-speed follower motion.
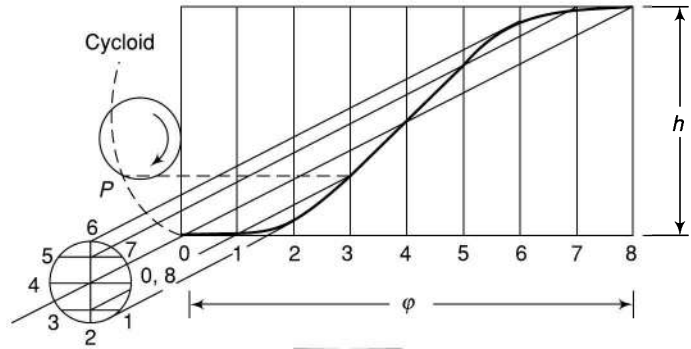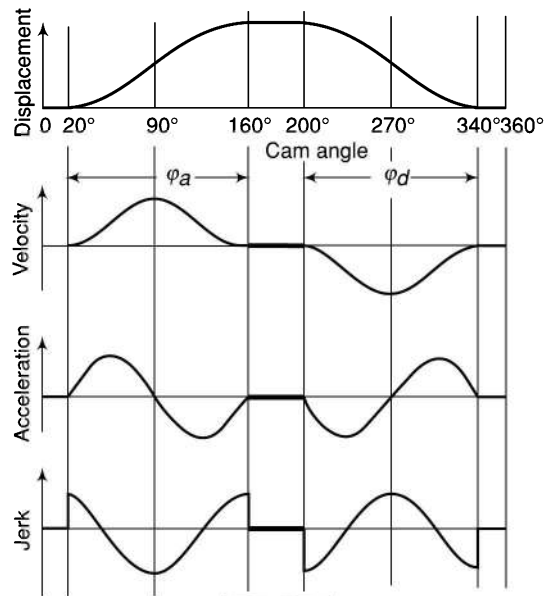

Fig. 7.22

7.9 LAYOUT OF CAM PROFILES

A cam profile is constructed on the principle of kinematic inversion, i.e., considering the cam to be stationary and the follower to be rotating about it in the opposite direction of the cam rotation. In general, the following procedure is adopted for laying out the profile for a reciprocating follower:

1. Draw the displacement diagram of the follower according to the given follower motion by dividing the cam displacement interval into n equal parts as has been discussed in the previous section. The usual number taken is 6, 8, 10 or 12 depending upon the angular displacement and convenience. Remember that the scale of the displacement interval does not affect the cam profile whereas the follower displacement does.

2. Draw the prime circle of the cam with radius
   (a) r_c if it is a knife-edge or mushroom follower (Ex. 7.1, 7.2 and 7.5)
   (b) $r_c + r_r$ if it is a roller follower (Ex. 7.3 and 7.4)

3. Divide the prime circle into segments as follows:
   (a) In case of a radial follower, divide the circle from the vertical position indicating the angles of ascent, dwell period and angle of descent, etc., in the opposite direction of the cam rotation (Ex. 7.1, 7.3 and 7.5).
   (b) In case of an offset follower, draw another circle with radius equal to the offset of the follower and assume the initial position on the prime circle where the tangent to the horizontal radius of the circle meets the prime circle (Ex. 7.2 and 7.4).

4. Further, divide each segment of ascent and descent into the same number of angular parts as is done in the displacement diagram.

5. On the radial lines produced, mark distances equal to the lift of the follower beyond the circumference of the prime circle (Ex. 7.1, 7.3 and 7.5). In case of offset follower, the distances are marked on the tangents drawn to the circle with radius equal to the offset (Ex. 7.2 and 7.4). It can be visualized that with rotation of the cam, each radial or tangential line so obtained merges with the axis of the follower at successive intervals of time and the marked points are the various positions of the tracing point of the follower.

6. Obtain the cam profile as follows:
   (a) For a knife-edge follower, draw a smooth curve passing through the marked points which is the required cam profile (Ex. 7.1 and 7.2).
   (b) In case of a roller follower, draw a series of arcs of radii equal to r_r on the inner side and draw a smooth curve tangential to all the arcs to get the required cam profile (Ex. 7.3 and 7.4).
   (c) For a mushroom follower, draw the follower in all the positions by drawing perpendiculars to the radial or tangent lines and draw a smooth curve tangential to the flat-faces of the follower representing the cam profile (Ex. 7.5).

Example 7.1 *Draw the profile of a cam operating a knife-edge follower having a lift of 30 mm. The cam raises the follower with SHM for 150° of the rotation followed by a period of dwell for 60°. The follower descends for the next 100° rotation of the cam with uniform velocity, again followed by a dwell period. The cam rotates at a uniform velocity of 120 rpm and has a least radius of 20 mm. What will be the maximum velocity and acceleration of the follower during the lift and the return?*

Solution:

$h = 30$ mm $\qquad \varphi_a = 150°$
$N = 120$ rpm $\qquad \delta_1 = 60°$

$r_c = 20$ mm $\qquad \varphi_d = 100°$

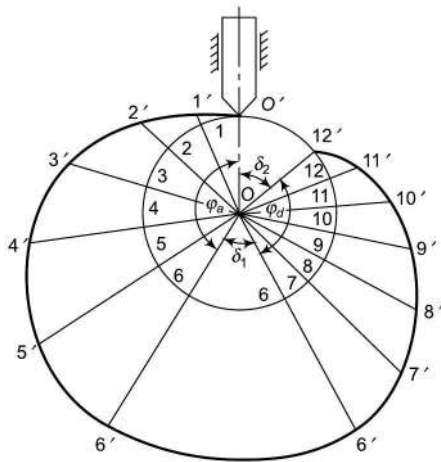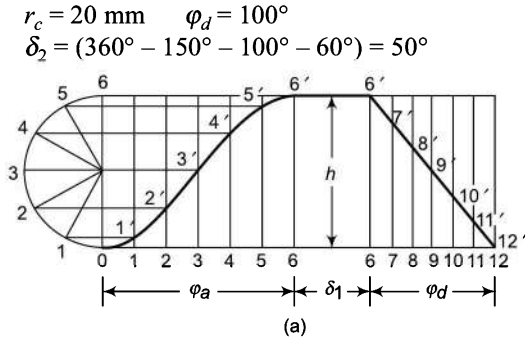$\delta_2 = (360° - 150° - 100° - 60°) = 50°$

(a)

Fig. 7.23

Draw the displacement diagram of the follower as discussed earlier [Fig. 7.23(a)] taking a convenient scale. Construct the cam profile as follows [refer Fig. 7.23(b)]:

(i) Draw a circle with radius r_c.

(ii) If the cam rotates clockwise and the follower remains in vertical direction, the cam profile can be drawn by assuming that the cam is stationary and the follower rotates about the cam in the counter-clockwise direction.

From the vertical position, mark angles φ_a, δ_1, φ_d, and δ_2 in the counter-clockwise direction, representing angles of ascent, rest or dwell, descent and rest respectively.

(iii) Divide the angles φ_a and φ_d into same number of parts as is done in the displacement

diagram. In this case, each has been divided into 6 equal parts.

(iv) Draw radial lines O-1, O-2, O-3, etc., O-1 represents that after an interval of $\varphi_d/6$ of the cam rotation in the clockwise direction it will take the vertical position of O-O'.

(v) On the radial lines produced, take distances equal to the lift of the follower beyond the circumference of the circle with radius r_c, i.e., 1-1', 2-2', 3-3', etc.

(vi) Draw a smooth curve passing through O', 1', 2',...., 10', 11' and 12'. Draw an arc of radius O-6' for the dwell period δ_1.

During ascent

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 120}{60} = 12.57 \text{ rad/s}$$

$$v_{max} = \frac{h}{2} \frac{\pi\omega}{\varphi_a} \qquad \text{[refer Eq. (7.3)]}$$

or

$$v_{max} = \frac{30}{2} \times \frac{\pi \times 12.57}{150 \times \dfrac{\pi}{180}} = 226.3 \text{ mm/s}$$

$$f_{max} = \frac{h}{2}\left(\frac{\pi\omega}{\varphi_a}\right)^2 \qquad \text{[refer Eq. (7.5)]}$$

or

$$f_{max} = \frac{30}{2} \times \left(\frac{\pi \times 12.57}{150 \times \dfrac{\pi}{180}}\right)^2$$

$$= 3413 \text{ mm/s}^2 \text{ or } \underline{3.413 \text{ m/s}^2}$$

During desent

$$v_{max} = h\frac{\omega}{\varphi_d} \qquad \text{[refer Eq. (7.10)]}$$

$$v_{max} = 30 \times \frac{12.57}{100 \times \dfrac{\pi}{180}} = \underline{216 \text{ mm/s}}$$

$$f_{max} = f = 0$$

Note that to draw the cam profile, it is not necessary that the interval δ_1 is taken in the displacement diagram. Also, the scales of φ_a and φ_d can be taken different and of any magnitudes.

Example 7.2 *A cam with a minimum radius of 25 mm is to be designed for a knife-edge follower with the following data:*

- *To raise the follower through 35 mm during 60° rotation of the cam*
- *Dwell for next 40° of the cam rotation*
- *Descending of the follower during the next 90° of the cam rotation*
- *Dwell during the rest of the cam rotation*

Draw the profile of the cam if the ascending and descending of the cam is with simple harmonic motion and the line of stroke of the follower is offset 10 mm from the axis of the cam shaft.

What is the maximum velocity and acceleration of the follower during the ascent and the descent if the cam rotates at 150 rpm?

Solution

$h = 35$ mm $\qquad \varphi_a = 60°$
$N = 150$ rpm $\qquad \delta_1 = 40°$
$r_c = 25$ mm $\qquad \varphi_d = 90°$
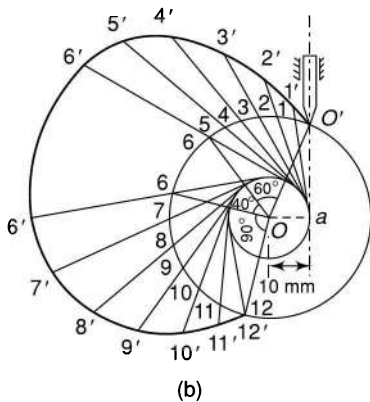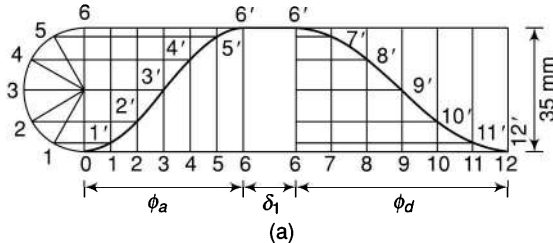$x = 10$ mm

(a)

(b)

Fig. 7.24

Draw the displacement diagram of the follower as discussed earlier [Fig. 7.24(a)]. Construct the cam profile as follows [refer Fig. 7.24(b)]:

(i) Draw a circle with radius r_c (= 25 mm).
(ii) Draw another circle concentric with the previous circle with radius x (= 10 mm). If the cam is assumed stationary, the follower will be tangential to this circle in all the positions. Let the initial position be a-O'.
(iii) Join O-O'. Divide the circle of radius r_c into four parts as usual with angles φ_a, δ_1, φ_d and δ_2 starting from O-O'.
(iv) Divide the angles φ_a and φ_d into same number of parts as is done in the displacement diagram and obtain the points 1, 2, 3, etc., on the circumference of circle with radius r_c.
(v) Draw tangents to the circle with radius x from the points 1, 2, 3, etc.
(vi) On the extension of the tangent lines, mark the distances from the displacement diagram.
(vii) Draw a smooth curve through O',1′, 2′, etc. This is the required pitch curve.

During ascent $\quad \omega = \dfrac{2\pi \times 150}{60} = 5\pi$ rad/s

$$v_{max} = \frac{h}{2}\frac{\pi\omega}{\varphi_a} \qquad \text{[refer Eq. (7.3)]}$$

or $\quad v_{max} = \dfrac{35}{2} \times \dfrac{\pi \times 5\pi}{60 \times \dfrac{\pi}{180}} = \underline{824.7 \text{ mm/s}}$

$$f_{max} = \frac{h}{2}\left(\frac{\pi\omega}{\varphi_a}\right)^2 \qquad \text{[refer Eq. (7.5)]}$$

or $\quad f_{max} = \dfrac{35}{2} \times \left(\dfrac{\pi \times 5\pi}{60 \times \dfrac{\pi}{180}}\right)^2$

$\qquad = 38\,862$ mm/s^2 or $\underline{38.882 \text{ m/s}^2}$

During descent

$$v_{max} = \frac{35}{2} \times \frac{\pi \times 5\pi}{90 \times \dfrac{\pi}{180}} = \underline{549.8 \text{ mm/s}}$$

$$f_{max} = \frac{35}{2} \times \left(\frac{\pi \times 5\pi}{90 \times \dfrac{\pi}{180}}\right)^2$$

$\qquad = 17\,272$ mm/s^2 or $\underline{17.272 \text{ m/s}^2}$

Example 7.3   *A cam is to give the following motion to a knife-edged follower:*

- *To raise the follower through 30 mm with uniform acceleration and deceleration during 120° rotation of the cam*
- *Dwell for next 30° of the cam rotation*
- *To lower the follower with simple harmonic motion during the next 90° rotation of the cam*
- *Dwell for the rest of the cam rotation*

The cam has a minimum radius of 30 mm and rotates counter-clockwise at a uniform speed of 800 rpm. Draw the profile of the cam if the line of stroke of the follower passes through the axis of the cam shaft. Also, draw the displacement, velocity and the acceleration diagrams for the motion of the follower for one complete revolution of the cam indicating main values.

Solution

$h = 30$ mm, $r_c = 30$ mm, $\varphi_a = 120°$,

$\delta_1 = 30°$, $\varphi_d = 90°$, $N = 800$ rpm

$\delta_2 = 360° - 120° - 30° - 90° = 120°$

(a)

(b)
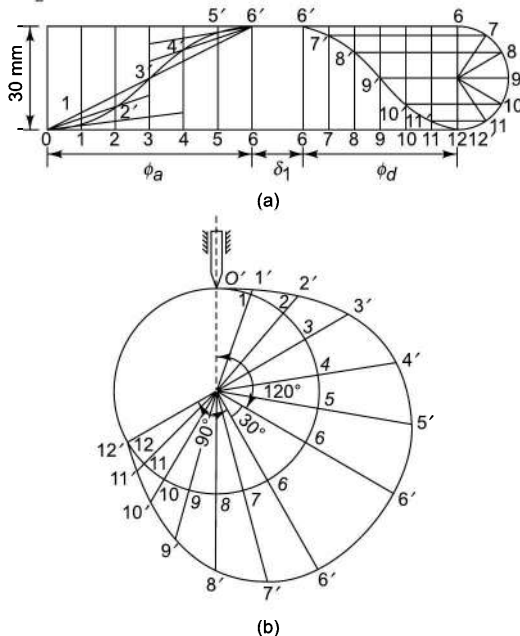
Fig. 7.25

Draw the displacement diagram of the follower as shown in Fig. 7.25(a). As the rotation of the cam shaft is counter-clockwise, the cam profile is to be drawn assuming the cam to be stationary and the follower rotating clockwise about the cam. Construct the cam profile as described below [Fig. 7.25(b)]:

(i) Draw a circle with radius r_c.

(ii) From the vertical position, mark angles φ_a, δ_1, φ_d and δ_2 in the clockwise direction.

(iii) Divide the angles φ_a and φ_d into same number of parts as is done in the displacement diagram. In this case, φ_a as well as φ_d have been divided into 6 equal parts.

(iv) On the radial lines produced, mark the distances from the displacement diagram.

(v) Draw a smooth curve tangential to end points of all the radial lines to obtain the required cam profile.
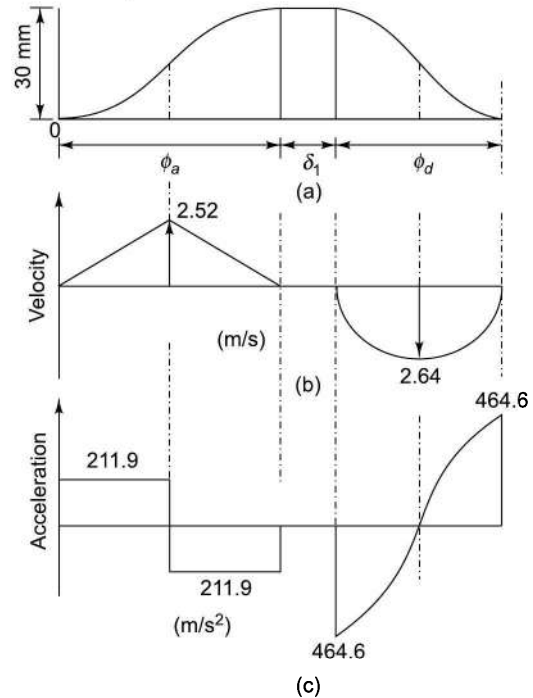
Fig. 7.26

The displacement diagram is reproduced in Fig. 7.26(a). The velocity and acceleration diagrams are to be drawn below this figure.

$$\omega = \frac{2\pi \times 840}{60} = 88 \text{ rad/s}$$

During ascent

During the ascent period, the acceleration and the deceleration are uniform. Thus, the velocity is linear and is given by

$$v = \frac{4h\omega}{\varphi_a^2} \cdot \theta \qquad \text{[Eq. (7.7a)]}$$

The maximum velocity is at the end of the acceleration period, i.e., when $\theta = \varphi_{a/2}$.

$$\therefore \quad v_{\max} = 2h \frac{\omega}{\varphi_a}$$

$$= 2 \times 0.03 \times \frac{88}{120\pi/180} = \underline{2.52 \text{ m/s}}$$

The plot of velocity variation during the ascent period is shown in Fig. 7.26(b).

$$f_{\text{uniform}} = \frac{4h\omega^2}{\varphi_a^2} \qquad \text{[Eq. (7.6a)]}$$

$$\text{or} \quad f_{\text{uniform}} = \frac{4 \times 0.03 \times 88^2}{(120\pi/180)^2} = 211.9 \text{ m/s}^2$$

This has been shown in Fig. 7.26(c).

During descent

During descent, it is simple harmonic motion. The variation of velocity is give by

$$v = \frac{h}{2} \frac{\pi\omega}{\varphi_d} \sin \frac{\pi\theta}{\varphi_d} \qquad \text{[Eq. (7.2)]}$$

Maximum value is at $\theta = \varphi_d/2$,

$$v_{\max} = \frac{h}{2} \frac{\pi\omega}{\varphi_d}$$

$$\therefore \quad = \frac{0.03}{2} \times \frac{\pi \times 88}{90\pi/180} = 2.64 \text{ mm/s}$$

The plot of velocity variation during the descent period is shown in Fig. 7.26(b).

The acceleration variation is given by,

$$f = \frac{h}{2} \left(\frac{\pi\omega}{\varphi} \right)^2 \cos \frac{\pi\theta}{\varphi} \qquad \text{[Eq. (7.4)]}$$

It is maximum at $\theta = 0$, i.e.,

$$f_{\max} = \frac{h}{2} \left(\frac{\pi\omega}{\varphi_d} \right)^2$$

$$= \frac{0.03}{2} \times \left(\frac{\pi \times 88}{90\pi/180} \right)^2 = \underline{464.6 \text{ m/s}^2}$$

This variation is shown in Fig. 7.26(c).

Example 7.4 Draw the profile of a cam operating a roller reciprocating follower and with the following data:

Minimum radius of cam = 25 mm
Lift = 30 mm
Roller diameter = 15 mm

The cam lifts the follower for 120° with SHM followed by a dwell period of 30°. Then the follower lowers down during 150° of the cam rotation with uniform acceleration and deceleration followed by a dwell period. If the cam rotates at a uniform speed of 150 rpm, calculate the maximum velocity and acceleration of the follower during the descent period.

Solution:

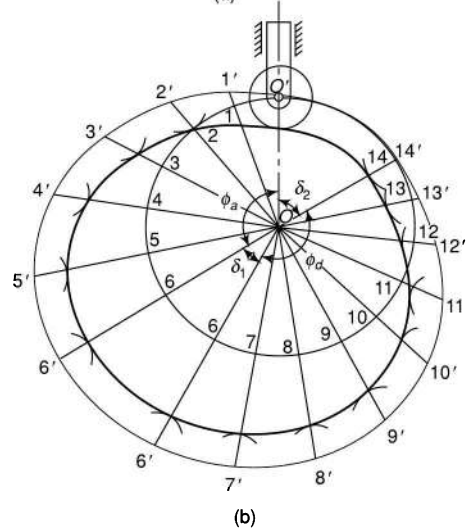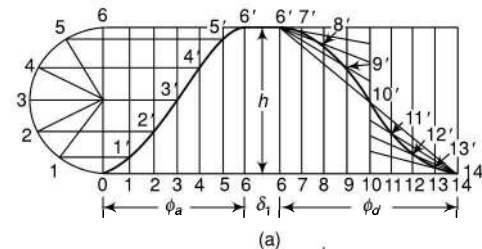|	
$h = 30$ mm	$\varphi_a = 120°$
$N = 150$ mm	$\delta_1 = 30°$
$r_c = 25$ mm	$\varphi_d = 150°$
$r_r = 7.5$ mm	$\delta_2 = 60°$

(a)

(b)

Fig. 7.27

Draw the displacement diagram of the follower as shown in Fig. 7.27(a). Construct the cam profile as described below [Fig. 7.27(b)].

(i) Draw a circle with radius $(r_c + r_r)$.

(ii) From the vertical position, mark angles φ_a, δ_1, φ_d and δ_2 in the counter-clockwise direction (assuming that the cam is to rotate in the clockwise direction).

(iii) Divide the angles φ_a and φ_d into the same number of parts as is done in the displacement diagram. In this case, φ_a has been divided into 6 equal parts whereas φ_d is divided into 8 equal parts.

(iv) On the radial lines produced, mark the distances from the displacement diagram.

(v) Draw a series of arcs of radii equal to r_r, as shown in the diagram from the points $1'$, $2'$, $3'$, etc.

(vi) Draw a smooth curve tangential to all the arcs which is the required cam profile.

During the descent period, the acceleration and the deceleration are uniform. Therefore, the maximum velocity is at the end of the acceleration period.

$$v_{max} = 2h \frac{\omega}{\varphi_d} \qquad \text{Eq. (7.8)}$$

or

$$v_{max} = 2 \times 30 \times \frac{\dfrac{2\pi \times 150}{60}}{150 \times \dfrac{\pi}{180}} = \underline{360 \text{ m/s}}$$

$$f_{max} = f_{uniform} = \frac{4h\omega^2}{\varphi_d^2} \qquad \text{Eq. (7.6)}$$

$$f_{max} = \frac{4 \times 30 \times \left(\dfrac{2\pi \times 150}{60}\right)^2}{\left(150 \times \dfrac{\pi}{180}\right)^2} = 4320 \text{ mm/s}^2$$

or $\underline{4.32 \text{ m/s}^2}$

Example 7.5 *The following data relate to a cam profile in which the follower moves with uniform acceleration and deceleration during ascent and descent.*

Minimum radius of cam = 25 mm
Roller diameter = 7.5 mm

Lift = 28 mm
Offset of follower axis = 12 mm towards right
Angle of ascent = 60°
Angle of descent = 90°
Angle of dwell between ascent and descent = 45°
Speed of the cam = 200 rpm

Draw the profile of the cam and determine the maximum velocity and the uniform acceleration of the follower during the outstroke and the return stroke.

Solution:

$h = 28$ mm $\qquad \varphi_a = 60°$

$r_c = 25$ mm $\qquad \delta_1 = 45°$

$r_r = 7.5$ mm $\qquad \varphi_d = 90°$

offset, $x = 12$ mm $\qquad \delta_2 = (360° - 60° - 45° - 90°)$

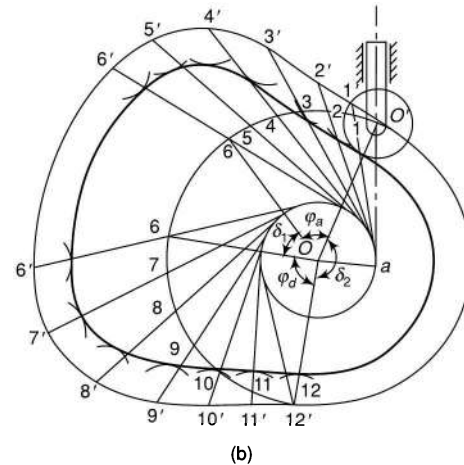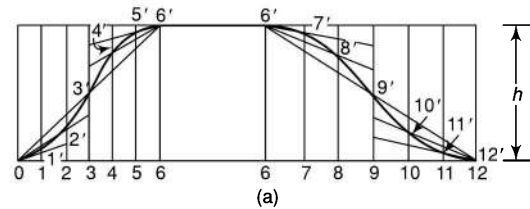$N = 200$ mm $\qquad = 165°$

(a)

(b)

Fig. 7.28

From the given data, construct the displacement diagram as usual [Fig. 7.28(a)]. For the cam profile [Fig. 7.28(b)], the procedure is as follows:

(i) Draw a circle with radius $(r_c + r_r)$.

(ii) Draw another circle concentric with the previous circle with radius x. If the cam is assumed stationary, the follower will be tangential to this circle in all the positions. Let the initial position be a-O'.

(iii) Join O-O'. Divide the circle of radius $(r_c + r_r)$ into four parts as usual with angles φ_a, δ_1, φ_d and δ_2 starting from O-O'.

(iv) Divide the angles φ_a and φ_d into same number of parts as is done in the displacement diagram and obtain the points 1, 2, 3, etc., on the circumference of circle with radius $(r_c + r_r)$.

(v) Draw tangents to the circle with radius x from the points 1, 2, 3, etc.

(vi) On the extension of the tangent lines, mark the distances from the displacement diagram.

(vii) Draw a smooth curve through $O',1', 2'$, etc. This is the pitch curve.

(viii) With $1', 2', 3'$, etc., as centres, draw a series of arcs of radii equal to r_r.

(ix) Draw a smooth curve tangential to all the arcs and obtain the required cam profile.

During outstroke

$$\omega = \frac{2\pi \times 200}{60} = 20.94 \text{ rad/s}$$

$$v_{\max} = 2h\frac{\omega}{\varphi_a}$$

$$= 2 \times 28 \times \frac{20.94}{60 \times \pi / 180}$$

$$= 1120 \text{ mm/s or } 1.12 \text{ m/s}$$

$$f_{\text{uniform}} = \frac{4h\omega^2}{\varphi_a^2}$$

$$= \frac{4 \times 28 \times (20.94)^2}{\left(60 \times \dfrac{\pi}{180}\right)^2}$$

$$= 44\,800 \text{ mm/s}^2 \text{ or } 44.8 \text{ m/s}^2$$

During return stroke

$$v_{\max} = \frac{2h\omega}{\varphi_d}$$

$$= \frac{2 \times 28 \times 20.94}{90 \times \pi / 180}$$

$$= 747 \text{ mm/s or } 0.747 \text{ m/s}$$

$$f_{\text{uniform}} = \frac{4 \times 28 \times (20.94)^2}{\left(90 \times \dfrac{\pi}{180}\right)^2}$$

$$= 19\,900 \text{ mm/s}^2 \text{ or } 19.9 \text{ m/s}^2$$

Example 7.6 *A flat-faced mushroom follower is operated by a uniformly rotating cam. The follower is raised through a distance of 25 mm in 120° rotation of the cam, remains at rest for the next 30° and is lowered during further 120° rotation of the cam. The raising of the follower takes place with cycloidal motion and the lowering with uniform acceleration and deceleration. However, the uniform acceleration is 2/3 of the uniform deceleration. The least radius of the cam is 25 mm which rotates at 300 rpm.*

Draw the cam profile and determine the values of the maximum velocity and maximum acceleration during rising, and maximum velocity and uniform acceleration and deceleration during lowering of the follower.

Solution:

$h = 25$ mm	$\varphi_a = 120°$
$r_c = 25$ mm	$\delta_1 = 30°$
$N = 300$ mm	$\varphi_d = 120°$
	$\delta_2 = 90°$

During the return stroke, as the uniform acceleration is 2/3 of the uniform deceleration, the uniform deceleration is 3/2 of the uniform acceleration.

Let the uniform acceleration be f so that the uniform deceleration be $(3/2)f$.

Time of acceleration

Final velocity,

$v = u + ft = ft$ (initial velocity is zero)

or $\quad t = \dfrac{v}{f}$

Time of deceleration

Initial velocity is v and final velocity zero. Therefore,

$0 = v - (3/2)\,ft'$

where t' is the time of deceleration and negative sign due to deceleration.

or $$t' = \frac{2}{3} \cdot \frac{v}{f}$$

Thus, the time of deceleration is 2/3 of the time of acceleration.

Displacement

During acceleration, $s = ut + \dfrac{1}{2} ft^2$

$$= \frac{1}{2} ft^2 \qquad \text{(i)}$$

During deceleration

$$= ft\left(\frac{2}{3}t\right) - \frac{1}{2}\left(\frac{3}{2}f\right)\left(\frac{2}{3}t\right)^2$$

[as initial velocity ft and time taken $(2/3)t$]

$$= \frac{2}{3} ft^2 - \frac{1}{3} ft^2$$

$$= \frac{2}{3}\left(\frac{1}{2} ft^2\right) \qquad \text{(ii)}$$

Comparison of (i) and (ii), shows that the distance travelled during deceleration period is 2/3 of the distance travelled during acceleration.
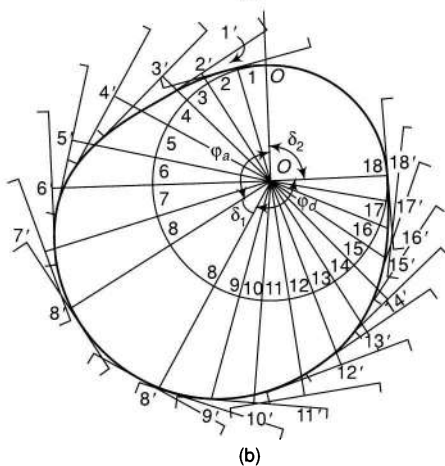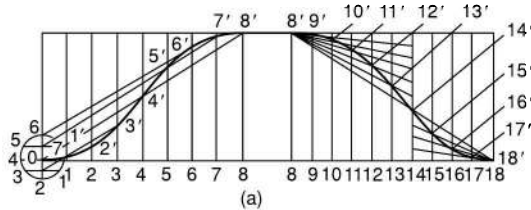

(a)

(b)

Fig. 7.29

The displacement diagram has been shown in Fig. 7.29(a). Note that during the return stroke, the time of acceleration and the displacement are 3/2 times of the corresponding values during the deceleration. (Time is measured along the *x*-axis and displacement along the *y*-axis). Thus, the time of acceleration is 3/5 of the total time of return and the displacement is 3/5 of the total displacement.

To draw the cam profile, proceed as follows [Fig. 7.29(b)]:

(i) Draw a circle with radius r_c.

(ii) Take angles φ_a, δ_1, φ_d, and δ_2 as before (in the counter-clockwise direction if the cam rotation is assumed clockwise).

(iii) Divide φ_a and φ_d into same number of parts as in the displacement diagram.

(iv) Draw radial lines and on them mark the distances 1-1', 2-2', 3-3', etc.

(v) Draw the follower in all the positions by drawing perpendiculars to the radial lines at 1', 2', 3', etc. In all the positions, the axis of the follower passes through the centre *O*.

(vi) Draw a curve tangential to the flat-faces of the follower representing the cam profile.

Remember that in case of mushroom followers, the contact point will rarely lie on the axis of the follower.

During ascent

$$v_{max} = \frac{2h\omega}{\varphi_a} \qquad \text{Eq. (7.14)}$$

where $\omega = \dfrac{2\pi \times 300}{60} = 31.4$ rad/s

$$v_{max} = \frac{2 \times 25 \times 31.4}{120 \times \pi / 180} = 750 \text{ mm/s or } 0.75 \text{ m/s}$$

$$f_{max} = \frac{2h\pi\omega^2}{\varphi_a^2} \qquad \text{Eq. (7.16)}$$

$$f_{max} = \frac{2 \times 25 \times \pi \times (31.4)^2}{(120 \times \pi / 180)^2}$$

$$= 35\,310 \text{ mm/s}^2 \text{ or } \underline{35.31 \text{ m/s}^2}$$

During descent

$$v = ft \qquad \text{Eq. (7.7)}$$

or $v = \left(\dfrac{2s}{t^2}\right)t = \dfrac{2s}{t}$ [Refer Eq. (7.6)]

v will be maximum at the end of the acceleration period. At the end of the acceleration period,

$$s = \frac{3}{5} \times 25 = 15 \text{ mm}$$

and the time taken to travel this distance is found as under,

Time for 300 rev. = 60 s

Time for 1 rev. $(= 360°) = \dfrac{60}{300} = 0.2$ s

Time for $\left(\dfrac{3}{5} \times 120°\right) = \dfrac{0.2}{360} \times 72 = 0.04$ s

$$v_{max} = \frac{2 \times 15}{0.04} = 750 \text{ mm/s}^2 \text{ or } 0.075 \text{ m/s}^2$$

Uniform acceleration

$$= \frac{v_{max}}{time} = \frac{0.75}{0.04} = 18.75 \text{ m/s}^2$$

Uniform deceleration

$$= 18.75 \times \frac{3}{2} = 28.13 \text{ m/s}^2$$

Example 7.7 *The following data relate to a cam operating an oscillating roller follower:*

Minimum radius of cam	*= 44 mm*
Diameter of roller	*= 14 mm*
Length of the follower arm	*= 40 mm*
Distance of fulcrum centre from cam centre	*= 50 mm*
Angle of ascent	*= 75°*
Angle of descent	*= 105°*
Angle of dwell for follower in the highest position	*= 60°*
Angle of oscillation of follower	*= 28°*

Draw the profile of the cam if the ascent and descent both take place with SHM.

Solution:

$r_c = 22$ mm	$\theta = 28°$
$r_r = 7$ mm	$\varphi_a = 75°$
Follower arm length $= 40$ mm	$\delta_1 = 60°$
$h = \theta \times$ arm length	$\varphi_d = 105°$
$= \left(28° \times \dfrac{\pi}{180°}\right) \times 40$	$\delta_2 = 120°$
$= 19.5$ mm	
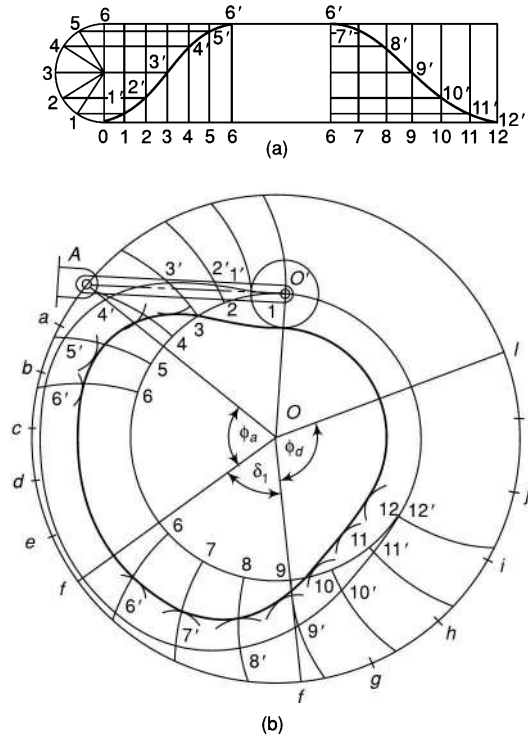
Fig. 7.30

The displacement diagram has been shown in Fig. 7.30(a). To draw the cam profile, proceed as follows:

(i) Draw a circle with radius $(r_c + r_r)$ [Fig. 7.30(b)].

(ii) Assuming the initial position of the roller centre vertically above the cam centre O, locate the fulcrum centre as its distances from the cam centre and the roller centre (equal to length of follower arm) arc known.

(iii) Draw a circle with radius OA and centre at O.

(iv) On the circle through A, starting form OA, take angles φ_a, δ_1 and φ_d as usual.

(v) Divide the angles φ_a and φ_d into same number of parts as is done in the displacement diagram and obtain the points a, b, c, d, etc., on this circle through A.

(vi) With centres A, a, b, etc., draw arcs with radii equal to length of the arm.

(vii) Mark distances 1-1', 2-2', 3-3', etc., on

these arcs as shown in the diagram. It is on the assumption that for small angular displacements, the linear displacements on the arcs and on the straight lines are the same.

(viii) With 1′, 2′, 3′, etc., draw a series of arcs of radii equal of r_r.

(ix) Draw a smooth curve tangential to all the arcs and obtain the required cam profile.

7.10 CAMS WITH SPECIFIED CONTOURS

It is always desired that a cam is made to provide a smooth motion of the follower and for that a follower motion programme is always selected first. However, sometimes it becomes difficult to manufacture the cams in large quantities of the specified contours. Under such circumstances, it becomes necessary that the cam is designed first and then some improvements are made in that if possible. Such cams are generally made up of some combination of curves such as straight lines, circular arcs, etc. In the present section, some cams with specified contours are analysed.

1. Tangent Cam (with Roller Follower)

A tangent cam is symmetrical about the centre line. It has straight flanks (such as *AK* in Fig. 7.31) with a circular nose. The centre of the cam is at *O* and that of the nose at *Q*. A tangent cam is used with a roller cam since there is no meaning of using flat-faced followers with straight flanks.

Let
r_c = least radius of cam
r_n = radius of nose
r_r = radius of roller
r = distance between the cam and the nose centres.

Fig. 7.31

Roller on the Flank When the roller is on the straight flank, the centre of the roller is at *C* on the pitch profile as shown in Fig. 7.31.

Let θ = angle turned by the cam from the beginning of the follower motion

Let,
$$x = OC - OD = OC - OB = \frac{OB}{\cos \theta} - OB = OB\left(\frac{1}{\cos \theta} - 1\right)$$

or
$$x = (r_c + r_r)\left(\frac{1}{\cos \theta} - 1\right) \tag{7.17}$$

$$v = \frac{dx}{dt} = \frac{dx}{d\theta}\frac{d\theta}{dt} = (r_c + r_r)\left(\frac{\sin \theta}{\cos^2 \theta} - 0\right)\omega$$

$$= \omega\,(r_c + r_r)\frac{\sin \theta}{\cos^2 \theta} \tag{7.18}$$

$\sin \theta$ increases with the increase in θ whereas $\cos \theta$ decreases. Hence the velocity increases with θ and it is maximum when θ is maximum. This will happen when the point of contact leaves the straight flank.

Let β = angle turned by the cam when the roller leaves the flank.

$\therefore$
$$v_{max} = \omega\,(r_c + r_r)\frac{\sin \beta}{\cos^2 \beta} \text{ and } v_{min} = 0 \text{ at } \theta = 0$$
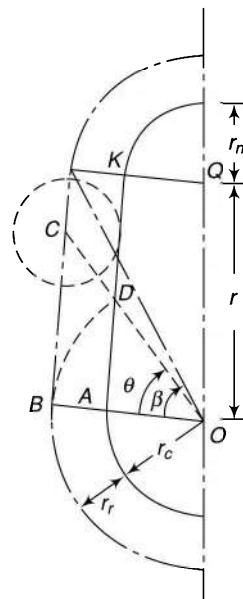
$$f = \frac{dv}{dt} = \frac{dv}{d\theta} \times \frac{d\theta}{dt}$$

$$= \omega \, (r_c + r_r) \left[\frac{\cos^2 \theta \cos \theta - \sin \theta \, (-2 \sin \theta \cos \theta)}{\cos^4 \theta} \right] \omega$$

$$= \omega^2 \, (r_c + r_r) \left[\frac{\cos^2 \theta + 2 \sin^2 \theta}{\cos^3 \theta} \right]$$

$$= \omega^2 \, (r_c + r_r) \left[\frac{2(\cos^2 \theta + \sin^2 \theta) - \cos^2 \theta}{\cos^3 \theta} \right] \qquad (7.19)$$

$$= \frac{\omega^2 \, (r_c + r_r) \, (2 - \cos^2 \theta)}{\cos^3 \theta}$$

Acceleration is minimum when $\dfrac{2 - \cos^2 \theta}{\cos^3 \theta}$ is minimum, i.e., when $(2 - \cos^2\theta)$ is minimum and $\cos^3\theta$ is maximum or when $\cos \theta$ is maximum.

This is possible when $\theta = 0°$ or when the roller touches the straight flank.

$$f_{\min} = \omega^2 \, (r_c + r_r) \qquad (7.20)$$

Roller on the Nose   Let the centre of the roller be at the point C on the pitch profile over the nose. Let QN be the perpendicular to OC (Fig. 7.32).

$$x = OC - OD$$

$$= (ON + NC) - OB$$

$$= OQ \cos \varphi + CQ \cos \psi - OB$$

$$= OQ \cos \varphi + CQ\sqrt{1 - \sin^2 \psi} \; - OB$$

$$= OQ \cos \varphi + CQ\sqrt{1 - \frac{(NQ)^2}{(CQ)^2}} \; - OB$$

$$= r \cos (\alpha - \theta) + \sqrt{l^2 - r^2 \sin^2 (\alpha - \theta)} \; - n \qquad (7.21)$$

where $\qquad l = CQ = r_n + r_r$

and $\qquad n = OB = r_c + r_r$

$$v = \frac{dx}{dt} = \frac{dx}{d\theta} \frac{d\theta}{dt}$$

$$= \left[\frac{d}{d\theta} \{ r \cos (\alpha - \theta) + \sqrt{(l^2 - r^2 \sin^2(\alpha - \theta))} \; - n \} \right] \omega$$

$$= \omega \left[-r \sin (\alpha - \theta) (-1) + \frac{1}{2} \frac{r^2 \, 2 \sin (\alpha - \theta) \cos (\alpha - \theta)}{\sqrt{(l^2 - r^2 \sin^2(\alpha - \theta))}} \right]$$

$$= \omega r \left[\sin (\alpha - \theta) + \frac{r \sin 2 (\alpha - \theta)}{2\sqrt{(l^2 - r^2 \sin^2(\alpha - \theta))}} \right] \qquad (7.22)$$

Fig. 7.32

$$f = \frac{dv}{dt} = \frac{dv}{d\theta}\frac{d\theta}{dt} = \omega r \frac{d}{d\theta}\left[\sin(\alpha-\theta) + \frac{r}{2}\frac{\sin 2(\alpha-\theta)}{\sqrt{(l^2-r^2\sin^2(\alpha-\theta))}}\right]\omega$$

$$= \omega^2 r \frac{d}{d\theta}\left[\sin(\alpha-\theta) + \frac{\omega^2 r^2}{2}\frac{d}{d\theta}[\sin 2(\alpha-\theta)]\{l^2-r^2\sin^2(\alpha-\theta)\}^{-1/2}\right]$$

$$= \omega^2 r[\cos(\alpha-\theta)(-1)] + \frac{\omega^2 r^2}{2}\left[\sin 2(\alpha-\theta)\left(-\frac{1}{2}\right)\right]$$

$$\{l^2-r^2\sin^2(\alpha-\theta)\}^{-3/2}(-r^2)\{2\sin(\alpha-\theta)\cos(\alpha-\theta)(-1)\}$$
$$+\{l^2-r^2\sin^2(\alpha-\theta)\}^{-1/2}2\cos 2(\alpha-\theta)(-1)]$$

$$= \omega^2 r\left[-\cos(\alpha-\theta) - \frac{r^3\sin^2 2(\alpha-\theta)}{4[l^2-r^2\sin^2(\alpha-\theta)]^{3/2}} - \frac{r\cos 2(\alpha-\theta)}{\sqrt{l^2-r^2\sin^2(\alpha-\theta)}}\right] \qquad (7.23)$$

2. Circular Arc (Convex) Cam (with Flat-faced Follower)

A circular arc cam is made up of three arcs of different radii (Fig. 7.33). In such cams, the acceleration may change abruptly at the blending points due to instantaneous change in the radius of curvature.

Follower Touching Circular Flank When the flat-faced follower touches point E on the circular flank, it is lifted through a distance AC.

Let P be the centre of the circular arc of the flank and r_f the radius of the circular flank ($= PD = PE$).

Fig. 7.33

$x = OC - OA = EF - OA = (PE - PF) - OA$

$\qquad = PE - OP\cos\theta - OA$

$\qquad = PE - (PD - OD)\cos\theta - OA$

or $\qquad x = r_f - (r_f - r_c)\cos\theta - r_c$

$\qquad = r_f - r_f\cos\theta + r_c\cos\theta - r_c$

$\qquad = r_f(1 - \cos\theta) - r_c(1 - \cos\theta)$

$\qquad = (r_f - r_c)(1 - \cos\theta) \qquad (7.24)$

$$v = \frac{dx}{dt} = \frac{dx}{d\theta}\frac{d\theta}{dt}$$
$$= [(r_f - r_c)(\sin\theta)]\omega$$
$$= \omega(r_f - r_c)\sin\theta \qquad (7.25)$$

It is zero when $\theta = 0$, i.e., when the follower starts ascending. It increases with θ and is maximum when θ is maximum or when the follower leaves the flank and $\theta = \beta$.

$$v_{\max} = \omega(r_f - r_c)\sin\beta$$

$$f = \frac{dv}{dt} = \frac{dv}{d\theta}\frac{d\theta}{dt} = \left[(r_f - r_c)\cos\theta\right]\omega = \omega^2(r_f - r_c)\cos\theta \qquad (7.26)$$
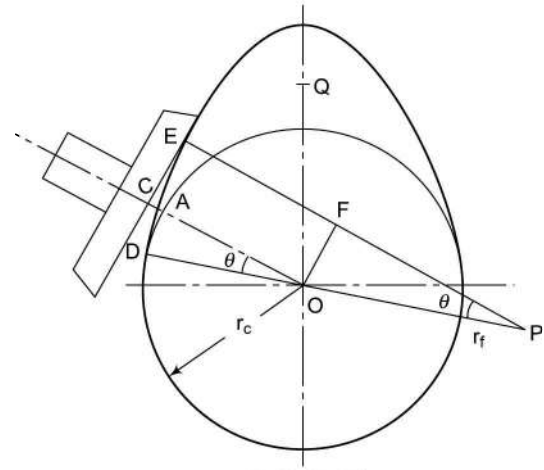
It is maximum at the beginning when $\theta = 0$, i.e when the rise commences.

$$f_{max} = \omega^2 \, (r_f - r_c)$$

$$f_{min} = \omega^2 \, (r_f - r_c) \cos \beta$$

Follower on the Nose (Fig. 7.34)

$$x = OC - OA = EF - OA$$

$$= QE + QF - OA$$

$$= QE + OQ \cos\varphi - OA$$

$$= r_n + r \cos (\alpha - \theta) - r_c$$

$$= r_n - r_c + r \cos (\alpha - \theta) \qquad (7.27)$$

$$v = \frac{dx}{dt} = \frac{dx}{d\theta} \frac{d\theta}{dt}$$

$$= [r \sin (\alpha - \theta)]\omega$$

Fig. 7.34

$$= \omega r \sin (\alpha - \theta) \qquad (7.28)$$

It is maximum when $(\alpha - \theta)$ is maximum, i.e when θ is maximum or when the follower just touches the nose of the cam.

Velocity is minimum when $(\alpha - \theta)$ is minimum or when θ is maximum, i.e., when the follower is at the apex of the circular nose or $\theta = \alpha$.

$$f = \frac{dv}{dt} = \frac{dv}{d\theta} \frac{d\theta}{dt} = \omega r \cos (\alpha - \theta)(-1)\omega = -\omega^2 r \cos (\alpha - \theta) \qquad (7.29)$$

i.e., it is retardation. It is maximum when $(\alpha - \theta)$ is minimum or when θ is maximum or when the follower is at the apex of the nose.

Similarly, it is minimum at the commencement of the nose travel.

3. Circular Arc (Convex) Cam (with Roller Follower)

Follower on the Flank (Fig. 7.35)

$$x = OC - OA = FC - FO - OB$$

$$= CP \cos\varphi - OP \cos \theta - OB$$

$$= CP \cos\varphi - (DP - DO) \cos \theta - OB$$

or $\qquad x = (r_f + r_r) \cos \varphi - (r_f + r_c) \cos \theta - (r_c + r_r)$

where, $\cos\varphi$ is given by,

$$\cos \varphi = \sqrt{1 - \sin^2 \varphi}$$

$$= \sqrt{1 - (FP / CP)^2}$$

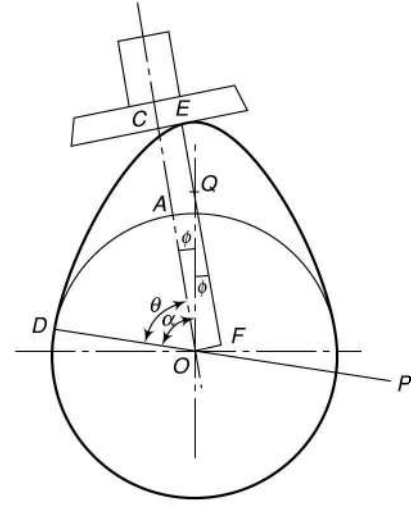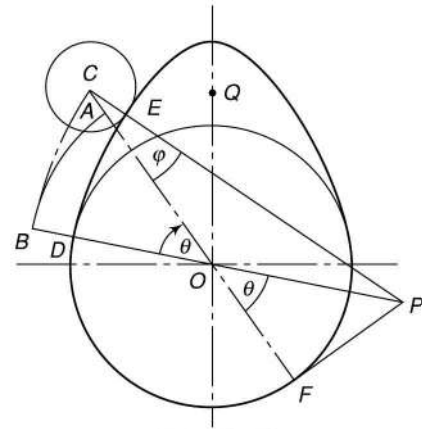$$= \sqrt{1 - \left(\frac{OP \sin \theta}{CP} \right)^2}$$

Fig. 7.35

$$= \sqrt{1 - \left[\frac{(r_f - r_c)\sin\theta}{(r_f + r_r)} \right]^2}$$

or $\quad \cos\varphi = \sqrt{1 - \left(\frac{A\sin\theta}{B} \right)^2} = \frac{1}{B}\sqrt{B^2 - A^2\sin^2\theta}$

where $A = r_f - r_c$ and $B = r_f + r_r$

$$x = \sqrt{B^2 - A^2\sin^2\theta} - A\cos\theta - (r_c + r_r) \tag{7.30}$$

$$v = \frac{dx}{dt} = \frac{dx}{d\theta}\frac{d\theta}{dt} = \left[\frac{1}{2}(B^2 - A^2\sin^2\theta)^{-1/2}(-A^2 2\sin\theta\cos\theta) + A\sin\theta \right]\omega$$

$$= \omega A\left[\sin\theta - \frac{A\sin 2\theta}{2\sqrt{B^2 - A^2\sin^2\theta}} \right] \tag{7.31}$$

$$f = \frac{dv}{dt} = \frac{dv}{d\theta}\frac{d\theta}{dt}$$

$$= \omega^2 A\left[\cos\theta - \frac{A}{2}\left\{ \frac{2\cos 2\theta}{\sqrt{B^2 - A^2\sin^2\theta}} + \sin 2\theta\left(-\frac{1}{2} \right) \atop (B^2 - A^2\sin^2\theta)^{-3/2}(-A^2)(2\sin\theta\cos\theta) \right\} \right]$$

$$= \omega^2 A\left[\cos\theta - \frac{A\cos 2\theta}{\sqrt{B^2 - A^2\sin^2\theta}} - \frac{A^3\sin 2\theta}{4(B^2 - A^2\sin^2\theta)^{3/2}} \right] \tag{7.32}$$

Follower on the Nose This case has already been discussed for the tangent cam when the roller follower is on the nose. Same expressions for the displacement, velocity and the acceleration hold good.

Example 7.8 A tangent cam with straight working faces tangential to a base circle of 120 mm diameter has a roller follower of 48-mm diameter. The line of stroke of the roller follower passes through the axis of the cam. The nose circle radius of the cam is 12 mm and the angle between the tangential faces of the cam is 90°. If the speed of the cam is 180 rpm, determine the acceleration of the follower when
(i) during the lift, the roller just leaves the straight flank
(ii) the roller is at the outer end of its lift, i.e., at the top of the nose

Solution:
$r_c = 60$ mm $\quad r_n = 12$ mm
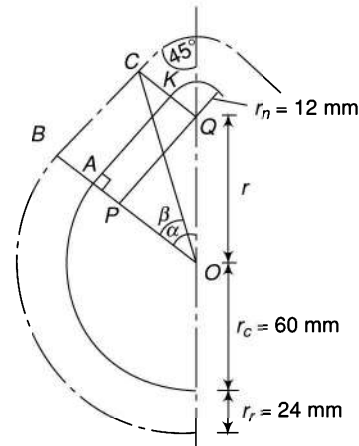$r_r = 24$ mm $\quad N = 180$ rpm

Fig. 7.36

$r_n = 12$ mm
$r_c = 60$ mm
$r_r = 24$ mm

$$\omega = \frac{2\pi \times 180}{60} = 6\pi \text{ rad/s}$$

Refer Fig. 7.36,

$\alpha = (180° - 45° - 90°) = 45°$

$OA = OP + PA$

$= OQ \cos \alpha + QK$

$r_c = r \cos \alpha + r_n$

$60 = r \cos 45° + 12$

$r = 67.9 \text{ mm}$

$\tan \beta = \dfrac{BC}{OB} = \dfrac{PQ}{OB}$

$= \dfrac{r \sin \alpha}{r_c + r_r}$

$= \dfrac{67.9 \sin 45°}{60 + 24}$

$= 0.571$

$\therefore \quad \beta = 29.74°$

(i) Acceleration when the roller just leaves the straight flank,

$f = \dfrac{\omega^2 (r_c + r_r) \ (2 - \cos^2 \theta)}{\cos^3 \theta}$

$= \dfrac{(6\pi)^2 \ (0.06 + 0.024) \ (2 - \cos^2 29.74°)}{\cos^3 29.74°}$

$= (6\pi)^2 \times 0.084 \times 1.9035 = \underline{56.8 \text{ m/s}^2}$

(ii) Acceleration when the roller is at the outer end of its lift, i.e., at the top of the nose,

$\theta = \alpha$

$f = \omega^2 r \left[-\cos(\alpha - \theta) - \dfrac{r^3 \sin^2 2(\alpha - \theta)}{4\left[l^2 - r^2 \sin^2 (\alpha - \theta)\right]^{3/2}} - \dfrac{r \cos 2(\alpha - \theta)}{\sqrt{l^2 - r^2 \sin^2 (\alpha - \theta)}} \right]$

$= \omega^2 r \left[-1 - \dfrac{r}{l} \right]$

$= (6\pi)^2 \times 0.0679 \left[-1 - \dfrac{0.0679}{0.036} \right]$

$(l = r_r + r_n = 24 + 12 = 36 \text{ mm})$

$= \underline{69.6 \text{ m/s}^2}$

Example 7.9 *A tangent cam with a base circle diameter of 50 mm operates a roller follower 20 mm in diameter. The line of stroke of the roller follower passes through the axis of the cam. The angle between the tangential faces of the cam is 60°, speed of the cam shaft is 200 rpm and the lift of the follower is 15 mm. Calculate the*

(i) *main dimensions of the cam*

(ii) *acceleration of the follower at*

(a) *the beginning of lift*

(b) *where the roller just touches the nose*

(c) *the apex of the circular nose*

Solution:

$r_c = 25 \text{ mm} \qquad h = 15 \text{ mm}$

$r_r = 10 \text{ mm} \qquad N = 200 \text{ rpm}$

$\alpha = (180° - 30° - 90°) = 60°$

(i) Refer Fig.7.37.

$r + r_n + r_r = r_c + r_r + h$

$r + r_n = r_c + h = 25 + 15 = 40 \qquad \text{(i)}$

Also, $OP + r_n = r_c$

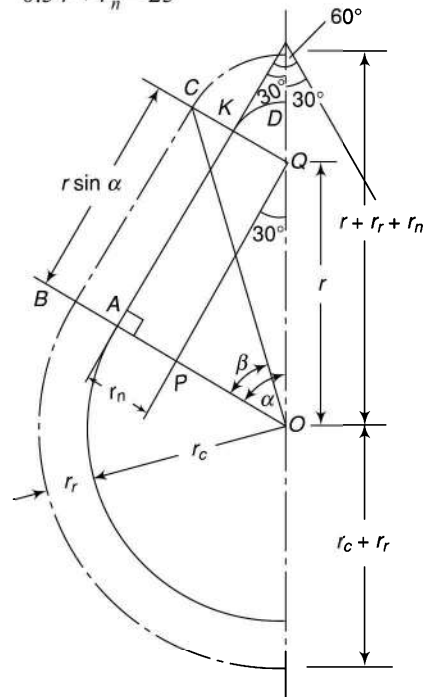or $r \cos 60° + r_n = 25$

or $0.5 \, r + r_n = 25 \qquad \text{(ii)}$

Fig. 7.37

Subtracting (ii) from (i),

$$r = 30 \text{ mm}$$
$$r_n = 10 \text{ mm}$$

$$\tan \beta = \frac{r \sin \alpha}{r_c + r_r}$$
$$= \frac{30 \sin 60°}{25 + 10} = 0.742$$
$$\therefore \qquad \beta = 36.6° \text{ or } 36°36'$$

(ii) **Acceleration of the follower**

$$\omega = \frac{2\pi \times 200}{60} = 20.94 \text{ rad/s}$$

(a) *At the beginning of Lift*, i.e., roller centre at B, $\theta = 0°$.

$$f = \frac{\omega^2 (r_c + r_r)\,(2 - \cos^2 \theta)}{\cos^3 \theta}$$
$$= \frac{(20.94)^2\,(0.025 + 0.01)\,(2 - \cos^2 0°)}{\cos^3 0°}$$
$$= 15.35 \text{ m/s}^2$$

(b) *The roller just touches the nose* ($\theta = \beta$), i.e., the roller centre at C.

When the contact is with straight flank, $\theta = 36.6° = \beta$

$$f = \frac{(20.94)^2\,(0.025 + 0.01)\,(2 - \cos^2 36.6°)}{\cos^3 36.6°}$$
$$= 40.2 \text{ m/s}^2$$

When the contact is on the circular nose,

$$l = r_n + r_r = 10 + 10 = 20 \text{ mm}$$

$$f = \omega^2 r \left[\begin{array}{c} -\cos(\alpha - \theta) \\[2mm] -\dfrac{r^3 \sin^2 2(\alpha - \theta)}{4\left[l^2 - r^2 \sin^2 (\alpha - \theta)\right]^{3/2}} \\[3mm] -\dfrac{r \cos 2(\alpha - \theta)}{\sqrt{l^2 - r^2 \sin^2 (\alpha - \theta)}} \end{array} \right]$$

$$= (20.94)^2\,(0.03)$$

$$\left[\begin{array}{c} -\cos(60° - 36.6°) \\[2mm] -\dfrac{(0.03)^3 \sin^2 2(60° - 36.6°)}{4\left[(0.02)^2 - (0.03)^2 \sin^2 (60° - 36.6°)\right]^{3/2}} \\[3mm] -\dfrac{(0.03)\cos 2(60° - 36.6°)}{\sqrt{(0.02)^2 - (0.03)^2 \sin^2 (60° - 36.6°)}} \end{array} \right]$$

$$= 13.15\,(-0.917 - 0.865 - 1.278) = \underline{-40.24 \text{ m/s}^2}$$

(c) *When the roller is at the apex of the circular nose*, i.e., at D, $\theta = \alpha$

and

$$f = \omega^2 r \left(-1 - \frac{r}{1} \right)$$
$$= (20.94)^2 \times 0.03 \left(-1 - \frac{0.03}{0.02} \right) = \underline{-32.9 \text{ m/s}^2}$$

Example 7.10   *The following data relate to a circular cam operating a flat-faced follower:*

Least diameter = 40 mm
Lift = 12 mm
Angle of action = 160°
Speed = 500 rpm

If the period of acceleration of the follower is 60° of the retardation during the lift, determine the

(i) *main dimensions of the cam*
(ii) *acceleration at the main points*

What is the maximum acceleration and deceleration during the lift?

Solution:

$$r_c = 20 \text{ mm} \qquad h = 15 \text{ mm}$$
$$2\alpha = 160° \qquad N = 500 \text{ rpm}$$
$$\alpha = 80°$$

During the lifting of the follower, the acceleration takes place when the follower is on the radial flank and the deceleration when the follower is on the nose. When the follower just touches the nose, the follower position will be as shown in Fig. 7.38. OC and PQE are parallel and the angles $\beta = 30°$ and $\varphi = 50°$ so that β is 60% of φ.
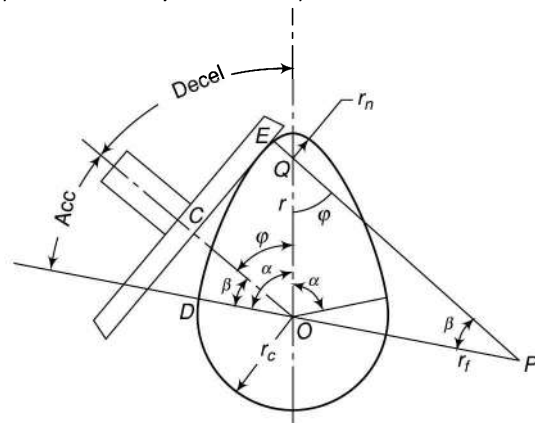

Fig. 7.38

(i) Apply sine rule to ΔPOQ,

$$\frac{OP}{\sin \varphi} = \frac{PQ}{\sin (180° - \alpha)} = \frac{OQ}{\sin \beta}$$

$$\frac{r_f - r_c}{\sin 50°} = \frac{r_f - r_n}{\sin 100°} = \frac{r}{\sin 30°}$$

But $\qquad r + r_n = r_c + h$

or $\qquad r = r_c + h - r_n$

$$= 20 + 12 - r_n$$

$$= 32 - r_n$$

$$\frac{r_f - 20}{\sin 50°} = \frac{r_f - r_n}{\sin 100°} = \frac{32 - r_n}{\sin 30°}$$

From first and last terms,

$$0.5\, r_f - 10 = 24.51 - 0.766\, r_n$$
$$0.5\, r_f = 34.51 - 0.766\, r_n$$

or $\qquad r_f = 69.02 - 1.532\, r_n$ $\qquad$ (i)

From second and last terms,

$$0.5\, r_f - 0.5\, r_n = 31.51 - 0.985\, r_n$$
$$r_f = 63.02 - 0.97\, r_n \qquad \text{(ii)}$$

From (i) and (ii),

$$69.02 - 1.532\, r_n = 63.02 - 0.97\, r_n$$
$$r_n = \underline{10.7 \text{ mm}}$$
$$r = 32.0 - 10.7 = \underline{21.3 \text{ mm}}$$
$$r_f = 69.02 - 1.532 \times 10.7 = \underline{52.6 \text{ mm}}$$

(ii) Accelerations

(a) At the beginning of contact, $\theta = 0°$,

$$f = \omega^2 (r_f - r_c) \cos 0°$$

or $\qquad f = \left(\frac{2\pi \times 500}{60}\right)^2 (52.6 - 20)$

$$= 2742 \times 32.6$$
$$= 89\,370 \text{ mm/s}^2 \text{ or } 89.37 \text{ m/s}^2$$

(b) Contact on circular flank when $\theta = \beta = 30°$,
$f = 2742 \times 32.6 \cos 30° = 77\,410 \text{ mm/s}^2 \text{ or } 77.41$ m/s^2

(c) Contact on circular nose, when $\theta = \beta = 30°$,
$f = -\omega^2 r \cos (\alpha - \theta) = -2742 \times 21.3 \cos (80° - 30°)$
$\qquad = -37\,540 \text{ mm/s}^2 \text{ or } -37.54 \text{ m/s}^2$

(d) Contact at the apex of nose, $\theta = \alpha = 80°$,

$$f = -\omega^2 r \cos (80° - 80°) = -2742 \times 21.3$$
$$= -58\,400 \text{ mm/s}^2 \text{ or } -58.4 \text{ m/s}^2$$

Maximum acceleration is when the contact is just made with the circular flank; it is 89.37 m/s^2 and the maximum retardation is at the end of the lifting period, i.e., when the contact is at the apex of the nose; it is 58.4 m/s^2.

Example 7.11 *The following data relate to a symmetrical circular cam operating a flat-faced follower:*

Minimum radius of the cam	*40 mm*
Lift	*24 mm*
Angle of lift	*75°*
Nose radius	*8 mm*
Speed of the cam	*420 rpm*

Determine the main dimensions of the cam and the acceleration of the follower at the
 (i) beginning of the lift
 (ii) end of contact with the circular flank
 (iii) beginning of contact with the nose
 (iv) apex of nose

Solution
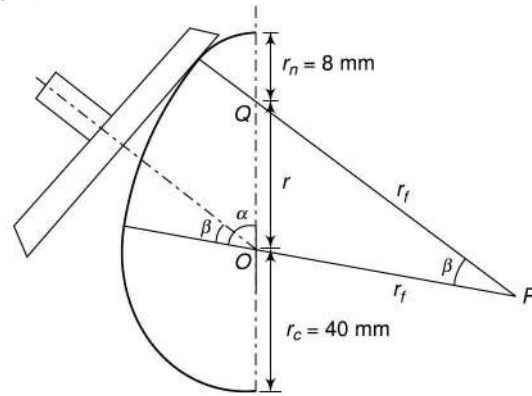
Fig. 7.39

$r_c = 40$ mm $\qquad\qquad N = 400$ rpm
$h = 24$ mm $\qquad\qquad r_n = 8$ mm
$\alpha = 75°$

$$\omega = \frac{2\pi \times 420}{60} = 44 \text{ rad/s}$$

Refer to Fig. 7.39.

$$r + r_n = r_c + h$$

or $\qquad r = 40 + 24 - 8 = 56$ mm

$$(PQ)^2 = (OP)^2 + (OQ)^2 - 2(OP)(OQ) \cos \angle POQ$$

$(r_f - 8)^2 = (r_f - 40)^2 + (56)^2 - 2(r_f - 40)(56) \cos$
$$(180° - 75°)$$

$r_f^2 + 64 - 16r_f = r_f^2 + 1600 - 80r_f$
$+ 3136 + 29r_f - 1160$

$35 \, r_f = 3512$
$r_f = 100.3$ mm
$OP = 100.3 - 40 = 60.3$ mm
$PQ = 100.3 - 8 = 92.3$ mm
Applying sine rule to ΔOPQ,

$$\frac{OQ}{\sin \beta} = \frac{PQ}{\sin (180° - \alpha)}$$

or $\quad \dfrac{r}{\sin \beta} = \dfrac{r_f - r_n}{\sin 105°}$

or $\quad \dfrac{56}{\sin \beta} = \dfrac{100.3 - 8}{\sin 105°}$

$\sin \beta = 0.586$

$\beta = 35.9°$

Acceleration when the follower is on the circular flank, $f = \omega^2 (r_f - r_c) \cos\theta$

(i) At the beginning of lift, $\theta = 0°$,

$f = \omega^2 (r_f - r_c) = 44^2 (100.3 - 40)$
$= 116\,740$ mm/s^2 = $\underline{116.74 \text{ m/s}^2}$

(ii) At the end of contact with the circular flank,

$f = \omega^2 (r_f - r_c) \cos \theta = 44^2 (100.3 - 40)$
$\cos 35.9° = 94\,565$ mm/s^2 = $\underline{94.565 \text{ m/s}^2}$
Acceleration when the follower is on the nose, $f = -\omega^2 r \cos (\alpha - \beta)$

(iii) At the beginning of contact with the nose

$f = - \omega^2 r \cos (\alpha - \beta) = - 44^2 \times 56$
$\times \cos (75° - 35.9°) = -84\,136$ mm/s^2
or $-\underline{84.136 \text{ m/s}^2}$

(iv) At the apex of the nose, $\alpha = \beta$

$f = -\omega^2 r = - 44^2 \times 56 = - 108\,416$ mm/s^2
or $\quad - \underline{108.416 \text{ m/s}^2}$

Example 7.12 *In a four-stroke petrol engine, the exhaust valve opens 45° before the t.d.c. and closes 15° after the b.d.c. The valve has a lift of 12 mm. The least radius of the circular-arc-type cam operating a flat-faced follower is 25 mm. The nose radius is 3 mm.*

The camshaft rotates at 1500 rpm. Calculate the maximum velocity of the valve and the minimum force exerted by the spring to overcome the inertia of the moving parts that weigh 300 g.

Solution:

$r_c = 25$ mm $\qquad N = 1500$ rpm
$h = 12$ mm $\qquad r_n = 3$ mm
$m = 0.3$ kg

Crank rotation during of the exhaust valve = 45° + 180° + 15° = 240°

In four-stroke engines, the camshaft speed is half that of the crankshaft.

Angle of action of the camshaft,

$$2\alpha = \frac{240}{2} = 120°$$

$\therefore \qquad \alpha = \dfrac{120}{2} = 60°$

Refer to Fig. 7.37,

$r + r_n = r_c + h$
or $\qquad r = 25 + 12 - 3 = 34$ mm

$(PQ)^2 = (OP)^2 + (OQ)^2 - 2 \, (OP) \, (OQ) \cos \angle POQ$
$(r_f - 3)^2 = (r_f - 25)^2 + (34)^2 - 2 \, (r_f - 25) \, (34)$
$$\cos (180° - 60°)$$
$r_f^2 + 9 - 6r_f = r_f^2 + 625 - 50r_f + 1156 + 34r_f - 850$
$r_f = 92.2$ mm

Applying sine rule to ΔOPQ,

$$\frac{OQ}{\sin \beta} = \frac{PQ}{\sin (180° - \alpha)}$$

$\dfrac{r}{\sin \beta} = \dfrac{r_f - r_n}{\sin 120°}$

$\dfrac{34}{\sin \beta} = -\dfrac{92.2 - 3}{\sin 120°}$

$\sin \beta = 0.33$
$\alpha = 19.27°$

Velocity is maximum when the contact is on the point where the circular flank meets the circular nose.

$v_{max} = \omega (r_f - r_c) \sin \beta$

$= \dfrac{2\pi \times 1500}{60} (92.2 - 25) \sin 19.27°$

$= 157.08 \times 67.2 \times 0.33$

$= 3480$ mm/s or 3.48 m/s

The same result is obtained if relation of Eq. (7.28) is used, i.e.,

$$v_{max} = \omega r \sin (\alpha - \theta)$$

Maximum acceleration is when $\theta = 0$,

$$f_{max} = \omega^2 (r_f - r_c) = (157.08)^2 (92.2 - 25)$$
$$= 1658\,090 \text{ mm/s}^2 \text{ or } 1658.09 \text{ m/s}^2$$

Maximum retardation is when $\alpha - \theta = 0$,

$$f_{max} = \omega^2 r = (157.08)^2 \times 34 = 838\,920 \text{ mm/s}^2 \text{ or } 838.92 \text{ m/s}^2$$

Spring force is needed to maintain contact during the retardation of the follower.

Minimum force, $F = m \times f$
$$= 0.3 \times 838.92 = 251.7 \text{ N}$$

7.11   ANALYSIS OF A RIGID ECCENTRIC CAM

Analysis of a rigid eccentric cam involves the determination of the contact force, the spring force and the cam shaft torque for one revolution of the cam. In simplified analysis, all the components of the cam system are assumed to be rigid and the results are applicable to low-speed systems. However, if the speeds are high and the members are elastic, an elastic body analysis must be made. The elasticity of the members may be due to extreme length of the follower or due to use of elastic materials in the system. In such cases, noise, excessive wear, chatter, fatigue failure of some of the parts are the usual things.

(a)   (b)

Fig. 7.40

A circular disc cam with the cam shaft hole drilled off centre is known as an *eccentric plate cam*. Figure 7.40(a) shows a simplified reciprocating eccentric cam system consisting of a plate cam, a flat face follower and a retaining spring.

Let e = eccentricity, the distance between the centre of the disc and of the shaft
          m = mass of the follower
          s = stiffness of the retaining spring
          ω = angular velocity of the cam rotation
          P = preload including the weight of the follower or the force on the cam at $x = 0$
 x = motion of the follower (zero at the bottom of the stroke)

If the disc is rotated through an angle θ [Fig. 7.40(b)], the mass m is displaced by a distance x, so that

$$x = e - e \cos \theta \tag{7.33}$$

Velocity, $\dot{x} = \dfrac{dx}{dt} = \dfrac{dx}{d\theta} \dfrac{d\theta}{dt} = e\omega \sin \theta$ (7.34)

Acceleration , $\ddot{x} = \dfrac{d\dot{x}}{dt} = \dfrac{d\dot{x}}{d\theta} \dfrac{d\theta}{dt} = e\omega^2 \cos \theta$ (7.35)

Now, as the follower is displaced in the upward direction, the acceleration $\ddot{x}$ of the follower is in the upward direction indicating the inertia force to be in the downward direction. The spring force is also exerted in the downward direction. The force exerted by the cam on the follower F, however, will be in the upward direction.
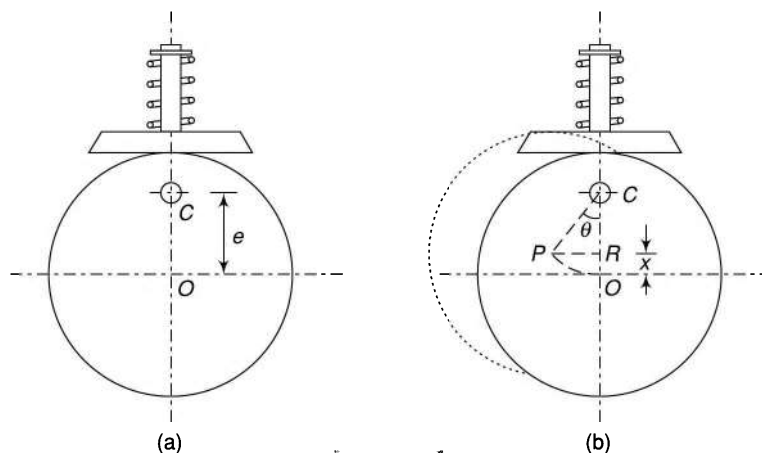
Thus, the various forces acting on the follower mass are

Inertia force	$= m\ddot{x}$	(downwards)
Spring force	$= sx$	(downwards)
Preload	$= P$	(downwards)
Force exerted by cam	$= F$	(upwards)

Thus the equilibrium equation becomes,

$$m\ddot{x} + sx + P - F = 0$$

$$F = m\ddot{x} + sx + P$$

$$= me\omega^2 \cos \omega t + se - se \cos \omega t + P$$

$$= (se + P) + (m\omega^2 - s)\, e \cos \omega t \tag{7.36}$$
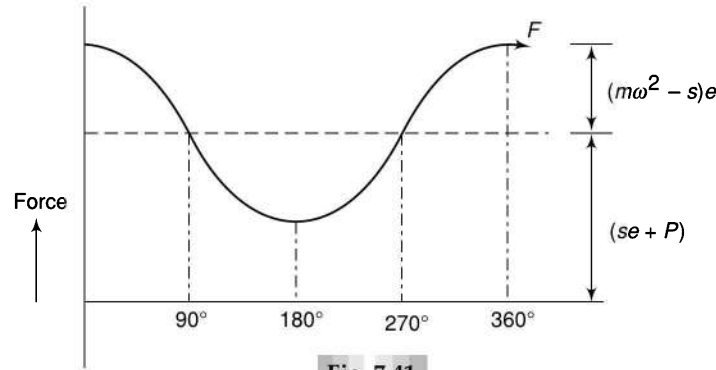

Fig. 7.41

Equation 7.36 shows that the force F exerted by a cam on the follower consists of a constant term $(se + P)$ with a cosine wave superimposed on it, the maximum value of which occurs at $\theta = 0°$ and minimum at $\theta = 180°$. Figure 7.41 shows the change of this force with the angular displacement of the cam. As the cam shaft velocity increases, the term involving square of the velocity increases at a faster rate and the force F becomes zero at a speed when

$$(se + P) + (m\omega^2 - s)\, e \cos \omega t = 0 \tag{7.37}$$

This can happen at $\theta = 180°$. At that speed, there will be some impact between the cam and the follower, resulting in a rattling, clicking and noisy operation. This is usually known as *jump*. However, this can be prevented to some extent by increasing the preload exerted by the spring or the spring stiffness.

At jump speed,

$$(se + P) - (m\omega^2 - s)\, e = 0$$

or

$$2se + P - me\omega^2 = 0$$

or

$$\omega = \sqrt{\frac{2se + P}{me}} \tag{7.38}$$

and jump will not occur if preload of the spring is increased such that $P > e(m\omega^2 - 2s)$

The torque applied by the shaft to the cam,

$$T = F.\, e \sin \omega t$$

$$= [(se + P) + (m\omega^2 - s)\, e \cos \omega t]\, e \sin \omega t$$

$$= e(se + P) \sin \omega t + \frac{e^2}{2} (m\omega^2 - s) \sin 2\omega t \tag{7.39}$$

Figure 7.42 shows the variation of torque with the cam rotation. It may be observed from the plots that area of the torque–displacement diagram above and below the *X*-axis is the same meaning that the energy required to raise the follower is recovered during the return. A flywheel may be used to handle this fluctuation of energy.
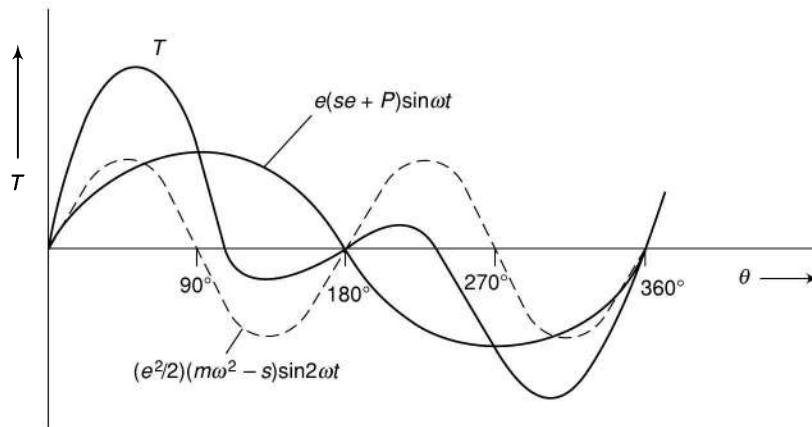

Fig. 7.42

Example 7.13 *A circular disc cam of 120 mm diameter with its centre displaced at 40 mm from the camshaft is used with a flat surface follower. The line of action of the follower is vertical and passes through the shaft axis. The mass of the follower is 3 kg and is pressed downwards with a spring of stiffness 5 N/mm. In the lowest position, the spring force is 60 N.*

Derive an expression for the acceleration of the follower as a function of cam rotation from the lowest position of the follower. Also, find the speed at which the follower begins to lift from the cam surface.

Solution:

$$e = 40 \text{ mm} \qquad m = 3 \text{ kg}$$
$$s = 5 \text{ N/mm} = 5\,000 \text{ N/m}$$
$$P = 60 \text{ N} + mg = (60 + 3 \times 9.81) \text{ N}$$

Consider the rotation of the cam through angle θ (Refer Fig. 7.43),

Now, $\quad x = 40 - 40\, e \cos \theta = 40 \,(1 - \cos \theta)$

$$\dot{x} = \frac{dx}{dt} = \frac{dx}{d\theta}\frac{d\theta}{dt} = 40\omega \sin \theta$$

$$\ddot{x} = \frac{dv}{dt} = \frac{dv}{d\theta}\frac{d\theta}{dt} = 40\omega^2 \cos \theta$$

which is the required expression for acceleration of the cam follower system.

To find the speed at which the follower begins to lift from the cam surface or the jump speed,
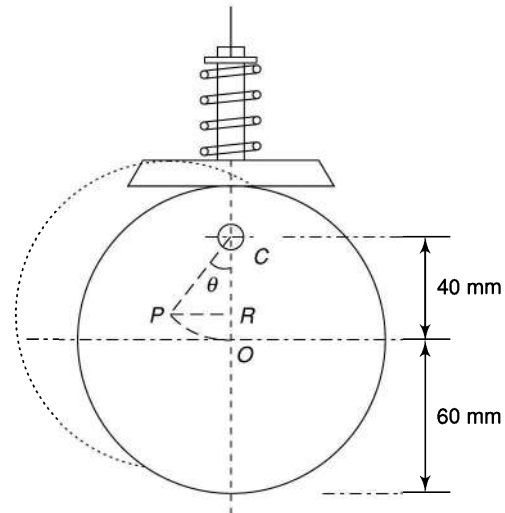

Fig. 7.43

$$\omega = \sqrt{\frac{2se + P}{me}} \qquad \text{(Eq. 7.38)}$$

$$= \sqrt{\frac{2 \times 5000 \times 0.04 + 60 + 3 \times 9.81}{3 \times 0.04}}$$

$$= 63.86 \text{ rad/s}$$

$$= \sqrt{4078.6}$$

or $\quad \dfrac{2\pi N}{60} = 63.86$

or $\quad N = \underline{609.9 \text{ rpm}}$

7.12   ANALYSIS OF AN ELASTIC CAM SYSTEM

To illustrate the effect of follower elasticity upon its displacement and velocity, consider a simplified model of a cam system with a linear motion used for low-speed cams (Fig. 7.44).

Let

    m = lumped mass of the follower

    s_1 = stiffness of the retaining spring

    s_2 = stiffness of the follower

    x = displacement of the lumped mass of the follower

    y = motion machined into the cam surface

    h = lift of the follower

Assume a cam profile that gives a uniform rise for an angle of rotation ϕ followed by a dwell. Thus,

$$y = h\,\frac{\theta}{\varphi} = h\,\frac{\omega t}{\varphi}$$
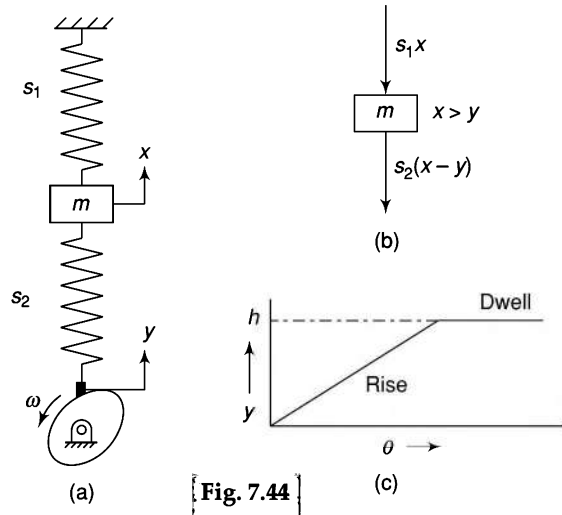

[Fig. 7.44]

As the follower is usually a rod, its stiffness s_2 is far greater than the stiffness s_1 of the spring. The spring is assembled in such a way that it exerts a preload force. The displacement x of the lumped mass is taken from the equilibrium position after the spring is assembled. In the equilibrium position, the spring and the follower exert equal and opposite preload forces on the mass.

Assuming the displacement x to be more than y, the various forces acting on the follower mass are

Inertia force   = $m\ddot{x}$ (downwards)

Spring force   = $s_1 x$ (downwards)

Force of elastic follower = $s_2(x - y)$ (downwards)

Thus

$$m\ddot{x} + s_1 x_1 + s_2(x - y) = 0$$

$$\ddot{x} + \frac{s_1 + s_2}{m}\,x = \frac{s_2}{m}\,y$$

$$\ddot{x} + \omega_n^{\,2} x = \frac{s_2}{m}\,y$$

During ascent, the displacement of the follower mass is given by the solution of the above equation, i.e.,

$$x = A\cos\omega_n t + B\sin\omega_n t + \frac{s_2}{m\omega_n^{\,2}}\,y \tag{i}$$

Differentiating with respect to t,

$$\dot{x} = -A\omega_n \sin\omega_n t + B\omega_n \cos\omega_n t + \frac{s_2}{m\omega_n^{\,2}}\,\dot{y} \tag{ii}$$

When $t = 0$, $x = \dot{x} = 0$ and $y = 0$

$\therefore$ from (i), $0 = A + 0 + 0$

or $A = 0$

From (ii),

$$0 = 0 + B\omega_n + \frac{s_2}{m\omega_n^2}\dot{y}$$

$$B = -\frac{s_2}{m\omega_n^2}\dot{y}$$

$\therefore$ (i) becomes,

$$x = 0 - \frac{s_2}{m\omega_n^2}\dot{y}\sin\omega_n t + \frac{s_2}{m\omega_n^2}y \tag{7.40}$$

The particular integral $\dfrac{s_2}{m\omega_n^2}y$ is called the *follower command*.

$$x = \frac{s_2}{m\omega_n^2}\left(y - \frac{\dot{y}}{\omega_n}\sin\omega_n t\right)$$

$$= \frac{s_2}{m\omega_n^2}\left(\frac{h}{\varphi}\theta - \frac{h\omega}{\varphi\omega_n}\sin\omega_n t\right) \cdots \left(\dot{y} = \frac{h}{\varphi}\omega\right)$$

$\therefore$

$$x = \frac{s_2 h}{m\omega_n^2\varphi}\left(\omega t - \frac{\omega}{\omega_n}\sin\omega_n t\right)$$

$$= \frac{\omega \cdot s_2 h}{m\omega_n^2\varphi}\left(t - \frac{\sin\omega_n t}{\omega_n}\right)$$

$$= \frac{s_2 h}{rm\omega_n\varphi}\left(t - \frac{\sin\omega_n t}{\omega_n}\right)$$

where $r = \omega_n/\omega$

$$\dot{x} = \frac{s_2 h}{rm\omega_n\varphi}\left(1 - \frac{\omega_n}{\omega_n}\cos\omega_n t\right)$$

$$= \frac{s_2 h}{rm\omega_n\varphi}(1 - \cos r\theta)$$

At the peak,

$$x = x_1, \quad \theta = \varphi$$

$$x_1 = \frac{s_2 h}{rm\omega_n^2\varphi}(r\varphi - \sin r\varphi)$$

$$\dot{x}_1 = \frac{s_2 h}{rm\omega_n\varphi}(1 - \cos\varphi)$$

These become the initial conditions for dwell period.

For dwell period,

$$x = A\cos\omega_n t + B\sin\omega_n t + \frac{s_2}{m\omega_n^2}h$$

$$\dot{x} = -A\omega_n \sin \omega_n t + B\omega_n \cos \omega_n t \qquad \left(\frac{s_2}{m\omega_n^{\,2}} h \text{ is constant} \right)$$

At $t = 0$, $x = x_1$, $y = h$, $\dot{x} = \dot{x}_1$

$$x_1 = A + \frac{s_2 h}{m\omega_n^{\,2}}, \qquad \therefore \qquad A = x_1 - \frac{s_2 h}{m\omega_n^{\,2}},$$

$$\dot{x}_1 = B\omega_n, \qquad \therefore \qquad B = \frac{\dot{x}_1}{\omega_n}$$

Therefore, during the dwell period,

$$x = \left(x_1 - \frac{s_2 h}{m\omega_n^{\,2}} \right) \cos \omega_n t + \frac{\dot{x}_1}{\omega_n} \sin \omega_n t + \frac{s_2}{m\omega_n^{\,2}} h \qquad (7.41)$$

Figure 2.45 shows the response of the follower x, the cam motion y, and the follower command. The follower command is different from the cam motion due to the elasticity of the follower.
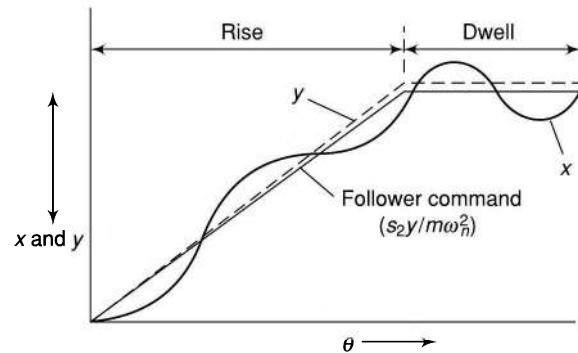

Fig. 7.45

7.13 SPRING SURGE, UNBALANCE AND WIND UP

Usually, the helical springs have a tendency to vibrate of their own when subjected to rapidly varying forces. This vibration of the retaining spring of a cam is called *spring surge*. Valve springs of the automobiles that operate near the critical frequency may keep the valve open for short periods when they are supposed to be closed. This results in lowering the efficiency of the engine apart from early fatigue failure of the springs.

If the mass of the cam is not uniformly distributed about the axis of the rotation, *unbalance* is produced on the shaft. This, along with the reaction of the follower against the cam, produce vibratory forces in a cam. Therefore, a face or end cam has better balance characteristics as compared to a disc cam.

It is observed from the plot of cam shaft torque (Fig.7.42) that it varies over a complete rotation of the cam. This tends to twist or *wind up* the shaft. During the rise of the follower, the angular cam velocity is decreased which also results in the slower follower velocity. As the follower reaches near the end of rise, the energy stored as a result of wind up is released resulting in velocity and acceleration of the follower to rise above average values. This may produce follower *jump* or impact. This phenomenon is more pronounced during the movement of heavy loads by the follower, when the follower moves at high speed or when the shaft is flexible.

Summary

1. A cam is a mechanical member used to impart desired motion to a follower by direct contact.

2. Complicated output motions which are otherwise difficult to achieve can easily be produced with the help of cams.

3. Cams are classified according to shape, follower movement, and the manner of constraint of the follower.

4. The motions of the followers are distinguished from each other by the dwells they have. A dwell is the zero displacement or the absence of motion of the follower during the motion of the cam.

5. Cam followers are classified according to shape, movement, and the location of line of movement.

6. *Base circle* is the smallest circle tangent to the cam profile (contour) drawn from the centre of rotation of a radial cam.

7. Pitch curve is the curve drawn by the trace point assuming the cam to be fixed and rotating the trace point of the follower around the cam.

8. The pressure angle, representing the steepness of the cam profile, is the angle between the normal to the pitch curve at a point and the direction of the follower motion. It varies in magnitude at all instants of the follower motion.

9. In simple harmonic motion, the maximum velocity of the follower is $v_{max} = \dfrac{h}{2} \dfrac{\pi\omega}{\varphi}$ at $\theta = \dfrac{\varphi}{2}$ and maximum acceleration

$$f_{max} = \frac{h}{2} \left(\frac{\pi\omega}{\varphi} \right)^2 \text{ at } \theta = 0° \cdot$$

10. In constant acceleration and deceleration, acceleration is given by, $f = \dfrac{4h\omega^2}{\varphi^2}$ and maximum velocity $v_{max} = \dfrac{2h\omega}{\varphi}$ at $\theta = \varphi/2$.

11. In constant velocity of the follower, the constant velocity is given by $v = \dfrac{h\omega}{\varphi}$.

12. In cycloidal motion, $v_{max} = \dfrac{2h\omega}{\varphi}$ at $\theta = \dfrac{\varphi}{2}$ and $f_{max} = \dfrac{2h\pi\omega^2}{\varphi^2}$ at $\theta = \dfrac{\varphi}{4}$

13. Cycloidal motion is the most ideal programme for high-speed follower motion.

14. A tangent cam is symmetrical about the centre line. It has straight flanks with circular nose.

15. A circular arc cam is made up of three arcs of different radii. In such cams, the acceleration may change abruptly at the blending points due to instantaneous change in the radius of curvature.

Exercises

1. What is a cam? What type of motion can be transmitted with a cam and follower combination? What are its elements?

2. How are the cams classified? Describe in detail.

3. Discuss various types of cams.

4. Compare the performance of knife-edge, roller and mushroom followers.

5. Define base circle, pitch circle, trace point, pitch curve and pressure angle.

6. What are the requirements of a high-speed cam?

7. Which follower programme do you recommend for a high-speed cam and why?

8. What is a displacement diagram? Why is it necessary to draw it before drawing a cam profile?

9. Deduce expressions for the velocity and acceleration of the follower when it moves with simple harmonic motion.

10. Why is a cycloidal motion programme most suitable for high-speed cams?

11. Explain the procedure to lay out the cam profile for a reciprocating follower.

12. What is a tangent cam? Find the expressions for the velocity and acceleration of a roller follower for such a cam.

13. Derive relations for velocity and acceleration for a convex cam with a flat-faced follower.

14. Draw the profile of a cam that gives a lift of 40 mm to a rod carrying a 20 mm diameter roller. The axis of the roller passes through the centre of the cam. The least radius of the cam is 50 mm. The rod is to be lifted with simple harmonic motion in a quarter revolution and is to be dropped suddenly at half

revolution. Determine the maximum velocity and maximum acceleration during the lifting. The cam rotates at 60 rpm.

(0.25 m/s; 3.155 m/s²)

15. Lay out the profile of a cam so that the follower
    • is moved outwards through 30 mm during 180° of cam rotation with cycloidal motion
    • dwells for 20° of the cam rotation
    • returns with uniform velocity during the remaining 160° of the cam rotation

    The base circle diameter of the cam is 28 mm and the roller diameter 8 mm. The axis of the follower is offset by 6 mm to the left. What will be the maximum velocity and acceleration of the follower during the outstroke if the cam rotates at 1500 rpm counter-clockwise?

(3 m/s; 471.2 m/s²)

16. Use the following data in drawing the profile of a cam in which a knife-edged follower is raised with uniform acceleration and deceleration and is lowered with simple harmonic motion:

    Least radius of cam = 60 mm
    Lift of follower = 45 mm
    Angle of ascent = 60°
    Angle of dwell between ascent
    and descent = 40°
    Angle of descent = 75°

    If the cam rotates at 180 rpm, determine the maximum velocity and acceleration during ascent and descent.

(*Ascent:* 1.62 m/s; 58.3 m/s²)
(*Descent:* 1.02 m/s; 46.05 m/s²)

17. Draw the profile of a cam which is to give oscillatory motion to the follower with uniform angular velocity about its pivot. The base circle diameter is 50 mm, angle of oscillation of the follower is 30° and the distance between the cam centre and the pivot of the follower is 60 mm. The oscillating lever is 60 mm long with a roller of 8-mm diameter at the end. One oscillation of the follower is completed in one revolution of the cam.

18. Set out the profile of a cam to give the following motion to a flat mushroom contact face follower:
    • Follower to rise through 24 mm during 150° of cam rotation with SHM
    • Follower to dwell for 30° of the cam rotation
    • Follower to return to the initial position during 90° of the cam rotation with SHM
    • Follower to dwell for the remaining 90° of cam rotation

    Take minimum radius of the cam as 30 mm.

19. A cam is required to give motion to a follower fitted with a roller that is 50 mm in diameter. The lift of the follower is 30 mm and is performed
    • with uniform acceleration for 12 mm, the cam turns through 45°
    • with uniform velocity for 12 mm, the cam turns through the next 30°
    • with uniform deceleration for the remainder of the lift, the cam turns through the next 45°

    The follower falls through immediately with simple harmonic motion while the cam turns through 120°. Then a period of dwell is followed for 120° of the cam angle. Construct a lift and fall diagram on a cam angle base. Also, draw the outline of the cam. The least radius of the cam is 35 mm. The line of motion of the follower passes through the centre of the cam axis.

20. Draw the profile of a cam operating a roller reciprocating follower having a lift of 35 mm. The line of stroke of the follower passes through the axis of the cam shaft. The radius of the roller is 10 mm and the minimum radius of the cam is 40 mm. The cam rotates at 630 rpm counter-clockwise. The follower is raised with simple harmonic motion for 90° of the cam rotation, dwells for next 60° and then lowers with uniform acceleration and deceleration for the next 150°. The follower dwells for the rest of the cam rotation.

    Also, draw the displacement, velocity and the acceleration diagrams for the motion of the follower for one complete revolution of the cam indicating main values.

(2.31 m/s, 304.9 m/s², 5.54 m/s, 878.2 m/s²)

21. A tangent cam with straight working faces is tangential to a base circle of 80 mm diameter. It operates a roller follower of 32 mm diameter. The line of stroke of the follower passes through the axis of the cam. The nose circle radius of the cam is 10 mm and the angle between the tangential faces of the cam is 90°. If the speed of the cam is 315 rpm, determine the acceleration of the follower when (i) during the lift, the roller just leaves the straight flank, and (ii) the roller is at the outer end of its lift, i.e., at the top of the nose.

(105.1 m/s², 799.1 m/s²)

22. The following particulars relate to a symmetrical tangent cam having a roller follower:

    Minimum radius of the cam = 40 mm
    Lift = 20 mm
    Speed = 360 rpm

Roller diameter = 44 mm

Angle of ascent = 60°

Calculate the acceleration of a follower at the beginning of the lift. Also, find its values when the roller just touches the nose and is at the apex of the circular nose. Sketch the variation of displacement, velocity and acceleration during ascent.

(88.12 m/s²; 164 m/s²; and −92.6 m/s²; −111 m/s²)

23. A flat-ended valve tappet is operated by a symmetrical cam with circular arcs for flank and nose profiles. The total angle of action is 150°, base circle diameter is 125 mm and the lift is 25 mm. During the lift, the period of acceleration is half that of the declaration. The speed of cam shift is 1250 rpm. The straight-line path of the tappet passes through the cam axis. Find
 (i) radii of the nose and the flank, and
 (ii) maximum acceleration and declaration during the lift.
    (40.3 mm, 148 mm; 1465 m/s²; 808.8 m/s²)

24. In a four-stroke petrol engine, the crank angle is 5° after t.d.c. when the suction valve opens and 53° after b.d.c. when the suction valve closes. The lift is 8 mm, the nose radius is 3 mm and the least radius of the cam is 18 mm. The shaft rotates at 800 rpm. The cam is of the circular type with a circular nose and flanks while the follower is flat-faced. Determine the maximum velocity and the maximum acceleration and retardation of the valve.

When is the minimum force exerted by the springs to overcome the inertia of moving parts weighting 250 g.

(1.3 m/s; 433.7 m/s; 161.4 m/s²; 40.35 N)

25. A symmetrical circular cam operates a flat-faced follower with a lift of 30 mm. The minimum radius of the cam is 50 mm and the nose radius is 12 mm. The angle of lift is 80°. If the speed of the cam is 210 rpm, find the main dimensions of the cam and the acceleration of the follower at (i) the beginning of the lift (ii) the end of contact with the circular flank (iii) the beginning of contact with the nose, and (iv) the apex of nose.

(*r* = 68 mm, 29.38 m/s², 23.8 m/s², 26.2 m/s², 32.9 m/s²)

26. A circular disc cam with diameter of 80 mm with its centre displaced at 30 mm from the camshaft is used with a flat surface follower. The line of action of the follower is vertical and passes through the shaft axis. The mass of the follower is 2.5 kg and is pressed downwards with a spring of stiffness 4 N/mm. In the lowest position, the spring force is 50 N. Derive an expression for the acceleration of the follower as a function of cam rotation from the lowest position of the follower. Also, find the speed at which the follower begins to lift from the cam surface.

(618.4 rpm)