1 MECHANISMS AND MACHINES

Introduction

If a number of bodies are assembled in such a way that the motion of one causes constrained and predictable motion to the others, it is known as a *mechanism*. A mechanism transmits and modifies a motion. A *machine* is a mechanism or a combination of mechanisms which, apart from imparting definite motions to the parts, also transmits and modifies the available mechanical energy into some kind of desired work. Thus, a mechanism is a fundamental unit and one has to start with its study.

The study of a mechanism involves its analysis as well as synthesis. *Analysis* is the study of motions and forces concerning different parts of an existing mechanism, whereas *synthesis* involves the design of its different parts. In a mechanism, the various parts are so proportioned and related that the motion of one imparts requisite motions to the others and the parts are able to withstand the forces impressed upon them. However, the study of the relative motions of the parts does not depend on the strength and the actual shapes of the parts.

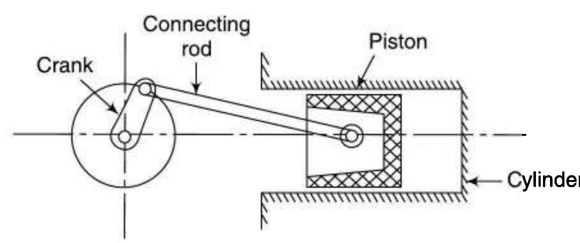

Fig. 1.1

In a reciprocating engine, the displacement of the piston depends upon the lengths of the connecting rod and the crank (Fig. 1.1). It is independent of the bearing strength of the parts or whether they are able to withstand the forces or not. Thus for the study of motions, it is immaterial if a machine part is made of mild steel, cast iron or wood. Also, it is not necessary to know the actual shape and area of the cross section of the part. Thus, for the study of motions of different parts of a mechanism, the study of forces is not necessary and can be neglected. The study of mechanisms, therefore, can be divided into the following disciplines:

Kinematics It deals with the relative motions of different parts of a mechanism without taking into consideration the forces producing the motions. Thus, it is the study, from a geometric point of view, to know the displacement, velocity and acceleration of a part of a mechanism.

Dynamics It involves the calculations of forces impressed upon different parts of a mechanism. The forces can be either static or dynamic. Dynamics is further subdivided into *kinetics* and *statics*. Kinetics is the study of forces when the body is in motion whereas statics deals with forces when the body is stationary.

1.1 MECHANISM AND MACHINE

As mentioned earlier, a combination of a number of bodies (usually rigid) assembled in such a way that the motion of one causes constrained and predictable motion to the others is known as a *mechanism*. Thus, the function of a mechanism is to transmit and modify a motion.

A *machine* is a mechanism or a combination of mechanisms which, apart from imparting definite motions to the parts, also transmits and modifies the available mechanical energy into some kind of desired work. It is neither a source of energy nor a producer of work but helps in proper utilization of the same. The motive power has to be derived from external sources.

A slider-crank mechanism (Fig. 1.2) converts the reciprocating motion of a slider into rotary motion of the crank or vice-versa. However, when it is used as an automobile engine by adding valve mechanism, etc., it becomes a machine which converts the available energy (force on the piston) into the desired energy (torque of the crank-shaft). The torque is used to move a vehicle. Reciprocating pumps, reciprocating compressors and steam engines are other examples of machines derived from the slider-crank mechanism.

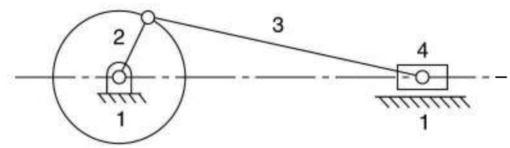

Fig. 1.2

Some other examples of mechanisms are typewriters, clocks, watches, spring toys, etc. In each of these, the force or energy provided is not more than what is required to overcome the friction of the parts and which is utilized just to get the desired motion of the mechanism and not to obtain any useful work.

1.2 TYPES OF CONSTRAINED MOTION

There are three types of constrained motion:

(i) ***Completely constrained motion*** When the motion between two elements of a pair is in a definite direction irrespective of the direction of the force applied, it is known as completely constrained motion. The constrained motion may be linear or rotary. The sliding pair of Fig. 1.3(a) and the turning pair of Fig. 1.3(b) are the examples of the completely constrained motion. In sliding pair, the inner prism can only slide inside the hollow prism. In case of a turning pair, the inner shaft can have only rotary motion due to collars at the ends. In each case the force has to be applied in a particular direction for the required motion.

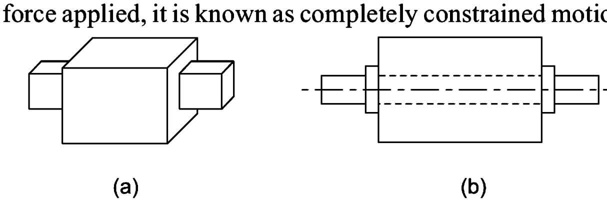

Fig. 1.3

(ii) ***Incompletely constrained motion*** When the motion between two elements of a pair is possible in more than one direction and depends upon the direction of the force applied, it is known as incompletely constrained motion. For example, if the turning pair of Fig. 1.4 does not have collars, the inner shaft may have sliding or rotary motion depending upon the direction of the force applied. Each motion is independent of the other.

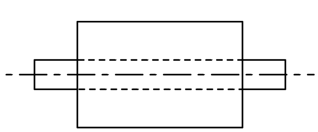
Fig. 1.4

(iii) ***Successfully constrained motion*** When the motion between two elements of a pair is possible in more than one direction but is made to have motion only in one direction by using some external means, it is a successfully constrained motion. For example, a shaft in a footstep bearing may have vertical motion apart from rotary motion (Fig. 1.5). But due to load applied on the shaft it is constrained to move in

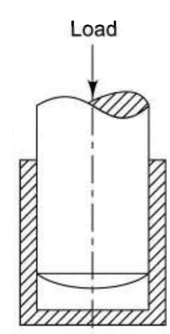

Fig. 1.5

that direction and thus is a successfully constrained motion. Similarly, a piston in a cylinder of an internal combustion engine is made to have only reciprocating motion and no rotary motion due to constrain of the piston pin. Also, the valve of an IC engine is kept on the seat by the force of a spring and thus has successfully constrained motion.

1.3 RIGID AND RESISTANT BODIES

A body is said to be *rigid* if under the action of forces, it does not suffer any distortion or the distance between any two points on it remains constant.

Resistant bodies are those which are rigid for the purposes they have to serve. Apart from rigid bodies, there are some semi-rigid bodies which are normally flexible, but under certain loading conditions act as rigid bodies for the limited purpose and thus are resistant bodies. A belt is rigid when subjected to tensile forces. Therefore, the belt-drive acts as a resistant body. Similarly, fluids can also act as resistant bodies when compressed as in case of a hydraulic press. For some purposes, springs are also resistant bodies.

These days, resistant bodies are usually referred as rigid bodies.

1.4 LINK

A mechanism is made of a number of resistant bodies out of which some may have motions relative to the others. A resistant body or a group of resistant bodies with rigid connections preventing their relative movement is known as a *link*. A link may also be defined as a member or a combination of members of a mechanism, connecting other members and having motion relative to them. Thus, a link may consist of one or more resistant bodies. A slider-crank mechanism consists of four links: frame and guides, crank, connecting-rod and slider. However, the frame may consist of bearings for the crankshaft. The crank link may have a crankshaft and flywheel also, forming one link having no relative motion of these.

A link is also known as *kinematic link* or *element*.

Links can be classified into *binary, ternary* and *quaternary* depending upon their ends on which revolute or turning pairs (Sec. 1.5) can be placed. The links shown in Fig. 1.6 are rigid links and there is no relative motion between the joints within the link.

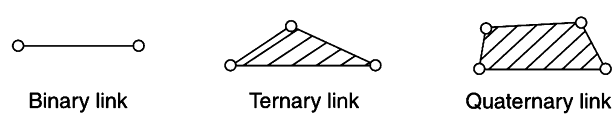

Fig. 1.6

1.5 KINEMATIC PAIR

A kinematic pair or simply a pair is a joint of two links having relative motion between them. In a slider-crank mechanism (Fig. 1.2), the link 2 rotates relative to the link 1 and constitutes a revolute or turning pair. Similarly, links 2, 3 and 3, 4 constitute turning pairs. Link 4 (slider) reciprocates relative to the link 1 and is a sliding pair.

Types of Kinematic Pairs Kinematic pairs can be classified according to

- nature of contact
- nature of mechanical constraint
- nature of relative motion

Kinematic Pairs according to Nature of Contact

(a) Lower Pair A pair of links having surface or area contact between the members is known as a lower pair. The contact surfaces of the two links are similar.

Examples Nut turning on a screw, shaft rotating in a bearing, all pairs of a slider-crank mechanism, universal joint, etc.

(b) Higher Pair When a pair has a point or line contact between the links, it is known as a higher pair. The contact surfaces of the two links are dissimilar.

Examples Wheel rolling on a surface, cam and follower pair, tooth gears, ball and roller bearings, etc.

Kinematic Pairs according to Nature of Mechanical Constraint

(a) Closed Pair When the elements of a pair are held together mechanically, it is known as a closed pair. The two elements are geometrically identical; one is solid and full and the other is hollow or open. The latter not only envelops the former but also encloses it. The contact between the two can be broken only by destruction of at least one of the members.

All the lower pairs and some of the higher pairs are closed pairs. A cam and follower pair (higher pair) shown in Fig. 1.7(a) and a screw pair (lower pair) belong to the closed pair category.

(b) Unclosed Pair When two links of a pair are in contact either due to force of gravity or some spring action, they constitute an unclosed pair. In this, the links are not held together mechanically, e.g., cam and follower pair of Fig. 1.7(b).

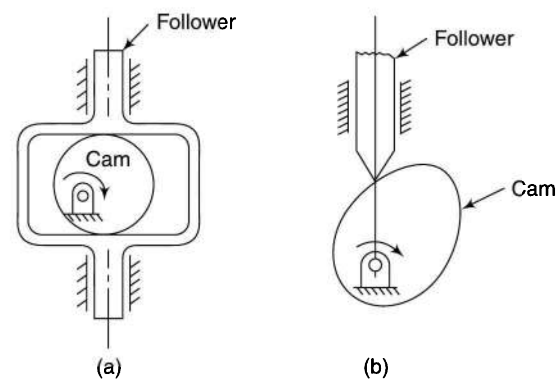

Fig. 1.7

Kinematic Pairs according to Nature of Relative Motion

(a) Sliding Pair If two links have a sliding motion relative to each other, they form a sliding pair.

A rectangular rod in a rectangular hole in a prism is a sliding pair [Fig. 1.8(a)].

(b) Turning Pair When one link has a turning or revolving motion relative to the other, they constitute a turning or revolving pair [Fig. 1.8(b)].

In a slider-crank mechanism, all pairs except the slider and guide pair are turning pairs. A circular shaft revolving inside a bearing is a turning pair.

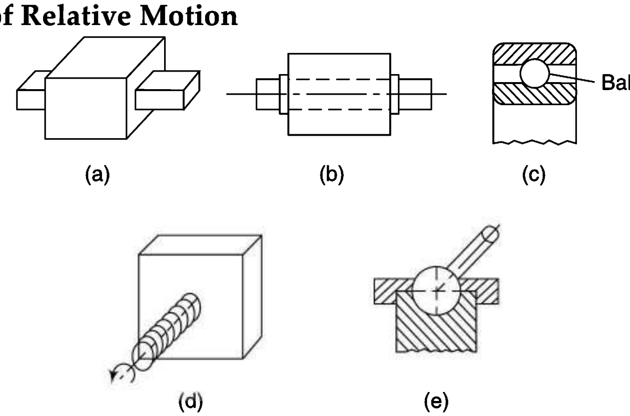

Fig. 1.8

(c) Rolling Pair When the links of a pair have a rolling motion relative to each other, they form a rolling pair, e.g., a rolling wheel on a flat surface, ball and roller bearings, etc. In a ball bearing [Fig. 1.8(c)], the ball and the shaft constitute one rolling pair whereas the ball and the bearing is the second rolling pair.

(d) Screw Pair (Helical Pair) If two mating links have a turning as well as sliding motion between them, they from a screw pair. This is achieved by cutting matching threads on the two links.

The lead screw and the nut of a lathe is a screw pair [Fig. 1.8(d)].

(e) Spherical Pair When one link in the form of a sphere turns inside a fixed link, it is a spherical pair. The ball and socket joint is a spherical pair [Fig. 1.8(e)].

1.6 TYPES OF JOINTS

The usual types of joints in a chain are

- Binary joint
- Ternary joint
- Quaternary joint

Binary Joint If two links are joined at the same connection, it is called a binary joint. For example, Fig. 1.9 shows a chain with two binary joints named *B*.

Ternary Joint If three links are joined at a connection, it is known as a ternary joint. It is considered equivalent to two binary joints since fixing of any one link constitutes two binary joints with each of the other two links. In Fig. 1.9 ternary links are mentioned as *T*.

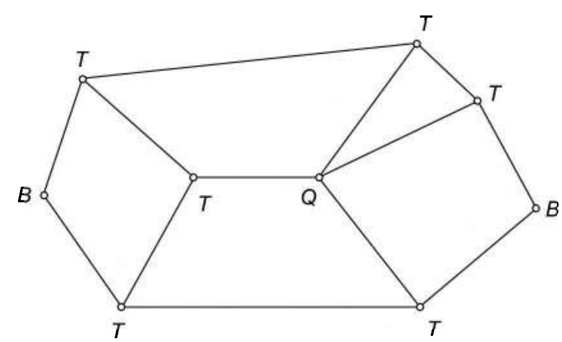

Fig. 1.9

Quaternary Joint If four links are joined at a connection, it is known as a quaternary joint. It is considered equivalent to three binary joints since fixing of any one link constitutes three binary joints. Figure 1.9 shows one quaternary joint.

In general, if n number of links are connected at a joint, it is equivalent to $(n - 1)$ binary joints.

1.7 DEGREES OF FREEDOM

An unconstrained rigid body moving in space can describe the following independent motions (Fig. 1.10):

1. Translational motions along any three mutually perpendicular axes x, y and z
2. Rotational motions about these axes

Thus, a rigid body possesses six degrees of freedom. The connection of a link with another imposes certain constraints on their relative motion. The number of restraints can never be zero (joint is disconnected) or six (joint becomes solid).

Degrees of freedom of a pair is defined as the number of independent relative motions, both translational and rotational, a pair can have.

$$\text{Degrees of freedom} = 6 - \text{Number of restraints}$$

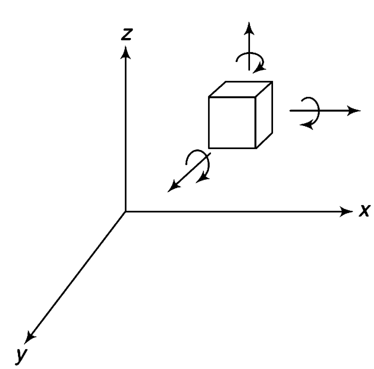

Fig. 1.10

1.8 CLASSIFICATION OF KINEMATIC PAIRS

Depending upon the number of restraints imposed on the relative motion of the two links connected together, a pair can be classified as given in Table 1.1 which gives the possible form of each class.

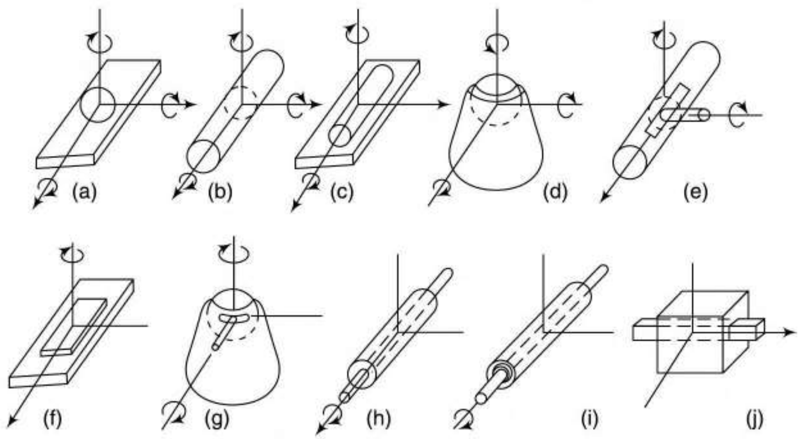

Fig. 1.11

Different forms of each class have also been shown in Fig. 1.11. Remember that a particular relative motion between two links of a pair must be independent of the other relative motions that the pair can have. A screw and nut pair permits translational and rotational motions. However, as the two motions cannot be accomplished independently, a screw and nut pair is a kinematic pair of the fifth class and not of the fourth class.

1.9 KINEMATIC CHAIN

A *kinematic chain* is an assembly of links in which the relative motions of the links is possible and the motion of each relative to the other is definite [Fig.1.12 (a), (b), and (c)].

Table 1.1

Class	Number of Restraints	Form	Restraints on Translatory motion	Restraints on Rotary motion	Kinematic pair	Fig. 1.11
I	1	1st	1	0	Sphere-plane	a
II	2	1st	2	0	Sphere-cylinder	b
		2nd	1	1	Cylinder-plane	c
III	3	1st	3	0	Spheric	d
		2nd	2	1	Sphere-slotted cylinder	e
		3rd	1	2	Prism-plane	f
IV	4	1st	3	1	Slotted-spheric	g
		2nd	2	2	Cylinder	h
V	5	1st	3	2	Cylinder (collared)	i
		2nd	2	3	Prismatic	j

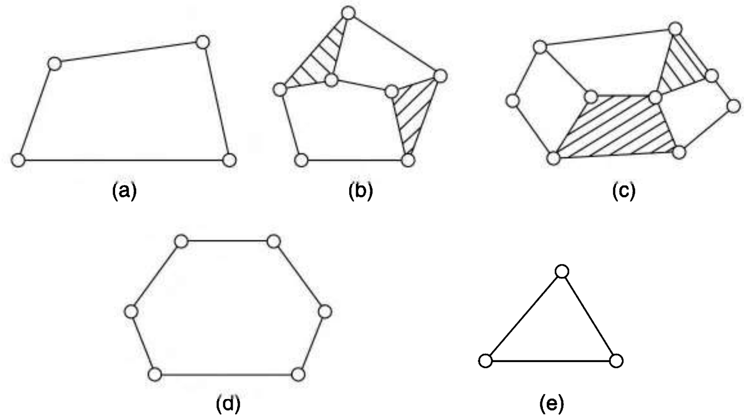

Fig. 1.12

In case the motion of a link results in indefinite motions of other links, it is a *non-kinematic chain* [Fig. 1.12(d)]. However, some authors prefer to call all chains having relative motions of the links as kinematic chains.

A *redundant chain* does not allow any motion of a link relative to the other [Fig. 1.12(e)].

1.10 LINKAGE, MECHANISM AND STRUCTURE

A *linkage* is obtained if one of the links of a kinematic chain is fixed to the ground. If motion of any of the moveable links results in definite motions of the others, the linkage is known as a *mechanism*. However, this distinction between a mechanism and a linkage is hardly followed and each can be referred in place of the other.

If one of the links of a redundant chain is fixed, it is known as a *structure* or *a locked system*. To obtain constrained or definite motions of some of the links of a linkage (or mechanism), it is necessary to know how many inputs are needed. In some mechanisms, only one input is necessary that determines the motions of other links and it is said to have one degree of freedom. In other mechanisms, two inputs may be necessary to get constrained motions of the other links and they are said to have two degrees of freedom, and so on.

The degree of freedom of a structure or a locked system is zero. A structure with negative degree of freedom is known as a *superstructure*.

1.11 MOBILITY OF MECHANISMS

A mechanism may consist of a number of pairs belonging to different classes having different number of restraints. It is also possible that some of the restraints imposed on the individual links are common or general to all the links of the mechanism. According to the number of these general or common restraints, a mechanism may be classified into a different order. A zero-order mechanism will have no such general restrain. Of course, some of the pairs may have individual restraints. A first-order mechanism has one general restraint; a second-order mechanism has two general restraints, and so on, up to the fifth order. A sixth-order mechanism cannot exist since all the links become stationary and no movement is possible.

Expressing the number of degrees of freedom of a linkage in terms of the number of links and the number of pair connections of different types is known as *number synthesis*. *Degrees of freedom* of a mechanism in space can be determined as follows:

Let

N = total number of links in a mechanism
F = degrees of freedom
P_1 = number of pairs having one degree of freedom
P_2 = number of pairs having two degrees of freedom, and so on
In a mechanism, one link is fixed.

Therefore,

Number of movable links = $N - 1$

Number of degrees of freedom of $(N - 1)$ movable links = $6(N - 1)$

Each pair having one degree of freedom imposes 5 restraints on the mechanism, reducing its degrees of freedom by $5P_1$.

Each pair having two degrees of freedom will impose 4 restraints, reducing the degrees of freedom of the mechanism by $4P_2$.

Similarly, other pairs having 3, 4 and 5 degrees of freedom reduce the degrees of freedom of the mechanism. Thus,

$$F = 6(N - 1) - 5P_1 - 4P_2 - 3P_3 - 2P_4 - P_5 \tag{1.1}$$

The above criterion is hardly necessary to find the degrees of freedom, as space mechanisms, especially of the zero order are not practical. Most of the mechanisms are two-dimensional such as a four-link or a slider-crank mechanism in which displacement is possible along two axes (one restraint) and rotation about only one axis (two restraints). Thus, there are three general restraints.

Therefore, for plane mechanisms, the following relation may be used to find the degrees of freedom

$$F = 3(N - 1) - 2P_1 - 1P_2 \tag{1.2}$$

This is known as *Gruebler's criterion* for degrees of freedom of plane mechanisms in which each movable link possesses three degrees of freedom. Each pair with one degree of freedom imposes two further restraints on the mechanisms, thus reducing its degrees of freedom. Similarly, each pair with two degrees of freedom reduces the degrees of freedom of the mechanism at the rate of one restraint each.

Some authors mention the above relation as *Kutzback's criterion* and a simplified relation [$F = 3(N-1) - 2P_1$] which is applicable to linkages with a single degree of freedom only as Gruebler's criterion. However, many authors make no distinction between Kutzback's criterion and Gruebler's criterion.

Thus, for linkages with a single degree of freedom only, $P_2 = 0$

$$F = 3(N - 1) - 2P_1 \tag{1.3}$$

Most of the linkages are expected to have one degree of freedom so that with one input to any of the links, a constrained motion of the others is obtained.

Then,

$$1 = 3(N - 1) - 2P_1$$

or

$$2P_1 = 3N - 4 \tag{1.4}$$

As P_1 and N are to be whole numbers, the relation can be satisfied only if N is even. For possible linkages made of binary links only,

$N = 4$,	$P_1 = 4$	No excess turning pair
$N = 6$,	$P_1 = 7$	One excess turning pair
$N = 8$,	$P_1 = 10$	Two excess turning pairs

and so on.

Thus, with the increase in the number of links, the number of excess turning pairs goes on increasing. Getting the required number of turning pairs from the required number of binary links is not possible. Therefore, the excess or the additional pairs or joints can be obtained only from the links having more than two joining points, i.e., ternary or quaternary links, etc.

For a six-link chain, some of the possible types are Watts six-bar chain, in which the ternary links are directly connected [Fig. 1.13(a)] and Stephenson's six-bar chain, in which ternary links are not directly connected [Fig. 1.13(b)]. Another possibility is also shown in Fig. 1.13(c). However, this chain is not a six-link chain but a four-link chain as links 1, 2 and 3 are, in fact, one link only with no relative motion of these links.

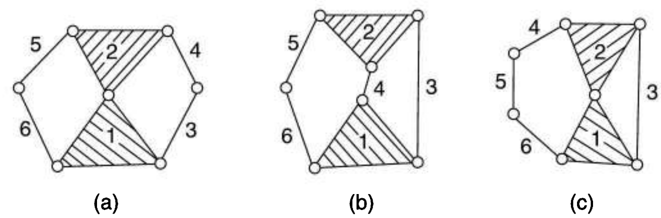

Fig. 1.13

Two excess turning pairs required for an eight-link chain can be obtained by using (apart from binary links):

four ternary links [Figs 1.14(a) and (b)]
two quaternary links [Fig.1.14(c)]
one quaternary and two ternary links [Fig. 1.14(d)].

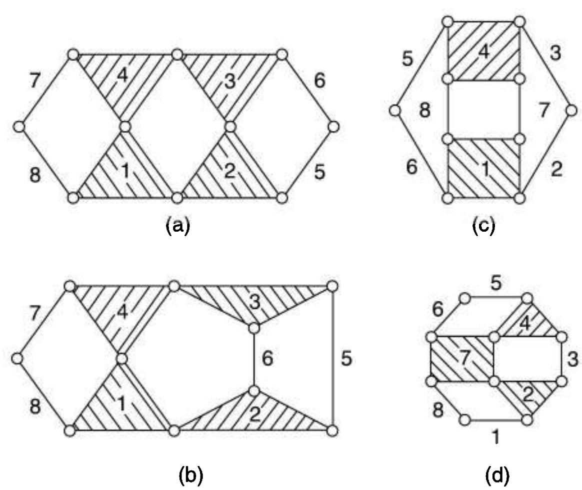

Fig. 1.14

Now, consider the kinematic chain shown in Fig. 1.15. It has 8 links, but only three ternary links. However, the links 6, 7 and 8 constitute a double pair so that the total number of pairs is again 10. The degree of freedom of such a linkage will be

$$F = 3\,(8 - 1) - 2 \times 10$$
$$= 1$$

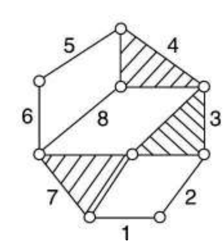

Fig. 1.15

This shows that the number of ternary or quaternary links in a chain can be reduced by providing double joints also.

The following empirical relations formulated by the author provide the degree of freedom and the number of joints in a linkage when the number of links and the number of loops in a kinematic chain are known. These relations are valid for linkages with turning pairs,

$$F = N - (2L + 1) \tag{1.5}$$

$$P_1 = N + (L - 1) \tag{1.6}$$

where

L = number of loops in a linkage.

Thus, for different number of loops in a linkage, the degrees of freedom and the number of pairs are as shown in Table 1.2.

Table 1.2

L	F	P_1
1	$N - 3$	N
2	$N - 5$	$N + 1$
3	$N - 7$	$N + 2$
4	$N - 9$	$N + 3$
5	$N - 11$	$N + 4$
and so on		

For example, if in a linkage, there are 4 loops and 11 links, its degree of freedom will be 2 and the number of joints, 14. Similarly, if a linkage has 3 loops, it will require 8 links to have one degree of freedom, 9 links to have 2 degrees of freedom, 7 links to have –1 degree of freedom, etc.

Sometimes, all the above empirical relations can give incorrect results, e.g., Fig.1.16(a) has 5 links, 6 turning pairs and 2 loops. Thus, it is a structure with zero degree of freedom. However, if the links are arranged in such a way as shown in Fig. 1.16(b), a *double parallelogram linkage* with one degree of freedom is obtained. This is due to the reason that the lengths of the links or other dimensional properties are not considered in these empirical relations. So, exceptions are bound to come with equal lengths or parallel links.

Sometimes, a system may have one or more links which do not introduce any extra constraint. Such links are known as *redundant links* and should not be counted to find the degree of freedom. For example, the mechanism of Fig. 1.16(b) has 5 links, but the function of the mechanism is not affected even if any one of the links 2, 4 or 5 are removed. Thus, the effective number of links in this case is 4 with 4 turning pairs, and thus has one degree of freedom.

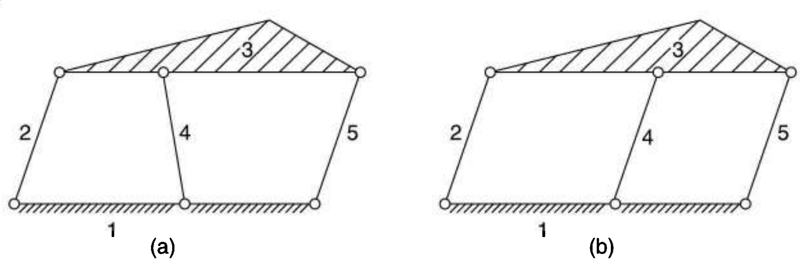

Fig. 1.16

Sometimes, one or more links of a mechanism can be moved without causing any motion to the rest of the links of the mechanism. Such a link is said to have a *redundant degree of freedom.* Thus in a mechanism, it is necessary to recognize such links prior to investigate the degree of freedom of the whole mechanism. For example, in the mechanism shown in Fig. 1.17, roller 3 can rotate about its axis without causing any movement to the rest of the mechanism. Thus, the mechanism represents a redundant degree of freedom.

In case of a mechanism possessing some redundant degree of freedom, the effective degree of freedom is given by

$$F = 3\,(N-1) - 2P_1 - 1P_2 - F_r$$

where F_r is the number of redundant degrees of freedom. Now, as the above mechanism has a cam pair, its degree of freedom must be found from Gruebler's criterion.

Total number of links = 4

Number of pairs with 1 degree of freedom = 3

Number of pairs with 2 degrees of freedom = 1

$$\begin{aligned} F &= 3\,(N-1) - 2P_1 - 1P_2 - F_r \\ &= 3\,(4-1) - 2\times 3 - 1\times 1 - 1 \\ &= 1 \end{aligned}$$

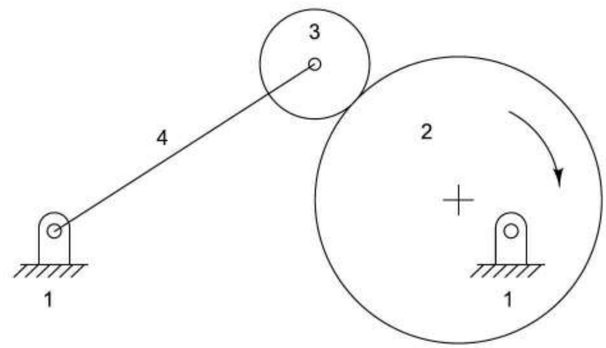

Fig. 1.17

Example 1.1 *For the kinematic linkages shown in Fig. 1.18, calculate the following:*

- *the number of binary links (N_b)*
- *the number of ternary links (N_t)*
- *the number of other (quaternary, etc.) links (N_o)*
- *the number of total links (N)*
- *the number of loops (L)*
- *the number of joints or pairs (P_1)*
- *the number of degrees of freedom (F)*

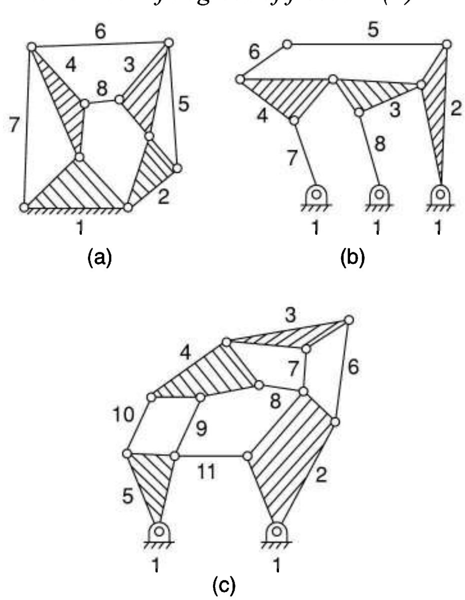

Fig. 1.18

Solution

(a) $N_b = 4;\ N_t = 4;\ N_o = 0;\ N = 8;\ L = 4$

P_1 = 11 by counting

or $P_1 = (N + L - 1) = 11$

$F = 3(N-1) - 2P_1$

$= 3\,(8-1) - 2\times 11 = -1$

or $F = N - (2L + 1)$

$= 8 - (2\times 4 + 1) = -1$

The linkage has negative degree of freedom and thus is a superstructure.

(b) $N_b = 4;\ N_t = 4;\ N_o = 0\ ;\ N = 8\ ;\ L = 3$

P_1 = 10 (by counting)

or $P_1 = (N + L - 1) = 10$

$F = N - (2L + 1) = 8 - (2\times 3 + 1) = 1$

or $F = 3(N-1) - 2P_1$

$= 3(8-1) - 2\times 10 = 1$

i.e., the linkage has a constrained motion when one of the seven moving links is driven by an external source.

(c) $N_b = 7\ ;\ N_t = 2\ ;\ N_o = 2\ ;\ N = 11$

$L = 5\ ;\ P_1 = 15$

$F = N - (2L + 1) = 11 - (2\times 5 + 1) = 0$

Therefore, the linkage is a structure.

Example 1.2 *State whether the linkages shown in Fig. 1.19 are mechanisms with one degree of freedom. If not, make suitable changes. The number of links should not be varied by more than 1.*

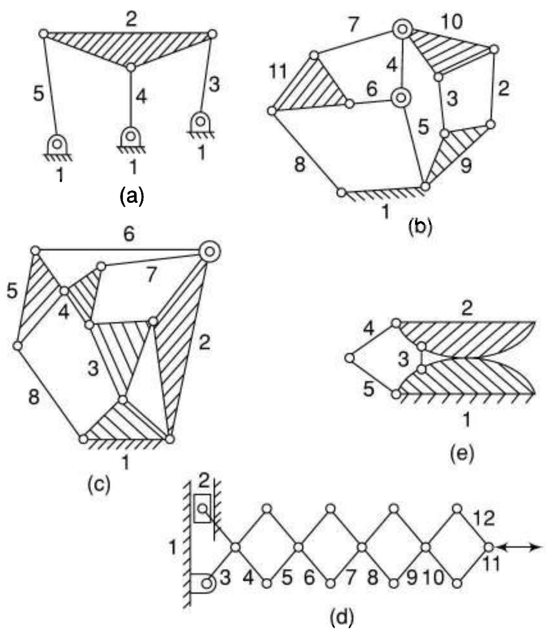

Fig. 1.19

Solution (a) The linkage has 2 loops and 5 links.

$F = N - (2L + 1) = 5 - (2 \times 2 + 1) = 0$

Thus, it is a structure. Referring Table 1.2, for a 2-loop mechanism, n should be six to have one degree of freedom. Thus, one more link should be added to the linkage to make it a mechanism of $F = 1$. One of the possible solutions has been shown in Fig. 1.20(a).

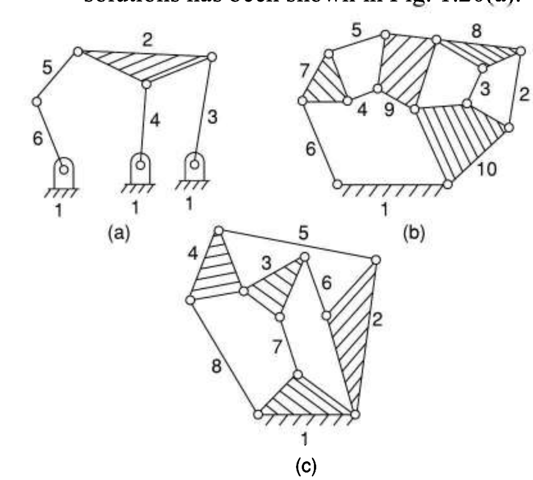

Fig. 1.20

(b) The linkage has 4 loops and 11 links. Referring Table 1.2, it has 2 degrees of freedom. With 4 loops and 1 degree of freedom, the number of links should be 10 and the number of joints 13. Three excess joints can be formed by

6 ternary links or

4 ternary links and 1 quaternary link or

2 ternary links, and 2 quaternary links, or

3 quaternary links, or

a combination of ternary and quaternary links with double joints.

Figure 1.20(b) shows one of the possible solutions.

(c) There are 4 loops and 8 links.

$F = N - (2L +) = 8 - (4 \times 2 + 1) = -1$

It is a superstructure. With 4 loops, the number of links must be 10 to obtain one degree of freedom. As the number of links is not to be increased by more than one, the number of loops has to be decreased. With 3 loops, 8 links and 10 joints, the required linkage can be designed. One of the many solutions is shown in Fig. 1.20(c).

(d) It has 5 loops and 12 links. Referring Table 1.2, it has 1 degree of freedom and thus is a mechanism.

(e) The mechanism has a cam pair, therefore, its degree of freedom must be found from Gruebler's criterion.

Total number of links = 5

Number of pairs with 1 degree of freedom = 5

Number of pairs with 2 degrees of freedom = 1

$F = 3\,(N - 1) - 2P_1 - P_2$

$= 3(5 - 1) - 2 \times 5 - 1 = 1$

Thus, it is a mechanism with one degree of freedom.

Example 1.3 *Determine the degree of freedom of the mechanisms shown in Fig. 1.21.*

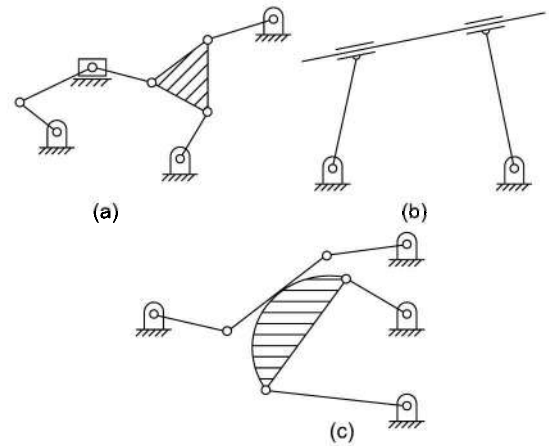

Fig. 1.21

Solution

(a) The mechanism has a sliding pair. Therefore, its degree of freedom must be found from Gruebler's criterion.

Total number of links = 8 (Fig. 1.22)

Number of pairs with 1 degree of freedom = 10

(At the slider, one sliding pair and two turning pairs)

$F = 3\,(N - 1) - 2P_1 - P_2$

$\quad = 3(8 - 1) - 2 \times 10 - 0 = 1$

Thus, it is a mechanism with a single degree of freedom.

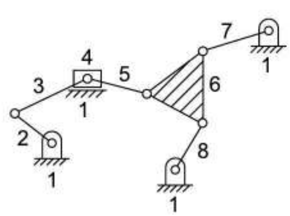

Fig. 1.22

(b) The system has a redundant degree of freedom as the rod of the mechanism can slide without causing any movement in the rest of the mechanism.

$\therefore$ effective degree of freedom

$= 3\,(N - 1) - 2P_1 - P_2 - F_r$

$= 3(4 - 1) - 2 \times 4 - 0 - 1 = 0$

As the effective degree of freedom is zero, it is a locked system.

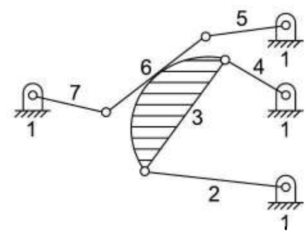

Fig. 1.23

(c) The mechanism has a cam pair. Therefore, its degree of freedom must be found from Gruebler's criterion.

Total number of links = 7 (Fig. 1.23)

Number of pairs with 1 degree of freedom = 8

Number of pairs with 2 degrees of freedom = 1

$F = 3\,(N - 1) - 2P_1 - P_2$

$\quad = 3(7 - 1) - 2 \times 8 - 1 = 1$

Thus, it is a mechanism with one degree of freedom.

Example 1.4 *How many unique mechanisms can be obtained from the 8-link kinematic chain shown in Fig. 1.24?*

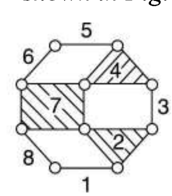

Fig. 1.24

Solution The kinematic chain has 8 links in all. A unique mechanism is obtained by fixing one of the links to the ground each time and retaining only one out of the symmetric mechanisms thus obtained.

The given kinematic chain is symmetric about links 3 or 7. Thus, identical inversions (mechanisms) are obtained if the links 2, 1, 8 or 4, 5, 6 are fixed. In addition, two more unique mechanisms can be obtained from the 8-link kinematic chain as shown in Fig. 1.25.

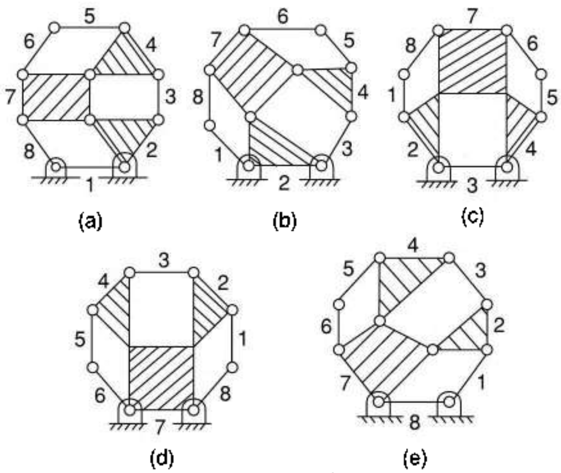

Fig. 1.25

Example 1.5 *A linkage has 11 links and 4 loops. Calculate its degree of freedom and the number of ternary and quaternary links it will have if it has only single turning pairs.*

Solution $F = N - (2L + 1) = 11 - (2 \times 4 + 1) = 2$

$P_1 = N + (L - 1) = 11 + (4 - 1) = 14$

The linkage has 3 excess joints and if all the joints are single turning pairs, the excess joints can be provided either by

- 6 ternary links or
- 4 ternary links and one quaternary link or
- 2 ternary links and two quaternary links or
- 3 quaternary links

1.12 EQUIVALENT MECHANISMS

It is possible to replace turning pairs of plane mechanisms by other types of pairs having one or two degrees of freedom, such as sliding pairs or cam pairs. This can be done according to some set rules so that the new mechanisms also have the same degrees of freedom and are kinematically similar.

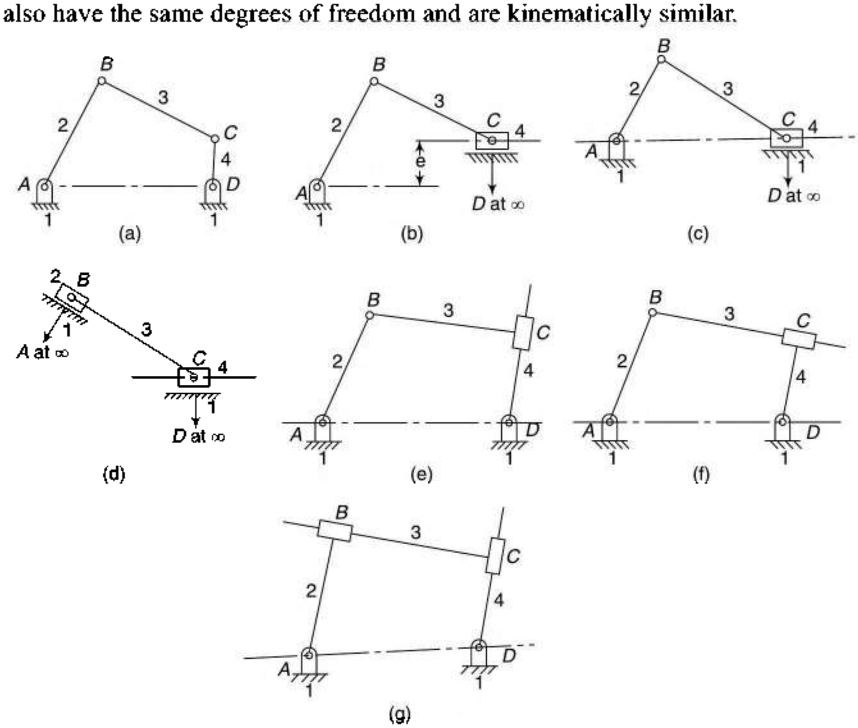

Fig. 1.26

1. Sliding Pairs in Place of Turning Pairs

Figure 1.26(a) shows a four-link mechanism. Let the length of the link 4 be increased to infinity so that *D* lies at infinity. Now, with the rotation of the link 2, *C* will have a linear motion perpendicular to the axis of the link 4. The same motion of *C* can be obtained if the link 4 is replaced by a slider, and guides are provided for its motion as shown in Fig. 1.26(b). In this case, the axis of the slider does not pass through *A* and there is an eccentricity. Figure 1.26(c) shows a slider-crank mechanism with no eccentricity. In this way, a binary link is replaced by a slider pair.

Note that the axis of the sliding pair must be in the plane of the linkage or parallel to it.

Similarly, the turning pair at *A* can also be replaced by a sliding pair by providing a slider with guides at *B* [Fig. 1.26(d)].

In case the axes of the two sliding pairs are in one line or parallel, the two sliders along with the link 3 act as one link with no relative motion among these links. Then the arrangement ceases to be a linkage. Thus, in order to replace two turning pairs in a linkage with sliding pairs, the axes of the sliding pairs must intersect.

In the same way, the turning pairs at *B* and *C* can be replaced by sliding pairs by fixing a slider to any of the two links forming the pair [Figs 1.26(e) and (f)]. Figure 1.26(g) shows both of the turning pairs at *B* and *C* replaced by sliding pairs.

2. Spring in Place of Turning Pairs

The action of a spring is to elongate or to shorten as it becomes in tension or in compression. A similar variation in length is accomplished by two binary links joined by a turning pair. In Fig. 1.27(a), the length *AB* varies as *OB* is moved away or towards point *A*. Figure 1.27(b) shows a 6-link mechanism in which links 4 and 5 have been shown replaced by a spring.

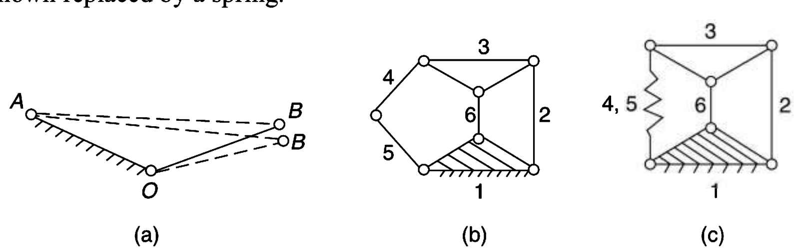

Fig. 1.27

Remember that the spring is not a rigid link but is simulating the action of two binary links joined by a turning pair. Therefore, to find the degree of freedom of such a mechanism, the spring has to be replaced by the binary links.

3. Cam Pair in Place of Turning Pair

A cam pair has two degrees of freedom. For linkages with one degree of freedom, application of Gruebler's equation yields,

$$F = 3(N-1) - 2P_1 - 1P_2$$

or $$1 = 3N - 3 - 2P_1 - 1 \times 1$$

or $$P_1 = \frac{3N-5}{2}$$

This shows that to have one cam pair in a mechanism with one degree of freedom, the number of links and turning pairs should be as below:

$N = 3, \quad P_1 = 2$
$N = 5, \quad P_1 = 5$
$N = 7, \quad P_1 = 8$
$N = 9, \quad P_1 = 11$ and so on.

A comparison of this with linkages having turning pairs only (Table 1.2) indicates that a cam pair can be replaced by one binary link with two turning pairs at each end.

Figure 1.28(a) shows link *CD* (of a four-link mechanism) with two turning pairs at its ends replaced by a cam pair. The centres of curvatures at the point of contact *X* of the two cams lie at *D* and *C*. Figures 1.28(b) and (c) show the link *BC* with turning pairs at *B* and *C* replaced by a cam pair. The centres of curvature at the point of contact *X* lie at *B* and *C* respectively. Figure 1.28(d) shows equivalent mechanism for a disc cam with reciprocating curved-face follower. The centres of curvature of the cam and the follower at the instant lie at *A* and *B* respectively.

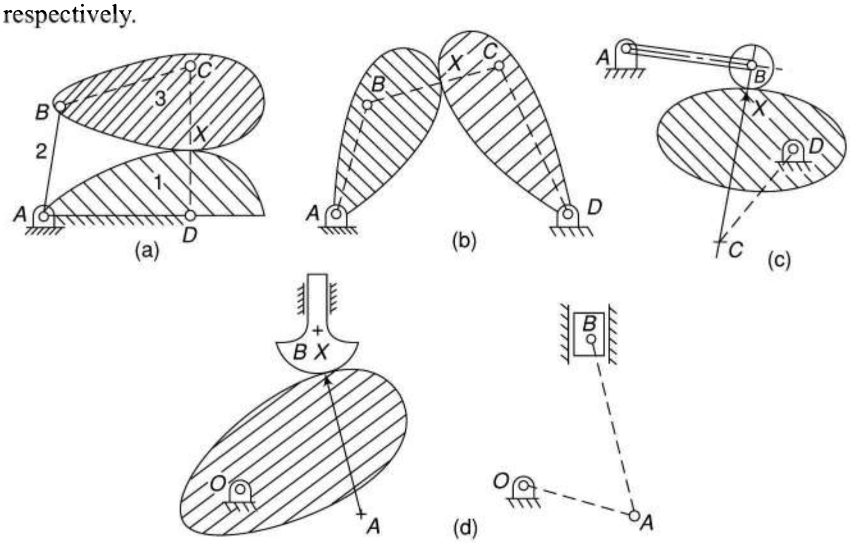

Fig. 1.28

Example 1.6 *Sketch a few slider-crank mechanisms derived from Stephenson's and Watt's six-bar chains.*

Solution Figure 1.29(a) shows a Stephenson's chain in which the ternary links are not directly connected. Thus, any of the binary links 3 or 6 can be replaced by a slider to obtain a slider-crank mechanism as shown in Fig. 1.29(b) and (c).

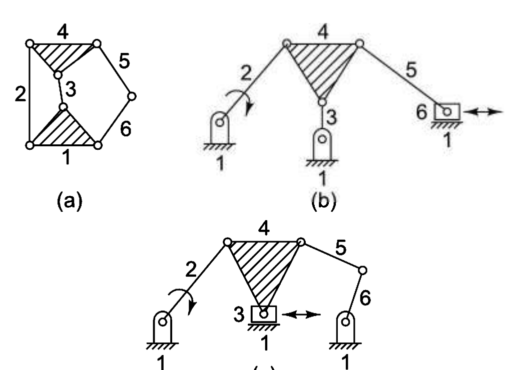

Fig. 1.29

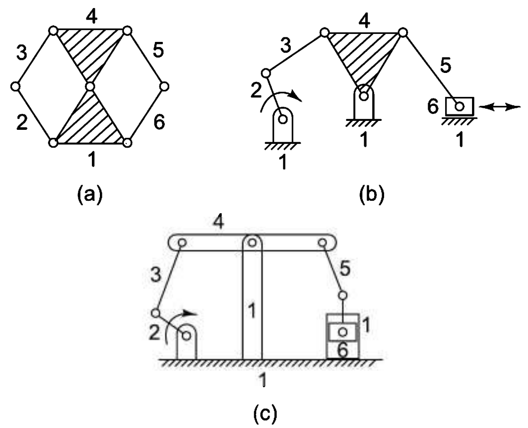

Fig. 1.30

Figure 1.30(a) shows a Watt's chain in which the ternary links are directly connected. Thus, any of the binary links 2 or 6 can be replaced by a slider to obtain a slider-crank mechanism. Figure 1.30 (b) and (c) show two variations of the slider obtained by replacing the binary link 6. The slider-crank mechanism of Fig. 1.30(c) is known as *beam engine.*

Example 1.7 *Sketch the equivalent kinematic chains with turning pairs for the chains shown in Fig. 1.31.*

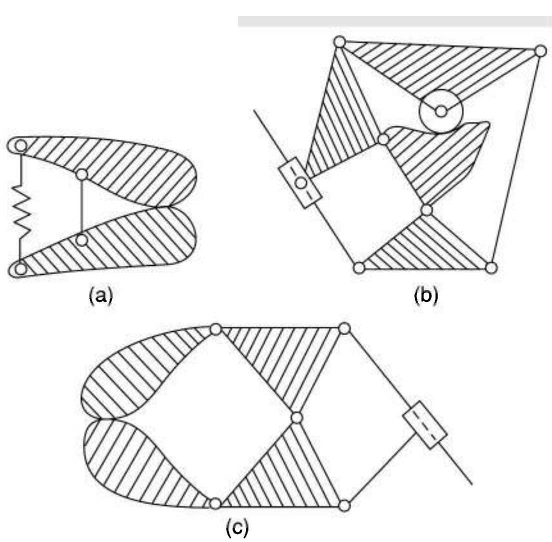

Fig. 1.31

Solution

(a) A spring is equivalent to two binary links connected by a turning pair. A cam pair is equivalent of one binary link with turning pairs at each end. The equivalent chain with turning pairs is shown in Fig. 1.32(a).

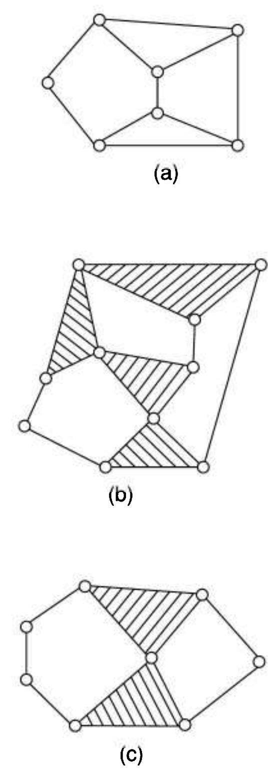

Fig. 1.32

(b) A slider pair can be replaced by one link with a turning pair at the other end. A cam pair with a roller follower can be replaced by a binary link with turning pairs at each end similar to the case of a curved-face follower of Fig. 1.28(d). the equivalent chain is shown in Fig. 1.32(b).

(c) The equivalent chain has been shown in Fig. 1.32(c).

1.13 THE FOUR-BAR CHAIN

A four-bar chain is the most fundamental of the plane kinematic chains. It is a much preferred mechanical device for the mechanisation and control of motion due to its simplicity and versatility. Basically, it consists of four rigid links which are connected in the form of a quadrilateral by four pin-joints. When one of the links is fixed, it is known as a *linkage* or *mechanism*. A link that makes complete revolution is called the *crank*, the link opposite to the fixed link is called the *coupler*, and the fourth link is called a *lever* or *rocker* if it oscillates or another crank, if it rotates.

Note that it is impossible to have a four-bar linkage if the length of one of the links is greater than the sum of the other three. This has been shown in Fig. 1.33 in which the length of link d is more than the sum of lengths of a, b and c, and therefore, this linkage cannot exist.

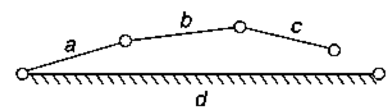

Fig. 1.33

Consider a four-link mechanism shown in Fig. 1.34(a) in which the length a of the link AB is more than d, the length of the fixed link AD. The linkage has been shown in various positions. It can be observed from these configurations that if the link a is to rotate through a full revolution, i.e., if it is to be a crank, then the following conditions must be met:

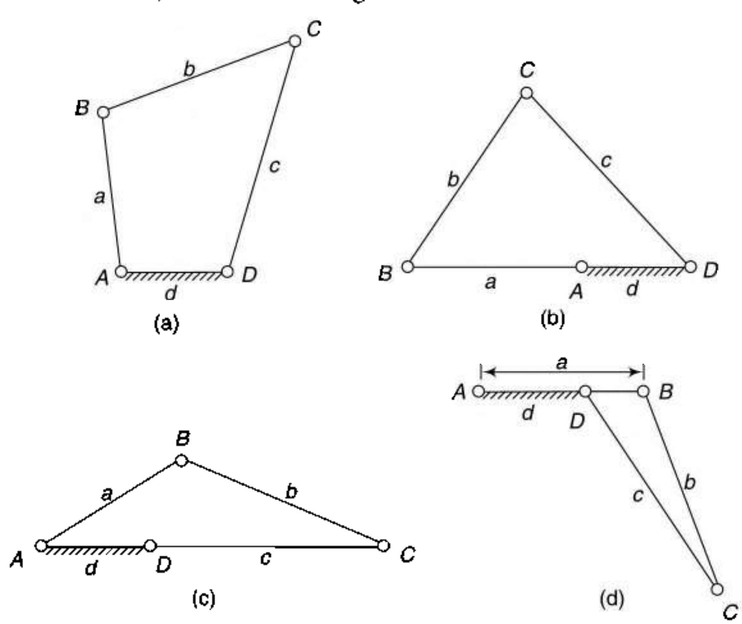

Fig. 1.34

From Fig. 1.34(b), $d + a < b + c$ (i)

From Fig. 1.34(c), $d + c < a + b$ (ii)

From Fig. 1.34(d), $b < c + (a - d)$ or $d + b < c + a$ (iii)

Adding (i) and (ii), $2d + a + c < 2b + a + c$

or $d < b$

Similarly, adding (ii) and (iii), and (iii) and (i) we get

$$d < a$$

and $d < c$

Thus, d is less than a, b and c, i.e., it is the shortest link if a is to rotate a full circle or act as a crank. The above inequalities also suggest that out of a, b and c, whichever is the longest, the sum of that with d, the shortest link will be less than the sum of the remaining two links. Thus, the necessary conditions for the link a to be a crank is

- the shortest link is fixed, and
- the sum of the shortest and the longest links is less than the sum of the other two links.

In a similar way, it can be shown that if the link c is to rotate through a full circle, i.e., if it is to be a crank then the conditions to be realised are the same as above. Also, it can be shown that if both the links a and c rotate through full circles, the link b also makes one complete revolution relative to the fixed link d.

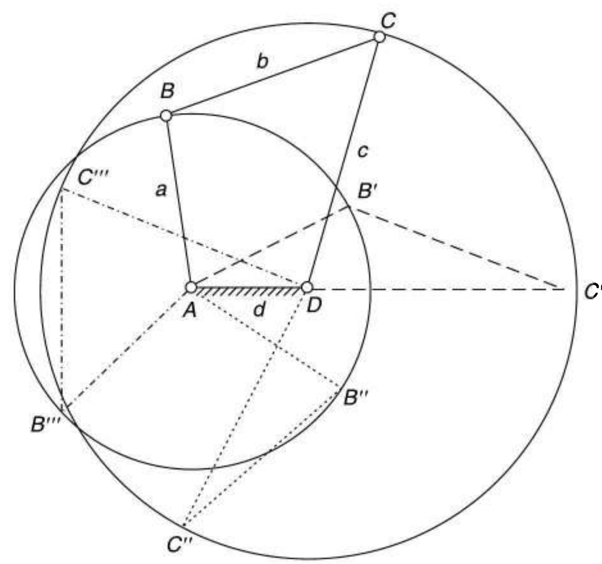

Fig. 1.35

The mechanism thus obtained is known as *crank-crank* or *double-crank* or *drag-crank mechanism* or *rotary-rotary converter.* Figure 1.35 shows all the three links a, b and c rotating through one complete revolution.

In the above consideration, the rotation of the links is observed relative to the fixed link d. Now, consider the movement of b relative to either a or c. The complete rotation of b relative to a is possible if the angle $\angle ABC$ can be more than 180° and relative to c if the angle $\angle DCB$ more than 180°. From the positions of the links in Fig. 1.35(b) and (c), it is clear that these angles cannot become more than 180° for the above stated conditions.

Now, as the relative motion between two adjacent links remains the same irrespective of which link is fixed to the frame, different mechanisms (known as *inversions*) obtained by fixing different links of this kind of chain will be as follows:

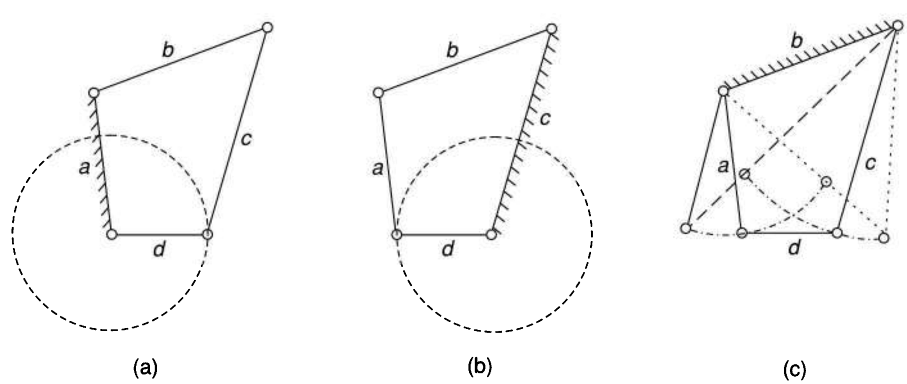

Fig. 1.36

1. If any of the adjacent links of link *d,* i.e., *a* or *c* is fixed, *d* can have a full revolution (crank) and the link opposite to it oscillates (rocks). In Fig. 1.36(a), *a* is fixed, *d* is the crank and *b* oscillates whereas in Fig. 1.36(b), *c* is fixed, *d* is the crank and *b* oscillates. The mechanism is known as *crank-rocker* or *crank-lever mechanism* or *rotary-oscillating converter.*
2. If the link opposite to the shortest link, i.e., link *b* is fixed and the shortest link *d* is made a coupler, the other two links *a* and *c* would oscillate [Fig. 1.36(c)]. The mechanism is known as a *rocker-rocker* or *double-rocker* or *double-lever mechanism* or *oscillating-oscillating converter.*

A linkage in which the sum of the lengths of the longest and the shortest links is less than the sum of the lengths of the other two links, is known as a *class-I,* four-bar linkage.

When the sum of the lengths of the largest and the shorted links is more than the sum of the lengths of the other two links, the linkage is known as a *class-II*, four-bar linkage. In such a linkage, fixing of any of the links always results in a *rocker-rocker* mechanism. In other words, the mechanism and its inversions give the same type of motion (of a *double-rocker* mechanism).

The above observations are summarised in *Grashof's law* which states *that a four-bar mechanism has at least one revolving link if the sum of the lengths of the largest and the shortest links is less than the sum of lengths of the other two links.*

Further, if the *shortest link is fixed,* the chain will act as a double-crank mechanism in which links adjacent to the fixed link will have complete revolutions. If the *link opposite to the shortest link is fixed*, the chain will act as double-rocker mechanism in which links adjacent to the fixed link will oscillate. If the *link adjacent to the shortest link is fixed*, the chain will act as crank-rocker mechanism in which the shortest link will revolve and the link adjacent to the fixed link will oscillate.

If the sum of the lengths of the largest and the shorted links is equal to the sum of the lengths of the other two links, i.e., when equalities exist, the four inversions, in general, result in mechanisms similar to those as given by Grashof's law, except that sometimes the links may become collinear and may have to be guided in the proper direction. Usually, the purpose is served by the inertia of the links. A few special cases may arise when equalities exist. For example, *parallel-crank four-bar linkage* and *deltoid* linkage.

Parallel-Crank Four-Bar Linkage If in a four-bar linkage, two opposite links are parallel and equal in length, then any of the links can be made fixed. The two links adjacent to the fixed link will always act as two cranks. The four links form a parallelogram in all the positions of the cranks, provided the cranks rotate in the same sense as shown in Fig. 1.37.

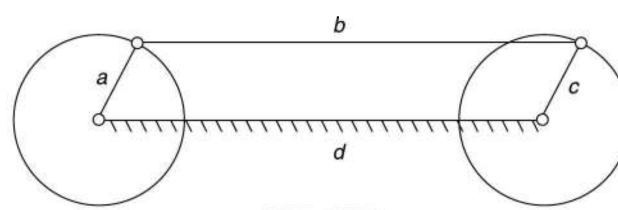

Fig. 1.37

The use of such a mechanism is made in the coupled wheels of a locomotive in which the rotary motion of one wheel is transmitted to the other wheel. For kinematic analysis, link *d* is treated as fixed and the relative motions of the other links are found. However, in fact, *d* has a translatory motion parallel to the rails.

Deltoid Linkage In a deltoid linkage (Fig. 1.38), the equal links are adjacent to each other. When any of the shorter links is fixed, a double-crank mechanism is obtained in which one revolution of the longer link causes two revolutions of the other shorter link. As shown in Fig. 1.38 (a), when the link *c* rotates through half a revolution and assumes the position *DC′*, the link *a* has completed a full revolution.

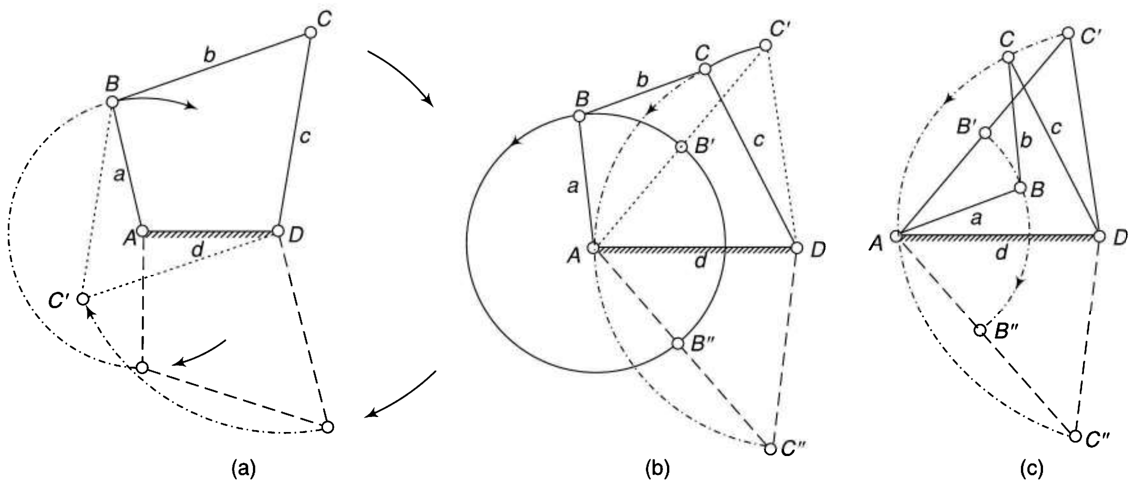

Fig. 1.38

When any of the longer links is fixed, two crank-rocker mechanisms are obtained [Fig. 1.38(b) and (c)]

Example 1.8 *Find all the inversion of the chain given in Fig. 1.39.*

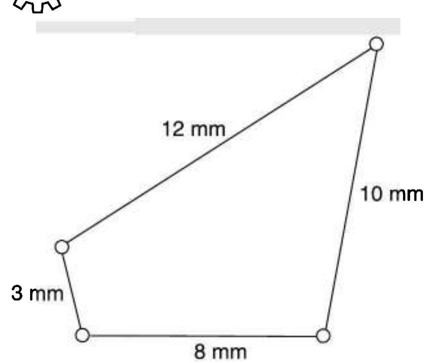

Fig. 1.39

Solution

(a) Length of the longest link = 12 mm
Length of the shortest link = 3 mm
Length of other links = 10 mm and 8 mm
Since 12 + 3 < 10 + 8, it belongs to the class-I mechanism and according to Grashoff's law, three distinct inversions are possible.

Shortest link fixed, i.e., when the link with 3-mm length is fixed, the chain will act as double-crank mechanism in which links with lengths of 12 mm and 8 mm will have complete revolutions.

Link opposite to the shortest link fixed, i.e., when the link with 10-mm length is fixed, the chain will act as double-rocker mechanism in which links with lengths of 12 mm and 8 mm will oscillate.

Link adjacent to the shortest link fixed, i.e., when any of the links adjacent to the shortest link, i.e., link with a length of 12-mm or 8 mm is fixed, the chain will act as crank-rocker mechanism in which the shortest link of 3-mm length will revolve and the link with 10-mm length will oscillate.

Example 1.9 *Figure 1.40 shows some four-link mechanisms in which the figures indicate the dimensions in standard units of length. Indicate the type of each mechanism whether crank-rocker or double-crank or double-rocker.*

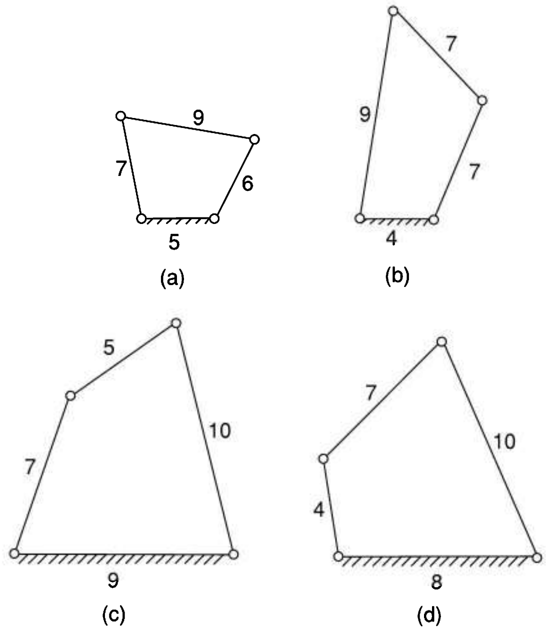

Fig. 1.40

Solution

(a) Length of the longest link = 9
Length of the shortest link = 5
Length of other links = 7 and 6
Since 9 + 5 > 7 + 6, it does not belong to the class-I mechanism. Therefore, it is a double-rocker mechanism.

(b) Length of the longest link = 9
Length of the shortest link = 4
Length of other links = 7 and 7
Since 9 + 4 < 7 + 7, it belongs to the class-I mechanism. In this case as the shortest link is fixed, it is a double-crank mechanism.

(c) Length of the longest link = 10
Length of the shortest link = 5
Length of other links = 9 and 7
Since 10 + 5 < 9 + 7, it belongs to the class-I mechanism. In this case as the link opposite to the shortest link is fixed, it is a double-rocker mechanism.

(d) Length of the longest link = 10
Length of the shortest link = 4
Length of other links = 8 and 7
Since 10 + 4 < 8 + 7, it belongs to the class-I mechanism. In this case as the link adjacent to the shortest link is fixed, it is a crank-rocker mechanism.

Example 1.10 *Figure 1.41 shows a plane mechanism in which the figures indicate the dimensions in standard units of length. The slider C is the driver. Will the link AG revolve or oscillate?*

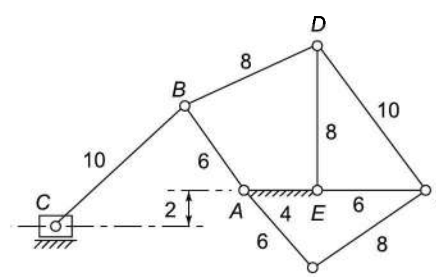

Fig. 1.41

Solution The mechanism has three sub-chains:

(i) *ABC*, a slider-crank chain
(ii) *ABDE*, a four-bar chain
(iii) *AEFG*, a four-bar chain

DEF is a locked chain as it has only three links.

- As the length *BC* is more than the length *AB* plus the offset of 2 units, *AB* acts as a crank and can revolve about *A*.
- In the chain *ABDE*,
  Length of the longest link = 8
  Length of the shortest link = 4
  Length of other links = 8 and 6

Since 8 + 4 < 8 + 6, it belongs to the class-I mechanism. In this case as the shortest link is fixed, it is a double-crank mechanism and thus *AB* and *ED* can revolve fully.

- In the chain *AEFG*,
  Length of the longest link = 8
  Length of the shortest link = 4
  Length of other links = 6 and 6

Since 8 + 4 = 6 + 6, it belongs to the class-I mechanism. As the shortest link is fixed, it is a double-crank mechanism and thus EF and AG can revolve fully.

As DEF is a locked chain with three links, the link EF revolves with the revolving of ED. With the revolving of ED, AG also revolves.

1.14 MECHANICAL ADVANTAGE

The *mechanical advantage* (MA) of a mechanism is the ratio of the output force or torque to the input force or torque at any instant. Thus for the linkage of Fig. 1.42, if friction and inertia forces are ignored and the input torque T_2 is applied to the link 2 to drive the output link 4 with a resisting torque T_4 then

Power input = Power output

$$T_2\,\omega_2 = T_4\,\omega_4$$

or $$\text{MA} = \frac{T_4}{T_2} = \frac{\omega_2}{\omega_4}$$

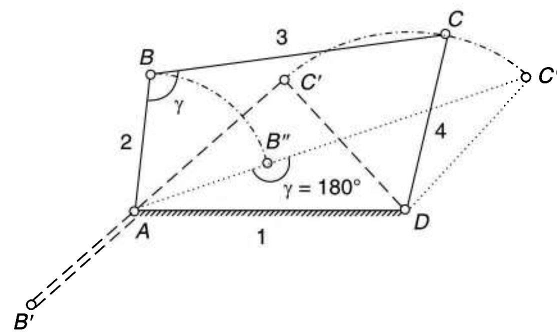

Fig. 1.42

Thus, it is the reciprocal of the velocity ratio. In case of crank-rocker mechanisms, the velocity ω_4 of the output link DC (rocker) becomes zero at the extreme positions ($AB'C''D$ and $AB''C''D$), i.e., when the input link AB is in line with the coupler BC and the angle γ between them is either zero or 180°, it makes the mechanical advantage to be infinite at such positions. Only a small input torque can overcome a large output torque load. The extreme positions of the linkage are known as *toggle* positions.

1.15 TRANSMISSION ANGLE

The angle μ between the output link and the coupler is known as *transmission angle.* In Fig. 1.43, if the link AB is the input link, the force applied to the output link DC is transmitted through the coupler BC. For a particular value of force in the coupler rod, the torque transmitted to the output link (about the point D) is maximum when the transmission angle μ is 90°. If links BC and DC become coincident, the *transmission angle* is zero and the mechanism would lock or jam. If μ deviates significantly from 90°, the torque on the output link decreases. Sometimes, it may not be sufficient to overcome the friction in the system and the mechanism may be locked or jammed. Hence μ is usually kept more than 45°. The best mechanisms, therefore, have a transmission angle that does not deviate much from 90°.

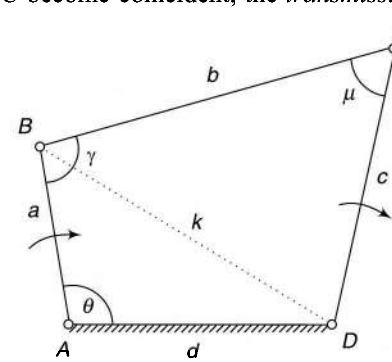

Fig. 1.43

Applying cosine law to triangles ABD and BCD (Fig. 1.43),

$$a^2 + d^2 - 2ad\cos\theta = k^2 \quad \text{(i)}$$

and $$b^2 + c^2 - 2bc\cos\mu = k^2 \quad \text{(ii)}$$

From (i) and (ii),

$$a^2 + d^2 - 2ad\cos\theta = b^2 + c^2 - 2bc\cos\mu$$

or $$a^2 + d^2 - b^2 - c^2 - 2ad\cos\theta + 2bc\cos\mu = 0$$

The maximum or minimum values of the transmission angle can be found by putting $d\mu/d\theta$ equal to zero.

Differentiating the above equation with respect to θ,

$$2\,ad\sin\theta - 2bc\sin\mu.\frac{d\mu}{d\theta} = 0$$

or $$\frac{d\mu}{d\theta} = \frac{ad\sin\theta}{bc\sin\mu}$$

Thus, if $d\mu/d\theta$ is to be zero, the term $ad \sin\theta$ has to be zero which means θ is either 0° or 180°. It can be seen that μ is maximum when θ is 180° and minimum when θ is 0°. However, this would be applicable to the mechanisms in which the link a is able to assume these angles, i.e., in double-crank or crank-rocker mechanisms. Figures 1.44(a) and (b) show a crank-rocker mechanism indicating the positions of the maximum and the minimum transmission angles. Figures 1.45(a) and (b) show the maximum and the minimum transmission angles for a double-rocker mechanism.

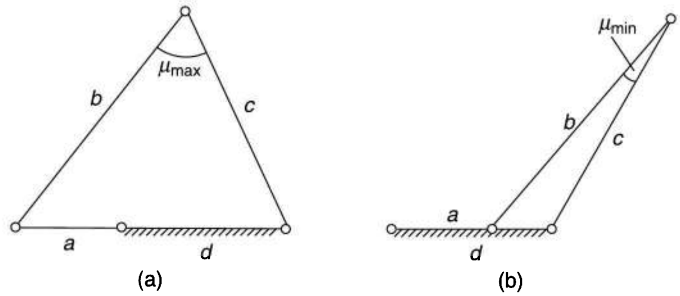

Fig. 1.44

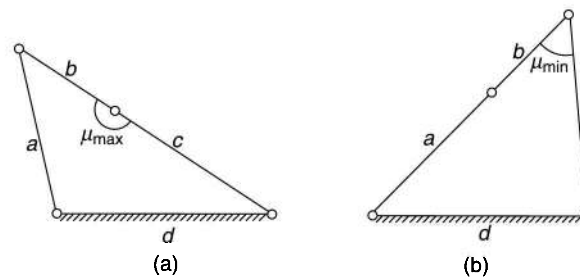

Fig. 1.45

Example 1.11 *Find the maximum and minimum transmission angles for the mechanisms shown in Fig. 1.46. The figures indicate the dimensions in standard units of length.*

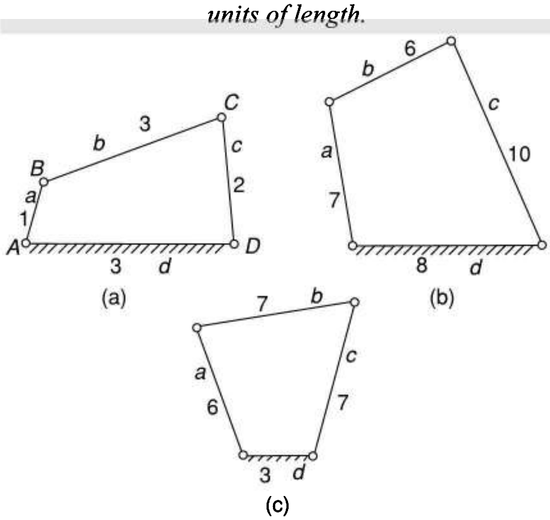

Fig. 1.46

Solution

(a) In this mechanism,

Length of the longest link = 3

Length of the shortest link = 1

Length of other links = 3 and 2

Since 3 + 1 < 3 + 2, it belongs to the class I mechanism. In this case as the link adjacent to the shortest link is fixed, it is a crank-rocker mechanism.

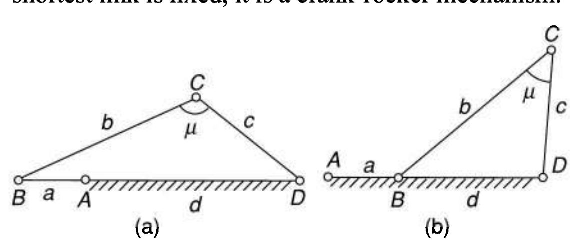

Fig. 1.47

Maximum transmission angle is when θ is 180° [Fig. 1.47(a)],

Thus $(a + d)^2 = b^2 + c^2 - 2bc\cos\mu$

$$(1 + 3)^2 = 3^2 + 2^2 - 2 \times 3 \times 2\cos\mu$$

$$16 = 9 + 4 - 12\cos\mu$$

$$\cos\mu = -\frac{3}{12} = -0.25$$

$$\mu = 104.5^{\circ}$$

Minimum transmission angle is when θ is 0° [Fig. 1.47(b)],

Thus $(d - a)^2 = b^2 + c^2 - 2bc\cos\mu$

$$(3 - 1)^2 = 3^2 + 2^2 - 2 \times 3 \times 2\cos\mu$$

$$4 = 9 + 4 - 12\cos\mu$$

$$\cos\mu = \frac{3}{4} = 0.75$$

$$\mu = 41.4^{\circ}$$

(b) In this mechanism,

Length of the longest link = 10

Length of the shortest link = 6

Length of other links = 8 and 7

Since 10 + 6 > 8 + 7, it belongs to the class-II mechanism and thus is a double-rocker mechanism.

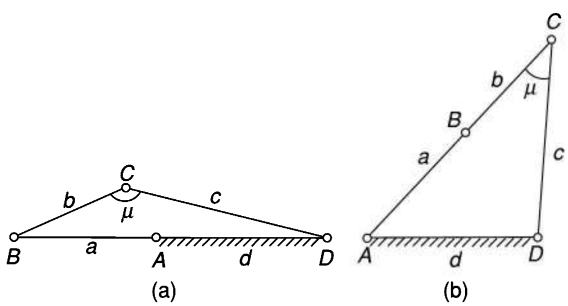

Fig. 1.48

Maximum transmission angle is when θ is 180° [Fig. 1.48(a)],

Thus, $(a + d)^2 = b^2 + c^2 - 2bc\cos\mu$

$$(7 + 8)^2 = 6^2 + 10^2 - 2 \times 6 \times 10\cos\mu$$

$$225 = 36 + 100 - 120\cos\mu$$

$$\cos\mu = -\frac{89}{120} = -0.742$$

$$\mu = 137.9^{\circ}$$

Minimum transmission angle is when the angle at B is 180° [Fig. 1.48(b)],

Thus, $d^2 = (a + b)^2 + c^2 - 2(a + b)c\cos\mu$

$$8^2 = (7 + 6)^2 + 10^2 - 2(7 + 6) \times 10 \times \cos\mu$$

$$64 = 169 + 100 - 260\cos\mu$$

$$\cos\mu = \frac{205}{260} = 0.788$$

$$\mu = 38^{\circ}$$

(c) In this mechanism,

Length of the longest link = 7

Length of the shortest link = 3

Length of other links = 6 and 6

Since 7 + 3 < 6 + 6, it belongs to the class-I mechanism. In this case as the shortest link is fixed, it is a double-crank or drag-link mechanism.

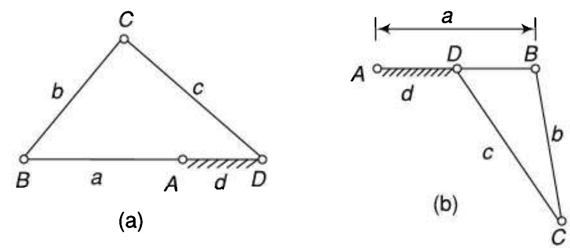

Fig. 1.49

Maximum transmission angle is when θ is 180° [Fig. 1.49(a)],

Thus $(a + d)^2 = b^2 + c^2 - 2bc\cos\mu$

$$(6 + 3)^2 = 6^2 + 7^2 - 2 \times 6 \times 7\cos\mu$$

$$81 = 36 + 49 - 84\cos\mu$$

$$\cos\mu = \frac{4}{84} = 0.476$$

$$\mu = 87.27^{\circ}$$

Minimum transmission angle is when θ is 0° [Fig. 1.49(b)],

Thus $(a - d)^2 = b^2 + c^2 - 2bc\cos\mu$

$$(6 - 3)^2 = 6^2 + 7^2 - 2 \times 6 \times 7\cos\mu$$

$$9 = 36 + 49 - 84\cos\mu$$

$$\cos\mu = \frac{76}{84} = 0.9048$$

$$\mu = 25.2^{\circ}$$

Example 1.12 *A crank-rocker mechanism has a 70-mm fixed link, a 20-mm crank, a 50-mm coupler, and a 70-mm rocker. Draw the mechanism and determine the maximum and minimum values of the transmission angle. Locate the two toggle positions and find the corresponding crank angles and the transmission angles.*

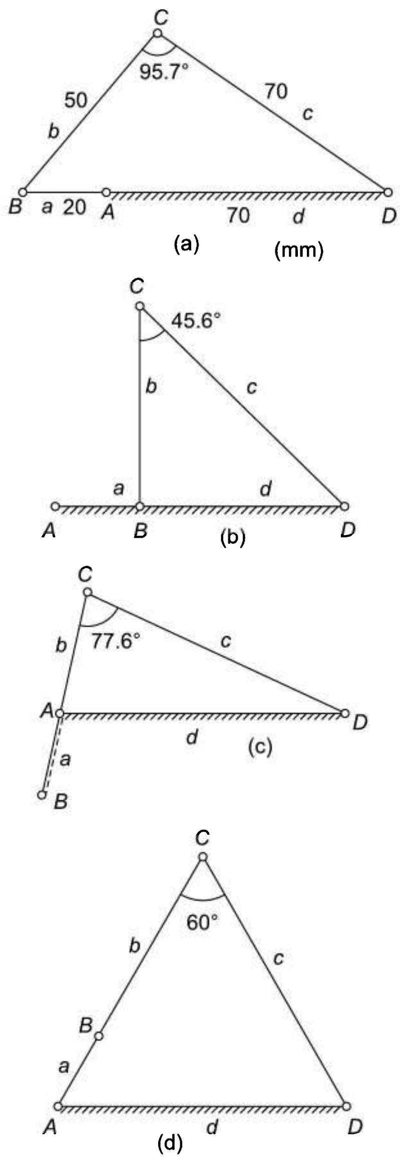

Fig. 1.50

Solution In this mechanism,

Length of the longest link = 70 mm
Length of the shortest link = 20 mm
Length of other links = 70 and 50 mm

Since 70 + 20 < 70 + 50, it belongs to the class-I mechanism. In this case as the link adjacent to the shortest link is fixed, it is a crank-rocker mechanism.

Maximum transmission angle is when θ is 180° [Fig. 1.50(a)],

Thus $(a + d)^2 = b^2 + c^2 - 2bc \cos \mu$

$$(20 + 70)^2 = 50^2 + 70^2 - 2 \times 50 \times 70 \cos \mu$$
$$8100 = 2500 + 4900 - 7000 \cos \mu$$
$$\cos \mu = -0.1$$
$$\mu = 95.7^\circ$$

Minimum transmission angle is when θ is 0° [Fig. 1.50(b)],

Thus $(70 - 20)^2 = 50^2 + 70^2 - 2 \times 50 \times 70 \cos \mu$

$$2500 = 2500 + 4900 - 7000 \cos \mu$$
$$\cos \mu = 0.7$$
$$\mu = 45.6^\circ$$

The two toggle positions are shown in Figs 1.50(c) and (d).

Transmission angle for first position,

$$d^2 = (b - a)^2 + c^2 - 2(b - a)c \cos \mu$$
$$70^2 = 30^2 + 70^2 - 2 \times 30 \times 70 \cos \mu$$
$$4900 = 900 + 4900 - 4200 \cos \mu$$
$$\cos \mu = 0.214$$
$$\mu = 77.6^\circ$$

As c and d are of equal length [Fig. 1.50(c)], it is an isosceles triangle and thus input angle $\theta = (77.6^\circ + 180^\circ) = 257.6^\circ$

Transmission angle for second position Fig. 1.50(d),

$$d^2 = (b + a)^2 + c^2 - 2(b + a)c \cos \mu$$
$$70^2 = 70^2 + 70^2 - 2 \times 70 \times 70 \cos \mu$$
$$4900 = 4900 + 4900 - 9800 \cos \mu$$
$$\cos \mu = 0.5$$
$$\mu = 60^\circ$$

(or as all the sides of the triangle of Fig. 1.50(d) are of equal length, it is an equilateral triangle and thus transmission angle is equal to 60°)

And the input angle, $\theta = 60^\circ$

- The above results can also be obtained graphically by drawing the figures to scale and measuring the angles.

1.16 THE SLIDER-CRANK CHAIN

When one of the turning pairs of a four-bar chain is replaced by a sliding pair, it becomes a *single slider-crank chain* or simply a *slider-crank chain*. It is also possible to replace two sliding pairs of a four-bar chain to get a *double slider-crank chain* (Sec. 1.17). Further, in a slider-crank chain, the straight line path of the slider may be passing through the fixed pivot O or may be displaced. The distance e between the fixed pivot O and the straight line path of the slider is called the *offset* and the chain so formed an *offset slider-crank chain.*

Different mechanisms obtained by fixing different links of a kinematic chain are known as its *inversions*. A slider-crank chain has the following inversions:

First Inversion

This inversion is obtained when link 1 is fixed and links 2 and 4 are made the crank and the slider respectively [Fig. 1.51(a)].

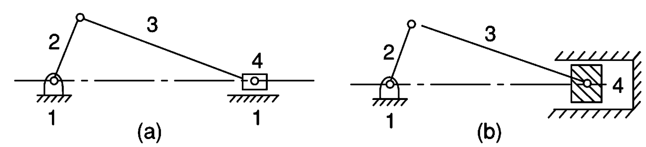

Fig. 1.51

Applications

1. Reciprocating engine
2. Reciprocating compressor

As shown in Fig. 1.51(b), if it is a reciprocating engine, 4 (piston) is the driver and if it is a compressor, 2 (crank) is the driver.

Second Inversion

Fixing of the link 2 of a slider-crank chain results in the second inversion.

The slider-crank mechanism of Fig. 1.51(a) can also be drawn as shown in Fig. 11.52(a). Further, when its link 2 is fixed instead of the link 1, the link 3 along with the slider at its end B becomes a crank. This makes the link 1 to rotate about O along with the slider which also reciprocates on it [Fig. 1.52(b)].

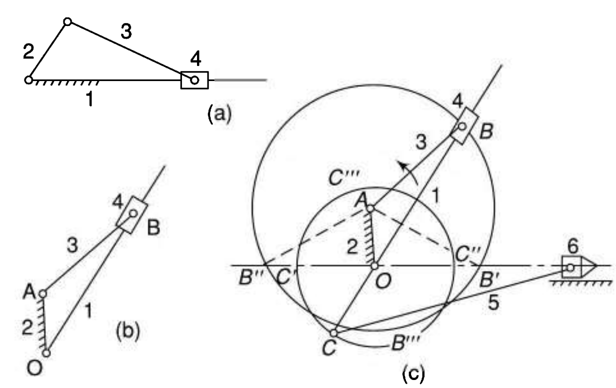

Fig. 1.52

Applications

1. Whitworth quick-return mechanism
2. Rotary engine

Whitworth Quick-Return Mechanism It is a mechanism used in workshops to cut metals. The forward stroke takes a little longer and cuts the metal whereas the return stroke is idle and takes a shorter period.

Slider 4 rotates in a circle about A and slides on the link 1 [Fig. 1.52(c)]. C is a point on the link 1 extended backwards where the link 5 is pivoted. The other end of the link 5 is pivoted to the tool, the forward stroke of which cuts the metal. The axis of motion of the slider 6 (tool) passes through O and is perpendicular to OA, the fixed link. The crank 3 rotates in the counter-clockwise direction.

Initially, let the slider 4 be at B' so that C be at C'. Cutting tool 6 will be in the extreme left position. With the movement of the crank, the slider traverses the path $B'BB''$ whereas the point C moves through $C'CC''$. Cutting tool 6 will have the forward stroke. Finally, the slider B assumes the position B'' and the cutting tool 6 is in the extreme right position. The time taken for the forward stroke of the slider 6 is proportional to the obtuse angle $B''AB'$ at A.

Similarly, the slider 4 completes the rest of the circle through the path $B''B'''B'$ and C passes through $C''C'''C'$. There is backward stroke of the tool 6. The time taken in this is proportional to the acute angle $B''AB'$ at A.

Let

θ = obtuse angle $B'AB''$ at A

β = acute angle $B'AB''$ at A

Then,

$$\frac{\text{Time of cutting}}{\text{Time of return}} = \frac{\theta}{\beta}$$

A nine-cylinder rotary engine

Rotary Engine Referring Fig. 1.52(b), it can be observed that with the rotation of the link 3, the link 1 rotates about O and the slider 4 reciprocates on it. This also implies that if the slider is made to reciprocate on the link 1, the crank 3 will rotate about A and the link 1 about O.

In a rotary engine, the slider is replaced by a piston and the link 1 by a cylinder pivoted at O. Moreover, instead of one cylinder, seven or nine cylinders symmetrically placed at regular intervals in the same plane or in parallel planes, are used. All the cylinders rotate about the same fixed centre and form a balanced system. The fixed link 2 is also common to all cylinders (Fig. 1.53).

Thus, in a rotary engine, the crank 2 is fixed and the body 1 rotates whereas in a reciprocating engine (1st inversion), the body 1 is fixed and the crank 2 rotates.

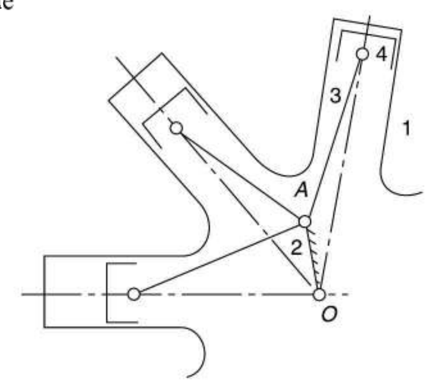

Fig. 1.53

Third Inversion

By fixing the link 3 of the slider-crank mechanism, the third inversion is obtained [Fig. 1.54(a)]. Now the link 2 again acts as a crank and the link 4 oscillates.

Applications

1. Oscillating cylinder engine
2. Crank and slotted-lever mechanism

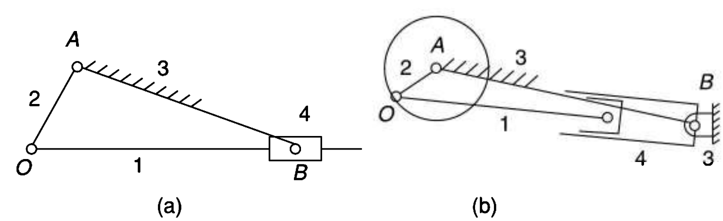

Fig. 1.54

Oscillating Cylinder Engine As shown in Fig. 1.54(b), the link 4 is made in the form of a cylinder and a piston is fixed to the end of the link 1. The piston reciprocates inside the cylinder pivoted to the fixed link 3. The arrangement is known as oscillating cylinder engine, in which as the piston reciprocates in the oscillating cylinder, the crank rotates.

Crank and Slotted-Lever Mechanism If the cylinder of an oscillating cylinder engine is made in the form of a guide and the piston in the form of a slider, the arrangement as shown in Fig. 1.55(a) is obtained. As the crank rotates about A, the guide 4 oscillates about B. At a point C on the guide, the link 5 is pivoted, the other end of which is connected to the cutting tool through a pivoted joint.

Figure 1.55(b) shows the extreme positions of the oscillating guide 4. The time of the forward stroke is proportional to the angle θ whereas for the return stroke, it is proportional to angle β, provided the crank rotates clockwise.

Comparing a crank and slotted-lever quick-return mechanism with a Whitworth quick-return mechanism, the following observations are made:

1. Crank 3 of the Whitworth mechanism is longer than its fixed link 2 whereas the crank 2 of the slotted-lever mechanism is shorter than its fixed link 3.
2. Coupler link 1 of the Whitworth mechanism makes complete rotations about its pivoted joint O with the fixed link. However, the coupler link 4 of the slotted-lever mechanism oscillates about its pivot B.
3. The coupler link holding the tool can be pivoted to the main coupler link at any convenient point C in both cases. However, for the same displacement of the tool, it is more convenient if the point C is taken on the extension of the main coupler link (towards the pivot with the fixed link) in case of the Whitworth mechanism and beyond the extreme position of the slider in the slotted-lever mechanism.

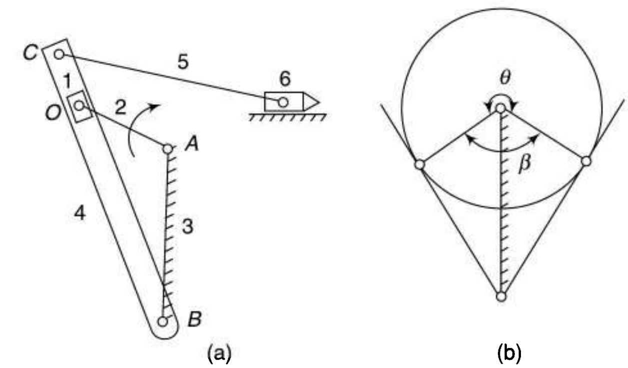

Fig. 1.55

A shaping machine. Shaping machines are fitted with quick-return mechanisms.

Fourth Inversion

If the link 4 of the slider-crank mechanism is fixed, the fourth inversion is obtained [Fig. 1.56(a)]. Link 3 can oscillate about the fixed pivot B on the link 4. This makes the end A of the link 2 to oscillate about B and the end O to reciprocate along the axis of the fixed link 4.

Application Hand-pump

Figure 1.56(b) shows a hand-pump. Link 4 is made in the form of a cylinder and a plunger fixed to the link 1 reciprocates in it.

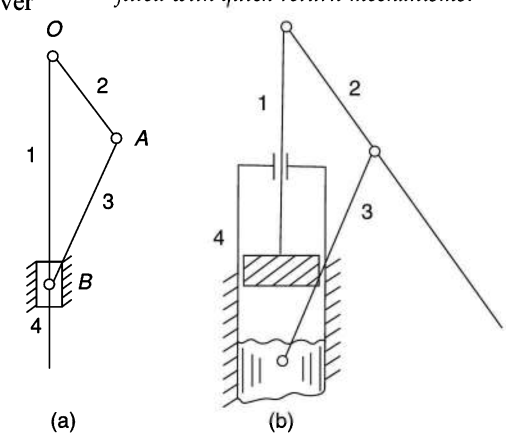

Fig. 1.56

Example 1.13 *The length of the fixed link of a crank and slotted-lever mechanism is 250 mm and that of the crank is 100 mm. Determine the*

(i) inclination of the slotted lever with the vertical in the extreme position,

(ii) ratio of the time of cutting stroke to the time of return stroke, and

(iii) *length of the stroke, if the length of the slotted lever is 450 mm and the line of stroke passes through the extreme positions of the free end of the lever.*

Solution Refer Fig. 1.57.

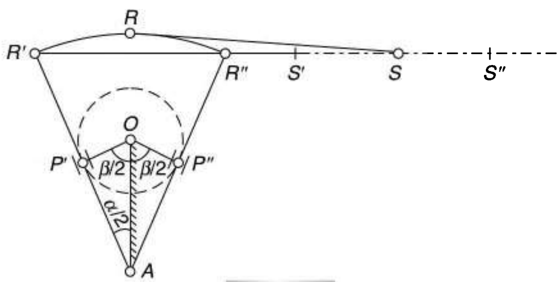

Fig. 1.57

$OA = 250$ mm $\quad OP' = OP'' = 100$ mm
$AR' = AR'' = AR = 450$ mm

$$\cos\frac{\beta}{2} = \frac{OP'}{OA} = \frac{100}{250} = 0.4$$

or $\quad \dfrac{\beta}{2} = 66.4°\quad$ or $\quad \beta = 132.8°$

(i) Angle of the slotted lever with the vertical
$\alpha/2 = 90° - 66.4° = 23.6°$

(ii) $\dfrac{\text{Time of cutting stroke}}{\text{Time of return stroke}}$

$$= \frac{360° - \beta}{\beta} = \frac{360° - 132.8°}{132.8°} = 1.71$$

(iii) Length of stroke $= S'S'' = R'R''$
$= 2\,AR'.\sin(\alpha/2)$
$= 2 \times 450 \sin 23.6°$
$= 360.3$ mm

1.17 DOUBLE SLIDER-CRANK CHAIN

A four-bar chain having two turning and two sliding pairs such that two pairs of the same kind are adjacent is known as a double-slider-crank chain [Fig. 1.58(a)]. The following are its inversions.

First Inversion

This inversion is obtained when the link 1 is fixed and the two adjacent pairs 23 and 34 are turning pairs and the other two pairs 12 and 41 sliding pairs.

Application Elliptical trammel

Elliptical Trammel Figure 1.58(b) shows an elliptical trammel in which the fixed link 1 is in the form of guides for sliders 2 and 4. With the movement of the sliders, any point *C* on the link 3, except the midpoint of *AB* will trace an ellipse on a fixed plate. The midpoint of *AB* will trace a circle.

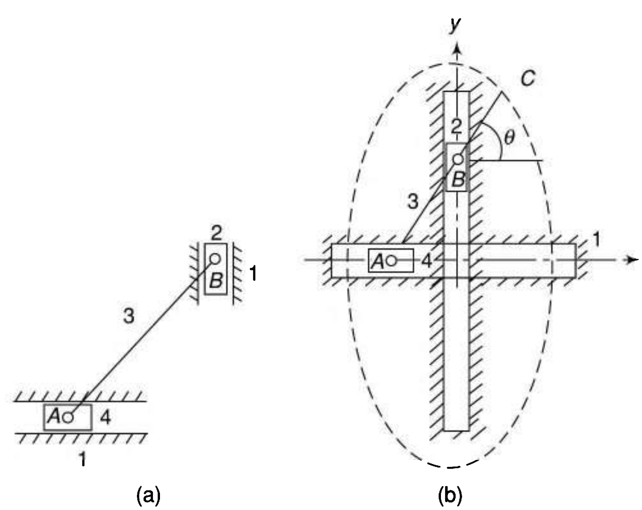

Fig. 1.58

Let at any instant, the link 3 make angle θ with the X-axis. Considering the displacements of the sliders from the centre of the trammel,

$$x = BC\cos\theta \text{ and } y = AC\sin\theta$$

$$\therefore \quad \frac{x}{BC} = \cos\theta \text{ and } \frac{y}{AC} = \sin\theta$$

Squaring and adding,

$$\frac{x^2}{(BC)^2}+\frac{y^2}{(AC)^2}=\cos^2\theta+\sin^2\theta=1$$

This is the equation of an ellipse. Therefore, the path traced by C is an ellipse with the semi-major and semi-minor axes being equal to AC and BC respectively.

When C is the midpoint of AB; $AC = BC$,

and

$$\frac{x^2}{(BC)^2}+\frac{y^2}{(AC)^2}=1 \quad \text{or} \quad x^2+y^2=(AC)^2$$

which is the equation of a circle with AC (=BC) as the radius of the circle.

Second Inversion

If any of the slide-blocks of the first inversion is fixed, the second inversion of the double-slider-crank chain is obtained. When the link 4 is fixed, the end B of the crank 3 rotates about A and the link 1 reciprocates in the horizontal direction.

Application Scotch yoke

Scotch Yoke A scotch-yoke mechanism (Fig. 1.59) is used to convert the rotary motion into a sliding motion. As the crank 3 rotates, the horizontal portion of the link 1 slides or reciprocates in the fixed link 4.

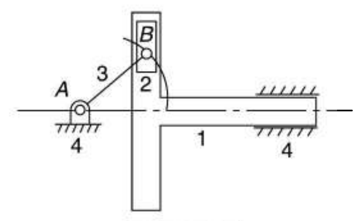

Fig. 1.59

Third Inversion

This inversion is obtained when the link 3 of the first inversion is fixed and the link 1 is free to move.

The rotation of the link 1 has been shown in Fig. 1.60 in which the full lines show the initial position. With rotation of the link 4 through 45° in the clockwise direction, the links 1 and 2 rotate through the same angle whereas the midpoint of the link 1 rotates through 90° in a circle with the length of link 3 as diameter. Thus, the angular velocity of the midpoint of the link 1 is twice that of links 2 and 4.

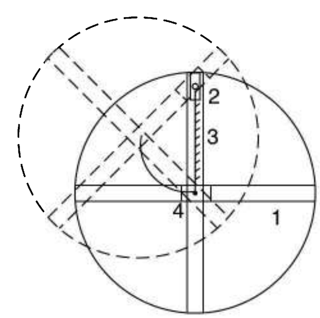

Fig. 1.60

The sliding velocity of the link 1 relative to the link 4 will be maximum when the midpoint of the link 1 is at the axis of the link 4. In this position, the sliding velocity is equal to the tangential velocity of the midpoint of the link 1.

Maximum sliding velocity = tangential velocity of midpoint of the link 1
= angular velocity of midpoint of the link 1 × radius
= (2 × angular velocity of the link 4) × (distance between axes of links 2 and 4)/2
= angular velocity of link 4 × distance between axes of links 2 and 4

The sliding velocity of the link 1 relative to the link 4 is zero when the midpoint of 1 is on the axis of the link 2.

Application Oldham's coupling

Oldham's Coupling If the rotating links 2 and 4 of the mechanism are replaced by two shafts, one can act as the driver and the other as the driven shaft with their axes at the pivots of links 2 and 4.

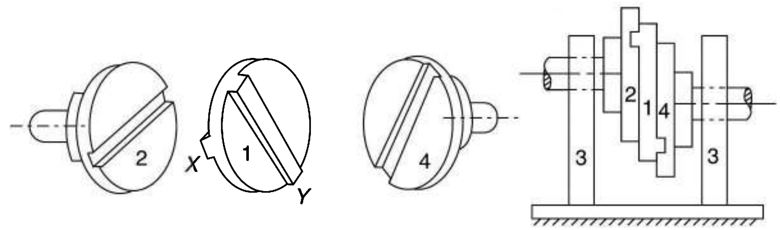

Fig. 1.61

Figure 1.61 shows an actual Oldham's coupling which is used to connect two parallel shafts when the distance between their axes is small. The two shafts have flanges at the ends and are supported in the fixed bearings representing the link 3. In the flange 2, a slot is cut in which the tongue X of the link 1 is fitted and has a sliding motion. Link 1 is made circular and has another tongue Y at right angles to the first and which fits in the recess of the flange of the shaft 4. Thus, the intermediate link 1 slides in the two slots in the two flanges while having the rotary motion.

As mentioned earlier, the midpoint of the intermediate piece describes a circle with distance between the axes of the shafts as diameter. The maximum sliding velocity of each tongue in the slot will be the peripheral velocity of the midpoint of the intermediate disc along the circular path.

Maximum sliding velocity = peripheral velocity along the circular path
= angular velocity of shaft × distance between shafts

Example 1.14 *The distance between two parallel shafts is 18 mm and they are connected by an Oldham's coupling. The driving shaft revolves at 160 rpm. What will be the maximum speed of sliding of the tongue of the intermediate piece along its groove?*

Solution $$\omega = \frac{2\pi \times 160}{60} = 16.75 \text{ rad/s}$$

Maximum velocity of sliding $= \omega \times d$
$= 16.75 \times 0.018$
$= 0.302$ m/s

1.18 MISCELLANEOUS MECHANISMS

Snap-Action Mechanisms

The mechanisms used to overcome a large resistance of a member with a small driving force are known as *snap action* or *toggle* mechanisms. They find their use in a variety of machines such as stone crushers, embossing presses, switches, etc. Figure 1.62(a) shows such a type of mechanism in which links of equal lengths 4 and 5 are connected by a pivoted joint at B. Link 4 is free to oscillate about the pivot C and the link 5 is connected to a sliding link 6. Link 3 joins links 4 and 5. When force is applied at the point B through the link 3, the angle α decreases and links 4 and 5 tend to become collinear. At this instant, the force is greatly multiplied at B, i.e., a very small force is required to overcome a great resistance R at the slider. This is because a large movement at B produces a relatively slight displacement of the slider at D. As the angle α approaches zero, reaction at the pivot becomes equal to R and for force balance in the link BC or BD,

$$\frac{F}{2\sin\alpha} = \frac{R}{\cos\alpha}$$

or $$2\tan\alpha = \frac{F}{R}$$

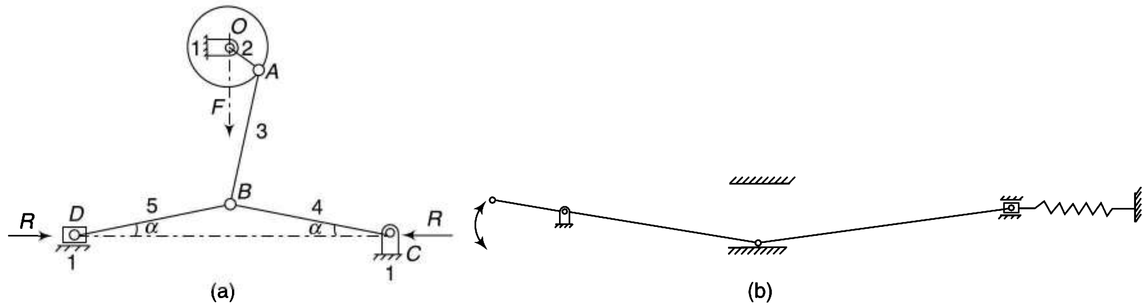

Fig. 1.62

As $\alpha \to 0$, $\tan\alpha \to 0$. Thus for a small value of the force F, R approaches infinity. In a stone crusher, a large resistance at D is overcome with a small force F in this way. Figure 1.62(b) shows another such mechanism.

Indexing Mechanisms

An *indexing mechanism* serves the purpose of dividing the periphery of a circular piece into a number of equal parts. Indexing is generally done on gear cutting or milling machines.

An indexing mechanism consists of an index head in which a spindle is carried in a headstock [Fig. 1.63(a)]. The work to be indexed is held either between centres or in a chuck attached to the spindle. A 40-tooth worm wheel driven by a single-threaded right-hand worm is also fitted to the spindle. At the end of the worm shaft an adjustable index crank with a handle and a plunger pin is also fitted. The plunger pin can be made to fit into any hole in the index plate which has a number of circles of equally spaced holes as shown in Fig. 1.63(b). An index head is usually provided with a number of interchangeable index plates to cover a wide range of work. However, the figure shows only the circle of 17 holes for sake of clarity.

As the worm wheel has 40 teeth, the number of revolutions of the index crank required to make one revolution of the work is also 40. The number of revolutions of the crank, needed for a proper division of the work into the desired number of divisions, can be calculated as follows:

- If a work is to be divided into 40 divisions, the crank should be given one complete revolution; if 20 divisions, two revolutions for each division, and so on.
- If the work is to be divided into 160 divisions, obviously the crank should be rotated through one-fourth of a rotation. For such cases, an index plate with a number of holes divisible by 4 such as with 16 or 20 holes can be chosen.
- If the work is to be divided into 136 parts, the use of the index plate will be essential since the rotation of the crank for each division will be 40/136 or five-seventeenth of a turn. Thus, a plate with 17 holes is selected in this case. To obviate the necessity of counting the holes at each partial turn of the crank, an index sector with two arms which can be set and clamped together at any angle is also available. In this case, this can be set to measure off 5 spaces. Starting with the crankpin in the hole *a*, a cut would be made in the work. The crank is rotated and the pin is made to enter into the hole *b*, 5 divisions apart and a second cut is made in the work. In a similar way, a third cut is made by rotating the crank again through five divisions with the help of an index sector, and so on. Usually, index tables are provided to ascertain the number of turns of the crank and the number of holes for the given case.

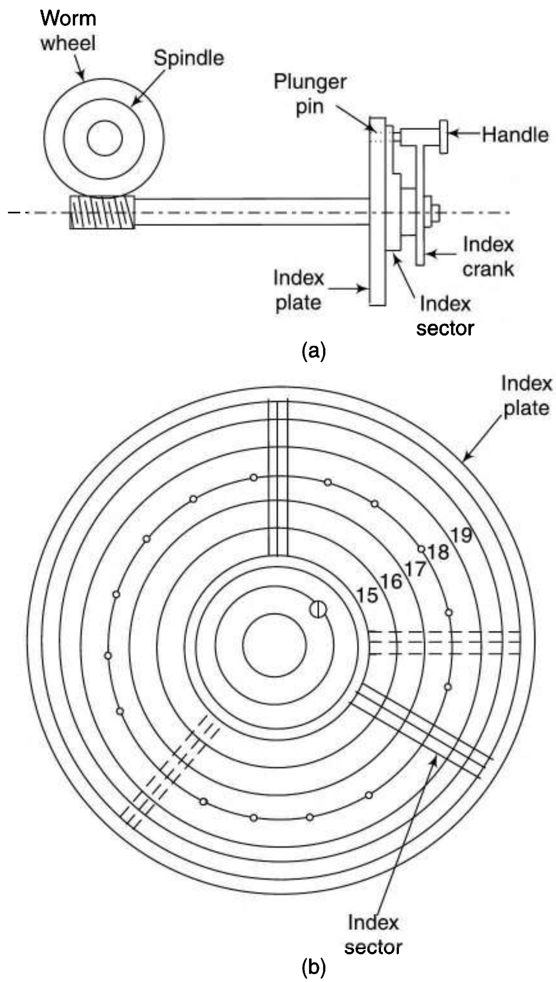

Fig. 1.63

Index plate of an indexing mechanism

Summary

1. *Kinematics* deals with the relative motions of different parts of a mechanism without taking into consideration the forces producing the motions whereas *dynamics* involves the calculation of forces impressed upon different parts of a mechanism.
2. *Mechanism* is a combination of a number of rigid bodies assembled in such a way that the motion of one causes constrained and predictable motion of the others whereas a *machine* is a mechanism or a combination of mechanisms which, apart from imparting definite motions to the parts, also transmits and modifies the available mechanical energy into some kind of useful work.
3. There are three types of constrained motion: *completely constrained, incompletely constrained* and *successfully constrained*.
4. A *link* is a resistant body or a group of resistant bodies with rigid connections preventing their

relative movement. A link may also be defined as a member or a combination of members of a mechanism, connecting other members and having motion relative to them.

5. A *kinematic pair* or simply a pair is a joint of two links having relative motion between them.
6. A pair of links having surface or area contact between the members is known as a *lower pair* and a pair having a point or line contact between the links, a *higher pair.*
7. When the elements of a pair are held together mechanically, it is known as a *closed pair*. The two elements are geometrically identical. When two links of a pair are in contact either due to force of gravity or some spring action, they constitute an *unclosed pair*.
8. Usual types of joints in a chain are binary joint, ternary joint and quaternary joint
9. *Degree of freedom of a pair* is defined as the number of independent relative motions, both translational and rotational, a pair can have.
10. A *kinematic chain* is an assembly of links in which the relative motions of the links is possible and the motion of each relative to the other is definite.
11. A *redundant chain* does not allow any motion of a link relative to the other.
12. A *linkage* or *mechanism* is obtained if one of the links of a kinematic chain is fixed to the ground.
13. *Degree of freedom of a mechanism* indicates how many inputs are needed to have a constrained motion of the other links.
14. *Kutzback's criterion* for the degree of freedom of plane mechanisms is
    $F = 3(N - 1) - 2P_1 - 1P_2$
15. *Gruebler's criterion* for degree of freedom of plane mechanisms with single-degree of freedom joints only is
    $F = 3(N - 1) - 2P_1$
16. *Author's criterion* for degree of freedom and the number of joints of plane mechanisms with turning pairs is
    $F = N - (2L + 1)$
    $P_1 = N + (L - 1)$
17. In a four-link mechanism, a link that makes a complete revolution is known as a *crank*, the link opposite to the fixed link is called the *coupler* and the fourth link is called a *lever* or *rocker* if it oscillates or another crank, if it rotates.
18. In a Watts six-bar chain, the ternary links are direct connected whereas in a Stephenson's six-bar chain, they are not direct connected.
19. If a system has one or more links which do not introduce any extra constraint, it is known as *redundant link* and is not counted to find the degree of freedom.
20. If a link of a mechanism can be moved without causing any motion to the rest of the links of the mechanism, it is said to have a *redundant degree of freedom.*
21. The *mechanical advantage* (MA) of a mechanism is the ratio of the output force or torque to the input force or torque at any instant.
22. The angle μ between the output link and the coupler is known as *transmission angle*.
23. Different mechanisms obtained by fixing different links of a kinematic chain are known as its *inversions*.
24. The mechanisms used to overcome a large resistance of a member with a small driving force are known as *snap action* or *toggle* mechanisms.
25. An *indexing mechanism* serves the purpose of dividing the periphery of a circular piece into a number of equal parts.

Exercises

1. Distinguish between
   (i) mechanism and machine
   (ii) analysis and synthesis of mechanisms
   (iii) kinematics and dynamics
2. Define: kinematic link, kinematic pair, kinematic chain.
3. What are rigid and resistant bodies? Elaborate.
4. How are the kinematic pairs classified? Explain with examples.
5. Differentiate giving examples:
   (i) lower and higher pairs
   (ii) closed and unclosed pairs
   (iii) turning and rolling pairs
6. What do you mean by degree of freedom of a

kinematic pair? How are pairs classified? Give examples.
7. Discuss various types of constrained motion.
8. What is a redundant link in a mechanism?
9. How do a Watt's six-bar chain and Stephenson's six-bar chain differ?
10. What is redundant degree of freedom of a mechanism?
11. What are usual types of joints in a mechanism?
12. What is the degree of freedom of a mechanism? How is it determined?
13. What is Kutzback's criterion for degree of freedom of plane mechanisms? In what way is Gruebler's criterion different from it?
14. How are the degree of freedom and the number of joints in a linkage can be found when the number of links and the number of loops in a kinematic chain are known?
15. What is meant by equivalent mechanisms?
16. Define Grashof's law. State how is it helpful in classifying the four-link mechanisms into different types.
17. Why are parallel-crank four-bar linkage and deltoid linkage considered special cases of four-link mechanisms?
18. Define mechanical advantage and transmission angle of a mechanism.
19. Describe various inversions of a slider-crank mechanism giving examples.
20. What are quick-return mechanisms? Where are they used? Discuss the functioning of any one of them.
21. How are the Whitworth quick-return mechanism and crank and slotted-lever mechanism different from each other?
22. Enumerate the inversions of a double-slider-crank chain. Give examples.
23. Describe briefly the functions of elliptical trammel and scotch yoke.
24. In what way is Oldham's coupling useful in connecting two parallel shafts when the distance between their axes is small?
25. What are snap-action mechanisms? Give examples.
26. What is an indexing mechanism? Describe how it is used to divide the periphery of a circular piece into a number of equal parts.
27. For the kinematic linkages shown in Fig. 1.64, find the degree of freedom (F).

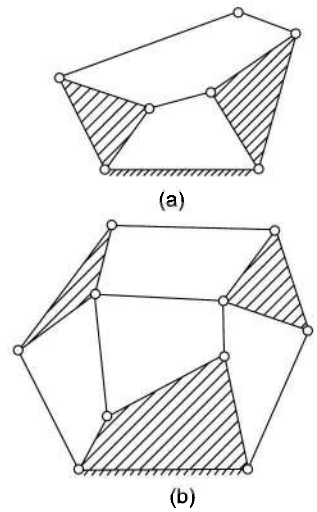

Fig. 1.64

[(a) 1 (b) 0]

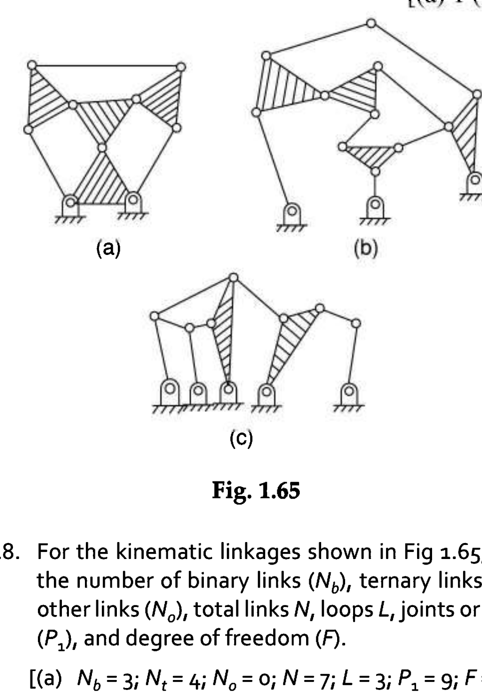

Fig. 1.65

28. For the kinematic linkages shown in Fig 1.65, find the number of binary links (N_b), ternary links (N_t), other links (N_o), total links N, loops L, joints or pairs (P_1), and degree of freedom (F).

[(a) $N_b = 3$; $N_t = 4$; $N_o = 0$; $N = 7$; $L = 3$; $P_1 = 9$; $F = 0$

(b) $N_b = 7$; $N_t = 5$; $N_o = 0$; $N = 12$; $L = 4$; $P_1 = 15$; $F = 3$

(c) $N_b = 8$; $N_t = 2$; $N_o = 1$; $N = 11$; $L = 5$; $P_1 = 15$; $F = 0$]

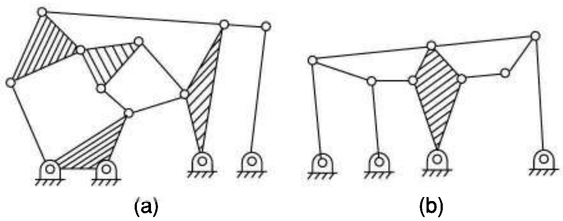

Fig. 1.66

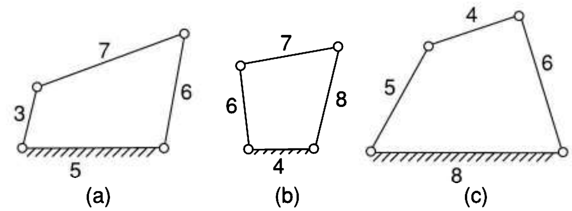

Fig. 1.67

29. Show that the linkages shown in Fig. 1.66 are structures. Suggest some changes to make them mechanisms having one degree of freedom. The number of links should not be changed by more than ±1.
30. A linkage has 14 links and the number of loops is 5. Calculate its
    (i) degrees of freedom
    (ii) number of joints
    (iii) maximum number of ternary links that can be had.

    Assume that all the pairs are turning pairs.

    (3; 18; 8)
31. Figure 1.67 shows some four-link mechanisms in which the figures indicate the dimensions in standard units of length. Indicate the type of each mechanism, whether it is crank-rocker or double-crank or double-rocker.

    [(a) crank-rocker (b) double-crank (c) double-rocker]
32. A crank-rocker mechanism *ABCD* has the dimensions *AB* = 30 mm, *BC* = 90 mm, *CD* = 75 mm and *AD* (fixed link) = 100 mm. Determine the maximum and the minimum values of the transmission angle. Locate the toggle positions and indicate the corresponding crank angles and the transmission angles.

    (103°, 49°, θ = 228°, μ = 92°, θ =38.5°, μ = 56°)