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CEE 598 Topic: Traffic Simulation Modelling and Applications

Homework part I

Transportation problem:

Given:
$a(i)$: the demand of city i, $i \in \{1,2\}$
$b(j)$: the capacity of airport j, $j \in \{1,2\}$
$c(i,j)$: the driving time from city i to airport j

To find:
$x(i,j)$: the passengers traveling from city i to airport j
z: the total travel cost

Model:

$$\min \sum_{i,j} \{x(i,j) \times c(i,j)\}$$

Subject to

$$\sum_{j} x(i,j) = a(i), \forall i$$
$$\sum_{i} x(i,j) = b(j), \forall i$$
$$x(i,j) \geq 0$$

Trial 1: The travel Times were assumed as following

From	To	Travel Time
1	1	1
1	2	2
2	1	2
2	2	2

Codes used in Gams:

```
$title Transportation problem

Set i /1*2/;

alias (i, j);

parameter a(i)/

1 100
```

```
2 200
/;

parameter b(j)/
1 150
2 150
/;

parameter c(i,j)/
1. 1   1
1. 2   2
2. 1   2
2. 2   2
/;

variable z;
positive variables  x(i,j);

equations
obj
demand(i)
supply(j)
;

obj.. z =e= sum((i,j),c(i,j)*x(i,j));
demand(i).. sum(j,x(i,j)) =e= a(i);
supply(j).. sum(i,x(i,j)) =e= b(j);

Model problem_2 /all/;
```

solve problem_2 using LP minimizing z;

display x.l;

display z.l;

The results:

```
----     39 VARIABLE x.L

            1           2

1     100.000
2      50.000     150.000


----     40 VARIABLE z.L                  =      500.000


EXECUTION TIME       =        0.578 SECONDS      4 MB  32.2.0 rc62c018 WEX-WEI
```

Trial 2: The travel Times were assumed as following

From	To	Travel Time
1	1	2
1	2	3
2	1	3
2	2	3

Codes used in Gams:

$title Transportation problem

Set i /1*2/;

alias (i, j);

```
parameter a(i)/
1 100
2 200
/;

parameter b(j)/
1 150
2 150
/;

parameter c(i,j)/
1. 1   2
1. 2   3
2. 1   3
2. 2   3
/;

variable z;
positive variables  x(i,j);

equations
obj
demand(i)
supply(j)
;

obj.. z =e= sum((i,j),c(i,j)*x(i,j));
demand(i).. sum(j,x(i,j)) =e= a(i);
supply(j).. sum(i,x(i,j)) =e= b(j);
```

Model problem_2 /all/;

solve problem_2 using LP minimizing z;

display x.l;

display z.l;

The results:

```
----     39 VARIABLE x.L

            1           2

1     100.000
2      50.000     150.000


----     40 VARIABLE z.L                   =      800.000


EXECUTION TIME       =        0.672 SECONDS      4 MB  32.2.0 rc62c018 WEX-WEI
```

Shortest path problem

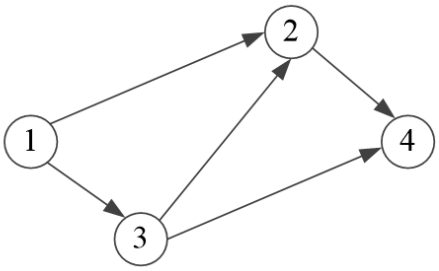
Minimize

$$\sum_{ij \in A} w_{ij} x_{ij}$$

Subject to

$$\sum_j x_{ij} - \sum_j x_{ji} = \begin{cases} 1, & \text{if } i = s; \\ -1, & \text{if } i = t; \\ 0, & \text{otherwise.} \end{cases}$$

$$x \geq 0$$



Trial 1: Assuming default length = 1 for all links

Codes used in Gams:

$title Shortest Path Problem

*LIMROW = 0, LIMCOL = 0

*OPTIONS ITERLIM=100000, RESLIM = 1000000, SYSOUT = OFF, SOLPRINT = OFF, lp = COINGLPK, mip = COINGLPK, OPTCR= 0.1;

set i nodes /1*4/;

alias (i, j);

parameter w(i,j) link length /

1. 2 1

1. 3 1

2. 4 1

3. 4 1

3. 2 1

/;

```
parameter origin(i);
origin('1') = 1;

parameter destination(i);
destination('4') = 1;

parameter intermediate_node(i);
intermediate_node(i) = (1- origin(i))*(1- destination(i));

variable z;
positive variables
x(i,j)  selection of flow between i and j;

equations
so_obj                  define objective function
flow_on_node_origin
flow_on_node_intermediate(i)
flow_on_node_destination
;
so_obj.. z =e= sum((i,j)$(w(i,j)),w(i,j)*x(i,j));
flow_on_node_origin.. sum(j$(w('1',j)), x('1',j)) =e= 1;
flow_on_node_intermediate(i)$(intermediate_node(i)=1).. sum(j$(w(i,j)), x(i,j))-sum(j$(w(j,i)), x(j,i))=e= 0;
flow_on_node_destination..  sum(j$(w(j,'4')), x(j,'4'))=e= 1;
Model shortest_path_problem /all/ ;
solve shortest_path_problem using LP minimizing z;
display x.l;
display z.l;
```

Results:

```
----     45 VARIABLE x.L  selection of flow between i and j

            2           4

1       1.000
2                   1.000


----     46 VARIABLE z.L                  =        2.000


EXECUTION TIME       =        0.422 SECONDS      4 MB  32.2.0 rc62c018 WEX-WEI
```

We can see that the shortest path in this case: 1-2-4

Trial 2: The length selected as following

From	To	length
1	2	4
1	3	2
2	4	2
3	4	4
3	2	5

Codes used in Gams:

```
$title Shortest Path Problem

*LIMROW = 0, LIMCOL = 0

*OPTIONS  ITERLIM=100000, RESLIM = 1000000, SYSOUT = OFF, SOLPRINT = OFF, lp = COINGLPK, mip = COINGLPK, OPTCR= 0.1;


set i nodes /1*4/;

alias (i, j);
```

```
parameter w(i,j) link length /
1. 2   4
1. 3   2
2. 4   2
3. 4   4
3. 2   5
/;

parameter origin(i);
origin('1') = 1;

parameter destination(i);
destination('4') = 1;

parameter intermediate_node(i);
intermediate_node(i) = (1- origin(i))*(1- destination(i));

variable z;
positive variables
x(i,j)  selection of flow between i and j;

equations
so_obj                          define objective function
flow_on_node_origin
flow_on_node_intermediate(i)
flow_on_node_destination
;
so_obj.. z =e= sum((i,j)$(w(i,j)),w(i,j)*x(i,j));
flow_on_node_origin.. sum(j$(w('1',j)), x('1',j)) =e= 1;
flow_on_node_intermediate(i)$(intermediate_node(i)=1).. sum(j$(w(i,j)), x(i,j))-sum(j$(w(j,i)), x(j,i))=e= 0;
flow_on_node_destination..  sum(j$(w(j,'4')), x(j,'4'))=e= 1;
Model shortest_path_problem /all/ ;
```

solve shortest_path_problem using LP minimizing z;

display x.l;

display z.l;

Results

```
----     45 VARIABLE x.L  selection of flow between i and j

            2           4

1       1.000
2                   1.000


----     46 VARIABLE z.L                   =        6.000

EXECUTION TIME       =        0.406 SECONDS      4 MB  32.2.0 rc62c018 WEX-WEI
```

We can see that the shortest path in this case: 1-2-4

Trial 3: The length selected as following

From	To	length
1	2	5
1	3	4
2	4	5
3	4	2
3	2	5

Codes used in Gams:

$title Shortest Path Problem

*LIMROW = 0, LIMCOL = 0

```
*OPTIONS  ITERLIM=100000, RESLIM = 1000000, SYSOUT = OFF, SOLPRINT = OFF, lp = COINGLPK, mip = COINGLPK, OPTCR= 0.1;

set i nodes /1*4/;

alias (i, j);

parameter w(i,j) link length /

1. 2   5

1. 3   4

2. 4   5

3. 4   2

3. 2   5

/;

parameter origin(i);

origin('1') = 1;

parameter destination(i);

destination('4') = 1;

parameter intermediate_node(i);

intermediate_node(i) = (1- origin(i))*(1- destination(i));

variable z;

positive variables

x(i,j)  selection of flow between i and j;

equations

so_obj                    define objective function

flow_on_node_origin

flow_on_node_intermediate(i)

flow_on_node_destination
```

;

so_obj.. z =e= sum((i,j)$(w(i,j)),w(i,j)*x(i,j));

flow_on_node_origin.. sum(j$(w('1',j)), x('1',j)) =e= 1;

flow_on_node_intermediate(i)$(intermediate_node(i)=1).. sum(j$(w(i,j)), x(i,j))-sum(j$(w(j,i)), x(j,i))=e= 0;

flow_on_node_destination.. sum(j$(w(j,'4')), x(j,'4'))=e= 1;

Model shortest_path_problem /all/ ;

solve shortest_path_problem using LP minimizing z;

display x.l;

display z.l;

Results

```
----     45 VARIABLE x.L  selection of flow between i and j

            3           4

1       1.000
3                   1.000


----     46 VARIABLE z.L                   =        6.000


EXECUTION TIME       =        0.188 SECONDS      4 MB  32.2.0 rc62c018 WEX-WEI
```

We can see that the shortest path in this case: 1-3-4