Matrix Theory EE5609 - Assignment 10

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***Abstract*—Find rank of the matrix.**

Download python code from

https://github.com/SANDHYA−A/Assignment10

1 Problem

Let J denote the matrix of order $n \times n$ with all entries 1 and let B be a $3n \times 3n$ matrix given by $B = \begin{pmatrix} 0 & 0 & J \\ 0 & J & 0 \\ J & 0 & 0 \end{pmatrix}$.
Find rank of matrix B.

2 Solution

Let J be an $n \times n$ with all entries 1. Matrix B is given as:-

$$B = \begin{pmatrix} 0 & 0 & J \\ 0 & J & 0 \\ J & 0 & 0 \end{pmatrix}$$

The rank of a matrix is given by the number of linearly independent row vectors in the matrix.We can observe that n rows of matrix B, formed by matrix J are identical, hence there are three linearly independent row vectors in matrix B.
$\therefore$ the rank of matrix B is 3.

Example:

$$\text{Let, } n = 3 \tag{2.0.1}$$

$$J = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \tag{2.0.2}$$

$$B = \begin{pmatrix} 0 & 0 & J \\ 0 & J & 0 \\ J & 0 & 0 \end{pmatrix} \tag{2.0.3}$$

$$= \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{2.0.4}$$

We can observe that there are three linearly independent row vectors in the above matrix.
Hence, rank of the matrix B is 3.