Matrix Theory EE5609 - Assignment 10

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***Abstract*—Find rank of the matrix.**

Download python code from

https://github.com/SANDHYA−A/Assignment10

1 Problem

Let J denote the matrix of order $n \times n$ with all entries 1 and let B be a $3n \times 3n$ matrix given by $B = \begin{pmatrix} 0 & 0 & J \\ 0 & J & 0 \\ J & 0 & 0 \end{pmatrix}$.
Find rank of matrix B.

2 Solution

Let J be an $n \times n$ with all entries 1. Matrix B is given as:-
$B = \begin{pmatrix} 0 & 0 & J \\ 0 & J & 0 \\ J & 0 & 0 \end{pmatrix}$
We can observe that n rows of matrix B, formed by the matrix J are identical. Hence, by row reduction, they can be reduced to single non-zero row. By row reduction on the matrix B, we obtain 3 unique non-zero rows.
$\therefore$ the rank of matrix B is 3.

Example:

$$\text{Let, } n = 3 \tag{2.0.1}$$

$$J = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \tag{2.0.2}$$

$$B = \begin{pmatrix} 0 & 0 & J \\ 0 & J & 0 \\ J & 0 & 0 \end{pmatrix} \tag{2.0.3}$$

$$= \begin{pmatrix} 0&0&0&0&0&0&1&1&1 \\ 0&0&0&0&0&0&1&1&1 \\ 0&0&0&0&0&0&1&1&1 \\ 0&0&0&1&1&1&0&0&0 \\ 0&0&0&1&1&1&0&0&0 \\ 0&0&0&1&1&1&0&0&0 \\ 1&1&1&0&0&0&0&0&0 \\ 1&1&1&0&0&0&0&0&0 \\ 1&1&1&0&0&0&0&0&0 \end{pmatrix} \tag{2.0.4}$$

By row reduction, we obtain,

$$\xrightarrow[R_2 \to R_2 - R_1]{\substack{R_9 \to R_9 - R_7 \\ R_8 \to R_8 - R_7 \\ R_5 \to R_5 - R_4 \\ R_6 \to R_6 - R_4 \\ R_3 \to R_3 - R_1}} \begin{pmatrix} 0&0&0&0&0&0&1&1&1 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&1&1&1&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 1&1&1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \end{pmatrix} \tag{2.0.5}$$

$$\xrightarrow{\substack{R_4 \leftrightarrow R_2 \\ R_3 \leftrightarrow R_7}} \begin{pmatrix} 0&0&0&0&0&0&1&1&1 \\ 0&0&0&1&1&1&0&0&0 \\ 1&1&1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \end{pmatrix} \tag{2.0.6}$$

Hence, rank of the matrix B is 3.