Matrix Theory EE5609 - Assignment 10

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***Abstract*—Find rank of the matrix.**

Download python code from

https://github.com/SANDHYA−A/Assignment10

1 Problem

Let J denote the matrix of order $n \times n$ with all entries 1 and let B be a $3n \times 3n$ matrix given by $B = \begin{pmatrix} 0 & 0 & J \\ 0 & J & 0 \\ J & 0 & 0 \end{pmatrix}$.
Find rank of matrix B.

2 Solution

Given	a) Matrix J of $n\times n$ dimension with all entries 1. b) Matrix B of $3n\times3n$ dimension $$B = \begin{pmatrix} 0 & 0 & J \\ 0 & J & 0 \\ J & 0 & 0 \end{pmatrix}$$
Transforming matrix B into Block diagonal matrix using transformation Matrix	$$M = \mathbf{T}(B)$$ $$M = \begin{pmatrix} 0 & 0 & I \\ 0 & I & 0 \\ I & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 & J \\ 0 & J & 0 \\ J & 0 & 0 \end{pmatrix}$$ $$M = \begin{pmatrix} J & 0 & 0 \\ 0 & J & 0 \\ 0 & 0 & J \end{pmatrix}$$
Rank of Block Diagonal matrix M	It is equal to the sum of rank of individual blocks in diagonal $$r(J) = 1$$ $$\therefore r(M) = 1 + 1 + 1 = 3$$
Rank of a matrix and its transformation are same.	$\therefore$ rank of matrix B is $$r(B) = r(M) = 3$$

Example	Let $n = 2$ $$J = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$ $$B = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$
Transforming matrix B into Block diagonal matrix using transformation Matrix	$$M = \mathbf{T}(B)$$ $$M = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$ $$M = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$
Rank of Block Diagonal matrix M	It is equal to the sum of rank of individual blocks in diagonal $$r(J) = 1$$ $$\therefore r(M) = 1 + 1 + 1 = 3$$
Rank of a matrix and its transformation are same.	$\therefore$ rank of matrix B is $$r(B) = r(M) = 3$$

Approach 2:

Example	Let $n = 2$ $$J = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$ $$B = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$
Representing matrix B as Jordan normal form	$$B = \mathbf{S}(N)\mathbf{S}^{-1}$$ $$B = \begin{pmatrix} -1 & 0 & 0 & 0 & 1 & -1 \\ 1 & 0 & 0 & 0 & 1 & -1 \\ 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 1 \\ 0 & 0 & -1 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & -2 \end{pmatrix} \begin{pmatrix} \frac{-1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{-1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{-1}{2} & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{-1}{4} & \frac{-1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$$
The matrix N in Jordan Normal form is	$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & -2 \end{pmatrix}$$
Rank of Block Diagonal matrix N	It is equal to the sum of rank of individual blocks in diagonal $$\therefore r(N) = 3$$
Rank of a matrix B and its Jordan Normal form are same.	$\therefore$ rank of matrix B is $$r(B) = r(N) = 3$$