

Matrix Theory EE5609 - Assignment 2

Find Inverse of a matrix using Elementary transformations

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Abstract—This document provides a solution for finding inverse of a matrix using elementary transformations.

Download all python codes from

<https://github.com/SANDHYA-A/Codes/blob/master/Assignment2.py>

I. PROBLEM STATEMENT

Using elementary transformations, find the inverse of the matrix $A = \begin{pmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{pmatrix}$, if it exists.

II. THEORY

For any $n \times n$ matrix A , if the augmented matrix $[A|I]$ is transformed into a matrix of the form $[I|B]$, then the matrix A is invertible and the inverse matrix A^{-1} is given by B . If the reduced row echelon form matrix for $[A|I]$ is not of the form $[I|B]$, then the matrix A is not invertible.

III. SOLUTION

The augmented matrix $[A|I]$ is as given below:-

$$\begin{pmatrix} 2 & 0 & -1 & 1 & 0 & 0 \\ 5 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 3 & 0 & 0 & 1 \end{pmatrix} \quad (1)$$

We apply the elementary row operations on $[A|I]$ as follows :-

$$[A|I] = \begin{pmatrix} 2 & 0 & -1 & 1 & 0 & 0 \\ 5 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 3 & 0 & 0 & 1 \end{pmatrix} \quad (2)$$

$$\xrightarrow{R_2 \leftarrow 2R_2 - 5R_1} \begin{pmatrix} 2 & 0 & -1 & 1 & 0 & 0 \\ 0 & 2 & 5 & -5 & 2 & 0 \\ 0 & 1 & 3 & 0 & 0 & 1 \end{pmatrix} \quad (3)$$

$$\xrightarrow{R_3 \leftarrow 2R_3 - R_2} \begin{pmatrix} 2 & 0 & -1 & 1 & 0 & 0 \\ 0 & 2 & 5 & -5 & 2 & 0 \\ 0 & 0 & 1 & 5 & -2 & 2 \end{pmatrix} \quad (4)$$

$$\xrightarrow{\begin{matrix} R_1 \leftarrow \frac{R_1}{2} \\ R_2 \leftarrow \frac{R_2}{2} \end{matrix}} \begin{pmatrix} 1 & 0 & -\frac{1}{2} & \frac{-1}{2} & 0 & 0 \\ 0 & 1 & \frac{5}{2} & \frac{-5}{2} & 1 & 0 \\ 0 & 0 & 1 & 5 & -2 & 2 \end{pmatrix} \quad (5)$$

$$\xrightarrow{\begin{matrix} R_2 \leftarrow R_2 - \frac{5}{2}R_3 \\ R_1 \leftarrow R_1 + \frac{R_3}{2} \end{matrix}} \begin{pmatrix} 1 & 0 & 0 & 3 & -1 & 1 \\ 0 & 1 & 0 & -15 & 6 & -5 \\ 0 & 0 & 1 & 5 & -2 & 2 \end{pmatrix} \quad (6)$$

IV. CONCLUSION

By performing elementary transformations on augmented matrix $[A|I]$, we obtained the augmented matrix in the form $[I|B]$. Hence we can conclude that the matrix A is invertible and inverse of the matrix is:-

$$A^{-1} = \begin{pmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{pmatrix} \quad (7)$$