

# Matrix Theory EE5609 - Assignment 2

## Find Inverse of a matrix using Elementary transformations

Sandhya Addetla  
PhD Artificial Intelligence Department  
15-Sep-2020  
AI20RESCH14001

**Abstract**—This document provides a solution for finding inverse of a matrix using elementary transformations.

### I. PROBLEM STATEMENT

Using elementary transformations, find the inverse of the matrix  $A = \begin{pmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{pmatrix}$ , if it exists.

### II. THEORY

For any  $n \times n$  matrix  $A$ , if the augmented matrix  $[A|I]$  is transformed into a matrix of the form  $[I|B]$ , then the matrix  $A$  is invertible and the inverse matrix  $A^{-1}$  is given by  $B$ . If the reduced row echelon form matrix for  $[A|I]$  is not of the form  $[I|B]$ , then the matrix  $A$  is not invertible.

### III. SOLUTION

The augmented matrix  $[A|I]$  is as given below:-

$$\begin{pmatrix} 2 & 0 & -1 & 1 & 0 & 0 \\ 5 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 3 & 0 & 0 & 1 \end{pmatrix} \quad (1)$$

We apply the elementary row operations on  $[A|I]$  as follows :-

$$[A|I] = \begin{pmatrix} 2 & 0 & -1 & 1 & 0 & 0 \\ 5 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 3 & 0 & 0 & 1 \end{pmatrix} \quad (2)$$

$$\xrightarrow{R_2 \rightarrow 2R_2 - 5R_1} \begin{pmatrix} 2 & 0 & -1 & 1 & 0 & 0 \\ 0 & 2 & 5 & -5 & 2 & 0 \\ 0 & 1 & 3 & 0 & 0 & 1 \end{pmatrix} \quad (3)$$

$$\xrightarrow{R_3 \rightarrow 2R_3 - R_2} \begin{pmatrix} 2 & 0 & -1 & 1 & 0 & 0 \\ 0 & 2 & 5 & -5 & 2 & 0 \\ 0 & 0 & 1 & 5 & -2 & 2 \end{pmatrix} \quad (4)$$

$$\xrightarrow{\begin{matrix} R_1 \rightarrow \frac{R_1}{2} \\ R_2 \rightarrow \frac{R_2}{2} \end{matrix}} \begin{pmatrix} 1 & 0 & -\frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 1 & \frac{5}{2} & -\frac{5}{2} & 1 & 0 \\ 0 & 0 & 1 & 5 & -2 & 2 \end{pmatrix} \quad (5)$$

$$\xrightarrow{\begin{matrix} R_1 \rightarrow R_1 + \frac{R_3}{2} \\ R_2 \rightarrow R_2 - \frac{5}{2}R_3 \end{matrix}} \begin{pmatrix} 1 & 0 & 0 & 3 & -1 & 1 \\ 0 & 1 & 0 & -15 & 6 & -5 \\ 0 & 0 & 1 & 5 & -2 & 2 \end{pmatrix} \quad (6)$$

$$(7)$$

### IV. CONCLUSION

By performing elementary transformations on augmented matrix  $[A|I]$ , we obtained the augmented matrix in the form  $[I|B]$ . Hence we can conclude that the matrix  $A$  is invertible and inverse of the matrix is:-

$$A^{-1} = \begin{pmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{pmatrix} \quad (8)$$