Matrix Theory EE5609 - Assignment 2 Find Inverse of a matrix using Elementary transformations

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***Abstract*—This document provides a solution for finding inverse of a matrix using elementary transformations.**

I. Problem Statement

Using elementary transformations, find the inverse of the matrix A = $\begin{pmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{pmatrix}$, if it exists.

II. Theory

For any n $\times$ n matrix A, if the augmented matrix $[A|I]$ is transformed into a matrix of the form$[I|B]$, then the matrix A is invertible and the inverse matrix A^{-1} is given by B. If the reduced row echelon form matrix for$[A|I]$ is not of the form $[I|B]$, then the matrix A is not invertible.

III. Solution

The augmented matrix $[A|I]$ is as given below:-

$$\begin{pmatrix} 2 & 0 & -1 & 1 & 0 & 0 \\ 5 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 3 & 0 & 0 & 1 \end{pmatrix} \tag{1}$$

We apply the elementary row operations on $[A|I]$ as follows :-

$$[A|I] = \begin{pmatrix} 2 & 0 & -1 & 1 & 0 & 0 \\ 5 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 3 & 0 & 0 & 1 \end{pmatrix} \tag{2}$$

$$\xrightarrow{R_2 \leftarrow 2R_2 - 5R_1} \begin{pmatrix} 2 & 0 & -1 & 1 & 0 & 0 \\ 0 & 2 & 5 & -5 & 2 & 0 \\ 0 & 1 & 3 & 0 & 0 & 1 \end{pmatrix} \tag{3}$$

$$\xrightarrow{R_3 \leftarrow 2R_3 - R_2} \begin{pmatrix} 2 & 0 & -1 & 1 & 0 & 0 \\ 0 & 2 & 5 & -5 & 2 & 0 \\ 0 & 0 & 1 & 5 & -2 & 2 \end{pmatrix} \tag{4}$$

$$\xleftrightarrow[R_2 \leftarrow \frac{R_2}{2}]{R_1 \leftarrow \frac{R_1}{2}} \begin{pmatrix} 1 & 0 & \frac{-1}{2} & \frac{-1}{2} & 0 & 0 \\ 0 & 1 & \frac{5}{2} & \frac{-5}{2} & 1 & 0 \\ 0 & 0 & 1 & 5 & -2 & 2 \end{pmatrix} \tag{5}$$

$$\xleftrightarrow[R_1 \leftarrow R_1 + \frac{R_3}{2}]{R_2 \leftarrow R_2 - \frac{5}{2}R_3} \begin{pmatrix} 1 & 0 & 0 & 3 & -1 & 1 \\ 0 & 1 & 0 & -15 & 6 & -5 \\ 0 & 0 & 1 & 5 & -2 & 2 \end{pmatrix} \tag{6}$$

IV. Conclusion

By performing elementary transormations on augmented matrix$[A|I]$, we obtained the augmented matrix in the form $[I|B]$. Hence we can conclude that the matrix A is invertible and inverse of the matrix is:-

$$A^{-1} = \begin{pmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{pmatrix} \tag{7}$$