Matrix Theory EE5609 - Assignment 4

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***Abstract*—This document finds the point at which a line touches a circle.**

Download all python codes from

https://github.com/SANDHYA−A/Assignment4/blob/master/Assignment4.py

1 Problem Statement

For what values of m does the line

$$\begin{pmatrix} m & -1 \end{pmatrix} \mathbf{x} = 0 \tag{1.0.1}$$

touch the circle

$$\mathbf{x}^T\mathbf{x} - \begin{pmatrix} 6 & 2 \end{pmatrix} \mathbf{x} + 8 = 0 \tag{1.0.2}$$

2 Solution

The general equation of a circle is

$$\implies \mathbf{x}^T\mathbf{x} + 2\mathbf{u}^T\mathbf{x} + f = 0 \tag{2.0.1}$$

$$\text{If } r \text{ is radius,} f = \mathbf{u}^T\mathbf{u} - r^2 \tag{2.0.2}$$

$$\text{center } \mathbf{c} = -\mathbf{u} \tag{2.0.3}$$

From equations 1.0.2 and 2.0.1,

$$\mathbf{u} = \begin{pmatrix} -3 \\ -1 \end{pmatrix} \tag{2.0.4}$$

$$\implies \text{Center of the cirlce } \mathbf{c} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \tag{2.0.5}$$

Also, radius can be determined as follows from 2.0.2

$$\implies 8 = \begin{pmatrix} -3 & -1 \end{pmatrix} \begin{pmatrix} -3 \\ -1 \end{pmatrix} - r^2 \tag{2.0.6}$$

$$\implies 8 = 10 - r^2 \tag{2.0.7}$$

$$\implies r = \sqrt{2} \tag{2.0.8}$$

Given equation of the line is

$$\begin{pmatrix} m & -1 \end{pmatrix} \mathbf{x} = 0 \tag{2.0.9}$$

It can be expressed as:-

$$L: \quad \mathbf{x} = \mathbf{q} + \mu\mathbf{m} \quad \mu \in \mathbb{R} \tag{2.0.10}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ mx \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ m \end{pmatrix} \tag{2.0.11}$$

$$\mathbf{q} = \begin{pmatrix} x \\ mx \end{pmatrix} \tag{2.0.12}$$

$$\mathbf{m} = \begin{pmatrix} 1 \\ m \end{pmatrix} \tag{2.0.13}$$

If the line L is a tangent to the circle, then:-

$$\mathbf{m}^T(\mathbf{V}\mathbf{q} + \mathbf{u}) = 0 \tag{2.0.14}$$

$$\mathbf{V} = \mathbf{I} \text{ for a circle} \tag{2.0.15}$$

$$\begin{pmatrix} 1 \\ m \end{pmatrix} \left[\begin{pmatrix} x \\ mx \end{pmatrix} + \begin{pmatrix} -3 \\ -1 \end{pmatrix} \right] = 0 \tag{2.0.16}$$

$$(x-3) + m(mx-1) = 0 \tag{2.0.17}$$

$$x(1+m^2) - m - 3 = 0 \tag{2.0.18}$$

The point $\mathbf{q}$ also satisfies the equation of the circle at 1.0.2.

$$\mathbf{q}^T\mathbf{q} + 2\mathbf{u}^T\mathbf{q} + f = 0 \tag{2.0.19}$$

$$\mathbf{q}^T\mathbf{q} - \begin{pmatrix} 6 & 2 \end{pmatrix} \mathbf{q} + 8 = 0 \tag{2.0.20}$$

$$\begin{pmatrix} x & mx \end{pmatrix} \begin{pmatrix} x \\ mx \end{pmatrix} - \begin{pmatrix} 6 & 2 \end{pmatrix} \begin{pmatrix} x \\ mx \end{pmatrix} + 8 = 0 \tag{2.0.21}$$

$$x^2 + m^2x^2 - 6x - 2m^2 + 8 = 0 \tag{2.0.22}$$

$$x^2(1+m^2) - 6x - 2m^2 + 8 = 0 \tag{2.0.23}$$

From equations 2.0.18 and 2.0.23, by substitution, we get

$$x = \frac{m+3}{1+m^2} \tag{2.0.24}$$

$$x^2(1+m^2) - 2x(m+3) + 8 = 0 \tag{2.0.25}$$

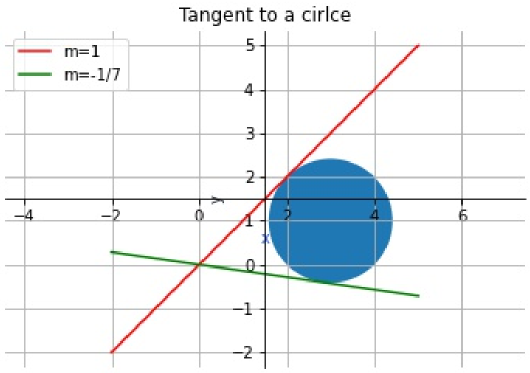

Fig. 1: Circle with tangent

$$\frac{(m+3)^2}{(1+m^2)^2}((1+m^2) - 2\frac{(m+3)}{(1+m^2)}(m+3)+8=0 \quad (2.0.26)$$

$$(m+3)^2 - 2(m+3)^2 + 8 = 0 \quad (2.0.27)$$

$$7m^2 - 6m - 1 = 0 \quad (2.0.28)$$

$$m = 1, -1/7 \quad (2.0.29)$$

For the line 1.0.1 to be a tangent to circle at equation 1.0.2, the values of m are $1, -\frac{1}{7}$