Matrix Theory EE5609 - Assignment 4

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***Abstract*—This document finds the point at which a line touches a circle.**

Download all python codes from

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https://github.com/SANDHYA-A/Assignment4/
    blob/master/Assignment4.py
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1 Problem Statement

For what values of m does the line

$$\begin{pmatrix} m & -1 \end{pmatrix} \mathbf{x} = 0 \tag{1.0.1}$$

touch the circle

$$\mathbf{x}^T\mathbf{x} - \begin{pmatrix} 6 & 2 \end{pmatrix} \mathbf{x} + 8 = 0 \tag{1.0.2}$$

2 Solution

The general equation of a circle is

$$\implies \mathbf{x}^T\mathbf{x} + 2\mathbf{u}^T\mathbf{x} + f = 0 \tag{2.0.1}$$

$$\text{If } r \text{ is radius,} f = \mathbf{u}^T\mathbf{u} - r^2 \tag{2.0.2}$$

$$\text{center } \mathbf{c} = -\mathbf{u} \tag{2.0.3}$$

From equations 1.0.2 and 2.0.1,

$$\mathbf{u} = \begin{pmatrix} -3 \\ -1 \end{pmatrix} \tag{2.0.4}$$

$$\implies \text{Center of the cirlce } \mathbf{c} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \tag{2.0.5}$$

Also, radius can be determined as follows from 2.0.2

$$\implies 8 = \begin{pmatrix} -3 & -1 \end{pmatrix} \begin{pmatrix} -3 \\ -1 \end{pmatrix} - r^2 \tag{2.0.6}$$

$$\implies 8 = 10 - r^2 \tag{2.0.7}$$

$$\implies r = \sqrt{2} \tag{2.0.8}$$

Given equation of the line is

$$\begin{pmatrix} m & -1 \end{pmatrix} \mathbf{x} = 0 \tag{2.0.9}$$

It can be expressed as:-

$$L: \quad \mathbf{x} = \mathbf{q} + \mu\mathbf{m} \quad \mu \in \mathbb{R} \tag{2.0.10}$$

$$\tag{2.0.11}$$

The normal vector to the line is obtained as

$$\mathbf{n} = \mathbf{q} + \mathbf{u} \tag{2.0.12}$$

$$\implies \mathbf{q} = \mathbf{n} - \mathbf{u} \tag{2.0.13}$$

$$\mathbf{n} = \begin{pmatrix} m & -1 \end{pmatrix}^T \text{ and } \mathbf{u} = \begin{pmatrix} -3 & -1 \end{pmatrix}^T \tag{2.0.14}$$

$$\mathbf{q} = \begin{pmatrix} m+3 \\ 0 \end{pmatrix} \tag{2.0.15}$$

The point $\mathbf{q}$ also satisfies the equation of the circle at 1.0.2.

$$\mathbf{q}^T\mathbf{q} + 2\mathbf{u}^T\mathbf{q} + f = 0 \tag{2.0.16}$$

$$\tag{2.0.17}$$

$$(\mathbf{n}-\mathbf{u})^T(\mathbf{n}-\mathbf{u}) + 2\mathbf{u}^T(\mathbf{n}-\mathbf{u}) + f = 0 \tag{2.0.18}$$

$$\|\mathbf{n}\|^2 - \mathbf{n}^T\mathbf{u} - \mathbf{u}^T\mathbf{n} + \|\mathbf{u}\|^2 + 2\mathbf{u}^T\mathbf{n} - 2\|\mathbf{u}\|^2 + f = 0 \tag{2.0.19}$$

$$\|\mathbf{n}\|^2 - \mathbf{n}^T\mathbf{u} + \mathbf{u}^T\mathbf{n} - \|\mathbf{u}\|^2 + f = 0 \tag{2.0.20}$$

$$(m^2+1) - (-3m+1) + (-3m+1) - 10 + 8 = 0 \tag{2.0.21}$$

$$m^2 - 1 = 0 \tag{2.0.22}$$

$$m = \pm 1 \tag{2.0.23}$$

For the line 1.0.1 to be a tangent to circle at equation 1.0.2, the values of m are ± 1

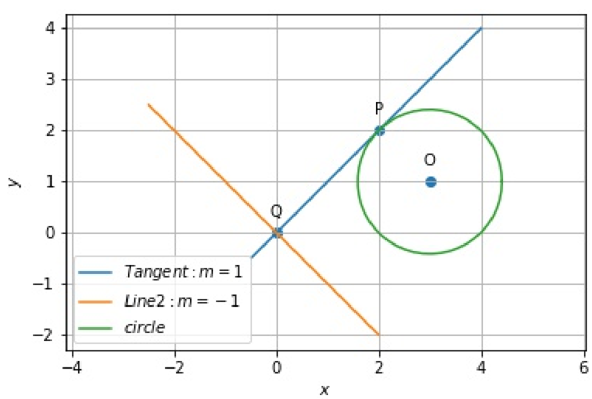

Fig. 1: Circle with tangent