Matrix Theory EE5609 - Assignment 4
Find point were the line is a tangent to Circle

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***Abstract*—This document finds the value of m for which the line**

$$\begin{pmatrix} m & -1 \end{pmatrix} \mathbf{x} = 0 \tag{1}$$

touches the circle

$$\mathbf{x}^T\mathbf{x} - \begin{pmatrix} 6 & 2 \end{pmatrix} \mathbf{x} + 8 = 0 \tag{2}$$

Download all python codes from

https://github.com/SANDHYA−A/Assignment4/blob/master/Assignment4.py

I. Problem Statement

For what values of m does the line

$$\begin{pmatrix} m & -1 \end{pmatrix} \mathbf{x} = 0 \tag{3}$$

touch the circle

$$\mathbf{x}^T\mathbf{x} - \begin{pmatrix} 6 & 2 \end{pmatrix} \mathbf{x} + 8 = 0 \tag{4}$$

II. Solution

The general equation of a circle is

$$\implies \mathbf{x}^T\mathbf{x} - 2\mathbf{O}^T\mathbf{x} + \|\mathbf{O}\|^2 - r^2 = 0 \tag{5}$$

Where $\mathbf{O}$ is the centre and r is the radius of the circle. Observing equation 4 and 5, we get , the center of the circle $\mathbf{O} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$. Also,

$$\|\mathbf{O}\|^2 - r^2 = 8 \tag{6}$$
$$(3^2 + 1^2) - r^2 = 8 \tag{7}$$
$$r^2 = 2 \tag{8}$$
$$r = \sqrt{2} \tag{9}$$

For the line $\begin{pmatrix} m & -1 \end{pmatrix} \mathbf{x} = 0$ to be a tangent, the distance between the center of the circle $\mathbf{O} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ and the tangent point on the line is equal to the radius of the circle.

Also, distance d between a point $P = (x_0, y_0)$ and line $L : ax + by + c = 0$ is given by

$$d = \frac{\|ax_0 + by_0 + c\|}{\sqrt{(a^2 + b^2)}} \tag{10}$$

Here, equation of the line(tangent) is

$$mx - y = 0 \tag{11}$$

using equation 10 format for the existing problem, we get values

$$a = m; b = -1; c = 0; \tag{12}$$
$$P = (x_0, y_0) = (3, 1) \tag{13}$$
$$d = \sqrt{2} \tag{14}$$

Substituting these values in equation 10, we get,

$$\sqrt{2} = \frac{\|m \times 3 + (-1) \times 1 + 0\|}{\sqrt{m^2 + (-1)^2}} \tag{15}$$

Applying square on both sides,

$$2 = \frac{\|3m - 1\|^2}{m^2 + 1} \tag{16}$$
$$7m^2 - 6m - 1 = 0 \tag{17}$$
$$m = 1, -\frac{1}{7} \tag{18}$$

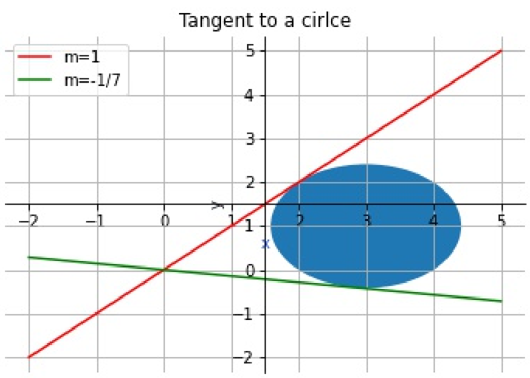

Fig. 1: Circle with tangent