

# Matrix Theory EE5609 - Assignment 4

## Find point where the line is a tangent to Circle

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**Abstract**—This document finds the value of  $m$  for which the line

$$(m - 1)x = 0 \quad (1)$$

touches the circle

$$\mathbf{x}^T \mathbf{x} - (6 \ 2) \mathbf{x} + 8 = 0 \quad (2)$$

Download all python codes from

<https://github.com/SANDHYA-A/Assignment4/blob/master/Assignment4.py>

### I. PROBLEM STATEMENT

For what values of  $m$  does the line

$$(m - 1)x = 0 \quad (3)$$

touch the circle

$$\mathbf{x}^T \mathbf{x} - (6 \ 2) \mathbf{x} + 8 = 0 \quad (4)$$

### II. SOLUTION

The general equation of a circle is

$$\Rightarrow \mathbf{x}^T \mathbf{x} - 2\mathbf{O}^T \mathbf{x} + \|\mathbf{O}\|^2 - r^2 = 0 \quad (5)$$

Where  $\mathbf{O}$  is the centre and  $r$  is the radius of the circle. Observing equation 4 and 5, we get, the center of the circle  $\mathbf{O} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ . Also,

$$\|\mathbf{O}\|^2 - r^2 = 8 \quad (6)$$

$$(3^2 + 1^2) - r^2 = 8 \quad (7)$$

$$r^2 = 2 \quad (8)$$

$$r = \sqrt{2} \quad (9)$$

For the line  $(m - 1)x = 0$  to be a tangent, the distance between the center of the circle  $\mathbf{O} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$  and the tangent point on the line is equal to the radius of the circle.

Also, distance  $d$  between a point  $P = (x_0, y_0)$  and line  $L : ax + by + c = 0$  is given by

$$d = \frac{\|ax_0 + by_0 + c\|}{\sqrt{a^2 + b^2}} \quad (10)$$

Here, equation of the line(tangent) is

$$mx - y = 0 \quad (11)$$

using equation 10 format for the existing problem, we get values

$$a = m; b = -1; c = 0; \quad (12)$$

$$P = (x_0, y_0) = (3, 1) \quad (13)$$

$$d = \sqrt{2} \quad (14)$$

Substituting these values in equation 10, we get,

$$\sqrt{2} = \frac{\|m \times 3 + (-1) \times 1 + 0\|}{\sqrt{m^2 + (-1)^2}} \quad (15)$$

Applying square on both sides,

$$2 = \frac{\|3m - 1\|^2}{m^2 + 1} \quad (16)$$

$$7m^2 - 6m - 1 = 0 \quad (17)$$

$$m = 1, -\frac{1}{7} \quad (18)$$

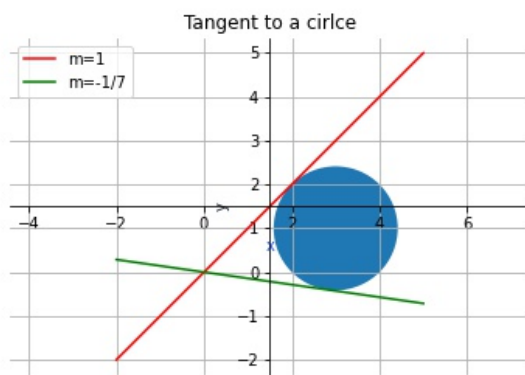


Fig. 1: Circle with tangent