

Matrix Theory EE5609 - Assignment 4

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Abstract—This document finds the point at which a line touches a circle.

Download all python codes from

<https://github.com/SANDHYA-A/Assignment4/blob/master/Assignment4.py>

1 PROBLEM STATEMENT

For what values of m does the line

$$(m \ -1) \mathbf{x} = 0 \quad (1.0.1)$$

touch the circle

$$\mathbf{x}^T \mathbf{x} - (6 \ 2) \mathbf{x} + 8 = 0 \quad (1.0.2)$$

2 SOLUTION

The general equation of a circle is

$$\Rightarrow \mathbf{x}^T \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (2.0.1)$$

$$\text{If } r \text{ is radius, } f = \mathbf{u}^T \mathbf{u} - r^2 \quad (2.0.2)$$

$$\text{center } \mathbf{c} = -\mathbf{u} \quad (2.0.3)$$

From equations 1.0.2 and 2.0.1,

$$\mathbf{u} = \begin{pmatrix} -3 \\ -1 \end{pmatrix} \quad (2.0.4)$$

$$\Rightarrow \text{Center of the circle } \mathbf{c} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \quad (2.0.5)$$

Also, radius can be determined as follows from 2.0.2

$$\Rightarrow 8 = (-3 \ -1) \begin{pmatrix} -3 \\ -1 \end{pmatrix} - r^2 \quad (2.0.6)$$

$$\Rightarrow 8 = 10 - r^2 \quad (2.0.7)$$

$$\Rightarrow r = \sqrt{2} \quad (2.0.8)$$

Given equation of the line is

$$(m \ -1) \mathbf{x} = 0 \quad (2.0.9)$$

It can be expressed as:-

$$L : \mathbf{x} = \mathbf{q} + \mu \mathbf{m} \quad \mu \in \mathbb{R} \quad (2.0.10)$$

$$(2.0.11)$$

The normal vector to the line is obtained as

$$\mathbf{n} = \mathbf{q} + \mathbf{u} \quad (2.0.12)$$

$$\Rightarrow \mathbf{q} = \mathbf{n} - \mathbf{u} \quad (2.0.13)$$

$$\mathbf{n} = (m \ -1)^T \text{ and } \mathbf{u} = (-3 \ -1)^T \quad (2.0.14)$$

$$\mathbf{q} = \begin{pmatrix} m+3 \\ 0 \end{pmatrix} \quad (2.0.15)$$

The point \mathbf{q} also satisfies the equation of the circle at 1.0.2.

$$\mathbf{q}^T \mathbf{q} + 2\mathbf{u}^T \mathbf{q} + f = 0 \quad (2.0.16)$$

$$(2.0.17)$$

$$(\mathbf{n} - \mathbf{u})^T (\mathbf{n} - \mathbf{u}) + 2\mathbf{u}^T (\mathbf{n} - \mathbf{u}) + f = 0 \quad (2.0.18)$$

$$\|\mathbf{n}\|^2 - \mathbf{n}^T \mathbf{u} - \mathbf{u}^T \mathbf{n} + \|\mathbf{u}\|^2 + 2\mathbf{u}^T \mathbf{n} - 2\|\mathbf{u}\|^2 + f = 0 \quad (2.0.19)$$

$$\|\mathbf{n}\|^2 - \mathbf{n}^T \mathbf{u} + \mathbf{n}^T \mathbf{u} - \|\mathbf{u}\|^2 + f = 0 \quad (2.0.20)$$

$$\|\mathbf{n}\|^2 - \|\mathbf{u}\|^2 + f = 0 \quad (2.0.21)$$

$$(m^2 + 1) - 10 + 8 = 0 \quad (2.0.22)$$

$$m^2 - 1 = 0 \quad (2.0.23)$$

$$m = \pm 1 \quad (2.0.24)$$

For the line 1.0.1 to be a tangent to circle at equation 1.0.2, the values of m are ± 1

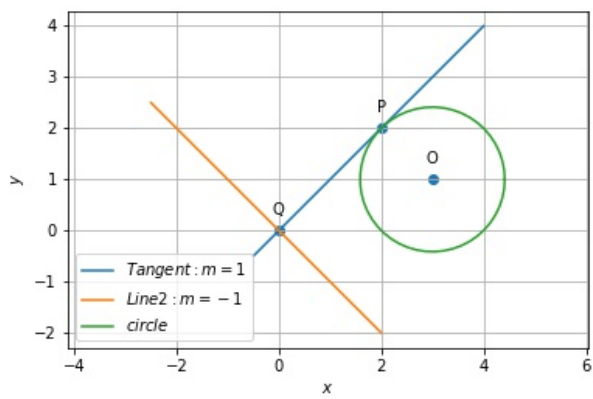


Fig. 1: Circle with tangent