Matrix Theory EE5609 - Assignment 6

Sandhya Addetla
PhD Artificial Inteligence Department
AI20RESCH14001

***Abstract*—Perform QR decomposition of a matrix.**

Download python code from

https://github.com/SANDHYA−A/Assignment6

1 Problem

Perform QR decomposition of the matrix

$$\mathbf{V} = \begin{pmatrix} 19 & 12 \\ 12 & 1 \end{pmatrix} \tag{1.0.1}$$

2 Solution

Any matrix A can be converted in the form

$$\mathbf{A} = \mathbf{QR} \tag{2.0.1}$$

Here $\mathbf{Q}$ is an orthogonal matrix and $\mathbf{R}$ is an upper triangular matrix. This is known as QR decomposition.

For the given matrix at 1.0.1, column vectors are,

$$\mathbf{a} = \begin{pmatrix} 19 \\ 12 \end{pmatrix} \quad \mathbf{b} = \begin{pmatrix} 12 \\ 1 \end{pmatrix} \tag{2.0.2}$$

Equation 2.0.1 can be written in $\mathbf{QR}$ form as:

$$\mathbf{QR} = \begin{pmatrix} \mathbf{q_1} & \mathbf{q_2} \end{pmatrix} \begin{pmatrix} u_1 & u_3 \\ 0 & u_2 \end{pmatrix} \tag{2.0.3}$$

Where,

$$u_1 = \|\mathbf{a}\| = \sqrt{19^2 + 12^2} = \sqrt{505} \tag{2.0.4}$$

$$\mathbf{q_1} = \frac{\mathbf{a}}{u_1} = \begin{pmatrix} \frac{19}{\sqrt{505}} \\ \frac{12}{\sqrt{505}} \end{pmatrix} \tag{2.0.5}$$

$$u_3 = \frac{\mathbf{q_1}^T \mathbf{b}}{\|\mathbf{q_1}\|^2} = \begin{pmatrix} \frac{19}{\sqrt{505}} & \frac{12}{\sqrt{505}} \end{pmatrix} \begin{pmatrix} 12 \\ 1 \end{pmatrix} = \frac{240}{\sqrt{505}} \tag{2.0.6}$$

$$\mathbf{q_2} = \frac{\mathbf{b} - u_3\mathbf{q_1}}{\|\mathbf{b} - u_3\mathbf{q_1}\|} = \frac{101}{25\sqrt{505}} \begin{pmatrix} \frac{300}{101} \\ -\frac{475}{101} \end{pmatrix} = \begin{pmatrix} \frac{12}{\sqrt{505}} \\ -\frac{19}{\sqrt{505}} \end{pmatrix} \tag{2.0.7}$$

$$u_2 = \mathbf{q_2}^T \mathbf{b} = \begin{pmatrix} \frac{12}{\sqrt{505}} & -\frac{19}{\sqrt{505}} \end{pmatrix} \begin{pmatrix} 12 \\ 1 \end{pmatrix} = \frac{125}{\sqrt{505}} \tag{2.0.8}$$

By substituting equation 2.0.4 to 2.0.8 in 2.0.3,we obtain the QR Decomposition of the given matrix as:

$$\begin{pmatrix} 19 & 12 \\ 12 & 1 \end{pmatrix} = \begin{pmatrix} \frac{19}{\sqrt{505}} & \frac{12}{\sqrt{505}} \\ \frac{12}{\sqrt{505}} & -\frac{19}{\sqrt{505}} \end{pmatrix} \begin{pmatrix} \sqrt{505} & \frac{240}{\sqrt{505}} \\ 0 & \frac{125}{\sqrt{505}} \end{pmatrix} \tag{2.0.9}$$