Matrix Theory EE5609 - Assignment 6

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***Abstract*—Perform QR decomposition of a 3×3 matrix.**

Download python code from

https://github.com/SANDHYA−A/Assignment8

1 Problem

Perform QR decomposition of the matrix

$$\mathbf{V} = \begin{pmatrix} 9 & 0 & -6 \\ 0 & -4 & 0 \\ -6 & 0 & 1 \end{pmatrix} \tag{1.0.1}$$

2 Solution

Any matrix A can be converted in the form

$$\mathbf{A} = \mathbf{QR} \tag{2.0.1}$$

Here $\mathbf{Q}$ is an orthogonal matrix and $\mathbf{R}$ is an upper triangular matrix. This is known as QR decomposition.

For the given matrix at 1.0.1, column vectors are,

$$\mathbf{a} = \begin{pmatrix} 9 \\ 0 \\ -6 \end{pmatrix} \quad \mathbf{b} = \begin{pmatrix} 0 \\ -4 \\ 0 \end{pmatrix} \quad \mathbf{c} = \begin{pmatrix} -6 \\ 0 \\ 1 \end{pmatrix} \tag{2.0.2}$$

Equation 2.0.1 can be written in $\mathbf{QR}$ form as:

$$\mathbf{QR} = \begin{pmatrix} \mathbf{q_1} & \mathbf{q_2} & \mathbf{q_3} \end{pmatrix} \begin{pmatrix} r_1 & r_2 & r_3 \\ 0 & r_4 & r_5 \\ 0 & 0 & r_6 \end{pmatrix} \tag{2.0.3}$$

Where,

$$r_1 = \|\mathbf{a}\| = \sqrt{9^2 + (-6)^2} = \sqrt{117} \tag{2.0.4}$$

$$\mathbf{q_1} = \frac{\mathbf{a}}{r_1} = \begin{pmatrix} \frac{9}{\sqrt{117}} \\ 0 \\ \frac{-6}{\sqrt{117}} \end{pmatrix} \tag{2.0.5}$$

The value of $\mathbf{q_2}$ can be obtained as,

$$r_2 = \mathbf{q_1}^T \mathbf{b} \tag{2.0.6}$$

$$= \begin{pmatrix} \frac{9}{\sqrt{117}} & 0 & \frac{-6}{\sqrt{117}} \end{pmatrix} \begin{pmatrix} 0 \\ -4 \\ 0 \end{pmatrix} = 0 \tag{2.0.7}$$

$$r_4 = \|\mathbf{b} - r_2\mathbf{q_1}\| = \sqrt{(-4)^2} = 4 \tag{2.0.8}$$

$$\mathbf{q_2} = \frac{\mathbf{b} - r_2\mathbf{q_1}}{\|\mathbf{b} - r_2\mathbf{q_1}\|} \tag{2.0.9}$$

$$\mathbf{q_2} = \frac{1}{4} \begin{pmatrix} 0 \\ -4 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} \tag{2.0.10}$$

The value of $\mathbf{q_3}$ can be obtained as,

$$r_3 = \mathbf{q_1}^T \mathbf{c} \tag{2.0.11}$$

$$= \begin{pmatrix} \frac{9}{\sqrt{117}} & 0 & \frac{-6}{\sqrt{117}} \end{pmatrix} \begin{pmatrix} -6 \\ 0 \\ 1 \end{pmatrix} = \frac{-60}{\sqrt{117}} \tag{2.0.12}$$

$$r_5 = \mathbf{q_2}^T \mathbf{c} \tag{2.0.13}$$

$$= \begin{pmatrix} 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} -6 \\ 0 \\ 1 \end{pmatrix} = 0 \tag{2.0.14}$$

$$r_6 = \|\mathbf{c} - r_3\mathbf{q_1} - r_5\mathbf{q_2}\| \tag{2.0.15}$$

$$= \left\| \begin{pmatrix} -6 \\ 0 \\ 1 \end{pmatrix} + \frac{60}{\sqrt{117}} \begin{pmatrix} \frac{9}{\sqrt{117}} \\ 0 \\ \frac{-6}{\sqrt{117}} \end{pmatrix} \right\| \tag{2.0.16}$$

$$= \sqrt{\left(\frac{-18}{13}\right)^2 + \left(\frac{-27}{13}\right)^2} = \frac{9}{\sqrt{13}} \tag{2.0.17}$$

$$\mathbf{q_3} = \frac{\mathbf{c} - r_3\mathbf{q_1} - r_5\mathbf{q_2}}{\|\mathbf{c} - r_3\mathbf{q_1} - r_5\mathbf{q_2}\|} \tag{2.0.18}$$

$$= \frac{\sqrt{13}}{9} \begin{pmatrix} \frac{-18}{13} \\ 0 \\ \frac{-27}{13} \end{pmatrix} = \begin{pmatrix} \frac{-2}{\sqrt{13}} \\ 0 \\ \frac{-3}{\sqrt{13}} \end{pmatrix} \tag{2.0.19}$$

By substituting equation (2.0.4) to (2.0.19) in (2.0.3),we obtain the QR Decomposition of the given matrix as:

$$\begin{pmatrix} 9 & 0 & -6 \\ 0 & -4 & 0 \\ -6 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \frac{9}{\sqrt{117}} & 0 & \frac{-2}{\sqrt{13}} \\ 0 & -1 & 0 \\ \frac{-6}{\sqrt{117}} & 0 & \frac{-3}{\sqrt{13}} \end{pmatrix} \begin{pmatrix} \sqrt{117} & 0 & \frac{-60}{\sqrt{117}} \\ 0 & 4 & 0 \\ 0 & 0 & \frac{9}{\sqrt{13}} \end{pmatrix} \tag{2.0.20}$$