Matrix Theory EE5609

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Assignment-9

Problem:

Let M_n denote the vector space of all $n \times n$ real matrices. Which of the following is a linear subspaces of M_n :-

1) $V_1 = \{A \in M_n :$ A is nonsingular$\}$
2) $V_2 = \{A \in M_n : det(A) = 0\}$
3) $V_3 = \{A \in M_n : trace(A) = 0\}$
4) $V_4 = \{BA : A \in M_n\}$, where B is some fixed matrix in M_n

Solution:

Vector space	Is it subspace to M_n?
1)V_1: All non-singular matrices of $n \times n$	The matrices $I_{n\times n}$ and $-I_{n\times n}$ are non-singular matrices, but the sum $I_{n\times n} - I_{n\times n}$ is zero matrix and it is singular. $\therefore V_1$ does not form subspace of M_n.
2)V_2: All singular matrices of $n \times n$	The matrices $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} and \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$ are singular matrices, but the sum is a non-singular matrix. $\therefore V_2$ does not form subspace M_n.
3)V_3: All matrices of $n \times n$ with trace =0	Let $\mathbf{v_1}$ and $\mathbf{v_2}$ be matrices with Trace = 0. $Tr(\mathbf{v_1} + \alpha\mathbf{v_2}) = Tr(\mathbf{v_1}) + \alpha Tr(\mathbf{v_2}) = 0$. $\therefore$ the vector space V_3 forms linear subspace of M_n.
4)V_4: F_A = BA, where B is some fixed matrix in M_n	Let $\mathbf{v_1}$ and $\mathbf{v_2}$ be matrices in the vector space V_4. $F_{v_1+\alpha v_2} = B(\mathbf{v_1} + \alpha\mathbf{v_2})$ $=B\mathbf{v_1} + \alpha B\mathbf{v_2} =$ $F_{v_1} + \alpha F_{v_2}$. $\therefore$ V_4 forms linear subspace of M_n.