# Matrix Theory EE5609 - Assignment 1

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***Abstract*—This document provides a solution for the problem of finding slopes of two lines, slope of one line being double of the slope of another line. and tangent of the angle between them is 1/3.**

## I. Problem Statement

The slope of a line is double of the slope of another line. If the tangent of the angle between them is 1/3 , find the slopes of the lines.

## II. Solution

Two vectors are considered to be passing through origin. Where $m_1$ and $m_2$ are slopes of the vectors $\mathbf{m_1}$ and $\mathbf{m_2}$ respectively. Let $m_1 = m$. Given $m_2 = 2m$

These vectors can be represented as below:-

$$\mathbf{m_1} = \begin{pmatrix} 1 \\ m \end{pmatrix} \mathbf{m_2} = \begin{pmatrix} 1 \\ 2m \end{pmatrix} \tag{1}$$

The dot product of the vectors is given by:-

$$\mathbf{m_1}^T \mathbf{m_2} = \left\| \mathbf{m_1} \right\| \left\| \mathbf{m_2} \right\| \cos\theta \tag{2}$$

Given that $tan\theta$ is $\frac{1}{3}$. By Pythagorus theorem, we can obtain $cos\theta$ as $\frac{3}{\sqrt{10}}$. Therefore,

$$\cos\theta = \frac{\mathbf{m_1}^T \mathbf{m_2}}{\left\| \mathbf{m_1} \right\| \left\| \mathbf{m_2} \right\|} \tag{3}$$

$$\frac{3}{\sqrt{10}} = \frac{1 \times 1 + m \times 2m}{\sqrt{1+m^2}\sqrt{1+4m^2}} \tag{4}$$

Applying square on both sides:-

$$9 \times (1+m^2)(1+4m^2) = 10(1+2m^2)^2 \tag{5}$$

$$4m^4 - 5m^2 + 1 = 0 \tag{6}$$

$$m_1 = m = 1, -1, \frac{1}{2}, \frac{-1}{2} \tag{7}$$

Substituting the value of $m_1$ we get value of $m_2 = 2, -2, 1, -1$.

## III. Conclusion

The slopes $m_1$ and $m_2$ of vectors $\mathbf{m_1}$ *and* $\mathbf{m_2}$ for the said conditions are: -

$$\begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} -1 \\ -2 \end{pmatrix}, \begin{pmatrix} \frac{1}{2} \\ 1 \end{pmatrix}, \begin{pmatrix} -\frac{1}{2} \\ -1 \end{pmatrix}$$