Matrix Theory EE5609 - Assignment 1

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***Abstract*—This document provides a solution for the problem of finding slopes of two lines, slope of one line being double of the slope of another line. and tangent of the angle between them is 1/3.**

I. Problem Statement

The slope of a line is double of the slope of another line. If the tangent of the angle between them is 1/3 , find the slopes of the lines.

II. Theory

Consider the lines as two directional vectors $\boldsymbol{v_1}$ and $\boldsymbol{v_2}$. Angle between the two vectors can be obtained by dot product of the two vectors.

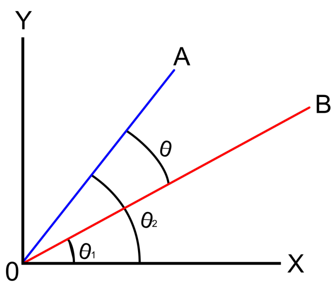

Fig. 1. Angle between two vectors

$$\theta = \boldsymbol{v_1}.\boldsymbol{v_2}$$

The dot product of the vectors is given by

$$\boldsymbol{v_1}.\boldsymbol{v_2} = \mid \boldsymbol{v_1} \mid\mid \boldsymbol{v_2} \mid \cos\theta$$

III. Solution

Here these two vectors are considered to be passing through origin. The equations for these two vectors are:-

$$y = m_1 x$$
$$y = m_2 x$$

Where m_1 and m_2 are slopes of the vectors $\boldsymbol{v_1}$ and $\boldsymbol{v_2}$ respectively. Let $m_1 = m$. Given $m_2 = 2m$

These vectors can be represented as below:-

$$\boldsymbol{v_1} = \begin{pmatrix} 1 \\ m \end{pmatrix} \boldsymbol{v_2} = \begin{pmatrix} 1 \\ 2m \end{pmatrix}$$

The dot product of the vectors is given by:-

$$\boldsymbol{v_1}.\boldsymbol{v_2} = \mid \boldsymbol{v_1} \mid\mid \boldsymbol{v_2} \mid \cos\theta$$

Given that $tan\theta$ is $\frac{1}{3}$. By Pythagorus theorem, we can obtain $cos\theta$ as $\frac{3}{\sqrt{10}}$. Therefore,

$$\cos\theta = \frac{\boldsymbol{v_1}.\boldsymbol{v_2}}{\mid \boldsymbol{v_1} \mid\mid \boldsymbol{v_2} \mid}$$
$$\frac{3}{\sqrt{10}} = \frac{1 \times 1 + m \times 2m}{\sqrt{1+m^2}\sqrt{1+4m^2}}$$

Applying square on both sides:-

$$9 \times (1+m^2)(1+4m^2) = 10(1+2m^2)^2$$
$$4m^4 - 5m^2 + 1 = 0$$
$$m_1 = m = 1, -1, \frac{1}{2}, \frac{-1}{2}$$

Substituting the value of m_1 we get value of $m_2 = 2, -2, 1, -1$.

IV. Conclusion

The slopes m_1 and m_2 of vectors $\boldsymbol{v_1}$ *and* $\boldsymbol{v_2}$ for the said conditions are: -

$$\begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} -1 \\ -2 \end{pmatrix}, \begin{pmatrix} \frac{1}{2} \\ 1 \end{pmatrix}, \begin{pmatrix} -\frac{1}{2} \\ -1 \end{pmatrix}$$