## Matrix Theory EE5609 - Assignment 1

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Abstract—This document provides a solution for the problem of finding slopes of two lines, slope of one line being double of the slope of another line. and tangent of the angle between them is 1/3.

## I. PROBLEM STATEMENT

The slope of a line is double of the slope of another line. If the tangent of the angle between them is 1/3, find the slopes of the lines.

## II. SOLUTION

Two vectors are considered to be passing through origin. Where  $m_1$  and  $m_2$  are slopes of the vectors  $\vec{m_1}$  and  $\vec{m_2}$ respectively. Let  $m_1 = m$ . Given  $m_2 = 2m$ 

These vectors can be represented as below:-

$$\vec{m_1} = \begin{pmatrix} 1 \\ m \end{pmatrix} \vec{m_2} = \begin{pmatrix} 1 \\ 2m \end{pmatrix} \tag{1}$$

The dot product of the vectors is given by:-

$$\vec{m_1}^T \vec{m_2} = \|\vec{m_1}\| \|\vec{m_2}\| \cos \theta \tag{2}$$

Given that  $tan\theta$  is  $\frac{1}{3}$ . By Pythagorus theorem, we can obtain  $\cos\theta$  as  $\frac{3}{\sqrt{10}}$ . Therefore,

$$\cos \theta = \frac{\vec{m_1}^T \vec{m_2}}{\|\vec{m_1}\| \|\vec{m_2}\|}$$

$$\frac{3}{\sqrt{10}} = \frac{1 \times 1 + m \times 2m}{\sqrt{1 + m^2} \sqrt{1 + 4m^2}}$$
(4)

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Applying square on both sides:-

$$9 \times (1 + m^2)(1 + 4m^2) = 10(1 + 2m^2)^2 \tag{5}$$

$$4m^4 - 5m^2 + 1 = 0 (6)$$

$$m_1 = m = 1, -1, \frac{1}{2}, \frac{-1}{2}$$
 (7)

Substituting the value of  $m_1$  we get value of  $m_2 = 2, -2, 1, -1.$ 

## III. CONCLUSION

The slopes  $m_1$  and  $m_2$  of vectors  $\vec{m_1}$  and  $\vec{m_2}$  for the said conditions are: -

$$\begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} -1 \\ -2 \end{pmatrix}, \begin{pmatrix} \frac{1}{2} \\ 1 \end{pmatrix}, \begin{pmatrix} -\frac{1}{2} \\ -1 \end{pmatrix}$$