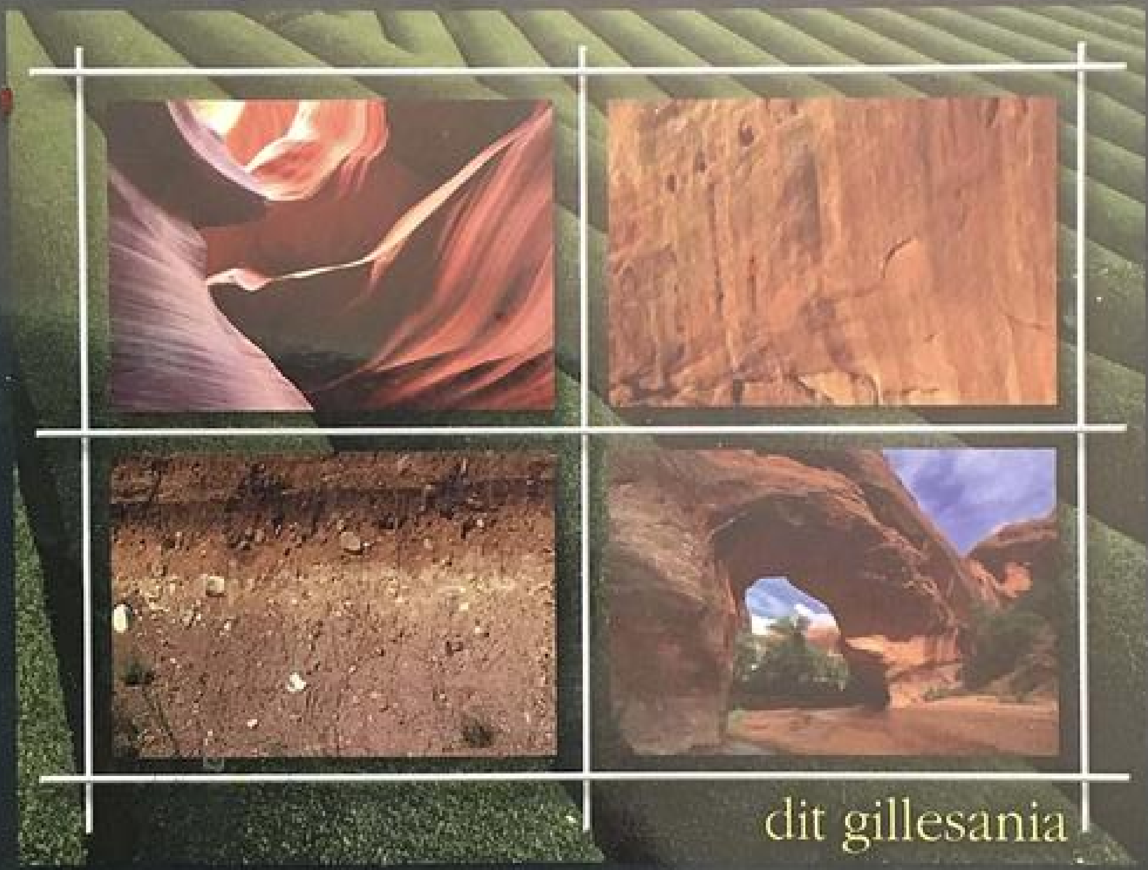
Fundamentals of Geotechnical Engineering



- Soil properties
- Classification of soil
- Flow of water through soil
- Stresses in soil
- Stress distribution in soil
- Compressibility of soil
- Shear strength of soil
- Lateral earth pressure
- Bearing capacity of soil
- Piles
- Slope stability
- Braced cuts

Fundamentals of Geotechnical Engineering

DIEGO INOCENCIO T. GILLESANIA
Civil Engineer

BSCE, EVSU (LIT) – Magna Cum Laude

5th Place, PICE National Students' Quiz, 1989

Awardee, Most Outstanding Student, 1989

3rd Place, CE Board November 1989

Review Director & Reviewer in all Subjects
Gillesania Engineering Review Center

Author of Various Engineering Books

Fundamentals of
Geotechnical Engineering

ISBN 971-8614-33-8

Cover design by the author.

Preface

The cardinal objective of this book is to facilitate preparation for the Civil Engineering Licensure examination given by the Professional Regulation Commission (PRC). Since this book includes complete discussion of the principles in geotechnical engineering, this may also serve as a guide to the civil engineering undergraduates.

The book is divided into 10 Chapters. Each chapter presents the formulas and principles in Geotechnical Engineering, followed by illustrative problems. Each step in the solution is carefully explained to ensure that it will be readily understood.

Most of the materials in this book have been used in my review classes. The choice of these materials was guided by their effectiveness as tested in my classes.

I wish to thank all my friends and relatives who inspired me in writing my books, especially to my wife Imelda who is very supportive to me.

I will appreciate any errors pointed out and will welcome any suggestion for further improvement.

DIEGO INOCENCIO T. GILLESANIA
Cebu City, Philippines

@seismicisolation

To my mother Iluminada,
my wife Imelda,
and our Children Kim Deunice,
Ken Dainiel,
and Karla Denise

Table of Contents

Chapter 02 – Classification of Soil (55 – 84)

Chapter 03 – Flow of Water through Soils (85 – 130)

Chapter 04 – Stresses in Soil (131 – 164)

Chapter 05 – Stress Distribution in Soil (165 – 190)

Chapter 06 – Compressibility of Soil (191 – 218)

Chapter 07 – Shear Strength of Soil (219 – 242)

Chapter 08 – Lateral Earth Pressure (243 – 284)

Chapter 09 – Bearing Capacity of Soils (285 – 334)

Chapter 10 – Miscellaneous Topics and Additional Problems (335 – 432)

Chapter 01

Properties of Soil

1.1 SYMBOLS AND NOTATIONS

e = void ratio
n = porosity
D_r = relative density
G = specific gravity of solids (usually in the range 2.67 ± 0.05)
GI = group index
LI = liquidity index
LL = liquid limit
MC = moisture content
PI = plasticity index
PL = plastic limit
S = degree of saturation
V = volume of soil mass
V_a = volume of air
V_S = volume of solids
V_w = volume of water
W = total weight of soil
W_s = weight of solids
W_w = weight of water
γ_{dry} = dry unit weight
γ_m = unit weight of soil mass
γ_s = unit weight of solids
γ_{sat} = saturated unit weight
γ_w = unit weight of water

1.2 DENSITY AND UNIT WEIGHT OF WATER

Density of water, $\rho_w = 1000\ kg/m^3$

$\rho_w = 1\ kg/liter = 1\ gram/cc$

Unit weight f water, $\gamma_w = 9.81\ kN/m^3$

1.3 BASIC FORMULAS

Unit weight of substance, $\gamma_s = G\,\gamma_w$ Eq. 1.1

Weight of water, $W_w = \gamma_w\,V_w$ Eq. 1.2

Weight of substance, $W_s = \gamma_s\,V_s = G\,\gamma_w\,V_s$ Eq. 1.3

Specific gravity of substance, $G_{subs} = \gamma_{subs}/\gamma_w$ Eq. 1.4

1.4 PHYSICAL PROPERTIES OF SOIL

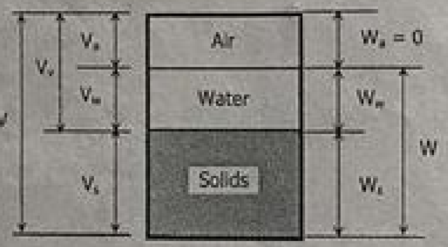


Figure 01.1 – Phase diagram of soil

The following relationships can be made from the phase diagram shown:

Total weight of soil, $W = W_w + W_s$ Eq. 1.5

Volume of voids, $V_v = V_a + V_w$ Eq. 1.6

Total volume, $V = V_s + V_v$ Eq. 1.7

1.4.1 VOID RATIO, e

Void ratio is the ratio between the volumes of voids to the volume of solids of a soil mass. It is usually expressed in percent.

$$e = \frac{V_v}{V_s} \qquad \text{Eq. 1.8}$$

Note: $0 < e < \infty$

1.4.2 POROSITY, n

Porosity is the ratio between the volumes of voids to the total volume of a soil mass. It is usually expressed in percent.

$$n = \frac{V_v}{V} \qquad \text{Eq. 1.9}$$

Note: $0 < n < 1$

1.4.3 RELATIONSHIP BETWEEN e AND n

$$n = \frac{e}{1+e} \quad \text{and} \quad e = \frac{n}{1-n} \qquad \text{Eq. 1.10}$$

1.4.4 WATER CONTENT OR MOISTURE CONTENT, MC OR w

The ratio of the weight of water to the weight of the solid particles.

$$MC \text{ or } w = \frac{W_w}{W_s} \times 100\% \qquad \text{Eq. 1.11}$$

Note: $0 < MC < \infty$

1.4.5 DEGREE OF SATURATION, S

The ratio of the volume of water to the volume of voids.

$$S = \frac{V_w}{V_v} \times 100 \qquad \text{Eq. 1.12}$$

Degree of saturation varies from $S = 0$ for completely dry soil and $S = 100\%$ for totally saturated soil.

1.4.6 RELATIONSHIP BETWEEN G, MC, S AND e

$$G \times MC = S \times e \qquad \text{Eq. 1.13}$$

1.4.7 UNIT WEIGHT (OR BULK UNIT WEIGHT) OF SOIL MASS, γ_m

$$\gamma_m = \frac{W}{V} \qquad \text{Eq. 1.14}$$

$$\gamma_m = \frac{G + Se}{1+e}\gamma_w \qquad \text{Eq. 1.15}$$

$$\gamma_m = \frac{G + G\,MC}{1+e}\gamma_w \qquad \text{Eq. 1.16}$$

1.4.8 DRY UNIT WEIGHT, γ_d

For dry soils, $S = 0$ and $MC = 0$

$$\gamma_d = \frac{W_s}{V} = \frac{G}{1+e}\gamma_w \qquad \text{Eq. 1.17}$$

$$W_s = \frac{W}{1+MC} \qquad \text{Eq. 1.18}$$

$$\gamma_d = \frac{\gamma_m}{1+MC} \qquad \text{Eq. 1.19}$$

1.4.9 SATURATED UNIT WEIGHT, γ_{SAT}

For saturated soils, $S = 1$, $V_v = V_w$

$$\gamma_{sat} = \frac{G+e}{1+e}\gamma_w \qquad \text{Eq. 1.20}$$

1.4.10 SUBMERGED OR BUOYANT UNIT WEIGHT, γ_b OR γ'

$$\gamma_b \text{ or } \gamma' = \gamma_{sat} - \gamma_w \qquad \text{Eq. 1.21}$$

$$\gamma_b \text{ or } \gamma' = \frac{G-1}{1+e}\gamma_w \qquad \text{Eq. 1.22}$$

1.4.11 CRITICAL HYDRAULIC GRADIENT

Critical hydraulic gradient is the hydraulic gradient that brings a soil mass (essentially, coarse-grained soils) to static liquefaction (quick condition).

$$i_{cr} = \frac{\gamma_b}{\gamma_w} = \frac{G-1}{1+e} \qquad \text{Eq. 1.23}$$

1.4.12 OTHER FORMULAS

These formulas may not be memorized. These can be derived from the previous formulas.

$$\text{Volume of voids, } V_v = \frac{e}{1+e}V \qquad \text{Eq. 1.24}$$

$$\text{Volume of solid, } V_s = \frac{V}{1+e} \qquad \text{Eq. 1.25}$$

$$\text{Volume of water, } V_w = \frac{Se}{1+e}V \qquad \text{Eq. 1.26}$$

$$\text{Weight of water, } W_w = \frac{Se}{1+e}V\gamma_w \qquad \text{Eq. 1.27}$$

$$\text{Weight of solid, } W_s = \frac{1}{1+e}V\,G_s\,\gamma_w \qquad \text{Eq. 1.28}$$

$$\text{Weight of soil, } W = \frac{G+Se}{1+e}V\gamma_w \qquad \text{Eq. 1.29}$$

$$\text{Dry unit weight, } \gamma_d = \frac{\gamma_m}{1+MC} \qquad \text{Eq. 1.30}$$

1.5 SPECIFIC GRAVITY OF SOME MINERALS

Mineral	Specific Gravity
Gypsum volcanic ash	2.32
Orthoclase	2.56
Kaolinite	2.61
Quartz	2.67
Calcite	2.72
Dolomite	2.87
Magnetite	5.17

1.6 RELATIVE DENSITY OF GRANULAR SOILS

The relative density, D_r expresses the state of compactness of a natural granular soil.

$$D_r = \frac{e_{max} - e}{e_{max} - e_{min}} \times 100 \qquad \text{Eq. 1.31}$$

$$\text{or } D_r = \frac{1/\gamma_{min} - 1/\gamma_d}{1/\gamma_{min} - 1/\gamma_{max}} \qquad \text{Eq. 1.32}$$

@seismicisolation

where:

e_{max} = void ratio of the soil in loosest state.
e_{min} = void ratio of the soil in densest state.
e = void ratio of the soil deposit (in-situ state)
$\gamma_{d\,max}$ = dry unit weight in densest state.
$\gamma_{d\,min}$ = dry unit weight in loosest state.
γ_d = dry unit weight (in-situ state)

1.7 DESIGNATION OF GRANULAR SOILS

Designation	D_r (%)
Very loose	0 – 15
Loose	15 – 35
Medium dense	35 – 70
Dense	70 – 85
Very dense	85 – 100

1.8 CONSISTENCY

Consistency is the term used to describe the degree of firmness (e.g., soft, medium, firm, or hard) of a soil.

The consistency of a cohesive soil is greatly affected by the water content of the soil. A gradual increase of the water content may transform a dry soil from *solid state* to a *semisolid state*, to a *plastic state*, and after further moisture increase, into a *liquid state*. The water content at the corresponding junction points of these states are known as the *shrinkage limit*, the *plastic limit*, and the *liquid limit*, respectively.

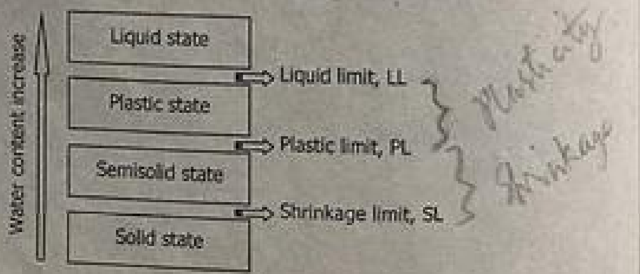


Figure 01.2

1.9 SOIL INDICES

Index	Definition	Correlation
Plasticity	$PI = LL - PL$	Strength, compressibility, compactibility, ...
Liquidity	$LI = \dfrac{MC - PL}{PI}$	Compressibility and stress rate
Shrinkage	$SI = PL - SL$	Shrinkage potential
Activity of clay	$A_c = \dfrac{PI}{\mu}$	Swell potential, and so forth

where μ = percent of soil finer than 0.002 mm (clay size)

Activity	Classification
$A_c < 0.7$	Inactive clay
$0.7 < A_c < 1.2$	Normal clay
$A_c > 1.2$	Active clay

1.10 DESCRIPTION OF SOIL BASED ON LIQUIDITY INDEX

$LI < 0$	*Semisolid state* – high strength, brittle (sudden) fracture is expected
$0 < LI < 1$	*Plastic state* – intermediate strength, soil deforms like a plastic material
$LI > 1$	*Liquid state* – low strength, soil deforms like a viscous fluid

1.11 DESCRIPTION OF SOIL BASED ON PLASTICITY INDEX

PI	Description
0	Nonplastic
1 - 5	Slightly plastic
5 - 10	Low plasticity
10 - 20	Medium plasticity
20 - 40	High plasticity
> 40	Very high plasticity

1.12 FALL CONE METHOD TO DETERMINE LIQUID AND PLASTIC LIMITS

Fall cone test (cone penetration test) offers more accurate method of determining both the liquid limit and the plastic limit. In this test, a cone with apex angle of 30° and total mass of 80 grams is suspended above, but just in contact with, the soil sample. The cone is permitted to fall freely for a period of 5 seconds. The water content corresponding to a cone penetration of 20 mm defines the liquid limit.

The liquid limit is difficult to achieve in just a single test. In this regard, four or more tests at different moisture content is required. The results are plotted as water content (ordinate, arithmetic scale) versos penetration (abscissa, logarithmic scale) and the best-fit straight line (liquid state line) linking the data points is drawn (see figure below). The liquid limit is read from the plot as the water content on the liquid state line corresponding to a penetration of 20 mm.

The plastic limit is found by repeating the test with a cone of similar geometry, but with a mass of (M_2) 240 grams. The liquid state line for this cone will be below the liquid state line for the 80-gram cone (M_1) and parallel to it.

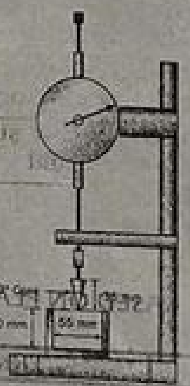


Figure 01.3 – Fall cone apparatus

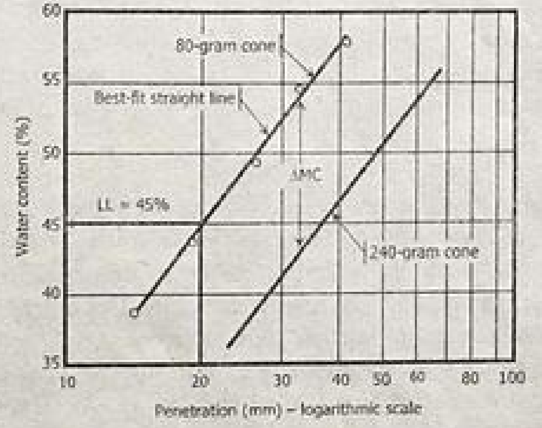


Figure 01.4 – Typical fall cone result

The plastic limit is given as:

$$PL = LL - \frac{2\Delta MC}{\log \frac{M_2}{M_1}} \qquad \text{Eq. 1.33}$$

1.13 CUP METHOD TO DETERMINE LIQUID LIMIT

The device used in this method consists of a brass cup and a hard rubber. The brass cup is dropped onto the base by a cam operated by a crank.

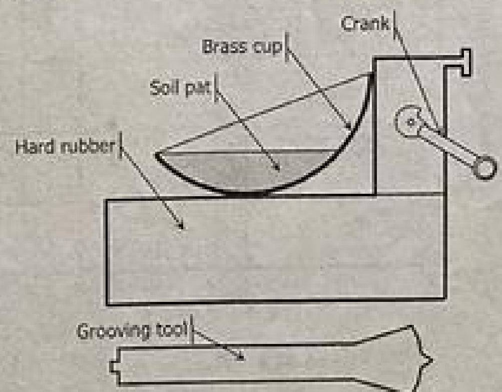


Figure 01.5 – Liquid limit device and grooving tool

The soil paste is placed in the cup. a groove is then cut at the center of the soil pat with the standard grooving tool. By the use of the crank-operated cam, the cup is lifted and dropped from a height of 10 mm. The moisture content required to close a distance of 12.7 mm along the bottom of the groove after 25 blows is defined as the liquid limit.

Since it is difficult to adjust the moisture content to meet the required closure after 25 blows, at least three tests for the same soil are conducted at varying moisture contents, with the number of blows required to achieve closure varying between 15 and 35. The results are plotted on a graph paper, with the moisture content along the vertical axis (algebraic scale) and the number of blows, N, along the horizontal axis (logarithmic scale). The graph is approximated as a straight line (called the *flow curve*). The moisture content corresponding to N = 25 is the liquid limit of the soil. The slope of the flow line is defined as the *flow index* and may be written as:

$$\text{Flow index, } FI = \frac{MC_1 - MC_2}{\log(N_2 / N_1)} \qquad \text{Eq. 1.34}$$

where MC_1 and MC_2 are the moisture contents, in percent, corresponding to number of blows N_1, and N_2, respectively.

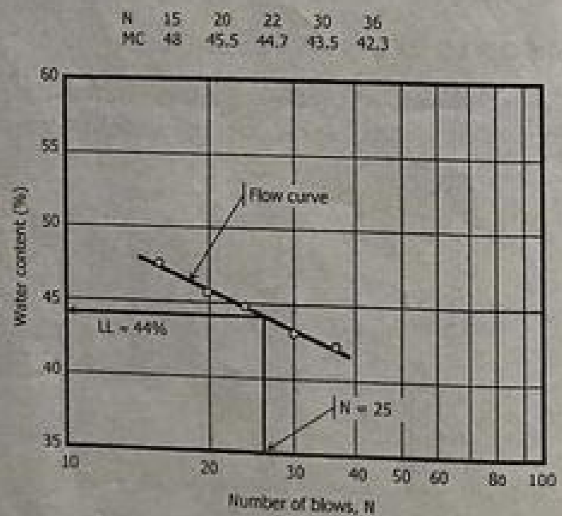


Figure 01.6 – Flow curve

1.14 ONE-POINT METHOD TO DETERMINE LIQUID LIMIT

This method may be used when only one test is run for a soil. This is established by the U.S. corps of Engineers in 1949 and was also adopted by ASTM under designation D-4318.

$$LL = MC_N \left(\frac{N}{25}\right)^{\tan\beta} \qquad \text{Eq. 1.35}$$

where:

N = number of blows in the liquid limit device for a 0.5-in groove closure
MC_N = corresponding moisture content
$\tan\beta$ = 0.121 (but note that $\tan\beta$ is not equal to 0.121 for all soils)

This method yields good results for the number of blows between 20 and 30.

1.15 SHRINKAGE LIMIT

Soil shrinks as moisture is gradually lost from it. With continuing loss of moisture, a stage of equilibrium is reached at which more loss of moisture will result in no further volume change. The moisture content, in percent, at which the volume of the soil mass ceases to change, is defined as the *shrinkage limit*.

The shrinkage limit is determined as follows. A mass of wet soil, m_1, is placed in a porcelain dish 44.5 mm in diameter and 12.5 mm high and then oven-dried. The volume of oven-dried soil is determined by using mercury to occupy the vacant spaces caused by shrinkage. The mass of mercury is determined and the volume decrease caused by shrinkage can be calculated from the known density of mercury. The shrinkage limit is calculated from

$$SL = \frac{m_1 - m_2}{m_2} - \frac{V_1 - V_2}{m_2}\rho_w \qquad \text{Eq. 1.36}$$

where:

m_1 = mass of wet (saturated) soil
m_2 = mass of oven-dried soil
V_1 = volume of wet soil
V_2 = volume of oven-dried soil
ρ_w = density of water

1.15.1 SHRINKAGE RATIO

$$SR = \frac{1}{\rho_w}\frac{m_2}{V_2} \qquad \text{Eq. 1.37}$$

1.15.2 SPECIFIC GRAVITY OF SOLIDS

$$G = \frac{1}{\dfrac{1}{SR} - \dfrac{SL}{100}} \qquad \text{Eq. 1.38}$$

1.16 LIQUIDITY INDEX & CONSISTENCY INDEX

Liquidity index (LI) defines the relative consistency of a cohesive soil in the natural state.

$$\text{Liquidity index, } LI = \frac{MC - PL}{LL - PL} \qquad \text{Eq. 1.39}$$

where MC = *in situ* or natural moisture content. If MC is greater than LL, $LI > 1$. If $MC < PL$, $LI < 0$.

$$\text{Consistency index, } CI = \frac{LL - MC}{LL - PI} = \frac{LL - MC}{LL - (LL - PL)} = \frac{LL - MC}{PL} \qquad \text{Eq. 1.40}$$

If MC is equal to LL, CI is zero. If $MC = PI$, $CI = 1$.

Atterberg's limits are also used to asses the potential *swell* of a given soil.

LL	PI	Potential swell classification
<50	<25	Low
50 - 60	25 - 35	Medium
>60	>35	High

ILLUSTRATIVE PROBLEMS

PROBLEM 01.1

A sample of saturated soil weighs 588 N and has a volume of 0.03 m³. If the voids ratio of the soil is 0.75, determine the specific gravity of the solids

SOLUTION

$\gamma = \frac{W}{V}$

$\gamma_m = \frac{G + Se}{1+e}\gamma_w$

$$[\gamma_m = \frac{W}{V}]$$

$$\gamma_m = \frac{588}{0.03}$$

$$\gamma_m = 19{,}600 \text{ N/m}^3$$

$\frac{(1+\omega)\gamma_w}{1+e}$ $\quad \frac{(G+Se)\gamma_w}{1+e}$

$$[\gamma_m = \frac{G + Se}{1+e}\gamma_w]$$

$$19{,}600 = \frac{G + 1(0.75)}{1 + 0.75}9{,}810$$

$$G = 2.75$$

PROBLEM 01.2

A clay sample has unit weight of 21.1 kN/m³ at moisture content of 9.8%. When completely saturated with water, its unit weight is 22.58 kN/m³. Determine the porosity of the soil.

SOLUTION

$\gamma = 21.1 \text{ kN/m}^3$

$\omega = 9.8\%$

$S = 1,\ \gamma = 22.58 \text{ kN/m}^3$

$$[\gamma_m = \frac{G + G\,MC}{1+e}\gamma_w]$$

$$21.1 = \frac{G + G(0.098)}{1+e}(9.81)$$

$$G = 1.959 + 1.959e$$

$$[\gamma_{sat} = \frac{G + e}{1+e}\gamma_w]$$

$$22.58 = \frac{(1.959 + 1.959e) + e}{1+e}(9.81)$$

$n =$

$2.302 + 2.302e = 1.959 + 2.959e$
$e = 0.5221$

$[n = \frac{e}{1+e}]$
$n = \frac{0.5221}{1+0.5221}$
$n = 0.343 = \mathbf{34.3\%}$

PROBLEM 01.3 (CE NOVEMBER 1998)

A specimen of moist clay has a mass of 183.4 grams. After oven drying, the mass is reduced to 157.7 grams. What is the moisture content of the sample:

SOLUTION

Moisture content, $MC = \frac{W_w}{W_s} \times 100\%$

Total weight of soil mass, $W = 183.4$ grams $= W_w + W_s$
Weight of solid (oven-dried weight), $W_s = 157.7$ grams
Weight of water, $W_w = W - W_s = 183.4 - 157.7 = 25.7$ grams

Moisture content, $MC = \frac{25.7}{157.7} \times 100\%$
Moisture content, $MC = \mathbf{16.3\%}$

PROBLEM 01.4

A sample of moist clay is found to have moisture content of 400% and degree of saturation of 85%. The specific gravity of the solids is 2.76. Determine the voids ratio of this soil.

SOLUTION

$[G\,MC = S\,e]$
$2.76(4) = 0.85\,e$
$e = \mathbf{12.99}$

PROBLEM 01.5 (CE MAY 1999)

A sample of moist soil has water content of 18% and moist unit weight of 17.3 kN/m³. The specific gravity of solids is 2.65. Compute the degree of saturation of the soil.

SOLUTION

Solve for e:
$[\gamma_m = \frac{G + G\,MC}{1+e}\gamma_w]$
$17.3 = \frac{2.65 + 2.65(0.18)}{1+e}(9.81)$
$e = 0.7732$

$[G\,MC = S\,e]$
$2.65(0.18) = S(0.7732)$
$S = 0.617$
$S = \mathbf{61.7\%}$

PROBLEM 01.6

Saturated silty clay encountered in a deep excavation is found to have a water content of 28%. Determine unit weight of the clay in kN/m³. Assume G = 2.7.

SOLUTION

S = 1 (saturated) $\quad G_s w = Se$
$[G\,MC = S\,e]$
$2.7(0.28) = 1(e)$
$e = 0.756$

$[\gamma_m = \frac{G + GMC}{1+e}\gamma_w]$
$\gamma_m = \frac{2.7 + 2.7(0.28)}{1+0.756}(9.81)$
$\gamma_m = \mathbf{19.31\ kN/m^3}$

PROBLEM 01.7

A hand-carved sample of soft saturated clay weighs 350 grams and has a volume of 200 cc. After oven-drying, it weighs 240 grams. Calculate the following:

a) moisture content in percent.
b) specific gravity of solids.
c) porosity in percent.

SOLUTION

$$[MC = \frac{W_w}{W_s}]$$

$$MC = \frac{350 - 240}{240}$$

$$MC = 0.45833$$

$$MC = 45.83\%$$

$$[G\ MC = S\ e] \qquad S = 1 \text{ (saturated)}$$

$$G(0.45833) = 1\ e$$

$$G = 2.182\ e$$

$$\gamma_{sat} = W/V = 350/200$$

$$\gamma_{sat} = 1.75 \text{ grams/cc}$$

$$[\gamma_{sat} = \frac{G+e}{1+e}\gamma_w]$$

$$1.75 = \frac{2.182e + e}{1+e}(1)$$

$$1.75 + 1.75e = 3.182e$$

$$e = 1.222$$

$$n = \frac{e}{1+e} = 0.55 = 55\% \quad \rightarrow \text{porosity}$$

$$G = 2.182(1.222) = 2.67 \quad \rightarrow \text{specific gravity of solids}$$

PROBLEM 01.8 (CE NOVEMBER 1999)

13. Nov 99: A soil sample was compacted. The result of the standard proctor test shows that at 100% compaction, the soil weighs 131.1 pcf with optimum moisture content of 14%. What is the saturated unit weight of the soil in pcf?

G = 2.67

SOLUTION

$$[\gamma_m = \frac{G + G\ MC}{1+e}\gamma_w]$$

$$131.1 = \frac{2.67 + 2.67(0.14)}{1+e}(62.4)$$

$$e = 0.449$$

$$[\gamma_{sat} = \frac{G+e}{1+e}\gamma_w]$$

$$\gamma_{sat} = \frac{2.67 + 0.449}{1 + 0.449}(62.4)$$

$$\gamma_{sat} = 134.32 \text{ pcf}$$

PROBLEM 01.9

A soil sample has a bulk unit weight of 19.6 kN/m³ at a water content of 10%. Assuming $G = 2.7$, determine the percentage air in the voids (air voids).

SOLUTION

$$[\gamma_m = \frac{G + G\ MC}{1+e}\gamma_w]$$

$$19.6 = \frac{2.7 + 2.7(0.10)}{1+e}(9.81)$$

$$e = 0.4865$$

$$[G\ MC = S\ e]$$

$$2.7(0.10) = S(0.4865)$$

$$S = 55.5\%$$

Percentage of air in the voids = 100% – S
Percentage of air in the voids = 100% - 55.5%
Percentage of air in the voids = **44.5%**

PROBLEM 01.10

The moist weight of 0.2 ft³ of a soil is 23 lb. The moisture content and the specific gravity of soil solids are determined in the laboratory to be 11% and 2.7, respectively.

a) What is the moist unit weight in lb/ft³?
b) What is the dry unit weight in lb/ft³?
c) What is the degree of saturation in percent?

SOLUTION

a) Moist unit weight:

$$\gamma_m = W/V = 23/0.2 = \mathbf{115\ lb/ft^3}$$

b) Dry unit weight:

$$[\gamma_m = \frac{G + GMC}{1+e}\gamma_w]$$

$$115 = \frac{2.7 + 2.7(0.11)}{1+e}(62.4)$$

$$e = 0.626$$

$$[\gamma_d = \frac{G}{1+e}\gamma_w]$$

$$\gamma_d = \frac{2.7}{1+0.626}(62.4)$$

$$\gamma_d = \mathbf{103.62\ lb/ft^3}$$

c) Degree of saturation

$$[G\,MC = S\,e]$$

$$2.7(0.11) = S(0.626)$$

$$S = 0.4744 = \mathbf{47.44\%}$$

PROBLEM 01.11

A specimen of sand has a porosity of 45%, and the specific gravity of its solids is 2.71. Compute the specific weight of this soil in the submerged state, in kN/m³.

SOLUTION

$$[e = \frac{n}{1-n}]$$

$$e = \frac{0.45}{1-0.45}$$

$$e = 0.8182$$

$$[\gamma_b = \frac{G-1}{1+e}\gamma_w]$$

$$\gamma_b = \frac{2.71-1}{1+0.8182}(9.81)$$

$$\gamma_b = \mathbf{9.23\ kN/m^3}$$

PROBLEM 01.12

The mass of a sample of saturated soil is 520 grams. The dry mass, after oven drying is 405 grams. Assuming $G = 2.7$, calculate the effective unit weight of the soil mass, in kN/m³?

SOLUTION

$$[MC = \frac{W_w}{W_s}]$$

$$MC = \frac{W - W_s}{W_s}$$

$$MC = \frac{520-405}{405}$$

$$MC = 28.4\%$$

$[G\,MC = S\,e]$ where $S = 1$ (saturated)

$$2.7(0.284) = (1)e$$

$$e = 0.7667$$

$$[\gamma_b = \frac{G-1}{1+e}\gamma_w]$$

$$\gamma_b = \frac{2.7-1}{1+0.7667}(9.81)$$

$$\gamma_b = \mathbf{9.44\ kN/m^3}$$

PROBLEM 01.13

Laboratory tests on a soil sample yielded the following information: $G = 2.71$, $G_m = 1.72$, $MC = 13\%$. Determine the following properties:

a) void ratio
b) degree of saturation
c) porosity

@seismicisolation

SOLUTION

a) Void ratio:

$[\gamma_m = \gamma_w G_m]$

$[\gamma_m = \frac{G + G\,MC}{1+e}\gamma_w]$

$\frac{G + G\,MC}{1+e}\gamma_w = \gamma_w G_m$

$\frac{2.71 + 2.71(0.13)}{1+e}\gamma_w = \gamma_w(1.72)$

$e = 0.78$

b) $[G\,MC = S\,e]$

$2.71(0.13) = S(0.78)$

$S = 0.452 = 45.2\%$

c) $[n = \frac{e}{1+e}]$

$n = \frac{0.78}{1+0.78}$

$n = 0.438 = 43.8\%$

PROBLEM 01.14

In its natural state, a moist soil has a volume of 9350 cc and weighs 18 kg. The oven-dried weight of the soil is 15.54 kg. Use G = 2.67.

a) Determine the moisture content in percent.
b) Determine the void ratio in percent.
c) Determine the degree of saturation in percent

SOLUTION

$\gamma_m = W/V = 18000/9350$

$\gamma_m = 1.925$ gram/cc

a) $MC = \frac{W_w}{W_s}$

$MC = \frac{18 - 15.54}{15.54}$

$MC = 0.1583 = 15.83\%$

b) $[\gamma_m = \frac{G + G\,MC}{1+e}\gamma_w]$

$1.925 = \frac{2.67 + 2.67(0.1583)}{1+e}(1)$

$e = 0.6066 = 60.66\%$

c) $[G\,MC = S\,e]$

$2.67(0.1583) = S(0.6066)$

$S = 0.697 = 69.7\%$

PROBLEM 01.15 (CE NOVEMBER 1999)

A soil sample was compacted. The result of the standard proctor test shows that at 100% compaction, the soil weighs 131.1 pcf with optimum moisture content of 14%. What is the maximum dry unit weight of the soil (at zero air voids) in pcf? G = 2.67

SOLUTION

At zero air voids, $V_v = V_w$ $(S = 1)$

$[G\,MC = S\,e]$

$2.67\,(0.14) = 1\,e$

$e = 0.3738$

$[\gamma_d = \frac{G}{1+e}\gamma_w]$

$\gamma_d = \frac{2.67}{1+0.3738}(62.4)$

$\gamma_d = 121.28$ pcf

PROBLEM 01.16

A compacted clay weighing 1.62 kg weighs 0.88 kg when immersed (suspended) in water. Determine the bulk specific gravity of the clay.

SOLUTION

$\text{Bulk sp. gr.} = \frac{\text{Weight in air}}{\text{Weight in air} - \text{Weight in water}}$

$\text{Bulk sp. gr.} = \frac{1.62}{1.62 - 0.88} = 2.19$

PROBLEM 01.17

A 50 cc of moist clay was obtained by pressing a sharpened hollow cylinder into the wall of a test pit. The extruded sample had an initial weight of 85 grams. After oven-drying it weighs 60 grams. If G = 2.72, determine the degree of saturation of the sample

SOLUTION

Given: $W = 85$ g
$V = 50$ cm³; $W_s = 60$ g

$$[\gamma_m = \frac{W}{V}]$$
$$\gamma_m = 85/50$$
$$\gamma_m = 1.7 \text{ g/cc}$$

$$[MC = \frac{W_w}{W_s}]$$
$$MC = \frac{85-60}{60}$$
$$MC = 0.4167$$

$$[\gamma_m = \frac{G + G\,MC}{1+e}\gamma_w]$$
$$1.7 = \frac{2.72 + 2.72(0.4167)}{1+e}(1)$$
$$e = 1.2667$$

$$[G\,MC = S\,e]$$
$$2.72(0.4167) = S(1.2667)$$
$$S = 0.895 = 89.5\%$$

PROBLEM 01.18

A cubic meter of soil in its natural state weighs 17.5 kN. After oven-drying the soil weighs 14.2 kN. Assume G = 2.7.

a) Calculate the void ratio of the soil.
b) Calculate the degree of saturation of the soil.
c) Calculate the saturated density of the soil in kN/m³.

SOLUTION

$$[MC = \frac{W_w}{W_s}]$$
$$MC = \frac{W - W_s}{W_s}$$
$$MC = \frac{17.5 - 14.2}{14.2}$$
$$MC = 0.2324 = 23.24\%$$

$$[\gamma_m = \frac{W}{V}]$$
$$\gamma_m = 17.5/1 = 17.5 \text{ kN/m}^3$$

$$[\gamma_m = \frac{G + G\,MC}{1+e}\gamma_w]$$
$$17.5 = \frac{2.7 + 2.7(0.2324)}{1+e}(9.81)$$
$$e = 0.865 = 86.5\% \quad \rightarrow \text{void ratio}$$

$$[G\,MC = S\,e]$$
$$2.7(0.2324) = S(0.865)$$
$$S = 0.7254$$
$$S = 72.54\% \quad \rightarrow \text{degree of saturation}$$

$$[\gamma_{sat} = \frac{G+e}{1+e}\gamma_w]$$
$$\gamma_{sat} = \frac{2.7 + 0.865}{1 + 0.865}(9.81)$$
$$\gamma_{sat} = 18.75 \text{ kN/m}^3$$

PROBLEM 01.19 (CE MAY 2000)

A soil sample has a moisture content of 30% and degree of saturation of 45%. The solids have specific gravity of 2.61. Determine the dry unit weight of the soil in kN/m³.

SOLUTION

$$[G\,MC = S\,e]$$
$$2.61(0.3) = 0.45\,e$$

@seismicisolation

$e = 1.74$

$$[\gamma_{dry} = \frac{G}{1+e}\gamma_w]$$

$$\gamma_{dry} = \frac{2.61}{1+1.74}(9.81)$$

$$\gamma_{dry} = \mathbf{9.34\ kN/m^3}$$

PROBLEM 01.20

The dry density of a sand with a porosity of 0.387 is 1600 kg/m³.

a) Calculate the void ratio of the soil.
b) Calculate the specific gravity of the soil solids.
c) Calculate the effective density of the soil, in kg/m³.

SOLUTION

Given: $\rho_d = 1{,}600\ kg/m^3$
$n = 0.387$

a) $$e = \frac{n}{1-n}$$

$$e = \mathbf{0.631}$$

b) $$\rho_d = \frac{G}{1+e}\rho_w$$

$$1{,}600 = \frac{G}{1+0.631}(1000)$$

$$G = \mathbf{2.61}$$

c) $$\rho_b = \frac{G-1}{1+e}\rho_w$$

$$\rho_b = \frac{2.61-1}{1+0.631}(1000)$$

$$\rho_b = \mathbf{987.12\ kg/m^3}$$

PROBLEM 01.21

The void ratio of a soil is 0.85. What is the percentage error of the bulk unit weight if the soil were 95% saturated and assumed to be totally saturated?

SOLUTION

Assumed S = 1:

$$\gamma_{sat} = \frac{G+e}{1+e}\gamma_w$$

$$\gamma_{sat} = \frac{2.7+0.85}{1+0.85}(9.81)$$

$$\gamma_{sat} = 18.82\ kN/m^3$$

Actual S = 95%:

$$\gamma_m = \frac{G+Se}{1+e}\gamma_w$$

$$\gamma_m = \frac{2.7+0.95(0.85)}{1+0.85}(9.81)$$

$$\gamma_m = 18.60\ kN/m^3$$

$$\text{Percentage Error} = \frac{18.82-18.60}{18.60}\times 100\%$$

Percentage Error = **1.18% more**

PROBLEM 01.22

A wet soil sample has a volume of 4.85×10^{-4} m³ and weighs 8.5 N. After oven drying the weight reduces to 7.5 N. Use G = 2.7. Determine the following:

a) unit weight,
b) moisture content,
c) degree of saturation

SOLUTION

a) $$[\gamma_m = \frac{W}{V}]$$

$$\gamma_m = \frac{8.5}{4.85\times 10^{-4}}$$

$$\gamma_m = 17{,}526\ N/m^3 = \mathbf{17.526\ kN/m^3}$$

b) $$[MC = \frac{W_w}{W_s}]$$

$$MC = \frac{8.5-7.5}{7.5}$$

@seismicisolation

$$MC = 0.1333 = 13.33\%$$

c) Degree of saturation:
Solving for e:

$$[\gamma_m = \frac{G + GMC}{1+e}\gamma_w]$$

$$17.526 = \frac{2.7 + 2.7(0.1333)}{1+e}(9.81)$$

$$e = 0.7128$$

$$[G\,MC = S\,e]$$

$$S = (2.7 \times 0.1333)/0.7128$$

$$S = 0.505 = 50.5\%$$

PROBLEM 01.23 (CE NOVEMBER 2000)

A clay sample has unit weight of 20.06 kN/m³ with moisture content of 8.2%. The saturated unit weight of the sample is 21.58 kN/m³. Determine the porosity of the soil.

SOLUTION

$$[\gamma_m = \frac{G + G\,MC}{1+e}\gamma_w]$$

$$20.06 = \frac{G + G(0.082)}{1+e}(9.81)$$

$$G = 1.89 + 1.89\,e$$

$$[\gamma_{sat} = \frac{G+e}{1+e}\gamma_w]$$

$$21.58 = \frac{1.89 + 1.89e + e}{1+e}(9.81)$$

$$2.2 + 2.2e = 1.89 + 2.89e$$

$$e = 0.45$$

$$[n = \frac{e}{1+e}]$$

$$n = \frac{0.45}{1+0.45}$$

$$n = 0.31 = 31\%$$

PROBLEM 01.24

Situation 4 – A sample of dry sand having a unit weight of 16.50 kN/m³ and a specific gravity of 2.70 is placed in the rain. During the rain the volume of the sample remains constant but the degree of saturation increases to 40%. Determine the following:

a) The voids ratio of the sample in percent.
b) The unit weight of the sample after being in the rain
c) The water content of the sample after being in the rain

SOLUTION

$$G = 2.70$$

$$\gamma_{dry} = 16.5 \text{ kN/m}^3$$

a) $[\gamma_{dry} = \frac{G}{1+e}\gamma_w]$

$$16.5 = \frac{2.7}{1+e} \times 9.81$$

$$e = 0.6053$$

$$e = 60.53\%$$

b) After being in the rain, $S = 0.40$

$$[\gamma_m = \frac{G + Se}{1+e}\gamma_w]$$

$$\gamma_m = \frac{2.70 + 0.40(0.6053)}{1 + 0.6053} \times 9.81$$

$$\gamma_m = 17.98 \text{ kN/m}^3$$

c) $[G\,MC = S\,e]$

$$2.70\,MC = (0.40)(0.6053)$$

$$MC = 0.0897 = 8.97\%$$

PROBLEM 01.25

The moist unit weights and degrees of saturation of a soil are given in the following table:

γ (pcf)	S (%)
105.73	50
112.67	75

@seismicisolation

a) Determine the void ratio of the soil in percent.
b) Determine the specific gravity of the soil solids.
c) Determine the porosity of the soil in percent

SOLUTION

$$[\gamma_m = \frac{G+Se}{1+e}\gamma_w]$$

First soil:

$$105.73 = \frac{G+0.5e}{1+e}(62.4)$$

$$1.694 + 1.694e = G + 0.5e$$

$$G = 1.694 + 1.194e \quad \rightarrow \text{Eq. (1)}$$

Second soil:

$$112.67 = \frac{G+0.75e}{1+e}(62.4)$$

$$1.806 + 1.806e = G + 0.75e$$

$$G = 1.806 + 1.056e \quad \rightarrow \text{Eq. (2)}$$

$[G = G]$

$$1.694 + 1.194e = 1.806 + 1.056e$$

$$e = 0.812 = 81.2\% \rightarrow \text{void ratio}$$

From Eq. (2)

$$G = 1.806 + 1.056(0.812) = 2.66 \rightarrow \text{specific gravity of solids}$$

$$\text{Porosity, } n = \frac{e}{1+e}$$

$$\text{Porosity, } n = \frac{0.812}{1+0.812}$$

$$\text{Porosity, } n = 0.4481 = 44.81\%$$

PROBLEM 01.26

In an experiment of determining the porosity and specific gravity of solids of a soil, a soil is dug out from the ground, weighed, then oven dried, and then saturated with water. The weight of soil taken from the test hole is 1346 grams. Its was then oven-dried and weighed 1076 grams. After saturating it

with water, it weighed 1462 grams. The volume of the test hole was then measured and found to be 792 cc. Determine the voids ratio of the soil.

SOLUTION

Given $W = 1346$ grams
$Ws = 1076$ grams
$V = 792$ cm^3

$[\gamma_m = W/V]$

$$\gamma_m = 1346/792$$

$$\gamma_m = 1.7 \text{ grams/cc}$$

$$[\gamma_{sat} = \frac{W_{sat}}{V}]$$

$$\gamma_{sat} = \frac{1462}{792} = 1.846 \text{ grams/cc}$$

$$[MC = \frac{W_w}{W_s} = \frac{W - W_s}{W_s}]$$

$$MC = \frac{1346 - 1076}{1076}$$

$$MC = 0.251$$

$$[\gamma_m = \frac{G + GMC}{1+e}\gamma_w]$$

$$1.7 = \frac{G + G(0.251)}{1+e}(1)$$

$$G = 1.36 + 1.36\,e \quad \rightarrow (1)$$

$$[\gamma_{sat} = \frac{G+e}{1+e}\gamma_w]$$

$$1.846 = \frac{G+e}{1+e}(1)$$

$$G = 1.846 + 0.846\,e \quad \rightarrow (2)$$

$[G = G]$

$$1.36 + 1.36e = 1.846 + 0.846\,e$$

$$e = 0.946 = 94.6\%$$

PROBLEM 01.27 (CE MAY 2001)

A clay sample has the following properties:

Porosity = 0.35 (in situ)
Maximum void ratio = 0.85
Minimum void ratio = 0.42
Specific gravity of solids = 2.72
Moisture content = 62%

Determine the dry unit weight of the soil in its natural state, in pcf.

SOLUTION

$$[e = \frac{n}{1-n}]$$

$$e = \frac{0.35}{1-0.35}$$

$$e = 0.538$$

$$[\gamma_{dry} = \frac{G}{1+e}\gamma_w]$$

$$\gamma_{dry} = \frac{2.72}{1+0.538}(62.4)$$

$$\gamma_{dry} = 110.36 \text{ pcf}$$

PROBLEM 01.28 (CE NOVEMBER 2001)

A 480 cc soil sample taken from the site weighs 850.5 grams. After oven drying, it weighed 594.4 grams. If the specific gravity of solids is 2.72, determine the void ratio of the soil.

SOLUTION

$$[\gamma_m = \frac{W}{V}]$$

$$\gamma_m = \frac{850.5}{480}$$

$$\gamma_m = 1.772 \text{ g/cc}$$

$$[MC = \frac{W_w}{W_s}]$$

$$MC = \frac{850.5-594.4}{594.4}$$

$$MC = 0.43$$

$$[\gamma_m = \frac{G+G\,MC}{1+e}\gamma_w]$$

$$1.772 = \frac{2.72+2.72(0.43)}{1+e}(1)$$

$$e = 1.195$$

PROBLEM 01.29

A sample of saturated clay was placed in a container and weighed. The weight was 6 N. The clay in its container was placed in an oven for 24 hours at 105°C. The weight was reduced to a constant value of 5 N. The weight of the container is 1 N. G = 2.7. Determine the following:

a) water content of the soil in percent.
b) void ratio in percent.
c) effective unit weight of the soil in kN/m³.

SOLUTION

Weight of moist soil (saturated), $W = 6 - 1 = 5$ N
Weight of dry soil, $W_s = 5 - 1 = 4$ N

a) Water content:

$$[MC = \frac{W_w}{W_s}]$$

$$MC = \frac{5-4}{4} = 0.25$$

$$MC = 25\%$$

b) Void ratio:

$$[G\,MC = S\,e]$$

$$2.7(0.25) = (1)e$$

$$e = 0.675 = 67.5\%$$

c) Effective unit weight:

$$[\gamma_b = \frac{G-1}{1+e}\gamma_w]$$

$$\gamma_b = \frac{2.7-1}{1+0.675}(9.81)$$

$$\gamma_b = 9.956 \text{ kN/m}^3$$

PROBLEM 01.30 (CE NOVEMBER 2001)

A 480 cc soil sample taken from the site weighs 850.5 grams. After oven drying, it weighed 594.4 grams. If the specific gravity of solids is 2.72, determine the degree of saturation of the soil.

SOLUTION

$$[\gamma_m = \frac{W}{V}]$$
$$\gamma_m = \frac{850.5}{480}$$
$$\gamma_m = 1.772 \text{ g/cc}$$

$$[MC = \frac{W_w}{W_s}]$$
$$MC = \frac{850.5 - 594.4}{594.4}$$
$$MC = 0.43$$

$$[\gamma_m = \frac{G + G\,MC}{1+e}\gamma_w]$$
$$1.772 = \frac{2.72 + 2.72(0.43)}{1+e}(1)$$
$$e = 1.195$$

$$[G \times MC = S \times e]$$
$$2.72(0.43) = S(1.195)$$
$$S = 0.979$$

PROBLEM 01.31 (CE NOVEMBER 2002)

Given the following characteristics of a soil sample:

Volume = 0.5 cubic ft.

Mass = 56.7 pound mass

The solids have specific gravity of 2.69. After oven drying, the mass of the soil was 48.7 pounds.

a) What is the density of the in-situ soil?

b) What is the porosity of the in-situ soil?

c) What is the degree of saturation of the in-situ soil?

SOLUTION

Given: $W = 56.7$ lbs

$Ws = 48.7$ lbs

$V = 0.5$ ft³

$G = 2.69$

a) Density of the in-situ soil

$$\gamma_m = \frac{W}{V}$$
$$\gamma_m = \frac{56.7}{0.5}$$
$$\gamma_m = 113.4 \text{ lbs/ft}^3$$

b) Porosity:

$$\text{Moisture content, } MC = \frac{W_w}{W_s}$$
$$\text{Moisture content, } MC = \frac{W - W_s}{W_s}$$
$$\text{Moisture content, } MC = \frac{56.7 - 48.7}{48.7} = 0.1643$$

$$[\gamma = \frac{G + GMC}{1+e}\gamma_w]$$
$$113.4 = \frac{2.69 + 2.69(0.1643)}{1+e}(62.4)$$
$$e = 0.7234$$

$$\text{Porosity, } n = \frac{e}{1+e}$$
$$\text{Porosity, } n = \frac{0.7234}{1 + 0.7234} = 0.4198$$
$$\text{Porosity, } n = \mathbf{41.98\%}$$

c) Degree of saturation

$$[G\,MC = S\,e]$$
$$2.69(0.1643) = S\,(0.7234)$$
$$S = 0.611$$
$$S = \mathbf{61.1\%}$$

PROBLEM 01.32

The saturated unit weight of a soil is 19.49 kN/m³, and the specific gravity of the soil solids is 2.7.

a) What is the void ratio of the soil.
b) What is the dry unit weight of the soil in kN/m³.
c) What is the effective unit weight of the soil in kN/m³.

SOLUTION

a) $[\gamma_{sat} = \frac{G+e}{1+e}\gamma_w]$

$$19.49 = \frac{2.7+e}{1+e}(9.81)$$
$$1.987 + 1.987e = 2.7 + e$$
$$e = 0.7226$$

b) $[\gamma_{dry} = \frac{G}{1+e}\gamma_w]$

$$\gamma_{dry} = \frac{2.7}{1+0.7226}(9.81)$$
$$\gamma_{dry} = 15.376 \text{ kN/m}^3$$

c) $[\gamma_b = \gamma_{sat} - \gamma_w]$

$$\gamma_b = 19.49 - 9.81$$
$$\gamma_b = 9.68 \text{ kN/m}^3$$

PROBLEM 01.33 (CE MAY 2003)

The following data was obtained from laboratory tests for a cohesive specimen: moisture content, w, was 22.5%; G_s = 2.60; and to determine the approximate unit weight, a sample having a mass of 224.0 g was placed in a 500 cm³ container with 382 cm³ of water required to fill the container.

a) What is the total unit weight of the soil sample in kN/m³?
b) What is the void ratio e?
c) What is the dry unit weight of the soil sample in kN/m³?

SOLUTION

Given: $MC = 22.5\%$
$G = 2.60$
Mass, $M = 224$ grams
Volume, $V = 500 - 382 = 118$ cc

a) Unit weight:

$$[\rho = \frac{M}{V}]$$
$$\rho = \frac{224}{500-382} = 1.898 \text{ g/cc}$$
$$[G_m = \frac{\rho}{\rho_w}]$$
$$G_m = \frac{1.898}{1} = 1.898$$
$$[\gamma_m = \gamma_w \times G_m]$$
$$\gamma_m = 9.81 \times 1.898 = \mathbf{18.62 \text{ kN/m}^3}$$

b) Void ratio

$$[\gamma_m = \frac{G+GMC}{1+e}\gamma_w]$$
$$18.62 = \frac{2.6+2.6(0.225)}{1+e} \times 9.81$$
$$e = 0.678$$

c) Dry unit weight

$$[\gamma_d = \frac{G}{1+e}\gamma_w]$$
$$\gamma_d = \frac{2.6}{1+0.678} \times 9.81$$
$$\gamma_d = \mathbf{15.2 \text{ kN/m}^3}$$

PROBLEM 01.34 (CE NOVEMBER 2003)

A fully saturated clay sample has a mass of 1526 grams. After oven-drying, its mass was reduced to 1,053 grams. The specific gravity soil particles is 2.7.

a) Calculate the natural water content of the sample in percent.
b) Calculate the void ratio in percent.
c) Calculate the porosity in percent.

SOLUTION

Given: $S = 1$
$M = 1526$ grams
$M_s = 1053$ grams
$G = 2.7$

a) Water content, MC

$$[MC = \frac{M_w}{M_s}]$$
$$MC = \frac{1,526 - 1,053}{1,053}$$
$$MC = 0.4492 = \mathbf{44.92\%}$$

b) Void ratio

$$[G\,MC = S\,e]$$
$$2.7(0.4492) = (1)e;$$
$$e = 1.213 = \mathbf{121.3\%}$$

c) Porosity

$$[n = \frac{e}{1+e}]$$
$$n = \frac{1.213}{1+1.213}$$
$$n = 0.5481 = \mathbf{54.81\%}$$

PROBLEM 01.35

A sandy soil has a natural water content of 27.5% and bulk unit weight of 19.2 kN/m³. The void ratios corresponding to the densest and loosest state are 0.51 and 0.87. Assume G = 2.7.

a) Which of the following gives the in situ void ratio of the soil.
b) Which of the following gives the degree of saturation of the soil in its natural state.
c) Which of the following gives the relative density of the soil.

SOLUTION

a) $[\gamma_m = \frac{G + G\,MC}{1+e}\gamma_w]$

$$19.2 = \frac{2.7 + 2.7(0.275)}{1+e}(9.81)$$
$$e = \mathbf{0.759}$$

b) $[G\,MC = S\,e]$

$$2.7(0.275) = S(0.759)$$
$$S = \mathbf{0.978}$$

c) $[D_r = \frac{e_{max} - e}{e_{max} - e_{min}}]$

$$D_r = \frac{0.87 - 0.759}{0.87 - 0.51}$$
$$D_r = 0.308 = \mathbf{30.8\%}$$

PROBLEM 01.36 (CE MAY 2000)

A sample of moist sand taken from the field was found to have a moisture content of 14% and a porosity of 38%. In a laboratory test that simulates field conditions, it was found that at its densest state, its void ratio is 85% and at its loosest state its void ratio is 40%. Determine the relative of the sand.

SOLUTION

$$[e = \frac{n}{1-n}]$$
$$e = \frac{0.38}{1 - 0.38}$$
$$e = 0.613$$

$$D_r = \frac{e_{max} - e}{e_{max} - e_{min}}$$
$$D_r = \frac{0.85 - 0.613}{0.85 - 0.40}$$
$$D_r = \mathbf{0.527}$$

PROBLEM 0 1.37

A test of the density of soil in place was performed by digging a small hole in the soil, weighing the extracted soil, and measuring the volume of the hole. The soil (moist) weighed 895 g; the volume of the hole was 426 cm³. After drying, the sample weighed 779 g. Of the dried soil, 400 g was poured into a vessel in a very loose state. Its volume was subsequently determined to be 276 cm³. That same 400 g was then vibrated and tamped to a volume of 212 cm³. G = 2.71. Determine the relative density of the soil.

SOLUTION

$$D_r = \frac{1/\gamma_{dmin} - 1/\gamma_d}{1/\gamma_{dmin} - 1/\gamma_{dmax}}$$

$\gamma_d = W_s / V$
$\gamma_d = 779 / 426 = 1.8286$ g/cc

$\gamma_{d\,max} = 400 / 212$
$\gamma_{d\,max} = 1.8868$ g/cc

$\gamma_{d\,min} = 400 / 276$
$\gamma_{d\,min} = 1.4493$ g/cc

$$D_r = \frac{\frac{1}{1.4493} - \frac{1}{1.8286}}{\frac{1}{1.4493} - \frac{1}{1.8868}}$$

$D_r = 0.8946 = 89.46\%$

PROBLEM 01.38

For a sandy soil, $e_{max} = 0.86$, $e_{min} = 0.43$, and $G_s = 2.66$. What is the required void ratio at $D_r = 56\%$?

SOLUTION

$$[D_r = \frac{e_{max} - e}{e_{max} - e_{min}}]$$

$$0.56 = \frac{0.86 - e}{0.86 - 0.43}$$

$e = 0.6192$

PROBLEM 01.39 (CE MAY 2000)

Field density test on a compacted fill of sandy clay gives the following results:

Weight of moist soil from the hole = 1038 grams
Oven-dried weight of the soil = 914 grams
Volume of test hole = 0.0169 ft³

Laboratory moisture density test on this soil indicated a maximum dry density of 120 pcf at an optimum moisture content of 11%. What is the percent compaction of the fill.

SOLUTION

$$\text{Percent compaction} = \frac{\text{Dry density of soil}}{\text{Maximum dry density}}$$

Maximum dry density = 120 pcf

Dry density, $\gamma_d = \frac{W_s}{V}$

$V = 0.0169\ \text{ft}^3 \times (12^3)(2.54^3)$
$V = 478.55$ cc

$$\text{Dry density, } \gamma_d = \frac{914}{478.55} = 1.91 \text{ gram/cc}$$

Dry density, $\gamma_d = 119.18$ pcf

$$\text{Percent compaction} = \frac{119.48}{120} \times 100\%$$

Percent compaction = 99.3%

PROBLEM 01.40 (CE MAY 2001)

A soil sample in its natural state has a wet density of 155.1 pcf and a moisture content of 36%. After compaction, its maximum dry density is 118.5 pcf. Determine the percent compaction of the soil. $G = 2.65$.

SOLUTION

$$\text{Percent compaction} = \frac{\text{Dry density of soil}}{\text{Maximum dry density}} \times 100\%$$

$$[\gamma = \frac{G + G\,MC}{1 + e}\gamma_w]$$

$$155.1 = \frac{2.65 + 2.65(0.36)}{1 + e}(62.4)$$

$e = 0.45$

$$\left[\gamma_{dry} = \frac{G}{1+e}\gamma_w\right]$$

$$\gamma_{dry} = \frac{2.65}{1+0.45}(62.4)$$

$$\gamma_{dry} = 114.04 \text{ pcf}$$

$$\text{Percent compaction} = \frac{114.04}{118.5} \times 100\%$$

$$\text{Percent compaction} = 96.24\%$$

PROBLEM 0 1.41

An embankment for a highway 30 m wide and 1.2 m in compacted thickness is to be constructed from a sandy soil trucked from a borrow pit. The water content of the sandy soil in the borrow pit is 15% and its void ratio is 0.75. The specification requires the embankment be compacted to a dry unit weight of 18.2 kN/m³. Length of embankment is 1.5 km. Assume G = 2.7. Determine the volume of borrow material required.

SOLUTION

Volume of finished embankment = (30)(1.2)(1500) = 54,000 m³

Dry unit weight of borrow material:

$$\left[\gamma_d = \frac{G}{1+e}\gamma_w\right]$$

$$\gamma_d = \frac{2.7}{1+0.75}(9.81)$$

$$\gamma_d = 15.135 \text{ kN/m}^3$$

$$\text{Volume of borrow material} = \frac{(\gamma_d)_{reqd}}{(\gamma_d)_{pit}}(V)_{\text{finished embankment}}$$

$$\text{Volume of borrow material} = \frac{18.2}{15.135}(54{,}000)$$

$$\text{Volume of borrow material} = \mathbf{64{,}936\ m^3}$$

PROBLEM 0 1.42

A building requires a 10,000-m³ fill at a void ratio of 20%. Material for earth fill was available from a borrow site at P320 per cubic meter. It was found that the average void ratio from the site is 80%. Estimate the total cost of fill.

SOLUTION

Porosity, $n = e/(1 + e)$

For the required soil:

$n = 0.2/(1 + 0.2)$

$n = 0.1667 = 16.67\%$ (% void)

Percent solid = $1 - n$ = 0.8333 = 83.33%

Volume of solid = 10,000 × 0.8333

Volume of solid = 8,333 m³

For the borrow soil:

$n = 0.8/(1 + 0.8)$

$n = 0.4444 = 44.44\%$

Percent solid = 100 − 44.44 = 55.56%

Volume of solid = Loose volume × %solid

$8{,}333 = V_{loose} \times 0.5556$

$V_{loose} = 15{,}000 \text{ m}^3$

Or:

$[V_{loose} = (1 + e)V_{solid}]$

$V_{loose} = (1 + 0.8)(8{,}333.33)$

$V_{loose} = 15{,}000 \text{ m}^3$

Total cost = 15,000 m³ × P320/m³

Total cost = **P4,800,000.00**

PROBLEM 0 1.43

Following are the results of a shrinkage limit test:

Initial volume of soil in saturated state = 24.6 cc

Final volume of soil in a dry state = 15.9 cc

Initial mass in a saturated state = 44 g

Final mass in a dry state = 30.1 g

a) Determine the dry density of the soil in grm/cc.

b) Determine the void ratio of the soil.

c) Determine the shrinkage limit of the soil
d) Determine the shrinkage ratio
e) Determine the specific gravity of the solids

SOLUTION

a) $\rho_d = M_s / V$

$\rho_d = 30.1/24.6$

$\rho_d = 1.2236$ g/cc

b) $\rho_{sat} = M_{sat} / V$

$\rho_{sat} = 44/24.6 = 1.7886$ g/cc

$$[\rho_d = \frac{G}{1+e}\rho_w]$$

$$1.2236 = \frac{G}{1+e}1$$

$$G = 1.2236(1+e)$$

$$\rho_{sat} = \frac{G+e}{1+e}\rho_w$$

$$1.7886 = \frac{G+e}{1+e}(1)$$

$$1.7886(1+e) = 1.2236(1+e) + e$$

$$0.565(1+e) = e$$

$$0.565 + 0.565e = e$$

$$e = 1.3$$

c) $$SL = \left(\frac{m_1 - m_2}{m_2}\right) - \left(\frac{V_1 - V_2}{m_2}\right)\rho_w$$ Note: $\rho_w = 1$ grm/cc

$$SL = \frac{44-30.1}{30.1} - \frac{24.6-15.9}{30.1}(1) = 0.1728 \approx 17.28\%$$

d) $$[SR = \frac{1}{\rho_w}\frac{m_2}{v_2}]$$

$$SR = \frac{1}{1}\frac{30.1}{15.9}$$

$$SR = 1.893$$

e) $$[G = \frac{1}{\frac{1}{SR} - \frac{SL}{100}}]$$

$$G = \frac{1}{\frac{1}{1.893} - \frac{17.28}{100}}$$

$$G = 2.813$$

PROBLEM 0 1.44

The following results were obtained from a liquid limit test on a clay using the Casagrande cup device. Use the graph in Figure 01.7. The natural water content of this clay is 38% and the plastic limit is 21%.

Number of blows	6	12	20	28	32
Water content (%)	52.5	47.1	42.3	38.6	37.5

a) What is the liquid limit of this clay?
b) What is the plasticity index of this clay?
c) What is the liquidity index of this clay?
d) What is the flow index?

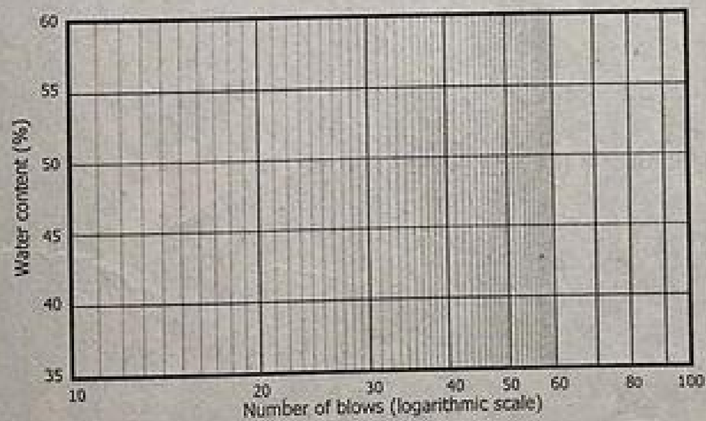
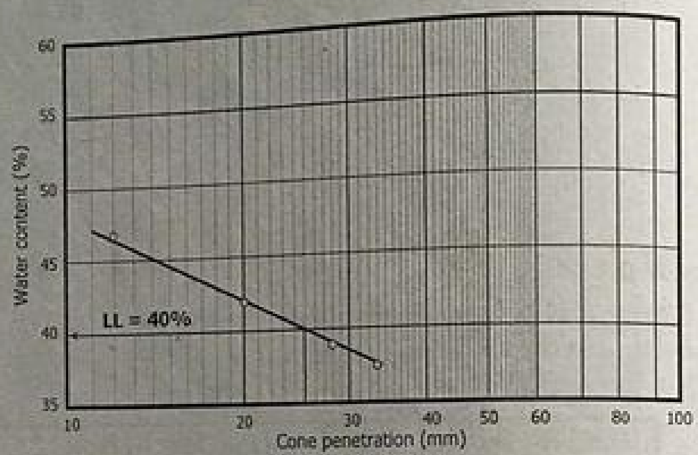


Figure 01.7

SOLUTION



The liquid limit is read from the graph as the water content on the liquid state line corresponding to 25 blows.

Thus, $LL = 40\%$

Plasticity index, $PI = LL - PL$
Plasticity index, $PI = 40 - 21 = 19\%$

$$\text{Liquidity index, } LI = \frac{MC_n - PL}{PI}$$

$$\text{Liquidity index, } LI = \frac{38 - 21}{19} = 0.895$$

$$\text{Flow index, } FI = \frac{MC_1 - MC_2}{\log(N_2 / N_1)}$$

PROBLEM 01.45

Given the laboratory results of the Atterberg Limits Test in Figure 01.8. Plot the water content versus the cone penetration in Figure 01.9.

a) Determine the nearest value to the Liquid Limit of the soil.
b) Determine the nearest value to the Plastic Limit of the soil.
c) Determine the nearest value to the Liquidity Index of the soil.

A. Liquid Limit				
Test Number →	1	2	3	4
Cone penetration, mm	16	18	28	33
Weight of Wet Soil + Container, g	35.62	36.91	41.26	45.70
Weight of Dry Soil + Container, g	28.84	29.89	31.42	33.69
Weight of Container, g	10.52	12.33	11.74	11.45
Weight of Water, g				
Weight of Dry Soil, g				
Water Content, %				

B. Plastic Limit and Natural Water Content				
	Plastic Limit		Natural Water Content	
Test Number →	1	2	1	2
Weight of Wet Soil + Container, g	30.18	31.78	27.77	30.04
Weight of Dry Soil + Container, g	25.76	27.18	25.39	27.23
Weight of Container, g	10.52	12.33	11.74	11.45
Weight of Water, g				
Weight of Dry Soil, g				
Water Content, %				
Average, %				

Figure 01.8

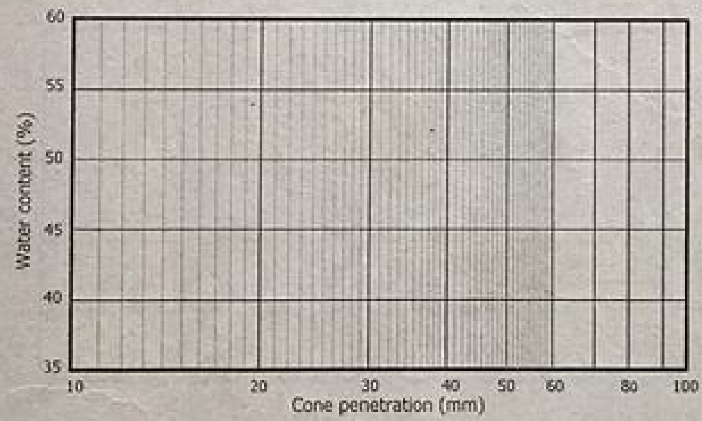


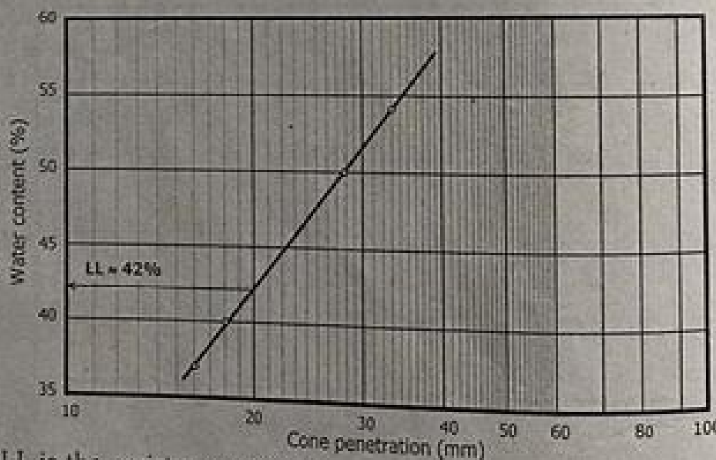
Figure 01.9

@seismicisolation

SOLUTION

A. Liquid Limit				
Test Number →	1	2	3	4
Cone penetration	16	18	28	33
Weight of Wet Soil + Container, g	35.62	36.91	41.26	45.70
Weight of Dry Soil + Container, g	28.84	29.89	31.42	33.69
Weight of Container, g	10.52	12.33	11.74	11.45
Weight of Water, g	6.78	7.02	9.84	12.01
Weight of Dry Soil, g	18.32	17.56	19.68	22.24
Water Content, %	37%	40%	50%	54%

B. Plastic Limit and Natural Water Content	Plastic Limit		Natural Water Content	
Test Number →	1	2	1	2
Weight of Wet Soil + Container, g	30.18	31.78	27.77	30.04
Weight of Dry Soil + Container, g	25.76	27.18	25.39	27.23
Weight of Container, g	10.52	12.33	11.74	11.45
Weight of Water, g	4.42	4.60	2.38	2.81
Weight of Dry Soil, g	15.24	14.85	13.65	15.78
Water Content, %	29%	31%	17.40%	17.80%
Average, %	30%		17.60%	



LL is the moisture content corresponding to 20 mm cone penetration. From the graph shown, $LL \approx 42\%$

From the table above, the Plastic Limit is the average of the MC of the two tests, and is equal to 30%

$$\text{Liquidity index, } LI = \frac{MC_n - PL}{LL - PL}$$

$$\text{Liquidity index, } LI = \frac{17.60 - 30}{42 - 30}$$

$$\text{Liquidity index, } LI = \mathbf{-1.033}$$

PROBLEM 01.46 (CE MAY 2004)

Given the laboratory results of the Atterberg Limits Test in Figure 01.10. Plot the water content versus the number of blows in Figure 01.11.

a) Determine the nearest value to the Liquid Limit of the soil.
b) Determine the nearest value to the Plastic Limit of the soil.
c) Determine the nearest value to the Liquidity Index of the soil.

A. Liquid Limit				
Test Number →	1	2	3	4
Number of Blows	38	29	20	14
Weight of Wet Soil + Container, g	22.47	21.29	21.27	26.12
Weight of Dry Soil + Container, g	19.44	18.78	18.75	22.10
Weight of Container, g	12.74	13.24	13.20	13.27
Weight of Water, g				
Weight of Dry Soil, g				
Water Content, %				

B. Plastic Limit and Natural Water Content	Plastic Limit		Natural Water Content	
Test Number →	1	2	1	2
Weight of Wet Soil + Container, g	23.20	22.80	17.53	16.97
Weight of Dry Soil + Container, g	20.42	20.19	14.84	14.36
Weight of Container, g	12.90	12.95	9.50	9.55
Weight of Water, g				
Weight of Dry Soil, g				
Water Content, %				
Average, %				

Figure 01.10

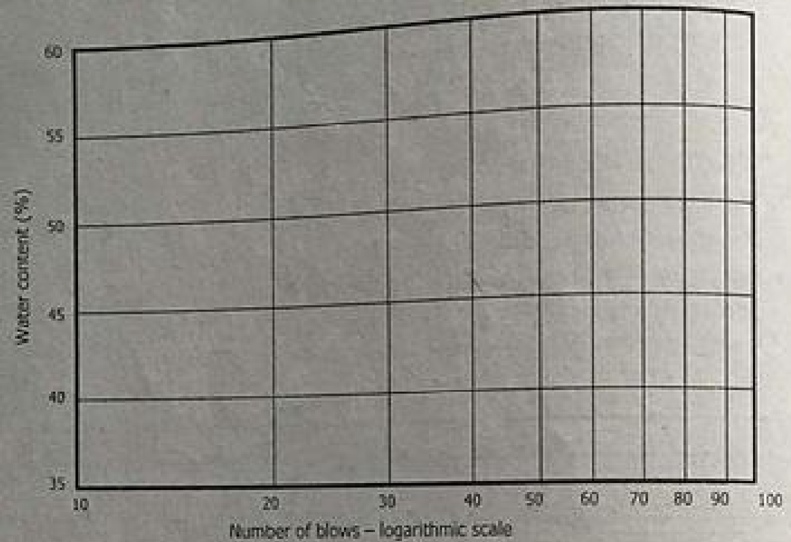


Figure 01.11

SOLUTION

A. Liquid Limit				
Test Number →	1	2	3	4
Number of Blows	38	29	20	14
Weight of Wet Soil + Container, g	22.47	21.29	21.27	26.12
Weight of Dry Soil + Container, g	19.44	18.78	18.75	22.10
Weight of Container, g	12.74	13.24	13.20	13.27
Weight of Water, g	3.03	2.51	2.52	4.02
Weight of Dry Soil, g	6.7	5.54	5.55	8.83
Water Content, %	45.22%	45.31%	45.41%	45.53%

B. Plastic Limit and Natural Water Content				
	Plastic Limit		Natural Water Content	
Test Number →	1	2	1	2
Weight of Wet Soil + Container, g	23.20	22.80	17.53	16.97
Weight of Dry Soil + Container, g	20.42	20.19	14.84	14.36
Weight of Container, g	12.90	12.95	9.50	9.55
Weight of Water, g	2.78	2.61	2.69	2.81
Weight of Dry Soil, g	7.52	7.24	5.34	4.81
Water Content, %	36.97%	36.05%	50.37%	54.26%
Average, %	36.51%		52.32%	

Figure 01.10 (a)

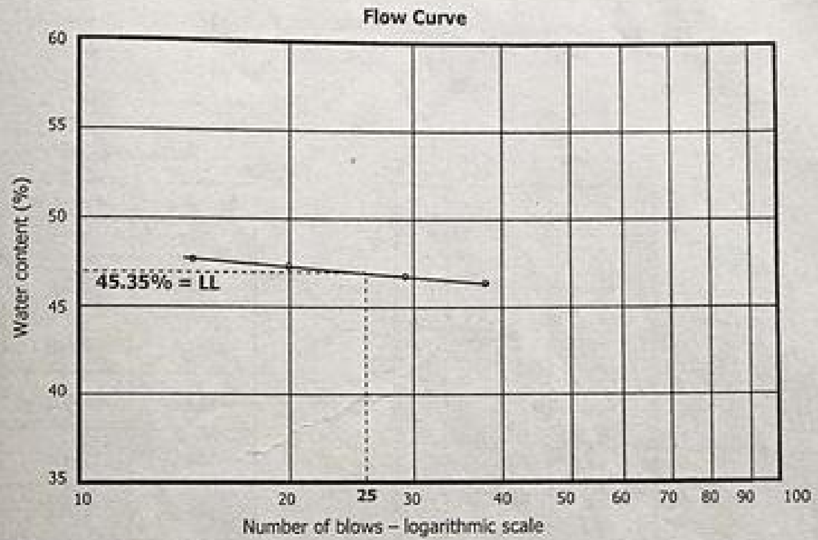


Figure 01.11 (a)

a) From the flow curve in Figure 01.11 (a), LL = **45.35%**

b) From Figure 01.10 (a)

$$PL = \frac{36.97\% + 36.05\%}{2}$$

$$PL = 36.51\%$$

c) $$LI = \frac{MC_n - PL}{LL - PL}$$

$$MC_n = \frac{50.37 + 54.26}{2}$$

$$MC_n = 52.32\%$$

$$LI = \frac{52.32 - 36.51}{45.35 - 36.51} = 1.788$$

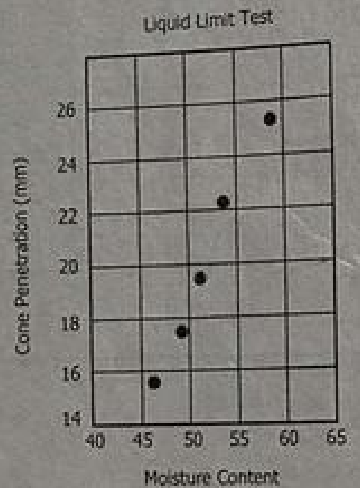
@seismicisolation

PROBLEM 01.47 (CE NOVEMBER 1998, MAY 2001)

The results of Liquid Limit and Plastic Limit tests are shown in Figure 01.12

a) Determine the Liquid Limit (LL) of the soil

b) Determine the Plasticity index



Plastic Limit Test

Weight of Moist Soil (g)	Weight of Oven Dried Soil (g)
128.6	105.4
141.4	116.8
132.6	109.6
134.6	111.2
136.0	113.4

Figure 01.12

SOLUTION

a) Liquid limit:

The liquid limit of the soil is the moisture content corresponding to 20 mm cone penetration.

From Figure 01.13, LL = 52%

b) Plasticity index:

Plasticity Index, $PI = LL - PL$

From the plastic limit test, PL is the average of all the tests.

$$LL = MC = \frac{W_w}{W_s}$$

$$LL = \frac{W_{moist} - W_{dry}}{W_{dry}}$$

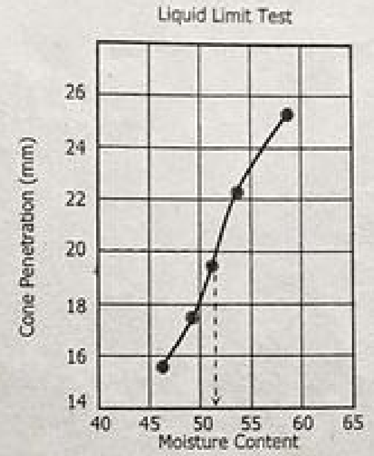


Figure 01.13

Plastic Limit Test

Weight of Moist Soil (g)	Weight of Oven Dried Soil (g)	LL (MC)
128.6	105.4	22.01%
141.4	116.8	21.06%
132.6	109.6	20.99%
134.6	111.2	21.04%
136.0	113.4	19.93%

$PL = (22.01 + 21.06 + 20.99 + 21.04 + 19.93) / 5$

$PL = 21.01\%$

$PI = 52\% - 21.01\% = 30.99\%$

PROBLEM 01.48 (CE MAY 2000)

The result of a Standard Proctor Test is as follows:

Water Content (%)	Weight of moist soil in Proctor mold (grams)
10	1,485
12	1,606
14	1,696
16	1,757
18	1,741
20	1,651

The volume of the mold for this test is 1/30 cubic feet (946,000 cubic millimeters).

a) Determine the maximum dry unit weight of the soil in grams/cc.
b) Determine the optimum moisture content in percent.

SOLUTION

$$W_s = \frac{W}{1+MC}$$

$$\gamma_{dry} = \frac{W_s}{V} = \frac{W}{(1+MC)V}$$

$V = 946$ cc

MC (%)	W (grams)	$\gamma_{dry} = \frac{W}{(1+MC)V}$
10	1,485	1.4271
12	1,606	1.5158
14	1,696	1.5726
16	1,757	1.5993
18	1,741	1.5596
20	1,651	1.4544

From the table shown, the maximum dry unit weight is 1.5993 grams/cm³ and the optimum moisture content is 16%.

PROBLEM 01.49 (CE MAY 2003)

The following data were obtained from the Atterberg Limits test for a soil:

Liquid Limit = 41.0 %
Plastic Limit = 21.1 %

a) What is the plasticity index of the soil?
b) If the in situ moisture content of the soil is 30%, what is the liquidity index of the soil?
c) What would be the nature of the soil?

SOLUTION

a) Plasticity Index:

$PI = LL - PL = 41 - 21.1$
$PI = \mathbf{19.9\%}$

b) Liquidity Index:

$$LI = \frac{MC - PL}{PI}$$

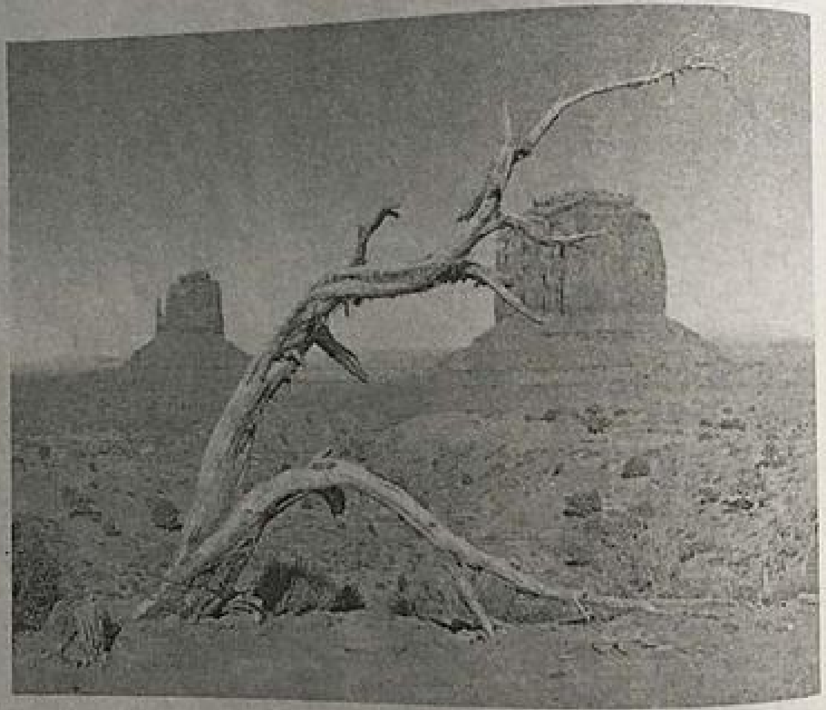
$$LI = \frac{30 - 21.1}{19.9}$$

$LI = \mathbf{0.447}$

c) Nature of soil:

Since $LI = 0.447$, i.e. $0 < LI < 1$, the nature of the soil is **PLASTIC**
See Section 1.10 in page 7.

@seismicisolation



Chapter 02

Classification of Soil

2.1 TEXTURAL CLASSIFICATION

In this classification system, the soils are named after their principal components, such as *sandy clay, silty clay, silty loam,* and so on. There are a number of classification systems developed by different organizations. Shown below is the one developed by the U.S. Department of Agriculture (USDA). This method is based on the following limits of particle size:

Sand size : 2.0 to 0.05 mm in diameter
Silt size : 0.05 to 0.002 mm in diameter
Clay size : smaller than 0.002 mm in diameter

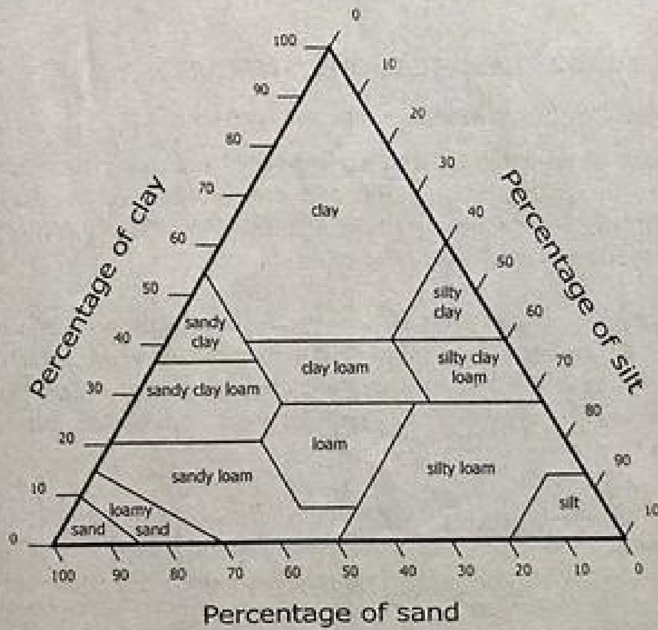


Figure 02.1 – USDA Triangular Textural Classification Chart

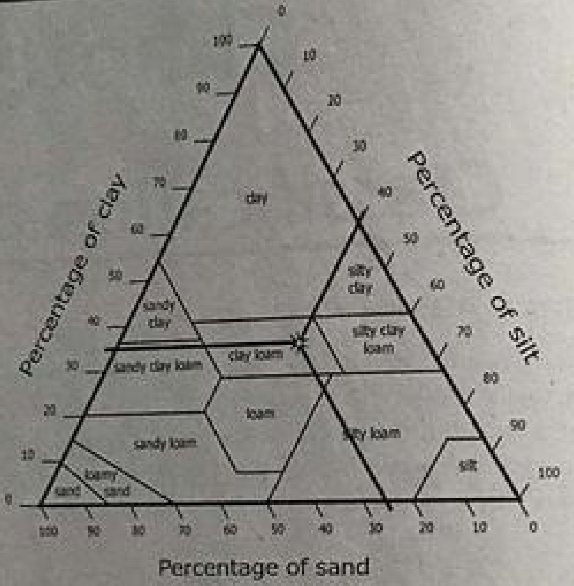


Figure 02.2 – Example: 35% clay, 40% sand, 26% sand (clay loam)

2.2 UNIFIED SOIL CLASSIFICATION SYSTEM (USCS)

This system classifies soils into two broad categories:

1. Coarse-grained soils that are gravelly and sandy in nature with less than 50% passing through the No. 200 Sieve. the group symbols start with prefixes of either G or S. G for gravel or gravelly soil, and S for sand or sandy soil.

2. Fine-grained soil with 50% or more passing through the No. 200 sieve. The groups symbol start with prefixes of M, which stands for inorganic silt, C for inorganic clay, and O for organic silts and clays. The symbol Pt is used for peat, muck, and other highly organic soils.

Other symbols used:

W - well graded
P - poorly graded
L - low plasticity (LL < 50)
H - high plasticity (LL > 50)

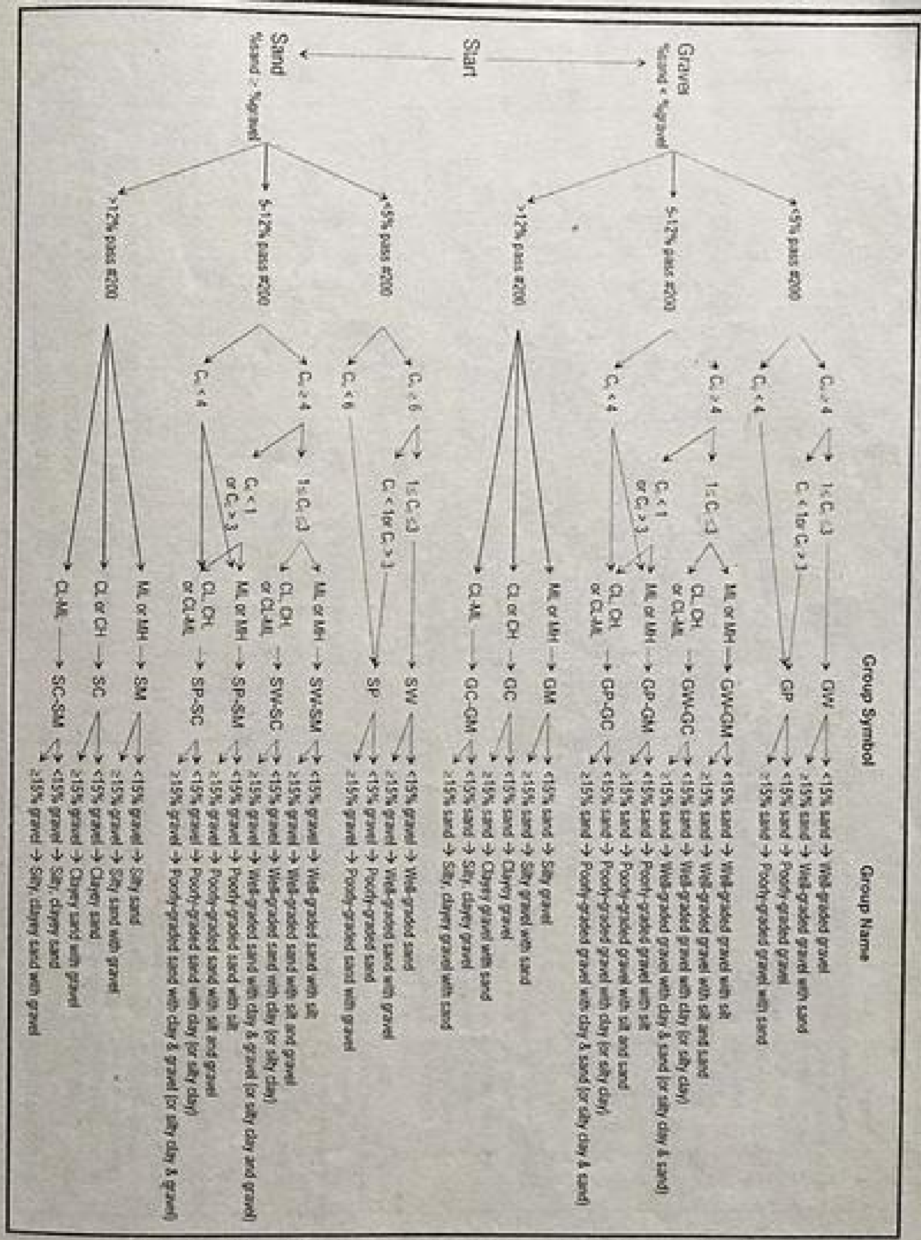


Figure 02.3 – Flow chart for classification of coarse-grained soils (<50% passing No. 200 sieve) (USCS)

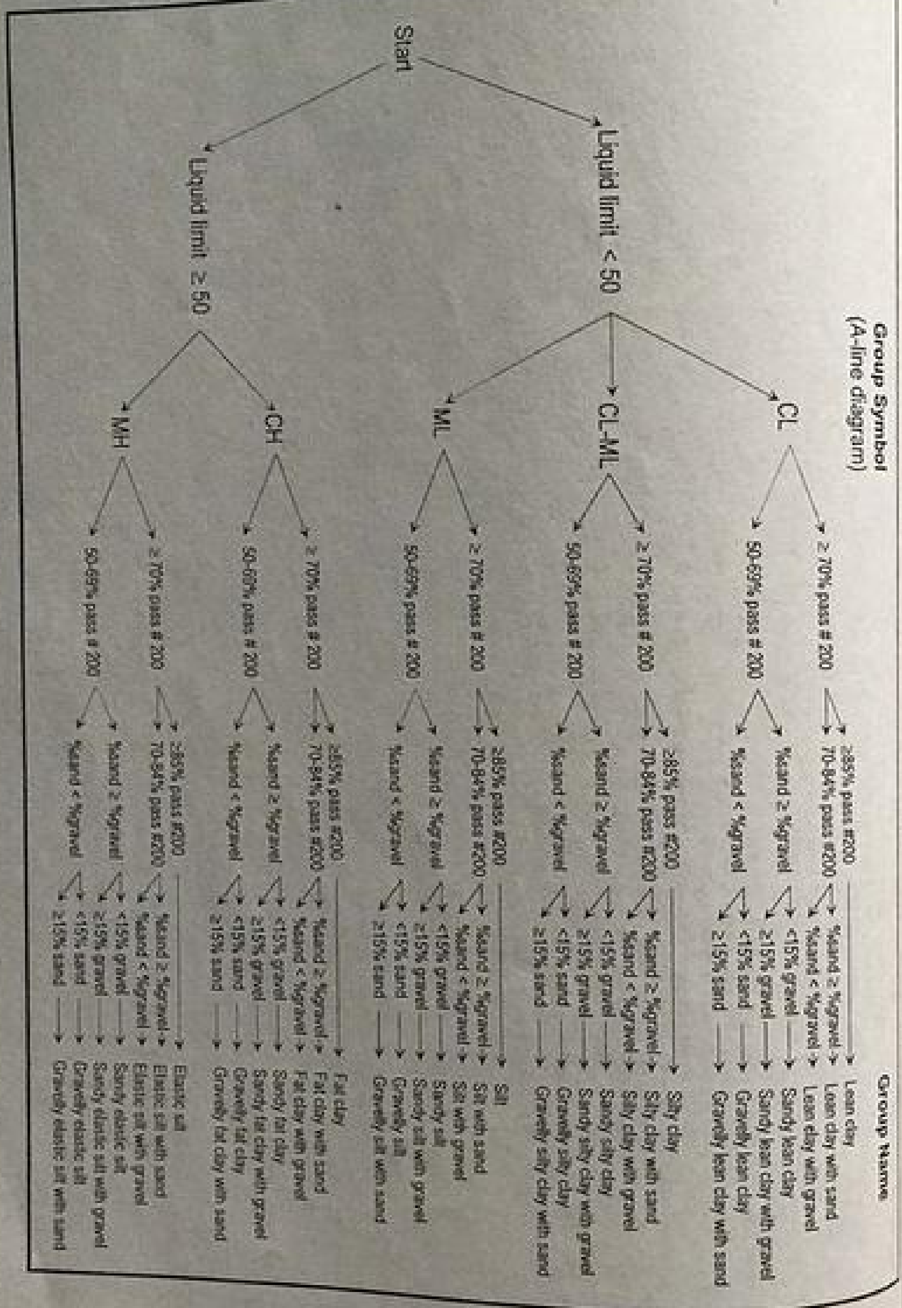
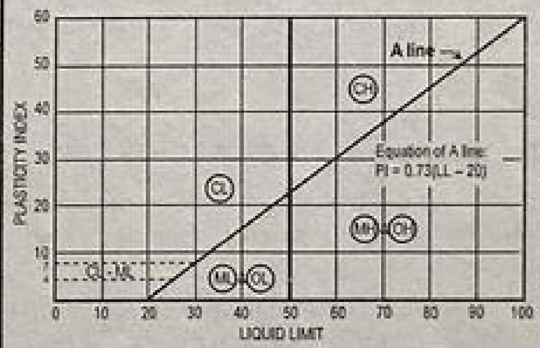


Figure 02.4 – Flow chart for classification of organic fine-grained soils (≥50% passing No. 200 sieve) (USCS)

Table 02.1 - UNIFIED SOIL CLASSIFICATION SYSTEM (USCS)

Major Divisions			Group Symbols	Typical Names		Classification Criteria	
Coarse-Grained Soils More than 50% retained on No. 200 sieve	Gravels 50% or more of coarse fraction retained on No. 4 sieve	Clean Gravels	GW	Well-graded gravels and gravel-sand mixtures, little or no fines	Classification on basis of percentage of fines. Less than 5% pass No. 200 sieve: GW, GP, SW, SP. More than 12% pass No. 200 sieve: GM, GC, SM, SC. 5% to 12% pass No. 200 sieve: Borderline Classification requiring use of dual symbols	$C_u = (D_{60}/D_{10}) > 4$ $C_c = (D_{30})^2/(D_{10} \times D_{60})$ Between 1 & 3	
			GP	Poorly graded gravels and gravel-sand mixtures, little or no fines		Not meeting both criteria for GW	
		Gravels with fines	GM	Silty gravels, gravel-sand-silt mixtures		Atterberg limits plot below "A" line or Plasticity index less than 4	Atterberg limits plotting in hatched area are borderline classifications requiring use of dual symbols
			GC	Clayey gravels, gravel-sand-clay mixtures		Atterberg limits plot above "A" line or Plasticity index greater than 7	
	Sands more than 50% of coarse fraction passes No. 4 sieve	Clean Sands	SW	Well graded sands and gravely sands, little or no fines		$C_u = (D_{60}/D_{10}) > 6$ $C_c = (D_{30})^2/(D_{10} \times D_{60})$ Between 1 & 3	
			SP	Poorly graded sands and gravely sands, little or no fines		Not meeting both criteria for SW	
		Sands with fines	SM	Silty sands, sand-silt mixtures		Atterberg limits plot below "A" line or Plasticity index less than 4	Atterberg limits plotting in hatched area are borderline classifications requiring use of dual symbols
			SC	Clayey sands, sand-clay mixtures		Atterberg limits plot above "A" line or Plasticity index greater than 7	
Fine Grained Soils 50% or more passes No. 200 sieve	Silts and Clays Liquid Limit 50% or less		ML	Inorganic silts, very fine sands, rock flour, silty or clayey fine sands			
			CL	Inorganic clays of low to medium plasticity, gravely clays, sandy clays, silty clays, lean clays			
			OL	Organic silts and organic silty clays of low plasticity			
	Silts and Clays Liquid Limit greater than 50%		MH	Inorganic silts, micaceous or diatomaceous fine sands or silts, elastic silts			
			CH	Inorganic clays of high plasticity, fat clays			
			OH	Organic clays of medium to high plasticity			
Highly Organic Soils			PT	Peat, muck, and other highly organic soils	Visual-Manual identification, See ASTM Designation D2488		

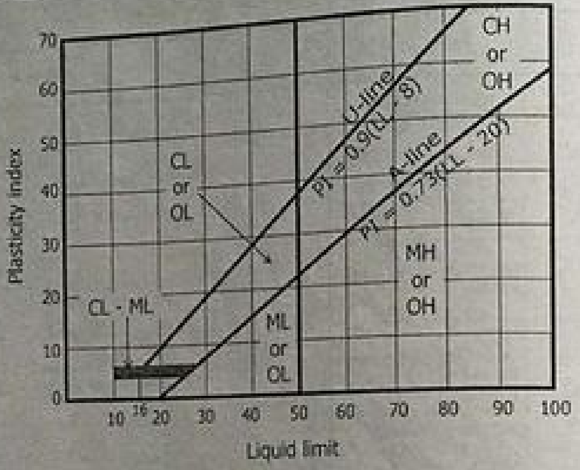


Figure 02.5 – Plasticity chart

- CL ‣ Inorganic; $LL < 50$; $PI > 7$; Atterberg limits plot on or above A-line
- ML ‣ Inorganic; $LL < 50$; $PI < 4$ or Atterberg limits plot below A-line
- OL ‣ Organic; $(LL - \text{ovendried})/(LL - \text{not dried}) < 0.75$; $LL < 50$
- CH ‣ Inorganic; $LL \geq 50$; Atterberg limits plot on or above A-line
- MH ‣ Inorganic; $LL \geq 50$; Atterberg limits plot below A-line
- OH ‣ Organic; $(LL - \text{ovendried})/(LL - \text{not dried}) < 0.75$; $LL \geq 50$
- CL – ML ‣ Inorganic, Atterberg limits plot in the hatched zone

2.3 PARTICLE-SIZE DISTRIBUTION CURVE (SIEVE ANALYSIS)

Sieve analysis consists of shaking the soil sample through a set of sieves that have progressively smaller openings. These sieves are generally 200 mm in diameter.

To conduct a sieve analysis, the soil is first oven-dried and then all lumps must be broken into small particles. The soil is then shaken through a stack of sieves with openings of decreasing size from top to bottom. A pan is placed below the stack.

The following example shows the calculation procedure for sieve analysis.

Sieve No	Diameter (mm)	Mass Retained (grams)	Cumulative mass retained above each sieve	Percent passing (percent finer)
4	4.76	5	5	99.37%
8	2.38	45	50	93.71%
10	2.00	65	115	85.53%
20	0.84	92	207	73.96%
40	0.42	152	359	54.84%
60	0.25	115	474	40.38%
80	0.180	212	686	13.71%
100	0.149	63	749	5.79%
200	0.074	32	781	1.76%
PAN		14	795	0.00%
	TOTAL →	795		

Sieve #4: Cumulative mass retained = 5 g
Percent passing = $(795 - 5)/795 \times 100\% = 99.37\%$

Sieve #8: Cumulative mass retained = 5 + 45 = 50
Percent passing = $(795 - 50)/795 \times 100\% = 93.71\%$

Sieve #10: Cumulative mass retained = 50 + 65 = 115
Percent passing = $(795 - 115)/795 \times 100\% = 85.53\%$

Sieve #20: Cumulative mass retained = 115 + 92 = 207
Percent passing = $(795 - 207)/795 \times 100\% = 73.96\%$

A particle-size distribution curve can be used to determine the following four parameters for a give soil:

2.3.1 EFFECTIVE SIZE, D_{10}

This parameter is the diameter in the curve corresponding to 10% finer. The effective size of a granular soil is a good measure to estimate the hydraulic conductivity and drainage through soil.

In the given example, $D_{10} = 0.17$ mm

2.3.2 UNIFORMITY COEFFICIENT, C_U

$$C_u = \frac{D_{60}}{D_{10}} \quad \text{Eq. 2.1}$$

where D_{60} = diameter corresponding to 60% finer

In the example, $C_u = \dfrac{0.51}{0.17} = 3$

2.3.3 COEFFICIENT OF GRADATION OR COEFFICIENT OF CURVATURE, C_C

$$C_c = \frac{(D_{30})^2}{D_{60} \times D_{10}} \quad \text{Eq. 2.2}$$

where D_{30} = diameter corresponding to 30% finer

In the example, $C_c = \dfrac{(0.21)^2}{0.51(0.17)} = 0.509$

2.3.4 SORTING COEFFICIENT, S_O

$$S_o = \sqrt{\frac{D_{75}}{D_{25}}} \quad \text{Eq. 2.3}$$

where

D_{75} = diameter corresponding to 75% finer

D_{25} = diameter corresponding to 25% finer

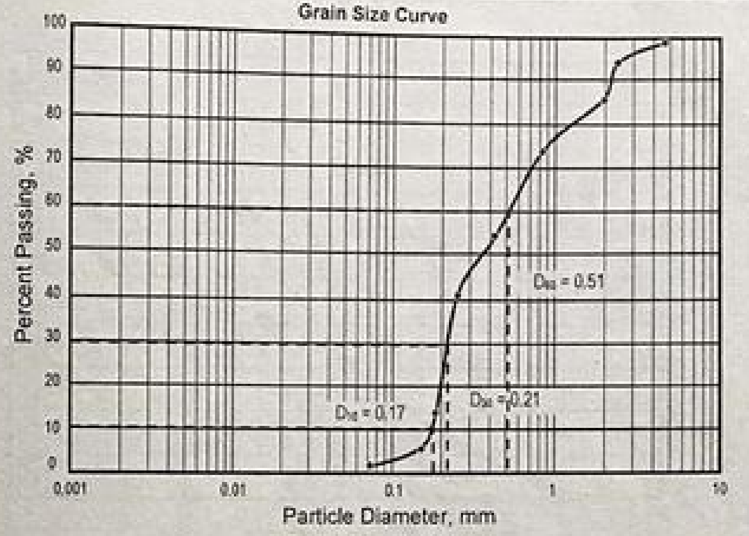


Figure 02.6 – Particle distribution curve for the above example

The particle-size distribution curve shows not only the range of particle sizes present in the soil, but also the type of distribution of various-size particles.

Poorly graded soil is one where most of the soil grains are the same size.

Well graded soil is one in which the particle sizes are distributed over a wide range. A well graded soil has C_u greater than about 4 for gravels and 6 for sands, and C_c between 1 and 3 for gravels and sands.

Gap graded soil is characterized by two ore more humps in the grading curve.

The average grain size of the soil is D_{50}.

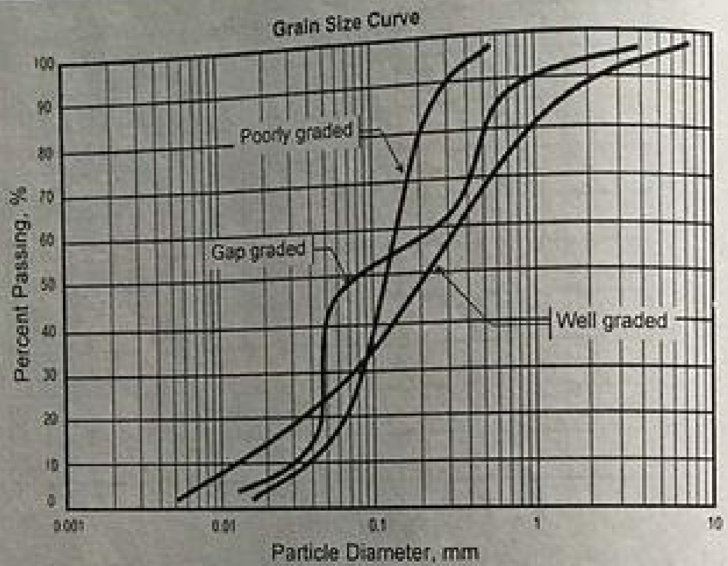


Figure 02.7 – Different types of particle-size distribution curve

2.4 AASHTO CLASSIFICATION SYSTEM

According to this system, soil is classified into seven major groups: A-1 through A-7. Soils classified under groups A-1, A-2, and A-3 are granular materials of which 35% or less of the particles pass through the No. 200 sieve. Soils of which more than 35% pass through the No. 200 sieve are classified under groups A-4, A-5, A-6, and A-7. These soils are mostly silt and clay-type materials.

To classify the soil using the tables below, one must apply the test data from left to right. By process of elimination, the first group from the left into which the test data fit is the correct classification.

To evaluate the quality of a soil as a highway subgrade material, one must also incorporate a number called the *group index* with the groups and subgroups of the soil. This index is written in parentheses after group or subgroup designation, example, A-7-5(35).

$$GI = (F_{200} - 35)[0.2 + 0.005(LL - 40)] + 0.01(F_{200} - 15)(PI - 10) \qquad \text{Eq. 2.4}$$

where:

F_{200} = percentage passing No. 200 sieve
LL = liquid limit, PI = plasticity index

Table 02.2 - Classification of Highway Subgrade Materials for Granular Materials (AASHTO)

General classification	Granular materials (35% or less of total sample passing No. 200)						
	A-1			A-2			
Group classification	A-1-a	A-1-b	A-3	A-2-4	A-2-5	A-2-6	A-2-7
Sieve analysis (percentage passing)							
No. 10	50 max.						
No. 40	30 max.	50 max.	51 min.				
No. 200	15 max.	25 max.	10 max.	35 max.	35 max.	35 max.	35 max.
Characteristics of fraction passing No. 40							
Liquid limit				40 max.	41 min.	40 max.	41 min.
Plasticity index	6 max.		NP	10 max.	10 max.	11 min.	11 min.
Usual types of significant constituent materials	Stone fragments, gravel, and sand		Fine sand	Silty or clayey gravel and sand			
General subgrade rating	Excellent to good						

Table 02.3 - Classification of Highway Subgrade Materials for Silt-Clay Materials (AASHTO)

General classification	Silt-clay materials (more than 35% of total sample passing No. 200)			
Group classification	A-4	A-5	A-6	A-7 A-7-5[a] A-7-6[b]
Sieve analysis (percentage passing)				
No. 10				
No. 40				
No. 200	36 min.	36 min.	36 min.	36 min.
Characteristics of fraction passing No. 40				
Liquid limit	40 max.	41 min.	40 max.	41 min.
Plasticity index	10 max.	10 max.	11 min.	11 min.
Usual types of significant constituent materials	Silty soils		Clayey soils	
General subgrade rating	Fair to poor			

[a] For A-7-5, $PI \le LL - 30$
[b] For A-7-6, $PI > LL - 30$

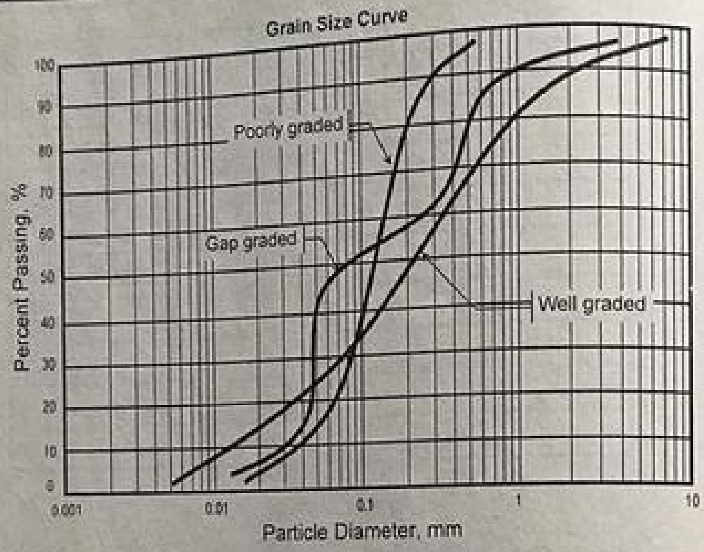


Figure 02.7 – Different types of particle-size distribution curve

2.4 AASHTO CLASSIFICATION SYSTEM

According to this system, soil is classified into seven major groups: A-1 through A-7. Soils classified under groups A-1, A-2, and A-3 are granular materials of which 35% or less of the particles pass through the No. 200 sieve. Soils of which more than 35% pass through the No. 200 sieve are classified under groups A-4, A-5, A-6, and A-7. These soils are mostly silt and clay-type materials.

To classify the soil using the tables below, one must apply the test data from left to right. By process of elimination, the first group from the left into which the test data fit is the correct classification.

To evaluate the quality of a soil as a highway subgrade material, one must also incorporate a number called the *group index* with the groups and subgroups of the soil. This index is written in parentheses after group or subgroup designation, example, A-7-5(35).

$$GI = (F_{200} - 35)[0.2 + 0.005(LL - 40)] + 0.01(F_{200} - 15)(PI - 10) \qquad \text{Eq. 2.4}$$

where:

F_{200} = percentage passing No. 200 sieve
LL = liquid limit, PI = plasticity index

Table 02.2 - Classification of Highway Subgrade Materials for Granular Materials (AASHTO)

General classification	Granular materials (35% or less of total sample passing No. 200)						
Group classification	A-1			A-2			
	A-1-a	A-1-b	A-3	A-2-4	A-2-5	A-2-6	A-2-7
Sieve analysis (percentage passing)							
No. 10	50 max.						
No. 40	30 max.	50 max.	51 min.				
No. 200	15 max.	25 max.	10 max.	35 max.	35 max.	35 max.	35 max.
Characteristics of fraction passing No. 40							
Liquid limit				40 max.	41 min.	40 max.	41 min.
Plasticity index	6 max.		NP	10 max.	10 max.	11 min.	11 min.
Usual types of significant constituent materials	Stone fragments, gravel, and sand		Fine sand	Silty or clayey gravel and sand			
General subgrade rating	Excellent to good						

Table 02.3 - Classification of Highway Subgrade Materials for Silt-Clay Materials (AASHTO)

General classification	Silt-clay materials (more than 35% of total sample passing No. 200)			
Group classification	A-4	A-5	A-6	A-7 A-7-5ᵃ A-7-6ᵇ
Sieve analysis (percentage passing)				
No. 10				
No. 40				
No. 200	36 min.	36 min.	36 min.	36 min.
Characteristics of fraction passing No. 40				
Liquid limit	40 max.	41 min.	40 max.	41 min.
Plasticity index	10 max.	10 max.	11 min.	11 min.
Usual types of significant constituent materials	Silty soils		Clayey soils	
General subgrade rating	Fair to poor			

[a] For A-7-5, PI ≤ LL − 30
[b] For A-7-6, PI > LL − 30

@seismicisolation

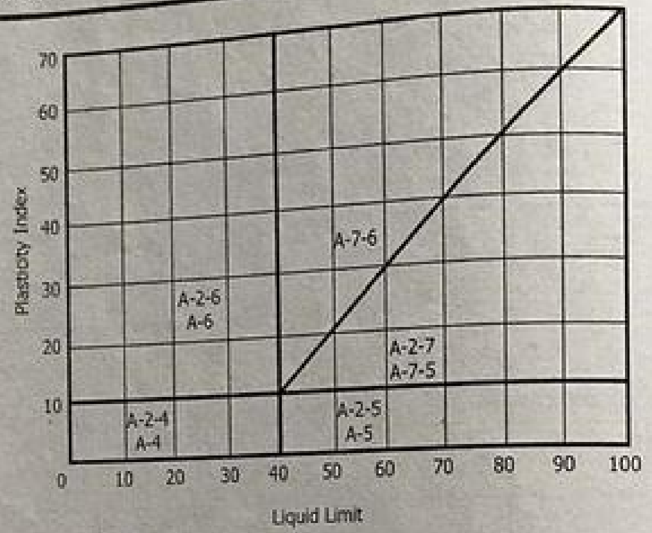


Figure 02.8 – Range of LL and PI for soils in groups A-2, A-4, A-5, A-6, and A-7

The first group of Eq. 2.4 is the partial group index determined from liquid limit. The second term is the partial group index determined from plasticity index.

- If Eq. 2.4 yields a negative value for GI, it is taken as 0.
- *GI* calculated from Eq. 2.4 is rounded-off to the nearest whole number.
- There is no upper limit for *GI*
- The *GI* of soils belonging to groups A-1-a, A-1-b, A-2-4, A-2-5, and A-3 is always zero.
- When calculating the *GI* for soils that belong to groups A-2-6 and A-2-7, use the partial *GI* for *PI*, or

$$GI = 0.01(F_{200} - 15)(PI - 10) \qquad \text{Eq. 2.5}$$

ILLUSTRATIVE PROBLEMS

PROBLEM 02.1

The result of the sieve analysis is shown below:

a) What percentage of the soil is retained in No. 200 sieve?
b) What is the effective grain size of the soil in mm?
c) Determine the uniformity coefficient.

Sieve No	Diameter (mm)	Mass Retained (grams)
4	4.76	25
8	2.38	80
10	2.00	110
20	0.84	160
40	0.42	180
60	0.25	220
80	0.180	380
100	0.149	590
200	0.074	110
Pan		85

SOLUTION

Sieve #	Size, mm	Mass Retained	Cumulative Mass Retained	Percent Finer
4	4.76	25	25	98.71%
8	2.38	80	105	94.59%
10	2	110	215	88.92%
20	0.84	160	375	80.67%
40	0.42	180	555	71.39%
60	0.25	220	775	60.05%
80	0.18	380	1155	40.46%
100	0.149	590	1745	10.05%
200	0.074	110	1855	4.38%
Pan		85	1940	
	Total	1940		

From the table shown:

a) Percent retained in No. 200 sieve = 100% - 4.38%
Percent retained in No. 200 sieve = **95.62%**

b) Effective grain size, $D_{10} = 0.149$ mm

c) Uniformity coefficient, $C_u = D_{60}/D_{10}$
Uniformity coefficient, $C_u = 0.25/0.149 = 1.68$

PROBLEM 02.2

A soil has the following particle-size distribution:

Gravel = 20%
Sand = 10%
Silt = 30%
Clay = 40%

Classify the soil according to USDA textural classification system.

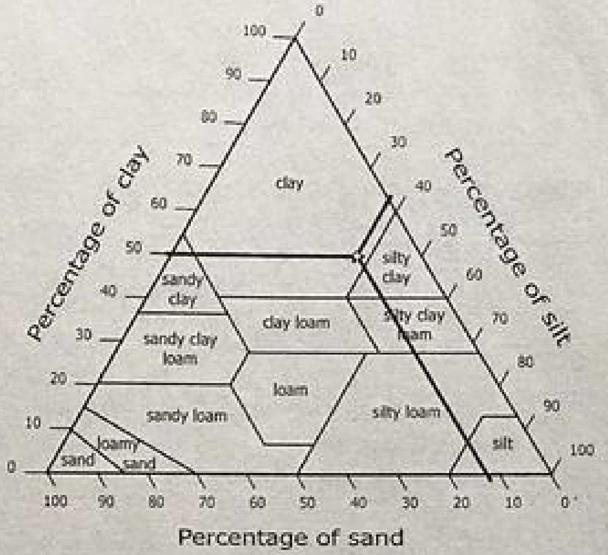
SOLUTION

Modified percentages of sand, silt, and clay:

$$\text{Modified \% sand} = \frac{\%sand}{100 - \%gravel} = \frac{10}{100-20} = 12.5\%$$

$$\text{Modified \% silt} = \frac{\%silt}{100 - \%gravel} = \frac{30}{100-20} = 37.5\%$$

$$\text{Modified \% clay} = \frac{\%clay}{100 - \%gravel} = \frac{40}{100-20} = 50.0\%$$



The lines correspond to each percentages intersect on the clay region, thus, the soil is clay.

PROBLEM 02.3

Classify the following soils by the AASHTO classification system.

Description	Soil A	Soil B	Soil C
Percent finer than No. 10 sieve	83	100	48
Percent finer than No. 40 sieve	48	92	28
Percent finer than No. 200 sieve	20	86	6
Liquid limit	20	70	—
Plasticity index	5	32	Nonplastic

SOLUTION

To classify a soil according to this table, one must apply the test data from left to right. By process of elimination, the first group from the left into which the test data fit is the correct classification.

Soil A: Percent passing No. 200 = 20% < 35%
The soil is either A-1, A-3, or A-2

% passing No. 10 = 83% > 50, it is not A-1-a
% passing No. 40 = 48 < 50
% passing No. 200 = 20 < 25
Thus, the soil is A-1-b, with GI = 0

Thus, the soil is **A-1-b(0)**

Soil B: Percent passing No. 200 = 86%> 35%
The soil is either A-4, A-5, A-6, or A-7
LL = 70 > 40, it is not A-4
PI = 30 > 10, it is not A-5
LL = 70 > 40, it is not A-6
LL – 30 = 40
PI < LL – 30, the soil is A-7-5

$GI = (86 - 35)[0.2 + 0.005(70 - 40)] + 0.01(86 - 15)(32 - 10)$
GI = 33.47 use 33
Therefore the soil is **A-7-5(33)**

Soil C: Percent passing No. 200 = 6%< 35%, it is A-1, A-3, or A-2
% passing No. 200 = 6% < 50
% passing No. 40 = 28% < 30
% passing No. 10 = 48 < 50
Thus, the soil is A-1-a and GI = 0
Thus, the soil is **A-1-a(0)**

PROBLEM 02.4

The table below shows the laboratory results of the sieve analysis of a sample. Plot the grain size curve of the soil in the attached Figure 02.10. The soil has a liquid limit of 35% and plasticity index of 26%. Classify the soil according to *a*) USCS, *b*) USDA, and *c*) AASHTO.

Size (mm)	Weight Retained
0.25	18.96
0.149	33.18
0.074	45.03
0.052	54.51
0.02	42.66
0.01	11.85
0.004	4.74
0.001	4.74
Pan	21.33

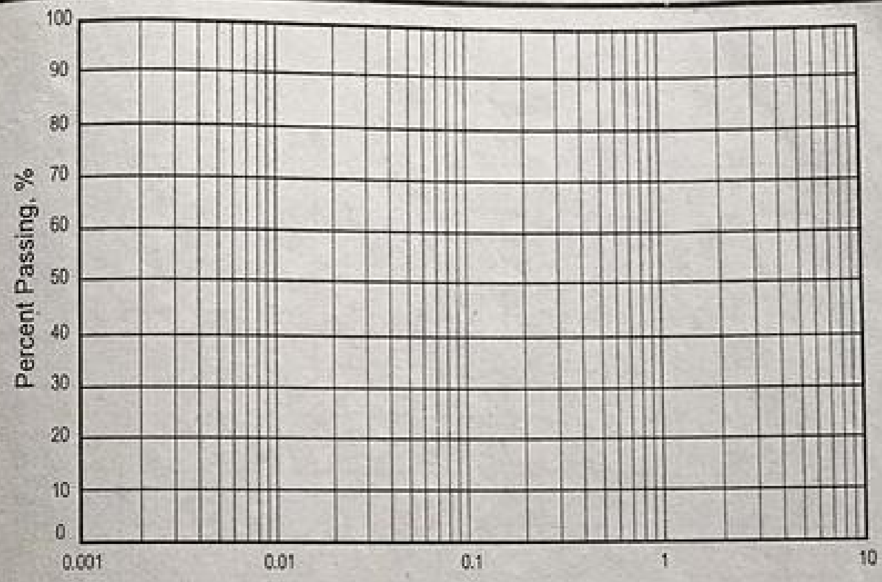
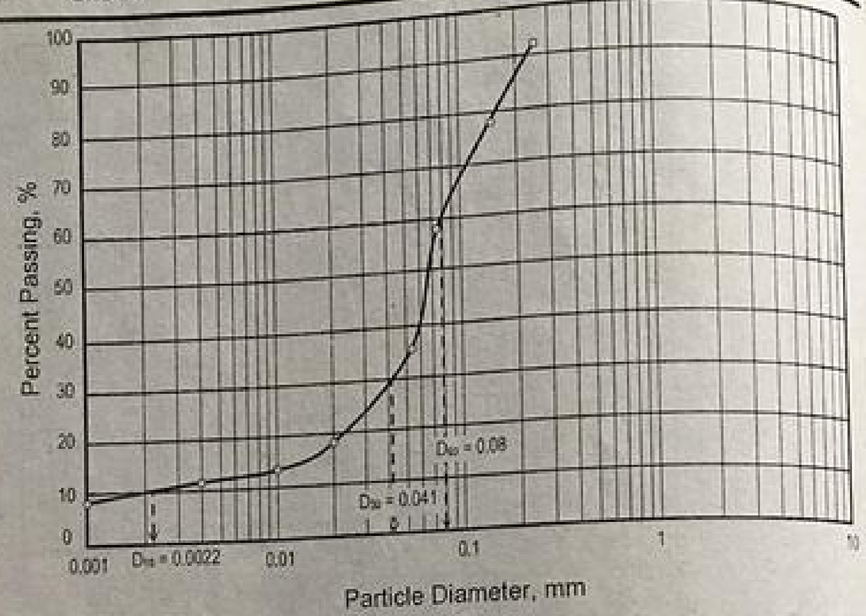


Figure 02.9 – Grain size curve

SOLUTION

Size (mm)	Weight Retained	Accumulated Weight	Percent Finer
0.25	18.96	18.96	92%
0.149	33.18	52.14	78%
0.074	45.03	97.17	59%
0.052	54.51	151.68	36%
0.02	42.66	194.34	18%
0.01	11.85	206.19	13%
0.004	4.74	210.93	11%
0.001	4.74	215.67	9%
Pan	21.33	237	
Total	237.00		



a) **USCS:**

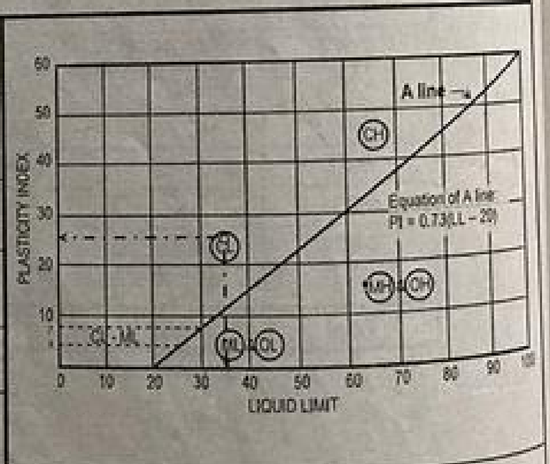
Percent passing No. 200 sieve (0.074 mm) = 59% > 50%

Therefore, the soil is FINE GRAINED.

LL = 35% < 50% (ML, CL, or OL)

From the plasticity chart, with LL = 35% and PI = 26%, the soil is CL

Fine Grained Soils 50% or more passes No. 200 sieve	Silts and Clays Liquid Limit 50% or less	ML	Inorganic silts, very fine sands, rock flour, silty or clayey fine sands
		CL	Inorganic clays of low to medium plasticity, gravelly clays, sandy clays, silty clays, lean clays
		OL	Organic silts and organic silty clays of low plasticity
	Silts and Clays Liquid Limit greater than 50%	MH	Inorganic silts, micaceous or diatomaceous fine sands or silts, elastic silts
		CH	Inorganic clays of high plasticity, fat clays
		OH	Organic clays of medium to high plasticity
Highly Organic Soils		PT	Peat, muck, and other highly organic soils



Visual-Manual identification. See ASTM Designation D2488

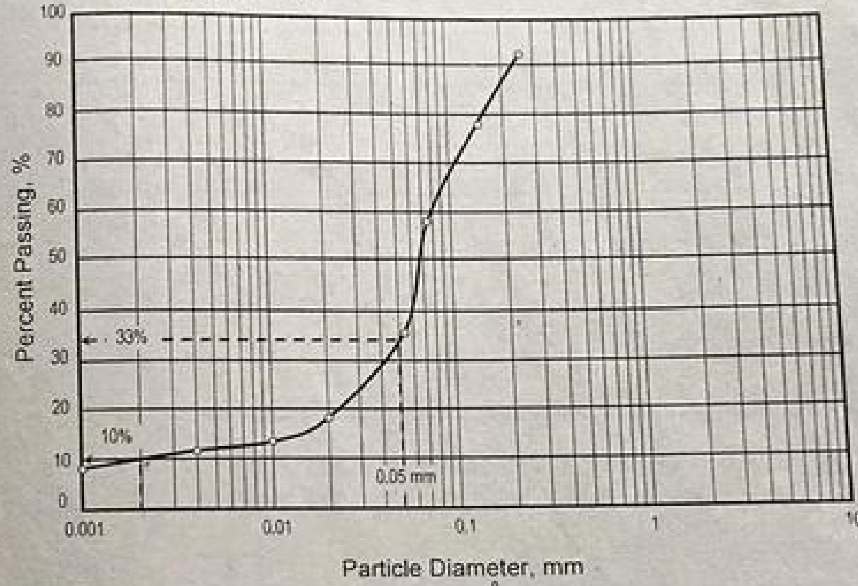
@seismicisolation

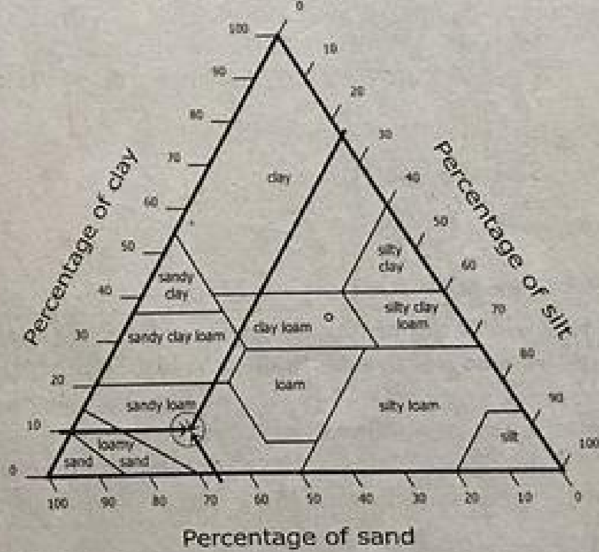
b) **USDA**

Percent sand (2.0 mm to 0.05 mm in diameter) = 100 – 33 = 67%

Percent silt (0.05 mm to 0.002 mm) = 33 – 10 = 23%

Percent clay = 10%





From the chart shown, the soil is **sandy loam**

c) ASSHTO

Percent passing No. 200 sieve (0.074 mm) = 59% > 35%
"Silt-clay materials". Use Table 02.3.

General classification	Silt-clay materials (more than 35% of total sample passing No. 200)			
Group classification	A-4	A-5	A-6	A-7 A-7-5[a] A-7-6[b]
Sieve analysis (percentage passing)				
No. 10				
No. 40				
No. 200	36 min. ✓	36 min. ✓	36 min. ✓	36 min.
Characteristics of fraction passing No. 40 Liquid limit Plasticity index	40 max. ✓ 10 max. ×	41 min. × 10 max. ×	40 max. ✓ 11 min. ✓	41 min. 11 min.
Usual types of significant constituent materials	Silty soils		Clayey soils	
General subgrade rating	Fair to poor			
[a] For A-7-5, PI ≤ LL – 30 [b] For A-7-6, PI > LL - 30				

The soil cannot be A-4 because its $PI = 26\% > 10\%$.
The soil cannot be A-5 because its $LL = 35\% > 41\%$.
The soil is A-6

Solving for GI:

$$GI = (F_{200} - 35)[0.2 + 0.005(LL - 40)] + 0.01(F_{200} - 15)(PI - 10)$$

$$GI = (59 - 35)[0.2 + 0.005(35 - 40)] + 0.01(59 - 15)(26 - 10)$$

$$GI = 11.24$$

Thus, the soil is **A-6(11)**

PROBLEM 02.5 (CE MAY 2003)

The table below shows the laboratory results of the sieve analysis of a sample. Plot the grain size curve of the soil in the attached Figure 02.10. Determine the following:

a) Determine the nearest value to the effective size.
b) Determine the nearest value to the coefficient, of uniformity, C_u.
c) Classify the soil according to the Unified Classification System, using Table 02.1

Sieve No	Diameter (mm)	Percent Passing, %
4	4.76	90
8	2.38	64
10	2.00	58
20	0.84	35
40	0.42	22
60	0.25	15
100	0.149	10
200	0.074	4

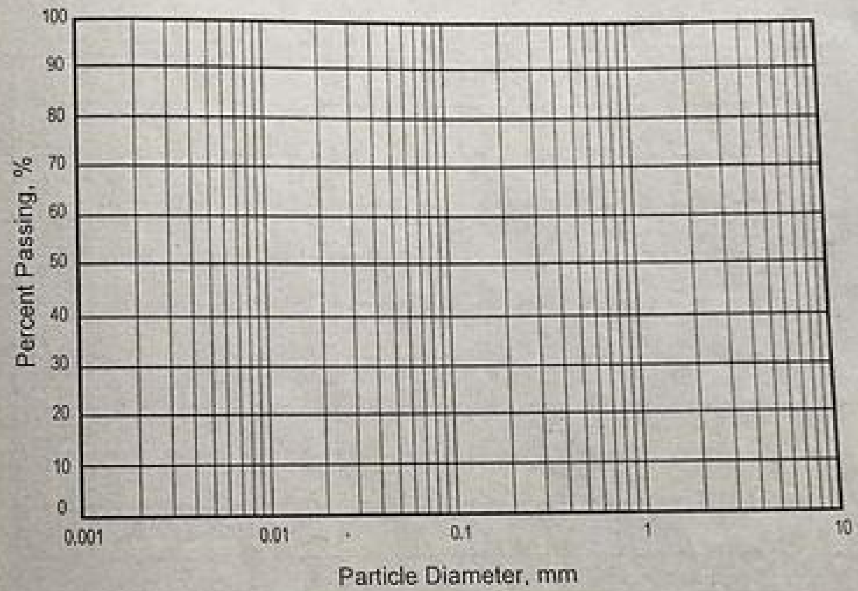


Figure 02.10 – Grain size curve

SOLUTION

Sieve No	Diameter (mm)	Percent Passing, % (or % Finer)
4	4.76	90
8	2.38	64
10	2.00	58
20	0.84	35
40	0.42	22
60	0.25	15
100	0.149	10
200	0.074	4

@seismicisolation

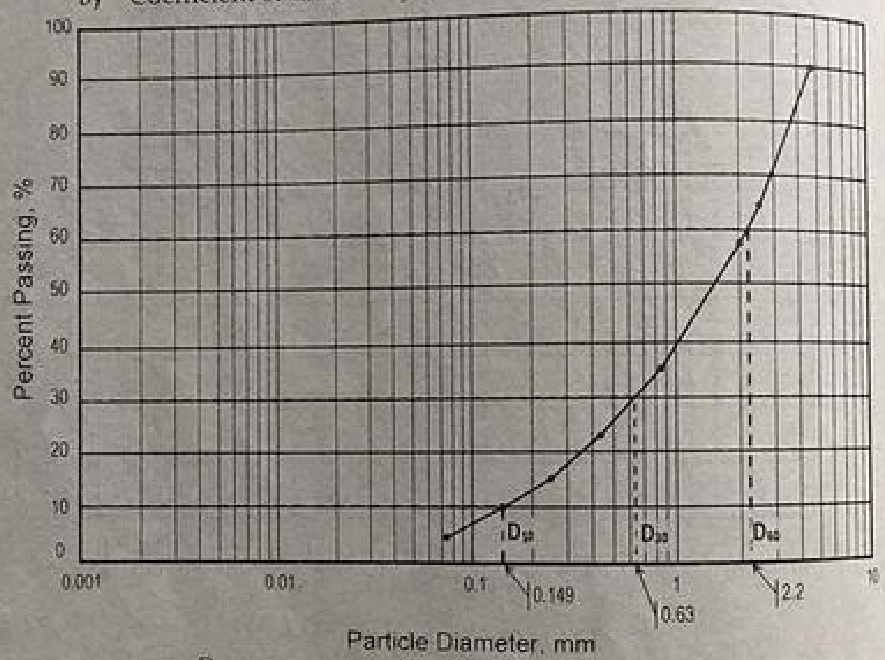
a) Effective Size:

The *Effective Size*, D_{10}, is the diameter of the particles of which 10% of the soil is finer. D_{10} is an important value in regulating flow through soils and can significantly influence the mechanical behavior of soils.

For this problem, D_{10} = **0.149 mm**

The *Average Grain Size* diameter of the soil is D_{50}.

b) Coefficient of uniformity:



$$C_u = \frac{D_{60}}{D_{10}}$$

$D_{10} = 0.149$

From the grain size curve shown, D_{60} = 2.2 mm

$$C_u = \frac{2.2}{0.149} = 14.8$$

Part 3 Classification of soil:

Percent gravel (retained in # 4 sieve) = 100% - 90%

Percent gravel (retained in # 4 sieve) = 10%

$$C_c = \frac{(D_{30})^2}{D_{10}\,D_{60}}$$

From the graph, D_{30} = 0.63 mm

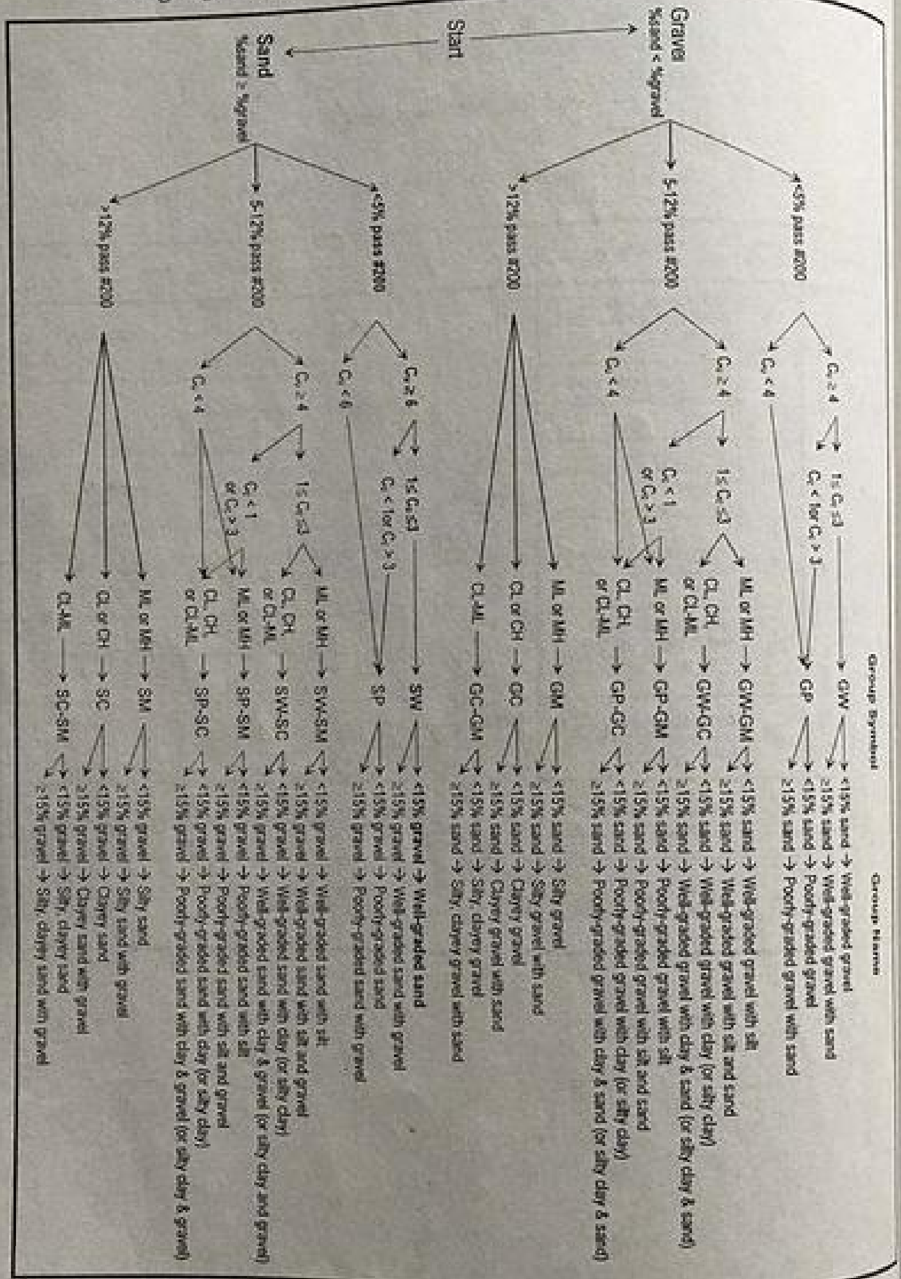
$$C_c = \frac{(0.63)^2}{0.149(2.2)} = 1.21 \text{ (Between 1 \& 3)}$$

Since $C_u > 6$, C_c is between 1 & 3, the soil is **SW (Well graded sand)**

<table>
<tr><th colspan="3">Major Divisions</th><th>Group Symbols</th><th>Typical Names</th><th></th><th colspan="2">Classification Criteria</th></tr>
<tr><td rowspan="8">Coarse-Grained Soils More than 50% retained on No. 200 sieve</td><td rowspan="4">Gravels 50% or more of coarse fraction retained on No. 4 sieve</td><td rowspan="2">Clean Gravels</td><td>GW</td><td>Well-graded gravels and gravel-sand mixtures, little or no fines</td><td rowspan="8">Classification on basis of percentage of fines
Less than 5% pass No. 200 sieve GW, GP, SW, SP
More than 12% pass No. 200 sieve GM, GC, SM, SC
12% pass No. 200 sieve Borderline Classification requiring use of dual symbols</td><td colspan="2">Cu = (D60/D10) > 4   Cc = (D30)²/(D10 x D60) Between 1 & 3</td></tr>
<tr><td>GP</td><td>Poorly graded gravels and gravel-sand mixtures, little or no fines</td><td colspan="2">Not meeting both criteria for GW</td></tr>
<tr><td rowspan="2">Gravels with fines</td><td>GM</td><td>Silty gravels, gravel-sand-silt mixtures</td><td>Atterberg limits plot below "A" line or Plasticity index less than 4</td><td rowspan="2">Atterberg limits plotting in hatched area are borderline classifications requiring use of dual symbols</td></tr>
<tr><td>GC</td><td>Clayey gravels, gravel-sand-clay mixtures</td><td>Atterberg limits plot above "A" line or Plasticity index greater than 7</td></tr>
<tr><td rowspan="4">Sands more than 50% of coarse fraction passes No. 4 sieve</td><td rowspan="2">Clean Sands</td><td>SW</td><td>Well graded sands and gravely sands, little or no fines</td><td colspan="2">Cu = (D60/D10) > 6   Cc = (D30)²/(D10 x D60) Between 1 & 3</td></tr>
<tr><td>SP</td><td>Poorly graded sands and gravely sands, little or no fines</td><td colspan="2">Not meeting both criteria for SW</td></tr>
<tr><td rowspan="2">Sands with fines</td><td>SM</td><td>Silty sands, sand-silt mixtures</td><td>Atterberg limits plot below "A" line or Plasticity index less than 4</td><td rowspan="2">Atterberg limits plotting in hatched area are borderline classifications requiring use of dual symbols</td></tr>
<tr><td>SC</td><td>Clayey sands, sand-clay mixtures</td><td>Atterberg limits plot above "A" line or Plasticity index greater than 7</td></tr>
</table>

@seismicisolation

Using Figure 02.3



PROBLEM 02.6 (CE NOVEMBER 2003)

The grain-size curves for soils A and B is shown in Figure 02.11. It is required to classify the soils according to the Unified Soil Classification System.

a) Determine the value of the coefficient of uniformity of soil A.
b) What is the classification of soil A?
c) What is the classification of soil B?

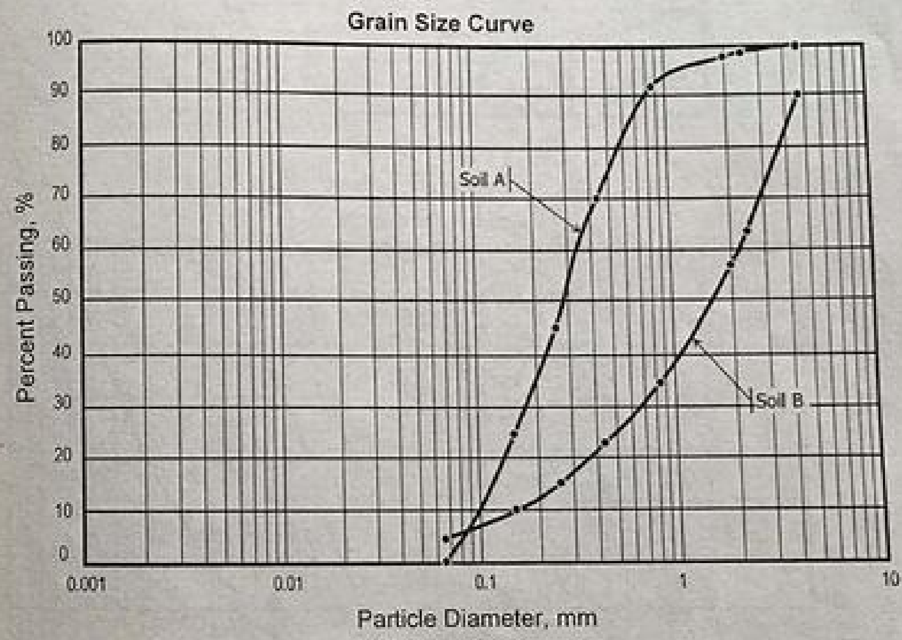
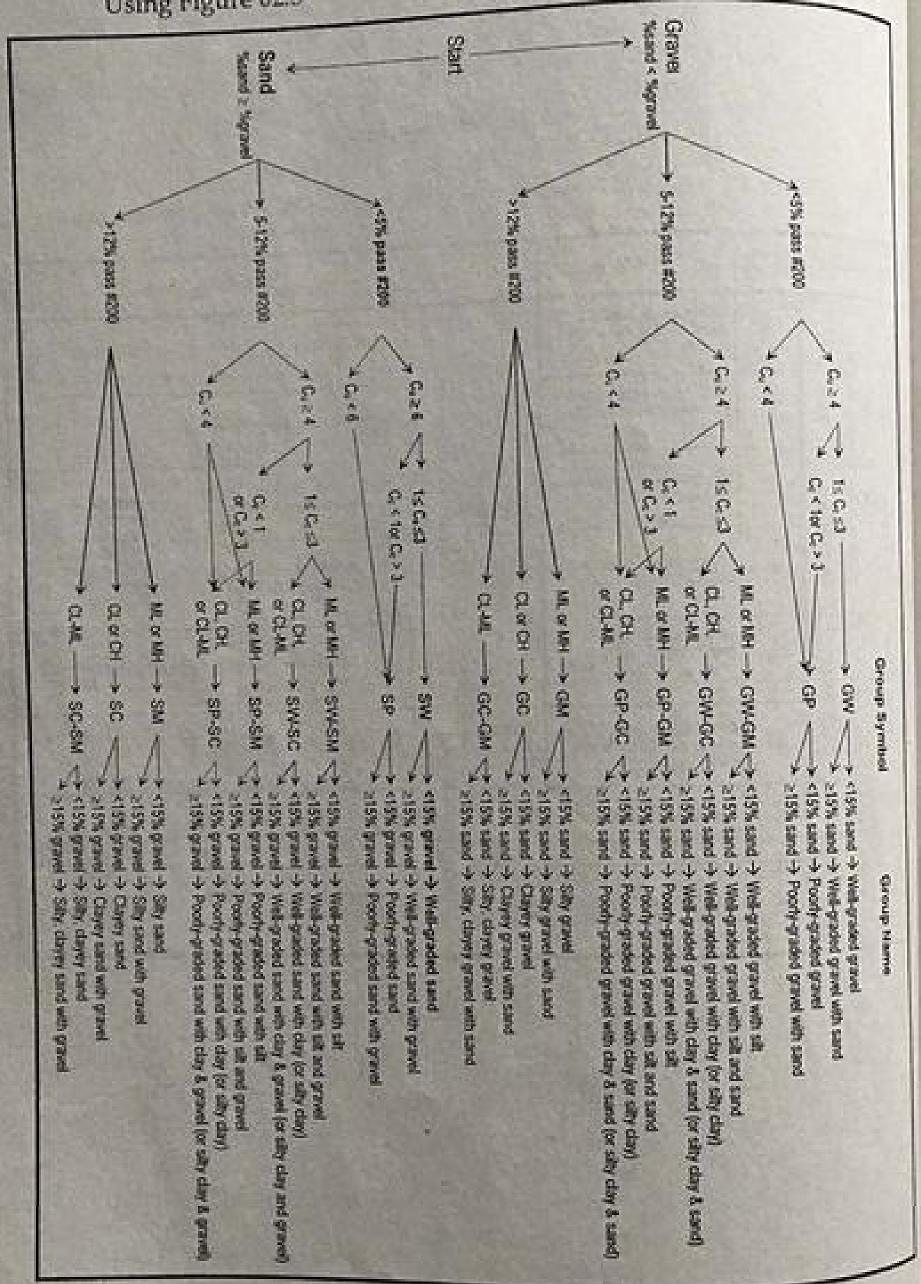


Figure 02.11 – Grain size curve

Using Figure 02.3



PROBLEM 02.6 (CE NOVEMBER 2003)

The grain-size curves for soils *A* and *B* is shown in Figure 02.11. It is required to classify the soils according to the Unified Soil Classification System.

a) Determine the value of the coefficient of uniformity of soil *A*.
b) What is the classification of soil *A*?
c) What is the classification of soil *B*?

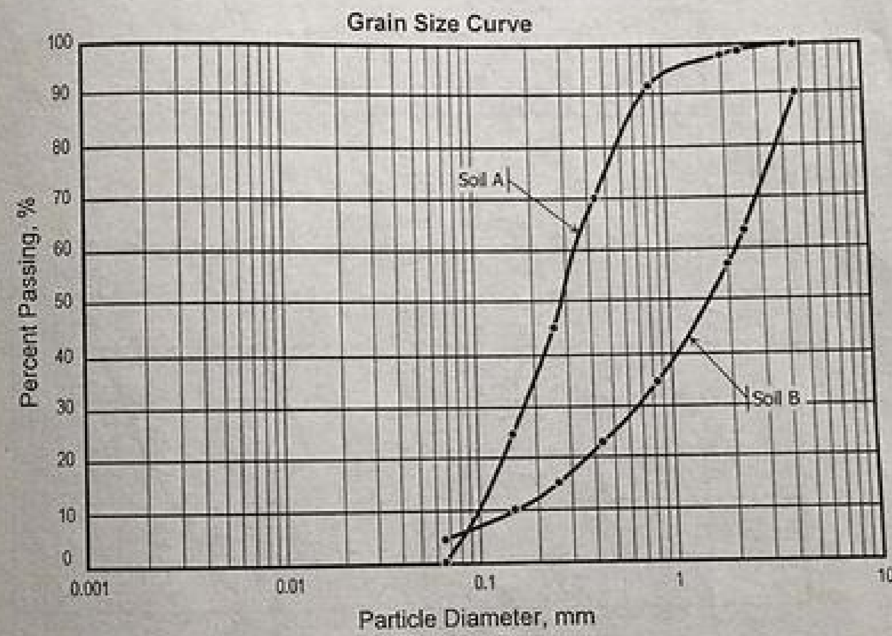
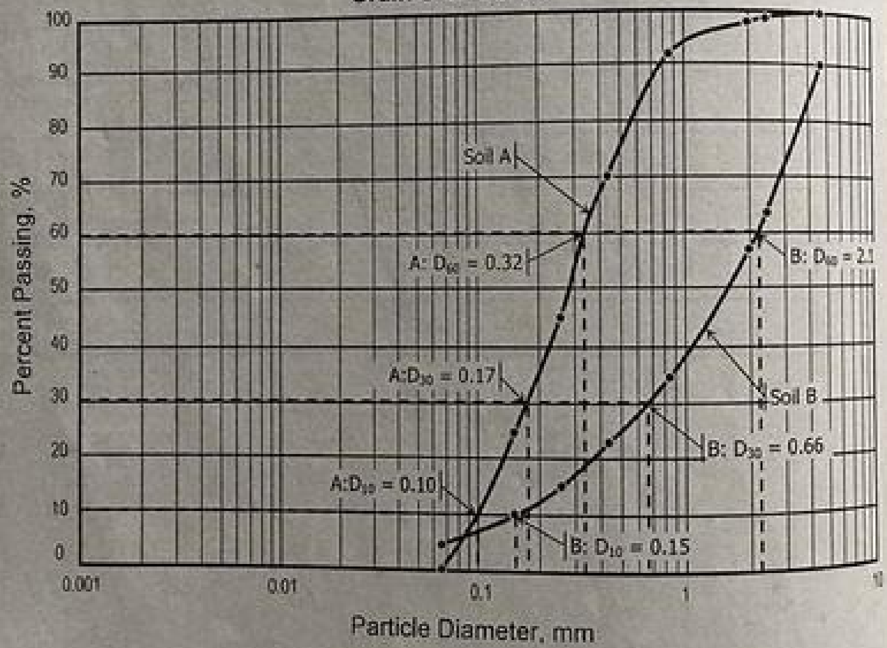


Figure 02.11 – Grain size curve

SIEVE ANALYSIS			
Sieve No.	Diameter mm	Percent Passing	
		Soil *A*	Soil *B*
4	4.760	100	90
8	2.380	99	64
10	2.000	98	58
20	0.850	92	35
40	0.425	70	22
60	0.250	46	15
100	0.150	25	10
200	0.074	0	4

SOLUTION

Grain Size Curve



Soil *A*:

$D_{10} = 0.10$; $D_{30} = 0.17$

$D_{60} = 0.32$

Uniformity coefficient, $C_u = \dfrac{D_{60}}{D_{10}}$

Uniformity coefficient, $C_u = \dfrac{0.32}{0.10} = 3.2$

Coefficient of curvature, $C_c = \dfrac{D_{30}^2}{D_{10}\,D_{60}}$

Coefficient of curvature, $C_c = \dfrac{0.17^2}{0.10(0.32)} = 0.903$

Classification:

Retained on No. 200 sieve = 100% (more than 50%)

Coarse-Grained Soil

Passing No. 4 sieve = 100% (more than 50%)

Sands

$C_u = 3.2 < 6$; $C_c = 0.903$ (not between 1 and 3)

Since the soil does not meet both criteria for SW, the soil is **SP**

Soil *B*:

$D_{10} = 0.15$; $D_{30} = 0.66$; $D_{60} = 2.1$

Uniformity coefficient, $C_u = \dfrac{D_{60}}{D_{10}} = \dfrac{2.1}{0.15}$

Uniformity coefficient, $C_u = 14$

Coefficient of curvature, $C_c = \dfrac{D_{30}^2}{D_{10}\,D_{60}} = \dfrac{0.66^2}{0.15(2.1)}$

Coefficient of curvature, $C_c = 1.38$

Classification:

Retained on No. 200 sieve = 96% (more than 50%)

Coarse-Grained Soil

Passing No. 4 sieve = 90% (more than 50%)

Sands

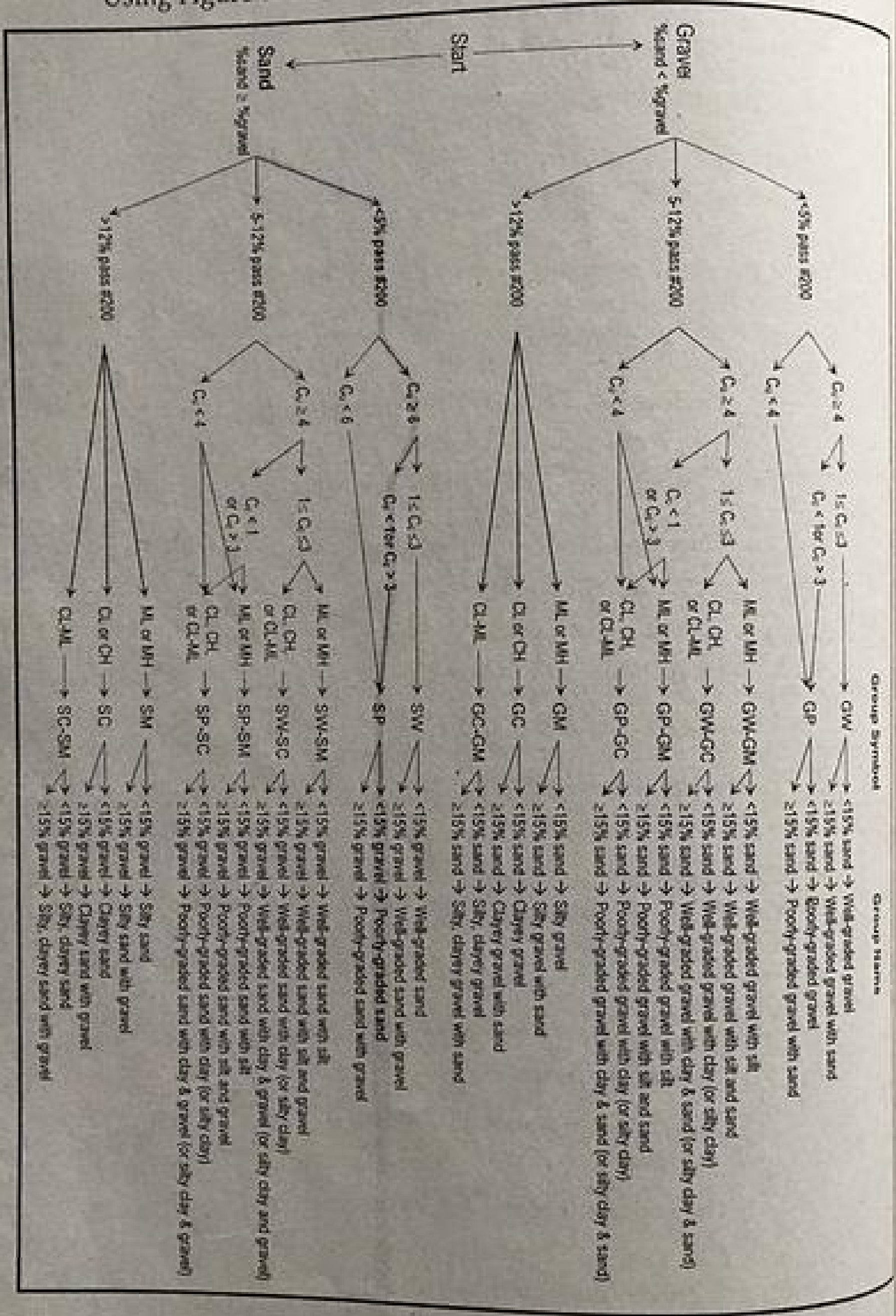
The soil is either SW, SP, SM, or SC

$C_u = 14 > 6$

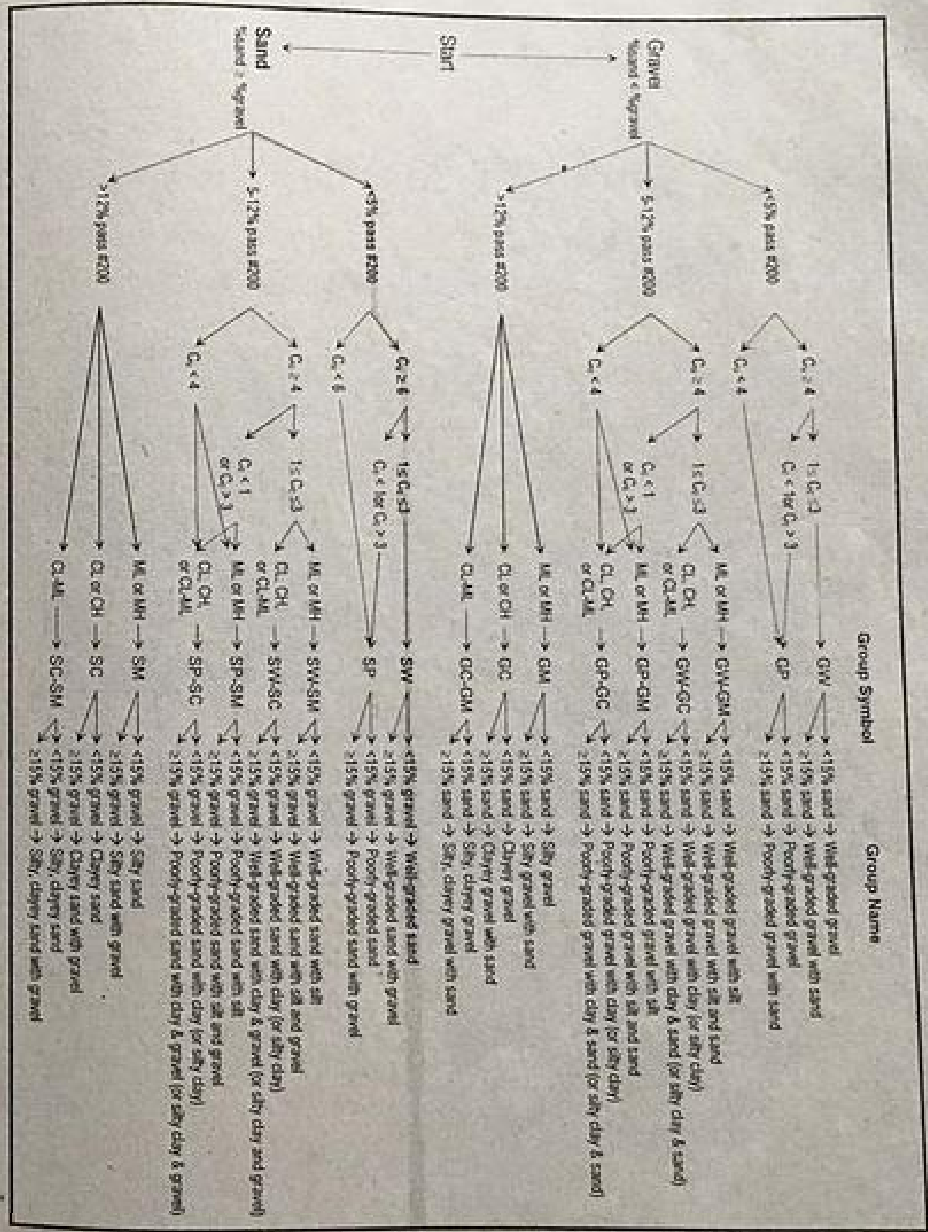
$C_c = 1.38$ (between 1 and 3)

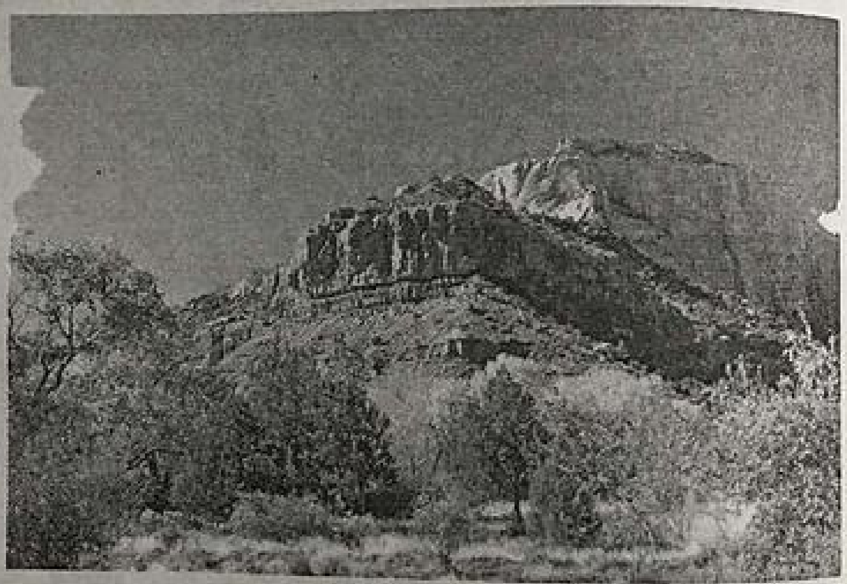
Therefore, the soil is **SW**

Using Figure 02.3 for Soil A:



Using Figure 02.3 for Soil B:





Chapter 03

Flow of Water through Soils

3.1 DARCY'S LAW

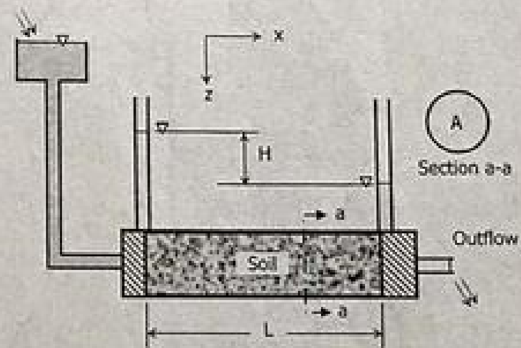


Figure 03. 1 – Flow of water through soil

Darcy's law governs the flow of water through soils. Darcy (1856) proposed that the average flow velocity through soils is proportional to the gradient of the total head. The velocity of flow is:

$$v = ki \qquad \text{Eq. 3.1}$$

$$\text{Seepage velocity, } v_s = v/n \qquad \text{Eq. 3.2}$$

where

$i = \dfrac{H}{L}$ = hydraulic gradient

k = coefficient of permeability or hydraulic conductivity, m/s or m/day

n = porosity

The flow of water is:

$$Q = k\,i\,A \qquad \text{Eq. 3.3}$$

3.2 DETERMINATION OF THE COEFFICIENT OF PERMEABILITY

3.2.1 CONSTANT-HEAD TEST

The constant-head test is used to determine the coefficient of permeability of coarse-grained soils.

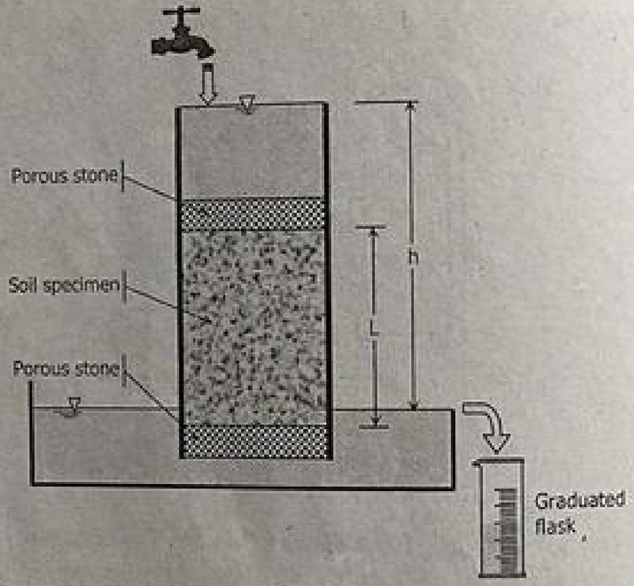


Figure 03.2 – Constant head permeability test

$$k = \frac{VL}{tAh} \qquad \text{Eq. 3.4}$$

where:

- V = volume of water collected in time t
- h = constant head
- A = cross-sectional area of the soil
- L = length of soil sample
- t = duration of water collection

3.2.2 FALLING-HEAD TEST

The falling head test is used for fine-grained soils because the flow of water through these soils is too slow to get reasonable measurement from the constant-head test

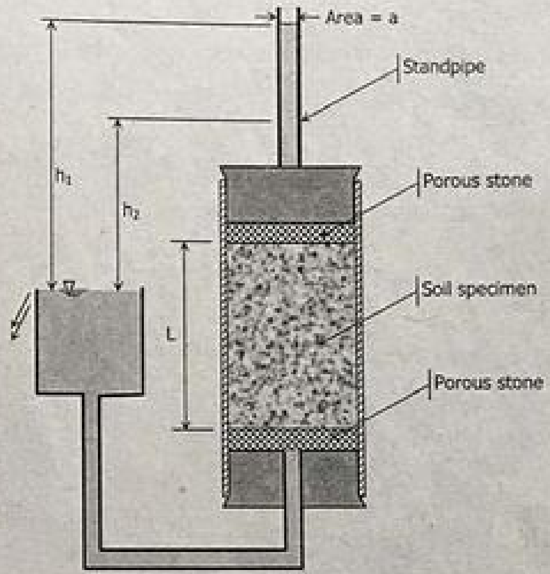


Figure 03.3 – Falling-head permeability test

$$k = \frac{aL}{A(t_2 - t_1)} \ln\left(\frac{h_1}{h_2}\right) \qquad \text{Eq. 3.5}$$

where:

- a = cross-sectional area of the standpipe
- h_1 = head at time t_1
- h_2 = head at time t_2

3.2.2 EFFECT OF WATER TEMPERATURE ON K

The hydraulic conductivity of soil is a function of unit weight of water, and thus, it is affected by water temperature. The relationship is given by:

$$\frac{k_{T_1}}{k_{T_2}} = \frac{\mu_{T_2}}{\mu_{T_1}} \cdot \frac{\gamma_{wT_1}}{\gamma_{wT_2}} \qquad \text{Eq. 3.6}$$

where:

k_{T_1}, k_{T_2} = hydraulic conductivities at temperatures T_1 and T_2, respectively

μ_{T_1}, μ_{T_2} = viscosity of water at temperatures T_1 and T_2, respectively

$\gamma_{wT_1}, \gamma_{wT_2}$ = unit weight of water at temperatures T_1 and T_2, respectively

3.3 FLOW THROUGH PERMEABLE LAYERS

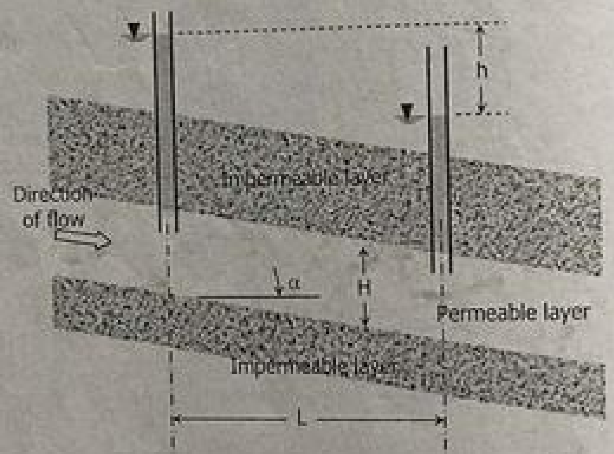


Figure 03.4 – Flow through permeable layer

With reference to Figure 03.4, the hydraulic gradient is:

$$\text{Hydraulic gradient, } i = \frac{h}{\dfrac{L}{\cos\alpha}} \qquad \text{Eq. 3.7}$$

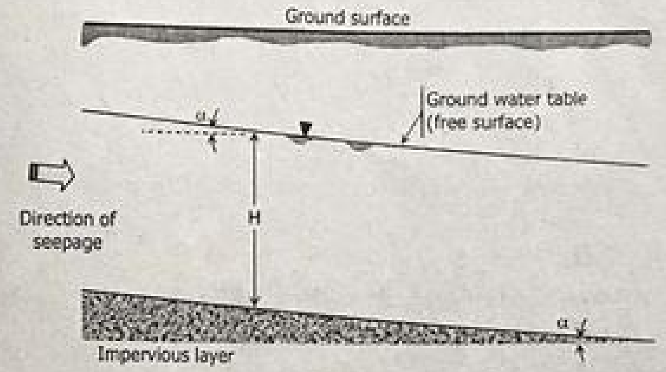


Figure 03.5 – Flow through permeable layer

With reference to Figure 03.5, the hydraulic gradient is:

$$\text{Hydraulic gradient, } i = \sin\alpha \qquad \text{Eq. 3.8}$$

3.4 EMPIRICAL RELATIONS FOR HYDRAULIC CONDUCTIVITY

3.4.1 HAZEN FORMULA (for fairly uniform sand):

$$k\ (\text{cm/sec}) = c\,(D_{10})^2 \qquad \text{Eq. 3.9}$$

where c = a constant that varies from 1 to1.5

D_{10} = effective size, mm

3.4.2 CASAGRANDE (FOR FINE TO MEDIUM CLEAN SAND):

$$k = 1.4\,e^2\,k_{0.85} \qquad \text{Eq. 3.10}$$

where k = hydraulic conductivity at void ratio e

$k_{0.85}$ = k at void ratio of 0.85

3.4.3 KOZENY-CARMAN EQUATION

$$k = C_1 \frac{e^3}{1+e} \qquad \text{Eq. 3.11}$$

where k is the hydraulic conductivity at a void ratio of e and C_1 is a constant.

3.4.4 SAMARASINHE, HUANG, AND DRNEVICH

$$k = C_3 \frac{e^n}{1+e} \qquad \text{Eq. 3.12}$$

where C_3 and n are constants to be determined experimentally.

3.5 EQUIVALENT HYDRAULIC CONDUCTIVITY IN STRATIFIED SOIL

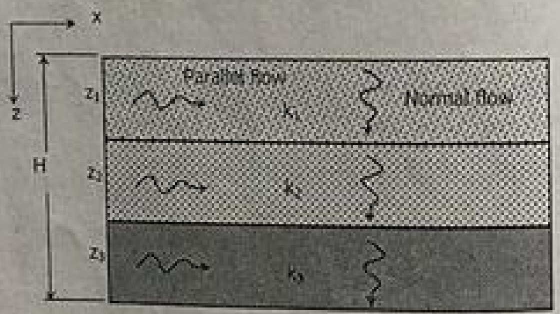


Figure 03.6 – Equivalent hydraulic conductivity in stratified soil

The equivalent permeability in the x-direction is (parallel flow):

$$(k_x)_{eq}\, H = \Sigma\, k_x\, z \qquad \text{Eq. 3.13}$$

$$(k_x)_{eq}\, H = k_{x1}\, z_1 + k_{x2}\, z_2 + \ldots + k_{xn}\, z_n \qquad \text{Eq. 3.14}$$

The equivalent permeability in the z-direction is (normal flow)

$$\frac{H}{(k_z)_{eq}} = \Sigma \frac{z}{k_z} \qquad \text{Eq. 3.15}$$

$$\frac{H}{(k_z)_{eq}} = \frac{z_1}{k_{z1}} + \frac{z_2}{k_{z2}} + \ldots + \frac{z_n}{k_{zn}} \qquad \text{Eq. 3.16}$$

3.6 FLOW THROUGH LAYERS OF AQUIFERS

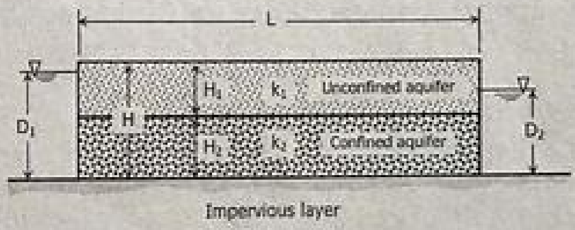


Figure 03.7 – Aquifers in horizontal layers

$$k_{eq}\,(H) = k_1\, H_1 + k_2\, H_2$$

Flow per unit width, $q = k_{eq}\, i\, a$

$$i = \frac{D_1 - D_2}{L}$$

$$a = \frac{D_1 + D_2}{2}(1)$$

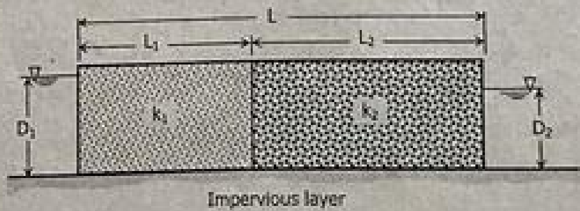


Figure 03.8 – Non-homogeneous unconfined aquifers

$$\frac{L}{k_{eq}} = \frac{L_1}{k_1} + \frac{L_2}{k_2}$$

Flow per unit width, $q = k_{eq} i a$

$$i = \frac{D_1 - D_2}{L}$$

$$a = \frac{D_1 + D_2}{2} (1)$$

3.7 CONTINUITY EQUATION FOR SIMPLE FLOW PROBLEMS

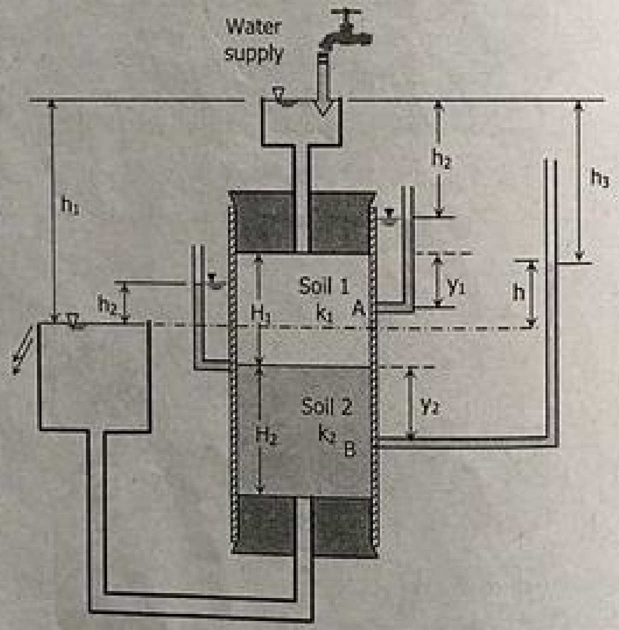


Figure 03.9

Consider the one-dimensional flow problem shown. The following calculations may be applied:

For the entire system:

Total head = h_1

Total length of soil = $H_1 + H_2$

$i = h_1/(H_1 + H_2)$

Equivalent k:

$$[\frac{H}{k_{eq}} = \Sigma \frac{h}{k}] \quad \frac{H_1 + H_2}{k_{eq}} = \frac{H_1}{k_1} + \frac{H_2}{k_2}$$

Total flow, $q = k_{eq}\, i\, A$

At point A:

Head, $H = h_2$

Length, $L = y_1$

$i_A = h_2/y_1$

Permeability = k_1

Flow, $q_A = k_1\, i_A\, A$

Continuity Equation; $q_A = q$

At point B:

Head, $h = h_3$

Length, $L = H_1 + y_2$

$i_B = h_3/(H_1 + y_2)$

Permeability, $k_{eq\,B}$:

$$\frac{H_1 + y_2}{k_{eq\,B}} = \frac{H_1}{k_1} + \frac{y_2}{k_2}$$

Flow, $q_B = k_{eq\,B}\, i_B\, A$

Continuity Equation; $q_B = q_A = q$

3.8 HYDRAULIC OF WELLS

Underground water constitutes an important source of water supply. The stratum of soil in which this water is present is known as an aquifer. On the basis of their hydraulic characteristics, wells are divided into two categories: *gravity* or *water-table wells*, and *artesian* or *pressure wells*. If the pressure at the surface of the surrounding underground water is atmospheric, the well is of the *gravity type*; if this pressure is above atmospheric because an impervious soil stratum overlies the aquifer, the well is *artesian*.

Assume that the water surrounding a well has a horizontal surface under static conditions. The lateral flow of water toward the well requires the

@seismicisolation

existence of a hydraulic gradient, this gradient being caused by a difference in pressure. To create this difference in pressure, the surface of the surrounding water assumes the shape of an inverted "cone" during pumping of the well, as shown in profile in the figure. This cone is known as the *cone of depression*, the cross section of the cone at the water surface is called the *circle of influence*, and the distance through which the water surface is lowered at the well is termed the *drawdown*. The discharge corresponding to a drawdown of 1 m is called *specific capacity* of the well.

3.8.1 GRAVITY WELL

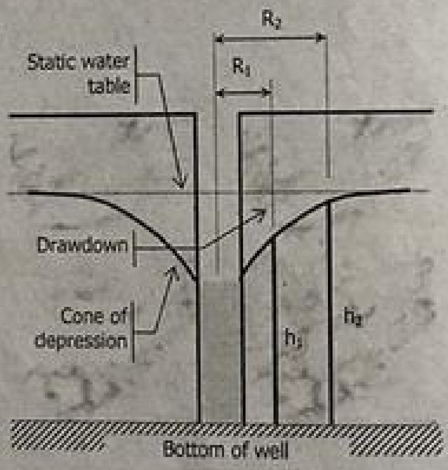


Figure 03.10 – Gravity well

$$Q = \frac{\pi k ({h_2}^2 - {h_1}^2)}{\ln(R_2 / R_1)} \qquad \text{Eq. 3.17}$$

3.8.2 ARTESSIAN WELL

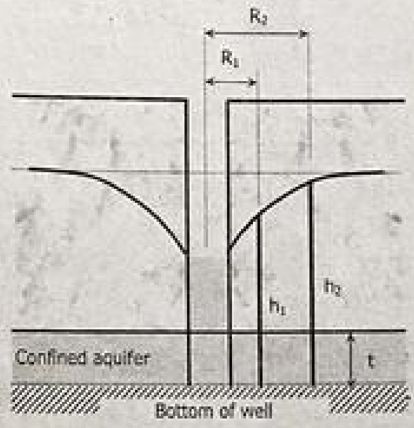


Figure 03.11 – Artesian well

$$Q = \frac{2\pi k t (h_2 - h_1)}{\ln(R_2 / R_1)} \qquad \text{Eq. 3.18}$$

where:

h_1, h_2, R_1, R_2 are in meters
k = coefficient of permeability in m/hr
Q = discharge in m^3/hr.

3.9 TWO-DIMENSIONAL FLOW OF WATER THROUGH SOILS

3.9.1 FLOW NETS

Seepage losses through the ground or through earth dams and levees and the related flow pattern and rate of energy loss, or dissipation of hydrostatic head, are frequently estimated by means of a graphical technique known as flow net.

Flow net is a graphical representation of a flow field that satisfies Laplace's equation and comprises a family of flow lines and equipotential lines.

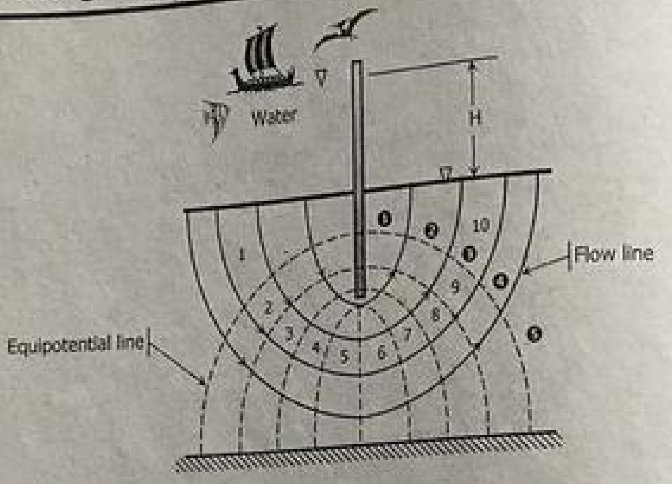


Figure 03.12 – Flow nets

A flow net must meet the following criteria:

1. The boundary conditions must be satisfied.
2. Flow lines must intersect equipotential lines at right angles.
3. The area between flow lines and equipotential lines must be curvilinear squares. A curvilinear square has the property that an inscribed circle can drawn to touch each side of the square and continuous bisection results, in the limit, in a point.
4. The quantity of flow through each flow channel id constant.
5. The head loss between each consecutive equipotential line is constant.
6. A flow line cannot intersect another flow line.
7. An equipotential line cannot intersect another equipotential line.

Flow line is the path followed by a particle of water as it moves through a saturated soil mass.

Equipotential line is a line connecting points of equal potential energy.

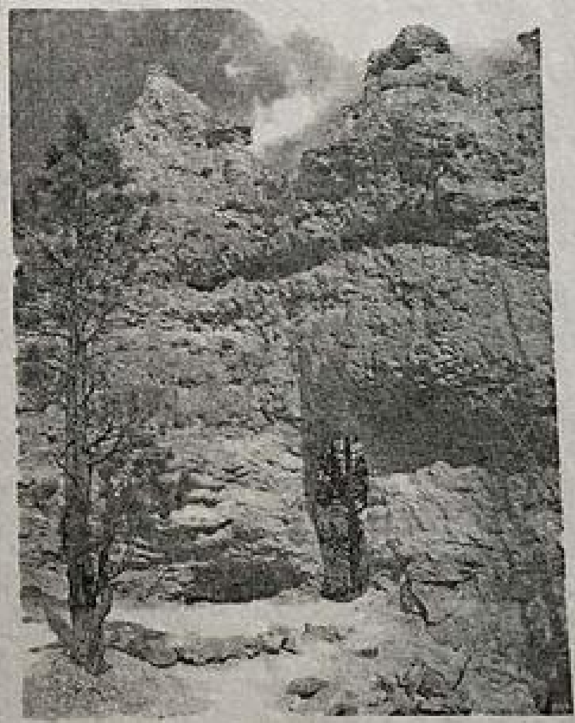
The flow of water through isotropic soil is:

$$q = kH\frac{N_f}{N_d} \qquad \text{Eq. 3.19}$$

where:
- k = coefficient of permeability
- H = head
- N_f = number of flow channels = Number of flow lines minus one
- N_d = number of equipotential (pressure) drops
- N_d = Number of equipotential lines minus one
- $\frac{N_f}{N_d}$ is called the shape factor

If the soil is anisotropic:

$$q = H\frac{N_f}{N_d}\sqrt{k_x k_z} \qquad \text{Eq. 3.20}$$



ILLUSTRATIVE PROBLEMS

PROBLEM 03.1

For a normally consolidated clay soil, the following values are given:

Void ratio	k (cm/sec)
1.1	0.302×10^{-7}
0.9	0.12×10^{-7}

The hydraulic conductivity for normally consolidated clay is given by the following equation:

$$k = C_3 \frac{e^n}{1+e}$$

where e is the void ratio, C_3 and n are constants to be determined experimentally.

a) Determine the value of n.
b) Determine the value of C_3.
c) Estimate the hydraulic conductivity of the clay at a void ratio of 0.75.

SOLUTION

a) Value of n:

$$k = C_3 \frac{e^n}{1+e}$$

$$\frac{k_1}{k_2} = \frac{C_3 \dfrac{e_1^{\,n}}{1+e_1}}{C_3 \dfrac{e_2^{\,n}}{1+e_2}}$$

$$\frac{k_1}{k_2} = \frac{\dfrac{e_1^{\,n}}{1+e_1}}{\dfrac{e_2^{\,n}}{1+e_2}}$$

$$\frac{k_1}{k_2} = \left(\frac{e_1}{e_2}\right)^n \frac{1+e_2}{1+e_1}$$

$$\frac{0.302 \times 10^{-7}}{0.12 \times 10^{-7}} = \left(\frac{1.1}{0.9}\right)^n \frac{1+0.9}{1+1.1}$$

$$(1.2222)^n = 2.781579$$

$$n \log (1.2222) = \log 2.781579$$

$$n = 5.1$$

b) Value of C_3:

$$[k = C_3 \frac{e^n}{1+e}]$$

$$0.302 \times 10^{-7} = C_3 \frac{1.1^{5.1}}{1+1.1}$$

$$C_3 = 0.39 \times 10^{-7}$$

c) Value of k when $e = 0.75$

$$k = 0.39 \times 10^{-7} \frac{0.75^{5.1}}{1+0.75}$$

$$k = 0.514 \times 10^{-8} \text{ cm/sec}$$

PROBLEM 03.2

For a constant head laboratory permeability test on a fine sand, the following values are given: (Refer to Figure Figure 03.13)

Length of specimen, L = 10 inches
Diameter of specimen = 2.5 inches
Head difference, h = 22 inches
Water collected in 2 min = 0.044 in³

The void ratio of the specimen is 0.34.

a) Determine the conductivity, k, of the soil in in/min.
b) Determine the discharge velocity through the soil in in/min.
c) Determine the seepage velocity in in/min.

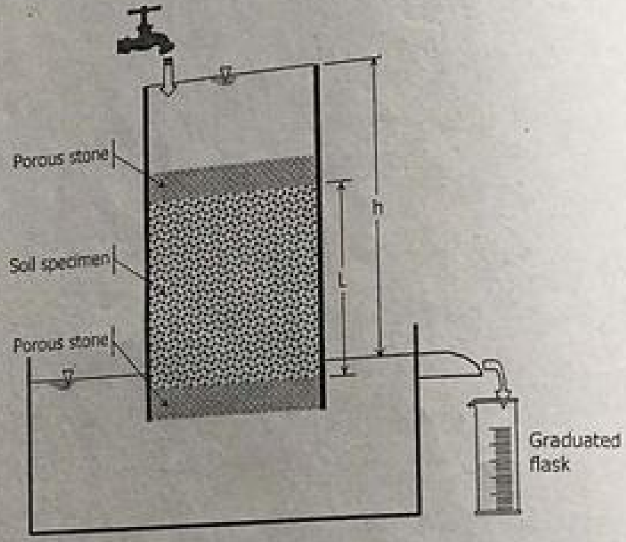


Figure 03.13

SOLUTION

$$\text{Discharge, } Q = \frac{Vol}{t} = \frac{0.044}{2}$$

$$\text{Discharge, } Q = 0.022 \text{ in}^3/\text{min}$$

$$[Q = kiA]$$

$$0.022 = k\frac{22}{10}\,\frac{\pi}{4}(2.5)^2$$

$$k = 0.2037 \times 10^{-2} \text{ in/min}$$

$$\text{Discharge velocity, } v = ki = \frac{Q}{A}$$

$$\text{Discharge velocity, } v = \frac{0.022}{\frac{\pi}{4}(2.5)^2}$$

$$\text{Discharge velocity, } v = 0.448 \times 10^{-2} \text{ in/min}$$

$$\text{Seepage velocity, } v_s = \frac{v}{n}$$

$$n = e/(1 + e)$$

$$n = 0.34/(1 + 0.34)$$

$$n = 0.2537$$

$$\text{Seepage velocity, } v_s = \frac{0.448 \times 10^{-2}}{0.2537}$$

$$\text{Seepage velocity, } v_s = 1.765 \times 10^{-2} \text{ in/min}$$

PROBLEM 03.3

A soil sample 10 cm in diameter is placed in a tube 1 m long. A constant supply of water is allowed to flow into one end of the soil at A and the outflow at B is collected by a beaker, as shown in Figure 03.14. The average amount of water collected is 1 cc for every 10 seconds. The tube is inclined as shown.

a) Determine the average velocity of flow through the soil in cm/s.

b) Determine the seepage velocity (velocity through the void spaces) in cm/s.

c) Determine the coefficient of permeability of the soil in cm/s.

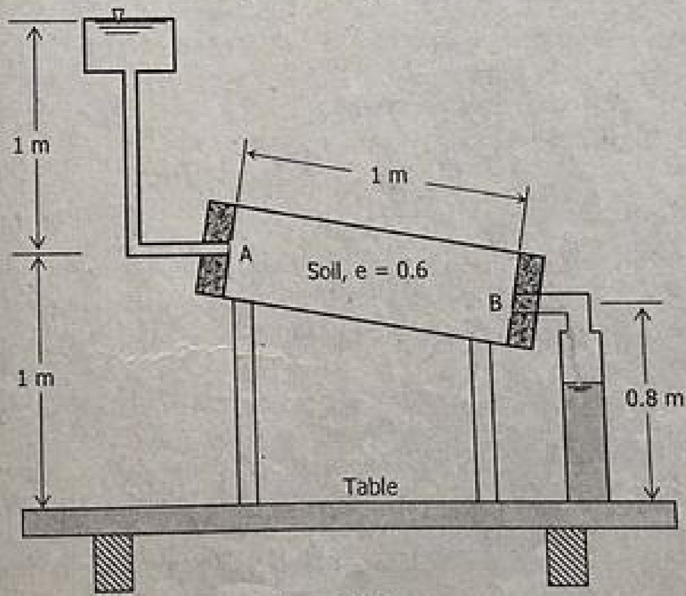


Figure 03.14

SOLUTION

a) Average velocity

Q = Volume/time = 1/10

Q = 0.1 cc/s

$$\text{Average velocity, } v = \frac{Q}{A}$$

$$\text{Average velocity, } v = \frac{0.1}{\frac{\pi}{4}(10)^2}$$

Average velocity, v = **0.00127 cm/s**

b) Seepage velocity:

$$\text{Seepage velocity, } v_s = \frac{v}{n}$$

$$n = \frac{e}{1+e}$$

$$n = \frac{0.6}{1+0.6}$$

$n = 0.375$

$$\text{Seepage velocity, } v_s = \frac{0.00127}{0.375}$$

Seepage velocity, v_s = **0.00339 cm/s**

c) Coefficient of permeability:

$[Q = k\,i\,A]$

$$i = \frac{h}{L} = \frac{(2-0.8)}{1}$$

$i = 1.2$

$0.1 = k\,(1.2)\ \frac{\pi}{4}(10)^2$

$k = 0.001061 = 10.61 \times 10^{-4}$ **cm/s**

PROBLEM 03.4 (CE MAY 1999)

A sand layer having the cross-section area as shown in Figure 03.15 has been determined to exist for a 350-meter length of the levee. The coefficient of permeability of the sand layer is 3.5 m/day. Determine the flow of water into the ditch in Lit/min?

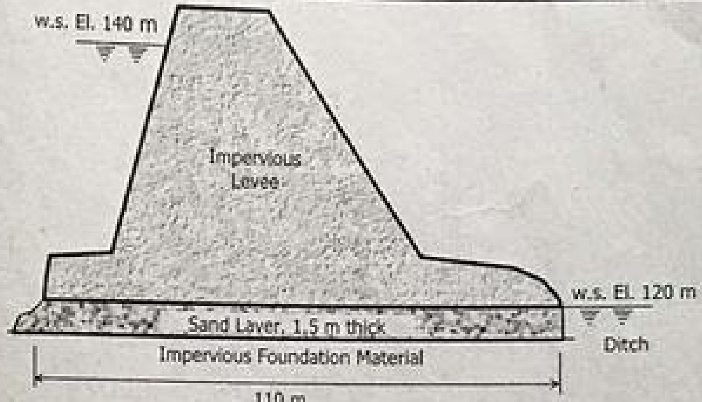


Figure 03.15

SOLUTION

$Q = kiA$

$k = 3.5$ m/day

$i = h/L = 20/110$

$A = 1.5(350) = 525\ \text{m}^2$

$Q = 3.5(20/110)(525)$

$Q = 334.1\ \text{m}^3/\text{day} \times 1000\ \text{Lit/m}^3 \times 1\ \text{day}/24\ \text{hrs} \times 1\ \text{hr}/60\ \text{min}$

Q = **232 Lit/min**

PROBLEM 03.5

A falling-head permeability test was run on a soil sample 9.6 cm in diameter and 10 cm long. The head at the start of the test was 90 cm. The coefficient of permeability of the soil was found to be 5×10^{-6} cm/s. The diameter of the stand pipe was 1 cm.

a) Determine the flow at the start of the test, in cm³/hr.
b) Determine how much head was lost during the first 30 min
c) Determine the flow after 30 minutes, in cm³/hr.

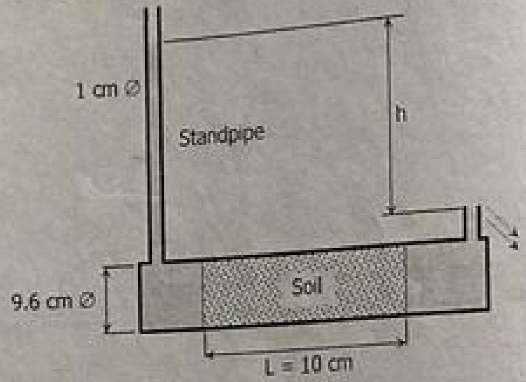


Figure 03.16

SOLUTION

Part *a*:

$h = 90$ cm

$Q = kiA = 5 \times 10^{-6}(90/10)\ \frac{\pi}{4}(9.6)^2$

$Q = 0.003257$ cm³/sec

$Q = \mathbf{11.72}$ **cm³/hr**

Part *b*:

$$t = \int_{h_2}^{h_1} \frac{A_s\,dh}{Q}$$

$A_s = \frac{\pi}{4}(1)^2 = \pi/4$

$Q = k\,i\,A = 5 \times 10^{-6}(h/10)\ \frac{\pi}{4}(9.6)^2$

$Q = \frac{\pi}{4}(0.00004608\ h)$

$h_1 = 90$ cm

$h_2 = ?$

$t = 30$ min $= 1800$ s

$$1800 = \int_{h_2}^{90} \frac{(\pi/4)dh}{(\pi/4)(0.00004608\,h)}$$

$$0.082944 = \Big[\ln h\Big]_{h_2}^{90} = \ln(90) - \ln(h_2)$$

$\ln(h_2) = 4.41686$

$h_2 = 82.84$ cm

Lost of head $= 90 - 82.84$

Lost of head $= \mathbf{7.16}$ **cm**

Using the formula:

$$k = \frac{aL}{A(t_2 - t_1)}\ \ln\left(\frac{h_1}{h_2}\right)$$

$$5 \times 10^{-6} = \frac{\frac{\pi}{4}(1)^2(10)}{\frac{\pi}{4}(9.6)^2(1800-0)}\ \ln\frac{90}{h_2}$$

$$\ln\frac{90}{h_2} = 0.0829$$

$$\frac{90}{h_2} = e^{0.0829}$$

$h_2 = 82.84$ cm

Lost of head $= h_1 - h_2$

Lost of head $= \mathbf{7.16}$ **cm**

Part *c*:

Head, $h = 90 - 7.16 = 82.84$

$[Q = kiA]$

$Q = 5 \times 10^{-6}(82.84/10)\ \frac{\pi}{4}(9.6)^2$

$Q = 0.002998$ cm³/sec

$Q = \mathbf{10.79}$ **cm³/hr**

PROBLEM 03.6

A permeable soil is underlain by an impervious layer, as shown in Figure 03.17. For the permeable layer, $k = 0.0048$ cm/sec. $H = 3$ m and $\alpha = 5°$.

a) Calculate the hydraulic gradient?
b) Calculate for the flow of water per meter width in m³/hr.
c) Calculate the total amount of water percolated per day per meter width, in cubic meter.

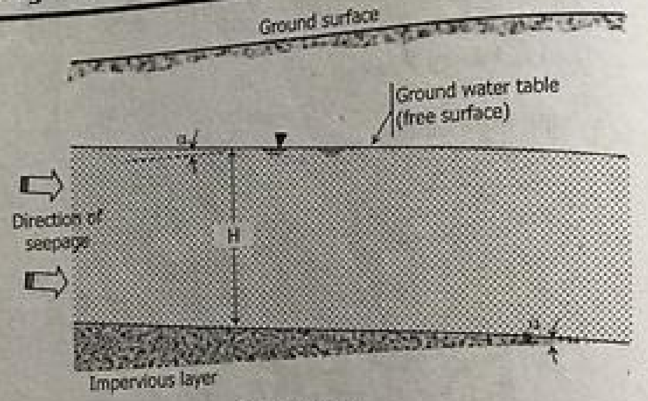
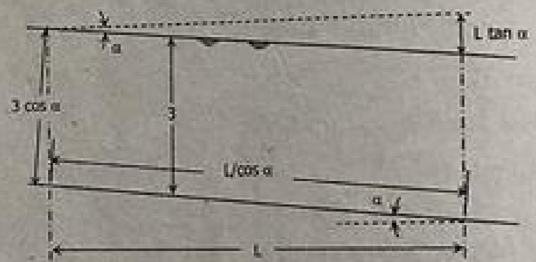


Figure 03.17

SOLUTION



a) Hydraulic gradient:

i = Head loss/Length

$i = L \tan\alpha \,/\, (L/\cos\alpha)$

$i = \sin\alpha$

$i = \sin 5° = 0.08716$

b) Flow per meter width:

$[q = kiA]$

$A = 3\cos\alpha \times 1$

$A = 3\cos 5°$

$A = 2.99\ m^2$

$k = 0.0048$ cm/sec

$k = 0.000048$ m/sec

$q = 0.000048(0.08716)(2.99)$

$q = 0.00001251\ m^3/s$ per meter

$q =$ **0.045 m³/hr per meter**

c) Volume percolated per day:

Volume $= qt$

Volume $= 0.045 \times 24$

Volume $=$ **1.08 m³**

PROBLEM 03.7

Water flows through the permeable layer as shown in Figure 03.18. Given H = 3.5 ft, h = 4.6 ft, L = 120 ft, α = 14°, and k = 0.0016 ft/sec. Consider 1 ft width perpendicular to the figure.

a) Calculate the hydraulic gradient in percent?

b) Calculate for the flow of water per ft width in ft³/hr.

c) Calculate the total amount of water percolated per day per ft width, in cubic ft.

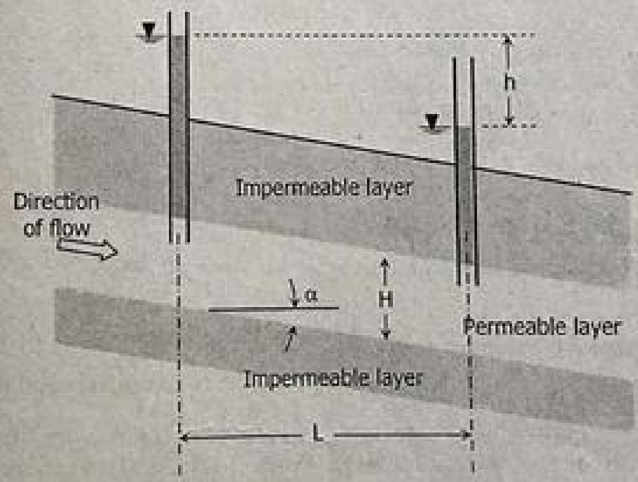


Figure 03.18

SOLUTION

a) Hydraulic gradient:

$$i = \frac{h}{\dfrac{L}{\cos\alpha}}$$

$$i = \frac{4.6}{\dfrac{120}{\cos 14°}}$$

$$i = 0.03719$$

$$i = 3.72\%$$

b) Flow per unit width

$[q = kiA]$

$$A = H\cos\alpha \times 1$$

$$A = 3.5\cos 14° \times 1$$

$$A = 3.396 \text{ ft}^2$$

$$q = 0.0016(0.03719)(3.396)$$

$$q = 2.02078 \times 10^{-4} \text{ ft}^3/\text{sec}$$

$$q = 0.7275 \text{ ft}^3/\text{hr}$$

c) Volume percolated per day:

$$\text{Volume} = 0.7275(24)$$

$$\text{Volume} = 17.46 \text{ ft}^3$$

PROBLEM 03.8 (CE MAY 2003)

Consider the stratified soil deposit shown in Figure 03.19.

where;

1. hydraulic gradient is equal; $i_{eq} = i_1 = i_2 = \ldots = i_n$
2. quantity of flow in each layer is added to make the total flow q

Note: Darcy's equation $v = ki$ and $q = vA$; k_{H1}, k_{H2}, k_{H3}, … k_{Hn} are the coefficients of permeability of the individual layers in the horizontal direction. Consider 1 unit width.

a) Derive the expression for the equivalent coefficient of permeability in the horizontal direction.

b) If there are four layers, 3 m thick each, and $k_{H1} = 2 \times 10^{-3}$, $k_{H2} = 1 \times 10^{-5}$, $k_{H3} = 2 \times 10^{-4}$, $k_{H4} = 1 \times 10^{-3}$ in cm/sec, determine the equivalent coefficient of permeability in the horizontal direction.

c) If $i = 0.70$, determine the total flow q in cm³/sec.

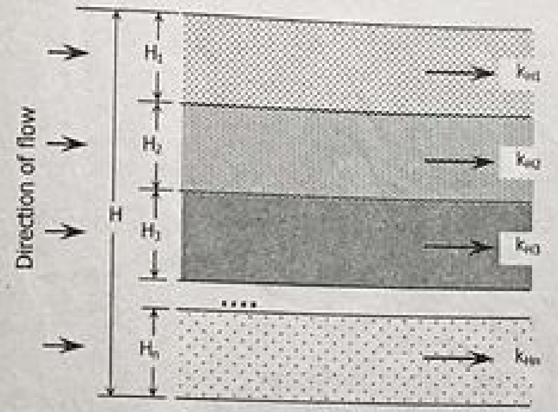


Figure 03.19

SOLUTION

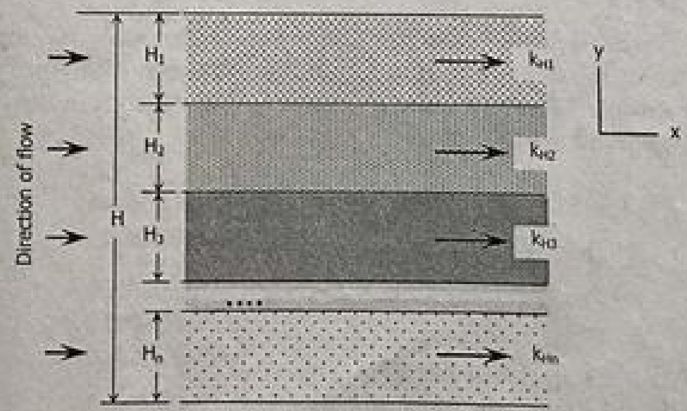
Part *a*:

$q = a\,v = k\,i\,a$

Considering 1 unit width perpendicular to the figure:

$$q = (1 \times H)(k_{x\,eq})i = (1 \times H_1)(k_{H1})i + (1 \times H_2)(k_{H2})i + (1 \times H_3)(k_{H3})i + \ldots + (1 \times H_n)(k_{Hn})i$$

$$k_{x\,eq} = \frac{1}{H}(k_{H1}H_1 + k_{H2}H_2 + k_{H3}H_3 + \ldots + k_{Hn}H_n)$$



Part b

$H_1 = H_2 = H_3 = H_4 = 3$ m

$k_{H1} = 2 \times 10^{-3}$ cm/sec

$k_{H2} = 1 \times 10^{-5}$ cm/sec

$k_{H3} = 2 \times 10^{-4}$ cm/sec

$k_{H4} = 1 \times 10^{-3}$ cm/sec

$$k_{x\,eq} = \frac{1}{H}(k_{H1} H_1 + k_{H2} H_2 + k_{H3} H_3 + \ldots + k_{Hn} H_n)$$

$$k_{x\,eq} = \frac{1}{12 \times 100}[2 \times 10^{-3}(300) + 1 \times 10^{-5}(300) + 2 \times 10^{-4}(300) + 1 \times 10^{-3}(300)]$$

$k_{x\,eq} = 0.0008025$ cm/s

$k_{x\,eq} = 8.025 \times 10^{-4}$ cm/s

Part III:

$i = 0.70$

$q = k_{x\,eq}\, i\, A$

Considering 1 cm width:

$H = 3\text{m} \times 4 = 12$ m

$H = 1200$ cm

$A = H \times 1 = (1200)(1)$

$A = 1{,}200$ cm^2

$q = (8.025 \times 10^{-4})(0.70)(1{,}200)$

$q = 0.674$ cm^3/s per cm width

PROBLEM 03.9

Figure 03.20 shows layers of soil in a tube that is 100 mm × 100 mm in cross-section. Water is supplied to maintain a constant head difference of 400 mm across the sample. The hydraulic conductivities of the soils in the direction of flow through them are as follows:

Soil	k (cm/sec)	Porosity, n
A	1×10^{-2}	25%
B	3×10^{-3}	32%
C	4.9×10^{-4}	22%

a) Calculate the equivalent k in cm/sec.
b) Calculate the rate of water supply in cm^3/hr.
c) Calculate the seepage velocity through soil C in m/sec.

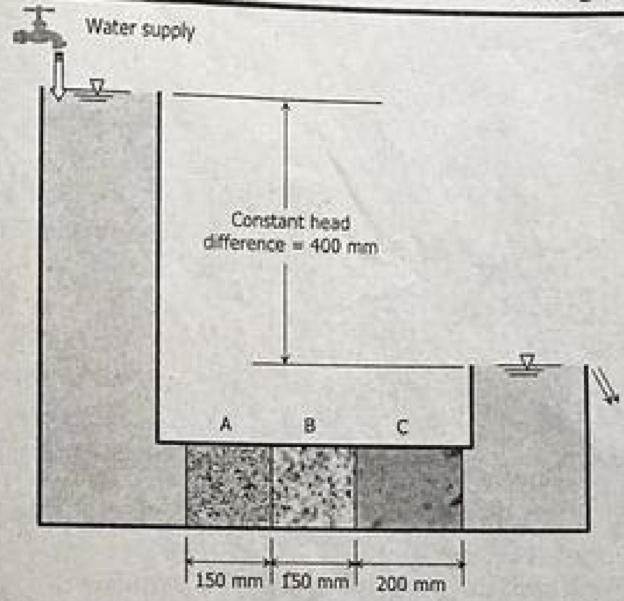


Figure 03.20

SOLUTION

a) Equivalent k:

$$\left[\frac{H_o}{k_{eq}} = \Sigma \frac{H}{k}\right] \quad \text{(parallel flow)}$$

$$\frac{500}{k_{eq}} = \frac{150}{10^{-2}} + \frac{150}{3 \times 10^{-3}} + \frac{200}{4.9 \times 10^{-4}}$$

$k_{eq} = 0.001057$ cm/sec

b) Flow rate:

$[Q = kiA]$

$i = h/L$

$i = 400/500 = 0.80$

$Q = 0.001057(0.80)(10 \times 10)$

$Q = 0.08456$ cm^3/s

$Q = 304.4$ cm^3/hr

c) Seepage velocity through soil C:

$[v_s = v_{ave}/n]$

$v_{ave} = Q/A$

$v_{ave} = 0.08456/(10 \times 10)$

$v_{ave} = 0.0008456 \text{ cm/sec}$

$v_{ave} = 0.08456 \text{ m/sec}$

$v_s = v_{ave}/n$

$v_s = 0.08456/0.22$

$v_s = 0.384 \text{ m/s}$

PROBLEM 03.10

Given the stratified soil shown in Figure 03.21. The properties of each soil are as follows:

Coefficient of permeability:

$k_1 = 6.25 \text{ cm/hr}$, $k_2 = 5.75 \text{ cm/hr}$, $k_3 = 4.50 \text{ cm/hr}$

$k_4 = 6.25 \text{ cm/hr}$, $k_5 = 8.15 \text{ cm/hr}$, $k_6 = 3.60 \text{ cm/hr}$

Thickness:

$H = 1.20 \text{ m}$, $H_3 = 0.30 \text{ m}$

$H_4 = 0.50 \text{ m}$, $H_5 = 0.40 \text{ m}$

Length:

$L_1 = 0.8 \text{ m}$, $L_2 = 0.7 \text{ m}$

$L_3 = 1.5 \text{ m}$, $L_4 = 0.9 \text{ m}$,

Head, $h = 1.8 \text{ m}$

a) Determine the total flow per meter
b) Determine the equivalent coefficient of permeability

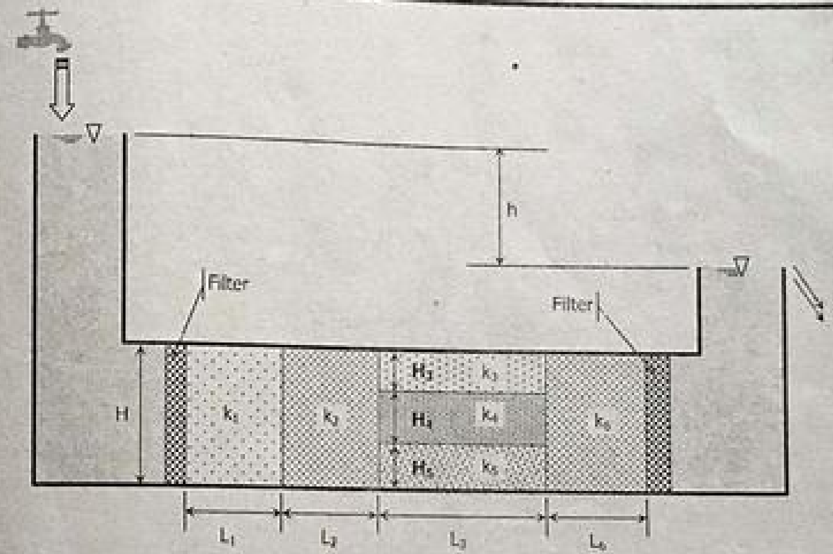
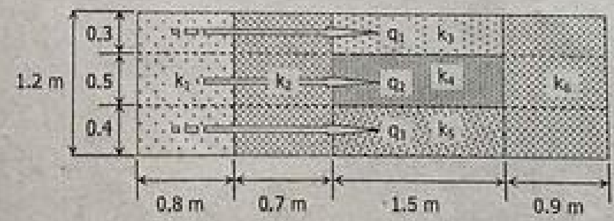


Figure 03.21

SOLUTION



$H_o = 0.8 + 0.7 + 1.5 + 0.9$

$H_o = 3.9 \text{ m}$

$[i = \frac{h}{L}]$

$L = L_1 + L_2 + L_3$ +L4

$L = 0.8 + 0.7 + 1.5 + 0.9$

$L = 3.9 \text{ m}$

$i = \frac{1.8}{3.9}$

$i = 0.4615$

$$\frac{H_o}{k_{eq}} = \Sigma \frac{H}{k}$$

$$k_{eq} = \frac{H_o}{\Sigma(H/k)}$$

a) Total flow:

Solving for q_1:

$[q_1 = k_{1\,eq}\; i\; a_1]$

$$k_{1\,eq} = \frac{3.9}{\frac{0.80}{6.25} + \frac{0.70}{5.75} + \frac{1.5}{4.5} + \frac{0.9}{3.6}}$$

$k_{1\,eq} = 4.681$ cm/hr

$k_{1\,eq} = 0.04681$ m/hr

$a = 0.3 \times 1 = 0.3$ m²

$q_1 = 0.04681(0.4615)(0.3)$

$q_1 = 0.00648$ m³/hr

Solving for q_2:

$[q_2 = k_{2\,eq}\; i\; a_2]$

$$k_{2\,eq} = \frac{3.9}{\frac{0.80}{6.25} + \frac{0.70}{5.75} + \frac{1.5}{6.25} + \frac{0.9}{3.6}}$$

$k_{2\,eq} = 5.272$ cm/hr

$k_{2\,eq} = 0.05272$ m/hr

$a_2 = 0.5(1) = 0.5$ m²

$q_2 = 0.05272(0.4615)(0.5)$

$q_2 = 0.01217$ m³/hr

Solving for q_3:

$[q_3 = k_{3\,eq}\; i\; a_3]$

$$k_{3\,eq} = \frac{3.9}{\frac{0.80}{6.25} + \frac{0.70}{5.75} + \frac{1.5}{8.15} + \frac{0.9}{3.6}}$$

$k_{3\,eq} = 5.7035$ cm/hr

$k_{3\,eq} = 0.057035$ m/hr

$a_3 = 0.4(1) = 0.4$ m²

$q_3 = 0.057035(0.4615)(0.4)$

$q_3 = 0.010529$ m³/hr

Total flow, $q = q_1 + q_2 + q_3$

Total flow, $q = 0.00648 + 0.01217 + 0.010529$

Total flow, $q =$ **0.029179 m³/hr**

b) Equivalent coefficient of permeability for all layers:

$[q = k_{eq}\; i\; a]$

$0.029179 = k_{eq}(0.4615)(1.2 \times 1)$

$k_{eq} = 0.05269$ m/hr

$k_{eq} =$ **5.269 cm/hr**

PROBLEM 03.11

A canal is cut into a soil with a stratigraphy shown in Figure 03.22. Assume flow takes place laterally and vertically through the sides of the canal and vertically below the canal. The values of $k = k_x = k_z$ in each layer are given.

a) What is the equivalent permeability in the horizontal direction through the sides of the canal, in cm/day.

b) What is the equivalent permeability in the vertical directions through the sides of the canal, in cm/day.

c) Determine the equivalent permeability in the vertical directions below the bottom of the canal, in cm/day.

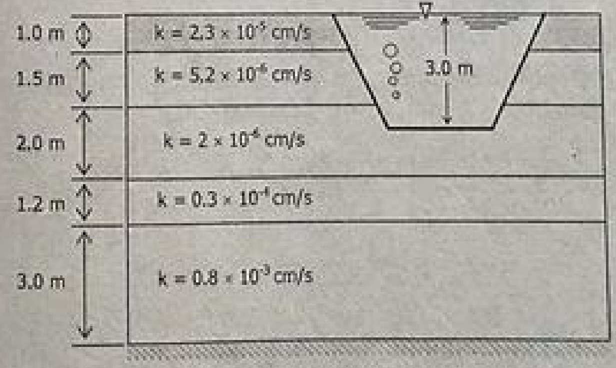
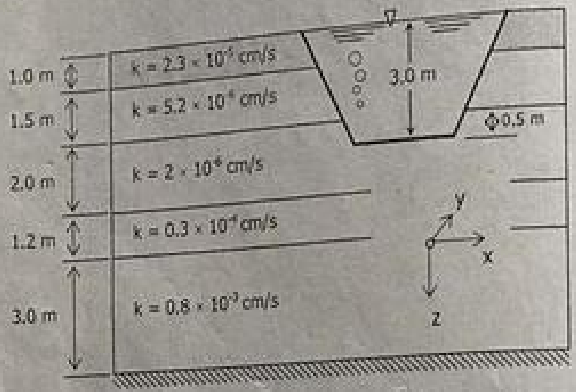


Figure 03.22

SOLUTION



a) Equivalent permeability in horizontal direction though the sides of the canal ($H_o = 3$ m)

$$k_{x\,(eq)} = \frac{1}{H_o}\left(z_1 k_{x1} + z_2 k_{x2} + \cdots + z_n k_{xn}\right)$$

$$k_{x\,(eq)} = \frac{1}{3}\left[1(2.3\times10^{-5}) + 1.5(5.2\times10^{-6}) + 0.5(2\times10^{-6})\right]$$

$k_{x\,(eq)} = 10.6 \times 10^{-6}$ cm/s

$k_{x\,(eq)} = 0.91584$ cm/day

b) Equivalent permeability in vertical direction though the sides of the canal ($H_o = 3$ m)

$$k_{z\,(eq)} = \frac{H_o}{\dfrac{z_1}{k_{z1}} + \dfrac{z_2}{k_{z2}} + \cdots + \dfrac{z_n}{k_{zn}}}$$

$$k_{z\,(eq)} = \frac{3}{\dfrac{1}{2.3\times10^{-5}} + \dfrac{1.5}{5.2\times10^{-6}} + \dfrac{0.5}{2\times10^{-6}}}$$

$k_{z\,(eq)} = 5.16 \times 10^{-6}$ cm/s

$k_{z\,(eq)} = 0.445824$ cm/day

c) Equivalent permeability in vertical direction though the bottom of the canal

$H_o = 1.5 + 1.2 + 3 = 5.7$ m

$$k_{z\,(eq)} = \frac{H_o}{\dfrac{z_1}{k_{z1}} + \dfrac{z_2}{k_{z2}} + \cdots + \dfrac{z_n}{k_{zn}}}$$

$$k_{z\,(eq)} = \frac{5.7}{\dfrac{1.5}{2\times10^{-6}} + \dfrac{1.2}{0.3\times10^{-4}} + \dfrac{3}{0.8\times10^{-3}}}$$

$k_{z\,(eq)} = 7.18 \times 10^{-6}$ cm/s

$k_{z\,(eq)} = 0.620352$ cm/day

PROBLEM 03.12 (NOVEMBER 2003/MAY 2009...)

A test is set-up as shown in Figure 03.23. A cylindrical mold 4″ in diameter is filled with silt to height $H_1 = 0.2$ ft, whose coefficient of permeability $k_1 = 3.6 \times 10^{-4}$ ft/min.

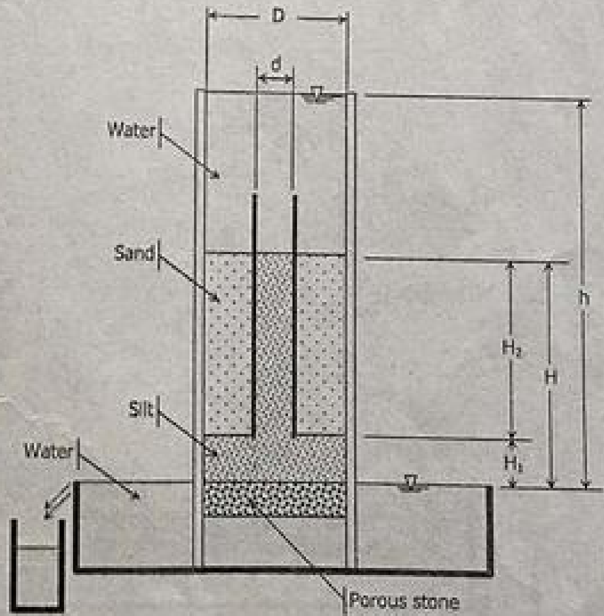


Figure 03.23

A second coaxial mold is placed on top of the first mold whose inside diameter is $d = 1.5''$ and whose height is $H_2 = 0.30$ ft. Its thickness is negligible. The inside of this second mold is filled with the same silt, but the annular ring outside the small tube and outer tube is filled with sand whose coefficient of permeability is $k_2 = 2.7 \times 10^{-3}$ ft/min.

The test set-up is a permeameter of constant head. Water is placed in the mold and maintained at a level $h = 1.25$ ft. above the level of the outlet. It may be considered that the system consists of a fictitious soil of thickness $H = H_1 + H_2$ and coefficient of permeability k_f.

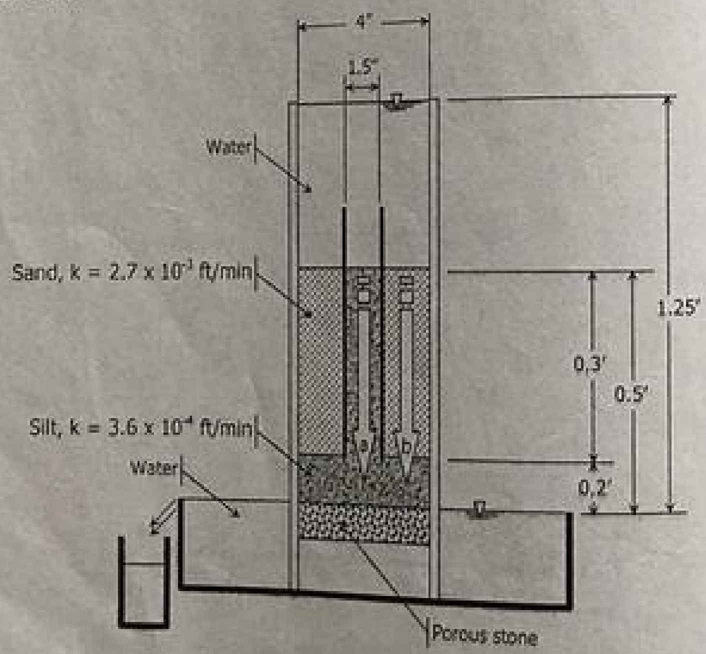
The following general equations may be useful:

$k_{v\,eq} = H / [\,(H_1/k_{v1}) + (H_2/k_{v2}) + \ldots + (H_n/k_{vn})]$

$k_{h\,eq} = (1/H)[\,k_{h1}H_1 + k_{h2}H_2 + \ldots + k_{hn}H_n\,]$

a) What is the total flow of water in ft³/min?
b) What is the equivalent coefficient of permeability, k_f, in ft/min?
c) What is the total amount of water that percolated after 55 minutes.

SOLUTION



Part a:

Path a:

$Q_a = k\,i\,A$

$k = 3.6 \times 10^{-4}$ ft/min

$i = h/L$

$i = 1.25/0.5 = 2.5$

$A = \frac{\pi}{4} d^2$

$A = \frac{\pi}{4}(1.5/12)^2$

$A = 0.01227$ ft²

$Q_a = 3.6 \times 10^{-4}\,(2.5)(0.01227)$

$\mathbf{Q_a = 1.1045 \times 10^{-5}}$ **ft³/min**

Path b:

$Q_b = k_{eq}\,i\,A$

$$k_{eq} = \frac{0.5}{\dfrac{0.3}{2.7 \times 10^{-3}} + \dfrac{0.2}{3.6 \times 10^{-4}}}$$

$k_{eq} = 7.5 \times 10^{-4}$ ft/min

$i = h/L$

$i = 1.25/0.5 = 2.5$

$A = \frac{\pi}{4}(D^2 - d^2)$

$A = \frac{\pi}{4}[(\frac{4}{12})^2 - (\frac{1.5}{12})^2]$

$A = 0.075$ ft²

$Q_b = (7.5 \times 10^{-4})(2.5)(0.075)$

$Q_b = 1.40625 \times 10^{-4}$ ft³/min

$Q = Q_a + Q_b$

$Q = 1.1045 \times 10^{-5} + 1.40625 \times 10^{-4}$

$\mathbf{Q = 1.5167 \times 10^{-4}}$ **ft³/min**

Part b:

$Q = k_f\,i\,A$

$i = h/L$

$i = 1.25/0.5 = 2.5$

$A = \frac{\pi}{4} D^2$

$A = \frac{\pi}{4}(\frac{4}{12})^2$

$A = 0.08727$ ft²

$Q = 1.5167 \times 10^{-4}$ ft³/min

$1.5167 \times 10^{-4} = k_f(2.5)(0.08727)$

$\mathbf{k_f = 6.9518 \times 10^{-4}}$ **ft/min**

Part c:

$$V = Q \times t = 1.5167 \times 10^{-4}\ \text{ft}^3/\text{min} \times 55\ \text{min}$$
$$V = 0.00834185\ \text{ft}^3 \times (100\ \text{cm}/3.28\ \text{ft})^3$$
$$V = 236.4\ \text{cm}^3$$

PROBLEM 03.13

Refer to Figure 03.24. Given that $H_1 = 300$ mm, $H_2 = 500$ mm, and $h_1 = 600$ mm, and that at $z = 200$ mm, $h = 500$ mm. It is required to determine h at $z = 600$ mm.

The following general equations may be useful:

$$k_{v\,eq} = H \,/\, [\,(H_1/k_{v1}) + (H_2/k_{v2}) + \ldots + (H_n/k_{vn})]$$
$$k_{h\,eq} = (1/H)[\,k_{h1} H_1 + k_{h2} H_2 + \ldots + k_{hn} H_n\,]$$

a) What is the ratio k_1/k_2?
b) What is the value of h at $z = 600$ mm?
c) If $k_1 = 5 \times 10^{-6}$ cm/s, what is the equivalent k of soils 1 and 2?

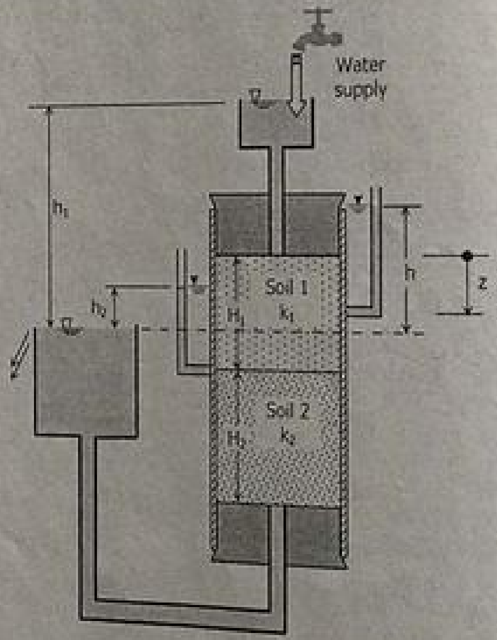
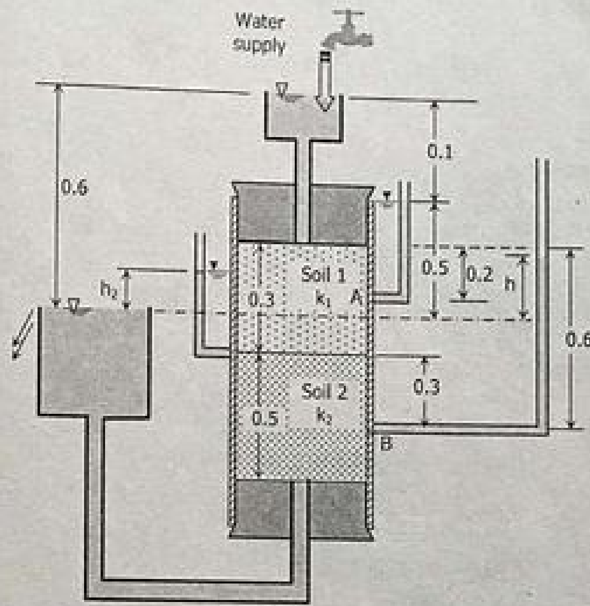


Figure 03.24

SOLUTION



Part *a*:

For the entire system of soil:

$$[q = k_{eq}\, i\, A]$$
$$\left[\frac{H}{k_{eq}} = \Sigma \frac{h}{k}\right]$$
$$\frac{0.8}{k_{eq}} = \frac{0.3}{k_1} + \frac{0.5}{k_2}$$
$$\frac{0.8}{k_{eq}} = \frac{0.3k_2 + 0.5k_1}{k_1 k_2}$$
$$k_{eq} = \frac{0.8k_1k_2}{0.3k_2 + 0.5k_1}$$

$$i = \frac{H}{L}$$
$$i = \frac{0.6}{0.8} = 0.75$$

$$q = \frac{0.8k_1k_2}{0.3k_2 + 0.5k_1}(0.75)A$$
$$q = \frac{0.6k_1k_2}{0.3k_2 + 0.5k_1}A$$

At point A:

Head, $H = 0.1$
Length of soil, $L = 0.2$
$i_1 = y/z$
$i_1 = 0.1/0.2$
$i_1 = 0.5$
$q_1 = k_1\,(0.5)A$
$q_1 = 0.5k_1A$

But $q_1 = q$ (continuity equation)

$$0.5k_1A = \frac{0.6k_1k_2}{0.3k_2 + 0.5k_1}A$$
$0.3k_2 + 0.5k_1 = 1.2k_2$
$0.5\,k_1 = 0.9\,k_2$
$k_1 = 1.8\,k_2$
$k_1/k_2 = 1.8$

Part b:

At point B:

Head, $H = ?$
$L = 0.3 + 0.3$
$L = 0.6$
Solve for k'_{eq}
$$\frac{0.6}{k'_{eq}} = \frac{0.3}{k_1} + \frac{0.3}{k_2}$$
$$\frac{0.6}{k'_{eq}} = \frac{0.3}{1.8k_2} + \frac{0.3}{k_2}$$
$$\frac{0.6}{k'_{eq}} = \frac{0.4667}{k_2}$$
$k'_{eq} = 1.2857k_2$
$k'_{eq} = 0.7143k_1$

$q_2 = k'_{eq}\,i\,L$
$q_1 = k'_{eq}\,i\,L$

$0.7143k_1(H/0.6)\,A = 0.5\,k_1\,A$
$H = 0.42$

$0.6 - h = 0.42$
$h = 0.18$ m
$h = 180$ mm

Part c:

$k_1 = 5 \times 10^{-6}$ cm/s
$k_2 = k_1/1.8$
$k_2 = 2.778 \times 10^{-6}$ cm/s
$$k_{eq} = \frac{0.8k_1k_2}{0.3k_2 + 0.5k_1}$$
$$k_{eq} = \frac{0.8(5\times10^{-6})(2.778\times10^{-6})}{0.3(2.778\times10^{-6}) + 0.5(5\times10^{-6})}$$
$k_{eq} = 3.33 \times 10^{-6}$ cm/s

PROBLEM 03.14 (MAY 2004)

A confined aquifer underlies an unconfined aquifer as shown in Figure 03.25. Given the following: $D_1 = 59$ m, $D_2 = 41$ m, $H_1 = 45$ m, $H_2 = 33$ m, $K_1 = 35$ m/day, $K_2 = 27$ m/day, $L = 2$ km.

a) Calculate the equivalent coefficient of permeability in horizontal direction.
b) Calculate the hydraulic gradient.
c) Calculate the flow of water from one stream to another per meter width.

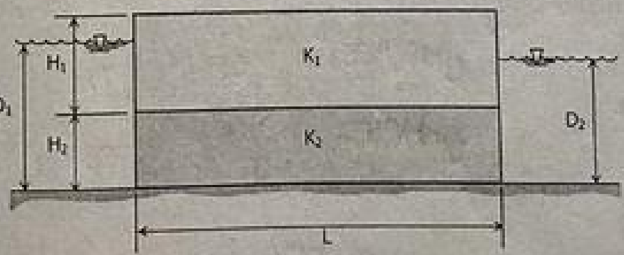
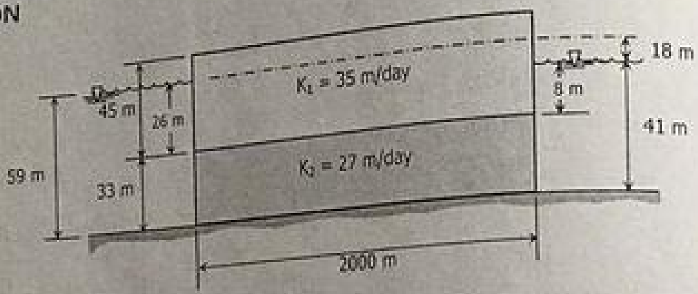


Figure 03.25

SOLUTION



Part *a*:

$K_{eq}H = \Sigma Kh$

$K_{eq}(33 + 45) = 35(45) + 27(33)$

$K_{eq} = \mathbf{31.615\ m/day}$

Part *b*:

$i = h/L = 18/2000 = 0.009$

Part *c*:

$Q = K_{eq}iA$

$A = h_{ave} \times 1 = \frac{59+41}{2}(1) = 50\ m^2$

$Q = 31.615(0.009)(50) = \mathbf{14.227\ m^3/day}$

PROBLEM 03.15 (NOVEMBER 2001)

The section of a cofferdam is as shown in Figure 03.26. If the coefficient of permeability of the soil is $k = 5 \times 10^{-3}$ m/s, determine the seepage into the ditches per meter length of the cofferdam.

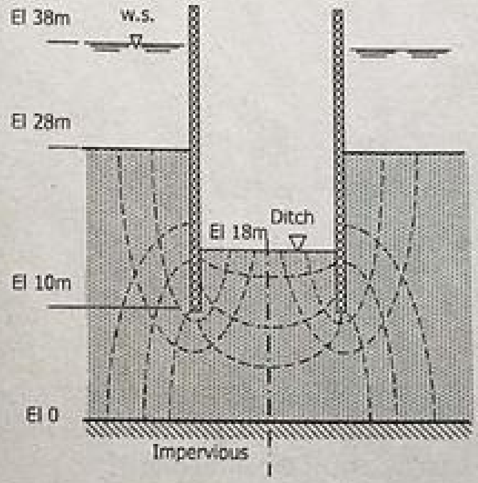
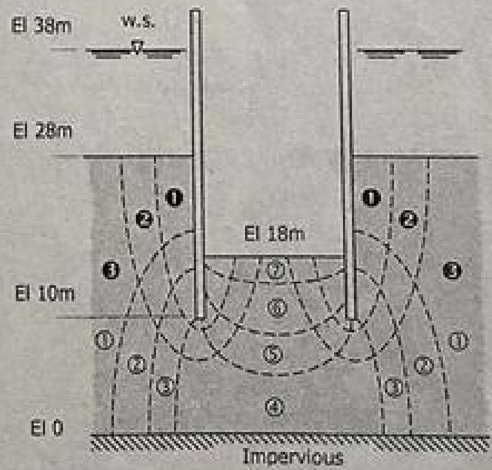


Figure 03.26

SOLUTION



$$q = kH\frac{N_f}{N_d}$$

$H = 38 - 18$

$H = 20$ m

N_f = number of flow channels

$N_f = 3$
N_d = number of pressure drops
$N_d = 7$

$$q = (5 \times 10^{-3})(20)\frac{3}{7}$$

$q = 0.043\ m^3/s$ per meter

Since there are two identical sides
$q = 0.043 \times 2$
$q = 0.086\ m^3/s$ **per meter**

PROBLEM 03.16

The section of a sheet pile is shown in Figure 03.27. The coefficient of permeability of the soil is $k = 4.2 \times 10^{-3}$ m/s. Determine the seepage into the downstream side per meter length of the sheet pile.

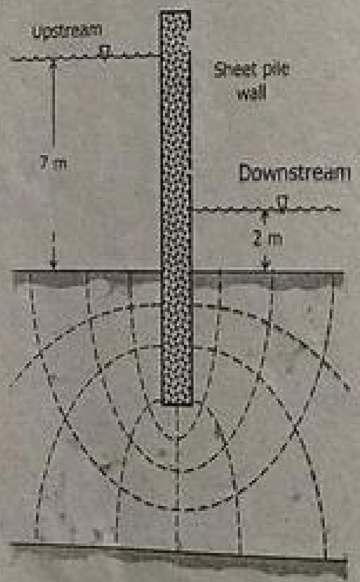
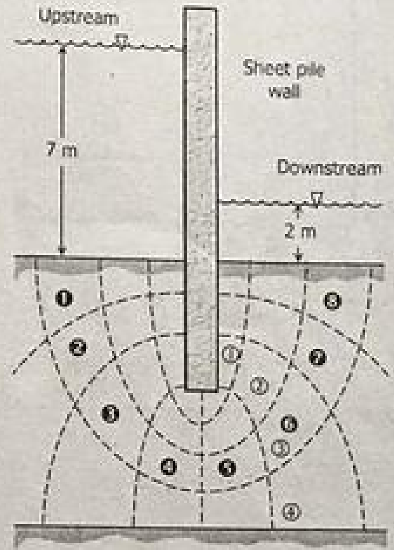


Figure 03.27

SOLUTION



$$[q = kH\frac{N_f}{N_d}]$$

$k = 4.2 \times 10^{-3}$ m/s
$N_f = 4$
$N_d = 8$
$H = 7 - 2 = 5$ m

$$q = 4.2 \times 10^{-3}(5)\frac{4}{8}$$

$q = 0.0105\ m^3/s$ per meter
$q =$ **10.5 L/s**

PROBLEM 03.17

For the masonry dam shown in Figure 03.28, $k = 5$ m/day.

a) Determine the seepage flow per meter width of dam in liters per minute
b) Determine the uplift pressure at A and B in kPa.
c) Determine the uplift force per meter of dam. Assume that the uplift pressure under the dam varies uniformly.

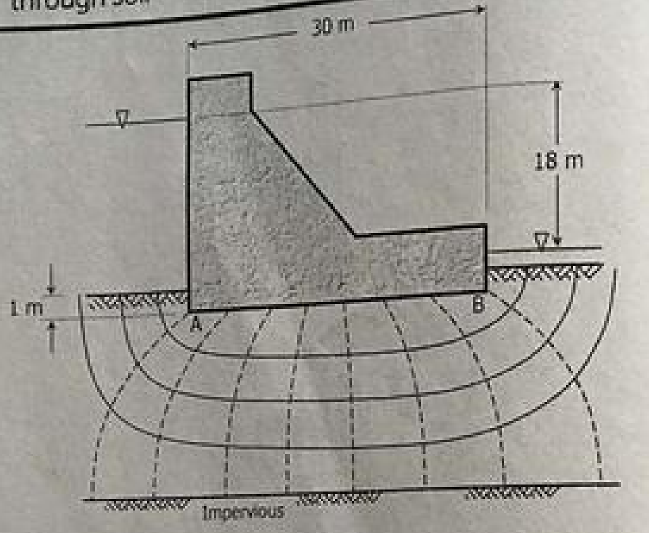
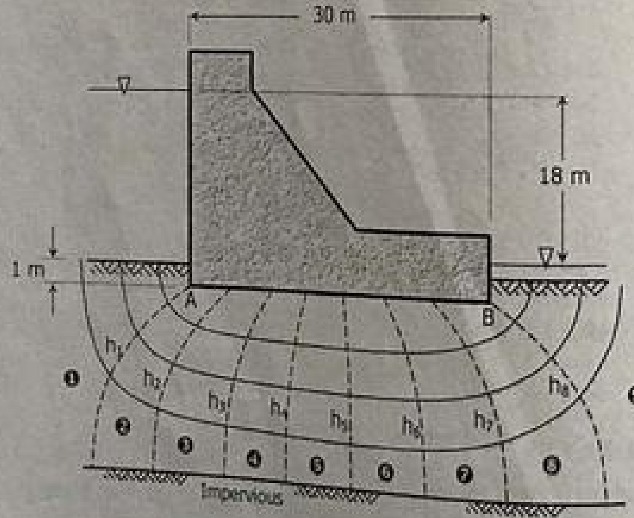


Figure 03.28

SOLUTION



a) Flow per unit width:
Number of pressure drops, $N_d = 9$
Number of flow channels, $N_f = 4$

Seepage flow, $q = kH\dfrac{N_f}{N_d}$

Seepage flow, $q = 5(18)\dfrac{4}{9}$

Seepage flow, $q = 40\ \text{m}^3/\text{day per meter}$

$= 40 \times \frac{1}{24} \times \frac{1}{60} \times 1000$

$= 27.78$ **Lit/min**

b) Pressure at *A* and *B*:

Pressure head drop $= \dfrac{H}{N_d}$

Pressure head drop $= \dfrac{18}{9}$

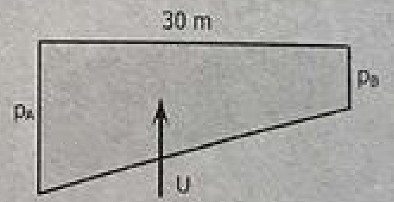
Pressure head drop = 2 m per drop

Pressure head, $h_1 = 18 - 2(1) = 16$ m
Pressure head, $h_8 = 18 - 2(8) = 2$ m

$p_A = \gamma_w h_1 = 9.81(16)$
$p_A = \mathbf{156.96\ kPa}$

$p_B = \gamma_w h_8 = 9.81(2)$
$p_B = \mathbf{19.62\ kPa}$

c) Uplift pressure per unit length of dam:



$U = \dfrac{p_A + p_B}{2}(30) \times 1$

$U = \dfrac{156.96 + 19.62}{2}(30)$

$U = \mathbf{2{,}648.7\ kN}$

Chapter 04

Stresses in Soil

4.1 INTERGRANULAR STRESS, p_E (EFFECTIVE STRESS)

Intergranular or effective stress is the stress resulting from particle-to-particle contact of soil.

$$p_E = p_T - p_w \quad \text{Eq. 4.1}$$

4.2 PORE WATER PRESSURE, p_w (NEUTRAL STRESS)

Pore water pressure or neutral stress is the stress induced by water-pressures.

$$p_w = \gamma_w h_w \quad \text{Eq. 4.2}$$

Note: For soils above water table, $p_w = 0$.

4.3 TOTAL STRESS, p_T

The sum of the effective and neutral stresses.

$$p_T = p_E + p_w \quad \text{Eq. 4.3}$$

4.4 STRESS IN SOIL WITHOUT SEEPAGE

Consider the soil layer shown in Figure 04.1.

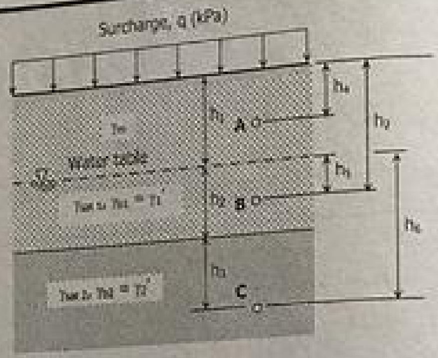


Figure 04.1 – Soil layers with surcharge and without seepage

At point A:

Total stress, $p_T = \gamma_w h_4 + q$

Neutral stress, $p_w = 0$

Effective stress, $p_E = p_T - p_w$

At point B:

Total stress, $p_T = \gamma_{sat1} h_5 + \gamma_w h_1 + q$

Neutral stress, $p_w = \gamma_w h_5$

Effective stress, $p_E = p_T - p_w$

or $p_E = \gamma_{b1} h_5 + \gamma_w h_1 + q$

At point C:

Total stress, $p_T = \gamma_{sat2} h_3 + \gamma_{sat1} h_2 + \gamma_w h_1 + q$

Neutral stress, $p_w = \gamma_w h_6$

Effective stress, $p_E = p_T - p_w$

or $p_E = = \gamma_{b2} h_3 + \gamma_{b1} h_2 + \gamma_w h_1 + q$

4.5 STRESS IN SATURATED SOIL WITH SEEPAGE

4.5.1 UPWARD SEEPAGE

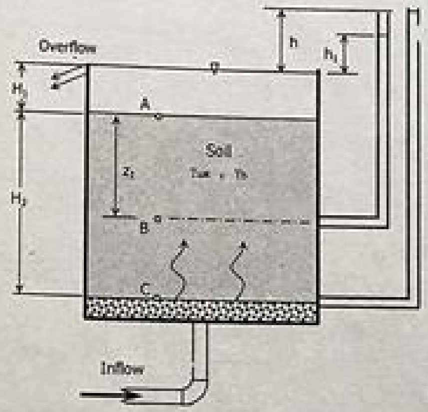


Figure 04.2 – Soil with upward seepage

Hydraulic gradient, $i = h/H_2$

$h_1 = i \times z_1 = i(h/H_2)$

At point A:

$p_T = \gamma_w H_1$ $\qquad$ $p_E = p_T - p_w = 0$

$p_w = \gamma_w H_1$

At point B:

$p_T = \gamma_{sat} z_1 + \gamma_w H_1$ $\qquad$ $p_E = p_T - p_w = \gamma_b z_1 - \gamma_w h_1$

$p_w = \gamma_w(z_1 + H_1 + h_1)$

At point C:

$p_T = \gamma_{sat} H_2 + \gamma_w H_1$ $\qquad$ $p_E = p_T - p_E = \gamma_b H_2 - \gamma_w h$

$p_w = \gamma_w(H_2 + H_1 + h)$

The seepage force per unit volume of soil is:

$$F = i\,\gamma_w \qquad \text{Eq. 4.4}$$

4.5.2 DOWNWARD SEEPAGE

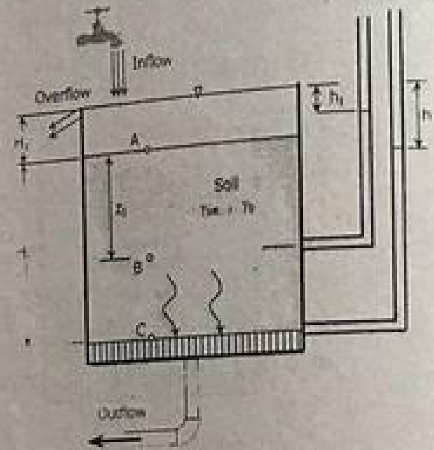


Figure 04.3 – Soil with downward seepage

Hydraulic gradient, $i = h/H$
$h_1 = i \times z_1 = i(h/H_2)$

At point A
$p_T = \gamma_w H$
$p_w = \gamma_w H_1$
$p_i = p_T - p_w = 0$

At point B
$p_T = \gamma_{sat}$ [illegible] H
$p_w = \gamma_w($ [illegible] $+ H_1 - h_1)$
[illegible] $= p_T - p_w =$ [illegible] $+ \gamma_w h_1$

At point C
$p_T = \gamma_{sat} H_2 + \gamma_w H$
$p_w = \gamma_w(H_2 + H - h)$
[illegible] $= \gamma' H_2 + \gamma_w h$

4.6 EFFECT OF CAPILLARY RISE TO SOIL STRESS

Capillary rise in soil is demonstrated on the following figure. A sandy soil is placed in contact with water. After a certain period, water rises and the variation of the degree of saturation with the height of the soil column caused by capillary rise is approximately given in the figure.

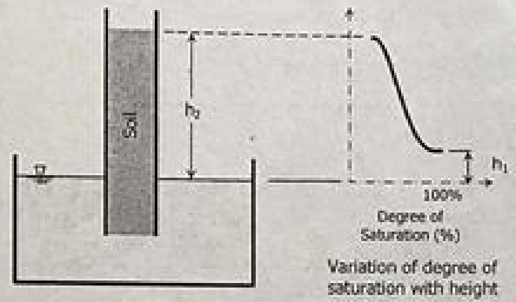


Figure 04.4 – Capillary rise in soil

The degree of saturation is about 100% up to a height h_1. Beyond the height h_1, water can occupy only the smaller voids, hence the degree of saturation is less than 100%.

The approximate height of capillary rise is given by Hazen as:

$$h_2 = \frac{C}{e D_{10}} \qquad \text{Eq. 4.5}$$

where D_{10} = effective grain size, e = void ratio, and C = a constant that varies from 10 to 50 mm².

The pore water pressure, p_w, at a point in the layer of soil fully saturated by capillary rise is:

$$p_w = -\gamma_w h \qquad \text{Eq. 4.6}$$

where h is the height of the point under consideration measured from the ground water table.

If a partial saturation is caused by capillary action, the pore water pressure, p_w, can be approximated as:

$$p_w = -S\,\gamma_w\,h \qquad \text{Eq. 4.7}$$

where S is the degree of saturation at the point under consideration.

Consider the soil layer shown in Figure 04.5:

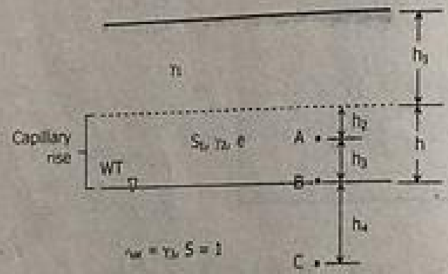


Figure 04.5

At point A:
Total stress, $p_T = \gamma_1 h_1 + \gamma_2 h_2$
Pore water stress, $p_w = -S_1\,\gamma_w\,h_3$

At point B:
Total stress, $p_T = \gamma_1 h_1 + \gamma_2 h$
Pore water stress, $p_w = 0$

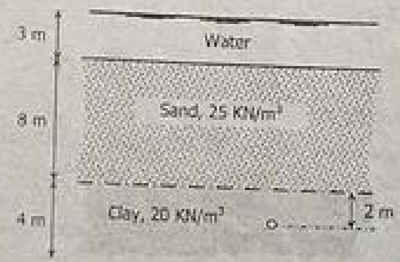
At point C:
Total stress, $p_T = \gamma_1 h_1 + \gamma_2 h + \gamma_3 h_4$
Pore water stress, $p_w = \gamma_w h_4$

ILLUSTRATIVE PROBLEMS

PROBLEM 04.1 (CE NOVEMBER 1998)

20. Nov 98: A clay layer 4 m thick rests beneath a deposit of submerged sand 8 m thick. The top of the sand is located 3 m below the surface of a lake. The saturated unit weight of sand is 25 kN/cu. m and of clay 20 kN/cu. m. Determine the total vertical pressure (P) at mid-height of the clay layer.

SOLUTION



$$[p_T = \sum \gamma_{sat}\,h + \gamma_w\,h_w]$$
$$p_T = 20(2) + 25(8) + 9.81(3)$$
$$p_T = \mathbf{269.43\ kPa}$$

PROBLEM 04.2 (CE NOVEMBER 2000)

A clay layer 25 feet thick is overlain with 50 feet thick of sand (G = 2.71). The water table is 20 feet below the sand (ground) surface. The saturated unit weight of clay is 141 pcf. The sand below the water table has a unit weight of 128 pcf. The sand above the water table has average moisture content of 20%. After drying, the sand was found to have a dry unit weight of 92 pcf. Determine the effective stress at the mid-height of the clay layer.

SOLUTION

For the sand above the water table

$$\left[\gamma_{dry} = \frac{G}{1+e}\gamma_w\right]$$

@seismicisolation

$$92 = \frac{2.71}{1+e}(62.4)$$
$$e = 0.838$$

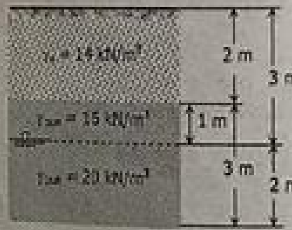
$$[\gamma_{sand} = \frac{G + GMC}{1+e}\gamma_w]$$
$$\gamma_{sand} = \frac{2.71 + 2.71(0.2)}{1 + 0.838}(62.4)$$
$$\gamma_{sand} = 110.4 \text{ pcf}$$

$$\gamma_t = (\sum \gamma_b\,h)_{below\ water\ table} + (\sum \gamma_m\,h)_{above\ water\ table}$$
$$\gamma_t = (141 - 62.4)(12.5) + (128 - 62.4)(30) + 110.4(20)$$
$$\sigma_t = 5158.5 \text{ psf}$$

PROBLEM 04.3 (CE NOVEMBER 2001)

A ground profile consists of 2 m of silty sand underlain by 3 m of clay. The ground water table is 3 m below the ground surface. The sand has a unit weight of 14 kN/m³. The clay has a unit weight of 16 kN/m³ above the water table and 20 kN/m³ below the water table. Determine the total stress at the bottom of the clay layer

SOLUTION



$$p_T = \gamma_{csat}\,h_{sat} + \gamma_c\,h_c + \gamma_s\,h_s$$
$$p_T = (20)(2) + 16(1) + 14(2)$$
$$p_T = 84 \text{ kPa}$$

PROBLEM 04.4 (CE MAY 2002)

The soil shown in Figure 04.6 has a void ratio of 0.50 and G = 2.70. h_1 = 1.5 m. h_2 = 3 m.

a) What is the effective unit weight of sand in kN/m³.
b) What is the effective stress at point *A* in kPa.
c) What is the critical hydraulic gradient of sand (for quick condition)

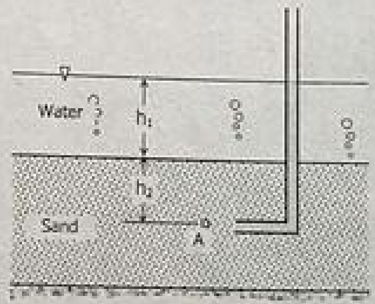


Figure 04.6

SOLUTION

a) Effective unit weight (Submerged unit weight, γ_b)

$$[\gamma_b = \frac{G-1}{1+e}\gamma_w]$$
$$\gamma_b = \frac{2.7-1}{1+0.5}(9.81)$$
$$\gamma_b = 11.118 \text{ kN/m}^3$$

b) Effective stress at *A*:

$$[p_{total} = \gamma_{sat}\,h_2 + \gamma_w\,h_1]$$
$$p_{total} = (20.928)(3) + 9.81(1.5)$$
$$p_{total} = 77.499 \text{ kPa}$$
$$[p_w = \gamma_w\,h_w]$$
$$p_w = 9.81(3 + 1.5)$$
$$p_w = 44.145 \text{ kPa}$$

$$[p_{eff} = p_{total} - p_w]$$
$$p_{eff} = 77.499 - 44.145$$
$$p_{eff} = 33.354 \text{ kPa}$$

c) Critical hydraulic gradient:

$$[i_c = \frac{G-1}{1+e}]$$

$$i_c = \frac{2.7-1}{1+0.5}$$

$$i_c = 1.133$$

PROBLEM 04.5 (CE MAY 2002)

A soil deposit is shown in Figure 04.7. The ground water table, initially at the ground surface, was lowered to a depth of 25 ft below the ground. After such lowering, the degree of saturation of the sand above water table was lowered to 20%.

a) What is the vertical effective pressure at the midheight of the clay layer before lowering of the water table, in psf.
b) What is the vertical effective pressure at the midheight of the clay layer after lowering of the water table, in psf.
c) What is the vertical effective pressure when there is no water in the sand layer, in psf.

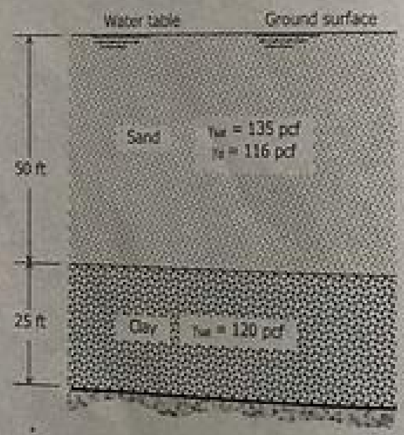


Figure 04.7

SOLUTION

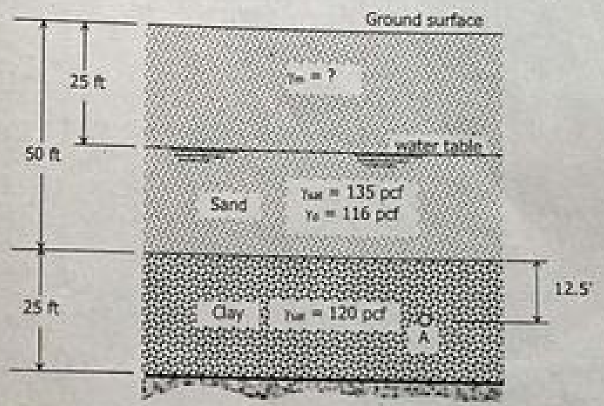
a) Vertical effective pressure at A before lowering of the water table:

$[p_A = \sum \gamma_b h]$

$p_A = (120 - 62.4)(12.5) + (135 - 62.4)(50)$

$p_A = \mathbf{4{,}350\ psf}$

b) After lowering of the water table:



Solving for γ_m (20% saturated)

$\gamma_m = 116 + 0.2(135 - 116)$

$\gamma_m = 119.8$ pcf

$p_A = (120 - 62.4)(12.5) + (135 - 62.4)(25) + 119.8(25)$

$p_A = \mathbf{5530\ psf}$

c) Vertical effective pressure at A when there is no water in the sand layer:

$p_A = (120 - 62.4)(12.5) + 119.8(50)$

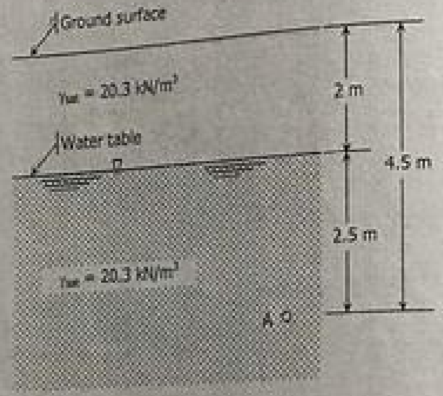
$p_A = \mathbf{6{,}710\ psf}$

PROBLEM 04.6 (CE MAY 2003)

The ground water level in a thick, very fine sand deposit is located 2.0 m below the ground surface. Above the free ground water line, the sand is saturated by capillary action. The unit weight of the saturated sand is 20.3 kN/m³.

a) What is the total stress in kPa on a horizontal plane A located 4.5 m below the ground surface?
b) What is the pore water pressure in kPa at this plane?
c) What is the effective vertical stress in kPa in plane A?

SOLUTION



a) Total stress in A:

$p_T = 20.3(2.5) + 20.3(2)$

$p_T = 91.35$ **kPa**

b) Pore water pressure in A:

$p_w = 9.81(2.5)$

$p_w = 24.525$ **kPa**

c) Effective stress in A:

$[p_T = p_E + p_w]$

$p_E = p_T - p_w$

$p_E = 91.35 - 24.525$

$p_E = 66.825$ **kPa**

PROBLEM 04.7 (CE MAY 2003)

A dense silt layer has the following properties: void ratio = 0.40, effective diameter d_{10} = 10 μm, capillary constant C = 0.20 cm². Free ground water level is 8.0 m below the ground surface.

a) Find the height of capillary rise the silt. Capillary rise is given as $h = C / (e \times d_{10})$.
b) Find the vertical effective stress in kPa at 5 m depth. Assume unit weight of solids = 26.5 kN/m³ and that the soil above the capillary action rise and ground surface is partially saturated at 50%.
c) Find the vertical effective stress at 10 m depth. Assume unit weight of solids = 26.5 kN/m³ and that the soil above the capillary action rise and ground surface is partially saturated at 50%.

SOLUTION

a) Capillary rise:

$$h = \frac{C}{e \times d_{10}}$$

$C = 0.20\ \text{cm}^2$

$d_{10} = 10\ \mu\text{m}$

$d_{10} = 10 \times 10^{-6}\ \text{m}$

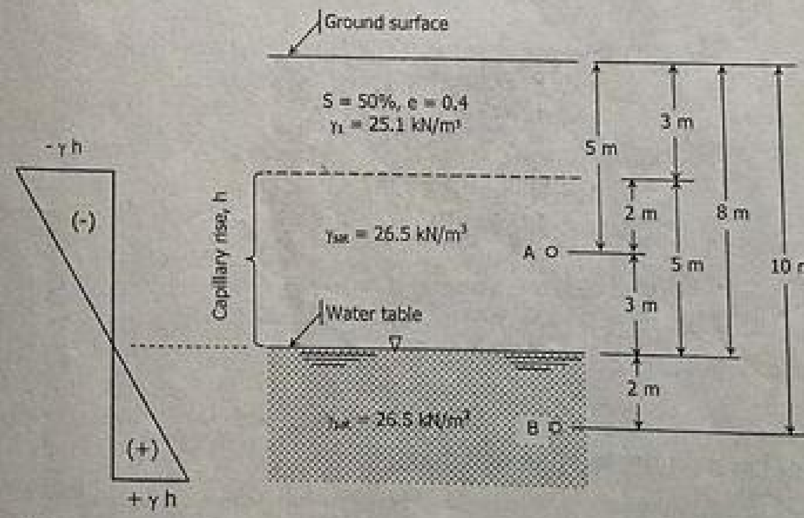
$d_{10} = 10 \times 10^{-4}\ \text{cm}$

$e = 0.4$

$$h = \frac{0.20}{0.4(10 \times 10^{-4})}$$

$h = 500$ cm

$h = 5$ **m**



Pore water stress distribution

b) Vertical effective stress at 5 m depth

Solving for γ_1 and γ_{sat}:

First we solve G:

$[\gamma_s = \gamma_w G]$

$26.5 = 9.81 \times G$

$G = 2.701$

$[\gamma_1 = \frac{G + Se}{1 + e} \gamma_w]$

$\gamma_1 = \frac{2.701 + (0.5)(0.4)}{1 + 0.4} \times 9.81$

$\gamma_1 = 20.33 \text{ kN/m}^3$

$[\gamma_{sat} = \frac{G + e}{1 + e} \gamma_w]$

$\gamma_{sat} = \frac{2.701 + 0.4}{1 + 0.4} \times 9.81$

$\gamma_{sat} = 21.73 \text{ kN/m}^3$

Vertical effective stress at *A*:

Total stress at *A*:

$p_T = 21.73(2) + 20.33(3)$

$p_T = 104.45$ kPa

Pore water stress at *A* (within capillary rise):

$p_w = -9.81(3)$

$p_w = -29.43$ kPa

$[p_T = p_E + p_w]$

$104.45 = p_E + (-29.43)$

$p_E = $ **133.88 kPa**

c) Vertical effective stress at *B*:

$p_E = (21.73 - 9.81)(2) + 21.73(5) + 20.33(3)$

$p_E = $ **193.48 kPa**

PROBLEM 04.8 (CE NOVEMBER 2003)

A 20-m thick submerged saturated clay layer has water content of 57%. The specific gravity of the solid particles is 2.84.

a) Determine the density of the clay in kg/m³.

b) Determine the total vertical stress at the bottom of the clay layer, in kPa

c) Determine the effective vertical stress at the bottom of the clay layer, in kPa.

SOLUTION

Given: $MC = 0.57$ $\quad S = 1$ (saturated)

$G = 2.84$

$[G\,MC = S\,e]$

$2.84(0.57) = 1(e)$

$e = 1.6188$

$[\gamma_m = \frac{G + Se}{1 + e} \gamma_w]$

$\gamma_m = \frac{2.84 + 1(1.6188)}{1 + 1.6188}(9.81)$

$\gamma_m = 16.7026 \text{ kN/m}^3$ (saturated unit weight)

a) Density, $\rho = \gamma/g = 16{,}702.6 / 9.81$

Density, $\rho = $ **1702.6 kg/m³**

b) Total stress, $p_T = \gamma_m h$

Total stress, $p_T = 16.7026(20)$

Total stress, $p_T = $ **334 kPa**

c) Effective stress $= p_T - p_w$

Effective stress $= 334 - 9.81(20)$

Effective stress = **137.85 kPa**

PROBLEM 04.9

A borehole at a site reveals the soil profile shown in Figure 04.8. Assume G = 2.7 for all soil types.

a) What is the unit weight of the soil in layer 1 in kN/m³?

b) What is the effective stress at a depth of 2 m below the ground surface, in kPa?

c) What is the effective stress at a depth of 20.6 m below the ground surface, in kPa?

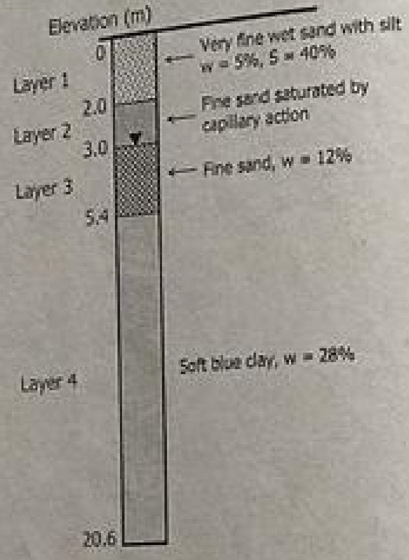
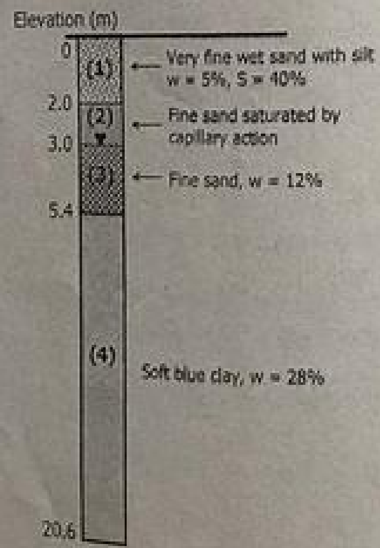


Figure 04.8

SOLUTION



Properties of each layer:

Layer (1):

$[G\,MC = S\,e]$

$2.7(0.05) = 0.4e$

$e = 0.3375$

$$\left[\gamma_m = \frac{G + GMC}{1+e}\gamma_w\right]$$

$$\gamma_m = \frac{2.7 + 2.7(0.05)}{1 + 0.3375}(9.81)$$

γ_m **= 20.79 kN/m³ → Part *a***

Layers (2) and (3):

$[G\,MC = S\,e]$

$2.7(0.12) = 1(e)$

$e = 0.324$

$$\left[\gamma_{sat} = \frac{G + e}{1+e}\gamma_w\right]$$

$$\gamma_{sat} = \frac{2.7 + 0.324}{1 + 0.324}(9.81)$$

$\gamma_{sat} = 22.405\ \text{kN/m}^3$

Layer (4):

$[G\,MC = S\,e]$

$2.7(0.28) = 1(e)$

$e = 0.756$

$$\left[\gamma_{sat} = \frac{G + e}{1+e}\gamma_w\right]$$

$$\gamma_{sat} = \frac{2.7 + 0.756}{1 + 0.756}(9.81)$$

$\gamma_{sat} = 19.307\ \text{kN/m}^3$

b) Stress at 2 m depth:

$p_{total} = 20.79(2)$

$p_{total} = 41.58$ kPa

$p_w = -9.81(1)$

$p_w = -9.81$

$p_{eff} = p_{total} - p_w$

$p_{eff} = 41.58 - (-9.81)$

$p_{eff} = 51.39$ kPa

c) Stress at 20.6 m depth:

$p_{eff} = (19.307 - 9.81)(20.6 - 5.4) + (22.405 - 9.81)(5.4 - 3) + 22.405(1) + 20.79(2)$

$p_{eff} = 238.567$ kPa

PROBLEM 04.10

Consider the upward flow of water through a layer of sand in a tank shown in Figure 04.9. For the sand, the following properties are given: $e = 0.40$, $G = 2.67$

a) Calculate the effective stress at point *A*
b) Calculate the effective stress at point *B*
c) Calculate the upward seepage force per unit volume of soil

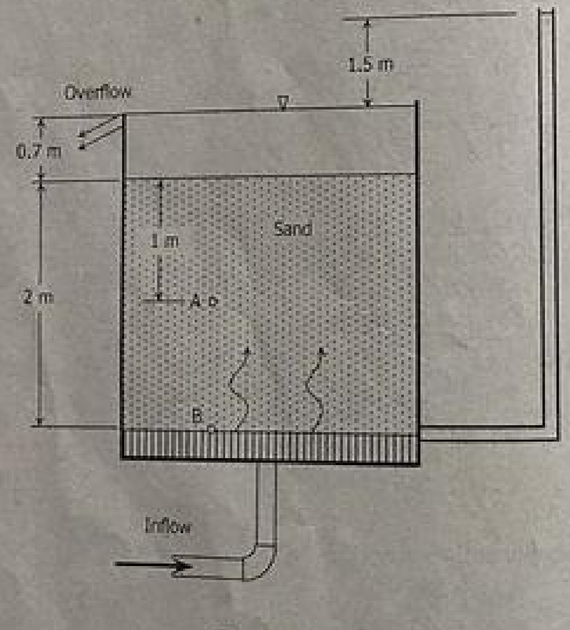
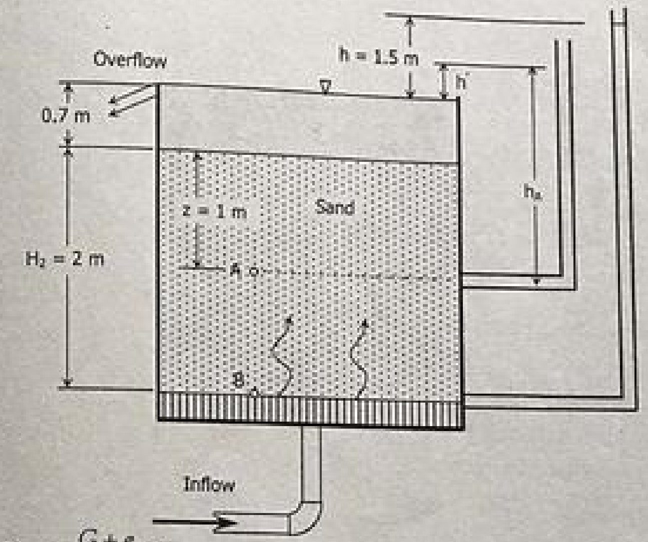


Figure 04.9

SOLUTION



$[\gamma_{sat} = \dfrac{G+e}{1+e}\gamma_w]$

$\gamma_{sat} = \dfrac{2.67+0.4}{1+0.4}(9.81)$

$\gamma_{sat} = 21.51\ \text{kN/m}^3$

$[i = \dfrac{h}{H_2}]$

$i = \dfrac{1.5}{2}$

$i = 0.75$

Note: $i = \dfrac{h'}{z} = \dfrac{h}{H_2}$ = constant

a) Effective stress at point *A*:

$p_T = (21.51)(1) + 9.81(0.7)$

$p_T = 28.377$ kPa

$p_w = 9.81\, h_A$

$h_A = 1 + 0.7 + h'$

$$\frac{h'}{z} = i = 0.75$$
$$h' = 0.75(1) = 0.75$$
$$h_A = 1.7 + 0.75$$
$$h_A = 2.45 \text{ m}$$

$$p_w = 9.81(2.45)$$
$$p_w = 24.0345 \text{ kPa}$$

$$[p_E = p_T - p_w]$$
$$p_E = 28.377 - 24.0345$$
$$p_E = \mathbf{4.343\ kPa}$$

b) Effective stress at point *B*

$$p_T = 21.51(2) + 9.81(0.7)$$
$$p_T = 49.887 \text{ kPa}$$

$$p_w = 9.81(2 + 0.7 + 1.5)$$
$$p_w = 41.202 \text{ kPa}$$

$$[p_E = p_T - p_w]$$

$$p_E = 49.887 - 41.202$$
$$p_E = \mathbf{8.685\ kPa}$$

c) Seepage Force per unit volume of soil = $i\,\gamma_w$

$$i = h/L = 1.5/2 = 0.75$$

Seepage Force per unit volume of soil = (0.75)(9.81)
Seepage Force per unit volume of soil = **7.3575 kN/m³**

PROBLEM 04.11

Consider the downward flow of water through a layer of sand in a tank shown in Figure 04.10. For the sand, the following properties are given: e = 0.48, G = 2.7

a) Determine the saturated unit weight of sand in kN/m³
b) Determine the effective stress at point *A* in kPa
c) Determine the effective stress at point *B* in kPa

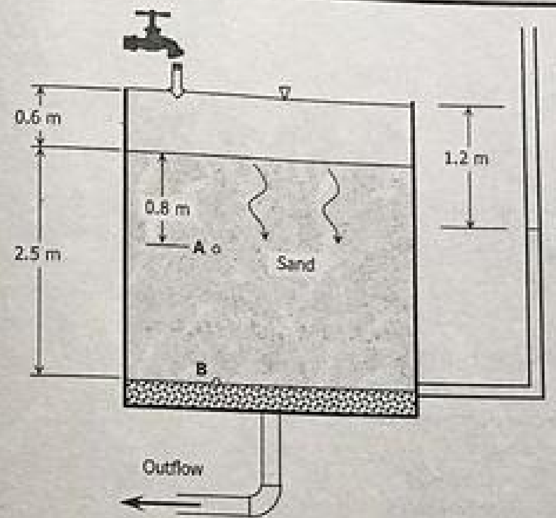
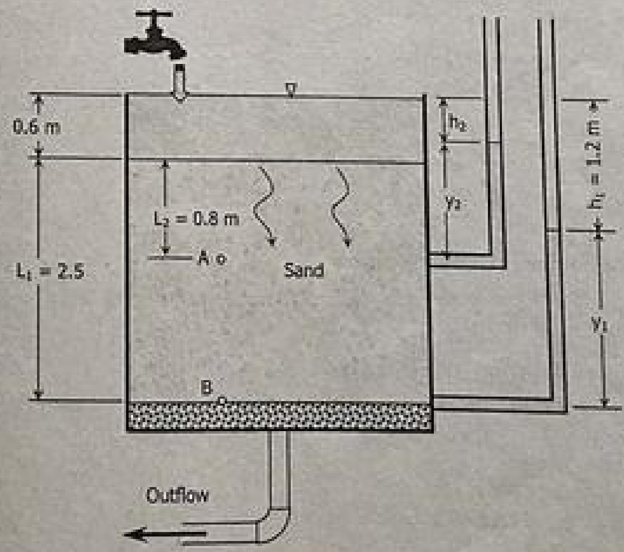


Figure 04.10

SOLUTION



$[i = \frac{h}{L}]$

$$i = \frac{h_1}{L_1} = \frac{1.2}{2.5}$$

$$i = 0.48$$

a) Saturated unit weight

$[\gamma_{sat} = \frac{G+e}{1+e}\gamma_w]$

$$\gamma_{sat} = \frac{2.7+0.48}{1+0.48}(9.81)$$

$$\gamma_{sat} = 21.078 \text{ kN/m}^3$$

b) Effective stress at *A*:

$[h_2 = i\,L_2]$

$$h_2 = 0.48(0.8)$$

$$h_2 = 0.384 \text{ m}$$

$$y_2 = 0.6 + 0.8 - 0.384$$

$$y_2 = 1.016 \text{ m}$$

$$p_T = 21.078(0.8) + 9.81(0.6)$$

$$p_T = 22.748 \text{ kPa}$$

$[p_w = \gamma_w\,y_2]$

$$p_w = 9.81(1.016)$$

$$p_w = 9.967 \text{ kPa}$$

$[p_E = p_T - p_w]$

$$p_E = 22.748 - 9.967$$

$$p_E = 12.78 \text{ kPa}$$

c) Effective stress at point *B*:

$$y_1 = 0.6 + 2.5 - 1.2$$

$$y_1 = 1.9 \text{ m}$$

$$p_T = 21.078(2.5) + 9.81(0.6)$$

$$p_T = 58.581 \text{ kPa}$$

$[p_w = \gamma_w\,y_2]$

$$p_w = 9.81(1.9)$$

$$p_w = 18.639 \text{ kPa}$$

$[p_E = p_T - p_w]$

$$p_E = 58.581 - 18.639$$

$$p_E = \mathbf{39.942\ kPa}$$

PROBLEM 04.12

A soil profile consists of a clay layer underlain by a sand layers as shown in Figure 38. A tube is inserted into the bottom sand layer and the water level rises to 1.2 m above the ground surface.

a) Determine the effective stress at point *A*.
b) Determine the effective stress at point *B*.
c) Determine the effective stress at point *C*.

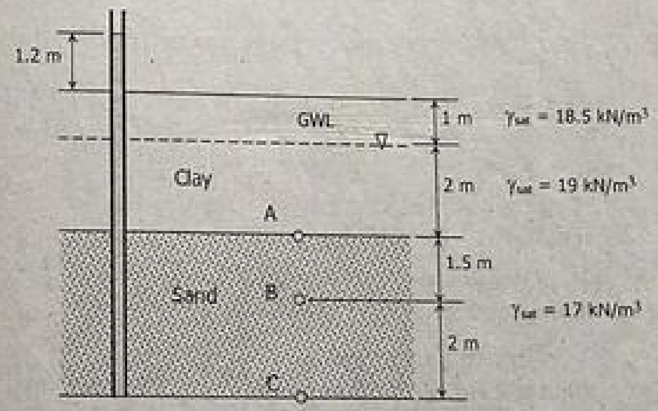


Figure 04.11

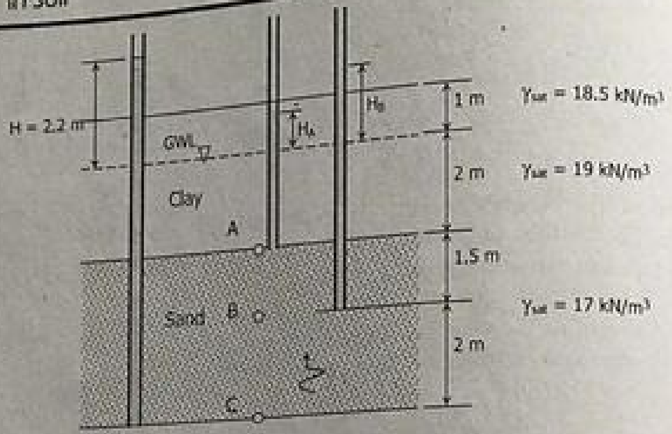
SOLUTION

Hydraulic gradient (based at point C)

$[i = \frac{H}{L}]$

$$i = \frac{2.2}{2 + 1.5 + 2}$$

$$i = 0.4$$



a) At point *A*:

$[i = \frac{H_A}{L_A}]$ $\quad 0.4 = \frac{H_A}{2}$

$H_A = 0.8$ m

Total stress, $p_T = 19(2) + 18.5(1)$
Total stress, $p_T = 56.5$ kPa

Pore water stress, $p_w = 9.81(2 + 0.8)$
Pore water stress, $p_w = 27.468$ kPa

Effective stress, $p_E = p_T - p_w$
Effective stress, $p_E = 56.5 - 27.468$
Effective stress, p_E = **29.032 kPa**

b) At point *B*:

$[i = \frac{H_B}{L_B}]$ $\quad 0.4 = \frac{H_B}{2 + 1.5}$

$H_B = 1.4$ m

Total stress, $p_T = 17(1.5) + 19(2) + 18.5(1)$
Total stress, $p_T = 82$ kPa

Pore water stress, $p_w = 9.81(3.5 + 1.4)$
Pore water stress, $p_w = 48.069$ kPa

Effective stress, $p_E = p_T - p_w$
Effective stress, $p_E = 82 - 48.069$

Effective stress, p_E = **33.931 kPa**

c) At point C:
Total stress, $p_T = 17(3.5) + 19(2) + 18.5(1)$
Total stress, $p_T = 116$ kPa

Pore water stress, $p_w = 9.81(5.5 + 2.2)$
Pore water stress, $p_w = 75.537$ kPa

Effective stress, $p_E = p_T - p_w$
Effective stress, $p_E = 116 - 75.537$
Effective stress, p_E = **40.463 kPa**

PROBLEM 04.13

A cut is made in a stiff, saturated clay that is underlain by a layer of sand as shown in Figure 04.12.

a) Calculate the total stress at point *A* as a function of *h*.
b) Calculate the neutral stress at point *A*.
c) Calculate the height of water *h*, in the cut so that the stability of the saturated clay is not lost.

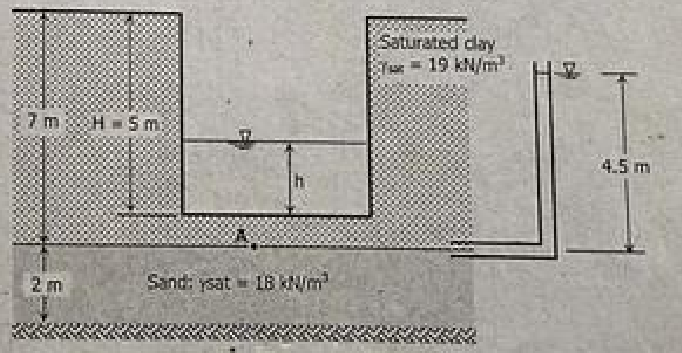


Figure 04.12

SOLUTION

a) Total stress at point *A*:
$p_T = 19(7 - 5) + 9.81h$
$p_T = 38 + 9.81h$

b) Neutral stress at A:

$p_w = 9.81(4.5)$

$p_w = 44.145$ kPa

c) For loss of stability, $p_E = 0$

$p_E = p_T - p_w$

$0 = 38 + 9.81h - 44.145$

$h = 0.626$ m

PROBLEM 04.14

For the system shown in Figure 04.13, $L = 1.2$ m, $h = 2.3$ m, and $y = 1.9$ m. What is the effective stress at point A where $z = 0.7$ m.

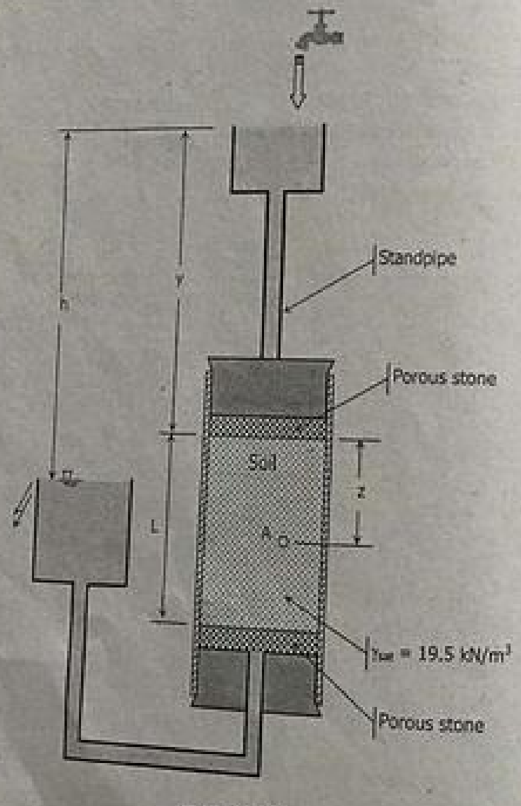
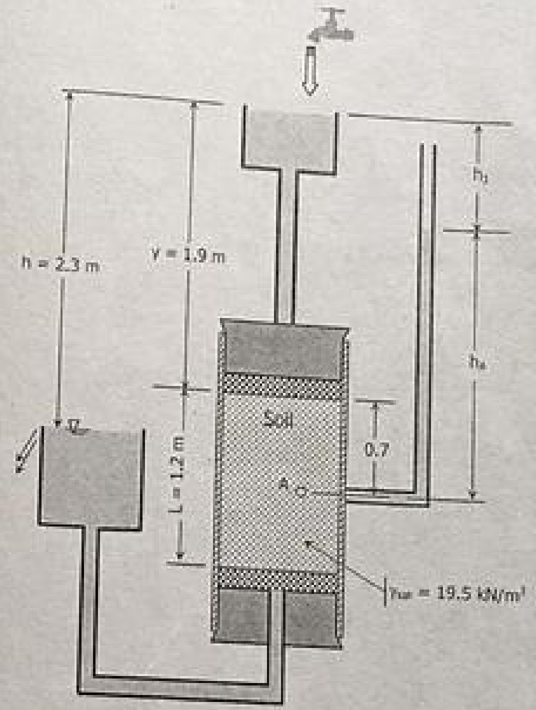


Figure 04.13

SOLUTION



Energy gradient, $i = \dfrac{h}{L} = \dfrac{h_1}{z}$

$\dfrac{2.3}{1.2} = \dfrac{h_1}{0.7}$

$h_1 = 1.3417$ m

$h_A = 0.7 + 1.9 - 1.3417$

$h_A = 1.2583$ m

Total stress at A, $p_T = 19.5(0.7) + 9.81(1.9)$

Total stress at A, $p_T = 32.289$ kPa

Pore water stress, $p_w = \gamma_w h_A$

Pore water stress, $p_w = 9.81(1.2583)$

Pore water stress, $p_w = 12.344$ kPa

Effective stress, $p_E = p_T - p_w$
Effective stress, $p_E = 32.289 - 12.344$
Effective stress, $p_E = 19.945$ kPa

PROBLEM 04.15

A soil element shown in Figure 04.14 is subjected to the following stresses:

$\sigma_x = 120$ kPa $\qquad \sigma_y = 300$ kPa
$\tau = 40$ kPa $\qquad \theta = 20°$

Calculate the normal and shear stress on plane AB.

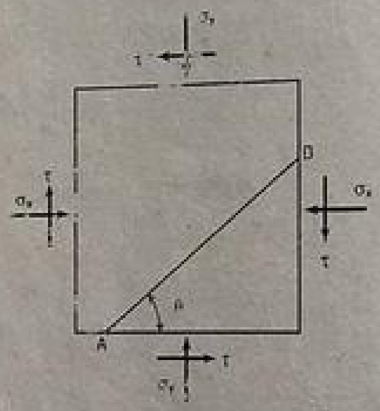
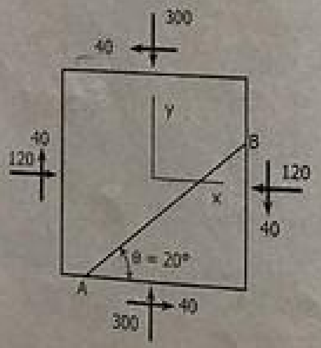


Figure 04.14

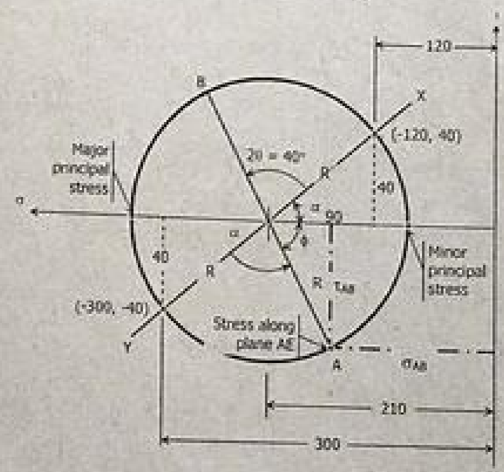
SOLUTION



$\sigma_x = -120$ kPa
$\tau_{xy} = 40$ kPa (-120, 40)

$\sigma_y = -300$ kPa
$\tau_{yx} = -40$ kPa (-300, -40)

Sign: Tensile normal stresses are taken as positive. Shear stresses are considered positive if they act on opposite faces of the element in such a way that tend to produce clockwise rotation



From the Mohr's circle:

$$R = \sqrt{90^2 + 40^2}$$
$$R = 98.49$$

$$\tan\alpha = 40/90$$
$$\alpha = 23.96°$$
$$\phi = 180° - 40° - \alpha$$
$$\phi = 116.04°$$

$$\sigma_{AB} = -(210 - R\cos\phi)$$
$$\sigma_{AB} = -253.23 \text{ kPa}$$

$$\tau_{AB} = -R\sin\phi$$
$$\tau_{AB} = -88.49 \text{ kPa}$$

PROBLEM 04.16

For the stressed soil element shown in Figure 04.15:

a) Calculate the major principal stress.
b) Calculate the minor principal stress.
c) Calculate the normal and shear stress on the plane *AE*.

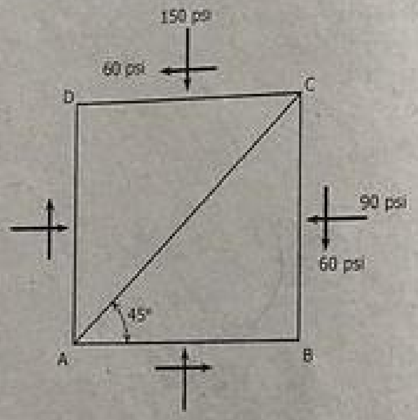
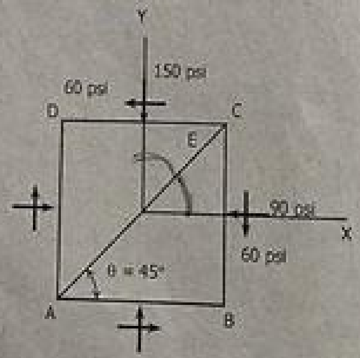


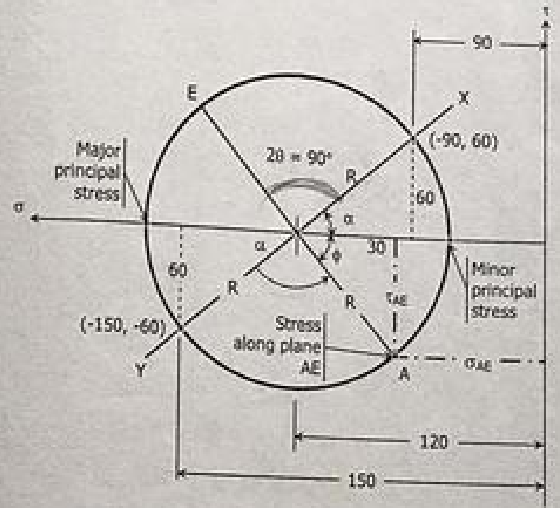
Figure 04.15

SOLUTION



$\sigma_x = -90$ psi
$\tau_{xy} = 60$ psi (-90, 60)

$\sigma_y = -150$ psi
$\tau_{yx} = -60$ psi (-150, -60)



From the Mohr's circle:

$R = \sqrt{30^2 + 60^2} = 67.082$

Major principal stress = 120 + R
Major principal stress = **187.08 psi**

Minor principal stress = 120 – R
Minor principal stress = **52.92 psi**

$\tan\alpha = 60/30$
$\alpha = 63.4349°$

$\phi = 180° - 90° - 63.4349°$
$\phi = 26.565°$

c) Normal and shear stress on the plane *AE*:
Normal stress:
$\sigma_{AE} = -(120 - 67.082 \cos 26.565°)$
$\sigma_{AE} = \mathbf{-60\ psi}$

Shearing stress:
$\tau_{AE} = -67.082 \sin 26.565$
$\tau_{AE} = \mathbf{-30\ psi}$

PROBLEM 04.17

For the soil element shown in Figure 04.16, determine the following

a) Maximum and minimum principal stresses
b) Normal and shear stresses on plane *CD*

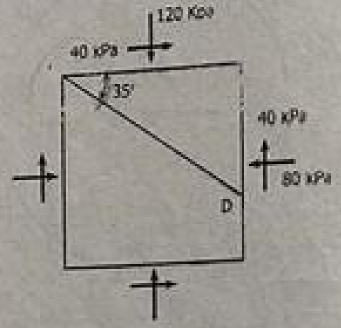
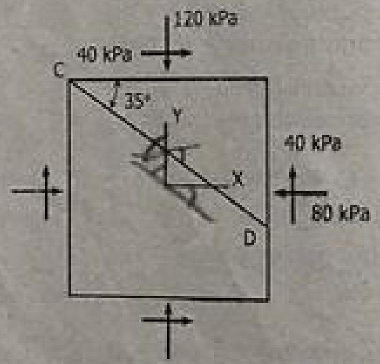


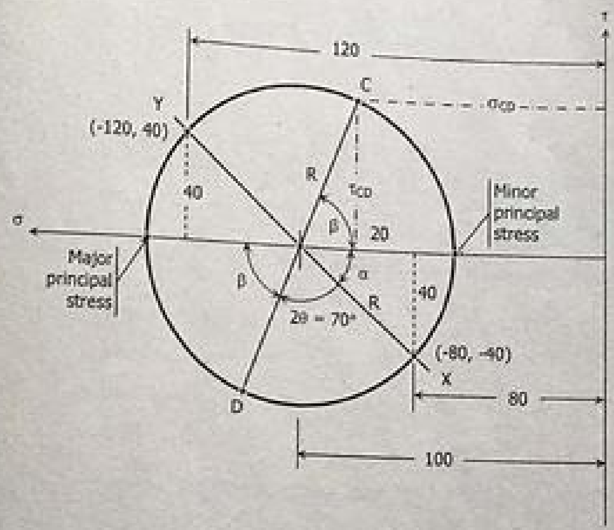
Figure 04.16

SOLUTION



σ_x = -80 kPa
τ_{xy} = -40 kPa (-80, -40)

σ_y = -120 kPa
τ_{yx} = 40 kPa (-120, 40)



a) Major and minor principal stresses:

$R = \sqrt{20^2 + 40^2}$
R = 44.721 kPa

Minor principal stress = 100 – R
Minor principal stress = **55.279 kPa**

Major principal stress = 100 + R
Major principal stress = **144.721 kPa**

b) Normal and shear stress on the plane *CD*:

$\tan \alpha = 40/20$
$\alpha = 63.435°$
$\beta = 180° - 70° - 63.435°$
$\beta = 46.565°$

Normal stress, $\sigma_{CD} = 100 - R \cos \beta$
Normal stress, $\sigma_{CD} = 100 - 44.721 \cos 46.565°$
Normal stress, σ_{CD} = **69.253 kPa**

Shearing stress, $\tau_{CD} = R \sin \beta$
Shearing stress, $\tau_{CD} = 44.721 \sin 46.565°$
Shearing stress, $\tau_{CD} = 32.474$ kPa

Chapter 05

Stress Distribution in Soil

5.1 STRESS CAUSED BY A POINT LOAD

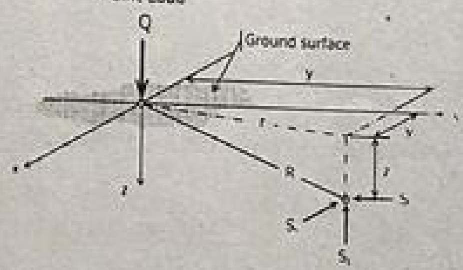


Figure 05.1 – Point load on ground surface

The Boussinesq equations for the stresses due to concentrated load are:

$$S_z = \frac{3Q}{2\pi}\frac{z^3}{R^5} = \frac{Q}{z^2}N_B \quad \text{Eq 5.1}$$

$$S_x = \frac{3Q}{2\pi}\left\{\frac{x^2 z}{R^5} + \frac{1-2\mu}{3}\left[\frac{1}{R(R+z)} - \frac{(2R+z)x^2}{R^3(R+z)^2} - \frac{z}{R^3}\right]\right\} \quad \text{Eq 5.2}$$

$$S_y = \frac{3Q}{2\pi}\left\{\frac{y^2 z}{R^5} + \frac{1-2\mu}{3}\left[\frac{1}{R(R+z)} - \frac{(2R+z)y^2}{R^3(R+z)^2} - \frac{z}{R^3}\right]\right\} \quad \text{Eq 5.3}$$

$$R = \sqrt{x^2 + y^2 + z^2} \quad \text{Eq 5.4}$$

where Q = load
μ = Poisson's ratio

Table 05.1 - Values of Boussinesq's Vertical Stress Coefficient, N_B

r/z	N_B	r/z	N_B	r/z	N_B
0.00	0.47746	2.50	0.00337	5.00	0.00014
0.25	0.41032	2.75	0.00223	5.25	0.00011
0.50	0.27332	3.00	0.00151	5.50	0.00009
0.75	0.15645	3.25	0.00105	5.75	0.00007
1.00	0.08440	3.50	0.00075	6.00	0.00006
1.25	0.04543	3.75	0.00054	6.25	0.00005
1.50	0.02508	4.00	0.00040	6.50	0.00004
1.75	0.01436	4.25	0.00030	6.75	0.00003
2.00	0.00854	4.50	0.00023	7.00	0.00003
2.25	0.00528	4.75	0.00018	7.25	0.00002

5.2 VERTICAL STRESS CAUSED BY A LINE LOAD

When a line load of infinite length having an intensity of q (kN/m) acts on the surface of a soil mass, the vertical stress, Δp, inside the soil mass is given by:

$$\Delta p = \frac{2qz^3}{\pi(x^2+z^2)^2} \qquad \text{Eq 5.5}$$

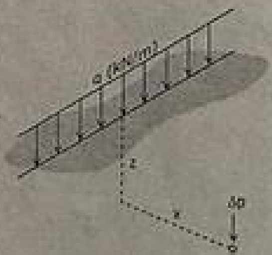


Figure 05.2 – Line load on ground surface

5.3 VERTICAL STRESS CAUSED BY A FLEXIBLE STRIP LOAD (FINITE WIDTH AND INFINITE LENGTH)

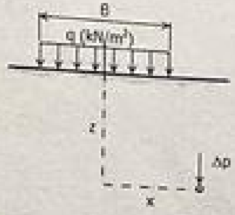


Figure 05.3 – Flexible strip load

$$\Delta p = \frac{q}{\pi}\left\{\tan^{-1}\left[\frac{z}{x-B/2}\right]-\tan^{-1}\left[\frac{z}{x+B/2}\right]-\frac{Bz[x^2-z^2-(B^2/4)]}{[x^2+z^2-(B^2/4)]+B^2z^2}\right\} \qquad \text{Eq 5.6}$$

5.4 VERTICAL STRESS CAUSED BY A RECTANGULARLY LOADED AREA

5.4.1 INCREASE IN PRESSURE BELOW THE CORNER OF RECTANGULAR AREA

The increase in vertical stress below the "corner" of a rectangular area of width B and length L is given as:

$$\Delta p = q_s \times I_i \qquad \text{Eq 5.7}$$

where $m = B/z$, $n = L/z$, and z = depth of the point

The value of I_i can be obtained from Figure 05.4, or from Table 05.2. It can also be computed from the following equation:

$$I_i = \frac{1}{4\pi}\left[\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2+m^2n^2+1}\left(\frac{m^2+n^2+2}{m^2+n^2+1}\right)+\tan^{-1}\left(\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2-m^2n^2+1}\right)\right] \qquad \text{Eq 5.8}$$

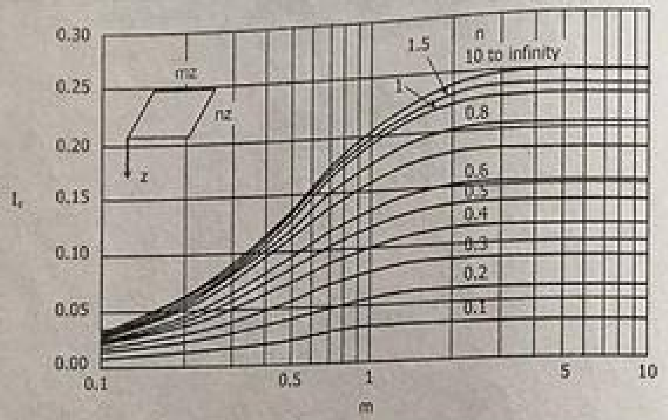


Figure 05.4 – Influence factor for calculating the vertical stress increase under the corner of a rectangle

To solve the stress at a point using Eq 5.7, divide the area into rectangles such that the point is at the corners of each rectangle.

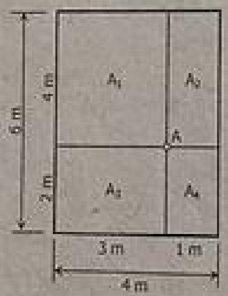


Figure 05.5 – Increase in stress under point A

Suppose that it is required to solve the increase in pressure below point A in the 4 m by 6 m footing shown in Figure 05.5. The area can be divided into four areas (A_1, A_2, A_3, and A_4). The values of Δp in the corner of each area is then computed. The increase in pressure is the sum of Δp's of each area.

Table 05.2 - Variation of I_r with m and n to be used in Eq 5.7

n	m									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.10	0.0047	0.0092	0.0132	0.0168	0.0198	0.0222	0.0242	0.0258	0.0270	0.0279
0.20	0.0092	0.0179	0.0259	0.0328	0.0387	0.0435	0.0473	0.0504	0.0528	0.0547
0.30	0.0132	0.0259	0.0374	0.0474	0.0559	0.0629	0.0686	0.0731	0.0766	0.0794
0.40	0.0168	0.0328	0.0474	0.0602	0.0711	0.0801	0.0873	0.0931	0.0977	0.1013
0.50	0.0198	0.0387	0.0559	0.0711	0.0840	0.0947	0.1034	0.1103	0.1158	0.1202
0.60	0.0222	0.0435	0.0629	0.0801	0.0947	0.1069	0.1168	0.1247	0.1311	0.1360
0.70	0.0242	0.0473	0.0686	0.0873	0.1034	0.1168	0.1277	0.1365	0.1436	0.1491
0.80	0.0258	0.0504	0.0731	0.0931	0.1103	0.1247	0.1365	0.1461	0.1537	0.1598
0.90	0.0270	0.0528	0.0766	0.0977	0.1158	0.1311	0.1436	0.1537	0.1618	0.1684
1.00	0.0279	0.0547	0.0794	0.1013	0.1202	0.1360	0.1491	0.1598	0.1684	0.1752
1.20	0.0293	0.0573	0.0832	0.1063	0.1263	0.1431	0.1570	0.1684	0.1777	0.1851
1.40	0.0301	0.0589	0.0856	0.1094	0.1300	0.1475	0.1620	0.1739	0.1836	0.1914
1.60	0.0306	0.0599	0.0871	0.1114	0.1324	0.1503	0.1652	0.1774	0.1874	0.1955
1.80	0.0309	0.0606	0.0880	0.1126	0.1339	0.1521	0.1672	0.1797	0.1899	0.1981
2.00	0.0311	0.0610	0.0887	0.1134	0.1350	0.1533	0.1686	0.1812	0.1915	0.1999
2.50	0.0314	0.0615	0.0895	0.1145	0.1363	0.1548	0.1704	0.1832	0.1937	0.2024
3.00	0.0315	0.0618	0.0898	0.1150	0.1368	0.1555	0.1711	0.1841	0.1947	0.2034
4.00	0.0316	0.0619	0.0901	0.1153	0.1372	0.1560	0.1717	0.1847	0.1954	0.2042
5.00	0.0316	0.0620	0.0901	0.1154	0.1374	0.1561	0.1718	0.1849	0.1956	0.2044
6.00	0.0316	0.0620	0.0902	0.1154	0.1374	0.1562	0.1719	0.1850	0.1957	0.2045

n	m									
	1.2	1.4	1.6	1.8	2	2.5	3	4	5	6
0.10	0.0293	0.0301	0.0306	0.0309	0.0311	0.0314	0.0315	0.0316	0.0316	0.0316
0.20	0.0573	0.0589	0.0599	0.0606	0.061	0.0615	0.0618	0.0619	0.062	0.062
0.30	0.0832	0.0856	0.0871	0.088	0.0887	0.0895	0.0898	0.0901	0.0901	0.0902
0.40	0.1063	0.1094	0.1114	0.1126	0.1134	0.1145	0.115	0.1153	0.1154	0.1154
0.50	0.1263	0.13	0.1324	0.1339	0.135	0.1363	0.1368	0.1372	0.1374	0.1374
0.60	0.1431	0.1475	0.1503	0.1521	0.1533	0.1548	0.1555	0.156	0.1561	0.1562
0.70	0.157	0.162	0.1652	0.1672	0.1686	0.1704	0.1711	0.1717	0.1718	0.1719
0.80	0.1684	0.1739	0.1774	0.1797	0.1812	0.1832	0.1841	0.1847	0.1849	0.185
0.90	0.1777	0.1836	0.1874	0.1899	0.1915	0.1937	0.1947	0.1954	0.1956	0.1957
1.00	0.1851	0.1914	0.1955	0.1981	0.1999	0.2024	0.2034	0.2042	0.2044	0.2045
1.20	0.1958	0.2028	0.2073	0.2103	0.2124	0.2151	0.2163	0.2172	0.2175	0.2176
1.40	0.2028	0.2102	0.2151	0.2184	0.2206	0.2236	0.225	0.226	0.2263	0.2264
1.60	0.2073	0.2151	0.2202	0.2237	0.2261	0.2294	0.2309	0.232	0.2324	0.2325
1.80	0.2103	0.2184	0.2237	0.2274	0.2299	0.2334	0.235	0.2362	0.2366	0.2367
2.00	0.2124	0.2206	0.2261	0.2299	0.2325	0.2361	0.2378	0.2391	0.2395	0.2397
2.50	0.2151	0.2236	0.2294	0.2334	0.2361	0.2401	0.242	0.2434	0.2439	0.2441
3.00	0.2163	0.225	0.2309	0.235	0.2378	0.242	0.2439	0.2455	0.2461	0.2463
4.00	0.2172	0.226	0.232	0.2362	0.2391	0.2434	0.2455	0.2473	0.2479	0.2482
5.00	0.2175	0.2263	0.2324	0.2366	0.2395	0.2439	0.2461	0.2479	0.2486	0.2489
6.00	0.2176	0.2264	0.2325	0.2367	0.2397	0.2441	0.2463	0.2482	0.2489	0.2492

5.4.2 INCREASE IN PRESSURE BELOW THE CENTER OF RECTANGULAR AREA

The increase in vertical stress below the "center" of a rectangular area of width B and length L is given as:

$$\Delta p = q_o \times I_4 \qquad \text{Eq 5.9}$$

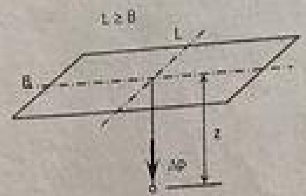


Figure 05.6 – Increase in stress below the center of a rectangular area

The value of I_4 can be obtained using Eq 5.10 or Table 05.3.

$$I_4 = \frac{2}{\pi}\left[\frac{m_1 n_1}{\sqrt{1+m_1^2+n_1^2}}\,\frac{1+m_1^2+2n_1^2}{(1+n_1^2)(m_1^2+n_1^2)} + \sin^{-1}\left(\frac{m_1}{\sqrt{m_1^2+n_1^2}\sqrt{1+n_1^2}}\right)\right] \qquad \text{Eq 5.10}$$

where $m_1 = L/b$, $n_1 = z/b$, and $b = B/2$

Table 05.3 - Variation of I_4 with m_1 and n_1 to be used in Eq 5.9

n_1	m_1									
	1	2	3	4	5	6	7	8	9	10
0.20	0.9943	0.9966	0.9967	0.9967	0.9968	0.9968	0.9968	0.9968	0.9968	0.9968
0.40	0.9604	0.9757	0.9769	0.9772	0.9772	0.9773	0.9773	0.9773	0.9773	0.9773
0.60	0.8916	0.9318	0.9357	0.9364	0.9367	0.9367	0.9368	0.9368	0.9368	0.9368
0.80	0.7997	0.8703	0.8784	0.8801	0.8806	0.8808	0.8809	0.8809	0.8810	0.8810
1.00	0.7009	0.7998	0.8136	0.8167	0.8176	0.8180	0.8181	0.8182	0.8182	0.8183
1.20	0.6064	0.7274	0.7478	0.7527	0.7542	0.7548	0.7551	0.7552	0.7553	0.7553
1.40	0.5220	0.6576	0.6849	0.6919	0.6942	0.6951	0.6955	0.6957	0.6958	0.6958
1.60	0.4492	0.5927	0.6266	0.6359	0.6391	0.6404	0.6410	0.6413	0.6415	0.6416
1.80	0.3877	0.5337	0.5736	0.5853	0.5896	0.5913	0.5921	0.5925	0.5927	0.5928
2.00	0.3361	0.4807	0.5254	0.5398	0.5451	0.5474	0.5484	0.5490	0.5493	0.5495
3.00	0.1789	0.2929	0.3460	0.3726	0.3837	0.3891	0.3918	0.3933	0.3942	0.3947
4.00	0.1081	0.1901	0.2410	0.2694	0.2848	0.2932	0.2979	0.3007	0.3023	0.3034
5.00	0.0716	0.1312	0.1739	0.2017	0.2189	0.2293	0.2358	0.2398	0.2423	0.2440
6.00	0.0507	0.0952	0.1301	0.1562	0.1724	0.1836	0.1915	0.1966	0.1999	0.2022
7.00	0.0377	0.0719	0.1003	0.1223	0.1386	0.1503	0.1586	0.1642	0.1683	0.1711
8.00	0.0291	0.0561	0.0794	0.0984	0.1133	0.1248	0.1330	0.1391	0.1436	0.1469
9.00	0.0231	0.0449	0.0642	0.0807	0.0941	0.1047	0.1129	0.1192	0.1240	0.1276
10.00	0.0188	0.0367	0.0529	0.0671	0.0791	0.0889	0.0968	0.1030	0.1079	0.1117

5.5 VERTICAL STRESS CAUSED BY A SQUARE AND CONTINUOUS FOOTING

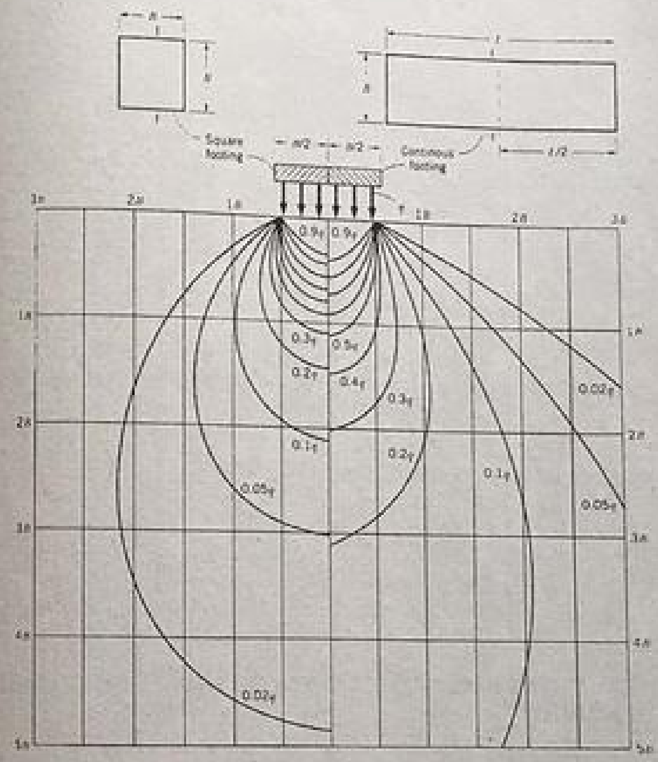


Figure 05.7 – Pressure isobars based on Boussinesq equation for square and continuous footings used to find the pressures along line 1-1

5.6 APPROXIMATE METHOD FOR RECTANGULAR LOADS

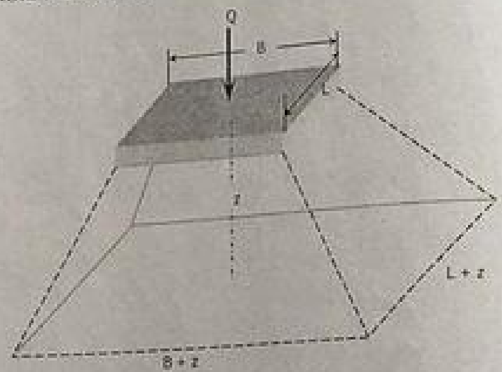


Figure 05.8 – Dispersion of load for approximate increase in vertical stress under a rectangle

The approximate increase in vertical stress is:

$$\Delta p = \frac{Q}{(B+z)(L+z)} \qquad \text{Eq 5.11}$$

5.7 VERTICAL STRESS BELOW THE CENTER OF A UNIFORMLY LOADED CIRCULAR AREA

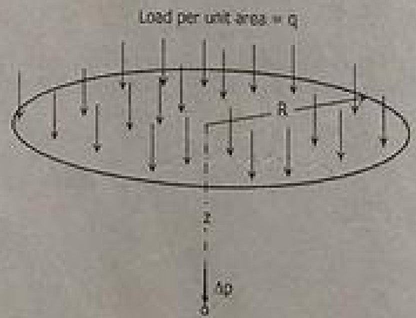


Figure 05.9 – Increase in stress below the center of a circular area

$$\Delta p = q\left\{1 - \frac{1}{[(R/z)^2 + 1]^{3/2}}\right\} \qquad \text{Eq 5.12}$$

$$\Delta p = q \times I_5 \qquad \text{Eq 5.13}$$

Table 05.4 - Variation of I_5 with z/R be used in Eq 5.13

z/R	I_5	z/R	I_5	z/R	I_5	z/R	I_5
0	1.00000	2.6	0.18693	5.6	0.04599	8.6	0.01994
0.02	0.99999	2.7	0.17537	5.7	0.04445	8.7	0.01950
0.04	0.99994	2.8	0.16479	5.8	0.04299	8.8	0.01906
0.06	0.99979	2.9	0.15509	5.9	0.04159	8.9	0.01864
0.08	0.99949	3	0.14619	6	0.04027	9	0.01824
0.1	0.99901	3.1	0.13799	6.1	0.03900	9.1	0.01784
0.2	0.99246	3.2	0.13044	6.2	0.03779	9.2	0.01746
0.3	0.97627	3.3	0.12347	6.3	0.03664	9.3	0.01710
0.4	0.94877	3.4	0.11702	6.4	0.03553	9.4	0.01674
0.5	0.91056	3.5	0.11104	6.5	0.03448	9.5	0.01639
0.6	0.86381	3.6	0.10550	6.6	0.03347	9.6	0.01606
0.7	0.81141	3.7	0.10035	6.7	0.03251	9.7	0.01573
0.8	0.75622	3.8	0.09556	6.8	0.03158	9.8	0.01542
0.9	0.70063	3.9	0.09109	6.9	0.03070	9.9	0.01511
1	0.64645	4	0.08692	7	0.02985	10	0.01481
1.1	0.59487	4.1	0.08303	7.1	0.02903	10.1	0.01453
1.2	0.54662	4.2	0.07938	7.2	0.02825	10.2	0.01425
1.3	0.50203	4.3	0.07597	7.3	0.02750	10.3	0.01397
1.4	0.46118	4.4	0.07276	7.4	0.02678	10.4	0.01371
1.5	0.42397	4.5	0.06975	7.5	0.02609	10.5	0.01345
1.6	0.39020	4.6	0.06692	7.6	0.02542	10.6	0.01320
1.7	0.35964	4.7	0.06425	7.7	0.02478	10.7	0.01296
1.8	0.33201	4.8	0.06174	7.8	0.02416	10.8	0.01272
1.9	0.30704	4.9	0.05937	7.9	0.02356	10.9	0.01249
2	0.28446	5	0.05713	8	0.02299	11	0.01227
2.1	0.26403	5.1	0.05502	8.1	0.02243	11.1	0.01205
2.2	0.24552	5.2	0.05302	8.2	0.02190	11.2	0.01184
2.3	0.22873	5.3	0.05112	8.3	0.02139	11.3	0.01163
2.4	0.21347	5.4	0.04932	8.4	0.02089	11.4	0.01143
2.5	0.19959	5.5	0.04761	8.5	0.02041	11.5	0.01124

5.8 INFLUENCE CHART FOR VERTICAL PRESSURE

Newmark (1942) presented an influence chart based on Boussinesq's theory that can be used to determine the vertical pressure at any point below a uniformly loaded flexible area of any shape. This chart is based on Eq 5.12. The influence value of the chart is $1/N$, where N is equal to the number of elements. The figure shows 200 elements, thus, the influence value is $i = 0.005$.

$$\Delta p = iqN \quad \text{Eq 5.14}$$

where i is the influence value, q is the pressure on the loaded area, and N is the number of elements of the chart enclosed by the plan of the loaded area.

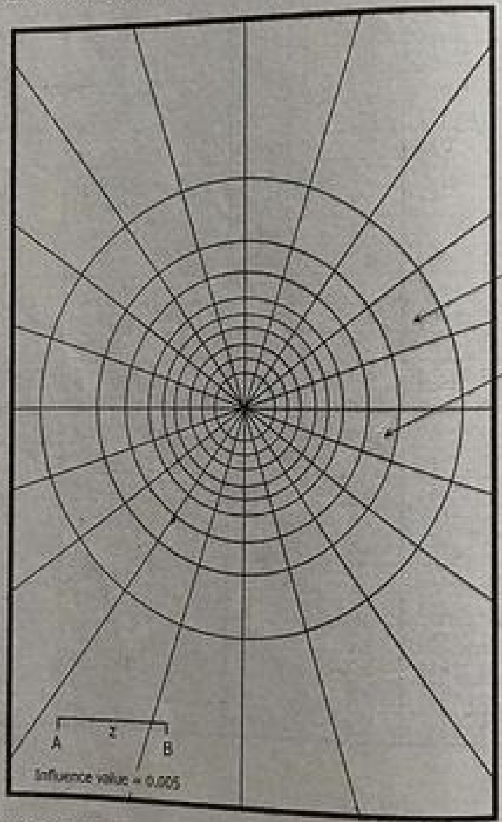


Figure 05.10 – Newmark's influence chart for vertical pressure based on Boussineq's theory

The procedure for obtaining the value of N to be used in Eq 5.14:

1. Determine the depth z below the loaded area where the stress increase is desired.
2. Plot the plan of the loaded area with a scale of z equal to the unit length AB of the chart. (i.e. if depth $z = 4$ m, then $AB = 4$ m)
3. Place the plan on the influence chart in a manner such that the point below which the stress is required is located at the center of the chart.
4. Count the number of elements (N) of the chart enclosed by the plan of the loaded area. If certain segments are not fully covered, you can estimate what fraction is covered.

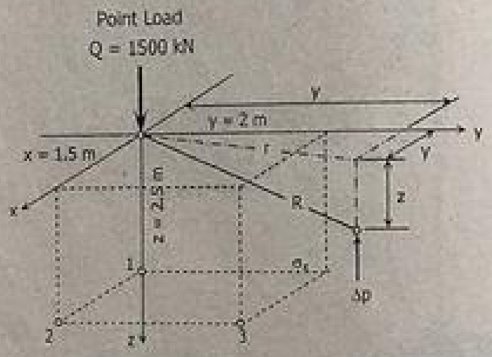
ILLUSTRATIVE PROBLEMS

PROBLEM 05.1

A concentrated load of 1,500 kN is applied at the ground surface at point *A* whose coordinate is (0, 0, 0).

a) Determine the vertical stress, in kN, at a point 2.5 m directly below *A*.
b) Determine the vertical stress, in kN, at a point whose coordinate is (1.5, 0, 2.5).
c) Determine the vertical stress, in kN, at a point whose coordinate is (1.5, 2, 2.5).

SOLUTION



Part *a*:

$x = 0;\ \ y = 0;\ \ z = 2.5$ m

$R = \sqrt{0^2 + 0^2 + 2.5^2}$

$R = 2.5$

$$\Delta p = \frac{3(1500)}{2\pi}\frac{2.5^3}{2.5^5}$$

$\Delta p = 114.6$ kN

Part *b*:

$x = 1.5;\ \ y = 0;\ \ z = 2.5$

$R = \sqrt{1.5^2 + 0^2 + 2.5^2}$

$R = 2.915$ m

$$\Delta p = \frac{3(1500)}{2\pi}\frac{2.5^3}{2.915^5}$$

$\Delta p = 53.17$ kN

Part *c*:

$x = 1.5;\ \ y = 2;\ \ z = 2.5$

$R = \sqrt{1.5^2 + 2^2 + 2.5^2}$

$R = 3.5355$

$$\Delta p = \frac{3(1500)}{2\pi}\frac{2.5^3}{3.5355^5}$$

$\Delta p = 20.26$ kN

PROBLEM 05.2

A line load and a point load acting on the ground surface is shown in Figure 05.11. Determine the increase in vertical stress at point *A*.

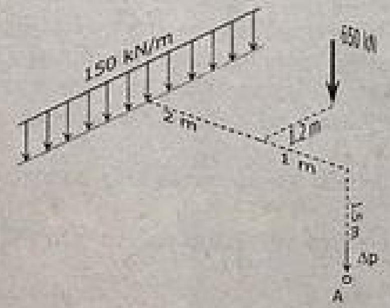


Figure 05.11

SOLUTION

Using Eq 5.1 for point load and Eq 5.5:

$\Delta p = (\Delta p)_{\text{line load}} + (\Delta p)_{\text{point load}}$

$$\Delta p = \frac{2qz^3}{\pi(x^2 + z^2)^2} + \frac{3Q}{2\pi}\frac{z^3}{R^5}$$

$q = 150$ kN/m

$x = 3$ m

$z = 1.5$ m

$Q = 650$ kN

$R = \sqrt{1.2^2 + 1^2 + 1.5^2}$

$R = 2.1656$ m

$$\Delta p = \frac{2(150)(1.5)^3}{\pi(3^2 + 1.5^2)^2} + \frac{3(650)}{2\pi}\frac{1.5^3}{2.1656^5}$$

$\Delta p = 2.5465 + 21.991$

$\Delta p = \mathbf{24.537\ kPa}$

PROBLEM 05.3

A rectangular concrete slab, 3 m × 4.5 m shown in Figure 05.12, rests on the surface of a soil mass. The load on the slab is 1620 kN.

a) Determine the soil stress below the slab using Eq 5.7

b) Determine the vertical stress increase at point *A* using Eq 5.7

c) Determine the vertical stress increase at point *A* using Eq 5.9

d) Determine the vertical stress increase at point *B* using Eq 5.7

e) Determine the vertical stress increase at points *A* and *B* using Eq 5.11

f) Determine the vertical stress increase at points *B* and *C* using pressure isobars based on Boussinesq equation for continuous footings shown in Figure 05.7

g) Determine the vertical stress increase at point *A* using Newmark's influence chart.

h) Determine the vertical stress increase at point *B* using Newmark's influence chart.

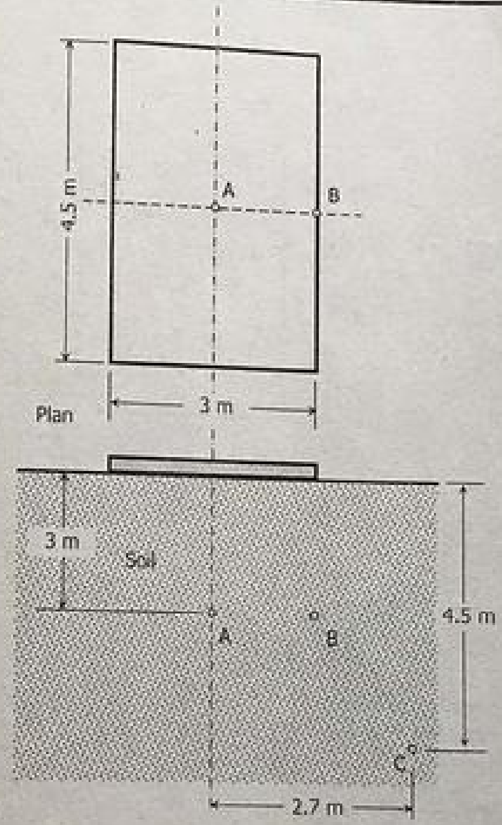
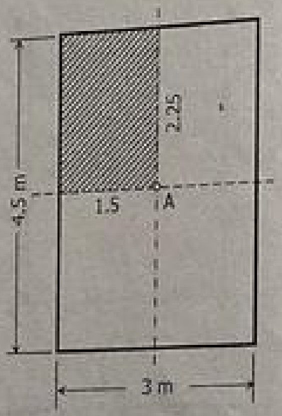


Figure 05.12

SOLUTION

a) $q_s = \dfrac{1620}{3(4.5)} = 120$ kPa

b) Divide the area into small rectangles such that A is at the corner of each rectangle.



$B = 2.25$
$L = 1.5$
$z = 3$ m
$m = B/z = 0.75$
$n = L/z = 0.5$

From Table 05.2:
For $m = 0.7$ and $n = 0.5$, $I_z = 0.1034$
For $m = 0.8$ and $n = 0.5$, $I_z = 0.1103$

$$I_z = \frac{0.1034 + 0.1103}{2} = 0.107$$

$\Delta p = q_s\, I_z \times 4$
$\Delta p = 120 \times 0.107 \times 4$
$\Delta p = \mathbf{51.36\ kPa}$

Note: Using Eq 5.8 for $m = 0.75$ and $n = 0.5$, $I_z = 0.1071$

c) $\Delta p = q_s\, I_4$
$b = B/2$
$b = 3/2$
$b = 1.5$

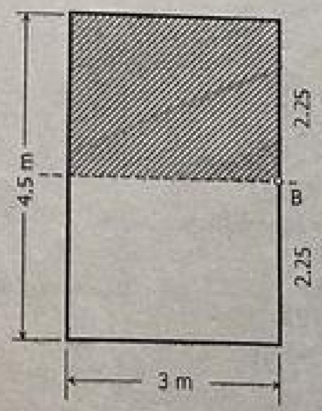
$m_1 = L/b$
$m_1 = 4.5/1.5$
$m_1 = 3$

$n_1 = z/b$
$n_1 = 3/1.5$
$n_1 = 2$

From Table 05.3, $I_4 = 0.5254$

$\Delta p = 120 \times 0.5254$
$\Delta p = \mathbf{63.048\ kPa}$

d) Divide the area into small rectangles such that point B is at the corner of each rectangle.



$B = 3$ m
$L = 2.5$m
$z = 3$ m
$m = B/z = 1$
$n = L/z = 0.75$

From Table 05.2:
For $m = 1$ and $n = 0.7$, $I_z = 0.1491$
For $m = 1$ and $n = 0.8$, $I_z = 0.1598$

$$I_z = \frac{0.1491 + 0.1598}{2} = 0.1545$$

$\Delta p = q_o I_z \times 2$
$\Delta p = 120 \times 0.1545 \times 2$
$\Delta p = 37.08$ kPa

Note: Using Eq 5.8 for $m = 1$ and $n = 0.75$, $I_z = 0.1547$

e) $$\Delta p = \frac{Q}{(B+z)(L+z)}$$

$$\Delta p = \frac{1620}{(3+3)(4.5+3)}$$

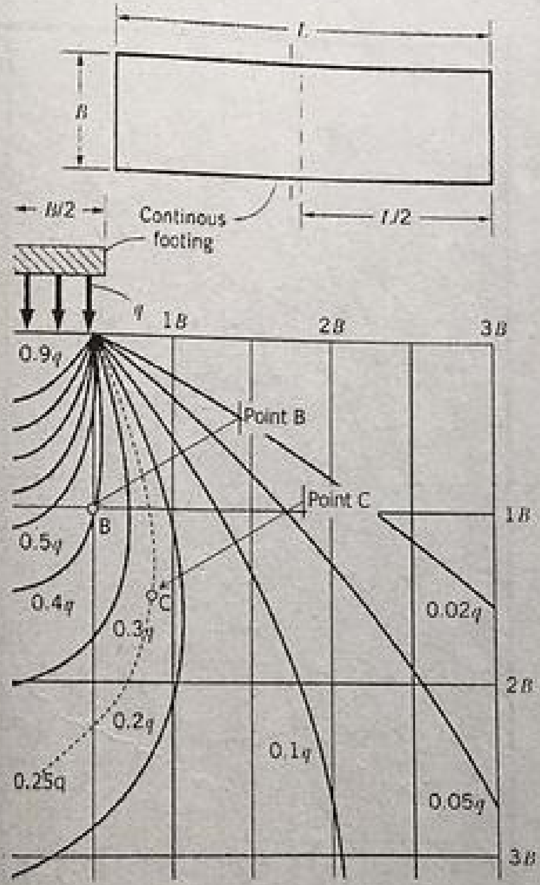
$\Delta p = 36$ kPa

f) At point B:
$x = 1.5/3 = 0.5B$
$z = 3/3 = 1B$

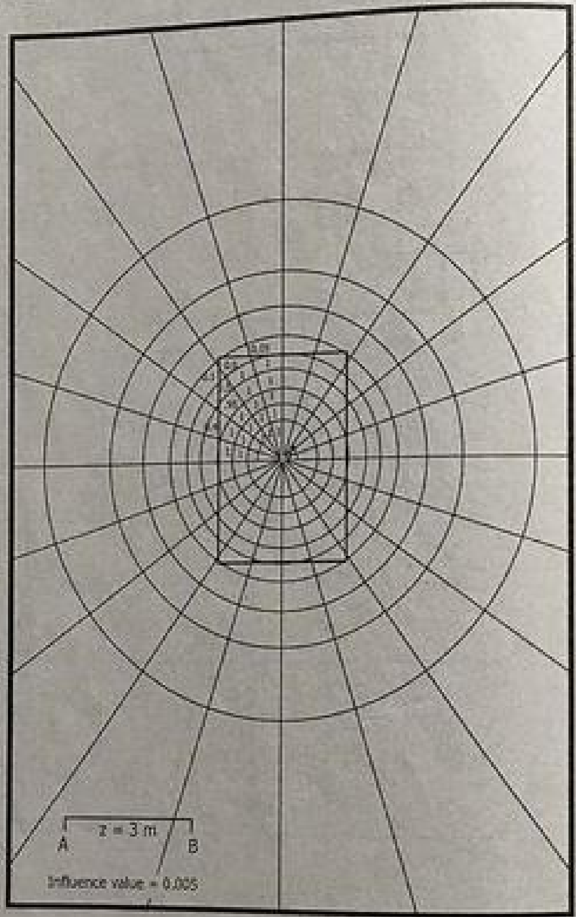
At point C:
$x = 2.7/3 = 0.9B$
$z = 4.5/3 = 1.5B$

From the figure shown below:
At point B:
$\Delta p = 0.4q$
$\Delta p = 0.4(120)$
$\Delta p = 48$ kPa

At point C:
$\Delta p = 0.25q$
$\Delta p = 0.25(120)$
$\Delta p = 30$ kPa



g) Vertical stress at *A* using Newmark's influence chart.
$z = 3$ m



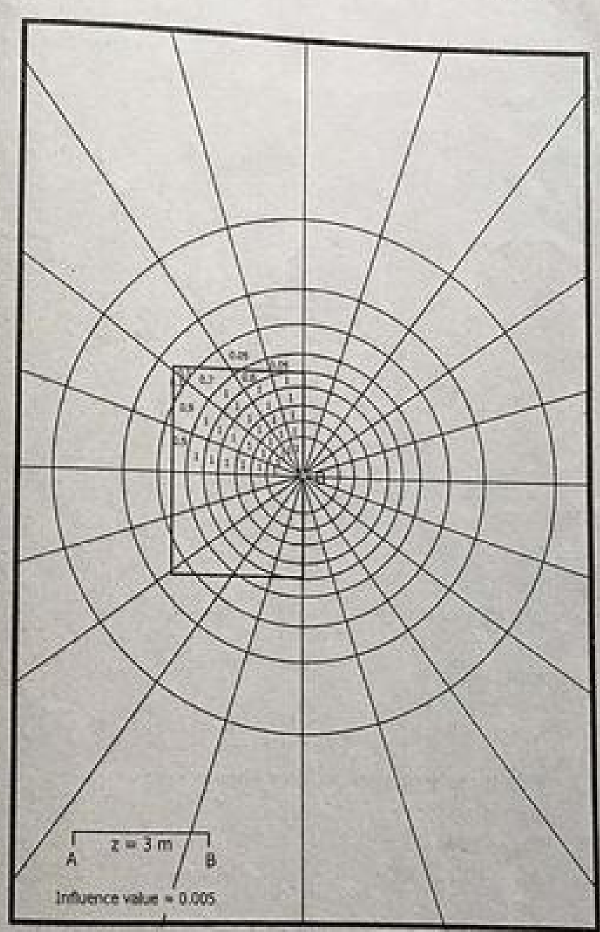
From the chart, the number of elements *N* is about $21.6 \times 4 = 86.4$

$\Delta p = iqN$

$\Delta p = 0.005(120)(86.4)$

$\Delta p =$ **51.84 kPa** (very much close to the answer in part *b*)

h) Vertical stress at *B* using Newmark's influence chart.
$z = 3$ m



From the chart, the number of elements *N* is about $31 \times 2 = 62$

$\Delta p = iqN$

$\Delta p = 0.005(120)(62)$

$\Delta p =$ **37.2 kPa** (very much close to the answer in part *d*)

PROBLEM 05.4

A square footing is loaded as shown in Figure 05.13. The center of the base of footing is at coordinate (0, 0, 0). Determine the increase in vertical stress at a point whose coordinate is (3, 0, 4) using pressure isobars based on Boussinesq equation for square footings shown in Figure 05.7.

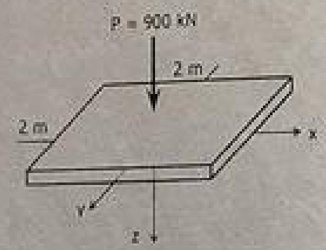


Figure 05.13

SOLUTION

$q = P/A$

$q = 900/(2 \times 2)$

$q = 225$ kPa

At point (3, 0, 4):

$x = 3\text{ m} = 1.5B$

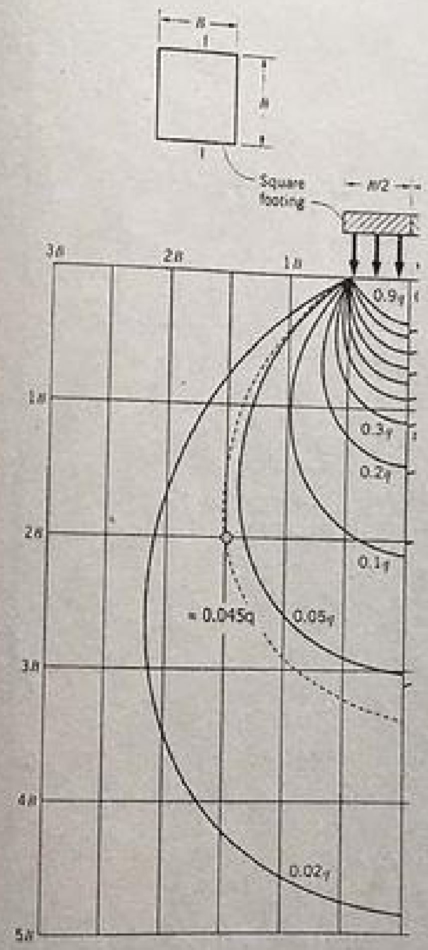
$z = 4\text{ m} = 2B$

From the pressure isobars shown below:

$\Delta p = 0.045q$

$\Delta p = 0.045(225)$

$\Delta p = \mathbf{10.125\ kPa}$



PROBLEM 05.5

Determine the increase in vertical pressure at point 2 meters below point C in the loaded area shown in Figure 05.14. The foundation applies a vertical stress of 250 kPa on the soil surface.

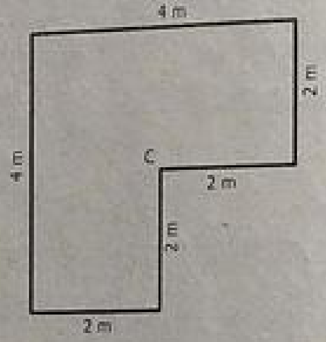


Figure 05.14

SOLUTION

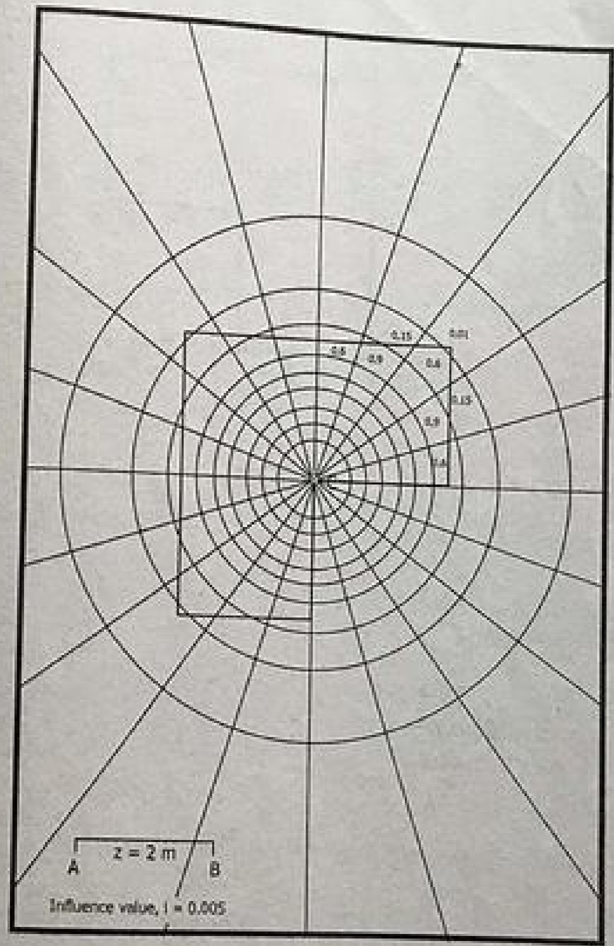
$N = 34.91 \times 3$

$N = 104.73$

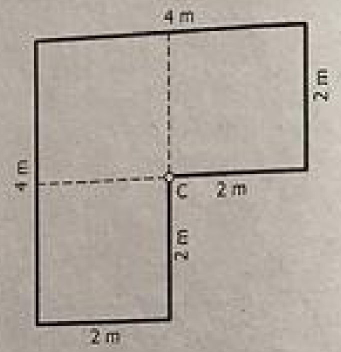
$\Delta p = iqN$

$\Delta p = 0.005(250)(104.73)$

$\Delta p = 130.91 \text{ kPa}$



Using Eq 5.7, divide the area into three rectangles 2 m × 2 m.



$z = 2$ m
$m = B/z$
$m = 2/2 = 1$
$n = L/z$
$n = 2/2 = 1$

From Table 05.2:
$I_z = 0.1752$

$\Delta p = q \times I_z \times 3$
$\Delta p = 250(0.1752)(3)$
$\Delta p = \mathbf{131.4\ kPa}$

Chapter 06

Compressibility of Soil

The increase in stress caused by foundation and other loads compresses a soil layer. This compression is caused by (1) deformation of soil particles, (2) relocations of soil particles, and (3) expulsion of water or air from the void spaces.

Soil settlement may be divided into three categories:

1. **Immediate settlement** – caused by the elastic deformation of dry, moist, and saturated soils, without any change in moisture content.
2. **Primary consolidation settlement** – caused by a volume change in saturated cohesive soils due to expulsion of water that occupies the void spaces.
3. **Secondary consolidation settlement** – caused by plastic adjustment of soil fabrics. It is an additional form of compression that occurs at constant effective stress.

6.1 SETTLEMENT FROM ONE DIMENSIONAL PRIMARY CONSOLIDATION

6.1.1 BASIC SETTLEMENT FORMULA

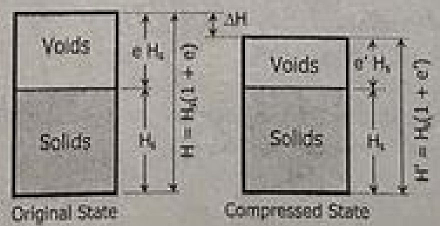


Figure 06.1

$$H = H_s(1 + e)$$

$$H_s = \frac{H}{1+e}$$

$$H' = H_s(1 + e')$$

$$H' = \frac{H}{1+e}(1 + e')$$

$$\Delta H = H - H'$$

$$\Delta H = H - \frac{H}{1+e}(1 + e')$$

$$\Delta H = H\frac{1+e-(1+e')}{1+e} = H\frac{e-e'}{1+e}$$

$$\Delta H = \frac{H(e_o - e')}{1+e_o} = H\frac{\Delta e}{1+e_o} \qquad \text{Eq. 6.1}$$

where:
- H = thickness of stratum
- e_o = void ratio before the vertical load is applied
- e' = void ratio after the vertical load is applied

6.1.2 PRIMARY CONSOLIDATION SETTLEMENT OF NORMALLY CONSOLIDATED FINE-GRAINED SOILS

$$\Delta H = H\frac{C_c}{1+e_o}\log\frac{p_f}{p_o} \qquad \text{Eq. 6.2}$$

where:
- H = thickness of stratum
- C_c = compression index
- e_o = initial void ratio
- p_o = initial vertical effective soil stress
- p_f = final vertical effective soil stress
- $p_f = p_o + \Delta p$

6.1.3 PRIMARY CONSOLIDATION SETTLEMENT OF OVERCONSOLIDATED FINE-GRAINED SOILS

When $p_f < p_c$:

$$\Delta H = H\frac{C_s}{1+e_o}\log\frac{p_f}{p_o} \qquad \text{Eq. 6.3}$$

When $p_f > p_c$

$$\Delta H = H\frac{C_s}{1+e_o}\log\frac{p_c}{p_o} + H\frac{C_c}{1+e_o}\log\frac{p_f}{p_c} \qquad \text{Eq. 6.4}$$

where:
- C_s = swell index
- p_c = preconsolidation pressure

6.2 OVERCONSOLIDATION RATIO, OCR

$$OCR = \frac{p_c}{p_o} \qquad \text{Eq. 6.5}$$

- p_c = presconsolidation stress (past maximum vertical effective stress)
- p_o = overburden effective stress (current vertical effective stress)

If OCR = 1, the soil normally consolidated soil.

6.3 COMPRESSION INDEX, C_c:

6.3.1 SKEMPTON:

For remolded clay:

$$C_c = 0.007(LL - 7\%) \qquad \text{Eq. 6.6}$$

For undisturbed clay:

$$C_c = 0.009(LL - 10\%) \qquad \text{Eq. 6.7}$$

6.3.2 RENDON-HERREO:

$$C_c = 0.141\, G^{1.2}\left(\frac{1+e_o}{G}\right)^{2.38} \qquad \text{Eq. 6.8}$$

6.3.3 NISHIDA:

All clays

$$C_c = 1.15(e_o - 0.27) \qquad \text{Eq. 6.9}$$

6.4 SWELL INDEX, C_S:

The swell index is smaller in magnitude than the compression index. In most cases.

$$C_s \cong \tfrac{1}{5}C_c \text{ to } \tfrac{1}{10}C_c \qquad \text{Eq. 6.10}$$

6.4.1 NAGARAJ AND MURTY:

$$C_s = 0.0463\frac{LL\%}{100} \times G \qquad \text{Eq. 6.11}$$

6.5 SETTLEMENT FROM SECONDARY CONSOLIDATION

Secondary consolidation can be calculated as:

$$\Delta H_s = C'_\alpha\, H \log\left(\frac{t_2}{t_1}\right) \qquad \text{Eq. 6.12}$$

$$C'_\alpha = \frac{C_\alpha}{1+e_p} \qquad \text{Eq. 6.13}$$

$$C_\alpha = \frac{\Delta e}{\log t_2 - \log t_1} \qquad \text{Eq. 6.14}$$

where

- C_α = secondary compression index
- Δe = change in void ratio
- t_1 = time for completion of primary settlement
- t_2 = time after completion of primary settlement, where settlement is required
- e_p = void ratio at the end of primary consolidation
- $e_p = e_o - \Delta e$
- H = thickness of clay layer

6.6 CALCULATION OF CONSOLIDATION SETTLEMENT UNDER A FOUNDATION

In Chapter 05, we have seen that the increase in vertical stress caused by a load applied over a limited area decreases by depth. To estimate the one-dimensional settlement of a foundation, we can use the equations of this section. However, the increase in of stress Δp should be the average increase in pressure below the center of the foundation.

Assuming the pressure increase varies parabolically, the average pressure may be estimated as:

$$\Delta p_{av} = \frac{\Delta p_t + 4\Delta p_m + \Delta p_b}{6} \qquad \text{Eq. 6.15}$$

where Δp_t = increase in pressure at the top of the layer
Δp_m = increase in pressure at the middle of the layer
Δp_b = increase in pressure at the bottom of the layer

6.7 TIME RATE OF CONSOLIDATION

The time t required to achieve a certain degree of consolidation U is evaluated as a function of the shortest drainage path within the compressible zone H_{dr}, coefficient of consolidation C_v, and the dimensionless time factor T_v.

$$t = T_v \frac{(H_{dr})^2}{C_v} \qquad \text{Eq. 6.16}$$

where
H_{dr} = one-half the thickness of the drainage layer if drainage occurs at the top and bottom of the layer (*two-way drainage*)
H_{dr} = thickness of the drainage layer if drainage occurs at the top or bottom only (*one-way layer*)

Table 06.1: Variation of T_v with U

U%	T_v	U%	T_v	U%	T_v	U%	T_v
1	-	26	0.0531	51	0.204	76	0.493
2	0.00008	27	0.0572	52	0.212	77	0.511
3	0.00030	28	0.0615	53	0.221	78	0.529
4	0.00071	29	0.0660	54	0.230	79	0.547
5	0.00126	30	0.0707	55	0.239	80	0.567
6	0.00196	31	0.0754	56	0.248	81	0.588
7	0.00283	32	0.0803	57	0.257	82	0.610
8	0.00502	33	0.0855	58	0.267	83	0.633
9	0.00636	34	0.0907	59	0.276	84	0.658
10	0.00785	35	0.0962	60	0.286	85	0.684
11	0.0096	36	0.102	61	0.297	86	0.712
12	0.0113	37	0.107	62	0.307	87	0.742
13	0.0133	38	0.113	63	0.318	88	0.774
14	0.0154	39	0.119	64	0.329	89	0.809
15	0.0177	40	0.126	65	0.304	90	0.848
16	0.0201	41	0.132	66	0.352	91	0.891
17	0.0227	42	0.138	67	0.364	92	0.938
18	0.0254	43	0.145	68	0.377	93	0.993
19	0.0283	44	0.152	69	0.390	94	1,055
20	0.0314	45	0.159	70	0.403	95	1,129
21	0.0346	46	0.417	71	0.417	96	1,219
22	0.0380	47	0.173	72	0.431	97	1.336
23	0.0415	48	0.181	73	0.446	98	1.500
24	0.0452	49	0.188	74	0.461	99	1.781
25	0.0491	50	0.197	75	0.477	100	infinity

The approximate values of time factor T_v are:

$$\text{For } U = 0 \text{ to } 60\%, \quad T_v = \frac{\pi}{4}\left(\frac{U\%}{100}\right)^2 \qquad \text{Eq. 6.17}$$

$$\text{For } U > 60\%, \quad T_v = 1.781 - 0.933 \log(100 - U\%) \qquad \text{Eq. 6.18}$$

The time factor T_v provides a useful expression to estimate the settlement in the field from the results of a laboratory consolidation.

$$\frac{t_{field}}{t_{lab}} = \frac{(H_{dr\,field})^2}{(H_{dr\,lab})^2} \qquad \text{Eq. 6.19}$$

$$\text{Also,} \quad \frac{t_1}{t_2} = \frac{U_1^{\,2}}{U_2^{\,2}} \qquad \text{Eq. 6.20}$$

where t_1 = time to reach a consolidation of U_1%
t_2 = time to reach a consolidation of U_2%

The degree of consolidation at a distance z at any time is:

$$U_z = 1 - \frac{p_{w_z}}{p_{w_o}} \qquad \text{Eq. 6.21}$$

The average degree of consolidation for the entire depth of layer at any time is:

$$U_z = \frac{\Delta H_t}{\Delta H_{max}} \qquad \text{Eq. 6.22}$$

where p_{w_z} = excess pore pressure at time t
p_{w_o} = initial excess pore water pressure
ΔH_t = settlement of the layer at time t
ΔH_{max} = ultimate settlement of the layer from primary consolidation

@seismicisolation

6.8 COEFFICIENT OF CONSOLIDATION

$$\text{Root time method,} \quad C_v = \frac{0.848(H_{dr})^2}{t_{90}} \qquad \text{Eq. 6.23}$$

$$\text{Log time method,} \quad C_v = \frac{0.197(H_{dr})^2}{t_{50}} \qquad \text{Eq. 6.24}$$

where t_{90} = time for 90% consolidation ($\sqrt{t}$ curve)
t_{50} = time for 50% consolidation (log t curve)

6.9 COEFFICIENT OF VOLUME COMPRESSIBILITY, m_v

$$m_v = \frac{a_v}{1+e_{ave}} = \frac{(e_o - e)/\Delta p}{1+e_{ave}} \qquad \text{Eq. 6.25}$$

$$e_{ave} = \frac{e+e_o}{2} \qquad \text{Eq. 6.26}$$

The hydraulic conductivity of the layer for the loading range is:

$$k = C_v \, m_v \, \gamma_w \qquad \text{Eq. 6.27}$$

where e_o = initial void ratio
e = final void ratio
Δp = rise in pressure

6.10 IMMEDIATE SETTLEMENT

Immediate or elastic settlement of foundations occurs directly after application of a load, without change in moisture content. This depends on the flexibility of the foundation and the type of material on which it is resting.

Immediate settlement of foundations resting on the ground surface of an elastic material of finite thickness is given by:

$$\Delta H_i = pB\,\frac{1-\mu^2}{E}\,I_f \qquad \text{Eq. 6.28}$$

where:
p = net pressure applied in kPa or psf
B = width or diameter of foundation in m or feet
μ = Poisson's ratio
E = modulus of elasticity of soil in kPa or psf
I_f = influence factor (dimensionless)

The influence factor for the *corner* of a flexible rectangular footing given as:

$$I_f = \frac{1}{\pi}\left[m_1 \ln\left(\frac{1+\sqrt{1+m_1^2}}{m_1}\right) + \ln\left(m_1 + \sqrt{1+m_1^2}\right)\right] \qquad \text{Eq. 6.29}$$

Table 06.2: Influence Factors for Foundations

Shape	m_1	I_f		
		Flexible		Rigid
		Center	Corner	
Circle	-	1.00	0.64	0.79
Rectangle	1	1.12	0.56	0.88
	1.5	1.36	0.68	1.07
	2	1.53	0.77	1.21
	3	1.78	0.89	1.42
	5	2.10	1.05	1.70
	10	2.54	1.27	2.10
	20	2.99	1.49	2.46
	50	3.57	1.8	3.00
	100	4.01	2.0	3.43

where m_1 = length of foundation / width of foundation.

Table 06.3: Values of Modulus of Elasticity

Type of soil	E psi	E kPa
Soft clay	250 – 500	1,725 – 3,450
Hard clay	850 – 2000	5,865 – 13,800
Loose sand	1,500 – 4,000	10,350 – 27,600
Dense sand	5,000 – 10,000	34,500 – 69,000

Table 06.4: Values of Poisson's Ratio

Type of Soil	Poisson's Ratio
Loose sand	0.2 – 0.4
Medium sand	0.25 – 0.4
Dense sand	0.3 – 0.45
Silty sand	0.2 – 0.4
Soft clay	0.15 – 0.25
Medium clay	0.2 – 0.5

6.11 TOTAL SETTLEMENT OF FOUNDATION

The total settlement of a foundation is the sum of the primary, secondary, and immediate settlement.

$$\Delta H_T = \Delta H + \Delta H_s + \Delta H_i \qquad \text{Eq. 6.30}$$

ILLUSTRATIVE PROBLEMS

PROBLEM 06.1

The soil profile shown in Figure 06.2 is to carry a surcharge of 60 kPa applied at the ground surface. The result of laboratory consolidation test conducted on a specimen collected from the middle of the clay layer is also shown. Calculate the settlement in the field caused by primary consolidation due to surcharge.

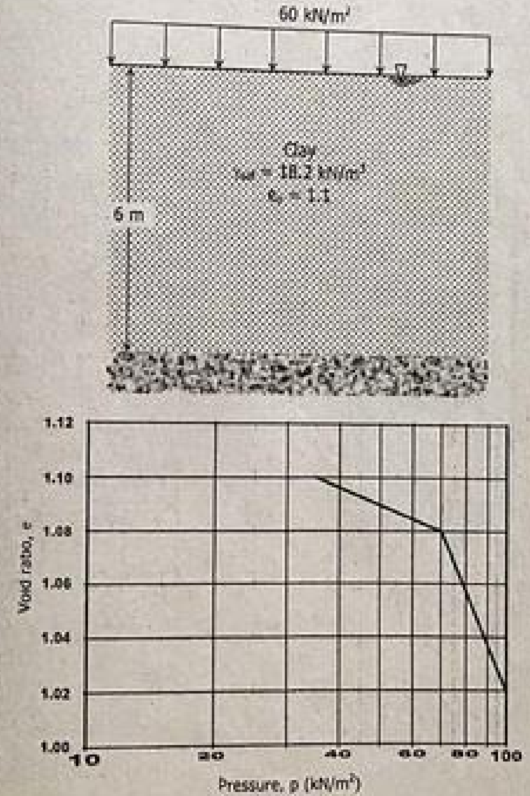


Figure 06.2

SOLUTION

Settlement, $\Delta H = H\dfrac{e_o - e}{1+e_o}$

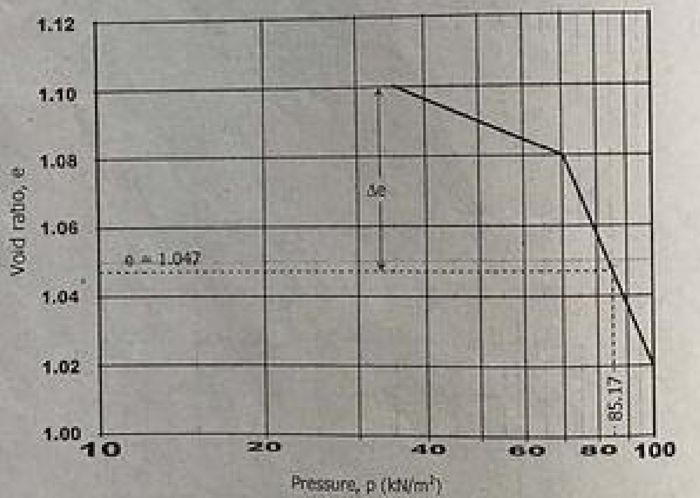
$H = 6$ m
$e_o = 1.1$

Solving for e corresponding to p_f:

Initial effective stress at midheight of clay

$p_o = \gamma_b(3)$
$p_o = (18.2 - 9.81)(3)$
$p_o = 25.17$ kPa

$p_f = p_o + \Delta p$
$p_f = 25.17 + 60$
$p_f = 85.17$ kPa



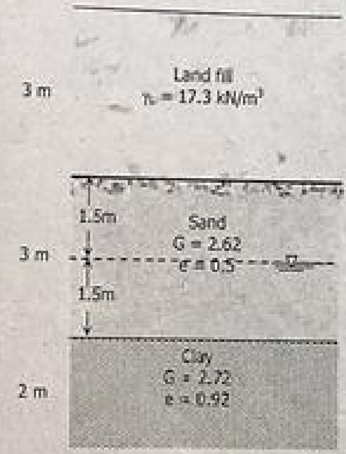
From the graph, $e = 1.047$

$\Delta H = 6\dfrac{1.1 - 1.047}{1+1.1}$
$\Delta H = 0.1514$ m
$\Delta H = 151.4$ mm

PROBLEM 06.2 (CE MAY 1999)

A 2-m clay layer ($e = 0.92$, $G = 2.72$, $C_c = 1/3$) is overlain with 3 m thick of sand layer ($e = 0.5$, $G = 2.62$, $MC = 0$). The water table is 1.5 m below the ground (sand) surface. If a 3-m thick land fill ($\gamma = 17.3$ kN/m³) is placed over the existing ground surface, compute the consolidation settlement of the clay layer.

SOLUTION



Unit weight of soil:

Sand above water table:

$[\gamma_s = \dfrac{G + GMC}{1+e}\gamma_w]$

$\gamma_s = \dfrac{2.62 + 0}{1 + 0.5}(9.81)$

$\gamma_s = 17.13$ kN/m³

Sand below water table (submerged):

$[\gamma_{bs} = \dfrac{G - 1}{1+e}\gamma_w]$

$\gamma_{bs} = \dfrac{2.62 - 1}{1 + 0.5}(9.81)$

$\gamma_{bs} = 10.59$ kN/m³

Clay: (submerged)

$$[\gamma_b = \frac{G-1}{1+e}\gamma_w]$$

$$\gamma_b = \frac{2.72-1}{1+0.92}(9.81)$$

$$\gamma_b = 8.79 \text{ kN/m}^3$$

$$\Delta H = H\frac{C_c}{1+e_o}\log\frac{p_f}{p_o}$$

$$H = 2 \text{ m}$$

$$e_o = 0.92$$

$$p_o = 8.79(1) + 10.59(1.5) + 17.13(1.5)$$

$$p_o = 50.37 \text{ kPa}$$

$$p_f = p_o + \Delta p$$

$$\Delta p = \gamma_L h_L$$

$$\Delta p = 17.3(3)$$

$$\Delta p = 51.9 \text{ kPa}$$

$$p_f = 50.37 + 51.9$$

$$p_f = 102.27 \text{ kPa}$$

$$\Delta H = 2\frac{1/3}{1+0.92}\log\frac{102.27}{50.37}$$

$$\Delta H = 0.1068 \text{ m}$$

$$\Delta H = 106.7 \text{ mm}$$

PROBLEM 06.3 (CE MAY 2004, NOVEMBER 2003)

Given in Figure 06.3 is the borehole log in a project site. The proposed building will exert a net stress of 12 Newtons per square centimeter.

a) Determine the buoyant unit weight of the clay
b) Determine the effective vertical stress at the mid-height of the clay layer
c) Determine the average settlement of the normally consolidated clay layer. Use compression index $C_c = 0.009(LL - 10)$

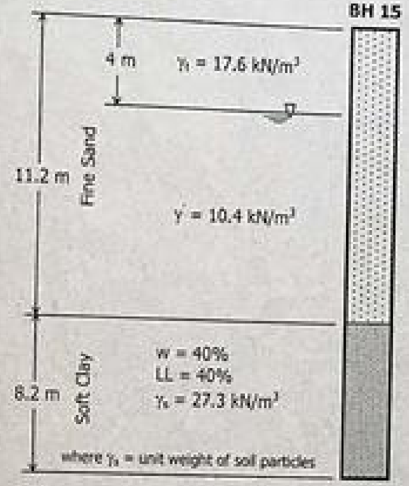


Figure 06.3

SOLUTION

Part *a*:

$$\gamma_b = \frac{G-1}{1+e}\gamma_w$$

$$G = \frac{\gamma_s}{\gamma_w}$$

$$G = \frac{27.3}{9.81}$$

$$G = 2.783$$

$$G\,MC = S\,e, \text{ where } S = 1$$

$$2.783(0.40) = 1e$$

$$e = 1.113$$

$$\gamma_b = \frac{2.783-1}{1+1.113}(9.81)$$

$$\gamma_b = 8.278 \text{ kN/m}^3$$

Part *b*:

$$p_c = (8.278)(4.1) + (10.4)(7.2) + 17.9(4)$$
$$p_c = \mathbf{180.42\ kPa}$$

Part *c*:

$$\Delta H = H\frac{C_c}{1+e_o}\log\frac{p_f}{p_o}$$
$$C_c = 0.009(40 - 10)$$
$$C_c = 0.27$$
$$e_o = 1.113$$
$$p_o = 180.42$$
$$p_f = p_o + \Delta p$$
$$\Delta p = 12\ \text{N/cm}^2 \times (100\ \text{cm/m})^2$$
$$\Delta p = 120{,}000\ \text{Pa}$$
$$\Delta p = 120\ \text{kPa}$$

$$p_f = 180.42 + 120$$
$$p_f = 300.42\ \text{kPa}$$
$$\Delta H = 8.2\frac{0.27}{1+1.113}\log\frac{300.42}{180.42}$$
$$\Delta H = 0.232\ \text{m}$$
$$\Delta H = \mathbf{23.2\ cm}$$

PROBLEM 06.4

A soil profile is shown in Figure 06.4. A uniformly distributed load, $\Delta p = 50$ kPa, is applied at the ground surface. Assume $C_s = \frac{1}{5}C_c$

Determine the settlement of the clay layer caused by primary consolidation if:

a) The clay is normally consolidated;
b) The preconsolidation pressure p_c is 210 kPa;
c) The preconsolidation pressure p_c is 150 kPa;

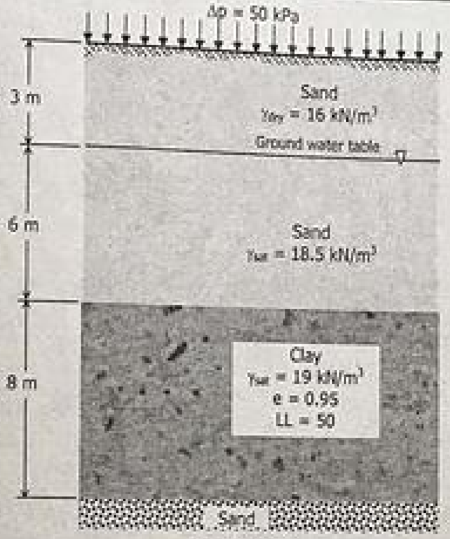


Figure 06.4

SOLUTION

Part *a*: (Normally consolidated clay)

$$\Delta H = H\frac{C_c}{1+e_o}\log\frac{p_f}{p_o}$$
$$C_c = 0.009(LL - 10)$$
$$C_c = 0.009(50 - 10) = 0.36$$

Compute the initial and final effective stresses at the midheight of clay:

$$p_o = (\gamma_b)_{clay}\,h_{clay} + (\gamma_b)_{sand}\,h_{sand\ sat} + (\gamma_{dry})_{sand}\,h_{sand\ dry}$$
$$p_o = (19 - 9.81)(4) + (18.5 - 9.81)(6) + 16(3)$$
$$p_o = 136.9\ \text{kPa}$$

$$p_f = p_o + \Delta p$$
$$p_f = 136.9 + 50$$
$$p_f = 186.9\ \text{kPa}$$

$$\Delta H = 8 \times \frac{0.36}{1+0.95}\log\frac{186.9}{136.9}$$
$$\Delta H = 0.2\ \text{m}$$
$$\Delta H = \mathbf{200\ mm}$$

Part *b*: (Overconsolidated clay, $p_c = 210$ kPa)

$p_f = 186.9 \text{ kPa} < p_c$

$$\Delta H = H\frac{C_s}{1+e_o}\log\frac{p_f}{p_o}$$

$$C_s = 0.36/5$$

$$C_s = 0.072$$

$$\Delta H = 8\frac{0.072}{1+0.95}\log\frac{186.9}{136.9}$$

$$\Delta H = 0.04 \text{ m}$$

$$\mathbf{\Delta H = 40 \text{ mm}}$$

Part *c*: (Overconsolidated clay, $p_c = 150$ kPa)

$p_f = 186.9 \text{ kPa} > p_c$

$$\Delta H = H\frac{C_s}{1+e_o}\log\frac{p_c}{p_o} + H\frac{C_c}{1+e_o}\log\frac{p_f}{p_c}$$

$$\Delta H = 8\frac{0.072}{1+0.95}\log\frac{150}{136.9} + 8\frac{0.36}{1+0.95}\log\frac{186.9}{150}$$

$$\Delta H = 0.01172 + 0.1411$$

$$\Delta H = 0.1528 \text{ m}$$

$$\mathbf{\Delta H = 152.8 \text{ mm}}$$

PROBLEM 06.5

It is desired to calculate the consolidation settlement of the 4-m thick clay layer shown in Figure 06.5 that will result from the load carried by the footing measuring 3 m × 1.5 m in plan. Assume the clay to be normally consolidated, and the load of footing results to an increase of pressure of 23.2 kPa, 12.22 kPa, and 6.1 kPa at the top, midheight, and bottom portion, respectively, of the clay layer.

Assume the pressure increase varies parabolically and use Simpson's rule to solve for the average pressure increase.

$$\Delta p = \frac{\Delta p_{top} + 4\Delta p_{mid} + \Delta p_{bot}}{6}$$

a) Calculate the overburden pressure at the base of the footing, in kPa.
b) Calculate the initial effective stress at the midheight of the clay layer.
c) Calculate the consolidation settlement of the clay layer.

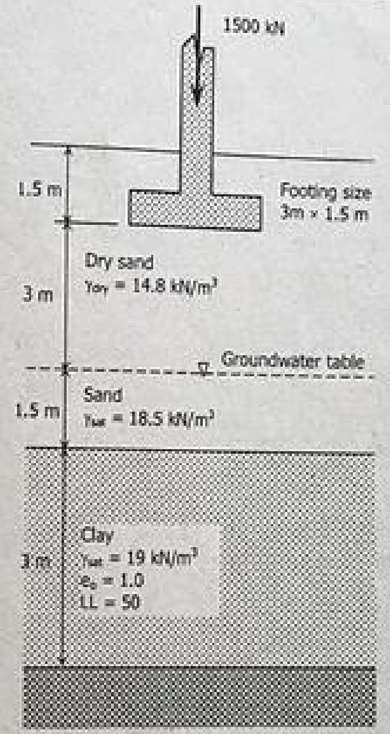


Figure 06.5

SOLUTION

Part *a*:

$$q = \gamma D_f$$

$$q = 14.8(1.5)$$

$$\mathbf{q = 22.2 \text{ kPa}}$$

Part *b*:

p_o = initial effective stress at midheight of the clay layer

$$p_o = (19 - 9.81)(3 \div 2) + (18.5 - 9.81)(1.5) + 14.8(3 + 1.5)$$

$$\mathbf{p_o = 93.42 \text{ kPa}}$$

Part c:

For normally consolidated clay:

$$\Delta H = H\frac{C_c}{1+e_o}\log\frac{p_f}{p_o}$$

$$C_c = 0.009(LL - 10)$$
$$C_c = 0.009(50 - 10)$$
$$C_c = 0.36$$
$$H = 3\text{ m}$$
$$e_o = 1.0$$

$$p_f = p_o + \Delta p$$

$$\Delta p = \frac{\Delta p_{top} + 4\Delta p_{mid} + \Delta p_{bot}}{6}$$

$$\Delta p = \frac{23.2 + 4(12.22) + 6.1}{6}$$

$$\Delta p = 13.03\text{ kPa}$$

$$p_f = 93.42 + 13.03$$
$$p_f = 106.45\text{ kPa}$$

$$\Delta H = 3\frac{0.36}{1+1}\log\frac{106.45}{93.42}$$
$$\Delta H = 0.0306\text{ m}$$
$$\Delta H = 30.6\text{ mm}$$

PROBLEM 06.6

A normally consolidated clay layer, 3 m thick, has the following properties:

Initial void ratio, $e_o = 0.8$
Compression index, $C_c = 0.25$
Average effective pressure, $p_o = 125$ kPa
Expected pressure increase, $\Delta p = 45$ kPa
Secondary compression index, $C_\alpha = 0.02$
Time for completion of primary settlement = 1.5 years

What is the total settlement of the clay layer five years after the completion of primary consolidation settlement?

SOLUTION

Total consolidation settlement, $\Delta H = \Delta H_{primary} + \Delta H_{secondary}$

Primary consolidation settlement:

$$\Delta H_p = H\frac{C_c}{1+e_o}\log\frac{p_f}{p_o} = H\frac{\Delta e}{1+e_o}$$

$$\Delta e = C_c\log\frac{p_f}{p_o}$$

$$p_o = 125\text{ kPa}$$
$$p_f = p_o + \Delta p$$
$$p_f = 125 + 45$$
$$p_f = 170\text{ kPa}$$

$$\Delta e = 0.25\log\frac{170}{125}$$

$$\Delta e = 0.0334$$

$$\Delta H_p = 3\frac{0.0334}{1+0.8}$$
$$\Delta H_p = 0.05564\text{ m}$$
$$\Delta H_p = 55.64\text{ mm}$$

Secondary consolidation settlement, Eq. 6.12:

$$\Delta H_s = C'_\alpha H\log\left(\frac{t_2}{t_1}\right)$$

$$C'_\alpha = \frac{C_\alpha}{1+e_p}$$

$$e_p = e_o - \Delta e$$
$$e_p = 0.8 - 0.0334$$
$$e_p = 0.7666$$

$$C'_\alpha = \frac{0.02}{1+0.7666}$$
$$C'_\alpha = 0.01132$$

$$\Delta H_s = (0.01132)(3)\log\frac{5}{1.5}$$

$$\Delta H_s = 0.01776\text{ m}$$
$$\Delta H_s = 17.76\text{ mm}$$

Total consolidation settlement, $\Delta H = 55.64 + 17.76$

Total consolidation settlement, $\Delta H = 73.4$ mm

PROBLEM 06.7

A surcharge of 120 kPa is applied on the ground surface on n the soil profile shown in Figure 06.6.

a) How high will the water rise in the piezometer immediately after the application of the load?

b) What is the degree of consolidation at point D when $h = 6$ m?

c) Find h when the degree of consolidation at D is 80%

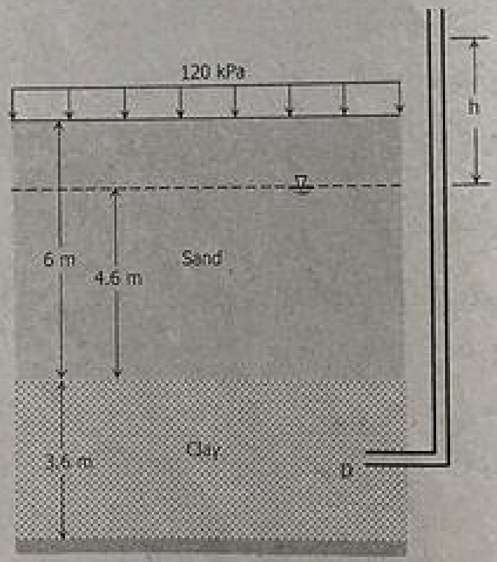


Figure 06.6

SOLUTION

a) $h = \dfrac{\Delta p}{\gamma_w}$

$h = \dfrac{120}{9.81}$

$h = 12.232$ m

b) $U_D = \left(1 - \dfrac{p_{wz}}{p_{wo}}\right)100\%$

p_{wo} = initial excess pore water pressure

$p_{wo} = \gamma_w h$

$p_{wo} = (9.81)(12.232)$

$p_{wo} = 120$ kPa

$p_{wz} = 9.81(6)$

$p_{wz} = 58.86$ kPa

$U_D = \left(1 - \dfrac{58.86}{120}\right)100\%$

$U_D = 50.95\%$

c) $U_D = \left(1 - \dfrac{p_{wz}}{p_{wo}}\right)100\%$

$80\% = \left(1 - \dfrac{p_{wz}}{120}\right)100\%$

$p_{wz} = 24$ kPa

$h = \dfrac{p_{wz}}{\gamma_w}$

$h = \dfrac{24}{9.81}$

$h = 2.446$ m

PROBLEM 06.8

Under a given surcharge, a 5-m thick clay layer has a consolidation settlement of 305 mm. Assume $C_v = 0.003$ cm²/sec.

a) What is the average degree of consolidation for the clay layer when the settlement is 75 mm?

b) How long will it take for 50% consolidation to occur if the layer is drained at the top only?

c) How long will it take for 50% consolidation to occur if the layer is drained on both ends?

SOLUTION

a) From Eq. 6.22:

$$U = \frac{\Delta H_t}{\Delta H_{max}}$$

$$U = \frac{75}{305}$$

$$U = 0.2459 = 24.59\%$$

b) From Eq. 6.16:

$$t = T_v \frac{(H_{dr})^2}{C_v}$$

From Table 06.1, for $U = 50\%$, $T_v = 0.197$

With single drainage, $H_{dr} = 5$ m = 500 cm

$$t = 0.197 \frac{500^2}{0.003}$$

$$t = 16{,}416{,}667 \text{ sec} \times \frac{1}{3600} \times \frac{1}{24}$$

$$t = \mathbf{190 \ days}$$

c) $t = T_v \dfrac{(H_{dr})^2}{C_v}$

From Table 06.1, for $U = 50\%$, $T_v = 0.197$

With single drainage, $H_{dr} = ½(5) = 2.5$ m

With single drainage, $H_{dr} = 250$ cm

$$t = 0.197 \frac{250^2}{0.003}$$

$$t = 4{,}104{,}167 \text{ sec} \times \frac{1}{3600} \times \frac{1}{24}$$

$$t = \mathbf{47.5 \ days}$$

PROBLEM 06.9

A 3.2-m thick layer of saturated clay under a surcharge loading underwent 90% primary consolidation in 80 days with double drainage.

a) Determine the coefficient of consolidation for the pressure range

b) For a 10-cm thick specimen of the said clay, how long will it take to undergo 90% consolidation in the laboratory for a similar pressure range?

SOLUTION

$$t = T_v \frac{(H_{dr})^2}{C_v}$$

$T_v = 0.848$ (from Table 06.1)

$H_{dr} = ½(3.2) = 1.6$ m

$H_{dr} = 160$ cm

$t = 80 \text{ days} \times 24 \times 3600$

$t = 6{,}912{,}000$ sec

$$6{,}912{,}000 = 0.848 \frac{160^2}{C_v}$$

$$C_v = \mathbf{0.00314 \ cm^2/sec}$$

b) From Eq. 6.19:

$$\frac{t_{field}}{t_{lab}} = \frac{(H_{dr\,field})^2}{(H_{dr\,lab})^2}$$

$$\frac{6{,}912{,}000}{t_{lab}} = \frac{160^2}{(10/2)^2}$$

$$t_{lab} = 6{,}750 \text{ sec}$$

$$t_{lab} = \mathbf{1.875 \ hours}$$

PROBLEM 06.10

Under a given load (surcharge), a 3-m thick clay layer undergoes a 180-mm of total primary consolidation settlement. If it took 80 days for the first 100-mm of settlement to occur, what is the estimated time for the first 50-mm of settlement?

SOLUTION

From Eq. 6.20, $\dfrac{t_1}{t_2} = \dfrac{U_1^{\,2}}{U_2^{\,2}}$

Degree of consolidation @ $t_1 = 80$ days, $U_1 = \dfrac{\Delta H_t}{\Delta H_{max}}$

Degree of consolidation, $U_1 = \dfrac{100}{180}$

Degree of consolidation, $U_1 = 0.5556 = 55.56\%$

Degree of consolidation @ t_2, $U_2 = \frac{50}{180}$

Degree of consolidation, $U_2 = 0.2778 = 27.78\%$

$$\frac{80}{t_2} = \frac{55.56^2}{27.78^2}$$

t_2 = **20 days**

PROBLEM 06.11

Under normal loading condition, a 3.6-m thick clay (normally consolidated) has p_o = 190 kPa and e_o = 1.22. A surcharge of 190 kPa reduces its void ratio to 0.98. The hydraulic conductivity of the clay for the loading range is 6.1 × 10^{-5} m/day.

a) What is the coefficient of volume compressibility of the clay?
b) What is the coefficient of consolidation of the clay?
c) How long will it take for this clay layer to reach 60% consolidation if it is drained on one side only?

SOLUTION

From Eq. 6.16:

$$t = T_v \frac{(H_{dr})^2}{C_v}$$

H_{dr} = 3.6 m (single drainage)

From Table 06.1, T_v = 0.286

$$C_v = \frac{k}{m_v \gamma_w} \qquad \text{(Eq. 6.27)}$$

$$m_v = \frac{(e_o - e)/\Delta p}{1 + e_{ave}} \qquad \text{(Eq. 6.25)}$$

$e_o = 1.22$

$e = 0.98$

Δp = 190 kPa

$$e_{ave} = \frac{1.22 + 0.98}{2}$$

$e_{ave} = 1.1$

$$m_v = \frac{(1.22 - 0.98)/190}{1 + 1.1}$$

m_v = **0.0006015 m²/kN** → Coeff. of compressibility (a)

$$C_v = \frac{6.1 \times 10^{-5}}{0.0006015(9.81)}$$

C_v = **0.01034 m²/day** → Coeff. of consolidation (b)

$$t_{60} = (0.286)\frac{(3.6)^2}{0.01034}$$

t_{60} = **358.5 days** → Time for 60% consolidation (c)

PROBLEM 06.12

The following data were obtained from a laboratory consolidation test on a 30 mm thick clay specimen drained on both sides:

Pressure (kPa)	Void ratio (%)
60	87
150	72

Time for 60% consolidation = 3.2 min

a) Determine the coefficient of volume compressibility in m²/kN
b) Determine the hydraulic conductivity of the clay for the loading range

SOLUTION

$k = C_v m_v \gamma_w$

$$C_v = T_v \frac{(H_{dr})^2}{t} \qquad \text{(Eq. 6.27)}$$

$T_v = 0.286$ (Table 06.1)

H_{dr} = 0.03/2 = 0.015 m (double drain)

t = 3.2 min

$$C_v = 0.286 \frac{(0.015)^2}{3.2}$$

C_v = 0.00002011 m²/min

$$m_v = \frac{(e_o - e)/\Delta p}{1 + e_{ave}}$$

$$m_v = \frac{(0.87 - 0.72)/(150 - 60)}{1 + \frac{0.87 + 0.72}{2}}$$

m_v = 0.0009285 m²/kN

$k = 0.00002011(0.0009285)(9.81)$

k = **1.83 × 10^{-7} m/min**

PROBLEM 06.13

A rigid column footing 1.2-m in diameter is constructed on unsaturated clay layer. The load on the footing is 170 kN. Estimate the immediate settlement. Assume the clay has E = 6900 kPa and μ = 0.2.

SOLUTION

Using Eq. 6.28:

$$\Delta H_i = pB\frac{1-\mu^2}{E}I_f$$

$$p = \frac{Q}{A}$$

$$p = \frac{170}{\frac{\pi}{4}(1.2)^2}$$

$$p = 150.31 \text{ kPa}$$

$$B = 1.2 \text{ m}$$

From Table 06.2, $I_f = 0.79$

$$\Delta H_i = 150.31(1.2)\frac{1-0.2^2}{6,900}(0.79)$$

$$\Delta H_i = 0.0198 \text{ m}$$

$$\Delta H_i = 19.8 \text{ mm}$$

Chapter 07

Shear Strength of Soil

The shear strength of soil may be attributed to three basic components:

1. Frictional resistance to sliding between solid particles
2. Cohesion and adhesion between particles
3. Interlocking and bridging of solid particles to resist deformation

7.1 MOHR – COULOMB FAILURE CRITERIA

A material fails because of a critical combination of normal stress and shearing stress, and not from either maximum normal or shear stress alone. This theory was presented by Mohr. Thus, a failure plane can be expressed as a function of normal and shearing stress as follows:

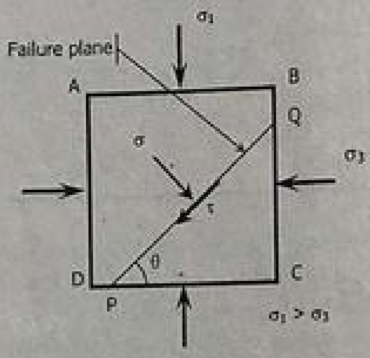
$$\tau_f = f(\sigma) \qquad \text{Eq. 7.1}$$

For most soil mechanics problem, Coulomb suggested that the shear stress on the failure plane can be expressed as a linear function of normal stress. This relationship is known as *Mohr-Coulumb failure criteria* and can be written as:

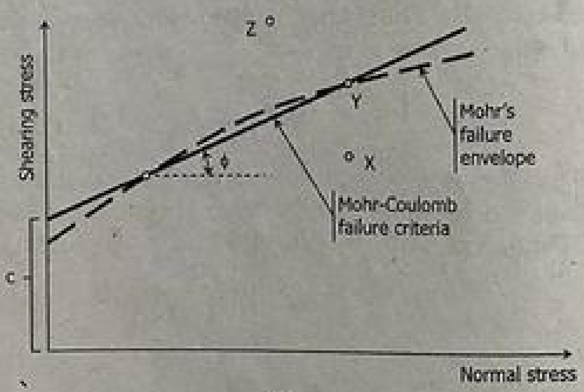
$$\tau_f = c + \sigma \tan \phi \qquad \text{Eq. 7.2}$$

where c = cohesion
ϕ = angle of internal friction

These functions are shown in Figure 7.1. The significance of the failure envelope is as follows. If the normal and shearing stress on a plane in a soil mass are such that they plot as point X, shear failure will not occur along that plane. If it plots at Y, shear failure will occur along that plane because it plots along that plane. Point Z cannot exist because it plots above the failure envelope and shear failure would have occurred already.



(a)



(b)

Figure 7.1 – Mohr's failure envelope and Mohr-Coulomb failure criteria

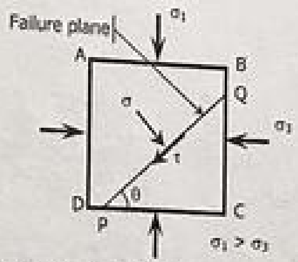


Figure 7.2 – Applied stresses on soil

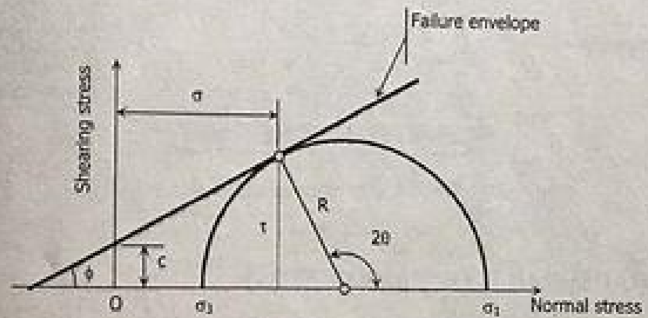


Figure 7.3 – Mohr's circle

$$\theta = 45° + \frac{\phi}{2} \qquad \text{Eq. 7.3}$$

$$\sigma_1 = \sigma_3 \tan^2\left(45° + \frac{\phi}{2}\right) + 2c \tan\left(45° + \frac{\phi}{2}\right) \qquad \text{Eq. 7.4}$$

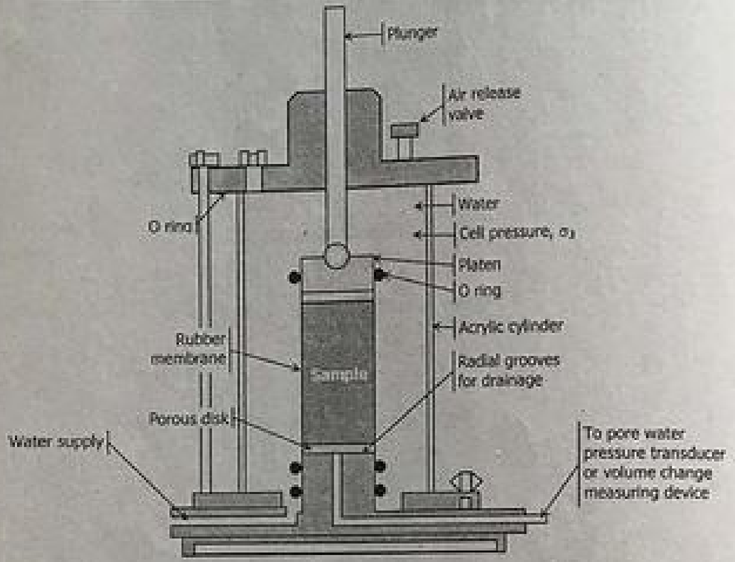


Figure 7.4 – Schematic of a triaxial cell

7.2 TRIAXIAL SHEAR TEST (SINGLE TEST)

7.2.1 COHESIONLESS SOIL

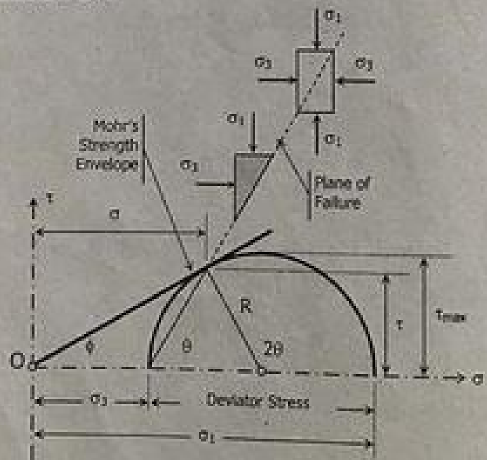


Figure 7.5 – Single test on cohesionless soil

The following equations can be derived from Figure 7.5:

$$\tau = \sigma \tan\phi = R \sin 2\theta \quad \text{Eq. 7.5}$$

$$R = \tfrac{1}{2}(\sigma_1 - \sigma_3) = \tau_{max} \quad \text{Eq. 7.6}$$

$$\sin\phi = \frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3} \quad \text{Eq. 7.7}$$

$$\theta = 45° + \phi/2 \quad \text{Eq. 7.8}$$

7.2.2 COHESIVE SOIL

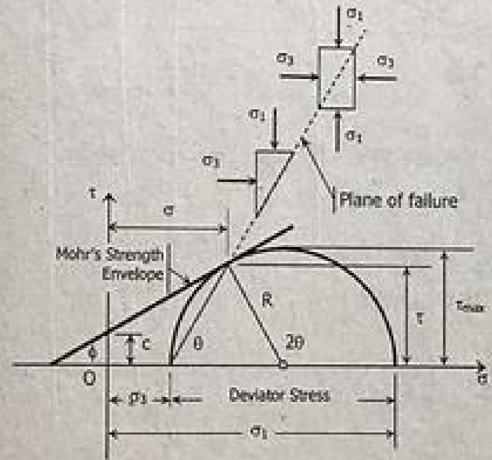


Figure 7.6 – Single test on cohesive soil

$$\tau = c + \sigma \tan\phi = R \sin 2\theta \quad \text{Eq. 7.9}$$

where:

σ_1 = Major principal stress at failure
σ_2 = Minor principal stress at failure
τ = Shear stress
c = Cohesion of soil
ϕ = Angle of internal friction
θ = Angle that the failure plane makes with the major principal plane.

7.3 DRAINED AND UNDRAINED TRIAXIAL TEST

For drained triaxial test σ'_1 and σ'_3 are taken as the effective principal stresses. For undrained triaxial test, σ_1 and σ_3 are taken as the total principal stresses.

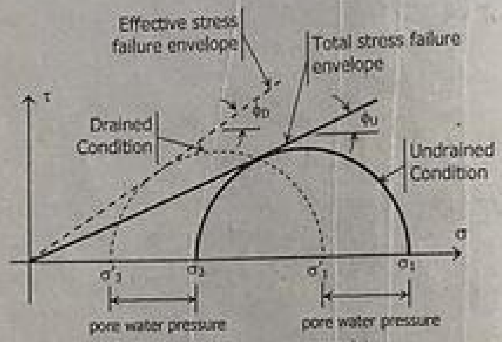


Figure 7.7 – Drained and undrained condition

where:

ϕ_D = drained friction angle

ϕ_U = undrained friction angle

7.4 TRIAXIAL TEST (SERIES)

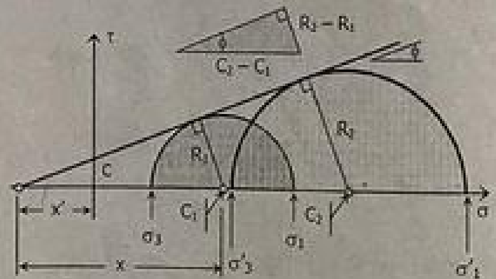


Figure 7.8

The following equations can be derived from Figure 7.8:

$$R_1 = \frac{\sigma_1 - \sigma_3}{2} \quad \text{Eq. 7.10}$$

$$R_2 = \frac{\sigma'_1 - \sigma'_3}{2} \quad \text{Eq. 7.11}$$

$$C_1 = \frac{\sigma_1 + \sigma_3}{2} \quad \text{Eq. 7.12}$$

$$C_2 = \frac{\sigma'_1 + \sigma'_3}{2} \quad \text{Eq. 7.13}$$

$$\sin\phi = \frac{R_2 - R_1}{C_2 - C_1} \quad \text{Eq. 7.14}$$

$$c = x' \tan\phi \quad \text{Eq. 7.15}$$

$$x' = x - C_1 \text{ and } x = \frac{R_1}{\sin\phi} \quad \text{Eq. 7.16}$$

$$c = R_1 \cos\phi - (C_1 - R_1 \sin\phi)\tan\phi \quad \text{Eq. 7.17}$$

$$c = R_2 \cos\phi - (C_2 - R_2 \sin\phi)\tan\phi \quad \text{Eq. 7.18}$$

7.5 UNCONFINED COMPRESSION TEST (UNIAXIAL)

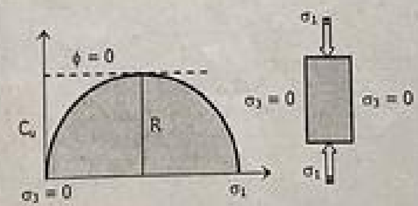


Figure 7.9 – Unconfined compression test

$$C_u = R = \frac{\sigma_1}{2} \quad \text{Eq. 7.19}$$

$$q_u = 2\,C_u \quad \text{Eq. 7.20}$$

where q_u = unconfined compression strength

7.6 DIRECT SHEAR TEST

Direct shear test is the simplest for of shear test. The test equipment consists of a metal shear box (See Figure 7.10) in which the soil sample is placed. The sizes of the sample used are usually 50 mm × 50 mm or 100 mm × 100 mm across and about 25 mm high. The box is split horizontally into halves. A normal force is applied from the top of the shear box. Shear force is applied by moving half of the box relative to the other to cause failure in the soil sample.

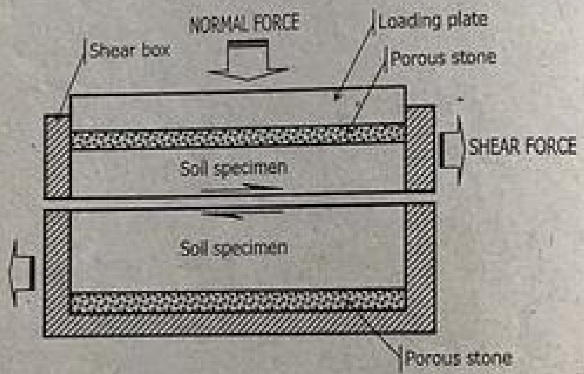


Figure 7.10 – Direct shear test arrangement

$$\text{Normal stress, } \sigma = \frac{\text{Normal force}}{\text{Cross-sectional area of specimen}} \qquad \text{Eq. 7.21}$$

$$\text{Shear stress, } \tau = \frac{\text{Resisting shear force}}{\text{Cross-sectional area of specimen}} \qquad \text{Eq. 7.22}$$

ILLUSTRATIVE PROBLEMS

PROBLEM 07.1

Direct shear tests were performed on a dry, sandy soil. The specimen is 50 mm in diameter and 25 mm in height. Test results were as follows:

Test No.	Normal Force, N	Shear Force, N
1	243	124
2	268	137
3	352	179
4	412	210

Determine the cohesion and angle of internal friction.

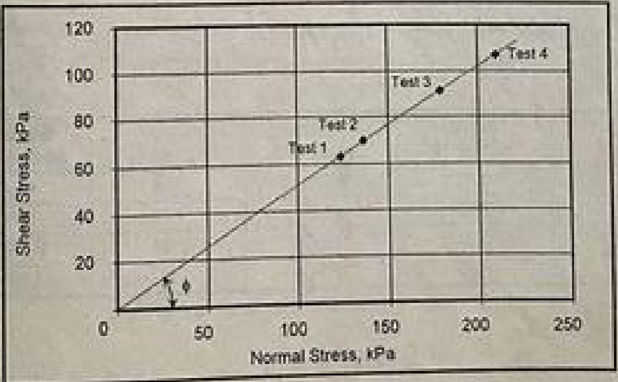
SOLUTION

Cross-sectional area of specimen:

$$A = \frac{\pi}{4}(0.05)^2$$

$$A = 0.001963 \text{ m}^2$$

Test No.	Normal Force, N	Normal Stress, kPa	Shear Force, N	Shearing Stress, kPa
1	243	123.8	124	63.1
2	268	136.5	137	69.8
3	352	179.3	179	91.2
4	412	209.8	210	106.9



The graph shows a straight line that passes through the origin, hence cohesion $c = 0$.

Solving for ϕ using the results of Test 1:

$$\tan\phi = \frac{\tau}{\sigma}$$

$$\tan\phi = \frac{63.1}{123.8}$$

$$\phi = 27^\circ$$

PROBLEM 07.2

The results of four drained direct shear tests on an over consolidated clay are as follows:

Test No.	Normal Force, N	Shear Force, N
1	120	114
2	220	179
3	320	239
4	420	286

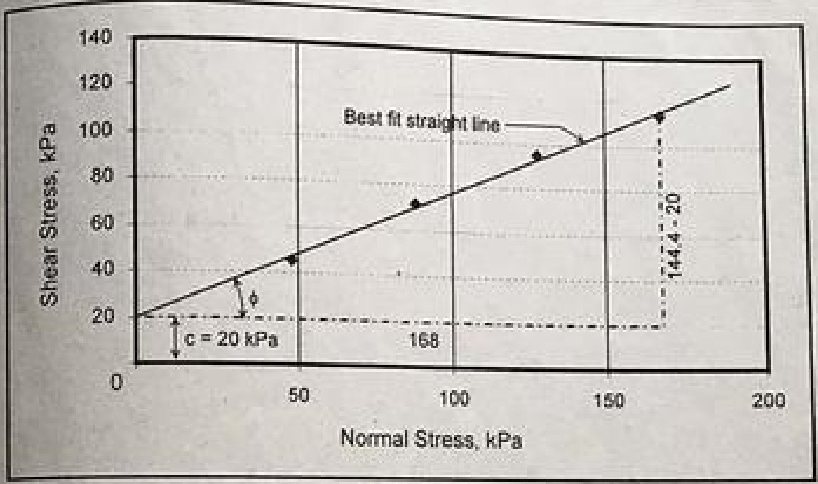
Size of specimen = 50 mm × 50 mm
Height of specimen = 25 mm

Determine the cohesion and angle of internal friction.

SOLUTION

Cross-sectional area of specimen, $A = 0.05(0.05)$
Cross-sectional area of specimen, $A = 0.0025\ m^2$

Test No.	Normal Force, N	Normal Stress, kPa	Shear Force, N	Shearing Stress, kPa
1	120	48.00	114	45.60
2	220	88.00	179	71.60
3	320	128.00	239	95.60
4	420	168.00	286	114.40



From the graph, we draw the best fit straight line.

Cohesion, $c = 20$ kPa

Angle of internal friction:
Using the result of Test 4 and the Y-intercept:

$$\tan\phi \approx \frac{114.4 - 20}{168}$$

$$\phi \approx 29.33^\circ$$

PROBLEM 07.3

A direct shear test is performed on a specimen of dry sand. The shear box is circular in cross-section with a diameter of 50 mm. The normal force imposed on the specimen is 250 N. The shears when the shear force is 150 N. Determine the angle of internal friction of this sand.

SOLUTION

From Eq. 7.2:

$$\tau_f = c + \sigma \tan\phi$$

$c = 0$ (dry sand is cohesionless)

@seismicisolation

$$\tau_f = \frac{V}{A}$$

$$\tau_f = \frac{150}{\frac{\pi}{4}(0.05)^2}$$

$$\tau_f = 76{,}394 \text{ Pa}$$

$$\sigma = \frac{F}{A} = \frac{250}{\frac{\pi}{4}(0.05)^2}$$

$$\sigma = 127{,}324 \text{ Pa}$$

$$76{,}394 = 0 + (127{,}324)\tan\phi$$

$$\tan\phi = 0.6$$

$$\phi = 30.96°$$

PROBLEM 07.4

A 7-m thick soil has water table 3 m below the ground surface. The soil above the water table has degree of saturation of 45%, void ratio of the soil is 0.4 and the solids have specific gravity of 2.70. Tests show that the soil have angle of internal friction of 32° and cohesion of 14.6 kPa. What is the potential shear strength on a horizontal plane at a depth of 2 m below the ground surface?

SOLUTION

Potential shear strength, $\tau = c + \sigma \tan\phi$

$c = 14.6$

$\phi = 32°$

σ = vertical effective stress

$$\sigma = \gamma_m h$$

$$\gamma_m = \frac{G + Se}{1+e}\gamma_w$$

$$\gamma_m = \frac{2.70 + 0.45(0.40)}{1 + 0.40}(9.81)$$

$$\gamma_m = 20.18 \text{ kN/m}^3$$

$$\sigma = 20.18(2)$$

$$\sigma = 40.36 \text{ kPa}$$

Potential shear strength, $\tau = 14.6 + 40.36 \tan 32°$

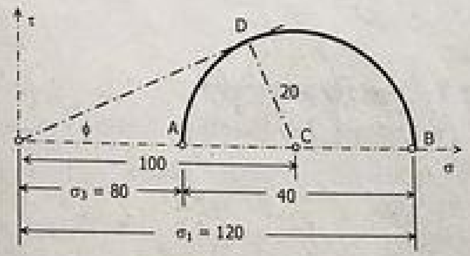
Potential shear strength, $\tau = 39.82$ **kPa**

PROBLEM 07.5 (CE NOVEMBER 1998)

In a triaxial test, a specimen of saturated (normally consolidated) clay was consolidated under a chamber confining pressure of 80 KiloPascals. The axial stress on the specimen was then increased through the allowing the drainage from the specimen. The specimen fails when the 120 KiloPascals. The pore water pressure (U) at that time was 50 KiloPascals. What is the consolidated undrained friction angle (phi):

SOLUTION

For undrained condition, we use the total principal stresses



$$\sin\phi = \frac{20}{100}$$

$$\phi = 11.54°$$

PROBLEM 07.6 (CE NOVEMBER 2001)

In a triaxial test, a specimen of saturated (normally consolidated) clay was consolidated under a chamber confining pressure of 90 KiloPascals. The axial stress on the specimen was then increased allowing the drainage from the specimen. The specimen fails when the deviator stress is 60 KiloPascals. The pore water pressure (p_w) at that time was 40 KiloPascals. What the consolidated drained friction angle (phi):

SOLUTION

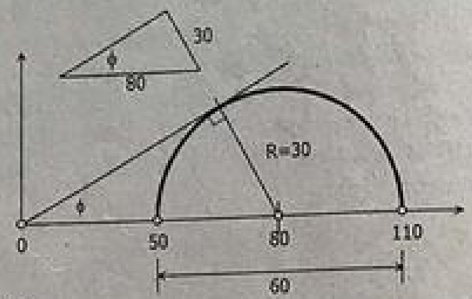
Total principal stress, $\sigma_3 = 90$ kPa

Effective principal stress, $\sigma'_3 = \sigma_3 - p_w$

Effective principal stress, $\sigma'_3 = 90 - 40$

Effective principal stress, $\sigma'_3 = 50$ kPa

Deviator stress = 60 kPa



$$\sin\phi = \frac{30}{80}$$
$$\phi = 22.024°$$

PROBLEM 07.7 (CE NOVEMBER 1999)

A triaxial test on a saturated soil has the following results:

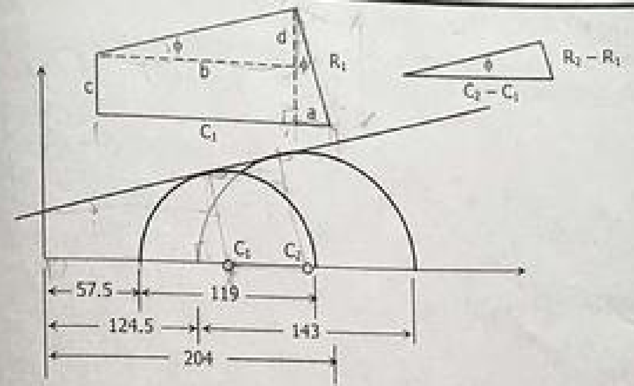
Cell pressure (kPa)	Deviator stress (kPa)	Pore pressure (kPa)
200	119	142.5
400	143	275.5
600	178	396

a) Determine the drained angle of internal friction of the soil.
b) Determine the cohesion of the soil in drained condition.

SOLUTION

For drained condition:

Effective stress, σ'_3	Deviator stress
200 – 142.5 = 57.5	119
400 – 275.5 = 124.5	143



From the figure:

$R_1 = 119/2$
$R_1 = 59.5$

$R_2 = 143/2$
$R_2 = 71.5$

$C_1 = 57.5 + 59.5$
$C_1 = 117$

$C_2 = 124.5 + 71.5$
$C_2 = 196$

a) Drained angle of friction
$\sin\phi = (R_2 - R_1) / (C_2 - C_1)$
$\sin\phi = (71.5 - 59.5) / (196 - 117)$
$\phi = 8.737°$

b) Cohesion in drained condition:
$b = C_1 - a$
$b = 117 - 59.5 \sin 8.737°$
$b = 107.96$

$c = R_1 \cos\phi - d$
$c = R_1 \cos\phi - b \tan\phi$
$c = 59.5 \cos 8.737° - 107.96 \tan 8.737°$
$c = \mathbf{42.22\ kPa}$

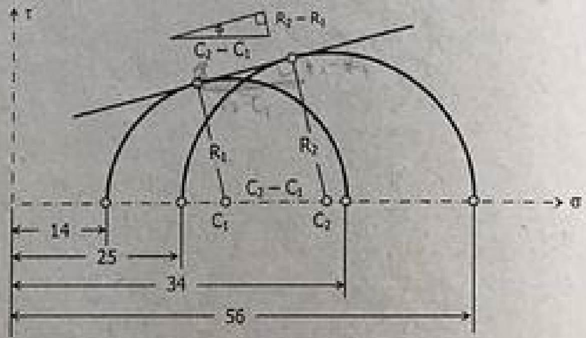
PROBLEM 07.8 (CE NOVEMBER 1999)

A sample of moist sand was subjected to a series of triaxial tests. The soil fails under the following stresses:

	Cell pressure, σ_3	Plunger stress, σ_d
Sample 1	14 kPa	20 kPa
Sample 2	25 kPa	31 kPa

What is the angle of internal friction of the soil in degrees.

SOLUTION



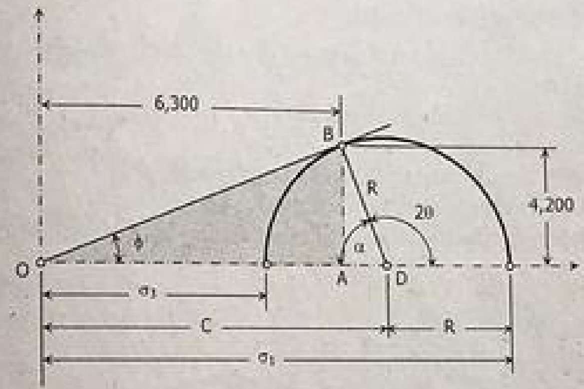
$$R_1 = \frac{34-14}{2}$$
$$R_1 = 10$$
$$C_1 = 14 + 10$$
$$C_1 = 24$$
$$R_2 = \frac{56-25}{2}$$
$$R_2 = 15.5$$
$$C_2 = 25 + 15.5$$
$$C_2 = 40.5$$
$$\sin\phi = \frac{R_2 - R_1}{C_2 - C_1}$$
$$\sin\phi = \frac{15.5-10}{40.5-24}$$
$$\phi = 19.47°$$

PROBLEM 07.9 (CE NOVEMBER 2002)

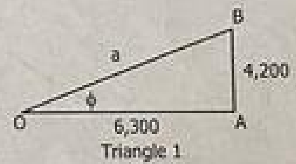
A triaxial shear test was performed on a well-drained sand sample. The normal stress on the failure plane and the shear stress on the failure plane, at failure were determined to be 6,300 psf and 4,200 psf, respectively.

a) Determine the angle of internal friction of the sand?
b) Determine the angle of the failure plane?
c) Determine the maximum principal stress?

SOLUTION



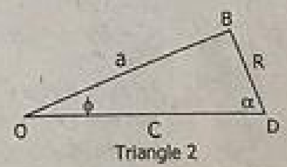
In triangle 1:



$$\tan\phi = \frac{4200}{6300}$$
$$\phi = 33.69°$$
$$a = \sqrt{(4{,}200)^2 + (6{,}300)^2}$$
$$a = 7{,}571.66$$

In triangle 2:

$$\alpha = 90° - \phi$$
$$\alpha = 90° - 33.69°$$
$$\alpha = 56.31°$$
$$R = a \tan\phi$$
$$R = 7{,}571.66 \tan 33.69°$$
$$R = 5{,}047.76$$
$$C = a \sec\phi$$
$$C = 7{,}571.66 \sec 33.69°$$
$$C = 9{,}100$$



Angle of failure plane, θ:

$\alpha + 2\theta = 180°$

$56.31° + 2\theta = 180°$

$\theta = \mathbf{61.84°}$

Maximum principal stress, $\sigma_1 = C + R$

Maximum principal stress, $\sigma_1 = 9{,}100 + 5{,}047.76$

Maximum principal stress, $\sigma_1 = \mathbf{14{,}147.76\ psf}$

PROBLEM 07.10 (CE MAY 2004)

The results of a consolidated-drained triaxial test conducted on normally consolidated clay, are as follows:

Chamber confining stress = 250 kPa

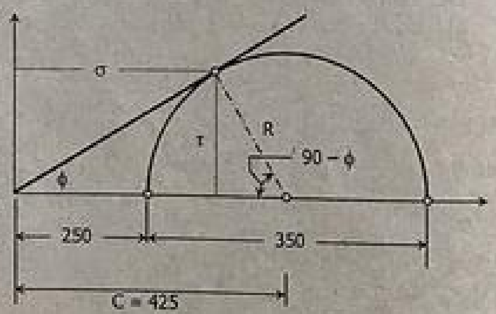
Deviator stress at failure = 350 kPa

a) Calculate the angle of friction of the soil sample.

b) Calculate the shear stress on the failure plane.

c) Calculate the effective normal stress on the plane of maximum shear.

SOLUTION



a) Angle of internal friction

$R = 350/2$

$R = 175$

$C = 250 + 175$

$C = 425$

$\sin\phi = R/C$

$\sin\phi = 175/425$

$\phi = \mathbf{24.316°}$

b) Shear stress on the failure plane

$\tau = R\sin(90 - \phi)$

$\tau = 175\sin(90 - 24.316)$

$\tau = \mathbf{159.5\ kPa}$

c) Effective normal stress at the point of maximum shear

$\sigma = C$

$\sigma = \mathbf{425\ kPa}$

PROBLEM 07.11

A consolidated undrained (CU) compression test was conducted on a saturated clay soil by isotropically consolidating the soil using a cell pressure of 150 kPa and then incrementally applying loads on the plunger while keeping the cell pressure constant. Failure was observed when the stress exerted by the plunger (deviator stress) was 160 kPa and the pore water pressure recorded was 54 kPa.

a) Determine the undrained shear strength of the clay

b) Determine the undrained friction angle

c) Determine the drained friction angle

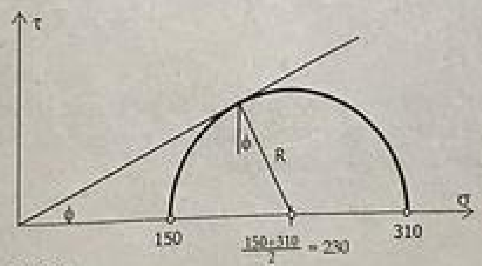
SOLUTION

a) Undrained friction angle:

Cell pressure, $\sigma_3 = 150$ kPa

Principal stress, $\sigma_1 = \sigma_3$ + plunger stress

Principal stress, $\sigma_1 = 150 + 160 = 310$ kPa



$\sin\phi = R/230$

$R = 230 - 150$

$R = 80$ kPa

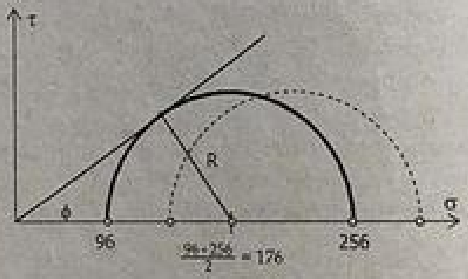
$\sin \phi_u = 80/230$
$\phi_u = 20.354°$

b) Undrained shear strength:

$\tau_u = R$
$\tau_u = 80$ **kPa**

c) Drained friction angle:

$\sigma'_3 = 150 - 54$
$\sigma'_3 = 96$ kPa
$\sigma'_1 = 310 - 54$
$\sigma'_1 = 256$ kPa



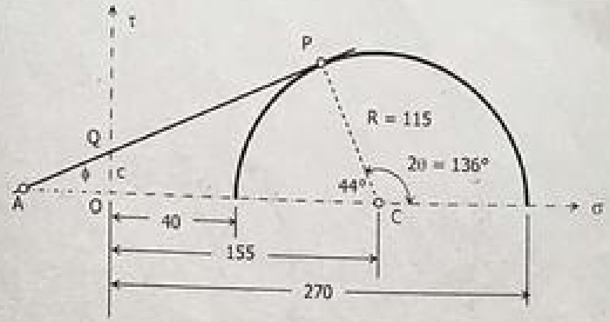
$R = 176 - 96$
$R = 80$

$\sin \phi_d = 80/176$
$\phi_d = 27.03°$

PROBLEM 07.12

In a triaxial test for a soil sample, when the principal stresses are 270 kPa and 40 kPa, the soil fails along a plane making an angle of 62° with the horizontal. What is the cohesion of the soil in kPa?

SOLUTION



$\theta = 68°$

In right triangle *APC*:

$\phi = 90° - 44°$
$\phi = 46°$

$$\sin \phi = \frac{R}{AC}$$

$$\sin 46° = \frac{115}{AC}$$

$AC = 159.87$

In right triangle *AOQ*:

$OA = AC - 155$
$OA = 4.869$

$$\tan \phi = \frac{c}{OA}$$

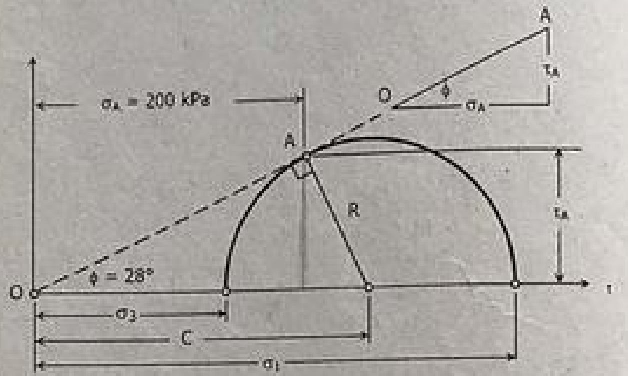
$c = 4.869 \tan 46°$
$c = 5.04$ **kPa**

PROBLEM 07.13

A cohesionless soil sample is subjected to a triaxial test. The critical state friction angle of the soil is 28° and the normal effective stress at failure is 200 kPa.

a) Determine the critical state shear stress
c) Determine the plunger stress
d) Determine the cell pressure

SOLUTION



a) Critical state shear stress, $\tau_A = \sigma_A \tan\phi$
Critical state shear stress, $\tau_A = 200 \tan 28°$
Critical state shear stress, τ_A = **106.34 kPa**

b) Plunger stress

$$\cos\phi = \frac{\sigma_A}{OA}$$

$$OA = \frac{200}{\cos 28°}$$

$$OA = 226.514$$

$$\tan\phi = \frac{R}{OA} = \frac{\tau_A}{\sigma_A}$$

$$\frac{R}{226.514} = \frac{106.34}{200}$$

$$R = 120.44 \text{ kPa}$$

$$C = \frac{OA}{\cos\phi}$$

$$C = \frac{226.514}{\cos 28°}$$

$$C = 256.54 \text{ kPa}$$

Plunger stress, $\sigma_d = 2R$
Plunger stress, $\sigma_d = 2(120.44)$
Plunger stress, σ_d = **240.88 kPa**

c) Cell pressure, $\sigma_3 = C - R$
Cell pressure, $\sigma_3 = 256.54 - 120.44$
Cell pressure, σ_3 = **136.1 kPa**

PROBLEM 07.14

An unconfined compression test was carried out on a saturated clay sample. The maximum load the clay sustained was 127 N and the vertical displacement is 0.8 mm. The size of the sample was 38 mm diameter × 76 mm long.

a) Calculate the axial strain of the soil sample
b) Calculate the major principal stress at failure
c) Calculate the undrained shear strength of the soil sample

SOLUTION

a) Axial strain, $\varepsilon = \dfrac{\Delta H}{H}$

Axial strain, $\varepsilon = \dfrac{0.8}{76}$

Axial strain, ε = **0.010526 mm/mm**

b) Sample area (initial), $A_o = \frac{\pi}{4}(0.038)^2$

Sample area (initial), $A_o = 0.001134 \text{ m}^2$

Sample area at failure, $A = \dfrac{A_o}{1-\varepsilon}$

Sample area at failure, $A = \dfrac{0.001134}{1-0.010526}$

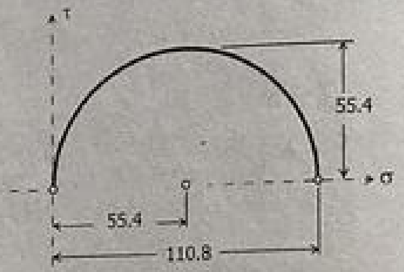
Sample area at failure, $A = 0.001146 \text{ m}^2$

Major principal stress at failure, $\sigma_1 = \dfrac{127}{0.001146}$

Major principal stress at failure, σ_1 = 110,814 Pa
Major principal stress at failure, $(\sigma_1)_U$ = **110.814 kPa**

c)

Note: $\sigma_3 = 0$



From the Mohr's circle shown, τ_u = **55.4 kPa**

Chapter 08

Lateral Earth Pressure

Chapter 04 was focused on the vertical stress caused by the weight of soil and structures above the soil surface. This chapter will focus on the lateral stress exerted by the soil mass on a structure, such as retaining walls, basement walls, and bulkheads.

Active earth pressure coefficient, K_a - the ratio between the lateral and vertical principal effective stresses when an earth retaining structure moves away (by a small amount) from a retained soil.

Passive earth pressure coefficient, K_p - the ratio between the lateral and vertical principal effective stresses when an earth retaining structure is forced against a soil mass.

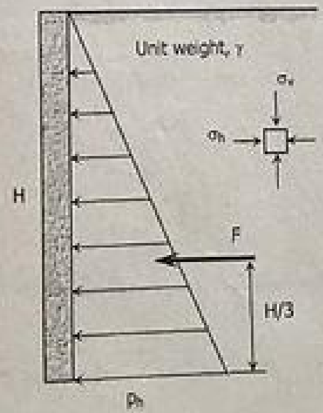


Figure 08.1

8.1 EARTH PRESSURE AT REST

If a retaining structure does not move either to the right or to the left of its initial position, the soil mass will be in a state of *elastic equilibrium*, meaning the horizontal strain is zero. The ratio of the horizontal stress to the vertical stress is called the *coefficient of earth pressure at rest*, K_o.

$$K_o = \frac{\sigma_h}{\sigma_v} = 1 - \sin\phi \qquad \text{Eq. 8.1}$$

where ϕ is the drained friction angle.

For dense sand backfill:

$$K_o = (1 - \sin\phi) + \left(\frac{\gamma_d}{\gamma_{d\,min}} - 1\right)5.5 \qquad \text{Eq. 8.2}$$

where

γ_d = actual compacted dry nit weight of the sand behind the wall

$\gamma_{d\,min}$ = dry nit weight of the sand in the loosest state

For fine-grained normally consolidated soils:

$$K_o = 0.44 + 0.42(PI\%/100) \qquad \text{Eq. 8.3}$$

For overconsolidated clays:

$$K_{o\,(overconsolidated)} = K_{o\,(normally\ consolidated)}\sqrt{OCR} \qquad \text{Eq. 8.4}$$

$$OCR = \frac{\text{Preconsolidation pressure}}{\text{Present effective overburden pressure}} \qquad \text{Eq. 8.5}$$

With reference to Figure 08.1:

$$p_o = K_o \gamma H \qquad \text{Eq. 8.6}$$

$$F = \tfrac{1}{2} K_o \gamma H^2 \qquad \text{Eq. 8.7}$$

8.2 RANKINES THEORY

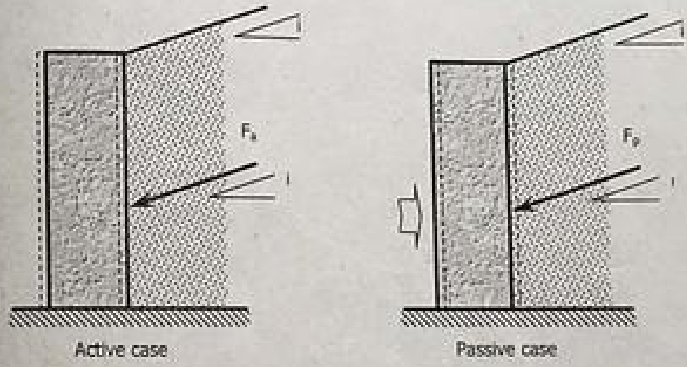


Figure 08.2 – Vertical face and inclined backfill

Coefficient of active pressure:

$$K_a = \frac{\cos i - \sqrt{\cos^2 i - \cos^2\phi}}{\cos i + \sqrt{\cos^2 i - \cos^2\phi}}\cos i \qquad \text{Eq. 8.8}$$

Coefficient of passive pressure:

$$K_p = \frac{\cos i + \sqrt{\cos^2 i - \cos^2\phi}}{\cos i - \sqrt{\cos^2 i - \cos^2\phi}}\cos i \qquad \text{Eq. 8.9}$$

8.2.1 RANKINE'S THEORY (FOR HORIZONTAL BACKFILL)

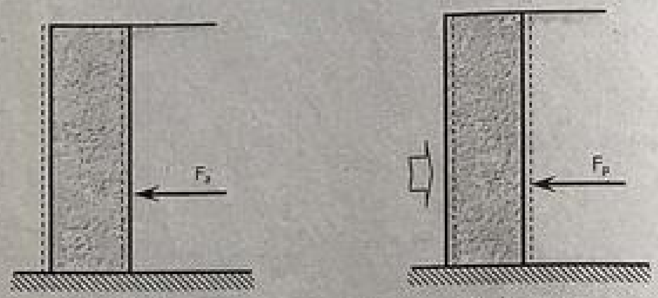


Figure 08.3 – Vertical face and horizontal backfill

Coefficient of active pressure:

When $i = 0$, Eq. 8.8 yields

$$K_a = \frac{1-\sin\phi}{1+\sin\phi} \qquad \text{Eq. 8.10}$$

Coefficient of passive pressure:

When $i = 0$, Eq. 8.9 yields

$$K_p = \frac{1+\sin\phi}{1-\sin\phi} \qquad \text{Eq. 8.11}$$

8.3 COULOMB'S THEORY

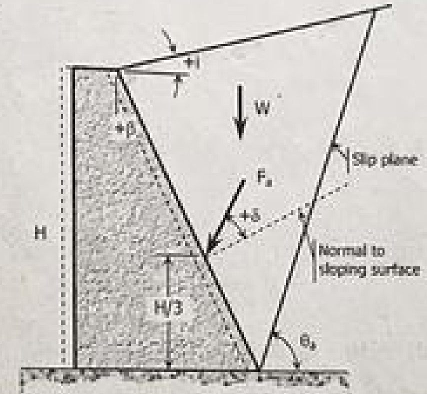


Figure 08.4 – Wall sloping face (active case)

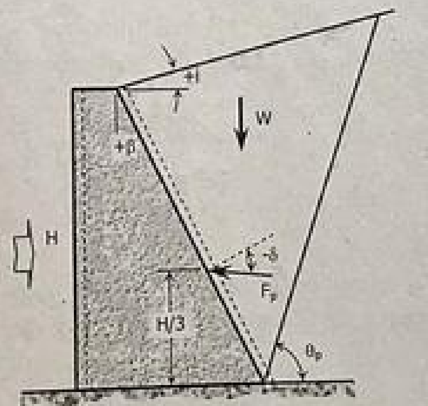


Figure 08.5 – Wall sloping face (passive case)

Because of frictional resistance to sliding at the face of the wall, F_a and F_p is inclined at an angle of δ with the normal to the wall, where δ is the angle of wall friction

8.3.1 ACTIVE PRESSURE COEFFICIENT

$$K_a = \frac{\cos^2(\phi-\beta)}{\cos^2\beta\cos(\beta+\delta)\left[1+\sqrt{\dfrac{\sin(\phi+\delta)\sin(\phi-i)}{\cos(\beta+\delta)\cos(\beta-i)}}\right]^2} \qquad \text{Eq. 8.12}$$

The inclination of the slip plane to the horizontal is:

$$\tan\theta_a = \frac{\sqrt{\sin\phi\cos\delta}}{\cos\phi\sqrt{\sin(\phi+\delta)}} + \tan\phi \qquad \text{Eq. 8.13}$$

The effect of wall friction on K_a is small, and is usually neglected. For $\delta = 0$:

$$K_a = \frac{\cos^2(\phi-\beta)}{\cos^3\beta\left[1+\sqrt{\dfrac{\sin\phi\sin(\phi-i)}{\cos\beta\cos(\beta-i)}}\right]^2} \qquad \text{Eq. 8.14}$$

When $i = 0$ and $\delta = 0$:

$$K_a = \frac{\cos^2(\phi-\beta)}{\cos^3\beta\left(1+\dfrac{\sin\phi}{\cos\beta}\right)^2} \qquad \text{Eq. 8.15}$$

For wall with vertical back face supporting granular soil backfill with horizontal surface (i.e. $i = 0°$ and $\beta = 0°$), Eq. 8.14 yields

$$K_a = \frac{1-\sin\theta}{1+\sin\theta} \qquad \text{Eq. 8.16}$$

Note that this is the same with Rankine's value given by Eq. 8.10.

8.3.2 PASSIVE PRESSURE COEFFICIENT

$$K_p = \frac{\cos^2(\phi+\beta)}{\cos^2\beta\cos(\beta-\delta)\left[1-\sqrt{\dfrac{\sin(\phi+\delta)\sin(\phi+i)}{\cos(\beta-\delta)\cos(\beta-i)}}\right]^2} \qquad \text{Eq. 8.17}$$

The inclination of the slip plane to the horizontal is:

$$\tan\theta_p = \frac{\sqrt{\sin\phi\cos\delta}}{\cos\phi\sqrt{\sin(\phi+\delta)}} - \tan\phi \qquad \text{Eq. 8.18}$$

For frictionless wall with vertical back face supporting granular soil backfill with horizontal surface (i.e. $\delta = 0°$, $i = 0°$ and $\beta = 0°$), Eq. 8.17 yields

$$K_a = \frac{1+\sin\theta}{1-\sin\theta} \qquad \text{Eq. 8.19}$$

Note that this is the same with Rankine's value given by Eq. 8.11.

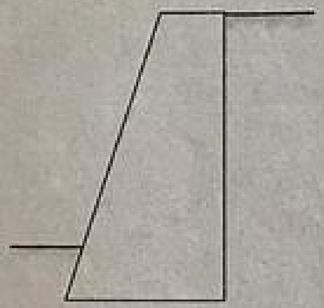
The critical value of θ is:

$$\theta = \theta_{cr} = 45° + \phi/2 \qquad \text{Eq. 8.20}$$

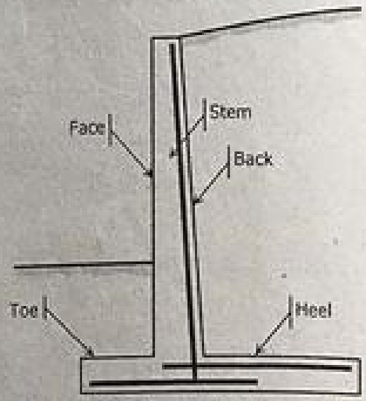
8.4 RETAINING WALLS

A retaining wall may be defined as a structure whose primary purpose is to prevent lateral movement of earth or some other material. For some special cases, as in basement walls or bridge abutments, a retaining wall may also have a function of supporting vertical loads.

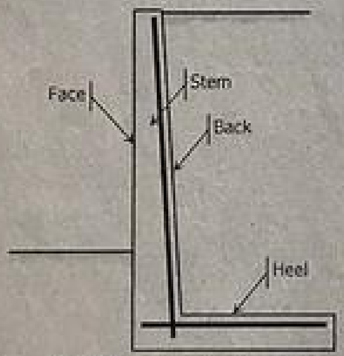
8.4.1 TYPES OF RETAINING WALLS



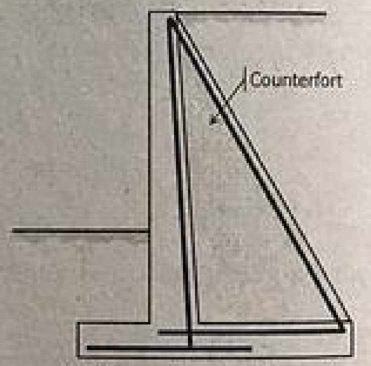
(a) Gravity retaining wall



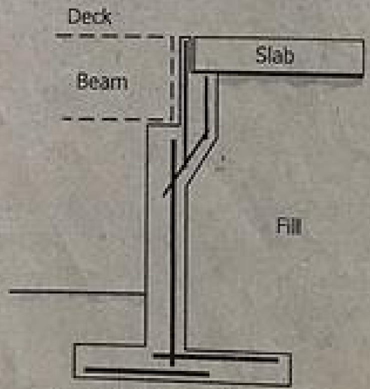
(b) T-shaped retaining wall



(c) L-shaped retaining wall



(d) Counterfort retaining wall



(e) Counterfort retaining wall

Figure 08.6 – Types of retaining walls

Gravity retaining wall, shown in Figure 08.6 (*a*), is usually built of plain concrete. This type of wall depends only on its own weight for stability, and hence, its height is subject to some definite practical limits.

Semi-gravity wall is in essence a gravity wall that has been given a wider base (a toe or heel or both) to increase its stability. Some reinforcement is usually necessary for this type of wall.

T-shaped wall as shown in Figure 08.6 (*b*) is perhaps the most common cantilever wall. For this type of wall, the weight of the earth in the back of the stem (the backfill) contributes to its stability.

L-shaped wall as shown in Figure 08.6 (*c*) is frequently used when property line restrictions forbid the use of a T-shaped wall. On the other hand, when it is not feasible (due to construction limitation) to excavate for a heel, a *reversed L-shape* may serve the need.

Counterfort retaining wall, as shown in Figure 08.6 (*d*) consists of three main components: base, stem, and intermittent vertical ribes called counterforts, which tie the base and the stem together. These ribs, which acts as tension ties, transform the stem and heel into continuous slabs supported on three sides - at two adjacent counterforts and at the base of the stem.

Buttressed wall is constructed by placing the ribs on the front face of the stem where they act in compression.

Bridge abutment as shown in Figure 08.6 (*e*) is a retaining wall, generally short and typically accompanied by wing walls.

8.5 ACTIVE PRESSURE ON WALL

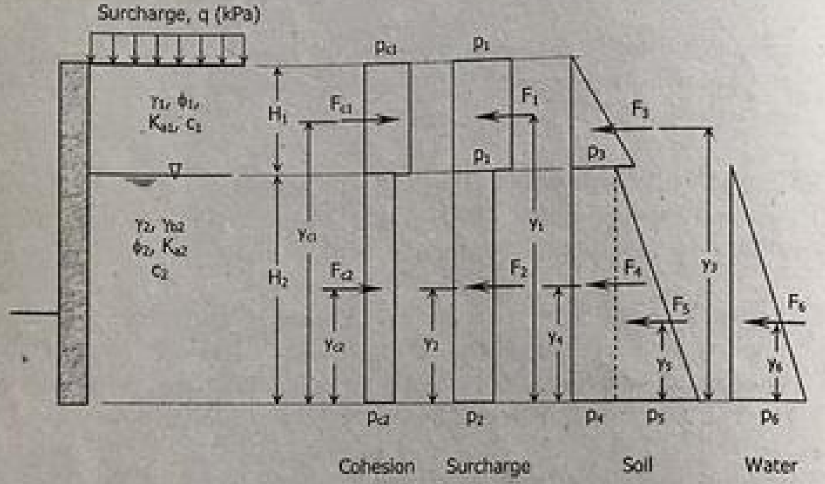


Figure 08.7 – Commonly assumed active pressure on retaining walls

With reference to Figure 08.7:

$$K_a = \frac{1-\sin\phi}{1+\sin\phi} \quad \text{(Rankine or Coulomb)}$$

Cohesion:

$$p_{c1} = 2c_1\sqrt{K_{a1}}$$
$$F_{C1} = p_{c1} \times H_1$$
$$p_{c2} = 2c_2\sqrt{K_{a2}}$$
$$F_{C2} = p_{c2} \times H_2$$

Surcharge:

$$p_1 = K_{a1}\, q$$
$$F_1 = p_1 \times H_1$$
$$p_2 = K_{a2}\, q$$
$$F_2 = p_2 \times H_2$$

Soil:

$$p_3 = K_{a1}\,\gamma_1 H_1$$
$$F_3 = \tfrac{1}{2} p_3 H_1$$
$$p_4 = K_{a2}\,\gamma_1 H_1$$
$$F_4 = p_4 H_2$$
$$p_5 = K_{a2}\,\gamma_{b2} H_2$$
$$F_5 = \tfrac{1}{2} p_5 H_2$$

Water:

$$p_6 = \gamma_w H_2$$
$$F_6 = \tfrac{1}{2} p_6 H_2$$

Total active force, $F_a = F_1 + F_2 + F_3 + F_4 + F_5 + F_6 - F_{C1} - F_{C2}$

Total active moment, $M_a = F_1 y_1 + F_2 y_2 + F_3 y_3 + F_4 y_4 + F_5 y_5 + F_6 y_6 - F_{C1} y_{c1} - F_{C2} y_{c2}$

8.6 PASSIVE PRESSURE ON WALL

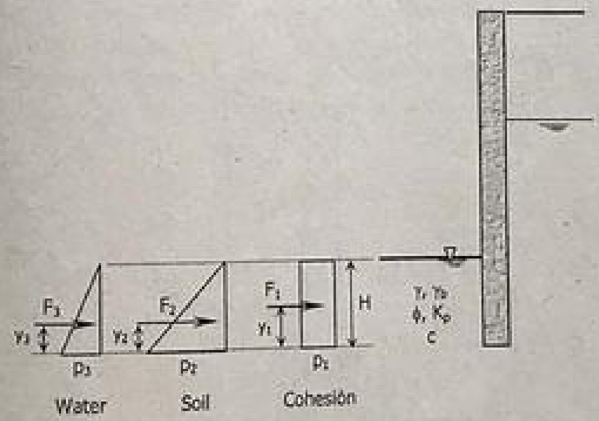


Figure 08.8 – Commonly assumed passive pressure on retaining walls

With reference to Figure 08.8:

$$K_p = \frac{1+\sin\phi}{1-\cos\phi} \quad \text{(Rankine or Coulomb)}$$

Cohesion:

$$p_1 = 2c\sqrt{K_p}$$

$$F_1 = p_1 H$$

Soil:

$$p_2 = K_p \gamma_b H$$

$$F_2 = \tfrac{1}{2} p_2 H$$

Water:

$$p_3 = \gamma_w H$$

$$F_3 = \tfrac{1}{2} p_3 H$$

Total passive resistance, $F_p = F_1 + F_2 + F_3$

Total passive moment, $M_p = F_1 y_1 + F_2 y_2 + F_3 y_3$

8.7 FACTORS OF SAFETY

The structural elements of the wall should be so proportioned that the following safety factors are realized:

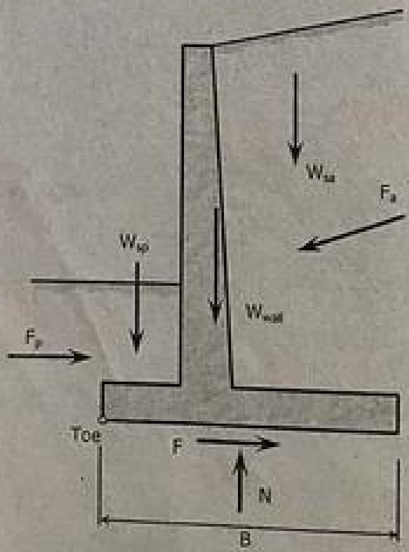


Figure 08.9 – Forces acting on wall

Factor of safety against sliding:

$$FS_s = \frac{\text{Resisting forces}}{\text{Active forces}} \qquad \text{Eq. 8.21}$$

For granular backfill, $FS_s \geq 1.5$
For cohesive backfill, $FS_s \geq 2.0$

Factor of safety against overturning about the toe:

$$FS_o = \frac{\text{Stabilizing moments}}{\text{Overturning moments}} \qquad \text{Eq. 8.22}$$

For granular backfill, $FS_s \geq 1.5$
For cohesive backfill, $FS_s \geq 2.0$

The horizontal components of the lateral forces tends to force the wall to slide along its base. The resisting force is povided by the horizontal forces composed of friction and adhesion, and by passive resistance of soil in front of the wall. The passive resistance is not to be counted on if there is a chance that the soil in front of the wall may be eroded or excavated during the life of the wall.

The force F at the base of the wall consist of the friction and cohesion. It is given by:

$$F = \mu N + c_b B \qquad \text{Eq. 8.23}$$

where N is the normal reaction, μ is the coefficient of friction, c_b is the base cohesion, and B is the base width of wall. Commonly assumed values of μ and c_b are as follows:

$$\tan\phi > \mu > (2/3)\tan\phi \qquad \text{Eq. 8.24}$$

$$0.5c \leq c_b \leq 0.75c \qquad \text{Eq. 8.25}$$

8.8 PRESSURE DISTRIBUTION AT BASE OF WALL

The actual bearing pressure on the base of the wall is a combination of normal forces and the effects of moments.

$$R_y = \Sigma F_y \qquad \text{Eq. 8.26}$$

$$R_y \bar{x} = RM - OM \qquad \text{Eq. 8.27}$$

$$e = \frac{B}{2} - \bar{x} \qquad \text{Eq. 8.28}$$

where:

RM = righting or stabilizing moments
OM = overturning moments

Note that in computing RM and R_y, the passive resistance is not to be counted on if there is a chance that the soil in front of the wall may be eroded or excavated during the life of the wall.

When $e \leq B/6$

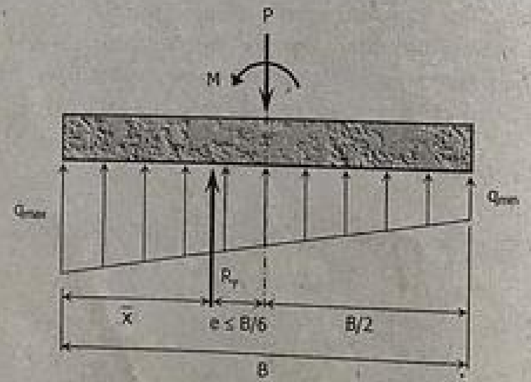


Figure 08.10 – Stress distribution at base of wall when e ≤ B/6

Considering 1 m length of wall:

$$q_{\substack{max \\ min}} = \frac{R_y}{B}\left(1 \pm \frac{6e}{B}\right) \qquad \text{Eq. 8.29}$$

When $e > B/6$

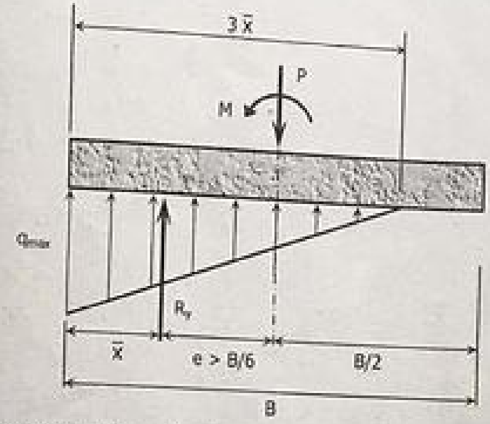


Figure 08.11 – Stress distribution at base of wall when e > B/6

Considering 1 m length of wall:

$$q_{max} = \frac{2R_y}{3\bar{x}} \qquad \text{Eq. 8.30}$$

8.9 LATERAL PRESSURE ON RETAINING WALLS DUE TO POINT-LOAD SURCHARGE

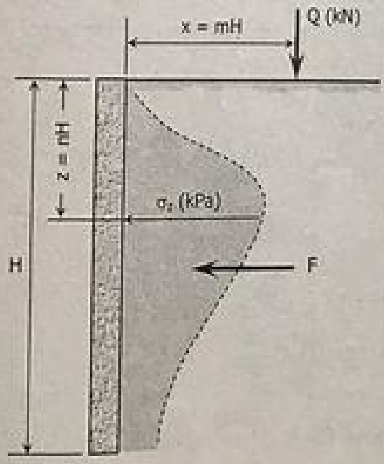


Figure 08.12 – Stress on wall caused by a point load

The lateral stress on the wall induced by a point-load surcharge is given by:

For $m > 0.4$

$$\sigma_x = \frac{1.77Q}{H^2}\frac{m^2n^2}{(m^2+n^2)^3} \qquad \text{Eq. 8.31}$$

For $m \le 0.4$

$$\sigma_x = \frac{0.28Q}{H^2}\frac{n^2}{(0.16+n^2)^3} \qquad \text{Eq. 8.32}$$

where Q is the point load (kN or lbs), H is the height of wall (m or feet), and σ_x is the stress (kPa or psf)

The force F per unit length of wall caused by the point load can be obtained by approximating the area of the shaded portion using trapezoidal rule or Simpson's one-third rule.

8.10 LATERAL PRESSURE ON RETAINING WALLS DUE TO LINE-LOAD SURCHARGE

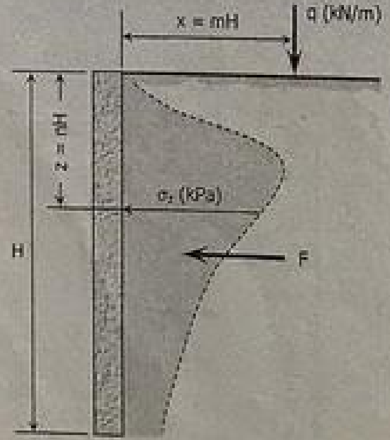


Figure 08.13 – Stress on wall caused by a line load

The lateral stress on the wall induced by a line-load surcharge is given by:

For $m > 0.4$

$$\sigma_x = \frac{4q}{\pi H}\frac{m^2n}{(m^2+n^2)^2} \qquad \text{Eq. 8.33}$$

For $m \le 0.4$

$$\sigma_x = \frac{0.203q}{H}\frac{n}{(0.16+n^2)^2} \qquad \text{Eq. 8.34}$$

where q is the line load (kN/m or lbs/ft), H is the height of wall (m or feet), and σ_x is the stress (kPa or psf)

The force F per unit length of wall caused by the strip load can be obtained by approximating the area of the shaded portion using trapezoidal rule or Simpson's one-third rule.

8.11 LATERAL PRESSURE ON RETAINING WALLS DUE TO STRIP-LOAD SURCHARGE

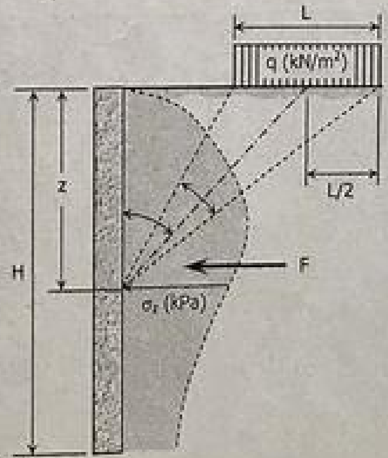


Figure 08.14 – Stress on wall caused by a strip load

The lateral stress on the wall induced by a strip-load surcharge is given by:

$$\sigma_x = \frac{2q}{H}(\beta - \sin\beta\cos 2\alpha) \qquad \text{Eq. 8.35}$$

The force F per unit length of wall caused by the strip load can be obtained by approximating the area of the shaded portion using trapezoidal rule or Simpson's one-third rule, or by integration of σ_x with limits from 0 to H.

ILLUSTRATIVE PROBLEMS

PROBLEM 08.1

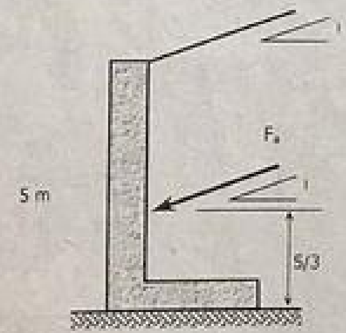
A 5-m tall cantilever retaining wall retains soil having the following properties:

Cohesion, $c = 0$
Unit weight = 19.8 kN/m³
Angle of internal friction, $\phi = 30°$

The ground surface behind the wall is inclined at a slope of 3 horizontal to 1 vertical, and the wall has moved sufficiently to develop the active condition. Use Rankine's Theory and consider 1 m length of wall.

a) Determine the coefficient of active pressure
b) Determine the total active force
c) Determine the overturning moment on the wall

SOLUTION



a) Coefficient of active pressure

Inclined Backfill (Rankine Theory)

$$K_a = \frac{\cos i - \sqrt{\cos^2 i - \cos^2\phi}}{\cos i + \sqrt{\cos^2 i - \cos^2\phi}}\cos i$$

$$\tan i = 1/3$$
$$i = 18.435°$$

$$K_a = \frac{\cos 18.435° - \sqrt{\cos^2 18.435° - \cos^2 30°}}{\cos 18.435° + \sqrt{\cos^2 18.435° - \cos^2 30°}}\cos 18.435°$$

$$K_a = \mathbf{0.42}$$

b) Active pressure

$F_a = ½ \gamma K_a H^2$

$F_a = ½(19.8)(0.42)(5)^2$

$F_a =$ **103.95 kN**

c) Overturning moment

$OM = F_a \cos i\ (5/3)$

$OM = 103.95 \cos 18.435°\ (5/3)$

$OM =$ **164.36 kN-m**

PROBLEM 08.2

A 3 m high vertical retaining wall is shown in Figure 08.15

a) What is the effective vertical stress at the base of the wall?
b) What is the total active force acting on the wall?
c) What is the total active moment on the wall?

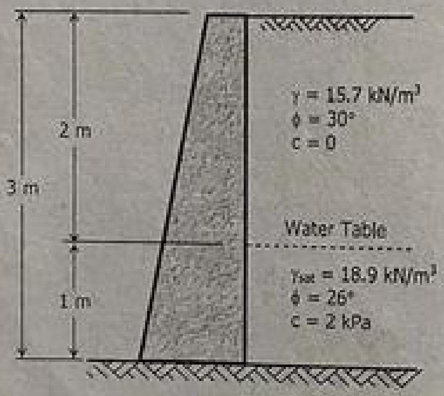


Figure 08.15

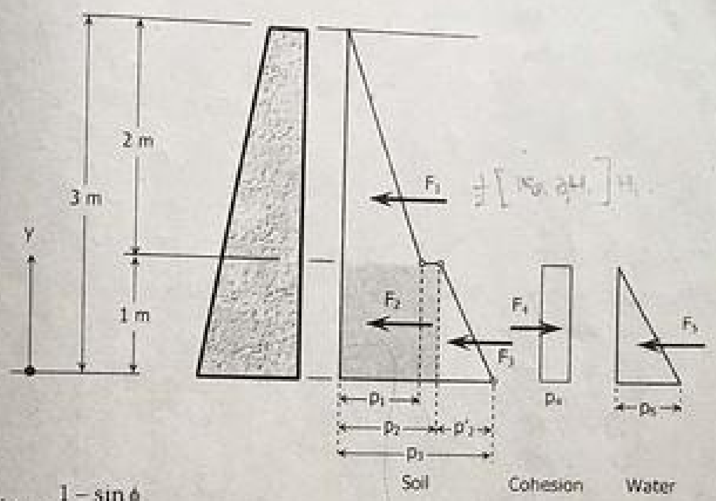
SOLUTION

a) Effective vertical stress at the bottom of wall:

$p_E = (18.9 - 9.81)(1) + 15.7(2)$

$p_E =$ **40.49 kPa**

b & c)



$K_{a1} = \dfrac{1 - \sin\phi}{1 + \sin\phi}$

$K_{a1} = \dfrac{1 - \sin 30°}{1 + \sin 30°}$

$K_{a1} = 1/3$

$K_{a2} = \dfrac{1 - \sin 26°}{1 + \sin 26°}$

$K_{a2} = 0.39$

$p_1 = K_{a1}\,\gamma\,H_1$

$p_1 = (1/3)(15.7)(2)$

$p_1 = 10.467$ kPa

$F_1 = ½\,p_1\,H_1$

$F_1 = ½(10.467)(2)$

$F_1 = 10.467$ kN

$y_1 = 1 + 2/3$

$y_1 = 5/3$

cohesion!
$2c\sqrt{K}$

$p_2 = K_{a2}\,\gamma\,H_1$

$p_2 = (0.39)(15.7)(2)$

$p_2 = 12.26$ kPa

$F_2 = p_2\,H_2$

$F_2 = 12.26(1)$

$F_2 = 12.26$ kN

$y_2 = 0.5$

$p'_2 = K_{a2}\,\gamma_{b2}\,H_2$
$p'_2 = (0.39)(18.9 - 9.81)(1)$
$p'_2 = 3.545$ kPa
$F_3 = ½\,p'_2\,H_2$
$F_3 = ½(3.545)(1)$
$F_3 = 1.773$ kN
$y_3 = 1/3$

$p_4 = -2c\sqrt{K_{a2}}$
$p_4 = -2(2)\sqrt{0.39}$
$p_4 = -2.498$ kPa
$F_4 = p_4\,H_2$
$F_4 = -2.498(1)$
$F_4 = -2.498$ kN
$y_4 = 0.5$

$p_5 = \gamma_w\,H_2$
$p_5 = 9.81(1)$
$p_5 = 9.81$ kPa
$F_5 = ½\,p_5\,H_2$
$F_5 = ½(9.81)(1)$
$F_5 = 4.905$ kN
$y_5 = 1/3$

Total active pressure, $F_a = F_1 + F_2 + F_3 + F_4 + F_5$
Total active pressure, $F_a = 10.467 + 12.26 + 1.77 - 2.498 + 4.905$
Total active pressure, $F_a =$ **26.904 kN**

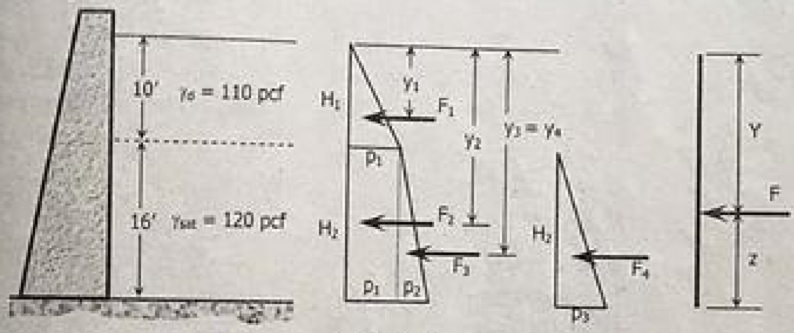
Total active moment:
$M_a = F_1\,y_1 + F_2\,y_2 + F_3\,y_3 + F_4\,y_4 + F_5\,y_5$
$M_a = 10.467(5/3) + 12.26(0.5) + 1.773(1/3) - 2.498(0.5) + 4.905(1/3)$
$M_a =$ **24.552 kN-m**

PROBLEM 08.3 (CE MAY 2002)

A vertical retaining wall retains 26-ft deep of soil. The soil has a dry unit weight of 110 pcf above water table and 120 pcf below water table. The ground water table is 10 feet below the ground surface. The angle of internal friction of the soil is 35°.

a) Calculate the total active pressure acting on the wall in pounds per foot width.
b) Determine the location of the resultant pressure from the ground surface
c) Determine the overturning moment caused by the active pressure acting on the wall per foot width.

SOLUTION



Effective stress diagram　Pore water stress diagram　Resultant force

$\gamma_b = \gamma_{sat} - \gamma_w$
$\gamma_b = 120 - 62.4$
$\gamma_b = 57.6$ pcf

$$K_a = \frac{1-\sin\phi}{1+\sin\phi}$$

$$K_a = \frac{1-\sin 35°}{1+\sin 35°}$$

$K_a = 0.271$

Part *a*:

Considering 1-foot length of wall:

Pressure:
$p_1 = \gamma\,K_a\,H_1$
$p_1 = 110(0.271)(10)$
$p_1 = 298.1$ psf

$p_2 = \gamma_b\,K_a\,H_2$
$p_2 = 57.6(0.271)(16)$
$p_2 = 249.75$ psf

$p_3 = \gamma_w H_2$
$p_3 = 62.4(16)$
$p_3 = 998.4$ psf

Total pressure:
$F_1 = ½\, p_1 H_1$
$F_1 = ½(298.1)(10)$
$F_1 = 1490.5$ lbs

$F_2 = p_1 H_2$
$F_2 = 298.1(16)$
$F_2 = 4769.6$ lbs

$F_3 = ½\, p_2 H_2$
$F_3 = ½(249.75)(16)$
$F_3 = 1998$ lbs

$F_4 = ½\, p_3 H_2$
$F_4 = ½(998.4)(16)$
$F_4 = 7987.2$ lbs

Total force, $F = F_1 + F_2 + F_3 + F_4$
$F = 1490.5 + 4769.6 + 1998 + 7987.2$
$F = $ **16,245.3 lbs**

Location of F:
$y_1 = (2/3)(10)$
$y_1 = 20/3$ ft.

$y_2 = 10 + 16/2$
$y_2 = 18$ ft.

$y_3 = y_4 = 10 + (2/3)(16)$
$y_3 = y_4 = 20.667$ ft.

Taking moment of forces about the top:
$F\,Y = F_1 y_1 + F_2 y_2 + F_3 y_3 + F_4 y_4$
$16{,}245.3\ Y = 1490.5(20/3) + 4769.6(18) + 1998(20.667) + 7987.2(20.667)$
$Y = 18.6$ ft

Part *c*:
$z = 26 - 18.6$
$z = 7.4$ ft

Overturning moment $= F\,z = 16{,}245.3\ (7.4)$
Overturning moment = **120,215.22 ft-lb**

PROBLEM 08.4 (CE NOVEMBER 2003, MAY 2004)

A retaining wall 8 m high supports a cohesionless soil having a dry density of 1600 kg/m³, angle of shearing resistance is 33° and void ratio of 0.68. The surface of the soil is horizontal and level with the top of the wall. Neglect wall friction and use Rankine's formula for active pressure of a cohesionless soil.

a) Determine the total earth thrust on the wall in kN per linear meter if the soil is dry.
b) Determine the thrust on the wall in kN per lineal meter if owing to inadequate drainage, it is waterlogged to a level 3.5 m below the surface.
c) Determine the height above the base of the wall where the thrust acts during the waterlogged condition.

SOLUTION

a) $K_a = \dfrac{1 - \sin\phi}{1 + \sin\phi}$

$K_a = \dfrac{1 - \sin 33°}{1 + \sin 33°}$

$K_a = 0.2948$

$F_a = ½\, K_a\, \gamma\, H^2$
$\gamma = \rho g$
$\gamma = 1600(9.81)$
$\gamma = 15{,}696\ \text{N/m}^3$
$\gamma = 15.696\ \text{kN/m}^3$

$F_a = ½(0.2948)(15.696)(8)^2$
$F_a = $ **148.1 kN**

b & c)
Solving for G and γ_b:

$\gamma_d = \dfrac{G}{1+e}\gamma_w$

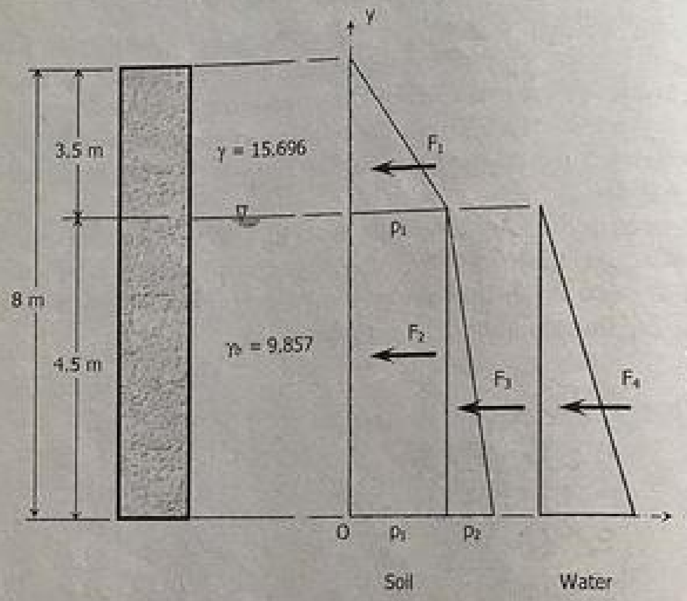
$15.696 = \dfrac{G}{1+0.68}(9.81)$

$G = 2.688$

$\gamma_b = \dfrac{G-1}{1+e}\gamma_w$

$\gamma_b = \dfrac{2.688-1}{1+0.68}(9.81)$

$\gamma_b = 9.857\ \text{kN/m}^3$



$p_1 = K_a \gamma H$
$p_1 = 0.2948(15.696)(3.5)$
$p_1 = 16.195$ kPa
$F_1 = ½(16.195)(3.5)(1)$
$F_1 = 28.34$ kN
$y_1 = 4.5 + 3.5/3$
$y_1 = 5.667$ m
$F_2 = 16.195(4.5)(1)$
$F_2 = 72.88$ kN
$y_2 = ½(4.5)$
$y_2 = 2.25$ m
$p_2 = K_a \gamma_b H$
$p_2 = 0.2948(9.857)(4.5)$
$p_2 = 13.076$ kPa
$F_3 = ½(13.076)(4.5)(1)$
$F_3 = 29.421$ kN
$y_3 = 4.5/3$
$y_3 = 1.5$ m
$F_4 = ½ \gamma_w H^2$
$F_4 = ½(9.81)(4.5)^2$
$F_4 = 99.33$ kN
$y_4 = 4.5/3$
$y_4 = 1.5$ m

Total thrust, $F_T = F_1 + F_2 + F_3 + F_4$
Total thrust, $F_T = 28.34 + 72.88 + 29.421 + 99.33$
Total thrust, $F_T =$ **229.97 kN**

Location:
$F_T \times \bar{y} = \Sigma Fy$
$229.97\,\bar{y} = 28.34(5.667) + 72.88(2.25) + 29.421(1.5) + 99.33(1.5)$
$\bar{y} =$ **2.251 m**

PROBLEM 08.5

A frictionless retaining wall is shown in Figure 08.16. Consider 1 m length of wall.

a) Determine the total horizontal passive pressure on the backfill at the bottom of the wall
b) Determine the passive resistance on the backfill
c) Determine the location of the resultant passive force from the bottom of the wall

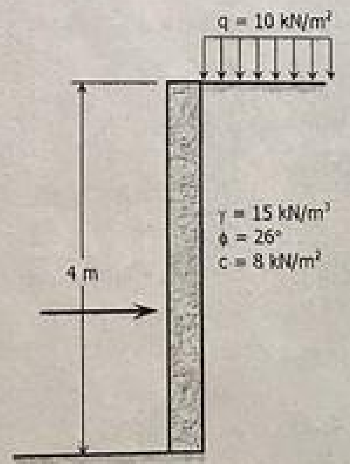
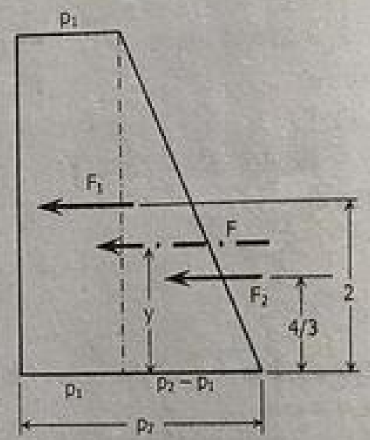


Figure 08.16

SOLUTION



Pressure diagram

$$K_p = \frac{1+\sin 26°}{1-\sin 26°}$$
$$K_p = 2.56$$

The passive pressure at any point is $p = K_p \gamma H + K_p q + 2c\sqrt{K_p}$

At $H = 0$:

$$p_1 = 2.56(10) + 2(8)\sqrt{2.56}$$
$$p_1 = 51.2 \text{ kPa}$$

@ $H = 4$ m

$$p_2 = 2.56(15)(4) + 2.56(10) + 2(8)\sqrt{2.56}$$
$$p_2 = \mathbf{204.8 \text{ kPa}}$$

$$p_2 - p_1 = 153.6 \text{ kPa}$$

$$F_1 = 51.2(4)(1)$$
$$F_1 = 204.8 \text{ kN}$$

$$F_2 = ½(153.6)(4)(1)$$
$$F_2 = 307.2 \text{ kN}$$

$$F = F_1 + F_2$$
$$F = \mathbf{512 \text{ kN}}$$

Location of F:

$$F \times y = F_1(2) + F_2(4/3)$$
$$512y = 204.8(2) + 307.2(4/3)$$
$$y = \mathbf{1.6 \text{ m}}$$

PROBLEM 08.6

A retaining wall is shown in Figure 08.17.

a) Determine the total active pressure before tensile crack occurs:
b) Determine the total active pressure after tensile crack occurs:

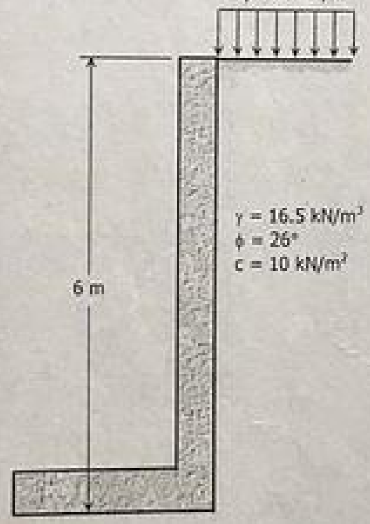


Figure 08.17

SOLUTION

$$K_a = \frac{1-\sin\phi}{1+\sin\phi}$$
$$K_a = 0.39$$

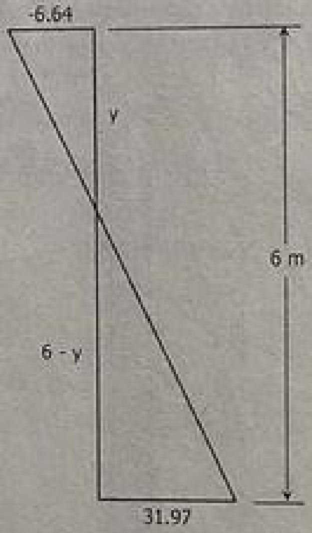
$$p_a = K_a \gamma H + K_a q - 2c\sqrt{K_a}$$

At $H = 0$ (at the ground surface)

$$p_a = 0 + 0.39(15) - 2(10)\sqrt{0.39}$$
$$p_a = -6.64 \text{ kN/m}^2$$

At $H = 6$:

$$p_a = 0.39(16.5)(6) + 0.39(15) - 2(10)\sqrt{0.39}$$
$$p_a = 31.97 \text{ kN/m}^2$$



$$\frac{y}{6.64} = \frac{6}{6.64 + 31.97}$$
$$y = 1.03 \text{ m}$$
$$6 - y = 4.97 \text{ m}$$

Depth of tensile crack = 1.03 m

Active pressure before tensile crack:

$$F_a = \tfrac{1}{2}K_a\gamma H^2 + K_a qH - 2c\sqrt{K_a}\ H$$
$$F_a = \tfrac{1}{2}(0.39)(16.5)(6)^2 + 0.39(15)(6) - 2(10)\sqrt{0.39}\ (6)$$
$$F_a = 76 \text{ kN}$$

or:

$$F_a = \tfrac{1}{2}(31.97)(4.97)(1) - \tfrac{1}{2}(6.64)(1.03)(1)$$
$$F_a = 76 \text{ kN}$$

Active pressure after tensile crack occurs

$$F_a = \tfrac{1}{2}(31.97)(4.97)(1)$$
$$F_a = 79.45 \text{ kN}$$

PROBLEM 08.7

Analyze the stability of the frictionless wall shown in Figure 08.18.

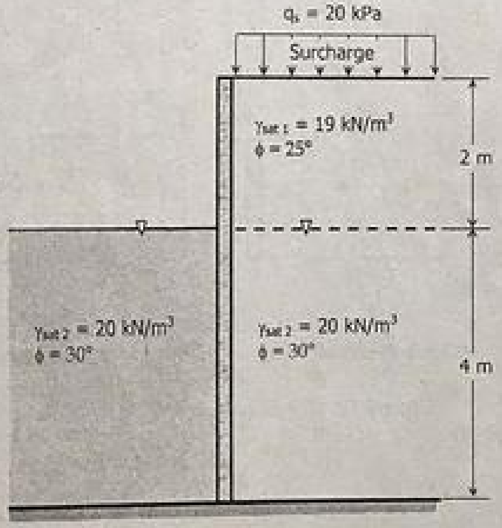


Figure 08.18

SOLUTION

With reference to Figure 08.19:
Note: y = distance of the force from the bottom of wall.

<u>Active Pressure</u>

$$p_1 = q_s \times K_{a1}$$
$$K_{a1} = \frac{1 - \sin 25^\circ}{1 + \sin 25^\circ}$$
$$K_{a1} = 0.40586$$

$p_1 = 20(0.40586)$
$p_1 = 8.12$ kPa

$F_1 = p_1(2)(1)$
$F_1 = 8.12(2)(1)$
$F_1 = 16.24$ kN
$y_1 = 4 + 1 = 5$ m

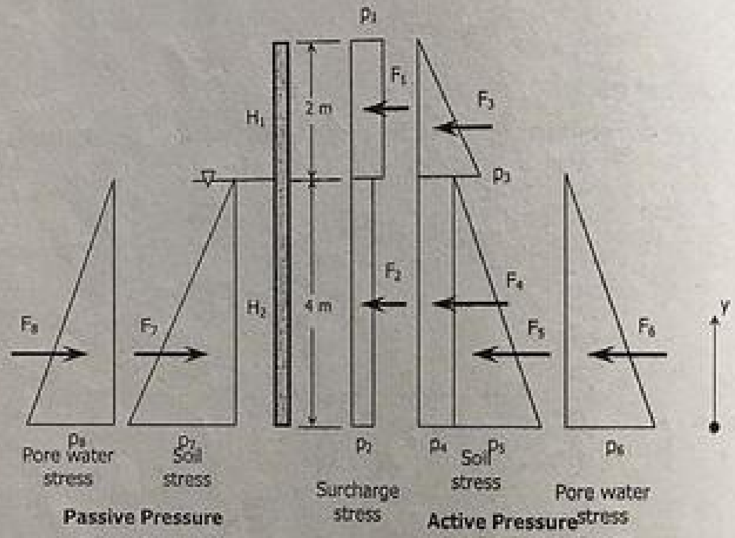


Figure 08.19 – Forces acting on the wall

$p_3 = \gamma_{sat\,1}\, K_{a1}\, H_1$
$p_3 = 19(0.40586)(2)$
$p_3 = 15.42$ kN

$F_3 = ½(15.42)(2)(1)$
$F_3 = 15.42$ kN
$y_3 = 4 + 2/3$
$y_3 = \frac{14}{3}$ m

$p_2 = q_s\, K_{a2}$

$$K_{a2} = \frac{1 - \sin 30°}{1 + \sin 30°}$$

$K_{a2} = \frac{1}{3}$

$p_2 = 20\,\frac{1}{3}$
$p_2 = 6.67$ kPa
$F_2 = 6.67(4)(1)$
$F_2 = 26.68$ kN
$y_2 = 2$ m

$p_4 = \gamma_{sat\,1}\, K_{a2}\, H_1$
$p_4 = 19(\frac{1}{3})(2)$
$p_4 = 12.67$ kPa
$F_4 = 12.67(4)(1)$
$F_4 = 50.68$ kN
$y_4 = 2$ m

$p_5 = \gamma_{b\,2}\, K_{a2}\, H_2$
$p_5 = (20 - 9.81)(\frac{1}{3})(4)$
$p_5 = 13.59$ kPa
$F_5 = ½(13.59)(4)(1)$
$F_5 = 27.18$ kN
$y_5 = \frac{4}{3}$ m

$p_6 = \gamma_w\, H_2$
$p_6 = 9.81(4)$
$p_6 = 39.24$ kPa
$F_6 = ½(39.24)(4)(1)$
$F_6 = 78.48$ kN
$y_6 = \frac{4}{3}$ m

Total active force:
$F_a = F_1 + F_2 + F_3 + F_4 + F_5 + F_6$
$F_a = 16.24 + 26.68 + 15.42 + 50.68 + 27.18 + 78.48$
$F_a = 214.68$ kN

Active moment
$M_a = F_1y_1 + F_2y_1 + F_3y_1 + F_4y_1 + F_5y_1 + F_6y_1$
$M_a = 16.24(5) + 26.68(2) + 15.42(\frac{14}{3}) + 50.68(2)$
$+ 27.18(\frac{4}{3}) + 78.48(\frac{4}{3})$
$M_a = 448.76$ kN-m

Passive Pressure

$p_7 = \gamma_{b2} K_{p2} H_2$

$K_{p2} = \dfrac{1+\sin 30°}{1-\sin 30°}$

$K_{p2} = 3$

$p_7 = (20 - 9.81)(3)(4)$

$p_7 = 122.28$ kPa

$F_7 = ½(122.28)(4)(1)$

$F_7 = 244.56$ kN

$y_7 = \frac{4}{3}$ m

$p_8 = p_6 = 39.24$ kPa

$F_8 = F_6 = 78.48$ kN

$y_8 = \frac{4}{3}$ m

Total passive force:

$F_p = F_7 + F_8$

$F_p = 323.04$ kN

Passive moments

$M_p = F_7\, y_7 + F_8\, y_8$

$M_p = 430.72$ kN-m

Ratio of forces $= \dfrac{F_p}{F_a}$

Ratio of forces $= \dfrac{323.04}{214.68}$

Ratio of forces $= 1.5 > 1$ (OK)

Ratio of moments $= \dfrac{M_p}{M_a}$

Ratio of moments $= \dfrac{430.72}{448.76}$

Ratio of moments $= 0.96 < 1$ (unstable)

PROBLEM 08.8 (CE MAY 2002)

A solid concrete retaining wall is shown in Figure 08.20. The fill behind the wall has a unit weight of 110 pcf whose active soil pressure may be assumed equivalent to a fluid pressure of 30 psf per foot. The passive pressure may be assumed equivalent to a fluid pressure of 300 psf per foot. The live load surcharge behind the wall is equivalent to an additional of 2 feet of fill. Assume unit weight of concrete = 150 pcf. Consider 1 ft. length of wall.

a) Determine the total active pressure acting on the wall in lbs.
b) Determine the overturning moment about the toe in ft-lb.
c) Determine the factor of safety against overturning.

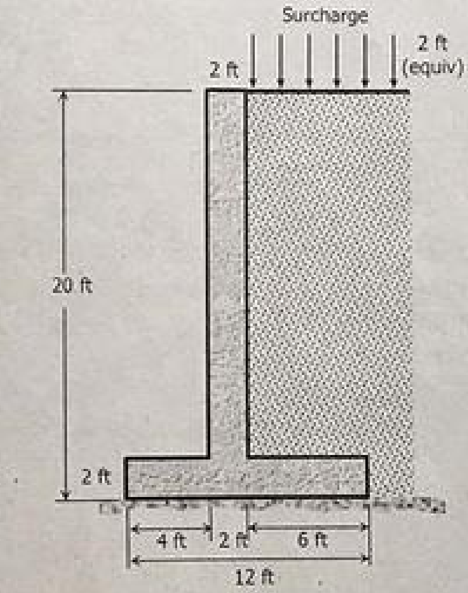


Figure 08.20

SOLUTION

$p_1 = 30(2)$

$p_1 = 60$ psf

$p_2 = 30(20)$

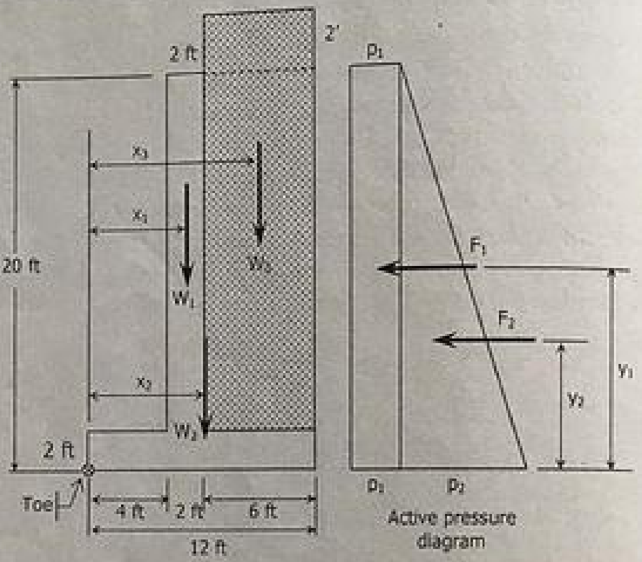
$p_2 = 600$ psf

Active pressure:

$F_1 = p_1(20)$

$F_1 = (60)(20)$

$F_1 = 1200$ lbs



$F_2 = ½ p_2 (20)$
$F_2 = ½(600)(20)$
$F_2 = 6000$ lbs

Total active pressure, $F = F_1 + F_2$
Total active pressure, $F =$ **7200 lbs**

Overturning moment:
$y_1 = 20/2$
$y_1 = 10$ ft
$y_2 = 20/3$ ft

$OM = F_1\, y_1 + F_2\, y_2$
$OM = 1200(10) + 6000(20/3)$
$OM =$ **52,000 ft-lb**

Righting moment:
$W_1 = \gamma_c\, V_1$
$W_1 = 150(18 \times 2 \times 1)$
$W_1 = 5400$ lbs

$W_2 = \gamma_c\, V_2$
$W_2 = 150(12 \times 2 \times 1)$
$W_2 = 3600$ lbs

$W_3 = \gamma_{soil}\, V_{soil}$
$W_3 = 110(20 \times 6 \times 1)$
$W_3 = 13200$ lbs

$x_1 = 4 + 1 = 5$ ft
$x_2 = ½(12) = 6$ ft.
$x_3 = 12 - ½(6) = 9$ ft.

$RM = W_1\, x_1 + W_2\, x_2 + W_3\, x_3$
$RM = 5400(5) + 3600(6) + 13200(9)$
$RM = 167{,}400$ ft-lb

Factor of safety against overturning $= \dfrac{RM}{OM}$

Factor of safety against overturning $= \dfrac{167{,}400}{52{,}000}$

Factor of safety against overturning = **3.22**

PROBLEM 08.9 (CE MAY 2001)

The ties for the anchored bulkhead shown in Figure 08.21 are located 1.2 m from the top of the sheet piling and are spaced 5 m on center. The end of the ties is secured to two anchored piles raked as shown. The active earth pressure may be assumed equivalent to a fluid pressure of 5 kN/m² per meter and the maximum passive pressure may be assumed equivalent to a fluid pressure of 65 kN/m² per meter. Calculate the compressive force in the anchor pile.

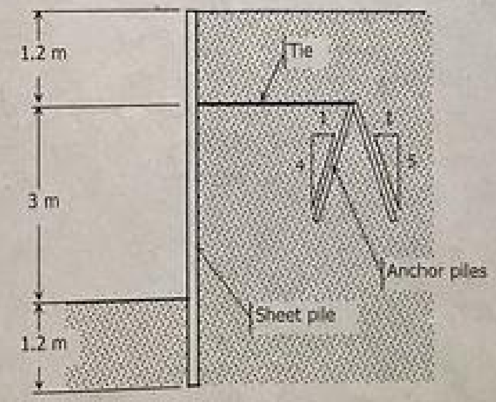
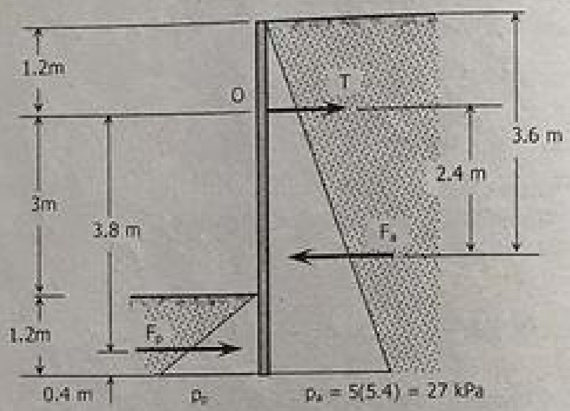


Figure 08.21

SOLUTION



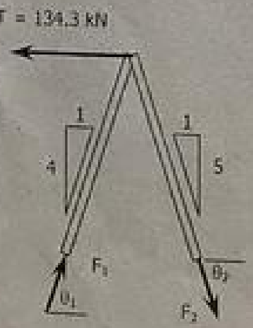
$F_a = ½(27)(5.4)(5)$
$F_a = 364.5$ kN

$[\Sigma M_O = 0]$
$F_p(3.8) = 364.27(2.4)$
$F_p = 230.2$ kN

Check for p_p:
$½(p_p)(1.2)(5) = 230.2$
$p_p = 76.73$ kPa

Maximum $p_p = 65(1.2) = > 76.73$ kPa (OK)

$[\Sigma F_H = 0]$
$T + F_p = F_a$
$T = 364.5 - 230.2$
$T = 134.3$ kN



From the diagram shown:
$\theta_1 = \tan^{-1}(4/1)$
$\theta_1 = 75.96°$
$\theta_2 = \tan^{-1}(5/1)$
$\theta_2 = 78.69°$

$[\Sigma F_v = 0]$
$F_1 \sin\theta_1 = F_2 \sin\theta_2$
$F_2 = 0.989\,F_1$

$[\Sigma F_H = 0]$
$F_1 \cos\theta_1 + F_2 \cos\theta_2 = T$
$F_1 \cos 75.96° + 0.989\,F_1 \cos 78.69° = 134.3$
$F_1 = 307.6$ **kN** (compression)

PROBLEM 08.10

A sheet pile wall is a shown in Figure 08.22.

a) Determine the minimum value of embedment depth d for stability, in meters.
b) Using the minimum value of d, determine the tension in the tie rod per meter length of pile.

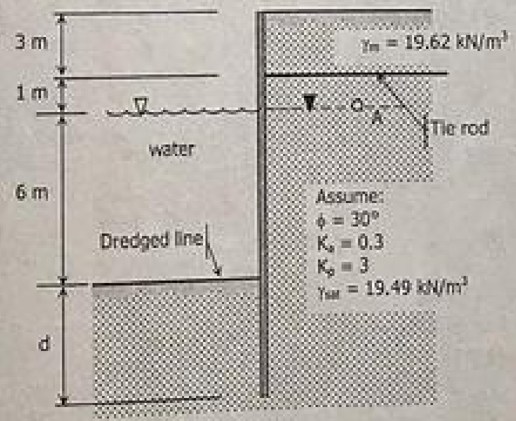
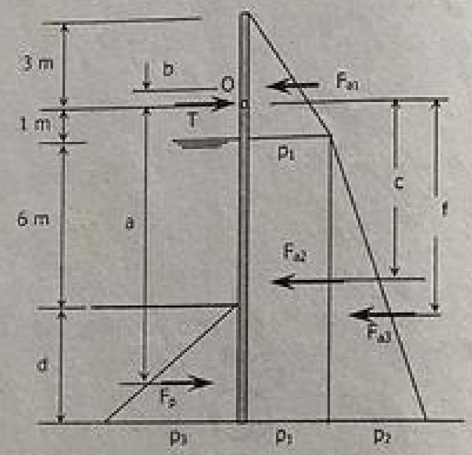


Figure 08.22

SOLUTION



Note: The water pressure may not be included in the analysis. It will cancel out because it appears at the same level on both sides of the wall

Active pressures

$p_1 = K_a \gamma H_1$

$p_1 = 0.3(19.62)(4)$

$p_1 = 23.544$ kPa

$F_{a1} = ½(23.544)(4 \times 1)$

$F_{a1} = 47.088$ kN

$b = 4/3 - 1$

$b = 1/3$

$F_{a2} = 23.544(6 + d) \times 1$

$c = 1 + ½(6 + d)$

$c = 4 + d/2$

$p_2 = K_a \gamma_b H_2$

$p_2 = 0.3(9.68)(6 + d)$

$p_2 = 2.904(6 + d)$ kPa

$F_{a3} = ½[2.904(6 + d)](6 + d) \times 1$

$F_{a3} = 1.452(36 + 12d + d^2)$

$f = 1 + (2/3)(6 + d)$

$f = 5 + 2d/3$

Passive pressures:

$p_3 = K_p \gamma_b H_3$

$p_3 = 3(9.68)(d)$

$p_3 = 29.04\, d$ kPa

$F_p = ½(29.04\, d)(d) = 14.52\, d^2$

$a = 7 + 2d/3$

$\Sigma M_O = 0$

$F_p\, a + F_{a1}\, b = F_{a2}\, c + F_{a3}\, f$

$14.52\, d^2\, (7 + 2d/3) + 47.088(1/3) = 23.544(6 + d)(4 + d/2)$
$+ 1.452(36 + 12d + d^2)(5 + 2d/3)$

$101.64\, d^2 + 9.68\, d^3 + 15.696 = 23.544(24 + 7d + 0.5d^2)$
$+ 1.452(180 + 24d + 60d + 8d^2 + 5d^2 + 0.67\, d^3)$

$8.71\, d^3 + 70.99\, d^2 - 286.78d - 810.72 = 0$

$d =$ **4.347 m**

$[\Sigma F_H = 0]$

$T = F_{a1} + F_{a2} + F_{a3} - F_p$

$T = 47.088 + 23.544(6 + 4.347) + 1.452[36 + 12(4.347) + (4.347)^2]$
$- 14.52(4.347)^2$

$T =$ **171.77 kN**

PROBLEM 08.11

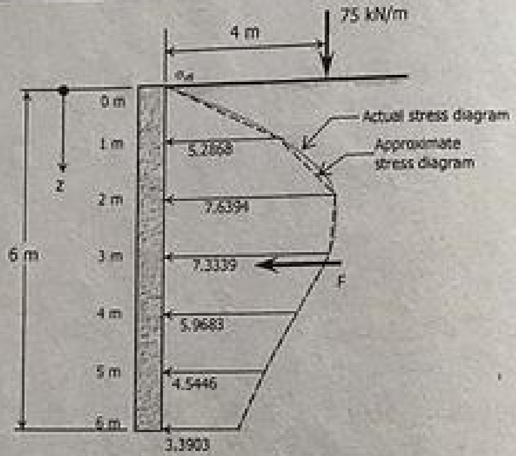
A line load of 75 kN/m is located at a distance of 4 m from the back face of a 6-m high retaining wall. Determine the lateral force on the wall due to the line load alone.

SOLUTION

$m = 4/6$

$m = 0.667 > 0.4$ Use Eq. 8.33

$$\text{Stress, } \sigma_x = \frac{4q}{\pi H}\,\frac{m^2 n}{(m^2 + n^2)^2}$$



Depth, z	n = z/H	σ_x
0	0	0
1	0.1667	5.2868
2	0.3333	7.6394
3	0.5000	7.3339
4	0.6667	5.9683
5	0.8333	4.5446
6	1.0000	3.3903

F = Area of stress diagram (using trapezoidal rule)

$$F = \frac{d}{2}\left[\sigma_0 + 2\sigma_1 + 2\sigma_2 + \ldots + 2\sigma_5 + \sigma_6\right],$$

where d = 1 m (interval of computed stress)

$$F = \frac{1}{2}[0 + 2(5.2868) + 2(7.6394) + 2(7.3339) + 2(5.9683) + 2(4.5446) + 3.3903]$$

F = **32.468 kN**

Or:

$$F = \frac{0+5.2868}{2}(1) + \frac{5.2868+7.6394}{2}(1) + \frac{7.6394+7.3339}{2}(1) + \frac{7.3339+5.9683}{2}(1) + \frac{5.9683+4.5446}{2}(1) + \frac{4.5446+3.3903}{2}(1)$$

F = **32.468 kN**

Note: The using d = 0.1 m, F = 33.049 kN

Chapter 09

Bearing Capacity of Soils

9.1 DEFINITIONS

Foundation is that part of a structure which transmits the building load directly into the underlying soil. If the soil conditions at the site are sufficiently strong and capable of supporting the required load, then shallow spread footings or mats can be used to transmit the load.

Footing is a foundation consisting of a small slab for transmitting the structure load to the underlying soil. Footings can be individual slabs supporting single columns or combined to support two or more columns, or be a long strip of concrete slab (width B to length L ratio is small, i.e., it approaches zero) supporting a load bearing wall, or a mat.

Shallow foundation is one in which the ratio of the embedment depth to the minimum plan dimension, which is usually the width, is $D_f/B \le 2.5$.

Embedment depth (D_f) is the depth below the ground surface where the base of the foundation rests.

Ultimate bearing capacity (q_u) is the maximum pressure that the soil can support.

Ultimate net bearing capacity (q_{ult}) is the maximum pressure that the soil can support above its current overburden pressure.

Allowable bearing capacity or **safe bearing capacity** (q_a) is the working pressure that would ensure a margin of safety against collapse of the structure from shear failure. The allowable bearing capacity is usually a fraction of the ultimate net bearing capacity.

Overburden Pressure q, is the pressure (effective stress) of the soil removed to place the footing.

Factor of safety or **safety factor** (FS) is the ratio of the ultimate net bearing capacity to the allowable bearing capacity or to the applied maximum vertical stress. In geotechnical engineering, a factor of safety between 1.5 and 5 is used to calculate the allowable bearing capacity.

9.2 VARIOUS TYPES OF FOOTING ON SOIL

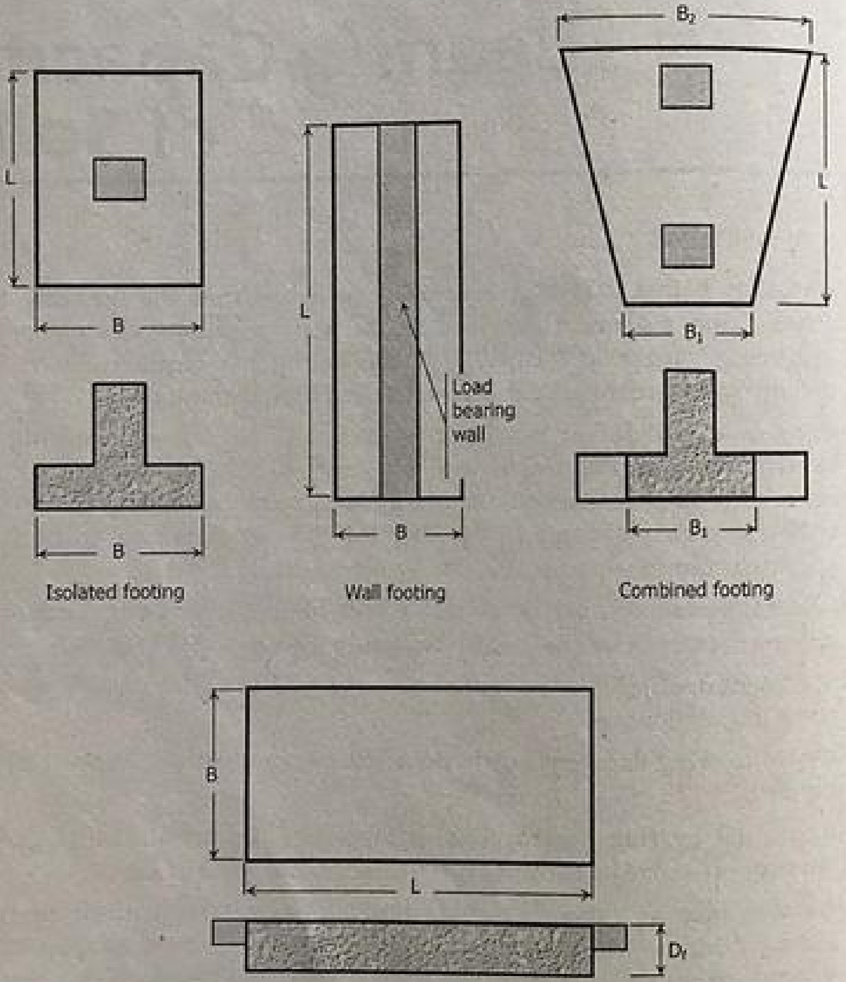


Figure 09.1 – Types of footing

9.3 BEARING CAPACITY ANALYSIS

Bearing capacity analysis is the method used to determine the ability of the soil to support the required load in a safe manner without gross distortion resulting from objectionable settlement. The ultimate bearing capacity (q_u) is defined as that pressure causing a shear failure of the supporting soil lying immediately below and adjacent to the footing. Generally three modes of failure have been identified:

General Shear Failure: a continuous failure surface develops between the edge of the footing and the ground surface. This type of failure is characterized by heaving at the ground surface accompanied by tilting of the footing. It occurs in soil of low compressibility such as dense sand or stiff clay.

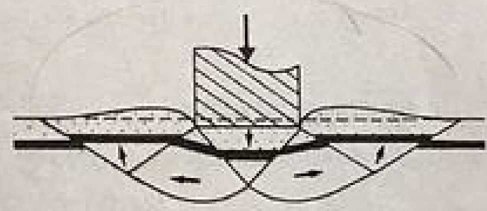


Figure 09.2 – General shear failure

2. *Local Shear Failure*: a condition where significant compression of the soil occurs but only slight heave occurs at the ground surface. Tilting of the foundation is not expected. This type of failure occurs in highly compressible soil and the ultimate bearing capacity is not well defined.

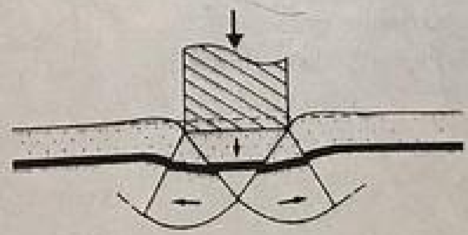


Figure 09.3 – Local shear failure

3. *Punching Shear Failure*: a condition that occurs where there is relatively high compression of the soil underlying the footing with neither heaving at the ground surface nor tilting of the foundation. Large settlement is expected without a clearly defined ultimate bearing capacity. Punching will occur in low compressible soil if the foundation is located at a considerable depth below ground surface.

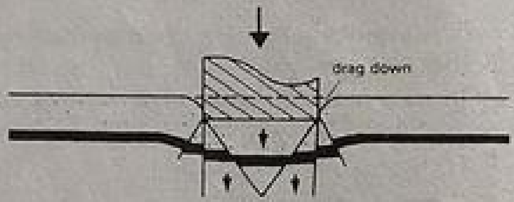


Figure 09.4 – Punching shear failure

9.4 ULTIMATE SOIL BEARING CAPACITY

In general, the ***ultimate bearing capacity*** of soil is given by:

$$q_u = K_c\, c\, N_c + K_q\, q\, N_q + K_\gamma\, \gamma_e\, B\, N_\gamma \qquad \text{Eq. 9.1}$$

where:

q_u = ultimate bearing capacity
γ_e = unit weight of soil in kPa or pcf
B = width of footing in meter or feet
c = cohesion of soil in kPa or psf
N_γ = factor for unit weight of soil
N_c = factor of soil cohesion
N_q = factor of overburden pressure
q = overburden pressure (effective stress)
K_c, K_q, & K_γ = constant

9.5 TERZAGHI'S BEARING CAPACITY EQUATIONS

Terzaghi's bearing capacity equations are based on the following assumptions:

- Depth of foundation is less than or equal to its width
- No sliding occurs between foundation and soil (rough foundation)
- Soil beneath foundation is homogeneous semi-infinite mass
- Mohr- Coulomb model for soil
- General shear failure mode is the governing mode (but not the only mode)
- No soil consolidation occurs
- Foundation is very rigid relative to the soil
- Soil above bottom of foundation has no shear strength; is only a surcharge load against the overturning load
- Applied load is compressive and applied vertically to the centroid of the foundation
- No applied moments present

9.5.1 GENERAL SHEAR FAILURE:

9.5.1.1 LONG FOOTINGS

$$q_u = c\, N_c + q\, N_q + \tfrac{1}{2}\, \gamma_e\, B\, N_\gamma \qquad \text{Eq. 9.2}$$

9.5.1.2 SQUARE FOOTINGS

$$q_u = 1.3\, c\, N_c + q\, N_q + 0.4\, \gamma_e\, B\, N_\gamma \qquad \text{Eq. 9.3}$$

9.5.1.3 CIRCULAR FOOTINGS

$$q_u = 1.3\, c\, N_c + q\, N_q + 0.3\, \gamma_e\, B\, N_\gamma \qquad \text{Eq. 9.4}$$

where:

γ_e = unit weight of soil at base of footing in kPa or pcf
B = width of footing in meter or feet
c = cohesion of soil in kPa or psf
N_γ = factor for unit weight of soil
N_c = factor of soil cohesion
N_q = factor of overburden pressure
q = overburden pressure (effective stress) at base of footing
D_f = depth of footing in meter or feet

See Table 09.1 or Figure 09.10 for the values of N_c, N_γ and N_q

9.5.2 LOCAL SHEAR FAILURE

For local shear failure, it may be assumed that:

$$\bar{c} = \tfrac{2}{3}c \qquad \text{Eq. 9.5}$$

$$\tan\bar{\phi} = \tfrac{2}{3}\tan\phi \qquad \text{Eq. 9.6}$$

9.5.2.1 LONG FOOTINGS (STRIP FOOTING)

$$q_u = \bar{c}N'_c + qN'_q + \tfrac{1}{2}\gamma_e BN'_\gamma \qquad \text{Eq. 9.7}$$

9.5.2.2 SQUARE FOOTINGS

$$q_u = 1.3\bar{c}N'_c + qN'_q + 0.4\gamma_e BN'_\gamma \qquad \text{Eq. 9.8}$$

9.5.2.3 CIRCULAR FOOTINGS

$$q_u = 1.3\bar{c}N'_c + qN'_q + 0.3\gamma_e BN'_\gamma \qquad \text{Eq. 9.9}$$

The modified bearing capacity factors N'_c, N'_q, and N'_γ are calculated using the same general equations at that for N_c, N_q, and N_γ but by substituting $\bar{\phi} = \tan^{-1}(\tfrac{2}{3}\tan\phi)$ for ϕ, and using Table 09.1 or Figure 09.10.

The values of bearing capacity factors for a local shear failure are given in Table 09.2 and Figure 09.11.

9.6 ALLOWABLE BEARING CAPACITY AND FACTOR OF SAFETY

The allowable bearing capacity, q_a, is calculated by dividing the ultimate bearing capacity, q_u, by a Factor of safety, FS. The factor of safety is intended to compensate for the assumptions made in developing the bearing capacity equations, soil variability, inaccurate soil data, and uncertainties of loads.

9.6.1 GROSS ALLOWABLE BEARING CAPACITY:

$$q_{all} = \frac{q_u}{FS} \qquad \text{Eq. 9.10}$$

9.6.2 NET ALLOWABLE BEARING CAPACITY:

$$q_{all\,(net)} = \frac{q_{u\,net}}{FS} \qquad \text{Eq. 9.11}$$

$$q_{u\,net} = q_u - q \qquad \text{Eq. 9.12}$$

9.6.3 GROSS ALLOWABLE BEARING CAPACITY WITH FS WITH RESPECT TO SHEAR

The gross allowable bearing capacity using a factor of safety on shear strength of soil may be computed using the developed cohesion c_d and values of N_c, N_q, and N_γ derived using the developed angle of friction ϕ_d.

$$\text{Developed cohesion, } c_d = c / FS \qquad \text{Eq. 9.13}$$

$$\text{Developed angle of friction, } \phi_d = \tan^{-1}\left(\frac{\tan\phi}{FS}\right) \qquad \text{Eq. 9.14}$$

For example, on strip footing, $q_u = c_d N_c + qN_q + ½ \gamma_e BN_\gamma$

Alternatively, if the maximum applied foundation stress, $(f_a)_{max}$, is known, the factor of safety can be computed by replacing q_u by $(f_a)_{max}$.

$$FS = \frac{q_u}{(f_a)_{max} - q}\ ; \quad q < (f_a)_{max} \qquad \text{Eq. 9.15}$$

9.7 EFFECT OF WATER TABLE ON BEARING CAPACITY

The unit weight of soil used in the equations for bearing capacity are effective unit weights. With the rising water table, the subsoil becomes saturated and the unit weight of submerged soil is greatly reduced. The reduction of this unit weight results in a decrease in the ultimate bearing capacity of the soil.

9.7.1 GROUNDWATER LEVEL ABOVE BASE OF FOOTING.

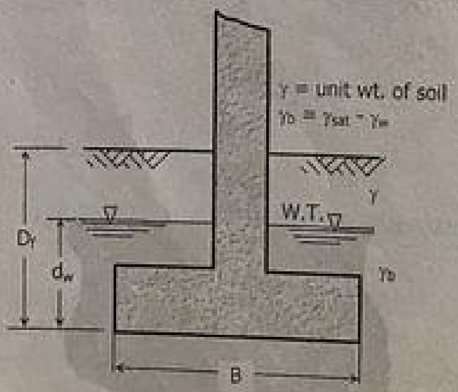


Figure 09.5 – Groundwater above base of footing

$$q = \gamma(D_f - d_w) + \gamma_b\, d_w$$

$$\text{Unit weight, } \gamma_e = \gamma_b$$

9.7.2 GROUNDWATER LEVEL AT THE BASE OF FOOTING.

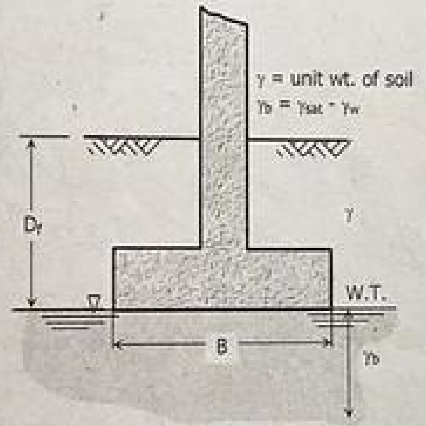


Figure 09.6 – Groundwater at the base of footing

$$\text{Overburden pressure, } q = \gamma\, D_f$$

$$\text{Unit weight, } \gamma_e = \gamma_b$$

9.7.2 GROUNDWATER LEVEL BELOW THE BASE OF FOOTING.

$$\text{Overburden pressure, } q = \gamma\, D_f$$

When $d_w < B$

$$\gamma_e = \gamma_b\,(1 + d_w/B) = \text{approx.}$$

When $d_w \geq B$

$\gamma_e = \gamma$

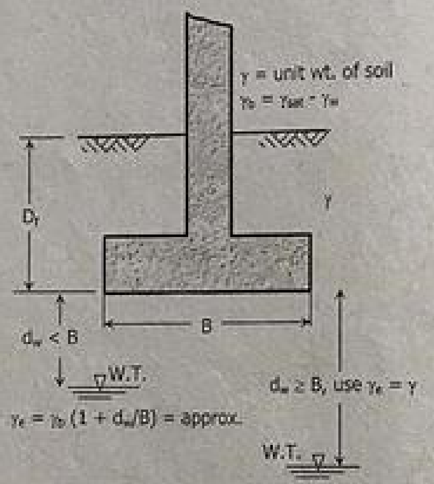


Figure 09.7 – Groundwater at the base of footing

Note: If the problem does not provide or mention about the position of the water table, use $q = \gamma D_f$ and $q_e = \gamma$.

9.8 MEYERHOF'S EQUATION (General Bearing Capacity equation)

9.8.1 VERTICAL LOAD:

$$q_u = c\,N_c\,s_c\,d_c + q\,N_q\,s_q\,d_q + 0.5\,\gamma\,B\,N_\gamma\,s_\gamma\,d_\gamma \qquad \text{Eq. 9.16}$$

9.8.2 INCLINED LOAD:

$$q_u = c\,N_c\,s_c\,d_c\,i_c + q\,N_q\,s_q\,d_q\,i_q + 0.5\,\gamma\,B\,N_\gamma\,s_\gamma\,d_\gamma\,i_\gamma \qquad \text{Eq. 9.17}$$

9.8.3 BEARING CAPACITY FACTORS:

$$N_q = e^{\pi \tan\phi} \tan^2(45° + \phi/2) \qquad \text{Eq. 9.18}$$

$$N_c = (N_q - 1)\cot\phi \qquad \text{Eq. 9.19}$$

$$N_\gamma = (N_q - 1)\tan(1.4\,\phi) \qquad \text{Eq. 9.20}$$

9.8.4 MEYERHOF'S FACTORS (shape s, depth d, and inclination i factors)

9.8.4.1 SHAPE AND DEPTH FACTORS

For $\phi = 0°$

$$s_c = 1 + 0.2\frac{B}{L} \qquad \text{Eq. 9.21}$$

$$s_q = s_\gamma = 1.0 \qquad \text{Eq. 9.22}$$

$$d_c = 1 + 0.2\frac{D_f}{B} \qquad \text{Eq. 9.23}$$

$$d_q = d_\gamma = 1.0 \qquad \text{Eq. 9.24}$$

For $\phi \geq 10°$

$$s_c = 1 + 0.2K_p\frac{B}{L} \qquad \text{Eq. 9.25}$$

$$s_q = s_\gamma = 1 + 0.1\,K_p\frac{B}{L} \qquad \text{Eq. 9.26}$$

$$d_c = 1 + 0.2\sqrt{K_a}\,\frac{D_f}{B} \qquad \text{Eq. 9.27}$$

$$d_q = d_\gamma = 1 + 0.1\sqrt{K_p}\,\frac{D_f}{B} \qquad \text{Eq. 9.28}$$

9.8.4.2 LOAD INCLINATION FACTORS

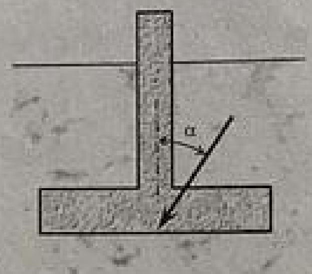


Figure 09.8

$$i_c = i_q = (1 - \alpha / 90°)^2 \qquad \text{Eq. 9.29}$$

$$i_\gamma = \left(1 - \frac{\alpha}{\phi}\right)^2 \qquad \text{Eq. 9.30}$$

where: $K_p = \tan^2\left(45° + \frac{\phi}{2}\right)$

α = angle of resultant measured from vertical axis.

9.9 HANSEN'S BEARING CAPACITY EQUATION (VESIC'S FACTORS)

Hansen (J. Brinch Hansen) proposed what is referred to as the *general bearing-capacity equation* given by the following equations. Hansen's equation is an extension of Meyerhof's equations. The N_c and N_q factors are identical. Both Meyerhof and Hansen provided expressions for the shape, depth, and inclination factors; however, Hansen also added what he called ground factors and base factors.

$$q_u = c\, N_c\, s_c\, d_c\, i_c\, b_c\, g_c + q\, N_q\, s_q\, d_q\, i_q\, b_q\, g_q + 0.5\gamma B N_\gamma\, s_\gamma\, d_\gamma\, i_\gamma\, b_\gamma\, g_\gamma \qquad \text{Eq. 9.31}$$

In the special case of a horizontal ground surface,

$$q_u = -c\cot\phi + (q + c\cot\phi)\, N_q\, s_q\, d_q\, i_q\, b_q + 0.5\gamma B N_\gamma\, s_\gamma\, d_\gamma\, i_\gamma\, b_\gamma \qquad \text{Eq. 9.32}$$

9.8.1 HANSEN'S FACTORS

9.8.1.1 SHAPE FACTORS:

For failure along base width B:

$$s_c = 0.2\, i_{cB}\, B/L \qquad \text{Eq. 9.33}$$

$$s_q = 1 + \sin\phi\, B\, i_{qL} / L \qquad \text{Eq. 9.34}$$

$$s_\gamma = 1 - 0.4\,(B\, i_{\gamma B}) / (L\, i_{\gamma L}) \qquad \text{Eq. 9.35}$$

For failure along base length L:

$$s_c = 0.2\, i_{cB}\, L/B \qquad \text{Eq. 9.36}$$

$$s_q = 1 + \sin\phi\, L\, i_{qL} / B \qquad \text{Eq. 9.37}$$

$$s_\gamma = 1 - 0.4\,(L\, i_{\gamma L}) / (B\, i_{\gamma B}) \qquad \text{Eq. 9.38}$$

9.8.1.2 BASE AND GROUND INCLINATION FACTORS:

$$b_c = \frac{2\nu}{\pi + 2} = \frac{\nu°}{147°} \qquad \text{Eq. 9.39}$$

$$b_q = e^{-2\nu\tan\phi} \qquad \text{Eq. 9.40}$$

$$b_\gamma = e^{-2.7\nu\tan\phi}$$

$$g_c = \frac{2\beta}{\pi + 2} = \frac{\beta°}{147°} \qquad \text{Eq. 9.41}$$

$$g_q = g_\gamma = [1 - 0.5\tan\beta]^5 \qquad \text{Eq. 9.42}$$

where:

ν = base (of footing) inclination

β = ground inclination

9.8.1.3 DEPTH FACTORS:

For $D_f \leq B$

$$d_c = 0.4\,(D_f/B) \qquad \text{Eq. 9.43}$$

$$d_q = 1 + 2\tan\phi\,(1 - \sin\phi)^2\,D_f/B \qquad \text{Eq. 9.44}$$

For $D_f > B$

$$d_c = 0.4\arctan\,(D_f/B) \qquad \text{Eq. 9.45}$$

$$d_q = 1 + 2\tan\phi\,(1 - \sin\phi)^2\arctan\,(D_f/B) \qquad \text{Eq. 9.46}$$

$$d_\gamma = 1 \qquad \text{Eq. 9.47}$$

Load inclination factors:

For $v = 0°$

$$i_c = 0.5 - 0.5\sqrt{1 - H/Ac_u} \qquad \text{Eq. 9.48}$$

$$i_q = [1 - 0.5H/(V + Ac\cot\phi)]^5 \qquad \text{Eq. 9.49}$$

For $v > 0°$

$$i_\gamma = [1 - (0.7 - v°/450°)H/(V + Ac\cot\phi)]^5 \qquad \text{Eq. 9.50}$$

where:

V = foundation load normal to the base

H = load parallel to the base of footing

9.10 ULTIMATE LOAD FOR SHALLOW FOUNDATION UNDER ECCENTRIC LOAD (ONE-WAY ECCENTRICITY)

Meyerhof introduced the following procedures to calculate the bearing capacity of footings under eccentric load.

The load is assumed to act at the center of the footing whose effective dimension is x by y, as shown in Figure 09.9.

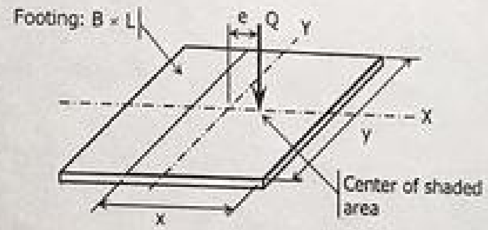


Figure 09.9 – Footing under eccentric load (one-way eccentricity)

The smaller value of x and y is the effective width (B') and the larger value is the effective length (L'), and the effective area is $B' \times L'$. Using the effective width, Eq. 9.16 may be rewritten as:

$$q_u = c\,N_c\,s_c\,d_c + q\,N_q\,s_q\,d_q + 0.5\,\gamma\,B'\,N_\gamma\,s_\gamma\,d_\gamma \qquad \text{Eq. 9.51}$$

$$\text{Ultimate load} = q_u\,(B'\,L') \qquad \text{Eq. 9.52}$$

The values of the shape and depth factors are computed by substituting B' for B and L' for L.

9.11 BEARING CAPACITY FROM STANDARD PENETRATION TEST (SPT)

Allowable bearing capacity:

$$q_a = 0.41\,N_{cor}\,\rho_a \text{ (kPa)} \qquad \text{Eq. 9.53}$$

$$N_{cor} = c_N\,c_W\,N \qquad \text{Eq. 9.54}$$

where:

N = standard penetration number (the number of blows required to drive the sampler an additional 300 mm)

c_N = correction factor for overburden pressure

$$c_N = \left(\frac{95.8}{\sigma'_{zo}}\right)^{1/2} \; ; c_N \leq 2 \text{ (Liao and Whitman, 1985)}$$

$c_N = 0.77 \log_{10}\left(\frac{1916}{\sigma'_{zo}}\right)$; $c_N \le 2$, $\sigma'_{zo} > 24$ kPa (Peck, 1974)

σ'_{zo} = effective overburden pressure, kPa

c_W = correction factor if the groundwater level is within a depth B below the base of the footing

$$c_W = \frac{1}{2} + \frac{z}{2(D_f + B)}$$

z = depth of the ground water table

D_f = depth of footing

B = width of footing

c_W = 1 if the depth of the groundwater level if beyond B from the footing base.

ρ_a = allowable settlement in mm

Table 09.1 - Terzaghi's Bearing-Capacity Factors for General Shear Failure

ϕ °	N_c	N_q	N_γ	ϕ °	N_c	N_q	N_γ
0	5.70	1.00	0.00	26	27.09	14.21	9.84
1	6.00	1.10	0.01	27	29.24	15.90	11.60
2	6.30	1.22	0.04	28	31.61	17.81	13.70
3	6.62	1.35	0.06	29	34.24	19.98	16.18
4	6.97	1.49	0.10	30	37.16	22.46	19.13
5	7.34	1.64	0.14	31	40.41	25.28	22.65
6	7.73	1.81	0.20	32	44.04	28.52	26.87
7	8.15	2.00	0.27	33	48.09	32.23	31.94
8	8.60	2.21	0.35	34	52.64	36.50	38.04
9	9.09	2.44	0.44	35	57.75	41.44	45.41
10	9.61	2.69	0.56	36	63.53	47.16	54.36
11	10.16	2.98	0.69	37	70.01	53.80	65.27
12	10.76	3.29	0.85	38	77.50	61.55	78.61
13	11.41	3.63	1.04	39	85.97	70.61	95.03
14	12.11	4.02	1.26	40	95.66	81.27	115.31
15	12.86	4.45	1.52	41	106.81	93.85	140.51
16	13.68	4.92	1.82	42	119.67	108.75	171.99
17	14.60	5.45	2.18	43	134.58	126.50	211.56
18	15.12	6.04	2.59	44	151.95	147.74	261.60
19	16.56	6.70	3.07	45	172.28	173.28	325.34
20	17.69	7.44	3.64	46	196.22	204.19	407.11
21	18.92	8.26	4.31	47	224.55	241.80	512.84
22	20.27	9.19	5.09	48	258.28	287.85	650.67
23	21.75	10.23	6.00	49	298.71	344.63	831.99
24	23.36	11.40	7.08	50	347.50	415.14	1072.80
25	25.13	12.72	8.34				

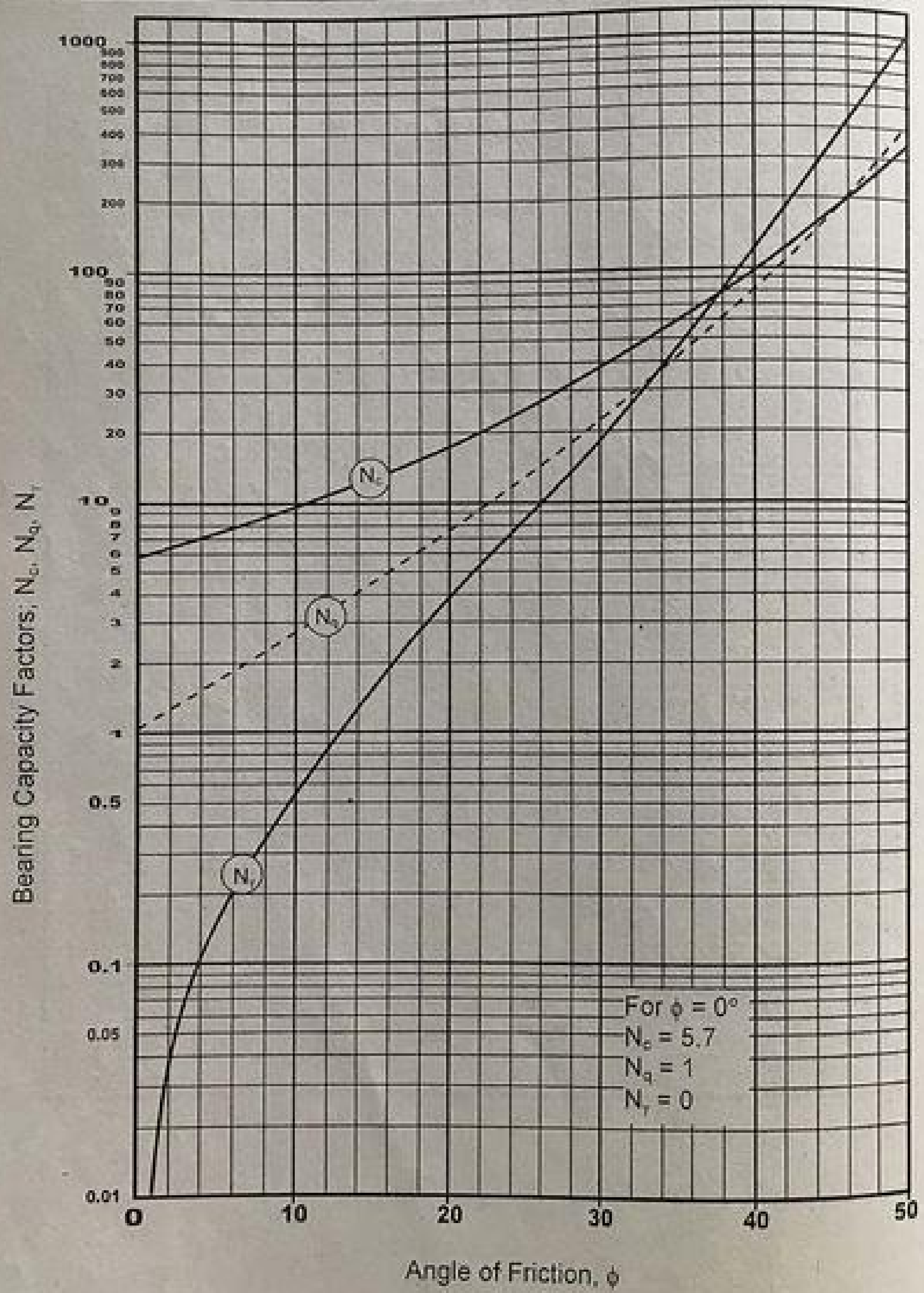


Figure 09.10 – Terzaghi Bearing capacity factors for General Shear Failure

Table 09.2 - Terzaghi's Modified Bearing Capacity Factors N'_c, N'_q, and N'_γ for Local Shear Failure

$\phi°$	N'_c	N'_q	N'_γ	$\phi°$	N'_c	N'_q	N'_γ
0	5.70	1.00	0	26	15.53	6.05	2.59
1	5.90	1.07	0.01	27	16.30	6.54	2.88
2	6.10	1.14	0.02	28	17.13	7.07	3.29
3	6.30	1.22	0.04	29	18.03	7.66	3.76
4	6.51	1.30	0.06	30	18.99	8.31	4.39
5	6.74	1.39	0.07	31	20.03	9.03	4.83
6	6.97	1.49	0.10	32	21.16	9.82	5.51
7	7.22	1.59	0.13	33	22.39	10.69	6.32
8	7.47	1.70	0.16	34	23.72	11.67	7.22
9	7.74	1.82	0.20	35	25.18	12.75	8.35
10	8.02	1.94	0.24	36	26.77	13.97	9.41
11	8.32	2.08	0.30	37	28.51	15.32	10.90
12	8.63	2.22	0.35	38	30.43	16.85	12.75
13	8.96	2.38	0.42	39	32.53	18.56	14.71
14	9.31	2.55	0.48	40	34.87	20.50	17.22
15	9.67	2.73	0.57	41	37.45	22.70	19.75
16	10.06	2.92	0.67	42	40.33	25.21	22.50
17	10.47	3.13	0.76	43	43.54	28.06	26.25
18	10.90	3.36	0.88	44	47.13	31.34	30.40
19	11.36	3.61	1.03	45	51.17	35.11	36.00
20	11.85	3.88	1.12	46	55.73	39.48	41.70
21	12.37	4.17	1.35	47	60.91	44.54	49.30
22	12.92	4.48	1.55	48	66.80	50.46	59.25
23	13.51	4.82	1.74	49	73.55	57.41	71.45
24	14.14	5.20	1.97	50	81.31	65.60	85.75
25	14.80	5.60	2.25				

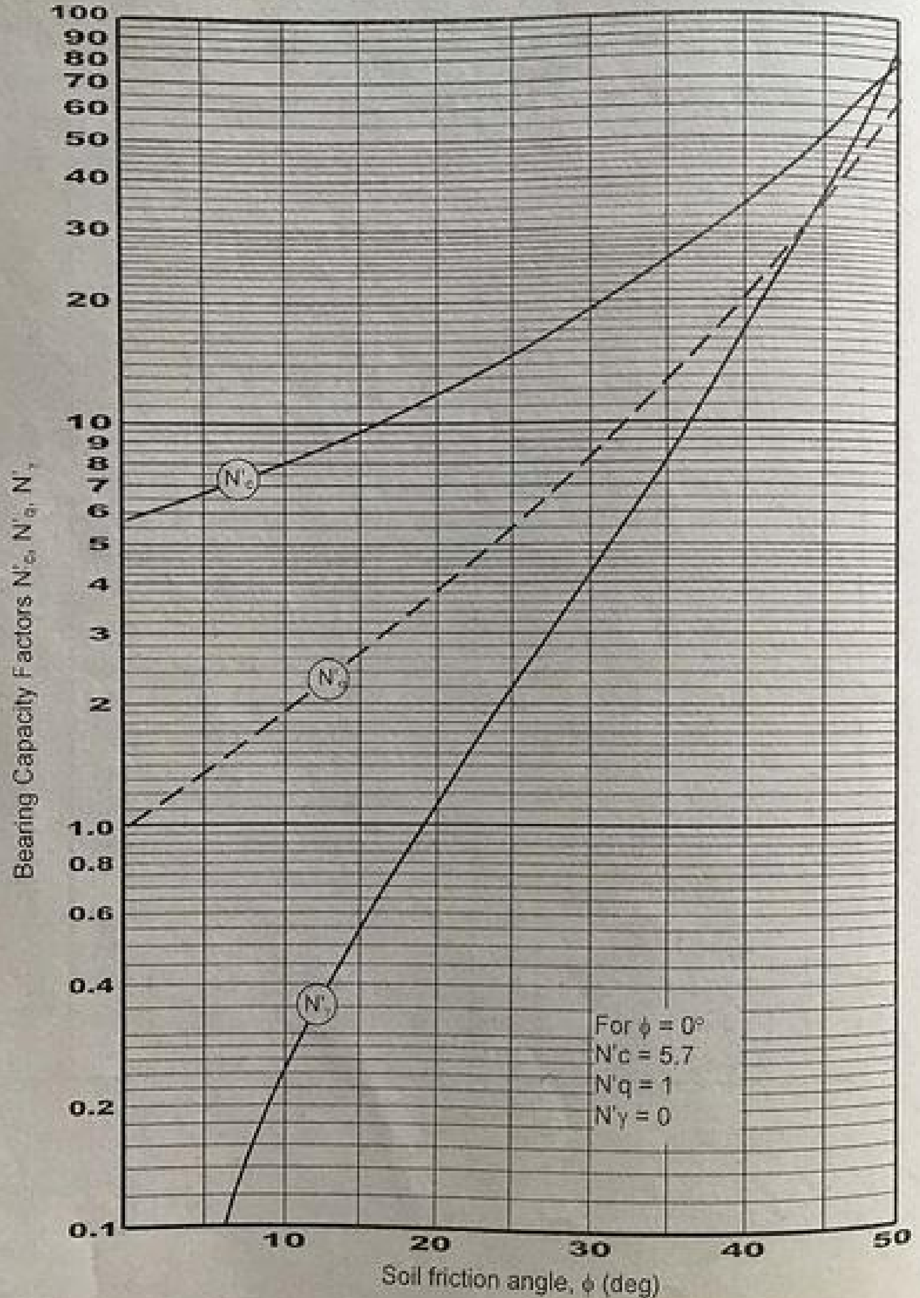


Figure 09.11 – Terzaghi Bearing capacity factors for Local Shear Failure

Table 09.3 - Meyerhof & Hansen Bearing-Capacity Coefficients

$\phi°$	Meyerhof			Hansen		
	N_c	N_q	N_γ	N_c	N_q	N_γ
0	5.10	1.00	0.00	5.10	1.00	0.00
2	5.63	1.20	0.01	5.63	1.20	0.01
4	6.19	1.43	0.04	6.19	1.43	0.05
6	6.81	1.72	0.11	6.81	1.72	0.11
8	7.53	2.06	0.21	7.53	2.06	0.22
10	8.34	2.47	0.37	8.34	2.47	0.39
12	9.28	2.97	0.60	9.28	2.97	0.63
14	10.37	3.59	0.92	10.37	3.59	0.97
16	11.63	4.34	1.37	11.63	4.34	1.43
18	13.10	5.26	2.00	13.10	5.26	2.08
20	14.33	6.40	2.37	14.83	6.40	2.95
22	16.88	7.82	4.07	16.88	7.82	4.13
24	19.32	9.60	5.72	19.32	9.60	5.75
26	22.25	11.85	8.00	22.25	11.85	7.94
28	25.80	14.72	11.19	25.80	14.72	10.94
30	30.14	18.40	15.67	30.14	18.40	15.07
32	35.49	23.18	22.02	35.49	23.18	20.79
34	42.16	29.44	31.15	42.16	29.44	28.77
36	50.59	37.75	44.43	50.59	37.75	40.05
38	61.35	48.93	64.08	61.35	48.93	56.18
40	75.32	64.20	93.69	75.32	64.20	79.54
42	93.71	85.38	139.32	93.71	85.38	113.96
44	118.37	115.31	211.41	118.37	115.31	165.58
46	152.10	158.51	329.74	152.10	158.51	244.65
48	199.27	222.31	526.47	199.27	222.31	368.68
50	266.89	319.07	873.89	266.89	319.07	568.59

For intermediate values of ϕ, the value of bearing coefficients may be computed by linear interpolation.

$$N_\phi = N_{\phi 1} + \frac{N_{\phi 2} - N_{\phi 1}}{\phi_2 - \phi_1}(N_{\phi 2} - N_{\phi 1}) \times (\phi - \phi_1) \qquad \text{Eq. 9.55}$$

For example, to find the value of N_q for ϕ = 22.32, using Table 09.1:

For $\phi_1 = 22°$:
$N_{q1} = 9.19$

For $\phi_2 = 23°$:
$N_{q2} = 10.23$

Thus, $N_q = 9.19 + \dfrac{10.23 - 9.19}{22 - 21}(22.32 - 22)$
$N_q = \underline{9.5228}$

ILLUSTRATIVE PROBLEMS

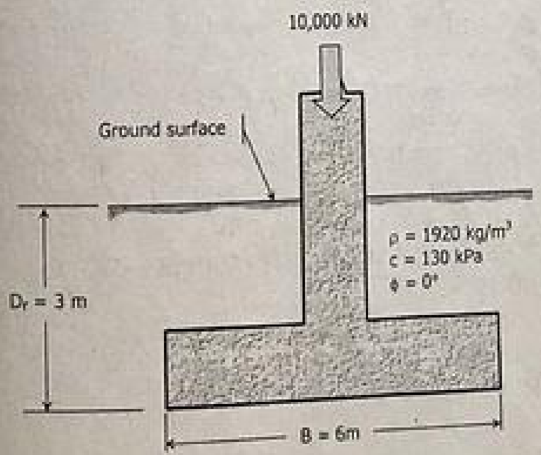
PROBLEM 09.1 (CE MAY 2003)

A footing 6 m square carries a total load, including its own weight, of 10,000 kN. The base of the footing is at a depth of 3 m below the ground surface. The soil strata at the site consist of a layer of stiff saturated clay 27.5 m thick overlying dense sand. The average bulk density of the clay is 1,920 kg/m³ and its average shear strength determined from undrained triaxial test is 130 kN/m² and ϕ = 0°. Given is Terzaghi's ultimate bearing capacity for square footings:

$$q_u = 1.3\,c\,N_c + \gamma\,D_f\,N_q + 0.40\,\gamma B\,N_\gamma$$

a) Determine the gross foundation pressure in kPa.
b) Determine the net foundation pressure.
c) Calculate the factor of safety of the foundation against complete shear failure under the undrained condition (both gross and net). Side cohesion on the foundation may be neglected.

SOLUTION



$$q_u = 1.3\,c\,N_c + \gamma\,D_f\,N_q + 0.4\,\gamma\,B\,N_\gamma$$

Part *a*:

Gross foundation pressure, $q_g = \dfrac{10{,}000}{6(6)}$

Gross foundation pressure, $q_g = \mathbf{277.8\ kPa}$

Part *b*:

Net foundation pressure, $q_n = q_g - \gamma D_f$

$\gamma = \rho g = 1920(9.81)$

$= 18{,}835.2\ \text{N/m}^3$

$\gamma = 18.835\ \text{kN/m}^3$

Net foundation pressure, $q_n = 277.8 - (18.835)(3)$

Net foundation pressure, $q_n = \mathbf{221.3\ kPa}$

Part *c*:

$q_u = 1.3\,c\,N_c + \gamma\,D_f\,N_q + 0.40\,\gamma B\,N_\gamma$

$c = 130$

$\gamma = \rho g$

$\gamma = 1920(9.81)$

$= 18{,}835.2\ \text{N/m}^3$

$\gamma = 18.835\ \text{kN/m}^3$

$D_f = 3\ \text{m}$

$B = 6$

From Table 09.1, for $\phi = 0°$

$N_c = 5.7$

$N_q = 1.0$

$N_\gamma = 0$

$q_u = 1.3(130)(5.7) + 18.835(3)(1) + 0.40(18.835)(6)(0)$

$q_u = 1{,}019.8\ \text{kPa}$

Factor of safety:

Gross:

$$FS_{gross} = \frac{q_u}{q_g}$$

$$FS_{gross} = \frac{1019.8}{277.8}$$

$FS_{gross} = \mathbf{3.67}$

Net:

$$FS_{net} = \frac{q_{u\,net}}{q_{net}}$$

$q_{u\,net} = q_u - \gamma D_f$

$q_{u\,net} = 1{,}019.8 - 18.835(3)$

$q_{u\,net} = 963.295\ \text{kPa}$

$$FS_{net} = \frac{963.295}{221.3}$$

$FS_{net} = \mathbf{4.35}$

PROBLEM 09.2

A continuous footing is shown in Figure 09.12. Use the Terzaghi's bearing capacity equation.

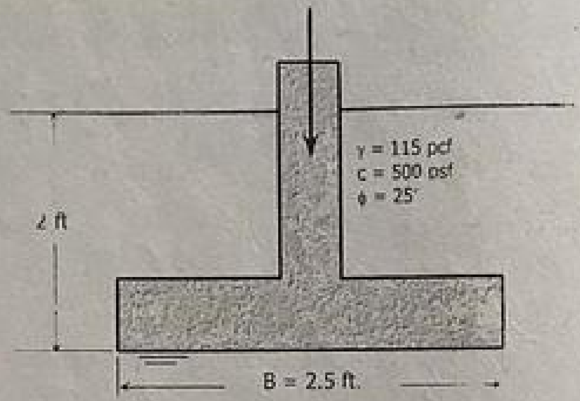
Given:

$\gamma = 115$ pcf	$D_f = 2$ ft
$c = 500$ psf	$B = 2.5$ ft
$\phi = 25°$	Factor of safety = 2

a) Determine the gross allowable load per unit area that the footing can carry, in psf.

b) Determine the net allowable bearing capacity with a factor of safety of 2, in psf.

c) Determine the gross allowable bearing capacity with a factor of safety of 2 with respect to shear failure, in psf.



$q_u = c\,N_c + q\,N_q + 0.5\,\gamma\,B\,N$

Figure 09.12

SOLUTION

Part *a*:

$q_u = c\,N_c + q\,N_q + 0.5\,\gamma\,B\,N$

For $\phi = 25°$

From the diagram shown below

$N_c = 25.1$

$N_q = 12.7$

$N_\gamma = 8.3$

$q = \gamma\,D_f$

$q = 115(2)$

$q = 230$ psf

$q_u = 500(25.1) + 230(12.7) + 0.5(115)(2.5)(8.3)$

$q_u = 16{,}664$ psf

$$q_{all} = \frac{q_u}{FS}$$

$$q_{all} = \frac{16{,}664}{2}$$

q_{all} = **8,332 psf**

Part *b*:

$(q_u)_{net} = q_u - q$

$(q_u)_{net} = 16{,}664 - 230$

$(q_u)_{net} = 16{,}434$ psf

$$(q_{all})_{net} = \frac{(q_u)_{net}}{FS}$$

$$(q_{all})_{net} = \frac{16{,}434}{2}$$

$(q_{all})_{net}$ = **8,217 psf**

Part *c*:

Calculate the developed cohesion:

$$c_d = \frac{c}{FS}$$

$$c_d = \frac{500}{2}$$

$c_d = 250$ psf

Calculate the developed angle of friction of soil:

$$\tan\phi_d = \frac{\tan\phi}{FS}$$

$$\tan\phi_d = \frac{\tan 25°}{2}$$

$\tan\phi_d = 0.23315$

$\phi_d = 13.124°$

From the diagram:

$N_c = 11.4$

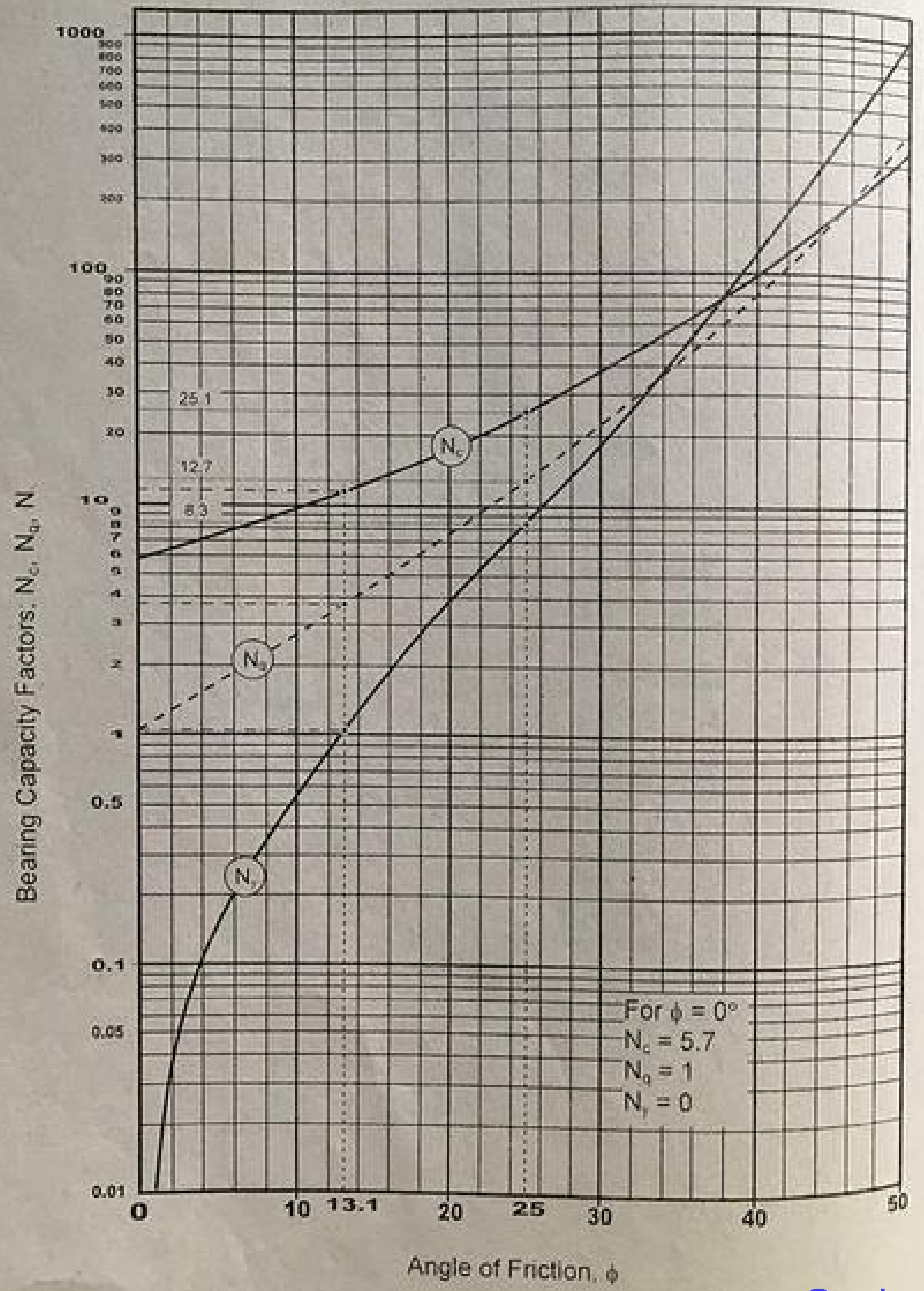
$N_q = 3.7$

$N_\gamma = 1.05$

$q_{all} = c_d\,N_c + q\,N_q + 0.5\,\gamma\,B\,N_\gamma$

$q_{all} = (250)(11.4) + 230(3.7) + 0.5(115)(2.5)(1.05)$

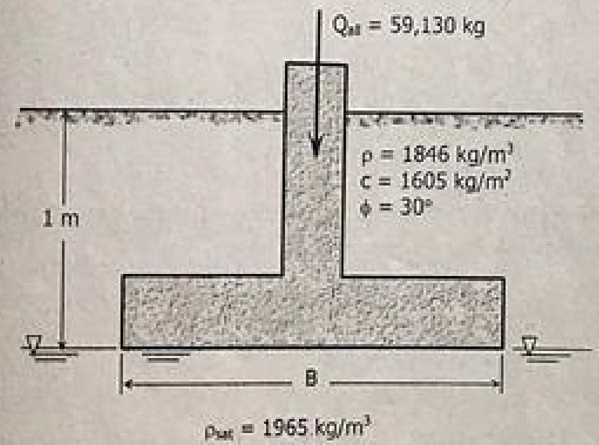
q_{all} = **3,852 psf**



PROBLEM 09.3

A footing 1 m square carries a total load, including its own weight, of 59,130 kg. The base of the footing is at a depth of 1 m below the ground surface. The soil strata at the site consist of a layer of stiff saturated clay 27.5 m thick overlying dense sand. The average density of the clay is 1,846 kg/m³. Given is Terzaghi's ultimate bearing capacity for square footings: $q_u = 1.3\,c\,N_c + \gamma D_f N_q + 0.40\,\gamma B\,N_\gamma$. See Figure 09.13.

a) Determine the gross foundation pressure.
b) Determine the overburden pressure.
c) Determine the ultimate bearing capacity of the soil.
d) Assuming local shear failure, determine the ultimate bearing capacity of the soil.



$q_u = 1.3cN_c + \gamma D_f N_q + 0.4\,\gamma B N_\gamma$

Figure 09.13

SOLUTION

a) Gross foundation pressure:

$$q_g = \frac{\text{Load}}{\text{Area}}$$

$$q_g = \frac{59130 \times 9.81 \times \frac{1}{1000}}{1(1)}$$

$$q_g = 580 \text{ kN/m}^2$$

@seismicisolation

b) Overburden Pressure:

$q = \gamma D_f$

$\gamma = \rho g$

$\gamma = 1846(9.81)$

$\gamma = 18109.26\ \text{N/m}^3$

$\gamma = 18.11\ \text{kN/m}^3$

$q = 18.11(1)$

$\mathbf{q = 18.11\ kN/m^2}$

c) Ultimate bearing capacity:

$q_u = 1.3\,c\,N_c + \gamma D_f\,N_q + 0.40\,\gamma_e B\,N_\gamma$

$c = 1{,}605\ \text{kg/m}^2$

$c = 15.745\ \text{kN/m}^2$

$B = 1\ \text{m}$

From Table 09.1, $\phi = 30°$:

$N_c = 37.16$

$N_q = 22.46$

$N_\gamma = 19.73$

Note: This is case 2, the water table is at the base of footing. For the third term, we will use $\gamma_e = \gamma_b$.

$\gamma_b = \gamma_{sat} - \gamma_w$

$\gamma_b = (1965 \times 9.81) - 9810$

$\gamma_b = 9466.65\ \text{N/m}^3$

$\gamma_b = 9.467\ \text{kN/m}^3$

$q_u = 1.3(15.745)(37.16) + 18.11(1)(22.46) + 0.4(9.467)(1)(19.73)$

$\mathbf{q_u = 1{,}242\ kN/m^2}$

Local shear failure:

$\bar{c} = \frac{2}{3}c$

$\bar{c} = \frac{2}{3}(15.745) = 10.497$

From Table 09.2, for $\phi = 30°$

$N'_c = 18.99$

$N'_q = 8.31$

$N'_\gamma = 4.39$

$q_u = 1.3\bar{c}\,N'_c + q\,N'_q + 0.4\,\gamma_e\,B\,N'_\gamma$

$q_u = 1.3(15.745)(18.99) + 18.11(1)(8.31) + 0.4(9.467)(1)(4.39)$

$\mathbf{q_u = 555.81\ kPa}$

?Bakit hindi ginamit yung 10.497?

Or, you may use Table 09.1 or Figure 09.10 using the modified value of ϕ as follows:

$\tan\bar{\phi} = \frac{2}{3}\tan\phi$

$\tan\bar{\phi} = \frac{2}{3}\tan 30°;\quad \bar{\phi} = 21.05°$

By interpolation between $\phi = 21°$ and $\phi = 22°$ in Table 09.1:

For $\phi = 21°$, $N_c = 18.92$, $N_q = 8.26$, $N_\gamma = 4.31$

For $\phi = 22°$, $N_c = 20.27$, $N_q = 9.19$, $N_\gamma = 5.09$

$$N_c = 18.92 + \frac{20.27 - 18.92}{22 - 21}(21.05 - 21) = \underline{18.9875}$$

$$N_q = 8.26 + \frac{9.19 - 8.26}{22 - 21}(21.05 - 21) = \underline{8.3065}$$

$$N_\gamma = 4.31 + \frac{5.09 - 4.31}{22 - 21}(21.05 - 21) = \underline{4.349}$$

PROBLEM 09.4 (CE MAY 2004)

A soil has the following properties:

Unit weight, $\gamma = 19.2\ \text{kN/m}^3$

Cohesion, $c = 50$ kPa

Angle of friction = 10°

Assume local shear failure and use Figure 09.14 to get the bearing capacity factors.

a) Calculate the net bearing capacity for a strip footing of width 1.25 m at a depth of 4.5 m. The Terzaghi's ultimate bearing capacity equation is given by:

$$q_u = c\,N_c + \gamma D_f N_q + ½\,\gamma B\,N_\gamma$$

b) Considering shear failure only, calculate the safe bearing pressure on a footing 6 m long by 1.25 m wide, using a load factor of 2.5. Given:

$$q_u = c\,N_c[1 + 0.3(B/L)] + \gamma D_f N_q + ½\,\gamma\,B\,N_\gamma[1 - 0.2(B/L)]$$

$$q_s = q_{u\,net} / \text{FS} + \gamma D_f$$

c) What is the safe total load of the footing?

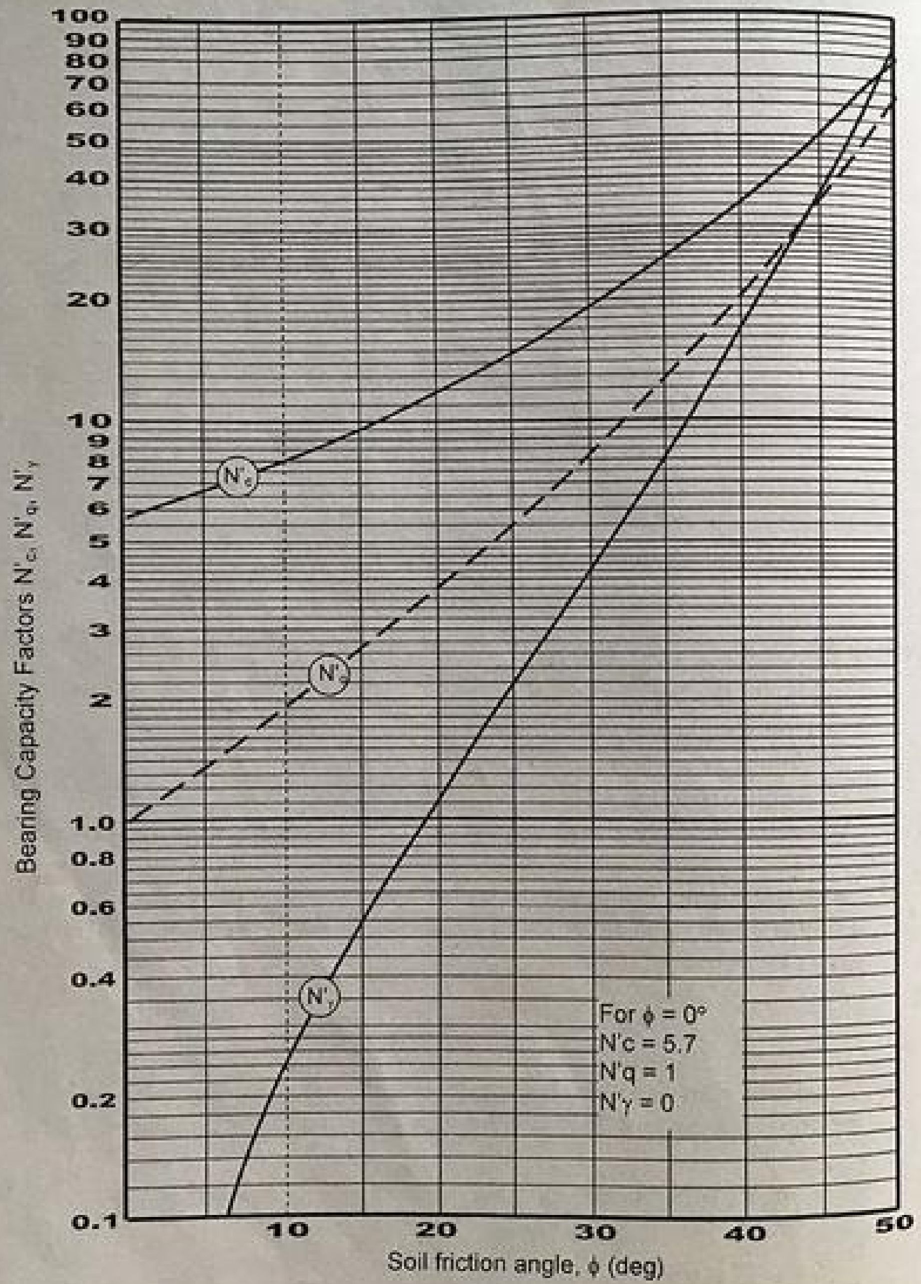


Figure 09.14

SOLUTION

Part *a*:

$$q_u = c\,N_c + \gamma D_f N_q + ½\,\gamma B\,N_\gamma$$

For local shear failure:

$$q_u = \bar{c}\,N'_c + \gamma D_f N'_q + ½\,\gamma B\,N'_\gamma$$

$$\bar{c} = \tfrac{2}{3}c$$

$$\bar{c} = \tfrac{2}{3}(50) = 33.33$$

$D_f = 4.5$ m

$\gamma = 19.2$ kN/m³

$B = 1.25$ m

From the figure, for $\phi = 10°$

$N'_c = 8$

$N'_q = 1.94$

$N'_\gamma = 0.25$

$$q_u = 33.33(8) + 19.2(4.5)(1.94) + ½(19.2)(1.25)(0.25)$$

$$q_u = \mathbf{437.256\ kPa}$$

Part *b*:

$$q_u = \bar{c}\,N'_c[1 + 0.3(B/L)] + \gamma D_f N'_q + ½\,\gamma\,B\,N'_\gamma[1 - 0.2(B/L)]$$

$$q_u = 33.33(8)[1 + 0.3(1.25/6)] + 19.2(4.5)(1.94) + ½(19.2)(1.25)(0.25)[1 + 0.2(1.25/6)]$$

$$q_u = 454.05\ \text{kPa}$$

$$q_{u\,net} = q_u - \gamma D_f$$

$$q_{u\,net} = 454.05 - 19.2(4.5)$$

$$q_{u\,net} = 367.65\ \text{kPa}$$

$$q_s = \frac{367.65}{2.5} + 19.2(4.5)$$

$$q_s = \mathbf{233.46\ kPa}$$

Part *c*:

$$Q = q_s \times A_{ftg}$$

$$Q = 233.46(1.25 \times 6)$$

$$Q = \mathbf{1750.95\ kPa}$$

PROBLEM 09.5

A circular footing 2.5 m in diameter is shown in Figure 09.15. Assume general shear failure and use a factor of safety of 3. Determine the following:

a) the gross allowable bearing capacity
b) the net allowable bearing capacity
c) the safe load that the footing can carry

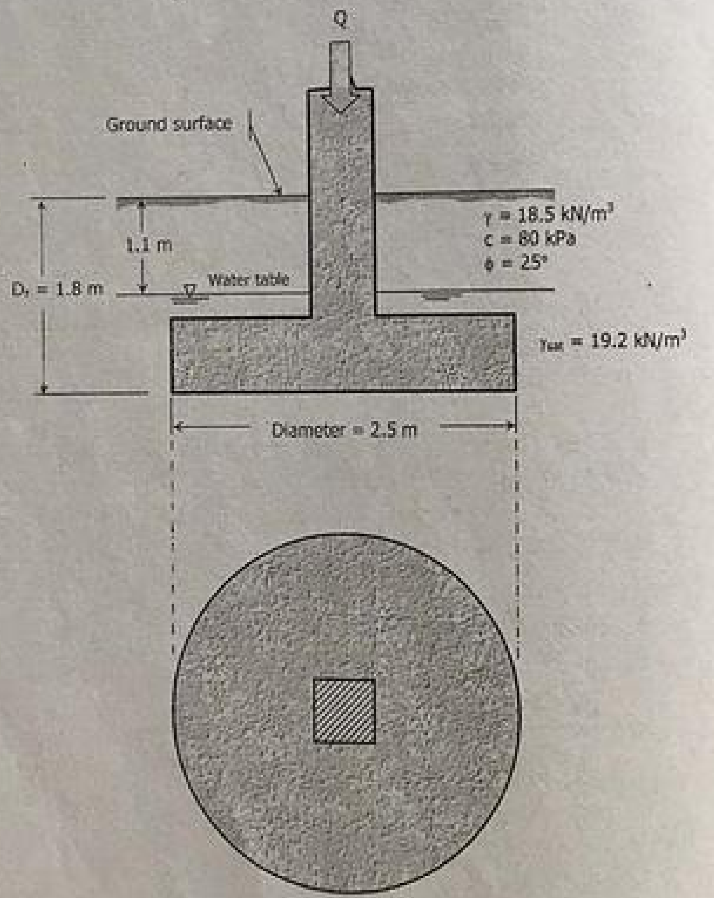


Figure 09.15

SOLUTION

General shear failure for circular footing:

$$q_u = 1.3\,c\,N_c + q\,N_q + 0.3\,\gamma_e\,B\,N_\gamma$$

From Table 09.1, for $\phi = 25°$:

$N_c = 25.13$
$N_q = 12.72$
$N_\gamma = 8.34$
$c = 80$ kPa
q = effective vertical stress at base of footing
$q = (19.2 - 9.81)(1.8 - 1.1) + 18.5(1.1)$
$q = 26.923$ kPa
$\gamma_e = \gamma_b = 19.2 - 9.81$
$\gamma_e = 9.39$ kN/m³
$B = 2.5$ m

$$q_u = 1.3(80)(25.13) + 26.923(12.72) + 0.3(9.39)(2.5)(8.34)$$
$$q_u = 3{,}014.72 \text{ kPa}$$

a) Gross allowable bearing capacity:

$$(q_{allow})_{gross} = \frac{q_u}{FS}$$
$$(q_{allow})_{gross} = \frac{3{,}014.72}{3}$$
$$(q_{allow})_{gross} = \mathbf{1{,}004.9\ kPa}$$

b) Net allowable bearing capacity:

$$q_{u\,net} = q_u - q$$
$$q_{u\,net} = 3{,}014.72 - 26.923$$
$$q_{u\,net} = 2{,}987.8 \text{ kPa}$$
$$(q_{allow})_{net} = \frac{q_{u\,net}}{FS}$$
$$(q_{allow})_{net} = \frac{2{,}987.8}{3}$$
$$(q_{allow})_{net} = \mathbf{995.9\ kPa}$$

c) Safe load:

$$Q_{allow} = (q_{allow})_{net} \times \text{Area of footing}$$
$$Q_{allow} = 995.9 \times \frac{\pi}{4}(2.5)^2$$
$$Q_{allow} = \mathbf{4{,}888.8\ kN}$$

PROBLEM 09.6

A square footing is shown in Figure 09.16. The footing will carry a gross load of 700 kN. Using a factor of safety of 3, determine the required value of B. Assume general shear failure.

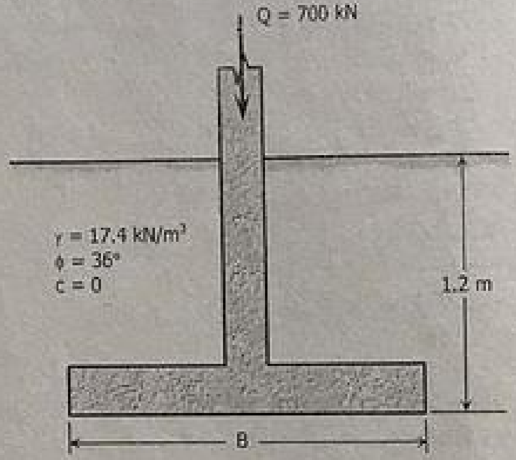


Figure 09.16

SOLUTION

$$q_{all} = \frac{q_u}{3} \geq \frac{Q}{A_{ftg}}$$

Assuming general shear failure:

$$q_u = 1.3cN_c + qN_q + 0.4\gamma_e BN_\gamma \quad \text{(Eq. 9.3)}$$

From Table 09.1:

$N_c = 63.53$
$N_q = 47.16$
$N_\gamma = 54.36$
$q = 17.4(1.2)$
$q = 20.88$ kPa
$\gamma_e = \gamma = 17.4$ kN/m³

$q_u = 1.3(0))63.53) + 20.88(47.16) + 0.4(17.4)B(54.36)$

$q_u = 984.7 + 378.3B$

$$[\frac{q_u}{3} = \frac{Q}{B^2}]$$

$$\frac{984.7 + 378.3B}{3} = \frac{700}{B^2}$$

$378.3B^3 + 984.7B^2 = 2100$

By trial and error, B = **1.207 m**

PROBLEM 09.7

A rectangular footing 5 ft × 2.5 ft is shown in Figure 09.17. Use the Meyerhof equation for the ultimate load for shallow foundation under eccentric load (one-way eccentricity). Calculate the value of the ultimate load Q_u in pounds.

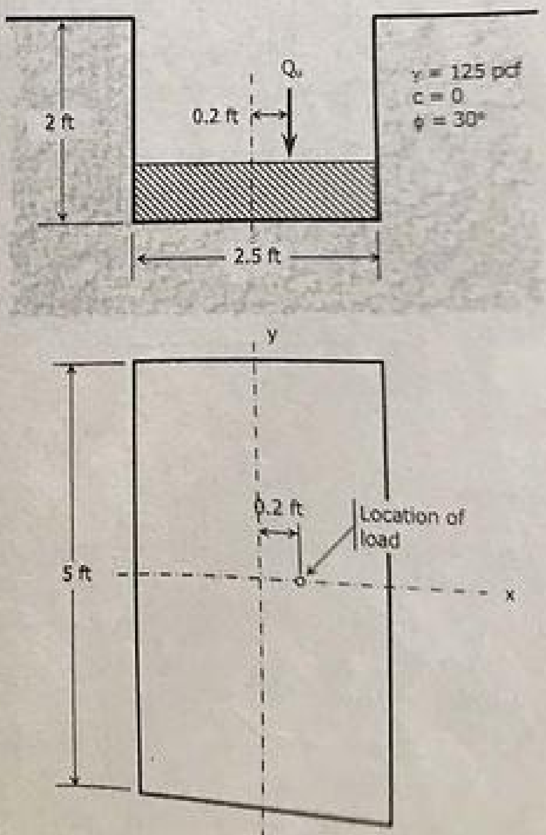


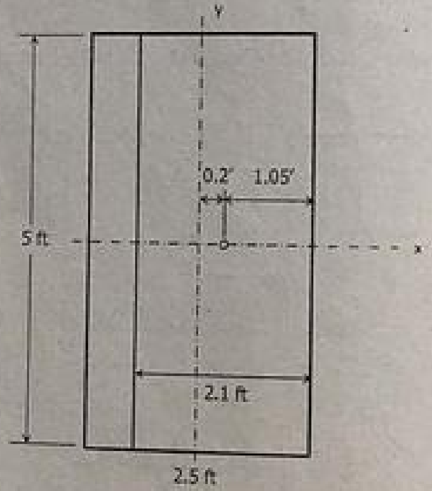
Figure 09.17

SOLUTION

$$q_u = c\,N_c\,s_c\,d_c\,i_c + qN_q\,s_q\,d_q\,i_q + 0.5\,\gamma\,B'\,N_\gamma\,s_\gamma\,d_\gamma\,i_\gamma$$

$c = 0$
$q = \gamma D_f$
$q = 125(2)$
$q = 250$ psf

From Table 09.3:
$N_q = 18.4$
$N_\gamma = 15.67$



$s_q = s_\gamma = 1 + 0.1\,K_p\,\dfrac{B}{L}$ since $\phi > 10°$

B' = smaller effective dimension = 2.1
$L' = 5'$
$K_p = \tan^2(45° + 30°/2)$
$K_p = 3$
$s_q = s_\gamma = 1 + 0.1(3)(2.1/5)$
$s_q = s_\gamma = 1.126$

$$d_q = d_\gamma = 1 + 0.1\sqrt{K_p}\,\frac{D_f}{B'}$$

$$d_q = d_\gamma = 1 + 0.1\sqrt{3}\,\frac{2}{2.1}$$

$d_q = d_\gamma = 1.165$

Note: No inclination factor.

$q_u = 0 + 250(18.4)(1.126)(1.165) + 0.5(125)(2.1)(15.67)(1.126)(1.165)$
$q_u = 8{,}732$ pcf

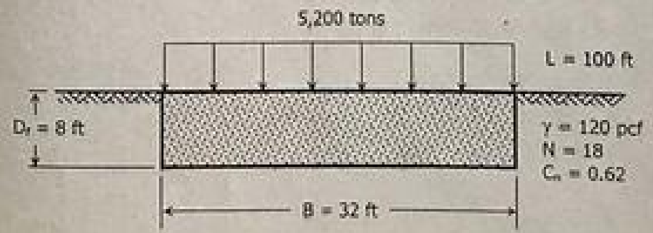
$Q_u = q_u\,(B'\,L')$
$Q_u = 8{,}732(2.1)(5)$
$Q_u =$ **91,686 lbs**

PROBLEM 09.8

A mat foundation 100 ft by 32 ft is to support a total load 5200 tons. The formation rests on sand with a density of 120 pcf and an average standard penetration resistance (SPT) of N-value of 18. The base of the mat is located at a depth of $D_f = 8$ feet. Allowable settlement is 2 inches.

a) Determine the overburden pressure at the base of the mat foundation in tons per square foot?
b) Assuming a correction factor $C_n = 0.62$, determine the allowable bearing capacity of the mat in tons per square foot?
c) Determine the factor of safety against bearing capacity failure?

SOLUTION



Overburden pressure, $q = \gamma\,D_f = 120(8)$
Overburden pressure = 960 lb/ft² × (1 ton/2000 lbs)
Overburden pressure = **0.48 tons/ft²**

For this problem:

$N = 18$

$c_N = 0.62$

$c_W = 1$ (the depth of groundwater table was not specified)

$N_{cor} = 0.62(1)(18)$

$N_{cor} = 11.16$

$\rho_a = 2$ inches

$\rho_a = 50.8$ mm

$q_a = 0.41(11.16)(50.8)$

$q_a = 232.44 \text{ kPa} \times \dfrac{14.7 \text{ psi}}{101.325 \text{ kPa}}$

$q_a = 33.72$ psi

$q_a = 33.72 \text{ psi} \times 144$

$q_a = 4{,}855.96$ psf

$q_a = 4{,}855.96 \text{ psf} \div 2{,}000$

$q_a =$ **2.43 tons/sq. ft.**

Factor of safety:

$$\text{F.S.} = \frac{q_a}{\sigma_a - \gamma D_f}$$

σ_a = maximum applied foundation stress

$$\sigma_a = \frac{5{,}200}{100(32)}$$

$\sigma_a = 1.625$ tons/sq. ft.

$\gamma D_f = 120(8)$

$\gamma D_f = 960 \text{ psf} \div 2000$

$\gamma D_f = 0.48$ tons/sq. ft

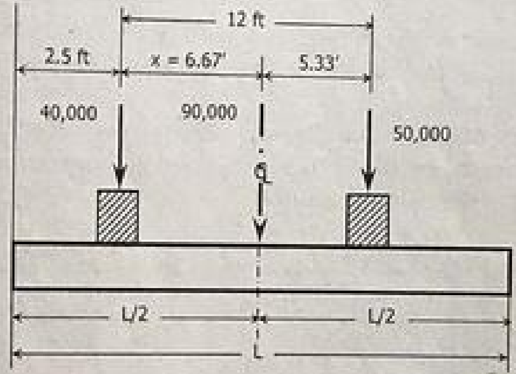
$$\text{F.S.} = \frac{2.43}{1.625 - 0.48}$$

F.S. = **2.12**

PROBLEM 09.9

A rectangular footing is to support two square columns each 12″ × 12″ and spaced 12 feet on centers. One column carries a load of 40 kips and the other carries a load of 50 kips. The footing is 2 ft thick and its length should extend 2.5 ft beyond the center of the column carrying the 40-kip load. The base of footing is 5 ft below the ground surface. Assume specific gravity of concrete and soil above the footing to be 2.4 and 1.78, respectively. Determine the length of the footing if the allowable soil bearing capacity is 2000 psf.

SOLUTION



For the pressure in the soil to be uniform, the resultant footing load should coincide with the centroid of the footing.

Locate the resultant load by taking moment about the 40-kip load:

$90x = 40(0) + 50(12)$

$x = 6.67$ feet

From the figure

$L/2 = 2.5 + 6.67$

L = **18.34 feet**

To compute the width of footing:

Effective soil pressure

$q_e = 2000 - (62.4 \times 2.4)(2) - (62.4 \times 1.78)(5 - 2)$

$q_e = 1367.264$ psf

Total load = 40 + 50

Total load = 90 kips

Total load = 90,000 lbs

$$A_{footing} = \frac{90{,}000}{1367.264}$$

$A_{footing} = 65.82\ ft^2$

$L \times W = 18.34 \times W$

$18.34 \times W = 65.82$

$W = \underline{3.59\ feet}$

PROBLEM 09. 10

A trapezoidal combined footing is shown in Figure 09.18. The base of the footing is 1.20 m below the ground and the allowable soil bearing pressure at that point is 250 kPa.

a) Determine the effective allowable bearing pressure.
b) Determine the required area of the footing in square meter.
c) Determine the value of *a* and *b*.

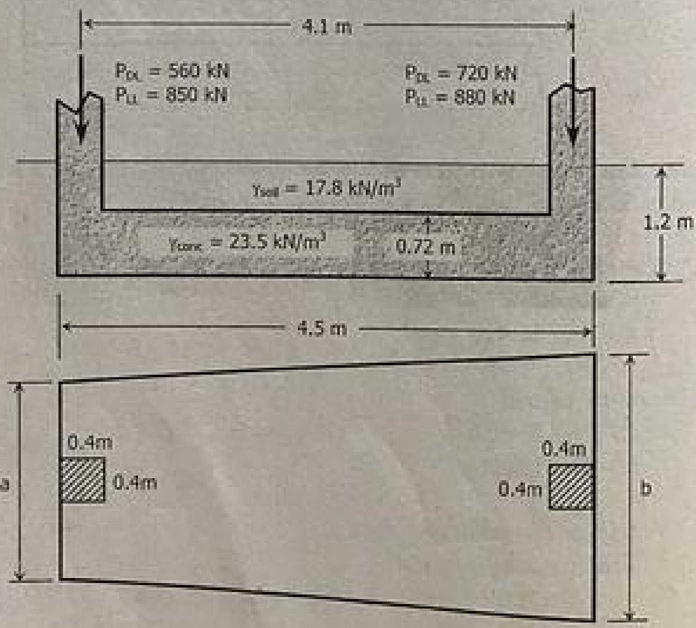


Figure 09.18

SOLUTION

$q_e = q_a - \Sigma\gamma h$

$q_e = 250 - 23.5(0.72) - 17.8(1.2 - 0.72)$

$q_e = \mathbf{224.536\ kPa}$

$P_1 = 560 + 850 = 1410\ kN$

$P_2 = 720 + 880 = 1600\ kN$

$P = 1410 + 1600 = 3010\ kN$

$A_{ftg} = P/q_e = 3010/224.536$

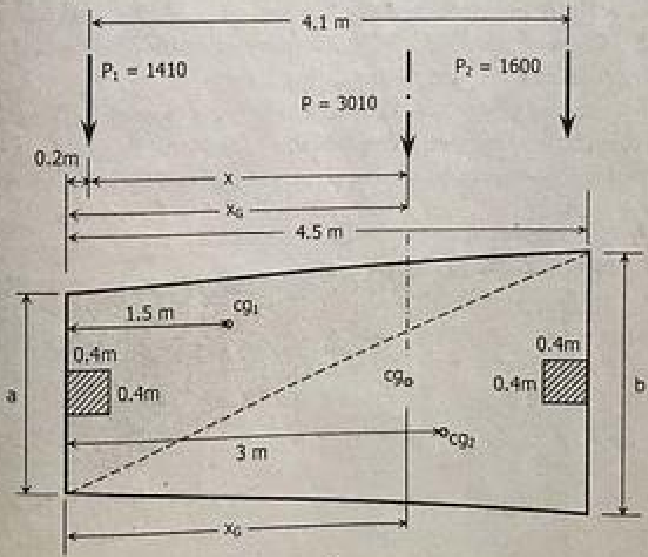
$A_{ftg} = \mathbf{13.405\ m^2}$

Solving for *a* and *b*:

$$A_{ftg} = \frac{a+b}{2}(4.5) = 13.405$$

$a + b = 5.958\ m \quad \rightarrow$ Eq. (1)

The centroid of the footing must coincide with the resultant load:



$3010x = 1410(0) + 1600(4.1)$

$x = 2.179\ m$

$x_G = 0.2 + x$

$x_G = 2.379\ m$

$A_{trig}\, x_G = A_1 x_1 + A_2 x_2$

$13.405(2.379) = ½(a)(4.5)(1.5) + ½(b)(4.5)(3)$

$a + 2b = 9.45 \rightarrow$ Eq. (2)

From Eq. (1):

$b = 5.958 - a$

$a + 2(5.958 - a) = 9.45$

a **= 2.45 m**

b **= 3.5 m**

PROBLEM 09. 11 (CE MAY 2003)

An exterior column with service dead load = 760 kN and service live load = 580 kN, and an interior column with service dead load = 1,100 kN and service live load = 890 kN are to be supported on a combined rectangular footing whose outer end cannot protrude beyond the outer face of the exterior column, as shown in Figure 09.19. The allowable bearing pressure of the soil is 290 kPa. The bottom of the footing is 1.80 m below grade and a surcharge of 4.8 kPa is specified on the surface. The footing thickness is 1.20 m. The unit weight of concrete is 24 kN per cubic meter and the unit weight of soil is 18 kN per cubic meter. The footing is to be designed such that the resulting pressure under service loads is uniform.

a) Determine the effective allowable bearing pressure (allowable bearing pressure minus the weights of concrete, soil and surcharge).

b) Determine the minimum dimensions of the combined footing.

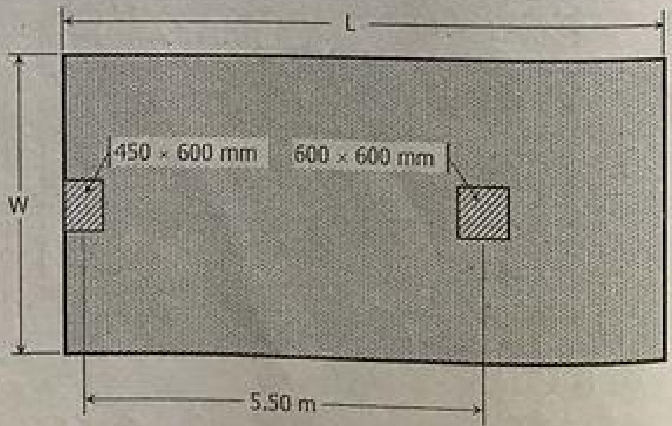


Figure 09.19

SOLUTION

Part *a*:

$q_e = q_a - \gamma_{soil} h_{soil} - \gamma_{conc} h_{conc}$ - Surcharge

$q_e = 290 - 18(0.6) - 24(1.2) - 4.8$

q_e **= 245.6 kPa**

Part *b*: See Figure 09.20

Value of a:

Solve for X by taking moment of force about P_1:

$P_1 = 760 + 580$

$P_1 = 1{,}340$ kN

$P_2 = 1{,}100 + 890$

$P_2 = 1{,}990$ kN

$P = P_1 + P_2$

$P = 1{,}340 + 1{,}990$

$P = 3{,}330$ kN

$[PX = P_1 x_1 + P_2 x_2]$

$3{,}330\,X = 1{,}340(0) + 1{,}990(5.5)$

$X = 3.317$ m

$a = X + 0.225$

$a = 3.317 + 0.225$

$a = 3.542$ m

Such that the resulting pressure under the service load is uniform, the resultant service load P must coincide with the centroid of the footing.

Thus, $a = L/2$

$3.542 = L/2$

L **= 7.084 m**

Area of footing, $A_f = \dfrac{P}{q_e}$

Area of footing, $A_f = \dfrac{3{,}330}{245.6}$

Area of footing, $A_f = 13.559\ \text{m}^2$

$[A_f = L \times W]$

$$13.559 = 7.084 \times W$$

$$W = 1.91 \text{ m}$$

Therefore, the footing dimension is 2m × 7m

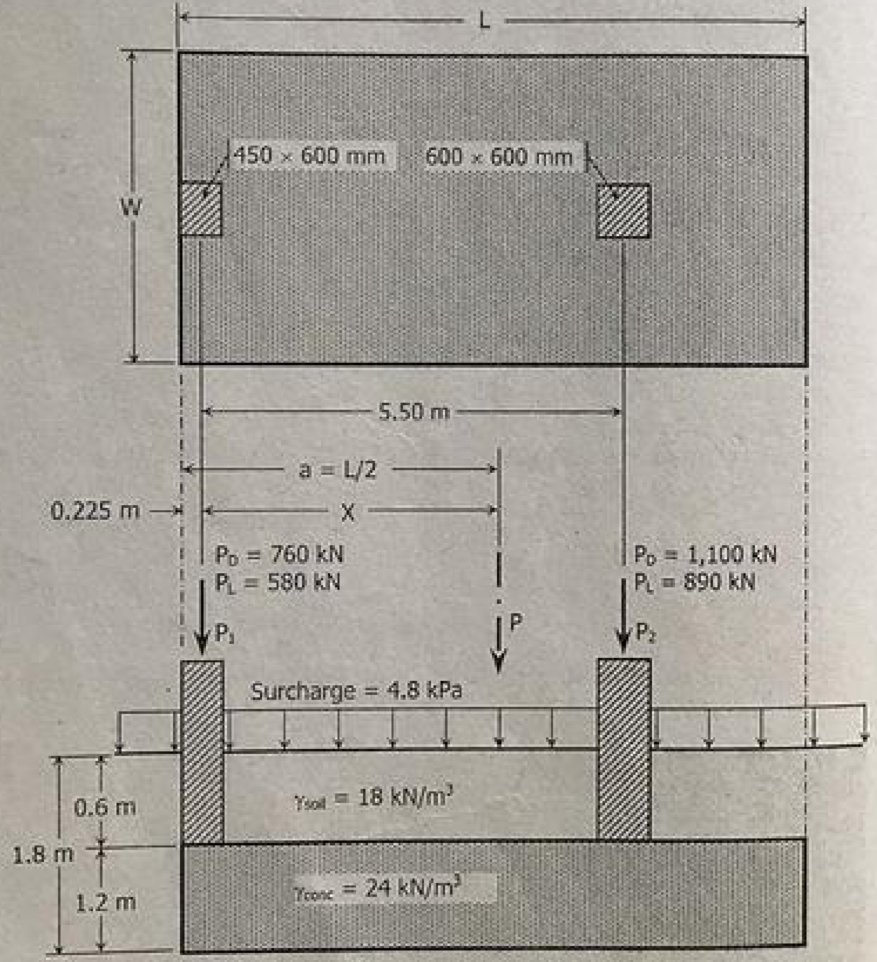


Figure 09.20

PROBLEM 09.12 (CE NOVEMBER 2002)

A line of four piles supporting a pier is shown in Figure 09.21. The vertical load of 200 kips includes the weight of the rigid pile cap. The piles may be assumed to be fixed at the depth H_1 = 3 ft below the bottom of sea. The height H_2 from the sea bead to the bottom of the pile cap is 17 feet.

a) Determine the axial force on pile D?
b) Determine the shear force on each pile?
c) Determine the maximum moment in pile D, assuming point of contraflexure at a depth of 10 feet below the pile cap?

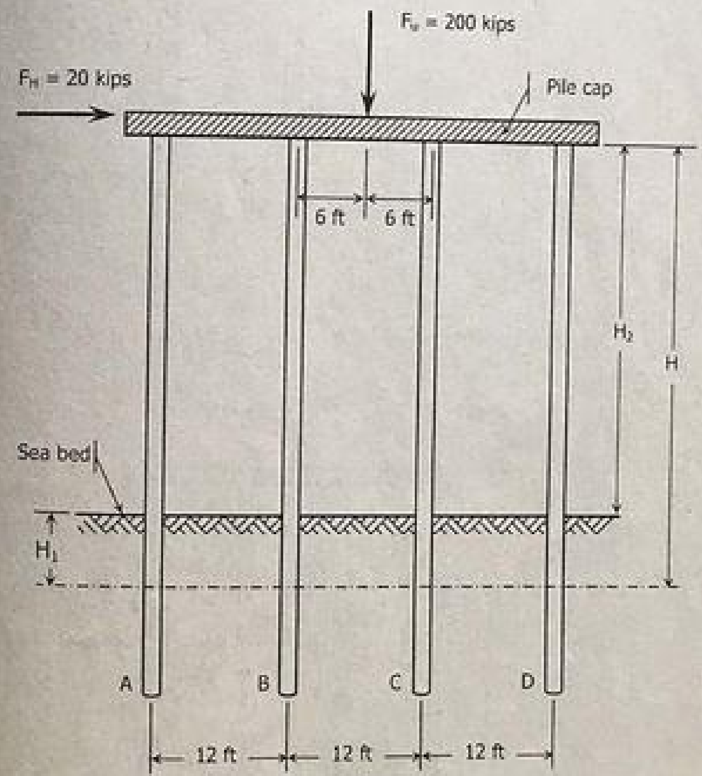
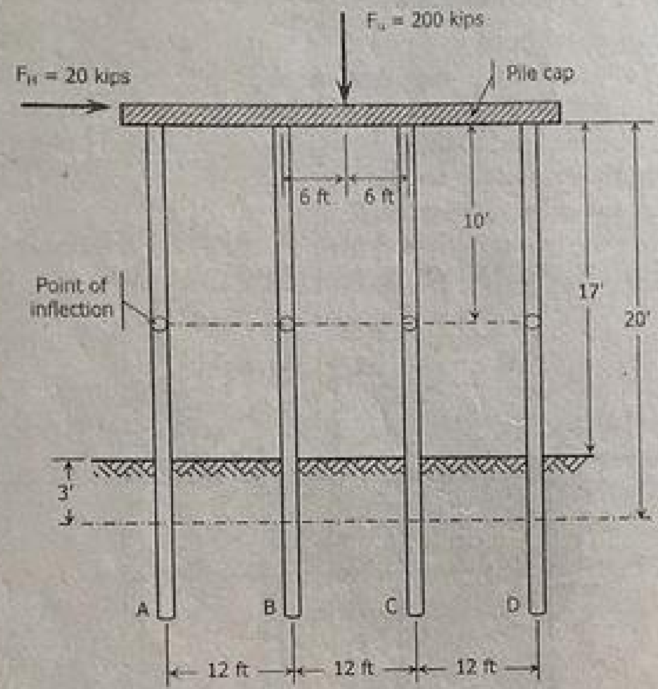
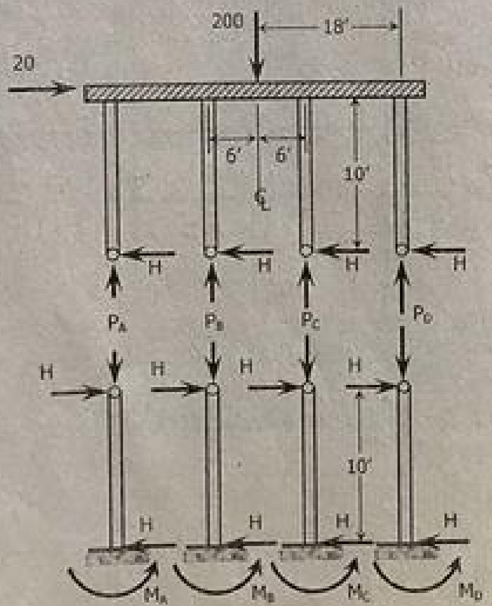


Figure 09.21

SOLUTION





$[\Sigma F_H = 0]$

$4H = 20$

$H = \mathbf{5\ kips}$ (shear force in each pile)

Maximum moment at D:

$M_D = H \times 10$

$M_D = 5(10)$

$M_D = \mathbf{50\ kip\text{-}ft}$

Axial force at D:

$$P = \frac{F}{N} \pm \frac{Tr}{J}$$

$F = 200$ kips

$N = 4$

$T = 20(10)$

$T = 200$ kip-ft

$r = 18$ ft

$J = \Sigma x^2$

$J = (6^2 + 18^2) \times 2$

$J = 720\ ft^2$

$$P_D = \frac{200}{4} + \frac{200(18)}{720}$$

$P_D = \mathbf{55\ kips}$

To solve for the other axial force:

$$P_C = \frac{200}{4} + \frac{200(6)}{720}$$

$P_C = 51.67$ kips

$$P_B = \frac{200}{4} - \frac{200(6)}{720}$$

$P_B = 48.33$ kips

$$P_A = \frac{200}{4} - \frac{200(18)}{720}$$

$P_A = 45$ kips

Check:

$\Sigma F_V = 0$

$P_A + P_B + P_C + P_D - 200 = 0$

$45 + 48.33 + 51.67 + 55 - 200 = 0$ (OK)

Chapter 10

Miscellaneous Topics & Additional Problems

10.1 PILES AND DEEP FOUNDATION

10.1.1 PILE CAPACITY FROM DRIVING DATA (DYNAMIC PILE FORMULAS)

10.1.1.1 AASHTO FORMULA

$$Q_u = \frac{2h(W_r + A_r p)}{s + 0.1}, \text{ lbs} \qquad \text{Eq. 10.1}$$

Recommended factor of safety = 6

10.1.1.2 Navy-McKay Formula

$$Q_u = \frac{e_h E_h}{s\left(1 + 0.3\dfrac{W_p}{W_r}\right)}, \text{ lbs} \qquad \text{Eq. 10.2}$$

Recommended factor of safety = 6

10.1.1.3 Eytelwein Formula

$$Q_u = \frac{e_h E_h}{s + 0.1(W_p / W_r)}, \text{ lbs} \qquad \text{Eq. 10.3}$$

Recommended factor of safety = 6

Where:

e_h = Efficiency of hammer

Recommended values:

Single-acting, $e_h = 0.8$

Double-acting, $e_h = 0.85$

Drop, $e_h = 0.85$

A_r = ram cross-section, in²

p = pressure, psi

E_h = rated hammer energy, in-lb

s = average penetration of pile during the last few blows, inches

W_p = Total weight of pile, pounds

Wr = Weight of ram, pounds

10.1.1.4 Engineer News Record (ENR or Engineering News)

Drop hammer

$$Q_u = \frac{2W_h h}{s+1}, \text{lbs} \qquad \text{Eq. 10.4}$$

Single-acting steam hammer

Driven weight < striking weight

$$Q_u = \frac{2W_h h}{s+0.1}, \text{lbs} \qquad \text{Eq. 10.5}$$

Driven weight > striking weight

$$Q_u = \frac{2W_h h}{s+0.1\left(\frac{W_{driven}}{W_h}\right)}, \text{lbs} \qquad \text{Eq. 10.6}$$

Double-acting steam hammer

Driven weight < striking weight

$$Q_u = \frac{2E}{s+0.1}, \text{lbs} \qquad \text{Eq. 10.7}$$

Driven weight > striking weight

$$Q_u = \frac{2E}{s+0.1\left(\frac{W_{driven}}{W_h}\right)}, \text{lbs} \qquad \text{Eq. 10.8}$$

Where:

Q_u = ultimate pile capacity, lbs

W_h = weight of hammer in lbs

h = height of fall, ft

s = average penetration during the last five blows

E = energy transferred to the pile on each stroke, ft-lb

10.1.1.5 Modified Engineer News Record

$$Q_u = \frac{1.25e_h E_h}{s+0.1}\,\frac{W_r + n^2 W_p}{W_r + W_p}, \text{lbs} \qquad \text{Eq. 10.9}$$

Where:

e_h = Efficiency of hammer

Recommended values:

Single-acting, $e_h = 0.8$

Double-acting, $e_h = 0.85$

Drop, $e_h = 0.85$

n = Coefficient of restitution

Recommended values:

Wood piles, $n = 0.25$

Wood cushion on steel, $n = 0.32$

Steel-on-steel anvil, $n = 0.5$

E_h = rated hammer energy, in-lb

s = average penetration of pile during the last five blows, inches

W_p = Total weight of pile, pounds

Wr = Weight of ram, pounds

10.1.1.6 Danish Formula

One of the few formulae considered to have a reasonable precision, based on a statistical study of pile load tests, is the Danish formula, which should be used with a factor of safety of 3.

$$Q_u = \frac{\alpha W_H H}{S + 0.5S_e} \quad \text{Eq. 10.10}$$

$$S_e = \sqrt{\frac{2\alpha W_H HL}{AE}} \quad \text{Eq. 10.11}$$

Where

Q_u = ultimate dynamic bearing capacity of driven pile
α = pile driving hammer efficiency (normally 1)
W_H = weight of hammer
H = hammer drop (note that $W_H H$ = Hammer energy)
S = Inelastic set of pile, in distance per hammer blow
S_e = Elastic set of pile, in distance per hammer blow
L = Pile length
A = Pile end area
E = Modulus of elasticity of pile material

10.1.2 THEORETICAL PILE CAPACITY

The ultimate load capacity Q_u consists of two parts. One part is due to friction, called skin friction or shaft friction or side shear Q_f, and the other is due to end bearing at the base or tip of the pile Q_b.

$$Q_u = Q_f + Q_b \quad \text{Eq. 10.12}$$

Where:

Q_f = skin/shaft friction or side shear (ultimate)
Q_b = end bearing or point resistance (ultimate)

10.1.2.1 THE ALPHA α METHOD

The α-method determines the adhesion factor, α, as the ratio of the skin friction factor, f_s, to the undrained shear strength (cohesion), c_u.

$$Q_f = \alpha\, c_u\, P\, L \quad \text{Eq. 10.13}$$

$$Q_b = f_b\, A_b = N_c\, (c_u)_b\, A_b \quad \text{Eq. 10.14}$$

$$c_u = \frac{q_u}{2} \text{ of } \frac{S_u}{2} \quad \text{Eq. 10.15}$$

Table 10.1 – Values of N_c for driven piles

L/B	N_c
0	6.3
1	7.8
2	8.5
≥ 3	9

Table 10.2 – Typical values of the adhesion factor α

Cohesion c (GPa)	α Range of values	Average
24	-	1.0
48	0.56 – 0.96	0.83
96	0.34 – 0.83	0.56
144	0.26 – 0.78	0.43

10.1.2.2 THE BETA β METHOD

In β-method, the friction capacity is estimated as a fraction of the average effective vertical stress (as evaluated halfway down the pile).

$$Q_f = \beta\, p_{eff}\, P\, L \quad \text{Eq. 10.16}$$

$$Q_b = N_q\, (p_{eff})_b\, A_b \quad \text{Eq. 10.17}$$

Table 10.3 – Meyerhof values of N_q for driven and drilled piles

φ	20°	25°	28°	30°	32°	34°	36°	38°	40°	42°	45°
Driven	8	12	20	25	35	45	60	80	120	160	230
Drilled	4	5	8	12	17	22	30	40	60	80	115

Table 10.4 – Typical values of β for driven piles in soft to medium clay with c < 96 GPa

Pile length, L (m) →	0	7.5	15	23	30	38	45	53	60
β →	0.3	0.3	0.3	0.27	0.23	0.20	0.18	0.17	0.16

Table 10.5 – Recommended values of adhesion & cohesion for piles in clay

Pile type	Consistency of clay	Cohesion, c (GPa)	Adhesion, c_A (GPa)
Timber or concrete	Very soft	0 - 12	0 - 12
	Soft	12 - 24	12 - 23
	Medium stiff	24 - 48	23 - 36
	Stiff	48 - 96	36 - 45
	Very stiff	96 - 192	45 - 62
Steel	Very soft	0 - 12	0 - 12
	Soft	12 - 24	12 - 22
	Medium stiff	28 - 48	22 - 34
	stiff	48 - 96	34 - 34.5
	Very stiff	96 - 192	34.5 - 36

Burland Formula (for clay)

$$\beta = (1 - \sin\phi)\tan\phi\,\sqrt{OCR} \qquad \text{Eq. 10.18}$$

Where:

B = diameter of width of pile
P = perimeter of cross-section of pile
L = length of pile, m
c_u = undrained shear strength (cohesion)
$(c_u)_b$ = undrained shear strength of soil at base of pile
A_b = cross-sectional area of pile base
f_b = base resistance
N_c = capacity factor
s_u or q_u = unconfined compressive strength
p_{eff} = average effective vertical stress at midheight of pile in a layer
N_q = capacity factor

Note: The actual values of α, β, and N_q are uncertain.

10.1.3 CAPACITY OF PILE GROUP

Some piles are installed in groups, spaced approximately 4 to 3.5 times the pile diameter apart. The piles function as a group due to the use of a concrete load-transfer cap encasing all of the pile heads. The weight of the cap subtracts from the gross group capacity. The capacity due to the pile cap resting on the ground (as a spread footing) is disregarded.

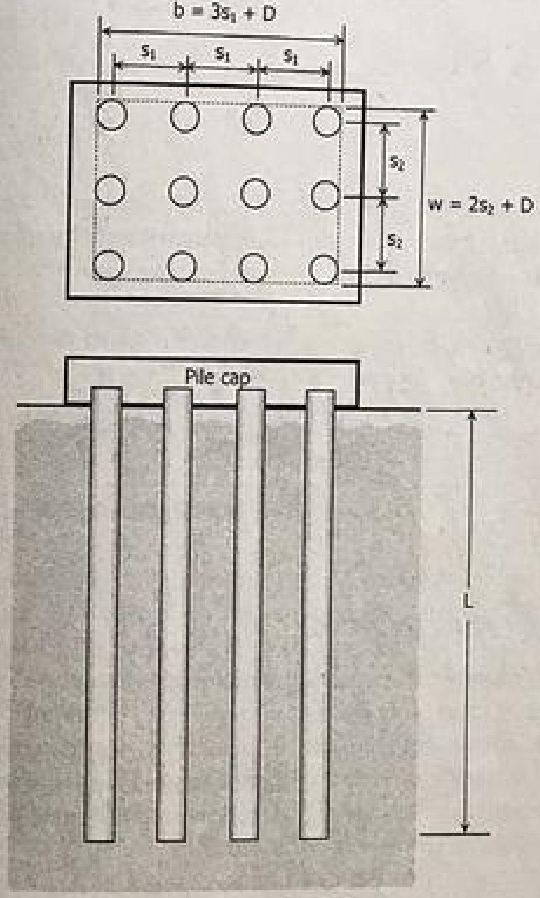


Figure 10.1 - Pile Group

For cohesionless (granular) soils, the capacity of a pile group is taken as the sum of the individual capacities, although the actual capacity will be greater. In-situ tests should be used to justify any increase.

For cohesive soils, the group capacity is taken as the smaller of (*a*) the sum of the individual capacities and (*b*) the capacity assuming block action. The block action capacity is calculated assuming that the piles form a large pier whose dimensions are the group's perimeter. The block depth, L, is the distance from the surface to the depth of the pile points. The width, w, and length, b, of the pier are the length and width of the pile group as measured from the outside (not centers) of the outermost piles.

$$\text{Perimeter, } p = 2(b + w) \qquad \text{Eq. 10.19}$$

$$\text{Area, } A_p = (b + w)^2 \qquad \text{Eq. 10.20}$$

The average undrained shear strength, c_u, along the depth of the piles is used to calculate the skin friction capacity. The average undrained shear strength at the pile tips, c_{ub}, is used to calculate the end-bearing capacity.

The group capacity can be more or less than the sum of the individual pile capacities pile capacities. The *pile group efficiency*, η_G, is:

$$\eta_G = \frac{\text{Group capacity}, Q_{ug}}{\sum \text{individual capacities}, Q_u} \qquad \text{Eq. 10.21}$$

10.1.4 TENSILE CAPACITY OF PILES

Tension piles are intended to resist upward forces. Basement and buried tanks below water level may require tension piles to prevent "floating away". However, tall buildings subjected to overturning moments also need to resist pile pull-out. Unlike piles loaded in compression, the *pull-out capacity* of piles does not include the tip capacity. The pullout capacity includes the weight of the pile and the shaft resistance (skin friction).

10.1.5 SETTLEMENT OF PILE GROUP

Piles bearing on rock essentially do not settle. Piles in sand experience minimal settlement. Piles in clay may experience significant settling. The settlement of a pile group can be estimated by assuming that the support block (used to calculate the group capacity) extends to a depth of only 2/3 of the pile

length. Settlement above $(2/3)L$ is assumed to be negligible. Below the $(2/3)L$ depth, the pressure distribution spreads out at a vertical:horizontal rate of 2:1. The presence of the lower $L/3$ pile length is disregarded. See Figure 10.2.

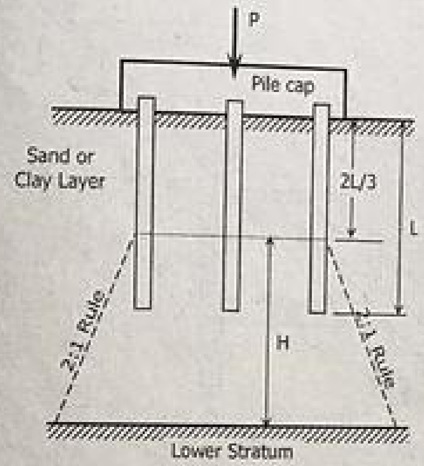


Figure 10.2 – Pile Group

10.2 BRACED CUTS

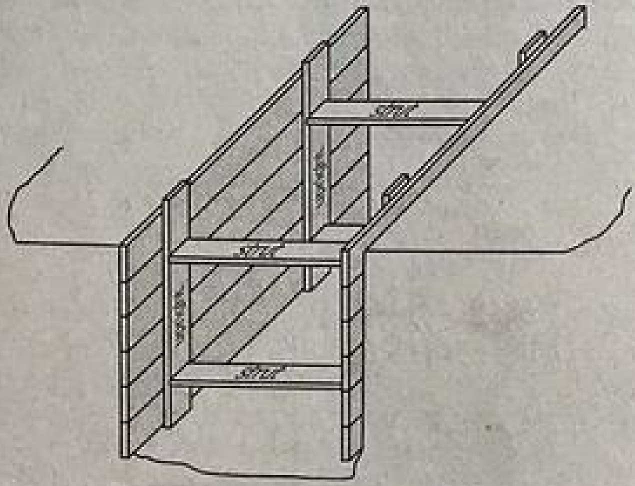
Bracing is used when temporary trenches for water, sanitary, and other lines are opened in soil. A *braced cut* is an excavation in which the active earth pressure from one bulkhead is used to support the facing bulkhead. The *box-shoring* and *close-sheeting* methods of support are shown in Figure 10.3.

The load is transferred to the struts at various points, so the triangular active pressure distribution does not develop. Since struts are installed as the excavation goes down, the upper part of the wall deflects very little due to the strut restraint. The pressure on the upper part of the wall is considerably higher than is predicted by the active earth pressure equations.

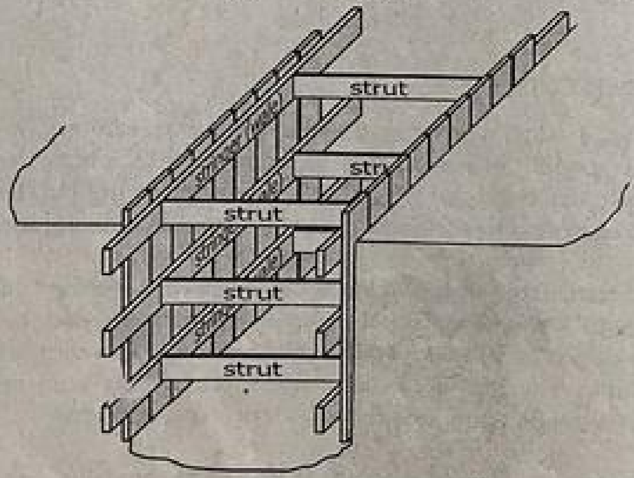
The soil removed from the excavation is known as the *spoils*. Spoils should be placed far enough from the edge of the cut so that they do not produce a surcharge lateral loading.

The bottom of the excavation is referred to as the *base of the cut*, *mudline*, *dredge line*, and *toe of the excavation*.

Excavations below the water table should be dewatered prior to cutting.



(*a*) Box Shoring

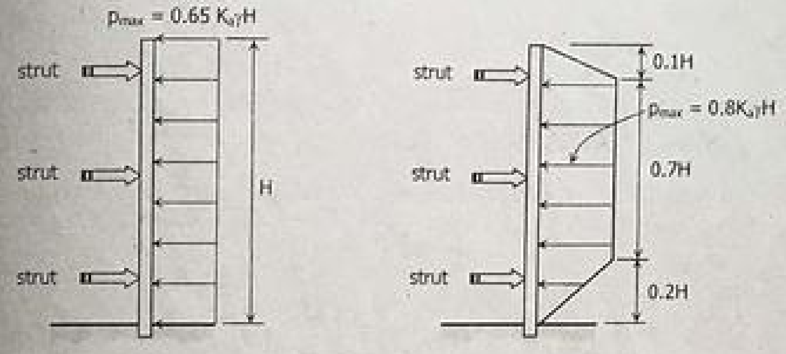


(*b*) Close Sheeting

Figure 10.3 – Shoring of Braced Cuts

10.2.1 BRACED CUTS IN SAND

The analysis of braced cuts is approximate due to the extensive bending of the sheeting. For drained sand, the pressure distribution is approximately uniform with depth.



(a) Pecks pressure diagram (b) Tschebotarioff

Figure 10.4 – Pressure diagram for design of bracing system

10.2.2 BRACED CUTS IN STIFF CLAY

For undrained clay, ϕ = 0°. In this case, the lateral pressure distribution depends on the average undrained shear strength (cohesion) of the clay. If $\gamma H/c \leq 4$, the clay is stiff and the pressure distribution is given in Figure 10.5.

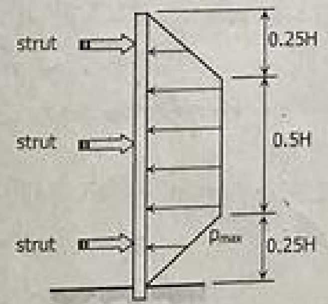


Figure 10.5 – Peck's pressure diagram for stiff clay

$$p_{max} = 0.2\gamma H \text{ to } 0.4\gamma H \qquad \text{Eq. 10.22}$$

Except when the cut is underlain by deep, soft, normally consolidated clay, the maximum pressure can be approximated as:

$$p_{max} = \left(1 - \frac{4c}{\gamma H}\right)\gamma H \qquad \text{Eq. 10.23}$$

10.2.3 BRACED CUTS IN SOFT CLAY

If $\gamma H/c \geq 6$, the clay is soft and the lateral pressure distribution will be as shown in Figure 10.6.

$$p_{max} = \left(1 - \frac{4c}{\gamma H}\right)\gamma H \qquad \text{Eq. 10.24}$$

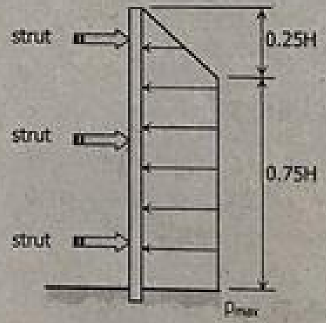


Figure 10.6 – Peck's pressure diagram for soft clay

For cuts underlain by deep, soft, normally consolidated clays, the maximum pressure is:

$$p_{max} = \gamma H - 4c \qquad \text{Eq. 10.25}$$

If $6 \leq \gamma H/c \leq 8$, the bearing capacity of the soil is probably sufficient to prevent shearing and upward heave. Simple braced cuts should not be attempted is $\gamma H/c > 8$.

10.2.4 BRACED CUTS IN MEDIUM CLAY

If $4 < \gamma H/c < 6$, the soft and stiff clay cases should both be evaluated. the case that results in greater pressure should be used when designing the bracing.

10.2.5 ANALYSIS OF STRUT REACTION

Since braced excavations with more than one strut are statically indeterminate, strut forces and sheet piling moments may be evaluated by assuming hinged beam action.

The strut load may be determined by assuming that the vertical members are hinged at each strut level except the topmost and the bottommost one.

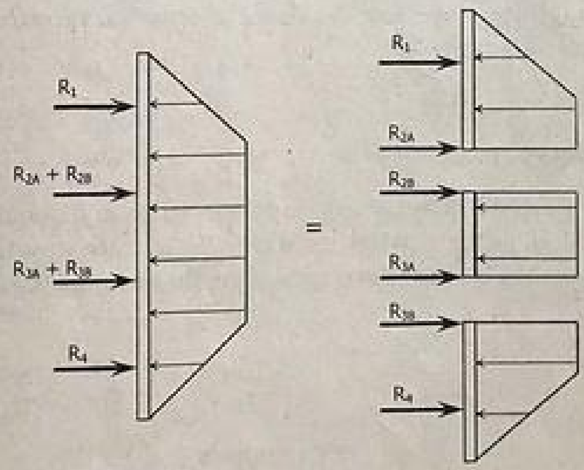


Figure 10.7 – Determination of strut loads

10.3 SLOPE STABILITY

The maximum slope for cuts in cohesionless (drained) sand is the *angle of internal friction* or *angle of repose*, ϕ. In cohesive soils such as clay however, the maximum slope for cuts is more difficult to determine.

The soil or rock in a slope exist in a state of equilibrium between gravity forces tending to move the material down the slope and the internal shearing resistance of the material. A slope failure occurs when the force tending to cause rupture exceeds the resisting force. The overstressing of a slope or reduction in shear strength may cause displacements that may be very slow or very rapid and progressive.

Extremely slow movements in soils are called *soil creep*. Rapid movements of intact or nearly intact soil or rock masses are called *slides*. Rock or soil that detaches from a nearly vertical slope and descends mainly through the air by falling, bouncing, or rolling is called a *fall*. Very soft cohesive soils can fail by lateral spreading or by *mud flows*.

The factors to be considered for stability of slope are the cohesion of the soil, c, shear strength, τ, soil stratification and its in-place shear strength parameters. Seepage through the slope and the choice of potential slip surface add up to the complexity of the problem.

10.3.1 FACTORS OF SAFETY

The primary purpose of analyzing slope stability is to determine the *factor of safety*. In general, factor of safety is the ratio the average shear strength of soil, τ, to the average shear stress developed along the potential failure surface, τ_d.

$$FS = \frac{\tau}{\tau_d} \qquad \text{Eq. 10.26}$$

$$\tau = c + \sigma \tan \phi \qquad \text{Eq. 10.27}$$

$$\tau_d = c_d + \sigma \tan \phi_d \qquad \text{Eq. 10.28}$$

Factor of safety with respect to strength:

$$FS_s = \frac{\tau}{\tau_d} = \frac{c + \sigma \tan \phi}{c_d + \sigma \tan \phi_d} \qquad \text{Eq. 10.29}$$

Factor of safety with respect to cohesion:

$$FS_c = \frac{c}{c_d} \qquad \text{Eq. 10.30}$$

Factor of safety with respect to friction:

$$FS_\phi = \frac{\tan \phi}{\tan \phi_d} \qquad \text{Eq. 10.31}$$

Relation of FS_s, FS_c, and FS_ϕ:

$$FS_s = FS_c = FS_\phi \qquad \text{Eq. 10.32}$$

When $F_s = 1$, the slope is in a state of impending failure

10.3.2 STABILITY OF INFINTE SLOPE WITHOUT SEEPAGE

Infinite slope analysis is used when a layer of firm soil or rock lies parallel to a thin layer of softer material and the potential slip surfaces are very long compared to their depth. This occurs when a rock surface is parallel to the slope and there is a thin layer of soil overlying the rock. In this analysis, the driving forces of the uphill wedges and the resisting forces of the downhill wedges are ignored, and only the remaining central wedge is considered.

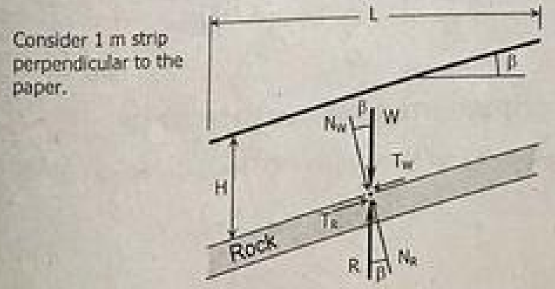


Figure 10.8 – Analysis of infinite slope

Normal stress:

$$\sigma = \frac{N_W}{\text{Area of base}} = \frac{W\cos\beta}{(1)(L/\cos\beta)}$$

$$\sigma = \frac{\gamma V\cos\beta}{L/\cos\beta} = \frac{\gamma LH(1)\cos\beta}{L/\cos\beta}$$

$$\sigma = \gamma H\cos^2\beta \qquad \text{Eq. 10.33}$$

Tangential stress:

$$\tau = \frac{T_W}{\text{Area of base}} = \frac{W\sin\beta}{(1)(L/\cos\beta)}$$

$$\tau = \frac{\gamma V\sin\beta}{L/\cos\beta} = \frac{\gamma LH(1)\sin\beta}{L/\cos\beta}$$

$$\tau = \gamma H\sin\beta\cos\beta \qquad \text{Eq. 10.34}$$

$$FS_s = \frac{c}{\gamma H\cos^2\beta\tan\beta} + \frac{\tan\phi}{\tan\beta} \qquad \text{Eq. 10.35}$$

If $FS_s = 1$, H = critical depth, H_{cr}

$$1 = \frac{c}{\gamma H_{cr}\cos^2\beta\tan\beta} + \frac{\tan\phi}{\tan\beta}$$

$$\frac{\tan\beta - \tan\phi}{\tan\beta} = \frac{c}{\gamma H_{cr}\cos^2\beta\tan\beta}$$

$$H_{cr} = \frac{c}{\gamma}\frac{1}{\cos^2\beta(\tan\beta - \tan\phi)} \qquad \text{Eq. 10.36}$$

10.3.3 STABILITY OF INFINTE SLOPE WITH SEEPAGE

For soils with seepage and ground water level coincides with the ground surface:

$$\tau = c + \sigma'\tan\phi \qquad \text{Eq. 10.37}$$

$$\tau_d = c_d + \sigma'\tan\phi_d \qquad \text{Eq. 10.38}$$

Normal stress:

$$\sigma = \gamma_{sat}H\cos^2\beta \qquad \text{Eq. 10.39}$$

Effective stress:

$$\sigma' = \gamma' H\cos^2\beta = (\gamma_{sat} - \gamma_w)H\cos^2\beta \qquad \text{Eq. 10.40}$$

Tangential stress:

$$\tau = \gamma_{sat}H\sin\beta\cos\beta \qquad \text{Eq. 10.41}$$

$$FS_s = \frac{c}{\gamma_{sat}H\cos^2\beta\tan\beta} + \frac{\gamma'}{\gamma_{sat}}\frac{\tan\phi}{\tan\beta} \qquad \text{Eq. 10.42}$$

10.3.4 FINITE SLOPE WITH PLANE FAILURE (CULMANN'S METHOD)

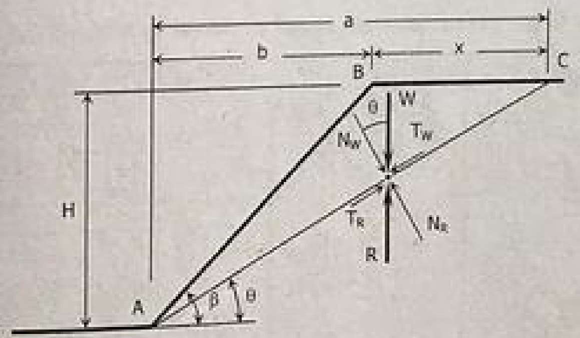


Figure 10.9 – Finite slope with plane failure

Normal stress:

$$\sigma = \frac{1}{2}\gamma H\left[\frac{\sin(\beta-\theta)}{\sin\beta}\right]\cos\theta \qquad \text{Eq. 10.43}$$

Critical angle of slip plane:

$$\theta_{cr} = \frac{\beta + \phi_d}{2} \qquad \text{Eq. 10.44}$$

$$c_d = \frac{1}{4}\gamma H\left(\frac{1-\cos(\beta-\phi_d)}{\sin\beta\cos\phi_d}\right) \qquad \text{Eq. 10.45}$$

Eq. 10.45 can also be written as

$$\frac{c_d}{\gamma H} = m = \frac{1-\cos(\beta-\phi_d)}{4\sin\beta\cos\phi_d} \qquad \text{Eq. 10.46}$$

where m = stability number

Based on Eq. 10.46, values of $1/m$ for various β and ϕ_d are shown in Table 10.6.

Table 10.6 – Stability Numbers Based on Culmann's Analysis (Eq. 10.46)

β (°)	ϕ_d	1/m	β (°)	ϕ_d	1/m	β (°)	ϕ_d	1/m	β (°)	ϕ_d	1/m
10	0	45.72	30	10	32.66	50	25	29.64	80	0	4.77
	5	181.84		15	56.70		30	44.00		5	5.29
15	0	30.38		20	123.71	60	0	6.93		10	5.90
	5	67.89		25	476.34		5	8.09		15	6.59
	10	267.93	40	0	10.99		10	9.55		20	7.40
20	0	22.69		5	14.16		15	11.42		25	8.37
	5	40.00		10	18.90		20	13.91		30	9.55
	10	88.68		15	26.51		25	17.36	90	0	4.00
	15	347.27		20	40.06		30	22.39		5	4.37
25	0	18.04		25	68.39	70	0	5.71		10	4.77
	5	27.92		30	146.57		5	6.49		15	5.21
	10	48.86	50	0	8.58		10	7.40		20	5.71
	15	107.48		5	10.42		15	8.51		25	6.28
	20	417.45		10	12.90		20	9.89		30	6.93
30	0	14.93		15	16.37		25	11.63			
	5	21.27		20	21.49		30	13.91			

When $c_d = c$ and $\phi_d = \phi$, then $H = H_{cr}$, from Eq. 10.45:

$$H_{cr} = \frac{4c}{\gamma}\left(\frac{\sin\beta\cos\phi}{1-\cos(\beta-\phi)}\right) \qquad \text{Eq. 10.47}$$

10.3.5 SLOPES WITH WATER IN THE TENSILE CRACK:

When tensile cracks are developed at the top of the slope and filled with water, the stability of such slope can be determined in the following manner.

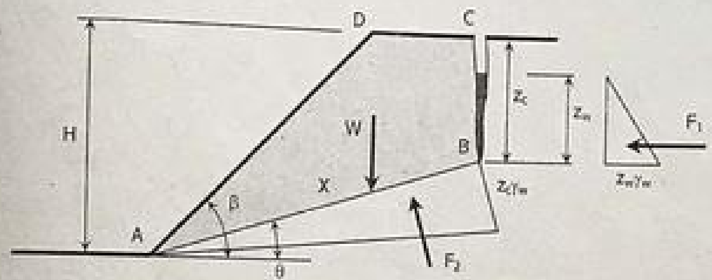


Figure 10.10 – Finite slope with water on tensile crack

z_c = depth of crack
z_w = depth of water in the crack
X = length $AB = (H - z_c)/\sin\theta$
θ = angle of failure plane
W = weight of soil wedge $ABCD$
F_1 = force due to water in the crack
F_2 = force due to pore water pressure along AB

$F_1 = \frac{1}{2}\gamma_w z_1^2$
$F_2 = \frac{1}{2}\gamma_w z_c X = \frac{1}{2}\gamma_w z_c (H - z_c)/\sin\theta$

Components of W and F_1 along AB:
$F = W\sin\theta + F_1\cos\theta$

Resisting force to F:
$R = cX + (W\cos\theta - F_1\sin\theta - F_2)\tan\phi$

Factor of safety with respect to strength:

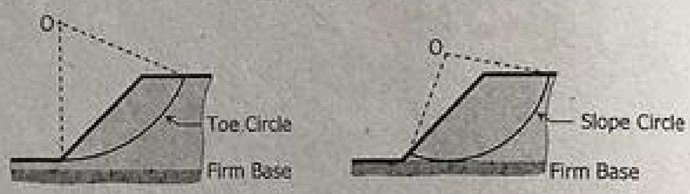
$$FS_s = \frac{R}{F} = \frac{cX + (W\cos\theta - F_1\sin\theta - F_2)\tan\phi}{W\sin\theta + F_1\cos\theta} \qquad \text{Eq. 10.48}$$

The magnitude of FS_s for various trial wedges can be calculated by varying the value of θ. The minimum value of FS_s is the factor of safety of the slope.

10.3.6 ANALYSIS OF THE FINITE SLOPES WITH CIRCULAR FAILURE SURFACES – GENERAL FAILURE SURFACES:

Modes of Failure:
Generally, finite slope failure occurs in one of the following diagrams:

1. Slope Failures



2. Shallow Slope Failure
3. Base Failure

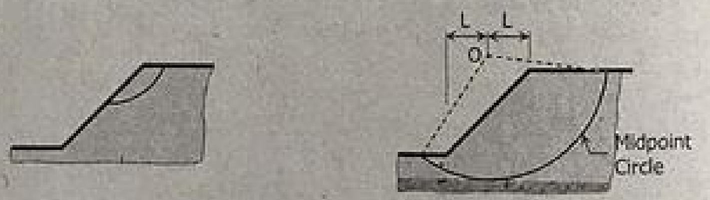


Figure 10.11 – Modes of slope failure

Two major classes of stability analysis procedure:

1. **Mass Procedure** – the soil that formed the slope is assumed to be homogeneous and the mass of soil above the surface of sliding is taken as a unit.

2. **Method of Slices** – In this procedure, the nonhomogeneity of the soil and the pore water pressure can be taken into consideration and the soil above the surface of sliding is divided into a number of vertical parallel slices.

10.3.7 MASS PROCEDURE (HOMOGENEOUS CLAY SOIL WITH $\phi = 0$)

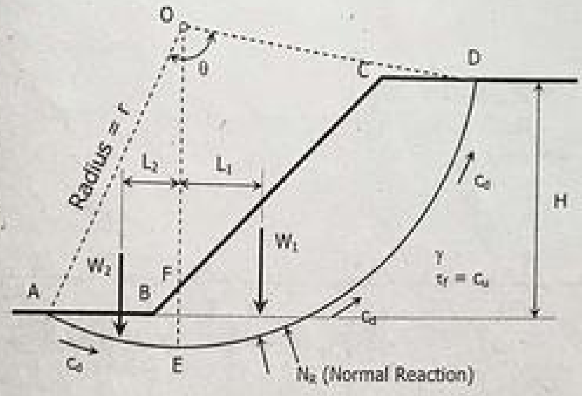


Figure 10.12 – Stability analysis in homogeneous clay ($\phi = 0$)

$W_1 = (\text{Area of } FCDEF)\,\gamma$
$W_2 = (\text{Area of ABFEA})\,\gamma$

Driving force about O to cause instability:

$$M_D = W_1L_1 - W_2L_2$$

Developed cohesion along the surface of sliding:

$$M_R = c_d\,(\text{arc } AED)(1)\,r$$
$$M_R = c_d\,r^2\,\theta$$

For equilibrium:

$$M_R = M_D$$
$$c_d\,r^2\,\theta = W_1L_1 - W_2L_2$$
$$c_d = \frac{W_1L_1 - W_2L_2}{r^2\,\theta}$$

Factor of safety against sliding:

$$FS_s = \frac{\tau}{c_d} = \frac{c_u}{c_d} \qquad \text{Eq. 10.49}$$

For critical circles

$$c_d = \gamma H m \quad \text{or} \quad m = \frac{c_d}{\gamma H} \qquad \text{Eq. 10.50}$$

For critical height; $FS_s = 1$, $H = H_{cr}$ and $c_d = c_u$:

$$H_{cr} = \frac{c_u}{\gamma m} \qquad \text{Eq. 10.51}$$

where m = stability number
c_u = undrained shear strength
c_d = developed cohesion

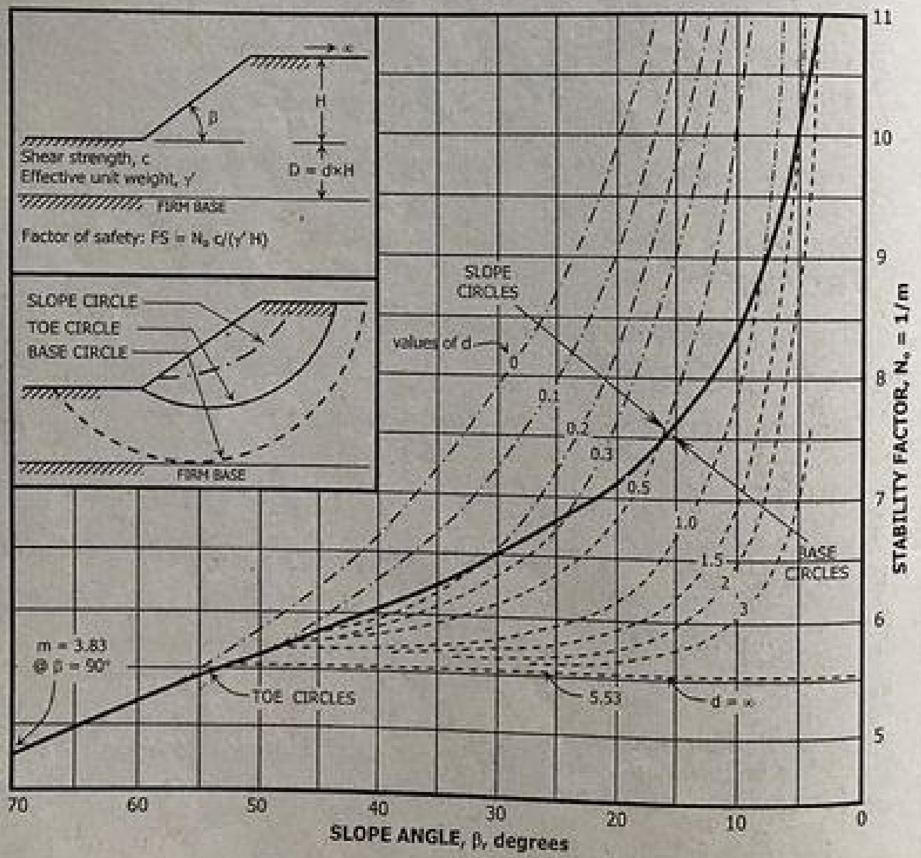


Figure 10.13 – Taylor Slope Stability Chart ($\phi = 0°$)

For saturated clay with $\phi = 0°$, the *Taylor slope stability chart* (Figure 10.13) can be used to determine the factor of safety against slope failure. The Taylor chart makes the following assumptions:

(*a*) There is no open water outside the slope
(*b*) There is no surcharge or tension cracks
(*c*) Shear strength is derived from cohesion only and is constant with depth
(*d*) Failure takes place as rotation on a circular arc

$$FS_s = N_o \frac{c}{\gamma' H} \qquad \text{Eq. 10.52}$$

The Taylor chart shows that *toe circle failures* occur in slopes steeper than 53°. For slopes less than 53°, *slope circle failure, toe circle failure,* or *base circle failure* may occur.

For $\beta > 53°$, all circles are toe circles. The location of the center of critical circle can be found using the graph shown in Figure 10.14.

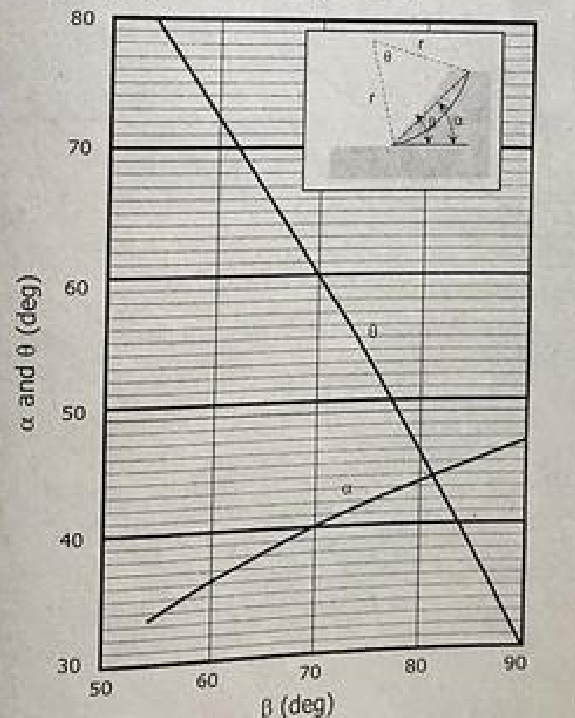


Figure 10.14 – Location of the center of critical center for $\beta > 53°$

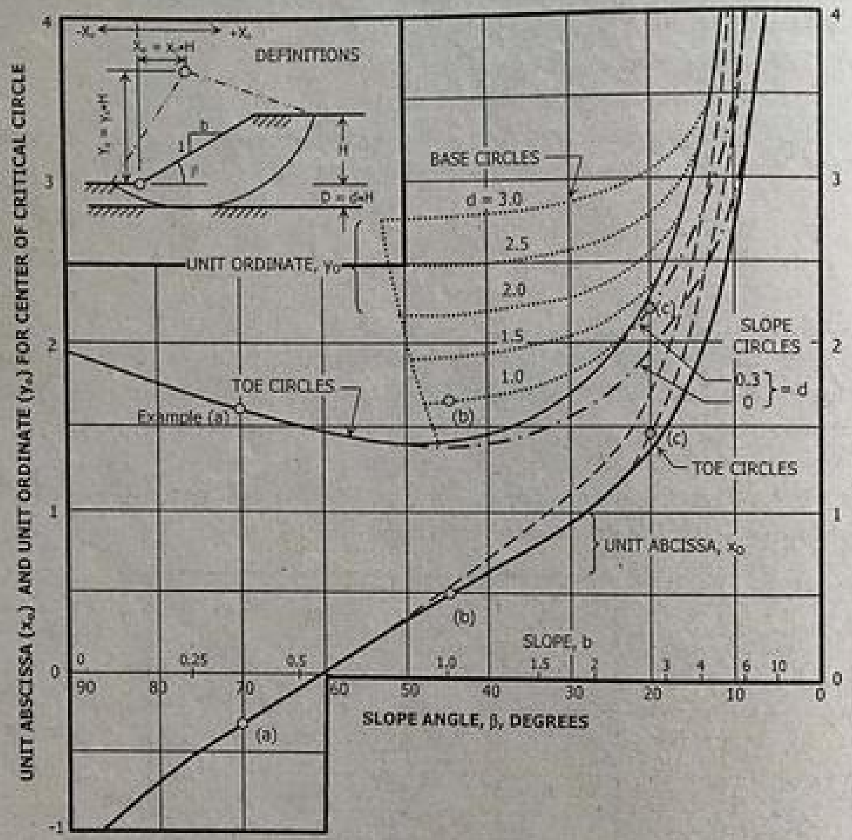


Figure 10.15 – Center of critical circle for homogeneous slope in cohesive soils, $\phi = 0°$

Consider the following examples on how to use the graph in Figure 10.15.

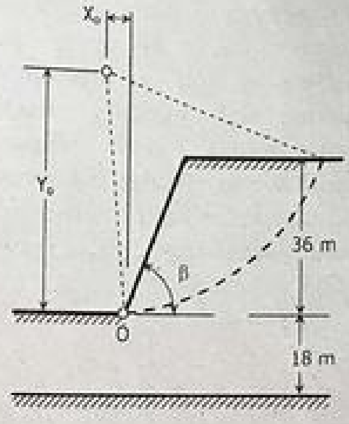
Example (*a*): Toe Circle

$\beta = 70°$
$d = 18/36 = 0.5$

From Figure 10.15:
$x_o = -0.30$
$y_o = 1.6$

$X_o = 0.3(36)$
$X_o = 10.8$ m to the left of O

$Y_o = 1.6(36) = 57.6$ m



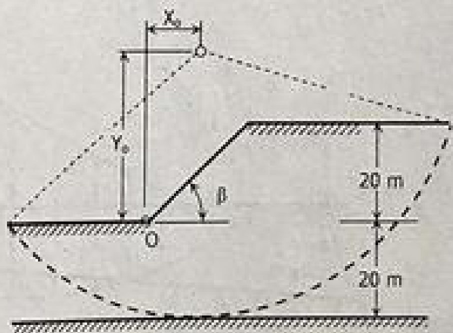
Example (*b*): Base Circle

$\beta = 45°$
$d = 20/20 = 1$

From Figure 10.15:
$x_o = 0.5$
$y_o = 1.65$

$X_o = 0.5(20)$
$X_o = 10$ m to the right of O

$Y_o = 1.65(20) = 33$ m



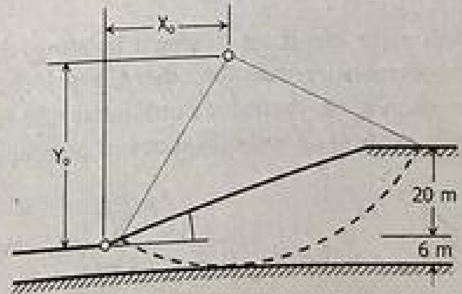
Example (*b*): Slope Circle

$\beta = 20°$
$d = 6/20 = 0.3$

From Figure 10.15:
$x_o = 1.45$
$y_o = 2.2$

$X_o = 1.45(20)$
$X_o = 29$ m to the right of O

$Y_o = 2.2(20) = 44$ m



10.3.8 METHOD OF SLICES

The method of slices was developed in the early 1920s in Sweden and was later refined by Bishop to consider interslice forces to some degree. This analysis method can accommodate complex slope geometries, variable soil layering and strengths, variable pore water pressure conditions, internal reinforcement, and the influence of external boundary loads, but it is only applicable to circular slip surfaces. It accomplishes this by dividing a slope into a series of vertical slices for analysis, with limiting equilibrium conditions evaluated for each slice, as shown in Figure 10.16.

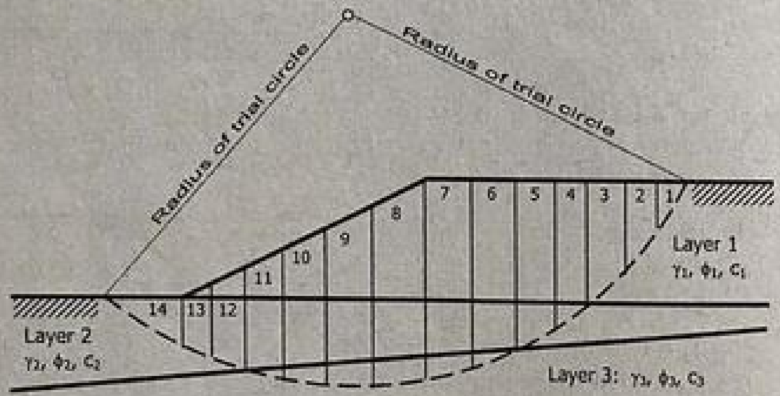


Figure 10.16 – Typical slope stability analysis using the method of slices

Each slice can have different layering, different strength, and different pore water pressure than the other slices. If the condition of equilibrium is satisfied for each slice, then it is considered that the entire mass is in equilibrium. The force system on a single slice is shown in Figure 10.17.

$$FS = \frac{\sum_{n=1}^{n=p}(cb_n + W_n \tan\phi)\frac{1}{m_{\alpha(n)}}}{\sum_{n=1}^{n=p} W_n \sin\alpha_n} \qquad \text{Eq. 10.53}$$

$$m_{\alpha(n)} = \cos\alpha_n + \frac{\tan\phi \sin\alpha_n}{FS} \qquad \text{Eq. 10.54}$$

Note that the FS is present on both sides of Eq. 10.53. Hence, a trial-and-error solution or a programmable calculator is necessary to find the value of FS.

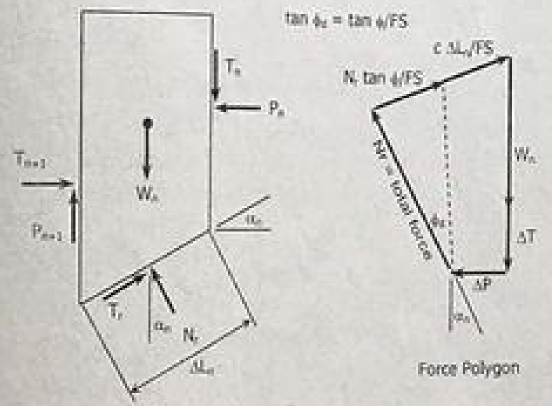


Figure 10.17 – Forces acting on the n^{th} slice in the Bishop simplified method of slices

ILLUSTRATIVE PROBLEMS

PROBLEM 10.1 (MAY 2005, NOVEMBER 2005)

A 0.36-m square prestressed concrete pile is to be driven in a clayey soil as shown in Figure 10.18. The design capacity of the pile is 360 kN, with a factor of safety of 2.0.

Given:

Undrained shear strength, q_u = 111 kN/m²

Unit weight of clayey soil = 18.5 kN/m³

(a) Compute the end bearing capacity of pile in if $N_c = 9$

(b) Compute the skin friction expected to develop along the shaft of the pile

(c) Compute the length of the pile if frictional constant α = 0.76

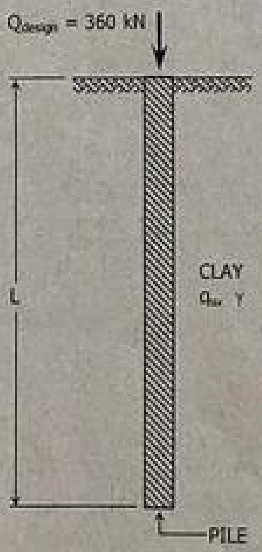


Figure 10.18

SOLUTION

Design load Q_a = 360 kN

Ultimate pile load capacity, $Q_u = Q_a \times FS$

Ultimate pile load capacity, Q_u = = 360 × 2 = 720 kN

$Q_u = Q_f + Q_b$]

Part (a):

$Q_b = f_b\,A_b = N_c\,(c_u)_b\,A_b$

$N_c = 9$

$(c_u)_b = q_u/2 = 111/2 = 55.5$ kPa

$A_b = 0.36^2 = 0.1296$ m²

$Q_b = 9(55.5)(0.1296)$

Q_b = **64.74 kN**

Part (b):

$[Q_f = Q_u - Q_b]$ $Q_f = 720 - 64.74$

Q_f = **655.26 kN**

Part (c):

$Q_f = \alpha\,c_u\,p\,L$

$\alpha = 0.76$

$c_u = q_u/2 = 111/2 = 55.5$ kPa

p = perimeter = 0.36 × 4 = 1.44 m

$655.26 = 0.76(55.5)(0.36 \times 4)L$

L = **10.79 m**

PROBLEM 10.2

A square concrete pile 0.3 m × 0.3 m is required to support a load of 175 kN with a factor of safety of 3. The soil stratification consists of 5 m of soft gray, normally consolidated clay (c_u = 25 kPa, ϕ = 26°, γ_{sat} = 18 kN/m³) underlain by a deep deposit of overconsolidated clay (c_u = 80 kPa, ϕ = 24°, OCR = 4, γ_{sat} = 18.5 kN/m³, N_c = 9). Groundwater level is at 2 m below the ground surface. Assume the soil above groundwater level is saturated. Assume α = 1 for soft clay and 0.5 for stiff clay. See Figure 10.19.

(a) What is the ultimate load capacity due to skin friction in soft clay?

(b) What is the value of L_1?

(c) Using the β-method, what is the value of Q_a if L_1 = 2.1 m? Assume N_q = 9.6. Use $\beta = (1 - \sin\phi)(OCR)^{0.5}\tan\phi$.

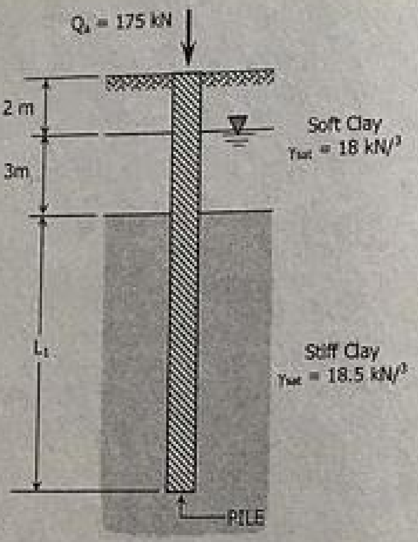


Figure 10.19

SOLUTION

Parts (*a*) and (*b*):

$Q_a = 175$ kN

$Q_u = Q_a \times FS$

$Q_u = 175 \times 3 = 525$ kN

$[Q_u = (Q_f)_{soft\ clay} + (Q_f)_{stiff\ clay} + Q_b]$

$(Q_f)_{soft\ clay} = \alpha\ c_u\ p\ L$

$(Q_f)_{soft\ clay} = 1(25)(0.3 \times 4)(5) = \mathbf{150\ kN}$

$(Q_f)_{stiff\ clay} = \alpha\ c_u\ p\ L_1$

$(Q_f)_{stiff\ clay} = 0.5(80)(0.3 \times 4)L_1 = 48L_1$

$Q_b = N_c\ (c_u)_b\ A_b$

$Q_b = 9(80)(0.3 \times 0.3) = 64.8$ kN

$525 = 150 + 48L_1 + 64.8$

$L_1 = \mathbf{6.46\ m}$

Part (*c*):

$Q_f = \beta\ p_{eff}\ P\ L$

Soft Clay (OCR = 1, normally consolidated)

$\beta = (1 - \sin 26°)(1)^{0.5} \tan 26°$

$\beta = 0.274$

p_{eff} = average effective vert. stress at midheight of pile in soft clay

$p_{eff} = 18(2) + (18 - 9.81)(0.5) = 40.095$ kPa

$(Q_f)_{soft\ clay} = 0.274(40.095)(1.2)(5)$

$(Q_f)_{soft\ clay} = 65.92$ kN

Stiff clay (OCR = 4)

$\beta = (1 - \sin 24°)(4)0.5 \tan 24° = 0.528$

p_{eff} = average effective vert. stress at midheight of pile in stiff clay

$p_{eff} = 18(2) + (18 - 9.81)(3) + (18.5 - 9.81)(1.05)$

$p_{eff} = 69.7$ kPa

$(Q_f)_{stiff\ clay} = 0.528(69.7)(1.2)(2.1)$

$(Q_f)_{stiff\ clay} = 92.74$ kN

$Q_b = N_q\ (p_{eff})_b\ A_b$

$(p_{eff})_b$ = effective vert. stress at the bottom of pile.

$(p_{eff})_b = 18(2) + (18 - 9.81)(3) + (18.5 - 9.81)(2.1)$

$(p_{eff})_b = 78.82$ kPa

$Q_b = 9.6(78.82)(0.3 \times 0.3)$

$Q_b = 68.1$ kN

$Q_u = 65.92 + 92.74 + 68.1 = 226.76$ kN

$Q_a = Q_u/FS = 226.76/3$

$Q_a = \mathbf{75.6\ kN}$

PROBLEM 10.3 (CE NOVEMBER 2005)

The pile group shown in Figure 10.20 consists of 12 piles, each 0.4 m in diameter, arranged in a 3 × 4 matrix. The pile penetrates a soft clay (L_1 = 2m, c_{u1} = 20 kPa), a medium dense clay (L_2 = 6 m, c_{u2} = 60 kPa), and a stiff clay (L_3 = 4 m, c_{u3} = 95 kPa). Assume N_c = 9 and use α = 1 for soft and medium dense clay, α = 0.5 for stiff clay.

(*a*) Determine the capacity of the pile group based on single pile failure mode.

(*b*) Determine the capacity of the pile group based on block failure mode.

(*c*) Compute the maximum center-to-center spacing of the piles for 100% efficiency.

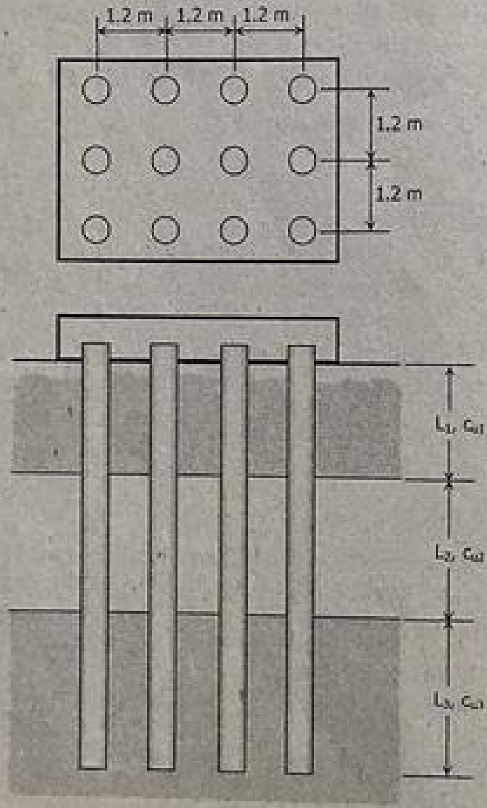


Figure 10.20

SOLUTION

Part 1: Single pile failure mode:

Perimeter, $p = \pi D = \pi(0.4) = 1.257$ m

Area, $A_b = \frac{\pi}{4} D^2 = \frac{\pi}{4}(0.4)^2 = 0.1257\ \text{m}^2$

$Q_f = \alpha_1 c_{u1}\, p\, L_1 + \alpha_2 c_{u2}\, p\, L_2 + \alpha_3 c_{u3}\, p\, L_3$

$Q_f = 1(20)(1.257)(2) + 1(60)(1.257)(6) + 0.5(95)(1.257)(4)$

$Q_f = 741.63$ kN

$Q_b = N_c\,(c_u)_b\,A_b = 9(95)(0.1257) = 107.4735$ kN

$Q_u = Q_f + Q_b = 741.63 + 107.4735 = 849.1$ kN

Group load capacity = $12 \times Q_u = 12 \times 849.1$

Group load capacity = **10,189.2 kN**

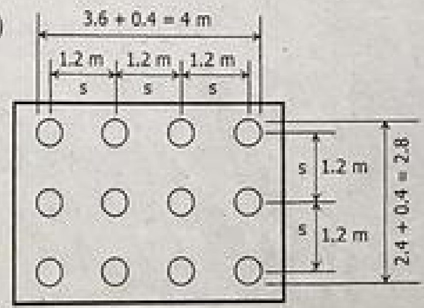
Part 2: Block failure mode:

Perimeter, $p = 2(4 + 2.8)$

Perimeter, $p = 13.6$ m

Area, $A_b = 4(2.8)$

Area, $A_b = 11.2\ \text{m}^2$



$Q_f = \alpha_1 c_{u1}\, p\, L_1 + \alpha_2 c_{u2}\, p\, L_2 + \alpha_3 c_{u3}\, p\, L_3$

$Q_f = 1(20)(13.6)(2) + 1(60)(13.6)(6) + 0.5(95)(13.6)(4)$

$Q_f = 8{,}024$ kN

$Q_b = N_c\,(c_u)_b\,A_b = 9(95)(11.2) = 9{,}576$ kN

$Q_{ug} = Q_f + Q_b = 8{,}024 + 9{,}576 =$ **17,600 kN**

Part 3:

Efficiency, $\eta = \dfrac{Q_{ug}}{nQ_u} = 100\%$

$Q_{ug} = n\,Q_u$

$n\,Q_u = 10{,}189.2$ kN

Solve for Q_{ug} in terms of spacing s:

$A_p = (2s + 0.4)(3s + 0.4) = 6s^2 + 2s + 0.16$

$p = 2[(2s + 0.4) + (3s + 0.4)] = 10s + 1.6$

$$Q_{ug} = Q_f + Q_b$$

$$Q_f = \alpha_1 c_{u1} p L_1 + \alpha_2 c_{u2} p L_2 + \alpha_3 c_{u3} p L_3$$

$$Q_f = 1(20)(10s + 1.6)(2) + 1(60)(10s + 1.6)(6) + 0.5(95)(10s + 1.6)(4)$$

$$Q_f = 590(10s + 1.6) = 5900s + 944$$

$$Q_b = N_c\,(c_u)_b\,A_b = 9(95)(6s^2 + 2s + 0.16)$$

$$Q_b = 5130s^2 + 1710s + 136.8$$

$$Q_{ug} = (5900s + 944) + (5130s^2 + 1710s + 136.8)$$

$$Q_{ug} = 5130s^2 + 7610s + 1080.8$$

$[Q_{ug} = n\,Q_u]$

$$5130s^2 + 7610s + 1{,}080.8 = 10{,}189.2$$

$$5130s^2 + 7610s - 9108.4 = 0$$

$$s = \frac{-7{,}610 \pm \sqrt{(7{,}610)^2 - 4(5{,}130)(-9{,}108.4)}}{2(5{,}130)}$$

$$s = 0.783 \text{ m} = \mathbf{783\ mm}$$

PROBLEM 10.4

The foundation shown in Figure 10.21 is supported by 9 piles. The foundation rests on a sand layer underlain with 3 m thick of normally consolidated clay.

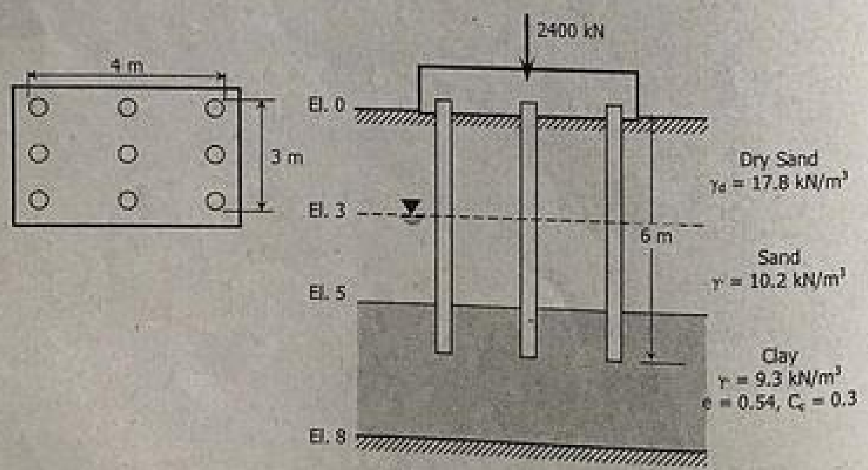


Figure 10.21

(*a*) Compute the effective stress at the midheight of the clay layer.
(*b*) Determine the increase in pressure at the midheight of the clay layer.

(*c*) Determine the consolidation settlement of the clay layer.

SOLUTION

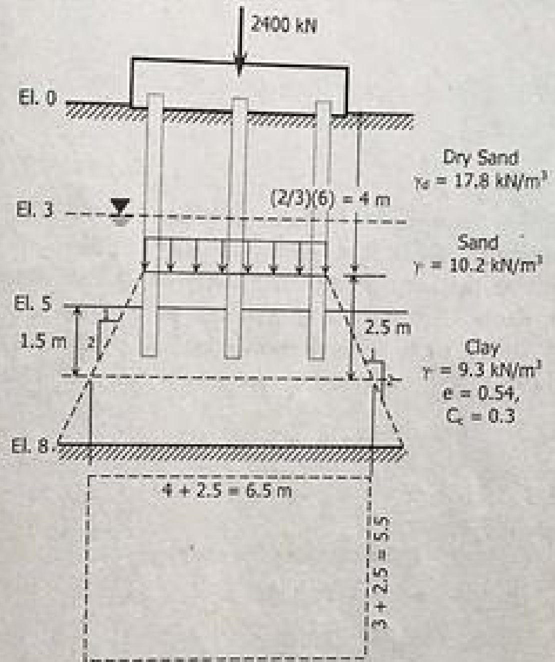


Figure 10.22

Part (*a*):

$$\Delta p = 2400/(6.5 \times 5.5)$$

$$\Delta p = \mathbf{67.13\ kPa}$$

Part (*b*):

$$p_o = 9.3(1.5) + 10.2(2) + 17.8(3)$$

$$p_o = \mathbf{87.75\ kPa}$$

Part (*c*):

$$\Delta H = H\frac{C_c}{1+e}\log\frac{p_f}{p_o}$$

$$p_f = p_o + \Delta p = 154.88 \text{ kPa}$$

$$H = 3 \text{ m} = 3000 \text{ mm}$$

$$\Delta H = 3000\,\frac{0.3}{1+0.54}\log\frac{154.88}{87.75}$$

$$\Delta H = \mathbf{144.2\ mm}$$

PROBLEM 10.5

A 1 ft × 1 ft concrete pile weighing 140 lb/ft, 40 feet long is driven on a sandy soil. Use a factor of safety of 6.

Given:
Unconfined compression strength, $q_u = 2{,}320$ psf
$N_c = 9$
$\alpha = 0.76$.

(a) Determine the ultimate frictional resistance of the pile.
(b) Determine the allowable bearing capacity of pile.
(c) Determine the allowable pile capacity using the Modified Engineering News Record formula. The average penetration during the last five blows is 0.2 inch. The hammer is DALMAG D-12 (double acting) weighing 2750 lbs and rated energy of 16,500 ft-lb. Use $e_h = 0.85$ and $n = 0.5$.

SOLUTION

Parts *a* & *b*:

$Q_u = Q_f + Q_b$
$Q_u = \alpha\, c_u\, p\, L + N_c\,(c_u)_b\, A_p$

$c_u = ½q_u = ½(2{,}320)$
$c_u = 1{,}160$ psf

$Q_f = \alpha\, c_u\, p\, L$
$Q_f = 0.76(1{,}160)(1 \times 4)(40)$
$Q_f = 141{,}056$ lbs = **141.06 kips → Part *a***

$Q_u = 141{,}056 + 9(1{,}160)(1) = 151{,}496$ lbs
$Q_a = Q_u/FS = 151{,}496/6 = 25{,}249$ lbs
$Q_a =$ **25.25 kips→ Part *b***

Part *c*:

$$P_u = \frac{1.25 e_h E_h}{s + 0.1}\,\frac{W_r + n^2 W_p}{W_r + W_p}$$

$W_r = 2{,}750$ lbs
$W_p = 40(140) = 5{,}600$ lbs
$E_h = 16{,}500$ ft-lb = 198,000 in-lb
$s = 0.2$ in
$e_h = 0.85,\ n = 0.5$

$$P_u = \frac{1.25(0.85)(198{,}000)}{0.2 + 0.1}\,\frac{2750 + (0.5)^2(5600)}{2750 + 5600}$$

$P_u = 348{,}525$ lbs
$P_a = P_u/FS$
$P_a = 348{,}525/6 = 58087.5$ lbs
$P_a =$ **58.09 kips**

PROBLEM 10.6 (CE MAY 2005)

A braced cut in sand 7 m deep is shown in Figure 10.23. In the plan, the struts are placed at s = 2.0 m center to center. Use Peck's empirical pressure diagram. Determine the load on each strut.

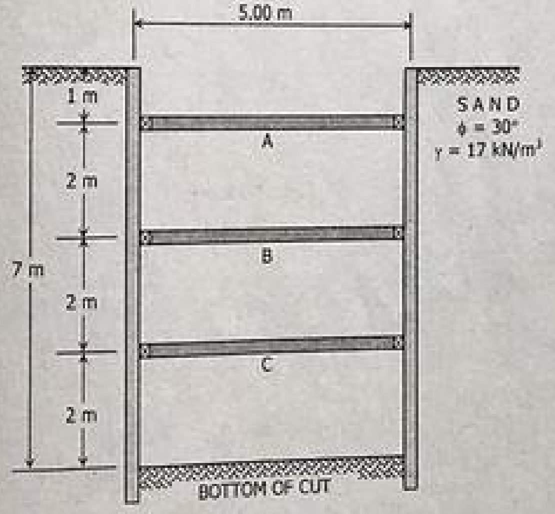


Figure 10.23

SOLUTION

$$K_a = \frac{1 - \sin\phi}{1 + \sin\phi} = \frac{1 - \sin 30°}{1 + \sin 30°} = 1/3$$

$p_a = 0.65 K_a\, \gamma\, H = 0.65(1/3)(17)(7) = 25.783$ kPa
$w = p_a\, s = 25.78 \times 2 = 51.57$ kN/m

With reference to Figure 10.4 (*a*):

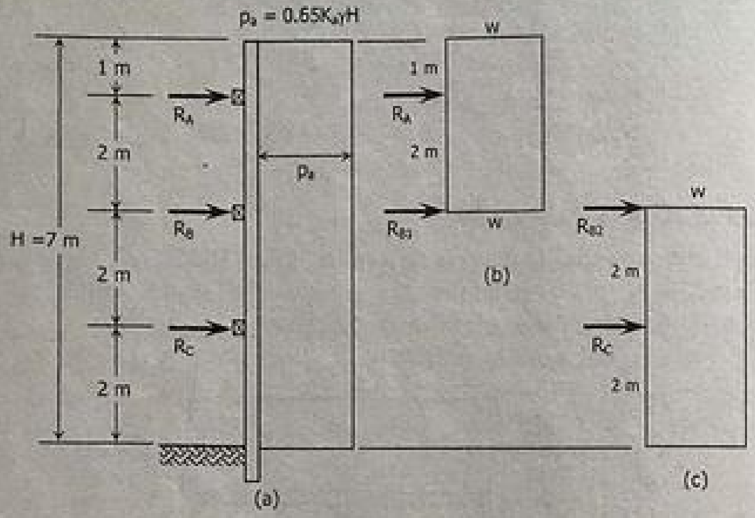


Figure 10.24

In Figure 10.24(*a*):

$[\Sigma M_{B1} = 0]$ $2R_A = 51.57(3)(1.5)$; $R_A = \mathbf{116.03\ kN}$

$[\Sigma M_A = 0]$ $2R_{B1} = 51.57(3)(0.5)$; $R_{B1} = 38.68$ kN

In Figure 10.24 (*b*):

$[\Sigma M_C = 0]$ $2R_{B2} = 51.57(4)(0)$; $R_{B2} = 0$

$[\Sigma M_{B2} = 0]$ $2R_C = 51.57(4)(2)$; $R_C = \mathbf{206.28\ kN}$

$R_B = R_{B1} + R_{B2} = 38.68 + 0$

$R_B = \mathbf{38.68\ kN}$

PROBLEM 10.7

The elevation and plan of a bracing system for an open cut in sand are shown in Figure 10.25. Assume γ_{sand} = 110 pcf and ϕ = 36°. Use Peck's empirical pressure diagram. Calculate the load on each strut.

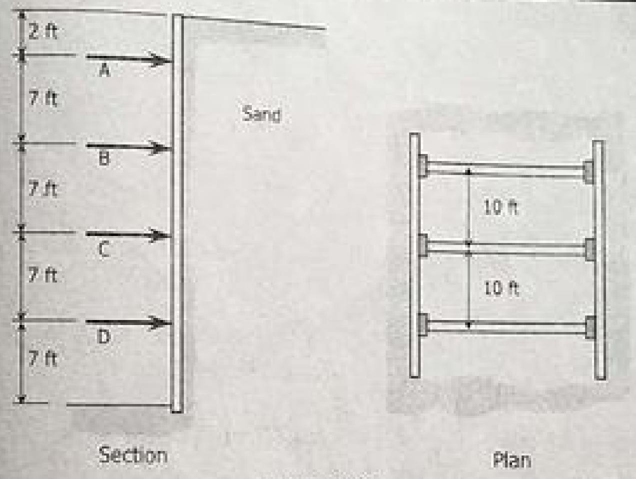


Figure 10.25

SOLUTION

With reference to Figure 10.4(*a*), the pressure diagram is as shown in Figure 10.26. The strut loads may be determined by assuming that the vertical members are hinged at each strut level except the topmost and bottommost ones.

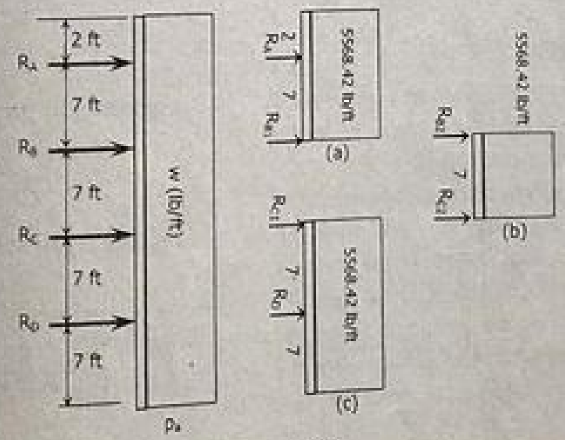


Figure 10.26

$p_a = 0.65K_a\gamma H$

$K_a = (1 - \sin 36°)/(1 + \sin 36°) = 0.2596$

$p_a = 0.65(0.2596)(110)(30)$

$p_a = 556.842$ psf

$w = p_a \times s = 556.842 \times 10$

$w = 5568.42$ lb/ft

Figure 10.26(*a*) $[\Sigma M_{RB1} = 0]$ $7R_A = 5{,}568.42(9)(4.5)$

$R_A =$ **32,217.3 lbs**

$[\Sigma F_H = 0]$ $R_{B1} = 5{,}568.42(9) - 32{,}217.3$

$R_{B1} = 17{,}898.48$ lbs

Figure 10.26(*b*) $R_{B2} = R_{C2} = 5{,}568.42(3.5) = 19{,}489.47$ lbs

$R_B = R_{B1} + R_{B2}$

$R_B =$ **37,387.95 lbs**

Figure 10.26(*c*) $R_{C1} = 0$

$R_C = R_{C1} + R_{C2}$

$R_C =$ **19,489.47 lbs**

$R_D = 5{,}568.42(14)$

$R_D = 77{,}957.88$ lbs

PROBLEM 10.8

A infinite slope is shown in Figure 10.27. The shear strength parameters at the interface of soil and rock are as follows:

Cohesion, $c = 16$ kN/m²

Angle of shearing resistance, $\phi = 25°$

(*a*) If $H = 8$ m and $\beta = 20°$, find the factor of safety against sliding on the rock surface. Assume no seepage.

(*b*) If $\beta = 30°$, find the critical height H. Assume no seepage.

(*c*) If there were seepage through the soil, and the groundwater table coincide with the ground surface, and $H = 5$ m, $\beta = 20°$, what would be the factor of safety. Assume $\rho_{sat} = 1900$ kg/m³.

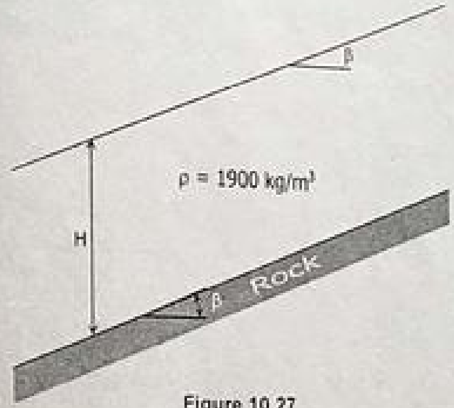


Figure 10.27

SOLUTION

Part *a*: $FS = \dfrac{c}{\gamma H \cos^2\beta \tan\beta} + \dfrac{\tan\phi}{\tan\beta}$

$$FS = \frac{16}{18.639(8)\cos^2 20° \tan 20°} + \frac{\tan 25°}{\tan 20°}$$

$FS =$ **1.615**

Part *b*: $H_{cr} = \dfrac{16}{18.639}\,\dfrac{1}{\cos^2 30°(\tan 30° - \tan 25°)}$

$H_{cr} =$ **10.31 m**

Part *c*:

$\gamma_{sat} = 1900 \times 9.81 = 18{,}639$ N/m³

$\gamma_{sat} = 18.639$ kN/m³

$\gamma' = \gamma_{sat} - \gamma_w = 18.639 - 9.81$

$\gamma' = 8.829$ kN/m³

$$FS = \frac{c}{\gamma_{sat} H \cos^2\beta \tan\beta} + \frac{\gamma'}{\gamma_{sat}}\frac{\tan\phi}{\tan\beta}$$

$$FS = \frac{16}{18.639(5)\cos^2 20° \tan 20°} + \frac{8.829}{18.639}\frac{\tan 25°}{\tan 20°}$$

$FS =$ **1.141**

PROBLEM 10.9

A cut is to be made in a soil that has γ = 16.5 kN/m³, c = 15 kN/m², and ϕ = 26°. The side of the cut slope will make an angle of 45° with the horizontal. Use a factor of safety of 3. Use Culmanns Method.

(a) What is the developed angle of friction?
(b) What is the maximum depth of cut?
(c) What is the critical angle of slip plane?

SOLUTION

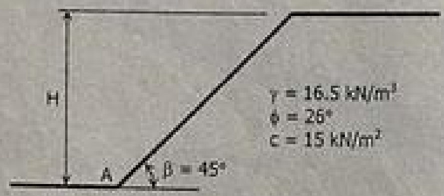


Figure 10.28

$$c_d = \frac{1}{4}\gamma H\left(\frac{1-\cos(\beta-\phi_d)}{\sin\beta\cos\phi_d}\right)$$

$$FS = \frac{c}{c_d};\; c_d = \frac{15}{3} = 5\ \text{kN/m}^2$$

$$FS = \frac{\tan\phi}{\tan\phi_d};\qquad \tan\phi_d = \frac{\tan\phi}{FS} = \frac{\tan 26^\circ}{3}$$

$$\phi_d = 9.23^\circ$$

$$c_d = \frac{1}{4}\gamma H\left(\frac{1-\cos(\beta-\phi_d)}{\sin\beta\cos\phi_d}\right)$$

$$5 = \frac{1}{4}(16.5)H\left(\frac{1-\cos(45^\circ-9.23^\circ)}{\sin 45^\circ\cos 9.23^\circ}\right)$$

$$H = 4.48\ \text{m}$$

Critical angle of slip plane:

$$\theta_{cr} = \frac{\beta+\phi_d}{2} = \frac{45^\circ+9.23^\circ}{2} = 27.115^\circ$$

PROBLEM 10.10

A cut slope is to be made in soft clay with its sides rising at an angle of 75° to the horizontal as shown in Figure 10.29. Given, c_u = 30.87 kN/m² and γ = 17.14 kN/m³. Stability number, m = 0.219. Assume the critical circle is a toe circle. Use Figure 15.15 for the values of θ and α.

(a) Determine the maximum depth up to which the excavation can be carried out?
(b) Find the radius, r, of the critical circle when the factor of safety is equal to 1.
(c) Find the distance BC.

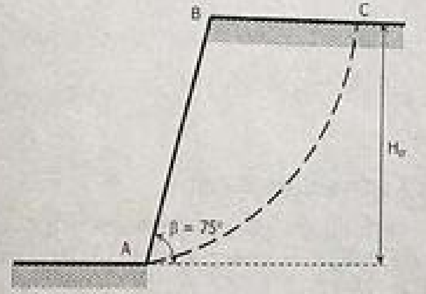


Figure 10.29

SOLUTION

Part a: $H_{cr} = \dfrac{c_u}{\gamma m} = \dfrac{30.87}{17.14(0.219)} = 8.22\ \text{m}$

Part b: From Figure 10.14, for $\beta = 75^\circ$, $\alpha = 41.8^\circ$ and $\theta = 53^\circ$

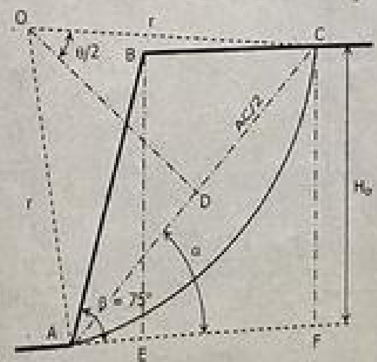


Figure 10.30

In right triangle AFC:

$\sin\alpha = H_{cr}/AC$

$AC = 12.33$ m

In right triangle ODC:

$\sin(\theta/2) = CD/r$

$$\sin(53°/2) = \frac{12.33/2}{r}$$

$r = 13.817$ m

Part c: $BC = H_{cr}\cot\alpha - H_{cr}\cot\beta$

$BC = H_{cr}(\cot\alpha - \cot\beta) = 8.22(\cot 41.8° - \cot 75°)$

$BC = 6.99$ m

PROBLEM 10.11 (CE NOVEMBER 2005)

The apparatus shown in Figure 10.31 maintains a constant head of 160 mm. The soil sample has a hydraulic conductivity of 5×10^{-3} cm/sec with moisture content of 18.5%. Use $G = 2.7$.

(a) Calculate the seepage velocity in.
(b) Calculate the flow of water.
(c) Calculate the time required for the red dye to pass through the soil assuming that there is no diffusion, or the red dye does not scatter as it passes through the soil.

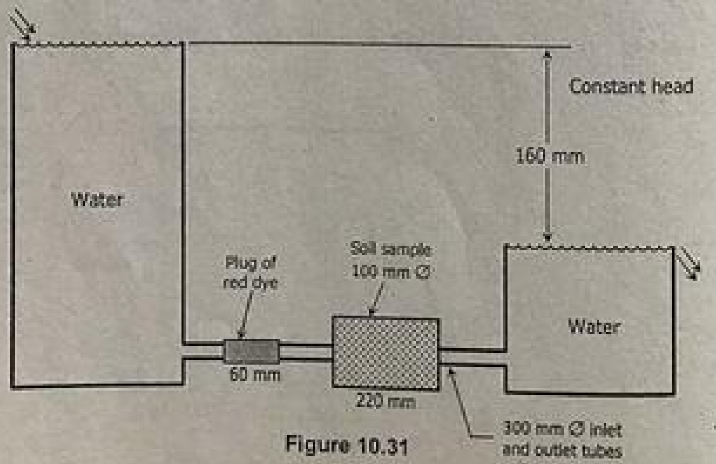


Figure 10.31

SOLUTION

$[G\,MC = S\,e]$ Note: the soil sample is saturated ($S = 1$)

$2.7(0.185) = 1(e)$

$e = 0.4995$

$[n = e/(1+e)]$ $n = 0.4995/(1 + 0.4995)$

$n = 0.333$

$[i = h/L]$ $i = 160/220 = 0.7273$

$[v = ki]$ $v = 5 \times 10^{-3}(0.7273)$

$v = 0.003636$ cm/sec $= 0.03636$ mm/sec

Part *a*:

$$\text{Seepage velocity, } v_s = \frac{v}{n} = \frac{0.03636}{0.333} = \mathbf{0.1092\ mm/sec}$$

Part *b*:

Flow of water, $Q = Av = \frac{\pi}{4}(100)^2(0.03636)$

$Q = 285.57\ \text{mm}^3/\text{sec} = \mathbf{0.28557\ cm^3/sec}$

Part *c*:

$$\text{Time, } t = \frac{L}{v_s} = \frac{220}{0.1092} = 2{,}014.6\ \text{sec} = \mathbf{33.6\ min}$$

PROBLEM 10.12 (CE MAY 2005)

A reservoir with a 3,400 m² area is underlain by layers of stratified soils as shown in Figure 10.32. The values of L_1, L_2, and L_3 are 2.0 m, 1.4 m, and 3.2 m, respectively.

(a) What the average vertical coefficient of permeability.
(b) Determine the interstitial (actual) velocity of water moving through the soil if it has a void ratio of 0.60.
(c) Compute the water loss from the reservoir in one year. Assume that the pore pressure at the bottom sand layer is zero.

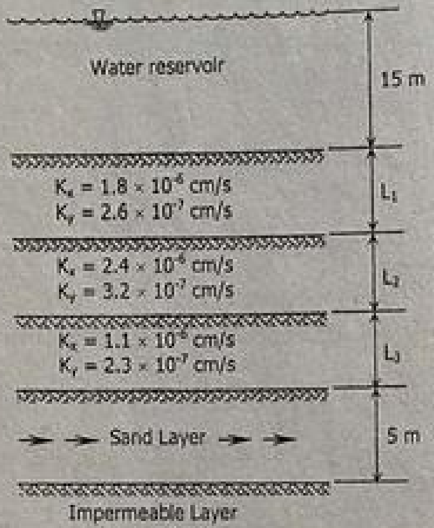
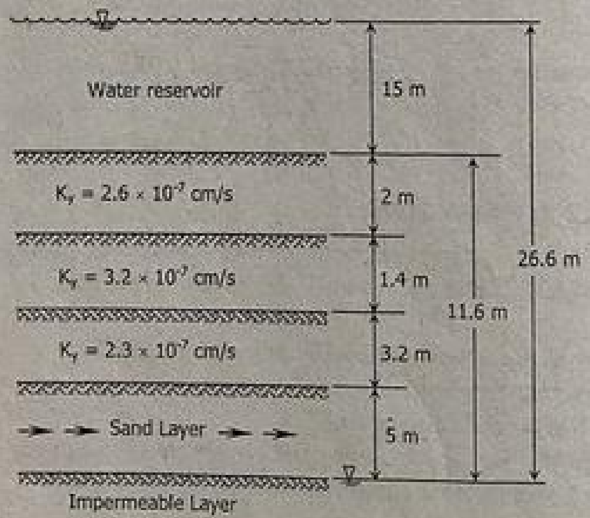


Figure 10.32

SOLUTION



$$\frac{H}{(k_y)_{eq}} = \Sigma\frac{h}{k_y};\ H = 2 + 1.4 + 3.2 = 6.6 \text{ m}$$

$$\frac{6.6}{(k_y)_{eq}} = \frac{2}{2.6\times10^{-7}} + \frac{1.4}{3.2\times10^{-7}} + \frac{3.2}{2.3\times10^{-7}}$$

$(k_y)_{eq} = 2.54038 \times 10^{-7}$ cm/s × (1 m/100 cm) × (3600 s/hr)

$(k_y)_{eq}$ = **9.145×10^{-6} m/hr**

Average velocity of flow through soil, $v = ki$

$i = h/L$

$h = 5 + 3.2 + 1.4 + 2 + 15 = 26.6$ m

$L = 5 + 3.2 + 1.4 + 2 = 11.6$ m

$i = 26.6/11.6 = 2.293$

$v = 2.54038 \times 10^{-7}\,(2.293) = 5.8251 \times 10^{-7}$ cm/s

Interstitial (or seepage) velocity, $v_s = \dfrac{v}{n}$

$$n = \frac{e}{1+e} = \frac{0.6}{1+0.6} = 0.375$$

Interstitial velocity, $v_s = 5.8251 \times 10^{-7} / 0.375$

Interstitial velocity, v_s = **1.55×10^{-6} cm/s**

$Q = kiA$

$k = 9.145 \times 10^{-6}$ m/hr

$i = 2.293$

$A = 3{,}400$ m²

$Q = 9.145 \times 10^{-6}\,(2.293)(3{,}400) = 0.071296249$ m³/hr

$Q = 0.071296249$ m³/hr × (24 hr/day) × (365 days/year)

Q = **625 m³/year**

PROBLEM 10.13 (CE MAY 2005)

Two footings rest in a layer of sand 2.7 m thick. The bottoms of the footings are 0.90 m below the ground surface. Beneath the sand layer is a 1.8-m thick clay layer. Underneath the clay layer is solid rock. Water table is at a depth of 1.8 m below the ground surface. See Figure 10.33.

(a) Compute the stress increase in kPa below footing A (1.5 m × 1.5 m) at the center of the clay layer. Assume that the pressure beneath footing A is spread at an angle of 2 vertical to 1 horizontal.

(b) Determine the size of footing B so that the settlement in the clay layer is the same beneath footings A and B.

(c) Determine the settlement in mm beneath footing A.

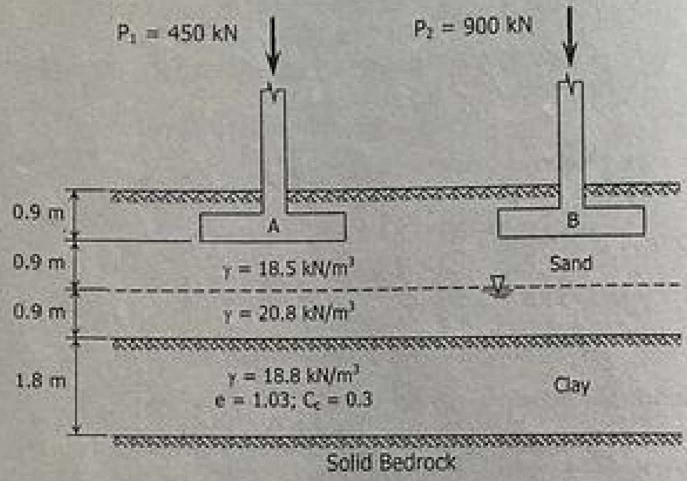


Figure 10.33

SOLUTION

$B' = B + 2(d/2)$
$B' = B + d$

Part *a*:
$B' = 1.5 + 2.7)$
$B' = 4.2$ m
$\Delta p = 450/4.2^2$
$\Delta p = \mathbf{25.51\ kPa}$

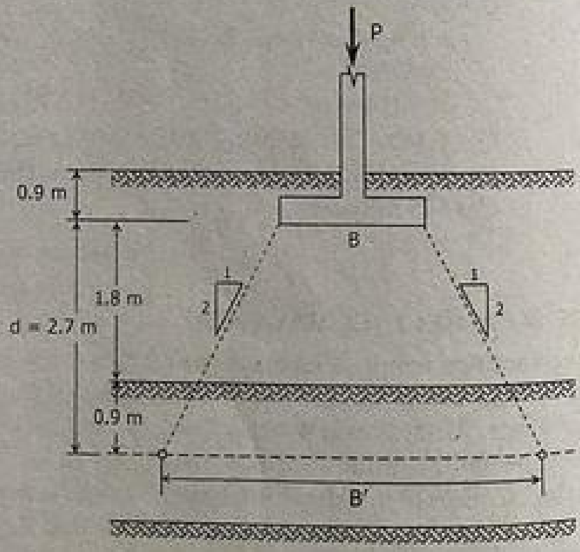


Figure 10.34

Part *b*:
In order that the settlement in both footings are equal, the increase in pressure at midheight of clay layer under each footing must be equal.

$900/B'^2 = 25.51$; $B' = 5.94$ m
$B' = B + 2.7 = 5.94$; $B = 3.24$ m
Size of footing B = **3.24 m × 3.24 m**

Part *c*:
$p_o = 18.5(1.8) + (20.8 - 9.81)(0.9) + (18.8 - 9.81)(0.9)$
$p_o = 51.282$ kPa
$p_f = p_o + \Delta p = 76.792$ kPa

$$\Delta H = H\frac{C_c}{1+e_o}\log\frac{p_f}{p_o}$$

$$\Delta H = 1800\frac{0.3}{1+1.03}\log\frac{76.792}{51.282} = \mathbf{46.65\ mm}$$

PROBLEM 10.14

In the profile shown in Figure 10.35, steady vertical seepage is occurring.

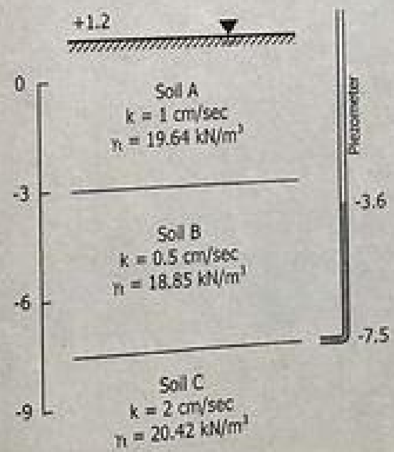


Figure 10.35

(a) Compute the average vertical coefficient of permeability in layers A and B.

(b) Compute the hydraulic gradient for flow through A and B.

(c) Compute the seepage in soil A per square meter.

SOLUTION

Equivalent K for layers A and B (Normal flow, series):

$H_A = 4.2$ m; $H_B = 4.5$ m

$$\left[\frac{H}{K_{eq}} = \Sigma\frac{h}{k}\right] \qquad \frac{8.7}{K_{eq}} = \frac{4.2}{1} + \frac{4.5}{0.5}$$

$$K_{eq} = \mathbf{0.006591\ m/s}$$

Hydraulic Gradient, $i = H/L = [1.2 - (-3.6)] / [1.2 - (-7.5)]$

Hydraulic Gradient, $i = \mathbf{0.5517}$

Flow of water per square meter of area:

$Q = KiA = 0.006591(0.5517)(1) = 0.003636\ m^3/s$

$Q = \mathbf{3.636\ L/s} = Q_A = Q_B$

PROBLEM 10.15

A layered soil is shown in Figure 10.36.

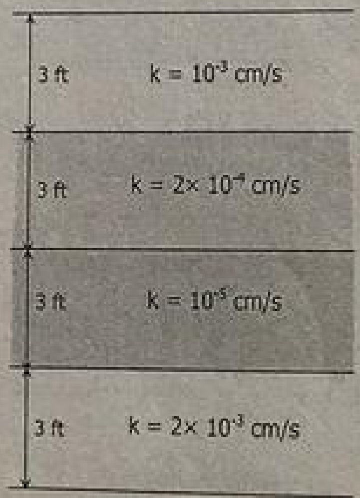


Figure 10.36

(a) Calculate the equivalent coefficient of permeability in the horizontal direction.

(b) Calculate the equivalent coefficient of permeability in the vertical direction.

(c) What is the ratio of $(k_H)_{eq}$ to $(k_V)_{eq}$?

SOLUTION

Part a: Parallel flow:

$[k_{H(eq)}\, H = \Sigma\, kh]$

$k_{H(eq)}\,(12) = (10^{-3})(3) + (2\times10^{-4})(3) + (10^{-5})(3) + (2\times10^{-3})(3)$

$k_{H(eq)} = 0.0008025$ cm/sec

$k_{H(eq)} = \mathbf{8.025\times10^{-4}\ cm/sec}$

Part b: Normal flow:

$$\left[\frac{H}{k_{V(eq)}} = \Sigma\frac{h}{k}\right]$$

$$\frac{12}{k_{V(eq)}} = \frac{3}{10^{-3}} + \frac{3}{2\times10^{-4}} + \frac{3}{10^{-5}} + \frac{3}{2\times10^{-3}}$$

$k_{V(eq)} = \mathbf{3.756\times10^{-5}\ cm/sec}$

Part c: Ratio, $k_{H(eq)}/k_{V(eq)} = \mathbf{21.37}$

PROBLEM 10.16

The setup shown in Figure 10.37 is 50 mm wide (perpendicular to the paper). The flow through the soil is known to be 4.08 liters per minute.

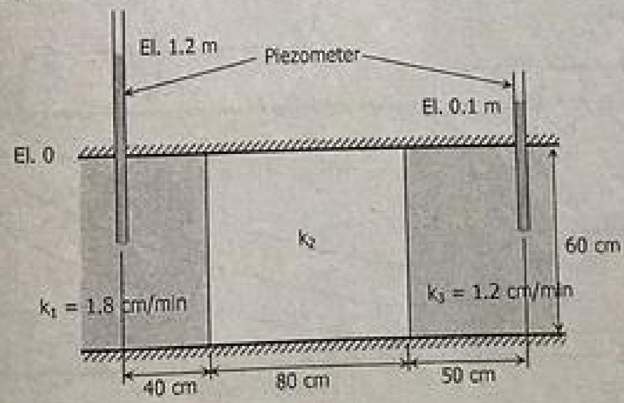


Figure 10.37

(*a*) What is the hydraulic gradient?
(*b*) Calculate the equivalent hydraulic conductivity in.
(*c*) Calculate the hydraulic conductivity k_2.

SOLUTION

$$Q = 4.08 \text{ Liters/minute} = 4{,}080 \text{ cm}^3/\text{min}$$

$$i = h/L = (1.2 - 0.1)/1.7 = \mathbf{0.647}$$
$$A = 60 \times 50 = 3000 \text{ cm}^2$$

$$[Q = K_{eq}\, i\, A] \qquad 4{,}080 = K_{eq}(0.647)(3000)$$
$$K_{eq} = \mathbf{2.1 \text{ cm/min}}$$

$$\left[\frac{H}{K_{eq}} = \Sigma h/k\right] \qquad \frac{170}{2.1} = \frac{40}{1.8} + \frac{80}{k_2} + \frac{50}{1.2}$$
$$k_2 = \mathbf{4.69 \text{ cm/min}}$$

PROBLEM 10.17

The section of a concrete gravity dam with its flow net diagram is shown in Figure 10.38. The coefficient of permeability of the soil (anistropic) in the vertical and horizontal directions are 0.25 m/day and 0.58 m/day, respectively. The length of dam perpendicular to the paper is 120 m.

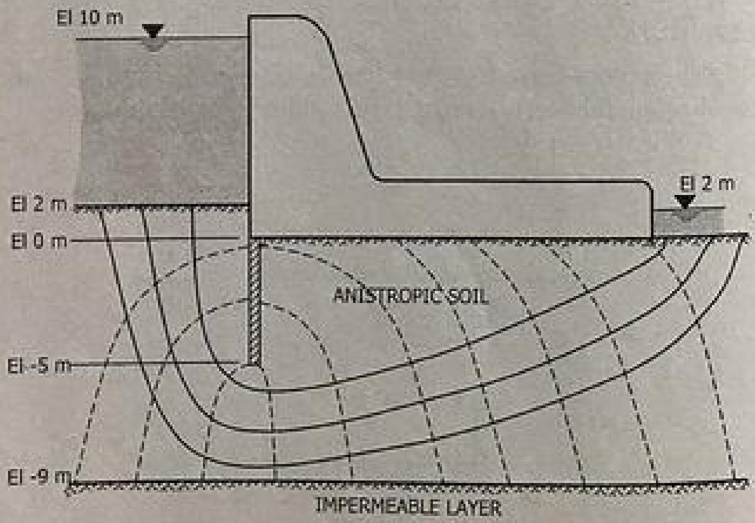


Figure 10.38

(*a*) Determine the number of pressure drops
(*b*) Determine the seepage loss.
(*c*) Determine the total amount of water percolated in one year

SOLUTION

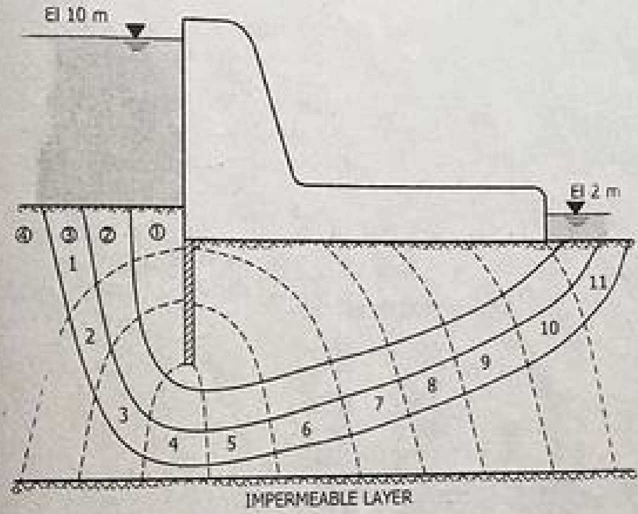


Figure 10.39

$$q = \sqrt{K_v K_h}\, H \frac{N_f}{N_d}$$

Number of flow channels, $N_f = 4$
Number of pressure drops, $N_d = 11$
Head, H = 10 – 2 = 8 m

$$q = \sqrt{0.25(0.58)}\ (8)(4/11) = 1.108 \text{ m}^3/\text{day per meter}$$
$$Q = qL = 1.108(120) = 132.96 \text{ m}^3/\text{day} = \mathbf{5.54 \text{ m}^3/\text{hr}}$$

In one year: Volume = 132.96 × 365 = **48,530 m³**

PROBLEM 10.18

A bulkhead is to be constructed of steel-sheet piling and tie rods as shown in Figure 10.40. Consider 1 foot strip only of sheet piling.

(*a*) Determine the nearest value to the total active pressure acting on the pile.
(*b*) Determine the nearest value to the total passive resistance.
(*c*) What is the tension in the tie rod?

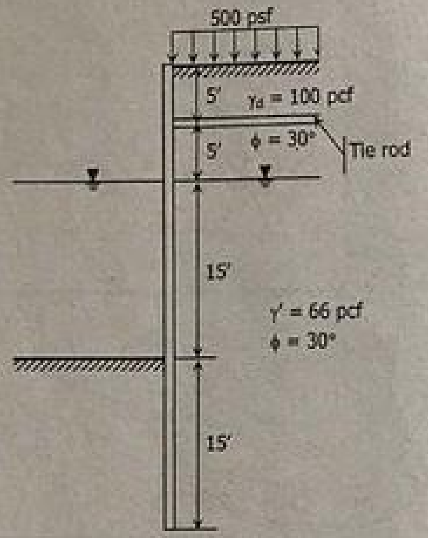


Figure 10.40

SOLUTION

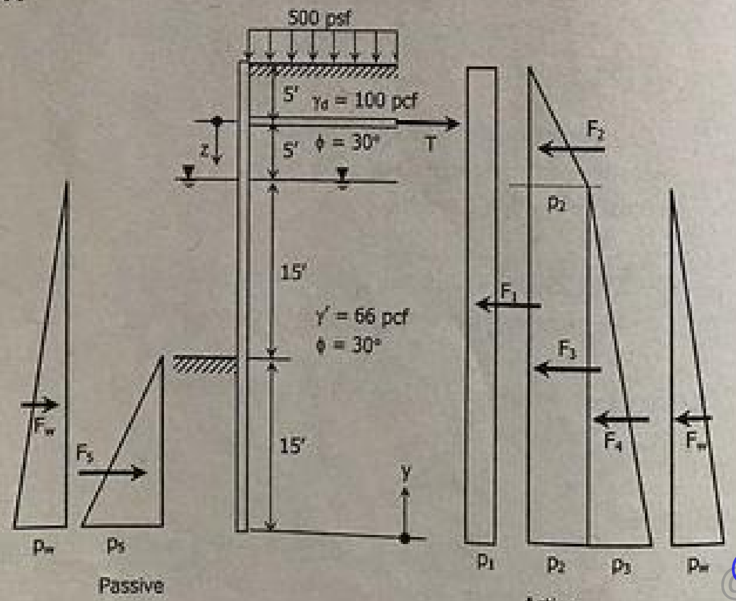


Figure 10.41

Active Pressure:

$K_a = (1 - \sin 30°)/(1 + \sin 30°) = 1/3$

$p_1 = K_a \times q = (1/3)(500) = 500/3$ psf

$F_1 = p_1(40) = 6667$ lbs; $\quad y_1 = 20' \quad z_1 = 15'$

$p_2 = K_a\,\gamma_d\,(10) = 333$ psf

$F_2 = ½(333)(10) = 1{,}667$ lbs; $\quad y_2 = 33.33' \quad z_2 = 1.67'$

$F_3 = (333)(30) = 9{,}990$ lbs; $\quad y_3 = 15' \quad z_3 = 20'$

$p_3 = K_a\,\gamma'(30) = 660$ psf

$F_4 = ½(660)(30) = 9{,}900$ lbs $\quad y_4 = 10' \quad z_4 = 25'$

$p_w = \gamma_w(30) = (62.4)(30) = 1{,}872$ psf

$F_w = ½(1{,}872)(30) = 28{,}080$ lbs

Total active force, $F_a = F_1 + F_2 + F_3 + F_4 + F_w$
Total active force, $F_a = 6667 + 1{,}667 + 9{,}990 + 9{,}900 + 28{,}080$
Total active force, F_a = **56,304 lbs**

Passive Pressure:

$K_p = (1 + \sin 30°)/(1 - \sin 30°) = 3$

$p_5 = K_p\,\gamma'(15) = 2{,}970$ psf

$F_{5\,max} = ½(2{,}970)(15)$

$F_{5\,max} = 22{,}275$ lbs $\quad y_5 = 5' \quad z_5 = 30'$

Total Passive Resistance = $F_{5\,max} + F_w$
Total Passive Resistance = 22,275 + 28,080
Total Passive Resistance = **50,355 lbs**

Note: The active and passive water pressures for this problem are equal, hence it may be disregarded in the analysis.

Solving for actual passive reaction F_5 by taking moments about T:

$F_5\,z_5 = F_1\,z_1 + F_2\,z_2 + F_3\,z_3 + F_4\,z_4$

$F_5(30) = 6667(15) + 1{,}667(1.67) + 9{,}990(20) + 9{,}900(25)$

$F_5 = 18{,}336.3 \text{ lbs} < F_{5\,max}$ (OK)

$[\Sigma F_H = 0] \quad T + F_5 = F_1 + F_2 + F_3 + F_4$

$T = 6667 + 1{,}667 + 9{,}990 + 9{,}900 - 18{,}336.3$

T = **9,887.7 lbs**

PROBLEM 10.19

After several years of service, the retaining wall shown in Figure 10.42 was found to have plugged drains and 7 feet of compound water above the footing level. After performing a forensic field exploration, laboratory testing was conducted on the backfill soil. The difference in strength parameters were found to be negligible in comparison to the original design conditions. Consider only 1 ft length of wall perpendicular to the paper.

(*a*) Determine the total active force on the wall before the waterlog condition.

(*b*) Determine the total active force on the wall after the waterlog condition.

(*c*) Determine the resulting overturning moment about point *A* after the waterlog condition.

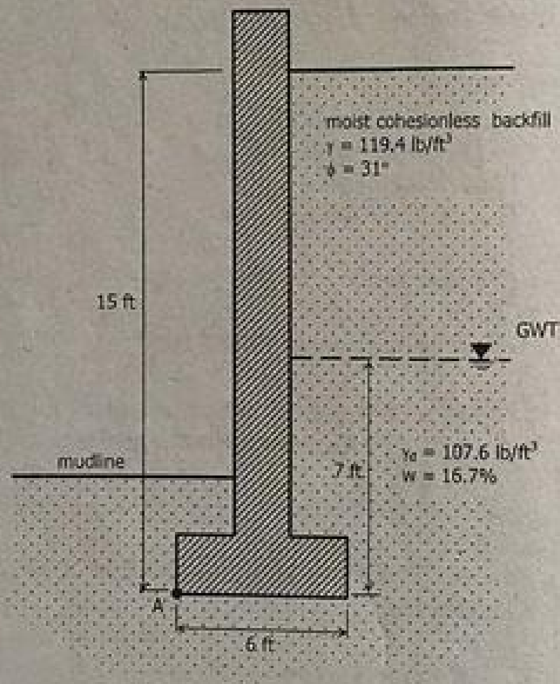


Figure 10.42

SOLUTION

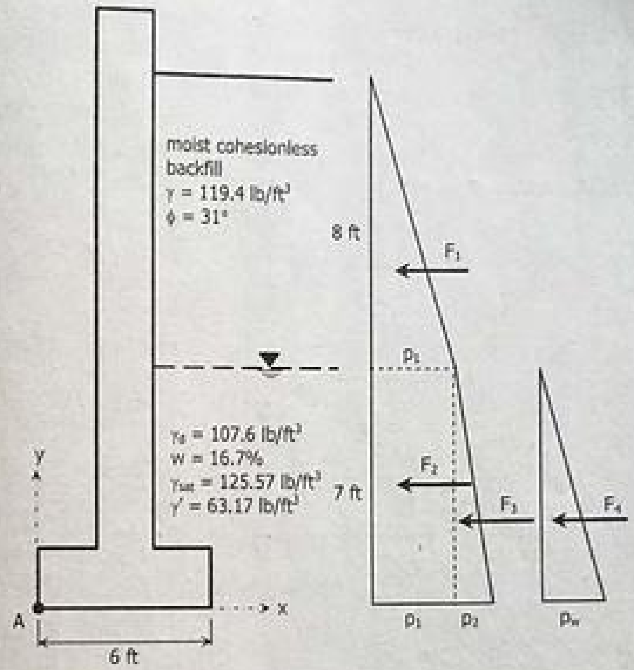


Figure 10.43

$K_a = (1 - \sin 31°)/(1 + \sin 31°) = 0.32$

Part *a*: before the waterlog condition:

$F_a = ½\, K_a\, \gamma H^2 = ½(0.32)(119.4)(15)^2 = \mathbf{4298.4\ lbs}$

Parts *b* & *c*:

$p_1 = K_a\gamma H = 0.32(119.4)(8) = 305.664$ psf

$F_1 = ½(305.664)(8) = 1222.66$ lbs; $\quad y_1 = 9.67$ ft

$F_2 = 305.664(7) = 2139.65$ lbs; $\quad y_2 = 3.5$ ft

$p_2 = K_a\gamma' H = 0.32(63.17)(7) = 141.5$ psf

$F_3 = ½(141.5)(7) = 495.25$ lbs; $\quad y_3 = 2.33$ ft

$F_4 = \gamma_w H^2/2 = 62.4(7)^2/2 = 1528.8$ lbs $\quad y_4 = 2.33$ ft

$F_a = 1222.66 + 2139.65 + 495.25 + 1528.8 = \mathbf{5{,}386.36\ lbs}$

$M_a = 1222.66{\times}9.67 + 2139.65{\times}3.5 + 495.25{\times}2.33 + 1528.8{\times}2.33$

$M_a = \mathbf{24{,}028\ ft\text{-}lb}$

PROBLEM 10.20

A 1.8-m square footing is shown in Figure 10.44. Use a factor of safety of 3.

(a) Determine the overburden pressure at the base of footing.
(b) Determine the gross allowable bearing capacity.
(c) Using the value of q_a obtained above, determine the settlement of the soil. Assume average standard penetration resistance (SPT) of N-value of 18 and use a correction factor $c_N = 0.62$.

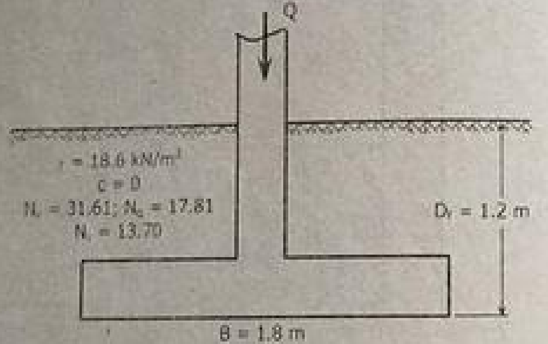


Figure 10.44

SOLUTION

Part a: Overburden pressure, $q = 18.6(1.2) = \mathbf{22.32\ kPa}$

Part b: $q_u = 1.3\,c\,N_c + q\,N_q + 0.4\,\gamma_s\,B\,N_\gamma$
$q_u = 0 + 18.6(1.2)(17.81) + 0.4(18.6)(1.8)(13.7)$
$q_u = 581$ kPa
$q_{a\ gross} = 581/3 = \mathbf{193.67\ kPa}$

Part c: Settlement from standard penetration test (SPT)

$$\text{Settlement, } S = \frac{q}{0.41\,N_{cor}}, \text{ in mm}$$

q = bearing capacity of soil, in kPa
N_{cor} = corrected SPT value (N value)
N_{cor} = correction factor × N

$$S = \frac{193.67}{0.41(0.62 \times 18)} = \mathbf{42.3\ mm}$$

PROBLEM 10.21

A raft foundation is to be designed for a 36 m × 60 m building with a total loading of 2.5 × 10⁵ kN. The clay specific weight is 1840 kg/m³, and the clay has an average unconfined compressive strength of 28.7 kPa. Neglect depth correction factors. Use a shape factor of 1.15. $\phi = 0°$. The bearing capacity factors are given in Table 09.1 Page 301.

(a) What is the actual foundation pressure?
(b) What should be the raft depth for full compensation?
(c) What should be the raft depth for partially compensated foundation using a factor of safety of 3?

SOLUTION

$\gamma = 1840(9.81) = 18050.4\ \text{N/m}^3 = 18.05\ \text{kN/m}^3$

$$\text{Part } a\text{: } p_a = \frac{\text{Total load}}{\text{Raft area}} = \frac{2.5\times10^5}{36(60)} = \mathbf{115.741\ kPa}$$

Part b: Depth for full compensation:

$$\gamma D_f = \frac{\text{Total load}}{\text{Raft area}}$$

$18.0504\,D_f = 115.741$
$D_f = \mathbf{6.4\ m}$

Part c:

$p_a - \gamma D_f = q_{a\ net}$
$q_u = 1.15\,c\,N_c + \gamma D_f N_q$
for $\phi = 0°$, $N_c = 5.7$, $N_q = 1$, $N_\gamma = 0$
$c = S_u/2 = 28.7/2 = 14.35$ kPa
$q_u = 1.15(14.35)(5.7) + 18.05D_f$
$q_u = 94.06 + 18.05D_f$
$q_{u\ net} = q_u - \gamma D_f = 94.06$ kPa

$$q_{a\ net} = \frac{q_{u\ net}}{FS} = \frac{94.06}{3} = 31.35\ \text{kPa}$$

$115.741 - 18.05\,D_f = 31.35$ kPa
$D_f = \mathbf{4.68\ m}$

PROBLEM 10.22

A municipal storage tank is to be supported by a circular raft (mat) foundation placed on the surface of the soil. The diameter of the foundation is 12.2 m. The maximum load exerted on the soil when the tank is full is 11,040 kN. The following data was taken from a boring log and other soil tests from the proposed site.

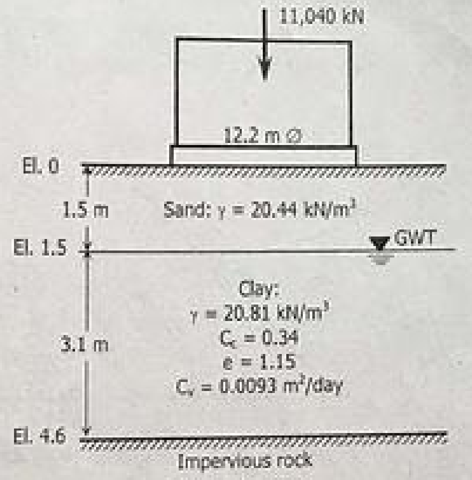
Elev. 0.0:
- ground surface
- well-graded sand and gravel
- unit weight: 20.44 kN/m³
- allowable bearing pressure: 143 kPa

Elev. 1.5:
- encountered GWT
- soft, brown clay
- unit weight: 20.81 kN/m³
- compression index: 0.34
- unloaded, original void ratio: 1.15
- coefficient of consolidation: 0.0093 m²/day

Elev. 4.6:
- encountered thick, impervious rock layer

(*a*) What is the factor of safety in bearing?
(*b*) If the total clay thickness is 3.1 m, what is the primary settlement?
(*c*) How long will it take for 80% of the primary settlement to occur? Time factor of 80% consolidation is 0.567.

SOLUTION



$$\Delta p = \frac{11040}{\frac{\pi}{4}(12.2)^2}$$

$$\Delta p = 94.44 \text{ kPa}$$

$$q_a = 143 \text{ kPa}$$

Part *a*:

$$FS = \frac{q_a}{\Delta p}$$

$$FS = \frac{143}{94.44}$$

$$FS = \mathbf{1.51}$$

Part *b*:

$$p_o = (20.81 - 9.81)(3.1/2) + 20.44(1.5) = 47.71 \text{ kPa}$$

$$p_f = p_o + \Delta p = 47.71 + 94.44 = 142.15 \text{ kPa}$$

$$\Delta H = H\frac{C_c}{1+e}\log\frac{p_f}{p_o}$$

$$\Delta H = 3100\frac{0.34}{1+1.15}\log\frac{142.15}{47.71}$$

$$\Delta H = \mathbf{232.4 \text{ mm}}$$

Part *c*: *Please refer to page 196 :*

This is a single-drainage layer. $H_{dr} = 3.1$ m

$$t = 0.567\frac{3.1^2}{0.0093} = \mathbf{586 \text{ days}}$$

PROBLEM 10.23

The earth dam shown in Figure 10.45 is to be compacted to a void ratio of 0.78 in place. A borrow pit nearby contains soil having a void ratio of 120%, a true specific gravity of 2.65, and a moisture content of 15%. The total cost of moving the soil from the borrow pit to the dam site is P40.00 per cu. m., based on the original volume at the borrow pit.

(a) What is the required loose volume of soil from the borrow pit?
(b) How many tons of water will be transported with the fill?
(c) What is the total cost of moving all the required soil to the dam site?

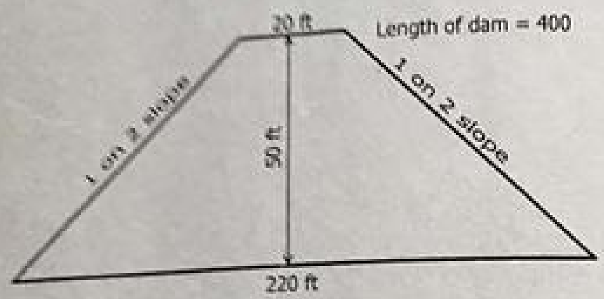


Figure 10.45

SOLUTION

$$V_{dam} = \frac{20+220}{2}(50) \times 400 = 2{,}400{,}000 \text{ ft}^3 \text{ (loose volume)}$$

Required volume of solids:

$$V = V_s + V_v; \quad \frac{V_v}{V_s} = e; \quad V_v = eV_s$$

$$V = V_s + eV_s = V_s(1+e)$$
$$2{,}400{,}000 = V_s(1 + 0.78)$$
$$V_s = 1{,}348{,}314.6 \text{ ft}^3$$

Part a: Required loose volume from borrow pit:

$$V = 1{,}348{,}314.6\,(1 + 1.2) = 2{,}966{,}292.13 \text{ ft}^3$$
$$V = 84{,}060.6 \text{ m}^3$$

Part b: Volume of water:

$$MC = \frac{W_w}{W_s}$$

$$W_s = G\,V_s = (62.4 \times 2.65)(1{,}348{,}314.6)$$
$$W_s = 222{,}957{,}302.256 \text{ lbs}$$
$$W_w = MC \times W_s = 0.15(222{,}957{,}302.256)$$
$$W_w = 33{,}443{,}595 \text{ lbs} / (2000) = \mathbf{16{,}722 \text{ tons}}$$

Part c:

$$\text{Cost} = 84{,}060.6 \times \text{P}40$$
$$\text{Cost} = \mathbf{P3{,}362{,}424}$$

PROBLEM 10.24

A 3,000,000 cubic meter earthen dam is constructed from borrow soil with the following original properties:

Void ratio 0.70
Degree of saturation......................0.40
Solids specific gravity 2.65

The borrow soil is compacted to a final specific weight of 17.29 kN/m³ and a 17% moisture content. The trucks used to transport the borrow soil from the borrow pit to the dam site each have maximum capacities of 10 cubic meters

(a) What is the weight of solids in 1 m³ of compacted fill?
(b) What is the weight of water in 1 m³ of compacted fill?
(c) Given a fluff factor of 10%, how many trips must the truck make to complete the dam?

SOLUTION

For compacted fill:

$$\gamma_d = \frac{\gamma_m}{1+MC} = \frac{17.29}{1+0.17}$$

$$\gamma_d = \mathbf{14.78 \text{ kN per } 1 \text{ m}^3} \qquad \rightarrow \text{Part } (a)$$

$$W_w = 17.29 - 14.78 = \mathbf{2.51 \text{ kN per } 1 \text{ m}^3} \qquad \rightarrow \text{Part } (b)$$

$$\left[\gamma_m = \frac{G + G\,MC}{1+e}\gamma_w\right] \quad 17.29 = \frac{2.65 + 2.65(0.17)}{1+e}(9.81)$$

$$e = 0.759$$

$$V = V_v + V_s = eV_s + V_s$$

$$V_s = \frac{V}{1+e} = \frac{3{,}000{,}000}{1+0.759} = 1{,}705{,}514 \text{ m}^3$$

For the borrow material:

$$V = V_s(1+e) = 1{,}705{,}514(1 + 0.7) = 2{,}899{,}375 \text{ m}^3$$

$$\text{Number of trips} = \frac{(1.1)(2{,}899{,}375)\text{m}^3}{10\text{m}^3/\text{trip}} = \mathbf{318{,}931 \text{ trips}} \qquad \rightarrow \text{Part } (c)$$

PROBLEM 10.25

A proctor test is performed on four samples. The mold volume and mass are 1/30 ft³ and 4200 g for each sample. The following data are collected

Sample	Mass of mold and soil	Water content
1	6100 g	8.2%
2	6300 g	10.1%
3	6425 g	11.7%
4	6330 g	14.8%

(*a*) What is the wet mass density for sample 4?
(*b*) What is the dry mass density for sample 1?
(*c*) What is the relative compaction of sample 2?

SOLUTION

Volume, $V = 1/30\ ft^3 = 944$ cc

	Mass of soil	MC	ρ_{wet} (g/cc)	$\rho_{dry} = \rho_{wet}/(1 + MC)$
1	1900	8.2%	2.0127	1.86
2	2100	10.1%	2.2246	2.021
3	2225	11.7%	2.357	2.11
4	2130	14.8%	2.256	1.965

$$\rho_4 = \frac{2130}{944} = 2.256\ g/cc = \mathbf{2256\ kg/m^3}$$

$$\rho_{d1} = 1.86\ g/cc = \mathbf{1860\ kg/m^3}$$

Relative compaction of sample 2:

Relative compaction $= \rho_{d2} / \rho_{d\,max}$

$\rho_{d\,max} = 2.11$ g/cc (sample 3)

$$\text{Relative compaction} = \frac{2.021}{2.11} \times 100\% = \mathbf{95.78\%}$$

PROBLEM 10.26 (CE NOVEMBER 2005)

The following data were obtained from a triaxial test on a cohesive soil.

Maximum shearing stress at failure plane = 70 kPa
Angle of friction = 28°
Cohesion = 30 kPa

(*a*) Determine the normal stress at failure plane
(*b*) Determine the maximum normal stress (plunger stress) applied on the soil
(*c*) Determine the minimum normal stress (confining pressure)

SOLUTION

[illegible] 70 kPa, ϕ = 28° [illegible] kPa

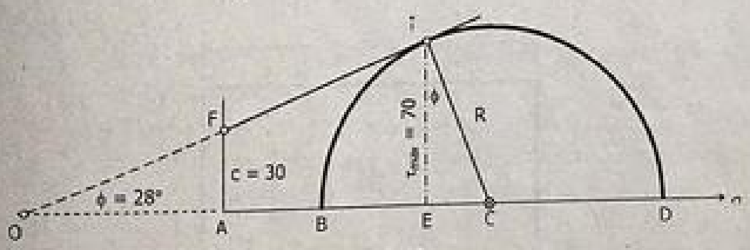


Figure 10.46

In right triangle TEC:

$\cos\phi = 70/R;\ R = 70 / \cos 28°$
$R = 79.28$ kPa
$\tan\phi = EC/70$
$EC = 70 \tan 28° = 37.22$ kPa

In right triangle OAF:

$\tan 28° = 30/OA$
$OA = 56.42$ kPa

In right triangle OTC:

$\sin 28° = R/OC$
$OC = 79.28/\sin 28° = 168.87$ kPa

$AC = OC - OA = 168.87 - 56.42 = 112.45$ kPa

Part *a*: Normal stress at failure plane:

$\sigma = AE = AC - EC = 112.45 - 37.22$

$\sigma = \mathbf{75.23\ kPa}$

Part *b*: Maximum normal stress (plunger stress)

$\sigma_1 = AC + R = 112.45 + 79.28 = \mathbf{191.73\ kPa}$

Part *c*: Minimum normal stress (confining stress)

$\sigma_3 = AB = AC - R = 112.45 - 79.28 = \mathbf{33.17\ kPa}$

PROBLEM 10.27

The tank shown in Figure 10.47 has an inside diameter of 10 m and is 6 m high. The tank is used for storage of oil having specific gravity of 0.82. The combined weight of empty tank and the concrete footing is 3200 kN. It is required to excavate an amount of soil such that it will compensate to the dead weight of the tank and concrete.

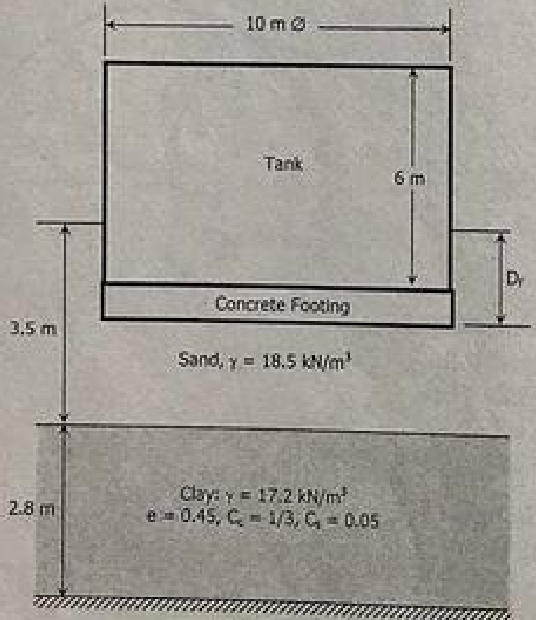


Figure 10.47

(*a*) Determine the required depth of footing.

(*b*) What pressure increase in the soil is expected when the tank is filled with oil?

(*c*) What is the expected settlement of the clay layer if it is overconsolidated with OCR = 2.2?

SOLUTION

Part *a*:

Weight of soil removed = Weight of tank and concrete

$\gamma_{soil}\ V_{soil} = 3{,}200$

$(18.5) \times \frac{\pi}{4}(10)^2 D_f = 3{,}200;\ D_f = \mathbf{2.2\ m}$

Part *b*:

Weight of oil $= \gamma_{oil}\ V_{oil} = (9.81 \times 0.82)\ \frac{\pi}{4}(10)^2(6) = 3790.74$ kN

$$\text{Pressure increase} = \frac{W}{A} = \frac{3790.74}{\frac{\pi}{4}(10)^2} = \mathbf{48.2652\ kPa}$$

or, Pressure increase $= \gamma_o\ h_{oil} = (9.81 \times 0.82)(6) = 48.2652$ kPa

Part *c*:

p_o = initial effective stress at midheight of clay layer

$p_o = 17.2(1.4) + 18.5(3.5) = 88.83$ kPa

Δp = Pressure increase due to oil = 48.2652 kPa

Note: The weight of the tank was no longer included because it was compensated by the weight of soil excavated.

$$\text{OCR} = \frac{p_c}{p_o} = 2.2;\ p_c = 195.426\ \text{kPa}$$

Note: p_c = preconsolidation stress

$p_f = p_o + \Delta p = 88.83 + 48.2652 = 137.0952$ kPa

Since $p_f < p_c$:

$$\Delta H = H\frac{C_s}{1+e_o}\log\frac{p_f}{p_o}$$

$$\Delta H = 2800\frac{0.05}{1+0.45}\log\frac{137.0952}{88.83} = \mathbf{18.196\ mm}$$

PROBLEM 10.28

The section of a retaining wall is shown in Figure 10.48. Consider 1 m length o wall and use Rankine's active state.

(*a*) Determine the total active lateral pressure at the bottom of the wall.

(*b*) Determine total active force acting on the wall.

(*c*) Determine the location of the total active force measured from the bottom of the wall.

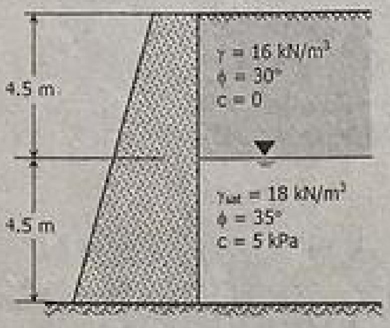
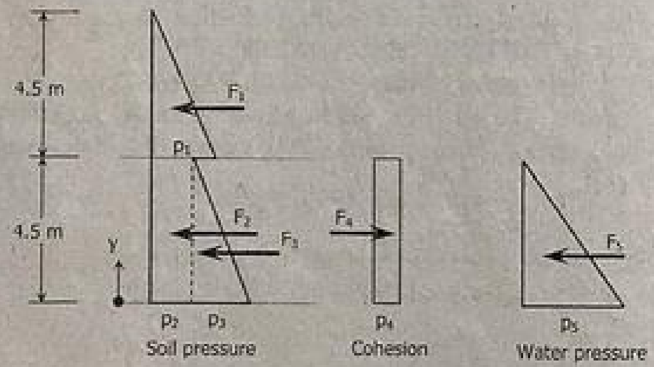


Figure 10.48

SOLUTION



For the upper soil layer:

$$K_a = \frac{1-\sin 30^\circ}{1+\sin 30^\circ} = \frac{1}{3}$$

For the lower soil layer:

$$K_a = \frac{1-\sin 35^\circ}{1+\sin 35^\circ} = 0.271$$

Soil pressure:

$p = K_a \gamma h$

$p_1 = (1/3)16(4.5) = 24$ kPa

$F_1 = ½(24)(4.5)(1) = 54$ kN $\qquad y_1 = 4.5 + 1.5 = 6$ m

$p_2 = 0.271(16)(4.5) = 19.512$ kPa

$F_2 = 19.512(4.5)(1) = 87.804$ kN $\qquad y_2 = 2.25$ m

$p_3 = 0.271(18 - 9.81)(4.5) = 9.988$ kPa

$F_3 = ½(9.988)(4.5)(1) = 22.473$ kN $\qquad y_3 = 1.5$ m

Cohesion.

$p = 2c\sqrt{K_a}$

$p_4 = 2(5)\sqrt{0.271} = 5.206$ kPa

$F_4 = 5.206(4.5)(1) = 23.427$ kN $\qquad y_4 = 2.25$ m

Water pressure

$p_5 = 9.81(4.5) = 44.145$ kPa

$F_5 = ½(44.145)(4.5)(1) = 99.33$ kN $\qquad y_5 = 1.5$ m

Active lateral pressure at the bottom of wall:

$p = 24 + 19.512 + 9.988 - 5.206 + 44.145$

$p = \mathbf{92.44\ kPa}$ → Part *a*

Total active force, $F_a = F_1 + F_2 + F_3 - F_4 + F_5$

Total active force, $F_a = \mathbf{240.18\ kN}$ → Part *b*

Location from the bottom:

$240.18\,\bar{y} = 54(6) + 87.804(2.25) + 22.473(1.5) - 23.427(2.25) + 99.33(1.5)$

$\bar{y} = \mathbf{2.713\ m}$ → Part *c*

PROBLEM 10.29

Classify the following soils by using the Unified Soil Classification System

(*a*) What is the classification of soil *A*?

(*a*) What is the classification of soil *B*?

(*a*) What is the classification of soil *C*?

Soil	Sieve analysis, % finer		Liquid limit	Plastic limit	C_u	C_c
	No. 4	No. 200				
A	95	70	48	24		
B	15	42	32	16	4.8	2.9
C	60	4	39	8	3.2	4.2

SOLUTION

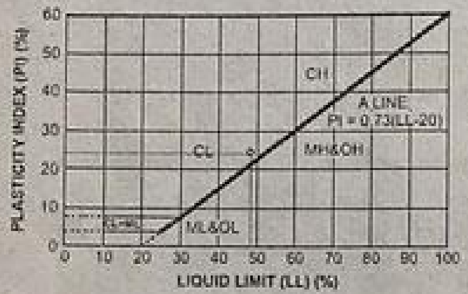
Using Table 2.01 in Page 59:

Soil *A*: %finer than No. 200 = 70% > 50% (Fine-grained soil)

$LL = 48 < 50\%$ (Silts and Clays, ML, CL, or OL)
$PI = LL - PL = 48 - 24 = 24$

From plasticity chart, since the point plot above A-line, the soil is CL

PLASTICITY CHART



- CL ▸ Inorganic; $LL < 50$; $PI > 7$; Atterberg limits plot on or above A-line
- ML ▸ Inorganic; $LL < 50$; $PI < 4$ or Atterberg limits plot below A-line
- OL ▸ Organic; $(LL$ – ovendried$)/(LL$ – not dried$) < 0.75$; $LL < 50$
- CH ▸ Inorganic; $LL \geq 50$; Atterberg limits plot on or above A-line
- MH ▸ Inorganic; $LL \geq 50$; Atterberg limits plot below A-line
- OH ▸ Organic; $(LL$ – ovendried$)/(LL$ – not dried$) < 0.75$; $LL \geq 50$
- CL - ML ▸ Inorganic, Atterberg limits plot in the hatched zone

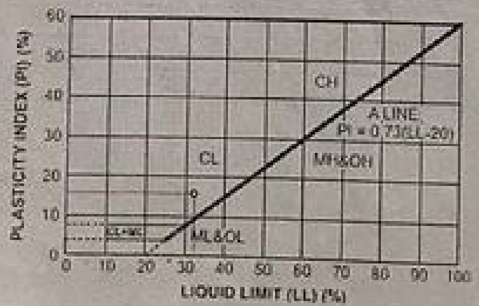
Soil *B*:

%finer than No. 200 = 22% < 50% (Coarse-grained soil)
%finer than No. 4 sieve = 15% < 50% (Gravels)

Since there is more than 12% fines, the soil is either GM or GC.

$PI = LL - PL = 32 - 16 = 16 > 7$
The point plots above A-line
Therefore the soil is GC

PLASTICITY CHART



Soil C:

% Passing No. 200 = 4 < 50 (Coarse-grained soil)
% Passing No. 4 = 60% (Sands)

Since there is less than 5% fines, the soil is either SW or SP.
Since $C_u < 6$, the soil is **SP**

PROBLEM 10.30

A footing rest in a layer of sand 6 m thick. The bottom of the footing is 1.5 m below the ground surface. Beneath the sand layer is a 4 m thick clay layer. Underneath the clay layer is solid rock. Water table is at a depth of 4.5 m below the ground surface. See Figure 10.49. Assume that the pressure beneath the footing is spread at an angle of 2 vertical to 1 horizontal. Assume OCR = 1.

(*a*) What is the vertical effective stress at the midheight of the clay layer?
(*a*) What is the average pressure increase in the clay layer due to the load on footing? Use Simpson's rule.
(*a*) Determine the settlement in mm beneath the footing.

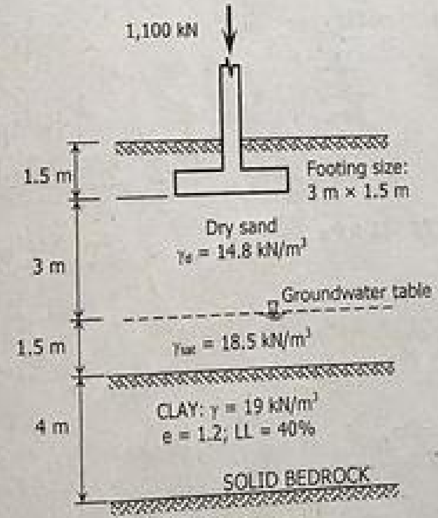
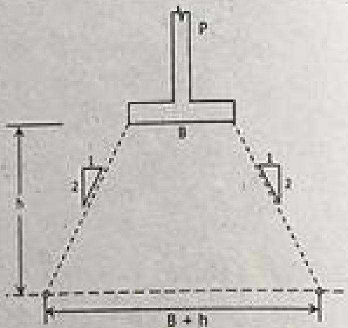


Figure 10.49

SOLUTION

Part *a*: $p_o = (19 - 9.81)(2) + (18.5 - 9.81)(1.5) + 14.8(4.5)$
$p_o = 98.015$ kPa

Part *b*: $\Delta p_{ave} = \dfrac{\Delta p_1 + 4\Delta p_m + \Delta p_2}{6}$



At any vertical distance h from the base of footing:

$b = 3 + (1/2h) \times 2 = 3 + h$

$d = 1.5 + (1/2h) \times 2 = 1.5 + h$

$A_h = (3 + h)(1.5 + h)$

$$\Delta p = \frac{1{,}100}{(3+h)(1.5+h)}$$

At top of clay, (h = 4.5):

$$\Delta p_1 = \frac{1{,}100}{(3+4.5)(1.5+4.5)} = 24.44 \text{ kPa}$$

At midheight of clay, (h = 6.5)

$$\Delta p_m = \frac{1{,}100}{(3+6.5)(1.5+6.5)} = 14.47 \text{ kPa}$$

At bottom of clay, (h = 8.5)

$$\Delta p_m = \frac{1{,}100}{(3+8.5)(1.5+8.5)} = 9.57 \text{ kPa}$$

$$\Delta p_{ave} = \frac{\Delta p_1 + 4\Delta p_m + \Delta p_2}{6} = \frac{24.44 + 4(14.47) + 9.57}{6}$$

Δp_{ave} = **15.315 kPa**

Part *c*:

$C_c = 0.009(LL - 10) = 0.27$

$$\Delta H = H\frac{C_c}{1+e_o}\log\frac{pf}{p_o}$$

$$\Delta H = 4000\frac{0.27}{1+1.2}\log\frac{98.015+15.315}{98.015}$$

ΔH = **30.95 mm**

PROBLEM 10.31 (CE NOVEMBER 2008)

A cut slope was excavated in a saturated clay. The slope made an angle of 60° with the horizontal. When slope failure occurs, distance BC is 8 m. Given: Stability number, m = 0.185; γ = 17 kN/m³; c_u = 20 kPa.

a) What is the stability factor?

b) Determine the critical depth of cut in meters.

c) What is the angle of failure plane in degrees?

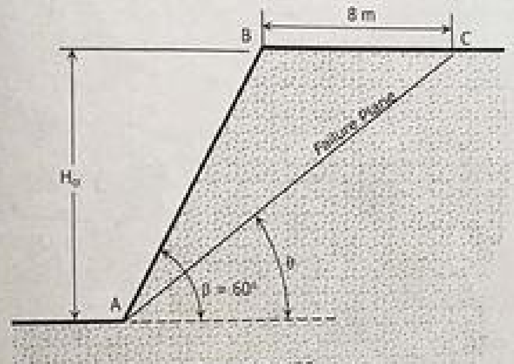


Figure SM-456

SOLUTION

Given: Stability number, m = 0.185

γ = 17 kN/m³

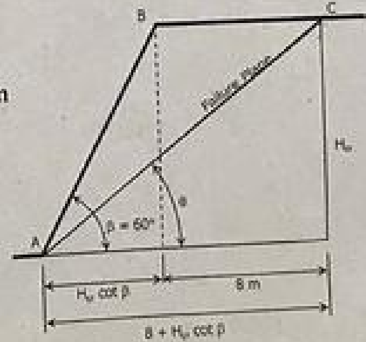
c_u = 20 kPa

Part *a*: Stability Factor

Stability Factor = $1/m$ = 5.405

Part *b*: Critical depth of excavation:

$$H_{cr} = \frac{c_u}{\gamma m} = \frac{20}{17(0.185)} = 6.359 \text{ m}$$



Part *c*: Angle of failure plane

$$\left[\tan\theta = \frac{H_{cr}}{8 + H_{cr}\cot\beta}\right]$$

$$\tan\theta = \frac{6.359}{8 + 6.359 \times \cot 60°}$$

θ = **28.58°**

PROBLEM 10.32 (CE NOVEMBER 2008)

A group of friction piles is shown in Figure 28.87 SM-57. The total load on the pile less the soil displaced by the footing is 1800 kN. The clay is 16 m thick and the silt is 2 m thick. Ground water table is located 2 m below the clay surface.

a) Compute for the effective pressure at the midheight of the consolidating clay layer, in kPa.

b) Compute for the compression index of the clay.

c) What is the approximate settlement of the foundation?

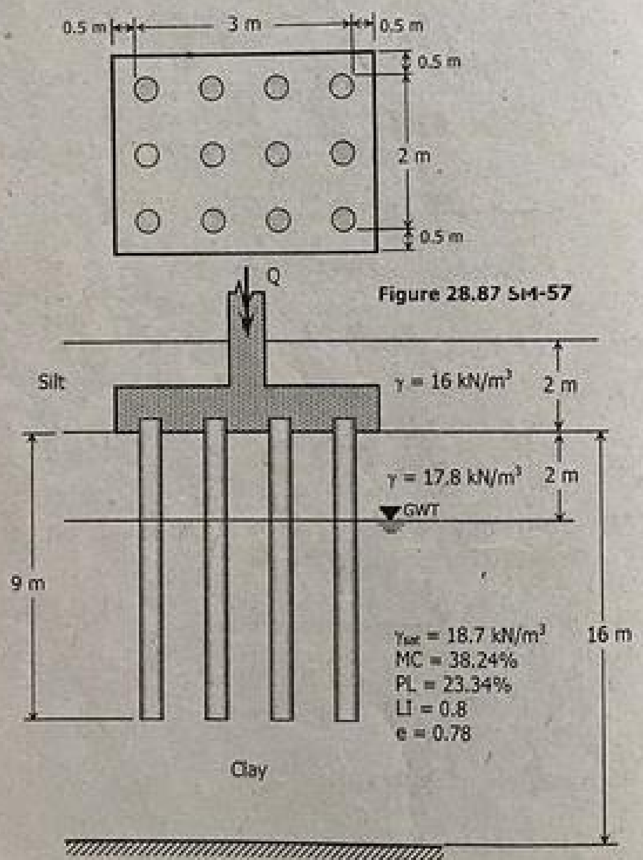
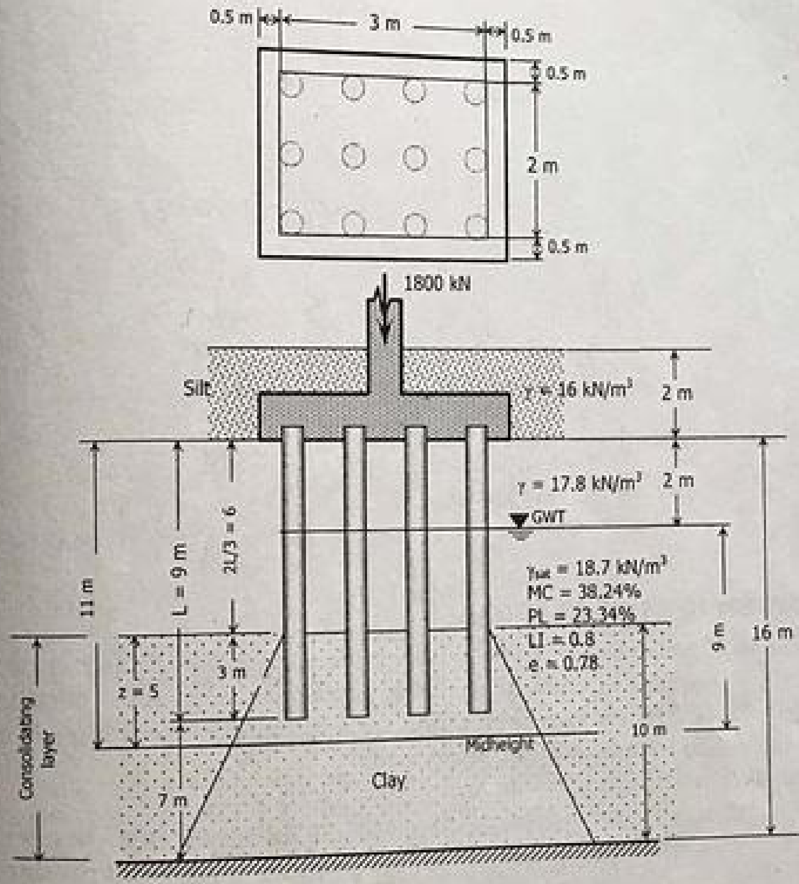


Figure 28.87 SM-57

SOLUTION



Part *a*:

$$p_o = 18.7(9) + 17.8(2) + 16(2) - 9.81(9)$$

$$p_o = \mathbf{147.61\ kPa}$$

Part *b*:

$$C_c = 0.009(LL - 10) \qquad \textit{undisturbed clay}$$

$$\left[LI = \frac{MC - PL}{LL - PL}\right] \quad 0.8 = \frac{38.24 - 23.34}{LL - 23.34}$$

$$LL = 41.965\%$$

$$C_c = 0.009(41.965 - 10) = 0.288$$

Part *c*:

$$\Delta H = H\frac{C_c}{1+e_o}\log\frac{p_f}{p_o}$$

Solve for Δp and p_f:

$$\Delta p = \frac{Q}{A_z}$$

$$A_z = (3 + 4.67)(2 + 4.67) = 51.159 \text{ m}^2$$

$$\Delta p = \frac{1800}{51.159} = 35.184 \text{ kPa}$$

$[p_f = p_o + \Delta p]$ $\quad p_f = 147.61 + 35.184$

$p_f = 182.794$ kPa

$$\Delta H = 10\frac{0.288}{1+0.78}\log\frac{182.794}{147.61}$$

$\Delta H = 0.150$ m = **150 mm**

PROBLEM 10.33 (CE MAY 2008)

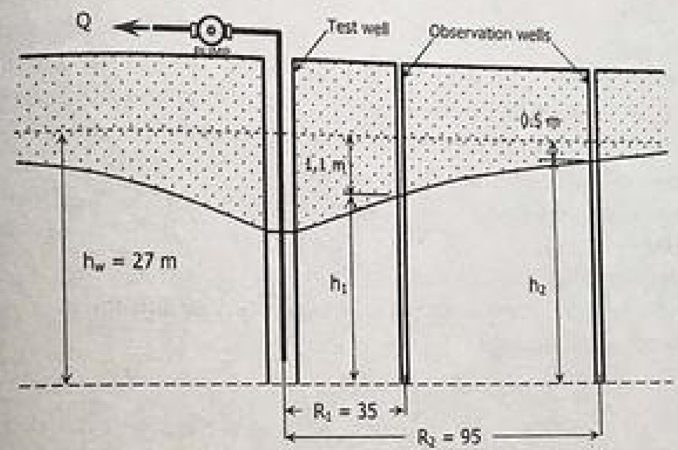
A test well penetrates 27 m below the static water table. Water is pumped from the test well at uniform rate of 69 liters per second for a period of 24 hours. Drawdown in two observation wells located 35 m and 95 m from the test well were 1.1 m and 0.5 m, respectively.

a) What is the flow rate in MLD?

b) What is the coefficient of permeability of the aquifer in meters per day?

c) What is the transmissibility of the aquifer in square meter per day.

SOLUTION



$Q = 69$ liters/sec $= 0.069$ m³/s

Part *a*:

$Q = 69$ lit/sec × (3600 sec/hr) × (24 hr/day) ÷ 10^6

$Q =$ **5.962 MLD**

Part *b*:

$h_1 = 27 - 1.1 = 25.9$ m; $\quad R_1 = 35$ m

$h_2 = 27 - 0.5 = 26.5$ m; $\quad R_2 = 95$ m

$$\left[Q = \frac{\pi k(h_2^2 - h_1^2)}{\ln(R_2/R_1)}\right] \quad 0.069 = \frac{\pi k[26.5^2 - 25.9^2]}{\ln(95/35)}$$

$k = 6.976 \times 10^{-4}$ m/sec × 3600 × 24

$k =$ **60.269 m/day**

Part *c*:

Transmissibility, $T = k \times h_w$

Transmissibility, $T = 60.269 \times 27$

Transmissibility, $T =$ **1,627.263 m²/day**

PROBLEM 10.34 (CE JANUARY 2008)

A confined aquifer is shown in Figure 36-56(78)S03. This aquifer has a source of recharge located as shown. The hydraulic conductivity of the aquifer is 40 m/day with a porosity of 25%. The piezometric (head) surface in the two observation wells 1200 m apart are at elevation 65 m and 60 m, respectively from the common datum. The aquifer has an average thickness of 25 m and an average width of 4 km

a) Determine the rate of flow of water through the aquifer, in cubic meters per day.
b) Determine the seepage velocity in m/day.
c) Determine the time of travel from the head of aquifer to a point 4 km downstream, in days.

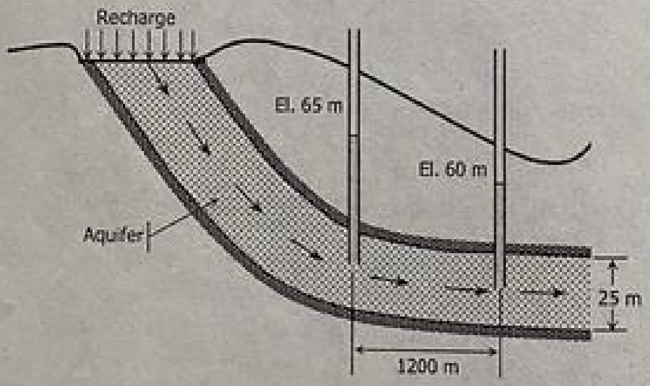


Figure 36-56(78) S03

SOLUTION

Considering the two observation wells:

Head, $h = 65 - 60 = 5$ m

$L = 1200$ m

$$i = \frac{h}{L} = 0.004167$$

A = width × thickness = 4000 × 25 = 100,000 m²

$K = 40$ m/day

$[Q = KiA]$ $\quad Q = 40(0.004167)(100,000)$

$Q = 16,667\text{ m}^3\text{/day}$ → Part *a*

$[v = Ki]$ $\quad v = 40(0.004167) = 0.1667$ m/day

$$\text{Seepage velocity, } v_s = \frac{v}{n}$$

$$\text{Seepage velocity, } v_s = \frac{0.1667}{0.25} = 0.667\text{ m/day}$$ → Part *b*

Part 3: Time to travel 4 km

$$\text{Time} = \frac{\text{Distance}}{v_s} = \frac{4000}{0.667}$$

Time = **6000 days** → Part *c*

PROBLEM 10.35 (CE JANUARY 2008)

A footing 1.2-m square have its base 1 m below the ground. The soil strata at the site consist of a layer of stiff saturated clay 27.5 m thick overlying dense sand. The average bulk density of the clay is 1,920 kg/m³, saturated density = 2,120 kg/m³, cohesion $c = 0$, and angle of friction $\phi = 30°$. Given is Terzaghi's ultimate bearing capacity for square footings: $q_u = 1.3\ c\ N_c + \gamma\ D_f\ N_q + 0.40\ \gamma B\ N_\gamma$. Use Figure 45-6a to get the bearing capacity factors.

a) Determine the ultimate bearing capacity when the water table is 0.5 m below the ground.
b) Determine the ultimate bearing capacity when the water table is at the base of the footing.
c) Determine the ultimate bearing capacity when the water table is 1.2 m below the ground.

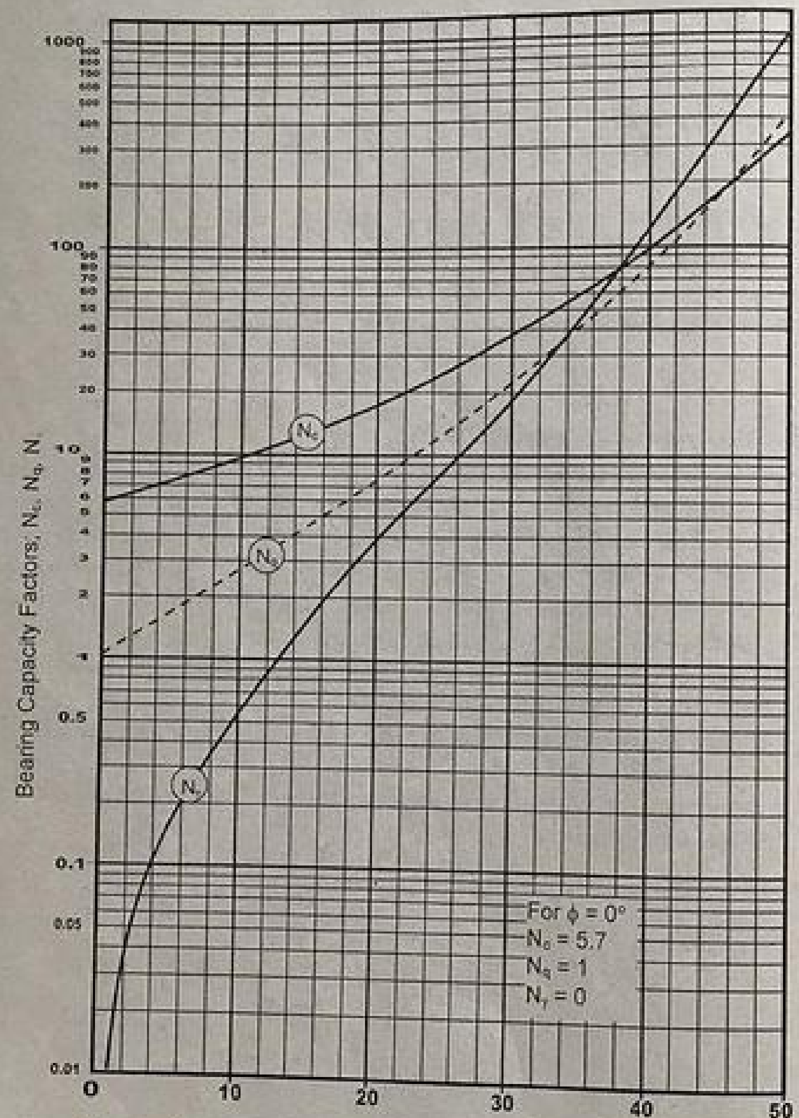
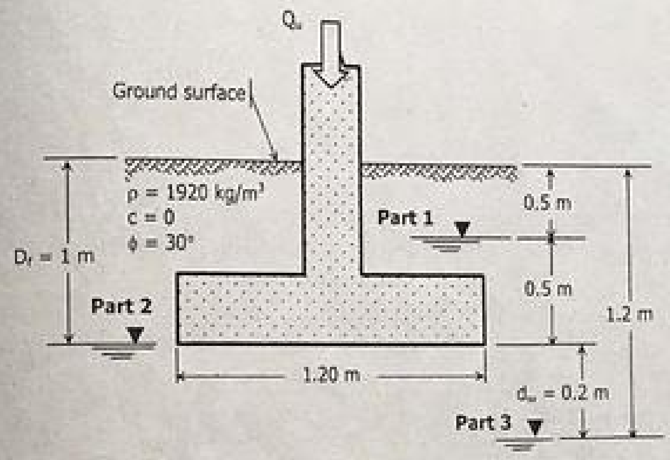


Figure 45-6(a)

SOLUTION

Bearing capacity factors for $\phi = 30°$.

$N_c = 37;\ N_q = 22;\ N_\gamma = 19$



$\gamma = \rho g = 1920 \times 9.81 / 1000 = 18.835\ \text{kN/m}^3$

$\gamma_{sat} = 2{,}120 \times 9.81 / 1000 = 20.797\ \text{kN/m}^3$

$\gamma_b = \gamma_{sat} - \gamma_w = 10.987\ \text{kN/m}^3$

$q_u = 1.3\,c\,N_c + q\,N_q + 0.40\,\gamma_e\,B\,N_\gamma$

Part *a*: Water table is 0.5 m below the ground.

$\gamma_e = \gamma_b = 10.987\ \text{kN/m}^3$

$q = \gamma(0.5) + \gamma_b(0.5) = 14.911\ \text{kPa}$

$q_u = 1.3\,(0)(37) + (14.911)(22) + 0.40(10.987)(1.2)(19)$

$q_u = \mathbf{428.25\ kPa}$

Part *b*: Water table is at the base of footing.

$\gamma_e = \gamma_b = 10.987\ \text{kN/m}^3$

$q = \gamma(1) = 18.835\ \text{kPa}$

$q_u = 1.3\,(0)(37) + (18.835)(22) + 0.40(10.987)(1.2)(19)$

$q_u = \mathbf{514.58\ kPa}$

Part *c*: Water table is 1.2 m below the ground.

$\gamma_e = \gamma_b\left(1 + \dfrac{d_w}{B}\right)$ where $d_w = 0.2$ m

(water table is 0.2 m below base of footing)

$$\gamma_c = \gamma_b\left(1+\frac{0.2}{1.2}\right) = 12.818 \text{ kN/m}^3$$

$$q = \gamma(1) = 18.835 \text{ kPa}$$

$$q_u = 1.3\,(0)(37) + (18.836)(22) + 0.40(12.818)(1.2)(19)$$

$$q_u = 531.28 \text{ kPa}$$

PROBLEM 10.36

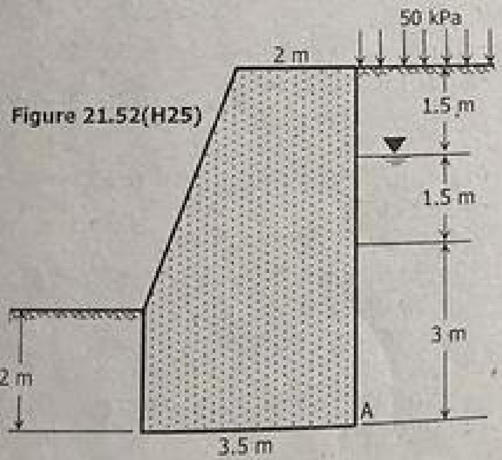
The concrete gravity retaining wall shown in Figure 21.52(H25) supports two layers of soil each having a thickness of 3 m. The properties of the layers are:

Upper layer: $c = 0$ kPa, $\phi = 30°$, $\gamma_d = 17.5$ kN/m³, $\gamma_{sat} = 19.5$ kN/m³

Lower layer: $c = 10$ kPa, $\phi = 18°$, $\gamma_{sat} = 19$ kN/m³

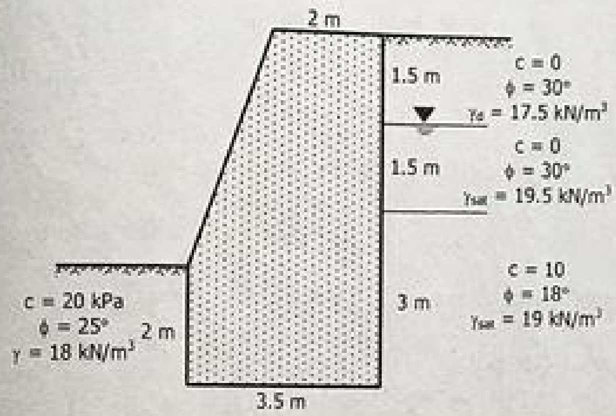
There is a surface load of 50 kPa and the water table is 1.5 m below the ground surface. The front of the wall is supported by soil with $c = 20$ kPa, $\phi = 25°$, and $\gamma = 18$ kN/m³. Take unit weight of concrete = 24 kN/m³. The cohesion between the base of the wall and the soil is 20 kPa, and the mobilized friction angle on this interface is 25°.

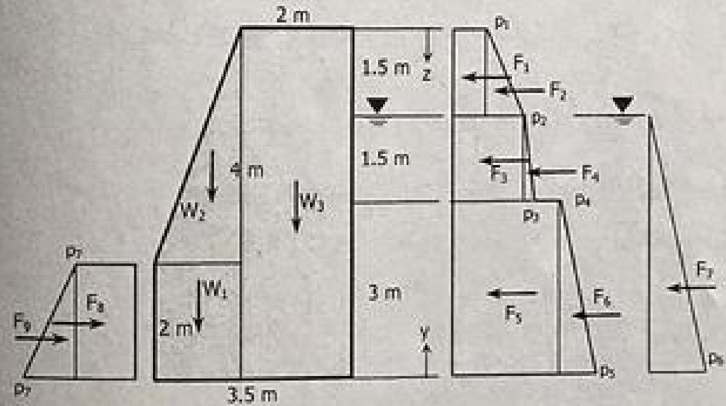
Figure 21.52(H25)



a) What is the total active force?
b) What is the total active moment?
c) What is total passive resistance?
d) What is the factor of safety against sliding?

SOLUTION





Upper layer: $K_a = (1 - \sin 30°)/(1 + \sin 30°) = 0.333$
Lower layer: $K_a = (1 - \sin 18°)/(1 + \sin 18°) = 0.528$

Effective lateral pressures:

At z = 0: $p_1 = 0.333(50) = 16.7$ kPa
At z = 1.5 $p_2 = 16.7 + 0.333(17.5)(1.5) = 25.42$ kPa
At z = 3 $p_3 = 25.42 + 0.333(19.5 - 9.81)(1.5) = 30.26$

$$p_4 = 0.258[50 + 17.5(1.5) + (19.5 - 9.81)(1.5)] - 2(10)\sqrt{0.528}$$

$$p_4 = 33.39 \text{ kPa}$$

At z = 6 $p_5 = 33.39 + 0.528(19 - 9.81)(3) = 47.94$ kPa
Water pressure, $p_6 = 9.81(4.5) = 44.15$ kPa

$F_1 = p_1(1.5) = 25$ kN	$y_1 = 6 - 0.5(1.5) = 5.25$ m
$F_2 = ½(p_2 - p_1)(1.5) = 6.56$ kN	$y_2 = 6 - (2/3)(1.5) = 5$ m
$F_3 = p_2(1.5) = 38.125$ kN	$y_3 = 3 + 0.75 = 3.75$
$F_4 = ½(p_3 - p_2)1.5 = 3.634$ kN	$y_4 = 3 + (1/3)(1.5) = 3.5$ m
$F_5 = p_4(3) = 100.174$ kN	$y_5 = ½(3) = 1.5$ m
$F_6 = ½(p_5 - p_4)(3) = 21.83$ kN	$y_6 = 3/3 = 1$ m
$F_7 = ½(p_6)(4.5) = 99.33$ kN	$y_2 = 1.5$ m

$F_a = F_1 + F_2 + F_3 + F_4 + F_4 + F_6 + F_7 = $ **294.65 kN**

Active Moment = $\Sigma F \times y$ = **640.83 kN-m**

Passive Pressure:

$K_p = (1 + \sin 25°)/(1 - \sin 25°) = 2.464$

At $z = 0$: $p_{p1} = 2c\sqrt{K_p} = 62.79$ kPa

At $z = 2$ $p_{p2} = 62.79 + 2.464(18)(2) = 151.49$ kPa

$F_8 = p_{p1} \times 2 = 125.575$	$y_8 = 1$ m
$F_9 = ½(p_{p2} - p_{p1})(2) = 88.7$ kN	$y_9 = 2/3$ m

Total Passive resistance, $F_p = F_8 + F_9 =$ **214.28 kN**

Passive Moment = $F_8\, y_8 + F_9\, y_9$ = **184.71 kN-m**

Weight of Wall:

$W_1 = 24(1.5)(2) = 72$ kN
$W_2 = 24 \times ½(1.5)(4) = 72$ kN
$W_3 = 24(2)(6) = 288$ kN
$W = W_1 + W_2 + W_3 = 432$ kN

Frictional resistance, $F_f = \mu N$
Frictional resistance, $F_f = \mu W = \tan 25°(432) = 201.445$ kN
Cohesion, $F_c = 20(3.5 \times 1) = 70$ kN

Factor of safety against sliding:

$$FS_s = \frac{F_p + F_f + F_c}{F_a} = 1.648$$

PROBLEM 10.37

A test is set-up as shown in Figure 36-45(01)S01. A cylindrical mold 10 cm in diameter is filled with silt to a height of $H_1 = 20$ cm, whose coefficient of permeability $k_1 = 0.011$ cm/min.

A second mold is placed on top of the first mold whose inside diameter is 20 cm is filled with sand whose coefficient of permeability $k_2 = 0.0823$ cm/min and whose height is $H_2 = 30$ cm.

The test set-up is a permeameter of constant head. Water is placed in the mold and maintained at a level $h = 70$ cm above the level of the outlet.

a) Calculate the ratio between the hydraulic gradients in silt and in sand.
b) Calculate the flow of water in cc/sec.
c) Calculate the pore water stress at the midheight of the silt in Pascals.

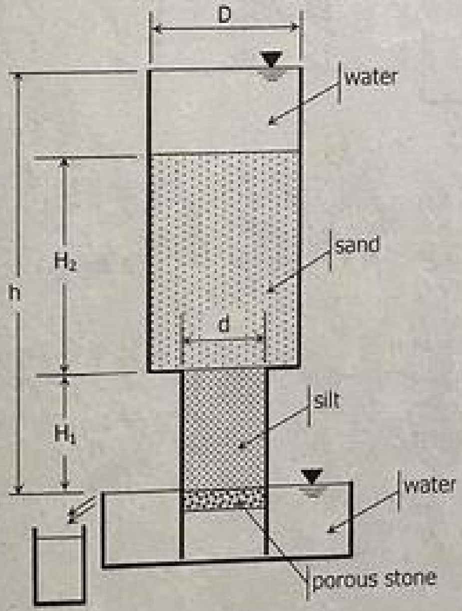
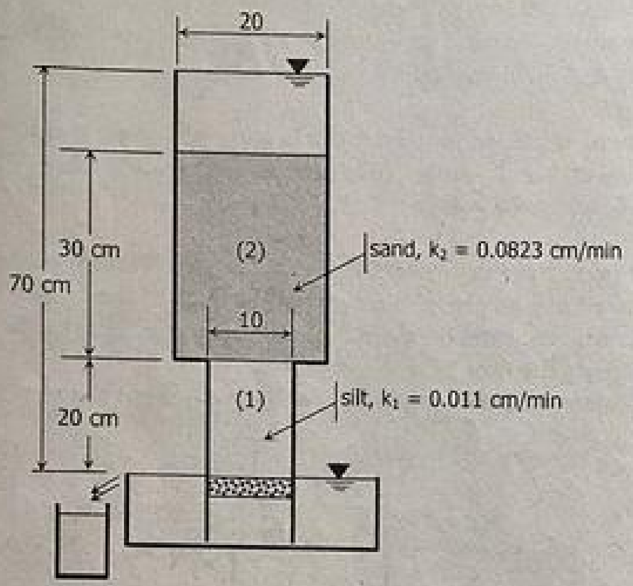


Figure 36-45(01)S01

SOLUTION



Total head, $h = h_1 + h_2 = 70$ cm; $i = h/L$; $h = iL$
Total head, $h = i_1 L_1 + i_2 L_2 = 70$
Total head, $h = 20\, i_1 + 30\, i_2 = 70$ → Eq. (1)

$Q_1 = Q_2$
$k_1\, i_1\, A_1 = k_2\, i_2\, A_2$
$0.011(i_1)\frac{\pi}{4}(10)^2 = 0.0823(i_2)\frac{\pi}{4}(20)^2$
$i_1 = 29.927\, i_2$ → Eq. (2)
$i_1/i_2 = \mathbf{29.927}$

In Eq. (1): $20(29.93\, i_2) + 30\, i_2 = 70$
$i_2 = 0.111$
$i_1 = 3.33$

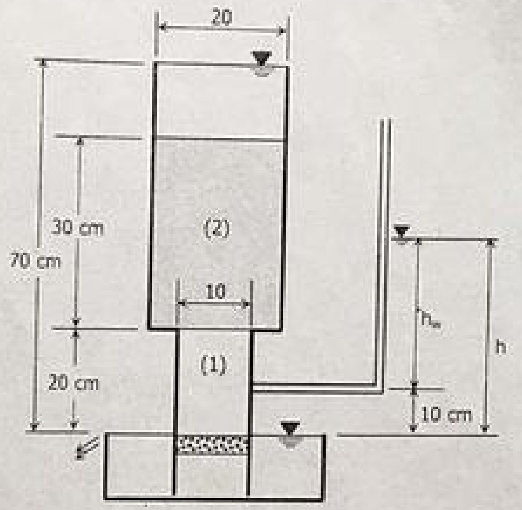
$Q = k_1\, i_1\, A_1 = 0.011(3.33)\,\frac{\pi}{4}(10)^2 = \mathbf{2.877\ cm^3/min}$

Part c: Pore water stress at midheight of silt:

$i_1 = 3.33$
$i_1 = h/10$
$h = 33.3$ cm

$h_w = 23.3$ cm
$h_w = 0.233$ m

$p_w = \gamma_w\, h_w$
$p_w = 9810(0.233)$
$p_w = \mathbf{2285.7\ Pa}$



PROBLEM 10.38

For a constant permeability test in sand as shown in Figure 85.36(S25), the following are given:

$L = 350$ mm
$A = 125\ cm^2$
$h = 420$ mm
Water collected in 3 minutes = 580 cm^3
Void ratio of sand = 0.61

a) Determine the hydraulic conductivity in m/min.
b) Determine the seepage velocity in m/min.
c) How long will it take for h to drop from 420 mm to 400 mm when the supply is cut-off?

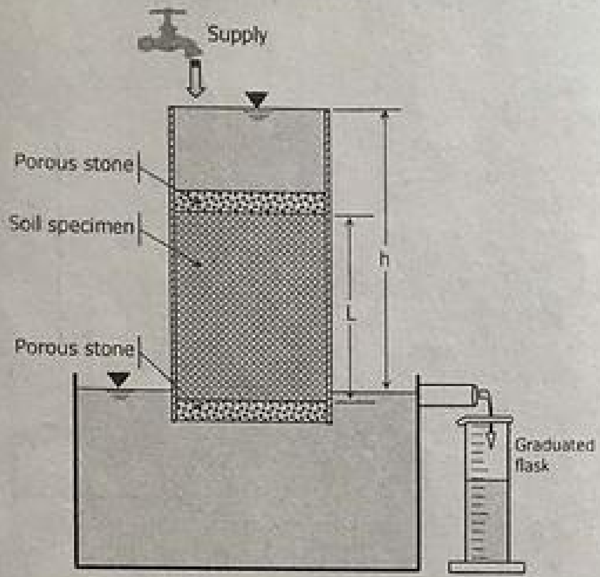


Figure 85.36(S25)

SOLUTION

$[Q = \dfrac{Volume}{time}]$ $\quad Q = \dfrac{580}{3} = 193.33$ cm/min

$[i = h/L]$ $\quad i = 420/350 = 1.2$

$[Q = KiA]$ $\quad 193.33 = K(1.2)(125)$

$\quad K = \mathbf{1.289\ cm/min}$

Average velocity, $v = Ki$

Average velocity, $v = 1.289(1.2) = 1.547$ m/min

Void ratio, $n = \dfrac{e}{1+e} = \dfrac{0.61}{1+0.61} = 0.3789$

Seepage velocity, $v_s = \dfrac{v}{n}$

Seepage velocity, $v_s = \dfrac{1.547}{0.3789} = \mathbf{4.08\ cm/min}$

Part c:

$$t = \int_{h_2}^{h_1} \frac{A_s\,dh}{Q_o}$$

$A_s = 125 \qquad h_1 = 420$ cm $\qquad h_2 = 400$ cm

$[Q_o = KiA] \qquad Q_o = 1.289(h/350)(125) = 0.4604h$

$$t = \int_{400}^{420} \frac{125\,dh}{0.4604\,h} = 271.5\Big[\ln(h)\Big]_{400}^{420}$$

$t = \mathbf{13.25\ minutes}$

PROBLEM 10.39

An exploration drill hole was made in stiff, saturated clay as shown in Figure 12.365(S74). The sand layer underlying the clay was observed to be under artesian pressure. Water in the drill hole rose to a height of $H_1 = 5.5$ m above the top of the sand layer.

a) Determine the porosity of the clay in percent.
b) What is the critical hydraulic gradient?
c) If an open excavation is to be made in the clay, how deep can the excavation proceed before the bottom heaves?

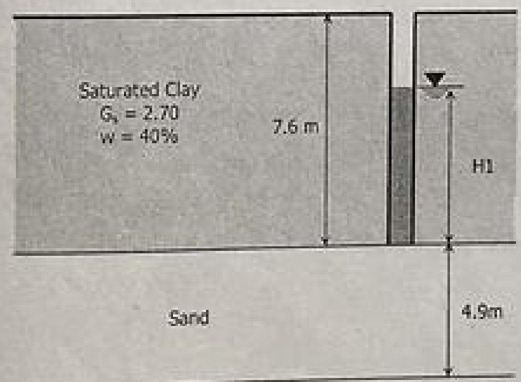


Figure 12.365(S74)

SOLUTION

$[G\,MC = s\,e]$ $\quad 2.70(0.4) = 1(e);\ e = 1.08$

$[n = \dfrac{e}{1+e}]$ $\quad n = 0.5192 = 51.92\%$

Critical hydraulic gradient, $i_{cr} = \dfrac{G-1}{1+e} = 0.8173$

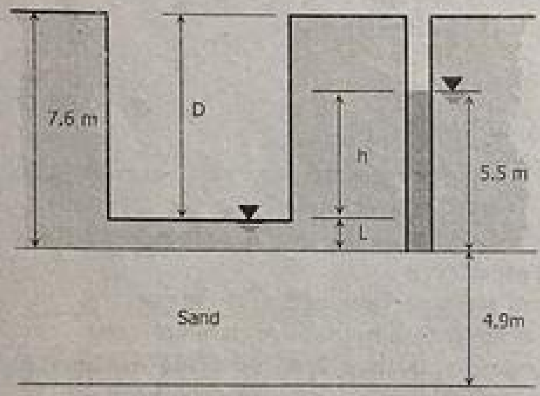


Figure 12.365(S74)

Critical hydraulic gradient, $i_{cr} = \dfrac{G-1}{1+e} = 0.81731$

Hydraulic gradient, $i = \dfrac{h}{L}$; $h = 0.81731L$

$[h + L = 5.5]$ $\quad 0.81731L + L = 5.5$

$L = 3.026$ m

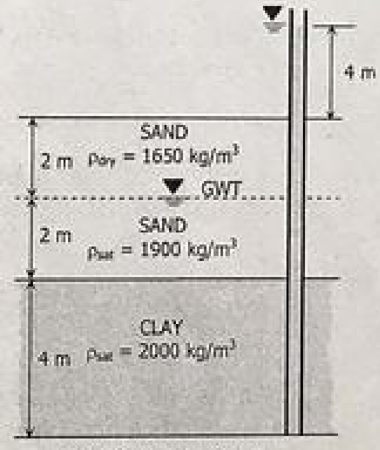
Depth of excavation, $D = 7.6 - L =$ **4.574 m**

PROBLEM 10.40

A clay layer of 4 m thick with ρ_{sat} = 2000 kg/m³ is overlain by a 4 m sand with ρ_{sat} = 1900 kg/m³ and ρ_{dry} = 1,650 kg/m³, the top of this layer being the ground surface. The water table is located 2 m below the ground surface. The clay layer is underlain by a sand stratum that is in artesian conditions with the water level in a standpipe being 4 m above the ground surface.

a) What is the effective stress at the base of the clay in kPa?
b) What is the effective stress at the top of the clay in kPa?
c) If the dry sand is excavated, in what depth the effective stress at the bottom of the clay layer will become zero?

SOLUTION



Part *a*: At the base of the clay layer:

$p_T = (9.81 \times 2)(4)$
$\quad + (9.81 \times 1.9)(2)$
$\quad + (9.81 \times 1.65)(2)$

$p_T = 148.131$

$p_w = 9.81(12)$

$p_w = = 117.72$ kPa

$p_E = p_T - p_w =$ **40.41 kPa**

Part *b*: At the top of the clay layer:

$p_T = (9.81 \times 1.9)(2)$
$\quad + (9.81 \times 1.65)(2)$

$p_T = 69.651$

$pw = 9.81(2) = 19.62$ kPa

$p_E =$ **50.03 kPa**

Part *c*:

$p_E = 0 = p_T - p_w$

$p_w = 117.72\text{ kPa} = p_T$

Assuming that the excavation on sand does not reach the water table:

$p_T = (9.81 \times 2)(4) + (9.81 \times 1.9)(2) + (9.81 \times 1.65)(2 - d) = 117.72$

$d =$ **1.879 m** < 2 m (ok)

PROBLEM 10.41

The foundations supporting two columns of a building are shown in Figure 32-89(45)S47. An extensive soil investigation was not carried out and it was assumed in the design of the footing that the clay layer has a uniform thickness of 1.2 m. Three years after construction, the building settled with a differential settlement of 10 mm. Walls of the building began to crack. The doors have not jammed but by measuring the out-of-vertical distance of the doors, it is estimated that they would become jammed if the differential settlement exceeded 24 mm. A subsequent soil investigation showed that the thickness of the clay layer was not uniform but varies as shown in the figure. Assume that the pressure below the footing is spread through a slope of 1 H to 2 V.

The following equations may be useful:

$\Delta H = H\, m_v\, \Delta p$

Degree of consolidation, $U = \dfrac{\delta_c}{\delta}$

δ_c = differential settlement at a time t_1

δ = maximum differential settlement

$T_v = \dfrac{4}{\pi} U^2$

$t = T_v \dfrac{(H_{dr})^2}{C_v}$

a) What is the increase in pressure at the midheight of the clay layer below footing *A* in kPa.

b) What is the degree of consolidation after three years?.

c) How long will it take before the doors become jammed?

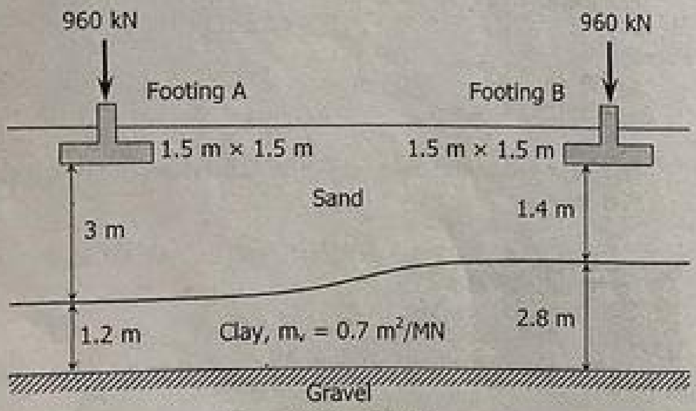


Figure 32-89(45)S47

SOLUTION

m_v = modulus of volume compressibility

$\Delta H = H\, m_v\, \Delta p$

$$\Delta p = \frac{P}{(B+z)(L+z)} \qquad \text{1H to 2V}$$

Footing *A*:

$$\Delta p = \frac{960}{(1.5+3.6)^2} = \mathbf{36.9\ kPa}$$

$\Delta H = 1200(0.7 \times 10^{-3})36.9 = 31.003$ mm

Footing *B*:

$$\Delta p = \frac{960}{(1.5+2.8)^2} = 51.92 \text{ kPa}$$

$\Delta H = 2800(0.7 \times 10^{-3})51.92 = 101.76$ mm

Differential settlement, $\Delta S = 101.76 - 30.003 = 70.76$ mm

Current differential settlement, $\Delta S_1 = 10$ mm (in $t = 3$ years)

Degree of consolidation, $U = \dfrac{\Delta S_1}{\Delta S} = \dfrac{10}{70.76} = \mathbf{0.14132}$

$T_v = (4/\pi)(0.14132)^2 = 0.02543$

$t = T_v \dfrac{(H_{dr})^2}{C_v} = K\,T_v$

$3 = K(0.02543)$; $K = 117.98$

Time when differential settlement becomes 24 mm:

$U = \dfrac{24}{70.76} = 0.33918$

$T_v = = (4/\pi)(0.33918)^2 = 0.14647$

$t = K\,T_v = 117.98(0.14647) = 17.28$ years

Therefore, the doors become jammed after 17.28 - 3 = **14.28 years**

Another solution:

$$t = T\frac{(H_{dr})^2}{C_v} = \frac{4}{\pi}U^2\frac{(H_{dr})^2}{C_v} = K\,U^2$$

or $\frac{t}{U^2}$ = constant

Thus,

$$\left[\frac{t_1}{U_1^2} = \frac{t_2}{U_2^2}\right] \quad 3/(0.14132)^2 = t/(0.33918)^2$$

t = 17.28 years

PROBLEM 10.42

A soil element is subjected to different stresses as shown in Figure 25.63(68)S08. Compute the following:

a) The normal stress on plane *AB* in kPa.

b) The shear stress on plane *AB* in kPa

c) The maximum shear stress in kPa

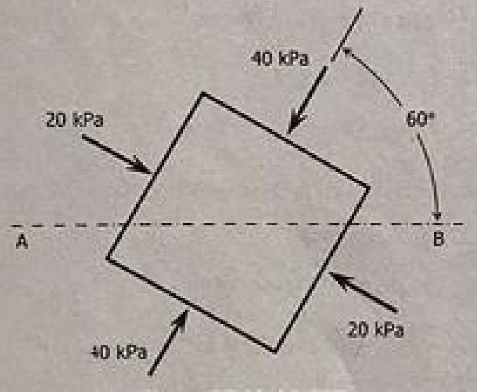


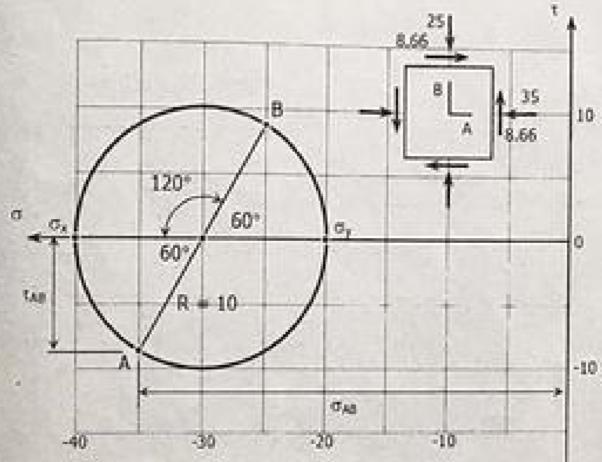
Figure 25.63(68)S08

SOLUTION

σ_{AB} = -35 kPa

τ_{AB} = -(10 sin 60°) = -8.66°

τ_{max} = R = 10 kPa



PROBLEM 10.43

A shallow footing rests on a normally consolidated soil as shown in Figure 56_69(5)S41. It is required to determine the primary settlement of the clay layer. Assume that the pressure under the footing base is spread on a slope of 2V:1H and the average increase in pressure in clay is given as:

$$\Delta p_{ave} = \frac{\Delta p_{top} + 4\Delta p_{mid} + \Delta p_{bot}}{6}$$

Use the following data:

Sand: H_1 = 3 m, G = 2.68, e = 0.52, S = 0.8

Clay: H_2 = 4 m, G = 2.72, e = 1.2, LL = 45%

Footing: B = 3 m, L = 4 m, D_f = 1.5, Q = 3200 kN

a) Determine the unit weight of the clay layer in kN/m³.

b) Determine the effective stress at the midheight of the clay layer in kPa

c) Determine the settlement of the clay layer in mm.

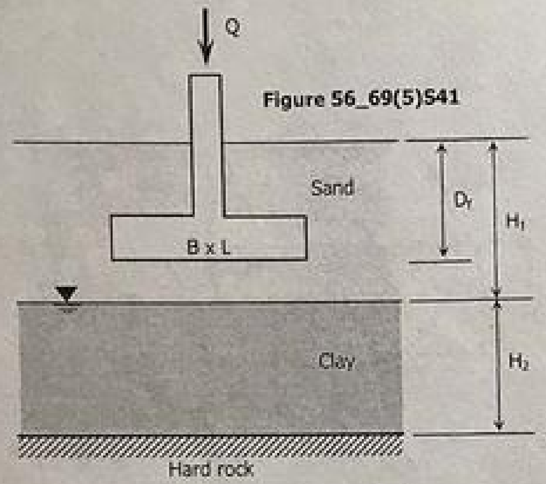
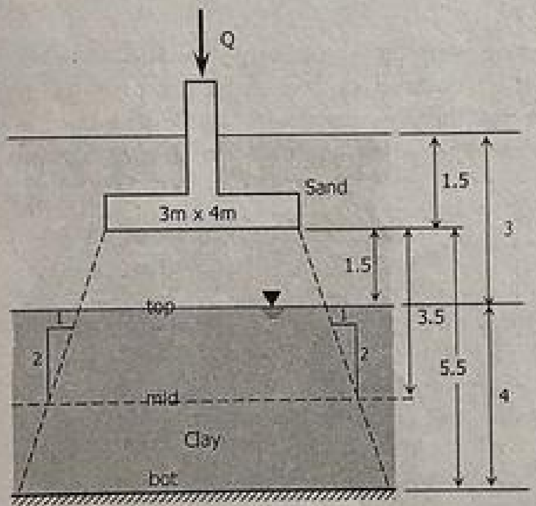


Figure 56_69(5)S41

SOLUTION



Sand: $\gamma_m = \dfrac{2.68 + 0.8(0.52)}{1 + 0.52}(9.81) = 19.98 \text{ kN/m}^3$

Clay: $\gamma_m = \dfrac{2.72 + 1(1.2)}{1 + 1.2}(9.81) = 17.48 \text{ kN/m}^3$

$\gamma_b = \dfrac{2.72 - 1}{1 + 1.2}(9.81) = 7.67 \text{ kN/m}^3$

$p_o = 7.67(2) + 19.98(3) = 75.28 \text{ kPa}$

Increase in pressure:

$$\Delta p_{top} = \frac{3200}{(3+1.5)(4+1.5)} = 129.29 \text{ kPa}$$

$$\Delta p_{mid} = \frac{3200}{(3+3.5)(4+3.5)} = 65.64 \text{ kPa}$$

$$\Delta p_{bot} = \frac{3200}{(3+5.5)(4+5.5)} = 39.63 \text{ kPa}$$

$$\Delta p_{ave} = \frac{129.29 + 4(65.64) + 39.63}{6} = 71.91 \text{ kPa}$$

$p_f = 75.28 + 71.91 = 147.19$ kPa

$C_c = 0.009(LL - 10) = 0.009(45 - 10) = 0.315$

$$\Delta H = H\frac{C_c}{1+e_o}\log\frac{p_f}{p_o} = 4000\frac{0.315}{1+1.2}\log\frac{147.19}{75.28}$$

$\Delta H = \mathbf{166.8 \text{ mm}}$

PROBLEM 10.44

A soil profile is shown in Figure 45.56(54)S85. A surcharge of $\Delta p = 119$ kPa is applied on the ground surface.

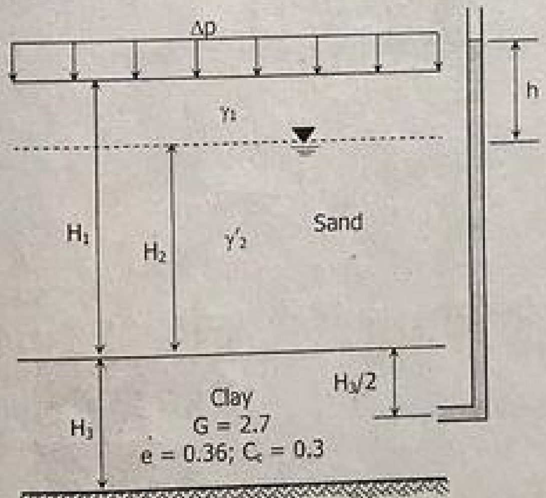


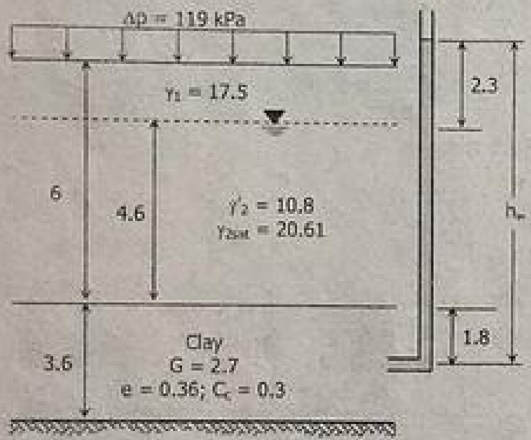
Figure 45.56(54)S85

Given:

$H_1 = 6$ m, $H_2 = 4.6$ m, $H_3 = 3.6$ m, $h = 2.3$ m
$\gamma_1 = 17.5$ kN/m³, $\gamma'_2 = 10.8$ kN/m³

a) Determine the unit weight of the clay layer.
b) Determine the effective stress at the midheight of the clay layer before the application of the surcharge.
c) Determine the settlement of the normally consolidated clay layer.

SOLUTION



Clay: $\gamma_{sat} = \dfrac{2.7 + 0.36}{1 + 0.36}(9.81) = \mathbf{22.073\ kN/m^3}$

$\gamma_{2sat} = \gamma'_2 + \gamma_w = 10.8 + 9.81 = 20.61$ kN/³
$h_w = 1.8 + 4.6 + 2.3 = 8.7$ m

$p_o = 22.073(1.8) + 20.61(4.6) + 17.5(6 - 4.6) - 9.81(8.7)$
$p_o = \mathbf{73.69\ kPa}$

$p_f = 73.69 + 119 = 192.69$ kPa

$$\Delta H = H\frac{C_c}{1+e_o}\log\frac{p_f}{p_o} = 3{,}600\frac{0.3}{1+0.36}\log\frac{192.69}{73.69}$$

$\Delta H = \mathbf{331.5\ mm}$

Index

Fundamentals of Geotechnical Engineering

GILLESANIA Engineering Review & Training Center

TACLOBAN: 3rd Floor, YKS Bldg., Zamora St. (053) 321-7065
CEBU: CEERS, Doña Esperanza Village 2, Tisa (032) 414-5058

Civil Engineering Review

Other books by DIT Gillesania

ISBN 971-8614-33-8