9.197 Determine the head loss in the system shown in Fig. 9-34 and the discharge in the pipe.

▮ $$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L \qquad 0+0+100 = 0 + v_2^2/[(2)(32.2)] + 0 + h_L$$

$$h_L = h_f + h_m \qquad h_f = (f)(L/D)(v^2/2g) = f[500/(\tfrac{6}{12})]\{v_2^2/[(2)(32.2)]\} = 15.53fv_2^2 \qquad h_m = (K)(v^2/2g) \qquad K = 0.5$$

$$h_m = 0.5\{v_2^2/[(2)(32.2)]\} = 0.007764v_2^2 \qquad h_L = 15.53fv_2^2 + 0.007764v_2^2$$

$$100 = 0.01553v_2^2 + 15.53fv_2^2 + 0.007764v_2^2 \qquad v_2 = \sqrt{100/(0.02329 + 15.53f)}$$

Try $v_2 = 10$ ft/s: $N_R = Dv/\nu = (\frac{6}{12})(10)/(1.21 \times 10^{-5}) = 4.13 \times 10^5$, $\epsilon/D = 0.00015/(\frac{6}{12}) = 0.00030$. From Fig. A-5, $f = 0.0165$. $v_2 = \sqrt{100/[(0.02329 + (15.53)(0.0165)]} = 18.91$ ft/s. Try $v_2 = 18.91$ ft/s: $N_R = (\frac{6}{12})(18.91)/(1.21 \times 10^{-5}) = 7.81 \times 10^5$, $f = 0.0157$, $v_2 = \sqrt{100/[0.02329 + (15.53)(0.0157)]} = 19.35$ ft/s. Try $v_2 = 19.35$ ft/s: $N_R = (\frac{6}{12})(19.35)/(1.21 \times 10^{-5}) = 8.00 \times 10^5$, $f = 0.0157$ (O.K.); $h_L = (15.53)(0.0157)(19.35)^2 + (0.007764)(19.35)^2 = 94.2$ ft of water, $Q = Av = [(\pi)(\frac{6}{12})^2/4](19.35) = 3.80$ ft³/s.

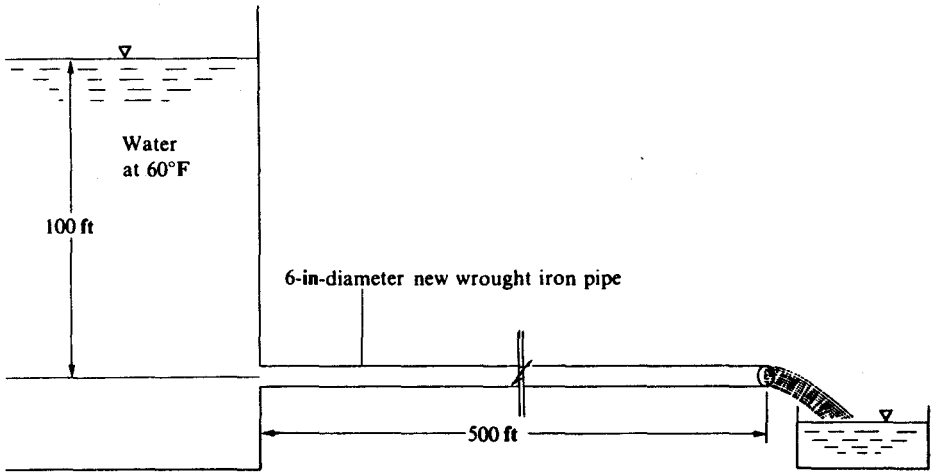


Fig. 9-34

9.198 Figure 9-35 shows two reservoirs containing water at 60 °F. The water in the upper reservoir is to be drained to the other reservoir at a lower level as shown. The total length of commercial steel pipe is 100 ft, and the diameter of the pipe is 4 in. What will be the flow rate of water through the pipe when the water surface elevations are as shown in Fig. 9-35?

▮ $$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L \qquad 0+0+335 = 0+0+300+h_L$$

$$h_L = h_f + h_m \qquad h_f = (f)(L/D)(v^2/2g) = f[100/(\tfrac{4}{12})]\{v_2^2/[(2)(32.2)]\} = 4.658fv_2^2 \qquad h_m = (K)(v^2/2g)$$

Due to entrance, take $K_1 = 0.45$. Due to elbow, $K_2 = 0.64$. Due to gate valve, $K_{open} = 0.11$. Take $K_3/K_{open} = 17$. Hence, $K_3 = (0.11)(17) = 1.87$. Due to exit, $K_4 = 1.0$.

$$h_m = (0.45 + 0.64 + 1.87 + 1.0)\{v^2/[(2)(32.2)]\} = 0.06149v^2 \qquad h_L = 4.658fv^2 + 0.06149v^2$$

$$335 = 300 + 4.658fv^2 + 0.06149v^2 \qquad v = \sqrt{35/(4.658f + 0.06149)}$$

Try $f = 0.019$: $v = \sqrt{35/[(4.658)(0.019) + 0.06149]} = 15.28$ ft/s, $N_R = Dv/\nu = (\frac{4}{12})(15.28)/(1.21 \times 10^{-5}) = 4.21 \times 10^5$. From Fig. A-5, $f = 0.0175$. Try $f = 0.0175$: $v = \sqrt{35/[(4.658)(0.0175) + 0.06149]} = 15.64$ ft/s, $N_R = (\frac{4}{12})(15.64)/(1.21 \times 10^{-5}) = 4.31 \times 10^5$, $f = 0.0175$ (O.K.); $Q = Av = [(\pi)(\frac{4}{12})^2/4](15.64) = 1.36$ ft³/s.

9.199 A 15-in-diameter new cast iron pipe connecting two reservoirs as shown in Fig. 9-36 carries water at 60 °F. The pipe is 120 ft long, and the discharge is 20 ft/s. Determine the difference in elevation between water surfaces in the two reservoirs.

▮ $$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L \qquad 0+0+z_1 = 0+0+z_2+h_L$$

Since $z_1 - z_2 = H$ and $h_L = h_f + h_m$, $H = h_f + h_m$, $h_f = (f)(L/D)(v^2/2g)$.

$$v = Q/A = 20/[(\pi)(\tfrac{15}{12})^2/4] = 16.30 \text{ ft/s}$$

$$N_R = Dv/\nu = (\tfrac{15}{12})(16.30)/(1.21 \times 10^{-5}) = 1.68 \times 10^6 \qquad \epsilon/D = 0.00085/(\tfrac{15}{12}) = 0.00068$$

From Fig. A-5, $f = 0.018$. $h_f = 0.018[120/(\frac{15}{12})]\{16.30^2/[(2)(32.2)]\} = 7.13$ ft, $h_m = (K)(v^2/2g)$. For entrance, take $K_1 = 0.45$. For exit, $K_2 = 1.0$. $h_m = (0.45 + 1.0)\{16.30^2/[(2)(32.2)]\} = 5.98$ ft, $H = 7.13 + 5.98 = 13.11$ ft.

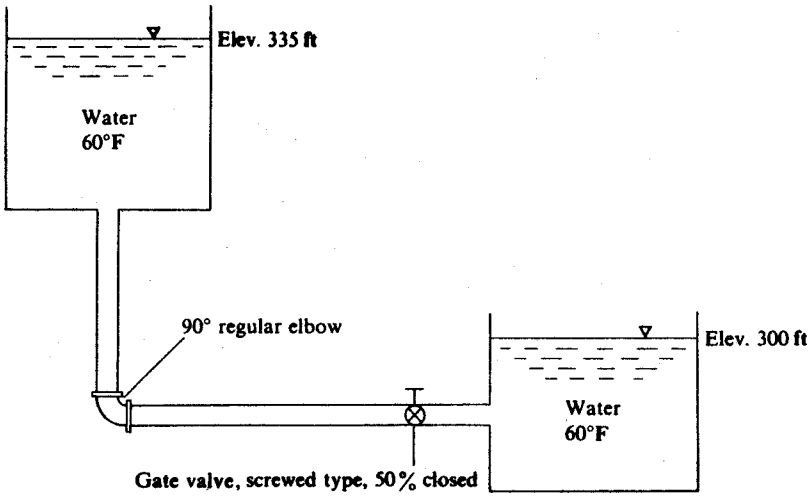


Fig. 9-35

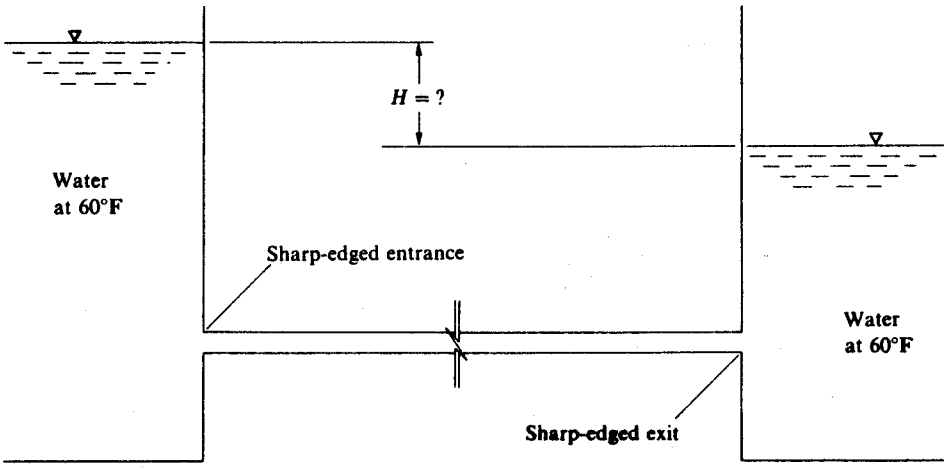


Fig. 9-36

9.200 Repeat Prob. 9.87 by including losses due to a sharp-edged entrance, the exit, and a fully open flanged globe valve.

For entrance, $K_1 = 0.5$. For exit, $K_2 = 1.0$. For globe valve, $K_3 = 8.5$. From Prob. 9.87, $h_L = h_f + h_f = [(f)(L/d) + K_1 + K_2 + K_3](v^2/2g)$, $98 = \{f[7000/0.050] + 0.5 + 1.0 + 8.5\}\{v^2/[(2)(9.807)]\}$, $1922 = (140\,000f + 10.0)(v^2)$, $v = [1922/(140\,000f + 10.0)]^{1/2}$. Try $f = 0.02$: $v = \{1922/[(140\,000)(0.02) + 10.0]\}^{1/2} = 0.8270$ m/s, $N_R = \rho dv/\mu = (998)(0.050)(0.8270)/(1.02 \times 10^{-3}) = 4.05 \times 10^4$. From Fig. A-5, $f = 0.022$. Try $f = 0.022$: $v = \{1922/[(140\,000)(0.022) + 10.0]\}^{1/2} = 0.7887$ m/s, $N_R = (998)(0.050)(0.7887)/(1.02 \times 10^{-3}) = 3.86 \times 10^4$, $f = 0.022$ (O.K.); $Q = Av = [(\pi)(0.050)^2/4](0.7887) = 0.001548$ m^3/s, or 5.57 m^3/h.

9.201 Repeat Prob. 9.95 by including losses due to a sharp entrance and a fully open screwed swing-check valve.

For sharp entrance, $K_1 = 0.5$. For swing-check valve, $K_2 = 5.1$. $h_m = (K_1 + K_2)(v^2/2g)$. From Prob. 9.95, $v = 14.67$ ft/s. $h_m = (0.5 + 5.1)\{14.67^2/[(2)(32.2)]\} = 18.71$ ft. From Prob. 9.95, $0 + 0 + h = 0 + 14.67^2/[(2)(32.2)] + 0 + (236.6 + 18.71)$, $h = 258.7$ ft.

9.202 Two lakes (Fig. 9-37) communicate via two 20-ft-long wrought iron pipes joined abruptly. The entrance and exit are sharp-edged. Including minor losses, compute the flow rate if lake 1 stands 59 ft higher than lake 2, and the average temperature of the system is 20 °C.

$$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L \qquad h_L = h_f + h_m$$

$$h_f = (f)(L/D)(v^2/2g) = (f_a)[20/(\tfrac{1}{12})]\{v_a^2/[(2)(32.2)]\} + f_b[20/(\tfrac{2}{12})]\{v_b^2/[(2)(32.2)]\}$$

$$A_a v_a = A_b v_b \qquad [(\pi)(\tfrac{1}{12})^2/4](v_a) = [(\pi)(\tfrac{2}{12})^2/4](v_b) \qquad v_b = v_a/4$$

$$h_f = 3.727 f_a v_a^2 + (1.863)(f_b)(v_a/4)^2 = 3.727 f_a v_a^2 + 0.01164 f_b v_a^2$$

For sharp entrance, $K_1 = 0.5$. For sharp exit, $K_2 = 1.0$. For sudden expansion, $K_2 = (1 - 0.5)^2 = 0.5625$. $h_m = (0.5 + 0.5625)\{v_a^2/[(2)(32.2)]\} + (1.0)\{v_b^2/[(2)(32.2)]\} = 0.01650v_a^2 + (0.01553)(v_a/4)^2 = 0.01747v_a^2$, $0 + 0 + 59 = 0 + 0 + 0 + [(3.727f_a v_a^2 + 0.1164f_b v_a^2) + 0.01747v_a^2]$. Try $f_a = f_b = 0.020$:

$$59 = (3.727)(0.020)(v_a^2) + (0.1164)(0.020)(v_a^2) + 0.01747v_a^2 \qquad v_a = 25.01 \text{ ft/s} \qquad v_b = 25.01/4 = 6.253 \text{ ft/s}$$

$$N_R = Dv/\nu \qquad (N_R)_a = (\tfrac{1}{12})(25.01)/(1.11 \times 10^{-5}) = 1.88 \times 10^5 \qquad (\epsilon/D)_a = 0.00015/(\tfrac{1}{12}) = 0.00180$$

From Fig. A-5, $f_a = 0.024$. $(N_R)_b = (\frac{2}{12})(6.253)/(1.11 \times 10^{-5}) = 9.39 \times 10^4$, $(\epsilon/D)_b = 0.00015/(\frac{2}{12}) = 0.000900$, $f_b = 0.022$. Try $f_a = 0.024$ and $f_b = 0.022$:

$$59 = (3.727)(0.024)(v_a^2) + (0.1164)(0.022)(v_a^2) + 0.01747v_a^2 \qquad v_a = 23.21 \text{ ft/s} \qquad v_b = 23.21/4 = 5.803 \text{ ft/s}$$

$$(N_R)_a = (\tfrac{1}{12})(23.41)/(1.11 \times 10^{-5}) = 1.76 \times 10^5 \qquad f_a = 0.024$$

$$(N_R)_b = (\tfrac{2}{12})(5.852)/(1.11 \times 10^{-5}) = 8.79 \times 10^4 \qquad f_b = 0.022$$

Therefore, $f_a = 0.024$ and $f_b = 0.022$ is O.K. $Q = Av = [(\pi)(\frac{1}{12})^2/4](23.21) = 0.127 \text{ ft}^3/\text{s}$.

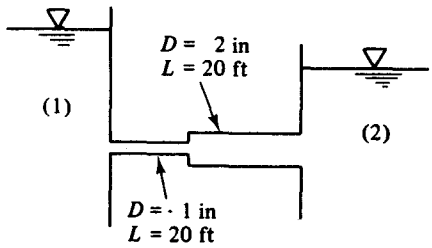


Fig. 9-37

9.203 Two reservoirs containing water at 20 °C are connected by 800 m of 180-mm cast iron pipe, including a sharp entrance, a submerged exit, a gate valve 75 percent open, two 1-m-radius bends, and six regular 90° elbows. If the flow rate is 9 m³/min, find the difference in reservoir elevations.

▮ $$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L \qquad h_L = h_f + h_m \qquad h_f = (f)(L/D)(v^2/2g)$$

$$v = Q/A = (9/60)/[(\pi)(0.180)^2/4] = 5.895 \text{ m/s} \qquad N_R = Dv/\nu = (0.180)(5.895)/(1.02 \times 10^{-6}) = 1.04 \times 10^6$$

$$\epsilon/D = 0.00026/0.180 = 0.00144$$

From Fig. A-5, $f = 0.0217$. $h_f = 0.0217[800/0.180]\{5.895^2/[(2)(9.807)]\} = 170.9$ m. For sharp entrance, $K_1 = 0.5$. For exit, $K_2 = 1.0$. For gate valve 75 percent open, $K_3 = 0.3$. For bends, $K_4 = (2)(0.15) = 0.30$. For elbows, $K_5 = (6)(0.27) = 1.62$. $h_m = (0.5 + 1.0 + 0.3 + 0.30 + 1.62)\{5.895^2/[(2)(9.807)]\} = 6.6$ m, $h_L = 170.9 + 6.6 = 177.5$ m, $0 + 0 + z_1 = 0 + 0 + z_2 + 177.5$, $z_1 - z_2 = 177.5$ m.

9.204 The system in Fig. 9-38 consists of 1000 m of 50-mm cast iron pipe, two 45° and four 90° flanged long-radius elbows, a fully open flanged globe valve, and a sharp exit into a reservoir. What gage pressure is required at point 1 to deliver 5 L/s of water at 20 °C into the reservoir, whose free surface lies 100 m above point 1?

▮ $$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L \qquad h_L = h_f + h_m \qquad h_f = (f)(L/D)(v^2/2g)$$

$$v = Q/A = 0.005/[(\pi)(0.050)^2/4] = 2.546 \text{ m/s} \qquad N_R = Dv/\nu = (0.050)(2.546)/(1.02 \times 10^{-6}) = 1.25 \times 10^5$$

$$\epsilon/D = 0.00026/0.050 = 0.00520$$

From Fig. A-5, $f = 0.0315$. $h_f = 0.0315[1000/0.050]\{2.546^2/[(2)(9.807)]\} = 208.2$ m. For 45° elbows, $K_1 = (2)(0.20) = 0.40$. For 90° elbows, $K_2 = (4)(0.30) = 1.20$. For the open valve, $K_3 = 8.5$. For exit, $K_4 = 1.0$. $h_m = (0.40 + 1.20 + 8.5 + 1.0)\{2.546^2/[(2)(9.807)]\} = 3.7$ m, $h_L = 208.2 + 3.7 = 211.9$ m, $p_1/9.79 + 2.546^2/[(2)(9.807)] + 0 = 0 + 0 + 100 + 211.9$, $p_1 = 3056$ kPa gage.

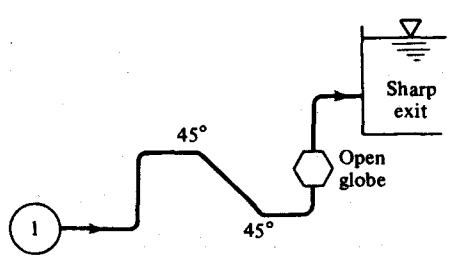


Fig. 9-38

9.205 A 1-in-diameter smooth water pipe slopes upward at 30° (Fig. 9-39); the flanged globe valve is fully open. Calculate the flow rate.

▮ $$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L \qquad v_1^2/2g = v_2^2/2g \qquad h_L = h_f + h_m$$

Therefore, $p_1/\gamma + z_1 = p_2/\gamma + z_2 + h_f + h_m$,

$$(p_1 - p_2)/\gamma = z_2 - z_1 + h_f + h_m \tag{1}$$

From manometer,

$$p_1 - p_2 = (\gamma_{Hg} - \gamma_{H_2O})(\tfrac{7}{12}) + (\gamma_{H_2O})(z_2 - z_1) \tag{2}$$

Combining Eqs. (*1*) and (*2*), $[(\gamma_{Hg} - \gamma_{H_2O})/\gamma_{H_2O}](\frac{7}{12}) = h_f + h_m$, $h_f = (f)(L/D)(v^2/2g) = (f)[(9/\cos 30°)/(\frac{1}{12})]\{v^2/[(2)(32.2)]\} = 1.936fv^2$, $h_m = (K)(v^2/2g)$. For globe valve, $K = 13$. $h_m = 13\{v^2/[(2)(32.2)]\} = 0.2019v^2$, $\{[(13.6)(62.4) - 62.4]/62.4\}(\frac{7}{12}) = 1.936fv^2 + 0.2019v^2$, $v = \sqrt{7.350/(1.936f + 0.2019)}$. Try $f = 0.02$: $v = \sqrt{7.350/[(1.936)(0.02) + 0.2019]} = 5.527$ ft/s, $N_R = Dv/\nu = (\frac{1}{12})(5.527)/(1.05 \times 10^{-5}) = 4.39 \times 10^4$. From Fig. A-5, $f = 0.0217$. Try $f = 0.0217$: $v = \sqrt{7.350/[(1.936)(0.0217) + 0.2019]} = 5.489$ ft/s, $N_R = (\frac{1}{12})(5.489)/(1.05 \times 10^{-5}) = 4.36 \times 10^4$, $f = 0.0217$ (O.K.); $Q = Av = [(\pi)(\frac{1}{12})^2/4](5.489) = 0.0299$ ft³/s.

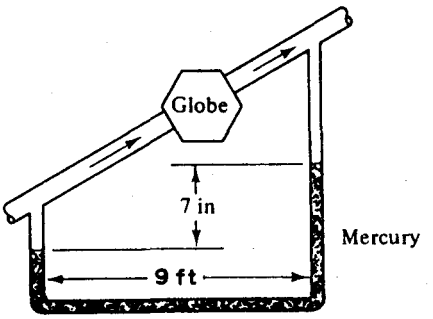


Fig. 9-39

9.206 A pipe system carries water from a reservoir and discharges it as a free jet, as shown in Fig. 9-40. How much flow is to be expected through a 200-mm steel commercial pipe with the fittings shown?

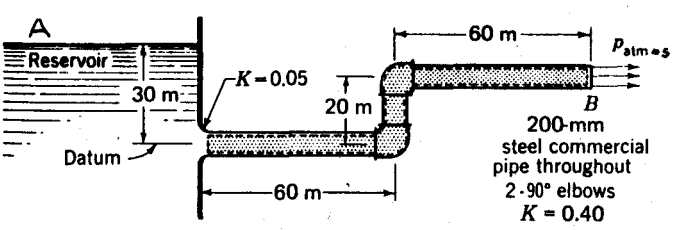
▮ $$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L \qquad h_L = h_f + h_m$$

$$h_f = (f)(L/D)(v^2/2g) = f[(60 + 20 + 60)/(\tfrac{200}{1000})]\{v_2^2/[(2)(9.807)]\} = 35.69fv_2^2$$

$$h_m = (K)(v^2/2g) = [0.05 + (2)(0.40)]\{v_2^2/[(2)(9.807)]\} = 0.04334v_2^2$$

$$0 + 0 + 30 = 0 + v_2^2/[(2)(9.807)] + 20 + (35.69fv_2^2 + 0.04334v_2^2) \qquad v_2 = \sqrt{10/(35.69f + 0.09432)}$$

Try $f = 0.014$: $v_2 = \sqrt{10/[(35.69)(0.014) + 0.09432]} = 4.103$ m/s, $N_R = Dv/\nu = (\frac{200}{1000})(4.103)/(0.0113 \times 10^{-4}) = 7.26 \times 10^5$, $\epsilon/D = 0.000046/(\frac{200}{1000}) = 0.000230$. From Fig. A-5, $f = 0.0152$. Try $f = 0.0152$: $v_2 = \sqrt{10/[(35.69)(0.0152) + 0.09432]} = 3.963$ m/s, $N_R = (\frac{200}{1000})(3.963)/(0.0113 \times 10^{-4}) = 7.01 \times 10^5$, $f = 0.0152$ (O.K.); $Q = Av = [(\pi)(\frac{200}{1000})^2/4](3.963) = 0.125$ m³/s.



$\nu = .0113 \times 10^{-4}$ m²/s
$\rho = 999$ kg/m³

Fig. 9-40

9.207 A pipe system having a given centerline geometry as shown in Fig. 9-41 is to be chosen to transport a maximum of 1 ft³/s of oil from tank *A* to tank *B*. What is a pipe size that will do the job?

▮ $$p_c/\gamma + v_c^2/2g + z_c = p_p/\gamma + v_p^2/2g + z_p + h_L \qquad h_L = h_f + h_m$$

$$h_f = (f)(L/D)(v^2/2g) = f[(100 + 130 + 300)/D]\{v_p^2/[(2)(32.2)]\} = 8.230fv_p^2/D$$

$$h_m = (K)(v^2/2g) = (0.05 + 0.5 + 0.5 + 1)\{v_p^2/[(2)(32.2)]\} = 0.03183v_p^2$$

$$(100)(144)/50 + 0 + 130 = (50)(144)/50 + v_p^2/[(2)(32.2)] + 0 + 8.230fv_p^2/D + 0.03183v_p^2$$

$$0.04736v_p^2 + 8.230fv_p^2/D = 274 \qquad v_p^2 = 274/(0.04736 + 8.230f/D) \qquad v_p = Q/A_p = 1/(\pi D^2/4) = 1.273/D^2$$

$$(1.273/D^2)^2 = 274/(0.04736 + 8.230f/D)$$

Try $f = 0.015$: $(1.273/D^2)^2 = 274/[0.04736 + (8.230)(0.015)/D]$. By trial and error, $D = 0.240$ ft.

$$v = 1.273/0.240^2 = 22.1 \text{ ft/s} \qquad N_R = \rho Dv/\mu = (\gamma/g)(Dv)/\mu = (50/32.2)(0.240)(22.1)/(50 \times 10^{-5}) = 1.65 \times 10^4$$

$$\epsilon/D = 0.00015/0.240 = 0.000625$$

From Fig. A-5, $f = 0.0285$. Try $f = 0.0285$: $(1.273/D^2)^2 = 274/[0.04736 + (8.230)(0.0285)/D]$. By trial and error, $D = 0.272$ ft.

$$v = 1.273/0.272^2 = 17.2 \text{ ft/s} \qquad N_R = (50/32.2)(0.272)(17.2)/(50 \times 10^{-5}) = 1.45 \times 10^4$$

$$f = 0.0285 \qquad \text{(O.K.)}$$

Hence, $D = 0.272$ ft, or 3.26 in.

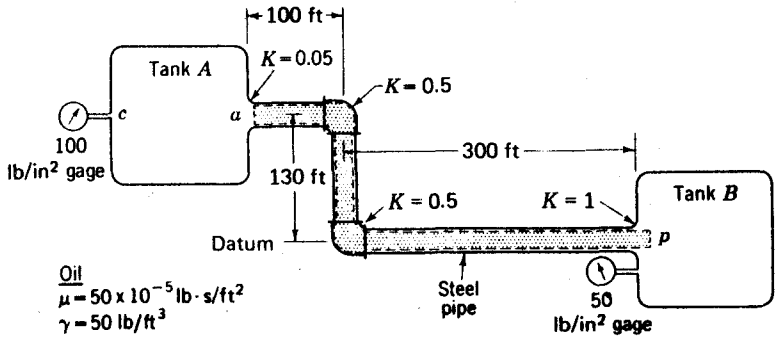


Fig. 9-41

9.208 What gage pressure p_1 is required to cause 5 ft³/s of water to flow through the system shown in Fig. 9-42? Assume that the reservoir is large, that minor losses are negligible, and that $\nu = 2.11 \times 10^{-5}$ ft²/s.

▮ $$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L \qquad v_2 = Q/A_2 = 5/[(\pi)(\tfrac{6}{12})^2/4] = 25.46 \text{ ft/s}$$

$$h_L = h_f = (f)(L/D)(v^2/2g) \qquad N_R = dv/\nu = (\tfrac{6}{12})(25.46)/(2.11 \times 10^{-5}) = 6.03 \times 10^5$$

$$\epsilon/d = 0.00015/(\tfrac{6}{12}) = 0.00030$$

From Fig. A-5, $f = 0.016$.

$$h_L = 0.016[(400 + 160 + 250)/(\tfrac{6}{12})]\{25.46^2/[(2)(32.2)]\} = 260.9 \text{ ft}$$

$$(p_1)(144)/62.4 + 0 + 120 = 0 + 25.46^2/[(2)(32.2)] + 160 + 260.9 \qquad p_1 = 135 \text{ lb/in}^2 \text{ gage}$$

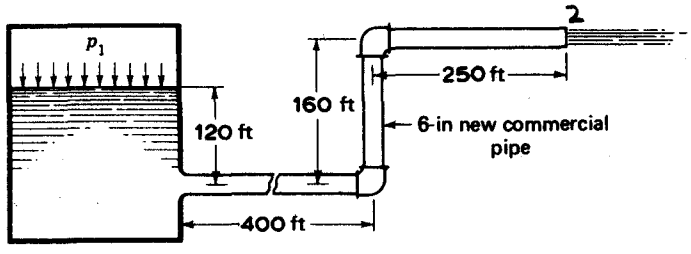


Fig. 9-42

9.209 In Prob. 9.208, let 6 in be the *nominal* diameter of the pipe. For the entrance fitting, $r/d = 0.06$. Calculate the pressure p_1. The elbows are screwed elbows and there is now an open globe valve in the pipe system. Include minor losses.

▮ Use $d = 6.065$ in

$$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L \qquad v_2 = Q/A_2 = 5/[(\pi)(6.065/12)^2/4] = 24.92 \text{ ft/s}$$

$$h_L = h_f + h_m \qquad h_f = (f)(L/d)(v^2/2g) \qquad N_R = dv/\nu = (6.065/12)(24.92)/(2.11 \times 10^{-5}) = 5.97 \times 10^5$$

$$\epsilon/d = 0.00015/(6.065/12) = 0.000297$$

From Fig. A-5, $f = 0.016$. $h_f = 0.016[(400 + 160 + 250)/(6.065/12)]\{24.92^2/[(2)(32.2)]\} = 247.3$ ft, $h_m = (K)(v^2/2g)$. For entrance, $K_1 = 0.15$. For elbows, $K_2 = (2)(0.45) = 0.90$. For globe valve, $K_3 = 5.1$.

$$h_m = (0.15 + 0.90 + 5.1)\{24.92^2/[(2)(32.2)]\} = 59.3 \text{ ft} \qquad h_L = 247.3 + 59.3 = 306.6 \text{ ft}$$

$$(p_1)(144)/62.4 + 0 + 120 = 0 + 24.92^2/[(2)(32.2)] + 160 + 306.6 \qquad p_1 = 154 \text{ lb/in}^2 \text{ gage}$$

9.210 The industrial scrubber B of Fig. 9-43 consumes water ($\nu = 0.113 \times 10^{-5}$ m^2/s) at the rate of 0.1 m^3/s. If the pipe is 150-mm commercial pipe, compute the necessary tank pressure p_1.

▮ $$p_A/\gamma + v_A^2/2g + z_A = p_B/\gamma + v_B^2/2g + z_B + h_L \qquad v_B = Q/A_B = 0.1/[(\pi)(0.150)^2/4] = 5.659 \text{ m/s}$$

$$h_L = h_f + h_m \qquad h_f = (f)(L/d)(v^2/2g) \qquad N_R = dv/\nu = (0.150)(5.659)/(0.113 \times 10^{-5}) = 7.51 \times 10^5$$

$$\epsilon/d = 0.000046/0.150 = 0.000307$$

From Fig. A-5, $f = 0.016$.

$$h_f = 0.016[(300 + 150 + 295)/0.150]\{5.659^2/[(2)(9.807)]\} = 129.7 \text{ m}$$

$$h_m = (K)(v^2/2g) = (0.4 + 0.9 + 0.9 + 1)\{5.659^2/[(2)(9.807)]\} = 5.2 \text{ m} \qquad h_L = 129.7 + 5.2 = 134.9 \text{ m}$$

$$p_1/9.79 + 0 + 25 = 40/9.79 + 5.659^2/[(2)(9.807)] + 150 + 134.9 \qquad p_1 = 2600 \text{ kPa gage}$$

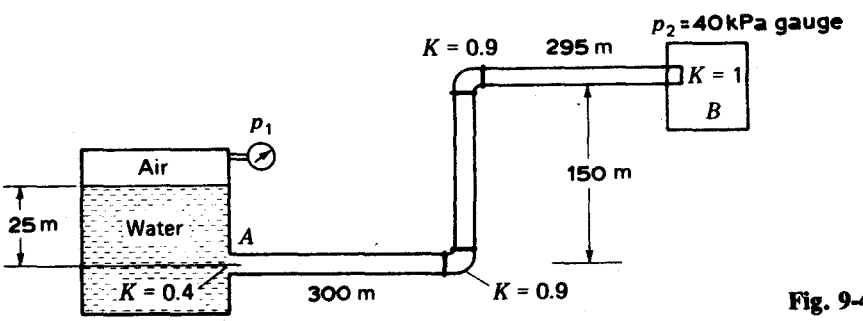


Fig. 9-43

9.211 What pressure p_1 is required in Fig. 9-44 to cause 1 ft^3/s of water to flow, given $p_2 = 5$ psig? Take $\mu = 2.11 \times 10^{-5}$ lb · s/ft^2 for water.

▮ $$p_A/\gamma + v_A^2/2g + z_A = p_B/\gamma + v_B^2/2g + z_B + h_L \qquad p_A/\gamma = (p_1)(144)/62.4 + (10)(0.8) = 2.308p_1 + 8.00$$

$$v_B = Q/A_B = 1/[(\pi)(\tfrac{6}{12})^2/4] = 5.093 \text{ ft/s} \qquad h_L = h_f + h_m \qquad h_f = (f)(L/d)(v^2/2g)$$

$$N_R = \rho dv/\mu = (1.94)(\tfrac{6}{12})(5.093)/(2.11 \times 10^{-5}) = 2.34 \times 10^5 \qquad \epsilon/d = 0.00015/(\tfrac{6}{12}) = 0.00030$$

From Fig. A-5, $f = 0.0175$.

$$h_f = 0.0175[2800/(\tfrac{6}{12})]\{5.093^2/[(2)(32.2)]\} = 39.47 \text{ ft} \qquad h_m = (K)(v^2/2g) = (0.5 + 1)\{5.093^2/[(2)(32.2)]\} = 0.60 \text{ ft}$$

$$h_L = 39.47 + 0.60 = 40.07 \text{ ft}$$

$$(2.308p_1 + 8.00) + 0 + 25 = (5)(144)/62.4 + 5.093^2/[(2)(32.2)] + 0 + 40.07 \qquad p_1 = 8.2 \text{ psig}$$

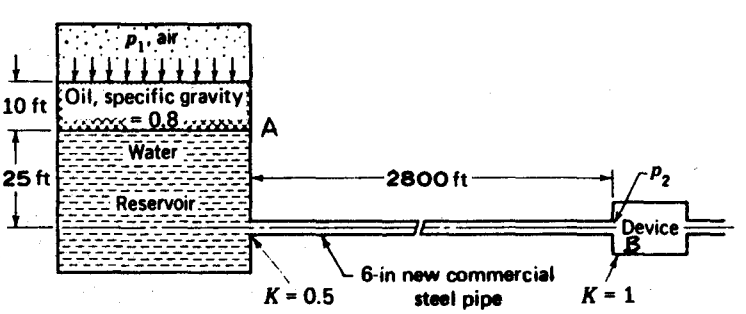


Fig. 9-44

9.212 Find the flow through the system of Fig. 9-45. Use $\epsilon = 0.00015$ ft.

▮ $p_A/\gamma + v_A^2/2g + z_A = p_B/\gamma + v_B^2/2g + z_B + h_L \qquad 0+0+(100+100) = 0+0+100+h_L \qquad h_L = 100\text{ ft} = h_f + h_m$

$$h_f = (f)(L/D)(v^2/2g) = f[(500+100+102)/(\tfrac{6}{12})]\{v^2/[(2)(32.2)]\} = 21.80fv^2$$

$$h_m = (K)(v^2/2g) = (0.4+0.8+0.8+1)\{v^2/[(2)(32.2)]\} = 0.04658v^2 \qquad h_L = 21.80fv^2 + 0.04658v^2$$

$$100 = 21.80fv^2 + 0.04658v^2 \qquad v = \sqrt{100/(21.80f + 0.04658)}$$

Try $f = 0.015$: $v = \sqrt{100/[(21.80)(0.015)+0.04658]} = 17.49$ ft/s, $N_R = Dv/\nu = (\frac{6}{12})(17.49)/(1.21\times10^{-5}) = 7.23\times10^5$, $\epsilon/D = 0.00015/(\frac{6}{12}) = 0.00030$. From Fig. A-5, $f = 0.016$. Try $f = 0.016$: $v = \sqrt{100/[(21.74)(0.016)+0.04658]} = 15.92$ ft/s, $N_R = (\frac{6}{12})(15.92)/(1.21\times10^{-5}) = 6.58\times10^5$, $f = 0.016$ (O.K.); $Q = Av = [(\pi)(\frac{6}{12})^2/4](15.92) = 3.13\text{ ft}^3/\text{s}$.

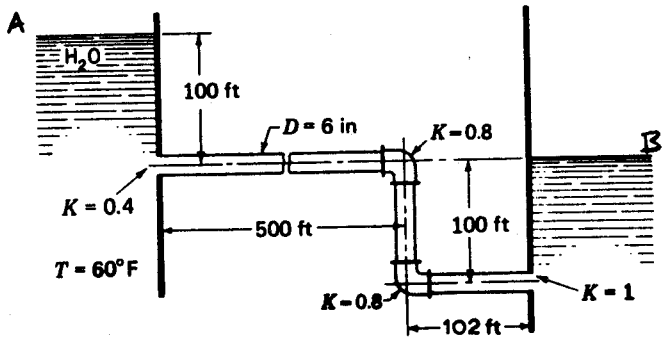


Fig. 9-45

9.213 A flow of 170 L/s is to go from tank A to tank B in Fig. 9-46. If $\nu = 0.113\times10^{-5}$ m²/s, what should the diameter be for the horizontal section of pipe?

▮ $p_A/\gamma + v_A^2/2g + z_A = p_B/\gamma + v_B^2/2g + z_B + h_L \qquad 0+0+(35+35) = 0+0+16+h_L \qquad h_L = 54\text{ m} = h_f + h_m$

$$v_1 = Q/A_1 = (\tfrac{170}{1000})/(\pi D^2/4) = 0.2165/D^2 \qquad v_2 = Q/A_2 = (\tfrac{170}{1000})/[(\pi)(\tfrac{150}{1000})^2/4] = 9.620\text{ m/s}$$

$$h_f = (f)(L/D)(v^2/2g) = (f_1)(65/D)\{(0.2165/D^2)^2/[(2)(9.807)]\}$$
$$+ f_2[35/(\tfrac{150}{1000})]\{9.620^2/[(2)(9.807)]\} = 0.1553f_1/D^5 + 1101f_2$$

$$h_m = (K)(v^2/2g) = (0.4+0.9)\{(0.2165/D^2)^2/[(2)(9.807)]\}$$
$$+ 1.0\{9.620^2/[(2)(9.807)]\} = 0.003107/D^4 + 4.718$$

$$h_L = 0.1553f_1/D^5 + 1101f_2 + 0.003107/D^4 + 4.718$$

$$54 = 0.1553f_1/D^5 + 1101f_2 + 0.003107/D^4 + 4.718$$

Try $f_1 = f_2 = 0.015$: $54 = (0.1553)(0.015)/D^5 + (1101)(0.015) + 0.003107/D^4 + 4.718$.
By trial and error, $D = 0.154$ m.

$$v_1 = 0.2165/0.154^2 = 9.129\text{ m/s} \qquad N_R = Dv/\nu$$

$$(N_R)_1 = (0.154)(9.129)/(0.113\times10^{-5}) = 1.24\times10^6 \qquad (\epsilon/D)_1 = 0.000046/0.154 = 0.00030$$

From Fig. A-5, $f_1 = 0.0155$. $(N_R)_2 = (\frac{150}{1000})(9.620)/(0.113\times10^{-5}) = 1.28\times10^6$, $(\epsilon/D)_2 = 0.000046/(\frac{150}{1000}) = 0.00031$, $f_2 = 0.0155$. Try $f_1 = f_2 = 0.0155$: $54 = (0.1553)(0.0155)/D^5 + (1101)(0.0155) + 0.003107/D^4 + 4.718$. By trial and error, $D = 0.155$ m.

$$v_1 = 0.2165/0.155^2 = 9.011\text{ m/s} \qquad (N_R)_1 = (0.155)(9.011)/(0.113\times10^{-5}) = 1.24\times10^6$$

$$(\epsilon/D)_1 = 0.000046/0.155 = 0.00030 \qquad f_1 = 0.0155$$

$$(N_R)_2 = (\tfrac{150}{1000})(9.620)/(0.113\times10^{-5}) = 1.28\times10^6 \qquad f_2 = 0.0155 \qquad \text{(O.K.)}$$

Therefore, $D = 0.155$ m, or 155 mm.

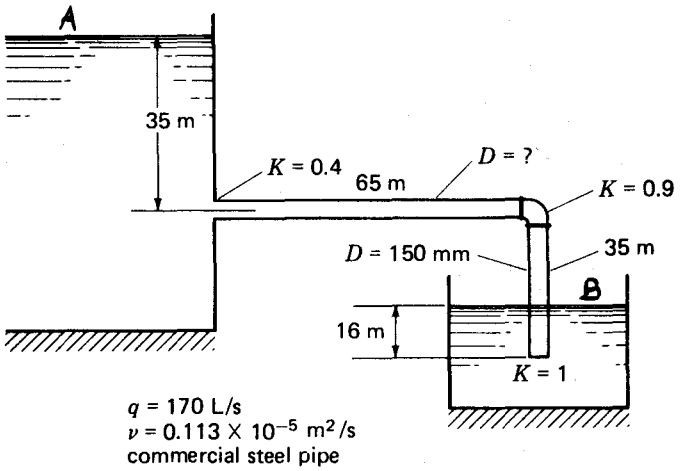


Fig. 9-46

9.214 In Prob. 9.208, if $p_1 = 200\ \text{lb/in}^2$ gage, what should the inside pipe diameter be to enable transport of $12\ \text{ft}^3/\text{s}$ of water? Again neglect minor losses.

$$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L \qquad v_2 = Q/A_2 = 12/(\pi d^2/4) = 15.28/d^2$$

$$h_L = h_f = (f)(L/d)(v^2/2g)$$

$$h_f = (f)[(400 + 160 + 250)/d]\{(15.28/d^2)^2/[(2)(32.2)]\} = 2937f/d^5$$

$$(200)(144)/62.4 + 0 + 120 = 0 + (15.28/d^2)^2/[(2)(32.2)] + 160 + 2937f/d^5 \qquad 421.5 = 3.625/d^4 + 2937f/d^5$$

Try $f = 0.015$: $421.5 = 3.625/d^4 + (2937)(0.015)/d^5$. By trial and error, $d = 0.643$ ft. $v = 15.28/0.643^2 = 36.96$ ft/s, $N_R = dv/\nu = (0.643)(36.96)/(2.11 \times 10^{-5}) = 1.12 \times 10^6$, $\epsilon/d = 0.00015/0.652 = 0.00023$. From Fig. A-5, $f = 0.015$ (O.K.). Therefore, $d = 0.643$ ft, or 7.72 in.

9.215 Under the pressures established in Prob. 9.210, what size pipe is needed to double the flow?

$$p_A/\gamma + v_A^2/2g + z_A = p_B/\gamma + v_B^2/2g + z_B + h_L \qquad v_B = Q/A_B = 0.2/(\pi d^2/4) = 0.2546/d^2$$

$$h_f = (f)(L/d)(v^2/2g) = f[(300 + 150 + 295)/d]\{(0.2546/d^2)^2/[(2)(9.807)]\} = 2.462f/d^5$$

$$h_m = (K)(v^2/2g) = (0.4 + 0.9 + 0.9 + 1)\{(0.2546/d^2)^2/[(2)(9.807)]\} = 0.01058/d^4$$

$$h_L = h_f + h_m = 2.462f/d^5 + 0.01058/d^4$$

$$2600/9.79 + 0 + 25 = 40/9.79 + (0.2546/d^2)^2/[(2)(9.807)] + 150 + 2.462f/d^5 + 0.01058/d^4$$

$$136.5 = 2.462f/d^5 + 0.01388/d^4$$

Try $f = 0.015$: $136.5 = (2.462)(0.015)/d^5 + 0.01388/d^4$. By trial and error, $d = 0.196$ m. $v = 0.2546/0.196^2 = 6.627$ m/s, $N_R = dv/\nu = (0.196)(6.627)/(0.113 \times 10^{-5}) \doteq 1.15 \times 10^6$, $\epsilon/d = 0.000046/0.196 = 0.000235$. From Fig. A-5, $f = 0.015$ (O.K.). Therefore, $d = 0.196$ m, or 196 mm.

9.216 A 12-in-diameter pipe with a friction factor of 0.02 conducts fluid between two tanks at 10 fps. The ends of the pipe are flush with the tank walls. Find the ratio of the minor losses to the pipe friction loss if the length of the pipe is **(*a*)** 10 ft, **(*b*)** 250 ft, and **(*c*)** 1500 ft.

$$h_f = (f)(L/d)(v^2/2g) \qquad h_m = (K)(v^2/2g)$$

(*a*)
$$h_f = 0.02[(10/\tfrac{12}{12})](v^2/2g) = 0.20v^2/2g \qquad h_m = (0.5 + 1.0)(v^2/2g) = 1.5v^2/2g$$

$$h_m/h_f = (1.5v^2/2g)/(0.20v^2/2g) = 7.5 \qquad (7.5:1)$$

(*b*)
$$h_f = 0.02[250/(\tfrac{12}{12})](v^2/2g) = 5.0v^2/2g \qquad h_m = (0.5 + 1.0)(v^2/2g) = 1.5v^2/2g$$

$$h_m/h_f = (1.5v^2/2g)/(5.0v^2/2g) = 0.3 \qquad (0.3:1)$$

(*c*)
$$h_f = 0.02[1500/(\tfrac{12}{12})](v^2/2g) = 30v^2/2g \qquad h_m = (0.5 + 1.0)(v^2/2g) = 1.5v^2/2g$$

$$h_m/h_f = (1.5v^2/2g)/(30v^2/2g) = 0.05 \qquad (0.05:1)$$

9.217 A smooth pipe 30 cm in diameter and 100 m long has a flush entrance and a submerged discharge. The velocity is 3 m/s. If the fluid is water at 15 °C, what is the total loss of head?

▮ $h_L = h_f + h_m \qquad h_f = (f)(L/d)(v^2/2g) \qquad N_R = dv/\nu = (0.30)(3)/(1.16 \times 10^{-6}) = 7.76 \times 10^5$

From Fig. A-5, $f = 0.0122$. $h_f = 0.0122[100/0.30]\{3^2/[(2)(9.807)]\} = 1.87$ m, $h_m = (K)(v^2/2g) = (0.5 + 1.0)\{3^2/[(2)(9.807)]\} = 0.69$ m, $h_L = 1.87 + 0.69 = 2.56$ m.

9.218 The water of Prob. 9.217 is replaced by oil with kinematic viscosity 9.3×10^{-5} m²/s and specific gravity 0.925. Determine the pressure loss.

▮ $h_L = h_f + h_m \qquad h_f = (f)(L/d)(v^2/2g) \qquad N_R = dv/\nu = (0.30)(3)/(9.3 \times 10^{-5}) = 9.68 \times 10^3$

From Fig. A-5, $f = 0.031$.

$$h_f = 0.031[100/0.30]\{3^2/[(2)(9.807)]\} = 4.74 \text{ m} \qquad h_m = (K)(v^2/2g) = (0.5 + 1.0)\{3^2/[(2)(9.807)]\} = 0.69 \text{ m}$$

$$h_L = 4.74 + 0.69 = 5.43 \text{ m of oil} \qquad \Delta p = \gamma h_L = [(0.925)(9.79)](5.43) = 49.2 \text{ kPa}$$

9.219 A smooth pipe consists of 100 ft of 8-in pipe followed by 200 ft of 24-in pipe, with an abrupt change of cross section at the junction. It has a flush entrance and a submerged exit. If it carries water at 60 °F in the smaller leg with a velocity of 18 fps, what is the total head loss?

▮ $h_L = h_f + h_m \qquad h_f = (f)(L/d)(v^2/2g) \qquad N_R = dv/\nu \qquad (N_R)_{d=8} = (\frac{8}{12})(18)/(1.21 \times 10^5) = 9.92 \times 10^5$

From Fig. A-5, $(f)_{d=8} = 0.0117$.

$$A_1 v_1 = A_2 v_2 \qquad A_1 = [(\pi)(\tfrac{8}{12})^2/4] = 0.3491 \text{ ft}^2 \qquad A_2 = [(\pi)(\tfrac{24}{12})^2/4] = 3.142 \text{ ft}^2$$

$$(0.3491)(18) = (3.142)(v_2) \qquad v_2 = 2.000 \text{ ft/s} \qquad (N_R)_{d=24} = (\tfrac{24}{12})(2.000)/(1.21 \times 10^{-5}) = 3.31 \times 10^5$$

$$(f)_{d=24} = 0.0142 \qquad h_f = 0.0117[100/(\tfrac{8}{12})]\{18^2/[(2)(32.2)]\} + 0.0142[200/(\tfrac{24}{12})]\{2.000^2/[(2)(32.2)]\} = 8.92 \text{ ft}$$

$$h_m = (K)(v^2/2g)$$

For entrance, $K_1 = 0.5$. For abrupt change, $K_2 = (1 - A_1/A_2)^2 = (1 - 0.3491/3.142)^2 = 0.7901$. For exit, $K_3 = 1.0$. $h_m = (0.5 + 0.7901)\{18^2/[(2)(32.2)]\} + 1.0\{2.000^2/[(2)(32.2)]\} = 6.55$ ft, $h_L = 8.92 + 6.55 = 15.47$ ft.

9.220 A 6-in-diameter pipe ($f = 0.032$) of length 110 ft connects two reservoirs whose water-surface elevations differ by 10 ft. The pipe entrance is flush, and the discharge is submerged. **(a)** Compute the flow rate. **(b)** If the last 10 ft of pipe were replaced with a conical diffuser with a cone angle of 10°, compute the flow rate.

▮

$$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L$$

(a)

$$h_f = (f)(L/d)(v^2/2g) = 0.032[110/(\tfrac{6}{12})]\{v^2/[(2)(32.2)]\} = 0.1093v^2$$

$$h_m = (K)(v^2/2g) = (0.5 + 1.0)\{v^2/[(2)(32.2)]\} = 0.02329v^2$$

$$h_L = h_f + h_m = 0.1093v^2 + 0.02329v^2 = 0.1326v^2$$

$$0 + 0 + 10 = 0 + 0 + 0 + 0.1326v^2 \qquad v = 8.684 \text{ ft/s}$$

$$Q = Av = [(\pi)(\tfrac{6}{12})^2/4](8.684) = 1.71 \text{ ft}^3/\text{s}$$

(b) $h_f = 0.032[(110 - 10)/(\frac{6}{12})]\{v^2/[(2)(32.2)]\} = 0.09938v^2 \qquad h_m = (0.5 + 0.40)\{v^2/[(2)(32.2)]\} = 0.01398v^2$

$$h_L = 0.09938v^2 + 0.01398v^2 = 0.1134v^2 \qquad 0 + 0 + 10 = 0 + 0 + 0 + 0.1134v^2$$

$$v = 9.391 \text{ ft/s} \qquad Q = Av = [(\pi)(\tfrac{6}{12})^2/4](9.391) = 1.84 \text{ ft}^3/\text{s}$$

9.221 Given two pipes in series with a diameter ratio of 1:2 and flow velocity of 15 fps in the smaller pipe, find the loss of head due to abrupt **(a)** contraction and **(b)** enlargement.

▮

$$h_m = (K)(v^2/2g)$$

(a) $d/D = 0.5 \qquad K = 0.33 \qquad h_m = 0.33\{15^2/[(2)(32.2)]\} = 1.15$ ft

(b) $K = 0.55 \qquad h_m = 0.55\{15^2/[(2)(32.2)]\} = 1.92$ ft

9.222 In a 50-ft length of 4-in-diameter wrought iron pipe there are one open globe valve ($K = 10$), one 45° regular elbow ($K = 0.75$), and one pipe bend with a radius of curvature of 40 in ($K = 0.10$). The bend is 90°, and its

length is not included in the 50 ft. No entrance or discharge losses are involved. If the fluid is water at 72 °F and the velocity is 6 fps, what is the total head loss?

▮ $$h_L = h_f + h_m \qquad h_f = (f)(L/d)(v^2/2g)$$

$$N_R = dv/\nu = (\tfrac{4}{12})(6)/(1.02 \times 10^{-5}) = 1.96 \times 10^5 \qquad \epsilon/d = 0.00015/(\tfrac{4}{12}) = 0.00045$$

From Fig. A-5, $f = 0.0185$.

$$L = 50 + \tfrac{1}{4}[(2)(\pi)(\tfrac{40}{12})] = 55.2\text{ ft} \qquad h_f = 0.0185[55.2/(\tfrac{4}{12})]\{6^2/[(2)(32.2)]\} = 1.71\text{ ft}$$

$$h_m = (K)(v^2/2g) = (10 + 0.75 + 0.10)\{6^2/[(2)(32.2)]\} = 6.07\text{ ft} \qquad h_L = 1.71 + 6.07 = 7.78\text{ ft}$$

9.223 Compute the friction head per 1000 ft of pipe for laminar flow at Reynolds number 50 000 (the empirical upper limit). Consider two situations: one where the fluid is water at 60 °F, the other where the fluid is SAE10 oil at 150 °F ($\nu = 0.00016\text{ ft}^2/\text{s}$). Pipe diameter is 2.0 in.

▮ $$h_f = (f)(L/d)(v^2/2g) \qquad f = 64/N_R = 64/50\,000 = 0.00128 \qquad N_R = dv/\nu$$

For water:

$$50\,000 = (2.0/12)(v)/(1.21 \times 10^{-5}) \qquad v = 3.630\text{ ft/s}$$

$$h_f = 0.00128[1000/(2.0/12)]\{3.630^2/[(2)(32.2)]\} = 1.57\text{ ft}$$

For oil:

$$50\,000 = (2.0/12)(v)/0.00016 \qquad v = 48.00\text{ ft/s}$$

$$h_f = 0.00128[1000/(2.0/12)]\{48.00^2/[(2)(32.2)]\} = 275\text{ ft}$$

9.224 Repeat Prob. 9.223 if the flow is turbulent in a smooth pipe.

▮ $h_f = (f)(L/d)(v^2/2g)$. From Fig. A-5, $f = 0.0207$.

For water: From Prob. 9.223, $v = 3.630$ ft/s. $h_f = 0.0207[1000/(2.0/12)]\{3.630^2/[(2)(32.2)]\} = 25.4$ ft.

For oil: From Prob. 9.223, $v = 48.00$ ft/s. $h_f = 0.0207[1000/(2.0/12)]\{48.00^2/[(2)(32.2)]\} = 4443$ ft.

9.225 Repeat Prob. 9.223 if the flow is turbulent in a rough pipe with $\epsilon/d = 0.05$.

▮ $h_f = (f)(L/d)(v^2/2g)$. From Fig. A-5, $f = 0.072$.

For water: From Prob. 9.223, $v = 3.630$ ft/s. $h_f = 0.072[1000/(2.0/12)]\{3.630^2/[(2)(32.2)]\} = 88.4$ ft.

For oil: From Prob. 9.223, $v = 48.00$ ft/s. $h_f = 0.072[1000/(2.0/12)]\{48.00^2/[(2)(32.2)]\} = 15\,455$ ft.

9.226 Water at 60 °F flows through 10 000 ft of 12-in-diameter pipe between two reservoirs whose water-surface elevation difference is 200 ft. Find the flow rate if $\epsilon = 0.0018$ in.

▮ $$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L \qquad 0 + 0 + 200 = 0 + 0 + 0 + h_L$$

$$h_L = 200\text{ ft} = h_f = (f)(L/d)(v^2/2g) = f[10\,000/(\tfrac{12}{12})]\{v^2/[(2)(32.2)]\} = 155.3fv^2$$

$$200 = 155.3fv^2 \qquad v = 1.135/\sqrt{f}$$

Try $f = 0.03$: $v = 1.135/\sqrt{0.03} = 6.553$ ft/s, $N_R = dv/\nu = (\tfrac{12}{12})(6.553)/(1.21 \times 10^{-5}) = 5.42 \times 10^5$, $\epsilon/d = 0.0018/12 = 0.00015$. From Fig. A-5, $f = 0.015$. Try $f = 0.015$: $v = 1.135/\sqrt{0.015} = 9.267$ ft/s, $N_R = (\tfrac{12}{12})(9.267)/(1.21 \times 10^{-5}) = 7.66 \times 10^5$, $f = 0.0145$. Try $f = 0.0145$: $v = 1.135/\sqrt{0.0145} = 9.426$ ft/s, $N_R = (\tfrac{12}{12})(9.426)/(1.21 \times 10^{-5}) = 7.79 \times 10^5$, $f = 0.0145$ (O.K.); $Q = Av = [(\pi)(\tfrac{12}{12})^2/4](9.426) = 7.40\text{ ft}^3/\text{s}$.

9.227 Repeat Prob. 9.226 if ϵ is twenty times larger than in Prob. 9.226.

▮ $v = 1.135/\sqrt{f}$ (from Prob. 9.226). Try $f = 0.03$: $v = 1.135/\sqrt{0.03} = 6.553$ ft/s, $N_R = dv/\nu = (\tfrac{12}{12})(6.553)/(1.21 \times 10^{-5}) = 5.42 \times 10^5$, $\epsilon/d = (20)(0.0018)/12 = 0.00300$. From Fig. A-5, $f = 0.027$. Try $f = 0.027$: $v = 1.135/\sqrt{0.027} = 6.907$ ft/s, $N_R = (\tfrac{12}{12})(6.907)/(1.21 \times 10^{-5}) = 5.71 \times 10^5$, $f = 0.027$ (O.K.); $Q = Av = [(\pi)(\tfrac{12}{12})^2/4](6.907) = 5.42\text{ ft}^3/\text{s}$.

9.228 How large a wrought iron pipe is required to convey oil (s.g. = 0.9, $\mu = 0.0008$ lb · s/ft^2) from one tank to another at a rate of 1.0 cfs if the pipe is 3000 ft long and the difference in elevation of the free liquid surfaces is 40 ft?

▮
$$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L \qquad 0+0+40 = 0+0+0+h_L$$
$$h_L = 40\text{ ft} = h_f = (f)(L/d)(v^2/2g) \qquad v = Q/A = 1.0/(\pi d^2/4) = 1.273/d^2$$
$$h_L = (f)(3000/d)\{(1.273/d^2)^2/[(2)(32.2)]\} = 75.49f/d^5 \qquad 40 = 75.49f/d^5 \qquad d = 1.135f^{1/5}$$

Try $f = 0.03$: $d = (1.135)(0.03)^{1/5} = 0.5629$ ft, $N_R = \rho dv/\mu = [(0.9)(1.94)](0.5629)(1.273/0.5629^2)/0.0008 = 4936$, $\epsilon/d = 0.00015/0.5629 = 0.00027$. From Fig. A-5, $f = 0.0375$. Try $f = 0.0375$: $d = (1.135)(0.0375)^{1/5} = 0.5886$ ft, $N_R = [(0.9)(1.94)](0.5886)(1.273/0.5886^2)/0.0008 = 4720$, $f = 0.038$. Try $f = 0.038$: $d = (1.135)(0.038)^{1/5} = 0.5901$ ft, $N_R = [(0.9)(1.94)](0.5901)(1.273/0.5901^2)/0.0008 = 4708$, $f = 0.038$ (O.K.). Therefore, $d = 0.5901$ ft, or 7.08 in.

9.229 If the diameter of a pipe is doubled, what effect does this have on the flow rate for a given head loss if the flow is laminar?

▮
$$h_L = (32)(\nu)(L/gd^2)(v) = (\text{constant})(v/d^2) \qquad v = kd^2 \qquad Q = Av = k'd^4$$

Thus, doubling the diameter will increase the flow rate by a factor of 2^4, or 16.

9.230 If the diameter of a pipe is doubled, what effect does this have on the flow rate for a given head loss if the flow is turbulent?

▮ If f = constant (complete turbulence): $h_L = k_1 v^2/d$, $v = k_2 d^{1/2}$, $Q = Av = k_3 d^{5/2}$. Thus, doubling the diameter will increase the flow rate by a factor of $2^{5/2}$, or 5.66.

For smooth pipe with $N_R < 100\,000$: $f = 0.316/N_R^{0.25}$, $h_L = (f)(L/d)(v^2/2g) = (k_1)(v^{7/4}/d^{5/4})$, $v = k_2 d^{5/7}$, $Q = Av = k_3 d^{19/7}$. Thus, doubling the diameter will increase the flow rate by a factor of $2^{19/7}$, or 6.56.

9.231 A 150-mm-diameter pipeline 100 m long discharges a 50-mm-diameter jet of water into the atmosphere at a point 60 m below the water surface at intake. The entrance to the pipe is a projecting one, with $K = 0.9$, and the nozzle loss coefficient is 0.05. Find the flow rate and the pressure head at the base of the nozzle, assuming $f = 0.03$.

▮
$$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L \qquad (1)$$
$$h_f = (f)(L/d)(v^2/2g) = 0.03[100/0.150]\{v^2/[(2)(9.807)]\} = 1.020v^2$$
$$h_m = (K)(v^2/2g) = (0.9)\{v^2/[(2)(9.907)]\} + (0.05)\{v_{\text{jet}}^2/[(2)(9.907)]\} = 0.04589v^2 + 0.002549v_{\text{jet}}^2$$
$$h_L = h_f + h_m = 1.020v^2 + 0.04589v^2 + 0.002549v_{\text{jet}}^2 = 1.066v^2 + 0.002549v_{\text{jet}}^2$$
$$0+0+60 = 0 + v_{\text{jet}}^2/[(2)(9.807)] + 0 + 1.066v^2 + 0.002549v_{\text{jet}}^2$$

Since velocity varies as the square of the diameter, $v_{\text{jet}} = (\frac{15}{5})^2(v) = 9v$, $60 = (9v)^2/[(2)(9.807)] + 1.066v^2 + (0.002549)(9v)^2$, $v = 3.333$ m/s; $Q = Av = [(\pi)(0.150)^2/4](3.333) = 0.0589$ m^3/s. Applying Eq. (*1*) to the nozzle, $p_1/9.79 + 3.333^2/[(2)(9.807)] + 0 = 0 + [(9)(3.333)]^2/[(2)(9.807)] + 0 + 0.05\{[(9)(3.333)]^2/[(2)(9.807)]\}$, $p_1 = 46.9$ kPa.

9.232 A 2.0-m-diameter, 1600-m-long concrete pipe ($\epsilon = 1.5$ mm) carries water at 12 °C between two reservoirs at 8.0 m^3/s. Find the difference in water-surface elevation between the two reservoirs, considering minor losses at entrance and exit.

▮
$$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L \qquad h_L = h_f + h_m \qquad h_f = (f)(L/d)(v^2/2g)$$
$$v = Q/A = 8.0/[(\pi)(2.0)^2/4] = 2.546\text{ m/s} \qquad N_R = dv/\nu = (2.0)(2.546)/(1.24\times10^{-6}) = 4.11\times10^6$$
$$\epsilon/d = 0.0015/2.0 = 0.00075$$

From Fig. A-5, $f = 0.0185$.

$$h_f = (0.0185)(1600/2.0)\{2.546^2/[(2)(9.807)]\} = 4.89\text{ m}$$
$$h_m = (K)(v^2/2g) = (0.5+1.0)\{2.546^2/[(2)(9.807)]\} = 0.50\text{ m}$$
$$h_L = 4.89 + 0.50 = 5.39\text{ m} \qquad 0+0+z_1 = 0+0+z_2+5.39\text{ m} \qquad z_1 - z_2 = 5.39\text{ m}$$

9.233 A pipe of mean diameter 5 ft and length 6000 ft delivers water to a facility 1300 ft below the water surface at intake. Assume $f = 0.025$. When the pipe delivers 300 cfs, what is the horsepower delivered?

▮
$$v_2 = Q/A_2 = 300/[(\pi)(\tfrac{60}{12})^2/4] = 15.28 \text{ ft/s}$$
$$h_f = (f)(L/d)(v^2/2g) = 0.025[6000/(\tfrac{60}{12})]\{15.28^2/[(2)(32.2)]\} = 108.76 \text{ ft}$$
$$h_m = (K)(v_2/2g) = 0.50\{15.28^2/[(2)(32.2)]\} = 1.81 \text{ ft}$$
$$h_L = h_f + h_m = 108.76 + 1.81 = 110.57 \text{ ft}$$
$$P = Q\gamma(\Delta z - h_L) = (300)(62.4)(1300 - 110.57) = 2.227 \times 10^7 \text{ ft} \cdot \text{lb/s} = (2.227 \times 10^7)/550 = 40\,491 \text{ hp}$$

9.234 Find the kilowatt loss in 500 m of 50-cm-diameter pipe for which $\epsilon = 0.05$ mm when dye at 45 °C (s.g. = 0.86, $\nu = 4.4 \times 10^{-6}$ ft²/s) flows at 0.22 m³/s. Neglect minor losses.

▮
$$h_f = (f)(L/d)(v^2/2g) \qquad v = Q/A = 0.22/[(\pi)(\tfrac{50}{100})^2/4] = 1.120 \text{ m/s}$$
$$N_R = dv/\nu = (0.50)(1.120)/(4.4 \times 10^{-6}) = 1.27 \times 10^5 \qquad \epsilon/d = (5 \times 10^{-5})/(5 \times 10^{-1}) = 0.0001$$

From Fig. A-5, $f = 0.018$. $h_f = 0.018[500/0.50]\{1.120^2/[(2)(9.807)]\} = 1.151$ m, $P = Q\gamma h_f = 0.22[(0.86)(9.79)](1.151) = 2.13$ kW.

9.235 Linseed oil, of kinematic viscosity 0.0005 ft²/s and weight density 59.8 lb/ft³, is pumped through a 3-in pipe ($\epsilon = 0.001$ in). **(*a*)** At what maximum velocity would the flow still be laminar? **(*b*)** What would then be the loss in energy head per 1000 ft of pipe?

▮ **(*a*)** Assume laminar flow exists for $N_R \le 2000$. $N_R = dv/\nu$, $2000 = (\tfrac{3}{12})(v)/0.0005$, $v = 4.00$ ft/s.

(*b*) $f = 64/N_R = \tfrac{64}{2000} = 0.032 \qquad h_f = (f)(L/d)(v^2/2g) = 0.032[1000/(\tfrac{3}{12})]\{4.00^2/[(2)(32.2)]\} = 31.80$ ft

$$p = \gamma h_f = (59.8)(31.80)/144 = 13.2 \text{ psi per 1000 ft}$$

9.236 Repeat Prob. 9.235 if the velocity is three times the maximum velocity for laminar flow.

▮
$$v = (3)(4.00) = 12.00 \text{ ft/s} \qquad h_f = (f)(L/d)(v^2/2g)$$
$$N_R = dv/\nu = (\tfrac{3}{12})(12.00)/0.0005 = 6000 \qquad \epsilon/d = 0.001/2 = 0.00050$$

From Fig. A-5, $f = 0.036$.

$$h_f = 0.036[1000/(\tfrac{3}{12})]\{12.00^2/[(2)(32.2)]\} = 322.0 \text{ ft}$$
$$p = \gamma h_f = (59.8)(322.0)/144 = 134 \text{ psi per 1000 ft}$$

9.237 Water flows upward at 3 m/s through a vertical 150-mm-diameter pipe standing in a body of water with its lower end 1.0 m below the surface. Considering all losses and with $f = 0.022$, find the pressure at a point 3 m above the surface of the water.

▮ $p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L$. Let point 1 be at the water surface and point 2 be 3 m above the water surface. $h_L = h_f + h_m$, $h_f = (f)(L/d)(v^2/2g) = 0.022[(3 + 1.0)/0.150]\{3^2/[(2)(9.807)]\} = 0.269$ m, $h_m = (K)(v^2/2g)$. For entrance loss, assume $K = 0.8$. $h_m = 0.8\{3^2/[(2)(9.807)]\} = 0.367$ m, $h_L = 0.269 + 0.367 = 0.636$ m, $0 + 0 + 0 = p_2/9.79 + 3^2/[(2)(9.807)] + 3 + 0.636$, $p_2 = -40.1$ kPa.

9.238 Work Prob. 9.237 if the flow is downward.

▮
$$p_2/\gamma + v_2^2/2g + z_2 = p_1/\gamma + v_1^2/2g + z_1 + h_L \qquad h_L = h_f + h_m$$
$$h_f = 0.269 \text{ m} \quad \text{(from Prob. 9.237)} \qquad h_m = (K)(v^2/2g)$$

For exit loss, assume $K = 1.0$.

$$h_m = 1.0\{3^2/[(2)(9.807)]\} = 0.459 \text{ m} \qquad h_L = 0.269 + 0.459 = 0.728 \text{ m}$$
$$p_2/9.79 + 3^2/[(2)(9.807)] + 3 = 0 + 0 + 0 + 0.728 \qquad p_2 = -26.7 \text{ kPa}$$

9.239 A horizontal pipe 20 cm in diameter and for which $f = 0.030$ projects into a pond 1 m below the surface. Considering all losses, find the pressure at a point 4 m from the end of the pipe if the flow is at 3 m/s from the pipe into the pond.

▮ $p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L$. Let point 1 be 4 m from the end of the pipe and point 2 be at the water surface. $h_L = h_f + h_m$, $h_f = (f)(L/d)(v^2/2g) = 0.030[4/0.20]\{3^2/[(2)(9.807)]\} = 0.275$ m, $h_m = (K)(v^2/2g)$. For exit loss, $K = 1.0$. $h_m = 1.0\{3^2/[(2)(9.807)]\} = 0.459$ m, $h_L = 0.275 + 0.459 = 0.734$ m, $p_1/9.79 + 3^2/[(2)(9.807)] + 0 = 0 + 0 + 1 + 0.734$, $p_1 = 12.5$ kPa.

9.240 Repeat Prob. 9.239 if the flow is from the pond into the pipe.

▮

$$p_2/\gamma + v_2^2/2g + z_2 = p_1/\gamma + v_1^2/2g + z_1 + h_L \qquad h_L = h_f + h_m$$

$$h_f = 0.275 \text{ m} \qquad \text{(from Prob. 9.239)} \qquad h_m = (K)(v^2/2g)$$

For entrance, $K = 0.8$.

$$h_m = 0.8\{3^2/[(2)(9.807)]\} = 0.367 \text{ m} \qquad h_L = 0.275 + 0.367 = 0.642 \text{ m}$$

$$0 + 0 + 1 = p_1/9.79 + 3^2/[(2)(9.807)] + 0 + 0.642 \qquad p_1 = -0.99 \text{ kPa}$$

9.241 A pipe runs from one reservoir to another, both ends of the pipe being under water. The intake is nonprojecting; the length of pipe is 480 ft; its diameter is 10.25 in; and the difference in the two water levels is 106 ft. If $f = 0.02$, what will be the pressure at a point 300 ft from the intake and 120 ft below the surface of the water in the upper reservoir?

▮

$$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L \qquad (1)$$

Let points 1 and 2 be at the water surface in the upper and lower reservoirs, respectively.

$$h_f = (f)(L/d)(v^2/2g) = 0.02[(480)/(10.25/12)]\{v^2/[(2)(32.2)]\} = 0.1745v^2$$

$$h_m = (K)(v^2/2g) = (0.5 + 1.0)\{v^2/[(2)(32.2)]\} = 0.02329v^2$$

$$h_L = h_f + h_m = 0.1745v^2 + 0.02329v^2 = 0.1978v^2 \qquad 0 + 0 + 106 = 0 + 0 + 0 + 0.1978v^2 \qquad v = 23.15 \text{ ft/s}$$

Now apply Eq. (*1*) between the upper reservoir (point 1) and the point 300 ft from the intake (point 2).

$$h_f = 0.02[(300)/(10.25/12)]\{23.15^2/[(2)(32.2)]\} = 58.46 \text{ ft} \qquad h_m = 0.5\{23.15^2/[(2)(32.2)]\} = 4.16 \text{ ft}$$

$$h_L = 58.46 + 4.16 = 62.62 \text{ ft} \qquad 0 + 0 + 120 = (p_2)(144)/62.4 + 23.15^2/[(2)(32.2)] + 0 + 62.62 \qquad p_2 = 21.3 \text{ lb/in}^2$$

9.242 A 9.5-in-diameter pipeline runs from one reservoir to another, both ends being under water, and the intake end is nonprojecting. The difference in water levels between the two reservoirs is 110 ft, and the length of pipe is 1000 ft. What is the discharge if $f = 0.06$?

▮

$$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L$$

$$h_f = (f)(L/d)(v^2/2g) = 0.06[(1000)/(9.5/12)]\{v^2/[(2)(32.2)]\} = 1.1769v^2$$

$$h_m = (K)(v^2/2g) = (0.5 + 1.0)\{v^2/[(2)(32.2)]\} = 0.02329v^2$$

$$h_L = h_f + h_m = 1.1769v^2 + 0.02329v^2 = 1.200v^2 \qquad 0 + 0 + 110 = 0 + 0 + 0 + 1.200v^2$$

$$v = 9.57 \text{ ft/s} \qquad Q = Av = [(\pi)(9.5/12)^2/4](9.57) = 4.71 \text{ ft}^3/\text{s}$$

9.243 A jet of water is discharged through a nozzle at a point 200 ft below the water level at intake. The jet is 4 in in diameter, and the loss coefficient of the nozzle is 0.15. If the pipeline is 12 in in diameter, 600 ft long, with a

nonprojecting entrance, what is the pressure at the base of the nozzle? Assume $f = 0.0125$.

$$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L \qquad (1)$$

$$h_f = (f)(L/d)(v^2/2g) = 0.0125[600/(\tfrac{12}{12})]\{v^2/[(2)(32.2)]\} = 0.1165v^2$$

$$h_m = (K)(v^2/2g) = 0.5\{v^2/[(2)(32.2)]\} + (0.15)\{v_{jet}^2/[(2)(32.2)]\} = 0.007764v^2 + 0.002329v_{jet}^2$$

$$h_L = h_f + h_m = 0.1165v^2 + 0.007764v^2 + 0.002329v_{jet}^2 = 0.1243v^2 + 0.002329v_{jet}^2$$

$$0 + 0 + 200 = 0 + v_{jet}^2/[(2)(32.2)] + 0 + 0.1243v^2 + 0.002329v_{jet}^2$$

Since velocity varies with the square of diameter, $v_{jet} = (\tfrac{12}{4})^2(v) = 9v$, $200 = (9v)^2/[(2)(32.2)] + 0.1243v^2 + (0.002329)(9v)^2$, $v = 11.28$ ft/s. Applying Eq. (*1*) to the nozzle, $(p_1)(144)/62.4 + 11.28^2/[(2)(32.2)] + 0 = 0 + [(9)(11.28)]^2/[(2)(32.2)] + 0 + (0.15)\{[(9)(11.28)]^2/[(2)(32.2)]\}$, $p_1 = 78.9$ lb/in^2.

9.244 Compute the losses due to flow of 25 m^3/min of air, $p = 1$ atm, $T = 20$ °C, through a sudden expansion from 300-mm pipe to 900-mm pipe. How much head would be saved by using a 10° conical diffuser?

For sudden expansion: $h_m = [1 - (D_1/D_2)^2]^2(v_1^2/2g)$, $v_1 = Q/A_1 = (\tfrac{25}{60})/[(\pi)(\tfrac{300}{1000})^2/4] = 5.895$ m/s, $h_m = [1 - (\tfrac{300}{900})^2]^2\{5.895^2/[(2)(9.807)]\} = 1.400$ m.

For conical diffuser: $h_m = 0.152[(v_1 - v_2)^2/2g]$, $v_2 = Q/A_2 = (\tfrac{25}{60})/[(\pi)(\tfrac{900}{1000})^2/4] = 0.655$ m/s, $h_m = 0.152\{(5.895 - 0.655)^2/[(2)(9.807)]\} = 0.213$ m.

$$\text{Saving in head} = 1.400 - 0.213 = 1.187 \text{ m} \quad \text{or} \quad 1.187 \text{ N}\cdot\text{m/N} \quad \text{or} \quad 1.187 \text{ J/N}$$

9.245 Calculate the value of H in Fig. 9-47 for 125 L/s of water at 15 °C through commercial steel pipe.

$$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L \qquad h_L = h_f + h_m \qquad h_f = (f)(L/d)(v^2/2g)$$

$$v = Q/A = (125 \times 10^{-3})/[(\pi)(0.30)^2/4] = 1.768 \text{ m/s}$$

$$N_R = dv/\nu = (0.30)(1.768)/(1.16 \times 10^{-6}) = 4.57 \times 10^5 \qquad \epsilon/d = 0.000046/0.30 = 0.00015$$

From Fig. A-5, $f = 0.015$.

$$h_f = 0.015[50/0.30]\{1.768^2/[(2)(9.807)]\} = 0.398 \text{ m}$$

$$h_m = (K)(v^2/2g) = (0.5 + 1.0)\{1.768^2/[(2)(9.807)]\} = 0.239 \text{ m}$$

$$h_L = 0.398 + 0.239 = 0.637 \text{ m} \qquad 0 + 0 + z_1 = 0 + 0 + z_2 + 0.637 \qquad z_1 - z_2 = H = 0.637 \text{ m}$$

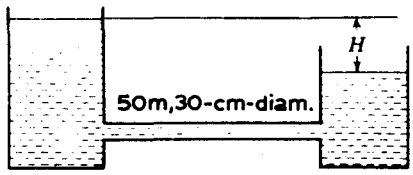


Fig. 9-47

9.246 In Fig. 9-47, for $H = 3$ m and a fluid with s.g. $= 0.8$ and $\mu = 0.007$ Pa · s, calculate the discharge through smooth pipe.

$$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L$$

$$h_f = (f)(L/d)(v^2/2g) = (f)[50/0.30]\{v^2/[(2)(9.807)]\} = 8.497fv^2$$

$$h_m = (K)(v^2/2g) = (0.5 + 1.0)\{v^2/[(2)(9.807)]\} = 0.07648v^2 \qquad h_L = h_f + h_m = 8.497fv^2 + 0.07648v^2$$

$$0 + 0 + 3 = 0 + 0 + 0 + 8.497fv^2 + 0.07648v^2 \qquad v = \sqrt{3/(8.497f + 0.07648)}$$

Try $f = 0.02$: $v = \sqrt{3/[(8.497)(0.02) + 0.07648]} = 3.489$ m/s, $N_R = \rho dv/\mu = [(0.8)(1000)](0.30)(3.489)/0.007 = 1.20 \times 10^5$. From Fig. A-5, $f = 0.017$. Try $f = 0.017$: $v = \sqrt{3/[(8.497)(0.017) + 0.07648]} = 3.685$ m/s, $N_R = [(0.8)(1000)](0.30)(3.685)/0.007 = 1.26 \times 10^5$, $f = 0.017$ (O.K.); $Q = Av = [(\pi)(0.30)^2/4](3.685) = 0.260$ m^3/s.

9.247 Evaluate K for a valve that, placed in the line in Prob. 9.246, would reduce the discharge by 50%.

▮ From Prob. 9.246, $H = 3 = [(f)(50)/0.30 + 0.5 + 1.0 + K_{valve}]\{(3.685/2)^2/[(2)(9.807)]\}$, $K_{valve} = 15.83 - 166.7f$, $N_R = \rho dv/\mu = [(0.8)(1000)](0.30)(3.685/2)/0.007 = 6.32 \times 10^4$. From Fig. A-5, $f = 0.0197$. $K_{valve} = 15.83 - (166.7)(0.0197) = 12.5$.

9.248 A water line connecting two reservoirs at 70 °F has 4860 ft of 24-in-diameter steel pipe, three standard elbows ($K = 0.9$), a globe valve ($K = 10$), a re-entrant pipe entrance ($K = 1.0$), and a submerged pipe exit ($K = 1.0$). What is the difference in reservoir elevations for 20 cfs?

▮
$$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L \qquad h_L = h_f + h_m \qquad h_f = (f)(L/d)(v^2/2g)$$
$$v = Q/A = 20/[(\pi)(\tfrac{24}{12})^2/4] = 6.366 \text{ ft/s} \qquad N_R = dv/\nu = (\tfrac{24}{12})(6.366)/(1.05 \times 10^{-5}) = 1.21 \times 10^6$$
$$\epsilon/d = 0.00015/(\tfrac{24}{12}) = 0.000075$$

From Fig. A-5, $f = 0.013$.

$$h_f = 0.013[4860/(\tfrac{24}{12})]\{6.366^2/[(2)(32.2)]\} = 19.88 \text{ ft}$$
$$h_m = (K)(v^2/2g) = [(3)(0.9) + 10 + 1.0 + 1.0]\{6.366^2/[(2)(32.2)]\} = 9.25 \text{ ft}$$
$$h_L = 19.88 + 9.25 = 29.13 \text{ ft} \qquad 0 + 0 + z_1 = 0 + 0 + z_2 + 29.13 \qquad z_1 - z_2 = 29.13 \text{ ft}$$

9.249 For the conditions given in Prob. 9.248, determine the discharge if the difference in elevations is 40 ft.

▮ From Prob. 9.248, $40 = h_f + h_m = \{v^2/[(2)(32.2)]\}\{(f)[4860/(\tfrac{24}{12})] + [(3)(0.9) + 10 + 1.0 + 1.0]\}$, $v = \sqrt{2576/(2430f + 14.7)}$. Try $f = 0.013$: $v = \sqrt{2576/[(2430)(0.013) + 14.7]} = 7.46$ ft/s, $N_R = dv/\nu = (\tfrac{24}{12})(7.46)/(1.05 \times 10^{-5}) = 1.42 \times 10^6$, $\epsilon/d = 0.000075$ (from Prob. 9.248). From Fig. A-5, $f = 0.0125$. Try $f = 0.0125$: $v = \sqrt{2576/[(2430)(0.0125) + 14.7]} = 7.560$ ft/s, $N_R = (\tfrac{24}{12})(7.560)/(1.05 \times 10^{-5}) = 14.4 \times 10^6$, $f = 0.0125$ (O.K.); $Q = Av = [(\pi)(\tfrac{24}{12})^2/4](7.560) = 23.8 \text{ ft}^3/\text{s}$.

9.250 What size commercial steel pipe is needed to convey 200 L/s of water at 20 °C a distance of 5000 m with a head drop of 4 m? The line connects two reservoirs, has a re-entrant ($K = 1.0$), a submerged outlet ($K = 1.0$), four standard elbows ($K = 0.9$), and a globe valve ($K = 10$).

▮
$$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L \qquad 0 + 0 + 4 = 0 + 0 + 0 + h_L \qquad h_L = h_f + h_m$$
$$v = Q/A = (\tfrac{200}{1000})/(\pi d^2/4) = 0.2546/d^2$$
$$h_f = (f)(L/d)(v^2/2g) = (f)(5000/d)\{(0.2546/d^2)^2/[(2)(9.807)]\} = 16.52f/d^5$$
$$h_m = (K)(v^2/2g) = [1.0 + 1.0 + (4)(0.9) + 10]\{(0.2546/d^2)^2/[(2)(9.807)]\} = 0.05156/d^4$$
$$4 = 16.52f/d^5 + 0.05156/d^4$$

Try $f = 0.02$: $4 = (16.52)(0.02)/d^5 + 0.05156/d^4$. By trial and error, $d = 0.619$ m.

$$v = 0.2546/0.619^2 = 0.6645 \text{ m/s} \qquad N_R = dv/\nu = (0.619)(0.6645)/(1.02 \times 10^{-6}) = 4.03 \times 10^5$$
$$\epsilon/d = 0.000046/0.619 = 0.000074$$

From Fig. A-5, $f = 0.0145$. Try $f = 0.0145$: $4 = (16.52)(0.0145)/d^5 + 0.05156/d^4$, $d = 0.588$ m, $v = 0.2546/0.588^2 = 0.7364$ m/s, $N_R = (0.588)(0.7364)/(1.02 \times 10^{-6}) = 4.25 \times 10^5$, $f = 0.0145$. Therefore, $d = 0.588$ m, or 588 mm.

9.251 Find the equivalent lengths of 167-mm-diameter pipe, $f = 0.024$, for **(a)** a re-entrant pipe entrance ($K = 1.0$), **(b)** a sudden expansion from 167 mm to 334 mm diameter, and **(c)** a globe valve and a standard tee?

▮
$$L_e = KD/f$$

(a) $$L_e = (1.0)(0.167)/0.024 = 6.94 \text{ m}$$

(b) $$K = [1 - (D_1/D_2)^2]^2 = [1 - (\tfrac{1}{2})^2]^2 = 0.5625 \qquad L_e = (0.5625)(0.167)/0.024 = 3.91 \text{ m}$$

(c) $$K = 10 + 1.8 = 11.8 \qquad L_e = (11.8)(0.167)/0.024 = 81.9 \text{ m}$$

9.252 Find H in Fig. 9-48 for 200 gpm of oil ($\mu = 1$ cP, $\gamma = 60$ lb/ft^3) for the angle valve wide open ($K = 5.0$).

▮ $$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L \qquad h_L = h_f + h_m \qquad h_f = (f)(L/d)(v^2/2g)$$

$$Q = (200)(0.002228) = 0.4456\ \text{ft}^3/\text{s} \qquad v = Q/A = 0.4456/[(\pi)(\tfrac{3}{12})^2/4] = 9.078\ \text{ft/s}$$

By Prob. 9.40, $\mu = 0.0002089$ lb · s/ft^2; $N_R = (\gamma/g)(dv)/\mu = (60/32.2)(\frac{3}{12})(9.078)/0.0002089 = 2.02 \times 10^4$, $\epsilon/d = 0.00015/(\frac{3}{12}) = 0.000600$. From Fig. A-5, $f = 0.0275$.

$$h_f = 0.0275[212/(\tfrac{3}{12})]\{9.078^2/[(2)(32.2)]\} = 29.84\ \text{ft}$$

$$h_m = (K)(v^2/2g) = [0.5 + 5.0 + 1.0]\{9.078^2/[(2)(32.2)]\} = 8.32\ \text{ft}$$

$$h_L = 29.84 + 8.32 = 38.16\ \text{ft} \qquad 0 + 0 + z_1 = 0 + 0 + z_2 + 38.16 \qquad z_1 - z_2 = H = 38.16\ \text{ft}$$

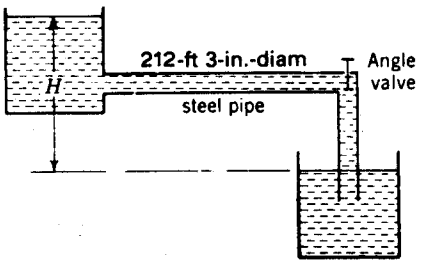


Fig. 9-48

9.253 Find K for the angle valve in Prob. 9.252 for a flow of 10 L/s at the same H.

▮ $$v = Q/A = [(\tfrac{10}{1000})/0.3048^3]/[(\pi)(\tfrac{3}{12})^2/4] = 7.194\ \text{ft/s}$$

$$N_R = \rho dv/\mu = (\gamma/g)(dv)/\mu = (60/32.2)(\tfrac{3}{12})(7.194)/0.0002089 = 1.60 \times 10^4$$

$$\epsilon/d = 0.000600 \qquad \text{(from Prob. 9.252)}$$

From Fig. A-5, $f = 0.0285$. From Prob. 9.252, $H = [(f)(L/d) + K_1 + K_2 + K_3](v^2/2g)$, $38.16 = \{(0.0285)[212/(\frac{3}{12})] + 0.5 + K_2 + 1.0\}\{7.194^2/[(2)(32.2)]\}$, $K_2 = 21.8$.

9.254 Calculate the discharge through the system of Fig. 9-48 for water at 25 °C and $H = 8$ m.

▮ $$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L$$

In British Engineering units, we have

$$h_f = (f)(L/d)(v^2/2g) = (f)[212/(\tfrac{3}{12})]\{v^2/[(2)(32.2)]\} = 13.17fv^2$$

$$h_m = (K)(v^2/2g) = (0.5 + 5.0 + 1.0)\{v^2/[(2)(32.2)]\} = 0.1009v^2 \qquad h_L = h_f + h_m = 13.17fv^2 + 0.1009v^2$$

$$0 + 0 + 8/0.3048 = 0 + 0 + 0 + 13.17fv^2 + 0.1009v^2 \qquad v = \sqrt{26.25/(13.17f + 0.1009)}$$

Try $f = 0.02$: $v = \sqrt{26.25/[(13.17)(0.02) + 0.1009]} = 8.489$ ft/s, $N_R = dv/\nu = (\frac{3}{12})(8.489)/(9.56 \times 10^{-6}) = 2.22 \times 10^5$, $\epsilon/d = 0.00015/(\frac{3}{12}) = 0.000600$. From Fig. A-5, $f = 0.019$. Try $f = 0.019$: $v = \sqrt{26.25/[(13.17)(0.019) + 0.1009]} = 8.646$ ft/s, $N_R = (\frac{3}{12})(8.677)/(9.56 \times 10^{-6}) = 2.27 \times 10^5$, $f = 0.019$ (O.K.); $Q = Av = [(\pi)(\frac{3}{12})^2/4](8.646) = 0.4244$ ft^3/s $= (0.4244)(0.3048)^3 = 0.0120$ m^3/s, or 12.0 L/s.

9.255 Find the discharge through the pipeline in Fig. 9-49 for $H = 10$ m, as shown. Use minor loss coefficients for the entrance, elbows, and globe valve of 0.5, 0.9 (each), and 10, respectively.

▮ $$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L$$

$$h_f = (f)(L/d)(v^2/2g) = f[(30 + 12 + 60)/(\tfrac{150}{1000})]\{v_2^2/[(2)(9.807)]\} = 34.67fv_2^2$$

$$h_m = (K)(v^2/2g) = [0.5 + (2)(0.9) + 10]\{v_2^2/[(2)(9.807)]\} = 0.6271v_2^2$$

$$h_L = h_f + h_m = 34.67fv_2^2 + 0.6271v_2^2 \qquad 0 + 0 + 10 = 0 + v_2^2/[(2)(9.807)] + 0 + 34.67fv_2^2 + 0.6271v_2^2$$

$$v = \sqrt{10/(34.67f + 0.6781)}$$

Try $f = 0.02$: $v = \sqrt{10/[(34.67)(0.02) + 0.6781]} = 2.700$ m/s, $N_R = dv/\nu = (\frac{150}{1000})(2.700)/(1.02 \times 10^{-6}) = 3.97 \times 10^5$, $\epsilon/d = 0.00026/(\frac{150}{1000}) = 0.00173$. From Fig. A-5, $f = 0.023$. Try $f = 0.023$: $v = \sqrt{10/[(34.67)(0.023) + 0.6781]} = 2.603$ m/s, $N_R = (\frac{150}{1000})(2.603)/(1.02 \times 10^{-6}) = 3.83 \times 10^5$, $f = 0.023$ (O.K.); $Q = Av = [(\pi)(\frac{150}{1000})^2/4](2.603) = 0.0460$ m^3/s, or 46.0 L/s.

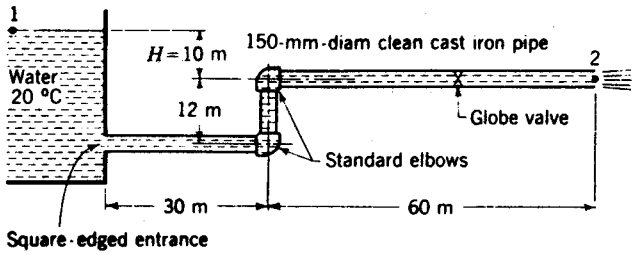


Fig. 9-49

9.256 Rework Prob. 9.255 to find H if $Q = 60$ L/s.

▮ $$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L \qquad h_L = h_f + h_m \qquad h_f = (f)(L/d)(v^2/2g)$$

$$v = Q/A = (\tfrac{60}{1000})/[(\pi)(\tfrac{150}{1000})^2/4] = 3.395 \text{ m/s} \qquad N_R = dv/\nu = (\tfrac{150}{1000})(3.395)/(1.02 \times 10^{-6}) = 4.99 \times 10^5$$

$$\epsilon/d = 0.00173 \qquad \text{(from Prob. 9.255)}$$

From Fig. A-5, $f = 0.0225$.

$$h_f = 0.0225[(30 + 12 + 60)/(\tfrac{150}{1000})]\{3.395^2/[(2)(9.807)]\} = 8.99 \text{ m}$$

$$h_m = (K)(v^2/2g) = [0.5 + (2)(0.9) + 10]\{3.395^2/[(2)(9.807)]\} = 7.23 \text{ m}$$

$$h_L = 8.99 + 7.23 = 16.22 \text{ m} \qquad 0 + 0 + z_1 = 0 + 3.395^2/[(2)(9.807)] + z_2 + 16.22 \qquad z_1 - z_2 = H = 16.81 \text{ m}$$

9.257 Assume that water at 10 °C is to be conveyed at 300 L/s through 500 m of commercial steel pipe with a total head drop of 6 m. Minor losses are $12v^2/2g$. Determine the required diameter.

▮ $$N_R = R_5/D \tag{1}$$

$$f = R_7/[\ln(R_3/D + R_2/N_R^{0.9})]^2 \tag{2}$$

$$x = R_6 + R_4 D/f \tag{3}$$

$$D = (R_0)(x^{4.75} + R_1 x^{5.2})^{0.04} \tag{4}$$

where $R_0 = (0.66)(\epsilon^{1.25}Q^{9.5})^{0.04}$, $R_1 = \nu/\epsilon^{1.25}Q^{0.1}$, $R_2 = 5.74$, $R_3 = \epsilon/3.7$, $R_4 = K/gh_f$, $R_5 = 4Q/\pi\nu$, $R_6 = L/gh_f$, $R_7 = 1.325$, $\epsilon = 0.000046$ m, $Q = 300$ L/s, or 0.300 m^3/s, $\nu = 1.30 \times 10^{-6}$ m^2/s.

$$K = 12 \qquad h_f = 6 \text{ mm} \qquad R_0 = (0.66)(0.000046^{1.25}0.300^{9.5})^{0.04} = 0.25351$$

$$R_1 = 1.30 \times 10^{-6}/(0.000046^{1.25}0.300^{0.1}) = 0.38707 \qquad R_3 = 0.000046/3.7 = 1.2432 \times 10^{-5}$$

$$R_4 = 12/[(9.807)(6)] = 0.20394 \qquad R_5 = (4)(0.300)/[(\pi)(1.30 \times 10^{-6})] = 2.9382 \times 10^5$$

$$R_6 = 500/[(9.807)(6)] = 8.4973$$

Assume $D = 1$ m. Substituting into Eqs. (*1*), (*2*), (*3*), and (*4*),

$$N_R = 2.9382 \times 10^5/1 = 2.9382 \times 10^5 \qquad f = 1.325/\{\ln[(1.2432 \times 10^{-5})/1 + 5.74/(2.9382 \times 10^5)^{0.9}]\}^2 = 0.014938$$

$$x = 8.4973 + (0.20394)(1)/0.014938 = 22.150 \qquad D = (0.25351)[22.150^{4.75} + (0.38707)(22.150^{5.2})]^{0.04} = 0.47418 \text{ m}$$

Try $D = 0.47418$ m: $N_R = 2.9382 \times 10^5/0.47418 = 6.1964 \times 10^5$, $f = 1.325/\{\ln[(1.2432 \times 10^{-5})/0.47418 + 5.74/(6.1964 \times 10^5)^{0.9}]\}^2 = 0.014086$, $x = 8.4973 + (0.20394)(0.47418)/0.014086 = 15.363$, $D = (0.25351)[15.363^{4.75} + (0.38707)(15.363^{5.2})]^{0.04} = 0.44063$ m. Try $D = 0.44063$ m: $N_R = 2.9382 \times 10^5/0.44063 = 6.6682 \times 10^5$, $f = 1.325/\{\ln[(1.2432 \times 10^{-5})/0.44063 + 5.74/(6.6682 \times 10^5)^{0.9}]\}^2 = 0.014075$, $x = 8.4973 + (0.20394)(0.44063)/0.014075 = 14.882$, $D = (0.25351)[14.882^{4.75} + (0.38707)(14.882^{5.2})]^{0.04} = 0.43783$ m. Try $D = 0.43783$ m: $N_R = 2.9382 \times 10^5/0.43783 = 6.7108 \times 10^5$, $f = 1.325/\{\ln[(1.2432 \times 10^{-5})/0.43783 + 5.74/(6.7108 \times 10^5)^{0.9}]\}^2 = 0.014074$, $x = 8.4973 + (0.20394)(0.43783)/0.014074 = 14.842$, $D = (0.25351)[14.842^{4.75} + (0.38707)(14.842^{5.2})]^{0.04} = 0.43759$ m. Therefore, $D = 0.438$ m, or 438 mm.

9.258 One equation for determining the friction factor is $1/\sqrt{f} = 0.869 \ln(N_R\sqrt{f}) - 0.8$. Compare the smooth pipe curve on the Moody diagram (Fig. A-5) with results from the equation above for values of Reynolds number of 10^5, 10^6, and 10^7.

▮ For $N_R = 10^5$, from Fig. A-5, $f = 0.0178$. From the equation, $1/\sqrt{f} = 0.869\{\ln[(10^5)(\sqrt{f})]\} - 0.8$. By trial and error, $f = 0.0183$. For $N_R = 10^6$, from Fig. A-5, $f = 0.0116$. From the equation, $1/\sqrt{f} = 0.869\{\ln[(10^6)(\sqrt{f})]\} - 0.8$. By trial and error, $f = 0.0116$. For $N_R = 10^7$, from Fig. A-5, $f = 0.0082$. From the equation, $1/\sqrt{f} = 0.869\{\ln[(10^7)(\sqrt{f})]\} - 0.8$. By trial and error, $f = 0.00810$.

9.259 An equation for determining the friction factor developed by Colebrook is $1/\sqrt{f} = -0.869 \ln [(\epsilon/D)/3.7 + 2.523/(N_R\sqrt{f})]$. Check the location of line $\epsilon/D = 0.0002$ on the Moody diagram (Fig. A-5) with the equation above for a Reynolds number of 10^5.

▮ From Fig. A-5, $f = 0.0190$. From the equation, $1/\sqrt{f} = -0.869 \ln \{0.0002/3.7 + 2.523/[(10^5)(\sqrt{f})]\}$. By trial and error, $f = 0.0192$.

9.260 Find the head loss in a pipeline consisting of 200 ft of 4-in steel pipe, a 90° bend on 24-in radius ($K = 0.15$), 4-in gate valve (fully open) ($K = 0.20$), 100 ft of 4-in steel pipe, expansion to 6 in with a 20° taper ($K = 0.4$), 300 ft of 6-in steel pipe, abrupt contraction to 3-in diameter ($K = 0.35$), and 50 ft of 3-in steel pipe. The discharge rate is 1.5 cfs.

▮ $h_f = (f)(L/d)(v^2/2g)$, $h_m = (K)(v^2/2g)$. For 200 ft of 4-in pipe: $v_1 = Q/A = 1.5/[(\pi)(\frac{4}{12})^2/4] = 17.19$ ft/s, $N_R = dv_1/\nu = (\frac{4}{12})(17.19)/(1.05 \times 10^{-5}) = 5.46 \times 10^5$, $\epsilon/d = 0.00015/(\frac{4}{12}) = 0.00045$. From Fig. A-5, $f = 0.0175$. $h_f = 0.0175[200/(\frac{4}{12})]\{17.19^2/[(2)(32.2)]\} = 48.2$ ft.

For bend: $h_m = 0.15\{17.19^2/[(2)(32.2)]\} = 0.7$ ft.

For gate valve: $h_m = 0.20\{17.19^2/[(2)(32.2)]\} = 0.9$ ft.

For 100 ft of 4-in pipe: $h_f = 0.0175[100/(\frac{4}{12})]\{17.19^2/[(2)(32.2)]\} = 24.1$ ft.

For expansion: $v_2 = 1.5/[(\pi)(\frac{6}{12})^2/4] = 7.639$ ft/s, $h_m = 0.4\{17.19 - 7.639)^2/[(2)(32.2)]\} = 0.6$ ft.

For 300 ft of 6-in pipe: $N_R = (\frac{6}{12})(7.639)/(1.05 \times 10^{-5}) = 3.64 \times 10^5$, $\epsilon/d = 0.00015/(\frac{6}{12}) = 0.00030$, $f = 0.0170$, $h_f = 0.0170[300/(\frac{6}{12})]\{7.639^2/[(2)(32.2)]\} = 9.2$ ft.

For abrupt contraction: $v_3 = 1.5/[(\pi)(\frac{3}{12})^2/4] = 30.56$ ft/s, $h_m = 0.35\{30.56^2/[(2)(32.2)]\} = 5.1$ ft.

For 50 ft of 3-in pipe: $N_R = (\frac{3}{12})(30.56)/(1.05 \times 10^{-5}) = 7.28 \times 10^5$, $\epsilon/d = 0.00015/(\frac{3}{12}) = 0.00060$, $f = 0.0180$, $h_f = 0.0180[50/(\frac{3}{12})]\{30.56^2/[(2)(32.2)]\} = 52.2$ ft, $h_L = 48.2 + 0.7 + 0.9 + 24.1 + 0.6 + 9.2 + 5.1 + 52.2 = 141.0$ ft.

9.261 Using the Darcy–Weisbach formula, find the head loss in 1000 ft of 6-ft-diameter smooth concrete pipe carrying 80 cfs of water at 50 °F.

▮
$$h_f = (f)(L/D)(v^2/2g) \qquad v = Q/A = 80/[(\pi)(6)^2/4] = 2.829 \text{ ft/s}$$
$$N_R = Dv/\nu = (6)(2.829)/(1.40 \times 10^{-5}) = 1.21 \times 10^6 \qquad \epsilon/D = 0.001/6 = 0.000167$$

From Fig. A-5, $f = 0.014$.

$$h_f = (0.014)(\tfrac{1000}{6})\{2.829^2/[(2)(32.2)]\} = 0.290 \text{ ft}$$

9.262 Solve Prob. 9.261 using the Manning formula.

▮
$$v = (1.486/n)(R)^{2/3}(s)^{1/2} = 2.829 \text{ ft/s} \qquad \text{(from Prob. 9.261)}$$
$$n = 0.013 \qquad \text{(from Table A-13)} \qquad R = D/4 = \tfrac{6}{4} = 1.500 \text{ ft}$$
$$2.829 = (1.486/0.013)(1.500)^{2/3}(s)^{1/2} \qquad s = 0.0003567 \qquad h_f = (0.0003567)(1000) = 0.357 \text{ ft}$$

9.263 Solve Prob. 9.261 using the Hazen–Williams formula.

▮
$$v = 1.318CR^{0.63}s^{0.54} = 2.829 \text{ ft/s} \qquad \text{(from Prob. 9.261)}$$
$$C = 120 \qquad \text{(from Table A-14)} \qquad R = 1.500 \text{ ft} \qquad \text{(from Prob. 9.262)}$$
$$2.829 = (1.318)(120)(1.500)^{0.63}(s)^{0.54} \qquad s = 0.0003618 \qquad h_f = (1000)(0.0003618) = 0.362 \text{ ft}$$

9.264 Using the Darcy–Weisbach formula, find the head loss in 100 ft of 3-ft-diameter welded steel pipe carrying 15 cfs of water at 60 °F.

▮
$$h_f = (f)(L/D)(v^2/2g) \qquad v = Q/A = 15/[(\pi)(3)^2/4] = 2.122 \text{ ft/s}$$
$$N_R = Dv/\nu = (3)(2.122)/(1.21 \times 10^{-5}) = 5.26 \times 10^5 \qquad \epsilon/D = 0.00015/3 = 0.000050$$

From Fig. A-5, $f = 0.0135$. $h_f = (0.0135)(\frac{100}{3})\{2.122^2/[(2)(32.2)]\} = 0.031$ ft.

9.265 Solve Prob. 9.264 using the Manning formula with $n = 0.012$.

▮ $$v = (1.486/n)(R)^{2/3}(s)^{1/2} = 2.122 \text{ ft/s} \quad \text{(from Prob. 9.264)}$$

$$R = D/4 = \tfrac{3}{4} = 0.7500 \text{ ft} \qquad 2.122 = (1.486/0.012)(0.7500)^{2/3}(s)^{1/2} \qquad s = 0.0004309$$

$$h_f = (0.0004309)(100) = 0.043 \text{ ft}$$

9.266 Solve Prob. 9.264 using the Hazen–Williams formula with $C = 120$.

▮ $$v = 1.318CR^{0.63}s^{0.54} = 2.122 \text{ ft/s} \quad \text{(from Prob. 9.264)}$$

$$R = 0.7500 \text{ ft} \quad \text{(from Prob. 9.265)}$$

$$2.122 = (1.318)(120)(0.7500)^{0.63}(s)^{0.54} \qquad s = 0.0004769 \qquad h_f = (100)(0.0004769) = 0.048 \text{ ft}$$

9.267 A 36-in-diameter concrete pipe is 4000 ft long and has a head loss of 12.7 ft. Find the discharge capacity of water for this pipe by the Hazen–Williams formula

▮ $$v = 1.318CR^{0.63}s^{0.54} = (1.318)(120)[(\tfrac{36}{12})/4]^{0.63}(12.7/4000)^{0.54} = 5.906 \text{ ft/s}$$

$$Q = Av = [(\pi)(\tfrac{36}{12})^2/4](5.906) = 41.7 \text{ ft}^3/\text{s}$$

9.268 Solve Prob. 9.267 using the Manning formula.

▮ $$v = (1.486/n)(R)^{2/3}(s)^{1/2} = (1.486/0.013)[(\tfrac{36}{12})/4]^{2/3}(12.7/4000)^{1/2} = 5.317 \text{ ft/s}$$

$$Q = Av = [(\pi)(\tfrac{36}{12})^2/4](5.317) = 37.6 \text{ ft}^3/\text{s}$$

9.269 A 1-m-diameter new cast iron pipe is 845 m long and has a head loss of 1.11 m. Find the discharge capacity of water for this pipe by the Hazen–Williams formula.

▮ $$v = 0.8492CR^{0.63}s^{0.54} = (0.8492)(130)(\tfrac{1}{4})^{0.63}(1.11/845)^{0.54} = 1.281 \text{ m/s}$$

$$Q = Av = [(\pi)(1)^2/4](1.281) = 1.01 \text{ m}^3/\text{s}$$

9.270 Solve Prob. 9.269 using the Manning formula.

▮ $$v = (1.0/n)(R)^{2/3}(s)^{1/2} = (1.0/0.013)(\tfrac{1}{4})^{2/3}(1.11/845)^{1/2} = 1.106 \text{ m/s}$$

$$Q = Av = [(\pi)(1)^2/4](1.106) = 0.869 \text{ m}^3/\text{s}$$

9.271 A riveted steel pipe must transport 2.4 ft^3/s of water a distance of 190 ft with a head loss of 2.7 ft. Find the necessary pipe diameter using the Hazen–Williams formula.

▮ $$v = 1.318CR^{0.63}s^{0.54} = Q/A = 2.4/(\pi D^2/4) = 3.056/D^2$$

$$3.056/D^2 = (1.318)(110)(D/4)^{0.63}(2.7/190)^{0.54} \qquad D = 0.7695 \text{ ft} \quad \text{or} \quad 9.23 \text{ in}$$

9.272 A square concrete conduit must transport 4.0 m^3/s of water a distance of 45 m with a head loss of 1.80 m. Find the necessary conduit size using the Hazen–Williams formula.

▮ $v = 0.8492CR^{0.63}s^{0.54}$. Let a = length of conduit side. $v = Q/A = 4.0/a^2$, $4.0/a^2 = (0.8492)(120)(a^2/4a)^{0.63}(1.80/45)^{0.54}$, $a = 0.788$ m.

9.273 Solve Prob. 9.267 using the Hazen–Williams pipe diagram.

▮ Let h_1 = unit head loss. $h_1 = 12.7/4000 = 0.003175$. From Fig. A-13 with $h_1 = 0.003175$ and $D = 3$ ft, or 36 in, $Q = 41.5$ ft^3/s.

9.274 Solve Prob. 9.269 using the Hazen–Williams pipe diagram.

▮ $h_1 = 1.11/845 = 0.001314$. From Fig. A-14 with $h_1 = 0.001314$ and $D = 1$ m, or 1000 mm, $Q = 0.91$ m^3/s. This value of Q is for $C = 120$ (since the pipe diagram is for $C = 120$). Inasmuch as $C = 130$ for new cast iron (Table A-14), this value of Q must be adjusted. Since Q varies directly with C, $0.91/(Q)_{C=130} = \frac{120}{130}$, $(Q)_{C=130} = 0.99$ m^3/s.

9.275 Water is flowing in a 500-mm-diameter new cast iron pipe at a velocity of 2.0 m/s. Using the Hazen–Williams pipe diagram, find the pipe friction loss per 100 m of pipe.

▮ $C = 130$ for new cast iron pipe. In order to use the pipe diagram (Fig. A-14) for which $C = 120$, the given velocity must be adjusted for a value of C of 120. Since velocity varies directly with C, $2.0/(v)_{C=120} = \frac{130}{120}$, $(v)_{c=120} = 1.85$ m/s. From Fig. A-14 with $v = 1.85$ m/s and $D = 500$ mm, $h_1 = 0.0067$ m/m, or 0.67 m per 100 m. $p = \gamma h = (9.79)(0.67) = 6.6$ kPa per 100 m.

9.276 A new cast iron pipe must carry 30 cfs of water at a head loss of 19.0 ft per mile of pipe length. Find the required pipe diameter using the Hazen–Williams pipe diagram.

▮ $h_1 = 19.0/5280 = 0.003598$. $C = 130$ for new cast iron pipe. In order to use the pipe diagram (Fig. A-13) for which $C = 120$, the given discharge must be adjusted for a value of C of 120. Since discharge varies directly with C, $30/(Q)_{C=120} = \frac{130}{120}$, $(Q)_{C=120} = 27.7$ cfs. With $Q = 27.7$ cfs and $h_1 = 0.003598$, a pipe diameter of 30 in is determined from Fig. A-13.

9.277 Solve Prob. 9.270 using the Manning pipe diagram with $n = 0.012$.

▮ $h_1 = 1.11/845 = 0.001314$. With $D = 1$ m, or 1000 mm, and $h_1 = 0.001314$, a discharge of 0.88 m^3/s is determined from Fig. A-16. Inasmuch as Fig. A-16 is for $n = 0.013$ and n is 0.012 in this problem, this discharge must be adjusted. Since discharge varies inversely with n, $0.88/(Q)_{n=0.012} = 0.012/0.013$, $(Q)_{n=0.012} = 0.95$ m^3/s.

9.278 A concrete pipe must carry 80 cfs of water at a head loss of 1.5 ft per 100 ft of pipe length. Find the required pipe diameter using the Manning pipe diagram.

▮ $h_1 = 1.5/100 = 0.015$. With $Q = 80$ cfs and $h_1 = 0.015$, a pipe diameter of about 36 in is determined from Fig. A-15.

9.279 Solve Prob. 9.278 for the same given conditions except that the pipe has an n value of 0.015.

▮ $$h_1 = 0.015 \qquad \text{(from Prob. 9.278)}$$

The Manning pipe diagram (Fig. A-15) is for a value of n of 0.013. The given discharge (80 cfs) is for a value of n of 0.015. In order to use Fig. A-15, the given discharge must be adjusted for a value of n of 0.013. Since discharge varies inversely with n, $80/(Q)_{n=0.013} = 0.013/0.015$, $(Q)_{n=0.013} = 92.3$ cfs. With $Q = 92.3$ cfs and $h_1 = 0.015$, a pipe diameter of about 38 in is determined from Fig. A-15.

9.280 A capillary tube of inside diameter 6 mm connects tank A and open container B, as shown in Fig. 9-50. The liquid in A, B, and capillary CD is water having a specific weight of 9780 N/m^3 and a viscosity of 0.0008 kg/(m · s). The pressure $p_A = 34.5$ kPa gage. Which direction will the water flow? What is the flow rate?

▮ Assume laminar flow from B to A.

$$\frac{p_B}{\gamma} + \frac{v_B^2}{2g} + z_B = \frac{p_A}{\gamma} + \frac{v_A^2}{2g} + z_A + h_L \qquad h_f = \frac{128\mu LQ}{\pi\gamma D^4} = \frac{(128)(0.0008)(4.3)(Q)}{(\pi)(9780)(\frac{6}{1000})^4} = 11\,058Q$$

$$0 + 0 + (1.4 + 4.3 \sin 45°) = 34.5/9.79 + 0 + 1 + 11\,058Q$$

$$Q = -7.55 \times 10^{-6}\ m^3/s \quad \text{or} \quad -7.55 \times 10^{-3}\ L/s$$

Since Q is negative, the flow must be from A to B. Check for laminar flow: $v = Q/A = (7.55 \times 10^{-6})/[(\pi)(\frac{6}{1000})^2/4] = 0.2670$ m/s, $N_R = \rho Dv/\mu = (9780/9.807)(\frac{6}{1000})(0.2670)/0.0008 = 1997$ (laminar).

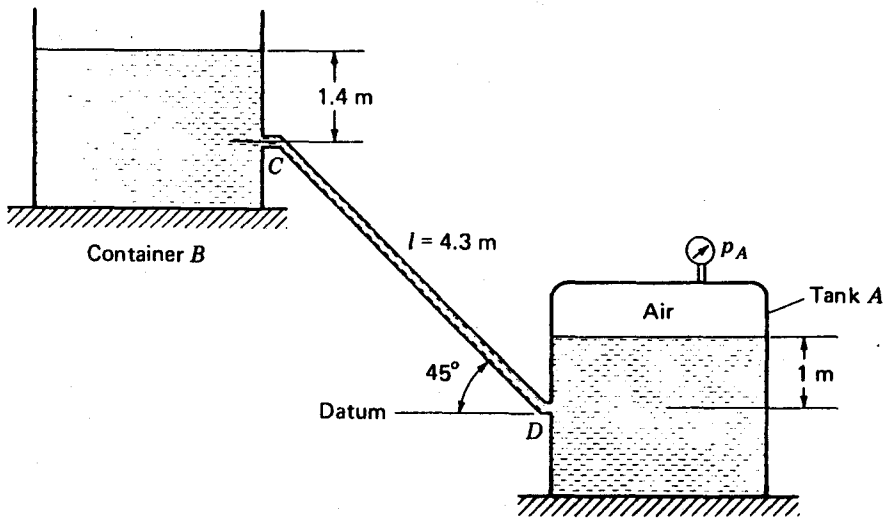


Fig. 9-50

9.281 An *equivalent length* of pipe is one whose head loss for the same value of flow is equal to that of some other system of different geometry for which it is equivalent. Consider a steel pipe of inside diameter 10.02 in having in it an open globe valve ($K = 4.8$) and four screwed 90° elbows ($K = 0.42$ each). The length of the pipe is 100 ft, and 5 ft^3/s of water at 60 °F flows through the pipe. What is the equivalent length of pipe with inside diameter 13.00 in?

▮ **For the given pipe:** $h_L = h_f + h_m$, $h_f = (f)(L/d)(v^2/2g)$, $v = Q/A = 5/[(\pi)(10.02/12)^2/4] = 9.131$ ft/s, $N_R = dV/\nu = (10.02/12)(9.131)/(1.21 \times 10^{-5}) = 6.30 \times 10^5$, $\epsilon/d = 0.00015/(10.02/12) = 0.00018$. From Fig. A-5, $f = 0.015$. $h_f = 0.015[100/(10.02/12)]\{9.131^2/[(2)(32.2)]\} = 2.33$ ft, $h_m = (K)(v^2/2g) = [4.8 + (4)(0.42)]\{9.131^2/[(2)(32.2)]\} = 8.39$ ft, $h_L = 2.33 + 8.39 = 10.72$ ft.

For the equivalent pipe: $v = 5/[(\pi)(13.00/12)^2/4] = 5.424$ ft/s, $N_R = (13.00/12)(5.424)/(1.21 \times 10^{-5}) = 4.86 \times 10^5$, $\epsilon/d = 0.00015/(13.00/12) = 0.000138$, $f = 0.015$, $10.72 = 0.015[L_e/(13.00/12)]\{5.424^2/[(2)(32.2)]\}$, $L_e = 1695$ ft.

9.282 A 100-ft-long duct transports air at 50 °F (assumed constant) at the rate of 8000 ft^3/min. The duct is made of galvanized iron and has a cross section of 2 ft by 1 ft. Find the outlet pressure, if the inlet pressure is 16.7 psia.

▮ Assume incompressible, isothermal, fully turbulent flow.

$$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L \qquad h_L = h_f = (f)(L/d)(v^2/2g) \qquad N_R = \rho d v/\mu$$

$$\rho = p/RT = (16.7)(144)/[(1716)(460 + 50)] = 0.002748 \text{ slug/ft}^3 \qquad \gamma = \rho g = (0.002748)(32.2) = 0.08849 \text{ lb/ft}^3$$

$$d = D_h = 4A/p_w = 4[(2)(1)]/[(2)(2 + 1)] = 1.333 \text{ ft} \qquad v = Q/A = (\tfrac{8000}{60})/[(2)(1)] = 66.67 \text{ ft/s}$$

$$N_R = (0.002748)(1.333)(66.67)/(3.68 \times 10^{-7}) = 6.64 \times 10^5 \quad \text{(O.K.)} \qquad \epsilon/d = 0.0005/1.333 = 0.000375$$

From Fig. A-5, $f = 0.0165$.

$$h_L = (0.0165)(100/1.333)\{66.67^2/[(2)(32.2)]\} = 85.43 \text{ ft}$$

$$p_1/0.08849 + v_1^2/2g + 0 = p_2/0.08849 + v_2^2/2g + 85.43$$

$$v_1^2/2g = v_2^2/2g \qquad p_1 - p_2 = 7.56 \text{ lb/ft}^2 \qquad p_2 = 16.7 - 7.56 = 9.1 \text{ psia}$$

9.283 In a heating system, there is a run of insulated duct of 50 m carrying air at a temperature of 35 °C at a pressure at the inlet of 100 kPa. The duct has a rectangular cross section of 650 mm by 320 mm. If there is a pressure drop from inlet to outlet of 5 mm of mercury, what is the volumetric flow? For such a small pressure drop in the duct, treat the flow as incompressible. Take $R = 287$ J/(kg · K). The duct is galvanized iron. Consider that the flow is entirely turbulent.

▮ $$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L \qquad h_L = h_f = (f)(L/d)(v^2/2g) \qquad N_R = \rho d v/\mu$$

$$\rho = p/RT = (100)(1000)/[(287)(273 + 35)] = 1.131 \text{ kg/m}^3 \qquad \gamma = \rho g = (1.131)(9.807) = 11.09 \text{ N/m}^3$$

$$d = D_h = 4A/p_w = 4[(\tfrac{650}{1000})(\tfrac{320}{1000})]/[(2)(\tfrac{650}{1000} + \tfrac{320}{1000})] = 0.4289 \text{ m} \qquad h_L = (f)(50/0.4289)\{v^2/[(2)(9.807)]\} = 5.944 f v^2$$

$$p_1 - p_2 = \tfrac{5}{1000}[(13.6)(9.79)] = 0.6657 \text{ kPa} \quad \text{or} \quad 665.7 \text{ Pa} \qquad p_1/\gamma - p_2/\gamma + v_1^2/2g + z_1 = v_2^2/2g + z_2 + h_L$$

$$v_1^2/2g = v_2^2/2g \qquad 665.7/11.09 + 0 = 0 + 5.944 f v^2 \qquad v = 3.178/\sqrt{f}$$

Try $f = 0.016$: $v = 3.178/\sqrt{0.016} = 25.12$ m/s, $N_R = (1.131)(0.4289)(25.12)/(1.88 \times 10^{-5}) = 6.48 \times 10^5$, $\epsilon/d = 0.00015/0.4289 = 0.000350$. From Fig. A-5, $f = 0.0165$. Try $f = 0.0165$: $v = 3.178/\sqrt{0.0165} = 24.74$ m/s, $N_R = (1.131)(0.4289)(24.74)/(1.88 \times 10^{-5}) = 6.38 \times 10^5$, $f = 0.0165$ (O.K.); $Q = Av = [(\tfrac{650}{1000})(\tfrac{320}{1000})](24.74) = 5.15$ m^3/s.

9.284 Water at 60 °F flows through a smooth pipe of 12 in diameter at 9 ft^3/s. Estimate the shear stress at the wall and the thickness of the viscous sublayer.

▮ $$v = Q/A = 9/[(\pi)(\tfrac{12}{12})^2/4] = 11.46 \text{ ft/s}$$

$$\tau_0 = 0.03325 \rho v^2 (\nu/Rv)^{1/4} = (0.03325)(1.94)(11.46)^2\{(1.21 \times 10^{-5})/[(\tfrac{6}{12})(11.46)]\}^{1/4} = 0.323 \text{ lb/ft}^2$$

$$\delta_1 = 5\nu/v_* = 5\nu\sqrt{\rho/\tau_0} = (5)(1.21 \times 10^{-5})\sqrt{1.94/0.323} = 0.000148 \text{ ft} \quad \text{or} \quad 0.00178 \text{ in}$$

9.285 Solve Prob. 9.284 if the shear stress is given by $\tau_0 = (f/4)(\rho v^2/2)$.

▮ $$N_R = dv/\nu = (\tfrac{12}{12})(11.46)/(1.21 \times 10^{-5}) = 9.47 \times 10^5$$

From Fig. A-5, $f = 0.0118$.

$$\tau_0 = (0.0118/4)[(1.94)(11.46)^2/2] = 0.376 \text{ lb/ft}^2$$

$$\delta_1 = 5\nu/v_* = 5\nu\sqrt{\rho/\tau_0} = (5)(1.21 \times 10^{-5})\sqrt{1.94/0.376} = 0.0001374 \text{ ft} \quad \text{or} \quad 0.00165 \text{ in}$$

9.286 What head H is needed in Fig. 9-51 to produce a discharge of 0.3 m³/s? Use minor loss coefficients of 0.5 for the entrance and 0.12 for the diffusers.

▮ $p_A/\gamma + v_A^2/2g + z_A = p_B/\gamma + v_B^2/2g + z_B + h_L \qquad h_L = h_f + h_m \qquad h_f = (f)(L/d)(v^2/2g) \qquad N_R = dv/(\mu/\rho)$

$v_1 = Q/A_1 = 0.3/[(\pi)(0.210)^2/4] = 9.549$ m/s $\qquad (N_R)_1 = (0.210)(9.549)/[(0.04)(0.1)/1000] = 5.01 \times 10^5$

From Fig. A-5, $f_1 = 0.013$.

$v_2 = Q/A_2 = 0.3/[(\pi)(0.305)^2/4] = 4.244$ m/s $\qquad (N_R)_2 = (0.305)(4.244)/[(0.04)(0.1)/1000] = 3.24 \times 10^5$

$f_2 = 0.014 \qquad h_f = 0.013[30/(0.210)]\{9.549^2/[(2)(9.907)]\} + 0.014[60/(0.305)]\{4.244^2/[(2)(9.907)]\} = 11.64$ m

For entrance: $h_m = (K)(v^2/2g)$, $(h_m)_1 = 0.5\{9.549^2/[(2)(9.907)]\} = 2.32$ m.

For first diffuser: $h_m = (K)(v_1 - v_2)^2/2g$, $(h_m)_2 = (0.12)(9.549 - 4.244)^2/[(2)(9.807)] = 0.17$ m.

For second diffuser: $v_3 = Q/A_3 = 0.3/[(\pi)(0.450)^2/4] = 1.886$ m/s, $(h_m)_3 = (0.12)(4.244 - 1.886)^2/[(2)(9.807)] = 0.03$ m.

For exit: $(h_m)_4 = 1.0\{1.886^2/[(2)(9.807)]\} = 0.18$ m,
$H = h_L = 11.64 + 2.32 + 0.17 + 0.03 + 0.18 = 14.34$ m.

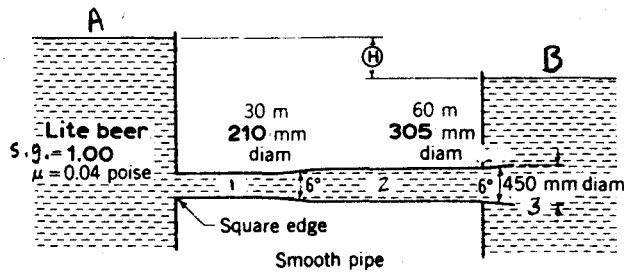


Fig. 9-51

9.287 Calculate the discharge through the siphon of Fig. 9-52 ($H = 4$ ft) without the conical diffuser. Use minor loss coefficients of 1.0 for both the entrance and exit, and 0.9 for each elbow.

▮ $$p_A/\gamma + v_A^2/2g + z_A = p_B/\gamma + v_B^2/2g + z_B + h_L$$

$$h_f = (f)(L/d)(v^2/2g) = f[(10 + 18 + 12)/(\tfrac{8}{12})]\{v^2/[(2)(32.2)]\} = 0.9317fv^2$$

$$h_m = (K)(v^2/2g) = [(1.0 + (2)(0.9) + 1.0]\{v^2/[(2)(32.2)]\} = 0.05901v^2 \qquad h_L = h_f + h_m = 0.9317fv^2 + 0.05901v^2$$

$$0 + 0 + 4 = 0 + 0 + 0 + 0.9317fv^2 + 0.05901v^2 \qquad v = \sqrt{4/(0.9317f + 0.05901)}$$

Try $f = 0.016$: $v = \sqrt{4/[(0.9317)(0.016) + 0.05901]} = 7.356$ ft/s, $N_R = dv/\nu = (\tfrac{8}{12})(7.356)/(1.21 \times 10^{-5}) = 4.05 \times 10^5$, $\epsilon/d = 0.00015/(\tfrac{8}{12}) = 0.000225$. From Fig. A-5, $f = 0.016$ (O.K.). $Q = Av = [(\pi)(\tfrac{8}{12})^2/4](7.356) = 2.57$ ft³/s.

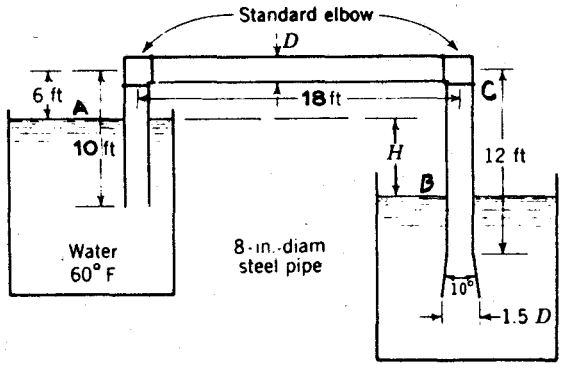


Fig. 9-52

9.288 Calculate the discharge in the siphon of Fig. 9-52 for $H = 8$ ft and with diffuser included. Use minor loss coefficients of 1.0 for both the entrance and exit, 0.9 for each elbow, and 0.15 for the diffuser.

▮
$$p_A/\gamma + v_A^2/2g + z_A = p_B/\gamma + v_B^2/2g + z_B + h_L \qquad (1)$$

$$h_L = h_f + h_m \qquad h_f = (f)(L/d)(v^2/2g) = f[(10+18+12)/(\tfrac{8}{12})]\{v_1^2/[(2)(32.2)]\} = 0.9317fv_1^2$$

$h_m = (K)(v^2/2g)$ for entrance, exit, and elbows. $h_m = (K)(v_1 - v_2)^2/2g$ for diffuser.

$$A_1v_1 = A_2v_2 \qquad [(\pi)(\tfrac{8}{12})^2/4](v_1) = \{(\pi)[(1.5)(\tfrac{8}{12})]^2/4\}(v_2) \qquad v_2 = 0.4444v_1$$

$$h_m = [1.0 + (2)(0.9)]\{v_1^2/[(2)(32.2)]\} + 0.15(v_1 - 0.4444v_1)^2/[(2)(32.2)]$$
$$+ 1.0\{(0.4444v_1)^2/[(2)(32.2)]\} = 0.04726v_1^2$$

$$h_L = 0.9317fv_1^2 + 0.04726v_1^2 \qquad 0+0+8 = 0+0+0+0.9317fv^2 + 0.04726v^2$$
$$v = \sqrt{8/(0.9317f + 0.04726)}$$

Try $f = 0.015$: $v = \sqrt{8/[(0.9317)(0.015) + 0.04726]} = 11.43$ ft/s, $N_R = dv/\nu = (\tfrac{8}{12})(11.43)/(1.21 \times 10^{-5}) = 6.30 \times 10^5$, $\epsilon/d = 0.00015/(\tfrac{8}{12}) = 0.000225$. From Fig. A-5, $f = 0.0155$. Try $f = 0.0155$: $v = \sqrt{8/[(0.9317)(0.0155) + 0.04726]} = 11.39$ ft/s, $N_R = (\tfrac{8}{12})(11.39)/(1.21 \times 10^{-5}) = 6.28 \times 10^5$, $f = 0.0155$ (O.K.); $Q = Av = [(\pi)(\tfrac{8}{12})^2/4](11.39) = 3.98$ ft³/s.

9.289 Find the discharge through the siphon of Fig. 9-53, as well as the minimum pressure in the system. Use minor loss coefficients of 1.0 for the entrance, 0.1 for the nozzle, and 2.2 for the bend.

▮ Let v = velocity of water in the pipe and v_2 = velocity through the nozzle. Since velocity varies with the square of diameter, $v_2 = 4v$.

$$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L \qquad (1)$$

$$h_f = (f)(L/d)(v^2/2g) = f[(6+1.8+3.6)/0.100]\{v^2/[(2)(9.807)]\} = 5.812fv^2$$

$$h_m = (K)(v^2/2g) = (1.0+2.2)\{v^2/[(2)(9.807)]\} + (0.1)\{v_2^2/[(2)(9.807)]\}$$
$$= (1.0+2.2)\{v^2/[(2)(9.807)]\} + (0.1)\{(4v)^2/[(2)(9.807)]\} = 0.2447v^2$$

$$h_L = h_f + h_m = 5.812fv^2 + 0.2447v^2 \qquad 0+0+(6-1.8) = 0+(4v)^2/[(2)(9.807)]$$
$$+ (6-1.8-3.6) + 5.812fv^2 + 0.2447v^2 \qquad v = \sqrt{3.6/(5.812f + 1.060)}$$

Try $f = 0.03$: $v = \sqrt{3.6/[(5.812)(0.03) + 1.060]} = 1.708$ m/s, $N_R = \rho dv/\mu = [(0.81)(1000)](0.100)(1.708)/0.01 = 1.38 \times 10^4$. From Fig. A-5, $f = 0.028$. Try $f = 0.028$: $v = \sqrt{3.6/[(5.812)(0.028) + 1.060]} = 1.716$ m/s, $N_R = [(0.81)(1000)](0.100)(1.716)/0.01 = 1.38 \times 10^4$, $f = 0.028$ (O.K.); $Q = Av = [(\pi)(0.100)^2/4](1.716) = 0.0135$ m³/s, or 13.5 L/s.

Minimum pressure would occur at point 3. Apply Eq. (*1*) between points 1 and 3: $h_f = 0.028[6/0.100]\{1.716^2/[(2)(9.807)]\} = 0.2522$ m, $h_m = (1.0+2.2)\{1.716^2/[(2)(9.807)]\} = 0.4804$ m, $h_L = 0.2522 + 0.4804 = 0.7326$ m, $0+0+0 = p_3/[(0.81)(9.79)] + 1.716^2/[(2)(9.807)] + 1.8 + 0.7326$, $p_3 = -21.3$ kPa.

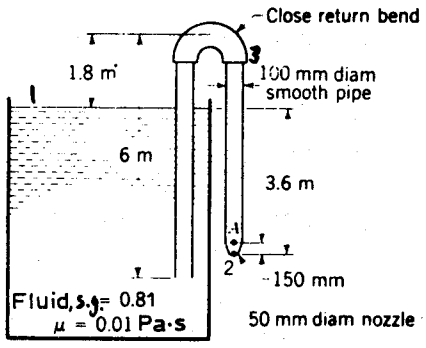


Fig. 9-53

9.290 With reference to Fig. 9-54, what is the maximum height of point A for no cavitation? Barometer reading is 29.5 in of mercury. Use minor loss coefficients of 1.0 for both entrance and exit, and 4.5 for the globe valve.

▮ First calculate the flow velocity v.

$$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L \qquad (1)$$

$$h_f = (f)(L/d)(v^2/2g) = f[(100+70+105)/(\tfrac{8}{12})]\{v^2/[(2)(32.2)]\} = 6.405fv^2$$

$$h_m = (K)(v^2/2g) = (1.0+4.5+1.0)\{v^2/[(2)(32.2)]\} = 0.1009v^2$$

$$h_L = h_f + h_m = 6.405fv^2 + 0.1009v^2 \qquad 0+0+12 = 0+0+0+6.405fv^2+0.1009v^2$$

$$v = \sqrt{12/(6.405f + 0.1009)}$$

Try $f = 0.013$: $v = \sqrt{12/[(6.405)(0.013)+0.1009]} = 8.072$ ft/s, $N_R = dv/\nu = (\frac{8}{12})(8.072)/(1.21\times10^{-5}) = 4.45\times10^5$. From Fig. A-5, $f = 0.0135$. Try $f = 0.0135$: $v = \sqrt{12/[(6.405)(0.0135)+0.1009]} = 8.003$ ft/s, $N_R = (\frac{8}{12})(8.003)/(1.21\times10^{-5}) = 4.41\times10^5$, $f = 0.0135$ (O.K.). Now, to find the maximum height of point A, apply Eq. (1) between points 1 and A: $h_f = 0.0135[100/(\frac{8}{12})]\{8.003^2/[(2)(32.2)]\} = 2.014$ ft, $h_m = 0$, $h_L = 2.014 + 0 = 2.014$ ft. From Table A-1, $p_v = 36.5$ lb/ft^2. $p_{atm} = [(13.6)(62.4)](29.5/12) = 2086.2$ lb/ft^2. At cavitation, $p = (2086.2 - 36.5)/62.4 = 32.85$ ft (vacuum), or -32.85 ft, $0+0+0 = -32.85 + 0.995 + z_A + 2.014$, $z_A = 29.8$ ft. (z_A is the height of point A above the water surface in the left reservoir.)

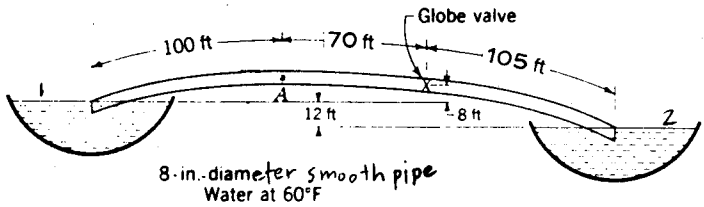


Fig. 9-54

9.291 What diameter smooth pipe is required to convey 8 L/s of kerosene ($\nu = 1.93\times10^{-6}$ m^2/s) at 32 °C, 150 m with a head of 5 m? There are a valve and other minor losses with total K of 7.6.

▮

$$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L \qquad v = Q/A = (\tfrac{8}{1000})/(\pi D^2/4) = 0.01019/D^2$$

$$h_f = (f)(L/d)(v^2/2g) = (f)(150/D)\{(0.01019/D^2)^2/[(2)(9.807)]\} = 0.0007941f/D^5$$

$$h_m = (K)(v^2/2g) = 7.6\{(0.01019/D^2)^2/[(2)(9.807)]\} = 0.00004023/D^4$$

$$h_L = h_f + h_m = 0.0007941f/D^5 + 0.00004023/D^4 \qquad p_1/\gamma + z_1 - p_2/\gamma - z_2 = 5$$

$$v_1^2/2g = v_2^2/2g \qquad 5 = 0.0007941f/D^5 + 0.00004023/D^4$$

Try $f = 0.02$: $5 = (0.0007941)(0.02)/D^5 + 0.00004023/D^4$. By trial and error, $D = 0.0826$ m. $N_R = Dv/\nu = (0.0826)(0.01019/0.0826^2)/(1.93\times10^{-6}) = 6.39\times10^4$. From Fig. A-5, $f = 0.0195$. Try $f = 0.0195$: $5 = (0.0007941)(0.0195)/D^5 + 0.00004023/D^4$, $D = 0.0822$ m, $N_R = (0.0822)(0.01019/0.0822^2)/(1.93\times10^{-6}) = 6.42\times10^4$, $f = 0.0195$. Therefore, $D = 0.0822$ m, or 82.2 mm.

9.292 Find the value of the Hazen–Williams coefficient for the water flow in Prob. 9.130.

▮

$$v = 1.318CR^{0.63}s^{0.54} = Q/A = 3.2/[(\pi)(\tfrac{24}{12})^2/4] = 1.0186 \text{ ft/s}$$

$$1.0186 = (1.318)(C)[(\tfrac{24}{12})/4]^{0.63}(0.0003/1)^{0.54} \qquad C = 95.5$$

9.293 Find the value of the Hazen–Williams coefficient for the case where water flows at 0.20 m^3/s in a 1-m-diameter pipeline with a head loss of 0.0012 m/m.

▮

$$v = 0.8492CR^{0.63}s^{0.54} = Q/A = 0.20/[(\pi)(1)^2/4] = 0.2546 \text{ m/s}$$

$$0.2546 = (0.8492)(C)[1/4]^{0.63}(0.0012)^{0.54} \qquad C = 27.1$$

9.294 Manning's n equals 0.0132 when water at 50 °F flows at a Reynolds number of 1.10×10^7 through a tunnel that is 16 ft in diameter. Determine the average value of ϵ.

▮

$$v = \left(\frac{1.486}{n}\right)(R^{2/3})(s^{1/2}) \qquad s = \left(\frac{nv}{1.486R^{2/3}}\right)^2 \qquad h_L = (f)\left(\frac{L}{d}\right)\left(\frac{v^2}{2g}\right) \qquad \frac{h_L}{L} = \left(\frac{f}{d}\right)\left(\frac{v^2}{2g}\right)$$

Equating s and h_L/L gives

$$\left(\frac{nv}{1.486R^{2/3}}\right)^2 = \left(\frac{f}{d}\right)\left(\frac{v^2}{2g}\right) \qquad \left[\frac{0.0132v}{(1.486)(\frac{16}{4})^{2/3}}\right]^2 = \left(\frac{f}{16}\right)\left\{\frac{v^2}{(2)(32.2)}\right\} \qquad f = 0.0128$$

From Fig. A-5 with $f = 0.0128$ and $N_R = 1.10 \times 10^7$, $\epsilon/d = 0.00015$; $\epsilon = (16)(0.00015) = 0.00240$ ft, or 0.0288 in.

9.295 Prepare a computer program that will determine for a single pipe with constant diameter either the flow rate or the required conduit diameter for closed conduit, incompressible flow. The program must be usable for problems in both the USCS and the SI.

```
C     THIS PROGRAM DETERMINES THE FLOW RATE OR THE REQUIRED CONDUIT
C     DIAMETER FOR CLOSED CONDUIT, INCOMPRESSIBLE FLOW.  IT CAN BE USED
C     FOR PROBLEMS IN BOTH THE ENGLISH SYSTEM OF UNITS AND THE INTER-
C     NATIONAL SYSTEM OF UNITS.  THE APPLICATION OF THE PROGRAM IS
C     LIMITED TO CASES INVOLVING A SINGLE PIPE WITH CONSTANT DIAMETER.
C
C     THIS PROGRAM IS BASED ON THE SOLUTION OF THE BERNOULLI EQUATION;
C     ACCORDINGLY, CERTAIN DATA MUST BE ENTERED FOR EACH OF TWO POINTS
C     IN A PIPE SYSTEM.  HOWEVER, BOTH THE VELOCITY AT POINT 1 AND THE
C     VELOCITY AT POINT 2 ARE NOT CONSIDERED AS KNOWN VALUES INITIALLY.
C     IF EITHER VALUE OF VELOCITY IS ESSENTIALLY ZERO (SUCH AS AT A
C     RESERVOIR OR TANK SURFACE), ENTER 0 (ZERO) FOR THAT VELOCITY (OR
C     FOR BOTH VELOCITIES IF BOTH ARE ESSENTIALLY ZERO).  OTHERWISE,
C     ENTER 1.0 FOR ALL OTHER VELOCITIES.
C
C     INPUT DATA MUST BE SET UP AS FOLLOWS.
C
C     CARD 1   COLUMN 1          ENTER 0 (ZERO) OR BLANK IF ENGLISH SYSTEM
C                                OF UNITS IS TO BE USED.  ENTER 1 (ONE)
C                                IF INTERNATIONAL SYSTEM OF UNITS IS TO
C                                BE USED.
C              COLUMNS 2-79      ENTER TITLE, DATE, AND OTHER INFORMATION,
C                                IF DESIRED.
C     CARD 2   COLUMNS 1-10      ENTER NUMBER INCLUDING DECIMAL GIVING
C                                GAUGE PRESSURE AT POINT 1  (IN POUNDS
C                                PER SQUARE INCH OR KILOPASCALS).
C              COLUMNS 11-20     ENTER NUMBER INCLUDING DECIMAL GIVING
C                                GAUGE PRESSURE AT POINT 2  (IN POUNDS
C                                PER SQUARE INCH OR KILOPASCALS).
C              COLUMNS 21-30     ENTER 0 (ZERO) OR BLANK IF VELOCITY AT
C                                POINT 1 IS ESSENTIALLY ZERO; OTHERWISE,
C                                ENTER 1.0.
C              COLUMNS 31-40     ENTER 0 (ZERO) OR BLANK IF VELOCITY AT
C                                POINT 2 IS ESSENTIALLY ZERO; OTHERWISE,
C                                ENTER 1.0.
C              COLUMNS 41-50     ENTER NUMBER INCLUDING DECIMAL GIVING
C                                ELEVATION AT POINT 1 (IN FEET OR METERS).
C              COLUMNS 51-60     ENTER NUMBER INCLUDING DECIMAL GIVING
C                                ELEVATION AT POINT 2 (IN FEET OR METERS).
C              COLUMNS 61-70     ENTER NUMBER INCLUDING DECIMAL GIVING
C                                ACTUAL ENERGY ADDED (IN HORSEPOWER OR
C                                KILOWATTS).
C              COLUMNS 71-80     ENTER NUMBER INCLUDING DECIMAL GIVING
C                                ACTUAL ENERGY REMOVED (IN HORSEPOWER OR
C                                KILOWATTS).
C     CARD 3   COLUMNS 1-10      ENTER NUMBER INCLUDING DECIMAL GIVING
C                                TOTAL "MINOR LOSSES" (IN FEET OR METERS).
C              COLUMNS 11-20     ENTER NUMBER INCLUDING DECIMAL GIVING
C                                DIAMETER OF CONDUIT (IN INCHES OR
C                                MILLIMETERS).
C              COLUMNS 21-30     ENTER NUMBER INCLUDING DECIMAL GIVING
C                                LENGTH OF CONDUIT (IN FEET OR METERS).
C              COLUMNS 31-40     ENTER NUMBER INCLUDING DECIMAL GIVING
C                                KINEMATIC VISCOSITY OF FLUID (IN SQUARE
C                                FEET PER SECOND OR SQUARE METERS PER
C                                SECOND).
C              COLUMNS 41-50     ENTER NUMBER INCLUDING DECIMAL GIVING
C                                ROUGHNESS OF CONDUIT MATERIAL (IN FEET
C                                OR METERS).  ENTER A VALUE OF 0 (ZERO)
C                                FOR "SMOOTH" CONDUITS.
```

```
C              COLUMNS 51-60   ENTER NUMBER INCLUDING DECIMAL GIVING
C                              SPECIFIC (OR UNIT) WEIGHT OF FLUID (IN
C                              POUNDS PER CUBIC FOOT OR KILO-
C                              NEWTONS PER CUBIC METER).
C              COLUMNS 61-70   ENTER NUMBER INCLUDING DECIMAL GIVING
C                              FLOW RATE OF FLUID (IN CUBIC FEET PER
C                              SECOND OR CUBIC METERS PER SECOND).
C     CARD 4   COLUMNS 1-24    ENTER TYPE OF FLUID
C              COLUMNS 25-48   ENTER TYPE OF CONDUIT.
C
C ***********************************************************************
C *                                                                     *
C *   NOTE WELL....EITHER THE FLOW RATE (COLUMNS 61-70) OR THE          *
C *   CONDUIT DIAMETER (COLUMNS 11-20), WHICHEVER ONE IS TO BE DETER-   *
C *   MINED BY THIS PROGRAM, SHOULD BE LEFT BLANK ON CARD 3.            *
C *                                                                     *
C ***********************************************************************
C
C     MULTIPLE DATA SETS FOR SOLVING ANY NUMBER OF PROBLEMS MAY BE
C     INCLUDED FOR PROCESSING.
C
      REAL L
      DIMENSION TITLE(13),FLUID(4),PIPE(4)
      INTEGER UNITS
      COMMON F,ED,RN
      PI=3.14159265
1     READ(5,100,END=2)UNITS,TITLE
      FACTOR=12.0
      IF(UNITS.EQ.1)FACTOR=1000.0
  100 FORMAT(I1,13A6)
      WRITE(6,101)TITLE
  101 FORMAT('1',13A6,////)
      READ(5,102)P1,P2,V1,V2,Z1,Z2,HA,HR,HM,D,L,VIS,E,SW,Q,FLUID,PIPE
  102 FORMAT(8F10.0/7F10.0/8A6)
      P1SW=P1/SW*FACTOR**2
      IF(UNITS.EQ.1)P1SW=P1/SW
      G=32.2
      IF(UNITS.EQ.1)G=9.807
      P2SW=P2/SW*FACTOR**2
      IF(UNITS.EQ.1)P2SW=P2/SW
      FF=0.02
      IF(Q.GT.0.0001)GO TO 117
      IF(V1.GT.0.0001)V1=1.0/2.0/G
      IF(V2.GT.0.0001)V2=1.0/2.0/G
  105 HF=FF*L/D*FACTOR/2.0/G
      HAT=550.*HA/SW
      IF(UNITS.EQ.1)HAT=HA/SW
      HRT=550.*HR/SW
      IF(UNITS.EQ.1)HRT=HR/SW
      Q=0.001
      VTRY=(Q/(PI*(D/FACTOR)**2/4.0))**2
      TRY1=P1SW+VTRY*V1+Z1+HAT/Q-HRT/Q-(P2SW+VTRY*V2+Z2+HF*VTRY)
  116 Q=Q+0.001
      VTRY=(Q/(PI*(D/FACTOR)**2/4.0))**2
      TRY2=P1SW+VTRY*V1+Z1+HAT/Q-HRT/Q-(P2SW+VTRY*V2+Z2+HF*VTRY)
      IF(TRY1*TRY2)114,114,115
  115 TRY1=TRY2
      GO TO 116
  114 Q=Q-0.0005
      V=Q/(PI*(D/FACTOR)**2/4.0)
      RN=D/FACTOR*V/VIS
      ED=E/D*FACTOR
      CALL ROUGH
      DIFF=ABS(F-FF)
      IF(DIFF.LT.0.0001)GO TO 104
      FF=F
      GO TO 105
  104 IF(V1.GT.0.0001)V1=V
      IF(V2.GT.0.0001)V2=V
      IF(UNITS.EQ.0)WRITE(6,106)P1,P2,Z1,Z2,HA,HR,HM,D,L,FLUID,PIPE,Q,
     *V1,V2
  106 FORMAT(1X,'GIVEN DATA FOR A CIRCULAR CLOSED CONDUIT CARRYING INCOM
     *PRESSIBLE FLOW',//5X,'PRESSURE AT POINT 1 =',F7.1,' PSI',//5X,'PRE
```

```
     *SSURE AT POINT 2 =',F7.1,' PSI',//5X,'ELEVATION AT POINT 1 =',
     *F7.1,' FT',//5X,'ELEVATION AT POINT 2 =',F7.1,' FT',//5X,'ACTUAL E
     *NERGY ADDED BETWEEN POINTS 1 AND 2 =',F5.1,' HP',//5X,'ACTUAL ENER
     *GY REMOVED BETWEEN POINTS 1 AND 2 =',F5.1,' HP',//5X,'MINOR HEAD L
     *OSSES BETWEEN POINTS 1 AND 2 =',F5.1,' FT',//5X,'DIAMETER OF CONDU
     *IT =',F6.2,' IN',//5X,'LENGTH OF CONDUIT =',F8.1,' FT',//5X,'FLUID
     * FLOWING IS ',4A6,//5X,'CONDUIT MATERIAL IS ',4A6,////1X,'THE FLOW
     * RATE WILL BE',F7.1,' CU FT/S',//5X,'VELOCITY AT POINT 1 =',F6.2,
     *' FT/S',//5X,'VELOCITY AT POINT 2 =',F6.2,' FT/S')
      IF(UNITS.EQ.1)WRITE(6,107)P1,P2,Z1,Z2,HA,HR,HM,D,L,FLUID,PIPE,Q,
     *V1,V2
107   FORMAT(1X,'GIVEN DATA FOR A CIRCULAR CLOSED CONDUIT CARRYING INCOM
     *PRESSIBLE FLOW',//5X,'PRESSURE AT POINT 1 =',F7.1,' KPA',//5X,'PRE
     *SSURE AT POINT 2 =',F7.1,' KPA',//5X,'ELEVATION AT POINT 1 =',
     *F7.1,' M ',//5X,'ELEVATION AT POINT 2 =',F7.1,' M ',//5X,'ACTUAL E
     *NERGY ADDED BETWEEN POINTS 1 AND 2 =',F5.1,' KW',//5X,'ACTUAL ENER
     *GY REMOVED BETWEEN POINTS 1 AND 2 =',F5.1,' KW',//5X,'MINOR HEAD L
     *OSSES BETWEEN POINTS 1 AND 2 =',F5.1,' M ',//5X,'DIAMETER OF CONDU
     *IT =',F7.1,' MM',//5X,'LENGTH OF CONDUIT =',F8.1,' M ',//5X,'FLUID
     * FLOWING IS ',4A6,//5X,'CONDUIT MATERIAL IS ',4A6,////1X,'THE FLOW
     * RATE WILL BE',F7.3,' CU M/S',//5X,'VELOCITY AT POINT 1 =',F6.2,
     *' M/S ',//5X,'VELOCITY AT POINT 2 =',F6.2,' M/S ')
      GO TO 1
117   V1=V1*(Q/PI*4.0)
      V2=V2*(Q/PI*4.0)
103   HF=FF*L*(Q/PI*4.0)**2/2.0/G
      HAT=550.*HA/SW/Q
      IF(UNITS.EQ.1)HAT=HA/SW/Q
      HRT=550.*HR/SW/Q
      IF(UNITS.EQ.1)HRT=HR/SW/Q
      D=0.001
      TRY1=HF/D**5+(V2**2/2.0/G)/D**4-(V1**2/2.0/G)/D**4-P1SW-Z1-HAT
     *+HRT*P2SW+Z2+HM
110   D=D+0.001
      TRY2=HF/D**5+(V2**2/2.0/G)/D**4-(V1**2/2.0/G)/D**4-P1SW-Z1-HAT
     *+HRT+P2SW+Z2+HM
      IF(TRY1*TRY2)108,108,109
109   TRY1=TRY2
      GO TO 110
108   D=D-0.0005
      RN=D*Q/(PI*D**2/4.0)/VIS
      ED=E/D
      CALL ROUGH
      DIFF=ABS(F-FF)
      IF(DIFF.LT.0.0001)GO TO 111
      FF=F
      GO TO 103
111   V1=V1/D**2
      V2=V2/D**2
      D=D*FACTOR
      IF(UNITS.EQ.0)WRITE(6,112)P1,P2,Z1,Z2,HA,HR,HM,Q,L,FLUID,PIPE,D,
     *V1,V2
112   FORMAT(1X,'GIVEN DATA FOR A CIRCULAR CLOSED CONDUIT CARRYING INCOM
     *PRESSIBLE FLOW',//5X,'PRESSURE AT POINT 1 =',F7.1,' PSI',//5X,'PRE
     *SSURE AT POINT 2 =',F7.1,' PSI',//5X,'ELEVATION AT POINT 1 =',
     *F7.1,' FT',//5X,'ELEVATION AT POINT 2 =',F7.1,' FT',//5X,'ACTUAL E
     *NERGY ADDED BETWEEN POINTS 1 AND 2 =',F5.1,' HP',//5X,'ACTUAL ENER
     *GY REMOVED BETWEEN POINTS 1 AND 2 =',F5.1,' HP',//5X,'MINOR LOSSES
     * BETWEEN POINTS 1 AND 2 =',F5.1,' FT',//5X,'FLOW RATE =',F7.1,
     *' CU FT/S',//5X,'LENGTH OF CONDUIT =',F8.1,' FT',//5X,'FLUID FLOWI
     *NG IS ',4A6,//5X,'CONDUIT MATERIAL IS ',4A6,///1X,'THE CONDUIT DIA
     *METER REQUIRED WILL BE',F6.2,' IN',//5X,'VELOCITY AT POINT 1 =',
     *F6.2,' FT/S',//5X,'VELOCITY AT POINT 2 =',F6.2,' FT/S')
      IF(UNITS.EQ.1)WRITE(6,113)P1,P2,Z1,Z2,HA,HR,HM,Q,L,FLUID,PIPE,D,
     *V1,V2
113   FORMAT(1X,'GIVEN DATA FOR A CIRCULAR CLOSED CONDUIT CARRYING INCOM
     *PRESSIBLE FLOW',//5X,'PRESSURE AT POINT 1 =',F7.1,' KPA',//5X,'PRE
     *SSURE AT POINT 2 =',F7.1,' KPA',//5X,'ELEVATION AT POINT 1 =',
     *F7.1,' M ',//5X,'ELEVATION AT POINT 2 =',F7.1,' M ',//5X,'ACTUAL E
     *NERGY ADDED BETWEEN POINTS 1 AND 2 =',F5.1,' KW',//5X,'ACTUAL ENER
     *GY REMOVED BETWEEN POINTS 1 AND 2 =',F5.1,' KW',//5X,'MINOR LOSSES
     * BETWEEN POINTS 1 AND 2 =',F5.1,' M ',//5X,'FLOW RATE =',F7.3,
     *' CU M/S ',//5X,'LENGTH OF CONDUIT =',F8.1,' M ',//5X,'FLUID FLOWI
```

```
     *NG IS ',4A6,//5X,'CONDUIT MATERIAL IS ',4A6,///1X,'THE CONDUIT DIA
     *METER REQUIRED WILL BE',F7.1,' MM',//5X,'VELOCITY AT POINT 1 =',
     *F6.2,' M/S ',//5X,'VELOCITY AT POINT 2 =',F6.2,' M/S ')
      GO TO 1
2     STOP
      END
      SUBROUTINE ROUGH
      COMMON F,ED,RN
      IF(RN.LE.2000.0)F=64.0/RN
      IF(RN.LE.2000.0)RETURN
      IF(RN.LT.4000.0)WRITE(6,103)
103   FORMAT(1X,'A REYNOLDS NUMBER IS IN THE CRITICAL ZONE, FOR WHICH TH
     *E FRICTION FACTOR IS UNCERTAIN.  HENCE, PROGRAM EXECUTION WAS TERM
     *INATED.')
      IF(RN.LT.4000.0)STOP
      F=0.006
      TRY1=1.0/SQRT(F)+2.0*ALOG10(ED/3.7+2.51/RN/SQRT(F))
102   F=F+0.00001
      TRY2=1.0/SQRT(F)+2.0*ALOG10(ED/3.7+2.51/RN/SQRT(F))
      IF(TRY1*TRY2)100,100,101
101   TRY1=TRY2
      GO TO 102
100   F=F-0.000005
      RETURN
      END
```

9.296 For the data given in Prob. 9.65, find the flow rate at which water is being discharged from the pipe, utilizing the computer program of Prob. 9.295.

INPUT

```
OSAMPLE ANALYSIS ØF INCØMPRESSIBLE FLØW
                                1.0        150.5      98.4
          24.0       130.0      0.0000105 0.00085    62.4
WATER                     NEW CAST IRØN
```

OUTPUT

```
SAMPLE ANALYSIS OF INCOMPRESSIBLE FLOW


GIVEN DATA FOR A CIRCULAR CLOSED CONDUIT CARRYING INCOMPRESSIBLE FLOW

    PRESSURE AT POINT 1 =     0.0 PSI

    PRESSURE AT POINT 2 =     0.0 PSI

    ELEVATION AT POINT 1 =  150.5 FT

    ELEVATION AT POINT 2 =   98.4 FT

    ACTUAL ENERGY ADDED BETWEEN POINTS 1 AND 2 =  0.0 HP

    ACTUAL ENERGY REMOVED BETWEEN POINTS 1 AND 2 =  0.0 HP

    MINOR HEAD LOSSES BETWEEN POINTS 1 AND 2 =  0.0 FT

    DIAMETER OF CONDUIT = 24.00 IN

    LENGTH OF CONDUIT =   130.0 FT

    FLUID FLOWING IS WATER

    CONDUIT MATERIAL IS NEW CAST IRON


THE FLOW RATE WILL BE  127.0 CU FT/S

    VELOCITY AT POINT 1 =  0.00 FT/S

    VELOCITY AT POINT 2 = 40.44 FT/S
```

9.297 For the data given in Prob. 9.66, find the pipe diameter, utilizing the computer program of Prob. 9.295.

INPUT

```
1SAMPLE ANALYSIS ØF INCØMPRESSIBLE FLØW
2.5                 1.0        1.0        82.65      66.66
                    965.5      .000000406.00050      7.05       .10
GASØLINE                 (RØUGHNESS = 0.500 MM)
```

OUTPUT

```
SAMPLE ANALYSIS OF INCOMPRESSIBLE FLOW


GIVEN DATA FOR A CIRCULAR CLOSED CONDUIT CARRYING INCOMPRESSIBLE FLOW

    PRESSURE AT POINT 1 =     2.5 KPA

    PRESSURE AT POINT 2 =     0.0 KPA

    ELEVATION AT POINT 1 =    82.7 M

    ELEVATION AT POINT 2 =    66.7 M

    ACTUAL ENERGY ADDED BETWEEN POINTS 1 AND 2 =  0.0 KW

    ACTUAL ENERGY REMOVED BETWEEN POINTS 1 AND 2 =  0.0 KW

    MINOR LOSSES BETWEEN POINTS 1 AND 2 =  0.0 M

    FLOW RATE =  0.100 CU M/S

    LENGTH OF CONDUIT =   965.5 M

    FLUID FLOWING IS GASOLINE

    CONDUIT MATERIAL IS (ROUGHNESS = 0.500 MM)


THE CONDUIT DIAMETER REQUIRED WILL BE  257.5 MM

    VELOCITY AT POINT 1 =  1.92 M/S

    VELOCITY AT POINT 2 =  1.92 M/S
```

CHAPTER 10
Series Pipeline Systems

10.1 For a 12-in-diameter concrete pipe 12 000 ft long, find the diameter of a 1000-ft-long equivalent pipe.

Assume a flow rate of 3.0 cfs. (The result should be the same regardless of the flow rate assumed.) From Fig. A-13, with $D = 12$ in and $Q = 3.0\ \text{ft}^3/\text{s}$, $h_1 = 0.0052$ ft/ft. Therefore, $h_f = (0.0052)(12\,000) = 62.40$ ft. For a 1000-ft-long equivalent pipe with the same head loss, $h_1 = 62.40/1000 = 0.06240$ ft/ft. From Fig. A-13, with $h_1 = 0.06240$ ft/ft and $Q = 3.0\ \text{ft}^3/\text{s}$, $D = 7.3$ in.

10.2 A 480-ft-long, 18-in-diameter concrete pipe and a 590-ft-long, 12-in-diameter concrete pipe are connected in series. Find the length of an equivalent pipe of 10 in diameter.

Assume a flow rate of 5 cfs through the two given pipes. For the 18-in-diameter pipe, from Fig. A-13, $h_1 = 0.00180$ ft/ft. For the 12-in-diameter pipe, $h_1 = 0.0137$ ft/ft. The total head loss for both pipes is $h_f = (0.00180)(480) + (0.0137)(590) = 8.947$ ft. For a 10-in-diameter pipe with $Q = 5.0$ cfs, from Fig. A-13, $h_1 = 0.032$ ft/ft. Since the equivalent pipe must have the same head loss as that of the system it replaces (i.e., 8.947 ft), the required length of a 10-in-diameter equivalent pipe can now be determined by $0.032L = 8.947$, $L = 280$ ft. Note that the required length of pipe can be determined in a single computation as follows: $L = (0.00180)(480)/0.032 + (0.0137)(590)/0.032 = 280$ ft.

10.3 A 225-m-long, 300-mm-diameter concrete pipe and a 400-m-long, 500-mm-diameter concrete pipe are connected in series. Find the diameter of a 625-m-long equivalent pipe.

Assume a flow rate of $0.1\ \text{m}^3/\text{s}$. For the 300-mm-diameter pipe, from Fig. A-14, $h_1 = 0.0074$ m/m. For the 500-mm-diameter pipe, $h_1 = 0.00064$ m/m. The total head loss for both pipes is $h_f = (0.0074)(225) + (0.00064)(400) = 1.921$ m. For a 625-m-long equivalent pipe with this head loss, $h_1 = 1.921/625 = 0.00307$ m/m. From Fig. A-14, $D = 360$ mm.

10.4 Water flows at a rate of $0.020\ \text{m}^3/\text{s}$ from reservoir A to reservoir B through three concrete pipes connected in series, as shown in Fig. 10-1. Find the difference in water-surface elevations in the reservoirs. Neglect minor losses.

$$p_A/\gamma + v_A^2/2g + z_A = p_B/\gamma + v_B^2/2g + z_B + h_L \qquad 0 + 0 + z_A = 0 + 0 + z_B + h_L \qquad h_L = h_f = z_A - z_B$$

With $Q = 0.020\ \text{m}^3/\text{s}$ and $D = 160$ mm, from Fig. A-14, $h_1 = 0.0082$ m/m. With $Q = 0.020\ \text{m}^3/\text{s}$ and $D = 200$ mm, $h_1 = 0.0028$ m/m. With $Q = 0.020\ \text{m}^3/\text{s}$ and $D = 180$ mm, $h_1 = 0.0046$ m/m. $H = h_f = (0.0082)(1000) + (0.0028)(1600) + (0.0046)(850) = 16.59$ m.

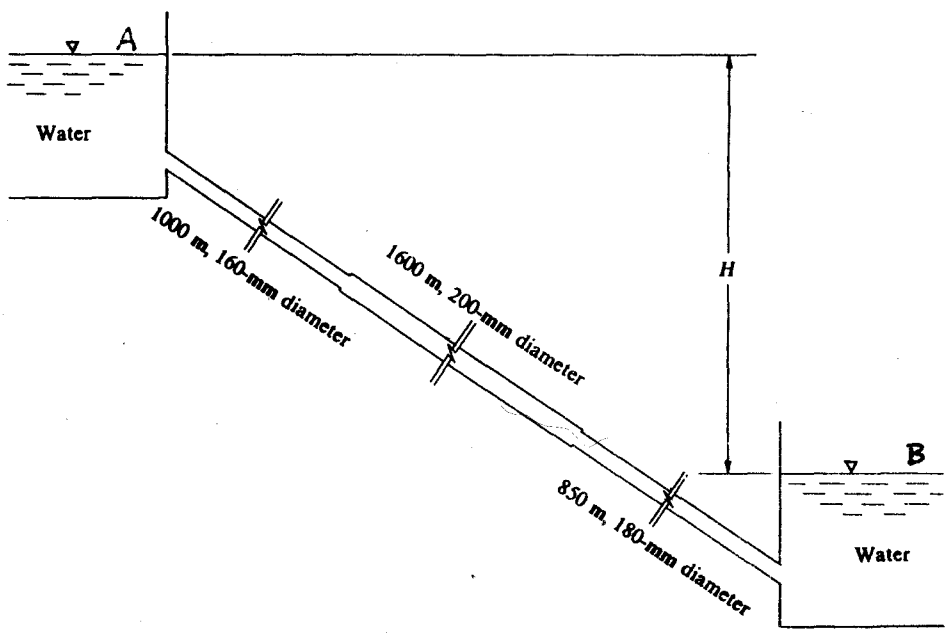


Fig. 10-1

10.5 Compute the flow rate of water through the three concrete pipes connected in series as shown in Fig. 10-2 by the equivalent length method.

▮ $h_f = z_A - z_B = 20$ ft/ft. Assume $Q = 1$ cfs. From Fig. A-13, $(h_1)_{12\text{-in}} = 0.00067$ ft/ft; $(h_1)_{10\text{-in}} = 0.0016$ ft/ft; $(h_1)_{8\text{-in}} = 0.00475$ ft/ft.

Consider a 10-in-diameter equivalent pipe: $L_e = 120 + (0.00475)(150)/0.0016 + (0.00067)(100)/0.0016 = 607$ ft. With $D = 10$ in and $h_1 = \frac{20}{607} = 0.0329$ ft/ft, from Fig. A-13, $Q = 5.1$ cfs.

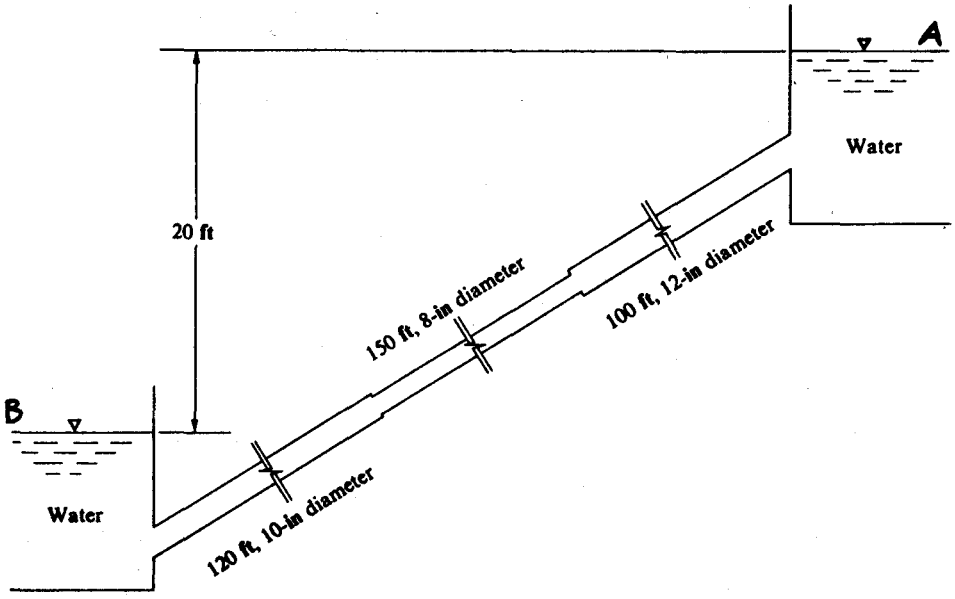


Fig. 10-2

10.6 Solve Prob. 10.5 by the equivalent diameter method.

▮ Using values obtained from Prob. 10.5, $h_f = (0.00067)(100) + (0.00475)(150) + (0.0016)(120) = 0.972$ ft. With $Q = 1$ cfs and $h_1 = 0.972/(100 + 150 + 120) = 0.00263$ ft/ft, $D = 8.9$ in. With $D = 8.9$ in and $h_1 = \frac{20}{370} = 0.0541$ ft/ft, $Q = 5.1$ cfs.

10.7 Two concrete pipes are connected in series. The flow rate of water through the pipes is 0.14 m^3/s with a total friction loss of 14.10 m for both pipes. Each pipe has a length of 300 m. If one pipe has a diameter of 300 mm, what is the diameter of the other one? Neglect minor losses.

▮ **For first pipe:** With $Q = 0.14$ m^3/s and $D = 300$ mm, $h_1 = 0.014$ m/m. $h_f = (0.014)(300) = 4.20$ m. **For second pipe:** $h_f = 14.10 - 4.20 = 9.90$ m, $h_1 = 9.90/300 = 0.033$ m/m. With $Q = 0.14$ m^3/s and $h_1 = 0.033$ m/m, $D = 250$ mm.

10.8 Three concrete pipes are connected in series, as shown in Fig. 10-3. Determine the length of an 8-in-diameter equivalent pipe.

▮ Assume $Q = 1$ cfs. $(h_1)_{8\text{-in}} = 0.00475$ ft/ft; $(h_1)_{6\text{-in}} = 0.0195$ ft/ft; $(h_1)_{10\text{-in}} = 0.0016$ ft/ft; $L_e = 1200 + (0.0195)(1000)/0.00475 + (0.0016)(2000)/0.00475 = 5979$ ft.

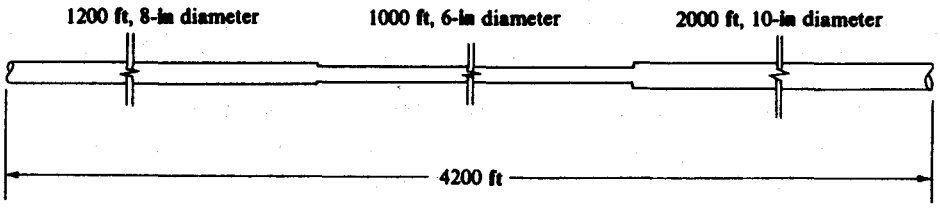


Fig. 10-3

10.9 For the three pipes in Fig. 10-3, determine the diameter of a 4200-ft-long equivalent pipe.

▮ Using values obtained from Prob. 10.8, for a 4200-ft-long equivalent pipe, $h_f = (0.00475)(1200) + (0.0195)(1000) + (0.0016)(2000) = 28.4$ ft, $h_1 = 28.4/4200 = 0.00676$ ft/ft. With $Q = 1$ cfs and $h_1 = 0.00676$ ft/ft, $D = 7.5$ in.

10.10 For three pipes in series, the total pressure drop is $p_A - p_B = 150\,000$ Pa, and the elevation drop is $z_A - z_B = 5$ m. The pipe data are

pipe	L, m	d, cm	ϵ, mm	ϵ/d
1	100	8	0.24	0.003
2	150	6	0.12	0.002
3	80	4	0.20	0.005

The fluid is water, $\rho = 1000\ \text{kg/m}^3$, and $\nu = 1.02 \times 10^{-6}\ \text{m}^2/\text{s}$. Calculate the flow rate in cubic meters per hour through the system. Neglect minor losses.

▮ The total head loss in the system is $h_L = (p_A - p_B)/\rho g + (z_A - z_B) = 150\,000/[(1000)(9.807)] + 5 = 20.30\ \text{m} = h_f = (f)(L/d)(v^2/2g)$. From the continuity relation, $v_2 = (d_1/d_2)^2(v_1) = (\frac{8}{6})^2(v_1) = 1.778v_1$, $v_3 = (d_1/d_3)^2(v_1) = (\frac{8}{4})^2(v_1) = 4.000v_1$, and

$$(N_R)_2 = \left(\frac{v_2 d_2}{v_1 d_1}\right)(N_R)_1 = \left[\frac{(1.778v_1)(6)}{(v_1)(8)}\right](N_R)_1 = (1.333)(N_R)_1$$

$$(N_R)_3 = \left(\frac{v_3 d_3}{v_1 d_1}\right)(N_R)_1 = \left[\frac{(4.000v_1)(4)}{(v_1)(8)}\right](N_R)_1 = (2.000)(N_R)_1$$

$$20.30 = f_1[100/(\tfrac{8}{100})]\{v_1^2/[(2)(9.807)]\} + f_2[150/(\tfrac{6}{100})]\{(1.778v_1)^2/[(2)(9.807)]\} + f_3[80/(\tfrac{4}{100})]\{(4.000v_1)^2/[(2)(9.807)]\}$$

$$20.30 = (63.73f_1 + 402.9f_2 + 1631f_3)(v_1)^2$$

From Fig. A-5 from the fully rough regime, estimate $f_1 = 0.0262$, $f_2 = 0.0234$, and $f_3 = 0.0304$.

$$20.35 = [(63.73)(0.0262) + (402.9)(0.0234) + (1631)(0.0304)](v_1)^2 \qquad v_1 = 0.5791\ \text{m/s} \qquad N_R = dv/\nu$$

$$(N_R)_1 = (\tfrac{8}{100})(0.5791)/(1.02 \times 10^{-6}) = 4.54 \times 10^4$$

$$(N_R)_2 = (1.333)(4.54 \times 10^{-4}) = 6.05 \times 10^4 \qquad (N_R)_3 = (2.000)(4.54 \times 10^{-4}) = 9.08 \times 10^4$$

From Fig. A-5, $f_1 = 0.0288$, $f_2 = 0.0260$, and $f_3 = 0.0314$. $20.35 = [(63.73)(0.0288) + (402.9)(0.0260) + (1631)(0.0314)](v_1)^2$, $v_1 = 0.5660$ m/s; $Q = Av = [(\pi)(\frac{8}{100})^2/4](0.5660) = 0.002845\ \text{m}^3/\text{s}$, or $10.2\ \text{m}^3/\text{h}$. An additional iteration (not shown) gives essentially the same result.

10.11 For a head loss of 5.0 ft/1000 ft, and using $C = 120$ for all pipes, how many 8-in pipes are equivalent to a 16-in pipe? To a 24-in pipe?

▮ From Fig. A-13, for $h_1 = 5.0/1000$, or 0.005, and $d = 8$ in, $Q = 1.0$ cfs. For $d = 16$ in, $Q = 6.6$ cfs. For $d = 24$ in, $Q = 17$ cfs. Thus it would take 6.6/1.0, or 6.6 eight-in pipes to be hydraulically equivalent to a 16-in pipe of the same relative roughness. Likewise, 17/1.0, or 17 eight-in pipes are equivalent to a 24-in pipe for a head loss of 5.0 ft/1000 ft, or for any other head loss condition.

10.12 A series piping system consists of 6000 ft of 20-in new cast iron pipe, 4000 ft of 16-in, and 2000 ft of 12-in pipe. Convert the system to an equivalent length of 16-in pipe.

▮ Assume a value of Q of 3.8 cfs. For new cast iron, $C = 130$. In order to use Fig. A-13, change Q_{130} to Q_{120}. $Q_{120} = (\frac{120}{130})(3.8) = 3.5$ cfs. From Fig. A-13, $(h_1)_1 = 0.00064$ ft/ft, $(h_1)_2 = 0.00187$ ft/ft, and $(h_1)_3 = 0.0070$ ft/ft. $h_L = (0.00064)(6000) + (0.00187)(4000) + (0.0070)(2000) = 25.32$ ft. The equivalent 16-in pipe must carry 3.8 cfs with a head loss of 25.32 ft ($C = 130$). Hence, $25.32/L_e = 0.00187$, $L_e = 13\,540$ ft.

10.13 Convert the system of Prob. 10.12 to an equivalent size pipe 12 000 ft long.

▮ The 12 000 ft of pipe, $C = 130$, must carry 3.8 cfs with a head loss of 25.32 ft. Hence, $h_1 = 25.32/12\,000 = 0.00211$. From Fig. A-13, using $Q = 3.5$ cfs, $d = 15.5$ in.

10.14 Suppose in Fig. 10-4 pipes 1, 2, and 3 are 300 m of 30-cm-diameter, 150 m of 20-cm-diameter, and 250 m of 25-cm-diameter, respectively, of new cast iron and are conveying water at 15 °C. If $h = 10$ m, find the rate of flow from A to B by the equivalent velocity method. Neglect minor losses.

▮ $(\epsilon/d)_1 = 0.00026/(\frac{30}{100}) = 0.000867 \qquad (\epsilon/d)_2 = 0.00026/(\frac{20}{100}) = 0.00130 \qquad (\epsilon/d)_3 = 0.00026/(\frac{25}{100}) = 0.00104$

From Fig. A-5, assume $f_1 = 0.019$, $f_2 = 0.021$, and $f_3 = 0.020$. $h_f = (f)(L/d)(v^2/2g) = 10 = 0.019[300/(\frac{30}{100})]\{v_1^2/[(2)(9.807)]\} + 0.021[150/(\frac{20}{100})]\{v_2^2/[(2)(9.807)]\} + 0.020[250/(\frac{25}{100})]\{v_3^2/[(2)(9.807)]\}$, $10 = 0.9687v_1^2 + 0.8030v_2^2 + 1.020v_3^2$. From the continuity relation, $v_2 = (d_1/d_2)^2(v_1) = (\frac{30}{20})^2(v_1) = 2.250v_1$, $v_3 = (d_1/d_3)^2(v_1) = (\frac{30}{25})^2(v_1) = 1.440v_1$, and

$$(N_R)_2 = \left(\frac{v_2 d_2}{v_1 d_1}\right)(N_R)_1 = \left[\frac{(2.250v_1)(20)}{(v_1)(30)}\right](N_R)_1 = (1.500)(N_R)_1$$

$$(N_R)_3 = \left(\frac{v_3 d_3}{v_1 d_1}\right)(N_R)_1 = \left[\frac{(1.440v_1)(25)}{(v_1)(30)}\right](N_R)_1 = (1.200)(N_R)_1$$

$$10 = 0.9687v_1^2 + (0.8030)(2.250v_1)^2 + (1.020)(1.440v_1)^2 \qquad v_1 = 1.183 \text{ m/s} \qquad N_R = dv/\nu$$

$$(N_R)_1 = (\tfrac{30}{100})(1.183)/(1.16 \times 10^{-6}) = 3.06 \times 10^5 \qquad (N_R)_2 = (1.500)(3.06 \times 10^5) = 4.59 \times 10^5$$

$$(N_R)_3 = (1.200)(3.06 \times 10^5) = 3.67 \times 10^5$$

From Fig. A-5, $f_1 = 0.021$, $f_2 = 0.021$, and $f_3 = 0.020$.

$$h_f = 10 = 0.021[300/(\tfrac{30}{100})]\{v_1^2/[(2)(9.807)]\} + 0.021[150/(\tfrac{20}{100})]\{v_2^2/[(2)(9.807)]\} + 0.020[250/(\tfrac{25}{100})]\{v_3^2/[(2)(9.807)]\}$$

$$10 = 1.071v_1^2 + 0.8030v_2^2 + 1.020v_3^2 = 1.071v_1^2 + (0.8030)(2.250v_1)^2 + (1.020)(1.440v_1)^2$$

$$v_1 = 1.174 \text{ m/s} \qquad Q = Av = [(\pi)(\tfrac{30}{100})^2/4](1.174) = 0.0830 \text{ m}^3/\text{s}$$

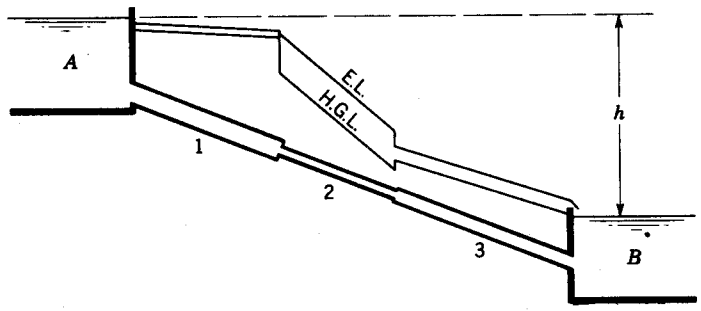


Fig. 10-4

10.15 Solve Prob. 10.14 by the equivalent length method.

▮ Using data from Prob. 10.14 and choosing a 30-cm pipe as the standard,

$$(L_e)_2 = (150)(0.021/0.020)(\tfrac{30}{20})^5 = 1196 \text{ m of 30-cm pipe}$$

$$(L_e)_3 = (250)(0.020/0.020)(\tfrac{30}{25})^5 = 622 \text{ m of 30-cm pipe}$$

$$(L_e)_{\text{total}} = 300 + 1196 + 622 = 2118 \text{ m of 30-cm pipe} \qquad h = 10 = 0.020[2118/(\tfrac{30}{100})]\{v^2/[(2)(9.807)]\}$$

$$v = 1.179 \text{ m/s} \qquad Q = [(\pi)(\tfrac{30}{100})^2/4](1.179) = 0.0833 \text{ m}^3/\text{s}$$

10.16 In Fig. 10-4 pipes 1, 2, and 3 are 500 ft of 3.068-in, 200 ft of 2.067-in, and 400 ft of 2.469-in wrought iron pipe. Given a head loss of 19.5 ft from A to B, find the flow of water at 60 °F.

▮ $$(\epsilon/d)_1 = 0.00015/(3.068/12) = 0.000587 \qquad (\epsilon/d)_2 = 0.00015/(2.067/12) = 0.000871$$

$$(\epsilon/d)_3 = 0.00015/(2.469/12) = 0.000729$$

From Fig. A-5, assume $f_1 = 0.0172$, $f_2 = 0.0190$, and $f_3 = 0.0181$.

$$h_f = (f)(L/d)(v^2/2g) = 19.5 = 0.0172[500/(3.068/12)]\{v_1^2/[(2)(32.2)]\} + 0.0190[200/(2.067/12)]\{v_2^2/[(2)(32.2)]\} + 0.0181[400/(2.469/12)]\{v_3^2/[(2)(32.2)]\}$$

$$19.5 = 0.5223v_1^2 + 0.3426v_2^2 + 0.5464v_3^2$$

From the continuity relation, $v_2 = (d_1/d_2)^2(v_1) = (3.068/2.067)^2(v_1) = 2.203v_1$, $v_3 = (d_1/d_3)^2(v_1) = (3.068/2.469)^2(v_1) = 1.544v_1$, and

$$(N_R)_2 = \left(\frac{v_2 d_2}{v_1 d_1}\right)(N_R)_1 = \left[\frac{(2.203v_1)(2.067)}{(v_1)(3.068)}\right](N_R)_1 = (1.484)(N_R)_1$$

$$(N_R)_3 = \left(\frac{v_3 d_3}{v_1 d_1}\right)(N_R)_1 = \left[\frac{(1.544v_1)(2.469)}{(v_1)(3.068)}\right](N_R)_1 = (1.243)(N_R)_1$$

$$20 = 0.5223v_1^2 + (0.3426)(2.203v_1)^2 + (0.5464)(1.544v_1)^2 \qquad v_1 = 2.365 \text{ ft/s}$$

$$N_R = dv/\nu \qquad (N_R)_1 = (3.068/12)(2.365)/(1.21 \times 10^{-5}) = 5.00 \times 10^4$$

$$(N_R)_2 = (1.484)(5.00 \times 10^4) = 7.42 \times 10^4 \qquad (N_R)_3 = (1.243)(5.00 \times 10^4) = 6.22 \times 10^4$$

From Fig. A-5, $f_1 = 0.0228$, $f_2 = 0.0225$, and $f_3 = 0.0225$.

$$h_f = 19.5 = 0.0228[500/(3.068/12)]\{v_1^2/[(2)(32.2)]\} + 0.0225[200/(2.067/12)]\{v_2^2/[(2)(32.2)]\}$$
$$+ 0.0225[400/(2.469/12)]\{v_3^2/[(2)(32.2)]\}$$
$$19.5 = 0.6924v_1^2 + 0.4057v_2^2 + 0.6792v_3^2 = 0.6924v_1^2 + (0.4057)(2.203v_1)^2 + (0.6792)(1.544v_1)^2 \qquad v_1 = 2.134 \text{ ft/s}$$
$$Q = Av = [(\pi)(3.068/12)^2/4](2.134) = 0.110 \text{ ft}^3/\text{s}$$

An additional iteration (not shown) produces an insignificant difference.

10.17 Suppose that 100 ft of 1-in ($\epsilon = 0.000005$ ft) pipe and 500 ft of 6-in ($\epsilon = 0.0008$ ft) pipe are connected in series. For a total head loss of 24 ft, find the flow of water at 60 °F.

▮ $$(\epsilon/d)_1 = 0.000005/(\tfrac{1}{12}) = 0.0000600 \qquad (\epsilon/d)_2 = 0.0008/(\tfrac{6}{12}) = 0.00160$$

From Fig. A-5, assume $f_1 = 0.0110$ and $f_2 = 0.0215$.

$$h_f = (f)(L/d)(v^2/2g) = 25 = 0.0110[100/(\tfrac{1}{12})]\{v_1^2/[(2)(32.2)]\} + 0.0215[500/(\tfrac{6}{12})]\{v_2^2/[(2)(32.2)]\}$$
$$25 = 0.2050v_1^2 + 0.3339v_2^2$$

From the continuity relation, $v_2 = (d_1/d_2)^2(v_1) = (\tfrac{1}{6})^2(v_1) = 0.02778v_1$ and

$$(N_R)_2 = \left(\frac{v_2 d_2}{v_1 d_1}\right)(N_R)_1 = \left[\frac{(0.02778v_1)(6)}{(v_1)(1)}\right](N_R)_1 = (0.1667)(N_R)_1$$
$$24 = 0.2050v_1^2 + (0.3339)(0.02778v_1)^2 \qquad v_1 = 11.04 \text{ ft/s} \qquad N_R = dv/\nu$$
$$(N_R)_1 = (\tfrac{1}{12})(11.04)/(1.21 \times 10^{-5}) = 7.60 \times 10^4 \qquad (N_R)_2 = (0.1667)(7.60 \times 10^4) = 1.27 \times 10^4$$

From Fig. A-5, $f_1 = 0.0195$ and $f_2 = 0.032$.

$$h_f = 24 = 0.0195[100/(\tfrac{1}{12})]\{v_1^2/[(2)(32.2)]\} + 0.032[500/(\tfrac{6}{12})]\{v_2^2/[(2)(32.2)]\}$$
$$24 = 0.3634v_1^2 + 0.4969v_2^2 = 0.3634v_1^2 + (0.4969)(0.02778v_1)^2 \qquad v_1 = 8.122 \text{ ft/s}$$
$$(N_R)_1 = (\tfrac{1}{12})(8.122)/(1.21 \times 10^{-5}) = 5.59 \times 10^4 \qquad (N_R)_2 = (0.1667)(5.59 \times 10^4) = 9.32 \times 10^3$$

From Fig. A-5, $f_1 = 0.0203$ and $f_2 = 0.033$.

$$h_f = 24 = 0.0203[100/(\tfrac{1}{12})]\{v_1^2/[(2)(32.2)]\} + 0.033[500/(\tfrac{6}{12})]\{v_2^2/[(2)(32.2)]\}$$
$$24 = 0.3783v_1^2 + 0.5124v_2^2 = 0.3783v_1^2 + (0.5124)(0.02778v_1)^2 \qquad v_1 = 7.961 \text{ ft/s}$$
$$Q = Av = [(\pi)(\tfrac{1}{12})^2/4](7.961) = 0.0434 \text{ ft}^3/\text{s}$$

An additional iteration (not shown) produces an insignificant difference.

10.18 Repeat Prob. 10.16 for the case where the fluid has s.g. = 0.9 and $\mu = 0.0008$ lb · s/ft^2.

▮ First apply $h_f = (f)(L/d)(v^2/2g)$, using values from Prob. 10.16 and assuming $f = 0.03$ for all pipes.

$$19.5 = 0.03[500/(3.068/12)]\{v_1^2/[(2)(32.2)]\} + 0.03[200/(2.067/12)]\{v_2^2/[(2)(32.2)]\}$$
$$+ 0.03[400/(2.469/12)]\{v_3^2/[(2)(32.2)]\}$$
$$= 0.9110v_1^2 + 0.5409v_2^2 + 0.9056v_3^2$$
$$= 0.9110v_1^2 + (0.5409)(2.203v_1)^2 + (0.9056)(1.544v_1)^2$$
$$v_1 = 1.850 \text{ ft/s} \qquad N_R = \rho dv/\mu \qquad (N_R)_1 = [(0.9)(1.94)](3.068/12)(1.850)/0.0008 = 1032$$
$$(N_R)_2 = (1.484)(1046) = 1532 \qquad (N_R)_3 = (1.243)(1046) = 1283$$

Therefore, the flow is laminar and $h_L = (32)(\mu/\gamma)(L/d^2)(v)$.

$$19.5 = 32\{0.0008/[(0.9)(62.4)]\}[500/(3.068/12)^2](v_1)$$
$$+ 32\{0.0008/[(0.9)(62.4)]\}[200/(2.067/12)^2](2.203v_1)$$
$$+ 32\{0.0008/[(0.9)(62.4)]\}[400/(2.469/12)^2](1.544v_1)$$
$$v_1 = 1.074 \text{ ft/s} \qquad Q = Av = [(\pi)(3.068/12)^2/4](1.074) = 0.0551 \text{ ft}^3/\text{s}$$

10.19 One end of a 150-m-long, 300-mm-diameter pipe is submerged in a reservoir; the other end abuts on a 90-m-long, 200-mm-diameter pipe at a point 30 m below the reservoir surface. Water discharges freely ($K = 1.0$) from the free end of the shorter pipe, which is 15 m below the junction. [This implies a 2° bend at the junction.]

Determine the pressure heads just above and just below the junction, if $f = 0.04$, $K = 0.8$ for the entrance, and $K = 0.24$ for the contraction-bend at the junction.

▮
$$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L \qquad (1)$$
$$h_f = (f)(L/d)(v^2/2g) = 0.04[150/0.300]\{v_1^2/[(2)(9.807)]\} + 0.04[90/0.200]\{v_2^2/[(2)(9.807)]\} = 1.020v_1^2 + 0.9177v_2^2$$
$$h_m = (K)(v^2/2g) = 0.8\{v_1^2/[(2)(9.807)]\} + 0.24\{v_2^2/[(2)(9.807)]\} = 0.04079v_1^2 + 0.01224v_2^2$$
$$h_L = h_f + h_m = (1.020v_1^2 + 0.9177v_2^2) + (0.04079v_1^2 + 0.01224v_2^2) = 1.061v_1^2 + 0.9299v_2^2$$
$$A_1v_1 = A_2v_2 \qquad [(\pi)(0.300)^2/4](v_1) = [(\pi)(0.200)^2/4](v_2) \qquad v_1 = 0.4444v_2$$
$$h_L = (1.061)(0.4444v_2)^2 + 0.9299v_2^2 = 1.139v_2^2 \qquad 0 + 0 + 45 = 0 + v_2^2/[(2)(9.807)] + 0 + 1.139v_2^2$$
$$v_2 = 6.149 \text{ m/s} \qquad v_1 = (0.4444)(6.149) = 2.733 \text{ m/s} \qquad Q = A_2v_2 = [(\pi)(0.200)^2/4](6.149) = 0.193 \text{ m}^3/\text{s}$$

Now apply Eq. (1) between the water level at intake and a point just above the junction. $0 + 0 + 30 = p_2/\gamma + 2.733^2/[(2)(9.807)] + 0 + (1.020v_1^2 + 0.04079v_1^2)$, $30 = p_2/\gamma + 0.3808 + (1.020 + 0.04079)(2.733^2)$, $p_2/\gamma = 21.70$ m. Finally apply Eq. (1) between the water level at intake and a point just below the junction. $0 + 0 + 30 = p_2/\gamma + 6.149^2/[(2)(9.807)] + 0 + (1.020v_1^2 + 0.04079v_1^2 + 0.01224v_2^2)$, $30 = p_2/\gamma + 1.928 + [(1.020 + 0.04079)(2.733^2) + (0.01224)(6.149^2)]$, $p_2/\gamma = 19.69$ m.

10.20 Repeat Prob. 10.19 neglecting minor losses.

▮
$$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L \qquad (1)$$
$$h_L = h_f + h_m \qquad h_m = 0$$

Using data from Prob. 10.19, $h_f = 1.020v_1^2 + 0.9177v_2^2$, $h_L = (1.020)(0.4444v_2)^2 + 0.9177v_2^2 = 1.119v_2^2$, $0 + 0 + 45 = 0 + v_2^2/[(2)(9.807)] + 0 + 1.119v_2^2$, $v_2 = 6.202$ m/s, $v_1 = (0.4444)(6.202) = 2.756$ m/s; $Q = A_2v_2 = [(\pi)(0.200)^2/4](6.202) = 0.195$ m^3/s. For the pressure head just above the junction, $0 + 0 + 30 = p_2/\gamma + 2.756^2/[(2)(9.807)] + 0 + 1.020v_1^2$, $30 = p_2/\gamma + 0.3873 + (1.020)(2.756^2)$, $p_2/\gamma = 21.87$ m. For the pressure head just below the junction, $0 + 0 + 30 = p_2/\gamma + 6.202^2/[(2)(9.807)] + 0 + 1.020v_1^2$, $30 = p_2/\gamma + 1.961 + (1.020)(2.756^2)$, $p_2/\gamma = 20.29$ m.

10.21 Three new cast iron pipes, having diameters of 30 in, 24 in, and 18 in, respectively, each 500 ft long, are connected in series. The 30-in pipe leads from a reservoir (flush entrance), and the 18-in pipe discharges into the air at a point 11.5 ft below the water surface in the reservoir. Assuming all changes in section to be abrupt, find the rate of discharge of water at 60 °F.

▮
$$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L \qquad h_L = h_f + h_m \qquad h_f = (f)(L/d)(v^2/2g)$$

Assume $f = 0.016$ for each pipe. v_2 = velocity for 18-in pipe, velocity for 24-in pipe $= (\frac{18}{24})^2(v_2) = 0.5625v_2$, velocity for 30-in pipe $= (\frac{18}{30})^2(v_2) = 0.3600v_2$.

$$h_f = 0.016[500/(\tfrac{30}{12})]\{(0.3600v_2)^2/[(2)(32.2)]\} + 0.016[500/(\tfrac{24}{12})]\{(0.5625v_2)^2/[(2)(32.2)]\}$$
$$+ 0.016[500/(\tfrac{18}{12})]\{v_2^2/[(2)(32.2)]\} = 0.1089v_2^2$$
$$h_m = (K)(v^2/2g)$$

For entrance, take $K = 0.5$ (Fig. A-7). For sudden contractions, with $d/D = \frac{24}{30}$, or 0.80, $K = 0.15$ and with $d/D = \frac{18}{24}$, or 0.75, $K = 0.18$ (Fig. A-9). For exit, $K = 1.0$ (Fig. A-7).

$$h_m = (0.5)(0.3600v_2)^2 + (0.15)(0.5625v_2)^2 + 0.18v_2^2 = 0.2923v_2^2 \qquad h_L = 0.1089v_2^2 + 0.2923v_2^2 = 0.4012v_2^2$$
$$0 + 0 + 11.5 = 0 + v_2^2/[(2)(32.2)] + 0 + 0.4012v_2^2 \qquad v_2 = 5.253 \text{ ft/s} \qquad N_R = dv/\nu$$
$$(N_R)_{30\text{-in}} = \tfrac{30}{12}[(0.3600)(5.253)]/(1.21 \times 10^{-5}) = 3.91 \times 10^5 \quad (N_R)_{24\text{-in}} = \tfrac{24}{12}[(0.5625)(5.253)]/(1.21 \times 10^{-5}) = 4.88 \times 10^5$$
$$(N_R)_{18\text{-in}} = (\tfrac{18}{12})(5.253)/(1.21 \times 10^{-5}) = 6.51 \times 10^5 \qquad (\epsilon/d)_{30\text{-in}} = 0.00085/(\tfrac{30}{12}) = 0.000340$$
$$(\epsilon/d)_{24\text{-in}} = 0.00085/(\tfrac{24}{12}) = 0.000425 \qquad (\epsilon/d)_{18\text{-in}} = 0.00085/(\tfrac{18}{12}) = 0.000567$$

From Fig. A-5, $f_{30\text{-in}} = 0.0168$, $f_{24\text{-in}} = 0.0172$, and $f_{18\text{-in}} = 0.0176$.

$$h_f = 0.0168[500/(\tfrac{30}{12})]\{(0.3600v_2)^2/[(2)(32.2)]\} + 0.0172[500/(\tfrac{24}{12})]\{(0.5625v_2)^2/[(2)(32.2)]\}$$
$$+ 0.0176[500/(\tfrac{18}{12})]\{v_2^2/[(2)(32.2)]\} = 0.1190v_2^2$$
$$h_L = 0.1190v_2^2 + 0.2923v_2^2 = 0.4113v_2^2 \qquad 0 + 0 + 11.5 = 0 + v_2^2/[(2)(32.2)] + 0 + 0.4113v_2^2 \qquad v_2 = 5.191 \text{ ft/s}$$

An additional iteration (not shown) gives no significant change in v_2. $Q = Av = [(\pi)(\frac{18}{12})^2/4](5.191) = 9.17$ ft^3/s.

10.22 In Fig. 10-5, $K_e = 0.5$, $L_1 = 300$ m, $D_1 = 600$ mm, $\epsilon_1 = 2$ mm, $L_2 = 240$ m, $D_2 = 1$ m, $\epsilon_2 = 0.3$ mm, $\nu = 3 \times 10^{-6}$ m²/s, and $H = 6$ m. Determine the discharge through the system.

▮ $$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L \qquad h_L = h_f + h_m \qquad h_f = (f)(L/d)(v^2/2g)$$

Assume $f_1 = 0.026$ and $f_2 = 0.015$. $v_2 = [(\frac{600}{1000})/1]^2(v_1) = 0.3600v_1$, $h_f = 0.026[300/(\frac{600}{1000})]\{v_1^2/[(2)(9.807)]\} + (0.015)(240/1)\{(0.3600v_1)^2/[(2)(9.807)]\} = 0.6866v_1^2$.
For entrance: $h_m = (K)(v^2/2g) = 0.5\{v_1^2/[(2)(9.807)]\} = 0.02549v_1^2$.
For exit: $h_m = 1.0\{v_2^2/[(2)(9.807)]\} = 0.05098v_2^2 = (0.05098)(0.3600v_1)^2 = 0.006607v_1^2$.
For sudden contraction:

$$h_m = (v_1 - v_2)^2/2g = (v_1 - 0.3600v_1)^2/[(2)(9.807)]$$
$$= 0.02088v_1^2 (h_m)_{\text{total}} = 0.02549v_1^2 + 0.006607v_1^2 + 0.02088v_1^2 = 0.05298v_1^2$$
$$h_L = 0.6866v_1^2 + 0.05298v_1^2 = 0.7396v_1^2 \qquad 0 + 0 + 6 = 0 + 0 + 0 + 0.7396v_1^2 \qquad v_1 = 2.848 \text{ m/s}$$
$$v_2 = (0.3600)(2.848) = 1.025 \text{ m/s}$$
$$N_R = dv/\nu \qquad (N_R)_1 = (\tfrac{600}{1000})(2.848)/(3 \times 10^{-6}) = 5.70 \times 10^5 \qquad (N_R)_2 = (1)(1.025)/(3 \times 10^{-6}) = 3.42 \times 10^5$$
$$(\epsilon/d)_1 = \tfrac{2}{600} = 0.00333 \qquad (\epsilon/d)_2 = (0.3/1000)/1 = 0.000300$$

From Fig. A-5, $f_1 = 0.0265$ and $f_2 = 0.0168$.

$$h_f = 0.0265[300/(\tfrac{600}{1000})]\{v_1^2/[(2)(9.807)]\} + (0.0168)(240/1)\{(0.3600v_1)^2/[(2)(9.807)]\} = 0.7022v_1^2$$
$$h_L = 0.7022v_1^2 + 0.05298v_1^2 = 0.7552v_1^2 \qquad 0 + 0 + 6 = 0 + 0 + 0 + 0.7552v_1^2 \qquad v_1 = 2.819 \text{ m/s}$$

An additional iteration (not shown) gives no significant change in v_1. $Q = Av = [(\pi)(\frac{600}{1000})^2/4](2.819) = 0.797$ m³/s.

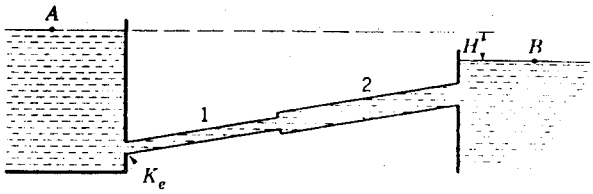


Fig. 10-5

10.23 Solve Prob. 10.22 by means of equivalent pipes.

▮ Expressing the minor losses in terms of equivalent lengths gives for pipe 1: $K_1 = 0.5 + [1 - (\frac{600}{1000})^2]^2 = 0.9096$, $(L_e)_1 = K_1D_1/f_1 = (0.9096)(\frac{600}{1000})/0.026 = 20.99$ m; and for pipe 2: $K_2 = 1.0$, $(L_e)_2 = K_2D_2/f_2 = (1.0)(1)/0.015 = 66.67$ m. The values of f_1 and f_2 are selected for the fully turbulent range as an approximation. The problem is now reduced to 300 + 20.99, or 320.99 m of 600-mm pipe and 240 + 66.67, or 306.67 m of 1-m pipe. Expressing the 1-m pipe in terms of an equivalent length of 600-m pipe, $L_e = (f_2/f_1)(L_2)(D_1/D_2)^5 = (0.015/0.026)(306.67)[(\frac{600}{1000})/1]^5 = 13.76$ m. By adding to the 600-mm pipe, the problem is reduced to finding the discharge through 320.99 + 13.76, or 334.75 m of 600-mm pipe, $\epsilon_1 = 2$ mm, $H = 6$ m: $h_f = (f)(L/D)(v^2/2g)$, $6 = (f)[334.75/(\frac{600}{1000})]\{v^2/[(2)(9.807)]\}$, $v = 0.4593/\sqrt{f}$. Try $f = 0.026$: $v = 0.4593/\sqrt{0.026} = 2.848$ m/s, $N_R = Dv/\nu = (\frac{600}{1000})(2.848)/(3 \times 10^{-6}) = 5.70 \times 10^5$. From Fig. A-5, with $N_R = 5.70 \times 10^5$ and $\epsilon/D = 0.00333$, $f = 0.0265$. Try $f = 0.0265$: $v = 0.4593/\sqrt{0.0265} = 2.821$ m/s, $N_R = (\frac{600}{1000})(2.821)/(3 \times 10^{-6}) = 5.64 \times 10^5$, $f = 0.0265$ (O.K.); $Q = Av = [(\pi)(\frac{600}{1000})^2/4](2.821) = 0.798$ m³/s.

10.24 Two reservoirs are connected by three clean cast iron pipes in series: $L_1 = 300$ m, $D_1 = 200$ mm; $L_2 = 400$ m, $D_2 = 300$ mm; $L_3 = 1200$ m, $D_3 = 450$ mm. If the flow is 360 m³/h of water at 20 °C, determine the difference in elevation of the reservoirs.

▮ $$v = Q/A \qquad v_1 = (360/3600)/[(\pi)(0.200)^2/4] = 3.183 \text{ m/s}$$
$$v_2 = (360/3600)/[(\pi)(0.300)^2/4] = 1.415 \text{ m/s} \qquad v_3 = (360/3600)/[(\pi)(0.450)^2/4] = 0.6288 \text{ m/s}$$
$$N_R = Dv/\nu \qquad (N_R)_1 = (0.200)(3.183)/(1.02 \times 10^{-6}) = 6.24 \times 10^5$$
$$(N_R)_2 = (0.300)(1.415)/(1.02 \times 10^{-6}) = 4.16 \times 10^5 \qquad (N_R)_3 = (0.450)(0.6288)/(1.02 \times 10^{-6}) = 2.77 \times 10^5$$
$$(\epsilon/D)_1 = 0.00026/0.200 = 0.00130 \qquad (\epsilon/D)_2 = 0.0026/0.300 = 0.000867$$
$$(\epsilon/D)_3 = 0.00026/0.450 = 0.000578$$

From Fig. A-5, $f_1 = 0.0215$, $f_2 = 0.020$, and $f_3 = 0.0185$. $H = h_f = (f)(L/D)(v^2/2g) = 0.0215[300/0.200]\{3.183^2/[(2)(9.807)]\} + 0.020[400/0.300]\{1.415^2/[(2)(9.807)]\} + 0.0185[1200/0.450]\{0.6288^2/[(2)(9.807)]\} = 20.37$ m.

10.25 Solve Prob. 10.24 by the method of equivalent lengths.

▮ Express pipes 2 and 3 in terms of pipe 1:

$$L_e = (f_2/f_1)(L_2)(D_1/D_2)^5$$

$$(L_e)_2 = (0.020/0.0215)(400)(\tfrac{200}{300})^5 = 49.00 \text{ m} \qquad (L_e)_3 = (0.0185/0.0215)(1200)(\tfrac{200}{450})^5 = 17.91 \text{ m}$$

$$(L_e)_{\text{total}} = 300 + 49.00 + 17.91 = 366.9 \text{ m}$$

$$H = h_f = (f)(L/D)(v^2/2g) = 0.0215[366.9/0.200]\{3.183^2/[(2)(9.807)]\} = 20.37 \text{ m}$$

10.26 Air at atmospheric pressure and 60 °F is carried through two horizontal pipes ($\epsilon = 0.06$ in) in series. The upstream pipe is 360 ft of 24 in diameter, and the downstream pipe is 120 ft of 36 in diameter. Estimate the equivalent length of 18-in ($\epsilon = 0.003$ in) pipe. Neglect minor losses.

▮

$$\epsilon/D_1 = 0.06/24 = 0.0025 \qquad \epsilon/D_2 = 0.06/36 = 0.00167$$

From Fig. A-5, assuming high Reynolds numbers, $f_1 = 0.025$ and $f_2 = 0.022$.

$$L_e = (0.025/f_3)(360)(\tfrac{18}{24})^5 + (0.022/f_3)(120)(\tfrac{18}{36})^5 = 2.218/f_3$$

$$\epsilon_3/D_3 = 0.003/18 = 0.000167 \qquad f_3 = 0.013 \qquad L_e = 2.218/0.013 = 171 \text{ ft}$$

10.27 What pressure drop is required for flow of 6000 ft³/min in Prob. 10.26? Include losses due to sudden expansion.

▮

$$h_L = h_f + h_m \qquad h_f = (f)(L/D)(v^2/2g)$$

From Prob. 10.26, $\epsilon/D_1 = 0.0025$, $\epsilon/D_2 = 0.00167$.

$$v_1 = Q/A_1 = (\tfrac{6000}{60})/[(\pi)(\tfrac{24}{12})^2/4] = 31.83 \text{ ft/s} \qquad v_2 = Q/A_2 = (\tfrac{6000}{60})/[(\pi)(\tfrac{36}{12})^2/4] = 14.15 \text{ ft/s}$$

$$N_R = Dv/\nu \qquad (N_R)_1 = (\tfrac{24}{12})(31.83)/(1.58\times10^{-4}) = 4.03\times10^5 \qquad (N_R)_2 = (\tfrac{36}{12})(14.15)/(1.58\times10^{-4}) = 2.69\times10^5$$

From Fig. A-5, $f_1 = 0.025$ and $f_2 = 0.022$.

$$h_f = 0.025[360/(\tfrac{24}{12})]\{31.83^2/[(2)(32.2)]\} + 0.022[120/(\tfrac{36}{12})]\{14.15^2/[(2)(32.2)]\} = 73.53 \text{ ft}$$

$$h_m = (v_1 - v_2)^2/2g = (31.83 - 14.15)^2/[(2)(32.2)] = 4.85 \text{ m} \qquad h_L = 73.53 + 4.85 = 78.38 \text{ ft}$$

$$\Delta p = \gamma h \qquad \gamma = p/RT$$

$$\gamma = (14.7)(144)/[(53.3)(460+60)] = 0.07637 \text{ lb/ft}^3 \qquad \Delta p = (0.07637)(78.38) = 5.99 \text{ lb/ft}^2$$

10.28 Two pipes, $D_1 = 3$ in, $L_1 = 300$ ft, and $D_2 = 2$ in, $L_2 = 180$ ft, are joined in series with a total pressure drop of 3 lb/in². What is the flow rate of syrup ($\rho = 1.78$ slugs/ft³, $\mu = 0.00606$ lb · s/ft²) at 20 °C?

▮

$$h_f = \Delta p/\gamma = \Delta p/\rho g = (3)(144)/[(1.78)(32.2)] = 7.537 \text{ ft}$$

Assume laminar flow

$$h_f = \frac{128\mu LQ}{\pi\rho g D^4} \qquad 7.537 = \frac{(128)(0.00606)(300)(Q)}{(\pi)(1.78)(32.2)(\frac{3}{12})^4} + \frac{(128)(0.00606)(180)(Q)}{(\pi)(1.78)(32.2)(\frac{2}{12})^4} \qquad Q = 0.00564 \text{ ft}^3/\text{s}$$

Computation of N_R (not shown) indicates the flow is indeed laminar.

10.29 Convert the piping system shown in Fig. 10-6 to an equivalent length of 6-in pipe.

▮

$$p_A/\gamma + V_A^2/2g + z_A = p_M/\gamma + V_M^2/2g + z_M + h_L \qquad h_L = h_f + h_m$$

$$h_f = (f)(L/d)(V^2/2g) = 0.025[150/(\tfrac{12}{12})]\{V_{12}^2/[(2)(32.2)]\} + 0.020[100/(\tfrac{6}{12})]\{V_6^2/[(2)(32.2)]\}$$
$$= 0.05823V_{12}^2 + 0.06211V_6^2$$

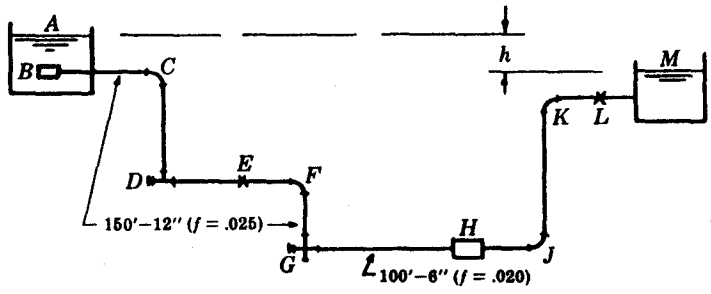
$$h_m = (K)(V^2/2g) = [8.0 + (2)(0.5) + 0.7 + 1.0]\{V_{12}^2/[(2)(32.2)]\}$$
$$+ [0.7 + 6.0 + (2)(0.5) + 3.0 + 1.0]\{V_6^2/[(2)(32.2)]\}$$
$$= 0.1661V_{12}^2 + 0.1817V_6^2$$

$$h_L = 0.05823V_{12}^2 + 0.06211V_6^2 + 0.1661V_{12}^2 + 0.1817V_6^2 = 0.2243V_{12}^2 + 0.2438V_6^2$$

$$0 + 0 + h = 0 + 0 + 0 + 0.2243V_{12}^2 + 0.2438V_6^2 \qquad V_{12} = (\tfrac{6}{12})^2(V_6) = 0.2500V_6$$

$$h = (0.2243)(0.2500V_6)^2 + 0.2438V_6^2 = 0.2578V_6^2$$

For a 6-in equivalent pipe, $h = 0.020[L_e/(\tfrac{6}{12})]\{V_6^2/[(2)(32.2)]\} = 0.0006211L_eV_6^2$, $0.2578V_6^2 = 0.0006211L_eV_6^2$, $L_e = 415$ ft.



Factors K

Strainer B	= 8.0
12″ Bends C, F (each)	= 0.5
12″ Tee D	= 0.7
12″ Valve E	= 1.0
12″ × 6″ Cross G ($\times V_6^2/2g$)	= 0.7
6″ Meter H	= 6.0
6″ Bends J, K (each)	= 0.5
6″ Valve L	= 3.0

Fig. 10-6

10.30 For the compound piping system described in Prob. 10.12, what flow will be produced for a total head loss of 70.0 ft?

▮ From Prob. 10.12, 13 540 ft of 16-in pipe is equivalent to the compound system. For a head loss of 70.0 ft, $h_1 = 70.0/13\,540 = 0.00517$. From Fig. A-13, $Q = 6.3$ cfs. This is for $C = 120$. For new cast iron pipe ($C = 130$), $Q = (\frac{130}{120})(6.3) = 6.8$ cfs.

10.31 For the system shown in Fig. 10-7, when the flow from reservoir A to main D is 3.25 mgd, the pressure at D is 20.0 psi. The flow to D must be increased to 4.25 mgd with the pressure at 40.0 psi. What size pipe, 5000 ft long, should be laid from B to C (shown dashed) parallel to the existing 12-in pipe to accomplish this?

▮ The elevation of reservoir A can be determined as follows: $Q = (3.25 \times 10^6)/[(7.48)(86\,400)] = 5.03$ cfs. From Fig. A-13, $(h_1)_{16} = 0.0038$ ft/ft, $(h_1)_{12} = 0.0150$ ft/ft. These values of h_1 are for $C = 120$; they must be adjusted for $C = 100$:

$$(h_1)_{16} = (\tfrac{120}{100})(0.0038) = 0.0046 \text{ ft/ft} \qquad (h_1)_{12} = (\tfrac{120}{100})(0.0150) = 0.018 \text{ ft/ft}$$

$$(h_f)_{16} = (0.0046)(8000) = 36.8 \text{ ft} \qquad (h_f)_{12} = (0.018)(5000) = 90.0 \text{ ft} \qquad (h_f)_{\text{total}} = 36.8 + 90.0 = 126.8 \text{ ft}$$

The hydraulic grade line drops 126.8 ft to an elevation of 46.2 ft above D (the equivalent of 20.0 psi). Thus reservoir A is 126.8 + 46.2, or 173.0 ft above point D. For a pressure of 40.0 psi, the elevation of the hydraulic grade line at D will be 92.4 ft above D, or the available head for the flow of 4.25 mgd will be 173.0 − 92.4, or 80.6 ft. In the 16-in pipe with $Q = 4.25$ mgd, or 6.58 cfs, $(h_1)_{16} = 0.0062$ ft/ft for $C = 120$.

For $C = 100$: $(h_1)_{16} = (\frac{120}{100})(0.0062) = 0.0074$ ft/ft, $(h_f)_{16} = (0.0074)(8000) = 59.2$ ft, $(h_f)_{B\text{ to }C} = 80.6 - 59.2 = 21.4$ ft.

For the existing 12-in pipe: $(h_1)_{12} = 21.4/5000 = 0.0043$ ft/ft, $Q_{12} = 2.7$ cfs for $C = 120$.

For $C = 100$, $Q_{12} = (\frac{100}{120})(2.7) = 2.3$ cfs. The flow in the new pipe must be 6.58 − 2.3, or 4.28 cfs with an available head (drop in the hydraulic grade line) of 21.4 ft from B to C. $h_1 = 21.4/5000 = 0.0043$ ft/ft. From Fig. A-13, $D = 14$ in, approximately.

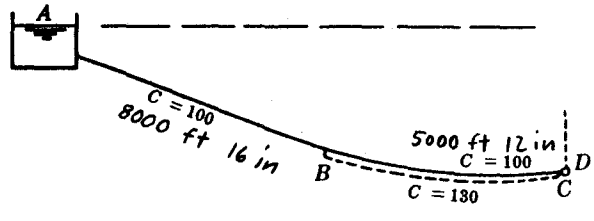


Fig. 10-7

CHAPTER 11
Parallel Pipeline Systems

11.1 Figure 11-1 shows a looping pipe system. Pressure heads at points A and E are 70.0 m and 46.0 m, respectively. Compute the flow rate of water through each branch of the loop. Assume $C = 120$ for all pipes.

▮ $(h_f)_{A-E} = 70.0 - 46.0 = 24.0$ m. From Fig. A-14, for pipe ABE, with $h_1 = 24.0/3000$, or 0.0080 m/m, and $D = 300$ mm, $Q_{ABE} = 0.105\ m^3/s$. For pipe ACE, with $h_1 = 24.0/1300$, or 0.0185 m/m, and $D = 200$ mm, $Q_{ACE} = 0.056\ m^3/s$. For pipe ADE, with $h_1 = 24.0/2600$, or 0.0092 m/m, and $D = 250$ mm, $Q_{ADE} = 0.070\ m^3/s$.

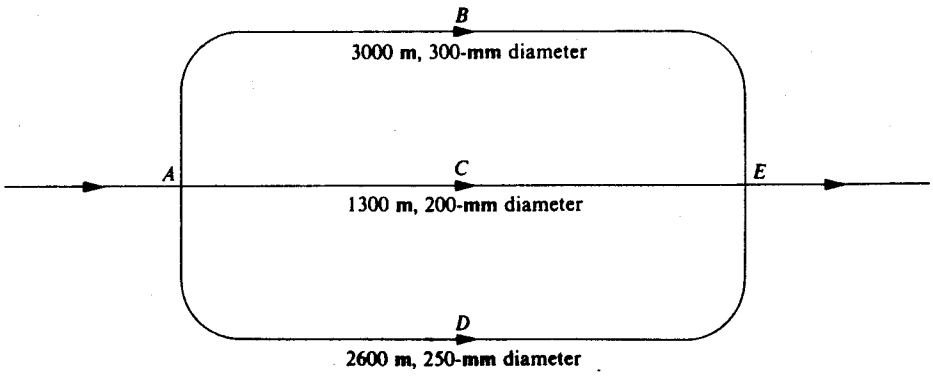


Fig. 11-1

11.2 A looping concrete pipe system is shown in Fig. 11-2. The total flow rate of water is 18.0 cfs. Determine the division of flow and the loss of head from point B to point E.

▮ Assume a head loss of 20 ft from point B to point E. With $(h_1)_{BCE} = \frac{20}{5000}$, or 0.0040 ft/ft and $D = 15$ in, $Q_{BCE} = 4.75$ cfs. With $(h_1)_{BDE} = \frac{20}{4000}$, or 0.0050 ft/ft and $D = 18$ in, $Q_{BDE} = 8.60$ cfs. Fraction of flow through pipe $BCE = 4.75/(4.75 + 8.60) = 0.356$. Fraction of flow through pipe $BDE = 8.60/(4.75 + 8.60) = 0.644$. $Q_{BCE} = (18.0)(0.356) = 6.4$ cfs, $Q_{BDE} = (18.0)(0.644) = 11.6$ cfs. With $Q_{BCE} = 6.4$ cfs and $D = 15$ in, $(h_1)_{B-E} = 0.0070$ ft/ft. $(h_f)_{B-E} = (0.0070)(5000) = 35.0$ ft.

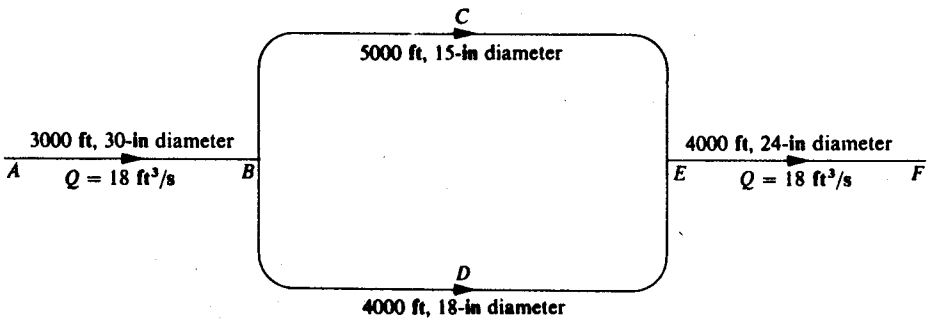


Fig. 11-2

11.3 The discharge of water in the concrete pipe looping system shown in Fig. 11-3 if 15.0 ft^3/s. Compute the head loss from point A to point G.

▮ With $Q_{A-B} = 15.0$ cfs and $D = 30$ in, $(h_1)_{A-B} = 0.00116$ ft/ft. With $Q_{FG} = 15.0$ cfs and $D = 24$ in, $(h_1)_{FG} = 0.00345$ ft/ft. $(h_f)_{AB} = (0.00116)(2500) = 2.90$ ft, $(h_f)_{FG} = (0.00345)(3000) = 10.35$ ft. Assume $(h_f)_{BF} = 30$ ft. With $D = 18$ in and $(h_1)_{BCF} = \frac{30}{1500} = 0.020$ ft/ft, $Q_{BCF} = 18.1$ cfs. With $D = 12$ in and $(h_1)_{BDF} = \frac{30}{1000} = 0.030$ ft/ft, $Q_{BDF} = 7.7$ cfs. With $D = 15$ in and $(h_1)_{BEF} = \frac{30}{2000} = 0.015$ ft/ft, $Q_{BEF} = 9.6$ cfs. Fraction of flow through pipe $BCF = 18.1/(18.1 + 7.7 + 9.6) = 0.511$, fraction of flow through pipe $BDF = 7.7/(18.1 + 7.7 + 9.6) = 0.218$, fraction of flow through pipe $BEF = 9.6/(18.1 + 7.7 + 9.6) = 0.271$; $Q_{BCF} = (15.0)(0.511) = 7.67$ cfs. With $Q_{BCF} = 7.67$ and $D = 18$ in, $(h_1)_{BCF} = 0.0040$ ft/ft. $(h_f)_{B-F} = (0.0040)(1500) = 6.00$ ft; $(h_f)_{A-G} = (h_f)_{AB} + (h_f)_{B-F} + (h_f)_{FG} = 2.90 + 6.00 + 10.35 = 19.25$ ft.

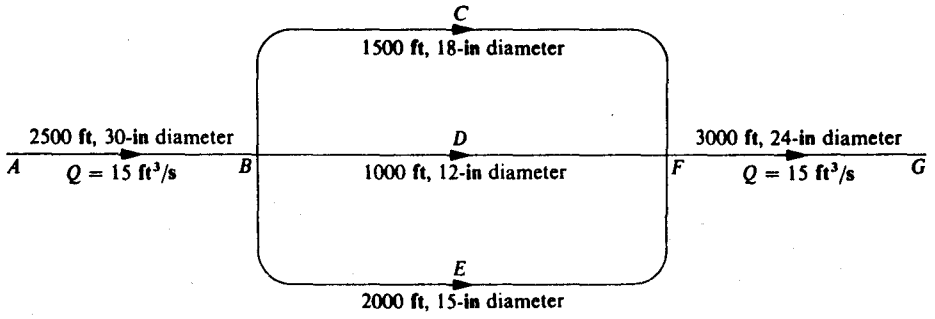


Fig. 11-3

11.4 If the flow rate of water through the pipe system shown in Fig. 11-4 is 0.050 m^3/s under total head loss of 9.0 m, determine the diameter of pipe C. Assume a C coefficient of 120 for all pipes.

▮ With $Q_A = 0.050\ m^3/s$ and $D_A = 200$ mm, $(h_1)_A = 0.015$ m/m and $(h_f)_A = (0.015)(300) = 4.50$ m. With $Q_D = 0.050\ m^3/s$ and $D_D = 250$ mm, $(h_1)_D = 0.0049$ m/m and $(h_f)_D = (0.0049)(510) = 2.50$ m. $(h_f)_B = (h_f)_C = 9.0 - 4.50 - 2.50 = 2.00$ m. With $(h_1)_B = 2.00/266 = 0.0075$ m/m, $Q_B = 0.019\ m^3/s$. $Q_C = 0.050 - 0.019 = 0.031\ m^3/s$. With $Q_C = 0.031\ m^3/s$ and $(h_1)_C = 2.0/190 = 0.0105$ m/m, $D_C = 180$ mm.

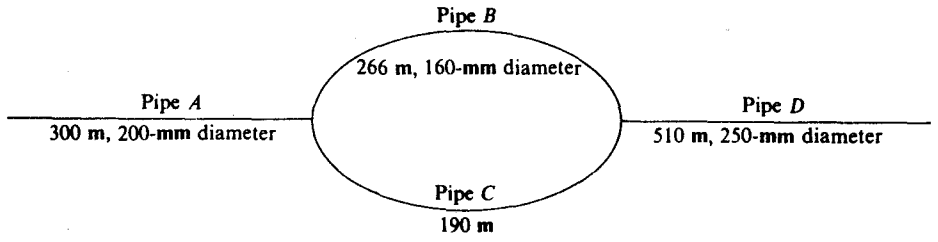


Fig. 11-4

11.5 For the looping pipe system shown in Fig. 11-5, the head loss between points A and F is 160.0 ft of water. Determine the flow rate of water in the system. Assume $C = 120$ for all pipes.

▮ Assume a head loss of 12 ft from point B to point E. With $(h_1)_{BCE} = \frac{12}{4000} = 0.0030$ ft/ft and $D = 15$ in, $Q_{BCE} = 4.0$ cfs. With $(h_1)_{BDE} = \frac{12}{3000} = 0.0040$ ft/ft and $D = 12$ in, $Q_{BDE} = 2.6$ cfs. Fraction of flow through pipe $BCE = 4.0/(4.0 + 2.6) = 0.606$; fraction of flow through pipe $BDE = 2.6/(4.0 + 2.6) = 0.394$; $Q_{AB} = Q_{EF} = 4.0 + 2.6 = 6.6$ cfs. With $Q_{AB} = 6.6$ cfs and $D = 24$ in, $(h_1)_{AB} = 0.00075$ ft/ft; $(h_f)_{AB} = (0.00075)(8000) = 6.0$ ft. With $Q_{EF} = 6.6$ cfs and $D = 18$ in, $(h_1)_{EF} = 0.0030$ ft/ft; $(h_f)_{EF} = (0.0030)(6000) = 18.0$ ft. Fraction of total head loss through pipe $AB = 6.0/(6.0 + 12.0 + 18.0) = 0.167$, fraction of total head loss from B to $E = 12.0/(6.0 + 12.0 + 18.0) = 0.333$, fraction of total head loss through pipe $EF = 18.0/(6.0 + 12.0 + 18.0) = 0.500$. Actual head losses are, therefore, $(h_f)_{AB} = (160)(0.167) = 26.72$ ft, $(h_f)_{B\text{-}E} = (160)(0.333) = 53.28$ ft, $(h_f)_{EF} = (160)(0.500) = 80.00$ ft. With $(h_1)_{AB} = 26.72/8000 = 0.00334$ ft/ft and $D = 24$ in, $Q_{AB} = 14.5$ cfs. With $(h_1)_{EF} = 80.00/6000 = 0.0133$ ft/ft and $D = 18$ in, $Q_{EF} = 14.5$ cfs. $Q_{BCE} = (14.5)(0.606) = 8.8$ cfs, $Q_{BDE} = (14.5)(0.394) = 5.7$ cfs.

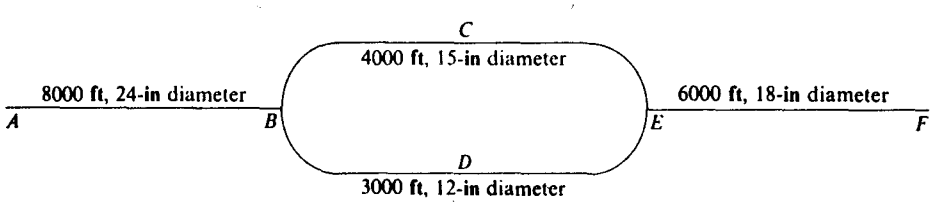


Fig. 11-5

11.6 A flow of 570 L/s is proceeding through the pipe network shown in Fig. 11-6. For a pressure of 690 kPa at node A, what pressure may be expected at node B? Neglect minor losses. Take $\rho = 1000\ kg/m^3$.

▮ $p_A/\gamma + v_A^2/2g + z_A = p_B/\gamma + v_B^2/2g + z_B + h_L$. Assume that a flow Q_1' of 170 L/s is proceeding through branch 1.

$$h_f = (f)(L/D)(v^2/2g) \qquad v_1 = Q_1' = (\tfrac{170}{1000})/[(\pi)(\tfrac{300}{1000})^2/4] = 2.405\ m/s \qquad N_R = Dv/\nu$$

$$(N_R)_1 = (\tfrac{300}{1000})(2.405)/(0.0113 \times 10^{-4}) = 6.38 \times 10^5 \qquad \epsilon/D_1 = 0.00026/(\tfrac{300}{1000}) = 0.00087$$

From Fig. A-5, $f = 0.0198$. $h_L = (h_f)_1 = 0.0198[600/(\frac{300}{1000})]\{2.405^2/[(2)(9.807)]\} = 11.68$ m. For branch 1,

$$p_A/\gamma + (v_1^2)_A/2g + 6 = p_B/\gamma + (v_1^2)_B/2g + 15 + 11.68 \qquad (v_1^2)_A/2g = (v_1^2)_B/2g \qquad [(p_A - p_B)/\gamma]_1 = 20.68\ m$$

Using this pressure head, which must be the same for each loop, for branch 2,

$$20.68 + (v_2^2)_A/2g + 6 = (v_2^2)_B/2g + 15 + (h_f)_2 \qquad (v_2^2)_A/2g = (v_2^2)_B/2g \qquad (h_f)_2 = 11.68\text{ m}$$

$$11.68 = f_2[460/(\tfrac{470}{1000})]\{v_2^2/[(2)(9.807)]\} = 49.90 f_2 v_2^2 \qquad (\epsilon/D)_2 = 0.00026/(\tfrac{470}{1000}) = 0.00055$$

Estimate $f_2 = 0.018$.

$$11.68 = (49.90)(0.018)(v_2^2) \qquad v_2 = 3.606\text{ m/s}$$

$$Q_2' = A_2 v_2 = [(\pi)(\tfrac{470}{1000})^2/4](3.606) = 0.6256\text{ m}^3/\text{s} \quad \text{or} \quad 625.6\text{ L/s}$$

Now the desired actual flows Q_1 and Q_2 may be computed so as to maintain the ratio Q_1'/Q_2' and to satisfy continuity so that $Q_1 + Q_2 = 570$ L/s.

$$Q_1 = [170/(625.6 + 170)](570) = 121.8\text{ L/s} \qquad Q_2 = [625.6/(625.6 + 170)](570) = 448.2\text{ L/s}$$

$$v_1 = (121.8/1000)/[(\pi)(\tfrac{300}{1000})^2/4] = 1.723\text{ m/s} \qquad v_2 = (448.2/1000)/[(\pi)(\tfrac{470}{1000})^2/4] = 2.583\text{ m/s}$$

$$(N_R)_1 = (\tfrac{300}{1000})(1.723)/(0.0113 \times 10^{-4}) = 4.57 \times 10^5 \qquad (N_R)_2 = (\tfrac{470}{1000})(2.583)/(0.0113 \times 10^{-4}) = 1.07 \times 10^6$$

$$f_1 = 0.0198 \qquad f_2 = 0.0180 \qquad (h_f)_1 = 0.0198[600/(\tfrac{300}{1000})]\{1.723^2/[(2)(9.807)]\} = 5.99\text{ m}$$

$$p_A/\gamma + (v_1^2)_A/2g + 6 = p_B/\gamma + (v_1^2)_B/2g + 15 + 5.99 \qquad [(p_A - p_B)/\gamma]_1 = 14.99\text{ m}$$

$$(h_f)_2 = 0.0180[460/(\tfrac{470}{1000})]\{2.583^2/[(2)(9.807)]\} = 5.99\text{ m}$$

$$p_A/\gamma + (v_2^2)_A/2g + 6 = p_B/\gamma + (v_2^2)_B/2g + 15 + 5.99$$

$$[(p_A - p_B)/\gamma]_2 = 14.99\text{ m} \qquad [(690 - p_B)/9.79] = 14.99 \qquad p_B = 543\text{ kPa}$$

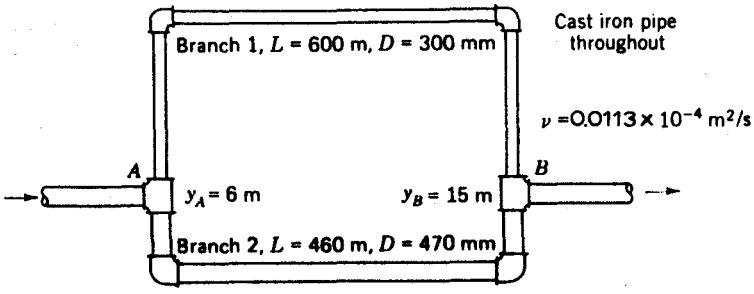


Fig. 11-6

11.7 The system shown in Fig. 11-7 receives at A 66 ft³/min of water at a pressure of 95 psig. Neglecting minor losses, find the pressure at B. The pipe is commercial steel and $\mu = 2.11 \times 10^{-5}$ lb · s/ft².

▮ $$p_A/\gamma + v_A^2/2g + z_A = p_C/\gamma + v_C^2/2g + z_C + h_L.$$

Assume that a flow Q_1' of 30 ft³/min = 0.5 ft³/s is proceeding through branch 1.

$$h_f = (f)(L/D)(v^2/2g) \qquad v_1 = Q_1'/A_1 = 0.5/[(\pi)(\tfrac{6}{12})^2/4] = 2.546\text{ ft/s} \qquad N_R = \rho D v/\mu$$

$$(N_R)_1 = (1.94)(\tfrac{6}{12})(2.546)/(2.11 \times 10^{-5}) = 1.17 \times 10^5 \qquad \epsilon/D_1 = 0.00015/(\tfrac{6}{12}) = 0.00030$$

From Fig. A-5, $f = 0.0195$. $h_L = (h_f)_1 = 0.0195[(35 + 1200 + 35)/(\tfrac{6}{12})]\{2.546^2/[(2)(32.2)]\} = 4.985$ ft.

For branch 1, $p_A/\gamma + (v_1^2)_A/2g + 0 = p_B/\gamma + (v_1^2)_B/2g + 0 + 4.985$, $(v_1^2)_A/2g = (v_1^2)_B/2g$, $[(p_A - p_B)/\gamma]_1 = 4.985$ ft. Using this pressure head, which must be the same for each loop, for branch 2,

$$4.985 + (v_2^2)_A/2g + 0 = (v_2^2)_B/2g + 0 + (h_f)_2 \qquad (v_2^2)_A/2g = (v_2^2)_B/2g \qquad (h_f)_2 = 4.985\text{ ft}$$

$$4.985 = f_2[(35 + 1200 + 35)/(\tfrac{8}{12})]\{v_2^2/[(2)(32.2)]\} = 29.58 f_2 v_2^2 \qquad (\epsilon/D)_2 = 0.00015/(\tfrac{8}{12}) = 0.000225$$

Estimate $f_2 = 0.018$.

$$4.985 = (29.58)(0.018)(v_2^2) \qquad v_2 = 3.060\text{ ft/s} \qquad Q_2' = A_2 v_2 = [(\pi)(\tfrac{8}{12})^2/4](3.060) = 1.068\text{ ft}^3/\text{s}$$

Now the desired actual flows Q_1 and Q_2 may be computed so as to maintain the ratio Q_1'/Q_2' and to satisfy continuity so that $Q_1 + Q_2 = 66$ ft³/min = 1.1 ft³/s.

$$Q_1 = [0.5/(0.5 + 1.068)](1.1) = 0.3508\text{ ft}^3/\text{s} \qquad Q_2 = [1.068/(0.5 + 1.068)](1.1) = 0.7492\text{ ft}^3/\text{s}$$

$$v_1 = (0.3508)/[(\pi)(\tfrac{6}{12})^2/4] = 1.787\text{ ft/s} \qquad v_2 = (0.7492)/[(\pi)(\tfrac{8}{12})^2/4] = 2.146\text{ ft/s}$$

$$(N_R)_1 = (1.94)(\tfrac{6}{12})(1.787)/(2.11 \times 10^{-5}) = 8.22 \times 10^4 \qquad (N_R)_2 = (1.94)(\tfrac{8}{12})(2.146)/(2.11 \times 10^{-5}) = 1.32 \times 10^5$$

$$f_1 = 0.0205 \qquad f_2 = 0.0190 \qquad (h_f)_1 = 0.0205[(35 + 1200 + 35)/(\tfrac{6}{12})]\{1.787^2/[(2)(32.2)]\} = 2.582\text{ ft}$$

$$p_A/\gamma + (v_1^2)_A/2g + 0 = p_C/\gamma + (v_1^2)_C/2g + 0 + 2.582 \qquad [(p_A - p_B)/\gamma]_1 = 2.582 \text{ ft}$$

$$(h_f)_2 = 0.0190[(35 + 1200 + 35)/(\tfrac{8}{12})]\{2.146^2/[(2)(32.2)]\} = 2.588 \text{ ft}$$

$$p_A/\gamma + (v_2^2)_A/2g + 0 = p_C/\gamma + (v_2^2)_C/2g + 0 + 2.588 \qquad [(p_A - p_C)/\gamma]_2 = 2.588 \text{ ft}$$

$$[(95 - p_C)(144)/62.4] = 2.588 \text{ ft} \qquad p_C = 93.9 \text{ lb/in}^2$$

For pipe CB,

$$v = Q/A = 1.1/[(\pi)(\tfrac{8}{12})^2/4] = 3.151 \text{ ft/s} \qquad N_R = (1.94)(\tfrac{8}{12})(3.151)/(2.11 \times 10^{-5}) = 1.93 \times 10^5 \qquad f = 0.0178$$

$$h_f = 0.0178[3200/(\tfrac{8}{12})]\{3.151^2/[(2)(32.2)]\} = 13.17 \text{ ft} \qquad p_B = 93.9 - (13.17)(62.4)/144 = 88.2 \text{ psig}$$

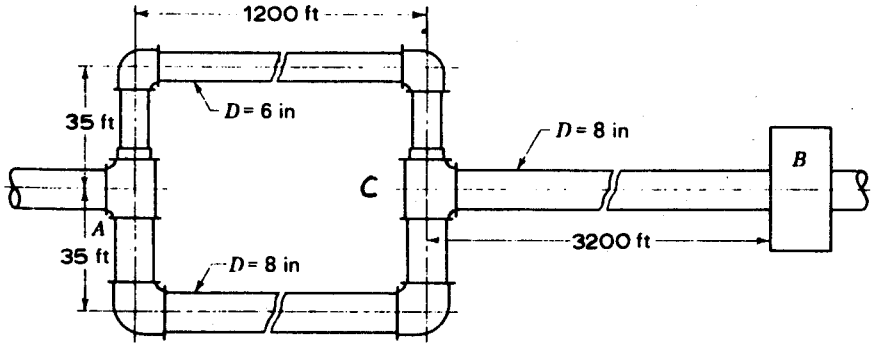


Fig. 11-7

11.8 Repeat Prob. 11.7 using *nominal* pipe sizes (6.065 in and 7.981 in diameters) and supposing an open globe valve in the 8-in pipe just before B. The fittings are all screwed fittings. Use $K = 0.45$ for a 6-in standard tee, 0.30 for elbows, and 4.8 for the globe valve.

▮ $p_A/\gamma + v_A^2/2g + z_A = p_C/\gamma + v_C^2/2g + z_C + h_L$. Assume that a flow Q_1' of 0.5 ft³/s is proceeding through branch 1.

$$h_L = h_f + h_m \qquad h_f = (f)(L/D)(v^2/2g) \qquad v_1 = Q_1'/A_1 = 0.5/[(\pi)(6.065/12)^2/4] = 2.492 \text{ ft/s}$$

$$N_R = \rho D v/\mu \qquad (N_R)_1 = (1.94)(6.065/12)(2.492)/(2.11 \times 10^{-5}) = 1.16 \times 10^5 \qquad \epsilon/D_1 = 0.00015/(\tfrac{6.065}{12}) = 0.00030$$

From Fig. A-5, $f = 0.0198$.

$$(h_f)_1 = 0.0198[(35 + 1200 + 35)/(6.065/12)]\{2.492^2/[(2)(32.2)]\} = 4.798 \text{ ft}$$

$$h_m = (K)(v^2/2g) \qquad (h_m)_1 = [(2)(0.30) + (2)(0.45)]\{2.492^2/[(2)(32.2)]\} = 0.145 \text{ ft}$$

$$(h_L)_1 = 4.798 + 0.145 = 4.943 \text{ ft}$$

For branch 1,

$$p_A/\gamma + (v_1^2)_A/2g + 0 = p_B/\gamma + (v_1^2)_B/2g + 0 + 4.943 \qquad (v_1^2)_A/2g = (v_1^2)_B/2g \qquad [(p_A - p_B)/\gamma]_1 = 4.943 \text{ ft}$$

Using this pressure head, which must be the same for each loop, for branch 2,

$$4.943 + (v_2^2)_A/2g + 0 = (v_2^2)_B/2g + 0 + (h_L)_2 \qquad (v_2^2)_A/2g = (v_2^2)_B/2g \qquad (h_L)_2 = 4.943 \text{ ft}$$

$$4.943 = f_2[(35 + 1200 + 35)/(7.981/12)]\{v_2^2/[(2)(32.2)]\} + [(2)(0.30) + (2)(0.45)]\{v_2^2/[(2)(32.2)]\}$$

$$= 29.65 f_2 v_2^2 + 0.02329 v_2^2$$

$$(\epsilon/D)_2 = 0.00015/(7.981/12) = 0.000226$$

Estimate $f_2 = 0.018$.

$$4.943 = (29.65)(0.018)(v_2^2) + 0.02329 v_2^2 \qquad v_2 = 2.979 \text{ ft/s} \qquad Q_2' = A_2 v_2 = [(\pi)(7.981/12)^2/4](2.979) = 1.035 \text{ ft}^3/\text{s}$$

Now the desired actual flows Q_1 and Q_2 may be computed so as to maintain the ratio Q_1'/Q_2' and to satisfy continuity so that $Q_1 + Q_2 = 1.1$ ft³/s.

$$Q_1 = [0.5/(0.5 + 1.035)](1.1) = 0.3583 \text{ ft}^3/\text{s} \qquad Q_2 = [1.035/(0.5 + 1.035)](1.1) = 0.7417 \text{ ft}^3/\text{s}$$

$$v_1 = (0.3583)/[(\pi)(6.065/12)^2/4] = 1.786 \text{ ft/s} \qquad v_2 = (0.7417)/[(\pi)(7.981/12)^2/4] = 2.135 \text{ ft/s}$$

$$(N_R)_1 = (1.94)(6.065/12)(1.786)/(2.11 \times 10^{-5}) = 8.30 \times 10^4$$

$$(N_R)_2 = (1.94)(7.981/12)(2.135)/(2.11 \times 10^{-5}) = 1.31 \times 10^5$$

$$f_1 = 0.0205 \qquad f_2 = 0.019$$

$$(h_f)_1 = 0.0205[(35 + 1200 + 35)/(6.065/12)]\{1.786^2/[(2)(32.2)]\} = 2.551 \text{ ft}$$

$$(h_m)_1 = [(2)(0.30) + (2)(0.45)]\{1.786^2/[(2)(32.2)]\} = 0.074 \text{ ft} \qquad (h_L)_1 = 2.551 + 0.074 = 2.625 \text{ ft}$$

$$p_A/\gamma + (v_1^2)_A/2g + 0 = p_C/\gamma + (v_1^2)_C/2g + 0 + 2.625 \qquad [(p_A - p_B)/\gamma]_1 = 2.625 \text{ ft}$$

$$(h_f)_2 = 0.019[(35 + 1200 + 35)/(7.981/12)]\{2.135^2/[(2)(32.2)]\} = 2.568 \text{ ft}$$

$$(h_m)_2 = [(2)(0.30) + (2)(0.45)]\{2.135^2/[(2)(32.2)]\} = 0.106 \text{ ft}$$

$$(h_L)_1 = 2.568 + 0.106 = 2.674 \text{ ft} \qquad p_A/\gamma + (v_2^2)_A/2g + 0 = p_C/\gamma + (v_2^2)_C/2g + 0 + 2.674$$

$$[(p_A - p_C)/\gamma]_2 = 2.674 \text{ ft} \qquad [(95 - p_C)(144)/62.4] = 2.674 \text{ ft} \qquad p_C = 93.8 \text{ lb/in}^2$$

For pipe CB,

$$v = Q/A = 1.1/[(\pi)(7.981/12)^2/4] = 3.166 \text{ ft/s} \qquad N_R = (1.94)(7.981/12)(3.166)/(2.11 \times 10^{-5}) = 1.94 \times 10^5$$

$$f = 0.0178 \qquad h_f = 0.0178[3200/(7.981/12)]\{3.166^2/[(2)(32.2)]\} = 13.33 \text{ ft}$$

$$h_m = 4.8\{3.166^2/[(2)(32.2)]\} = 0.75 \text{ ft} \qquad h_L = 13.33 + 0.75 = 14.08 \text{ ft}$$

$$p_B = 93.8 - (14.08)(62.4)/144 = 87.7 \text{ lb/in}^2$$

11.9 A two-branch, commercial steel pipe system (Fig. 11-8) delivers 0.42 m³/s of water at 5 °C. The pressure at B is 22 kPa gage. What is the pressure at A? Neglect minor losses.

▮ $p_A/\gamma + v_A^2/2g + z_A = p_B/\gamma + v_B^2/2g + z_B + h_L$. Assume that a flow Q_1' of 0.20 m³/s is proceeding through branch 1.

$$h_f = (f)(L/D)(v^2/2g) \qquad (v_1)_{250} = Q_1'/A_1 = 0.20/[(\pi)(0.250)^2/4] = 4.074 \text{ m/s}$$

$$(v_1)_{200} = Q_1'/A_1 = 0.20/[(\pi)(0.200)^2/4] = 6.366 \text{ m/s} \qquad N_R = Dv/\nu$$

$$[(N_R)_1]_{250} = (0.250)(4.074)/(1.52 \times 10^{-6}) = 6.70 \times 10^5 \qquad (\epsilon/D_1)_{250} = 0.000046/0.250 = 0.000184$$

From Fig. A-5, $(f_1)_{250} = 0.015$.

$$[(N_R)_1]_{200} = (0.200)(6.366)/(1.52 \times 10^{-6}) = 8.38 \times 10^5 \qquad (\epsilon/D_1)_{200} = 0.000046/0.200 = 0.00023 \qquad (f_1)_{200} = 0.015$$

$$h_L = 0.015[(60 + 110 + 40)/0.250]\{4.074^2/[(2)(9.807)]\} + 0.015[(110 + 40)/0.200]\{6.366^2/[(2)(9.807)]\} = 33.91 \text{ m}$$

For branch 1,

$$p_A/\gamma + (v_1^2)_A/2g + 0 = p_B/\gamma + (v_1^2)_B/2g + 0 + 33.91 \qquad (v_1^2)_A/2g = (v_1^2)_B/2g \qquad [(p_A - p_B)/\gamma]_1 = 33.91 \text{ m}$$

Using this pressure head, which must be the same for each loop, for branch 2,

$$33.91 + (v_2^2)_A/2g + 0 = (v_2^2)_B/2g + 0 + (h_f)_2 \qquad (v_2^2)_A/2g = (v_2^2)_B/2g \qquad (h_f)_2 = 33.91 \text{ m}$$

$$33.91 = f_2[(20 + 220)/0.200]\{v_2^2/[(2)(9.807)]\} = 61.18 f v_2^2 \qquad (\epsilon/D)_2 = 0.000046/0.200 = 0.00023$$

Estimate $f_2 = 0.015$.

$$33.91 = (61.18)(0.015)(v_2^2) \qquad v_2 = 6.079 \text{ m/s} \qquad Q_2' = A_2 v_2 = [(\pi)(0.200)^2/4](6.079) = 0.1910 \text{ m}^3/\text{s}$$

Now the desired actual flows Q_1 and Q_2 may be computed so as to maintain the ratio Q_1'/Q_2' and to satisfy continuity so that $Q_1 + Q_2 = 0.42$ m³/s.

$$Q_1 = [0.20/(0.20 + 0.1910)](0.42) = 0.2148 \text{ m}^3/\text{s} \qquad Q_2 = [0.1910/(0.20 + 0.1910)](0.42) = 0.2052 \text{ m}^3/\text{s}$$

$$(v_1)_{250} = 0.2148/[(\pi)(0.250)^2/4] = 4.376 \text{ m/s} \qquad (v_1)_{200} = (0.2148)/[(\pi)(0.200)^2/4] = 6.837 \text{ m/s}$$

$$v_2 = (205.2/1000)/[(\pi)(0.200)^2/4] = 6.532 \text{ m/s}$$

Since the assumed value of Q_1 is close to 0.2148 m³/s, the values of f of 0.015 should be close enough.

$$(h_f)_1 = 0.015[(60 + 110 + 40)/0.250]\{4.376^2/[(2)(9.807)]\}$$
$$+ 0.015[(110 + 40)/0.200]\{6.837^2/[(2)(9.807)]\} = 39.11 \text{ m}$$

$$p_A/\gamma + (v_1^2)_A/2g + 0 = p_B/\gamma + (v_1^2)_B/2g + 0 + 39.11 \qquad [(p_A - p_B)/\gamma]_1 = 39.11 \text{ m}$$

$$(h_f)_2 = 0.015[(20 + 220)/0.200]\{6.532^2/[(2)(9.807)]\} = 39.16 \text{ m}$$

$$p_A/\gamma + (v_2^2)_A/2g + 0 = p_B/\gamma + (v_2^2)_B/2g + 0 + 39.16 \qquad [(p_A - p_B)/\gamma]_2 = 39.16 \text{ m}$$

These values of $[(p_A - p_B)/\gamma]$ are close enough, so take an average value of (39.11 + 39.16)/2, or 39.14 m: $(p_A - 22)/9.79 = 39.14$, $p_A = 405$ kPa.

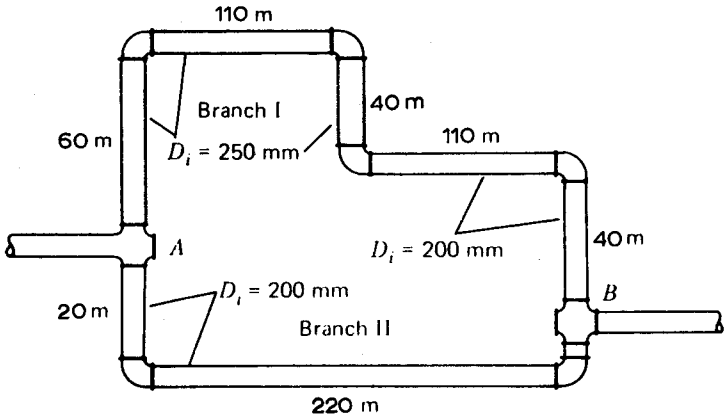


Fig. 11-8

11.10 The vertical, commercial steel pipe system shown in Fig. 11-9 ($D = 300$ mm throughout) carries 0.82 m³/s of water. Find the pressure drop between A and B if B is 100 m higher than A. Neglect minor losses. The water is at 5 °C.

▮ $$p_A/\gamma + v_A^2/2g + z_A = p_B/\gamma + v_B^2/2g + z_B + h_L$$

For branch I: Estimate $Q_I' = 0.25\ \text{m}^3/\text{s}$, $h_f = (f)(L/D)(v^2/2g)$. Let $f = 0.014$: $v_I' = Q_I'/A_I = 0.25/[(\pi)(0.300)^2/4] = 3.537$ m/s, $h_f = 0.014[(250 + 550 + 250)/0.300]\{3.537^2/[(2)(9.807)]\} = 31.25$ m, $p_A/\gamma + v_A^2/2g + 0 = p_B/\gamma + v_B^2/2g + 100 + 31.25$, $v_A^2/2g = v_B^2/2g$, $(p_A - p_B)/\gamma = 131.25$ m.
For branch II: Let $f = 0.015$: $h_f = 0.015[550/0.300]\{(v_{II}')^2/[(2)(9.807)]\} = (1.402)(v_{II}')^2$, $p_A/\gamma + v_A^2/2g + 0 = p_B/\gamma + v_B^2/2g + 100 + (1.402)(v_{II}')^2$, $(p_A - p_B)/\gamma = 100 + (1.402)(v_{II}')^2$. Use $(p_A - p_B)/\gamma = 131.25$ m. $131.25 = 100 + (1.402)(v_{II}')^2$, $v_{II}' = 4.721$ m/s.
For branch III: Let $f = 0.013$: $h_f = 0.013[(350 + 550 + 350)/0.300]\{(v_{III}')^2/[(2)(9.807)]\} = (2.762)(v_{III}')^2$, $p_A/\gamma + v_A^2/2g + 0 = p_B/\gamma + v_B^2/2g + 100 + (2.762)(v_{III}')^2$, $(p_A - p_B)/\gamma = 100 + (2.762)(v_{III}')^2$. Use $(p_A - p_B)/\gamma = 131.25$ m. $131.25 = 100 + (2.762)(v_{III}')^2$, $v_{III}' = 3.364$ m/s. The new values of Q are $Q_I = 250$ L/s; $Q_{II} = [(\pi)(0.300)^2/4](4.721) = 0.3337$ m³/s, or 333.7 L/s; $Q_{III} = [(\pi)(0.300)^2/4](3.364) = 0.2378$ m³/s, or 237.8 L/s. Find new values of Q: $Q_I = [250/(250 + 333.7 + 237.8)](820) = 249.5$ L/s, $Q_{II} = [333.7/(250 + 333.7 + 237.8)](820) = 333.1$ L/s, $Q_{III} = [237.8/(250 + 383.4 + 237.8)](820) = 237.4$ L/s.
For branch I: $v_I = (249.5/1000)/[(\pi)(0.300)^2/4] = 3.530$ m/s, $N_R = Dv/\nu = (0.300)(3.530)/(1.52 \times 10^{-6}) = 6.97 \times 10^5$, $\epsilon/D = 0.000046/0.300 = 0.00015$. From Fig. A-5, $f = 0.0146$.
For branch II: $v_{II} = (333.1/1000)/[(\pi)(0.300)^2/4] = 4.712$ m/s, $N_R = (0.300)(4.712)/(1.52 \times 10^{-6}) = 9.30 \times 10^5$, $f = 0.0141$.
For branch III: $v_{III} = (237.4/1000)/[(\pi)(0.300)^2/4] = 3.359$ m/s, $N_R = (0.300)(3.359)/(1.52 \times 10^{-6}) = 6.63 \times 10^5$, $f = 0.0146$. Compute $(p_A - p_B)/\gamma$ for each branch.
For branch I: $h_f = 0.0146[(250 + 550 + 250)/0.300]\{3.530^2/[(2)(9.807)]\} = 32.46$ m, $p_A/\gamma + v_A^2/2g + 0 = p_B/\gamma + v_B^2/2g + 100 + 32.46$, $(p_A - p_B)/\gamma = 132.46$ m.
For branch II: $h_f = 0.0141[550/0.300]\{4.712^2/[(2)(9.807)]\} = 29.26$ m, $p_A/\gamma + v_A^2/2g + 0 = p_B/\gamma + v_B^2/2g + 100 + 29.26$, $(p_A - p_B)/\gamma = 129.26$ m.
For branch III: $h_f = 0.0146[(350 + 550 + 350)/0.300]\{3.359^2/[(2)(9.807)]\} = 34.99$ m, $p_A/\gamma + v_A^2/2g + 0 = p_B/\gamma + v_B^2/2g + 100 + 34.99$, $(p_A - p_B)/\gamma = 134.99$ m.
We could go through another cycle. Instead, we stop here and take an average value of $(p_A - p_B)/\gamma$: $(132.46 + 129.26 + 134.99)/3$, or 132.24 m. $(p_A - p_B) = (132.24)(9.79) = 1295$ kPa gage.

11.11 In Fig. 11-10, $L_1 = 3000$ ft, $D_1 = 1$ ft, $\epsilon_1 = 0.001$ ft; $L_2 = 2000$ ft, $D_2 = 8$ in, $\epsilon_2 = 0.0001$ ft; $L_3 = 4000$ ft, $D_3 = 16$ in, $\epsilon_3 = 0.0008$ ft; $\rho = 2.00$ slugs/ft³, $\nu = 0.00003$ ft²/s, $p_A = 80$ psi, $z_A = 100$ ft, $z_B = 80$ ft. For a total flow of 12 cfs, determine the flow through each pipe and the pressure at B.

▮ $h_f = (f)(L/D)(v^2/2g)$. Assume $Q_1' = 3$ cfs.

$$v_1' = Q_1'/A_1 = 3/[(\pi)(1)^2/4] = 3.820\ \text{ft/s} \qquad N_R = Dv/\nu$$
$$(N_R)_1' = (1)(3.820)/0.00003 = 1.27 \times 10^5 \qquad \epsilon/D_1 = 0.001/1 = 0.0010$$

From Fig. A-5, $f_1' = 0.022$. $(h_f)_1' = (0.022)(\frac{3000}{1})\{3.820^2/[(2)(32.2)]\} = 14.95$ ft. For pipe 2, assume $f_2' = 0.020$.

$$14.95 = 0.020[2000/(\tfrac{8}{12})]\{v_2'^2/[(2)(32.2)]\} \qquad v_2' = 4.006\ \text{ft/s} \qquad (N_R)_2' = (\tfrac{8}{12})(4.006)/0.00003 = 8.90 \times 10^4$$
$$\epsilon_2/D_2 = 0.0001/(\tfrac{8}{12}) = 0.00015 \qquad f_2' = 0.019 \qquad 14.95 = 0.019[2000/(\tfrac{8}{12})]\{v_2'^2/[(2)(32.2)]\}$$
$$v_2' = 4.110\ \text{ft/s} \qquad Q_2' = [(\pi)(\tfrac{8}{12})^2/4](4.110) = 1.43\ \text{ft}^3/\text{s}$$

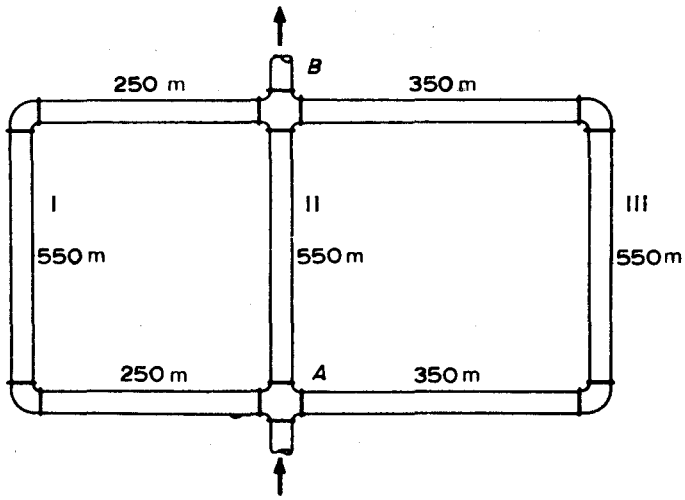


Fig. 11-9

For pipe 3, assume $f_3' = 0.020$.

$$14.95 = 0.020[4000/(\tfrac{16}{12})]\{v_3'^2/[(2)(32.2)]\} \qquad v_3' = 4.006\ \text{ft/s} \qquad (N_R)_2' = (\tfrac{16}{12})(4.006)/0.00003 = 1.78 \times 10^5$$

$$\epsilon_3/D_3 = 0.0008/(\tfrac{16}{12}) = 0.00060 \qquad f_3' = 0.020 \qquad Q_3' = [(\pi)(\tfrac{16}{12})^2/4](4.006) = 5.59\ \text{ft}^3/\text{s}$$

$$\Sigma Q = 3 + 1.43 + 5.59 = 10.02\ \text{ft}^3/\text{s} \qquad Q_1 = (3/10.02)(12) = 3.59\ \text{ft}^3/\text{s}$$

$$Q_2 = (1.43/10.02)(12) = 1.71\ \text{ft}^3/\text{s} \qquad Q_3 = (5.59/10.02)(12) = 6.69\ \text{ft}^3/\text{s}$$

Check the values of h_1, h_2, and h_3:

$$v_1 = 3.59/[(\pi)(1)^2/4] = 4.571\ \text{ft/s} \qquad (N_R)_1 = (1)(4.571)/0.00003 = 1.52 \times 10^5$$

$$f_1 = 0.021 \qquad (h_f)_1 = (0.021)(\tfrac{3000}{1})\{4.571^2/[(2)(32.2)]\} = 20.44\ \text{ft} \qquad v_2 = 1.71/[(\pi)(\tfrac{8}{12})^2/4] = 4.899\ \text{ft/s}$$

$$(N_R)_2 = (\tfrac{8}{12})(4.899)/0.00003 = 1.09 \times 10^5 \qquad f_2 = 0.018 \qquad (h_f)_2 = 0.018[2000/(\tfrac{8}{12})]\{4.899^2/[(2)(32.2)]\} = 20.12\ \text{ft}$$

$$v_3 = 6.69/[(\pi)(\tfrac{16}{12})^2/4] = 4.791\ \text{ft/s} \qquad (N_R)_3 = (\tfrac{16}{12})(4.791)/0.00003 = 2.13 \times 10^5$$

$$f_3 = 0.019 \qquad (h_f)_3 = 0.019[4000/(\tfrac{16}{12})]\{4.791^2/[(2)(32.2)]\} = 20.32\ \text{ft}$$

Since these values of h_f are close, accept the flow values determined above and take an average value of $(20.44 + 20.12 + 20.32)/3$, or 20.29 ft for calculating p_B: $p_A/\gamma + v_A^2/2g + z_A = p_B/\gamma + v_B^2/2g + z_B + h_L$, $(80)(144)/62.4 + v_A^2/2g + 100 = (p_B)(144)/62.4 + v_B^2/2g + 80 + 20.29$, $v_A^2/2g = v_B^2/2g$, $p_B = 79.9\ \text{lb/in}^2$.

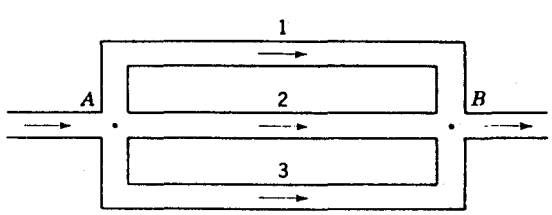


Fig. 11-10

11.12 Prepare a computer program to solve for the head drop and flow distribution through N pipes in parallel. Check it out with the data in Prob. 11.11.

```
10 REM B:                                   SOLUTION OF PARALLEL PIPE PROBLEMS
20 DEFINT I,N:     DEF FNQ(D1,D2,D3,DH)=D1*DH*LOG(D2+D3/DH)
30 READ N,QQ,RHO,KVIS,PA,ZA,ZB,G,II
40 DATA 3,12.,2.,3E-5,80.,100.,80.,32.2,15
50 LPRINT:     LPRINT"N,QQ,RHO,KVIS=";N;QQ;RHO;KVIS
60 LPRINT:     LPRINT"PA,ZA,ZB,G,II=";PA;ZA;ZB;G;II:     LPRINT
70 FOR I=1 TO N:     READ L(I),D(I),EP(I):     NEXT I
80 DATA 3000.,1.,.001,2000.,.6667,.0001,4000.,1.3333,.0008
90 FOR I=1 TO N:     LPRINT"I,L,D,EPS=";I;L(I);D(I);EP(I):     NEXT I:     LPRINT
100 FOR I=1 TO N:     C=SQR(G*D(I)/L(I)):     E1(I)=-.9650001*D(I)^2*C
110 E2(I)=EP(I)/(3.7*D(I)):     E3(I)=1.784*KVIS/(D(I)*C):     NEXT I
120 X=.02*L(1)*QQ^2/(.7854*D(1)^5*2!*G) ' MAX POSS. HEAD WITH FLOW THRU PIPE 1
130 HMA=SQR(X):     HMI=0!
140 FOR II=1 TO II:     S=0!:     HF=. *(HMA+HMI)     ' START OF BISECTION METHOD
150 FOR I=1 TO N:     S=S+FNQ(E1(I),E (I),E3(I),HF):     NEXT I
160 IF S-QQ>0! THEN HMA=HF ELSE HMI= F
170 PRINT"S,QQ,HF=";S;QQ;HF
```

```
180 NEXT I1:     HFR=.5*(HMA+HMI):     HFF=HFR^2        ' FINAL VALUE OF HEAD DROP
190 FOR I=1 TO N:    LPRINT"I,HFF,Q=";I;HFF;FNQ(E1(I),E2(I),E3(I),HFR):  NEXT I
200 LPRINT:  LPRINT"PB=";:    LPRINT USING" ###.### ";PA+(ZA-ZB-HFF)*RHO*G/144'

N,QQ,RHO,KVIS= 3  12  2  .00003

PA,ZA,ZB,G,II= 80  100  80  32.2  15

I,L,D,EPS= 1  3000  1  .001
I,L,D,EPS= 2  2000  .6667  .0001
I,L,D,EPS= 3  4000  1.3333  .0008

I,HFF,Q= 1  20.68776  3.576553
I,HFF,Q= 2  20.68776  1.710832
I,HFF,Q= 3  20.68776  6.713136

PB=  79.692
```

11.13 Two pipes are connected in parallel between two reservoirs: $L_1 = 2600$ m, $D_1 = 1.3$ m, $C = 90$; $L_2 = 2400$ m, $D_2 = 0.9$ m, $C = 100$. For a difference in elevation of 3.8 m, determine the total flow of water at 20 °C.

▮

$$v = 0.8492CR^{0.63}s^{0.54}$$

$$v_1 = (0.8492)(90)(1.3/4)^{0.63}(3.8/2600)^{0.54} = 1.109 \text{ m/s}$$

$$v_2 = (0.8492)(100)(\tfrac{0.9}{4})^{0.63}(3.8/2400)^{0.54} = 1.020 \text{ m/s}$$

$$Q = Av = [(\pi)(1.3)^2/4](1.109) + [(\pi)(0.9)^2/4](1.020) = 2.12 \text{ m}^3/\text{s}$$

11.14 For 5.4 m³/s of flow in the system of Prob. 11.13, determine the difference in elevation of reservoir surfaces.

▮

$$Q = Av = 0.8492ACR^{0.63}s^{0.54}$$

$$5.4 = 0.8492[(\pi)(1.3)^2/4](90)(1.3/4)^{0.63}s^{0.54} + 0.8492[(\pi)(0.9)^2/4](100)(\tfrac{0.9}{4})^{0.63}s^{0.54}$$

$$s = 0.008455 \text{ m/m} \qquad \Delta \text{ elevation} = (0.008455)(2400) = 20.3 \text{ m}$$

11.15 Three smooth tubes are connected in parallel: $L_1 = 45$ ft, $D_1 = \frac{1}{2}$ in; $L_2 = 55$ ft, $D_2 = 1$ in; $L_3 = 60$ ft, $D_3 = \frac{3}{4}$ in. Given $Q = 33$ gpm, $\gamma = 55$ lb/ft³, and $\mu = 0.13$ Pa · s, what is the drop in hydraulic grade line between junctions?

▮

$$h_f = \frac{128\mu LQ}{\pi\gamma D^4}$$

Therefore, $L_1Q_1/D_1^4 = L_2Q_2/D_2^4 = L_3Q_3/D_3^4$.

$$Q = Q_1 + Q_2 + Q_3 = (33)(0.002228) = 0.07352 \text{ ft}^3/\text{s} \qquad 45Q_1/[(\tfrac{1}{2})/12]^4 = 55Q_2/(\tfrac{1}{12})^4 \qquad Q_1 = 0.07639Q_2$$

$$55Q_2/(\tfrac{1}{12})^4 = 60Q_3/[(\tfrac{3}{4})/12]^4 \qquad Q_3 = 0.2900Q_2 \qquad 0.07639Q_2 + Q_2 + 0.2900Q_2 = 0.07352 \qquad Q_2 = 0.05381 \text{ ft}^3/\text{s}$$

$$\mu = \frac{(0.13)(0.3048)^2}{4.448} = 0.002716 \text{ lb} \cdot \text{s/ft}^2 \qquad h_f = \frac{(128)(0.002716)(55)(0.05381)}{(\pi)(55)(\tfrac{1}{12})^4} = 123.4 \text{ ft}$$

11.16 If $H = 11$ m, $\mu = 8 \times 10^{-3}$ Pa · s, and s.g. = 0.9 in Fig. 11-16, find the discharges through pipes 1, 2, and 3.

▮

$$h_f = (f)(L/D)(v^2/2g) \qquad (\epsilon/D)_1 = 0.006/0.050 = 0.120 \qquad f_1 = 0.114$$

$$(\epsilon/D)_2 = 0.009/0.120 = 0.075 \qquad f_2 = 0.088 \qquad (\epsilon/D)_3 = 0.012/0.100 = 0.120 \qquad f_3 = 0.114$$

$$(h_f)_1 = (h_f)_2 \qquad v_1 = Q_1/A_1 = Q_1/[(\pi)(0.050)^2/4] = 509.3Q_1 \qquad v_2 = Q_2/A_2 = Q_2/[(\pi)(0.120)^2/4] = 88.42Q_2$$

$$0.114[70/0.050]\{(509.3Q_1)^2/[(2)(9.807)]\} = 0.088[80/0.120]\{(88.42Q_2)^2/[(2)(9.807)]\}$$

$$Q_2 = 9.500Q_1 \qquad Q_3 = Q_1 + Q_2 = Q_1 + 9.500Q_1 = 10.500Q_1$$

$$v_3 = Q_3/A_3 = 10.500Q_1/[(\pi)(0.100)^2/4] = 1337Q_1 \qquad H = (h_f)_1 + (h_f)_3$$

$$11 = 0.114[70/0.050]\{(509.3Q_1)^2/[(2)(9.807)]\} + 0.114[110/0.100]\{1337Q_1)^2/[(2)(9.807)]\}$$

$$Q_1 = 0.000901 \text{ m}^3/\text{s} = 0.901 \text{ L/s} \qquad Q_2 = (9.500)(0.901) = 8.56 \text{ L/s} \qquad Q_3 = (10.500)(0.901) = 9.46 \text{ L/s}$$

$$N_R = \frac{4Q\rho}{\pi D\mu} = \frac{(4)(0.00946)(0.9)(1000)}{(\pi)(0.100)(8 \times 10^{-3})} = 13\,550$$

Hence, both f's are O.K.

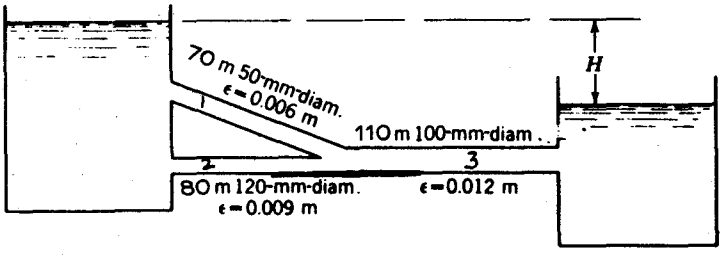


Fig. 11-11

11.17 Find H in Fig. 11-11 for $Q_3 = 20$ L/s, $\mu = 5 \times 10^{-3}$ Pa · s, s.g. = 0.9.

▮ Q_1 and Q_2 will be in the same proportion as in Prob. 11.16. Hence, $Q_1 = (0.901/9.46)(20) = 1.90$ L/s, $Q_2 = (9.500)(1.90) = 18.0$ L/s.

$$H = (h_f)_1 + (h_f)_3 \qquad h_f = (f)(L/D)(v^2/2g)$$

$$v_1 = Q_1/A_1 = 0.00190/[(\pi)(0.050)^2/4] = 0.9677 \text{ m/s} \qquad v_3 = Q_3/A_3 = 0.020/[(\pi)(0.100)^2/4] = 2.546 \text{ m/s}$$

Using values of f from Prob. 11.16,

$$H = 0.114[70/0.050]\{0.9677^2/[(2)(9.907)]\} + 0.114[110/0.100]\{2.546^2/[(2)(9.907)]\} = 49.06 \text{ m}$$

11.18 What length of 300-mm-diameter, clean cast-iron pipe could replace the system of Fig. 11-12?

▮ $(\epsilon/D)_1 = 0.00026/0.200 = 0.0013 \qquad (\epsilon/D)_2 = 0.00026/0.300 = 0.00087 \qquad (\epsilon/D)_3 = 0.00026/0.500 = 0.00052$

$$(\epsilon/D)_4 = 0.00026/0.300 = 0.00087 \qquad (\epsilon/D)_5 = 0.00026/0.300 = 0.00087$$

Try $f_1 = 0.023$, $f_2 = 0.021$, $f_3 = 0.018$, $f_4 = 0.021$, and $f_5 = 0.021$: Assume a head loss in pipes 1 and 2 of 3 m.

$$h_f = (f)(L/D)(v^2/2g) \qquad 3 = 0.023[310/0.200]\{v_1^2/[(2)(9.807)]\} \qquad v_1 = 1.285 \text{ m/s}$$

$$Q_1 = A_1 v_1 = [(\pi)(0.200)^2/4](1.285) = 0.04037 \text{ m}^3/\text{s} \qquad 3 = 0.021[310/0.300]\{v_2^2/[(2)(9.807)]\} \qquad v_2 = 1.647 \text{ m/s}$$

$$Q_2 = A_2 v_2 = [(\pi)(0.300)^2/4](1.647) = 0.1164 \text{ m}^3/\text{s}$$

To find equivalent for 1 and 2, $v_e = (0.04037 + 0.1164)/[(\pi)(0.300)^2/4] = 2.218$ m/s, $3 = 0.021[L_e/0.300]\{2.218^2/[(2)(9.807)]\}$, $L_e = 170.9$ m (for pipes 1 and 2).
For pipe 3:

$$0.018\left(\frac{290}{0.500}\right)\frac{\{Q/[(\pi)(0.500)^2/4]\}^2}{(2)(9.807)} = 0.021\left(\frac{L_e}{0.300}\right)\frac{\{Q/[(\pi)(0.300)^2/4]\}^2}{(2)(9.807)}$$

$$L_e = 53.7 \text{ m} \qquad \text{(for pipe 3)}$$

Assume a head loss in pipes 4 and 5 of 3 m.

$$3 = 0.021[620/0.300]\{v_4^2/[(2)(9.807)]\} \qquad v_4 = 1.164 \text{ m/s}$$

$$Q_4 = [(\pi)(0.300)^2/4](1.164) = 0.08228 \text{ m}^3/\text{s} \qquad 3 = 0.021[790/0.300]\{v_5^2/[(2)(9.807)]\}$$

$$v_5 = 1.032 \text{ m/s} \qquad Q_5 = [(\pi)(0.300)^2/4](1.032) = 0.07295 \text{ m}^3/\text{s}$$

To find equivalent for 4 and 5,

$$v_e = (0.08228 + 0.07295)/[(\pi)(0.300)^2/4] = 2.196 \text{ m/s} \qquad 3 = 0.021[L_e/0.300]\{2.196^2/[(2)(9.807)]\}$$

$$L_e = 174.3 \text{ m} \qquad \text{(for pipes 4 and 5)} \qquad (L_e)_{\text{total}} = 170.9 + 53.7 + 174.3 = 398.9 \text{ m}$$

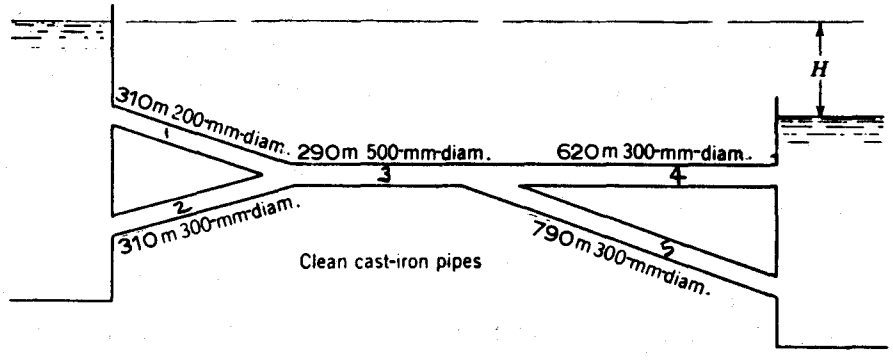


Fig. 11-12

11.19 For a velocity of 1.1 m/s in pipe 1 of Fig. 11-12, calculate the flow through the system and the head H required. The fluid is water at 15 °C.

▮ Using values from Prob. 11.18, $h_f = (f)(L/D)(v^2/2g)$, $(h_f)_1 = 0.023[310/0.200]\{1.1^2/[(2)(9.807)]\} = 2.199$ m, $(h_f)_2 = (h_f)_1 = 0.021[310/0.300]\{v_2^2/[(2)(9.807)]\} = 2.199$ m, $v_2 = 1.410$ m/s; $Q = Av = [(\pi)(0.200)^2/4](1.1) + [(\pi)(0.300)^2/4](1.410) = 0.134$ m³/s. (This is the flow through the system.) $v_3 = Q_3/A_3 = 0.134/[(\pi)(0.500)^2/4] = 0.6825$ m/s, $(h_f)_3 = 0.018[290/0.500]\{0.6825^2/[(2)(9.807)]\} = 0.363$ m. The flow of 0.134 m³/s will split in pipes 4 and 5 in the same proportions as determined in Prob. 11.18. Hence, $Q_4 = [0.08228/(0.08228 + 0.07295)](0.134) = 0.0710$ m³/s, $v_4 = 0.0710/[(\pi)(0.300)^2/4] = 1.004$ m/s, $(h_f)_4 = 0.021[620/0.300]\{1.004^2/[(2)(9.807)]\} = 2.230$ m, $H = 2.199 + 0.363 + 2.230 = 4.792$ m.

11.20 Three pipes are in parallel with a total head loss of 20.3 m. The pipe data are

pipe	L, m	d, cm	ϵ, mm	ϵ/d
1	100	8	0.24	0.003
2	150	6	0.12	0.002
3	80	4	0.20	0.005

The fluid is water, $\rho = 1000$ kg/m³, and $\nu = 1.02 \times 10^{-6}$ m²/s. Calculate the total flow rate in cubic meters per hour, neglecting minor losses.

▮ $h_f = (f)(L/D)(v^2/2g)$, $(h_f)_1 = (h_f)_2 = (h_f)_3$. Guess fully rough flow in pipe 1: $f_1 = 0.0262$.

$$(h_f)_1 = 20.3 = 0.0262[100/(\tfrac{8}{100})]\{v_1^2/[(2)(9.807)]\} \qquad v_1 = 3.487 \text{ m/s}$$

$$N_R = Dv/\nu \qquad (N_R)_1 = (\tfrac{8}{100})(3.487)/(1.02 \times 10^{-6}) = 2.73 \times 10^5$$

From Fig. A-5, $f_1 = 0.0267$.

$$20.3 = 0.0267[100/(\tfrac{8}{100})]\{v_1^2/[(2)(9.807)]\} \qquad v_1 = 3.454 \text{ m/s}$$

$$Q_1 = A_1 v_1 = [(\pi)(\tfrac{8}{100})^2/4](3.454) = 0.01736 \text{ m}^3/\text{s}$$

Guess $f_2 = 0.0234$:

$$(h_f)_2 = 20.3 = 0.0234[150/(\tfrac{6}{100})]\{v_2^2/[(2)(9.807)]\} \qquad v_2 = 2.609 \text{ m/s}$$

$$(N_R)_2 = (\tfrac{6}{100})(2.609)/(1.02 \times 10^{-6}) = 1.53 \times 10^5 \qquad f_2 = 0.0246 \qquad 20.3 = 0.0246[150/(\tfrac{6}{100})]\{v_2^2/[(2)(9.807)]\}$$

$$v_2 = 2.544 \text{ m/s} \qquad Q_2 = [(\pi)(\tfrac{6}{100})^2/4](2.544) = 0.007193 \text{ m}^3/\text{s}$$

Guess $f_3 = 0.0304$:

$$(h_f)_3 = 20.3 = 0.0304[80/(\tfrac{4}{100})]\{v_3^2/[(2)(9.807)]\} \qquad v_3 = 2.559 \text{ m/s}$$

$$(N_R)_3 = (\tfrac{4}{100})(2.559)/(1.02 \times 10^{-6}) = 1.00 \times 10^5$$

$$f_3 = 0.0313 \qquad 20.3 = 0.0313[80/(\tfrac{4}{100})]\{v_3^2/[(2)(9.807)]\} \qquad v_3 = 2.522 \text{ m/s}$$

$$Q_3 = [(\pi)(\tfrac{4}{100})^2/4](2.522) = 0.003169 \text{ m}^3/\text{s}$$

This is satisfactory convergence. Hence, $Q_{\text{total}} = 0.01736 + 0.007193 + 0.003169 = 0.02772$ m³/s, or 99.8 m³/h.

11.21 For the parallel pipe system of Fig. 11-13, each pipe is cast iron, and the pressure drop $p_1 - p_2$ equals 2.5 psi. Compute the total flow rate between 1 and 2 if the fluid properties are $\gamma = 57.32$ lb/ft³, $\mu = 0.00606$ lb · s/ft².

▮ $h_f = (p_1 - p_2)/\gamma = (2.5)(144)/57.32 = 6.281$ ft. Assume laminar flow.

$$h_f = \frac{128\mu LQ}{\pi\gamma D^4}$$

For the 3-in-diameter pipe:

$$6.281 = \frac{(128)(0.00606)(225)(Q_3)}{(\pi)(57.32)(\frac{3}{12})^4} \qquad Q_3 = 0.02531 \text{ ft}^3/\text{s} \qquad N_R = \frac{\gamma D v}{\mu g}$$

$$v_3 = Q_3/A_3 = 0.02531/[(\pi)(\tfrac{3}{12})^2/4] = 0.5156 \text{ ft/s} \qquad (N_R)_3 = (57.32)(\tfrac{3}{12})(0.5156)/(0.00606)(32.2) = 38 \quad \text{(laminar)}$$

For the 2-in-diameter pipe:

$$6.281 = \frac{(128)(0.00606)(185)(Q_2)}{(\pi)(57.32)(\frac{2}{12})^4} \qquad Q_2 = 0.006081\ \text{ft}^3/\text{s} \qquad v_2 = 0.006081/[(\pi)(\tfrac{2}{12})^2/4] = 0.2787\ \text{ft/s}$$

$$(N_R)_3 = (57.32)(\tfrac{2}{12})(0.2787)/(0.00606)(32.2) = 14 \quad \text{(laminar)} \qquad Q_{\text{total}} = 0.02531 + 0.006081 = 0.0314\ \text{ft}^3/\text{s}$$

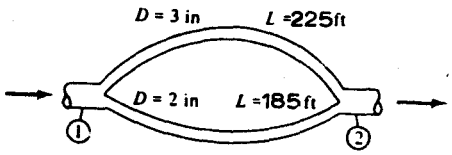


Fig. 11-13

11.22 The system of Fig. 11-14 consists of 60-mm-diameter, wrought iron pipes containing water at 20 °C. If the total flow rate from 1 to 2 is 13 L/s, compute the total pressure drop $p_1 - p_2$. Neglect minor losses.

▮ **For pipe *c*:**

$$v_c = Q_C/A_C = 0.013/[(\pi)(0.060)^2/4] = 4.598\ \text{m/s} \qquad N_R = dv/\nu$$

$$(N_R)_c = (0.060)(4.598)/(1.02 \times 10^{-6}) = 2.70 \times 10^5 \qquad \epsilon/d = 0.000046/0.060 = 0.00077$$

From Fig. A-5, $f_c = 0.020$.

$$h_f = (f)(L/d)(v^2/2g) \qquad (h_f)_c = 0.020[175/0.060]\{4.598^2/[(2)(9.807)]\} = 62.88\ \text{m}$$

$$(\Delta p)_c = (\gamma)(h_f)_c = (9.79)(62.88) = 616\ \text{kPa}$$

For pipes *a* and *b*: $(h_f)_a = (h_f)_b$, $v_a = Q_a/A_a = Q_a/[(\pi)(\frac{6}{100})^2/4] = 353.7Q_a$, $v_b = Q_b/A_b = Q_b/[(\pi)(\frac{6}{100})^2/4] = 353.7Q_b$.

$$f_a[240/0.060]\{(353.7Q_a)^2/[(2)(9.807)]\} = f_b[120/0.060]\{(353.7Q_b)^2/[(2)(9.807)]\} \qquad (1)$$

If $f_a = f_b$, $Q_a = 0.7071Q_b$, $Q_a + Q_b = 1.7071Q_b = 0.013\ \text{m}^3/\text{s}$, $Q_b = 0.007615\ \text{m}^3/\text{s}$, $Q_a = (0.7071)(0.007615) = 0.005385\ \text{m}^3/\text{s}$; $(N_R)_a = \frac{6}{100}[(353.7)(0.005385)]/(1.02 \times 10^{-6}) = 1.12 \times 10^5$, $f_a = 0.0219$; $(N_R)_b = \frac{6}{100}[(353.7)(0.007615)]/(1.02 \times 10^{-6}) = 1.58 \times 10^5$, $f_b = 0.0208$. Substituting these values of f into Eq. (*1*), $0.0219[240/(\frac{6}{100})]\{353.7Q_a)^2/[(2)(9.807)]\} = 0.0208[120/(\frac{6}{100})]\{(353.7Q_b)^2/[(2)(9.807)]\}$, $Q_a = 0.6891Q_b$, $Q_a + Q_b = 0.6891Q_b + Q_b = 0.013$, $Q_b = 0.007696\ \text{m}^3/\text{s}$, $Q_a = (0.6891)(0.007696) = 0.005303\ \text{m}^3/\text{s}$.

$$(N_R)_a = 0.060[(353.7)(0.005303)]/(1.02 \times 10^{-6}) = 1.10 \times 10^5 \qquad f_a = 0.0219 \qquad \text{(O.K.)}$$

$$(N_R)_b = 0.060[(353.7)(0.007696)]/(1.02 \times 10^{-6}) = 1.60 \times 10^5 \qquad f_b = 0.0208 \qquad \text{(O.K.)}$$

Hence, $Q_a = 0.005303\ \text{m}^3/\text{s}$ and $Q_b = 0.007696\ \text{m}^3/\text{s}$.

$$(h_f)_a = (h_f)_b = 0.0219[240/0.060]\{[(353.7)(0.005303)]^2/[(2)(9.807)]\} = 15.71\ \text{m}$$

$$(\Delta p)_{a,b} = (9.79)(15.71) = 154\ \text{kPa} \qquad (\Delta p)_{\text{total}} = 616 + 154 = 770\ \text{kPa}$$

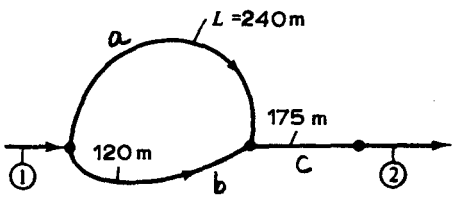


Fig. 11-14

11.23 Reconsider Prob. 11.22 with the flow rate unknown but with the total pressure drop given as 850 kPa. Find the resulting flow rate Q.

▮ Using data from Prob. 11.22, guess $Q_a = 0.006\ \text{m}^3/\text{s}$: $Q_b = 0.006/0.6891 = 0.008707\ \text{m}^3/\text{s}$, $Q_c = 0.006 + 0.008707 = 0.014707\ \text{m}^3/\text{s}$, $v_c = Q_c/A_c = 0.014707/[(\pi)(0.060)^2/4] = 5.202\ \text{m/s}$, $N_R = dv/\nu$, $(N_R)_c = (0.060)(5.202)/(1.02 \times 10^{-6}) = 3.06 \times 10^5$, $\epsilon/d = 0.000046/0.060 = 0.00077$. From Fig. A-5, $f_c = 0.0195$: $h_f = (f)(L/d)(v^2/2g)$, $(h_f)_c = 0.0195[175/0.060]\{5.202^2/[(2)(9.807)]\} = 78.47\ \text{m}$.

For pipes *a* and *b*:

$$(h_f)_a = (h_f)_b \qquad v_a = Q_a/A_a = 0.006/[(\pi)(0.060)^2/4] = 2.122 \text{ m/s}$$

$$v_b = Q_b/A_b = 0.008707/[(\pi)(0.060)^2/4] = 3.079 \text{ m/s}$$

$$(N_R)_a = (0.060)(2.122)/(1.02 \times 10^{-6}) = 1.25 \times 10^5 \qquad f_a = 0.0205$$

$$(N_R)_b = (0.060)(3.079)/(1.02 \times 10^{-6}) = 1.81 \times 10^5 \qquad f_b = 0.020$$

$$(h_f)_a = 0.0205[240/0.060]\{2.122^2/[(2)(9.807)]\} = 18.83 \text{ m}$$

$$(h_f)_b = 0.020[120/0.060]\{3.079^2/[(2)(9.807)]\} = 19.33 \text{ m}$$

Since these are fairly close, forego another iteration and take an average value of $(h_f)_{a,b}$: $(h_f)_{a,b} = (18.83 + 19.33)/2 = 19.08$ m, $(h_f)_{\text{total}} = 78.47 + 19.08 = 97.55$ m, $(\Delta p)_{\text{total}} = (\gamma)(h_f)_{\text{total}} = (9.79)(97.55) =$ 955 kPa. Since this is slightly larger than the required 850 kPa, reduce Q_a a small amount; e.g., try $Q_a = 0.00565$ m^3/s:

$$Q_b = 0.00565/0.6891 = 0.008199 \text{ m}^3/\text{s} \qquad Q_c = 0.00565 + 0.008199 = 0.013849 \text{ m}^3/\text{s}$$

$$v_c = Q_c/A_c = 0.013849/[(\pi)(0.060)^2/4] = 4.898 \text{ m/s} \qquad (N_R)_c = (0.060)(4.898)/(1.02 \times 10^{-6}) = 2.88 \times 10^5$$

$$f_c = 0.0195 \qquad (h_f)_c = 0.0195[175/0.060]\{4.898^2/[(2)(9.807)]\} = 69.57 \text{ m}$$

For pipes *a* and *b*:

$$v_a = Q_a/A_a = 0.00565/[(\pi)(0.060)^2/4] = 1.998 \text{ m/s} \qquad v_b = Q_b/A_b = 0.008199/[(\pi)(0.060)^2/4] = 2.900 \text{ m/s}$$

$$(N_R)_a = (0.060)(1.998)/(1.02 \times 10^{-6}) = 1.18 \times 10^5 \qquad f_a = 0.0205$$

$$(N_R)_b = (0.060)(2.900)/(1.02 \times 10^{-6}) = 1.71 \times 10^5 \qquad f_b = 0.020$$

$$(h_f)_a = 0.0205[240/0.060]\{1.998^2/[(2)(9.807)]\} = 16.69 \text{ m}$$

$$(h_f)_b = 0.020[120/0.060]\{2.900^2/[(2)(9.807)]\} = 17.15 \text{ m}$$

Since these are fairly close, forego another iteration and take an average value of $(h_f)_{a,b}$: $(h_f)_{a,b} = (16.69 + 17.15)/2 = 16.92$ m, $(h_f)_{\text{total}} = 69.57 + 16.92 = 86.49$ m, $(\Delta p)_{\text{total}} = (\gamma)(h_f)_{\text{total}} = (9.79)(86.49) =$ 847 kPa (close enough). Hence, $Q = Q_c = 0.0138$ m^3/s = 13.8 L/s.

11.24 For the piping system of Fig. 11-15, all pipes are concrete with roughness of 0.03 in. If the flow rate is 23 cfs of water at 20 °C, compute the pressure drop $p_1 - p_2$ and the three volumetric flows.

For pipe *a*:

$$v_a = Q_a/A_a = 23/[(\pi)(\tfrac{12}{12})^2/4] = 29.28 \text{ ft/s} \qquad N_R = Dv/\nu$$

$$(N_R)_a = (\tfrac{12}{12})(29.28)/(1.08 \times 10^{-5}) = 2.71 \times 10^6 \qquad \epsilon/D = 0.03/12 = 0.0025$$

From Fig. A-5, $f_a = 0.0249$. $h_f = (f)(L/D)(v^2/2g)$, $(h_f)_a = 0.0249[1200/(\tfrac{12}{12})]\{29.28^2/[(2)(32.2)]\} = 397.8$ ft.

For pipes *b*, *c*, and *d*:

$$(h_f)_b = (h_f)_c = (h_f)_d \qquad v_b = Q_b/A_b = Q_b/[(\pi)(\tfrac{8}{12})^2/4] = 2.865Q_b$$

$$v_c = Q_c/A_c = Q_c/[(\pi)(\tfrac{12}{12})^2/4] = 1.273Q_c \qquad v_d = Q_d/A_d = Q_d/[(\pi)(\tfrac{15}{12})^2/4] = 0.8149Q_d$$

$$f_b\left(\frac{1400}{\frac{8}{12}}\right)\left[\frac{(2.865Q_b)^2}{(2)(32.2)}\right] = f_c\left(\frac{900}{\frac{12}{12}}\right)\left[\frac{(1.273Q_c)^2}{(2)(32.2)}\right] = f_d\left(\frac{1300}{\frac{15}{12}}\right)\left[\frac{(0.8149Q_d)^2}{(2)(32.2)}\right] \qquad (1)$$

If $f_b = f_c = f_d$,

$$Q_b = 0.2209Q_c \qquad Q_d = 1.453Q_c \qquad Q_b + Q_c + Q_d = 0.2209Q_c + Q_c + 1.453Q_c = 23 \qquad Q_c = 8.602 \text{ ft}^3/\text{s}$$

$$Q_b = (0.2209)(8.602) = 1.900 \text{ ft}^3/\text{s} \qquad Q_d = (1.453)(8.602) = 12.50 \text{ ft}^3/\text{s}$$

$$(N_R)_b = \tfrac{8}{12}[(2.865)(1.900)]/(1.08 \times 10^{-5}) = 3.36 \times 10^5 \qquad (\epsilon/D)_b = 0.03/8 = 0.0038 \qquad f_b = 0.028$$

$$(N_R)_c = \tfrac{12}{12}[(1.273)(8.602)]/(1.08 \times 10^{-5}) = 1.01 \times 10^6 \qquad (\epsilon/D)_c = 0.03/12 = 0.0025 \qquad f_c = 0.025$$

$$(N_R)_d = \tfrac{15}{12}[(0.8149)(12.50)]/(1.08 \times 10^{-5}) = 1.18 \times 10^6 \qquad (\epsilon/D)_d = 0.03/15 = 0.0020 \qquad f_d = 0.0235$$

Substituting these values of f into Eq. (1),

$$0.028[1400/(\tfrac{8}{12})]\{(2.865Q_b)^2/[(2)(32.2)]\} = 0.025[900/(\tfrac{12}{12})]\{(1.273Q_c)^2/[(2)(32.2)]\}$$
$$= 0.0235[1300/(\tfrac{15}{12})]\{(0.8149Q_d)^2/[(2)(32.2)]\}$$

$$Q_b = 0.2749Q_c \qquad Q_d = 1.499Q_c \qquad Q_b + Q_c + Q_d = 0.2749Q_c + Q_c + 1.499Q_c = 23 \qquad Q_c = 8.292 \text{ ft}^3/\text{s}$$

$$Q_b = (0.2749)(8.292) = 2.279 \text{ ft}^3/\text{s} \qquad Q_d = (1.499)(8.292) = 12.43 \text{ ft}^3/\text{s}$$

$$(N_R)_b = \tfrac{8}{12}[(2.865)(2.279)]/(1.08\times10^{-5}) = 4.03\times10^5 \qquad f_b = 0.028 \qquad \text{(O.K.)}$$
$$(N_R)_c = \tfrac{12}{12}[(1.273)(8.292)]/(1.08\times10^{-5}) = 9.77\times10^5 \qquad f_c = 0.025 \qquad \text{(O.K.)}$$
$$(N_R)_d = \tfrac{15}{12}[(0.8149)(12.43)]/(1.08\times10^{-5}) = 1.17\times10^6 \qquad f_d = 0.0235 \qquad \text{(O.K.)}$$

Hence, $Q_b = 2.28\ \text{ft}^3/\text{s}$, $Q_c = 8.29\ \text{ft}^3/\text{s}$, and $Q_d = 12.43\ \text{ft}^3/\text{s}$.

$$(h_f)_b = (h_f)_c = (h_f)_d \qquad (h_f)_b = 0.028[1400/(\tfrac{8}{12})]\{[(2.865)(2.28)]^2/[(2)(32.2)]\} = 39.0\ \text{ft}$$
$$(h_f)_{\text{total}} = 397.8 + 39.0 = 436.8\ \text{ft} \qquad (\Delta p)_{\text{total}} = (\gamma)(h_f)_{\text{total}} = (62.4)(436.8) = 27\,256\ \text{lb/ft}^2 \quad \text{or} \quad 189\ \text{lb/in}^2$$

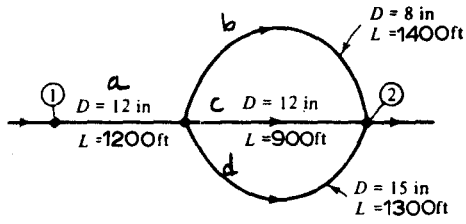


Fig. 11-15

11.25 For the system of Fig. 11-15, with vegetable oil ($\nu = 4.62\times10^{-6}\ \text{ft}^2/\text{s}$, $\gamma = 42.5\ \text{lb/ft}^3$) as the fluid, compute the flow rate in all pipes if the pressure drop $p_1 - p_2$ is 44 psi. Neglect minor losses.

▮ Using data from Prob. 11.24, note that pipe a takes $397.8/436.8 = 0.9107$, or 91.07 percent of the total pressure drop. Hence, take $(\Delta p)_a = (44)(144)(0.9107) = 5770\ \text{lb/ft}^2$, $(h_f)_a = (\Delta p)_a/\gamma = 5770/42.5 = 135.8\ \text{ft}$, $h_f = (f)(L/d)(v^2/2g)$, $135.8 = f_a[1200/(\tfrac{12}{12})]\{v_a^2/[(2)(32.2)]\}$, $v_a = \sqrt{7.288/f_a}$. Try $f_a = 0.025$: $v_a = \sqrt{7.288/0.025} = 17.07\ \text{ft/s}$, $N_R = dv/\nu$, $(N_R)_a = (\tfrac{12}{12})(17.07)/(4.62\times10^{-6}) = 3.69\times10^6$, $(\epsilon/d)_a = 0.03/12 = 0.0025$. From Fig. A-5, $f_a = 0.0245$. Try $f_a = 0.0245$: $v_a = \sqrt{7.288/0.0245} = 17.25\ \text{ft/s}$, $(N_R)_a = (\tfrac{12}{12})(17.25)/(4.62\times10^{-6}) = 3.73\times10^6$, $f_a = 0.0245$ (O.K.); $Q_a = A_a v_a = [(\pi)(\tfrac{12}{12})^2/4](17.25) = 13.55\ \text{ft}^3/\text{s}$. This flow of $13.55\ \text{ft}^3/\text{s}$ for Q_a will divide among the three parallel pipes in the same proportions as found in Prob. 11.24. Hence,

$$Q_b = [2.28/(2.28 + 8.29 + 12.43)](13.55) = 1.343\ \text{ft}^3/\text{s} \qquad Q_c = [8.29/(2.28 + 8.29 + 12.43)](13.55) = 4.884\ \text{ft}^3/\text{s}$$
$$Q_d = [12.43/(2.28 + 8.29 + 12.43)](13.55) = 7.323\ \text{ft}^3/\text{s} \qquad v_b = 1.343/[(\pi)(\tfrac{8}{12})^2/4] = 3.847\ \text{ft/s}$$
$$(\epsilon/d)_b = 0.03/8 = 0.0038 \qquad (N_R)_b = (\tfrac{8}{12})(3.847)/(4.62\times10^{-6}) = 5.55\times10^5 \qquad f_b = 0.028$$
$$(h_f)_{b,c,d} = 0.028[1400/(\tfrac{8}{12})]\{3.847^2/[(2)(32.2)]\} = 13.51\ \text{ft} \qquad (\Delta p)_{b,c,d} = (42.5)(13.51) = 574\ \text{lb/ft}^2$$
$$(\Delta p)_{\text{total}} = 5770 + 574 = 6344\ \text{lb/ft}^2 \quad \text{or} \quad 44.1\ \text{lb/in}^2 \quad \text{(O.K.)}$$

Hence, $Q_a = 13.55$ cfs, $Q_b = 1.34$ cfs, $Q_c = 4.88$ cfs, and $Q_d = 7.32$ cfs.

11.26 In Fig. 11-16 all pipes are cast iron and $p_1 - p_2 = 49$ psi. Compute the total flow rate of water at 20 °C.

▮
$$h_f = (49)(144)/62.4 = 113.1\ \text{ft} = (f)(L/d)(v^2/2g)$$

For pipe c: $(h_f)_c = 113.1 = f_c[2000/(\tfrac{3}{12})]\{v_c^2/[(2)(32.2)]\}$, $v_c = \sqrt{0.9105/f_c}$. Try $f_c = 0.03$: $v_c = \sqrt{0.9105/0.03} = 5.509\ \text{ft/s}$, $N_R = dv/\nu$, $(N_R)_c = (\tfrac{3}{12})(5.509)/(1.08\times10^{-5}) = 1.28\times10^5$, $(\epsilon/d)_c = 0.00085/(\tfrac{3}{12}) = 0.0034$. From Fig. A-5, $f_c = 0.0283$. Try $f_c = 0.0283$: $v_c = \sqrt{0.9105/0.0283} = 5.672\ \text{ft/s}$, $(N_R)_c = (\tfrac{3}{12})(5.672)/(1.08\times10^{-5}) = 1.31\times10^5$, $f_c = 0.0283$ (O.K.); $Q_c = A_c v_c = [(\pi)(\tfrac{3}{12})^2/4](5.672) = 0.2784\ \text{ft}^3/\text{s}$.

For pipes a and b: $(h_f)_{a,b} = 113.1 = f_a[1200/(\tfrac{2}{12})]\{v_a^2/[(2)(32.2)]\} + f_b[1300/(\tfrac{4}{12})]\{v_b^2/[(2)(32.2)]\}$. From continuity, $v_b = v_a/4$. $113.1 = f_a[1200/(\tfrac{2}{12})]\{v_a^2/[(2)(32.2)]\} + f_b[1300/(\tfrac{4}{12})]\{(v_a/4)^2/[(2)(32.2)]\}$, $7284 = (v_a^2)(7200f_a + 243.8f_b)$. Try $f_a = f_b = 0.03$:

$$7284 = (v_a^2)[(7200)(0.03) + (243.8)(0.03)] \qquad v_a = 5.711\ \text{ft/s} \qquad (N_R)_a = (\tfrac{2}{12})(5.711)/(1.08\times10^{-5}) = 8.81\times10^4$$
$$(\epsilon/d)_a = 0.00085/(\tfrac{2}{12}) = 0.0051 \qquad f_a = 0.031 \qquad v_b = 5.711/4 = 1.428\ \text{ft/s}$$
$$(N_R)_b = (\tfrac{4}{12})(1.428)/(1.08\times10^{-5}) = 4.41\times10^4$$
$$(\epsilon/d)_b = 0.00085/(\tfrac{4}{12}) = 0.0026 \qquad f_b = 0.028$$

Try $f_a = 0.031$ and $f_b = 0.028$:

$$7284 = (v_a^2)[(7200)(0.031) + (243.8)(0.028)] \qquad v_1 = 5.627\ \text{ft/s}$$
$$(N_R)_a = (\tfrac{2}{12})(5.627)/(1.08\times10^{-5}) = 8.68\times10^4 \qquad f_a = 0.031 \qquad \text{(O.K.)}$$
$$v_b = 5.627/4 = 1.407\ \text{ft/s} \qquad (N_R)_b = (\tfrac{4}{12})(1.407)/(1.08\times10^{-5}) = 4.34\times10^4 \qquad f_b = 0.028 \qquad \text{(O.K.)}$$
$$Q_a = [(\pi)(\tfrac{2}{12})^2/4](5.627) = 0.1228\ \text{ft}^3/\text{s} \qquad Q_{\text{total}} = 0.2784 + 0.1228 = 0.4012\ \text{ft}^3/\text{s}$$

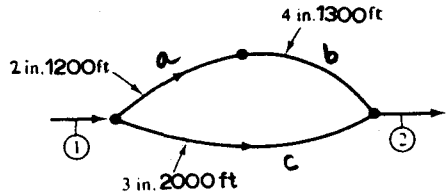


Fig. 11-16

11.27 Refer to Prob. 11.26. Given $Q_{total} = 0.7\ \text{ft}^3/\text{s}$, compute the pressure drop $p_1 - p_2$ and the division of flow between the two paths.

▮ The flow will divide between the two paths in the same proportions as found in Prob. 11.26. Hence,

$$Q_c = [0.2784/(0.2784 + 0.1228)](0.7) = 0.4857\ \text{ft}^3/\text{s}$$

$$Q_a = Q_b = [0.1228/(0.2784 + 0.1228)](0.7) = 0.2143\ \text{ft}^3/\text{s}$$

$$h_f = (f)(L/d)(v^2/2g) \qquad v_c = Q_c/A_c = 0.4857/[(\pi)(\tfrac{3}{12})^2/4] = 9.895\ \text{ft/s} \qquad N_R = dv/\nu$$

$$(N_R)_c = (\tfrac{3}{12})(9.895)/(1.08 \times 10^{-5}) = 2.29 \times 10^5 \qquad (\epsilon/d)_c = 0.00085/(\tfrac{3}{12}) = 0.0034$$

From Fig. A-5, $f_c = 0.028$. $(h_f)_c = 0.028[2000/(\tfrac{3}{12})]\{9.895^2/[(2)(32.2)]\} = 340.6$ ft, $p_1 - p_2 = (\gamma)(h_f)_c = (62.4)(340.6) = 21\,253\ \text{lb/ft}^2$, or $147.6\ \text{lb/in}^2$.

11.28 Two cisterns are connected by cast iron pipes as in Fig. 11-17. Neglecting minor losses, compute the flow rate in each pipe for water at 20 °C.

▮ Assume $(h_f)_c = 20$ ft.

$$\alpha = gd^3h_f/L\nu^2 \qquad N_R = -\sqrt{8\alpha}\log[(\epsilon/d)/3.7 + 2.51/\sqrt{2\alpha}]$$

$$\alpha_c = (32.2)(\tfrac{4}{12})^3(20)/[(1300)(1.08 \times 10^{-5})^2] = 1.573 \times 10^8 \qquad (\epsilon/d)_c = 0.00085/(\tfrac{4}{12}) = 0.00255$$

$$(N_R)_c = -\sqrt{(8)(1.573 \times 10^8)}\log[0.00255/3.7 + 2.51/\sqrt{(2)(1.573 \times 10^8)}] = 1.093 \times 10^5 \qquad N_R = 4Q/\pi\nu d$$

$$1.093 \times 10^5 = 4Q_c/[(\pi)(1.08 \times 10^{-5})(\tfrac{4}{12})] \qquad Q_c = 0.3090\ \text{ft}^3/\text{s} \qquad (h_f)_a = (h_f)_b = 59 - 20 = 39\ \text{ft}$$

$$\alpha_a = (32.2)(\tfrac{3}{12})^3(39)/[(900)(1.08 \times 10^{-5})^2] = 1.869 \times 10^8 \qquad (\epsilon/d)_a = 0.00085/(\tfrac{3}{12}) = 0.00340$$

$$(N_R)_a = -\sqrt{(8)(1.869 \times 10^8)}\log[0.00340/3.7 + 2.51/\sqrt{(2)(1.869 \times 10^8)}] = 1.152 \times 10^5$$

$$1.152 \times 10^5 = 4Q_a/[(\pi)(1.08 \times 10^{-5})(\tfrac{3}{12})] \qquad Q_a = 0.2443\ \text{ft}^3/\text{s}$$

$$\alpha_b = (32.2)(\tfrac{2}{12})^3(39)/[(600)(1.08 \times 10^{-5})^2] = 8.308 \times 10^7 \qquad (\epsilon/d)_b = 0.00085/(\tfrac{2}{12}) = 0.00510$$

$$(N_R)_b = -\sqrt{(8)(8.308 \times 10^7)}\log[0.00510/3.7 + 2.51/\sqrt{(2)(8.308 \times 10^7)}] = 7.227 \times 10^4$$

$$7.227 \times 10^4 = 4Q_b/[(\pi)(1.08 \times 10^{-5})(\tfrac{2}{12})] \qquad Q_b = 0.1022\ \text{ft}^3/\text{s} \qquad Q_a + Q_b = 0.2443 + 0.1022 = 0.3465\ \text{ft}^3/\text{s}$$

Since this value of $Q_a + Q_b = 0.3465$ ft/s is not equal to $Q_c = 0.3090\ \text{ft}^3/\text{s}$, the assumed value of $(h_f)_c = 20$ ft is incorrect. Try $(h_f)_c = 23.3$ ft:

$$\alpha_c = (32.2)(\tfrac{4}{12})^3(23.3)/[(1300)(1.08 \times 10^{-5})^2] = 1.833 \times 10^8$$

$$(N_R)_c = -\sqrt{(8)(1.833 \times 10^8)}\log[0.00255/3.7 + 2.51/\sqrt{(2)(1.833 \times 10^8)}] = 1.182 \times 10^5$$

$$1.182 \times 10^5 = 4Q_c/[(\pi)(1.08 \times 10^{-5})(\tfrac{4}{12})] \qquad Q_c = 0.3341\ \text{ft}^3/\text{s}$$

$$(h_f)_a = (h_f)_b = 59 - 23.3 = 35.7\ \text{ft} \qquad \alpha_a = (32.2)(\tfrac{3}{12})^3(35.7)/[(900)(1.08 \times 10^{-5})^2] = 1.711 \times 10^8$$

$$(N_R)_a = -\sqrt{(8)(1.711 \times 10^8)}\log[0.00340/3.7 + 2.51/\sqrt{(2)(1.711 \times 10^8)}] = 1.101 \times 10^5$$

$$1.101 \times 10^5 = 4Q_a/[(\pi)(1.08 \times 10^{-5})(\tfrac{3}{12})] \qquad Q_a = 0.2336\ \text{ft}^3/\text{s}$$

$$\alpha_b = (32.2)(\tfrac{2}{12})^3(35.7)/[(600)(1.08 \times 10^{-5})^2] = 7.604 \times 10^7$$

$$(N_R)_b = -\sqrt{(8)(7.604 \times 10^7)}\log[0.00510/3.7 + 2.51/\sqrt{(2)(7.604 \times 10^7)}] = 6.908 \times 10^4$$

$$6.908 \times 10^4 = 4Q_b/[(\pi)(1.08 \times 10^{-5})(\tfrac{2}{12})] \qquad Q_b = 0.0977\ \text{ft}^3/\text{s} \qquad Q_a + Q_b = 0.2336 + 0.0977 = 0.3313\ \text{ft}^3/\text{s}$$

This value of $Q_a + Q_b = 0.3313\ \text{ft}^3/\text{s}$ is practically the same as $Q_c = 0.3341\ \text{ft}^3/\text{s}$. Hence, take $Q_a = 0.235\ \text{ft}^3/\text{s}$, $Q_b = 0.098\ \text{ft}^3/\text{s}$, $Q_c = 0.333\ \text{ft}^3/\text{s}$.

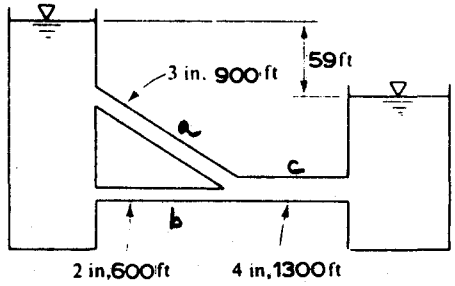


Fig. 11-17

11.29 For the pipe system in Fig. 11-18, the pressure head at A is 120.0 ft of water and the pressure head at E is 72.0 ft of water. Assuming the pipes are in a horizontal plane, what are the flows in each branch of the loop? Neglect minor losses.

▮

$$h_f = 120.0 - 72.0 = 48.0 \text{ ft} \qquad Q = 1.318ACR^{0.63}s^{0.54}$$

$$Q_B = 1.318[(\pi)(\tfrac{12}{12})^2/4](100)[(\tfrac{12}{12})/4]^{0.63}(48.0/12\,000)^{0.54} = 2.19 \text{ ft}^3/\text{s}$$

$$Q_C = 1.318[(\pi)(\tfrac{8}{12})^2/4](100)[(\tfrac{8}{12})/4]^{0.63}(48.0/4000)^{0.54} = 1.37 \text{ ft}^3/\text{s}$$

$$Q_B = 1.318[(\pi)(\tfrac{10}{12})^2/4](100)[(\tfrac{10}{12})/4]^{0.63}(48.0/8000)^{0.54} = 1.69 \text{ ft}^3/\text{s}$$

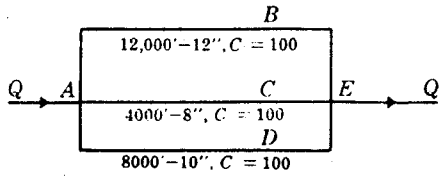


Fig. 11-18

11.30 In Fig. 11-18, if the total flow is 6.50 mgd, how much head loss occurs between A and E and how does Q divide in the loop?

▮ The total flow will divide in the loop in the same proportions as in Prob. 11.29. Hence, $Q_B = [2.19/(2.19 + 1.37 + 1.69)](6.50) = 2.71$ mgd, or 4.20 cfs; $Q_C = [1.37/(2.19 + 1.37 + 1.69)](6.50) = 1.70$ mgd, or 2.62 cfs; $Q_D = [1.69/(2.19 + 1.37 + 1.69)](6.50) = 2.09$ mgd, or 3.24 cfs; $Q = 1.318ACR^{0.63}s^{0.54}$. For pipe B, $4.20 = 1.318[(\pi)(\frac{12}{12})^2/4](100)[(\frac{12}{12})/4]^{0.63}s^{0.54}$, $s = 0.01334$; $(h_f)_B = (0.01334)(12\,000) = 160$ ft. This is the head loss between A and E. It should, of course, be the same when computed by pipes C and D. To confirm this, for pipe C, $2.62 = 1.318[(\pi)(\frac{8}{12})^2/4](100)[(\frac{8}{12})/4]^{0.63}s^{0.54}$, $s = 0.04010$; $(h_f)_C = (0.04010)(4000) = 160$ ft. For pipe D, $3.24 = 1.318[(\pi)(\frac{10}{12})^2/4](100)][(\frac{10}{12})/4]^{0.63}s^{0.54}$, $s = 0.02005$; $(h_f)_B = (0.02005)(8000) = 160$ ft.

11.31 For the system shown in Fig. 11-19, what flow will occur when the drop in the hydraulic grade line from A to B is 200 ft?

▮ Assume $(h_f)_{WZ} = 30$ ft. Using Fig. A-13: For pipe 2, with $d = 12$ in and $h_1 = \frac{30}{5000} = 0.0060$, $Q_2 = 3.2$ ft^3/s. For pipe 3, with $d = 16$ in and $h_1 = \frac{30}{3000} = 0.0100$, $Q_3 = 9.0$ ft^3/s. For pipe 1, with $d = 24$ in and $Q = 3.2 + 9.0 = 12.2$ ft^3/s, $(h_1)_1 = 0.0024$ ft/ft; $(h_f)_{AW} = (0.0024)(10\,000) = 24.0$ ft. For pipe 4, with $d = 20$ in and $Q = 12.2$ ft^3/s, $(h_1)_4 = 0.0060$ ft/ft; $(h_f)_{ZB} = (0.0060)(8000) = 48.0$ ft, $(h_f)_{A-B} = 24.0 + 30 + 48.0 = 102.0$ ft. This value of $(h_f)_{A-B} = 24.0 + 30 + 48.0 = 102.0$ ft is not equal to the given value of 200 ft, but the actual head losses will be in the same proportions as those above. Hence, $(h_f)_{AW} = (24.0/102.0)(200) = 47.06$ ft. For pipe 1, with $d = 24$ in and $h_1 = 47.06/10\,000 = 0.0047$, $Q = 18$ cfs.

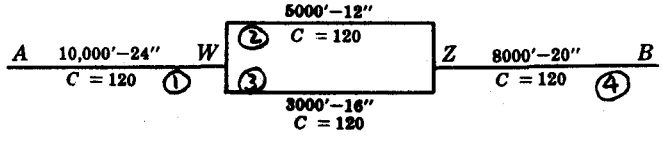


Fig. 11-19

11.32 In Prob. 11.31, what length of 20-in pipe with $C = 120$ is equivalent to section A–B?

▮ From Prob. 11.31, it is known that a flow of 18 cfs is produced when the drop in the hydraulic grade line from A to B is 200 ft. For 18 cfs in a 20-in pipe, from Fig. A-13, $h_1 = 0.012$ ft/ft, $0.012L_e = 200$, $L_e = 16\,700$ ft.

11.33 In Fig. 11-20, when pump YA delivers 5.00 cfs, find the pressure heads at A and B. Draw the hydraulic grade lines.

▮ Reduce loop BC to an equivalent pipe, 16 in in diameter, $C = 100$. By so doing, a single-size pipe of the same relative roughness is readily handled for all conditions of flow. Assume a drop in the grade line of 22 ft ftom B to C. $Q = 1.318AR^{0.63}s^{0.54}$.
For the 10-in pipe: $Q_{10} = 1.318[(\pi)(\frac{10}{12})^2/4](90)[(\frac{10}{12})/4]^{0.63}(22/10\,000)^{0.54} = 0.8843$ cfs.
For the 8-in pipe: $Q_8 = 1.318[(\pi)(\frac{8}{12})^2/4](100)[(\frac{8}{12})/4]^{0.63}(22/11\,000)^{0.54} = 0.5190$ cfs. $Q_{\text{total}} = 0.8843 + 0.5190 = 1.403$ cfs. For a 16-in-diameter equivalent pipe with $C = 100$, $1.403 = 1.318[(\pi)(\frac{16}{12})^2/4](100)[(\frac{16}{12})/4]^{0.63}s^{0.54}$, $s = 0.0004313$ ft/ft; $0.0004313L_e = 22$, $L_e = 51\,000$ ft. For a 16-in-diameter pipe from A to C with a length of $16\,000 + 51\,000 = 67\,000$ ft and carrying 5.00 cfs, $5.00 = 1.318[(\pi)(\frac{16}{12})^2/4](100)[(\frac{16}{12})/4]^{0.63}s^{0.54}$, $s = 0.004537$ ft/ft; $h_f = (0.004537)(67\,000) = 304.0$ ft. Thus the elevation of the hydraulic grade line at A is $217.0 + 304.0 = 521.0$ ft, as shown in the figure. The drop from A to B is $(0.004537)(16\,000) = 72.6$ ft and the elevation of the hydraulic grade line at B becomes $521.0 - 72.6 = 448.4$ ft. Pressure head at $A = 521.0 - 50.0 = 471.0$ ft, pressure head at $B = 448.4 - 50.0 = 398.4$ ft.

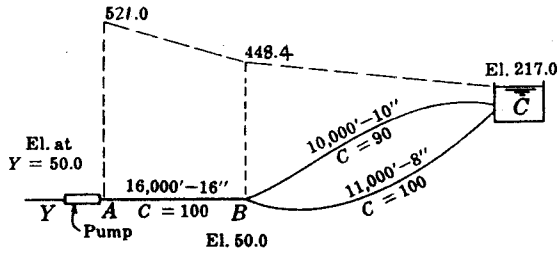


Fig. 11-20

11.34 In Fig. 11-21, which system has the greater capacity, $ABCD$ or $EFGH$? Use $C = 120$ for all pipes.

▮ Assume $Q = 3$ cfs in $ABCD$. Using Fig. A-13,

$$(h_1)_{AB} = 0.0014 \text{ ft/ft} \qquad (h_f)_{AB} = (0.0014)(9000) = 12.6 \text{ ft}$$
$$(h_1)_{BC} = 0.0053 \text{ ft/ft} \qquad (h_f)_{BC} = (0.0053)(6000) = 31.8 \text{ ft}$$

$(h_1)_{CD} = 0.013$ ft/ft $\quad (h_f)_{AB} = (0.013)(3000) = 39.0$ ft $\quad (h_f)_{\text{total}} = 12.6 + 31.8 + 39.0 = 83.4$ ft $\quad$ (for $ABCD$)

For $EFGH$, assume $(h_f)_{FG} = 24$ ft.

$$(h_1)_{FIG} = \tfrac{24}{5000} = 0.00480 \text{ ft/ft} \qquad (h_1)_{FJG} = \tfrac{24}{7000} = 0.00343 \text{ ft/ft} \qquad Q_{FIG} = 0.97 \text{ cfs} \qquad Q_{FJG} = 1.6 \text{ cfs}$$

Hence, pipe FIG carries $0.97/(0.97 + 1.6) = 0.3774$, or 37.74 percent of the flow and pipe FJG carries $1.6/(0.97 + 1.6) = 0.6226$, or 62.26 percent. For $Q = 3.0$ cfs in pipe EF,

$$(h_1)_{EF} = 0.00074 \text{ ft/ft} \qquad (h_f)_{EF} = (0.00074)(11\,000) = 8.1 \text{ ft} \qquad Q_{FIG} = (0.3774)(3.0) = 1.13 \text{ cfs}$$
$$(h_1)_{FIG} = 0.0060 \text{ ft/ft} \qquad (h_f)_{FIG} = (0.0060)(5000) = 30.0 \text{ ft}$$

For $Q = 3.0$ cfs in pipe GH,

$(h_1)_{GH} = 0.013$ ft/ft $\quad (h_f)_{GH} = (0.013)(2500) = 32.5$ ft $\quad (h_f)_{\text{total}} = 8.1 + 30.0 + 32.5 = 70.6$ ft $\quad$ (for $EFGH$)

Since $EFGH$ carries the assumed flow of 3.0 cfs with a lesser head loss than $ABCD$, it ($EFGH$) has the greater capacity.

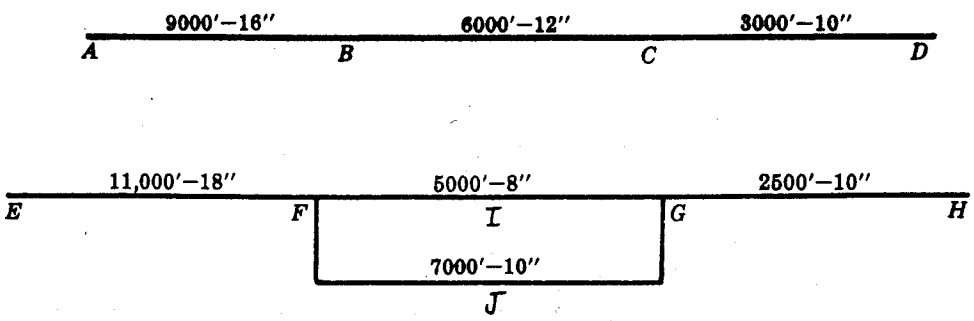


Fig. 11-21

11.35 Three pipes A, B, and C are interconnected as shown in Fig. 11-22. The pipe characteristics are as follows:

pipe	D, in	L, ft	f
A	6	2000	0.020
B	4	1600	0.032
C	8	4000	0.024

Find the rate at which water will flow in each pipe. Find also the pressure at point P. Neglect minor losses.

▮
$$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L \qquad (1)$$

$$h_L = h_f = (f)(L/d)(v^2/2g) = 0.020[2000/(\tfrac{6}{12})]\{v_A^2/[(2)(32.2)]\} + 0.024[4000/(\tfrac{8}{12})]\{v_C^2/[(2)(32.2)]\} = 1.242v_A^2 + 2.236v_C^2$$

$$0 + 0 + 200 = 0 + v_C^2/[(2)(32.2)] + 50 + 1.242v_A^2 + 2.236v_C^2$$

$$150 = 1.242v_A^2 + 2.252v_C^2 \qquad (2)$$

$$Q_A + Q_B = Q_C \qquad [(\pi)(\tfrac{6}{12})^2/4](v_A) + [(\pi)(\tfrac{4}{12})^2/4](v_B) = [(\pi)(\tfrac{8}{12})^2/4](v_C)$$

$$0.1963v_A + 0.08727v_B = 0.3491v_C \qquad (3)$$

$$(h_f)_A = (h_f)_B \qquad 0.020[2000/(\tfrac{6}{12})]\{v_A^2/[(2)(32.2)]\} = 0.032[1600/(\tfrac{4}{12})]\{v_B^2/[(2)(32.2)]\} \qquad v_B = 0.7217v_A$$

Substituting into Eq. (3), $0.1963v_A + (0.08727)(0.7217v_A) = 0.3491v_C$, $v_A = 1.346v_C$. Substituting into Eq. (2), $150 = (1.242)(1.346v_C)^2 + 2.252v_C^2$, $v_C = 5.772$ ft/s; $Q_C = A_Cv_C = [(\pi)(\tfrac{8}{12})^2/4](5.772) = 2.01$ ft^3/s, $v_A = (1.346)(5.772) = 7.769$ ft/s, $Q_A = [(\pi)(\tfrac{6}{12})^2/4](7.769) = 1.53$ ft^3/s. Substituting into Eq. (3), $(0.1963)(7.769) + 0.08727v_B = (0.3491)(5.772)$, $v_B = 5.614$ ft/s; $Q_B = [(\pi)(\tfrac{4}{12})^2/4](5.614) = 0.490$ ft^3/s. To find p_P, apply Eq. (1) between points P and 2. $(p_P)(144)/62.4 + v_P^2/2g + 120 = 0 + v_2^2/2g + 50 + 0.024[4000/(\tfrac{8}{12})]\{5.772^2/[(2)(32.2)]\}$, $v_P^2/2g = v_2^2/2g$, $p_P = 1.95$ lb/in^2.

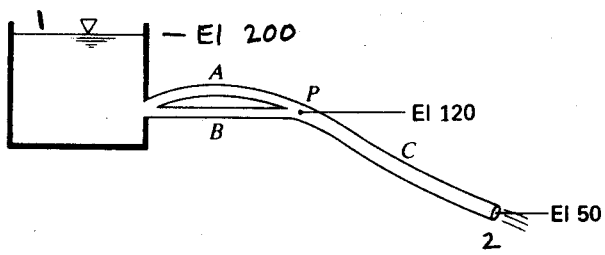


Fig. 11-22

11.36 Figure 11-23 shows an element of a sugar refinery. Smooth brass pipes 1, 2, and 3 are 550 ft of 2-in diameter, 350 ft of 3-in diameter, and 600 ft of 4-in diameter, respectively. If the net flow of molasses ($\nu = 6.5 \times 10^{-5}$ ft^2/s) is 0.7 ft^3/s, find the head loss from A to B and the flow in each pipe.

▮
$$h_f = (f)(L/d)(v^2/2g) \qquad (h_f)_1 = (h_f)_2 = (h_f)_3$$

$$f_1[550/(\tfrac{2}{12})]\{v_1^2/[(2)(32.2)]\} = f_2[350/(\tfrac{3}{12})]\{v_2^2/[(2)(32.2)]\} = f_3[600/(\tfrac{4}{12})]\{v_3^2/[(2)(32.2)]\} \qquad (1)$$

If $f_1 = f_2 = f_3$,

$$51.24v_1^2 = 21.74v_2^2 = 27.95v_3^2 \qquad v_2 = 1.535v_1 \qquad v_3 = 1.354v_1 \qquad Q_{\text{total}} = Q_1 + Q_2 + Q_3 = A_1v_1 + A_2v_2 + A_3v_3$$

$$0.7 = [(\pi)(\tfrac{2}{12})^2/4](v_1) + [(\pi)(\tfrac{3}{12})^2/4](1.535v_1) + [(\pi)(\tfrac{4}{12})^2/4](1.354v_1)$$

$$v_1 = 3.251 \text{ ft/s} \qquad v_2 = (1.535)(3.251) = 4.990 \text{ ft/s} \qquad v_3 = (1.354)(3.251) = 4.402 \text{ ft/s}$$

$N_R = dv/\nu$, $(N_R)_1 = (\tfrac{2}{12})(3.251)/(6.5 \times 10^{-5}) = 8336$, $(N_R)_2 = (\tfrac{3}{12})(4.990)/(6.5 \times 10^{-5}) = 19\,192$, $(N_R)_3 = (\tfrac{4}{12})(4.402)/(6.5 \times 10^{-5}) = 22\,547$. For Reynolds numbers in this range the Blasius formula can be used to determine friction factors: $f = 0.316/N_R^{0.25}$.

$$f_1 = 0.316/8336^{0.25} = 0.0331 \qquad f_2 = 0.316/19\,192^{0.25} = 0.0268 \qquad f_3 = 0.316/22\,547^{0.25} = 0.0258$$

Substituting these values of f into Eq. (1),

$$0.0331[550/(\tfrac{2}{12})]\{v_1^2/[(2)(32.2)]\} = 0.0268[350/(\tfrac{3}{12})]\{v_2^2/[(2)(32.2)]\} = 0.0258[600/(\tfrac{4}{12})]\{v_3^2/[(2)(32.2)]\}$$

$$1.696v_1^2 = 0.5826v_2^2 = 0.7211v_3^2 \qquad v_2 = 1.706v_1 \qquad v_3 = 1.534v_1$$

$$0.7 = [(\pi)(\tfrac{2}{12})^2/4](v_1) + [(\pi)(\tfrac{3}{12})^2/4](1.706v_2) + [(\pi)(\tfrac{4}{12})^2/4](1.534v_1)$$

$$v_1 = 2.918 \text{ ft/s} \qquad v_2 = (1.706)(2.918) = 4.978 \text{ ft/s} \qquad v_3 = (1.534)(2.918) = 4.476 \text{ ft/s}$$

$$(N_R)_1 = (\tfrac{2}{12})(2.918)/(6.5 \times 10^{-5}) = 7482 \qquad (N_R)_2 = (\tfrac{3}{12})(4.978)/(6.5 \times 10^{-5}) = 19\,146$$

$$(N_R)_3 = (\tfrac{4}{12})(4.476)/(6.5 \times 10^{-5}) = 22\,954$$

$$f_1 = 0.316/7482^{0.25} = 0.0340 \qquad f_2 = 0.316/19\,146^{0.25} = 0.0269 \qquad f_3 = 0.316/22\,954^{0.25} = 0.0257$$

$$0.0340[550/(\tfrac{2}{12})]\{v_1^2/[(2)(32.2)]\} = 0.0269[350/(\tfrac{3}{12})]\{v_2^2/[(2)(32.2)]\}$$
$$= 0.0257[600/(\tfrac{4}{12})]\{v_3^2/[(2)(32.2)]\}$$

$$1.742v_1^2 = 0.5848v_2^2 = 0.7183v_3^2 \qquad v_2 = 1.726v_1 \qquad v_3 = 1.557v_1$$

$$0.7 = [(\pi)(\tfrac{2}{12})^2/4](v_1) + [(\pi)(\tfrac{3}{12})^2/4](1.726v_1) + [(\pi)(\tfrac{4}{12})^2/4](1.557v_1)$$

$$v_1 = 2.888 \text{ ft/s} \qquad v_2 = (1.726)(2.888) = 4.985 \text{ ft/s} \qquad v_3 = (1.557)(2.888) = 4.497 \text{ ft/s}$$

$$(N_R)_1 = (\tfrac{2}{12})(2.888)/(6.5 \times 10^{-5}) = 7405 \qquad (N_R)_2 = (\tfrac{3}{12})(4.985)/(6.5 \times 10^{-5}) = 19\,173$$

$$(N_R)_3 = (\tfrac{4}{12})(4.497)/(6.5 \times 10^{-5}) = 23\,062$$

$$f_1 = 0.316/7405^{0.25} = 0.0341 \quad \text{(O.K.)} \qquad f_2 = 0.316/19\,173^{0.25} = 0.0269 \quad \text{(O.K.)}$$

$$f_3 = 0.316/23\,062^{0.25} = 0.0256 \quad \text{(O.K.)}$$

$$Q_1 = [(\pi)(\tfrac{2}{12})^2/4](2.888) = 0.0630 \text{ ft}^3/\text{s} \qquad Q_2 = [(\pi)(\tfrac{3}{12})^2/4](4.985) = 0.245 \text{ ft}^3/\text{s}$$

$$Q_3 = [(\pi)(\tfrac{4}{12})^2/4](4.497) = 0.392 \text{ ft}^3/\text{s}$$

$$(h_f)_{AB} = (h_f)_1 = 0.0341[550/(\tfrac{2}{12})]\{2.888^2/[(2)(32.2)]\} = 14.6 \text{ ft}$$

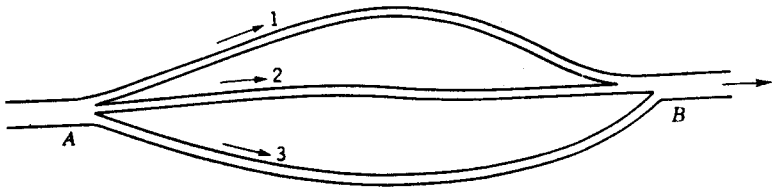


Fig. 11-23

11.37 Repeat Prob. 11.36 for a flow rate of 0.07 ft³/s.

▮ Assume this flow will be laminar. $h_f = (32\nu)(L/gD^2)(v)$. Since $(h_f)_1 = (h_f)_2 = (h_f)_3$,

$$L_1v_1/D_1^2 = L_2v_2/D_2^2 = L_3v_3/D_3^2 \qquad 550v_1/(\tfrac{2}{12})^2 = 350v_2/(\tfrac{3}{12})^2 = 600v_3/(\tfrac{4}{12})^2 \qquad v_2 = 3.536v_1 \qquad v_3 = 3.667v_1$$

$$Q_{\text{total}} = Q_1 + Q_2 + Q_3 = A_1v_1 + A_2v_2 + A_3v_3$$

$$0.07 = [(\pi)(\tfrac{2}{12})^2/4](v_1) + [(\pi)(\tfrac{3}{12})^2/4](3.536v_1) + [(\pi)(\tfrac{4}{12})^2/4](3.667v_1)$$

$$v_1 = 0.1358 \text{ ft/s} \qquad v_2 = (3.536)(0.1358) = 0.4802 \text{ ft/s} \qquad v_3 = (3.667)(0.1358) = 0.4980 \text{ ft/s} \qquad N_R = dv/\nu$$

From Fig. A-2, $\nu = 6.5 \times 10^{-5}$ ft²/s;

$$(N_R)_1 = (\tfrac{2}{12})(0.1358)/(6.5 \times 10^{-5}) = 348 \quad \text{(laminar)}$$

$$(N_R)_2 = (\tfrac{3}{12})(0.4802)/(6.5 \times 10^{-5}) = 1847 \quad \text{(laminar)}$$

$$(N_R)_3 = (\tfrac{4}{12})(0.4980)/(6.5 \times 10^{-5}) = 2554 \quad \text{(laminar)}$$

$$Q_1 = [(\pi)(\tfrac{2}{12})^2/4](0.1358) = 0.00296 \text{ ft}^3/\text{s} \qquad Q_2 = [(\pi)(\tfrac{3}{12})^2/4](0.4802) = 0.0236 \text{ ft}^3/\text{s}$$

$$Q_3 = [(\pi)(\tfrac{4}{12})^2/4](0.4980) = 0.0435 \text{ ft}^3/\text{s}$$

$$(h_f)_{AB} = (h_f)_1 = (32)(6.5 \times 10^{-5})\{550/[(32.2)(\tfrac{2}{12})^2]\}(0.1358) = 0.174 \text{ ft}$$

11.38 Rework Prob. 11.36 if the parameters are changed to: 90 m of 30-mm (pipe 1), 120 m of 40-mm (pipe 2), 90 m of 50-mm (pipe 3); $\nu = 4.8 \times 10^{-6}$ m²/s; 6 L/s net flow.

▮ Assume Reynolds numbers between 3000 and 100 000, in which case the Blasius equation can be used to determine f: $h_f = (f)(L/d)(v^2/2g)$, $f = 0.316/N_R^{0.25}$, $N_R = dv/\nu$. Therefore,

$$h_f = \left[\frac{0.316}{(dv/\nu)^{0.25}}\right]\left(\frac{L}{d}\right)\left(\frac{v^2}{2g}\right) = \frac{0.316\nu^{0.25}Lv^{1.75}}{2gd^{1.25}} \qquad h_f \propto \frac{Lv^{1.75}}{d^{1.25}} \qquad (h_f)_1 = (h_f)_2 = (h_f)_3$$

$$(90)(v_1^{1.75})/(0.030)^{1.25} = (120)(v_2^{1.75})/(0.040)^{1.25} = (90)(v_3^{1.75})/(0.050)^{1.25} \qquad v_1 = 0.9597v_2 \qquad v_3 = 1.382v_2$$

$$Q_{\text{total}} = Q_1 + Q_2 + Q_3 = A_1v_1 + A_2v_2 + A_3v_3$$

$$6 \times 10^{-3} = [(\pi)(0.030)^2/4](0.9597v_2) + [(\pi)(0.040)^2/4](v_2) + [(\pi)(0.050)^2/4](1.382v_2)$$

$$v_2 = 1.291 \text{ m/s} \qquad v_1 = (0.9597)(1.291) = 1.239 \text{ m/s} \qquad v_3 = (1.382)(1.291) = 1.784 \text{ m/s}$$

$$(N_R)_1 = (0.030)(1.239)/(4.8 \times 10^{-6}) = 7.74 \times 10^3 \qquad (N_R)_2 = (0.040)(1.291)/(4.8 \times 10^{-6}) = 1.08 \times 10^4$$

$$(N_R)_3 = (0.050)(1.784)/(4.8 \times 10^{-6}) = 1.86 \times 10^4$$

Hence, use of the Blasius equation is O.K.

$$Q_1 = [(\pi)(0.030)^2/4](1.239) = 0.00088 \text{ m}^3/\text{s} \quad \text{or} \quad 0.88 \text{ L/s}$$

$$Q_2 = [(\pi)(0.040)^2/4](1.291) = 0.00162 \text{ m}^3/\text{s} \quad \text{or} \quad 1.62 \text{ L/s}$$

$$Q_3 = [(\pi)(0.050)^2/4](1.784) = 0.00350 \text{ m}^3/\text{s} \quad \text{or} \quad 3.50 \text{ L/s}$$

$$(h_f)_{AB} = (h_f)_1 = \frac{(0.316)(4.8 \times 10^{-6})^{0.25}(90)(1.239)^{1.75}}{(2)(9.807)(0.030)^{1.25}} = 7.91 \text{ m}$$

11.39 Repeat Prob. 11.38 for a net flow of 0.5 L/s; other parameters as in Prob. 11.38.

▮ Assume laminar flow, in which case the following equation can be used to determine head loss: $h_f = (32\nu)(L/gd^2)(v)$, $h_f \propto Lv/d^2$.

$$(h_f)_1 = (h_f)_2 = (h_f)_3 \qquad 90v_1/(0.030)^2 = 120v_2/(0.040)^2 = 90v_3/(0.050)^2 \qquad v_1 = 0.7500v_2 \qquad v_3 = 2.083v_2$$

$$Q_{\text{total}} = Q_1 + Q_2 + Q_3 = A_1v_1 + A_2v_2 + A_3v_3$$

$$0.5 \times 10^{-3} = [(\pi)(0.030)^2/4](0.7500v_2) + [(\pi)(0.040)^2/4](v_2) + [(\pi)(0.050)^2/4](2.083v_2)$$

$$v_2 = 0.08508 \text{ m/s} \qquad v_1 = (0.7500)(0.08508) = 0.06381 \text{ m/s} \qquad v_3 = (2.083)(0.08508) = 0.1772 \text{ m/s}$$

$$(N_R)_1 = (0.030)(0.06381)/(4.8 \times 10^{-6}) = 399 \qquad \text{(laminar)}$$

$$(N_R)_2 = (0.040)(0.08508)/(4.8 \times 10^{-6}) = 709 \qquad \text{(laminar)}$$

$$(N_R)_3 = (0.050)(0.1772)/(4.8 \times 10^{-6}) = 1846 \qquad \text{(laminar)}$$

$$Q_1 = [(\pi)(0.030)^2/4](0.06381) = 0.0000451 \text{ m}^3/\text{s} \quad \text{or} \quad 0.0451 \text{ L/s}$$

$$Q_2 = [(\pi)(0.040)^2/4](0.08508) = 0.0001069 \text{ m}^3/\text{s} \quad \text{or} \quad 0.1069 \text{ L/s}$$

$$Q_3 = [(\pi)(0.050)^2/4](0.1772) = 0.0003479 \text{ m}^3/\text{s} \quad \text{or} \quad 0.3479 \text{ L/s}$$

$$(h_f)_{AB} = (h_f)_1 = (32)(4.8 \times 10^{-6})\{90/[(9.807)(0.030)^2]\}(0.06381) = 0.0999 \text{ m} \quad \text{or} \quad 1 \text{ cm}$$

11.40 Figure 11-24 shows a network of cast iron pipes. Given a flow of 24 cfs, find the head loss from A to D. Neglect minor losses.

▮ $Q_n = A_n\sqrt{2gh_L/(f_n)(L_n/d_n)}$ or $Q_n = C_n\sqrt{h_L}$, where $C_n = A_n\sqrt{2gd_n/f_nL_n}$. Assume $f_1 = f_2 = f_3 = 0.019$.

$$C_2 = [(\pi)(\tfrac{14}{12})^2/4]\sqrt{(2)(32.2)(\tfrac{14}{12})/[(0.019)(3300)]} = 1.170$$

$$C_3 = [(\pi)(\tfrac{12}{12})^2/4]\sqrt{(2)(32.2)(\tfrac{12}{12})/[(0.019)(2600)]} = 0.8967$$

$$C_4 = [(\pi)(\tfrac{16}{12})^2/4]\sqrt{(2)(32.2)(\tfrac{16}{12})/[(0.019)(3200)]} = 1.659$$

$$Q_2 = [1.170/(1.170 + 0.8967 + 1.659)](24) = 7.537 \text{ ft}^3/\text{s}$$

$$Q_3 = [0.8967/(1.170 + 0.8967 + 1.659)](24) = 5.776 \text{ ft}^3/\text{s}$$

$$Q_4 = [1.659/(1.170 + 0.8967 + 1.659)](24) = 10.687 \text{ ft}^3/\text{s}$$

$$v_2 = Q_2/A_2 = 7.537/[(\pi)(\tfrac{14}{12})^2/4] = 7.050 \text{ ft/s} \qquad v_3 = 5.776/[(\pi)(\tfrac{12}{12})^2/4] = 7.354 \text{ ft/s}$$

$$v_4 = 10.687/[(\pi)(\tfrac{16}{12})^2/4] = 7.654 \text{ ft/s} \qquad N_R = dv/\nu$$

$$(N_R)_2 = (\tfrac{14}{12})(7.050)/(1.05 \times 10^{-5}) = 7.83 \times 10^5$$

$$(N_R)_3 = (\tfrac{12}{12})(7.354)/(1.05 \times 10^{-5}) = 7.00 \times 10^5$$

$$(N_R)_4 = (\tfrac{16}{12})(7.654)/(1.05 \times 10^{-5}) = 9.72 \times 10^5$$

$$(\epsilon/d)_2 = 0.00085/(\tfrac{14}{12}) = 0.00073 \qquad (\epsilon/d)_3 = 0.00085/(\tfrac{12}{12}) = 0.00085 \qquad (\epsilon/d)_4 = 0.00085/(\tfrac{16}{12}) = 0.00064$$

From Fig. A-5, $f_2 = 0.0185$, $f_3 = 0.0195$, and $f_4 = 0.0185$.

$$C_2 = [(\pi)(\tfrac{14}{12})^2/4]\sqrt{(2)(32.2)(\tfrac{14}{12})/[(0.0185)(3300)]} = 1.186$$
$$C_3 = [(\pi)(\tfrac{12}{12})^2/4]\sqrt{(2)(32.2)(\tfrac{12}{12})/[(0.0195)(2600)]} = 0.8852$$
$$C_4 = [(\pi)(\tfrac{16}{12})^2/4]\sqrt{(2)(32.2)(\tfrac{16}{12})/[(0.0185)(3200)]} = 1.585$$
$$Q_2 = [1.186/(1.186 + 0.8852 + 1.585)](24) = 7.785\ \text{ft}^3/\text{s}$$
$$Q_3 = [0.8852/(1.186 + 0.8852 + 1.585)](24) = 5.811\ \text{ft}^3/\text{s}$$
$$Q_4 = [1.585/(1.186 + 0.8852 + 1.585)](24) = 10.404\ \text{ft}^3/\text{s}$$
$$v_2 = Q_2/A_2 = 7.785/[(\pi)(\tfrac{14}{12})^2/4] = 7.282\ \text{ft/s} \qquad v_3 = 5.811/[(\pi)(\tfrac{12}{12})^2/4] = 7.399\ \text{ft/s}$$
$$v_4 = 10.404/[(\pi)(\tfrac{16}{12})^2/4] = 7.451\ \text{ft/s}$$
$$(N_R)_2 = (\tfrac{14}{12})(7.282)/(1.05 \times 10^{-5}) = 8.09 \times 10^5$$
$$(N_R)_3 = (\tfrac{12}{12})(7.399)/(1.05 \times 10^{-5}) = 7.05 \times 10^5$$
$$(N_R)_4 = (\tfrac{16}{12})(7.451)/(1.05 \times 10^{-5}) = 9.46 \times 10^5$$
$$f_2 = 0.0185 \qquad f_3 = 0.0195 \qquad \text{and} \qquad f_4 = 0.0185 \qquad \text{(O.K.)}$$
$$Q_{\text{total}} = \sqrt{h_L}(C_2 + C_3 + C_4) \qquad 24 = \sqrt{(h_L)_{BC}}(1.186 + 0.8852 + 1.585)$$
$$(h_L)_{BC} = 43.09\ \text{ft} \qquad h_L = h_f = (f)(L/d)(v^2/2g)$$
$$v_1 = 24/[(\pi)(\tfrac{24}{12})^2/4] = 7.639\ \text{ft/s} \qquad (N_R)_1 = (\tfrac{24}{12})(7.639)/(1.05 \times 10^{-5}) = 1.46 \times 10^6$$
$$(\epsilon/d)_1 = 0.00085/(\tfrac{24}{12}) = 0.00042 \qquad f_1 = 0.0168$$
$$(h_L)_1 = 0.0168[3300/(\tfrac{24}{12})]\{7.639^2/[(2)(32.2)]\} = 25.12\ \text{ft} \qquad v_5 = 24/[(\pi)(\tfrac{30}{12})^2/4] = 4.889\ \text{ft/s}$$
$$(N_R)_5 = (\tfrac{30}{12})(4.889)/(1.05 \times 10^{-5}) = 1.16 \times 10^6 \qquad (\epsilon/d)_5 = 0.00085/(\tfrac{30}{12}) = 0.00034 \qquad f_5 = 0.016$$
$$(h_L)_5 = 0.016[5200/(\tfrac{30}{12})]\{4.889^2/[(2)(32.2)]\} = 12.35\ \text{ft} \qquad (h_L)_{AD} = 25.12 + 43.09 + 12.35 = 80.56\ \text{ft}$$

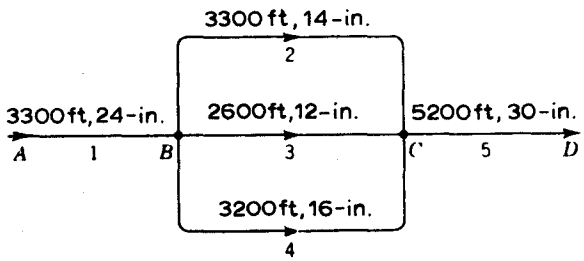


Fig. 11-24

11.41 Find the diameter of the series equivalent of the three parallel pipes in Prob. 11.40.

▮ $h_f = (f)(L/d)(v^2/2g)$. Assume $f = 0.016$:

$$v = Q/A = 24/(\pi d^2/4) = 30.56/d^2 \qquad 43.09 = (0.016)(2600/d)\{(30.56/d^2)^2/[(2)(32.2)]\} \qquad d = 1.695\ \text{ft}$$
$$N_R = dv/\nu = (1.695)(30.56/1.695^2)/(1.05 \times 10^{-5}) = 1.72 \times 10^6 \qquad \epsilon/d = 0.00085/1.707 = 0.00050$$

From Fig. A-5, $f = 0.017$. Try $f = 0.017$:

$$43.09 = (0.017)(2600/d)\{(30.56/d^2)^2/[(2)(32.2)]\} \qquad d = 1.716\ \text{ft}$$
$$N_R = (1.716)(30.56/1.716^2)/(1.05 \times 10^{-5}) = 1.70 \times 10^6 \qquad f = 0.017 \qquad \text{(O.K.)}$$

Therefore, $d = 1.716\ \text{ft} = 20.6\ \text{in}$.

11.42 For the network of Fig. 11-24, determine the five pipe flows, given the head loss from A to D is 91 ft and all pipes have $f = 0.017$.

▮ $$h_f = (f)(L/d)(v^2/2g) = (fL/d)(Q^2/2gA^2) = (fL/2gd)[Q/(\pi d^2/4)]^2 = (8f/\pi^2 g)(LQ^2/d^5)$$

Hence,

$$h_f \propto LQ^2/d^5 \qquad (h_f)_2 = (h_f)_3 = (h_f)_4 \qquad 3300Q_2^2/(\tfrac{14}{12})^5 = 2600Q_3^2/(\tfrac{12}{12})^5 = 3200Q_4^2/(\tfrac{16}{12})^5$$
$$Q_2 = 1.305Q_3 \qquad Q_4 = 1.850Q_3 \qquad Q_1 = Q_5 = Q_2 + Q_3 + Q_4 = 1.305Q_3 + Q_3 + 1.850Q_3 = 4.155Q_3$$
$$(h_f)_{AD} = (h_f)_1 + (h_f)_3 + (h_f)_5$$

$$91 = \{(8)(0.017)/[(\pi)^2(32.2)]\}[(3300)(4.155Q_3)^2/(\tfrac{24}{12})^5 + (2600)(Q_3)^2/(\tfrac{12}{12})^5 + (5200)(4.155Q_3)^2/(\tfrac{30}{12})^5]$$

$$Q_3 = 6.33\ \text{ft}^3/\text{s} \qquad Q_2 = (1.305)(6.33) = 8.26\ \text{ft}^3/\text{s}$$

$$Q_4 = (1.850)(6.33) = 11.71\ \text{ft}^3/\text{s} \qquad Q_1 = Q_5 = (4.155)(6.33) = 26.30\ \text{ft}^3/\text{s}$$

11.43 In Prob. 11.42, what percentage increase in the capacity of the system would be achieved by adding another 12-in pipe 2600 ft long between B and C?

▮ The additional pipe from B to C is identical to pipe 3. As in Prob. 11.42,

$$h_f = (8f/\pi^2 g)(LQ^2/d^5) \qquad Q_2 = 1.305Q_3 \qquad Q_4 = 1.850Q_3$$

$$Q_1 = Q_5 = Q_2 + 2Q_3 + Q_4 = 1.305Q_3 + 2Q_3 + 1.850Q_3 = 5.155Q_3 \qquad (h_f)_{AD} = (h_f)_1 + (h_f)_3 + (h_f)_5$$

$$91 = \{(8)(0.017)/[(\pi)^2(32.2)]\}[(3300)(5.155Q_3)^2/(\tfrac{24}{12})^5 + (2600)(Q_3)^2/(\tfrac{12}{12})^5 + (5200)(5.155Q_3)^2/(\tfrac{30}{12})^5]$$

$$Q_3 = 5.61\ \text{ft}^3/\text{s} \qquad Q_1 = Q_5 = (5.155)(5.61) = 28.92\ \text{ft}^3/\text{s}$$

Increase in capacity $= (28.92 - 26.30)/26.30 = 0.100$, or 10.0 percent.

11.44 Compute the flows in all pipes of the system shown in Fig. 11-25. Pipe AB is 800 ft long, 6 in. in diameter, $f = 0.03$; pipe u is 500 ft long, 4 in. in diameter, $f = 0.02$; pipe l is 700 ft long, 2 in. in diameter, $f = 0.04$; pipe CD is 300 ft long, 4 in. in diameter, $f = 0.02$. The tank surface is 84 ft above the free outlet D.

▮ $h_f = (f)(L/d)(v^2/2g)$. For B to C, $(h_f)_u = (h_f)_l$.

$$0.02[500/(\tfrac{4}{12})]\{v_u^2/[(2)(32.2)]\} = 0.04[700/(\tfrac{2}{12})]\{v_l^2/[(2)(32.2)]\} \qquad v_u = 2.366v_l$$

$$Q = Q_u + Q_l = A_u v_u + A_l v_l = [(\pi)(\tfrac{4}{12})^2/4](v_u) + [(\pi)(\tfrac{2}{12})^2/4](v_l) = [(\pi)(\tfrac{4}{12})^2/4](2.366v_l) + [(\pi)(\tfrac{2}{12})^2/4](v_l)$$

$$= 0.2283v_l \qquad Q = Q_u + A_l v_l = Q_u + [(\pi)(\tfrac{2}{12})^2/4](Q/0.2283)$$

$$Q_u = 0.9044Q \qquad Q_l = (1 - 0.9044)(Q) = 0.0956Q \qquad v_u = Q_u/A_u = 0.9044Q/[(\pi)(\tfrac{4}{12})^2/4] = 10.36Q$$

$$v_{AB} = Q/A_{AB} = Q/[(\pi)(\tfrac{6}{12})^2/4] = 5.093Q \qquad v_u = (10.36)(v_{AB}/5.093) = 2.034v_{AB} \qquad h_f = (h_f)_{AB} + (h_f)_u + (h_f)_{CD}$$

$$84 = 0.03[800/(\tfrac{6}{12})]\{v_{AB}^2/[(2)(32.2)]\} + 0.02[500/(\tfrac{4}{12})]\{v_u^2/[(2)(32.2)]\} + 0.02[300/(\tfrac{4}{12})]\{v_{CD}^2/[(2)(32.2)]\}$$

$$[(\pi)(\tfrac{6}{12})^2/4](v_{AB}) = [(\pi)(\tfrac{4}{12})^2/4](v_{CD}) \qquad v_{CD} = 2.250v_{AB}$$

$$84 = 0.03[800/(\tfrac{6}{12})]\{v_{AB}^2/[(2)(32.2)]\} + 0.02[500/(\tfrac{4}{12})]\{(2.034v_{AB})^2/[(2)(32.2)]\}$$
$$+ 0.02[300/(\tfrac{4}{12})]\{(2.250v_{AB})^2/[(2)(32.2)]\}$$

$$v_{AB} = 4.533\ \text{ft/s} \qquad Q = Q_{AB} = [(\pi)(\tfrac{6}{12})^2/4](4.533) = 0.890\ \text{ft}^3/\text{s} \qquad Q_{CD} = Q_{AB} = 0.890\ \text{ft}^3/\text{s}$$

$$Q_u = (0.9044)(0.890) = 0.805\ \text{ft}^3/\text{s} \qquad Q_l = (0.0956)(0.890) = 0.085\ \text{ft}^3/\text{s}$$

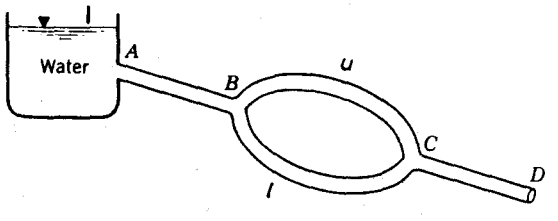


Fig. 11-25

11.45 Rework Prob. 11.44 if pipe AB is 400 m long, 200 mm in diameter, and $f = 0.03$; pipe u is 300 m long, 100 mm in diameter, and $f = 0.02$; pipe l is 200 m long, 150 mm in diameter, and $f = 0.025$; pipe CD is 700 m long, 300 mm in diameter, and $f = 0.018$; the tank surface is 94 ft above D.

▮ $(h_f)_u = (h_f)_l$. $\quad 0.02[300/(0.100)]\{v_u^2/[(2)(9.807)]\} = 0.025[200/(0.150)]\{v_l^2/[(2)(9.807)]\} \qquad v_u = 0.7454v_l$

$$Q = Q_u + Q_l = A_u v_u + A_l v_l = [(\pi)(0.100)^2/4](v_u) + [(\pi)(0.150)^2/4](v_l)$$
$$= [(\pi)(0.100)^2/4](0.7454v_l) + [(\pi)(0.150)^2/4](v_l)$$

$$Q = 0.02353v_l \qquad Q = Q_u + A_l v_l = Q_u + [(\pi)(0.150)^2/4](Q/0.02353)$$

$$Q_u = 0.2490Q \qquad Q_l = (1 - 0.2490)(Q) = 0.7510Q \qquad v_u = Q_u/A_u = 0.2490Q/[(\pi)(0.100)^2/4] = 31.70Q$$

$$v_{AB} = Q/A_{AB} = Q/[(\pi)(0.200)^2/4] = 31.83Q \qquad v_u = (31.70)(v_{AB}/31.83) = 0.9959v_{AB}$$

$$h_f = (h_f)_{AB} + (h_f)_u + (h_f)_{CD}$$

$$94 = 0.03[400/0.200]\{v_{AB}^2/[(2)(9.807)]\} + 0.02[300/0.100]\{v_u^2/[(2)(9.807)]\}$$
$$+ 0.018[700/0.300]\{v_{CD}^2/[(2)(9.807)]\}$$
$$[(\pi)(0.200)^2/4](v_{AB}) = [(\pi)(0.300)^2/4](v_{CD}) \qquad v_{CD} = 0.4444v_{AB}$$
$$94 = 0.03[400/0.200]\{v_{AB}^2/[(2)(9.807)]\} + 0.02[300/0.100]\{(0.9959v_{AB})^2/[(2)(9.807)]\}$$
$$+ 0.018[700/0.300]\{(0.4444v_{AB})^2/[(2)(9.807)]\}$$
$$v_{AB} = 3.798 \text{ m/s} \qquad Q = Q_{AB} = [(\pi)(0.200)^2/4](3.798) = 0.1193 \text{ m}^3/\text{s} \qquad Q_{CD} = Q_{AB} = 0.01193 \text{ m}^3/\text{s}$$
$$Q_u = (0.2490)(0.1193) = 0.0297 \text{ m}^3/\text{s} \qquad Q_l = (0.7510)(0.1193) = 0.0896 \text{ m}^3/\text{s}$$

Here, as in Prob. 11.44, the question remains open as to whether the computed flows are physically realizable. It all depends on the elevations of junctions B and C.

11.46 Pump P in Fig. 11-26 develops 27 ft of head, producing a velocity of flow in pipe C of 5 fps. Neglecting minor losses, find the flow rates in pipes A and B. The parameters are: pipe A—4200 ft long, 2 ft diameter, $f = 0.03$; pipe B—4200 ft long, 1 ft in diameter, $f = 0.03$; pipe C—4200 ft long, 2 ft in diameter, $f = 0.02$.

▮ $h_f = (f)(L/d)(v^2/2g) \qquad (h_f)_A = (0.03)(\frac{4200}{2})\{v_A^2/[(2)(32.2)]\} = 0.9783v_A^2 \qquad (h_L)_C = (h_f)_C - h_{\text{pump}}$

$$(h_f)_C = (0.02)(\tfrac{4200}{2})\{5^2/[(2)(32.2)]\} = 16.30 \text{ ft} \qquad (h_L)_C = 16.30 - 27 = -10.70 \text{ ft}$$

Hence the energy is greater at the right end of C and flow will be to the left in A.

$$0.9783v_A^2 = 10.70 \qquad v_A = 3.307 \text{ ft/s} \qquad Q_A = A_A v_A = [(\pi)(2)^2/4](3.307) = 10.39 \text{ ft}^3/\text{s} \qquad \text{(to the left)}$$
$$Q_C = [(\pi)(2)^2/4](5) = 15.71 \text{ ft}^3/\text{s} \qquad \text{(to the right)}$$
$$Q_B = Q_A + Q_C = -10.39 + 15.71 = -5.32 \text{ ft}^3/\text{s} \qquad \text{(to the right)}$$

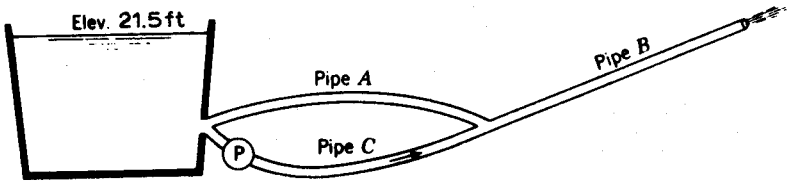


Fig. 11-26

11.47 In Prob. 11.46, find the elevation (El) of pipe B at discharge.

▮
$$\text{El}_B = \text{El}_{\text{reservoir surface}} + h_{\text{pump}} - \sum h_f - v_B^2/2g$$
$$v_B = Q_B/A_B = 5.32/[(\pi)(1)^2/4] = 6.774 \text{ ft/s}$$
$$(h_f)_B = (0.03)(\tfrac{4200}{1})\{6.774^2/[(2)(32.2)]\} = 89.78 \text{ ft} \qquad \sum h_f = 16.30 + 89.78 = 106.08 \text{ ft}$$
$$\text{El}_B = 21.5 + 27 - 106.08 - 6.774^2/[(2)(32.2)] = -58.3 \text{ ft}$$

11.48 Repeat Prob. 11.46 given that the velocity in pipe C is 3 fps and all other data remain the same.

▮ $h_f = (f)(L/d)(v^2/2g) \qquad (h_f)_A = (0.03)(\frac{4200}{2})\{v_A^2/[(2)(32.2)]\} = 0.9783v_A^2 \qquad (h_L)_C = (h_f)_C - h_{\text{pump}}$

$$(h_f)_C = (0.02)(\tfrac{4200}{2})\{3^2/[(2)(32.2)]\} = 5.87 \text{ ft} \qquad (h_L)_C = 5.87 - 27 = -21.13 \text{ ft}$$

Hence the energy is greater at the right end of C and flow will be to the left in A.

$$0.9783v_A^2 = 21.13 \qquad v_A = 4.647 \text{ ft/s} \qquad Q_A = A_A v_A = [(\pi)(2)^2/4](4.647) = 14.60 \text{ ft}^3/\text{s} \qquad \text{(to the left)}$$
$$Q_C = [(\pi)(2)^2/4](3) = 9.42 \text{ ft}^3/\text{s} \qquad \text{(to the right)}$$
$$Q_B = Q_A + Q_C = -14.60 + 9.42 = -5.18 \text{ ft}^3/\text{s} \qquad \text{(to the left)}$$

11.49 In Prob. 11.48, find the elevation of pipe B at discharge.

▮
$$\text{El}_B = \text{El}_{\text{reservoir surface}} + h_{\text{pump}} - \sum h_f + v_B^2/2g \qquad v_B = Q_B/A_B = 5.18/[(\pi)(1)^2/4] = 6.595 \text{ ft/s}$$
$$(h_f)_B = (0.03)(\tfrac{4200}{1})\{6.595^2/[(2)(32.2)]\} = 85.10 \text{ ft} \qquad \sum h_f = 5.87 - 85.10 = -79.23 \text{ ft}$$
$$\text{El}_B = 21.5 + 27 - (-79.23) + 6.595^2/[(2)(32.2)] = 127.8 \text{ ft}$$

11.50 Refer to Fig. 11-26. Assume the water surface in the reservoir is at elevation 94 m. Pipes A, B, and C are all 840 m long, and they all have diameter of 0.7 m, with $f = 0.022$. Neglecting minor losses, find the flow rate in all pipes, supposing that the pump develops 9 m of head when the velocity in pipe C is 3.6 m/s.

▮ $h_f = (f)(L/d)(v^2/2g)$ $\quad (h_f)_A = 0.022[840/(0.7)]\{v_A^2/[(2)(9.807)]\} = 1.346v_A^2$ $\quad (h_L)_C = (h_f)_C - h_{pump}$

$$(h_f)_C = 0.022[840/(0.7)]\{3.6^2/[(2)(9.807)]\} = 17.44 \text{ m} \qquad (h_L)_C = 17.44 - 9 = 8.44 \text{ m}$$

$$1.346v_A^2 = 8.44 \qquad v_A = 2.50 \text{ m/s}$$

$$Q_A = A_A v_A = [(\pi)(0.7)^2/4](2.50) = 0.962 \text{ m}^3/\text{s} \qquad \text{(to the right)}$$

$$Q_C = [(\pi)(0.7)^2/4](3.6) = 1.385 \text{ m}^3/\text{s} \qquad \text{(to the right)}$$

$$Q_B = Q_A + Q_C = 0.962 + 1.385 = 2.347 \text{ m}^3/\text{s} \qquad \text{(to the right)}$$

11.51 In Prob. 11.50, find the elevation of pipe B at discharge.

▮ $\text{El}_B = \text{El}_{\text{reservoir surface}} + h_{\text{pump}} - \sum h_f - v_B^2/2g$ $\quad v_B = Q_B/A_B = 2.347/[(\pi)(0.7)^2/4] = 6.099$ m/s

$$(h_f)_B = 0.022[840/(0.7)]\{6.099^2/[(2)(9.807)]\} = 50.07 \text{ m} \qquad \sum h_f = 17.44 + 50.07 = 67.51 \text{ m}$$

$$\text{El}_B = 94 + 9 - 67.51 - 6.099^2/[(2)(9.807)] = 33.6 \text{ m}$$

11.52 Using $n = 0.013$ and neglecting minor losses, express the head loss through the pipe system of Fig. 11-27 in the form of $h_L = KQ^x$.

▮ $h_L = \dfrac{n^2v^2L}{(1.486)^2(R^{2/3})^2} = \dfrac{n^2Q^2L}{(1.486)^2R^{4/3}A^2}$ $\quad (h_L)_{AB} = \dfrac{(0.013)^2(Q)^2(500)}{(1.486)^2[(\frac{24}{12})/4]^{4/3}[(\pi)(\frac{24}{12})^2/4]^2} = 0.009770Q^2$

$$(h_L)_{CD} = \frac{(0.013)^2(Q)^2(1000)}{[illegible]} = 0.09063Q^2 \qquad (h_L)_{BFC} = (h_L)_{BFC}$$

$$\frac{Q_1^2L_1}{R_1^{4/3}A_1^2} = \frac{Q_2^2L_2}{R_2^{4/3}A_2^2} \qquad \frac{(Q_1^2)(4000)}{[(\frac{18}{12})/4]^{4/3}[(\pi)(\frac{18}{12})^2/4]^2} = \frac{(Q_2^2)(5000)}{[(\frac{12}{12})/4]^{4/3}[(\pi)(\frac{12}{12})^2/4]^2}$$

$$Q_1 = 3.269Q_2 \qquad Q = Q_1 + Q_2 = 3.269Q_2 + Q_2 \qquad Q_2 = 0.2342Q$$

$$(h_L)_{BFC} = \frac{(0.013)^2(Q_2)^2(5000)}{(1.486)^2[(\frac{12}{12})/4]^{4/3}[(\pi)(\frac{12}{12})^2/4]^2} = 3.939Q_2^2$$

$$(h_L)_{BFC} = (3.939)(0.2342Q)^2 = 0.05485Q^2 \qquad (h_L)_{AD} = 0.009770Q^2 + 0.09063Q^2 + 0.05485Q^2 = 0.155Q^2$$

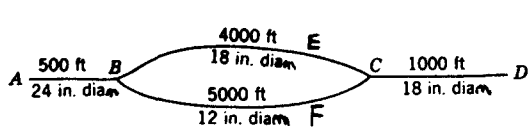


Fig. 11-27

11.54 Refer to Fig. 11-27. Suppose $p_A/\gamma = 6.5$ ft, $p_D/\gamma = 20$ ft, and $z_A = z_D$. A pump in the 4000-ft pipe (flow from left to right) develops 30 ft of head. Find the flow rate in each pipe. Assume $n = 0.013$ for all pipes.

▮

$$Q_{AB} = Q_{BEC} - Q_{BFC} = Q_{CD} \tag{1}$$

$$6.5 - (h_f)_{AB} = p_B/\gamma \tag{2}$$

$$p_B/\gamma + h_{\text{pump}} - (h_f)_{BEC} = p_C/\gamma \tag{3}$$

$$p_C/\gamma - (h_f)_{BFC} = p_B/\gamma \tag{4}$$

$$p_C/\gamma - (h_f)_{CD} = 20 \tag{5}$$

$$h_f = n^2v^2L/(1.486R^{2/3})^2 \qquad v_{AB} = Q_{AB}/A_{AB} = Q_{AB}/[(\pi)(\tfrac{24}{12})^2/4] = 0.3183Q_{AB}$$

$$(h_f)_{AB} = (0.013)^2(0.3183Q_{AB})^2(500)/\{(1.486)[(\tfrac{24}{12})/4]^{2/3}\}^2 = 0.009769Q_{AB}^2 \tag{6}$$

$$v_{BEC} = Q_{BEC}/[(\pi)(\tfrac{18}{12})^2/4] = 0.5659Q_{BEC}$$

$$(h_f)_{BEC} = (0.013)^2(0.5659Q_{BEC})^2(4000)/\{(1.486)[(\tfrac{18}{12})/4]^{2/3}\}^2 = 0.3625Q_{BEC}^2 \tag{7}$$

$$v_{BFC} = Q_{BFC}/[(\pi)(\tfrac{12}{12})^2/4] = 1.273Q_{BFC}$$

$$(h_f)_{BFC} = (0.013)^2(1.273Q_{BFC})^2(5000)/\{(1.486)[(\tfrac{12}{12})/4]^{2/3}\}^2 = 3.938Q_{BFC}^2 \tag{8}$$

$$v_{CD} = Q_{CD}/[(\pi)(\tfrac{18}{12})^2/4] = 0.5659Q_{CD}$$

$$(h_f)_{CD} = (0.013)^2(0.5659Q_{CD})^2(1000)/\{(1.486)[(\tfrac{18}{12})/4]^{2/3}\}^2 = 0.09063Q_{CD}^2 \tag{9}$$

Substitute Eqs. (*6*), (*7*), (*8*), and (*9*) into Eqs. (*2*), (*3*), (*4*), and (*5*), respectively, and then solve simultaneously between Eqs. (*2*), (*3*), (*4*), and (*5*), introducing also Eq. (*1*). Two equations result, such as

$$Q_{BEC}^2 - 2Q_{BEC}^2Q_{BFC} - 37.40Q_{BFC}^2 = -132 \qquad Q_{BEC}^2 + 10.7Q_{BFC}^2 = 82$$

By trial and error, $Q_{BEC} = 6.25$ cfs and $Q_{BFC} = 2.00$ cfs. Hence, $Q_{AB} = 6.25 - 2.00 = 4.25$ cfs.

CHAPTER 12
Branching Pipeline Systems

12.1 In Fig. 12-1, find the flows for the following data: $L_1 = 200$ m, $D_1 = 300$ mm, $\epsilon_1/D_1 = 0.0002$, $z_1 = 700$ m, $p_1 = 7$ atm; $L_2 = 300$ m, $D_2 = 350$ mm, $\epsilon_2/D_2 = 0.00015$, $z_2 = 400$ m, $p_2 = 2$ atm; $L_3 = 400$ m, $D_3 = 400$ mm, $\epsilon_3/D_3 = 0.0001$, $z_3 = 100$ m, $p_3 = 3$ atm. The fluid is water with $\nu = 0.113 \times 10^{-5}$ m²/s.

I We first calculate H_1, H_2, and H_3. Thus, $H_1 = z_1 + (7)(p_{atm})/\gamma = 700 + (7)(101\,325)/9806 = 772.3$ m, $H_2 = z_2 + (2)(p_{atm})/\gamma = 400 + (2)(101\,325)/9806 = 420.7$ m, $H_3 = z_3 + (3)(p_{atm})/\gamma = 100 + (3)(101\,325)/9806 = 131.0$ m. Now estimate $H_J = 380$ m. We will hence use the continuity equation for test purposes. We can then say that $(772.3 - 380) = (1/g)f_1(200/0.300)(V_1^2/2)$, $(420.7 - 380) = (1/g)f_2(300/0.350)(V_2^2/2)$, $(380 - 131.0) = (1/g)f_3(400/0.400)(V_3^2/2)$. Estimating $f_1 = 0.014$, $f_2 = 0.013$, and $f_3 = 0.012$, we can compute velocities and the flows q_i. We get

$$V_1 = 28.72 \text{ m/s} \qquad q_1 = (\pi/4)(0.300^2)(28.72) = 2.03 \text{ m}^3/\text{s}$$

$$V_2 = 8.465 \text{ m/s} \qquad q_2 = (\pi/4)(0.350^2)(8.465) = 0.814 \text{ m}^3/\text{s}$$

$$V_3 = 20.18 \text{ m/s} \qquad q_3 = (\pi/4)(0.400^2)(20.18) = 2.536 \text{ m}^3/\text{s}$$

We see that $(q_1 + q_2) > q_3$. As a second estimate, we increase H_J and use more accurate friction factors. Using the preceding velocities, we now find a second set of friction factors: $(N_R)_1 = (28.72)(0.300)/(0.0113 \times 10^{-4}) = 7.625 \times 10^6$, $f_1 = 0.014$; $(N_R)_2 = (8.465)(0.350)/(0.0113 \times 10^{-4}) = 2.622 \times 10^6$, $f_2 = 0.0134$; $(N_R)_3 = (20.18)(0.400)/(0.0113 \times 10^{-4}) = 7.143 \times 10^6$, $f_3 = 0.012$. Suppose we next choose H_J to be 400 m. Thus, we still use the continuity equation. We get the following results: $V_1 = 27.98$ m/s, $q_1 = 1.977$ m³/s; $V_2 = 5.95$ m/s, $q_2 = 0.5721$ m³/s; $V_3 = 20.97$ m/s, $q_3 = 2.635$ m³/s.

Note that $(q_1 + q_2) < q_3$. We now *interpolate* to get the final result. That is, $H_J = 380$ m gave us a value $(q_1 + q_2) - q_3 = 0.308$, while $H_J = 400$ m gave us a value $(q_1 + q_2) - q_3 = -0.0859$. Hence we choose a final H_J to be $H_J = 380 + [0.308/(0.308 + 0.0859)](400 - 380) = 396$ m. For this we get $q_1 = 1.988$ m³/s $= 1988$ L/s, $q_2 = 0.6249$ m³/s $= 624.9$ L/s, $q_3 = 2.616$ m³/s $= 2616$ L/s. We come very close to satisfying the continuity equation so the above are the desired flows.

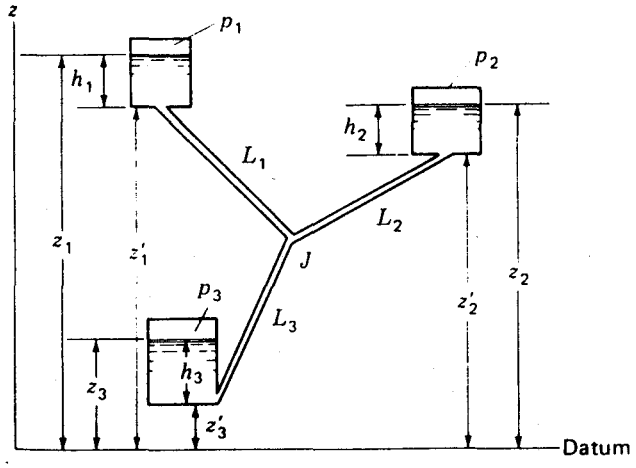


Fig. 12-1

12.2 The three-cistern system of Fig. 12-2 contains water at 5 °C, compute the three volumetric flows, given: $L_1 = 2000$ m, $L_2 = 2300$ m, $L_3 = 2500$ m; $D_1 = 1$ m, $D_2 = 0.60$ m, $D_3 = 1.20$ m; $\epsilon_1/D_1 = 0.00015$, $\epsilon_2/D_2 = 0.001$, $\epsilon_3/D_3 = 0.002$.

I We assume initially a junction head $h_j = z_j + (p_j/\gamma) = 80$ m. Using $h_f = f(L/d)(v^2/2g)$ for each branch we get:

$$(120 - 80) = f_1(2000/1)(v_1^2/2g) \qquad (100 - 80) = f_2(2300/0.60)(v_2^2/2g) \qquad (80 - 28) = f_3(2500/1.20)(v_3^2/2g)$$

Guessing $f_1 = 0.013$, $f_2 = 0.020$, $f_3 = 0.023$, we obtain

$$v_1 = 5.494 \text{ m/s} \qquad Q_1 = (\pi/4)(1^2)(5.494) = 4.315 \text{ m}^3/\text{s}$$
$$v_2 = 2.262 \text{ m/s} \qquad Q_2 = (\pi/4)(0.60)^2(2.262) = 0.6396 \text{ m}^3/\text{s}$$
$$v_3 = 4.6136 \text{ m/s} \qquad Q_3 = (\pi/4)(1.2)^2(4.6136) = 5.218 \text{ m}^3/\text{s}$$

Checking with the continuity equation,

$$\left.\begin{aligned} Q_1 + Q_2 &= 4.955 \\ Q_3 &= 5.2179 \end{aligned}\right\} \qquad \Delta = (Q_1 + Q_2) - Q_3 - 0.2629$$

we decrease h_J to 76 m. Then

$$(120 - 76) = 0.013(\tfrac{2000}{1})(v_1^2/2g) \qquad v_1 = 5.7613 \text{ m/s} \qquad Q_1 = 4.5249 \text{ m}^3/\text{s}$$
$$(100 - 76) = 0.020(2300/0.6)(v_2^2/2g) \qquad v_2 = 2.4779 \text{ m/s} \qquad Q_2 = 0.7006 \text{ m}^3/\text{s}$$
$$(76 - 28) = 0.023(2500/1.20)(v_3^2/2g) \qquad v_3 = 4.4326 \text{ m/s} \qquad Q_3 = 5.0132 \text{ m}^3/\text{s}$$

$$\left.\begin{aligned} Q_1 + Q_2 &= 5.226 \\ Q_3 &= 5.013 \end{aligned}\right\} \qquad \Delta = 0.213 \text{ m/s}$$

Interpolation now yields:

$$h_J = 80 - [0.2629/(0.213 + 0.2629)](4) = 77.7903 \text{ m}$$
$$v_1 = 5.643 \text{ m/s} \qquad Q_1 = 4.432 \text{ m}^3/\text{s}$$
$$v_2 = 2.3837 \text{ m/s} \qquad Q_2 = 0.674 \text{ m}^3/\text{s}$$
$$v_3 = 4.5145 \text{ m/s} \qquad Q_3 = 5.106 \text{ m}^3/\text{s}$$

Checking:

$$Q_1 + Q_2 = 4.432 + 0.674 = 5.106 \qquad q_3 = 5.106 \qquad \text{O.K.}$$

Therefore, $Q_1 = 4.39 \text{ m}^3/\text{s}$, $Q_2 = 0.661 \text{ m}^3/\text{s}$, $Q_3 = 5.048 \text{ m}^3/\text{s}$.

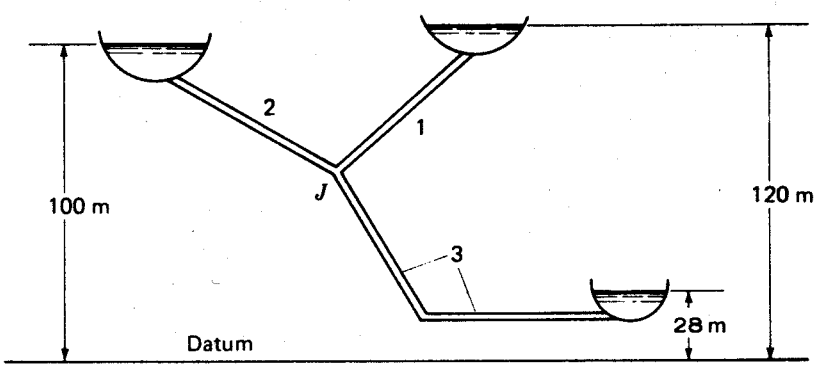


Fig. 12-2

12.3 The system of Fig. 12-3(*a*) holds water at 60 °F. Data are: $p = 50$ psig; $z_1 = 700$ ft, $L_1 = 2000$ ft, $D_1 = 3$ ft, $(\epsilon/D)_1 = 0.001$; $z_2 = 650$ ft, $L_2 = 2500$ ft $D_2 = 3.5$ ft, $(\epsilon/D)_2 = 0.002$; $z_3 = 100$ ft, $L_3 = 2200$ ft, $D_3 = 4$ ft, $(\epsilon/D)_3 = 0.002$. Determine the three pipe flows.

I The heads for the free surfaces are $h_1 = 700$ ft, $h_2 = 650$ ft, $h_3 = 100 + (50)(144)/62.4 = 215.4$ ft. Initially we take h_J to be 450 ft. Applying $h_f = f(L/d)(v^2/2g)$,

$$(700 - 450) = f_1(2000/3)(v_1^2/2g) \qquad (650 - 450) = f_2(2500/3.5)(v_2^2/2g)$$
$$(450 - 215.4) = f_3(2200/4)(v_3^2/2g)$$

Guessing $f_1 = 0.02$, $f_2 = 0.023$, $f_3 = 0.023$, we solve for the velocities and flows:

$$v_1 = 34.75 \text{ ft/s} \qquad Q_1 = 245 \text{ cfs} \qquad v_2 = 28.0 \text{ ft/s} \qquad Q_2 = 269.4 \text{ cfs}$$
$$v_3 = 34.56 \text{ ft/s} \qquad Q_3 = 434.3 \text{ cfs}$$

Checking continuity: $Q_1 + Q_2 = 245 + 269.4 = 514$ cfs, $Q_3 = 434.3$ cfs; $\Delta = 79.7$ cfs. We increase h_J to 475 ft.

$$(700 - 475) = (0.02)(2000/3)(v_1^2/2g) \qquad (650 - 475) = (0.023)(2500/3.5)v_2^2/2g$$

$$(475 - 215.4) = (0.023)(2200/4)(v_3^2/2g)$$

from which

$$v_1 = 32.96 \text{ ft/s} \qquad Q_1 = 233 \text{ cfs} \qquad v_2 = 26.19 \text{ ft/s} \qquad Q_2 = 252 \text{ cfs} \qquad v_3 = 36.35 \text{ ft/s} \qquad Q_3 = 456 \text{ cfs}$$

Check continuity again: $Q_1 + Q_2 = 485$, $Q_3 = 456$ ft^3/s; $\Delta = 29.0$ cfs. Extrapolating Δ to zero [Fig. 12-3(*b*)], we get $h_J = 487$ ft:

$$(700 - 487) = (0.02)(2000/3)(v_1^2/2g) \qquad (650 - 487) = (0.023)(2500/3.5)(v_2^2/2g)$$

$$(487 - 215.4) = (0.023)(2200/4)(v_3^2/2g)$$

$$v_1 = 32.07 \text{ ft/s} \qquad Q_1 = 226.7 \text{ cfs} \qquad v_2 = 25.27 \text{ ft/s} \qquad Q_2 = 243.2 \text{ cfs}$$

$$v_3 = 37.185 \text{ ft/s} \qquad Q_3 = 467.3 \text{ cfs}$$

Check continuity: $Q_1 + Q_2 = 469.9$, $Q_3 = 467.3$ ft^3/s; close enough. Therefore, $Q_1 = 227$ cfs, $Q_2 = 243$ cfs, $Q_3 = 467$ cfs.

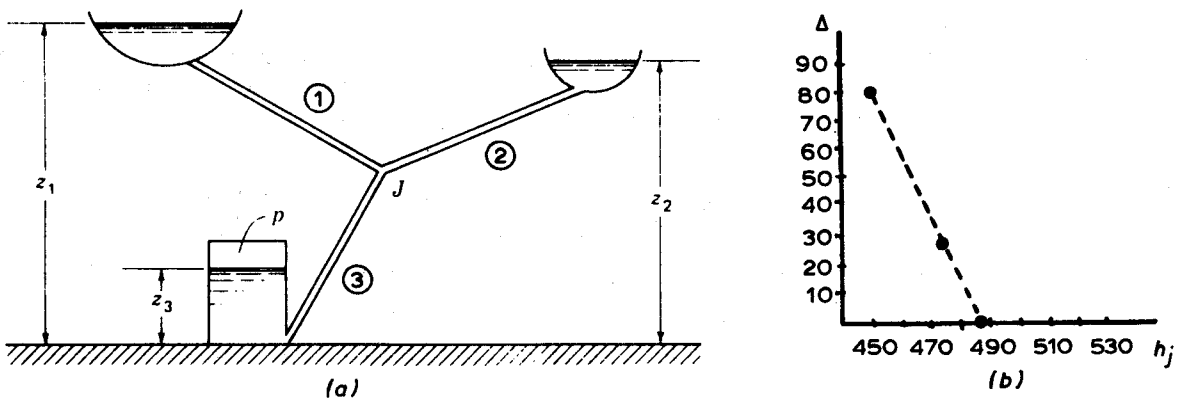


Fig. 12-3

12.4 In Fig. 12-4, valve *F* is partly closed, creating a head loss of 3.60 ft when the flow through the valve is 0.646 mgd (1.00 cfs). What is the length of 10-in pipe to reservoir *A*?

▮

$$Q = 1.318ACR^{0.63}s^{0.54} \qquad 1.00 = 1.318[(\pi)(\tfrac{12}{12})^2/4](80)[(\tfrac{12}{12})/4]^{0.63}(s_{DB})^{0.54}$$

$$s_{DB} = 0.001414 \text{ ft/ft}$$

$$(h_L)_{DB} = (0.001414)(1000) + 3.60 = 5.01 \text{ ft}$$

If $\text{El}_E = 0$, the grade line elevation at $B = 20 - 5.01 = 14.99$ ft.

$$s_{BE} = (14.99 - 0)/5000 = 0.002998 \text{ ft/ft}$$

$$Q_{BE} = 1.318[(\pi)(\tfrac{12}{12})^2/4](120)[(\tfrac{12}{12})/4]^{0.63}(0.002998)^{0.54} = 2.25 \text{ ft}^3/\text{s}$$

$$Q_{AB} = Q_{BE} - Q_{DB} = 2.25 - 1.00 = 1.25 \text{ ft}^3/\text{s}$$

$$1.25 = 1.318[(\pi)(\tfrac{10}{12})^2/4](100)[(\tfrac{10}{12})/4]^{0.63}(s_{AB})^{0.54}$$

$$s_{AB} = 0.003436 \text{ ft/ft} = (h_f)_{AB}/L_{AB} \qquad 0.003436 = (20 - 15 - 2)/L_{AB} \qquad L_{AB} = 873 \text{ ft}$$

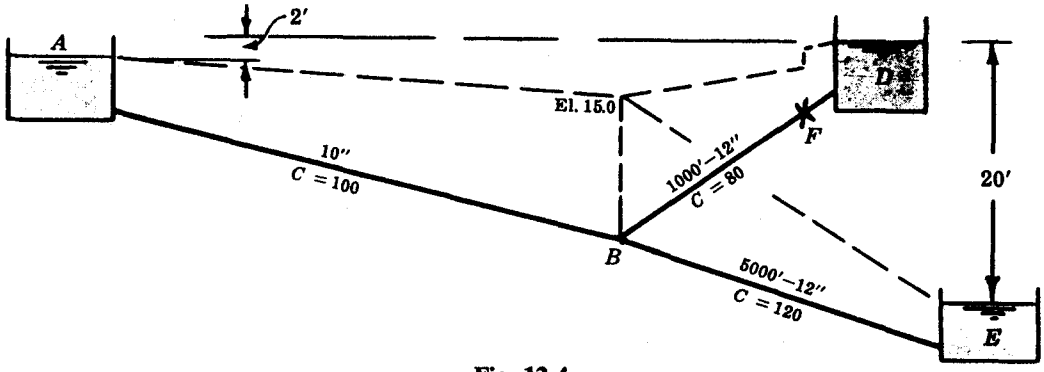


Fig. 12-4

12.5 For the constant elevations of the water surfaces shown in Fig. 12-5*a*, what flows will occur?

▮ Because the elevation of the hydraulic grade line at C cannot be computed (all flows unknown), the problem will be solved by successive trials. A convenient assumption is to choose the elevation of the hydraulic grade line at C at 190.0 ft. By so assuming, flow to or from reservoir B will be zero, reducing the number of calculations thereby. With this assumption,

$$Q = 1.318ACR^{0.63}s^{0.54} \qquad s_{AC} = (212.0 - 190.0)/8000 = 0.002750 \text{ ft/ft}$$

$$Q_{AB} = 1.318[(\pi)(\tfrac{24}{12})^2/4](100)[(\tfrac{24}{12})/4]^{0.63}(0.002750)^{0.54} = 11.1 \text{ ft}^3/\text{s}$$

$$s_{CD} = (190.0 - 100.0)/4000 = 0.02250 \text{ ft/ft}$$

$$Q_{CD} = 1.318[(\pi)(\tfrac{12}{12})^2/4](100)[(\tfrac{12}{12})/4]^{0.63}(0.02250)^{0.54} = 5.57 \text{ ft}^3/\text{s} \qquad \text{(away from } C\text{)}$$

Examination of these values of flow indicates that the grade line at C must be higher, thereby reducing the flow from A and increasing the flow to D as well as adding flow to B. In an endeavor to "straddle" the correct elevation at C, assume a value of 200.0 ft. Thus, for elevation at $C = 200.0$ ft,

$$s_{AC} = (212.0 - 200.0)/8000 = 0.001500 \text{ ft/ft}$$

$$Q_{AB} = 1.318[(\pi)(\tfrac{24}{12})^2/4](100)[(\tfrac{24}{12})/4]^{0.63}(0.001500)^{0.54} = 7.99 \text{ ft}^3/\text{s}$$

$$s_{CD} = (200.0 - 100.0)/4000 = 0.02500 \text{ ft/ft}$$

$$Q_{CD} = 1.318[(\pi)(\tfrac{12}{12})^2/4](100)[(\tfrac{12}{12})/4]^{0.63}(0.02500)^{0.54} = 5.90 \text{ ft}^3/\text{s} \qquad \text{(away from } C\text{)}$$

$$s_{CB} = (200.0 - 190.0)/4000 = 0.002500 \text{ ft/ft}$$

$$Q_{CB} = 1.318[(\pi)(\tfrac{16}{12})^2/4](120)[(\tfrac{16}{12})/4]^{0.63}(0.002500)^{0.54} = 4.35 \text{ ft}^3/\text{s} \qquad \text{(away from } C\text{)}$$

The flow away from C is 5.90 + 4.35, or 10.25 ft^3/s compared with the flow to C of 7.90 ft^3/s. Using Fig. 12-5*b* to obtain a guide regarding a reasonable third assumption, connect plotted points R and S. The line so drawn intersects the $(Q_{to} - Q_{away})$ zero abscissa at approximately $Q_{to} = 8.8$ ft^3/s. Inasmuch as the values plotted do not vary linearly, use a flow to C slightly larger, say 9.2 ft^3/s. For $Q = 9.2$ ft^3/s to C (i.e., $Q_{AB} = 9.2$ ft^3/s), $9.2 = 1.318[(\pi)(\tfrac{24}{12})^2/4](100)[(\tfrac{24}{12})/4]^{0.63}s^{0.54}$, $s_{AC} = 0.001948$ ft/ft; $(h_f)_{AC} = (0.001948)(8000) = 15.6$ ft. Then, the hydraulic grade line at C will be 212.0 − 15.6 = 196.4 ft.

$$s_{CD} = (196.4 - 100.0)/4000 = 0.02410 \text{ ft/ft}$$

$$Q_{CD} = 1.318[(\pi)(\tfrac{12}{12})^2/4](100)[(\tfrac{12}{12})/4]^{0.63}0.02410^{0.54} = 5.78 \text{ ft}^3/\text{s} \qquad \text{(away from } C\text{)}$$

$$s_{CB} = (196.4 - 190.0)/4000 = 0.001600 \text{ ft/ft}$$

$$Q_{CB} = 1.318[(\pi)(\tfrac{16}{12})^2/4](120)[(\tfrac{16}{12})/4]^{0.63}0.001600^{0.54} = 3.42 \text{ ft}^3/\text{s} \qquad \text{(away from } C\text{)}$$

The flow away from C is 5.78 + 3.42, or 9.20 ft^3/s compared with the flow to C of 9.2 ft^3/s; hence these values of Q_{AC}, Q_{CD}, and Q_{CB} must be O.K.

12.6 Three pipes connect three reservoirs as shown in Fig. 12-6 at these surface elevations: $z_1 = 20$ m, $z_2 = 100$ m, and $z_3 = 40$ m. The pipe data are

pipe	L, m	d, m	ϵ, mm	ϵ/d
1	100	8	0.24	0.003
2	150	6	0.12	0.002
3	80	4	0.20	0.005

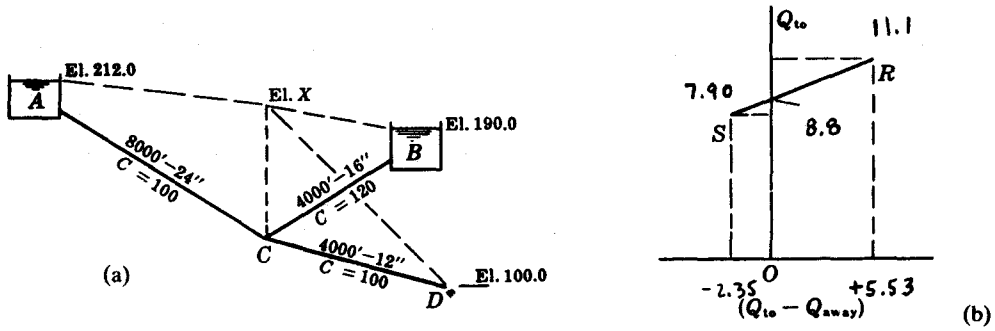


Fig. 12-5

The fluid is water, $\rho = 1000\ \text{kg/m}^3$, and $\nu = 1.02 \times 10^{-6}\ \text{m}^2/\text{s}$. Find the resulting flow in each pipe, neglecting minor losses.

▮ As a first guess, take h_J equal to the middle reservoir height, $z_3 = h_J = 40$ m. This saves one calculation ($Q_3 = 0$) and enables us to get the lay of the land:

reservoir	h_J, m	$z_i - h_J$, m	f_i	V_i, m/s	Q_i, m³/h	L_i/d_i
1	40	−20	0.0267	−3.43	−62.1	1250
2	40	60	0.0241	4.42	45.0	2500
3	40	0		0	0	2000
					$\Sigma Q = -17.1$	

Since the sum of the flow rates toward the junction is negative, we guessed h_J too high. Reduce h_J to 30 m and repeat:

reservoir	h_J, m	$z_i - h_J$, m	f_i	V_i, m/s	Q_i, m³/h
1	30	−10	0.0269	−2.42	−43.7
2	30	70	0.0241	4.78	48.6
3	30	10	0.0317	1.76	8.0
					$\Sigma Q =$ 12.9

This is positive ΣQ, and so we can linearly interpolate to get an accurate guess: $h_J \approx 34.3$ m. Make one final list:

reservoir	h_J, m	$z_i - h_J$, m	f_i	V_i, m/s	Q_i, m³/h
1	34.3	−14.3	0.0268	−2.90	−52.4
2	34.3	65.7	0.0241	4.63	47.1
3	34.3	5.7	0.0321	1.32	6.0
					$\Sigma Q =$ 0.7

This is close enough; hence we calculate that the flow rate is 52.4 m³/h toward reservoir 3, balanced by 47.1 m³/h away from reservoir 1 and 6.0 m³/h away from reservoir 3.

One further iteration with this problem would give $h_J = 34.53$ m, resulting in $Q_1 = -52.8$, $Q_2 = 47.0$, and $Q_3 = 5.8\ \text{m}^3/\text{h}$, so that $\Sigma Q = 0$ to three-place accuracy.

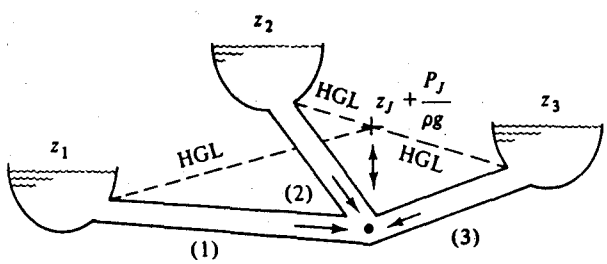


Fig. 12-6

12.7 For the three-reservoir system of Fig. 12-7, $z_1 = 29$ m, $L_1 = 80$ m, $z_2 = 129$ m, $L_2 = 150$ m, $z_3 = 69$ m, and $L_3 = 110$ m. All pipes are 250-mm-diameter concrete with roughness height 0.5 mm. Compute the flow rates for water at 20 °C.

▮ $h_f = (f)(L/d)(v^2/2g)$. Guess $h_a = 69$ m: $(h_f)_1 = z_1 - h_a = 29 - 69 = -40$ m, $\epsilon/d = 0.5/250 = 0.00200$ (for all pipes), $N_R = dv/\nu$. Try $v_1 = 10$ m/s: $(N_R)_1 = (0.250)(10)/(1.02 \times 10^{-6}) = 2.45 \times 10^6$. From Fig. A-5, $f_1 = 0.0235$.

$$40 = 0.0235[80/0.250]\{v_1^2/[(2)(9.807)]\} \qquad v_1 = 10.21 \text{ m/s}$$
$$(N_R)_1 = (0.250)(10.21)/(1.02 \times 10^{-6}) = 2.50 \times 10^6 \qquad f_1 = 0.0235 \qquad \text{(O.K.)}$$
$$Q_1 = A_1 v_1 = [(\pi)(0.250)^2/4](10.21) = 0.5012 \text{ m}^3/\text{s} \qquad \text{(away from } a\text{)}$$
$$(h_f)_2 = z_2 - h_a = 129 - 69 = 60 \text{ m}$$

Try $v_2 = 10$ m/s:

$$(N_R)_2 = (0.250)(10)/(1.02 \times 10^{-6}) = 2.45 \times 10^6 \qquad f_2 = 0.0235$$
$$60 = 0.0235[150/0.250]\{v_2^2/[(2)(9.807)]\} \qquad v_2 = 9.136 \text{ m/s}$$
$$(N_R)_2 = (0.250)(9.136)/(1.02 \times 10^{-6}) = 2.24 \times 10^6 \qquad f_1 = 0.0235 \qquad \text{(O.K.)}$$
$$Q_2 = [(\pi)(0.250)^2/4](9.136) = 0.4485 \text{ m}^3/\text{s} \qquad \text{(toward } a\text{)}$$
$$(h_f)_3 = z_3 - h_a = 69 - 69 = 0 \text{ m}$$

Hence, $Q_3 = 0$; $Q_{\text{to } a} - Q_{\text{from } a} = 0.4485 - 0.5012 = -0.0527$ m^3/s. Hence h_a must be a little lower. Try $h_a = 68.5$ m: $(h_f)_1 = z_1 - h_a = 29 - 68.5 = -39.5$ m. Try $v_1 = 10$ m/s:

$$(N_R)_1 = (0.250)(10)/(1.02 \times 10^{-6}) = 2.45 \times 10^6 \qquad f_1 = 0.0235$$
$$39.5 = (0.0235)[80/0.250]\{v_1^2/[(2)(9.807)]\} \qquad v_1 = 10.15 \text{ m/s}$$
$$(N_R)_1 = (0.250)(10.15)/(1.02 \times 10^{-6}) = 2.49 \times 10^6 \qquad f_1 = 0.0235 \qquad \text{(O.K.)}$$
$$Q_1 = [(\pi)(0.250)^2/4](10.15) = 0.4982 \text{ m}^3/\text{s} \qquad \text{(away from } a\text{)}$$
$$(h_f)_2 = z_2 - h_a = 129 - 68.5 = 60.5 \text{ m}$$

Try $v_2 = 10$ m/s:

$$(N_R)_2 = (0.250)(10)/(1.02 \times 10^{-6}) = 2.45 \times 10^6 \qquad f_2 = 0.0235$$
$$60.5 = 0.0235[150/0.250]\{v_2^2/[(2)(9.807)]\} \qquad v_2 = 9.174 \text{ m/s}$$
$$(N_R)_2 = (0.250)(9.174)/(1.02 \times 10^{-6}) = 2.25 \times 10^6 \qquad f_2 = 0.0235 \qquad \text{(O.K.)}$$
$$Q_2 = [(\pi)(0.250)^2/4](9.174) = 0.4503 \text{ m}^3/\text{s} \qquad \text{(toward } a\text{)}$$
$$(h_f)_3 = z_3 - h_a = 69 - 68.5 = 0.5 \text{ m}$$

Try $v_3 = 1$ m/s:

$$(N_R)_3 = (0.250)(1)/(1.02 \times 10^{-6}) = 2.45 \times 10^5 \qquad f_3 = 0.0244$$
$$0.5 = 0.0244[110/0.250]\{v_3^2/[(2)(9.807)]\} \qquad v_3 = 0.9558 \text{ m/s}$$
$$(N_R)_3 = (0.250)(0.9558)/(1.02 \times 10^{-6}) = 2.34 \times 10^5 \qquad f_3 = 0.0244 \qquad \text{(O.K.)}$$
$$Q_3 = [(\pi)(0.250)^2/4](0.9558) = 0.0469 \text{ m}^3/\text{s} \qquad \text{(toward } a\text{)}$$
$$Q_{\text{to } a} - Q_{\text{from } a} = (0.4503 + 0.0469) - 0.4982 = -0.001 \text{ m}^3/\text{s}$$

Hence, $Q_1 = 0.498$ m^3/s from a, and $Q_2 = 0.450$ m^3/s and $Q_3 = 0.047$ m^3/s toward a.

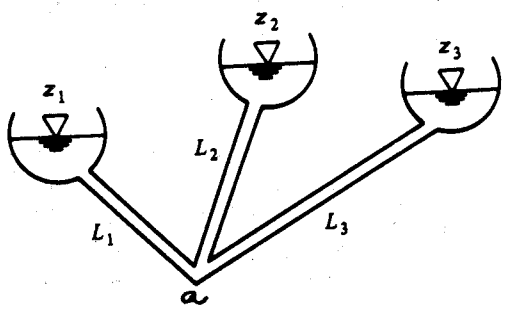


Fig. 12-7

12.8 The three pipes in Fig. 12-8 are cast iron: $d_1 = 7$ in, $L_1 = 2000$ ft; $d_2 = 5$ in, $L_2 = 1000$ ft; $d_3 = 8$ in, $L_3 = 1500$ ft. Compute the flow rates for water at 68 °F.

▮ $h_f = (f)(L/d)(v^2/2g)$. Guess $h_J = 48$ ft: $(h_f)_1 = z_1 - h_J = 18 - 48 = -30$ m, $(\epsilon/d)_1 = 0.00085/(\frac{7}{12}) = 0.00146$, $N_R = dv/\nu$. Try $v_1 = 5$ ft/s: $(N_R)_1 = (\frac{7}{12})(5)/(1.02 \times 10^{-5}) = 2.86 \times 10^5$. From Fig. A-5, $f_1 = 0.0225$.

$$30 = 0.0225[2000/(\tfrac{7}{12})]\{v_1^2/[(2)(32.2)]\} \qquad v_1 = 5.004 \text{ ft/s} \qquad \text{(value of } f_1 \text{ O.K.)}$$
$$Q_1 = A_1 v_1 = [(\pi)(\tfrac{7}{12})^2/4](5.004) = 1.337 \text{ ft}^3/\text{s} \qquad \text{(away from } J\text{)}$$
$$(h_f)_2 = z_2 - h_J = 98 - 48 = 50 \text{ ft}$$

Try $v_2 = 7$ ft/s:

$$(N_R)_2 = (\tfrac{5}{12})(7)/(1.02 \times 10^{-5}) = 2.86 \times 10^5$$
$$(\epsilon/d)_2 = 0.00085/(\tfrac{5}{12}) = 0.00204 \qquad f_2 = 0.024$$
$$50 = 0.024[1000/(\tfrac{5}{12})]\{v_2^2/[(2)(32.2)]\} \qquad v_2 = 7.477 \text{ ft/s}$$
$$(N_R)_2 = (\tfrac{5}{12})(7.477)/(1.02 \times 10^{-5}) = 3.05 \times 10^5 \qquad f_2 = 0.024 \qquad \text{(O.K.)}$$
$$Q_2 = [(\pi)(\tfrac{5}{12})^2/4](7.477) = 1.020 \text{ ft}^3/\text{s} \qquad \text{(toward } J\text{)} \qquad (h_f)_3 = z_3 - h_J = 48 - 48 = 0 \text{ ft}$$

Hence, $Q_3 = 0$; $Q_{\text{to}\,J} - Q_{\text{from}\,J} = 1.020 - 1.337 = -0.317$ ft^3/s. Hence h_J must be a little lower. Try $h_J = 47.35$ ft: $(h_f)_1 = z_1 - h_J = 18 - 47.35 = -29.35$ m. Try $v_1 = 5$ ft/s:

$$(N_R)_1 = (\tfrac{7}{12})(5)/(1.02 \times 10^{-5}) = 2.86 \times 10^5 \qquad f_1 = 0.0225$$
$$29.35 = 0.0225[2000/(\tfrac{7}{12})]\{v_1^2/[(2)(32.2)]\} \qquad v_1 = 4.950 \text{ ft/s} \qquad \text{(value of } f_1 \text{ O.K.)}$$
$$Q_1 = [(\pi)(\tfrac{7}{12})^2/4](4.950) = 1.323 \text{ ft}^3/\text{s} \qquad \text{(away from } J\text{)}$$
$$(h_f)_2 = z_2 - h_J = 98 - 47.35 = 50.65 \text{ ft}$$

Try $v_2 = 7.5$ ft/s:

$$(N_R)_2 = (\tfrac{5}{12})(7.5)/(1.02 \times 10^{-5}) = 3.06 \times 10^5 \qquad f_2 = 0.024$$
$$50.65 = 0.024[1000/(\tfrac{5}{12})]\{v_2^2/[(2)(32.2)]\} \qquad v_2 = 7.525 \text{ ft/s} \qquad \text{(value of } f_2 \text{ O.K.)}$$
$$Q_2 = [(\pi)(\tfrac{5}{12})^2/4](7.525) = 1.026 \text{ ft}^3/\text{s} \qquad \text{(toward } J\text{)}$$
$$(h_f)_3 = z_3 - h_J = 48 - 47.35 = 0.65 \text{ ft}$$

Try $v_3 = 1$ ft/s:

$$(N_R)_3 = (\tfrac{8}{12})(1)/(1.02 \times 10^{-5}) = 6.54 \times 10^4$$
$$(\epsilon/d)_3 = 0.00085/(\tfrac{8}{12}) = 0.00128 \qquad f_3 = 0.024$$
$$0.65 = 0.024[1500/(\tfrac{8}{12})]\{v_3^2/[(2)(32.2)]\} \qquad v_3 = 0.8804 \text{ ft/s}$$
$$(N_R)_3 = (\tfrac{8}{12})(0.8804)/(1.02 \times 10^{-5}) = 5.75 \times 10^4 \qquad f_3 = 0.024 \qquad \text{(O.K.)}$$
$$Q_3 = [(\pi)(\tfrac{8}{12})^2/4](0.8804) = 0.307 \text{ ft}^3/\text{s} \qquad \text{(toward } J\text{)}$$
$$Q_{\text{to}\,J} - Q_{\text{from}\,J} = (1.026 + 0.307) - 1.323 = 0.010 \text{ ft}^3/\text{s}$$

Hence, $Q_1 = 1.323$ ft^3/s from J, and $Q_2 = 1.026$ ft^3/s and $Q_3 = 0.307$ ft^3/s toward J.

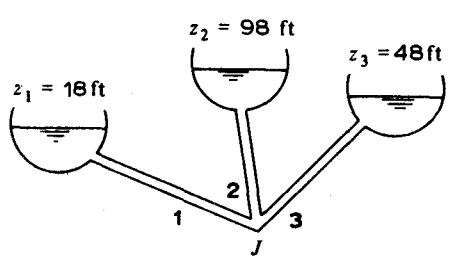


Fig. 12-8

12.9 In Fig. 12-9 find the discharges for water at 20 °C with the following pipe data and reservoir elevations: $L_1 = 3000$ m, $D_1 = 1$ m, $\epsilon_1/D_1 = 0.0002$; $L_2 = 600$ m, $D_2 = 0.45$ m, $\epsilon_2/D_2 = 0.002$; $L_3 = 1000$ m, $D_3 = 0.6$ m, $\epsilon_3/D_3 = 0.001$; $z_1 = 30$ m, $z_2 = 18$ m, $z_3 = 9$ m.

▮ Assume $z_J + p_J/\gamma = 23$ m. Then $7 = f_1(\frac{3000}{1})(V_1^2/2g)$, $f_1 = 0.014$, $V_1 = 1.75$ m/s, $Q_1 = 1.380$ m^3/s; $5 = f_2(600/0.45)(V_2^2/2g)$, $f_2 = 0.024$, $V_2 = 1.75$ m/s, $Q_2 = 0.278$ m^3/s; $14 = f_3(1000/0.60)(V_3^2/2g)$, $f_3 = 0.020$, $V_3 =$

2.87 m/s, $Q_3 = 0.811\ \text{m}^3/\text{s}$; so that the inflow is greater than the outflow by $1.380 - 0.278 - 0.811 = 0.291\ \text{m}^3/\text{s}$. Assume $z_J + p_J/\gamma = 24.6$ m. Then $5.4 = f_1(\frac{3000}{1})(V_1^2/2g)$, $f_1 = 0.015$, $V_1 = 1.534$ m/s, $Q_1 = 1.205\ \text{m}^3/\text{s}$; $6.6 = f_2(600/0.45)(V_2^2/2g)$, $f_2 = 0.024$, $V_2 = 2.011$ m/s, $Q_2 = 0.320\ \text{m}^3/\text{s}$; $15.6 = f_3(1000/0.60)(V_3^2/2g)$, $f_3 = 0.020$, $V_3 = 3.029$ m/s, $Q_3 = 0.856\ \text{m}^3/\text{s}$. The inflow is still greater by $0.029\ \text{m}^3/\text{s}$. By extrapolating linearly, $z_J + p_J/\gamma = 24.8$ m, $Q_1 = 1.183$, $Q_2 = 0.325$, $Q_3 = 0.862\ \text{m}^3/\text{s}$.

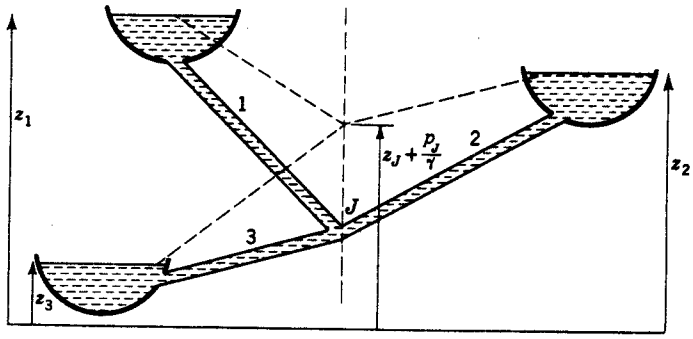


Fig. 12-9

12.10 Prepare a computer program to balance the flows in a system of N reservoirs connected by a common junction. Use the data of Prob. 12.9 to check out the program.

```
10 REM B:   N reservoirs with a common junction
20 DEFINT I,N: DEF FNQ(D1,D2,D3,DH)=D1*DH*LOG(D2+D3/DH)
30 READ N,G,KVIS,II:     DATA 3,9.806,1.007E-6,15
40 LPRINT:     LPRINT"N,G,KVIS,II=";N;G;KVIS;II
50 ZMA=-1000!:     ZMI=1000!
60 FOR I=1 TO N:     READ L(I),D(I),EP(I),Z(I)
70 LPRINT"L,D,EP,Z=";L(I);D(I);EP(I);Z(I)
80 NEXT I
90 DATA 3000.,1.,.0002,30.,600.,.45,.0009,18.,1000.,.6,.0006,9.
100 FOR I=1 TO N:     C=SQR(G*D(I)/L(I)):     E1(I)=-.9650001*D(I)^2*C
110 E2(I)=EP(I)/(3.7*D(I)):     E3(I)=1.784*KVIS/(D(I)*C)     ' FOR EQ.(5.8.15)
120 IF Z(I)>ZMA THEN ZMA=Z(I)                                  ' ELEV. OF HIGHEST RESERVOIR
130 IF Z(I)<ZMI THEN ZMI=Z(I)                                  ' ELEV. OF LOWEST RESERVOIR
140 NEXT I
150 FOR I1=1 TO 15:     HJUN=.5*(ZMA+ZMI):     S=0!     ' START BISECTION METHOD
160 FOR I=1 TO N     :HF=Z(I)-HJUN:     HFS=SQR(ABS(HF))
170 Q(I)=FNQ(E1(I),E2(I),E3(I),HFS)*SGN(HF)
180 S=S+Q(I):     PRINT"HF,Q,S=";HF;Q(I);S
190 NEXT I
200 IF S>0! THEN ZMI=HJUN ELSE ZMA=HJUN
210 NEXT I1
220 LPRINT"ELEVATION OF JUNCTION IS ";HJUN
230 LPRINT"PIPE DISCHARGES ARE (POSITIVE INTO THE JUNCTION)"
240 FOR I=1 TO N:     LPRINT"Q(";I;")=";:     LPRINT USING" ###.### ";Q(I);
250 NEXT I:     LPRINT

N,G,KVIS,II= 3  9.806001  1.007E-06  15
L,D,EP,Z= 3000  1  .0002  30
L,D,EP,Z= 600  .45  .0009  18
L,D,EP,Z= 1000  .6  .0006  9
ELEVATION OF JUNCTION IS  24.8801
PIPE DISCHARGES ARE (POSITIVE INTO THE JUNCTION)
Q( 1 )=   1.198 Q( 2 )=  -0.329 Q( 3 )=  -0.869
```

12.11 Calculate the three volumetric flows in Fig. 12-10.

$$(\epsilon/D)_{AJ} = 1/200 = 0.0050 \qquad (\epsilon/D)_{BJ} = 1/200 = 0.0050 \qquad (\epsilon/D)_{CJ} = 3/300 = 0.0100$$

$$f_{AJ} = 0.032 \qquad f_{BJ} = 0.032 \qquad f_{CJ} = 0.038 \qquad h_f = (f)(L/D)(v^2/2g) = (f)(L/D^5)(8Q^2/g\pi^2)$$

$$28 - h_J = 0.032[1000/(0.200)^5]\{8Q_{AJ}^2/[(9.807)(\pi)^2]\} = 8265Q_{AJ}^2$$

$$25 - h_J = 0.032[300/(0.200)^5]\{8Q_{BJ}^2/[(9.807)(\pi)^2]\} = 2480Q_{BJ}^2$$

$$h_J - 15 = 0.038[600/(0.300)^5]\{8Q_{CJ}^2/[(9.807)(\pi)^2]\} = 775.5Q_{CJ}^2$$

Try $h_J = 19$ m:

$$28 - 19 = 8265Q_{AJ}^2 \qquad Q_{AJ} = 0.03300\ \text{m}^3/\text{s} \qquad 25 - 19 = 2480Q_{BJ}^2 \qquad Q_{BJ} = 0.04919\ \text{m}^3/\text{s}$$

$$19 - 15 = 775.5Q_{CJ}^2 \qquad Q_{CJ} = 0.07182\ \text{m}^3/\text{s}$$

$$Q_{\text{to}\,J} - Q_{\text{from}\,J} = 0.03300 + 0.04919 - 0.07182 = 0.01037\ \text{m}^3/\text{s}$$

Try $h_J = 20$ m:

$$28 - 20 = 8265Q_{AJ}^2 \qquad Q_{AJ} = 0.03111\ \text{m}^3/\text{s} \qquad 25 - 20 = 2480Q_{BJ}^2 \qquad Q_{BJ} = 0.04490\ \text{m}^3/\text{s}$$
$$20 - 15 = 775.5Q_{CJ}^2 \qquad Q_{CJ} = 0.08030\ \text{m}^3/\text{s}$$
$$Q_{\text{to}\,J} - Q_{\text{from}\,J} = 0.03111 + 0.04490 - 0.08030 = -0.00429\ \text{m}^3/\text{s}$$

Try $h_J = 19.7$ m:

$$28 - 19.7 = 8265Q_{AJ}^2 \qquad Q_{AJ} = 0.03169\ \text{m}^3/\text{s} \qquad 25 - 19.7 = 2480Q_{BJ}^2 \qquad Q_{BJ} = 0.04623\ \text{m}^3/\text{s}$$
$$19.7 - 15 = 775.5Q_{CJ}^2 \qquad Q_{CJ} = 0.07785\ \text{m}^3/\text{s}$$
$$Q_{\text{to}\,J} - Q_{\text{from}\,J} = 0.03169 + 0.04623 - 0.07785 = 0.00007\ \text{m}^3/\text{s}$$

Hence, $Q_{CJ} = 77.8$ L/s from J, and $Q_{AJ} = 31.7$ L/s and $Q_{BJ} = 46.2$ L/s toward J.

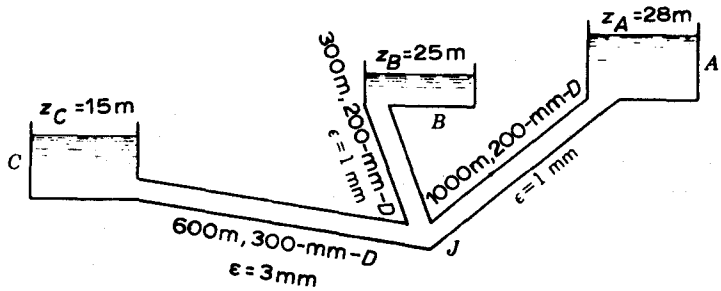


Fig. 12-10

12.12 Calculate the five volumetric flows in Fig. 12-11.

▮

$$h_f = (f)(L/D)(v^2/2g) = (f)(L/D^5)(8Q^2/g\pi^2)$$
$$h_J - 78 = 0.025[600/(0.600)^5]\{8Q_A^2/[(9.807)(\pi)^2]\} = 15.945Q_A^2$$
$$h_J - 88 = 0.030[600/(0.450)^5]\{8Q_B^2/[(9.807)(\pi)^2]\} = 80.624Q_B^2$$
$$h_j - h_J = 0.030[900/(0.450)^5]\{8Q_{Jj}^2/[(9.807)(\pi)^2]\} = 120.94Q_{Jj}^2$$
$$98 - h_j = 0.030[300/(0.450)^5]\{8Q_C^2/[(9.807)(\pi)^2]\} = 40.312Q_C^2$$
$$88 - h_j = 0.030[300/(0.450)^5]\{8Q_B^2/[(9.807)(\pi)^2]\} = 40.312Q_D^2$$

Try $h_J = 82$ m:

$$82 - 78 = 15.945Q_A^2 \qquad Q_A = 0.5009\ \text{m}^3/\text{s} \qquad 82 - 88 = 80.624Q_B^2 \qquad Q_B = -0.2728\ \text{m}^3/\text{s}$$
$$h_j - 82 = (120.94)(0.5009 - 0.2728)^2 \qquad h_j = 88.29\ \text{m}$$
$$98 - 88.29 = 40.312Q_C^2 \qquad Q_C = 0.4908\ \text{m}^3/\text{s}$$
$$88 - 88.29 = 40.312Q_D^2 \qquad Q_D = -0.0848\ \text{m}^3/\text{s}$$

If the above values are correct, Q_{Jj} ($0.5009 - 0.2728 = 0.2281$ m^3/s) must equal $Q_C + Q_D$ ($0.4908 - 0.0848 = 0.4060$ m^3/s). Since they are not equal, try $h_J = 83$ m and note that $Q_{Jj} < Q_C + Q_D$:

$$83 - 78 = 15.945Q_A^2 \qquad Q_A = 0.5600\ \text{m}^3/\text{s} \qquad 83 - 88 = 80.624Q_B^2 \qquad Q_B = -0.2490\ \text{m}^3/\text{s}$$
$$h_j - 83 = (120.94)(0.5600 - 0.2490)^2 \qquad h_j = 94.70\ \text{m}$$
$$98 - 94.70 = 40.312Q_C^2 \qquad Q_C = 0.2861\ \text{m}^3/\text{s}$$
$$88 - 94.70 = 40.312Q_D^2 \qquad Q_D = -0.4077\ \text{m}^3/\text{s}$$

If these values are correct, Q_{Jj} ($0.5600 - 0.2490 = 0.3110$ m^3/s) must equal $Q_C + Q_D$ ($0.2861 - 0.4077 = -0.1216$ m^3/s). Since this time, $Q_{Jj} > Q_C + Q_D$, the correct value of h_J must be between 82 m and 83 m. Try $h_J = 82.24$ m:

$$82.24 - 78 = 15.945Q_A^2 \qquad Q_A = 0.5157\ \text{m}^3/\text{s} \qquad 82.24 - 88 = 80.624Q_B^2 \qquad Q_B = -0.2673\ \text{m}^3/\text{s}$$
$$h_j - 82.24 = (120.94)(0.5157 - 0.2673)^2 \qquad h_j = 89.70\ \text{m}$$
$$98 - 89.70 = 40.312Q_C^2 \qquad Q_C = 0.4538\ \text{m}^3/\text{s}$$
$$88 - 89.70 = 40.312Q_D^2 \qquad Q_D = -0.2054\ \text{m}^3/\text{s}$$

If these values are correct, Q_{Jj} ($0.5157 - 0.2673 = 0.2484$ m^3/s) must equal $Q_C + Q_D$

$(0.4538 - 0.2054 = 0.2484\ \text{m}^3/\text{s})$. Since they are in fact equal, the correct values must be $Q_A = 516$ L/s, $Q_B = 267$ L/s, $Q_{Jj} = 248$ L/s, $Q_C = 454$ L/s, and $Q_D = 205$ L/s.

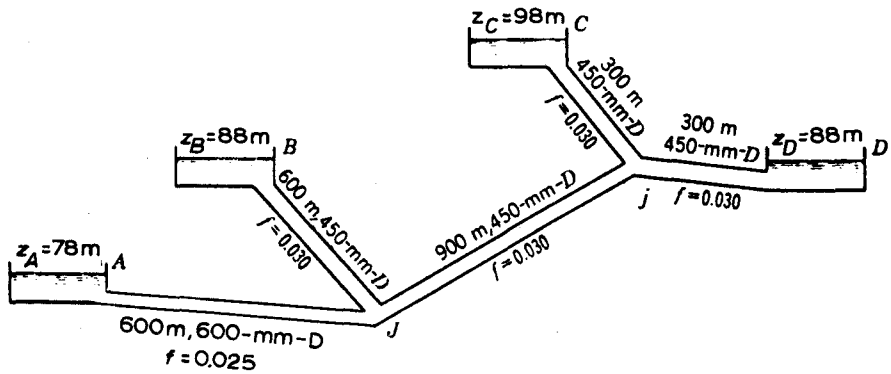


Fig. 12-11

12.13 In Fig. 12-12, pipe 1 is 36-in smooth concrete, 5000 ft long; pipe 2 is 24-in cast iron, 1500 ft long; and pipe 3 is 18-in cast iron, 4000 ft long. The elevations of water surfaces in reservoirs A and B are 300 ft and 250 ft, respectively, and discharge Q_1 is 50 ft^3/s. Find the elevation of the surface of reservoir C.

▮ $h_f = (f)(L/d)(v^2/2g) \qquad (\epsilon/d)_1 = 0.001/(\frac{36}{12}) = 0.000333 \qquad N_R = dv/\nu$

$$v_1 = Q_1/A_1 = 50/[(\pi)(\tfrac{36}{12})^2/4] = 7.074\ \text{ft/s}$$

$$(N_R)_1 = (\tfrac{36}{12})(7.074)/(1.05 \times 10^{-5}) = 2.02 \times 10^6$$

From Fig. A-5, $f_1 = 0.0157$. $(h_f)_1 = 0.0157[5000/(\frac{36}{12})]\{7.074^2/[(2)(32.2)]\} = 20.33$ ft, $h_J = 300 - 20.33 = 279.67$ ft, $(h_f)_2 = 279.67 - 250 = 29.67$ ft. Assume $f_2 = 0.0162$:

$$29.67 = 0.0162[1500/(\tfrac{24}{12})]\{v_2^2/[(2)(32.2)]\} \qquad v_2 = 12.54\ \text{ft/s}$$

$$(N_R)_2 = (\tfrac{24}{12})(12.54)/(1.05 \times 10^{-5}) = 2.39 \times 10^6$$

$$(\epsilon/d)_2 = 0.00085/(\tfrac{24}{12}) = 0.000425 \qquad f_2 = 0.0162 \qquad \text{(O.K.)}$$

$$Q_2 = A_2 v_2 = [(\pi)(\tfrac{24}{12})^2/4](12.54) = 39.40\ \text{ft}^3/\text{s} \qquad Q_3 = 50 - 39.40 = 10.6\ \text{ft}^3/\text{s}$$

$$v_3 = 10.6/[(\pi)(\tfrac{18}{12})^2/4] = 5.998\ \text{ft/s} \qquad (N_R)_3 = (\tfrac{18}{12})(5.998)/(1.05 \times 10^{-5}) = 8.57 \times 10^5$$

$$(\epsilon/d)_3 = 0.00085/(\tfrac{18}{12}) = 0.000567 \qquad f_3 = 0.0177$$

$$(h_f)_3 = 0.0177[4000/(\tfrac{18}{12})]\{5.998^2/[(2)(32.2)]\} = 26.37\ \text{ft}$$

$$z_C = 279.67 - 26.37 = 253.30\ \text{ft}$$

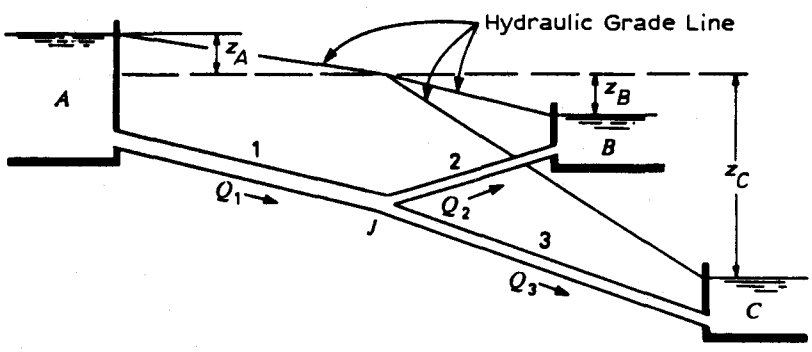


Fig. 12-12

12.14 In Prob. 12.13, suppose that the surface elevations of reservoirs A and C are 300 ft and 225 ft, respectively, and discharge Q_2 is 20 cfs into B; other data are unchanged. Find the surface elevation of reservoir B.

▮ $h_f = (f)(L/d)(v^2/2g) \qquad v_2 = Q_2/A_2 = 20/[(\pi)(\frac{24}{12})^2/4] = 6.366\ \text{ft/s} \qquad N_R = dv/\nu$

$$(N_R)_2 = (\tfrac{24}{12})(6.366)/(1.05 \times 10^{-5}) = 1.21 \times 10^6$$

$$(\epsilon/d)_2 = 0.000425 \qquad \text{(from Prob. 12.13)}$$

From Fig. A-5, $f_2 = 0.0168$. $(h_f)_2 = 0.0168[1500/(\frac{24}{12})]\{6.366^2/[(2)(32.2)]\} = 7.93$ ft.

$$Q_1 - Q_3 = 20 \qquad (1)$$

$$(h_f)_1 + (h_f)_3 = 300 - 225 = 75 \text{ ft} \qquad (2)$$

We need to find an h_f in line 1 or 3 such that both Eqs. (*1*) and (*2*) are satisfied. Try $(h_f)_3 = 64$ ft and $f_3 = 0.0175$:

$$64 = 0.0175[4000/(\tfrac{18}{12})]\{v_3^2/[(2)(32.2)]\} \qquad v_3 = 9.398 \text{ ft/s}$$

$$(N_R)_3 = \tfrac{18}{12}(9.398)/(1.05 \times 10^{-5}) = 1.34 \times 10^6$$

$$(\epsilon/d)_3 = 0.000567 \qquad \text{(from Prob. 12.13)}$$

$$f_3 = 0.0175 \qquad \text{(O.K.)} \qquad Q_3 = A_3 v_3 = [(\pi)(\tfrac{18}{12})^2/4](9.398) = 16.61 \text{ ft}^3/\text{s}$$

$$Q_1 - 16.61 = 20 \qquad Q_1 = 36.61 \text{ ft}^3/\text{s}$$

$$v_1 = 36.61/[(\pi)(\tfrac{36}{12})^2/4] = 5.179 \text{ ft/s} \qquad (N_R)_1 = (\tfrac{36}{12})(5.179)/(1.05 \times 10^{-5}) = 1.48 \times 10^6$$

$$(\epsilon/d)_1 = 0.000333 \qquad \text{(from Prob. 12.13)} \qquad f_1 = 0.0158$$

$$(h_f)_1 = 0.0158[5000/(\tfrac{36}{12})]\{5.179^2/[(2)(32.2)]\} = 10.97 \text{ ft}$$

$$z_{BB} = z_A - (h_f)_1 - (h_f)_2 = 300 - 10.97 - 7.93 = 281.1 \text{ ft}$$

12.15 Given, in Prob. 12.14, that discharge Q_2 is 20 cfs *out of* reservoir B, find the elevation of the surface of B.

▮ As in Prob. 12.14, $(h_f)_2 = 7.93$ ft, but Eq. (*1*) becomes

$$Q_3 - Q_1 = 20 \qquad (1)$$

Equation (*2*) is the same as in Prob. 12.14; that is,

$$(h_f)_1 + (h_f)_3 = 300 - 225 = 75 \text{ ft} \qquad (2)$$

We need to find an h_f in line 1 or 3 such that both Eqs. (*1*) and (*2*) are satisfied. Try $(h_f)_3 = 77$ ft and $f_3 = 0.0175$:

$$77 = 0.0175[4000/(\tfrac{18}{12})]\{v_3^2/[(2)(32.2)]\} \qquad v_3 = 10.31 \text{ ft/s}$$

$$(N_R)_3 = (\tfrac{18}{12})(10.31)/(1.05 \times 10^{-5}) = 1.47 \times 10^6$$

$$(\epsilon/d)_3 = 0.000567 \qquad \text{(from Prob. 12.13)}$$

$$f_3 = 0.0175 \qquad \text{(O.K.)}$$

$$Q_3 = A_3 v_3 = [(\pi)(\tfrac{18}{12})^2/4](10.31) = 18.22 \text{ ft}^3/\text{s} \qquad 18.22 - Q_1 = 20$$

$$Q_1 = -1.78 \text{ ft}^3/\text{s} \qquad \text{(i.e., flow is into reservoir } A\text{)}$$

$$v_1 = 1.78/[(\pi)(\tfrac{36}{12})^2/4] = 0.2518 \text{ ft/s}$$

$$(N_R)_1 = (\tfrac{36}{12})(0.2518)/(1.05 \times 10^{-5}) = 7.19 \times 10^4$$

$$(\epsilon/d)_1 = 0.000333 \qquad \text{(from Prob. 12.13)} \qquad f_1 = 0.0208$$

$$(h_f)_1 = 0.0208[5000/(\tfrac{36}{12})]\{0.2518^2/[(2)(32.2)]\} = 0.03 \text{ ft}$$

$$z_B = z_A + (h_f)_1 + (h_f)_2 = 300 + 0.03 + 7.93 = 308 \text{ ft}$$

12.16 In Fig. 12-12, assume that 1, 2, and 3 represent 900 m of 60-cm, 300 m of 45-cm, and 1200 m of 40-cm, respectively, of new welded steel pipe. The surface elevations of A, B, and C are 32 m, 20 m, and 2 m, respectively. Estimate the water flows in all pipes.

▮

$$(\epsilon/d)_1 = 0.000046/(0.60) = 0.0000767 \qquad (\epsilon/d)_2 = 0.000046/(0.45) = 0.0001022$$

$$(\epsilon/d)_3 = 0.000046/(0.40) = 0.0001150$$

Assuming complete turbulence (high Reynolds numbers), friction factors for these values of (ϵ/d) will be $f_1 = 0.0115$, $f_2 = 0.0122$, and $f_3 = 0.0126$.

$$h_f = (f)(L/d)(v^2/2g) = (f)(L/d^5)(8Q^2/g\pi^2)$$

$$(h_f)_1 = 0.0115[900/(0.60)^5]\{8Q_1^2/[(9.807)(\pi)^2]\} = 11.00Q_1^2$$

$$(h_f)_2 = 0.0122[300/(0.45)^5]\{8Q_2^2/[(9.807)(\pi)^2]\} = 16.39Q_2^2$$

$$(h_f)_3 = 0.0126[1200/(0.40)^5]\{8Q_3^2/[(9.807)(\pi)^2]\} = 122.0Q_3^2$$

$$32 - 11.00Q_1^2 = h_J \qquad (1)$$

$$h_J - 16.39Q_2^2 = 20 \qquad \text{or} \qquad 20 - 16.39Q_2^2 = h_J \tag{2}$$

$$h_J - 122.0Q_3^2 = 2 \tag{3}$$

$$Q_1 = Q_2 + Q_3 \qquad \text{or} \qquad Q_1 + Q_2 = Q_3 \tag{4}$$

Assume no flow in pipe 2, in which case $h_J = 20$ m. $32 - 11.00Q_1^2 = 20$, $Q_1^2 = 1.091$; $20 - 122.0Q_3^2 = 2$, $Q_3^2 = 0.1475$. Since $Q_1 > Q_3$, flow must be into reservoir B, and $h_J > 20$ m. By trial and error, $h_J = 23.5$ m. $32 - 11.00Q_1^2 = 23.5$, $Q_1 = 0.879\ \text{m}^3/\text{s}$; $23.5 - 16.39Q_2^2 = 20$, $Q_2 = 0.462\ \text{m}^3/\text{s}$; $23.5 - 122.0Q_3^2 = 2$, $Q_3 = 0.420\ \text{m}^3/\text{s}$. Check Eq. (4): $0.879 = 0.462 + 0.420$, $0.879 = 0.882$ (close enough). Check the Reynolds numbers for normal temperature:

$$N_R = dv/\nu \qquad v_1 = 0.879/[(\pi)(0.60)^2/4] = 3.109\ \text{m/s}$$

$$(N_R)_1 = (0.60)(3.109)/(1.02 \times 10^{-6}) = 1.83 \times 10^6$$

$$v_2 = 0.462/[(\pi)(0.45)^2/4] = 2.905\ \text{m/s}$$

$$(N_R)_2 = (0.45)(2.905)/(1.02 \times 10^{-6}) = 1.28 \times 10^6$$

$$v_3 = 0.420/[(\pi)(0.40)^2/4] = 3.342\ \text{m/s}$$

$$(N_R)_1 = (0.40)(3.342)/(1.02 \times 10^{-6}) = 1.31 \times 10^6$$

From Fig. A-5, $f = 0.0135$ approximately for all pipes; hence, the above flows should be multiplied by $(0.012/0.0135)$, giving $Q_1 = (0.012/0.0135)(0.879) = 0.781\ \text{m}^3/\text{s}$, $Q_2 = (0.012/0.0135)(0.462) = 0.411\ \text{m}^3/\text{s}$, $Q_3 = (0.012/0.0135)(0.420) = 0.373\ \text{m}^3/\text{s}$. Further refinement of these approximations is not justified.

12.17 Suppose that, in Fig. 12-12, pipe 1 is 1500 ft of 12-in new cast iron, pipe 2 is 800 ft of 6-in wrought iron, and pipe 3 is 1200 ft of 8-in wrought iron. The water surface of reservoir B is 20 ft below that of A, while junction J is 40 ft below the surface of A. When the pressure head at J is 30 ft, find the flow in each pipe.

▮ $(h_f)_1 = 40 - 30 = 10$ ft $\qquad (h_f)_2 = (40 - 20) - 30 = -10$ ft $\qquad h_f = (f)(L/d)(v^2/2g)$

Assume $f_1 = 0.019$:

$$10 = 0.019[1500/(\tfrac{12}{12})]\{v_1^2/[(2)(32.2)]\} \qquad v_1 = 4.754\ \text{ft/s} \qquad N_R = dv/\nu$$

$$(N_R)_1 = (\tfrac{12}{12})(4.754)/(1.05 \times 10^{-5}) = 4.53 \times 10^5 \qquad (\epsilon/d)_1 = 0.00085/(\tfrac{12}{12}) = 0.00085$$

From Fig. A-5, $f_1 = 0.0198$. Try $f_1 = 0.0198$:

$$10 = 0.0198[1500/(\tfrac{12}{12})]\{v_1^2/[(2)(32.2)]\} \qquad v_1 = 4.657\ \text{ft/s}$$

$$(N_R)_1 = (\tfrac{12}{12})(4.657)/(1.05 \times 10^{-5}) = 4.44 \times 10^5 \qquad f_1 = 0.0198 \qquad \text{(O.K.)}$$

$$Q_1 = A_1 v_1 = [(\pi)(\tfrac{12}{12})^2/4](4.657) = 3.658\ \text{ft}^3/\text{s}$$

Assume $f_2 = 0.015$:

$$10 = 0.015[800/(\tfrac{6}{12})]\{v_2^2/[(2)(32.2)]\} \qquad v_2 = 5.180\ \text{ft/s}$$

$$(N_R)_2 = (\tfrac{6}{12})(5.180)/(1.05 \times 10^{-5}) = 2.47 \times 10^5 \qquad (\epsilon/d)_2 = 0.00015/(\tfrac{6}{12}) = 0.00030 \qquad f = 0.0175$$

Try $f_2 = 0.0175$:

$$10 = 0.0175[800/(\tfrac{6}{12})]\{v_2^2/[(2)(32.2)]\} \qquad v_2 = 4.796\ \text{ft/s}$$

$$(N_R)_2 = (\tfrac{6}{12})(4.796)/(1.05 \times 10^{-5}) = 2.28 \times 10^5 \qquad f_2 = 0.0175 \qquad \text{(O.K.)}$$

$$Q_2 = [(\pi)(\tfrac{6}{12})^2/4](4.796) = 0.942\ \text{ft}^3/\text{s} \qquad Q_3 = Q_1 - Q_2 = 3.658 - 0.942 = 2.716\ \text{ft}^3/\text{s}$$

12.18 In the reservoir system of Fig. 12-13, $z_A = 210$ ft, $z_B = 240$ ft, $z_C = 130$ ft, $z_D = 150$ ft, $BD = 3000$ ft of 4-in cast iron pipe, $AD = 2000$ ft of 1-in steel pipe, and $DC = 500$ ft of 6-in cast iron pipe. Using $f = 0.025$ and neglecting minor losses, determine the flow in each pipe.

▮ $h_f = (f)(L/d)(v^2/2g)$. Let $p_D/\gamma = -15$ ft:

$$(h_f)_{AD} = 210 - 150 - (-15) = 0.025[2000/(\tfrac{1}{12})]\{v_{AD}^2/[(2)(32.2)]\} \qquad v_{AD} = 2.837\ \text{ft/s}$$

$$(h_f)_{BD} = 240 - 150 - (-15) = 0.025[3000/(\tfrac{4}{12})]\{v_{BD}^2/[(2)(32.2)]\} \qquad v_{BD} = 5.482\ \text{ft/s}$$

$$(h_f)_{CD} = 130 - 150 - (-15) = 0.025[500/(\tfrac{6}{12})]\{v_{CD}^2/[(2)(32.2)]\} \qquad v_{CD} = 3.589\ \text{ft/s} \qquad \text{(away from } D\text{)}$$

If these velocities are correct, $Q_{AD} + Q_{BD}$ must equal Q_{CD}: $Q_{AD} + Q_{BD} = [(\pi)(\tfrac{1}{12})^2/4](2.837) + [(\pi)(\tfrac{4}{12})^2/4](5.482) = 0.4939\ \text{ft}^3/\text{s}$, $Q_{CD} = [(\pi)(\tfrac{6}{12})^2/4](3.589) = 0.7047\ \text{ft}^3/\text{s}$. Since they are unequal, try

$p_D/\gamma = -17$ ft:

$$(h_f)_{AD} = 210 - 150 - (-17) = 0.025[2000/(\tfrac{1}{12})]\{v_{AD}^2/[(2)(32.2)]\} \qquad v_{AD} = 2.875 \text{ ft/s}$$
$$(h_f)_{BD} = 240 - 150 - (-17) = 0.025[3000/(\tfrac{4}{12})]\{v_{BD}^2/[(2)(32.2)]\} \qquad v_{BD} = 5.534 \text{ ft/s}$$
$$(h_f)_{CD} = 130 - 150 - (-17) = 0.025[500/(\tfrac{6}{12})]\{v_{CD}^2/[(2)(32.2)]\} \qquad v_{CD} = 2.789 \text{ ft/s} \qquad \text{(away from } D\text{)}$$
$$Q_{AD} + Q_{BD} = [(\pi)(\tfrac{1}{12})^2/4](2.875) + [(\pi)(\tfrac{4}{12})^2/4](5.534) = 0.4986 \text{ ft}^3/\text{s}$$
$$Q_{CD} = [(\pi)(\tfrac{6}{12})^2/4](2.789) = 0.5476 \text{ ft}^3/\text{s}$$

$Q_{AD} + Q_{BD}$ is still not equal to Q_{CD}; try $p_D/\gamma = -17.4$ ft:

$$(h_f)_{AD} = 210 - 150 - (-17.5) = 0.025[2000/(\tfrac{1}{12})]\{v_{AD}^2/[(2)(32.2)]\} \qquad v_{AD} = 2.884 \text{ ft/s}$$
$$(h_f)_{BD} = 240 - 150 - (-17.5) = 0.025[3000/(\tfrac{4}{12})]\{v_{BD}^2/[(2)(32.2)]\} \qquad v_{BD} = 5.547 \text{ ft/s}$$
$$(h_f)_{CD} = 130 - 150 - (-17.5) = 0.025[500/(\tfrac{6}{12})]\{v_{CD}^2/[(2)(32.2)]\} \qquad v_{CD} = 2.538 \text{ ft/s} \qquad \text{(away from } D\text{)}$$
$$Q_{AD} + Q_{BD} = [(\pi)(\tfrac{1}{12})^2/4](2.884) + [(\pi)(\tfrac{4}{12})^2/4](5.547) = 0.4998 \text{ ft}^3/\text{s}$$
$$Q_{CD} = [(\pi)(\tfrac{6}{12})^2/4](2.538) = 0.4983 \text{ ft}^3/\text{s}$$

Since $Q_{AD} + Q_{BD}$ is close enough to Q_{CD}, take

$$Q_{AD} = [(\pi)(\tfrac{1}{12})^2/4](2.884) = 0.015 \text{ ft}^3/\text{s} \qquad \text{(toward } D\text{)}$$
$$Q_{BD} = [(\pi)(\tfrac{4}{12})^2/4](5.547) = 0.483 \text{ ft}^3/\text{s} \qquad \text{(toward } D\text{)}$$
$$Q_{CD} = 0.498 \text{ ft}^3/\text{s} \qquad \text{(away from } D\text{)}$$

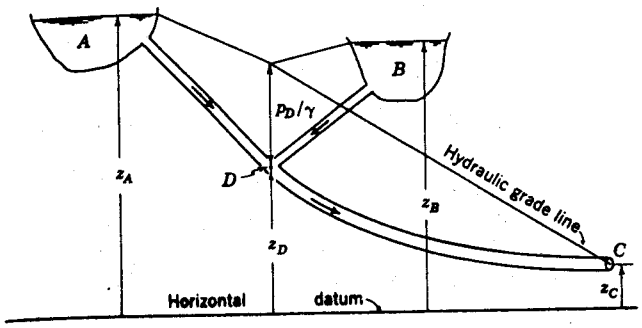


Fig. 12-13

12.19 Three reservoirs A, B, and C whose water-surface elevations are $z_A = 60$ ft, $z_B = 50$ ft, and $z_C = 32$ ft, are interconnected by a pipe system with a common junction D, $z_D = 35$ ft. The pipes are as follows: from A to junction, $L = 800$ ft, $d = 3$ in; from B to junction, $L = 500$ ft, $d = 10$ in; from C to junction, $L = 1000$ ft, $d = 4$ in. Assume $f = 0.02$ for all pipes and neglect minor losses and velocity heads. Determine whether the flow is into or out of reservoir B.

▌ Assume no flow between B and D.

$$h_f = (f)(L/d)(v^2/2g) \qquad (h_f)_A = 0.02[800/(\tfrac{3}{12})]\{v_A^2/[(2)(32.2)]\} = 0.9938v_A^2$$
$$(h_f)_C = 0.02[1000/(\tfrac{4}{12})]\{v_C^2/[(2)(32.2)]\} = 0.9317v_C^2$$
$$A_A v_A = A_C v_C \qquad [(\pi)(\tfrac{3}{12})^2/4](v_A) = [(\pi)(\tfrac{4}{12})^2/4](v_C)$$
$$v_A = 1.778v_C \qquad (h_f)_A/(h_f)_C = 0.9938v_A^2/0.9317v_C^2 = (0.9938)(1.778v_C)^2/0.9317v_C^2 = 3.372$$

Hence, $(h_f)_A = [3.372/(3.372 + 1)](60 - 32) = 21.60$ ft, and $z_D = 60 - 21.60 = 38.40$ ft. Since our no-flow assumption makes D too high (by 3.40 ft) relative to B, the flow must be out of B into D.

CHAPTER 13
Pipe Networks

13.1 Develop the expression utilized to study flows in a pipe network (Hardy Cross method).

I The method of attack, developed by Professor Hardy Cross, consists of assuming flows throughout the network, and then balancing the calculated head losses (LH). In the simple looping pipe system shown in Fig. 13-1, for the correct flow in each branch of the loop,

$$\mathrm{LH}_{ABC} = \mathrm{LH}_{ADC} \qquad \text{or} \qquad \mathrm{LH}_{ABC} - \mathrm{LH}_{ADC} = 0 \tag{1}$$

In order to utilize this relationship, the flow formula to be used must be written in the form $\mathrm{LH} = kQ^n$. For the Hazen–Williams formula, this expression is $\mathrm{LH} = kQ^{1.85}$.

But, since we are assuming flows Q_o, the correct flow Q in any pipe of a network can be expressed as $Q = Q_o + \Delta$, where Δ is the correction to be applied to Q_o. Then, using the binomial theorem, $kQ^{1.85} = k(Q_o + \Delta)^{1.85} = k(Q_o^{1.85} + 1.85Q_o^{1.85-1}\,\Delta + \cdots)$. Terms beyond the second can be neglected since Δ is small compared with Q_o.

For the loop above, substituting in expression (*1*) we obtain $k(Q_o^{1.85} + 1.85Q_o^{0.85}\,\Delta) - k(Q_{o'}^{1.85} + 1.85Q_{o'}^{0.85}\,\Delta) = 0$, $k(Q_o^{1.85} - Q_{o'}^{1.85}) + 1.85k(Q_o^{0.85} - Q_{o'}^{0.85})\,\Delta = 0$. Solving for Δ,

$$\Delta = -k(Q_o^{1.85} - Q_{o'}^{1.85})/[1.85k(Q_o^{0.85} - Q_{o'}^{0.85})] \tag{2}$$

In general, we may write for a more complicated loop,

$$\Delta = -\sum kQ_o^{1.85}/(1.85\sum kQ_o^{0.85}) \tag{3}$$

But $kQ_o^{1.85} = \mathrm{LH}$ and $kQ_o^{0.85} = (\mathrm{LH})/Q_o$. Therefore

$$\Delta = -\sum(\mathrm{LH})/[1.85\sum(\mathrm{LH}/Q_o)] \qquad \text{for each loop of a network} \tag{4}$$

In utilizing expression (*4*), care must be exercised regarding the sign of the numerator. Expression (*1*) indicates that clockwise flows may be considered as producing clockwise losses, and counterclockwise flows, counterclockwise losses. This means that the minus sign is assigned to all counterclockwise conditions in a loop, namely flow Q and lost head LH. Hence to avoid mistakes, this sign notation must be observed in carrying out a solution. On the other hand, the denominator of (*4*) is always positive.

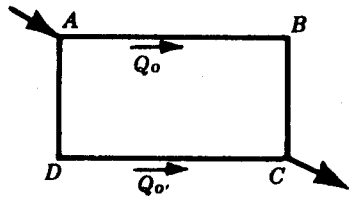


Fig. 13-1

13.2 In Fig. 13-2, for $Q = 11.7$ mgd total, how much flow occurs in each branch of the loop, using the Hardy Cross method?

I Values of Q_{12} and Q_{16} are assumed to be 4.0 mgd and 7.7 mgd, respectively. The tabulation below is prepared, (note the −7.70 mgd), the values of S calculated by Fig. A-17, then $\mathrm{LH} = S \times L$, and LH/Q_o can be calculated. Note that the large $\sum$ LH indicates that the Q's are not well-balanced. (The values were assumed deliberately to produce this large $\sum$ LH, to illustrate the procedure.)

D	L	assumed Q_o mgd	S, ft/1000 ft	LH, ft	LH/Q_o	Δ	Q_1
12 in	5000 ft	4.00	19.5	97.5	24.4	−0.85	3.15
16 in	3000 ft	−7.70	−16.3	−48.9	6.4	−0.85	−8.55
		$\lvert\Sigma\rvert = 11.70$		$\Sigma = +48.6$	30.8		11.70

$$\Delta = -\sum \mathrm{LH}/[1.85\sum(\mathrm{LH}/Q)] = -(+48.6)/[1.85(30.8)] = -0.85 \text{ mgd}$$

Then the Q_1 values become $(4.00 - 0.85) = 3.15$ mgd and $(-7.70 - 0.85) = -8.55$ mgd. Repeating the calculation produces

S	LH	LH/Q_1	Δ	Q_2
12.5	62.5	19.80	−0.06	3.09
−19.8	−59.4	6.95	−0.06	−8.61
	$\Sigma = +3.1$	26.75		11.70

No further calculation is necessary, since the slide rule or chart cannot be read to the accuracy of 0.06 mgd. Ideally, Σ LH should equal zero, but this goal is seldom attained.

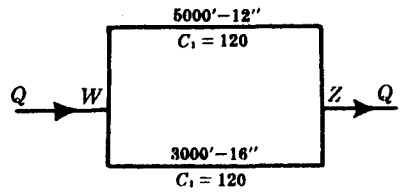


Fig. 13-2

13.3 For the pipe network giving pipe diameters and lengths and external flows entering and leaving the network as shown in Fig. 13-3*a*, find the flow rate in each pipe in the network.

▮ The first iteration in carrying out the Hardy Cross analysis is given in the table below. A detailed explanation corresponding to steps in carrying out the procedure is given after the table.

Iteration No. 1

Loop I

(1) pipe no.	(2) length, ft	(3) diameter, in	(4) flow rate (Q), ft³/s	(5) unit head loss, ft/ft	(6) head loss (h_f), ft	(7) h_f/Q s/ft²
given	given	given	estimated or assumed	from pipe diagram	(2) × (5)	(6)/(4)
1	2000	18	7.00	0.0034	6.800	0.971
2	900	24	3.50	0.000235	0.212	0.061
3	2800	12	−2.19	−0.0029	−8.120	3.708
4	1100	24	−7.00	−0.00085	−0.935	0.134
					−2.043	4.874

Loop II

(1) pipe no.	(2) length, ft	(3) diameter, in	(4) flow rate (Q), ft³/s	(5) unit head loss, ft/ft	(6) head loss (h_f), ft	(7) h_f/Q, s/ft²
given	given	given	estimated or assumed	from pipe diagram	(2) × (5)	(6)/(4)
5	2200	12	3.50	0.0070	15.400	4.400
6	750	18	−3.14	−0.00078	−0.585	0.186
7	2600	24	−5.69	−0.00057	−1.482	0.260
2	900	24	−3.50	−0.00023	−0.207	0.059
					13.126	4.905

Step 1: Columns (1), (2), and (3) in the table above record given data. Flow rates recorded in column (4) are the initial flow rate estimates. These are listed on the sketch of Fig. 13-3*b*, but they are not "given" data. The flow rate of 14.00 cfs entering joint *A* was estimated to separate such that 7.00 cfs goes through pipe 1 and an equal amount goes through pipe 4. At joint *F*, since 7.00 cfs enters the joint from pipe 4 and 4.81 cfs leaves the joint externally, the flow rate in pipe 3 has to be 2.19 cfs (that is, 7.00 cfs − 4.81 cfs) in the direction away from joint *F* in order to satisfy the principle that the total flow entering a joint must be equal to the total flow leaving that joint. Remaining flow rates were estimated in a similar manner, making sure that for each joint the total flow entering equaled the total flow leaving. It should be noted that clockwise flows in each loop (such as in pipes 1 and 5) are indicated as positive, while counterclockwise flows (such as in pipes 3 and 6) are indicated as negative. It should particularly be noted that the flow in pipe 2 is clockwise with respect to loop I but counterclockwise with respect to loop II; hence, it is indicated as positive when listed in loop I and negative when listed in loop II.

Step 2: Unit head losses in column (5) are determined from Fig. A-13, based on diameters [column (3)] and flow rates [column (4)]. For example, pipe 1 has a diameter of 18 in and flow rate of 7.00 cfs; hence, the unit head loss is determined from Fig. A-13 to be 0.0034 ft/ft. Head losses in column (6) are determined by multiplying pipe lengths [column (2)] by unit head losses [column (5)]. For example, pipe 1 has a length of 2000 ft and unit head loss of 0.0034 ft/ft; hence, the head loss is (2000 ft)(0.0034 ft/ft), or 6.800 ft. It should be noted that head losses are positive if their corresponding flow rates are positive and negative if their corresponding flow rates are negative. The h_f/Q fractions in column (7) are determined by dividing head losses [column (6)] by flow rates [column (4)]. For example, pipe 1 has a head loss of 6.800 ft and a flow rate of 7.00 cfs; hence, h_f/Q is 6.800/7.00, or 0.971 s/ft^2.

Step 3: Algebraic sums of head losses are observed from the table to be −2.043 ft in loop I and 13.126 ft in loop II. Since these are not zero, the original estimated flows are not correct.

Step 4: A flow rate correction (ΔQ) can be computed for each loop: $\Delta Q = -\Sigma\, h_f/[n\, \Sigma\, (h_f/Q)]$. (Since the Hazen–Williams formula is being used, $n = 1.85$.) $(\Delta Q)_{\text{loop I}} = -(-2.043)/[(1.85)(4.874)] = 0.23$ cfs, $(\Delta Q)_{\text{loop II}} = -(13.126)/[(1.85)(4.905)] = -1.45$ cfs.

Step 5: Adjusted flow rates for each pipe are determined by adding flow rate corrections to the previous rate for each pipe. These are as follows:

Loop I

pipe	old Q, cfs	ΔQ, cfs	new Q, cfs
1	7.00	+0.23	7.23
2	3.50	+0.23 + 1.45	5.18
3	−2.19	+0.23	−1.96
4	−7.00	+0.23	−6.77

Loop II

pipe	old Q, cfs	ΔQ, cfs	new Q, cfs
5	3.50	−1.45	2.05
6	−3.14	−1.45	−4.59
7	−5.69	−1.45	−7.14
2	−3.50	−1.45 − 0.23	−5.18

These adjusted flow rates are shown in Fig. 13-3*c*. It should be noted that the flow rate in pipe 2 was adjusted using flow rate corrections for both loops, since this pipe is common to both. It should be particularly noted that the sign of the flow rate correction for loop II was reversed when it was applied in loop I (and vice versa).

Step 6: Return to step 2 and repeat the entire procedure using adjusted flow rates. This is given in the table below.

Iteration No. 2

Loop I

(1) pipe no.	(2) length, ft	(3) diameter, in	(4) flow rate (Q), ft^3/s	(5) unit head loss, ft/ft	(6) head loss (h_f), ft	(7) h_f/Q, s/ft^2
given	given	given	estimate or assumed	from pipe diagram	(2) × (5)	(6)/(4)
1	2000	18	7.23	0.0036	7.200	0.996
2	900	24	5.18	0.00047	0.423	0.082
3	2800	12	−1.96	−0.0024	−6.720	3.429
4	1100	24	−6.77	−0.00080	−0.880	0.130
					0.023	4.637

Loop II

(1) pipe no.	(2) length, ft	(3) diameter, in	(4) flow rate (Q), ft^3/s	(5) unit head loss, ft/ft	(6) head loss (h_f), ft	(7) h_f/Q s/ft
given	given	given	estimate or assumed	from pipe diagram	(2) × (5)	(6)/(4)
5	2200	12	2.05	0.0026	5.720	2.790
6	750	18	−4.59	−0.00156	−1.170	0.255
7	2600	24	−7.14	−0.00088	−2.288	0.320
2	900	24	−5.18	−0.00047	−0.423	0.082
					1.839	3.447

Since the algebraic sums of head losses are not both zero, new flow rate corrections must be computed. $(\Delta Q)_{\text{loop I}} = -(0.023)/[(1.85)(4.637)] = 0.00$ cfs, $(\Delta Q)_{\text{loop II}} = -(1.839)/[(1.85)(3.447)] = -0.29$ cfs. Revised flow rates for each pipe are determined using these flow rate corrections. These are as follows:

Loop I

pipe	old Q, cfs	ΔQ, cfs	new Q, cfs
1	7.23	0.00	7.23
2	5.18	0.00 + 0.29	5.47
3	−1.96	0.00	−1.96
4	−6.77	0.00	−6.77

Loop II

pipe	old Q, cfs	ΔQ, cfs	new Q, cfs
5	2.05	−0.29	1.76
6	−4.59	−0.29	−4.88
7	−7.14	−0.29	−7.43
2	−5.18	−0.29	−5.47

These adjusted flow rates are shown in Fig. 13-3*d*. Return to step 2 and repeat the entire procedure using these revised flow rates. This is given in the table below.

Iteration No. 3

Loop I

(1) pipe no.	(2) length, ft	(3) diameter, in	(4) flow rate (Q), ft³/s	(5) unit head loss, ft/ft	(6) head loss (h_f), ft	(7) h_f/Q, s/ft²
given	given	given	estimate or assumed	from pipe diagram	(2) × (5)	(6)/(4)
1	2000	18	7.23	0.0036	7.200	0.996
2	900	24	5.47	0.00053	0.477	0.087
3	2800	12	−1.96	−0.0024	−6.720	3.429
4	1100	24	−6.77	−0.00080	−0.880	0.130
					0.777	4.642

Loop II

(1) pipe no.	(2) length, ft	(3) diameter, in	(4) flow rate (Q), ft³/s	(5) unit head loss, ft/ft	(6) head loss (h_f), ft	(7) h_f/Q, s/ft²
given	given	given	estimate or assumed	from pipe diagram	(2) × (5)	(6)/(4)
5	2200	12	1.76	0.0019	4.180	2.375
6	750	18	−4.88	−0.00175	−1.312	0.269
7	2600	24	−7.43	−0.00094	−2.444	0.329
2	900	24	−5.47	−0.00053	−0.477	0.087
					−0.053	3.060

Since the algebraic sums of head losses are not both zero, new flow rate corrections must be computed. $(\Delta Q)_{\text{loop I}} = -(0.077)/[(1.85)(4.642)] = -0.01$ cfs, $(\Delta Q)_{\text{loop II}} = -(-0.053)/[(1.85)(3.060)] = 0.01$ cfs. Revised flow rates for each pipe are determined using these flow rate corrections. These are as follows:

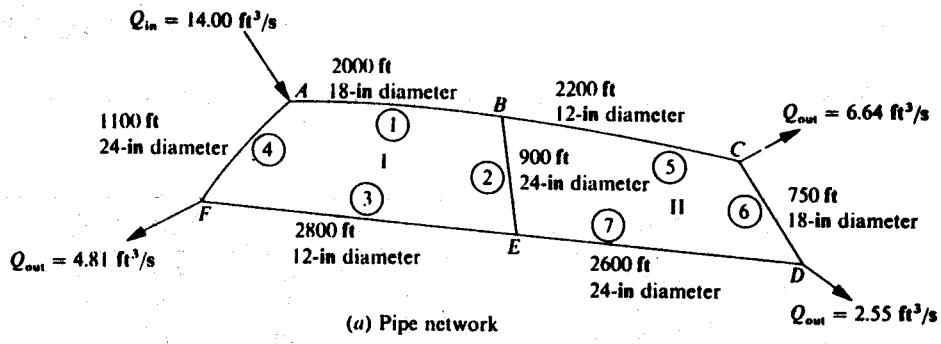
Loop I

pipe	old Q, cfs	ΔQ, cfs	new Q, cfs
1	7.23	−0.01	7.22
2	5.47	−0.01 − 0.01	5.45
3	−1.96	−0.01	−1.97
4	−6.77	−0.01	−6.78

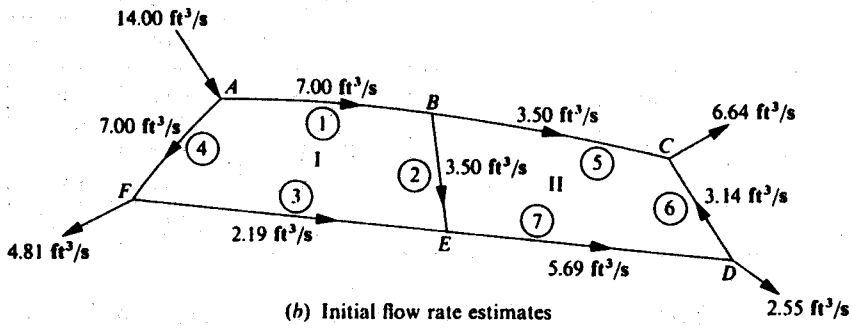
Loop II

pipe	old Q, cfs	ΔQ, cfs	new Q, cfs
5	1.76	+0.01	1.77
6	−4.88	+0.01	−4.87
7	−7.43	+0.01	−7.42
2	−5.47	+0.01 + 0.01	−5.45

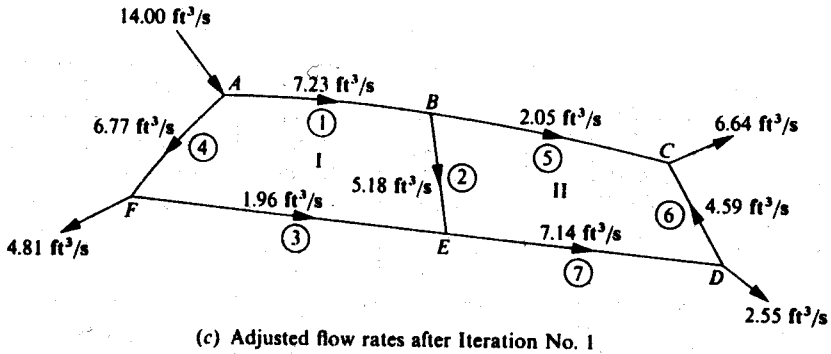
These adjusted flow rates are shown in Fig. 13-3*e*. It would be appropriate to return to step 2 and repeat the entire procedure using these revised flow rates. However, an additional iteration (not shown) indicates that the next flow rate corrections would be no greater than 0.01 cfs, and further computation would appear to be wasted effort. Hence, the "new Q" values just before this paragraph are taken to be the correct flow rates for these pipes.



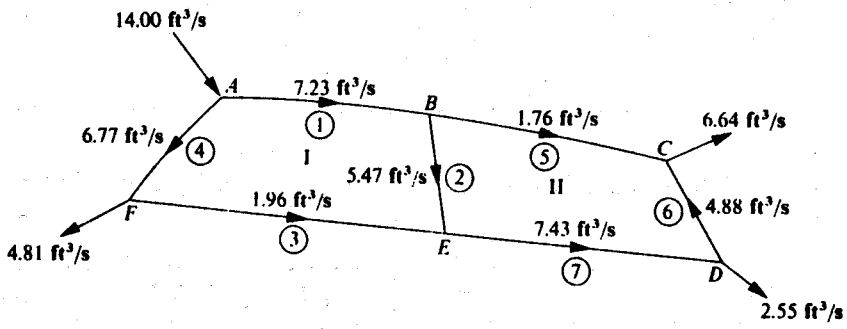
(*a*) Pipe network



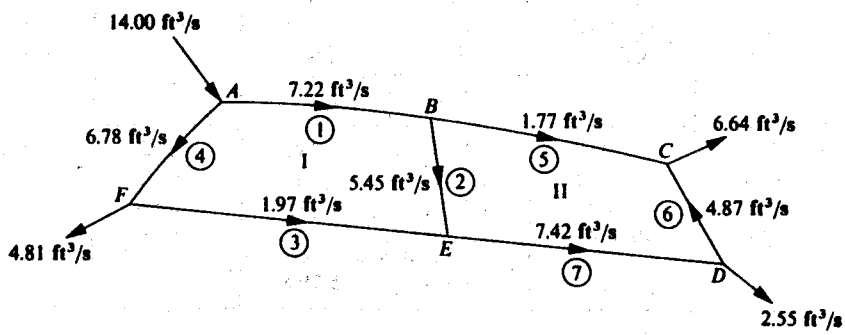
(*b*) Initial flow rate estimates



(*c*) Adjusted flow rates after Iteration No. 1



(*d*) Adjusted flow rates after Iteration No. 2



(*e*) Adjusted flow rates after Iteration No. 3

Fig. 13-3

13.4 For the pipe network shown in Fig. 13-4, find the rate of flow of water in each pipe. Assume $C = 120$ for all pipes.

▮

pipe	D, in	L, ft	$Q_{assumed}$, cfs	h_1, ft/ft	h_f, ft	h_f/Q	ΔQ	Q_{new}
AB	18	1500	8.0	0.0044	6.60	0.825	−0.11	7.89
BC	12	500	3.0	0.0053	2.65	0.883	−0.11	2.89
CF	15	1500	−4.0	−0.00295	−4.42	1.105	2.23	−1.77
FA	15	500	−7.0	−0.0083	−4.15	0.593	−0.11	−7.11
					0.68	3.406		
FC	15	1500	4.0	0.00295	4.42	1.105	−2.23	1.77
CD	10	500	7.0	0.059	29.50	4.214	−2.34	4.66
DE	12	1500	2.0	0.0025	3.75	1.875	−2.34	−0.34
EF	12	500	−3.0	−0.0053	−2.65	0.883	−2.34	−5.34
					35.02	8.077		

$\Delta Q_{\mathrm{I}} = -\sum h_f/[n \sum (h_f/Q)] = -0.68/[(1.85)(3.406)] = -0.11$ cfs $\qquad \Delta Q_{\mathrm{II}} = -35.02/[(1.85)(8.077)] = -2.34$ cfs

pipe	h_1	h_f	h_f/Q	ΔQ	Q_{new}
AB	0.0042	6.30	0.798	−0.66	7.23
BC	0.0049	2.45	0.848	−0.66	2.23
CF	0.00067	−1.00	0.565	0.08	−1.69
FA	0.0086	−4.30	0.605	−0.66	−7.77
		3.45	2.816		
FC	0.00067	1.00	0.565	−0.08	1.69
CD	0.028	14.00	3.000	−0.74	3.92
DE	0.000095	−0.14	0.412	−0.74	−1.08
EF	0.015	−7.50	1.404	−0.74	−6.08
		7.36	5.381		

$\Delta Q_{\mathrm{I}} = -3.45/[(1.85)(2.816)] = -0.66$ cfs $\qquad \Delta Q_{\mathrm{II}} = -7.36/[(1.85)(5.381)] = -0.74$ cfs

pipe	h_1	h_f	h_f/Q	ΔQ	Q_{new}
AB	0.0036	5.40	0.747	−0.20	7.03
BC	0.00295	1.48	0.664	−0.20	2.03
CF	−0.00060	−0.90	0.533	−0.20	−1.89
FA	−0.0100	−5.00	0.644	−0.20	−7.97
		0.98	2.588		
FC	0.00060	0.90	0.533	0.20	1.89
CD	0.0200	10.00	2.551	0.00	3.92
DE	−0.00080	−1.20	1.111	0.00	−1.08
EF	−0.0195	−9.75	1.604	0.00	−6.08
		−0.05	5.799		

$\Delta Q_{\mathrm{I}} = -0.98/[(1.85)(2.588)] = -0.20$ cfs $\qquad \Delta Q_{\mathrm{II}} = -(-0.05)/[(1.85)(5.799)] = 0.00$ cfs

pipe	h_1	h_f	h_f/Q	ΔQ	Q_{new}
AB	0.0034	5.10	0.725	0.02	7.05
BC	0.0026	1.30	0.640	0.02	2.05
CF	−0.00075	−1.11	0.587	0.03	−1.86
FA	−0.0108	−5.40	0.678	0.02	−7.95
		−0.11	2.630		
FC	0.00074	1.11	0.587	−0.03	1.86
CD	0.0200	10.00	2.551	−0.01	3.91
DE	−0.00080	−1.20	1.111	−0.01	−1.09
EF	−0.0195	−9.75	1.604	−0.01	−6.09
		0.16	5.853		

$\Delta Q_{\mathrm{I}} = -(-0.11)/[(1.85)(2.630)] = 0.02$ cfs $\qquad \Delta Q_{\mathrm{II}} = -0.16/[(1.85)(5.853)] = -0.01$ cfs

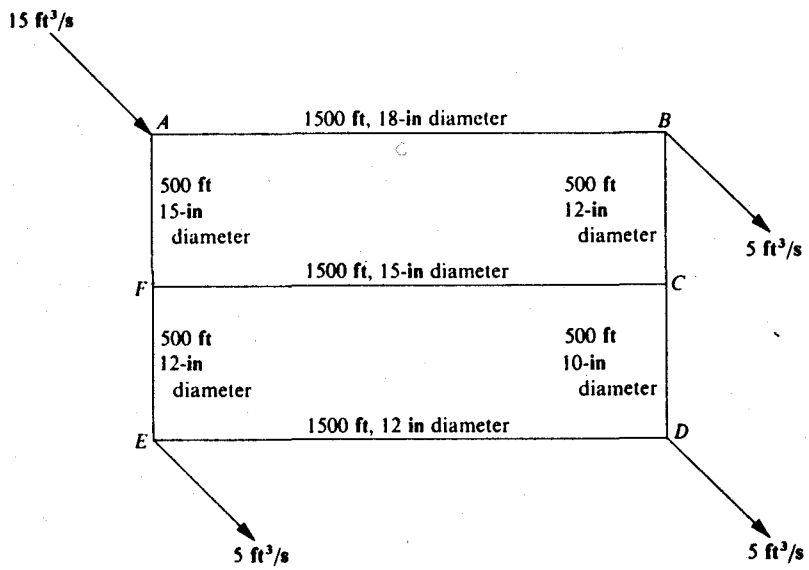


Fig. 13-4

13.5 The pipe network shown in Fig. 13-5 represents a spray rinse system. Find the flow rate of water in each pipe. Assume $C = 120$ for all pipes.

I

pipe	D, mm	L, m	$Q_{assumed}$, m³/s	h_1, m/m	h_f, m	h_f/Q	ΔQ	Q_{new}
AB	300	600	0.200	0.027	16.20	81.0	0.011	0.211
BG	250	400	0.100	0.0175	7.00	70.0	−0.003	0.097
GH	300	600	−0.100	−0.0074	−4.44	44.4	0.011	−0.089
HA	250	400	−0.200	−0.064	−25.60	128.0	0.011	−0.189
					−6.84	323.4		
BC	300	600	0.100	0.0074	4.44	44.4	0.014	0.114
CF	250	400	0.050	0.0049	1.96	39.2	0.014	0.064
FG	300	600	−0.100	−0.0074	−4.44	44.4	0.014	−0.086
GB	250	400	−0.100	−0.0175	−7.00	70.0	0.003	−0.097
					−5.04	198.0		

pipe	D, mm	L, m	$Q_{assumed}$, m³/s	h_1, m/m	h_f, m	h_f/Q	ΔQ	Q_{new}
CD	300	600	0.050	0.0020	1.20	24.0	0.000	0.050
DE	250	400	0.050	0.0049	1.96	39.2	0.000	0.050
EF	300	600	−0.050	−0.0020	−1.20	24.0	0.000	−0.050
FC	250	400	−0.050	−0.0049	−1.96	39.2	−0.014	−0.064
					0.00	126.4		

$$\Delta Q_{\mathrm{I}} = -\sum h_f/[n \sum (h_f/Q)] = -(-6.84)/[(1.85)(323.4)] = 0.011$$

$$\Delta Q_{\mathrm{II}} = -(-5.04)/[(1.85)(198.0)] = 0.014 \qquad \Delta Q_{\mathrm{III}} = -0.00/[(1.85)(126.4)] = 0.00$$

pipe	h_1	h_f	h_f/Q	ΔQ	Q_{new}
AB	0.0295	17.70	83.9	0.004	0.215
BG	0.017	6.80	70.1	0.001	0.098
GH	−0.0059	−3.54	39.8	0.004	−0.085
HA	−0.058	−23.20	122.8	0.004	−0.185
		−2.24	316.6		
BC	0.0095	5.70	50.0	0.003	0.117
CF	0.0079	3.16	49.4	−0.002	0.062
FG	−0.0056	−3.36	39.1	0.003	−0.083
GB	−0.017	−6.80	70.1	−0.001	−0.098
		−1.30	208.6		
CD	0.0020	1.20	24.0	0.005	0.055
DE	0.0049	1.96	39.2	0.005	0.055
EF	−0.0020	−1.20	24.0	0.005	−0.045
FC	−0.0079	−3.16	49.4	0.002	−0.062
		−1.20	136.6		

$$\Delta Q_{\mathrm{I}} = -(-2.24)/[(1.85)(316.6)] = 0.004 \qquad \Delta Q_{\mathrm{II}} = -(-1.30)/[(1.85)(208.6)] = 0.003$$

$$\Delta Q_{\mathrm{III}} = -(-1.20)/[(1.85)(136.6)] = 0.005$$

pipe	h_1	h_f	h_f/Q	ΔQ	Q_{new}
AB	0.031	18.60	86.5	0.000	0.215
BG	0.0172	6.88	70.2	−0.003	0.095
GH	0.0055	−3.30	38.8	0.000	−0.085
HA	0.056	−22.40	121.1	0.000	−0.185
		−0.22	316.6		
BC	0.010	6.00	51.3	0.003	0.120
CF	0.0075	3.00	48.4	0.002	0.064
FG	0.0052	−3.12	37.6	0.003	−0.080
GB	0.0172	−6.88	70.2	0.003	−0.095
		−1.00	207.5		
CD	0.0024	1.44	26.2	0.001	0.056
DE	0.0059	2.36	42.9	0.001	0.056
EF	0.0017	−1.02	22.7	0.001	−0.044
FC	0.0075	−3.00	48.4	−0.002	−0.064
		−0.22	140.2		

$$\Delta Q_{\mathrm{I}} = -(-0.22)/[(1.85)(316.6)] = 0.000 \qquad \Delta Q_{\mathrm{II}} = -(-1.00)/[(1.85)(207.5)] = 0.003$$

$$\Delta Q_{\mathrm{III}} = -(-0.22)/[(1.85)(140.2)] = 0.001$$

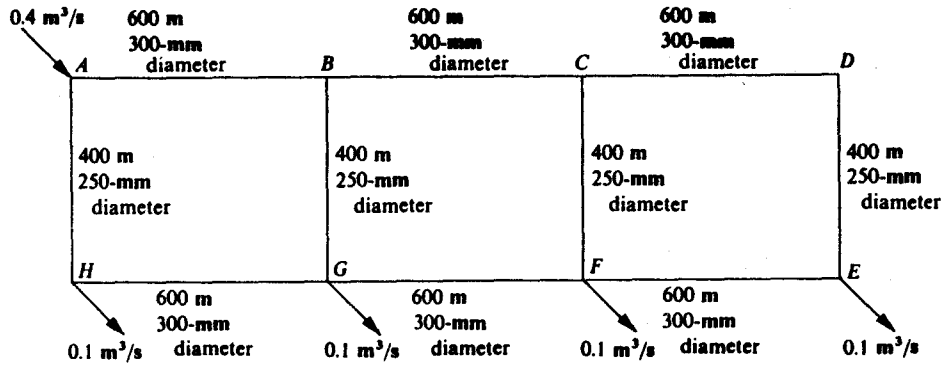


Fig. 13-5

13.6 The pipe network shown in Fig. 13-6 represents a small industrial park. Find the flow rate of water in each pipe. Assume $C = 120$ for all pipes.

pipe	D, mm	L, m	$Q_{assumed}$, m³/s	h_1, m/m	h_f, m	h_f/Q	ΔQ	Q_{new}
AB	500	1000	0.250	0.0034	3.40	13.6	0.028	0.278
BE	400	1200	0.120	0.0026	3.12	26.0	−0.005	0.115
EF	300	1000	−0.130	−0.012	−12.00	92.3	0.063	−0.067
FA	600	1200	−0.250	−0.0014	−1.68	6.7	0.028	−0.222
					−7.16	138.6		
BC	500	1000	0.130	0.00102	1.02	7.8	0.033	0.163
CD	400	1200	0.030	0.00020	0.24	8.0	0.033	0.063
DE	400	1000	−0.100	−0.0018	−1.8	18.0	0.050	−0.050
EB	400	1200	−0.120	−0.0026	−3.12	26.0	0.005	−0.115
					−3.66	59.8		
ED	400	1000	0.100	0.0018	1.80	18.0	−0.050	0.050
DI	300	1200	0.080	0.0048	5.76	72.0	−0.017	0.063
IH	300	1000	−0.020	−0.00037	−0.37	18.5	−0.017	−0.037
HE	300	1200	−0.050	−0.0020	−2.40	48.0	0.018	−0.032
					4.79	156.5		
FE	300	1000	0.130	0.012	12.00	92.3	−0.063	0.067
EH	300	1200	0.050	0.0020	2.40	48.0	−0.018	0.032
HG	400	1000	−0.020	−0.000095	−0.10	5.0	−0.035	−0.055
GF	400	1200	−0.120	−0.0026	−3.12	26.0	−0.035	−0.155
					11.18	171.3		

$$\Delta Q_{\mathrm{I}} = -\sum h_f/[n \sum (h_f/Q)] = -(-7.16)/[(1.85)(138.6)] = 0.028 \qquad \Delta Q_{\mathrm{II}} = -(-3.66)/[(1.85)(59.8)] = 0.033$$

$$\Delta Q_{\mathrm{III}} = -4.79/[(1.85)(156.5)] = -0.017 \qquad \Delta Q_{\mathrm{IV}} = -11.18/[(1.85)(171.3)] = -0.035$$

pipe	h_1	h_f	h_f/Q	ΔQ	Q_{new}
AB	0.0043	4.30	15.5	−0.013	0.265
BE	0.0023	2.76	24.0	−0.020	0.095
EF	−0.0034	−3.40	50.7	−0.018	−0.085
FA	−0.0011	−1.32	5.9	−0.013	−0.235
		2.34	96.1		

pipe	h_1	h_f	h_f/Q	ΔQ	Q_{new}
BC	0.0016	1.60	9.8	0.007	0.170
CD	0.00078	0.94	14.9	0.007	0.070
DE	−0.00050	−0.50	10.0	0.015	−0.035
EB	−0.0023	−2.76	24.0	0.020	−0.095
		−0.72	58.7		
ED	0.00050	0.50	10.0	−0.015	0.035
DI	0.0031	3.72	59.0	−0.008	0.055
IH	−0.00115	−1.15	31.1	−0.008	−0.045
HE	−0.00087	−1.04	32.5	−0.013	−0.045
		2.03	132.6		
FE	0.0034	3.40	50.7	0.018	0.085
EH	0.00087	1.04	32.5	0.013	0.045
HG	−0.00060	−0.60	10.9	0.005	−0.050
GF	−0.0041	−4.92	31.7	0.005	−0.150
		−1.08	125.8		

$\Delta Q_{\mathrm{I}} = -2.34/[(1.85)(96.1)] = -0.013$ $\Delta Q_{\mathrm{II}} = -(-0.72)/[(1.85)(58.7)] = 0.007$

$\Delta Q_{\mathrm{III}} = -2.03/[(1.85)(132.6)] = -0.008$ $\Delta Q_{\mathrm{IV}} = -(-1.08)/[(1.85)(125.8)] = 0.005$

pipe	h_1	h_f	h_f/Q	ΔQ	Q_{new}
AB	0.0039	3.90	14.7	0.006	0.271
BE	0.00165	1.98	20.8	0.012	0.107
EF	−0.0055	−5.50	64.7	0.014	−0.071
FA	−0.00125	−1.50	6.4	0.006	−0.229
		−1.12	106.6		
BC	0.0017	1.70	10.0	−0.006	0.164
CD	0.00094	1.13	16.1	−0.006	0.064
DE	−0.000265	−0.26	7.4	−0.008	−0.043
EB	−0.00165	−1.98	20.8	−0.012	−0.107
		0.59	54.3		
ED	0.000265	0.26	7.4	0.008	0.043
DI	0.0024	2.88	52.4	0.002	0.057
IH	−0.0017	−1.70	37.8	0.002	−0.043
HE	−0.0017	−2.04	45.3	0.010	−0.035
		−0.60	142.9		
FE	0.0055	5.50	64.7	−0.014	0.071
EH	0.0017	2.04	45.3	−0.010	0.035
HG	−0.00051	−0.51	10.2	−0.008	−0.058
GF	−0.0039	−4.68	31.2	−0.008	−0.158
		2.35	151.4		

$\Delta Q_{\mathrm{I}} = -(-1.12)/[(1.85)(106.6)] = 0.006$ $\Delta Q_{\mathrm{II}} = -0.59/[(1.85)(54.3)] = -0.006$

$\Delta Q_{\mathrm{III}} = -(-0.60)/[(1.85)(142.9)] = 0.002$ $\Delta Q_{\mathrm{IV}} = -2.35/[(1.85)(151.4)] = -0.008$

pipe	h_1	h_f	h_f/Q	ΔQ	Q_{new}
AB	0.0040	4.00	14.8	−0.006	0.265
BE	0.0021	2.52	23.6	−0.009	0.098
EF	−0.0039	−3.90	54.9	−0.008	−0.079
FA	−0.0012	−1.44	6.3	−0.006	−0.235
		1.18	99.6		
BC	0.0016	1.60	9.8	0.003	0.167
CD	0.00080	0.96	15.0	0.003	0.067
DE	−0.00038	−0.38	8.8	0.006	−0.037
EB	−0.0021	−2.52	23.6	0.009	−0.098
		−0.34	57.2		
ED	0.00038	0.38	8.8	−0.006	0.037
DI	0.0026	3.12	54.7	−0.003	0.054
IH	−0.0015	−1.50	34.9	−0.003	−0.046
HE	−0.00105	−1.26	36.0	−0.005	−0.040
		0.74	134.4		
FE	0.0039	3.90	54.9	0.008	0.079
EH	0.00105	1.26	36.0	0.005	0.040
HG	−0.00067	−0.67	11.6	0.002	−0.056
GF	−0.0041	−4.92	31.1	0.002	−0.156
		−0.43	133.6		

$\Delta Q_{\mathrm{I}} = -1.18/[(1.85)(99.6)] = -0.006$ $\Delta Q_{\mathrm{II}} = -(-0.34)/[(1.85)(57.2)] = 0.003$

$\Delta Q_{\mathrm{III}} = -0.74/[(1.85)(134.4)] = -0.003$ $\Delta Q_{\mathrm{IV}} = -(-0.43)/[(1.85)(133.6)] = 0.002$

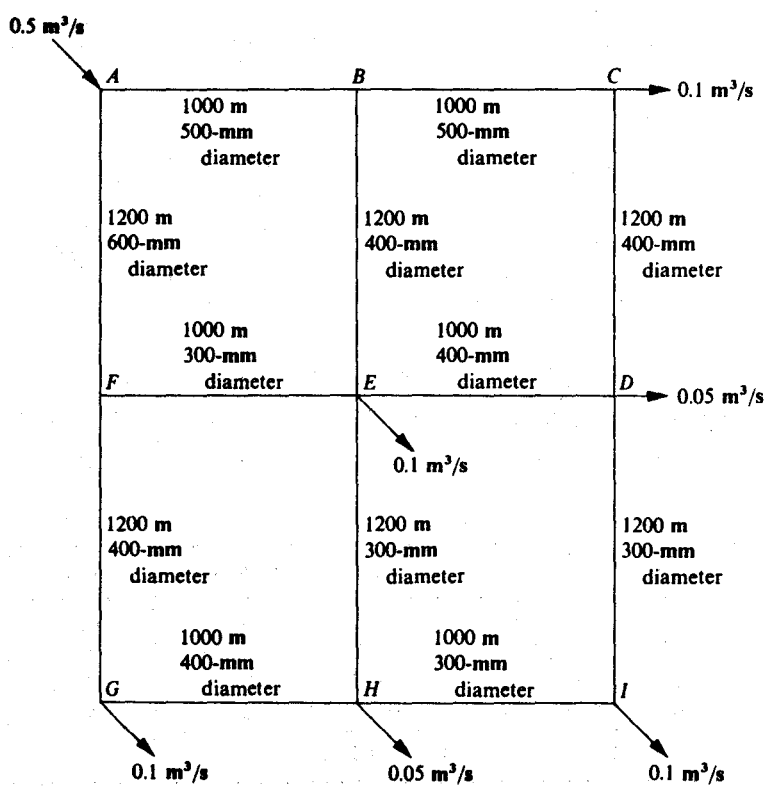


Fig. 13-6

13.7 Compute the flow rate of water in each pipe of the network shown in Fig. 13-7. Assume $C = 120$ for all pipes.

pipe	D, in.	L, ft	$Q_{assumed}$, cfs	h_1, ft/ft	h_f, ft	h_f/Q	ΔQ	Q_{new}
AB	24	2000	5.0	0.00045	0.90	0.180	0.29	5.29
BG	15	3000	1.0	0.00022	0.66	0.660	0.42	1.42
GH	18	2000	−2.5	−0.00051	−1.02	0.408	0.34	−2.16
HA	24	3000	−5.0	−0.00045	−1.35	0.270	0.29	−4.71
					−0.81	1.518		
BC	24	2000	4.0	0.00030	0.60	0.150	−0.13	3.87
CF	15	3000	1.0	0.00022	0.66	0.660	0.53	1.53
FG	18	2000	−1.0	−0.000095	−0.19	0.190	−0.48	−1.48
GB	15	3000	−1.0	−0.00022	−0.66	0.660	−0.42	−1.42
					0.41	1.660		
CD	15	2000	3.0	0.0017	3.40	1.133	−0.66	2.34
DE	12	3000	1.0	0.00067	2.01	2.010	−0.66	0.34
EF	18	2000	−0.3	−0.000010	−0.02	0.067	−0.33	−0.63
FC	15	3000	−1.0	−0.00022	−0.66	0.660	−0.53	−1.53
					4.73	3.870		
FE	18	2000	0.3	0.000010	0.02	0.067	0.33	0.63
EL	12	3000	1.3	0.0011	3.30	2.538	−0.33	0.97
LK	15	2000	−1.7	−0.00059	−1.18	0.694	−0.33	−2.03
KF	15	3000	−0.2	−0.000011	−0.03	0.150	−0.68	−0.88
					2.11	3.449		
GF	18	2000	1.0	0.000095	0.19	0.190	0.48	1.48
FK	15	3000	0.2	0.000011	0.03	0.150	0.68	0.88
KJ	18	2000	−1.5	−0.00019	−0.38	0.253	0.35	−1.15
JG	15	3000	−1.0	−0.00022	−0.66	0.660	0.40	−0.60
					−0.82	1.253		
HG	18	2000	2.5	0.00051	1.02	0.408	−0.34	2.16
GJ	15	3000	1.0	0.00022	0.66	0.660	−0.40	0.60
JI	24	2000	−0.5	−0.0000063	−0.01	0.020	−0.05	−0.55
IH	18	3000	−2.5	−0.00050	−1.50	0.600	−0.05	−2.55
					0.17	1.688		

$\Delta Q_{\mathrm{I}} = -(-0.81)/[(1.85)(1.518)] = 0.29$ $\quad$ $\Delta Q_{\mathrm{II}} = -0.41/[(1.85)(1.660)] = -0.13$

$\Delta Q_{\mathrm{III}} = -4.73/[(1.85)(3.870)] = -0.66$ $\quad$ $\Delta Q_{\mathrm{IV}} = -2.11/[(1.85)(3.449)] = -0.33$

$\Delta Q_{\mathrm{V}} = -(-0.82)/[(1.85)(1.253)] = 0.35$ $\quad$ $\Delta Q_{\mathrm{VI}} = -0.17/[(1.85)(1.688)] = -0.05$

pipe	h_1	h_f	h_f/Q	ΔQ	Q_{new}
AB	0.00050	1.00	0.189	−0.10	5.19
BG	0.00043	1.29	0.908	0.00	1.42
GH	−0.00039	−0.78	0.361	−0.31	−2.47
HA	−0.00040	−1.20	0.255	−0.10	−4.81
		0.31	1.713		

pipe	h_1	h_f	h_f/Q	ΔQ	Q_{new}
BC	0.00029	0.58	0.150	−0.10	3.77
CF	0.00050	1.50	0.980	0.07	1.60
FG	−0.00019	−0.38	0.257	0.04	−1.44
GB	−0.00043	−1.29	0.908	0.00	−1.42
		0.41	2.295		
CD	0.0011	2.20	0.940	−0.17	2.17
DE	0.000090	0.27	0.794	−0.17	0.17
EF	−0.000039	−0.08	0.127	−0.20	−0.83
FC	−0.00050	−1.50	0.980	−0.07	−1.60
		0.89	2.841		
FE	0.000039	0.08	0.127	0.20	0.83
EL	0.00064	1.92	1.979	0.03	1.00
LK	−0.00082	−1.64	0.808	0.03	−2.00
KF	−0.00018	−0.54	0.614	0.17	−0.71
		−0.18	3.528		
GF	0.00019	0.38	0.257	−0.04	1.44
FK	0.00018	0.54	0.614	−0.17	0.71
KJ	−0.00012	−0.24	0.209	−0.14	−1.29
JG	−0.000090	−0.27	0.450	−0.35	−0.95
		0.41	1.530		
HG	0.00039	0.78	0.361	0.31	2.47
GJ	0.000090	0.27	0.450	0.35	0.95
JI	−0.0000075	−0.02	0.036	0.21	−0.34
IH	−0.00053	−1.59	0.624	0.21	−2.34
		−0.56	1.471		

$\Delta Q_{\mathrm{I}} = -0.31/[(1.85)(1.713)] = -0.10$ $\Delta Q_{\mathrm{II}} = -0.41/[(1.85)(2.295)] = -0.10$

$\Delta Q_{\mathrm{III}} = -0.89/[(1.85)(2.841)] = -0.17$ $\Delta Q_{\mathrm{IV}} = -(-0.18)/[(1.85)(3.528)] = 0.03$

$\Delta Q_{\mathrm{V}} = -0.41/[(1.85)(1.530)] = -0.14$ $\Delta Q_{\mathrm{VI}} = -(-0.56)/[(1.85)(1.471)] = 0.21$

pipe	h_1	h_f	h_f/Q	ΔQ	Q_{new}
AB	0.00048	0.96	0.185	0.00	5.19
BG	0.00043	1.29	0.908	0.11	1.53
GH	−0.00050	−1.00	0.405	0.08	−2.39
HA	−0.00042	−1.26	0.262	0.00	−4.81
		−0.01	1.760		
BC	0.00027	0.54	0.143	−0.11	3.66
CF	0.00053	1.59	0.994	−0.05	1.55
FG	−0.00018	−0.36	0.250	−0.17	−1.61
GB	−0.00043	−1.29	0.908	−0.11	−1.53
		0.48	2.295		
CD	0.00097	1.94	0.894	−0.06	2.11
DE	0.000025	0.08	0.471	−0.06	0.11
EF	−0.000066	−0.13	0.157	−0.04	−0.87
FC	−0.00053	−1.59	0.994	0.05	−1.55
		0.30	2.156		

pipe	h_1	h_f	h_f/Q	ΔQ	Q_{new}
FE	0.000066	0.13	0.157	0.04	0.87
EL	0.00067	2.01	2.010	−0.02	0.98
LK	−0.00081	−1.62	0.810	−0.02	−2.02
KF	−0.00012	−0.36	0.507	−0.08	−0.79
		0.16	3.484		
GF	0.00018	0.36	0.250	0.17	1.61
FK	0.00012	0.36	0.507	0.08	0.79
KJ	−0.00015	−0.30	0.233	0.06	−1.23
JG	−0.00020	−0.60	0.632	0.14	−0.81
		−0.18	1.622		
HG	0.00050	1.00	0.405	−0.08	2.39
GJ	0.00020	0.60	0.632	−0.14	0.81
JI	−0.0000031	−0.01	0.029	−0.08	−0.42
IH	−0.00045	−1.35	0.577	−0.08	−2.42
		0.24	1.643		

$\Delta Q_{\mathrm{I}} = -(-0.01)/[(1.85)(1.760)] = 0.00$ $\Delta Q_{\mathrm{II}} = -0.48/[(1.85)(2.295)] = -0.11$

$\Delta Q_{\mathrm{III}} = -0.30/[(1.85)(2.516)] = -0.06$ $\Delta Q_{\mathrm{IV}} = -0.16/[(1.85)(3.484)] = -0.02$

$\Delta Q_{\mathrm{V}} = -(-0.18)/[(1.85)(1.622)] = 0.06$ $\Delta Q_{\mathrm{VI}} = -0.24/[(1.85)(1.643)] = -0.08$

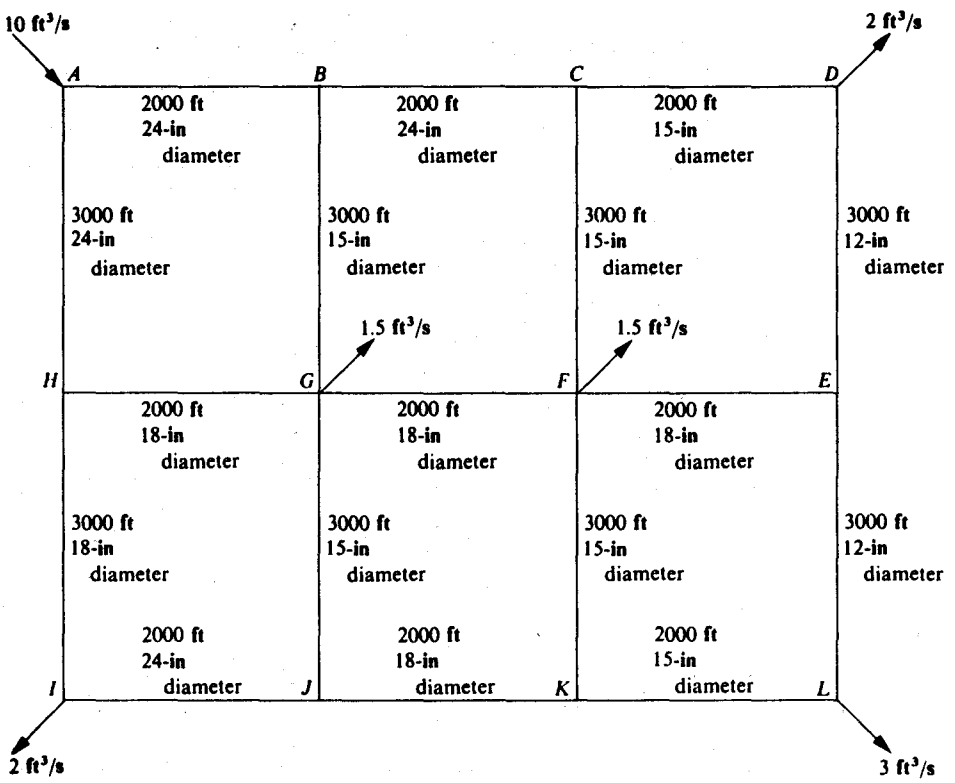


Fig. 13-7

13.8 Water flows through the piping system shown in Fig. 13-8 with certain measured flows indicated on the sketch. Determine the flows throughout the network. Use $C = 120$ throughout.

I

line	D, in	L, ft	assumed Q_1, mgd	S, ft/1000 ft	LH, ft	$\frac{LH}{Q_1}$	Δ	Q_2
AB	20	3000	4.0	1.62	4.86	1.22	+0.31	4.31
BE	16	4000	1.0	0.37	1.48	1.48	+0.31 − (0.13) = +0.18	1.18
EF	16	3000	−2.0	−1.33	−3.99	2.00	+0.31 − (0.50) = −0.19	−2.19
FA	24	4000	−6.0	−1.41	−5.64	0.94	+0.31	−5.69
					$\Sigma = -3.29$	5.64		
BC	20	3000	3.0	0.95	2.85	0.95	+0.13	3.13
CD	16	4000	2.0	1.33	5.32	2.66	+0.13	2.13
DE	12	3000	−1.5	−3.15	−9.45	6.30	+0.13 − (−0.12) = +0.25	−1.25
EB	16	4000	−1.0	−0.37	−1.48	1.48	+0.13 − (0.31) = −0.18	−1.18
					$\Sigma = -2.76$	11.39		
FE	16	3000	2.0	1.33	3.99	2.00	+0.50 − (+0.31) = +0.19	2.19
EH	12	4000	1.0	1.48	5.92	5.92	+0.50 − (−0.12) = +0.62	1.62
HG	16	3000	−2.0	−1.33	−3.99	2.00	+0.50	−1.50
GF	16	4000	−4.0	−4.85	−19.40	4.85	+0.50	−3.50
					$\Sigma = -13.48$	14.77		
ED	12	3000	1.5	3.15	9.45	6.30	−0.12 − (0.13) = −0.25	1.25
DI	12	4000	1.0	1.48	5.92	5.92	−0.12	0.88
IH	12	3000	−1.0	−1.48	−4.44	4.44	−0.12	−1.12
HE	12	4000	−1.0	−1.48	−5.92	5.92	−0.12 − (0.50) = −0.62	−1.62
					$\Sigma = +5.01$	22.58		

$$\Delta_{\mathrm{I}} = -(-3.29)/[1.85(5.64)] = +0.31 \qquad \Delta_{\mathrm{II}} = -(-2.76)/[1.85(11.39)] = +0.13$$

$$\Delta_{\mathrm{III}} = -(-13.48)/[1.85(14.77)] = +0.50 \qquad \Delta_{\mathrm{IV}} = -(+5.01)/[1.85(22.58)] = -0.12$$

For line *EF* in loop I, its net Δ term is $(\Delta_{\mathrm{I}} - \Delta_{\mathrm{III}})$ or $[+0.31 - (+0.50)] = -0.19$. It will be observed that the Δ for loop I is combined with that of the Δ for loop III since the line *EF* occurs in each loop. In similar fashion, for line *FE* in loop III, the net Δ term is $(\Delta_{\mathrm{III}} - \Delta_{\mathrm{I}})$ or $[+0.50 - (+0.31)] = +0.19$. Note that the net Δ's have the same magnitude but *opposite signs*. This can readily be understood since flow in *EF* is counterclockwise for loop I, whereas flow in *FE* in loop III is clockwise.

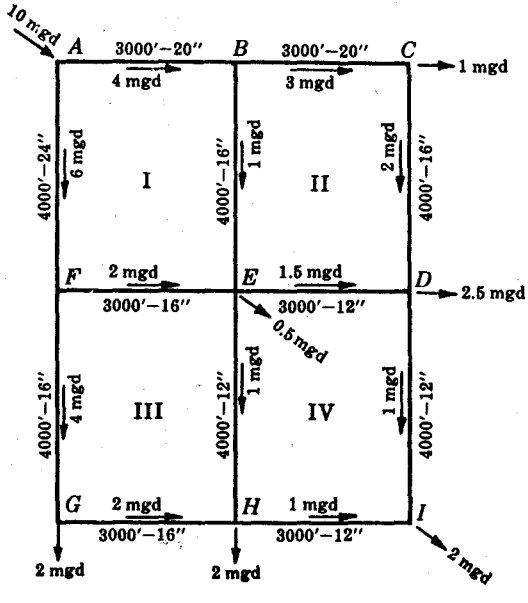


Fig. 13-8

In determining the values of Q_2 for the second calculation, $Q_{AB} = (4.00 + 0.31) = 4.31$ mgd whereas $Q_{EF} = (-2.00 - 0.19) = -2.19$ mgd and $Q_{FA} = (-6.00 + 0.31) = -5.69$ mgd.

line	Q_2	S	LH	LH/Q	Δ
AB	4.31	1.86	5.58	1.29	+0.20
BE	1.18	0.51	2.04	1.72	+0.20 + negl = +0.20
EF	−2.19	−1.57	−4.71	2.15	+0.20 − (−0.06) = +0.26
FA	−5.69	−1.28	−5.12	0.90	+0.20
			Σ = −2.21	6.06	
BC	3.13	1.02	3.06	0.98	negl
CD	2.13	1.48	5.92	2.79	negl
DE	−1.25	−2.28	−6.84	5.50	negl − 0.19 = −0.19
EB	−1.18	−0.51	−2.04	1.72	negl − 0.20 = −0.20
			Σ = +0.10	10.99	
FE	2.19	1.57	4.71	2.15	−0.06 − 0.20 = −0.26
EH	1.62	3.65	14.60	9.02	−0.06 − 0.19 = −0.25
HG	−1.50	−0.79	−2.37	1.58	−0.06
GF	−3.50	−3.75	−15.00	4.28	−0.06
			Σ = +1.94	17.03	
ED	1.25	2.28	6.84	5.42	+0.19 + negl = +0.19
DI	0.88	1.18	4.72	5.38	+0.19
IH	−1.12	−1.83	−5.49	4.90	+0.19
HE	−1.62	−3.65	−14.60	9.02	+0.19 − (−0.06) = +0.25
			Σ = −8.53	24.72	

line	Q_3	S	LH	LH/Q	Δ	Q_4
AB	4.51	2.02	6.06	1.34	−0.02	4.49
BE	1.39	0.68	2.72	1.95	−0.02 − 0.12 = −0.14	1.25
EF	−1.93	−1.25	−3.75	1.94	−0.02 − 0.12 = −0.14	−2.07
FA	−5.49	−1.20	−4.80	0.88	−0.02	−5.51
			Σ = +0.23	6.11		
BC	3.12	1.02	3.06	0.98	+0.12	3.24
CD	2.12	1.49	5.96	2.81	+0.12	2.24
DE	−1.45	−2.97	−8.91	6.15	+0.12 + 0.02 = +0.14	−1.31
EB	−1.39	−0.68	−2.72	1.95	+0.12 + 0.02 = +0.14	−1.25
			Σ = −2.61	11.89		
FE	1.93	1.25	3.75	1.94	+0.12 + 0.02 = +0.14	2.07
EH	1.37	2.68	10.72	7.83	+0.12 + 0.02 = +0.14	1.51
HG	−1.56	−0.84	−2.52	1.62	+0.12	−1.44
GF	−3.56	−3.90	−15.60	4.38	+0.12	−3.44
			Σ = −3.65	15.77		
ED	1.45	2.97	8.91	6.15	−0.02 − 0.12 = −0.14	1.31
DI	1.07	1.68	6.72	6.28	−0.02	1.05
IH	−0.93	−1.31	−3.93	4.23	−0.02	−0.95
HE	−1.37	−2.68	−10.72	7.83	−0.02 − 0.12 = −0.14	−1.51
			Σ = +0.98	24.49		

13.9 For the piping system of Prob. 13.8, if the elevation at point A is 200.0 ft and the pressure head is 150.0 ft and the elevation at I is 100.0 ft, find the pressure head at I.

▮ The elevation of the hydraulic grade line at A is (200.0 + 150.0) = 350.0. The lost head to I can be calculated by any route from A to I, adding the losses in the usual manner, i.e., in the direction of flow. Using $ABEHI$ we obtain $\mathrm{LH}_{A\ \mathrm{to}\ I}$ = (6.06 + 2.72 + 10.72 + 3.93) = 23.43 ft. As a check, using $ABEDI$, LH = (6.06 + 2.72 + 8.91 + 6.72) = 24.41 ft. Using a value of 24 ft, the elevation of the hydraulic grade line at I = (350.0 − 24.0) = 326.0 ft. Hence the pressure head at I = (326.0 − 100.0) = 226.0 ft.

13.10 If the flows into and out of a two-loop pipe system are as shown in Fig. 13-9, determine the flow in each pipe. The K values for each pipe were calculated from the pipe and minor loss characteristics and from an assumed value of f.

▮ As a first step, assume flow in each pipe such that continuity is satisfied at all junctions. Calculate ΔQ for each loop, make corrections to the assumed Q's and repeat several times until the ΔQ's are quite small. As a final step the values of f for each pipe should be checked against the Moody diagram and modified, if necessary.

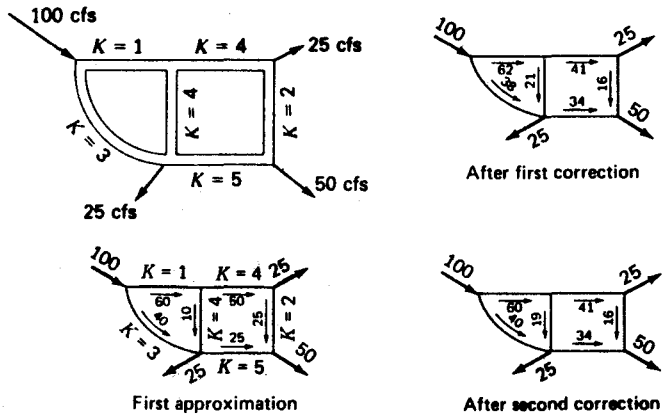


Fig. 13-9

left loop		right loop	
ΣKQ_0^n	$\Sigma \lvert KnQ_0^{n-1}\rvert$	ΣKQ_0^n	$\Sigma \lvert KnQ_0^{n-1}\rvert$
$1\times 60^2 = 3\,600$ $4\times 10^2 = 400$ 4 000↓ $3\times 40^2 = 4\,800$↑ 800↑	$1\times 2\times 60 = 120$ $4\times 2\times 10 = 80$ $3\times 2\times 40 = 240$ 440	$4\times 50^2 = 10\,000$ $2\times 25^2 = 1\,250$ 11 250↓ $4\times 10^2 = 400$ $5\times 25^2 = 3\,125$ 3 525↑ 7 725↓	$4\times 2\times 50 = 400$ $2\times 2\times 25 = 100$ $4\times 2\times 10 = 80$ $5\times 2\times 25 = 250$ 830
$\Delta Q_1 = \dfrac{800}{440} \approx 2$↓		$\Delta Q_1 = \dfrac{7\,725}{830} \approx 9$↑	
$1\times 62^2 = 3\,844$ $4\times 21^2 = 1\,764$ 5 608↓ $3\times 38^2 = 4\,332$↑ 1 276↓	$1\times 2\times 62 = 124$ $4\times 2\times 21 = 168$ $3\times 2\times 38 = 228$ 520	$4\times 41^2 = 6\,724$ $2\times 16^2 = 512$ 7 236↓ $4\times 21^2 = 1\,764$ $5\times 34^2 = 5\,780$ 7 544↑ 308↑	$4\times 2\times 41 = 328$ $2\times 2\times 16 = 64$ $4\times 2\times 21 = 168$ $5\times 2\times 34 = 340$ 900
$\Delta Q_2 = \dfrac{1\,276}{520} \approx 2$↑		$\Delta Q_2 = \dfrac{308}{900} \approx 0$	

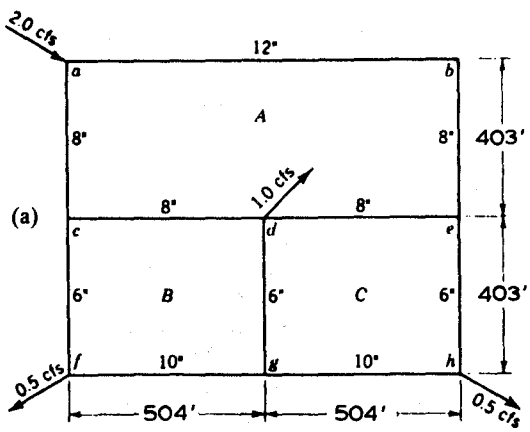
13.11 Determine the flows in the cast iron piping of Fig. 13-10. Take $n = 2.0$ and use the values of f for complete turbulence, as given in Fig. A-5. Use four stages of approximation.

▌ With $V = Q/(\pi D^2/4)$: $h_L = f(L/D)(16Q^2/\pi^2 D^4)/2g$, that is, $h_L = KQ^n$ with $K = 16fL/(2\pi^2 gD^5) = 0.025fL/D^5$, $n = 2$. For complete turbulence, f = const, K = const (for a given pipe). Table A-9 for cast iron gives $\epsilon = 0.00085$ ft. Thus for pipe ab, $\epsilon/D = 0.00085$. Figure A-5 for complete turbulence gives $f = 0.0189$; $K = 0.476$. Similarly $K = 1.606$ for ac and be, $K = 2.01$ for cd and de, $K = 1.722$ for cf, dg, and eh, $K = 0.620$ for fg and gh.

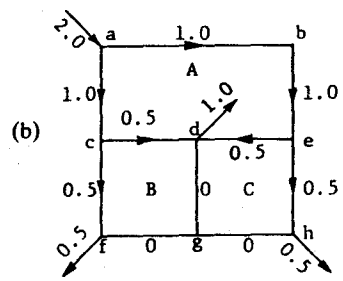
First Approximation and Correction

loop *A*			
pipe	***K***	KQ_0^n	$\lvert KnQ_0^{n-1}\rvert$
ab	0.476	0.476	0.952
be	1.606	1.606	3.21
ed	2.01	0.502	2.01
		2.584	
ac	1.606	1.606	3.21
cd	2.01	0.502	2.01
		2.108	11.39
$\Delta Q = (2.584 - 2.108)/11.39 = 0.0418\uparrow$			

loop *B*				loop *C*			
pipe	***K***	KQ_0^n	$\lvert KnQ_0^{n-1}\rvert$	**pipe**	***K***	KQ_0^n	$\lvert KnQ_0^{n-1}\rvert$
cd	2.01	0.502↓	2.01	*eh*	7.22	1.804↓	7.22
cf	7.22	1.804↑	7.22	*ed*	2.01	0.502↑	2.01
			9.22				9.22
$\Delta Q = (1.804 - 0.502)/9.22 = 0.1412\downarrow$				$\Delta Q = (1.804 - 0.502)/9.22 = 0.1412\uparrow$			



Initial flow assumptions:



Resulting final flows:

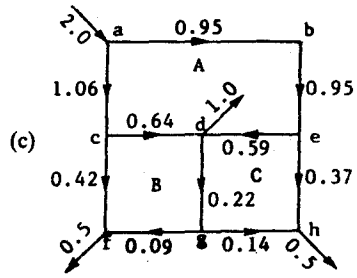


Fig. 13-10

pipe	Q(2d approximation), cfs	Q(4th and final approx), cfs
ab	$1.00 - 0.04 = 0.96$	0.95
be	$1.00 - 0.04 = 0.96$	0.95
ed	$0.50 - 0.04 + 0.14 = 0.60$	0.59
ac	$1.00 + 0.04 = 1.04$	1.06
cd	$0.50 + 0.04 + 0.14 = 0.68$	0.64
dg	$0 + 0.14 + 0.14 = 0.28$	0.22
gf	$0 + 0.14 = 0.14$	0.09
cf	$0.50 - 0.14 = 0.36$	0.42
eh	$0 + 0.14 = 0.14$	0.37
gh	$0.50 - 0.14 = 0.36$	0.14

13.12 If in Prob. 13.11 the pressure head at *a* is 200 ft, find the pressure head at *d*. Neglect velocity heads.

▮ From the final approximations in Prob. 13.11,

In pipe *ac* $\quad h_L = KQ^2 = 1.606 \times 1.06^2 = 1.805$ ft
In pipe *cd*, $\quad h_L = 2.01 \times 0.64^2 = 0.823$ ft
$\qquad\qquad\qquad

loop A				loop B			
$\Delta Q = \dfrac{1.026 - 1.829}{2 \times 0.765} = -0.525$				$\Delta Q = \dfrac{0.489 - 1.030}{2 \times 0.801} = -0.338$			
loop C				**loop D**			
pipe	Q_0	KQ_0^2	$\lvert h_L/Q_0 \rvert$	**pipe**	Q_0	KQ_0^2	$\lvert h_L/Q_0 \rvert$
cd	0	0.000	0.000	*ie*	4	0.989	0.247
gf	1	0.237	0.237	*ef*	2	0.947	0.473
		0.237				1.935	
cf	2	0.335	0.1673	*ij*	3	0.1543	0.0514
gd	2	0.335	0.1673	*jg*	3	0.556	0.1854
		0.669	0.571	*gf*	1	0.237	0.237
						0.947	1.194
$\Delta Q = \dfrac{0.237 - 0.669}{2 \times 0.573} = -0.379$				$\Delta Q = \dfrac{1.935 - 0.947}{2 \times 1.194} = +0.414$			

After five trials, the flows (cfs) within approximately 1 percent are $ab = 3.54$; $bc = 2.48$; $cd = 0.40$; $ef = 1.47$; $gf = 1.45$; $hi = 6.46$; $ij = 3.05$; $ha = 3.54$; $be = 1.06$; $ie = 3.41$; $cf = 2.08$; $gd = 1.60$; $jg = 3.05$.

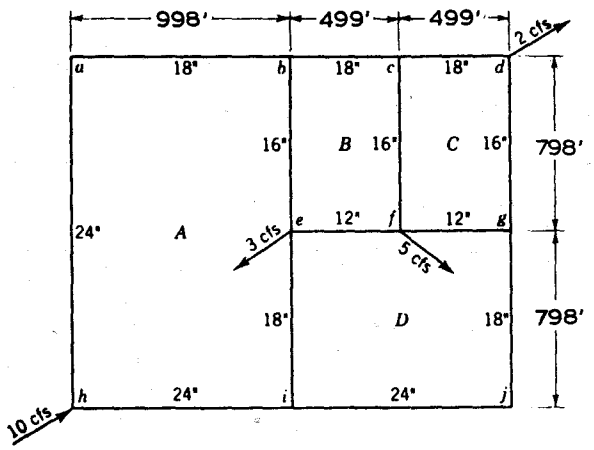


Fig. 13-11

13.14 Calculate the pressure drop from h to f in Prob. 13.13.

▮ Head loss from h to f (any path):

$$\begin{aligned} hi &= 0.716 \text{ ft} \\ ie &= 0.719 \text{ ft} \\ ef &= 0.511 \text{ ft} \\ \hline & \quad 1.946 \text{ ft} \end{aligned}$$

Hence $\Delta p = (1.946)(62.4)/144 = 0.8$ psi

13.15 Fit an equation of the form $h_f = KQ^n$ to flow of 60-°F water through 1000 ft of 10-in cast iron pipe. The equation should hold over the velocity range 2 to 8 ft/s.

▮ $h_f = (f)(L/D)(v^2/2g) \qquad \epsilon/D = 0.00085/(\frac{10}{12}) = 0.00128 \qquad N_R = Dv/\nu$

For $v = 2$ fps: $N_R = (\frac{10}{12})(2)/(1.21 \times 10^{-5}) = 1.38 \times 10^5$. From Fig. A-5, $f = 0.0230$: $h_f = 0.0230[1000/(\frac{10}{12})]\{2^2/[(2)(32.2)]\} = 1.714$ ft, $Q = Av = [(\pi)(\frac{10}{12})^2/4](2) = 1.091$ ft^3/s. For $v = 8$ fps: $N_R = (\frac{10}{12})(8)/(1.21 \times 10^{-5}) = 5.51 \times 10^5$, $f = 0.0213$, $h_f = 0.0213[1000/(\frac{10}{12})]\{8^2/[(2)(32.2)]\} = 25.40$ ft, $Q = Av = [(\pi)(\frac{10}{12})^2/4](8) = 4.363$ ft^3/s. Given $h_f = KQ^n$, at 2 fps, $1.714 = (K)(1.091)^n$. At 8 fps, $25.4 = (K)(4.363)^n$:

$$\log 1.714 = \log K + n \log 1.091 \qquad (1)$$

$$\log 25.4 = \log K + n \log 4.363 \qquad (2)$$

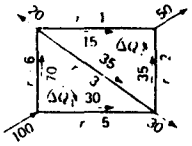
Subtracting Eq. (*1*) from Eq. (*2*) gives $1.171 = 0.6022n$, $n = 1.945$. Substituting into Eq. (*1*), $\log 1.714 = \log K + (1.945)(\log 1.091)$, $K = 1.447$; $h_f = 1.447Q^{1.945}$.

13.16 The distribution of flow through the network of Fig. 13-12 is desired for the inflows and outflows as given. For simplicity n has been given the value 2.0.

I The assumed distribution is shown in Fig. 13-12*a*. At the upper left the term $\sum rQ_0\,|Q_0|^{n-1}$ is computed for the lower circuit number 1. Next to the diagram on the left is the computation of $\sum nr\,|Q_0|^{n-1}$ for the same circuit. The same format is used for the second circuit in the upper right of the figure. The corrected flow after the first step for the top horizontal pipe is determined as $15 + 11.06 = 26.06$ and for the diagonal as $35 + (-21.17) + (-11.06) = 2.77$. Figure 13-12*b* shows the flows after one correction and Fig. 13-12*c* the values after four corrections.

$70^2 \times 6 = 29{,}400 \quad 2 \times 70 \times 6 = 840$
$35^2 \times 3 = 3{,}675 \quad 2 \times 35 \times 3 = 210$
$-30^2 \times 5 = -4{,}500 \quad 2 \times 30 \times 5 = 300$
$28{,}575 \quad 1{,}350$

$\Delta Q_1 = -\dfrac{28{,}575}{1{,}350} = -21.17$



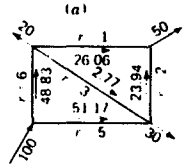
$15^2 \times 1 = 225 \quad 2 \times 15 \times 1 = 30$
$-35^2 \times 2 = -2{,}450 \quad 2 \times 35 \times 2 = 140$
$-13.83^2 \times 3 = -574 \quad 2 \times 13.83 \times 3 = 83$
$-2{,}799 \quad 253$

$\Delta Q_2 = \dfrac{2{,}799}{253} = 11.06$

(*a*)

$48.83^2 \times 6 = 14{,}308 \quad 2 \times 48.83 \times 6 = 586$
$2.77^2 \times 3 = 23 \quad 2 \times 2.77 \times 3 = 17$
$-51.17^2 \times 5 = -13{,}090 \quad 2 \times 51.17 \times 5 = 511$
$1{,}241 \quad 1{,}114$

$\Delta Q_1 = -\dfrac{1{,}241}{1{,}114} = -1.114$

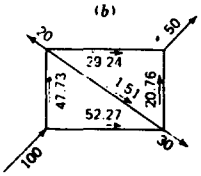


$26.06^2 \times 1 = 679 \quad 2 \times 26.06 \times 1 = 52$
$-23.94^2 \times 2 = -1{,}146 \quad 2 \times 23.94 \times 2 = 96$
$-1.656^2 \times 3 = -8 \quad 2 \times 1.656 \times 3 = 10$
$-475 \quad 158$

$\Delta Q_2 = \dfrac{475}{158} = 3.006$

(*b*)

$\Delta Q_1 = 0.0079$
$\Delta Q_1 = 0.0013$



$\Delta Q_2 = 0.169$
$\Delta Q_2 = 0.0003$

(*c*)

Fig. 13-12

13.17 Calculate the flow through each of the pipes of the network shown in Fig. 13-13 ($n = 2$).

I

$1 \times 60^2 = 3600 \quad 2 \times 1 \times 60 = 120$ $3 \times 5^2 = 75 \quad 2 \times 3 \times 5 = 30$ $2 \times 40^2 = -3200 \quad 2 \times 2 \times 40 = 160$ $475 \quad 310$ $\Delta Q = -\dfrac{475}{310} \approx -1.5$	$2 \times 30^2 = 1800 \quad 2 \times 2 \times 30 = 120$ $3 \times 5^2 = -75 \quad 2 \times 3 \times 5 = 30$ $1 \times 45^2 = -2025 \quad 2 \times 1 \times 45 = 90$ $-300 \quad 240$ $\Delta Q = \dfrac{300}{240} \approx 1$	Fig. 13-13*b*
$1 \times 58.5^2 = 3422.25 \quad 2 \times 1 \times 58.5 = 117$ $3 \times 2.5^2 = 18.75 \quad 2 \times 3 \times 2.5 = 15$ $2 \times 41.5^2 = -3444.5 \quad 2 \times 2 \times 41.5 = 166$ $-3.5 \quad 298$ $\Delta Q = \dfrac{3.5}{298} = 0.012$	$2 \times 31^2 = 1922 \quad 2 \times 2 \times 31 = 124$ $3 \times 2.5^2 = -19 \quad 2 \times 3 \times 2.5 = 15$ $1 \times 44^2 = -1936 \quad 2 \times 1 \times 44 = 88$ $-33 \quad 227$ $\Delta Q = \dfrac{33}{227} = 0.145$	Fig. 13.13*c*

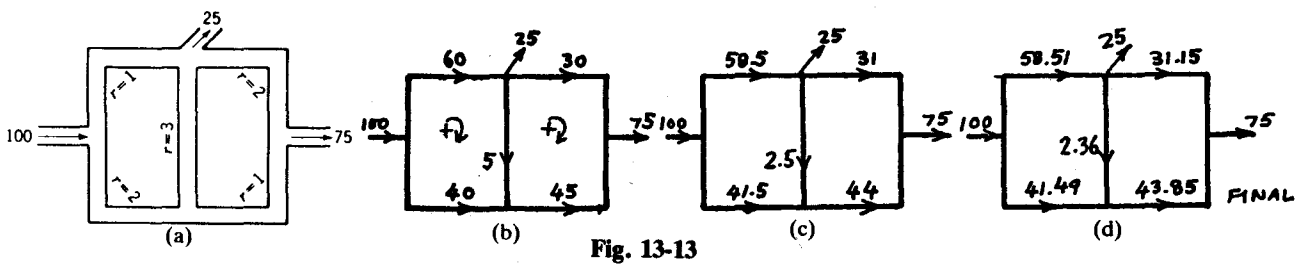


Fig. 13-13

13.18 Determine the flow through each line of Fig. 13-14 ($n = 2$).

I

$2\times19^2 = 722$ $1\times32^2 = 1024$ $2\times31^2 = -1922$ -176	$2\times2\times19 = 76$ $2\times1\times32 = 64$ $2\times2\times31 = 124$ 264	$4\times37^2 = 5476$ $1\times32^2 = -1024$ $3\times38^2 = -4332$ 120	$2\times4\times37 = 296$ $2\times1\times32 = 64$ $2\times3\times38 = 228$ 588
$\Delta Q = \frac{176}{264} = 0.67$		$\Delta Q = \frac{-120}{588} = -0.20$	Fig. 13-14*b*
$2\times19.67^2 = 773.82$ $1\times32.87^2 = 1080.44$ $2\times30.33^2 = -1839.82$ 14.4	$2\times2\times19.67 = 78.68$ $2\times1\times32.87 = 65.74$ $2\times2\times30.33 = 121.32$ 265.74	$4\times36.8^2 = 5417.0$ $1\times32.87^2 = -1080.4$ $3\times38.2^2 = -4377.7$ -41.12	$2\times4\times36.8 = 294.4$ $2\times1\times32.87 = 65.74$ $2\times3\times38.2 = 229.2$ 589.3
$\Delta Q = -\frac{14.44}{265.74} = -0.05$		$\Delta Q = -\frac{41.12}{589.3} = 0.07$	

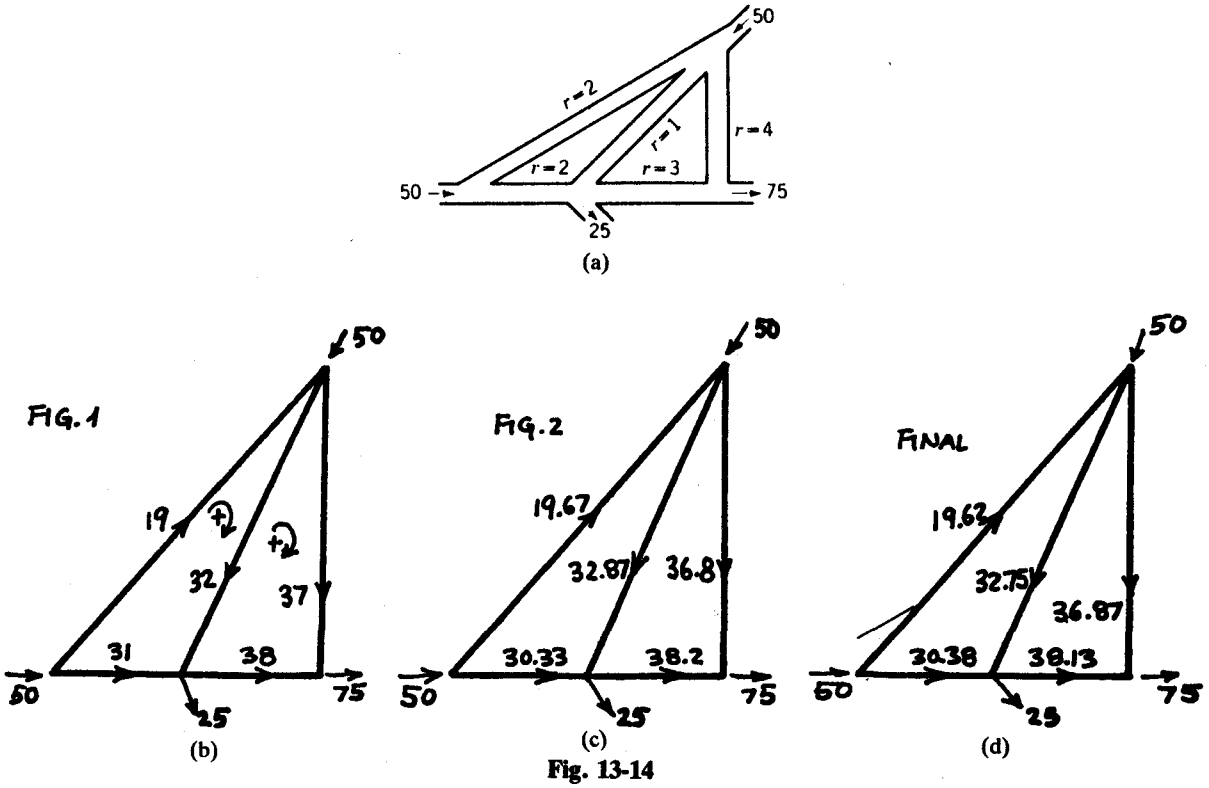


Fig. 13-14

13.19 Determine the flow rates in Fig. 13-15. The fluid is water at 20 °C and all five pipes have $f = 0.0201$.

I

$$h_f = (f)\left(\frac{L}{D}\right)\left(\frac{v^2}{2g}\right) = \frac{8fLQ^2}{\pi^2 g D^5} = KQ^2 \qquad K_1 = \frac{8f_1L_1}{\pi^2 g D_1^5} = \frac{(8)(0.0201)(3000)}{(\pi)^2(32.2)(\frac{6}{12})^5} = 48.57$$

$$K_2 = \frac{(8)(0.0201)(4000)}{(\pi)^2(32.2)(\frac{8}{12})^5} = 15.37 \qquad K_3 = \frac{(8)(0.0201)(5000)}{(\pi)^2(32.2)(\frac{9}{12})^5} = 10.66$$

$$K_4 = \frac{(8)(0.0201)(4000)}{(\pi)^2(32.2)(\frac{8}{12})^5} = 15.37 \qquad K_5 = \frac{(8)(0.0201)(3000)}{(\pi)^2(32.2)(\frac{3}{12})^5} = 1554$$

Two head-loss loops:

Loop *ABC*: $$15.37Q_2^2 + 10.66Q_3^2 - 48.57Q_1^2 = 0 \qquad (1)$$

Loop *BCD*: $$10.66Q_3^2 + 15.37Q_4^2 - 1554Q_5^2 = 0 \qquad (2)$$

Three junctions:

Junction A: $$Q_1 + Q_2 = 2.0 \tag{3}$$

Junction B: $$Q_2 = Q_3 + Q_5 \tag{4}$$

Junction C: $$Q_1 + Q_3 = Q_4 \tag{5}$$

Solving these simultaneous equations by trial and error gives $Q_1 = 0.81$ cfs, $Q_2 = 1.19$ cfs, $Q_3 = 0.99$ cfs, $Q_4 = 1.80$ cfs, $Q_5 = 0.20$ cfs.

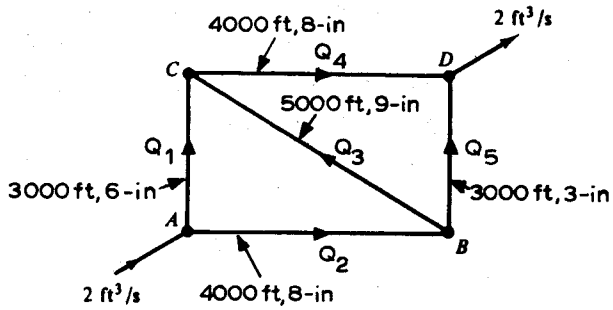


Fig. 13-15

13.20 In Prob. 13.19 the pressure at A is given as 200 psig. Find the pressures at B, C, and D.

▮ $$p_B = p_A - \rho g K_2 Q_2^2 = (200)(144) - (1.94)(32.2)(15.37)(1.19)^2 = 27\,440 \text{ lb/ft}^2 \quad \text{or} \quad 190.6 \text{ psig}$$

$$p_C = p_A - \rho g K_1 Q_1^2 = (200)(144) - (1.94)(32.2)(48.57)(0.81)^2 = 26\,809 \text{ lb/ft}^2 \quad \text{or} \quad 186.2 \text{ psig}$$

$$p_D = p_C - \rho g K_4 Q_4^2 = (186.2)(144) - (1.94)(32.2)(15.37)(1.80)^2 = 23\,702 \text{ lb/ft}^2 \quad \text{or} \quad 164.6 \text{ psig}$$

13.21 Repeat Prob. 13.19 for the boundary flows indicated in Fig. 13-16.

▮ $$h_f = (f)(L/D)(v^2/2g) = 8fLQ^2/\pi^2 g D^5 = KQ^2$$

From Prob. 13.19, $K_1 = 48.57$, $K_2 = 15.37$, $K_3 = 10.66$, $K_4 = 15.37$, and $K_5 = 1554$.

Two head-loss loops:

Loop ABC: $$15.37Q_2^2 + 10.66Q_3^2 - 48.57Q_1^2 = 0 \tag{1}$$

Loop BCD: $$10.66Q_3^2 + 15.37Q_4^2 - 1554Q_5^2 = 0 \tag{2}$$

Three junctions:

Junction A: $$Q_1 + Q_2 = 2.0 \tag{3}$$

Junction B: $$Q_2 = Q_3 + Q_5 + 1.0 \tag{4}$$

Junction C: $$Q_1 + Q_3 = Q_4 + 0.5 \tag{5}$$

Solving these simultaneous equations by trial and error gives $Q_1 = 0.725$ cfs, $Q_2 = 1.275$ cfs, $Q_3 = 0.226$ cfs, $Q_4 = 0.451$ cfs, $Q_5 = 0.049$ cfs.

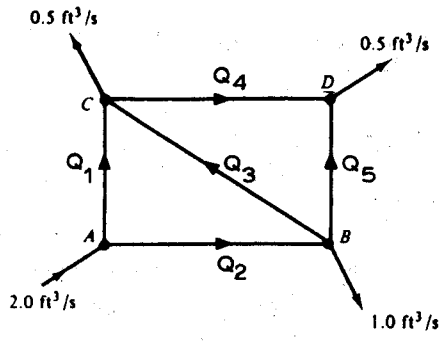


Fig. 13-16

13.22 Repeat Prob. 13.20 for the flows found in Prob. 13.21.

▮ $p_B = p_A - \rho g K_2 Q_2^2 = (200)(144) - (1.94)(32.2)(15.37)(1.275)^2 = 27\,239\ \text{lb/ft}^2$ or $189.2\ \text{lb/in}^2$

$p_C = p_A - \rho g K_1 Q_1^2 = (200)(144) - (1.94)(32.2)(48.57)(0.725)^2 = 27\,205\ \text{lb/ft}^2$ or $188.9\ \text{lb/in}^2$

$p_D = p_C - \rho g K_4 Q_4^2 = (188.9)(144) - (1.94)(32.2)(15.37)(0.451)^2 = 27\,006\ \text{lb/ft}^2$ or $187.5\ \text{lb/in}^2$

13.23 Compute the junction flows in Fig. 13-17, taking flow *into* the junction as positive. Each pipe is 30 m of 60-mm-diameter cast iron, with $f = 0.0294$.

▮ $$h_f = \Delta p/\gamma = 8fLQ^2/\pi^2 g D^5$$

$(800 - 500)/9.79 = (8)(0.0294)(30)(Q_1)^2/(\pi)^2(9.807)(0.060)^5$ $\quad Q_1 = +0.0181\ \text{m}^3/\text{s}$

$(500 - 200)/9.79 = (8)(0.0294)(30)(Q_4)^2/(\pi)^2(9.807)(0.060)^5$ $\quad Q_4 = -0.0181\ \text{m}^3/\text{s}$

$(500 - 400)/9.79 = (8)(0.0294)(30)(Q_2)^2/(\pi)^2(9.807)(0.060)^5$ $\quad Q_2 = -0.0104\ \text{m}^3/\text{s}$

$(600 - 500)/9.79 = (8)(0.0294)(30)(Q_3)^2/(\pi)^2(9.807)(0.060)^5$ $\quad Q_3 = +0.0104\ \text{m}^3/\text{s}$

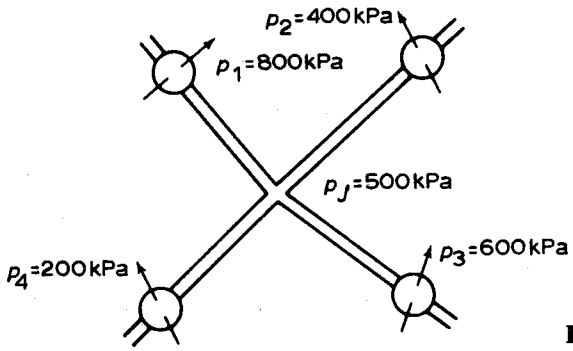


Fig. 13-17

13.24 Determine the flow in each pipe of the network shown in Fig. 13-18, using $f = 0.02$ throughout.

▮ Taking $x = 2$, $h_L = f(L/D)(V^2/2g) = f(L/D)(1/2g)(4Q/\pi D^2)^2 = (8fL/\pi^2 g D^5)(Q^2) = KQ^2$. Hence $K = 0.81fL/gD^5$, and the K value for each pipe is

Diameter, in	3	4	5	6	7	8
K	1030	368	160	80.4	22.4	11.5

The assumed flows are indicated on the figure in parentheses. For loop $AEDB$,

$$\Delta_1 = -[(1030 \times 0.5^2) + (11.5 \times 0.1^2) - (22.4 \times 0.2^2) - (368 \times 0.7^2)]/$$
$$2[(1030 \times 0.5) + (11.5 \times 0.1) + (22.4 \times 0.2) + (368 \times 0.7)] = -0.05\ \text{cfs}$$

and for loop BDC,

$$\Delta_2 = -[(22.4 \times 0.2^2) + (80.4 \times 0.3^2) - (160 \times 0.5^2)]/2[(22.4 \times 0.2) + (80.4 \times 0.3) + (160 \times 0.5)] = +0.15\ \text{cfs}$$

The corrected flows appear on the figure below the first assumed flows. Recomputing Δ for each loop yields $\Delta_1 = +0.001$ cfs and $\Delta_2 = -0.001$ cfs.

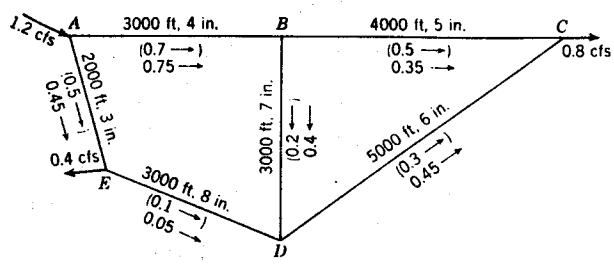


Fig. 13-18

13.25 Replace the quadrilateral network of Fig. 13-19 by a single 500-ft-long pipe AC. Assume f is the same for all pipes.

▮

$$(h_f)_{ABC} = (h_f)_{ADC} \qquad h_f = (f)(L/D)(v^2/2g)$$

$$f(500/\tfrac{8}{12})(V_8^2/2g) + f(\tfrac{500}{2})(V_{24}^2/2g) = f(\tfrac{800}{1})(V_{12}^2/2g) + f(\tfrac{1000}{3})(V_{36}^2/2g)$$

$$750V_8^2 + 250V_{24}^2 = 800V_{12}^2 + 333V_{36}^2 \qquad V_8 = 9V_{24} \qquad V_{12} = 9V_{36}$$

$$(750 \times 81 + 250)V_{24}^2 = (800 \times 81 + 333)V_{36}^2 \qquad V_{36} = 0.96V_{24}$$

$$D^2V = 2^2V_{24} + 3^2V_{36} = 4V_{24} + 9(0.96V_{24}) \qquad D^2V = 12.63V_{24}$$

$$(500/\tfrac{8}{12})V_8^2 + \tfrac{500}{2}V_{24}^2 = (1000/D)V^2$$

$$750V_8^2 + 250V_{24}^2 = 61\,000V_{24}^2 = (1000/D)V^2 \qquad (1)$$

But

$$V = 12.63V_{24}/D^2 \qquad (2)$$

Substituting (2) into (1) yields $D^5 = 1.308$. Hence $D = 1.055$ ft.

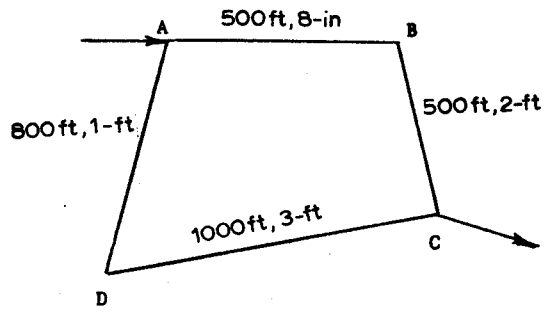


Fig. 13-19

13.26 The pipe network in Fig. 13-20 consists of pipes as follows: AB, 5000 ft, 12 in; BC, 3000 ft, 6 in; CD, 8000 ft, 24 in; DE, 7000 ft, 8 in; EA, 4000 ft, 10 in; BD, 7000 ft, 12 in. A flow of 4000 gpm enters the system at D, while outflow at the junctions is as follows: A, 500 gpm; B, 300 gpm; C, 1000 gpm; E, 2200 gpm. Find the flow in each pipe and the pressure at each junction if the head at D is 400 ft. Take $f = 0.023$.

▮

$$h_f = 8fLQ^2/\pi^2gD^5 = (8)(0.023)(LQ^2)/(\pi)^2(32.2)D^5 = 0.000579LQ^2/D^5$$

D^5	pipe	D, in	k
1.0	AB	12	2.89
0.031	BC	6	56.0
32	CD	24	0.143
0.132	DE	8	30.6
1.0	BD	12	4.05
0.402	AE	10	5.75

Loop 1:

$$\Delta_1 = [-(2.9)(3.62)^2 - (4.05)(3.40)^2 + (30.6)(2.43)^2 - (5.75)(2.5)^2]/$$
$$2[(2.9)(3.62) + (4.05)(3.40) + (30.6)(2.43) + (5.75)(2.5)] = -0.27$$

Loop 2:

$$\Delta_2 = -[-(56)(0.89)^2 - (0.143)(3.13)^2 + (4.05)(3.4)^2]/2[(56)(0.89) + (0.143)(3.13) + (4.05)(3.4)] = -0.01$$

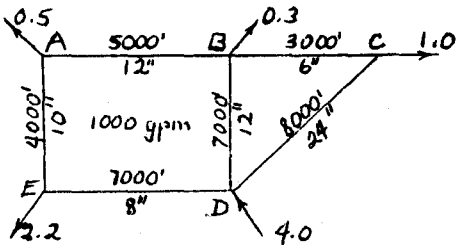
$$\Delta_1 = [-(2.9)(3.89)^2 - (4.05)(3.66)^2 + (30.6)(2.16)^2 - (5.75)(2.77)^2]/$$
$$2[(2.9)(3.89) + (4.05)(3.66) + (30.6)(2.16) + (5.75)(2.77)] = 0$$

$$\Delta_2 = -(-45.3 - 1.4 + 54.1)/2(50.5 + 0.45 + 14.8) = -0.056$$

$$\Delta_1 = -(-44 - 52.5 + 142.5 - 44)/2(11.3 + 14.6 + 66.1 + 15.9) = -0.01$$

$$\Delta_2 = -[-(56)(0.96)^2 - (0.143)(3.20)^2 + (4.05)(3.6)^2]/2(53.6 + 0.46 + 14.6) = +0.01$$

$$\Delta_1 = -44.1 - 53.1 + 140.2 - 44.2 = -1.2 \qquad \Delta_2 = -50.5 - 1.46 + 53.2 = +1.2$$

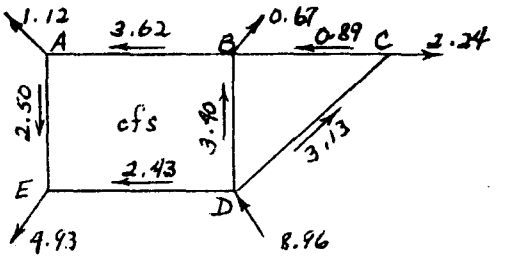


Flow in 1000's gpm

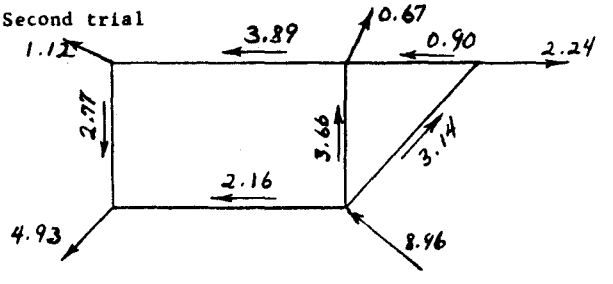
f = 0.023

1000 gpm = 2.24 cfs

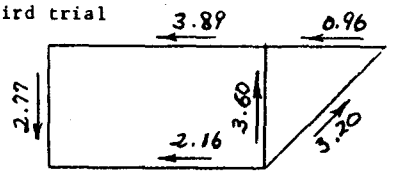
First trial



Second trial



Third trial



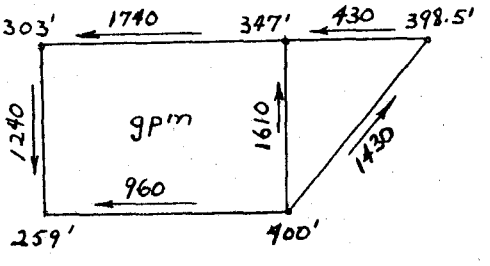


Fig. 13-20

13.27 The pipe network in Fig. 13-21 consists of the following pipes: *AB*, 3000 ft, 8 in; *BC*, 3000 ft, 12 in; *CD*, 10 000 ft, 36 in; *DE*, 8000 ft, 24 in; *EF*, 5000 ft, 6 in; *FA*, 4000 ft, 8 in; *BE*, 10 000 ft, 6 in; *BF*, 8000 ft, 12 in. Inflow at $D = 6$ cfs. Outflows are *A*, 2 cfs; *B*, 1 cfs; *E*, 1.5 cfs; *C*, 0.5 cfs; *F*, 1 cfs. Assume Manning $n = 0.015$, and find the flow in each pipe and the pressure at each junction if the pressure at *D* is 120 psi.

▮ $n = 0.015$ $\quad S = n^2V^2/2.21R^{4/3} = n^2Q^2/[2.21(D/4)^{4/3}(\pi^2D^4/16)]$

$$= [(0.015)^2Q^2(16)(6.4)]/(2.21\pi^2D^{16/3}) = 0.00106(Q^2/D^{16/3})$$

$$h_L = kQ^2 = (0.00106L/D^{16/3})(Q^2) \qquad k = 0.00106L/D^{16/3}$$

	D	$D^{16/3}$	$0.00106L$	k
AB	0.67	0.116	3.18	27.4
BC	1.00	1.0	3.18	3.2
CD	3.0	355	10.60	0.03
DE	2.0	40	8.48	0.21
EF	0.5	0.0245	5.30	216
FA	0.67	0.116	4.24	36.5
BF	1.0	1.0	8.48	8.48
BE	0.5	0.0245	10.60	432

Loop 1:

$$\Delta_1 = -\frac{-(27.4)(1.3)^2 + (8.48)(1.6)^2 + (36.5)(0.7)^2}{2[(27.4)(1.3) + (8.48)(1.6) + (36.5)(0.7)]} = +0.044$$

Loop 2:

$$\Delta_2 = -\frac{(216)(0.1)^2 - (8.48)(1.6)^2}{2[0 + (216)(0.1) + (8.48)(1.6)]} = +0.277$$

Loop 3:

$$\Delta_3 = -\frac{-(3.2)(3.9)^2 - (0.03)(4.4)^2 + (0.21)(1.6)^2}{2[(3.2)(3.9) + (0.03)(4.4) + (0.21)(1.6)]} = +1.88$$

$$\Delta_1 = -\frac{-43.5 + 15.8 + 20}{2[(27.4)(1.26) + (8.48)(1.36) + (36.5)(0.74)]} = +0.053$$

$$\Delta_2 = -\frac{-1110 + 31 - 15.6}{2[432(1.60) + 216(0.38) + (8.48)(1.36)]} = +0.695$$

$$\Delta_3 = -\frac{-13.1 - 0.19 + 2.56 + 1110}{2[(3.2)(2.02) + (0.03)(2.52) + (0.21)(3.48) + (432)(1.6)]} = -0.72$$

$$\Delta_1 = -\frac{-40 + 4.2 + 22.8}{2[(27.4)(1.21) + (8.48)(0.71) + (36.5)(0.79)]} = +0.096$$

$$\Delta_2 = -\frac{-14.1 + 251 - 4.2}{2[(432)(0.18) + (216)(1.08) + (8.48)(0.71)]} = -0.37$$

$$\Delta_3 = -\frac{-24.2 - 0.3 + 1.6 + 14.1}{2[(3.2)(2.74) + (0.03)(3.24) + (0.21)(2.76) + (432)(0.18)]} = +0.05$$

$$\Delta_1 = -\frac{-33.8 + 11.8 + 28.9}{2[(27.4)(1.11) + (8.48)(1.18) + (36.5)(0.89)]} = -0.05$$

$$\Delta_2 = -\frac{-155 + 109 - 11.8}{2[(432)(0.60) + (216)(0.71) + (8.48)(1.18)]} = +0.07$$

$$\Delta_3 = -\frac{-23 - 0.3 + 1.6 + 155}{2[(3.2)(2.69) + (0.03)(3.19) + (0.21)(2.81) + (432)(0.6)]} = -0.25$$

$$\Delta_1 = -\frac{-37 + 9.5 + 25.7}{2[(27.4)(1.16) + (8.5)(1.06) + (36.5)(0.84)]} = +0.01$$

$$\Delta_2 = -\frac{-33.8 + 132 - 9.6}{2[(432)(0.28) + (216)(0.78) + (8.5)(1.06)]} = -0.15$$

$$\Delta_3 = -\frac{-27.5 - 0.4 + 1.4 + 33.8}{2[(3.2)(2.94) + (0.03)(3.44) + (0.21)(2.56) + (432)(0.28)]} = -0.03$$

And after several more trials, final results are as shown on the last sketch of Fig. 13-21.

pressures (psi)	
A	87.5
B	104.5
C	119.8
D	120.0
E	119.5
F	98.0

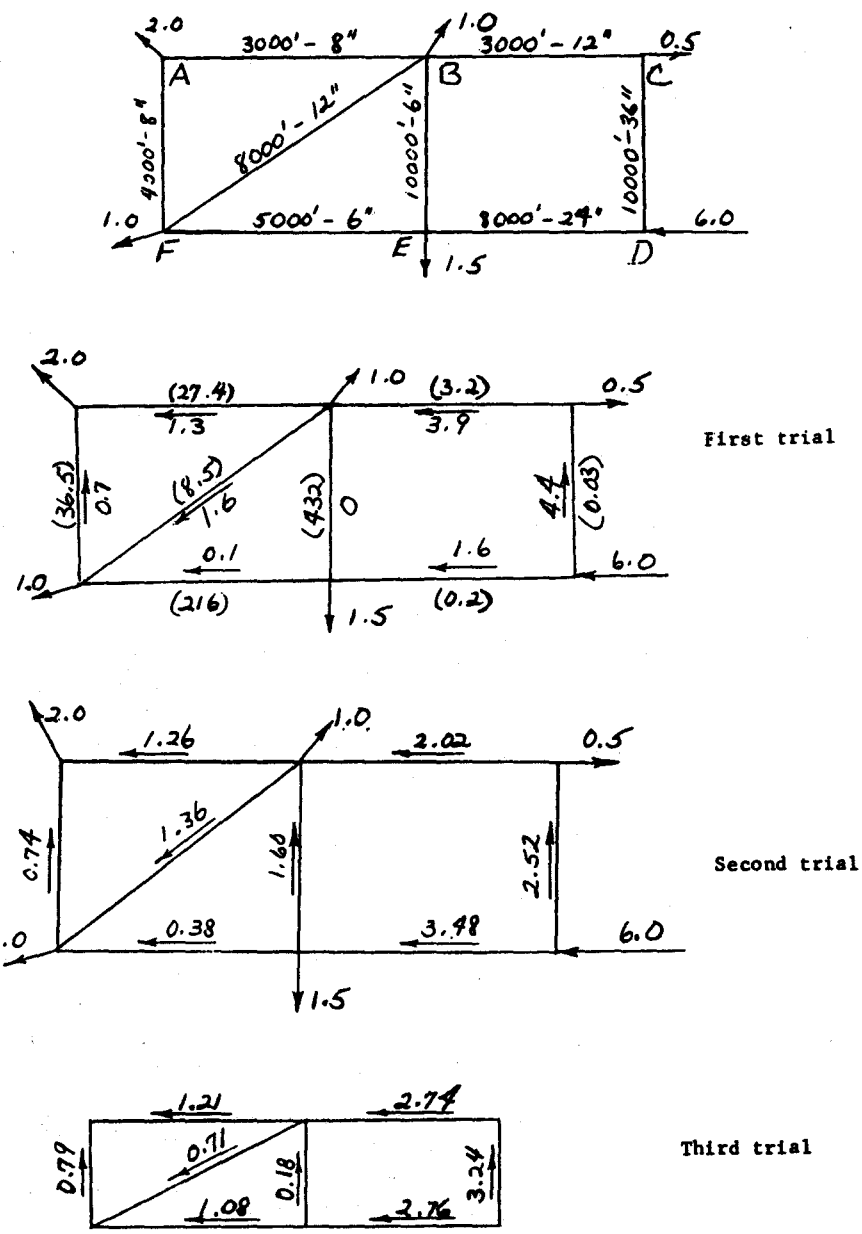
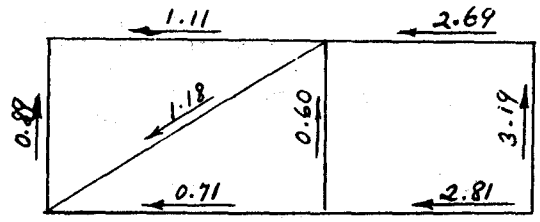
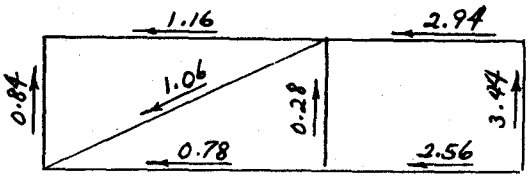


Fig. 13-21



Fourth trial



Fifth trial

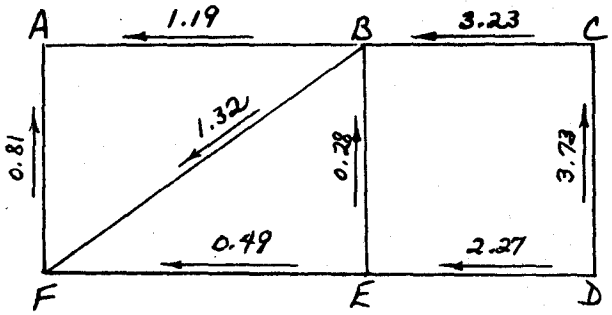


Fig. 13-21 ***(continued)***

13.28 Prepare a computer program written in Fortran to determine the flow in each pipe in a pipe network by the Hardy Cross method. The program must be usable in both the English system of units and the International system of units.

```
C     THIS PROGRAM DETERMINES THE FLOW RATE IN EACH PIPE IN A PIPE NET-
C     WORK BY THE HARDY CROSS METHOD.  IT CAN BE USED FOR PROBLEMS IN
C     BOTH THE ENGLISH SYSTEM OF UNITS AND THE INTERNATIONAL SYSTEM OF
C     UNITS.  EACH LOOP MUST BE NUMBERED AND EACH INDIVIDUAL PIPE MUST
C     ALSO BE NUMBERED.
C
C     INPUT DATA MUST BE SET UP AS FOLLOWS.
C
C     CARD 1   COLUMN 1          ENTER 0 (ZERO) OR BLANK IF ENGLISH SYSTEM
C                                OF UNITS IS TO BE USED.  ENTER 1 (ONE) IF
C                                INTERNATIONAL SYSTEM OF UNITS IS TO BE
C                                USED.
C              COLUMN 2          ENTER 0 (ZERO) OR BLANK IF ONLY THE
C                                RESULTS AFTER THE FINAL ITERATION ARE TO
C                                BE PRINTED.  ENTER 1 (ONE) IF THE RESULTS
C                                AFTER EACH ITERATION ARE TO BE PRINTED.
C              COLUMNS 3-5       ENTER INTEGER NUMBER (RIGHT ADJUSTED)
C                                TELLING HOW MANY LOOPS ARE IN THE PIPE
C                                NETWORK.
C              COLUMNS 6-80      ENTER TITLE, DATE, AND OTHER INFORMATION,
C                                IF DESIRED.
C     CARD 2   COLUMNS 1-2       ENTER INTEGER NUMBER (RIGHT ADJUSTED)
C                                TELLING HOW MANY PIPES ARE IN THE FIRST
C                                LOOP.
C              COLUMNS 3-4       ENTER INTEGER NUMBER (RIGHT ADJUSTED)
C                                TELLING HOW MANY PIPES ARE IN THE SECOND
C                                LOOP.
C              COLUMNS 5-6       ENTER INTEGER NUMBER (RIGHT ADJUSTED)
C                                TELLING HOW MANY PIPES ARE IN THE THIRD
C                                LOOP.
C              (CONTINUE PATTERN FOR ALL LOOPS.)
C     CARD 3   COLUMNS 1-3       ENTER INTEGER NUMBER (RIGHT ADJUSTED) TO
C                                IDENTIFY AND TO REPRESENT A PARTICULAR
C                                PIPE IN THE FIRST LOOP.
```

```
C              COLUMNS 4-6      IF THIS PIPE IS IN COMMON WITH ANY OTHER
C                               LOOP, ENTER INTEGER NUMBER (RIGHT AD-
C                               JUSTED) IDENTIFYING THAT LOOP.  OTHER-
C                               WISE, ENTER 0 (ZERO) OR BLANK.
C              COLUMNS 7-16     ENTER NUMBER INCLUDING DECIMAL GIVING
C                               DIAMETER OF THIS PIPE (IN INCHES OR
C                               MILLIMETERS).
C              COLUMNS 17-26    ENTER NUMBER INCLUDING DECIMAL GIVING
C                               LENGTH OF THIS PIPE (IN FEET OR METERS).
C              COLUMNS 27-36    ENTER NUMBER INCLUDING DECIMAL GIVING
C                               INITIAL ESTIMATE OF FLOW RATE FOR THIS
C                               PIPE (IN CUBIC FEET PER SECOND OR CUBIC
C                               METERS PER SECOND).  (ENTER CLOCKWISE
C                               FLOW RATES WITH RESPECT TO THIS LOOP AS
C                               POSITIVE AND COUNTERCLOCKWISE FLOW RATES
C                               AS NEGATIVE.)
C              COLUMNS 37-46    ENTER NUMBER INCLUDING DECIMAL GIVING
C                               HAZEN-WILLIAMS ROUGHNESS COEFFICIENT FOR
C                               THIS PIPE.
C              (ENTER ADDITIONAL CARDS LIKE CARD 3 FOR EACH REMAINING
C              PIPE IN THE FIRST LOOP, THEN FOR EACH PIPE IN THE SECOND
C              LOOP, ETC. FOR ALL LOOPS.)
C
C     MULTIPLE DATA SETS FOR SOLVING ANY NUMBER OF PROBLEMS MAY BE
C     INCLUDED FOR PROCESSING.
C
      DIMENSION TITLE(13),DIAM(100,10),Q(100,10),ROUGH(100,10),
     *     HLOSS(100,10),QDEL(100)
      REAL LENGTH(100,10)
      INTEGER UNITS,LOOPS,PIPES(100),PPLOOP,PIPENO(100,10),OLOOP(100,10)
1     READ(5,100,END=2)UNITS,IWRITE,LOOPS,TITLE
  100 FORMAT(2I1,I3,12A6,A3)
      WRITE(6,105)TITLE
  105 FORMAT('1',12A6,A3,////)
      COEFF=1.318
      FACTOR=12.0
      ERROR=.01
      IF(UNITS.EQ.1)COEFF=.8492
      IF(UNITS.EQ.1)FACTOR=1000.0
      IF(UNITS.EQ.1)ERROR=.001
      READ(5,101)(PIPES(J),J=1,LOOPS)
  101 FORMAT(40I2)
      DO 200 J=1,LOOPS
      PPLOOP=PIPES(J)
      DO 200 K=1,PPLOOP
  200 READ(5,102)PIPENO(J,K),OLOOP(J,K),DIAM(J,K),LENGTH(J,K),Q(J,K),
     *     ROUGH(J,K)
  102 FORMAT(2I3,4F10.0)
      NIT=1
  205 IF(IWRITE.EQ.1)WRITE(6,106)NIT
  106 FORMAT(//38X,'ITERATION NO.',I3,/,38X,'========= === ==',//)
      IF(IWRITE.EQ.1.AND.UNITS.EQ.0)WRITE(6,107)
  107 FORMAT(' LOOP NO.  PIPE NO.  DIAMETER (IN)  LENGTH (FT)  ROUGHNESS
     *  FLOW RATE (CFS)  HEAD LOSS (FT)',/' --------  --------  --------
     *-----  -----------  ---------  ---------------  --------------'/)
      IF(IWRITE.EQ.1.AND.UNITS.EQ.1)WRITE(6,108)
  108 FORMAT(' LOOP NO.  PIPE NO.  DIAMETER (MM)  LENGTH ( M)  ROUGHNESS
     *  FLOW RATE (CMS)  HEAD LOSS ( M)',/' --------  --------  --------
     *-----  -----------  ---------  ---------------  --------------'/)
      DO 201 J=1,LOOPS
      IF(J.EQ.1)NIT=NIT+1
      SHLOSS=0.0
      SHQ=0.0
      PPLOOP=PIPES(J)
      DO 202 K=1,PPLOOP
      HLOSS(J,K)=(ABS(Q(J,K))*4.0**1.63/3.14159265/(DIAM(J,K)/FACTOR)
     *     **2.63/COEFF/ROUGH(J,K))**(1.0/.54)*LENGTH(J,K)
      IF(Q(J,K).LT.0.0)HLOSS(J,K)=-HLOSS(J,K)
      HQ=HLOSS(J,K)/Q(J,K)
      SHLOSS=SHLOSS+HLOSS(J,K)
      SHQ=SHQ+HQ
      IF(IWRITE.EQ.1)WRITE(6,109)J,PIPENO(J,K),DIAM(J,K),LENGTH(J,K),
     *     ROUGH(J,K),Q(J,K),HLOSS(J,K)
```

```
109 FORMAT(3X,I3,7X,I3,7X,F7.1,6X,F8.0,7X,F5.0,7X,F8.3,7X,F9.3)
202 CONTINUE
    QDEL(J)=-SHLOSS/1.85/SHQ
    IF(IWRITE.EQ.1.AND.UNITS.EQ.0)WRITE(6,110)SHLOSS,J,QDEL(J)
110 FORMAT(78X,'------------',/,78X,F9.3,/1X,'FLOW RATE ADJUSTMENT FOR
   * LOOP',I3,' = ',F7.3,' CFS',/)
    IF(IWRITE.EQ.1.AND.UNITS.EQ.1)WRITE(6,111)SHLOSS,J,QDEL(J)
111 FORMAT(78X,'------------',/,78X,F9.3,/1X,'FLOW RATE ADJUSTMENT FOR
   * LOOP',I3,' = ',F7.3,' CMS',/)
201 CONTINUE
    DO 203 J=1,LOOPS
    PPLOOP=PIPES(J)
    DO 203 K=1,PPLOOP
    Q(J,K)=Q(J,K)+QDEL(J)
    L=OLOOP(J,K)
    IF(OLOOP(J,K).NE.O)Q(J,K)=Q(J,K)-QDEL(L)
203 CONTINUE
    DO 204 J=1,LOOPS
    IF(ABS(QDEL(J)).GT.ERROR)GO TO 205
204 CONTINUE
    IF(IWRITE.EQ.1)GO TO 1
    IF(UNITS.EQ.0)WRITE(6,107)
    IF(UNITS.EQ.1)WRITE(6,108)
    DO 206 J=1,LOOPS
    PPLOOP=PIPES(J)
    DO 206 K=1,PPLOOP
    IF(K.NE.PPLOOP)WRITE(6,109)J,PIPENO(J,K),DIAM(J,K),LENGTH(J,K),
   *     ROUGH(J,K),Q(J,K),HLOSS(J,K)
    IF(K.EQ.PPLOOP)WRITE(6,112)J,PIPENO(J,K),DIAM(J,K),LENGTH(J,K),
   *     ROUGH(J,K),Q(J,K),HLOSS(J,K)
112 FORMAT(3X,I3,7X,I3,7X,F7.1,6X,F8.0,7X,F5.0,7X,F8.3,7X,F9.3,/)
206 CONTINUE
    NIT=NIT-1
    WRITE(6,104)NIT
104 FORMAT(//,1X,I3,' ITERATIONS WERE REQUIRED.')
    GO TO 1
2   STOP
    END
```

13.29 Use the computer program written for Prob. 13.28 to solve for the flow rate in each pipe of the network of Prob. 13.3.

Input

```
01  2SAMPLE PIPE NETWØRK ANALYSIS
 4 4
  1          18.     2000.      7.00      120.
  2  2       24.      900.      3.50      120.
  3          12.     2800.     -2.19      120.
  4          24.     1100.     -7.00      120.
  5          12.     2200.      3.50      120.
  6          18.      750.     -3.14      120.
  7          24.     2600.     -5.69      120.
  2  1       24.      900.     -3.50      120.
```

Output

```
SAMPLE PIPE NETWORK ANALYSIS

                                   ITERATION NO.  1
                                   ========= === ==

LOOP NO.  PIPE NO.  DIAMETER (IN)  LENGTH (FT)  ROUGHNESS  FLOW RATE (CFS)  HEAD LOSS (FT)
--------  --------  -------------  -----------  ---------  ---------------  --------------

   1         1         18.0         2000.       120.        7.000             6.803
   1         2         24.0          900.       120.        3.500             0.209
   1         3         12.0         2800.       120.       -2.190            -7.978
   1         4         24.0         1100.       120.       -7.000            -0.922
                                                                           -------------
                                                                             -1.888
```

FLOW RATE ADJUSTMENT FOR LOOP 1 = 0.212 CFS

2	5	12.0	2200.	120.	3.500	14.936
2	6	18.0	750.	120.	-3.140	-0.578
2	7	24.0	2600.	120.	-5.690	-1.484
2	2	24.0	900.	120.	-3.500	-0.209
						12.665

FLOW RATE ADJUSTMENT FOR LOOP 2 = -1.435 CFS

ITERATION NO. 2

LOOP NO.	PIPE NO.	DIAMETER (IN)	LENGTH (FT)	ROUGHNESS	FLOW RATE (CFS)	HEAD LOSS (FT)
1	1	18.0	2000.	120.	7.212	7.190
1	2	24.0	900.	120.	5.147	0.427
1	3	12.0	2800.	120.	-1.978	-6.605
1	4	24.0	1100.	120.	-6.788	-0.870
						0.141

FLOW RATE ADJUSTMENT FOR LOOP 1 = -0.017 CFS

2	5	12.0	2200.	120.	2.065	5.624
2	6	18.0	750.	120.	-4.575	-1.160
2	7	24.0	2600.	120.	-7.125	-2.251
2	2	24.0	900.	120.	-5.147	-0.427
						1.786

FLOW RATE ADJUSTMENT FOR LOOP 2 = -0.286 CFS

ITERATION NO. 3

LOOP NO.	PIPE NO.	DIAMETER (IN)	LENGTH (FT)	ROUGHNESS	FLOW RATE (CFS)	HEAD LOSS (FT)
1	1	18.0	2000.	120.	7.196	7.159
1	2	24.0	900.	120.	5.416	0.469
1	3	12.0	2800.	120.	-1.994	-6.709
1	4	24.0	1100.	120.	-6.804	-0.874
						0.044

FLOW RATE ADJUSTMENT FOR LOOP 1 = -0.005 CFS

2	5	12.0	2200.	120.	1.779	4.267
2	6	18.0	750.	120.	-4.861	-1.298
2	7	24.0	2600.	120.	-7.411	-2.421
2	2	24.0	900.	120.	-5.416	-0.469
						0.079

FLOW RATE ADJUSTMENT FOR LOOP 2 = -0.014 CFS

ITERATION NO. 4

LOOP NO.	PIPE NO.	DIAMETER (IN)	LENGTH (FT)	ROUGHNESS	FLOW RATE (CFS)	HEAD LOSS (FT)
1	1	18.0	2000.	120.	7.190	7.149
1	2	24.0	900.	120.	5.425	0.470
1	3	12.0	2800.	120.	-2.000	-6.742
1	4	24.0	1100.	120.	-6.810	-0.876
						0.002

```
FLOW RATE ADJUSTMENT FOR LOOP  1 =  -0.000 CFS

    2        5        12.0       2200.      120.       1.765        4.206
    2        6        18.0        750.      120.      -4.875       -1.305
    2        7        24.0       2600.      120.      -7.425       -2.429
    2        2        24.0        900.      120.      -5.425       -0.470
                                                                  ----------
                                                                     0.001

FLOW RATE ADJUSTMENT FOR LOOP  2 =  -0.000 CFS
```

13.30 Use the computer program written for Prob. 13.28 to solve for the flow rate in each pipe of the network shown in Fig. 13-22*a*.

I It is necessary to assume an initial value of flow rate for each pipe in the network. The values assumed for this example are shown in Fig. 13-22*b*.

Input

```
10  7SAMPLE PIPE NETWØRK ANALYSIS
 4 4 5 6 3 4 4
  1  0      1000.      1000.      -0.6       100.
  2  2       750.       925.       0.4       100.
  3  3       750.      1000.      1.15       100.
  4  0       750.       925.      -2.6       100.
 10  0      1000.      1000.      -2.0       120.
 11  0      1000.       925.      10.0       120.
  9  4       750.      1000.       4.0       120.
  2  1       750.       925.      -0.4       100.
  3  1       750.      1000.     -1.15       100.
  8  4       500.       650.      2.25       100.
  7  0       500.       400.     -0.75       100.
  6  0       500.       671.     -1.75       100.
  5  0       500.       350.     -2.75       100.
  9  2       750.      1000.      -4.0       120.
 12  0      1000.       800.       5.0       120.
 14  5      1000.       650.       2.0       120.
 21  6       500.       800.      0.75       120.
 22  0       500.      1000.      -2.0       120.
  8  3       500.       650.     -2.25       100.
 13  0       750.       763.       2.0       120.
 15  7       750.       400.       0.5       120.
 14  4      1000.       650.      -2.0       120.
 21  4       500.       800.     -0.75       120.
 18  7       500.       125.      0.75       120.
 19  0       500.       800.     -0.75       120.
 20  0       500.       125.     -1.75       120.
 15  5       750.       400.      -0.5       120.
 16  0       750.       125.       0.5       120.
 17  0       750.       400.      -0.5       120.
 18  6       500.       125.     -0.75       120.
```

Output

```
SAMPLE PIPE NETWORK ANALYSIS

LOOP NO.  PIPE NO.  DIAMETER (MM)  LENGTH ( M)  ROUGHNESS  FLOW RATE (CMS)  HEAD LOSS ( M)
--------  --------  -------------  -----------  ---------  ---------------  --------------

   1         1        1000.0        1000.       100.        -0.532          -0.655
   1         2         750.0         925.       100.         2.537          44.438
   1         3         750.0        1000.       100.         0.211           0.481
   1         4         750.0         925.       100.        -2.532         -44.269
```

continued

LOOP NO.	PIPE NO.	DIAMETER (MM)	LENGTH (M)	ROUGHNESS	FLOW RATE (CMS)	HEAD LOSS (M)
2	10	1000.0	1000.	120.	-4.068	-20.247
2	11	1000.0	925.	120.	7.932	64.482
2	9	750.0	1000.	120.	0.152	0.187
2	2	750.0	925.	100.	-2.537	-44.438
3	3	750.0	1000.	100.	-0.211	-0.481
3	8	500.0	650.	100.	1.478	82.694
3	7	500.0	400.	100.	0.258	1.998
3	6	500.0	671.	100.	-0.742	-23.884
3	5	500.0	350.	100.	-1.742	-60.446
4	9	750.0	1000.	120.	-0.152	-0.187
4	12	1000.0	800.	120.	6.780	41.703
4	14	1000.0	650.	120.	3.932	12.357
4	21	500.0	800.	120.	0.940	31.420
4	22	500.0	1000.	120.	-0.220	-2.676
4	8	500.0	650.	100.	-1.478	-82.694
5	13	750.0	763.	120.	1.848	14.537
5	15	750.0	400.	120.	-0.942	-2.191
5	14	1000.0	650.	120.	-3.932	-12.357
6	21	500.0	800.	120.	-0.940	-31.420
6	18	500.0	125.	120.	1.050	6.025
6	19	500.0	800.	120.	0.840	25.500
6	20	500.0	125.	120.	-0.160	-0.186
7	15	750.0	400.	120.	0.942	2.191
7	16	750.0	125.	120.	1.790	2.246
7	17	750.0	400.	120.	0.790	1.580
7	18	500.0	125.	120.	-1.050	-6.025

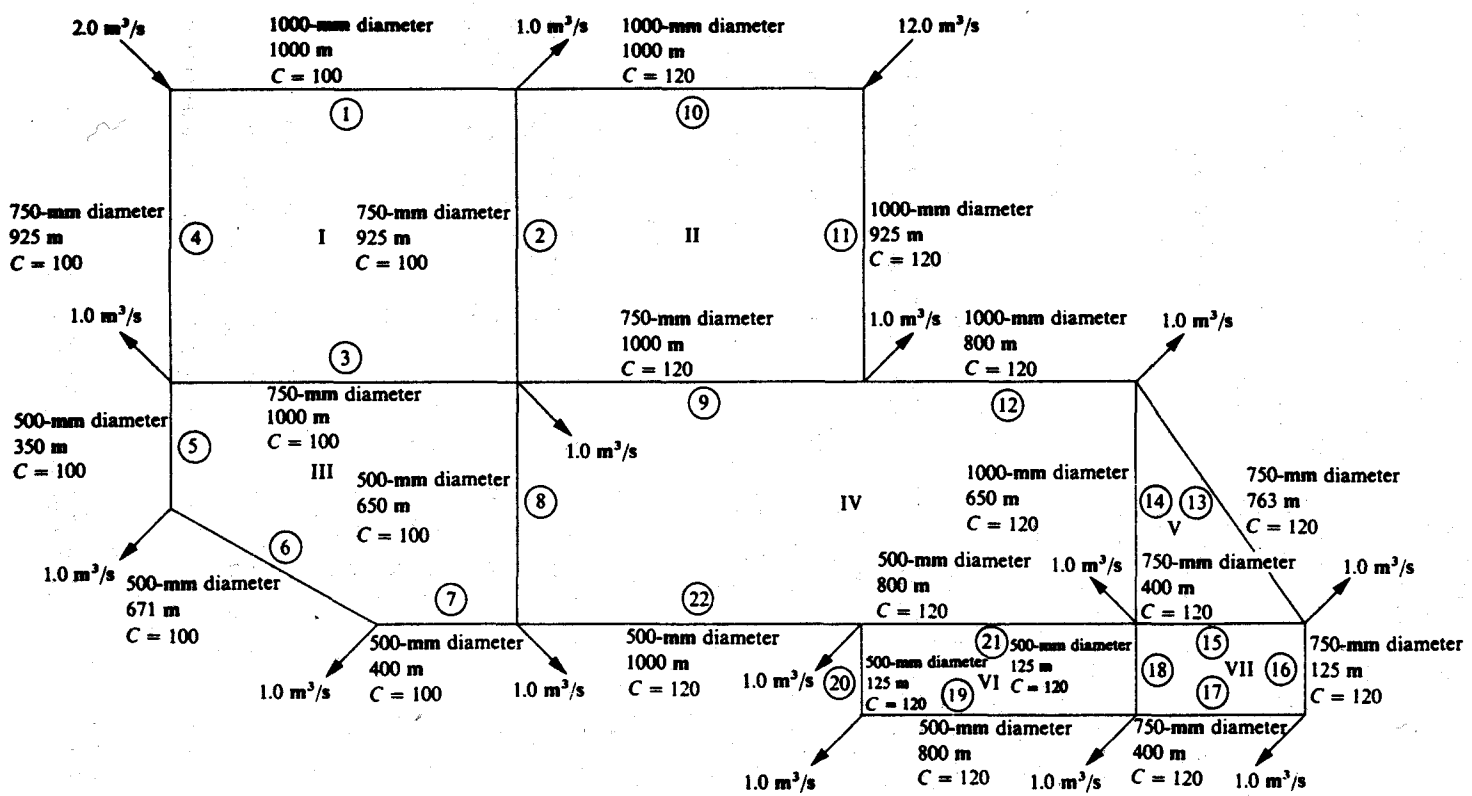
ITERATIONS WERE REQUIRED.

For illustrative purposes, the actual flow rates as determined by the computer program are shown on Fig. 13-22*c*.

13.31 Prepare a compute program written in Basic to determine the flow in each pipe in a pipe network by the Hardy Cross method.

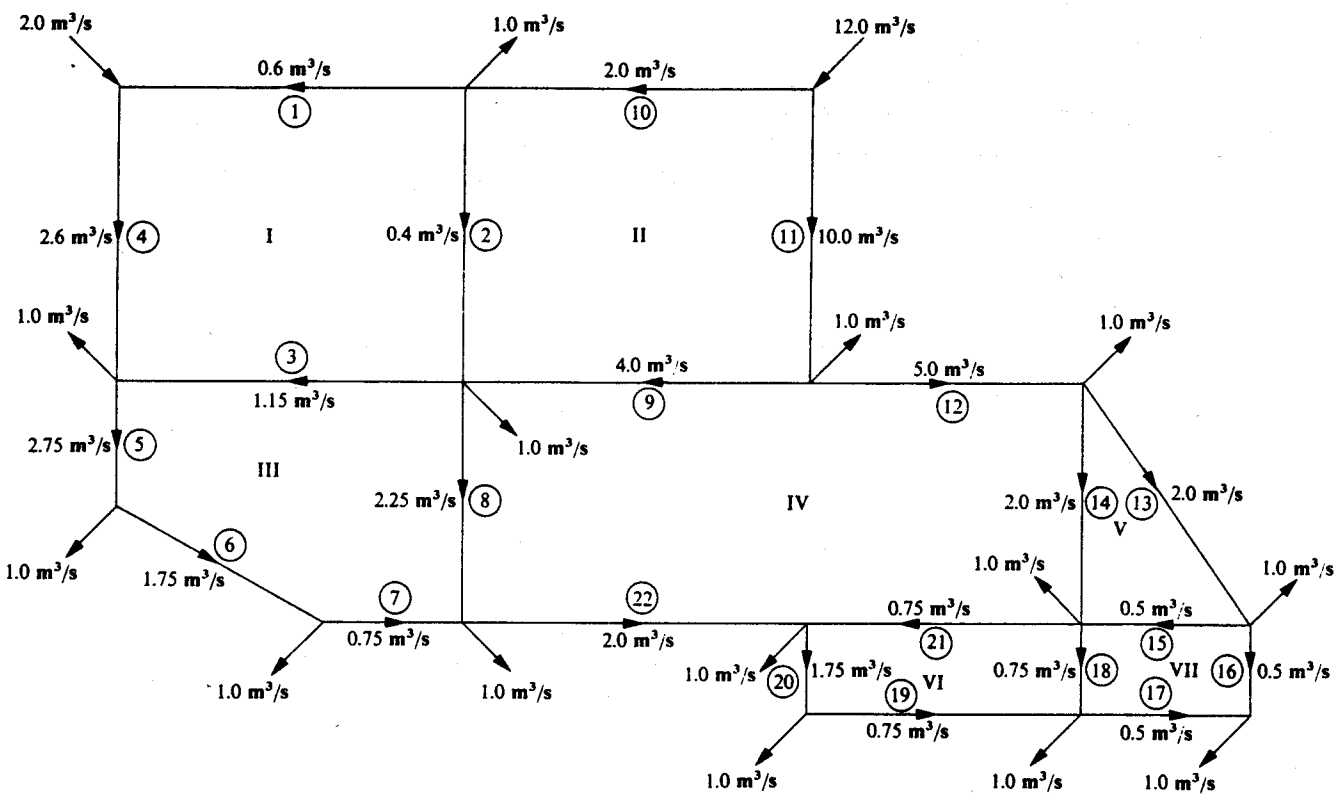
```
10  'HARDY-CROSS LOOP BALANCING NETWORK PROGRAM,                NET.BAS
20  'U.S. CUSTOMARY(USC) OR SI UNITS(SI) MAY BE USED.
30  'HAZEN-WILLIAMS(HW) OR DARCY-WEISBACH(DW) EQUATION MAY BE USED FOR PIPES.
40  'DATA ENTRY VIA READ AND DATA STATEMENTS.
50  '*******READ DATA AND PRINT NETWORK INFORMATION***************************
60  DEFINT I,J,K,N
70  DIM ITYPE(1000),ELEM(500),IND(500),Q(100),H(100),S(20),IX(240)
80  FOR J=1 TO 100:            ITYPE(J)=5:          H(J)=-1000!:   NEXT J
90  READ TITLE$:               LPRINT:              LPRINT"   ";TITLE$
100 READ TT$,KK,TOL,VNU,DEFA 'NT$=USC OR SI, KK=NO. OF ITER., VNU=KIN. VISC.
110 'TOL=TOLERANCE IN ITERATION, DEFA=DEFAULT COEF.- EITHER C OR EPS
120 IF(TT$="SI" OR TT$="si") THEN GOTO 150
130 UNITS=4.727:               G=32.174:
140 LPRINT" US CUSTOMARY UNITS SPEC.,VISCOSITY IN FT^2/S=";VNU: GOTO 170
150 UNITS=10.674:              G=9.806:
160 LPRINT" SI UNITS SPEC.,VISCOSITY IN  M^2/S=";VNU
170 LPRINT" DESIRED TOLERANCE=";TOL;   " NO. OF ITERATIONS=";KK
180 LPRINT"   PIPE Q(CFS OR M^3/S)  L(FT OR M)      D(FT OR M)  HW C OR EPS"
190 READ NPI,TT$:              IF NPI=0 THEN 310
200 'NPI=NO. OF PIPES IN NETWORK, TT$="HW" OR "DW"
210 FOR I1=1 TO NPI:           READ I,QQ,L,D,X3:          IF X3=0! THEN X3=DEFA
220 'I=PIPE NO.,QQ=FLOW,L=LENGTH,D=DIAMETER,X3=C OR EPS
230 Q(I)=QQ:                   KP=4*(I-1)+1
240 IF TT$="HW" OR TT$="hw" THEN 280
250 ITYPE(I)=2:                ELEM(KP)=L/(2!*G*D^5*.7854^2):                 'DW
260 ELEM(KP+1)=1!/(.7854*D*VNU): ELEM(KP+2)=X3/(3.7*D):
270 EX=2!:                     GOTO 290
280 ITYPE(I)=1:                EX=1.852:   ELEM(KP)=UNITS*L/(X3^EX*D^4.8704)  'HW
290 EN=EX-1!:                  LPRINT"   ";I;:
300 LPRINT USING"   #####.######";Q(I);L;D;X3:                    NEXT I1
310 READ NPS,TT$:              IF NPS=0 THEN 360
320 'NPS=NO. OF PSEUDO ELEMENTS, TT$="PS"
330 FOR I1=1 TO NPS:           READ I,DH:          ITYPE(I)=3:      KP=4*(I-1)+1
```

```
340 ELEM(KP)=DH:                LPRINT"   ";I;" RESERVOIR ELEV. DIFFERENCE=";DH
350 NEXT I1
360 READ NPU,TT$:               IF NPU=0 THEN 480
370 'NPU=NO. OF PUMPS, TT$="PU"
380 FOR I1=1 TO NPU:            READ I,QQ,DQ,H1,H2,H3,H4:            ITYPE(I)=4
390 'I=PUMP NO.,QQ=FLOW,DQ=DEL Q, H1,H2,H3,H4=EQUALLY SPACED PTS ON PUMP CURVE
400 KP=4*(I-1)+1:                Q(I)=QQ:            ELEM(KP)=H1
410 ELEM(KP+3)=(H4-3!*(H3-H2)-H1)/(6!*DQ^3)
420 ELEM(KP+2)=(H3-2!*H2+H1)/(2!*DQ^2)-3!*ELEM(KP+3)*DQ
430 ELEM(KP+1)=(H2-H1)/DQ-ELEM(KP+2)*DQ-ELEM(KP+3)*DQ^2
440 LPRINT"   ";I;" PUMP CURVE, DQ=";DQ;" H=";H1,H2,H3,H4
450 LPRINT"        COEF. IN PUMP EQ.=";ELEM(KP);ELEM(KP+1);ELEM(KP+2);ELEM(KP+3)
460 NEXT I1
470 '*******READ LOOP INDEXING DATA AND BALANCE ALL LOOPS*********************
480 READ NI,TT$:                IF NI=0 THEN 820
490 'NI=NO. OF ITEMS IN VECTOR IND, TT$="IND"
500 FOR I=1 TO NI:              READ IND(I):        NEXT I:        IND(NI+1)=0
510 LPRINT " IND=";:            FOR I=1 TO NI:      LPRINT IND(I);: NEXT I
520 FOR K=1 TO KK:              DDQ=0!:             IP=1
530 I1=IND(IP):                 IF I1=0 THEN 780
540 DH=0!:                      HDQ=0!
550 FOR J=1 TO I1:              I=IND(IP+J):        IF I<0 THEN 570
560 IF I=0 THEN 710 ELSE GOTO 580
570 S(J)=-1!:                   I=-I:               GOTO 590
580 S(J)=1!
590 NTY=ITYPE(I):               KP=4*(I-1)+1
600 ON NTY GOTO 610,620,680,690
610 R=ELEM(KP):                 GOTO 660
620 REY=ELEM(KP+1)*ABS(Q(I)):                       IF REY<1! THEN REY=1!
630 IF REY<2000! THEN 640 ELSE GOTO 650
640 R=ELEM(KP)*64!/REY:         GOTO 660
650 R=ELEM(KP)*1.325/(LOG(ELEM(KP+2)+5.74/REY^.9))^2
660 DH=DH+S(J)*R*Q(I)*ABS(Q(I))^EN
670 HDQ=HDQ+EN*R*ABS(Q(I))^EN:                      GOTO 710
680 DH=DH+S(J)*ELEM(KP):                            GOTO 710
690 DH=DH-S(J)*(ELEM(KP)+Q(I)*(ELEM(KP+1)+Q(I)*(ELEM(KP+2)+Q(I)*ELEM(KP+3))))
700 HDQ=HDQ-(ELEM(KP+1)+2!*ELEM(KP+2)*Q(I)+3!*ELEM(KP+3)*Q(I)^2)
710 NEXT J
720 IF ABS(HDQ)<.0001 THEN HDQ=1!
730 DQ=-DH/HDQ:                 DDQ=DDQ+ABS(DQ)
740 FOR J=1 TO I1:              I=ABS(IND(IP+J)):   IF ITYPE(I)=3 THEN 760
750 Q(I)=Q(I)+S(J)*DQ
760 NEXT J
770 IP=IP+I1+1:                 GOTO 530
780 LPRINT:                 LPRINT" ITERATION NO.";K;" SUM OF FLOW CORRECTIONS=";
790 LPRINT USING "###.####";DDQ
800 IF DDQ<TOL THEN 820
810 NEXT K
820 LPRINT" ELEMENT FLOW":   FOR I=1 TO 100:     NTY=ITYPE(I)
830 ON NTY GOTO 840,840,850,840,850
840 LPRINT"   ";I;:             LPRINT USING"   ###.####";Q(I)
850 NEXT I
855 '******* DATA FOR PATH THRU SYSTEM TO COMPUTE HGL*************************
860 READ NU,TT$:    IF NU=0 THEN 80  'NU=NO. OF NODES WITH GIVEN HGL,TT$=NODES
865 FOR I=1 TO NU:              READ I1,H2:         H(I1)=H2:      NEXT I
870 READ NI,TT$:                IF NI=0 THEN 80  'NI=NO. OF ITEMS IN PATH,TT$="IX"
875 FOR I=1 TO NI:              READ IX(I):         NEXT I:        IX(NI+2)=0
880 LPRINT" IX=";:              FOR I=1 TO NI:      LPRINT IX(I);: NEXT I: LPRINT
885 IP=1
890 FOR J=1 TO 238 STEP 2:   IF J=1 THEN K=IX(IP)
895 I=IX(IP+J):                 N=IX(IP+J+1)
900 IF I<1 THEN SS=-1!:         I=-I:               GOTO 910
905 IF I=0 THEN 955 ELSE SS=1!
910 NTY=ITYPE(I):               KP=4*(I-1)+1
915 ON NTY GOTO 920,925,945,950,955
920 R=ELEM(KP):                 GOTO 940
925 REY=ELEM(KP+1)*ABS(Q(I)):                       IF REY<1! THEN REY=1!
930 IF REY<2000! THEN R=ELEM(KP)*64!/REY:           GOTO 940
935 IF REY>=2000! THEN R=ELEM(KP)*1.325/(LOG(ELEM(KP+2)+5.74/REY^.9))^2
940 H(N)=H(K)-SS*R*Q(I)*ABS(Q(I))^EN:               GOTO 955
945 H(N)=H(K)-SS*ELEM(KP):                          GOTO 955
950 H(N)=H(K)+SS*(ELEM(KP)+Q(I)*(ELEM(KP+1)+Q(I)*(ELEM(KP+2)+Q(I)*ELEM(KP+3))))
955 IF IX(J+IP+3)=0 THEN 975
960 IF IX(J+IP+2)=0 THEN 970
965 K=N:                        NEXT J
970 IP=IP+J+3:                  GOTO 890
975 LPRINT" JUNCTION HEAD"
980 FOR N=1 TO 100:             IF H(N)=-1000! THEN 990
985 LPRINT"   ";N;:             LPRINT USING" ###.###";H(N)
990 NEXT N
995 GOTO 80
```



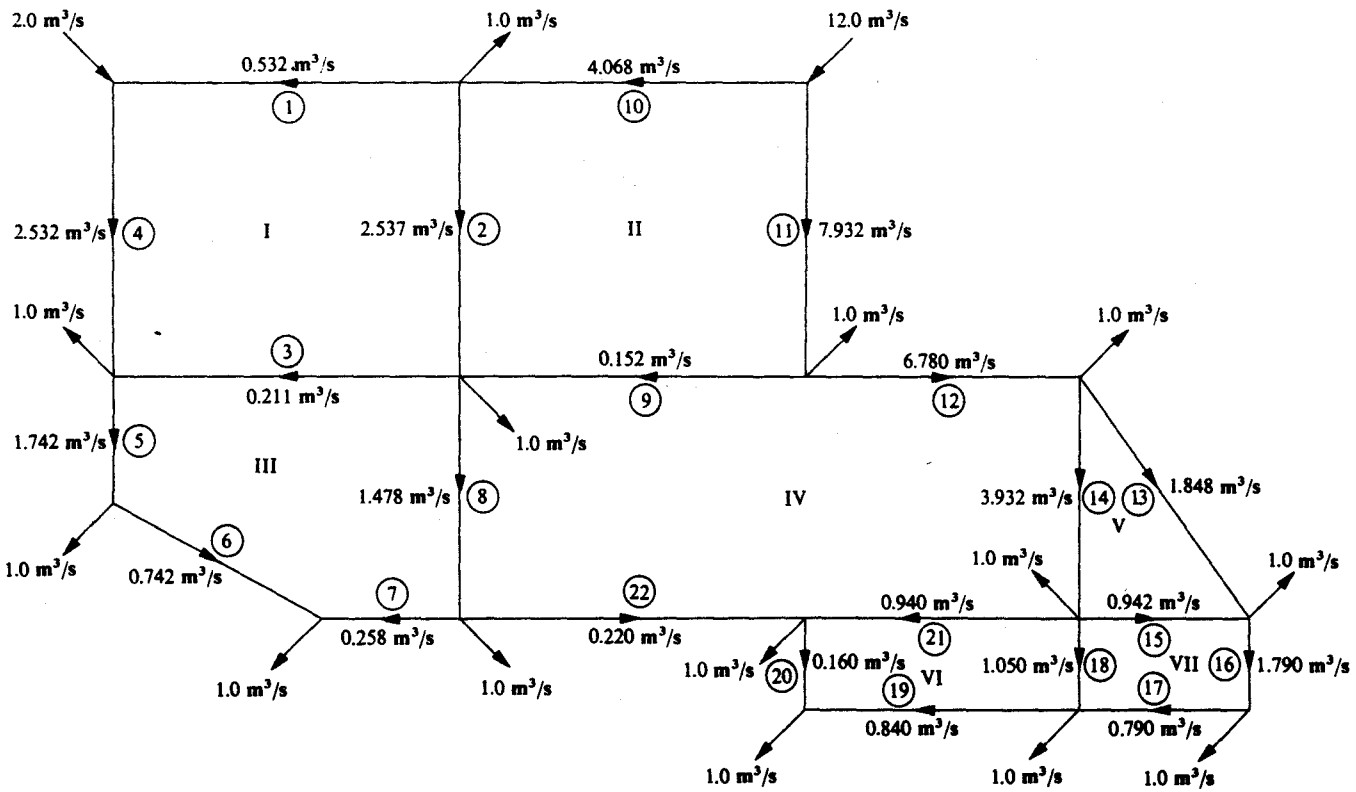
(*a*) Pipe network

Fig. 13-22



(*b*) Initial flow rate estimates

Fig. 13-22 (*continued*)



(c) Adjusted flow rates after 16 iterations

Fig. 13-22 (*continued*)

13.32 Use the computer program written for Prob. 13.31 to solve the network problem displayed in Fig. 13-23. The pump data are as follows:

Q, m³/s	0	0.03	0.06	0.09
H, m	30	29	26	20

Input

```
DATA EXAMPLE
DATA SI,30,.001,.000001,100.
DATA 5,HW
DATA 1,.12,600.,0.3,.0
DATA 2,.03,300.,.15,.0
DATA 3,.0,500.,.6,.0
DATA 4,.03,400.,.3,.0
DATA 5,.03,300.,.3,.0
DATA 2,PS
DATA 6,15.0
DATA 7,18.0
DATA 1,PU
DATA 8,.06,.03,30.,29.,26.,20.
DATA 16,IND
DATA 3,2,1,-3,3,4,-5,3,3,6,-4,-1,3,5,7,8
DATA 1,NODES
DATA 5,117.
DATA 9,IX
DATA 5,8,4,2,2,1,1,4,3
```

Output

```
  EXAMPLE
SI UNITS SPEC.,VISCOSITY IN  M^2/S= .000001
DESIRED TOLERANCE= .001  NO. OF ITERATIONS= 30
  PIPE Q(CFS OR M^3/S)  L(FT OR M)      D(FT OR M)  HW C OR EPS
   1        0.12000     600.00000       0.30000     100.00000
   2        0.03000     300.00000       0.15000     100.00000
   3        0.00000     500.00000       0.60000     100.00000
   4        0.03000     400.00000       0.30000     100.00000
   5        0.03000     300.00000       0.30000     100.00000
   6  RESERVOIR ELEV. DIFFERENCE= 15
   7  RESERVOIR ELEV. DIFFERENCE= 18
   8  PUMP CURVE, DQ= .03  H= 30          29            26            20
      COEF. IN PUMP EQ.= 30 -11.11112 -555.5555 -6172.841
IND= 3  2  1 -3  3  4 -5  3  3  6 -4 -1  3  5  7  8
ITERATION NO. 1  SUM OF FLOW CORRECTIONS=  0.1385

ITERATION NO. 2  SUM OF FLOW CORRECTIONS=  0.1040

ITERATION NO. 3  SUM OF FLOW CORRECTIONS=  0.0372

ITERATION NO. 4  SUM OF FLOW CORRECTIONS=  0.0034

ITERATION NO. 5  SUM OF FLOW CORRECTIONS=  0.0006
ELEMENT   FLOW
   1      0.143
   2     -0.034
   3      0.027
   4      0.080
   5      0.094
   8      0.087
IX= 5  8  4  2  2  1  1  4  3
JUNCTION HEAD
   1  137.811
   2  150.044
   3  135.044
   4  137.797
   5  117.000
```

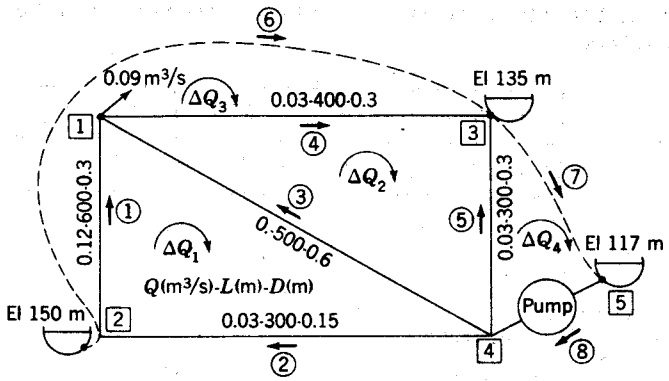

⑥
0.09 m³/s
ΔQ₃
0.03-400-0.3
El 135 m
1
3
④
ΔQ₂
⑦
0.12-600-0.3
①
③
⑤
0.03-300-0.3
ΔQ₄
El 117 m
ΔQ₁
0.-500-0.6
Q(m³/s)-L(m)-D(m)
Pump
5
El 150 m
2
0.03-300-0.15
4
⑧
②

Fig. 13–23

CHAPTER 14
Flow in Open Channels

14.1 Water flows in a rectangular, concrete, open channel that is 12.0 m wide at a depth of 2.5 m. The channel slope is 0.0028. Find the water velocity and the flow rate.

$$v = (1.0/n)(R^{2/3})(s^{1/2}) \qquad n = 0.013 \qquad \text{(from Table A-13)}$$

$$R = A/p_w = (12.0)(2.5)/(2.5 + 12.0 + 2.5) = 1.765 \text{ m}$$

$$v = (1.0/0.013)(1.765)^{2/3}(0.0028)^{1/2} = 5.945 \text{ m/s} \qquad Q = Av = [(12.0)(2.5)](5.945) = 178 \text{ m}^3/\text{s}$$

14.2 Water flows in the symmetrical trapezoidal channel lined with asphalt shown in Fig. 14-1. The channel bottom drops 0.1 ft vertically for every 100 ft of length. What are the water velocity and flow rate?

$$v = (1.486/n)(R^{2/3})(s^{1/2}) \qquad n = 0.015 \qquad \text{(from Table A-13)}$$

$$R = A/p_w \qquad A = (16.0)(4.5) + (2)\{(4.5)[(3)(4.5)]/2\} = 132.8 \text{ ft}^2$$

$$p_w = 16.0 + 2\sqrt{(4.5)^2 + [(3)(4.5)]^2} = 44.46 \text{ ft} \qquad R = 132.8/44.46 = 2.987 \text{ ft}$$

$$s = 0.1/100 = 0.00100 \qquad v = (1.486/0.015)(2.987)^{2/3}(0.00100)^{1/2} = 6.498 \text{ ft/s}$$

$$Q = Av = (132.8)(6.498) = 863 \text{ ft}^3/\text{s}$$

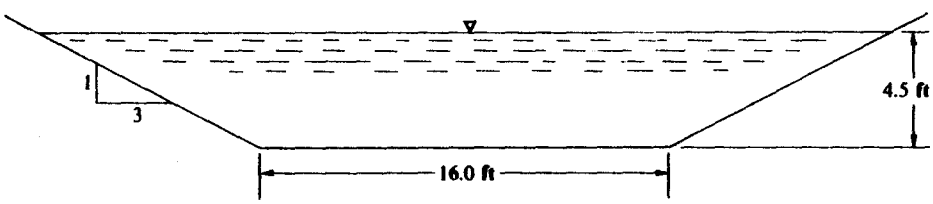


Fig. 14-1

14.3 Water is to flow at a rate of 30 m³/s in the concrete channel shown in Fig. 14-2. Find the required vertical drop of the channel bottom per kilometer of length.

$$A = (3.6)(2.0) + (4.0 - 2.0)[(1.6 + 3.6)/2] = 12.40 \text{ m}^2 \qquad v = (1.0/n)(R^{2/3})(s^{1/2}) = Q/A = 30/12.40 = 2.419 \text{ m/s}$$

$$p_w = 3.6 + 2.0 + \sqrt{(4.0 - 2.0)^2 + (3.6 - 1.6)^2} + 1.6 = 10.03 \text{ m} \qquad R = A/p_w = 12.40/10.03 = 1.236 \text{ m}$$

$$2.419 = (1.0/0.013)(1.236)^{2/3}(s)^{1/2} \qquad s = 0.000746$$

This slope represents a drop of the channel bottom of 0.000746 m per meter of length, or 0.746 m per kilometer of length.

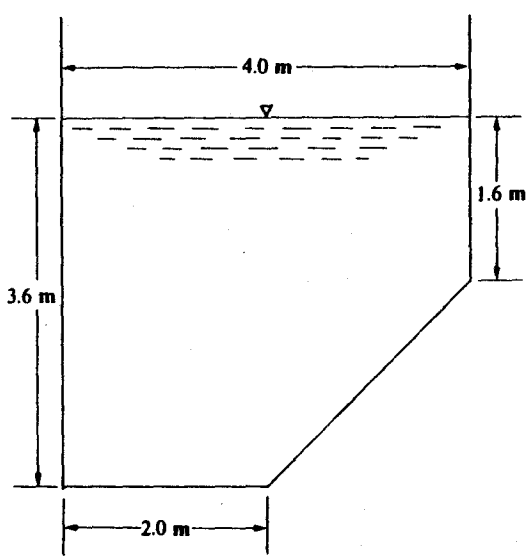


Fig. 14-2

14.4 Water flows in the triangular steel channel shown in Fig. 14-3 at a velocity of 2.9 ft/s. Find the depth of flow if the channel slope is 0.0015.

▮ $$v = (1.486/n)(R^{2/3})(s^{1/2}) \qquad R = A/p_w = 2\{[(d)(d \tan 27.5°)/2]/(2d/\cos 27.5°)\} = 0.2309d$$

$$2.9 = (1.486/0.014)(0.2309d)^{2/3}(0.0015)^{1/2} \qquad d = 2.57 \text{ ft}$$

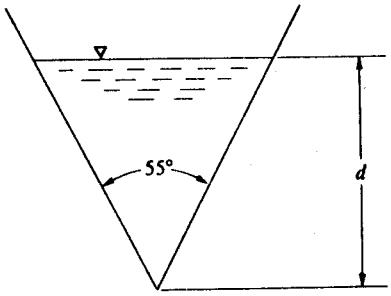


Fig. 14-3

14.5 After a flood had passed an observation station on a river, an engineer visited the site and, by locating flood marks, performing appropriate surveying, and doing necessary computations, determined that the cross-sectional area, wetted perimeter, and water-surface slope at the time of the peak flooding were 2960 m^2, 341 m, and 0.00076, respectively. The engineer also noted that the channel bottom was "earth with grass and weeds" ($n = 0.030$). Estimate the peak flood discharge.

▮ $$v = (1.0/n)(R^{2/3})(s^{1/2}) = (1.0/0.030)(2960/341)^{2/3}(0.00076)^{1/2} = 3.881 \text{ m/s}$$

$$Q = Av = (2960)(3.881) = 11\,490 \text{ m}^3/\text{s}$$

14.6 A rectangular, concrete channel 50 ft wide is to carry water at a flow rate of 800 cfs. The channel slope is 0.00025. Find the depth of flow.

▮ $$v = (1.486/n)(R^{2/3})(s^{1/2}) = Q/A = 800/50d = 16.00/d \qquad R = A/p_w = 50d/(50 + 2d)$$

$$16.00/d = (1.486/0.013)[50d/(50 + 2d)]^{2/3}(0.00025)^{1/2}$$

This equation is not readily solvable, but a trial-and-error solution (not shown here) reveals that $d = 3.92$ ft.

14.7 Prepare a computer program that will determine the depth of flow of water in a rectangular channel (as in Prob. 14.6).

▮
```
C     THIS PROGRAM DETERMINES THE DEPTH OF FLOW FOR OPEN CHANNEL FLOW
C     IN RECTANGULAR SECTIONS.  IT CAN BE USED FOR PROBLEMS IN BOTH THE
C     ENGLISH SYSTEM OF UNITS AND THE INTERNATIONAL SYSTEM OF UNITS.
C
C     INPUT DATA MUST BE SET UP AS FOLLOWS.
C
C     CARD 1   COLUMN 1          ENTER 0 (ZERO) OR BLANK IF ENGLISH SYSTEM
C                                OF UNITS IS TO BE USED.  ENTER 1 (ONE) IF
C                                INTERNATIONAL SYSTEM OF UNITS IS TO BE
C                                USED.
C              COLUMNS 2-79      ENTER TITLE, DATE, AND OTHER INFORMATION,
C                                IF DESIRED.
C     CARD 2   COLUMNS 1-10      ENTER NUMBER INCLUDING DECIMAL GIVING
C                                WIDTH OF CHANNEL (IN FEET OR METERS).
C              COLUMNS 11-20     ENTER NUMBER INCLUDING DECIMAL GIVING
C                                FLOW RATE (IN CUBIC FEET PER SECOND OR
C                                CUBIC METERS PER SECOND).
C              COLUMNS 21-30     ENTER NUMBER INCLUDING DECIMAL GIVING
C                                SLOPE.
C              COLUMNS 31-40     ENTER NUMBER INCLUDING DECIMAL GIVING
C                                MANNING N-VALUE.
C
C     MULTIPLE DATA SETS FOR SOLVING ANY NUMBER OF PROBLEMS MAY BE
C     INCLUDED FOR PROCESSING.
C
      DIMENSION TITLE(13)
      REAL N
      INTEGER UNITS
1     READ(5,100,END=2)UNITS,TITLE
```

```
100 FORMAT(I1,13A6)
    WRITE(6,105)TITLE
105 FORMAT('1',13A6,////)
    COEFF=1.486
    IF(UNITS.EQ.1)COEFF=1.0
    READ(5,101)W,Q,S,N
101 FORMAT(4F10.0)
    D=0.001
    TRY1=COEFF/N*(W*D/(W+2.0*D))**(2.0/3.0)*SQRT(S)-Q/W/D
104 D=D+0.001
    TRY2=COEFF/N*(W*D/(W+2.0*D))**(2.0/3.0)*SQRT(S)-Q/W/D
    IF(TRY1*TRY2)102,102,103
103 TRY1=TRY2
    GO TO 104
102 D=D-0.0005
    IF(UNITS.EQ.0)WRITE(6,106)W,Q,S,N,D
106 FORMAT(1X,'GIVEN DATA FOR AN OPEN CHANNEL FLOW IN A RECTANGULAR SE
   *CTION',//5X,'WIDTH =',F7.1,' FT',//5X,'FLOW RATE =',F7.1,' CU FT/S
   *',//5X,'SLOPE =',F10.7,//5X,'MANNING N-VALUE =',F6.3,////1X,'THE D
   *EPTH OF FLOW WILL BE',F7.2,' FT')
    IF(UNITS.EQ.1)WRITE(6,107)W,Q,S,N,D
107 FORMAT(1X,'GIVEN DATA FOR AN OPEN CHANNEL FLOW IN A RECTANGULAR SE
   *CTION',//5X,'WIDTH =',F7.1,' M ',//5X,'FLOW RATE =',F7.1,' CU M/S
   *',//5X,'SLOPE =',F10.7,//5X,'MANNING N-VALUE =',F6.3,////1X,'THE D
   *EPTH OF FLOW WILL BE',F7.2,' M')
    GO TO 1
2   STOP
    END
```

14.8 Solve Prob. 14.6 using the computer program of Prob. 14.7.

Input

```
0SAMPLE ANALYSIS ØF ØPEN CHANNEL FLØW IN A RECTANGULAR SECTIØN
50.0        800.0       0.00025    0.013
```

Output

```
SAMPLE ANALYSIS OF OPEN CHANNEL FLOW IN A RECTANGULAR SECTION


GIVEN DATA FOR AN OPEN CHANNEL FLOW IN A RECTANGULAR SECTION

    WIDTH =    50.0 FT

    FLOW RATE =  800.0 CU FT/S

    SLOPE = 0.0002500

    MANNING N-VALUE = 0.013



THE DEPTH OF FLOW WILL BE  3.92 FT
```

14.9 A rectangular channel ($n = 0.016$) 20 m wide is to carry water at a flow rate of 30 m^3/s at a slope of 0.00032. Find the depth of flow using the computer program of Prob. 14.7.

Input

```
1SAMPLE ANALYSIS ØF ØPEN CHANNEL FLØW IN A RECTANGULAR SECTIØN
20.0        30.0        0.00032    0.016
```

Output

```
SAMPLE ANALYSIS OF OPEN CHANNEL FLOW IN A RECTANGULAR SECTION


GIVEN DATA FOR AN OPEN CHANNEL FLOW IN A RECTANGULAR SECTION

    WIDTH =    20.0 M

    FLOW RATE =    30.0 CU M/S

    SLOPE = 0.0003200

    MANNING N-VALUE = 0.016


THE DEPTH OF FLOW WILL BE  1.25 M
```

14.10 A corrugated metal pipe of 500 mm diameter flows half-full at a slope of 0.0050 (see Fig. 14-4). What is the flow rate for this condition?

▮ $$v = (1.0/n)(R^{2/3})(s^{1/2}) = (1.0/0.024)[(\tfrac{500}{1000})/4]^{2/3}(0.0050)^{1/2} = 0.7366 \text{ m/s}$$
$$Q = Av = \{[(\pi)(\tfrac{500}{1000})^2/4]/2\}(0.7366) = 0.0723 \text{ m}^3/\text{s}$$

(*Note:* The hydraulic radius for both a circular cross section and a semicircular cross section is one-fourth the diameter.)

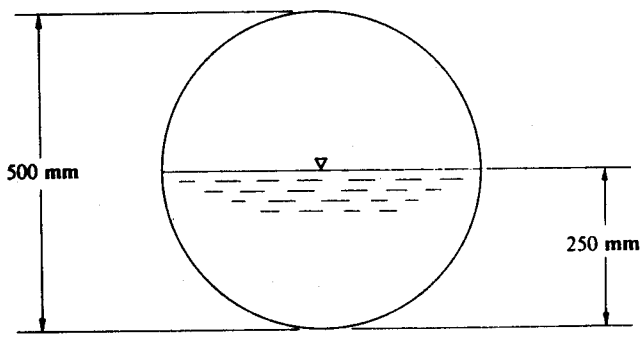
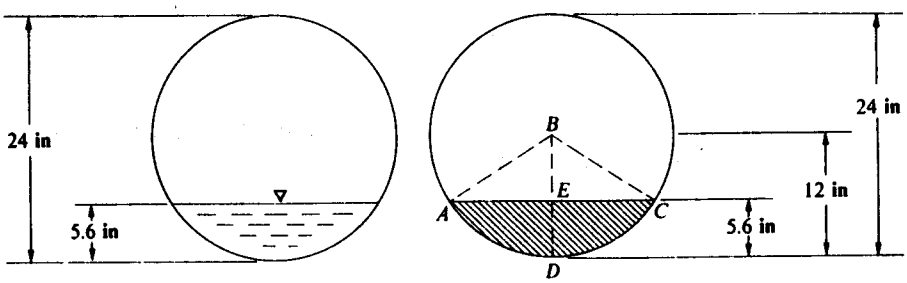


Fig. 14-4

14.11 A 24-in-diameter cast iron pipe on a $\frac{1}{400}$ slope carries water at a depth of 5.6 in, as shown in Fig. 14-5*a*. What is the flow rate?

▮ $v = (1.486/n)(R^{2/3})(s^{1/2})$, $R = A/p_w$. The applicable area in this problem is the shaded area (*AECDA*) in Fig. 14-5*b*: $AB = BC = 12$ in (both are radii), $BE = 12 - 5.6 = 6.4$ in. Therefore, $AE = EC = \sqrt{(12)^2 - (6.4)^2} = 10.15$ in and $\measuredangle ABE = \measuredangle EBC = \arccos(6.4/12) = 57.77°$, $(\text{Area})_{ABCDA} = [(\pi)(24)^2/4][(2)(57.77°)/360°] = 145.19 \text{ in}^2$, $(\text{Area})_{ABEA} = (\text{Area})_{BCEB} = (6.4)(10.15)/2 = 32.48 \text{ in}^2$, $(\text{Area})_{AECDA} = (\text{Area})_{ABCDA} - (2)(\text{Area})_{ABEA} = 145.19 - (2)(32.48) = 80.23 \text{ in}^2$. The applicable wetted perimeter in this problem is the arc distance *ADC* in Fig. 14-5: $p_w = ADC = (\pi)(24)[(2)(57.77°)/360°] = 24.20$ in. $R = 80.23/24.20 = 3.315$ in, or 0.2763 ft, $v = (1.486/0.012)(0.2763)^{2/3}(\frac{1}{400})^{1/2} = 2.627$ ft/s, $Q = Av = (80.23/144)(2.627) = 1.46 \text{ ft}^3/\text{s}$.



(*a*) (*b*)

Fig. 14-5

14.12 A 500-mm-diameter concrete pipe on a $\frac{1}{500}$ slope is to carry water at a flow rate of 0.040 m^3/s. Find the depth of flow. See Fig. 14-6*a*.

▮ $v = (1.0/n)(R^{2/3})(s^{1/2}) = Q/A = 0.040/A \qquad 0.040/A = (1.0/0.013)(R^{2/3})(\frac{1}{500})^{1/2} \qquad AR^{2/3} = 0.01163$

Since $R = A/p_w$,

$$A^{5/3}/p_w^{2/3} = 0.01163 \tag{1}$$

Equation (*1*) contains two unknowns, A and p_w; however, both unknowns can be expressed in terms of the unknown depth of flow, d. The applicable area in this problem is the shaded area (*AECDA*) in Fig. 14-6*b*: $AB = BC = 0.25$ m (both are radii), $BE = 0.25 - d$. Therefore, $AE = CE = \sqrt{(0.25)^2 - (0.25 - d)^2}$, $\measuredangle ABE = \measuredangle EBC = \arccos[(0.25 - d)/0.25]$,

$$(\text{Area})_{ABCDA} = \left[\frac{(\pi)(0.50)^2}{4}\right]\left\{\frac{(2)\arccos[(0.25-d)/0.25]}{360°}\right\} = (0.001091)\left(\arccos\frac{0.25-d}{0.25}\right)$$

$$(\text{Area})_{ABEA} = (\text{Area})_{BCEB} = \frac{(0.25-d)\sqrt{(0.25)^2-(0.25-d)^2}}{2}$$

$$(\text{Area})_{AECDA} = (\text{Area})_{ABCDA} - 2(\text{Area})_{ABEA} = (0.001091)\left(\arccos\frac{0.25-d}{0.25}\right) - (2)\left[\frac{(0.25-d)\sqrt{(0.25)^2-(0.25-d)^2}}{2}\right]$$

$$p_w = ADC = (\pi)(0.50)\left\{\frac{(2)\arccos[(0.25-d)/0.25]}{360°}\right\} = (0.008727)\left(\arccos\frac{0.25-d}{0.25}\right)$$

Therefore, substituting into Eq. (*1*),

$$\frac{[(0.001091)\{\arccos[(0.25-d)/0.25]\} - (0.25-d)\sqrt{(0.25)^2-(0.25-d)^2}]^{5/3}}{[(0.008727)\{\arccos[(0.25-d)/0.25]\}]^{2/3}} = 0.01163\,\frac{m^3}{s}$$

This equation is not readily solvable, but a trial-and-error solution (not shown here) reveals that $d = 0.166$ m or 166 mm.

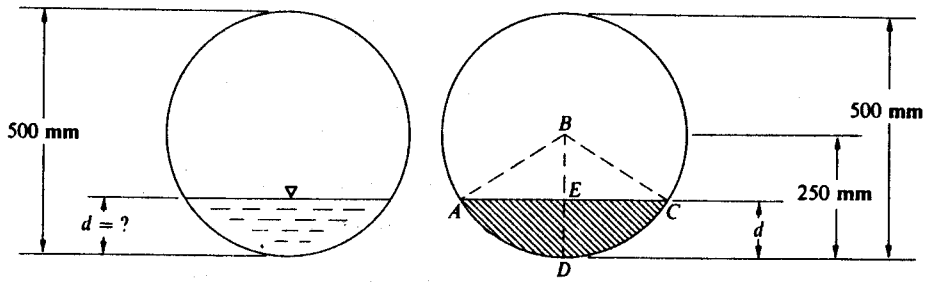


Fig. 14-6

14.13 Solve Prob. 14.11 utilizing Fig. A-18.

▮ $s = \frac{1}{400} = 0.0025$ ft/ft. From Fig. A-15, $Q_{full} = 11.4$ ft^3/s and $v_{full} = 3.6$ ft/s. These values of Q_{full} and v_{full} must be adjusted for an n value of 0.012 for the given cast iron pipe (Fig. A-15 is based on an n value of 0.013): $(Q_{full})_{n=0.012}/11.4 = 0.013/0.012$, $(Q_{full})_{n=0.012} = 12.4$ ft^3/s, $(v_{full})_{n=0.012}/3.6 = 0.013/0.012$, $(v_{full})_{n=0.012} = 3.9$ ft/s, $d/d_{full} = 5.6/24 = 0.23$, or 23 percent. Enter the ordinate of Fig. A-18 with a value of d/d_{full} of 23 percent, move horizontally to the line marked "discharge," and then vertically downward to the abscissa to read $Q/Q_{full} = 12$ percent. In a similar manner using the "velocity" line, read $v/v_{full} = 63$ percent. Therefore, $Q = (0.12)(12.4) = 1.5$ ft^3/s, $v = (0.63)(3.9) = 2.5$ ft/s.

14.14 Solve Prob. 14.12 utilizing Fig. A-18.

▮ $s = \frac{1}{500} = 0.00200$. From Fig. A-16, $Q_{full} = 0.169$ m^3/s: $Q/Q_{full} = 0.040/0.169 = 0.24$, or 24 percent. From Fig. A-18, $d/d_{full} = 34$ percent; $d = (0.34)(500) = 170$ mm.

14.15 A 30-in-diameter concrete storm sewer pipe must carry a flow rate of 9.0 cfs at a minimum velocity of 2.5 ft/s. Find the required slope and water depth.

▮

$$A = Q/v = 9.0/2.5 = 3.600\ ft^2 \qquad A_{full} = [(\pi)(\tfrac{30}{12})^2/4] = 4.909\ ft^2$$

$$A/A_{full} = 3.600/4.909 = 0.73 \quad \text{or} \quad 73 \text{ percent}$$

From Fig. A-18, $d/d_{full} = 69$ percent and $R/R_{full} = 116$ percent.

$$d = (0.69)(30) = 20.7 \text{ in} \qquad v = (1.486/n)(R^{2/3})(s^{1/2})$$
$$2.5 = (1.486/0.013)\{(1.16)[(\tfrac{30}{12})/4]\}^{2/3}(s^{1/2}) \qquad s = 0.000734$$

14.16 A concrete pipe must carry water at a slope of 0.0075, at a velocity of 0.76 m/s, and at a depth of flow equal to one-tenth its diameter. What is the required pipe diameter?

▮ $$v = (1.0/n)(R^{2/3})(s^{1/2}) \qquad 0.76 = (1.0/0.013)(R^{2/3})(0.0075)^{1/2} \qquad R = 0.03853 \text{ ft}$$

From Fig. A-18 with $d/d_{full} = 10$ percent, $R/R_{full} = 25$ percent.

$$0.03853/R_{full} = 0.25 \qquad R_{full} = 0.1541 \text{ m} \qquad 0.1541 = d/4 \qquad d = 0.616 \text{ m} \quad \text{or} \quad 616 \text{ mm}$$

14.17 Prepare a computer program that will determine either the depth of flow or the flow rate for open channel flow in circular sections.

▮
```
C     THIS PROGRAM DETERMINES EITHER THE DEPTH OF FLOW OR THE FLOW RATE
C     FOR OPEN CHANNEL FLOW IN CIRCULAR SECTIONS.  IT CAN BE USED FOR
C     PROBLEMS IN BOTH THE ENGLISH SYSTEM OF UNITS AND THE INTERNATIONAL
C     SYSTEM OF UNITS.
C
C     INPUT DATA MUST BE SET UP AS FOLLOWS.
C
C     CARD 1   COLUMN 1         ENTER 0 (ZERO) OR BLANK IF ENGLISH SYSTEM
C                               OF UNITS IS TO BE USED.  ENTER 1 (ONE) IF
C                               INTERNATIONAL SYSTEM OF UNITS IS TO BE
C                               USED.
C              COLUMNS 2-79     ENTER TITLE, DATE, AND OTHER INFORMATION,
C                               IF DESIRED.
C     CARD 2   COLUMNS 1-10     ENTER NUMBER INCLUDING DECIMAL GIVING
C                               DIAMETER OF CHANNEL (IN INCHES OR MILLI-
C                               METERS).
C              COLUMNS 11-20    ENTER NUMBER INCLUDING DECIMAL GIVING
C                               DEPTH OF FLOW (IN INCHES OR MILLIMETERS).
C              COLUMNS 21-30    ENTER NUMBER INCLUDING DECIMAL GIVING
C                               SLOPE.
C              COLUMNS 31-40    ENTER NUMBER INCLUDING DECIMAL GIVING
C                               MANNING N-VALUE.
C              COLUMNS 41-50    ENTER NUMBER INCLUDING DECIMAL GIVING
C                               FLOW RATE (IN CUBIC FEET PER SECOND OR
C                               CUBIC METERS PER SECOND).
C
C ***********************************************************************
C *                                                                     *
C *   NOTE WELL....EITHER THE DEPTH OF FLOW (COLUMNS 11-20) OR THE      *
C *   FLOW RATE (COLUMNS 41-50), WHICHEVER ONE IS TO BE DETERMINED BY   *
C *   THIS PROGRAM, SHOULD BE LEFT BLANK ON CARD 2.                     *
C *                                                                     *
C ***********************************************************************
C
C     MULTIPLE DATA SETS FOR SOLVING ANY NUMBER OF PROBLEMS MAY BE
C     INCLUDED FOR PROCESSING.
C
      DIMENSION TITLE(13)
      COMMON D,R,D1,DIAM,PI,FACTOR,AREA,WP
      REAL N
      INTEGER UNITS
      PI=3.14159265
 1    READ(5,100,END=2)UNITS,TITLE
  100 FORMAT(I1,13A6)
      WRITE(6,105)TITLE
  105 FORMAT('1',13A6,////)
      COEFF=1.486
      FACTOR=12.0
      IF(UNITS.EQ.1)COEFF=1.0
      IF(UNITS.EQ.1)FACTOR=1000.0
      READ(5,101)DIAM,D,S,N,Q
  101 FORMAT(5F10.0)
      R=DIAM/2.0
      IF(Q.GT.0.0001)GO TO 102
      D1=D
```

```
      CALL AREAWP
      HR=AREA/WP
      Q=AREA*COEFF/N*HR**(2.0/3.0)*SQRT(S)
      IF(UNITS.EQ.0)WRITE(6,103)DIAM,D,S,N,Q
  103 FORMAT(1X,'GIVEN DATA FOR AN OPEN CHANNEL FLOW IN A CIRCULAR SECTI
     *ON'//5X,'DIAMETER =',F7.2,' IN',//5X,'DEPTH OF FLOW =',F7.2,' IN',
     *//5X,'SLOPE =',F10.7,//5X,'MANNING N-VALUE =',F6.3,////1X,'THE FLO
     *W RATE WILL BE',F8.3,' CU FT/S')
      IF(UNITS.EQ.1)WRITE(6,104)DIAM,D,S,N,Q
  104 FORMAT(1X,'GIVEN DATA FOR AN OPEN CHANNEL FLOW IN A CIRCULAR SECTI
     *ON'//5X,'DIAMETER =',F7.1,' MM',//5X,'DEPTH OF FLOW =',F7.1,' MM',
     *//5X,'SLOPE =',F10.7,//5X,'MANNING N-VALUE =',F6.3,////1X,'THE FLO
     *W RATE WILL BE',F8.3,' CU M/S')
      GO TO 1
  102 AWP=Q*N/COEFF/SQRT(S)
      D=0.01
      D1=D
      CALL AREAWP
      TRY1=AREA**(5.0/3.0)/WP**(2.0/3.0)-AWP
  108 D=D+0.01
      IF(D.GT.DIAM)GO TO 112
      D1=D
      CALL AREAWP
      TRY2=AREA**(5.0/3.0)/WP**(2.0/3.0)-AWP
      IF(TRY1*TRY2)106,106,107
  107 TRY1=TRY2
      GO TO 108
  106 D=D-0.005
      IF(UNITS.EQ.0)WRITE(6,109)DIAM,Q,S,N,D
  109 FORMAT(1X,'GIVEN DATA FOR AN OPEN CHANNEL FLOW IN A CIRCULAR SECTI
     *ON',//5X,'DIAMETER =',F7.2,' IN',//5X,'FLOW RATE =',F8.3,' CU FT/S
     *',//5X,'SLOPE =',F13.7,//5X,'MANNING N-VALUE =',F6.3,////1X,'THE D
     *EPTH OF FLOW WILL BE',F7.2,' IN')
      IF(UNITS.EQ.1)WRITE(6,110)DIAM,Q,S,N,D
  110 FORMAT(1X,'GIVEN DATA FOR AN OPEN CHANNEL FLOW IN A CIRCULAR SECTI
     *ON',//5X,'DIAMETER =',F7.1,' MM',//5X,'FLOW RATE =',F8.3,' CU M/S
     *',//5X,'SLOPE =',F13.7,//5X,'MANNING N-VALUE =',F6.3,////1X,'THE D
     *EPTH OF FLOW WILL BE',F7.1,' MM')
      GO TO 1
  112 WRITE(6,116)
  116 FORMAT(1X,'THIS CIRCULAR CONDUIT CANNOT CARRY THIS GREAT A FLOW AS
     * OPEN CHANNEL FLOW.')
      GO TO 1
2     STOP
      END
      SUBROUTINE AREAWP
      COMMON D,R,D1,DIAM,PI,FACTOR,AREA,WP
      IF(D.GT.R)D1=DIAM-D
      ABCDA=DIAM**2/4.0*ARCOS((R-D1)/R)
      ABEA=(R-D1)*SQRT(R**2-(R-D1)**2)/2.0
      AREA=ABCDA-2.0*ABEA
      WP=DIAM*ARCOS((R-D1)/R)
      IF(D.GT.R)AREA=PI*DIAM**2/4.0-AREA
      IF(D.GT.R)WP=PI*DIAM-WP
      AREA=AREA/FACTOR**2
      WP=WP/FACTOR
      RETURN
      END
```

14.18 Solve Prob. 14.11 utilizing the computer program of Prob. 14.17.

Input

```
OSAMPLE ANALYSIS ØF AN ØPEN CHANNEL FLØW IN A CIRCULAR SECTIØN
24.0        5.6         0.0025      0.012
```

Output

```
SAMPLE ANALYSIS OF AN OPEN CHANNEL FLOW IN A CIRCULAR SECTION



GIVEN DATA FOR AN OPEN CHANNEL FLOW IN A CIRCULAR SECTION

    DIAMETER =  24.00 IN

    DEPTH OF FLOW =   5.60 IN

    SLOPE = 0.0025000

    MANNING N-VALUE = 0.012



THE FLOW RATE WILL BE   1.463 CU FT/S
```

14.19 Solve Prob. 14.12 utilizing the computer program of Prob. 14.17.

Input

```
1SAMPLE ANALYSIS ØF AN ØPEN CHANNEL FLØW IN A CIRCULAR SECTIØN
500.                0.002     0.013     0.040
```

Output

```
SAMPLE ANALYSIS OF AN OPEN CHANNEL FLOW IN A CIRCULAR SECTION


GIVEN DATA FOR AN OPEN CHANNEL FLOW IN A CIRCULAR SECTION

    DIAMETER =  500.0 MM

    FLOW RATE =   0.040 CU M/S

    SLOPE =    0.0020000

    MANNING N-VALUE = 0.013


THE DEPTH OF FLOW WILL BE  165.7 MM
```

14.20 An open channel is to be designed to carry 1.0 m^3/s at a slope of 0.0065. The channel material has an n value of 0.011. Find the most efficient cross section for a semicircular section.

$$v = (1.0/n)(R^{2/3})(s^{1/2}) \qquad Q/A = (1.0/n)(A/p_w)^{2/3}(s^{1/2})$$

$$A^{5/3}/p_w^{2/3} = Qn/s^{1/2} = (1.0)(0.011)/0.0065^{1/2} = 0.1364$$

$$A = \pi d^2/8 \qquad p_w = \pi d/2 \qquad (\pi d^2/8)^{5/3}/(\pi d/2)^{2/3} = 0.1364 \qquad d = 0.951 \text{ m} \quad \text{or} \quad 951 \text{ mm}$$

(d is the diameter of the semicircular section; the depth of flow would, of course, be half of d.)

14.21 Find the most efficient cross section for Prob. 14.20 for a rectangular section.

$A^{5/3}/p_w^{2/3} = 0.1364$ (from Prob. 14.20). The most efficient rectangular section has a width equal to twice its depth. Letting d = depth, $A = (d)(2d) = 2d^2$, $p_w = d + 2d + d = 4d$, $(2d^2)^{5/3}/(4d)^{2/3} = 0.1364$, $d = 0.434$ m, or 434 mm; width $= 2d = (2)(434) = 868$ mm.

14.22 Find the most efficient cross section for Prob. 14.20 for a triangular section.

$A^{5/3}/p_w^{2/3} = 0.1364$ (from Prob. 14.20). The most efficient triangular section has a 90° angle and 1:1 side slopes (see Fig. 14-7). $A = (\frac{1}{2})(d\sqrt{2})(d\sqrt{2}) = d^2$, $p_w = (2)(d\sqrt{2}) = 2.828d$, $(d^2)^{5/3}/(2.828d)^{2/3} = 0.1364$, $d =$ 0.614 m, or 614 mm; sides $= d\sqrt{2} = (614)(\sqrt{2}) = 868$ mm.

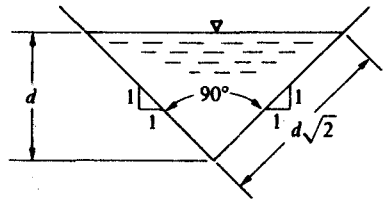


Fig. 14-7

14.23 Find the most efficient cross section for Prob. 14.20 for a trapezoidal section.

▮ $A^{5/3}/p^{2/3} = 0.1364$ (from Prob. 14.20). The most efficient trapezoidal section is half a regular hexagon (see Fig. 14-8). $A = (1.155d)(d) + (2)[(d)(d \tan 30°)/2] = 1.732d^2$, $p_w = (3)(1.155d) = 3.465d$, $(1.732d^2)^{5/3}/(3.465d)^{2/3} = 0.1364$, $d = 0.459$ m; sides and bottom: each $= 1.155d = (1.155)(0.459) = 0.530$ m.

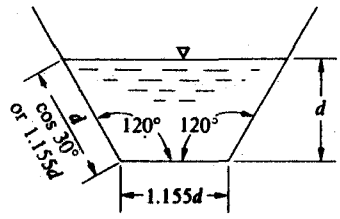


Fig. 14-8

14.24 For the same conditions given in Prob. 14.2, determine the status of flow (i.e., is it critical, subcritical, or supercritical?).

▮ $N_F = v/\sqrt{gd_m}$ $\quad v = 6.498$ ft/s (from Prob. 14.2) $\quad d_m = A/B \quad A = 132.8$ ft^2 (from Prob. 14.2)

$$B = (3)(4.5) + 16.0 + (3)(4.5) = 43.0 \text{ ft} \qquad d_m = 132.8/43.0 = 3.088 \text{ ft}$$

$$N_F = 6.498/\sqrt{(32.2)(3.088)} = 0.652$$

Since $N_F < 1.0$, the flow is subcritical.

14.25 The triangular channel ($n = 0.012$) shown in Fig. 14-9 is to carry water at a flow rate of 10 m^3/s. Find the critical depth, critical velocity, and critical slope of the channel.

▮ $B/A^3 = g/Q^2 \qquad B = 6d_c \qquad A = 2[(d_c)(3d_c)/2] = 3d_c^2 \qquad 6d_c/(3d_c^2)^3 = 9.807/10^2 \qquad d_c = 1.178$ m

$$v_c = Q/A = 10/[(3)(1.178)^2] = 2.402 \text{ m/s} \qquad s_c = \{nv_c/[(1.0)(R_c^{2/3})]\}^2 \qquad R = A/p_w$$

$$R_c = [(3)(1.178)^2]/[(2)(\sqrt{10})(1.178)] = 0.5588 \text{ m} \qquad s_c = \{(0.012)(2.402)/[(1.0)(0.5588)^{2/3}]\}^2 = 0.00181$$

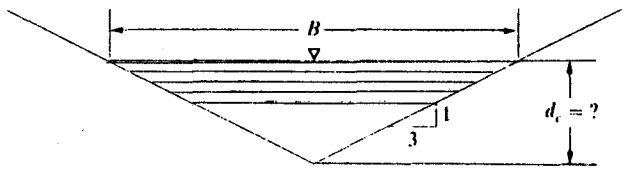


Fig. 14-9

14.26 The semicircular channel ($n = 0.010$) shown in Fig. 14-10 is to carry water at a depth of 1.0 ft. Find the velocity, slope, and discharge at the critical stage.

▮ $d_m = A/B = [(\frac{1}{2})(\pi)(2.0)^2/4]/2.0 = 0.7854$ ft $\qquad v_c = \sqrt{gd_m} = \sqrt{(32.2)(0.7854)} = 5.029$ ft/s

$$s_c = \{nv_c/[(1.486)(R_c^{2/3})]\}^2 = \{(0.010)(5.029)/[(1.486)(2.0/4)^{2/3}]\}^2 = 0.00289$$

$$Q = Av_c = [(\tfrac{1}{2})(\pi)(2.0)^2/4](5.029) = 7.90 \text{ ft}^3/\text{s}$$

14.27 A flow rate of 2.1 m^3/s is to be carried in an open channel at a velocity of 1.3 m/s. Determine the dimensions of the channel cross section and required slope if the cross section is rectangular with depth equal to one-half the width. Use $n = 0.020$.

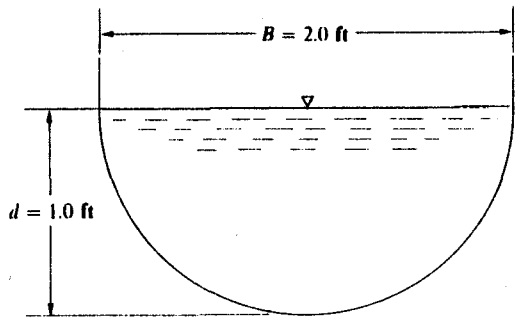


Fig. 14-10

▮ $Q = Av$. Let B = channel width. Then channel depth = $B/2$; $2.1 = [(B)(B/2)](1.3)$, $B = 1.797$ m. Hence, the required width is 1.797 m and depth is 1.797/2, or 0.898 m.

$$R = A/p_w = (1.797)(0.898)/[1.797 + (2)(0.898)] = 0.4491 \text{ m}$$
$$s = \{nv/[(1.0)(R^{2/3})]\}^2 = \{(0.020)(1.3)/[(1.0)(0.4491)^{2/3}]\}^2 = 0.00197$$

14.28 Repeat Prob. 14.27 if the depth must be equal to twice the width. Compare answers with Prob. 14.27.

▮ $Q = Av$. Let B = channel width. Then channel depth = $2B$; $2.1 = [(B)(2B)](1.3)$, $B = 0.899$ m. Hence, the required width is 0.899 m and depth is (2)(0.899), or 1.798 m.

$$R = A/p_w = (1.798)(0.899)/[0.899 + (2)(1.798)] = 0.3596 \text{ m}$$
$$s = \{nv/[(1.0)(R^{2/3})]\}^2 = \{(0.020)(1.3)/[(1.0)(0.3596)^{2/3}]\}^2 = 0.00264$$

The channel area is the same (neglecting round-off errors) but a steeper slope is required for the narrower channel.

14.29 Repeat Prob. 14.27 if the channel cross section is semicircular.

▮ $Q = Av \qquad 2.1 = [(\frac{1}{2})(\pi d^2/4)](1.3) \qquad d = 2.028 \text{ m} \qquad r = 2.028/2 = 1.014 \text{ m}$

$$s = \{nv/[(1.0)(R^{2/3})]\}^2 = \{(0.020)(1.3)/[(1.0)(2.028/4)^{2/3}]\}^2 = 0.00167$$

14.30 Repeat Prob. 14.27 if the channel cross section is trapezoidal, with depth equal to the width of the channel bottom and side slopes of 1:1.

▮ $Q = Av$. Let depth and channel bottom width = B (see Fig. 14-11). Then surface width = $3B$; $A = (3B)(B) - (2)[(\frac{1}{2})(B)(B)] = 2B^2$, $2.1 = (2B^2)(1.3)$, $B = 0.899$ m.

$$R = A/p_w = (2)(0.899)^2/[0.899 + (2)(0.899)\sqrt{2}] = 0.4696 \text{ m}$$
$$s = \{nv/[(1.0)(R^{2/3})]\}^2 = \{(0.020)(1.3)/[(1.0)(0.4696)^{2/3}]\}^2 = 0.00185$$

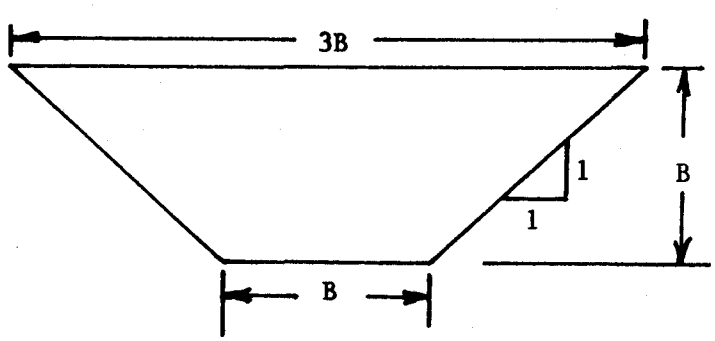


Fig. 14-11

14.31 For each of the channel cross sections shown in Fig. 14-12, compute the area, wetted perimeter, and hydraulic radius.

▮ **(a)** $A = \frac{1}{2}[(\pi)(4.0)^2/4] = 6.283 \text{ m}^2 \qquad p_w = \frac{1}{2}[(\pi)(4.0)] = 6.283 \text{ m} \qquad R = A/p_w = 6.283/6.283 = 1.000 \text{ m}$

(b) $A = (5.0)(2.5) = 12.50 \text{ m}^2 \qquad p_w = 2.5 + 5.0 + 2.5 = 10.00 \text{ m} \qquad R = 12.50/10.00 = 1.250 \text{ m}$

(c) $A = (5.0)(1.2) + (2)[(\frac{1}{2})(1.2)(1.2)] = 7.440 \text{ m}^2 \qquad p_w = 5.0 + (2)[(1.2)(\sqrt{2})] = 8.394 \text{ m}$

$$R = 7.440/8.394 = 0.886 \text{ m}$$

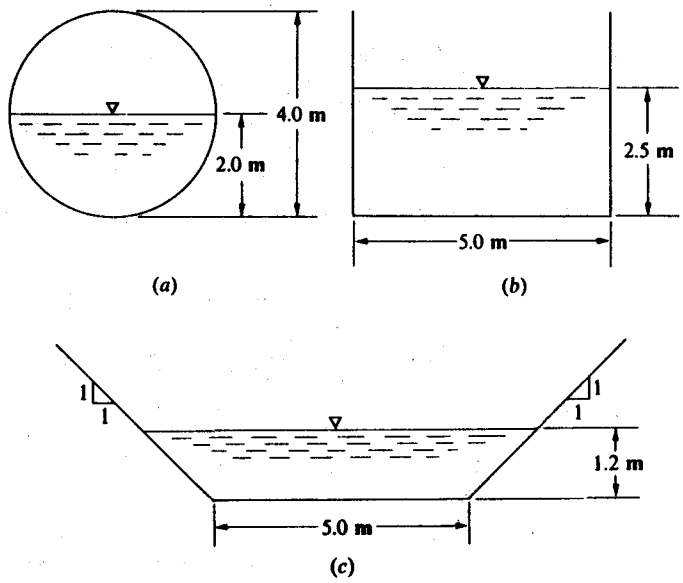


Fig. 14-12

14.32 Water is to flow in a rectangular flume at a rate of 1.42 m^3/s and at a slope of 0.0028. Determine the dimensions of the channel cross section if width must be equal to twice the depth. Use $n = 0.017$.

▮ $Q = (A)(1.0/n)(R^{2/3})(s^{1/2})$. Let B = channel width and $B/2$ = channel depth; $1.42 = [(B)(B/2)](1.0/0.017)[(B)(B/2)/(B/2 + B + B/2)]^{2/3}(0.0028)^{1/2}$, $B = 1.366$ m. Hence, required channel width = 1.366 m and depth = 1.366/2, or 0.683 m.

14.33 Rework Prob. 14.32, assuming width must be equal to the depth. Note which solution gives the smaller (and therefore more efficient) cross section.

▮ $Q = (A)(1.0/n)(R^{2/3})(s^{1/2})$. Let B = channel width and depth; $1.42 = [(B)(B)](1.0/0.017)[(B)(B)/(B + B + B)]^{2/3}(0.0028)^{1/2}$, $B = 0.981$ m. Hence, required channel width and depth are each 0.981 m. $A = (0.981)(0.981) = 0.962\ m^2$. For Prob. 14.32, $A = (1.366)(0.683) = 0.933\ m^2$. The cross section of Prob. 14.32 has the smaller cross-sectional area.

14.34 A rectangular channel ($n = 0.011$) 18 m wide is to carry water at a flow rate of 35 cfs. The slope of the channel is 0.00078. Determine the depth of flow.

▮ $$Q = (A)(1.0/n)(R^{2/3})(s^{1/2}) \qquad 35 = (18d)(1.0/0.011)[18d/(18 + 2d)]^{2/3}(0.00078)^{1/2}$$

$$d = 0.885\ \text{m} \qquad \text{(by trial and error)}$$

14.35 The trapezoidal channel shown in Fig. 14-13 is laid on a slope of 0.00191. The channel must carry 60 cfs. Determine the depth of flow. Use $n = 0.015$.

▮ $$Q = (A)(1.486/n)(R^{2/3})(s^{1/2}) \qquad A = 4.0d + (2)[(d)(d)/2] = 4.0d + d^2$$

$$p_w = 4.0 + (2)(d\sqrt{2}) = 4.0 + 2d\sqrt{2} \qquad 60 = (4.0d + d^2)(1.486/0.015)[(4.0d + d^2)/(4.0 + 2d\sqrt{2})]^{2/3}(0.00191)^{1/2}$$

$$d = 2.00\ \text{ft} \qquad \text{(by trial and error)}$$

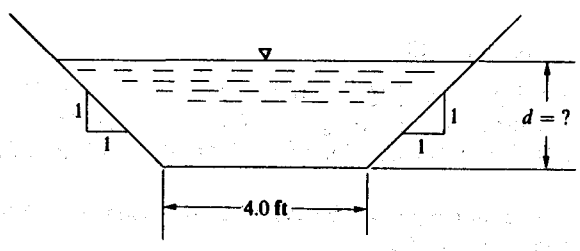


Fig. 14-13

14.36 A 36-in-diameter concrete pipe on a 0.0015 slope carries water at a depth of 26 in. Determine the flow rate for this pipe.

▮ See Fig. 14-14.

$$Q = (A)(1.486/n)(R^{2/3})(s^{1/2}) \qquad OE = 26 - \tfrac{36}{2} = 8 \text{ in} \quad \text{or} \quad 0.6667 \text{ ft} \qquad \measuredangle COE = \arccos[8/(\tfrac{36}{2})] = 63.61°$$

$$EC = \sqrt{[(\tfrac{36}{2})/12]^2 - 0.6667^2} = 1.344 \text{ ft}$$

$$A = \{[360 - (2)(63.61)]/360\}[(\pi)(\tfrac{36}{12})^2/4] + (2)[(\tfrac{1}{2})(1.344)(0.6667)] = 5.467 \text{ ft}^2$$

$$p_w = \{[360 - (2)(63.61)]/360\}[(\pi)(\tfrac{36}{12})] = 6.094 \text{ ft}$$

$$Q = (5.467)(1.486/0.013)(5.467/6.094)^{2/3}(0.0015)^{1/2} = 22.5 \text{ ft}^3/\text{s}$$

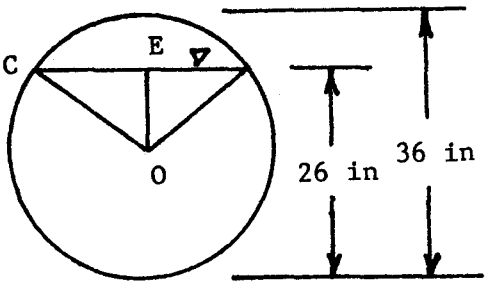


Fig. 14-14

14.37 Rework Prob. 14.36 using Fig. A-18.

▮ From Fig. A-15, $Q_{full} = 25.8 \text{ ft}^3/\text{s}$; $d/d_{full} = \frac{26}{36} = 0.722$, or 72.2 percent. From Fig. A-18, $Q/Q_{full} = 87.5$ percent; $Q = (0.875)(25.8) = 22.6 \text{ ft}^3/\text{s}$.

14.38 A sewer pipe, for which $n = 0.014$, is laid on a slope of 0.00018 and is to carry 2.76 m^3/s when the pipe flows at 80 percent of full depth. Determine the required diameter of pipe.

▮ See Fig. 14-15.

$$Q = (A)(1.0/n)(R^{2/3})(s^{1/2}) \qquad OE = 0.80D - D/2 = 0.3000D \qquad \alpha = \arccos[0.30D/(D/2)] = 53.13°$$

$$CE = (0.3000D)(\tan 53.13°) = 0.4000D$$

$$A = \{[360 - (2)(53.13)]/360\}(\pi D^2/4) + 2[(\tfrac{1}{2})(0.3000D)(0.4000D)] = 0.6736D^2$$

$$p_w = \{[360 - (2)(53.13)]/360\}(\pi D) = 2.214D$$

$$2.76 = (0.6736D^2)(1.0/0.014)(0.6736D^2/2.214D)^{2/3}(0.00018)^{1/2} \qquad D = 2.32 \text{ m}$$

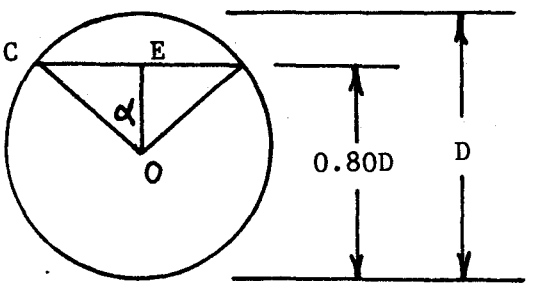


Fig. 14-15

14.39 Rework Prob. 14.38 using Fig. A-18.

▮ $D/D_{full} = 0.80$. From Fig. A-18, $Q/Q_{full} = 0.96$, $A/A_{full} = 0.84$, and $R/R_{full} = 1.21$: $Q_{full} = 2.76/0.96 = 2.88 \text{ m}^3/\text{s}$. Figure A-16 cannot be used for $Q_{full} > 1.0 \text{ m}^3/\text{s}$: $Q = (A)(1.0/n)(R^{2/3})(s^{1/2})$, $A = (0.84)(\pi D^2/4) = 0.6597D^2$, $R = (1.21)(D/4) = 0.3025D$, $2.76 = (0.6597D^2)(1.0/0.014)(0.3025D)^{2/3}(0.00018)^{1/2}$, $D = 2.34$ m.

14.40 A 72-in-diameter vitrified sewer pipe ($n = 0.014$) is laid on a slope of 0.00025 and carries wastewater at a flow rate of 50 cfs. What is the depth of flow?

▮ From Fig. A-15, $Q_{full} = 67 \text{ ft}^3/\text{s}$: $[(Q_{full})_{n=0.014}]/67 = 0.013/0.014$, $(Q_{full})_{n=0.014} = 62.2 \text{ ft}^3/\text{s}$; $Q/Q_{full} = 50/62.2 = 0.804$, or 80.4 percent. From Fig. A-18, $D/D_{full} = 69$ percent, $D = (0.69)(72) = 49.7$ in.

14.41 A 1.0-m-diameter pipe must carry a discharge of 0.40 m^3/s at a velocity of 0.80 m/s. Determine the slope and the depth of water.

▮ $A = Q/v = 0.40/0.80 = 0.5000\ m^2 \qquad A_{full} = (\pi)(1.0)^2/4 = 0.7854\ m^2$

$A/A_{full} = 0.5000/0.7854 = 0.64$ or 64 percent

From Fig. A-18, $D/D_{full} = 0.63$ and $R/R_{full} = 1.12$.

$D = (0.63)(1.0) = 0.630\ m \qquad v = (1.0/n)(R^{2/3})(s^{1/2}) \qquad R_{full} = 1.0/4 = 0.2500\ m$

$R = (1.12)(0.2500) = 0.2800\ m \qquad 0.80 = (1.0/0.013)(0.2800)^{2/3}(s^{1/2}) \qquad s = 0.000590$

14.42 The trapezoidal channel of Fig. 14-16 is to carry 500 cfs of water. The maximum allowable velocity of flow is 3.0 fps to avoid scouring. Determine the depth of flow, d, and the width of the channel bottom, B, if the hydraulic radius of the channel is one-half the depth of flow. Also, determine the slope of the channel bottom. Use $n = 0.025$.

▮ $R = d/2 = A/p_w \qquad A = Bd + 2[(\frac{1}{2})(1.5d)(d)] = Bd + 1.5d^2 \qquad p_w = B + 2\sqrt{d^2 + (1.5d)^2} = B + 3.606d$

$d/2 = (Bd + 1.5d^2)/(B + 3.606d) \qquad B + 3.606d = 2B + 3.0d \qquad B = 0.606d \qquad A = Q/v$

$Bd + 1.5d^2 = 500/3.0$

Substituting $B = 0.606d$, $(0.606d)(d) + 1.5d^2 = 166.7$, $d = 8.90$ ft; B = (0.606)(8.90) = 5.39 ft.

$v = (1.486/n)(R^{2/3})(s^{1/2}) \qquad R = 8.90/2 = 4.45\ ft \qquad 3.0 = (1.486/0.025)(4.45)^{2/3}(s^{1/2}) \qquad s = 0.000348$

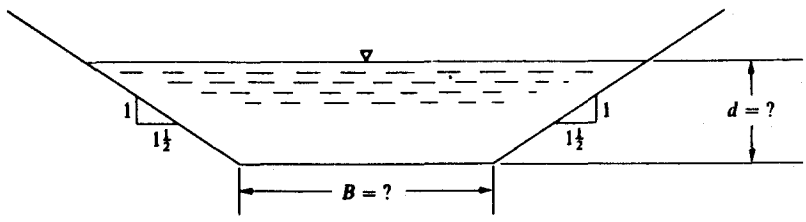


Fig. 14-16

14.43 An open channel to be made of concrete is to be designed to carry 1.5 m^3/s at a slope of 0.00085. Find the most efficient cross section for a semicircular section.

▮ $v = (1.0/n)(R^{2/3})(s^{1/2}) \qquad Q/A = (1.0/n)(A/p_w)^{2/3}(s^{1/2})$

$A^{5/3}/p_w^{2/3} = Qn/s^{1/2} = (1.5)(0.013)/0.00085^{1/2} = 0.6688 \qquad A = \pi d^2/8$

$p_w = \pi d/2 \qquad (\pi d^2/8)^{5/3}/(\pi d/2)^{2/3} = 0.6688 \qquad d = 1.727\ m$

(d is the diameter of the semicircular section; the depth of flow would, of course, be half of d.)

14.44 Find the most efficient cross section for Prob. 14.43 for a rectangular section.

▮ $A^{5/3}/p_w^{2/3} = 0.6688$ (from Prob. 14.43). The most efficient rectangular section has a width equal to twice its depth. Letting d = depth, $A = (d)(2d) = 2d^2$, $p_w = d + 2d + d = 4d$, $(2d^2)^{5/3}/(4d)^{2/3} = 0.6688$, $d = 0.789$ m; width $= 2d = (2)(0.789) = 1.578$ m.

14.45 Find the most efficient cross section for Prob. 14.43 for a triangular section.

▮ $A^{5/3}/p_w^{2/3} = 0.6688$ (from Prob. 14.43). The most efficient triangular section has a 90° angle and 1:1 side slopes (see Fig. 14-7): $A = (\frac{1}{2})(d\sqrt{2})(d\sqrt{2}) = d^2$, $p_w = (2)(d\sqrt{2}) = 2.828d$, $(d^2)^{5/3}/(2.828d)^{2/3} = 0.6688$, $d = 1.115$ m; sides $= d\sqrt{2} = (1.115)(\sqrt{2}) = 1.577$ m.

14.46 Find the most efficient cross section for Prob. 14.43 for a trapezoidal section.

▮ $A^{5/3}/p_w^{2/3} = 0.6688$ (from Prob. 14.43). The most efficient trapezoidal section is half a regular hexagon (see Fig. 14-8): $A = (1.155d)(d) + 2[(d)(d \tan 30°)/2] = 1.732d^2$, $p_w = (3)(1.155d) = 3.465d$, $(1.732d^2)^{5/3}/(3.465d)^{2/3} = 0.6688$, $d = 0.832$ m. Sides and bottom: each $= 1.155d = (1.155)(0.832) = 0.961$ m.

14.47 For the conditions given in Prob. 14.32, determine whether the flow is critical, subcritical, or supercritical.

▮ $v = Q/A = 1.42/[(1.366)(0.683)] = 1.522\ m/s \qquad N_F = v/\sqrt{gd_m} = 1.522/\sqrt{(9.807)(0.683)} = 0.588$

Since $N_F < 1.0$, the flow is subcritical.

14.48 A rectangular channel with a width of 3.0 m and an n value of 0.014 is to carry water at a flow rate of 13.4 m^3/s. Determine the critical depth, velocity, and channel slope.

▮ $d_c = [(Q/B)^2/g]^{1/3} = [(13.4/3.0)^2/9.807]^{1/3} = 1.267$ m $\qquad v_c = Q/A = 13.4/[(1.267)(3.0)] = 3.525$ m/s

$$R = A/p_w = (1.267)(3.0)/(1.267 + 3.0 + 1.267) = 0.6868 \text{ m}$$

$$s_c = \{nv_c/[(1.0)(R^{2/3})]\}^2 = \{(0.014)(3.525)/[(1.0)(0.6868)^{2/3}]\}^2 = 0.00402$$

14.49 The semicircular channel ($n = 0.013$) shown in Fig. 14-17 is to carry water while flowing full (i.e., at a depth of 1.5 ft). Determine the velocity, slope, and discharge when flow is critical.

▮ $d_m = A/B = \frac{1}{2}[(\pi)(3.0)^2/4]/3.0 = 1.178$ ft $\qquad v_c = \sqrt{gd_m} = \sqrt{(32.2)(1.178)} = 6.159$ ft/s

$$s_c = \{nv_c/[(1.486)(R^{2/3})]\}^2 = \{(0.013)(6.159)/[(1.486)(3.0/4)^{2/3}]\}^2 = 0.00426$$

$$Q = Av = \{\tfrac{1}{2}[(\pi)(3.0)^2/4]\}(6.159) = 21.8 \text{ ft}^3/\text{s}$$

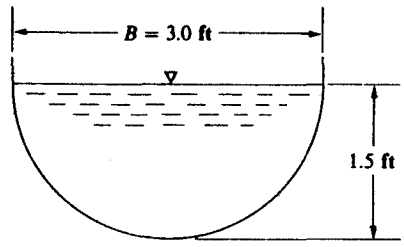


Fig. 14-17

14.50 Determine the dimensions of the most economical trapezoidal brick-lined ($n = 0.016$) channel to carry 200 m^3/s with a slope of 0.0004.

▮ The most economical trapezoidal channel has a cross section as shown in Fig. 14-8 and $R = d/2$ and $A = \sqrt{3}d^2$; $Q = (A)(1.0/n)(R^{2/3})(s^{1/2})$, $200 = (\sqrt{3}d^2)(1.0/0.016)(d/2)^{2/3}(0.0004)^{1/2}$, $d = 6.491$ m. Bottom width = $(1.155)(6.491) = 7.497$ m.

14.51 Determine the discharge for a trapezoidal channel with a bottom width of 8 ft and side slopes 1:1. The depth is 6 ft, and the slope of the bottom is 0.0009. The channel has a finished concrete lining ($n = 0.012$).

▮ $A = (8)(6) + (2)[(\frac{1}{2})(6)(6)] = 84.00$ ft^2 $\qquad p_w = 8 + (2)[(6)(\sqrt{2})] = 24.97$ ft

$$Q = (A)(1.486/n)(R^{2/3})(s^{1/2}) = (84.00)(1.486/0.012)(84.00/24.97)^{2/3}(0.0009)^{1/2} = 701 \text{ ft}^3/\text{s}$$

14.52 What depth is required for 4-m^3/s flow in a rectangular planed-wood ($n = 0.012$) channel 2 m wide with a bottom slope of 0.002?

▮ $Q = (A)(1.0/n)(R^{2/3})(s^{1/2})$ $\qquad 4 = (2d)(1.0/0.012)[2d/(2 + 2d)]^{2/3}(0.002)^{1/2}$

$d^{5/3}/(2 + 2d)^{2/3} = 0.3381$ $\qquad d = 0.888$ m $\qquad$ (by trial and error)

14.53 A developer has been required by environmental regulatory authorities to line an open channel to prevent erosion. The channel is trapezoidal in cross section and has a slope of 0.0009. The bottom width is 10 ft and side slopes are 2:1 (horizontal to vertical). If he uses rubble ($\gamma_s = 135$ lb/ft^3) for the lining, what is the minimum D_{50} of the rubble that can be used? The design flow is 1000 cfs. Assume the shear that rubble can withstand is described by $\tau = (0.040)(\gamma_s - \gamma)(D_{50})$ (lb/ft^2), in which γ_s is the unit weight of rock and D_{50} is the average rock diameter in feet.

▮ A Manning n of 0.03 is appropriate for rubble.

$$Q = (A)(1.486/n)(R^{2/3})(s^{1/2})$$

$$1000 = [(d)(10 + 2d)](1.486/0.03)\{(d)(10 + 2d)/[10 + (2)(\sqrt{5})(d)]\}^{2/3}(0.0009)^{1/2}$$

$[(d)(10 + 2d)]^{5/3}/[10 + (2)(\sqrt{5})(d)]^{2/3} = 672.9$ $\qquad d = 8.63$ ft $\qquad$ (by trial and error) $\qquad \tau_0 = \gamma Rs$

$R = 8.63[10 + (2)(8.63)]/[10 + (2)(\sqrt{5})(8.63)] = 4.841$ ft $\qquad \tau_0 = (62.4)(4.841)(0.0009) = 0.2719$ lb/ft^2

To find the D_{50} size for incipient movement $\tau = \tau_0$ and $0.2719 = (0.040)(135 - 62.4)(D_{50})$, $D_{50} = 0.0936$ ft.

14.54 A metal-lined rectangular sluiceway is to carry 1.0 m^3/s at a slope of 0.010. Determine the minimum area of galvanized iron ($n = 0.011$) needed per meter of length. Neglect freeboard.

▮ For minimum area, $D = B/2$ and $R = B/4$.

$$Q = (A)(1.0/n)(R^{2/3})(s^{1/2}) \qquad 1.0 = [(B)(B/2)](1.0/0.011)(B/4)^{2/3}(0.010)^{1/2} \qquad B = 0.8015\text{ m}$$

$$D = 0.8015/2 = 0.4008\text{ m} \qquad A_{\text{metal}}/L = 0.4008 + 0.8015 + 0.4008 = 1.6\text{ m}^2/\text{m}$$

14.55 The sides of a trapezoidal channel are inclined 63.4° to the vertical; the channel is to carry 18 m³/s with a bottom slope of 0.0009. Determine the bottom width, depth, and velocity for the best hydraulic section ($n = 0.026$).

▮
$$Q = (A)(1.0/n)(R^{2/3})(s^{1/2}) \qquad \text{and} \qquad m = \tan 63.4° = 2.000$$

$$p_w = 4d\sqrt{1+m^2} - 2md = (4)(d)\sqrt{1+(2.000)^2} - (2)(2.000)(d) = 4.944d = B + 2\sqrt{5}d$$

$$B = 0.4719d \qquad A = Bd + 2d^2 = (0.4719d)(d) + 2d^2 = 2.472d^2$$

$$18 = (2.472d^2)(1.0/0.026)(2.472d^2/4.944d)^{2/3}(0.0009)^{1/2} \qquad d = 2.373\text{ m}$$

$$B = (0.4719)(2.373) = 1.120\text{ m} \qquad v = Q/A = 18/[(2.472)(2.373)^2] = 1.293\text{ m/s}$$

14.56 A semicircular corrugated-metal ($n = 0.025$) channel must transport 2.4 m³/s a distance of 1000 m with a head loss of 2 m. Compute the required radius.

▮
$$Q = (A)(1.0/n)(R^{2/3})(s^{1/2}) \qquad 2.4 = (\pi r^2/2)(1.0/0.025)(r/2)^{2/3}(\tfrac{2}{1000})^{1/2} \qquad r = 1.121\text{ m}$$

14.57 Determine the best hydraulic trapezoidal section to convey 86 m³/s with a bottom slope of 0.002. The lining is finished concrete ($n = 0.012$).

▮
$$Q = (A)(1.0/n)(R^{2/3})(s^{1/2}) \qquad 86 = (A)(1.0/0.012)(A/p_w)^{2/3}(0.002)^{1/2} \qquad A^{5/3}/p_w^{2/3} = 23.08$$

The best trapezoidal section is half a regular hexagon (see Fig. 14-8) for which $A = 1.732d^2$ and $p_w = 3.465d$ (from Prob. 14.23). $(1.732d^2)^{5/3}/(3.465d)^{2/3} = 23.08$, $d = 3.141$ m. Sides and bottom: each $= 1.155d = (1.155)(3.141) = 3.628$ m.

14.58 Calculate the discharge in steady flow through the channel and floodway of Fig. 14-18; take $s = 0.0010$ and $y = 2.438$ m.

▮
$$Q = (A)(1.0/n)(R^{2/3})(s^{1/2})$$

$$A_1 = (12)(5+2.438) + (2)(5+2.438)(5+2.438)/2 - (2.438)(2.438)/2 = 141.6\text{ m}^2$$

$$(p_w)_1 = \sqrt{(5+2.438)^2 + (5+2.438)^2} + 12 + \sqrt{5^2+5^2} = 29.59\text{ m}$$

$$Q_1 = (141.6)(1.0/0.025)(141.6/29.59)^{2/3}(0.0010)^{1/2} = 508.6\text{ m}^3/\text{s}$$

$$A_2 = (120)(2.438) + (2.438)(2.438)/2 = 295.5\text{ m}^2$$

$$(p_w)_2 = 120 + \sqrt{2.438^2 + 2.438^2} = 123.4\text{ m} \qquad Q_2 = (295.5)(1.0/0.040)(295.5/123.4)^{2/3}(0.0010)^{1/2} = 418.1\text{ m}^3/\text{s}$$

$$Q = Q_1 + Q_2 = 508.6 + 418.1 = 926.7\text{ m}^3/\text{s}$$

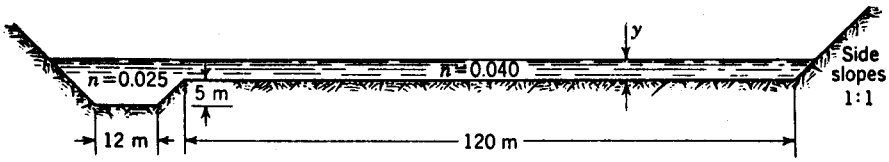


Fig. 14-18

14.59 For 25 000 cfs through the section of Fig. 14-18, find the depth of flow in the floodway (i.e., evaluate y) in feet when the slope of the energy grade line is 0.0004.

▮
$$Q = (A)(1.0/n)(R^{2/3})(s^{1/2}) \qquad A_1 = (12)(5+y) + (2)(5+y)(5+y)/2 - (y)(y)/2 = y^2/2 + 22y + 85$$

$$(p_w)_1 = \sqrt{(5+y)^2 + (5+y)^2} + 12 + \sqrt{y^2+y^2} = (5)(\sqrt{2}) + 12 + (2y)(\sqrt{2})$$

$$A_2 = 120y + (y)(y)/2 = 120y + y^2/2$$

$$(p_w)_2 = 120 + \sqrt{y^2+y^2} = 120 + (y)(\sqrt{2}) \qquad Q = (25\,000)(0.3048)^3 = 707.9\text{ m}^3/\text{s}$$

$$707.9 = (y^2/2 + 22y + 85)(1.0/0.025)\{(y^2/2 + 22y + 85)/[(5)(\sqrt{2}) + 12 + (2y)(\sqrt{2})]\}^{2/3}(0.0004)^{1/2}$$
$$+ (120y + y^2/2)(1.0/0.040)\{(120y + y^2/12)/[120 + (y)(\sqrt{2})]\}^{2/3}(0.0004)^{1/2}$$

$$y = 2.79\text{ m} \quad \text{or} \quad 9.15\text{ ft} \qquad \text{(by trial and error)}$$

14.60 Find the critical depth for flow at 1.6 m^3/s per meter of width.

$$y_c = (q^2/g)^{1/3} = (1.6^2/9.807)^{1/3} = 0.639 \text{ m}$$

14.61 Compute the critical depth for flow at 0.4 m^3/s through the cross section of Fig. 14-19.

$$Q^2T/gA^3 = 1 \qquad T = (2)(y \tan 60°/2) = 1.155y$$

$$A = (y)(y \tan 60°/2) = 0.5774y^2 \qquad (0.4)^2(1.155y)/[(9.807)(0.5774y^2)^3] = 1 \qquad y = 0.628 \text{ m}$$

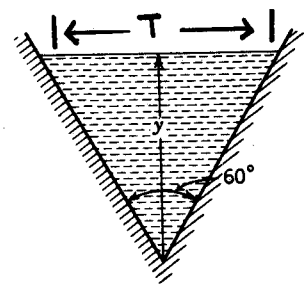


Fig. 14-19

14.62 Determine the critical depth for flow at 8.49 m^3/s through a trapezoidal channel with bottom 2.5 m wide and with 45° sides.

$$Q^2T/gA^3 = 1 \qquad T = 2.5 + 2y \qquad A = By + my^2 = 2.5y + (1)(y)^2$$

$$(8.49)^2(2.5 + 2y)/[(9.807)(2.5y + y^2)^3] = 1 \qquad y = 0.928 \text{ m} \qquad \text{(by trial and error)}$$

14.63 Design a transition from a trapezoidal section, 8 ft bottom width and side slopes 1 on 1, depth 4 ft, to a rectangular section, 6 ft wide and 6 ft deep, for a flow of 250 cfs. The transition is to be 20 ft long, and the loss is one-tenth the difference between velocity heads. Show the bottom profile, and do not make any sudden changes in cross-sectional area.

$A_1 = 8 \times 4 + 4^2 = 48 \text{ ft}^2$, $A_2 = 36 \text{ ft}^2$, loss $= 0.1[(v_2^2/2g) - (v_1^2/2g)]$, and $y_1 + (v_1^2/2g) + z_1 = y_2 + (v_2^2/2g) + z_2 +$ loss. Assume a linear change in area, b, and T: $b = 8 - 2(x/L)$ and $T = 16 - 10(x/L)$. Hence: $A = (b + T)(y/2) = 48 - 12(x/L)$ and $y = 2\{[4 - (x/L)]/[2 - (x/L)]\}$.

x/L	A	$v^2/2g$	loss	EGL	y	z
0	48	0.421		4.421	4.0	0
			0.008			
0.333	44	0.501		4.413	4.4	−0.488
			0.011			
0.667	40	0.607		4.402	5.001	−1.206
			0.014			
1.0	36	0.749		4.388	6.0	−2.361

The profile is shown in Fig. 14-20.

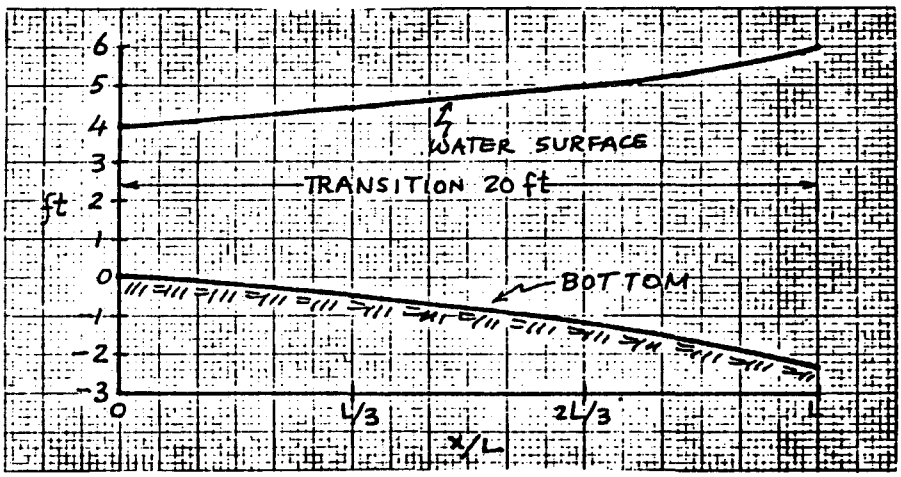


Fig. 14-20

14.64 In a transition from a rectangular channel (2.5 m wide, 2 m deep) to a trapezoidal channel (bottom width 4 m, side slopes 2 on 1, depth 1.3 m) the energy loss is equal to 0.4 times the difference between velocity heads. The discharge is 5.8 m^3/s. Determine the difference between elevations of channel bottoms.

▮ $$0 + v_1^2/2g + y_1 = \Delta z + v_2^2/2g + y_2 + (0.4)(v_1^2/2g - v_2^2/2g) \qquad A_1 = (2.5)(2) = 5.00\ m^2$$

$$A_2 = (4)(1.3) + (2)\{[(1.3)(2)](1.3)/2\} = 8.580\ m^2 \qquad v_1 = Q/A_1 = 5.8/5.00 = 1.160\ m/s$$

$$v_2 = 5.8/8.580 = 0.6760\ m/s$$

$$v_1^2/2g = 1.160^2/[(2)(9.807)] = 0.06860\ m \qquad v_2^2/2g = 0.6760^2/[(2)(9.807)] = 0.02330\ m$$

$$0 + 0.06860 + 2 = \Delta z + 0.02330 + 1.3 + (0.4)(0.06860 - 0.02330) \qquad \Delta z = 0.727\ m$$

14.65 A dam gate (Fig. 14-21) admits water to a horizontal canal. Considering the pressure distribution hydrostatic at section O, compute the depth at O and the discharge per meter of width when $y = 1.0$ m.

▮ $$d_0 = C_c y = (0.85)(1.0) = 0.85\ m \qquad p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L$$

$$0 + 0 + 6 = 0 + v_t^2/[(2)(9.807)] + 0.85 + 0 \qquad v_t = 10.05\ m/s \qquad v_a = C_v v_t = (0.95)(10.05) = 9.548\ m/s$$

$$Q = Av = [(0.85)(1)](9.548) = 8.12\ m^3/s \text{ per meter of width}$$

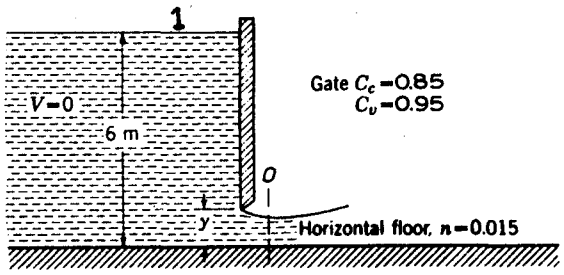


Fig. 14-21

14.66 A discharge of 4.5 m^3/s occurs in a rectangular channel 1.83 m wide with $s = 0.002$ and $n = 0.012$. Find the normal depth for uniform flow and determine the critical depth. Is the flow subcritical or supercritical?

▮ $$Q = (A)(1.0/n)(R^{2/3})(s^{1/2}) \qquad 4.5 = (1.83y_n)(1.0/0.012)[1.83y_n/(y_n + 1.83 + y_n)]^{2/3}(0.002)^{1/2}$$

$$y_n = 1.060\ m \qquad \text{(by trial and error)}$$

$$Q^2/g = A^3/B \qquad 4.5^2/9.807 = (1.83y_c)^3/1.83 \qquad y_c = 0.851\ m$$

Since $y_c < y_n$, the flow is subcritical.

14.67 Determine the rate of a 17-in-deep flow in a 48-in circular corrugated-metal ($n = 0.022$) pipe on a slope of 0.003.

▮ $$Q = (A)(1.486/n)(R^{2/3})(s^{1/2})$$

$$Q_{full} = [(\pi)(\tfrac{48}{12})^2/4](1.486/0.022)[(\tfrac{48}{12})/4]^{2/3}(0.003)^{1/2} = 46.49\ \text{ft}^3/\text{s} \qquad d/d_{full} = \tfrac{17}{48} = 0.35 \quad \text{or} \quad 35\ \text{percent}$$

From Fig. A-18, $Q/Q_{full} = 25$ percent, $Q = (0.25)(46.49) = 11.6\ \text{ft}^3/\text{s}$.

14.68 At what depth will 9.0 ft³/s flow in a 3-ft diameter concrete pipe on a slope of 0.004?

▮ $$Q = (A)(1.486/n)(R^{2/3})(s^{1/2}) \qquad Q_{full} = [(\pi)(3)^2/4](1.486/0.013)[3/4]^{2/3}(0.004)^{1/2} = 42.18\ \text{ft}^3/\text{s}$$

$$Q/Q_{full} = 9.0/42.18 = 0.21 \quad \text{or} \quad 21\%$$

From Fig. A-18, $d/d_{full} = 32\%$, $d = (0.32)(3) = 0.96$ ft.

14.69 Find the flow rate in a 400-mm-diameter concrete pipe on a slope of 0.004, if the depth is 85 mm?

▮ $$Q = (A)(1.0/n)(R^{2/3})(s^{1/2}) \qquad Q_{full} = [(\pi)(0.400)^2/4](1.0/0.013)[0.400/4]^{2/3}(0.004)^{1/2} = 0.1317\ \text{m}^3/\text{s}$$

$$d/d_{full} = 85/400 = 0.21 \quad \text{or} \quad 21\%$$

From Fig. A-18, $Q/Q_{full} = 10\%$, $Q = (0.10)(0.1317) = 0.0132\ \text{m}^3/\text{s}$.

14.70 Repeat Prob. 14.68 if the pipe is corrugated metal ($n = 0.024$).

▮ $$Q = (A)(1.486/n)(R^{2/3})(s^{1/2}) \qquad Q_{full} = [(\pi)(3)^2/4](1.486/0.024)[3/4]^{2/3}(0.004)^{1/2} = 22.85\ \text{ft}^3/\text{s}$$

$$Q/Q_{full} = 9.0/22.85 = 0.39 \quad \text{or} \quad 39\%$$

From Fig. A-18, $d/d_{full} = 43\%$, $d = (0.43)(3) = 1.29$ ft.

14.71 Find the smallest downslope of 3-ft-diameter cast iron piping ($n = 0.015$) that will yield an 18 ft³/s, 2.4-ft-deep flow.

▮ $d/d_{full} = 2.4/30.8$. From Fig. A-18, $Q/Q_{full} = 96\%$, $Q_{full} = 18/0.96 = 18.75\ \text{ft}^3/\text{s}$, $Q = (A)(1.486/n)(R^{2/3})(s^{1/2})$, $18.75 = [(\pi)(3)^2/4](1.486/0.015)[3/4]^{2/3}(s)^{1/2}$, $s = 0.00105$.

14.72 Find the slope of a 10-ft-wide rectangular channel having $n = 0.012$ that makes critical flow occur at a depth of 4.0 ft.

▮ $$Q = \sqrt{A^3g/B} = \sqrt{[(4)(10)]^3(32.2)/10} = 454.0\ \text{ft}^3/\text{s} = (A)(1.486/n)(R^{2/3})(s^{1/2})$$

$$454.0 = [(4)(10)](1.486/0.012)[(4)(10)/(4.0 + 10 + 4.0)]^{2/3}(s^{1/2}) \qquad s = 0.00290$$

14.73 Is the flow of Prob. 14.68 subcritical or supercritical?

▮ For critical flow, $Q^2/g = A^3/B$. For a 3-ft pipe transporting at a depth of 0.96 ft, $B \approx 2.8$ ft and $A \approx 1.94\ \text{ft}^2$. Because $9.0^2/32.2 = 2.5 < 1.94^3/2.8 = 2.6$, the flow is barely subcritical.

14.74 Water flows steadily at 16.0 ft³/s in a triangular sluice with side slopes 1 on 1; the bottom has slope 0.0039. At a certain section, the depth of flow is 2.00 ft. Characterize the flow at this section as subcritical or supercritical.

▮ $Q^2/g = A^3/B$. At critical flow, $A = (2)[(y_c)(y_c)/2] = y_c^2$, $16.0^2/32.2 = (y_c^2)^3/(2y_c)$, $y_c = 1.74$ ft. Since $y_c <$ 2.00 ft, the flow at this section is subcritical.

14.75 Water flows at 8.5 m³/s in a 3.0-m-wide open channel of rectangular cross section. The bottom slopes up, rising 2 mm per meter in the direction of flow. If the water depth decreases from 2.10 m to 1.65 m in a 155 m length of channel, determine Manning's n.

▮ $$z_1 + y_1 + v_1^2/2g = z_2 + y_2 + v_2^2/2g + h_L$$

$$v_1 = Q/A_1 = 8.5/[(2.10)(3.0)] = 1.349 \text{ m/s} \qquad v_2 = 8.5/[(1.65)(3.0)] = 1.717 \text{ m/s}$$

$$0 + 2.10 + 1.349^2/[(2)(9.807)] = (0.002)(155) + 1.65 + 1.717^2/[(2)(9.807)] + h_L \qquad h_L = 0.0825 \text{ m}$$

$$v = (1.0/n)(R^{2/3})(s^{1/2}) = v_{avg} = (1.349 + 1.717)/2 = 1.533 \text{ m/s} \qquad R = A/p_w$$

$$R_1 = (2.10)(3.0)/(2.10 + 3.0 + 2.10) = 0.8750 \text{ m} \qquad R_2 = (1.65)(3.0)/(1.65 + 3.0 + 1.65) = 0.7857 \text{ m}$$

$$R_{avg} = (0.8750 + 0.7857)/2 = 0.8304 \text{ m} \qquad 1.533 = (1.0/n)(0.8304)^{2/3}(0.0825/155)^{1/2} \qquad n = 0.013$$

14.76 A flow of 1000 ft^3/s occurs in a long 10-ft-wide rectangular channel of constant bottom slope. The Manning equation yields 7.0 ft as the normal depth of flow for this flow rate. Will the depth of flow increase, decrease, or remain the same as one proceeds downstream from a point where the depth is 3.0 ft?

▮ $$y_c = (q^2/g)^{1/3} = [(\tfrac{1000}{10})^2/32.2]^{1/3} = 6.77 \text{ ft}$$

which is between 3.0 ft and 7.0 ft. Hence, the depth of flow will increase.

14.77 Determine the cross section of greatest hydraulic efficiency for the trapezoidal channel of Fig. 14-22, if the discharge is 10.0 m^3/s and the channel slope is 0.0005. Use $n = 0.020$.

▮ $R = A/p_w = (1.5y + x)(y)/\{x + (2)[\sqrt{y^2 + (1.5y)^2}]\}$. For greatest hydraulic efficiency, $R = y/2$. Hence, $(1.5y + x)(y)/[x + (2)\sqrt{y^2 + (1.5y)^2}] = y/2$, $x = 0.606y$.

$$Q = (A)(1.0/n)(R^{2/3})(s^{1/2}) \qquad A = (1.5y + x)(y) = (1.5y + 0.606y)(y) = 2.106y^2$$

$$10.0 = (2.106y^2)(1.0/0.020)(y/2)^{2/3}(0.0005)^{1/2} \qquad y = 2.05 \text{ m} \qquad x = (0.606)(2.05) = 1.24 \text{ m}$$

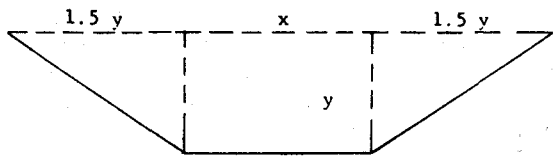


Fig. 14-22

14.78 Specify a canal with $n = 0.0182$ that will carry 500 ft^3/s of water a distance of 1 mile, if the total drop must not exceed 53 ft.

▮ Use a trapezoidal channel with side slopes of 1:1, and assume $v_{max} = 4.0$ ft/s.

$$s_{max} = 53/5280 = 0.0100 \qquad A_{min} = Q/v_{max} = 500/4.0 = 125.0 \text{ ft}^2 \qquad Q = (A)(1.486/n)(R^{2/3})(s^{1/2})$$

Try $b = 8$ ft and $y = 8$ ft: $A = (8)(8 + 8) = 128.0$ ft^2, $p_w = 8 + (2)(\sqrt{8^2 + 8^2}) = 30.63$ ft, $500 = (128.0)(1.486/0.0182)(128.0/30.63)^{2/3}(s)^{1/2}$, $s = 0.000340$. Because $s < s_{max}$ this design is acceptable.

14.79 Given an open channel with a parabolic cross section ($x = 1.0$ m and $y = 1.0$ m in Fig. 14-23) on a slope of 0.02 with $n = 0.015$, find the normal depth and the critical depth for a flow rate of 2.0 m^3/s.

▮ $Q = (A)(1.0/n)(R^{2/3})(s^{1/2})$. Equation of parabola: $x^2 = y$.

$$A = (2)(\tfrac{2}{3})(xy) = (\tfrac{4}{3})(y^{1/2}y) = 4y^{3/2}/3$$

$$p_w = (2x)[1 + (\tfrac{2}{3})(y/x)^2 - (\tfrac{2}{5})(y/x)^4 + \cdots] = (2y^{1/2})[1 + 2y/3 - 2y^2/5 + \cdots]$$

Try $y_n = 0.5$ m: $A = (4)(0.5)^{3/2}/3 = 0.4714$ m^2, $p_w = (2)(0.5)^{1/2}[1 + (2)(0.5)/3 - (2)(0.5)^2/5 + \cdots] = 1.744$ m, $Q = (0.4714)(1.0/0.015)(0.4714/1.744)^{2/3}(0.02)^{1/2} = 1.86$ m^3/s. Since this value of Q (1.86 m^3/s) is slightly less than the given value (2.0 m^3/s), try a slightly higher value of y_n, say 0.52 m: $A = (4)(0.52)^{3/2}/3 = 0.5000$ m^2, $p_w = (2)(0.52)^{1/2}[1 + (2)(0.52)/3 - (2)(0.52)^2/5 + \cdots] = 1.786$ m, $Q = (0.5000)(1.0/0.015)(0.5000/1.786)^{2/3}(0.02)^{1/2} = 2.02$ m^3/s. An additional iteration (not shown) indicates an appropriate value of y_n of 0.518 m. Critical depth occurs when $Q^2/g = A^3/B$: $2.0^2/9.807 = (4y_c^{3/2}/3)^3/[(2)(\sqrt{y_c})]$, $y_c = 0.766$ m.

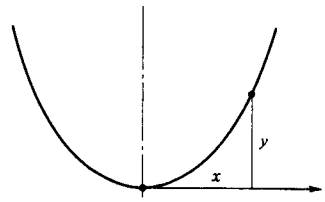


Fig. 14-23

14.80 Consider the frictionless open-channel flow of Fig. 14-24*a*, in which the water depth y_∞ is 2 ft and the volume flow rate per unit width $\dot{Q}/w$ is 5 ft²/s. Discuss what happens as the height of the obstacle on the channel floor is increased.

I We first find out whether the flow upstream of the obstacle is subcritical or supercritical by calculating the critical depth y_c, using $y_c = [(\dot{Q}/w)^2/g]^{1/3} = [(5)^2/32.17]^{1/3} = 0.9194$ ft. The upstream Froude number F_∞ is, then, $F_\infty = (y_c/y_\infty)^{3/2} = (0.9194/2)^{3/2} = 0.3117$. F_∞ is less than 1, showing that the upstream flow is subcritical.

Next we calculate the height of an obstacle which results in critical flow. The specific head of the flow upstream of the obstacle is $H_\infty = (\dot{Q}/w)^2/(2gy_\infty^2) + y_\infty = (5)^2/[(2)(32.17)(2)^2] + 2 = 2.097$ ft. The specific head of the flow at the critical point is the minimum specific head for that $\dot{Q}/w$: $H_{min} = \frac{3}{2}y_c = (\frac{3}{2})(0.9194) = 1.379$ ft. Then the height of an obstacle resulting in critical flow h_c is $h_c = H_\infty - H_{min} = 2.097 - 1.379 = 0.718$ ft.

Let us calculate the surface profile of a flow over an obstacle which is not high enough to cause critical flow, say an obstacle 0.5 ft high. Since the flow is subcritical, the water surface must dip or depress over the obstacle since dy/dx is opposite in sign to dh/dx. The water surface passes through a minimum depth right over the crest of the obstacle. The specific head of the flow over the crest is $H_{crest} = H_\infty - h_{crest} = 2.097 - 0.5 = 1.597$ ft. A dimensionless specific head of $H_{crest}/y_c = 1.597/0.9194 = 1.737$ corresponds to a dimensionless depth for subcritical flow of $y_{crest}/y_c = 1.52$. The depth of the flow over the crest is $y_{crest} = (1.52)(0.9194) = 1.4$ ft. The Froude number of the flow over the crest is $F_{crest} = (y_c/y_{crest})^{3/2} = (1/1.52)^{3/2} = 0.534$.

The local surface depth all along the obstacle is calculated in exactly the same way using the local height $h(x)$ of the obstacle. Figure 14-24*a* shows the water surface and Froude-number variation for this flow over a circular obstacle. The flow is symmetrical about the crest of the obstacle. Obstacles of the same crest height but different shape change the shape of the water-surface depression but not its minimum depth.

Suppose the height of the crest of the obstacle is increased to precisely $h_c = 0.718$ ft, the height at which critical flow occurs. The flow decreases in depth from y_∞ upstream of the obstacle to y_c at the crest with a corresponding increase in Froude number from $N_F = 0.3117$ to 1. Downstream of the crest there are two possible surface profiles. The first possibility is that the flow remains subcritical and its depth increases from y_c back to y_∞ as the Froude number decreases from 1 to $N_F = 0.3117$. This is the limiting case of subcritical flow. The flow is symmetrical about the crest. The second possibility is that the flow passes through the critical point and its Froude number continues to increase while its depth continues to decrease. The flow becomes supercritical. Downstream of the obstacle the flow has a new depth corresponding to the supercritical branch for the same specific head as upstream. Although the specific head is the same on either side of the obstacle, there is a different distribution between the kinetic and potential energy. Upstream of the obstacle the flow is subcritical, and most of the energy is potential, whereas downstream of the obstacle the flow is supercritical and a larger proportion of the energy is kinetic. The transition from subcritical to supercritical flow downstream of the critical point depends on whether the conditions downstream are favorable for maintaining supercritical flow.

Let us calculate the depth downstream of the obstacle for supercritical flow $y_{\infty,sup}$. The specific head is the same as upstream, $H_\infty = 2.097$ ft or $H/y_c = 2.281$. $y/y_c = 0.535$ is on the supercritical branch for this value of H/y_c. The downstream depth is then $y_{\infty,sup} = 0.535(0.9194) = 0.492$ ft. The Froude number there is $(N)_{\infty,sup} = (y_c/y_{\infty,sup})^{3/2} = (N)(1/0.535)^{3/2} = 2.56$.

The surface elevation undergoes its largest changes when the Froude number is in the vicinity of 1. At low subcritical Froude numbers $1 - N_F^2$ is nearly 1, and the decrease in surface elevation is about the same as the increase in obstacle elevation. As the Froude number in supercritical flow increases, $1 - N_F^2$ becomes increasingly more negative, causing the surface elevation to change less and less with changes in obstacle elevation.

Figure 14-24*b* shows the surface profile and Froude-number variation for subcritical and supercritical flow downstream of the crest.

What happens if the obstacle height is increased above h_c, the obstacle height which results in critical flow? The flow can no longer take place with the same values of y_∞ and $\dot{Q}/w$ as before. There must be an adjustment in either y_∞ or $\dot{Q}/w$ or both to raise H_∞ to at least the value which results in critical flow at the crest. This is the minimum specific head required to sustain the flow over the obstacle.

14.81 A large reservoir 5 m deep has a rectangular sluice gate 1 m wide. How does the volume flow rate change as the sluice gate is raised (Fig. 14-25)?

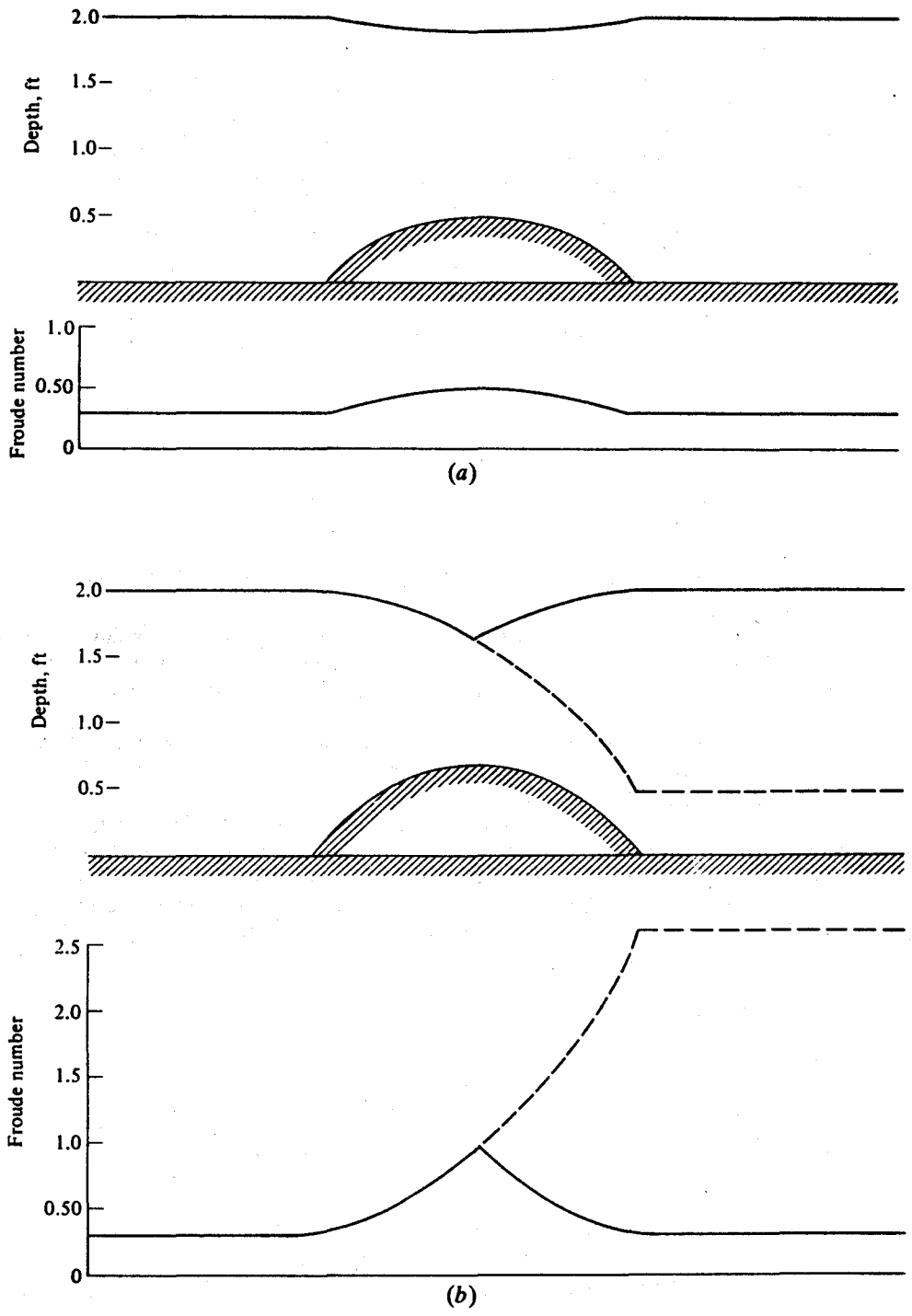


Fig. 14-24

Apply the ideal Bernouilli equation in head form at the water surface and far enough up and downstream of the gate for the streamlines to be parallel: $(p_{atm}/\rho g) + (V_\infty^2/2g) + y_\infty = (p_{atm}/\rho g) + (V^2/2g) + y$. In terms of specific head $H_\infty = H$. This is a constant-specific-head flow. The sluice gate is not an obstacle which changes the specific head of the flow; it simply changes the distribution of energy between kinetic and potential.

Now we examine H_∞, $H_\infty = (V_\infty^2/2g) + y_\infty$. Assume that the reservoir is so large that y_∞ upstream of the gate does not change with changes in sluice-gate opening. Furthermore, V_∞ is very small. Then, $H_\infty \approx y_\infty$ and is the same for all sluice-gate openings. Thus, the outflow has the same specific head regardless of the height of the sluice-gate opening.

The flow rate is then given by $\dot{Q} = wy\sqrt{2g(y_\infty - y)}$. The critical depth y_c changes with the flow rate and

consequently with y: $y_c = [(\dot{Q}/w)^2/g]^{1/3} = [2y^2(y_\infty - y)]^{1/3}$. The corresponding Froude number is $N_F = (y_c/y)^{3/2} = \sqrt{2[(y_\infty/y) - 1]}$.

What happens as the sluice gate is raised? First, consider a small opening which results in a y of, say, 0.5 m. The flow rate is $\dot{Q} = (1)(0.5)\sqrt{2(9.8)(5 - 0.5)} = 4.696\ \text{m}^3/\text{s}$ and the corresponding Froude number is $N_F = \sqrt{2[(5/0.5) - 1]} = 4.243$. The flow is supercritical.

As the gate is raised further, y increases, causing the Froude number to decrease and the flow rate to increase. Finally, at a gate height which gives $y = \frac{2}{3}y_\infty = (\frac{2}{3})(5) = 3.333$ m, The Froude number is 1 and the maximum flow rate is passed: $\dot{Q}_{max} = w\sqrt{\frac{8}{27}gy_\infty^3} = (1)\sqrt{\frac{8}{27}(9.8)(5)^3} = 19.05\ \text{m}^3/\text{s}$.

If the gate is raised farther, the flow becomes subcritical, the Froude number passes through 1 and decreases, and the flow rate starts to decrease.

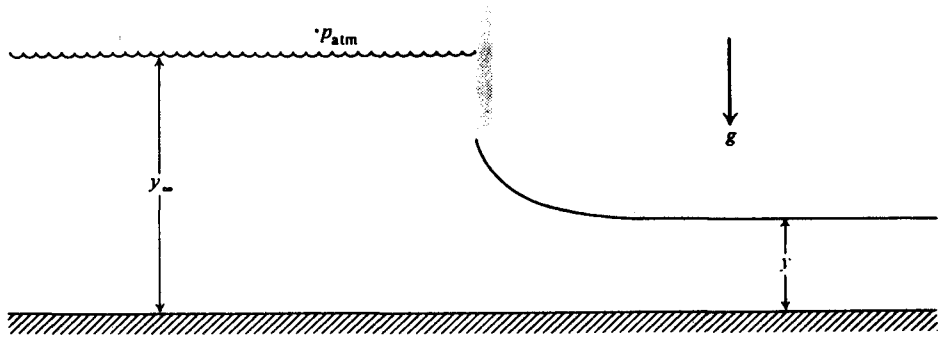


Fig. 14-25

14.82 Reconsider the flow leaving the sluice gate in Prob. 14-81 in which $y_\infty = 5$ m. The flow can be made subcritical by placing a step downstream of the gate. Suppose the flow leaving the sluice gate is $y_1 = 0.5$ m deep. Estimate the height of the step h required to cause a subcritical flow $y_3 = 2$ m deep over the step.

I The problem is solved by considering the flow in two parts (Fig. 14-26). First, a hydraulic jump occurs in front of the step, resulting in subcritical flow and a dissipation of mechanical energy. Then the level of the flow coming over the step decreases in a frictionless flow.

First we calculate the characteristics of the hydraulic jump. From Prob. 13.2 $(N_F)_1 = 4.243$. The depth ratio across the hydraulic jump is calculated from $y_2/y_1 = \frac{1}{2}(\sqrt{1 + 8(N_F)_1^2} - 1) = \frac{1}{2}[\sqrt{1 + (8)(4.243)^2} - 1] = 5.521$. The mechanical-energy dissipation across the hydraulic jump is $(h_f)_{1\to2}/y_1 = [(y_2/y_1) - 1]^3/(4y_2/y_1) = (5.521 - 1)^3/(4)(5.521) = 4.184$, $(h_f)_{1\to2} = 2.092$ m. The frictionless flow over the step is described by $H_2 = H_3 + h$. Now, $h_{t_2} = h_{t_1} - (h_f)_{1\to2}$, or $H_2 = H_1 - (h_f)_{1\to2}$. $H_1 = H_\infty = y_\infty = 5$ m. Then $H_2 = H_1 - (h_f)_{1\to2} = 5 - 2.092 = 2.908$ m, $H_3 = [(\dot{Q}/w)^2/2gy_3^2] + y_3 = \{(4.696)^2/[2(9.8)(2)^2]\} + 2 = 2.281$ m. Finally, $h = H_2 - H_3 = 2.908 - 2.281 = 0.627$ m.

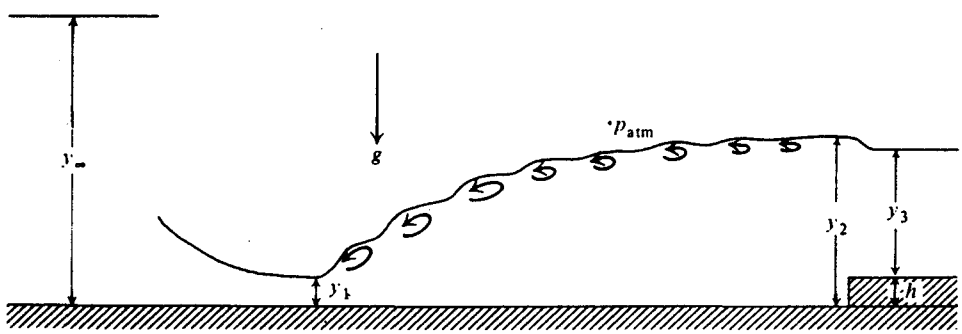


Fig. 14-26

14.83 A long channel with a rectangular cross section and an unfinished concrete surface ($n = 0.017$) is 35 ft wide and has a constant slope of 0.5°. What is the water depth when the channel carries 3500 cfs? Is the channel slope mild or steep?

I $Q = (A)(1.486/n)(R^{2/3})(s^{1/2})$ $\qquad 3500 = (35d)(1.486/0.017)[35d/(35 + 2d)]^{2/3}(\tan 0.5°)^{1/2}$

$d = 4.97$ ft (by trial and error) $\qquad d_c = (q^2/g)^{1/3} = [(\frac{3500}{35})^2/32.2]^{1/3} = 6.77$ ft

Since $d < d_c$, flow is supercritical and the channel slope is steep.

14.84 Find the depth for uniform flow in Fig. 14-27 when the flow rate is 225 cfs if $s = 0.0006$ and n is assumed to be 0.016. Compute the corresponding value of ϵ.

▮ $Q = (A)(1.486/n)(R^{2/3})(s^{1/2})$ $\quad A = (y_0)(10 + 2y_0)$ $\quad p_w = 10 + (2)(\sqrt{5})(y_0)$

$$225 = [(y_0)(10 + 2y_0)](1.486/0.016)\{[(y_0)(10 + 2y_0)]/[10 + (2)(\sqrt{5})(y_0)]\}^{2/3}(0.0006)^{1/2}$$

$$y_0 = 3.41 \text{ ft} \quad \text{(by trial and error)} \quad n = 0.093f^{1/2}R^{1/6}$$

$$R = 3.41[10 + (2)(3.41)]/[10 + (2)(\sqrt{5})(3.41)] = 2.272 \text{ ft} \quad 0.016 = (0.093)(f^{1/2})(2.272)^{1/6} \quad f = 0.0225$$

$$1/\sqrt{f} = 2\log(14.8R/\epsilon) \quad 1/\sqrt{0.0225} = 2\log[(14.8)(2.272)/\epsilon] \quad \epsilon = 0.0156 \text{ ft}$$

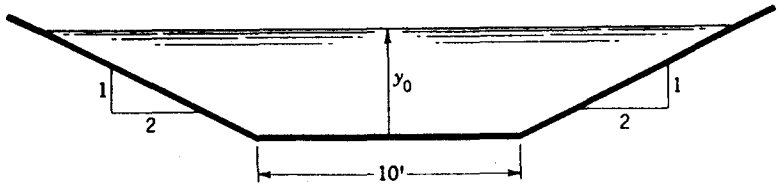


Fig. 14-27

14.85 In Fig. 14-28, water flows uniformly at a steady rate of 14.0 cfs in a very long triangular flume which has side slopes of 1:1. The bottom of this flume is on a slope of 0.006, and $n = 0.012$. **(*a*)** Is the flow subcritical or supercritical? **(*b*)** Find the relation between $v_c^2/2g$ and y_c for this channel.

▮ **(*a*)** $Q = (A)(1.486/n)(R^{2/3})(s^{1/2})$ $\quad A = (y)(2y)/2 = y^2$ $\quad p_w = (2)(\sqrt{2})(y) = 2.828y$

$$14.0 = (y^2)(1.486/0.012)(y^2/2.828y)^{2/3}(0.006)^{1/2} \quad y = 1.49 \text{ ft} \quad \text{(by trial and error)}$$

$$Q^2/g = A^3/B \quad 14.0^2/32.2 = (y_c^2)^3/2y_c \quad y_c = 1.65 \text{ ft}$$

Since $y < y_c$, flow is supercritical.

(*b*) $v_c^2/g = A_c/B_c = y_c^2/2y_c = y_c/2 \quad y_c^2/2g = y_c/4$

Consequently, we see that the relation between $v^2/2g$ and y for critical-flow conditions depends on the geometry of the flow section. If the vertex angle of the triangle had been different, the relation would have been different.

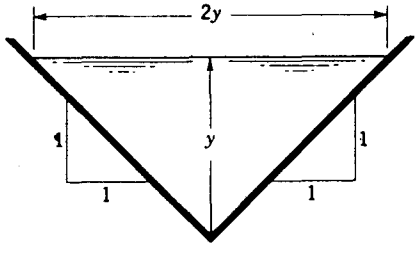


Fig. 14-28

14.86 In Fig. 14-29, uniform flow of water occurs at 27 cfs in a 4-ft-wide rectangular flume at a depth of 2.00 ft. **(*a*)** Is the flow subcritical or supercritical? **(*b*)** If a hump of height $\Delta t = 0.30$ ft is placed in the bottom of the flume, calculate the water depth on the hump. Neglect head loss in flow over the hump. **(*c*)** If the hump height is raised to $\Delta z = 0.60$ ft, what then are the water depths upstream and downstream of the hump? Once again neglect head loss over the hump.

▮ **(*a*)** First find critical depth: $y_c = (q^2/g)^{1/3} = [(\frac{27}{4})^2/32.2]^{1/3} = 1.12$ ft. Since the normal depth (2.00 ft) is greater than the critical depth, the flow is subcritical and the channel slope is mild.

(*b*) Find the critical hump height. Write the energy equation between sections 1 and 2, assume critical flow on the hump and apply continuity.

$$2.00 + (V_1^2/2g) = (\Delta z)_{\text{crit}} + 1.12 + (V_2^2/2g) \tag{1}$$

$$V_2 = 27/(4 \times 1.12) = 6.03 \text{ fps} \tag{2}$$

$$V_1 = 27/(4 \times 2) = 3.38 \text{ fps} \tag{3}$$

Substituting (*2*) and (*3*) in (*1*) gives $(\Delta z)_{\text{crit}} = 0.49$ ft. Thus the minimum-height hump that will produce critical depth on the hump is 0.49 ft.

Since the actual hump height, $\Delta z = 0.30$ ft, is less than the critical hump height, 0.49 ft, critical flow does not occur on the hump and there is no damming action.

Let us now find the depth y_2 on the hump:

Energy: $$2.00 + (V_1^2/2g) = 0.30 + y_2 + (V_2^2/2g) \tag{4}$$

Continuity: $$(4 \times 2)V_1 = 4y_2V_2 = 27 \text{ cfs} \tag{5}$$

Eliminating V_1 and V_2 from Eqs. (4) and (5) gives three roots for y_2; $y_2 = 1.60$ ft, 0.82 ft, or a negative answer that has no physical meaning. Since the hump height is less than $(\Delta z)_{crit}$, the flow on the hump must be subcritical (that is, $y_2 > y_c$). Hence $y_2 = 1.60$ ft and the drop in the water surface on the hump $= 2.00 - (0.30 + 1.60) = 0.10$ ft.

(c) In this case the hump height $\Delta z = 0.60$ ft which is greater than the critical hump height. Hence critical depth ($y_c = 1.12$ ft) will occur on the hump. Writing the energy equation for this case, we have

$$y_1 + (V_1^2/2g) = 0.60 + 1.12 - (V_2^2/2g) \tag{6}$$

From continuity,

$$(4 \times y_1)V_1 = 27 \text{ cfs} \tag{7}$$

and, for critical flow,

$$V_2^2/2g = \tfrac{1}{2}y_2 = 0.56 \text{ ft} \tag{8}$$

Combining Eqs. (6), (7), and (8) gives $y_1 + [(\frac{27}{4})^2/(2g)y_1^2] = 2.28$ from which $y_1 = 2.12$ ft, 0.66 ft, or a negative answer which has no physical meaning. In this case, damming action occurs and the depth y, upstream of the hump, is 2.12 ft. On the hump the depth passes through critical depth of 1.12 ft and just downstream of the hump the depth will be 0.66 ft. The depth will then increase in the downstream direction following an M_2 water-surface profile until a hydraulic jump occurs to return the depth to the normal uniform flow depth of 2.00 ft.

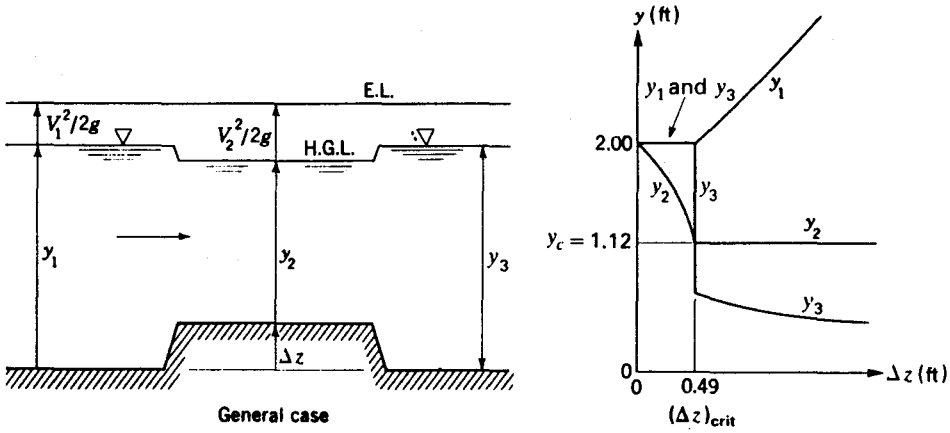


Fig. 14-29

14.87 For the channel of Prob. 14.84, compute the "open-channel Reynolds number" assuming that water at 50 °F is flowing. Refer to Fig. A-5 to verify whether or not the flow is wholly rough. Determine ϵ from Fig. A-5 and compare it with the value computed in Prob. 14.84.

▮ $N_R = Rv/\nu$. From Prob. 14.84, $Q = 225$ cfs, $R = 2.272$ ft, $f = 0.0225$, and $A = 3.41[10 + (2)(3.41)] = 57.36$ ft^2; $v = Q/A = 225/57.36 = 3.923$ ft/s.

Open channel: $N_R = (2.272)(3.923)/(1.40 \times 10^{-5}) = 6.37 \times 10^5$

Equivalent pipe: $N_R = (4)(6.37 \times 10^5) = 2.55 \times 10^6$

From Fig. A-5 with $N_R = 2.55 \times 10^6$ and $f = 0.0225$, $\epsilon/D = 0.0018$ and the flow is wholly rough: $\epsilon = 0.0018D = (0.0018)(4R) = 0.0018[(4)(2.272)] = 0.0164$ ft. This value of ϵ (0.0164) is close to the value of 0.0156 computed in Prob. 14.84.

14.88 On the assumption that Fig. A-5 applies also to open channels, find the rate of discharge of water at 60 °F in a 100-in-diameter smooth concrete pipe flowing half full ($R = D/4$), if the pipe is laid on a grade of 1.8 ft/mile.

▮ $\epsilon/D = 0.001/(\frac{100}{12}) = 0.00012$. Try turbulent flow with $f = 0.0135$:

$$v = \sqrt{(8g/f)(Rs)} \qquad R = D/4 = (\tfrac{100}{12})/4 = 2.083 \text{ ft} \qquad s = \tfrac{1.8}{5280} = 0.0003409$$

$$v = \sqrt{[(8)(32.2)/0.0135][(2.083)(0.0003409)]} = 3.681 \text{ ft/s}$$

$$N_R = Dv/\nu = [(4)(2.083)](3.681)/(1.21 \times 10^{-5}) = 2.53 \times 10^6$$

From Fig. A-5, $f = 0.013$. Try $f = 0.013$:

$$v = \sqrt{[(8)(32.2)/0.013][(2.083)(0.0003409)]} = 3.751 \text{ ft/s} \qquad N_R = [(4)(2.083)](3.751)/(1.21 \times 10^{-5}) = 2.58 \times 10^6$$

$$f = 0.013 \quad \text{(O.K.)} \qquad Q = Av = [(\tfrac{1}{2})(\pi)(\tfrac{100}{12})^2/4](3.751) = 102 \text{ ft}^3/\text{s}$$

14.89 For the channel of Prob. 14.84, compute the flow rate for depth 8 ft.

▮ $Q = (A)(1.486/n)(R^{2/3})(s^{1/2})$. For $y_0 = 8$ ft, $A = 8[10 + (2)(8)] = 208$ ft^2, $p_w = 10 + (2)(\sqrt{5})(8) = 45.78$ ft, $Q = (208)(1.486/0.016)(208/45.78)^{2/3}(0.0006)^{1/2} = 1298$ ft^3/s.

14.90 Figure 14-30 shows a cross section of a canal designed to carry 1590 cfs. The canal is lined with concrete ($n = 0.014$). Find the grade of the canal and the drop in elevation per mile.

▮

$$Q = (A)(1.486/n)(R^{2/3})(s^{1/2}) \qquad A = (10.2)(50.6 + 20)/2 = 360.1 \text{ ft}^2$$

$$p_w = 18.39 + 20 + 18.39 = 56.78 \text{ ft}$$

$$1590 = (360.1)(1.486/0.014)(360.1/56.78)^{2/3}(s)^{1/2} \qquad s = 0.000147$$

$$\text{Drop in elevation} = (0.000147)(5280) = 0.776 \text{ ft/mile}$$

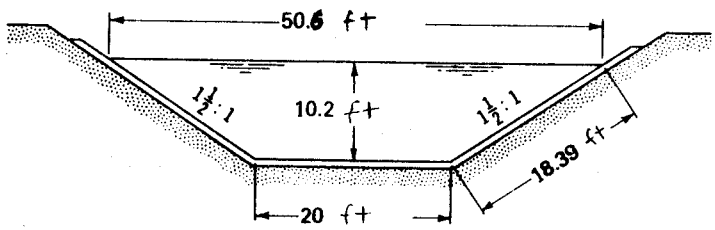


Fig. 14-30

14.91 If the flow in the canal of Prob. 14.90 were halved, all other data, including the slope, being the same, what would be the depth of water?

▮

$$Q = (A)(1.486/n)(R^{2/3})(s^{1/2}) \qquad A = y[20 + (2)(1.5)(y) + 20]/2 = 20y + 1.5y^2$$

$$p_w = 20 + (2)[\sqrt{y^2 + (1.5y)^2}] = 20 + 3.606y$$

$$795 = (20y + 1.5y^2)(1.486/0.014)[(20y + 1.5y^2)/(20 + 3.606y)]^{2/3}(0.000147)^{1/2}$$

$$y = 7.10 \text{ ft} \qquad \text{(by trial and error)}$$

14.92 Evaluate ϵ for Prob. 14.90.

▮ $1/\sqrt{f} = 2\log(14.8R/\epsilon)$, $n = 0.093f^{1/2}R^{1/6}$. Therefore,

$$2\log(14.8R/\epsilon) = 0.093R^{1/6}/n \qquad R = 360.1/56.78 = 6.342 \text{ ft} \qquad \text{(from Prob. 14.90)}$$

$$2\log[(14.8)(6.342)/\epsilon] = (0.093)(6.342)^{1/6}/0.014 \qquad \epsilon = 0.00284 \text{ ft}$$

14.93 Find the capacity of the canal of Prob. 14.90, assuming the grade to be 1.5 ft/mile.

▮ $Q = (A)(1.486/n)(R^{2/3})(s^{1/2})$. From Prob. 14.90, $A = 360.1$ ft^2, $n = 0.014$, $R = 360.1/56.78 = 6.342$ ft, $Q = (360.1)(1.486/0.014)(6.342)^{2/3}(1.5/5280)^{1/2} = 2207$ ft^3/s.

14.94 Water flows uniformly in a 2-m-wide rectangular channel at a depth of 0.5 m. The channel slope is 0.0025 and $n = 0.015$. Find the flow rate.

▮

$$Q = (A)(1.0/n)(R^{2/3})(s^{1/2}) \qquad R = (2)(0.5)/(0.5 + 2 + 0.5) = 0.3333 \text{ m}$$

$$Q = [(2)(0.5)](1.0/0.015)(0.3333)^{2/3}(0.0025)^{1/2} = 1.60 \text{ m}^3/\text{s}$$

14.95 At what depth will 4 m^3/s of water flow in a 3-m-wide rectangular channel if $n = 0.016$ and $s = 0.0009$.

▮

$$Q = (A)(1.0/n)(R^{2/3})(s^{1/2}) \qquad 4 = (3d)(1.0/0.016)[3d/(d + 3 + d)]^{2/3}(0.0009)^{1/2}$$

$$d = 1.00 \text{ m} \qquad \text{(by trial and error)}$$

14.96 The water cross-sectional area in Fig. 14-31 measures 191 ft^2, and the wetted perimeter is 39.1 ft. If the flow is 1580 ft^3/s and $n = 0.012$, find the slope.

▮ $Q = (A)(1.486/n)(R^{2/3})(s^{1/2})$ $\quad 1580 = (191)(1.486/0.012)(191/39.1)^{2/3}(s)^{1/2}$ $\quad s = 0.000538$

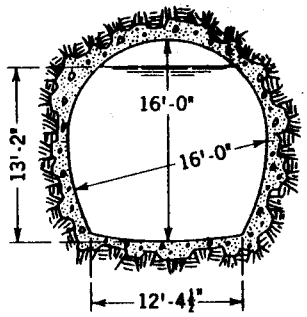


Fig. 14-31

14.97 Rework Prob. 14.96 for a completely filled conduit.

▮ $Q = (A)(1.486/n)(R^{2/3})(s^{1/2})$ $\quad 1580 = [(\pi)(16)^2/4](1.486/0.012)(\frac{16}{4})^{2/3}(s)^{1/2}$ $\quad s = 0.000634$

14.98 A 30-in-diameter pipe is known to have a Manning's n of 0.01800. Calculate Manning's n for a 96-in-diameter pipe that has the same ϵ-value as a 30-in pipe for which $n = 0.01800$.

▮ $1/\sqrt{f} = 2\log(14.8R/\epsilon)$, $n = 0.093f^{1/2}R^{1/6}$. Therefore, $2\log(14.8R/\epsilon) = 0.093R^{1/6}/n$. For 30-in pipe: $R = (\frac{30}{12})/4 = 0.6250$ ft, $2\log[(14.8)(0.6250)/\epsilon] = (0.093)(0.6250)^{1/6}/0.01800$, $\epsilon = 0.03780$ ft. For 96-in pipe: $R = (\frac{96}{12})/4 = 2.000$ ft, $2\log[(14.8)(2.000)/0.03780] = (0.093)(2.000)^{1/6}/n$, $n = 0.01804$.

14.99 The dimensions indicated in Fig. 14-32 pertain to a flow of 30 cfs with $n = 0.018$. Compute the required slope.

▮ $Q = (A)(1.0/n)(R^{2/3})(s^{1/2})$ $\quad A = (2.5)(5) + (2.5-2)[(25-5)/2]/2 = 15.00$ ft^2

$$p_w = 2.5 + 5 + 2 + \sqrt{(2.5-2)^2 + [(25-5)/2]^2} = 19.51 \text{ ft}$$

$$30 = (15.00)(1.0/0.018)(15.00/19.51)^{2/3}(s)^{1/2} \qquad s = 0.00184$$

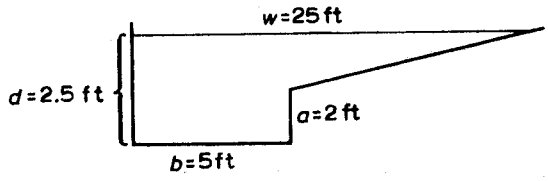


Fig. 14-32

14.100 Water flows at 10 ft/s in a rectangular trough 6 ft wide for which $n = 0.013$. Find the slope needed for a water depth of 3 ft.

▮ $v = (1.486/n)(R^{2/3})(s^{1/2})$ $\quad 10 = (1.486/0.013)[(6)(3)/(3+6+3)]^{2/3}(s)^{1/2}$ $\quad s = 0.00446$

14.101 In Fig. 14-33, area A_1 is 100 ft by 2 ft, A_2 is 30 ft by 10 ft, and A_3 is 200 ft by 3 ft. Compute the flow rate if $s = 0.0018$, $n_1 = n_3 = 0.03$, and $n_2 = 0.020$.

▮ $Q = (A)(1.486/n)(R^{2/3})(s^{1/2})$

$$Q_1 = [(100)(2)](1.486/0.03)[(100)(2)/(2+100)]^{2/3}(0.0018)^{1/2} = 658 \text{ ft}^3/\text{s}$$

$$Q_2 = [(30)(10)](1.486/0.02)\{(30)(10)/[(10-2)+30+(10-3)]\}^{2/3}(0.0018)^{1/2} = 3350 \text{ ft}^3/\text{s}$$

$$Q_3 = [(200)(3)](1.486/0.03)[(200)(3)/(3+200)]^{2/3}(0.0018)^{1/2} = 2597 \text{ ft}^3/\text{s}$$

$$Q = Q_1 + Q_2 + Q_3 = 658 + 3350 + 2597 = 6605 \text{ ft}^3/\text{s}$$

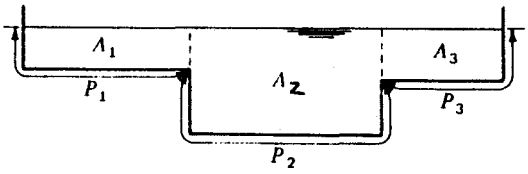


Fig. 14-33

14.102 In Prob. 14.99 the parameters are changed as follows: $a = 3$ ft, $b = 6$ ft, $d = 5$ ft, $w = 36$ ft; $n = 0.020$. Given the slope $s = 0.0015$, determine the rate of flow.

▮ $Q = (A)(1.486/n)(R^{2/3})(s^{1/2})$. For depth = 5 ft: $A_1 = (6)(5) = 30.00\ \text{ft}^2$, $A_2 = (5-3)[(36-6)]/2 = 30.00\ \text{ft}^2$, $(p_w)_1 = 5+6+3 = 14.00$ ft, $(p_w)_2 = \sqrt{(5-3)^2 + [(36-6)]^2} = 30.07$ ft, $Q = (30.00)(1.486/0.020)(30.00/14.00)^{2/3}(0.0015)^{1/2} + (30.00)(1.486/0.020)(30.00/30.07)^{2/3}(0.0015)^{1/2} = 230\ \text{ft}^3/\text{s}$.

14.103 Determine the depth below the surface of clear water at which the velocity (u) as given by the von Karman equation is equal to the mean velocity (v).

▮ $u = v + (1/K)(\sqrt{gy_0s})[1 + 2.3\log(y/y_0)]$. Where $u = v$, $v = v + (1/K)(\sqrt{gy_0s})[1 + 2.3\log(y/y_0)]$, $y/y_0 = 0.367$. Hence, the velocity (u) is equal to the mean velocity (v) when $y/y_0 = 0.367$, or at a depth below the surface of $1 - 0.367$, or 0.633 times the channel depth.

14.104 Figure 14-34 is the longitudinal section of a very wide channel. The fluid is clear water ($K = 0.40$). Given $a = 2.50$ ft and $n = 0.020$, find b.

▮ $v = v + (1/K)(\sqrt{gy_0s})[1 + 2.3\log(y/y_0)]$. Working with a 1-ft width of channel,

$$v = (1.486/0.020)[(2+4+3)(1)/1]^{2/3}(s)^{1/2} \qquad s = 0.000009676v^2 \qquad u_a^2/2g = 2.80\ \text{ft}$$

$$u_a = \sqrt{(2)(32.2)(2.50)} = 12.69\ \text{ft/s} \qquad u = v + (1/K)(\sqrt{gy_0s})[1 + 2.3\log(y/y_0)]$$

$$12.69 = v + (1/0.40)[\sqrt{(32.2)(2+4+3)(0.000009676v^2)}][1 + 2.3\log(2+4)/(2+4+3)] \qquad v = 11.76\ \text{ft/s}$$

$$u_b = 11.76 + (1/0.40)[\sqrt{(32.2)(2+4+3)(0.000009676)(12.45^2)}]\{1 + 2.3\log[2/(4+3+2)]\} = 10.98\ \text{ft/s}$$

$$b = 10.98^2/[(2)(32.2)] = 1.87\ \text{ft}$$

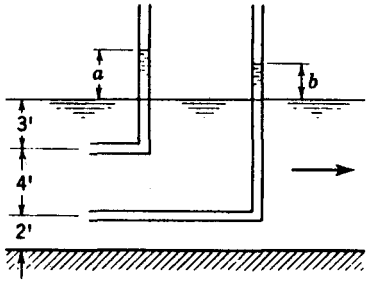


Fig. 14-34

14.105 Water flows uniformly in a very wide rectangular channel at a depth of 1.5 m ($s = 0.006$ and $n = 0.015$). Calculate the velocities at two-thirds and at full depth.

▮ $v = (1.0/n)(R^{2/3})(s^{1/2})$. For a very wide channel, $R = y_0 = 1.5$ m: $v = (1.0/0.015)(1.5)^{2/3}(0.006)^{1/2} = 6.767$ m/s, $u = v + (1/K)(\sqrt{gy_0s})[1 + 2.3\log(y/y_0)]$. At $y = 1.0$ m, $u = 6.767 + (1/0.40)[\sqrt{(9.807)(1.5)(0.006)}][1 + 2.3\log(1.0/1.5)] = 7.21$ m/s. At $y = 1.5$ m, $u = 6.767 + (1/0.40)[\sqrt{(9.807)(1.5)(0.006)}][1 + 2.3\log(1.5/1.5)] = 7.51$ m/s.

14.106 For a rectangular channel cross section of area 5 m^2, tabulate hydraulic radius versus width over the range of widths from 1 m to 5 m.

▮ $R = A/p_w = A/(w + 2d) = Aw(w^2 + 2A)$; see following table.

w, m	R, m
1	0.455
2	0.715
3	0.790
4	0.769
5	0.715

14.107 Set up a general expression for the wetted perimeter p_w of a trapezoidal channel in terms of the cross-sectional area A, depth y, and angle of side slope ϕ. Then differentiate p_w with respect to y with A and ϕ held constant. From this, prove that $R = y/2$ for the section of greatest hydraulic efficiency (i.e., smallest p_w for a given A).

▮ Let B = bottom width.

$$A = By + (y)(y \tan\phi) = By + y^2 \tan\phi \qquad B = A/y - y\tan\phi$$

$$p_w = B + 2y\sec\phi = A/y - y\tan\phi + 2y\sec\phi$$

$$dp_w/dy = -A/y^2 - \tan\phi + 2\sec\phi = -(By + y^2\tan\phi)/y^2 - \tan\phi + 2\sec\phi$$

Setting $dp_w/dy = 0$, $(By + y^2\tan\phi)/y^2 = 2\sec\phi - \tan\phi$, $B = 2y\sec\phi - 2y\tan\phi = (2y)(\sec\phi - \tan\phi)$.

$$R = \frac{A}{p_w} = \frac{By + y^2\tan\phi}{B + 2y\sec\phi} = \frac{(2y)(\sec\phi - \tan\phi)(y) + y^2\tan\phi}{(2y)(\sec\phi - \tan\phi) + 2y\sec\phi} = \frac{y}{2}$$

14.108 Prove that the most efficient triangular section is the one with a 90° vertex angle.

▮ See Fig. 14-35.

$$A = a^2\sin\phi\cos\phi \qquad p_w = 2a \qquad R = A/p_w$$

$$R = (a^2\sin\phi\cos\phi)/2a = (a/2)(\sin\phi\cos\phi) \qquad dR/d\phi = (a/2)(\cos^2\phi - \sin^2\phi) = 0$$

Hence, $\cos\phi = \sin\phi$; or $\phi = 45°$ and the vertex angle = (2)(45), or 90°.

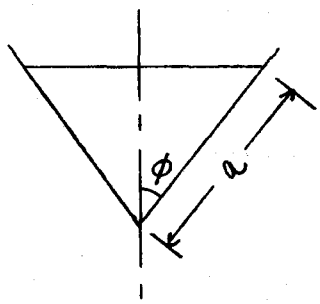


Fig. 14-35

14.109 A canal cut in smooth earth ($n = 0.03123$) must transport 9.0 m^3/s of water at a depth of at most 1.5 m (see Fig. 14-36). If the side slopes are 2:1 and the channel slope is 0.0004, what must be the width at the bottom?

$$Q = (A)(1.0/n)(R^{2/3})(s^{1/2}) \qquad A = 1.5[b + b + (2)(1.5) + (2)(1.5)]/2 = 1.5b + 4.5$$

$$p_w = b + (2)\sqrt{1.5^2 + [(2)(1.5)]^2} = b + 6.708$$

$$9.0 = (1.5b + 4.5)(1.0/0.03123)[(1.5b + 4.5)/(b + 6.708)]^{2/3}(0.0004)^{1/2}$$

$$b = 6.00 \text{ m} \qquad \text{(by trial and error)}$$

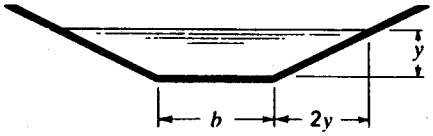


Fig. 14-36

14.110 Refer to Fig. 14-36. If the discharge in the canal ($n = 0.03123$) is to be 6 m³/s at a depth of 1.5 m and if the velocity is not to exceed 1.0 m/s, what must be the width at the bottom and the slope?

$$A = Q/v = 6/(1.0) = 6.00 \text{ m}^2 \qquad (b)(1.5) + (2)[(2)(1.5)(1.5)]/2 = 6.00 \qquad b = 1.0 \text{ m}$$

$$v = (1.0/n)(R^{2/3})(s^{1/2}) \qquad p_w = 1.00 + (2)\sqrt{1.5^2 + [(2)(1.5)]^2} = 7.708 \text{ m}$$

$$\tfrac{60}{60} = (1.0/0.03123)(6.00/7.708)^{2/3}(s)^{1/2} \qquad s = 0.001362$$

14.111 A rectangular flume of smooth wood ($n = 0.012$) slopes 1 ft per 1000 ft. **(*a*)** Compute the rate of discharge if the width is 4 ft and the depth of water is 2 ft. **(*b*)** What would be the rate of discharge if the width were 2 ft and the depth of water 4 ft? **(*c*)** Which of the two forms would have the greater capacity and which would require less lumber?

$$Q = (A)(1.486/n)(R^{2/3})(s^{1/2})$$

(*a*) $Q = [(4)(2)](1.468/0.012)[(4)(2)/(2 + 4 + 2)]^{2/3}(\tfrac{1}{1000})^{1/2} = 31.3 \text{ ft}^3/\text{s}$

(*b*) $Q = [(4)(2)](1.468/0.012)[(4)(2)/(4 + 2 + 4)]^{2/3}(\tfrac{1}{1000})^{1/2} = 27.0 \text{ ft}^3/\text{s}$

(*c*) Lumber ratio $= (2 + 4 + 2)/(4 + 2 + 4) = 0.80$ Flow ratio $= 31.3/27.0 = 1.16$

Hence, the first design provides 16 percent more flow capacity while requiring only 80 percent as much lumber.

14.112 What diameter of semicircular channel will provide the same capacity as a rectangular channel of width 6 m and depth 3 m? Assume s and n are the same for both channels.

$Q = (A)(1.486/n)(R^{2/3})(s^{1/2})$. Since $Q_s = Q_r$ and $(1.486/n)(s^{1/2})$ is constant, $A_sR_s^{2/3} = A_rR_r^{2/3}$.

$$[(\pi d^2/4)/2](d/4)^{2/3} = [(6)(3)][(6)(3)/(3 + 6 + 3)]^{2/3} \qquad d = 6.57 \text{ m}$$

14.113 Consider steady flow of water in a circular concrete pipe ($n = 0.016$) of diameter 10 ft at a depth of 4 ft. Using Fig. A-18, determine the flow rate and the average velocity of flow ($s = 0.0004$).

$$Q = (A)(1.486/n)(R^{2/3})(s^{1/2}) \qquad Q_{\text{full}} = [(\pi)(10)^2/4](1.486/0.016)(\tfrac{10}{4})^{2/3}(0.0004)^{1/2} = 269 \text{ ft}^3/\text{s}$$

$$v_{\text{full}} = Q_{\text{full}}/A_{\text{full}} = 269/[(\pi)(10)^2/4] = 3.43 \text{ ft/s} \qquad d/d_{\text{full}} = \tfrac{4}{10} = 0.40 \quad \text{or} \quad 40 \text{ percent}$$

From Fig. A-18, $Q/Q_{\text{full}} = 32$ percent and $v/v_{\text{full}} = 88$ percent: $Q = (0.32)(269) = 86.1 \text{ ft}^3/\text{s}$, $v = (0.88)(3.43) = 3.02$ ft/s.

14.114 At what depth will water flow at 0.25 m³/s in a 1.0-m-diameter concrete pipe ($n = 0.012$) on a slope of 0.0004?

$$Q = (A)(1.0/n)(R^{2/3})(s^{1/2})$$

$$Q_{\text{full}} = [(\pi)(1.0)^2/4](1.0/0.012)[1.0/4]^{2/3}(0.0004)^{1/2} = 0.519 \text{ m}^3/\text{s}$$

$Q/Q_{\text{full}} = 0.25/0.519 = 0.48$, or 48 percent. From Fig. A-18, $d/d_{\text{full}} = 51$ percent: $d = (0.51)(1.0) = 0.51$ m.

14.115 Evaluate the friction factor f for laminar flow in terms of the Reynolds number, and compare with the equation for pipe flow. (*Note:* Recall that for a wide channel the hydraulic radius is approximately equal to the depth.)

▮ $N_R = (4R)(v)/\nu = 4y_0 v/\nu \qquad q = (g/\nu)(y_0^3/3)(s) \qquad v = Q/A = q/y_0 = gsy_0^2/3\nu = [(8g/f)(R)(s)]^{1/2}$

Therefore, $[(8g/f)(R)(s)] = vgsy_0^2/3\nu$. With $R = y_0$, $f = 24\nu/y_0 v = 96\nu/4y_0 v = 96/N_R$. This compares with $f = 64/N_R$ for pipe flow.

14.116 A viscous fluid ($\nu = 0.0015\ \text{ft}^2/\text{s}$) flows down a flat plate 10 ft wide. Find the maximum rate of discharge for laminar flow, assuming a critical Reynolds number of 500.

▮ $N_R = Rv/\nu = y_0 q/\nu y_0 = q/\nu \qquad q = (500)(0.0015) = 0.7500\ \text{ft}^3/\text{s per ft} \qquad Q = (10)(0.7500) = 7.50\ \text{ft}^3/\text{s}$

14.117 At what rate will water at 60 °F flow in a wide rectangular channel on a slope of 0.00018 if the depth is 0.01 ft?

▮ Assuming laminar flow, $q = (g/\nu)(y_0^3/3)(s) = [32.2/(1.21 \times 10^{-5})](0.01^3/3)(0.00018) = 1.60 \times 10^{-4}\ \text{ft}^3/\text{s/ft}$, $N_R = Rv/\nu = y_0 v/\nu = (y_0)(q/y_0)/\nu = q/\nu = 1.60 \times 10^{-4}/(1.21 \times 10^{-5}) = 13$. Since $N_R < 500$, the assumption of laminar flow is justified.

14.118 At what rate will water at 15 °C flow in a wide, smooth, rectangular channel on a slope of 0.0002, if the depth is 8.0 mm?

▮ Assuming laminar flow, $q = (g/\nu)(y_0^3/3)(s) = [9.807/(1.16 \times 10^{-6})][(0.008)^3/3](0.0002) = 2.89 \times 10^{-4}$ $(\text{m}^3/\text{s})/\text{m}$. Checking the assumption: $N_R = Rv/\nu = y_0 v/\nu = (y_0)(q/y_0)/\nu = q/\nu = 2.89 \times 10^{-4}/(1.16 \times 10^{-6}) =$ 249 (laminar).

14.119 Water flows with a velocity of 4 fps and at a depth of 2 ft in a wide rectangular channel. Is the flow subcritical or supercritical? Find the alternate depth for the same discharge and specific energy.

▮ $$v^2/2g = 4^2/[(2)(32.2)] = 0.2484\ \text{ft} \qquad y/2 = \tfrac{2}{2} = 1.000\ \text{ft}$$

Since $0.2484 < 1.000$, the flow is subcritical.

$$E = y + (1/2g)(q^2/y^2) = 2 + 0.2484 = 2.2484\ \text{ft} \qquad q = (4)(2) = 8.000\ (\text{ft}^2/\text{s})/\text{ft}$$

$$2.2484 = y + \{1/[(2)(32.2)]\}(8.00^2/y^2) \qquad 2.2484y^2 - y^3 - 0.9938 = 0$$

Since $y = 2$ is one known solution, divide by $(y - 2)$ to yield $y^2 - 0.2484y - 0.4968 = 0$, $y = [-(-0.2484) \pm \sqrt{(-0.2484)^2 - (4)(1)(-0.4968)}]/[(2)(1)] = 0.840$ ft.

14.120 Water flows down a wide rectangular channel of concrete ($n = 0.014$) laid on a slope of 2.4 mm/m. Find the depth and rate of flow for critical conditions.

▮ $$v = (1.0/n)(R^{2/3})(s^{1/2}) \qquad v_c = \sqrt{gy_c} \qquad \sqrt{(9.807)(y_c)} = (1.0/0.014)(y_c)^{2/3}(0.0024)^{1/2}$$

$$y_c = 0.514\ \text{m} \qquad q = y_c v_c = (y_c)(\sqrt{gy_c}) = (0.514)[\sqrt{(9.807)(0.514)}] = 1.15\ (\text{m}^3/\text{s})/\text{m}$$

14.121 Water flows at 15 ft/s in a rectangular channel at a depth of 2 ft. Find the critical depth for (***a***) this specific energy, (***b***) this rate of discharge.

▮ (***a***) $E = y + v^2/2g = 2 + 15^2/[(2)(32.2)] = 5.494\ \text{ft} \qquad y_c = (\tfrac{2}{3})(E) = (\tfrac{2}{3})(5.494) = 3.66\ \text{ft}$

(***b***) $q = yv = (2)(15) = 30.00\ \text{ft}^3/\text{s/ft} \qquad y_c = (q^2/g)^{1/3} = (30.00^2/32.2)^{1/3} = 3.03\ \text{ft}$

14.122 A flow of 120 ft^3/s is carried in a rectangular channel 10 ft wide at a depth of 1.5 ft, the channel is made of smooth concrete ($n = 0.013$). Find (***a***) the necessary slope, (***b***) the roughness coefficient needed to produce uniform critical flow for the given rate of discharge on this slope.

▮ (a) $Q = (A)(1.486/n)(R^{2/3})(s^{1/2})$ $\quad 120 = [(1.5)(10)](1.486/0.013)[(1.5)(10)/(1.5 + 10 + 1.5)]^{2/3}(s)^{1/2}$

$$s = 0.00405$$

(b) $y_c = (q^2/g)^{1/3} \qquad y_c = [(\frac{120}{10})^2/32.2]^{1/3} = 1.648 \text{ ft}$

$$v_c = \sqrt{gy_c} = \sqrt{(32.2)(1.648)} = 7.285 \text{ ft/s}$$

$$7.285 = (1.486/n)[(1.648)(10)/(1.648 + 10 + 1.648)]^{2/3}(0.00405)^{1/2} \qquad n = 0.0150$$

14.123 A rectangular channel 4 m wide shows a wavy surface at depth 2.5 m. Estimate the rate of discharge.

▮ The wavy surface indicates a near-critical depth; hence $y_c \approx 2.5$ m. $q_{max} = \sqrt{gy_c^3} = \sqrt{(9.807)(2.5)^3} = 12.38 \text{ m}^3/\text{s/m}$, $Q = (4)(12.38) = 49.5 \text{ m}^3/\text{s}$.

14.124 Probing for oil, geologists drive a small pipe vertically down into the bed of a fast-running stream. The upstream "wake" of the pipe has a spread $\theta = 120°$. Estimate the stream velocity.

▮ $\sin(\theta/2) = c/v \qquad c = \sqrt{gy} = \sqrt{(32.2)(2)} = 8.025 \text{ ft/s} \qquad \sin 60° = 8.025/v \qquad v = 9.3 \text{ ft/s}$

14.125 A speedboat in shallow water lifts a 1-ft wave (height above undisturbed surface), which travels at 9 mph. Find the approximate depth of the water.

▮ $c = \sqrt{(g)(y + \Delta y)[(y + \Delta y/2)/y]} = (9)(5280)/3600 = 13.2 \text{ ft/s} \qquad 13.2 = \sqrt{(32.2)(y + 1)[(y + \frac{1}{2})/y]}$

$$174.2 = 32.2y + 48.30 + 16.1/y \qquad 32.2y^2 - 125.9y + 16.1 = 0$$

The larger root is

$$y = [-(-125.9) + \sqrt{(-125.9)^2 - (4)(32.2)(16.1)}]/[(2)(32.2)] = 3.78 \text{ ft}$$

14.126 A rectangular channel 10 ft wide carries a flow of 180 cfs. Find the critical depth and critical velocity for this flow.

▮ $y_c = (q^2/g)^{1/3} = [(\frac{180}{10})^2/32.2]^{1/3} = 2.16 \text{ ft} \qquad v_c = \sqrt{gy_c} = \sqrt{(32.2)(2.16)} = 8.34 \text{ ft/s}$

14.127 Water flows at 600 ft³/min in a isosceles right-triangular flume for which $n = 0.012$. Find the critical depth and critical slope.

▮ $Q^2/g = A^3/B$. If y = depth, $A = y^2$ and $B = 2y$.

$$(600/60)^2/32.2 = (y^2)^3/2y \qquad y = y_c = 1.44 \text{ ft} \qquad Q = (A)(1.486/n)(R^{2/3})(s^{1/2})$$

$$R = A/p_w = 1.44^2/[(2)(\sqrt{1.44^2 + 1.44^2})] = 0.5091 \text{ ft} \qquad 10 = (1.44^2)(1.486/0.012)(0.5091)^{2/3}(s)^{1/2}$$

$$s = s_c = 0.00373$$

14.128 A trapezoidal canal with side slopes of 2:1 has a bottom width of 4 m and carries a flow of 23 m³/s. Calculate the critical depth.

▮ $Q^2/g = A^3/B$. If y = depth, $A = 4y + 2y^2$ and $B = 4 + 4y$.

$$23^2/9.807 = (4y + 2y^2)^3/(4 + 4y) \qquad y = y_c = 1.22 \text{ m} \qquad \text{(by trial and error)}$$

14.129 Find the specific energy at depth 3 ft for flow of 100 ft³/s through a 10-ft-diameter tunnel.

▮ $E = y + v^2/2g$. For $y = 3$ ft:

$y/y_{full} = \frac{3}{10} = 0.30$ or 30 percent From Fig. A-18 $A/A_{full} = 25$ percent $A = [(\pi)(10)^2/4](0.25) = 19.6 \text{ ft}^2$

$$v = Q/A = 100/19.6 = 5.10 \text{ ft/s} \qquad E = 3 + 5.10^2/[(2)(32.2)] = 3.40 \text{ ft}$$

14.130 A circular conduit flowing half full carries 500 ft³/s at velocity 10 ft³/s. If $n = 0.13$, will the flow be subcritical or supercritical?

▮ $$A = Q/v = (\pi d^2/4)/2 \qquad \tfrac{500}{10} = (\pi d^2/4)/2 \qquad d = 11.28\text{ ft}$$

$$A = [(\pi)(11.28)^2/4]/2 = 49.97\text{ ft}^2 \qquad N_F = v/\sqrt{gy} = v/\sqrt{(g)(A/B)} = 10/\sqrt{(32.2)(49.97/11.28)} = 0.837$$

Since $N_F < 1.0$, the flow is subcritical.

14.131 Figure 14-37 describes the cross section of an open channel for which $s_0 = 0.02$ and $n = 0.015$. The sketch is drawn to the scale shown. When the flow rate is 100 cfs, find (*a*) the depth for uniform flow and (*b*) the critical depth.

▮ The cross-sectional area is found by planimetry and the wetted perimeter by use of dividers.

(*a*)

y_0, ft	A, ft	p_w, ft	R, ft	$Q = (A)(1.486/n)(R^{2/3})(s^{1/2})$, cfs
1	1.30	3.29	0.395	10
2	4.90	6.12	0.801	59
3	10.48	8.96	1.170	163

A plot of Q versus y_0 (not shown) indicates that $y_0 = 2.50$ ft for $Q = 100$ cfs.

(*b*)

y_c, ft	A, ft²	$B(x_L + x_R)$, ft	$Q = (gA^3/B)^{1/2}$, cfs
1	1.30	2.60	5
2	4.90	4.60	29
3	10.48	6.55	75
4	18.30	9.10	147

A plot of Q versus y_c (not shown) indicates that $y_c = 3.35$ ft for $Q = 100$ cfs.

14.132 Refer to Fig. 14-37 and replace feet dimensions with meters. Let the slope be 0.007 with $n = 0.015$. When the flow rate is 50 m³/s, find (*a*) the depth for uniform flow and (*b*) the critical depth.

y ft	x_L ft	x_R ft
0	0	0
1	1.5	1.1
2	2.6	2.0
3	3.25	3.3
4	3.6	5.5
5	3.85	8.2

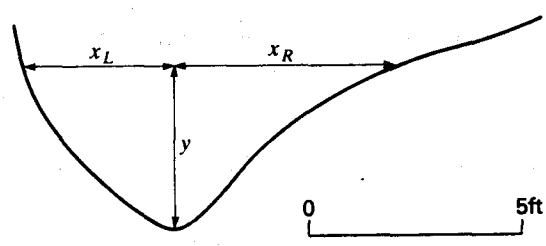


Fig. 14-37

▮ The cross-sectional area is found by planimetry and the wetted perimeter by use of dividers.

(*a*)

y_0, m	A, m²	p_w, m	R, m	$Q = (A)(1.0/n)(R^{2/3})(s^{1/2})$, m³/s
1	1.30	3.29	0.395	4
2	4.90	6.12	0.801	24
3	10.48	8.96	1.170	65

A plot of Q versus y_0 (not shown) indicates that $y_0 = 2.70$ m for $Q = 50$ m^3/s.

(*b*)

y_c, m	A, m^2	$B(x_L + x_R)$, m	$Q = (gA^3/B)^{1/2}$, m^3/s
1	1.30	2.60	3
2	4.90	4.60	16
3	10.48	6.55	42
4	18.30	9.10	81

A plot of Q versus y_c (not shown) indicates that $y_c = 3.25$ m for $Q = 50$ m^3/s.

14.133 A rectangular channel 2 m wide carries 2.2 m^3/s of water in subcritical uniform flow at a depth of 1.0 m. What is the lowest transverse hump in the bottom such that y_c is attained at the peak?

▮ $(\Delta z)_c = E_0 - E_{min}$ $\quad E = y + v^2/2g$ $\quad v = Q/A = 2.2/[(2)(1.0)] = 1.10$ m/s

$$E_0 = 1.0 + 1.10^2/[(2)(9.807)] = 1.0617 \text{ m}$$

$$E_{min} = (\tfrac{3}{2})(y_c) \qquad y_c = (q^2/g)^{1/3} = [(\tfrac{2.2}{2})^2/9.807]^{1/3} = 0.4978 \text{ m}$$

$$E_{min} = (\tfrac{3}{2})(0.4978) = 0.7467 \text{ m} \qquad (\Delta z)_c = 1.0617 - 0.7467 = 0.315 \text{ m}$$

14.134 Rework Prob. 14.86 ($y_0 = 2.0$ ft) for the case where the flow rate is 50 cfs.

▮ (*a*) $q = Q/B = \frac{50}{4} = 12.50$ (ft^3/s)/ft $\quad y_c = (q^2/g)^{1/3} = (12.50^2/32.2)^{1/3} = 1.693$ ft

Since $y_0 > y_c$, the flow is subcritical (and slope is mild).

(*b*) Install hump with $\Delta z = 0.30$ ft. First find critical hump height:

$$E = y + (\tfrac{1}{2}g)(q^2/y^2) \qquad E_0 = 2.00 + \{1/[(2)(32.2)]\}(12.50^2/2.00^2) = 2.607 \text{ ft}$$

$$E_{min} = (\tfrac{3}{2})(y_c) = (\tfrac{3}{2})(1.693) = 2.540 \text{ ft} \qquad E_0 = \Delta z_{crit} + E_{min} \qquad 2.607 = \Delta z_{crit} + 2.540 \qquad \Delta z_{crit} = 0.067 \text{ ft}$$

Since $\Delta z > \Delta z_{crit}$, y_c occurs on the hump and damming action occurs. Hence, water depth on hump = 1.693 ft.

(*c*) Increase hump height to 0.60 ft. Still, $\Delta z > \Delta z_{crit}$, so y_c occurs on the hump. Let y = upstream or downstream depth, with specific energy E.

$$E = \Delta z + E_{min} = y + (1/2g)(q^2/y^2) \qquad \Delta z + E_{min} = y + (1/2g)(q^2/y^2)$$

$$0.60 + 2.540 = y + \{1/[(2)(32.2)]\}(12.50^2/y^2) \qquad y = 2.84 \text{ ft or } 1.09 \text{ ft} \quad \text{(by trial and error)}$$

This hump causes damming action, and the depths just upstream and downstream of the hump are 2.84 ft (subcritical) and 1.09 ft (supercritical), respectively.

14.135 A flow of 2.0 m^3/s is carried in a rectangular channel 1.8 m wide at a depth of 1.0 m. Will critical depth occur at a section where (*a*) a frictionless hump 15 cm high is installed across the bed? (*b*) a frictionless sidewall reduces the channel width to 1.3 m? (*c*) the hump and the sidewall constriction are installed together?

▮ (*a*) $\Delta z_{crit} = E_0 - E_{min}$ $\quad E = y + (1/2g)(q^2/y^2)$ $\quad q = 2.0/1.8 = 1.111$ (m^3/s)/m

$$E_0 = 1.0 + \{1/[(2)(9.807)]\}(1.111^2/1.0^2) = 1.063 \text{ m} \qquad E_{min} = (\tfrac{3}{2})(y_c) \qquad y_c = (q^2/g)^{1/3}$$

$$y_c = (1.111^2/9.807)^{1/3} = 0.5011 \text{ m} \qquad E_{min} = (\tfrac{3}{2})(0.5011) = 0.7517 \text{ m}$$

$$\Delta z_{crit} = 1.063 - 0.7517 = 0.3113 \text{ m}$$

Since $\Delta z = 0.15 < 0.3113 = \Delta z_{crit}$, y_c does not occur at the hump.

(b) $q = 2.0/1.3 = 1.538$ m $\quad y_c = (1.538^2/9.807)^{1/3} = 0.6225$ m $\quad E_{min} = (\frac{3}{2})(0.6225) = 0.9338$ m

Since $E_{min} = 0.9338 < 1.063 = E_0$, y_c does not occur at the constriction.

(c) $\Delta z_{crit} = E_0 - E_{min}$. With constriction, $E_{min} = 0.9338$ m; hence, $\Delta z_{crit} = 1.063 - 0.9338 = 0.1292$ m. Since $\Delta z = 0.15 > 0.1292 = \Delta z_{crit}$, y_c does occur at the hump with constriction.

14.136 Rework Prob. 14.86 ($y_0 = 2.0$ ft) for the case where the flow rate is 16 cfs.

▮ **(a)** $q = Q/B = \frac{16}{4} = 4.000$ ft²/s/ft $\quad y_c = (q^2/g)^{1/3} = (4.000^2/32.2)^{1/3} = 0.7921$ ft

Since $y_0 > y_c$, the flow is subcritical (and slope is mild).

(b) Install hump with $\Delta z = 0.30$ ft. First find critical hump height.

$$E = y + (\tfrac{1}{2}g)(q^2/y^2) \qquad E_0 = 2.00 + \{1/[(2)(32.2)]\}(4.000^2/2.00^2) = 2.062 \text{ ft}$$

$$E_{min} = (\tfrac{3}{2})(y_c) = (\tfrac{3}{2})(0.7921) = 1.188 \text{ ft} \qquad E_0 = \Delta z_{crit} + E_{min} \qquad 2.062 = \Delta z_{crit} + 1.188 \qquad \Delta z_{crit} = 0.874 \text{ ft}$$

Since $\Delta z < \Delta z_{crit}$, y_c does not occur on the hump and damming action does not result.

$$E_h = E_0 - \Delta z \qquad E = y + (1/2g)(q^2/y^2) = y_h + \{1/[(2)(32.2)]\}(4.000^2/y_h^2)$$

$$y_h + \{1/[(2)(32.2)]\}(4.000^2/y_h^2) = 2.062 - 0.30$$

By trial and error, $y_h = 1.67$ ft (subcritical) and 0.432 ft (supercritical). The flow does not pass through y_c, so it cannot become supercritical. Therefore, the water depth on the 0.30-ft hump is 1.67 ft.

(c) Increase hump height to 0.60 ft. Still, $\Delta z < \Delta z_{crit}$, so y_c does not occur on the hump and damming action does not result. $y_h + \{1/[(2)(32.2)]\}(4.000^2/y_h^2) = 2.062 - 0.60$. By trial and error $y_h = 1.32$ ft (subcritical) and 0.511 ft (supercritical). The flow does not pass through y_c, so it cannot become supercritical. Therefore, the water depth on the 0.60-ft hump is 1.32 ft.

14.137 A 4-ft-wide rectangular ditch carries 40 ft³/s of water at a depth of 2.80 ft. A man standing in the middle of the ditch presents a 1-ft width to the stream. Find the local change in the water-surface elevation.

▮ $v_0 = Q/A = 40/[(2.80)(4)] = 3.571$ ft/s. At the man.

$$v = 40/[(4-1)(y)] = 13.33/y \qquad E = y + v^2/2g = y + (13.33/y)^2/[(2)(32.2)] = y + 2.759/y^2$$

$$E_0 = 2.80 + 3.571^2/[(2)(32.2)] = 2.998 \text{ ft} \qquad y + 2.759/y^2 = 2.998$$

By trial and error, $y = 2.59$ ft or 1.26 ft. Since locally supercritical conditions are impossible, $y = 2.59$ ft. Change in water depth $= 2.80 - 2.59 = 0.21$ ft drop.

14.138 A rectangular channel 10 ft wide carries 20 ft³/s in uniform flow at depth 1.0 ft. Find the local change in water-surface elevation caused by an obstruction 0.20 ft high on the floor of the channel.

▮ $y_c = (q^2/g)^{1/3} = [(\frac{20}{10})^2/32.2]^{1/3} = 0.499$ ft. Since $y_c < 1.0$ ft, the flow is subcritical.

$$E = y + v^2/2g \qquad v = Q/A = 20/[(1.0)(10)] = 2.000 \text{ ft/s} \qquad E_1 = 1.0 + 2.000^2/[(2)(32.2)] = 1.062 \text{ ft}$$

$$E_2 = E_1 - \Delta z = 1.062 - 0.20 = 0.862 \text{ ft} \qquad E = y + (1/2g)(q^2/y^2)$$

$$E_2 = y_2 + \{1/[(2)(32.2)]\}[(\tfrac{20}{10})^2/y_2^2)] \qquad 0.862 = y_2 + 0.06211/y_2^2 \qquad y_2 = 0.752 \text{ ft} \quad \text{(by trial and error)}$$

Change in water-surface elevation $= 1.00 - (0.20 + 0.752) = 0.048$ ft (drop).

14.139 Rework Prob. 14.138 for a flow at depth 0.31 ft.

▮ Since $y_c = 0.499$ ft > 0.31 ft, the flow is supercritical.

$$E = y + v^2/2g \qquad v = Q/A = 20/[(0.31)(10)] = 6.452 \text{ ft/s} \qquad E_1 = 0.31 + 6.452^2/[(2)(32.2)] = 0.9564 \text{ ft}$$

$$E_2 = E_1 - \Delta z = 0.9564 - 0.20 = 0.7564 \text{ ft} \qquad E = y + (1/2g)(q^2/y^2) \qquad E_2 = y_2 + \{1/[(2)(32.2)]\}[(\tfrac{20}{10})^2/y_2^2)]$$

$$0.7564 = y_2 + 0.06211/y_2^2 \qquad y_2 = 0.45 \text{ ft} \quad \text{(by trial and error)}$$

Change in water-surface elevation $= (0.20 + 0.45) - 0.31 = 0.34$ ft (rise).

14.140 Repeat Prob. 14.137 for flow depth 0.90 ft.

▮ $E = y + v^2/2g \qquad v_0 = Q/A = 40/[(4)(0.90)] = 11.11 \text{ ft/s} \qquad E_0 = 0.90 + 11.11^2/[(2)(32.2)] = 2.817 \text{ ft}$

As in Prob. 14.137, $v = 13.33/y$.

$$E = y + (13.33/y)^2/[(2)(32.2)] = y + 2.759/y^2 \qquad 2.817 = y + 2.759/y^2$$

$$y = 1.39 \text{ ft} \qquad \text{(by trial and error)}$$

Change in water-surface elevation = 1.39 − 0.90 = 0.49 ft (rise).

14.141 If 1.4 m³/s of water flows uniformly in a channel of width 1.8 m at a depth of 0.75 m, what is the change in water-surface elevation at a section contracted to a 1.2 m width with a 6-cm depression in the bottom?

▮ $y_c = (q^2/g)^{1/3} \qquad (y_c)_2 = [(1.4/1.2)^2/9.807]^{1/3} = 0.5177 \text{ m} \qquad E_2 = E_1 + \Delta z \qquad E = y + v^2/2g$

$$v_1 = Q/A = 1.4/[(0.75)(1.8)] = 1.037 \text{ m/s} \qquad E_1 = 0.75 + 1.037^2/[(2)(9.807)] = 0.8048 \text{ m}$$

$$v_2 = 1.4/(1.2y_2) = 1.167/y_2$$

$$E_2 = y_2 + (1.167/y_2)^2/[(2)(9.807)] = y_2 + 0.06943/y_2^2 \qquad y_2 + 0.06943/y_2^2 = 0.8048 + \tfrac{6}{100}$$

$y_2 = 0.737$ m (subcritical) or 0.377 m (supercritical) (by trial and error). y_2 cannot be less than $(y_c)_2$; hence, $y_2 = 0.737$ m. Change in water-surface elevation $= 0.75 - (0.737 - \frac{6}{100}) = 0.073$ m (drop).

14.142 A finished-concrete channel 8 ft wide has a slope of 0.5° and a water depth of 4 ft. Predict the uniform flow rate by (*a*) Manning's formula with $n = 0.012$ and (*b*) the friction-factor analysis with $\epsilon = 0.0032$ ft.

▮ (*a*)

$$R = A/p_w = (8)(4)/(4 + 8 + 4) = 2.000 \text{ ft}$$

$$Q = (A)(1.486/n)(R^{2/3})(s^{1/2}) = [(8)(4)](1.486/0.012)(2.000)^{2/3}(\tan 0.5°)^{1/2} = 588 \text{ ft}^3/\text{s}$$

(*b*)

$$C = (8g/f)^{1/2} \qquad 1/f^{1/2} = 2.0 \log (3.7D_h/\epsilon) \qquad D_h = 4R = (4)(2.000) = 8.000 \text{ ft}$$

$$1/f^{1/2} = (2.0)\log [(3.7)(8.000)/(0.0032)] \qquad f = 0.01589 \qquad C = [(8)(32.2)/0.01589]^{1/2} = 127.3$$

$$Q = CA(Rs)^{1/2} = (127.3)[(4)(8)][(2.000)(\tan 0.5°)]^{1/2} = 538 \text{ ft}^3/\text{s}$$

14.143 The asphalt-lined trapezoidal channel in Fig. 14-38 carries 300 cfs of water under uniform flow conditions when $s = 0.0015$. What is the normal depth? Use $n = 0.016$.

▮ $Q = (A)(1.486/n)(R^{2/3})(s^{1/2}) \qquad A = (\frac{1}{2})(6 + b_0)(y_n) = 6y_n + y_n^2 \cot 50° \qquad p_w = 6 + 2y_n \csc 50°$

$$300 = (6y_n + y_n^2 \cot 50°)(1.486/0.016)\ [(6y_n + y_n^2 \cot 50°)/(6 + 2y_n \csc 50°)]^{2/3}(0.0015)^{1/2}$$

$$y_n = 4.58 \text{ ft} \qquad \text{(by trial and error)}$$

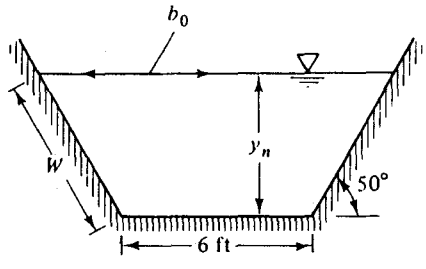


Fig. 14-38

14.144 What are the best dimensions for a rectangular brick ($n = 0.015$) channel designed to carry 5 m³/s of water in uniform flow with $s = 0.001$?

▮ The best dimensions are for width (b) equal to twice the depth (y): $Q = (A)(1.0/n)(R^{2/3})(s^{1/2})$, $5 = [(y)(2y)](1.0/0.015)[(y)(2y)/(y + 2y + y)]^{2/3}(0.001)^{1/2}$, $y = 1.268$ m, $b = (2)(1.268) = 2.536$ m.

14.145 A wide rectangular clean-earth channel ($n = 0.022$) has a flow rate q of 50 cfs/ft (*a*) What is the critical depth? (*b*) What type of flow exists if $y = 3$ ft? (*c*) What is the critical slope?

▮ (*a*)

$$y_c = (q^2/g)^{1/3} = (50^2/32.2)^{1/3} = 4.27 \text{ ft}$$

(*b*) For $y < y_c$, the flow will be supercritical.

(*c*)

$$s_c = gn^2/(2.208y_c^{1/3}) = (32.2)(0.022)^2/[(2.208)(4.27)^{1/3}] = 0.00435$$

14.146 The 50° triangular channel in Fig. 14-39 has a flow rate Q of 16 m³/s. Compute (*a*) y_c, (*b*) v_c, and (*c*) s_c if $n = 0.018$.

▮ (*a*)
$$gA^3 = b_0Q^2 \qquad A = y^2 \cot 50° \qquad b_0 = 2y \cot 50°$$
$$(9.807)(y^2 \cot 50°)^3 = (2y \cot 50°)(16^2) \qquad y = y_c = 2.37 \text{ m}$$

(*b*)
$$v_c = Q/A = 16/[(2.37)^2(\cot 50)] = 3.39 \text{ m/s}$$

(*c*)
$$R = A/p_w = y^2 \cot 50°/(2y \csc 50°) = (2.37)^2(\cot 50°)/[(2)(2.37)(\csc 50°)] = 0.7617 \text{ m}$$
$$v = (1.0/n)(R^{2/3})(s^{1/2}) \qquad 3.39 = (1.0/0.018)(0.7617)^{2/3}(s)^{1/2} \qquad s = s_c = 0.00535$$

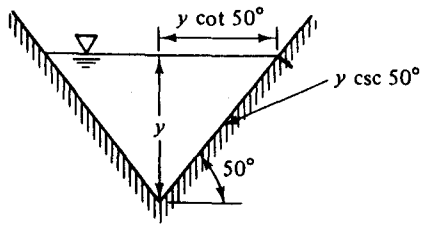


Fig. 14-39

14.147 The formula for shallow-water wave-propagation speed, $c_0 = \sqrt{gy}$, is independent of the physical properties of the liquid, i.e., density, viscosity, or surface tension. Does this mean that waves propagate at the same speed in water, mercury, gasoline, and glycerin? Explain.

▮ $c_0 = \sqrt{gy}$ is correct for any fluid except for viscosity and surface tension (very small wave) effects. It would be accurate for water, mercury, and gasoline but inaccurate for glycerin (too viscous).

14.148 A 10-cm-high wave travels over (shallow) water of depth 1.2 m. Compute the wave speed c and the velocity δv induced by the wave.

▮
$$c = \sqrt{gy(1 + \delta y/y)[1 + (\tfrac{1}{2})(\delta y/y)]} = \sqrt{(9.79)(1.2)(1 + 1/12)[1 + (\tfrac{1}{2})(1/12)]} = 2.60 \text{ m/s}$$
$$\delta v = \frac{(c)(\delta y/y)}{1 + (\delta y/y)} = \frac{(2.60)(1/12)}{13/12} = 0.20 \text{ m/s}$$

14.149 Water flows rapidly in a flat wide channel 0.4 m deep. Pebbles dropped successively in the water at the same spot create two circular ripples which are shown from above in Fig. 14-40. Determine the current speed V.

▮ The centers of the circles move at current V; hence,

$$(x_0)(c_0) = 4V \qquad \text{(smaller circle)}$$
$$(x_0 + 4 + 6 + 9)(c_0) = 9V \qquad \text{(larger circle)}$$

Subtracting these equations, $19c_0 = 5V$, $V = 19c_0/5$, $c_0 = \sqrt{gy} = \sqrt{(9.807)(0.4)} = 1.981$ m/s, $V = (19)(1.981)/5 = 7.53$ m/s.

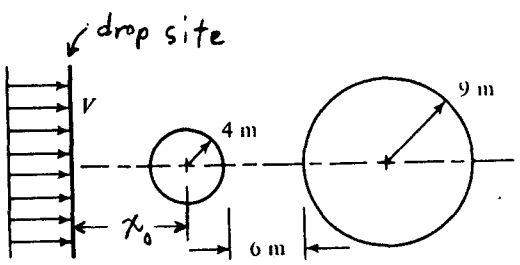


Fig. 14-40

14.150 Rework Prob. 14.149 if the channel depth is 0.6 m and Fig. 14-41 applies.

▮ The centers of the circles move at current V. If the pebble drop site is at distance x_0 ahead of the small circle,

$$(x_0)(c_0) = 3V \qquad \text{(smaller circle)}$$
$$(x_0 + 4)(c_0) = 9V \qquad \text{(larger circle)}$$

Subtracting these equations, $4c_0 = 6V$, $V = 2c_0/3$, $c_0 = \sqrt{gy} = \sqrt{(9.807)(0.6)} = 2.426$ m/s, $V = (2)(2.426)/3 = 1.62$ m/s.

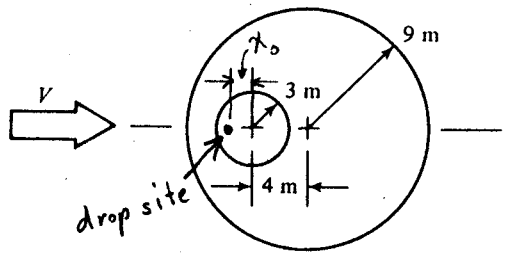


Fig. 14-41

14.151 Consider flow in a wide channel over a bump, as in Fig. 14-42. One can estimate the water-depth change or *transition* with frictionless flow. Use continuity and the Bernoulli equation to show that $dy/dx = -(dh/dx)/(1 - V^2/gy)$. Explain under what conditions the surface might rise above its upstream position y_0.

▮ From the Bernoulli equation, $p_0/\gamma + V_0^2/2g + y_0 = p_1/\gamma + V_1^2/2g + y_1 + h$, $0 + V_0^2/2g + y_0 = 0 + V_1^2/2g + y_1 + h$,

$$V\,dV/g + dy + dh = 0 \qquad (1)$$

From continuity, $V_0 y_0 = V_1 y_1 = \text{constant}$, $V\,dy + y\,dV = 0$, $dV = -V\,dy/y$. Substituting this value of dV into Eq. (1), $(V)(-V\,dy/y)/g + dy + dh = 0$, $dy/dx = -(dh/dx)/(1 - V^2/gy)$. If $dh/dx > 0$ (a bump) and $V^2 < gy$ (subcritical Froude number), dy/dx will be negative and the water level will drop across the bump. If $V^2 > gy$ (supercritical Froude number), the water level will rise.

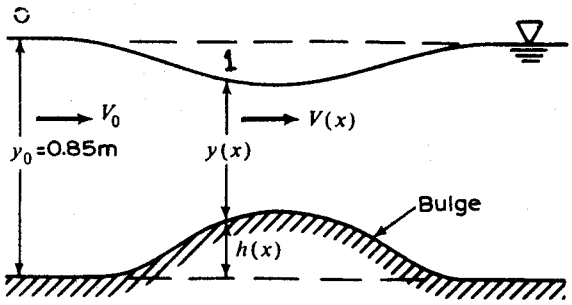


Fig. 14-42

14.152 In Fig. 14-42, the flow rate is 1.2 m³/s per meter of channel width. Compute the velocity and water depth at the top of the bulge ($h_{max} = 0.1$ m). Characterize the flow there.

▮
$$p_0/\gamma + V_0^2/2g + y_0 = p_1/\gamma + V_1^2/2g + y_1 + h \qquad 0 + V_0^2/2g + y_0 = 0 + V_1^2/2g + y_1 + h_{max}$$
$$q = (V_0)(0.85) = V_1 y_1 = 1.2 \qquad V_0 = 1.412 \text{ m/s} \qquad y_1 = 1.2/V_1$$
$$1.412^2/[(2)(9.807)] + 0.85 = V_1^2/[(2)(9.807)] + 1.2/V_1 + 0.1 \qquad 0.05098V_1^3 - 0.8516V_1 + 1.2 = 0$$
$$V_1 = 1.7 \text{ m/s} \quad \text{(by trial and error)} \qquad y_1 = 1.2/1.7 = 0.706 \text{ m}$$
$$(N_F)_1 = V_1/\sqrt{gy_1} = 1.7/\sqrt{(9.807)(0.706)} = 0.646 \quad \text{(subcritical).}$$

14.153 Rework Prob. 14.152 for a flow rate three times as large.

▮
$$p_0/\gamma + V_0^2/2g + y_0 = p_1/\gamma + V_1^2/2g + y_1 + h \qquad 0 + V_0^2/2g + y_0 = 0 + V_1^2/2g + y_1 + h$$
$$q = (V_0)(0.85) = V_1 y_1 = 3.6 \qquad V_0 = 4.24 \text{ m/s} \qquad y_1 = 3.6/V_1$$
$$4.24^2/[(2)(9.807)] + 0.85 = V_1^2/[(2)(9.807)] + 3.6/V_1 + 0.1 \qquad 0.05098V_1^3 - 1.666V_1 + 3.6 = 0$$
$$V_1 = 3.6 \text{ m/s} \quad \text{(by trial and error)} \qquad y_1 = 3.6/3.6 = 1.00 \text{ m}$$
$$(N_F)_1 = V_1/\sqrt{gy_1} = 3.6/\sqrt{(9.807)(1.00)} = 1.15 \quad \text{(supercritical).}$$

14.154 Given the flow of a channel of large width b under a sluice gate, as shown in Fig. 14-43 and assuming frictionless steady flow with negligible upstream kinetic energy, derive a formula for the dimensionless ratio $Q^2/y_1^3b^2g$ as a function of the ratio y_2/y_1.

▮ From the Bernoulli equation, $y_1 + V_1^2/2g = y_2 + V_2^2/2g$. Assuming $V_1^2/2g$ to be negligible, $y_1 = y_2 + V_2^2/2g$. From continuity: $V_1 y_1 = V_2 y_2 = Q/b$, $V_2 = Q/by_2$, $y_1 = y_2 + (Q/by_2)^2/2g$. Multiplying by y_2^2/y_1^3, $y_2^2/y_1^2 = y_2^3/y_1^3 + Q^2/2gb^2y_1^3$, $Q^2/2gb^2y_1^3 = (y_2/y_1)^2 - (y_2/y_1)^3$.

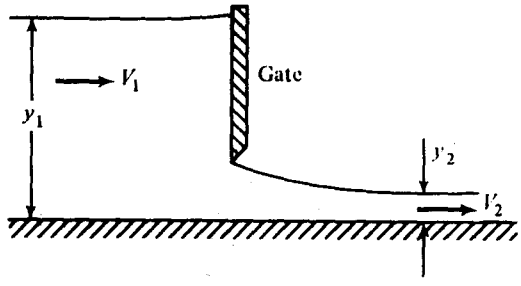


Fig. 14-43

14.155 With reference to Prob. 14.154, take $y_1 = 1.00$ m and $y_2 = 0.60$ m, $b = 10$ m. Compute flow rate Q if the upstream kinetic energy is (*a*) neglected and (*b*) considered.

▮ (*a*) If $V_1^2/2g$ is neglected, the equation developed in Prob. 14.154 applies: $Q^2/2gb^2y_1^3 = (y_2/y_1)^2 - (y_2/y_1)^3$, $Q^2/[(2)(9.807)(10)^2(1.00)^3] = [(0.60)/(1.00)]^2 - [(0.60)/(1.00)]^3$, $Q = 16.8\ \text{m}^3/\text{s}$.

(*b*) From the Bernoulli equation,

$$y_1 + V_1^2/2g = y_2 + V_2^2/2g \qquad V_1 y_1 = V_2 y_2 \qquad (V_1)(1.00) = (V_2)(0.60) \qquad V_2 = 1.667V_1$$

$$1.00 + V_1^2/[(2)(9.807)] = 0.60 + (1.667V_1)^2/[(2)(9.807)] \qquad V_1 = 2.10\ \text{m/s}$$

$$Q = Av = [(10)(1.00)](2.10) = 21.0\ \text{m}^3/\text{s}$$

14.156 For laminar = sheet draining, the flow may become turbulent if N_R exceeds 500. If $s = 0.0018$, what is the maximum sheet thickness y_0 to ensure laminar flow of water at 20 °C?

▮ $$N_R = gy_0^3 s/3\nu^2 \qquad 500 = (9.807)(y_0^3)(0.0018)/[(3)(1.02 \times 10^{-6})^2] \qquad y_0 = 4.45\ \text{mm}$$

14.157 A rectangular channel 3 m wide contains water 2 m deep. If the slope is 0.5°, compute the discharge for uniform flow by the Manning formula ($n = 0.014$).

▮ $$Q = (A)(1.0/n)(R^{2/3})(s^{1/2}) = [(3)(2)](1.0/0.014)[(3)(2)/(2+3+2)]^{2/3}(\tan 0.5°)^{1/2} = 36.13\ \text{m}^3/\text{s}$$

14.158 Solve Prob. 14.157 by the Moody formula ($\epsilon = 0.0024$ m).

▮ $$C = \sqrt{8g/f} \qquad 1/f^{1/2} = 2\log(3.7D_h/\epsilon) \qquad D_h = 4R = (4)[(3)(2)/(2+3+2)] = 3.429\ \text{m}$$

$$1/f^{1/2} = 2\log[(3.7)(3.429)/(0.0024)] \qquad f = 0.01804 \qquad C = \sqrt{(8)(9.807)/0.01804} = 65.95$$

$$Q = CA(Rs)^{1/2} = (65.95)[(3)(2)]\{[(3)(2)/(2+3+2)](\tan 0.5°)\}^{1/2} = 34.22\ \text{m}^3/\text{s}$$

14.159 A large metal trough ($n = 0.014$) slopes at 1:500, its section corresponds to Fig. 14-35 with $\phi = 30°$. Find the water depth for uniform flow at 222.2 L/s.

▮ $$Q = (A)(1.0/n)(R^{2/3})(s^{1/2}) \qquad A = (y)[y\tan 30°] = 0.5774y^2 \qquad p_w = 2y/\cos 30° = 2.309y$$

$$R = A/p_w = 0.5774y^2/2.309y = 0.2501y \qquad 0.2222 = (0.5774y^2)(1.0/0.014)(0.2501y)^{2/3}(\tfrac{1}{500})^{1/2} \qquad y = 0.639\ \text{m}$$

14.160 A trapezoidal channel similar to that of Fig. 14-38 has a bottom width of 10 ft, a side angle of 45°, and a depth of 3 ft. Compute the uniform flow discharge for a clean earth channel ($n = 0.021$) with $s = 0.0004$.

▮ $$A = (10)(3) + (3)[3\tan(90° - 45°)] = 39\ \text{ft}^2 \qquad p_w = 10 + (2)[3/\cos(90° - 45°)] = 18.49\ \text{ft}$$

$$Q = (A)(1.486/n)(R^{2/3})(s^{1/2}) = (39)(1.486/0.021)(39/18.49)^{2/3}(0.0004)^{1/2} = 90.8\ \text{ft}^3/\text{s}$$

14.161 For the channel of Prob. 14.160, compute the normal depth if the flow rate is 170 cfs.

▮ $$Q = (A)(1.486/n)(R^{2/3})(s^{1/2}) \qquad A = 10y + (y)[y\tan(90° - 45°)] = 10y + y^2$$

$$p_w = 10 + (2)[y/\cos(90° - 45°)] = 10 + 2.828y$$

$$170 = (10y + y^2)(1.486/0.021)[(10y + y^2)/(10 + 2.828y)]^{2/3}(0.0004)^{1/2}$$

$$y = 5.63\ \text{ft} \qquad \text{(by trial and error)}$$

14.162 Use Moody's formula to express Manning's n as a function of channel size R.

▮ Combining $C = \sqrt{8g/f} = (1.486/n)(R^{1/6})$, $1/f^{1/2} = 2\log(3.7D_h/\epsilon)$, and $D_h = 4R$, we obtain $n = R^{1/6}/(\alpha + \beta \log R)$, for suitable constants α and β.

14.163 A 7-ft-diameter metal ($n = 0.020$) conduit is flowing half-full on a slope of 1:880. Estimate the discharge.

▮ $$Q = (A)(1.486/n)(R^{2/3})(s^{1/2}) = [(\tfrac{1}{2})(\pi)(7)^2/4](1.486/0.020)(7/4)^{2/3}(1/880)^{1/2} = 70.0\ \text{ft}^3/\text{s}$$

14.164 A trapezoidal conduit similar to that of Fig. 14-38 has a bottom width of 6 m, a side angle of 35°, and carries 60 m³/s of water at a depth of 4 m. If $n = 0.014$, find the slope.

▮ $$Q = (A)(1.0/n)(R^{2/3})(s^{1/2}) \qquad A = (6)(4) + 4[4\tan(90° - 35°)] = 46.85\ \text{m}^2$$

$$p_w = 6 + (2)[4/\cos(90° - 35°)] = 19.95\ \text{m} \qquad 60 = (46.85)(1.0/0.014)(46.85/19.95)^{2/3}(s)^{1/2} \qquad s = 0.000103$$

14.165 For the conduit of Prob. 14.164, determine the water depth given a slope of 0.000512 and a discharge of 40 m³/s.

▮ $$Q = (A)(1.0/n)(R^{2/3})(s^{1/2}) \qquad A = 6y + y[y\tan(90° - 35°)] = 6y + 1.428y^2$$

$$p_w = 6 + 2[y/\cos(90° - 35°)] = 6 + 3.487y$$

$$40 = (6y + 1.428y^2)(1.0/0.014)[(6y + 1.428y^2)/(6 + 3.487y)]^{2/3}(0.000512)^{1/2}$$

$$y = 2.14\ \text{m} \qquad \text{(by trial and error)}$$

14.166 Uniform water flow in a wide brick channel ($n = 0.015$) of slope 0.02° moves over a 10-cm bump as in Fig. 14-44. A slight depression in water surface results. If the minimum water depth over the bump is 50 cm, compute the velocity over the bump and the flow rate per meter of width.

▮ $$v = (1.0/n)(R^{2/3})(s^{1/2}) \qquad v_1 = (1.0/0.015)(y_1)^{2/3}(\tan 0.02°)^{1/2} = 1.246y_1^{2/3}$$

From continuity, $v_1y_1 = v_2y_2$, $(1.246y_1^{2/3})(y_1) = (v_2)(\frac{50}{100})$, $v_2 = 2.492y_1^{5/3}$. Applying the Bernoulli equation and neglecting bottom slope,

$$y_1 + v_1^2/2g = y_2 + v_2^2/2g + h_{\max} \qquad y_1 + (1.246y_1^{2/3})^2/[(2)(9.807)] = \tfrac{50}{100} + (2.492y_1^{5/3})^2/[(2)(9.807)] + \tfrac{10}{100}$$

$$y_1 = 0.623\ \text{m} \qquad \text{(by trial and error)} \qquad v_1 = (1.246)(0.623)^{2/3} = 0.9089\ \text{m/s}$$

$$v_2 = (2.492)(0.623)^{5/3} = 1.13\ \text{m/s} \qquad q = v_1y_1 = (0.9089)(0.623) = 0.566\ (\text{m}^3/\text{s})/\text{m}$$

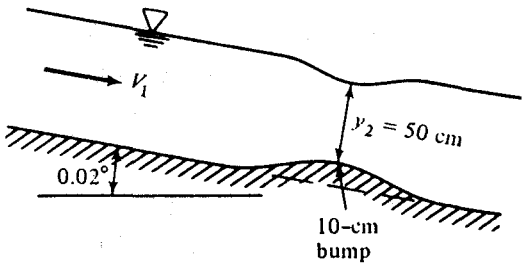


Fig. 14-44

14.167 A tar-coated ($n = 0.014$) triangular channel has 45° sides and a slope of 0.0004. Compute the normal depth for a discharge of 140 ft³/s.

▮ $$Q = (A)(1.486/n)(R^{2/3})(s^{1/2}) \qquad A = (y)(y\cot 45°) = 1.000y^2$$

$$p_w = (2y)(\csc 45°) = 2.828y \qquad R = A/p_w = 1.000y^2/2.828y = 0.3536y$$

$$140 = (1.000y^2)(1.486/0.014)(0.3536y)^{2/3}(0.0004)^{1/2} \qquad y = 6.24\ \text{ft}$$

14.168 A brick ($n = 0.015$) rectangular channel with $s = 0.002$ is designed to carry 230 cfs of water in uniform flow. There is an argument over whether the channel width should be 4 ft or 8 ft. Which design needs fewer bricks? By what percentage?

▮ $Q = (A)(1.486/n)(R^{2/3})(s^{1/2})$. For 4-ft width: $230 = (4y)(1.486/0.015)[4y/(y + 4 + y)]^{2/3}(0.002)^{1/2}$, $y = 9.31$ ft (by trial and error), $p_w = 9.31 + 4 + 9.31 = 22.62$ ft. For 8-ft width: $230 = (8y)(1.486/0.015)[8y/(y + 8 + y)]^{2/3}(0.002)^{1/2}$, $y = 4.07$ ft (by trial and error), $p_w = 4.07 + 8 + 4.07 = 16.14$ ft. Hence, the 8-ft-width design needs fewer bricks by $(22.62 - 16.14)/22.62 = 0.286$, or 28.6 percent.

14.169 In the T-shaped channel of Fig. 14-45, the two arms have rougher walls than does the trunk. Approximate the discharge, if $y_1 = 20$ ft, $y_2 = 5$ ft, $b_1 = 50$ ft, $b_2 = 100$ ft, $n_1 = 0.018$, $n_2 = 0.036$, with a slope of 0.0004.

▮

$$Q = (A)(1.486/n)(R^{2/3})(s^{1/2})$$

$$Q_1 = [(50)(20 + 5)](1.486/0.018)[(50)(20 + 5)/(20 + 50 + 20)]^{2/3}(0.0004)^{1/2} = 11\,925 \text{ ft}^3/\text{s}$$

$$Q_2 = [(100)(5)](1.486/0.036)[(100)(5)/(100 + 5)]^{2/3}(0.0004)^{1/2} = 1168 \text{ ft}^3/\text{s}$$

$$Q = 11\,925 + (2)(1168) = 14\,261 \text{ ft}^3/\text{s}$$

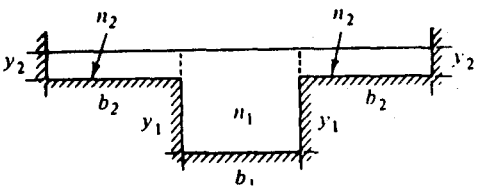


Fig. 14-45

14.170 Repeat Prob. 14.169 for the following parameters: $y_1 = b_1 = 9$ m, $n_1 = 0.018$, $y_2 = 1$ m, $b_2 = 100$ m, $n_2 = 0.036$. The angle of downslope is 0.1°.

▮

$$Q = (A)(1.0/n)(R^{2/3})(s^{1/2})$$

$$Q_1 = [(9)(9 + 1)](1.0/0.018)[(9)(9 + 1)/(9 + 9 + 9)]^{2/3}(\tan 0.1°)^{1/2} = 466 \text{ m}^3/\text{s}$$

$$Q_2 = [(100)(1)](1.0/0.036)[(100)(1)/(100 + 1)]^{2/3}(\tan 0.1°)^{1/2} = 115 \text{ m}^3/\text{s}$$

$$Q = 466 + (2)(115) = 696 \text{ m}^3/\text{s}$$

14.171 A 1-m-diameter clay tile ($n = 0.014$) sewer pipe runs half-full on a slope of 0.004. Compute the flow rate by the Manning formula.

▮

$$Q = (A)(1.0/n)(R^{2/3})(s^{1/2}) = [(\tfrac{1}{2})(\pi)(1)^2/4](1.0/0.014)(\tfrac{1}{4})^{2/3}(0.004)^{1/2} = 0.704 \text{ m}^3/\text{s}$$

14.172 Solve Prob. 14.171 by the Moody formula, with $\epsilon = 0.0026$ m.

▮

$$1/f^{1/2} = 2\log(3.7D/\epsilon) = 2\log[(3.7)(1)/0.0026] \qquad f = 0.02514$$

$$C = \sqrt{8g/f} = \sqrt{(8)(9.807)/0.02514} = 55.86$$

$$Q = CA(Rs)^{1/2} = 55.86[(\tfrac{1}{2})(\pi)(1)^2/4][(\tfrac{1}{4})(0.004)]^{1/2} = 0.694 \text{ m}^3/\text{s}$$

14.173 Four of the sewer pipes from Prob. 14.171 empty into a single finished-cement ($n = 0.012$) pipe, also sloping at 0.004°. If the large pipe is also to run at half-full, what should its diameter be?

▮ $Q = (A)(1.0/n)(R^{2/3})(s^{1/2})$ $\quad (4)(0.704) = [(\tfrac{1}{2})(\pi)(D)^2/4](1.0/0.012)(D/4)^{2/3}(0.004)^{1/2}$ $\quad D = 1.59$ m

14.174 For the circular channel of Fig. 14-46, if $n = 0.016$, $D = 3$ m, and the slope is 0.2°, find the normal depth for a discharge of 20 m³/s.

▮

$$Q = (A)(1.0/n)(R^{2/3})(s^{1/2}) \qquad y = (D/2)(1 + \sin\theta) = (\tfrac{3}{2})(1 + \sin\theta)$$

$$A = (\tfrac{1}{2})(D/2)^2(\pi + 2\theta + \sin 2\theta) = (\tfrac{1}{2})(\tfrac{3}{2})^2(\pi + 2\theta + \sin 2\theta) = (1.125)(\pi + 2\theta + \sin 2\theta)$$

$$p_w = (D/2)(\pi + 2\theta) = (\tfrac{3}{2})(\pi + 2\theta)$$

$$20 = [(1.125)(\pi + 2\theta + \sin 2\theta)](1.0/0.016)[(1.125)(\pi + 2\theta + \sin 2\theta)/(\tfrac{3}{2})(\pi + 2\theta)]^{2/3}(\tan 0.2°)^{1/2}$$

$$\theta = 31.57° \quad \text{(by trial and error)} \qquad y = (\tfrac{3}{2})(1 + \sin 31.57°) = 2.29 \text{ m}$$

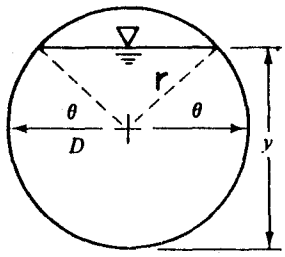


Fig. 14-46

14.175 For fixed n and s, find the diameter of a semicircular channel that will have the same discharge as a rectangular channel with $b = 4$ m and $y = 2$ m.

▮ $$Q = (A)(1.0/n)(R^{2/3})(s^{1/2}) \qquad Q_1 = Q_2$$

$$[(4)(2)](1.0/n)[(4)(2)/(2+4+2)]^{2/3}(s)^{1/2} = [(\tfrac{1}{2})(\pi)(D)^2/4](1.0/n)(D/4)^{2/3}(s)^{1/2} \qquad D = 4.38\text{ m}$$

14.176 A trapezoidal channel has $n = 0.020$ and $s = 0.0004$ and is made in the shape of a half-hexagon for maximum efficiency (see Fig. 14-47). For what length of side will the channel carry 250 cfs of water?

▮ $$Q = (A)(1.486/n)(R^{2/3})(s^{1/2}) \qquad A = (\tfrac{3}{2})(b^2)(\sin 60°) = 1.299b^2$$

$$250 = (1.299b^2)(1.486/0.020)(1.299b^2/3b)^{2/3}(0.0004)^{1/2} \qquad b = 7.64\text{ ft}$$

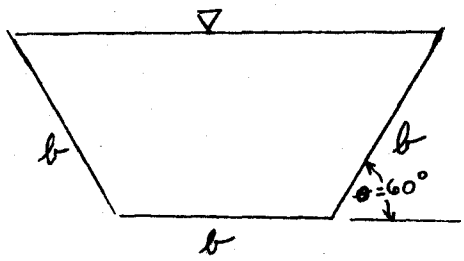


Fig. 14-47

14.177 Compute the discharge of a semicircular channel of the same area, slope, and n-value as the channel of Prob. 14.176.

▮ $A = (1.299)(7.64)^2 = 75.82\text{ ft}^2$ (from Prob. 14.176) $\quad 75.82 = (\tfrac{1}{2})(\pi)(d)^2/4 \quad d = 13.90\text{ ft}$

$$Q = (A)(1.486/n)(R^{2/3})(s^{1/2}) = 75.82(1.486/0.020)(13.9/4)^{2/3}(0.0004)^{1/2} = 258\text{ ft}^3/\text{s}$$

14.178 Find the optimal dimensions of a wooden ($n = 0.012$) rectangular channel that will carry $4.0\text{ m}^3/\text{s}$ at $s = 0.0008$.

▮ For an optimum, width = twice depth ($w = 2y$): $Q = (A)(1.0/n)(R^{2/3})(s^{1/2})$, $4.0 = [(y)(2y)](1.0/0.012)[(y)(2y)/(y+2y+y)]^{2/3}(0.0008)^{1/2}$, $y = 1.118$ m, $w = (2)(1.118) = 2.236$ m.

14.179 How deep should an asphalt ($n = 0.015$) trapezoidal channel, with sides sloping at 45°, be to carry $5\text{ m}^3/\text{s}$ at $s = 0.0006$?

▮ In an optimal design, $R = y/2$. $Q = (A)(1.0/n)(R^{2/3})(s^{1/2})$, $A = (y^2)[(2)(1+\cot 45°)^{1/2} - \cot 45°] = 1.828y^2$, $5 = (1.828y^2)(1.0/0.015)(y/2)^{2/3}(0.0006)^{1/2}$, $y = 1.44$ m.

14.180 Compare the channel of Prob. 14.179, with a semicircular channel of the same slope and area.

▮ $A = (1.828)(1.44)^2 = 3.79\text{ m}^2$ (from Prob. 14.179) $\quad 3.79 = (\tfrac{1}{2})(\pi)(D)^2/4 \quad D = 3.107$ m

$$Q = (A)(1.0/n)(R^{2/3})(s^{1/2}) = (3.79)(1.0/0.015)(3.107/4)^{2/3}(0.0006)^{1/2} = 5.23\text{ m}^3/\text{s}$$

The semicircular channel carries $(5.23 - 5)/5 = 0.046$, or 4.6 percent more than the trapezoidal channel.

14.181 If the side angles of the trapezoidal channel of Prob. 14.176 are reduced to $\theta = 20°$ and if the bottom flat width is 8 ft, find the normal depth of the new channel.

▮ $Q = (A)(1.486/n)(R^{2/3})(s^{1/2})$ $\quad A = by + y^2 \cot 20° = 8y + 2.747y^2$ $\quad p_w = b + 2y \csc 20° = 8 + 5.848y$

$$250 = (8y + 2.747y^2)(1.486/0.020)[(8y + 2.747y^2)/(8 + 5.848y)]^{2/3}(0.0004)^{1/2}$$

$$y = 4.40 \text{ ft} \qquad \text{(by trial and error)}$$

14.182 A 16-ft-deep, clean-earth river ($n = 0.030$) has a flow rate $q = 140$ cfs/ft. Calculate the Froude number of the river.

▮ $y_c = (q^2/g)^{1/3} = (140^2/32.2)^{1/3} = 8.47$ ft $\quad v = 140/16 = 8.750$ ft/s

$$N_F = v/\sqrt{gy_c} = 8.75/\sqrt{(32.2)(8.47)} = 0.530$$

14.183 Find the critical slope in Prob. 14.182 by the Moody method, using $\epsilon = 0.78$ ft.

▮ $s_c = f/8 \quad 1/f^{1/2} = 2 \log (3.7D/\epsilon)$

Using data from Prob. 14.182, $D = 4R = 4y_c = (4)(8.47) = 33.88$ ft, $1/f^{1/2} = 2 \log [(3.7)(33.88)/0.78]$, $f = 0.05137$, $s_c = 0.05137/8 = 0.00642$.

14.184 Are the normal flows corresponding to the two widths in Prob. 14.168 subcritical or supercritical?

▮ $y_c = (Q^2/b^2g)^{1/3}$. For $w = 4$ ft, $y_c = \{230^2/[(4)^2(32.2)]\}^{1/3} = 4.68$ ft < 9.31 ft; so the flow is subcritical. For $w = 8$ ft, $y_c = \{230^2/[(8)^2(32.2)]\}^{1/3} = 2.95$ ft < 4.07 ft; so the flow is subcritical.

14.185 In a 250-mm-deep test channel, a (stationary) ship model sets up the bow wave shown in Fig. 14-48. Determine the flow velocity in the channel.

▮ $\sin \theta = 1/N_F = \sqrt{gh}/v \quad \sin 30° = \sqrt{(9.807)(0.250)}/v \quad v = 3.13$ m/s

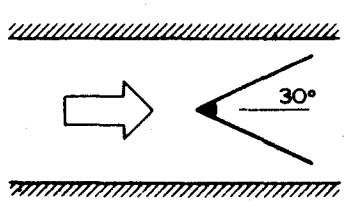


Fig. 14-48

14.186 Suppose that the wave of Fig. 14-48 is seen instead on the surface of water flowing half-full in a circular channel of diameter 1.0 m. What is the flow rate if the surface is finished cement?

▮ Flow is supercritical.

$$v = N_F v_c \qquad N_F = \csc 30° = 2.00 \qquad A_c = (\tfrac{1}{2})(\pi)(1.0)^2/4 = 0.3927 \text{ m}^2$$

$$v_c = (gA_c/b_0)^{1/2} = [(9.807)(0.3927)/1.0]^{1/2} = 1.96 \text{ m/s} \qquad v = (2.00)(1.96) = 3.92 \text{ m/s}$$

$$Q = Av = (0.3927)(3.92) = 1.539 \text{ m}^3/\text{s}$$

14.187 Suppose that the wave of Fig. 14-48 is seen instead on the surface of water flowing full in a half-hexagon of side length 300 mm. What is the flow rate if the sides are of planed wood?

▮ Flow is supercritical.

$$v = N_F v_c \qquad N_F = \csc 30° = 2.00 \qquad v_c = (gA_c/b_0)^{1/2}$$

$$b_0 = 0.300 + (2)(0.300)(\cos 60°) = 0.600 \text{ m} \qquad y = (0.300)(\sin 60°) = 0.260 \text{ m}$$

$$A_c = (\tfrac{1}{2})(0.600 + 0.300)(0.260) = 0.1169 \text{ m}^2 \qquad v_c = [(9.807)(0.1169)/0.600]^{1/2} = 1.38 \text{ m/s}$$

$$v = (2.00)(1.38) = 2.76 \text{ m/s} \qquad Q = Av = (0.1169)(2.76) = 0.323 \text{ m}^3/\text{s}$$

14.188 Find the critical depth and the critical slope for the conduit of Prob. 14.164.

▮ $A = (b_0Q^2/g)^{1/3}$ $\quad b_0 = 6 + 2y_c \cot 35° = 6 + 2.856y_c$ $\quad A = (y_c)(6 + y_c \cot 35°) = 6y_c + 1.428y_c^2$

$6y_c + 1.428y_c^2 = [(6 + 2.856y_c)(50^2)/9.807]^{1/3}$ $\quad y_c = 1.86$ m (by trial and error)

$v_c = (1.0/n)(R^{2/3})(s^{1/2})$ $\quad A = (6)(1.86) + (1.428)(1.86)^2 = 16.10\ \text{m}^2$ $\quad v_c = Q/A = 60/16.10 = 3.727$ m/s

$p_w = 6 + (2)(1.86)/\sin 35° = 12.49$ m $\quad 3.727 = (1.0/0.014)(16.10/12.49)^{2/3}(s_c)^{1/2}$ $\quad s_c = 0.00194$

14.189 For the river of Prob. 14.182, find the depth y_2 at which the specific energy is equal to that at the bottom, $y_1 = 16$ ft.

▮ $E = y + v^2/2g$ $\quad v_1 = 8.750$ ft/s (from Prob. 14.182)

$E_1 = 16 + 8.750^2/[(2)(32.2)] = 17.19$ ft $\quad E_2 = E_1 = 17.19 = y_2 + (140/y_2)^2/[(2)(32.2)]$

$17.19y_2^2 - y_2^3 - 304.3 = 0$ $\quad y_2 = 5.00$ ft (by trial and error)

14.190 Determine the critical slope of a clay-tile ($n = 0.014$) triangular channel with sides sloping at 60° and carrying 12 m³/s.

▮ $gA_c^3 = b_0Q^2$ $\quad A_c = y_c^2 \cot 60° = 0.5774y_c^2$ $\quad b_0 = 2y_c \cot 60° = 1.155y_c$

$(9.807)(0.5774y_c^2)^3 = (1.155y_c)(12)^2$ $\quad y_c = 2.45$ m

$v_c = Q/A_c = 12/[(0.5774)(2.45)^2] = 3.46$ m/s

$v_c = (1.0/n)(R^{2/3})(s^{1/2})$

$3.46 = (1.0/0.014)\{(0.5774)(2.45)^2/[(2)(2.45)/\sin 60°]\}^{2/3}(s_c)^{1/2}$ $\quad s_c = 0.00451$

14.191 A triangular duct flows partly full as shown in Fig. 14-49. If the critical depth is 0.6 m and $n = 0.016$, compute **(*a*)** the critical flow rate and **(*b*)** the critical slope.

▮ **(*a*)** $b_0 = \{[(\sqrt{3}/2) - 0.6]/(\sqrt{3}/2)\}(1) = 0.3072$ m $\quad A = (\frac{1}{2})(0.3072 + 1)(0.6) = 0.3922\ \text{m}^2$

$Q = A\sqrt{gA/b_0} = (0.3922)\sqrt{(9.807)(0.3922)/0.3072} = 1.39\ \text{m}^3/\text{s}$

(*b*) $Q = (A)(1.0/n)(R^{2/3})(s^{1/2})$ $\quad p_w = 1 + (2)[0.6/\sin 60°] = 2.386$ m

$1.39 = (0.3922)(1.0/0.016)(0.3922/2.386)^{2/3}(s)^{1/2}$ $\quad s = 0.0357$

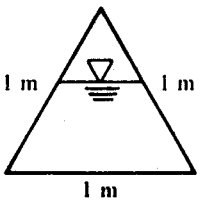


Fig. 14-49

14.192 For the triangular duct of Prob. 14.191, if the critical flow rate is 1.0 m³/s, compute **(*a*)** the critical depth and **(*b*)** the critical slope.

▮ **(*a*)** $Q = A\sqrt{gA/b_0}$ $\quad b_0 = [((\sqrt{3}/2) - y_c)/(\sqrt{3}/2)](1) = 1 - 1.155y_c$

$A = \frac{1}{2}[(1 - 1.155y_c) + 1](y_c) = y_c - 0.5775y_c^2$

$1.0 = (y_c - 0.5775y_c^2)\sqrt{(9.807)(y_c - 0.5775y_c^2)/(1 - 1.155y_c)}$ $\quad y_c = 0.493$ m (by trial and error)

(*b*) $Q = (A)(1.0/n)(R^{2/3})(s^{1/2})$ $\quad p_w = 1 + (2)[0.493/\sin 60°] = 2.139$ m

$A = 0.493 - (0.5775)(0.493)^2 = 0.3526\ \text{m}^2$ $\quad 1.0 = (0.3526)(1.0/0.016)(0.3526/2.139)^{2/3}(s)^{1/2}$

$s = 0.228$

14.193 A rectangular channel 2 m wide and 1 m deep has critical slope of 0.009. Estimate the surface roughness height.

▮ $s_c = gn^2p_w/(1.0b_0R^{1/3})$ $\quad R = A/p_w = (2)(1)/(1 + 2 + 1) = 0.5000$ m

$0.009 = (9.807)(n)^2(1 + 2 + 1)/[(1.0)(2)(0.5000)^{1/3}]$ $\quad n = 0.0191$

$n = 0.0382\epsilon^{1/6}$ $\quad 0.0191 = 0.0382\epsilon^{1/6}$ $\quad \epsilon = 0.0156$ m $= 15.6$ mm

14.194 For a water flow in a rectangular channel 10 m wide, the critical slope is 0.003491. Find the critical depth and discharge, given $n = 0.014$.

▮ $s_c = gn^2p_w/(1.0b_0R^{1/3})$ $\quad 0.003491 = (9.807)(0.014)^2(10 + 2y_c)/\{(1.0)(10)[10y_c/(10 + 2y_c)]^{1/3}\}$

$y_c = 0.196$ m (by trial and error) $\quad Q_c = (gA^3/b_0)^{1/3} = \{(9.807)[(10)(0.196)]^3/10\}^{1/3} = 1.95\ \text{m}^3/\text{s}$

14.195 A circular aluminium ($n = 0.020$) channel 8 ft in diameter has Froude number 0.5 in uniform half-full flow. Compute the channel slope.

▮
$$v_c = (gA/b_0)^{1/2} = \{(32.2)[(\tfrac{1}{2})(\pi)(8)^2/4]/8\}^{1/2} = 10.06\ \text{ft/s}$$
$$v = (1.486/n)(R^{2/3})(s^{1/2}) = v_c/2 = 10.06/2 = 5.03\ \text{ft/s}$$
$$5.03 = (1.486/0.020)(\tfrac{8}{4})^{2/3}(s)^{1/2} \qquad s = 0.00182$$

14.196 Water is in steady flow through the finished-concrete, semicircular channel shown in Fig. 14-50. If the bed slope is 0.0016, what is the flow rate?

▮ $A = (\tfrac{1}{2})(\pi)(10)^2 + (3)(10 + 10) = 217.1\ \text{ft}^2 \qquad R_h = A/p_w = 217.1/[(\pi)(10) + 3 + 3] = 5.802$ ft

$$Q = (A)(1.486/n)(R_h^{2/3})(s_0^{1/2}) = (217.1)(1.486/0.012)(5.802)^{2/3}(0.0016)^{1/2} = 3472\ \text{ft}^3/\text{s}$$

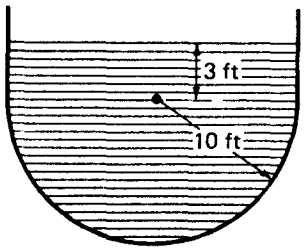


Fig. 14-50

14.197 In a planed-wood ($n = 0.012$) rectangular channel of width 4 m, water is flowing at the rate of 20 m^3/s. If the slope of the channel is 0.0012, what is the depth of uniform flow?

▮ $Q = (A)(1.0/n)(R^{2/3})(s^{1/2}) \qquad 20 = (4D)(1.0/0.012)[4D/(D + 4 + D)]^{2/3}(0.0012)^{1/2}$

$D = 1.80$ m (by trial and error)

14.198 In a planed-wood rectangular channel of width 4 m, water is flowing at a rate of 5 m^3/s. The slope of the channel is 0.0001. The roughness coefficient is 0.5 mm. What is the height h of the cross section of flow for normal, steady flow? The water has a temperature of 10 °C.

▮ $v = Q/A = (8gsR/f)^{1/2} \qquad 5/4y = \{(8)(9.807)(0.0001)[4y/(y + 4 + y)]/f\}^{1/2}$

Like problems in Chap. 9, this requires a trial-and-error solution whereby a value of the friction factor (f) is guessed and subsequent computations and work with Fig. A-5 are done to find the right combination of parameter values to satisfy the problem. Inasmuch as a number of trial-and-error problems of this type were presented in detail in Chap. 9, we shall save time and space in problems of this type in this chapter by "guessing" the correct value of f on the first try! Guess $f = 0.0132$.

$$5/4y = \{(8)(9.807)(0.0001)[4y/(y + 4 + y)]/0.0132\}^{1/2} \qquad y = 1{:}69\ \text{m} \quad \text{(by trial and error)}$$
$$v = Q/A = 5/[(4)(1.69)] = 0.7396\ \text{m/s} \qquad R = (4)(1.69)/(1.69 + 4 + 1.69) = 0.9160\ \text{m}$$
$$N_R = D_hv/\nu = (4R)(v)/\nu = (4)(0.9160)(0.7396)/(1.30 \times 10^{-6}) = 2.08 \times 10^6$$
$$\epsilon/D_h = \epsilon/4R = (0.5/1000)/[(4)(0.9160)] = 0.00014$$

From Fig. A-5, $f = 0.0132$ (O.K.). Therefore, $y = 1.69$ m.

14.199 Do Prob. 14.196 using friction-factor formulas with $\epsilon = 0.0032$ ft. Assume a temperature of 60 °F.

▮ $v^*\epsilon/\nu = (\sqrt{gRs})(\epsilon)/\nu$. Using values from Prob. 14.196, $v^*\epsilon/\nu = [\sqrt{(32.2)(5.802)(0.0016)}](0.0032)/(1.21 \times 10^{-5}) = 145$. Since $v^*\epsilon/\nu > 100$, we are in the fully rough zone and $1/f^{1/2} = 2.16 - 2\log(\epsilon/R) = 2.16 -$

$2\log(0.0032/5.802)$, $f = 0.01328$, $v = (8gsR/f)^{1/2} = \{(8)(32.2)(0.0016)(5.802)/0.01328\}^{1/2} = 13.42$ ft/s, $Q = Av = (217.1)(13.42) = 2913$ ft^3/s.

14.200 A wide rectangular section has flow $q = 80$ cfs/ft. Find the critical depth and the critical slope, if $\epsilon = 0.0035$ ft.

▮
$$y_c = (q^2/g)^{1/3} = (80^2/32.2)^{1/3} = 5.84 \text{ ft}$$
$$s_c = f/8 \qquad N_R = D_h v/\nu \qquad D_h = 4R = (4)(5.84) = 23.36 \text{ ft}$$
$$N_R = (23.36)\{80/[(5)(1)]\}/(1.21\times10^{-5}) = 3.09\times10^7$$
$$\epsilon/D_h = 0.0035/23.36 = 0.000150$$

From Fig. A-5, $f = 0.0128$. $s_c = 0.0128/8 = 0.0016$.

14.201 For steady laminar flow in a thin sheet over a flat surface (see Fig. 14-51), $V_z = [(\gamma\sin\theta)/\mu][(3q\mu/\gamma\sin\theta)^{1/3}y - y^2/2]$, where q is the volumetric flow per unit width. Find the thickness t of such a flow of water at 5 °C for $\theta = 25°$ and $q = 18$ (L/min)/m.

▮ When $y = t$, $dV_z/dy = 0$. $dV_z/dy = 0 = [(\gamma\sin\theta)/\mu][(3q\mu/\gamma\sin\theta)^{1/3} - t]$, $t = (3q\mu/\gamma\sin\theta)^{1/3}$:

$$t = \left\{\frac{(3)[(18\times10^{-3})/60](1.52\times10^{-3})}{(9.79\times10^{-3})(\sin 25°)}\right\}^{1/3} = 0.692 \text{ mm}$$

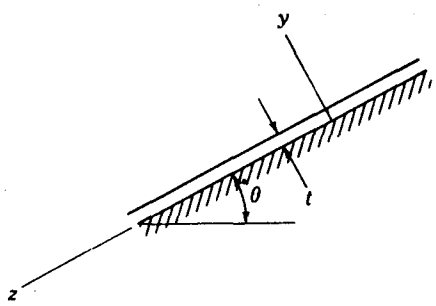


Fig. 14-51

14.202 A film of oil ($\nu = 1.664\times10^{-4}$ ft^2/s) of thickness 0.0022 ft moves at uniform speed down an inclined surface having an angle $\theta = 25°$ (see Fig. 14-51). Compute the volume flow per unit width.

▮ Assuming laminar flow,

$$t = (3q\mu/\gamma\sin\theta)^{1/3} = (3q\nu/g\sin\theta)^{1/3} \qquad \text{(from Prob. 14.201)}$$
$$0.0022 = \{(3)(q)(1.664\times10^{-4})/[(32.17)(\sin 25°)]\}^{1/3} \qquad q = 2.9\times10^{-4} \text{ cfs/ft}$$

14.203 Check the assumption of laminar flow in **(a)** Prob. 14.201, **(b)** Prob. 14.202.

▮ **(a)**
$$N_R = q/\nu = (3\times10^{-4})/(1.52\times10^{-6}) \approx 2 < 500 \quad \text{(O.K.)}.$$

(b)
$$N_R = q/\nu = (2.9\times10^{-4})/(1.664\times10^{-4}) \approx 2 < 500 \quad \text{(O.K.)}.$$

14.204 Water at 5 °C is flowing 1 m deep in a finished-concrete rectangular channel of width 8 m with a slope of 0.001. Find the volume of flow.

▮
$$Q = (A)(1.0/n)(R^{2/3})(s^{1/2}) = [(8)(1)](1.0/0.012)[(8)(1)/(1+8+1)]^{2/3}(0.001)^{1/2} = 18.2 \text{ m}^3/\text{s}$$

14.205 A wide rectangular channel dug from clean earth ($n = 0.024$) is to conduct a flow of 5 m^3/s per meter of width. The slope of the bed is 0.0016. What would be the depth of flow for normal flow?

▮
$$s = (n/1.0)^2(q^2/y_N^{10/3}) \qquad 0.0016 = (0.024/1.0)^2(5^2/y_N^{10/3}) \qquad y_N = 1.93 \text{ m}$$

14.206 The channel of Fig. 14-52 has $n = 0.020$ and carries 8 ft^3/s of water through a total drop of 10 ft. How long should the channel be for normal flow?

$$Q = (A)(1.486/n)(R^{2/3})(s^{1/2}) \qquad A = (2)(2\tan 45°) = 4.000\ \text{ft}^2$$
$$R = A/p_w = 4.000/[(2)(2/\cos 45°)] = 0.7071\ \text{ft}$$
$$8 = (4.000)(1.486/0.020)(0.7071)^{2/3}[10/L]^{1/2} \qquad L = 8694\ \text{ft}$$

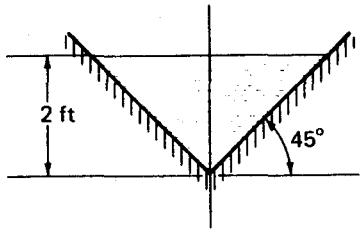


Fig. 14-52

14.207 Find the depth of flow in a 2-m-wide rectangular channel ($n = 0.016$) that carries 5 m^3/s of water 1 km with a head loss of 7.5 m.

$$s = 7.5/1000 = 0.0075 \qquad Q = (A)(1.0/n)(R^{2/3})(s^{1/2})$$
$$5 = (2y_N)(1.0/0.016)[2y_N/(y_N + 2 + y_N)]^{2/3}(0.0075)^{1/2} \qquad y_N = 0.795\ \text{m} \qquad \text{(by trial and error)}$$

14.208 The channel of Fig. 14-53 has a slope of 0.0018 and carries 57 m^3/s. Determine the flow depth, if $n = 0.015$.

$$Q = (A)(1.0/n)(R^{2/3})(s^{1/2}) \qquad A = 10y_N - (\tfrac{10}{2})[(\tfrac{10}{2})(\tan 10°)] = 10y_N - 4.408$$
$$p_w = 2[y_N - (\tfrac{10}{2})(\tan 10°) + (\tfrac{10}{2})/\cos 10°] = 2y_N + 8.391 \qquad R = A/p_w = (10y_N - 4.408)/(2y_N + 8.391)$$
$$57 = (10y_N - 4.408)(1.0/0.015)[(10y_N - 4.408)/(2y_N + 8.391)]^{2/3}(0.0018)^{1/2}$$
$$y_N = 2.11\ \text{m} \qquad \text{(by trial and error)}$$

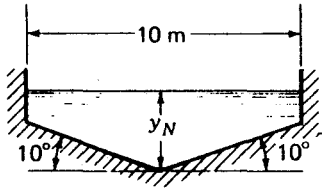


Fig. 14-53

14.209 Shown in Fig. 14-54 is a partially filled pipe. If Manning's n is 0.020, what slope is necessary for a steady flow of 60 cfs?

$$Q = (A)(1.486/n)(R^{2/3})(s^{1/2}) \qquad \theta = \arcsin \tfrac{2}{4} = 30.0°$$
$$A = \{[180° + (2)(30.0°)]/360°\}[(\pi)(8)^2/4] + (2)(4\cos 30.0°) = 40.44\ \text{ft}^2$$
$$p_w = \{[180° + (2)(30.0°)]/360°\}[(\pi)(8)] = 16.76\ \text{ft} \qquad R = A/p_w = 40.44/16.76 = 2.413\ \text{ft}$$
$$60 = (40.44)(1.486/0.020)(2.413)^{2/3}(s)^{1/2} \qquad s = 0.0001232$$

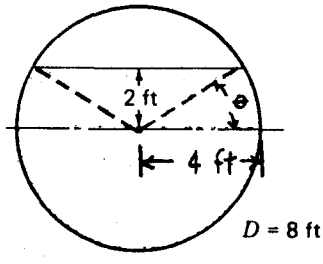


Fig. 14-54

14.210 Find the flow in Fig. 14-55 if the slope of the channel is 0.0004 and $n = 0.024$. Side slopes are all 1:1.

▮ $A = (3)(3)/2 + (3)(20) + (5)(3+3+5)/2 + (8)(3+5) + (5)(3+3+5)/2 + (3)(25) + (3)(3)/2 = 263.0\ \text{m}^2$

$p_w = \sqrt{3^2+3^2} + 20 + \sqrt{5^2+5^2} + 8 + \sqrt{5^2+5^2} + 25 + \sqrt{3^2+3^2} = 75.63\ \text{m}$ $\qquad R = A/p_w = 263.0/75.63 = 3.477\ \text{m}$

$$Q = (A)(1.0/n)(R^{2/3})(s^{1/2}) = (263.0)(1.0/0.024)(3.477)^{2/3}(0.0004)^{1/2} = 503\ \text{m}^3/\text{s}$$

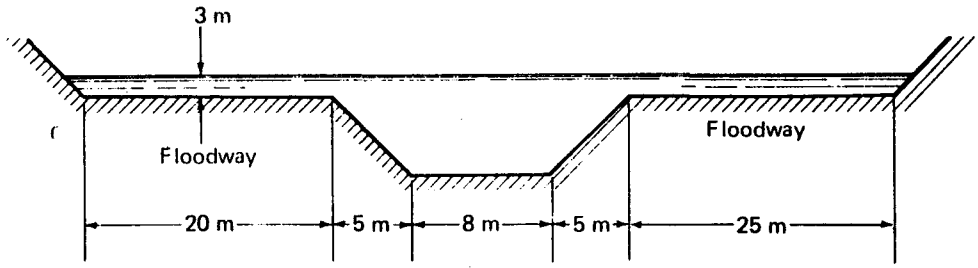


Fig. 14-55

14.211 The channel in Prob. 14.196 is to be replaced by a rectangular channel of width 16 ft. What is the ratio of cost of the concrete allowing 2 ft of freeboard (distance above the free surface) for the walls of the channels?

▮ From Prob. 14.196, $Q = 3472\ \text{ft}^3/\text{s}$, $s = 0.0016$, and $n = 0.012$.

$$Q = (A)(1.486/n)(R^{2/3})(s^{1/2}) \qquad 3472 = (16y)(1.486/0.012)[16y/(y+16+y)]^{2/3}(0.0016)^{1/2}$$

$$y = 14.65\ \text{ft} \quad \text{(by trial and error)} \qquad (p_w)_1 = 16 + (2)(14.65) + (2)(2) = 49.30\ \text{ft}$$

For the channel of Prob. 14.196, $(p_w)_2 = (\pi)(10) + 3 + 3 + (2)(2) = 41.42$ ft. Ratio of cost $= (p_w)_1/(p_w)_2 = 49.30/41.42 = 1.19$.

14.212 Repeat Prob. 14.204 using the friction-factor approach. Check your assumption of rough-zone flow.

▮ $R = A/p_w = (1)(8)/(1+8+1) = 0.8\ \text{m} \qquad 1/f^{1/2} = 2.16 - 2\log(\epsilon/R) = 2.16 - 2\log(0.001/0.8)$

$$f = 0.01576 \qquad v = (8gsR/f)^{1/2} = [(8)(9.807)(0.001)(0.8)/0.01576]^{1/2} = 1.996\ \text{m/s}$$

$$Q = Av = [(1)(8)](1.996) = 16.0\ \text{m}^3/\text{s}$$

$$N_R = D_h v/\nu = (4R)(v)/\nu = [(4)(0.8)](1.996)/(1.52\times10^{-6}) = 4.20\times10^6$$

The values $f = 0.01576$ and $N_R = 4.20\times10^6$ fall in in the rough zone of Fig. A-5.

14.213 For a rectangular channel with a flow of $20\ \text{m}^3/\text{s}$ at a velocity of 5 m/s, what should width b and depth y be for the best hydraulic section?

▮ $A = Q/v = \frac{20}{5} = 4.000\ \text{m}^2$. For a rectangular section, the best hydraulic section has a width equal to twice the depth. Hence, $(b)(b/2) = 4.000$, $b = 2.828$ m, $y = 2.828/2 = 1.414$ m.

14.214 For steady flow at $6\ \text{m}^3/\text{s}$ on a slope of 0.0018, what is the width of the rectangular channel of least wetted perimeter? Take $n = 0.0015$.

▮ $Q = (A)(1.0/n)(R^{2/3})(s^{1/2})$. The best hydraulic section has a width b equal to twice the depth. Hence, $A = (b)(b/2) = b^2/2$, $R = A/p_w = (b^2/2)/(b/2 + b + b/2) = b/4$, $6 = (b^2/2)(1.0/0.015)(b/4)^{2/3}(0.0018)^{1/2}$, $b = 2.43$ m.

14.215 A stream has a speed of 12 ft/s and is 2 ft deep. Find the angular spread of the wave set up by a thin stationary obstruction.

▮ $$\sin\theta/2 = \sqrt{gy}/v = \sqrt{(32.2)(2)}/12 \qquad \theta = 84°$$

14.216 A small boat is moving in shallow still water where the depth is 2 m; its bow wave makes an angle of 60° with the line of motion (see Fig. 14-56). Compute the speed of the boat.

▮ $$\sin\alpha = \sqrt{gy}/v \qquad \sin 60° = \sqrt{(9.807)(2)}/v \qquad v = 5.11\ \text{m/s}$$

14.217 A stone is dropped into a pond; the ripples have an amplitude of about 1 in and travel at 4 ft/s. About how deep is the pond?

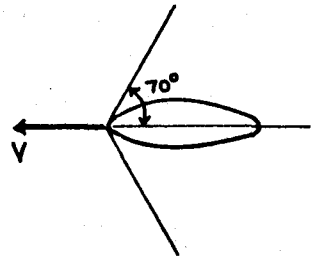


Fig. 14-56

▮ $(g)(y+\Delta y/2)(y+\Delta y)=c^2y \qquad 32.2[y+(\frac{1}{12})/2](y+\frac{1}{12})=(4)^2y \qquad y^2-0.3719y+0.003472=0$

Choosing the larger root,

$$y=[-(-0.3719)+\sqrt{(-0.3719)^2-(4)(1)(0.003472)}]/[(2)(1)]=0.362\text{ ft}$$

14.218 Two fishermen are stationed a distance D apart along a straight running stream (depth y, speed v). At a certain moment, a small gravity wave is initiated midway between them. How much later than the downstream man does the upstream man feel the wave?

▮ With $c=\sqrt{gy}$ as the wave velocity,

$$\Delta t=\frac{D/2}{v-c}-\frac{D/2}{v+c}=\frac{Dc}{v^2-c^2}=\frac{D\sqrt{gy}}{v^2-gy}$$

14.219 A wide rectangular channel excavated from clean earth has a flow of 4 $(\text{m}^3/\text{s})/\text{m}$. Determine the minimum specific energy.

▮ $y_c=(q^2/g)^{1/3}=(4^2/9.807)^{1/3}=1.177\text{ m} \qquad E_{\min}=(\frac{3}{2})(y_c)=(\frac{3}{2})(1.177)=1.766\text{ m}$

14.220 Find the critical slope in Prob. 14.219, assuming water at 5 °C ($\epsilon=0.047$ m).

▮ $s_c=f/8 \qquad D_h=4R=4y_c=(4)(1.177)=4.708\text{ m} \qquad v=q/y_c=4/1.177=3.398\text{ m/s}$

$N_R=D_hv/\nu=(4.708)(3.398)/(1.52\times10^{-6})=1.05\times10^7 \qquad \epsilon/D_h=0.047/4.708=0.010$

From Fig. A-5, $f=0.038$ and so $s_c=0.038/8=0.00475$.

14.221 Find the flow in a wide rectangular channel for which the critical depth is 3 m.

▮ $y_c=(q^2/g)^{1/3} \qquad 3=(q^2/9.807)^{1/3} \qquad q=16.27\ (\text{m}^3/\text{s})/\text{m}$

14.222 By what factor does the Froude number of a wide rectangular channel increase when the depth is decreased by a factor of 3/4, keeping the mass flow fixed?

▮ Since $N_F\propto y^{-3/2}$,

$$\text{factor}=\left(\frac{4}{3}\right)^{3/2}=\frac{8}{3\sqrt{3}}=1.54$$

14.223 At a section in the rectangular channel shown in Fig. 14-57, the average velocity is 28.28 fps. Is the flow tranquil?

▮ $q=(Av)/b=[(10)(50)](28.28)/50=282.8\text{ cfs/ft} \qquad y_c=(q^2/g)^{1/3}=(282.8^2/32.2)^{1/3}=13.54\text{ ft}$

Since $y_c>10$ ft, flow is not tranquil but shooting.

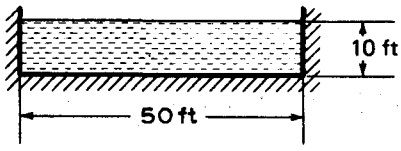


Fig. 14-57

14.224 How deep is a flow of 5 (m^3/s)/m in a wide rectangular channel having a critical slope of 0.002? The fluid is water at 5 °C.

▮ Assume rough-flow zone:

$$v = (8gsR/f)^{1/2} \qquad s_c = f/8 \qquad 0.002 = f/8 \qquad f = 0.016$$
$$5/[(1)(y)] = [(8)(9.807)(0.002)(y)/0.016]^{1/2} \qquad y = 1.366 \text{ m}$$

14.225 Compute the minimum specific energy in an 8-ft-wide channel through which 150 ft^3/s of water is flowing.

▮
$$gA^3 = bQ^2 \qquad (32.2)(8y_c)^3 = (8)(150)^2 \qquad y_c = 2.218 \text{ ft} \qquad E_{min} = A_c/2b + y_c$$
$$E_{min} = (8)(2.218)/[(2)(8)] + 2.218 = 3.327 \text{ ft}$$

14.226 Find the critical slope of a rectangular finished-concrete channel of width 4 m that carries 3 m^3/s of water at 5 °C ($\epsilon = 0001$ m).

▮
$$gA^3 = bQ^2 \qquad (9.807)(4y_c)^3 = (4)(3)^2 \qquad y_c = 0.386 \text{ m} \qquad s_c = fp_w/8b \qquad \epsilon/D_h = \epsilon/4R$$
$$R = A/p_w = (4)(0.386)/(0.386 + 4 + 0.386) = 0.3236 \text{ m}$$
$$\epsilon/D_h = 0.001/[(4)(0.3236)] = 0.00077$$
$$N_R = D_h v/\nu = (4R)(Q/A)/\nu = (4)(0.3236)\{3/[(4)(0.386)]\}/(1.52 \times 10^{-6}) = 1.65 \times 10^6$$

From Fig. A-5, $f = 0.0186$. $s_c = (0.0186)(0.386 + 4 + 0.386)/[(8)(4)] = 0.00277$.

14.227 What is the critical depth for a right triangular cross section for a flow of 4 m^3/s?

▮ See Fig. 14-58.
$$gA^3 = bQ^2 \qquad b = (2)(y_c \tan 45°) = 2.0y_c \qquad A = (y_c)(y_c \tan 45°) = 1.0y_c^2$$
$$(9.807)(1.0y_c^2)^3 = (2.0y_c)(4)^2 \qquad y_c = 1.267 \text{ m}$$

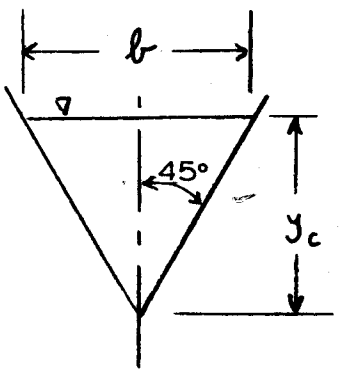


Fig. 14-58

14.228 What is the critical depth of a trapezoidal cross section for a flow of 10 m^3/s? The width at the base is 3 m and angle θ at the sides is 60°.

▮
$$gA^3 = bQ^2 \qquad b = 3 + (2)(y_c/\tan 60°) = 3 + 1.155y_c$$
$$A = 3y_c + (y_c)(y_c/\tan 60°) = 3y_c + 0.5774y_c^2$$
$$(9.807)(3y_c + 0.5774y_c^2)^3 = (3 + 1.155y_c)(10)^2 \qquad y_c = 0.976 \text{ m} \qquad \text{(by trial and error)}$$

14.229 Shown in Fig. 14-59 is a partially filled pipe discharging 450 cfs. What is the critical depth?

▮
$$gA^3 = bQ^2 \qquad b = (2)(4 \cos \theta) = 8 \cos \theta$$
$$A = [(\pi)(8^2/4)]/2 + [(\pi)(8^2/4)](2\theta/360) + (\tfrac{8}{2})(\cos \theta)(\tfrac{8}{2})(\sin \theta)$$
$$= 25.13 + 0.2793\theta + (16)(\cos \theta)(\sin \theta)$$
$$(32.2)[25.13 + 0.2793\theta + (16)(\cos \theta)(\sin \theta)]^3 = (8 \cos \theta)(450)^2$$
$$\theta = 20.5° \qquad \text{(by trial and error)}$$
$$y_c = \tfrac{8}{2} + (\tfrac{8}{2})(\sin 20.5°) = 5.40 \text{ ft}$$

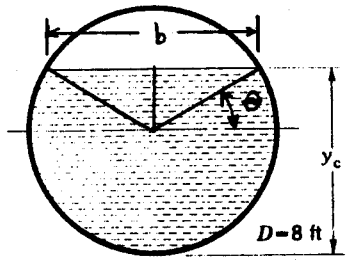


Fig. 14-59

14.230 Calculate the critical depth of the parabolic channel of Fig. 14-60, if the flow is 4 m³/s.

▮

$$gA^3 = bQ^2$$

$$A_c = (2)\int_0^{y_c} x\,dy = (2)\int_0^{y_c} (y/2)^{1/2}\,dy = 1.414[(\tfrac{2}{3})(y)^{3/2}]_0^{y_c}$$

$$= 1.414[(\tfrac{2}{3})(y_c)^{3/2}] = 0.9427y_c^{3/2}$$

$$b = 2x = (2)(\sqrt{y_c/2}) = 1.414y_c^{1/2}$$

$$(9.807)(0.9427y_c^{3/2})^3 = (1.414y_c^{1/2})(4)^2 \qquad y_c = 1.288 \text{ m}$$

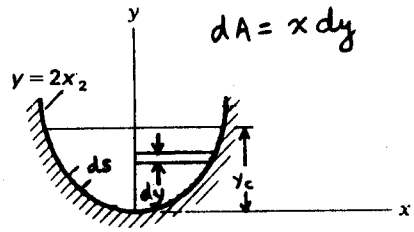


Fig. 14-60

14.231 In Prob. 14.230, what is the critical slope for normal flow? The friction factor is 0.014.

▮ $s_c = (f)(p_w)_c/8b$. To evaluate $(p_w)_c$, consider a differential length along the wetted perimeter (ds in Fig. 14-60): $ds = \sqrt{dx^2 + dy^2} = \sqrt{1 + (dy/dx)^2}\,dx$. From Prob. 14.230, when $y = y_c$, $x = \sqrt{y_c/2} = \sqrt{1.288/2} = 0.8025$ m.

$$(p_w)_c = \int ds = (2)\int_0^{0.7470} \sqrt{1 + (dy/dx)^2}\,dx \qquad y = 2x^2 \qquad dy/dx = 4x$$

$$(p_w)_c = (2)\int_0^{0.7470} \sqrt{1 + (4x)^2}\,dx = (8)\int_0^{0.8025} \sqrt{(\tfrac{1}{16} + x^2)}\,dx$$

$$= (\tfrac{8}{2})[(x)(\sqrt{\tfrac{1}{16} + x^2}) + \tfrac{1}{16}\{\log(x + \sqrt{\tfrac{1}{16} + x^2})\}]_0^{0.8025} = 2.711 \text{ m}$$

$$b = (2)(0.8025) = 1.605 \text{ m} \qquad s_c = (0.014)(2.711)/[(8)(1.605)] = 0.00296$$

14.232 At a section in the triangular channel of Fig. 14-61 the average velocity is 8 ft/s. Is the flow tranquil or shooting?

▮

$$\tan(45°/2) = (b/2)/10 \qquad b = 8.284 \text{ ft} \qquad A = 10b/2 = (10)(8.282)/2 = 41.41 \text{ ft}^2$$

$$Q = Av = (41.41)(8) = 331.3 \text{ ft}^3/\text{s} \qquad gA_c^3 = b_cQ^2$$

$$\tan(45°/2) = (b_c/2)/y_c \qquad b_c = 0.8284y_c$$

$$A_c = y_cb_c/2 = (y_c)(0.8284y_c)/2 = 0.4142y_c^2$$

$$(32.2)(0.4142y_c^2)^3 = (0.8284y_c)(331.3)^2 \qquad y_c = 8.31 \text{ ft} < 10 \text{ ft} = y$$

The flow is tranquil.

14.233 Water is moving in Fig. 14-62 with a velocity of 1 ft/s and a depth of 3 ft. It approaches a smooth rise in the channel bed of 1 ft. What should the estimated depth be after the rise? The channel is rectangular.

▮

$$E = q^2/2y^2g + y \qquad Q = Av = (3b)(1) = 3b \qquad q = Q/b = 3b/b = 3 \text{ cfs/ft}$$

$$E_1 = 3^2/[(2)(3)^2(32.2)] + 3 = 3.016 \text{ ft}$$

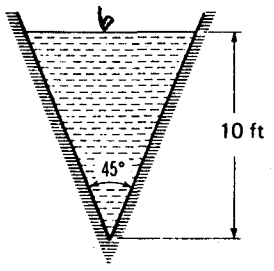


Fig. 14-61

Assuming no losses, $E_1 = E_2 + 1$, $3.016 = E_2 + 1$, $E_2 = 2.016$ ft; $y_c = (q^2/g)^{1/3} = (3^2/32.2)^{1/3} = 0.654$ ft. With surface elevation increasing, E must decrease. We must have one value of y which must be for tranquil flow. Hence, $2.016 = 3^2/[(2)(y_2)^2(32.2)] + y_2$, $y_2 = 1.98$ ft (by trial and error).

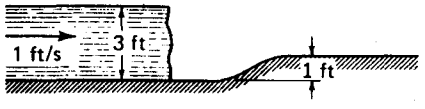


Fig. 14-62

14.234 In Fig. 14-63, a flow of 0.2 cfs flows over the rectangular channel of width 3 ft. If there is a smooth drop of 2 in, what is the elevation of the free surface above the bed of the channel after the drop? The velocity before the drop is 0.3 fps.

▮

$$Q = Av \qquad 0.2 = (3y_1)(0.3) \qquad y_1 = 0.2222 \text{ ft}$$
$$y_c = (q^2/g)^{1/3} = [(0.2/3)^2/32.2]^{1/3} = 0.05168 \text{ ft}$$

Since $y_c < y_1$, we have tranquil flow: $E = q^2/2y^2g + y$, $E_1 = (0.2/3)^2/[(2)(0.2222)^2(32.2)] + 0.2222 = 0.2236$ ft. Assuming no losses, $E_2 = E_1 + \frac{2}{12} = 0.2236 + \frac{2}{12} = 0.3903$ ft. We must have one depth downstream greater than y_1: $0.3903 = (0.2/3)^2/[(2)(y_2)^2(32.2)] + y_2$, $y_2 = 0.390$ ft (by trial and error).

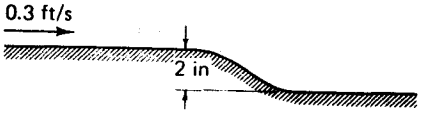


Fig. 14-63

14.235 Using the Powell equation, what quantity of liquid will flow in a smooth rectangular channel 2 ft wide, on a slope of 0.10 if the depth is 1.00 ft? Use $\nu = 0.00042$ ft^2/s.

▮ $C = -42 \log (C/N_R + \epsilon/R)$. For smooth channels, ϵ/R is small and can be neglected; hence,

$$C = 42 \log (N_R/C) \qquad v = C\sqrt{Rs} \qquad N_R = 4Rv/\nu = 4RC\sqrt{Rs}/\nu$$
$$N_R/C = (4)[(2)(1)/(1+2+1)]^{3/2}(0.010)^{1/2}/0.00042 = 336.7$$
$$C = (42)(\log 336.7) = 106.1$$
$$Q = CA\sqrt{Rs} = (106.1)[(2)(1)]\sqrt{[(2)(1)/(1+2+1)](0.010)} = 15.0 \text{ ft}^3/\text{s}$$

14.236 Determine C by the Powell equation for a 2 ft by 1 ft rectangular channel, if $v = 5.50$ fps, $\epsilon/R = 0.0020$, and $\nu = 0.00042$ ft^2/s.

▮

$$N_R = 4Rv/\nu = (4)[(2)(1)/(1+2+1)](5.50)/0.00042 = 26\,190$$
$$C = -42 \log (C/N_R + \epsilon/R) = (-42)[\log (C/26\,190 + 0.0020)] \qquad C = 95 \qquad \text{(by trial and error)}$$

14.237 Show a correlation between roughness factor f and roughness factor n.

▮ Taking the Manning formula as a basis of correlation, $C = \sqrt{8g/f} = 1.486R^{1/6}/n$, $1/\sqrt{f} = 1.486R^{1/6}/(n\sqrt{8g})$, $f = 8gn^2/(2.208R^{1/3})$.

14.238 What is the average shear stress at the sides and bottom of a rectangular flume 12 ft wide, flowing 4 ft deep, and laid on a slope of 1.60 ft per 1000 ft?

▮

$$\tau_0 = \gamma Rs = 62.4[(4)(12)/(4+12+4)](1.60/1000) = 0.240 \text{ lb/ft}^2$$

14.239 What flow can be expected in a 4-ft-wide rectangular, concrete-lined channel laid on a slope of 4 ft in 10 000 ft, if the water flows 2 ft deep. Solve using the Manning formula with $n = 0.015$.

▮ $$Q = (A)(1.486/n)(R^{2/3})(s^{1/2}) = [(4)(2)](1.486/0.015)[(4)(2)/(2+4+2]^{2/3}(4/10\,000)^{1/2} = 15.9\ \text{ft}^3/\text{s}$$

14.240 Solve Prob. 14.239 using Kutter's C.

▮ $$Q = AC\sqrt{Rs} \qquad s = 4/10\,000 = 0.0004 \qquad R = A/p_w = (4)(2)/(2+4+2) = 1.000\ \text{ft}$$

From Table A-15, with $s = 0.0004$, $R = 1.000$ ft, and $n = 0.015$, $C = 98$. $Q = [(4)(2)](98)\sqrt{(1.000)(0.0004)} = 15.7\ \text{ft}^3/\text{s}$.

14.241 In a hydraulics laboratory, a flow of 14.56 cfs was measured from a rectangular channel flowing 4 ft wide and 2 ft deep. If the slope of the channel was 0.00040, what is the roughness factor for the lining of the channel? Solve using Kutter's formula.

▮ $$Q = AC\sqrt{Rs} \qquad 14.56 = [(4)(2)](C)\sqrt{[(4)(2)/(2+4+2)](0.00040)} \qquad C = 91$$

From Table A-15, by interpolation, $n = 0.016$.

14.242 Solve Prob. 14.241 using Manning's formula.

▮ $$Q = (A)(1.486/n)(R^{2/3})(s^{1/2})$$
$$14.56 = [(4)(2)](1.486/n)[(4)(2)/(2+4+2)]^{2/3}(0.00040)^{1/2} \qquad n = 0.0163$$

14.243 On what slope should a 24-in vitrified sewer pipe be laid in order that 6.00 cfs will flow when the sewer is half-full? Use $n = 0.013$.

▮ $$Q = (A)(1.486/n)(R^{2/3}) \qquad 6.00 = [(\tfrac{1}{2})(\pi)(\tfrac{24}{12})^2/4](1.486/0.013)[(\tfrac{24}{12})/4]^{2/3}(s)^{1/2} \qquad s = 0.00281$$

14.244 What would the slope be in Prob. 14.243 if the sewer flows full?

▮ $$Q = (A)(1.486/n)(R^{2/3})(s^{1/2}) \qquad 6.00 = [(\pi)(\tfrac{24}{12})^2/4](1.486/0.013)[(\tfrac{24}{12})/4]^{2/3}(s)^{1/2} \qquad s = 0.000703$$

14.245 A trapezoidal channel, bottom width 20 ft and side slopes 1 to 1, flows 4 ft deep on a slope of 0.0009. For a value of $n = 0.025$, what is the uniform discharge?

▮ $$Q = (A)(1.486/n)(R^{2/3})(s^{1/2}) \qquad A = (20)(4) + (4)(4) = 96.00\ \text{ft}^2$$
$$p_w = 20 + (2)(4)(\sqrt{2}) = 31.31\ \text{ft}$$
$$Q = (96.00)(1.486/0.025)(96.00/31.31)^{2/3}(0.0009)^{1/2} = 361\ \text{ft}^3/\text{s}$$

14.246 Two concrete pipes ($C = 100$) must carry the flow from an open channel of half-square section 6 ft wide and 3 ft deep ($C = 120$). The slope of both structures is 0.00090. ***(a)*** Determine the diameter of the pipes. ***(b)*** Find the depth of water in the rectangular channel after it has become stabilized, if the slope is changed to 0.00160, using $C = 120$.

▮ ***(a)*** $$Q_{\text{channel}} = Q_{\text{pipes}} \qquad (AC\sqrt{Rs})_{\text{channel}} = (2)(AC\sqrt{Rs})_{\text{pipe}}$$
$$[(3)(6)](120)\sqrt{[(3)(6)/(3+6+3)](0.00090)} = (2)[(\pi)(D)^2/4](100)\sqrt{(D/4)(0.00090)} \qquad D = 4.08\ \text{ft}$$

(b) $$Q = [(3)(6)](120)\sqrt{[(3)(6)/(3+6+3)](0.00090)} = 79.36\ \text{ft}^3/\text{s}$$
$$79.36 = [(y)(6)](120)\sqrt{[(y)(6)/(y+6+y)](0.001600)} \qquad y = 2.39\ \text{ft} \qquad \text{(by trial and error)}$$

14.247 How deep will water flow at the rate of 240 cfs in a rectangular channel 20 ft wide, laid on a slope of 0.00010? Use $n = 0.0149$.

▮ $$Q = (A)(1.486/n)(R^{2/3})(s^{1/2}) \qquad 240 = (20y)(1.486/0.0149)[20y/(y+20+y)]^{2/3}(0.00010)^{1/2}$$
$$y = 5.27\ \text{ft} \qquad \text{(by trial and error)}$$

14.248 How wide must a rectangular channel be constructed in order to carry 500 cfs at a depth of 6 ft on a slope of 0.00040? Use $n = 0.010$.

▮ $$Q = (A)(1.486/n)(R^{2/3})(s^{1/2}) \qquad 500 = (6b)(1.486/0.010)[6b/(6+b+6)]^{2/3}(0.00040)^{1/2}$$
$$b = 13.1\ \text{ft} \qquad \text{(by trial and error)}$$

14.249 A rectangular channel carries 200 cfs. Find the critical depth and the critical velocity for **(a)** a width of 12 ft and **(b)** a width of 9 ft. **(c)** What slope will produce the critical velocity in **(a)** if $n = 0.020$?

▮ **(a)** $y_c = (q^2/g)^{1/3} = [(\frac{200}{12})^2/32.2]^{1/3} = 2.05$ ft $\quad v_c = \sqrt{gy_c} = \sqrt{(32.2)(2.05)} = 8.13$ ft/s

(b) $y_c = [(\frac{200}{9})^2/32.2]^{1/3} = 2.48$ ft $\quad v_c = \sqrt{(32.2)(2.48)} = 8.94$ ft/s

(c) $v = (1.486/n)(R^{2/3})(s^{1/2}) \quad 8.13 = (1.486/0.020)[(12)(2.05)/(2.05 + 12 + 2.05)]^{2/3}(s)^{1/2} \quad s = 0.00680$

14.250 A trapezoidal channel with side slopes of 2 horizontal to 1 vertical is to carry a flow of 590 cfs. For a bottom width of 12 ft, calculate the **(a)** critical depth and **(b)** critical velocity.

▮ **(a)** $Q^2/g = A^3/b \quad b = 12 + 4y \quad A = 12y + 2y^2$

$590^2/32.2 = (12y + 2y^2)^3/(12 + 4y) \quad y = y_c = 3.46$ ft (by trial and error)

(b) $v_c = \sqrt{gA/b} = \sqrt{(32.2)[(12)(3.46) + (2)(3.46)^2]/[12 + (4)(3.46)]} = 9.03$ ft/s

14.251 A trapezoidal channel has a bottom width of 20 ft, side slopes of 1 to 1, and flows at a depth of 3.00 ft. For $n = 0.015$, and a discharge of 360 cfs, calculate **(a)** the normal slope, **(b)** the critical slope and critical depth for 360 cfs, and **(c)** the critical slope at the normal depth of 3.00 ft.

▮ **(a)** $Q = (A)(1.486/n)(R^{2/3})(s^{1/2}) \quad A = (20)(3) + (3)(3) = 69.00$ ft^2 $\quad p_w = 20 + (2)(\sqrt{3^2 + 3^2}) = 28.49$ ft

$360 = (69.00)(1.486/0.015)(69.00/28.49)^{2/3}(s)^{1/2} \quad s = 0.000853$

(b) $v_c = \sqrt{gA/b} = \sqrt{(32.2)(20y_c + y_c^2)/(20 + 2y_c)}$

Also,

$v_c = Q/A_c = 360/(20y_c + y_c^2) \quad \sqrt{(32.2)(20y_c + y_c^2)/(20 + 2y_c)} = 360/(20y_c + y_c^2)$

$y_c = 2.08$ ft (by trial and error)

$360 = [(20)(2.08) + 2.08^2](1.486/0.015)\{[(20)(2.08) + 2.08^2]/[20 + (2)(\sqrt{2.08^2 + 2.08^2}]\}^{2/3}(s_c)^{1/2} \quad s_c = 0.00291$

(c) $v_c = \sqrt{(32.2)[(20)(3.00) + 3.00^2]/[20 + (2)(3.00)]} = 9.24$ ft/s $\quad v_c = (1.486/n)(R^{2/3})(s^{1/2})$

$9.24 = (1.486/0.015)\{[(20)(3.00) + 3.00^2]/[20 + (2)(\sqrt{3.00^2 + 3.00^2}]\}^{2/3}(s_c)^{1/2} \quad s_c = 0.00267$

14.252 A rectangular channel, 30 ft wide, carries 270 cfs when flowing 3.00 ft deep. **(a)** What is the specific energy? **(b)** Is the flow subcritical or supercritical?

▮ **(a)** $v = Q/A = 270/[(3.00)(30)] = 3.00$ ft/s $\quad E = v^2/2g + y = 3.000^2/[(2)(32.2)] + 3.00 = 3.14$ ft

(b) $y_c = (q^2/g)^{1/3} = [(\frac{270}{30})^2/32.2]^{1/3} = 1.36$ ft

Since $[y_c = 1.36] < [y = 3.00]$, the flow is subcritical.

14.253 A trapezoidal channel has a bottom width of 20 ft and side slopes of 2 horizontal to 1 vertical. When the depth of water is 3.50 ft, the flow is 370 cfs. **(a)** What is the specific energy? **(b)** Is the flow subcritical or supercritical?

▮ **(a)** $A = (20)(3.50) + (3.50)[(2)(3.50)] = 94.50$ ft^2 $\quad v = Q/A = 370/94.50 = 3.915$ ft/s

$E = v^2/2g + y = 3.915^2/[(2)(32.2)] + 3.50 = 3.74$ ft

(b) $Q^2/g = A_c^3/b \quad 370^2/32.2 = \{20y_c + (y_c)[(2)(y_c)]\}^3/[20 + (2)(2y_c)] \quad y_c = 2.05$ ft (by trial and error)

Since $[y_c = 2.05] < [y = 3.50$ ft$]$, the flow is subcritical.

14.254 The discharge through a rectangular channel ($n = 0.012$) 15 ft wide is 400 cfs when the slope is 1 ft in 100 ft. Is the flow subcritical or supercritical?

▮ $y_c = (q^2/g)^{1/3} = [(\frac{400}{15})^2/32.2]^{1/3} = 2.81$ ft $\quad Q = (A)(1.486/n)(R^{2/3})(s^{1/2})$

$400 = [(15)(2.81)](1.486/0.012)[(15)(2.81)/(2.81 + 15 + 2.81)]^{2/3}(s_c)^{1/2} \quad s_c = 0.00226$

Since $[s_c = 0.00226] < [s = \frac{1}{100}]$, the flow is supercritical.

14.255 A rectangular channel, 10 ft wide, carries 400 cfs. **(a)** Tabulate depth of flow against specific energy for depths from 1 ft to 8 ft. **(b)** Determine the minimum specific energy. **(c)** What type of flow exists when the depth is 2 ft and when it is 8 ft? **(d)** For $C = 100$, what slopes are necessary to maintain the depths in **(c)**?

▮ (*a*) $E = v^2/2g + y = (Q/A)^2/2g + y = (Q/10y)^2/2g + y$. For $y = 1$ ft, $E = \{400/[(10)(1)]\}^2/[(2)(32.2)] + 1 =$ 25.8 ft. For succeeding depths,

y, ft	E, ft
1	25.8
2	8.21
3	5.76
4	5.55
5	5.99
6	6.69
7	7.51
8	8.39

(*b*) $y_c = (q^2/g)^{1/3} = [(\frac{400}{10})^2/32.2]^{1/3} = 3.676$ ft $\quad E_{min} = \{400/[(10)(3.676)]\}^2/[(2)(32.2)] + 3.676 = 5.51$ ft

(*c*) Since $[y = 2] < [y_c = 3.676]$, the flow is supercritical for a 2-ft depth. Since $[y = 8] > [y_c = 3.676]$, the flow is subcritical for an 8-ft depth.

(*d*) $Q = CA\sqrt{Rs}$. For a 2-ft depth: $400 = (100)[(2)(10)]\sqrt{[(2)(10)/(2 + 10 + 2)](s)}$, $s = 0.0280$. For an 8-ft depth: $400 = (100)[(8)(10)]\sqrt{[(8)(10)/(8 + 10 + 8)](s)}$, $s = 0.000812$.

14.256 A rectangular flume ($n = 0.012$) is laid on a slope of 0.0036 and carries 580 cfs. For critical-flow conditions, what width is required?

▮ $y_c = (q^2/g)^{1/3} = [(Q/b)^2/g]^{1/3}$. Try $b = 8.0$ ft:

$$y_c = [(580/8.0)^2/32.2]^{1/3} = 5.465 \text{ ft}$$

$$Q = (A)(1.486/n)(R^{2/3})(s^{1/2}) = [(5.465)(8.0)](1.486/0.012) \times [(5.465)(8.0)/(5.465 + 8.0 + 5.465)]^{2/3}(0.0036)^{1/2} = 568 \text{ ft}^3/\text{s}$$

Try $b = 8.5$ ft:

$$y_c = [(580/8.5)^2/32.2]^{1/3} = 5.249 \text{ ft}$$

$$Q = [(5.249)(8.5)](1.486/0.012)[(5.249)(8.5)/(5.249 + 8.5 + 5.249)]^{2/3}(0.0036)^{1/2} = 586 \text{ ft}^3/\text{s}$$

Try $b = 8.33$ ft:

$$y_c = [(580/8.33)^2/32.2]^{1/3} = 5.320 \text{ ft}$$

$$Q = [(5.320)(8.33)](1.486/0.012)[(5.320)(8.33)/(5.320 + 8.33 + 5.320)]^{2/3}(0.0036)^{1/2} = 580 \text{ ft}^3/\text{s}$$

Hence, $b = 8.33$ ft.

14.257 For a constant specific energy of 6.60 ft, what maximum flow may occur in a rectangular channel 10.0 ft wide?

▮ $$y_c = (\tfrac{2}{3})(E) = (\tfrac{2}{3})(6.60) = 4.40 \text{ ft} \qquad v_c = \sqrt{gy_c} = \sqrt{(32.2)(4.40)} = 11.90 \text{ ft/s}$$

$$Q_{max} = A_c v_c = [(10.0)(4.40)](11.90) = 524 \text{ ft}^3/\text{s}$$

14.258 A rectangular channel, 20 ft wide, $n = 0.025$, flows 5 ft deep on a slope of 14.7 ft in 10 000 ft. A suppressed weir C, 2.45 ft high, is built across the channel ($m = 3.45$). Taking the elevation of the bottom of the channel just upstream from the weir to be 100.00 ft, estimate (using one reach) the elevation of the water surface at a point A, 1000 ft upstream from the weir. See Fig. 14-64.

▮ $$Q = (A)(1.486/n)(R^{2/3})(s^{1/2}) = [(20)(5)](1.486/0.025)[(20)(5)/(5 + 20 + 5)]^{2/3}(14.7/10\,000)^{1/2} = 509 \text{ ft}^3/\text{s}$$

Calculate the new elevation of the water surface at B (before dropdown). Note that the flow is nonuniform since the depths, velocities, and areas are not constant after the weir is installed. Estimate a depth of 6 ft just upstream from the weir (i.e., at B). $v_{approach} = Q/A = 509/[(20)(6)] = 4.24$ ft. The applicable weir formula is $Q = mb[(H + v^2/2g)^{3/2} - (v^2/2g)^{3/2}]$.

$$509 = (3.45)(20)\left\{\left[H + \frac{4.24^2}{(2)(32.2)}\right]^{3/2} - \left[\frac{4.24^2}{(2)(32.2)}\right]^{3/2}\right\} \qquad H = 3.56 \text{ ft}$$

$$y_B = 3.56 + 2.45 = 6.01 \text{ ft} \qquad \text{(estimated depth of 6 ft O.K.)}$$

The new elevation at A must lie between $101.47 + 5 = 106.47$ ft and $101.47 + 6 = 107.47$ ft. Try an elevation of 106.90 ft:

$$(A_A)_{new} = (20)(106.90 - 101.47) = 108.6 \text{ ft}^2 \qquad (v_A)_{new} = Q/A = 509/108.6 = 4.69 \text{ ft/s}$$

$$v_{mean} = (4.24 + 4.69)/2 = 4.46 \text{ ft/s} \qquad y_A = (106.90 - 101.47) = 5.43 \text{ ft}$$

$$R_{mean} = \frac{\frac{1}{2}[(6)(20) + 108.6]}{\frac{1}{2}[(6 + 20 + 6) + (5.43 + 20 + 5.43)]} = 3.637 \text{ ft}$$

$$h_L = (vn/1.486R^{2/3})^2(L) = \{(4.46)(0.025)/[(1.486)(3.637)^{2/3}]\}^2(1000) = 1.01 \text{ ft}$$

Check by the Bernoulli equation: $v_A^2/2g + z_A = v_B^2/2g + z_B + h_L$, $4.69^2/[(2)(32.2)] + 106.90 = 4.24^2/[(2)(32.2)] + 106.00 + 1.01$, $107.24 = 107.29$ (approximately). Further refinement is not necessary. Hence, use an elevation of 106.90 ft at A.

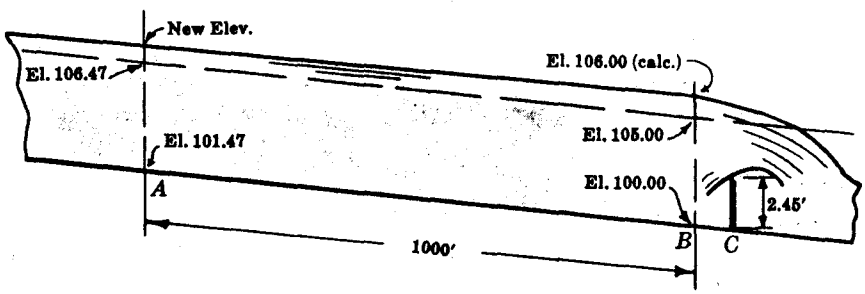


Fig. 14-64

14.259 Develop a formula for the length–energy–slope relationship for nonuniform flow problems similar to Prob. 14.258.

▮ Energy at 1 − head loss = energy at 2, $(z_1 + y_1 + v_1^2/2g) - h_L = (z_2 + y_2 + v_2^2/2g)$. Let s = slope of the energy line and s_0 = slope of the channel bottom: $s = h_L/L$, $h_L = sL$, $s_0 = (z_1 - z_2)/L$, $z_1 - z_2 = s_0L$. Therefore, $s_0L + (y_1 - y_2) + (v_1^2/2g - v_2^2/2g) = sL$, $L = [(y_1 + v_1^2/2g) - (y_2 + v_2^2/2g)]/(s - s_0) = (E_1 - E_2)/(s - s_0)$.

14.260 A rectangular flume ($n = 0.013$) is 6 ft wide and carries 66 cfs of water. At a certain section F, the depth is 3.20 ft. If the slope of the channel bed is constant at 0.000400, determine the distance from F where the depth is 2.70 ft. (Use one reach.)

▮ Assume the depth is upstream from F. Let subscript 2 refer to point F and subscript 1 to the other point.

$$L = [(y_1 + v_1^2/2g) - (y_2 + v_2^2/2g)]/(s - s_0) \qquad A_1 = (6)(2.70) = 16.20 \text{ ft}^2$$

$$v_1 = Q/A_1 = 66/16.20 = 4.074 \text{ ft/s}$$

$$R = A/p_w \qquad R_1 = 16.20/(2.70 + 6 + 2.70) = 1.421 \text{ ft}$$

$$A_2 = (6)(3.20) = 19.20 \text{ ft}^2 \qquad v_2 = Q/A_2 = 66/19.20 = 3.438 \text{ ft/s}$$

$$R_2 = 19.20/(3.20 + 6 + 3.20) = 1.548 \text{ ft} \qquad v_{mean} = (4.074 + 3.438)/2 = 3.756 \text{ ft/s}$$

$$R_{mean} = (1.421 + 1.548)/2 = 1.484 \text{ ft}$$

$$v = (1.486/n)(R^{2/3})(s^{1/2}) \qquad 3.756 = (1.486/0.013)(1.484)^{2/3}(s)^{1/2} \qquad s = 0.000638$$

$$L = \frac{\{2.70 + 4.074^2/[(2)(32.2)]\} - \{3.20 + 3.438^2/[(2)(32.2)]\}}{0.000638 - 0.000400} = -1789 \text{ ft}$$

The minus sign signifies that the section with the 2.70-ft depth is downstream from F, not upstream as assumed.

14.261 A rectangular channel, 40 ft wide, carries 900 cfs of water. The slope of the channel is 0.00283. At section 1 the depth is 4.50 ft and at section 2, 300 ft downstream, the depth is 5.00 ft. What is the average value of roughness factor n?

▮

$$L = [(y_1 + v_1^2/2g) - (y_2 + v_2^2/2g)]/(s - s_0) \qquad A_1 = (40)(4.50) = 180.0 \text{ ft}^2$$

$$v_1 = Q/A_1 = 900/180.0 = 5.000 \text{ ft/s}$$

$$R = A/p_w \qquad R_1 = 180.0/(4.50 + 40 + 4.50) = 3.673 \text{ ft}$$

$$A_2 = (40)(5.00) = 200.0 \text{ ft}^2 \qquad v_2 = Q/A = 900/200.0 = 4.500 \text{ ft/s}$$

$$R_2 = 200.0/(5.00 + 40 + 5.00) = 4.000 \text{ ft} \qquad v_{mean} = (5.000 + 4.500)/2 = 4.750 \text{ ft/s}$$

$$R_{mean} = (3.673 + 4.000)/2 = 3.836 \text{ ft}$$

$$300 = \frac{\{4.50 + 5.000^2/[(2)(32.2)]\} - \{5.00 + 4.500^2/[(2)(32.2)]\}}{s - 0.00283} \qquad s = 0.001409$$

$$v = (1.486/n)(R^{2/3})(s^{1/2}) \qquad 4.750 = (1.486/n)(3.836)^{2/3}(0.001409)^{1/2} \qquad n = 0.0288$$

14.262 A rectangular channel, 20 ft wide, has a slope of 1 ft per 1000 ft. The depth at section 1 is 8.50 ft and at section 2, 2000 ft downstream, the depth is 10.25 ft. If $n = 0.011$, determine the probable flow.

▮ $$E = y + v^2/2g + z \qquad E_1 = 8.50 + v_1^2/2g + (2000)(\tfrac{1}{1000}) \qquad E_2 = 10.25 + v_2^2/2g + 0$$

$$s = \text{head loss}/L = [(10.50 - 10.25) + (v_1^2/2g - v_2^2/2g)]/2000$$

$$Q = (A)(1.486/n)(R^{2/3})(s^{1/2})$$

$$A_1 = (20)(8.50) = 170.0 \text{ ft}^2 \qquad R = A/p_w$$

$$R_1 = 170.0/(8.50 + 20 + 8.50) = 4.595 \text{ ft} \qquad A_2 = (20)(10.25) = 205.0 \text{ ft}^2$$

$$R_2 = 205.0/(10.25 + 20 + 10.25) = 5.062 \text{ ft} \qquad A_{mean} = (170.0 + 205.0)/2 = 187.5 \text{ ft}^2$$

$$R_{mean} = (4.595 + 5.062)/2 = 4.828 \text{ ft}$$

Assume $s = 0.000144$: $Q = (187.5)(1.486/0.011)(4.828)^{2/3}(0.000144)^{1/2} = 868.3 \text{ ft}^3/\text{s}$. Check on s:

$$v_1 = Q/A_1 = 868.3/170.0 = 5.108 \text{ ft/s} \qquad v_1^2/2g = 5.108^2/[(2)(32.2)] = 0.4052 \text{ ft}$$

$$v_2 = 868.3/205.0 = 4.236 \text{ ft/s}$$

$$v_2^2/2g = 4.236^2/[(2)(32.2)] = 0.2786 \text{ ft} \qquad s = [(10.50 - 10.25) + (0.4052 - 0.2786)]/2000 = 0.000188$$

This value of s (0.000188) does not equal the assumed value (0.000144); hence, try $s = 0.000210$: $Q = (187.5)(1.486/0.011)(4.828)^{2/3}(0.000210)^{1/2} = 1049 \text{ ft}^3/\text{s}$. Check on s:

$$v_1 = Q/A_1 = 1049/170.0 = 6.171 \text{ ft/s} \qquad v_1^2/2g = 6.171^2/[(2)(32.2)] = 0.5913 \text{ ft}$$

$$v_2 = 1049/205.0 = 5.117 \text{ ft/s}$$

$$v_2^2/2g = 5.117^2/[(2)(32.2)] = 0.4066 \text{ ft} \qquad s = [(10.50 - 10.25) + (0.5913 - 0.4066)]/2000 = 0.000217$$

This is close to the assumed value of s of 0.000210; hence, approximate $Q = 1050 \text{ ft}^3/\text{s}$.

14.263 A reservoir feeds a rectangular channel, 15 ft wide, $n = 0.015$, as shown in Fig. 14-65. At the entrance, the depth of water is 6.22 ft above the channel bottom. The flume is 800 ft long and drops 0.72 ft in this length. The depth behind a weir at the discharge end of the channel is 4.12 ft. Determine, using one reach, the capacity of the channel assuming the loss at the entrance to be $0.25v_1^2/2g$.

▮ $$p_A/\gamma + v_A^2/2g + z_A = p_1/\gamma + v_1^2/2g + z_1 + h_m \qquad 0 + 0 + 6.22 = 0 + v_1^2/2g + y_1 + 0.25v_1^2/2g$$

$$L = [(v_2^2/2g + y_2) - (v_1^2/2g + y_1)]/(s_0 - s) \qquad s = (nv/1.486R^{2/3})^2$$

Solve these equations by successive trials until L approximates or equals 800 ft. Try $y_1 = 5.0$ ft:

$$6.22 = v_1^2/[(2)(32.2)] + 5.0 + 0.25v_1^2/[(2)(32.2)] \qquad v_1 = 7.928 \text{ ft/s}$$

$$q = y_1v_1 = (5.0)(7.928) = 39.64 \text{ ft}^3/\text{s/ft} \qquad v_2 = q/y_2 = 39.64/4.12 = 9.621 \text{ ft/s}$$

$$v_{mean} = (7.928 + 9.621)/2 = 8.774 \text{ ft/s}$$

$$R_1 = (15)(5.0)/(5.0 + 15 + 5.0) = 3.000 \text{ ft} \qquad R_2 = (15)(4.12)/(4.12 + 15 + 4.12) = 2.659 \text{ ft}$$

$$R_{mean} = (3.000 + 2.659)/2 = 2.830 \text{ ft}$$

$$s = \{(0.015)(8.774)/[(1.486)(2.830)^{2/3}]\}^2 = 0.001960$$

$$L = \frac{\{(9.621)^2/[(2)(32.2)] + 4.12\} - \{(7.928)^2/[(2)(32.2)] + 5.0\}}{0.72/800 - 0.001960} = 395 \text{ ft}$$

Since L is not equal to 800 ft, try $y_1 = 5.21$ ft:

$$6.22 = v_1^2/[(2)(32.2)] + 5.21 + 0.25v_1^2/[(2)(32.2)] \qquad v_1 = 7.214 \text{ ft/s}$$

$$q = y_1v_1 = (5.21)(7.214) = 37.58 \text{ ft}^3/\text{s/ft}$$

$$v_2 = q/y_2 = 37.58/4.12 = 9.121 \text{ ft/s} \qquad v_{mean} = (7.214 + 9.121)/2 = 8.168 \text{ ft/s}$$

$$R_1 = (15)(5.21)/(5.21 + 15 + 5.21) = 3.074 \text{ ft}$$

$$R_2 = (15)(4.12)/(4.12 + 15 + 4.12) = 2.659 \text{ ft} \qquad R_{mean} = (3.074 + 2.659)/2 = 2.866 \text{ ft}$$

$$s = \{(0.015)(8.168)/[(1.486)(2.866)^{2/3}]\}^2 = 0.001670$$

$$L = \frac{\{(9.121)^2/[(2)(32.2)] + 4.12\} - \{(7.214)^2/[(2)(32.2)] + 5.21\}}{0.72/800 - 0.001670} = 787 \text{ ft}$$

L is not exactly equal to 800 ft, but additional computations (not shown here) show that 5.21 ft is the best value of y_1 to the nearest hundredth of a foot. Hence, $Q = (37.58)(15) = 564 \text{ ft}^3/\text{s}$.

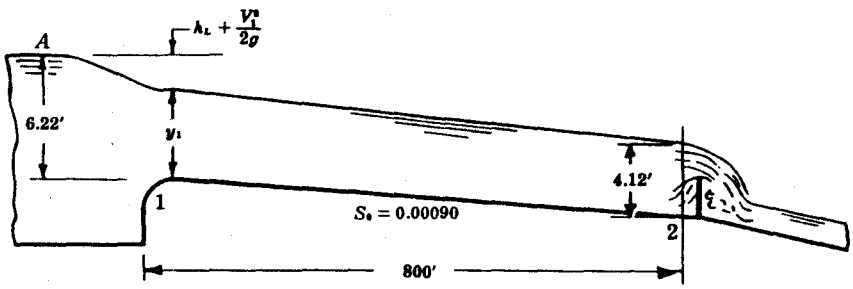


Fig. 14-65

14.264 A rectangular concrete channel 15 ft wide carries water as shown in Fig. 14-66. The channel bed slope is 0.0010. Find the theoretical rate of flow in the channel.

▮

$$v_1^2/2g + d_1 + s_0L = v_2^2/2g + d_2 + sL \qquad v_1 = Q/[(15)(5.1)] = 0.01307Q$$

$$v_2 = Q/[(15)(3.9)] = 0.01709Q$$

$$s = (nv_m/1.486R_m^{2/3})^2 \qquad v_m = (0.01307Q + 0.01709Q)/2 = 0.01508Q \qquad R = A/p_w$$

$$R_1 = (15)(5.1)/(5.1 + 15 + 5.1) = 3.036 \text{ ft} \qquad R_2 = (15)(3.9)/(3.9 + 15 + 3.9) = 2.566 \text{ ft}$$

$$R_m = (3.036 + 2.566)/2 = 2.801 \text{ ft}$$

$$s = \{(0.013)(0.01508Q)/[(1.486)(2.801)^{2/3}]\}^2 = 4.408 \times 10^{-9}Q^2$$

$$(0.01307Q)^2/[(2)(32.2)] + 5.1 + (0.0010)(1000)$$

$$= (0.01709Q)^2/[(2)(32.2)] + 3.9 + (4.408 \times 10^{-9}Q^2)(1000) \qquad Q = 591 \text{ ft}^3/\text{s}$$

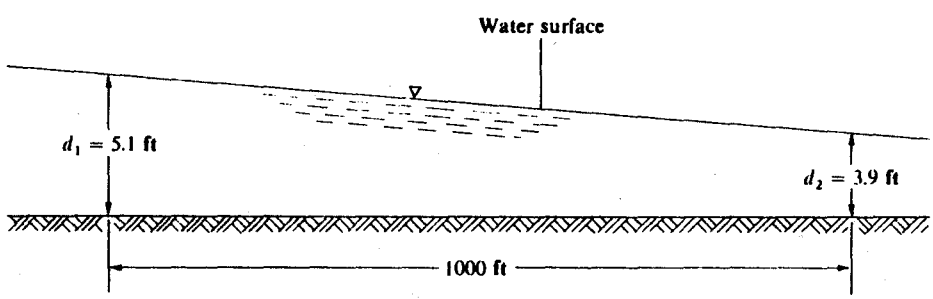


Fig. 14-66

14.265 Water flowing at the normal depth in a rectangular concrete channel that is 12.0 m wide encounters an obstruction, as shown in Fig. 14-67, causing the water level to rise above the normal depth at the obstruction and for some distance upstream. The water discharge is 126 m^3/s and the channel bottom slope is 0.00086. The depth of water just upstream from the obstruction (d_0) is 4.55 m. Find the distance upstream to the point where the water surface is at the normal depth.

▮

$$d_c = (q^2/g)^{1/3} = [(126/12.0)^2/9.807]^{1/3} = 2.24 \text{ m} \qquad Q = (A)(1.0/n)(R^{2/3})(s^{1/2})$$

$$126 = (12.0d)(1.0/0.013)[12.0d/(d + 12.0 + d)]^{2/3}(0.00086)^{1/2}$$

$$2.256[12.0d/(d + 12.0 + d)]^{2/3} - 10.5/d = 0$$

$$d = 2.95 \text{ m} \qquad \text{(by trial and error)}$$

Since $d > d_c$, flow is subcritical, and computations should proceed upstream. The problem now is to determine the distance from the point where the depth is 4.55 m to the point upstream where the depth is 2.95 m. This will be done in ten equal depth increments of 0.16 m. The computations are given in the table below.

(1) d, m	(2) v, m/s $\frac{126}{12.0\times(1)}$	(3) v_m, m/s	(4) $v^2/2g$, m $\frac{(2)^2}{2\times 9.807}$	(5) R, m $\frac{12.0\times(1)}{12.0+2\times(1)}$	(6) R_m, m	(7) s $\left[\frac{0.013\times(3)^2}{(6)^{2/3}}\right]^2$	(8) L, m $\frac{[(4)+(1)]_2-[(4)+(1)]_1}{0.00086-(7)}$
4.55	2.308		0.2716	2.588			
		2.350			2.562	0.0002662	−236
4.39	2.392		0.2917	2.535			
		2.437			2.508	0.0002946	−243
4.23	2.482		0.3141	2.481			
		2.531			2.453	0.0003272	−253
4.07	2.580		0.3394	2.425			
		2.633			2.396	0.0003654	−266
3.91	2.685		0.3676	2.367			
		2.743			2.338	0.0004098	−284
3.75	2.800		0.3997	2.308			
		2.863			2.277	0.0004626	−311
3.59	2.925		0.4362	2.246			
		2.993			2.214	0.0005246	−353
3.43	3.061		0.4777	2.182			
		3.136			2.150	0.0005989	−429
3.27	3.211		0.5257	2.117			
		3.294			2.083	0.0006893	−613
3.11	3.376		0.5811	2.048			
		3.468			2.013	0.0007997	−1580
2.95	3.559		0.6458	1.978			
							−4568 m

Hence, the answer to the problem is 4568 m.

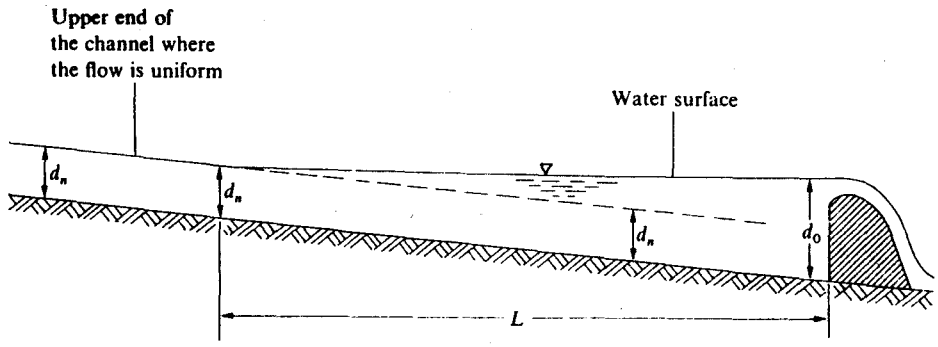


Fig. 14-67

14.266 Water flows in a rectangular concrete channel that is 5.0 ft wide, as shown in Fig. 14-68*a*, at a discharge of 16.5 cfs. Find the water-surface profile through the channel.

▮ $d_c = (q^2/g)^{1/3} = [(16.5/5.0)^2/32.2]^{1/3} = 0.70$ ft. In segment AB,

$$Q = (A)(1.486/n)(R^{2/3})(s^{1/2})$$

$$16.5 = (5.0d)(1.486/0.013)[5.0d/(d+5.0+d)]^{2/3}(0.00040)^{1/2}$$

$$2.286[5.0d/(d+5.0+d)]^{2/3} - 3.300/d = 0 \qquad d = 1.50 \text{ ft} \qquad \text{(by trial and error)}$$

Since $d > d_c$, the flow in segment AB is subcritical.

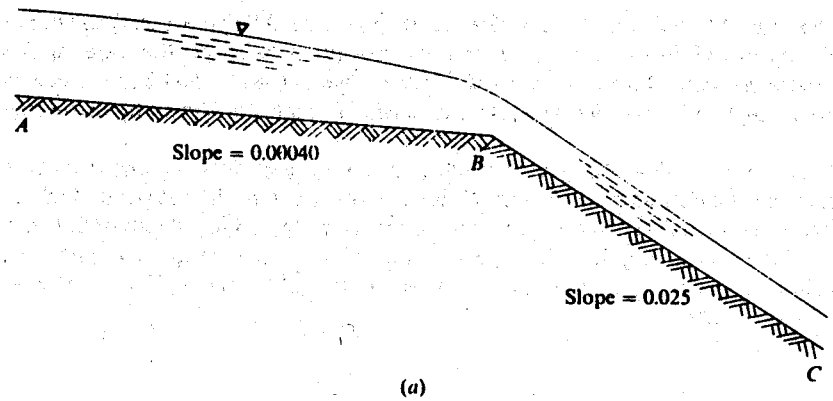
In segment BC,

$$16.5 = (5.0d)(1.486/0.013)[5.0d/(d+5.0+d)]^{2/3}(0.025)^{1/2}$$

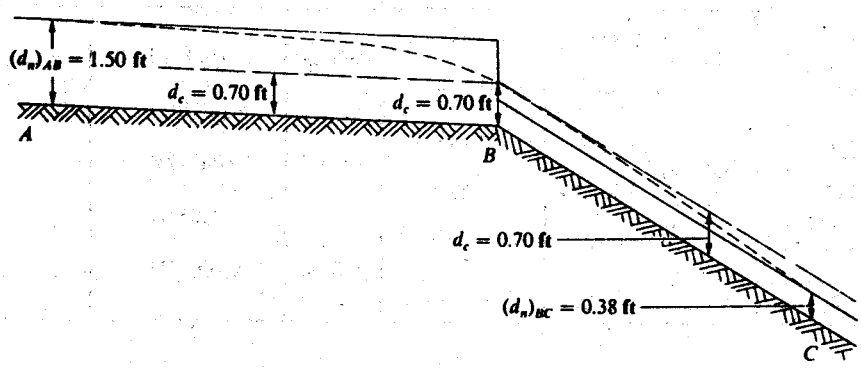
$$18.07[5.0d/(d+5.0+d)]^{2/3} - 3.300/d = 0$$

$$d = 0.38 \text{ ft} \qquad \text{(by trial and error)}$$

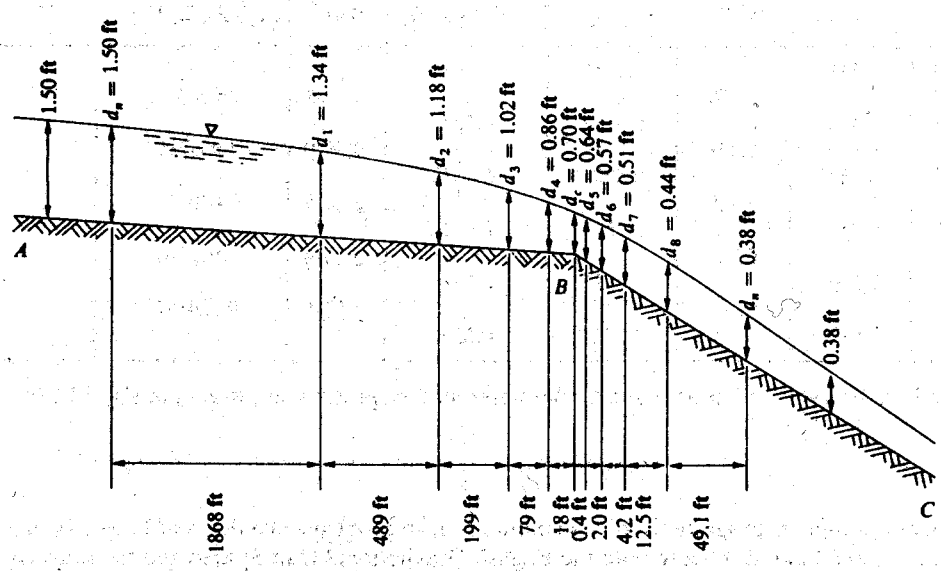
Since $d < d_c$, the flow in segment BC is supercritical.


A
Slope = 0.00040
B
Slope = 0.025
C

(a)


$(d_n)_{AB}$ = 1.50 ft
d_c = 0.70 ft
d_c = 0.70 ft
A
B
d_c = 0.70 ft
$(d_n)_{BC}$ = 0.38 ft
C

(b)


1.50 ft
d_n = 1.50 ft
d_1 = 1.34 ft
d_2 = 1.18 ft
d_3 = 1.02 ft
d_4 = 0.86 ft
d_c = 0.70 ft
d_5 = 0.64 ft
d_6 = 0.57 ft
d_7 = 0.51 ft
d_8 = 0.44 ft
d_n = 0.38 ft
0.38 ft
A
B
C
1868 ft
489 ft
199 ft
79 ft
18 ft
0.4 ft
2.0 ft
4.2 ft
12.5 ft
49.1 ft

(c)

Fig. 14-68

Figure 14-68*b* shows the location of the critical depth (the dashed line), which is constant throughout, along with the normal depths of flow in segment *AB* and segment *BC*. Obviously, the water-surface profile cannot drop instantaneously at point *B* from the normal depth in segment *AB* (1.50 ft) to the normal depth in segment *BC* (0.38 ft); there must be a transition zone on both sides of point *B* as shown by the dashed line in Fig. 14-68*b*.

As a matter of fact, as the flow changes from subcritical to supercritical in going from segment *AB* to segment *BC*, it passes through the critical state at point *B*. Hence, the depth of flow at point *B* will be 0.70 ft. The problem now becomes one of determining the flow profile from the critical depth of 0.70 ft at point *B* upstream to the point where the normal depth of 1.50 ft is reached and downstream to the point where the normal depth of 0.38 ft is reached. These computations are carried out in the tables below. Each profile will be analyzed using five equal depth increments.

For Segment *AB*

(1) d, ft	(2) v, ft/s $\frac{16.5}{5.0\times(1)}$	(3) v_m, ft/s	(4) $v^2/2g$, ft $\frac{(2)^2}{2\times 32.2}$	(5) R, ft $\frac{5.0\times(1)}{5.0+2\times(1)}$	(6) R_m, ft	(7) s $\left[\frac{0.013\times(3)}{1.486\times(6)^{2/3}}\right]^2$	(8) L, ft $\frac{[(4)+(1)]_2-[(4)+(1)]_1}{0.00040-(7)}$
0.70	4.714		0.3451	0.5469			
		4.276			0.5934	0.0028063	−18
0.86	3.837		0.2286	0.6399			
		3.536			0.6822	0.0015934	−79
1.02	3.235		0.1625	0.7244			
		3.016			0.7630	0.0009985	−199
1.18	2.797		0.1215	0.8016			
		2.630			0.8370	0.0006711	−489
1.34	2.463		0.0942	0.8724			
		2.332			0.9050	0.0004755	−1868
1.50	2.200		0.0752	0.9375			

For Segment *BC*

(1) d, ft	(2) v, ft/s $\frac{16.5}{5.0\times(1)}$	(3) v_m, ft/s	(4) $v^2/2g$, ft $\frac{(2)^2}{2\times 32.2}$	(5) R, ft $\frac{5.0\times(1)}{5.0+2\times(1)}$	(6) R_m, ft	(7) s $\left[\frac{0.013\times(3)}{1.486\times(6)^{2/3}}\right]^2$	(8) L, ft $\frac{[(4)+(1)]_2-[(4)+(1)]_1}{0.025-(7)}$
0.70	4.714		0.3451	0.5469			
		4.935			0.5283	0.004364	0.4
0.64	5.156		0.4128	0.5096			
		5.473			0.4869	0.005985	2.0
0.57	5.789		0.5204	0.4642			
		6.130			0.4439	0.008493	4.2
0.51	6.471		0.6502	0.4236			
		6.986			0.3989	0.012720	12.5
0.44	7.500		0.8734	0.3741			
		8.092			0.3520	0.020164	49.1
0.38	8.684		1.1710	0.3299			

Based on the values computed above, the water-surface profile is illustrated in Fig. 14-68*c*.

14.267 Prepare a computer program to solve nonuniform flow problems like those of Probs. 14.265 and 14.266. The program should handle data in both the English Gravitational Unit System and the International System of Units.

```
C     THIS PROGRAM DETERMINES THE FLOW PROFILE FOR A NON-UNIFORM FLOW
C     IN A RECTANGULAR OPEN CHANNEL.  IT CAN BE USED FOR PROBLEMS IN
C     BOTH THE ENGLISH SYSTEM OF UNITS AND THE INTERNATIONAL SYSTEM OF
C     UNITS.
C
C     THE PROGRAM CONSIDERS TWO ADJACENT CHANNEL SEGMENTS.  CONSIDER
C     ONE SEGMENT TO GO FROM POINT "A" TO POINT "B" AND THE OTHER
C     SEGMENT TO GO FROM POINT "B" TO POINT "C" IN THE DOWNSTREAM
C     DIRECTION.  THE PROGRAM COMPUTES THE FLOW PROFILE IN SEGMENT AB
C     IN THE UPSTREAM DIRECTION (I.E., FROM "B" TO "A") BASED ON SUB-
C     CRITICAL FLOW IN SEGMENT AB AND THE FLOW PROFILE IN SEGMENT BC IN
C     THE DOWNSTREAM DIRECTION (I.E., FROM "B" TO "C") BASED ON SUPER-
C     CRITICAL FLOW IN SEGMENT BC.  THE PROGRAM CAN BE USED EITHER FOR
C     TWO SEGMENTS AS DESCRIBED ABOVE OR FOR A SINGLE SEGMENT WITH
C     COMPUTATIONS TO PROCEED UPSTREAM (SEGMENT AB) OR DOWNSTREAM (SEG-
C     MENT BC), AS DESIRED.  IN THE CASE OF A SINGLE SEGMENT, ENTER DATA
C     FOR SEGMENT AB IF FLOW IS SUBCRITICAL AND FOR SEGMENT BC IF FLOW
C     IS SUPERCRITICAL.
C
C     THIS PROGRAM IS BASED ON CONSTANT CHANNEL WIDTH, CONSTANT FLOW
C     RATE, AND CONSTANT MANNING N-VALUE THROUGHOUT AND ON A SEPARATE
C     CONSTANT CHANNEL SLOPE IN EACH OF SEGMENTS AB AND BC.
C
C     INPUT DATA MUST BE SET UP AS FOLLOWS.
C
C     CARD 1   COLUMN 1          ENTER 0 (ZERO) OR BLANK IF THE ENGLISH
C                                SYSTEM OF UNITS IS TO BE USED.  ENTER 1
C                                (ONE) IF THE INTERNATIONAL SYSTEM OF
C                                UNITS IS TO BE USED.
C              COLUMN 2          ENTER 1 (ONE) IF ONLY A SINGLE UPSTREAM
C                                COMPUTATION IS DESIRED.  (IN THIS CASE,
C                                ENTER DEPTHS AT POINTS A AND B AND SLOPE
C                                IN SEGMENT AB.  LEAVE DEPTH AT POINT C
C                                AND SLOPE IN SEGMENT BC BLANK.)  ENTER 2
C                                (TWO) IF ONLY A SINGLE DOWNSTREAM COM-
C                                PUTATION IS DESIRED.  (IN THIS CASE, ENTER
C                                DEPTHS AT POINTS B AND C AND SLOPE IN
C                                SEGMENT BC.  LEAVE DEPTH AT POINT A AND
C                                SLOPE IN SEGMENT AB BLANK.)  ENTER 3
C                                (THREE) IF COMPUTATIONS FOR BOTH SEGMENTS
C                                ARE DESIRED. (IN THIS CASE, ENTER DEPTHS
C                                AT POINTS A, B, AND C AND SLOPES IN SEG-
C                                MENTS AB AND BC.)
C              COLUMNS 3-5       ENTER INTEGER NUMBER (RIGHT ADJUSTED)
C                                GIVING NUMBER OF LENGTH INCREMENTS TO BE
C                                USED IN COMPUTING THE FLOW PROFILE IN
C                                EACH SEGMENT.
C              COLUMNS 6-80      ENTER TITLE, DATE, AND OTHER INFORMATION,
C                                IF DESIRED.
C     CARD 2   COLUMNS 1-10      ENTER NUMBER INCLUDING DECIMAL GIVING
C                                WIDTH OF RECTANGULAR CHANNEL (IN FEET OR
C                                METERS).
C              COLUMNS 11-20     ENTER NUMBER INCLUDING DECIMAL GIVING
C                                FLOW RATE (IN CUBIC FEET PER SECOND OR
C                                CUBIC METERS PER SECOND).
C              COLUMNS 21-30     ENTER NUMBER INCLUDING DECIMAL GIVING
C                                DEPTH AT POINT A (IN FEET OR METERS).
C                                THIS VALUE MAY BE LEFT BLANK, IN WHICH
C                                CASE THE "NORMAL DEPTH" IN SEGMENT AB
C                                WILL AUTOMATICALLY BE USED.
C              COLUMNS 31-40     ENTER NUMBER INCLUDING DECIMAL GIVING
C                                DEPTH AT POINT B (IN FEET OR METERS).
C                                THIS VALUE MAY BE LEFT BLANK, IN WHICH
C                                CASE THE "CRITICAL DEPTH" WILL AUTO-
C                                MATICALLY BE USED.
C              COLUMNS 41-50     ENTER NUMBER INCLUDING DECIMAL GIVING
C                                DEPTH AT POINT C (IN FEET OR METERS).
C                                THIS VALUE MAY BE LEFT BLANK, IN WHICH
C                                CASE THE "NORMAL DEPTH" IN SEGMENT BC
C                                WILL AUTOMATICALLY BE USED.
C              COLUMNS 51-60     ENTER NUMBER INCLUDING DECIMAL GIVING
C                                CHANNEL SLOPE IN SEGMENT AB.
C              COLUMNS 61-70     ENTER NUMBER INCLUDING DECIMAL GIVING
```

```
C                                CHANNEL SLOPE IN SEGMENT BC.
C                COLUMNS 71-80   ENTER NUMBER INCLUDING DECIMAL GIVING
C                                MANNING N-VALUE.
C
C     MULTIPLE DATA SETS FOR SOLVING ANY NUMBER OF PROBLEMS MAY BE
C     INCLUDED FOR PROCESSING.
C
      COMMON DMID,DUP,NSEG,Q,W,N,COEFF,G,SUP
      DIMENSION TITLE(13)
      REAL N
      INTEGER UNITS,CODE
1     READ(5,100,END=2)UNITS,CODE,NSEG,TITLE
  100 FORMAT(2I1,I3,12A6,A3)
      WRITE(6,106)TITLE
  106 FORMAT('1',12A6,A3,////)
      READ(5,101)W,Q,DUP,DMID,DDOWN,SUP,SDOWN,N
  101 FORMAT(8F10.0)
      COEFF=1.486
      IF(UNITS.EQ.1)COEFF=1.0
      G=32.2
      IF(UNITS.EQ.1)G=9.807
      IF(DMID.LT.0.0001)DMID=((Q/W)**2/G)**(1.0/3.0)
      IF(CODE.EQ.2)GO TO 110
      IF(DUP.LT.0.0001)DUP=DNORM(W,Q,SUP,N,COEFF)
      IF(CODE.EQ.1)GO TO 109
  110 IF(DDOWN.LT.0.0001)DDOWN=DNORM(W,Q,SDOWN,N,COEFF)
      IF(CODE.EQ.2)GO TO 105
  109 X1=' UPST'
      X2='REAM '
      IF(UNITS.EQ.0)WRITE(6,107)X1,X2,W,Q,DUP,DMID,SUP,N
  107 FORMAT(1X,'GIVEN DATA FOR ' ,2A5,' FLOW PROFILE FOR A RECTANGULAR
     *OPEN CHANNEL',//5X,'WIDTH OF CHANNEL =',F7.1,' FT',6X,'FLOW RATE O
     *F WATER =',F7.1,' CU FT/S',//5X,'DEPTH OF WATER AT UPSTREAM END OF
     * SEGMENT =',F7.2,' FT',//5X,'DEPTH OF WATER AT DOWNSTREAM END OF S
     *EGMENT =',F7.2,' FT',//5X,'SLOPE =',F10.7,20X,'MANNING N-VALUE =',
     *F6.3,///1X,'THE FLOW PROFILE WITHIN THE SEGMENT IS GIVEN IN THE TA
     *BLE BELOW'///4X,'DEPTH (FT)',10X,'VELOCITY (FT/S)',10X,'LENGTH OF
     *SUBSEGMENT (FT)')
      IF(UNITS.EQ.1)WRITE(6,108)X1,X2,W,Q,DUP,DMID,SUP,N
  108 FORMAT(1X,'GIVEN DATA FOR ' ,2A5,' FLOW PROFILE FOR A RECTANGULAR
     *OPEN CHANNEL',//5X,'WIDTH OF CHANNEL =',F7.1,' M',6X,'FLOW RATE OF
     * WATER =',F7.1,' CU M/S',//5X,'DEPTH OF WATER AT UPSTREAM END OF S
     *EGMENT =',F7.2,' M',//5X,'DEPTH OF WATER AT DOWNSTREAM END OF SEGM
     *ENT =',F7.2,' M',//5X,'SLOPE =',F10.7,20X,'MANNING N-VALUE =',
     *F6.3,///1X,'THE FLOW PROFILE WITHIN THE SEGMENT IS GIVEN IN THE TA
     *BLE BELOW'///4X,'DEPTH (M)',10X,'VELOCITY (M/S)',10X,'LENGTH OF SU
     *BSEGMENT (M)')
      CALL LENGTH
      IF(CODE.EQ.1)GO TO 1
  105 X1='DOWNS'
      X2='TREAM'
      IF(UNITS.EQ.0)WRITE(6,107)X1,X2,W,Q,DMID,DDOWN,SDOWN,N
      IF(UNITS.EQ.1)WRITE(6,108)X1,X2,W,Q,DMID,DDOWN,SDOWN,N
      DUP=DDOWN
      SUP=SDOWN
      CALL LENGTH
      GO TO 1
2     STOP
      END
      FUNCTION DNORM(W,Q,S,N,COEFF)
      D=0.001
      TRY1=COEFF/N*(W*D/(W+2.0*D))**(2.0/3.0)*SQRT(S)-Q/W/D
  104 D=D+0.001
      TRY2=COEFF/N*(W*D/(W+2.0*D))**(2.0/3.0)*SQRT(S)-Q/W/D
      IF(TRY1*TRY2)102,102,103
  103 TRY1=TRY2
      GO TO 104
  102 DNORM=D-0.0005
      RETURN
      END
      SUBROUTINE LENGTH
      COMMON DMID,DUP,NSEG,Q,W,N,COEFF,G,SUP
      DINC=(DMID-DUP)/FLOAT(NSEG)
```

```
      D1=DMID
      TOTAL=0.0
      DO 102  J=1,NSEG
      D2=D1-DINC
      V1=Q/W/D1
      V2=Q/W/D2
      VMEAN=(V1+V2)/2.0
      V2G1=V1**2/2.0/G
      V2G2=V2**2/2.0/G
      HR1=W*D1/(W+2.0*D1)
      HR2=W*D2/(W+2.0*D2)
      HRMEAN=(HR1+HR2)/2.0
      SMEAN=(N*VMEAN/COEFF/HRMEAN**(2.0/3.0))**2
      SEGL=(V2G2+D2-(V2G1+D1))/(SUP-SMEAN)
      TOTAL=TOTAL+SEGL
      WRITE(6,103)D1,V1,SEGL
  103 FORMAT(1X,F10.2,11X,F12.3,/55X,F8.1)
  102 D1=D2
      WRITE(6,103)D2,V2
      WRITE(6,104)TOTAL
  104 FORMAT(41X,'TOTAL LENGTH =',F8.1,//)
      RETURN
      END
```

14.268 Solve Prob. 14.265 using the computer program developed in Prob. 14.267.

Input

```
11 10SAMPLE ANALYSIS ØF PRØFILE IN NØN-UNIFØRM FLØW
12.0      126.0                4.55                0.00086              0.013
```

Output

```
SAMPLE ANALYSIS OF PROFILE IN NON-UNIFORM FLOW

GIVEN DATA FOR  UPSTREAM  FLOW PROFILE FOR A RECTANGULAR OPEN CHANNEL

    WIDTH OF CHANNEL =   12.0 M      FLOW RATE OF WATER =  126.0 CU M/S

    DEPTH OF WATER AT UPSTREAM END OF SEGMENT =   2.95 M

    DEPTH OF WATER AT DOWNSTREAM END OF SEGMENT =   4.55 M

    SLOPE = 0.0008600                    MANNING N-VALUE = 0.013

THE FLOW PROFILE WITHIN THE SEGMENT IS GIVEN IN THE TABLE BELOW

   DEPTH (M)          VELOCITY (M/S)          LENGTH OF SUBSEGMENT (M)
     4.55                 2.308
                                                     -235.2
     4.39                 2.392
                                                     -242.8
     4.23                 2.482
                                                     -252.6
     4.07                 2.579
                                                     -265.6
     3.91                 2.685
                                                     -283.6
     3.75                 2.799
                                                     -309.9
     3.59                 2.924
                                                     -351.8
     3.43                 3.060
                                                     -427.9
     3.27                 3.209
                                                     -607.7
     3.11                 3.374
                                                    -1530.9
     2.95                 3.556

                                        TOTAL LENGTH = -4508.0
```

14.269 Solve Prob. 14.266 using the computer program developed in Prob. 14.267.

Input

```
03  5SAMPLE ANALYSIS ØF PRØFILE IN NØN-UNIFØRM FLØW
5.0       16.5                                    0.00040   0.025     0.013
```

Output

```
SAMPLE ANALYSIS OF PROFILE IN NON-UNIFORM FLOW


GIVEN DATA FOR  UPSTREAM  FLOW PROFILE FOR A RECTANGULAR OPEN CHANNEL

    WIDTH OF CHANNEL =     5.0 FT       FLOW RATE OF WATER =    16.5 CU FT/S

    DEPTH OF WATER AT UPSTREAM END OF SEGMENT =    1.50 FT

    DEPTH OF WATER AT DOWNSTREAM END OF SEGMENT =    0.70 FT

    SLOPE = 0.0004000                    MANNING N-VALUE = 0.013

THE FLOW PROFILE WITHIN THE SEGMENT IS GIVEN IN THE TABLE BELOW

   DEPTH (FT)          VELOCITY (FT/S)          LENGTH OF SUBSEGMENT (FT)
     0.70                  4.736
                                                          -17.6
     0.86                  3.845
                                                          -78.9
     1.02                  3.236
                                                         -201.2
     1.18                  2.793
                                                         -501.2
     1.34                  2.457
                                                        -1986.4
     1.50                  2.193

                                        TOTAL LENGTH =  -2785.2

GIVEN DATA FOR DOWNSTREAM FLOW PROFILE FOR A RECTANGULAR OPEN CHANNEL

    WIDTH OF CHANNEL =     5.0 FT       FLOW RATE OF WATER =    16.5 CU FT/S

    DEPTH OF WATER AT UPSTREAM END OF SEGMENT =    0.70 FT

    DEPTH OF WATER AT DOWNSTREAM END OF SEGMENT =    0.38 FT

    SLOPE = 0.0250000                    MANNING N-VALUE = 0.013

THE FLOW PROFILE WITHIN THE SEGMENT IS GIVEN IN THE TABLE BELOW

   DEPTH (FT)          VELOCITY (FT/S)          LENGTH OF SUBSEGMENT (FT)
     0.70                  4.736
                                                           0.5
     0.63                  5.208
                                                           1.9
     0.57                  5.783
                                                           4.5
     0.51                  6.501
                                                          11.0
     0.44                  7.423
                                                          46.1
     0.38                  8.650

                                        TOTAL LENGTH =    63.9
```

14.270 A rectangular channel 12.0 m wide is laid on a slope of 0.0028. The depth of flow at one section is 1.50 m, while the depth of flow at another section 500 ft downstream is 1.80 m. Determine the probable rate of flow, if $n = 0.026$.

▮

$$Q = A_m v_m = (A_m)(1.0/n)R_m^{2/3}s^{1/2} \qquad A_m = [(12.0)(1.50) + (12.0)(1.80)]/2 = 19.8\ \text{m}^2$$

$$R_m = [(12.0)(1.50)/(1.50 + 12.0 + 1.50) + (12.0)(1.80)/(1.80 + 12.0 + 1.80)]/2 = 1.29\ \text{m}$$

$$Q = (19.8)(1.0/0.026)(1.29)^{2/3}s^{1/2} = 902.44s^{1/2} \qquad (1)$$

$$v_1^2/2g + d_1 + s_0L = v_2^2/2g + d_2 + sL \qquad v_1 = Q/A_1 = Q/[(12.0)(1.50)] = Q/18.0$$

$$v_2 = Q/A_2 = Q/[(12.0)(1.80)] = Q/21.6$$

$$(Q/18.0)^2/[(2)(9.807)] + 1.50 + (0.0028)(500) = (Q/21.6)^2/[(2)(9.807)] + 1.80 + 500s \qquad (2)$$

Substituting (*1*) into (*2*) gives $(902.44s^{1/2}/18.0)^2/[(2)(9.807)] + 1.50 + 1.40 = (902.44s^{1/2}/21.6)^2[(2)(9.807)] + 1.80 + 500s$, $460.8s = 1.10$, $s = 0.002387$; $Q = (902.44)(0.002387)^{1/2} = 44.1\ \text{m}^3/\text{s}$.

14.271 Solve Prob. 14.265 using five equal depth increments to determine the distance upstream to the point where the water surface is at the normal depth. Compare the answer with the one obtained in Prob. 14.265.

▮ Using data found in Prob. 14.265,

d	v	v_m	$v^2/2g$	R	R_m	s_g	L, m
4.55	2.308		0.2716	2.588			
		2.395			2.535	0.0002805	−479
4.23	2.482		0.3141	2.481			
		2.584			2.424	0.0003465	−519
3.91	2.685		0.3676	2.367			
		2.805			2.307	0.0004362	−593
3.59	2.925		0.4362	2.246			
		3.068			2.182	0.0005621	−774
3.27	3.211		0.5257	2.117			
		3.385			2.048	0.0007446	−1732
2.95	3.559		0.6458	1.978			
							−4097

The answer using five increments is $(4568 - 4097)/4568 = 0.103$, or 10.3 percent smaller than that using ten increments.

14.272 Water flows in a rectangular concrete channel similar to the one depicted in Fig. 14-68*a*. If the channel width is 3.0 ft and the discharge is 12.0 cfs, determine the water-surface profile throughout the channel shown.

▮

$$d_c = [(Q/B)^2/g]^{1/3} = [(12.0/3.0)^2/32.2]^{1/3} = 0.79\ \text{ft} \qquad Q = Av = (A)(1.486/n)(R^{2/3})(s^{1/2})$$

$$12.0 = (3.0d_{AB})(1.486/0.013)[3.0d_{AB}/(3.0 + 2d_{AB})]^{2/3}(0.00052)^{1/2}$$

By trial and error, $d_{AB} = 1.76$ ft. $12.0 = (3.0d_{BC})(1.486/0.013)[3.0d_{BC}/(3.0 + 2d_{BC})]^{2/3}(0.030)^{1/2}$. By trial and error, $d_{BC} = 0.42$ ft. Hence, the water-surface profile changes from the normal depth in segment AB (1.76 ft) to the critical depth at B (0.79 ft) to the normal depth in segment BC (0.42 ft).

For Segment *AB* (*B* to *A*)

d	v	v_m	$v^2/2g$	R	R_m	s_g	L, ft
0.79	5.063		0.398	0.517			
		4.572			0.555	0.003508	−17
0.98	4.082		0.259	0.593			
		3.736			0.626	0.001995	−81
1.18	3.390		0.178	0.660			
		3.155			0.688	0.001254	−196
1.37	2.920		0.132	0.716			
		2.734			0.742	0.0008516	−510
1.57	2.548		0.101	0.767			
		2.410			0.788	0.0006107	−1863
1.76	2.273		0.080	0.810			

For Segment *BC* (*B* to *C*)

d	v	v_m	$v^2/2g$	R	R_m	s_g	L, ft
0.79	5.063		0.398	0.517			
		5.310			0.502	0.005409	0.4
0.72	5.556		0.479	0.486			
		5.903			0.468	0.007340	2.1
0.64	6.250		0.607	0.449			
		6.634			0.431	0.010346	4.5
0.57	7.018		0.765	0.413			
		7.591			0.391	0.015425	13.0
0.49	8.163		1.035	0.369			
		8.844			0.349	0.024362	53.7
0.42	9.524		1.408	0.328			

14.273 In Fig. 14-69, 400 cfs flows through the transition. The rectangular section is 8 ft wide and $y_1 = 8$ ft. The trapezoidal section is 6 ft wide at the bottom with side slopes 1:1, and $y_2 = 7.5$ ft. Determine the rise z in the bottom through the transition.

▮ $$v_1^2/2g + y_1 = v_2^2/2g + y_2 + z + E_1 \qquad v_1 = Q/A_1 = 400/[(8)(8)] = 6.250 \text{ ft/s}$$

$$v_2 = 400/[(6)(7.5) + (7.5)(7.5)] = 3.951 \text{ ft/s}$$

$$E_1 = (0.3)(v_1^2/2g - v_2^2/2g) = (0.3)\{6.250^2/[(2)(32.2)] - 3.951^2/[(2)(32.2)]\} = 0.109 \text{ ft}$$

$$6.250^2/[(2)(32.2)] + 8 = 3.951^2/[(2)(32.2)] + 7.5 + z + 0.109 \qquad z = 0.755 \text{ ft}$$

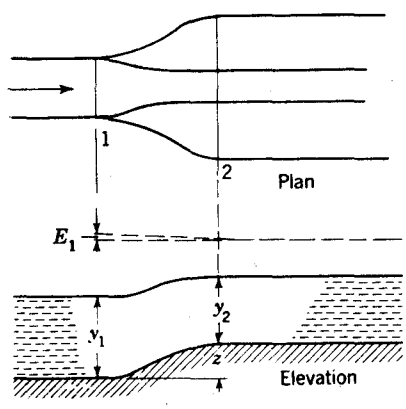


Fig. 14-69

14.274 In a critical-depth meter 2 m wide with $z = 0.3$ m, the depth y_1 is measured to be 0.75 m. Find the discharge.

▮ $q = (0.517)(g^{1/2})[y_1 - z + (0.55/g)(q^2/y_1^2)]^{3/2}$. Initially, neglect the last term in the equation above. $q = (0.517)(9.807)^{1/2}(0.75 - 0.3)^{3/2} = 0.489$ (m³/s)/m. Now, try $q = 0.500$ (m³/s)/m in the whole equation:

$q = (0.517)(9.807)^{1/2}[0.75 - 0.3 + (0.55/9.807)(0.500^2/0.75^2)]^{3/2} = 0.530\ (\text{m}^3/\text{s})/\text{m}$. Try $q = 0.536\ (\text{m}^3/\text{s})/\text{m}$: $q = (0.517)(9.807)^{1/2}[0.75 - 0.3 + (0.55/9.807)(0.536^2/0.75^2)]^{3/2} = 0.536\ (\text{m}^3/\text{s})/\text{m}$, $Q = (0.536)(2) = 1.07\ \text{m}^3/\text{s}$.

14.275 At section 1 of a canal, the cross section is trapezoidal with bottom width $b_1 = 10$ m, depth $y_1 = 7$ m, and side slopes of 2 horizontal to 1 vertical. At section 2, 200 m downstream, the bottom is 0.08 m higher than at section 1, $b_2 = 15$ m, and side slopes are 3 horizontal to 1 vertical ($Q = 200\ \text{m}^3/\text{s}$ and $n = 0.035$). Determine the depth of water at section 2.

▮ $$\Delta L = [(v_1^2 - v_2^2)/2g + y_1 - y_2]/(s - s_0) \qquad s = (nQ/1.0AR^{2/3})^2$$

Try $y = 6.92$ m:

$$A_1 = (10)(7) + (7)[(2)(7)] = 168.0\ \text{m}^2 \qquad A_2 = (15)(6.92) + (6.92)[(3)(6.92)] = 247.5\ \text{m}^2$$

$$A_{avg} = (168.0 + 247.5)/2 = 207.8\ \text{m}^2 \qquad R = A/p_w \qquad (p_w)_1 = 10 + (2)(7)(\sqrt{2^2+1}) = 41.30\ \text{m}$$

$$(p_w)_2 = 15 + (2)(6.92)(\sqrt{3^2+1}) = 58.77\ \text{m} \qquad (p_w)_{avg} = (41.30 + 58.77)/2 = 50.04\ \text{m}$$

$$R = 207.8/50.04 = 4.153\ \text{m}$$

$$s = \{(0.035)(200)/[(1.0)(207.8)(4.153)^{2/3}]\}^2 = 0.0001700$$

$$v_1 = Q/A_1 = 200/168.0 = 1.190\ \text{m/s} \qquad v_2 = 200/247.5 = 0.8081\ \text{m/s}$$

$$\Delta L = \frac{(1.190^2 - 0.8081^2)/[(2)(9.807)] + 7 - 6.92}{0.0001700 - (-0.08/200)} = 209\ \text{m}$$

This value of ΔL (209 m) is not equal to the given value (200 m). However, reworking the problem with $y_2 = 6.93$ m (not shown) yields a value of ΔL of 191 m. Thus y_2 must be approximately 6.925 m.

14.276 A trapezoidal channel, $b = 3$ m, side slopes of 1:1, $n = 0.014$, $s_0 = 0.001$, carries 28 m³/s. If the depth is 3 m at section 1, determine the water-surface profile for the next 700 m downstream.

▮ To determine whether the depth increases or decreases, the slope of the energy grade line must be computed.

$$A = (3)(3) + (3)(3) = 18.00\ \text{m}^2 \qquad R = A/p_w = 18.00/[3 + (2)(3)(\sqrt{1^2+1^2})] = 1.567\ \text{m}$$

$$s = (nQ/1.0AR^{2/3})^2 = \{(0.014)(28)/[(1.0)(18.00)(1.567)^{2/3}]\}^2 = 0.0002606$$

$$(Q^2/gA_c^3)(T_c) = \{28^2/[(9.807)(18.00)^3]\}(9) = 0.1234$$

Since $(Q^2/gA_c^3)(T_c) < 1.0$, the depth is above critical. With the depth greater than critical and the energy grade line less steep than the bottom of the channel, the specific energy is increasing. When the specific energy increases above critical, the depth of flow increases, y is then positive.

$$L = \int_{y_1}^{y_2} \frac{1 - Q^2T/gA^3}{s_0 - (nQ/1.0AR^{2/3})^2}\,dy$$

$$L = \int_3^y \frac{1 - (28^2)(T)/[(9.807)(A)^3]}{0.001 - \{(0.014)(28)/[(1.0)(AR^{2/3})]\}^2}\,dy$$

$$L = \int_3^y \frac{1 - 79.94T/A^3}{0.001 - [0.1537/(A^2R^{4/3})]}\,dy$$

The following table evaluates the terms of the integrand:

y	A	P	R	T	numerator	$10^6 \times$ denominator	$F(y)$	L
3	18	11.48	1.57	9	0.8766	739	1185	0
3.2	19.84	12.05	1.65	9.4	0.9038	799	1131	231.6
3.4	21.76	12.62	1.72	9.8	0.9240	843	1096	454.3
3.6	23.76	13.18	1.80	10.2	0.9392	876	1072	671.1
3.8	25.84	13.75	1.88	10.6	0.9509	901	1056	883.9

The integral $\int F(y)\,dy$ can be evaluated by plotting the curve and taking the area under it between $y = 3$ and the following values of y. As $F(y)$ does not vary greatly in this example, the average of $F(y)$ can be used for each reach (the trapezoidal rule); and when it is multiplied by Δy, the length of reach is obtained. Between $y = 3$ and $y = 3.2$, $[(1185 + 1131)/2](0.2) = 231.6$. Between $y = 3.2$ and $y = 3.4$, $[(1131 + 1096)/2](0.2) = 222.7$ and so on. Five points on it are known, so the water surface can be plotted.

14.277 After contracting below a sluice gate, water flows onto a wide horizontal floor with a velocity of 15 m/s and a depth of 0.7 m. Find the equation for the water-surface profile ($n = 0.015$).

$$x = -(\tfrac{3}{13})(1.0/nq)^2(y^{13/3} - y_1^{13/3}) + (3/4g)(1.0/n)^2(y^{4/3} - y_1^{4/3})$$

$$q = (0.7)(15) = 10.5\ (\mathrm{m^3/s})/\mathrm{m}$$

$$x = -(\tfrac{3}{13})\{1.0/[(0.015)(10.5)]\}^2(y^{13/3} - 0.7^{13/3}) + \{3/[(4)(9.807)]\}(1.0/0.015)^2(y^{4/3} - 0.7^{4/3})$$

$$= -9.303y^{13/3} + 339.9y^{4/3} - 209.3$$

$$y_c = (q^2/g)^{1/3} = (10.5^2/9.807)^{1/3} = 2.240\ \mathrm{m}$$

The depth must increase downstream, since the specific energy decreases, and the depth must move toward the critical value for less specific energy. The equation does not hold near the critical depth because of vertical accelerations that have been neglected in the derivation of gradually varied flow. If the channel is long enough for critical depth to be attained before the end of the channel, the high-velocity flow downstream from the gate may be drowned or a jump may occur. The water-surface calculation for the subcritical flow must begin with critical depth at the downstream end of the channel.

14.278 Prepare a computer program in BASIC to calculate the steady gradually varied water-surface profile in any prismatic rectangular, symmetric trapezoidal, or triangular channel.

```
10 REM B:PROFILES            WATER SURFACE PROFILES--STEADY STATE
20 ' WATER SURFACE PROFILE IN RECT, TRAPEZOIDAL, OR TRIANGULAR CHANNEL.
30 ' XL=LENGTH, B=BOT WIDTH, Z=SIDE SLOPE, RN=MANNING N,SO=BOT SLOPE,Q=FLOW.
40 ' YCONT=CONTROL DEPTH. IF YCONT=0 IN DATA, YCONT IS SET EQUAL TO YC.
50 ' IN SUBCRITICAL FLOW, CONTROL IS DOWNSTREAM AND DISTANCES ARE MEASURED
60 ' IN THE UPSTREAM DIRECTION
70 ' IN SUPERCRITICAL FLOW, CONTROL IS U.S. AND DISTANCES ARE MEASURED D.S.
80 LPRINT CHR$(14);"STEADY-STATE WATER-SURFACE PROFILES"
90 LPRINT CHR$(14);"     DATE=";DATE$;" TIME=";TIME$
100 DEF FNAREA(YY)=YY*(B+Z*YY): DEF FNPER(YY)=B+2!*YY*SQR(1'+Z^2)
110 DEF FNYCRIT(YY)=1!-Q^2*(B+2!*Z*YY)/(G*FNAREA(YY)^3)
120 DEF FNYNORM(YY)=1!-Q^2*CON/(FNAREA(YY)^3.333/FNPER(YY)^1.333)
130 DEFINT I:     DEF FNDL(YY)=FNYCRIT(YY)/(FNYNORM(YY)*SO)
140 DEF FNFPM(YY)=GAM*(YY^2*(.5*B+Z*YY/3!)+Q^2/(G*FNAREA(YY)))
150 ISI$="SI":     DEF FNENERGY(YY)=YY+Q^2/(2!*G*FNAREA(YY)^2)
160 READ IUNIT$,XL,B,Z,RN,SO,Q,YCONT:  DATA "SI",200.,2.5,.8,.012,.025,25.,.0
170 IF IUNIT$=ISI$ THEN 190
180 GAM=62.4:G=32.2:CON=(RN/1.486)^2/SO:LPRINT"USC UNITS":LPRINT: GOTO 200
190 GAM=9802!:    G=9.806001:    CON=RN^2/SO:    LPRINT"SI UNITS":LPRINT
200 LPRINT" CHANNEL LENGTH=";XL;" DISCHARGE=";Q;" B=";B;" Z=";Z;"RN=";RN;
    " SO=";SO
210 ' DETERMINATION OF CRITICAL AND NORMAL DEPTHS
220 NN=30:    DN=0!:   UP=30!: YC=15!: FOR I= 1 TO 15: IF FNYCRIT(YC)=0! THEN 250
230 IF FNYCRIT(YC)<0! THEN DN=YC ELSE UP=YC
240 YC=.5*(UP+DN):    NEXT I
250 IF YCONT=0! THEN YCONT=YC
260 IF SO<=0! THEN 320
270 UP=40!:    DN=0!:    YN=20!:    FOR I= 1 TO 15
280 X=FNYNORM(YN):    IF X<0! THEN DN=YN:    GOTO 300
290 IF X>0! THEN UP=YN ELSE GOTO 310
300 YN=.5*(UP+DN):    NEXT I
310 LPRINT:    LPRINT" NORMAL DEPTH=";YN;" CRITICAL DEPTH=";YC:    GOTO 330
320 YN=3!*YC:    LPRINT:    LPRINT" CRITICAL DEPTH=";YC
330 IF YN<YC THEN 410
340 ' MILD,ADVERSE,OR HORIZONTAL CHANNEL YN>YC
350 IF YCONT<YC THEN 390
360 ' SUBCRITICAL FLOW, YCONT>=YC
370 SIGN=-1!:    DY=(YCONT-YN)*.998/NN:    LPRINT:
    LPRINT"CONTROL IS DOWNSTREAM,DEPTH=";YCONT:    GOTO 460
380 ' SUPERCRITICAL FLOW
390 SIGN=1!:    DY=(YC-YCONT)/NN:    LPRINT:
    LPRINT"CONTROL IS UPSTREAM, DEPTH=";YCONT:    GOTO 460
400 ' STEEP CHANNEL,YN<YC
410 IF YCONT<=YC THEN 450
420 ' SUBCRITICAL FLOW, YCONT>YC
430 SIGN=-1!:    DY=(YCONT-YC)/NN:    LPRINT:
    LPRINT" CONTROL IS DOWNSTREAM, DEPTH=";YCONT:    GOTO 460
440 ' SUPERCRITICAL FLOW, YCONT<=YC
450 SIGN=1!:    NN=2*NN:    DY=(YN-YCONT)*.998/NN:    LPRINT:
    LPRINT" CONTROL IS UPSTREAM, DEPTH=";YCONT
460 SL=0!:    Y=YCONT:    E=FNENERGY(Y):    FM=FNFPM(Y):    LPRINT
470 LPRINT"        DISTANCE        DEPTH        ENERGY        F+M":    GOSUB 550
480 ' WATER-SURFACE PROFILE CALCULATION
490 FOR I=1 TO NN STEP 2:    Y2=YCONT+SIGN*DY*(I+1)
500 DX=DY*(FNDL(Y)+FNDL(Y2)+4!*FNDL(YCONT+SIGN*I*DY))/3!
510 SL=SL+DX:    IF SL>XL THEN 540
520 Y=Y2:    E=FNENERGY(Y):    FM=FNFPM(Y):    IF (I=NN-1) AND (SL<0!) THEN SL=XL
530 GOSUB 550:    NEXT I:    GOTO 160
540 Y=Y2-SIGN*2!*DY*(SL-XL)/DX:    E=FNENERGY(Y):    FM=FNFPM(Y):    SL=XL:
    GOSUB 550:    GOTO 160
550 LPRINT SPC(5) USING"    ####.#";SL;:    LPRINT USING"     ###.###";Y;E;:
    LPRINT USING"    ########.";FM:    RETURN
560 DATA "SI",600.,2.5,.8,.012,.0002,25.,.907
570 DATA "SI",600.,2.5,.8,.012,.0002,25.,2.
```

Input data include the specification of the system of units (SI or USC) in the first columns of the data, followed by the channel dimensions, discharge, and water-surface control depth. If the control depth is set to zero in data, it is automatically assumed to be the critical depth in the program. For subcritical flow the control is downstream, and distances are measured in the upstream direction. For supercritical flow the control depth is upstream, and distances are measured in the downstream direction.

The program begins with several line functions to compute the various variables and functions in the problem. After the necessary data input, critical depth is computed, followed by the normal-depth calculation if normal depth exists. The bisection method is used in these calculations. The type of profile is then categorized, and finally the water-surface profile, specific energy, and $F + M$ are calculated and printed. Simpson's rule is used in the integration for the water-surface profile.

The program begins with several line functions to compute the various variables and functions in the problem. After the necessary data input, critical depth is computed, followed by the normal-depth calculation if normal.

14.279 A trapezoidal channel, $B = 2.5$ m, side slope = 0.8, has two bottom slopes. The upstream portion is 200 m long, $S_0 = 0.025$, and the downstream portion, 600 m long, $S_0 = 0.0002$, $n = 0.012$. A discharge of 25 m^3/s enters at critical depth from a reservoir at the upstream end, and at the downstream end of the system the water depth is 2 m. Determine the water-surface profiles throughout the system, including jump location, using the computer program of Prob. 14.278.

I Three separate sets of data are included in the program and are needed to obtain the results used to plot the solution as shown in Fig. 14-70. The first set for the steep upstream channel has a control depth equal to zero since it will be automatically assumed critical depth in the program. The second set is for the supercritical flow in the mild channel. It begins at a control depth equal to the end depth from the upstream channel and computes the water surface downstream to the critical depth. The third set of data uses the 2-m downstream depth as the control depth and computes in the upstream direction. Computer output from the last two data sets are given below. The jump is located by finding the position of equal $F + M$ from the output of the last two data sets.

```
SI UNITS

 CHANNEL LENGTH= 600  DISCHARGE= 25  B= 2.5  Z= .8 RN= .012  SO= .0002

 NORMAL DEPTH= 3.190308  CRITICAL DEPTH= 1.780243

CONTROL IS UPSTREAM, DEPTH= .9070001

        DISTANCE        DEPTH       ENERGY        F+M
             0.0        0.907        4.630      225573
            25.8        0.965        4.160      211573
            51.2        1.023        3.786      199573
            76.0        1.082        3.487      189272
           100.1        1.140        3.247      180431
           123.3        1.198        3.054      172859
           145.5        1.256        2.900      166400
           166.5        1.315        2.777      160926
           186.0        1.373        2.679      156334
           204.0        1.431        2.603      152534
           220.0        1.489        2.544      149455
           233.9        1.547        2.500      147035
           245.4        1.606        2.469      145221
           254.1        1.664        2.448      143970
           259.6        1.722        2.437      143242
           261.5        1.780        2.433      143007
SI UNITS

 CHANNEL LENGTH= 600  DISCHARGE= 25  B= 2.5  Z= .8 RN= .012  SO= .0002

 NORMAL DEPTH= 3.190308  CRITICAL DEPTH= 1.780243

CONTROL IS DOWNSTREAM,DEPTH= 2

        DISTANCE        DEPTH       ENERGY        F+M
             0.0        2.000        2.474      146109
            33.0        2.079        2.504      148634
            80.5        2.158        2.541      151844
           145.4        2.238        2.583      155711
           231.3        2.317        2.630      160211
           343.0        2.396        2.681      165326
           486.6        2.475        2.734      171040
           600.0        2.524        2.769      174857
```

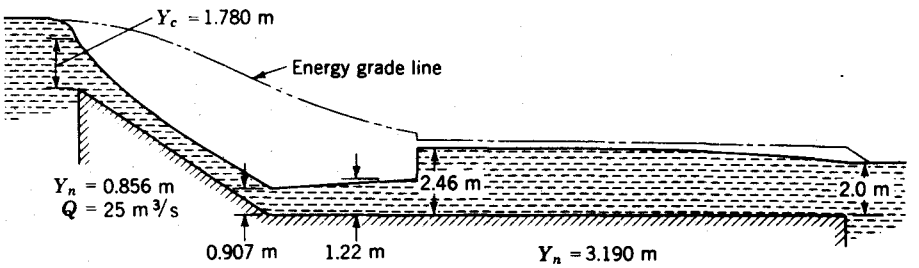


Fig. 14-70

14.280 A rectangular channel 3 m wide and 2 m deep, discharging 18 m³/s, suddenly has the discharge reduced to 12 m³/s at the downstream end. Compute the height and speed of the surge wave.

▮ $$(v_1+c)(y_1)=(v_2+c)(y_2) \qquad (\gamma/2)(y_1^2-y_2^2)=(\gamma/g)(y_1)(v_1+c)(v_2+c-v_1-c)$$

$$v_1=Q/A_1=18/[(3)(2)]=3\text{ m/s} \qquad v_2=12/(3y_2) \qquad v_2y_2=4\text{ m}^2/\text{s}$$

$$(3+c)(2)=4+cy_2 \qquad 6=4+(c)(y_2-2)$$

$$(9.79/2)(2^2-y_2^2)=(9.79/9.807)(2)(3+c)(v_2+c-3-c)$$

Eliminating c and v_2 gives $y_2^2-4=(4/9.807)[2/(y_2-2)+3](3-4/y_2)$, $y_2=2.75$ m (by trial and error), $v_2=4/2.75=1.455$ m/s. The height of the surge wave is $2.75-2=0.75$ m, and the speed of the wave is $c=2/(y_2-2)=2/(2.75-2)=2.67$ m/s.

14.281 In Fig. 14-71 find the Froude number of the undisturbed flow such that the depth y_1 at the gate is just zero when the gate is suddenly closed. For $v_0=20$ ft/s, find the liquid-surface elevation.

▮ It is required that $v_1=0$ when $y_1=0$ at $x=0$ for any time after $t=0$.

$$v=v_0-(2)(\sqrt{g})(\sqrt{y_0}-\sqrt{y}) \qquad 0=v_0-(2)(\sqrt{g})(\sqrt{y_0}-\sqrt{0})$$

$$v_0=(2)(\sqrt{gy_0}) \qquad N_F=v_0/\sqrt{gy_0}=2$$

$$x=[v_0-(2)(\sqrt{gy_0})+(3)(\sqrt{gy})](t) \qquad y_0=v_0^2/4g=20^2/[(4)(32.2)]=3.106\text{ ft}$$

$$x=\{20-(2)[\sqrt{(32.2)(3.106)}]+(3)(\sqrt{32.2y})\}(t)=17.02ty^{1/2}$$

The liquid surface is a parabola with vertex at the origin and surface concave upward.

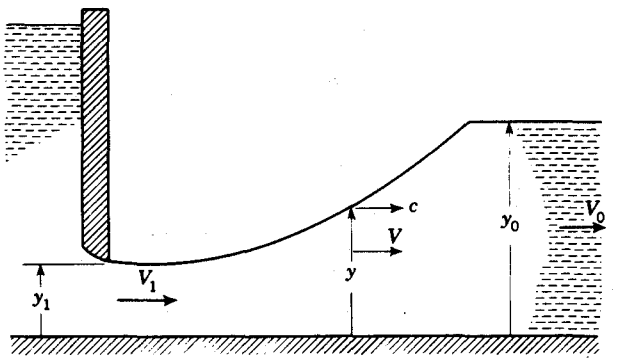


Fig. 14-71

14.282 In Fig. 14-71 the gate is partially closed at the instant $t=0$ so that the discharge is reduced by 50 percent ($v_0=6$ m/s, $y_0=3$ m). Find v_1, y_1, and the surface profile.

▮ The new discharge is

$$q=(6)(3)/2=9=v_1y_1 \qquad v=v_0-(2)(\sqrt{g})(\sqrt{y_0}-\sqrt{y})$$

$$v_1=6-(2)(\sqrt{9.807})(\sqrt{3}-\sqrt{y_1})$$

$$v_1=4.25\text{ m/s} \quad \text{and} \quad y_1=2.11\text{ m} \quad \text{(by trial and error)}$$

$$x=[v_0-(2)(\sqrt{gy_0})+(3)(\sqrt{gy})](t)=\{6-(2)[\sqrt{(9.807)(3)}]+(3)[\sqrt{(9.807)(y)}]\}(t)$$
$$=[(9.39)(\sqrt{y})-4.85](t)$$

This holds for the range of values of y between 2.12 m and 3 m.

14.283 A discharge of 160 cfs occurs in a rectangular open channel 6 ft wide with $s_0 = 0.002$ and $n = 0.012$. If the channel ends in a free outfall, calculate the depth at the brink, y_n, and y_c. Determine the shape of the water-surface profile for a distance of 100 ft upstream from the brink.

▮ $Q = (A)(1.486/n)(R^{2/3})(s^{1/2})$ $\quad 160 = (6y_n)(1.486/0.012)[6y_n/(y_n + 6 + y_n)]^{2/3}(0.002)^{1/2}$

$y_n = 3.5$ ft (by trial and error)

$Q^2/g = A^3/B \quad 160^2/32.2 = (6y_c)^3/6 \quad y_c = 2.81$ ft

Since $y_n > y_c$, the flow is subcritical and the water-surface profile is M_2 (see Fig. A-19). The depth at the outfall is approximately $0.7y_c = (0.7)(2.81) = 2.0$ ft. Critical depth occurs at about $4y_c = (4)(2.81) = 11$ ft upstream from the brink. Computations for the water-surface profile are given below.

y, ft	A, ft^2	$B+2y$, ft	R, ft	V, ft/s	$\frac{V^2}{2g}$, ft	$y+\frac{V^2}{2g}$, ft	$\Delta\left(y+\frac{V^2}{2g}\right)$, ft	V_{avg}, ft/s	R_{avg}, ft	S	$S-S_0$	x, ft	Σx,* ft
2.81	16.86	11.62	1.451	9.49	1.398	4.208							
							0.005	9.34	1.463	0.00341	0.00141	4	4
2.90	17.40	11.80	1.475	9.20	1.313	4.213							
							0.014	9.04	1.488	0.00312	0.00112	12	16
3.00	18.00	12.00	1.500	8.89	1.227	4.227							
							0.022	8.74	1.512	0.00284	0.00084	26	42
3.10	18.60	12.20	1.525	8.60	1.149	4.249							
							0.029	8.47	1.536	0.00262	0.00062	47	89
3.20	19.20	12.40	1.548	8.33	1.078	4.278							

* Summation x is measured from the point of critical depth 11 ft upstream from the brink.

The water-surface profile is sketched in Fig. 14-72.

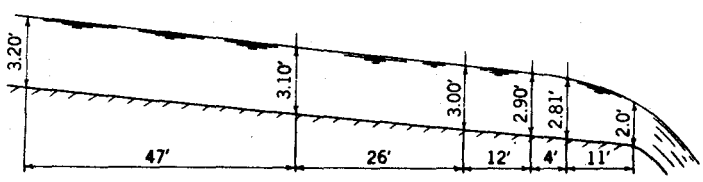


Fig. 14-72

14.284 Examine the flow conditions in a 10-ft-wide open rectangular channel of rubble masonry with $n = 0.017$ when the flow rate is 400 cfs. The channel slope is 0.020, and an ogee weir 5.0 ft high with $C_w = 3.8$ is located in the downstream end of the channel (see Fig. 14-73).

▮ $Q = (A)(1.486/n)(R^{2/3})(s^{1/2}) \quad 400 = (10y_n)(1.486/0.017)[10y_n/(y_n + 10 + y_n)]^{2/3}(0.020)^{1/2}$

$y_n = 2.36$ ft (by trial and error) $\quad y_c = (q^2/g)^{1/3} \quad y_c = [(\frac{400}{10})^2/32.2]^{1/3} = 3.68$ ft

Since $y_n < y_c$, the flow is supercritical. The head required at the weir to discharge the given flow is found from the equation $Q = (C_wL)(h + V_0^2/2g)^{3/2}$:

$$V_0 = Q/A = 400/[(5+h)(10)] = 400/(50+10h) \qquad 400 = (3.8)(10)\{h + [400/(50+10h)]^2/[(2)(32.2)]\}^{3/2}$$

$$h = 4.53 \text{ ft} \quad \text{(by trial and error)}$$

Hence, the depth of water just upstream of the weir is $5 + 4.53 = 9.53$ ft, which is greater than y_c. The flow at this point is subcritical, and a hydraulic jump must occur upstream. The depth y_2 after the jump is found from $y_2 = -y_1/2 + (y_1^2/4 + 2V_1^2y_1/g)^{1/2}$, $V_1 = Q/A_1 = 400/[(2.36)(10)] = 16.95$ ft/s, $y_2 = -2.36/2 + [2.36^2/4 + (2)(16.95)^2(2.36)/32.2]^{1/2} = 5.42$ ft. The distance from the weir to the jump is determined by the equation $x = [(y_A + V_A^2/2g) - (y_B + V_B^2/2g)]/(s - s_0)$:

$$V_A = Q/A = 400/[(5.42)(10)] = 7.380 \text{ ft/s} \qquad V_B = 400/[(9.53)(10)] = 4.197 \text{ ft/s}$$

$$s = (nV/1.486R^{2/3})^2 \qquad V_m = (7.380 + 4.197)/2 = 5.788 \text{ ft/s}$$

$$(p_w)_A = 5.42 + 10 + 5.42 = 20.84 \text{ ft} \qquad (p_w)_B = 9.53 + 10 + 9.53 = 29.06 \text{ ft}$$

$$R_m = [(5.42)(10)/20.84 + (9.53)(10)/29.06]/2 = 2.940 \text{ ft}$$

$$s = \{(0.017)(5.788)/[(1.486)(2.940)^{2/3}]\}^2 = 0.001041$$

$$x = \{5.42 + 7.380^2/[(2)(32.2)] - 9.53 - 4.197^2/[(2)(32.2)]\}/(0.001041 - 0.020) = 187 \text{ ft}$$

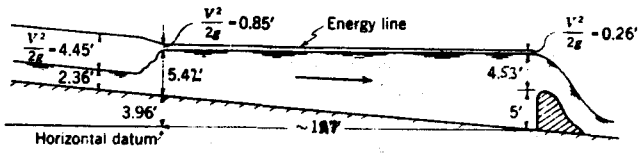


Fig. 14-73

14.285 A rectangular channel is 8 ft wide, has an 0.008 slope, discharge of 150 cfs, and $n = 0.014$. Find y_n and y_c. If the actual depth of flow is 5 ft, what type of profile exists?

▮ $Q = (A)(1.486/n)(R^{2/3})(s^{1/2})$ $\quad 150 = (8y_n)(1.486/0.014)[8y_n/(y_n + 8 + y_n)]^{2/3}(0.008)^{1/2}$

$y_n = 1.74$ ft (by trial and error) $\quad y_c = (q^2/g)^{1/3} = [(\frac{150}{8})^2/32.2]^{1/3} = 2.22$ ft

From Fig. A-19, this is an S_1 water-surface profile.

14.286 A rectangular channel is 3 m wide and ends in a free outfall. If the discharge is 10 m³/s, slope is 0.0025, and $n = 0.016$, find y_n, y_c, and the water-surface profile for a distance of 150 m upstream from the outfall.

▮ $Q = (A)(1.0/n)(R^{2/3})(s^{1/2})$ $\quad 10 = (3y_n)(1.0/0.016)[3y_n/(y_n + 3 + y_n)]^{2/3}(0.0025)^{1/2}$

$y_n = 1.34$ m (by trial and error) $\quad y_c = (q^2/g)^{1/3} = [(\frac{10}{3})^2/9.807]^{1/3} = 1.04$ m

The depth at the outfall is approximately $0.7y_c = (0.7)(1.04) = 0.73$ m. Critical depth occurs at about $4y_c = (4)(1.04) = 4$ m upstream from the brink. Computations for the water-surface profile are given below $[s = (nv/1.0R^{2/3})^2,\ x = E/(s - s_0)]$.

y_n, m	v, m/s	$v^2/2g$, m	E, m	v_m, m/s	R_m, m	s	Σx, m	x, m
1.04	3.205	0.524	1.546					
				3.06	0.630	0.00443	6	6
1.14	2.924	0.436	1.576					
				2.80	0.662	0.00346	33	39
1.24	2.688	0.368	1.608					
				2.58	0.694	0.00276	185	224
1.34	2.488	0.316	1.656					

14.287 A rectangular drainage channel is 15 ft wide and is to carry 500 cfs. The channel is lined with rubble masonry ($n = 0.017$) and has a bottom slope of 0.0015. It discharges into a stream which may reach a stage 10 ft above the channel bottom during floods. Calculate y_n, y_c, and the distance from the channel outlet to the point where normal depth would occur under this condition. Use one step.

▮ $Q = (A)(1.486/n)(R^{2/3})(s^{1/2})$ $\quad 500 = (15y_n)(1.486/0.017)[15y_n/(y_n + 15 + y_n)]^{2/3}(0.0015)^{1/2}$

$y_n = 4.81$ ft (by trial and error) $\quad y_c = (q^2/g)^{1/3} = [(\frac{500}{15})^2/32.2]^{1/3} = 3.26$ ft

$\Delta x = [(y_1 + V_1^2/2g) - (y_2 + V_2^2/2g)]/(s - s_0)$ $\quad V_1 = Q/A = 500/[(4.81)(15)] = 6.930$ ft/s

$V_2 = 500/[(10)(15)] = 3.333$ ft/s

$s = (nV/1.486R^{2/3})^2$ $\quad V_m = (6.930 + 3.333)/2 = 5.132$ ft/s

$(p_w)_1 = 4.81 + 15 + 4.81 = 24.62$ ft $\quad (p_w)_2 = 10 + 15 + 10 = 35.00$ ft

$R_m = [(4.81)(15)/24.62 + (10)(15)/35.00]/2 = 3.608$ ft

$s = \{(0.017)(5.132)/[(1.486)(3.608)^{2/3}]\}^2 = 0.0006229$

$\Delta x = \{4.81 + 6.930^2/[(2)(32.2)] - 10 - 3.333^2/[(2)(32.2)]\}/(0.0006229 - 0.0015) = 5264$ ft

14.288 Solve Prob. 14.287 using three steps.

▮ Using data from Prob. 14.287, $s = (nv/1.486R^{2/3})^2$, $x = \Delta E/(s - s_0)$.

y_n, ft	V, ft/s	$V^2/2g$, ft	E, ft	V_m, ft/s	R_m, ft	s	x, ft
10	3.33	0.17	10.17				
				3.75	4.09	0.000285	1560
8	4.17	0.27	8.27				
				4.86	3.60	0.000559	1900
6	5.55	0.48	6.48				
				6.28	3.12	0.000112	2420
4.8	7.00	0.76	5.56				
							5880

14.289 A trapezoidal channel with a bottom width of 10 ft and side slopes of 2 horizontal to 1 vertical has a horizontal curve with a radius of 100 ft without superelevation. If the discharge is 800 cfs and the water surface at the inside of the curve is 5 ft above the channel bottom, find the water-surface elevation at the outside of the curve. Assume the flow is subcritical.

▮ See Fig. 14-74. $y_2 - y_1 = v^2B/gr$, $x - 5 = \{800/[(x+5)(10+x) - 25 - x^2]\}^2(10 + 10 + 2x)/[(32.2)(100)]$, $x = 5.53$ ft (by trial and error).

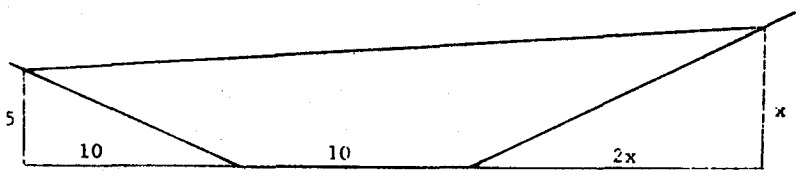


Fig. 14-74

14.290 A rectangular flume of planed timber ($n = 0.011$) is 1.5 m wide and carries 1.7 m³/s of water. The bed slope is 0.0005, and at section 1 the depth is 0.9 m. Find the distance to section 2, where the depth is 0.75 m. Is 2 upstream or downstream of 1?

▮ $\Delta x = (y_1 + v_1^2/2g - y_2 - v_2^2/2g)/(s - s_0)$. Assume the 0.75 m depth is downstream.

$$v_1 = Q/A_1 = 1.7/[(0.9)(1.5)] = 1.259 \text{ m/s}$$

$$v_2 = Q/A_2 = 1.7/[(0.75)(1.5)] = 1.511 \text{ m/s} \qquad s = (nv/1.0R^{2/3})^2$$

$$v_m = (1.259 + 1.511)/2 = 1.385 \text{ m/s} \qquad (p_w)_1 = 0.9 + 1.5 + 0.9 = 3.30 \text{ m}$$

$$(p_w)_2 = 0.75 + 1.5 + 0.75 = 3.00 \text{ m}$$

$$R_m = [(0.9)(1.5)/3.30 + (0.75)(1.5)/3.00]/2 = 0.3920 \text{ m}$$

$$s = \{(0.011)(1.385)/[(1.0)(0.3920)^{2/3}]\}^2 = 0.000809$$

$$\Delta x = \{0.9 + 1.259^2/[(2)(9.807)] - 0.75 - 1.511^2/[(2)(9.807)]\}/(0.000809 - 0.0005) = 370 \text{ m}$$

Since Δx is positive, the 0.75 m depth is downstream, as assumed.

14.291 The flume of Prob. 14.290 still carries 1.7 m³/s, but now the depth varies from 1.2 m at one section to 0.9 m at a section 200 m downstream. Find the new bed slope.

▮

$$\Delta x = (y_1 + v_1^2/2g - y_2 - v_2^2/2g)/(s - s_0) \qquad v_1 = Q/A_1 = 1.7/[(1.2)(1.5)] = 0.9444 \text{ m/s}$$

$$v_2 = Q/A_2 = 1.7/[(0.9)(1.5)] = 1.259 \text{ m/s} \qquad s = (nv/1.0R^{2/3})^2$$

$$v_m = (0.9444 + 1.259)/2 = 1.102 \text{ m/s}$$

$$(p_w)_1 = 1.2 + 1.5 + 1.2 = 3.90 \text{ m} \qquad (p_w)_2 = 0.9 + 1.5 + 0.9 = 3.30 \text{ m}$$

$$R_m = [(1.2)(1.5)/3.90 + (0.9)(1.5)/3.30]/2 = 0.4353 \text{ m}$$

$$s = \{(0.012)(1.102)/[(1.0)(0.4353)^{2/3}]\}^2 = 0.0005301$$

$$200 = \{1.2 + 0.9444^2/[(2)(9.807)] - 0.9 - 1.259^2/[(2)(9.807)]\}/(0.0005301 - s_0) \qquad s_0 = -0.000793$$

See Fig. 14-75.

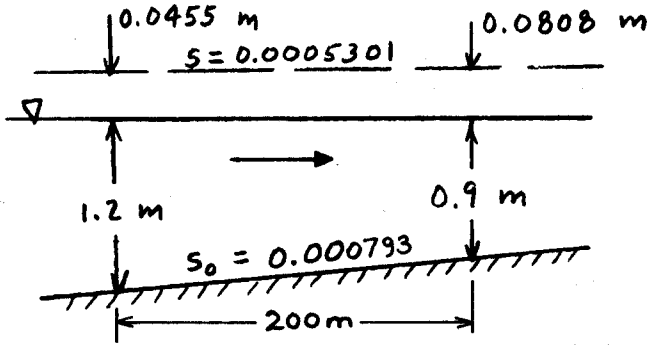


Fig. 14-75

14.292 The slope of the flume in Prob. 14.290 is now increased to 0.01. For the same flow as before, find the depth 35 m downstream from a section where the flow is 0.40 m deep.

▮ $y_c = (q^2/g)^{1/3} = [(1.7/1.5)^2/9.807]^{1/3} = 0.508$ m. Since $y < y_c$, the flow is supercritical.

$$\Delta x = (y_1 + v_1^2/2g - y_2 - v_2^2/2g)/(s - s_0) \qquad v_1 = Q/A_1 = 1.7/[(0.40)(1.5)] = 2.833 \text{ m/s}$$

$$v_2 = Q/A_2 = 1.7/(1.5y_2) = 1.133/y_2 \qquad s = (nv/1.0R^{2/3})^2$$

$$v_m = (2.833 + 1.133/y_2)/2 = 1.417 + 0.5665/y_2$$

$$(p_w)_1 = 0.4 + 1.5 + 0.4 = 2.30 \text{ m} \qquad (p_w)_2 = y_2 + 1.5 + y_2 = 1.5 + 2y_2$$

$$R_m = [(0.4)(1.5)/2.30 + 1.5y_2/(1.5 + 2y_2)]/2 = 0.2609 + 0.75y_2/(1.5 + 2y_2)$$

$$s = \left\{\frac{(0.011)(1.417 + 0.5665/y_2)}{1.0[0.2609 + 0.75y_2/(1.5 + 2y_2)]^{2/3}}\right\}^2$$

$$35 = \{0.40 + 2.833^2/[(2)(9.807)] - y_2 - (1.133/y_2)^2/[(2)(9.807)]\}/(s - 0.01)$$

$$y_2 = 0.31 \text{ m} \qquad \text{(by trial and error)}$$

14.293 A rectangular flume 12 in wide and 30 ft long yielded the following test results: with still water, $z_1 - z_2 = 0.010$ ft; with a measured flow of 0.20 cfs, $y_1 = 0.400$ ft, $y_2 = 0.405$ ft. Evaluate the roughness coefficient n.

▮ $$Q = (A)(1.486/n)(R^{2/3})(s^{1/2}) \qquad A_1 = (0.400)(\tfrac{12}{12}) = 0.400 \text{ ft}^2 \qquad A_2 = (0.405)(\tfrac{12}{12}) = 0.405 \text{ ft}^2$$

$$A_m = (0.400 + 0.405)/2 = 0.4025 \text{ ft}^2 \qquad (p_w)_1 = 0.40 + \tfrac{12}{12} + 0.40 = 1.800 \text{ ft}$$

$$(p_w)_2 = 0.405 + \tfrac{12}{12} + 0.405 = 1.810 \text{ ft}$$

$$R_m = (0.40/1.80 + 0.405/1.81)/2 = 0.2230 \text{ ft} \qquad x = (y_1 + v_1^2/2g - y_2 - v_2^2/2g)/(s - s_0)$$

$$v_1 = Q/A_1 = 0.20/0.40 = 0.500 \text{ ft/s} \qquad v_2 = Q/A_2 = 0.2/0.405 = 0.4938 \text{ ft/s}$$

$$30 = \{0.40 + 0.500^2/[(2)(32.2)] - 0.405 - 0.4938^2/[(2)(32.2)]\}/(s - 0.01/30) \qquad s = 0.0001699$$

$$0.20 = (0.4025)(1.486/n)(0.2230)^{2/3}(0.0001699)^{1/2} \qquad n = 0.0143$$

14.294 A rectangular flume 10 ft wide is built of planed timber ($n = 0.012$) on a bed slope of 0.2 ft per 1000 ft ending in a free overfall. If the measured depth at the fall is 1.82 ft, find **(*a*)** the rate of flow and **(*b*)** the distance upstream from the fall to where the depth is 4 ft.

▮ **(*a*)** $$y_{\text{brink}} = 0.72y_c \qquad 1.82 = 0.72y_c \qquad y_c = 2.528 \text{ ft}$$

$$y_c = (q^2/g)^{1/3} \qquad 2.528 = (q^2/32.2)^{1/3} \qquad q = 22.81 \text{ cfs/ft} \qquad Q = (22.81)(10) = 228 \text{ ft}^3/\text{s}$$

(*b*) Between y_{brink} and y_c the flow is rapidly varying. So Manning's equation is not valid there and cannot be used there to determine Q. $s = (nV/1.486R^{2/3})^2$, $\Delta x = (y_1 + V_1^2/2g - y_2 - V_2^2/2g)/(s - s_0)$.

y, ft	V, fps	$V^2/2g$, ft	E, ft	A, ft^2	P, ft	R, ft	R_m, ft	V_m, fps	s	Δx, ft
4.0	5.70	0.505	4.505	40	18.0	2.22				
							2.12	6.20	0.000915	568
3.4	6.71	0.698	4.095	34	16.8	2.02				
							1.949	7.15	0.001363	173
3.0	7.60	0.897	3.897	30	16.0	1.875				
							1.814	8.02	0.001888	53
2.7	8.44	1.108	3.808	27	15.4	1.753				
							1.717	8.73	0.00240	8
2.53	9.01	1.261	3.791	25.3	15.1	1.680				
										$4y_c = 10$
y_b										$\Sigma(\Delta x) = 812$

14.295 For a wide rectangular channel dredged in earth ($n = 0.030$), of slope 0.002273 and carrying 100 (ft^3/s)/ft, find the water depth 1.5 mi upstream of a location (2) where the depth is 30 ft.

▮
$$\Delta x = (y_1 + v_1^2/2g - y_2 - v_2^2/2g)/(s - s_0) \qquad v_1 = q/y_1 = 100/y_1$$
$$v_2 = q/y_2 = 100/30 = 3.333 \text{ ft/s}$$
$$v_m = (100/y_1 + 3.333)/2 = 50.0/y_1 + 1.667 \qquad s = (nv/1.486R^{2/3})^2$$
$$R_m = (y_1 + 30)/2 = y_1/2 + 15.00$$
$$s = \{(0.030)(50.0/y_1 + 1.667)/[(1.486)(y_1/2 + 15.00)^{2/3}]\}^2$$
$$(1.5)(5280) = \{y_1 + (100/y_1)^2/[(2)(32.2)] - 30 - 3.333^2/[(2)(32.2)]\}/(s - 0.002273)$$
$$y_1 = 11.9 \text{ ft} \qquad \text{(by trial and error)}$$

14.296 The slope of a stream of rectangular cross section is 0.00022, the width is 150 ft, and the value of the Chezy C is 80 ft$^{1/2}$/s. Find the depth for a uniform flow of 128.45 cfs per foot of width of the stream.

▮
$$v_0 = q/y_0 = 88.55/y_0 \qquad v_0 = C\sqrt{Rs} = (80.0)\{\sqrt{[150y_0/(y_0 + 150 + y_0)](0.00022)}\}$$
$$128.45/y_0 = 80\{\sqrt{[150y_0/(y_0 + 150 + y_0)](0.00022)}\} \qquad y_0 = 25.0 \text{ ft} \qquad \text{(by trial and error)}$$

14.297 If the stream of Prob. 14.296 is dammed, find the distance between a section where the increase in depth is 5 ft and one where the increase is 1 ft. Use reaches with 1-ft depth increments.

▮
$$v = C\sqrt{Rs} \qquad \Delta x = (y_1 + v_1^2/2g - y_2 - v_2^2/2g)/(s - s_0)$$

y, ft	A (150y), ft^2	p, ft	R (A/p), ft	v (q/y), fps	$v^2/2g$, ft	E, ft	ΔE ($E_1 - E_2$), ft	v_m, fps	R_m, ft	s	Δx, ft
30	4500	210	21.43	4.28	0.284	30.284					
							−0.979	4.36	21.17	0.0001403	12 284
29	4350	208	20.91	4.43	0.305	29.305					
							−0.978	4.51	20.65	0.0001539	14 796
28	4200	206	20.39	4.59	0.327	28.327					
							−0.975	4.68	20.12	0.0001701	19 539
27	4050	204	19.85	4.76	0.352	27.352					
							−0.973	4.85	19.58	0.0001877	30 124
26	3900	202	19.31	4.94	0.379	26.379					
											$\Sigma(\Delta x) = 76\,743$

14.298 A portion of an outfall sewer is approximately a circular conduit 5 ft in diameter and with a slope of 1 ft in 1100 ft. It is of brick, for which $n = 0.013$. What would be its maximum capacity for uniform flow?

▮
$$Q = (A)(1.486/n)(R^{2/3})(s^{1/2}) \qquad Q_{\text{full}} = [(\pi)(5)^2/4](1.486/0.013)(\tfrac{5}{4})^{2/3}(\tfrac{1}{1100})^{1/2} = 78.53 \text{ ft}^3/\text{s}$$

From Fig. A-18, $(Q_{\max}/Q_{\text{full}}) = 1.06$. $Q_{\max} = (78.53)(1.06) = 83.2$ ft^3/s.

14.299 If the outfall sewer in Prob. 14.298 discharges 120 cfs with a depth at the end of 2.90 ft, how far back from the end must it become a pressure conduit? Proceeding from the mouth upstream, find by tabular solution the length of sewer that is not flowing full. Use three reaches with equal depth increments.

▮ $$s = (nV/1.486R^{2/3})^2 \qquad \Delta x = (y_1 + v_1^2/2g - y_2 - v_2^2/2g)/(s - s_0)$$

y, ft	θ, deg	A, ft^2	P, ft	R (A/P), ft	V (Q/A), fps	$V^2/2g$, ft	E, ft	ΔE $(E_1 - E_2)$, ft	V_m, fps	R_m, ft	S	Δx, ft
2.9	99.2	11.81	8.66	1.364	10.16	1.603	4.503					
								0.073	9.05	1.429	0.00387	24.7
3.6	116.1	15.13	10.13	1.494	7.93	0.976	4.576					
								0.417	7.31	1.504	0.00236	287
4.3	136.1	17.96	11.87	1.513	6.68	0.693	4.993					
								0.587	6.40	1.382	0.00202	527
5.0	180	19.63	15.71	1.250	6.11	0.580	5.580					
												$\Sigma(\Delta x) = 839$

The sewer must become a pressure conduit 839 ft back from the end.

14.300 For the channel of Prob. 14.131, find the separation of a section where the depth is 2.0 ft and one where the depth is 3.0 ft.

▮ Assume the 2.0 ft depth is upstream. Then $y_1 = 2.0$ ft and $y_2 = 3.0$ ft. From Prob. 14.131, $Q = 100$ cfs, $s = 0.02$, and $n = 0.015$.

$$\Delta x = (y_1 + v_1^2/2g - y_2 - v_2^2/2g)/(s - s_0) \qquad v_1 = Q/A_1 = 100/4.90 = 20.41 \text{ ft/s}$$

$$v_2 = Q/A_2 = 100/10.48 = 9.542 \text{ ft/s}$$

$$s = (nv/1.486R^{2/3})^2 \qquad v_m = (20.41 + 9.542)/2 = 14.98 \text{ ft/s} \qquad R = A/p_w$$

$$R_1 = 4.90/6.12 = 0.801 \text{ ft}$$

$$R_2 = 10.48/8.96 = 1.170 \text{ ft} \qquad R_m = (0.801 + 1.170)/2 = 1.9855 \text{ ft}$$

$$s = \{(0.015)(14.98)/[(1.486)(0.9855)^{2/3}]\}^2 = 0.02331$$

$$\Delta x = \{2.0 + 20.41^2/[(2)(32.2)] - 3.0 - 9.542^2/[(2)(32.2)]\}/(0.02331 - 0.02) = 1225 \text{ ft}$$

Since Δx is positive, the assumption that the 2.0 ft depth was upstream is correct.

14.301 Sketch the water-surface profile in a long rectangular channel ($n = 0.014$), if the channel is 10 ft wide; the flow rate is 340 cfs; and there is an abrupt change in slope from 0.0016 to 0.0150.

▮ See Fig. 14-76. $y_c = (q^2/g)^{1/3} = [(340/10)^2/32.2]^{1/3} = 3.30$ ft, $Q = (A)(1.486/n)(R^{2/3})(s^{1/2})$. In upstream segment, $340 = (10y_n)(1.486/0.014)[10y_n/(y_n + 10 + y_n)]^{2/3}(0.0016)^{1/2}$, $y_n = 4.50$ ft (by trial and error). In downstream segment, $340 = (10y_n)(1.486/0.014)[10y_n/(y_n + 10 + y_n)]^{2/3}(0.0150)^{1/2}$, $y_n = 2.04$ ft (by trial and error). Thus, the flow is subcritical before the break and supercritical after the break.

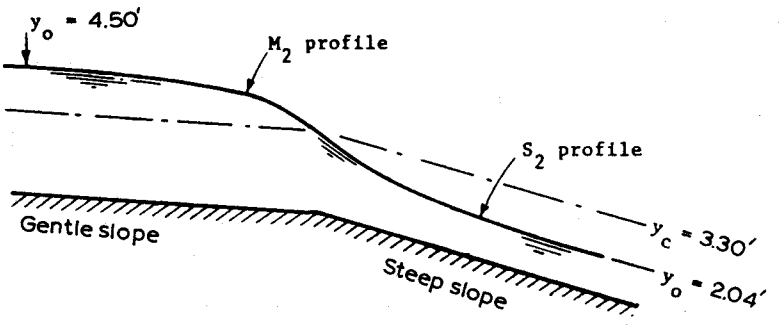


Fig. 14-76

14.302 Repeat Prob. 14.301 for the case where the flow rate is 130 cfs.

▮ $y_c = (q^2/g)^{1/3} = [(130/10)^2/32.2]^{1/3} = 1.74$ ft, $Q = (A)(1.486/n)(R^{2/3})(s^{1/2})$. In upstream segment, $130 = (10y_n)(1.486/0.014)[10y_n/(y_n + 10 + y_n)]^{2/3}(0.0016)^{1/2}$, $y_n = 2.28$ ft (by trial and error). In downstream segment,

$130 = (10y_n)(1.486/0.014)[10y_n/(y_n + 10 + y_n)]^{2/3}(0.0150)^{1/2}$, $y_n = 1.08$ ft (by trial and error). Thus, the flow is subcritical before the break and supercritical after the break, so that Fig. 14-76 still applies.

14.303 Repeat Prob. 14.302 if the slope abruptly changes from 0.0016 to 0.0006. Estimate the distance upstream from the break to the point where normal depth occurs.

▮ $y_c = 1.74$ ft (from Prob. 14.302). In upstream segment, $y_n = 2.28$ ft (from Prob. 14.302). In downstream segment, $130 = (10y_n)(1.486/0.014)[10y_n/(y_n + 10 + y_n)]^{2/3}(0.0006)^{1/2}$, $y_n = 3.20$ ft (by trial and error). Thus, the flow is subcritical before and after the break. A water surface on a gentle slope cannot deviate from y_n in the upstream direction. Therefore, the depth at the break is 3.20 ft. See Fig. 14-77.

$$\Delta x = (y_1 + v_1^2/2g - y_2 - v_2^2/2g)/(s - s_0) \qquad v_1 = Q/A_1 = 130/[(2.28)(10)] = 5.702 \text{ ft/s}$$

$$v_2 = Q/A_2 = 130/[(3.20)(10)] = 4.063 \text{ ft/s} \qquad s = (nv/1.486R^{2/3})^2$$

$$v_m = (5.702 + 4.063)/2 = 4.883 \text{ ft/s}$$

$$(p_w)_1 = 2.28 + 10 + 2.28 = 14.56 \text{ ft} \qquad (p_w)_2 = 3.20 + 10 + 3.20 = 16.40 \text{ ft/s}$$

$$R_m = [(2.28)(10)/14.56 + (3.20)(10)/16.40]/2 = 1.759 \text{ ft}$$

$$s = \{(0.014)(4.883)/[(1.486)(1.759)^{2/3}]\}^2 = 0.0009967$$

$$\Delta x = \{2.28 + 5.702^2/[(2)(32.2)] - 3.20 - 4.063^2/[(2)(32.2)]\}/(0.0009967 - 0.0016) = 1113 \text{ ft}$$

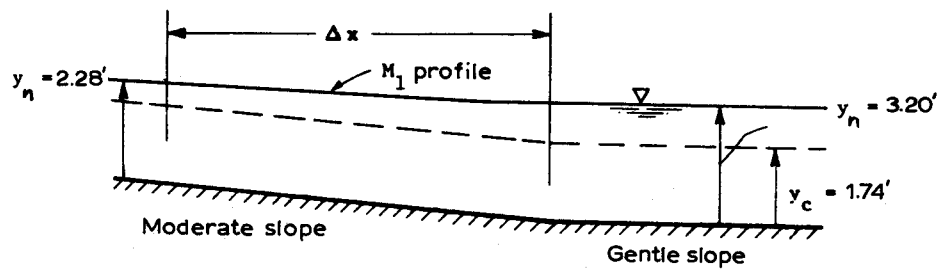


Fig. 14-77

14.304 In a 6-ft-wide rectangular channel ($s = 0.002$, $n = 0.014$), water flows at 240 cfs. A temporary dam increases the depth to 9.1 ft. Examine the water-surface profile upstream from the dam.

▮ $y_c = (q^2/g)^{1/3} = [(240/6)^2/32.2]^{1/3} = 3.68$ ft, $Q = (A)(1.486/n)(R^{2/3})(s^{1/2})$. In upstream segment, $240 = (6y_n)(1.486/0.014)[6y_n/(y_n + 6 + y_n)]^{2/3}(0.002)^{1/2}$, $y_n = 5.44$ ft (by trial and error). Since $y_n > y_c$, the channel slope is gentle. Also, since $9.1 \text{ ft} = y > y_n > y_c$, upstream of the dam is an M_1 profile (see Fig. A-19) with the depth gradually decreasing to the normal depth (5.44 ft). Critical depth (3.68 ft) occurs on the dam.

14.305 Solve Prob. 14.304 if the channel slope is 0.0004.

▮ $y_c = 3.68$ ft (from Prob. 14.304), $Q = (A)(1.486/n)(R^{2/3})(s^{1/2})$. In upstream segment, $240 = (6y_n)(1.486/0.014)[6y_n/(y_n + 6 + y_n)]^{2/3}(0.0004)^{1/2}$, $y_n = 10.7$ ft (by trial and error). Since $y_n > y_c$, the channel slope is gentle. But insertion of a dam cannot lower the water surface below y_n to 9.1 ft, so this situation is impossible.

14.306 Solve Prob. 14.304 if the channel slope is 0.0010.

▮ $y_c = 3.68$ ft (from Prob. 14.304), $Q = (A)(1.486/n)(R^{2/3})(s^{1/2})$. In upstream segment, $240 = (6y_n)(1.486/0.014)[6y_n/(y_n + 6 + y_n)]^{2/3}(0.0010)^{1/2}$, $y_n = 7.22$ ft (by trial and error). The conclusions follow Prob. 14.304.

14.307 Solve Prob. 14.304 if the channel slope is 0.006.

▮ $y_c = 3.68$ ft (from Prob. 14.304), $Q = (A)(1.486/n)(R^{2/3})(s^{1/2})$. In upstream segment, $240 = (6y_n)(1.486/0.014)[6y_n/(y_n + 6 + y_n)]^{2/3}(0.006)^{1/2}$, $y_n = 3.53$ ft (by trial and error). Since $y_n < y_c$, the channel slope is steep. Also, since $9.1 \text{ ft} = y > y_c > y_n$, upstream of the dam is an S_1 profile (see Fig. A-19), preceded by a hydraulic jump. Upstream of the jump is straight supercritical uniform flow with depth of 3.53 ft.

14.308 A rectangular channel changes in width from 3 ft to 5 ft, as shown in Fig. 14-78. Measurements indicate that $y_1 = 2.00$ ft and $Q = 60$ cfs. Determine the depth y_2, neglecting head loss.

▮ $E_1 = E_2 + h_L \qquad E = y + v^2/2g \qquad v_1 = Q/A_1 = 60/[(3)(2.00)] = 10.00$ ft/s

$$E_1 = 2.0 + 10.00^2/[(2)(32.2)] = 3.553 \text{ ft} \qquad v_2 = Q/A_2 = 60/(5y_2) = 12.0/y_2$$

$$E_2 = y_2 + (12.0/y_2)^2/[(2)(32.2)] = y_2 + 2.236/y_2^2$$

$$h_L = 0 \qquad 3.553 = y_2 + 2.236/y_2^2 + 0 \qquad y_2 = 3.36 \text{ ft} \qquad \text{(by trial and error)}$$

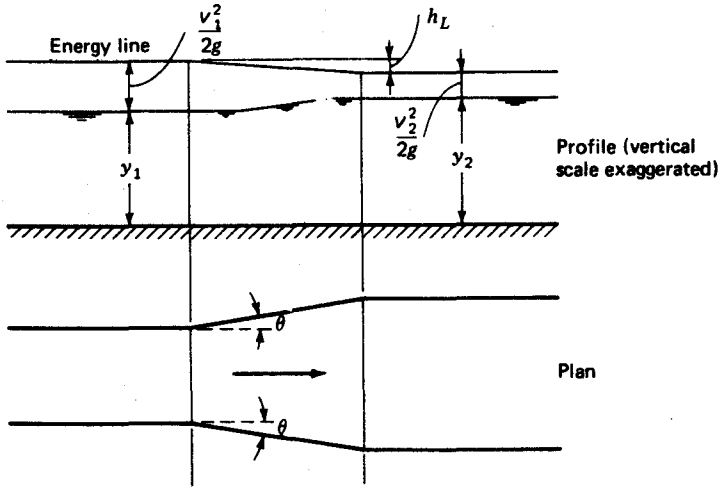


Fig. 14-78

14.309 Solve Prob. 14.308, assuming $h_L = (0.3)(v_1^2/2g - v_2^2/2g)$.

▮ Using data from Prob. 14.308, $h_L = 0.3\{10.00^2/[(2)(32.2)] - (12/y_2)^2/[(2)(32.2)]\} = 0.4658 - 0.6708/y_2^2$, $3.553 = y_2 + 2.236/y_2^2 + 0.4658 - 0.6708/y_2^2$, $y_2 = 2.90$ ft (by trial and error).

14.310 Consider a rectangular flume 4.5 m wide, built of unplaned planks ($n = 0.014$), leading from a reservoir in which the water surface is maintained constant at a height of 1.8 m above the bed of the flume at entrance (see Fig. 14-79). The flume is on a slope of 0.001. The depth 300 m downstream from the head end of the flume is 1.20 m. Assuming an entrance loss of $0.2V_1^2/2g$, find the flow rate for the given conditions.

▮ For a first approximate answer we shall consider the entire flume as one reach. The equations to be satisfied are

Energy at entrance: $$y_1 + (1.2V_1^2/2g) = 1.80 \qquad (1)$$

Energy equation for the entire reach:

$$y_1 + (V_1^2/2g) = 1.20 + (V_2^2/2g) + (S - 0.001)L \qquad (2)$$

where S is given by $$S = (nV_m/R_m^{2/3})^2 \qquad (3)$$

The procedure is to make successive trials of the upstream depth y_1. This determines corresponding values of V_1, q, V_2, V_m, R_m, and S. The trials are repeated until the value of Δx from Eq. (2) is close to 300 m. The solution is conveniently set in tabular form as follows:

Trial y_1, m	V_1, Eq. (1), m/s	$q = y_1V_1$, m³/s/m	$V_2 = q/1.20$, m/s	V_m, m/s	R_{h_1}, m	R_{h_2}, m	R_{h_m}, m	S, Eq. (3)	Δx, Eq. (2), m
1.50	2.22	3.33	2.78	2.50	0.90	0.78	0.89	0.00143	358
1.48	2.29	3.39	2.82	2.56	0.89	0.78	0.835	0.00163	226

Thus $y_1 \approx 1.49$ m and the flow rate $Q = qB \approx 3.36 \times 4.5 \approx 15.1$ m³/s. The accuracy of the result, of course, depends on one's ability to select the correct value for Manning's n. If n was assumed to be 0.015, for example,

rather than 0.014, the result would have been quite different. Also, a more accurate result can be obtained by dividing the flume into reaches in which the depth change is about 10 percent of the depth.

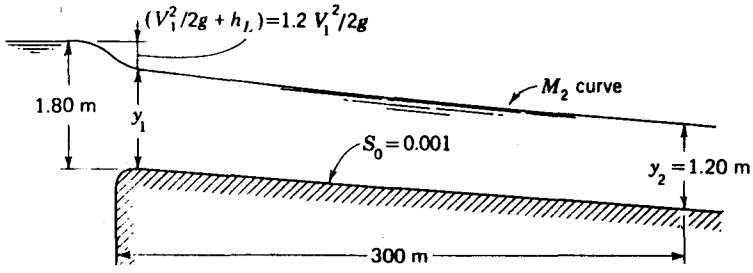


Fig. 14-79

14.311 A rectangular flume of planed timber ($n = 0.012$) 20 ft wide, 1000 ft long, with horizontal bed leads from a reservoir in which the still-water surface is 10 ft above the flume bed. Assume that the depth of the downstream end of the flume is fixed at 8 ft by some control section downstream. Allowing an entrance loss of 0.2 times the velocity head, find the flow rate in the flume using one reach.

▮ Energy at entrance:

$$y_1 + 1.2V_1^2/2g = 10 \tag{1}$$

Energy equation for the entire reach:

$$y_1 + V_1^2/2g = 8 + V_2^2/2g + (S - 0)(L) \tag{2}$$

where S is given by

$$S = (nV_m/1.486R_m^{2/3})^2 \tag{3}$$

This problem involves a trial-and-error solution exactly like that of Prob. 14.310. After successive trials (not shown), the solution is reached with $y_1 = 8.82$ ft and $Q = qB = (70.25)(20) = 1405\ \text{ft}^3/\text{s}$.

14.312 Find the flow rate in the flume of Prob. 14.311 if it ends in a free fall, all other conditions remaining the same. The critical depth may be supposed to occur at about $6y_c$ back from the fall. Thus the length of the reach is $1000 - 6y_c$, and $y_2 = y_c$.

▮ Take the flume in one reach with y_1 and V_1 at the upstream end and y_c and V_c at the downstream end at a distance $L = (1000 - 6y_c)$ ft. Then, energy at entrance:

$$y_1 + 1.2V_1^2/2g = 10 \tag{1}$$

Energy equation for the entire reach:

$$y_1 + V_1^2/2g = y_c + V_c^2/2g + (S - 0)(L) \tag{2}$$

where S is given by

$$S = (nV_m/1.486R_m^{2/3})^2 \tag{3}$$

First trial: Assume $y_1 = 7$ ft.

$$7 + 1.2V_1^2/[(2)(32.2)] = 10 \qquad V_1 = 12.69\ \text{ft/s} \qquad q = V_1 y_1 = (12.69)(7) = 88.83\ \text{cfs/ft}$$

$$y_c = (q^2/g)^{1/3} = (88.83^2/32.2)^{1/3} = 6.26\ \text{ft}$$

$$V_c = q/y_c = 88.83/6.26 = 14.19\ \text{ft/s} \qquad V_m = (12.69 + 14.19)/2 = 13.44\ \text{ft/s}$$

$$R_m = [(20)(7)/(7 + 20 + 7) + (20)(6.26)/(6.26 + 20 + 6.26)]/2 = 3.984\ \text{ft}$$

$$S = \{(0.012)(13.44)/[(1.486)(3.984)^{2/3}]\}^2 = 0.001865$$

$$7 + 12.69^2/[(2)(32.2)] = 6.26 + 14.19^2/[(2)(32.2)] + 0.001865L \qquad L = 61.1\ \text{ft}$$

But $L = 1000 - (6)(6.26) = 962$ ft. Since the computed value of L (61.1 ft) is not equal to the actual length of approximately 962 ft, the assumed value of y_1 of 7 ft is incorrect. As a second trial, y_1 should be increased. After successive trials (not shown), a value of $y_1 = 8.46$ ft is found to be appropriate. Then, $8.46 + 1.2V_1^2/[(2)(32.2)] = 10$, $V_1 = 9.091$ ft/s, $q = (9.091)(8.46) = 76.91$ cfs/ft, $Q = (76.91)(20) = 1538\ \text{ft}^3/\text{s}$.

14.313 An 8-ft-wide rectangular channel has bottom slope $s_0 = 0.0015$ and roughness $n = 0.013$; it carries 250 ft³/s of water. Compute the separation between sections where depths are 4.00 ft and 4.20 ft.

▮ $$\Delta x = (y_1 + v_1^2/2g - y_2 - v_2^2/2g)/(s - s_0) \qquad s = (nv_{avg}/1.486R_{avg}^{2/3})^2$$

The calculations in the following table give the separation as 512 ft.

y, ft	A, ft^2	p $(8+2y)$, ft	R_h, ft	v, fps	$v^2/2g$, ft	$y+(v^2/2g)$	Numerator $\Delta[y+(v^2/2g)]$	v_{avg}, fps	R_{avg}, ft	s	Denominator $s-s_0$	Δx, ft
4.00	32.00	16.00	2.00	7.81	0.947	4.947						
							0.055	7.72	2.013	0.00179	0.00029	190
4.10	32.80	16.20	2.025	7.62	0.902	5.002						
							0.058	7.53	2.037	0.00168	0.00018	322
4.20	33.60	16.40	2.049	7.44	0.860	5.060						
												$\Sigma(\Delta x) = 512$

14.314 Assume the channel of Prob. 14.145 has a bottom slope of 0.0048. If $y_0 = 3$ ft at $x = 0$, how far along the channel $x = L$ does it take the depth to rise to $y_L = 4$ ft (see Fig. 14-80). Use increments of $y = 0.2$ ft and Manning's formula. Is the 4-ft-depth position upstream or downstream in Fig. 14-80?

▮ $y_c = 4.27$ ft (from Prob. 14.145). Since $[y = 3\text{ ft}] < [y_c = 4.27\text{ ft}]$, the flow is supercritical; and the given channel slope (0.0048) is greater than s_c of 0.00435 (from Prob. 14.145). Therefore, we must be on an S profile. $Q = (A)(1.486/n)(R^{2/3})(s^{1/2})$. Since the channel is "wide", use $R = y_n$: $50 = [(1)(y_n)](1.486/0.022)(y_n)^{2/3}(0.0048)^{1/2}$, $y_n = 4.14$ ft. Thus y_0 and y_L are less than y_n, which is less than y_c and we *must* be on an S_3 curve. For the numerical solution, we tabulate $y = 3.0$ to 4.0 in intervals of 0.2, computing six values of $V = q/y$, $E = y + (V^2/2g)$, and $S = n^2V^2/2.208y^{4/3}$, from which S_{av} and Δx follow. The slope $S_0 = 0.0048$ is constant.

y, ft	$V = 50/y$	$E = y + (V^2/2g)$	S	S_{av}	Δx	$x = \Sigma\,\Delta x$
3.0	16.67	7.313	0.01407			0.0
				0.01271	40.7	
3.2	15.62	6.991	0.01135			40.7
				0.01031	42.3	
3.4	14.71	6.758	0.00927			83.0
				0.00847	44.4	
3.6	13.89	6.595	0.00766			127.4
				0.00703	48.0	
3.8	13.16	6.488	0.00640			175.4
				0.00590	56.4	
4.0	12.50	6.426	0.00539			$L = 231.8$ ft

For this depth increment of 0.2 ft, gradually-varied-flow theory predicts that a length of about 232 ft is required for the depth to rise from 3 to 4 ft in this supercritical flow. Using 10 increments by reducing Δy to 0.1 ft would give an estimate of $L = 235.2$ ft (calculations not shown). It should be clear that the $y = 4$ ft position is downstream.

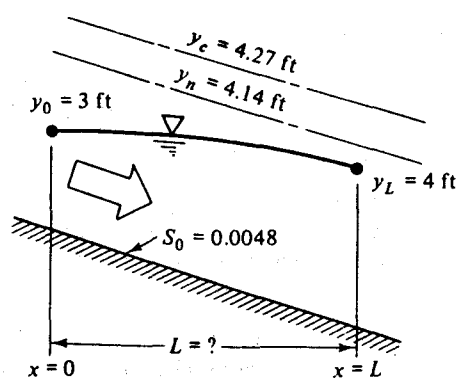


Fig. 14-80

14.315 If a rectangular channel 10 ft wide, with $n = 0.014$ and $s_0 = 0.0009$, carries $Q = 300$ cfs, is the slope steep or mild?

▮ $Q = (A)(1.486/n)(R^{2/3})(s^{1/2})$ $\quad 300 = (10y_n)(1.486/0.014)[10y_n/(y_n + 10 + y_n)]^{2/3}(0.0009)^{1/2}$

$y_n = 5.09$ ft (by trial and error) $\quad y_c = (q^2/g)^{1/3} = [(\frac{300}{10})^2/32.2]^{1/3} = 3.03$ ft

Since $y_c < y_n$, the slope is mild.

14.316 Repeat Prob. 14.315 if the slope is 0.006.

▮ $Q = (A)(1.486/n)(R^{2/3})(s^{1/2})$ $\quad 300 = (10y_n)(1.486/0.014)[10y_n/(y_n + 10 + y_n)]^{2/3}(0.006)^{1/2}$

$y_n = 2.57$ ft (by trial and error) $\quad y_c = (q^2/g)^{1/3} = [(\frac{300}{10})^2/32.2]^{1/3} = 3.03$ ft

Since $y_c > y_n$, the slope is steep.

14.317 Water flows in a wide clean-earth ($n = 0.020$) channel at $s_0 = 0.006$ and $q = 10$ (m³/s)/m. Give the ranges of depths corresponding to flow on a type 1, 2, or 3 curve.

▮ $Q/B = (A/B)(1.0/n)(R^{2/3})(s^{1/2})$ $\quad 10 = (y_n)(1.0/0.020)(y_n)^{2/3}(0.006)^{1/2}$

$y_n = 1.77$ m $\quad y_c = (q^2/g)^{1/3} = (10^2/9.807)^{1/3} = 2.17$ m

Since $y_c > y_n$, the slope is steep. Flow will be S_1 for $y > 2.17$ m (see Fig. A-19); S_2 for $1.77\text{ m} < y < 2.17$ m; and S_3 for $y < 1.77$ m.

14.318 Repeat Prob. 14.317 if the channel slope is 0.0014.

▮ $Q/B = (A/B)(1.0/n)(R^{2/3})(s^{1/2})$ $\quad 10 = (y_n)(1.0/0.020)(y_n)^{2/3}(0.0014)^{1/2}$

$y_n = 2.73$ m $\quad y_c = (q^2/g)^{1/3} = (10^2/9.807)^{1/3} = 2.17$ m

Since $y_c < y_n$, the slope is mild. Flow will be M_1 for $y > 2.73$ m (see Fig. A-19); M_2 for $2.17\text{ m} < y < 2.73$ m; and M_3 for $y < 2.17$ m.

14.319 Water flows at 130 cfs in an isosceles right-triangular sluice with $n = 0.015$ and $s_0 = 0.010$. For what ranges of depths will the flow be a type 1, 2, or 3 curve?

▮ $Q = (A)(1.486/n)(R^{2/3})(s^{1/2})$ $\quad A = y_n y_n = y_n^2$ $\quad p_w = 2y_n/(\cos 45°)$

$R = A/p_w = y_n^2/[2y_n/(\cos 45°)] = 0.3536y_n$

$130 = (y_n^2)(1.486/0.015)(0.3536y_n)^{2/3}(0.010)^{1/2}$ $\quad y_n = 3.41$ ft

$A_c = (bQ^2/g)^{1/3}$ $\quad y_c^2 = [(2y_c)(130)^2/32.2]^{1/3}$ $\quad y_c = 4.02$ ft

Since $y_c > y_n$, the slope is steep. Flow will be S_1 for $y > 4.02$ ft (see Fig. A-19); S_2 for $3.41\text{ ft} < y < 4.02$ ft; and S_3 for $y < 3.41$ ft.

14.320 A riveted-steel ($n = 0.014$) duct is 2 m in diameter and laid on a slope of 0.000758. The duct is half-full of water flowing at 2.5 m³/s. Specify the type of the flow curve.

▮ $Q = (A)(1.0/n)(R^{2/3})(s^{1/2})$, $Q_{full} = [(\pi)(2)^2/4](1.0/0.014)(\frac{2}{4})^{2/3}(0.000758)^{1/2} = 3.89$ m³/s, $Q/Q_{full} = 2.5/3.89 = 0.64$. From Fig. A-18, $D/D_{full} = 58$ percent. $y_n = (0.58)(2) = 1.16$ m, $A_c = (bQ^2/g)^{1/3}$. Try $y_c = 0.70$ m: $y_c/y_{full} = 0.70/2 = 0.35$. From Fig. A-18, $A_c/A_{full} = 30\%$.

$A_c = 0.30[(\pi)(2)^2/4] = 0.9425$ m² $\quad b = 2[\sqrt{1^2 - (1 - y_c)^2}]$ (see Fig. 14-81)

$b = 2[\sqrt{1^2 - (1 - 0.70)^2}] = 1.908$ m $\quad 0.9425 = (1.908Q^2/9.807)^{1/3}$ $\quad Q = 2.07$ m³/s

Since the computed value of Q (2.07 m³/s) is not equal to the given value (2.5 m³/s), try $y_c = 0.75$ m: $y_c/y_{full} = 0.75/2 = 0.375$. From Fig. A-18, $A_c/A_{full} = 34\%$.

$A_c = 0.34[(\pi)(2)^2/4] = 1.068$ m² $\quad b = 2[\sqrt{1^2 - (1 - 0.75)^2}] = 1.936$ m

$1.068 = (1.936Q^2/9.807)^{1/3}$ $\quad Q = 2.48$ m³/s

This value of Q is close enough to the given value of 2.5 m^3/s so that y_c can be taken as 0.75 m. Since $y_c = 0.75\ m < 1.16\ m = y_n$, the slope is mild, and an M_2 curve (Fig. A-19) is appropriate.

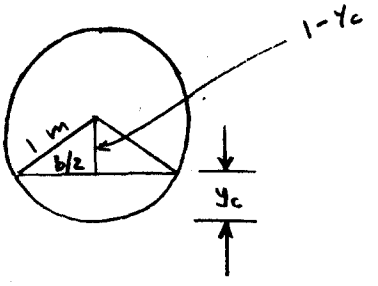


Fig. 14-81

14.321 Consider the gradual change from the profile beginning at point a in Fig. 14-82a, on a mild slope to a mild but steeper slope downstream. Sketch and label the curve $y(x)$ expected.

▮ See Fig. 14-82b.

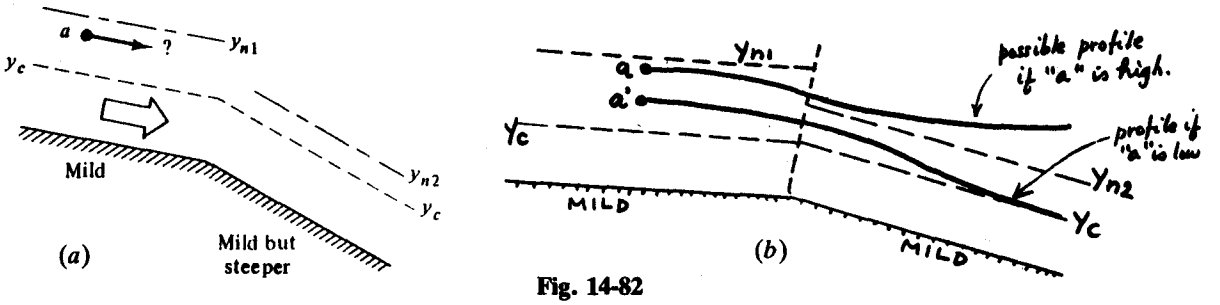


Fig. 14-82

14.322 Consider the wide-channel flow in Fig. 14-83a, which changes from a mild to a steep slope. Beginning at point a, sketch the water-surface profile $y(x)$ which is expected for gradually varied flow.

▮ See Fig. 14-83b.

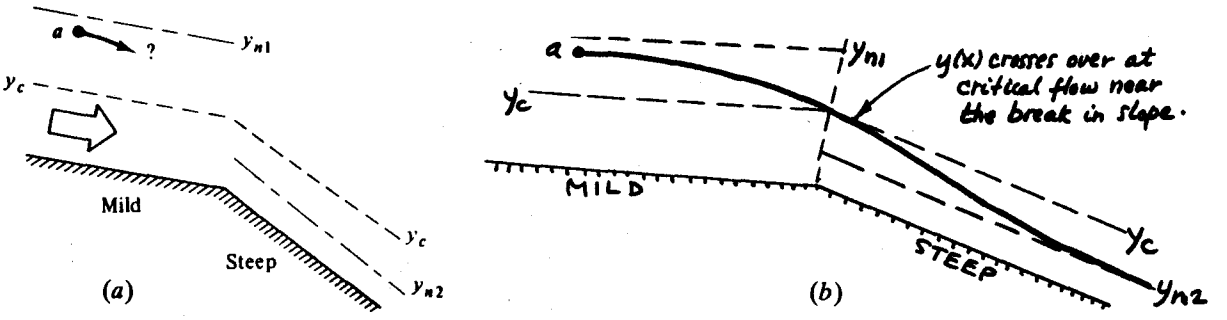


Fig. 14-83

14.323 In Fig. 14-84a the channel slope changes from steep to less steep. Beginning at point a, sketch and label the expected surface curve $y(x)$ from gradually-varied-flow theory.

▮ See Fig. 14-84b.

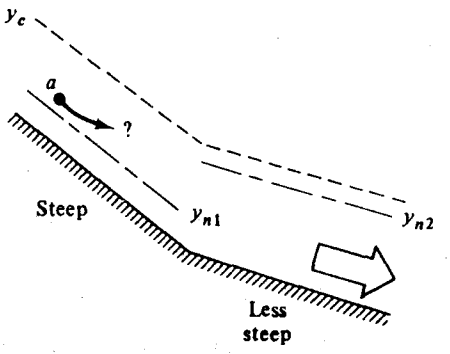


Fig. 14-84a

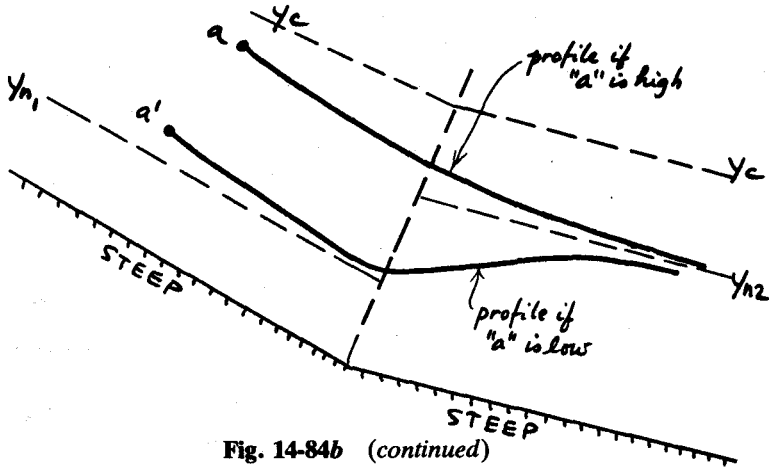


Fig. 14-84b *(continued)*

14.324 Repeat Prob. 14.314 to find the length L using only two depth increments of 0.5 ft each. Is the accuracy sufficient?

▮ Using data from Prob. 14.314,

y	V	E	S	S_{av}	Δx
3.0	16.67	7.313	0.01407		
				0.01125	100
3.5	14.29	6.669	0.00842		
				0.00691	115
4.0	12.50	6.426	0.00539		$L = 215$ ft

Using only two depth increments gives $(231.8 - 215)/231.8 = 0.07$, or 7 percent smaller length than that using five increments.

14.325 A wide-channel flow of $q = 4$ (m³/s)/m, with $n = 0.016$, climbs an adverse slope $s_0 = -0.0009$. Using distance increments of 125 mm, approximate the distance required for the depth to change from 2.5 m to 2.0 m.

▮ $y_c = (q^2/g)^{1/3} = (4^2/9.807)^{1/3} = 1.18$ m. Hence we have an A_2 profile (see Fig. A-19).

y	$v = 4/y$	E	$S = n^2v^2/y^{4/3}$	s_{avg}	$\Delta x = \Delta E/(s_0 - s_{avg})$
2.5	1.600	2.6305	0.0001931		
				0.0002111	99.8
2.375	1.684	2.5196	0.0002291		
				0.0002518	94.2
2.25	1.778	2.4111	0.0002745		
				0.0003032	87.7
2.125	1.882	2.3056	0.0003319		
				0.0003692	80.1
2.0	2.000	2.2039	0.0004064		$\Sigma(\Delta x) = L \approx 362$ m

14.326 Figure 14-85 illustrates "free overfall" or "dropdown," where a channel flow accelerates down a slope and falls freely over a sharp edge. As shown, the flow becomes critical just before the overfall. Between y_c and the edge the flow fluctuates rapidly and does not obey gradually-varied-flow theory. If the flow rate is $q = 1.1$ (m³/s)/m and the surface is unfinished concrete ($n = 0.014$), estimate the water depth 300 m upstream.

▮ $$q = (1.0/n)(y)(y^{2/3})(s^{1/2}) \qquad 1.1 = (1.0/0.014)(y_n)(y_n)^{2/3}(0.001047°)^{1/2} \qquad y_n = 0.64 \text{ m}$$

$$y_c = (q^2/g)^{1/3} = (1.1^2/9.807)^{1/3} = 0.50 \text{ m}$$

Hence we have an M_2 profile (see Fig. A-19). Use $\Delta y = 0.02$ m.

y	$v = 1.1/y$	E	s	s_{av}	Δx
0.50	2.200	0.74669	0.0023904		
				0.0022439	1.2
0.52	2.115	0.74808	0.0020975		
				0.0019735	3.7
0.54	2.037	0.75149	0.0018495		
				0.0017439	7.4
0.56	1.964	0.75666	0.0016384		
				0.0015480	13.3
0.58	1.897	0.76333	0.0014575		
				0.0013796	24.0
0.60	1.833	0.77131	0.0013018		
				0.0012344	48.0
0.62	1.774	0.78044	0.0011670		
				0.0011367	55.2
0.63	1.746	0.78538	0.0011064		
				0.0010920	57.3
0.635	1.732	0.78795	0.0010776		
				0.0010692	70.9
0.638	1.724	0.78951	0.0010608		
				0.0010594	21.6
0.6385	1.723	0.78977	0.0010580		
					$\Sigma(\Delta x) \approx 300$ m

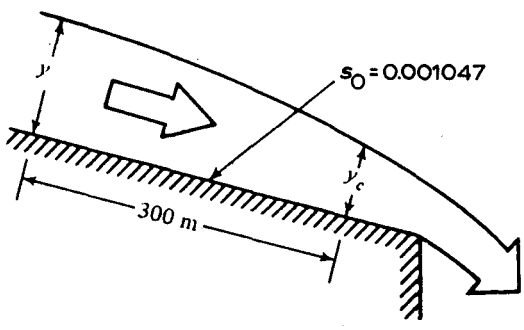


Fig. 14-85

14.327 The clean-earth ($n = 0.020$) channel in Fig. 14-86 is 6 m wide and laid on a slope of 0.005236. Water flows at 30 m³/s in the channel and enters a reservoir so that the channel depth is 3 m just before the entry. Assuming gradually varied flow, calculate the distance L.

▮ $Q = (A)(1.0/n)(R^{2/3})(s^{1/2})$ $\quad 30 = (6y_n)(1.0/0.022)[6y_n/(y_n + 6 + y_n)]^{2/3}(0.005236)^{1/2}$

$y_n = 1.52$ m (by trial and error) $\quad y_c = (q^2/g)^{1/3} = [(30/6)^2/9.807]^{1/3} = 1.37$ m

Hence we have an M_1 profile (see Fig. A-19). Use $\Delta y = 0.25$ m.

y	$v = 30/6y$	E	R_h	$s = n^2v^2/R_h^{4/3}$	s_{av}	Δx
3.0	1.667	3.1416	1.500	0.0006474		
					0.0007321	50
2.75	1.818	2.9185	1.435	0.0008168		
					0.0009373	50
2.50	2.000	2.7039	1.364	0.0010577		
					0.001235	51
2.25	2.222	2.5017	1.286	0.0014122		
					0.0016864	52
2.0	2.500	2.3186	1.200	0.0019605		
						$L = \Sigma(\Delta x) \approx 203$ m

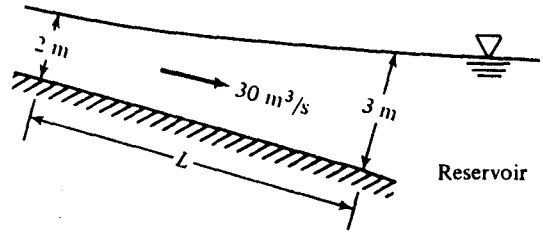


Fig. 14-86

14.328 The channel of Fig. 14-87 is sometimes called a *venturi flume,* because measurements of y_1 and y_2 can be used to meter the flow rate. Suppose that $b_1 = 4.5$ m, $b_2 = 3$ m, $y_1 = 2$ m, and $y_2 = 1.4$ m. Assuming no losses, compute the flow rate Q.

▮ $$y_1 + v_1^2/2g = y_2 + v_2^2/2g \qquad Q = y_1 b_1 v_1 = y_2 b_2 v_2$$

Solve these equations simultaneously: $v_2 = \{(2g)(y_1 - y_2)/[1 - (y_2 b_2/y_1 b_1)^2]\}^{1/2}$, $Q = y_2 b_2 v_2 = \{(2g)(y_1 - y_2)/[(1/b_2^2 y_2^2) - (1/b_1^2 y_1^2)]\}^{1/2} = \{(2)(9.807)(2 - 1.4)/[1/(3^2)(1.4)^2 - 1/(4.5)^2(2)^2]\}^{1/2} = 16.3$ m³/s.

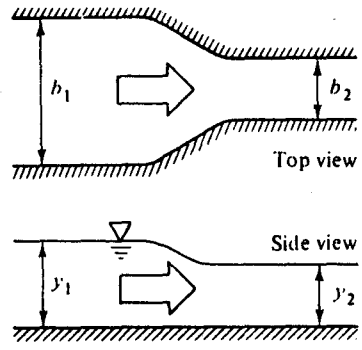


Fig. 14-87

14.329 In Prob. 14.328, holding y_1, b_1, and b_2 fixed, find y_2 for critical exit flow v_2.

▮ $y_c = [(Q/b)^2/g]^{1/3}$. y_2 must equal y_c. $Q = \{(2g)(y_1 - y_2)/[(1/b_2^2 y_2^2) - (1/b_1^2 y_1^2)]\}^{1/2}$ (from Prob. 14.328). Try $y_2 = 1.5$ m: $Q = \{(2)(9.807)(2 - 1.5)/[1/(3^2)(1.5)^2 - 1/(4.5)^2(2)^2]\}^{1/2} = 16.27$ m³/s, $y_c = [(16.27/3)^2/9.807]^{1/3} = 1.44$ m. Since $y_c = 1.44$ m is not equal to the assumed $y_2 = 1.5$ m, try $y_2 = 1.45$ m: $Q = \{(2)(9.807)(2 - 1.45)/[1/(3^2)(1.45)^2 - 1/(4.5)^2(2)^2]\}^{1/2} = 16.32$ m³/s, $y_c = [(16.32/3)^2/9.807]^{1/3} = 1.45$ m (O.K.). Hence critical flow of 16.32 m³/s will occur at $y_2 = 1.45$ m.

14.330 In Prob. 14.328, holding y_2, b_1, and b_2 fixed, find y_1 for a flow rate of 12.2 m³/s.

▮ $$Q = \{(2g)(y_1 - y_2)/[(1/b_2^2 y_2^2) - (1/b_1^2 y_1^2)]\}^{1/2} \qquad \text{(from Prob. 14.328)}$$

$$12.2 = \{(2)(9.807)(y_1 - 1.4)/[(1/(3^2)(1.4)^2 - 1/(4.5)^2(y_1)^2]\}^{1/2} \qquad y_1 = 1.70 \text{ m} \qquad \text{(by trial and error)}$$

14.331 In the venturi flume of Fig. 14-87, let $b_1 = 5$ ft, $b_2 = 4$ ft, $y_1 = 3$ ft, and $Q = 75.9$ cfs. Compute y_2, assuming no losses.

▮ $$Q = \{(2g)(y_1 - y_2)/[(1/b_2^2 y_2^2) - (1/b_1^2 y_1^2)]\}^{1/2} \qquad \text{(from Prob. 14.328)}$$

$$75.9 = \{(2)(32.2)(2 - y_2)/[(1/(3^2)(y_2)^2 - 1/(4)^2(2)^2]\}^{1/2} \qquad y_2 = 2.00 \text{ ft} \qquad \text{(by trial and error)}$$

14.332 A flow of 35 m³/s flows along a trapezoidal concrete channel where (see Fig. 14-88) the base a is 4 m and β is 45°. If at section 1, the depth of the flow is 3 m, what is the water-surface profile up to a distance 600 m downstream. The channel is finished concrete and has a constant slope S_0 of 0.001.

▮ $$\Delta L = \left([1 - (Q^2 b/gA^3)]/\{S_0 - (n/\kappa)^2[Q^2/(R_H^{4/3}A^2)]\}\right)\Delta y$$

We start with $y_1 = 3$ m. At this section we know that

$$A_1 = (3)(4) + \tfrac{1}{2}(3)(3)(2) = 21 \text{ m}^2$$

$$(R_H)_1 = A_1/(p_w)_1 = 21/[4 + (2)(3/0.707)] = 1.6818 \text{ m} \qquad (1)$$

$$b_1 = 4 + (2)(3) = 10 \text{ m}$$

We take $n = 0.012$ and $\kappa = 1.00$ and let $y_2 = 3.1$ m. Now we compute A_2, $(R_H)_2$, and b_2.*

$$A_2 = (3.1)(4) + \tfrac{1}{2}(3.1)(3.1)(2) = 22.01 \text{ m}^2$$
$$(R_H)_2 = 22.01/[4 + 2(3.1/0.707)] = 1.7236 \text{ m} \qquad (2)$$
$$b_2 = 4 + (2)(3.1) = 10.20 \text{ m}$$

In the first interval, the average values of A, R_H, and b are

$$(A_{1\text{-}2})_{\text{av}} = 21.505 \text{ m}^2$$
$$[(R_H)_{1\text{-}2}]_{\text{av}} = 1.7027 \text{ m} \qquad (3)$$
$$(b_{1\text{-}2})_{\text{av}} = 10.10 \text{ m}$$
$$(\Delta L)_{1\text{-}2} = \left\{\frac{1 - (35^2)(10.10)/[(9.81)(21.505^3)]}{0.01 - (0.012/1)^2(35^2)/[(1.7027^{4/3})(21.505^2)]}\right\}(0.1) = 107.4 \text{ m}$$

We thus have two points on the free-surface profile. Next we compute A_3, R_{H_3}, and b_3 for $y = 3.2$ m. Using Eqs. (2) we now find the average values of these quantities in the interval 2–3. For instance, $(A_{2\text{-}3})_{\text{av}} = \tfrac{1}{2}\{(22.01) + [(3.2)(4) + (3.2^2)]\} = 22.525 \text{ m}^2$. We then proceed as indicated for the first interval.

The following table gives the results using six sections so that $L_{\text{total}} \approx 600$ m.

section	y, m	Δy, m	ΔL, m	L_{total}, m
1	3.0	0.1	0	0
2	3.1	0.1	107.4	107.4
3	3.2	0.1	106.9	214.3
4	3.3	0.1	105.6	319.9
5	3.4	0.1	104.7	424.6
6	3.5	0.1	104.1	528.7
7	3.6	0.1	103.8	632.5

We can now plot y versus L, that is, the second and last columns, starting with $y = 3$ for $L = 0$ m and going on to $y = 3.6$ m for $L = 632.5$ m. A smooth curve through these parts gives the approximate desired profile.

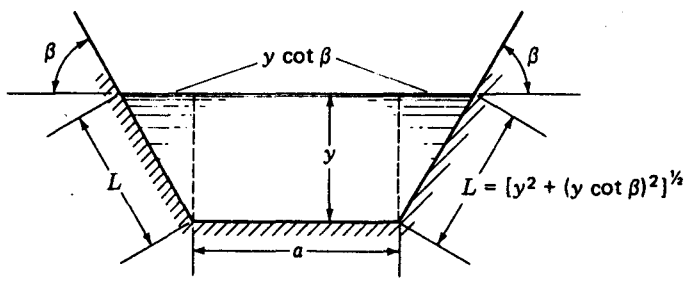


Fig. 14-88

14.333 Water enters a rectangular channel which is 3 ft wide at an average velocity of 0.8 ft/s and a depth of 3 in. The channel is laid on a slope of 0.003499. If Manning's n is 0.012, how far must the water travel before its depth becomes 4 in? Use four intermediary depths ($\Delta y = 0.2$ in) in the calculation.

▮
$$\Delta x = \left([1 - (Q^2 b/gA^3)]/\{s_0 - (n/\kappa)^2[Q^2/(R_h^{4/3}A^2)]\}\right)\Delta y$$
$$s_0 = 0.003499 \qquad R_h = by/(b + 2y) = y$$

Point 1:

$$v_1 = 0.8 \text{ ft/s} \qquad (f_R)_1 = v\sqrt{b}/\sqrt{gA} = (0.8)(\sqrt{3})/\sqrt{(g)(3)(\tfrac{1}{4})} = 0.282 \qquad \Delta y = 0.2/12 \text{ ft}$$

Point 2:

$$v_2 = (y_0/y_1)v_1 = (3/3.2)(0.8) = 0.750 \text{ ft/s} \qquad (f_R)_2 = (0.750)(\sqrt{3})/\sqrt{(g)(3)(3.2/12)} = 0.256$$

* We are assuming that the depth y relative to the channel is increasing. If we get a positive result for ΔL, we know that our assumption is correct. If ΔL is negative, then the depth must be decreasing along the channel flow. Use y_2 less than 3 m in that case.

1–2:

$$(v_{1\text{–}2})_{av} = (0.8 + 0.750)/2 = 0.775 \text{ ft/s} \qquad (f_{R,1\text{–}2})_{av} = (0.282 + 0.256)/2 = 0.269$$

$$(\Delta x)_{1\text{–}2} = \frac{1 - 0.269^2}{0.00349 - (0.012/1.486)^2[0.775^2/(3.1/12)^{4/3}]}\left(\frac{0.2}{12}\right) = 4.75 \text{ ft}$$

Point 3:

$$v_3 = (3/3.4)(0.8) = 0.706 \text{ ft/s} \qquad (f_R)_3 = 0.706/[g(3.4/12)]^{1/2} = 0.234$$

2–3:

$$(v_{2\text{–}3})_{av} = (0.750 + 0.706)/2 = 0.728 \text{ ft/s} \qquad (f_{R,2\text{–}3})_{av} = (0.256 + 0.234)/2 = 0.245$$

$$(\Delta x)_{2\text{–}3} = \frac{1 - 0.245^2}{0.00349 - (0.012/1.486)^2[0.728^2/(3.3/12)^{4/3}]}\left(\frac{0.2}{12}\right) = 4.75 \text{ ft}$$

Point 4:

$$v_4 = (3/3.6)(0.8) = 0.667 \text{ ft/s} \qquad (f_R)_4 = 0.667/\sqrt{(g)(3.6/12)} = 0.215$$

3–4:

$$(v_{3\text{–}4})_{av} = (0.706 + 0.667)/2 = 0.686 \text{ ft/s} \qquad (f_{R,3\text{–}4})_{av} = (0.234 + 0.215)/2 = 0.224$$

$$(\Delta x)_{3\text{–}4} = \frac{1 - 0.224^2}{0.00349 - (0.012/1.486)^2[0.686^2/(3.5/12)^{4/3}]}\left(\frac{0.2}{12}\right) = 4.75 \text{ ft}$$

Point 5:

$$v_5 = (3/3.8)(0.8) = 0.632 \text{ ft/s} \qquad (f_R)_5 = 0.632/\sqrt{(g)(3.8/12)} = 0.198$$

4–5:

$$(v_{4\text{–}5})_{av} = (0.667 + 0.632)/2 = 0.650 \text{ ft/s} \qquad (f_{R,4\text{–}5})_{av} = (0.215 + 0.198)/2 = 0.206$$

$$(\Delta x)_{4\text{–}5} = \frac{1 - 0.206^2}{0.00349 - (0.012/1.486)^2[0.650^2/(3.7/12)^{4/3}]}\left(\frac{0.2}{12}\right) = 4.75 \text{ ft}$$

Point 6:

$$v_6 = (\tfrac{3}{4})(0.8) = 0.600 \text{ ft/s} \qquad (f_R)_6 = 0.600/[g(4.0/12)]^{1/2} = 0.183$$

5–6:

$$(v_{5\text{–}6})_{av} = (0.632 + 0.600)/2 = 0.616 \text{ ft/s} \qquad (f_{R,5\text{–}6})_{av} = (0.198 + 0.183)/2 = 0.190$$

$$(\Delta x)_{5\text{–}6} = \frac{1 - 0.190^2}{0.00349 - (0.012/1.486)^2[0.616^2/(3.9/12)^{4/3}]}\left(\frac{0.2}{12}\right) = 4.75 \text{ ft}$$

$$x = \sum(\Delta x) = 23.75 \text{ ft}$$

14.334 In Prob. 14.332, compute the distance L in one calculation where the free surface has a depth of 3.6 m. Do not average.

▮

$$\Delta L = \left([1 - (Q^2b/gA^3)]/\{S_0 - (n/\kappa)^2[Q^2/(R_H^{4/3}A^2)]\}\right)\Delta y$$

$$A = (3)(4) + (3)(3) = 21.00 \text{ m}^2 \qquad p_w = 4 + (2)(\sqrt{3^2 + 3^2}) = 12.49 \text{ m}$$

$$R_H = 21.00/12.49 = 1.681 \text{ m} \qquad b = 3 + 4 + 3 = 10 \text{ m}$$

$$\Delta L = \left\{\frac{1 - (35)^2(10)/(9.807)(21.00)^3}{0.001 - (0.012/1)^2[35^2/(1.681)^{4/3}(21.00)^2]}\right\}(3.6 - 3) = 649 \text{ m}$$

14.335 Do Prob. 14.334 using a linear average in the calculations. What is the percent error in not averaging?

▮

$$\Delta L = \left([1 - (Q^2b/gA^3)]/\{S_0 - (n/\kappa)^2[Q^2/(R_H^{4/3}A^2)]\}\right)\Delta y$$

At the section where $y = 3.6$ m,

$$A = (3.6)(4) + (3.6)(3.6) = 27.36 \text{ m}^2 \qquad p_w = 4 + (2)(\sqrt{3.6^2 + 3.6^2}) = 14.18 \text{ m}$$

$$R_H = 27.36/14.18 = 1.929 \text{ m} \qquad b = 3.6 + 4 + 3.6 = 11.20 \text{ m}$$

Using corresponding values where $y = 3$ m from Prob. 14.334,

$$A_{av} = (21.00 + 27.36)/2 = 24.18 \text{ m}^2 \qquad (R_H)_{av} = (1.681 + 1.929)/2 = 1.805 \text{ m}$$

$$b_{av} = (10 + 11.20)/2 = 10.60 \text{ m}$$

$$\Delta L = \left\{\frac{1 - (35)^2(10.60)/(9.807)(24.18)^3}{0.001 - (0.012/1)^2[35^2/(1.805)^{4/3}(24.18)^2]}\right\}(3.6 - 3) = 630 \text{ m}$$

Error = (649 − 630)/630 = 0.03, or 3 percent.

14.336 A wide sluice, made of finished concrete and having slope $s_0 = 0.0004$, is fed from a large reservoir through a sharp-edged gate (Fig. 14-89). The coefficient of contraction C_c is 0.82 and the coefficient of friction $C_f = 0.86$. How far from the vena contracta does the water increase depth by 30 mm? Make a one-stage calculation with linear averages.

▮

$$v_1^2/2g + y_1 = v_2^2/2g + y_2 \qquad y_2 = (0.30)(0.82) = 0.2460 \text{ m}$$

$$0 + 12 = v_2^2/[(2)(9.807)] + 0.2460 \qquad v_2 = 15.18 \text{ m/s}$$

$$q_{\text{theoretical}} = y_2 v_2 = (0.2460)(15.18) = 3.734 \text{ (m}^3\text{/s)/m} \qquad q_{\text{actual}} = (0.86)(3.734) = 3.21 \text{ (m}^3\text{/s)/m}$$

$$\Delta x = \left([1 - (Q^2 b/gA^3)]/\{s_0 - (n/\kappa)^2[Q^2/(R_h^{4/3}A^2)]\}\right)\Delta y$$

$$\Delta y_{\text{downstream}} = 0.246 + 0.03 = 0.276 \text{ m}$$

$$\Delta x = \left\{\frac{1 - (3.21)^2/(9.807)(0.276)^3}{0.0004 - (0.012/1)^2[3.21^2/(0.276)^{4/3}(0.276)^2]}\right\}(0.030) = 13.60 \text{ m}$$

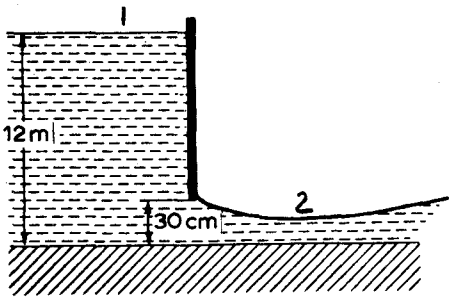


Fig. 14-89

14.337 Water is moving at 5 m/s in a very wide horizontal channel at a depth of 1 m. If $n = 0.024$, find analytically the distance x downstream to the section of depth 1.1 m.

▮

$$x = (\kappa/n)^2\left[\frac{3}{4g}(y^{4/3} - y_1^{4/3}) - \frac{3}{13q^2}(y^{13/3} - y_1^{13/3})\right]$$

$$= \left(\frac{1}{0.024}\right)^2\left\{\left[\frac{3}{(4)(9.807)}\right](1.1^{4/3} - 1^{4/3}) - \left[\frac{3}{(13)(5)^2}\right](1.1^{13/3} - 1^{13/3})\right\} = 9.80 \text{ m}$$

14.338 In a wide rectangular earth channel obstructed with stones ($n = 0.032$), the depth of flow increases from 1 m to 1.1 m over a 10 m distance. Compute the volumetric flow per unit width.

▮

$$x = \left(\frac{\kappa}{n}\right)^2\left[\frac{3}{4g}(y^{4/3} - y_1^{4/3}) - \frac{3}{13q^2}(y^{13/3} - y_1^{13/3})\right]$$

$$10 = \left(\frac{1}{0.032}\right)^2\left\{\left[\frac{3}{(4)(9.807)}\right](1.1^{4/3} - 1^{4/3}) - \left(\frac{3}{13q^2}\right)(1.1^{13/3} - 1^{13/3})\right\}$$

$$q = 30.96 \text{ (m}^3\text{/s)/m}$$

14.339 In Prob. 14.196, for a flow of 4900 cfs, calculate the distance downstream over which the depth increases from 13 ft to 14 ft. Make a one-step calculation without averaging.

$$\Delta x = \left([1 - (Q^2b/gA^3)]/\{s_0 - (n/\kappa)^2[Q^2/(R_h^{4/3}A^2)]\}\right)\Delta y$$

From Prob. 14.196, $A = 217.1\ \text{ft}^2$, $R_h = 5.802$ ft, $n = 0.012$, $s_0 = 0.0016$, $b = 20$ ft, and $\Delta y = 1$ ft.

$$\Delta x = \left\{\frac{1 - (4900)^2(20)/(32.2)(217.1)^3}{0.0016 - (0.012/1.486)^2[4900^2/(5.802)^{4/3}(217.1)^2]}\right\}(1) = 288\ \text{ft}$$

14.340 Repeat Prob. 14.339, using a linear average over the interval.

$$\Delta x = \left([1 - (Q^2b/gA^3)]/\{s_0 - (n/\kappa)^2[Q^2/(R_h^{4/3}A^2)]\}\right)\Delta y$$

At $y = 14$ ft, $A = (\frac{1}{2})(\pi)(10)^2 + (4)(10 + 10) = 237.1\ \text{ft}^2$ and $R_h = 237.1/[(\pi)(10) + 4 + 4] = 6.015$ ft. Therefore,

$$A_{av} = (217.1 + 237.1)/2 = 227.1\ \text{ft}^2 \qquad (R_h)_{av} = (5.802 + 6.015)/2 = 5.908\ \text{ft}$$

$$\Delta x = \left\{\frac{1 - (4900)^2(20)/(32.2)(227.1)^3}{0.0016 - (0.012/1.486)^2[4900^2/(5.908)^{4/3}(227.1)^2]}\right\}(1) = 220\ \text{ft}$$

14.341 Water flows at a rate of 500 ft³/s through a rectangular section 10.0 ft wide from a "steep" slope to a "mild" slope creating a hydraulic jump, in the manner illustrated in Fig. 14-90. The upstream depth of flow (d_1) is 3.1 ft. Find the **(*a*)** downstream depth, **(*b*)** energy (head) loss in the hydraulic jump, and **(*c*)** upstream and downstream velocities.

(*a*) $d_2 = (d_1/2)(\sqrt{1 + 8q^2/gd_1^3} - 1) = (3.1/2)\{\sqrt{1 + (8)(500/10.0)^2/[(32.2)(3.1)^3]} - 1\} = 5.7$ ft

(*b*) $E_j = (d_2 - d_1)^3/4d_1d_2 = (5.7 - 3.1)^3/[(4)(3.1)(5.7)] = 0.25$ ft of water

(*c*) $v_1 = Q/A_1 = 500/[(3.1)(10.0)] = 16.1$ ft/s $\qquad v_2 = 500/[(5.7)(10.0)] = 8.8$ ft/s

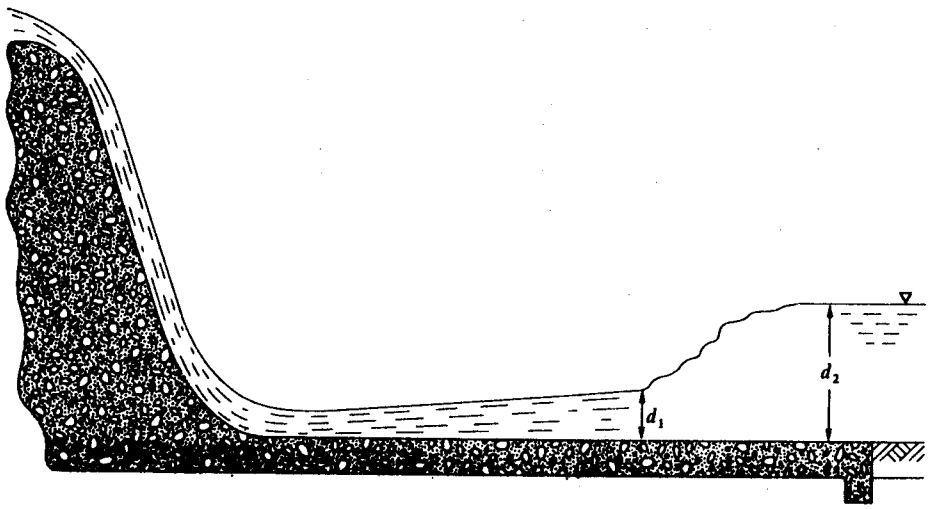


Fig. 14-90

14.342 Rework Prob. 14.341 for a flow rate of 20.0 m³/s, a channel width of 4.0 m, and an upstream depth of flow of 1.20 m.

(*a*) $d_2 = (d_1/2)(\sqrt{1 + 8q^2/gd_1^3} - 1) = (1.20/2)\{\sqrt{1 + (8)(20.0/4.0)^2/[(9.807)(1.20)^3]} - 1\} = 1.55$ m

(*b*) $E_j = (d_2 - d_1)^3/4d_1d_2 = (1.55 - 1.20)^3/[(4)(1.20)(1.55)] = 0.006$ m of water

(*c*) $v_1 = Q/A_1 = 20.0/[(1.20)(4.0)] = 4.17$ m/s $\qquad v_2 = 20.0/[(1.55)(4.0)] = 3.23$ m/s

14.343 Water flows over a concrete spillway into a rectangular channel 9.0 m wide through a hydraulic jump, in the manner illustrated in Fig. 14-90. The depths before and after the jump are 1.55 m and 3.08 m, respectively. Find the rate of flow in the channel.

$$q = \{[(d_1 + d_2)/2](gd_1d_2)\}^{1/2} = \{[(1.55 + 3.08)/2][(9.807)(1.55)(3.08)]\}^{1/2} = 10.41\ (\text{m}^3/\text{s})/\text{m}$$
$$Q = (10.41)(9.0) = 93.7\ \text{m}^3/\text{s}$$

14.344 Rework Prob. 14.343 if the channel width is 20 ft and the depths of flow before and after the jump are 4.5 ft and 8.2 ft, respectively.

$$q = \{[(d_1 + d_2)/2](gd_1d_2)\}^{1/2} = \{[(4.5 + 8.2)/2][(32.2)(4.5)(8.2)]\}^{1/2} = 86.86\ (\text{ft}^3/\text{s})/\text{ft}$$
$$Q = (86.86)(20) = 1737\ \text{ft}^3/\text{s}$$

14.345 A hydraulic jump occurs downstream from a 15-m-wide sluice gate. The depth is 1.5 m, and the velocity is 20 m/s. Determine **(*a*)** the Froude number and the Froude number corresponding to the conjugate depth, **(*b*)** the depth and velocity after the jump, and **(*c*)** the power dissipated by the jump.

(*a*) $N_F = v/\sqrt{gy}$ $\qquad (N_F)_1 = 20/\sqrt{(9.807)(1.5)} = 5.21$

$$(N_F)_2 = (2)(\sqrt{2})(N_F)_1/[\sqrt{1 + (8)(N_F)_1^2} - 1]^{3/2} = (2)(\sqrt{2})(5.21)/[\sqrt{1 + (8)(5.21)^2} - 1]^{3/2} = 0.288$$

(*b*) $(N_F)_2 = v_2/\sqrt{gy}$ $\qquad v_2y_2 = v_1y_1 = (1.5)(20) = 30.00\ \text{m}^2/\text{s}$

$$v_2^2 = (N_F)_2^2 gy_2 = (N_F)_2^2(g)(30.00/v_2) = (0.288)^2(9.807)(30.00/v_2)$$
$$v_2 = 2.90\ \text{m/s} \qquad y_2 = 30.00/2.90 = 10.34\ \text{m}$$

(*c*) $E_j = (y_2 - y_1)^3/4y_1y_2 = (10.34 - 1.5)^3/[(4)(1.5)(10.34)] = 11.13$ m of water

$$P = \gamma QE_j \qquad Q = Av = [(15)(1.5)](20) = 450\ \text{m}^3/\text{s} \qquad P = (9.79)(450)(11.13) = 49.0 \times 10^3\ \text{kW}$$

14.346 Water flows at a rate of 16 m^3/s at half critical depth in a trapezoidal channel with $b = 4$ m and $m = 0.4$ (see Fig. 14-91), before a hydraulic jump occurs. Prepare and execute a computer program to find the height after the jump and the energy loss in kilowatts.

```
10 REM B:EX124                          JUMP IN A TRAPEZOIDAL CHANNEL
20 DEFINT I:     DEF FNC(DY)=Q^2*(B+2!*M*DY)-G*(DY*(B+M*DY))^3   '
30 DEF FNFM(DY)=.5*B*DY^2+M*DY^3/3!+Q^2/(G*DY*(B+M*DY))          '
40 READ G,Q,B,M,GAM:     DATA 9.806,16.,4.,.4,9806.
50 YMAX=16!:     YMIN=0!:     LPRINT:     LPRINT"G,Q,B,M,GAM=";G;Q;B;M;GAM
60 FOR I=1 TO 15:     YC=.5*(YMAX+YMIN)
70 IF FNC(YC)>0! THEN YMIN=YC ELSE YMAX=YC
80 PRINT YMAX;YMIN;YC
90 NEXT I:     LPRINT"Y1,YC=";.5*YC;YC
100 Y1=.5*YC:     YMIN=YC:     YMAX=3!*YC:     FM=FNFM(Y1)
110 FOR I=1 TO 15:     Y2=.5*(YMAX+YMIN)
120 IF FNFM(Y2)-FM>0! THEN YMAX=Y2 ELSE YMIN=Y2
130 PRINT"YMAX,YMIN,Y2=";YMAX;YMIN;Y2:     NEXT I
140 A1=Y1*(B+M*Y1):     A2=Y2*(B+M*Y2):     V1=Q/A1:     V2=Q/A2
150 LOSS=(V1^2-V2^2)/(2!*G)+Y1-Y2:     POWER=GAM*Q*LOSS/1000!
160 LPRINT"Y1,Y2,V1,V2,LOSS,POWER=";
170 LPRINT USING"###.### ";Y1;Y2;V1;V2;LOSS;POWER

G,Q,B,M,GAM= 9.806001  16  4  .4  9806
Y1,YC= .5661621  1.132324
Y1,Y2,V1,V2,LOSS,POWER=  0.566   1.973   6.687   1.693   0.726 113.970
```

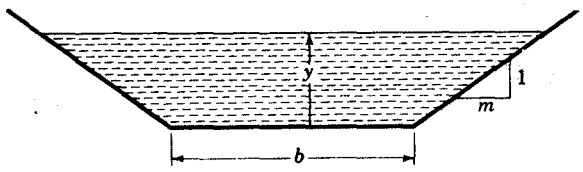


Fig. 14-91

14.347 A rectangular channel 10 ft wide carries 100 cfs in uniform flow at a depth of 1.67 ft. Suppose that an obstruction such as a submerged weir is placed across the channel, rising to a height of 6 in above the bottom. **(*a*)** Will this obstruction cause a hydraulic jump to form upstream? Why? **(*b*)** Find the water depth over the obstruction, and classify the surface profile, if possible, to be found upstream of the weir.

(*a*) $y_c = (q^2/g)^{1/3} = [(\frac{100}{10})^2/32.2]^{1/3} = 1.459$ ft. Since $[y_0 = 1.67\ \text{ft}] > [y_c = 1.459\ \text{ft}]$, the slope is mild, flow is subcritical, and no jump can occur.

(*b*) $E = y + (1/2g)(q^2/y^2)$ $\qquad E_0 = 1.67 + \{1/[(2)(32.2)]\}[(\frac{100}{10})^2/1.67^2] = 2.227$ ft

$$E_{\min} = (\tfrac{3}{2})(y_c) = (\tfrac{3}{2})(1.459) = 2.188\ \text{ft} \qquad \Delta z_{\text{crit}} = E_0 - E_{\min} = 2.227 - 2.188 = 0.039\ \text{ft}$$

Since $[\Delta z = 0.5\ \text{ft}] > [\Delta z_{\text{crit}} = 0.039\ \text{ft}]$, y_c occurs over the obstruction. Hence, the water depth over the

obstruction = 1.459 ft. Let y_1 = water depth just upstream of the obstruction. Assuming no energy loss over the obstruction, $E_1 = E_{min} + \Delta z$, or $y_1 + \{1/[(2)(32.2)]\}[(\frac{100}{10})^2/y_1^2] = 2.188 + 0.5 = 2.688$, $y_1 = 2.42$ ft or 0.943 ft (by trial and error). There is no mild profile that enables normal flow to become supercritical downstream, so $y_1 = 0.943$ ft is not possible here. Thus $y_1 = 2.42$ ft. Upstream of y_1, the water surface tends asymptotically to $y_0 = 1.67$ ft.

14.348 Suppose that the slope and roughness of the channel in Prob. 14.347 are such that uniform flow of 120 cfs occurs at 1.00 ft. Consider an obstruction rising 3 in above the bottom of the channel. Will a hydraulic jump form upstream? As in Prob. 14.347, classify the surface profile found just upstream from the obstruction.

▮ Since $y_0 = 1.000$ ft < 1.459 ft $= y_c$, the slope is steep and for $y = y_0$, the flow is supercritical.

$$E = y + (1/2g)(q^2/y^2) \qquad E_0 = 1.000 + \{1/[(2)(32.2)]\}[(\tfrac{120}{10})^2/1.00^2] = 3.236 \text{ ft}$$

$$E_{min} = 2.188 \text{ ft} \quad \text{(from Prob. 14.347)} \qquad \Delta z_{crit} = E_0 - E_{min} = 3.236 - 2.188 = 1.048 \text{ ft}$$

Since $\Delta z = 0.250$ ft < 1.048 ft $= \Delta z_{crit}$, the obstruction is not sufficiently high to produce critical flow. Hence, upstream the flow is straight supercritical uniform flow, and a hydraulic jump cannot form.

14.349 In a rectangular channel 4 m wide with a flow of 7.53 m^3/s the depth is 0.3 m. If a hydraulic jump occurs, find the depth downstream of it. Calculate the loss of specific energy through the jump.

▮
$$y_2 = (y_1/2)(\sqrt{1 + 8q^2/gy_1^3} - 1) = (0.3/2)\{\sqrt{1 + (8)(7.53/4)^2/[(9.807)(0.3)^3]} - 1\} = 1.410 \text{ m}$$

$$E_j = (y_2 - y_1)^3/4y_1y_2 = (1.410 - 0.3)^3/[(4)(0.3)(1.410)] = 0.808 \text{ m}$$

14.350 In a horizontal rectangular channel 8 ft wide the depths before and after a hydraulic jump are 0.82 ft and 3.16 ft. Find the rate of flow and the specific energy loss.

▮
$$q = \{[(d_1 + d_2)/2](gd_1d_2)\}^{1/2} = \{[(0.82 + 3.16)/2][(32.2)(0.82)(3.16)]\}^{1/2} = 18.18 \text{ (ft}^3\text{/s)/m}$$

$$Q = (18.18)(8) = 145 \text{ ft}^3/\text{s} \qquad E_j = (d_2 - d_1)^3/4d_1d_2 = (3.16 - 0.82)^3/[(4)(0.82)(3.16)] = 1.24 \text{ ft}$$

14.351 A tidal *bore* (progressive hydraulic jump) rises 13 ft above the normal low-tide river depth of 8 ft. The bore travels upstream at 16 mph (relative to the bank). Find the velocity of the undisturbed river. Is this subcritical or supercritical flow?

▮
$$q = \{[(d_1 + d_2)/2](gd_1d_2)\}^{1/2} = \{[(13 + 8 + 8)/2][(32.2)(13 + 8)(8)]\}^{1/2} = 280.1 \text{ cfs/ft}$$

$$v_1 = q/d_1 = 280.1/8 = 35.01 \text{ ft/s} \quad \text{(relative to the bore)} \qquad v_{jump} = (16)(\tfrac{5280}{3600}) = 23.47 \text{ ft/s}$$

The bore is moving upstream at a velocity of 23.47 ft/s while the river is moving into the bore at 35.01 ft/s. Therefore, the river is moving downstream at 35.01 − 23.47 = 11.54 ft/s; $v^2/2g = 11.54^2/[(2)(32.2)] = 2.07$ ft. Since $v^2/2g = 2.07$ ft < 4 ft $= y/2$, the flow is subcritical.

14.352 A hydraulic jump occurs in a triangular flume having side slopes 1:1. The flow rate is 17.72 cfs and the depth before the jump is 1.0 ft. Find the specific-energy loss in the jump.

▮ $Q^2/gA_1 + (h_c)_1A_1 = Q^2/gA_2 + (h_c)_2A_2$. For the triangular section, $h_c = y/3$, $A = y^2$, and $y_1 = 1.0$ ft.

$$17.72^2/[(32.2)(1.0)^2] + (1.0/3)(1.0)^2 = 17.72^2/[(32.2)(y_2)^2] + (y_2/3)(y_2)^2$$

$$y_2 = 3.00 \text{ ft} \quad \text{(by trial and error)}$$

$$v_1 = Q/A_1 = 17.72/1.0^2 = 17.72 \text{ ft/s} \qquad v_2 = 17.72/3.00^2 = 1.97 \text{ ft}$$

$$E_j = (y_1 + v_1^2/2g) - (y_2 + v_2^2/2g) = 1.0 + 17.72^2/[(2)(32.2)] - 3.00 - 1.97^2/[(2)(32.2)] = 2.815 \text{ ft}$$

14.353 A hydraulic jump occurs in a 4-m-wide rectangular channel carrying 5 m^3/s on a slope of 0.004. The depth after the jump is 1.2 m. Find **(*a*)** the depth before the jump, **(*b*)** the losses of energy and power in the jump.

▮ **(*a*)**
$$y_1 = (y_2/2)(\sqrt{1 + 8q^2/gy_2^3} - 1) = (1.2/2)\{\sqrt{1 + (8)(\tfrac{5}{4})^2/[(9.807)(1.2)^3]} - 1\} = 0.191 \text{ m}$$

(*b*)
$$E_j = (y_2 - y_1)^3/4y_1y_2 = (1.2 - 0.191)^3/[(4)(0.191)(1.2)] = 1.120 \text{ m}$$

$$P = Q\gamma E_j = (5)(9.79)(1.120) = 54.8 \text{ kW}$$

14.354 Analyze the water-surface profile in a long rectangular channel with concrete lining ($n = 0.013$). The channel is 10 ft wide, the flow rate is 400 cfs, and there is an abrupt change in channel slope from 0.0150 to 0.0016. Find also the horsepower loss in the jump.

▮ $Q = (1.486/n)(A)(R^{2/3})(s^{1/2})$ $\quad 400 = (1.486/0.013)(10y_{0_1})[10y_{0_1}/(10 + 2y_{0_1})]^{2/3}(0.015)^{1/2}$

By trial, $y_{0_1} = 2.17$ ft (normal depth on upper slope). Using a similar procedure, the normal depth y_{0_2} on the lower slope is found to be 4.80 ft. $y_c = (q^2/g)^{1/3} = [(\frac{400}{10})^2/32.2]^{1/3} = 3.68$ ft. Thus flow is supercritical ($y_{0_1} < y_c$) before break in slope and subcritical ($y_{0_2} > y_c$) after break, so a hydraulic jump must occur. $y_2 = (y_1/2)[-1 + \sqrt{1 + (8q^2/gy_1^3)}]$, $y_2' = (2.17/2)(-1 + \{1 + [8(40)^2/32.2(2.17)^3]\}^{1/2}) = 5.75$ ft. Therefore the depth conjugate to the upper-slope normal depth of 2.17 ft is 5.75 ft. This jump cannot occur because the normal depth y_{0_2} on the lower slope is less than 5.75 ft. $y_1' = (4.80/2)(-1 + \{1 + [8(40)^2/32.2(4.8)^3]\}^{1/2}) = 2.76$ ft. The lower conjugate depth of 2.76 ft will occur downstream of the break in slope. The location of the jump (i.e., its distance below the break in slope) may be found by $\Delta x = (E_1 - E_2)/(S - S_0)$, $E_1 = 2.17 + [(400/21.7)^2/64.4] = 7.45$ ft, $E_2 = 2.76 + [(400/27.6)^2/64.4] = 6.02$ ft, $V_m = (\frac{1}{2})[(400/21.7) + (400/27.6)] = 16.46$ fps, $R_m = (\frac{1}{2})[(21.7/14.34) + (27.6/15.52)] = 1.645$ ft, $S = (nv/1.486R^{2/3})^2 = [(0.013)(16.45)/1.49(1.645)^{2/3}]^2 = 0.01060$, $\Delta x = (7.45 - 6.02)/(0.0106 - 0.0016) = 160$ ft. Thus depth on the upper slope is 2.17 ft; downstream of the break the depth increases gradually (M_3 curve) to 2.76 ft over a distance of approximately 160 ft; then a hydraulic jump occurs from a depth of 2.76 ft to 4.80 ft; downstream of the jump the depth remains constant (i.e., normal) at 4.80 ft. HP loss $= \gamma Q h_{L_j}/550$ where $h_{L_j} = \Delta E$.

Before jump: $E_1' = 2.76 + [(400/27.6)^2/64.4] = 6.02$ ft. After jump: $E_{0_2} = 4.80 + [(400/48.0)^2/64.4] = 5.88$ ft. Hence HP loss $= 62.4(400)(6.02 - 5.88)/550 = 6.35$.

14.355 A very wide rectangular channel with bed slope $S_0 = 0.0003$ and roughness $n = 0.020$ carries a steady flow of 50 cfs/ft of width. If a sluice gate is so adjusted as to produce a minimum depth of 1.5 ft in the channel, determine whether a hydraulic jump will occur downstream, and if so, find (using one reach) the distance from the gate to the jump.

▮ $q = (y)(1.486/n)(y^{2/3})(s^{1/2})$ $\quad 50 = (y_0)(1.486/0.020)(y_0)^{2/3}(0.0003)^{1/2}$ $\quad y_0 = 8.99$ ft

$$y_c = (q^2/g)^{1/3} = (50^2/32.2)^{1/3} = 4.27 \text{ ft}$$

Since $y_c < y_0$, the slope is mild. Since $y = 1.5$ ft caused by the gate $< y_c < y_0$, downstream of the sluice gate is an M_3 profile, which must be followed by a hydraulic jump to enable the flow to return to normal depth.

$$y_1 = (y_2/2)(\sqrt{1 + 8q^2/gy_2^3} - 1) = (8.99/2)\{\sqrt{1 + (8)(50)^2/[(32.2)(8.99)^3]} - 1\} = 1.63 \text{ ft}$$

$$\Delta x = (E_1 - E_2)/(S - S_0) \qquad E = y + v^2/2g$$

For $y = 1.5$ ft, $v = q/y = 50/1.5 = 33.33$ ft/s, $E = 1.5 + 33.33^2/[(2)(32.2)] = 18.75$ ft. For $y = 1.63$ ft, $v = q/y = 50/1.63 = 30.67$ ft/s, $E = 1.63 + 30.67^2/[(2)(32.2)] = 16.24$ ft.

$$v_m = (33.33 + 30.67)/2 = 32.00 \text{ ft/s} \qquad R_m = (1.5 + 1.63)/2 = 1.565 \text{ ft}$$

$$S = (nv/1.486R^{2/3})^2 = \{(0.020)(32.00)/[(1.486)(1.565)^{2/3}]\}^2 = 0.1021$$

$$\Delta x = (18.75 - 16.24)/(0.1021 - 0.0003) = 24.7 \text{ ft}$$

14.356 A rectangular channel 8 ft wide carries 280 cfs in uniform flow at a depth of 5 ft. Around a 40-ft-radius bend, how much higher should the outside wall be than the inside wall?

▮ $v = Q/A = 280/[(5)(8)] = 7.000$ ft/s, $N_F = v/\sqrt{gy} = 7.000/\sqrt{(32.2)(5)} = 0.552$. Since $N_F < 1.0$, the flow is subcritical. $\Delta y = v^2B/gr = (7.000)^2(10)/[(32.2)(40)] = 0.380$ ft.

14.357 Repeat Prob. 14.356 if normal depth is 2 ft.

▮ $v = Q/A = 280/[(2)(8)] = 17.5$ ft/s, $N_F = v/\sqrt{gy} = 17.5/\sqrt{(32.2)(2)} = 2.18$. Since $N_F > 1.0$, the flow is supercritical. $\Delta y = v^2B/gr = (17.5)^2(8)/[(32.2)(40)] = 1.90$ ft. Because of (supercritical) wave action, maximum water depth at inside wall $= y_0$, and maximum water depth at outside wall $= (y_0 + \Delta y)$. Therefore, the required difference in wall elevations $= 1.90$ ft.

14.358 Rework Prob. 14.356 for these data: $b = 4$ m, $Q = 8$ m^3/s, $y = 2.0$ m, $r = 30$ m.

▮ $v = Q/A = 8/[(4)(2.0)] = 1.000$ m/s, $N_F = v/\sqrt{gy} = 1.000/\sqrt{(9.807)(2.0)} = 0.226$. Since $N_F < 1.0$, the flow is subcritical. $\Delta y = v^2B/gr = (1.000)^2(4)/[(9.807)(30)] = 0.0136$ m.

14.359 Find the maximum volumetric flow in a 3-ft by 3-ft concrete box culvert ($n = 0.013$) with a rounded entrance ($k_e = 0.05$, $C_d = 0.95$) if the culvert slope is 0.0065, the length 100 ft, and the headwater level 5 ft above the culvert invert? Assume free outlet conditions. Neglect headwater and tailwater velocity heads.

▮ Headwater/$d = \frac{5}{3} = 1.7$. Since $1.7 > 1.2$, conditions are those of Fig. 14-92*b* or 14-92*c*. Assume case *b*.

$$R = A/p_w = (3)(3)/(3+3+3+3) = 0.750 \text{ ft}$$

$$(h_L)_{1\text{-}3} = (y_1 - y_3) + (z_1 - z_3) = y_1 - y_3 + s_0 L$$

$$\Delta h = (k_e + 29n^2L/R^{4/3} + 1)(v^2/2g) \qquad y_1 - y_3 + s_0 L = (k_e + 29n^2L/R^{4/3} + 1)(v^2/2g)$$

$$5 - 3 + (0.0065)(100) = [0.05 + (29)(0.013)^2(100)/0.75^{4/3} + 1]\{v^2/[(2)(32.2)]\} \qquad v = 9.82 \text{ ft/s}$$

$$Q = Av = [(3)(3)](9.82) = 88.38 \text{ ft}^3/\text{s} = (A)(1.486/n)(R^{2/3})(s^{1/2})$$

Now find the depth y_0 which occurs with normal uniform flow at this flow rate: $88.38 = (3y_0)(1.486/0.013)[3y_0/(y_0 + 3 + y_0)]^{2/3}(0.0065)^{1/2}$, $y_0 = 3.16$ ft (by trial and error). Since $y_0 > d$, the culvert flows full. Free discharge at the outlet is given; therefore, the preceding assumption and computations are valid, and $Q = 88.38 \text{ ft}^3/\text{s}$.

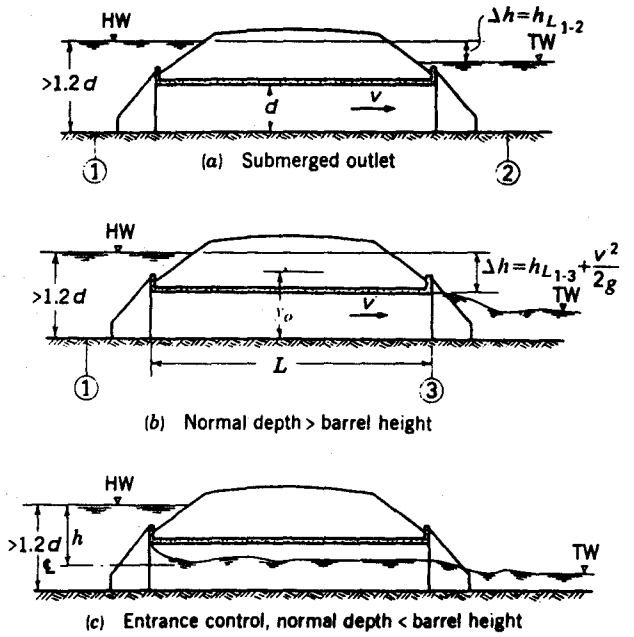


Fig. 14-92

14.360 Solve Prob. 14.359 given that the tailwater elevation is 1 ft above the top of the box at the outlet.

▮ Headwater/$d = \frac{5}{3} = 1.7$. Since $1.7 > 1.2$, conditions are those of Fig. 14-92*a* with $y_2 = 4$ ft.

$$R = A/p_w = (3)(3)/(3+3+3+3) = 0.75 \text{ ft}$$

$$(h_L)_{1\text{-}3} = (y_1 - y_3) + (z_1 - z_3) = y_1 - y_3 + s_0 L$$

$$\Delta h = (k_e + 29n^2L/R^{4/3} + 1)(v^2/2g) \qquad y_1 - y_3 + s_0 L = (k_e + 29n^2L/R^{4/3} + 1)(v^2/g)$$

$$5 - 4 + (0.0065)(100) = [0.05 + (29)(0.013)^2(100)/0.75^{4/3} + 1]\{v^2/[(2)(32.2)]\} \qquad v = 7.75 \text{ ft/s}$$

$$Q = Av = [(3)(3)](7.75) = 69.8 \text{ ft}^3/\text{s}$$

14.361 Solve Prob. 14.359 given that the tailwater elevation is 2 ft above the top of the box at the outlet.

▮ Headwater/$d = \frac{6}{4} = 1.5$. Since $1.5 > 1.2$, conditions are those of Fig. 14-92*a* with $y_2 = 6$ ft.

$$R = A/p_w = (4)(4)/(4+4+4+4) = 1.000 \text{ ft}$$

$$(h_L)_{1\text{-}3} = (y_1 - y_3) + (z_1 - z_3) = y_1 - y_3 + s_0 L$$

$$\Delta h = (k_e + 29n^2L/R^{4/3} + 1)(v^2/2g) \qquad y_1 - y_3 + s_0 L = (k_e + 29n^2L/R^{4/3} + 1)(v^2/2g)$$

$$5 - 5 + (0.0065)(100) = [0.05 + (29)(0.013)^2(100)/0.75^{4/3} + 1]\{v^2/[(2)(32.2)]\} \qquad v = 4.864 \text{ ft/s}$$

$$Q = Av = [(3)(3)](4.864) = 43.8 \text{ ft}^3/\text{s}$$

14.362 Repeat Prob. 14.359 if the culvert slope is 0.043.

▮ Headwater/$d = \frac{5}{3} = 1.7$. Since $1.7 > 1.2$, conditions are those of Fig. 14-92*b* or 14-92*c*. Assume case *b*.

$$R = A/p_w = (3)(3)/(3+3+3+3) = 0.75 \text{ ft}$$

$$(h_L)_{1\text{-}3} = (y_1 - y_3) + (z_1 - z_3) = y_1 - y_3 + s_0 L$$

$$\Delta h = (k_e + 29n^2L/R^{4/3} + 1)(v^2/2g) \qquad y_1 - y_3 + s_0L = (k_e + 29n^2L/R^{4/3} + 1)(v^2/2g)$$

$$5 - 3 + (0.043)(100) = [0.05 + (29)(0.013)^2(100)/0.75^{4/3} + 1]\{v^2/[(2)(32.2)]\} \qquad v = 15.14 \text{ ft/s}$$

$$Q = Av = [(3)(3)](15.14) = 136 \text{ ft}^3/\text{s} = (A)(1.486/n)(R^{2/3})(s^{1/2})$$

Now find the depth y_0 which occurs with normal uniform flow at this flow rate: $136 = (3y_0)(1.486/0.013)[3y_0/(y_0+3+y_0)]^{2/3}(0.043)^{1/2}$, $y_0 = 2.1$ ft (by trial and error). Since $y_0 < d$, the situation is actually case *c*; therefore, $Q = C_dA\sqrt{2gh} = (0.95)[(3)(3)]\sqrt{(2)(32.2)(3)} = 119 \text{ ft}^3/\text{s}$.

14.363 Repeat Prob. 14.359 if the culvert slope is 0.08.

▮ Headwater/$d = \frac{5}{3} = 1.7$. Since $1.7 > 1.2$, conditions are those of Fig. 14-92*b* or 14-92*c*. Assume case *b*.

$$R = A/p_w = (3)(3)/(3+3+3+3) = 0.75 \text{ ft}$$

$$(h_L)_{1\text{-}3} = (y_1 - y_3) + (z_1 - z_3) = y_1 - y_3 + s_0 L$$

$$\Delta h = (k_e + 29n^2L/R^{4/3} + 1)(v^2/2g) \qquad y_1 - y_3 + s_0L = (k_e + 29n^2L/R^{4/3} + 1)(v^2/2g)$$

$$5 - 3 + (0.08)(100) = [0.05 + (29)(0.013)^2(100)/0.75^{4/3} + 1]\{v^2/[(2)(32.2)]\} \qquad v = 19.08 \text{ ft/s}$$

$$Q = Av = [(3)(3)](19.08) = 172 \text{ ft}^3/\text{s} = (A)(1.486/n)(R^{2/3})(s^{1/2})$$

Now find the depth y_0 which occurs with normal uniform flow at this flow rate: $172 = (3y_0)(1.486/0.013)[3y_0/(y_0+3+y_0)]^{2/3}(0.08)^{1/2}$, $y_0 = 1.98$ ft (by trial and error). Since $y_0 < d$, our assumption was wrong and we have, as in Prob. 14.362, $Q = C_dA\sqrt{2gh} = (0.95)[(3)(3)]\sqrt{(2)(32.2)(3)} = 119 \text{ ft}^3/\text{s}$.

14.364 A culvert under a road must carry 4.3 m³/s. The culvert length will be 30 m and the slope will be 0.003. If the maximum permissible headwater level is 3.6 m above the culvert invert, what size corrugated-pipe culvert ($n = 0.025$) would you select? The outlet will discharge freely. Neglect velocity of approach. Assume square-edged entrance with $k_e = 0.5$ and $C_d = 0.65$.

▮ Assume $d < 3.0$ m, so that [headwater/d] > [3.6/3.0 = 1.2] and the conditions are those of Fig. 14-92*b* or 14-92*c*. Assume case *b*.

$$(h_L)_{1\text{-}3} = (y_1 - y_3) + (z_1 - z_3) = y_1 - y_3 + s_0L \qquad \Delta h = (k_e + 19.62n^2L/R^{4/3} + 1)(v^2/2g)$$

$$y_1 - y_3 + s_0L = (k_e + 19.62n^2L/R^{4/3} + 1)(v^2/2g) \qquad v = Q/A = 4.3/(\pi d^2/4) = 5.475/d^2$$

$$3.6 - d + (0.003)(30) = [0.5 + (19.62)(0.025)^2(30)/(d/4)^{4/3} + 1]\{(5.475/d^2)^2/[(2)(9.807)]\}$$

$$d = 1.20 \text{ m} \quad \text{(by trial and error)} \qquad Q = (A)(1.0/n)(R^{2/3})(s^{1/2})$$

Now find the diameter d_0 which just flows full with normal uniform flow: $4.3 = (\pi d_0^2/4)(1.0/0.025)(d_0/4)^{2/3}(0.003)^{1/2}$, $d_0 = 1.99$ m. Since $d_0 > d$, the culvert flows full, and free discharge at the outlet is given. Therefore, the above assumptions and analysis are valid, and $d = 1.20$ m.

14.365 Repeat Prob. 14.364 for a culvert length of 100 m.

▮ Assume $d < 3.0$ m, so that [headwater/d] > [3.6/3.0 = 1.2] and the conditions are those of Fig. 14-92*b* or 14-92*c*. Assume case *b*.

$$(h_L)_{1\text{-}3} = (y_1 - y_3) + (z_1 - z_3) = y_1 - y_3 + s_0L \qquad \Delta h = (k_e + 19.62n^2L/R^{4/3} + 1)(v^2/2g)$$

$$y_1 - y_3 + s_0L = (k_e + 19.62n^2L/R^{4/3} + 1)(v^2/2g) \qquad v = Q/A = 4.3/(\pi d^2/4) = 5.475/d^2$$

$$3.6 - d + (0.003)(100) = [0.5 + (19.62)(0.025)^2(100)/(d/4)^{4/3} + 1]\{(5.475/d^2)^2/[(2)(9.807)]\}$$

$$d = 1.41 \text{ m} \quad \text{(by trial and error)} \qquad Q = (A)(1.0/n)(R^{2/3})(s^{1/2})$$

Now find the diameter d_0 which just flows full with normal uniform flow: $4.3 = (\pi d_0^2/4)(1.0/0.025)(d_0/4)^{2/3}(0.003)^{1/2}$, $d_0 = 1.99$ m. Since $d_0 > d$, the culvert flows full, and free discharge at the

outlet is given. Therefore, the above assumptions and analysis are valid, and $d = 1.41$ m. Use standard diameter 1.50 m.

14.366 Water flows in a wide channel at $q = 18\ (\text{ft}^3/\text{s})/\text{ft}$, $y_1 = 1$ ft, and then undergoes a hydraulic jump. Compute y_2, v_2, $(N_F)_2$, E_j, and the power dissipated per unit width.

▮

$$N_F = v/\sqrt{gy} \qquad v_1 = q/y_1 = \tfrac{18}{1} = 18.00\ \text{ft/s}$$

$$(N_F)_1 = 18.00/\sqrt{(32.2)(1)} = 3.17 \qquad E = y + v^2/2g$$

$$E_1 = 1 + 18.00^2/[(2)(32.2)] = 5.03\ \text{ft}$$

$$y_2 = (y_1/2)(\sqrt{1 + 8q^2/gy_1^3} - 1) = (\tfrac{1}{2})\{\sqrt{1 + (8)(18)^2/[(32.2)(1)^3]} - 1\} = 4.01\ \text{ft}$$

$$v_2 = q/y_2 = 18/4.01 = 4.49\ \text{ft/s} \qquad (N_F)_2 = 4.49/\sqrt{(32.2)(4.01)} = 0.395$$

$$E_j = (y_2 - y_1)^3/4y_1y_2 = (4.01 - 1)^3/[(4)(4.01)(1)] = 1.70\ \text{ft}$$

$$p_{\text{dissipated}} = q\gamma E_j = (18)(62.4)(1.7) = 1909\ (\text{ft-lb/s})/\text{ft} = 3.47\ \text{hp/ft}$$

14.367 A wide-channel flow at depth 60 cm passes through a hydraulic jump and emerges at a depth of 3.0 m. Compute the velocities on either side of the jump and the critical depth of flow.

▮

$$2y_2/y_1 = -1 + [1 + 8(N_F)_1^2]^{1/2} \qquad (2)(3.0)/(0.60) = -1 + [1 + (8)(N_F)_1^2]^{1/2} \qquad (N_F)_1 = 3.87$$

$$N_F = v/\sqrt{gy} \qquad 3.87 = v_1/\sqrt{(9.807)(0.60)} \qquad v_1 = 9.39\ \text{m/s}$$

$$v_1y_1 = v_2y_2 \qquad (9.39)(0.60) = (v_2)(3.0) \qquad v_2 = 1.88\ \text{m/s}$$

$$q = yv = (3.0)(1.88) = 5.64\ (\text{m}^3/\text{s})/\text{m} \qquad y_c = (q^2/g)^{1/3} = (5.64^2/9.807)^{1/3} = 1.48\ \text{m}$$

14.368 The flow downstream of a hydraulic jump in a rectangular channel is 8 m deep and has a velocity of 3.6 m/s. Obtain the velocity and depth upstream of the jump.

▮

$$2y_1/y_2 = -1 + [1 + 8(N_F)_2^2]^{1/2} \qquad N_F = v/\sqrt{gy} \qquad (N_F)_2 = 3.6/\sqrt{(9.807)(8)} = 0.406$$

$$2y_1/8 = -1 + [1 + (8)(0.406)^2]^{1/2} \qquad y_1 = 2.09\ \text{m} \qquad y_1v_1 = y_2v_2$$

$$2.09v_1 = (8)(3.6) \qquad v_1 = 13.78\ \text{m/s}$$

14.369 Water in a horizontal channel accelerates smoothly over a bump and then undergoes a hydraulic jump, as in Fig. 14-93. If $y_1 = 1$ m and $y_3 = 30$ cm, estimate v_1, v_3, and y_4. Neglect friction.

▮

$$E_1 = E_3 \qquad y_1 + v_1^2/2g = y_3 + v_3^2/2g \qquad 1 + v_1^2/[(2)(9.807)] = 0.30 + v_3^2/[(2)(9.807)]$$

$$y_1v_1 = y_3v_3 \qquad v_1 = y_3v_3/y_1 = (0.30)(v_3)/1 = 0.3000v_3$$

$$1 + (0.3000v_3)^2/[(2)(9.807)] = 0.30 + v_3^2/[(2)(9.807)]$$

$$v_3 = 3.88\ \text{m/s} \qquad v_1 = (0.3000)(3.88) = 1.16\ \text{m/s}$$

$$2y_4/y_3 = -1 + [1 + 8(N_F)_3^2]^{1/2} \qquad N_F = v/\sqrt{gy}$$

$$(N_F)_3 = 3.88/\sqrt{(9.807)(0.30)} = 2.26 \qquad 2y_4/(0.30) = -1 + [1 + (8)(2.26)^2]^{1/2} \qquad y_4 = 82\ \text{cm}$$

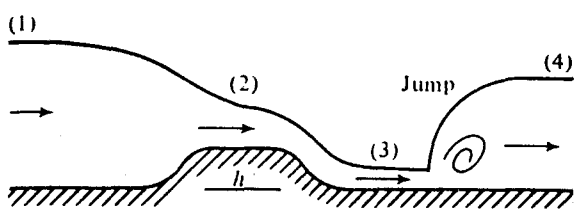


Fig. 14-93

14.370 Evaluate the bump height h in Prob. 14.369.

▮

$$y_1 + v_1^2/2g = y_2 + v_2^2/2g + h$$

$$1 + 1.16^2/[(2)(9.807)] = y_2 + v_2^2/[(2)(9.807)] + h$$

$$y_1v_1 = y_2v_2 \qquad (1)(1.16) = y_2v_2 \qquad v_2 = 1.16/y_2$$

Since flow over the bump is critical,

$$v_2 = v_c = \sqrt{gy_2} \qquad 1.16/y_2 = \sqrt{9.807y_2} \qquad y_2 = 0.516\text{ m} \qquad v_2 = 1.16/0.516 = 2.248\text{ m/s}$$

$$1 + 1.16^2/[(2)(9.807)] = 0.516 + 2.248^2/[(2)(9.807)] + h \qquad h = 29.5\text{ m}$$

14.371 Repeat Prob. 14.369 if $y_1 = 3$ ft and $y_3 = 1$ ft.

▮

$$E_1 = E_3 \qquad y_1 + v_1^2/2g = y_3 + v_3^2/2g \qquad 3 + v_1^2/[(2)(32.2)] = 1 + v_3^2/[(2)(32.2)]$$

$$y_1v_1 = y_3v_3 \qquad v_1 = y_3v_3/y_1 = (1)(v_3)/3 = 0.3333v_3$$

$$3 + (0.3333v_3)^2/[(2)(32.2)] = 1 + v_3^2/[(2)(32.2)]$$

$$v_3 = 12.04\text{ ft/s} \qquad v_1 = (0.3333)(12.04) = 4.01\text{ ft/s} \qquad 2y_4/y_3 = -1 + [1 + 8(N_F)_3^2]^{1/2}$$

$$N_F = v/\sqrt{gy} \qquad (N_F)_3 = 12.04/\sqrt{(32.2)(1)} = 2.12$$

$$2y_4/(1) = -1 + [1 + (8)(2.12)^2]^{1/2} \qquad y_4 = 2.54\text{ ft}$$

14.372 Evaluate the bump height h in Prob. 14.371.

▮

$$y_1 + v_1^2/2g = y_2 + v_2^2/2g + h$$

$$3 + 4.01^2/[(2)(32.2)] = y_2 + v_2^2/[(2)(32.2)] + h$$

$$y_1v_1 = y_2v_2 \qquad (3)(4.01) = y_2v_2 \qquad v_2 = 12.03/y_2$$

Since flow over the bump is critical,

$$v_2 = v_c = \sqrt{gy_2} \qquad 12.03/y_2 = \sqrt{32.2y_2} \qquad y_2 = 1.65\text{ ft} \qquad v_2 = 12.03/1.65 = 7.29\text{ ft/s}$$

$$3 + 4.01^2/[(2)(32.2)] = 1.65 + 7.29^2/[(2)(32.2)] + h \qquad h = 0.775\text{ ft} = 9.3\text{ in.}$$

14.373 Find the power loss in a 100-ft-wide hydraulic jump from depth 3 ft to depth 12 ft.

▮

$$E_j = (y_2 - y_1)^3/4y_1y_2 = (12-3)^3/[(4)(12)(3)] = 5.063\text{ ft} \qquad P = Q\gamma E_j \qquad N_F = v/\sqrt{gy}$$

$$2y_2/y_1 = -1 + [1 + 8(N_F)_1^2]^{1/2} \qquad (2)(12)/3 = -1 + [1 + (8)(N_F)_1^2]^{1/2} \qquad (N_F)_1 = 3.16$$

$$3.16 = v_1/\sqrt{(32.2)(3)} \qquad v_1 = 31.06\text{ ft/s} \qquad Q = A_1v_1 = [(100)(3)](31.06) = 9318\text{ ft}^3\text{/s}$$

$$P = (9318)(62.4)(5.063) = 2.944 \times 10^6\text{ ft-lb/s} = 5353\text{ hp}$$

14.374 At a certain section of a rectangular channel 10 ft wide, the depth of flow is 2 ft and the flow rate is 400 ft³/s. If a hydraulic jump occurs, will it be upstream or downstream of this section?

▮

$$v_c = \sqrt{gy} = \sqrt{(32.2)(2)} = 8.02\text{ ft/s} \qquad v = Q/A = 400/[(10)(2)] = 20.00\text{ ft/s}$$

Since $v > v_c$, the flow is supercritical, and a hydraulic jump must occur downstream.

14.375 A still canal is 2 m deep and the water behind a bore (Prob. 14.351) is 4 m deep. Obtain the propagation speed of the bore.

▮

$$N_F = v/\sqrt{gy} \qquad 2y_2/y_1 = -1 + [1 + 8(N_F)_1^2]^{1/2} \qquad (2)(4)/2 = -1 + [1 + (8)(N_F)_1^2]^{1/2}$$

$$(N_F)_1 = 1.73 \qquad 1.73 = v_1/\sqrt{(9.807)(2)} \qquad v_1 = c_{\text{bore}} = 7.66\text{ m/s}$$

14.376 In Prob. 14.375 suppose that the water upstream flows toward the bore at 4 m/s ground speed. What is the ground speed of the bore?

▮

$$v_{\text{bore/ground}} = v_{\text{bore/water}} + v_{\text{water/ground}} = -7.66 + 4 = -3.66\text{ m/s.}$$

14.377 Consider the flow under the sluice gate of Fig. 14-94. If $y_1 = 9$ ft and all losses are neglected except the dissipation in the jump, calculate y_2 and y_3 and the percentage dissipation. The channel is horizontal and wide.

▮

$$y_1 + V_1^2/2g = y_2 + V_2^2/2g \qquad 10 + 2^2/[(2)(32.2)] = y_2 + V_2^2/[(2)(32.2)]$$

$$V_1y_1 = V_2y_2 \qquad (2.2)(9) = V_2y_2 \qquad y_2 = 19.8/V_2$$

$$9 + 2.2^2/[(2)(32.2)] = 19.8/V_2 + V_2^2/[(2)(32.2)] \qquad V_2 = 23.0\text{ ft/s} \qquad \text{(by trial and error)}$$

$$y_2 = 19.8/23.0 = 0.861\text{ ft} \qquad 2y_3/y_2 = -1 + [1 + 8(N_F)_2^2]^{1/2} \qquad N_F = V/\sqrt{gy}$$

$$(N_F)_2 = 23.0/\sqrt{(32.2)(0.861)} = 4.37 \qquad 2y_3/0.861 = -1 + [1 + (8)(4.37)^2]^{1/2} \qquad y_3 = 4.91\text{ ft}$$

$$E_j = (y_3 - y_2)^3/4y_3y_2 = (4.91 - 0.861)^3/[(4)(4.91)(0.861)] = 3.93\text{ ft}$$

$$E_2 = y_2 + V_2^2/2g = 0.861 + 23^2/[(2)(32.2)] = 9.08\text{ ft}$$

$$\text{Percentage dissipation} = (3.93/9.08)(100\%) = 43.3\%$$

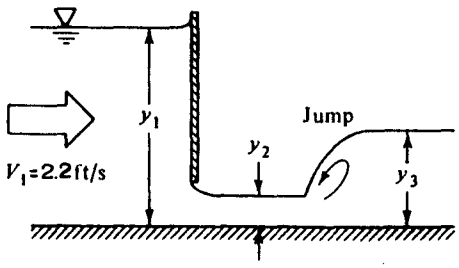


Fig. 14-94

14.378 If bottom friction is included in the sluice-gate flow of Fig. 14-94, the depths y_1, y_2, and y_3 will vary with x. What type of solution curves will we have in regions 1, 2, and 3?

▮ See Fig. 14-95.

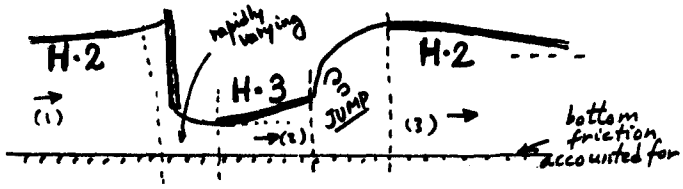


Fig. 14-95

14.379 A 10-cm-high bump in a wide horizontal water channel creates a hydraulic jump just upstream and the flow pattern in Fig. 14-96. Neglecting losses except in the jump, for the case $y_3 = 30$ cm, estimate V_4, y_4, V_1, and y_1.

▮ Since $y_4 < y_3$, assume flow over the hump is critical.

$$V = \sqrt{gy} \qquad V_3 = \sqrt{(9.807)(0.30)} = 1.715\text{ m}$$

$$E_2 = E_3 = E_4 = y_3 + V_3^2/2g + h = E_4 = 0.30 + 1.715^2/[(2)(9.807)] + 0.10 = E_4$$

$$E_2 = E_3 = E_4 = 0.5500\text{ m} \qquad E_2 = y_2 + V_2^2/2g = 0.5500\text{ m}$$

$$q = V_2y_2 = V_3y_3 = V_4y_4 = (1.715)(0.30) = 0.5145\text{ m}^2/\text{s} \qquad V_2 = 0.5145/y_2$$

$$y_2 + (0.5145/y_2)^2/[(2)(9.807)] = 0.5500$$

$$y_2 = 0.495\text{ m} \qquad \text{(by trial and error)} \qquad V_2 = 0.5145/0.495 = 1.039\text{ m/s}$$

$$N_F = V/\sqrt{gy} \qquad (N_F)_2 = 1.039/\sqrt{(9.807)(0.495)} = 0.472 \qquad \text{(subcritical)}$$

$$2y_1/y_2 = -1 + [1 + 8(N_F)_2^2]^{1/2} \qquad 2y_1/0.495 = -1 + [1 + (8)(0.472)^2]^{1/2} \qquad y_1 = 0.165\text{ m}$$

$$V_1 = q/y_1 = 0.5145/0.165 = 3.12\text{ m/s} \qquad E_3 = E_4 = 0.5500\text{ m} \qquad y_4 + V_4^2/2g = 0.5500\text{ m}$$

$$V_4y_4 = 0.5145\text{ m}^2/\text{s} \qquad V_4 = 0.5145/y_4 \qquad y_4 + (0.5145/y_4)^2/[(2)(9.807)] = 0.5500\text{ m}$$

$$y_4 = 0.195\text{ m} \qquad \text{(by trial and error)} \qquad V_4 = 0.5145/0.195 = 2.64\text{ m/s}$$

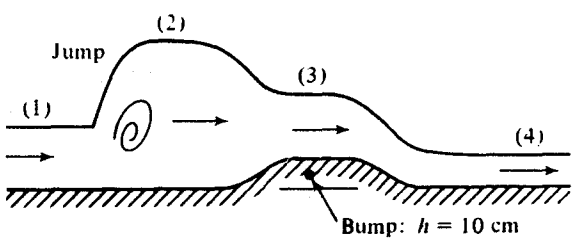


Fig. 14-96

14.380 Water flows in a wide channel at $q = 10\,(\text{m}^3/\text{s})/\text{m}$ and $y_1 = 1.25$ m. If the flow undergoes a hydraulic jump, compute **(a)** y_2, **(b)** V_2, **(c)** $(N_F)_2$, **(d)** E_j, **(e)** the percentage dissipation, **(f)** the power dissipated per unit width, and **(g)** the temperature rise due to dissipation if $c_p = 4200\,J/(\text{kg-K})$.

▮ (*a*) $V_1 = q/y_1 = 10/1.25 = 8.00\ \text{m/s}$ $\quad N_F = V/\sqrt{gy}$ $\quad (N_F)_1 = 8.00/\sqrt{(9.807)(1.25)} = 2.285$

$$2y_2/y_1 = -1 + [1 + 8(N_F)_1^2]^{1/2} \qquad 2y_2/1.25 = -1 + [1 + (8)(2.285)^2]^{1/2} \qquad y_2 = 3.46\ \text{m}$$

(*b*) $V_1 y_1 = V_2 y_2 \quad (8.00)(1.25) = (V_2)(3.46) \quad V_2 = 2.89\ \text{m/s}$

(*c*) $(N_F)_2 = 2.89/\sqrt{(9.807)(3.46)} = 0.496$

(*d*) $E_j = (y_2 - y_1)^3/4y_2y_1 = (3.46 - 1.25)^3/[(4)(3.46)(1.25)] = 0.624\ \text{m}$

(*e*) $E_1 = y_1 + v_1^2/2g = 1.25 + 8.00^2/[(2)(9.807)] = 4.51\ \text{m}$

Percentage dissipation $= E_j/E_1 = 0.624/4.51 = 0.138$ or 13.8 percent

(*f*) $P = q\gamma E_j \quad P_{\text{dissipated}} = (10)(9.79)(0.624) = 61.1\ \text{kW/m}$

(*g*) $P = \dot{m}c_p\,\Delta T = \rho q c_p\,\Delta T \quad (61.1)(1000) = (1000)(10)(4200)(\Delta T) \quad \Delta T = 0.0015\ \text{K}$

14.381 Water flows over a spillway into a sluice 10 m wide. Before the jump the water has a depth of 1 m and a velocity of 18 m/s. Determine the Froude number before the jump and the depth of flow after the jump.

▮ $N_F = V/\sqrt{gy} \quad (N_F)_1 = 18/\sqrt{(9.807)(1)} = 5.75$

$$y_2 = [-y_1 + \sqrt{y_1^2 + (8Q^2/gb^2)(1/y_1)}]/2 \qquad Q = A_1V_1 = [(10)(1)](18) = 180\ \text{ft}^3/\text{s}$$

$$y_2 = \{-1 + \sqrt{1^2 + [(8)(180)^2/(9.807)(10)^2](\tfrac{1}{1})}\}/2 = 7.64\ \text{m}$$

14.382 A rectangular channel has a width of 5 ft and a flow of 10 cfs; the depth in front of a hydraulic jump is 3 in. Determine the specific energy (in ft-lb/slug) behind the jump.

▮ $y_2 = [-y_1 + \sqrt{y_1^2 + (8Q^2/gb^2)(1/y_1)}]/2$

$$= \{-\tfrac{3}{12} + \sqrt{(\tfrac{3}{12})^2 + [(8)(10)^2/(32.2)(5)^2][1/(\tfrac{3}{12})]}\}/2 = 0.880\ \text{ft}$$

$\text{KE} = v^2/2 \quad v_2 = Q/A_2 = 10/[(0.880)(5)] = 2.27\ \text{ft/s} \quad (\text{KE})_2 = 2.27^2/2 = 2.58\ \text{ft-lb/slug}$

14.383 Water in a rectangular, finished-concrete channel overflows a dam, as shown in Fig. 14-97, and goes into a stilling basin at which there is a hydraulic jump. If the channel is of width 6 m, has slope 0.003, and carries 18 m³/s, calculate the depth y at O.

▮ $\Delta x = ([1 - (Q^2b/gA^3)]/\{s_0 - (n/\kappa)^2[Q^2/(R_h^{4/3}A^2)]\})\,\Delta y$

$$A = (5)(6) = 30.00\ \text{m}^2 \qquad R_h = A/p_w = 30.00/(5 + 6 + 5) = 1.875\ \text{m}$$

$$60 = \left\{\frac{1 - (18^2)(6)/(9.807)(30.00)^3}{0.003 - (0.012/1.0)^2[18^2/(1.875)^{4/3}(30.00)^2]}\right\}(\Delta y)$$

$$\Delta y = 0.180\ \text{m} \qquad y = 5 + 0.180 = 5.180\ \text{m}$$

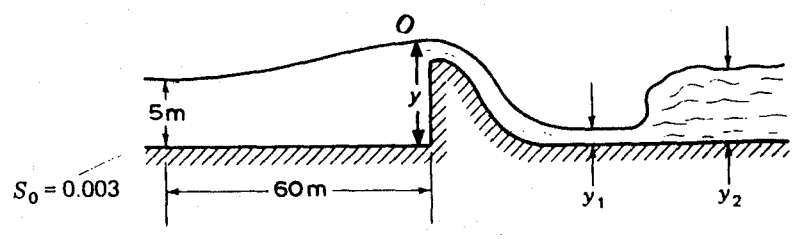


Fig. 14-97

14.384 Does linear averaging significantly improve the solution of Prob. 14.383?

▮ $\Delta x = ([1 - (Q^2b/gA^3)]/\{s_0 - (n/\kappa)^2[Q^2/(R_h^{4/3}A^2)]\})\,\Delta y$

At O, $A = (5.180)(6) = 31.08\ \text{m}^2$ and $R_h = A/p_w = 31.08/(5.180 + 6 + 5.180) = 1.900\ \text{m}$; therefore,

$$A_{av} = (30.00 + 31.08)/2 = 30.54\ \text{m}^2 \qquad (R_h)_{av} = (1.875 + 1.900)/2 = 1.888\ \text{m}$$

$$60 = \left\{\frac{1 - (18^2)(6)/(9.807)(30.54)^3}{0.003 - (0.012/1.0)^2[18^2/(1.888)^{4/3}(30.54)^2]}\right\}(\Delta y)$$

$$\Delta y = 0.180\ \text{m}$$

14.385 Determine the depth y_2 at the hydraulic jump of Prob. 14.383.

▮ Neglecting friction at the dam,

$$E_1 = E_0 \qquad E = y + q^2/2y^2g \qquad E_0 = 5.180 + (\tfrac{18}{6})^2/[(2)(5.180)^2(9.807)] = 5.197\ \text{m}$$

$$E_1 = y_1 + (\tfrac{18}{6})^2/[(2y_1^2)(9.807)] = 5.197\ \text{m} \qquad y_1 = 0.306\ \text{m} \qquad \text{(by trial and error)}$$

$$y_2 = [-y_1 + \sqrt{y_1^2 + (8Q^2/gb^2)(1/y_1)}]/2$$

$$= \{-0.306 + \sqrt{0.306^2 + [(8)(18)^2/(9.807)(6)^2](1/0.306)}\}/2 = 2.301\ \text{m}$$

14.386 Water flows in a 4-m-wide rectangular channel at Froude number $\sqrt{10}$; the depth of flow is 1 m. If the water undergoes a hydraulic jump, what is the Froude number downstream of the jump?

▮

$$N_F = v/\sqrt{gy} \qquad \sqrt{10} = v/\sqrt{(9.807)(1)} \qquad v = 9.903\ \text{m/s}$$

$$Q = Av = [(4)(1)](9.903) = 39.61\ \text{m}^3/\text{s}$$

$$y_2 = [-y_1 + \sqrt{y_1^2 + (8Q^2/gb^2)(1/y_1)}]/2$$

$$= \{-1 + \sqrt{1^2 + [(8)(39.61)^2/(9.807)(4)^2](1/1)}\}/2 = 4.000\ \text{m}$$

$$v_2 = Q/A_2 = 39.61/[(4.000)(4)] = 2.476\ \text{m/s} \qquad (N_F)_2 = 2.476/\sqrt{(9.807)(4.000)} = 0.395$$

14.387 Water in a 10-m-wide rectangular channel experiences a jump in depth from 2 m to 6 m. Find the Froude numbers on either side of the jump.

▮

$$y_2 = [-y_1 + \sqrt{y_1^2 + (8Q^2/gb^2)(1/y_1)}]/2 \qquad 6 = \{-2 + \sqrt{2^2 + [8Q^2/(9.807)(10)^2](\tfrac{1}{2})}\}/2$$

$$Q = 217\ \text{m}^3/\text{s}$$

$$N_F = v/\sqrt{gy} \qquad v_1 = Q/A_1 = 217/[(10)(2)] = 10.85\ \text{m/s} \qquad (N_F)_1 = 10.85/\sqrt{(9.807)(2)} = 2.45$$

$$v_2 = Q/A_2 = 217/[(10)(6)] = 3.62\ \text{m/s} \qquad (N_F)_2 = 3.62/\sqrt{(9.807)(6)} = 0.472$$

14.388 Water flows in a rectangular concrete channel and undergoes a hydraulic jump such that 60 percent of its mechanical energy is to be dissipated. If the volume flow rate is $100\ \text{m}^3/\text{s}$ and the width of the channel is 5 m, what must the Froude number be just before the jump? Set up the proper equations but do not actually solve.

▮

$$N_F = v/\sqrt{gy} \qquad E = y + v^2/2g \qquad v_1 = Q/A_1 = 100/(5y_1) = 20.00/y_1$$

$$E_1 = y_1 + (20.00/y_1)^2/[(2)(9.807)] = y_1 + 20.39/y_1^2 \qquad E_j = [y_2^3 - y_1^3 + y_1y_2(y_1 - y_2)]/4y_1y_2$$

$$0.60E_1 = (0.60)(y_1 + 20.39/y_1^2) = [y_2^3 - y_1^3 + y_1y_2(y_1 - y_2)]/4y_1y_2 \qquad (1)$$

$$y_2 = [-y_1 + \sqrt{y_1^2 + (8Q^2/gb^2)(1/y_1)}]/2 = \{-y_1 + \sqrt{y_1^2 + [(8)(100)^2/(9.807)(5)^2](1/y_1)}\}/2 \qquad (2)$$

Solve Eqs. (*1*) and (*2*) to find y_1 and y_2. Then, $v_1 = Q/A_1 = 100/5y_1$, $(F_R)_1 = (100/5y_1)/\sqrt{(9.807)(y_1)} = 6.386y^{-3/2}$.

14.389 Water is flowing from a spillway into a stilling basin, as shown in Fig. 14-98. The elevation y_A ahead of the spillway is 8 m. The width of the rectangular channel is 10 m. If the stilling basin dissipates half of the mechanical energy, what is the volume flow rate? Set up three simultaneous equations for y_1, y_2, and Q.

$$E = y + v^2/2g \qquad E_A = E_1$$

$$8 + Q^2/[(80)^2(2g)] = y_1 + Q^2/[(10y_1)^2(2g)] \tag{1}$$

$$0.50E_1 = E_j$$

$$0.50\{8 + Q^2/[(80)^2(2g)]\} = [y_2^3 - y_1^3 + y_1 y_2(y_1 - y_2)]/4y_1 y_2 \tag{2}$$

$$y_2 = [-y_1 + \sqrt{y_1^2 + (8Q^2/gb^2)(1/y_1)}]/2 = [-y_1 + \sqrt{y_1^2 + [8Q^2/(9.807)(10)^2](1/y_1)}]/2 \tag{3}$$

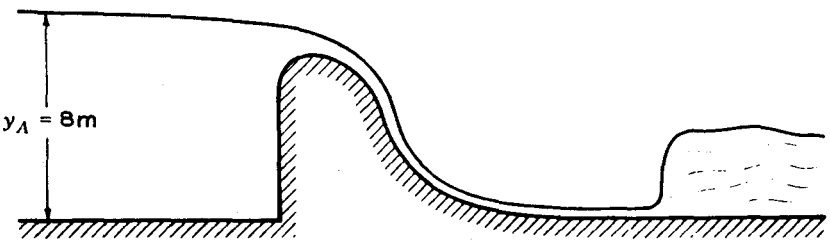


Fig. 14-98

14.390 Water having a steady known volumetric flow rate Q is moving in a rectangular channel at supercritical speed up an adverse slope. It undergoes a hydraulic jump as shown in Fig. 14-99. If we know y_B and y_A at positions L_1 and L_2 apart, how do we approximately locate the position of the hydraulic jump, i.e., how do we get L_1 and L_2? The channel has a known value of n. Explain the simplest method. The width is b.

Use the equation $\Delta L = ([1 - (Q^2b/gA^3)]/\{S_0 - (n/\kappa)^2[Q^2/(R_H^{4/3}A^2)]\})\,\Delta y$ and guess at a value L_1 and solve for $(\Delta y)_A$. Now compute $y_1 = y_A + (\Delta y)_A$ before the jump. Again go to the equation above and take L_2 and again solve for $(\Delta y)_B$ in the flow upstream of the jump: $y_2 = y_B - (\Delta y)_B$. Now insert y_1 and y_2 into the jump equation $y_2 = [-y_1 + \sqrt{y_1^2 + (8Q^2/gb^2)(1/y_1)}]/2$ to see whether this equation is satisfied for the given volumetric flow Q. If not, go back and choose a different value of L_1 proceeding in this way until the jump equation is satisfied.

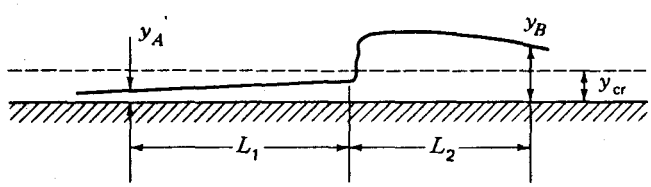


Fig. 14-99

14.391 The distance L_1 in Fig. 14-99 is 200 m and y_A is half the critical value. The channel is finished concrete with an adverse slope of -0.005. What is y_B at a distance $L_2 = 100$ m from the hydraulic jump? Solve in the simplest manner to get an approximate result. The flow rate is 200 m^3/s and the width of the channel is 7 m.

$$y_c = (q^2/g)^{1/3} = [(\tfrac{200}{7})^2/9.807]^{1/3} = 4.366\text{ m} \qquad y_A = 4.366/2 = 2.183\text{ m}$$

$$\Delta L = ([1 - (Q^2b/gA^3)]/\{S_0 - (n/\kappa)^2[Q^2/(R_H^{4/3}A^2)]\})\,\Delta y$$

$$A_1 = (7)(2.183) = 15.28\text{ m}^2 \qquad R_H = A/p_w \qquad (R_H)_1 = 15.28/(2.183 + 7 + 2.183) = 1.344\text{ m}$$

$$200 = \left\{\frac{1 - (200)^2(7)/[(9.807)(15.28)^3]}{-0.005 - (0.012/1)^2[200^2/(1.344)^{4/3}(15.28)^2]}\right\}(\Delta y)_A \qquad (\Delta y)_A = 0.618\text{ m}$$

$$y_1 = 2.183 + 0.618 = 2.801\text{ m}$$

$$y_2 = [-y_1 + \sqrt{y_1^2 + (8Q^2/gb^2)(1/y_1)}]/2 = \{-2.801 + \sqrt{2.801^2 + [(8)(200)^2/(9.807)(7)^2](1/2.801)}\}/2 = 6.435\text{ m}$$

$$A_2 = (7)(6.435) = 45.04\text{ m}^2 \qquad (R_H)_2 = 45.04/(6.435 + 7 + 6.435) = 2.267\text{ m}$$

$$100 = \left\{\frac{1 - (200)^2(7)/[(9.807)(45.04)^3]}{-0.005 - (0.012/1)^2[200^2/(2.267)^{4/3}(45.04)^2]}\right\}(\Delta y)_B \qquad (\Delta y)_B = -0.866\text{ m}$$

$$(\Delta y)_B = y_B - y_2 \qquad -0.866 = y_B - 6.435 \qquad y_B = 5.569\text{ m}$$

14.392 For a rectangular channel, develop an expression for the relation between the depths before and after a hydraulic jump. Refer to Fig. 14-100.

For the free body between sections 1 and 2, considering a unit width of channel and unit flow q, $P_1 = \gamma\bar{h}A = \gamma(\frac{1}{2}y_1)y_1 = \frac{1}{2}\gamma y_1^2$ and, similarly, $P_2 = \frac{1}{2}\gamma y_2^2$. From the principle of impulse and momentum, $\Delta P_x\,dt = \Delta$ linear momentum $= (W/g)(\Delta V_x)$, $\frac{1}{2}\gamma(y_2^2 - y_1^2)\,dt = (\gamma q\,dt/g)(V_1 - V_2)$.

Since $V_2 y_2 = V_1 y_1$ and $V_1 = q/y_1$, the above equation becomes

$$q^2/g = \tfrac{1}{2} y_1 y_2 (y_1 + y_2) \qquad (1)$$

Since $q^2/g = y_c^3$,

$$y_c^3 = \tfrac{1}{2} y_1 y_2 (y_1 + y_2) \qquad (2)$$

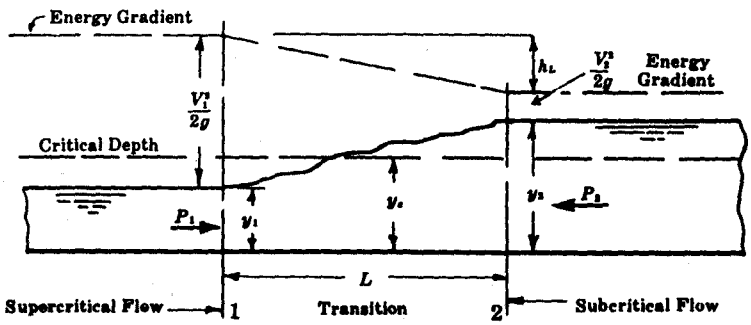


Fig. 14-100

14.393 A rectangular channel, 20 ft wide, carries 400 cfs and discharges onto a 20-ft-wide apron with no slope with a mean velocity of 20 fps. What is the height of the hydraulic jump? What energy is lost in the jump?

▮ $q^2/g = y_1 y_2 (y_1 + y_2)/2 \qquad y_1 = q/v_1 = (\frac{400}{20})/20 = 1.00\text{ ft} \qquad (\frac{400}{20})^2/32.2 = (1.00y_2)(1.00 + y_2)/2$

$0.500y_2^2 + 0.500y_2 - 12.42 = 0 \qquad y_2 = 4.51\text{ ft} \qquad \text{Height of jump} = 4.51 - 1.00 = 3.51\text{ ft}$

$E_j = (y_2 - y_1)^3/4y_1 y_2 = (4.51 - 1.00)^3/[(4)(4.51)(1.00)] = 2.40\text{ ft}$

$\text{Energy lost} = Q\gamma E_j = (400)(62.4)(2.40) = 59\,900\text{ ft-lb/s}$

14.394 A rectangular channel, 16 ft wide, carries a flow of 192 cfs. The depth of water on the downstream side of a hydraulic jump is 4.20 ft. What is the upstream depth? What is the loss of head?

▮ $q^2/g = y_1 y_2 (y_1 + y_2)/2 \qquad (\frac{196}{16})^2/32.2 = (y_1)(4.20)(y_1 + 4.20)/2$

$2.10y_1^2 + 8.82y_1 - 4.472 = 0 \qquad y_1 = 0.457\text{ ft}$

$E_j = (y_2 - y_1)^3/4y_1 y_2 = (4.20 - 0.457)^3/[(4)(0.457)(4.20)] = 6.83\text{ ft}$

14.395 After flowing over the concrete spillway of a dam, 9000 cfs then passes over a level concrete apron ($n = 0.013$). The velocity of the water at the bottom of the spillway is 42.0 ft/s and the width of the apron is 180 ft. Conditions will produce a hydraulic jump, the depth in the channel below the apron being 10.0 ft. In order that the jump be contained on the apron, (*a*) how long should the apron be built? (*b*) How much energy is lost from the foot of the spillway to the downstream side of the jump?

▮ Refer to Fig. 14-101.

(*a*) $q^2/g = y_1 y_2 (y_1 + y_2)/2 \qquad (\frac{9000}{180})^2/32.2 = 10y_2(10 + y_2)/2$

$5y_2^2 + 50y_2 - 77.64 = 0 \qquad y_2 = 1.37\text{ ft}$

$y_1 = q/V_1 = (\frac{9000}{180})/42.0 = 1.19\text{ ft} \qquad L = [(V_2^2/2g + y_2) - (V_1^2/2g + y_1)]/(S_0 - S)$

$V_2 = q/y_2 = (\frac{9000}{180})/1.37 = 36.50\text{ ft/s} \qquad S = (nV/1.486R^{2/3})^2$

$V_{av} = (42.0 + 36.40)/2 = 39.20\text{ ft/s}$

$R = A/p_w \qquad R_1 = (180)(1.19)/(1.19 + 180 + 1.19) = 1.174\text{ ft}$

$R_2 = (180)(1.37)/(1.37 + 180 + 1.37) = 1.349\text{ ft}$

$R_{av} = (1.174 + 1.349)/2 = 1.262\text{ ft} \qquad S = \{(0.013)(39.20)/[(1.486)(1.262)^{2/3}]\}^2 = 0.08623$

$L = \{36.50^2/[(2)(32.2)] + 1.37 - 42.0^2/[(2)(32.2)] - 1.19\}/(0 - 0.08623) = 75.7\text{ ft}$

The length of the jump L_3 from B to C is from $4.3y_3$ to $5.2y_3$. Assuming the conservative value of $5.0y_3$, $L_3 = (5.0)(10.0) = 50$ ft. Hence, $L_{ABC} = 76 + 50 = 126$ ft (approximately).

(*b*) $E = y + V^2/2g \qquad E_A = 1.19 + 42.0^2/[(2)(32.2)] = 28.58\text{ ft}$

$V_3 = Q/A_3 = 9000/[(10.0)(180)] = 5.00\text{ ft/s}$

$E_C = 10.0 + 5.00^2/[(2)(32.2)] = 10.39\text{ ft} \qquad E_{lost} = 28.58 - 10.39 = 18.19\text{ ft}$

$P = Q\gamma E \qquad P_{lost} = (9000)(62.4)(18.19) = 10.22 \times 10^6\text{ ft-lb/s}$

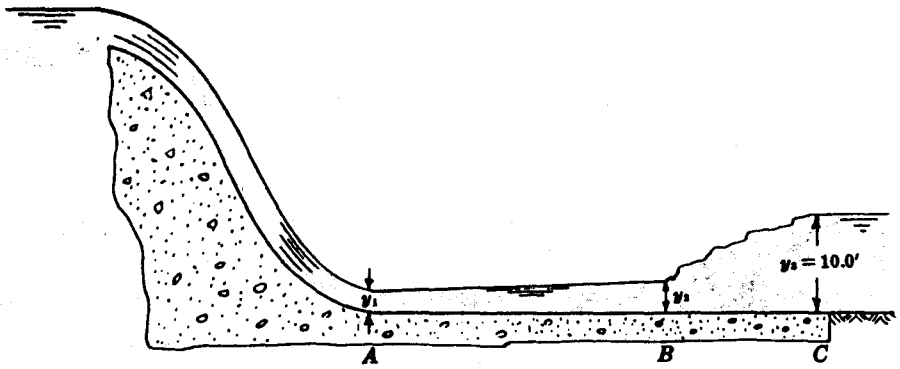


Fig. 14-101

14.396 Determine the elevation of the spillway apron if $q = 50$ cfs/ft, $h = 9$ ft, $D = 63$ ft, and the spillway crest is at elevation 200.0 ft.

▮
$$(\pi)_1(\pi_3 - \pi_2)(\pi_3 + 1)^{1/2} + 0.353 = \sqrt{\tfrac{1}{8} + (2.828)(\pi_1)(\pi_3 + 1)^{3/2}}$$
$$\pi_1 = g^{1/2}h^{3/2}/q = (32.2)^{1/2}(9)^{3/2}/50 = 3.06$$
$$\pi_2 = D/h = \tfrac{63}{9} = 7.00 \qquad \pi_3 = d/h = d/9$$
$$(3.06)(d/9 - 7.00)(d/9 + 1)^{1/2} + 0.353 = \sqrt{\tfrac{1}{8} + (2.828)(3.06)(d/9 + 1)^{3/2}}$$
$d = 77.9$ ft (by trial and error) Elevation of spillway apron $= 200.0 - 77.9 = 122.1$ ft

14.397 Establish the equation for flow over a broad-crested weir assuming no lost head. See Fig. 14-102.

▮ At the section where critical flow occurs, $q = V_c y_c$. But $y_c = V_c^2/g = \frac{2}{3}E$, and $V_c = \sqrt{g(\frac{2}{3}E_c)}$. Hence the theoretical value of flow q becomes $q = \sqrt{g(\frac{2}{3}E_c)} \times \frac{2}{3}E_c = 3.09E_c^{3/2}$. However, the value of E_c is difficult to measure accurately, because the critical depth is difficult to locate. The practical equation becomes $q = CH^{3/2} \approx 3H^{3/2}$. The weir should be calibrated in place to obtain accurate results.

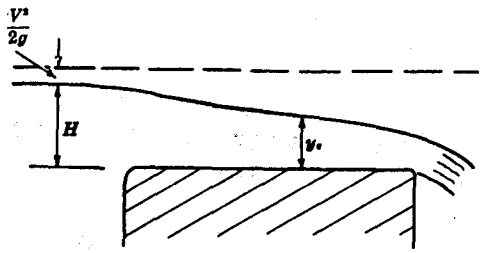


Fig. 14-102

14.398 Develop an expression for a critical-flow meter (Fig. 14-103).

▮ An excellent method of measuring flow in open channels is by means of a constriction. The measurement of the critical depth is not required. The depth y_1 is measured a short distance upstream from the constriction. The raised floor should be about $3y_c$ long and of such height as to have the critical velocity occur on it.

For a rectangular channel of constant width, the Bernoulli equation is applied between sections 1 and 2, in which the lost head in accelerated flow is taken as one-tenth of the difference in velocity heads, i.e., $y_1 + (V_1^2/2g) - (\frac{1}{10})[(V_c^2/2g) - (V_1^2/2g)] = [y_c + (V_c^2/2g) + z]$, which neglects the slight drop in the channel bed between 1 and 2. Recognizing that $E_c = y_c + V_c^2/2g$, we rearrange as follows: $[y_1 + (1.10V_1^2/2g)] = z + 1.0E_c + (\frac{1}{10})(\frac{1}{3}E_c)$, $[y_1 - z + (1.10V_1^2/2g)] = 1.033E_c = (1.033)(\frac{2}{3}\sqrt[1]{q^2/g})$, or

$$q = (2.94)(y_1 - z + 1.10V_1^2/2g)^{3/2} \tag{1}$$

Since $q = V_1 y_1$,
$$q = (2.94)(y_1 - z + 0.0171q^2/y_1^2)^{3/2} \tag{2}$$

14.399 Consider a rectangular channel 10 ft wide with the critical-flow meter of Fig. 14-103 having dimension $z = 1.10$ ft. If the measured depth y_1 is 2.42 ft, what is the discharge?

▮ $q = (2.94)(y_1 - z + 0.0171q^2/y_1^2)^{3/2} = (2.94)(2.42 - 1.10 + 0.0171q^2/2.42^2)^{3/2}$. As a first approximation, neglect the last term involving q. $q = (2.94)(2.42 - 1.10)^{3/2} = 4.46$ cfs/ft. Try $q = 4.80$ cfs/ft: $q = 2.94[2.42 - 1.10 + (0.0171)(4.80)^2/2.42^2]^{3/2} = 4.80$ cfs/ft (O.K.), $Q = (4.80)(10) = 48.0$ ft^3/s.

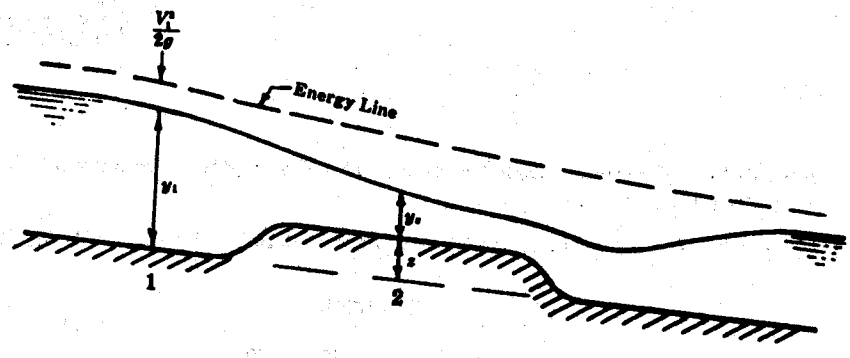


Fig. 14-103

14.400 In a 100-ft-wide rectangular channel, the depth upstream from a hydraulic jump is 4 ft. What flow rate is required in order for the downstream depth to be (*a*) twice the upstream depth and (*b*) ten times the upstream depth?

▮

$$q = \{[(d_1 + d_2)/2](gd_1d_2)\}^{1/2}$$

(*a*) $d_2 = (2)(4) = 8$ ft $\quad q = \{[(4+8)/2][(32.2)(4)(8)]\}^{1/2} = 78.63$ cfs/ft

$$Q = (100)(78.63) = 7863 \text{ ft}^3/\text{s}$$

(*b*) $d_2 = (10)(4) = 40$ ft $\quad q = \{[(4+40)/2][(32.2)(4)(40)]\}^{1/2} = 336.7$ cfs/ft

$$Q = (100)(336.7) = 33\,670 \text{ ft}^3/\text{s}$$

CHAPTER 15
Flood Routing

15.1 If the channel width for the stream described by Fig. 15-1 is $b = 28$ ft at stage $y = 3.5$ ft, determine the local velocity of a small-amplitude wave.

▮ $u = (1/b)(dQ/dy)$. From Fig. 15-1, $dQ/dy = 300$ cfs/ft when $y = 3.5$ ft. $u = (1/28)(300) = 10.7$ ft/s.

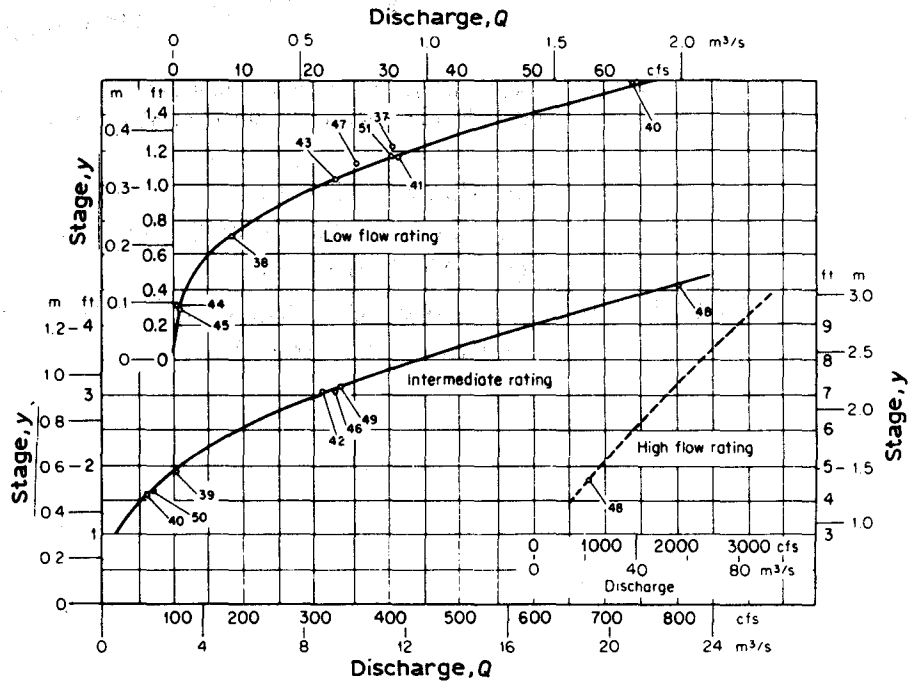


Fig. 15-1

15.2 Assume the stage-discharge relation $Q = (k)(y - a)^b$, where, in SI units, $a = 0.059$, $b = 2.55$, and $k = 10.30$. Compute the velocity of a monoclinal wave of small height at stage 1.5 m, where the channel is 19 m wide.

▮ $u = (1/w)(dQ/dy)$ $\quad Q = (10.30)(y - 0.059)^{2.55}$ $\quad dQ/dy = (2.55)(10.30)(y - 0.059)^{1.55}$

For $y = 1.5$ m, $dQ/dy = (2.55)(10.30)(1.5 - 0.059)^{1.55} = 46.27$ (m³/s)/m, $u = (1/19)(46.27) = 2.44$ m/s.

15.3 Repeat Prob. 15.1 if $b = 8$ m at $y = 0.8$ m.

▮ $u = (1/w)(dQ/dy)$. From Fig. 15-1, $dQ/dy = 20$ (m³/s)/m when $y = 0.8$ m. $u = (1/8)(20) = 2.50$ m/s.

15.4 Given the hydrographs tabulated below [I = inflow, O = outflow; M = midnight, N = noon], find the 12-hourly storage values for the reach. Ignore local inflow.

day	hour	I, m³/s	O, m³/s	day	hour	I, m³/s	O, m³/s
1	M	36	58	7	N	196	341
2	N	43	46		M	153	272
	M	121	42	8	N	128	219
3	N	348	64		M	101	180
	M	575	149	9	N	86	153
4	N	719	329		M	71	124
	M	741	536	10	N	60	104
5	N	615	678		M	55	88
	M	440	681	11	N	46	73
6	N	328	560		M	41	62
	M	255	439	12	N	37	52

▮ $[(I_1 + I_2)/2]t - [(O_1 + O_2)/2]t = S_2 - S_1$. Taking minimum storage equal to zero, and noting that $t = 0.5$ day $= 0.432 \times 10^5$ s, computations are carried out as in the following table. The last two columns are for use in Prob. 15.8.

day	hour	I, m³/s	O, m³/s	$\bar{I} = (I_1+I_2)/2$	$\bar{O} = (O_1+O_2)/2$	$\Delta S/t = \bar{I} - \bar{O}$	$S/t = \Sigma(\bar{I} - \bar{O})$	S, 10^5, m³	$xI + (1-x)O$	
									$x = 0.2$	$x = 0.3$
1	M	36	58				13	6	54	51
				39	52	−13				
2	N	43	46				0	0	45	45
				82	44	38				
	M	121	42				38	16	58	66
				234	53	181				
3	N	348	64				219	95	121	149
				462	106	356				
	M	575	149				575	248	234	277
				647	239	408				
4	N	719	329				983	425	407	446
				730	432	298				
	M	741	536				1281	553	577	598
				678	607	71				
5	N	615	678				1352	584	665	659
				528	679	−151				
	M	440	681				1201	519	633	609
				384	620	−236				
6	N	328	560				965	417	514	490
				292	499	−207				
	M	255	439				758	327	402	384
				226	390	−164				
7	N	196	341				594	257	312	298
				174	306	−132				
	M	153	272				462	200	248	236
				140	245	−105				
8	N	128	219				357	154	201	192
				114	199	−85				
	M	101	180				272	118	164	156
				94	166	−72				
9	N	86	153				200	86	140	133
				78	138	−60				
	M	71	124				140	60	113	108
				66	114	−48				
10	N	60	104				92	40	95	91
				58	96	−38				
	M	55	88				54	23	81	78
				50	80	−30				
11	N	46	73				24	10	68	65
				44	68	−24				
	M	41	62				0	0	58	56

15.5 Given the following inflow data to a reservoir and the reservoir relationships of Fig. 15-2, route the flood through the reservoir. The initial outflow from the reservoir is 1.7 m³/s, and the initial value of $2S/t + O$ is 9.0 m³/s.

day	hour	I, m³/s
1	Noon	2.0
	Midnight	5.2
2	Noon	10.1
	Midnight	12.2
3	Noon	8.5
	Midnight	4.7
4	Noon	2.3

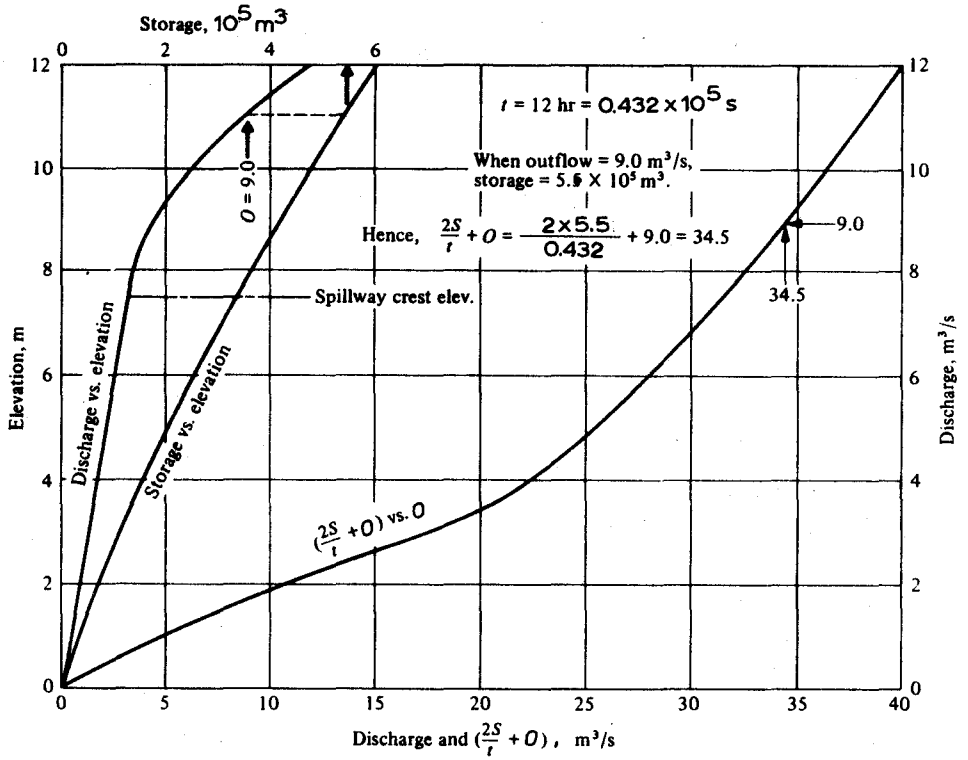


Fig. 15-2

▮ $$I_1 + I_2 + (2S_1/t - O_1) = 2S_2/t + O_2 \qquad 2S_1/t - O_1 = 2S_2/t + O_2 - 2O_2$$

$$2S_1/t - O_1 = 9.0 - (2)(1.7) = 5.6 \text{ m}^3/\text{s} \qquad 2.0 + 5.2 + 5.6 = 2S_2/t + O_2 \qquad 2S_2/t + O_2 = 12.8 \text{ m}^3/\text{s}$$

From Fig. 15-2, $O_2 = 2.3$ m³/s. Succeeding computations are carried out in the same manner with the results as tabulated below.

day	hour	I, m³/s	$(2S/t) - O$, m³/s	$(2S/t) + O$, m³/s	O, m³/s
1	Noon	2.0	5.6	9.0	1.7
	Midnight	5.2	8.2	12.8	2.3
2	Noon	10.1	14.9	23.5	4.3
	Midnight	12.2	16.2	37.2	10.5
3	Noon	8.5	16.3	36.9	10.3
	Midnight	4.7	16.3	29.5	6.6
4	Noon	2.3		23.3	4.2

15.6 A reservoir has an area of 295 acres at spillway level, and its banks are vertical for several feet above spillway level. The spillway is 16 ft long and has a coefficient of 3.75. If the inflow data of Prob. 15.4, with the units changed to cfs, apply to this reservoir, compute the maximum pool level and maximum discharge to be expected if the reservoir is initially at the spillway level at midnight on the first.

▮ The discharge over the spillway is ($C = 3.75$ and $L = 16$ ft): $Q = CLy^{3/2} = (3.75)(16)y^{3/2} = 60y^{3/2}$ cfs. The storage is equal to the reservoir area multiplied by the height of water above the spillway crest: $S = 295y$ acre-ft $= 149y$ cfs-day. From these equations and $t = 0.5$ day, we obtain:

y, ft	Q, cfs	S, cfs-day	$(2S/t) + O$, cfs
1	60	149	656
2	170	298	1362
3	312	447	2100
4	480	596	2864
5	671	745	3651

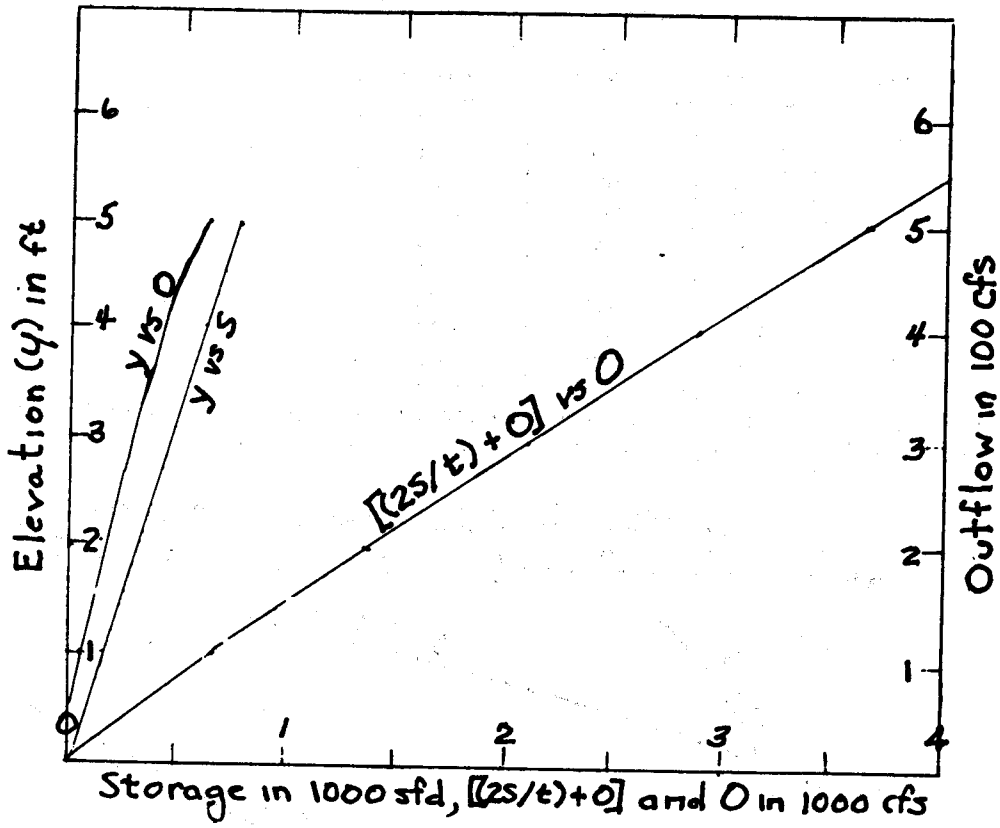


Fig. 15-3

These data provide the basis for deriving the routing curves of Fig. 15-3. The inflow hydrograph of Prob. 15.4 is routed as follows: $I_1 + I_2 + [(2S_1/t) - O_1] = (2S_2/t) + O_2$.

day	hour	I, cfs	$(2S_1/t) - O$, cfs	$(2S_2/t) + O$, cfs	O, cfs
1	M	36	0	0	0
2	N	43	71	79	4
	M	121	207	235	14
3	N	348	562	676	57
	M	575	1131	1485	177
4	N	719	1715	2425	355
	M	741	2145	3175	515
5	N	615	2321	3501	590
	M	440	2256	3376	560
6	N	328		3024	480

The peak flow is approximately 590 cfs and the maximum pool elevation is $y = (590/60)^{0.6667} = 4.59$ ft.

15.7 Tabulated below are the elevation-storage and elevation-discharge data for a small reservoir. The inflow hydrograph of Prob. 15.4, in units of cfs, applies to this reservoir. Assuming the pool elevation to be 875 ft at midnight on the first, find the maximum pool elevation and peak outflow rate.

elevation, ft	storage, acre-ft	discharge, cfs	elevation, ft	storage, acre-ft	discharge, cfs
862	0	0	882	1220	101
865	41	0	884	1633	232
870	202	0	886	2270	395
875	502	0	888	3152	600
880	1003	0			

From the data provided, and noting that $t = 0.5$ day and 1 acre-foot = 0.504 cfs-day:

elevation, ft	Q, cfs	S, cfs-day	$(2S/t) + O$
862	0	0	0
865	0	21	84
870	0	102	408
875	0	253	1012
880	0	506	2024
882	101	615	2561
884	232	823	3524
886	395	1144	4971
888	600	1589	6956

These data provide the basis for deriving the routing curves of Fig. 15-4. The inflow hydrograph of Prob. 15.4 is routed as follows: $I_1 + I_2 + [(2S_1/t) - O_1] = (2S_2/t) + O_2$.

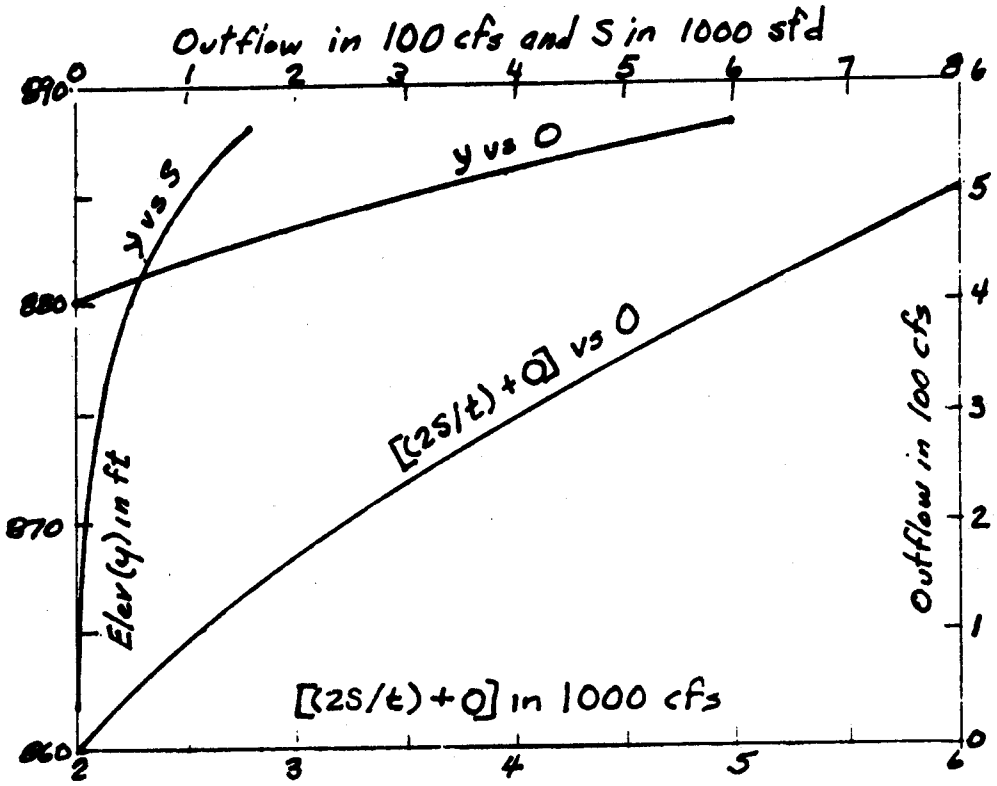


Fig. 15-4

day	hour	I, cfs	$(2S_1/t) - O$, cfs	$(2S_2/t) + O$, cfs	O, cfs
1	M	36	1010	1010	0
2	N	43	1089	1089	0
	M	121	1253	1253	0
3	N	348	1722	1722	0
	M	575	2419	2645	113
4	N	719	3207	3713	253
	M	741	3941	4667	363
5	N	615	4439	5297	429
	M	440	4594	5494	450
6	N	328	4488	5362	437
	M	255		5071	405

The peak flow is approximately 450 cfs and the maximum pool elevation is 886.6 ft.

15.8 The Muskingum method of flood routing involves a time parameter K and a weighting factor x. Find these quantities for the flood of Prob. 15.4 (flows in m^3/s).

▮ Columns 4, 10, and 11 of the solution tableau of Prob. 15.4 give the weighted flows corresponding to $x = 0$, 0.2, and 0.3; these flows are graphed in Fig. 15-5. It will be seen that increasing the value of x from 0.2 to 0.3 reverses the "loop" in the lower range of flow. Even so, a slight additional increase in x might provide a slightly better fit, without materially affecting K. Take $K = 26.7$ h, $x = 0.3$, and $t = 12$ h.

15.9 For a flood with $K = 25.9$ h and $x = 0.25$, determine the Muskingum routing coefficients for $t = 12$ h and $t = 24$ h.

▮
$$c_0 = (Kx - 0.5t)/(K - Kx + 0.5t) \qquad c_1 = (Kx + 0.5t)/(K - Kx + 0.5t)$$
$$c_2 = (K - Kx - 0.5t)/(K - Kx + 0.5t)$$

For $t = 12$ h, with $K = 25.9$ h and $x = 0.25$,

$$c_0 = -\frac{(25.9)(0.25) - (0.5)(12)}{25.9 - (25.9)(0.25) + (0.5)(12)} = -0.019 \qquad c_1 = \frac{(25.9)(0.25) + (0.5)(12)}{25.9 - (25.9)(0.25) + (0.5)(12)} = 0.491$$
$$c_2 = \frac{25.9 - (25.9)(0.25) - (0.5)(12)}{25.9 - (25.9)(0.25) + (0.5)(12)} = 0.528$$

For $t = 24$ h, with $K = 25.9$ h and $x = 0.25$,

$$c_0 = -\frac{(25.9)(0.25) - (0.5)(24)}{25.9 - (25.9)(0.25) + (0.5)(24)} = 0.176 \qquad c_1 = \frac{(25.9)(0.25) + (0.5)(24)}{25.9 - (25.9)(0.25) + (0.5)(24)} = 0.588$$
$$c_2 = \frac{25.9 - (25.9)(0.25) - (0.5)(24)}{25.9 - (25.9)(0.25) + (0.5)(24)} = 0.236$$

The 24-h routing period is to be preferred, since negative coefficients lead to instability when flow changes rapidly.

15.10 For a river reach with $K = 26$ h and $x = 0.22$, determine the Muskingum routing coefficients for $t = 12$ h.

▮
$$c_0 = (Kx - 0.5t)/(K - Kx + 0.5t) \qquad c_1 = (Kx + 0.5t)/(K - Kx + 0.5t)$$
$$c_2 = (K - Kx - 0.5t)/(K - Kx + 0.5t)$$
$$c_0 = -\frac{(26)(0.22) - (0.5)(12)}{26 - (26)(0.22) + (0.5)(12)} = 0.011 \qquad c_1 = \frac{(26)(0.22) + (0.5)(12)}{26 - (26)(0.22) + (0.5)(12)} = 0.446$$
$$c_2 = \frac{26 - (26)(0.22) - (0.5)(12)}{26 - (26)(0.22) + (0.5)(12)} = 0.543$$

15.11 The outflow hydrograph of Prob. 15.4 becomes the inflow hydrograph for a reach. Using the routing coefficients determined in Prob. 15.10, find the peak reach outflow by the Muskingum method.

▮ $O_2 = c_0 I_2 + c_1 I_1 + c_2 O_1$. The routing computations are as follows:

day	hour	I, m^3/s	$0.011I_2$, m^3/s	$0.446I_1$, m^3/s	$0.543O_1$, m^3/s	O, m^3/s
1	M	58				60 (est.)
2	N	46	1	26	33	60
	M	42	0	21	33	54
3	N	64	1	19	29	49
	M	149	2	29	27	58
4	N	329	4	66	31	101
	M	536	6	147	55	208
5	N	678	7	239	113	359
	M	681	7	302	195	504
6	N	560	6	304	274	584
	M	439	5	250	317	572
7	N	341	4	196	311	511
	M	272	3	152	277	432

The routed peak flow is 584 m^3/s.

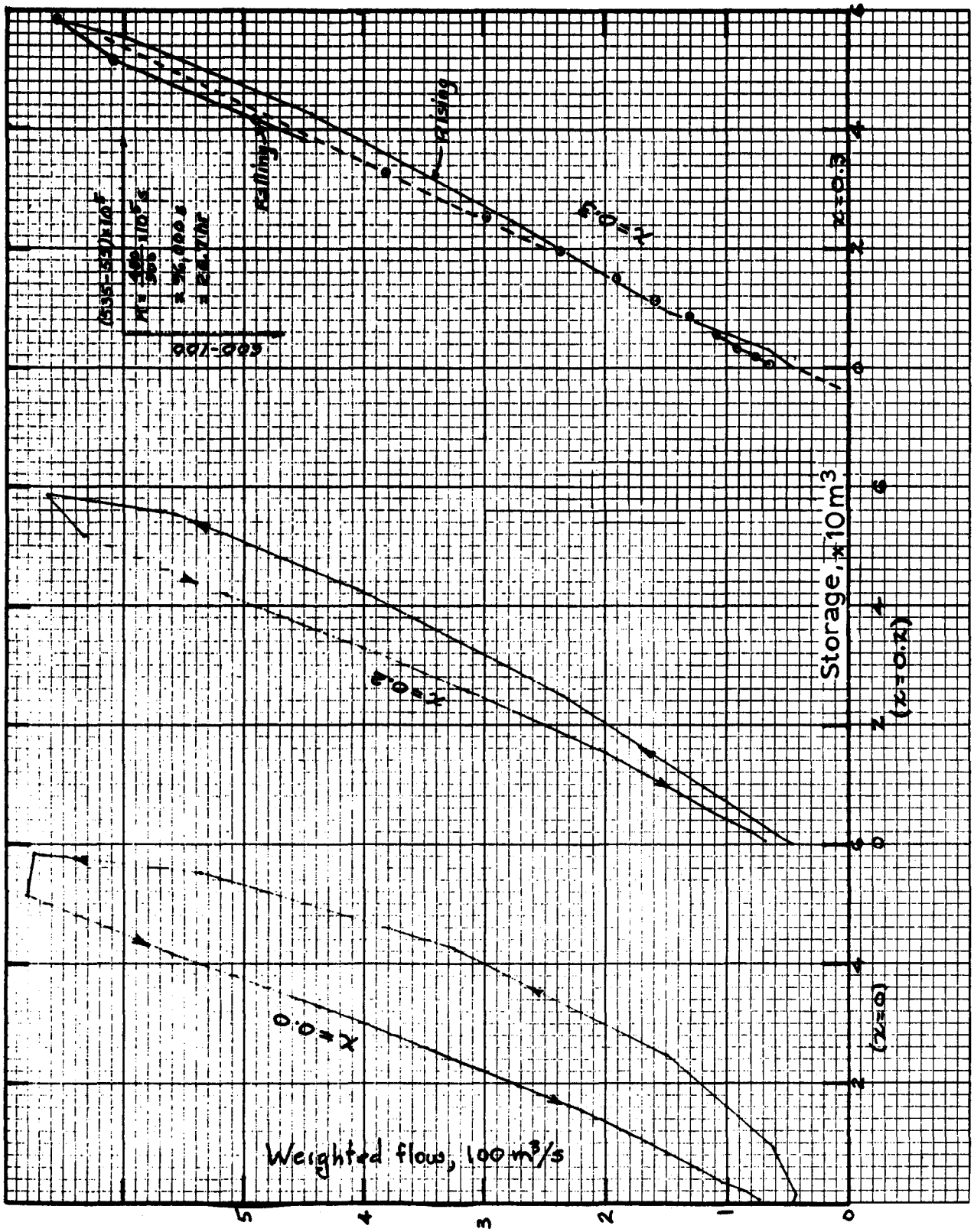

Weighted flow, 100 m³/s
Storage, ×10 m³
(x=0)
(x=0.2)
(x=0.3)
x = 0.0
x = 0.2
x = 0.3
Falling
Rising
600-700
0
1
2
3
4
5
6

Fig. 15-5

15.12 For the inflow hydrograph given below for a reach, route the flood through the reach by the Muskingum method. Take $K = 11$ h, $t = 6$ h, and $x = 0.13$; hence, $c_0 = 0.124$, $c_1 = 0.353$, and $c_2 = 0.523$.

day	hour	I, cfs
1	6 a.m.	10
	Noon	30
	6 p.m.	68
	Midnight	50
2	6 a.m.	40
	Noon	31
	6 p.m.	23

Assume an initial outflow of 10 cfs.

▮ $$O_2 = c_0 I_2 + c_1 I_1 + c_2 O_1$$

day	hour	I	$c_0 I_2$	$c_1 I_1$	$c_2 O_1$	O
1	6 a.m.	10	—	—	—	10
	Noon	30	3.7	3.5	5.2	12.4
	6 p.m.	68	8.4	10.6	6.5	25.5
	Midnight	50	6.2	24.0	13.3	43.5
2	6 a.m.	40	5.0	17.7	22.7	45.4
	Noon	31	3.8	14.1	23.7	41.6
	6 p.m.	23	2.9	10.9	21.8	35.6

All tabular values are in cubic feet per second.

15.13 Refer to Fig. 15-6. At time t_1, an infinitesimal gravity wave is initiated at the dam (station x_1). Obtain an analytical expression for the time t at which the wavefront passes through station x. Assume still water and a constant bed slope.

▮ $u = dx/dt = \sqrt{gy}$. But the depth equation is

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1} \equiv s_0 \tag{1}$$

so that $dx/dt = (1/s_0)(dy/dt)$ and

$$\frac{1}{s_0}\frac{dy}{dt} = \sqrt{gy}$$

$$\int_{t_1}^{t} dt = \frac{1}{s_0\sqrt{g}}\int_{y_1}^{y}\frac{dy}{\sqrt{y}} \tag{2}$$

$$t - t_1 = \frac{2}{s_0\sqrt{g}}(\sqrt{y} - \sqrt{y_1})$$

Finally, substitute for y in (*2*) from (*1*), obtaining

$$t - t_1 = \frac{2}{s_0\sqrt{g}}[\sqrt{y_1 + s_0(x - x_1)} - \sqrt{y_1}] \tag{3}$$

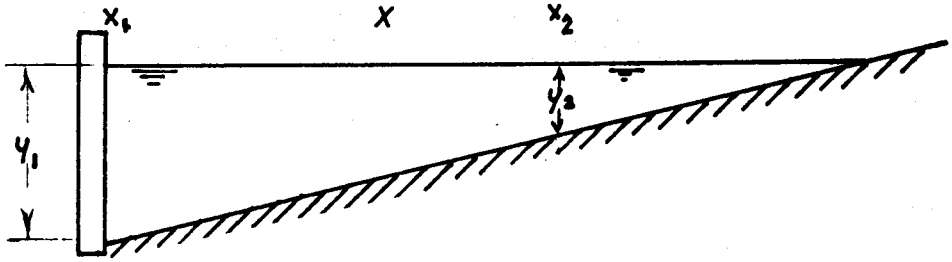


Fig. 15-6

15.14 Represent the wave of Prob. 15.13 in the xt plane, for the following assignment of parameters: $t_1 = 0$ s, $x_1 = 0$ ft, $y_1 = 55$ ft, $x_2 = 16\,000$ ft, $y_2 = 22$ ft.

▮ Using $s_0 = (22 - 55)/(6000 - 0) = -0.0055$ and $g = 32.2$ ft/s^2 in (*3*) of Prob. 15.13, we generate Table 15-1 and the rising curve of Fig. 15-7.

TABLE 15-1

x, ft	t, s	x, ft	t, s
0	0	8 000	207
1 000	24	9 000	236
2 000	48	10 000	265
3 000	73	11 000	296
4 000	99	12 000	327
5 000	125	13 000	360
6 000	152	14 000	394
7 000	179	15 000	429
		16 000	466

15.15 Refer to Probs. 15.13 and 15.14. Suppose that a similar wave is initiated at x_2 (= 16 000 ft) at time t_1 (= 0 s) and propagates toward the dam. Superimpose this second wave on Fig. 15-7. Where and when will the two waves meet?

▮ Making the appropriate replacements in (*3*) of Prob. 15.13, we obtain for the second wavefront

$$t - t_1 = \frac{2}{s^*\sqrt{g}}\left[\sqrt{y_2 + s^*(x_2 - x)} - \sqrt{y_2}\right] \qquad \text{where} \qquad s^* = -s_0$$

This equation generates Table 15-2 and the falling curve of Fig. 15-7. The two curves intersect at approximately $x = 8900$ ft, $t = 234$ s.

TABLE 15-2

x, ft	t, s	x, ft	t, s
0	466	8 000	259
1 000	442	9 000	230
2 000	417	10 000	200
3 000	392	11 000	170
4 000	367	12 000	138
5 000	341	13 000	106
6 000	314	14 000	72
7 000	287	15 000	37
		16 000	0

15.16 A uniform rectangular channel 4 m wide with $n = 0.014$ and slope of 0.0003 is flowing at a depth of 1.7 m. A sudden gate opening increases the depth to 2.0 m. Calculate the propagation speed c and the ground velocity v of the resulting abrupt wave.

▮

$$c = \sqrt{(g)(J_2 - J_1)/[(A_1)(1 - A_1/A_2)]} \qquad J = Ad/2 \qquad A_1 = (4)(1.7) = 6.80 \text{ m}^2$$

$$J_2 = (6.80)(1.7/2) = 5.780 \text{ m}^3$$

$$A_2 = (4)(2.0) = 8.00 \text{ m}^2 \qquad J_1 = (8.00)(2.0/2) = 8.000 \text{ m}^3$$

$$c = \sqrt{(9.807)(8.000 - 5.780)/[(6.80)(1 - 6.80/8.00)]} = 4.62 \text{ m/s}$$

$$u = v_1 + c \qquad v = (1.0/n)(R^{2/3})(s^{1/2}) \qquad v_1 = (1.0/0.014)[6.80/(1.7 + 4 + 1.7)]^{2/3}(0.0003)^{1/2} = 1.17 \text{ m/s}$$

$$u = 1.17 + 4.62 = 5.79 \text{ m/s}$$

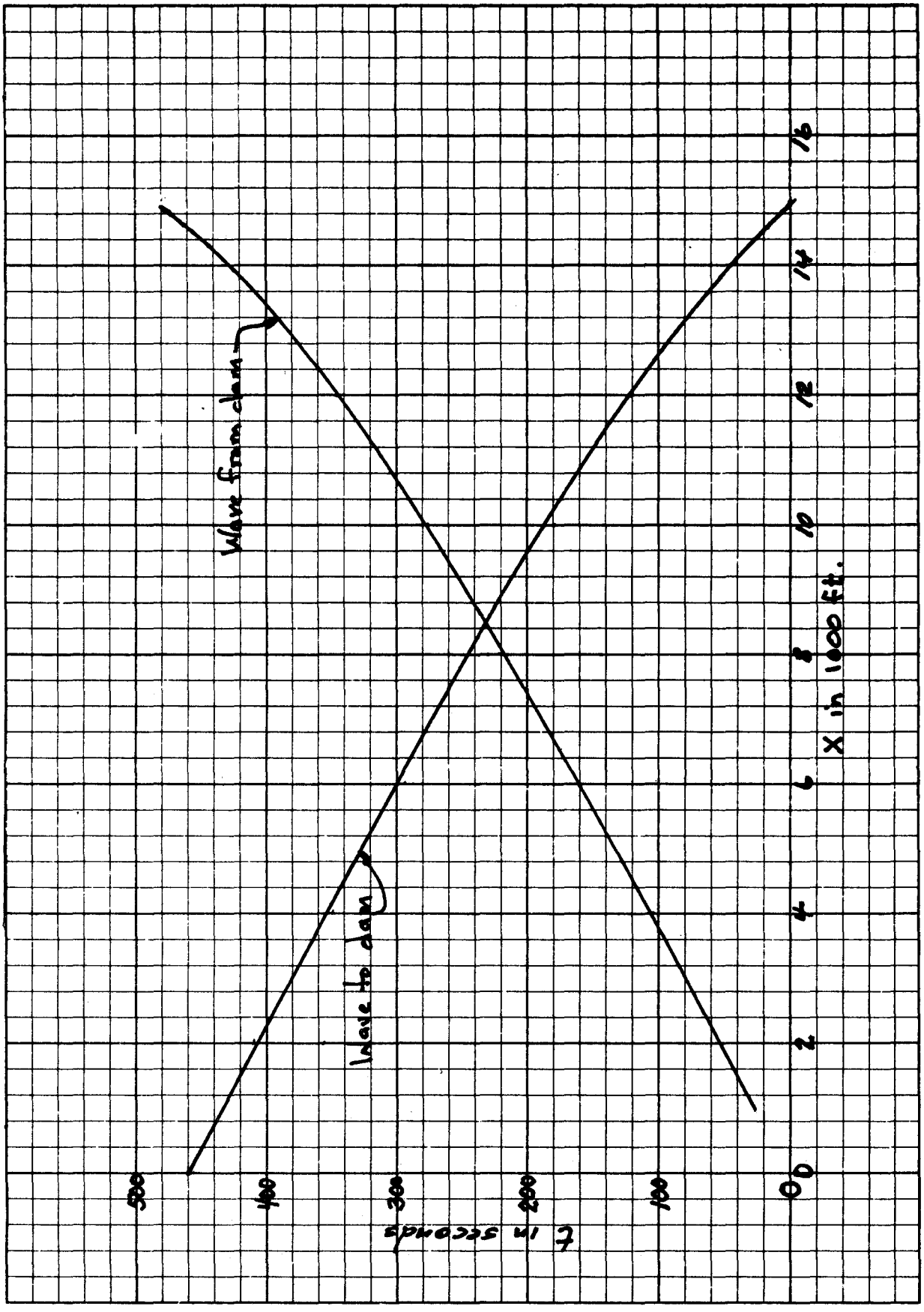

Wave from dam
Wave to dam
X in 1000 ft.
t in seconds
0
2
4
6
8
10
12
14
16
100
200
300
400
500

Fig. 15-7

CHAPTER 16
Flow of Compressible Fluids

16.1 For air at a temperature of 30 °C and pressure of 470 kPa abs, find the specific weight of the air.

$$\gamma = p/RT = 470/[(29.3)(30+273)] = 0.0529\ \text{kN/m}^3$$

16.2 Find the mass density of helium at a temperature of 39 °F and a pressure of 26.9 psig when the atmospheric pressure is 14.9 psig.

$$\rho = p/RT = (26.9+14.9)(144)/[(12\,420)(39+460)] = 0.000971\ \text{slug/ft}^3$$

16.3 Methane at 22 °C flows through a pipeline at a velocity of 416.4 m/s. Is the flow subsonic, sonic, supersonic, or hypersonic?

$N_M = v/\sqrt{kgRT} = 416.4/\sqrt{(1.32)(9.807)(52.9)(22+273)} = 0.93$. Since $N_M < 1.0$, the flow is subsonic.

16.4 Chlorine gas at 51 °F flows through a pipeline. Find the velocity at which the flow will be sonic.

$$N_M = v/\sqrt{kgRT} \qquad 1.0 = v/\sqrt{(1.34)(32.2)(21.8)(51+460)} \qquad v = 693\ \text{ft/s}$$

16.5 Compute the change in internal energy and the change in enthalpy of 15 kg of air if its temperature is raised from 20 to 30 °C. The initial pressure is 95 kPa abs.

$\Delta i = c_v(T_2 - T_1) = (0.716)(30-20) = 7.16$ kJ/kg Change in internal energy $= (\Delta i)(15) = 107.4$ kJ

$\Delta h = c_p(T_2 - T_1) = 1.003(10) = 10.03$ kJ/kg Change in enthalpy $= (\Delta h)(15) = 150.0$ kJ

16.6 Suppose that 15 kg of air ($T_1 = 20$ °C) of Prob. 16.5 were compressed isentropically to 40 percent of its original volume. Find the final temperature and pressure, the work required, and the changes in internal energy and enthalpy.

The following relations apply: $pv = RT$ and $pv^k =$ constant, where $k = 1.40$ for air. $pv^k = pv(v^k/v) = (RT/v)v^k = RTv^{k-1} =$ constant. Since $R =$ constant, $Tv^{k-1} =$ constant, and $T_2 = T_1(v_1/v_2)^{k-1} = (273+20)(1.0/0.4)^{1.40-1} = 422$ K $= 149$ °C, $pv/T = R =$ constant, $p_1 = 95$ kPa abs (from Prob. 16.5), $p_1v_1/T_1 = p_2v_2/T_2$ and $v_2 = 0.4v_1$, $95v_1/293 = [p_2(0.4v_1)/422]$, $p_2 = 342$ kPa abs. Since this is an isentropic process, the work required is equal to the change in internal energy. This can be confirmed by computing the values of the pressure and corresponding volumes occupied by the gas during the isentropic process, plotting a pressure-vs.-volume curve, and finding the area under the curve and thereby determining the work done on the fluid. Thus the work required is

$$\int_{s_1}^{s_2} F\,ds = \int_{s_1}^{s_2} \left(\frac{F}{A}\right) A\,ds = \int_{\text{vol }1}^{\text{vol }2} p\,d(\text{vol})$$

or $\Delta i = c_v(T_2 - T_1) = 0.716(422-293) = 92.4$ kJ/kg, $(\Delta i)(15\text{ kg}) = 1385$ kJ = work required = change in internal energy, $\Delta h = c_p(T_2 - T_1) = 1.003(129) = 129.4$ kJ/kg, $(\Delta h)(15\text{ kg}) = 1941$ kJ = change in enthalpy.

16.7 Using the data of Prob. 16.6 compute $\Delta(p/\rho)$ and thus show that $\Delta h = \Delta i + \Delta(p/\rho)$.

From Prob. 16.6, $\Delta h = 129.4$ kJ/kg, $\Delta i = 92.4$ kJ/kg, $R = 0.287$ kJ/kg · K; $p_1/\rho_1 = RT_1 = (0.287)(293) = 84.1$ kJ/kg; $p_2/\rho_2 = RT_2 = (0.287)(422) = 121.1$ kJ/kg; $\Delta(p/\rho) = p_2/\rho_2 - p_1/\rho_1 = 37.0$ kJ/kg. $\Delta i + \Delta(p/\rho) = 92 + 37 = 129.4$ which checks accurately with the value of Δh.

16.8 Using the data of Prob. 16.6, determine the work done in compressing the air by finding the area under a pressure-vs.-volume curve. Compute and tabulate volumes and pressures using volume increments which are 10 percent of the original volume.

$R = 0.287$ kJ/kg · K, $k = 1.4$. Initially, given: $T_1 = 273 + 20 = 293$ K; $p_1 = 95$ kPa; $v_1 = RT_1/p_1 = (0.287)(293)/95 = 0.885$ m^3/kg. After compression, from Prob. 16.6 $T = T_1(v_1/v)^{1-k} = (293)(v_1/v)^{0.4}$; $v = v_1/(v_1/v)$; $V = mv = 15v$; $p = RT/v$.

v_1/v	T, K	v, m³/kg	V, m³	p, kPa
$\frac{100}{100}$	293	0.885	13.28	95.0
$\frac{100}{90}$	306	0.797	11.95	110.1
$\frac{100}{80}$	320	0.708	10.62	129.9
$\frac{100}{70}$	338	0.619	9.29	156.6
$\frac{100}{60}$	359	0.531	7.97	194.1
$\frac{100}{50}$	387	0.443	6.64	251.0
$\frac{100}{40}$	423	0.354	5.31	343.0

These data are plotted in Fig. 16-1. Measure or calculate area A: Work done in compression = A = 1.395 MJ (by calculation).

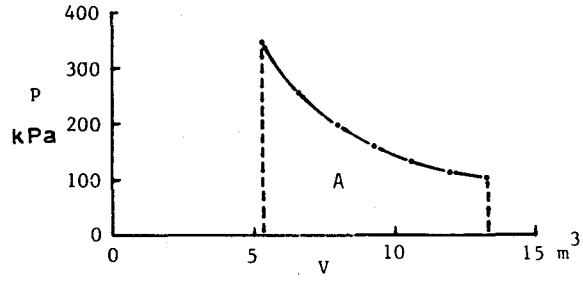


Fig. 16-1

16.9 Compute the change in enthalpy of 500 lb of oxygen if its temperature is increased from 125 °F to 160 °F.

▮ $c_p = 5437$ ft-lb/(slug-°R) $\Delta h = c_p(T_2 - T_1) = (5437)(160 - 125) = 190\,000$ ft-lb/slug

Five hundred pounds is equivalent to 15.53 slugs ($m = W/g$). Thus change in enthalpy = (190 000)(15.53) = 2.95×10^6 ft-lb.

16.10 Suppose the 500 lb of oxygen of Prob. 16.9 was compressed isentropically to 75 percent of its original volume. Find the final temperature and pressure, the work required, and the change in enthalpy. Assume $T_1 = 120$ °F and $p_1 = 200$ psia.

▮ pv^k = constant and $pv = RT$. Eliminating p: $(RT/v)V^k$ = constant = RTv^{k-1}. Since R = constant also, Tv^{k-1} = constant; $k = 1.4$. From which: $T_2 = T_1(v_1/v_2)^{k-1} = (120 + 460)(1/0.75)^{0.4} = 650.7$ °R = 190.7 °F. $p_1v_1/T_1 = p_2v_2/T_2$; so $200v_1/580 = p_2(0.75v_1)/651$; $p_2 = 299$ psia; $c_v = 3883$, $c_p = 5437$ ft²/(s²-°R). Work required = change of internal energy (as isentropic). Work required = $(\Delta i)(W) = c_v(\Delta T)(500/32.2)$ where $\Delta T = 190.7 - 120 = 70.7$ °F. Work required = (3883)(70.7)(500/32.2) = 4.26×10^6 ft-lb; increase in enthalpy = (5437)(70.7)(500/32.2) = 5.97×10^6 ft-lb.

16.11 Determine the sonic velocity in air at sea level and at 10 000, 20 000, and 30 000 ft. Assume standard atmosphere (Table A-7).

▮ $c = (kRT)^{1/2}$. In all cases $k = 1.4$ and $R = 1715$ ft-lb/(slug-°R). From Table A-7, temperatures = 59 °F, 23.4 °F, −12.3 °F, and −47.8 °F. Substitute for T = (460 + temperature) and compute the resulting c's.

elevation ft	temperature °F	c, fps
0	59	1 116
10 000	23.4	1 077
20 000	−12.3	1 037
30 000	−47.8	995

16.12 Repeat Prob. 16.11 for sea level, 6 km, and 10 km.

▮ $c = (kRT)^{1/2}$, $k = 1.4$, and $R = 0.287$ kJ/kg · K = 287 m²/s² · K; $g = 9.81$ m/s². From Table A-8: temperatures = 15.0, −24.0, and −49.9 °C. Substitute for T = (273 + temp. °C) and compute the resulting c's.

elevation, m	temperature, °C	c, m/s
0	15.0	340
6 000	−24.0	316
10 000	−49.9	299

16.13 Find the stagnation pressure and temperature in nitrogen flowing at 600 fps if the pressure and temperature in the undisturbed flow field are 100 psia and 200 °F, respectively.

▮ $$c = \sqrt{kRT} = \sqrt{(1.40)(1774)(460+200)} = 1280 \text{ ft/s} \qquad N_M = v/c = \tfrac{600}{1280} = 0.469$$

$$\rho_0 = kp_0/c_0^2 = (1.40)[(100)(144)]/1280^2 = 0.0123 \text{ slug/ft}^3$$

$$p_s = p_0 + (\rho_0)(v_0^2/2)[1 + (N_M)_0^2/4 + \cdots] = (100)(144) + (0.0123)(600^2/2)(1 + 0.469^2/4)$$

$$= 16\,736 \text{ lb/ft}^2 \quad \text{or} \quad 116 \text{ lb/in}^2$$

$$(c_p/g)(T_1) + V_1^2/2g = (c_p/g)(T_s)$$

$$(6210/32.2)(460+200) + 600^2/[(2)(32.3)] = (6210/32.2)(T_s) \qquad T_s = 689 \text{ °R} \quad \text{or} \quad 229 \text{ °F}$$

16.14 Find the stagnation pressure and temperature in air flowing at 90 fps if the pressure and temperature in the undisturbed flow field are 14.7 psia and 60 °F, respectively.

▮ $$c = \sqrt{kRT} = \sqrt{(1.40)(1716)(460+60)} = 1118 \text{ ft/s} \qquad N_M = v/c \qquad (N_M)_0 = \tfrac{90}{1118} = 0.0805$$

$$(c_p/g)(T_0) + V_0^2/2g = (c_p/g)(T_s)$$

$$(6000/32.2)(460+60) + 90^2/[(2)(32.2)] = (6000/32.2)(T_s) \qquad T_s = 521 \text{ °R} \quad \text{or} \quad 61 \text{ °F}$$

$$p_s = p_0 + (\rho_0)(v_0^2/2)[1 + (N_M)_0^2/4 + \cdots] \qquad \rho_0 = p_0/RT_0 = (14.7)(144)/[(1716)(460+60)] = 0.00237 \text{ slug/ft}^3$$

$$p_s = 14.7 + [(0.00237)(90^2/2)/144](1 + 0.0805^2/4) = 14.77 \text{ psia}$$

16.15 Air flows past an object at 500 fps. Determine the stagnation pressures and temperatures in the standard atmosphere at elevations of sea level, 5000 ft, and 30 000 ft.

▮ From Table A-7: at sea level, 59 °F, $\rho = 0.00238$ slug/ft³; at 5000 ft, 41.2 °F, $\rho = 0.00205$ slug/ft³; at 30 000, −47.8 °F, $\rho = 0.00089$ slug/ft³.

$$c = \sqrt{kRT} \qquad [R = 1716 \text{ lb-ft/(slug-°R)}] \qquad N_M = v_0/c \qquad (\text{where } v_0 = 500 \text{ fps})$$

$$T_s = T_0 + (V_0^2/2c_p) \qquad [\text{where } c_p = 6000 \text{ lb-ft/(slug-°R)}]$$

$$p_s = p_0 + [(\rho_0)(v_0^2/2)/144][1 + (N_M)_0^2/4 + \cdots]$$

elevation, ft	p_0, psi	c, fps	N_M	T_s, °F	p_s, psi
0	14.70	1 116	0.448	79.8	16.87
5 000	12.24	1 097	0.456	62.0	14.11
30 000	4.37	995	0.503	−27.0	5.21

16.16 Repeat Prob. 16.15 for an air speed of 180 m/s and elevation 2 km.

▮ From Table A-8: at 2000 m, 2 °C, $p = 79.50$ kPa, $\rho = 1.007$ kg/m³.

$$c = \sqrt{kRT} = \sqrt{(1.40)(287)(2+273)} = 332 \text{ m/s} \qquad N_M = v_0/c = \tfrac{180}{332} = 0.542$$

$$T_s = T_0 + (v_0^2/2c_p) = 2 + 180^2/[(2)(1003)] = 18 \text{ °C}$$

$$p_s = p_0 + (\rho_0)(v_0^2/2)[1 + (N_M)_0^2/4 + \cdots] = 79.50 + (1.007)(180^2/2)(1 + 0.542^2/4)/(1000) = 97.0 \text{ kPa abs}$$

16.17 Air at 250 psia is moving at 550 fps in a high-pressure wind tunnel at a temperature of 100 °F. Find the stagnation pressure and temperature. Note the magnitude of the sonic velocity for the 250-psia 100 °F air.

▮ $$c = \sqrt{kRT} = \sqrt{(1.40)(1716)(460+100)} = 1160 \text{ ft/s} \qquad N_M = v/c = \tfrac{550}{1160} = 0.474$$

Since $[v_0 = 550 \text{ ft/s}] < [c = 1160 \text{ ft/s}]$,

$$p_s/p_0 = [1 + (v_0/c_0)^2(k-1)/2]^{k/(k-1)} \qquad p_s/250 = [1 + (\tfrac{550}{1160})^2(1.40-1)/2]^{1.40/(1.40-1)}$$

$$p_s = 292 \text{ psia} \qquad c_pT_0 + v_0^2/2g = c_pT_s \qquad (6000/32.2)(460+100) + 550^2/[(2)(32.2)] = (6000/32.2)(T_s)$$

$$T_s = 585\ ^\circ\text{R} \quad \text{or} \quad 125\ ^\circ\text{F}$$

16.18 Show that the equation $p_s = p_0 + (\rho_0)(v_0^2/2)[1 + (N_M)_0^2/4 + \cdots]$ results from the binomial expansion of the equation $p_s/p_0 = [1 + (v_0/c)^2(k-1)/2]^{k/(k-1)}$.

▮ $p_s/p_0 = \{1 + (v_0/c)^2[(k-1)/2]\}^{k/(k-1)}$; substitute $k = \rho_0c_0^2/p_0$:

$$\frac{p_s}{p_0} = \left[1 + N_M^2\left(\frac{\rho_0c_0^2 - p_0}{2p_0}\right)\right]^{\rho_0c_0^2/(\rho_0c_0^2 - p_0)}$$

Expanding by the binomial theorem: $(a+b)^n = a^n + (na^{n-1})(b) + [n(n-1)/2!]a^{n-1}b^2 + \cdots$ and then simplifying leads to $p_s = p_0 + (\rho_0v_0^2/2)[1 + (N_M^2/4) + \cdots]$ Q.E.D.

16.19 Compute the value of R from the values of k and c_p for air.

▮ $$R = [(k-1)/k](c_p) = [(1.40-1)/1.40](6000) = 1714 \text{ ft-lb/(slug-°R)}$$

16.20 Compute the enthalpy change in 5 kg of oxygen when the initial conditions are $p_1 = 130$ kPa abs and $T_1 = 10\ ^\circ\text{C}$, and the final conditions are $p_2 = 500$ kPa abs and $T_2 = 95\ ^\circ\text{C}$ $[c_p = 0.917 \text{ kJ/kg}\cdot\text{K}]$.

▮ $$h_2 = h_1 = (c_p)(T_2 - T_1) \qquad H_2 - H_1 = (5)(c_p)(T_2 - T_1) = (5)(0.917)(95-10) = 390 \text{ kJ}$$

16.21 A cylinder containing 2 kg nitrogen at 0.14 MPa abs and 5 °C is compressed isentropically to 0.30 MPa abs. Find the final temperature and the work required $[c_v = 0.741 \text{ kJ/kg}\cdot\text{K}]$.

▮ $T_2 = (T_1)(p_2/p_1)^{(k-1)/k} = (273+5)(0.30/0.14)^{(1.40-1)/1.40} = 346$ K, or 73 °C. From the principle of conservation of energy, the work done on the gas must equal its increase in internal energy, since there is no heat transfer in an isentropic process; that is, $u_2 - u_1 = (c_v)(T_2 - T_1)$ = work per kilogram: Work = (2)(0.741)(73 − 5) = 101 kJ.

16.22 If 3.0 slugs of air are involved in a reversible polytropic process in which the initial conditions $p_1 = 12$ psia and $T_1 = 60\ ^\circ\text{F}$ change to $p_2 = 20$ psia and volume $V = 1011 \text{ ft}^3$, determine the (*a*) formula for the process and (*b*) work done on the air.

▮ (*a*) $\rho_1 = p_1/RT_1 = (12)(144)/(53.3)(32.17)(460+60) = 0.00194 \text{ slug/ft}^3$. R was converted to foot-pounds per slug and degree Rankine by multiplying by 32.17. Also, $\rho_2 = \frac{3}{1011} = 0.002967 \text{ slug/ft}^3$; $p_1/\rho_1^n = p_2/\rho_2^n$, $n = [\ln(p_2/p_1)]/[\ln(\rho_2/\rho_1)] = [\ln(\frac{20}{12})]/[\ln(0.002967/0.00194)] = 1.20$; hence $p/\rho^{1.2} = \text{const}$ describes the polytropic process.

(*b*) Work of expansion is

$$W = \int_{V_1}^{V_2} p\,dV$$

This is the work done by the gas on its surroundings. Since $p_1V_1^n = p_2V_2^n = pV^n$, by substituting into the integral,

$$W = p_1V_1^n\int_{V_1}^{V_2}\frac{dV}{V^n} = \frac{p_2V_2 - p_1V_1}{1-n} = \frac{mR}{1-n}(T_2 - T_1)$$

if m is the mass of gas. $V_2 = 1011 \text{ ft}^3$ and $V_1 = V_2(p_2/p_1)^{1/n} = (1011)(\frac{20}{12})^{1/1.2} = 1547 \text{ ft}^3$. Then $W = [(20)(144)(1011) - (12)(144)(1548)]/(1-1.2) = -1\,184\,000$ ft-lb. Hence, the work done on the gas is 1 184 000 ft-lb.

16.23 For Prob. 16.22, find the (*a*) amount of heat transfer and (*b*) entropy change.

▮ (*a*) From the first law of thermodynamics the heat added minus the work done by the gas must equal the increase in internal energy; that is, $Q_H - W = U_2 - U_1 = c_vm(T_2 - T_1)$. First $T_2 = p_2/\rho_2R = (20)(144)/(0.002965)(53.3)(32.17) = 566\ ^\circ\text{R}$. Then $Q_H = -(1\,184\,000/778) + (0.17)(32.17)[3(566-520)] = -761$ Btu and 761 Btu was transferred from the mass of air.

(*b*) $s_2 - s_1 = c_v \ln[(p_2/p_1)(\rho_1/\rho_2)^k]$ $\quad s_2 - s_1 = (0.171)\ln[(\frac{20}{12})(0.00194/0.002967)^{1.4}] = -0.01436$ Btu/(lbm-°R)

and $$S_2 - S_1 = -0.01436(3)(32.17) = -1.386 \text{ Btu/°R}$$

A rough check on the heat transfer can be made using an average temperature $T = (520 + 566)/2 = 543$ and by remembering that the losses are zero in a reversible process: $Q_H = T(S_2 - S_1) = (543)(-1.386) = -753$ Btu.

16.24 Carbon tetrachloride has a bulk modulus of elasticity of 1.124 GPa and a density of 1593 kg/m³. What is the speed of sound in the medium?

▮ $$c = \sqrt{K/\rho} = \sqrt{(1.124 \times 10^9)/1593} = 840 \text{ m/s}$$

16.25 What is the speed of sound in dry air at sea level when $T = 68$ °F and in the stratosphere when $T = -67$ °F?

▮ $c = \sqrt{kRT}$. At sea level, $c = \sqrt{(1.40)(1716)(460 + 68)} = 1126$ ft/s. In the stratosphere, $c = \sqrt{(1.40)(1716)(460 - 67)} = 972$ ft/s.

16.26 If 4 kg of a perfect gas, molecular weight 36, had its temperature increased 2 K when 6.4 kJ of work was done on it in an insulated constant-volume chamber, determine c_v and c_p.

▮ $$c_v = \Delta u/\Delta t = 6.4/(4)(2) = 0.8 \text{ kJ/kg}\cdot\text{K} \qquad c_p = c_v + R$$
$$R = 8.312/M = 8.312/36 = 0.231 \text{ kJ/kg}\cdot\text{K}$$
$$c_p = 0.8 + 0.231 = 1.03 \text{ kJ/kg}\cdot\text{K}$$

16.27 A gas of molecular weight 46 has $c_p = 1.558$ kJ/kg · K. What is c_v for this gas?

▮ $$R = 8.312/46 = 0.181 \text{ kJ/kg}\cdot\text{K} \qquad c_v = c_p - R = 1.558 - 0.181 = 1.377 \text{ kJ/kg}\cdot\text{K}$$

16.28 Calculate the specific heat ratio k for Probs. 16.26 and 16.27.

▮ $k = c_p/c_v$. For Prob. 16.26: $k = 1.03/0.8 = 1.288$. For Prob. 16.27: $k = 1.558/1.377 = 1.131$.

16.29 The enthalpy of a gas is increased by 0.5 Btu/(lbm-°R) when heat is added at constant pressure and the internal energy is increased by 0.4 Btu/(lbm-°R) when the volume is maintained constant and heat is added. Calculate the molecular weight.

▮ $$R = 1545/M = c_p - c_v \qquad c_p = 0.5 \text{ Btu/(lbm-°R)} \qquad c_v = 0.4 \text{ Btu/(lbm-°R)}$$
$$R = (0.5 - 0.4)(778) = 77.8 \text{ ft-lb/(lbm-°R)} \qquad 77.8 = 1545/M \qquad M = 19.86$$

16.30 Calculate the enthalpy change (in kcal) of 3 kg carbon monoxide from $p_1 = 16$ kPa abs and $T_2 = 5$ °C to $p_2 = 30$ kPa abs and $T_2 = 170$ °C.

▮ $$H = mc_p\Delta T = (3)(0.249)(170 - 5) = 123.3 \text{ kcal}$$

16.31 Calculate the entropy change in Prob. 16.30.

▮ $$\Delta S = mc_v \ln[(T_2/T_1)^k(p_2/p_1)^{1-k}] \qquad T_2 = 170 + 273 = 443 \text{ K} \qquad T_1 = 5 + 273 = 278 \text{ K}$$
$$\Delta S = (3)(0.178)\{\ln[(\tfrac{443}{278})^{1.40}(\tfrac{30}{16})^{1-1.40}]\} = 0.215 \text{ kcal/K}$$

16.32 In an isentropic process, 1 kg of oxygen at 17 °C has its absolute pressure doubled. What is the final temperature?

▮ $$T_2/T_1 = (p_2/p_1)^{(k-1)/k} \qquad T_2/(17 + 273) = (2)^{(1.40-1)/1.40} \qquad T_2 = 354 \text{ K} \quad \text{or} \quad 84 \text{ °C}$$

16.33 Work out the expression for density change with temperature for a reversible polytropic process.

▮ $$p/\rho^n = \text{constant} \qquad p/\rho T = \text{constant} \qquad \rho^{n-1}/T = \text{constant} \qquad T_1/T_2 = (\rho_1/\rho_2)^{n-1}$$

16.34 Hydrogen at 50 psia and 40 °F has its temperature increased to 120 °F by a reversible polytropic process with $n = 1.20$. Calculate the final pressure.

▮ $$T_2/T_1 = (p_2/p_1)^{(n-1)/n} \qquad (120 + 460)/(40 + 460) = (p_2/50)^{(1.20-1)/1.20} \qquad p_2 = 122 \text{ psia}$$

16.35 A gas has a density decrease of 15 percent in a reversible polytropic process when the temperature decreases from 50 °C to 10 °C. Compute the exponent n for the process.

▮ From Prob. 16.33, $T_1/T_2 = (\rho_1/\rho_2)^{n-1}$, $(50+273)/(10+273) = (1/0.85)^{n-1}$, $\log 1.141 = (n-1)\log 1.176$, $n = 1.81$.

16.36 A projectile moves through water at 80 °F at 1800 fps. What is the Mach number?

▮ $$c = \sqrt{K/\rho} = \sqrt{(322\,000)(144)/1.93} = 4902 \text{ ft/s} \qquad N_M = v/c = \tfrac{1800}{4902} = 0.367.$$

16.37 If an airplane travels at 1350 km/h at sea level, $p = 101$ kPa abs, $T = 25$ °C, and at the same speed in the stratosphere where $T = -55$ °C, how much greater is the Mach number in the latter case?

▮ $(N_M)_2/(N_M)_1 = c_1/c_2 = \sqrt{T_1/T_2} = \sqrt{(273+25)/(273-55)} = 1.169$. Thus, $(N_M)_2$ is 16.9 percent greater than $(N_M)_1$.

16.38 What is the speed of sound through hydrogen at 75 °F?

▮ $$c = \sqrt{kRT} = \sqrt{(1.40)(24\,649)(75+460)} = 4297 \text{ ft/s}$$

16.39 Argon ($M = 39.944$, $k = 1.67$) flows through a tube such that its initial condition is $p_1 = 250$ psia and $\rho_1 = 1.16$ lbm/ft³ and its final condition is $p_2 = 30$ psia and $T_2 = 265$ °F. Estimate the (*a*) initial temperature, (*b*) final density, (*c*) change in enthalpy, and (*d*) change in entropy.

▮ (*a*) $$R = 49\,720/39.944 = 1245 \text{ ft}^2/(\text{s}^2\text{-°R}) \qquad \rho_1 = 1.16/32.2 = 0.03602 \text{ slug/ft}^3$$
$$T_1 = p_1/R_1\rho_1 = (250)(144)/[(1245)(0.03602)] = 803 \text{ °R} \quad \text{or} \quad 343 \text{ °F}$$

(*b*) $$\rho_2 = p_2/RT_2 = (30)(144)/[(1245)(460+265)] = 0.00479 \text{ slug/ft}^3$$

(*c*) $$c_p = kR/(k-1) = (1.67)(1245)/(1.67-1) = 3103 \text{ ft}^2/(\text{s}^2\text{-°R})$$
$$h_2 - h_1 = (c_p)(T_2 - T_1) = (3103)[(460+265) - 803] = -242\,000 \text{ ft}^2/\text{s}^2$$

(*d*) $$s_2 - s_1 = c_p \ln(T_2/T_1) - R\ln(p_2/p_1) = (3103)\{\ln[(460+265)/803]\} - (1245)\{\ln[(30)(144)/(250)(144)]\} = 2323 \text{ ft}^2/(\text{s}^2\text{-°R})$$

16.40 Estimate the speed of sound of carbon monoxide [$k = 1.40$, $R = 0.297$ kJ/kg · K] at 200 °C.

▮ $$c = \sqrt{kRT} = \sqrt{(1.40)(297)(200+273)} = 443 \text{ m/s}$$

16.41 A gas flows adiabatically through a duct. At section 1, $p_1 = 195$ psia, $T_1 = 480$ °F, and $V_1 = 240$ fps, while farther downstream $V_2 = 1000$ fps and $p_2 = 35$ psia. Calculate T_2 and $s_2 - s_1$ if the gas is air [$c_p = 6010$ ft²/(s²-°R)].

▮ $$c_pT_1 + v_1^2/2 = c_pT_2 + v_2^2/2 \qquad (6010)(480) + 240^2/2 = 6010T_2 + 1000^2/2 \qquad T_2 = 402 \text{ °F}$$
$$s_2 - s_1 = c_p \ln(T_2/T_1) - R\ln(p_2/p_1) = (6010)\{\ln[(402+460)/(480+460)]\} - (1716)[\ln(\tfrac{35}{195})] = 2427 \text{ ft}^2/(\text{s}^2\text{-°R})$$

16.42 Rework Prob. 16.41 if the gas is argon [$c_p = 3103$ ft²/(s²-°R), $k = 1.67$, $R = 1245$ ft²/(s²-°R)].

▮ $$c_pT_1 + v_1^2/2 = c_pT_2 + v_2^2/2 \qquad (3103)(480) + 240^2/2 = 3103T_2 + 1000^2/2 \qquad T_2 = 328 \text{ °F}$$
$$s_2 - s_1 = c_p \ln(T_2/T_1) - R\ln(p_2/p_1) = (3103)\{\ln[(328+460)/(480+460)]\} - (1245)[\ln(\tfrac{35}{195})] = 1591 \text{ ft}^2/(\text{s}^2\text{-°R})$$

16.43 Solve Prob. 16.41 if the gas is steam. Assume an ideal gas, with $M = 18.02$ and $k = 1.33$.

▮ $$c_pT_1 + v_1^2/2 = c_pT_2 + v_2^2/2 \qquad R = 49\,720/M = 49\,720/18.02 = 2759 \text{ ft}^2/(\text{s}^2\text{-°R})$$
$$c_p = kR/(k-1) = (1.33)(2759)/(1.33-1) = 11\,120 \text{ ft}^2/(\text{s}^2\text{-°R})$$
$$(11\,120)(480) + 240^2/2 = 11\,120T_2 + 1000^2/2 \qquad T_2 = 438 \text{ °F}$$
$$s_2 - s_1 = c_p \ln(T_2/T_1) - R\ln(p_2/p_1) = (11\,120)\{\ln[(438+460)/(480+460)]\} - (2759)[\ln(\tfrac{35}{195})] = 4231 \text{ ft}^2/(\text{s}^2\text{-°R})$$

16.44 Solve Prob. 16.41 if the gas is steam. Assume a real gas and use the steam tables.

▮ $h_1 + v_1^2/2 = h_2 + v_2^2/2$. With $p_1 = 195$ psi and $T_1 = 480$ °F, $h_1 = 1258.2$ Btu/lbm and $s_1 = 1.6153$Btu/(lbm-°F) (from the steam tables). $1258.2 + 240^2/[(2)(32.2)(778)] = h_2 + 1000^2/[(2)(32.2)(778)]$, $h_2 = 1239.4$ Btu/lbm. With $p_2 = 35$ psi and $h_2 = 1239.4$ Btu/lbm, $T_2 = 406$ °F and $s_2 = 1.7641$Btu/(lbm-°F) (from the steam tables). $s_2 - s_1 = 1.7641 - 1.6153 = 0.149$Btu/(lbm-°F).

16.45 If 5 kg of oxygen in a closed tank at 240 kPa is heated from 100 °C to 300 °C, calculate the new pressure, the heat added, and the change in entropy.

▮ $p_2/p_1 = T_2/T_1 \qquad p_2/240 = (300+273)/(100+273) \qquad p_2 = 369\text{ kPa} \qquad Q = mc_v\,\Delta T$

$$c_p = kR/(k-1) = (1.40)(0.260)/(1.40-1) = 0.910\text{ kJ/kg}\cdot\text{K} \qquad c_v = c_p/k = 0.910/1.40 = 0.650\text{ kJ/kg}\cdot\text{K}$$

$$Q = (5)(0.650)(300-100) = 650\text{ kJ}$$

$$s_2 - s_1 = mc_v \ln(T_2/T_1) = (5)(0.650)\{\ln[(300+273)/(100+273)]\} = 1.395\text{ kJ/K}$$

16.46 Steam at 450 °F and 80 psia is compressed isentropically to 100 psia. What is the new temperature? Assume an ideal gas with $k = 1.33$.

▮ $T_2/T_1 = (p_2/p_1)^{(k-1)/k} \qquad T_2/(450+460) = (\frac{100}{80})^{(1.33-1)/1.33} \qquad T_2 = 962\text{ °R}$ or 502 °F

16.47 Solve Prob. 16.46 assuming a real gas and using the steam tables.

▮ With $p_1 = 80$ psia and $T_1 = 450$ °F, $s_1 = 1.7078$Btu/(lbm-°F) (from the steam tables). With $p_2 = 100$ psia and $s_2 = s_1 = 1.7078$Btu/(lbm-°F), $T_2 = 500$ °F (from the steam tables).

16.48 Carbon dioxide ($k = 1.30$, $M = 44.01$) enters a constant-area duct at 340 °F, 100 psia, and 500 fps. Farther downstream the properties are $V_2 = 1000$ fps and $T_2 = 940$ °F. Compute **(a)** p_2, **(b)** the heat added between sections, **(c)** the entropy change between sections, and **(d)** the mass flow per unit area.

▮ **(a)** $p_2/p_1 = (T_2/T_1)(V_1/V_2) \qquad p_2/100 = [(940+460)/(340+460)](\frac{500}{1000}) \qquad p_2 = 87.5$ psia

(b) $q = (c_p)(T_2 - T_1) + (v_2^2 - v_1^2)/2 \qquad c_p = kR/(k-1) \qquad R = 49\,720/M = 49\,720/44.01 = 1130\text{ ft}^2/(\text{s}^2\text{-°R})$

$$c_p = (1.30)(1130)/(1.30-1) = 4897\text{ ft}^2/(\text{s}^2\text{-°R})$$

$$q = (4897)(940-340) + (1000^2 - 500^2)/2 = 3.313\times10^6\text{ ft-lb/siug} = 3.313\times10^6/[(32.2)(778)] = 132\text{ Btu/lbm}$$

(c) $s_2 - s_1 = c_p \ln(T_2/T_1) - R\ln(p_2/p_1) = (4897)\{\ln[(940+460)/(340+460)]\} - (1130)[\ln(87.5/100)]$

$$= 2891\text{ ft-lb/(slug-°R)}$$

(d) $\dot{m} = \rho_1 A_1 v_1 \qquad \rho_1 = p_1/RT_1 = (100)(144)/[(1130)(340+460)] = 0.01593\text{ slug/ft}^3$

$$\dot{m}/A_1 = (0.01593)(500) = 7.97\text{ (slugs/s)/ft}^2$$

16.49 Steam enters a duct at $p_1 = 60$ psia, $T_1 = 350$ °F, $V_1 = 200$ fps, and leaves at $p_2 = 120$ psia, $T_2 = 850$ °F, and $V_2 = 1200$ fps. How much heat was added?

▮ $q = (h_2 - h_1) + (v_2^2 - v_1^2)/2$. From the steam tables, $h_1 = 1208.2$ Btu/lbm and $h_2 = 1453.4$ Btu/lbm. $q = (1453.4 - 1208.2) + [(1200^2 - 200^2)/2]/[(32.2)(778)] = 273$ Btu/lbm.

16.50 The internal energy of a hypothetical perfect gas is given as $u = T^{1/2}/50 + 100$. Determine c_v and c_p. Take $R = 50$ ft-lb/(lbm-°F).

▮

$$c_v = du/dt = (\tfrac{1}{2})(T^{-1/2})/50 = 0.0100T^{-1/2}\text{ Btu/(lbm-°R)}$$

$$c_p = R + c_v = \tfrac{50}{778} + 0.0100T^{-1/2} = 0.0643 + 0.0100T^{-1/2}\text{ Btu/(lbm-°R)}$$

16.51 Air at 15 °C and 101 325 Pa is compressed to a pressure of 345 000 Pa. If the compression is adiabatic and reversible, what is the final specific volume? How much work is done per kilogram of the gas?

▮ $p_1/p_2 = [(V_s)_2/(V_s)_1]^k \qquad V_s = RT/p \qquad (V_s)_1 = (287)(15+273)/101\,325 = 0.8158\text{ m}^3/\text{kg}$

$$101\,325/345\,000 = [(V_s)_2/(0.8158)]^{1.40} \qquad (V_s)_2 = 0.340\text{ m}^3/\text{kg} \qquad W = \int p\,dv$$

Since $p(V_s)^k = \text{constant}$,

$$W = \int_{0.8158}^{0.340} \frac{\text{constant}}{(V_s)^k}\,dv = (\text{constant})\left[\frac{1/(V_s)^{k-1}}{1-k}\right]_{0.8158}^{0.340}$$

$$= (\text{constant})\left[\frac{(1/0.340)^{1.40-1} - (1/0.8158)^{1.40-1}}{1-1.40}\right] = (-1.137)(\text{constant})$$

$$\text{Constant} = (101\,325)(0.8158)^{1.40} = 76\,196 \qquad W = (-1.137)(76\,196) = -86\,600\text{ J/kg}$$

16.52 Do Prob. 16.51 for an isothermal compression.

▮ $(p_1)(V_s)_1 = (p_2)(V_s)_2$. Using data from Prob. 16.51, $(101\,325)(0.8158) = (345\,000)(V_s)_2$, $(V_s)_2 = 0.240\ \text{m}^3/\text{kg}$.

$$W = \int p\,dv = \text{constant}\int_{0.8158}^{0.240} \frac{dV_s}{V_s} = \text{constant}\,[\ln V_s]_{0.8158}^{0.240} = (\text{constant})\left(\ln\frac{0.240}{0.8158}\right) = (\text{constant})(-1.224)$$

$$\text{Constant} = (101\,325)(0.8158) = 82\,661 \qquad W = (82\,661)(-1.224) = -101\,000\ \text{J/kg}$$

16.53 An airplane is capable of attaining a flight Mach number of 0.9. When it is flying at an altitude of 1000 ft in standard atmosphere, what is the ground speed if the air is not moving relative to the ground? What is the ground speed if the plane is at an altitude of 35 000 ft in standard atmosphere?

▮ At altitude 1000 ft, $c = 1113$ ft/s, $v_{\text{plane}} = (0.9)(1113) = 1002$ ft/s. At altitude 35 000 ft, $c = 973$ ft/s, $v_{\text{plane}} = (0.9)(973) = 876$ ft/s.

16.54 Do the first part of Prob. 16.53 if the air is moving at 30 mph directly opposite to the direction of flight.

▮ $$v_{\text{plane}} = 1002 - (30)(5280)/3600 = 958\ \text{ft/s}$$

16.55 What is the value of k for standard atmosphere at an altitude of 30 000 ft?

▮ $$c = \sqrt{kp/\rho} \qquad 995 = \sqrt{(k)(628)/(0.374)(0.00238)} \qquad k = 1.40$$

16.56 Suppose that a plane is moving horizontally relative to the ground at a speed of twice the velocity of sound (340.5 m/s) and that the air is moving in the opposite direction at a speed of one-fourth the velocity of sound relative to the ground. What is the Mach angle?

▮ Relative to the ground: $V_{\text{source}} = (2)(340.5) = 681.0$ m/s, $V_{\text{air}} = 340.5/4 = 85.1$ m/s. Velocity of source relative to the air is $681.0 + 85.1 = 766.1$ m/s. Hence, $N_M = 766.1/340.5 = 2.25$, $\sin\alpha = 1/2.25$, $\alpha = 26.4°$.

16.57 Suppose that a cruise missile under test is moving horizontally at $N_m = 2$ in the atmosphere at an elevation of 300 m above the earth's surface. How long does it take for an observer to hear the disturbance from the instant when it is directly overhead? Assume standard atmosphere.

▮ See Fig. 16-2. $\sin\alpha = 1/2$, $\alpha = 30.0°$; $\tan 30° = 300/d$, $d = 519.6$ m. Speed of shell is $(2)(340) = 680$ m/s. $t = d/V = 519.6/680 = 0.764$ s.

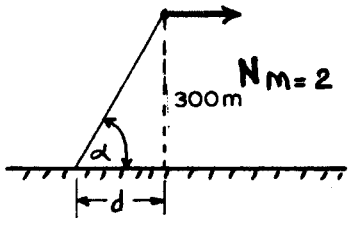


Fig. 16-2

16.58 Suppose in Prob. 16.57 that an observer in a plane is moving in the same direction as the missile at a speed of one-half the speed of sound at an elevation of 300 m above the missile. What is the time elapsed between the instant when the missile is directly below and the instant when the observer hears the sound? Neglect change of c from 300 m to 600 m elevation.

▮ See Fig. 16-3. Using data from Prob. 16.57, $680t = 519.6 + (\frac{680}{4})(t)$, $t = 1.02$ s.

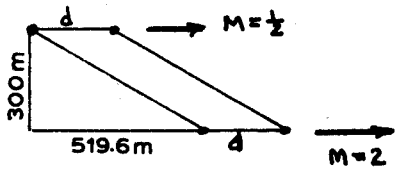


Fig. 16-3

16.59 A certain aircraft flies at the same Mach number regardless of its altitude. It flies 80 km/h slower at 10 km than at sea level. What is its Mach number?

▮ $c = \sqrt{kRT}$. At sea level, $c_1 = \sqrt{kRT_1} = 340.3$ m/s. At 10 000 m, $c_2 = \sqrt{kRT_2} = 299.4$ m/s, $\Delta v = (N_M)(c_2 - c_1)$, $(80)(1000)/3600 = (N_M)(340.3 - 299.4)$, $N_M = 0.543$.

16.60 At 150 °C and 1 atm, compute the speed of sound of (*a*) air, (*b*) oxygen, (*c*) hydrogen, (*d*) steam, (*e*) carbon monoxide.

▮ $$c = \sqrt{kRT}$$

(*a*) $c = \sqrt{(1.40)(287)(150+273)} = 412$ m/s (*b*) $c = \sqrt{(1.40)(260)(150+273)} = 392$ m/s

(*c*) $c = \sqrt{(1.41)(4127)(150+273)} = 1569$ m/s (*d*) $c = \sqrt{(1.33)(461)(150+273)} = 509$ m/s

(*e*) $c = \sqrt{(1.40)(297)(150+273)} = 419$ m/s

16.61 Assuming that water follows the liquid equation of state $K = (n)(B+1)(p_a)(\rho/\rho_a)^n$ with $n = 7$ and $B = 3000$, compute the bulk modulus and speed of sound at (*a*) 1 atm and (*b*) 1000 atm.

▮ (*a*) At 1 atm (101 350 Pa), $\rho = \rho_a$, $K = (7)(3000+1)(101\,350)(1)^7 = 2.13$ GPa, $c = \sqrt{K/\rho} = \sqrt{(2.13\times10^9)/998} = 1461$ m/s.

(*b*) At 1100 atm, $\rho/\rho_a = [(1000+3000)/3001]^{1/7} = 1.042$, $K = (K_{atm})(1.042)^7 = (2.13\times10^9)(1.042)^7 = 2.84\times10^9$ Pa, $c = \sqrt{(2.84\times10^9)/[(998)(1.042)]} = 1653$ m/s.

16.62 The measured value of the speed of sound in water at 20 °C and 9000 atm is 2650 m/s.* Compare this with the value computed by the analysis of Prob. 16.61.

▮ $c = \sqrt{K/\rho}$, $K = (n)(B+1)(p_a)(\rho/\rho_a)^n$, $n = 7$, $B = 3000$, $\rho_a = 998$ kg/m³. From Prob. 16.61, $9000 = (3001)(\rho/998)^7 - 3000$, $\rho = 1216.5$ kg/m³; $K = (7)(3000+1)(101\,350)(1216.5/998)^7 = 8.513$ GPa, $c = \sqrt{(8.513\times10^9)/1216.5} = 2645$ m/s. This is $(2650-2645)/2650 = 0.0019$, or 0.19 percent less than the measured value.

16.63 Mercury at 1 atm has a bulk modulus of about 28 GPa. It has also $n = 7$ in the equation of state of Prob. 16.61. What value of B in that equation best fits the measured bulk modulus? Estimate the bulk modulus and speed of sound of mercury at 2000 atm.

▮ From Prob. 16.61, $K_a = (n)(B+1)(p_a) = 2.8\times10^{10} = (7)(B+1)(101\,350)$, $B = 39\,466$. At 2000 atm,

$$p/p_a = 2000 = (39\,466+1)(\rho/\rho_a)^7 - 39\,466 \qquad \rho/\rho_a = 1.007$$

$$K = (K_a)(\rho/\rho_a)^7 = (2.8\times10^{10})(1.007)^7 = 29.4 \text{ GPa}$$

$$\rho = 1.007\rho_a = (1.007)(13\,550) = 13\,645 \text{ kg/m}^3 \qquad c = \sqrt{K/\rho} = \sqrt{(2.94\times10^{10})/13\,645} = 1468 \text{ m/s}$$

16.64 Why do (*a*) water and mercury and (*b*) aluminum and steel have nearly equal speeds of sound in spite of the fact that the second material of each pair is much heavier than the first? Can this behavior be predicted from molecular theory?

▮ In both cases, the lighter material has a proportionately smaller bulk modulus.

16.65 An airplane flies at 360 m/s through air at −10 °C and 40 kPa. Is the airplane supersonic?

▮ $c = \sqrt{kRT} = \sqrt{(1.40)(287)(273-10)} = 325$ m/s, $N_M = v/c = \frac{360}{325} = 1.1$. Since $N_M > 1.0$, it is supersonic.

16.66 A weak pressure wave (sound wave) with a pressure change $\Delta p = 50$ Pa propagates through air at 20 °C and 1 atm. Estimate the (*a*) velocity change across the wave, (*b*) density change, and (*c*) temperature change.

▮ (*a*) $\Delta p = \rho c\,\Delta v \qquad 50 = (1.205)(343)(\Delta v) \qquad \Delta v = 0.1210$ m/s

(*b*) $\Delta\rho = (\rho + \Delta\rho)(\Delta v/c) = (1.205 + \Delta\rho)(0.1210/343) = 0.0004253$ kg/m³

(*c*) Approximately isentropic:

$$(T + \Delta T)/T = [(p + \Delta p)/p]^{(k-1)/k}$$

$$(273 + 20 + \Delta T)/(273 + 20) = [(101\,350 + 50)/101\,350]^{(1.40-1)/1.40} \qquad \Delta T = 0.041\ °\text{C}$$

16.67 Air at 65 °F flows isothermally through a 6-in-diameter pipe. The pressure at one section is 82 psia, and that at a section 550 ft downstream is 65 psia. If the pipe surface is "smooth," find the weight flow rate of the air.

* A. H. Smith and A. W. Larson, *J. Chem. Phys.*, vol. 22, p. 351, 1954.

▮ $$p_1^2 - p_2^2 = (G^2RT/g^2A^2)[fL/D + 2\ln(p_1/p_2)] \qquad A = (\pi)(\tfrac{6}{12})^2/4 = 0.1963\ \text{ft}^2$$

Assume $f = 0.007$: $(82^2 - 65^2)(144)^2 = (G^2)(53.3)(65 + 460)/[(32.2)^2(0.1963)^2][(0.007)(550)/(\tfrac{6}{12}) + (2)(\ln \tfrac{82}{65})]$, $G = 95.2$ lb/s. This is the weight flow rate of the air if the assumed value of f of 0.007 is correct. This must be checked.

$$N_R = \rho D/\mu \qquad \rho = p/RT = (82)(144)/[(1716)(65 + 460)] = 0.01311\ \text{lb-s}^2/\text{ft}^4 \qquad G = \gamma A v$$

$$\gamma = \rho g = (0.01311)(32.2) = 0.4221\ \text{lb/ft}^3 \qquad 95.2 = (0.4221)(0.1963)(v) \qquad v = 1149\ \text{ft/s}$$

$$N_R = (0.01311)(\tfrac{6}{12})(1149)/(3.78 \times 10^{-7}) = 1.99 \times 10^7$$

$$f = 0.007 \qquad \text{(from Fig. A-5, using the "smooth pipes" line)}$$

Since this value of f is the same as the assumed value, the computed weight flow rate of 95.2 lb/s is taken as the correct value.

16.68 Air at 18 °C flows isothermally through a 300-mm-diameter pipe at of 0.450 kN/s (weight flow). The pressure at one section is 550 kPa abs, and the pipe surface is smooth. Find the pressure at a section 200 m downstream.

▮ $$p_1^2 - p_2^2 = (G^2RT/g^2A^2)[fL/D + 2\ln(p_1/p_2)] \qquad A = (\pi)(0.300)^2/4 = 0.07069\ \text{m}^2$$

$$N_R = DG/gA\mu = (300 \times 10^{-3})(0.450 \times 10^3)/(9.807)(0.07069)(1.81 \times 10^{-8})$$

From Fig. A-5, $f = 0.0080$. Substituting into the equation given above and neglecting temporarily the second term inside the brackets, $550^2 - p_2^2 = \{(0.450)^2(29.3)(273 + 18)/[(9.807)^2(0.07069)^2]\}[(0.0080)(200)/0.300]$, $p_2 = 532$ kPa. Substituting this value of p_2 into the term that was neglected and solving for the p_2 on the left side of the equation, $550^2 - p_2^2 = \{(0.450)^2(29.3)(273 + 18)/[(9.807)^2(0.07069)^2]\}[(0.0080)(200)/0.300 + (2)(\ln \tfrac{550}{532})]$, $p_2 = 532$ kPa (O.K.).

16.69 Air at 100 °F flows isothermally through a 4-in-diameter pipe. Pressures at sections 1 and 2 are 120 psia and 80 psia, respectively. Section 2 is located 400 ft downstream from section 1. Determine the weight flow rate of the air. Assume the pipe has a smooth surface.

▮ $$p_1^2 - p_2^2 = (G^2RT/g^2A^2)[fL/D + 2\ln(p_1/p_2)] \qquad A = (\pi)(\tfrac{4}{12})^2/4 = 0.08727\ \text{ft}^2$$

Assume $f = 0.007$: $(120^2 - 80^2)(144)^2 = (G^2)(53.3)(100 + 460)/[(32.2)^2(0.08727)^2][(0.007)(400)/(\tfrac{4}{12}) + (2)(\ln \tfrac{120}{80})]$, $G = 69.0$ lb/s. This is the weight flow rate of the air if the assumed value of f of 0.007 is correct. Further checking (not shown here but following the pattern of Prob. 16.67) indicates that the assumed value of f of 0.007 is correct, and therefore $G = 69.0$ lb/s.

16.70 Air at 85 °F flows isothermally through a 6-in-diameter pipe at a flow rate of 10 lb/s. The pipe surface is very smooth. If the pressure at one section is 70 psia, determine the pressure at a section 600 ft downstream from the first section.

▮ $$p_1^2 - p_2^2 = (G^2RT/g^2A^2)[fL/D + 2\ln(p_1/p_2)] \qquad A = (\pi)(\tfrac{6}{12})^2/4 = 0.1963\ \text{ft}^2 \qquad N_R = \rho D v/\mu$$

$$\gamma = p/RT = (70)(144)/[(53.3)(460 + 85)] = 0.3470\ \text{lb/ft}^3 \qquad \rho = \gamma/g = 0.3470/32.2 = 0.01078\ \text{slug/ft}^3$$

$$G = \gamma A v \qquad 10 = (0.3470)(0.1963)(v) \qquad v = 146.8\ \text{ft/s} \qquad N_R = (0.01708)(\tfrac{6}{12})(146.8)/(3.78 \times 10^{-7}) = 2.09 \times 10^6$$

From Fig. A-5, $f = 0.0103$. Substituting into the equation given above and neglecting temporarily the second term inside the brackets, $(70^2)(144)^2 - (p_2)^2(144)^2 = (10)^2(53.3)(460 + 85)/[(32.2)^2(0.1963)^2][(0.0103)(600)/(\tfrac{6}{12})]$, $p_2 = 69.69$ lb/in^2. Substituting this value of p_2 into the term that was neglected and solving for the p_2 on the left side of the equation, $(70^2)(144)^2 - (p_2)^2(144)^2 = \{(10)^2(53.3)(460 + 85)/[(32.2)^2(0.1963)^2]\} \times \{(0.0103)(600)/(\tfrac{6}{12}) + (2)[\ln(70/69.69)]$, $p_2 = 69.69$ lb/in^2 (O.K.).

16.71 Prepare a computer program to solve for either the weight flow rate or the pressure at a section downstream for isothermal flow of a compressible fluid.

▮
```
C     THIS PROGRAM DETERMINES EITHER THE FLOW RATE OR THE PRESSURE AT A
C     SECOND (DOWNSTREAM) POINT FOR CLOSED CONDUIT, COMPRESSIBLE FLOW.
C     IT IS BASED ON AN ISOTHERMAL FLOW ANALYSIS AND IS THEREFORE SUB-
C     JECT TO THE RESTRICTIONS AND/OR ASSUMPTIONS IMPLICIT IN ISOTHERMAL
C     FLOW.  IT CAN BE USED FOR PROBLEMS IN BOTH THE ENGLISH SYSTEM OF
```

```
C     UNITS AND THE INTERNATIONAL SYSTEM OF UNITS.  THE APPLICATION OF
C     THIS PROGRAM IS LIMITED TO CASES INVOLVING A SINGLE CONDUIT WITH
C     A CONSTANT DIAMETER.
C
C     INPUT DATA MUST BE SET UP AS FOLLOWS.
C
C     CARD 1   COLUMN 1          ENTER 0 (ZERO) OR BLANK IF ENGLISH SYSTEM
C                                OF UNITS IS TO BE USED.  ENTER 1 (ONE) IF
C                                INTERNATIONAL SYSTEM OF UNITS IS TO BE USED.
C              COLUMNS 2-79      ENTER TITLE, DATE, AND OTHER INFORMATION,
C                                IF DESIRED.
C     CARD 2   COLUMNS 1-10      ENTER NUMBER INCLUDING DECIMAL GIVING
C                                ABSOLUTE PRESSURE AT UPSTREAM POINT (IN
C                                POUNDS PER SQUARE INCH OR KILO-
C                                PASCALS).
C              COLUMNS 11-20     ENTER NUMBER INCLUDING DECIMAL GIVING
C                                ABSOLUTE PRESSURE AT DOWNSTREAM POINT (IN
C                                POUNDS PER SQUARE INCH OR KILO-
C                                PASCALS).
C              COLUMNS 21-30     ENTER NUMBER INCLUDING DECIMAL GIVING GAS
C                                CONSTANT (IN FEET PER DEGREE-RANKINE OR
C                                METERS PER DEGREE-KELVIN).
C              COLUMNS 31-40     ENTER NUMBER INCLUDING DECIMAL GIVING
C                                TEMPERATURE (IN DEGREES-FAHRENHEIT OR
C                                DEGREES-CELSIUS).
C              COLUMNS 41-50     ENTER NUMBER INCLUDING DECIMAL GIVING
C                                DIAMETER OF CONDUIT (IN INCHES OR MILLI-
C                                METERS).
C              COLUMNS 51-60     ENTER NUMBER INCLUDING DECIMAL GIVING
C                                LENGTH OF CONDUIT (IN FEET OR METERS).
C              COLUMNS 61-70     ENTER NUMBER INCLUDING DECIMAL GIVING
C                                ROUGHNESS (IN FEET OR METERS).  ENTER A
C                                VALUE OF 0 (ZERO) FOR "SMOOTH" CONDUITS.
C              COLUMNS 71-80     ENTER NUMBER INCLUDING DECIMAL GIVING
C                                VISCOSITY OF FLUID (IN POUNDS-SECONDS
C                                PER SQUARE FOOT OR KILONEWTONS-SECONDS PER
C                                SQUARE METER).
C     CARD 3   COLUMNS 1-10      ENTER NUMBER INCLUDING DECIMAL GIVING
C                                WEIGHT FLOW RATE (IN POUNDS PER
C                                SECOND OR KILONEWTONS PER SECOND).
C              COLUMNS 11-34     ENTER TYPE OF FLUID.
C              COLUMNS 35-58     ENTER TYPE OF CONDUIT.
C
C ***********************************************************************
C *                                                                     *
C *    NOTE WELL....EITHER THE FLOW RATE (COLUMNS 1-10 ON CARD 3) OR    *
C *    THE PRESSURE AT DOWNSTREAM POINT (COLUMNS 11-20 ON CARD 2),      *
C *    WHICHEVER ONE IS TO BE DETERMINED BY THIS PROGRAM, SHOULD BE     *
C *    LEFT BLANK.                                                      *
C *                                                                     *
C ***********************************************************************
C
C     MULTIPLE DATA SETS FOR SOLVING ANY NUMBER OF PROBLEMS MAY BE
C     INCLUDED FOR PROCESSING.
C
      DIMENSION TITLE(13),FLUID(4),PIPE(4)
      COMMON F,ED,RN
      PI=3.14159265
      REAL L
 1    READ(5,100,END=2)UNITS,TITLE
  100 FORMAT(I1,13A6)
      WRITE(6,105)TITLE
  105 FORMAT('1',13A6,//)
      READ(5,101)P1,P2,R,T,D,L,E,VIS,Q,FLUID,PIPE
  101 FORMAT(8F10.0,/,F10.0,8A6)
      FACTOR=12.0
      IF(UNITS.EQ.1)FACTOR=1000.0
      PFACT=144.0
      IF(UNITS.EQ.1)PFACT=1.0
      TFACT=460.0
      IF(UNITS.EQ.1)TFACT=273.0
      G=32.2
      IF(UNITS.EQ.1)G=9.807
```

```
      A=PI*D**2/FACTOR**2/4.0
      SW=P1*PFACT/R/(T+TFACT)
      RHO=SW/G
      ED=E/D*FACTOR
      IF(Q.GT.0.0001) GO TO 102
      FF=0.02
  104 Q=SQRT((P1**2-P2**2)*PFACT**2*G*G*A*A/(R*(T+TFACT)*(FF*L/D*FACTOR+
     *2.0*ALOG(P1/P2))))
      V=Q/SW/A
      RN=RHO*D/FACTOR*V/VIS
      CALL ROUGH
      DIFF=ABS(F-FF)
      IF(DIFF.LT.0.0001)GO TO 103
      FF=F
      GO TO 104
  103 IF (UNITS.EQ.0)WRITE(6,106)P1,P2,D,E,R,T,L,FLUID,PIPE,Q
  106 FORMAT(1X,'GIVEN DATA FOR A COMPRESSIBLE FLOW IN A CIRCULAR CLOSED
     * CONDUIT',//5X,'PRESSURE AT POINT 1 =',F8.1,' PSI',//5X,'PRESSURE
     *AT POINT 2 =',F8.1,' PSI',//5X,'DIAMETER OF CONDUIT =',F7.2,
     *' IN',//5X,'ROUGHNESS =',F10.7,' FT',//5X,'GAS CONSTANT =',F7.1,
     *' FT/DEG-R',//5X,'TEMPERATURE =',F7.1,' DEG-F',//5X,'LENGTH OF CON
     *DUIT =',F8.1,' FT',//5X,'FLUID FLOWING IS ',4A6,//5X,'CONDUIT MATE
     *RIAL IS ',4A6,//1X,'THE FLOW RATE WILL BE',F7.1,' LB/S')
      IF (UNITS.EQ.1)WRITE(6,107)P1,P2,D,E,R,T,L,FLUID,PIPE,Q
  107 FORMAT(1X,'GIVEN DATA FOR A COMPRESSIBLE FLOW IN A CIRCULAR CLOSED
     * CONDUIT',//5X,'PRESSURE AT POINT 1 =',F8.1,' KPA',//5X,'PRESSURE
     *AT POINT 2 =',F8.1,' KPA',//5X,'DIAMETER OF CONDUIT =',F7.1,
     *' MM',//5X,'ROUGHNESS =',F10.7,' M ',//5X,'GAS CONSTANT =',F7.1,
     * ' M/DEG-K',//5X,'TEMPERATURE =',F7.1,' DEG-C',//5X,'LENGTH OF CON
     *DUIT =',F8.1,' M ',//5X,'FLUID FLOWING IS ',4A6,//5X,'CONDUIT MATE
     *RIAL IS ',4A6,//1X,'THE FLOW RATE WILL BE',F7.3,' KN/S')
      GO TO 1
  102 V=Q/SW/A
      RN=RHO*D/FACTOR*V/VIS
      CALL ROUGH
      P2=SQRT(P1**2-Q**2*R*(T+TFACT/G**2/A**2*F*L/D*FACTOR/PFACT**2)
  109 P22=SQRT(P1**2-Q**2**R*(T+TFACT)/G**2/A**2*(F*L/D*FACTOR+2.0*
     *ALOG(P1/P2))/PFACT**2)
      DIFF=ABS(P22-P2)
      IF(DIFF.LT.0.01)GO TO 108
      P2=P22
      GO TO 109
  108 IF(UNITS.EQ.0)WRITE(6,111)P1,D,E,R,T,L,Q,FLUID,PIPE,P2
  111 FORMAT(1X,'GIVEN DATA FOR A COMPRESSIBLE FLOW IN A CIRCULAR CLOSED
     * CONDUIT',//5X,'PRESSURE AT POINT 1 =',F8.1,' PSI',//5X,'DIAMETER
     *OF CONDUIT =',F7.2,' IN',//5X,'ROUGHNESS =',F10.7,' FT',//5X,'GAS
     *CONSTANT =',F7.1,' FT/DEG-R',//5X,'TEMPERATURE =',F7.1,' DEG-F',
     *//5X,'LENGTH OF CONDUIT =',F8.1,' FT',//5X,'FLOW RATE =',F7.1,
     *' LB/S ',//5X,'FLUID FLOWING IS ',4A6,//5X,'CONDUIT MATERIAL IS ',
     *4A6,//1X,'THE PRESSURE AT POINT 2 WILL BE',F8.1,' PSI')
      IF(UNITS.EQ.1)WRITE(6,110)P1,D,E,R,T,L,Q,FLUID,PIPE,P2
  110 FORMAT(1X,'GIVEN DATA FOR A COMPRESSIBLE FLOW IN A CIRCULAR CLOSED
     * CONDUIT',//5X,'PRESSURE AT POINT 1 =',F8.1,' KPA',//5X,'DIAMETER
     *OF CONDUIT =',F7.1,' MM',//5X,'ROUGHNESS =',F10.7,' M ',//5X,'GAS
     *CONSTANT =',F7.1,' M/DEG-K ',//5X,'TEMPERATURE =',F7.1,' DEG-C',
     *//5X,'LENGTH OF CONDUIT =',F8.1,' M ',//5X,'FLOW RATE =',F7.3,
     *' KN/S ',//5X,'FLUID FLOWING IS ',4A6,//5X,'CONDUIT MATERIAL IS ',
     *4A6,//1X,'THE PRESSURE AT POINT 2 WILL BE ',F8.1,'KPA')
      GO TO 1
2     STOP
      END
      SUBROUTINE ROUGH
      COMMON F,ED,RN
      IF(RN.LE.2000.0)F=64.0/RN
      IF(RN.LE.2000.0)RETURN
      IF(RN.LT.4000.0)WRITE(6,103)
  103 FORMAT(1X,'A REYNOLDS NUMBER IS IN THE CRITICAL ZONE, FOR WHICH TH
     *E FRICTION FACTOR IS UNCERTAIN.  HENCE, PROGRAM EXECUTION WAS TERM
     *INATED.')
      IF(RN.LT.4000.0)STOP
      F=0.006
      TRY1=1.0/SQRT(F)+2.0*ALOG10(ED/3.7+2.51/RN/SQRT(F))
```

```
102 F=F+0.00001
    TRY2=1.0/SQRT(F)+2.0*ALOG10(ED/3.7+2.51/RN/SQRT(F))
    IF(TRY1*TRY2)100,100,101
101 TRY1=TRY2
    GO TO 102
100 F=F-0.000005
    RETURN
    END
```

16.72 Solve Prob. 16.67 using the computer program of Prob. 16.71.

Input

```
0SAMPLE ANALYSIS ØF CØMPRESSIBLE FLØW
82.0      65.0      53.3      65.0      6.0       550.0                .000000378
          AIR                         "SMØØTH"
```

Output

```
SAMPLE ANALYSIS OF COMPRESSIBLE FLOW

GIVEN DATA FOR A COMPRESSIBLE FLOW IN A CIRCULAR CLOSED CONDUIT

    PRESSURE AT POINT 1 =    82.0 PSI

    PRESSURE AT POINT 2 =    65.0 PSI

    DIAMETER OF CONDUIT =   6.00 IN

    ROUGHNESS = 0.0000000 FT

    GAS CONSTANT =   53.3 FT/DEG-R

    TEMPERATURE =   65.0 DEG-F

    LENGTH OF CONDUIT =   550.0 FT

    FLUID FLOWING IS AIR

    CONDUIT MATERIAL IS "SMOOTH"

THE FLOW RATE WILL BE  95.2 LB/S
```

16.73 Solve Prob. 16.68 using the computer program of Prob. 16.71.

Input

```
1SAMPLE ANALYSIS ØF CØMPRESSIBLE FLØW
550.0                29.3      18.0      300.0     200.0                .000000018
0.450     AIR                         "SMØØTH"
```

Output

```
SAMPLE ANALYSIS OF COMPRESSIBLE FLOW

GIVEN DATA FOR A COMPRESSIBLE FLOW IN A CIRCULAR CLOSED CONDUIT

    PRESSURE AT POINT 1 =   550.0 KPA

    DIAMETER OF CONDUIT =  300.0 MM

    ROUGHNESS = 0.0000000 M

    GAS CONSTANT =   29.3 M/DEG-K

    TEMPERATURE =   18.0 DEG-C

    LENGTH OF CONDUIT =   200.0 M
```

```
FLOW RATE =  0.450 KN/S

FLUID FLOWING IS AIR

CONDUIT MATERIAL IS "SMOOTH"

THE PRESSURE AT POINT 2 WILL BE   532.1 KPA
```

16.74 Air flows isothermally at 65 °F through a horizontal 10- by 14-in rectangular duct at 100 lb/s. If the pressure at a section is 80 psia, find the pressure at a second section 500 ft downstream from the first. Assume the duct surface is very smooth; hence the lowest curve of Fig. A-5 may be used to determine f.

▮ $(N_M)_1^2/(N_M)_2^2 = 1 - k(N_M)_1^2\{2\ln[(N_M)_2/(N_M)_1] + f(L/D)\}$ $\qquad R_h = A/p_w = \frac{140}{48} = 2.92\text{ in} = 0.243\text{ ft}$

$$R_1 = DV\rho/\mu = GD/\mu gA = G(4R)/\mu gA = [100(4\times 0.243)]/[(3.78\times 10^{-7})(32.2)\tfrac{140}{144}] = 8.2\times 10^6$$

$R_2 = R_1$ since $\rho_1 V_1 = \rho_2 V_2$ and $\mu_1 = \mu_2$. From Fig. A-5, $f = 0.0083$.

$$\gamma_1 = pg/RT = [(80\times 144)32.2]/[1715(460+65)] = 0.41\text{ lb/ft}^3 \qquad V_1 = G/\gamma_1 A = 100/[(0.41)\tfrac{140}{144}] = 250\text{ fps}$$

$$c = \sqrt{kRT} = \sqrt{(1.4)(1715)(525)} = 1123\text{ fps} \qquad (N_M)_1 = V_1/c = \tfrac{250}{1123} = 0.222$$

The limiting value of $(N_M)_1$ is $1/\sqrt{1.4} = 0.845$. Therefore,

$$(0.222)^2/(0.845)^2 = 1 - 1.4(0.222)^2\{2\ln(0.845/0.222) + 0.0083[L/4(0.243)]\} \qquad L = 1260\text{ ft}$$

Thus the equation $p_1^2 - p_2^2 = (G^2RT/g^2A^2)[f(L/D) + 2\ln(p_1/p_2)]$ applies for all values of $L < 1260$ ft. Substituting $L = 500$ ft and neglecting the usually small logarithmic term, $[(80)(144)]^2 - p_2^2 = [(100)^2(1715)(460+65)/(32.2)^2(\frac{140}{144})^2]\{(0.0083)[500/4(0.243)]\}$ from which $p_2 = 67.1$ psia. Substituting this value of p_2 and considering the logarithmic term yields $p_2 = 66.6$ psia. Repeating the process again will give a more accurate answer.

16.75 Refer to Prob. 16.74. Neglecting the logarithmic term, find the pressures at sections 100, 300, and 800 ft downstream of the section where the pressure is 80 psia.

▮
$$p_1 - p_2 = (G^2RT/g^2A^2)[f(L/D) + 2\ln(p_1/p_2)]$$
$$[(80)(144)]^2 - p_2^2 = [(100)^2 1715(460+65)/32.2^2(\tfrac{140}{144})^2]\{(0.0085)[L/4(0.243)]\}$$
$$p_2^2 = 1.327\times 10^8 - 80\,300L$$

At $L = 100$ ft, $p_2 = 11\,170$ psfa $= 77.5$ psia. At $L = 300$ ft, $p_2 = 10\,420$ psfa $= 72.4$ psia. At $L = 800$ ft, $p_2 = 8270$ psfa $= 57.5$ psia.

Express in terms of $p + \gamma(V^2/2g)$:

$$\text{At } L = 100\text{ ft},\ \gamma = 0.41(77.5/80) = 0.397\text{ pcf.}$$
$$\text{At } L = 300\text{ ft},\ \gamma = 0.41(72.4/80) = 0.371\text{ pcf.}$$
$$\text{At } L = 800\text{ ft},\ \gamma = 0.41(57.5/80) = 0.295\text{ pcf.}$$
$$\gamma_1 V_1 = \gamma_2 V_2 = \gamma_3 V_3$$

At $L = 100$ ft, $V = 250(0.41/0.397) = 258$ fps. At $L = 300$ ft, $V = 250(0.41/0.371) = 276$ fps. At $L = 800$ ft, $V = 250(0.41/0.295) = 347$ fps.

L, ft	p, psia	V, fps	$V^2/2g$	$\gamma V^2/2g$, psi	$p + \gamma V^2/2g$, psia
0	80.0	250	970	2.76	82.8
100	77.5	258	1034	2.86	80.4
300	72.4	276	1183	3.05	75.5
800	57.5	347	1870	3.83	61.3

$\Delta p = 80 - 57.5 = 22.5$ psi over the first 800 ft of length. *Note:* Greater accuracy can be achieved by considering the logarithmic term.

16.76 Air flows isothermally through a long horizontal pipe of uniform diameter. At a section where the pressure is 90 psia, the velocity is 110 fps. Because of fluid friction the pressure at a distant point is 30 psia. **(*a*)** What is the

increase in kinetic energy per pound of air? (*b*) What is the amount of thermal energy in Btu per pound of air that must be transferred in order to maintain the temperature constant? (*c*) Is this heat transferred to the air in the pipe or removed from it? (*d*) If the temperature of the air is 95 °F and the diameter of the pipe is 6 in find the total heat transferred in Btu per hour.

▮ pV = const for isothermal flow, therefore $V_2 = p_1V_1/p_2 = 90(110)/30 = 330$ fps, $V_1^2/2g = 188$ ft-lb/lb, $V_2^2/2g = 1691$ ft-lb/lb.

(*a*) $V_1^2/2g + Q_B = V_2^2/2g$. Increase in KE of air = 1691 − 188 = 1503 ft-lb/lb.
(*b*) Thermal energy/lb transferred = $Q_H = \Delta\text{KE} = \frac{1503}{778} = 1.932$ Btu/lb.
(*c*) KE increases, therefore heat is transferred *to* the air in the pipe.
(*d*)

$$\gamma_1 = gp_1/RT_1 = [(32.2)(90)(144)]/[(1715)(460+95)] = 0.438 \text{ pcf}$$

$$G = \gamma_1 V_1 A = (0.438)(110)[(\pi/4)(\tfrac{6}{12})^2] = 9.46 \text{ lb/s}$$

$$(1.932 \times 9.46) = 18.28 \text{ Btu/s} = 65\,808 \text{ Btu/h}$$

16.77 For the case of Prob. 16.74 with a duct length of 500 ft, compute the thermal energy (heat) that must be added to the fluid to maintain isothermal conditions.

▮ Since the flow is isothermal, $p_1/\rho_1 = p_2/\rho_2 = RT$ = constant; $p_1 = 80$ psia and $p_2 = 66.6$ psia. Thus $\rho_1/\rho_2 = 80/66.6 = 1.20$ and $V_2/V_1 = 1.20$ since ρV = constant from continuity.

So $V_2 = 1.20(250) = 300$ fps; $V_1^2/2g + Q_H = V_2^2/2g$, $Q_H = (300)^2/[(2)(32.2)] - (250)^2/[(2)(32.2)] = 427$ ft-lb/lb of air. Since $G = 100$ lb/s, the rate at which heat must be added to the fluid is (100)(427) = 42 700 ft-lb/s. *Note*: if $Q_H > 427$ ft-lb/lb of air, $T_2 > T_1$, and if $Q_H < 427$ ft-lb/lb of air, $T_2 < T_1$.

16.78 Carbon dioxide flows isothermally at 100 °F through a horizontal 6-in-diameter pipe. At this temperature $\mu = 4.0 \times 10^{-7}$ lb-s/ft². The pressure changes from 160.0 to 150.0 psig in a 120-ft length of pipe. Determine the flow rate if the atmospheric pressure is 14.5 psia and ϵ for the pipe is 0.002 ft.

▮ At section 1 (inlet), $p_1 = 160.0$ psig + 14.5 psi = 174.5 psia, $R = 1123$ ft²/(s²-°R), $\gamma = gp/RT$, $\gamma_1 = [(32.2)(174.5)(144)]/[(1123)(460+100)] = 1.287$ pcf; $\epsilon/D = 0.002/0.5 = 0.004$. Assume $N_R > 10^6$, Fig. A-5: $f = 0.0285$. At section 2, $p_2 = 150$ psig + 14.5 psi = 164.5 psia.

$$p_1^2 - p_2^2 = (G^2RT/g^2A^2)[f(L/D) + 2\ln(p_1/p_2)]$$

$$[(174.5)(144)]^2 - [(164.5)(144)]^2 = [G^2(1123)(460+100)]/[32.2^2(0.1963)^2][0.0285(120/0.5) + 2\ln(174.5/164.5)]$$

$$G = 25.3 \text{ lb/s} \quad \text{(based on the above assumption)} \qquad V_1 = G/\gamma_1 A = 25.3/[1.287(0.1963)] = 100.1 \text{ fps}$$

$$N_R = (DV\gamma/g)/\mu \simeq [(\tfrac{6}{12})100.1(1.287/32.2)]/(4.0 \times 10^{-7}) = 5.00 \times 10^6$$

Hence assumption for N_R was O.K., and $G = 25.3$ lb/s.

16.79 Methane gas is to be pumped through a 24-in-diameter welded-steel pipe connecting two compressor stations 24 mi apart. At the upstream station the pressure is not to exceed 65 psia, and at the downstream station it is to be at least 25 psia. Determine the maximum possible rate of flow (in cubic feet per day at 70 °F and 65 psia). Assume isothermal flow at 70 °F.

▮ For methane: $R = 3100$ ft²/(s²-°R), $\mu = 2.40 \times 10^{-7}$ lb-s/ft². Absolute viscosity is independent of pressure over a wide range of pressures but kinematic viscosity varies with pressure: $\nu_1 = \mu/\rho_1 = \mu/(p_1/RT_1) = 2.40 \times 10^{-7}/[(65)(144)/(3100)(530)] = 4.21 \times 10^{-5}$ ft²/s, $\epsilon = 0.00015$ ft; $\epsilon/D = 0.000075$. As a first trial, assume $N_R > 10^7$, in which case from Fig. A-5 $f = 0.0115$.

$$p_1^2 - p_2^2 = (G^2RT/g^2A^2)[f(L/D) + 2\ln(p_1/p_2)]$$

$$[(65)(144)]^2 - [(25)(144)]^2 = \{G^2(3100)(460+70)/32.2^2[(\pi)(\tfrac{24}{12})^2/4]\}\{(0.0115)[(24)(5280)/2] + 2\ln\tfrac{65}{25}\}$$

$$G = 25.2 \text{ lb/s}$$

$$N_R = GD/\mu gA = (25.2)(2)/(2.40 \times 10^{-7})[(32.2)(\pi)] = 2.08 \times 10^6$$

Thus, from Fig. A-5: $f = 0.0125$; initially assumed f was too low. Substituting $f = 0.0125$ gives $G = 24.2$ lb/s, $\gamma_1 = gp_1/RT_1 = (32.2)(65)(144)/[(3100)(530)] = 0.1834$ lb/ft³, $Q = G/\gamma = 24.2/0.1834 = 132.0$ ft³/s = 11.40×10^6 ft³/day at 70 °F and 65 psia.

16.80 Air flows isothermally in a long pipe. At one section the pressure is 84 psia, the temperature is 70 °F, and the velocity is 100 fps. At a second section some distance from the first the pressure is 14 psia. Find the energy head

loss due to friction, and determine the thermal energy that must have been added to or taken from the fluid between the two sections. The diameter of the pipe is constant.

▮ For isothermal flow: $\rho_1/\rho_2 = p_1/p_2 = \frac{84}{14} = 6$. But from continuity $\rho_1 A_1 V_1 = \rho_2 A_2 V_2$, thus $V_2 = 6V_1$, $V_1^2/2g + Q_H = V_2^2/2g$, $Q_H = 600^2/64.4 - 100^2/64.4 = 5430$ ft-lb/lb of air. For head loss: $dp/\gamma + d(V^2/2g) = -(2\tau\, ds/\gamma r) = -dh_L$, $\gamma = g\rho = gp/RT$; substituting for γ, $(RT/g)(dp/p) + d(V^2/2g) = -dh_L$. Integrating

$$h_L = (RT/g)\ln(p_1/p_2) + (V_1^2 - V_2^2)/2g = [1715(460+70)/32.2]\ln 6 + (100^2 - 600^2)/2(32.2)$$
$$= 50\,580 - 5430 = 45\,150 \text{ ft-lb/lb of air}$$

Note: The total energy at section 2 is greater than that at section 1 because external heat is added. Thus h_L represents a degradation of mechanical energy, but not a net loss of energy, because it is converted to another form of energy, namely internal heat.

16.81 Air is flowing at constant temperature through a 3-in-diameter horizontal pipe, $f = 0.018$. At the entrance, $V_1 = 320$ ft/s, $T = 110$ °F, and $p_1 = 30$ psia. What is the maximum pipe length for this flow, and how much heat is transferred to the air per pound mass?

▮
$$L_{max} = (D/f)[(1 - kN_M^2)/kN_M^2 + \ln(kN_M^2)] \qquad N_M = V/\sqrt{kRT}$$
$$(N_M)_1 = 320/\sqrt{(1.40)(1716)(110+460)} = 0.273$$
$$L_{max} = [(\tfrac{3}{12})/0.018]\{[1-(1.40)(0.273)^2]/(1.40)(0.273)^2 + \ln[(1.40)(0.273)^2]\} = 87.8 \text{ ft} \qquad q_H = (V_2^2 - V_1^2)/2g$$
$$V_2 = (N_M)_2(c) \qquad (N_M)_2 = 1/\sqrt{k} = 1/\sqrt{1.40} = 0.845$$
$$V_2 = (0.845)(1180) = 997.1 \text{ ft/s} \qquad q_H = (997.1^2 - 320^2)/[(2)(32.2)(778)] = 17.8 \text{ Btu/lbm}$$

16.82 Air at 15 °C flows through a 20-mm-diameter pipe at constant temperature. At the entrance $V_1 = 60$ m/s, and at the exit $V_2 = 90$ m/s ($f = 0.018$). What is the length of the pipe?

▮
$$(f/D)(\Delta L) = [(1 - kN_M^2)/kN_M^2 + \ln(kN_M^2)]_1 - [(1 - kN_M^2)/k_M^2 + \ln(kN_M^2)]_2 \qquad N_M = V/\sqrt{kRT}$$
$$(N_M)_1 = 60/\sqrt{(1.40)(287)(15+273)} = 0.176 \qquad (N_M)_2 = 90/\sqrt{(1.40)(287)(15+273)} = 0.265$$
$$[0.018/0.020](\Delta L) = \{[1-(1.40)(0.176)^2]/(1.40)(0.176)^2 + \ln[(1.40)(0.176)^2]\}$$
$$- \{[1-(1.40)(0.265)^2]/(1.40)(0.265)^2 + \ln[(1.40)(0.265)^2]\} \qquad \Delta L = 13.4 \text{ m}$$

16.83 If the pressure at the entrance of the pipe of Prob. 16.82 is 2.0 atm, find the pressure at the exit.

▮
$$p^{*t}/p = (\sqrt{k})(N_M) \qquad p^{*t}/p_1 = (\sqrt{k})(N_M)_1 \qquad p^{*t}/p_2 = (\sqrt{k})(N_M)_2$$

Using data from Prob. 16.82,

$$p_2/p_1 = (N_M)_1/(N_M)_2 = 0.176/0.265 = 0.664 \qquad p_2 = (0.664)(2.0) = 1.328 \text{ atm}$$

16.84 Hydrogen enters a pipe from a converging nozzle at $N_M = 1$, $p = 5$ psia, and $T = 0$ °F. Determine, for isothermal flow, the maximum length of pipe, in diameters, and the pressure change over this length ($f = 0.018$).

▮
$$L = (D/f)[(1 - kN_M^2)/kN_M^2 + \ln(kN_M^2)]$$
$$L/D = (1/0.018)\{[1-(1.40)(1)^2]/(1.40)(1)^2 + \ln[(1.40)(1)^2]\} = 2.82$$
$$p^{*t}/p = (\sqrt{k})(N_M) \qquad p^{*t}/5 = (\sqrt{1.40})(1) \qquad p^{*t} = 5.916 \text{ psia} \qquad \Delta p = 5.916 - 5 = 0.916 \text{ psia}$$

16.85 Oxygen flows at constant temperature of 20 °C from a pressure tank, $p = 130$ atm, through 3-mm-ID tubing to another tank where $p = 110$ atm, $f = 0.016$. Determine the mass rate of flow.

▮
$$\dot{m} = \rho A v \qquad \rho = p/RT \qquad \rho_1 = (130)(101\,310)/[(260)(20+273)] = 172.9 \text{ kg/m}^3$$
$$A_1 = (\pi)(0.003)^2/4 = 0.000007069 \text{ m}^2 \qquad N_M = v/\sqrt{kRT}$$
$$(f/D)(\Delta L) = [(1 - kN_M^2)/kN_M^2 + \ln(kN_M^2)]_1 - [(1 - kN_M^2)/kN_M^2 + \ln(kN_M^2)]_2$$
$$p_2/p_1 = (N_M)_1/(N_M)_2 \qquad \text{(from Prob. 16.83)}$$

$$p_2/p_1 = \tfrac{110}{130} = 0.8462 \qquad (N_M)_2 = (N_M)_1/0.8462 = 1.182(N_M)_1$$

$$[0.016/0.003][(10)(0.3048)] = \{[1-(1.40)(N_M)_1]^2/(1.40)(N_M)_1^2 + \ln[(1.40)(N_M)_1^2]\}$$
$$-(\{1-(1.40)[1.182(N_M)_1]^2\}/(1.40)[1.182(N_M)_1]^2 + \ln(1.40)[1.182(N_M)_1]^2)$$

$$(N_M)_1 = 0.111 \qquad 0.111 = v_1/\sqrt{(1.40)(260)(15+273)} \qquad v_1 = 35.94\ \text{m/s}$$

$$\dot{m} = (172.9)(0.000007069)(35.94) = 0.0439\ \text{kg/s}$$

16.86 In isothermal flow of nitrogen at 80 °F, 3 lbm/s is to be transferred 100 ft from a tank where $p = 200$ psia to a tank where $p = 160$ psia. What minimum size of tubing, $f = 0.016$, is needed?

▮ $$(f/D)(\Delta L) = [(1-kN_M^2)/kN_M^2 + \ln(kN_M^2)]_1 - [(1-kN_M^2)/k_M^2 + \ln(kN_M^2)]_2 \qquad N_M = v/c$$

$$\dot{m} = \rho_1 A_1 v_1 \qquad \rho = p/RT \qquad \rho_1 = (200)(144)/[(1773)(80+460)] = 0.03008\ \text{slug/ft}^3$$

$$\rho_2 = (\rho_1)(p_2/p_1) = (0.03008)(\tfrac{160}{200}) = 0.02406\ \text{slug/ft}^3 \qquad 3 = (0.03008)(32.17)(\pi D^2/4)(v_1) \qquad v_1 = 3.947/D^2$$

$$v_2 = (\tfrac{200}{160})(3.947/D^2) = 4.934/D^2 \qquad c = \sqrt{kRT} = \sqrt{(1.40)(55.2)(32.17)(80+460)} = 1159\ \text{ft/s}$$

$$(N_M)_1 = (3.947/D^2)/1159 = 0.003406/D^2 \qquad (N_M)_2 = (4.934/D^2)/1159 = 0.004257/D^2$$

$$(0.016/D)(100) = \{[1-(1.40)(0.003406/D^2)^2]/(1.40)(0.003406/D^2)^2 + \ln[(1.40)(0.003406/D^2)^2]\}$$
$$-\{[1-(1.40)(0.004257/D^2)^2]/(1.40)(0.004257/D^2)^2 + \ln[(1.40)(0.004257/D^2)^2]\}$$

$$D = 0.150\ \text{ft} \quad \text{or} \quad 1.80\ \text{in} \qquad \text{(by trial and error)}$$

16.87 Air enters a pipe of 1 in diameter at subsonic velocity and $p_1 = 30$ psia, $T_1 = 550$ °R. If the pipe is 10 ft long, $f = 0.025$, and the exit pressure $p_2 = 20$ psia, compute the mass flow for isothermal flow.

▮ $$G^2 = (\dot{m}/A)^2 = (p_1^2 - p_2^2)/\{RT[fL/D + 2\ln(p_1/p_2)]\}$$

$$fL/D + 2\ln(p_1/p_2) = [(0.025)(10)/\tfrac{1}{12}] + 2\ln\tfrac{30}{20} = 3.81$$

$$G^2 = [30(144)]^2 - [20(144)]^2/[(1717)(550)(3.81)] = 2.88\ \text{lb}^2\text{-s}^2/\text{ft}^6 \qquad \text{or} \qquad G = 1.70\ \text{lb-s/ft}^3 = 1.70\ \text{slugs/(s-ft}^2)$$

$$A = \tfrac{1}{4}\pi D^2 = \tfrac{1}{4}\pi(\tfrac{1}{12})^2 = 0.00545\ \text{ft}^2 \qquad \text{Hence} \qquad \dot{m} = GA = (1.70)(0.00545) = 0.00926\ \text{slug/s}$$

Check the Mach number at inlet and exit:

$$a_1 = a_2 \approx 49(550)^{1/2} = 1149\ \text{ft/s} \qquad \rho_1 = p_1/RT = (30)(144)/(1717)(550) = 0.00457\ \text{slug/ft}^3$$

$$\rho_2 = p_2/RT = (20)(144)/(1717)(550) = 0.00305\ \text{slug/ft}^3$$

Then

$$V_1 = G/\rho_1 = 1.70/0.00457 = 371\ \text{ft/s} \qquad (N_M)_1 = V_1/a_1 = \tfrac{371}{1149} = 0.323 \qquad V_2 = G/\rho_2 = 557\ \text{ft/s}$$

$$(N_M)_2 = \tfrac{557}{1149} = 0.484$$

Since these are well below choking, the solution is accurate.

16.88 Isentropic flow of nitrogen occurs in a 2-in-diameter pipe. At one point the velocity of flow, pressure, and specific weight are 409 fps, 85 psia, and 0.655 lb/ft^3, respectively. At a second point a short distance away, the pressure is 83 psia. What is the velocity at the second point?

▮ $$(v_2^2 - v_1^2)/2g = (p_1/\gamma_1)[k/(k-1)][1-(p_2/p_1)^{(k-1)/k_1}]$$

$$(v_2^2 - 409^2)/[(2)(32.2)] = [(85)(144)/0.655][1.40/(1.40-1)][1-(\tfrac{83}{85})^{(1.40-1)/1.40}] \qquad v_2 = 443\ \text{ft/s}$$

16.89 At one point on a streamline in an isentropic airflow, the velocity, pressure, and specific weight are 30.5 m/s, 350 kPa, and 0.028 kN/m^3, respectively. At a second point on the streamline, the velocity is 150 m/s. Find the pressure at the second point.

▮ $$(v_2^2 - v_1^2)/2g = (p_1/\gamma_1)[k/(k-1)][1-(p_2/p_1)^{(k-1)/k}]$$

$$(150^2 - 30.5^2)/[(2)(9.807)] = (350/0.028)[1.40/(1.40-1)][1-(p_2/350)^{(1.40-1)/(1.40)}] \qquad p_2 = 320\ \text{kPa}$$

16.90 Isentropic flow of oxygen occurs in a 100-m-diameter pipe. At one point the velocity of flow, pressure, and specific weight are 125 m/s, 450 kPa, and 0.058 kN/m^3, respectively. At a second point a short distance away, the pressure is 360 kPa. What is the velocity at the second point?

▮ $$(v_2^2 - v_1^2)/2g = (p_1/\gamma)[k/(k-1)][1-(p_2/p_1)^{(k-1)/k}]$$

$$(v_2^2 - 125^2)/[(2)(9.807)] = (450/0.058)[1.40/(1.40-1)][1-(\tfrac{360}{450})^{(1.40-1)/1.40}] \qquad v_2 = 220\ \text{m/s}$$

16.91 At one point on a streamline in an isentropic airflow, the velocity, pressure, and specific weight are 80 m/s, 405 kPa, and 0.046 kN/m^3, respectively. At a second point on the streamline, the velocity is 165 m/s. Find the pressure at the second point.

▮

$$(v_2^2 - v_1^2)/2g = (p_1/\gamma)[k/(k-1)][1-(p_2/p_1)^{(k-1)/k}]$$

$$(165^2 - 80^2)/[(2)(9.807)] = (405/0.046)[1.40/(1.40-1)][1-(p_2/405)^{(1.40-1)/1.40}] \qquad p_2 = 358 \text{ kPa}$$

16.92 Prepare a computer program that determines the pressure either at point 1 or point 2 or the velocity at point 1 or point 2 for closed conduit, compressible, isentropic flow.

▮

```
C     THIS PROGRAM DETERMINES EITHER THE PRESSURE AT POINT 1 OR
C     POINT 2 OR THE VELOCITY AT POINT 1 OR POINT 2 FOR CLOSED CONDUIT,
C     COMPRESSIBLE FLOW.  IT IS BASED ON AN ISENTROPIC FLOW ANALYSIS
C     AND IS THEREFORE SUBJECT TO THE RESTRICTIONS AND/OR ASSUMPTIONS
C     IMPLICIT IN ISENTROPIC FLOW.  IT CAN BE USED FOR PROBLEMS IN BOTH
C     THE ENGLISH SYSTEM OF UNITS AND THE INTERNATIONAL SYSTEM OF
C     UNITS.
C
C     INPUT DATA MUST BE SET UP AS FOLLOWS.
C
C     CARD 1   COLUMN 1          ENTER 0 (ZERO) OR BLANK IF ENGLISH SYSTEM
C                                OF UNITS IS TO BE USED.  ENTER 1 (ONE) IF
C                                INTERNATIONAL SYSTEM OF UNITS IS TO BE
C                                USED.
C              COLUMNS 2-79      ENTER TITLE, DATE, AND OTHER INFORMATION,
C                                IF DESIRED.
C     CARD 2   COLUMNS 1-10      ENTER NUMBER INCLUDING DECIMAL GIVING
C                                ABSOLUTE PRESSURE AT UPSTREAM POINT
C                                (POINT 1) (IN POUNDS PER SQUARE
C                                INCH OR KILOPASCALS).
C              COLUMNS 11-20     ENTER NUMBER INCLUDING DECIMAL GIVING
C                                ABSOLUTE PRESSURE AT DOWNSTREAM POINT
C                                (POINT 2) (IN POUNDS PER SQUARE
C                                INCH OR KILOPASCALS).
C              COLUMNS 21-30     ENTER NUMBER INCLUDING DECIMAL GIVING
C                                VELOCITY AT POINT 1 (IN FEET PER SECOND
C                                OR METERS PER SECOND).
C              COLUMNS 31-40     ENTER NUMBER INCLUDING DECIMAL GIVING
C                                VELOCITY AT POINT 2 (IN FEET PER SECOND
C                                OR METERS PER SECOND).
C              COLUMNS 41-50     ENTER NUMBER INCLUDING DECIMAL GIVING
C                                SPECIFIC (OR UNIT) WEIGHT AT POINT 1 (IN
C                                POUNDS PER CUBIC FOOT OR KILO-
C                                NEWTONS PER CUBIC METER).
C              COLUMNS 51-60     ENTER NUMBER INCLUDING DECIMAL GIVING
C                                SPECIFIC (OR UNIT) WEIGHT AT POINT 2 (IN
C                                POUNDS PER CUBIC FOOT OR KILO-
C                                NEWTONS PER CUBIC METER).
C              COLUMNS 61-70     ENTER NUMBER INCLUDING DECIMAL GIVING
C                                SPECIFIC HEAT RATIO.
C              COLUMNS 71-80     ENTER TYPE OF FLUID.
C
C ***********************************************************************
C *                                                                     *
C *   NOTE WELL....EITHER THE PRESSURE AT POINT 1 (COLUMNS 1-10 ON      *
C *   CARD 2) OR AT POINT 2 (COLUMNS 11-20) OR THE VELOCITY AT POINT 1  *
C *   (COLUMNS 21-30) OR AT POINT 2 (COLUMNS 31-40), WHICHEVER ONE IS   *
C *   TO BE DETERMINED BY THIS PROGRAM, SHOULD BE LEFT BLANK.  IF       *
C *   EITHER THE PRESSURE OR THE VELOCITY AT POINT 1 IS BEING DETER-    *
C *   MINED, ENTER A VALUE FOR THE SPECIFIC (OR UNIT) WEIGHT OF THE     *
C *   FLUID AT POINT 2 (COLUMNS 51-60) AND LEAVE SPECIFIC (OR UNIT)     *
C *   WEIGHT OF THE FLUID AT POINT 1 (COLUMNS 41-50) BLANK.  IF EITHER  *
C *   THE PRESSURE OR THE VELOCITY AT POINT 2 IS BEING DETERMINED,      *
C *   ENTER A VALUE FOR THE SPECIFIC (OR UNIT) WEIGHT OF THE FLUID AT   *
C *   POINT 1 (COLUMNS 41-50) AND LEAVE SPECIFIC (OR UNIT) WEIGHT OF    *
C *   THE FLUID AT POINT 2 (COLUMNS 51-60) BLANK.                       *
C *                                                                     *
C ***********************************************************************
C
C     MULTIPLE DATA SETS FOR SOLVING ANY NUMBER OF PROBLEMS MAY BE
C     INCLUDED FOR PROCESSING.
```

```
C
      REAL K
      DIMENSION TITLE(13),FLUID(2)
1     READ(5,100,END=2)UNITS,TITLE
  100 FORMAT(I1,13A6)
      WRITE(6,105)TITLE
  105 FORMAT('1',13A6,//)
      READ(5,102)P1,P2,V1,V2,SW1,SW2,K,FLUID
  102 FORMAT(7F10.0,2A5)
      G=32.2
      IF(UNITS.EQ.1)G=9.807
      PFACT=144.0
      IF(UNITS.EQ.1)PFACT=1.0
      IF(P1.LT.0.0001)GO TO 103
      IF(P2.LT.0.0001)GO TO 104
      IF(V1.LT.0.0001)GO TO 106
      V2=SQRT(2.0*G*P1*PFACT/SW1*K/(K-1.0)*(1.0-(P2/P1)**((K-1.0)/K))
     *+V1**2)
      IF(UNITS.EQ.0)WRITE(6,107)P1,P2,V1,SW1,FLUID,V2
  107 FORMAT(1X,'GIVEN DATA FOR A COMPRESSIBLE FLOW IN A CIRCULAR CLOSED
     * CONDUIT',//5X,'PRESSURE AT POINT 1 =',F8.1,' PSI',//5X,'PRESSURE
     *AT POINT 2 =',F8.1,' PSI',//5X,'VELOCITY AT POINT 1 =',F7.1,' FT/S
     *',//5X,'SPECIFIC (OR UNIT) WEIGHT OF FLUID AT POINT 1 =',F8.3,
     * ' LB/CU FT',//5X,'FLUID FLOWING IS ',2A5,//1X,'THE VELOCITY AT PO
     *INT 2 WILL BE',F7.1,' FT/S')
      IF(UNITS.EQ.1)WRITE(6,108)P1,P2,V1,SW1,FLUID,V2
  108 FORMAT(1X,'GIVEN DATA FOR A COMPRESSIBLE FLOW IN A CIRCULAR CLOSED
     * CONDUIT',//5X,'PRESSURE AT POINT 1 =',F8.1,' KPA',//5X,'PRESSURE
     *AT POINT 2 =',F8.1,' KPA',//5X,'VELOCITY AT POINT 1 =',F7.1,' M/S
     *',//5X,'SPECIFIC (OR UNIT) WEIGHT OF FLUID AT POINT 1 =',F8.3,
     *' KN/CU M  ',//5X,'FLUID FLOWING IS ',2A5,//1X,'THE VELOCITY AT PO
     *INT 2 WILL BE',F7.1,' M/S')
      GO TO 1
  103 P1=(((V2**2-V1**2)/2.0/G/(P2*PFACT)*SW2/K*(K-1.0)+1.0)*(P2*PFACT)
     ***((K-1.0)/K))**(K/(K-1.0))/PFACT
      IF(UNITS.EQ.0)WRITE(6,109)P2,V1,V2,SW2,FLUID,P1
  109 FORMAT(1X,'GIVEN DATA FOR A COMPRESSIBLE FLOW IN A CIRCULAR CLOSED
     * CONDUIT',//5X,'PRESSURE AT POINT 2 =',F8.1,' PSI',//5X,'VELOCITY
     *AT POINT 1 =',F7.1,' FT/S',//5X,'VELOCITY AT POINT 2 =',F7.1,' FT/
     *S',//5X,'SPECIFIC (OR UNIT) WEIGHT OF FLUID AT POINT 2 =',F8.3,
     * ' LB/CU FT',//5X,'FLUID FLOWING IS ',2A5,//1X'THE PRESSURE AT POI
     *NT 1 WILL BE ',F8.1,' PSI')
      IF(UNITS.EQ.1)WRITE(6,110)P2,V1,V2,SW2,FLUID,P1
  110 FORMAT(1X,'GIVEN DATA FOR A COMPRESSIBLE FLOW IN A CIRCULAR CLOSED
     * CONDUIT',//5X,'PRESSURE AT POINT 2 =',F8.1,' KPA',//5X,'VELOCITY
     *AT POINT 1 =',F7.1,' M/S ',//5X,'VELOCITY AT POINT 2 =',F7.1,' M/S
     * ',//5X,'SPECIFIC (OR UNIT) WEIGHT OF FLUID AT POINT 2 =',F8.3,
     *' KN/CU M  ',//5X,'FLUID FLOWING IS ',2A5,//1X'THE PRESSURE AT POI
     *NT 1 WILL BE ',F8.1,' KPA')
      GO TO 1
  104 P2=((1.0-(V2**2-V1**2)/2.0/G/(P1*PFACT)*SW1/K*(K-1.0))*(P1*PFACT)
     ***((K-1.0)/K))**(K/(K-1.0))/PFACT
      IF(UNITS.EQ.0)WRITE(6,111)P1,V1,V2,SW1,FLUID,P2
  111 FORMAT(1X,'GIVEN DATA FOR A COMPRESSIBLE FLOW IN A CIRCULAR CLOSED
     * CONDUIT',//5X,'PRESSURE AT POINT 1 =',F8.1,' PSI',//5X,'VELOCITY
     *AT POINT 1 =',F7.1,' FT/S',//5X,'VELOCITY AT POINT 2 =',F7.1,' FT/
     *S',//5X,'SPECIFIC (OR UNIT) WEIGHT OF FLUID AT POINT 1 =',F8.3,
     * ' LB/CU FT',//5X,'FLUID FLOWING IS ',2A5,//1X'THE PRESSURE AT POI
     *NT 2 WILL BE ',F8.1,' PSI')
      IF(UNITS.EQ.1)WRITE(6,112)P1,V1,V2,SW1,FLUID,P2
  112 FORMAT(1X,'GIVEN DATA FOR A COMPRESSIBLE FLOW IN A CIRCULAR CLOSED
     * CONDUIT',//5X,'PRESSURE AT POINT 1 =',F8.1,' KPA',//5X,'VELOCITY
     *AT POINT 1 =',F7.1,' M/S ',//5X,'VELOCITY AT POINT 2 =',F7.1,' M/S
     * ',//5X,'SPECIFIC (OR UNIT) WEIGHT OF FLUID AT POINT 1 =',F8.3,
     *' KN/CU M  ',//5X,'FLUID FLOWING IS ',2A5,//1X'THE PRESSURE AT POI
     *NT 2 WILL BE ',F8.1,' KPA')
      GO TO 1
  106 V1=SQRT(V2**2-2.0*G*P2*PFACT/SW2*K/(K-1.0)*((P1/P2)**((K-1.0)/K)
     *-1.0))
      IF(UNITS.EQ.0)WRITE(6,113)P1,P2,V2,SW2,FLUID,V1
  113 FORMAT(1X,'GIVEN DATA FOR A COMPRESSIBLE FLOW IN A CIRCULAR CLOSED
     * CONDUIT',//5X,'PRESSURE AT POINT 1 =',F8.1,' PSI',//5X,'PRESSURE
     *AT POINT 2 =',F8.1,' PSI',//5X,'VELOCITY AT POINT 2 =',F7.1,' FT/S
```

```
     *',//5X,'SPECIFIC (OR UNIT) WEIGHT OF FLUID AT POINT 2 =',F8.3,
     * ' LB/CU FT',//5X,'FLUID FLOWING IS ',2A5,//1X,'THE VELOCITY AT PO
     *INT 1 WILL BE',F7.1,' FT/S')
      IF(UNITS.EQ.1)WRITE(6,114)P1,P2,V2,SW2,FLUID,V1
  114 FORMAT(1X,'GIVEN DATA FOR A COMPRESSIBLE FLOW IN A CIRCULAR CLOSED
     * CONDUIT',//5X,'PRESSURE AT POINT 1 =',F8.1,' KPA',//5X,'PRESSURE
     *AT POINT 2 =',F8.1,' KPA',//5X,'VELOCITY AT POINT 2 =',F7.1,' M/S
     *',//5X,'SPECIFIC (OR UNIT) WEIGHT OF FLUID AT POINT 2 =',F8.3,
     *' KN/CU M  ',//5X,'FLUID FLOWING IS ',2A5,//1X,'THE VELOCITY AT PO
     *INT 1 WILL BE',F7.1,' M/S')
      GO TO 1
2     STOP
      END
```

16.93 Solve Prob. 16.88 using the computer program of Prob. 16.92.

Input

```
OSAMPLE ANALYSIS ØF CØMPRESSIBLE FLØW
85.0      83.0      409.0                0.655                1.40      NITRØGEN
```

Output

```
SAMPLE ANALYSIS OF COMPRESSIBLE FLOW


GIVEN DATA FOR A COMPRESSIBLE FLOW IN A CIRCULAR CLOSED CONDUIT

    PRESSURE AT POINT 1 =    85.0 PSI

    PRESSURE AT POINT 2 =    83.0 PSI

    VELOCITY AT POINT 1 =  409.0 FT/S

    SPECIFIC (OR UNIT) WEIGHT OF FLUID AT POINT 1 =   0.655 LB/CU FT

    FLUID FLOWING IS NITROGEN

THE VELOCITY AT POINT 2 WILL BE  442.5 FT/S
```

16.94 Solve Prob. 16.89 using the computer program of Prob. 16.92.

Input

```
1SAMPLE ANALYSIS ØF CØMPRESSIBLE FLØW
350.0                30.5      150.0     0.028                1.40      AIR
```

Output

```
SAMPLE ANALYSIS OF COMPRESSIBLE FLOW


GIVEN DATA FOR A COMPRESSIBLE FLOW IN A CIRCULAR CLOSED CONDUIT

    PRESSURE AT POINT 1 =   350.0 KPA

    VELOCITY AT POINT 1 =   30.5 M/S

    VELOCITY AT POINT 2 =  150.0 M/S

    SPECIFIC (OR UNIT) WEIGHT OF FLUID AT POINT 1 =   0.028 KN/CU M

    FLUID FLOWING IS AIR

THE PRESSURE AT POINT 2 WILL BE    320.2 KPA
```

16.95 Air at a pressure of 200 psia and a temperature of 140 °F expands in a suitable nozzle to 20 psia. (*a*) If the flow is frictionless and adiabatic and the initial velocity is negligible, find the final velocity. (*b*) Find the final temperature at the end of the expansion.

▮ (*a*) $V_1 \simeq 0 \qquad V_2^2/2g = (RT_1/g)[k/(k-1)][1-(p_2/p_1)^{(k-1)/k}]$

$V_2^2/2g = [(1715)(600)/32.2](1.4/0.4)[1-(\frac{20}{200})^{0.2860}] \qquad V_2 = 1864$ fps

(*b*) $V_2^2 - V_1^2 = 2c_p(T_1 - T_2) \qquad 1864^2 - 0 = 2(6000)(600 - T_2) \qquad T_2 = 310\ °\text{R} = -150\ °\text{F}$

16.96 Derive the equation for isentropic flow: $(V_2^2 - V_1^2)/2g = (p_1/\gamma_1)[k/(k-1)][1-(p_2/p_1)^{(k-1)/k}]$ by integrating the Euler equation.

▮ Euler equation: $$dp/\gamma + (V\,dV)/g = 0 \qquad (1)$$

$pv^k = \text{constant} = p/\rho^k$ because $v = 1/\rho$; so $p/\gamma^k = C$:

$p = C\gamma^k$: $$dp = Ck\gamma^{k-1}\,d\gamma \qquad (2)$$

Substitute (*2*) into (*1*) and integrate: $Ck\gamma^{k-2}\,d\gamma + (1/g)V\,dV = 0$.

$$\left[\frac{Ck\gamma^{k-1}}{k-1}\right]_{\gamma_1}^{\gamma_2} + \left[\frac{V^2}{2g}\right]_{V_1}^{V_2} = 0 \qquad \frac{V_2^2 - V_1^2}{2g} = \frac{k}{k-1}\left[\frac{p}{\gamma}\right]_2^1 = \frac{k}{k-1}\left(\frac{p_1}{\gamma_1} - \frac{p_2}{\gamma_2}\right)$$

Note that $p_1/\gamma_1 - p_2/\gamma_2 = p_1/\gamma_1[1-(p_2/\gamma_2)/(p_1/\gamma_1)]$ where $\gamma_1 = (1/C)p_1^{1/k}$ and $\gamma_2 = (1/C)p_2^{1/k}$, $p_2\gamma_1/p_1\gamma_2 = (p_2^{1-(1/k)})/(p_1^{1-(1/k)}) = (p_2/p_1)^{(k-1/k}$. Thus $(V_2^2 - V_1^2)/2g = (p_1/\gamma_1)[k/(k-1)][1-(p_2/p_1)^{(k-1)/k}]$ Q.E.D.

16.97 Carbon dioxide flows isentropically. At a point in the flow the velocity is 45 fps and the temperature is 120 °F. At a second point on the same streamline the temperature is 80 °F. What is the velocity at the second point?

▮ For carbon dioxide: $k = 1.28$; $R = 1123$ ft-lb/(slug-°R). Flow is isentropic, therefore adiabatic. $V_2^2 - V_1^2 = [2k/(k-1)](RT_1)[1-(T_2/T_1)]$, $V_2^2 - 45^2 = [2(1.28)/0.28](1123)(460+120)[1-(460+80)/(460+120)]$; $V_2 = 642$ fps. Check to see if sonic velocity is exceeded: $c = (kRT)^{1/2} = [1.28(1123)(460+80)]^{1/2} = 881$ fps. Since $V_2 < c$, flow is subsonic, $V_2 = 642$ fps is correct.

16.98 Refer to Prob. 16.97. If the pressure at the first point were 20 psi, determine the pressure and temperature on the nose of a streamlined object placed in the flow at that point.

▮ $\rho_0 = p_0/RT_0 = (20)(144)/(1123)(580) = 0.00442$ slug/ft^3 $\qquad c = (kRT_1)^{1/2} = [1.29(1123)(580)]^{1/2} = 917$ fps

$N_M = V/c = \frac{45}{917} = 0.0491 \qquad p_s = p_0 + \rho_0(V_0^2/2)(1 + N_M^2/4)(\frac{1}{144}) = 20.0$ psia $\qquad (c_p/g)T_1 + V_1^2/2g = (c_p/g)T_2$

$(5132/32.2)(580) + (45^2/64.4) = (5132/32.2)T_s \qquad T_s = 580\ °\text{R} = 120\ °\text{F}$

Temperature increase is negligible.

16.99 From the equation $s_2 - s_1 = c_v \ln[(T_2/T_1)(\rho_1/\rho_2)^{k-1}]$ and the perfect gas law, derive the equation of state for isentropic flow.

▮ Since $s_2 - s_1 = c_v = 0$ for isentropic flow,

$\ln[(T_2/T_1)(\rho_1/\rho_2)^{k-1}] = 0 \qquad (T_2/T_1)(\rho_1/\rho_2)^{1-k} = 1 \qquad T_2/T_1 = (p_2/\rho_2)/(p_1/\rho_1)$ (from perfect gas law)

Eliminating T's, $p_2/\rho_2^k = p_1/\rho_1^k$.

16.100 Compute the enthalpy change per slug for helium from $T_1 = 0$ °F, $p_1 = 15$ psia to $T_2 = 140$ °F in an isentropic process [$c_p = 1.25$ Btu/(lbm-°R)].

▮ $$H = mc_p\,\Delta T = (1)[(1.25)(32.17)](140 - 0) = 5630\ \text{Btu}$$

16.101 Isentropic flow of air occurs at a section of a pipe where $p = 50$ psia, $T = 100$ °F, and $V = 540$ ft/s. An object is immersed in the flow, which brings the velocity to zero. What are the temperature and pressure at the stagnation point?

▮ $V^2/2 = [kR/(k-1)](T_0 - T) \qquad 540^2/2 = [(1.40)(1716)/(1.40-1)][T_0 - (100+460)] \qquad T_0 = 584\ °\text{R}$ or $124\ °\text{F}$

$T_2/T_1 = (p_2/p_1)^{(k-1)/k} \qquad 584/(100+460) = (p_2/50)^{(1.40-1)/1.40} \qquad p_2 = 57.9$ psia

16.102 What is the Mach number for Prob. 16.101?