SCHAUM'S SOLVED PROBLEMS SERIES

2500 SOLVED PROBLEMS IN FLUID MECHANICS & HYDRAULICS

Revised First Edition

- Learn the best strategies for solving tough problems in step-by-step detail.
- Prepare effectively for exams and save time in doing homework problems.
- Choose from the largest selection of solved problems yet published on this subject.
- Use the index to quickly locate the types of problems you most need help with.
- Save this book for reference in other courses and even for your professional library.

JACK B. EVETT / CHENG LIU

SCHAUM'S SOLVED PROBLEMS SERIES

2500 SOLVED PROBLEMS IN

FLUID MECHANICS AND HYDRAULICS

by

Jack B. Evett, Ph.D

Cheng Liu, M.S.

The University of North Carolina at Charlotte

McGRAW-HILL, INC.

New York St. Louis San Francisco Auckland Bogotá Caracas Hamburg Lisbon London Madrid Mexico Milan Montreal New Delhi Paris San Juan São Paulo Singapore Sydney Tokyo Toronto

Jack B. Evett, Ph.D., *Professor of Civil Engineering*, and Cheng Liu, M.S., *Associate Professor of Civil Engineering Technology*, both at the *University of North Carolina at Charlotte*.

Both authors have extensive teaching experience in the domain of fluid mechanics and hydraulics. They are coauthors of a textbook in fluid mechanics for the McGraw-Hill College Division.

Project supervision by The Total Book.
Cover design by Wanda Siedlecka.
Index by Hugh C. Maddocks, Ph.D.

Library of Congress Cataloging-in-Publication Data

Evett, Jack B.
2500 solved problems in fluid mechanics and hydraulics / by Jack B. Evett, Cheng Liu.
p. cm. — (Schaum's solved problems series)
ISBN 0-07-019783-0
1. Fluid mechanics—Problems, exercises, etc. 2. Hydraulics—Problems, exercises, etc. I. Liu, Cheng. II. Title. III. Title: Twenty-five hundred solved problems in fluid mechanics and hydraulics. IV. Series.
TA357.3.E84 1988
620.1'06'076—dc 19 88-13373
CIP

3 4 5 6 7 8 9 0 SHP/SHP 9 3 2 1

ISBN 0-07-019784-9 (Formerly published under ISBN 0-07-019783-0)

CONTENTS

To the Student		v
List of Abbreviations		vii
List of Conversion Factors		xi
Chapter 1	PROPERTIES OF FLUIDS	**1**
Chapter 2	FLUID STATICS	**25**
Chapter 3	FORCES ON SUBMERGED PLANE AREAS	**53**
Chapter 4	DAMS	**77**
Chapter 5	FORCES ON SUBMERGED CURVED AREAS	**85**
Chapter 6	BUOYANCY AND FLOTATION	**108**
Chapter 7	KINEMATICS OF FLUID MOTION	**132**
Chapter 8	FUNDAMENTALS OF FLUID FLOW	**157**
Chapter 9	FLOW IN CLOSED CONDUITS	**197**
Chapter 10	SERIES PIPELINE SYSTEMS	**269**
Chapter 11	PARALLEL PIPELINE SYSTEMS	**278**
Chapter 12	BRANCHING PIPELINE SYSTEMS	**302**
Chapter 13	PIPE NETWORKS	**315**
Chapter 14	FLOW IN OPEN CHANNELS	**356**
Chapter 15	FLOOD ROUTING	**459**
Chapter 16	FLOW OF COMPRESSIBLE FLUIDS	**469**
Chapter 17	FLOW MEASUREMENT	**520**
Chapter 18	DIMENSIONAL ANALYSIS AND SIMILITUDE	**574**
Chapter 19	UNSTEADY FLOW	**589**
Chapter 20	PUMPS AND FANS	**610**
Chapter 21	TURBINES	**638**
Chapter 22	HYDRAULIC AND ENERGY GRADE LINES	**657**

Chapter 23 FORCES DEVELOPED BY FLUIDS IN MOTION 664

Chapter 24 DYNAMIC DRAG AND LIFT 684

Chapter 25 BASIC HYDRODYNAMICS 703

Appendix 709

Index 787

To the Student

This book contains precisely 2500 completely solved problems in the areas of fluid mechanics and hydraulics. Virtually all types of problems ordinarily encountered in study and practice in these areas are covered. Not only you, but teachers, practitioners, and graduates reviewing for engineering licensing examinations should find these problems valuable.

To acquaint you with our "approach," particular steps taken in presenting the problems and their solutions are itemized below.

- First and most important of all, each problem and its solution are essentially independent and self-contained. That is to say, each contains all the data, equations, and computations necessary to find the answers. Thus, you should be able to pick a problem anywhere and follow its solution without having to review whatever precedes it. The exception to this is the occasional problem that specifically refers to, and carries over information from, a previous problem.
- In the solutions, our objective has been to present any needed equation first and then clearly to evaluate each term in the equation in order to find the answer. The terms may be evaluated separately or within the equation itself. For example, when solving an equation that has the parameter "area" as one of its terms, the area term (A) may be evaluated separately and its value substituted into the equation [as in Prob. 14.209], or it may be evaluated within the equation itself [as in Prob. 14.94].
- Virtually every number appearing in a solution is either "given" information (appearing as data in the statement of the problem or on an accompanying illustration), a previously computed value within the problem, a conversion factor (obtainable from the List of Conversion Factors), or a physical property (obtainable from a table or illustration in the Appendix). For example, in Prob. 1.77, the number 1.49, which does not appear elsewhere in the problem, is the dynamic viscosity (μ) of glycerin; it was obtained from Table A-3 in the Appendix.
- We have tried to include all but the most familiar items in the List of Abbreviations and Symbols. Hence, when an unknown sign is encountered in a problem or its solution, a scan of that list should prove helpful. Thus, the infrequently used symbol ψ is encountered in Prob. 25.6. According to the list, ψ represents the stream function, and you are quickly on your way to a solution.

Every problem solution in this book has been checked, but, with 2500 in all, it is inevitable that some mistakes will slip through. We would appreciate it if you would take the time to communicate any mistakes you find to us, so that they may be corrected in future printings. We wish to thank Bill Langley, of The University of North Carolina at Charlotte, who assisted us with some of the problem selection and preparation.

Abbreviations and Symbols

a	acceleration or area
A	area
abs	absolute
α (alpha)	angle between absolute velocity of fluid in hydraulic machine and linear velocity of a point on a rotating body or coefficient of thermal expansion or dimensionless ratio of similitude
atm	atmosphere
atmos	atmospheric
β (beta)	angle between relative velocity in hydraulic machines and linear velocity of a point on a rotating body or coefficient of compressibility or ratio of obstruction diameter to duct diameter
b	surface width or other width
B	surface width or other width
bhp	brake horsepower
bp	brake power
Btu	British thermal unit
c	speed of sound or wave speed (celerity)
C	Celsius or discharge coefficient or speed of propagation
cal	calorie
c.b. or CB	center of buoyancy
C_c	coefficient of contraction
C_d	coefficient of discharge
C_D	drag coefficient
C_f	friction-drag coefficient
C_F	force coefficient
cfs	cubic foot per second
c.g. or CG	center of gravity
C_I	Pitot tube coefficient
C_L	lift coefficient
cm	centimeter (10^{-2} m)
cP	centipoise
c.p.	center of pressure
c_P	specific heat at constant pressure
c_v	specific heat at constant volume
C_v	coefficient of velocity
C_w	weir coefficient
d	depth or diameter
D	depth or diameter or drag force
δ (delta)	thickness of boundary layer
δ_1 (delta)	thickness of the viscous sublayer
Δ (Delta)	change in (or difference between)
d_c	critical depth
D_{eff}	effective diameter
D_h	hydraulic diameter
d_m	mean depth
d_n	normal depth
d_N	normal depth
E	modulus of elasticity or specific energy or velocity approach factor
e_h	hydraulic efficiency
el	elevation
η (eta)	pump or turbine efficiency
ϵ (epsilon)	height or surface roughness
E_p	pump energy
E_t	turbine energy
exp	exponential
f	frequency of oscillation (cycles per second) or friction factor

F	Fahrenheit or force
F_b	buoyant force
F_D	drag force
F_H	horizontal force
F_L	lift force
fps	foot per second
F.S.	factor of safety
ft	foot
F_U	uplift force on a dam
F_V	vertical force
g	acceleration due to gravity or gage height or gram
G	weight flow rate
gal	gallon
γ (gamma)	specific (or unit) weight
Γ (Gamma)	circulation
GN	giganewton (10^9 N)
GPa	gigapascal (10^9 Pa)
gpm	gallons per minute
h	enthalpy per unit mass or height or depth or pressure head or hour
$\bar{h}$	average height or depth or head
$\hat{h}$	enthalpy per unit weight
H	energy head or total energy head
h_1	unit head loss
h_{cg}	vertical depth to center of gravity
h_{cp}	vertical depth to center of pressure
h_f	head loss due to friction
Hg	mercury
HGL	hydraulic grade line
h_L	total head loss
h_m	head loss due to minor losses
hp	horsepower
Hz	hertz (cycles per second)
I	inflow or moment of inertia
ID	inside diameter
in	inch
∞ (infinity)	sometimes used as a subscript to indicate upstream
J	joule
K	bulk modulus of elasticity or Kelvin or minor loss coefficient
k	specific heat ratio
kcal	kilocalorie (10^3 cal)
kg	kilogram (10^3 g)
kJ	kilojoule (10^3 J)
km	kilometer (10^3 m)
kN	kilonewton (10^3 N)
kPa	kilopascal (10^3 Pa)
kW	kilowatt (10^3 W)
L	length or lift force or liter
λ (lambda)	model ratio or wave length
lb	pound
lb_m	pound mass
L_e	equivalent length
L_m	linear dimension in model
L_p	linear dimension in prototype
m	mass or meter
$\dot{m}$	mass flow rate
M	mass flow rate or molecular weight or moment or torque
$\overline{MB}$	distance from center of buoyancy to metacenter
mbar	millibar (10^{-3} bar)
mc	metacenter
mgd	million gallons per day

ml	milliliter (10^{-3} L)
min	minute
mm	millimeter (10^{-3} meter)
MN	meganewton (10^6 N)
MPa	megapascal (10^6 Pa)
mph	mile per hour
MR	manometer reading
μ (mu)	absolute or dynamic viscosity
MW	megawatt (10^6 W)
n	Manning roughness coefficient or number of moles
N	newton or rotational speed
N_B	Brinkman number
N_F	Froude number
N_M	Mach number
NPSH	net positive suction head
N_R	Reynolds number
N_s	specific speed of pump or turbine
ν (nu)	kinematic viscosity
N_W	Weber number
O	outflow
OD	outside diameter
Ω (ohm)	rotational rate
ω (omega)	angular velocity
p	pressure or poise
P	force (usually resulting from an applied pressure) or power
Pa	pascal
ϕ (phi)	peripheral-velocity factor
π (pi)	constant = 3.14159265
Π (pi)	dimensionless parameter
P_r	power ratio
p_s	stagnation pressure
psi	pound per square inch
ψ (psi)	stream function
psia	pound per square inch absolute
psig	pound per square inch gage
p^{*t}	pressure for condition at $N_M = 1/\sqrt{k}$
p_v	vapor pressure
p_w	wetted perimeter
q	flow rate per unit width or heat per unit mass
Q	discharge or heat or volume flow rate
Q_H	heat transferred per unit weight of fluid
Q/w	volume flow rate per unit width of channel
qt	quart
r	radius
R	gas constant or Rankine or resultant force or hydraulic radius
R'	manometer reading
rad	radian
R_c	critical hydraulic radius
R_h	hydraulic radius
ρ (rho)	mass density
r_i	inside radius
r_o	outside radius
rpm	revolutions per minute
R_u	universal gas constant
s	entropy of a substance or second or slope
S	slope or storage
s_c	critical slope
s.g.	specific gravity
s.g.$_M$	specific gravity of manometer fluid
s.g.$_F$	specific gravity of flowing fluid

σ (sigma)	pump cavitation parameter or stress or surface tension
σ'	cavitation index
Σ (sigma)	summation
S	specific gravity of flowing fluid
S_0	specific gravity of manometer fluid
t	thickness or time
T	surface width or temperature or torque or tension
τ (tau)	shear stress
τ_0 (tau)	shear stress at the wall
T_s	stagnation temperature
u	velocity
$u_{\text{℄}}$	centerline velocity
U	velocity
v	velocity
v_c	critical velocity
V	velocity or volume
v_{av}	average velocity
$V_{\text{℄}}$	centerline velocity
V_d	volume of fluid displaced
V_m	velocity in model
V_p	velocity in prototype
V_s	specific volume
v_*	shear velocity
v_t	tangential velocity
v_T	terminal velocity
w	width
W	watt or weight or weight flow rate or work
x_{cp}	distance from center of gravity to center of pressure in x direction
ξ (xi)	vorticity
y	depth
y_c	critical depth
y_{cp}	distance from center of gravity to center of pressure in y direction
y_n	normal depth
y_N	normal depth
z_{cg}	inclined distance from liquid surface to center of gravity
z_{cp}	inclined distance from liquid surface to center of pressure

Conversion Factors

$0.00001667\ m^3/s = 1\ L/min$
$0.002228\ ft^3/s = 1\ gal/min$
$0.0145\ lb/in^2 = 1\ mbar$
0.3048 m = 1 ft
2.54 cm = 1 in
3.281 ft = 1 m
4 qt = 1 gal
4.184 kJ = 1 kcal
4.448 N = 1 lb
$6.894\ kN/m^2 = 1\ lb/in^2$
$7.48\ gal = 1\ ft^3$
12 in = 1 ft
14.59 kg = 1 slug
25.4 mm = 1 in
60 min = 1 h
60 s = 1 min
100 cm = 1 m
100 kPa = 1 bar

101.3 kPa = 1 atm
$144\ in^2 = 1\ ft^2$
550 ft-lb/s = 1 hp
778 ft-lb = 1 Btu
1000 N = 1 kN
$1000\ L = 1\ m^3$
1000 mm = 1 m
1000 Pa = 1 kPa
$1728\ in^3 = 1\ ft^3$
2000 lb = 1 ton
3600 s = 1 h
4184 J = 1 kcal
5280 ft = 1 mile
86 400 s = 1 day
1 000 000 N = 1 MN
1 000 000 Pa = 1 MPa
1 000 000 000 N = 1 GN
1 000 000 000 Pa = 1 GPa

CHAPTER 1
Properties of Fluids

Note: For many problems in this chapter, values of various physical properties of fluids are obtained from Tables A-1 through A-8 in the Appendix.

1.1 A reservoir of glycerin (glyc) has a mass of 1200 kg and a volume of 0.952 m^3. Find the glycerin's weight (W), mass density (ρ), specific weight (γ), and specific gravity (s.g.).

$$F = W = ma = (1200)(9.81) = 11\,770\text{ N} \quad \text{or} \quad 11.77\text{ kN}$$
$$\rho = m/V = 1200/0.952 = 1261\text{ kg/m}^3$$
$$\gamma = W/V = 11.77/0.952 = 12.36\text{ kN/m}^3$$
$$\text{s.g.} = \gamma_{\text{glyc}}/\gamma_{\text{H}_2\text{O at 4}\,^\circ\text{C}} = 12.36/9.81 = 1.26$$

1.2 A body requires a force of 100 N to accelerate it at a rate of 0.20 m/s^2. Determine the mass of the body in kilograms and in slugs.

$$F = ma$$
$$100 = (m)(0.20)$$
$$m = 500\text{ kg} = 500/14.59 = 34.3\text{ slugs}$$

1.3 A reservoir of carbon tetrachloride (CCl_4) has a mass of 500 kg and a volume of 0.315 m^3. Find the carbon tetrachloride's weight, mass density, specific weight, and specific gravity.

$$F = W = ma = (500)(9.81) = 4905\text{ N} \quad \text{or} \quad 4.905\text{ kN}$$
$$\rho = m/V = 500/0.315 = 1587\text{ kg/m}^3$$
$$\gamma = W/V = 4.905/0.315 = 15.57\text{ kN/m}^3$$
$$\text{s.g.} = \gamma_{\text{CCl}_4}/\gamma_{\text{H}_2\text{O at 4}\,^\circ\text{C}} = 15.57/9.81 = 1.59$$

1.4 The weight of a body is 100 lb. Determine (*a*) its weight in newtons, (*b*) its mass in kilograms, and (*c*) the rate of acceleration [in both feet per second per second (ft/s^2) and meters per second per second (m/s^2)] if a net force of 50 lb is applied to the body.

(*a*) $$W = (100)(4.448) = 444.8\text{ N}$$

(*b*) $$F = W = ma \qquad 444.8 = (m)(9.81) \qquad m = 45.34\text{ kg}$$

(*c*) $$m = 45.34/14.59 = 3.108\text{ slugs}$$

$$F = ma \qquad 50 = 3.108a \qquad a = 16.09\text{ ft/s}^2 = (16.09)(0.3048) = 4.904\text{ m/s}^2$$

1.5 The specific gravity of ethyl alcohol is 0.79. Calculate its specific weight (in both pounds per cubic foot and kilonewtons per cubic meter) and mass density (in both slugs per cubic foot and kilograms per cubic meter).

$$\gamma = (0.79)(62.4) = 49.3\text{ lb/ft}^3 \qquad \gamma = (0.79)(9.79) = 7.73\text{ kN/m}^3$$
$$\rho = (0.79)(1.94) = 1.53\text{ slugs/ft}^3 \qquad \rho = (0.79)(1000) = 790\text{ kg/m}^3$$

1.6 A quart of water weights about 2.08 lb. Compute its mass in slugs and in kilograms.

$$F = W = ma \qquad 2.08 = (m)(32.2)$$
$$m = 0.0646\text{ slug} \qquad m = (0.0646)(14.59) = 0.943\text{ kg}$$

1.7 One cubic foot of glycerin has a mass of 2.44 slugs. Find its specific weight in both pounds per cubic foot and kilonewtons per cubic meter.

$F = W = ma = (2.44)(32.2) = 78.6$ lb. Since the glycerin's volume is 1 ft^3, $\gamma = 78.6\text{ lb/ft}^3 = (78.6)(4.448)/(0.3048)^3 = 12\,350\text{ N/m}^3$, or 12.35 kN/m^3.

1.8 A quart of SAE 30 oil at 68 °F weighs about 1.85 lb. Calculate the oil's specific weight, mass density, and specific gravity.

$$V = 1/[(4)(7.48)] = 0.03342\ \text{ft}^3$$
$$\gamma = W/V = 1.85/0.03342 = 55.4\ \text{lb/ft}^3$$
$$\rho = \gamma/g = 55.4/32.2 = 1.72\ \text{slugs/ft}^3$$
$$\text{s.g.} = \gamma_{\text{oil}}/\gamma_{\text{H}_2\text{O at 4}\,°\text{C}} = 55.4/62.4 = 0.888$$

1.9 The volume of a rock is found to be 0.00015 m³. If the rock's specific gravity is 2.60, what is its weight?

$$\gamma_{\text{rock}} = (2.60)(9.79) = 25.5\ \text{kN/m}^3 \qquad W_{\text{rock}} = (25.5)(0.00015) = 0.00382\ \text{kN} \quad \text{or} \quad 3.82\ \text{N}$$

1.10 A certain gasoline weighs 46.5 lb/ft³. What are its mass density, specific volume, and specific gravity?

$$\rho = \gamma/g = 46.5/32.2 = 1.44\ \text{slugs/ft}^3 \qquad V_s = 1/\rho = 1/1.44 = 0.694\ \text{ft}^3/\text{slug}$$
$$\text{s.g.} = 1.44/1.94 = 0.742$$

1.11 If the specific weight of a substance is 8.2 kN/m³, what is its mass density?

$$\rho = \gamma/g = 8200/9.81 = 836\ \text{kg/m}^3$$

1.12 An object at a certain location has a mass of 2.0 kg and weighs 19.0 N on a spring balance. What is the acceleration due to gravity at this location?

$$F = W = ma \qquad 19.0 = 2.0a \qquad a = 9.50\ \text{m/s}^2$$

1.13 If an object has a mass of 2.0 slugs at sea level, what would its mass be at a location where the acceleration due to gravity is 30.00 ft/s²?

Since the mass of an object does not change, its mass will be 2.0 slugs at that location.

1.14 What would be the weight of a 3-kg mass on a planet where the acceleration due to gravity is 10.00 m/s²?

$$F = W = ma = (3)(10.00) = 30.00\ \text{N}$$

1.15 Determine the weight of a 5-slug boulder at a place where the acceleration due to gravity is 31.7 ft/s².

$$F = W = ma = (5)(31.7) = 158\ \text{lb}$$

1.16 If 200 ft³ of oil weighs 10 520 lb, calculate its specific weight, density, and specific gravity.

$$\gamma = W/V = 10\,520/200 = 52.6\ \text{lb/ft}^3 \qquad \rho = \gamma/g = 52.6/32.2 = 1.63\ \text{slugs/ft}^3$$
$$\text{s.g.} = \gamma_{\text{oil}}/\gamma_{\text{H}_2\text{O at 4}\,°\text{C}} = 52.6/62.4 = 0.843$$

1.17 Find the height of the free surface if 0.8 ft³ of water is poured into a conical tank (Fig. 1-1) 20 in high with a base radius of 10 in. How much additional water is required to fill the tank?

$$V_{\text{cone}} = \pi r^2 h/3 = \pi(10)^2(20)/3 = 2094\ \text{in}^3 \qquad V_{\text{H}_2\text{O}} = 0.8\ \text{ft}^3 = 1382\ \text{in}^3$$

Additional water needed = 2095 − 1382 = 713 in³. From Fig. 1-1, $r_o/10 = h_o/20$, or $r_o = h_o/2.0$; $V_{\text{empty (top) cone}} = \pi(h_o/2.0)^2 h_o/3 = 713$; $h_o = 13.96$ in. Free surface will be 20 − 13.96, or 6.04 in above base of tank.

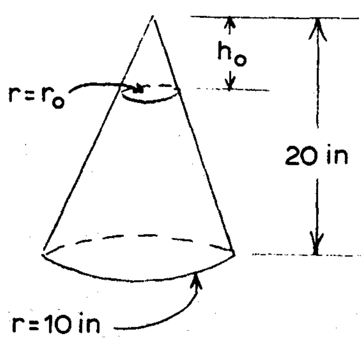


Fig. 1-1

1.18 If the tank of Prob. 1.17 holds 30.5 kg of salad oil, what is the density of the oil?

▮
$$V_{\text{cone}} = 2094 \text{ in}^3 \qquad \text{(from Prob. 1.17)}$$
$$= \tfrac{2094}{1728}(0.3048)^3 = 0.03431 \text{ m}^3$$
$$\rho = m/V = 30.5/0.03431 = 889 \text{ kg/m}^3$$

1.19 Under standard conditions a certain gas weighs 0.14 lb/ft³. Calculate its density, specific volume, and specific gravity relative to air weighing 0.075 lb/ft³.

▮
$$\rho = \gamma/g = 0.14/32.2 = 0.00435 \text{ slug/ft}^3 \qquad V_s = 1/\rho = 1/0.00435 = 230 \text{ ft}^3\text{/slug}$$
$$\text{s.g.} = 0.14/0.075 = 1.87$$

1.20 If the specific volume of a gas is 360 ft³/slug, what is its specific weight?

▮
$$\rho = 1/V_s = \tfrac{1}{360} = 0.002778 \text{ slug/ft}^3 \qquad \gamma = \rho g = (0.002778)(32.2) = 0.0895 \text{ lb/ft}^3$$

1.21 A vertical glass cylinder contains 900.00 mL of water at 10 °C; the height of the water column is 90.00 cm. The water and its container are heated to 80 °C. Assuming no evaporation, what will be the height of the water if the coefficient of thermal expansion (α) for the glass is $3.6 \times 10^{-6}\,°\text{C}^{-1}$?

▮ Mass of water $= \rho V = \rho_{10} V_{10} = \rho_{80} V_{80}$ $\qquad (1000)(900 \times 10^{-6}) = 971 V_{80} \qquad V_{80} = 926.9 \times 10^{-6} \text{ m}^3 = 926.9 \text{ cm}^3$

$$A_{10} = V_{10}/h_{10} = 900.00/90.00 = 10.000 \text{ cm}^2$$
$$A_{10} = \pi r_{10}^2 \qquad 10.000 = \pi r_{10}^2 \qquad r_{10} = 1.7841 \text{ cm}$$
$$r_{80} = r_{10}[1 + (\Delta T)(\alpha)] = (1.7841)[1 + (80 - 10)(3.6 \times 10^{-6})] = 1.7845 \text{ cm}$$
$$A_{80} = \pi r_{80}^2 = \pi(1.7845)^2 = 10.004 \text{ cm}^2 \qquad h_{80} = V_{80}/A_{80} = 926.9/10.004 = 92.65 \text{ cm}$$

1.22 If a vessel that contains 3.500 ft³ of water at 50 °F and atmospheric pressure is heated to 160 °F, what will be the percentage change in its volume? What weight of water must be removed to maintain the original volume?

▮
$$\text{Weight of water} = \gamma V = \gamma_{50} V_{50} = \gamma_{160} V_{160} \qquad (62.4)(3.500) = 61.0 V_{160} \qquad V_{160} = 3.5803 \text{ ft}^3$$

Change in volume = (3.5803 − 3.500)/3.000 = 0.027, or 2.7% (increase). Must remove (3.5803 − 3.500)(61.0), or 4.90 lb.

1.23 A vertical, cylindrical tank with a diameter of 12.00 m and a depth of 4.00 m is filled to the top with water at 20 °C. If the water is heated to 50 °C, how much water will spill over?

▮
$$V_{\text{tank}} = (V_{H_2O})_{20} = \pi(12.00/2)^2(4.00) = 452.4 \text{ m}^3$$
$$W_{H_2O} = (9.79)(452.4) = 4429 \text{ kN} \qquad (V_{H_2O})_{50} = 4429/9.69 = 457.1 \text{ m}^3$$
$$\text{Volume of water spilled} = 457.1 - 452.4 = 4.7 \text{ m}^3$$

1.24 A thick, closed, steel chamber is filled with water at 50 °F and atmospheric pressure. If the temperature of water and chamber is raised to 100 °F, find the new pressure of the water. The coefficient of thermal expansion of steel is 6.5×10^{-6} per °F.

▮ The volume of water would attempt to increase as the cube of the linear dimension; hence, $V_{90} = V_{50}[1 + (100 - 50)(6.5 \times 10^{-6})]^3 = 1.000975 V_{50}$; weight of water $= \gamma V = \gamma_{50} V_{50} = \gamma_{90} V_{90}$, $62.4 V_{50} = \gamma_{90}(1.000975 V_{50})$, $\gamma_{90} = 62.34 \text{ lb/ft}^3$. From Fig. A-3, $p_{90} = 1300$ psia (approximately).

1.25 A liquid compressed in a cylinder has a volume of 1000 cm³ at 1 MN/m² and a volume of 995 cm³ at 2 MN/m². What is its bulk modulus of elasticity (K)?

▮
$$K = -\frac{\Delta p}{\Delta V/V} = -\frac{2 - 1}{(995 - 1000)/1000} = 200 \text{ MPa}$$

1.26 Find the bulk modulus of elasticity of a liquid if a pressure of 150 psi applied to 10 ft³ of the liquid causes a volume reduction of 0.02 ft³.

▮
$$K = -\frac{\Delta p}{\Delta V/V} = -\frac{(150 - 0)(144)}{-0.02/10} = 10\,800\,000 \text{ lb/ft}^2 \quad \text{or} \quad 75\,000 \text{ psi}$$

1.27 If $K = 2.2$ GPa is the bulk modulus of elasticity for water, what pressure is required to reduce a volume by 0.6 percent?

$$K = -\frac{\Delta p}{\Delta V/V} \qquad 2.2 = -\frac{p_2 - 0}{-0.006} \qquad p_2 = 0.0132\ \text{GPa} \quad \text{or} \quad 13.2\ \text{MPa}$$

1.28 Find the change in volume of 1.00000 ft³ of water at 80 °F when subjected to a pressure increase of 300 psi. Water's bulk modulus of elasticity at this temperature is 325 000 psi.

$$K = -\frac{\Delta p}{\Delta V/V} \qquad 325\,000 = -\frac{300 - 0}{\Delta V/1.00000} \qquad \Delta V = -0.00092\ \text{ft}^3$$

1.29 From the following test data, determine the bulk modulus of elasticity of water: at 500 psi the volume was 1.000 ft³, and at 3500 psi the volume was 0.990 ft³.

$$K = -\frac{\Delta p}{\Delta V/V} = -\frac{500 - 3500}{(1.000 - 0.990)/1.000} = 300\,000\ \text{psi}$$

1.30 A rigid steel container is partially filled with a liquid at 15 atm. The volume of the liquid is 1.23200 L. At a pressure of 30 atm, the volume of the liquid is 1.23100 L. Find the average bulk modulus of elasticity of the liquid over the given range of pressure if the temperature after compression is allowed to return to its initial value. What is the coefficient of compressibility (β)?

$$K = -\frac{\Delta p}{\Delta V/V} = -\frac{(30 - 15)(101.3)}{(1.23100 - 1.23200)/1.23200} = 1.872 \times 10^6\ \text{kPa} \quad \text{or} \quad 1.872\ \text{GPa}$$

$$\beta = 1/K = 1/1.872 = 0.534\ \text{GPa}^{-1}$$

1.31 A heavy tank contains oil (A) and water (B) subject to variable air pressure; the dimensions shown in Fig. 1-2 correspond to 1 atm. If air is slowly added from a pump to bring pressure p up to 1 MPa gage, what will be the total downward movement of the free surface of oil and air? Take average values of bulk moduli of elasticity of the liquids as 2050 MPa for oil and 2075 MPa for water. Assume the container does not change volume. Neglect hydrostatic pressures.

$$K = -\frac{\Delta p}{\Delta V/V} \qquad 2050 = -\frac{1 - 0}{\Delta V_{\text{oil}}/[600\pi(300)^2/4]} \qquad \Delta V_{\text{oil}} = -20\,690\ \text{mm}^3$$

$$2075 = -\frac{1 - 0}{\Delta V_{H_2O}/[700\pi(300)^2/4]} \qquad \Delta V_{H_2O} = -23\,850\ \text{mm}^3$$

$$\Delta V_{\text{total}} = -44\,540\ \text{mm}^3$$

Let x = distance the upper free surface moves. $-44\,540 = -[\pi(300)^2/4]x$, $x = 0.630$ mm.

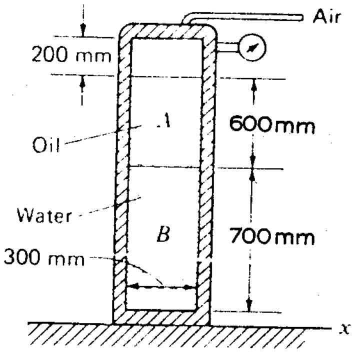


Fig. 1-2

1.32 A thin-walled spherical tank is filled with water at a pressure of 4666 psig; the tank's volume is then 805.407 in³. If the water is released from the tank, how many pounds will be collected at atmospheric pressure? 805.4069 in³ when the pressure is 4666 psig. Use 305 000 psi as an average value of the bulk modulus of elasticity.

$$K = -\frac{\Delta p}{\Delta V/V} \qquad 305\,000 = -\frac{0-4666}{(V_2 - 805.407)/805.407} \qquad V_2 = 817.73\ \text{in}^3$$

$$W = (62.4)(817.73/1728) = 29.5\ \text{lb}$$

1.33 Water in a hydraulic press, initially at 20 psia, is subjected to a pressure of 17 000 psia at 68 °F. Determine the percentage decrease in specific volume if the average bulk modulus of elasticity is 365 000 psi.

$$K = -\frac{\Delta p}{\Delta V/V} \qquad 365\,000 = -\frac{17\,000 - 20}{\Delta V/V_1} \qquad \frac{\Delta V}{V_1} = -0.0465 \quad \text{or} \quad 4.65\%\ \text{decrease}$$

1.34 At a depth of 7 km in the ocean, the pressure is 71.6 MPa. Assume a specific weight at the surface of 10.05 kN/m³ and an average bulk modulus of elasticity of 2.34 GPa for that pressure range. Find **(a)** the change in specific volume between the surface and 7 km; **(b)** the specific volume at 7 km; **(c)** the specific weight at 7 km.

(a)
$$(V_s)_1 = 1/\rho_1 = g/\gamma_1 = 9.81/10\,050 = 0.0009761\ \text{m}^3/\text{kg}$$

$$K = -\frac{\Delta p}{\Delta V_s/V_s} \qquad 2.34 \times 10^9 = -\frac{71.6\times 10^6 - 0}{\Delta V_s/0.0009761} \qquad \Delta V_s = -0.0000299\ \text{m}^3/\text{kg}$$

(b)
$$(V_s)_2 = (V_s)_1 + \Delta V_s = 0.0009761 - 0.0000299 = 0.000946\ \text{m}^3/\text{kg}$$

(c)
$$\gamma_2 = g/V_2 = 9.81/0.000946 = 10\,370\ \text{N/m}^3$$

1.35 Approximately what pressure must be applied to water at 60 °F to reduce its volume 2.5 percent?

$$K = -\frac{\Delta p}{\Delta V/V} \qquad 311\,000 = -\frac{p_2 - 0}{0.025} \qquad p_2 = 7775\ \text{psi}$$

1.36 A gas at 20 °C and 0.21 MPa abs has a volume of 41 L and a gas constant (R) of 210 m · N/(kg · K). Determine the density and mass of the gas.

$$\rho = p/RT = 0.21 \times 10^6/[(210)(20+273)] = 3.41\ \text{kg/m}^3 \qquad m = \rho V = (3.41)(0.041) = 0.140\ \text{kg}$$

1.37 What is the specific weight of air at 70 psia and 70 °F?

$\gamma = p/RT$. From Table A-6, $R = 53.3$ ft/°R; $\gamma = (70)(144)/[(53.3)(70+460)] = 0.357$ lb/ft³.

Note: p/RT gives ρ (Prob. 1.36) or γ (Prob. 1.37), depending on the value of R used. Corresponding values of R in Table A-6 differ by a factor of g.

1.38 Calculate the density of water vapor at 350 kPa abs and 20 °C if its gas constant (R) is 0.462 kPa · m³/kg · K.

$$\rho = p/RT = 350/[(0.462)(20+273)] = 2.59\ \text{kg/m}^3$$

1.39 Nitrogen gas (molecular weight 28) occupies a volume of 4.0 ft³ at 2500 lb/ft² abs and 750 °R. What are its specific volume and specific weight?

$$R = R_u/M = 49\,709/28 = 1775\ \text{ft}\cdot\text{lb/(slug}\cdot{}^\circ\text{R)}$$

[where R_u, the universal gas constant, $= 49\,709$ ft · lb/(slug · °R)]

$$\rho = 1/V_s = p/RT = 2500/[(1775)(750)] \qquad V_s = 532.5\ \text{ft}^3/\text{slug}$$

$$\gamma = \rho g = (1/V_s)(g) = (1/532.5)(32.2) = 0.0605\ \text{lb/ft}^3$$

1.40 One kilogram of hydrogen is confined in a volume of 200 L at −45 °C. What is the pressure if R is 4.115 kJ/kg · K?

$$p = \rho RT = (m/V)RT = (1/0.200)(4115)(-45+273) = 4.691 \times 10^6\ \text{Pa} \quad \text{or} \quad 4.691\ \text{MPa abs}$$

1.41 What is the specific weight of air at a temperature of 30 °C and a pressure of 470 kPa abs?

$$\gamma = p/RT = 470/[(29.3)(30+273)] = 0.0529\ \text{kN/m}^3$$

1.42 Find the mass density of helium at a temperature of 39 °F and a pressure of 26.9 psig, if atmospheric pressure is 14.9 psia.

I
$$\rho = p/RT = (14.9 + 26.9)(144)/[(12\,420)(39 + 460)]$$
$$= 0.000971 \text{ lb} \cdot \text{s}^2/\text{ft}^4 \quad \text{or} \quad 0.000971 \text{ slug/ft}^3$$

1.43 The temperature and pressure of nitrogen in a tank are 28 °C and 600 kPa abs, respectively. Determine the specific weight of the nitrogen.

I
$$\gamma = p/RT = 600/[(30.3)(28 + 273)] = 0.0658 \text{ kN/m}^3$$

1.44 The temperature and pressure of oxygen in a container are 60 °F and 20.0 psig, respectively. Determine the oxygen's mass density if atmospheric pressure is 14.7 psia.

I
$$\rho = p/RT = (20.0 + 14.7)(144)/[(1552)(60 + 460)] = 0.00619 \text{ slug/ft}^3$$

1.45 Calculate the specific weight and density of methane at 100 °F and 120 psia.

I
$$\gamma = p/RT = (120)(144)/[(96.2)(100 + 460)] = 0.321 \text{ lb/ft}^3$$
$$\rho = \gamma/g = 0.321/32.2 = 0.00997 \text{ slug/ft}^3$$

1.46 At 90 °F and 30.0 psia, the specific weight of a certain gas was 0.0877 lb/ft³. Determine the gas constant and density of this gas.

I
$$\gamma = p/RT \qquad 0.0877 = (30.0)(144)/[(R)(90 + 460)] \qquad R = 89.6 \text{ ft/°R}$$
$$\rho = \gamma/g = 0.0877/32.2 = 0.00272 \text{ slug/ft}^3$$

1.47 A cylinder contains 12.5 ft³ of air at 120 °F and 40 psia. The air is then compressed to 2.50 ft³. ***(a)*** Assuming isothermal conditions, what are the pressure at the new volume and the bulk modulus of elasticity? ***(b)*** Assuming adiabatic conditions, what are the final pressure and temperature and the bulk modulus of elasticity?

I ***(a)***
$$p_1V_1 = p_2V_2 \qquad \text{(for isothermal conditions)}$$
$$(40)(12.5) = (p_2')(2.50)$$
$$p_2' = 200 \text{ psia}$$
$$K = -\frac{\Delta p}{\Delta V/V} = -\frac{40 - 200}{(12.5 - 2.5)/12.5} = 200 \text{ psi}$$

(b) $p_1V_1^k = p_2V_2^k$ (for adiabatic conditions). From Table A-6, $k = 1.40$. $(40)(12.5)^{1.40} = (p_2')(2.50)^{1.40}$, $p_2' = 381$ psia; $T_2/T_1 = (p_2/p_1)^{(k-1)/k}$, $T_2/(120 + 460) = (\frac{381}{40})^{(1.40-1)/1.40}$, $T_2 = 1104$ °R, or 644 °F; $K = kp' = (1.40)(381) = 533$ psi.

1.48 Air is kept at a pressure of 200 kPa and a temperature of 30 °C in a 500-L container. What is the mass of the air?

I
$$\rho = p/RT = [(200)(1000)]/[(287)(30 + 273)] = 2.300 \text{ kg/m}^3 \qquad m = (2.300)(\tfrac{500}{1000}) = 1.15 \text{ kg}$$

1.49 An ideal gas has its pressure doubled and its specific volume decreased by two-thirds. If the initial temperature is 80 °F, what is the final temperature?

I
$$\rho = 1/V_s = p/RT \qquad pV_s = RT \qquad p_1(V_s)_1 = RT_1 \qquad p_2(V_s)_2 = RT_2$$
$$(p_2/p_1)[(V_s)_2/(V_s)_1] = (R/R)(T_2/T_1) \qquad (2)(\tfrac{1}{3}) = T_2/(80 + 460) \qquad T_2 = 360 \text{ °R} \quad \text{or} \quad -100 \text{ °F}$$

1.50 The tank of a leaky air compressor originally holds 90 L of air at 33 °C and 225 kPa. During a compression process, 4 grams of air is lost; the remaining air occupies 42 L at 550 kPa. What is the temperature of the remaining air?

▮ $\rho_1 = p_1/RT_1 = (225 \times 10^3)/[(287)(33+273)] = 2.562 \text{ kg/m}^3 \qquad m = (2.562)(0.090) = 0.2306 \text{ kg}$

$\rho_2 = p_2/RT_2 \qquad (0.2306 - 0.004)/0.042 = (550 \times 10^3)/(287T_2) \qquad T_2 = 355 \text{ K} \quad \text{or} \quad 82\text{ °C}$

1.51 In a piston-and-cylinder apparatus the initial volume of air is 90 L at a pressure of 130 kPa and temperature of 26 °C. If the pressure is doubled while the volume is decreased to 56 L, compute the final temperature and density of the air.

▮ $\rho_1 = p_1/RT_1 = (130 \times 10^3)/[(287)(26+273)] = 1.515 \text{ kg/m}^3 \qquad m = (1.515)(0.090) = 0.1364 \text{ kg}$

$\rho_2 = p_2/RT_2 \qquad 0.1364/\frac{56}{1000} = (2)(130 \times 10^3)/(287T_2) \qquad T_2 = 372 \text{ K} \quad \text{or} \quad 99\text{ °C}$

$$\rho = 0.1364/(0.056) = 2.44 \text{ kg/m}^3$$

1.52 For 2 lb mol of air with a molecular weight of 29, a temperature of 90 °F, and a pressure of 2.5 atm, what is the volume?

▮ $pV/nM = RT \qquad [(2.5)(14.7)(144)]\{V/[(2)(29)]\} = (53.3)(90+460) \qquad V = 321 \text{ ft}^3$

1.53 If nitrogen has a molecular weight of 28, what is its density according to the perfect gas law when $p = 0.290$ MPa and $T = 30$ °C?

▮ $R = R_u/M = 8312/28 = 297 \text{ J/(kg}\cdot\text{K)} \qquad [\text{where } R_u = 8312 \text{ J/(kg}\cdot\text{K)}]$

$$\rho = p/RT = 290\,000/[(297)(30+273)] = 3.22 \text{ kg/m}^3$$

1.54 If a gas occupies 1 m^3 at 1 atm pressure, what pressure is required to reduce the volume of the gas by 2 percent under isothermal conditions if the fluid is (*a*) air, (*b*) argon, and (*c*) hydrogen?

▮ $pV = nRT$ = constant for isothermal conditions. Therefore, if V drops to $0.98V_o$, p must rise to $(1/0.98)p_o$, or $1.020p_o$. This is true for any perfect gas.

1.55 (*a*) Calculate the density, specific weight, and specific volume of oxygen at 100 °F and 15 psia. (*b*) What would be the temperature and pressure of this gas if it were compressed isentropically to 40 percent of its original volume? (*c*) If the process described in (*b*) had been isothermal, what would the temperature and pressure have been?

▮ (*a*) $\rho = p/RT = (15)(144)/[(1552)(100+460)] = 0.00248 \text{ slug/ft}^3$

$\gamma = \rho g = (0.00248)(32.2) = 0.0799 \text{ lb/ft}^3 \qquad V_s = 1/\rho = 1/0.00248 = 403 \text{ ft}^3\text{/slug}$

(*b*) $p_1(V_s)_1^k = p_2(V_s)_2^k \quad [(15)(144)](403)^{1.40} = [(p_2)(144)][(0.40)(403)]^{1.40} \quad p_2 = 54.1 \text{ psia}$

$p_2 = \rho_2 RT_2 \qquad (54.1)(144) = (0.00248/0.40)(1552)(T_2 + 460) \qquad T_2 = 350\text{ °F}$

(*c*) If isothermal, $T_2 = T_1 = 100$ °F and pV = constant.

$$[(15)(144)](403) = [(p_2)(144)][(0.40)(403)] \qquad p_2 = 37.5 \text{ psia}$$

1.56 Calculate the density, specific weight, and volume of chloride gas at 25 °C and pressure of 600 000 N/m^2 abs.

▮ $\rho = p/RT = 600\,000/[(118)(25+273)] = 17.1 \text{ kg/m}^3$

$\gamma = \rho g = (17.1)(9.81) = 168 \text{ N/m}^3 \qquad V_s = 1/\rho = 1/17.1 = 0.0585 \text{ m}^3\text{/kg}$

1.57 If methane gas has a specific gravity of 0.55 relative to air at 14.7 psia and 68 °F, what are its specific weight and specific volume at that same pressure and temperature? What is the value of R for the gas?

▮ $\gamma_{\text{air}} = p/RT = (14.7)(144)/[(53.3)(68+460)] = 0.07522 \text{ lb/ft}^3$

$\gamma_{\text{gas}} = (0.55)(0.07522) = 0.0414 \text{ lb/ft}^3$

$V_s = 1/\rho = g/\gamma \qquad (V_s)_{\text{gas}} = 32.2/0.0414 = 778 \text{ ft}^3\text{/slug}$

Since R varies inversely with density for fixed pressure and temperature, $R_{\text{gas}} = 53.3/0.55 = 96.9$ ft/°R.

1.58 A gas at 40 °C under a pressure of 21.868 bar abs has a unit weight of 362 N/m³. What is the value of R for this gas? What gas might this be?

$$\gamma = p/RT \qquad 362 = (21.868 \times 10^5)/[(R)(40+273)] \qquad R = 19.3\ \text{m/K}$$

This gas might be carbon dioxide, since its gas constant is 19.3 m/K (from Table A-6).

1.59 If water vapor ($R = 85.7$ ft/°R) in the atmosphere has a partial pressure of 0.60 psia and the temperature is 80 °F, what is its specific weight?

$$\gamma = p/RT = (0.60)(144)/[(85.7)(80+460)] = 0.00187\ \text{lb/ft}^3$$

1.60 Refer to Prob. 1.59. If the barometer reads 14.60 psia, calculate the partial pressure of the air, its specific weight, and the specific weight of the atmosphere (air plus water vapor).

$$p_{air} = 14.60 - 0.60 = 14.00\ \text{psia} \qquad \gamma = p/RT$$

$$\gamma_{air} = (14.00)(144)/[(53.3)(80+460)] = 0.0700\ \text{lb/ft}^3 \qquad \gamma_{atm} = \gamma_{air} + \gamma_{H_2O(vap)}$$

$$\gamma_{H_2O(vap)} = 0.00187\ \text{lb/ft}^3 \quad \text{(from Prob. 1.59)} \qquad \gamma_{atm} = 0.0700 + 0.00187 = 0.0719\ \text{lb/ft}^3$$

1.61 (*a*) Calculate the density, specific weight, and specific volume of oxygen at 20 °C and 40 kPa abs. (*b*) If the oxygen is enclosed in a rigid container, what will be the pressure if the temperature is reduced to −100 °C?

(*a*)

$$\rho = p/RT = (40)(1000)/[(260)(20+273)] = 0.525\ \text{kg/m}^3$$

$$\gamma = \rho g = (0.525)(9.81) = 5.15\ \text{N/m}^3 \qquad V_s = 1/\rho = 1/0.525 = 1.90\ \text{m}^3/\text{kg}$$

(*b*) $\rho = 1/V_s = p/RT$. Since V_s and R are constants, $V_s/R = T/p =$ constant, $(20+273)/40 = (-100+273)/p_2$, $p_2 = 23.6$ kPa.

1.62 Helium at 149 kPa abs and 10 °C is isentropically compressed to one-fourth of its original volume. What is its final pressure?

$$p_1V_1^k = p_2V_2^k \qquad 149V_1^{1.66} = (p_2)(V_1/4)^{1.66} \qquad p_2 = 1488\ \text{kPa abs}$$

1.63 (*a*) If 9 ft³ of an ideal gas at 75 °F and 22 psia is compressed isothermally to 2 ft³, what is the resulting pressure? (*b*) What would the pressure and temperature have been if the process had been isentropic?

(*a*)

$$p_1V_1 = p_2V_2 \qquad (22)(9) = (p_2)(2) \qquad p_2 = 99\ \text{psia}$$

(*b*)

$$p_1V_1^k = p_2V_2^k \qquad (22)(9)^{1.30} = (p_2)(2)^{1.30} \qquad p_2 = 155\ \text{psia}$$

$$T_2/T_1 = (p_2/p_1)^{(k-1)/k} \qquad T_2/(75+460) = (\tfrac{155}{22})^{(1.30-1)/1.30} \qquad T_2 = 840\ \text{°R} \quad \text{or} \quad 380\ \text{°F}$$

1.64 (*a*) If 12 m³ of nitrogen at 30 °C and 125 kPa abs is permitted to expand isothermally to 30 m³, what is the resulting pressure? (*b*) What would the pressure and temperature have been if the process had been isentropic?

(*a*)

$$p_1V_1 = p_2V_2 \qquad (125)(12) = (p_2)(30) \qquad p_2 = 50.0\ \text{kPa abs}$$

(*b*)

$$p_1V_1^k = p_2V_2^k \qquad (125)(12)^{1.40} = (p_2)(30)^{1.40} \qquad p_2 = 34.7\ \text{kPa abs}$$

$$T_2/T_1 = (p_2/p_1)^{(k-1)/k} \qquad T_2/(30+273) = (34.7/125)^{(1.40-1)/1.40} \qquad T_2 = 210\ \text{K} \quad \text{or} \quad -63\ \text{°C}$$

1.65 If the viscosity of water at 68 °F is 0.01008 poise, compute its absolute viscosity (μ) in pound-seconds per square foot. If the specific gravity at 68 °F is 0.998, compute its kinematic viscosity (ν) in square feet per second.

The poise is measured in dyne-seconds per square centimeter. Since 1 lb = 444 800 dynes and 1 ft = 30.48 cm, 1 lb · s/ft² = 444 800 dyne · s/(30.48 cm)² = 478.8 poises

$$\mu = \frac{0.01008}{478.8} = 2.11 \times 10^{-5}\ \text{lb}\cdot\text{s/ft}^2 \qquad \nu = \frac{\mu}{\rho} = \frac{\mu}{\gamma/g} = \frac{\mu g}{\gamma} = \frac{(2.11\times 10^{-5})(32.2)}{(0.998)(62.4)} = 1.09 \times 10^{-5}\ \text{ft}^2/\text{s}$$

1.66 Convert 15.14 poises to kinematic viscosity in square feet per second if the liquid has a specific gravity of 0.964.

$$1\ \text{lb}\cdot\text{s/ft}^2 = 478.8\ \text{poises} \quad \text{(from Prob. 1.65)}$$

$$\mu = 15.14/478.8 = 0.03162\ \text{lb}\cdot\text{s/ft}^2 \qquad \nu = \mu g/\gamma = (0.03162)(32.2)/[(0.964)(62.4)] = 0.0169\ \text{ft}^2/\text{s}$$

1.67 The fluid flowing in Fig. 1-3 has an absolute viscosity (μ) of 0.0010 lb · s/ft^2 and specific gravity of 0.913. Calculate the velocity gradient and intensity of shear stress at the boundary and at points 1 in, 2 in, and 3 in from the boundary, assuming (*a*) a straight-line velocity distribution and (*b*) a parabolic velocity distribution. The parabola in the sketch has its vertex at A and origin at B.

▌ (*a*) For the straight-line assumption, the relation between velocity v and distance y is $v = 15y$, $dv = 15dy$. The velocity gradient $= dv/dy = 15$. Since $\mu = \tau/(dv/dy)$, $\tau = \mu\,(dv/dy)$. For $y = 0$ (i.e., at the boundary), $v = 0$ and $dv/dy = 15\ \mathrm{s}^{-1}$; $\tau = (0.0010)(15) = 0.015\ \mathrm{lb/ft^2}$. For $y = 1$ in, 2 in, and 3 in, dv/dy and τ are also $15\ \mathrm{s}^{-1}$ and $0.015\ \mathrm{lb/ft^2}$, respectively. (*b*) For the parabolic assumption, the parabola passes through the points $v = 0$ when $y = 0$ and $v = 45$ when $y = 3$. The equation of this parabola is $v = 45 - 5(3-y)^2$, $dv/dy = 10(3-y)$, $\tau = 0.0010\,(dv/dy)$. For $y = 0$ in, $v = 0$ in/s, $dv/dy = 30\ \mathrm{s}^{-1}$, and $\tau = 0.030\ \mathrm{lb/ft^2}$. For $y = 1$ in, $v = 25$ in/s, $dv/dy = 20\ \mathrm{s}^{-1}$, and $\tau = 0.020\ \mathrm{lb/ft^2}$. For $y = 2$ in, $v = 40$ in/s, $dv/dy = 10\ \mathrm{s}^{-1}$, and $\tau = 0.010\ \mathrm{lb/ft^2}$. For $y = 3$ in, $v = 45$ in/s, $dv/dy = 0\ \mathrm{s}^{-1}$, and $\tau = 0\ \mathrm{lb/ft^2}$.

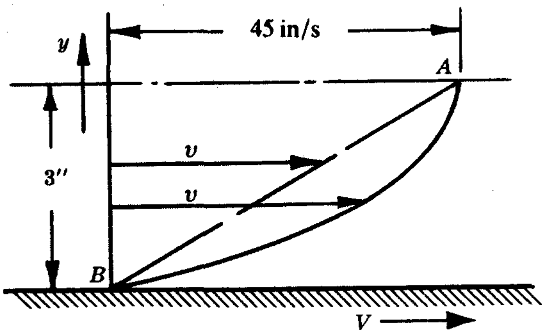


Fig. 1-3

1.68 A cylinder of 0.40-ft radius rotates concentrically inside a fixed cylinder of 0.42-ft radius. Both cylinders are 1.00 ft long. Determine the viscosity of the liquid that fills the space between the cylinders if a torque of 0.650 lb · ft is required to maintain an angular velocity of 60 rpm.

▌ The torque is transmitted through the field layers to the outer cylinder. Since the gap between the cylinders is small, the calculations may be made without integration. The tangential velocity v_t of the inner cylinder $= r\omega$, where $r = 0.40$ ft and $\omega = 2\pi$ rad/s. Hence, $v_t = (0.40)(2\pi) = 2.51$ ft/s. For the small space between cylinders, the velocity gradient may be assumed to be a straight line and the mean radius can be used. Then, $dv/dy = (2.51 - 0)/(0.42 - 0.40) = 125.5\ \mathrm{s}^{-1}$. Since applied torque equals resisting torque, applied torque = (τ)(area)(arm), $0.650 = \tau[(1.00)(2\pi)(0.40 + 0.42)/2][(0.40 + 0.42)/2]$, $\tau = 0.615\ \mathrm{lb/ft^2} = \mu\,(dv/dy)$, $0.615 = (\mu)(125.5)$, $\mu = 0.00490\ \mathrm{lb \cdot s/ft^2}$.

1.69 Water is moving through a pipe. The velocity profile at some section is shown in Fig. 1-4 and is given mathematically as $v = (\beta/4\mu)(d^2/4 - r^2)$, where v = velocity of water at any position r, β = a constant, μ = viscosity of water, d = pipe diameter, and r = radial distance from centerline. What is the shear stress at the wall of the pipe due to the water? What is the shear stress at a position $r = d/4$? If the given profile persists a distance L along the pipe, what drag is induced on the pipe by the water in the direction of flow over this distance?

▌
$$v = (\beta/4\mu)(d^2/4 - r^2) \qquad dv/dr = (\beta/4\mu)(-2r) = -2\beta r/4\mu$$
$$\tau = \mu\,(dv/dr) = \mu(-2\beta r/4\mu) = -2\beta r/4$$

At the wall, $r = d/2$. Hence,

$$\tau_{\mathrm{wall}} = \frac{-2\beta(d/2)}{4} = -\frac{\beta d}{4} \qquad \tau_{r=d/4} = \frac{-2\beta(d/4)}{4} = -\frac{\beta d}{8}$$

$$\mathrm{Drag} = (\tau_{\mathrm{wall}})(\mathrm{area}) = (\tau_{\mathrm{wall}})(\pi dL) = (\beta d/4)(\pi dL) = \beta d^2\pi L/4$$

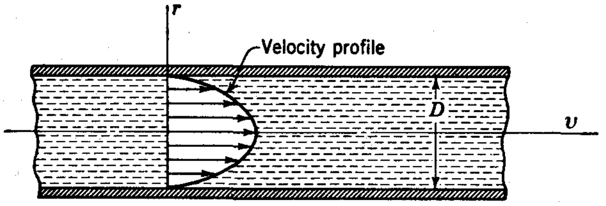


Fig. 1-4

1.70 A large plate moves with speed v_0 over a stationary plate on a layer of oil (see Fig. 1-5). If the velocity profile is that of a parabola, with the oil at the plates having the same velocity as the plates, what is the shear stress on the moving plate from the oil? If a linear profile is assumed, what is the shear stress on the upper plate?

For a parabolic profile, $v^2 = ay$. When $y = d$, $v = v_0$. Hence, $v_0^2 = ad$, $a = v_0^2/d$. Therefore,

$$v^2 = (v_0^2/d)(y) = (v_0^2)(y/d) \qquad v = v_0\sqrt{y/d} \qquad dv/dy = [(v_0)(1/\sqrt{d})(\tfrac{1}{2})(y^{-1/2})]$$

$$\tau = \mu\,(dv/dy) = \mu[(v_0)(1/\sqrt{d})(\tfrac{1}{2})(y^{-1/2})]$$

For $y = d$, $\tau = \mu[(v_0)(1/\sqrt{d})(\tfrac{1}{2})(d^{-1/2})] = \mu v_0/(2d)$. For a linear profile, $dv/dy = v_0/d$, $\tau = \mu(v_0/d)$.

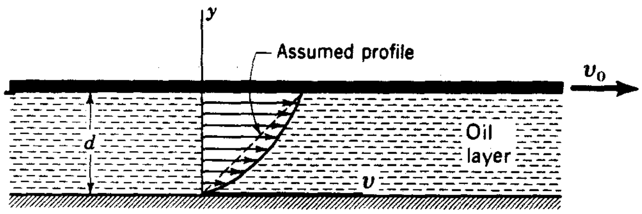


Fig. 1-5

1.71 A square block weighing 1.1 kN and 250 mm on an edge slides down an incline on a film of oil 6.0 μm thick (see Fig. 1-6). Assuming a linear velocity profile in the oil, what is the terminal speed of the block? The viscosity of the oil is 7 mPa · s.

$$\tau = \mu\,(dv/dy) = (7\times10^{-3})[v_T/(6.0\times10^{-6})] = 1167v_T \qquad F_f = \tau A = (1167v_T)(0.250)^2 = 72.9v_T$$

At the terminal condition, equilibrium occurs. Hence, $1100 \sin 20° = 72.9v_T$, $v_T = 5.16$ m/s.

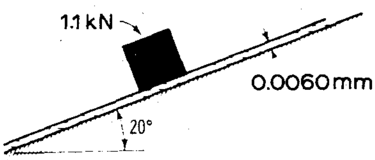


Fig. 1-6(*a*)

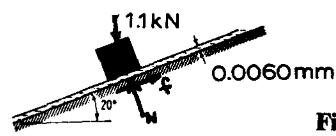


Fig. 1-6(*b*)

1.72 A piston of weight 21 lb slides in a lubricated pipe, as shown in Fig. 1-7. The clearance between piston and pipe is 0.001 in. If the piston decelerates at 2.1 ft/s^2 when the speed is 21 ft/s, what is the viscosity of the oil?

$$\tau = \mu\,(dv/dy) = \mu[v/(0.001/12)] = 12\,000\mu v$$

$$F_f = \tau A = 12\,000\mu v[(\pi)(\tfrac{6}{12})(\tfrac{5}{12})] = 7854\mu v$$

$$\Sigma F = ma \qquad 21 - (7854)(\mu)(21) = (21/32.2)(-2.1) \qquad \mu = 1.36\times10^{-4}\ \text{lb}\cdot\text{s/ft}^2$$

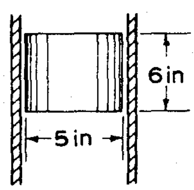


Fig. 1-7

1.73 A piston is moving through a cylinder at a speed of 19 ft/s, as shown in Fig. 1-8. The film of oil separating the piston from the cylinder has a viscosity of 0.020 lb · s/ft^2. What is the force required to maintain this motion?

Assume a cylindrically symmetric, linear velocity profile for the flow of oil in the film. To find the frictional resistance, compute the shear stress at the piston surface.

$$\tau = \mu\frac{dv}{dr} = 0.020\left[\frac{19}{(5.000-4.990)/2}\right](12) = 912\ \text{lb/ft}^2 \qquad F_f = \tau A = 912\left[\pi\left(\frac{4.990}{12}\right)\left(\frac{3}{12}\right)\right] = 298\ \text{lb}$$

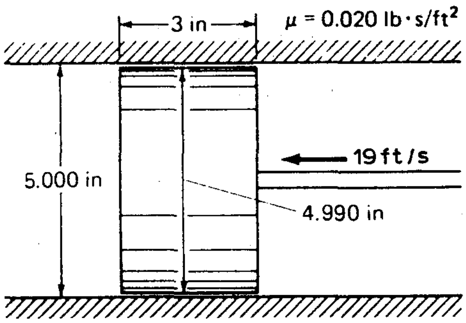


Fig. 1-8(a)

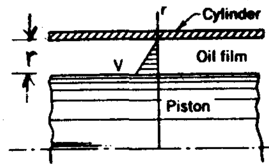


Fig. 1-8(b)

1.74 To damp oscillations, the pointer of a galvanometer is fixed to a circular disk which turns in a container of oil (see Fig. 1-9). What is the damping torque for $\omega = 0.3$ rad/s if the oil has a viscosity of 8×10^{-3} Pa · s? Neglect edge effects.

▮ Assume at any point that the velocity profile of the oil is linear $dv/dn = r\omega/(0.5/1000) = (r)(0.3)/(0.5/1000) = 600r$; $\tau = \mu\,(dv/dn) = \mu(600r) = (8 \times 10^{-3})(600r) = 4.80r$. The force dF_f on dA on the upper face of the disc is then $dF_f = \tau\, dA = (4.80r)(r\, d\theta\, dr) = 4.80r^2\, d\theta\, dr$. The torque dT for dA on the upper face is then $dT = r\, dF_f = r(4.80r^2\, d\theta\, dr) = 4.80r^3\, d\theta\, dr$. The total resisting torque on both faces is

$$T = 2\left[\int_0^{0.075/2}\int_0^{2\pi} 4.80r^3\, d\theta\, dr\right] = (9.60)(2\pi)\left[\frac{r^4}{4}\right]_0^{0.075/2} = 2.98 \times 10^{-5}\ \text{N}\cdot\text{m}$$

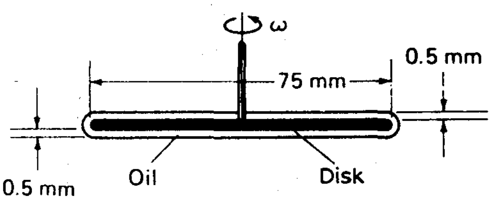


Fig. 1-9(a)

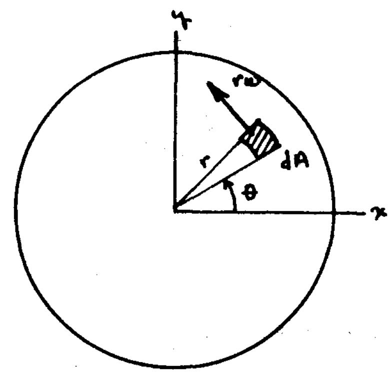


Fig. 1-9(b)

1.75 For angular velocity 0.3 rad/s of the mechanism of Prob. 1.74, express the damping torque (in N · m) as a function of displacement x (in mm) of the disk from its center position (Fig. 1-10).

▮ Assume at any point that the velocity profile of the oil is linear; $\tau = \mu\,(dv/dn)$. For the upper face, $dv/dn = r\omega/[(0.5 - x)/1000] = (r)(0.3)/[(0.5 - x)/1000]$; $\tau = (8 \times 10^{-3})\{(r)(0.3)/[(0.5 - x)/1000]\} = 2.40r/(0.5 - x)$. The force dF_f on dA on the upper face of the disc is then $dF_f = \tau\, dA = [2.40r/(0.5 - x)](r\, d\theta\, dr) = [2.40r^2/(0.5 - x)]\,(d\theta\, dr)$. The torque dT for dA on the upper face is then $dT = r\, dF_f = r[2.40r^2/(0.5 - x)]\,(d\theta\, dr) = [2.40r^3/(0.5 - x)]\,(d\theta\, dr)$. For the lower face, $dv/dn = r\omega/[(0.5 + x)/1000] = r(0.3)/[(0.5 + x)/1000]$; $\tau = (8 \times 10^{-3})\{r(0.3)/[(0.5 + x)/1000]\} = 2.40r/(0.5 + x)$. The force dF_f on dA on the lower face of the disc is then $dF_f = \tau\, dA = [2.40r/(0.5 + x)](r\, d\theta\, dr) = [2.40r^2/(0.5 + x)]\,(d\theta\, dr)$. The torque dT for dA on the lower face is then $dT = r\, dF_f = r[2.40r^2/(0.5 + x)]\,(d\theta\, dr) = [2.40r^3/(0.5 + x)]\,(d\theta\, dr)$. The total resisting torque on both faces is

$$T = \int_0^{0.075/2}\int_0^{2\pi} \frac{2.40r^3}{0.5 - x}\, d\theta\, dr + \int_0^{0.075/2}\int_0^{2\pi} \frac{2.40r^3}{0.5 + x}\, d\theta\, dr$$

$$= \left(\frac{1}{0.5 - x} + \frac{1}{0.5 + x}\right)(2.40)(2\pi)\left[\frac{r^4}{4}\right]_0^{0.075/2} = \left(\frac{0.5 + x + 0.5 - x}{0.25 - x^2}\right)(7.46 \times 10^{-6})$$

$$= \frac{7.46 \times 10^{-6}}{0.25 - x^2}$$

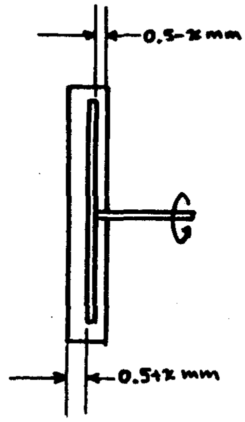


Fig. 1-10

1.76 A conical body turns in a container, as shown in Fig. 1-11, at constant speed 11 rad/s. A uniform 0.01-in film of oil with viscosity 3.125×10^{-7} lb · s/in^2 separates the cone from the container. What torque is required to maintain this motion, if the cone has a 2-in radius at its base and is 4 in tall?

▮ Consider the conical surface first ($r/2 = z/4$, $r = z/2$). The stress on this element is $\tau = \mu\,(dv/dx) = \mu(r\omega/0.01) = (3.125 \times 10^{-7})[(z/2)(11)/0.01] = 1.719 \times 10^{-4}z$. The area of the strip shown is $dA = 2\pi r\,ds = (2\pi z/2)[dz/(4/\sqrt{20})] = 3.512z\,dz$. The torque on the strip is $dT = \tau\,(dA)(r) = (1.719 \times 10^{-4}z)(3.512z\,dz)(z/2) = 3.019 \times 10^{-4}z^3\,dz$.

$$T_1 = \int_0^4 3.019 \times 10^{-4}z^3\,dz = 3.019 \times 10^{-4}\left[\frac{z^4}{4}\right]_0^4 = 0.01932 \text{ in} \cdot \text{lb}$$

Next consider the base: $dF_f = \tau\,dA$, $\tau = \mu(r\omega/0.01) = (3.125 \times 10^{-7})[(r)(11)/0.01] = 3.438 \times 10^{-4}r$, $dF_f = (3.438 \times 10^{-4}r)(r\,d\theta\,dr) = 3.438 \times 10^{-4}r^2\,d\theta\,dr$, $dT_2 = (3.438 \times 10^{-4}r^2\,d\theta\,dr)(r) = 3.438 \times 10^{-4}r^3\,d\theta\,dr$.

$$T_2 = \int_0^2 \int_0^{2\pi} 3.438 \times 10^{-4}r^3\,d\theta dr = (3.438 \times 10^{-4})(2\pi)\left[\frac{r^4}{4}\right]_0^2 = 0.00864 \text{ in} \cdot \text{lb}$$

$$T_{\text{tot}} = 0.01932 + 0.00864 = 0.0280 \text{ in} \cdot \text{lb}$$

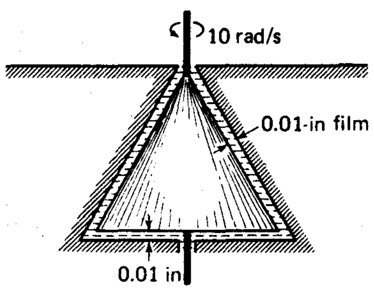


Fig. 1-11(a)

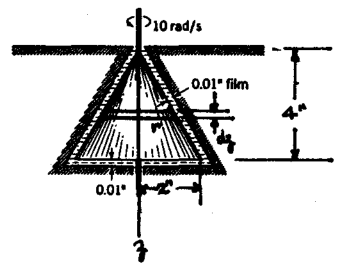


Fig. 1-11(b)

1.77 In Fig. 1-12, if the fluid is SAE 30 oil at 20 °C and $D = 7$ mm, what shear stress is required to move the upper plate at 3.5 m/s? Compute the Reynolds number based on D.

▮
$$\tau = \mu\,(dv/dh) = (0.440)[3.5/(\tfrac{7}{1000})] = 220 \text{ Pa}$$
$$N_R = \rho D v/\mu = (888)(\tfrac{7}{1000})(3.5)/0.440 = 49.4$$

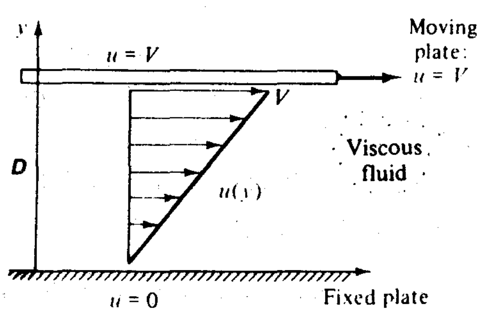


Fig. 1-12

1.78 Benzene at 20 °C has a viscosity of 0.000651 Pa · s. What shear stress is required to deform this fluid at a strain rate of 4900 s^{-1}?

$$\tau = \mu\,(dv/dx) = (0.000651)(4900) = 3.19\ \text{Pa}$$

1.79 SAE 30 oil at 20 °C is sheared between two parallel plates 0.005 in apart with the lower plate fixed and the upper plate moving at 13 ft/s. Compute the shear stress in the oil.

$$\tau = \mu\,(dv/dh) = (9.20 \times 10^{-3})[13/(0.005/12)] = 287\ \text{lb/ft}^2$$

1.80 An 18-kg slab slides down a 15° inclined plane on a 3-mm-thick film of SAE 10 oil at 20 °C; the contact area is 0.3 m^2. Find the terminal velocity of the slab.

See Fig. 1-13.

$$\Sigma F_x = 0 \qquad W \sin\theta - \tau A_{\text{bottom}} = 0$$

$$\tau = \mu\,(dv/dy) = (8.14 \times 10^{-2})(v_T/0.003) = 27.1 v_T$$

$$[(18)(9.81)](\sin 15°) - (27.1 v_T)(0.3) = 0 \qquad v_T = 5.62\ \text{m/s}$$

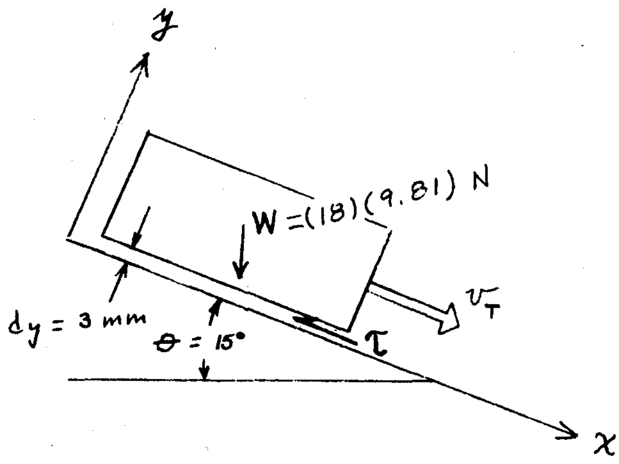


Fig. 1-13

1.81 A shaft 70.0 mm in diameter is being pushed at a speed of 400 mm/s through a bearing sleeve 70.2 mm in diameter and 250 mm long. The clearance, assumed uniform, is filled with oil at 20 °C with $\nu = 0.005\ m^2/s$ and s.g. = 0.9. Find the force exerted by the oil on the shaft.

$$F = \tau A \qquad \tau = \mu\,(dv/dr) \qquad \mu = \rho\nu = [(0.9)(998)](0.005) = 4.49\ \text{kg/(m}\cdot\text{s)}$$

$$dr = (0.0702 - 0.0700)/2 = 0.0001\ \text{m} \qquad \tau = (4.49)(0.4/0.0001) = 17\,960\ \text{N/m}^2$$

$$A = (\pi)(7.00/100)(25/100) = 0.05498\ \text{m}^2 \qquad F = (17\,960)(0.05498) = 987\ \text{N}$$

1.82 If the shaft in Prob. 1.81 is fixed axially and rotated inside the sleeve at 2000 rpm, determine the resisting torque exerted by the oil and the power required to rotate the shaft.

$$T = \tau A r \qquad \tau = \mu\,(dv/dr)$$

$$v = r\omega = [(7.00/2)/100][(2000)(2\pi/60)] = 7.330\ \text{m/s} \qquad dr = 0.0001\ \text{m}$$

$$\tau = (4.49)(7.330/0.0001) = 329.1 \times 10^3\ \text{N/m}^2 \qquad A = (\pi)(7.00/100)(\tfrac{25}{100}) = 0.05498\ \text{m}^2$$

$$T = (329.1 \times 10^3)(0.05498)[(7.00/2)/100] = 633\ \text{N}\cdot\text{m}$$

$$P = \omega T = [(2000)(2\pi/60)](633) = 132.6 \times 10^3\ \text{W} \quad \text{or} \quad 132.6\ \text{kW}$$

1.83 A steel (7850-kg/m^3) shaft 40.0 mm in diameter and 350 mm long falls of its own weight inside a vertical open tube 40.2 mm in diameter. The clearance, assumed uniform, is a film of SAE 30 oil at 20 °C. What speed will the cylinder ultimately reach?

▌ $$W_{\text{shaft}} = \tau A = [(7850)(9.81)][(0.350)(\pi)(0.0400)^2/4] = 33.87\ \text{N}$$
$$dr = (0.0402 - 0.0400)/2 = 0.0001\ \text{m}$$
$$\tau = \mu\,(dy/dr) = (0.440)(v_T/0.0001) = 4400v_T$$
$$A = (\pi)(4.00/100)(\tfrac{35}{100}) = 0.04398\ \text{m}^2 \qquad 33.87 = (4400v_T)(0.04398) \qquad v_T = 0.1750\ \text{m/s}$$

1.84 Air at 20 °C forms a boundary layer near a solid wall, in which the velocity profile is sinusoidal (see Fig. 1-14). The boundary-layer thickness is 7 mm and the peak velocity is 9 m/s. Compute the shear stress in the boundary layer at y equal to (*a*) 0, (*b*) 3.5 mm, and (*c*) 7 mm.

▌ $$\tau = \mu\,(dv/dy) \qquad v = v_{\max} \sin[\pi y/(2\delta)]$$
$$dv/dy = [\pi v_{\max}/(2\delta)] \cos[\pi y/(2\delta)] = \{(\pi)(9)/[(2)(0.007)]\} \cos\{\pi y/[(2)(0.007)]\} = 2020 \cos(224.4y)$$

Note: "224.4y" in the above equation is in radians.

$$\tau = (1.81 \times 10^{-5})[2020 \cos(224.4y)] = 0.03656 \cos(224.4y)$$

(*a*) At $y = 0$, $\tau = 0.03656 \cos[(224.4)(0)] = 0.0366$ Pa. (*b*) At $y = 0.0035$ m, $\tau = 0.03656 \cos[(224.4)(0.0035)] = 0.0259$ Pa. (*c*) At $y = 0.007$ m, $\tau = 0.03656 \cos[(224.4)(0.007)] = 0$.

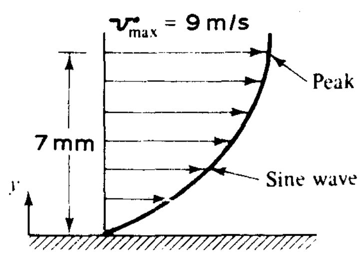


Fig. 1-14

1.85 A disk of radius r_0 rotates at angular velocity ω inside an oil bath of viscosity μ, as shown in Fig. 1-15. Assuming a linear velocity profile and neglecting shear on the outer disk edges, derive an expression for the viscous torque on the disk.

▌ $$\tau = \mu\,(dv/dy) = \mu(r\omega/h) \qquad \text{(on both sides)}$$
$$dT = (2)(r\tau\,dA) = (2)\{(r)[\mu(r\omega/h)](2\pi r\,dr)\} = (4\mu\omega\pi/h)(r^3\,dr)$$
$$T = \int_0^{r_0} \frac{4\mu\omega\pi}{h}(r^3\,dr) = \frac{4\mu\omega\pi}{h}\left[\frac{r^4}{4}\right]_0^{r_0} = \frac{\pi\mu\omega r_0^4}{h}$$

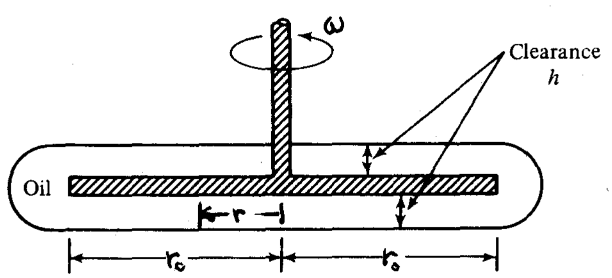


Fig. 1-15

1.86 A 35-cm-by-55-cm block slides on oil ($\mu = 0.81$ Pa · s) over a large plane surface. What force is required to drag the block at 3 m/s, if the separating oil film is 0.6 mm thick?

▌ $$\tau = \mu\,(dv/dx) = (0.81)[3/(0.6/1000)] = 4050\ \text{N/m}^2 \qquad F = \tau A = (4050)[(\tfrac{35}{100})(\tfrac{55}{100})] = 780\ \text{N}$$

1.87 The 1.5-in (0.125-ft) gap between two large plane surfaces is filled with SAE 30 oil at 80 °F ($\mu = 0.0063$ lb · s/ft^2). What force is required to drag a very thin plate of 5-ft^2 area between the surfaces at a speed of 0.5 ft/s if this plate is equally spaced between the two surfaces?

▌ $$\tau = \mu\,(dv/dx) = (0.0063)[0.5/(0.125/2)] = 0.0504\ \text{lb/ft}^2 \qquad F = \tau A = (0.0504)(5) = 0.252\ \text{lb}$$

Since there are two sides, $F_{\text{required}} = (2)(0.252)$, or 0.504 lb.

1.88 Rework Prob. 1.87 if the plate is at a distance of 0.50 in (0.0417 ft) from one surface.

▌ $\tau = \mu\,(dv/dx)$ $\quad \tau_1 = (0.0063)(0.5/0.0417) = 0.0755\ \text{lb/ft}^2$

$F = \tau A$ $\quad F_1 = (0.0755)(5) = 0.3775\ \text{lb}$ $\quad \tau_2 = (0.0063)[0.5/(0.125 - 0.0417)] = 0.0378\ \text{lb/ft}^2$

$F_2 = (0.0378)(5) = 0.1890\ \text{lb}$ $\quad F_{\text{required}} = F_1 + F_2 = 0.3775 + 0.1890 = 0.566\ \text{lb}$

1.89 A 10.000-in-diameter plunger slides in a 10.006-in-diameter cylinder, the annular space being filled with oil having a kinematic viscosity of 0.004 ft^2/s and specific gravity of 0.85. If the plunger moves at 0.6 ft/s, find the frictional resistance when 9 ft is engaged in the cylinder.

▌ $\tau = \mu\,(dv/dx)$ $\quad \rho = \gamma/g = [(0.85)(62.4)]/32.2 = 1.647\ \text{slugs/ft}^3$

$\mu = \rho\nu = (1.647)(0.004) = 0.006588\ \text{lb}\cdot\text{s/ft}^2$ $\quad dx = [(10.006 - 10.000)/2]/12 = 0.000250\ \text{ft}$

$\tau = (0.006588)(0.6/0.000250) = 15.81\ \text{lb/ft}^2$ $\quad F_f = \tau A = (15.81)[(9)(\pi)(\tfrac{10}{12})] = 373\ \text{lb}$

1.90 A 6.00-in shaft rides in a 6.01-in sleeve 8 in long, the clearance space (assumed to be uniform) being filled with lubricating oil at 100 °F ($\mu = 0.0018\ \text{lb}\cdot\text{s/ft}^2$). Calculate the rate at which heat is generated when the shaft turns at 90 rpm.

▌

$$dv = \omega(\text{circumference}) = \tfrac{90}{60}[\pi(6.00/12)] = 2.356\ \text{ft/s}$$

$$dx = [(6.01 - 6.00)/2]/12 = 0.0004167\ \text{ft}$$

$$\tau = \mu\,(dv/dx) = (0.0018)(2.356/0.0004167) = 10.18\ \text{lb/ft}^2$$

$$F_f = \tau A = 10.18[\pi(8.00/12)(\tfrac{6}{12})] = 10.66\ \text{lb}$$

$$\text{Rate of energy loss} = F_f v = (10.66)(2.356) = 25.11\ \text{ft}\cdot\text{lb/s}$$

$$\text{Rate of heat generation} = (25.11)(3600)/778 = 116\ \text{Btu/h}$$

1.91 A 10.00-cm shaft rides in an 10.03-cm sleeve 12 cm long, the clearance space (assumed to be uniform) being filled with lubricating oil at 40 °C ($\mu = 0.11\ \text{Pa}\cdot\text{s}$). Calculate the rate at which heat is generated when the shaft turns at 100 rpm.

▌ $dv = \omega(\text{circumference}) = \tfrac{100}{60}[\pi(0.10)] = 0.5236\ \text{m/s}$ $\quad dx = (0.1003 - 0.1000)/2 = 0.00015\ \text{m}$

$$\tau = \mu\,(dv/dx) = (0.11)(0.5236)/0.00015) = 384.0\ \text{N/m}^2$$

$$F_f = \tau A = 384.0[\pi(0.12)(0.10)] = 14.48\ \text{N}$$

$$\text{Rate of energy loss} = F_f v = (14.48)(0.5236) = 7.582\ \text{N}\cdot\text{m/s} = 7.582\ \text{W}$$

1.92 In using a rotating-cylinder viscometer, a bottom correction must be applied to account for the drag on the flat bottom of the cylinder. Calculate the theoretical amount of this torque correction, neglecting centrifugal effects, for a cylinder of diameter d, rotated at a constant angular velocity ω, in a liquid of viscosity μ, with a clearance Δh between the bottom of the inner cylinder and the floor of the outer one.

▌ Let r = variable radius. $T = \int r\tau\, dA$, $\tau = \mu\,(dv/dx) = \mu(r\omega/\Delta h)$, $dA = 2\pi r\, dr$.

$$T = \int_0^{d/2} r\left[\mu\left(\frac{r\omega}{\Delta h}\right)\right](2\pi r\, dr) = \frac{2\pi\mu\omega}{\Delta h}\int_0^{d/2} r^3\, dr = \frac{2\pi\mu\omega}{\Delta h}\left[\frac{r^4}{4}\right]_0^{d/2} = \frac{\pi\mu\omega d^4}{32\,\Delta h}$$

1.93 Assuming a boundary-layer velocity distribution as shown in Fig. 1-16, which is a parabola having its vertex 3 in from the wall, calculate the shear stresses for $y = 0$, 1 in, 2 in, and 3 in. Use $\mu = 0.00835\ \text{lb}\cdot\text{s/ft}^2$.

▌ $\tau = \mu\,(dv/dy)$. At $y = 0$, $v = 0$ and at $y = 3$ in, $v = 6$ ft/s, or 72 in/s. The equation of the parabola is $v = 72 - (8)(3 - y)^2$ (y in inches gives v in inches per second); $dv/dy = (16)(3 - y)$; $\tau = (0.00835)[(16)(3 - y)] = 0.4008 - 0.1336y$. At $y = 0$, $\tau = 0.4008 - (0.1336)(0) = 0.401\ \text{lb/ft}^2$. At $y = 1$ in, $\tau = 0.4008 - (0.1336)(1) = 0.267\ \text{lb/ft}^2$. At $y = 2$ in, $\tau = 0.4008 - (0.1336)(2) = 0.134\ \text{lb/ft}^2$. At $y = 3$ in, $\tau = 0.4008 - (0.1336)(3) = 0$.

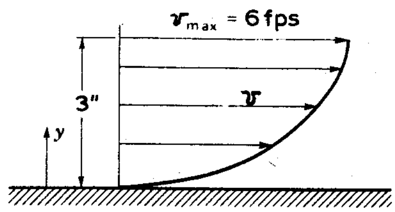


Fig. 1-16

1.94 In Fig. 1-17*a*, oil of viscosity μ fills the small gap of thickness Y. Determine an expression for the torque T required to rotate the truncated cone at constant speed ω. Neglect fluid stress exerted on the circular bottom.

▌ See Fig. 1-17*b*. $\tau = \mu\,(dv/dy)$, $v = r\omega = (y \tan\alpha)(\omega)$, $dv/dy = (y\tan\alpha)(\omega)/Y$.

$$\tau = \mu\left[\frac{(y\tan\alpha)(\omega)}{Y}\right] = \frac{\mu y\omega\tan\alpha}{Y}$$

$$dA = 2\pi r\,ds = 2\pi(y\tan\alpha)(dy/\cos\alpha) = 2\pi y(\tan\alpha/\cos\alpha)(dy)$$

$$dF = \tau\,dA = \left(\frac{\mu y\omega\tan\alpha}{Y}\right)\left[2\pi y\left(\frac{\tan\alpha}{\cos\alpha}\right)(dy)\right] = \left(\frac{2\pi\mu\omega\tan^2\alpha}{Y\cos\alpha}\right)y^2\,dy$$

$$dT = r\,dF = (y\tan\alpha)\left(\frac{2\pi\mu\omega\tan^2\alpha}{Y\cos\alpha}\right)y^2\,dy = \left(\frac{2\pi\mu\omega\tan^3\alpha}{Y\cos\alpha}\right)y^3\,dy$$

$$T = \int_a^{a+b}\left(\frac{2\pi\mu\omega\tan^3\alpha}{Y\cos\alpha}\right)y^3\,dy = \left(\frac{2\pi\mu\omega\tan^3\alpha}{Y\cos\alpha}\right)\left[\frac{y^4}{4}\right]_a^{a+b} = \left(\frac{2\pi\mu\omega\tan^3\alpha}{Y\cos\alpha}\right)\left[\frac{(a+b)^4}{4} - \frac{a^4}{4}\right]$$

$$= \left(\frac{\pi\mu\omega\tan^3\alpha}{2Y\cos\alpha}\right)[(a+b)^4 - a^4]$$

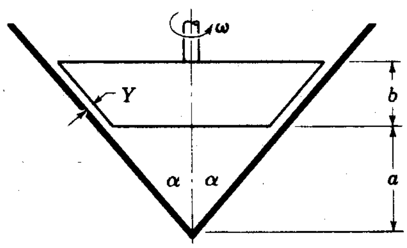


Fig. 1-17(*a*)

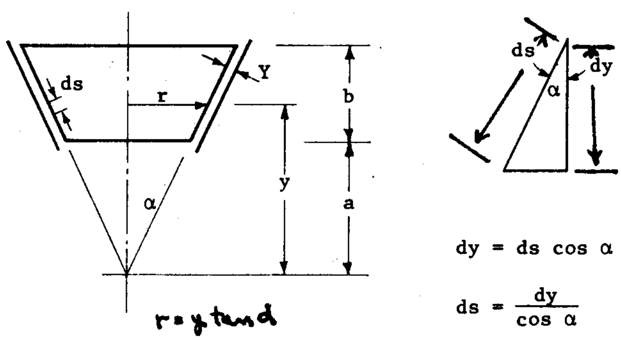


Fig. 1-17(*b*)

1.95 A Newtonian fluid fills the gap between a shaft and a concentric sleeve. When a force of 788 N is applied to the sleeve parallel to the shaft, the sleeve attains a speed of 2 m/s. If a 1400-N force is applied, what speed will the sleeve attain? The temperature of the sleeve remains constant.

▌ $\tau = F/A = \mu\,(dv/dx)$; $F/dv = \mu A/dx =$ constant. Therefore, $F_1/dv_1 = F_2/dv_2$, $\frac{788}{2} = 1400/dv_2$, $dv_2 = 3.55$ m/s.

1.96 A plate separated by 0.5 mm from a fixed plate moves at 0.50 m/s under a force per unit area of 4.0 N/m^2. Determine the viscosity of the fluid between the plates.

▌ $\tau = \mu\,(dv/dx)$ $4.0 = \mu[0.50/(0.0005)]$ $\mu = 0.00400\ \text{N}\cdot\text{s/m}^2 = 4.00\ \text{mPa}\cdot\text{s}$

1.97 Determine the viscosity of fluid between shaft and sleeve in Fig. 1-18.

▌ $\tau = F/A = \mu\,(dv/dx)$ $25/[(\pi)(\frac{4}{12})(\frac{9}{12})] = \mu[0.5/(0.004/12)]$ $\mu = 0.0212\ \text{lb}\cdot\text{s/ft}^2$

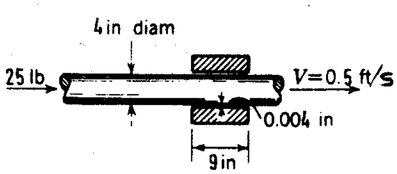


Fig. 1-18

1.98 A 1-in-diameter steel cylinder 10 in long falls at 0.6 ft/s inside a tube of slightly larger diameter. A castor-oil film of constant thickness is between the cylinder and the tube. Determine the clearance between the cylinder and the tube, if the temperature is 100 °F, s.g. = 7.85 for steel, and $\mu = 6 \times 10^{-3}$ lb · s/ft² for castor oil.

▮

$$\tau = F/A = \mu\,(dv/dx)$$

$$F = W = \gamma V = [(7.85)(62.4)][(\tfrac{10}{12})(\pi)(\tfrac{1}{12})^2/4] = 2.226 \text{ lb} \quad 2.226/[(\tfrac{10}{12})(\pi)(\tfrac{1}{12})] = (6 \times 10^{-3})(0.6/dx)$$

$$dx = 0.0003528 \text{ ft} \quad \text{or} \quad 0.00423 \text{ in}$$

1.99 A piston of diameter 70.00 mm moves inside a cylinder of diameter 70.10 mm. Determine the percent decrease in force necessary to move the piston when the lubricant warms from 0 to 120 °C. Values of μ for the lubricant are 0.01820 Pa · s at 0 °C and 0.00206 Pa · s at 120 °C.

▮ $\tau = F/A = \mu\,(dv/dx)$; $F/\mu = A\,(dv/dx) =$ constant. Therefore, $\Delta F/F_{0\,^\circ\mathrm{C}} = \Delta\mu/\mu_{0\,^\circ\mathrm{C}} = (0.01820 - 0.00206)/0.01820 = 0.887$, or 88.7%.

1.100 A body weighing 100 lb with a flat surface area of 3 ft² slides down a lubricated inclined plane making a 35° angle with the horizontal. For viscosity of 0.002089 lb · s/ft² and a body speed of 3.5 ft/s, determine the lubricant film thickness.

▮

$$F = \text{weight of body along inclined plane} = 100 \sin 35° = 57.4 \text{ lb}$$

$$\tau = F/A = \mu\,(dv/dx) \qquad 57.4/3 = (0.002089)(3.5/dx) \qquad dx = 0.0003821 \text{ ft} \quad \text{or} \quad 0.00459 \text{ in}$$

1.101 A small drop of water at 80 °F is in contact with the air and has a diameter of 0.0200 in. If the pressure within the droplet is 0.082 psi greater than the atmosphere, what is the value of the surface tension?

▮

$$p(\pi d^2/4) = (\pi d)(\sigma) \qquad \sigma = pd/4 = [(0.082)(144)](0.0200/12)/4 = 0.00492 \text{ lb/ft}$$

1.102 Estimate the height to which water at 70 °F will rise in a capillary tube of diameter 0.120 in.

▮ $h = 4\sigma \cos\theta/(\gamma d)$. From Table A-1, $\sigma = 0.00500$ lb/ft and $\gamma = 62.3$ lb/ft³ at 70 °F. Assume $\theta = 0°$ for a clean tube. $h = (4)(0.00500)(\cos 0°)/[(62.3)(0.120/12)] = 0.0321$ ft, or 0.385 in.

1.103 The shape of a hanging drop of liquid is expressible by the following formulation developed from photographic studies of the drop: $\sigma = (\gamma - \gamma_0)(d_e)^2/H$, where σ = surface tension, i.e., force per unit length, γ = specific weight of liquid drop, γ_0 = specific weight of vapor around it, d_e = diameter of drop at its equator, and H = a function determined by experiment. For this equation to be dimensionally homogeneous, what dimensions must H possess?

▮ Dimensionally, $(F/L) = (F/L^3)(L^2)/\{H\}$, $\{H\} = (1)$. Therefore, H is dimensionless.

1.104 Two clean, parallel glass plates, separated by a distance $d = 1.5$ mm, are dipped in a bath of water. How far does the water rise due to capillary action, if $\sigma = 0.0730$ N/m?

▮ Because the plates are clean, the angle of contact between water and glass is taken as zero. Consider the free-body diagram of a unit width of the raised water (Fig. 1-19). Summing forces in the vertical direction gives $(2)[(\sigma)(0.0015)] - (0.0015)^2(h)(\gamma) = 0$, $(2)[(0.0730)(0.0015)] - (0.0015)^2(h)(9790) = 0$, $h = 0.00994$ m, or 9.94 mm.

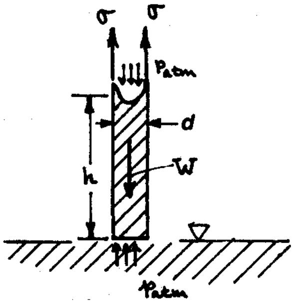


Fig. 1-19(c)

1.105 A glass tube is inserted in mercury (Fig. 1-20); the common temperature is 20 °C. What is the upward force on the glass as a result of surface effects?

▌ $F = (\sigma)(\pi d_o)(\cos 50°) + (\alpha)(\pi d_i)(\cos 50°) = (0.514)[(\pi)(0.035)](\cos 50°) + (0.514)[(\pi)(0.025)](\cos 50°) = 0.0623$ N

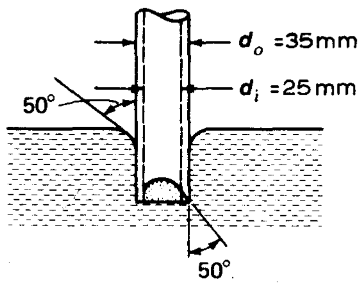


Fig. 1-20

1.106 In Fig. 1-21*a* estimate the depression *h* for mercury in the glass capillary tube. Angle θ is 40°.

▌ Consider the meniscus of the mercury as a free body (see Fig. 1-21*b*) of negligible weight. Summing forces in the vertical direction gives $-(\sigma)(\pi d)(\cos\theta) + (p)(\pi d^2/4) = 0$, $-(0.514)[(\pi)(0.002)](\cos 40°) + [(13.6)(9790)(h)][(\pi)(0.002)^2/4] = 0$, $h = 0.00591$ m, or 5.91 mm. Actual *h* must be larger because the weight of the meniscus was neglected.

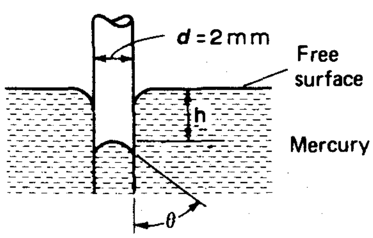


Fig. 1-21(*a*)

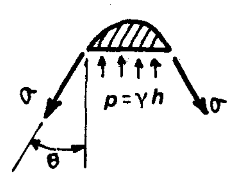


Fig. 1-21(*b*)

1.107 A narrow trough (Fig. 1-22) is filled with water at 20 °C to the maximum extent. If the gage measures a gage pressure of 2.8458 kPa, what is the radius of curvature of the water surface (away from the ends)?

▌
$$p = \sigma/r = p_{\text{gage}} - \gamma d = 2845.8 - (9790)(0.290) = 6.70 \text{ Pa gage}$$
$$6.70 = 0.0728/r \qquad r = 0.01087 \text{ m} \quad \text{or} \quad 10.87 \text{ mm}$$

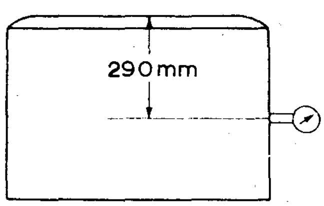


Fig. 1-22

1.108 Water at 10 °C is poured into a region between concentric cylinders until water appears above the top of the open end (see Fig. 1-23). If the pressure measured by a gage 42 cm below the open end is 4147.38 Pa gage, what is the curvature of the water at the top?

▌
$$p = \sigma/r = p_{\text{gage}} - \gamma d = 4147.38 - (9810)(0.42) = 27.18 \text{ Pa gage}$$
$$27.18 = 0.0742/r \qquad r = 0.00273 \text{ m} \quad \text{or} \quad 2.73 \text{ mm}$$

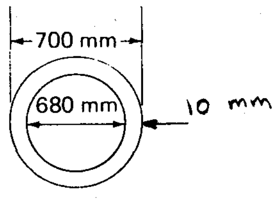


Fig. 1-23

1.109 The rate of twist α of a shaft of any shape may be found by using *Prandtl's soap-film analogy*. A soap film is attached to a sharp edge having the shape of the outside boundary of the shaft cross section (a rectangle here, as shown in Fig. 1-24). Air pressure is increased under the film so that it forms an elevated curved surface above the boundary. Then

$$\alpha = \frac{M_x \Delta p}{4\sigma G V} \qquad \text{(radians per unit length)}$$

where Δp = gage air pressure under the soap film, M_x = torque transmitted by actual shaft, G = shear modulus of actual shaft, and V = volume of air under the soap film and above the cross section formed by the sharp edge. For the case at hand, $\Delta p = 0.4\ \text{lb/ft}^2$ gage and $V = 0.5\ \text{in}^3$. The angle θ along the long edge of the cross section is measured optically to be 30°. For a torque of 600 lb · ft on a shaft having $G = 10 \times 10^6\ \text{lb/in}^2$, what angle of twist does this analogy predict?

▌ $$\alpha = \frac{M_x \Delta p}{4\sigma G V}$$

To get σ, consider a unit length of the long side of the shaft cross section away from the ends (see Fig. 1-24*c*). For equilibrium of the film in the vertical direction (remembering there are two surfaces on each side) $(-4)[(\sigma)(L)(\cos\theta)] + pA = 0$, $(-4)[(\sigma)(\frac{1}{12})(\cos 30°)] + (0.4)[(0.5)(1)/144] = 0$, $\sigma = 0.00481\ \text{lb/ft}$;

$$\alpha = \frac{(600)(0.4)}{(4)(0.00481)[(10\times10^6)(144)](0.5/1728)} = 0.0299\ \text{rad/ft}$$

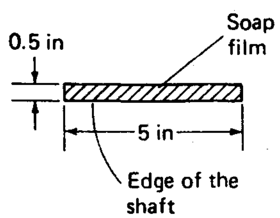


Fig. 1-24(*a*)

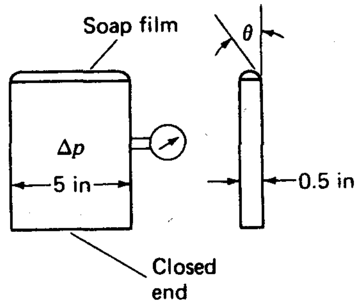


Fig. 1-24(*b*)

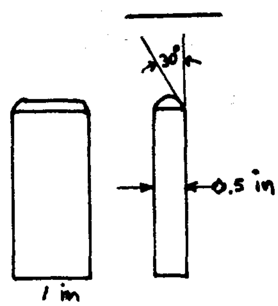


Fig. 1-24(*c*)

1.110 In using Prandtl's soap-film analogy (see Prob. 1.109), we wish to check the mechanism for measuring the pressure Δp under the soap film. Accordingly, we use a circular cross section (Fig. 1-25) for which we have an accurate theory for determining the rate of twist α. The surface tension for the soap film is 0.1460 N/m and volume V under the film is measured to be $0.001120\ \text{m}^3$. Compute Δp from consideration of the soap film and from solid mechanics using the equation given in Prob. 1.109 and the well-known formula from strength of materials

$$\alpha = \frac{M_x}{GJ}$$

where J, the polar moment of inertia, is $\pi r^4/2$. Compare the results.

▮ From consideration of the film (see Fig. 1-25), $-2\sigma\pi\, d \cos 45° + (\Delta p)(\pi d^2)/4 = 0$, $-(2)(0.1460)(\pi)(\frac{200}{1000})(\cos 45°) + (\Delta p)[(\pi)(\frac{200}{1000})^2/4] = 0$, $\Delta p = 4.13$ Pa gage. From strength of materials, equate α's for the equations given in this problem and in Prob. 1.109.

$$\frac{M_x \Delta p}{4\sigma GV} = \frac{M_x}{GJ} \qquad J = \frac{\pi[(\frac{200}{2})/1000]^4}{2} = 0.0001571\ \text{m}^4 \qquad \frac{\Delta p}{(4)(0.1460)(0.001120)} = \frac{1}{0.0001571} \qquad \Delta p = 4.16\ \text{Pa gage}$$

The pressure measurement is quite close to what is expected from theory.

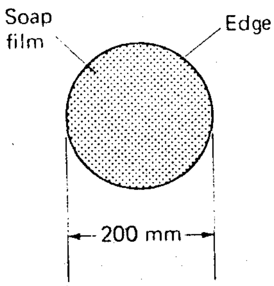


Fig. 1-25(*a*)

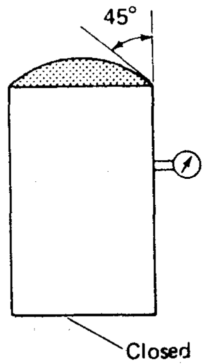


Fig. 1-25(*b*)

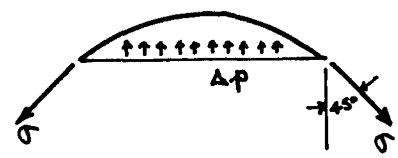


Fig. 1-25(*c*)

1.111 Find the capillary rise in the tube shown in Fig. 1-26 for a water–air–glass interface ($\theta = 0°$) if the tube radius is 1 mm and the temperature is 20 °C.

▮
$$h = \frac{2\sigma \cos\theta}{\rho g r} = \frac{(2)(0.0728)(\cos 0°)}{(1000)(9.81)(\frac{1}{1000})} = 0.0148\ \text{m} \quad \text{or} \quad 14.8\ \text{mm}$$

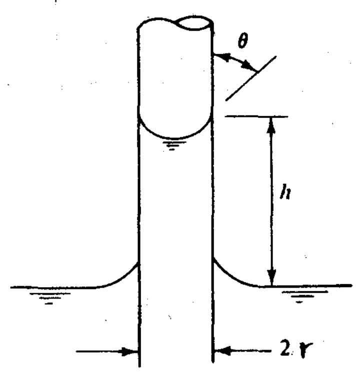


Fig. 1-26

1.112 Find the capillary rise in the tube shown in Fig. 1-26 for a mercury–air–glass interface with $\theta = 130°$ if the tube radius is 1 mm and the temperature is 20 °C.

▮
$$h = \frac{2\sigma \cos\theta}{\rho g r} = \frac{(2)(0.514)(\cos 130°)}{(13\,570)(9.81)(\frac{1}{1000})} = -0.0050\ \text{m} \quad \text{or} \quad -5.0\ \text{mm}$$

1.113 If a bubble is equivalent to an air–water interface with $\sigma = 0.005$ lb/ft, what is the pressure difference between the inside and outside of a bubble of diameter 0.003 in?

▮
$$p = 2\sigma/r = (2)(0.005)/[(0.003/2)/12] = 80.0\ \text{lb/ft}^2$$

1.114 A small circular jet of mercury 200 μm in diameter issues from an opening. What is the pressure difference between the inside and outside of the jet at 20 °C?

▮ See Fig. 1-27. Equating the force due to surface tension ($2\sigma L$) and the force due to pressure (pDL), $2\sigma L = pDL$, $p = 2\sigma/D = (2)(0.514)/(200 \times 10^{-6}) = 5140$ Pa.

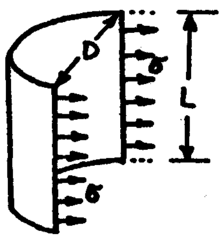


Fig. 1-27

1.115 The surface tensions of mercury and water at 60 °C are 0.47 N/m and 0.0662 N/m, respectively. What capillary-height changes will occur in these two fluids when they are in contact with air in a glass tube of radius 0.30 mm? Use $\theta = 130°$ for mercury, and 0° for water; $\gamma = 132.3\ \mathrm{kN/m^3}$ for mercury, and $9.650\ \mathrm{kN/m^3}$ for water.

▌
$$h = \frac{2\sigma \cos\theta}{\rho g r}$$

For mercury:

$$h = \frac{(2)(0.47)(\cos 130°)}{(132\,300)(0.30/1000)} = -0.0152\ \mathrm{m} \quad \text{or} \quad -15.2\ \mathrm{mm}$$

For water:

$$h = \frac{(2)(0.0662)(\cos 0°)}{(9650)(0.30/1000)} = 0.0457\ \mathrm{m} \quad \text{or} \quad 45.7\ \mathrm{mm}$$

1.116 At 30 °C what diameter glass tube is necessary to keep the capillary-height change of water less than 2 mm?

▌
$$h = \frac{2\sigma\cos\theta}{\rho g r} \qquad \frac{2}{1000} = \frac{(2)(0.0712)(\cos 0°)}{(996)(9.81)(r)}$$

$$r = 0.00729\ \mathrm{m} \quad \text{or} \quad 7.29\ \mathrm{mm} \qquad d = (2)(7.29) = 14.6\ \mathrm{mm}\ \text{(or greater)}$$

1.117 A 1-in-diameter soap bubble has an internal pressure $0.0045\ \mathrm{lb/in^2}$ greater than that of the outside atmosphere. Compute the surface tension of the soap–air interface. Note that a soap bubble has two interfaces with air, an inner and outer surface of nearly the same radius.

▌
$$p = 4\sigma/r \qquad (0.0045)(144) = (4)(\sigma)/[(\tfrac{1}{2})/12] \qquad \sigma = 0.00675\ \mathrm{lb/ft}$$

1.118 What force is required to lift a thin wire ring 6 cm in diameter from a water surface at 20 °C?

▌ Neglecting the weight of the wire, $F = \sigma L$. Since there is resistance on the inside and outside of the ring, $F = (2)(\sigma)(\pi d) = (2)(0.0728)[(\pi)(0.06)] = 0.0274\ \mathrm{N}$.

1.119 The glass tube in Fig. 1-28 is used to measure pressure p_1 in the water tank. The tube diameter is 1 mm and the water is at 30 °C. After correcting for surface tension, what is the true water height in the tube?

▌
$$h = \frac{2\sigma\cos\theta}{\rho g r} = \frac{(2)(0.0712)(\cos 0°)}{(996)(9.81)[(\tfrac{1}{2})/1000]} = 0.029\ \mathrm{m} \quad \text{or} \quad 2.9\ \mathrm{cm}$$

True water height in the tube = 17 − 2.9 = 14.1 cm.

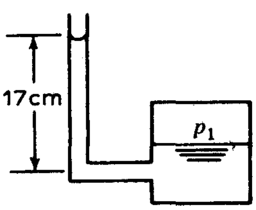


Fig. 1-28

1.120 An atomizer forms water droplets 45 μm in diameter. Find the excess pressure within these droplets for water at 30 °C?

$$p = 2\sigma/r = (2)(0.0712)/[(45 \times 10^{-6})/2] = 6329 \text{ Pa}$$

1.121 Rework Prob. 1.120 for droplets of 0.0018 in diameter and at 68 °F.

$$p = 2\sigma/r = (2)(0.005)/[(0.0018/2)/12] = 133 \text{ lb/ft}^2 \quad \text{or} \quad 0.93 \text{ lb/in}^2$$

1.122 What is the pressure difference between the inside and outside of a cylindrical water jet when the diameter is 2.2 mm and the temperature is 10 °C? (See Fig. 1-27.)

$$p = \sigma/r = 0.0742/0.0011 = 67.5 \text{ Pa}$$

1.123 Find the angle the surface tension film leaves the glass for a vertical tube immersed in water if the diameter is 0.25 in and the capillary rise is 0.08 in. Use $\sigma = 0.005$ lb/ft.

$$h = \frac{2\sigma \cos\theta}{\rho g r} \qquad \frac{0.08}{12} = \frac{(2)(0.005)(\cos\theta)}{(1.94)(32.2)[(0.25/2)/12]} \qquad \cos\theta = 0.433806 \qquad \theta = 64.3°$$

1.124 Develop a formula for capillary rise between two concentric glass tubes of radii r_o and r_i and contact angle θ.

See Fig. 1-29. Equating the force due to pressure and the force due to surface tension,

$$(h)(\gamma)(\pi r_o^2 - \pi r_i^2) = \sigma(2\pi r_i + 2\pi r_o)(\cos\theta)$$

$$h = \frac{(2)(\sigma)(r_i + r_o)(\cos\theta)}{\gamma(r_o^2 - r_i^2)} = \frac{2\sigma\cos\theta}{\gamma(r_o - r_i)}$$

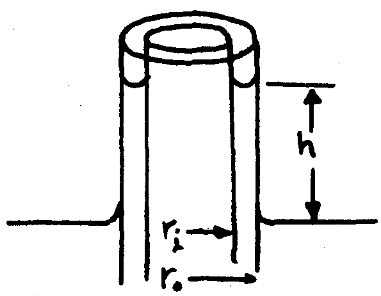


Fig. 1-29

1.125 Distilled water at 10 °C stands in a glass tube of 9.0-mm diameter at a height of 24.0 mm. What is the true static height?

$$h = \frac{2\sigma\cos\theta}{\rho g r} = \frac{(2)(0.0742)(\cos 0°)}{(1000)(9.81)[(9.0/2)/1000]} = 0.0034 \text{ m} \quad \text{or} \quad 3.4 \text{ mm}$$

True static height = 24.0 − 3.4 = 20.6 mm.

1.126 What capillary depression of mercury ($\theta = 140°$) may be expected in a 0.08-in-diameter tube at 68 °F?

$$h = \frac{2\sigma\cos\theta}{\rho g r} = \frac{(2)(0.0352)(\cos 140°)}{(26.34)(32.2)[(0.08/2)/12]} = -0.01908 \text{ ft} \quad \text{or} \quad -0.23 \text{ in}$$

1.127 At the top of Mount Olympus water boils at 85 °C. Approximately how high is the mountain?

From Table A-2, water boiling at 85 °C corresponds to a vapor pressure of 58.8 kPa. From Table A-8, this corresponds to a standard atmosphere elevation of approximately 4200 m.

1.128 At approximately what temperature will water boil at an elevation of 12 500 ft?

▎ From Table A-7, the pressure of the standard atmosphere at 12 500-ft evevation is 9.205 psia, or 1326 lb/ft^2 abs. From Table A-1, the saturation pressure of water is 1326 lb/ft^2 abs at about 189 °F. Hence, the water will boil at 193 °F; this explains why it takes longer to cook at high altitudes.

1.129 At approximately what temperature will water boil in Denver (elevation 5280 ft)?

▎ From Table A-7, the pressure of the standard atmosphere at 5280-ft elevation is 12.12 psia, or 1745 lb/ft^2 abs. From Table A-1, the saturation pressure of water is 1745 lb/ft^2 abs at about 202 °F. Hence, the water will boil at 198 °F.

1.130 Water at 105 °F is placed in a beaker within an airtight container. Air is gradually pumped out of the container. What reduction below standard atmospheric pressure of 14.7 psia must be achieved before the water boils?

▎ From Table A-1, $p_v = 162$ lb/ft^2 abs, or 1.12 psia at 105 °F. Hence, pressure must be reduced by $14.7 - 1.12$, or 13.58 psi.

1.131 At what pressure will 50 °C water boil?

▎ From Table A-2, $p_v = 12.3$ kPa at 50 °C. Hence, water will boil at 12.3 kPa.

1.132 At what pressure will cavitation occur at the inlet of a pump that is drawing water at 25 °C?

▎ Cavitation occurs when the internal pressure drops to the vapor pressure. From Table A-2, the vapor pressure of water at 25 °C is 3.29 kPa.

1.133 For low-speed (laminar) flow through a circular pipe, as shown in Fig. 1-30, the velocity distribution takes the form $v = (B/\mu)(r_0^2 - r^2)$, where μ is the fluid viscosity. What are the units of the constant B?

▎ Dimensionally, $(L/T) = [\{B\}/(M/LT)](L^2)$, $\{B\} = ML^{-2}T^{-2}$. In SI units, B could be kg/(m$^2\cdot$s^2), or Pa/m.

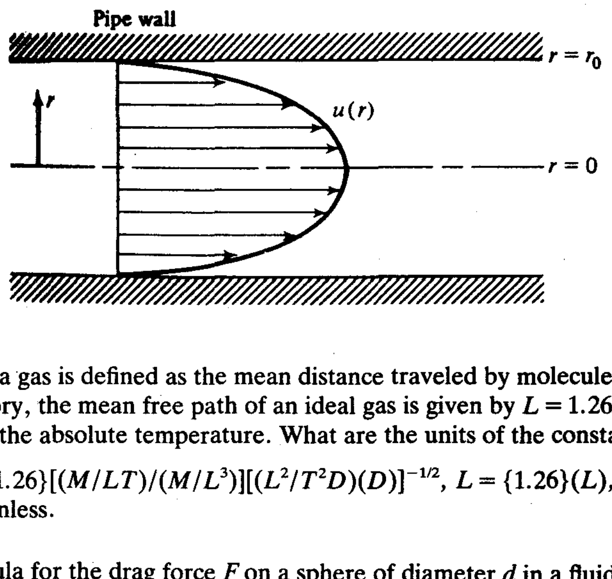


Fig. 1-30

1.134 The mean free path L of a gas is defined as the mean distance traveled by molecules between collisions. According to kinetic theory, the mean free path of an ideal gas is given by $L = 1.26(\mu/\rho)(RT)^{-1/2}$, where R is the gas constant and T is the absolute temperature. What are the units of the constant 1.26?

▎ Dimensionally, $L = \{1.26\}[(M/LT)/(M/L^3)][(L^2/T^2D)(D)]^{-1/2}$, $L = \{1.26\}(L)$, $\{1.26\} = 1$. Therefore, the constant 1.26 is dimensionless.

1.135 The Stokes–Oseen formula for the drag force F on a sphere of diameter d in a fluid stream of low velocity v is $F = 3\pi\mu dv + (9\pi/16)(\rho v^2 d^2)$. Is this formula dimensionally consistent?

▎ Dimensionally, $(F) = (1)(M/LT)(L)(L/T) + (1)(M/L^3)(L/T)^2(L)^2 = (ML/T^2) + (ML/T^2) = (F) + (F)$. Therefore, the formula is dimensionally consistent.

1.136 The speed of propagation C of waves traveling at the interface between two fluids is given by $C = (\pi\sigma/\rho_a\lambda)^{1/2}$, where λ is the wavelength and ρ_a is the average density of the two fluids. If the formula is dimensionally consistent, what are the units of σ? What might it represent?

▎ Dimensionally, $(L/T) = [(1)\{\sigma\}/(M/L^3)(L)]^{1/2} = [\{\sigma\}(L^2/M)]^{1/2}$, $\{\sigma\} = M/T^2 = F/L$. In SI units, σ could be N/m. (In this formula, σ is actually the surface tension.)

1.137 Is the following equation dimensionally homogeneous? $a = 2d/t^2 - 2v_0/t$, where a = acceleration, d = distance, v_0 = velocity, and t = time.

▌ $L/T^2 = (L)/(T^2) - (L/T)/(T) = (L/T^2) - (L/T^2)$. Therefore, the equation is homogeneous.

1.138 A popular formula in the hydraulics literature is the Hazen-Williams formula for volume flow rate Q in a pipe of diameter D and pressure gradient dp/dx: $Q = 61.9D^{2.63}(dp/dx)^{0.54}$. What are the dimensions of the constant 61.9?

▌ $$\frac{L^3}{T} = \{61.9\}(L)^{2.63}\left(\frac{M}{L^2T^2}\right)^{0.54} \qquad \{61.9\} = L^{1.45}T^{0.08}M^{-0.54}$$

CHAPTER 2
Fluid Statics

2.1 For the dam shown in Fig. 2-1, find the horizontal pressure acting at the face of the dam at 20-ft depth.

$$p = \gamma h = (62.4)(20) = 1248\ \text{lb/ft}^2$$

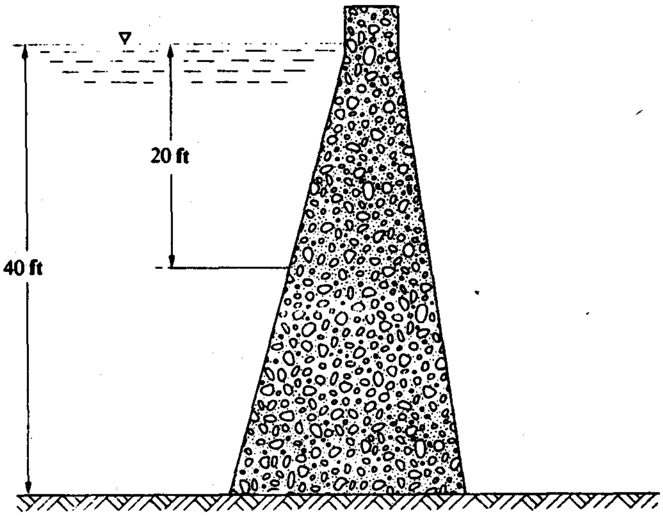


Fig. 2-1. Dam.

2.2 For the vessel containing glycerin under pressure as shown in Fig. 2-2, find the pressure at the bottom of the tank.

$$p = 50 + \gamma h = 50 + (12.34)(2.0) = 74.68\ \text{kN/m}^2 \quad \text{or} \quad 74.68\ \text{kPa}$$

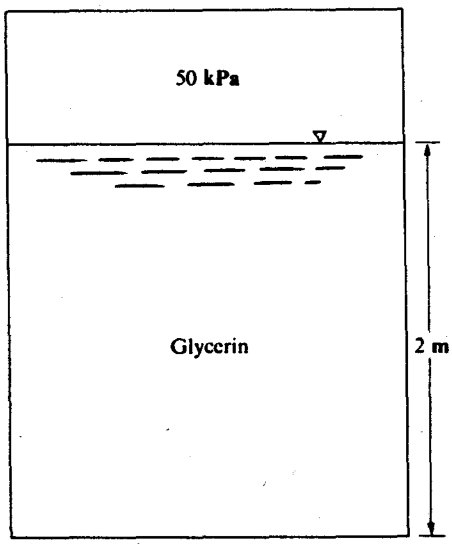


Fig. 2-2

2.3 If the pressure in a tank is 50 psi, find the equivalent pressure head of (*a*) water, (*b*) mercury, and (*c*) heavy fuel oil with a specific gravity of 0.92.

$$h = p/\gamma$$

(*a*) $h = [(50)(144)]/62.4 = 115.38\ \text{ft}$

(*b*) $h = [(50)(144)]/847.3 = 8.50\ \text{ft}$

(*c*) $h = [(50)(144)]/[(0.92)(62.4)] = 125.42\ \text{ft}$

2.4 A weather report indicates the barometric pressure is 29.75 in of mercury. What is the atmospheric pressure in pounds per square inch?

$$p = \gamma h = [(13.6)(62.4)][(29.75/12)]/144 = 14.61 \text{ lb/in}^2 \quad \text{or} \quad 14.61 \text{ psi}$$

2.5 Find the atmospheric pressure in kilopascals if a mercury barometer reads 742 mm.

$$p = \gamma h = (133.1)(\tfrac{742}{1000}) = 98.8 \text{ kN/m}^2 \quad \text{or} \quad 98.8 \text{ kPa}$$

2.6 A pressure gage 7.0 m above the bottom of a tank containing a liquid reads 64.94 kPa; another gage at height 4.0 m reads 87.53 kPa. Compute the specific weight and mass density of the fluid.

$$\gamma = \Delta p/\Delta h = (87.53 - 64.94)/(7.0 - 4.0) = 7.53 \text{ kN/m}^3 \quad \text{or} \quad 7530 \text{ N/m}^3$$

$$\rho = \gamma/g = 7530/9.81 = 786 \text{ kg/m}^3$$

2.7 A pressure gage 19.0 ft above the bottom of a tank containing a liquid reads 13.19 psi; another gage at height 14.0 ft reads 15.12 psi. Compute the specific weight, mass density, and specific gravity of the liquid.

$$\Delta p = \gamma(\Delta h) \qquad (15.12 - 13.19)(144) = (\gamma)(19.0 - 14.0) \qquad \gamma = 55.6 \text{ lb/ft}^3$$

$$\rho = \gamma/g = 55.6/32.2 = 1.73 \text{ slug/ft}^3 \qquad \text{s.g.} = 55.6/62.4 = 0.891$$

2.8 An open tank contains 5.7 m of water covered with 2.8 m of kerosene ($\gamma = 8.0 \text{ kN/m}^3$). Find the pressure at the interface and at the bottom of the tank.

$$p_{int} = \gamma h = (8.0)(2.8) = 22.4 \text{ kPa}$$

$$p_{bot} = 22.4 + (9.79)(5.7) = 78.2 \text{ kPa}$$

2.9 An open tank contains 9.4 ft of water beneath 1.8 ft of oil (s.g. = 0.85). Find the pressure at the interface and at the bottom of the tank.

$$p_{int} = \gamma h = [(0.85)(62.4)](1.8)/144 = 0.663 \text{ psi}$$

$$p_{bot} = 0.663 + (62.4)(9.4)/144 = 4.74 \text{ psi}$$

2.10 If air had a constant specific weight of 0.076 lb/ft^3 and were incompressible, what would be the height of the atmosphere if sea-level pressure were 14.92 psia?

$$h = p/\gamma = (14.92)(144)/0.076 = 28\,270 \text{ ft}$$

2.11 If the weight density of mud is given by $\gamma = 65.0 + 0.2h$, where γ is in lb/ft^3 and depth h is in ft, determine the pressure, in psi, at a depth of 17 ft.

$dp = \gamma\, dh = (65.0 + 0.2h)\, dh$. Integrating both sides: $p = 65.0h + 0.1h^2$. For $h = 17$ ft: $p = (65.0)(17)/144 + (0.1)(17)^2/144 = 7.87$ psi.

2.12 If the absolute pressure in a gas is 40.0 psia and the atmospheric pressure is 846 mbar abs, find the gage pressure in (*a*) lb/in^2; (*b*) kPa; (*c*) bar.

(*a*) $p_{atm} = (846)(0.0145) = 12.3 \text{ lb/in}^2 \qquad p_{gage} = 40.0 - 12.3 = 27.7 \text{ lb/in}^2$

(*b*) $p_{abs} = (40.0)(6.894) = 276 \text{ kPa} \qquad p_{atm} = (846)(0.100) = 85 \text{ kPa} \qquad p_{gage} = 276 - 85 = 191 \text{ kPa}$

(*c*) $p_{abs} = 40.0/14.5 = 2.759 \text{ bar} \qquad p_{gage} = 2.759 - 0.846 = 1.913 \text{ bar}$

2.13 If the atmospheric pressure is 0.900 bar abs and a gage attached to a tank reads 390 mmHg vacuum, what is the absolute pressure within the tank?

$$p = \gamma h \qquad p_{atm} = 0.900 \times 100 = 90.0 \text{ kPa}$$

$$p_{gage} = [(13.6)(9.79)](\tfrac{390}{1000}) = 51.9 \text{ kPa vacuum} \quad \text{or} \quad -51.9 \text{ kPa}$$

$$p_{abs} = 90.0 + (-51.9) = 38.1 \text{ kPa}$$

2.14 If atmospheric pressure is 13.99 psia and a gage attached to a tank reads 7.4 inHg vacuum, find the absolute pressure within the tank.

▮ $p = \gamma h \qquad p_{\text{gage}} = [(13.6)(62.4)][(7.4/12)/144] = 3.63 \text{ psi vacuum} \quad \text{or} \quad -3.63 \text{ psi}$

$$p_{\text{abs}} = 13.99 + (-3.63) = 10.36 \text{ psia}$$

2.15 The closed tank in Fig. 2-3 is at 20 °C. If the pressure at point A is 98 kPa abs, what is the absolute pressure at point B? What percent error results from neglecting the specific weight of the air?

▮ $p_A + \gamma_{\text{air}} h_{AC} - \gamma_{H_2O} h_{DC} - \gamma_{\text{air}} h_{DB} = p_B$, $98 + (0.0118)(5) - (9.790)(5-3) - (0.0118)(3) = p_B = 78.444$ kPa. Neglecting air, $p_B = 98 - (9.790)(5-3) = 78.420$ kPa; error $= (78.444 - 78.420)/78.444 = 0.00031$, or 0.031%.

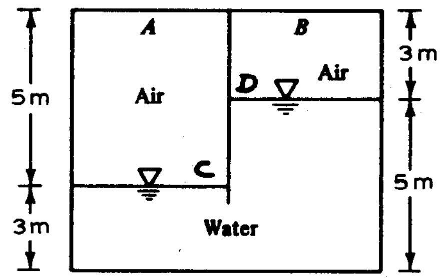


Fig. 2-3

2.16 The system in Fig. 2-4 is at 70 °F. If the pressure at point A is 2900 lb/ft², determine the pressures at points B, C, and D.

▮ $p_B = 2900 - (62.4)(4-3) = 2838 \text{ lb/ft}^2 \qquad p_D = 2900 + (62.4)(6) = 3274 \text{ lb/ft}^2$

$$p_C = 2900 + (62.4)(6-2) - (0.075)(5+3) = 3149 \text{ lb/ft}^2$$

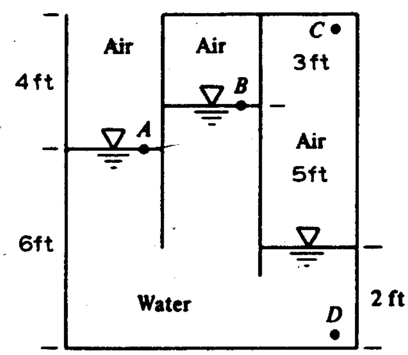


Fig. 2-4

2.17 The system in Fig. 2-5 is at 20 °C. If atmospheric pressure is 101.03 kPa and the absolute pressure at the bottom of the tank is 231.3 kPa, what is the specific gravity of olive oil?

▮ $101.03 + (0.89)(9.79)(1.5) + (9.79)(2.5) + (\text{s.g.})(9.79)(2.9) + (13.6)(9.79)(0.4) = 231.3 \qquad \text{s.g.} = 1.39$

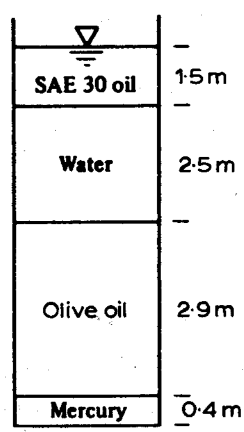


Fig. 2-5

2.18 Find the pressures at A, B, C, and D in Fig. 2-6.

▌ $p_A = (62.4)(4+2) = 374\ \text{lb/ft}^2$, $p_B = -(62.4)(2) = -125\ \text{lb/ft}^2$. Neglecting air, $p_C = p_B = -125\ \text{lb/ft}^2$; $p_D = -125 - (62.4)(4+2+2) = -624\ \text{lb/ft}^2$.

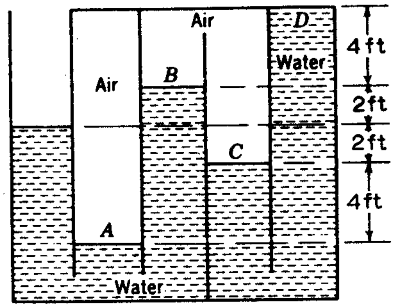


Fig. 2-6

2.19 The tube shown in Fig. 2-7 is filled with oil. Determine the pressure heads at A and B in meters of water.

▌ $(h_{H_2O})(\gamma_{H_2O}) = (h_{oil})(\gamma_{oil}) = (h_{oil})[(\text{s.g.}_{oil})(\gamma_{H_2O})]$; therefore, $h_{H_2O} = (h_{oil})(\text{s.g.}_{oil})$. Thus, $h_A = -(2.2+0.6)(0.85) = -2.38\ \text{m}\ H_2O$ and $h_B = (-0.6)(0.85) = -0.51\ \text{m}\ H_2O$.

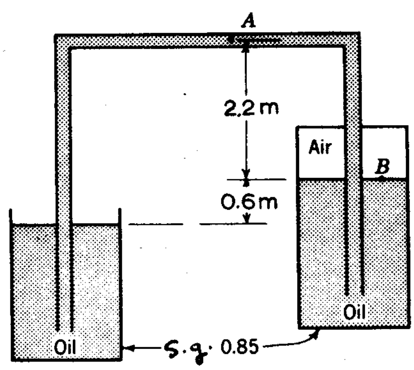


Fig. 2-7

2.20 Calculate the pressure, in kPa, at A, B, C, and D in Fig. 2-8.

▌ $p_A = -(0.4+0.4)(9.790) = -7.832\ \text{kPa}$; $p_B = (0.5)(9.790) = 4.895\ \text{kPa}$. Neglecting air, $p_C = p_B = 4.895\ \text{kPa}$; $p_D = 4.895 + (0.9)(9.790)(1+0.5+0.4) = 21.636\ \text{kPa}$.

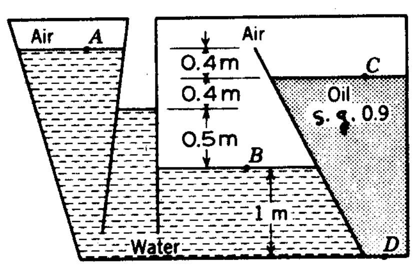


Fig. 2-8

2.21 Convert 9 psi to **(a)** inches of mercury, **(b)** feet of water, **(c)** feet of ichor (s.g. = 2.94).

▌ **(a)** $h = p/\gamma = [(9)(144)]/[(13.6)(62.4)] = 1.527\ \text{ft}$, or 18.33 inHg
(b) $h = [(9)(144)]/62.4 = 20.77$ ft of water
(c) $h = [(9)(144)]/[(2.94)(62.4)] = 7.06$ ft ichor

2.22 Express an absolute pressure of 5 atm in meters of water gage when the barometer reads 760 mmHg.

▌ $$p_{abs} = (5)(101.3)/9.79 = 51.74\ \text{m of water} \qquad p_{atm} = (0.760)(13.6) = 10.34\ \text{m of water}$$

$$p_{gage} = 51.74 - 10.34 = 41.40\ \text{m of water}$$

2.23 Figure 2-9 shows one pressurized tank inside another. If the sum of the readings of Bourdan gages A and B is 34.1 psi, and an aneroid barometer reads 29.90 inHg, what is the absolute pressure at A, in inHg?

▮ $$h = p/\gamma \qquad h_A + h_B = 34.1/[(13.6)(62.4)/(12)^3] = 69.44 \text{ inHg}$$

$$(h_A)_{abs} = 29.90 + 69.44 = 99.34 \text{ inHg}$$

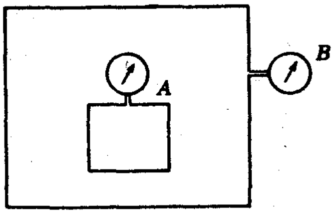


Fig. 2-9

2.24 Determine the heights of columns of water, kerosene (ker), and nectar (s.g. = 2.94) equivalent to 277 mmHg.

▮ $$(h_{Hg})(\gamma_{Hg}) = (H_{H_2O})(\gamma_{H_2O}) = (h_{ker})(\gamma_{ker}) = (h_{nectar})(\gamma_{nectar})$$

$$0.277[(13.6)(9.79)] = (h_{H_2O})(9.79) \qquad h_{H_2O} = 3.77 \text{ m}$$

$$0.277[(13.6)(9.79)] = (h_{ker})[(0.82)(9.79)] \qquad h_{ker} = 4.59 \text{ m}$$

$$0.277[(13.6)(9.79)] = (h_{nectar})[(2.94)(9.79)] \qquad h_{nectar} = 1.28 \text{ m}$$

2.25 In Fig. 2-10, if $h = 25.5$ in, determine the pressure at A. The liquid has a specific gravity of 1.85.

▮ $$p = \gamma h = [(1.85)(62.4)][25.5/12] = 245.3 \text{ lb/ft}^2 \quad \text{or} \quad 1.70 \text{ psi}$$

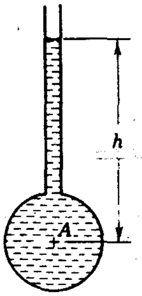


Fig. 2-10

2.26 For the pressure vessel containing glycerin, with piezometer attached, as shown in Fig. 2-11, what is the pressure at point A?

▮ $$p = \gamma h = [(1.26)(62.4)](40.8/12) = 267 \text{ lb/ft}^2$$

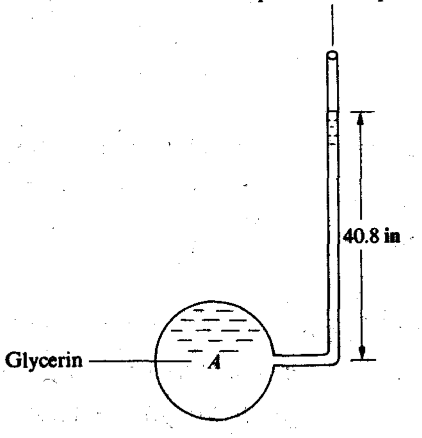


Fig. 2-11

2.27 For the open tank, with piezometers attached on the side, containing two different immiscible liquids, as shown in Fig. 2-12, find the (*a*) elevation of the liquid surface in piezometer A, (*b*) elevation of the liquid surface in piezometer B, and (*c*) total pressure at the bottom of the tank.

I (*a*) Liquid A will simply rise in piezometer A to the same elevation as liquid A in the tank (i.e., to elevation 2 m). (*b*) Liquid B will rise in piezometer B to elevation 0.3 m (as a result of the pressure exerted by liquid B) plus an additional amount as a result of the overlying pressure of liquid A. The overlying pressure can be determined by $p = \gamma h = [(0.72)(9.79)](2-0.3) = 11.98\ \text{kN/m}^2$. The height liquid B will rise in piezometer B as a result of the overlying pressure of liquid A can be determined by $h = p/\gamma = 11.98/[(2.36)(9.79)] = 0.519\ \text{m}$. Hence, liquid B will rise in piezometer B to an elevation of 0.3 m + 0.519 m, or 0.819 m.
(*c*) $p_{\text{bottom}} = [(0.72)(9.79)](2-0.3) + [(2.36)(9.79)](0.3) = 18.9\ \text{kPa}$.

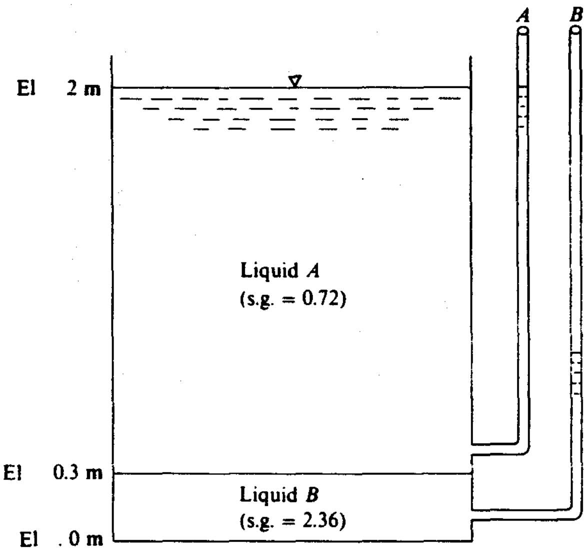


Fig. 2-12

2.28 The air–oil–water system shown in Fig. 2-13 is at 70 °F. If gage A reads 16.1 lb/in² abs and gage B reads 2.00 lb/in² less than gage C, compute (*a*) the specific weight of the oil and (*b*) the reading of gage C.

I (*a*) $(16.1)(144) + (0.0750)(3) + (\gamma_{\text{oil}})(2) = p_B$, $p_B + (\gamma_{\text{oil}})(2) + (62.4)(3) = p_C$. Since $p_C - p_B = 2.00$, $(\gamma_{\text{oil}})(2) + (62.4)(3) = (2.00)(144)$, $\gamma_{\text{oil}} = 50.4\ \text{lb/ft}^3$. (*b*) $(16.1)(144) + (0.0750)(3) + (50.4)(2) = p_B$, $p_B = 2419\ \text{lb/ft}^2$; $p_C = 2419 + (2.00)(144) = 2707\ \text{lb/ft}^2$, or $18.80\ \text{lb/in}^2$.

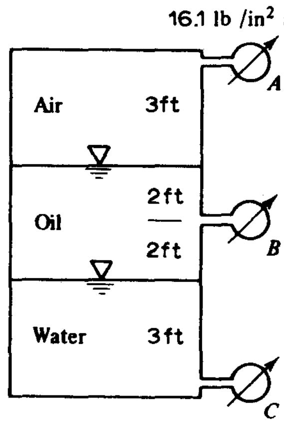


Fig. 2-13

2.29 For a gage reading at A of -2.50 psi, determine the (*a*) elevations of the liquids in the open piezometer columns E, F, and G and (*b*) deflection of the mercury in the U-tube gage in Fig. 2-14. Neglect the weight of the air.

I (*a*) The liquid between the air and the water would rise to elevation 49.00 ft in piezometer column E as a result of its weight. The actual liquid level in the piezometer will be lower, however, because of the vacuum in the air above the liquid. The amount the liquid level will be lowered (h in Fig. 2-14) can be determined by

$(-2.50)(144)+[(0.700)(62.4)](h)=0$, $h=8.24$ ft. Elevation at $L=49.00-8.24=40.76$ ft; $(-2.50)(144)+[(0.700)(62.4)][49.00-38.00]=p_M$, $p_M=120.5\ \text{lb/ft}^2$. Hence, pressure head at $M=120.5/62.4=1.93$ ft of water. Elevation at $N=38.00+1.93=39.93$ ft; $120.5+(62.4)(38.00-26.00)=p_O$, $p_O=869.3\ \text{lb/ft}^2$. Hence, pressure head at $O=869.3/[(1.600)(62.4)]=8.71$ ft (of the liquid with s.g. = 1.600). Elevation at $Q=26.00+8.71=34.71$ ft. **(b)** $869.3+(62.4)(26.00-14.00)-[(13.6)(62.4)](h_1)=0$, $h_1=1.91$ ft.

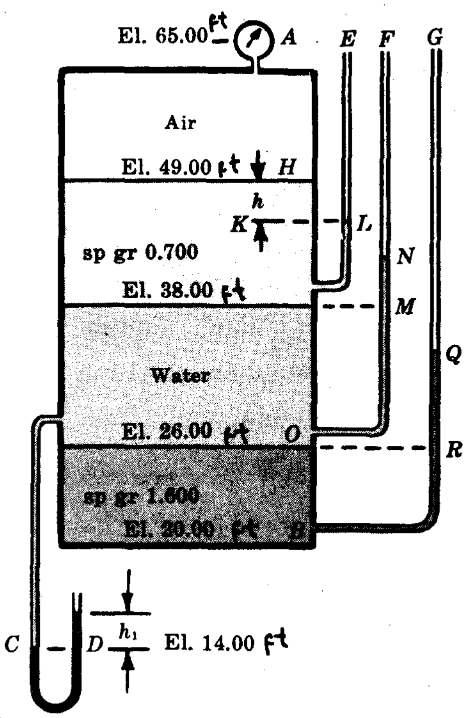


Fig. 2-14

2.30 A vessel containing oil under pressure is shown in Fig. 2-15. Find the elevation of the oil surface in the attached piezometer.

I Elevation of oil surface in piezometer $=2+35/[(0.83)(9.79)]=6.31$ m

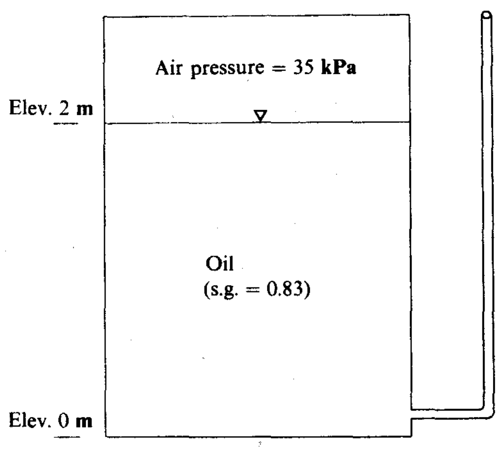


Fig. 2-15

2.31 The reading of an automobile fuel gage is proportional to the gage pressure at the bottom of the tank (Fig. 2-16). If the tank is 32 cm deep and is contaminated with 3 cm of water, how many centimeters of air remains at the top when the gage indicates "full"? Use $\gamma_{\text{gasoline}}=6670\ \text{N/m}^3$ and $\gamma_{\text{air}}=11.8\ \text{N/m}^3$.

I When full of gasoline, $p_{\text{gage}}=(6670)(0.32)=2134$ Pa. With water added, $2134=(9790)(0.03)+(6670)[(0.32-0.03)-h]+(11.8)(h)$, $h=0.0141$ m, or 1.41 cm.

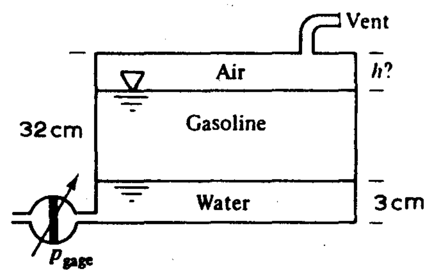


Fig. 2-16

2.32 The hydraulic jack shown in Fig. 2-17 is filled with oil at 55 lb/ft^3. Neglecting the weight of the two pistons, what force F on the handle is required to support the 2200-lb weight?

▌ The pressure against the large and the small piston is the same. $p = W/A_{\text{large}} = 2200/[\pi(\frac{3}{12})^2/4] =$ 44 818 lb/ft^2. Let P be the force from the small piston onto the handle. $P = pA_{\text{small}} = (44\,818)[\pi(\frac{1}{12})^2/4] = 244$ lb. For the handle, $\Sigma M_A = 0 = (16+1)(F) - (1)(244)$, $F = 14.4$ lb.

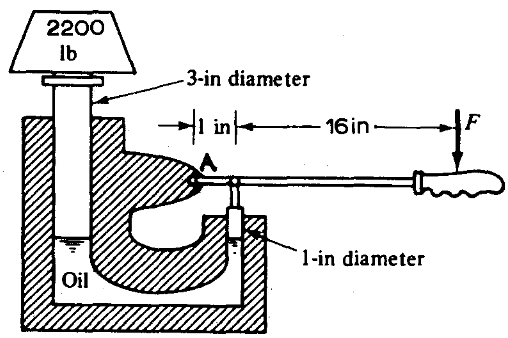


Fig. 2-17

2.33 Figure 2-18 shows a setup with a vessel containing a plunger and a cylinder. What force F is required to balance the weight of the cylinder if the weight of the plunger is negligible?

▌ $$10\,000/500 - [(0.78)(62.4)](15)/144 = F/5 \qquad F = 74.6 \text{ lb}$$

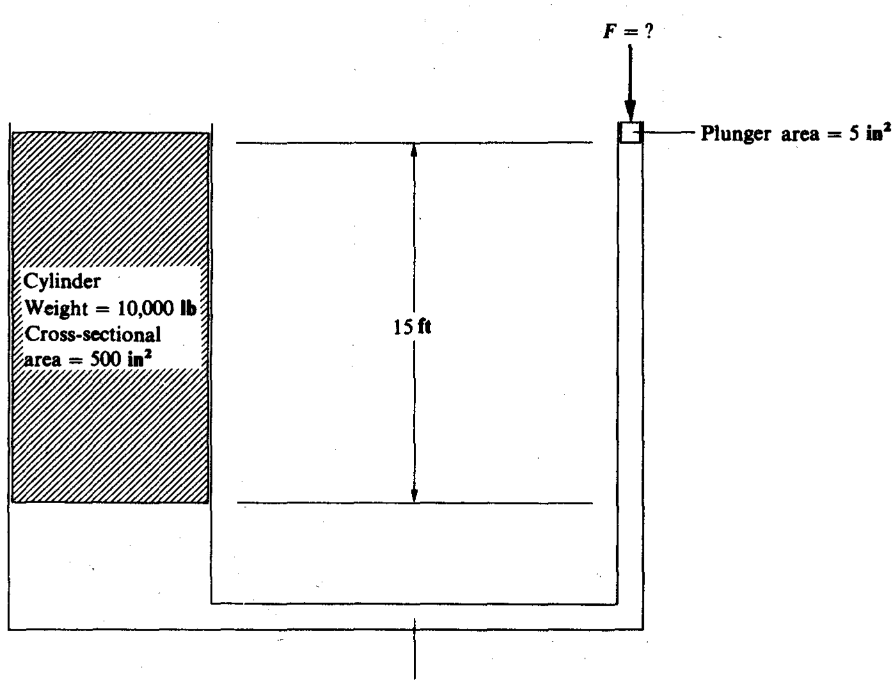


Fig. 2-18

2.34 For the vertical pipe with manometer attached, as shown in Fig. 2-19, find the pressure in the oil at point A.

▌ $$p_A + [(0.91)(62.4)](7.22) - [(13.6)(62.4)](1.00) = 0 \qquad p_A = 438.7 \text{ lb/ft}^2 \quad \text{or} \quad 3.05 \text{ lb/in}^2$$

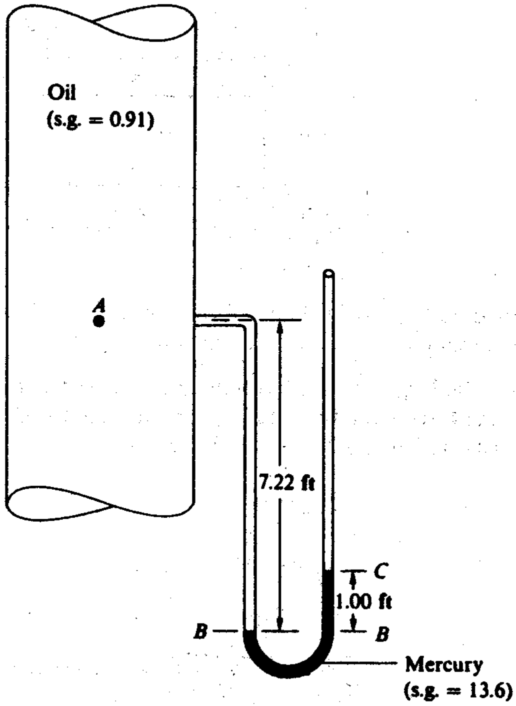


Fig. 2-19

2.35 A monometer is attached to a tank containing three different fluids, as shown in Fig. 2-20. What will be the difference in elevation of the mercury column in the manometer (i.e., y in Fig. 2-20)?

$$30+[(0.82)(9.79)](5-2)+(9.79)(2-0)+(9.79)(1.00)-[(13.6)(9.79)]y=0 \qquad y=0.627\text{ m}$$

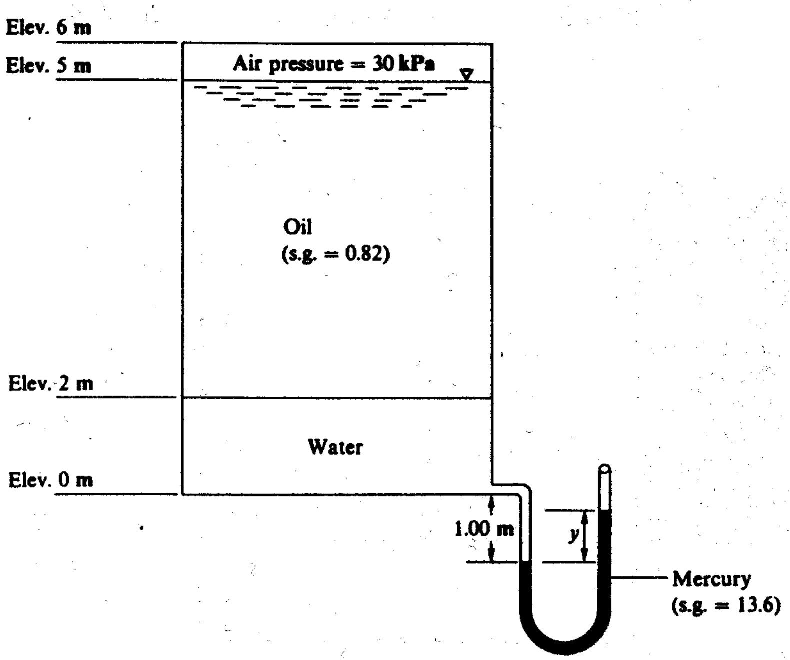


Fig. 2-20

2.36 Oil of specific gravity 0.750 flows through the nozzle shown in Fig. 2-21 and deflects the mercury in the U-tube gage. Determine the value of h if the pressure at A is 20.0 psi.

$$20.0+[(0.750)(62.4)](2.75+h)/144-[(13.6)(62.4)](h)/144=0 \qquad h=3.75\text{ ft}$$

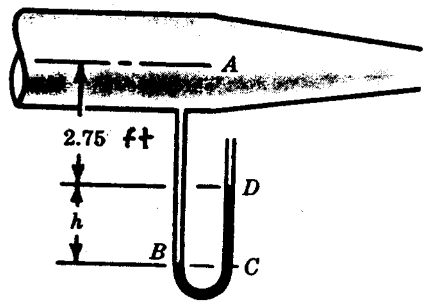


Fig. 2-21

2.37 Determine the reading h in Fig. 2-22 for $p_A = 39$ kPa vacuum if the liquid is kerosene (s.g. = 0.83).

▮

$$-39 + [(0.83)(9.79)]h = 0 \qquad h = 4.800 \text{ m}$$

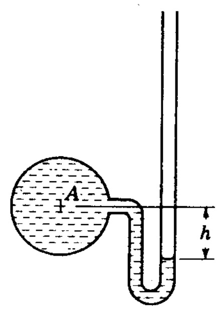


Fig. 2-22

2.38 In Fig. 2-22, the liquid is water. If $h = 9$ in and the barometer reading is 29.8 inHg, find p_A in feet of water absolute.

▮

$$p_A + \tfrac{9}{12} = (13.6)(29.8/12) \qquad p_A = 33.0 \text{ ft of water absolute}$$

2.39 In Fig. 2-23, s.g.$_1 = 0.84$, s.g.$_2 = 1.0$, $h_2 = 96$ mm, and $h_1 = 159$ mm. Find p_A in mmHg gage. If the barometer reading is 729 mmHg, what is p_A in mmH_2O absolute?

▮

$$p_A + (0.84)(96) - (1.0)(159) = 0$$

$$p_A = 78.4 \text{ mmH}_2\text{O gage} = 78.4/13.6 = 5.76 \text{ mmHg gage}$$

$$= 78.4 + (13.6)(729) = 9993 \text{ mmH}_2\text{O absolute}$$

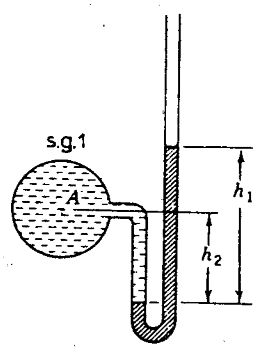


Fig. 2-23

2.40 At 20 °C, gage A in Fig. 2-24 reads 290 kPa abs. What is the height h of water? What does gage B read?

▮

$$290 - [(13.6)(9.79)](\tfrac{70}{100}) - 9.79h = 175 \qquad h = 2.227 \text{ m}$$

$$p_B - (9.79)(\tfrac{70}{100} + 2.227) = 175 \qquad p_B = 204 \text{ kPa}$$

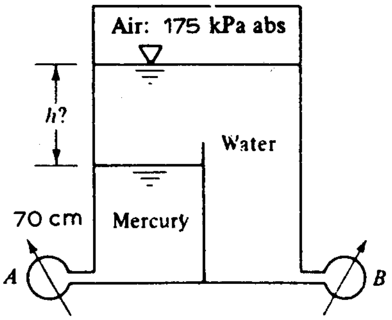


Fig. 2-24

2.41 The U-tube shown in Fig. 2-25*a* is 10 mm in diameter and contains mercury. If 12.0 mL of water is poured into the right-hand leg, what are the ultimate heights in the two legs?

▌ After the water is poured, the orientation of the liquids will be as shown in Fig. 2-25*b*; $h = (12.0 \times 10^3\ \text{mm}^3)/\pi(5\ \text{mm})^2 = 152.8\ \text{mm}$, $(13.6)(240 - L) = 13.6L + 152.8$, $L = 114.4\ \text{mm}$. Left leg height above bottom of U-tube $= 240 - 114.4 = 125.6\ \text{mm}$; right leg height above bottom of U-tube $= 114.4 + 152.8 = 267.2\ \text{mm}$.

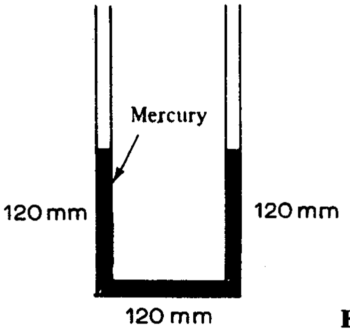


Fig. 2-25(*a*)

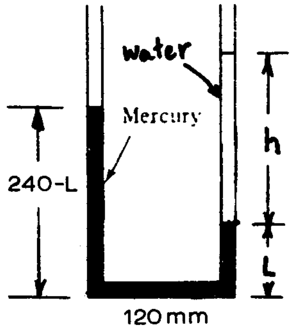


Fig. 2-25(*b*)

2.42 Assuming sea water to have a constant specific weight of 10.05 kN/m³, what is the absolute pressure at a depth of 10 km?

▌

$$p = 1 + (10.05)(10\,000)/101.3 = 993\ \text{atm}$$

2.43 In Fig. 2-26, fluid 2 is carbon tetrachloride and fluid 1 is benzene. If p_{atm} is 101.5 kPa, determine the absolute pressure at point *A*.

▌

$$101.5 + (15.57)(0.35) - (8.62)(0.12) = p_A \qquad p_A = 105.9\ \text{kPa}$$

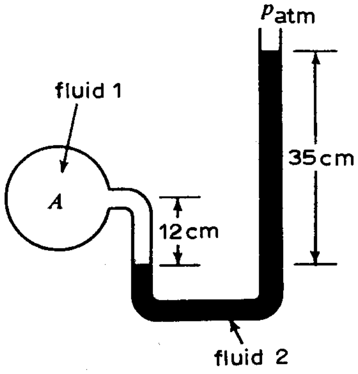


Fig. 2-26

2.44 In Fig. 2-27*a*, the manometer reads 4 in when atmospheric pressure is 14.7 psia. If the absolute pressure at *A* is doubled, what is the new manometer reading?

▌ $p_A + (62.4)(3.5) - [(13.6)(62.4)](\frac{4}{12}) = (14.7)(144)$, $p_A = 2181\ \text{lb/ft}^2$. If p_A is doubled to $4362\ \text{lb/ft}^2$, the mercury level will fall x inches on the left side of the manometer and will rise by that amount on the right side of the manometer (see Fig. 2-27*b*). Hence, $4362 + (62.4)(3.5 + x/12) - [(13.6)(62.4)][(4 + 2x)/12] = (14.7)(144)$, $x = 16.0\ \text{in}$. New manometer reading $= 4 + (2)(16.0) = 36.0\ \text{in}$.

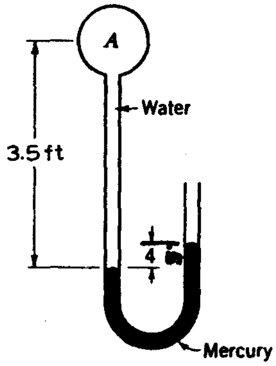


Fig. 2-27(a)

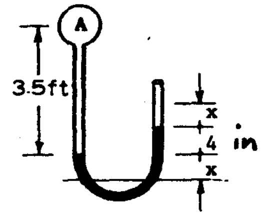


Fig. 2-27(b)

2.45 In Fig. 2-28*a*, *A* contains water, and the manometer fluid has density 2900 kg/m^3. When the left meniscus is at zero on the scale, $p_A = 100$ mm of water. Find the reading of the right meniscus for $p_A = 10$ kPa with no adjustment of the U-tube or scale.

▌ First, determine the reading of the right meniscus for $p_A = 100$ mm of water (see Fig. 2-28*b*): $100 + 500 - 2.90h = 0$, $h = 206.9$ mm. When $p_A = 10$ kPa, the mercury level will fall some amount, *d*, on the left side of the manometer and will rise by that amount on the right side of the manometer (see Fig. 2-28*b*). Hence, $10/9.79 + (500 + d)/1000 - [(206.9 + 2d)/1000](2.90) = 0$, $d = 192.0$ mm. Scale reading for $p_A = 10$ kPa is $206.9 + 192.0$, or 398.9 mm.

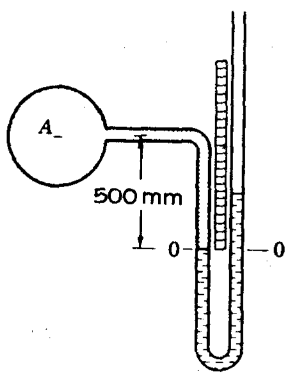


Fig. 2-28(a)

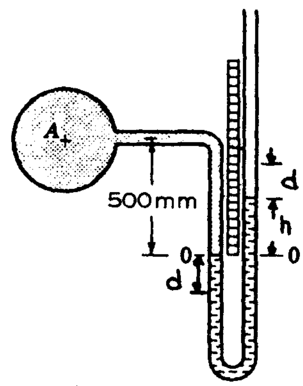


Fig. 2-28(b)

2.46 A manometer is attached to a conduit, as shown in Fig. 2-29. Calculate the pressure at point *A*.

▌ $$p_A + (62.4)[(5 + 15)/12] - [(13.6)(62.4)](\tfrac{15}{12}) = 0 \qquad p_A = 957 \text{ lb/ft}^2$$

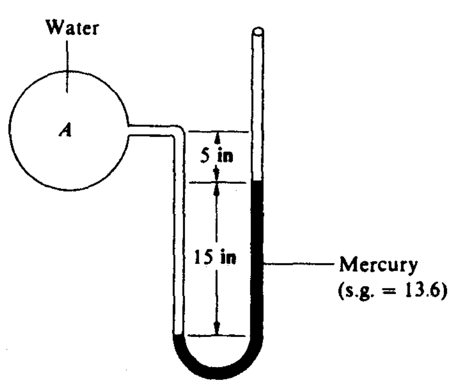


Fig. 2-29

2.47 A manometer is attached to a pipe containing oil, as shown in Fig. 2-30. Calculate the pressure at point *A*.

▌ $$p_A + [(0.85)(9.79)](0.2) - (9.79)(1.5) = 0 \qquad p_A = 13.02 \text{ kN/m}^2$$

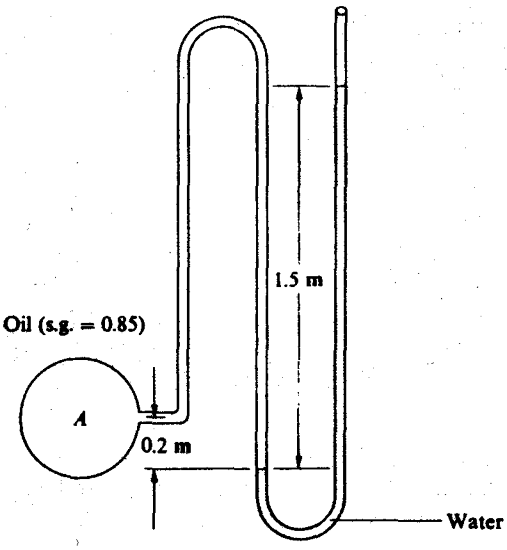


Fig. 2-30

2.48 A monometer is attached to a pipe to measure pressure, as shown in Fig. 2-31. Calculate the pressure at point A.

▮ $$p_A + (62.4)(\tfrac{18}{12}) - [(13.6)(62.4)](\tfrac{6}{12}) = 0 \qquad p_A = 331\ \text{lb/ft}^2$$

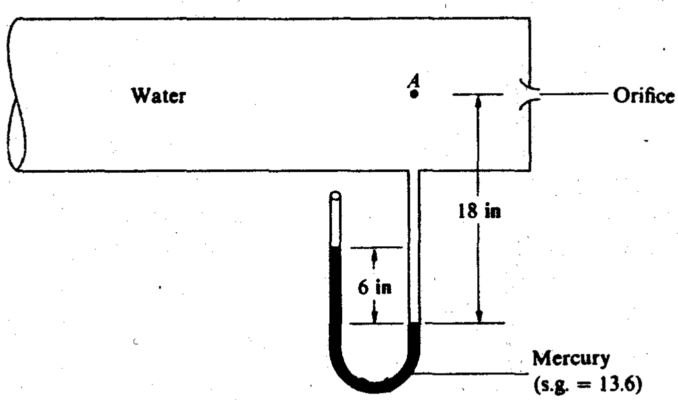


Fig. 2-31

2.49 A glass U-tube open to the atmosphere at both ends is shown in Fig. 2-32. if the U-tube contains oil and water as shown, determine the specific gravity of the oil.

▮ $$[(\text{s.g.}_{\text{oil}})(9.79)](0.35) - (9.79)(0.30) = 0 \qquad \text{s.g.}_{\text{oil}} = 0.86$$

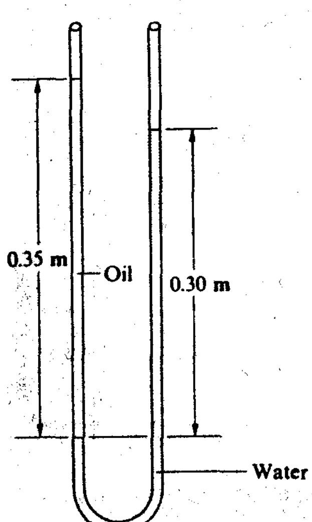


Fig. 2-32

2.50 A differential manometer is shown in Fig. 2-33. Calculate the pressure difference between points A and B.

▌ $$p_A + [(0.92)(62.4)][(x+12)/12] - [(13.6)(62.4)](\tfrac{12}{12}) - [(0.92)(62.4)][(x+24)/12] = p_B$$

$$p_A - p_B = 906\ \text{lb/ft}^2$$

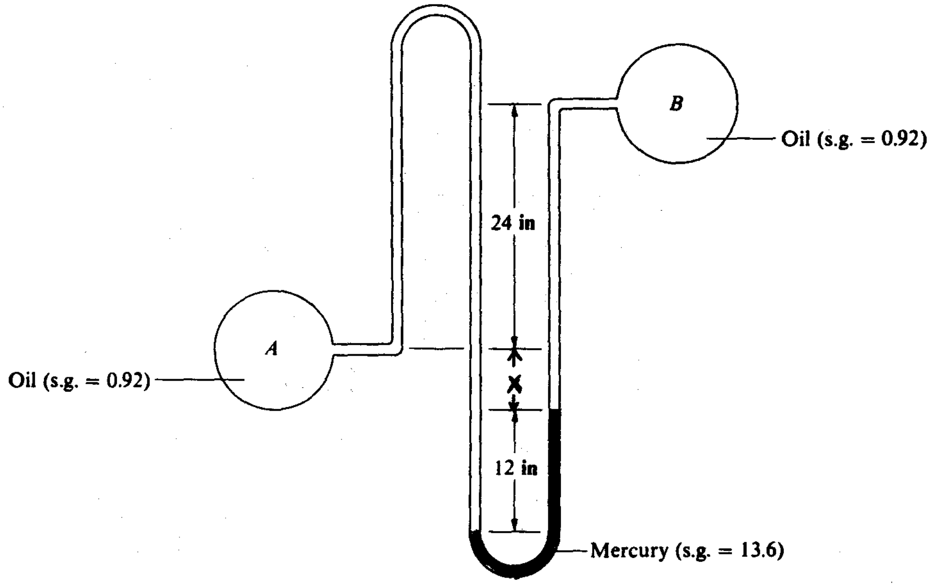


Fig. 2-33

2.51 A differential manometer is attached to a pipe, as shown in Fig. 2.34. Calculate the pressure difference between points A and B.

▌ $$p_A + [(0.91)(62.4)](y/12) - [(13.6)(62.4)](\tfrac{4}{12}) - [(0.91)(62.4)][(y-4)/12] = p_B$$

$$p_A - p_B = 264\ \text{lb/ft}^2$$

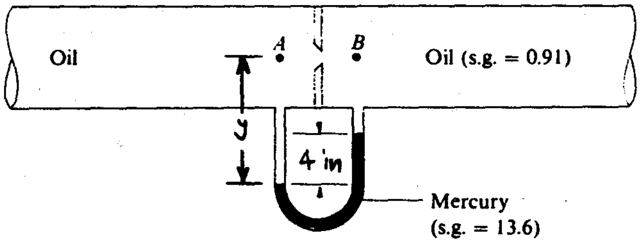


Fig. 2-34

2.52 A differential manometer is attached to a pipe, as shown in Fig. 2-35. Calculate the pressure difference between points A and B.

▌ $$p_A - [(0.91)(62.4)](y/12) - [(13.6)(62.4)](\tfrac{4}{12}) + [(0.91)(62.4)][(y+4)/12] = p_B$$

$$p_A - p_B = 264\ \text{lb/ft}^2$$

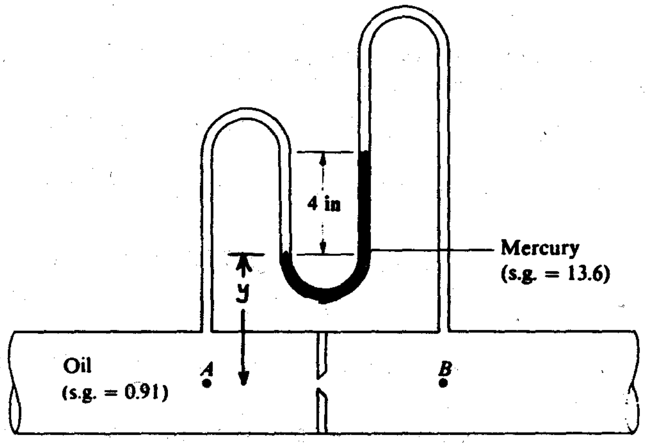


Fig. 2-35

2.53 For the configuration shown in Fig. 2-36, calculate the weight of the piston if the gage pressure reading is 70.0 kPa.

▌ Let W = weight of the piston. $W/[(\pi)(1)^2/4] - [(0.86)(9.79)](1) = 70.0$, $W = 61.6$ kN.

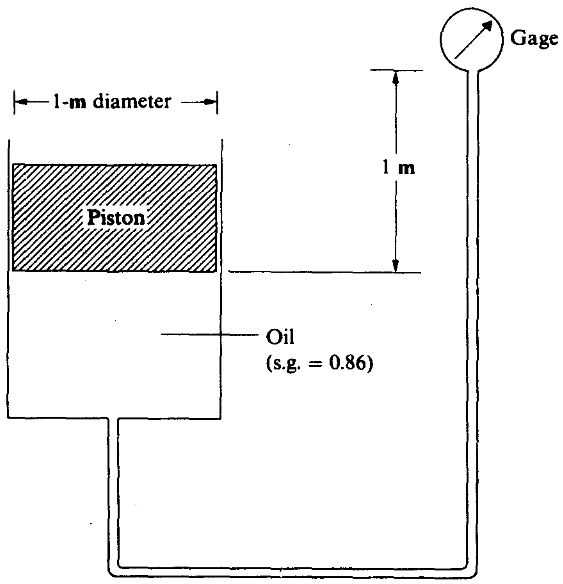


Fig. 2-36

2.54 A manometer is attached to a horizontal oil pipe, as shown in Fig. 2-37. If the pressure at point A is 10 psi, find the distance between the two mercury surfaces in the manometer (i.e., determine the distance y in Fig. 2-37).

▌ $$(10)(144) + [(0.90)(62.4)](3 + y) - [(13.6)(62.4)]y = 0 \qquad y = 2.03 \text{ ft} \quad \text{or} \quad 24.4 \text{ in}$$

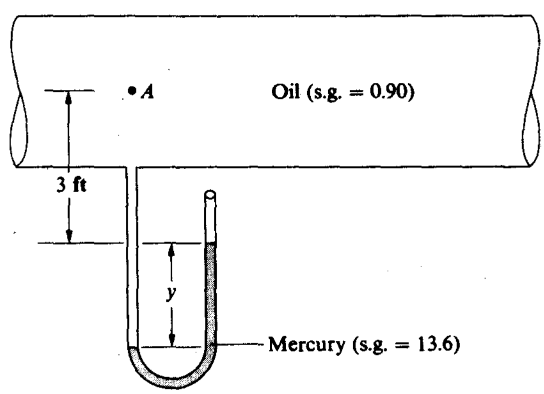


Fig. 2-37

2.55 A vertical pipe with attached gage and manometer is shown in Fig. 2-38. What will be the gage reading in pounds per square inch if there is no flow in the pipe?

▌ $$\text{Gage reading} + [(0.85)(62.4)](2 + 8)/144 - [(13.6)(62.4)](\tfrac{18}{12})/144 = 0 \qquad \text{Gage reading} = 5.16 \text{ psi}$$

2.56 A monometer is attached to a vertical pipe, as shown in Fig. 2-39. Calculate the pressure difference between points A and B.

▌ $$p_A - (62.4)(5 + 1) - [(13.6)(62.4)](2) + (62.4)(2 + 1) = p_B$$
$$p_A - p_B = 1884 \text{ lb/ft}^2 \quad \text{or} \quad 13.1 \text{ lb/in}^2$$

2.57 A manometer is attached to a water tank, as shown in Fig. 2-40. Find the height of the free water surface above the bottom of the tank.

▌ $$(9.79)(H - 0.15) - [(13.6)(9.79)](0.20) = 0 \qquad H = 2.87 \text{ m}$$

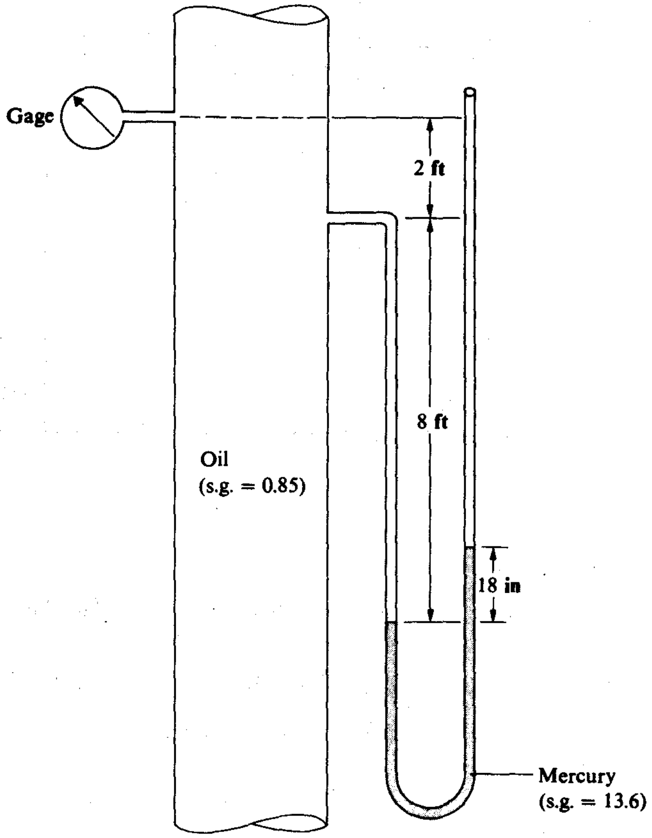


Fig. 2-38

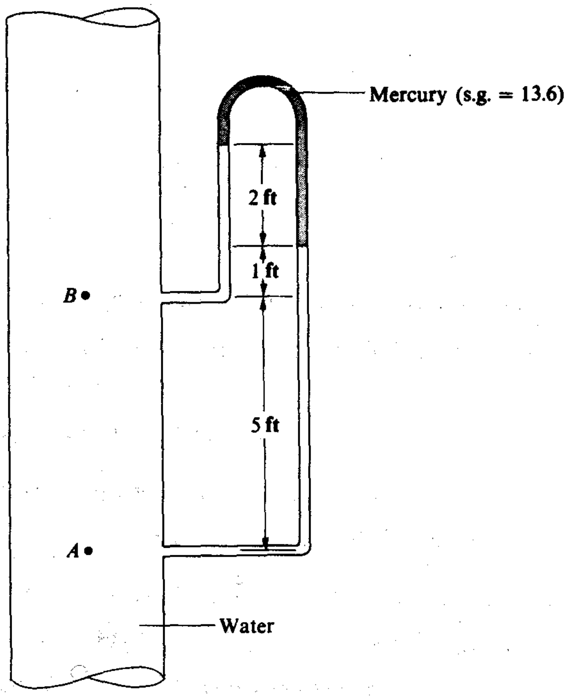


Fig. 2-39

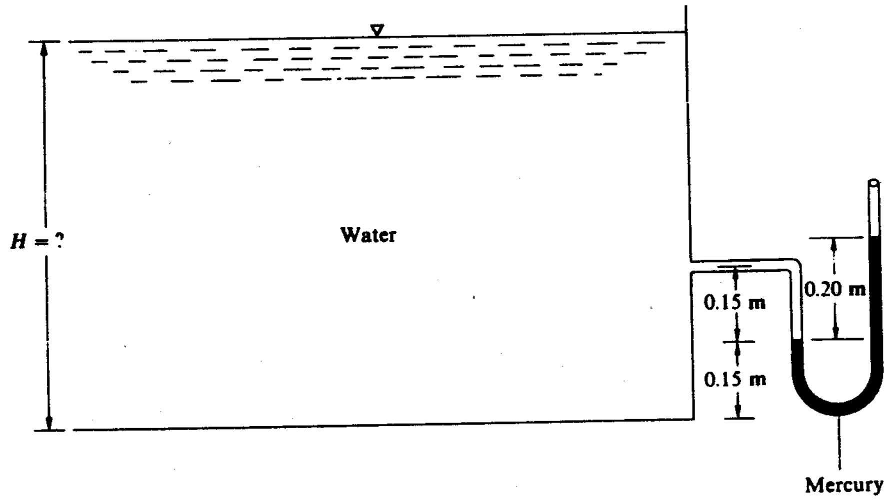


Fig. 2-40

2.58 A differential manometer is attached to two tanks, as shown in Fig. 2-41. Calculate the pressure difference between chambers A and B.

▮ $$p_A + [(0.89)(9.79)](1.1) + [(13.6)(9.79)](0.3) - [(1.59)(9.79)](0.8) = p_B$$

$$p_A - p_B = -37.1 \text{ kN/m}^2 \qquad (\text{i.e., } p_B > p_A)$$

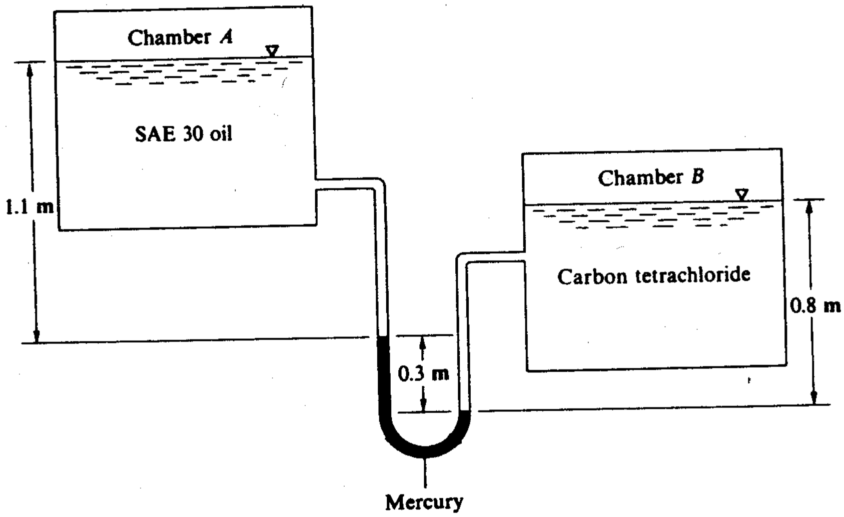


Fig. 2-41

2.59 Calculate the pressure difference between A and B for the setup shown in Fig. 2-42.

▮ $$p_A + (62.4)(66.6/12) - [(13.6)(62.4)](40.3/12) + (62.4)(22.2/12) - [(13.6)(62.4)](30.0/12) - (62.4)(10.0/12) = p_B$$

$$p_A - p_B = 4562 \text{ lb/ft}^2 \quad \text{or} \quad 31.7 \text{ lb/in}^2$$

2.60 Calculate the pressure difference between A and B for the setup shown in Fig. 2-43.

▮ $$p_A - (9.79)x - [(0.8)(9.79)](0.70) + (9.79)(x - 0.80) = p_B \qquad p_A - p_B = 13.3 \text{ kN/m}^2$$

2.61 Calculate the pressure difference between A and B for the setup shown in Fig. 2-44.

▮ $$p_A + (62.4)(x + 4) - [(13.6)(62.4)](4) + (62.4)(7 - x) = p_B$$

$$p_A - p_B = 2708 \text{ lb/ft}^2 \quad \text{or} \quad 18.8 \text{ lb/in}^2$$

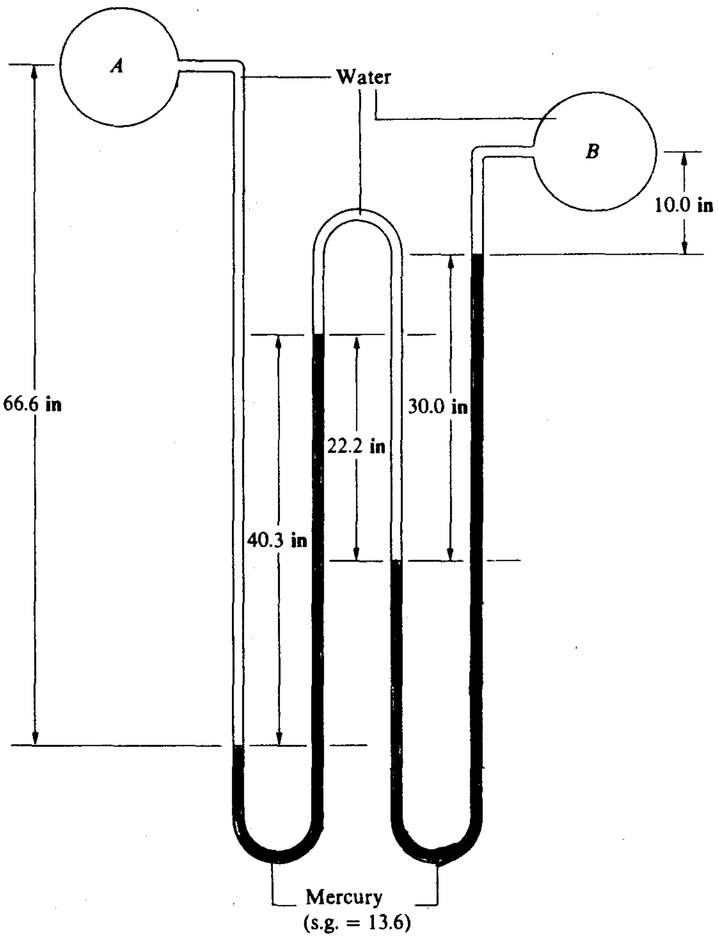


Fig. 2-42

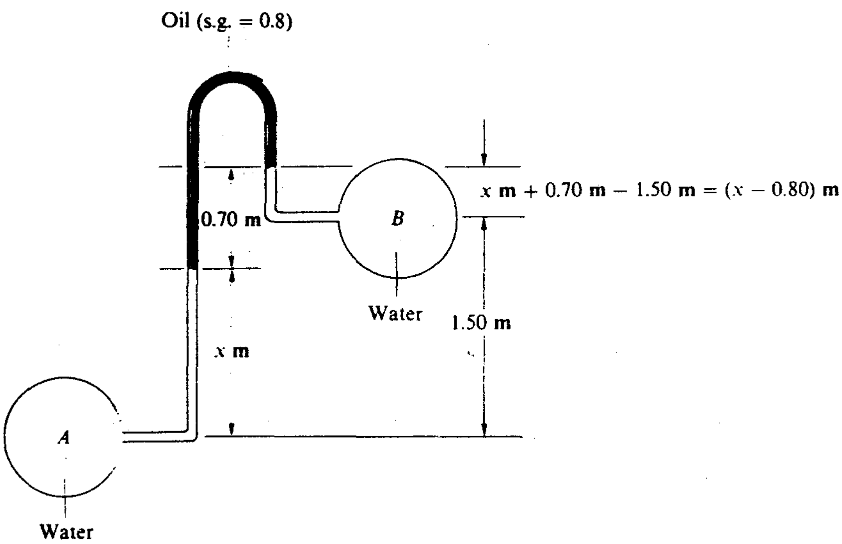


Fig. 2-43

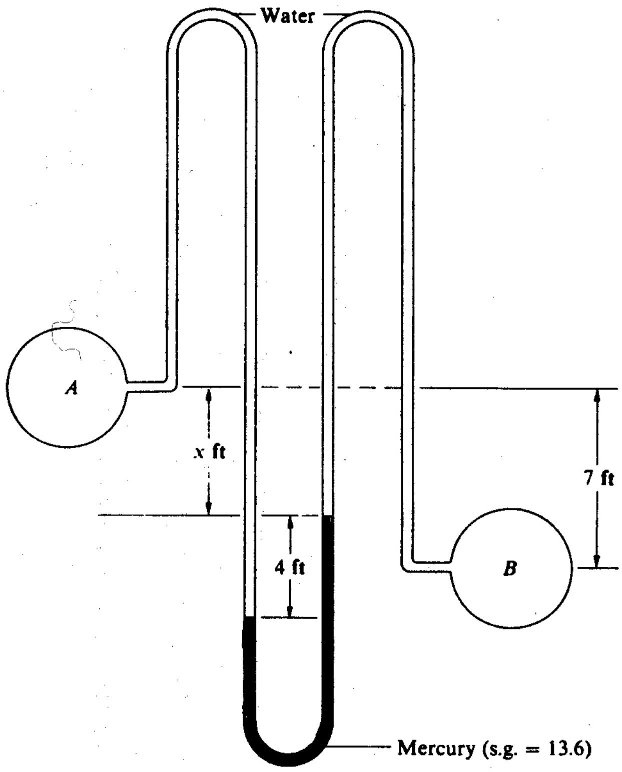


Fig. 2-44

2.62 Vessels A and B in Fig. 2-45 contain water under pressures of 40.0 psi and 20.0 psi, respectively. What is the deflection of the mercury in the differential gage?

$(40.0)(144)+(62.4)(x+h)-[(13.6)(62.4)]h+62.4y=(20.0)(144)$. Since $x+y=16.00-10.00$, or 6.00 ft, $h=4.14$ ft.

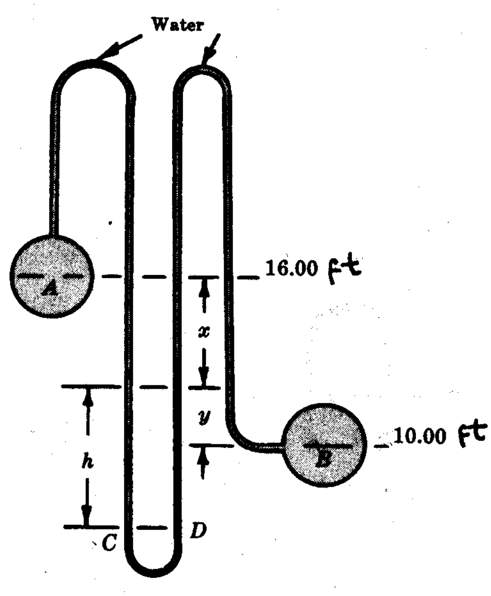


Fig. 2-45

2.63 For a gage pressure at A in Fig. 2-46 of -1.58 psi, find the specific gravity of gage liquid B.

$(-1.58)(144)+[(1.60)(62.4)](10.50-9.00)-(0.0750)(11.25-9.00)+[(\text{s.g.}_{\text{liq. }B})(62.4)](11.25-10.00)=0$

$$\text{s.g.}_{\text{liq. }B}=1.00$$

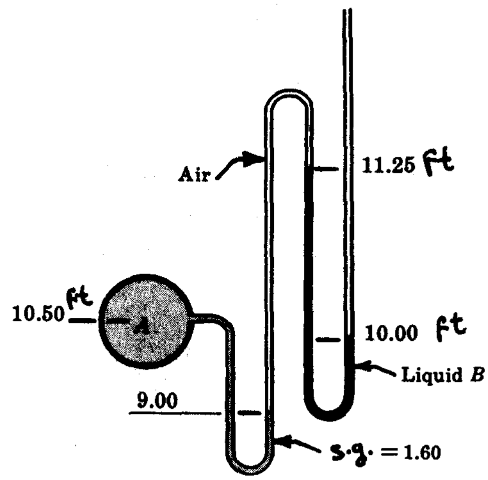


Fig. 2-46

2.64 In Fig. 2-47, liquid A weighs 53.5 lb/ft^3 and liquid B weighs 78.8 lb/ft^3. Manometer liquid M is mercury. If the pressure at B is 30 psi, find the pressure at A.

$$p_A - (53.5)(6.5 + 1.3) + [(13.6)(62.4)](1.3) + (78.8)(6.5 + 10.0) = (30)(144)$$

$$p_A = 2334\ lb/ft^2 \quad \text{or} \quad 16.2\ lb/in^2$$

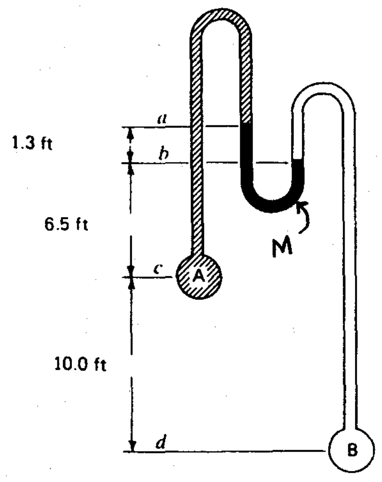


Fig. 2-47

2.65 What would be the manometer reading in Fig. 2-47 if $p_B - p_A$ is 165 kPa?

Converting to lb/ft^2, $p_B - p_A = 3446\ lb/ft^2$. The mercury level will rise some amount, x, on the left side of the manometer and will fall by that amount on the right side of the manometer (see Fig. 2.48). Hence, taking weight densities from Prob. 2.64, $p_A - (53.5)(6.5 + 1.3 + x) + [(13.6)(62.4)](1.3 + 2x) + (78.8)(6.5 + 10.0 - x) = p_B$, $1644x + 1986 = p_B - p_A = 3446$, $x = 0.89$ ft; manometer reading $= 1.3 + (2)(0.89) = 3.08$ ft.

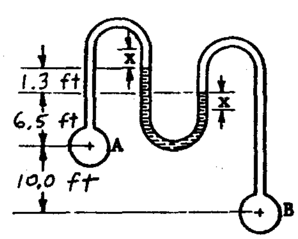


Fig. 2-48

2.66 In Fig. 2-49, water is contained in A and rises in the tube to a level 85 in above A; glycerin is contained in B. The inverted U-tube is filled with air at 23 psi and 70 °F. Atmospheric pressure is 14.6 psia. Determine the difference in pressure (psi) between A and B if y is 16 in. What is the absolute pressure in B in inches of mercury and in feet of glycerin?

$$p_A - (62.4)(\tfrac{85}{12}) = (23)(144) \qquad p_A = 3754.0 \text{ lb/ft}^2$$
$$p_B - [(1.26)(62.4)][(85-16)/12] = (23)(144) \qquad p_B = 3764.1 \text{ lb/ft}^2$$
$$p_A - p_B = 3754.0 - 3764.1 = -10.1 \text{ lb/ft}^2 \quad \text{or} \quad -0.070 \text{ lb/in}^2$$
$$(p_{abs})_B = (3764.1/144 + 14.6)/[(13.6)(62.4)/(12)^3] = 83.0 \text{ inHg}$$
$$(p_{abs})_B = (3764.1/144 + 14.6)/[(1.26)(62.4)/(12)^3] = 895.4 \text{ in} \quad \text{or} \quad 74.6 \text{ ft of glycerin}$$

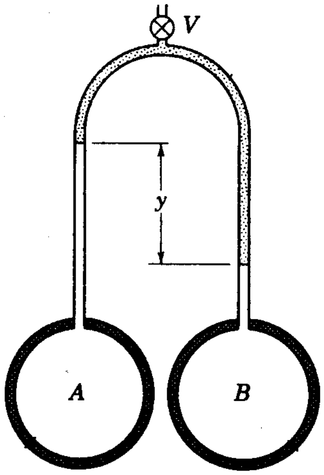


Fig. 2-49

2.67 Gas confined in a rigid container exerts a gage pressure of 150 kPa when its temperature is 7 °C. What pressure would the gas exert at 67 °C? Barometric pressure remains constant at 719 mmHg.

$$p_{atm} = [(13.6)(9.79)](0.719) = 95.7 \text{ kPa} \qquad p_{abs} = 95.7 + 150 = 245.7 \text{ kPa}$$
$$p_1V_1/T_1 = p_2V_2/T_2 \qquad (245.7)(V)/(273+7) = (p_2)(V)/(273+67) \qquad [V \text{ (volume) is constant}]$$
$$p_2 = 298.4 \text{ kPa (absolute)} = 298.4 - 95.7 = 202.7 \text{ kPa (gage)}$$

2.68 In Fig. 2-50, atmospheric pressure is 14.6 psia, the gage reading at A is 6.1 psi, and the vapor pressure of the alcohol is 1.7 psia. Compute x and y.

Working in terms of absolute pressure heads, $[(6.1 + 14.6)(144)]/[(0.90)(62.4)] - x = (1.7)(144)/[(0.90)(62.4)]$, $x = 48.72$ ft; $[(6.1 + 14.6)(144)]/[(0.90)(62.4)] + (y + 4.2) - (4.2)(13.6/0.90) = 0$, $y = 6.19$ ft.

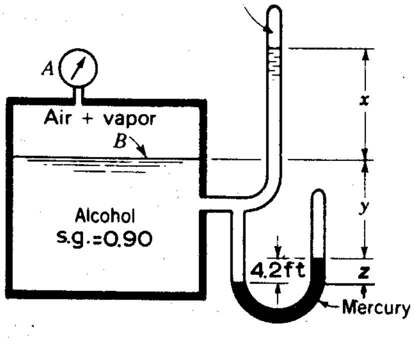


Fig. 2-50

2.69 In Fig. 2-50, assume the following: atmospheric pressure = 858 mbar abs, vapor pressure of the alcohol = 160 mbar abs, $x = 2.90$ m, $y = 2.10$ m. Compute the reading on the pressure gage (p_A) and on the manometer (z).

▌ Working in terms of absolute pressure heads, $[(p_A)_{gage} + 858](0.100)/[(0.90)(9.79)] - 2.90 = (160)(0.100)/[(0.90)(9.79)]$, $(p_A)_{gage} = -442$ mbar; $[(-442 + 858)(0.100)]/[(0.90)(9.79)] + (2.10 + z) - (z)(13.6/0.90) = 0$, $z = 0.483$ m.

2.70 A pipeline contains an incompressible gas ($\gamma = 0.05$ lb/ft^3) at rest; at point A the pressure is 4.69 in of water. What is the pressure, in inches of water, at point B, 492 ft higher than A?

▌ The change in pressure in the atmosphere must be considered; assume, however, that $\gamma_{air} = 0.076$ lb/ft^3 is constant.

$$(p_A/\gamma)_{abs} = (p_A/\gamma)_{atm} + 4.69/12 \text{ ft of water} \tag{1}$$

$$(p_B/\gamma)_{abs} = (p_B/\gamma)_{atm} + x/12 \text{ ft of water} \tag{2}$$

Subtracting Eq. (2) from Eq. (1),

$$(p_A/\gamma)_{abs} - (p_B/\gamma)_{abs} = (p_A/\gamma)_{atm} - (p_B/\gamma)_{atm} + 4.69/12 - x/12 \tag{3}$$

$$(p_A/\gamma)_{atm} - (p_B/\gamma)_{atm} = 492 \text{ ft of air} = (492)(0.076/62.4) = 0.599 \text{ ft of water}$$

$$(p_A/\gamma)_{abs} - (p_B/\gamma)_{abs} = 492 \text{ ft of gas} = (492)(0.05/62.4) = 0.394 \text{ ft of water}$$

Substituting these relationships into Eq. (3), $0.394 = 0.599 + 4.69/12 - x/12$, $x = 7.15$ in of water.

2.71 Determine the pressure difference between points A and B in Fig. 2-51.

▌ $$p_A + [(0.88)(9.79)](0.21) - [(13.6)(9.79)](0.09) - [(0.82)(9.79)](0.41 - 0.09) + (9.79)(0.41 - 0.15) - (0.0118)(0.10) = p_B$$

$$p_A - p_B = 10.2 \text{ kPa}$$

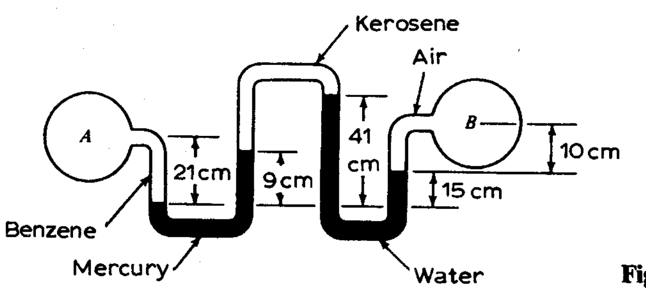


Fig. 2-51

2.72 In Fig. 2-52, if $p_B - p_A = 97.4$ kPa, calculate H.

▌ $$p_A - (9.79)(H/100) - [(0.827)(9.79)](\tfrac{17}{100}) + [(13.6)(9.79)][(34 + H + 17)/100] = p_B$$

$$1.234H + 66.53 = p_B - p_A = 97.4 \qquad H = 25.0 \text{ cm}$$

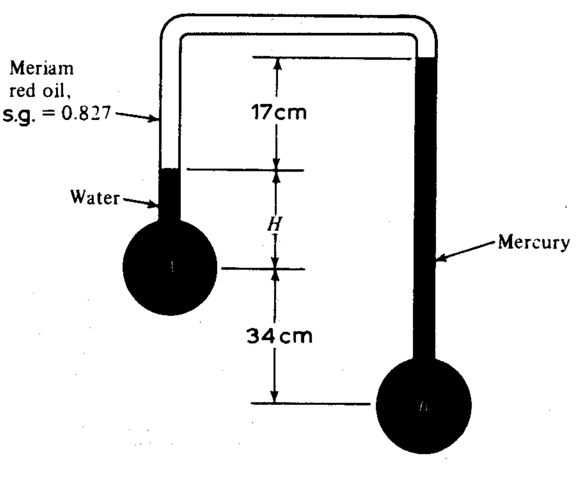


Fig. 2-52

2.73 For Fig. 2-53, if fluid 1 is water and fluid 2 is mercury, and $z_A = 0$ and $z_1 = -11$ cm, what is level z_2 at which $p_A = p_{\text{atm}}$?

▮ $$0 + (9.79)[0 - (-11)]/100 - [(13.6)(9.79)][z_2 - (-11)]/100 = 0 \qquad z_2 = -10.19\text{ cm}$$

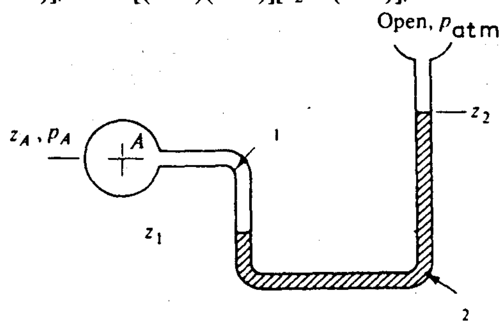


Fig. 2-53

2.74 The inclined manometer in Fig. 2-54*a* contains Meriam red manometer oil (s.g. = 0.827). Assume the reservoir is very large. What should the angle θ be if each inch along the scale is to represent a change of 0.8 lb/ft² in gage pressure p_A?

▮ From Fig. 2-54*b*, $\Delta p = \gamma \Delta z$, or

$$0.8\text{ lb/ft}^2 = [(0.827)(62.4\text{ lb/ft}^3)](\tfrac{1}{12}\text{ ft})(\sin\theta)$$

from which $\theta = 10.72°$.

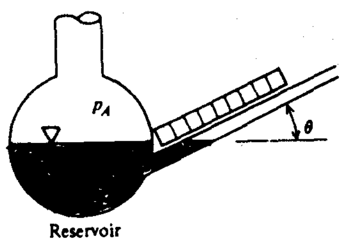


Fig. 2-54(*a*)

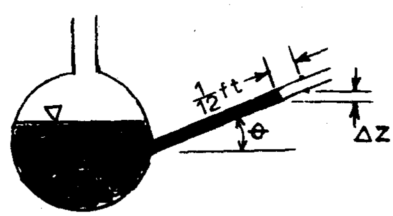


Fig. 2-54(*b*)

2.75 The system in Fig. 2-55 is at 20 °C. Compute the absolute pressure at point A.

▮ $$p_A + [(0.85)(62.4)](\tfrac{7}{12}) - [(13.6)(62.4)](\tfrac{9}{12}) + (62.4)(\tfrac{6}{12}) = (14.7)(144) \qquad p_A = 2691\text{ lb/ft}^2\text{ abs}$$

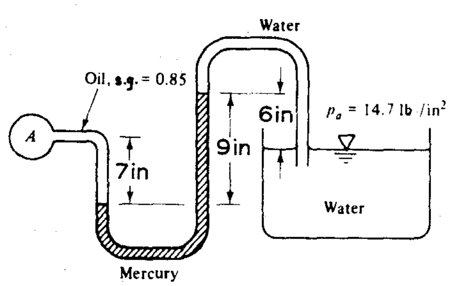


Fig. 2-55

2.76 Very small pressure differences $p_A - p_B$ can be measured accurately by the two-fluid differential manometer shown in Fig. 2-56. Density ρ_2 is only slightly larger than the upper fluid ρ_1. Derive an expression for the proportionality between h and $p_A - p_B$ if the reservoirs are very large.

▮ $p_A + \rho_1 g h_1 - \rho_2 g h - \rho_1 g(h_1 - h) = p_B$, $p_A - p_B = (\rho_2 - \rho_1)gh$. If $(\rho_2 - \rho_1)$ is small, h will be large (sensitive).

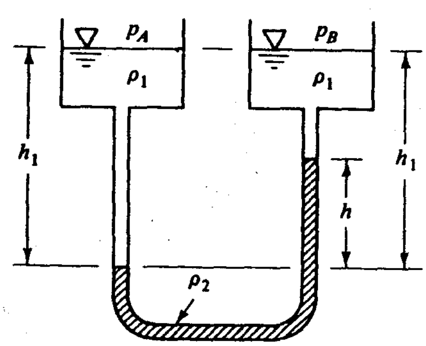


Fig. 2-56

2.77 Water flows downward in a pipe at 35°, as shown in Fig. 2-57. The pressure drop $p_1 - p_2$ is partly due to gravity and partly due to friction. The mercury manometer reads a 5-in height difference. What is the total pressure drop $p_1 - p_2$? What is the pressure drop due to friction only between 1 and 2? Does the manometer reading correspond only to friction drop?

▮
$$p_1 + (62.4)(6 \sin 35° + x/12 + \tfrac{5}{12}) - [(13.6)(62.4)](\tfrac{5}{12}) - (62.4)(x/12) = p_2$$
$$p_1 - p_2 = 112.9\ \text{lb/ft}^2 \qquad \text{(total pressure drop)}$$
$$\text{Pressure drop due to friction only} = [(13.6)(62.4) - 62.4](\tfrac{5}{12}) = 327.6\ \text{lb/ft}^2$$

Manometer reads only the friction loss.

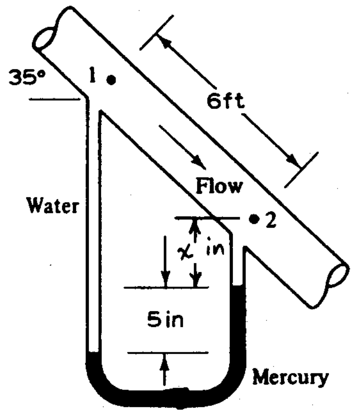


Fig. 2-57

2.78 Determine the gage pressure at point A in Fig. 2-58.

▮ $p_A - (9.79)(0.50) + (0.0118)(0.33) + [(13.6)(9.79)](0.17) - [(0.83)(9.79)](0.44) = 0 \qquad p_A = -14.17\ \text{kPa}$

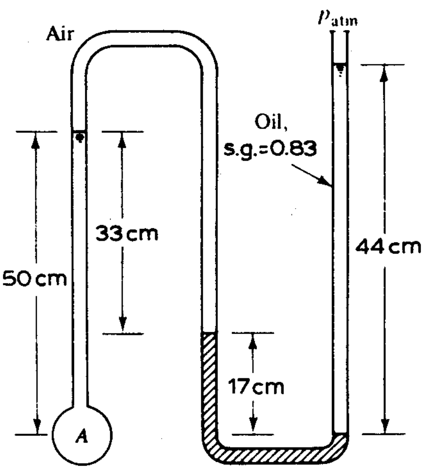


Fig. 2-58

2.79 In Fig. 2-59, calculate level h of the oil in the right-hand tube. Both tubes are open to the atmosphere.

▮
$$0 + (9.79)(0.110 + 0.240) - [(0.83)(9.79)](0.240 + h) = 0 \qquad h = 0.1817\ \text{m} = 181.7\ \text{mm}$$

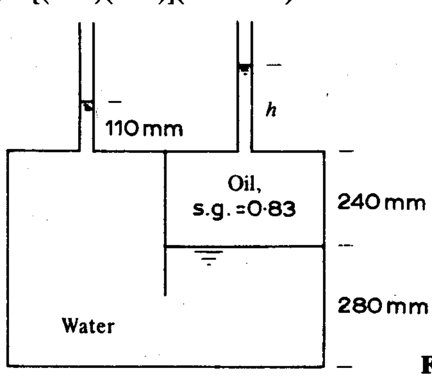


Fig. 2-59

2.80 In Fig. 2-60*a* the inclined manometer measures the excess pressure at *A* over that at *B*. The reservoir diameter is 2.5 in and that of the inclined tube is $\frac{1}{4}$ in. For $\theta = 32°$ and gage fluid with s.g. = 0.832, calibrate the scale in psi per ft.

$$p_A = \gamma(\Delta h + \Delta y) + p_B \qquad \text{(see Fig. 2-60}b\text{)} \qquad p_A - p_B = \gamma(\Delta h + \Delta y)$$

From Fig. 2-60*b*, $(A_A)(\Delta y) = (A_B)(R)$ or $\Delta y = A_B R/A_A$, $\Delta h = R \sin\theta$, $p_A - p_B = \gamma(R\sin\theta + A_B R/A_A) = \gamma R(\sin\theta + A_B/A_A)$, $A_B/A_A = [\pi(\frac{1}{4})^2/4]/[\pi(2.5)^2/4] = \frac{1}{100}$; $p_A - p_B = [(0.832)(62.4)](R)(\sin 32° + \frac{1}{100})/144 = 0.1947R$. The scale factor is thus 0.1947 psi/ft.

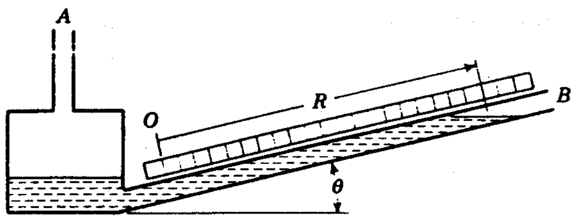


Fig. 2-60(*a*)

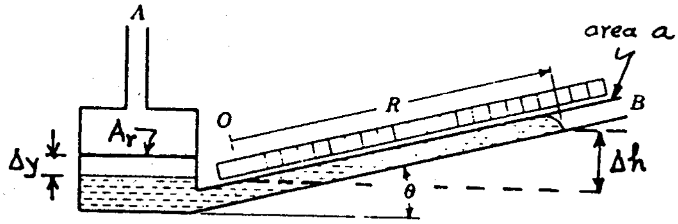


Fig. 2-60(*b*)

2.81 Determine the weight *W* that can be equilibrated by the force acting on the piston of Fig. 2-61.

$$p_1 = p_2 = F_1/A_1 = F_2/A_2 \qquad 1.25/[\pi(35)^2/4] = W/[\pi(250)^2/4] \qquad W = 63.8 \text{ kN}$$

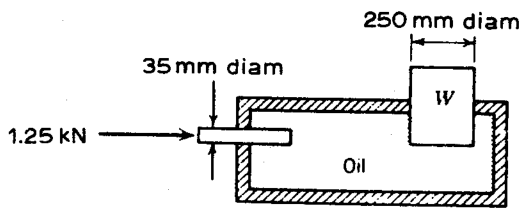


Fig. 2-61

2.82 Neglecting the container's weight in Fig. 2-62, find the force tending to lift the circular top *CD*.

$$p_{CD} - [(0.8)(62.4)](4) = 0 \qquad p_{CD} = 199.7 \text{ lb/ft}^2 \qquad F = pA = (199.7)[\pi(2.5)^2/4] = 980 \text{ lb}$$

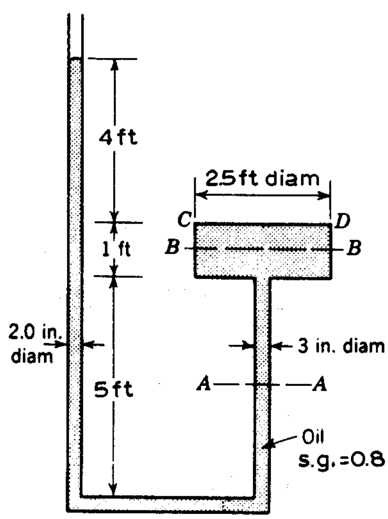


Fig. 2-62

2.83 Find the force of oil on the top surface CD of Fig. 2-62 if the liquid level in the open pipe is reduced by 1.3 m.

▌ $p_{CD} - [(0.8)(62.4)][4 - (1.3)(3.281)] = 0 \qquad p_{CD} = -13.24\ \text{lb/ft}^2$ (i.e., a downward pressure by CD)

$$F = pA = (-13.24)[\pi(2.5)^2/4] = -65.0\ \text{lb}$$

2.84 A drum 2.25 ft in diameter filled with water has a vertical pipe of 0.70-in diameter attached to the top. How many pounds of water must be poured into the pipe to exert a force of 1500 lb on the top of the drum?

▌
$$p = F/A = 1500/[\pi(2.25)^2/4] = 377.3\ \text{lb/ft}^2 \qquad h = p/\gamma = 377.3/62.4 = 6.05\ \text{ft}$$
$$W_{H_2O} = (6.05)[\pi(0.70/12)^2/4](62.4) = 1.01\ \text{lb}$$

2.85 In Fig. 2-63, the liquid at A and B is water and the manometer liquid is oil with s.g. $= 0.80$, $h_1 = 300$ mm, $h_2 = 200$ mm, and $h_3 = 600$ mm. **(a)** Determine $p_A - p_B$. **(b)** If $p_B = 50$ kPa and the barometer reading is 730 mmHg, find the absolute pressure at A in meters of water.

▌ **(a)**
$$p_A - (9.79)(\tfrac{300}{1000}) - [(0.80)(9.79)](\tfrac{200}{1000}) + (9.79)(\tfrac{600}{1000}) = p_B \qquad p_A - p_B = -1.37\ \text{kPa}$$

(b)
$$p_A - (9.79)(\tfrac{300}{1000}) - [(0.80)(9.79)](\tfrac{200}{1000}) + (9.79)(\tfrac{600}{1000}) = 50$$
$$p_A = 48.63\ \text{kPa}\quad (\text{gage}) = 48.63/9.79 + \tfrac{730}{1000}(13.6) = 14.90\ \text{m water}\quad (\text{absolute})$$

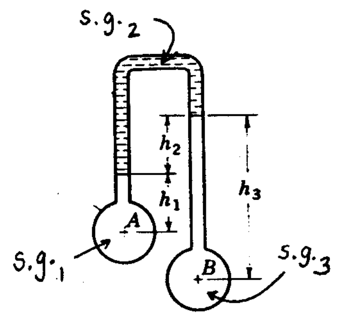


Fig. 2-63

2.86 In Fig. 2-63, s.g.$_1 = 1.0$, s.g.$_2 = 0.96$, s.g.$_3 = 1.0$, $h_1 = h_2 = 269$ mm, and $h_3 = 1.2$ m. Compute $p_A - p_B$ in millimeters of water.

▌
$$p_A - (1.0)(269) - (0.96)(269) + (1.0)(1200) = p_B \qquad p_A - p_B = -673\ \text{mm of water}$$

2.87 In Fig. 2-63, s.g.$_1 = 1.0$, s.g.$_2 = 0.94$, s.g.$_3 = 1.0$, $h_1 = 300$ mm, $h_3 = 1.1$ m, and $p_A - p_B = -360$ mm of water. Find the gage difference (h_2).

▌ $p_A - (1.0)(300) - (0.94)(h_2) + (1.0)(1100) = p_B \qquad p_A - p_B = -360 = -800 + (0.94)(h_2) \qquad h_2 = 468$ mm

2.88 What is the pressure difference, in pounds per square inch, of a 1000-ft water column?

▌
$$p = \gamma h = (62.4)(1000)/144 = 433\ \text{psi}$$

2.89 Find the pressure at a point 9.5 m below the free surface in a fluid whose density varies with depth h (in m) according to

$$\rho = (450\ \text{kg/m}^3) + (11\ \text{kg/m}^4)h$$

▌ $dp = \gamma\, dh = \rho g\, dh = (g)(450 + 11h)\, dh$. Integrating both sides: $p = (g)(450h + 11h^2/2)$. For $h = 9.5$ m: $p = (9.81)[(450)(9.5) + (11)(9.5)^2/2] = 46.807$ kPa.

2.90 If atmospheric pressure is 29.72 inHg, what will be the height of water in a water barometer if the temperature of the water is **(a)** 50 °F, **(b)** 100 °F, and **(c)** 150 °F?

▌
$$p = \gamma h = [(13.6)(62.4)](29.72/12) = 2102\ \text{lb/ft}^2 \quad \text{or} \quad 14.60\ \text{lb/in}^2$$

(a) At 50 °F, $\gamma = 62.4\ \text{lb/ft}^3$ and $p_{vapor} = 25.7/144$, or $0.178\ \text{lb/in}^2$, $h_{H_2O} = (14.60 - 0.178)(144)/62.4 = 33.28$ ft.
(b) At 100 °F, $\gamma = 62.0\ \text{lb/ft}^3$ and $p_{vapor} = \tfrac{135}{144}$, or $0.938\ \text{lb/in}^2$, $h_{H_2O} = (14.60 - 0.938)(144)/62.0 = 31.73$ ft.
(c) At 150 °F, $\gamma = 61.2\ \text{lb/ft}^3$ and $p_{vapor} = \tfrac{545}{144}$, or $3.78\ \text{lb/in}^2$, $h_{H_2O} = (14.60 - 3.78)(144)/61.2 = 25.46$ ft.

2.91 A bicycle tire is inflated at sea level (where atmospheric pressure is 14.6 psia and the temperature is 69 °F) to 65.0 psi. Assuming the tire does not expand, what is the gage pressure within the tire on the top of Everest (altitude 30 000 ft), where atmospheric pressure is 4.3 psia and the temperature is −38 °F?

▌ Let subscript 1 indicate sea level and subscript 2 indicate altitude 30 000 ft.

$$(p_1)_{abs} = 14.6 + 65.0 = 79.6 \text{ psia} \qquad p_1V_1/T_1 = p_2V_2/T_2$$

$$(79.6)(V)/(460+69) = (p_2)(V)/[460+(-38)] \qquad (V \text{ is constant})$$

$$(p_2)_{abs} = 63.5 \text{ psia} \qquad (p_2)_{gage} = 63.5 - 4.3 = 59.2 \text{ psi}$$

2.92 Find the difference in pressure between tanks A and B in Fig. 2-64 if $d_1 = 330$ mm, $d_2 = 160$ mm, $d_3 = 480$ mm, and $d_4 = 230$ mm.

▌ $$p_A + (9.79)(0.330) - [(13.6)(9.79)](0.480 + 0.230 \sin 45°) = p_B \qquad p_A - p_B = 82.33 \text{ kPa}$$

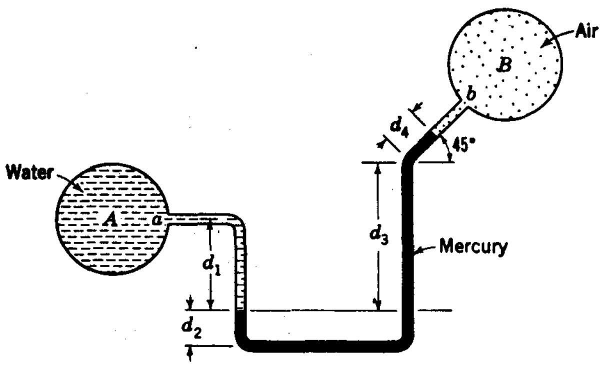


Fig. 2-64

2.93 A cylindrical tank contains water at a height of 55 mm, as shown in Fig. 2-65. Inside is a smaller open cylindrical tank containing cleaning fluid (s.g. = 0.8) at height h. If $p_B = 13.40$ kPa gage and $p_C = 13.42$ kPa gage, what are gage pressure p_A and height h of cleaning fluid? Assume that the cleaning fluid is prevented from moving to the top of the tank.

▌ $$p_A + (9.79)(0.055) = 13.42 \qquad p_A = 12.88 \text{ kPa}$$

$$12.88 + (9.79)(0.055 - h) + [(0.8)(9.79)]h = 13.40 \qquad h = 0.00942 \text{ m} = 9.42 \text{ mm}$$

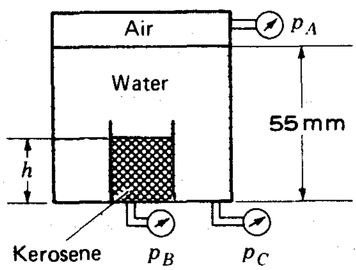


Fig. 2-65

2.94 An open tube is attached to a tank, as shown in Fig. 2-66. If the water rises to a height of 800 mm in the tube, what are the pressures p_A and p_B of the air above the water? Neglect capillary effects in the tube.

▌ $$p_A - (9.79)[(800 - 300 - 100)/1000] = 0 \qquad p_A = 3.92 \text{ kPa}$$

$$p_B - (9.79)[(800 - 300)/1000] = 0 \qquad p_B = 4.90 \text{ kPa}$$

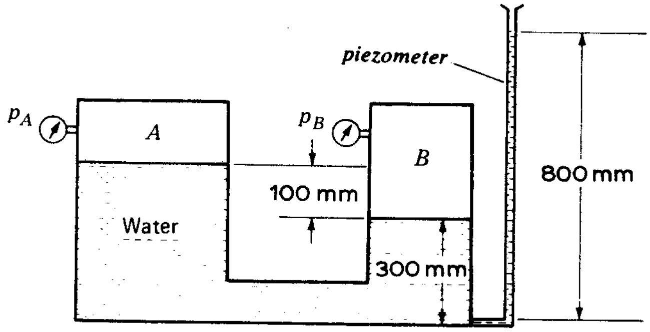


Fig. 2-66

2.95 For the setup shown in Fig. 2-67, what is the pressure p_A if the specific gravity of the oil is 0.82?

▌ $$p_A + [(0.82)(9.79)](3) + (9.79)(4-3) - [(13.6)(9.79)](0.320) = 0 \qquad p_A = 8.73 \text{ kPa}$$

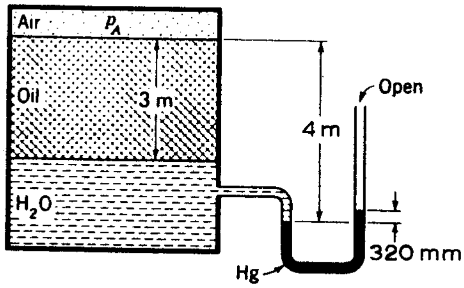


Fig. 2-67

2.96 For the setup shown in Fig. 2-68, calculate the absolute pressure at a. Assume standard atmospheric pressure, 101.3 kPa.

▌ $$101.3 + (9.79)(0.600 - 0.200) - [(13.6)(9.79)](0.140) + [(0.83)(9.79)](0.140 + 0.090) = p_A$$

$$p_A = 88.44 \text{ kPa}$$

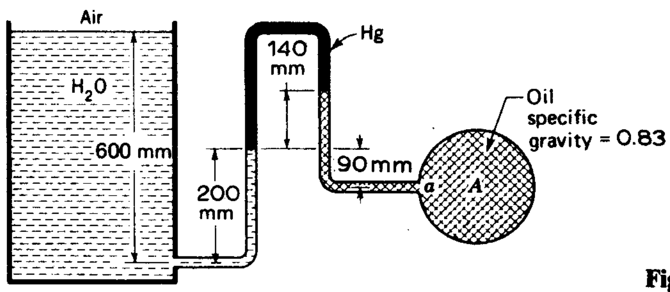


Fig. 2-68

2.97 A force of 460 N is exerted on lever AB, as shown in Fig. 2-69. End B is connected to a piston which fits into a cylinder having a diameter of 60 mm. What force F_D acts on the larger piston, if the volume between C and D is filled with water?

▌ Let F_C = force exerted on smaller piston at C: $F_C = (460)(\frac{220}{120}) = 843$ N. $F_C/A_C = F_D/A_D$, $(843)/[\pi(\frac{60}{1000})^2/4] = F_D/[\pi(\frac{260}{1000})^2/4]$, $F_D = 15\,830$ N, or 15.83 kN.

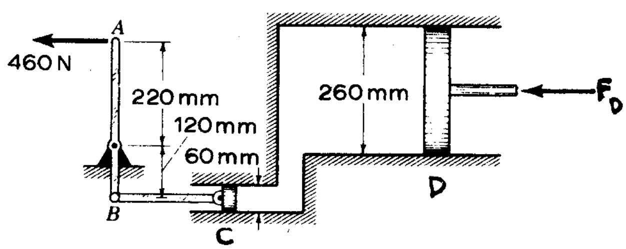


Fig. 2-69

CHAPTER 3
Forces on Submerged Plane Areas

3.1 If a triangle of height d and base b is vertical and submerged in liquid with its vertex at the liquid surface (see Fig. 3-1), derive an expression for the depth to its center of pressure.

$$h_{cp} = h_{cg} + \frac{I_{cg}}{h_{cg}A} = \frac{2d}{3} + \frac{bd^3/36}{(2d/3)(bd/2)} = \frac{3d}{4}$$

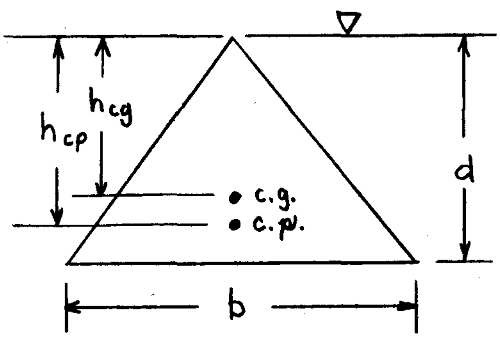


Fig. 3-1

3.2 If a triangle of height d and base b is vertical and submerged in liquid with its vertex a distance a below the liquid surface (see Fig. 3-2), derive an expression for the depth to its center of pressure.

$$h_{cp} = h_{cg} + \frac{I_{cg}}{h_{cg}A} = \left(a + \frac{2d}{3}\right) + \frac{bd^3/36}{(a + 2d/3)(bd/2)} = \left(a + \frac{2d}{3}\right) + \frac{d^2}{18(a + 2d/3)}$$
$$= \frac{18(a^2 + 4ad/3 + 4d^2/9) + d^2}{18(a + 2d/3)} = \frac{6a^2 + 8ad + 3d^2}{6(a + 2d/3)}$$

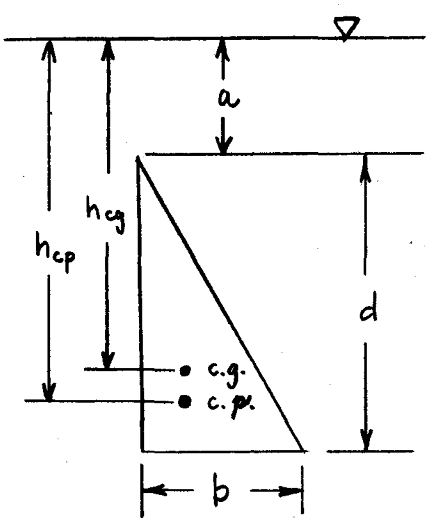


Fig. 3-2

3.3 If a triangle of height d and base b is vertical and submerged in liquid with its base at the liquid surface (see Fig. 3-3), derive an expression for the depth to its center of pressure.

$$h_{cp} = h_{cg} + \frac{I_{cg}}{h_{cg}A} = \frac{d}{3} + \frac{bd^3/36}{(d/3)(bd/2)} = \frac{d}{3} + \frac{d}{6} = \frac{d}{2}$$

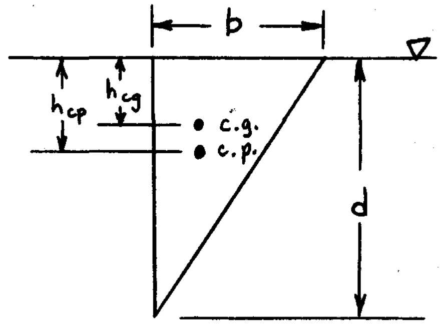


Fig. 3-3

3.4 A circular area of diameter d is vertical and submerged in a liquid. Its upper edge is coincident with the liquid surface (see Fig. 3-4). Derive an expression for the depth to its center of pressure.

▌

$$h_{cp} = h_{cg} + \frac{I_{cg}}{h_{cg}A} = \frac{d}{2} + \frac{\pi d^4/64}{(d/2)(\pi d^2/4)} = \frac{d}{2} + \frac{d}{8} = \frac{5d}{8}$$

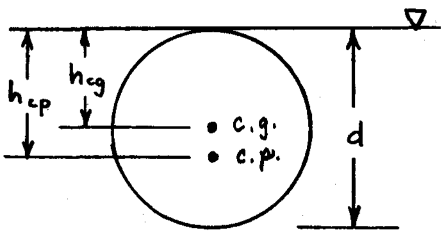


Fig. 3-4

3.5 A vertical semicircular area of diameter d and radius r is submerged and has its diameter in a liquid surface (see Fig. 3-5). Derive an expression for the depth to its center of pressure.

▌

$$h_{cp} = h_{cg} + \frac{I_{cg}}{h_{cg}A} \qquad h_{cg} = \frac{4r}{3\pi} \qquad I_x = \frac{1}{2}\left(\frac{\pi d^4}{64}\right) = \frac{1}{2}\left[\frac{\pi(2r)^4}{64}\right] = \frac{\pi r^4}{8}$$

$$I_{cg} = \frac{\pi r^4}{8} - \left(\frac{\pi r^2}{2}\right)\left(\frac{4r}{3\pi}\right)^2 = \left(\frac{\pi}{8} - \frac{8}{9\pi}\right)(r^4) \qquad h_{cp} = \frac{4r}{3\pi} + \frac{[\pi/8 - 8/(9\pi)](r^4)}{[4r/(3\pi)][(\pi r^2/2)]} = 0.589r$$

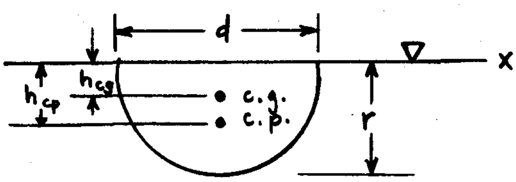


Fig. 3-5

3.6 A dam 20 m long retains 7 m of water, as shown in Fig. 3-6. Find the total resultant force acting on the dam and the location of the center of pressure.

▌ $F = \gamma hA = (9.79)[(0+7)/2][(20)(7/\sin 60°)] = 5339$ kN. The center of pressure is located at two-thirds the total water depth of 7 m, or 4.667 m below the water surface (i.e., $h_{cp} = 4.667$ m in Fig. 3-6).

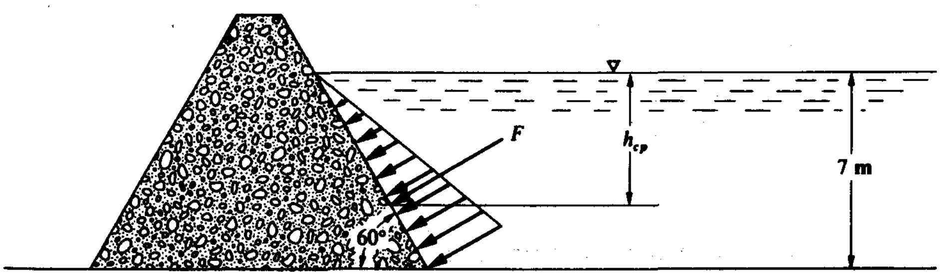


Fig. 3-6

3.7 A vertical, rectangular gate with water on one side is shown in Fig. 3-7. Determine the total resultant force acting on the gate and the location of the center of pressure.

▌

$$F = \gamma h_{cg} A = (9.79)(3 + 1.2/2)[(2)(1.2)] = 84.59 \text{ kN}$$

$$h_{cp} = h_{cg} + \frac{I_{cg}}{h_{cg} A} = \left(3 + \frac{1.2}{2}\right) + \frac{(2)(1.2)^3/12}{(3 + 1.2/2)[(2)(1.2)]} = 3.633 \text{ m}$$

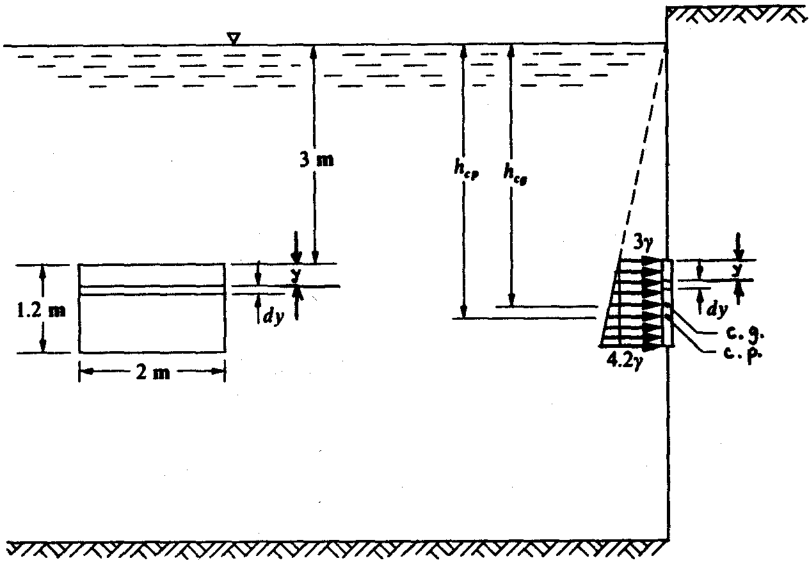


Fig. 3-7

3.8 Solve Prob. 3.7 by the integration method.

▌

$$F = \int \gamma h \, dA = \int_0^{1.2} (9.79)(3 + y)(2\,dy) = (19.58)\left[3y + \frac{y^2}{2}\right]_0^{1.2} = 84.59 \text{ kN}$$

$$h_{cp} = \frac{\int \gamma h^2 \, dA}{F} = \frac{\int_0^{1.2} (9.79)(3 + y)^2(2\,dy)}{84.59} = \frac{(19.58)[9y + 3y^2 + y^3/3]_0^{1.2}}{84.59} = 3.633 \text{ m}$$

3.9 A vertical, triangular gate with water on one side is shown in Fig. 3-8. Determine the total resultant force acting on the gate and the location of the center of pressure.

▌

$$F = \gamma h_{cg} A = (62.4)(6 + 3/3)[(2)(3)/2] = 1310 \text{ lb}$$

$$h_{cp} = h_{cg} + \frac{I_{cg}}{h_{cg} A} = \left(6 + \frac{3}{3}\right) + \frac{(2)(3)^3/36}{(6 + 3/3)[(2)(3)/2]} = 7.07 \text{ ft}$$

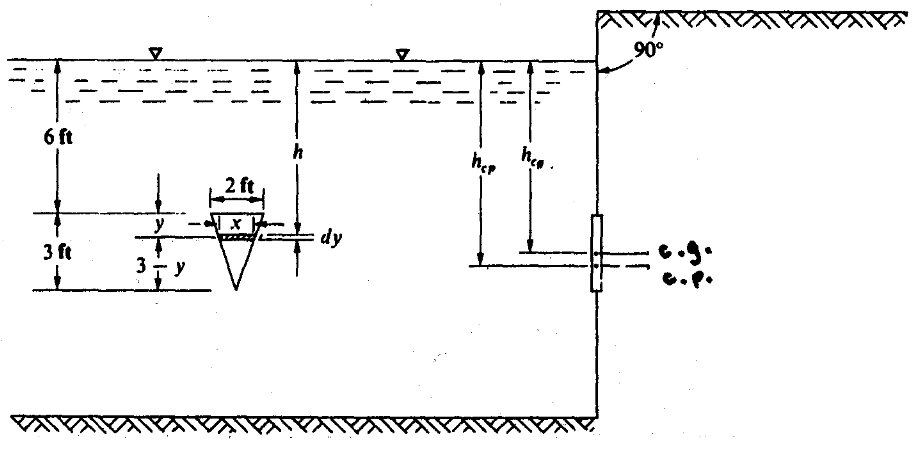


Fig. 3-8

3.10 Solve Prob. 3.9 by the integration method.

▮ $$F = \gamma h_{cg} A = [(0.82)(9.79)][4 + (1 + 1.2/2)(\sin 40°)][(0.8)(1.2)] = 38.75 \text{ kN}$$

$$F = \int_0^3 (62.4)(6+y)[(2 - 2y/3)\,dy] = \int_0^3 (62.4)(12 - 2y - 2y^2/3)\,dy = (62.4)[12y - y^2 - 2y^3/9]_0^3 = 1310 \text{ lb}$$

$$h_{cp} = \frac{\int \gamma h^2\,dA}{F} = \frac{\int_0^3 (62.4)(6+y)^2(2 - 2y/3)\,dy}{1310} = \frac{\int_0^3 (62.4)(72 - 6y^2 - 2y^3/3)\,dy}{1310}$$

$$= \frac{(62.4)[72y - 2y^3 - y^4/6]_0^3}{1310} = 7.07 \text{ ft}$$

3.11 An inclined, rectangular gate with water on one side is shown in Fig. 3-9. Determine the total resultant force acting on the gate and the location of the center of pressure.

▮ $$F = \gamma h_{cg} A = (62.4)[8 + \tfrac{1}{2}(4 \cos 60°)][(4)(5)] = 11\,230 \text{ lb}$$

$$z_{cp} = z_{cg} + \frac{I_{cg}}{z_{cg} A} = \left(\frac{8}{\cos 60°} + \frac{4}{2}\right) + \frac{(5)(4)^3/12}{(8/\cos 60° + \frac{4}{2})[(4)(5)]} = 18.07 \text{ ft}$$

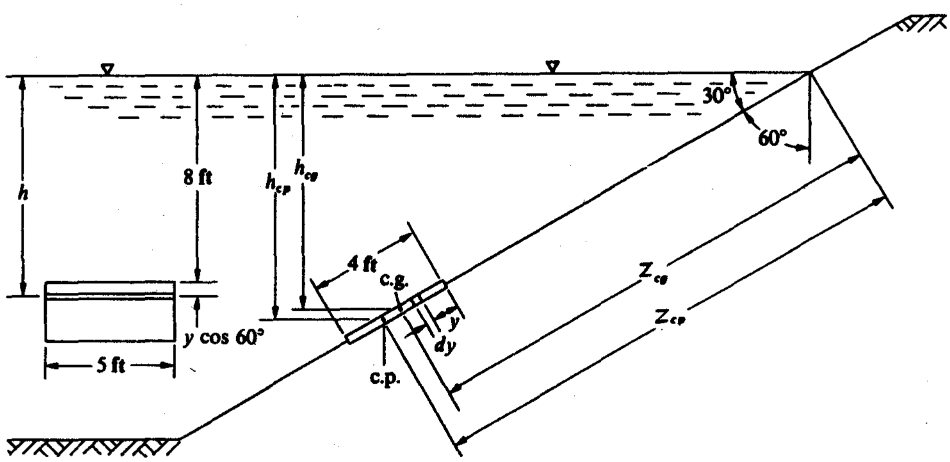


Fig. 3-9

3.12 Solve Prob. 3.11 by the integration method.

▮ $$F = \int \gamma h\,dA = \int_0^4 (62.4)(8 + y \cos 60°)(5\,dy) = (312)\left[8y + \frac{y^2}{4}\right]_0^4 = 11\,230 \text{ lb}$$

$$h_{cp} = \frac{\int \gamma h^2\,dA}{F} = \frac{\int_0^4 (62.4)(8 + y\cos 60°)^2(5\,dy)}{11\,230} = \frac{\int_0^4 (312)(64 + 8y + y^2/4)\,dy}{11\,230}$$

$$= \frac{(312)[64y + 4y^2 + y^3/12]_0^4}{11\,230} = 9.04 \text{ ft}$$

Note: h_{cp} is the vertical distance from the water surface to the center of pressure. The distance from the water surface to the center of pressure as measured along the inclination of the gate (z_{cp}) would be 9.04/cos 60°, or 18.08 ft.

3.13 An inclined, circular gate with water on one side is shown in Fig. 3-10. Determine the total resultant force acting on the gate and the location of the center of pressure.

▮ $$F = \gamma h_{cg} A = (9.79)[1.5 + \tfrac{1}{2}(1.0 \sin 60°)][\pi(1.0)^2/4] = 14.86 \text{ kN}$$

$$z_{cp} = z_{cg} + \frac{I_{cg}}{z_{cg} A} = \left[\frac{1.5}{\sin 60°} + \frac{1}{2}(1.0)\right] + \frac{\pi(1.0)^4/64}{[1.5/\sin 60° + \frac{1}{2}(1.0)][\pi(1.0)^2/4]} = 2.260 \text{ m}$$

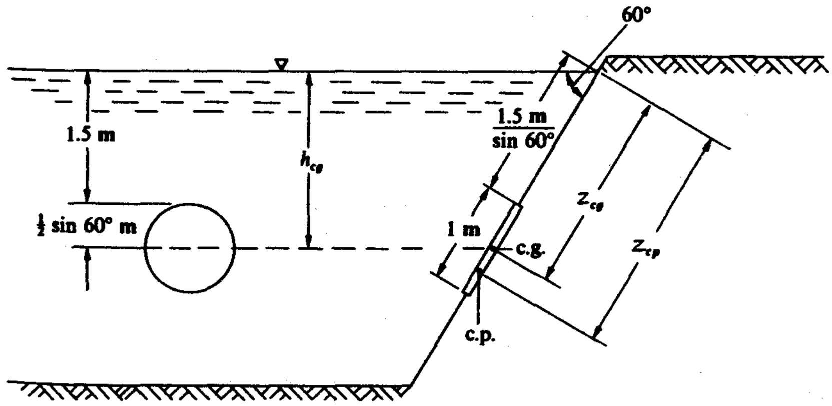


Fig. 3-10

3.14 A vertical, triangular gate with water on one side is shown in Fig. 3-11. Determine the total resultant force acting on the gate and the location of the center of pressure.

$$F = \gamma h_{cg} A = (9.79)[3 + \tfrac{2}{3}(1)][(1.2)(1)/2] = 21.54 \text{ kN}$$

$$h_{cp} = h_{cg} + \frac{I_{cg}}{h_{cg}A} = [3 + (\tfrac{2}{3})(1)] + \frac{(1.2)(1)^3/36}{[3 + \frac{2}{3}(1)][(1.2)(1)/2]} = 3.68 \text{ m}$$

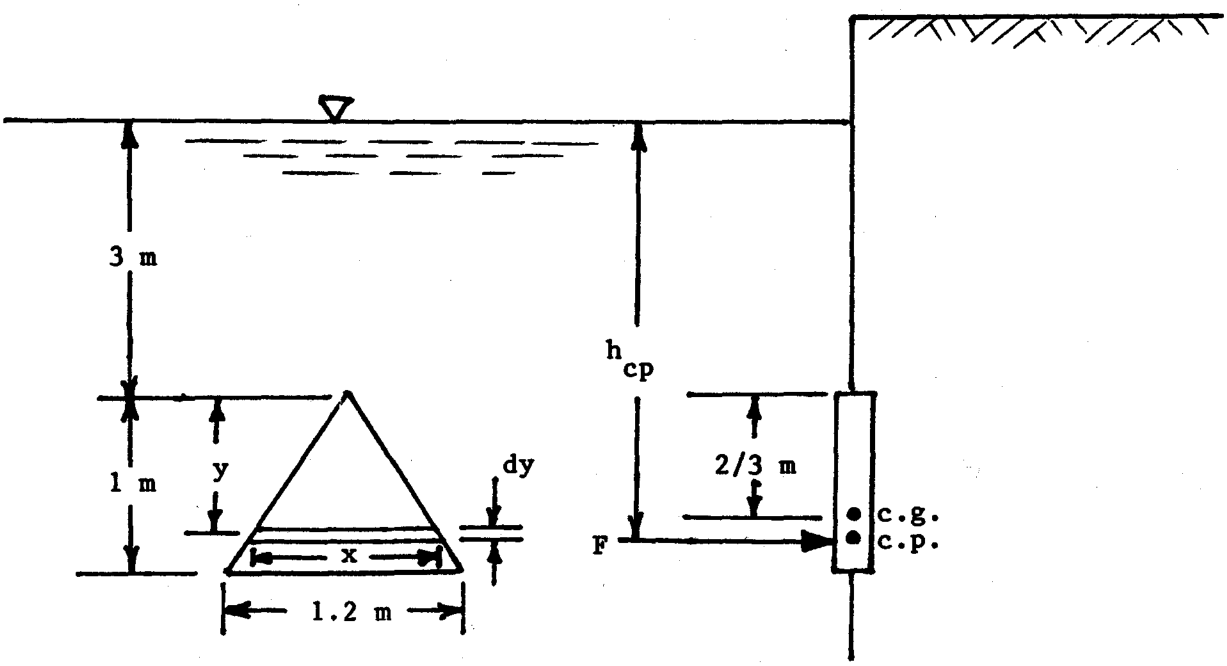


Fig. 3-11

3.15 Solve Prob. 3.14 by the integration method.

$F = \int \gamma h \, dA$. From Fig. 3-11, $y/x = 1/1.2$. Therefore, $x = 1.2y$.

$$F = \int_0^1 (9.79)(3 + y)(1.2y \, dy) = \int_0^1 (11.75)(3y + y^2) \, dy = (11.75)\left[\frac{3y^2}{2} + \frac{y^3}{3}\right]_0^1 = 21.54 \text{ kN}$$

$$h_{cp} = \frac{\int \gamma h^2 \, dA}{F} = \frac{\int_0^1 (9.79)(3 + y)^2(1.2y \, dy)}{21.54} = \frac{\int_0^1 (11.75)(9y + 6y^2 + y^3) \, dy}{21.54}$$

$$= \frac{(11.75)[9y^2/2 + 2y^3 + y^4/4]_0^1}{21.54} = 3.68 \text{ m}$$

3.16 A tank containing water is shown in Fig. 3-12. Calculate the total resultant force acting on side $ABCD$ of the container and the location of the center of pressure.

▮ $$F = \gamma h A = (62.4)[(0+6)/2][(20)(6)] = 22\,500 \text{ lb}$$

$$h_{cp} = (\tfrac{2}{3})(6) = 4.00 \text{ ft} \qquad \text{(vertically below the water surface)}$$

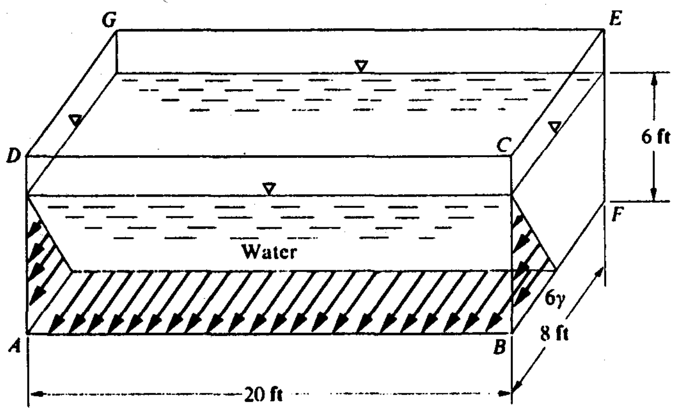


Fig. 3-12

3.17 The gate in Fig. 3-13 is 4 ft wide, is hinged at point B, and rests against a smooth wall at A. Compute (*a*) the force on the gate due to seawater pressure, (*b*) the (horizontal) force P exerted by the wall at point A, and (*c*) the reaction at hinge B.

▮ (*a*) $$F = \gamma h_{cg} A = (64)(17 - \tfrac{7.2}{2})[(4)(12)] = 30\,106 \text{ lb}$$

(*b*) $$y_{cp} = \frac{-I_{xx}\sin\theta}{h_{cg}A} = \frac{-[(4)(12)^3/12](\frac{7.2}{12})}{(17-\frac{7.2}{2})[(4)(12)]} = -0.537 \text{ ft}$$

$$\sum M_B = 0 \qquad (P)(7.2) - (30\,106)(12 - 6 - 0.537) = 0 \qquad P = 22\,843 \text{ lb}$$

(*c*) $$\sum F_x = 0 \qquad B_x + (30\,106)(\tfrac{7.2}{12}) - 22\,843 = 0 \qquad B_x = 4779 \text{ lb}$$

$$\sum F_y = 0 \qquad B_y - (30\,106)(\tfrac{9.6}{12}) = 0 \qquad B_y = 24\,085 \text{ lb}$$

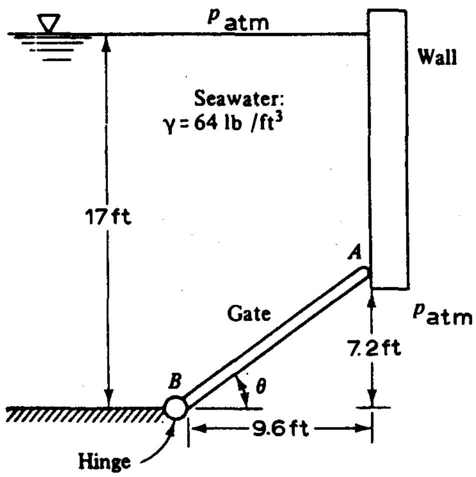


Fig. 3-13(*a*)

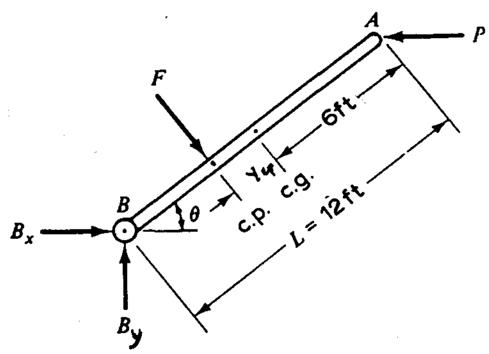


Fig. 3-13(*b*)

3.18 Repeat Prob. 3.17, but instead let the hinge be at point A and let point B rest against a smooth bottom.

▮ (*a*) From Prob. 3.17, $F = 30\,106$ lb. (*b*) From Prob. 3.17, $y_{cp} = -0.537$ ft; $\sum M_A = 0$; $(B_y)(9.6) - (30\,106)(6 + 0.537) = 0$, $B_y = 20\,500$ lb.

(*c*) $$\sum F_x = 0 \qquad (30\,106)(\tfrac{7.2}{12}) - A_x = 0 \qquad A_x = 18\,064 \text{ lb}$$

$$\sum F_y = 0 \qquad A_y - (30\,106)(\tfrac{9.6}{12}) + 20\,500 = 0 \qquad A_y = 3585 \text{ lb}$$

3.19 A tank of dye has a right-triangular panel near the bottom as shown in Fig. 3-14*a*. Calculate the resultant force on the panel and locate its center of pressure.

▮
$$F = \gamma h_{cg} A = \rho g h_{cg} A = (820)(9.81)(6+8)[\tfrac{1}{2}(8+16)(8+4)] = 16.22\ \text{MN}$$

$$I_{xx} = \frac{bh^3}{36} = \frac{(4+8)(8+16)^3}{36} = 4608\ \text{m}^4 \qquad y_{cp} = \frac{-I_{xx}\sin\theta}{h_{cg}A} = \frac{-(4608)(\sin 30°)}{(6+8)[\tfrac{1}{2}(8+16)(8+4)]} = -1.143\ \text{m}$$

$$I_{xy} = b(b-2s)(h)^2/72 = (4+8)[(4+8)-(2)(4+8)](8+16)^2/72 = -1152\ \text{m}^4$$

$$x_{cp} = \frac{-I_{xy}\sin\theta}{h_{cg}A} = \frac{-(-1152)(\sin 30°)}{(6+8)[\tfrac{1}{2}(8+16)(8+4)]} = +0.286\ \text{m}$$

(The resultant force acts at 1.143 m down and 0.286 m to the right of the centroid.)

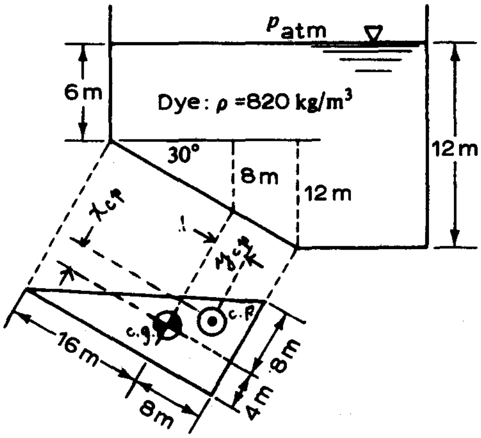
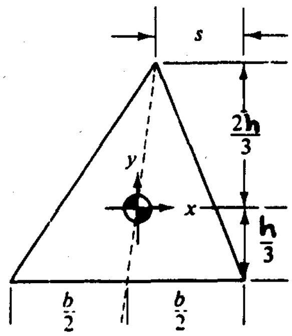


Fig. 3-14(*a*)



$$I_{xx} = \frac{bh^3}{36}$$

$$I_{xy} = \frac{b(b-2s)h^2}{72}$$

Fig. 3-14(*b*)

3.20 Gate *AB* in Fig. 3-15 is 1.0 m long and 0.9 m wide. Calculate force *F* on the gate and the position *X* of its center of pressure.

▮
$$F = \gamma h_{cg} A = [(0.81)(9.79)][3 + (1 + 1.0/2)(\sin 50°)][(0.9)(1.0)] = 29.61\ \text{kN}$$

$$y_{cp} = \frac{-I_{XX}\sin\theta}{h_{cg}A} = \frac{-[(0.9)(1.0)^3/12](\sin 50°)}{[3 + (1 + 1.0/2)(\sin 50°)][(0.9)(1.0)]}$$

$$= -0.015\ \text{m from the centroid}$$

$$X = 1.0/2 + 0.015 = 0.515\ \text{m from point } A$$

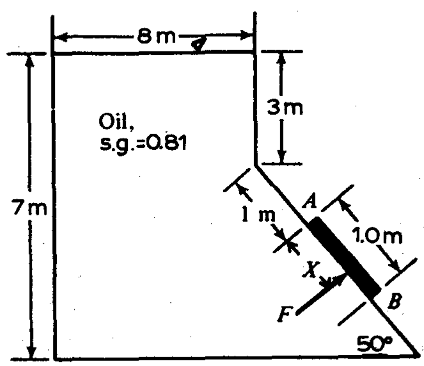


Fig. 3-15

3.21 A fishpond gate 6 ft wide and 9 ft high is hinged at the top and held closed by water pressure as shown in Fig. 3-16. What horizontal force applied at the bottom of the gate is required to open it?

$$F = \gamma h_{cg} A = (62.4)(8 + 4.5)[(6)(9)] = 42\,120 \text{ lb}$$

$$h_{cp} = h_{cg} + \frac{I_{cg}}{h_{cg}A} = (8 + 4.5) + \frac{(6)(9)^3/12}{(8 + 4.5)[(6)(9)]} = 13.04 \text{ ft}$$

$$\sum M_A = 0 \qquad (P)(9) - (42\,120)(13.04 - 8) = 0 \qquad P = 23\,587 \text{ lb}$$

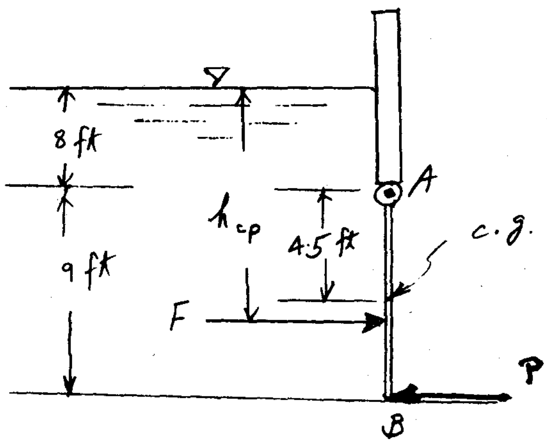


Fig. 3-16

3.22 A vat holding paint (s.g. = 0.80) is 8 m long and 4 m deep and has a trapezoidal cross section 3 m wide at the bottom and 5 m wide at the top (see Fig. 3-17). Compute (*a*) the weight of the paint, (*b*) the force on the bottom of the vat, and (*c*) the force on the trapezoidal end panel.

(*a*) $$W = \gamma V = [(0.80)(9.79)][(8)(4)(5 + 3)/2] = 1002 \text{ kN}$$

(*b*) $$F = \gamma h_{cg} A \qquad F_{bottom} = [(0.80)(9.79)](4)[(3)(8)] = 752 \text{ kN}$$

(*c*) $$F_{end} = F_{square} + 2F_{triangle} = [(0.80)(9.79)][(0 + 4)/2][(4)(3)] + (2)[(0.80)(9.79)](\tfrac{4}{3})[(4)(1)/2] = 230 \text{ kN}$$

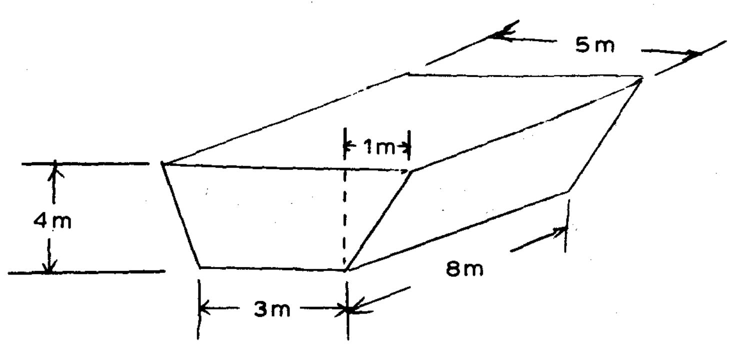


Fig. 3-17

3.23 Gate *AB* in Fig. 3-18 is 5 ft wide, hinged at point *A*, and restrained by a stop at point *B*. Compute the force on the stop and the components of the reaction at *A* if water depth *h* is 9 ft.

$$F = \gamma h_{cg} A = (62.4)(9 - \tfrac{3}{2})[(3)(5)] = 7020 \text{ lb}$$

$$y_{cp} = \frac{-I_{xx} \sin\theta}{h_{cg}A} = \frac{-[(5)(3)^3/12](\sin 90°)}{(9 - 3/2)[(3)(5)]} = -0.100 \text{ ft}$$

$$\sum M_A = 0 \qquad (B_x)(3) - (7020)(1.5 + 0.100) = 0 \qquad B_x = 3744 \text{ lb}$$

$$\sum F_x = 0 \qquad 7020 - 3744 - A_x = 0 \qquad A_x = 3276 \text{ lb}$$

If gate weight is neglected, $A_y = 0$.

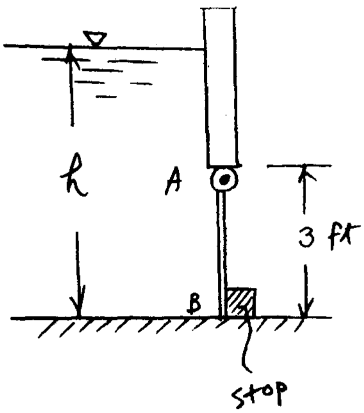


Fig. 3-18(a)

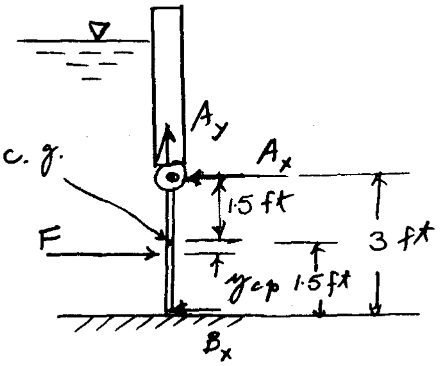


Fig. 3-18(b)

3.24 In Fig. 3-18, stop *B* will break if the force on it reaches 9000 lb. Find the critical water depth.

$$F = \gamma h_{cg} A = (62.4)(h_{cg})[(3)(5)] = 936h_{cg} \qquad y_{cp} = \frac{-I_{xx}\sin\theta}{h_{cg}A} = \frac{-[(5)(3)^3/12](\sin 90°)}{(h_{cg})[(3)(5)]} = -\frac{0.750}{h_{cg}}$$

$$\sum M_A = 0 \qquad (9000)(3) - (936h_{cg})(1.5 + 0.750/h_{cg}) = 0$$

$$h_{cg} = 18.73\text{ ft} \qquad h_{crit} = 18.73 + 1.5 = 20.23\text{ ft}$$

3.25 In Fig. 3-18, hinge *A* will break if its horizontal reaction becomes equal to 8000 lb. Find the critical water depth.

From Prob. 3.24, $F = 936h_{cg}$ and $y_{cp} = -0.750/h_{cg}$; $\sum M_B = 0$; $(936h_{cg})(1.5 - 0.750/h_{cg}) - (8000)(3) = 0$, $h_{cg} = 17.59$ ft; $h_{crit} = 17.59 + 1.5 = 19.09$ ft.

3.26 Calculate the resultant force on triangular window *ABC* in Fig. 3-19 and locate its center of pressure.

$$F = \gamma h_{cg} A = (10.08)[0.25 + (\tfrac{2}{3})(0.60)][(0.40)(0.60)/2] = 0.786\text{ kN}$$

$$I_{xx} = bh^3/36 = (0.40)(0.60)^3/36 = 0.00240\text{ m}^4$$

$$y_{cp} = \frac{-I_{xx}\sin\theta}{h_{cg}A} = \frac{-(0.00240)(\sin 90°)}{[0.25 + (\frac{2}{3})(0.60)][(0.40)(0.60)/2]} = -31\text{ mm} \qquad \text{(i.e., below the centroid)}$$

$$I_{xy} = b(b - 2s)(h)^2/72 = 0.40[0.40 - (2)(0.40)](0.60)^2/72 = -0.000800\text{ m}^4$$

$$x_{cp} = \frac{-I_{xy}\sin\theta}{h_{cg}A} = \frac{-(-0.000800)(\sin 90°)}{[0.25 + (\frac{2}{3})(0.60)][(0.40)(0.60)/2]} = +10\text{ mm} \qquad \text{(i.e., right of the centroid)}$$

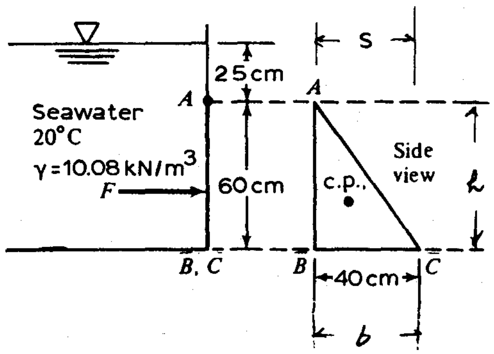


Fig. 3-19

3.27 Freshly poured concrete approximates a fluid with s.g. = 2.40. Figure 3-20 shows a slab poured between wooden forms which are connected by four corner bolts A, B, C, and D. Neglecting end effects, compute the forces in the four bolts.

▮

$$F = \gamma h_{cg} A = [(2.40)(62.4)](\tfrac{12}{2})[(9)(12)] = 97\,044 \text{ lb}$$

$$y_{cp} = \frac{-I_{xx}\sin\theta}{h_{cg}A} = \frac{-[(9)(12)^3/12](\sin 90°)}{[(\frac{12}{2})][(9)(12)]} = -2.00 \text{ ft}$$

$$\sum M_A = 0 \qquad (2)(F_C)(12) - (97\,044)(6 + 2.00) = 0 \qquad F_C = F_D = 32\,348 \text{ lb}$$

$$\sum M_C = 0 \qquad (97\,044)(6 - 2.00) - (2)(F_A)(12) = 0 \qquad F_A = F_B = 16\,174 \text{ lb}$$

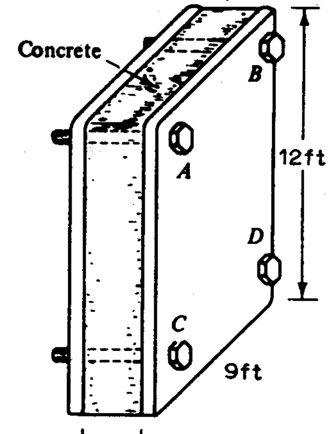


Fig. 3-20

3.28 Find the net hydrostatic force per unit width on rectangular panel AB in Fig. 3-21 and determine its line of action.

▮

$$F_{H_2O} = (9.79)(2 + 1 + \tfrac{2}{2})[(2)(1)] = 78.32 \text{ kN} \qquad F_{glyc} = (12.36)(1 + \tfrac{2}{2})[(2)(1)] = 49.44 \text{ kN}$$

$$F_{net} = F_{H_2O} - F_{glyc} = 78.32 - 49.44 = 28.88 \text{ kN}$$

$$y_{cp} = \frac{-I_{xx}\sin\theta}{h_{cg}A}$$

$$(y_{cp})_{H_2O} = \frac{-[(1)(2)^3/12](\sin 90°)}{(2 + 1 + \frac{2}{2})[(2)(1)]} = -0.0833 \text{ m}$$

$$(y_{cp})_{glyc} = \frac{-[(1)(2)^3/12](\sin 90°)}{[(1 + \frac{2}{2})][(2)(1)]} = -0.1667 \text{ m}$$

$$\sum M_B = 0 \qquad (78.32)(1 - 0.0833) - (49.44)(1 - 0.1667) = 28.88D$$

$$D = 1.059 \text{ m} \qquad \text{(above point } B\text{, as shown in Fig. 3-21}c\text{)}$$

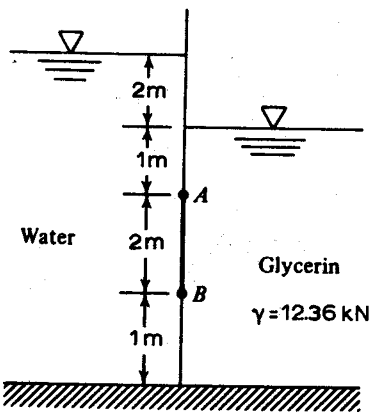


Fig. 3-21(a)

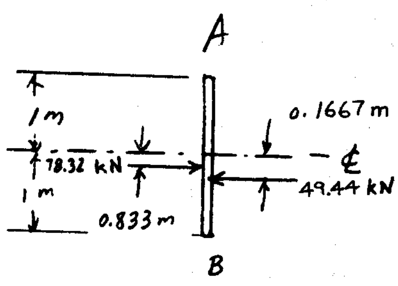


Fig. 3-21(b)

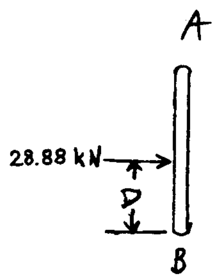


Fig. 3-21(c)

3.29 A cylindrical, wooden-stave barrel is 3.5 ft in diameter and 5 ft high, as shown in Fig. 3-22. It is held together by steel hoops at the top and bottom, each with a cross section of 0.40 in². If the barrel is filled with orange juice (s.g. = 1.04), compute the tension stress in each hoop.

▌

$$F = \gamma h_{cg} A = [(1.04)(62.4)](\tfrac{5}{2})[(3.5)(5)] = 2839 \text{ lb}$$

$$y_{cp} = \frac{-I_{xx}\sin\theta}{h_{cg}A} = \frac{-[(3.5)(5)^3/12](\sin 90°)}{\frac{5}{2}[(3.5)(5)]} = -0.833 \text{ ft}$$

$$\sum M_B = 0 \qquad 2839(\tfrac{5}{2} - 0.833) - 2(F_{\text{upper}})(5) = 0 \qquad F_{\text{upper}} = 473 \text{ lb}$$

$$\sum M_A = 0 \qquad 2(F_{\text{lower}})(5) - 2839(\tfrac{5}{2} + 0.833) = 0 \qquad F_{\text{lower}} = 946 \text{ lb}$$

$$\sigma_{\text{upper}} = 473/0.40 = 1182 \text{ psi} \qquad \sigma_{\text{lower}} = 946/0.40 = 2365 \text{ psi}$$

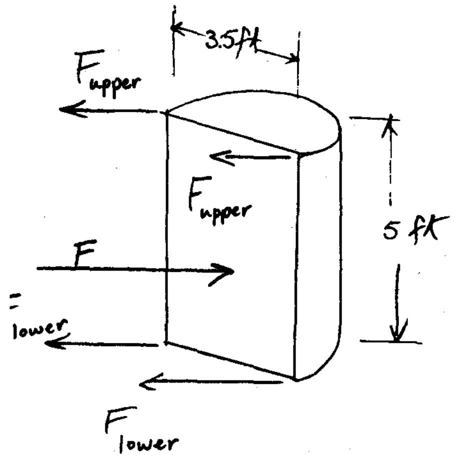


Fig. 3-22(*a*)

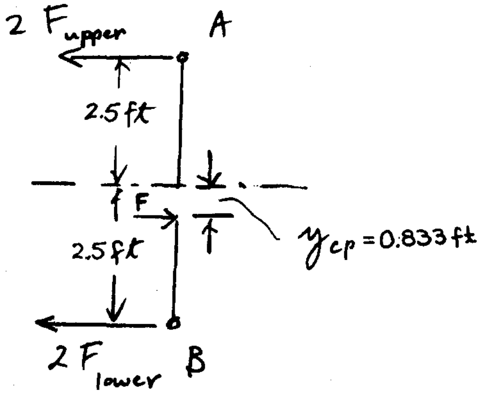


Fig. 3-22(*b*)

3.30 Gate *AB* in Fig. 3-23*a* is 16 ft long and 8 ft wide. Neglecting the weight of the gate, compute the water level *h* for which the gate will start to fall.

▌

$$F = \gamma h_{cg} A = (62.4)(h/2)[(8)(h/\sin 60°)] = 288.2h^2$$

$$y_{cp} = \frac{-I_{xx}\sin\theta}{h_{cg}A} = \frac{-[8(h/\sin 60°)^3/12](\sin 60°)}{(h/2)[8(h/\sin 60°)]} = -0.1925h$$

$$\sum M_B = 0 \qquad (11\,000)(16) - (288.2h^2)[(h/\sin 60°)/2 - 0.1925h] = 0 \qquad h = 11.7 \text{ ft}$$

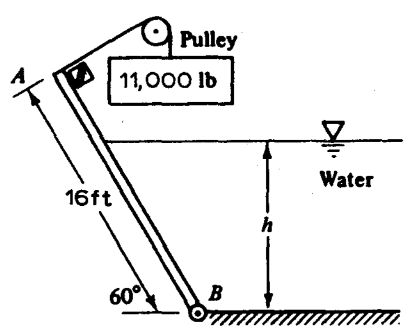


Fig. 3-23(*a*)

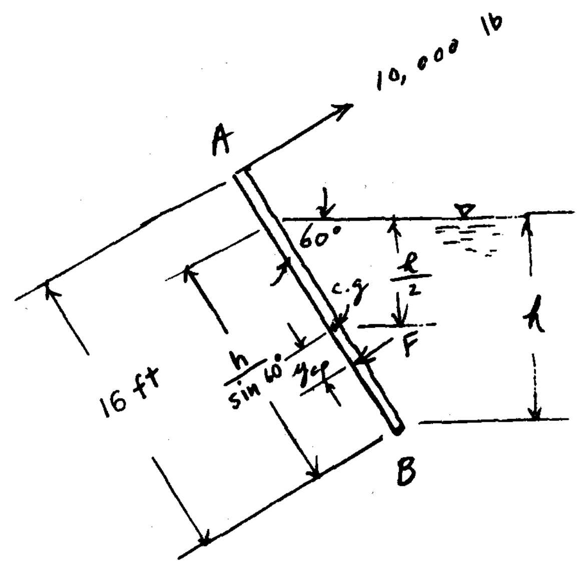


Fig. 3-23(*b*)

3.31 Repeat Prob. 3.30, including the weight of the 2-in-thick steel (s.g. = 7.85) gate. (See Fig. 3-24.)

▮ $W_{gate} = [(7.85)(62.4)][(16)(8)(\frac{2}{12})] = 10\,450$ lb. From Prob. 3.30, $F = 288.2h^2$; $\sum M_B = 0$, $(11\,000)(16) - (288.2h^2)[(h/\sin 60°)/2 - 0.1925h] - 10\,450(\frac{16}{2}\cos 60°) = 0$, $h = 10.7$ ft.

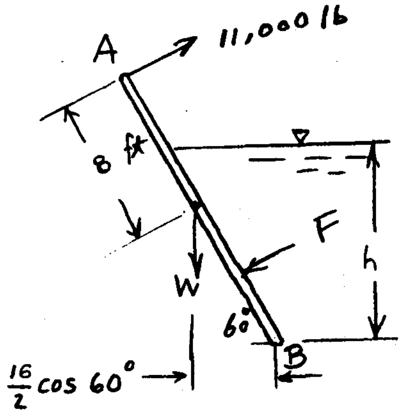


Fig. 3-24

3.32 A horizontal duct coming from a large dam is 2.5 m in diameter; it is closed by a circular door whose center or centroid is 45 m below the dam's water level. Compute the force on the door and locate its center of pressure.

▮ $$F = \gamma h_{cg} A = (9.79)(45)[\pi(2.5)^2/4] = 2163 \text{ kN} \qquad I_{xx} = \pi r^4/4 = \pi(\tfrac{2.5}{2})^4/4 = 1.917 \text{ m}^4$$

$$y_{cp} = \frac{-I_{xx}\sin\theta}{h_{cg}A} = \frac{-(1.917)(\sin 90°)}{(45)[\pi(2.5)^2/4)]} = -0.0087 \text{ m}$$

Line of action of F is 8.7 mm below the centroid of the door.

3.33 Gate AB in Fig. 3-25 is semicircular, hinged at B. What horizontal force P is required at A for equilibrium?

▮ $$4r/(3\pi) = (4)(4)/(3\pi) = 1.698 \text{ m} \qquad F = \gamma h_{cg} A = (9.79)(6 + 4 - 1.698)[\pi(4)^2/2)] = 2043 \text{ kN}$$

$$y_{cp} = \frac{-I_{xx}\sin\theta}{h_{cg}A} = \frac{-[(0.10976)(4)^4](\sin 90°)}{(6 + 4 - 1.698)[\pi(4)^2/2)]} = -0.1347 \text{ m}$$

$$\sum M_B = 0 \qquad (2043)(1.698 - 0.1347) - 4P = 0 \qquad P = 798 \text{ kN}$$

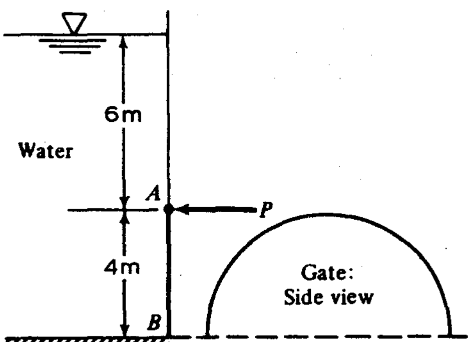


Fig. 3-25(a)

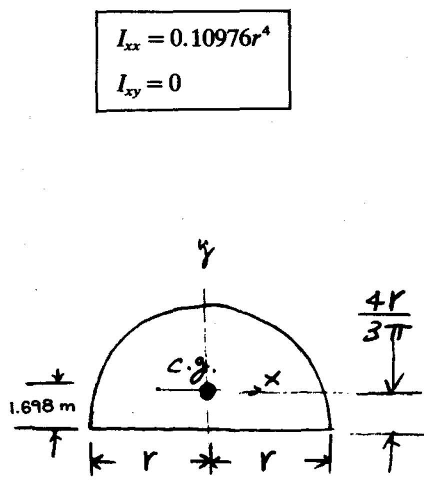


Fig. 3-25(b)

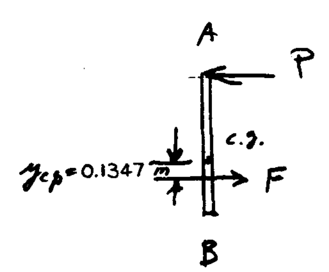


Fig. 3-25(c)

3.34 Dam ABC in Fig. 3-26 is 38 m wide and made of concrete weighing 22 kN/m³. Find the hydrostatic force on surface AB and its moment about C. Could this force tip the dam over?

▮ $F = \gamma h_{cg} A = (9.79)(\frac{64}{2})[(38)(80)] = 952\,371$ kN. F acts at $(\frac{2}{3})(80)$, or 53.33 m from A along surface AB (see Fig. 3-26b). For the given triangular shape, the altitude from C to AB intersects AB 51.2 m from A (see Fig. 3-26b). Hence, $M_C = (952\,371)(53.33 - 51.2) = 2\,028\,550$ kN. Since the moment of F about point C is counterclockwise, there is no danger of tipping.

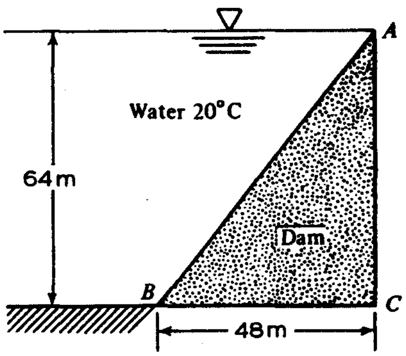


Fig. 3-26(a)

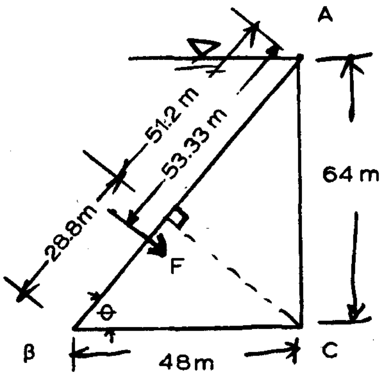


Fig. 3-26(b)

3.35 Isosceles triangular gate AB in Fig. 3-27 is hinged at A. Compute the horizontal force P required at point B for equilibrium, neglecting the weight of the gate.

▮ $AB = 3/\sin 60° = 3.464$ m $\qquad F = \gamma h_{cg} A = [(0.82)(9.79)](2 + 1.00)[(1.2)(3.464)/2] = 50.05$ kN

$$y_{cp} = \frac{-I_{xx} \sin\theta}{h_{cg} A} = \frac{-[(1.2)(3.464)^3/36](\sin 60°)}{(2 + 1.00)[(1.2)(3.464)/2]} = -0.1924 \text{ m}$$

$$\sum M_A = 0 \qquad 3P - (50.05)(3.464/3 + 0.1924) = 0 \qquad P = 22.47 \text{ kN}$$

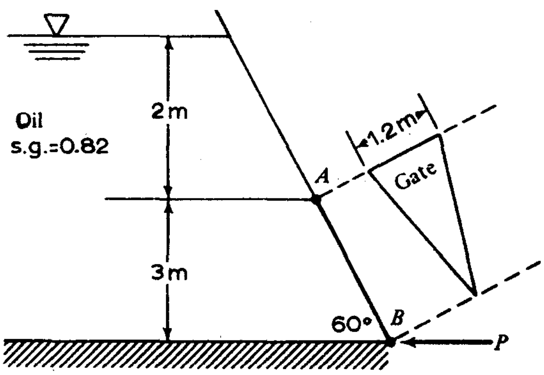


Fig. 3-27(a)

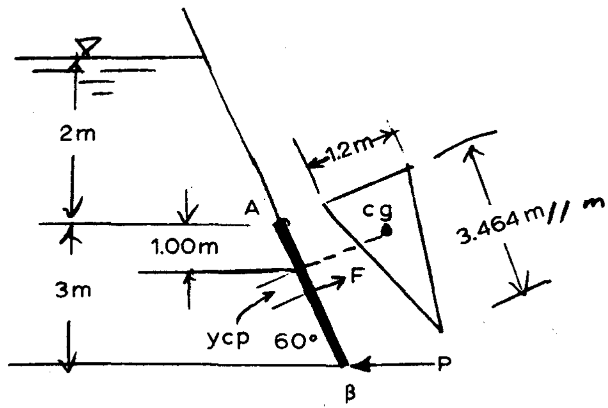


Fig. 3-27(b)

3.36 The tank in Fig. 3-28 is 40 cm wide. Compute the hydrostatic forces on horizontal panels BC and AD. Neglect atmospheric pressure.

▮ $p = \gamma h \qquad p_{BC} = [(0.84)(9.79)](0.35 + 0.40) + (9.79)(0.25) = 8.615$ kPa

$$F = pA \qquad F_{BC} = (8.615)[(1.20)(0.40)] = 4.135 \text{ kN}$$

$$p_{AD} = [(0.84)(9.79)](0.40) = 3.289 \text{ kPa} \qquad F_{AD} = (3.289)[(0.55)(0.40)] = 0.724 \text{ kN}$$

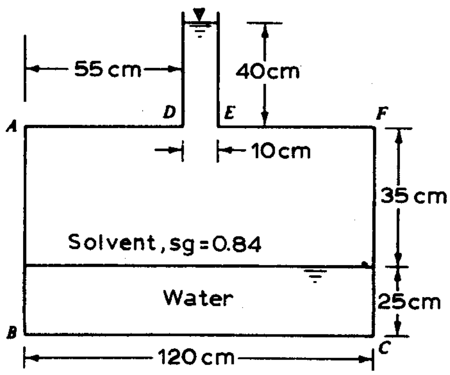


Fig. 3-28

3.37 Water in a tank is pressurized to 85 cmHg (Fig. 3-29). Determine the hydrostatic force per meter width on panel *AB*.

▌ On panel *AB*, $p_{cg} = [(13.6)(9.79)](0.85) + (9.79)(4 + \frac{3}{2}) = 167.0$ kPa, $F_{AB} = (167.0)[(3)(1)] = 501$ kN.

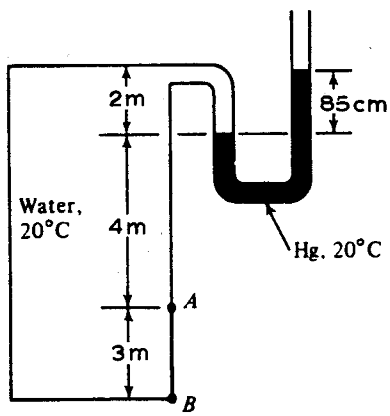


Fig. 3-29

3.38 Calculate the force and center of pressure on one side of the vertical triangular panel *ABC* in Fig. 3-30.

▌
$$F = \gamma h_{cg} A = (62.4)(1+6)[(9)(6)/2] = 11\,794 \text{ lb} \qquad I_{xx} = (6)(9)^3/36 = 121.5 \text{ ft}^4$$

$$y_{cp} = \frac{-I_{xx}\sin\theta}{h_{cg}A} = \frac{-(121.5)(\sin 90°)}{(1+6)[(9)(6)/2]} = -0.64 \text{ ft}$$

$$I_{xy} = \frac{6[6-(2)(6)](9)^2}{72} = -40.5 \text{ ft}^4 \qquad x_{cp} = \frac{-I_{xy}\sin\theta}{h_{cg}A} = \frac{-(-40.5)(\sin 90°)}{(1+6)[(9)(6)/2]} = 0.21 \text{ ft}$$

Thus, the center of pressure is 6 + 0.64, or 6.64 ft below point *A* and 2 + 0.21, or 2.21 ft to the right of point *B*.

3.39 In Fig. 3-31, gate *AB* is 4 m wide and is connected by a rod and pulley to a massive sphere (s.g. = 2.40). What is the smallest radius that will keep the gate closed?

▌
$$F = \gamma h_{cg} A = (9.79)(9 + \tfrac{3}{2})[(4)(3)] = 1234 \text{ kN}$$

$$y_{cp} = \frac{-I_{xx}\sin\theta}{h_{cg}A} = \frac{-[(4)(3)^3/12](\sin 90°)}{(9+\frac{3}{2})[(4)(3)]} = -0.071 \text{ m}$$

$$\sum M_B = 0 \qquad (W_{sphere})(7+9+3) - (1234)(3-1.5-0.071) = 0 \qquad W_{sphere} = 92.8 \text{ kN}$$

$$W_{sphere} = \gamma(4\pi r^3/3) \qquad 92.8 = [(2.40)(9.79)](4\pi r^3/3) \qquad r = 0.98 \text{ m}$$

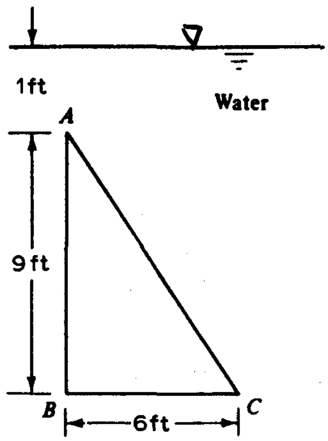


Fig. 3-30(a)

$$I_{xx} = \frac{bh^3}{36}$$

$$I_{xy} = \frac{b(b-2s)h^2}{72}$$

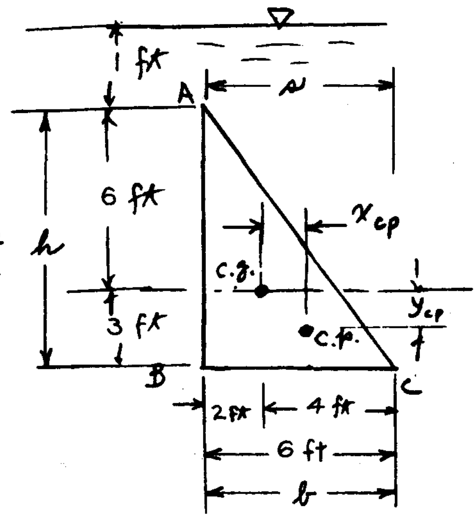


Fig. 3-30(b)

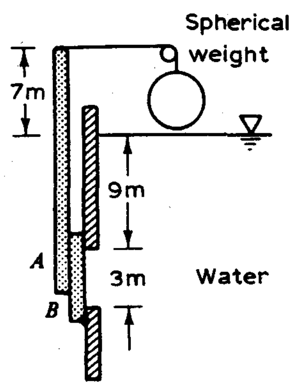


Fig. 3-31

3.40 The triangular trough in Fig. 3-32 is hinged at A and held together by cable BC at the top. If cable spacing is 1 m into the paper, what is the cable tension?

$$F = \gamma h_{cg} A = (9.79)(\tfrac{5}{2})[(8.717)(1)] = 213.3 \text{ kN}$$

$$y_{cp} = \frac{-I_{xx}\sin\theta}{h_{cg}A} = \frac{-[(1)(8.717)^3/12](\sin 35°)}{\frac{5}{2}[(8.717)(1)]} = -1.453 \text{ m}$$

$$\sum M_A = 0 \qquad (T)(2+5) - (213.3)(4.359 - 1.453) = 0 \qquad T = 88.5 \text{ kN}$$

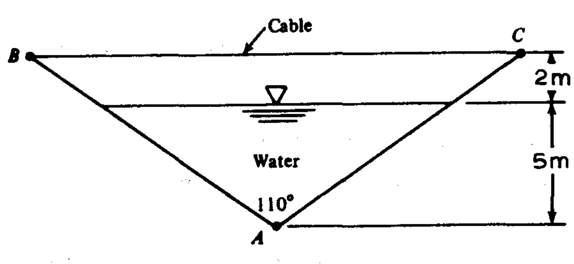


Fig. 3-32(a)

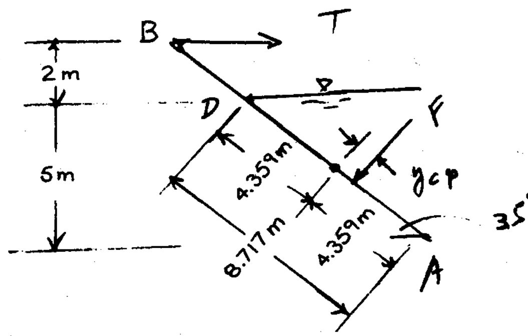


Fig. 3-32(b)

3.41 In Fig. 3-33, gate AB is 4 ft wide and opens to let fresh water out when the ocean tide is falling. The hinge at A is 3 ft above the fresh-water surface. At what ocean depth h will the gate open? Neglect the gate's weight.

▮ $F = \gamma h_{cg} A \qquad F_1 = (62.4)(\frac{12}{2})[(12)(4)] = 17\,971 \text{ lb} \qquad F_2 = [(1.025)(62.4)](h/2)[(4)(h)] = 127.9h^2$

$$\sum M_A = 0 \qquad (127.9h^2)(12 + 3 - h/3) - (17\,971)(3 + 8) = 0 \qquad h = 11.8 \text{ ft}$$

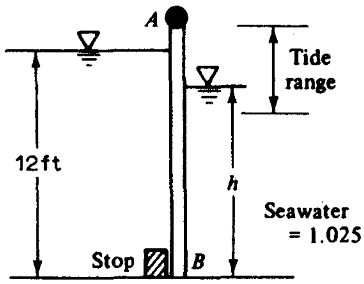


Fig. 3-33(*a*)

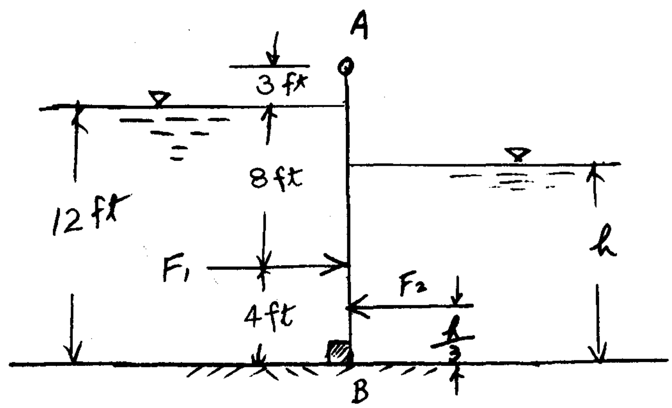


Fig. 3-33(*b*)

3.42 Show that depth h in Prob. 3.41 is independent of gate width b (perpendicular to the paper).

▮ Areas, and hence pressure forces, are directly proportional to b. Thus b will cancel out of the equation $\sum M_A = 0$ that determines h.

3.43 Compute the force on one side of parabolic panel ABC in Fig. 3-34 and the vertical distance down to the center of pressure.

▮
$$F = \gamma h_{cg} A = (9.79)(1 + 6)[(\tfrac{2}{3})(10)(6)] = 2741 \text{ kN}$$

$$I_{xx} = I_{x'} - A(\Delta h)^2 = \tfrac{2}{7}(bh^3) - [\tfrac{2}{3}(bh)][\tfrac{3}{5}(h)]^2 = (\tfrac{2}{7})(6)(10)^3 - [(\tfrac{2}{3})(6)(10)][(\tfrac{3}{5})(10)]^2 = 274.3 \text{ m}^4$$

$$y_{cp} = \frac{-I_{xx} \sin \theta}{h_{cg} A} = \frac{-(274.3)(\sin 90^\circ)}{(1 + 6)[(\tfrac{2}{3})(10)(6)]} = -0.980 \text{ m}$$

Hence, the center of pressure is 6 + 0.980, or 6.980 m below point A.

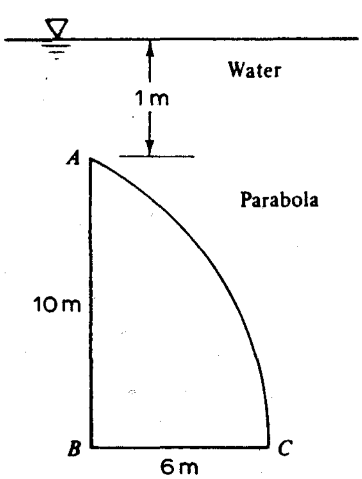


Fig. 3-34(*a*)

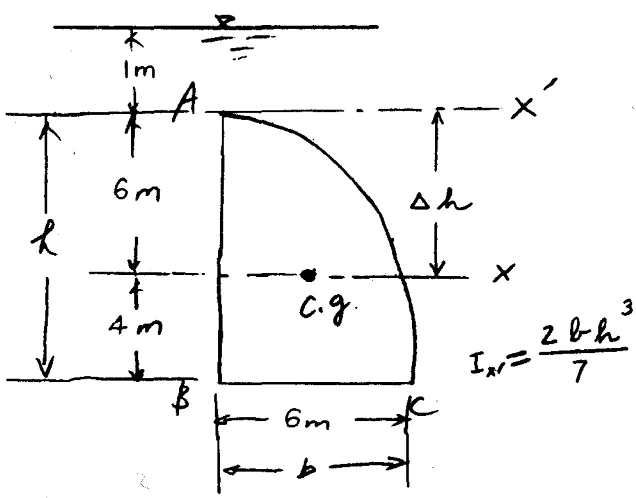


Fig. 3-34(*b*)

3.44 Circular gate ABC in Fig. 3-35 is 4 m in diameter and is hinged at B. Compute the force P just sufficient to keep the gate from opening when h is 8 m.

▌ $F = \gamma h_{cg}A = (9.79)(8)[\pi(4)^2/4] = 984.2\ \text{kN} \qquad I_{xx} = \pi d^4/64 = \pi(4)^4/64 = 12.57\ \text{m}^4$

$$y_{cp} = \frac{-I_{xx}\sin\theta}{h_{cg}A} = \frac{-(12.57)(\sin 90°)}{(8)[(\pi)(2)^2]} = -0.125\ \text{m}$$

$$\sum M_B = 0 \qquad (P)(2) - (984.2)(0.125) = 0 \qquad P = 61.5\ \text{kN}$$

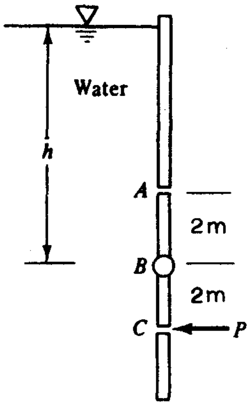


Fig. 3-35(a)

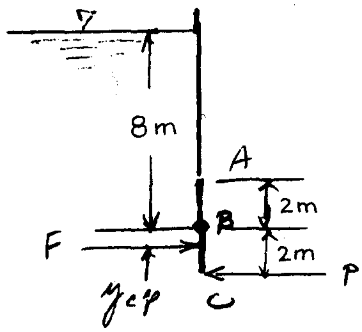


Fig. 3-35(b)

3.45 For the conditions given in Prob. 3.44, derive an analytical expression for P as a function of h.

▌ $F = \gamma h_{cg}A = \gamma h_{cg}[\pi(r)^2] \qquad I_{xx} = \pi(r)^4/4$

$$y_{cp} = \frac{-I_{xx}\sin\theta}{h_{cg}A} = \frac{-[(\pi)(r)^4/4](\sin 90°)}{h[(\pi)(r)^2]} = \frac{-r^2}{4h}$$

$$\sum M_B = 0 \qquad Pr - [\gamma h_{cg}(\pi)(r)^2][(r)^2/(4r)] = 0 \qquad P = \gamma\pi r^3/4$$

(Note that force P is independent of depth h.)

3.46 Gate ABC in Fig. 3-36 is 2 m square and hinged at B. How large must h be for the gate to open?

▌ The gate will open when resultant force F acts above point B—i.e., when $|y_{cp}| < 0.2$ m. (Note in Fig. 3-36b that y_{cp} is the distance between F and the centroid of gate ABC.)

$$y_{cp} = \frac{-I_{xx}\sin\theta}{h_{cg}A} = \frac{-[(2)(2)^3/12](\sin 90°)}{(h+1.0)[(2)(2)]} = \frac{-1.333}{4h+4}$$

For $|y_{cp}| < 0.2$, $1.333/(4h+4) < 0.2$, $h > 0.666$ m. (Note that this result is independent of fluid weight.)

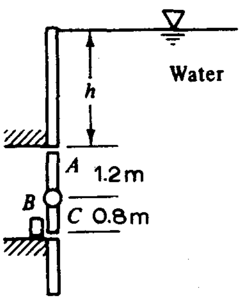


Fig. 3-36(a)

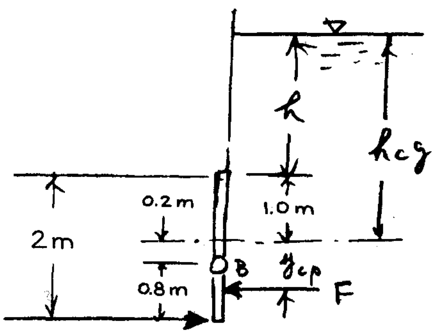


Fig. 3-36(b)

3.47 Gate AB in Fig. 3-37 is 6 ft wide and weighs 2000 lb when submerged. It is hinged at B and rests against a smooth wall at A. Determine the water level h which will just cause the gate to open.

▌ $F = \gamma h_{cg} A \qquad F_1 = 62.4(h + \frac{8}{2})[(10)(6)] = 3744h + 14\,976 \qquad F_2 = 62.4(5 + \frac{8}{2})[(10)(6)] = 33\,696 \text{ lb}$

$$y_{cp} = \frac{-I_{xx} \sin\theta}{h_{cg} A} \qquad (y_{cp})_1 = \frac{-[(6)(10)^3/12](\frac{8}{10})}{(h + \frac{8}{2})[(10)(6)]} = \frac{-6.67}{h+4}$$

$$(y_{cp})_2 = \frac{-[(6)(10)^3/12](\frac{8}{10})}{(5 + \frac{8}{2})[(10)(6)]} = -0.741 \text{ ft}$$

$\sum M_B = 0 \qquad (3744h + 14\,976)[5 - 6.67/(h+4)] - (33\,696)(5 - 0.741) - (2000)(\frac{6}{2}) = 0 \qquad h = 5.32 \text{ ft}$

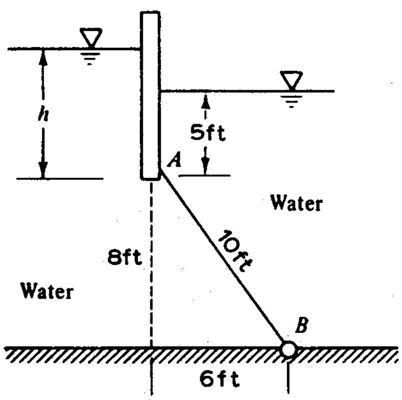


Fig. 3-37(a)

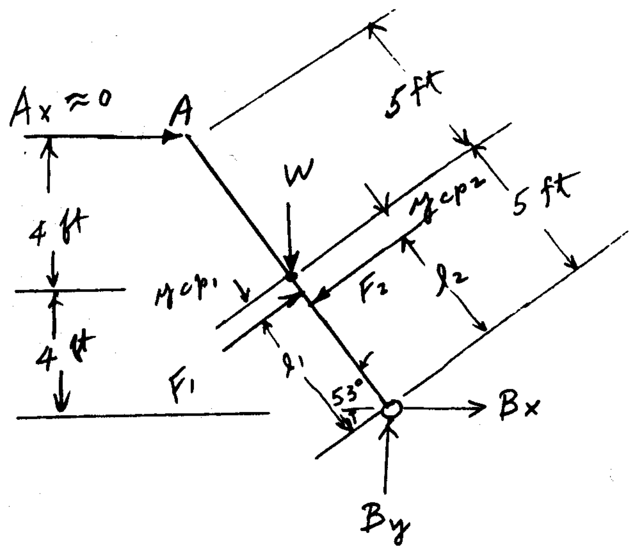


Fig. 3-37(b)

3.48 The tank in Fig. 3-38 contains oil and water as shown. Find the resultant force on side ABC, which is 4 ft wide.

▌ $F = \gamma h_{cg} A \qquad F_{AB} = [(0.80)(62.4)](\frac{10}{2})[(10)(4)] = 9980 \text{ lb}$

F_{AB} acts at a point $(\frac{2}{3})(10)$, or 6.67 ft below point A. Water is acting on area BC, and any superimposed liquid can be converted to an equivalent depth of water. Employ an imaginary water surface (IWS) for this calculation, locating IWS by changing 10 ft of oil to (0.80)(10), or 8 ft of water. Thus, $F_{BC} = (62.4)(8 + \frac{6}{2})[(6)(4)] = 16\,470$ lb.

$$y_{cp} = \frac{-I_{xx} \sin\theta}{h_{cg} A} = \frac{-[(4)(6)^3/12](\sin 90°)}{(8 + \frac{6}{2})[(6)(4)]} = -0.27 \text{ ft} \qquad \text{(i.e., below the centroid of } BC\text{)}$$

F_{BC} acts at a point $(2 + 8 + \frac{6}{2} + 0.27)$, or 13.27 ft below A. $\sum M_A = 0$; $(9980 + 16\,470)(h_{cp}) - (9980)(6.67) - (16\,470)(13.27) = 0$, $h_{cp} = 10.78$ ft from A. Thus, the total resultant force on side ABC is 9980 + 16 470, or 26 450 lb acting 10.78 ft below A.

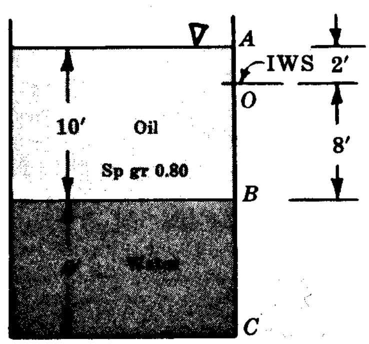


Fig. 3-38

3.49 Gate AB in Fig. 3-39 is 4 ft wide and hinged at A. Gage G reads -2.17 psi, while oil (s.g. $= 0.75$) is in the right tank. What horizontal force must be applied at B for equilibrium of gate AB?

▌ $$F = \gamma h_{cg} A \qquad F_{oil} = [(0.75)(62.4)](\tfrac{6}{2})[(6)(4)] = 3370 \text{ lb}$$

F_{oil} acts $(\tfrac{2}{3})(6)$, or 4.0 ft from A. For the left side, the negative pressure due to the air can be converted to its equivalent head in feet of water. $h = p/\gamma = (-2.17)(144)/62.4 = -5.01$ ft. This negative pressure head is equivalent to having 5.01 ft less water above A. Hence, $F_{H_2O} = (62.4)(6.99 + \tfrac{6}{2})[(6)(4)] = 14\,960$ lb.

$$y_{cp} = \frac{-I_{xx} \sin\theta}{h_{cg}A} = \frac{-[(4)(6)^3/12](\sin 90°)}{(6.99 + \tfrac{6}{2})[(6)(4)]} = -0.30 \text{ ft}$$

F_{H_2O} acts at $(0.30 + \tfrac{6}{2})$, or 3.30 ft below A. $\sum M_A = 0$; $(3370)(4.0) + 6F - (14\,960)(3.30) = 0$, $F = 5980$ lb (acting leftward).

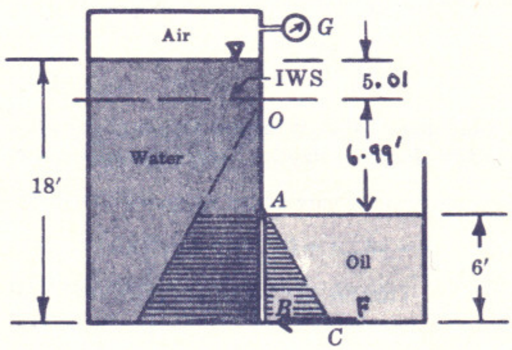


Fig. 3-39

3.50 A vertical circular disk 1.1 m in diameter has its highest point 0.4 m below the surface of a pond. Find the magnitude of the hydrostatic force on one side and the depth to the center of pressure.

▌ $$F = \gamma h_{cg} A = (9.79)(0.4 + 1.1/2)[(\pi)(1.1)^2/4] = 8.84 \text{ kN}$$

$$h_{cp} = h_{cg} + \frac{I_{cg}}{h_{cg}A} = \left(0.4 + \frac{1.1}{2}\right) + \frac{(\pi)(1.1)^4/64}{(0.4 + 1.1/2)[(\pi)(1.1)^2/4]} = 1.03 \text{ m}$$

3.51 The vertical plate shown in Fig. 3-40 is submerged in vinegar (s.g. $= 0.80$). Find the magnitude of the hydrostatic force on one side and the depth to the center of pressure.

▌ $$F = \gamma h_{cg} A \qquad F_1 = [(0.80)(9.79)](2 + \tfrac{7}{2})[(3)(7)] = 905 \text{ kN} \qquad h_{cp} = h_{cg} + \frac{I_{cg}}{h_{cg}A}$$

$$(h_{cp})_1 = 2 + \tfrac{7}{2} + \frac{(3)(7)^3/12}{(2 + \tfrac{7}{2})[(3)(7)]} = 6.24 \text{ m} \qquad F_2 = [(0.80)(9.79)][2 + 3 + 4/2][(2)(4)] = 439 \text{ kN}$$

$$(h_{cp})_2 = [2 + 3 + 4/2] + \frac{(2)(4)^3/12}{(2 + 3 + 4/2)[(2)(4)]} = 7.19 \text{ m}$$

$$F = 905 + 439 = 1344 \text{ kN} \qquad 1344h_{cp} = (905)(6.24) + (439)(7.19) \qquad h_{cp} = 6.55 \text{ m}$$

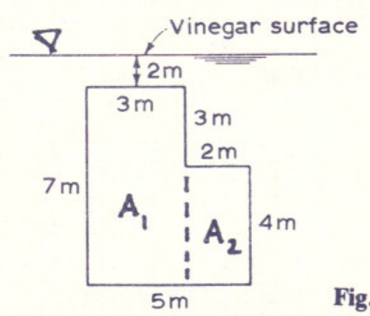


Fig. 3-40

3.52 The irrigation head gate shown in Fig. 3-41*a* is a plate which slides over the opening to a culvert. The coefficient of friction between the gate and its sliding ways is 0.5. Find the force required to slide open this 1000-lb gate if it is set **(*a*)** vertically and **(*b*)** on a 2 : 1 slope ($n = 2$ in Fig. 3-41*a*), as is common.

▮ **(a)** $F = \gamma h_{cg} A = (62.4)[14 + (\frac{60}{12})/2][(\frac{60}{12})(\frac{60}{12})] = 25\,740$ lb. Let T = force parallel to gate required to open it. $\sum F_y = 0$; $T - 1000 - (0.5)(25\,740) = 0$, $T = 13\,870$ lb. **(b)** See Fig. 3-41*b*. $F = (62.4)[14 + \frac{60}{12}(1/\sqrt{5})/2][(\frac{60}{12})(\frac{60}{12})] = 23\,584$ lb. Let N = total force normal to gate; $N = 23\,584 + (1000)(2/\sqrt{5}) = 24\,478$ lb. $\sum F_y = 0$; $T - (1000)(1/\sqrt{5}) - (0.5)(24\,478) = 0$, $T = 12\,686$ lb.

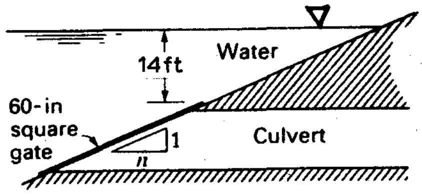
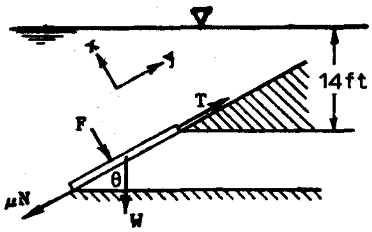


Fig. 3-41(a)



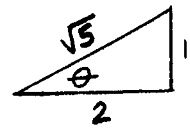


Fig. 3-41(b)

3.53 A 65-in-square floodgate, weighing 2200 lb, is hinged 44.5 in above the center, as shown in Fig. 3-42, and the face is inclined 5° to the vertical. Find the depth to which water will rise behind the gate before it will open.

▮ $$\text{Closing moment of gate about hinge} = (2200)[(\tfrac{44.5}{12})(\sin 5°)] = 711 \text{ lb}\cdot\text{ft}$$

$$F = \gamma h_{cg} A = (62.4)(h/2)[(\tfrac{65}{12})(h)/\cos 5°] = 169.6h^2$$

$$\sum M_{\text{hinge}} = 0 \qquad (169.6h^2)[(65+12)/12 - (h/\cos 5°)/3] - 711 = 0 \qquad h = 0.826 \text{ ft}$$

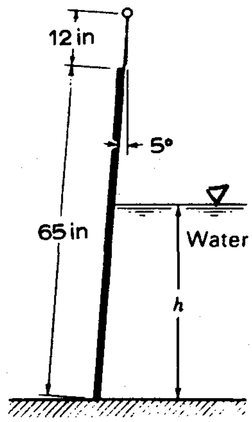


Fig. 3-42

3.54 Gate MN in Fig. 3-43 rotates about an axis through N. If the width of the gate is 5 ft, what torque applied to the shaft through N is required to hold the gate closed?

▮ $F = \gamma h_{cg} A \qquad F_1 = 62.4[6 + (3+4)/2][(3+4)(5)] = 20\,748 \text{ lb} \qquad F_2 = (62.4)(\tfrac{4}{2})[(5)(4)] = 2496 \text{ lb}$

$$y_{cp} = \frac{-I_{xx} \sin\theta}{h_{cg} A} \qquad (y_{cp})_1 = \frac{-[(5)(3+4)^3/12](\sin 90°)}{[6 + (3+4)/2][(3+4)(5)]} = 0.430 \text{ ft}$$

F_2 acts at $(\frac{1}{3})(4)$, or 1.333 ft from N. $\sum M_N = 0$; $(20\,748)[(3+4)/2 - 0.430] - (2496)(1.333) - \text{torque}_N = 0$, $\text{torque}_N = 60\,369 \text{ lb}\cdot\text{ft}$.

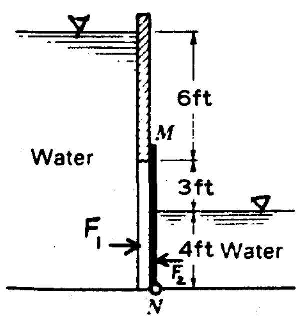


Fig. 3-43

3.55 Find the minimum depth of z for which the gate in Fig. 3-44 will open, if the gate is **(a)** square and **(b)** isosceles triangular, with base = height.

▌ (a)

$$F = \gamma h_{cg} A \qquad F_{H_2O} = (62.4)(z - \tfrac{3}{2})[(3)(3)] = (561.6)(z - 1.5)$$

$$y_{cp} = \frac{-I_{xx}\sin\theta}{h_{cg}A} \qquad (y_{cp})_{H_2O} = \frac{-[(3)(3)^3/12](\sin 90°)}{(z-\tfrac{3}{2})[(3)(3)]} = \frac{-0.750}{z-1.5}$$

$$\text{Moment due to water} = [(561.6)(z-1.5)][\tfrac{3}{2} + 0.750/(z-1.5)] = (561.6)(1.5z - 1.500)$$

$F_{gas} = pA = [(5)(144)][(3)(3)] = 6480$ lb. F_{gas} acts at $\frac{3}{2}$, or 1.5 ft below hinge. Moment due to gas = (6480)(1.5) = 9720 lb · ft. Equating moments gives $(561.6)(1.5z - 1.500) = 9720$, $z = 12.54$ ft.

(b)

$$F_{H_2O} = (62.4)[z - (\tfrac{2}{3})(3)][(3)(3)/2] = (280.8)(z - 2.000)$$

$$(y_{cp})_{H_2O} = \frac{-[(3)(3)^3/36](\sin 90°)}{[z-(\tfrac{2}{3})(3)][(3)(3)/2]} = \frac{0.500}{z-2.000}$$

$$\text{Moment due to water} = [(280.8)(z-2.000)][\tfrac{3}{3} + 0.500/(z-2.000)] = 280.8z - 421.2$$

$F_{gas} = [(5)(144)][(3)(3)/2] = 3240$ lb. F_{gas} acts at $\frac{3}{3}$, or 1.000 ft below hinge. Moment due to gas = (3240)(1.000) = 3240 lb · ft. Equating moments gives $(280.8z - 421.2) = 3240$, $z = 13.04$ ft.

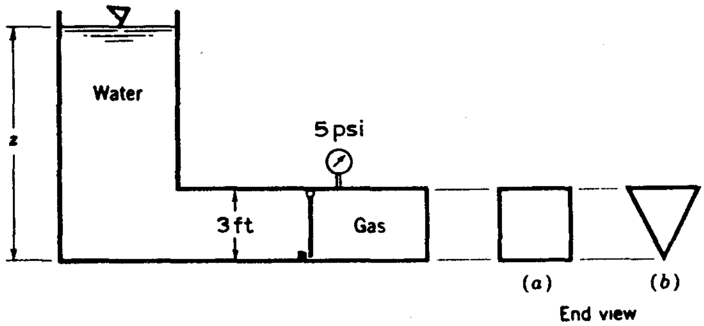


Fig. 3-44

3.56 The triangular gate *CDE* in Fig. 3-45 is hinged along *CD* and is opened by a normal force *P* applied at *E*. It holds a liquid of specific gravity 0.82 above it and is open to the atmosphere on its lower side. Neglecting the weight of the gate, find **(a)** the magnitude of force exerted on the gate, by direct integration; **(b)** the location of the center of pressure; and **(c)** the force *P* needed to open the gate.

▌ (a) $F = \int \gamma h\, dA = \int \gamma(y \sin\theta)(x\, dy)$. When $y = 8$, $x = 0$, and when $y = 8 + \frac{12}{2}$, or 14, $x = 6$, with x varying linearly with y. Hence, $x = y - 8$. When $y = 14$, $x = 6$, and when $y = 8 + 12$, or 20, $x = 0$, with x varying linearly with y. Hence, $x = 20 - y$.

$$F = \int_8^{14} [(0.82)(62.4)](y \sin 30°)[(y-8)\, dy] + \int_{14}^{20} [(0.82)(62.4)](y \sin 30°)[(20-y)\, dy]$$

$$= [(0.82)(62.4)](\sin 30°)\left\{\left[\frac{y^3}{3} - 4y^2\right]_8^{14} + \left[10y^2 - \frac{y^3}{3}\right]_{14}^{20}\right\} = 12\,894 \text{ lb}$$

(b)

$$x_{cp} = \frac{-I_{xy}\sin\theta}{h_{cg}A}$$

Since $I_{xy} = 0$, $x_{cp} = 0$,

$$y_{cp} = \frac{-I_{xx}\sin\theta}{h_{cg}A} = \frac{-[(2)(6)(\tfrac{12}{2})^3/12](\sin 30°)}{[(8+\tfrac{12}{2})(\sin 30°)][(12)(6)/2]} = -0.43 \text{ ft}$$

(i.e., the pressure center is 0.43 ft below the centroid, measured in the plane of the area).

(c)

$$\sum M_{CD} = 0 \qquad 6P = (12\,894)(\tfrac{6}{3}) \qquad P = 4298 \text{ lb}$$

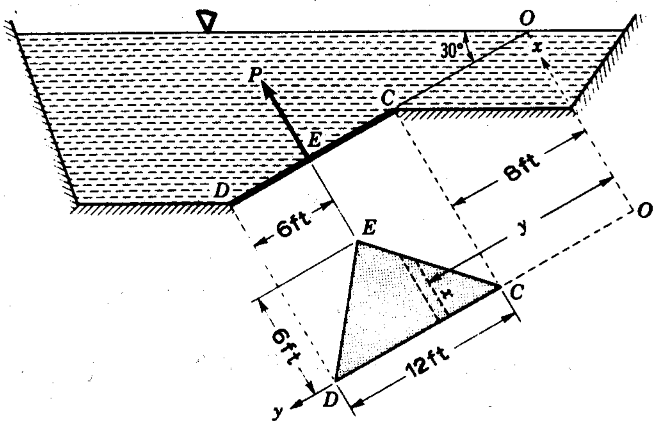


Fig. 3-45

3.57 Determine the force acting on one side of vertical surface $OACO$ in Fig. 3-46 and the location of the center of pressure, if $\gamma = 8.4\ \text{kN/m}^3$. The curved edge is an arc of the parabola $y = x^2/8$.

$$F = \int \gamma y\, dA = \int_0^1 (8.4)(y)(2x\, dy) = \int_0^1 (8.4)(y)(2\sqrt{8y})\, dy = \int_0^1 47.52y^{3/2}\, dy = [19.01y^{5/2}]_0^1 = 19.01\ \text{kN}$$

$$y_{cp} = \frac{\int \gamma y^2\, dA}{F} = \frac{\int_0^1 (8.4)(y)^2(2x\, dy)}{19.01} = \frac{\int_0^1 (8.4)(y)^2(2\sqrt{8y})\, dy}{19.01} = \frac{\int_0^1 47.52y^{5/2}\, dy}{19.01} = \frac{[13.58y^{7/2}]_0^1}{19.01} = 0.714\ \text{m}$$

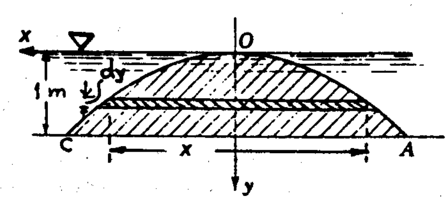


Fig. 3-46

3.58 Find the force exerted by water on one side of the vertical annular disk shown in Fig. 3-47. Also locate the center of pressure.

$$F = \gamma h_{cg} A = (9.79)(3)[(\pi)(1)^2 - (\pi)(\tfrac{600}{1000})^2] = 59.05\ \text{kN}$$

$$I_{cg} = (\pi)(1)^4/4 - (\pi)(\tfrac{600}{1000})^4/4 = 0.6836\ \text{m}^4$$

$$h_{cp} = h_{cg} + \frac{I_{cg}}{h_{cg}A} = 3 + \frac{0.6836}{3[(\pi)(1)^2 - (\pi)(\tfrac{600}{1000})^2]} = 3.113\ \text{m}$$

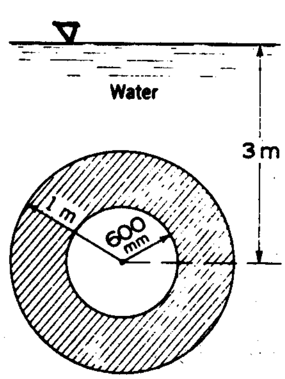


Fig. 3-47

3.59 Determine y in Fig. 3-48 so that the flashboards will tumble only when the water reaches their top.

The flashboards will tumble when y is at the center of pressure. Hence, $y = \frac{4}{3}$, or 1.333 m.

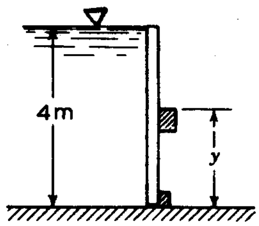


Fig. 3-48

3.60 Determine the pivot location y of the square gate in Fig. 3-49 so that it will rotate open when the liquid surface is as shown.

The gate will open when the pivot location is at the center of pressure.

$$h_{cp} = h_{cg} + \frac{I_{cg}}{h_{cg}A} = (3 - \tfrac{2}{2}) + \frac{(1)(2)^3/12}{(3 - \frac{2}{2})[(2)(1)]} = 2.167 \text{ m} \qquad y = 3 - 2.167 = 0.833 \text{ m}$$

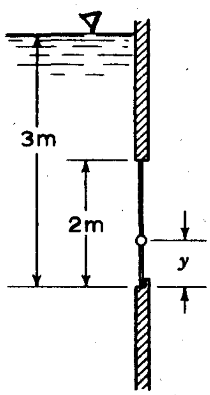


Fig. 3-49

3.61 The gate in Fig. 3-50*a* (shown in raised position) weighs 350 lb for each foot normal to the paper. Its center of gravity is 1.5 ft from the left face and 2.0 ft above the lower face. For what water level below the hinge at O does the gate just begin to swing up (rotate counterclockwise)?

Refer to Fig. 3-50*b* and consider 1 ft of length. $F = \gamma hA = (62.4)[(h_O/2)][(h_O)(1)] = 31.2h_O^2$; $\sum M_O = 0$; $(2)(350) - (5 - h_O/3)(31.2h_O^2) = 0$, $h_O = 2.30$ ft.

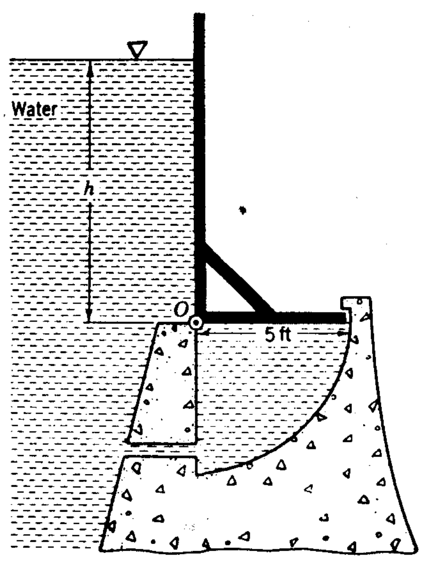


Fig. 3-50(*a*)

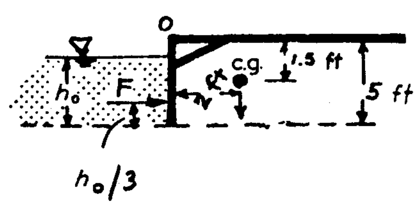


Fig. 3-50(*b*)

3.62 For the gate described in Prob. 3.61 and Fig. 3-50*a*, find *h* for the gate just to come up to the vertical position shown in Fig. 3-50*a*.

▌ See Fig. 3-51. $F_1 = \gamma hA = (62.4)(h)[(5)(1)] = 312h$, $F_2 = (62.4)(h/2)[(h)(1)] = 31.2h^2$; $\sum M_O = 0$; $(1.5)(350) + (h/3)(31.2h^2) - (2.5)(312h) = 0$, $h = 0.68$ ft.

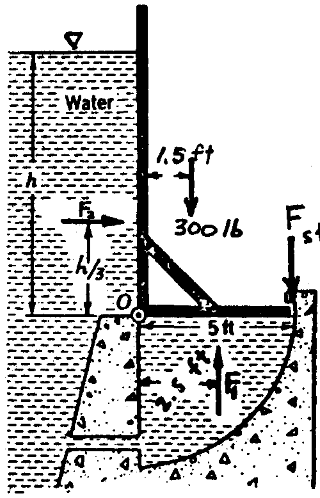


Fig. 3-51

3.63 For the gate described in Prob. 3.61 and Fig. 3-50*a*, find *h* and the force against the stop when this force is a maximum for the gate.

▌ See Fig. 3-51. $F_1 = \gamma hA = (62.4)(h)[(5)(1)] = 312h$, $F_2 = (62.4)(h/2)[(h)(1)] = 31.2h^2$; $\sum M_O = 0$; $(1.5)(350) + (h/3)(31.2h^2) - (2.5)(312h) + (5)(F_{stop}) = 0$, $F_{stop} = 156h - 2.08h^3 - 105$.

$$\frac{dF_{stop}}{dh} = 156 - 6.24h^2 = 0 \qquad h = 5.00\text{ ft}$$

$$F_{stop} = (156)(5.00) - (2.08)(5.00)^3 - 105 = 415\text{ lb}$$

3.64 Compute the air pressure required to keep the gate of Fig. 3-52 closed. The gate is a circular plate of diameter 0.8 m and weight 2.0 kN.

▌
$$F = \gamma hA \qquad F_{liq} = [(2)(9.79)][1.7 + (\tfrac{1}{2})(0.8)(\sin 45°)][\pi(0.8)^2/4] = 19.52\text{ kN}$$

$$z_{cp} = z_{cg} + \frac{I_{cg}}{z_{cg}A} = \left[\frac{1.7}{\cos 45°} + \left(\frac{1}{2}\right)(0.8)\right] + \frac{\pi[(\frac{1}{2})(0.8)]^4/4}{[1.7/\cos 45° + (\frac{1}{2})(0.8)][\pi(0.8)^2/4]} = 2.818\text{ m}$$

$$\sum M_{hinge} = 0 \qquad (19.52)(2.818 - 1.7/\cos 45°) + 2.0[(\tfrac{1}{2})(0.8)(\cos 45°)] - [\pi(0.8)^2/4](p_{air})[(\tfrac{1}{2})(0.8)] = 0$$

$$p_{air} = 42.99\text{ kPa}$$

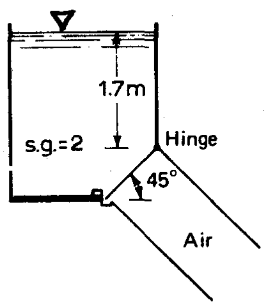


Fig. 3-52

CHAPTER 4
Dams

4.1 In Fig. 4-1, calculate the width of concrete dam that is necessary to prevent the dam from sliding. The specific weight of the concrete is 150 lb/ft^3, and the coefficient of friction between the base of the dam and the foundation is 0.42. Use 1.5 as the factor of safety (F.S.) against sliding. Will it also be safe against overturning?

Working with a 1-ft "slice" (i.e., dimension perpendicular to the paper) of the dam, $W_{dam} = (20)(w)(1)(150) = 3000w$, $F = \gamma hA$, $F_H = (62.4)[(0+15)/2][(15)(1)] = 7020$ lb.

$$\text{F.S.}_{\text{sliding}} = \frac{\text{sliding resistance}}{\text{sliding force}} \qquad 1.5 = \frac{(0.42)(3000w)}{7020} \qquad w = 8.36\text{ ft}$$

$$\text{F.S.}_{\text{overturning}} = \frac{\text{total righting moment}}{\text{overturning moment}} = \frac{[(3000)(8.36)](8.36/2)}{(7020)(\frac{15}{3})} = 2.99$$

Therefore, it should be safe against overturning.

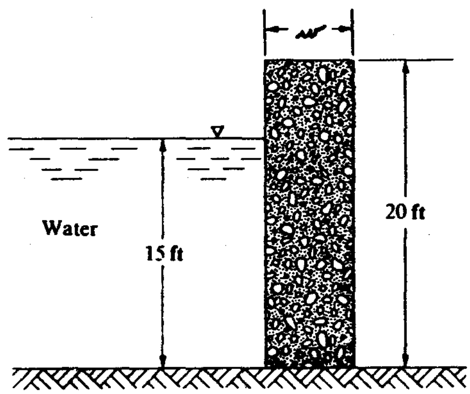


Fig. 4-1

4.2 Figure 4-2 is the cross section of an earthwork (s.g. = 2.5) dam. Assuming that hydrostatic uplift varies linearly from one-half the hydrostatic head at the upstream edge of the dam to zero at the downstream edge, find the maximum and minimum pressure intensity in the base of the dam.

$$F = \gamma hA \qquad F_H = (62.4)[(0+97)/2][(97)(1)] = 293\,561\text{ lb}$$

For equilibrium, $R_x = 293\,561$ lb.

$$W_1 = [(2.5)(62.4)][(1)(10)(90+30)] = 187\,200\text{ lb} \qquad W_2 = [(2.5)(62.4)][(1)(60)(90)/2] = 421\,200\text{ lb}$$

$$F_U = [(62.4)(48.5+0)/2][(60+10)(1)] = 105\,924\text{ lb} \qquad R_y = 187\,200 + 421\,200 - 105\,924 = 502\,476\text{ lb}$$

$$\sum M_0 = 0 \qquad (293\,561)(32.33) + (187\,200)(5) + (421\,200)(30) - (105\,924)[(60+10)/3] - 502\,476x = 0$$

$$x = 40.98\text{ ft} \qquad \text{Eccentricity} = 40.98 - (60+10)/2 = 5.98\text{ ft}$$

Since the eccentricity is less than one-sixth the base of the dam, the resultant acts within the middle third of the base.

$$p = \frac{F}{A} \pm \frac{M_y x}{I_y} \pm \frac{M_x y}{I_x} = \frac{502\,476}{(60+10)(1)} \pm \frac{[(502\,476)(5.98)](60+10)/2}{(1)(60+10)^3/12} \pm 0 = 7178 \pm 3679$$

$$p_{max} = 7178 + 3679 = 10\,857\text{ lb/ft}^2 \qquad p_{min} = 7178 - 3679 = 3499\text{ lb/ft}^2$$

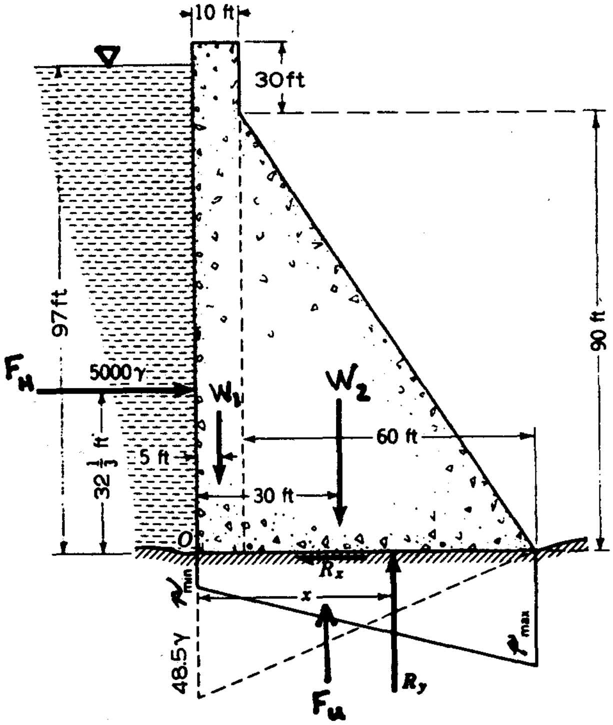


Fig. 4-2

4.3 For linear stress variation over the base of the dam of Fig. 4-3*a*, find where the resultant crosses the base and compute the maximum and minimum pressure intensity at the base. Neglect hydrostatic uplift.

▌ Figure 4-3*b* shows the forces acting on the dam. $F_1 = \gamma[(19+6)/2][(19+6)(1)] = 312\gamma$, $F_2 = \gamma[(6)(3)(1)] = 18\gamma$, $F_3 = \gamma[(1)(19)(3)/2] = 28.5\gamma$, $F_4 = [(2.5)(\gamma)][(4)(19+6)(1)] = 250\gamma$, $F_5 = [(2.5)(\gamma)][(1)(19)(3)/2] = 71.25\gamma$, $F_6 = [(2.5)(\gamma)][(1)(19)(11)/2] = 261\gamma$; $R_y = 18\gamma + 28.5\gamma + 250\gamma + 71.25\gamma + 261\gamma = 628.75\gamma$. $\sum M_A = 0$; $(628.75\gamma)(x) - (312\gamma)[(19+6)/3] - (18\gamma)(1.5) - (28.5\gamma)(1) - (250\gamma)(3+2) - (71.25\gamma)(3-1) - (261\gamma)(4+3+\frac{11}{3})$; $x = 10.87$ m. Eccentricity $= 10.87 - (11+4+3)/2 = 1.87$ ft. Since the eccentricity is less than one-sixth the base of the dam, the resultant acts within the middle third of the base.

$$p = \frac{F}{A} \pm \frac{M_y x}{I_y} \pm \frac{M_x y}{I_x} = \frac{(628.75)(9.79)}{(11+4+3)(1)} \pm \frac{[(628.75)(9.79)(1.87)](11+4+3)/2}{(1)(11+4+3)^3/12} \pm 0 = 342 \pm 213 \text{ kPa}$$

$$p_{max} = 342 + 213 = 555 \text{ kPa} \qquad p_{min} = 342 - 213 = 129 \text{ kPa}$$

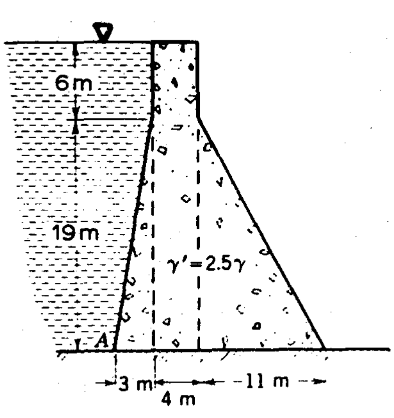


Fig. 4-3(*a*)

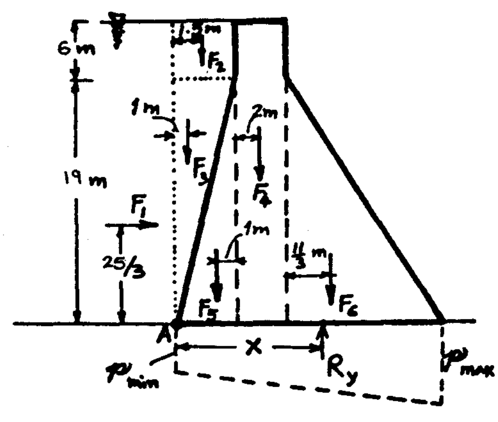


Fig. 4-3(*b*)

4.4 For the conditions given in Prob. 4.3 with the addition that hydrostatic uplift varies linearly from 19 m at A to zero at the toe of the dam, would the resultant still act within the middle third of the base?

$F_U = \gamma[(19+0)/2][(4+3+11)(1)] = 171\gamma \qquad R_y = 18\gamma + 28.5\gamma + 250\gamma + 71.25\gamma + 261\gamma - 171\gamma = 457.75\gamma$

$$\sum M_A = 0 \qquad (457.75\gamma)(x) - (312\gamma)[(19+6)/3] - (18\gamma)(1.5) - (28.5\gamma)(1) - (250\gamma)(3+2)$$
$$- (71.25\gamma)(3-1) - (261\gamma)(4+3+\tfrac{11}{3}) + (171\gamma)[(4+3+11)/3] = 0$$
$$x = 12.68\text{ m} \qquad \text{Eccentricity} = 12.68 - (11+4+3)/2 = 3.68\text{ ft}$$

Since the eccentricity is greater than one-sixth the base of the dam, the resultant acts outside the middle third of the base.

4.5 A concrete dam retaining water is shown in Fig. 4-4a. If the specific weight of the concrete is 150 lb/ft^3, find the factor of safety against sliding, the factor of safety against overturning, and the pressure intensity on the base. Assume the foundation soil is impermeable and that the coefficient of friction between dam and foundation soil is 0.45.

The forces acting on the dam are shown in Fig. 4.4b. $F = \gamma hA$, $F_x = (62.4)[(0+42)/2][(42)(1)] = 55\,040$ lb. From Fig. 4-4b, $CD/42 = \frac{10}{50}$, $CD = 8.40$ ft; $F_y = (62.4)[(8.40)(42)/2](1) = 11\,010$ lb.

component	weight of component (kips)	moment arm from toe, B (ft)	righting moment about toe, B (kip · ft)
1	$(\frac{1}{2})(10\times 50)(0.15)(1) = 37.50$	$20 + \frac{10}{3} = 23.33$	875
2	$(10\times 50)(0.15)(1) = 75.00$	$10 + \frac{10}{2} = 15.00$	1125
3	$(\frac{1}{2})(10\times 50)(0.15)(1) = 37.50$	$(\frac{2}{3})(10) = 6.67$	250
F_y	11.01	$30 - (\frac{1}{3})(8.40) = 27.20$	299
	$\sum V = 161.01$ **kips**		$\sum M_r = 2549$ **kip · ft**

$$M_{\text{overturning}} = (55.04)(\tfrac{42}{3}) = 771\text{ kip-ft} \qquad \text{F.S.}_{\text{sliding}} = \frac{\text{sliding resistance}}{\text{sliding force}} = \frac{(0.45)(161.01)}{55.04} = 1.32$$

$$\text{F.S.}_{\text{overturning}} = \frac{\text{total righting moment}}{\text{overturning moment}} = \frac{2549}{771} = 3.31$$

$R_x = F_x = 55.04$ kips and $R_y = \sum V = 161.01$ kips; hence, $R = \sqrt{55.04^2 + 161.01^2} = 170.16$ kips.

$$x = \frac{\sum M_B}{R_y} = \frac{\sum M_r - M_0}{\sum V} = \frac{2549 - 771}{161.01} = 11.04\text{ ft} \qquad \text{Eccentricity} = \tfrac{30}{2} - 11.04 = 3.96\text{ ft}$$

Since the eccentricity is less than one-sixth the base of the dam, the resultant acts within the middle third of the base.

$$p = \frac{F}{A} \pm \frac{M_y x}{I_y} \pm \frac{M_x y}{I_x} = \frac{161.01}{(30)(1)} \pm \frac{[(161.01)(3.96)](15)}{(1)(30)^3/12} \pm 0 = 5.37 \pm 4.25$$

$$p_B = 5.37 + 4.25 = 9.62\text{ kips/ft}^2 \qquad p_A = 5.37 - 4.25 = 1.12\text{ kips/ft}^2$$

The complete pressure distribution on the base of the dam is given in Fig. 4-4c.

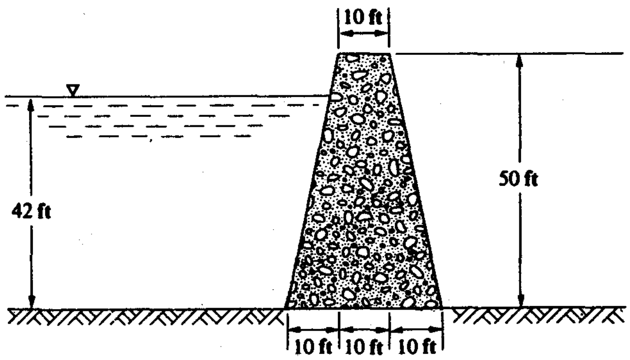


Fig. 4-4(a)

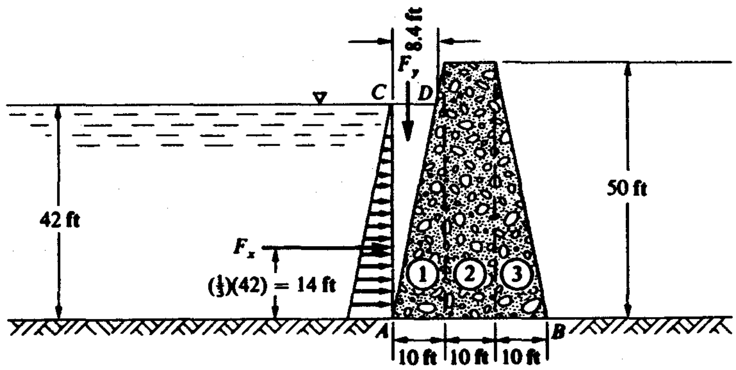


Fig. 4-4(b)

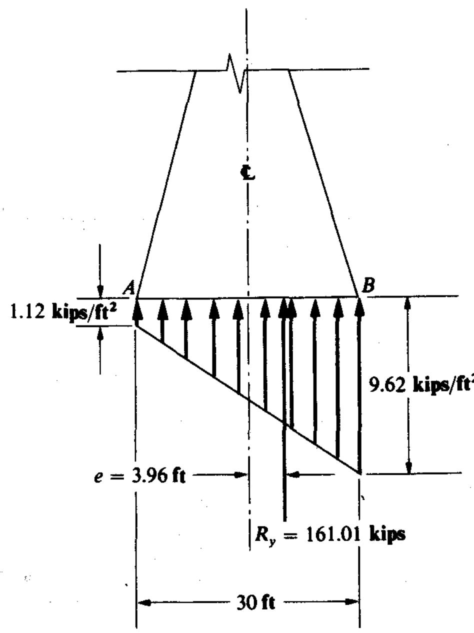


Fig. 4-4(c)

4.6 A concrete dam retaining water is shown in Fig. 4-5*a*. If the specific weight of the concrete is 23.5 kN/m³, find the factor of safety against sliding, the factor of safety against overturning, and the pressure intensity on the base. Assume there is a hydrostatic uplift that varies uniformly from full hydrostatic head at the heel of the dam to zero at the toe and that the coefficient of friction between dam and foundation soil is 0.45.

▌ The forces acting on the dam are shown in Fig. 4-5*b*. $F = \gamma h A$, $F_x = (9.79)[(0 + 14)/2][(14)(1)] = 959.4$ kN, $F_y = (9.79)[(3)(14 - 3)(1)] = 323.1$ kN. Hydrostatic uplift varies from (14)(9.79), or 137.1 kN/m² at the heel to zero at the toe, as shown in Fig. 4-5*b*. $F_U = (137.1/2)(15)(1) = 1028$ kN. It acts at $(\frac{1}{3})(15)$, or 5.0 m from point *A*, as shown in Fig. 4-5*b*.

component	weight of component (kN)	moment arm from toe, B (m)	righting moment about toe, B (kN · m)
1	$(\frac{1}{2})(15-3-4)(12)(23.5)(1) = 1128$	$(\frac{2}{3})(15-3-4) = 5.333$	6 016
2	$(4)(12+3)(23.5)(1) = 1410$	$(15-3-\frac{4}{2}) = 10.000$	14 100
3	$(15)(3)(23.5)(1) = 1058$	$\frac{15}{2} = 7.500$	7 935
F_y	$= 323$	$(15-\frac{3}{2}) = 13.500$	4 360
	$\sum V = 3919$ **kN**		$\sum M_r = 32\,411$ **kN · m**

$$M_{\text{overturning}} = (959.4)(\tfrac{14}{3}) + (1028)(10) = 14\,760 \text{ kN}$$

$$\text{F.S.}_{\text{sliding}} = \frac{\text{sliding resistance}}{\text{sliding force}} = \frac{(0.45)(3919-1028)}{959.4} = 1.36$$

$$\text{F.S.}_{\text{overturning}} = \frac{\text{total righting moment}}{\text{overturning moment}} = \frac{32\,411}{14\,760} = 2.20$$

$R_x = F_x = 959.4$ kN and $R_y = \sum V - F_U = 3919 - 1028 = 2891$ kN; hence, $R = \sqrt{959.4^2 + 2891^2} = 3046$ kN.

$$x = \frac{\sum M_B}{R_y} = \frac{\sum M_r - M_0}{\sum V} = \frac{32\,411 - 14\,760}{2891} = 6.105 \text{ m} \qquad \text{Eccentricity} = \tfrac{15}{2} - 6.105 = 1.395 \text{ m}$$

Since the eccentricity is less than one-sixth the base of the dam, the resultant acts within the middle third of the base.

$$p = \frac{F}{A} \pm \frac{M_y x}{I_y} \pm \frac{M_x y}{I_x} = \frac{2891}{(15)(1)} \pm \frac{[(2891)(1.395)](\frac{15}{2})}{(1)(15)^3/12} \pm 0 = 192.7 \pm 107.5$$

$$p_B = 192.7 + 107.5 = 300.2 \text{ kN/m}^2 \qquad p_A = 192.7 - 107.5 = 85.2 \text{ kN/m}^2$$

The complete pressure distribution on the base of the dam is given in Fig. 4-5*c*.

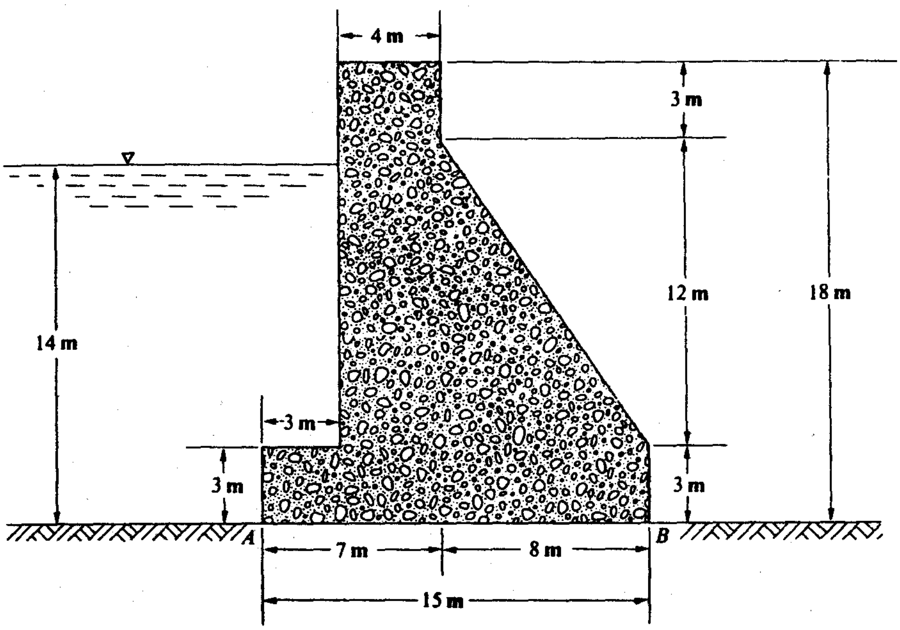


Fig. 4-5(*a*)

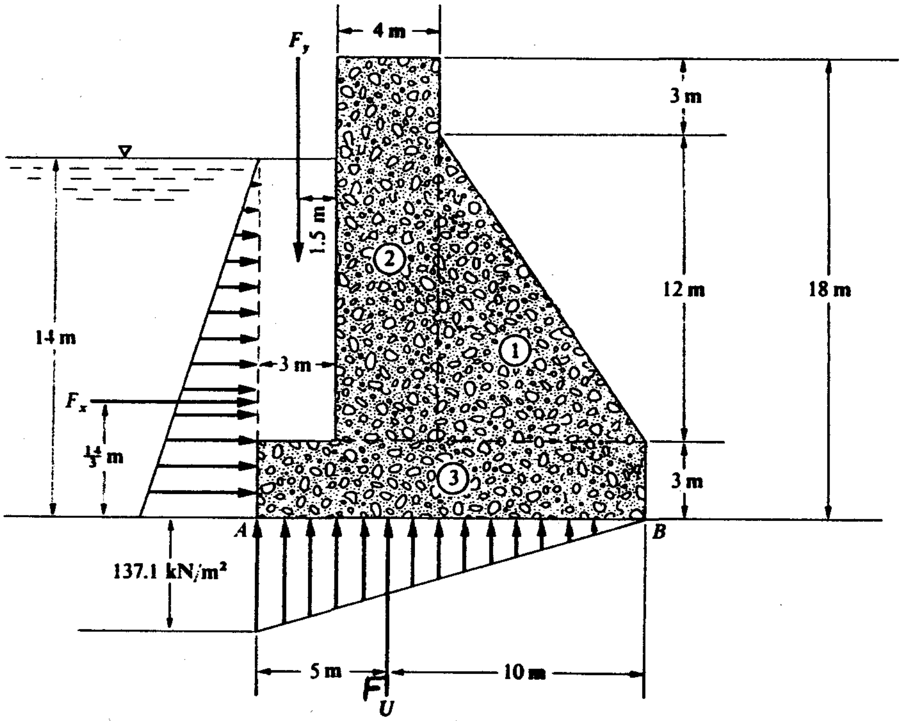


Fig. 4-5(b)

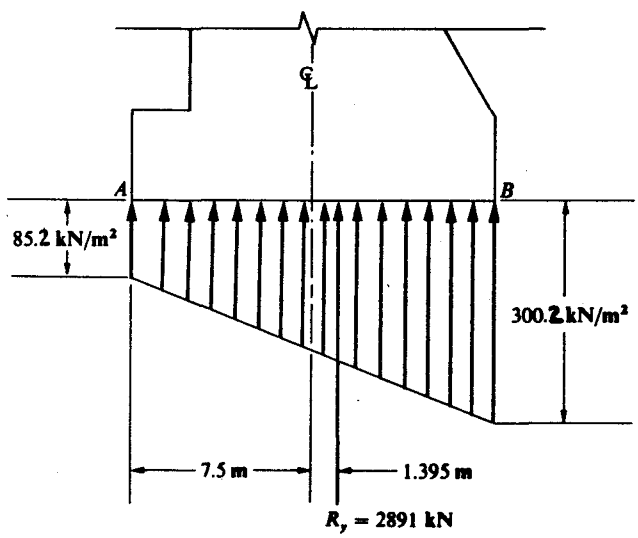


Fig. 4-5(c)

4.7 A concrete dam retaining water is shown in Fig. 4-6*a*. If the specific weight of the concrete is 23.5 kN/m³, find the factor of safety against sliding, the factor of safety against overturning, and the maximum and minimum pressure intensity on the base. Assume there is no hydrostatic uplift and that the coefficient of friction between dam and foundation soil is 0.48.

▮ The forces acting on the dam are shown in Fig. 4-6*b*. $F = \gamma hA$, $F_H = (9.79)[(0 + 6)/2][(6)(1)] = 176.2$ kN.

component	weight of component (kN)	moment arm from toe, *A* (m)	righting moment about toe, *A* (kN · m)
1	$(\frac{1}{2})(2)(7)(23.5) = 164.5$	$(\frac{2}{3})(2) = 1.333$	219
2	$(2)(7)(23.5) = 329.0$	$2 + \frac{2}{2} = 3.000$	987
	$\sum V = 493.5$ kN		$\sum M_r = 1206$ kN · m

$$M_{\text{overturning}} = (176.2)(\tfrac{6}{3}) = 352.4 \text{ kN}$$

$$\text{F.S.}_{\text{sliding}} = \frac{\text{slinding resistance}}{\text{sliding force}} = \frac{(0.48)(493.5)}{176.2} = 1.34$$

$$\text{F.S.}_{\text{overturning}} = \frac{\text{total righting moment}}{\text{overturning moment}} = \frac{1206}{352.4} = 3.42$$

$R_x = F_H = 176.2$ kN and $R_y = \sum V = 493.5$ kN; hence, $R = \sqrt{176.2^2 + 493.5^2} = 524$ kN.

$$x = \frac{\sum M_A}{R_y} = \frac{\sum M_r - M_0}{\sum V} = \frac{1206 - 352.4}{493.5} = 1.730 \text{ m} \qquad \text{Eccentricity} = \tfrac{4}{2} - 1.730 = 0.270 \text{ m}$$

Since the eccentricity is less than one-sixth the base of the dam, the resultant acts within the middle third of the base.

$$p = \frac{F}{A} \pm \frac{M_y x}{I_y} \pm \frac{M_x y}{I_x} = \frac{493.5}{(4)(1)} \pm \frac{[(493.5)(0.270)](\tfrac{4}{2})}{(1)(4)^3/12} \pm 0 = 123.4 \pm 50.0$$

$$p_{\max} = 123.4 + 50.0 = 173.4 \text{ kN/m}^2 \qquad p_{\min} = 123.4 - 50.0 = 73.4 \text{ kN/m}^2$$

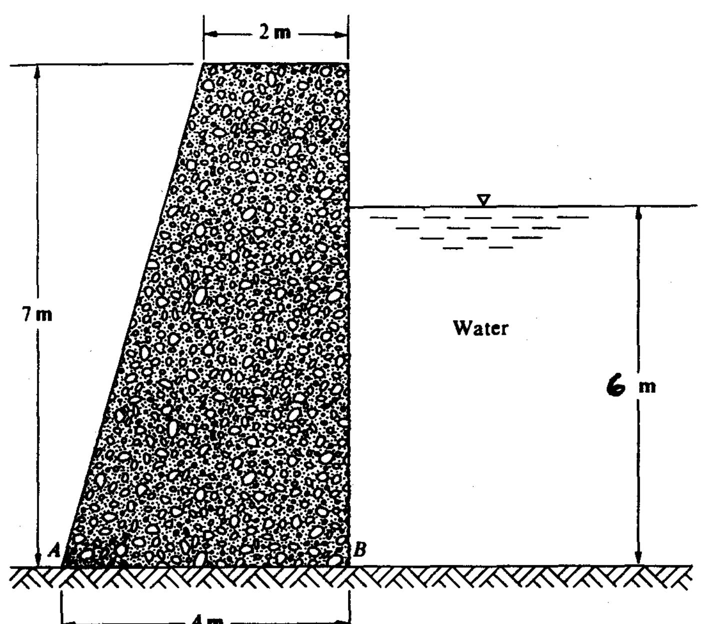


Fig. 4-6(*a*)

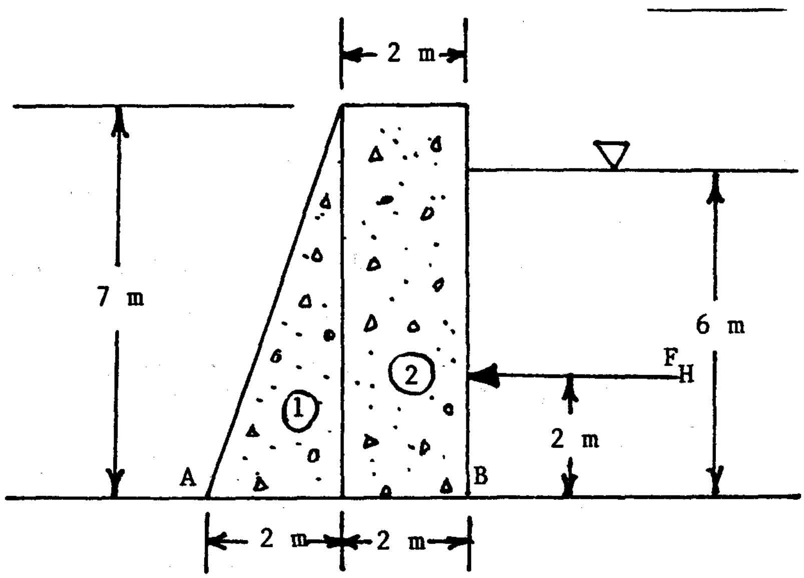


Fig. 4-6(*b*)

4.8 For the dam shown in Fig. 4-7, what is the minimum width b for the base of a dam 100 ft high if hydrostatic uplift is assumed to vary uniformly from full hydrostatic head at the heel to zero at the toe, and also assuming an ice thrust P_1 of 12 480 lb per linear foot of dam at the top? For this study, make the resultant of the reacting forces cut the base at the downstream edge of the middle third of the base (i.e., at O in Fig. 4-7) and take the weight of the masonry as 2.50γ.

▌ $F = \gamma h A \qquad F_H = (62.4)[(100+0)/2][(100)(1)] = 312\,000 \text{ lb} \qquad F_U = [(100)(62.4)/2][(1)(b)] = 3120b$

$W_1 = [(2.50)(62.4)][(20)(100)(1)] = 312\,000 \text{ lb} \qquad W_2 = [(2.50)(62.4)][(b-20)(100)(1)/2] = 7800b - 156\,000$

$$\sum M_O = 0$$

$$(312\,000)(\tfrac{100}{3}) + (3120b)(b/3) - (312\,000)[(\tfrac{2}{3})(b) - \tfrac{20}{2}] - (7800b - 156\,000)[(\tfrac{2}{3})(b-20) - b/3] + (12\,480)(100) = 0$$

$$3b^2 + 100b - 24\,400 = 0 \qquad b = 75.0 \text{ ft}$$

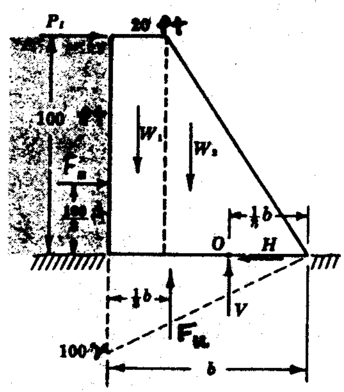


Fig. 4-7

CHAPTER 5
Forces on Submerged Curved Areas

5.1 The submerged, curved surface *AB* in Fig. 5-1*a* is one-quarter of a circle of radius 4 ft. The tank's length (distance perpendicular to the plane of the figure) is 6 ft. Find the horizontal and vertical components of the total resultant force acting on the curved surface and their locations.

▌ The horizontal component of the total resultant force acting on the curved surface is equal to the total resultant force, F_H, acting on the vertical projection of curved surface *AB* (i.e., *BF* in Fig. 5-1*b*). This projection is a rectangle 6 ft long and 4 ft high. For the portion of F_H resulting from horizontal pressure of *BHEF* in Fig. 5-1*b*, $p_1 = (8)(62.4) = 499\ \text{lb/ft}^2$, $A = (6)(4) = 24\ \text{ft}^2$, $F_1 = (499)(24) = 11\,980\ \text{lb}$. For the portion of F_H resulting from horizontal pressure of *HGE* in Fig. 5-1*b*, $p_2 = (62.4)[(0+4)/2] = 125\ \text{lb/ft}^2$, $F_2 = (125)(24) = 3000\ \text{lb}$; $F_H = F_1 + F_2 = 11\,980 + 3000 = 14\,980\ \text{lb}$. The vertical component of the total resultant force acting on the curved surface is equal to the weight of the volume of water vertically above curved surface *AB*. This volume consists of a rectangular area (*AFCD* in Fig. 5-1*c*) 4 ft by 8 ft and a quarter-circular area (*ABF* in Fig. 5-1*c*) of radius 4 ft, both areas being 6 ft long. This volume (*V*) is $V = [(4)(8) + (\pi)(4)^2/4](6) = 267.4\ \text{ft}^3$, F_V = weight of water in $V = (267.4)(62.4) = 16\,690\ \text{lb}$. The location of the horizontal component (F_H) is along a (horizontal) line through the center of pressure for the vertical projection (i.e., the center of gravity of *EFBG* in Fig. 5-1*b*). This can be determined by equating the sum of the moments of F_1 and F_2 about point *C* to the moment of F_H about the same point. $(11\,980)(8 + \frac{4}{2}) + (3000)[8 + (\frac{2}{3})(4)] = 14\,980 h_{cp}$, $h_{cp} = 10.13\ \text{ft}$. (This is the depth from the water surface to the location of the horizontal component. Stated another way, the horizontal component acts at a distance of $12 - 10.13$, or 1.87 ft above point *B* in Fig. 5-1*b*.) The location of the vertical component (F_V) is

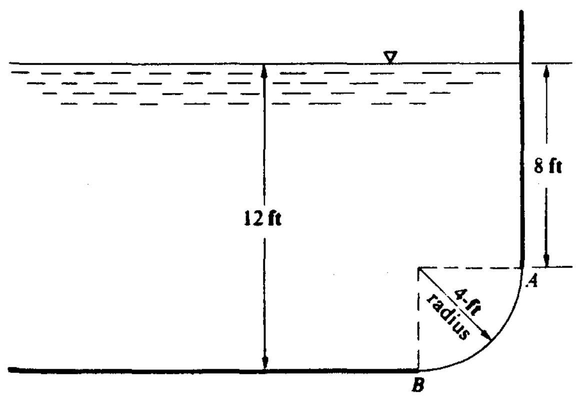


Fig. 5-1(*a*)

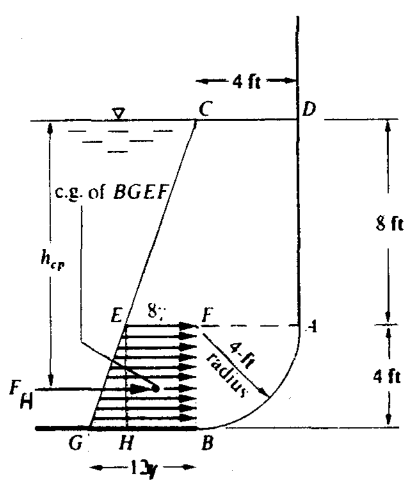


Fig. 5-1(*b*)

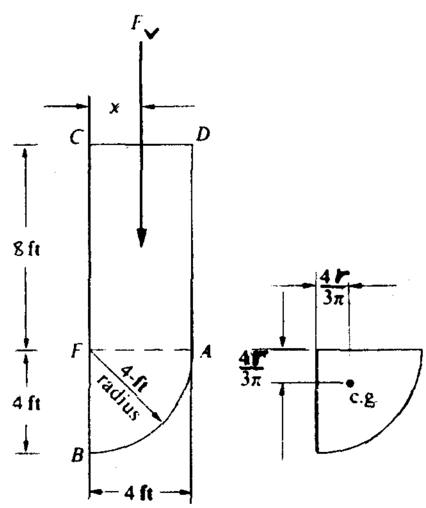


Fig. 5-1(*c*)

along a (vertical) line through the center of gravity of the liquid volume vertically above surface AB (i.e., the center of gravity of $ABCD$ in Fig. 5-1c). This can be determined by referring to Fig. 5-1c and equating the sum of the moments of the rectangular area ($AFCD$ in Fig. 5-1c) and of the quarter-circular area (ABF in Fig. 5-1c) about a vertical line through point B to the moment of the total area about the same line. $(x)[(8)(4)+(\pi)(4)^2/4]=[(8)(4)](\frac{4}{2})+[(\pi)(4)^2/4][(4)(4)/(3\pi)]$, $x=1.91$ ft. (This is the distance from point B to the line of action of the vertical component.)

5.2 Solve Prob. 5.1 for the same given conditions except that water is on the other side of curved surface AB, as shown in Fig. 5-2.

▮ If necessary, refer to the solution of Prob. 5.1 for a more detailed explanation of the general procedure for solving this type of problem. $p=p_{avg}=(\gamma)[(h_1+h_2)/2]=(62.4)[(8+12)/2]=624\ \text{lb/ft}^2$, $A=(6)(4)=24\ \text{ft}^2$, $F_H=pA=(624)(24)=14\,980$ lb. The vertical component (F_V) is equal to the weight of the imaginary volume of water vertically above surface AB. Hence, $F_V=[(4)(8)+(\pi)(4)^2/4](6)(62.4)=16\,690$ lb. The location of the horizontal component is 10.13 ft below the water surface (same as in Prob. 5.1 except that F_H acts toward the left). The location of the vertical component is 1.91 ft from point B (same as in Prob. 5.1 except that F_V acts upward).

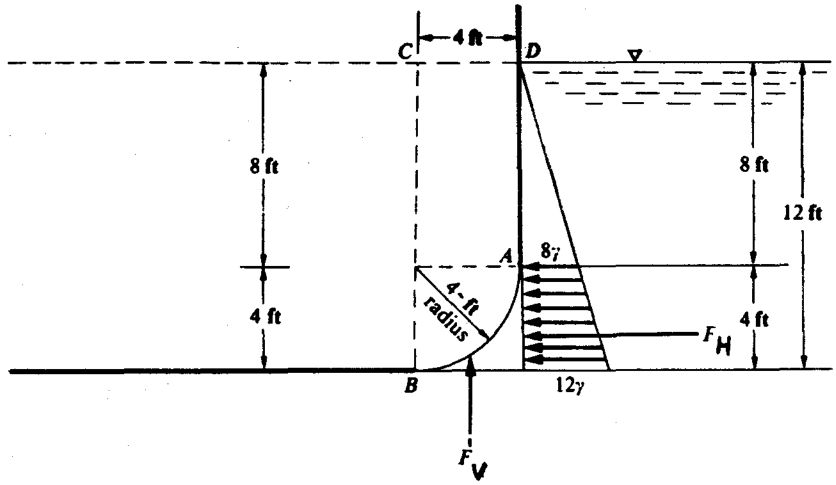
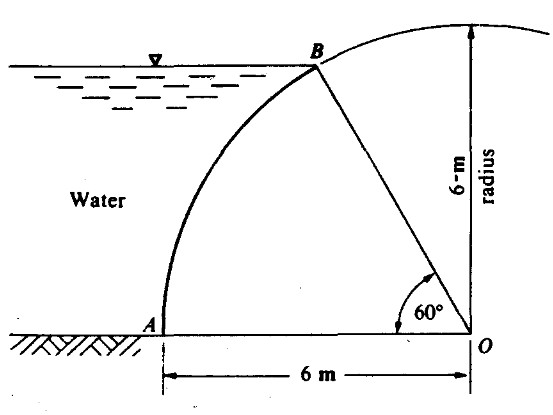


Fig. 5-2

5.3 The submerged sector gate AB shown in Fig. 5-3a is one-sixth of a circle of radius 6 m. The length of the gate is 10 m. Determine the amount and location of the horizontal and vertical components of the total resultant force acting on the gate.

▮ If necessary, refer to the solution of Prob. 5.1 for a more detailed explanation of the general procedure for solving this type of problem. Refer to Fig. 5-3b. $F_H=\gamma\bar{h}A=(9.79)[(0+5.196)/2][(10)(5.196)]=1322$ kN, $\text{Area}_{ABC}=\text{area}_{ACBD}+\text{area}_{BDO}-\text{area}_{ABO}=(5.196)(3)+(3.000)(5.196)/2-(\pi)(6)^2/6=4.532\ \text{m}^2$, $F_V=(\text{area}_{ABC})(\text{length of gate})(\gamma)=(4.532)(10)(9.79)=444$ kN. The location of the horizontal component (F_H) is along a (horizontal) line 5.196/3, or 1.732 m above the bottom of the gate (A). The location of the vertical component (F_V) is along a (vertical) line through the center of gravity of section ABC. Taking area moments about AC, $4.532x=[(5.196)(3)](\frac{3}{2})+[(\frac{1}{2})(3.000)(5.196)](3+3.000/3)-[(\pi)(6)^2/6]\{6-[\cos(60°/2)](2)(6)/\pi\}$, $x=0.842$ m.



(a)

Fig. 5-3(a)

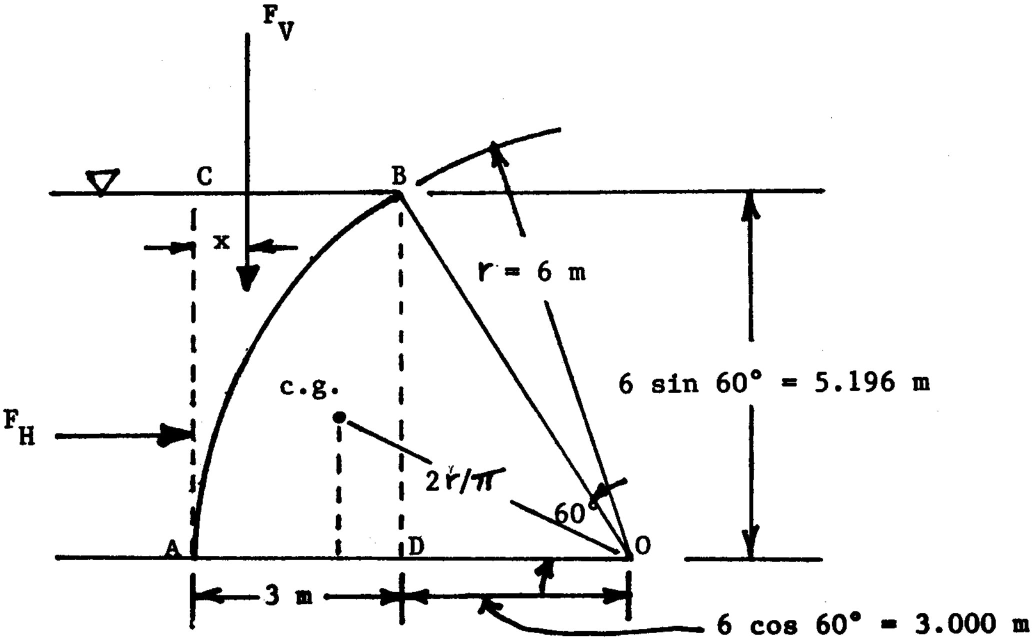


Fig. 5-3(*b*)

5.4 The curved surface AB shown in Fig. 5-4*a* is a quarter of a circle of radius 5 ft. Determine, for an 8-ft length perpendicular to the paper, the amount and location of the horizontal and vertical components of the total resultant force acting on surface AB.

I If necessary, refer to the solution of Prob. 5.1 for a more detailed explanation of the general procedure for solving this type of problem. Refer to Fig. 5-4*b*. $F_H = \gamma \bar{h} A = (62.4)[(0+5)/2][(5)(8)] = 6240$ lb, $\text{area}_{ABD} = \text{area}_{ACBD} - \text{area}_{ABC} = (5)(5) - (\pi)(5)^2/4 = 5.365\ \text{ft}^2$, $F_V = (\text{area}_{ABD})(\text{length})(\gamma) = (5.365)(8)(62.4) = 2678$ lb. F_H is located at $\frac{5}{3}$, or 1.67 ft above C. F_V is located at x from line AD. $5.365x = [(5)(5)](\frac{5}{2}) - [(\pi)(5)^2/4][5 - (4)(5)/(3\pi)]$, $x = 1.12$ ft.

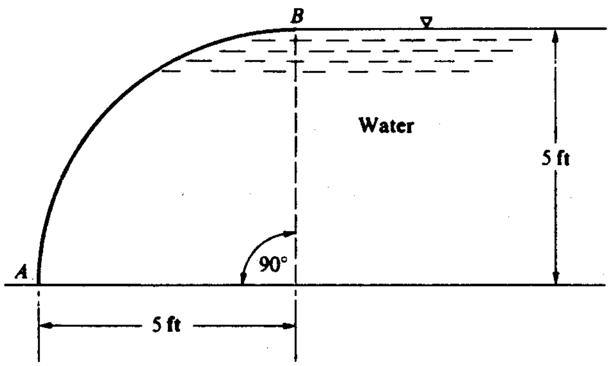


Fig. 5-4(*a*)

5.5 Determine the value and location of the horizontal and vertical components of the force due to water acting on curved surface AB in Fig. 5-5, per foot of its length.

I If necessary, refer to the solution of Prob. 5.1 for a more detailed explanation of the general procedure for solving this type of problem. $F_H = \gamma \bar{h} A = (62.4)[(0+6)/2][(6)(1)] = 1123$ lb, $F_V = (\text{area})(\text{length})(\gamma) = [(\pi)(6)^2/4](1)(62.4) = 1764$ lb. F_H is located at $(\frac{2}{3})(6)$, or 4.00 ft below C. F_V is located at the center of gravity of area ABC, or distance x from line CB. $x = 4r/(3\pi) = (4)(6)/(3\pi) = 2.55$ ft.

5.6 The 6-ft-diameter cylinder in Fig. 5-6 weighs 5000 lb and is 5 ft long. Determine the reactions at A and B, neglecting friction.

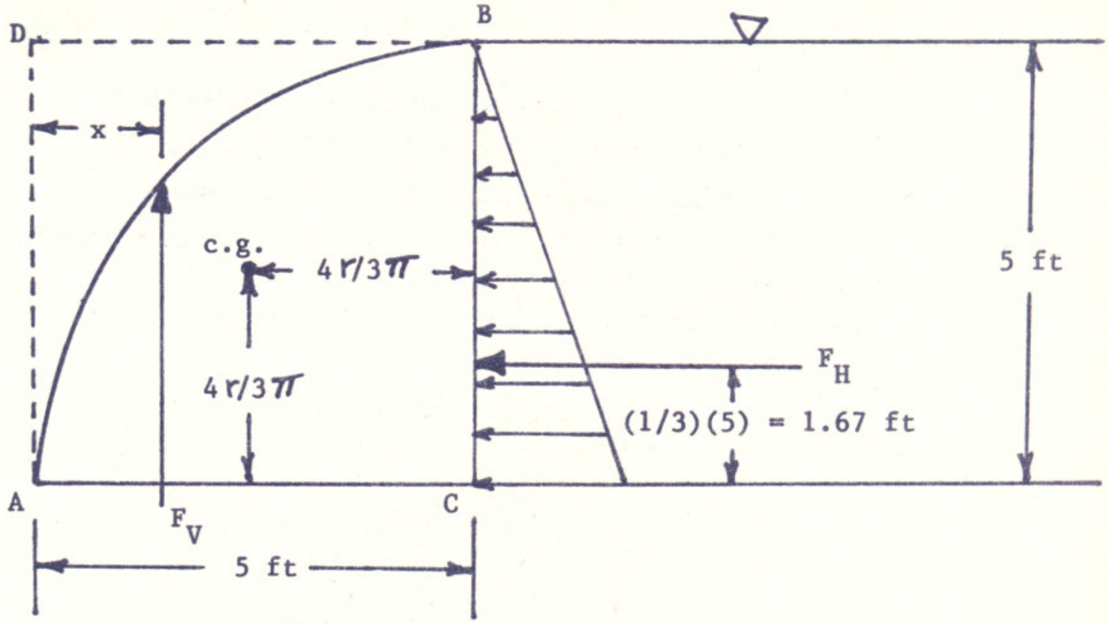


Fig. 5-4(*b*)

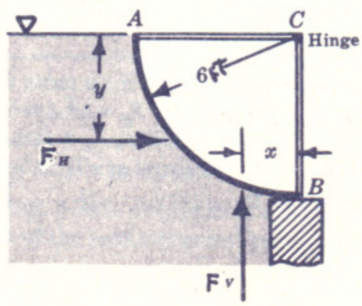


Fig. 5-5

▮ The reaction at A is due to the horizontal component of the liquid force acting on the cylinder (F_H). $F_H = \gamma \bar{h} A = [(0.800)(62.4)][(0+6)/2][(3+3)(5)] = 4493$ lb. F_H acts to the right; hence, the reaction at A is 4493 lb to the left. The reaction at B is the algebraic sum of the weight of the cylinder and the net vertical component of the force due to the liquid. $(F_V)_{\text{up}} = (\text{area}_{ECOBDE})(\text{length})(\gamma)$, $(F_V)_{\text{down}} = (\text{area}_{ECDE})(\text{length})(\gamma)$, $(F_V)_{\text{net}} = (F_V)_{\text{up}} - (F_V)_{\text{down}} = (\text{area}_{COBDC})(\text{length})(\gamma) = [(\pi)(3)^2/2](5)[(0.800)(62.4)] = 3529$ lb (upward). The reaction at B is $5000 - 3529$, or 1471 lb upward.

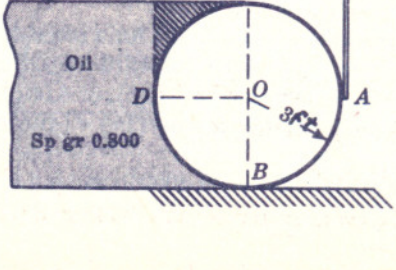


Fig. 5-6

5.7 Referring to Fig. 5-7, determine the horizontal and vertical forces due to the water acting on the cylinder per foot of its length.

$$(F_H)_{CDA} = 62.4\{[4 + (4 + 4.24 + 0.88)]/2\}[(2.12 + 3)(1)] = 2096 \text{ lb}$$

$$(F_H)_{AB} = (62.4)\{[(4 + 4.24) + (4 + 4.24 + 0.88)]/2\}[(0.88)(1)] = 477 \text{ lb}$$

$$(F_H)_{\text{net}} = (F_H)_{CDA} - (F_H)_{AB} = 2096 - 477 = 1619 \text{ lb (right)}$$

$$(F_V)_{\text{net}} = (F_V)_{DAB} - (F_V)_{DC} = \text{weight of volume}_{DABFED} - \text{weight of volume}_{DCGED} = \text{weight of volume}_{DABFGCD}$$
$$= \text{weight of (rectangle}_{GFJC} + \text{triangle}_{CJB} + \text{semicircle}_{CDAB})$$
$$= 62.4[(4)(4.24) + (4.24)(4.24)/2 + (\pi)(3)^2/2](1) = 2501 \text{ lb (upward)}$$

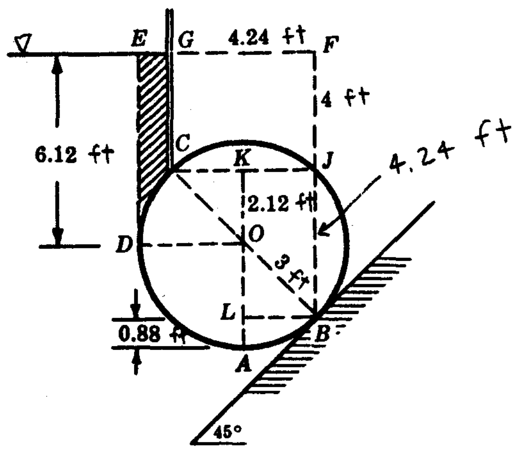


Fig. 5-7

5.8 In Fig. 5-8, an 8-ft-diameter cylinder plugs a rectangular hole in a tank that is 3 ft long. With what force is the cylinder pressed against the bottom of the tank due to the 9-ft depth of water?

$$(F_V)_{\text{net}} = (F_V)_{CDE} - (F_V)_{CA} - (F_V)_{BE} = 62.4[(4 + 4)(7) - (\pi)(4)^2/2](3)$$
$$- 62.4[(7)(0.54) + (\tfrac{30}{360})(\pi)(4)^2 - (2)(3.46)/2](3)$$
$$- 62.4[(7)(0.54) + (\tfrac{30}{360})(\pi)(4)^2 - (2)(3.46)/2](3) = 4090 \text{ lb (downward)}$$

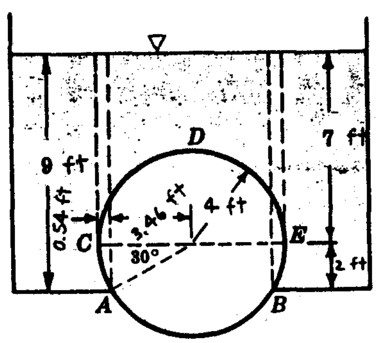


Fig. 5-8

5.9 In Fig. 5-9, the 8-ft-diameter cylinder weighs 500 lb and rests on the bottom of a tank that is 3 ft long. Water and oil are poured into the left- and right-hand portions of the tank to depths of 2 ft and 4 ft, respectively. Find the magnitudes of the horizontal and vertical components of the force that will keep the cylinder touching the tank at B.

$$(F_H)_{\text{net}} = (F_H)_{AB} - (F_H)_{CB} = [(0.750)(62.4)][(0 + 4)/2][(4)(3)] - (62.4)[(0 + 2)/2][(2)(3)] = 749 \text{ lb (left)}$$

$$(F_V)_{\text{net}} = (F_V)_{AB} + (F_V)_{CB} = [(0.750)(62.4)][(\pi)(4)^2/4](3) + (62.4)[(\tfrac{60}{360})(\pi)(4)^2 - (2)(\sqrt{12})/2](3)$$
$$= 2684 \text{ lb (upward)}$$

The components to hold the cylinder in place are 749 lb to the right and 2684 − 500, or 2184 lb down.

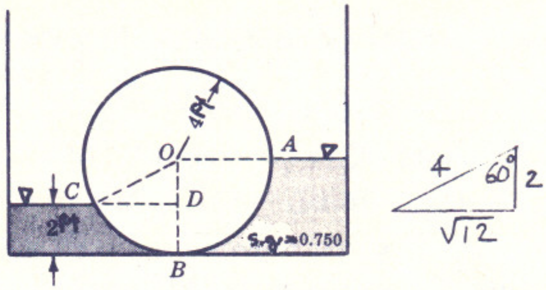


Fig. 5-9

5.10 The half-conical buttress ABE shown in Fig. 5-10 is used to support a half-cylindrical tower $ABCD$. Calculate the horizontal and vertical components of the force due to water acting on the buttress.

▮

$$F_H = \gamma h_{cg} A = (62.4)(3 + \tfrac{6}{3})[(6)(2+2)/2] = 3744 \text{ lb (right)}$$

$$F_V = \text{weight of (imaginary) volume of water above curved surface}$$

$$= (62.4)[(\tfrac{1}{2})(6)(\pi)(2)^2/3 + (\tfrac{1}{2})(\pi)(2)^2(3)] = 1960 \text{ lb (up)}$$

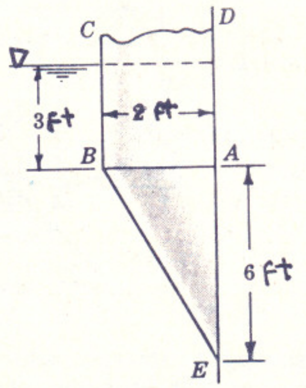


Fig. 5-10

5.11 A dam has a parabolic shape $z = z_0(x/x_0)^2$, as shown in Fig. 5-11*a*. The fluid is water and atmospheric pressure may be neglected. If $x_0 = 10$ ft and $z_0 = 24$ ft, compute forces F_H and F_V on the dam and the position c.p. where they act. The width of the dam is 50 ft.

▮ $F_H = \gamma \bar{h} A = 62.4[(24+0)/2][(24)(50)] = 898\,600$ lb. The location of F_H is along a (horizontal) line $\frac{24}{3}$, or 8.00 ft above the bottom of the dam. $F_V = (\text{area}_{0AB})(\text{width of dam})(\gamma)$. (See Fig. 5-11*b*.) $\text{Area}_{0AB} = 2x_0Z_0/3 = (2)(10)(24)/3 = 160 \text{ ft}^2$, $F_V = (160)(50)(62.4) = 499\,200$ lb. The location of F_V is along a (vertical) line through the center of gravity of area_{0AB}. From Fig. 5-11*b*, $x = 3x_0/8 = (3)(10)/8 = 3.75$ ft, $z = 3z_0/5 = (3)(24)/5 = 14.4$ ft, $F_{\text{resultant}} = \sqrt{499\,200^2 + 898\,600^2} = 1\,028\,000$ lb. As seen in Fig. 5-11*c*, $F_{\text{resultant}}$ acts down and to the right at an angle of arctan (499 200/898 600), or 29.1°. $F_{\text{resultant}}$ passes through the point $(x, z) = (3.75 \text{ ft}, 8 \text{ ft})$. If we move down alone the 29.1° line until we strike the dam, we find an equivalent center of pressure on the dam at $x = 5.43$ ft and $z = 7.07$ ft. This definition of c.p. is rather artificial, but this is an unavoidable complication of dealing with a curved surface.

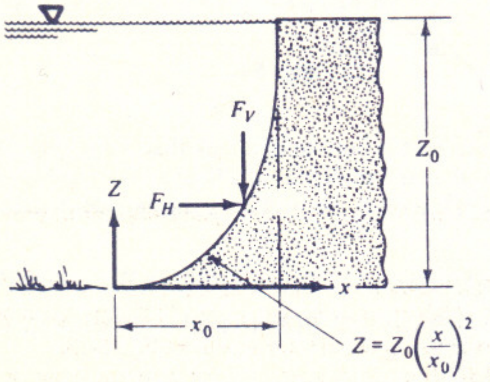


Fig. 5-11(*a*)

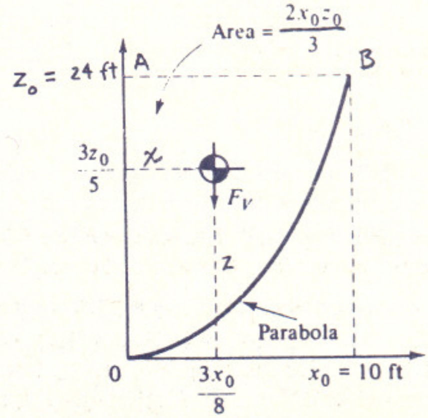


Fig. 5-11(*b*)

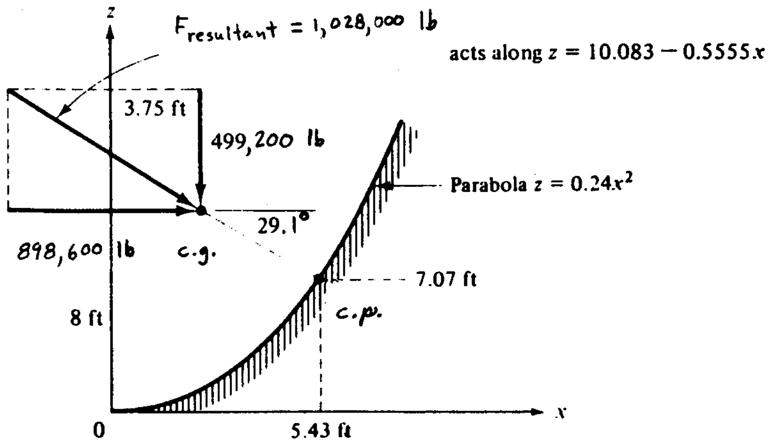


Fig. 5-11(c)

5.12 The canal shown in cross section in Fig. 5-12*a* runs 40 m into the paper. Determine the horizontal and vertical components of the hydrostatic force against the quarter-circle wall and the point c.p. where the resultant strikes the wall.

▎ $F_H = \gamma \bar{h} A = 9.79[(18 + 0)/2][(18)(40)] = 63\,439$ kN. The location of F_H is along a (horizontal) line $\frac{18}{3}$, or 6.00 m above the bottom of the wall. $F_V = 9.79[(40)(\pi)(18)^2/4] = 99\,650$ kN. The location of F_V is along a (vertical) line through the center of gravity of area$_{OAB}$. $x = 4r/(3\pi) = (4)(18)/(3\pi) = 7.64$ m, $F_{resultant} = \sqrt{63\,439^2 + 99\,650^2} = 118\,130$ kN. As seen in Fig. 5-12*b*, $F_{resultant}$ acts down and to the right at an angle of arctan (99 650/63 439), or 57.5°. $F_{resultant}$ passes through the point $(x, z) = (7.64\text{ m}, 6.00\text{ m})$. If we move down along the 57.5° line until we strike the wall, we find an equivalent center of pressure at $x = 8.33$ m and $z = 2.82$ m.

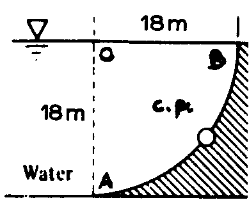


Fig. 5-12(*a*)

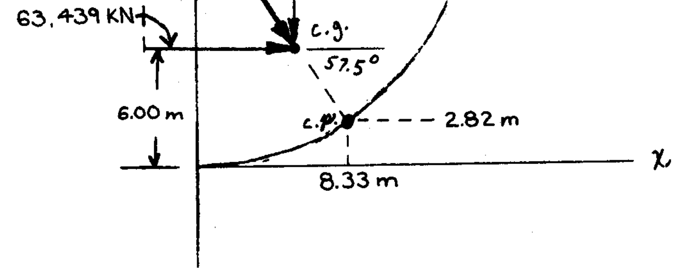


Fig. 5-12(*b*)

5.13 Gate *AB* in Fig. 5-13*a* is a quarter circle 8 ft wide into the paper. Find the force *F* just sufficient to prevent rotation about hinge *B*. Neglect the weight of the gate.

▎ $F_H = \gamma \bar{h} A = 62.4[(7 + 0)/2][(7)(8)] = 12\,230$ lb (left). The location of F_H is along a (horizontal) line $\frac{7}{3}$, or 2.333 ft above point *B*. (See Fig. 5-13*b*.) $F_V = F_1 - F_2 = 62.4[(8)(7)(7)] - 62.4[(8)(\pi)(7)^2/4] = 24\,461 - 19\,211 = 5250$ lb (up). The location of F_V can be determined by taking moments about point *B* in Fig. 5-13*b*. $5250x = (24\,461)(\frac{7}{2}) - (19\,211)[7 - (4)(7)/(3\pi)]$, $x = 1.564$ ft. The forces acting on the gate are shown in Fig. 5-13*c*. $\sum M_B = 0$; $7F - (2.333)(12\,230) - (1.564)(5250) = 0$, $F = 5249$ lb (down).

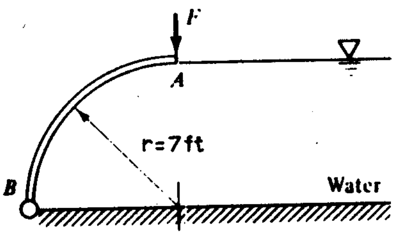


Fig. 5-13(a)

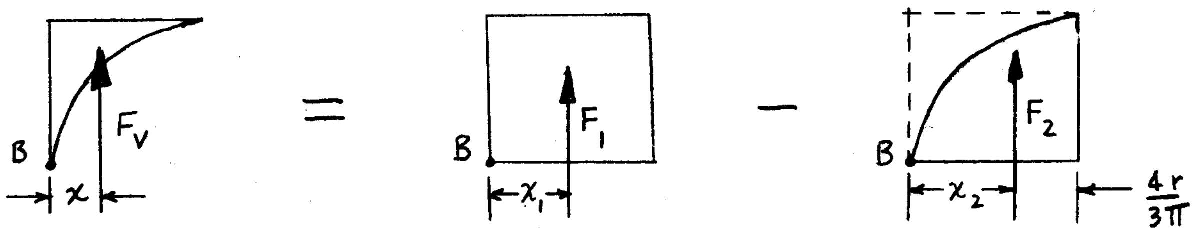


Fig. 5-13(b)

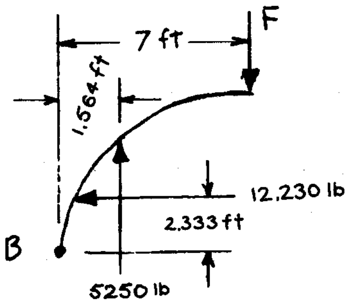


Fig. 5-13(c)

5.14 Repeat Prob. 5.13 if the gate is steel weighing 3000 lb.

▮ The weight of the gate acts at the center of gravity of the gate shown in Fig. 5-14. $2r/\pi = (2)(7)/\pi = 4.456$ ft; $\sum M_B = 0$. From Prob. 5.14, $7F - (2.333)(12\,230) - (1.564)(5250) + (3000)(7 - 4.456) = 0$, $F = 4159$ lb.

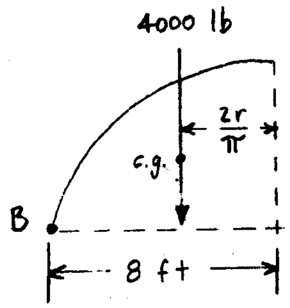


Fig. 5-14

5.15 Compute the horizontal and vertical components of the hydrostatic force on the quarter-circle face of the tank shown in Fig. 5-15*a*.

▮

$$F_H = \gamma h_{cg} A = 9.79[4 + \tfrac{1}{2}][(1)(7)] = 308 \text{ kN}$$

$$F_V = F_1 - F_2 = (9.79)[(7)(1)(5)] - (9.79)[(7)(\pi)(1)^2/4] = 289 \text{ kN} \qquad \text{(See Fig. 5-15}b\text{.)}$$

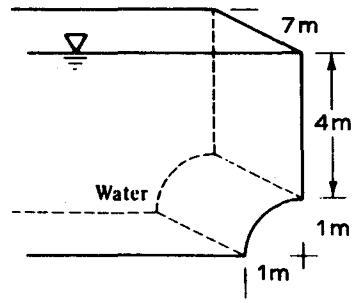


Fig. 5-15(a)

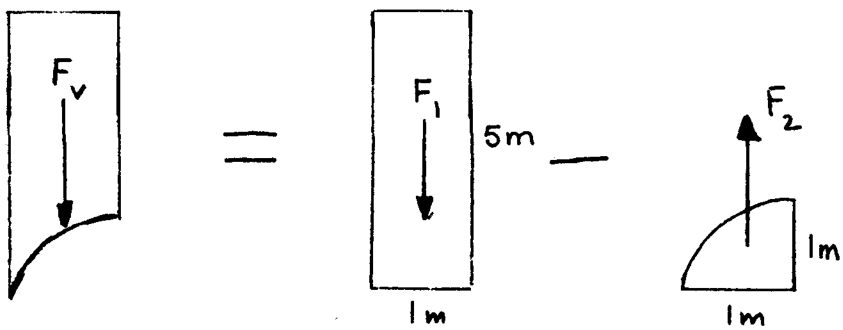


Fig. 5-15(b)

5.16 Compute the horizontal and vertical components of the hydrostatic force on the hemispherical boulder shown in Fig. 5-16*a*.

▌ From symmetry, $F_H = 0$, $F_V = F_1 - F_2$ (see Fig. 5-16*b*). $F_V = 62.4[(\pi)(3)^2(12)] - (62.4)[(\frac{1}{2})(\frac{4}{3})(\pi)(3)^3] =$ 17 643 lb.

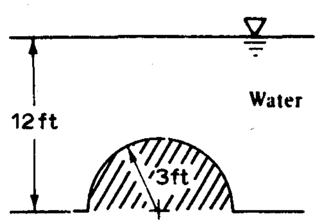


Fig. 5-16(a)

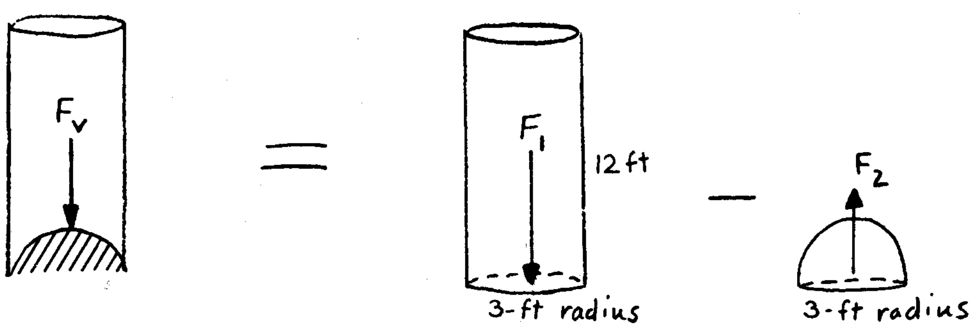


Fig. 5-16(b)

5.17 The bottled cider (s.g. = 0.96) in Fig. 5-17 is under pressure, as shown by the manometer reading. Compute the net force on the 2-in-radius concavity in the bottom of the bottle.

▌ From symmetry, $F_H = 0$, $p_{AA} + [(0.96)(62.4)](\frac{3}{12}) - [(13.6)(62.4)](\frac{5}{12}) = p_{atm} = 0$, $p_{AA} = 339\ \text{lb/ft}^2$ (gage); $F_V = p_{AA}A_{bottom}$ + weight of liquid below $AA = 339[(\pi)(\frac{4}{12})^2/4] + [(0.96)(62.4)][(\frac{7}{12})(\pi)(\frac{4}{12})^2/4] - [(0.96)(62.4)][(\frac{1}{2})(\frac{4}{3})(\pi)(\frac{2}{12})^3] = 32.1$ lb.

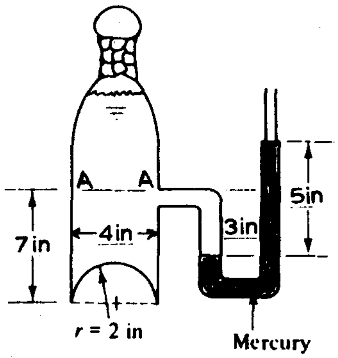


Fig. 5-17

5.18 Half-cylinder *ABC* in Fig. 5-18*a* is 9 ft wide into the paper. Calculate the net moment of the pressure forces on the body about point *C*.

▌ From symmetry, the horizontal forces balance and produce no net moment about point *C*. (See Fig. 5-18*b*.) $F_V = F_1 - F_2 = F_{\text{buoyancy of body } ABC} = [(0.85)(62.4)][(9)(\pi)(\frac{9}{2})^2/2] = 15\ 184$ lb, $x = 4r/(3\pi) = (4)(\frac{9}{2})/(3\pi) = 1.910$ ft, $M_C = (15\ 184)(1.910) = 29\ 001$ lb · ft (clockwise).

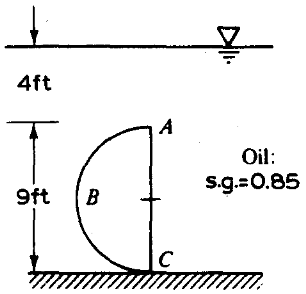


Fig. 5-18(*a*)

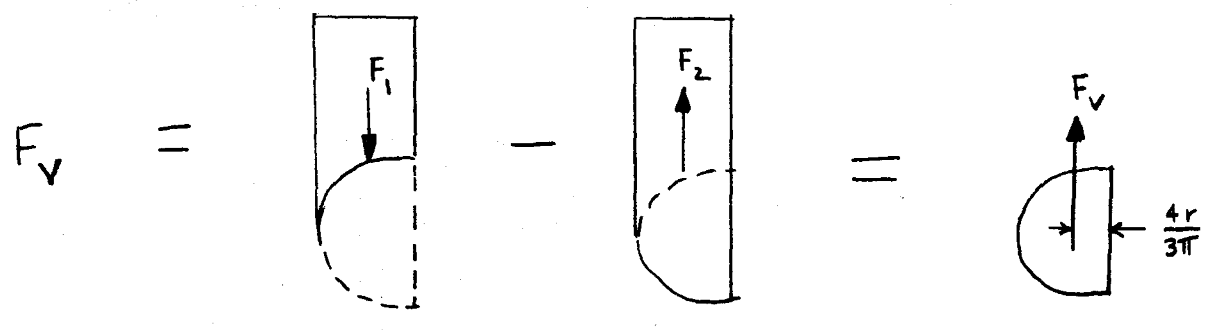


Fig. 5-18(*b*)

5.19 Compute the hydrostatic force and its line of action on semicylindrical indentation *ABC* in Fig. 5-19*a* per meter of width into the paper.

▌

$$F_H = \gamma h_{cg} A = [(0.88)(9.79)](2 + 2 + \tfrac{2.5}{2})[(2.5)(1)] = 113.1 \text{ kN}$$

$$y_{cp} = \frac{-I_{xx}\sin\theta}{h_{cg}A} = \frac{-[(1)(2.5)^3/12](\sin 90°)}{(2 + 2 + \frac{2.5}{2})[(2.5)(1)]} = -0.099 \text{ m}$$

As demonstrated in Prob. 5.18, $F_V = F_{\text{buoyancy of body } ABC}$ and it acts at $4r/(3\pi)$ from point *C*. $F_V = [(0.88)(9.79)][(1)(\pi)(\frac{2.5}{2})^2/2] = 21.14$ kN, $x = 4r/(3\pi) = (4)(\frac{2.5}{2})/(3\pi) = 0.531$ m. The forces acting on the indentation are shown in Fig. 5-19*b*. $F_{\text{resultant}} = \sqrt{21.14^2 + 113.1^2} = 115.1$ kN. As shown in Fig. 5-19*b*, $F_{\text{resultant}}$ passes through point *O* and acts up and to the right at an angle of arctan (21.14/113.1), or 10.59°.

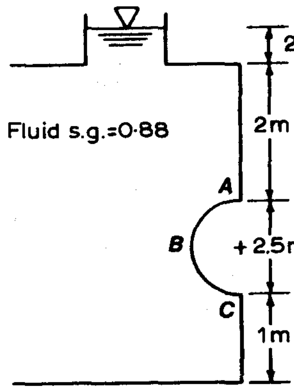


Fig. 5-19(a)

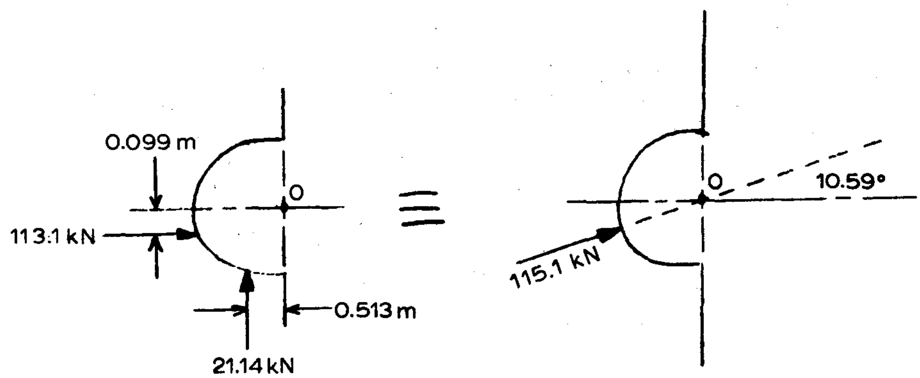


Fig. 5-19(b)

5.20 Find the force on the conical plug in Fig. 5-20. Neglect the weight of the plug.

▮ $F_V = pA_{\text{hole}} +$ weight of water above cone $= [(4.5)(144)][(\pi)(1)^2/4] + (62.4)[(4)(\pi)(1)^2/4]$
$- (62.4)[(\frac{1}{3})(1.207)(\pi)(1)^2/4] = 685$ lb

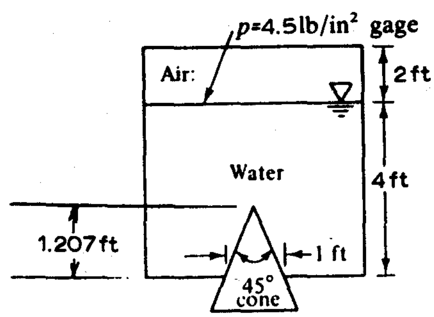


Fig. 5-20

5.21 The hemispherical dome in Fig. 5-21 is filled with water and is attached to the floor by two diametrically opposed bolts. What force in either bolt is required to hold the dome down, if the dome weighs 25 kN?

▮ $F_V =$ weight of (imaginary) water above the container

$= 9.79[(5 + 1.5)(\pi)(1.5)^2] - 9.79[(5)(\pi)(0.04)^2/4] - 9.79[(\frac{1}{2})(\frac{4}{3})(\pi)(1.5)^3] = 380.5$ kN (up)

net upward force on dome $= 380.5 - 25 = 355.5$ kN

force per bolt $= 355.5/2 = 177.7$ kN

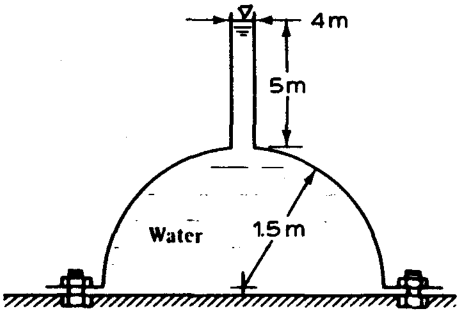


Fig. 5-21

5.22 A 3-m-diameter water tank consists of two half-cylinders, each weighing 3.5 kN/m, bolted together as shown in Fig. 5-22*a*. If support of the end caps is neglected, determine the force induced in each bolt.

▌ See Fig. 5-22*b*. Assuming the bottom half is properly supported, only the top half affects the bolt force. $p_1 = (9.79)(1.5 + 1) = 24.48\ \text{kN/m}^2$; $\sum F_y = p_1 A_1 - 2F_{\text{bolt}} - W_{\text{H}_2\text{O}} - W_{\text{tank half}} = 0$, $24.48[(3)(\frac{25}{100})] - 2F_{\text{bolt}} - 9.79[(\frac{25}{100})(\pi)(1.5)^2/2] - 3.5/4 = 0$, $F_{\text{bolt}} = 4.42\ \text{kN}$.

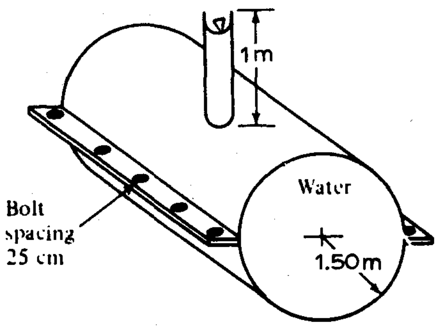


Fig. 5-22(*a*)

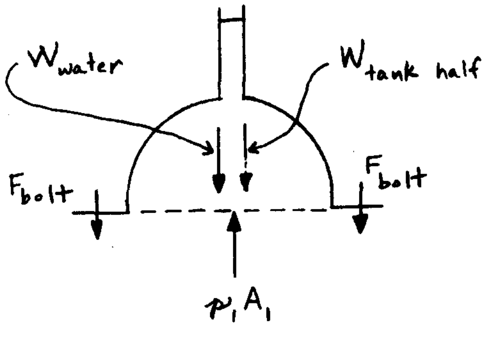


Fig. 5-22(*b*)

5.23 The cylinder in Fig. 5-23*a* extends 5 ft into the paper. Compute the horizontal and vertical components of the pressure force on the cylinder.

▌ See Fig. 5-23*b*. Note that the net horizontal force is based on the projected vertical area with depth *AB*. $F_H = \gamma h_{cg} A = 62.4[(4 + 2.828)/2][(4 + 2.828)(5)] = 7273\ \text{lb}$; F_V = equivalent weight of fluid in regions 1, 2, 3, and 4 = $(62.4)(5)[(\pi)(4)^2/2 + (2.828)(4) + (2.828)(2.828)/2 + (\pi)(4)^2/8] = 14\,579\ \text{lb}$.

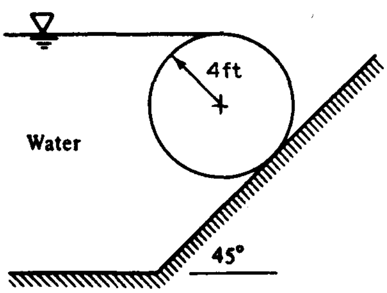


Fig. 5-23(*a*)

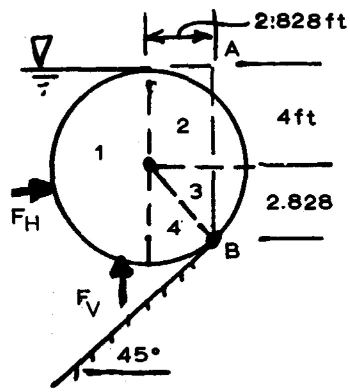


Fig. 5-23(*b*)

5.24 A 3-ft-diameter log (s.g. = 0.82) divides two shallow ponds as shown in Fig. 5-24*a*. Compute the net vertical and horizontal reactions at point *C*, if the log is 12 ft long.

▮ $F = \gamma hA$. Figure 5-24*b* shows the forces acting on the log.

$$(F_H)_1 = 62.4[(0+3)/2][(1.5+1.5)(12)] = 3370 \text{ lb} \qquad (F_H)_2 = 62.4[(0+1.5)/2][(1.5)(12)] = 842 \text{ lb}$$

$$(F_V)_1 = 62.4[(12)(\pi)(1.5)^2/2] = 2646 \text{ lb} \qquad (F_V)_2 = 62.4[(12)(\pi)(1.5)^2/4] = 1323 \text{ lb}$$

$$\sum F_x = 0 \qquad 3370 - 842 - C_x = 0 \qquad C_x = 2528 \text{ lb (left)}$$

$$\sum F_y = 0 \qquad 2646 + 1323 - [(0.82)(62.4)][(12)(\pi)(1.5)^2] + C_y = 0 \qquad C_y = 371 \text{ lb (up)}$$

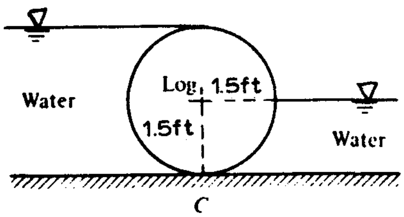


Fig. 5-24(*a*)

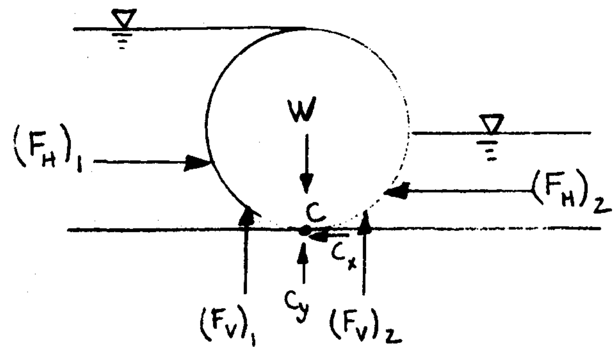


Fig. 5-24(*b*)

5.25 The 1-m-diameter cylinder in Fig. 5-25*a* is 8 m long into the paper and rests in static equilibrium against a frictionless wall at point *B*. Compute the specific gravity of the cylinder.

▮ See Fig. 5-25*b*. The wall reaction at *B* is purely horizontal. Then the log weight must exactly balance the vertical hydrostatic force, which equals the equivalent weight of water in the shaded area. $W_{\text{log}} = F_V = (9.79)(8)[(\frac{3}{4})(\pi)(0.5)^2 + (0.5)(0.5)] = 65.71$ kN, $\gamma_{\text{log}} = 65.71/[(8)(\pi)(0.5)^2] = 10.46$ kN/m³, s.g. $= 10.46/9.79 = 1.07$.

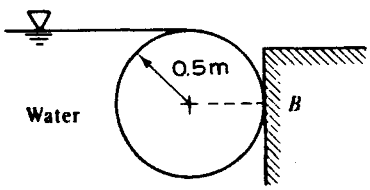


Fig. 5-25(*a*)

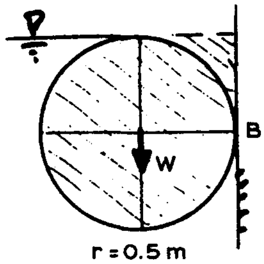


Fig. 5-25(*b*)

5.26 The tank in Fig. 5-26*a* is 3 m wide into the paper. Neglecting atmospheric pressure, compute the hydrostatic horizontal, vertical, and resultant force on quarter-circle panel *BC*.

▮ $F_H = \gamma h_{cg} A = (9.79)(4 + \frac{5}{2})[(5)(3)] = 954.5$ kN, F_V = weight of water above panel $BC = (9.79)[(3)(5)(4)] + (9.79)[(3)(\pi)(5)^2/4] = 1164$ kN, $F_{\text{resultant}} = \sqrt{954.5^2 + 1164^2} = 1505$ kN. As seen in Fig. 5-26*b*, $F_{\text{resultant}}$ passes through point *O* and acts down and to the right at an angle of arctan (1164/954.5), or 50.6°.

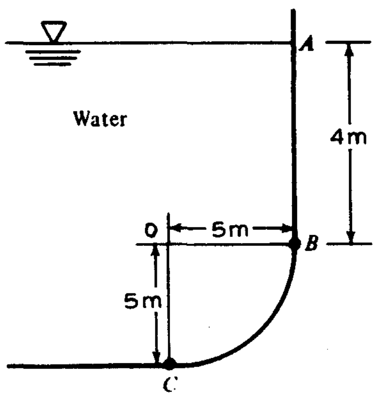


Fig. 5-26(*a*)

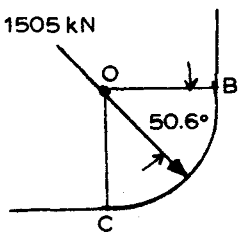


Fig. 5-26(*b*)

5.27 Gate AB in Fig. 5-27a is a quarter circle 7 ft wide, hinged at B and resting against a smooth wall at A. Compute the reaction forces at A and B.

$$F_H = \gamma h_{cg} A = (64)(11 - \tfrac{6}{2})[(7)(6)] = 21\,504 \text{ lb} \qquad y_{cp} = \frac{-I_{xx} \sin\theta}{h_{cg}A} = \frac{-[(7)(6)^3/12](\sin 90°)}{(11-\frac{6}{2})[(7)(6)]} = -0.375 \text{ ft}$$

Thus, F_H acts at $\frac{6}{2} - 0.375$, or 2.625 ft above point B. F_V = weight of seawater above gate AB = $(64)(7)[(11)(6)] - (64)(7)[(\pi)(6)^2/4] = 29\,568 - 12\,667 = 16\,901$ lb. The location of F_V can be determined by taking moments about point A in Fig. 5-27b. $(29\,568)(\frac{6}{2}) - (12\,667)[(4)(6)/(3\pi)] = 16\,901x$, $x = 3.340$ ft. The forces acting on the gate are shown in Fig. 5-27c.

$$\sum M_B = 0 \qquad (21\,504)(2.625) + (16\,901)(6 - 3.340) - 6A_x = 0 \qquad A_x = 16\,901 \text{ lb}$$

$$\sum F_x = 0 \qquad 21\,504 - B_x - 16\,901 = 0 \qquad B_x = 4603 \text{ lb}$$

$$\sum F_y = 0 \qquad B_y - 16\,901 = 0 \qquad B_y = 16\,901 \text{ lb}$$

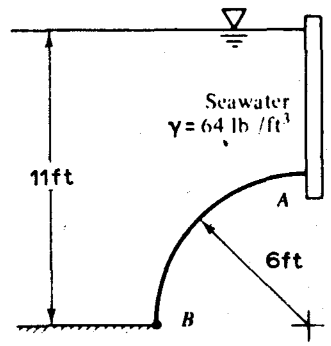


Fig. 5-27(a)

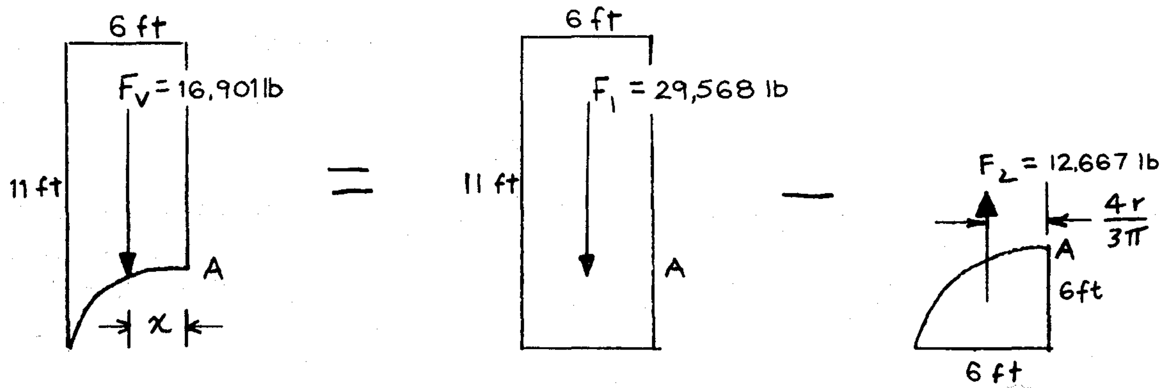


Fig. 5-27(b)

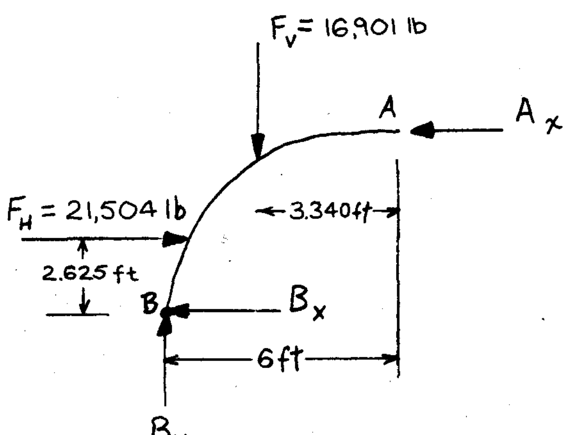
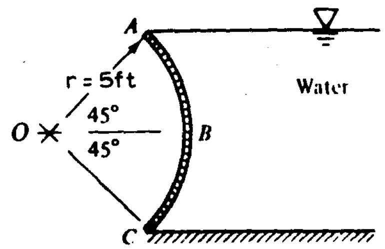


Fig. 5-27(c)

5.28 Curved wall ABC in Fig. 5-28a is a quarter circle 9 ft wide into the paper. Compute the horizontal and vertical hydrostatic forces on the wall and the line of action of the resultant force.

▌ See Fig. 5-28b. $F_H = \gamma h_{cg} A = (62.4)(3.536)[(7.072)(9)] = 14\,044$ lb, F_V = weight of (imaginary) water in crosshatched area in Fig. 5-28b $= (62.4)(9)[(\pi)(5)^2/4 - (2)(5 \sin 45°)(5 \cos 45°)/2] = 4007$ lb; $F_{\text{resultant}} = \sqrt{4007^2 + 14\,044^2} = 14\,604$ lb. $F_{\text{resultant}}$ passes through point O and acts at an angle of arctan $\frac{4007}{14\,044}$, or 15.9°, as shown in Fig. 5-28c.



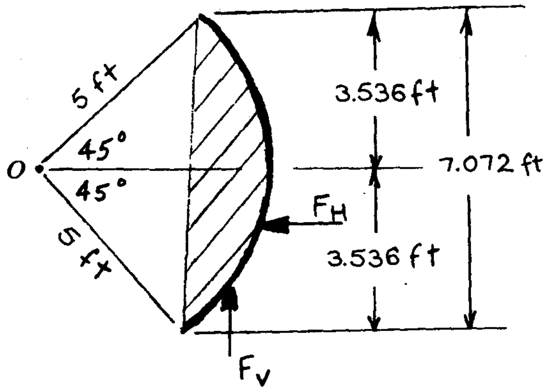


Fig. 5-28(b)

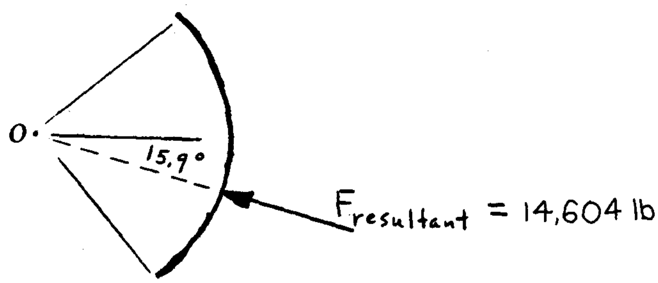


Fig. 5-28(c)

5.29 Pressurized water fills the tank in Fig. 5-29a. Compute the net hydrostatic force on conical surface ABC.

▌ From symmetry, $F_H = 0$. The gage pressure of 100 kPa corresponds to a fictitious water level at 100/9.79, or 10.215 m above the gage or $10.215 - 7$, or 3.215 m above AC (see Fig. 5-29b). F_V = weight of fictitious water above cone $ABC = 9.79[(3.215)(\pi)(3)^2/4 + (\frac{1}{3})(6)(\pi)(3)^2/4] = 361$ kN (up).

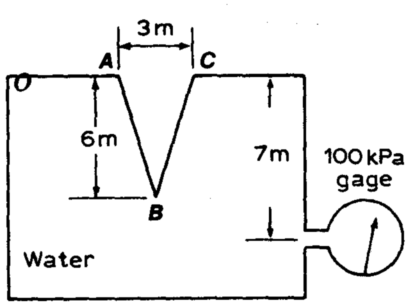


Fig. 5-29(a)

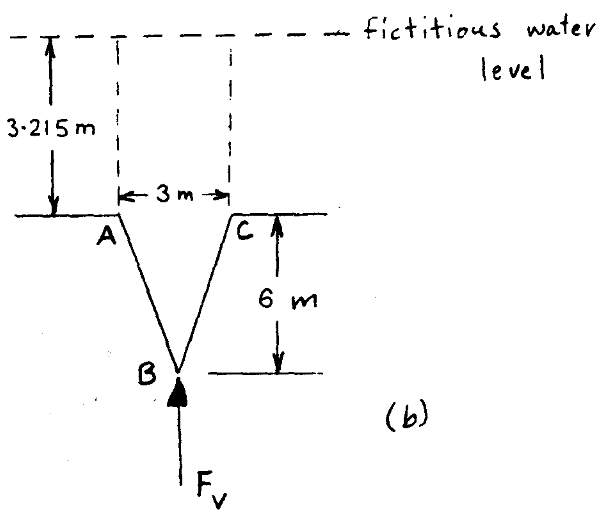


Fig. 5-29(b)

5.30 Gate AB in Fig. 5-30a is 7 m wide into the paper. Compute the force F required to prevent rotation about the hinge at B. Neglect atmospheric pressure.

▮ $F_H = \gamma \bar{h} A = 9.79[(9.6+0)/2][(9.6)(7)] = 3158$ kN. F_H acts at $\frac{9.6}{3}$, or 3.200 m above B (see Fig. 5-30b). F_V = weight of water above the gate $= 9.79[(\frac{2}{3})(6)(9.6)(7)] = 2632$ kN. F_V acts at $\frac{18}{8}$, or 2.250 m right of B (see Fig. 5-30b). $\sum M_B = 0$; $(3.200)(3158) + (2.250)(2632) - 9.6F = 0$, $F = 1670$ kN.

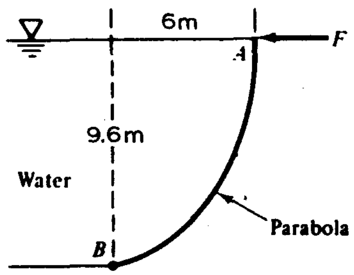


Fig. 5-30(a)

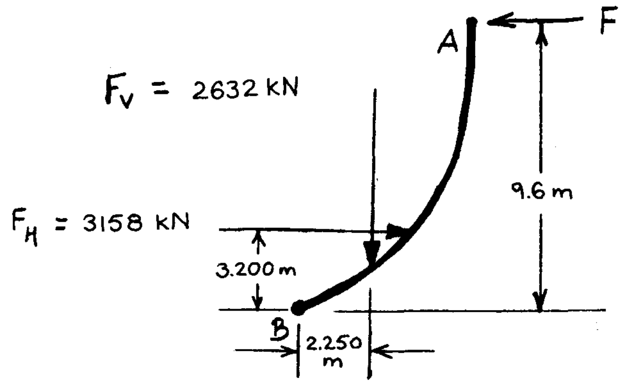


Fig. 5-30(b)

5.31 The cylindrical tank in Fig. 5-31 has a hemispherical end cap ABC. Compute the total horizontal and vertical forces exerted on ABC by the oil and water.

▮
$$F = \gamma h_{cg} A \qquad (F_H)_1 = [(0.9)(9.79)](3+\tfrac{2}{2})[(\pi)(2)^2/2] = 221 \text{ kN} \quad \text{(left)}$$
$$(F_H)_2 = \{[(0.9)(9.79)](3+2) + (9.79)(\tfrac{2}{2})\}[(\pi)(2)^2]/2 = 338 \text{ kN} \quad \text{(left)}$$
$$(F_H)_{\text{total}} = 221 + 338 = 559 \text{ kN} \quad \text{(left)}$$
$$F_V = \text{weight of fluid within hemisphere} = [(0.9)(9.79)][(\tfrac{1}{4})(\tfrac{4}{3})(\pi)(2)^3] + (9.79)[(\tfrac{1}{4})(\tfrac{4}{3})(\pi)(2)^3] = 156 \text{ kN} \quad \text{(down)}$$

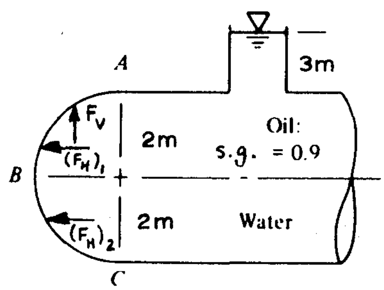


Fig. 5-31

5.32 A cylindrical barrier holds water, as shown in Fig. 5-32. The contact between cylinder and wall is smooth. Consider a 1-m length of cylinder and determine its weight and the force exerted against the wall.

▮
$$(F_V)_{BCD} = (9.79)(1)[(\pi)(2)^2/2 + (2)(2) + (2)(2)] = 139.8 \text{ kN} \quad \text{(up)}$$
$$(F_V)_{AB} = (9.79)(1)[(2)(2) - (\pi)(2)^2/4] = 8.4 \text{ kN} \quad \text{(down)}$$
$$\sum F_y = 0 \qquad 139.8 - W_{\text{cylinder}} - 8.4 = 0 \qquad W_{\text{cylinder}} = 131.4 \text{ kN}$$
$$F_H = \gamma h_{cg} A \qquad (F_H)_{ABC} = (9.79)(2)[(2+2)(1)] = 78.3 \text{ kN} \quad \text{(right)}$$
$$(F_H)_{DC} = (9.79)(2+\tfrac{2}{2})[(2)(1)] = 58.7 \text{ kN} \quad \text{(left)} \qquad (F_H)_{\text{against wall}} = 78.3 - 58.7 = 19.6 \text{ kN} \quad \text{(right)}$$

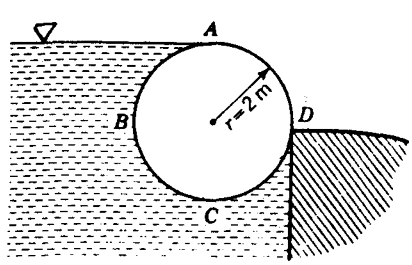


Fig. 5-32

5.33 The revolving gate in Fig. 5-33 is a quarter-cylinder with pivot through O. What force F is required to open it? (Treat the gate as weightless.)

At each point of $\widehat{ABC}$ the line of action of the pressure force passes through O; hence the pressure has no moment about O. It follows that any F, no matter how small, suffices to produce a net opening moment.

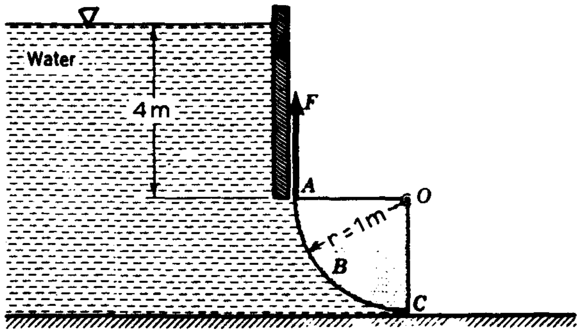


Fig. 5-33

5.34 Find the vertical component of force on the parabolic gate of Fig. 5-34a and its line of action.

$$F_V = \text{weight of imaginary liquid above gate} = \gamma L \int (H-y)\,dx \qquad \text{(see Fig. 5-34}b\text{)}$$

$$= (9.00)(3)\int_0^{0.8} (2-\sqrt{5x})\,dx = (9.00)(3)\left[2x - \frac{\sqrt{5}x^{3/2}}{\frac{3}{2}}\right]_0^{0.8} = 14.40\text{ kN}$$

$$x_{cp} = \frac{\gamma L \int (H-y)x\,dx}{F_V} \qquad \text{(see Fig. 5-34}b\text{)}$$

$$= \frac{(9.00)(3)\int_0^{0.8}(2-\sqrt{5x})x\,dx}{14.40} = \frac{(9.00)(3)\int_0^{0.8}(2x-\sqrt{5}x^{3/2})\,dx}{14.40}$$

$$= (9.00)(3)[x^2 - (\sqrt{5}x^{5/2})/\tfrac{5}{2}]_0^{0.8}/14.40 = 0.240\text{ m}$$

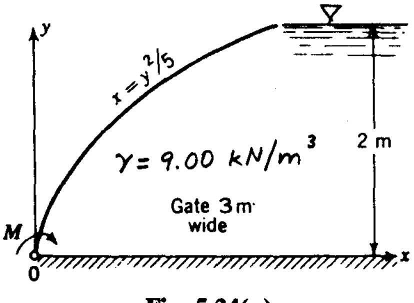


Fig. 5-34(a)

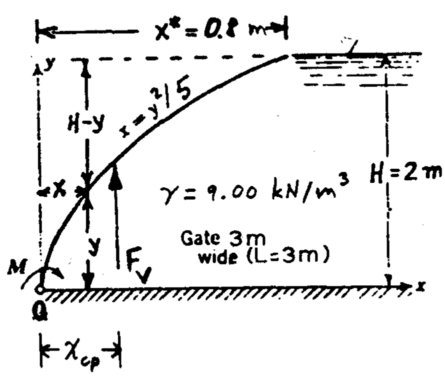


Fig. 5-34(b)

5.35 Determine the moment M needed to hold the gate of Fig. 5-34a shut. Neglect its weight.

$F_H = \gamma \bar{h} A = 9.00[(0+2)/2][(2)(3)] = 54.0$ kN (left). F_H acts at $\frac{1}{3}$, or 0.667 m above point 0. $F_V = 14.40$ kN (up) and $x_{cp} = 0.240$ m (from Prob. 5.34 and Fig. 5-34b). $\sum M_0 = 0$; $M - (14.40)(0.240) - (54.0)(0.667) = 0$, $M = 39.5$ kN · m.

5.36 Find the force on the body (part of a parabolic cylinder) of Fig. 5-35. The length normal to the paper is $L = 4.5$ m, and γ is 9.20 kN/m³.

$$F_H = \gamma \bar{h} A = (9.20)(\tfrac{1}{2})[(1)(4.5)] = 20.70 \text{ kN}$$

$$F_V = \text{weight of liquid above } OA = \int \gamma L y\, dx = \int_0^{\sqrt{8}} (9.20)(4.5)\left(\frac{x^2}{8}\right) dx = (9.20)(4.5)\left[\frac{x^3}{24}\right]_0^{\sqrt{8}} = 39.03 \text{ kN}$$

$$F_{\text{resultant}} = \sqrt{39.03^2 + 20.70^2} = 44.18 \text{ kN}$$

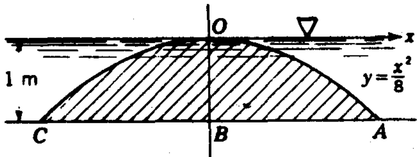


Fig. 5-35

5.37 The curved plate in Fig. 5-36 is an octant of a sphere. Find the resultant force, including its line of action, acting on the outer surface, if the radius of the sphere is 600 mm and its center is 2 m below the water surface.

See Fig. 5-36. $F_H = \gamma \bar{h} A = \gamma[H - 4r/(3\pi)](\pi r^2/4)$, $F_x = F_z = F_H = 9.79[2 - (4)(0.6)/(3\pi)][(\pi)(0.6)^2/4] =$ 4.831 kN (both F_x and F_z act toward 0); $F_y = F_V$ = weight of water above curved surface $= \gamma[(H)(\pi)(r)^2/4 - (\tfrac{4}{3})(\pi)(r)^3/8] = 9.79[(2)(\pi)(0.6)^2/4 - (\tfrac{4}{3})(\pi)(0.6)^3/8] = 4.429$ kN. $F_{\text{resultant}}$ acts on a line through 0 making a 45° angle with the x and z axes because of symmetry; $F_{\text{resultant}} = \sqrt{4.429^2 + 4.831^2 + 4.831^2} = 8.142$ kN. It acts at an angle $\theta = \arccos(4.429/8.142) = 57.0°$.

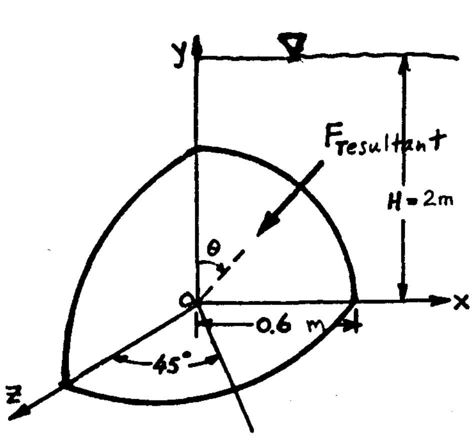


Fig. 5-36

5.38 Find the horizontal and vertical components of the force per unit width exerted by fluids on the horizontal cylinder in Fig. 5-37*a* if the fluid to the left of the cylinder is (*a*) a gas confined in a closed tank at a pressure of 35.0 kN/m² and (*b*) water with a free surface at an elevation coincident with the uppermost part of the cylinder. Assume in both instances that atmospheric pressure occurs to the right of the cylinder.

(*a*) The "net vertical projection" (see Fig. 5-37*a*) of the portion of the cylinder surface under consideration is $4 - (2 - 2\cos 30°)$, or 3.732 m. $F_H = pA = 35.0[(1)(3.732)] = 130.6$ kN (right). Note that the vertical force of the gas on surface *ab* is equal and opposite to that on surface *bc*. Hence, the "net horizontal projection" with regard to the gas is *ae* (see Fig. 5-38*b*), which is $2 \sin 30°$, or 1.000 m. $F_V = 35.0[(1)(1.000)] = 35.0$ kN (up).

(*b*) $$F_H = \gamma \bar{h} A = (9.79)(3.732/2)[(1)(3.732)] = 68.2 \text{ kN} \quad \text{(right)}$$

F_V = weight of crosshatched volume of water (Fig. 5-37*b*)

$$= (9.79)(1)[(\tfrac{210}{360})(\pi)(4)^2/4 + (\tfrac{1}{2})(1.000)(3.732 - \tfrac{4}{2}) + (1)(\tfrac{4}{2})] = 99.8 \text{ kN} \quad \text{(up)}$$

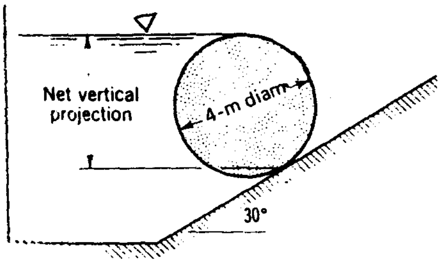


Fig. 5-37(a)

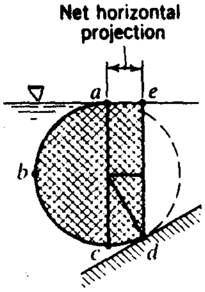


Fig. 5-37(b)

5.39 A vertical-thrust bearing consists of an 8-in-radius bronze hemisphere mating into a steel hemispherical shell. At what pressure must grease be supplied to the bearing so that an unbroken film is present when the vertical thrust on the bearing is 600 000 lb?

▌ Projected area $= \pi r^2 = (\pi)(8)^2 = 201.1\ \text{in}^2$ $\quad p = F/A = 600\,000/201.1 = 2984\ \text{lb/in}^2$

5.40 Find horizontal and vertical forces per foot of width on the Tainter gate shown in Fig. 5-38.

▌ $F_H = \gamma \bar{h} A = (62.4)[(0+25)/2][(25)(1)] = 19\,500$ lb. F_H acts at $(\frac{2}{3})(25)$, or 16.67 ft below the water surface. F_V = weight of imaginary water in $ACBA = (62.4)(1)[(\pi)(25)^2/5 - (2)(25\cos 36°)(25\sin 36°)/2] = 5959$ lb. F_V acts through the centroid of segment $ABCA$.

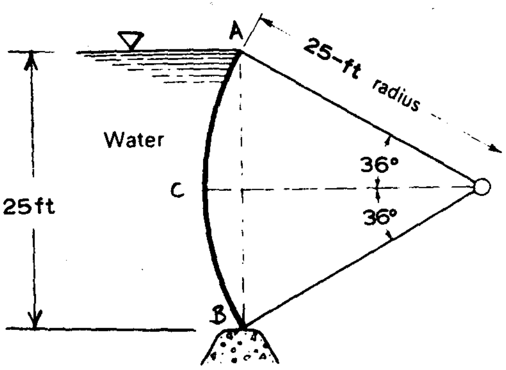


Fig. 5-38

5.41 The tank indicated in cross section in Fig. 5-39 is 6 m long normal to the paper. Curved panel MN is one-quarter of an ellipse with semiaxes b and d. If $b = 5$ m, $d = 7$ m, and $a = 1.0$ m, calculate the horizontal and vertical components of force and the resultant force on the panel.

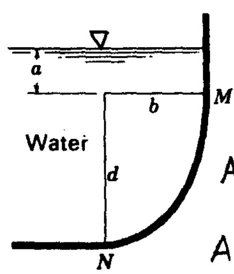
▌
$$F_H = \gamma h_{cg} A = 9.79(1.0 + \tfrac{7}{2})[(6)(7)] = 1850\ \text{kN}$$

$$h_{cp} = h_{cg} + \frac{I_{cg}}{h_{cg}A} = (1.0 + \tfrac{7}{2}) + \frac{(6)(7)^3/12}{(1.0+\tfrac{7}{2})[(6)(7)]} = 5.407\ \text{m below water surface}$$

$$F_V = \text{weight of water above surface } MN = (9.79)(6)[(\pi)(5)(7)/4 + (1.0)(5)] = 1908\ \text{kN}$$

$$x_{cp} = 4b/(3\pi) = (4)(5)/(3\pi) = 2.122\ \text{m to the right of } N \qquad F_{\text{resultant}} = \sqrt{1908^2 + 1850^2} = 2658\ \text{kN}$$

$F_{\text{resultant}}$ acts through the intersection of F_H and F_V at an angle of arctan (1908/1850), or 45.9°.



$\text{Area}_{\text{one quarter of elipse}} = \dfrac{\pi b d}{4}$

$\text{Area}_{\text{parabola}} = \dfrac{2bd}{3}$

Fig. 5-39

5.42 Solve Prob. 5.41 if $a = 1.0$ ft, $b = 5$ ft, $d = 7$ ft, the tank is 6 ft long, and MN represents a parabola with vertex at N.

$$F_H = \gamma h_{cg} A = (62.4)(1.0 + \tfrac{7}{2})[(6)(7)] = 11\,794 \text{ lb}$$

$$h_{cp} = h_{cg} + \frac{I_{cg}}{h_{cg}A} = (1.0 + \tfrac{7}{2}) + \frac{(6)(7)^3/12}{(1.0 + \tfrac{7}{2})[(6)(7)]} = 5.407 \text{ ft below water surface}$$

$$F_V = \text{weight of water above surface } MN = (62.4)(6)[(\tfrac{2}{3})(7)(5) + (1.0)(5)] = 10\,608 \text{ lb}$$

$$x_{cp} = (\tfrac{3}{8})(b) = (\tfrac{3}{8})(5) = 1.88 \text{ ft to the right of } N \qquad F_{\text{resultant}} = \sqrt{10\,608^2 + 11\,794^2} = 15\,863 \text{ lb}$$

$F_{\text{resultant}}$ acts through the intersection of F_H and F_V at an angle of arctan (10 608/11 794), or 42.0°.

5.43 In the cross section shown in Fig. 5-40, BC is a quarter-circle. If the tank contains water to a depth of 6 ft, determine the magnitude and location of the horizontal and vertical components on wall ABC per 1 ft width.

$$F_H = \gamma \bar{h} A = (62.4)[(0+6)/2][(1)(6)] = 1123 \text{ lb} \qquad h_{cp} = (\tfrac{2}{3})(6) = 4.00 \text{ ft}$$

$$F_V = \text{weight of water above surface } BC = (62.4)(1)[(6)(5)] - (62.4)(1)[(\pi)(5)^2/4] = 1872 - 1225 = 647 \text{ lb}$$

The location of F_V can be determined by taking moments about point B. $(1872)(\tfrac{5}{2}) - (1225)[(4)(5)/(3\pi)] = 647x_{cp}$, $x_{cp} = 3.22$ ft.

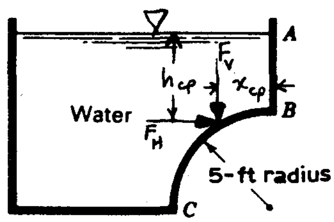


Fig. 5-40

5.44 Rework Prob. 5.43 where the tank is closed and contains gas at a pressure of 10 psi.

$$F_H = pA_V = [(10)(144)][(1)(6)] = 8640 \text{ lb} \qquad h_{cp} = \tfrac{6}{2} = 3.00 \text{ ft}$$

$$F_V = pA_H = [(10)(144)][(1)(5)] = 7200 \text{ lb} \qquad x_{cp} = \tfrac{5}{2} = 2.50 \text{ ft}$$

5.45 A spherical steel tank of 22 m diameter contains gas under a pressure of 300 kPa. The tank consists of two half-spheres joined together with a weld. What will be the tensile force across the weld? If the steel is 25.0 mm thick, what is the tensile stress in the steel?

$$F = pA = 300[(\pi)(22)^2/4] = 114\,040 \text{ kN} \qquad \sigma = \frac{\text{force/length}}{\text{thickness}} = \frac{114\,040/(22\pi)}{25.0/1000} = 66\,000 \text{ kPa}$$

5.46 Determine the force required to hold the cone shown in Fig. 5-41a in position.

Figure 5-41b shows the vertical projection above the opening. $p_{\text{air}} = 0.6 - [(0.83)(62.4)](5.1)/144 = -1.23$ psi, $F_{\text{air}} = [(1.23)(144)][(\pi)(0.804)^2] = 360$ lb, $F_{\text{cylinder}} = (62.4)(0.83)[(\pi)(0.804)^2(5.1 + 3)] = 852$ lb, $F_{\text{cone}} = (62.4)(0.83)[(3)(\pi)(0.804)^2/3] = 105$ lb; $\sum F_y = 0$, $360 - 852 + 105 + F = 0$, $F = 387$ lb.

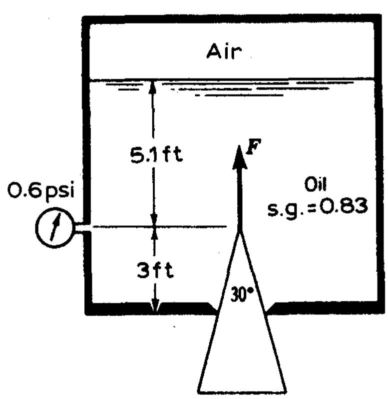


Fig. 5-41(a)

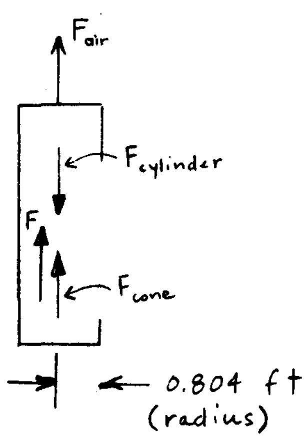


Fig. 5-41(b)

5.47 The cross section of the gate in Fig. 5-42 is given by $10x = 3y^2$; its dimension normal to the plane of the paper is 7 m. The gate is pivoted about O. Find the horizontal and vertical forces and the clockwise moment acting on the gate if the water depth is 1.8 m.

▮

$$F_H = \gamma \bar{h} A = 9.79[(0 + 1.8)/2][(7)(1.8)] = 111.0 \text{ kN}$$

$$F_V = \text{weight of water above the gate} = \int_0^{1.8} (9.79)(7)(x\,dy) = (9.79)(7)\int_0^{1.8} 0.3y^2\,dy = (9.79)(7)\left[\frac{0.3y^3}{3}\right]_0^{1.8} = 40.0 \text{ kN}$$

$$M_O = (111.0)(\tfrac{1.8}{3}) + \int_0^{1.8} (9.79)(7)\left(\frac{x}{2}\right)(x\,dy) = 66.6 + (9.79)(7)\int_0^{1.8} \frac{(0.3y^2)^2}{2}\,dy$$

$$= 66.6 + (9.79)(7)\left[\frac{0.09y^5}{10}\right]_0^{1.8} = 78.3 \text{ kN} \cdot \text{m}$$

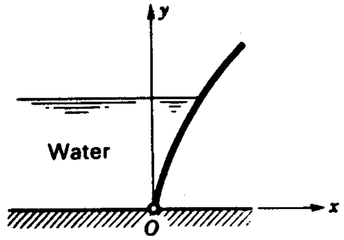


Fig. 5-42

5.48 Find the wall thickness of steel pipe needed to resist the static pressure in a 36-in-diameter steel pipe carrying water under a head of 750 ft of water. Use an allowable working stress for steel pipe of 16 000 psi.

▮

$$p = \gamma h = (62.4)(750) = 46\,800 \text{ lb/ft}^2 \quad \text{or} \quad 325 \text{ lb/in}^2$$

$$T = pd/2 = (325)(36)/2 = 5850 \text{ lb/in of pipe length} \qquad t = 5850/16\,000 = 0.366 \text{ in}$$

5.49 A vertical cylindrical tank is 6 ft in diameter and 10 ft high. Its sides are held in position by means of two steel hoops, one at the top and one at the bottom. The tank is filled with water up to 9 ft high. Determine the tensile stress in each hoop.

▮ See Fig. 5-43. $F = \gamma \bar{h} A = 62.4[(0 + 9)/2][(9)(6)] = 15\,163$ lb, $T = F/2 = 15\,163/2 = 7582$ lb; stress in top hoop $= (7582)(\frac{3}{10}) = 2275$ lb, stress in bottom hoop $= (7582)[(10 - 3)/10] = 5307$ lb.

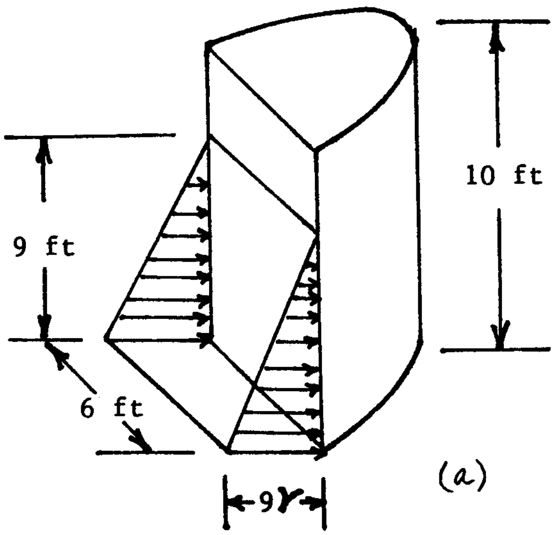


Fig. 5-43(*a*)

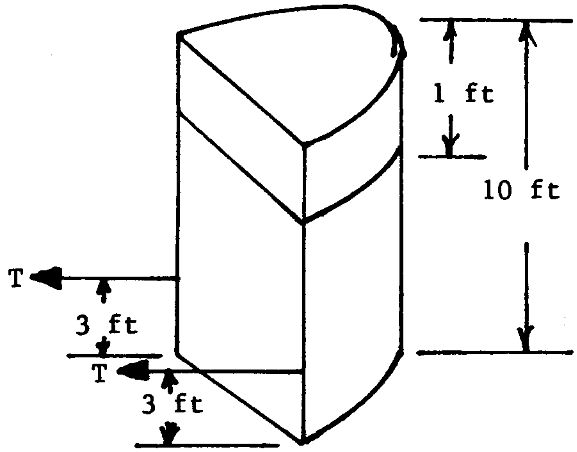


Fig. 5-43(*b*)

5.50 A 48-in-diameter steel pipe, $\frac{1}{4}$ in thick, carries oil of s.g. = 0.822 under a head of 400 ft of oil. Compute the (*a*) stress in the steel and (*b*) thickness of steel required to carry a pressure of 250 psi with an allowable stress of 18 000 psi.

$$p = \gamma h = [(0.822)(62.4)](400) = 20\,517 \text{ lb/ft}^2 \quad \text{or} \quad 142.5 \text{ lb/in}^2 \qquad \sigma = \frac{pr}{t}$$

(*a*) $$\sigma = \frac{(142.5)(48/2)}{\frac{1}{4}} = 13\,680 \text{ psi}$$

(*b*) $$18\,000 = \frac{(250)(48/2)}{t} \qquad t = 0.333 \text{ in}$$

5.51 A wooden storage vat, 20 ft in outside diameter, is filled with 24 ft of brine, s.g. = 1.06. The wood staves are bound by flat steel bands, 2 in wide by $\frac{1}{4}$ in thick, whose allowable stress is 16 000 psi. What is the spacing of the bands near the bottom of the vat, neglecting any initial stress? Refer to Fig. 5-44.

Force P represents the sum of all the horizontal components of small forces dP acting on length y of the vat, and forces T represent the total tension carried in a band loaded by the same length y.

$$\sum F_x = 0 \qquad 2T - P = 0 \qquad T = A_{\text{steel}}\sigma_{\text{steel}} = [(2)(\tfrac{1}{4})](16\,000) = 8000 \text{ lb}$$

$$p = \gamma hA = [(1.06)(62.4)](24)(20y) = 31\,749y \qquad (2)(8000) - 31\,749y = 0 \qquad y = 0.504 \text{ ft} \quad \text{or} \quad 6.05 \text{ in}$$

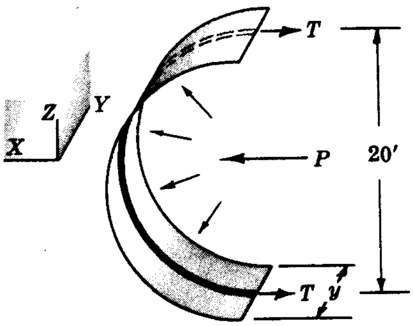


Fig. 5-44

5.52 A 4.0-in-ID steel pipe has a $\frac{1}{4}$-in wall thickness. For an allowable tensile stress of 10 000 psi, what is the maximum pressure?

$$\sigma = \frac{pr}{t} \qquad 10\,000 = \frac{(p)(4.0/2)}{\frac{1}{4}} \qquad p = 1250 \text{ lb/in}^2$$

5.53 A thin-walled hollow sphere 3.5 m in diameter holds gas at 1700 kPa. For an allowable stress of 50 000 kPa, determine the minimum wall thickness.

Considering half a sphere of diameter d (3.5 m) and thickness t, $(\pi dt)(\sigma) = (p)(\pi d^2/4)$, $[(\pi)(3.5)(t)](50\,000) = 1700[(\pi)(3.5)^2/4]$, $t = 0.02975$ m, or 29.75 mm.

5.54 A cylindrical container 8 ft high and 3 ft in diameter is reinforced with two hoops a foot from each end. When it is filled with water, what is the tension in each hoop due to the water?

See Fig. 5-45. $F = \gamma \bar{h} A = 62.4[(0 + 8)/2][(8)(3)] = 5990$ lb. F acts at $(\frac{2}{3})(8)$, or 5.333 ft from the top of the container.

$$\sum F_x = 0$$

$$2T_1 + 2T_2 - 5990 = 0 \tag{1}$$

$$\sum M_A = 0 \qquad (2T_2)(1.667) - (2T_1)(4.333) = 0$$

$$T_2 = 2.60T_1 \tag{2}$$

Solve simultaneous equations (1) and (2). $2T_1 + (2)(2.60T_1) - 5990 = 0$, $T_1 = 832$ lb, $T_2 = (2.60)(832) = 2163$ lb.

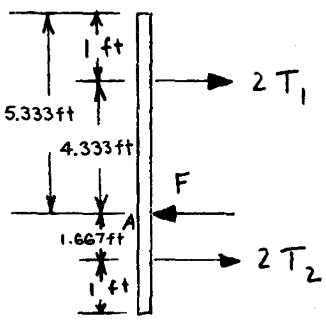


Fig. 5-45

5.55 A 30-mm-diameter steel ($\gamma = 77.0\ \mathrm{kN/m^3}$) ball covers a 15-mm-diameter hole in the bottom of a chamber in which gas pressure is 27 000 kPa. What force is required to push the ball up into the chamber?

▌ $$F = pA + \text{weight of ball} = 27\,000[(\pi)(\tfrac{15}{1000})^2/4] + [(\tfrac{4}{3})(\pi)(\tfrac{30}{1000})^3](77.0) = 4.780\ \mathrm{kN}$$

CHAPTER 6
Buoyancy and Flotation

6.1 A stone weighs 105 lb in air. When submerged in water, it weighs 67.0 lb. Find the volume and specific gravity of the stone.

▌ Buoyant force (F_b) = weight of water displaced by stone (W) = $105 - 67.0 = 38.0$ lb

$$W = \gamma V = 62.4V \qquad 38.0 = 62.4V \qquad V = 0.609\ \text{ft}^3$$

$$\text{s.g.} = \frac{\text{weight of stone in air}}{\text{weight of equal volume of water}} = \frac{105}{(0.609)(62.4)} = 2.76$$

6.2 A piece of irregularly shaped metal weighs 300.0 N in air. When the metal is completely submerged in water, it weighs 232.5 N. Find the volume of the metal.

▌ $$F_b = W \qquad 300.0 - 232.5 = [(9.79)(1000)](V) \qquad V = 0.00689\ \text{m}^3$$

6.3 A cube of timber 1.25 ft on each side floats in water as shown in Fig. 6-1. The specific gravity of the timber is 0.60. Find the submerged depth of the cube.

▌ $$F_b = W \qquad 62.4[(1.25)(1.25)(D)] = [(0.60)(62.4)][(1.25)(1.25)(1.25)] \qquad D = 0.750\ \text{ft}$$

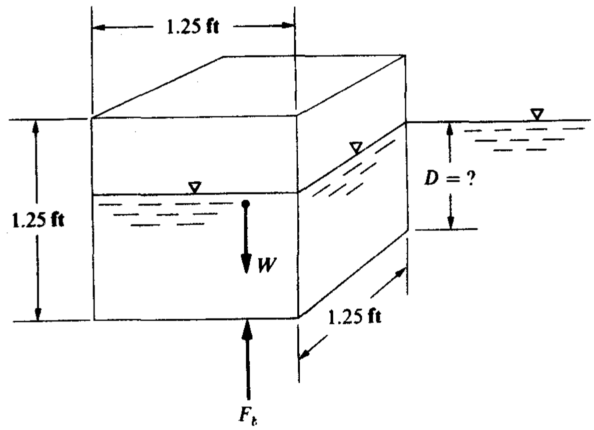


Fig. 6-1

6.4 Determine the magnitude and direction of the force necessary to hold a concrete cube, 0.300 m on each side, in equilibrium and completely submerged (*a*) in mercury (Hg) and (*b*) in water. Use $\text{s.g.}_{\text{concrete}} = 2.40$.

▌ (*a*) Since $\text{s.g.}_{\text{Hg}} = 13.6$ and $\text{s.g.}_{\text{concrete}} = 2.40$, it is evident that the concrete will float in mercury. Therefore, a force F acting downward will be required to hold the concrete in equilibrium and completely submerged in mercury. The forces acting on the concrete are shown in Fig. 6-2*a*, where F is the force required to hold the concrete cube in equilibrium and completely submerged, W is the weight of the concrete cube in air, and F_b is the buoyant force. $\sum F_y = 0$, $F + W - F_b = 0$, $F + [(2.40)(9.79)][(0.300)(0.300)(0.300)] - [(13.6)(9.79)][(0.300)(0.300)(0.300)] = 0$, $F = 2.96$ kN (downward). (*b*) Since $\text{s.g.}_{\text{concrete}} = 2.40$, it will sink in water. Therefore, a force F acting upward will be required to hold the concrete in equilibrium and completely submerged in water. The forces acting on the concrete in this case are shown in Fig. 6-2*b*. $\sum F_y = 0$, $W - F - F_b = 0$, $[(2.40)(9.79)][(0.300)(0.300)(0.300)] - F - 9.79[(0.300)(0.300)(0.300)] = 0$, $F = 0.370$ kN (upward).

6.5 A concrete cube 10.0 in on each side is to be held in equilibrium under water by attaching a lightweight foam buoy to it, as shown in Fig. 6-3. (In theory, the attached foam buoy and concrete cube, when placed under water, will neither rise nor sink.) If the specific weight of concrete and foam are 150 lb/ft³ and 5.0 lb/ft³, respectively, what minimum volume of foam is required?

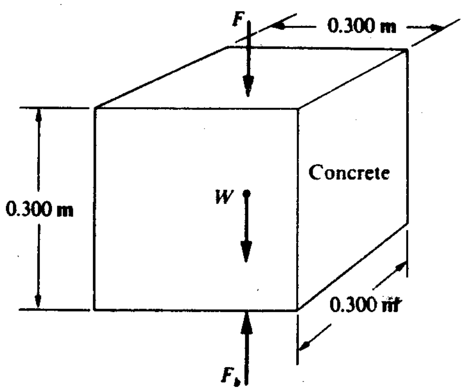


Fig. 6-2(a)

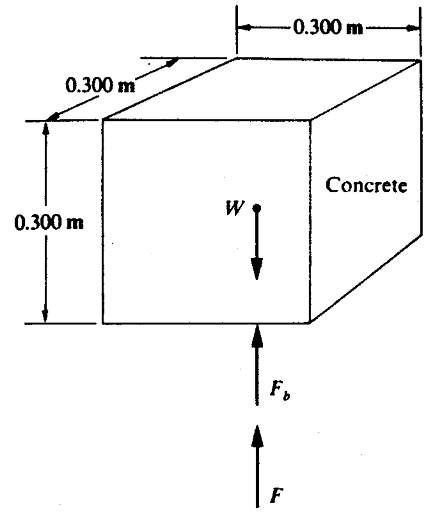


Fig. 6-2(b)

▌ The forces acting in this problem are shown in Fig. 6-3, where W_f and W_c are the respective weights of the foam and the concrete, and F_{bf} and F_{bc} are the respective buoyant forces on the foam and the concrete. $\sum F_y = 0$, $W_f - F_{bf} + W_c - F_{bc} = 0$, $5.0V_{\text{foam}} - 62.4V_{\text{foam}} + 150[(\frac{10}{12})(\frac{10}{12})(\frac{10}{12})] - 62.4[(\frac{10}{12})(\frac{10}{12})(\frac{10}{12})] = 0$, $V_{\text{foam}} = 0.883\ \text{ft}^3$.

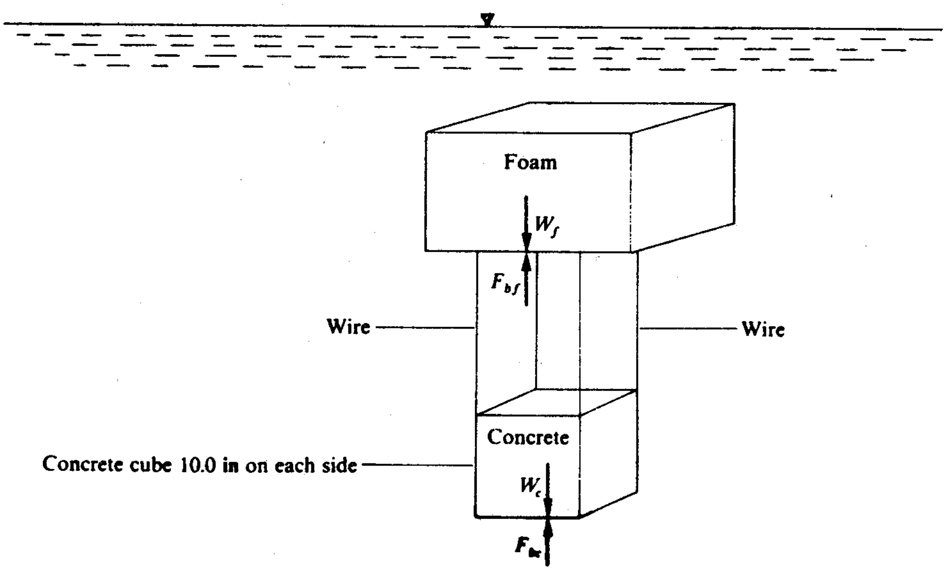


Fig. 6-3

6.6 A barge is loaded with 150 tons of coal. The weight of the empty barge in air is 35 tons. If the barge is 18 ft wide, 52 ft long, and 9 ft high, what is its draft (i.e., its depth below the water surface)?

▌ $$F_b = W \qquad 62.4[(18)(52)(D)] = (150 + 35)(2000) \qquad D = 6.33\ \text{ft}$$

6.7 Determine the submerged depth of a cube of steel 0.30 m on each side floating in mercury. The specific gravities of steel and mercury are 7.8 and 13.6, respectively.

▌ $$F_b = W \qquad [(13.6)(9.79)][(0.3)(0.3)(D)] = [(7.8)(9.79)][(0.3)(0.3)(0.3)] \qquad D = 0.172\ \text{m}$$

6.8 A cube of wood (s.g. = 0.60) has 9-in sides. Compute the magnitude and direction of the force F required to hold the wood completely submerged in water.

▌ Since s.g.$_{\text{wood}} = 0.60$, it is evident that the wood will float in water. Therefore, a force F acting downward will be required to hold the wood in equilibrium and completely submerged. The forces acting on the wood are essentially the same as those shown acting on the concrete cube in Fig. 6-2a: $\sum F_y = 0$, $F + W - F_b = 0$, $F + [(0.60)(62.4)][(\frac{9}{12})(\frac{9}{12})(\frac{9}{12})] - 62.4[(\frac{9}{12})(\frac{9}{12})(\frac{9}{12})] = 0$, $F = 10.5\ \text{lb}$ (downward).

6.9 A hollow cube 1.0 m on each side weighs 2.4 kN. The cube is tied to a solid concrete block weighing 10.0 kN. Will these two objects tied together float or sink in water? The specific gravity of the concrete is 2.40.

▌ Let W = weight of hollow cube plus solid concrete block, $(F_b)_1$ = buoyant force on hollow cube, and $(F_b)_2$ = buoyant force on solid concrete block. $W = 2.4 + 10.0 = 12.4$ kN, $(F_b)_1 = 9.79[(1)(1)(1)] = 9.79$ kN, $V_{block} = 10/[(2.40)(9.79)] = 0.4256\ \text{m}^3$, $(F_b)_2 = (9.79)(0.4256) = 4.17$ kN, $(F_b)_1 + (F_b)_2 = 9.79 + 4.17 = 13.96$ kN. Since $[W = 12.4] < [(F_b)_1 + (F_b)_2 = 13.96\ \text{kN}]$, the two objects tied together will float in water.

6.10 A concrete cube 0.5 m on each side is to be held in equilibrium under water by attaching a light foam buoy to it. What minimum volume of foam is required? The specific weights of concrete and foam are 23.58 kN/m³ and 0.79 kN/m³, respectively.

▌ Let W_f = weight of foam in air, $(F_b)_f$ = buoyant force on foam, W_c = weight of concrete in air, and $(F_b)_c$ = buoyant force on concrete. $\sum F_y = 0$, $W_f - (F_b)_f + W_c - (F_b)_c = 0$, $0.79V_{foam} - 9.79V_{foam} + 23.58[(0.5)(0.5)(0.5)] - 9.79[(0.5)(0.5)(0.5)] = 0$, $V_{foam} = 0.192\ \text{m}^3$.

6.11 A prismatic object 8 in thick by 8 in wide by 16 in long is weighed in water at a depth of 20 in and found to weigh 11.0 lb. What is its weight in air and its specific gravity?

▌ The forces acting on the object are shown in Fig. 6-4. $\sum F_y = 0$, $T + F_b - W = 0$, F_b = weight of displaced water $= 62.4[(8)(8)(16)/1728] = 37.0$ lb, $11.0 + 37.0 - W = 0$, $W = 48.0$ lb, s.g. $= 48.0/37.0 = 1.30$.

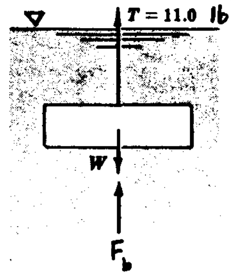


Fig. 6-4

6.12 A hydrometer weighs 0.00485 lb and has a stem at the upper end which is cylindrical and 0.1100 in in diameter. How much deeper will it float in oil of s.g. 0.780 than in alcohol of s.g. 0.821?

▌ $W_{hydrometer} = W_{displaced\ liquid}$. For position 1 in Fig. 6-5 in the alcohol, $0.00485 = [(0.821)(62.4)](V_1)$, $V_1 = 0.0000947\ \text{ft}^3$ (in alcohol). For position 2 in Fig. 6-5 in the oil, $0.00485 = [(0.780)(62.4)][0.0000947 + (h)(\pi)(0.1100/12)^2/4]$, $h = 0.0750$ ft, or 0.900 in.

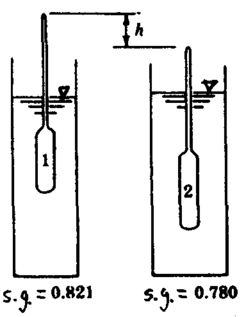


Fig. 6-5

6.13 A piece of wood of s.g. 0.651 is 3 in square and 5 ft long. How many pounds of lead weighing 700 lb/ft³ must be fastened at one end of the stick so that it will float upright with 1 ft out of water?

▌
$$W_{wood\ and\ lead} = W_{displaced\ water}$$
$$[(0.651)(62.4)][(5)(\tfrac{3}{12})(\tfrac{3}{12})] + 700V = 62.4[(5-1)(\tfrac{3}{12})(\tfrac{3}{12}) + V]$$
$$V = 0.00456\ \text{ft}^3 \qquad W_{lead} = (0.00456)(700) = 3.19\ \text{lb}$$

6.14 What fraction of the volume of a solid piece of metal of s.g. 7.25 floats above the surface of a container of mercury?

▌ Let V = volume of the metal and V' = volume of mercury displaced. $F_b = W$, $[(13.6)(62.4)](V') = [(7.25)(62.4)](V)$, $V'/V = 0.533$. Fraction of volume above mercury $= 1 - 0.533 = 0.467$.

6.15 A rectangular open box 25 ft by 10 ft in plan and 12 ft deep weighs 40 tons and is launched in fresh water. **(a)** How deep will it sink? **(b)** If the water is 12 ft deep, what weight of stone placed in the box will cause it to rest on the bottom?

▮ $$F_b = W$$

(a) $$62.4[(25)(10)(D)] = (40)(2000) \qquad D = 5.13\text{ ft}$$

(b) $$62.4[(25)(10)(12)] = (40 + W_{\text{stone}})(2000) \qquad W_{\text{stone}} = 53.6\text{ tons}$$

6.16 A block of wood floats in water with 2.0 in projecting above the water surface. When placed in glycerin of s.g. 1.35, the block projects 3.0 in above the liquid surface. Determine the specific gravity of the wood.

▮ Let A = area of block and h = height of block. $W_{\text{block}} = [(\text{s.g.})(62.4)](Ah/12)$, $W_{\text{displaced water}} = 62.4[(A)(h-2)/12]$, $W_{\text{displaced glycerin}} = [(1.35)(62.4)][(A)(h-3)/12]$. Since the weight of each displaced liquid equals the weight of the block, $W_{\text{displaced water}} = W_{\text{displaced glycerin}}$: $62.4[(A)(h-2)/12] = [(1.35)(62.4)][(A)(h-3)/12]$, $h = 5.86$ in. Also, $W_{\text{block}} = W_{\text{displaced water}}$: $[(\text{s.g.})(62.4)][(A)(5.86/12)] = 62.4[(A)(5.86-2)/12]$, s.g. $= 0.659$.

6.17 To what depth will an 8-ft-diameter log 15 ft long and of s.g. 0.425 sink in fresh water?

▮ The log is sketched in Fig. 6-6 with center O of the log above the water surface because the specific gravity is less than 0.5. (Had the specific gravity been equal to 0.5, the log would be half submerged.) $F_b = W$, F_b = weight of displaced liquid $= 62.4\{(15)[(2\theta/360)(\pi 4^2) - (2)(\frac{1}{2})(4\sin\theta)(4\cos\theta)]\} = 261.4\theta - (14\,976)(\sin\theta)(\cos\theta)$, $W = [(0.425)(62.4)][(15)(\pi 4^2) = 19\,996$.

$$261.4\theta - (14\,976)(\sin\theta)(\cos\theta) = 19\,996$$

This equation can be solved by successive trials.

Try $\theta = 85°$: $(261.4)(85) - (14\,976)(\sin 85°)(\cos 85°) = 20\,919 \quad (\neq 19\,996)$

Try $\theta = 83°$: $(261.4)(83) - (14\,976)(\sin 83°)(\cos 83°) = 19\,885 \quad (\neq 19\,996)$

Try $\theta = 83.2°$: $(261.4)(83.2) - (14\,976)(\sin 83.2°)(\cos 83.2°) = 19\,988 \quad (\neq 19\,996)$

Try $\theta = 83.22°$: $(261.4)(83.22) - (14\,976)(\sin 83.22°)(\cos 83.22°) = 19\,998$ (close enough)

$$\text{Depth of flotation} = DC = OC - OD = 4.00 - 4.00\cos 83.22° = 3.53\text{ ft}$$

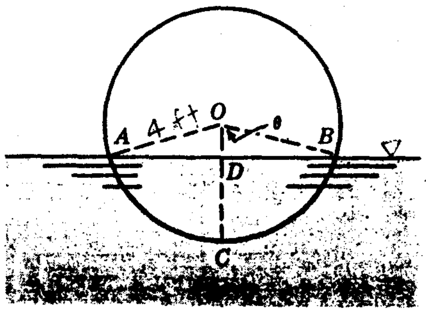


Fig. 6-6

6.18 **(a)** Neglecting the thickness of the tank walls in Fig. 6-7*a*, if the tank floats in the position shown what is its weight? **(b)** If the tank is held so that the top is 10 ft below the water surface, as shown in Fig. 6-7*b*, what is the force on the inside top of the tank? Use an atmospheric pressure equivalent to a 34.0-ft head of water.

▮ **(a)** $$W_{\text{tank}} = W_{\text{displaced liquid}} = 62.4[(1)(\pi 4^2/4)] = 784\text{ lb}$$

(b) The space occupied by the air will be less at the new depth shown in Fig. 6-7*b*. Assuming that the temperature of the air is constant, then for positions *a* and *b*, $p_A V_A = p_D V_D$, $[62.4(34.0+1)][(4)(\pi 4^2/4)] = [(62.4)(34.0+10+y)][(y)(\pi 4^2/4)]$, $y^2 + 44.0y - 140 = 0$, $y = 2.98$ ft. The pressure at D is $10 + 2.98$, or 12.98 ft of water (gage), which is essentially the same as the pressure on the inside top of the cylinder. Hence, the force on the inside top of the cylinder is given by $F = \gamma hA = (62.4)(12.98)(\pi 4^2/4) = 10\,180$ lb.

6.19 A ship, with vertical sides near the water line, weighs 4000 tons and draws 22 ft in salt water ($\gamma = 64.0\text{ lb/ft}^3$)(see Fig. 6-8). Discharge of 200 tons of water ballast decreases the draft to 21 ft. What would be the draft d of the ship in fresh water?

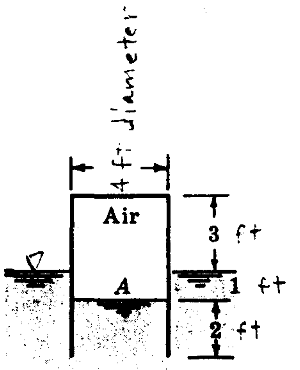


Fig. 6-7(a)

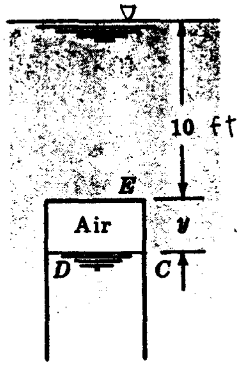


Fig. 6-7(b)

▮ Because the shape of the underwater section of the ship is not known, it is best to solve this problem on the basis of volumes displaced. A 1-ft decrease in draft was caused by a reduction in weight of 200 tons, or $\gamma V_d = 64.0[(1)(A)] = (200)(2000)$ where V_d represents the volume between drafts 22 ft and 21 ft, and $[(1)(A)]$ represents the water-line area times 1 ft, or the same volume V_d. From the equation above, $V_d = (200)(2000)/64.0 = 6250\ \text{ft}^3$ (this is per foot depth), F_b = weight of displaced liquid $= \gamma V_d$, $V_d = F_b/\gamma$. In Fig. 6-8, the vertically crosshatched volume is the difference in displaced fresh water and salt water. This difference in volume can be expressed as $W/\gamma_{\text{fresh H}_2\text{O}} - W/\gamma_{\text{salt H}_2\text{O}}$, or $(4000 - 200)(2000)/62.4 - (4000 - 200)(2000)/64.0$. Since $V_d = 6250\ \text{ft}^3/\text{ft}$ depth, the vertically crosshatched volume can also be expressed as $6250y$. Hence, $6250y = (4000 - 200)(2000)/62.4 - (4000 - 200)(2000)/64.0$, $y = 0.49$ ft; $d = 21 + 0.49 = 21.49$ ft.

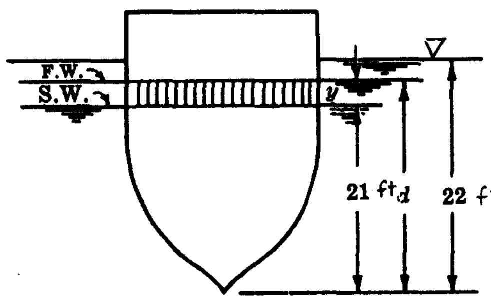


Fig. 6-8

6.20 A barrel containing water weighs 283.5 lb. What will be the reading on the scales if a 2 in by 2 in piece of wood is held vertically in the water to a depth of 2.0 ft?

▮ For every acting force, there must be an equal and opposite reacting force. The buoyant force exerted by the water upward against the bottom of the piece of wood is opposed by the 2 in by 2 in area of wood acting downward on the water with equal magnitude. This force will measure the increase in scale reading. $F_b = 62.4[(2)(\frac{2}{12})(\frac{2}{12})] = 3.5$ lb, new scale reading $= 283.5 + 3.5 = 287.0$ lb.

6.21 Find the weight of the floating can in Fig. 6-9.

▮ $$F_b = W \qquad 9.79[(\tfrac{7}{100})(\pi)(\tfrac{8}{100})^2/4] = W \qquad W = 0.00344\ \text{kN} \quad \text{or} \quad 3.44\ \text{N}$$

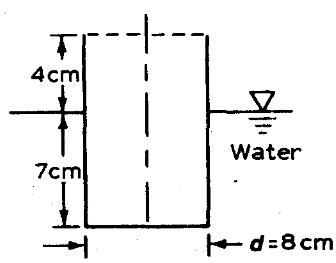


Fig. 6-9

6.22 The weight of a certain crown in air was found to be 14.0 N and its weight in water, 12.7 N. Was it gold (s.g. = 19.3)?

▮ $F_b = 14.0 - 12.7 = 1.3\ \text{N} \qquad V_{\text{displaced } H_2O} = V_{\text{crown}} = 1.3/[(9.79)(1000)] = 0.0001328\ \text{m}^3$

$\gamma_{\text{crown}} = 14.0/0.0001328 = 105\,422\ \text{N/m}^3 \quad \text{or} \quad 105.4\ \text{kN/m}^3 \qquad \text{s.g.}_{\text{crown}} = 105.4/9.79 = 10.77$

Thus the crown was not pure gold.

6.23 Repeat Prob. 6.22 assuming the crown is an alloy of gold (s.g. = 19.3) and silver (s.g. = 10.5). For the same measured weights, compute the fraction of silver in the crown.

▮ From Prob. 6.22, $\text{s.g.}_{\text{crown}} = 10.77$. Let α = fraction of silver in crown. $(\alpha)(10.5) + (1 - \alpha)(19.3) = 10.77$, $10.5\alpha + 19.3 - 19.3\alpha = 10.77$, $\alpha = 0.969$.

6.24 A plastic sphere is immersed in sea water ($\gamma = 64.0\ \text{lb/ft}^3$) and moored at the bottom. The sphere radius is 15 in. The mooring line has a tension of 160 lb. What is the specific weight of the sphere?

▮ The mooring line tension (T) and sphere weight (W) act downward on the sphere, while the buoyant force (F_b) acts upward. $\sum F_y = 0$; $F_b - T - W = 0$, $64.0[(\frac{4}{3})(\pi)(\frac{15}{12})^3] - 160 - (\gamma_{\text{sphere}})[(\frac{4}{3})(\pi)(\frac{15}{12})^3] = 0$, $\gamma_{\text{sphere}} = 44.4\ \text{lb/ft}^3$.

6.25 If the total weight of the hydrometer in Fig. 6-10 is 0.035 lb and the stem diameter is 0.35 in, compute the elevation h for a fluid of specific gravity 1.4.

▮ Let ΔV = submerged volume between s.g. = 1 and s.g. = 1.4, V_0 = submerged total volume when s.g. = 1.0, γ = specific weight of pure water, and W = weight of hydrometer. $W = \gamma V_0 = (\text{s.g.})(\gamma)(V_0 - \Delta V) = (\text{s.g.})(\gamma)(V_0) - (\text{s.g.})(\gamma)(\Delta V)$. Since $(\gamma)(V_0) = W$ and $\Delta V = hA = h(\pi d^2/4)$, $W = (\text{s.g.})(W) - (\text{s.g.})(\gamma)[(h)(\pi d^2/4)]$, $0.035 = (1.4)(0.035) - (1.4)(62.4)[(h)(\pi)(0.35/12)^2/4]$, $h = 0.240$ ft, or 2.88 in.

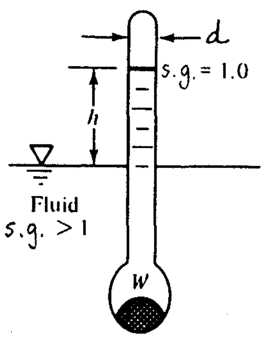


Fig. 6-10

6.26 For the hydrometer of Fig. 6-10, derive a formula for float position h as a function of s.g., W, d, and the specific weight γ of pure water. Are the scale markings linear or nonlinear as a function of s.g.?

▮ From Prob. 6.25, $W = (\text{s.g.})(W) - (\text{s.g.})(\gamma)[(h)(\pi d^2/4)]$.

$$h = \frac{(\text{s.g.})(W) - W}{(\text{s.g.})(\gamma)(\pi d^2/4)} = \frac{(W)(\text{s.g.} - 1)}{(\text{s.g.})(\gamma)(\pi d^2/4)}$$

When plotted in Fig. 6-11 (in arbitrary units), it is slightly nonlinear.

6.27 A hydrometer weighs of 0.17 N and has a stem diameter of 11 mm. What is the distance between scale markings for s.g. = 1.0 and s.g. = 1.1? Between 1.1 and 1.2?

▮ Let h_1 = distance between markings for s.g. = 1.0 and s.g. = 1.1 and h_2 = distance between scale markings for s.g. = 1.1 and s.g. = 1.2. From Prob. 6.26,

$$h = \frac{(W)(\text{s.g.} - 1)}{(\text{s.g.})(\gamma)(\pi d^2/4)} \qquad h_1 = \frac{(0.17)(1.1 - 1)}{1.1[(9.79)(1000)][(\pi)(0.011)^2/4]} = 0.0166\ \text{m} \quad \text{or} \quad 16.6\ \text{mm}$$

$$h_1 + h_2 = \frac{0.17(1.2 - 1)}{1.2[(9.79)(1000)][(\pi)(0.011)^2/4]} = 0.0305\ \text{m} \quad \text{or} \quad 30.5\ \text{mm}$$

$$h_2 = 30.5 - 16.6 = 13.9\ \text{mm}$$

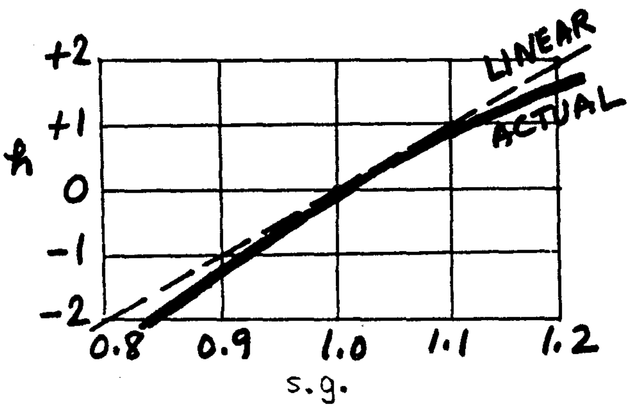


Fig. 6-11

6.28 A square pole (s.g. = 0.68), 80 mm by 80 mm by 6 m long, is suspended by a wire so that 4 m is submerged in water and 2 m is above the surface. What is the tension in the wire?

▌ Tension (T) and buoyant force (F_b) act upward on the pole, while pole weight (W) acts downward. $\sum F_y = 0$; $T + F_b - W = 0$, $T + 9.79[(0.080)(0.080)(4)] - [(0.68)(9.79)][(0.080)(0.080)(6)] = 0$, $T = 0.00501$ kN, or 5.01 N.

6.29 The spar in Fig. 6-12 is wood (s.g. = 0.62), 2 in by 2 in by 10 ft, and floats in sea water (s.g. = 1.025). How many pounds of steel (s.g. = 7.85) should be attached to the bottom to make a buoy that floats with exactly $h = 1.5$ ft of the spar exposed?

▌

$$V_{spar} = (\tfrac{2}{12})(\tfrac{2}{12})(10) = 0.2778 \text{ ft}^3 \qquad V_{submerged} = (\tfrac{2}{12})(\tfrac{2}{12})(8.5) = 0.2361 \text{ ft}^3$$

$$V_{steel} = W_{steel}/[(7.85)(62.4)] = 0.002041W_{steel} \qquad F_b = W_{wood} + W_{steel}$$

$$[(1.025)(62.4)](0.2361 + 0.002041W_{steel}) = [(0.62)(62.4)](0.2778) + W_{steel} \qquad W_{steel} = 5.01 \text{ lb}$$

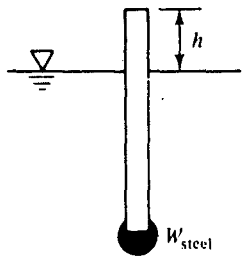


Fig. 6-12

6.30 A right circular cone is 50 mm in radius and 170 mm high and weighs 1.5 N in air. How much force is required to push this cone vertex-downward into ethanol so that its base is exactly at the surface? How much additional force will push the base 6.5 mm below the surface?

▌ Downward force (F) and cone weight (W) act downward on the cone, while buoyant force (F_b) acts upward. $\sum F_y = 0$; $F_b - F - W = 0$, $[(0.79)(9.79)(1000)][(\pi)(0.050)^2(0.170)/3] - F - 1.5 = 0$, $F = 1.94$ N. Once the cone is fully submerged, F is constant at 1.94 N.

6.31 A 2-in by 2-in by 10-ft spar has 7 lb of steel weight attached (Fig. 6-12); the buoy has lodged against a rock 7 ft deep, as depicted in Fig. 6-13. Compute the angle θ at which the buoy will lean, assuming the rock exerts no moment on the buoy.

▌ From Prob. 6.29, $V_{spar} = 0.2778$ ft³. $W_{wood} = [(0.62)(62.4)](0.2778) = 10.75$ lb and $F_b = 62.4[(\tfrac{2}{12})(\tfrac{2}{12})(L)] = 1.733L$. W_{wood} acts downward at a distance of $5 \sin\theta$ to the right of A, and F_b acts upward at a distance of $(L/2)(\sin\theta)$ to the right of A; while the steel force passes through point A. Hence, $\sum M_A = 0$, $10.75(5\sin\theta) - (1.733L)[(L/2)(\sin\theta)] = 0$, $L = 7.876$ ft; $\cos\theta = 7/L = 7/7.876 = 0.88878$, $\theta = 27.3°$.

6.32 The submerged brick in Fig. 6-14 is balanced by a 2.54-kg mass on the beam scale. What is the specific weight of the brick, if it displaces 2.197 liters of water?

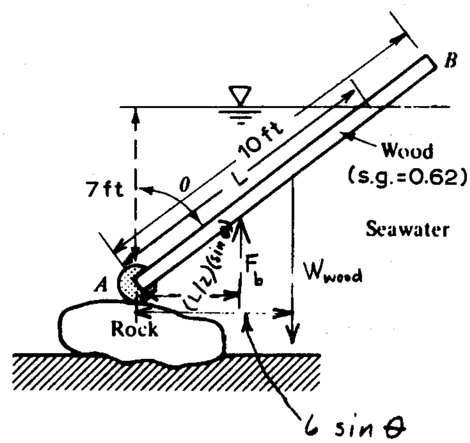


Fig. 6-13

▮ $F = mg = (2.54)(9.81) = 24.92$ N. Upward force (F) and buoyant force (F_b) act upward on the brick, while its weight (W) acts downward. $\sum F_y = 0$; $F_b + F - W = 0$, $[(9.79)(1000)](2.197 \times 10^{-3}) + 24.92 - W = 0$, $W = 46.43$ N; $\gamma = 46.43/(2.197 \times 10^{-3}) = 21\,133$ N/m³, or 21.13 kN/m³.

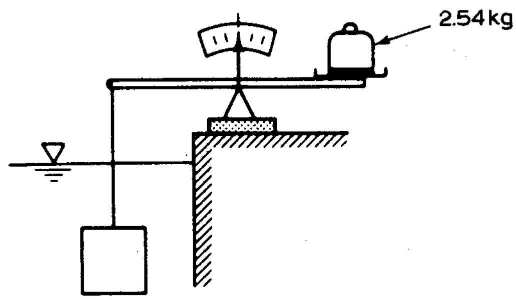


Fig. 6-14

6.33 The balloon in Fig. 6-15 is filled with helium pressurized to 111 kPa. Compute the tension in the mooring line.

▮ $\gamma = p/RT$; $\gamma_{air} = [(101)(1000)]/[(29.3)(273+20)] = 11.76$ N/m³, $\gamma_{He} = [(111)(1000)]/[(212.0)(273+20)] = 1.787$ N/m³. Weight of helium (W) and tension in mooring line (T) act downward on the balloon, while buoyant force (F_b) acts upward. $\sum F_y = 0$; $F_b - W - T = 0$, $11.76[(\frac{4}{3})(\pi)(\frac{9}{2})^3] - 1.787[(\frac{4}{3})(\pi)(\frac{9}{2})^3] - T = 0$, $T = 3807$ N.

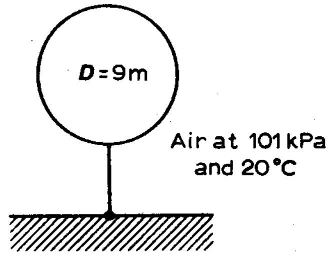


Fig. 6-15

6.34 A 1.1-ft-diameter hollow sphere is made of steel (s.g. = 7.85) with 0.015-ft wall thickness. How deep will the sphere sink in water (i.e., find h in Fig. 6-16)? How much weight must be added inside to make the sphere neutrally buoyant?

▮
$$F_b = W = \text{weight of displaced water} = \gamma[(\pi/3)(h)^2(3r - h)]$$
$$= 62.4\{(\pi/3)(h^2)[(3)(\tfrac{1.1}{2}) - h)]\} = 107.8h^2 - 65.35h^3$$
$$W = (\gamma_{steel})(V_{steel}) \qquad \gamma_{steel} = (7.85)(62.4) = 489.8 \text{ lb/ft}^3$$
$$V_{steel} = (\tfrac{4}{3})(\pi)(\tfrac{1.1}{2})^3 - (\tfrac{4}{3})(\pi)\{[1.1 - (2)(0.01500)]/2\}^3 = 0.05548 \text{ ft}^3$$
$$W = (489.8)(0.05548) = 27.17 \text{ lb} \qquad 107.8h^2 - 65.35h^3 = 27.17$$

Two roots of this equation are complex. The other, obtained by trial and error, is $h = 0.643$ ft. For neutral buoyancy, the total weight of the sphere plus added weight must equal the weight of water displaced by the entire sphere. Hence, $27.17 + W_{\text{added}} = 62.4[(\frac{4}{3})(\pi)(\frac{1.1}{2})^3]$, $W_{\text{added}} = 16.32$ lb.

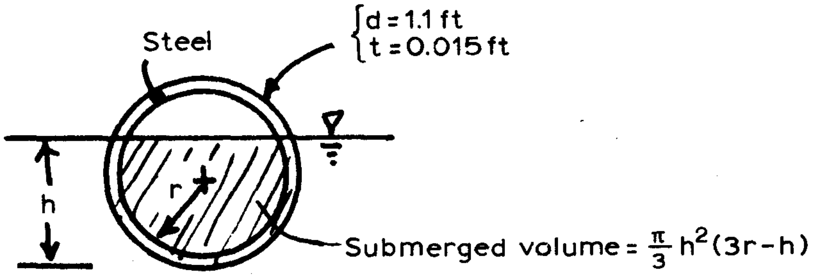


Fig. 6-16

6.35 When a 6-lb weight is placed on the end of a floating 5-in by 5-in by 10-ft wooden beam, the beam tilts at 1.6° with the weight at the surface, as shown in Fig. 6-17. What is the specific weight of the wood?

▮ $\tan 1.6° = h/10 \qquad h = 0.2793 \text{ ft} \qquad V_{\text{wood}} = (\frac{5}{12})(\frac{5}{12})(10) = 1.736 \text{ ft}^3$

$$F_b = W = 62.4[1.736 - (\tfrac{1}{2})(0.2793)(\tfrac{5}{12})(10)] = 72.02 \text{ lb}$$

$$W = (\gamma_{\text{wood}})(1.736) + 6 \qquad 72.02 = (\gamma_{\text{wood}})(1.736) + 6 \qquad \gamma_{\text{wood}} = 38.0 \text{ lb/ft}^3$$

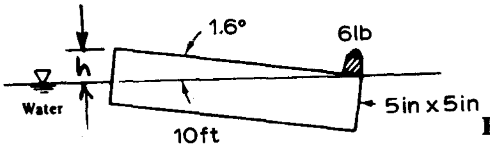


Fig. 6-17

6.36 A wooden beam (s.g. = 0.64) is 140 mm by 140 mm by 5 m and is hinged at A, as shown in Fig. 6-18. At what angle θ will the beam float in water?

▮ The forces acting on the beam are shown in Fig. 6-18. $W_{\text{beam}} = [(0.64)(9.79)][(0.140)(0.140)(5)] = 0.6140$ kN and $F_b = 9.79[(0.140)(0.140)(L)] = 0.1919L$. $\sum M_A = 0$; $(0.1919L)[(5 - L/2)(\cos\theta)] - (0.6140)[(\frac{5}{2})(\cos\theta)] = 0$, $-0.0960L^2 + 0.9595L - 1.535 = 0$, $L = 2.000$ m; $\sin\theta = 1/(5 - 2.000) = 0.33333$, $\theta = 19.5°$.

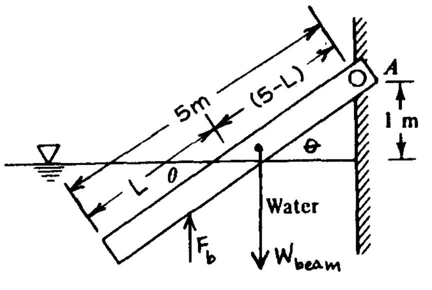


Fig. 6-18

6.37 A barge weighs 45 tons empty and is 18 ft wide, 45 ft long, and 9 ft high. What will be its draft when loaded with 125 tons of gravel and floating in sea water (s.g. = 1.025)?

▮ $F_b = W \qquad [(1.025)(62.4)][(18)(45)(h)] = (45 + 125)(2000) \qquad h = 6.56 \text{ ft}$

6.38 A block of steel (s.g. = 7.85) will "float" at a mercury–water interface as in Fig. 6-19. What will be the ratio of distances a and b for this condition?

▮ Let w = width of block and L = length of block. $F_b = W$, $(\gamma_{H_2O})(aLw) + (13.6)(\gamma_{H_2O})(bLw) = (7.85)(\gamma_{H_2O})(a + b)(Lw)$, $a + 13.6b = 7.85a + 7.85b$, $a/b = 0.839$.

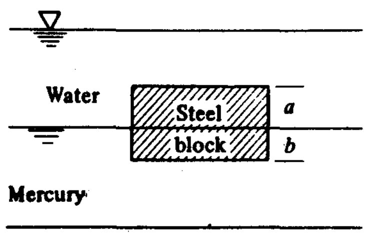


Fig. 6-19

6.39 A balloon weighing 3.2 lb is 5.5 ft in diameter. Upon release it is filled with hydrogen at 17 psia and 65° F. At what altitude in the standard atmosphere will this balloon be neutrally buoyant?

$$F_b = W \qquad \gamma = p/RT \qquad \gamma_H = (17)(144)/[(765.5)(460+65)] = 0.006091\ \text{lb/ft}^3$$

$$(\gamma_{air})[(\tfrac{4}{3})(\pi)(\tfrac{5.5}{2})^3] = 3.2 + (0.006091)[(\tfrac{4}{3})(\pi)(\tfrac{5.5}{2})^3] \qquad \gamma_{air} = 0.04282\ \text{lb/ft}^3$$

From Table A-7, altitude = approximately 18 600 ft.

6.40 A rectangular barge 18 ft wide by 46 ft long by 9 ft deep floats empty with a draft of 4 ft in a canal lock 28 ft wide by 56 ft long and water depth 7 ft when the empty barge is present. If 170 000 lb of steel is loaded onto the barge, what are the new draft of the barge (h) and water depth in the lock (H)?

The weight of the barge (W_b) is equal to the buoyant force when the draft is 4 ft. $W_b = 62.4[(18)(46)(4)] = 206\,669$ lb; $F_b = W$, $62.4[(18)(46)(h)] = 206\,669 + 170\,000$, $h = 7.290$ ft. Volume of water in lock $= (7)(28)(56) - (4)(18)(46) = 7664\ \text{ft}^3$. After steel is added, $(H)(28)(56) - (7.290)(18)(46) = 7664$, $H = 8.74$ ft.

6.41 A 4-in-diameter solid cylinder of height 3.75 in weighing 0.85 lb is immersed in liquid ($\gamma = 52.0\ \text{lb/ft}^3$) contained in a tall, upright metal cylinder having a diameter of 5 in. Before immersion the liquid was 3.0 in deep. At what level will the solid cylinder float? See Fig. 6-20.

Let x = distance solid cylinder falls below original liquid surface, y = distance liquid rises above original liquid surface, and $x + y$ = depth of submergence. $V_A = V_B$, $x[(\pi)(4)^2/4] = y[(\pi)(5)^2/4] - y[(\pi)(4)^2/4]$, $x = 0.5625y$. $F_b = W$, $52.0[(\pi)(\tfrac{4}{12})^2/4][(x+y)/12] = 0.85$, $x + y = 2.248$, $0.5625y + y = 2.248$, $y = 1.44$ in, $x = (0.5625)(1.44) = 0.81$ in. The bottom of the solid cylinder will be $3.0 - 0.81$, or 2.19 in above the bottom of the hollow cylinder.

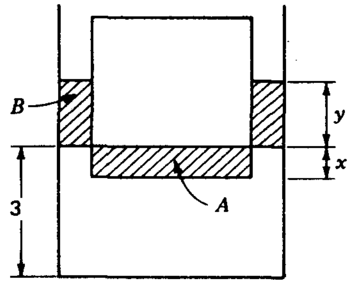


Fig. 6-20

6.42 An iceberg in the ocean floats with one-seventh of its volume above the surface. What is its specific gravity relative to ocean water? What portion of its volume would be above the surface if ice were floating in pure water? $\gamma_{\text{ocean}\ H_2O} = 64.0\ \text{lb/ft}^3$.

$$(V_{iceberg})_{submerged} = (1 - \tfrac{1}{7})V_{iceberg} = 0.857V_{iceberg}$$

$$F_b = W \qquad (\gamma_{\text{ocean}\ H_2O})(V_{iceberg})_{submerged} = (\gamma_{iceberg})(V_{iceberg})$$

$$\text{s.g.}_{iceberg} = \gamma_{iceberg}/\gamma_{\text{ocean}\ H_2O} = (V_{iceberg})_{submerged}/V_{iceberg}$$

$$= 00.857V_{iceberg}/V_{iceberg} = 0.857 \qquad \text{(relative to ocean water)}$$

$$\gamma_{iceberg} = (0.857)(64.0) = 54.85\ \text{lb/ft}^3 \qquad \text{s.g.}_{iceberg} = 54.85/62.4 = 0.879 \qquad \text{(relative to pure water)}$$

Therefore, $1 - 0.879 = 0.121$, or 12.1 percent of its volume would be above the water surface in pure water.

6.43 A hydrometer consists of an 11-mm-diameter cylinder of length 220 mm attached to a 26-mm-diameter weighted sphere. The cylinder has a mass of 1.5 g, and the mass of the sphere is 13.0 g. At what level will this device float in liquids having specific gravities of 0.8, 1.0, and 1.2? Is the scale spacing on the cylindrical stem uniform? Why or why not?

▮ Let y = submerged length of cylinder in millimeters. $V_{sphere} = (\frac{4}{3})(\pi)(\frac{26}{2})^3 = 9203\ mm^3$, $V_{submerged\ cylinder} = (y)(\pi)(\frac{11}{2})^2 = 95.03y$, $F_b = W = (s.g.)[(9.79)(1000)/1000^3](9203 + 95.03y) = (0.09010)(s.g.) + (0.0009303)(s.g.)(y)$, $W = mg = (1.5 + 13.0)(9.81) = 142.2\ g \cdot m/s^2$, or 0.1422 N, $(0.09010)(s.g.) + (0.0009303)(s.g.)(y) = 0.1422$, $y = [0.1422 - (0.09010)(s.g.)]/[(0.0009303)(s.g.)]$.

For s.g. = 0.8 $\quad y = [0.1422 - (0.09010)(0.8)]/[(0.0009303)(0.8)] = 94.2\ mm$

For s.g. = 1.0 $\quad y = [0.1422 - (0.09010)(1.0)]/[(0.0009303)(1.0)] = 56.0\ mm$

For s.g. = 1.2 $\quad y = [0.1422 - (0.09010)(1.2)]/[(0.0009303)(1.2)] = 30.5\ mm$

Scale spacing is not uniform because buoyant force is not directly proportional to submergence.

6.44 A typewriter weighs 6 lb in water and 8 lb in oil of specific gravity 0.86. Find its specific weight.

▮ $F_b = W$, $62.4V = W - 6$, $[(0.86)(62.4)](V) = W - 8$. Subtracting the second equation from the first gives $62.4V - [(0.86)(62.4)](V) = -6 - (-8)$, $V = 0.229\ ft^3$; $(62.4)(0.229) = W - 6$, $W = 20.3$ lb. $\gamma = 20.3/0.229 = 88.6\ lb/ft^3$.

6.45 A balloon weighs 270 lb and has a volume of 14 900 ft³. It is filled with helium, which weighs 0.0112 lb/ft³ at the temperature and pressure of the air, which weighs 0.0807 lb/ft³. What load will the balloon support?

$$F_b = W \qquad (0.0807)(14\,900) = 270 + (0.0112)(14\,900) + \text{load} \qquad \text{Load} = 766\ \text{lb}$$

6.46 A small cylindrical drum 32 cm in diameter and 52 cm high, weighing 27.0 N, contains perfume (s.g. = 0.83) to a depth of 22 cm. (*a*) When placed in water, what will be the depth y to the bottom of the drum? (*b*) How much perfume can the drum hold and still float?

▮ (*a*) $$F_b = W \qquad 9.79[(y)(\pi)(\tfrac{32}{100})^2/4] = 27.0/1000 + [(0.83)(9.79)][(0.22)(\pi)(0.32)^2/4]$$

$$y = 0.217\ \text{m} \quad \text{or} \quad 21.7\ \text{cm}$$

(*b*) $$9.79[(0.52)(\pi)(0.32)^2/4] = 27.0/1000 + [(0.83)(9.79)][(h)(\pi)(0.32)^2/4] \qquad h = 0.585\ \text{m} \quad \text{or} \quad 58.5\ \text{cm}$$

Since $h = 58.5$ cm is greater than the height of the drum (52 cm), the drum will float when full. Therefore, $V_{max} = (0.52)(\pi)(0.32)^2/4 = 0.0418\ m^3$, or 41.8 L.

6.47 A block ($\gamma = 124\ lb/ft^3$) 1 ft square and 9 in deep floats on a stratified liquid composed of a 7-in layer of water above a layer of mercury. (*a*) Determine the position of the bottom of the block.
(*b*) If a downward vertical force of 260 lb is applied to the center of mass of this block, what is the new position of the bottom of the block?

▮ (*a*) $F_b = W$. Let x = depth into mercury below water–mercury interface. $[(13.6)(62.4)][(1)(1)(x)] + 62.4[(1)(1)(\frac{7}{12})] = (124)[(1)(1)(\frac{9}{12})]$, $x = 0.0667$ ft, or 0.800 in. (*b*) In this case the top of the block will be below the water surface. Hence, $[(13.6)(62.4)][(1)(1)(x)] + 62.4[(1)(1)(\frac{9}{12} - x)] = 124[(1)(1)(\frac{9}{12})] + 260$, $x = 0.389$ ft, or 4.67 in.

6.48 Two spheres, each 1.3 m in diameter, weigh 5 kN and 13 kN, respectively. They are connected with a short rope and placed in water. What is the tension (T) in the rope and what portion of the lighter sphere protrudes from the water?

▮ For the lower (heavier) sphere, the buoyant force and T act upward and its weight acts downward. Hence, $\sum F_y = 0$, $F_b = 9.79[(\frac{4}{3})(\pi)(1.3/2)^3] = 11.26$ kN, $11.26 + T - 13 = 0$, $T = 1.74$ kN. For the upper (lighter) sphere, the buoyant force acts upward and its weight and T act downward. Hence, $F_b - 5 - 1.74 = 0$, $F_b = 6.74$ kN. Portion above water = $(11.26 - 6.74)/11.26 = 0.401$, or 40.1 percent of volume.

6.49 A board weighing 2.2 lb/ft and of cross-sectional area 8 in² dips into oil as shown in Fig. 6-21. If the hinge is frictionless, find θ.

▮ The forces acting on the board are shown in Fig. 6-21. $W = (2.2)(11) = 24.2$ lb; $F_b = (53)[(\frac{8}{144})(x)] = 2.944x$, $\sum M_{hinge} = 0$, $(24.2)[(\frac{11}{2})(\sin\theta)] - (2.944x)[(11 - x/2)(\sin\theta)] = 0$, $1.472x^2 - 32.38x + 133.1 = 0$; $x_1 = 16.53$ ft and

$x_2 = 5.47$ ft. Using $x = 5.47$ ft (since $x = 16.53$ ft is impossible for this situation), $\cos\theta = 5/(11 - 5.47) = 0.90416$, $\theta = 25.3°$.

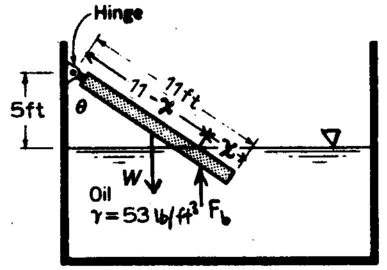


Fig. 6-21

6.50 A cube 2.2 ft on an edge has its lower half of s.g. = 1.6 and upper half of s.g. = 0.7. It rests in a two-layer fluid, with lower s.g. = 1.4 and upper s.g. = 0.8. Determine the height h of the top of the cube above the interface (see Fig. 6-22).

▌

$$F_b = W$$

$$[(1.4)(62.4)][(2.2)(2.2)(2.2 - h)] + [(0.8)(62.4)][(2.2)(2.2)(h)]$$
$$= [(1.6)(62.4)][(2.2)(2.2)(\tfrac{2.2}{2})] + [(0.7)(62.4)][(2.2)(2.2)(\tfrac{2.2}{2})]$$
$$h = 0.917 \text{ ft}$$

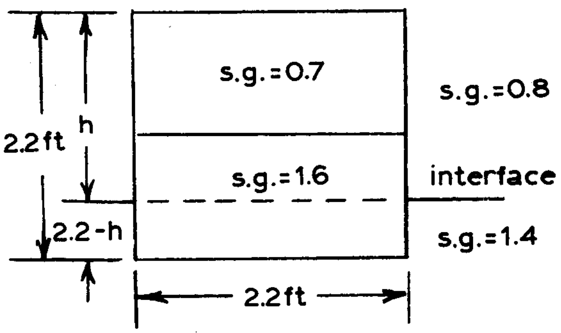


Fig. 6-22

6.51 Determine the volume and density of an object that weighs 4 N in water and 5 N in an alcohol of s.g. 0.80.

▌ $F_b = W$ $\quad [(9.79)(1000)](V_{\text{object}}) = W_{\text{object}} - 4 \quad [(0.80)(9.79)(1000)](V_{\text{object}}) = W_{\text{object}} - 5$

Subtracting the second equation from the first gives $9790V_{\text{object}} - 7832V_{\text{object}} = 1$, $V_{\text{object}} = 0.0005107\ \text{m}^3$. $[(9.79)(1000)](0.0005107) = W_{\text{object}} - 4$, $W_{\text{object}} = 9.000$ N; $\gamma = 9.000/0.0005107 = 17\,623\ \text{N/m}^3$.

$$\rho = \frac{\gamma}{g} = \frac{17\,623}{9.81} = 1796\ \text{kg/m}^3 \qquad V_s = \frac{1}{\rho} = \frac{1}{1796} = 0.000557\ \text{m}^3/\text{kg}$$

6.52 With how many pounds of concrete ($\gamma = 25\ \text{kN/m}^3$) must a beam of volume of 0.2 m³ and s.g. = 0.67 be coated to insure that it sinks in water?

▌ $F_b = W \quad (9.79)(0.2) + 9.79V_{\text{concrete}} = [(0.67)(9.79)](0.2) + 25V_{\text{concrete}} \quad V_{\text{concrete}} = 0.04248\ \text{m}^3$

$$W_{\text{concrete}} = (0.04248)(25) = 1.062\ \text{kN} \quad \text{or} \quad 1062\ \text{N} \quad \text{or} \quad 1062/4.448 = 239\ \text{lb}$$

6.53 The gate of Fig. 6-23 weighs 160 lb/ft normal to the page. It is in equilibrium as shown. Neglecting the weight of the arm and brace supporting the counterweight, find W (weight in air). The weight is made of concrete, s.g. = 2.50.

▌ $F_H = \gamma hA = (62.4)(\tfrac{6}{2})[(6)(1)] = 1123$ lb $\quad \sum M_{\text{hinge}} = 0 \quad (1123)(\tfrac{6}{3}) - (W)(5\sin 30°) = 0$

$$W = 898\ \text{lb}$$

This is the submerged weight.

$$F_b = W \qquad 62.4V_{\text{concrete}} = [(2.50)(62.4)](V_{\text{concrete}}) - 898 \qquad V_{\text{concrete}} = 9.594\ \text{ft}^3$$
$$W_{\text{concrete}} = [(2.50)(62.4)](9.594) = 1497\ \text{lb}$$

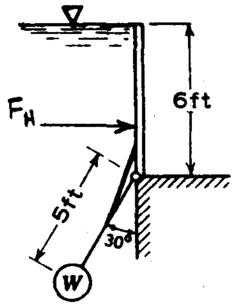


Fig. 6-23

6.54 A wooden pole (s.g. = 0.55), 550 mm in diameter, has a concrete cylinder (s.g. = 2.50), 550 mm long and of the same diameter, attached to one end. Determine the minimum length of pole for the system to float vertically in static equilibrium.

▌ The system will float at minimum length of wooden cylinder as shown in Fig. 6-24. $F_b = W$, $(\gamma)(A)(L + 0.550) = [(0.55)(\gamma)](L)(A) + [(2.50)(\gamma)](A)(0.550)$, $L = 1.833$ m.

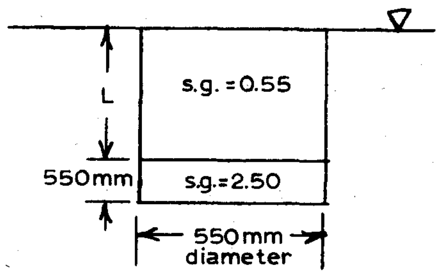


Fig. 6-24

6.55 A hydrometer weighs 0.040 N and has a stem 7 mm in diameter. Compute the distance between specific gravity markings 1.0 and 1.1.

▌ From Prob. 6.26,

$$h = \frac{(W)(\text{s.g.} - 1)}{(\text{s.g.})(\gamma)(\pi d^2/4)} = \frac{(0.040)(1.1 - 1)}{(1.1)[(9.79)(1000)][(\pi)(0.007)^2/4]} = 0.0097 \text{ m} \quad \text{or} \quad 9.7 \text{ mm}$$

6.56 What is the weight of the loaded barge in Fig. 6-25? The barge is 7 m in width.

▌ $$F_b = W \qquad 9.79\{(7)[(14)(2.4) + (2)(2.4)(2.4)/2]\} = W \qquad W = 2359 \text{ kN}$$

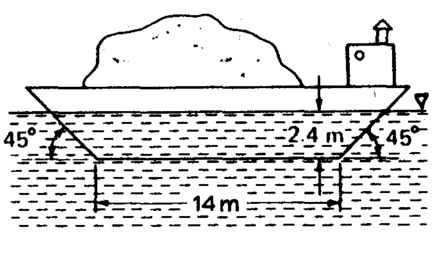


Fig. 6-25

6.57 In Fig. 6-26, a wedge of wood having specific gravity 0.66 supports a 160-lb mermaid (not shown). The wedge is 3 ft in width. What is depth d?

▌ The 160-lb force and the weight of the wood (W) act downward on the wedge, while the buoyant force (F_b) acts upward. $\sum F_y = 0$, $F_b - 160 - W = 0$, $62.4[(2)(3)(d)(d \tan 30°)/2] - 160 - [(0.66)(62.4)]\{(2)(3)(\frac{3}{2})[(\frac{3}{2})/\tan 30°]/2\} = 0$, $d = 2.44$ ft.

6.58 The tank in Fig. 6-27 is filled brimfull with water. If a cube 700 mm on an edge and weighing 530 N is lowered slowly into the water until it floats, how much water flows over the edge of the tank? Neglect sloshing, etc.

▌ $F_b = W$. Let h = the depth to which the cube will sink in the water. $[(9.79)(1000)][(0.700)(0.700)(h)] = 530$, $h = 0.120$ m, $V_{\text{displaced}} = [(0.700)(0.700)(0.120)] = 0.0588 \text{ m}^3$. This is the amount of water that will overflow.

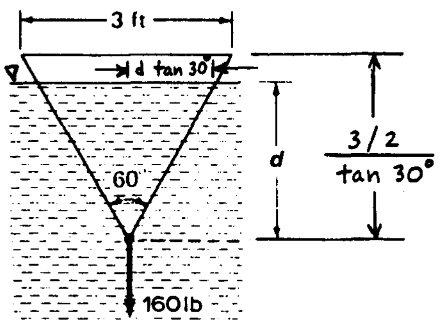


Fig. 6-26

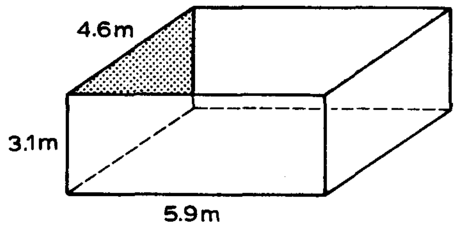


Fig. 6-27

6.59 A stone cube 280 mm on a side and weighing 425 N is lowered into a tank containing a layer of water over a layer of mercury. Determine the position of the block when it has reached equilibrium.

▮ $\gamma_{cube} = 425/(0.280)^3 = 19.360\ \text{kN/m}^3$. Since the cube is heavier than water but lighter than mercury, it will sink beneath the water surface and come to rest at the water–mercury interface, as shown in Fig. 6-28. $F_b = W$, $9.79[(0.280)(0.280)(0.280 - x)] + [(13.6)(9.79)][(0.280)(0.280)(x)] = 0.425$, $x = 0.0217$ m, or 21.7 mm. Thus, the bottom of the cube will come to rest 21.7 mm below the water–mercury interface.

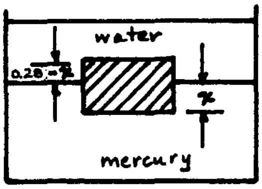


Fig. 6-28

6.60 An iceberg ($\gamma = 9\ \text{kN/m}^3$) floats in ocean water ($\gamma = 10\ \text{kN/m}^3$) with 3000 m^3 of the iceberg protruding above the free surface. What is the volume of the iceberg below the free surface?

▮ $$F_b = W \qquad 10\,000V_{below} = 9000(V_{below} + 3000) \qquad V_{below} = 27\,000\ \text{m}^3$$

6.61 A rectangular tank of internal width 7 m, partitioned as shown in Fig. 6-29, contains oil and water. (*a*) If the oil's specific gravity is 0.84, find its depth h. (*b*) If a 900-N block of wood is floated in the oil, what is the rise in free surface of the water in contact with air?

▮ (*a*) $p_{atm} + [(0.84)(9.79)](h) + (9.79)(3) - (9.79)(4) = p_{atm}$, $h = 1.190$ m. (*b*) Let h' = the new value of h with the 900-N block in flotation. Since the volume of oil does not change, $(1.190)(0.5)(7) = (h')(0.5)(7) - 900/[(0.84)(9.79)(1000)]$, $h' = 1.221$ m. If the oil–water interface drops by a distance δ, the free surface of water with air will rise by $\delta/2$. $p_{atm} + [(0.84)(9.79)](1.221) + 9.79(3 - \delta) - 9.79(4 + \delta/2) = p_{atm}$, $\delta = 0.01709$ m, or 17.09 mm. The free surface of the water will rise by 17.09/2, or 8.54 mm.

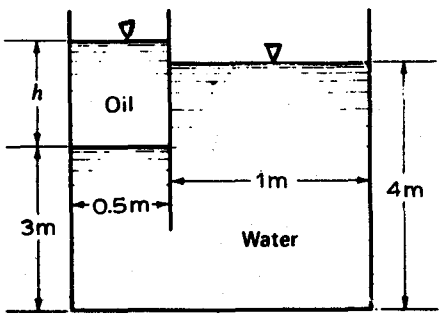


Fig. 6-29

6.62 A balloon is filled with 3000 m^3 of hydrogen having specific weight 1.1 N/m^3. What lift does the balloon exert (*a*) at the earth's surface, if the balloon weighs 1400 N and the temperature is 15 °C; (*b*) at an elevation of 10 km, assuming that the volume has increased 6 percent?

From Table A-8, $\gamma_{air} = 12.01$ N/m^3 at elevation 0 and 4.04 N/m^3 at elevation 10 000 m. $\sum F_y = 0$, $F_b - W_{balloon} - W_H = 0$.

(*a*) $(12.01)(3000) - 1400 - (1.1)(3000) - \text{lift} = 0 \qquad \text{Lift} = 31\,330\text{ N} \quad \text{or} \quad 31.33\text{ kN}$

(*b*) $4.04[(1.06)(3000)] - 1400 - (1.1)(3000) - \text{lift} = 0 \qquad \text{Lift} = 8147\text{ N} \quad \text{or} \quad 8.15\text{ kN}$

6.63 A wooden rod weighing 4 lb is hinged at one end (Fig. 6-30). The rod is 9 ft long and uniform in cross section, and the support is 4 ft below the free surface of a freshwater pond. At what angle α will it come to rest when allowed to drop from a vertical position? The cross section of the stick is 1.4 in^2 in area.

The forces acting on the beam are shown in Fig. 6-30.

$$F_b = 62.4[(9-e)(1.4/144)] = 5.460 - 0.6067e \qquad \sum M_A = 0$$

$$4(4.5\cos\alpha) - (5.460 - 0.6067e)[(9-e)/2](\cos\alpha) = 0 \qquad -0.303e^2 + 5.46e - 6.57 = 0 \qquad e = 1.297\text{ ft}$$

$$\sin\alpha = 4/(9-e) = 4/(9-1.297) = 0.51928 \qquad \alpha = 31.3°$$

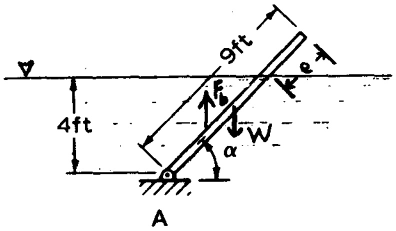


Fig. 6-30

6.64 A block of wood having a volume of 0.034 m^3 and weighing 300 N is suspended in water as shown in Fig. 6-31. A wooden rod of length 3.4 m and cross section of 2000 mm^2 is attached to the weight and also to the wall. If the rod weighs 16 N, what will angle θ be for equilibrium?

$$(F_b)_{block} = [(9.79)(1000)](0.034) = 333\text{ N} \qquad (F_b)_{rod} = [(9.79)(1000)][(AC)(2000/10^6)] = 19.58AC\text{ N}$$

$$\sum M_B = 0$$

$$333(3.4\cos\theta) + (19.58AC)[(AC/2) + (\tfrac{310}{1000})/\sin\theta](\cos\theta) - 300(3.4\cos\theta) - (16)(3.4/2)(\cos\theta) = 0$$

$$AC = 3.4 - (\tfrac{310}{1000})/\sin\theta$$

$$333(3.4\cos\theta) + 19.58[3.4 - (\tfrac{310}{1000})/\sin\theta]$$
$$\times\{[3.4 - (\tfrac{310}{1000})/\sin\theta]/2 + (\tfrac{310}{1000})/\sin\theta\}(\cos\theta) - 300(3.4\cos\theta) - (16)(3.4/2)(\cos\theta) = 0$$

$$4.341 = [3.4 - (\tfrac{310}{1000})/\sin\theta][1.700 + (\tfrac{310}{1000})/(2\sin\theta)] \qquad 4.341 = 5.780 - 0.048/\sin^2\theta$$

$$\sin^2\theta = 0.033357 \qquad \sin\theta = 0.18264 \qquad \theta = 10.5°$$

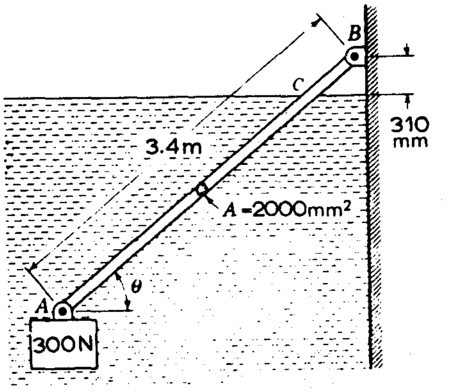


Fig. 6-31

6.65 A barge with a flat bottom and square ends has a draft of 6.0 ft when fully loaded and floating in an upright position, as shown in Fig. 6-32*a*. The center of gravity (CG) of the barge when fully loaded is on the axis of symmetry and 1.0 ft above the water surface. Is the barge stable? If it is stable, what is the righting moment when the angle of heel is 12°?

▮ $\overline{MB} = I/V_d = [(42)(25)^3/12]/[(25)(42)(6)] = 8.68$ ft. Therefore, the metacenter (mc) is located 8.68 ft above the center of buoyancy (CB), as shown in Fig. 6-32*b*. Hence, it (the metacenter) is located $8.68 - 3 - 1$, or 4.68 ft above the barge's center of gravity and the barge is stable. The end view of the barge when the angle of heel is 12° is shown in Fig. 6-32*c*. Righting moment = $(F_b)(x)$, $F_b = 62.4[(25)(42)(6)] = 393\,120$ lb, $x = (\sin 12°)(\text{distance from mc to } CG) = (\sin 12°)(4.68) = 0.973$ ft, righting moment = $(393\,120)(0.973) =$ 382 500 lb · ft.

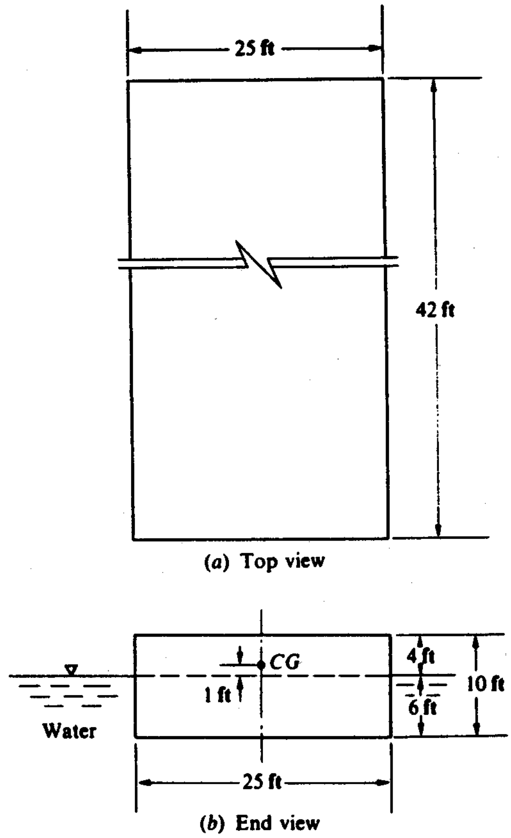


Fig. 6-32(*a*)

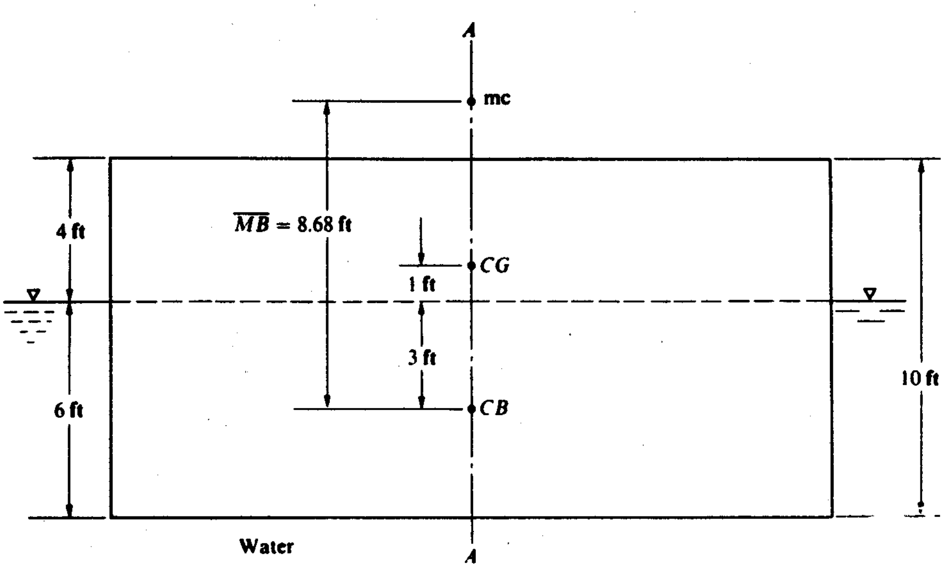


Fig. 6-32(*b*)

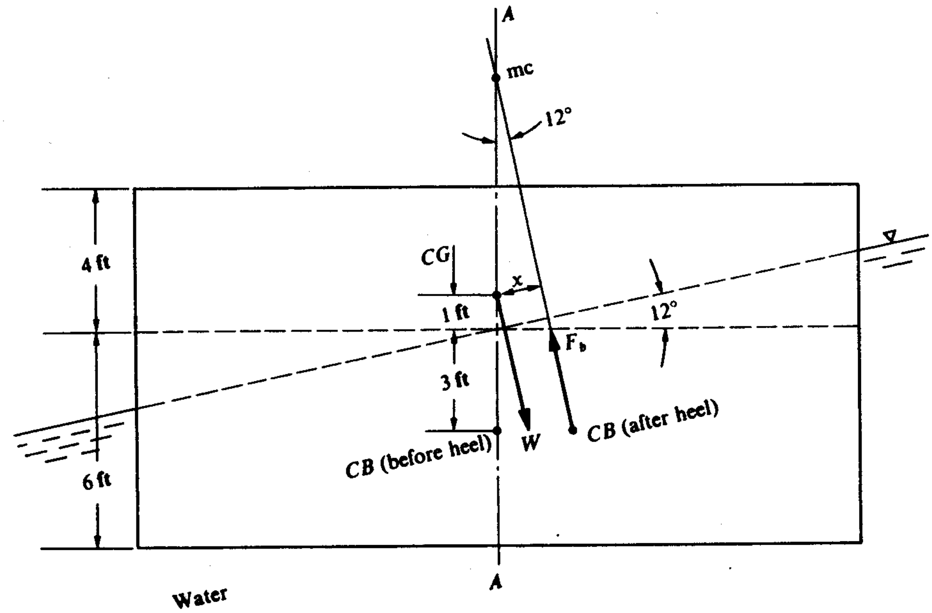


Fig. 6-32(c)

6.66 Would the wooden cylinder (s.g. = 0.61) in Fig. 6-33*a* be stable if placed vertically in oil as shown in the figure?

▌ The first step is to determine the submerged depth of the cylinder when placed in the oil. $F_b = W$, $[(0.85)(9.79)][(D)(\pi)(0.666)^2/4)] = [(0.61)(9.79)][(1.300)(\pi)(0.666)^2/4)]$, $D = 0.9333$ m. The center of buoyancy is located at a distance of 0.933/2, or 0.466 m from the bottom of the cylinder (see Fig. 6-33*b*). $\overline{MB} = I/V_d = [(\pi)(0.666)^4/64]/[(0.933)(\pi)(0.666)^2/4] = 0.030$ m. The metacenter is located 0.030 m above the center of buoyancy, as shown in Fig. 6-33*b*. This places the metacenter 1.300/2 − 0.466 − 0.030, or 0.154 m below the center of gravity. Therefore, the cylinder is not stable.

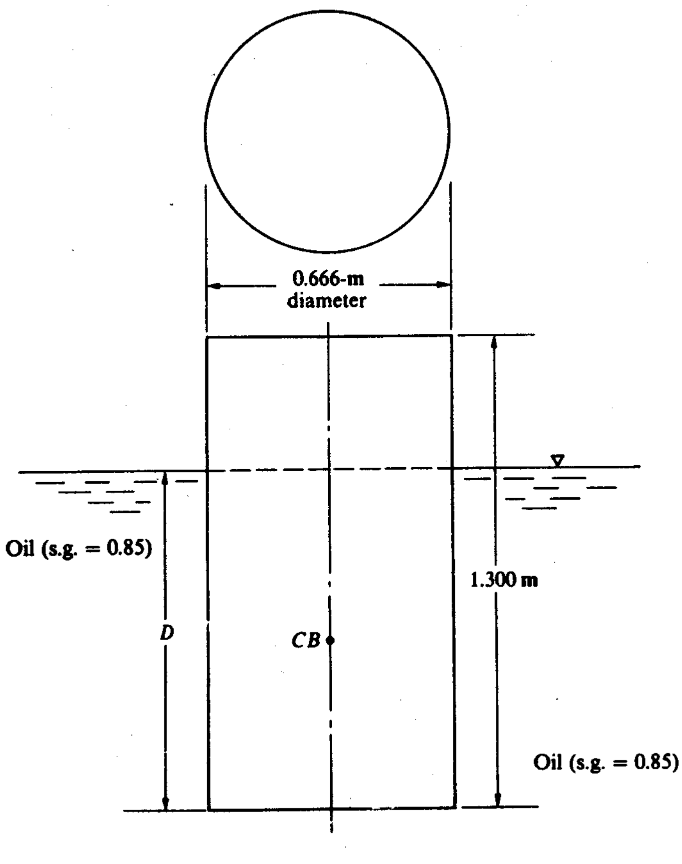


Fig. 6-33(*a*)

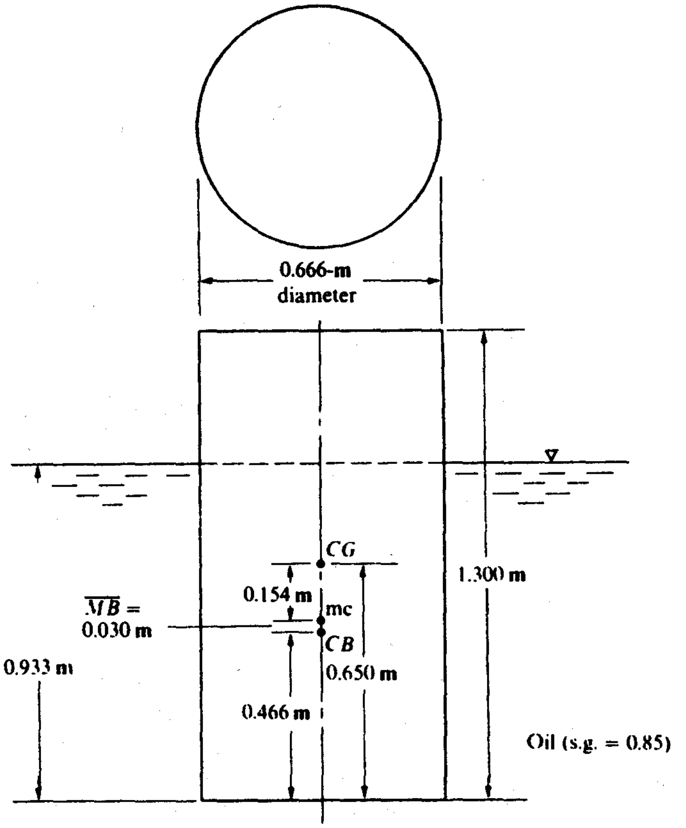


Fig. 6-33(b)

6.67 Figure 6-34*a* shows the cross section of a boat. The hull of the boat is solid. Show if the boat is stable or not. If the boat is stable, compute the righting moment when the angle of heel is 10°?

$\overline{MB} = I/V_d = [(20)(10)^3/12]/[(10)(5)(20)] = 1.67$ ft. Therefore, the metacenter is located $1.67 - 0.5$, or 1.17 ft above the center of gravity, as shown in Fig. 6-34*b*, and the barge is stable. The end view of the barge when the angle of heel is 10° is shown in Fig. 6-34*c*. Righting moment $= (F_b)(x)$, $F_b = 62.4[(10)(5)(20)] = 62\,400$ lb, $x = (\sin 10°)(1.17) = 0.203$ ft, righting moment $= (62\,400)(0.203) = 12\,670$ lb · ft.

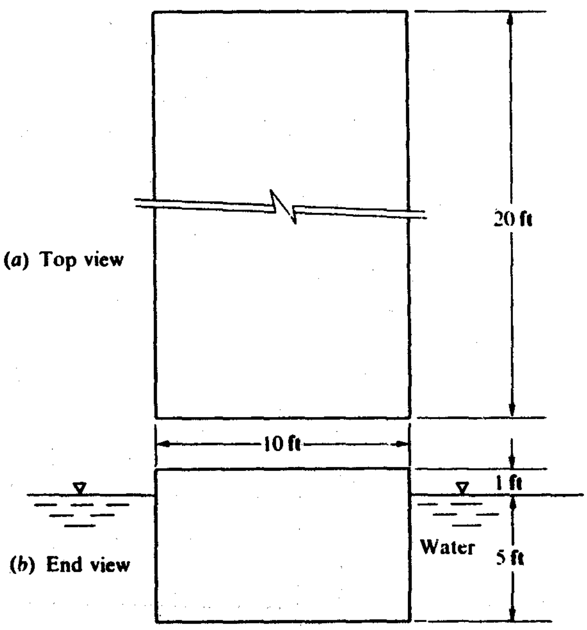


Fig. 6-34(a)

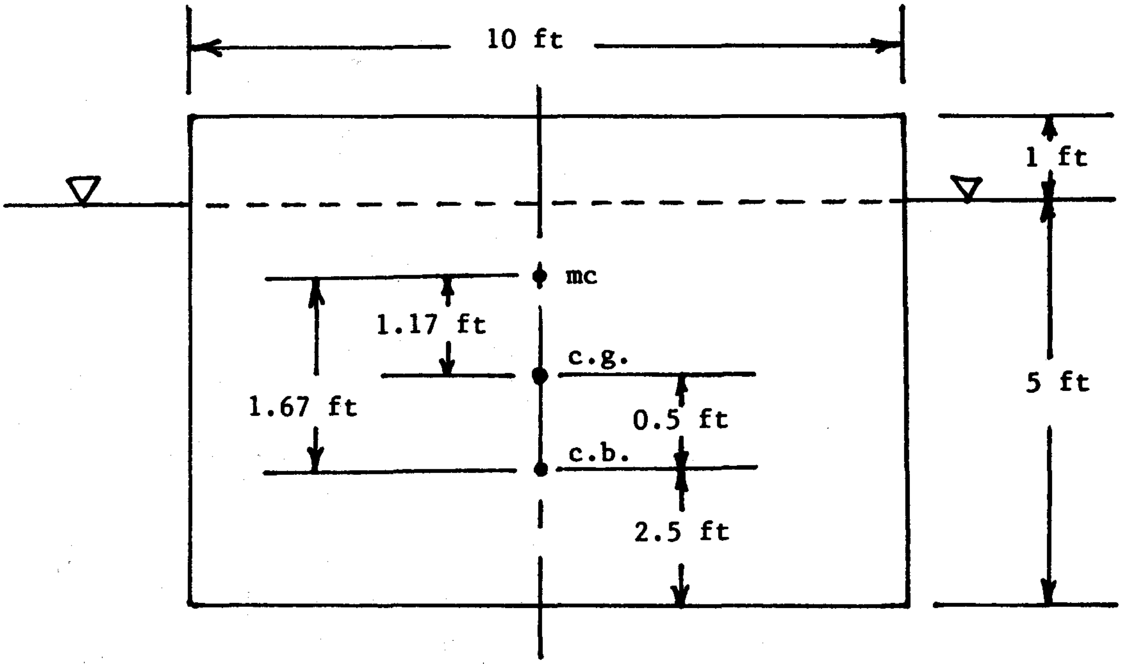


Fig. 6-34(b)

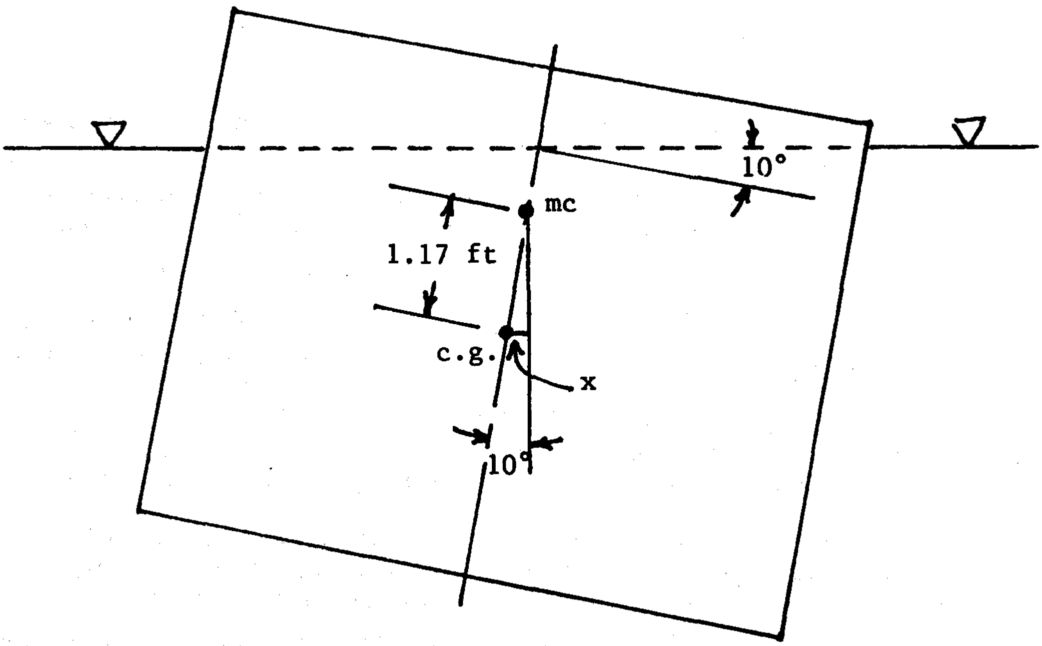


Fig. 6-34(c)

6.68 A solid wood cylinder has a diameter of 2.0 ft and a height of 4.0 ft. The specific gravity of the wood is 0.60. If the cylinder is placed vertically in oil (s.g. = 0.85), would it be stable?

▌ $F_b = W$, $[(0.85)(62.4)][(D)(\pi)(2)^2/4] = [(0.60)(62.4)][(4)(\pi)(2)^2/4]$, $D = 2.82$ ft. The center of buoyancy is located at a distance of 2.82/2, or 1.41 ft from the bottom of the cylinder (see Fig. 6-35). $\overline{MB} = I/V_d = [(\pi)(2)^4/64]/[(2.82)(\pi)(2)^2/4] = 0.09$ ft. The metacenter is located $2 - 1.41 - 0.09$, or 0.50 ft below the center of gravity, as shown in Fig. 6-35. Therefore, the cylinder is not stable.

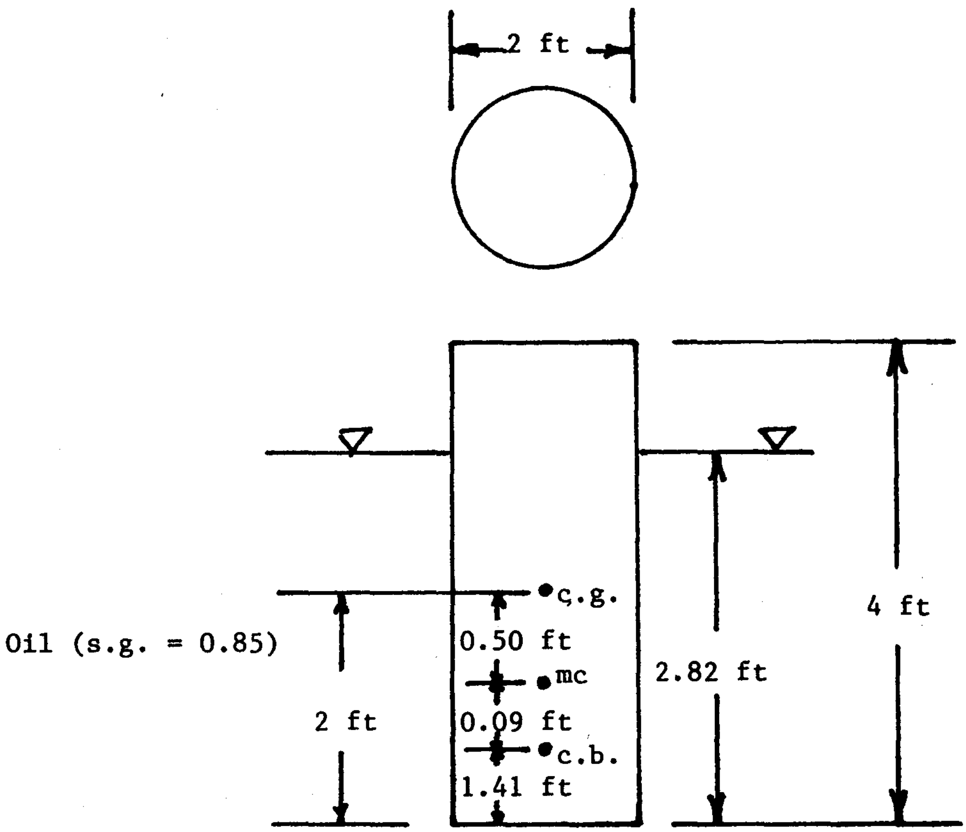


Fig. 6-35

6.69 A wood cone floats in water in the position shown in Fig. 6-36*a*. The specific gravity of the wood is 0.60. Would it be stable?

▮ The center of gravity is located $\frac{10}{4}$, or 2.50 in from the base of the cone or 7.50 in from the tip, as shown in Fig. 6-36*b*. $W_{cone} = [(0.60)(62.4)][(10)(\pi)(7)^2/12]/1728 = 2.779$ lb. Let x = submerged depth.

$$D_x = 0.700x \qquad V_d = (x)(\pi)(D_x)^2/12 = (x)(\pi)(0.700x)^2/12 = 0.1283x^3$$

$$F_b = W \qquad 62.4(0.1283x^3) = 2.779 \qquad x = 0.703 \text{ ft} \quad \text{or} \quad 8.44 \text{ in}$$

$$D_x = (0.700)(8.44) = 5.91 \text{ in} \qquad V_d = (0.1283)(0.703)^3 = 0.0446 \text{ ft}^3 \quad \text{or} \quad 77.1 \text{ in}^3$$

$$\overline{MB} = I/V_d = [(\pi)(5.91)^4/64]/77.1 = 0.78 \text{ in}$$

The metacenter is located 0.78 in above the center of buoyancy. Hence, the metacenter is located 7.50 − 6.33 − 0.78, or 0.39 in below the cone's center of gravity, and the cone is not stable.

6.70 A block of wood 6 ft by 8 ft floats on oil of specific gravity 0.751. A clockwise couple holds the block in the position shown in Fig. 6-37. Determine the **(*a*)** buoyant force acting on the block and its position, **(*b*)** magnitude of the couple acting on the block, and **(*c*)** location of the metacenter for the tilted position.

▮ **(*a*)** $F_b = W = [(0.751)(62.4)][(10)(4 + 4)(4.618)/2] = 8656$ lb. F_b acts upward through the center of gravity O' of the displaced oil. The center of gravity lies 5.333 ft from A and 1.540 ft from D, as shown in Fig. 6-37. $AC = AR + RC = AR + LO' = (5.333)(\cos 30°) + (1.540)(\sin 30°) = 5.388$ ft. Hence, the buoyant force of 8650 lb acts upward through the center of gravity of the displaced oil, which is 5.388 ft to the right of A.

(*b*) One method of obtaining the magnitude of the righting couple (which must equal the magnitude of the external couple for equilibrium) is to find the eccentricity e. This dimension is the distance between the two parallel, equal forces W and F_b, which form the righting couple. $e = FC = AC - AF$, $AF = AR + RF = (5.333)(\cos 30°) + GR \sin 30° = 4.619 + (0.691)(\sin 30°) = 4.964$ ft, $e = 5.388 - 4.964 = 0.424$ ft; couple $= (8656)(0.424) = 3670$ ft · lb.

(*c*) Metacentric distance $MG = MR - GR = RC/\sin 30° - GR = 0.770/\sin 30° - 0.691 = 0.85$ ft

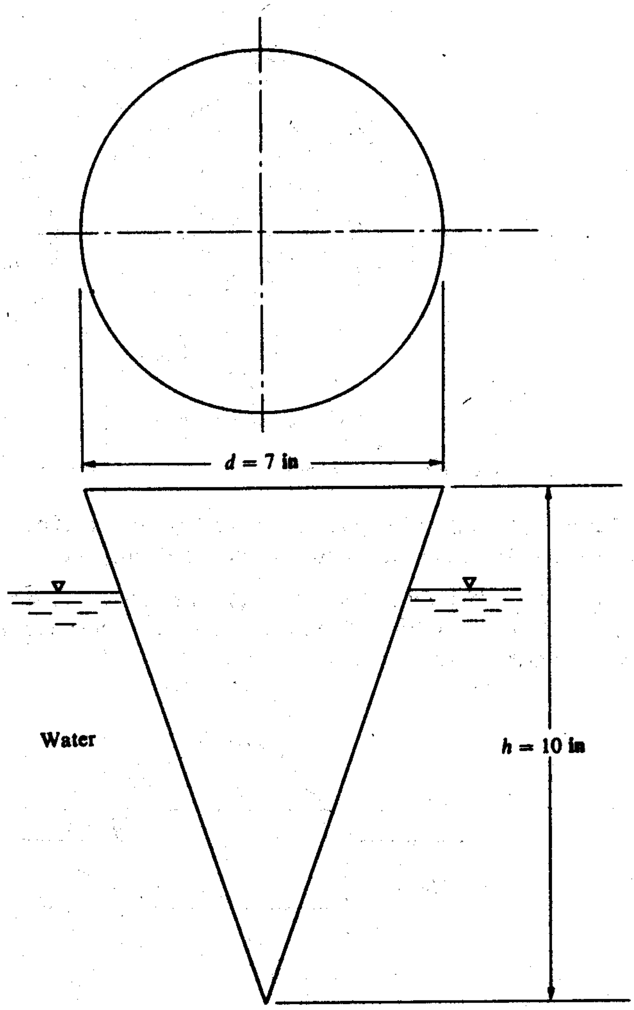


Fig. 6-36(a)

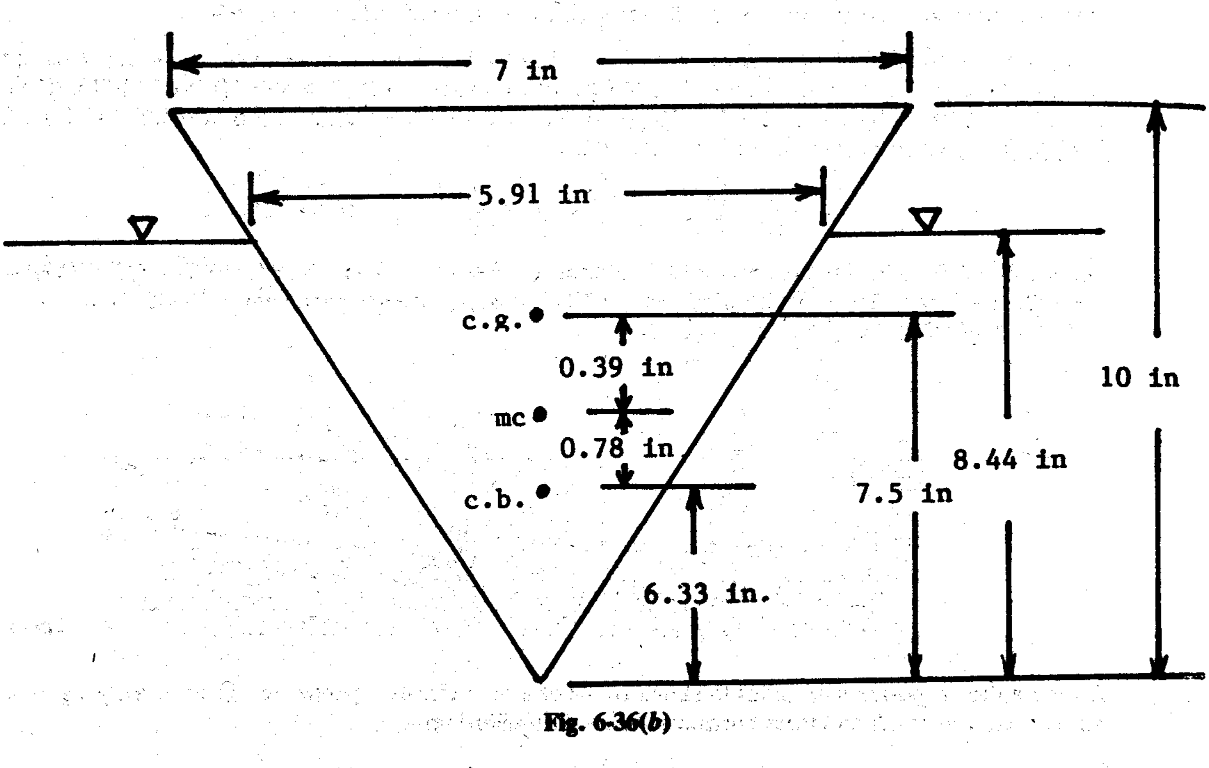
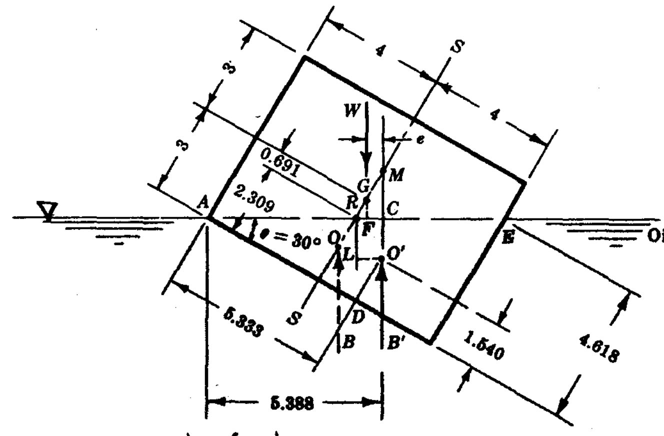


Fig. 6-36(b)



All lengths in feet.

Fig. 6-37

6.71 A rectangular scow 7 ft by 18 ft by 32 ft long; its load of garbage has its center of gravity 2 ft above the waterline, as shown in Fig. 6-38. Is the scow stable for this configuration?

$\overline{MB} = I/V_d = [(32)(18)^3/12]/[(5)(18)(32)] = 5.40$ ft. The metacenter is located 5.40 ft above the center of buoyancy, which is 2.5 ft above the bottom of the scow. Hence, the metacenter is located at $5.40 - 4.5$, or 0.90 ft above the center of gravity, and the scow is stable.

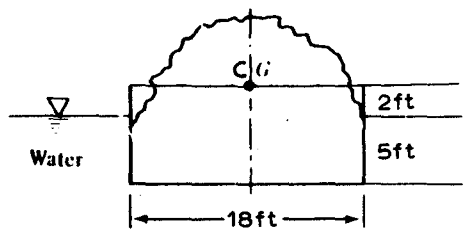


Fig. 6-38

6.72 In Fig. 6-39, a scow 20 ft wide and 60 ft long has a gross weight of 225 tons. Its center of gravity is 1.0 ft above the water surface. Find the metacentric height and restoring couple when $\Delta y = 1.0$ ft.

$F_b = W$, $62.4[(60)(20)(h)] = (225)(2000)$, $h = 6.01$ ft. To locate CB', the center of buoyancy in the tipped position, take moments about AB and BC. $(6.01)(20)(x) = (6.01 - 1.0)(20)(10) + [(1.0 + 1.0)(20)/2](\frac{20}{3})$, $x = 9.45$ ft; $(6.01)(20)(y) = (6.01 - 1.0)(20)[(6.01 - 1.0)/2] + [(1.0 + 1.0)(20)/2][(6.01 - 1.0) + (1.0 + 1.0)/3]$, $y = 3.03$ ft. By similar triangles AEO and $\overline{CB'}PM$,

$$\frac{\Delta y}{b/2} = \frac{\overline{CB'P}}{MP} \qquad \frac{1.0}{20/2} = \frac{10 - 9.45}{MP} \qquad MP = 5.50 \text{ ft}$$

CG is $6.01 + 1.0$, or 7.01 ft from the bottom. Hence, $\overline{CGP} = 7.01 - 3.03 = 3.98$ ft, $\overline{MCG} = \overline{MP} - \overline{CGP} = 5.50 - 3.98 = 1.52$ ft. The scow is stable, since $\overline{MCG}$ is positive. Righting moment $= W(\overline{MCG}) \sin\theta = [(225)(2000)](1.52)[1/\sqrt{(\frac{20}{2})^2 + 1.0^2}] = 68\,060$ ft · lb.

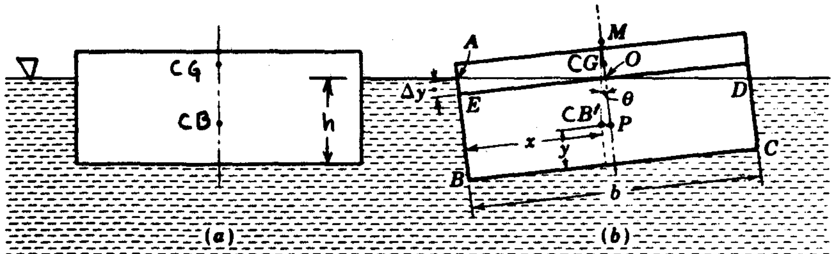


Fig. 6-39

6.73 What are the proportions of radius to height (r_0/h) of a right-circular cylinder of specific gravity s.g. so that it will float in water with end faces horizontal in stable equilibrium?

▌ See Fig. 6-40.

$$h_G = h/2 \qquad h_B = h_1/2 \qquad F_b = W \qquad \gamma(h_1\pi r_0^2) = [(\text{s.g.})(\gamma)](h\pi r_0^2) \qquad h_1 = (\text{s.g.})(h)$$

$$h_B = (\text{s.g.})(h)/2 \qquad MG = MB - GB$$

$$\overline{MB} = I/V_d = (\pi r_0^4/4)/(h_1\pi r_0^2) = r_0^2/(4h_1) = r_0^2/[(4)(\text{s.g.})(h)]$$

$$GB = h_G - h_B = h/2 - (\text{s.g.})(h)/2 = (h)(1-\text{s.g.})/2 \qquad MG = r_0^2[(4)(\text{s.g.})(h)] - (h)(1-\text{s.g.})/2$$

For stable equilibrium, $MG \geq 0$, in which case $r_0^2/[(4)(\text{s.g.})(h)] \geq (h)(1-\text{s.g.})/2$, $r_0/h \geq \sqrt{(2)(\text{s.g.})(1-\text{s.g.})}$.

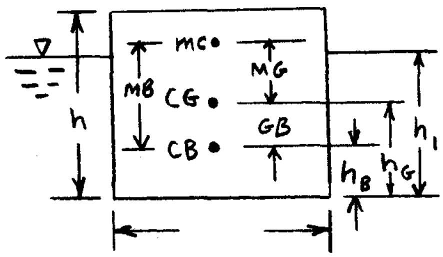


Fig. 6-40

6.74 The plane gate in Fig. 6-41*a* weighs 2.1 kN/m normal to the paper, and its center of gravity is 2 m from the hinge at *O*. (*a*) Find *h* as a function of θ for equilibrium of the gate. (*b*) Is the gate in stable equilibrium for any values of θ?

▌ Consider a unit width of gate, as shown in Fig. 6-41*b*.

(*a*) $\qquad F = \gamma\bar{h}A \qquad F_x = [(9.79)(1000)](h/2)(h/\sin\theta) = 4895h^2/\sin\theta \qquad \sum M_O = 0$

$$(4895h^2/\sin\theta)[(h/\sin\theta)/3] - 2100[(\tfrac{4}{2})(\cos\theta)] = 0 \qquad h^3 = 2.574\sin^2\theta\cos\theta \qquad h = 1.370(\sin^2\theta\cos\theta)^{1/3}$$

(*b*) From part (*a*) $\sum M_O = (1632h^3)/\sin^2\theta - 4200\cos\theta$, $dM/d\theta = -3264h^3\sin^{-3}\theta\cos\theta + 4200\sin\theta$. Substituting $h = 1.370(\sin^2\theta\cos\theta)^{1/3}$ [from part (*a*)], $dM/d\theta = -(3264)(1.370)^3(\cos^2\theta/\sin\theta) + 4200\sin\theta = -(8393)(\cos^2\theta/\sin\theta) + 4200\sin\theta$. For stability, $dM/d\theta < 0$, in which case $4200\sin\theta < 8393(\cos^2\theta/\sin\theta)$, $0.500\sin\theta < \cos^2\theta/\sin\theta$, $\tan^2\theta < (1/0.500 = 2.00)$. This occurs for $\theta \leq 54.7°$ (upper limit). For the lower limit (when water spills over the top of the gate), $h = 4\sin\theta$, $\sum M_O = (1632h^3)/\sin^2\theta - 4200\cos\theta$. Substituting $h = 4\sin\theta$, $\sum M_O = 1632(4\sin\theta)^3/\sin^2\theta - 4200\cos\theta = 104\,448\sin\theta - 4200\cos\theta$. In this case, $\sum M_O = 0$, $\tan\theta = 4200/104\,448 = 0.040211$, $\theta = 2.3°$. Thus for stable equilibrium, θ must be between 2.3° and 54.7°.

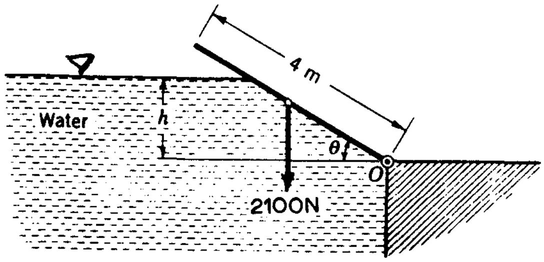


Fig. 6-41(*a*)

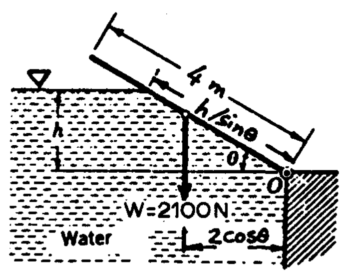


Fig. 6-41(*b*)

6.75 The barge shown in Fig. 6-42 has the form of a parallelopiped having dimensions 10 m by 26.7 m by 3 m. The barge weighs 4450 kN when loaded and has a center of gravity 4 m from the bottom. Find the metacentric height for a rotation about its longest centerline, and determine whether or not the barge is stable.

▌ First, find the center of buoyancy of the barge. $F_b = W$, $9.79[(10)(26.7)(D)] = 4450$, $D = \underline{1.702}$ m. Hence, the center of buoyancy (CB) is at a distance 1.702/2, or 0.851 m above the bottom of the barge. $\overline{MB} = I/V_d = [(26.7)(10)^3/12]/[(10)(26.7)(1.702)] = 4.896$. The distance from CB to CG is $4 - 0.851$, or 3.149 m. Therefore, the metacenter is located $4.896 - 3.149$, or 1.747 m above the CG, and the barge is stable.

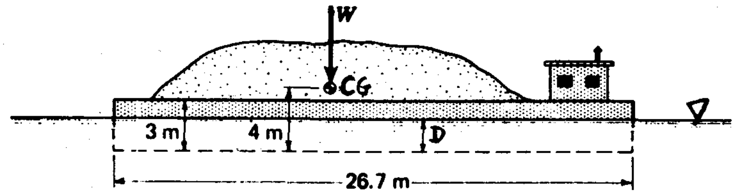


Fig. 6-42

6.76 A plastic cube of dimension L and specific gravity 0.82 floats in water. Is the cube stable?

▌ The cube's center of gravity is at $0.5L$ above its bottom. If the cube has a s.g. $= 0.82$, it will float at a submerged depth of $0.82L$, and its center of buoyancy will be at $0.41L$ above its bottom. $\overline{MB} = I/V_d = [(L)(L)^3/12]/[(L)(L)(0.82L) = 0.1016L$. Therefore, the metacenter is located $0.1016L$ above the center of buoyancy and $0.1016L + 0.41L - 0.5L$, or $0.0116L$ above the center of gravity, and the cube is stable (although just barely).

6.77 For the cube specified in Prob. 6.76, determine the range of values of specific gravity between 0 and 1.0 for which the cube is stable.

▌ The cube's center of gravity is at $0.5L$ above its bottom. For any specific gravity s.g., the cube will float at a submerged depth of (s.g.)(L), and its center of buoyancy will be at (s.g.)(L)/2 above its bottom. $\overline{MB} = I/V_d = [(L)(L)^3/12]/\{(L)(L)[(\text{s.g.})(L)]\} = 0.08333L/(\text{s.g.})$. Therefore, if the cube is stable, the metacenter must be located $0.08333L/(\text{s.g.})$ above the center of buoyancy and $0.08333L/(\text{s.g.}) + (\text{s.g.})(L)/2 - 0.5L$ above the center of gravity. For this to occur, $0.08333L/(\text{s.g.}) + (\text{s.g.})(L)/2 - 0.5L > 0$, $(\text{s.g.})^2/2 - (0.5)(\text{s.g.}) + 0.08333 > 0$. This condition is true (i.e., the cube is stable) for s.g. > 0.789 and s.g. < 0.211.

CHAPTER 7
Kinematics of Fluid Motion

7.1 A nozzle with base diameter 75-mm and a 35-mm-diameter tip discharges 12 L/s of fluid. Derive an expression for fluid velocity along the nozzle's axis. Measure distance x along the axis from the plane of the larger diameter.

▮ Let L = length of nozzle and D = diameter of nozzle at any point. $D = \frac{75}{1000} - (\frac{75}{1000} - \frac{35}{1000})(x/L) = 0.075 - 0.040x/L$, $v = Q/A = 0.012/[\pi(0.075 - 0.040x/L)^2/4] = 1.528/(0.70 - 0.40x/L)^2$. *Note*: x and L in millimeters gives v in m/s.

7.2 What angle α of jet is required to reach the roof of the building in Fig. 7-1 with minimum jet velocity v_0 at the nozzle? What is the value of v_0?

▮ $d^2y/dt^2 = -g$, $dy/dt = -gt + c_1$. At $t = 0$, $dy/dt = v_0 \sin\alpha$. Therefore, $c_1 = v_0 \sin\alpha$, and $dy/dt = -gt + v_0 \sin\alpha$, $y = -gt^2/2 + tv_0 \sin\alpha + c_2$. At $t = 0$, $y = 0$. Therefore $c_2 = 0$, and $y = -gt^2/2 + tv_0 \sin\alpha$, $L = tv_0 \cos\alpha$, $t = L/(v_0 \cos\alpha)$.

$$H = -g[L/(v_0 \cos\alpha)]^2/2 + [L/(v_0 \cos\alpha)](v_0 \sin\alpha) \quad (1)$$

Let $F = gL^2/(2v_0^2)$. Then, from Eq. (*1*), $F = (\cos\alpha)(L \sin\alpha - H \cos\alpha) = L \cos\alpha \sin\alpha - H \cos^2\alpha$. Find maximum F for minimum v_0.

$$dF/d\alpha = L(\cos^2\alpha - \sin^2\alpha) + 2H \sin\alpha \cos\alpha = 0 \qquad 2H/L = -(\cos^2\alpha - \sin^2\alpha)/(\sin\alpha \cos\alpha) = -2 \cot 2\alpha$$

$$(2)(28)/24 = -2 \cot 2\alpha \qquad \alpha = 69.7^\circ$$

Substituting into Eq. (*1*), $28 = -(9.807)[24/(v_0 \cos 69.7^\circ)]^2/2 + [24/(v_0 \cos 69.7^\circ)](v_0 \sin 69.7^\circ)$, $v_0 = 25.2$ m/s.

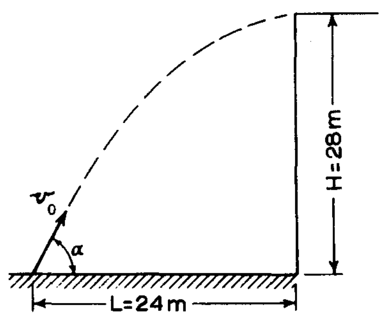


Fig. 7-1

7.3 Given the velocity field, $\mathbf{V}(x, y, z, t) = (5xy^2 + t)\mathbf{i} + (2z + 8)\mathbf{j} + 18\mathbf{k}$ m/s, with x, y, z in meters and t in seconds. Calculate $\mathbf{V}(9, -2, 1, 4)$. What is the magnitude of this velocity?

▮
$$\mathbf{V} = [(5)(9)(-2)^2 + 4]\mathbf{i} + [(2)(1) + 8]\mathbf{j} + 18\mathbf{k} = 184\mathbf{i} + 10\mathbf{j} + 18\mathbf{k} \quad \text{m/s}$$
$$|\mathbf{V}| = \sqrt{184^2 + 10^2 + 18^2} = 185 \text{ m/s}$$

Note: Boldface letters are used herein and hereafter to denote vectors.

7.4 The velocity components in a flow of fluid are specified as $v_x = 4xt + y^2z + 14$ m/s, $v_y = 2xy^2 + t^2 + y$ m/s, and $v_z = 3 + 2ty$ m/s, where x, y, and z are given in meters and t in seconds. What is the velocity vector at (2, 4, 3) m at time $t = 4$ s? What is the magnitude of this vector at this point and time?

▮
$$\mathbf{V} = [(4)(2)(4) + (4)^2(3) + 14]\mathbf{i} + [(2)(2)(4)^2 + 4^2 + 4]\mathbf{j} + [3 + (2)(4)(4)]\mathbf{k} = 94\mathbf{i} + 84\mathbf{j} + 35\mathbf{k} \text{ m/s}$$
$$|\mathbf{V}| = \sqrt{94^2 + 84^2 + 35^2} = 131 \text{ m/s}$$

7.5 Given the velocity field $\mathbf{V} = (5x)\mathbf{i} + (15y + 11)\mathbf{j} + (19t^2)\mathbf{k}$ m/s, determine the path of a particle which is at (4, 6, 2) m at time $t = 3$ s.

$$v_x = dx/dt = 5x \tag{1}$$
$$v_y = dy/dt = 15y + 11 \tag{2}$$
$$v_z = dz/dt = 19t^2 \tag{3}$$

From (1), $dx/x = 5\,dt$, $\ln x = 5t + c_1$. At $t = 3$, $x = 4$. Hence, $\ln 4 = (5)(3) + c_1$, $c_1 = -13.6$.

$$\ln x = 5t - 13.6 \tag{4}$$

From (2), $dy/(15y + 11) = dt$, $\ln(15y + 11) = 15t + c_2$. At $t = 3$, $y = 6$. Hence, $\ln[(15)(6) + 11] = (15)(3) + c_2$, $c_2 = -40.4$.

$$\ln(15y + 11) = 15t - 40.4 \tag{5}$$

From (3), $dz = 19t^2\,dt$, $z = 19t^3/3 + c_3$. At $t = 3$, $z = 2$. Hence, $2 = (19)(3)^3/3 + c_3$, $c_3 = -169$.

$$z = 19t^3/3 - 169 \tag{6}$$

Add Eqs. (4) and (5) to get

$$\ln x + \ln(15y + 11) = 20t - 54.0 \tag{7}$$

Solve for t in Eq. (6): $t = [(z + 169)(\frac{3}{19})]^{1/3}$. Substitute this value of t into Eq. (7): $\ln x + \ln(15y + 11) = 20[(z + 169)(\frac{3}{19})]^{1/3} - 54.0$, $\ln[(x)(15y + 11)] = 10.81(z + 169)^{1/3} - 54.0$.

7.6 An incompressible ideal fluid flows at 0.5 cfs through a circular pipe into a conically converging nozzle, as shown in Fig. 7-2. Determine the average velocity of flow at sections A and B.

As a first step, an approximate flow net is sketched to provide a general picture of the flow. Since this is an axially symmetric flow, the net is not a true two-dimensional flow net. At section A, the streamlines are parallel; hence, the area at right angles to the velocity vectors is a circle. Thus, $v_A = Q/A_A = 0.5/[(\pi)(\frac{8}{12})^2/4] = 1.43$ ft/s. At section B, however, the area at right angles to the streamlines is not clearly defined; it is a curved, dish-shaped section. As a rough approximation, it might be assumed to be the portion of the surface of a sphere of radius 2.0 in that is intersected by a circle of diameter 2.82 in. $v_B = Q/A_B = Q/(2\pi rh) = 0.5/[(2)(\pi)(\frac{2}{12})(0.59/12)] = 9.71$ ft/s.

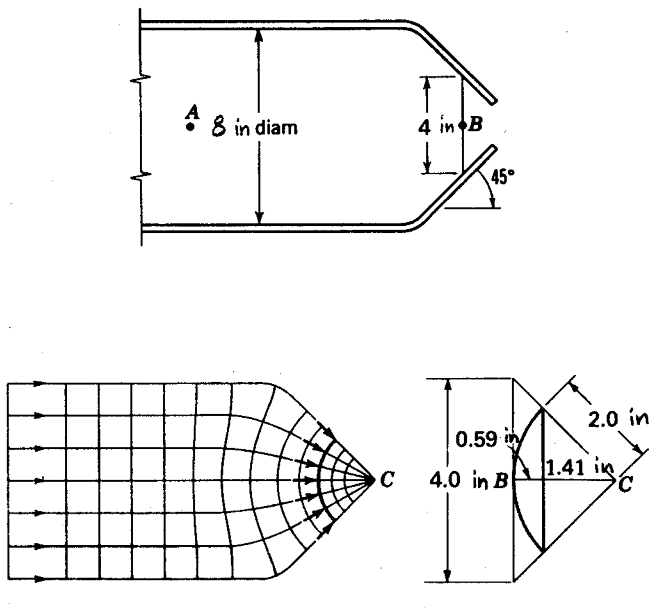


Fig. 7-2

7.7 Water flows at 6 gal/min through a small circular hole in the bottom of a large tank. Assuming the water in the tank approaches the hole radially, find the velocity in the tank at 2, 4, and 8 in from the hole.

The area through which flow occurs is a hemispherical surface, with $A = 2\pi r^2$. $Q = 6/[(7.48)(60)] = 0.01337$ ft³/s, $v = Q/A$. At 2 in from the hole, $v = 0.01337/[(2)(\pi)(\frac{2}{12})^2] = 0.0766$ ft/s. At 4 in from the hole, $v = 0.01337/[(2)(\pi)(\frac{4}{12})^2] = 0.0192$ ft/s. At 8 in from the hole, $v = 0.01337/[(2)(\pi)(\frac{8}{12})^2] = 0.00479$ ft/s.

7.8 Given the eulerian velocity-vector field $\mathbf{V}(x, y, z, t) = 3t\mathbf{i} + xz\mathbf{j} + ty^2\mathbf{k}$, find the acceleration of a particle.

$$\frac{dV}{dt} = \frac{\partial V}{\partial t} + \left(u\frac{\partial V}{\partial x} + v\frac{\partial V}{\partial y} + w\frac{\partial V}{\partial z}\right) \qquad u = 3t \qquad v = xz \qquad w = ty^2$$

$$\frac{\partial V}{\partial t} = \mathbf{i}\frac{\partial u}{\partial t} + \mathbf{j}\frac{\partial v}{\partial t} + \mathbf{k}\frac{\partial w}{\partial t} = 3\mathbf{i} + y^2\mathbf{k} \qquad \frac{\partial V}{\partial x} = z\mathbf{j} \qquad \frac{\partial V}{\partial y} = 2ty\mathbf{k} \qquad \frac{\partial V}{\partial z} = x\mathbf{j}$$

$$\frac{dV}{dt} = (3\mathbf{i} + y^2\mathbf{k}) + (3t)(z\mathbf{j}) + (xz)(2ty\mathbf{k}) + (ty^2)(x\mathbf{j}) = 3\mathbf{i} + (3tz + txy^2)\mathbf{j} + (y^2 + 2xyzt)\mathbf{k}$$

If V is valid everywhere as given, this acceleration applies to all positions and times within the flow field.

7.9 Flow through a converging nozzle can be approximated by a one-dimensional velocity distribution $u = u(x)$. For the nozzle shown in Fig. 7-3, assume the velocity varies linearly from $u = v_0$ at the entrance to $u = 3v_0$ at the exit: $u(x) = v_0(1 + 2x/L)$; $\partial u/\partial x = 2v_0/L$. **(a)** Compute the acceleration du/dt as a general function of x, and **(b)** evaluate du/dt at the entrance and exit if $v_0 = 10$ ft/s and $L = 1$ ft.

(a)

$$\frac{du}{dt} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} \qquad \frac{\partial u}{\partial t} = v\frac{\partial u}{\partial y} = w\frac{\partial u}{\partial z} = 0 \qquad u = v_0\left(1 + \frac{2x}{L}\right) \qquad \frac{\partial u}{\partial x} = \frac{2v_0}{L}$$

$$\frac{du}{dt} = 0 + \left[v_0\left(1 + \frac{2x}{L}\right)\right]\left(\frac{2v_0}{L}\right) + 0 + 0 = \left(\frac{2v_0^2}{L}\right)\left(1 + \frac{2x}{L}\right)$$

(b) At the entrance, where $x = 0$, $du/dt = [(2)(10)^2(1)][1 + (2)(0)/(1)] = 200\ \text{ft/s}^2$. At the exit, where $x = 1$ ft, $du/dt = [(2)(10)^2(1)][1 + (2)(1)/(1)] = 600\ \text{ft/s}^2$.

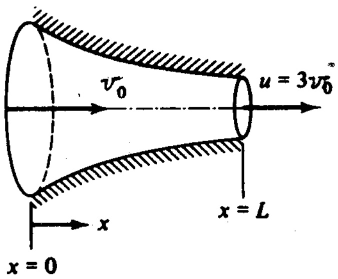


Fig. 7-3

7.10 A two-dimensional velocity field is given by $u = 2y^2$, $v = 3x$, $w = 0$. At $(x, y, z) = (1, 2, 0)$, compute the **(a)** velocity, **(b)** local acceleration, and **(c)** convective acceleration.

(a)

$$\mathbf{V} = \mathbf{i}[(2)(2)^2] + \mathbf{j}[(3)(1)] = 4\mathbf{i} + 3\mathbf{j}$$

(b)

$$\frac{\partial V}{\partial t} = 0$$

(c)

$$\frac{du}{dt} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = 0 + (2y^2)(0) + (3x)(4y) + (0)(0) = 12xy$$

$$\frac{dv}{dt} = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = 0 + (2y^2)(3) + (3x)(0) + (0)(0) = 6y^2$$

$$\mathbf{a} = (12)(1)(2)\mathbf{i} + (6)(2)^2\mathbf{j} = 24\mathbf{i} + 24\mathbf{j}$$

7.11 For the velocity field described in Prob. 7.10, at $(1, 2, 0)$ compute the **(a)** acceleration component parallel to the velocity vector and **(b)** component normal to the velocity vector.

From Prob. 7.10, $\mathbf{V} = 4\mathbf{i} + 3\mathbf{j}$ and $\mathbf{a} = 24\mathbf{i} + 24\mathbf{j}$ at $(1, 2, 0)$.

(a) Tangential acceleration:

$$\mathbf{n}_v = \mathbf{V}/|\mathbf{V}| = \tfrac{4}{5}\mathbf{i} + \tfrac{3}{5}\mathbf{j} \qquad a_t = \mathbf{a}\cdot\mathbf{n}_v = (24\mathbf{i} + 24\mathbf{j})\cdot(\tfrac{4}{5}\mathbf{i} + \tfrac{3}{5}\mathbf{j}) = 19.2 + 14.4 = 33.6 \text{ units parallel to } \mathbf{V}$$

(**b**) From Fig. 7-4, the angle θ between **V** and **a** is determined by $\cos\theta = a_t/|\mathbf{a}| = 33.6/(24^2 + 24^2)^{1/2} = 0.98995$, $\theta = 8.13°$, $a_n = |\mathbf{a}| \sin\theta = (24^2 + 24^2)^{1/2}(\sin 8.13°) = 4.80$ units normal to **V**.

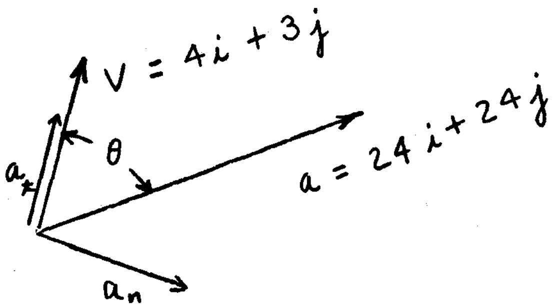


Fig. 7-4

7.12 Is the flow with velocity field $\mathbf{V} = 2tx\mathbf{i} - t^2y\mathbf{j} + 3xz\mathbf{k}$ steady or unsteady? Is it two- or three-dimensional? At the point $(x, y, z) = (2, -2, 0)$, compute the (**a**) total acceleration vector and (**b**) unit vector normal to the acceleration.

▌ Flow is unsteady because time t appears explicitly. Flow is three-dimensional because $u, v, w \neq 0$.

$$\frac{du}{dt} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = 2x + (2tx)(2t) + (-t^2y)(0) + (3xz)(0) = 2x + 4t^2x$$

$$\frac{dv}{dt} = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -2ty + (2tx)(0) + (-t^2y)(-t^2) + (3xz)(0) = -2ty + t^4y$$

$$\frac{dw}{dt} = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = 0 + (2tx)(3z) + (-t^2y)(0) + (3xz)(3x) = 6txz + 9x^2z$$

(**a**)
$$\mathbf{a} = \mathbf{i}\frac{du}{dt} + \mathbf{j}\frac{dv}{dt} + \mathbf{k}\frac{dw}{dt}$$

At point $(2, -2, 0)$, $du/dt = (2)(2) + (4)(t^2)(2) = 4 + 8t^2$, $dv/dt = -(2)(t)(-2) + (t^4)(-2) = 4t - 2t^4$, $dw/dt = (6)(t)(2)(0) + (9)(2)^2(0) = 0$. Hence, $\mathbf{a} = (4 + 8t^2)\mathbf{i} + (4t - 2t^4)\mathbf{j}$. (**b**) The unit vector normal to **a** must satisfy $\mathbf{a}\cdot\mathbf{n} = 0 = n_x(4 + 8t^2) + n_y(4t - 2t^4) + n_z(0)$ plus $n_x^2 + n_y^2 + n_z^2 = 1$. A special case solution is $n = \pm\mathbf{k}$.

7.13 For steady flow through a conical nozzle, the axial velocity is approximately $u = U_0(1 - x/L)^{-2}$, where U_0 is the entrance velocity and L is the distance to the geometrical vertex of the cone. Compute (**a**) a general expression for the axial acceleration du/dt and (**b**) its values at the entrance and at $x = 2$ m, if $U_0 = 4$ m/s and $L = 3$ m.

▌ (**a**)
$$\frac{du}{dt} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = 0 + \left[U_0\left(1 - \frac{x}{L}\right)^{-2}\right]\left[\left(1 - \frac{x}{L}\right)^{-3}(-2U_0)\left(-\frac{1}{L}\right)\right] + 0 + 0 = \left(1 - \frac{x}{L}\right)^{-5}\left(\frac{2U_0^2}{L}\right)$$

(**b**) At entrance ($x = 0$):
$$\frac{du}{dt} = (1 - \tfrac{0}{3})^{-5}[(2)(4)^2/3] = 10.7\ \text{m/s}^2$$

At $x = 2$ m:
$$\frac{du}{dt} = (1 - \tfrac{2}{3})^{-5}[(2)(4)^2/3] = 2592\ \text{m/s}^2$$

7.14 A two-dimensional velocity field is given by $\mathbf{V} = (x^2 - 2y^2 + 2x)\mathbf{i} - (3xy + y)\mathbf{j}$. At $x = 2$ and $y = 2$, compute the (**a**) accelerations a_x and a_y, (**b**) velocity component in the direction $\theta = 32°$, and (**c**) directions of maximum acceleration and maximum velocity.

▌
$$\frac{du}{dt} = a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = 0 + (x^2 - 2y^2 + 2x)(2x + 2) + (-3xy - y)(-4y) + 0$$

$$\frac{dv}{dt} = a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = 0 + (x^2 - 2y^2 + 2x)(-3y) + (-3xy - y)(-3x - 1) + 0$$

(a)
$$a_x = [2^2 - (2)(2)^2 + (2)(2)][(2)(2) + 2] + [(-3)(2)(2) - 2][(-4)(2)] = 112$$

$$a_y = [2^2 - (2)(2)^2 + (2)(2)][(-3)(2)] + [(-3)(2)(2) - 2][(-3)(2) - 1] = 98$$

(b)
$$v_{30°} = \mathbf{V} \cdot \mathbf{n}_{30°} \qquad \mathbf{V} = [2^2 - (2)(2)^2 + (2)(2)]\mathbf{i} - [(3)(2)(2) + 2]\mathbf{j} = 0\mathbf{i} - 14\mathbf{j}$$

$$n_{32°} = 0.848\mathbf{i} + 0.530\mathbf{j} \qquad v_{32°} = (0\mathbf{i} - 14\mathbf{j})(0.848\mathbf{i} + 0.530\mathbf{j}) = -7.42$$

(c) Direction of **a**: $\alpha = \arctan(98/112) = \arctan 0.87500 = 41.2°$. Direction of **V** (direction of $-\mathbf{j}$): $\beta = -90°$.

7.15 The velocity field in the neighborhood of a stagnation point is given by $u = U_0x/L$, $v = -U_0y/L$, $w = 0$. **(a)** Show that the acceleration vector is purely radial. **(b)** If $L = 3$ ft, what is the magnitude of U_0 if the total acceleration at $(x, y) = (L, L)$ is 29 ft/s²?

▮
$$\frac{du}{dt} = a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = 0 + \left(\frac{U_0x}{L}\right)\left(\frac{U_0}{L}\right) + 0 + 0 = \frac{U_0^2x}{L^2}$$

$$\frac{dv}{dt} = a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = 0 + 0 + \left(\frac{-U_0y}{L}\right)\left(\frac{-U_0}{L}\right) + 0 = \frac{U_0^2y}{L^2}$$

(a) $\mathbf{a} = a_x\mathbf{i} + a_y\mathbf{j} = (U_0^2/L^2)(x\mathbf{i} + y\mathbf{j}) = (U_0^2/L^2)(\mathbf{r})$. (Hence, purely radial.)
(b) $|\mathbf{a}| = a(L, L) = (U_0^2/L^2)\,|L\mathbf{i} + L\mathbf{j}| = U_0^2\sqrt{2}/L$. If $L = 3$ ft and $|\mathbf{a}| = 29$ ft/s², $29 = U_0^2\sqrt{2}/3$, $U_0 = 7.84$ ft/s.

7.16 A particle moves around the circular path $x^2 + y^2 = 9$ m² at a uniform speed of 4 m/s. Express the u and v components as functions of time, assuming $\theta = 0$ at $t = 0$. See Fig. 7-5.

▮ $u = u_r \cos\theta - u_\theta \sin\theta = -4\sin\theta$ m/s $\qquad v = v_r \sin\theta + v_\theta \cos\theta = +4\cos\theta$ m/s

But $v_\theta = r\dot{\theta}$, $4 = 3\dot{\theta}$, $\theta = \frac{4}{3}t$; hence $u = -4\sin\frac{4}{3}t$ and $v = +4\cos\frac{4}{3}t$.

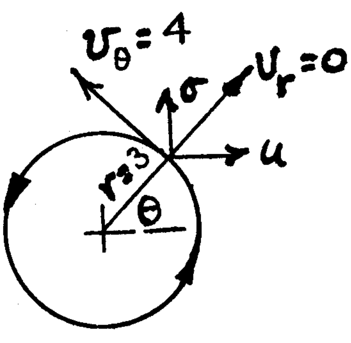


Fig. 7-5

7.17 A perfect fluid flows from the bottom of a large tank through a small hole at the rate $Q = 0.9$ L/s. If the fluid flows radially toward the hole with the same volume flow across every section, compute the convective acceleration at points 100 mm and 200 mm from the hole.

▮ Consider the radial velocity (v_r): $v_r = -Q/A_r = -Q/(2\pi r^2)$ (A_r is the area of a hemisphere).

$$a_r = v_r\frac{\partial v_r}{\partial r} = \left(\frac{-Q}{2\pi r^2}\right)\left(\frac{Q}{\pi r^3}\right) = -\frac{Q^2}{2\pi^2 r^5}$$

At $r = 0.100$ m, $a_r = -(0.9 \times 10^{-3})^2/[(2)(\pi)^2(0.100)^5] = -0.0041$ m/s², or -4.1 mm/s². At $r = 0.200$ m, $a_r = -0.0041/32 = -0.000128$ m/s², or -0.128 mm/s².

7.18 Given the velocity field $\mathbf{V}(x, y, z, t) = 10x^2\mathbf{i} - 20yx\mathbf{j} + 100t\mathbf{k}$, determine the velocity and acceleration of a particle at position $x = 1$ m, $y = 2$ m, $z = 5$ m, and $t = 0.1$ s.

$$\mathbf{V} = (10)(1)^2\mathbf{i} - (20)(2)(1)\mathbf{j} + (100)(0.1)\mathbf{k} = 10\mathbf{i} - 40\mathbf{j} + 10\mathbf{k}\ \text{m/s}$$

$$\mathbf{a}(x, y, z, t) = \frac{\partial V}{\partial t} + \left(v_x\frac{\partial V}{\partial x} + v_y\frac{\partial V}{\partial y} + v_z\frac{\partial V}{\partial z}\right)$$

$$\mathbf{a} = 100\mathbf{k} + [(10x^2)(20x\mathbf{i} - 20y\mathbf{j}) + (-20yx)(-20x\mathbf{j}) + (100t)(0)] = 200x^3\mathbf{i} + (-200x^2y + 400yx^2)\mathbf{j} + 100\mathbf{k}$$

At position $x = 1$ m, $y = 2$ m, $z = 5$ m, and $t = 0.1$ s, $\mathbf{a} = (200)(1)^3\mathbf{i} + [(-200)(1)^2(2) + (400)(2)(1)^2]\mathbf{j} + 100\mathbf{k} = 200\mathbf{i} + 400\mathbf{j} + 100\mathbf{k}\ \text{m/s}^2$.

7.19 If the flow in Fig. 7-2 is steady at 0.50 cfs, find the acceleration in the flow at sections A and B.

Since the flow at section A is uniform and also steady, $\mathbf{a}_A = 0$.

$$\mathbf{a} = \frac{\partial \mathbf{V}}{\partial t} + u\frac{\partial \mathbf{V}}{\partial x} + v\frac{\partial \mathbf{V}}{\partial y} + w\frac{\partial \mathbf{V}}{\partial z} \qquad \mathbf{a}_B = 0 + u\frac{\partial \mathbf{V}}{\partial x} + v\frac{\partial \mathbf{V}}{\partial y} + 0 = u\frac{\partial \mathbf{V}}{\partial x} + v\frac{\partial \mathbf{V}}{\partial y}$$

For point B on the axis of the pipe at section B, $v = 0$; hence,

$$\mathbf{a}_B = u\frac{\partial \mathbf{V}}{\partial x}$$

The effective area through which the flow is occurring in the converging section of the nozzle may be expressed approximately as $A = 2\pi hr$, where $h = r(1 - \cos 45°) = 0.293r$ and r is the distance from point C. Thus $A = (2\pi)(0.293r^2) = 1.84r^2$, and the velocity in the converging nozzle (assuming the streamlines flow radially toward C) may be expressed approximately as $v = Q/A = 0.50/(1.84r^2)$. At section B, $r = 2$ in $= 0.167$ ft; hence, $v = 0.50/[(1.84)(0.167)^2] = 9.744$ fps.

$$\frac{\partial \mathbf{V}}{\partial x} = -\frac{\partial \mathbf{V}}{\partial r} = -\left[\frac{(-2)(0.50)}{1.84r^3}\right] = \frac{(2)(0.50)}{(1.84)(0.167^3)} = 116.7\ \text{fps/ft} \qquad a_B = u\frac{\partial \mathbf{V}}{\partial x} = (9.744)(116.7) = 1137\ \text{ft/s}^2$$

7.20 A two-dimensional flow field is given by $u = 2y$, $v = x$. Sketch the flow field. Derive a general expression for the velocity and acceleration (x and y are in units of length L; u and v are in units of L/T). Find the acceleration in the flow field at point A ($x = 3.5$, $y = 1.2$).

The flow field is sketched in Fig. 7-6a. Velocity components u and v are plotted to scale, and streamlines are sketched tangentially to the resultant velocity vectors. This gives a general picture of the flow field.

$$V = (u^2 + v^2)^{1/2} = (4y^2 + x^2)^{1/2} \qquad a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = 2y(0) + x(2) = 2x \qquad a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = 2y(1) + x(0) = 2y$$

$$a = (a_x^2 + a_y^2)^{1/2} = (4x^2 + 4y^2)^{1/2} \qquad (a_A)_x = 2x = 7.0L/T^2 \qquad (a_A)_y = 2y = 2.4L/T^2$$

$$a_A = [(a_A)_x^2 + (a_A)_y^2]^{1/2} = [(7.0)^2 + (2.4)^2]^{1/2} = 7.4L/T^2$$

To get a rough check on the acceleration imagine a velocity vector at point A. This vector would have a magnitude approximately midway between that of the adjoining vectors, or $V_A \approx 4L/T$. The radius of curvature of the sketched streamline at A is roughly $3L$. Thus $(a_A)_n \approx 4^2/3 \approx 5.3L/T^2$. The tangential acceleration of the particle at A may be approximated by noting that the velocity along the streamline increases from about $3.2L/T$, where it crosses the x axis, to about $8L/T$ at B. The distance along the streamline between these two points is roughly $4L$. Hence a very approximate value of the tangential acceleration at A is

$$(a_A)_t = V\frac{\partial V}{\partial s} \approx 4\left(\frac{8 - 3.2}{4}\right) \approx 4.8L/T^2$$

Vector diagrams of these roughly computed normal and tangential acceleration components are plotted (Fig. 7-6b) for comparison with the true acceleration as given by the analytic expressions (Fig. 7-6c).

7.21 The velocity along a streamline coincident with the x axis is $u = 9 + x^{1/3}$. What is the convective acceleration at $x = 3.2$? Specify units in terms of L and T. Assuming the fluid is incompressible, is the flow converging or diverging?

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

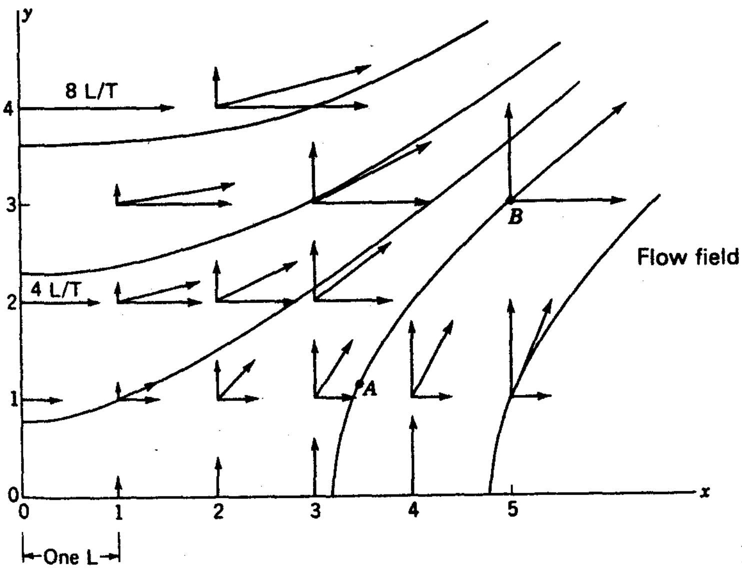


Fig. 7-6(a)

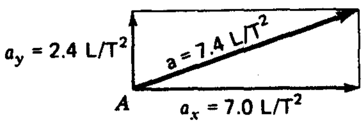


Fig. 7-6(b)

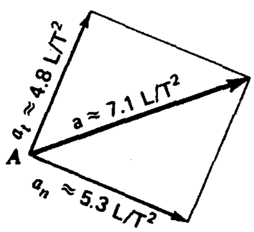


Fig. 7-6(c)

Since

$$\frac{\partial u}{\partial t} = v\frac{\partial u}{\partial y} = w\frac{\partial u}{\partial z} = 0 \qquad a_x = u\frac{\partial u}{\partial x} = (9 + x^{1/3})\left(\frac{x^{-2/3}}{3}\right) = 3x^{-2/3} + \tfrac{1}{3}x^{-1/3}$$

At $x = 3.2$, $a_x = (3)(3.2)^{-2/3} + (\frac{1}{3})(3.2)^{-1/3} = 1.61L/T^2$. For incompressible flow, the flow is converging.

7.22 A large hemispherical vat has a small taphole centered on its lowest point. Ideal liquid drains through the hole according to $Q = 11 - 0.5t$, where Q is in cubic feet per second and t is in seconds. Find the total acceleration at a point 3 ft from the center of the hole at $t = 16$ s. Assume that liquid approaches the center of the hole radially.

▮ $v = Q/A$. The area through which flow occurs is a hemispherical surface, so $v = (11 - 0.5t)/(2\pi r^2)$.

$$a_{\text{total}} = v\frac{\partial v}{\partial r} + \frac{\partial v}{\partial t} = \left[\frac{(11 - 0.5t)}{2\pi r^2}\right]\left[\frac{-(11 - 0.5t)}{\pi r^3}\right] - \frac{0.5}{2\pi r^2}$$

At $r = 3$ ft and $t = 16$ s, $a_{\text{total}} = \{[11 - (0.5)(16)]/[(2)(\pi)(3)^2]\}\{-[11 - (0.5)(16)]/[(\pi)(3)^3]\} - 0.5/[(2)(\pi)(3)^2] = -0.0107\ \text{ft/s}^2$.

7.23 Under what conditions does the velocity field $\mathbf{V} = (a_1x + b_1y + c_1z)\mathbf{i} + (a_2x + b_2y + c_2z)\mathbf{j} + (a_3x + b_3y + c_3z)\mathbf{k}$, where a_1, a_2, etc. = constant, represent an incompressible flow which conserves mass?

▮ $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \qquad \frac{\partial}{\partial x}(a_1x + b_1y + c_1z) + \frac{\partial}{\partial y}(a_2x + b_2y + c_2z) + \frac{\partial}{\partial z}(a_3x + b_3y + c_3z) = 0$$

or $a_1 + b_2 + c_3 = 0$. At least two of the constants a_1, b_2, and c_3 must have opposite signs. Continuity imposes no restrictions whatever on the constants b_1, c_1, a_2, c_2, a_3, and b_3, which do not contribute to a mass increase or decrease of a differential element.

7.24 An incompressible velocity field is given by $u = a(x^2 - y^2)$, v unknown, $w = b$, where a and b are constants. What must the form of the velocity component v be?

▌ $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \qquad \frac{\partial}{\partial x}(ax^2 - ay^2) + \frac{\partial v}{\partial y} + \frac{\partial b}{\partial z} = 0 \qquad \frac{\partial v}{\partial y} = -2ax$$

This is integrated partially with respect to y: $v(x, y, z, t) = -2axy + f(x, z, t)$. This is the only possible form for v that satisfies the incompressible continuity equation. The function of integration f is entirely arbitrary since it vanishes when v is differentiated with respect to y.

7.25 An incompressible flow field has $u = xz^3$ and $w = xe^{-y}$ (dimensional factors omitted). What form does continuity imply for the velocity component v?

▌ $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \qquad z^3 + \frac{\partial v}{\partial y} + 0 = 0 \qquad \frac{\partial v}{\partial y} = -z^3 \qquad v = -z^3y + f(x, z)$$

7.26 A two-dimensional incompressible velocity field has $u = K(1 - e^{-ay})$, for $x \le L$ and $0 \le L \le \infty$. What is the most general form of $v(x, y)$ for which continuity is satisfied and $v = v_0$ at $y = 0$? What are the proper dimensions for the constants K and a?

▌ Dimensions of constants: $\{K\} = \{L/T\}$, $\{a\} = \{1/L\}$.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \qquad 0 + \frac{\partial v}{\partial y} + 0 = 0 \qquad \frac{\partial v}{\partial y} = 0 \qquad v = f(x) \quad \text{only}$$

If $v = v_0$ at $y = 0$ for all x, then $v = v_0$ everywhere.

7.27 Which of the following velocity fields satisfies conservation of mass for incompressible plane flow?

(*a*) $u = -x$, $v = y$ (*b*) $u = 3y$, $v = 3x$ (*c*) $u = 4x$, $v = -4y$
(*d*) $u = 3xt$, $v = 3yt$ (*e*) $u = xy + y^2t$, $v = xy + x^4t$ (*f*) $u = 4x^2y^3$, $v = -2xy^4$

Ignore dimensional inconsistencies.

▌ In order to satisfy continuity,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial t} = 0 \qquad \text{or} \qquad \frac{\partial u}{\partial x} = -\frac{\partial v}{\partial y}$$

(*a*) $\frac{\partial u}{\partial x} = -1$ and $\frac{\partial v}{\partial y} = 1$ therefore, it does satisfy continuity.

(*b*) $\frac{\partial u}{\partial x} = 0$ and $\frac{\partial v}{\partial y} = 0$ therefore, it does satisfy continuity.

(*c*) $\frac{\partial u}{\partial x} = 4$ and $\frac{\partial v}{\partial y} = -4$ therefore, it does satisfy continutiy.

(*d*) $\frac{\partial u}{\partial x} = 3t$ and $\frac{\partial v}{\partial y} = 3t$ therefore, it does not satisfy continuity.

(*e*) $\frac{\partial u}{\partial x} = y$ and $\frac{\partial v}{\partial y} = x$ therefore, it does not satisfy continuity.

(*f*) $\frac{\partial u}{\partial x} = 8xy^3$ and $\frac{\partial v}{\partial y} = -8xy^3$ therefore, it does satisfy continuity.

7.28 If the radial velocity for incompressible flow is given by $v_r = b \cos\theta / r^2$, b = constant, what is the most general form of $v_\theta(r, \theta)$ that satisfies continuity?

▌ $$\frac{1}{r}\frac{\partial}{\partial r}(rv_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(v_\theta) + \frac{\partial v_z}{\partial z} = 0 \qquad \frac{1}{r}\frac{\partial}{\partial r}\left[(r)\left(\frac{b\cos\theta}{r^2}\right)\right] + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + 0 = 0$$

$$\frac{\partial v_\theta}{\partial \theta} = \frac{b\cos\theta}{r^2} \qquad v_\theta = \frac{b\sin\theta}{r^2} + f(r)$$

7.29 A two-dimensional velocity field is given by

$$u = -\frac{Ky}{x^2+y^2} \qquad v = \frac{Kx}{x^2+y^2}$$

where K is constant. Does this field satisfy incompressible continuity? Transform these velocities into polar components v_r and v_θ. What might the flow represent?

▮ $$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \qquad \frac{2xKy}{(x^2+y^2)^2}-\frac{2yKx}{(x^2+y^2)^2}+0=0$$

Therefore, continuity is satisfied. $x^2+y^2=r^2$, $\cos\theta = x/r$, $\sin\theta = y/r$.

$$v_r = u\cos\theta + v\sin\theta = -\left(\frac{Ky}{x^2+y^2}\right)\left(\frac{x}{r}\right)+\left(\frac{Kx}{x^2+y^2}\right)\left(\frac{y}{r}\right) = -\frac{Kyx}{r^3}+\frac{Kxy}{r^3}=0$$

$$v = -u\sin\theta + v\cos\theta = -\left[-\left(\frac{Ky}{x^2+y^2}\right)\right]\left(\frac{y}{r}\right)+\left(\frac{Kx}{x^2+y^2}\right)\left(\frac{x}{r}\right)=\frac{Ky^2}{r^3}+\frac{Kx^2}{r^3}=(y^2+x^2)\left(\frac{K}{r^3}\right)=\frac{K}{r}$$

Hence, in polar coordinates $v_r(r,\theta)$ and $v_\theta(r,\theta)$, $v_r=0$, $v_\theta = K/r$. (This represents a potential vortex.)

7.30 For incompressible polar coordinate flow, what is the most general form of a purely circulatory motion, $v_\theta = v(r,\theta,t)$ and $v_r=0$, which satisfies continuity?

▮ $$\frac{1}{r}\frac{\partial}{\partial r}(rv_r)+\frac{1}{r}\frac{\partial}{\partial\theta}(v_\theta)+\frac{\partial v_z}{\partial z}=0 \qquad \frac{1}{r}\frac{\partial}{\partial r}[(r)(0)]+\frac{1}{r}\frac{\partial}{\partial\theta}(v_\theta)+0=0 \qquad \frac{1}{r}\frac{\partial}{\partial\theta}(v_\theta)=0 \qquad v_\theta = f(r)$$

7.31 What is the most general form of a purely radial polar coordinate incompressible flow pattern, $v_r = v_r(r,\theta,t)$ and $v_\theta=0$ that satisfies continuity?

▮ $$\frac{1}{r}\frac{\partial}{\partial r}(rv_r)+\frac{1}{r}\frac{\partial}{\partial\theta}(v_\theta)+\frac{\partial v_z}{\partial z}=0 \qquad \frac{1}{r}\frac{\partial}{\partial r}(rv_r)+\left(\frac{1}{r}\right)(0)+0=0 \qquad \frac{1}{r}\frac{\partial}{\partial r}(rv_r)=0 \qquad v_r=\left(\frac{1}{r}\right)f(\theta)$$

7.32 An incompressible steady flow pattern is given by $u = x^4+3z^4$ and $w=y^4-3yz$. What is the most general form of the third component, $v(x,y,z)$, that satisfies continuity?

▮ $$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \qquad 4x^3+\frac{\partial v}{\partial y}-3y=0 \qquad \frac{\partial v}{\partial y}=3y-4x^3 \qquad v=\tfrac{3}{2}y^2-4x^3y+f(x,z)$$

7.33 A certain two-dimensional shear flow near a wall, as in Fig. 7-7, has the velocity component

$$u = U\left(\frac{3y}{ax}-\frac{y^2}{a^2x^2}\right)$$

where a and U are constants. Derive from continuity the velocity component $v(x,y)$ assuming that $v=0$ at the wall, $y=0$.

▮ $$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \qquad \frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+0=0 \qquad \frac{\partial v}{\partial y}=-\frac{\partial u}{\partial x}=-U\left(\frac{-3y}{ax^2}+\frac{2y^2}{a^2x^3}\right) \qquad v=U\left(\frac{3y^2}{2ax^2}-\frac{2y^3}{3a^2x^3}\right)+f(x)$$

Enforce no-slip condition: $v(x,0)=U(0-0)+f(x)=0$, $f(x)=0$.

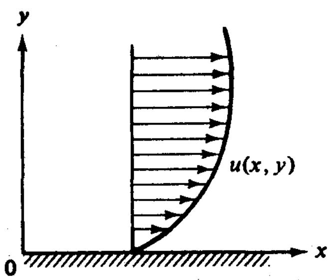


Fig. 7-7

7.34 Consider the flat-plate boundary-layer flow in Fig. 7-8. From the no-slip condition $v=0$ all along the wall $y=0$, and $u=U=$ constant outside the layer. If the layer thickness δ increases with x as shown, prove with incompressible two-dimensional continuity that **(a)** the component $v(x, y)$ is everywhere positive within the layer; **(b)** v increases parabolically with y very near the wall; and **(c)** v reaches a positive maximum at $y = \delta$.

(a) If δ increases with x, the streamlines in the shear layer must everywhere move upward to satisfy continuity. Therefore, $\partial u/\partial x < 0$ everywhere inside the shear layer. Since continuity requires

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

everywhere, it follows that $\partial v/\partial y > 0$ everywhere in the shear layer.
(b) Near the wall, $u = yf(x)$, $\partial v/\partial y = -\partial u/\partial x = -f'(x)$; therefore,

$$v = -\frac{y^2}{2}f'(x) \qquad \text{(parabolic)}$$

(c) At $y \geq \delta(x)$, $\partial u/\partial x \approx 0$; therefore $\partial v/\partial y = 0$, and v = maximum.

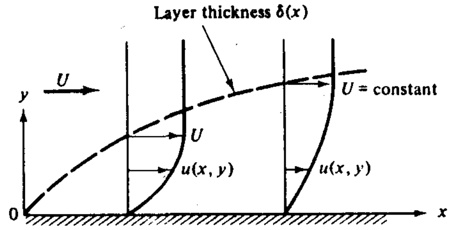


Fig. 7-8

7.35 The axial velocity field for fully developed laminar flow in a pipe is $v_z = u_{\max}(1 - r^2/R^2)$ and there is no swirl, $v_\theta = 0$. Determine the radial velocity field $v_r(r, z)$ from the incompressible relation if $u_{\max}$ is constant and $v_r = 0$ at $r = R$. (r denotes radial distance from the pipe's center; R denotes the pipe's radius.)

$$\frac{1}{r}\frac{\partial}{\partial r}(rv_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(v_\theta) + \frac{\partial v_z}{\partial z} = 0 \qquad \frac{1}{r}\frac{\partial}{\partial r}(rv_r) + 0 + 0 = 0 \qquad \frac{\partial}{\partial r}(rv_r) = 0 \qquad v_r = \frac{f(\theta, z)}{r}$$

if $v_r(R) = 0$ for all θ, z, $v_r = 0$.

7.36 An incompressible flow field has the cylindrical components $v_\theta = Cr$, $v_z = K(R^2 - r^2)$, $v_r = 0$, where C and K are constants and $r \leq R$, $z \leq L$. Does this flow satisfy continuity? What might it represent physically?

$$\frac{1}{r}\frac{\partial}{\partial r}(rv_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(v_\theta) + \frac{\partial v_z}{\partial z} = 0 \qquad \frac{1}{r}\frac{\partial}{\partial r}[(r)(0)] + \frac{1}{r}\frac{\partial}{\partial \theta}(Cr) + \frac{\partial}{\partial z}[K(R^2 - r^2)] = 0$$

$$0 + 0 + 0 = 0 \qquad \text{(satisfies continuity)}$$

This flow represents pressure-driven, laminar, steady flow in a rotating tube (fully developed).

7.37 An incompressible flow in polar coordinates is given by $v_r = K\cos\theta(1 - b/r^2)$, $v_\theta = -K\sin\theta(1 + b/r^2)$. Does this field satisfy continuity? For consistency, what should the dimensions of the constants K and b be?

$$\frac{1}{r}\frac{\partial}{\partial r}(rv_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(v_\theta) + \frac{\partial v_z}{\partial z} = 0 \qquad \frac{1}{r}\frac{\partial}{\partial r}\left[rK\cos\theta\left(1 - \frac{b}{r^2}\right)\right] + \frac{1}{r}\frac{\partial}{\partial \theta}\left[-K\sin\theta\left(1 + \frac{b}{r^2}\right)\right] + 0 = 0$$

$$\frac{1}{r}K\cos\theta\left(1 + \frac{b}{r^2}\right) - \frac{1}{r}K\cos\theta\left(1 + \frac{b}{r^2}\right) = 0 \qquad 0 = 0 \qquad \text{(satisfies continuity)}$$

Dimensions of constants: $\{K\} = \{L/T\}$, $\{b\} = \{L^2\}$.

7.38 The x component of velocity is $u = x^3 + z^4 + 6$, and the y component is $v = y^3 + z^4$. Find the simplest z component of velocity that satisfies continuity.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \qquad 3x^2 + 3y^2 + \frac{\partial w}{\partial z} = 0 \qquad \frac{\partial w}{\partial z} = -3(x^2 + y^2) \qquad w = -3z(x^2 + y^2)$$

7.39 Is the continuity equation for steady, incompressible flow satisfied if the following velocity components are involved?

$$u = 2x^2 - xy + z^2 \qquad v = x^2 - 4xy + y^2 \qquad w = -2xy - yz + y^2$$

▮ $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \qquad (4x - y) + (-4x + 2y) + (-y) = 0 \qquad \text{(satisfies continuity)}$$

7.40 For steady, incompressible flow, are the following values of u and v possible?
(*a*) $u = 4xy + y^2$, $v = 6xy + 3x$ (*b*) $u = 2x^2 + y^2$, $v = -4xy$

▮ $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

(*a*) $4y + 6x + 0 \neq 0$ (Flow is not possible.)
(*b*) $4x - 4x + 0 = 0$ (Flow is possible.)

7.41 Determine whether the velocity field $\mathbf{V} = 3t\mathbf{i} + xz\mathbf{j} + ty^2\mathbf{k}$ is incompressible, irrotational, both, or neither.

▮ The divergence of this velocity field is

$$\nabla \cdot V = \frac{\partial}{\partial x}(3t) + \frac{\partial}{\partial y}(xz) + \frac{\partial}{\partial z}(ty^2) = 0$$

Therefore, this velocity field is incompressible. The curl of this velocity field is

$$\nabla \times \mathbf{V} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ 3t & xz & ty^2 \end{vmatrix} = (2ty - x)\mathbf{i} + z\mathbf{k}$$

This is not zero; hence, the flow field is rotational, not irrotational.

7.42 If a velocity potential exists for the velocity field $u = a(x^2 - y^2)$, $v = -2axy$, $w = 0$, find it and plot it.

▮ Since $w = 0$, the curl of $\mathbf{V}$ has only one (z) component, and we must show that it is zero.

$$(\nabla \times \mathbf{V})_z = 2\omega_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \frac{\partial}{\partial x}(-2axy) - \frac{\partial}{\partial y}(ax^2 - ay^2) = -2ay + 2ay = 0 \qquad \text{checks}$$

The flow is indeed irrotational. A potential exists. To find $\phi(x, y)$, set

$$u = \frac{\partial \phi}{\partial x} = ax^2 - ay^2 \tag{1}$$

$$v = \frac{\partial \phi}{\partial y} = -2axy \tag{2}$$

Integrate (*1*)

$$\phi = \frac{ax^3}{3} - axy^2 + f(y) \tag{3}$$

Differentiate (*3*) and compare with (*2*)

$$\frac{\partial \phi}{\partial y} = -2axy + f'(y) = -2axy \tag{4}$$

Therefore $f' = 0$, or $f =$ constant. The velocity potential is $\phi = ax^3/3 - axy^2 + C$. Letting $C = 0$, we can plot the ϕ lines as shown in Fig. 7-9.

7.43 Given the velocity field $\mathbf{V} = 13x^2y\mathbf{i} + 18(yz + x)\mathbf{j} + 15\mathbf{k}$, find the angular velocity vector of a fluid particle at (2, 3, 4) m.

▮ $$\omega_x = \frac{1}{2}\left(\frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z}\right) = \tfrac{1}{2}(0 - 18y) = -9y \qquad \omega_y = \frac{1}{2}\left(\frac{\partial V_x}{\partial z} - \frac{\partial V_z}{\partial x}\right) = \tfrac{1}{2}(0 - 0) = 0$$

$$\omega_z = \frac{1}{2}\left(\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}\right) = \tfrac{1}{2}(18 - 13x^2) = 9 - 6.5x^2 \qquad \boldsymbol{\omega} = -9y\mathbf{i} + 0\mathbf{j} + (9 - 6.5x^2)\mathbf{k}$$

At point (2, 3, 4) m, $\boldsymbol{\omega} = (-9)(3)\mathbf{i} + [9 - (6.5)(2)^2]\mathbf{k} = -27\mathbf{i} - 17\mathbf{k}$ rad/s.

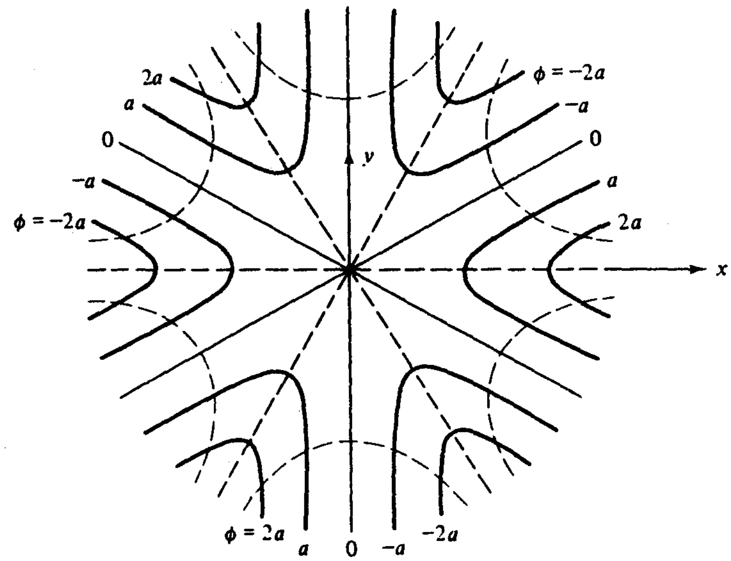


Fig. 7-9

7.44 Given the velocity field $\mathbf{V} = 6x^2y\mathbf{i} - (4x - 4z)\mathbf{j} + 12z^2\mathbf{k}$ m/s, compute the angular velocity field $\boldsymbol{\omega}(x, y, z)$.

$$\omega_x = \frac{1}{2}\left(\frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z}\right) = \tfrac{1}{2}(0 - 4) = -2 \qquad \omega_y = \frac{1}{2}\left(\frac{\partial V_x}{\partial z} - \frac{\partial V_z}{\partial x}\right) = \tfrac{1}{2}(0 - 0) = 0$$

$$\omega_z = \frac{1}{2}\left(\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}\right) = \tfrac{1}{2}(-4 - 6x^2) = -(2 + 3x^2) \qquad \boldsymbol{\omega} = -2\mathbf{i} + 0\mathbf{j} - (2 + 3x^2)\mathbf{k}\ \text{rad/s}$$

7.45 Show that any velocity field $\mathbf{V}$ expressible as the gradient of a scalar ϕ must be an irrotational field.

Show curl (grad ϕ) = $\mathbf{0}$

$$\text{curl}\left(\frac{\partial \phi}{\partial x}\mathbf{i} + \frac{\partial \phi}{\partial y}\mathbf{j} + \frac{\partial \phi}{\partial z}\mathbf{k}\right) = \mathbf{0}$$

$$\left[\frac{\partial}{\partial y}\left(\frac{\partial \phi}{\partial z}\right) - \frac{\partial}{\partial z}\left(\frac{\partial \phi}{\partial y}\right)\right]\mathbf{i} + \left[\frac{\partial}{\partial z}\left(\frac{\partial \phi}{\partial x}\right) - \frac{\partial}{\partial x}\left(\frac{\partial \phi}{\partial z}\right)\right]\mathbf{j} + \left[\frac{\partial}{\partial x}\left(\frac{\partial \phi}{\partial y}\right) - \frac{\partial}{\partial y}\left(\frac{\partial \phi}{\partial x}\right)\right]\mathbf{k} = 0$$

Since $\partial^2\phi/\partial y\,\partial z = \partial^2\phi/\partial z\,\partial y$, etc., we see that we have proven our point provided the partial derivatives of ϕ are continuous.

7.46 Is the following flow field irrotational or not? $\mathbf{V} = 12x^3y\mathbf{i} + 3x^4\mathbf{j} + 10\mathbf{k}$ ft/s.

$$\frac{\partial V_x}{\partial y} - \frac{\partial V_y}{\partial x} = 12x^3 - 12x^3 = 0 \qquad \frac{\partial V_y}{\partial z} - \frac{\partial V_z}{\partial y} = 0 \qquad \frac{\partial V_z}{\partial x} - \frac{\partial V_x}{\partial z} = 0$$

Therefore, the flow is irrotational.

7.47 For the velocity vector $\mathbf{V} = 3t\mathbf{i} + xz\mathbf{j} + ty^2\mathbf{k}$ evaluate the volume flow and the average velocity through the square surface whose vertices are at (0, 1, 0), (0, 1, 2), (2, 1, 2), and (2, 1, 0). See Fig. 7-10.

The surface S is shown in Fig. 7-10 and is such that $\mathbf{n} = \mathbf{j}$ and $dA = dx\,dz$ everywhere. The velocity field is $\mathbf{V} = 3t\mathbf{i} + xz\mathbf{j} + ty^2\mathbf{k}$. The normal component to S is $\mathbf{V} \cdot \mathbf{n} = \mathbf{V} \cdot \mathbf{j} = v$, the y component, which equals xz. The limits on the integral for Q are 0 to 2 for both dx and dz. The volume flow is thus

$$Q = \int_S V_n\,dA = \int_0^2 \int_0^2 xz\,dx\,dz = 4.0\ \text{units}$$

The area of the surface is (2)(2) = 4 units. Then the average velocity is $V_{av} = Q/A = 4.0/4.0 = 1.0$ unit.

7.48 At low velocities, the flow through a long circular tube has a paraboloid velocity distribution $u = u_{max}(1 - r^2/R^2)$, where R is the tube radius and u_{max} is the maximum velocity, which occurs at the tube centerline. (*a*) Find a general expression for volume flow and average velocity through the tube; (*b*) compute the volume flow if $R = 3$ cm and $u_{max} = 8$ m/s; and (*c*) compute the mass flow if $\rho = 1000$ kg/m^3.

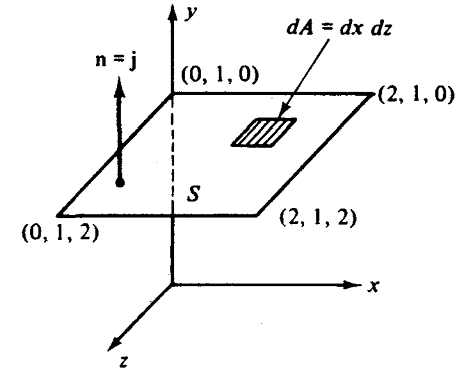


Fig. 7-10

(*a*) The area S is the cross section of the tube, and $\mathbf{n} = \mathbf{i}$. The normal component $\mathbf{V} \cdot \mathbf{n} = \mathbf{V} \cdot \mathbf{i} = u$. Since u varies only with r, the element dA can be taken to be the annular strip $dA = 2\pi r\,dr$. The volume flow becomes

$$Q = \int_S u\,dA = \int_0^R u_{max}\left(1 - \frac{r^2}{R^2}\right) 2\pi r\,dr$$

Carrying out the integration over r, we obtain $Q = \frac{1}{2}u_{max}\pi R^2$. The average velocity is $u_{av} = Q/A = \frac{1}{2}u_{max}\pi R^2/\pi R^2 = \frac{1}{2}u_{max}$. The average velocity is half the maximum, which is an accepted result for low-speed, or *laminar*, flow through a long tube. (*b*) For the given numerical values $Q = \frac{1}{2}(8)\pi(0.03)^2 = 0.0113\ \text{m}^3/\text{s}$. (*c*) For the given density, assumed constant, $\dot{m} = \rho Q = (1000)(0.0113) = 11.3\ \text{kg/s}$.

7.49 For low-speed (laminar) flow through a circular pipe, as shown in Fig. 7-11, the velocity distribution takes the form $u = (B/\mu)(r_0^2 - r^2)$, where μ is the fluid viscosity. Determine (*a*) the maximum velocity in terms of B, μ, and r_0 and (*b*) the mass flow rate in terms of B, μ, and r_0.

(*a*) u_{max} occurs when $du/dr = 0$. $du/dr = -2Br/\mu = 0$, $r = 0$, $u_{max} = Br_0^2/\mu$.

(*b*)
$$\dot{m} = \int \rho v_n\,dA = \int_0^{r_0} \rho\frac{B}{\mu}(r_0^2 - r^2)(2\pi r\,dr) = 2\pi\rho\frac{B}{\mu}\left[\frac{r_0^2 r^2}{2} - \frac{r^4}{4}\right]_0^{r_0}$$
$$= \frac{\rho}{2}\left(\frac{Br_0^2}{\mu}\right)\pi r_0^2 = \left(\frac{\rho}{2}\right)u_{max}(\pi r_0^2)$$

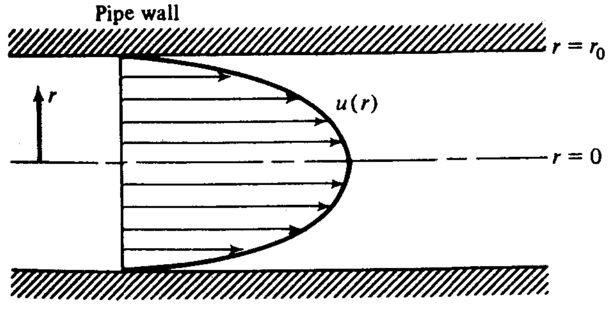


Fig. 7-11

7.50 If the fluid in Fig. 7-11 is water at 20 °C and 1 atm, what is the centerline velocity U_0 if the tube radius is 20 mm and the mass flow through the tube is 1.3 kg/s?

From Prob. 7.49, $\dot{m} = (\rho/2)u_{max}(\pi r_0^2)$, $1.3 = (\frac{998}{2})(u_{max})[(\pi)(0.020)^2]$, $u_{max} = U_0 = 2.07\ \text{m/s}$. (Actually, this is unrealistic. At this μ, $N_R > 2000$, so the flow is probably turbulent.)

7.51 A velocity field in arbitrary units is given by $\mathbf{V} = 3x^2\mathbf{i} - xy\mathbf{j} - 6xz\mathbf{k}$. Find the volume flow Q passing through the square with corners $(x, y, z) = (1, 0, 0)$, $(1, 1, 0)$, $(1, 1, 1)$, and $(1, 0, 1)$. See Fig. 7-12.

$Q = \iint (u)_{x=1}\,dy\,dz$. Since $\mathbf{n} = \mathbf{i}$, $\mathbf{V} \cdot \mathbf{n} = u = 3x^2$. $Q = \int_0^1\int_0^1 (3)\,dy\,dz = 3$ units.

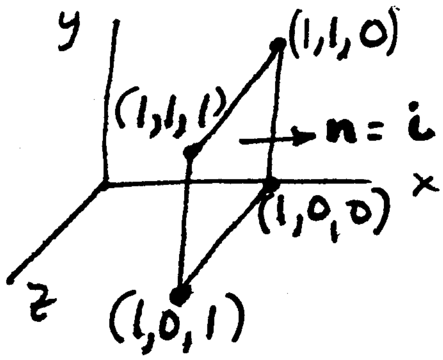


Fig. 7-12

7.52 For an incompressible fluid, express the volume flow Q across the upper surface in Fig. 7-13 in terms of the inlet velocity U_0 and the height δ of the fluid region shown in the figure?

▮

$$Q = \int_0^\delta U_0 b\,dy - \int_0^\delta U_0 \sin\left(\frac{\pi y}{\delta}\right) b\,dy = U_0 b[y]_0^\delta - U_0 b\left(\frac{\delta}{\pi}\right)\left[-\cos\left(\frac{\pi y}{\delta}\right)\right]_0^\delta$$

$$= U_0 b\delta - U_0 b\left(\frac{\delta}{\pi}\right)[1+1] = U_0 b\delta - 2U_0 b\left(\frac{\delta}{\pi}\right)$$

$$= U_0 b\delta(1 - 2/\pi) = 0.363 U_0 b\delta$$

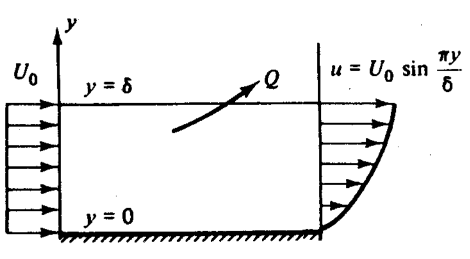


Plate (width b into paper)

Fig. 7-13

7.53 The velocity profile in water flow down a spillway is given approximately by $u = (U_0)(y/h)^{1/7}$ where $y = 0$ denotes the bottom and the depth is h (see Fig. 7-14). If $U_0 = 1.4$ m/s, $h = 3$ m, and the width is 17 m, how long will it take 10^5 m^3 of water to pass this section of the spillway?

▮

$$Q = \int u\,dA = \int_0^h (U_0)\left(\frac{y}{h}\right)^{1/7}(b\,dy) = U_0 b h^{-1/7}\left[\frac{y^{8/7}}{\frac{8}{7}}\right]_0^h = \tfrac{7}{8}U_0 bh = (\tfrac{7}{8})(1.4)(17)(3) = 62.5\ \text{m}^3/\text{s}$$

$$t = V/Q = 10^5/62.5 = 1603\ \text{s} \quad \text{or} \quad 26.7\ \text{min}$$

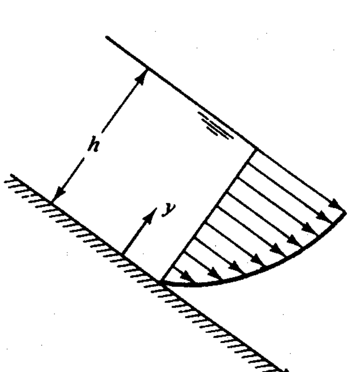


Fig. 7-14

7.54 Find the total derivative, $d\rho/dt$, at $x = 1$ and $y = 3$ of the density field $\rho = 3x^3 - 4y^2$ corresponding to the velocity field $\mathbf{V} = (x^2 - y^2 + x)\mathbf{i} - (3xy + y)\mathbf{j}$.

▮
$$\frac{d\rho}{dt} = \frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y} + w\frac{\partial \rho}{\partial z} = 0 + (x^2 - y^2 + x)(9x^2) - (3xy + y)(-8y) + 0$$

At $x = 1$ and $y = 3$, $d\rho/dt = (1^2 - 3^2 + 1)[(9)(1)^2] - [(3)(1)(3) + 3][(-8)(3)] = 279$ units.

7.55 A frictionless, incompressible ($\rho = \rho_0$) steady flow field is given by $\mathbf{V} = 3xy\mathbf{i} - 2y^2\mathbf{j}$ in arbitrary units. Neglecting gravity, calculate the pressure gradient and evaluate this gradient at (3, 1, 0).

▮ $\rho\dfrac{d\mathbf{V}}{dt} = \rho_0\left(u\dfrac{\partial \mathbf{V}}{\partial x} + v\dfrac{\partial \mathbf{V}}{\partial y} + w\dfrac{\partial \mathbf{V}}{\partial z}\right) = \rho_0[(3xy)(3y\mathbf{i}) + (-2y^2)(3x\mathbf{i} - 4y\mathbf{j}) + 0] = -\nabla p \qquad \nabla p = \rho_0(6xy^2\mathbf{i} + 4y^3\mathbf{j})$

At (3, 1, 0), $\nabla p = \rho_0[(6)(3)(1)^2\mathbf{i} + (4)(1)^3\mathbf{j}] = \rho_0(18\mathbf{i} + 4\mathbf{j})$.

7.56 A temperature field $T = 5xy^2$ is associated with a velocity field given by $u = 2y^2$, $v = 3x$, $w = 0$. Compute the rate of change dT/dt at the point $(x, y) = (3, 4)$.

▮
$$\frac{dT}{dt} = u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} = (2y^2)(5y^2) + (3x)(10xy) + 0 = 10y^4 + 30x^2y$$

At (3, 4),

$$\frac{dT}{dt} = (10)(4)^4 + (30)(3)^2(4) = 3640 \text{ units}$$

7.57 Take the velocity field $u = a(x^2 - y^2)$, $v = -2axy$, $w = 0$ and determine under what conditions it is a solution to the Navier–Stokes momentum equation. Assuming that these conditions are met, determine the resulting pressure distribution when z is "up" ($g_x = 0$, $g_y = 0$, $g_z = -g$).

▮
$$\rho g_x - \frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) = \rho\frac{du}{dt} \qquad \rho g_y - \frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) = \rho\frac{dv}{dt}$$

$$\rho g_z - \frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) = \rho\frac{dw}{dt}$$

Make a direct substitution of u, v, w.

$$\rho(0) - \frac{\partial p}{\partial x} + \mu(2a - 2a) = 2a^2\rho(x^3 + xy^2) \qquad (1)$$

$$\rho(0) - \frac{\partial p}{\partial y} + \mu(0) = 2a^2\rho(x^2y + y^3) \qquad (2)$$

$$\rho(-g) - \frac{\partial p}{\partial z} + \mu(0) = 0 \qquad (3)$$

The viscous terms vanish identically (although μ is *not* zero). Equation (3) can be integrated partially to obtain

$$p = -\rho g z + f_1(x, y) \qquad (4)$$

i.e., the pressure is hydrostatic in the z direction, which follows anyway from the fact that the flow is two-dimensional ($w = 0$). Now the question is: Do Eqs. (1) and (2) show that the given velocity field *is* a solution? One way to find out is to form the mixed derivative $\partial^2 p/(\partial x\,\partial y)$ from (1) and (2) separately and then compare them.

Differentiate Eq. (1) with respect to y

$$\frac{\partial^2 p}{\partial x\,\partial y} = -4a^2\rho xy \qquad (5)$$

Now differentiate Eq. (2) with respect to x

$$-\frac{\partial^2 p}{\partial x\,\partial y} = \frac{\partial}{\partial x}[2a^2\rho(x^2y + y^3)] = -4a^2\rho xy \qquad (6)$$

Since these are identical, the given velocity field is an *exact* solution to the Navier–Stokes equation.

To find the pressure distribution, substitute Eq. (4) into Eqs. (1) and (2), which will enable us to find $f_1(x, y)$

$$\frac{\partial f_1}{\partial x} = -2a^2\rho(x^3 + xy^2) \tag{7}$$

$$\frac{\partial f_1}{\partial y} = -2a^2\rho(x^2y + y^3) \tag{8}$$

Integrate Eq. (7) partially with respect to x

$$f_1 = -\tfrac{1}{2}a^2\rho(x^4 + 2x^2y^2) + f_2(y) \tag{9}$$

Differentiate this with respect to y and compare with Eq. (8)

$$\frac{\partial f_1}{\partial y} = -2a^2\rho x^2y + f_2'(y) \tag{10}$$

Comparing (8) and (10), we see they are equivalent if

$$f_2'(y) = -2a^2\rho y^3$$

or

$$f_2(y) = \tfrac{1}{2}a^2\rho y^4 + C \tag{11}$$

where C is a constant. Combine Eqs. (4), (9), and (11) to give the complete expression for pressure distribution

$$p(x, y, z) = -\rho gz - \tfrac{1}{2}a^2\rho(x^4 + y^4 + 2x^2y^2) + C \tag{12}$$

This is the desired solution. Do you recognize it? Not unless you go back to the beginning and square the velocity components:

$$u^2 + v^2 + w^2 = V^2 = a^2(x^4 + y^4 + 2x^2y^2) \tag{13}$$

Comparing with Eq. (12), we can rewrite the pressure distribution as

$$p + \tfrac{1}{2}\rho V^2 + \rho gz = C \tag{14}$$

7.58 The sprinkler shown in Fig. 7-15 on p. 148 discharges water upward and outward from the horizontal plane so that it makes an angle of $\theta°$ with the t axis when the sprinkler arm is at rest. It has a constant cross-sectional flow area of A_0 and discharges q cfs starting with $\omega = 0$ and $t = 0$. The resisting torque due to bearings and seals is the constant T_0, and the moment of inertia of the rotating empty sprinkler head is I_s. Determine the equation for ω as a function of time.

I The control volume is the cylindrical area enclosing the rotating sprinkler head. The inflow is along the axis, so that it has no moment of momentum; hence, the torque $-T_0$ due to friction is equal to the time rate of change of moment of momentum of sprinkler head and fluid within the sprinkler head plus the net efflux of moment of momentum from the control volume. Let $V_r = q/2A_0$.

$$-T_0 = 2\frac{d}{dt}\int_0^{r_0} A_o\rho\omega r^2\,dr + I_s\frac{d\omega}{dt} - \left[\frac{2\rho q r_0}{2}(V_r\cos\theta - \omega r_0)\right]$$

The total derivative can be used. Simplifying gives

$$\frac{d\omega}{dt}(I_s + \tfrac{2}{3}\rho A_0 r_0^3) = \rho q r_0(V_r\cos\theta - \omega r_0) - T_0$$

For rotation to start, $\rho q r_0 V_r \cos\theta$ must be greater than T_0. The equation is easily integrated to find ω as a function of t. The final value of ω is obtained by setting $d\omega/dt = 0$ in the equation.

7.59 A turbine discharging 10 m^3/s is to be so designed that a torque of 10 000 N · m is to be exerted on an impeller turning at 200 rpm that takes all the moment of momentum out of the fluid. At the outer periphery of the impeller, $r = 1$ m. What must the tangential component of velocity be at this location?

I $T = \rho Q[(rv_t)_2 - (rv_t)_1]$ $\quad 10\,000 = (1000)(10)[(1)(v_t)_{in} - 0]$ $\quad (v_t)_{in} = 1.00$ m/s

7.60 The sprinkler of Fig. 7-16 discharges 0.01 cfs through each nozzle. Neglecting friction, find its speed of rotation. The area of each nozzle opening is 0.001 ft^2.

I The fluid entering the sprinkler has no moment of momentum, and no torque is exerted on the system externally; hence the moment of momentum of fluid leaving must be zero. Let ω be the speed of rotation; then

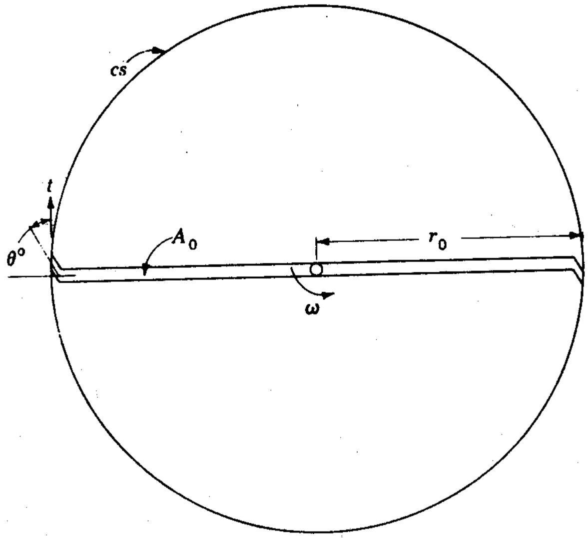


Fig. 7-15

the moment of momentum leaving is $\rho Q_1 r_1 v_{t1} + \rho Q_2 r_2 v_{t2}$ in which v_{t1} and v_{t2} are absolute velocities. Then $v_{t1} = v_{r1} - \omega r_1 = Q_1/0.001 - \omega r_1 = 10 - \omega$ and $v_{t2} = v_{r2} - \omega r_2 = 10 - \frac{2}{3}\omega$. For the moment of momentum to be zero, $\rho Q(r_1 v_{t1} + r_2 v_{t2}) = 0$ or $(1)(10 - \omega) + (\frac{2}{3})(10 - \frac{2}{3}\omega) = 0$ and $\omega = 11.54$ rad/s, or 110.2 rpm.

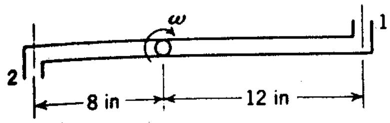


Fig. 7-16

7.61 The velocity profile for laminar flow between two plates, as in Fig. 7-17, is

$$u = \frac{4u_{\max} y(h - y)}{h^2} \qquad v = w = 0$$

If the wall temperature is T_w at both walls, use the incompressible-flow energy equation to solve for the temperature distribution $T(y)$ between the walls for steady flow.

▌

$$\rho c_v \frac{dT}{dt} = K\frac{d^2T}{dy^2} + \mu\left(\frac{du}{dy}\right)^2 \qquad 0 = K\frac{d^2T}{dy^2} + \mu\left(\frac{du}{dy}\right)^2$$

$$\frac{d^2T}{dy^2} = -\frac{\mu}{K}\left(\frac{du}{dy}\right)^2 = -\frac{\mu}{K}\left[\frac{4u_{\max}}{h^2}(h - 2y)\right]^2 = \left(\frac{-16\mu u_{\max}^2}{Kh^4}\right)(h^2 - 4hy + 4y^2)$$

$$\frac{dT}{dy} = \left(\frac{-16\mu u_{\max}^2}{Kh^4}\right)[h^2y - 2hy^2 + (\tfrac{4}{3})(y)^3 + C_1]$$

Since $dT/dy = 0$ at $y = h/2$, $C_1 = -h^3/6$.

$$\frac{dT}{dy} = \left(\frac{-16\mu u_{\max}^2}{Kh^4}\right)\left[h^2y - 2hy^2 + (\tfrac{4}{3})(y)^3 - \frac{h^3}{6}\right] \qquad T = \left(\frac{-16\mu u_{\max}^2}{Kh^4}\right)\left[\frac{h^2y^2}{2} - (\tfrac{2}{3})(hy^3) + \frac{y^4}{3} - \frac{yh^3}{6} + C_2\right]$$

If $T = T_w$ at $y = 0$ and $y = h$, then $C_2 = T_w$.

$$T = T_w + \left(\frac{8\mu u_{\max}^2}{K}\right)\left[\left(\frac{1}{3}\right)\left(\frac{y}{h}\right) - \frac{y^2}{h^2} + \left(\frac{4}{3}\right)\left(\frac{y^3}{h^3}\right) - \left(\frac{2}{3}\right)\left(\frac{y^4}{h^4}\right)\right]$$

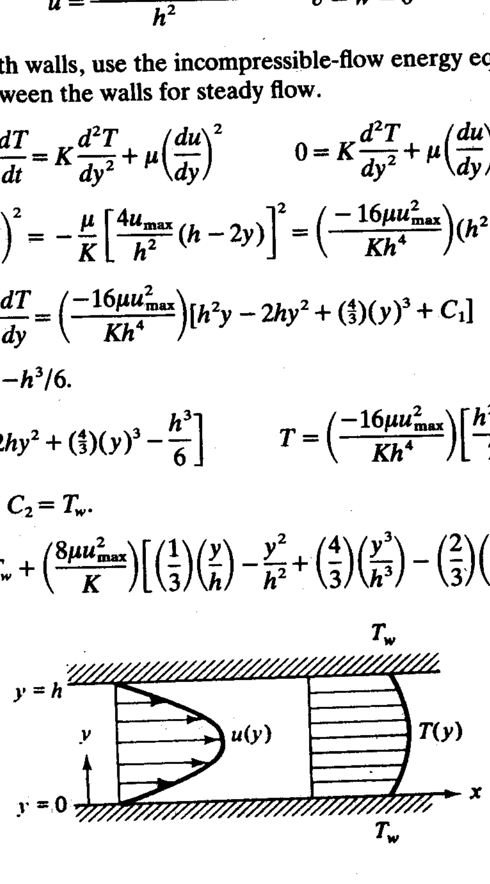


Fig. 7-17

7.62 Consider a viscous, steady flow through a pipe (Fig. 7-18*a*). The velocity profile forms a paraboloid about the pipe centerline, given as

$$V = -C(r^2 - D^2/4) \qquad \text{m/s} \tag{1}$$

where C is a constant. (*a*) What is the flow of mass through the left end of the control surface shown dashed? (*b*) What is the flow of kinetic energy through the left end of the control surface? Assume that the velocity profile does not change along the pipe.

▮ In Fig. 7-18*b*, we have shown a cross section of the pipe. For an infinitesimal strip, we can say noting that $\mathbf{V}$ and $d\mathbf{A}$ are collinear but of opposite sense: $\rho\mathbf{V}\cdot d\mathbf{A} = \rho[C(r^2 - D^2/4)]2\pi r\, dr$. For the whole cross section, we have

$$\iint \rho\mathbf{V}\cdot d\mathbf{A} = \rho\int_0^{D/2} C\left(r^2 - \frac{D^2}{4}\right)2\pi r\, dr = 2\pi\rho C\left[\frac{r^4}{4} - \frac{D^2}{4}\frac{r^2}{2}\right]_0^{D/2} = -\frac{\rho C\pi D^4}{32} \qquad \text{kg/s} \tag{2}$$

We now turn to the flow of kinetic energy through the left end of the control surface. The kinetic energy for an element of fluid is $\frac{1}{2}\, dmV^2$. This corresponds to an infinitesimal amount of an extensive property N. To get η, the corresponding intensive property, we divide by dm to get

$$\eta = \tfrac{1}{2}V^2 \tag{3}$$

We accordingly wish to compute $\iint \eta\rho\mathbf{V}\cdot d\mathbf{A} = \iint (\frac{1}{2}V^2)\{\rho\mathbf{V}\cdot d\mathbf{A}\}$. Employing Eq. (*1*) for V, and noting again that $\mathbf{V}$ and $d\mathbf{A}$ are collinear but of opposite sense, we get

$$\iint \eta\rho\mathbf{V}\cdot d\mathbf{A} = \int_0^{D/2}\frac{1}{2}C^2\left(r^2 - \frac{D^2}{4}\right)^2\left\{\rho\left[C\left(r^2 - \frac{D^2}{4}\right)2\pi r\, dr\right]\right\}$$

$$= \rho C^3\pi\int_0^{D/2}\left(r^2 - \frac{D^2}{4}\right)^3 r\, dr = \frac{\rho C^3\pi D^8}{2048}\ \text{N}\cdot\text{m/s} \tag{4}$$

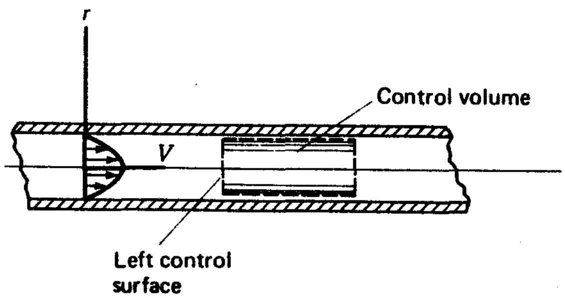


Fig. 7-18(*a*)

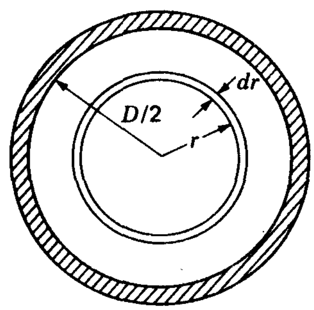


Fig. 7-18(*b*)

7.63 In Prob. 7.62, assume a *one-dimensional* model with the same mass flow. Compute the kinetic energy flow through a section of the pipe for this model. That is, compute kinetic energy flow with an average constant velocity. What is the ratio of the actual kinetic energy to the kinetic energy flow for the one-dimensional model flow?

▮ We first compute the constant velocity at a section for the one-dimensional model. Hence, using Eq. (*2*) of Prob. 7.62,

$$-(V_{av})\left(\frac{\rho\pi D^2}{4}\right) = -\frac{\rho CD^4\pi}{32}$$

$$V_{av} = \frac{CD^2}{8} \qquad \text{m/s} \tag{1}$$

The kinetic energy flow for the one-dimensional model is then

$$\iint \frac{V^2}{2}(\rho\mathbf{V}\cdot d\mathbf{A}) = -\frac{\rho}{2}\left(\frac{CD^2}{8}\right)^3\left(\frac{\pi D^2}{4}\right) = -\frac{\rho C^3D^8\pi}{4096} \qquad \text{N}\cdot\text{m/s} \tag{2}$$

We now define the *kinetic-energy correction* factor α as the ratio of the actual flow of kinetic energy through a cross section to the flow of kinetic energy for one-dimensional model for the same mass flow. That is

$$\alpha = \frac{\text{KE flow for section}}{\text{KE flow for 1-}D\text{ model}} \tag{3}$$

For the case at hand, we have from Eq. (2) of this problem and Eq. (4) of Prob. 7.62

$$\alpha = \frac{-\rho C^3 \pi D^8/2048}{-\rho C^3 \pi D^8/4096} = 2 \tag{4}$$

The factor α exceeds unity, so there is an underestimation of kinetic energy flow for a one-dimensional model.

7.64 The velocity field in a diffuser is $u = U_0 e^{-2x/L}$, and the density field is $\rho = \rho_0 e^{-x/L}$. Find the rate of change of density at $x = L$.

■
$$\frac{d\rho}{dt} = \frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y} + w\frac{\partial \rho}{\partial z} = 0 + (U_0 e^{-2x/L})\left(\frac{-\rho_0 e^{-x/L}}{L}\right) + 0 + 0 = -\frac{\rho_0 U_0}{L}e^{-3x/L}$$

At $x = L$,

$$\frac{d\rho}{dt} = -\frac{\rho_0 U_0}{L}e^{-3L/L} = -\frac{0.0498\rho_0 U_0}{L}$$

7.65 Gas is flowing in a long 4-in-diameter pipe from A to B. At section A the flow is 0.30 lb/s, while at the same instant at section B the flow is 0.33 lb/s. The distance between A and B is 700 ft. Find the mean value of the time rate of change of the specific weight of the gas between sections A and B at that instant.

■ $\gamma_1 A_1 v_1 - \gamma_2 A_2 v_2 = \left(\frac{\partial \gamma}{\partial t}\right)_{\text{avg}}$ (volume of section). Since $G = \gamma A v$,

$$0.30 - 0.33 = \left(\frac{\partial \gamma}{\partial t}\right)_{\text{avg}} [(700)(\pi)(\tfrac{4}{12})^2/4] \qquad \left(\frac{\partial \gamma}{\partial t}\right)_{\text{avg}} = -0.000491 \quad \text{lb/ft}^3/\text{s}$$

7.66 An incompressible flow field is given by $\mathbf{V} = x^2\mathbf{i} - z^2\mathbf{j} - 3xz\mathbf{k}$ with V in meters per second and (x, y, z) in meters. If the fluid viscosity is 0.04 Pa · s, evaluate the entire viscous stress tensor at the point $(x, y, z) = (3, 2, 1)$.

■
$$\tau_{ij} = \begin{vmatrix} \tau_{xx} & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \tau_{yy} & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \tau_{zz} \end{vmatrix} \qquad \tau_{xx} = 2\mu\frac{\partial u}{\partial x} = 4\mu x \qquad \tau_{yy} = 2\mu\frac{\partial v}{\partial y} = 0 \qquad \tau_{zz} = 2\mu\frac{\partial w}{\partial z} = -6\mu x$$

$$\tau_{xy} = \tau_{yx} = (\mu)\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = 0 \qquad \tau_{yz} = \tau_{zy} = (\mu)\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) = -2\mu z \qquad \tau_{xz} = \tau_{zx} = (\mu)\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) = -3\mu z$$

At $(x, y, z) = (3, 2, 1)$ for $\mu = 0.04$ kg/(m · s):

$$\tau_{ij} = \begin{vmatrix} 0.48 & 0 & -0.12 \\ 0 & 0 & -0.08 \\ -0.12 & -0.08 & -0.72 \end{vmatrix} \quad \text{Pa}$$

7.67 Given the velocity distribution

$$u = Kx \qquad v = -Ky \qquad w = 0 \tag{1}$$

where k is constant, compute and plot the streamlines of flow, including directions, and give some possible interpretations of the pattern.

■ Since time does not appear explicitly in Eqs. (1), the motion is steady, so that streamlines, path lines, and streaklines will coincide. Since $w = 0$ everywhere, the motion is two-dimensional, in the xy plane. The streamlines can be computed by substituting the expressions for u and v into

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w} = \frac{dr}{V} \qquad \frac{dx}{Kx} = -\frac{dy}{Ky} \qquad \text{or} \qquad \int\frac{dx}{x} = -\int\frac{dy}{y}$$

Integrating, we obtain $\ln x = -\ln y + \ln C$, or

$$xy = C \tag{2}$$

This is the general expression for the streamlines, which are hyperbolas. The complete pattern is plotted in Fig. 7-19 by assigning various values to the constant C. The arrowheads can be determined only by returning to Eqs. (*1*) to ascertain the velocity component directions, assuming K is positive. For example, in the upper right quadrant ($x>0$, $y>0$), u is positive and v is negative; hence the flow moves down and to the right, establishing the arrowheads as shown.

Note that the streamline pattern is entirely independent of the constant K. It could represent the impingement of two opposing streams, or the upper half could simulate the flow of a single downward stream against a flat wall. Taken in isolation, the upper right quadrant is similar to the flow in a 90° corner.

Finally note the peculiarity that the two streamlines ($C=0$) have opposite directions and intersect each other. This is possible only at a point where $u=v=w=0$, which occurs at the origin in this case. Such a point of zero velocity is called a *stagnation point*.

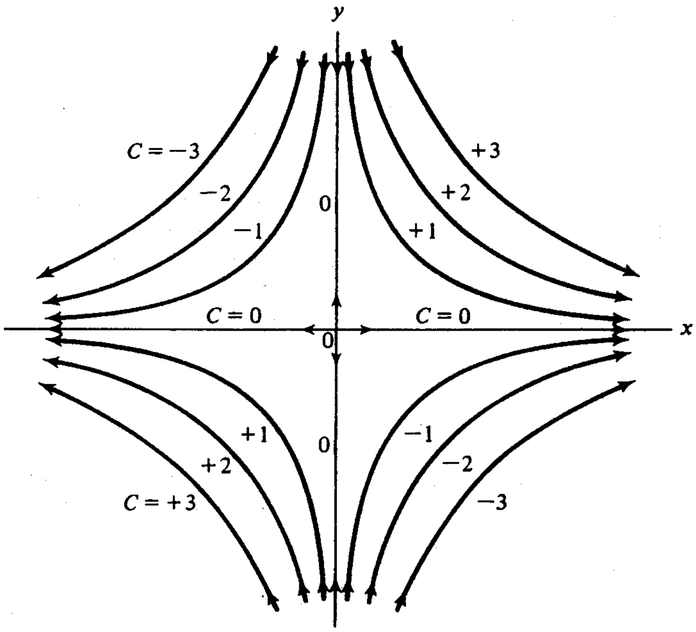


Fig. 7-19

7.68 A velocity field is given by $u=V\cos\theta$, $v=V\sin\theta$, and $w=0$, where V and θ are constants. Find an expression for the streamlines of this flow.

$$\frac{dx}{u}=\frac{dy}{v}=\frac{dz}{w}=\frac{dr}{V} \qquad \frac{dx}{V\cos\theta}=\frac{dy}{V\sin\theta}=\frac{dz}{0}$$

(*Note*: $dz/0$ indicates that the streamlines do not vary with z.)

$$\frac{dy}{dx}=\frac{V\sin\theta}{V\cos\theta}=\tan\theta \qquad y=x\tan\theta+C$$

Hence, the streamlines are straight and inclined at angle θ, as illustrated in Fig. 7-20.

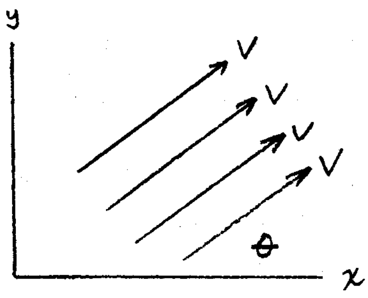


Fig. 7-20

7.69 A two-dimensional steady velocity field is given by $u = 3x^2 - 2y^2$, $v = -6xy$. Derive the streamline pattern and sketch a few streamlines in the upper half-plane.

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w} = \frac{dr}{V} \qquad \frac{dx}{3x^2 - 2y^2} = \frac{dy}{-6xy}$$

$$-6xy\,dx = (3x^2 - 2y^2)\,dy \qquad df = 6xy\,dx + (3x^2 - 2y^2)\,dy \qquad f(x, y) = 3x^2y - 2y^3/3 = \text{const.}$$

Hence, the streamlines represent inviscid flow in three corners, as illustrated in Fig. 7-21.

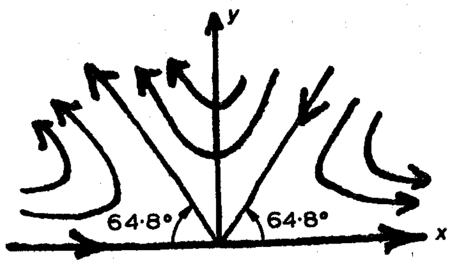


Fig. 7-21

7.70 A two-dimensional unsteady velocity field is given by $u = x(1 + 3t)$, $v = y$. Determine the one-parameter (t) family of streamlines through the point (x_0, y_0).

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w} = \frac{dr}{V} \qquad \frac{dx}{x(1 + 3t)} = \frac{dy}{y}$$

Integrate, holding t constant.

$$\frac{\ln x}{1 + 3t} = \ln y + C \qquad y = Cx^{1/(1+3t)}$$

If $y = y_0$ at $x = x_0$, $y_0 = Cx_0^{1/(1+3t)}$.

$$C = \frac{y_0}{x_0^{1/(1+3t)}} \qquad y = \left[\frac{y_0}{x_0^{1/(1+3t)}}\right][x^{1/(1+3t)}] = (y_0)\left(\frac{x}{x_0}\right)^{1/(1+3t)}$$

Some streamlines of the family are sketched in Fig. 7-22.

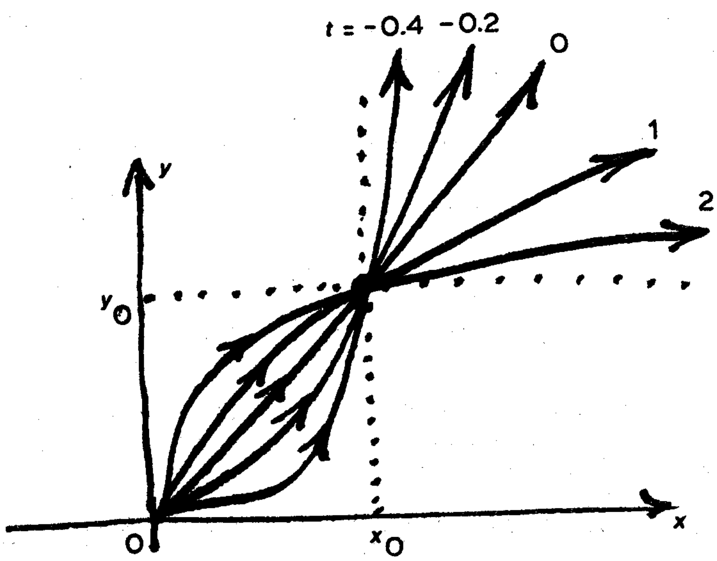


Fig. 7-22

7.71 Investigate the stream function in polar coordinates

$$\psi = U \sin\theta(r - R^2/r) \tag{1}$$

where U and R are constants, a velocity and a length, respectively. Plot the streamlines. What does the flow represent? Is it a realistic solution to the basic equations?

▌ The streamlines are lines of constant ψ, which has units of square meters per second. Note that ψ/UR is dimensionless. Rewrite Eq. (1) in dimensionless form

$$\psi/UR = \sin\theta(\eta - 1/\eta) \qquad \eta = r/R \tag{2}$$

Of particular interest is the special line $\psi = 0$. From Eq. (1) or (2) this occurs when (a) $\theta = 0°$ or $180°$ and (b) $r = R$. Case (a) is the x axis and case (b) is a circle of radius R, both of which are plotted in Fig. 7-23.

For any other nonzero value of ψ it is easiest to pick a value of r and solve for θ:

$$\sin\theta = \frac{\psi/UR}{r/R - R/r} \tag{3}$$

In general, there will be two solutions for θ because of the symmetry about the y axis. For example take $\psi/UR = +1.0$:

Guess r/R	3.0	2.5	2.0	1.8	1.7	1.618
Compute θ	22° 158°	28° 152°	42° 138°	54° 126°	64° 116°	90°

This line is plotted in Fig. 7-23 and passes over the circle $r = R$. You have to watch it, though, because there is a second curve for $\psi/UR = +1.0$ for small $r < R$ below the x axis:

Guess r/R	0.618	0.6	0.5	0.4	0.3	0.2	0.1
Compute θ	−90°	−70° −110°	−42° −138°	−28° −152°	−19° −161°	−12° −168°	−6° −174°

This second curve plots as a closed curve inside the circle $r = R$. There is a singularity of infinite velocity and indeterminate flow direction at the origin. Figure 7-23 shows the full pattern.

The given stream function, Eq. (1), is an exact and classic solution to the momentum equation for frictionless flow. Outside the circle $r = R$ it represents two-dimensional inviscid flow of a uniform stream past a circular cylinder. Inside the circle it represents a rather unrealistic trapped circulating motion of what is called a *line doublet*.

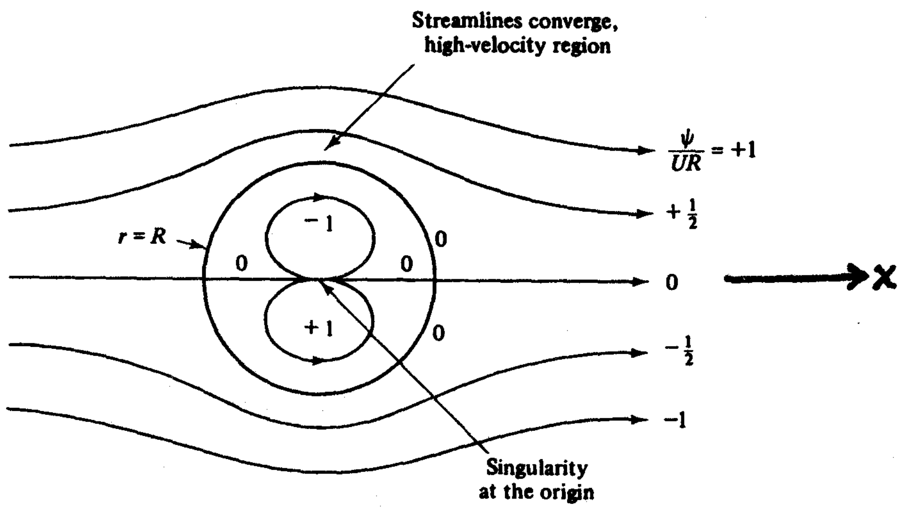


Fig. 7-23

7.72 In two-dimensional, incompressible steady flow around an airfoil, the streamlines are drawn so that they are 10 mm apart at a great distance from the airfoil, where the velocity is 40 m/s. What is the velocity near the airfoil, where the streamlines are 7.5 mm apart?

▌ $$Q = Av = (\tfrac{10}{1000})(40) = 0.40\,\frac{\text{m}^3/\text{s}}{\text{m}} \qquad 0.40 = (7.5/1000)(v) \qquad v = 53.3\ \text{m/s}$$

7.73 A three-dimensional velocity distribution is given by $u = -x$, $v = 2y$, $w = 6 - z$. Find the equation of the streamline through (1, 2, 3).

▌ $$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w} \qquad \frac{dx}{-x} = \frac{dy}{2y} \qquad -\ln x = \ln \sqrt{y} + \ln C_1 \qquad x\sqrt{y} = -C_1$$

At $x = 1$, $y = 2$, $1\sqrt{2} = -C_1$, $C_1 = -1.414$; $x\sqrt{y} = 1.414$.

$$\frac{dx}{-x} = \frac{dz}{5 - z} \qquad -\ln x = -\ln(6 - z) + \ln C_2 \qquad \frac{6 - z}{x} = C_2$$

At $x = 1$, $z = 3$, $(6 - 3)/1 = C_2$, $C_2 = 3$; $(6 - z)/x = 3$. Therefore, $x\sqrt{y} = 1.414$ and $(6 - z)/x = 3$ is the equation of the streamline.

7.74 A two-dimensional flow can be described by $u = -y/b^2$, $v = x/a^2$. Verify that this is the flow of an incompressible fluid and that the ellipse $x^2/a^2 + y^2/b^2 = 1$ is a streamline.

▌ $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 + 0 = 0 \qquad \text{(Therefore, continuity is satisfied.)}$$

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w} \qquad \frac{dx}{-y/b^2} = \frac{dy}{x/a^2} \qquad \frac{x\,dx}{a^2} = \frac{-y\,dy}{b^2}$$

$x^2/a^2 + y^2/b^2 = \text{constant}$ (Therefore, ellipse $x^2/a^2 + y^2/b^2 = 1$ is a streamline.)

7.75 A velocity potential in two-dimensional flow is $\phi = xy + x^2 - y^2$. Find the stream function for this flow.

▌ $$\frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y} \qquad \frac{\partial \phi}{\partial x} = y + 2x \qquad \frac{\partial \psi}{\partial y} = y + 2x \qquad \psi = \tfrac{1}{2}y^2 + 2xy + f(x)$$

$$\frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x} \qquad \frac{\partial \phi}{\partial y} = x - 2y \qquad -\frac{\partial \psi}{\partial x} = x - 2y \qquad -2y - f'(x) = x - 2y$$

Therefore, $f(x) = -\tfrac{1}{2}x^2(+C)$ and $\psi = 2xy + \tfrac{1}{2}y^2 - \tfrac{1}{2}x^2(+C)$.

7.76 For the steady two-dimensional flow shown in Fig. 7-24, the scalar components of the velocity field are $V_x = -x$, $V_y = y$, $V_z = 0$. Find the equations of the streamlines and the components of acceleration.

▌ $(dy/dx)_{\text{stream}} = V_y/V_x = -y/x$, $dy/y = -dx/x$, $\ln y = -\ln x + \ln C$. Hence, $xy = C$. Note that the streamlines form a family of rectangular hyperbolas. The wetted boundaries are part of the family, as is to be expected.

$$a_x = \frac{\partial V_x}{\partial t} + \left(V_x\frac{\partial V_x}{\partial x} + V_y\frac{\partial V_x}{\partial y} + V_z\frac{\partial V_x}{\partial z}\right) = 0 + (-x)(-1) + (y)(0) + 0 = x$$

$$a_y = \frac{\partial V_y}{\partial t} + \left(V_x\frac{\partial V_y}{\partial x} + V_y\frac{\partial V_y}{\partial y} + V_z\frac{\partial V_y}{\partial z}\right) = 0 + (-x)(0) + (y)(1) + 0 = y$$

$$a_z = 0 \qquad \mathbf{a} = x\mathbf{i} + y\mathbf{j}$$

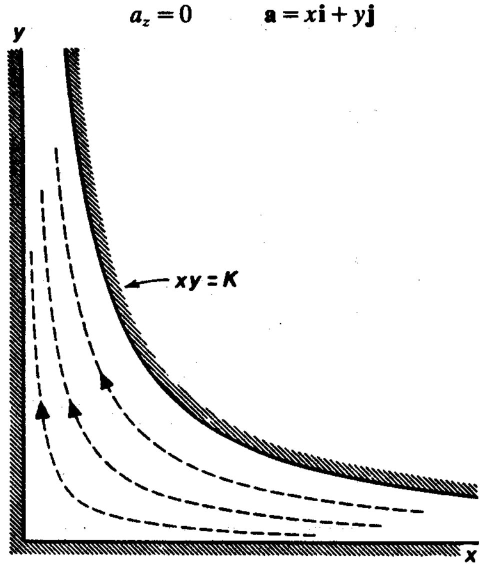


Fig. 7-24

7.77 A two-dimensional dipole source at the origin produces steady incompressible flow with stream function

$$\psi = \frac{y}{x^2 + y^2}$$

Find the direction of motion of a fluid particle at the point $x = 6$, $y = 9$.

▌ Along the streamline $\psi = \text{const.}$ through the given point,

$$0 = d\psi = \frac{(x^2 + y^2)\,dy - y\,(2x\,dx + 2y\,dy)}{(x^2 + y^2)^2}$$

from which

$$\frac{dy}{dx} = \frac{2xy}{x^2 - y^2} = \frac{2(6)(9)}{6^2 - 9^2} = -2.4$$

Therefore the particle is moving at an angle of $\arctan(-2.4) = -67.4°$ with the positive x-axis.

7.78 Sketch the streamlines for Prob. 7.77.

▌ The streamline $\psi = 1/2\lambda$ has the equation

$$\frac{y}{x^2 + y^2} = \frac{1}{2\lambda} \qquad \text{or} \qquad x^2 + (y - \lambda)^2 = \lambda^2$$

It is thus a circle of radius $|\lambda|$ centered at $(0, \lambda)$; see Fig. 7-25.

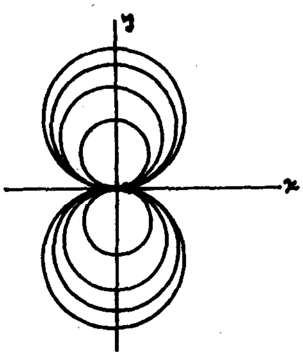


Fig. 7-25

7.79 In Prob. 7.76 find the normal acceleration of a fluid particle as it moves through the position $x = 3$, $y = 5$.

▌ At (3, 5), $\mathbf{V} = -3\mathbf{i} + 5\mathbf{j}$ and $\mathbf{a} = 3\mathbf{i} + 5\mathbf{j}$. Therefore (see Fig. 7-26),

$$\mathbf{a_V} = \frac{\mathbf{a}\cdot\mathbf{V}}{|\mathbf{V}|^2}\mathbf{V} = \frac{(3)(-3) + (5)(5)}{(-3)^2 + (5)^2}(-3\mathbf{i} + 5\mathbf{j}) = -\tfrac{24}{17}\mathbf{i} + \tfrac{40}{17}\mathbf{j}$$

$$\mathbf{a_N} = \mathbf{a} - \mathbf{a_V} = \tfrac{15}{17}(5\mathbf{i} + 3\mathbf{j})$$

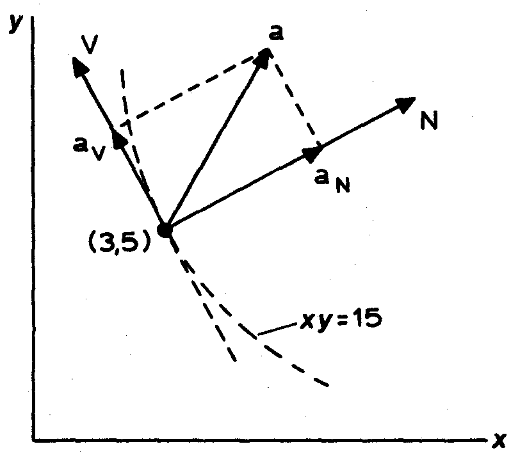


Fig. 7-26

7.80 Repeat Prob. 7.70 to find the equation of the path line that passes through (x_0, y_0) at time $t = 0$. Sketch it.

$$u = dx/dt = x(1+3t) \qquad dx/x = (1+3t)\,dt \qquad \ln x = t + 1.5t^2 + C_1$$
$$x = \exp(t + 1.5t^2 + C_1) = [\exp(t + 1.5t^2)](C_2)$$

At $x = x_0$ and $t = 0$, $x_0 = e^0 C_2$, $C_2 = x_0$; $x = x_0 \exp(t + 1.5t^2)$, $v = dy/dt = y$, $dy/y = dt$, $\ln y = t + C_3$, $y = \exp(t + C_3) = e^t C_4$.

At $y = y_0$ and $t = 0$, $y_0 = e^0 C_4$, $C_4 = y_0$; $y = y_0 e^t$, $t = \ln(y/y_0)$, $x = x_0 \exp(t + 1.5t^2) = x_0 \exp[\ln(y/y_0) + 1.5 \ln^2(y/y_0)]$. This pathline is sketched in Fig. 7-27.

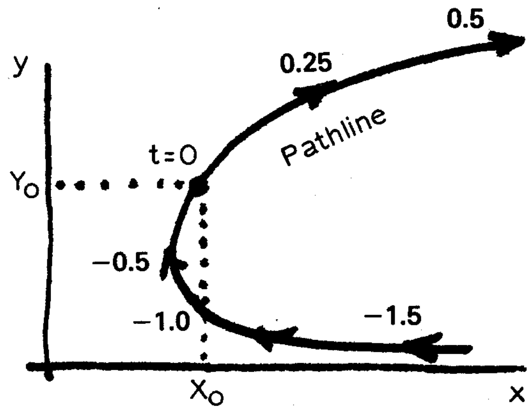


Fig. 7-27

CHAPTER 8
Fundamentals of Fluid Flow

8.1 Water flows through a 3-in-diameter pipe at a velocity of 10 ft/s. Find the (*a*) volume flow rate in cfs and gpm, (*b*) weight flow rate, and (*c*) mass flow rate.

(*a*) $$Q = Av = [(\pi)(\tfrac{3}{12})^2/4](10) = 0.4909 \text{ cfs} = 0.4909/0.002228 = 220 \text{ gpm}$$

(*b*) $$W = \gamma Av = 62.4[(\pi)(\tfrac{3}{12})^2/4](10) = 30.6 \text{ lb/s}$$

(*c*) $$M = \rho Av = 1.94[(\pi)(\tfrac{3}{12})^2/4](10) = 0.952 \text{ slug/s}$$

8.2 Benzene flows through a 100-mm-diameter pipe at a mean velocity of 3.00 m/s. Find the (*a*) volume flow rate in m^3/s and L/min, (*b*) weight flow rate, and (*c*) mass flow rate.

(*a*) $$Q = Av = [(\pi)(\tfrac{100}{1000})^2/4](3.00) = 0.0236 \text{ m}^3/\text{s} = 0.0236/0.00001667 = 1416 \text{ L/min}$$

(*b*) $$W = \gamma Av = 8.62[(\pi)(\tfrac{100}{1000})^2/4](3.00) = 0.203 \text{ kN/s}$$

(*c*) $$M = \rho Av = 879[(\pi)(\tfrac{100}{1000})^2/4](3.00) = 20.7 \text{ kg/s}$$

8.3 The flow rate of air moving through a square 0.50-m by 0.50-m duct is 160 m^3/min. What is the mean velocity of the air?

$$v = Q/A = 160/[(0.50)(0.50)] = 640 \text{ m/min} \quad \text{or} \quad 10.7 \text{ m/s}$$

8.4 Assume the conduit shown in Fig. 8-1 has (inside) diameters of 12 in and 18 in at sections 1 and 2, respectively. If water is flowing in the conduit at a velocity of 16.6 ft/s at section 2, find the (*a*) velocity at section 1, (*b*) volume flow rate at section 1, (*c*) volume flow rate at section 2, (*d*) weight flow rate, and (*e*) mass flow rate.

(*a*) $$A_1v_1 = A_2v_2 \qquad [(\pi)(\tfrac{12}{12})^2/4](v_1) = [(\pi)(\tfrac{18}{12})^2/4](16.6) \qquad v_1 = 37.3 \text{ ft/s}$$

(*b*) $$Q_1 = A_1v_1 = [(\pi)(\tfrac{12}{12})^2/4](37.3) = 29.3 \text{ ft}^3/\text{s}$$

(*c*) $Q_2 = A_2v_2 = [(\pi)(\tfrac{18}{12})^2/4](16.6) = 29.3 \text{ ft}^3/\text{s}$. (Since the flow in incompressible, the flow rate is the same at sections 1 and 2.)

(*d*) $$W = \gamma A_1v_1 = 62.4[(\pi)(\tfrac{12}{12})^2/4](37.3) = 1828 \text{ lb/s}$$

(*e*) $$M = \rho A_1v_1 = 1.94[(\pi)(\tfrac{12}{12})^2/4](37.3) = 56.8 \text{ slugs/s}$$

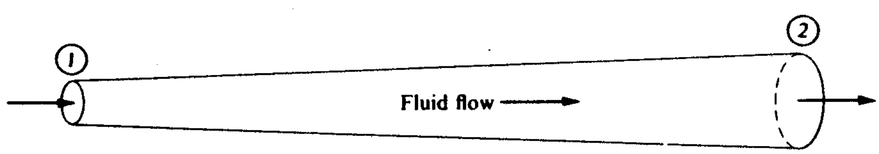


Fig. 8-1

8.5 A gas flows through a square conduit. At one point along the conduit, the conduit sides are 0.100 m, the velocity is 7.55 m/s, and the gas's mass density is (for its particular pressure and temperature) 1.09 kg/m^3. At a second point, the conduit sides are 0.250 m, and the velocity is 2.02 m/s. Find the mass flow rate of the gas and its mass density at the second point.

$$M = \rho_1A_1v_1 = 1.09[(0.100)(0.100)](7.55) = 0.0823 \text{ kg/s} \qquad \rho_1A_1v_1 = \rho_2A_2v_2$$

$$1.09[(0.100)(0.100)](7.55) = (\rho_2)[(0.250)(0.250)](2.02) \qquad \rho_2 = 0.652 \text{ kg/m}^3$$

8.6 Water enters the mixing device shown in Fig. 8-2 at 150 L/s through pipe *A*, while oil with specific gravity 0.8 is forced in at 30 L/s through pipe *B*. If the liquids are incompressible and form a homogeneous mixture of oil globules in water, find the average velocity and density of the mixture leaving through the 30-cm-diameter pipe *C*.

$$M = \rho Av = \rho Q \qquad \sum (\text{mass flow in unit time})_{\text{in}} = \sum (\text{mass flow in unit time})_{\text{out}}$$

$$(1000)(0.15) + [(0.8)(1000)](0.03) = (\rho)[(\pi)(0.30)^2/4](v) \qquad \rho v = 2462 \text{ kg/m}^2 \cdot \text{s}$$

We can assume no chemical reaction between oil and water and its mixture is incompressible; it is clear that volume is conserved. Hence, $Q = 0.15 + 0.03 = 0.18\ \mathrm{m^3/s}$; $Q = Av$, $0.18 = [(\pi)(0.30)^2/4](v)$, $v_C = 2.55\ \mathrm{m/s}$; $\rho_C = 2462/2.55 = 965\ \mathrm{kg/m^3}$.

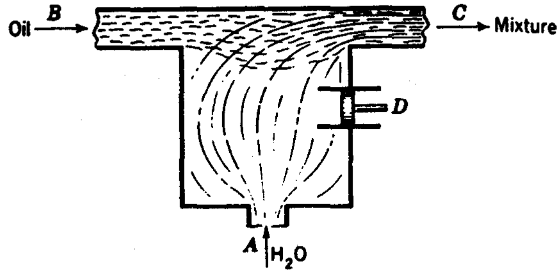


Fig. 8-2

8.7 Water flows into a cylindrical tank (Fig. 8-3) through pipe 1 at the rate of 25 ft/s and leaves through pipes 2 and 3 at 10 ft/s and 12 ft/s, respectively. At 4 is an open air vent. Inside pipe diameters are: $D_1 = 3$ in, $D_2 = 2$ in, $D_3 = 2.5$ in, $D_4 = 2$ in. Calculate **(a)** dh/dt; **(b)** the average velocity of airflow through vent 4, assuming that the flow is incompressible.

▌ **(a)** With the entire volume of the tank as control volume,

$$M = \rho A v = \rho Q \qquad \sum (\text{mass flow in unit time})_{in} = \sum (\text{mass flow in unit time})_{out}$$

$$(\rho)[(\pi)(\tfrac{3}{12})^2/4](25) = (\rho)[(\pi)(\tfrac{2}{12})^2/4](10) + (\rho)[(\pi)(2.5/12)^2/4](12) + (\rho)[(\pi)(2)^2/4](dh/dt)$$

$$dh/dt = 0.1910\ \mathrm{ft/s}$$

(b) Consider only air in the control volume. It must be conserved. Hence, $(\rho_{air})[(\pi)(\tfrac{2}{12})^2/4](v) = (\rho_{air})[(\pi)(2)^2/4](0.1484)$, $v = 21.4\ \mathrm{ft/s}$.

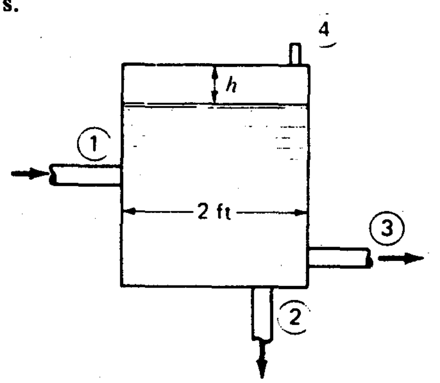


Fig. 8-3

8.8 The piston of a hypodermic apparatus (Fig. 8-4) is being withdrawn at 0.30 in/s; air leaks in around the piston at the rate $0.0012\ \mathrm{in^3/s}$. What is the average speed of blood flow in the needle?

▌ Choose as a control volume the region between the piston and the tip of the needle.

$$M = \rho A v = \rho Q \qquad \sum (\text{mass flow in unit time})_{in} = \sum (\text{mass flow in unit time})_{out}$$

$$(\rho_{blood})[(\pi)(0.02/12)^2/4](v) + (\rho_{blood})(0.0012/1728) = (\rho_{blood})[(\pi)(0.2/12)^2/4](0.30/12) \qquad v = 2.18\ \mathrm{ft/s}$$

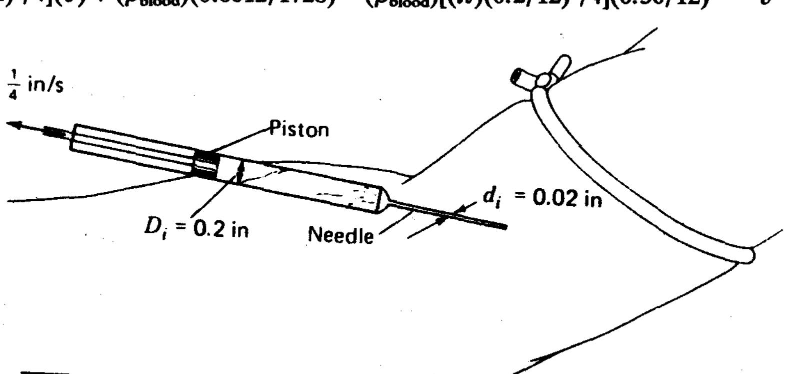


Fig. 8-4

8.9 Air at 30 °C and 110 kPa flows at 16 N/s through a rectangular duct that measures 160 mm by 320 mm. Compute the average velocity and volume flux.

$$W = \gamma A v \qquad \gamma = p/RT = (110)(1000)/[(29.3)(30+273)] = 12.39\ \text{N/m}^3$$
$$16 = 12.39[(0.160)(0.320)](v) \qquad v = 25.2\ \text{m/s} \qquad Q = Av = [(0.160)(0.320)](25.2) = 1.29\ \text{m}^3/\text{s}$$

8.10 Oil (s.g. = 0.86) flows through a 30-in-diameter pipeline at 8000 gpm. Compute the **(*a*)** volume flux, **(*b*)** average velocity, and **(*c*)** mass flux.

(*a*)
$$Q = 8000/[(7.48)(60)] = 17.8\ \text{ft}^3/\text{s}$$

(*b*)
$$Q = Av \qquad 17.8 = [(\pi)(\tfrac{30}{12})^2/4](v) \qquad v = 3.63\ \text{ft/s}$$

(*c*)
$$M = \rho A v = [(0.86)(1.94)][(\pi)(\tfrac{30}{12})^2/4](3.63) = 29.7\ \text{slugs/s}$$

8.11 In the rectilinear chamber of Fig. 8-5, section 1 has a diameter of 4 in and the flow in is 2 cfs. Section 2 has a diameter of 3 in and the flow out is 36 fps average velocity. Compute the average velocity and volume flux at section 3 if $D_3 = 1$ in. Is the flow at 3 in or out?

$$Q_1 = Q_2 + Q_3 \qquad \text{(assuming } Q_3 \text{ is out)}$$
$$2 = [(\pi)(\tfrac{3}{12})^2/4](36) + Q_3 \qquad Q_3 = 0.233\ \text{cfs} \qquad \text{(out)} \qquad v = Q/A = 0.233/[(\pi)(\tfrac{1}{12})^2/4] = 42.7\ \text{fps}$$

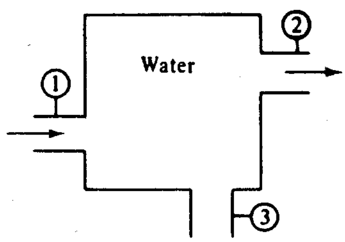


Fig. 8-5

8.12 The water tank in Fig. 8-6 is being filled through section 1 at $v_1 = 5$ m/s and through section 3 at $Q_3 = 0.012\ \text{m}^3/\text{s}$. If water level h is constant, determine exit velocity v_2.

$$Q_1 + Q_3 = Q_2 \qquad [(\pi)(0.040)^2/4](5) + 0.012 = Q_2$$
$$Q_2 = 0.01828\ \text{m}^3/\text{s} \qquad v_2 = Q_2/A_2 = 0.01828/[(\pi)(0.060)^2/4] = 6.47\ \text{m/s}$$

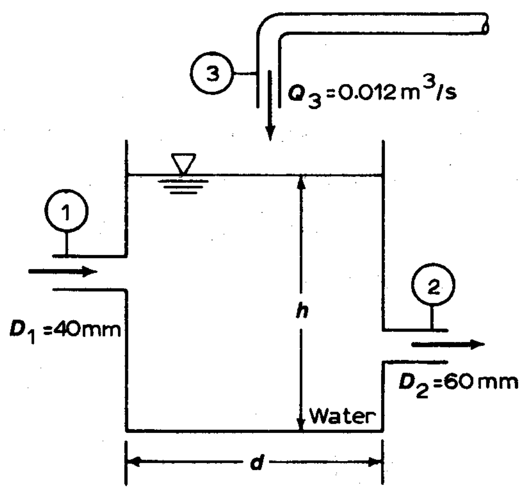


Fig. 8-6

8.13 If the water level varies in Prob. 8.12 and $v_2 = 8$ m/s, find rate of change dh/dt. Assume $d = 1.0$ m.

$$Q_1 + Q_3 = Q_2 + \frac{d}{dt}\frac{h\pi d^2}{4} \qquad [(\pi)(\tfrac{4}{100})^2/4](5) + 0.012 = [(\pi)(\tfrac{6}{100})^2/4](8) + \frac{dh}{dt}[(\pi)(1.0)^2/4]$$
$$dh/dt = -5.52\ \text{mm/s} \qquad \text{(i.e., falling)}$$

8.14 For the general case of the flow depicted in Fig. 8-6, derive an expression for dh/dt in terms of tank size and volume flows Q_1, Q_2, and Q_3 at the three ports.

▮ $$Q_1 + Q_3 = Q_2 + \frac{d}{dt}\frac{h\pi d^2}{4} \qquad \frac{dh}{dt} = \frac{4(Q_1 - Q_2 + Q_3)}{\pi d^2}$$

8.15 Water at 20 °C flows steadily through the nozzle in Fig. 8-7 at 60 kg/s. The diameters are $D_1 = 220$ mm and $D_2 = 80$ mm. Compute the average velocities at sections 1 and 2.

▮ $$Q = M/\rho = \tfrac{60}{998} = 0.0601 \text{ m}^3/\text{s}$$

$$v_1 = Q/A_1 = 0.0601/[(\pi)(0.220)^2/4] = 1.58 \text{ m/s} \qquad v_2 = Q/A_2 = 0.0601/[(\pi)(0.080)^2/4] = 12.0 \text{ m/s}$$

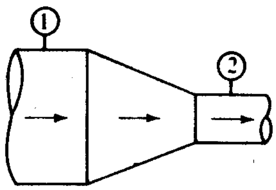


Fig. 8-7

8.16 The inseminator in Fig. 8-8 contains fluid of s.g. = 1.04. If the plunger is pushed in steadily at 1.0 in/s, what is exit velocity V_2? Assume no leakage past the plunger.

▮ $$\gamma_1 A_1 V_1 = \gamma_2 A_2 V_2$$

$$[(1.04)(62.4)][(\pi)(0.80/12)^2/4](1.0/12) = [(1.04)(62.4)][(\pi)(0.04/12)^2/4](V_2)$$

$$V_2 = 33.3 \text{ ft/s}$$

(Note that the answer is independent of the fluid's specific gravity.)

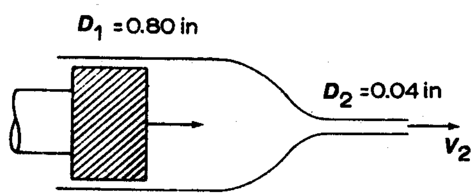


Fig. 8-8

8.17 Repeat Prob. 8.16 assuming there is leakage back past the plunger equal to 1/4 of the volume flux out of the needle. Compute V_2 and the average leakage velocity relative to the needle walls if the plunger diameter is 0.796 in.

▮ $$Q = Q_1 = A_1 V_1 = [(\pi)(0.750/12)^2/4](1.0/12) = 0.000256 \text{ ft}^3/\text{s}$$

$$Q_2 = (\tfrac{3}{4})(0.000256) = 0.000192 \text{ ft}^3/\text{s}$$

$$V_2 = Q_2/A_2 = 0.000192/[(\pi)(0.040/12)^2/4] = 22.0 \text{ ft/s}$$

$$Q_{\text{leak}} = (\tfrac{1}{4})(0.000256) = 0.000064 \text{ ft}^3/\text{s}$$

$$V_{\text{leak}} = Q_{\text{leak}}/A_{\text{leak}} = 0.000064/[(\pi)(0.80/12)^2/4 - (\pi)(0.796/12)^2/4] = 1.84 \text{ ft/s}$$

8.18 A 100-mm-diameter plunger (1) is being pushed at 60 mm/s into a tank filled with a fluid of s.g. = 0.68. If the fluid is incompressible, how many pounds per second is being forced out at section 2, $D_2 = 20$ mm?

▮ $$A_1 v_1 = A_2 v_2 \qquad [(\pi)(0.100)^2/4](0.060) = [(\pi)(0.020)^2/4](v_2) \qquad v_2 = 1.500 \text{ m/s}$$

$$W = \gamma A v = [(0.68)(9.79)][(\pi)(0.020)^2/4](1.500) = 0.003137 \text{ kN/s} \quad \text{or} \quad 3.137 \text{ N/s}$$

$$= 3.137/4.448 = 0.705 \text{ lb/s}$$

8.19 A gasoline pump fills a 80-L tank in 1 min 15 s. If the pump exit diameter is 4 cm, what is the average pump-flow exit velocity?

▮ $$Q = V/t = (\tfrac{80}{1000})/(60 + 15) = 0.001067 \text{ m}^3/\text{s} \qquad v = Q/A = 0.001067/[(\pi)(0.04)^2/4] = 0.85 \text{ m/s}$$

8.20 The tank in Fig. 8-9 is admitting water at 100 N/s and ejecting gasoline (s.g. = 0.69) at 52 N/s. If all three fluids are incompressible, how much air is passing through the vent? In which direction?

$$Q_1 = Q_2 + Q_3 \qquad \text{(assuming airflow is out)}$$

$$Q_1 = W_1/\gamma_{H_2O} = 100/[(9.79)(1000)] = 0.01021\ \text{m}^3/\text{s}$$

$$Q_2 = W_2/\gamma_{gas} = 52/[(0.69)(9.79)(1000)] = 0.007698\ \text{m}^3/\text{s}$$

$$0.01021 = 0.007698 + Q_3 \qquad Q_3 = 0.002512\ \text{m}^3/\text{s} \qquad \text{(out)}$$

$$\gamma_{air} = p/RT = (1)(101.3)/[(29.3)(20+273)] = 0.01180\ \text{kN/m}^3$$

$$W_3 = (\gamma_{air})(Q_3) = (0.01180)(0.002512) = 0.00002964\ \text{kN/s} \quad \text{or} \quad 0.0296\ \text{N/s}$$

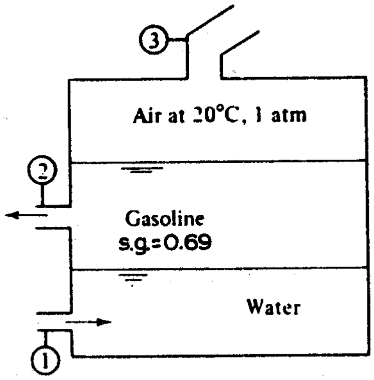


Fig. 8-9

8.21 Air at 72 °F and 16 psia enters a chamber at section 1 at velocity 210 fps and leaves section 2 at 1208 °F and 202 psia. What is the exit velocity if $D_1 = 8$ in and $D_2 = 3$ in? Assume the flow is steady.

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2 \qquad \rho = p/RT \qquad \rho_1 = (16)(144)/[(1716)(460+72)] = 0.002524\ \text{slug/ft}^3$$

$$\rho_2 = (202)(144)/[(1716)(460+1208)] = 0.01016\ \text{slug/ft}^3$$

$$0.002524[(\pi)(\tfrac{8}{12})^2/4](210) = 0.01016[(\pi)(\tfrac{3}{12})^2/4](v_2) \qquad v_2 = 371\ \text{fps}$$

8.22 Kerosene (s.g. = 0.88) enters the cylindrical arrangement of Fig. 8-10 at section 1, at 0.08 N/s. The 80-mm-diameter plates are 2 mm apart. Assuming steady flow, compute the inlet average velocity v_1, outlet average velocity v_2 assuming radial flow, and outlet volume flux.

$$W_1 = \gamma_{oil} A_1 v_1 \qquad 0.08 = [(0.88)(9.79)(1000)][(\pi)(0.004)^2/4](v_1) \qquad v_1 = 0.739\ \text{m/s}$$

$$Q_1 = A_1 v_1 = [(\pi)(0.004)^2/4](0.739) = 0.00000929\ \text{m}^3/\text{s} \qquad Q_2 = Q_1 = 0.00000929\ \text{m}^3/\text{s} \quad \text{or} \quad 9.29\ \text{mL/s}$$

$$v_2 = Q_2/A_2 = 0.00000929/[(\pi)(0.080)(0.003)] = 0.0123\ \text{m/s} \quad \text{or} \quad 12.3\ \text{mm/s}$$

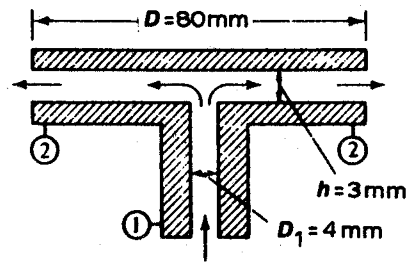


Fig. 8-10

8.23 In Fig. 8-11, pipes 1 and 2 are of diameter 3 cm; $D_3 = 4$ cm. Alcohol (s.g. = 0.80) enters section 1 at 6 m/s while water enters section 2 at 10 m/s. Assuming ideal mixing of incompressible fluids, compute the exit velocity and density of the mixture at section 3. The temperature is 20 °C.

$$Q_1 + Q_2 = Q_3 \qquad [(\pi)(0.03)^2/4](6) + [(\pi)(0.03)^2/4](10) = Q_3 \qquad Q_3 = 0.01131\ \text{m}^3/\text{s}$$

$$v_3 = Q_3/A_3 = 0.01131/[(\pi)(0.04)^2/4] = 9.00\ \text{m/s} \qquad M_1 + M_2 = M_3$$

$$\rho_{alcohol} A_1 v_1 + \rho_{H_2O} A_2 v_2 = \rho_{mixture} A_3 v_3$$

$$[(0.80)(998)][(\pi)(0.03)^2/4](6) + 998[(\pi)(0.03)^2/4](10) = (\rho_{mixture})[(\pi)(0.04)^2/4](9.00) \qquad \rho_{mixture} = 923\ \text{kg/m}^3$$

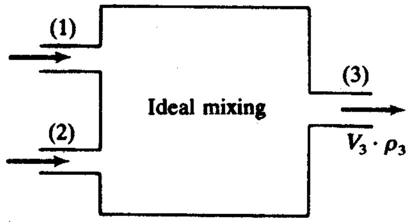


Fig. 8-11

8.24 In the wind tunnel of Fig. 8-12, the test-section wall is porous; fluid is sucked out to provide a thin viscous boundary layer. The wall contains 800 holes of 7-mm diameter per square meter of area. The suction velocity out each hole is $V_s = 10$ m/s, and the test section entrance velocity is $V_1 = 46$ m/s. Assuming incompressible flow of air at 20 °C and 1 atm, compute **(a)** V_0, **(b)** the total wall suction volume flow, **(c)** V_2, and **(d)** V_f.

(a) $A_0V_0 = A_1V_1 \qquad [(\pi)(2.6)^2/4](V_0) = [(\pi)(0.9)^2/4](45) \qquad V_0 = 5.4\text{ m/s}$

(b) $Q_{\text{suction}} = N_{\text{holes}}Q_{\text{hole}} \qquad N_{\text{holes}} = 800[(\pi)(0.9)(4)] = 9048$

$$Q_{\text{hole}} = A_{\text{hole}}V_{\text{hole}} = [(\pi)(\tfrac{7}{1000})^2/4](10) = 0.0003848\text{ m}^3/\text{s}$$

$$Q_{\text{suction}} = (9048)(0.0003848) = 3.48\text{ m}^3/\text{s}$$

(c) $Q_1 = Q_2 + Q_{\text{suction}} \qquad [(\pi)(0.9)^2/4](46) = Q_2 + 3.48 \qquad Q_2 = 25.78\text{ m}^3/\text{s}$

$$v_2 = Q_2/A_2 = 25.78/[(\pi)(0.9)^2/4] = 40.5\text{ m/s}$$

(d) $A_fV_f = A_2V_2 \qquad [(\pi)(2.4)^2/4](V_f) = [(\pi)(0.9)^2/4](40.5) \qquad V_f = 5.70\text{ m/s}$

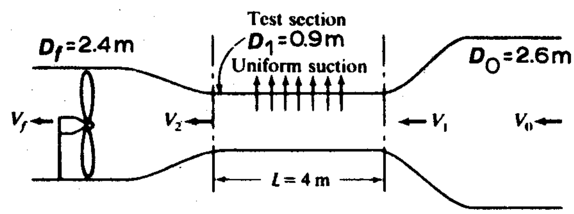


Fig. 8-12

8.25 A rocket motor is operating steadily as shown in Fig. 8-13. The exhaust products may be considered an ideal gas of molecular weight 26. Calculate v_2.

$$M_2 = M_1 + M_3 = 0.7 + 0.1 = 0.8\text{ slug/s} = \rho_2A_2v_2 \qquad R = 49\,709/26 = 1912\text{ lb}\cdot\text{ft/(slug}\cdot{}^\circ\text{R)}$$

$$\rho_2 = p/RT = (16)(144)/[(1912)(1105 + 460)] = 0.000770\text{ slug/ft}^3$$

$$0.8 = 0.000770[(\pi)(6.0/12)^2/4](v_2) \qquad v_2 = 5291\text{ ft/s}$$

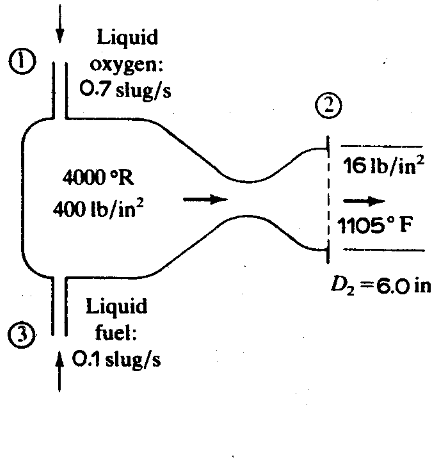


Fig. 8-13

8.26 For the solid-propellant rocket in Fig. 8-14, compute the rate of mass loss of the propellant, assuming the exit gas has a molecular weight of 30.

▌ $$M_{in} = M_{out} + \frac{d}{dt}(m_{propellant}) \qquad 0 = M_{out} + \frac{d}{dt}(m_{propellant})$$

$$\frac{d}{dt}(m_{propellant}) = -M_{out} = -\rho_e A_e V_e$$

$$R = \tfrac{8312}{30} = 277\ \text{N}\cdot\text{m}/(\text{kg}\cdot\text{K}) \qquad \rho_e = p_e/RT_e = (105)(1000)/[(277)(800)] = 0.4738\ \text{kg/m}^3$$

$$\frac{d}{dt}(m_{propellant}) = -(0.4738)\left[\frac{(\pi)(0.200)^2}{4}\right](1100) = -16.4\ \text{kg/s}$$

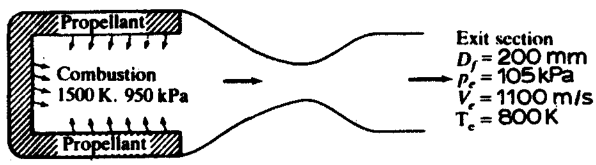


Fig. 8-14

8.27 The water-jet pump in Fig. 8-15 injects water at $U_1 = 80$ ft/s through a 4-in pipe which is surrounded by a secondary flow of water at $U_2 = 8$ ft/s. The two flows become fully mixed downstream, where U_3 is approximately constant. If the flow is steady and incompressible, compute U_3.

▌ $Q_1 + Q_2 = Q_3 \qquad [(\pi)(\frac{4}{12})^2/4](80) + \{(\pi)[(\frac{12}{12})^2 - (\frac{4}{12})^2]/4\}(8) = [(\pi)(\frac{12}{12})^2/4](U_3) \qquad U_3 = 16.0$ ft/s

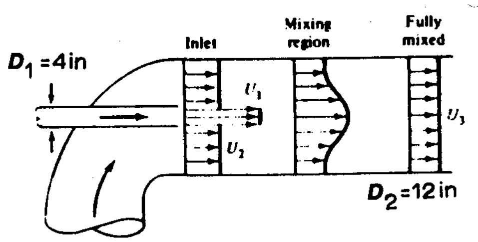


Fig. 8-15

8.28 The flow in the inlet between parallel plates in Fig. 8-16 is uniform at $U_0 = 50$ mm/s, while downstream the flow develops into the parabolic laminar profile $u = az(z_0 - z)$, where a is a constant. If $z_0 = 20$ mm, compute v_{max}.

▌ Let b = width of plates (into paper).

$$Q_{in} = Q_{out} \qquad z_0 b U_0 = \int u\, dA$$

$$z_0 b U_0 = \int_0^{z_0} az(z_0 - z)b\, dz = ab\left[\frac{z_0 z^2}{2} - \frac{z^3}{3}\right]_0^{z_0} = ab\left(\frac{z_0 z_0^2}{2} - \frac{z_0^3}{3}\right) = \frac{abz_0^3}{6}$$

$$a = 6U_0/z_0^2 \qquad u = az(z_0 - z) = (6U_0/z_0^2)(z)(z_0 - z)$$

u_{max} occurs at $z = z_0/2 = 0.020/2 = 0.010$ m: $u_{max} = [(6)(0.050)/(0.020)^2](0.01)(0.020 - 0.010) = 0.0750$ m/s or 75.0 mm/s.

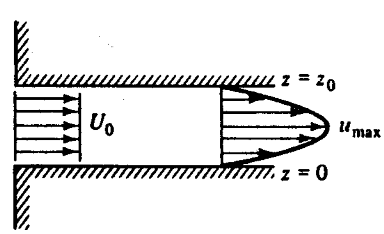


Fig. 8-16

8.29 Assuming the container in Fig. 8-17*a* is large and losses are negligible, derive an expression for the distance X where the free jet leaving horizontally will strike the floor, as a function of h and H. Sketch the three trajectories for $h/H = 0.25$, 0.50, and 0.75.

$$v_0 = \sqrt{2g(H-h)} \qquad h = gt^2/2 \qquad t = \sqrt{2h/g} \qquad X = v_0 t = \sqrt{2g(H-h)}\,\sqrt{2h/g} = 2\sqrt{h(H-h)}$$

For $h/H = 0.25$, or $h = 0.25H$, $X = 2\sqrt{(0.25H)(H-0.25H)} = 0.866H$. For $h/H = 0.50$, or $h = 0.50H$, $X = 2\sqrt{(0.50H)(H-0.50H)} = H$. For $h/H = 0.75$, or $h = 0.75H$, $X = 2\sqrt{(0.75H)(H-0.75)} = 0.866H$. These three trajectories are sketched in Fig. 8-17*b*.

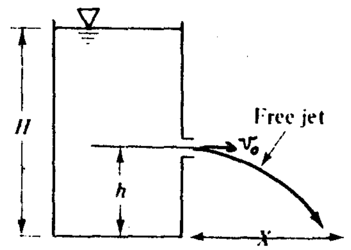


Fig. 8-17(*a*)

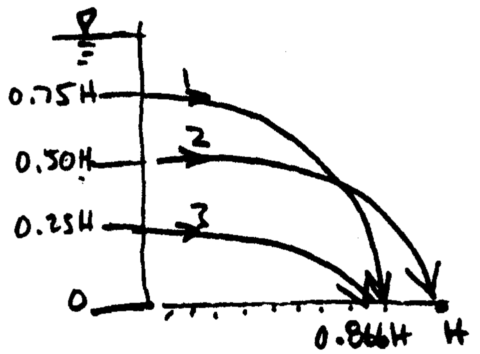


Fig. 8-17(*b*)

8.30 In Fig. 8-18 what should the water level h be for the free jet just to clear the wall?

$$v_0 = \sqrt{2gh} \qquad \text{Fall distance} = gt^2/2 = 0.40 \qquad t = 0.8944/\sqrt{g}$$

$$\text{Horizontal distance} = v_0 t = (\sqrt{2gh})(0.8944/\sqrt{g}) = 0.50 \qquad h = 0.156\text{ m} = 15.6\text{ cm}$$

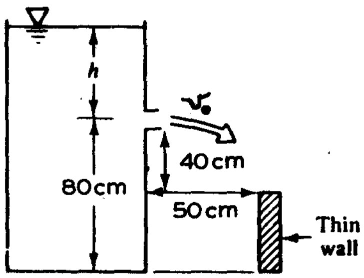


Fig. 8-18

8.31 When 500 gpm flows through a 12-in pipe which later reduces to a 6-in pipe, calculate the average velocities in the two pipes.

$$Q = 500/[(7.48)(60)] = 1.114\text{ ft}^3/\text{s}$$

$$v_{12} = Q/A_{12} = 1.114/[(\pi)(\tfrac{12}{12})^2/4] = 1.42\text{ ft/s} \qquad v_6 = Q/A_6 = 1.114/[(\pi)(\tfrac{6}{12})^2/4] = 5.67\text{ ft/s}$$

8.32 If the velocity in a 12-in pipe is 1.65 ft/s, what is the velocity in a 3-in-diameter jet issuing from a nozzle attached to the pipe?

$$A_1v_1 = A_2v_2 \qquad [(\pi)(\tfrac{12}{12})^2/4](1.65) = [(\pi)(\tfrac{3}{12})^2/4](v_2) \qquad v_2 = 26.4\text{ ft/s}$$

8.33 Air flows in a 6-in pipe at a pressure of 30.0 psig and a temperature of 100 °F. If barometric pressure is 14.7 psia and velocity is 10.5 ft/s, how many pounds of air per second are flowing?

$$\gamma = p/RT = (30.0 + 14.7)(144)/[(53.3)(100+460)] = 0.2157\text{ lb/ft}^3$$

$$W = \gamma A v = 0.2157[(\pi)(\tfrac{6}{12})^2/4](10.5) = 0.445\text{ lb/s}$$

8.34 Carbon dioxide passes point A in a 3-in pipe at a velocity of 15.0 ft/s. The pressure at A is 30 psig and the temperature is 70 °F. At point B downstream, the pressure is 20 psig and the temperature is 90 °F. For a barometric pressure reading of 14.7 psia, calculate the velocity at B and compare the flows at A and B.

$$\gamma = p/RT \qquad \gamma_A = (30 + 14.7)(144)/[(35.1)(70+460)] = 0.3460\text{ lb/ft}^3$$

$$\gamma_B = (20 + 14.7)(144)/[(35.1)(90+460)] = 0.2588\text{ lb/ft}^3 \qquad \gamma_A A_A v_A = \gamma_B A_B v_B$$

Since $A_A = A_B$, $(0.3460)(15.0) = (0.2588)(v_B)$, $v_B = 20.1$ ft/s. The number of pounds per second flowing is constant, but the flow in cubic feet per second will differ because the specific weight is not constant. $Q_A = A_A v_A = [(\pi)(\frac{3}{12})^2/4](15.0) = 0.736$ ft³/s; $Q_B = A_B v_B = [(\pi)(\frac{3}{12})^2/4](20.1) = 0.987$ ft³/s.

8.35 What minimum diameter of pipe is necessary to carry 0.500 lb/s of air with a maximum velocity of 18.5 ft/s? The air is at 80 °F and under an absolute pressure of 34.0 psi.

$$W = \gamma A v \qquad \gamma_{air} = p/RT = (34.0)(144)/[(53.3)(80+460)] = 0.170 \text{ lb/ft}^3$$

$$0.500 = (0.170)[(\pi)(d)^2/4](18.5) \qquad d = 0.450 \text{ ft} \quad \text{or} \quad 5.40 \text{ in}$$

8.36 In the laminar flow of a fluid in a circular pipe, the velocity profile is exactly a true parabola. The rate of discharge is then represented by the volume of a paraboloid. Prove that for this case the ratio of the mean velocity to the maximum velocity is 0.5.

See Fig. 8-19. For a paraboloid, $u = u_{max}[1 - (r/r_0)^2]$.

$$Q = \int u\,dA = \int_0^{r_0} u_{max}\left[1 - \left(\frac{r}{r_0}\right)^2\right](2\pi r\,dr) = 2\pi u_{max}\left[\frac{r^2}{2} - \frac{r^4}{4r_0^2}\right]_0^{r_0} = 2\pi u_{max}\left[\frac{r_0^2}{2} - \frac{r_0^4}{4r_0^2}\right] = u_{max}\left(\frac{\pi r_0^2}{2}\right)$$

$$V_{mean} = Q/A = u_{max}(\pi r_0^2/2)/(\pi r_0^2) = u_{max}/2. \text{ Thus } V_{mean}/u_{max} = 0.5.$$

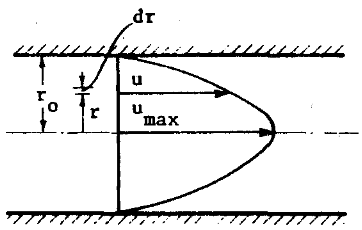


Fig. 8-19

8.37 A gas ($\gamma = 0.05$ lb/ft³) flows at the rate of 1.6 lb/s past section A through a long rectangular duct of uniform cross section 2 ft by 2 ft. At section B, the gas weighs 0.060 lb/ft³. Find the average velocities of flow at sections A and B.

$W = \gamma A v \qquad 1.6 = (0.05)[(2)(2)](v_A) \qquad v_A = 8.00$ ft/s $\qquad 1.6 = (0.060)[(2)(2)](v_B) \qquad v_B = 6.67$ ft/s

8.38 The velocity of a liquid (s.g. = 1.26) in a 6-in pipeline is 1.6 ft/s. Calculate the flow in: (*a*) ft³/s, (*b*) slug/s.

(*a*) $$Q = Av = [(\pi)(\frac{6}{12})^2/4](1.6) = 0.314 \text{ ft}^3/\text{s}$$

(*b*) $$M = \rho A v = [(1.26)(1.94)][(\pi)(\frac{6}{12})^2/4](1.6) = 0.768 \text{ slug/s}$$

8.39 Oxygen flows in a 3-in by 3-in duct at a pressure of 42 psi and a temperature of 105 °F. If atmospheric pressure is 13.4 psia and the velocity of flow is 18 fps, calculate the weight-flow rate.

$$\gamma = p/RT = (42 + 13.4)(144)/[(48.2)(460+105)] = 0.2929 \text{ lb/ft}^3$$

$$W = \gamma A v = (0.2929)[(\tfrac{3}{12})(\tfrac{3}{12})](18) = 0.330 \text{ lb/s}$$

8.40 Air at 42 °C and at 3 bar absolute pressure flows in a 200-mm-diameter conduit at a mean velocity of 12 m/s. Find the mass flow rate.

$\rho = p/RT = 3 \times 10^5/[(287)(273+42)] = 3.318 \text{ kg/m}^3 \qquad M = \rho A v = 3.318[(\pi)(0.200)^2/4](12) = 1.25 \text{ kg/s}$

8.41 A 120-mm-diameter pipe enlarges to a 180-mm-diameter pipe. At section 1 of the smaller pipe, the density of a gas in steady flow is 200 kg/m³ and the velocity is 20 m/s; at section 2 of the larger pipe the velocity is 14 m/s. Find the density of the gas at section 2.

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2 \qquad 200[(\pi)(0.120)^2/4](20) = (\rho_2)[(\pi)(0.180)^2/4](14) \qquad \rho_2 = 127 \text{ kg/m}^3$$

8.42 The Peconic River is spanned by Noyack Bridge and Smith's Bridge. At noon on July 4, the measured flows under the two bridges were $Q_N = 50\ \mathrm{m^3/s}$ and $Q_S = 40\ \mathrm{m^3/s}$. Neglecting losses, compute the instantaneous rate of water storage between the two bridges.

$$Q_N - Q_S = dS/dt \qquad 50 - 40 = dS/dt \qquad dS/dt = 10\ \mathrm{m^3/s}$$

8.43 A worker in a children's playground is cleaning a slide with a hose. She observes that a horizontal stream directed into the low end climbs to a point 12 ft above the nozzle. What is the nozzle velocity of the stream?

$$v = \sqrt{2gh} = \sqrt{(2)(32.2)(12)} = 27.8\ \mathrm{ft/s}$$

8.44 At section 1 of a pipe system carrying water the velocity is 3.0 fps and the diameter is 2.0 ft. At section 2 the diameter is 3.0 ft. Find the discharge and velocity at section 2.

$$Q_1 = Q_2 = Av = [(\pi)(2.0)^2/4](3.0) = 9.42\ \mathrm{cfs} \qquad v_2 = Q_2/A_2 = 9.42/[(\pi)(3.0)^2/4] = 1.33\ \mathrm{fps}$$

8.45 In two-dimensional flow around a circular cylinder (Fig. 8-20), the discharge between streamlines is $34.56\ \mathrm{in^3/s}$ per foot of depth. At a great distance the streamlines are 0.25 in. apart, and at a point near the cylinder they are 0.12 in. apart. Calculate the magnitudes of the velocity at these two points.

$v = (Q/d)/w$. At great distance, $v = (34.56/12)/0.25 = 11.52\ \mathrm{in/s}$. Near the cylinder, $v = (34.56/12)/0.12 = 24.0\ \mathrm{in/s}$.

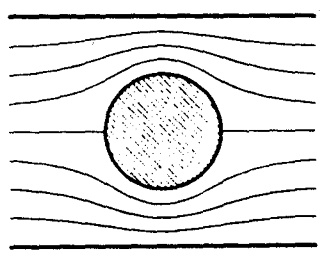


Fig. 8-20

8.46 A pipeline carries oil (s.g. = 0.86) at $v = 2\ \mathrm{m/s}$ through a 20-cm-ID pipe. At another section the diameter is 8 cm. Find the velocity at this section and the mass flow rate.

$$Q = A_1v_1 = [(\pi)(0.20)^2/4](2) = 0.06283\ \mathrm{m^3/s} \qquad v_2 = Q/A_2 = 0.06283/[(\pi)(0.08)^2/4] = 12.5\ \mathrm{m/s}$$

$$M = \rho Av = [(0.86)(1000)][(\pi)(0.20)^2/4](2) = 54.0\ \mathrm{kg/s}$$

8.47 Hydrogen is flowing in a 3.0-in-diameter pipe at the steady rate of 0.03 lbm/s. Calculate the average velocity over a section where the pressure is 30 psia and the temperature is 80 °F.

$$M = \rho Av \qquad \rho = p/RT = (30)(144)/[(765.5)(460 + 80)] = 0.01045\ \mathrm{lbm/ft^3}$$

$$0.03 = (0.01045)[(\pi)(3.0/12)^2/4](v) \qquad v = 58.5\ \mathrm{ft/s}$$

8.48 If a jet is inclined upward 30° from the horizontal, what must be its velocity to reach over a 10-ft wall at a horizontal distance of 60 ft, neglecting friction?

$(v_x)_0 = v_0 \cos 30° = 0.8660v_0$, $(v_z)_0 = v_0 \sin 30° = 0.5000v_0$. From Newton's laws, $x = (0.8660v_0)(t) = 60$, $z = 0.5000v_0t - 32.2t^2/2 = 10$. From the first equation, $t = 69.28/v_0$. Substituting this into the second equation, $(0.5000)(v_0)(69.28/v_0) - (32.2)(69.28/v_0)^2/2 = 10$, $v_0 = 56.0$ fps.

8.49 Water flows at $10\ \mathrm{m^3/s}$ in a 150-cm-diameter pipe; the head loss in a 1000-m length of this pipe is 20 m. Find the rate of energy loss due to pipe friction.

$$\text{Rate of energy loss} = \gamma QH = (9.79)(10)(20) = 1958\ \mathrm{kW}$$

8.50 Oil with specific gravity 0.750 is flowing through a 6-in pipe under a pressure of 15.0 psi. If the total energy relative to a datum plane 8.00 ft below the center of the pipe is 58.6 ft · lb/lb, determine the flow rate of the oil.

$$H = z + v^2/2g + p/\gamma \qquad 58.6 = 8.00 + v^2/[(2)(32.2)] + (15)(144)/[(0.750)(62.4)]$$

$$v = 16.92\ \mathrm{ft/s} \qquad Q = Av = [(\pi)(\tfrac{6}{12})^2/4](16.92) = 3.32\ \mathrm{ft^3/s}$$

8.51 In Fig. 8-21, water flows from A, where the diameter is 12 in, to B, where the diameter is 24 in, at the rate of 13.2 cfs. The pressure head at A is 22.1 ft. Considering no loss of energy from A to B, find the pressure head at B.

$$p_A/\gamma + v_A^2/2g + z_A = p_B/\gamma + v_B^2/2g + z_B \qquad v_A = Q/A_A = 13.2/[(\pi)(\tfrac{12}{12})^2/4] = 16.81 \text{ ft/s}$$

$$v_B = Q/A_B = 13.2/[(\pi)(\tfrac{24}{12})^2/4] = 4.202 \text{ ft/s}$$

$$22.1 + 16.81^2/[(2)(32.2)] + 0 = p_B/\gamma + 4.202^2/[(2)(32.2)] + (25.0 - 10.0) \qquad p_B/\gamma = 11.2 \text{ ft of water}$$

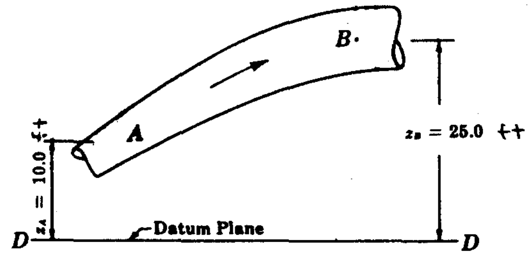


Fig. 8-21

8.52 A pipe carrying oil with specific gravity 0.877 changes in size from 6 in at section E to 18 in at section R. Section E is 12 ft lower than R, and the pressures are 13.2 psi and 8.75 psi, respectively. If the discharge is 5.17 cfs, determine the lost head and the direction of flow.

$H = z + v^2/2g + p/\gamma$. Use the lower section (E) as the datum plane. $v_E = Q/A_E = 5.17/[(\pi)(\tfrac{6}{12})^2/4] = 26.33$ ft/s, $v_R = Q/A_R = 5.17/[(\pi)(\tfrac{18}{12})^2/4] = 2.926$ ft/s; $H_E = 0 + 26.33^2/[(2)(32.2)] + (13.2)(144)/[(0.877)(62.4)] = 45.50$ ft, $H_R = 12 + 2.926^2/[(2)(32.2)] + (8.75)(144)/[(0.877)(62.4)] = 35.16$ ft. Since the energy at E exceeds that at R, flow occurs from E to R. The lost head is $45.50 - 35.16$, or 10.34 ft, E to R.

8.53 A horizontal air duct is reduced in cross-sectional area from 0.75 ft² to 0.20 ft². Assuming no losses, what pressure change will occur when 1.50 lb/s of air flows? Use $\gamma = 0.200$ lb/ft³ for the pressure and temperature conditions involved.

$$Q = 1.50/0.200 = 7.500 \text{ ft}^3/\text{s} \qquad p_A/\gamma + v_A^2/2g + z_A = p_B/\gamma + v_B^2/2g + z_B$$

$$p_A/\gamma + (7.500/0.75)^2/[(2)(32.2)] + 0 = p_B/\gamma + (7.500/0.20)^2/[(2)(32.2)] + 0$$

$$p_A/\gamma - p_B/\gamma = 20.28 \text{ ft of air} \qquad p_A - p_B = (20.28)(0.200)/144 = 0.0282 \text{ psi}$$

8.54 A turbine is rated at 600 hp when the flow of water through it is 21.5 cfs. Assuming an efficiency of 87 percent, what head is acting on the turbine?

$$\text{Rated horsepower} = (\text{extracted horsepower})(\text{efficiency}) = (\gamma QH/550)(\text{efficiency})$$

$$600 = [(62.4)(21.5)(H)/550](0.87) \qquad H = 283 \text{ ft}$$

8.55 A standpipe 20 ft in diameter and 40 ft high is filled with water. Calculate the potential energy of the water if the elevation datum is taken 10 ft below the base of the standpipe.

$$\text{PE} = Wz = [(62.4)(40)(\pi)(20)^2/4](10 + 40/2) = 2.35 \times 10^7 \text{ ft} \cdot \text{lb}$$

8.56 How much work could be obtained from the water in Prob. 8.55 if run through a 50-percent-efficient turbine that discharged into a reservoir 30 ft below the base of the standpipe?

$$\text{Work} = \eta\text{PE} = \eta Wz = (0.50)[(62.4)(40)(\pi)(20)^2/4](30 + \tfrac{40}{2}) = 1.96 \times 10^7 \text{ ft} \cdot \text{lb}$$

8.57 Determine the kinetic-energy flux of 0.01 m³/s of oil (s.g. = 0.80) discharging through a 40-mm-diameter nozzle.

$$v = Q/A = 0.01/[(\pi)(0.040)^2/4] = 7.96 \text{ m/s}$$

$$\dot{\text{KE}} = \dot{m}v^2/2 = \rho Qv^2/2 = [(0.80)(1000)](0.01)(7.96)^2/2 = 253 \text{ W}$$

8.58 Neglecting air resistance, determine the height a vertical jet of water will rise if projected with velocity 58 ft/s.

$$\text{PE} = \text{KE} \qquad Wz = mv^2/2 = (W/32.2)(58)^2/2 \qquad z = 52.2 \text{ ft}$$

8.59 If the water jet of Prob. 8.58 is directed upward 45° with the horizontal and air resistance is neglected, how high will it rise?

▌ At 45°, $v_H = v_V = (58)(0.7071) = 41.01$ ft/s; $Wz = mv^2/2 = (W/32.2)(41.01)^2/2$, $z = 26.1$ ft.

8.60 Show that the work a liquid can do by virtue of its pressure is $\int p\,dV$, in which V is the volume of liquid displaced.

▌ Work $= \int F\,ds$. Since $F = pA$, work $= \int pA\,ds$. Since $A\,ds = dV$, work $= \int p\,dV$.

8.61 A fluid is flowing in a 6-in-diameter pipe at a pressure of 4.00 lb/in² with a velocity of 8.00 ft/s. As shown in Fig. 8-22, the elevation of the center of the pipe above a given datum is 10.0 ft. Find the total energy head above the given datum if the fluid is (*a*) water, (*b*) oil with a specific gravity of 0.82, and (*c*) gas with a specific weight of 0.042 lb/ft³.

▌
$$H = z + v^2/2g + p/\gamma$$

(*a*)
$$H = 10.0 + 8.00^2/[(2)(32.2)] + (4.00)(144)/62.4 = 20.22 \text{ ft}$$

(*b*)
$$H = 10.0 + 8.00^2/[(2)(32.2)] + (4.00)(144)/[(0.82)(62.4) = 22.25 \text{ ft}$$

(*c*)
$$H = 10.0 + 8.00^2/[(2)(32.2)] + (4.00)(144)/(0.042) = 13\,725 \text{ ft}$$

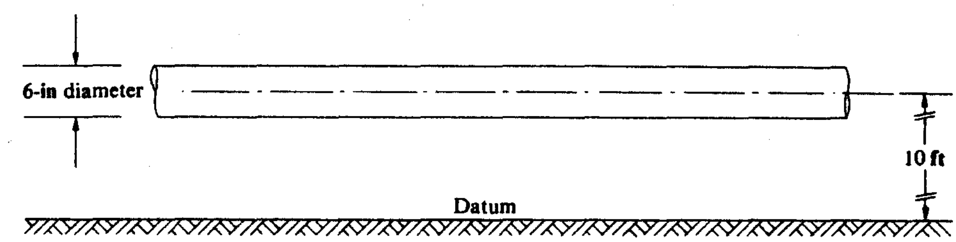


Fig. 8-22

8.62 A 100-mm-diameter suction pipe leading to a pump, as shown in Fig. 8-23, carries a discharge of 0.0300 m³/s of oil (s.g. = 0.85). If the pressure at point A in the suction pipe is a vacuum of 180 mmHg, find the total energy head at point A with respect to a datum at the pump.

▌
$$v = Q/A = 0.0300/[(\pi)(\tfrac{100}{1000})^2/4] = 3.820 \text{ m/s} \qquad p = \gamma h = [(13.6)(9.79)](-\tfrac{180}{1000}) = -23.97 \text{ kN/m}^2$$

$$H = z + v^2/2g + p/\gamma = -1.200 + 3.820^2/[(2)(9.807)] + (-23.97)/[(0.85)(9.79)] = -3.337 \text{ m}$$

8.63 Figure 8-24 shows a pump drawing water from a reservoir and discharging it into the air at point B. The pressure at point A in the suction pipe is a vacuum of 10 in mercury, and the discharge is 3.00 ft³/s. Determine the total head at point A and at point B with respect to a datum at the base of the reservoir.

▌
$$H = z + v^2/2g + p/\gamma \qquad v_A = Q/A_A = 3.00/[(\pi)(\tfrac{10}{12})^2/4] = 5.50 \text{ ft/s}$$

$$H_A = 25 + 5.50^2/[(2)(32.2)] + [(13.6)(62.4)](-\tfrac{10}{12})/62.4 = 14.14 \text{ ft}$$

$$v_B = Q/A_B = 3.00/[(\pi)(\tfrac{8}{12})^2/4] = 8.59 \text{ ft/s} \qquad H_B = (25 + 15 + 40) + 8.59^2/[(2)(32.2)] + 0 = 81.15 \text{ ft}$$

8.64 If the total available head of a stream flowing at a rate of 300 ft³/s is 25.0 ft, what is the theoretical horsepower available?

▌
$$P = Q\gamma H = (300)(62.4)(25.0) = 468\,000 \text{ ft}\cdot\text{lb/s} = 468\,000/550 = 851 \text{ hp}$$

8.65 A 150-mm-diameter jet of water is discharging from a nozzle into the air at a velocity of 36.0 m/s. Find the power in the jet with respect to a datum at the jet.

▌
$$Q = Av = [(\pi)(\tfrac{150}{1000})^2/4](36.0) = 0.6262 \text{ m}^3/\text{s}$$

$$H = z + v^2/2g + p/\gamma = 0 + 36.0^2/[(2)(9.807)] + 0 = 66.08 \text{ m}$$

$$P = Q\gamma H = (0.6362)(9.79)(66.08) = 412 \text{ kN}\cdot\text{m/s} \quad \text{or} \quad 412 \text{ kW}$$

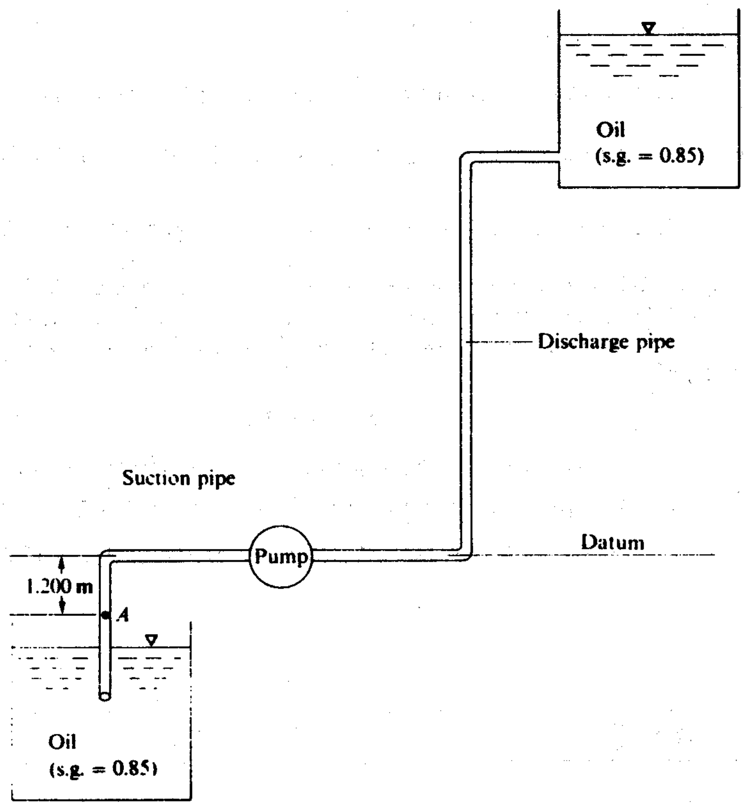


Fig. 8-23

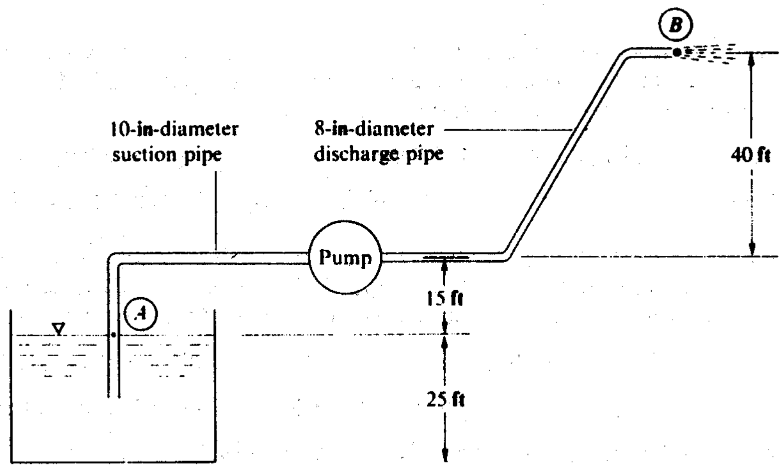


Fig. 8-24

8.66 Oil (s.g. = 0.84) is flowing in a pipe under the conditions shown in Fig. 8-25. If the total head loss (h_L) from point 1 to point 2 is 3.0 ft, find the pressure at point 2.

■

$$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L$$

$$v_1 = Q/A_1 = 2.08/[(\pi)(\tfrac{6}{12})^2/4] = 10.59 \text{ ft/s} \qquad v_2 = Q/A_2 = 2.08/[(\pi)(\tfrac{9}{12})^2/4] = 4.71 \text{ ft/s}$$

$$(65)(144)/[(0.84)(62.4)] + 10.59^2/[(2)(32.2)] + 10.70 = p_2/\gamma + 4.71^2/[(2)(32.2)] + 4.00 + 3.00$$

$$p_2/\gamma = 183.67 \text{ ft} \qquad p_2 = [(0.84)(62.4)](183.67) = 9627 \text{ lb/ft}^2 \quad \text{or} \quad 66.9 \text{ lb/in}^2$$

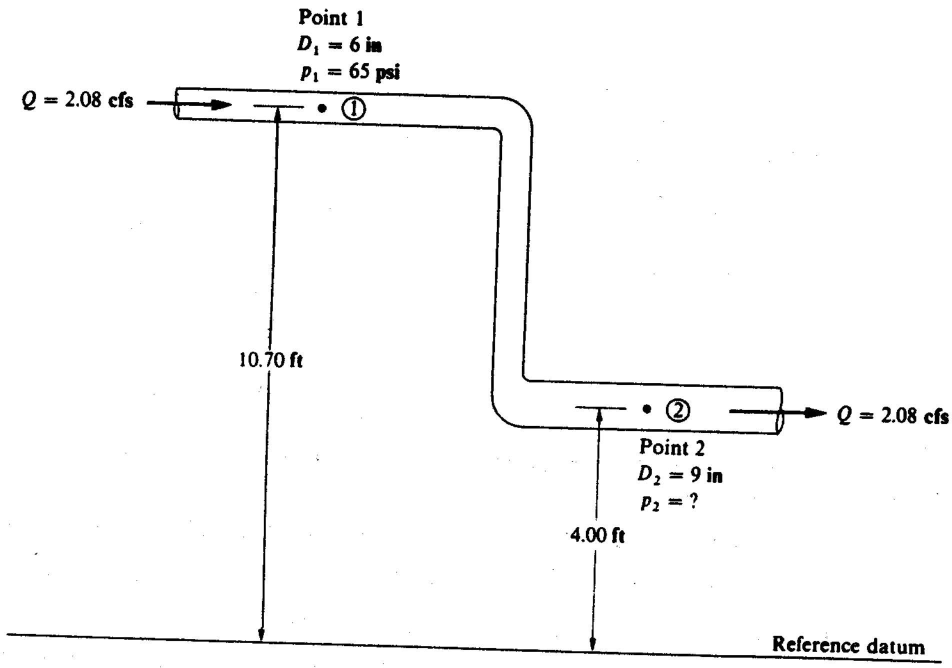


Fig. 8-25

8.67 An 8-in-diameter horizontal pipe is attached to a reservoir, as shown in Fig. 8-26. If the total head loss between the water surface in the reservoir and the water jet at the end of the pipe is 6.0 ft, what are the velocity and flow rate of the water being discharged from the pipe?

I $\quad p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L \qquad 0 + 0 + 15 = 0 + v_2^2/[(2)(32.2)] + 0 + 6.0$

$$v_2 = 24.1 \text{ ft/s} \qquad Q = A_2 v_2 = [(\pi)(\tfrac{8}{12})^2/4](24.1) = 8.41 \text{ ft}^3/\text{s}$$

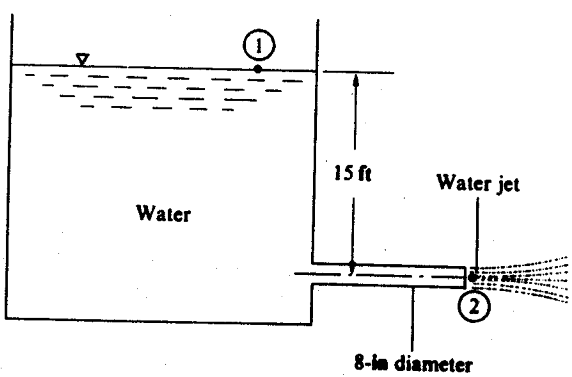


Fig. 8-26

8.68 A 50-mm-diameter siphon is drawing oil (s.g. = 0.82) from an oil reservoir, as shown in Fig. 8-27. If the head loss from point 1 to point 2 is 1.50 m and from point 2 to point 3 is 2.40 m, find the discharge of oil from the siphon and the oil pressure at point 2.

I $\quad p_1/\gamma + v_1^2/2g + z_1 = p_3/\gamma + v_3^2/2g + z_3 + h_L \qquad 0 + 0 + 5.00 = 0 + v_3^2/[(2)(9.807)] + 0 + 3.90$

$$v_3 = 4.645 \text{ m/s} \qquad Q = A_3 v_3 = [(\pi)(\tfrac{50}{1000})^2/4](4.645) = 0.00912 \text{ m}^3/\text{s}$$

$$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L \qquad 0 + 0 + 5.00 = p_2/\gamma + 4.645^2/[(2)(9.807)] + 7.00 + 1.50$$

$$p_2/\gamma = -4.60 \text{ m} \qquad p_2 = [(0.82)(9.79)](-4.60) = -36.9 \text{ kN/m}^2 \quad \text{or} \quad -36.9 \text{ kPa}$$

8.69 Figure 8-28 shows a siphon discharging oil (s.g. = 0.84) from a reservoir into open air. If the velocity of flow in the pipe is v, the head loss from point 1 to point 2 is $2.0v^2/2g$, and the head loss from point 2 to point 3 is

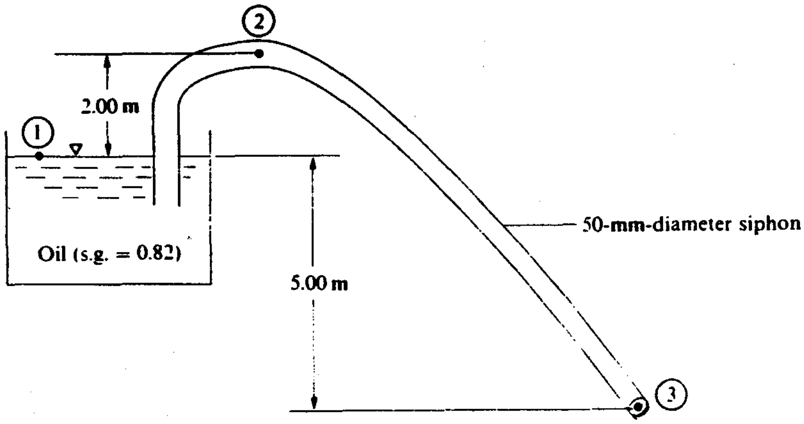


Fig. 8-27

$3.0v^2/2g$, determine the volume flow rate in the siphon pipe and the absolute pressure at point 2. Assume an atmospheric pressure of 14.70 psia.

▮ $p_1/\gamma + v_1^2/2g + z_1 = p_3/\gamma + v_3^2/2g + z_3 + h_L \qquad 0+0+10 = 0 + v_3^2/[(2)(32.2)] + 0 + 5\{v_3^2/[(2)(32.2)]\}$

$v_3 = 10.36\text{ ft/s} \qquad Q = A_3v_3 = [(\pi)(\frac{3}{12})^2/4](10.36) = 0.509\text{ ft}^3\text{/s} \qquad p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L$

$0+0+10 = p_2/\gamma + 10.36^2/[(2)(32.2)] + 15 + 2\{10.36^2/[(2)(32.2)]\} \qquad p_2/\gamma = -10.0\text{ ft of oil}$

$p_2 = [(0.84)(62.4)](-10.0) = -524\text{ lb/ft}^2 \quad \text{or} \quad -3.64\text{ lb/in}^2 \qquad p_2 = 14.70 - 3.64 = 11.06\text{ lb/in}^2\text{ abs}$

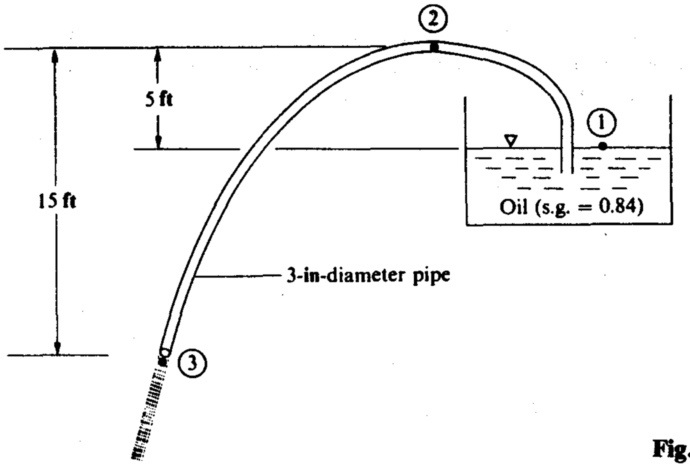


Fig. 8-28

8.70 Once it has been started by sufficient suction, the siphon in Fig. 8-29 will run continuously as long as reservoir fluid is available. Using Bernoulli's equation with no losses, show (*a*) that the exit velocity v_2 depends only upon gravity and the distance H and (*b*) that the lowest (vacuum) pressure occurs at point 3 and depends on the distance $L + H$.

▮ $p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L$, $0+0+z_1 = 0 + v_2^2/2g + z_2 + 0$, $v_2 = \sqrt{2g(z_1 - z_2)} = \sqrt{2gH}$. For any point B in the tube, $p_B/\gamma + v_B^2/2g + z_B = p_2/\gamma + v_2^2/2g + z_2 + h_L$. Since $v_B = v_2$ and $p_2 = p_{\text{atm}}$, $p_B = p_{\text{atm}} - \gamma(z_B - z_2)$. The lowest pressure occurs at the highest z_B, or $p_{\min} = p_3 = p_{\text{atm}} - \gamma(L + H)$.

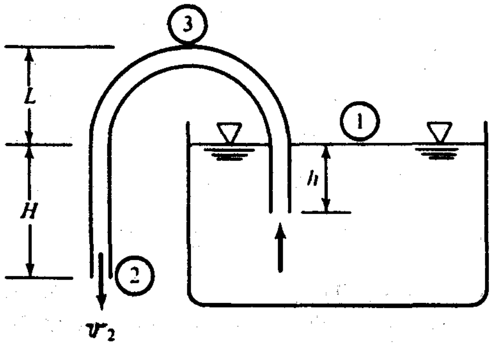


Fig. 8-29

8.71 The siphon of Fig. 8-30 is filled with water and discharging at 150 L/s. Find the losses from point 1 to point 3 in terms of velocity head $v^2/2g$. Find the pressure at point 2 if two-thirds of the losses occur between points 1 and 2.

▌ $p_1/\gamma + v_1^2/2g + z_1 = p_3/\gamma + v_3^2/2g + z_3 + h_L \qquad 0+0+1.5 = 0 + v_3^2/2g + 0 + (K)(v_3^2/2g)$

$v_3 = Q/A_3 = (\frac{150}{1000})/[(\pi)(\frac{200}{1000})^2/4] = 4.775\text{ m/s} \qquad 1.5 = 4.775^2/[(2)(9.807)] + K\{4.775^2/[(2)(9.807)]\}$

$K = 0.2904 \qquad h_L = (0.2904)\{4.775^2/[(2)(9.807)]\} = 0.338\text{ m}$

$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L \qquad 0+0+0 = p_2/\gamma + 4.775^2/[(2)(9.807)] + (\frac{2}{3})(0.338)$

$p_2/\gamma = -3.388\text{ m of water} \qquad p_2 = (-3.388)(9.79) = -33.2\text{ kN/m}^2$

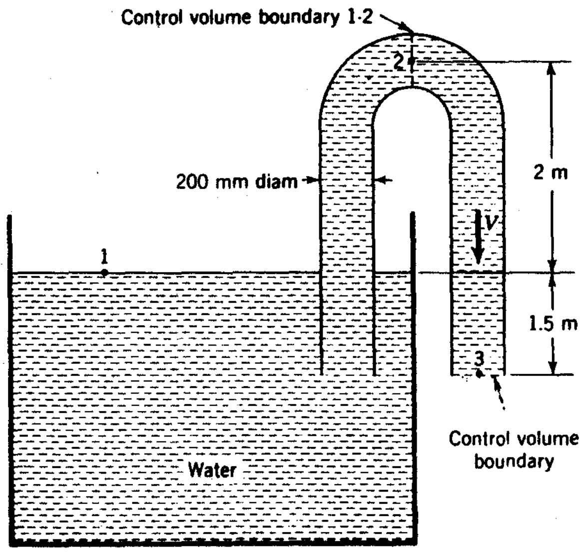


Fig. 8-30

8.72 For the water shooting out of the pipe and nozzle under the conditions shown in Fig. 8-31, find the height above the nozzle to which the water jet will "shoot" (i.e., distance *h* in Fig. 8-31). Assume negligible head loss.

▌ $p_A/\gamma + v_A^2/2g + z_A = p_{top}/\gamma + v_{top}^2/2g + z_{top} + h_L$

$55.0/9.79 + v_A^2/2g + 0 = 0 + 0 + (1.00 + h) + 0 \qquad h = 4.518 + v_A^2/2g$

$p_A/\gamma + v_A^2/2g + z_A = p_{nozzle}/\gamma + v_{nozzle}^2/2g + z_{nozzle} + h_L \qquad 55.0/9.79 + v_A^2/2g + 0 = 0 + v_{nozzle}^2/2g + 1.100 + 0$

$A_A v_A = A_{nozzle} v_{nozzle} \qquad [(\pi)(\frac{200}{1000})^2/4]v_A = [(\pi)(\frac{100}{1000})^2/4]v_{nozzle} \qquad v_{nozzle} = 4.00v_A$

$55.0/9.79 + v_A^2/[(2)(9.807)] + 0 = 0 + (4.00v_A)^2/[(2)(9.807)] + 1.100 + 0$

$v_A = 2.431\text{ m/s} \qquad h = 4.518 + 2.431^2/[(2)(9.807)] = 4.82\text{ m}$

8.73 Water flows from section 1 to section 2 in the pipe shown in Fig. 8-32. Determine the velocity of flow and the fluid pressure at section 2. Assume the total head loss from section 1 to section 2 is 3.00 m.

▌ $Q = A_1v_1 = A_2v_2 \qquad [(\pi)(\frac{100}{1000})^2/4](2.0) = [(\pi)(\frac{50}{1000})^2/4](v_2) \qquad v_2 = 8.00\text{ m/s}$

$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L$

$300/9.79 + 2.0^2/[(2)(9.807)] + 2 = p_2/9.79 + 8.00^2/[(2)(9.807)] + 0 + 3.00 \qquad p_2 = 260\text{ kPa}$

8.74 A nozzle is attached to a pipe as shown in Fig. 8-33. The inside diameter of the pipe is 100 mm, while the water jet exiting from the nozzle has a diameter of 50 mm. If the pressure at section 1 is 500 kPa, determine the water jet's velocity. Assume head loss in the jet is negligible.

▌ $Q = A_1v_1 = A_2v_2 \qquad [(\pi)(\frac{100}{1000})^2/4](v_1) = [(\pi)(\frac{50}{1000})^2/4](v_2) \qquad v_1 = 0.250v_2$

$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L$

$500/9.79 + (0.250v_2)^2/[(2)(9.807)] + 0 = 0 + v_2^2/[(2)(9.807)] + 0 + 0 \qquad v_2 = 32.7\text{ m/s}$

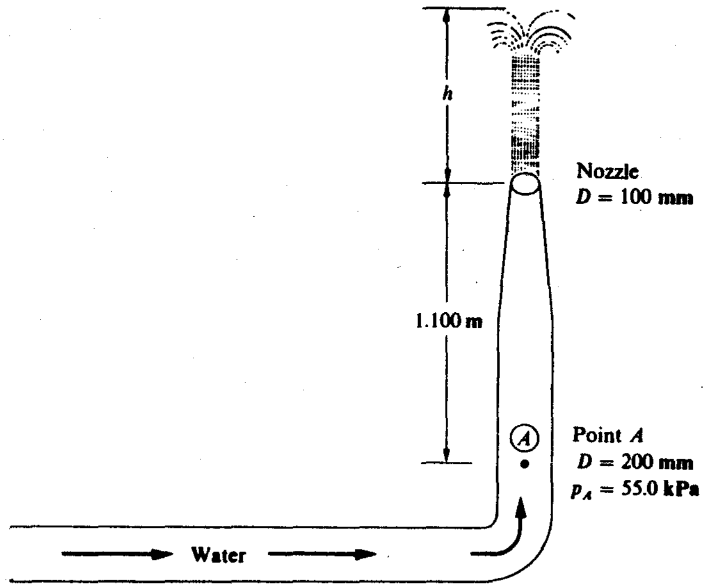


Fig. 8-31

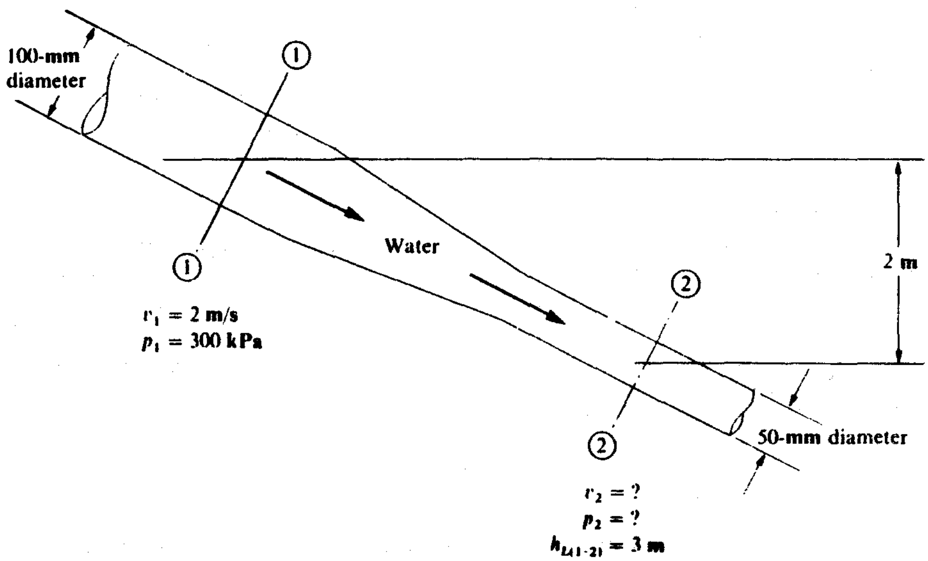


Fig. 8-32

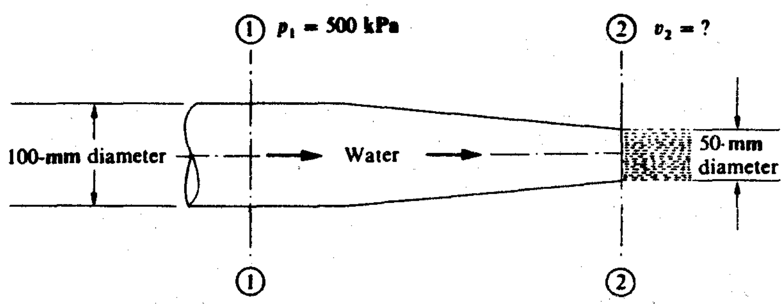


Fig. 8-33

8.75 Oil flows from a tank through 500 ft of 6-in-diameter pipe and then discharges into the air, as shown in Fig. 8-34. If the head loss from point 1 to point 2 is 1.95 ft of oil, determine the pressure needed at point 1 to cause 0.60 ft³ of oil to flow.

▌ $v_2 = Q/A = 0.60/[(\pi)(\frac{6}{12})^2/4] = 3.06\text{ ft/s} \qquad p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L$

$$p_1/\gamma + 0 + 80 = 0 + 3.06^2/[(2)(32.2)] + 100 + 1.95 \qquad p_1/\gamma = 22.10\text{ ft of oil}$$

$$p_1 = [(0.84)(62.4)](22.10)/144 = 8.04\text{ lb/in}^2$$

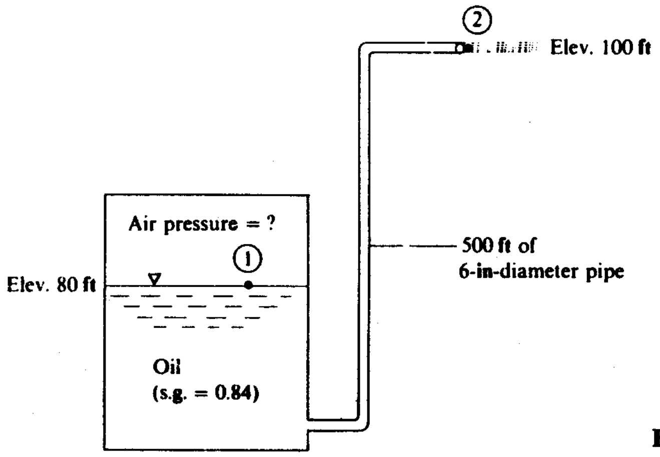


Fig. 8-34

8.76 Water is to be delivered from a reservoir through a pipe to a lower level and discharged into the air, as shown in Fig. 8-35. If head loss in the entire system is 11.58 m, determine the vertical distance between the point of water discharge and the water surface in the reservoir.

▌ $v_2 = Q/A_2 = 0.00631/[(\pi)(\frac{50}{1000})^2/4] = 3.214\text{ m/s} \qquad p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L$

$$0 + 0 + z_1 = 0 + 3.214^2/[(2)(9.807)] + 0 + 11.58 \qquad z_1 = 12.11\text{ m}$$

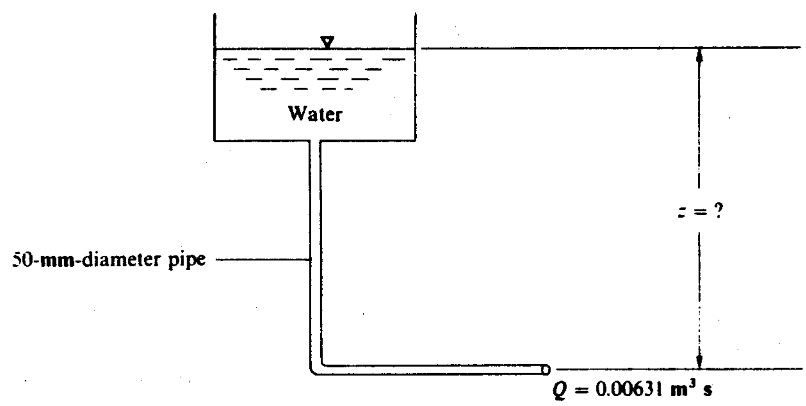


Fig. 8-35

8.77 Determine the velocity and pressure at section 2 and section 3 if water flows steadily through the pipe system shown in Fig. 8-36. Assume a head loss of 6.0 ft from section 1 to section 2 and of 15.0 ft from section 2 to section 3.

▌ $A_1v_1 = A_2v_2 \qquad [(\pi)(\frac{16}{12})^2/4](5.0) = [(\pi)(\frac{8}{12})^2/4](v_2) \qquad v_2 = 20.0\text{ ft/s}$

$$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L$$

$$(25)(144)/62.4 + 5.0^2/[(2)(32.2)] + 20 = (p_2)(144)/62.4 + 20.0^2/[(2)(32.2)] + 15 + 6.0$$

$$p_2 = 22.0\text{ lb/in}^2 \qquad A_1v_1 = A_3v_3$$

$$[(\pi)(\tfrac{16}{12})^2/4](5.0) = [(\pi)(\tfrac{12}{12})^2/4](v_3) \qquad v_3 = 8.99\text{ ft/s}$$

$$p_1/\gamma + v_1^2/2g + z_1 = p_3/\gamma + v_3^2/2g + z_3 + h_L$$

$$(25)(144)/62.4 + 5.0^2/[(2)(32.2)] + 20 = (p_3)(144)/62.4 + 8.89^2/[(2)(32.2)] + 10 + (15.0 + 6.0)$$

$$p_3 = 19.9\text{ lb/in}^2$$

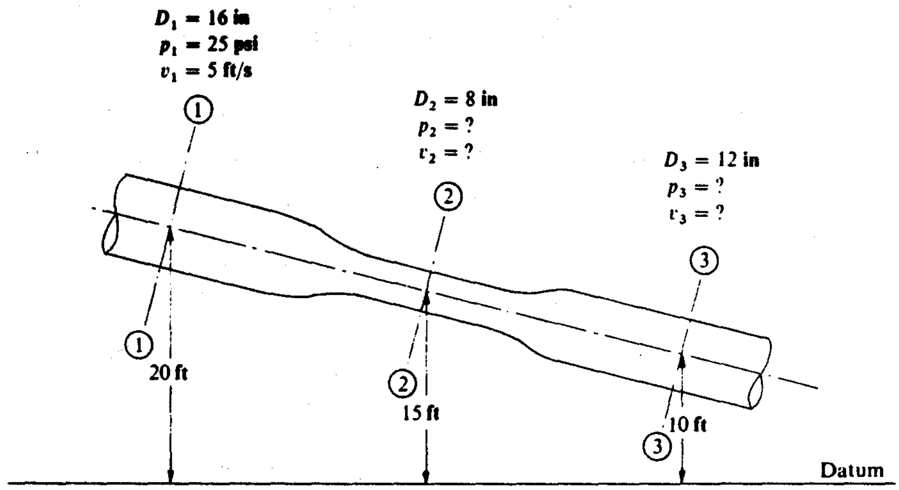


Fig. 8-36

8.78 Compute the ideal flow rate through the pipe system shown in Fig. 8-37.

▮ $p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L \qquad p_1/\gamma + v_1^2/[(2)(9.807)] + 0.6\sin 30° = p_2/\gamma + 0 + 0 + 0$

$$v_1^2/[(2)(9.807)] = p_2/\gamma - p_1/\gamma - 0.300$$

From the manometer reading, $p_1 - 9.79(1.2\sin 60°) = p_2$, $p_1 - p_2 = 10.17\ \text{kN/m}^2$; $v_1^2/[(2)(9.807)] = 10.17/9.79 - 0.300$, $v_1 = 3.807$ m/s; $Q = A_1v_1 = [(\pi)(\frac{200}{1000})^2/4](3.807) = 0.120\ \text{m}^3/\text{s}$.

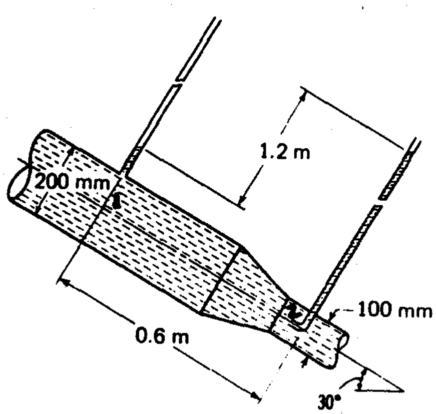


Fig. 8-37

8.79 A large tank with a well-rounded, small opening as an outlet is shown in Fig. 8-38. What is the velocity of a jet issuing from the tank?

▮ $p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L \qquad 0 + 0 + h = 0 + v_2^2/2g + 0 + 0 \qquad v_2 = \sqrt{2gh}$

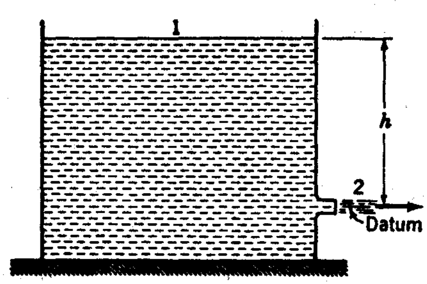


Fig. 8-38

8.80 Neglecting friction, find the velocity and volumetric discharge at the exit 2 in Fig. 8-39.

▮ $p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L \qquad 0 + 0 + (3.0 + 0.6 + 1.5) = 0 + v_2^2/[(2)(9.807)] + 0 + 0$

$$v_2 = 10.0\ \text{m/s} \qquad Q = Av = [(\pi)(\tfrac{150}{1000})^2/4](10.0) = 0.177\ \text{m}^3/\text{s}$$

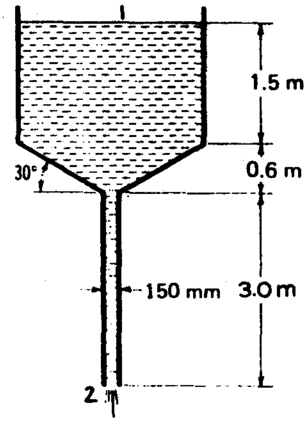


Fig. 8-39

8.81 One end of a U-tube is oriented directly into the flow (Fig. 8-40) so that the velocity of the stream is zero at this point. The pressure at a point in the flow that has been stopped in this way is called the *stagnation pressure*. The other end of the U-tube measures the undisturbed pressure at that section in the flow. Neglecting friction, determine the volume flow of water in the pipe.

▌ $p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L$, $p_1/\gamma + 0 + 0 = p_2/\gamma + v_2^2/2g + 0 + 0$, $v_2^2/2g = p_1/\gamma - p_2/\gamma$. From the manometer reading, $p_1 + (62.4)(\frac{2.5}{12}) - [(13.6)(62.4)](\frac{2.5}{12}) = p_2$, $p_1 - p_2 = 163.8\ \text{lb/ft}^2$; $v_2^2/[(2)(32.2)] = 163.8/62.4$, $v_2 = 13.00\ \text{ft/s}$; $Q = Av = [(\pi)(\frac{8}{12})^2/4](13.00) = 4.54\ \text{ft}^3/\text{s}$.

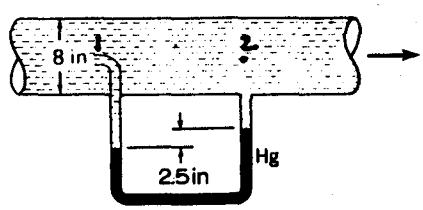


Fig. 8-40

8.82 A cylindrical tank contains air, oil, and water, as shown in Fig. 8-41; the air is under gage pressure $p = 4\ \text{lb/in}^2$. Find the exit velocity at 2, neglecting any friction and the kinetic energy of the fluid above elevation A. The jet of water leaving has a diameter of 1 ft.

▌ $$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L \qquad p_1 = (4)(144) + [(0.8)(62.4)](2) = 675.8\ \text{lb/ft}^2$$

$$675.8/62.4 + 0 + 8 = 0 + v_2^2/[(2)(32.2)] + 0 + 0 \qquad v_2 = 34.8\ \text{ft/s}$$

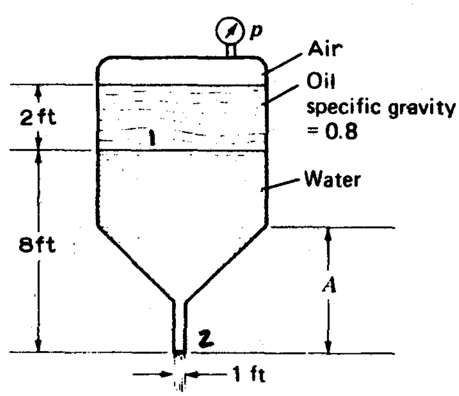


Fig. 8-41

8.83 A large tank contains compressed air, gasoline at specific gravity 0.68, light oil at specific gravity 0.80, and water, as shown in Fig. 8-42. The pressure p of the air is 120 kPa gage. If we neglect friction, what is the mass flow of oil from a 20-mm-diameter jet?

▌ $$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L \qquad p_1 = 120 + [(0.68)(9.79)](2) = 133.3\ \text{kN/m}^2$$

$$133.3/[(0.80)(9.79)] + 0 + 0 = 0 + v_2^2/[(2)(9.807)] + 4 + 0 \qquad v_2 = 15.98\ \text{m/s}$$

$$M = \rho A v = [(0.80)(1000)][(\pi)(\tfrac{20}{1000})^2/4](15.98) = 4.02\ \text{kg/s}$$

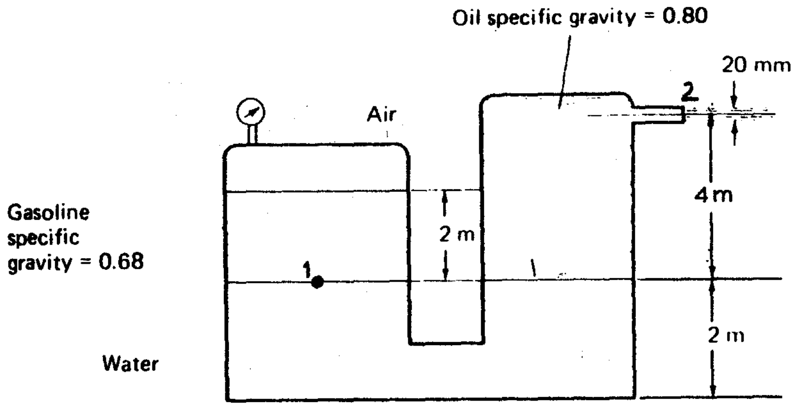


Fig. 8-42

8.84 A flow nozzle is a device inserted into a pipe as shown in Fig. 8-43. If A_2 is the exit area of the flow nozzle, show that for incompressible flow we get for Q,

$$Q = C_d\left[\frac{A_2}{\sqrt{1-(A_2/A_1)^2}}\sqrt{2g\left(\frac{p_1-p_2}{\gamma}\right)}\right]$$

where C_d is the *coefficient of discharge*, which takes into account frictional effects and is determined experimentally.

▌ $$\frac{p_A}{\gamma}+\frac{v_A^2}{2g}+z_A=\frac{p_B}{\gamma}+\frac{v_B^2}{2g}+z_B+h_L \qquad \frac{p_A}{\gamma}+\frac{v_1^2}{2g}+0=\frac{p_B}{\gamma}+\frac{v_2^2}{2g}+0+0 \qquad v_1^2=v_2^2+2g\left(\frac{p_B-p_A}{\gamma}\right)$$

But $p_B-p_A=p_2-p_1$ and $v_2=(v_1)(A_1/A_2)$; hence,

$$v_1^2=\left[v_1\left(\frac{A_1}{A_2}\right)\right]^2+2g\left(\frac{p_2-p_1}{\gamma}\right) \qquad v_1=\sqrt{\frac{1}{1-(A_1/A_2)^2}}\sqrt{2g\left(\frac{p_2-p_1}{\gamma}\right)}$$

$$Q=Av=C_dA_1\sqrt{\frac{1}{1-(A_1/A_2)^2}}\sqrt{2g\left(\frac{p_2-p_1}{\gamma}\right)}=C_d\sqrt{\frac{A_1^2}{1-(A_1/A_2)^2}}\sqrt{2g\left(\frac{p_2-p_1}{\gamma}\right)}$$

Multiplying by A_2^2/A_1^2 in the numerator and denominator of the radical gives

$$Q = C_d\left[\frac{A_2}{\sqrt{1-(A_2/A_1)^2}}\sqrt{2g\left(\frac{p_1-p_2}{\gamma}\right)}\right]$$

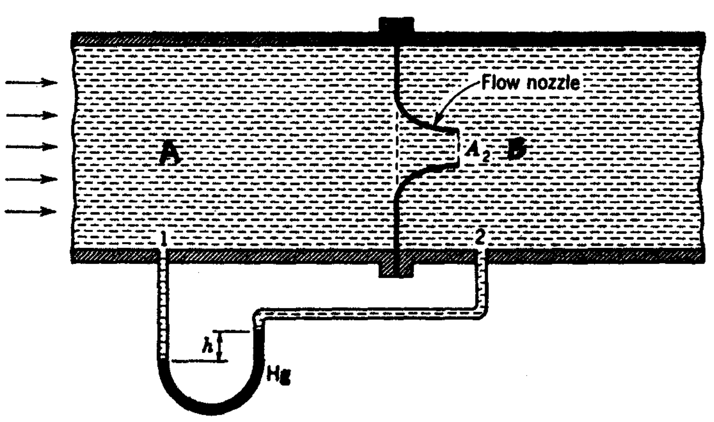


Fig. 8-43

8.85 In Prob. 8.84, express Q in terms of h, the height of the mercury column (Fig. 8-43), and the diameters of the pipe and flow nozzle.

▌ From Prob. 8.84,

$$Q = C_d\left[\frac{A_2}{\sqrt{1-(A_2/A_1)^2}}\sqrt{2g\left(\frac{p_1-p_2}{\gamma}\right)}\right]$$

From the manometer, $p_1 - p_2 = (\gamma_{Hg} - \gamma_{H_2O})(h)$.

$$Q = C_d\left[\frac{\pi d_2^2/4}{\sqrt{1-(d_2^2/d_1^2)^2}}\sqrt{\frac{2g(\gamma_{Hg}-\gamma_{H_2O})(h)}{\gamma_{H_2O}}}\right]$$

8.86 A hump of height δ is placed on the channel bed in a rectangular channel of uniform width over its entire width (see Fig. 8-44). The free surface has a dip d as shown. If we neglect friction, we can consider that we have one-dimensional flow. Compute the flow q for the channel per unit width. This system is called a *venturi flume.*

$$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L \qquad 0 + v_1^2/2g + h = 0 + v_2^2/2g + (h-d) + 0$$

$$v_1^2 = v_2^2 - 2gd \qquad A_1v_1 = A_2v_2 \qquad [(1)(h)](v_1) = [(1)(h-d-\delta)](v_2)$$

$$v_2 = \left(\frac{h}{h-d-\delta}\right)(v_1) \qquad v_1^2 = (v_1)^2\left(\frac{h}{h-d-\delta}\right)^2 - 2gd \qquad v_1^2\left[1-\left(\frac{h}{h-d-\delta}\right)^2\right] = -2gd$$

$$v_1 = \sqrt{\frac{-2gd}{1-[h/(h-d-\delta)]^2}} \qquad q = hv_1 = \sqrt{\frac{-2gd}{1/h^2 - [1/(h-d-\delta)]^2}}$$

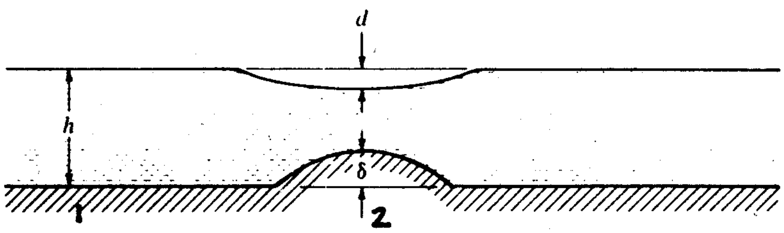


Fig. 8-44

8.87 In the fountain of Fig. 8-45, water flows steadily up the vertical pipe, enters the annular region between the circular plates, and emerges as a free sheet. Find the volume flow of water through the pipe, if the pressure at A is 70 kPa gage and friction is negligible.

$$p_A/\gamma + v_A^2/2g + z_A = p_E/\gamma + v_E^2/2g + z_E + h_L$$

$$70/9.79 + v_A^2/[(2)(9.807)] + 0 = 0 + v_E^2/[(2)(9.807)] + 1.5 + 0$$

$$A_Av_A = A_Ev_E \qquad [(\pi)(0.200)^2/4](v_A) = [(0.013)(\pi)(0.3+0.3)](v_E) \qquad v_A = 0.780v_E$$

$$70/9.79 + (0.780v_E)^2/[(2)(9.807)] + 0 = 0 + v_E^2/[(2)(9.807)] + 2.0 + 0 \qquad v_E = 16.06\text{ m/s}$$

$$Q = A_Ev_E = [(0.015)(\pi)(0.5+0.5)](16.06) = 0.757\text{ m}^3/\text{s}$$

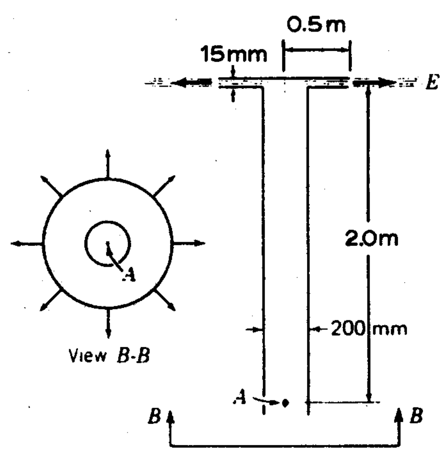


Fig. 8-45

8.88 If the velocity at point A in Fig. 8-46 is 18 m/s, what is the pressure at point B if we neglect friction?

$$p_B/\gamma + v_B^2/2g + z_B = p_A/\gamma + v_A^2/2g + z_A + h_L$$

$$p_B/9.79 + v_B^2/[(2)(9.807)] + 0 = 0 + 18^2/[(2)(9.807)] + (0.5 + 15) + 0$$

$$p_B = -0.4991v_B^2 + 313.5 \qquad p_C/\gamma + v_C^2/2g + z_C = p_A/\gamma + v_A^2/2g + z_A + h_L$$

$$0 + v_C^2/[(2)(9.807)] + 0 = 0 + 18^2/[(2)(9.807)] + 15 + 0 \qquad v_C = 24.86 \text{ m/s} \qquad A_B v_B = A_C v_C$$

$$[(\pi)(\tfrac{200}{1000})^2/4](v_B) = [(\pi)(\tfrac{75}{1000})^2/4](24.86) \qquad v_B = 3.496 \text{ m/s}$$

$$p_B = (-0.4991)(3.496)^2 + 313.5 = 319.6 \text{ kN/m}^2$$

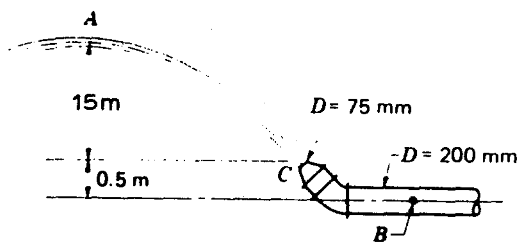


Fig. 8-46

8.89 A rocket-powered sled is used in the training of astronauts (Fig. 8-47). For braking, scoops are lowered to deflect water from a stationary tank of water. To what height h does a sled traveling at 100 km/h deflect water?

In an inertial frame fixed to the sled, apply Bernoulli's equation between the scoop (point 1) and the highest point in the trajectory (point 2); $p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L$, $0 + v_1^2/[(2)(9.807)] + 0 = 0 + (v_1 \cos 20°)^2/[(2)(9.807)] + (h - 0.150) + 0$. From the data, $v_1 = (100)(1000)/3600 = 27.78$ m/s, $27.78^2/[(2)(9.807)] = (27.78 \cos 20°)^2/[(2)(9.807)] + (h - 0.150)$, $h = 4.80$ m.

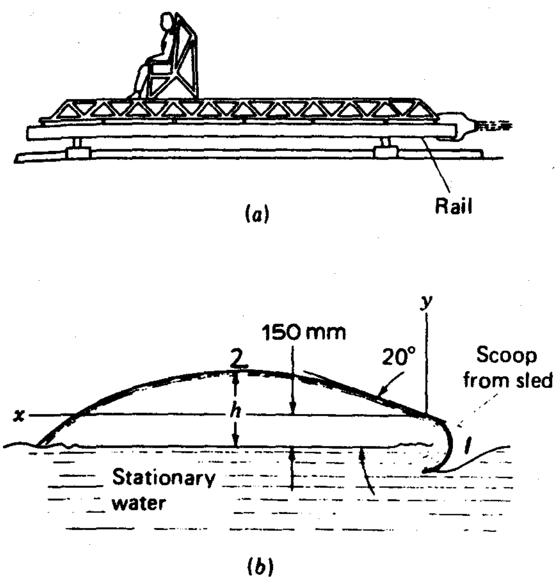


Fig. 8-47

8.90 A *venturi meter* is a device which is inserted into a pipe line to measure incompressible flow rates. As shown in Fig. 8-48, it consists of a convergent section which reduces the diameter to between one-half and one-fourth the pipe diameter. This is followed by a divergent section. The pressure difference between the position just before the venturi and at the throat of the venturi is measured by a differential manometer as shown. Show that

$$Q = C_d\left[\frac{A_2}{\sqrt{1-(A_2/A_1)^2}}\sqrt{2g\left(\frac{p_1 - p_2}{\gamma}\right)}\right]$$

where C_d is the *coefficient of discharge,* which takes into account frictional effects and is determined experimentally.

▌ $$\frac{p_1}{\gamma}+\frac{v_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{v_2^2}{2g}+z_2+h_L \qquad \frac{p_1}{\gamma}+\frac{v_1^2}{2g}+0=\frac{p_2}{\gamma}+\frac{v_2^2}{2g}+0+0 \qquad v_1^2-v_2^2=2g\left(\frac{p_2-p_1}{\gamma}\right)$$

$$A_1v_1=A_2v_2 \qquad v_1=(v_2)\left(\frac{A_2}{A_1}\right) \qquad \left[(v_2)\left(\frac{A_2}{A_1}\right)\right]^2-v_2^2=2g\left(\frac{p_2-p_1}{\gamma}\right) \qquad \left[\left(\frac{A_2}{A_1}\right)^2-1\right](v_2^2)=2g\left(\frac{p_2-p_1}{\gamma}\right)$$

$$v_2=\sqrt{\frac{1}{1-(A_2/A_1)^2}}\sqrt{2g\left(\frac{p_1-p_2}{\gamma}\right)}$$

$$Q=Av=C_dA_2\sqrt{\frac{1}{1-(A_2/A_1)^2}}\sqrt{2g\left(\frac{p_1-p_2}{\gamma}\right)}=C_d\left[\frac{A_2}{\sqrt{1-(A_2/A_1)^2}}\sqrt{2g\left(\frac{p_1-p_2}{\gamma}\right)}\right]$$

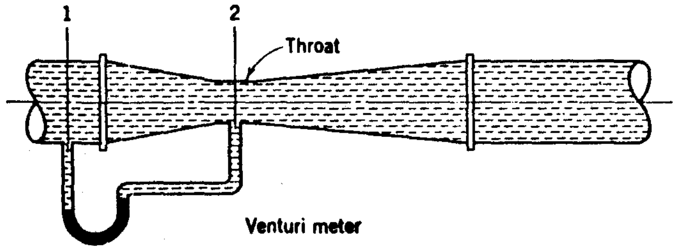


Fig. 8-48

8.91 A necked-down, or venturi, section of a pipe flow develops a low pressure which can be used to aspirate fluid upward from a reservoir, as shown in Fig. 8-49. Using Bernoulli's equation with no losses, derive an expression for the exit velocity v_2 that is just sufficient to cause the reservoir fluid to rise in the tube up to section 1.

▌ $$p_1/\gamma+v_1^2/2g+z_1=p_2/\gamma+v_2^2/2g+z_2+h_L \qquad p_1/\gamma+v_1^2/2g+0=p_{atm}/\gamma+v_2^2/2g+0+0$$

$$A_1v_1=A_2v_2 \qquad (\pi d_1^2/4)(v_1)=(\pi d_2^2/4)(v_2) \qquad v_1=(v_2)(d_2/d_1)^2$$

$$p_1/\gamma+[(v_2)(d_2/d_1)^2]^2/2g=p_{atm}/\gamma+v_2^2/2g \qquad p_{atm}-p_1=(\gamma/2g)(v_2^2)[(d_2/d_1)^4-1]$$

For fluid to rise in the tube, $p_{atm}-p_1 \geq \gamma h$; hence, $(\gamma/2g)(v_2^2)[(d_2/d_1)^4-1] \geq \gamma h$,

$$v_2 \geq \sqrt{\frac{2gh}{(d_2/d_1)^4-1}}$$

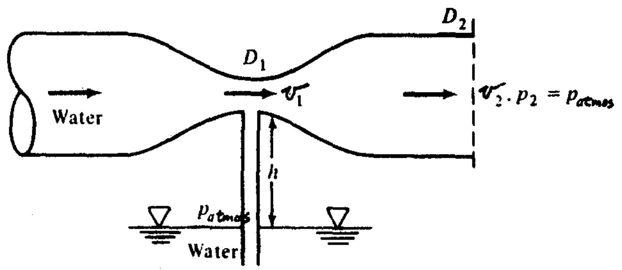


Fig. 8-49

8.92 Neglecting losses, find the discharge through the venturi meter of Fig. 8-50.

▌ $p_1/\gamma+v_1^2/2g+z_1=p_2/\gamma+v_2^2/2g+z_2+h_L$. From the manometer, $p_1/\gamma-(k+0.200)+(z_1-z_2+k)=p_2/\gamma$; $A_1v_1=A_2v_2$, $[(\pi)(0.300)^2/4](v_1)=[(\pi)(0.150)^2/4](v_2)$, $v_1=0.250v_2$; $p_1/\gamma+(0.250v_2)^2/[(2)(9.807)]+z_1=[p_1/\gamma-(k+0.250)+(z_1-z_2+k)]+v_2^2/[(2)(9.807)]+z_2+0$, $v_2=2.287$ m/s; $Q=A_2v_2=[(\pi)(0.150)^2/4](2.287)=0.0404$ m^3/s.

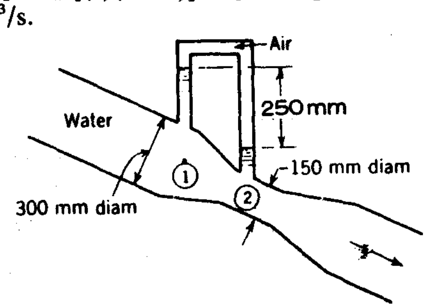


Fig. 8-50

8.93 With losses of $0.2v_1^2/2g$ between sections 1 and 2 of Fig. 8-50, calculate the flow in gallons per minute.

From Prob. 8.92, $v_1 = 0.250v_2 = (0.250)(2.287) = 0.5718$ m/s; $p_1/\gamma + (0.250v_2)^2/[(2)(9.807)] + z_1 = [p_1/\gamma - (k+0.250) + (z_1 - z_2 + k)] + v_2^2/[(2)(9.807)] + z_2 + 0$. For Prob. 8.93, add a term $0.2v_1^2/2g$ to the previous equation, giving $p_1/\gamma + (0.250v_2)^2/[(2)(9.807)] + z_1 = [p_1/\gamma - (k+0.250) + (z_1 - z_2 + k)] + v_2^2/[(2)(9.807)] + z_2 + (0.2)\{0.5718^2/[(2)(9.807)]\}$, $v_2 = 2.272$ m/s; $Q = A_2v_2 = [(\pi)(0.150)^2/4](2.272) = 0.0401$ m^3/s $= [0.0401/(0.3048)^3](7.48)(60) = 636$ gpm.

8.94 The device shown in Fig. 8-51 is used to determine the velocity of liquid at point 1. It is a tube with its lower end directed upstream and its other leg vertical and open to the atmosphere. The impact of liquid against opening 2 forces liquid to rise in the vertical leg to the height z above the free surface. Determine the velocity at 1.

$$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L \qquad k + v_1^2/2g + 0 = 0 + (k + \Delta z) + 0 + 0 \qquad v_1 = \sqrt{2g\,\Delta z}$$

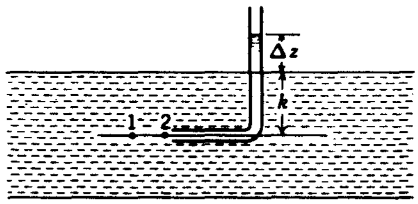


Fig. 8-51

8.95 In Fig. 8-52 the losses in the exit pipe equal $Kv^2/2g$, where $K = 5.0$. The tank reservoir is large. Compute the flow rate in cubic feet per minute.

$$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L$$

$$(15)(144)/[(0.86)(62.4)] + 0 + 6 = (14.7)(144)/[(0.86)(62.4)] + v_2^2/[(2)(32.2)] + 0 + (5.0)\{v_2^2/[(2)(32.2)]\}$$

$$v_2 = 8.54 \text{ ft/s} \qquad Q = Av = [(\pi)(\tfrac{1}{12})^2/4](8.54) = 0.0466 \text{ ft}^3/\text{s} \quad \text{or} \quad 2.80 \text{ ft}^3/\text{min}$$

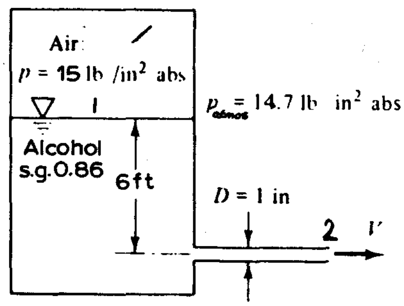


Fig. 8-52

8.96 The manometer fluid in Fig. 8-53 is mercury. Neglecting losses, calculate the flow rate in the tube if the flowing fluid is **(a)** water, **(b)** air. Use 60 °F as the fluid temperature.

$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L$, $p_1/\gamma + v_1^2/2g + 0 = p_2/\gamma + 0 + 0 + 0$, $p_2 - p_1 = (\gamma)(v_1^2/2g)$. From the manometer, $p_1 + (62.4)(y/12) + [(13.6)(62.4)](\tfrac{1}{12}) - (62.4)(\tfrac{1}{12}) - (62.4)(y/12) = p_2$, $p_2 - p_1 = 65.52$ lb/ft^2; $(\gamma)(v_1^2/2g) = 65.52$.

(a) $(62.4)\{v_1^2/[(2)(32.2)]\} = 65.52 \qquad v_1 = 8.223$ ft/s $\qquad Q = A_1v_1 = [(\pi)(\tfrac{4}{12})^2/4](8.223) = 0.718$ ft^3/s

(b) $(0.0763)\{v_1^2/[(2)(32.2)]\} = 65.52 \qquad v_1 = 235.2$ ft/s $\qquad Q = A_1v_1 = [(\pi)(\tfrac{4}{12})^2/4](235.2) = 20.5$ ft^3/s

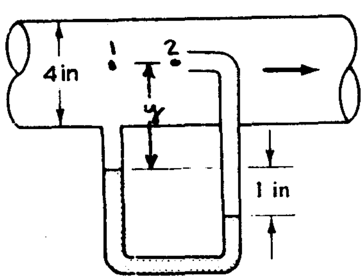


Fig. 8-53

8.97 In Fig. 8-54 the fluid is air ($\gamma = 12\ \mathrm{N/m^3}$), and the manometer fluid has s.g. = 0.827. Assuming no losses, compute the flow rate in L/s.

▮ $p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L$, $p_1/\gamma + 0 + 0 = p_2/\gamma + v_2^2/2g + 0 + 0$, $p_1 - p_2 = (\gamma)(v_2^2/2g)$. From the manometer, $p_1 + (12 \times 10^{-3})(y + 0.080) - [(0.827)(9.79)](0.080) - (12 \times 10^{-3})y = p_2$, $p_1 - p_2 = 0.6467\ \mathrm{kN/m^2}$; $(\gamma)(v_2^2/2g) = 0.6467$, $(12 \times 10^{-3})\{v_2^2/[(2)(9.807)]\} = 0.6467$, $v_2 = 32.51\ \mathrm{m/s}$; $Q = A_2v_2 = [(\pi)(0.050)^2/4](32.51) = 0.0638\ \mathrm{m^3/s} = 63.8\ \mathrm{L/s}$.

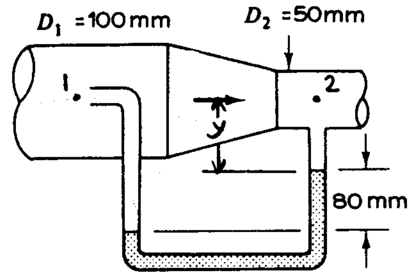


Fig. 8-54

8.98 The flow from two reservoirs mixes together and flows through a common pipe. The elevations and pipe diameters are indicated in Fig. 8-55. Both reservoirs contain the same liquid and are open to the atmosphere. The common pipe empties to the atmosphere. Neglecting any frictional effects, find the flow rate through the common pipe.

▮
$$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L \qquad p_{\mathrm{atm}}/\gamma + 0 + h_1 = p_2/\gamma + v_2^2/2g + 0 + 0$$
$$p_{\mathrm{atm}} + \gamma h_1 = p_2 + \gamma v_2^2/2g \tag{1}$$
$$p_3/\gamma + v_3^2/2g + z_3 = p_4/\gamma + v_4^2/2g + z_4 + h_L \qquad p_{\mathrm{atm}}/\gamma + 0 + h_2 = p_4/\gamma + v_4^2/2g + 0 + 0$$
$$p_{\mathrm{atm}} + \gamma h_2 = p_4 + \gamma v_4^2/2g \tag{2}$$
$$p_5/\gamma + v_5^2/2g + z_5 = p_6/\gamma + v_6^2/2g + z_6 + h_L \qquad p_5/\gamma + v_5^2/2g + 0 = p_{\mathrm{atm}}/\gamma + v_6^2/2g + (-h_3) + 0$$

Since $v_5 = v_6$,

$$p_5 = p_{\mathrm{atm}} - \gamma h_3 \tag{3}$$

Assume $p_5 = p_2 = p_4$. Substituting this common value of pressure back into Eqs. (1) and (2) and solving for the velocity in each branch, we get $v_2 = \sqrt{2g(h_1 + h_3)}$, $v_4 = \sqrt{2g(h_2 + h_3)}$ $Q = Av = (\pi d_1^2/4)\sqrt{2g(h_1 + h_3)} + (\pi d_2^2/4)\sqrt{2g(h_2 + h_3)} = (\pi/4)[d_1^2\sqrt{2g(h_1 + h_3)} + d_2^2\sqrt{2g(h_2 + h_3)}]$.

8.99 A steady jet of water comes from a hydrant and hits the ground some distance away, as shown in Fig. 8-56. If the water outlet is 1 m above the ground and the hydrant water pressure is 862 kPa, what distance from the hydrant does the jet hit the ground? Atmospheric pressure is 101 kPa.

▮ The magnitude of v_x can be obtained by noting that at the hydrant outlet the flow is entirely in the x direction, $v_x = V_2$. Applying the Bernoulli equation between the interior of the hydrant and the outlet gives $p_1 + \rho g y_1 + \frac{1}{2}\rho V_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho V_2^2$. The pressure in the hydrant p_1 is given, and the outlet is open to the atmosphere, $p_2 = p_{\mathrm{atm}}$. The elevation of points 1 and 2 is the same, $y_1 = y_2$. We assume that the outlet area is small enough compared with the hydrant cross-sectional area for the hydrant to be essentially a reservoir, $V_1^2 \ll V_2^2$. Neglecting V_1, we get $v_x = \sqrt{(2/\rho)(p_1 - p_{\mathrm{atm}})}$. Since v_x is constant, it can be brought outside the integral for l, giving $l = v_xT$.

To find the time T required for a fluid particle to hit the ground, we apply the Bernoulli equation between point 1 and some arbitrary point on the jet having elevation y and velocity V: $p_1 + \rho g h = p_{\mathrm{atm}} + \rho g y + \frac{1}{2}\rho V^2$. Now $V^2 = v_x^2 + v_y^2$. When we use the previously determined value of v_x and note that $v_y = dy/dt$, the Bernoulli equation becomes $(dy/dt)^2 = 2g(h - y)$. We take the square root (the negative root is the appropriate one since dy/dt must be negative): $dy/dt = -\sqrt{2g(h - y)}$. Then we separate variables and integrate between the limits of $y = h$, $t = 0$ and $y = 0$, $t = T$:

$$\int_h^0 \frac{dy}{\sqrt{h - y}} = -\sqrt{2g}\int_0^T dt$$

Integrating and solving for T gives $T = \sqrt{2h/g}$. The y component of the fluid motion is that of a body freely falling under the influence of gravity.

Finally, we substitute numerical values to get $v_x = \sqrt{\frac{2}{1000}[(862 - 101)(1000)]} = 39.0\ (\mathrm{m \cdot N/kg})^{1/2}$, or 39.0 m/s; $T = \sqrt{(2)(1.0)/9.807} = 0.452\ \mathrm{s}$; $L = vt = (39.0)(0.452) = 17.6\ \mathrm{m}$.

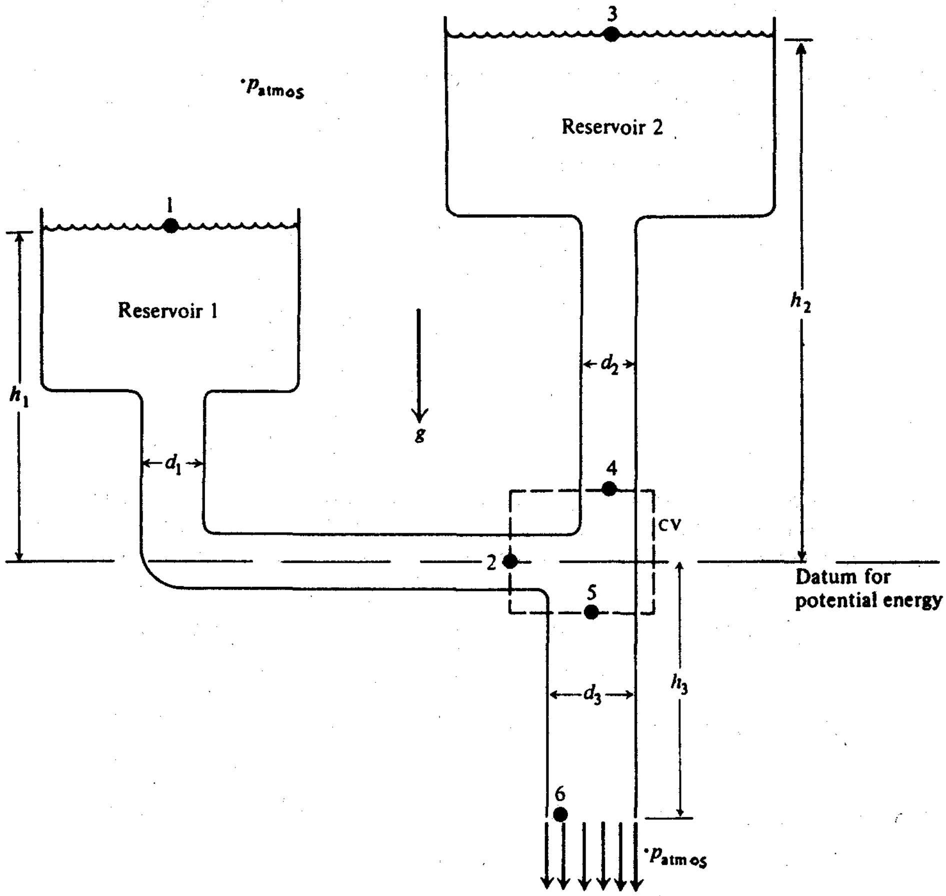


Fig. 8-55

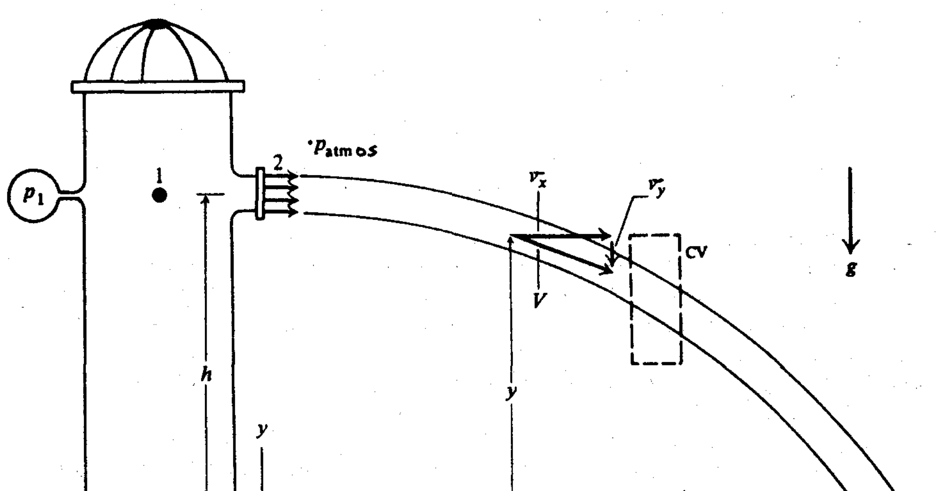


Fig. 8-56

8.100 Water flows between the two reservoirs in Fig. 8-57 at the rate of 16 L/s. What is the head loss in the pipe? If atmospheric pressure is 100 kPa and the vapor pressure is 8 kPa, for what constriction diameter d will cavitation occur? Assume no additional losses due to changes in the constriction.

▌ $p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L$, $0 + 0 + 20 = 0 + 0 + 5 + h_L$, $h_L = 15$ m; $v_{throat} = Q/A_{throat} = 0.016/(\pi d^2/4)$. Assume a central constriction, with $\frac{15}{2}$, or 7.5-m head loss on each side. Apply Bernoulli's equation between point 1 and the constriction, with $p_1 = p_{atm} = 100$ kPa and $p_v = 8$ kPa at the constriction.

$100/9.79 + 0 + 20 = 8/9.79 + v_{max}^2/[(2)(9.807)] + 0 + 7.5$, $v_{max} = 20.7$ m/s; $20.7 = 0.016/(\pi d^2/4)$, $d = 0.0314$ m, or 2.98 cm.

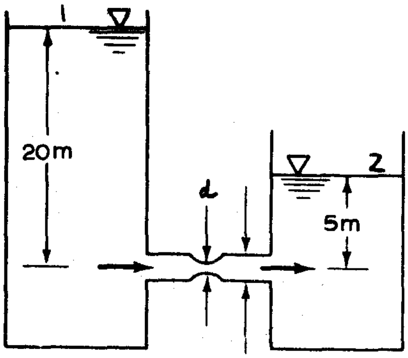


Fig. 8-57

8.101 The horizontal wye fitting in Fig. 8-58 splits Q_1 into two equal flow rates. At section 1, $Q_1 = 4$ ft³/s and $p_1 = 20$ psig. Neglecting losses, compute pressures p_2 and p_3.

$$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L \qquad v_1 = Q_1/A_1 = 4/[(\pi)(\tfrac{6}{12})^2/4] = 20.37 \text{ ft/s}$$

$$v_2 = Q_2/A_2 = \tfrac{4}{2}/[(\pi)(\tfrac{4}{12})^2/4] = 22.92 \text{ ft/s}$$

$$(20)(144)/62.4 + 20.37^2/[(2)(32.2)] + 0 = (p_2)(144)/62.4 + 22.92^2/[(2)(32.2)] + 0 + 0$$

$$p_2 = 19.3 \text{ psig} \qquad p_1/\gamma + v_1^2/2g + z_1 = p_3/\gamma + v_3^2/2g + z_3 + h_L \qquad v_3 = Q_3/A_3 = \tfrac{4}{2}/[(\pi)(\tfrac{3}{12})^2/4] = 40.74 \text{ ft/s}$$

$$(20)(144)/62.4 + 20.37^2/[(2)(32.2)] + 0 = (p_3)(144)/62.4 + 40.74^2/[(2)(32.2)] + 0 + 0 \qquad p_3 = 11.6 \text{ psig}$$

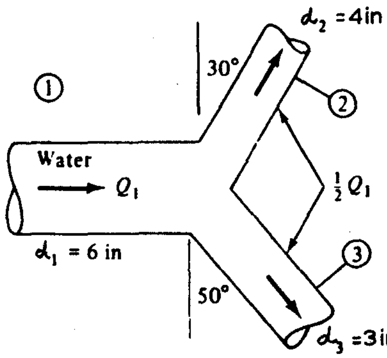


Fig. 8-58

8.102 A cylindrical tank of diameter d_0 contains liquid to an initial height h_0. At time $t = 0$ a small stopper of diameter d is removed from the bottom. Using Bernoulli's equation with no losses, derive a differential equation for the free-surface height h during draining and an expression for the time t_0 to drain the entire tank.

Letting point 1 be the liquid surface and point 2 at the exit, $p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L$, $0 + v_1^2/2g + h = 0 + v_2^2/2g + 0 + 0$, $v_1^2 = v_2^2 - 2gh$; $A_1v_1 = A_2v_2$, $(\pi d_0^2/4)(v_1) = (\pi d^2/4)(v_2)$, $v_2 = (d_0/d)^2(v_1)$; $v_1^2 = [(d_0/d)^2(v_1)]^2 - 2gh$, $v_1 = \sqrt{2gh/(d_0^4/d^4 - 1)}$. Or $v_1 = \sqrt{Kh}$ where $K = 2g/[(d_0/d)^4 - 1]$. But also, $v_1 = -dh/dt$, $dh/dt = -\sqrt{Kh}$

$$\int_{h_0}^{h} \frac{dh}{\sqrt{h}} = \int_0^t -\sqrt{K}\,dt \qquad (2)[h^{1/2}]_{h_0}^{h} = -\sqrt{K}\,[t]_0^t \qquad (2)(h^{1/2} - h_0^{1/2}) = -\sqrt{K}\,t \qquad h = \left(h_0^{1/2} - \frac{\sqrt{K}\,t}{2}\right)^2$$

Or, $h = \{h_0^{1/2} - [g/(2)(d_0^4/d^4 - 1)]^{1/2}t\}^2$.

8.103 In the water flow over the spillway in Fig. 8-59, the velocity is uniform at sections 1 and 2 and the pressure approximately hydrostatic. Neglecting losses, compute v_1 and v_2. Assume unit width.

$$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L \qquad 0 + v_1^2/2g + 6 = 0 + v_2^2/2g + 1 + 0$$

$$A_1v_1 = A_2v_2 \qquad [(6)(1)](v_1) = [(1)(1)](v_2) \qquad v_2 = 6v_1$$

$$v_1^2/[(2)(9.807)] + 6 = (6v_1)^2/[(2)(9.807)] + 1 \qquad v_1 = 1.67 \text{ m/s} \qquad v_2 = (6)(1.67) = 10.02 \text{ m/s}$$

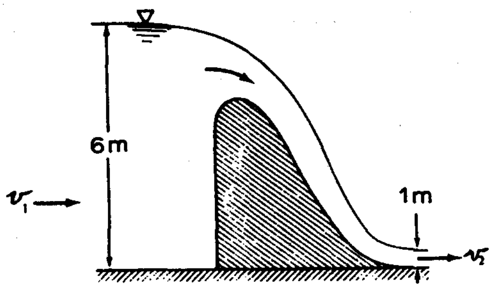


Fig. 8-59

8.104 For the water channel flow down the sloping ramp of Fig. 8-60, $h_1 = 1$ m, $H = 3$ m, and $v_1 = 4$ m/s. The flow is uniform at 1 and 2. Neglecting losses, find the downstream depth h_2 and show that three solutions are possible, of which only two are realistic. Neglect friction.

▌ $p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L \qquad 0 + 4^2/[(2)(9.807)] + (3+1) = 0 + v_2^2/[(2)(9.807)] + h_2 + 0$

$$A_1v_1 = A_2v_2 \qquad [(1)(1)](4) = [(h_2)(1)](v_2) \qquad v_2 = 4/h_2$$

$$4^2/[(2)(9.807)] + 4 = (4/h_2)^2/[(2)(9.807)] + h_2 \qquad h_2^3 - 4.816h_2^2 + 0.8157 = 0$$

There are three mathematical solutions to this equation:

$h_2 = 4.78$ m (subcritical)
$h_2 = 0.432$ m (supercritical)
$h_2 = -0.396$ m (impossible)

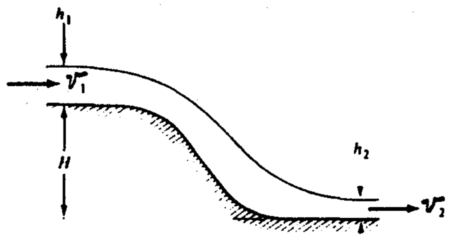


Fig. 8-60

8.105 For water flow up the sloping channel in Fig. 8-61, $h_1 = 0.5$ ft, $v_1 = 15$ ft/s, and $H = 2$ ft. Neglect losses and assume uniform flow at 1 and 2. Find the downstream depth h_2 and show that three solutions are possible, of which only two are realistic.

▌ $p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L \qquad 0 + 15^2/[(2)(32.2)] + 0.5 = 0 + v_2^2/[(2)(32.2)] + (2 + h_2) + 0$

$$A_1v_1 = A_2v_2 \qquad [(0.5)(1)](15) = [(h_2)(1)](v_2) \qquad v_2 = 7.5/h_2$$

$$15^2/[(2)(32.2)] + 0.5 = (7.5/h_2)^2/[(2)(32.2)] + (2 + h_2) \qquad h_2^3 - 1.994h_2^2 + 0.8734 = 0$$

There are three mathematical solutions to this equation:

$h_2 = 1.69$ ft (subcritical)
$h_2 = 0.887$ ft (supercritical)
$h_2 = -0.582$ ft (impossible)

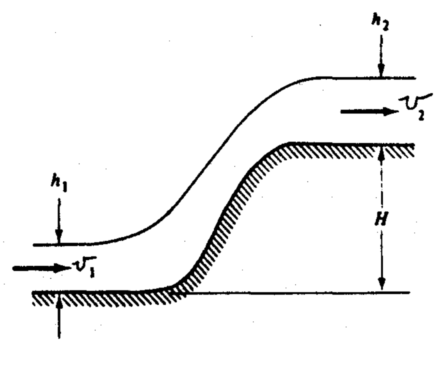


Fig. 8-61

8.106 A constant 12-lb force is applied to the piston in Fig. 8-62, and flow losses are negligible. Compute the water-jet exit velocity v_2.

$p_1/\gamma + v_1^2/2g + z_1 = p_{atm}/\gamma + v_2^2/2g + z_2 + h_L$. Considering the force acting on the piston, $12 = (p_1 - p_{atm})[(\pi)(\frac{8}{12})^2/4]$, $p_1 - p_{atm} = 34.38$ lb/ft^2; $A_1v_1 = A_2v_2$, $[(\pi)(\frac{8}{12})^2/4](v_1) = [(\pi)(\frac{4}{12})^2/4](v_2)$, $v_1 = 0.250v_2$; $(p_1 - p_{atm})/\gamma + (0.250v_2)^2/[(2)(32.2)] + 0 = v_2^2/[(2)(32.2)] + 0 + 0$, $34.38/62.4 + (0.250v_2)^2/[(2)(32.2)] + 0 = v_2^2/[(2)(32.2)] + 0 + 0$, $v_2 = 6.15$ ft/s.

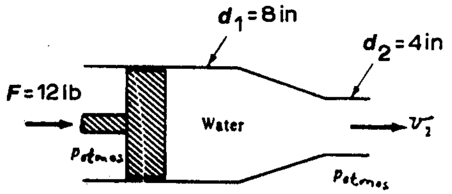


Fig. 8-62

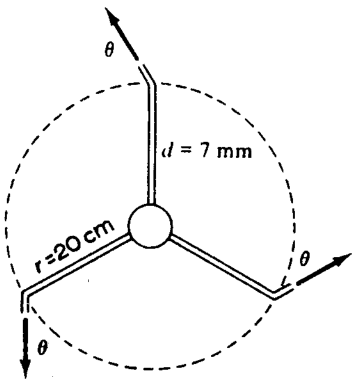


Fig. 8-63

8.107 The horizontal lawn sprinkler of Fig. 8-63 is fed water through the center at 1.2 L/s. If collar friction is negligible, what is the steady rotation rate for (*a*) $\theta = 0°$ and (*b*) $\theta = 30°$?

$$v_0 = \frac{Q}{A} = \frac{(1.2 \times 10^{-3})/3}{(\pi)(0.007)^2/4} = 10.39 \text{ m/s}$$

Choose an inertial (nonrotating) frame with origin at the center of the sprinkler; let (ρ, ϕ) be polar coordinates relative to this frame.

An emergent water jet has velocity components

$$v_\rho = v_0 \sin\theta \qquad v_\phi = v_0 \cos\theta - r\omega$$

For zero reactive torque—the criterion for the steady state—$v_\phi = 0$, or $\omega = (v_0 \cos\theta)/r$.

(*a*) $\omega = (10.39)(\cos 0°)/0.20 = 51.95$ rad/s or 496 rpm

(*b*) $\omega = (10.39)(\cos 30°)/0.20 = 44.99$ rad/s or 430 rpm

8.108 Water flows at 6 ft/s through a pipe 500 ft long with diameter 1 in. The inlet pressure $p_1 = 200$ psig, and the exit section is 100 ft higher than the inlet. What is the exit pressure p_2 if the friction head loss is 350 ft?

$$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L$$

$$(200)(144)/62.4 + v_1^2/2g + 0 = (p_2)(144)/62.4 + v_2^2/2g + 100 + 350 \qquad v_1^2/2g = v_2^2/2g \qquad p_2 = 5.00 \text{ psig}$$

8.109 A 30-in-diameter pipeline carries oil (s.g. = 0.86) at 600 000 barrels per day. The friction head loss is 10 ft per 1000 ft of pipe. Compute the pressure drop per mile.

$$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L$$

$$p_1/[(0.86)(62.4)] + v_1^2/2g + 0 = p_2/[(0.86)(62.4)] + v_2^2/2g + 0 + (10/1000)(5280)$$

$$v_1^2/2g = v_2^2/2g \qquad p_1 - p_2 = 2833 \text{ lb/ft}^2 \text{ or } 19.7 \text{ lb/in}^2$$

8.110 The long pipe in Fig. 8-64 is filled with water. When valve A is closed, $p_2 - p_1 = 12$ psi. When the valve is open and water flows at 10 ft^3/s, $p_1 - p_2 = 25$ psi. What is the friction head loss between 1 and 2 for the flowing condition?

$$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L \qquad v_1^2/2g = v_2^2/2g$$

Valve closed:

$$(p_2 - p_1)(144)/62.4 = z_1 - z_2 \qquad z_1 - z_2 = (12)(144)/62.4 = 27.69\ \text{ft}$$

Valve open:

$$(p_1 - p_2)(144)/62.4 + (z_1 - z_2) = h_L \qquad (25)(144)/62.4 + 27.69 = h_L \qquad h_L = 85.4\ \text{ft}$$

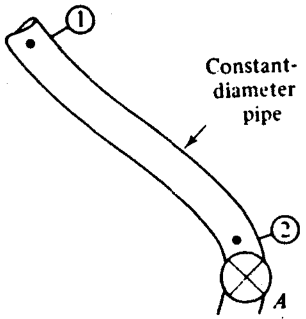


Fig. 8-64

8.111 Find the manometer reading in the lossless system of Fig. 8-65.

▮

$$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L$$

$$A_1v_1 = A_2v_2 \qquad [(\pi)(\tfrac{3}{12})^2/4](2) = [(\pi)(\tfrac{1}{12})^2/4](v_2) \qquad v_2 = 18.0\ \text{ft/s}$$

$$p_1/\gamma + 2^2/[(2)(32.2)] + 0 = 0 + 18.0^2/[(2)(32.2)] + 8 + 0 \qquad p_1/\gamma = 12.97\ \text{ft}$$

For the manometer, $12.97 + 2.5 - 13.6h = 0$, $h = 1.14$ ft.

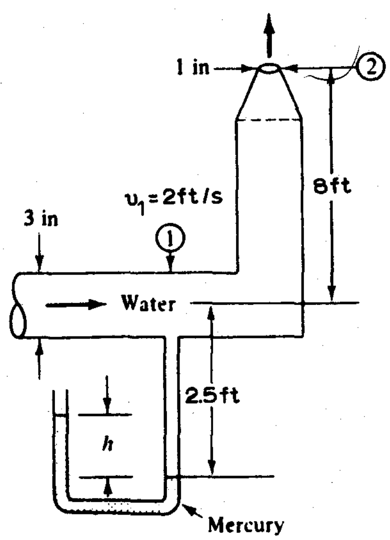


Fig. 8-65

8.112 In Fig. 8-66 on p. 188 the pipe exit losses are $(1.5)v^2/2g$, where v is the exit velocity. What is the exit weight flux of water?

▮ $$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L \qquad p_1 = (20)(144) + [(0.68)(62.4)](4) = 3050\ \text{lb/ft}^2$$

$$3050/62.4 + 0 + 5 = 0 + v_2^2/[(2)(32.2)] + 0 + (1.5)\{v_2^2/[(2)(32.2)]\} \qquad v_2 = 37.25\ \text{ft/s}$$

$$W = \gamma A v = (62.4)[(\pi)(\tfrac{2}{12})^2/4](37.25) = 50.7\ \text{lb/s}$$

8.113 In Fig. 8-67 the fluid is water, and the pressure gage reads $p_1 = 180$ kPa gage. If the mass flux is 15 kg/s, what is the head loss between 1 and 2?

▮

$$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L$$

$$M = \rho A v \qquad 15 = 1000[(\pi)(0.08)^2/4](v_1) \qquad v_1 = 2.984\ \text{m/s} \qquad 15 = 1000[(\pi)(0.05)^2/4](v_2) \qquad v_2 = 7.639\ \text{m/s}$$

$$180/9.79 + 2.984^2/[(2)(9.807)] + 0 = 0 + 7.639^2/[(2)(9.807)] + 12 + h_L \qquad h_L = 3.86\ \text{m}$$

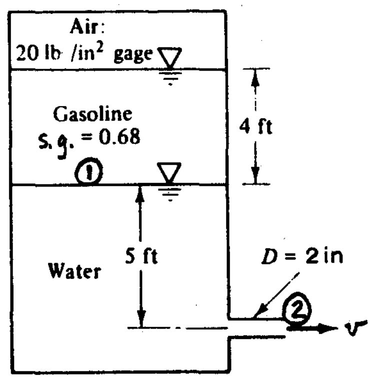


Fig. 8-66

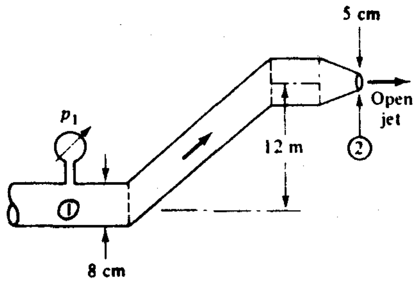


Fig. 8-67

8.114 Oil at specific gravity 0.761 flows from tank A to tank E, as shown in Fig. 8-68. Lost head items may be assumed to be as follows: A to $B = 0.60v_{12}^2/2g$; B to $C = 9.0v_{12}^2/2g$; C to $D = 0.40v_6^2/2g$; D to $E = 9.0v_6^2/2g$. Find the flow rate and the pressure at C.

▮

$$p_A/\gamma + v_A^2/2g + z_A = p_E/\gamma + v_E^2/2g + z_E + h_L$$

$$0 + 0 + 40.0 = 0 + 0 + 0 + (0.60v_{12}^2 + 9.0v_{12}^2 + 0.40v_6^2 + 9.0v_6^2)/[(2)(32.2)]$$

$$9.60v_{12}^2 + 9.40v_6^2 = 2576 \qquad A_1v_1 = A_2v_2 \qquad [(\pi)(\tfrac{12}{12})^2/4](v_{12}) = [(\pi)(\tfrac{6}{12})^2/4](v_6) \qquad v_6 = 4.00v_{12}$$

$$9.60v_{12}^2 + (9.40)(4.00v_{12})^2 = 2576 \qquad v_{12} = 4.012 \text{ ft/s}$$

$$Q = Av = [(\pi)(\tfrac{12}{12})^2/4](4.012) = 3.15 \text{ ft}^3/\text{s}$$

$$p_A/\gamma + v_A^2/2g + z_A = p_C/\gamma + v_C^2/2g + z_C + h_L$$

$$0 + 0 + 40.0 = (p_C)(144)/[(0.761)(62.4)] + 4.012^2/[(2)(32.2)]$$
$$+ (40.0 + 2) + [(0.60)(4.012)^2 + (9.0)(4.012)^2]/[(2)(32.2)]$$

$$p_C = -1.53 \text{ lb/in}^2$$

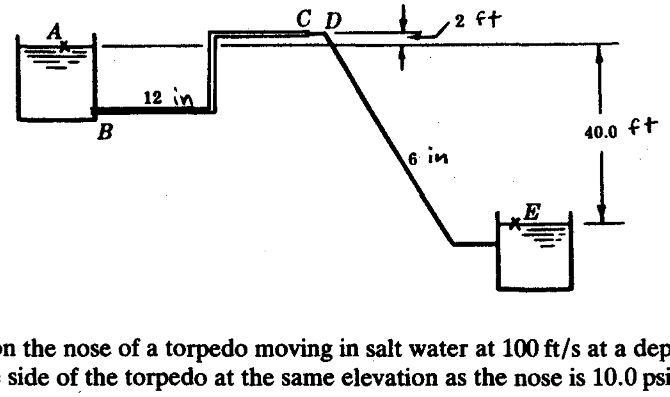


Fig. 8-68

8.115 (*a*) What is the pressure on the nose of a torpedo moving in salt water at 100 ft/s at a depth of 30.0 ft? (*b*) If the pressure at point C on the side of the torpedo at the same elevation as the nose is 10.0 psig, what is the relative velocity at that point?

▮ (*a*) In this case, greater clarity in the application of the Bernoulli equation may be attained by considering the relative motion of a stream of water past the stationary torpedo. The velocity of the nose of the torpedo will then be zero. Assume no lost head in the streamtube from a point A in the undisturbed water just ahead of the torpedo to a point B on the nose of the torpedo: $p_A/\gamma + v_A^2/2g + z_A = p_B/\gamma + v_B^2/2g + z_B + h_L$, $30.0 + 100^2/[(2)(32.2)] + 0 = p_B/\gamma + 0 + 0 + 0$, $p_B/\gamma = 185.3$ ft, $p_B = (64.2)(185.3)/144 = 82.6$ psi. This pressure is called the stagnation pressure and may be expressed as $p_s = p_0 + \rho v_0^2/2$.

(b)
$$p_A/\gamma + v_A^2/2g + z_A = p_C/\gamma + v_C^2/2g + z_C + h_L$$
$$30.0 + 100^2/[(2)(32.2)] + 0 = (10.0)(144)/64.2 + v_C^2/[(2)(32.2)] + 0 + 0 \qquad v_C = 102.4 \text{ ft/s}$$

8.116 A sphere is placed in an air stream which is at atmospheric pressure and is moving at 100.0 ft/s. Using the density of air constant at 0.00238 slug/ft³, calculate the stagnation pressure and the pressure on the surface of the sphere at a point B, 75° from the stagnation point, if the velocity there is 220.0 ft/s.

▌ From Prob. 8.115, $p_s = p_0 + \rho v_0^2/2 = (14.7)(144) + (0.00238)(100.0)^2/2 = 2129 \text{ lb/ft}^2$, or 14.8 lb/in^2. $p_S/\gamma + v_S^2/2g + z_S = p_B/\gamma + v_B^2/2g + z_B + h_L$, $2129/[(0.00238)(32.2)] + 0 + 0 = p_B/\gamma + 220.0^2/[(2)(32.2)] + 0 + 0$, $p_B/\gamma = 27\,029$ ft of air, $p_B = [(0.00238)(32.2)](27\,029) = 2071 \text{ lb/ft}^2$, or 14.4 lb/in^2.

8.117 A large closed tank is filled with ammonia (NH_3) under a pressure of 5.30 psig and at 65 °F. The ammonia discharges into the atmosphere through a small opening in the side of the tank. Neglecting friction losses, calculate the velocity of the ammonia leaving the tank assuming constant density. The gas constant for ammonia is 89.5 ft/°R.

▌ Apply Bernoulli's equation between the tank (1) and the atmosphere (2). $p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L$, $\gamma = p/RT$, $\gamma_{NH_3} = (5.30 + 14.7)(144)/[(89.5)(460 + 65)] = 0.06129 \text{ lb/ft}^3$, $(5.30)(144)/0.06129 + 0 + 0 = 0 + v_2^2/[(2)(32.2)] + 0 + 0$, $v_2 = 896$ ft/s.

8.118 Water at 90 °F is to be lifted from a sump at a velocity of 6.50 ft/s through the suction pipe of a pump. Calculate the theoretical maximum height of the pump setting under the following conditions: $p_{atm} = 14.25$ psia, $p_v = 0.70$ psia, and h_L in the suction pipe = 3 velocity heads.

▌ The minimum pressure at the entrance to the pump cannot be less than the vapor pressure of the liquid. Apply Bernoulli's equation between the water surface outside the suction pipe and the entrance to the pump: $p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L$, $(14.25)(144)/62.1 + 0 + 0 = (0.70)(144)/62.1 + 6.50^2/[(2)(32.2)] + z_2 + 3\{6.50^2/[(2)(32.2)]\}$, $z_2 = 28.8$ ft. (Under these conditions, serious damage due to cavitation will probably occur.)

8.119 For the Venturi meter shown in Fig. 8-69, the deflection of mercury in the differential gage is 14.3 in. Determine the flow of water through the meter if no energy is lost between A and B.

▌
$$p_A/\gamma + v_A^2/2g + z_A = p_B/\gamma + v_B^2/2g + z_B + h_L$$
$$p_A/\gamma + v_A^2/[(2)(32.2)] + 0 = p_B/\gamma + v_B^2/[(2)(32.2)] + 30.0/12 + 0 \qquad p_A/\gamma - p_B/\gamma = 0.01553(v_B^2 - v_A^2) + 2.500$$
$$A_A v_A = A_B v_B \qquad [(\pi)(\tfrac{12}{12})^2/4](v_A) = [(\pi)(\tfrac{6}{12})^2/4](v_B) \qquad v_A = 0.250 v_B$$

From the manometer, $p_A/\gamma + z + 14.3/12 - (13.6)(14.3/12) - z - 30.0/12 = p_B/\gamma$, $p_A/\gamma - p_B/\gamma = 17.52$ ft, $17.52 = 0.01553[v_B^2 - (0.250 v_B)^2] + 2.500$, $v_B = 32.12$ ft/s; $Q = Av = [(\pi)(\tfrac{6}{12})^2/4](32.12) = 6.31 \text{ ft}^3/\text{s}$.

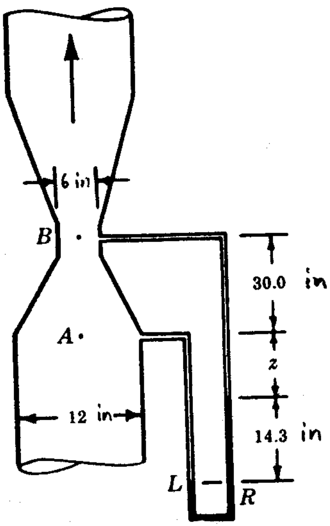


Fig. 8-69

8.120 For the meter in Fig. 8-69, consider air at 80 °F with the pressure at $A = 37.5$ psig. Consider a deflection of the gage of 14.3 in of water. Assuming that the specific weight of the air does not change between A and B and that the energy loss is negligible, determine the amount of air flowing in pounds per second.

▌ $$p_A/\gamma + v_A^2/2g + z_A = p_B/\gamma + v_B^2/2g + z_B + h_L$$

$$p_A/\gamma + v_A^2/[(2)(32.2)] + 0 = p_B/\gamma + v_B^2/[(2)(32.2)] + 30.0/12 + 0 \qquad p_A/\gamma - p_B/\gamma = 0.01553(v_B^2 - v_A^2) + 2.500$$

$$A_A v_A = A_B v_B \qquad [(\pi)(\tfrac{12}{12})^2/4](v_A) = [(\pi)(\tfrac{6}{12})^2/4](v_B) \qquad v_A = 0.250 v_B$$

$$\gamma = p/RT \qquad \gamma_{\text{air}} = (37.5 + 14.7)(144)/[(53.3)(460 + 80)] = 0.2612\ \text{lb/ft}^3$$

From the manometer, $p_A/\gamma + z + 14.3/12 - (62.4/0.2612)(14.3/12) - z - 30.0/12 = p_B/\gamma$, $p_A/\gamma - p_B/\gamma =$ 286.0 ft of air; $286.0 = 0.01553[v_B^2 - (0.250v_B)^2] + 2.500$, $v_B = 139.5$ ft/s; $W = \gamma A v =$ $0.2612[(\pi)(\tfrac{6}{12})^2/4](139.5) = 7.15$ lb/s.

8.121 Given a frictionless flow of water at 125.6 ft^3/s in a long, horizontal, conical pipe, of diameter 2 ft at one end and 6 ft at the other. The pressure head at the smaller end is 18 ft of water. Find the velocities at the two ends and the pressure head at the larger end.

▌ $$v_1 = Q/A_1 = 125.6/[(\pi)(2)^2/4] = 39.98\ \text{ft/s} \qquad v_2 = Q/A_2 = 125.6/[(\pi)(6)^2/4] = 4.44\ \text{ft/s}$$

$$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L$$

$$18 + 39.98^2/[(2)(32.2)] + 0 = p_2/\gamma + 4.44^2/[(2)(32.2)] + 0 + 0 \qquad p_2/\gamma = 42.5\ \text{ft of water}$$

8.122 Water flows through a long, horizontal, conical diffuser at the rate of 4.0 m^3/s. The diameter of the diffuser varies from 1.0 m to 2.0 m; the pressure at the smaller end is 8.0 kPa. Find the pressure at the downstream end of the diffuser, assuming frictionless flow and no separation from the walls.

▌ $$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L$$

$$v_1 = Q/A_1 = 4.0/[(\pi)(1.0)^2/4] = 5.093\ \text{m/s} \qquad v_2 = Q/A_2 = 4.0/[(\pi)(2.0)^2/4] = 1.273\ \text{m/s}$$

$$8.0/9.79 + 5.093^2/[(2)(9.807)] + 0 = p_2/9.79 + 1.273^2/[(2)(9.807)] + 0 + 0 \qquad p_2 = 20.13\ \text{kPa}$$

8.123 A vertical pipe 3 ft in diameter and 30 ft long has a pressure head at the upper end of 22 ft of water. When water flows through it with mean velocity 15 fps, the friction loss is 6 ft. Find the pressure head at the lower end of the pipe when the flow is ***(a)*** downward and ***(b)*** upward.

▌ ***(a)***
$$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L$$

$$22 + 15^2/[(2)(32.2)] + 30 = p_2/\gamma + 15^2/(2)(32.2)] + 0 + 6 \qquad p_2/\gamma = 46.0\ \text{ft}$$

(b)
$$p_2/\gamma + v_2^2/2g + z_2 = p_1/\gamma + v_1^2/2g + z_1 + h_L$$

$$p_2/\gamma + 15^2/[(2)(32.2)] + 0 = 22 + 15^2/[(2)(32.2)] + 30 + 6 \qquad p_2/\gamma = 58.0\ \text{ft}$$

8.124 A vertical conical pipe has diameter 1.5 ft at the top and 3.0 ft at the bottom, and is 60 ft long. The friction loss is 10 ft for flow in either direction when the velocity at the top is 30 fps and the pressure head there is 6.5 ft of water. Find the pressure head at the bottom when the flow is ***(a)*** downward and ***(b)*** upward.

▌ $$Q = A_1 v_1 = [(\pi)(1.5)^2/4](30) = 53.01\ \text{ft}^3/\text{s} \qquad v_2 = Q/A_2 = 53.01/[(\pi)(3.0)^2/4] = 7.50\ \text{ft/s}$$

(a)
$$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L$$

$$6.5 + 30^2/[(2)(32.2)] + 60 = p_2/\gamma + 7.50^2/[(2)(32.2)] + 0 + 10 \qquad p_2/\gamma = 69.6\ \text{ft}$$

(b)
$$p_2/\gamma + v_2^2/2g + z_2 = p_1/\gamma + v_1^2/2g + z_1 + h_L$$

$$p_2/\gamma + 7.5^2/[(2)(32.2)] + 0 = 6.5 + 30^2/[(2)(32.2)] + 60 + 10 \qquad p_2/\gamma = 89.6\ \text{ft}$$

8.125 The inclined pipe in Fig. 8-70 is of uniform diameter. The pressure at A is 20 psi and at B, 30 psi. In which direction is the flow, and what is the friction loss of the fluid, if the liquid has specific weight ***(a)*** 30 lb/ft^3 and ***(b)*** 100 lb/ft^3?

▌ $p_A/\gamma + v_A^2/2g + z_A = p_B/\gamma + v_B^2/2g + z_B + h_L$. Assume flow is from A to B.

(a) $(20)(144)/30 + v_A^2/2g + 25 = (30)(144)/30 + v_B^2/2g + 0 + h_L$, $v_A^2/2g = v_B^2/2g$, $h_L = -23.0$ ft. Since h_L is negative, flow is actually from B to A.
(b) $(20)(144)/100 + v_A^2/2g + 25 = (30)(144)/100 + v_B^2/2g + 0 + h_L$, $v_A^2/2g = v_B^2/2g$, $h_L = 10.6$ ft. Since h_L is positive, flow is from A to B, as assumed.

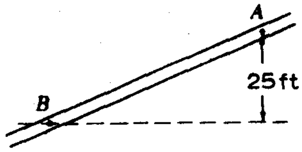


Fig. 8-70

8.126 In Fig. 8-70, if the difference in elevation between A and B is 10 m and the pressures at A and B are 150 kPa and 250 kPa, respectively, find the direction of flow and the head loss. The liquid has specific gravity 0.85.

▌ $p_A/\gamma + v_A^2/2g + z_A = p_B/\gamma + v_B^2/2g + z_B + h_L$. Assume flow is from A to B. $150/[(0.85)(9.79)] + v_A^2/2g + 10 = 250/[(0.85)(9.79)] + v_B^2/2g + 0 + h_L$, $v_A^2/2g = v_B^2/2g$, $h_L = -2.02$ m. Since h_L is negative, flow is actually from B to A.

8.127 An irrigation line carries water from a lake down into an arid valley floor 810 ft below the surface of the lake. The water is discharged through a nozzle with a jet velocity of 220 fps; the diameter of the jet is 4 in. Find the power of the jet and the power lost in friction.

▌
$$p_A/\gamma + v_A^2/2g + z_A = p_B/\gamma + v_B^2/2g + z_B + h_L \qquad 0 + 0 + 810 = 0 + 220^2/[(2)(32.2)] + 0 + h_L$$
$$h_L = 58.45 \text{ ft} \qquad Q = Av = [(\pi)(\tfrac{4}{12})^2/4](220) = 19.2 \text{ ft}^3/\text{s}$$
$$P_{\text{jet}} = Q\gamma v_2^2/2g = (19.2)(62.4)\{220^2/[(2)(32.2)]\} = 900\,000 \text{ ft}\cdot\text{lb/s} = 900\,000/550 = 1636 \text{ hp}$$
$$P_{\text{lost}} = Q\gamma h_L = (19.2)(62.4)(58.45) = 70\,000 \text{ ft}\cdot\text{lb/s} = 70\,000/550 = 127 \text{ hp}$$

8.128 Water is flowing in a channel, as shown in Fig. 8-71. Neglecting all losses, determine the two possible depths of flow y_1 and y_2.

▌
$$p_A/\gamma + v_A^2/2g + z_A = p_B/\gamma + v_B^2/2g + z_B + h_L$$
$$Q = A_A v_A = [(4)(10)](16.1) = 644 \text{ ft}^3/\text{s} \qquad v_B = Q/A_B = 644/(10y) = 64.4/y$$
$$0 + 16.1^2/[(2)(32.2)] + (8+4) = 0 + (64.4/y)^2/[(2)(32.2)] + y + 0 \qquad 64.4/y^2 + y - 16.02 = 0$$
$$y^3 - 16.02y^2 + 64.4 = 0 \qquad y_1 = 2.16 \text{ ft} \qquad y_2 = 15.8 \text{ ft}$$

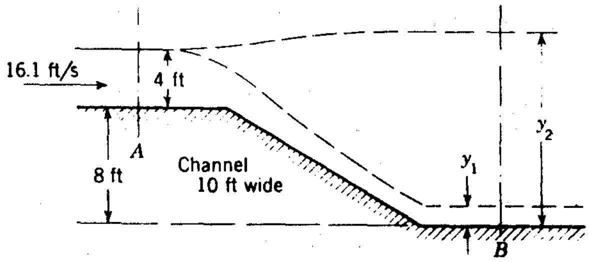


Fig. 8-71

8.129 Neglecting all losses, in Fig. 8-71 the channel narrows in the drop to 6 ft wide at section B. For uniform flow across section B, determine the two possible depths of flow.

▌
$$p_A/\gamma + v_A^2/2g + z_A = p_B/\gamma + v_B^2/2g + z_B + h_L$$
$$Q = A_A v_A = [(4)(10)](16.1) = 644 \text{ ft}^3/\text{s} \qquad v_B = Q/A_B = 644/(6y) = 107.3/y$$
$$0 + 16.1^2/[(2)(32.2)] + (8+4) = 0 + (107.3/y)^2/[(2)(32.2)] + y + 0 \qquad 178.8/y^2 + y - 16.02 = 0$$
$$y^3 - 16.02y^2 + 178.8 = 0 \qquad y_1 = 3.83 \text{ ft} \qquad y_2 = 15.3 \text{ ft}$$

8.130 If the losses from section A to section B of Fig. 8-71 are 1.9 ft, determine the two possible depths at section B.

▌ $$p_A/\gamma + v_A^2/2g + z_A = p_B/\gamma + v_B^2/2g + z_B + h_L$$

$$Q = A_A v_A = [(4)(10)](16.1) = 644 \text{ ft}^3/\text{s} \qquad v_B = Q/A_B = 644/(10y) = 64.4/y$$

$$0 + 16.1^2/[(2)(32.2)] + (8+4) = 0 + (64.4/y)^2/[(2)(32.2)] + y + 1.9 \qquad 64.4/y^2 + y - 14.12 = 0$$

$$y^3 - 14.12y^2 + 64.4 = 0 \qquad y_1 = 2.34 \text{ ft} \qquad y_2 = 13.8 \text{ ft}$$

8.131 High-velocity water flows up an inclined plane, as shown in Fig. 8-72. Neglecting all losses, calculate the two possible depths of flow at section B.

▌ $$p_A/\gamma + v_A^2/2g + z_A = p_B/\gamma + v_B^2/2g + z_B + h_L$$

$$Q = A_A v_A = [(\tfrac{500}{1000})(2)](9.806) = 9.806 \text{ m}^3/\text{s} \qquad v_B = Q/A_B = 9.806/(2y) = 4.903/y$$

$$0 + 9.806^2/[(2)(9.807)] + \tfrac{500}{1000} = 0 + (4.903/y)^2/[(2)(9.807)] + (2.5 + y) + 0 \qquad 1.226/y^2 + y - 2.903 = 0$$

$$y^3 - 2.903y^2 + 1.226 = 0 \qquad y_1 = 0.775 \text{ m} \qquad y_2 = 2.74 \text{ m}$$

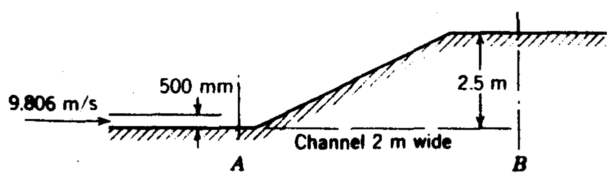


Fig. 8-72

8.132 In Fig. 8-72, the channel changes in width from 2 m at section A to 3 m at section B. For losses of 0.3 m between sections A and B, find the two possible depths at section B.

▌ $$p_A/\gamma + v_A^2/2g + z_A = p_B/\gamma + v_B^2/2g + z_B + h_L \qquad Q = A_A v_A = [(\tfrac{500}{1000})(2)](9.806) = 9.806 \text{ m}^3/\text{s}$$

$$v_B = Q/A_B = 9.806/(3y) = 3.269/y$$

$$0 + 9.806^2/[(2)(9.807)] + \tfrac{500}{1000} = 0 + (3.269/y)^2/[(2)(9.807)] + (2.5 + y) + 0.3$$

$$0.5448/y^2 + y - 2.603 = 0 \qquad y^3 - 2.603y^2 + 0.5448 = 0 \qquad y_1 = 0.510 \text{ m} \qquad y_2 = 2.52 \text{ m}$$

8.133 For losses of $0.05H$ through the nozzle of Fig. 8-73, what is the gage difference R in terms of H?

▌ $$1.2H + 1.2y + 1.2R - 3.0R - 1.2y = (0.95)(1.2)(H) \qquad R = 0.0333H$$

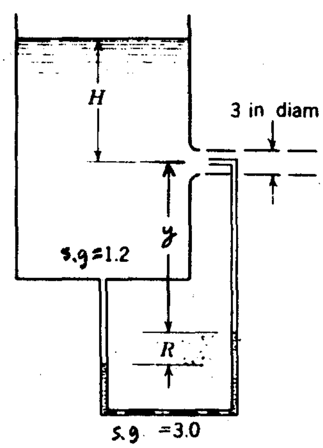


Fig. 8-73

8.134 Neglecting losses, calculate H in terms of R for Fig. 8-73.

▌ $1.2H + 1.2y + 1.2R - 3.0R - 1.2y = 1.2H$. Therefore, $R = 0$ for all H.

8.135 At point A in a pipeline carrying water, the diameter is 1 m, the pressure 100 kPa, and the velocity 1 m/s. At point B, 2 m higher than A, the diameter is 0.5 m and the pressure is 20 kPa. Determine the head loss and the direction of flow.

▌ Assume the direction of flow is from A to B. $p_A/\gamma + v_A^2/2g + z_A = p_B/\gamma + v_B^2/2g + z_B + h_L$, $Q = A_1 v_1 = [(\pi)(1)^2/4](1) = 0.7854\ \text{m}^3/\text{s}$, $v_2 = Q/A_2 = 0.7854/[(\pi)(0.5)^2/4] = 4.00\ \text{m/s}$, $100/9.79 + 1^2/[(2)(9.807)] + 0 = 20/9.79 + 4.00^2/[(2)(9.807)] + 2 + h_L$, $h_L = 5.40\ \text{m}$. Since h_L is positive, flow is from A to B as assumed.

8.136 Water is flowing in an open channel at a depth of 2 m and velocity of 3 m/s, as shown in Fig. 8-74. It then flows down a contracting chute into another channel where the depth is 1 m and the velocity is 10 m/s. Assuming frictionless flow, determine the difference in elevation of the channel floors.

▌ The velocities are assumed to be uniform over the cross sections, and the pressures hydrostatic. $p_A/\gamma + v_A^2/2g + z_A = p_B/\gamma + v_B^2/2g + z_B + h_L$, $0 + 3^2/[(2)(9.807)] + (y + 2) = 0 + 10^2/[(2)(9.807)] + 1 + 0$, $y = 3.64\ \text{m}$.

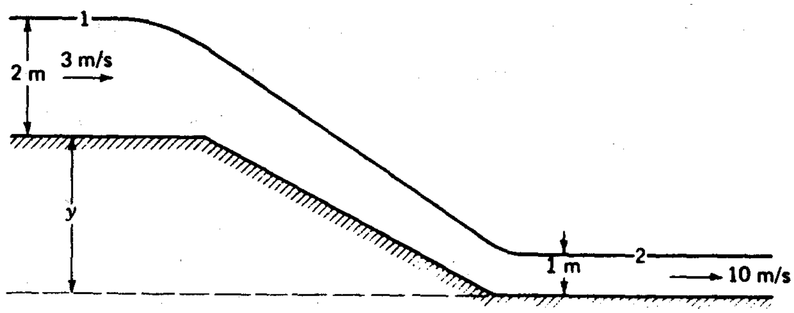


Fig. 8-74

8.137 For losses of 0.1 m, find the velocity at A in Fig. 8-75. The barometer reading is 750 mmHg.

▌
$$p_B/\gamma + v_B^2/2g + z_B = p_A/\gamma + v_A^2/2g + z_A + h_L$$
$$75/9.79 + 0 + 3 = [(13.6)(9.79)](0.750)/9.79 + v_A^2/[(2)(9.807)] + 0 + 0.1 \qquad v_A = 2.66\ \text{m/s}$$

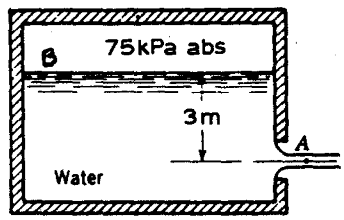


Fig. 8-75

8.138 For flow of 375 gpm in Fig. 8-76, determine H for losses of $5v^2/2g$.

▌
$$\frac{p_B}{\gamma} + \frac{v_B^2}{2g} + z_B = \frac{p_A}{\gamma} + \frac{v_A^2}{2g} + z_A + h_L \qquad v = \frac{Q}{A} = \frac{(375/7.48)/60}{(\pi)(\frac{6}{12})^2/4} = 4.255\ \text{ft/s}$$
$$0 + 0 + H = 0 + 4.255^2/[(2)(32.2)] + 0 + 5\{4.255^2/[(2)(32.2)]\} \qquad H = 1.69\ \text{ft}$$

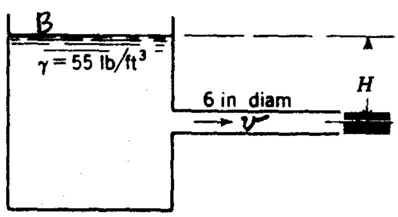


Fig. 8-76

8.139 For 1500-gpm flow and $H = 30$ ft in Fig 8-76, calculate the losses through the system in velocity heads, $Kv^2/2g$.

▌
$$\frac{p_B}{\gamma} + \frac{v_B^2}{2g} + z_B = \frac{p_A}{\gamma} + \frac{v_A^2}{2g} + z_A + h_L \qquad v = \frac{Q}{A} = \frac{(1500/7.48)/60}{(\pi)(\frac{6}{12})^2/4} = 17.02\ \text{ft/s}$$
$$0 + 0 + 30 = 0 + 17.02^2/[(2)(32.2)] + 0 + K\{17.02^2/[(2)(32.2)]\} \qquad K = 5.67 \qquad \text{(i.e., 5.67 velocity heads)}$$

8.140 The losses in Fig. 8-76 for $H = 20$ ft are $8(v^2/2g)$. What is the discharge?

▌ $p_B/\gamma + v_B^2/2g + z_B = p_A/\gamma + v_A^2/2g + z_A + h_L \qquad 0+0+20 = 0 + v_A^2/[(2)(32.2)] + 0 + 8\{v_A^2/[(2)(32.2)]\}$

$$v_A = 11.96\ \text{ft/s} \qquad Q = Av_A = [(\pi)(\tfrac{6}{12})^2/4](11.96) = 2.35\ \text{ft}^3/\text{s}$$

8.141 In Fig. 8-77, the losses up to section A are $5v_1^2/2g$ and the nozzle losses are $0.05v_2^2/2g$. Determine the discharge and pressure at A, if $H = 8$ m.

▌
$$p_B/\gamma + v_B^2/2g + z_B = p_C/\gamma + v_C^2/2g + z_C + h_L$$
$$0+0+8 = 0 + v_2^2/[(2)(9.807)] + 0 + 5\{v_1^2/[(2)(9.807)]\} + 0.05\{v_2^2/[(2)(9.807)]\}$$
$$0.05353v_2^2 + 0.2549v_1^2 - 8 = 0 \qquad A_1v_1 = A_2v_2 \qquad [(\pi)(\tfrac{150}{1000})^2/4](v_1) = [(\pi)(\tfrac{50}{1000})^2/4](v_2) \qquad v_1 = 0.1111v_2$$
$$0.05353v_2^2 + (0.2549)(0.1111v_2)^2 - 8 = 0 \qquad v_2 = 11.88\ \text{m/s} \qquad v_1 = (0.1111)(11.88) = 1.320\ \text{m/s}$$
$$Q = A_2v_2 = [(\pi)(\tfrac{50}{1000})^2/4](11.88) = 0.0233\ \text{m}^3/\text{s} \qquad p_B/\gamma + v_B^2/2g + z_B = p_A/\gamma + v_A^2/2g + z_A + h_L$$
$$0+0+8 = p_A/9.79 + 1.320^2/[(2)(9.807)] + 0 + 5\{1.320^2/[(2)(9.807)]\} \qquad p_A = 73.1\ \text{kN/m}^2$$

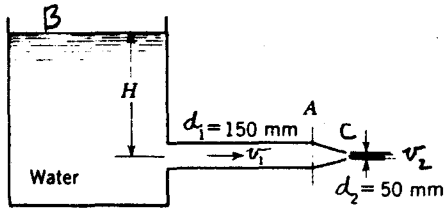


Fig. 8-77

8.142 For pressure at A of 25 kPa in Fig. 8-77 with the losses given in Prob. 8.141, determine the discharge and the head H.

▌
$$p_B/\gamma + v_B^2/2g + z_B = p_A/\gamma + v_A^2/2g + z_A + h_L$$
$$0+0+H = 25/9.79 + v_1^2/[(2)(9.807)] + 0 + 5\{v_1^2/[(2)(9.807)]\}$$
$$H = 0.3059v_1^2 + 2.554 \tag{1}$$
$$p_B/\gamma + v_B^2/2g + z_B = p_C/\gamma + v_C^2/2g + z_C + h_L$$
$$0+0+H = 0 + v_2^2/[(2)(9.807)] + 0 + 5\{v_1^2/[(2)(9.807)]\} + 0.05\{v_2^2/[(2)(9.807)]\}$$
$$0.05353v_2^2 + 0.2549v_1^2 - H = 0 \qquad A_1v_1 = A_2v_2 \qquad [(\pi)(\tfrac{150}{1000})^2/4](v_1) = [(\pi)(\tfrac{50}{1000})^2/4](v_2) \qquad v_2 = 9.000v_1$$
$$(0.05353)(9.000v_1)^2 + 0.2549v_1^2 = H \tag{2}$$

Solving Eqs. (*1*) and (*2*) simultaneously, $(0.05353)(9.000v_1)^2 + 0.2549v_1^2 = 0.3059v_1^2 + 2.554$, $v_1 = 0.7720$ m/s; $Q = A_1v_1 = [(\pi)(\tfrac{150}{1000})^2/4](0.7720) = 0.0136\ \text{m}^3/\text{s}$, $H = (0.3059)(0.7720)^2 + 2.554 = 2.736$ m.

8.143 The system shown in Fig. 8-78 involves 6-in.-i.d. pipe. The exit nozzle diameter is 3 in. What is the velocity v_e of flow leaving the nozzle? Neglect losses.

▌
$$p_1/\gamma + v_1^2/2g + z_1 = p_A/\gamma + v_A^2/2g + z_A + h_L$$
$$(14.7)(144)/62.4 + 0 + 40 = 4000/62.4 + v_A^2/[(2)(32.2)] + 0 + 0$$
$$v_A = 25.14\ \text{ft/s} \qquad A_Av_A = A_Dv_D \qquad [(\pi)(\tfrac{6}{12})^2](25.14) = [(\pi)(\tfrac{3}{12})^2](v_e) \qquad v_e = 100.56\ \text{ft/s}$$

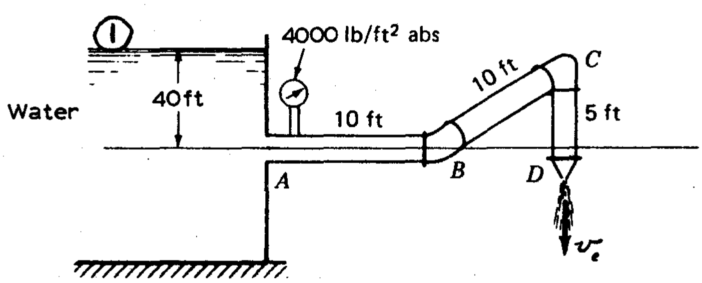


Fig. 8-78

8.144 In Fig. 8-79, $H = 6$ m and $h = 5.75$ m. Calculate the head loss.

▌ $$v_2 = \sqrt{2gh} = \sqrt{(2)(9.807)(5.75)} = 10.62 \text{ m/s} \qquad p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L$$
$$0 + 0 + 6 = 0 + 10.62^2/[(2)(9.807)] + 0 + h_L \qquad h_L = 0.250 \text{ m}$$

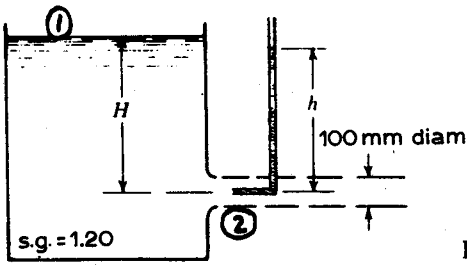


Fig. 8-79

8.145 In Fig. 8-80, 0.1 m³/s of water flows from section 1 to section 2 with losses of $0.4(v_1 - v_2)^2/2g$. If $p_1 = 100$ kPa, find p_2.

▌ $$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L$$
$$v_1 = Q/A_1 = (0.1)/[(\pi)(0.300)^2/4] = 1.415 \text{ m/s} \qquad v_2 = Q/A_2 = (0.1)/[(\pi)(0.450)^2/4] = 0.629 \text{ m/s}$$
$$100/9.79 + 1.415^2/[(2)(9.807)] + 0 = p_2/9.79 + 0.629^2/[(2)(9.807)] + 0 + \frac{0.4(1.415 - 0.629)^2}{(2)(9.807)} \qquad p_2 = 100.8 \text{ kPa}$$

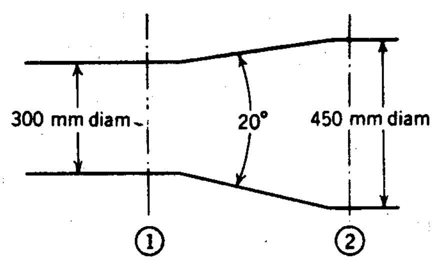


Fig. 8-80

8.146 Neglecting losses, determine the discharge in Fig. 8-81.

▌ $$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L \qquad [(0.86)(62.4)](3)/62.4 + 0 + 4 = 0 + v_2^2/[(2)(32.2)] + 0 + 0$$
$$v_2 = 20.59 \text{ ft/s} \qquad Q = Av = [(\pi)(\tfrac{2}{12})^2/4](20.59) = 0.45 \text{ ft}^3/\text{s}$$

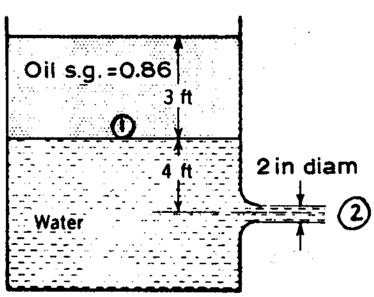


Fig. 8-81

8.147 A pipeline leads from one reservoir to another which has its water surface 10 m lower. For a discharge of 1.0 m³/s, determine the losses in meters and in kilowatts.

▌ $$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L \qquad 0 + 0 + 10 = 0 + 0 + 0 + h_L \qquad h_L = 10 \text{ m}$$
$$\text{Losses} = Q\gamma h_L = (1.0)(9.79)(10) = 97.9 \text{ kW}$$

8.148 In the siphon of Fig. 8-82, $h_1 = 1$ m, $h_2 = 3$ m, $d_1 = 3$ m, and $d_2 = 5$ m, and the losses to section 2 are $2.6v_2^2/2g$, with 10 percent of the losses occurring before section 1. Find the discharge and the pressure at section 1.

$$p_B/\gamma + v_B^2/2g + z_B = p_2/\gamma + v_2^2/2g + z_2 + h_L$$

$$0 + 0 + (1 + 3) = 0 + v_2^2/[(2)(9.807)] + 0 + 2.6v_2^2/[(2)(9.807)] \qquad v_2 = 4.668 \text{ m/s}$$

$$Q = A_2v_2 = [(\pi)(5)^2/4](4.668) = 91.7 \text{ m}^3/\text{s} \qquad v_1 = Q/A_1 = 91.7/[(\pi)(3)^2/4] = 12.97 \text{ m/s}$$

$$p_B/\gamma + v_B^2/2g + z_B = p_1/\gamma + v_1^2/2g + z_1 + h_L$$

$$0 + 0 + (1 + 3) = p_1/9.79 + 12.97^2/[(2)(9.807)] + 3 + 0.10\{(2.6)(4.668)^2/[(2)(9.807)]\}$$

$$p_1 = -77.0 \text{ kPa}$$

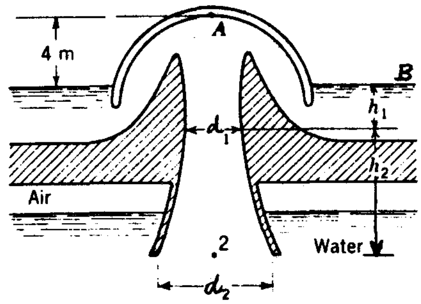


Fig. 8-82

8.149 Find the pressure at A of Prob. 8.148 if it is a stagnation point (velocity zero).

$$p_B/\gamma + v_B^2/2g + z_B = p_A/\gamma + v_A^2/2g + z_A + h_L \qquad 0 + 0 + 0 = p_A/9.79 + 0 + 4 + 0 \qquad p_A = -39.2 \text{ kPa}$$

8.150 In the friction-free siphon shown in Fig. 8-83, what are the pressures of the water in the tube at B and at A?

$$p_D/\gamma + v_D^2/2g + z_D = p_C/\gamma + v_C^2/2g + z_C + h_L$$

$$0 + 0 + 3.0 = 0 + v_C^2/[(2)(9.807)] + 0 + 0 \qquad v_C = 7.67 \text{ m/s} = v_B = v_A$$

$$p_D/\gamma + v_D^2/2g + z_D = p_B/\gamma + v_B^2/2g + z_B + h_L$$

$$0 + 0 + 3.0 = p_B/9.79 + 7.67^2/[(2)(9.807)] + (3.0 + 1.5) + 0 \qquad p_B = -44.0 \text{ kPa}$$

$$p_D/\gamma + v_D^2/2g + z_D = p_A/\gamma + v_A^2/2g + z_A + h_L$$

$$0 + 0 + 3.0 = p_A/9.79 + 7.67^2/[(2)(9.807)] + 3.0 + 0 \qquad p_A = -29.4 \text{ kPa}$$

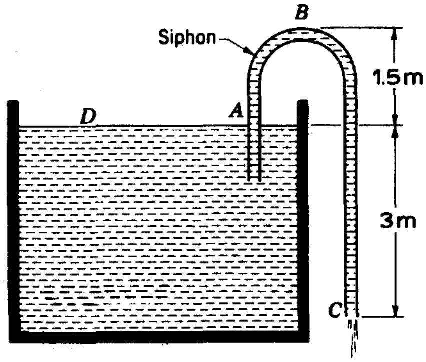


Fig. 8-83

8.151 If the vapor pressure of water is 0.1799 m of water, how high (h) above the free surface can point B be in Prob. 8.150 before the siphon action breaks down? Assume atmospheric pressure is 101 kPa.

$p_D/\gamma + v_D^2/2g + z_D = p_B/\gamma + v_B^2/2g + z_B + h_L$. Using absolute pressures and considering that $v_B = 0$ at maximum h when the siphon action breaks down, $101/9.79 + 0 + 3.0 = 0.1799 + 0 + (3.0 + h) + 0$, $h = 10.14$ m.

CHAPTER 9
Flow in Closed Conduits

9.1 Water at 10 °C flows in a 150-mm-diameter pipe at a velocity of 5.5 m/s. Is this flow laminar or turbulent?

$N_R = dv/\nu = (0.150)(5.5)/(1.30 \times 10^{-6}) = 634\,615$. Since $634\,615 > 4000$, the flow is turbulent.

9.2 SAE10 oil at 68 °F flows in a 9-in-diameter pipe. Find the maximum velocity for which the flow will be laminar.

$N_R = \rho dv/\mu$. For laminar flow, assume $N_R \le 2000$. $2000 = (1.68)(\frac{9}{12})(v)/(1.70 \times 10^{-3})$, $v = 2.70$ ft/s.

9.3 The accepted transition Reynolds number for flow past a smooth sphere is 250 000. At what velocity will this occur for airflow at 20 °C past a 10-cm-diameter sphere?

$$N_R = dv/\nu \qquad 250\,000 = (0.10)(v)/(1.51 \times 10^{-5}) \qquad v = 37.8 \text{ m/s}$$

9.4 Repeat Prob. 9.3 if the fluid is (*a*) water at 20° and (*b*) hydrogen at 20 °C ($\nu = 1.08 \times 10^{-4}$ m²/s).

(*a*) $$N_R = dv/\nu \qquad 250\,000 = (0.10)(v)/(1.02 \times 10^{-6}) \qquad v = 2.55 \text{ m/s}$$
(*b*) $$N_R = dv/\nu \qquad 250\,000 = (0.10)(v)/(1.08 \times 10^{-4}) \qquad v = 270 \text{ m/s}$$

9.5 A $\frac{1}{2}$-in-diameter water pipe is 60 ft long and delivers water at 5 gpm at 20 °C. What fraction of this pipe is taken up by the entrance region?

$$Q = (5)(0.002228) = 0.01114 \text{ ft}^3/\text{s} \qquad V = Q/A = 0.01114/[(\pi)(0.5/12)^2/4] = 8.170 \text{ ft/s} \qquad N_R = dv/\nu$$

From Table A-2, $\nu = 1.02 \times 10^{-6}$ m²/s at 20 °C, which equals 1.10×10^{-5} ft²/s; hence, $N_R = (0.5/12)(8.170)/(1.10 \times 10^{-5}) = 30\,947$. Since $30\,947 > 4000$, the flow is turbulent and for entrance length, $L_e/d = 4.4N_R^{1/6} = (4.4)(30\,947)^{1/6} = 25$. The actual pipe has $L/d = 60/[(\frac{1}{2})/12] = 1440$; hence,

$$\frac{L_e/d}{L/d} = \frac{L_e}{L} = \frac{25}{1440} = 0.017 \quad \text{or} \quad 1.7 \text{ percent}$$

9.6 An oil with $\rho = 900$ kg/m³ and $\nu = 0.0002$ m²/s flows upward through an inclined pipe as shown in Fig. 9-1. Assuming steady laminar flow, (*a*) verify that the flow is up and find the (*b*) head loss between section 1 and section 2, (*c*) flow rate, (*d*) velocity, and (*e*) Reynolds number. Is the flow really laminar?

(*a*) $$\text{HGL} = z + p/\rho g \qquad \text{HGL}_1 = 0 + 350\,000/[(900)(9.807)] = 39.65 \text{ m}$$
$$\text{HGL}_2 = (10)(\sin 40°) + 250\,000/[(900)(9.807)] = 34.75 \text{ m}$$

Since $\text{HGL}_1 > \text{HGL}_2$, the flow is upward.

(*b*) $$h_f = \text{HGL}_1 - \text{HGL}_2 = 39.65 - 34.75 = 4.90 \text{ m}$$
(*c*) $$\mu = \rho\nu = (900)(0.0002) = 0.180 \text{ kg/(m}\cdot\text{s)}$$
$$Q = \frac{\pi\rho g d^4 h_f}{128\mu L} = \frac{(\pi)(900)(9.807)(\frac{6}{100})^4(4.90)}{(128)(0.180)(10)} = 0.00764 \text{ m}^3/\text{s}$$
(*d*) $$v = Q/A = 0.00764/[(\pi)(\frac{6}{100})^2/4] = 2.70 \text{ m/s}$$
(*e*) $$N_R = dv/\nu = (\frac{6}{100})(2.70)/0.0002 = 810$$

This value of N_R is well within the laminar range; hence, the flow is most likely laminar.

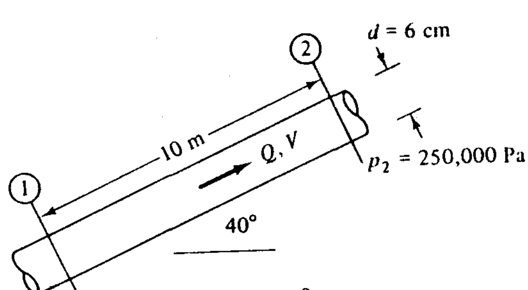


Fig. 9-1

9.7 For flow of SAE10 oil through a 100-mm-diameter pipe, for what flow rate in cubic meters per hour would we expect transition to turbulence at (*a*) 20 °C [$\mu = 0.104$ Pa · s], and (*b*) 100 °C [$\mu = 0.0056$ Pa · s]?

▌ Assume transition to turbulence occurs at $N_R = 2300$. $N_R = \rho dv/\mu$.

(*a*) $2300 = (869)(0.100)(v)/0.104 \qquad v = 2.753$ m/s

$$Q = Av = [(\pi)(0.100)^2/4](2.753) = 0.0216 \text{ m}^3/\text{s} \quad \text{or} \quad 77.76 \text{ m}^3/\text{h}$$

(*b*) $2300 = (869)(0.100)(v)/0.0056 \qquad v = 0.1482$ m/s

$$Q = Av = [(\pi)(0.100)^2/4](0.1482) = 0.00116 \text{ m}^3/\text{s} \quad \text{or} \quad 4.18 \text{ m}^3/\text{h}$$

9.8 A fluid at 20 °C flows at 0.8 L/s through an 100-mm-diameter pipe. Determine whether the flow is laminar or turbulent if the fluid is (*a*) hydrogen ($\nu = 1.08 \times 10^{-4}$ m²/s), (*b*) air, (*c*) gasoline ($\nu = 4.06 \times 10^{-7}$ m²/s), (*d*) water, (*e*) mercury ($\nu = 1.15 \times 10^{-7}$ m²/s), or (*f*) glycerin.

▌ $$N_R = dv/\nu \qquad v = Q/A = (0.8 \times 10^{-3})/[(\pi)(0.100)^2/4] = 0.1019 \text{ m/s}$$

(*a*) $N_R = (0.100)(0.1019)/(1.08 \times 10^{-4}) = 94$ (laminar)

(*b*) $N_R = (0.100)(0.1019)/(1.51 \times 10^{-5}) = 675$ (laminar)

(*c*) $N_R = (0.100)(0.1019)/(4.06 \times 10^{-7}) = 25\,099$ (turbulent)

(*d*) $N_R = (0.100)(0.1019)/(1.02 \times 10^{-6}) = 9990$ (turbulent)

(*e*) $N_R = (0.100)(0.1019)/(1.15 \times 10^{-7}) = 88\,609$ (turbulent)

(*f*) $N_R = (0.100)(0.1019)/(1.18 \times 10^{-3}) = 9$ (laminar)

9.9 Oil (s.g. = 0.9, $\nu = 0.0003$ m²/s) enters a 50-mm-diameter tube. Estimate the entrance length if the flow rate is 1 L/s.

▌ $N_R = dv/\nu$

$$v = Q/A = 0.001/[(\pi)(0.050)^2/4] = 0.5093 \text{ m/s} \qquad N_R = (0.050)(0.5093)/0.0003 = 85 \quad \text{(laminar)}$$

$$L_e/d = 0.06N_R \qquad L_e = (0.050)(0.06)(85) = 0.255 \text{ m}$$

9.10 What is the Reynolds number for a flow of oil (s.g. = 0.8, $\mu = 0.00200$ lb · s/ft²) in a 6-in-diameter pipe at a flow rate of 10 ft³/s. Is the flow laminar or turbulent?

▌ $$v = Q/A = 10/[(\pi)(\tfrac{6}{12})^2/4] = 50.9 \text{ ft/s}$$

$$N_R = \rho dv/\mu = [(0.8)(1.94)](\tfrac{6}{12})(50.9)/0.00200 = 19\,749 \quad \text{(turbulent)}$$

9.11 Gasoline at a temperature of 20 °C flows at the rate of 2 L/s through a pipe of inside diameter 60 mm. Find the Reynolds number.

▌ $$v = Q/A = (2 \times 10^{-3})/[(\pi)(0.060)^2/4] = 0.707 \text{ m/s}$$

$$N_R = \rho dv/\mu = (719)(0.060)(0.707)/(2.92 \times 10^{-4}) = 104\,452$$

9.12 The Reynolds number for fluid in a pipe of 10 in diameter is 2000. What will be the Reynolds number in a 6-in-diameter pipe forming an extension of the 10-in pipe? Take the flow as incompressible.

▌ $N_R = dv/\nu$. Since ν is constant, $[N_R/(dv)]_1 = [N_R/(dv)]_2$, $A_1v_1 = A_2v_2$, $[(\pi)(\tfrac{10}{12})^2/4](v_1) = [(\pi)(\tfrac{6}{12})^2/4](v_2)$, $v_1 = 0.360v_2$, $2000/[(\tfrac{10}{12})(0.360v_2)] = (N_R)_2/[(\tfrac{6}{12})(v_2)]$, $(N_R)_2 = 3333$.

9.13 Water is flowing through capillary tubes A and B into tube C, as shown in Fig. 9-2. If $Q_A = 3$ mL/s in tube A, what is the largest Q_B allowable in tube B for laminar flow in tube C? The water is at a temperature of 40 °C. With the calculated Q_B, what kind of flow exists in tubes A and B?

▌ For laminar flow, assume $N_R \leq 2300$. $N_R = dv/\nu$. In tube C, $2300 = (0.006)(v_C)/(6.56 \times 10^{-7})$, $v_C = 0.2515$ m/s; $Q_C = A_Cv_C = [(\pi)(0.006)^2/4](0.2515) = 7.11 \times 10^{-6}$ m³/s, or 7.11 mL/s, $Q_B = 7.11 - 3 = 4.11$ mL/s. In tube A, $v_A = Q_A/A_A = (3 \times 10^{-6})/[(\pi)(0.005)^2/4] = 0.1528$ m/s,

$$N_R = (0.005)(0.1528)/(6.56 \times 10^{-7}) = 1165 \quad \text{(laminar)}$$

In tube B, $v_B = Q_B/A_B = (4.11 \times 10^{-6})/[(\pi)(0.004)^2/4] = 0.3271$ m/s,

$$N_R = (0.004)(0.3271)/(6.56 \times 10^{-7}) = 1995 \qquad \text{(turbulent)}$$

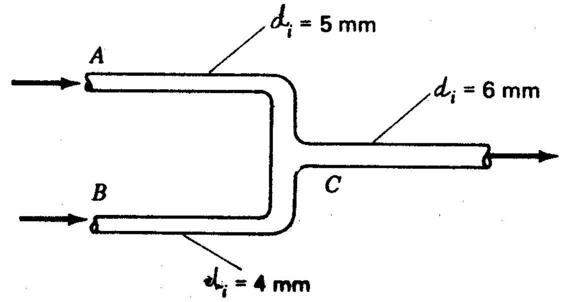


Fig. 9-2

9.14 Incompressible steady flow of water occurs in a tube of constant cross section, as shown in Fig. 9-3. What is the head loss between sections A and B?

$$p_A/\gamma + v_A^2/2g + z_A = p_B/\gamma + v_B^2/2g + z_B + h_L$$

$$(90)(144)/62.4 + v_A^2/2g + 0 = (30)(144)/62.4 + v_B^2/2g + 100 + h_L$$

$$v_A^2/2g = v_B^2/2g \qquad h_L = 38.5 \text{ ft}$$

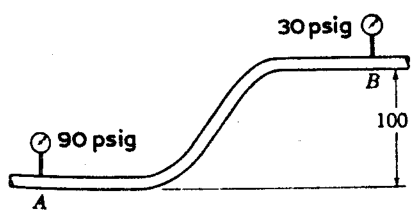


Fig. 9-3

9.15 Water flows through a pipe at 5 L/s, as shown in Fig. 9-4. If gage pressures of 12.5 kPa, 11.5 kPa, and 10.3 kPa are measured for p_1, p_2 and p_3, respectively, what are the head losses between 1 and 2 and 1 and 3?

$$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + (h_L)_{1-2} \qquad 12.5/9.79 + v_1^2/2g + 10 = 11.5/9.79 + v_2^2/2g + 10 + (h_L)_{1-2}$$

$$v_1^2/2g = v_2^2/2g \qquad (h_L)_{1-2} = 0.1021 \text{ m} \qquad p_1/\gamma + v_1^2/2g + z_1 = p_3/\gamma + v_3^2/2g + z_3 + (h_L)_{1-3}$$

$$v_1 = Q/A_1 = (5 \times 10^{-3})/[(\pi)(0.050)^2/4] = 2.546 \text{ m/s} \qquad v_3 = Q/A_3 = (5 \times 10^{-3})/[(\pi)(0.030)^2/4] = 7.074 \text{ m/s}$$

$$12.5/9.79 + 2.546^2/[(2(9.807)] + 10 = 10.3/9.79 + 7.074^2/[(2)(9.807)] + 0 + (h_L)_{1-3} \qquad (h_L)_{1-3} = 8.00 \text{ m}$$

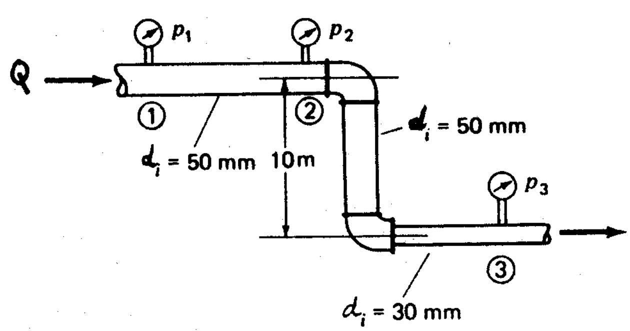


Fig. 9-4

9.16 A large oil reservoir has a pipe of 3 in diameter and 7000-ft length connected to it, as shown in Fig. 9-5. Assuming laminar flow through the pipe, compute the amount of oil issuing out of the pipe as a free jet.

Compute the velocity and Reynolds number to see if the flow is laminar. $\nu_{oil} = 1 \times 10^{-4}\ \text{ft}^2/\text{s}$. Neglect entrance losses to the pipe.

▮ $p_2/\gamma + v_2^2/2g + z_2 = p_3/\gamma + v_3^2/2g + z_3 + h_L \qquad p_2/62.4 + v_2^2/2g + 0 = 0 + v_3^2/2g + 0 + h_f \qquad v_2^2/2g = v_3^2/2g$

$$p_2/62.4 = h_f \tag{1}$$

$$Q = \frac{\pi \rho g d^4 h_f}{128\mu L}$$

$$h_f = \frac{128Q\mu L}{\pi\rho g d^4} = \frac{128[(\pi d^2/4)(v_2)]\mu L}{\pi \rho g d^4} = \frac{32 v_2 \nu L}{g d^2} = \frac{(32)(v_2)(1\times 10^{-4})(7000)}{(32.2)(3/12)^2} = 11.13 v_2 \tag{2}$$

$$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L \qquad 0 + 0 + 10 = p_2/62.4 + v_2^2/[(2)(32.2)] + 0 + 0$$

$$p_2/62.4 = 10 - v_2^2/[(2)(32.2)] \tag{3}$$

Equating h_f from Eqs. (1) and (2),

$$p_2/62.4 = 11.13 v_2 \tag{4}$$

Equating $p_2/62.4$ from Eqs. (3) and (4), $11.13v_2 = 10 - v_2^2/[(2)(32.2)]$, $v_2^2 + 716.8v_2 - 644 = 0$, $v_2 = 0.8973$ ft/s; $Q = Av = [(\pi)(\frac{3}{12})^2/4](0.8973) = 0.0440\ \text{ft}^3/\text{s}$; $N_R = dv/\nu = (\frac{3}{12})(0.8973)/(1\times 10^{-4}) = 2243$ (barely laminar).

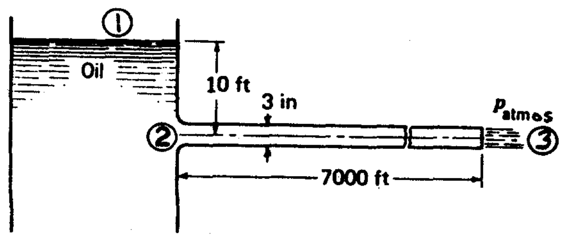


Fig. 9-5

9.17 If 140 L/s of water flows through the system shown in Fig. 9-6, calculate the total head loss between 2 and 3.

▮ $$p_2/\gamma + v_2^2/2g + z_2 = p_3/\gamma + v_3^2/2g + z_3 + h_L$$

$v_2 = Q/A_2 = (140\times 10^{-3})/[(\pi)(0.300)^2/4] = 1.981$ m/s $\qquad v_3 = Q/A_3 = (140\times10^{-3})/[(\pi)(0.150)^2/4] = 7.922$ m/s

$$p_2/9.79 + 1.981^2/[(2)(9.807)] + 0 = 0 + 7.922^2/[(2)(9.807)] + 15 + h_L \qquad h_L = p_2/9.79 - 18.00$$

$$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L \qquad 0 + 0 + 30 = p_2/9.79 + 1.981^2/[(2)(9.807)] + 0 + 0$$

$$p_2/9.79 = 29.80\ \text{m} \qquad h_L = 29.80 - 18.00 = 11.80\ \text{m}$$

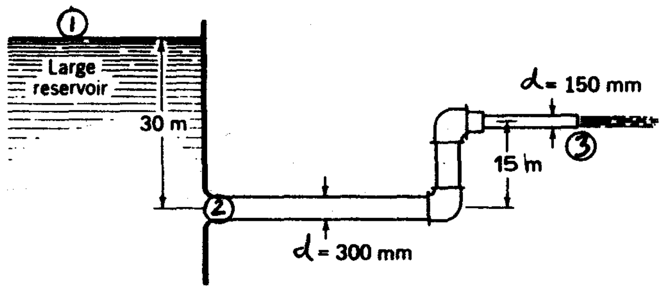


Fig. 9-6

9.18 Determine the maximum velocity for laminar flow for (*a*) medium fuel oil at 60 °F ($\nu = 4.75 \times 10^{-5}\ \text{ft}^2/\text{s}$) flowing through a 6-in pipe and (*b*) water at 60 °F flowing in the 6-in pipe.

▮ For laminar flow, assume $N_R \le 2000$. $N_R = dv/\nu$.

(*a*) $\qquad 2000 = (\frac{6}{12})(v)/(4.75\times 10^{-5}) \qquad v = 0.190$ ft/s

(*b*) $\qquad 2000 = (\frac{6}{12})(v)(1.21\times 10^{-5}) \qquad v = 0.0484$ ft/s

9.19 Determine the type of flow occurring in a 12-in pipe when (*a*) water at 60 °F flows at a velocity of 3.50 ft/s and (*b*) heavy fuel oil at 60 °F ($\nu = 221 \times 10^{-5}$ ft²/s) flows at the same velocity.

▮ $N_R = dv/\nu$

(*a*) $$N_R = (\tfrac{12}{12})(3.50)/(1.21 \times 10^{-5}) = 289\,256 \qquad \text{(turbulent)}$$

(*b*) $$N_R = (\tfrac{12}{12})(3.50)/(221 \times 10^{-5}) = 1584 \qquad \text{(laminar)}$$

9.20 For laminar flow conditions, what size pipe will deliver 90 gpm of medium fuel oil at 40 °F ($\nu = 6.55 \times 10^{-5}$ ft²/s)?

▮ $Q = (90)(0.002228) = 0.2005$ ft³/s. For laminar flow, assume $N_R \le 2000$. $N_R = dv/\nu$, $v = Q/A = 0.2005/(\pi d^2/4) = 0.2553d^{-2}$, $2000 = (d)(0.2553d^{-2})/(6.55 \times 10^{-5})$, $d = 1.95$ ft.

9.21 What is the Reynolds number of flow of 0.4 m³/s of oil (s.g. = 0.86, $\mu = 0.025$ Pa · s) through a 450-mm-diameter pipe?

▮ $$v = Q/A = 0.4/[(\pi)(0.450)^2/4] = 2.515 \text{ m/s}$$

$$N_R = \rho\, dv/\mu = [(0.86)(1000)](0.450)(2.515)/0.025 = 38\,932$$

9.22 An oil with s.g. = 0.85 and $\nu = 1.8 \times 10^{-5}$ m²/s flows in a 10-cm-diameter pipe at 0.50 L/s. Is the flow laminar or turbulent?

▮ $$v = Q/A = (0.50)(1000)/[(\pi)(\tfrac{10}{100})^2/4] = 0.06366 \text{ m/s}$$

$$N_R = dv/\nu = (\tfrac{10}{100})(0.06366)/(1.8 \times 10^{-5}) = 354 \qquad \text{(laminar)}$$

9.23 Fluid with kinematic viscosity 0.00015 ft²/s flows through a pipe of diameter 9 in. What is the maximum velocity for laminar flow?

▮ For laminar flow, assume $N_R \le 2000$. $N_R = dv/\nu$, $2000 = (\tfrac{9}{12})(v)/0.00015$, $v = 0.400$ ft/s.

9.24 An oil with $\nu = 0.005$ ft²/s flows through a 6-in-diameter pipe at 10 ft/sec. Is the flow laminar or turbulent?

▮ $$N_R = dv/\nu = (\tfrac{6}{12})(10)/0.005 = 1000 \qquad \text{(laminar)}$$

9.25 Hydrogen at atmospheric pressure and 50 °F has a kinematic viscosity of 0.0011 ft²/s. Determine the maximum mass flow rate for laminar flow in a 3-in-diameter pipe. $\gamma = 0.00540$ lb/ft³.

▮ For laminar flow, assume $N_R \le 2000$. $N_R = dv/\nu$, $2000 = (\tfrac{3}{12})(v)/0.0011$, $v = 8.80$ ft/s; $W = \gamma A v = (0.00540)[(\pi)(\tfrac{2}{12})^2/4](8.80) = 0.001037$ lb/s.

9.26 Air at 1500 kPa abs and 100 °C flows in a 20-mm-diameter tube. What is the maximum laminar flow rate?

▮ For laminar flow, assume $N_R \le 2000$. $N_R = \rho dv/\mu$, $\rho = p/RT = (1.5 \times 10^6)/[(287)(273 + 100)] = 14.01$ kg/m³, $2000 = (14.01)(0.020)(v)/(2.17 \times 10^{-5})$, $v = 0.1549$ m/s; $Q = Av = [(\pi)(0.020)^2/4](0.1549) = 0.0000487$ m³/s, or 0.0487 L/s.

9.27 What is the hydraulic radius of a rectangular air duct 8 in by 14 in?

▮ $$R_h = A/p_w = [(8)(14)]/(8 + 8 + 14 + 14) = 2.55 \text{ in} \quad \text{or} \quad 2.55 \text{ ft}$$

9.28 What is the percentage difference between the hydraulic radii of 30-cm-diameter circular and 30-cm square ducts?

▮ $$R_h = A/p_w$$

$$(R_h)_{\text{circular}} = [(\pi)(30)^2/4]/[(\pi)(30)] = 7.50 \text{ cm} \qquad (R_h)_{\text{square}} = (30)(30)/(30 + 30 + 30 + 30) = 7.50 \text{ cm}$$

Since they are equal, the percentage difference is zero. Note that the hydraulic radius of a circular section is one-fourth its diameter.

9.29 Two pipes, one circular and one square, have the same cross-sectional area. Which has the larger hydraulic radius, and by what percentage?

I Let d = diameter of the circular pipe and a = the side of the square one. Since they have the same cross-sectional area, $\pi d^2/4 = a^2$, $a = \sqrt{\pi}d/2$; $(R_h)_{circular} = d/4 = 0.2500d$, $(R_h)_{square} = A/p_w = a^2/4a = a/4$. Since $a = \sqrt{\pi}d/2$, $(R_h)_{square} = (\sqrt{\pi}d/2)/4 = 0.2216d$, hence, the circular pipe has the larger hydraulic radius by $(0.2500 - 0.2216)/0.2216 = 0.128$, or 12.8 percent.

9.30 Steam of weight density 0.26 lb/ft³ flows at 100 fps through a circular pipe. What is the shearing stress at the wall, if the friction factor is 0.015?

I
$$\tau_0 = (f/4)(\gamma)(v^2/2g) = (0.015/4)(0.26)\{100^2/[(2)(32.2)]\} = 0.151 \text{ lb/ft}^2$$

9.31 Glycerin at 68 °F flows 120 ft through a 6-in-diameter new wrought iron pipe at a velocity of 10.0 ft/s. Determine the head loss due to friction.

I
$$h_f = (f)(L/d)(v^2/2g) \qquad N_R = \rho dv/\mu = (2.44)(\tfrac{6}{12})(10.0)/(3.11\times10^{-2}) = 392$$

Since $N_R < 2000$, the flow is laminar and $f = 64/N_R = \frac{64}{392} = 0.1633$, $h_f = 0.1633[120/(\frac{6}{12})]\{10.0^2/[(2)(32.2)]\} = 60.9$ ft.

9.32 SAE10 oil flows through a cast iron pipe at a velocity of 1.0 m/s. The pipe is 45.0 m long and has a diameter of 150 mm. Find the head loss due to friction.

I
$$h_f = (f)(L/d)(v^2/2g) \qquad N_R = \rho dv/\mu = (869)(\tfrac{150}{1000})(1.0)/0.0814 = 1601$$

Since $N_R < 2000$, the flow is laminar and $f = 64/N_R = \frac{64}{1601} = 0.0400$, $h_f = 0.0400[45.0/(\frac{150}{1000})]\{1.0^2/[(2)(9.807)]\} = 0.612$ m.

9.33 A 60-mm-diameter pipe (Fig. 9-7) contains glycerin at 20 °C flowing at 8.5 m³/h. Verify that the flow is laminar. For the pressure measurements shown, is the flow ascending or descending? What is the head loss for these pressures?

I
$$v = Q/A = (8.5/3600)/[(\pi)(0.060)^2/4] = 0.835 \text{ m/s}$$
$$N_R = \rho dv/\mu = (1258)(0.060)(0.835)/1.49 = 63 \qquad \text{(laminar)}$$
$$\text{HGL} = z + p/\rho g \qquad \text{HGL}_A = 0 + (2.0)(101\,400)/[(1258)(9.807)] = 16.44 \text{ m}$$
$$\text{HGL}_B = 12 + (3.8)(101\,400)/[(1258)(9.807)] = 43.23 \text{ m}$$

Hence, the flow is from B to A (i.e., descending). Head loss = 43.23 − 16.44 = 26.79 m.

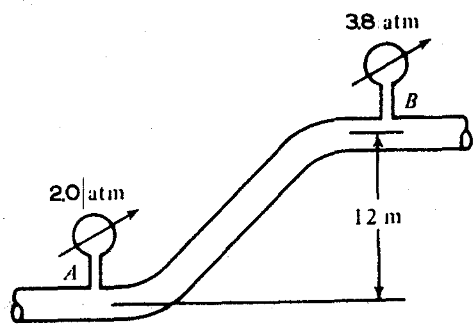


Fig. 9-7

9.34 For the data of Prob. 9.33, compute the theoretical head loss if the pipe length is 30 m between A and B. Compare with the head loss corresponding to the measured pressures.

I
$$Q = \frac{\pi\rho g d^4 h_f}{128\mu L} \qquad \frac{8.5}{3600} = \frac{(\pi)(1258)(9.807)(0.060)^4(h_f)}{(128)(1.49)(30)} \qquad h_f = 26.89 \text{ m}$$

which is only 10 cm greater than the value found in Prob. 9.33.

9.35 Two horizontal infinite plates keep a distance h apart as the upper plate moves at speed V, as in Fig. 9-8. There is a fluid of constant viscosity and constant pressure between the plates. If $V = 5$ m/s and $h = 20$ mm, compute the shear stress at the plates, given that the fluid is SAE 30 oil at 20 °C.

$N_R = \rho h V/\mu = (888)(0.020)(5)/0.440 = 202$. Since the flow is laminar,

$$\tau = \mu \frac{\partial u}{\partial y}$$

where $u = (V/h)(y)$, $\tau = (\mu)(V/h) = 0.440[5/(2.0/100)] = 110$ Pa.

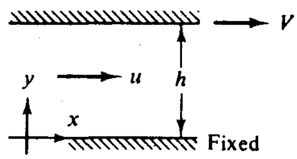


Fig. 9-8

9.36 Find the head loss per unit length when a fluid of s.g. 0.86 and kinematic viscosity 0.008 ft²/s flows in a 3-in-diameter pipe at a rate of 5 gpm.

$Q = (5)(0.002228) = 0.01114\ \text{ft}^3/\text{s}$ $\quad h_f = (f)(L/d)(v^2/2g)$ $\quad v = Q/A = 0.01114/[(\pi)(\frac{3}{12})^2/4] = 0.2269$ ft/s

$$N_R = dv/\nu = (\tfrac{3}{12})(0.2269)/0.008 = 7.09$$

Since $N_R < 2000$, the flow is laminar and $f = 64/N_R = 64/7.09 = 9.03$, $h_f = 9.03[1/(\frac{3}{12})]\{0.2269^2/[(2)(32.2)]\} = 0.0289$ ft per foot of length.

9.37 Tests made on a certain 12-in-diameter pipe showed that, when $V = 10$ fps, $f = 0.015$. The fluid used was water at 60 °F. Find the unit shear at the wall and at radii of 0, 0.2, 0.3, 0.5, and 0.75 times the pipe radius.

$$\tau_0 = (f/4)(\gamma)(V^2/2g) = (0.015/4)(62.4)\{10^2/[(2)(32.2)]\} = 0.3634\ \text{lb/ft}^2$$

The stress distribution is linear; hence,

r/r_o	τ, lb/ft²
0	0
0.2	0.0727
0.3	0.1090
0.5	0.1817
0.75	0.2726

9.38 If oil with a kinematic viscosity of 0.005 ft²/s weighs 54 lb/ft³, what will be the flow rate and head loss in a 3600-ft length of 4-in-diameter pipe when the Reynolds number is 800?

$N_R = dv/\nu$ $\quad 800 = (\frac{4}{12})(v)/0.005$ $\quad v = 12.00$ ft/s $\quad Q = Av = [(\pi)(\frac{4}{12})^2/4](12.00) = 1.047\ \text{ft}^3/\text{s}$

$f = 64/N_R = 64/800 = 0.0800$ $\quad h_f = (f)(L/d)(v^2/2g) = 0.0800[3600/(\frac{4}{12})]\{12.00^2/[(2)(32.2)]\} = 1932$ ft

9.39 How much power is lost per kilometer of length when a viscous fluid ($\mu = 0.20$ Pa · s) flows in a 200-mm-diameter pipeline at 1.00 L/s? The fluid has a density of 840 kg/m³.

$$v = Q/A = (1.00 \times 10^{-3})/[(\pi)(0.200)^2/4] = 0.03183\ \text{m/s}$$

$$N_R = \rho dv/\mu = (840)(0.200)(0.03183)/0.20 = 26.74 \qquad h_f/L = (f)(1/d)(v^2/2g)$$

Since $N_R < 2000$, the flow is laminar and $f = 64/N_R = 64/26.74 = 2.393$, $h_f/L = 2.393[1/(0.200)]\{0.03183^2/[(2)(9.807)]\} = 0.0006180$ m, $P/L = Q\gamma h_f/L = Q\rho g h_f/L = (1.00 \times 10^{-3})(840)(9.807)(0.0006180) = 0.00509$ W/m $= 5.09$ W/km.

9.40 Calculate the discharge of the system in Fig. 9-9, neglecting all losses except through the pipe.

Assume laminar flow and use the conversion 1.0 centipoise = 0.0002089 lb · s/ft².

$$v = \frac{\gamma\, \Delta h\, d^2}{32\mu L} = \frac{(55)(18)[(\frac{1}{4})/12]^2}{(32)(0.0002089)(16)} = 4.017\ \text{ft/s}$$

$$N_R = \rho dv/\mu = (\gamma/g)(d)(v)/\mu = (55/32.2)[(\tfrac{1}{4})/12](4.017)/0.0002089 = 684 \qquad \text{(laminar)}$$

$$Q = Av = \{(\pi)[(\tfrac{1}{4})/12]^2/4\}(4.017) = 0.00137\ \text{ft}^3/\text{s}$$

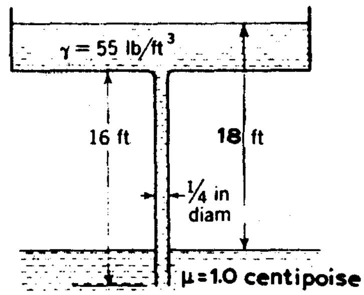


Fig. 9-9

9.41 In Fig. 9-10, $H = 25$ m, $L = 40$ m, $\theta = 30°$, $d = 8$ mm, $\gamma = 10$ kN/m^3, and $\mu = 0.08$ Pa · s. Find the head loss per unit length of pipe and the discharge in liters per minute.

▌ Assuming laminar flow,

$$v = \frac{\gamma\,\Delta h\, d^2}{32\mu L} = \frac{(10^4)(25)(8\times 10^{-3})^2}{(32)(0.08)(40)} = 0.1563 \text{ m/s}$$

$$N_R = \rho dv/\mu = (\gamma/g)(d)(v)/\mu = [10^4/9.807](8\times 10^{-3})(0.1563)/0.08 = 16 \qquad \text{(laminar)}$$

$$Q = Av = [(\pi)(8\times 10^{-3})^2/4](0.1563) = 7.857\times 10^{-6}\text{ m}^3/\text{s} = 0.471 \text{ L/min}$$

$$\Delta h/L = \tfrac{25}{40} = 0.625 \text{ m/m}$$

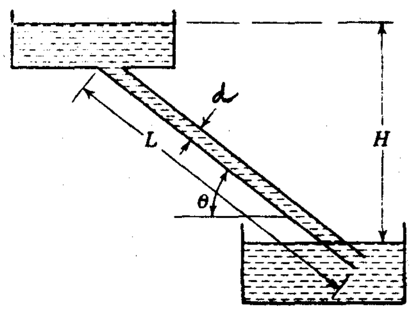


Fig. 9-10

9.42 For the data of Prob. 9.41, find H if the velocity is 0.1 m/s.

▌ By proportion, $H = (0.1/0.1563)(25) = 16.0$ m.

9.43 Water flows at 0.20 m^3/s through a 300-mm-diameter, 120-m-long pipe, under a pressure difference of 280 mmHg. Find the friction factor.

▌ $h_f = (f)(L/d)(v^2/2g)$. From the pressure gradient, $h_f = (13.6/1)(0.280) = 3.808$ m; $v = Q/A = 0.20/[(\pi)(0.300)^2/4] = 2.829$ m/s, $3.808 = (f)[120/(0.300)]\{2.829^2/[(2)(9.807)]\}$, $f = 0.0233$.

9.44 Use the Blasius equation for determination of friction factor to find the horsepower per mile required to pump 3.0 ft^3/s of liquid ($\nu = 3.3\times 10^{-4}$ ft^2/s, $\gamma = 60$ lb/ft^3) through an 18-in pipeline.

▌

$$h_f = (f)(L/d)(v^2/2g) \qquad f = 0.316/N_R^{1/4} \qquad v = Q/A = 3.0/[(\pi)(\tfrac{18}{12})^2/4] = 1.698 \text{ ft/s}$$

$$N_R = dv/\nu = (\tfrac{18}{12})(1.698)/(3.3\times 10^{-4}) = 7718 \qquad f = 0.316/7718^{1/4} = 0.03371$$

$$h_f = 0.03371[5280/(\tfrac{18}{12})]\{1.698^2/[(2)(32.2)]\} = 5.312 \text{ ft}$$

$$P = Q\gamma h_f = (3.0)(60)(5.312) = 956.2 \text{ ft}\cdot\text{lb/s per mile} \qquad 956.2/550 = 1.74 \text{ hp per mile}$$

9.45 Determine the head loss per kilometer required to maintain a velocity of 3 m/s in a 20-mm-diameter pipe, if $\nu = 4\times 10^{-5}$ m^2/s.

▌

$$h_f = (f)(L/d)(v^2/2g) \qquad N_R = dv/\nu = (0.020)(3)/(4\times 10^{-5}) = 1500 \qquad \text{(laminar)}$$

$$f = 64/N_R = 64/1500 = 0.04267 \qquad h_f = 0.04267[1000/(0.020)]\{3^2/[(2)(9.807)]\} = 979.0 \text{ m per km}$$

9.46 Fluid flows through a 10-mm-diameter tube at a Reynolds number of 1800. The head loss is 30 m in a 120-m length of tubing. Calculate the discharge in liters per minute.

▮ $h_f = (f)(L/d)(v^2/2g)$. Since $N_R < 2000$, flow is laminar and $f = 64/N_R = 64/1800 = 0.03556$, $30 = 0.03556[120/(0.010)]\{v^2/[(2)(9.807)]\}$, $v = 1.174$ m/s; $Q = Av = [(\pi)(0.010)^2/4](1.174) = 92.21 \times 10^{-6}$ m^3/s $= 5.53$ L/min.

9.47 Oil of absolute viscosity 0.00210 lb · s/ft^2 and specific gravity 0.850 flows through 10 000 ft of 12-in-diameter cast iron pipe at the rate of 1.57 cfs. What is the lost head in the pipe?

▮
$$h_f = (f)(L/d)(v^2/2g) \qquad v = Q/A = 1.57/[(\pi)(\tfrac{12}{12})^2/4] = 1.999 \text{ ft/s}$$
$$N_R = \rho dv/\mu = [(0.850)(1.94)](\tfrac{12}{12})(1.999)/0.00210 = 1570 \qquad \text{(laminar)}$$
$$f = 64/N_R = \tfrac{64}{1570} = 0.04076 \qquad h_f = 0.04076[10\,000/(\tfrac{12}{12})]\{1.999^2/[(2)(32.2)]\} = 25.3 \text{ ft}$$

9.48 When first installed between two reservoirs, a 4-in-diameter metal pipe of length 6000 ft conveyed 0.20 cfs of water. **(a)** If after 15 years a chemical deposit had reduced the effective diameter of the pipe to 3.0 in, what then would be the flow rate? Assume f remains constant. Assume no change in reservoir levels. **(b)** What would be the flow rate if in addition to the diamater change, f had doubled in value?

▮ **(a)** $(f_1)(L_1/d_1)(v_1^2/2g) = (f_2)(L_2/d_2)(v_2^2/2g)$. Since f, L, and g are constant and $v = Q/A = Q/(\pi d^2/4)$, $Q_1^2/d_1^5 = Q_2^2/d_2^5$, $0.20^2/4^5 = Q_2^2/3.0^5$, $Q_2 = 0.0974$ cfs.
(b) $(f_1)(Q_1^2/d_1^5) = (f_2)(Q_2^2/d_2^5)$. Since $f_2 = 2f_1$, $Q_1^2/d_1^5 = (2)(Q_2^2/d_2^5)$, $0.20^2/4^5 = (2)(Q_2^2/3.0^5)$, $Q_2 = 0.0689$ cfs.

9.49 A liquid with $\gamma = 58$ lb/ft^3 flows by gravity through a 1-ft tank and a 1-ft capillary tube at a rate of 0.15 ft^3/h, as shown in Fig. 9-11. Sections 1 and 2 are at atmospheric pressure. Neglecting entrance effects, compute the viscosity of the liquid in slugs per foot-second.

▮ $$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L \qquad v_2 = Q/A_2 = (0.15/3600)/[(\pi)(0.004)^2/4] = 3.316 \text{ ft/s}$$
$$0 + 0 + (1 + 1) = 0 + 3.316^2/[(2)(32.2)] + 0 + h_f \qquad h_f = 1.829 \text{ ft}$$

Assuming laminar flow,

$$h_f = \frac{32\mu Lv}{\gamma d^2} \qquad 1.829 = \frac{(32)(\mu)(1)(3.316)}{(58)(0.004)^2} \qquad \mu = 1.600 \times 10^{-5} \text{ slug/(ft} \cdot \text{s)}$$
$$N_R = \rho dv/\mu = (\gamma/g)(d)(v)/\mu = (58/32.2)(0.004)(3.316)/(1.600 \times 10^{-5}) = 1493 \qquad \text{(laminar)}$$

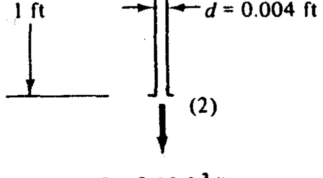


Fig. 9-11

9.50 In Prob. 9.49, suppose the flow rate is unknown but the liquid viscosity is 2.1×10^{-5} slug/(ft · s). What will be the flow rate in cubic feet per hour? Is the flow still laminar?

▮
$$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L$$
$$0 + 0 + (1 + 1) = 0 + v^2/[(2)(32.2)] + 0 + h_f \qquad h_f = 2 - 0.01553v^2$$

Assuming laminar flow,

$$h_f = 32\mu Lv/\gamma d^2 \qquad 2 - 0.01553v^2 = (32)(2.1\times10^{-5})(1)(v)/[(58)(0.004)^2]$$

$$v^2 + 46.63v - 128.8 = 0 \qquad v = 2.616\text{ ft/s}$$

$$Q = Av = [(\pi)(0.004)^2/4](2.616) = 0.00003287\text{ ft}^3/\text{s} = 0.118\text{ ft}^3/\text{h}$$

$$N_R = \rho dv/\mu = (\gamma/g)(d)(v)/\mu = (58/32.2)(0.004)(2.616)/(2.1\times10^{-5}) = 897 \qquad \text{(laminar)}$$

9.51 In the syringe of Fig. 9-12 the drug has $\rho = 900\text{ kg/m}^3$ and $\mu = 0.002\text{ Pa}\cdot\text{s}$. What steady force F is required to produce a flow of 0.4 mL/s through the needle? Neglect head loss in the larger cylinder.

▮

$$p_A/\rho g + v_A^2/2g + z_A = p_B/\rho g + v_B^2/2g + z_B + h_L$$

$$v_B = Q/A_B = 0.4\times10^{-6}/[(\pi)(0.25\times10^{-3})^2/4] = 8.149\text{ m/s}$$

$$N_R = \rho dv/\mu = (900)(0.25\times10^{-3})(8.149)/0.002 = 917 \qquad \text{(laminar)}$$

$$h_f = \frac{32\mu Lv}{\rho g d^2} = \frac{(32)(0.002)(0.020)(8.149)}{(900)(9.807)(0.25\times10^{-3})^2} = 18.91\text{ m}$$

$$p_A/[(900)(9.807)] + 0 + 0 = p_B/[(900)(9.807)] + (8.149)^2/[(2)(9.807)] + 0 + 18.91$$

$$p_A - p_B = 196\,788\text{ N/m}^2$$

$$F = (p_A - p_B)(A_{\text{piston}}) = 196\,788[(\pi)(0.010)^2/4] = 15.5\text{ N}$$

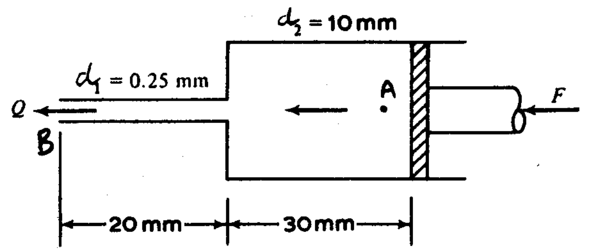


Fig. 9-12

9.52 Paint issues from the tank in Fig. 9-13 at $Q = 45\text{ ft}^3/\text{h}$. Find the kinematic viscosity. Is the flow laminar?

▮ $p_A/\rho g + v_A^2/2g + z_A = p_B/\rho g + v_B^2/2g + z_B + h_L \qquad v_B = Q/A_B = (\frac{45}{3600})/[(\pi)(0.5/12)^2/4] = 9.167\text{ ft/s}$

$$0 + 0 + 9 = 0 + 9.167^2/[(2)(32.2)] + 0 + h_f \qquad h_f = 7.695\text{ ft}$$

Assuming laminar flow,

$$h_f = \frac{128\nu LQ}{\pi g d^4} \qquad 7.695 = \frac{(128)(\nu)(6)(\frac{45}{3600})}{(\pi)(32.2)(0.5/12)^4} \qquad \nu = 0.0002444\text{ ft}^2/\text{s}$$

$$N_R = dv/\nu = (0.5/12)(9.167)/0.0002444 = 1563 \qquad \text{(laminar)}$$

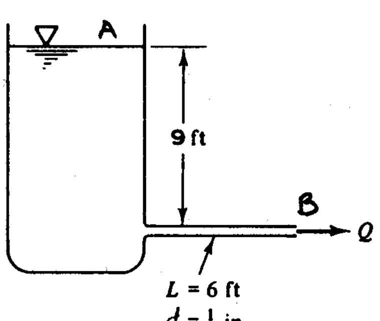


Fig. 9-13

9.53 In Prob. 9.52, what will the flow rate be if the paint properties are $\rho = 1.78\text{ slugs/ft}^3$ and $\mu = 0.00217\text{ lb}\cdot\text{s/ft}^2$?

▮ $p_A/\rho g + v_A^2/2g + z_A = p_B/\rho g + v_B^2/2g + z_B + h_L \qquad 0 + 0 + 9 = 0 + v^2/[(2)(32.2)] + 0 + h_f$

Assuming laminar flow,

$$h_f = \frac{32\mu L v}{\rho g d^2} = \frac{(32)(0.00217)(6)(v)}{(1.78)(32.2)(0.5/12)^2} = 4.187v$$

$$0+0+9 = 0 + v^2/[(2)(32.2)] + 0 + 4.187v \qquad v^2 + 269.6v - 579.6 = 0 \qquad v = 2.133 \text{ ft/s}$$

$$Q = Av = [(\pi)(0.5/12)^2/4](2.133) = 0.002908 \text{ ft}^3\text{/s} \quad \text{or} \quad 10.47 \text{ ft}^3\text{/h}$$

$$N_R = \rho d v/\mu = (1.78)(0.5/12)(2.133)/0.00217 = 73 \qquad \text{(laminar)}$$

9.54 The smaller tank in Fig. 9-14 is 50 m in diameter. If the fluid is ethanol at 20 °C, find the flow rate.

▌ $$p_A/\gamma + v_A^2/2g + z_A = p_B/\gamma + v_B^2/2g + z_B + h_L \qquad 0+0+(0.4+0.6) = 0+0+0+h_f$$

$$h_f = 1.000 \text{ m} = \frac{128\mu L Q}{\pi \rho g d^4} \qquad 1.000 = \frac{(128)(1.20\times 10^{-3})(0.8+0.4)(Q)}{(\pi)(788)(9.807)(0.002)^4}$$

$$Q = 2.107 \times 10^{-6} \text{ m}^3\text{/s} \quad \text{or} \quad 7.59 \text{ L/h}$$

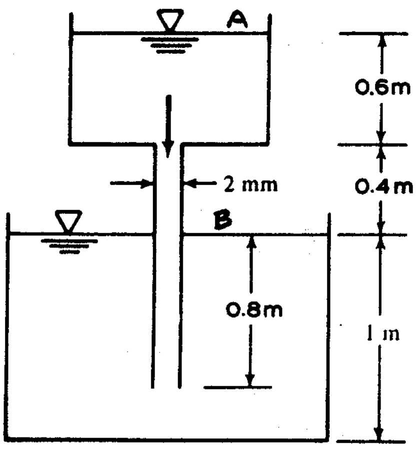


Fig. 9-14

9.55 For the system in Fig. 9-14, if the fluid has density of 920 kg/m³ and the flow rate is unknown, for what value of viscosity will the capillary Reynolds number exactly equal the critical value 2300?

▌ $$h_f = 1.000 \text{ m} = \frac{32\mu L v}{\rho g d^2} \qquad \text{(from Prob. 9.54)}$$

$$1.000 = \frac{(32)(\mu)(0.8+0.4)(v)}{(920)(9.807)(0.002)^2} \qquad v = \frac{0.0009398}{\mu}$$

$$N_R = \rho d v/\mu \qquad 2300 = (920)(0.002)(0.0009398/\mu)/\mu \qquad \mu = 0.000867 \text{ Pa}\cdot\text{s}$$

9.56 For the pressure measurements shown in Fig. 9-15, determine (*a*) whether the flow is up or down, and (*b*) the flow rate. Use $\rho = 917$ kg/m³ and $\mu = 0.290$ Pa · s.

▌ HGL $= z + p/\rho g$

(*a*)

$$\text{HGL}_B = 15 + (200)(1000)/[(917)(9.807)] = 37.24 \text{ m}$$

$$\text{HGL}_A = 0 + (600)(1000)/[(917)(9.807)] = 66.72 \text{ m}$$

Since $\text{HGL}_A > \text{HGL}_B$, the flow is from A to B (i.e., up).

(*b*) Assume flow is laminar.

$$h_f = \frac{128\mu L Q}{\pi \rho g d^4} = 66.72 - 37.24 = 29.48 \text{ m} \qquad L = \sqrt{15^2+20^2} = 25.00 \text{ m} \qquad 29.48 = \frac{(128)(0.290)(25.00)(Q)}{(\pi)(917)(9.807)(0.030)^4}$$

$$Q = 0.000727 \text{ m}^3\text{/s} \quad \text{or} \quad 2.617 \text{ m}^3\text{/h} \qquad v = Q/A = 0.000727/[(\pi)(0.030)^2/4] = 1.028 \text{ m/s}$$

$$N_R = \rho d v/\mu = (917)(0.030)(1.028)/0.290 = 98 \qquad \text{(laminar)}$$

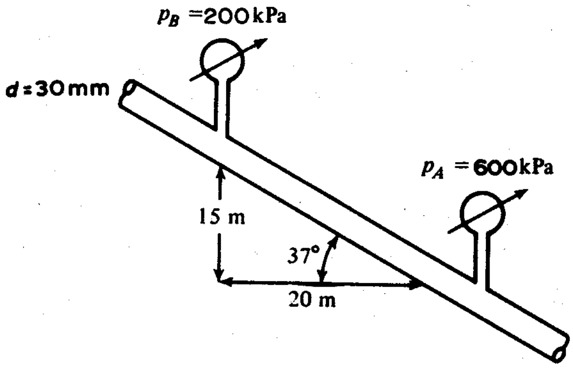


Fig. 9-15

9.57 Repeat Prob. 9.56 if the pressures are the same but there is a pump between A and B which adds a 10-m head rise in the flow direction. Is the flow still laminar?

▌ $h_f = \mathrm{HGL}_A - \mathrm{HGL}_B + h_{\mathrm{pump}}$. Using values of HGL_A and HGL_B from Prob. 9.56,

$$h_f = 66.72 - 37.24 + 10 = 39.48\ \mathrm{m} = \frac{128\mu LQ}{\pi\rho g d^4} \qquad L = 25.00\ \mathrm{m} \qquad \text{(from Prob. 9.56)}$$

$$39.48 = \frac{(128)(0.290)(25.00)(Q)}{(\pi)(917)(9.807)(0.030)^4} \qquad Q = 0.000974\ \mathrm{m^3/s} \quad \text{or} \quad 3.51\ \mathrm{m^3/h}$$

$$v = Q/A = 0.000974/[(\pi)(0.030)^2/4] = 1.378\ \mathrm{m/s}$$

$$N_R = \rho dv/\mu = (917)(0.030)(1.378)/0.290 = 131 \qquad \text{(laminar)}$$

9.58 Water at 40 °C flows from tank A to tank B as shown in Fig. 9-16. Find the volumetric flow, neglecting entrance losses to the capillary tube as well as exit losses.

▌ $p_A/\gamma + v_A^2/2g + z_A = p_B/\gamma + v_B^2/2g + z_B + h_L \qquad 0 + 0 + (0.22 + 0.1) = 0 + 0 + 0 + h_f \qquad h_f = 0.32$

Assume laminar flow.

$$h_f = \frac{128\mu LQ}{\pi\rho g d^4} \qquad 0.32 = \frac{(128)(6.51\times10^{-4})(0.22+0.08)(Q)}{(\pi)(992)(9.807)(0.001)^4} \qquad Q = 3.912\times10^{-7}\ \mathrm{m^3/s} = 1.41\ \mathrm{L/h}$$

$$v = Q/A = 3.912\times10^{-7}/[(\pi)(0.001)^2/4] = 0.4981\ \mathrm{m/s}$$

$$N_R = \rho dv/\mu = (992)(0.001)(0.4981)/(6.51\times10^{-4}) = 759 \qquad \text{(laminar)}$$

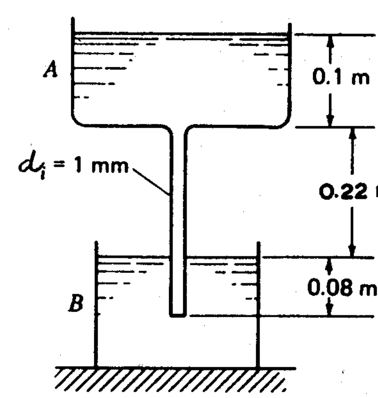


Fig. 9-16

9.59 In Prob. 9.58, what should the internal diameter of the tube be to permit a flow of 2.16 L/h?

▌ Assuming laminar flow, $Q \propto d^4$ or $d \propto Q^{1/4}$; hence

$$d = \left(\frac{2.16}{1.41}\right)^{1/4}(1\ \mathrm{mm}) = 1.534\ \mathrm{mm}$$

Computation of N_R shows the flow to be indeed laminar.

9.60 A hypodermic needle has an inside diameter of 0.3 mm and is 60 mm in length, as shown in Fig. 9-17. If the piston moves to the right at a speed of 18 mm/s and there is no leakage, what force F is needed on the piston? The medicine in the hypodermic has a viscosity μ of 0.980×10^{-3} Pa · s and its density ρ is 800 kg/m³. Consider flows in both needle and cylinder. Neglect exit losses from the needle as well as losses at the juncture of the needle and cylinder.

▮ For cylinder:

$$Q = Av = [(\pi)(0.005)^2/4](0.018) = 3.534 \times 10^{-7}\ \text{m}^3/\text{s}$$

$$N_R = \rho dv/\mu = (800)(0.005)(0.018)/(0.980 \times 10^{-3}) = 73 \qquad \text{(laminar)}$$

$$p = \frac{128\mu LQ}{\pi d^4} \qquad p_1 = \frac{(128)(0.980 \times 10^{-3})(0.050)(3.534 \times 10^{-7})}{(\pi)(0.005)^4} = 1.129\ \text{Pa}$$

For needle:

$$v = Q/A = 3.534 \times 10^{-7}/[(\pi)(0.3/1000)^2/4] = 5.000\ \text{m/s}$$

$$N_R = (800)(0.3/1000)(5.000)/(0.980 \times 10^{-3}) = 1224 \qquad \text{(laminar)}$$

$$p_2 = \frac{(128)(0.980 \times 10^{-3})(0.060)(3.534 \times 10^{-7})}{(\pi)(0.3/1000)^4} = 104\,525\ \text{Pa}$$

$$F = (\Delta p)(A_{\text{cylinder}}) = (104\,525 - 1.129)[(\pi)(0.005)^2/4] = 2.05\ \text{N}$$

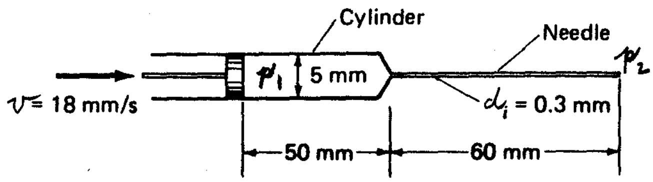


Fig. 9-17

9.61 In Prob. 9.60, suppose that medicine is drawn from a bottle at atmospheric pressure. What is the largest flow of fluid if the fluid has a vapor pressure of 4.8 kPa abs? Neglect losses in the cylinder.

▮
$$\Delta p = 101\,400 - 4800 = 96\,600\ \text{Pa} = \frac{128\mu LQ}{\pi d^4} \qquad 96\,600 = \frac{(128)(0.980 \times 10^{-3})(0.060)(Q_{\max})}{(\pi)(3 \times 10^{-4})^4}$$

$$Q_{\max} = 3.27 \times 10^{-7}\ \text{m}^3/\text{s} = 0.327\ \text{mL/s}$$

9.62 In Prob. 9.60, it took a force of 2.05 N to move the piston to the right at a speed of 18 mm/s. What should the inside diameter be for the cylinder if the force needed is only 1.2 N for the same piston speed? Neglect losses in cylinder.

▮
$$p_1 = \frac{F}{A} = \frac{1.2}{(\pi)(0.005)^2/4} = 61\,116\ \text{Pa} \qquad p = \frac{128\mu LQ}{\pi d^4} \qquad 61\,116 = \frac{(128)(0.980 \times 10^{-3})(0.060)(3.534 \times 10^{-7})}{\pi d^4}$$

$$d = 0.343\ \text{mm} \qquad N_R = \rho dv/\mu = (800)(0.343/1000)(0.018)/(0.980 \times 10^{-3}) = 5 \qquad \text{(laminar)}$$

9.63 Water at 70 °F flows through a new cast iron pipe at a velocity of 9.7 ft/s. The pipe is 1200 ft long and has a diameter of 6 in. Find the head loss due to friction.

▮
$$h_f = (f)(L/d)(v^2/2g) \qquad N_R = dv/\nu = (\tfrac{6}{12})(9.7)/(1.05 \times 10^{-5}) = 461\,905$$

From Table A-9, $\epsilon = 0.00085$ ft for new cast iron pipe; $\epsilon/d = 0.00085/(\frac{6}{12}) = 0.0017$. From Fig. A-5, $f = 0.0230$; $h_f = 0.0230[1200/(\frac{6}{12})]\{9.7^2/[(2)(32.2)]\} = 80.6$ ft.

9.64 A 96-in-diameter new cast iron pipe carries water at 60 °F. The head loss due to friction is 1.5 ft per 1000 ft of pipe. What is the discharge capacity of the pipe?

▮
$$h_f = (f)(L/d)(v^2/2g) \qquad 1.5 = (f)[1000/(\tfrac{96}{12})]\{v^2/[(2)(32.2)]\} \qquad fv^2 = 0.7728$$

Assume $f = 0.0150$; $(0.0150)(v^2) = 0.7728$, $v = 7.178$ ft/s; $N_R = dv/\nu = (\frac{96}{12})(7.178)/(1.21 \times 10^{-5}) = 4.75 \times 10^6$.

From Table A-9, $\epsilon = 0.00085$ ft for new cast iron pipe, $\epsilon/d = 0.00085/(\frac{96}{12}) = 0.000106$. From Fig. A-5, $f = 0.0124$. Evidently, the assumed value of f of 0.0150 was not the correct one. Try a value of f of 0.0124. $(0.0124)(v^2) = 0.7728$, $v = 7.894$ ft/s; $N_R = (\frac{96}{12})(7.894)/(1.21 \times 10^{-5}) = 5.22 \times 10^6$. From Fig. A-5, $f = 0.0124$. Hence, 0.0124 must be the correct value of f, and $v = 7.894$ ft/s. $Q = Av = [(\pi)(\frac{96}{12})^2/4](7.894) = 397$ ft³/s.

9.65 Water at 70 °F is being drained from an open tank through a 24-in-diameter, 130-ft-long new cast iron pipe, as shown in Fig. 9-18. Find the flow rate at which water is being discharged from the pipe. Neglect minor losses.

$$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L$$

$$h_L = h_f = (f)(L/d)(v^2/2g) = (f)[130/(\tfrac{24}{12})]\{v_2^2/[(2)(32.2)]\} = 1.009fv_2^2$$

$$0 + 0 + 150.5 = 0 + v_2^2/[(2)(32.2)] + 98.4 + 1.009fv_2^2$$

Assume $f = 0.0240$.

$$150.5 = v_2^2/[(2)(32.2)] + 98.4 + (1.009)(0.0240)(v_2^2) \qquad v_2 = 36.21 \text{ ft/s}$$

$$N_R = dv/\nu = (\tfrac{24}{12})(36.21)/(1.05 \times 10^{-5}) = 6.90 \times 10^6$$

From Table A-9, $\epsilon = 0.00085$, $\epsilon/d = 0.00085/(\frac{24}{12}) = 0.000425$. From Fig. A-5, $f = 0.0162$. Evidently, the assumed value of f of 0.0240 was not the correct one. Try a value of f of 0.0162.

$$150.5 = v_2^2/[(2)(32.2)] + 98.4 + (1.009)(0.0162)(v_2^2) \qquad v_2 = 40.43 \text{ ft/s}$$

$$N_R = (\tfrac{24}{12})(40.43)/(1.05 \times 10^{-5}) = 7.70 \times 10^6$$

From Fig. A-5, $f = 0.0162$. Hence, 0.0162 must be the correct value of f, and $v = 40.43$ ft/s. $Q = Av = [(\pi)(\frac{24}{12})^2/4](40.43) = 127$ ft³/s.

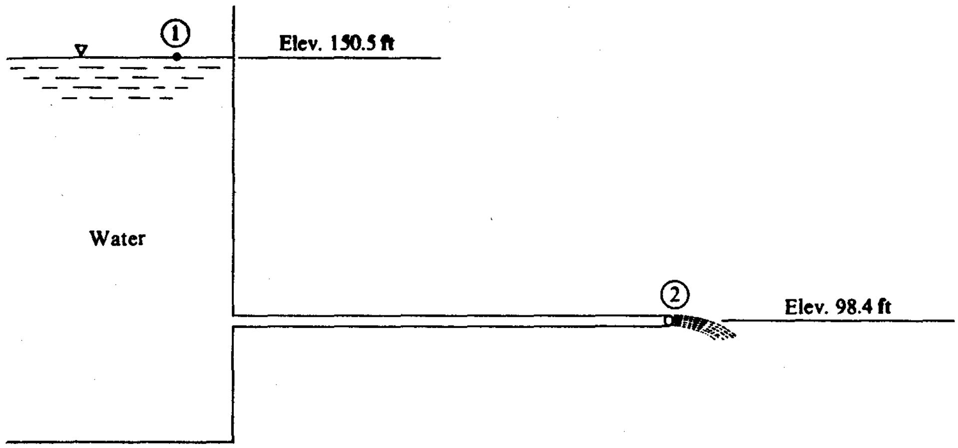


Fig. 9-18

9.66 Gasoline is being discharged from a pipe, as shown in Fig. 9-19. The pipe roughness (ϵ) is 0.500 mm, and the pressure at point 1 is 2500 kPa. Find the pipe diameter needed to discharge gasoline at a rate of 0.10 m³/s. Neglect any minor losses.

$$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L$$

$$h_L = h_f = (f)(L/d)(v^2/2g) = (f)(965.5/d)\{v_2^2/[(2)(9.807)]\} = 49.23fv_2^2/d$$

$$2.500/7.05 + v_1^2/2g + 82.65 = 0 + v_2^2/2g + 66.66 + 49.23fv_2^2/d \qquad v_1^2/2g = v_2^2/2g$$

$$fv_2^2/d = 0.3320 \qquad v_2 = Q/A_2 = 0.10/(\pi d^2/4) = 0.1273/d^2 \qquad (f)(0.1273/d^2)^2/d = 0.3320 \qquad d = (0.04881f)^{1/5}$$

Assume $f = 0.0200$. $d = [(0.04881)(0.0200)]^{1/5} = 0.2500$ m, $v_2 = 0.1273/0.2500^2 = 2.037$ m/s; $N_R = \rho dv/\mu = (719)(0.2500)(2.037)/(2.92 \times 10^{-4}) = 1.25 \times 10^6$. From Table A-9, $\epsilon = 0.00050$ m. $\epsilon/d = 0.00050/0.2500 = 0.0020$. From Fig. A-5, $f = 0.0235$. Evidently, the assumed value of f of 0.0200 was not the correct one. Try a value of f of 0.0235.

$$d = [(0.04881)(0.0235)]^{1/5} = 0.2582 \text{ m} \qquad v = 0.1273/0.2582^2 = 1.909 \text{ m/s}$$

$$N_R = (719)(0.2582)(1.909)/(2.92 \times 10^{-4}) = 1.21 \times 10^6$$

$$\epsilon/d = 0.00050/0.2582 = 0.00194 \qquad f = 0.0235$$

Hence, 0.0235 must be the correct value of f, and $d = 0.2582$ m.

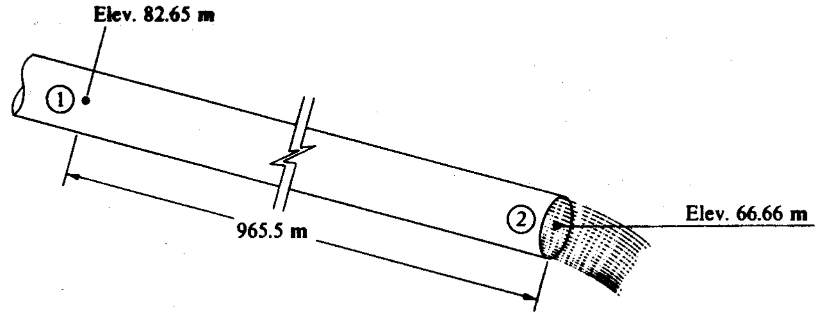


Fig. 9-19

9.67 Water at 20 °C flows through a new cast iron pipe at a velocity of 4.2 m/s. The pipe is 400 m long and has a diameter of 150 mm. Determine the head loss due to friction.

▌ $h_f = (f)(L/d)(v^2/2g) \qquad N_R = dv/\nu = (\frac{150}{1000})(4.2)/(1.02 \times 10^{-6}) = 6.18 \times 10^5$

From Table A-9, $\epsilon = 0.00026$ m. $\epsilon/d = 0.00026/0.150 = 0.00173$. From Fig. A-5, $f = 0.0226$. $h_f = 0.0226[400/(\frac{150}{1000})]\{4.2^2/[(2)(9.807)]\} = 54.20$ m.

9.68 SAE10 oil at 68 °F is to be pumped at a flow rate of 2.0 ft³/s through a level 6-in-diameter new wrought iron pipe. Determine the pressure loss in pounds per square inch per mile of pipe and compute the horsepower lost to friction.

▌ $$h_f = (f)(L/d)(v^2/2g) \qquad v = Q/A = 2.0/[(\pi)(\tfrac{6}{12})^2/4] = 10.19 \text{ ft/s}$$
$$N_R = \rho dv/\mu = (1.68)(\tfrac{6}{12})(10.19)/(1.70 \times 10^{-3}) = 5035 \qquad \text{(turbulent)}$$

From Table A-9, $\epsilon = 0.00015$ ft. $\epsilon/d = 0.00015/(\frac{6}{12}) = 0.00030$. From Fig. A-5, $f = 0.038$. $h_f = 0.038[5280/(\frac{6}{12})]\{10.19^2/[(2)(32.2)]\} = 647$ ft of oil; $p = \gamma h = (54.2)(647)/144 = 244$ psi/mile.

9.69 Water at 20 °C flows in a 100-mm-diameter new cast iron pipe with a velocity of 5.0 m/s. Determine the pressure drop in kilopascals per 100 m of pipe and the power lost to friction.

▌ $h_f = (f)(L/d)(v^2/2g) \qquad N_R = dv/\nu = (\frac{100}{1000})(5.0)/(1.02 \times 10^{-6}) = 4.90 \times 10^5$

From Table A-9, $\epsilon = 0.00026$ m. $\epsilon/d = 0.00026/(\frac{100}{1000}) = 0.0026$. From Fig. A-5, $f = 0.0252$.

$$h_f = 0.0252[100/(\tfrac{100}{1000})]\{5.0^2/[(2)(9.807)]\} = 32.12 \text{ m} \qquad p = (9.79)(32.12) = 314 \text{ kN/m}^2 \text{ per 100 m of pipe}$$
$$Q = Av = [(\pi)(\tfrac{100}{1000})^2/4](5.0) = 0.03927 \text{ m}^3\text{/s}$$
$$\text{Power lost} = Q\gamma h_f = (0.03927)(9.79)(32.12) = 12.35 \text{ kW per 100 m of pipe}$$

9.70 Determine the discharge capacity of a 150-mm-diameter new wrought iron pipe to carry water at 20 °C if the pressure loss due to friction may not exceed 35 kPa per 100 m of level pipe.

▌ $$N_R = dv/\nu = (\tfrac{150}{1000})(v)/(1.02 \times 10^{-6}) = 1.47 \times 10^5 v$$

Trial No. 1
Assume $v = 3.0$ m/s: $N_R = (1.47 \times 10^5)(3.0) = 4.41 \times 10^5$, $\epsilon/d = 0.000046/(\frac{150}{1000}) = 0.000307$. From Fig. A-5, $f = 0.0164$, $h_f = (f)(L/d)(v^2/2g) = p/\gamma = 35/9.79 = 3.575$ m, $3.575 = 0.0164[100/(\frac{150}{1000})]\{v^2/[(2)(9.807)]\}$, $v = 2.53$ m/s.

Trial No. 2
Assume $v = 2.53$ m/s: $N_R = (1.47 \times 10^5)(2.53) = 3.72 \times 10^5$, $f = 0.0166$, $3.575 = 0.0166[100/(\frac{150}{1000})]\{v^2/[(2)(9.807)]\}$, $v = 2.52$ m/s; $Q = Av = [(\pi)(\frac{150}{1000})^2/4](2.52) = 0.0445$ m³/s.

9.71 SAE30 oil at 68 °F is to be pumped at a flow rate of 3.0 ft³/s through a level new cast iron pipe. Allowable pipe friction loss is 10 psi per 1000 ft of pipe. What size commercial pipe should be used?

▌ $p = \gamma h \qquad (10)(144) = 55.4h_f \qquad h_f = 26.0$ ft of oil per 1000 ft of pipe

Trial No. 1
Assume $v = 5.0$ ft/s:

$$Q = A/v \qquad 3.0 = (\pi d^2/4)(5.0) \qquad d = 0.874 \text{ ft}$$
$$N_R = \rho d v/\mu = (1.72)(0.874)(5.0)/(9.2 \times 10^{-3}) = 817 \qquad \text{(laminar)}$$
$$h_f = (f)(L/d)(v^2/2g) \qquad f = 64/N_R = \tfrac{64}{817} = 0.0783$$
$$26.0 = (0.0783)(1000/0.874)\{v^2/[(2)(32.2)]\} \qquad v = 4.32 \text{ ft/s}$$

Trial No. 2
Assume $v = 4.32$ ft/s:

$$3.0 = (\pi d^2/4)(4.32) \qquad d = 0.940 \text{ ft} \qquad N_R = (1.72)(0.940)(4.32)/(9.2 \times 10^{-3}) = 759 \qquad \text{(laminar)}$$
$$f = \tfrac{64}{759} = 0.0843 \qquad 26.0 = (0.0843)(1000/0.940)\{v^2/[(2)(32.2)]\} \qquad v = 4.32 \text{ ft/s}$$

Hence, a pipe diameter of 0.940 ft, or 11.28 in, would be required. A 12-in-diameter commercial pipe should be used, which would result in a pipe friction loss somewhat less than the allowable 10 psi per 1000 ft of pipe.

9.72 SAE10 oil at 20 °C is to flow through a 300-m level concrete pipe. What size pipe will carry 0.0142 m³/s with a pressure drop due to friction of 23.94 kPa?

▮ $$p = \gamma h \qquad 23.94 = 8.52 h_f \qquad h_f = 2.81 \text{ m}$$

Trial No. 1
Assume $v = 1.5$ m/s:

$$Q = A/v \qquad 0.0142 = (\pi d^2/4)(1.5) \qquad d = 0.110 \text{ m}$$
$$N_R = \rho d v/\mu = (869)(0.110)(1.5)/(8.14 \times 10^{-2}) = 1761 \qquad \text{(laminar)}$$
$$h_f = (f)(L/d)(v^2/2g) \qquad f = 64/N_R = \tfrac{64}{1761} = 0.0363$$
$$2.81 = (0.0363)(300/0.110)\{v^2/[(2)(9.807)]\} \qquad v = 0.746 \text{ m/s}$$

Trial No. 2
Assume $v = 0.746$ m/s:

$$0.0142 = (\pi d^2/4)(0.746) \qquad d = 0.156 \text{ m} \qquad N_R = (869)(0.156)(0.746)/(8.14/10^{-2}) = 1242 \qquad \text{(laminar)}$$
$$f = \tfrac{64}{1242} = 0.0515 \qquad 2.81 = (0.0515)(300/0.156)\{v^2/[(2)(9.807)]\} \qquad v = 0.746 \text{ m/s}$$

Hence, a pipe diameter of 0.156 m, or 156 mm, would be required.

9.73 Compute the friction factor for flow having a Reynolds number of 5×10^3 and relative roughness (ϵ/d) of 0.015 (transition zone). Use the Colebrook formula, the Swamee–Jain formula, and the Moody diagram.

▮ **Colebrook formula:**

$$1/\sqrt{f} = 1.14 - 2.0 \log[\epsilon/d + 9.35/(N_F\sqrt{f})] = 1.14 - 2.0 \log[0.015 + 9.35/(5 \times 10^3\sqrt{f})]$$
$$f = 0.0515 \qquad \text{(by trial and error)}$$

Swamee–Jain formula:

$$f = 0.25/[\log(\epsilon/3.7d) + (5.47/N_R^{0.9})]^2 = 0.25/\{\log(0.015/3.7) + [5.47/(5 \times 10^3)^{0.9}]\}^2 = 0.0438$$

Moody diagram (Fig. A-5):

$$f = 0.0512$$

9.74 Repeat Prob. 9.73 for flow having a Reynolds number of 4×10^6 and relative roughness (ϵ/d) of 0.0001 (rough-pipe zone).

▮ **Colebrook formula:**

$$f = 1/[1.14 - 2.0 \log(\epsilon/d)]^2 = 1/[1.14 - 2.0 \log(0.0001)]^2 = 0.0120$$

Swamee–Jain formula:

$$f = 0.25/[\log(\epsilon/3.7d) + (5.74/N_R^{0.9})]^2 = 0.25/\{\log(0.0001/3.7) + [5.74/(4 \times 10^6)^{0.9}]\}^2 = 0.0120$$

Moody diagram (Fig. A-5):

$$f = 0.0125$$

9.75 We have oil of kinematic viscosity $8 \times 10^{-5}\ \text{ft}^2/\text{s}$ going through an 80-ft horizontal pipe. If the initial pressure is 5.0 psig and the final pressure is 3.5 psig, compute the mass flow if the pipe has a diameter of 3 in. At a point 10 ft from the end of the pipe a vertical tube is attached to be flush with the inside radius of the pipe. How high will the oil rise in the tube? $\rho = 50\ \text{lbm/ft}^3$. Pipe is commercial steel ($\epsilon = 0.000145$ ft).

▌

$$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L$$

$$(5.0)(144)/(50/32.2) + v_1^2/2g + 0 = (3.5)(144)/(50/32.2) + v_2^2/2g + 0 + h_f$$

$$v_1^2/2g = v_2^2/2g \qquad h_f = 139.1\ \text{ft} = (f)(L/d)(v^2/2) \qquad 139.1 = (f)[80/(\tfrac{3}{12})](v^2/2) \qquad fv^2 = 0.8694$$

Try $f = 0.020$: $0.020v^2 = 0.8694$, $v = 6.593$ ft/s; $N_R = dv/\nu = (\tfrac{3}{12})(6.593)/(8 \times 10^{-5}) = 2.06 \times 10^4$; $\epsilon/d = 0.000145/(\tfrac{3}{12}) = 0.000580$. From Fig. A-5, $f = 0.0265$. Try $f = 0.0265$: $0.0265v^2 = 0.8694$, $v = 5.728$ ft/s; $N_R = (\tfrac{3}{12})(5.728)/(8 \times 10^{-5}) = 1.79 \times 10^4$; $f = 0.0267$, $0.0267v^2 = 0.8694$, $v = 5.706$ ft/s; $M = \rho A v = 50[(\pi)(\tfrac{3}{12})^2/4](5.706) = 14.0$ lbm/s. To find the pressure at the point 10 ft from the end of the pipe (call it point A), apply the Bernoulli equation between point 1 and point A:

$$(5.0)(144)/(50/32.2) + v_1^2/2g + 0 = p_A/(50/32.2) + v_2^2/2g + 0 + h_f$$

$$h_f = 463.7 - 0.6440p_A = 0.0267[70/(\tfrac{3}{12})](5.706^2/2) = 121.7\ \text{ft}$$

$$121.7 = 463.7 - 0.6440p_A \qquad p_A = 531.1\ \text{lbm/ft}^2 \qquad h = p/\rho = 531.1/50 = 10.62\ \text{ft}$$

9.76 How much water is flowing through the pipe shown in Fig. 9-20? Take $\nu = 0.114 \times 10^{-5}\ \text{m}^2/\text{s}$ and $\epsilon = 0.0000442$ m.

▌

$$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L \qquad 1.6 + v_1^2/2g + 0 = 0.3 + v_2^2/2g + 0 + h_f$$

$$v_1^2/2g = v_2^2/2g \qquad h_f = 1.3\ \text{m} = (f)(L/d)(v^2/2g)$$

Try $f = 0.015$:

$$1.3 = 0.015[10/(0.150)]\{v^2/[(2)(9.807)]\} \qquad v = 5.050\ \text{m/s}$$

$$N_R = dv/\nu = (0.150)(5.050)/(0.114 \times 10^{-5}) = 6.64 \times 10^5 \qquad \epsilon/d = 0.0000442/(0.150) = 0.000295$$

From Fig. A-5, $f = 0.016$.

$$1.3 = 0.016[10/(0.150)]\{v^2/[(2)(9.807)]\} \qquad v = 4.889\ \text{m/s}$$

$$M = \rho A v = 1000[(\pi)(0.150)^2/4](4.889) = 86.4\ \text{kg/s}$$

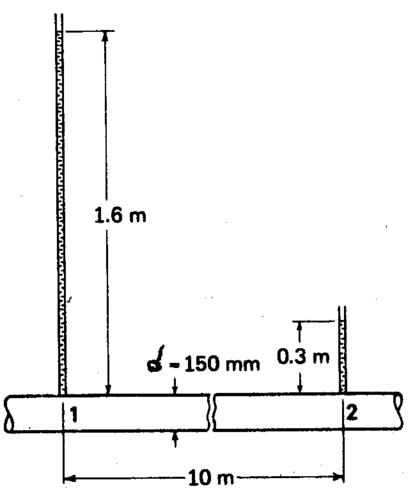


Fig. 9-20

9.77 Whiskey (s.g. = 0.6, $\nu = 5.0 \times 10^{-7}\ \text{m}^2/\text{s}$) is drawn from a tank through a hose of inside diameter 25 mm (see Fig. 9-21). The relative roughness for the hose is 0.0004. Calculate the volumetric flow and the minimum pressure in the hose. The total length of hose is 9 m and the length of hose to point A is 3.25 m. Neglect minor losses at head entrance.

▌

$$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L \qquad 0 + 0 + (5 - 1.5) = 0 + v_2^2/[(2)(9.807)] + 0 + h_f$$

$$h_f = 3.5 - 0.05098v_2^2 = (f)(L/d)(v^2/2g)$$

Try $f = 0.016$:

$$h_f = 0.016[9/(0.025)]\{v_2^2/[(2)(9.807)]\} = 0.2937v_2^2 \qquad 0.2937v_2^2 = 3.5 - 0.05098v_2^2 \qquad v = 3.187 \text{ m/s}$$

$$N_R = dv/\nu = (0.025)(3.187)/(5.0 \times 10^{-7}) = 1.59 \times 10^5$$

From Fig. A-5 with $\epsilon/d = 0.0004$, $f = 0.019$. Try $f = 0.019$:

$$h_f = 0.019[9/(0.025)]\{v_2^2/[(2)(9.807)]\} = 0.3487v_2^2 \qquad 0.3487v_2^2 = 3.5 - 0.05098v_2^2 \qquad v = 2.959 \text{ m/s}$$

$$N_R = dv/\nu = (0.025)(2.959)/(5.0 \times 10^{-7}) = 1.48 \times 10^5 \qquad f = 0.019$$

$$Q = Av = [(\pi)(0.025)^2/4](2.959) = 1.45 \text{ L/s} \qquad p_1/\gamma + v_1^2/2g + z_1 = p_A/\gamma + v_A^2/2g + z_A + h_L$$

$$0 + 0 + (5 - 1.5) = p_A/[(0.6)(9.79)] + 2.959^2/[(2)(9.807)] + 5 + h_f \qquad p_A = -11.43 - 5.874h_f$$

$$h_f = 0.019[3.25/(0.025)]\{2.959^2/[(2)(9.807)]\} = 1.103 \text{ m} \qquad p_A = -11.43 - (5.874)(1.103) = -17.91 \text{ kPa}$$

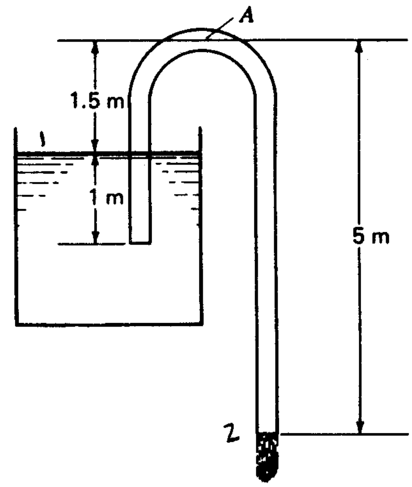


Fig. 9-21

9.78 In using the Darcy–Weisbach equation for flow in a pressure conduit, what percentage error is introduced in Q when f is misjudged by 20 percent?

▮ $$h_f = (f)(L/d)(v^2/2g) = K_1 f Q^2 \qquad \text{(where } K_1 \text{ is a constant)}$$

Assume h_f is constant. $Q = K_2/\sqrt{f} = K_2 f^{-1/2}$, $dQ = -\frac{1}{2}(K_2)(f^{-3/2})(df)$

$$\frac{dQ}{Q} = \frac{-\frac{1}{2}(K_2)(f^{-3/2})(df)}{K_2 f^{-1/2}} = -\frac{1}{2}\frac{df}{f} = -(\tfrac{1}{2})(0.20) = -0.10 \quad \text{or} \quad -10 \text{ percent}$$

9.79 For the system in Fig. 9-13, find the flow rate if the liquid is water at 68 °F.

▮ Assume smooth-wall turbulent flow. $p_A/\gamma + v_A^2/2g + z_A = p_B/\gamma + v_B^2/2g + z_B + h_L$, $0 + 0 + 10 = 0 + v_B^2/[(2)(32.2)] + 0 + h_f$, $h_f = 10 - 0.01553v_B^2 = (f)(L/d)(v^2/2g)$. Try $f = 0.02$:

$$h_f = 0.02[6/(0.5/12)]\{v_B^2/[(2)(32.2)]\} = 0.04472v_B^2 \qquad 10 - 0.01553v_B^2 = 0.04472v_B^2 \qquad v_B = 12.88 \text{ ft/s}$$

$$N_R = \rho dv/\mu = (1.93)(0.5/12)(12.88)/(2.04 \times 10^{-5}) = 5.08 \times 10^4 \qquad \text{(turbulent)}$$

From Fig. A-5, $f = 0.0208$. Try $f = 0.0208$:

$$h_f = 0.0208[6/(0.5/12)]\{v_B^2/[(2)(32.2)]\} = 0.04651v_B^2 \qquad 10 - 0.01553v_B^2 = 0.04651v_B^2 \qquad v_B = 12.70 \text{ ft/s}$$

$$N_R = \rho dv/\mu = (1.93)(0.5/12)(12.70)/(2.04 \times 10^{-5}) = 5.01 \times 10^4 \qquad \text{(turbulent)}$$

$$f = 0.0208 \qquad Q = Av = [(\pi)(0.5/12)^2/4](12.70) = 0.0173 \text{ ft}^3/\text{s}$$

9.80 If 1 mile of 3-in-diameter wrought iron pipe carries water at 68 °F and $v = 23$ ft/s, compute the head loss and the pressure drop.

▮ $$h_f = (f)(L/d)(v^2/2g) \qquad N_R = \rho dv/\mu = (1.93)(\tfrac{3}{12})(7/0.3048)/(2.04 \times 10^{-5}) = 5.43 \times 10^5$$

$$\epsilon/d = 0.00015/(\tfrac{3}{12}) = 0.000600$$

From Fig. A-5, $f = 0.0182$.

$$h_f = 0.0182[5280/(\tfrac{3}{12})]\{(23)^2/[(2)(32.2)]\} = 3157 \text{ ft} \qquad p = \gamma h_f = (62.4)(3157)/144 = 1368 \text{ lb/in}^2$$

9.81 Mercury at 20 °C flows through 3 m of 6-mm-diameter glass tubing with average velocity 2.0 m/s. Compute the head loss and the pressure drop.

$$h_f = (f)(L/d)(v^2/2g) \qquad N_R = \rho dv/\mu = (13\,570)(0.006)(2.0)/(1.56 \times 10^{-3}) = 1.04 \times 10^5$$

From Fig. A-5, $f = 0.0180$ (assuming glass to be "smooth").

$$h_f = 0.0180[3/(0.006)]\{2.0^2/[(2)(9.807)]\} = 1.835 \text{ m} \qquad p = \gamma h_f = [(13.6)(9.79)](1.835) = 244 \text{ kPa}$$

9.82 Gasoline at 20 °C is pumped at 0.2 m³/s through 16 km of 180-mm-diameter cast iron pipe. Compute the power required if the pumps are 75 percent efficient.

$$h_f = (f)(L/d)(v^2/2g) \qquad v = Q/A = 0.2/[(\pi)(0.180)^2/4] = 7.860 \text{ m/s}$$

$$N_R = \rho dv/\mu = (719)(0.180)(7.860)/(2.92 \times 10^{-4}) = 3.48 \times 10^6 \qquad \epsilon/d = 0.00026/(0.180) = 0.00144$$

From Fig. A-5, $f = 0.0216$.

$$h_f = 0.0216[(16)(1000)/(0.180)]\{7.860^2/[(2)(9.807)]\} = 6048 \text{ m}$$

$$P = \rho g Q h_f/\eta = (719)(9.807)(0.2)(6048)/0.75 = 11.37 \times 10^6 \text{ W} \quad \text{or} \quad 11.37 \text{ MW}$$

9.83 Vinegar (s.g. = 0.86, $\nu = 0.00003$ ft²/s) flows at 1 ft³/s through a 6-in asphalted cast iron pipe. The pipe is 2000 ft long and slopes upward at 10° in the flow direction. Compute the head loss and the pressure change.

$$h_f = (f)(L/d)(v^2/2g) \qquad v = Q/A = 1/[(\pi)(\tfrac{6}{12})^2/4] = 5.093 \text{ ft/s}$$

$$N_R = dv/\nu = (\tfrac{6}{12})(5.093)/0.00003 = 8.49 \times 10^4 \qquad \epsilon/d = 0.0004/(\tfrac{6}{12}) = 0.000800$$

From Fig. A-5, $f = 0.0219$.

$$h_f = 0.0219[2000/(\tfrac{6}{12})]\{5.093^2/[(2)(32.2)]\} = 35.28 \text{ ft} \qquad h_L = 35.28 + 2000 \sin 10° = 382.6 \text{ ft}$$

$$p = \gamma h_f = [(0.86)(62.4)](382.6)/144 = 142.6 \text{ lb/in}^2$$

9.84 The pipe flow in Fig. 9-22 is driven by pressurized air in the tank. What gage pressure p_1 is needed to provide a flow rate of 50 m³/h of water? Assume a "smooth" pipe.

$$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L \qquad v_2 = Q/A_2 = (50/3600)/[(\pi)(0.050)^2/4] = 7.074 \text{ m/s}$$

$$h_L = h_f = (f)(L/d)(v^2/2g) \qquad N_R = \rho dv/\mu = (998)(0.050)(7.074)/(1.02 \times 10^{-3}) = 3.46 \times 10^5$$

From Fig. A-5, $f = 0.0140$.

$$h_L = 0.0140[(40 + 80 + 20)/0.050]\{7.074^2/[(2)(9.807)]\} = 100.0 \text{ m}$$

$$p_1/9.79 + 0 + 10 = 0 + 7.074^2/[(2)(9.807)] + 80 + 100.0 \qquad p_1 = 1689 \text{ kPa gage}$$

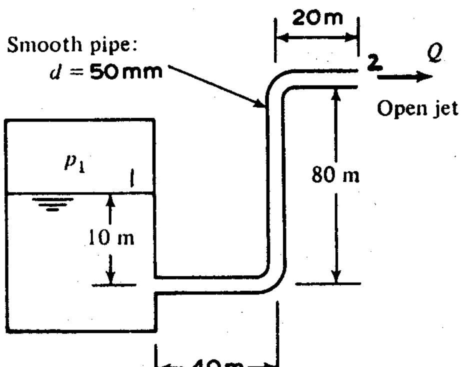


Fig. 9-22

9.85 In Fig. 9-22 suppose the fluid is methanol at 20 °C and $p_1 = 900$ kPa gage. What flow rate Q results?

▮ $p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L$ $\quad 900/7.73 + 0 + 10 = 0 + v_2^2/[(2)(9.807)] + 80 + h_L$

$$h_L = 46.43 - 0.05098v_2^2 = h_f = (f)(L/d)(v^2/2g)$$

Try $f = 0.02$:

$h_L = 0.02[(40+80+20)/0.050]\{v_2^2/[(2)(9.807)]\} = 2.855v_2^2 \quad 2.855v_2^2 = 46.43 - 0.05098v_2^2 \quad v_2 = 4.000$ m/s

$$N_R = \rho dv/\mu = (788)(0.050)(4.000)/(5.98\times 10^{-4}) = 2.64\times 10^5$$

From Fig. A-5, $f = 0.0150$. Try $f = 0.0150$:

$h_L = 0.0150[(40+80+20)/0.050]\{v_2^2/[(2)(9.807)]\} = 2.141v_2^2 \quad 2.141v_2^2 = 46.43 - 0.05098v_2^2 \quad v_2 = 4.602$ m/s

$$N_R = (788)(0.050)(4.602)/(5.98\times 10^{-4}) = 3.03\times 10^5$$

Try $f = 0.0145$:

$h_L = 0.0145[(40+80+20)/0.050]\{v_2^2/[(2)(9.807)]\} = 2.070v_2^2 \quad 2.070v_2^2 = 46.43 - 0.05098v_2^2 \quad v_2 = 4.679$ m/s

$$N_R = (788)(0.050)(4.679)/(5.98\times 10^{-4}) = 3.08\times 10^5 \qquad f = 0.0145$$

At this Reynolds number, $Q = Av = [(\pi)(0.050)^2/4](4.679) = 0.00919\ \text{m}^3/\text{s}$ or $33.1\ \text{m}^3/\text{h}$.

9.86 In Fig. 9-22 suppose the fluid is carbon tetrachloride at 20 °C and $p_1 = 1300$ kPa. Calculate the pipe diameter needed for a volumetric flow of 5.555 L/s.

▮ $p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L \quad p_1/\gamma + 0 + z_1 = 0 + v_2^2/2g + z_2 + (f)(L/d)(v_2^2/2g)$

$$v_2^2 = \frac{(2g)(p_1/\gamma + z_1 - z_2)}{1 + fL/d} = \frac{(2)(9.807)(1300/15.57 + 10 - 80)}{1 + (f)(40+80+20)/d} = \frac{264.7}{1 + 140f/d}$$

$$= (Q/A_2)^2 = [(5.555\times 10^{-3})/(\pi d^2/4)]^2 = 0.00005004/d^4$$

$$v_2 = 0.007074/d^2$$

$$\frac{0.00005004}{d^4} = \frac{264.7}{1+140f/d} \qquad d = \frac{(1+140f/d)^{1/4}}{47.96}$$

Try $d = 50$ mm, or 0.050 m: $N_R = \rho dv/\mu = (1588)(0.050)(0.007074/0.050^2)/(9.67\times 10^{-4}) = 2.32\times 10^5$. From Fig. A-5, $f = 0.0151$. $d = [1 + (140)(0.0151/0.050)]^{1/4}/47.96 = 0.0535$ m. Try $d = 0.0535$ m:

$$N_R = (1588)(0.0535)(0.007074/0.0535^2)/(9.67\times 10^{-4}) = 2.17\times 10^5 \qquad f = 0.0155$$

$$d = [1 + (140)(0.0155/0.0535)]^{1/4}/47.96 = 0.0529\ \text{m}$$

Try $d = 0.0529$ m:

$$N_R = (1588)(0.0529)(0.007074/0.0529^2)/(9.67\times 10^{-4}) = 2.20\times 10^5 \qquad f = 0.0155$$

$$d = [1 + (140)(0.0155/0.0529)]^{1/4}/47.96 = 0.0531\ \text{m}$$

Try $d = 0.0531$ m:

$$N_R = (1588)(0.0531)(0.007074/0.0531^2)/(9.67\times 10^{-4}) = 2.19\times 10^5 \qquad f = 0.0155$$

$$d = [1 + (140)(0.0155/0.0531)]^{1/4}/47.96 = 0.05301\ \text{m}$$

Hence, use $d = 0.053$ m, or 53 mm.

9.87 The reservoirs in Fig. 9-23 contain water at 20 °C. If the pipe is smooth, with $L = 7$ km and $d = 50$ mm, what will the flow rate be for $\Delta z = 98$ m?

▮ $p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L \quad 0 + 0 + 100 = 0 + 0 + 0 + h_f \quad h_f = 98\ \text{m} = (f)(L/d)(v^2/2g)$

$$98 = (f)[7000/0.050]\{v^2/[(2)(9.807)]\} \qquad v = 0.1172/\sqrt{f}$$

Try $f = 0.02$: $v = 0.1172/\sqrt{0.02} = 0.8287$ m/s, $N_R = \rho dv/\mu = (998)(0.050)(0.8287)/(1.02\times 10^{-3}) = 4.05\times 10^4$. From Fig. A-5, $f = 0.022$. Try $f = 0.022$:

$$v = 0.1172/\sqrt{0.022} = 0.7902\ \text{m/s} \qquad N_R = (998)(0.050)(0.7902)/(1.02\times 10^{-3}) = 3.87\times 10^4$$

$$f = 0.022 \qquad \text{(O.K.)}$$

$$Q = Av = [(\pi)(0.050)^2/4](0.7902) = 0.00155\ \text{m}^3/\text{s} \quad \text{or} \quad 5.58\ \text{m}^3/\text{h}$$

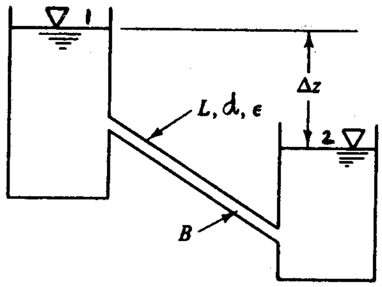


Fig. 9-23

9.88 Repeat Prob. 9.87 to find Q if $L = 2500$ ft, $d = 3$ in, and $\Delta z = 82$ ft.

$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L \qquad 0+0+82 = 0+0+0+h_f \qquad h_f = 82\text{ ft} = (f)(L/d)(v^2/2g)$

$$82 = (f)[2500/(\tfrac{3}{12})]\{v^2/[(2)(32.2)]\} \qquad v = 0.7267/\sqrt{f}$$

Try $f = 0.02$:

$$v = 0.7267/\sqrt{0.02} = 5.139\text{ ft/s} \qquad N_R = \rho dv/\mu = (1.93)(\tfrac{3}{12})(5.139)/(2.04\times10^{-5}) = 1.22\times10^5$$

From Fig. A-5, $f = 0.0175$. Try $f = 0.0175$:

$$v = 0.7267/\sqrt{0.0175} = 5.493\text{ ft/s} \qquad N_R = (1.93)(\tfrac{3}{12})(5.493)/(2.04\times10^{-5}) = 1.30\times10^5 \qquad f = 0.0170$$

Try $f = 0.0170$:

$$v = 0.7267/\sqrt{0.0170} = 5.574\text{ ft/s} \qquad N_R = (1.93)(\tfrac{3}{12})(5.574)/(2.04\times10^{-5}) = 1.32\times10^5$$

$$f = 0.0170 \quad \text{(O.K.)} \qquad Q = Av = [(\pi)(\tfrac{3}{12})^2/4](5.574) = 0.2736\text{ ft}^3/\text{s} \quad \text{or} \quad 985\text{ ft}^3/\text{h}$$

9.89 Repeat Prob. 9.88 if the pipe has a roughness of 0.2 mm.

From Prob. 9.88, $v = 0.7267/\sqrt{f}$. Try $f = 0.02$:

$$N_R = 1.22\times10^5 \qquad \epsilon/d = 0.2/[(3)(25.4)] = 0.00262$$

From Fig. A-5, $f = 0.0265$. Try $f = 0.0265$:

$$v = 0.7267/\sqrt{0.0265} = 4.464\text{ ft/s} \qquad N_R = (1.93)(\tfrac{3}{12})(4.464)/(2.04\times10^{-5}) = 1.06\times10^5$$

$$f = 0.0265 \quad \text{(O.K.)} \qquad Q = Av = [(\pi)(\tfrac{3}{12})^2/4](4.464) = 0.2191\text{ ft}^3/\text{s} \quad \text{or} \quad 789\text{ ft}^3/\text{h}$$

This is $(985 - 789)/985 = 0.199$, or 19.9 percent less than when the pipe is smooth.

9.90 Water at 20 °C flows through a 598-m pipe 150 mm in diameter at 60 L/s. Determine the pipe roughness if the head loss is 49 m.

$$h_L = (f)(L/d)(v^2/2g) \qquad v = Q/A = 0.06/[(\pi)(0.150)^2/4] = 3.395\text{ m/s}$$

$$49 = (f)[598/(0.150)]\{3.395^2/[(2)(9.807)]\} \qquad f = 0.0209$$

$$N_R = \rho dv/\mu = (998)(0.150)(3.395)/(1.02\times10^{-3}) = 4.98\times10^5$$

From Fig. A-5 with $f = 0.0209$ and $N_R = 4.98\times10^5$, $\epsilon/d = 0.0012$; $\epsilon = (150)(0.0012) = 0.180$ mm.

9.91 A 4-in-diameter commercial steel pipe is to be sloped so that 198 gpm of water at 20 °C passes through it in gravity flow. Find the declination θ of the pipe.

$$Q = (198)(0.002228) = 0.4411\text{ ft}^3/\text{s} \qquad v = Q/A = 0.4411/[(\pi)(\tfrac{4}{12})^2/4] = 5.055\text{ ft/s}$$

$$\epsilon/d = 0.00015/(\tfrac{4}{12}) = 0.00045 \qquad N_R = dv/\nu = (\tfrac{4}{12})(5.055)/(1.05\times10^{-5}) = 1.60\times10^5$$

From Fig. A-5, $f = 0.0190$. $\sin\theta = \Delta z/L = h_f/L = (f)(1/d)(v^2/2g)$, $\sin\theta = 0.0190[1/(\frac{4}{12})]\{5.055^2/[(2)(32.2)]\}$, $\theta = 1.30°$.

9.92 In Prob. 9.91 find the volume flow corresponding to $\theta = 3°$.

▮ From Prob. 9.91, $(f)(1/d)(v^2/2g) = \sin\theta$, $(f)[1/(\frac{4}{12})]\{v^2/[(2)(32.2)]\} = \sin 3°$, $v = 1.060/\sqrt{f}$. Try $f = 0.02$:

$$v = 1.060/\sqrt{0.02} = 7.495\ \text{ft/s} \qquad N_R = dv/\nu = (\tfrac{4}{12})(7.495)/(1.05\times10^{-5}) = 2.38\times10^5$$

$$\epsilon/d = 0.00045 \qquad \text{(from Prob. 9.91)}$$

From Fig. A-5, $f = 0.0182$. Try $f = 0.0182$:

$$v = 1.060/\sqrt{0.0182} = 7.857\ \text{ft/s} \qquad N_R = (\tfrac{4}{12})(7.857)/(1.05/10^{-5}) = 2.49\times10^5$$

$$f = 0.0182 \quad \text{(O.K.)} \qquad Q = Av = [(\pi)(\tfrac{4}{12})^2/4](7.857) = 0.6857\ \text{ft}^3/\text{s} = 0.6857/0.002228 = 308\ \text{gpm}$$

9.93 A tank containing 1 m³ of water at 20 °C has an outlet tube at the bottom, as shown in Fig. 9-24. Find the instantaneous volume flux Q, if the roughness of the tank is $\epsilon = 1.5\ \mu\text{m}$.

▮
$$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L$$

$$h_L = h_f = (f)(L/d)(v^2/2g) = (f)[0.82/0.04]\{v_2^2/[(2)(9.807)]\} = 1.045fv_2^2$$

$$0 + 0 + (1 + 0.82) = 0 + v_2^2/[(2)(9.807)] + 0 + 1.045fv_2^2 \qquad v_2 = 1.349/\sqrt{0.05098 + 1.045f}$$

Try $f = 0.02$:

$$v_2 = 1.349/\sqrt{0.05098 + (1.045)(0.02)} = 5.032\ \text{m/s}$$

$$N_R = dv/\nu = (0.04)(5.032)/(1.02\times10^{-6}) = 1.97\times10^5 \qquad \epsilon/d = 0.0000015/0.04 = 0.0000375$$

From Fig. A-5, $f = 0.016$. Try $f = 0.016$:

$$v_2 = 1.349/\sqrt{0.05098 + (1.045)(0.016)} = 5.185\ \text{m/s} \qquad N_R = (0.04)(5.185)/(1.02\times10^{-6}) = 2.03\times10^5$$

$$f = 0.016 \quad \text{(O.K.)} \qquad Q = Av = [(\pi)(0.04)^2/4](5.185) = 0.006516\ \text{m}^3/\text{s} = 6.516\ \text{L/s}$$

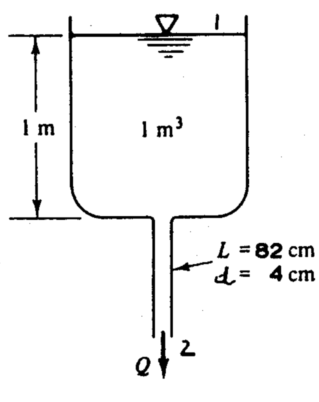


Fig. 9-24

9.94 Repeat Prob. 9.93 for a fluid with $\rho = 917\ \text{kg/m}^3$ and $\mu = 0.29\ \text{Pa}\cdot\text{s}$.

▮ $p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L$. Assume laminar flow.

$$h_L = h_f = \frac{32\mu Lv}{\rho g d^2} = \frac{(32)(0.29)(0.82)(v)}{(917)(9.807)(0.04)^2} = 0.5288v_2$$

$$0 + 0 + (1 + 0.82) = 0 + v_2^2/[(2)(9.807)] + 0 + 0.5288v_2 \qquad v_2^2 + 10.37v_2 - 35.70 = 0 \qquad v = 2.726\ \text{m/s}$$

$$N_R = \rho dv/\mu = (917)(0.04)(2.726)/0.29 = 345 \qquad \text{(laminar)}$$

$$Q = Av = [(\pi)(0.04)^2/4](2.726) = 0.003426\ \text{m}^3/\text{s} = 3.426\ \text{L/s}$$

9.95 What depth of water behind a dam (Fig. 9-25) will yield a flow rate of 0.02 ft³/s through the 0.5-in commercial steel exit pipe?

$$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L \qquad v_2 = Q/A = 0.02/[(\pi)(0.5/12)^2/4] = 14.67 \text{ ft/s}$$

$$h_L = h_f = (f)(L/d)(v^2/2g) \qquad N_R = dv/\nu = (0.5/12)(14.67)/(1.05\times10^{-5}) = 5.82\times10^4$$

$$\epsilon/d = 0.00015/(0.5/12) = 0.00360$$

From Fig. A-5, $f = 0.0295$: $h_L = 0.0295[100/(0.5/12)]\{14.67^2/[(2)(32.2)]\} = 236.6$ ft, $0 + 0 + H = 0 + 14.67^2/[(2)(32.2)] + 0 + 236.6$, $H = 239.9$ ft.

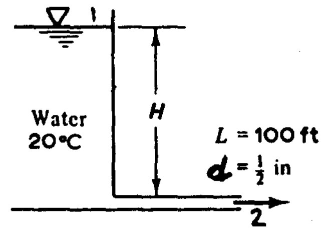


Fig. 9-25

9.96 In Fig. 9-25, imagine the fluid to be benzene at 20 °C and $H = 100$ ft. What pipe diameter is required for the flow rate to be 0.02 ft³/s?

$$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L$$

$$h_L = h_f = (f)(L/d)(v^2/2g) = (f)(100/d)\{v_2^2/[(2)(32.2)]\} = 1.553fv_2^2/d$$

$$0 + 0 + 100 = 0 + v_2^2/[(2)(32.2)] + 0 + 1.553fv_2^2/d \qquad v_2 = Q/A_2 = 0.02/(\pi d^2/4) = 0.02546/d^2$$

$$100 = (0.02546/d^2)^2/[(2)(32.2)] + (1.553)(f)(0.02546/d^2)^2/d \qquad d = [(1.007\times10^{-7}) + (1.007\times10^{-5}f/d)]^{1/4}$$

Try $d = 0.05$ ft: $N_R = \rho dv/\mu = (1.70)(0.05)(0.02546/0.05^2)/(1.36\times10^{-5}) = 6.36\times10^4$, $\epsilon/d = 0.00015/0.05 = 0.00300$. From Fig. A-5, $f = 0.028$. $d = [(1.007\times10^{-7}) + (1.007\times10^{-5})(0.028)/0.05]^{1/4} = 0.0489$ ft. Try $d = 0.00489$ ft:

$$N_R = (1.70)(0.0489)(0.02546/0.0489^2)/(1.36\times10^{-5}) = 6.51\times10^4 \qquad f = 0.028 \qquad \text{(O.K.)}$$

$$d = 0.0489 \text{ ft} \quad \text{or} \quad 0.59 \text{ in}$$

9.97 Water at 70 °F flows through 102 ft of 0.5-in-diameter piping whose roughness is 0.01 in. Find the volumetric flow if the pressure drop is 60 psi.

$$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L$$

$$h_L = h_f = (f)(L/d)(v^2/2g) = (f)[102/(0.5/12)]\{v^2/[(2)(32.2)]\} = 38.01fv^2$$

$$(60)(144)/62.3 + v_1^2/2g + 0 = 0 + v_2^2/2g + 0 + 38.01fv^2 \qquad v_1^2/2g = v_2^2/2g \qquad v_2 = 1.910/\sqrt{f}$$

Try $f = 0.05$:

$$v_2 = 1.910/\sqrt{0.05} = 8.542 \text{ ft/s} \qquad N_R = dv/\nu = (0.5/12)(8.542)/(1.05\times10^{-5}) = 3.39\times10^4$$

$$\epsilon/d = 0.01/0.5 = 0.0200$$

$$f = 0.050 \qquad \text{(O.K.)} \qquad Q = Av = [(\pi)(0.5/12)^2/4](8.542) = 0.0116 \text{ ft}^3/\text{s}$$

9.98 Ethanol at 20 °C flows at 3.2 m/s through a 10-cm-diameter pipeline ($\epsilon = 1.5\ \mu$m). Compute **(a)** the head loss per kilometer of tube and **(b)** the wall shear stress.

(a)

$$h_f = (f)(L/d)(v^2/2g) \qquad N_R = \rho dv/\mu = (788)(0.10)(3.2)/(1.20\times10^{-3}) = 2.10\times10^5$$

$$\epsilon/d = 0.0000015/0.10 = 0.0000150$$

From Fig. A-5, $f = 0.0158$. $h_f = 0.0158[1000/0.10]\{(3.2)^2/[(2)(9.807)]\} = 82.5$ m per 1000 m.

(b)

$$u^* = v\sqrt{f/8} = (3.2)\sqrt{0.0158/8} = 0.1422 \text{ m/s} \qquad \tau_{\text{wall}} = (\rho)(u^*)^2 = (788)(0.1422)^2 = 15.9 \text{ Pa}$$

9.99 In Fig. 9-26, the 50-m duct is 60 mm in diameter. Compute the flow rate if the fluid has $\rho = 917\ \text{kg/m}^3$ and $\mu = 0.29\ \text{Pa}\cdot\text{s}$.

▮ Assume laminar flow from 1 to 2.

$$p_1/\rho g + v_1^2/2g + z_1 = p_2/\rho g + v_2^2/2g + z_2 + h_L$$

$$h_L = h_f = \frac{128\mu LQ}{\pi\rho g d^4} = \frac{(128)(0.29)(50)(Q)}{(\pi)(917)(9.807)(0.060)^4} = 5069Q$$

$$0 + 0 + 12 = (205)(1000)/[(917)(9.807)] + 0 + 0 + 5069Q \qquad Q = -0.002130\ \text{m}^3/\text{s} \quad \text{or} \quad -7.67\ \text{m}^3/\text{h}$$

Since Q is negative, flow is from 2 to 1.

$$v = Q/A = 0.002130/[(\pi)(0.060)^2/4] = 0.7533\ \text{m/s}$$

$$N_R = \rho dv/\mu = (917)(0.060)(0.7533)/0.29 = 143 \qquad \text{(laminar)}$$

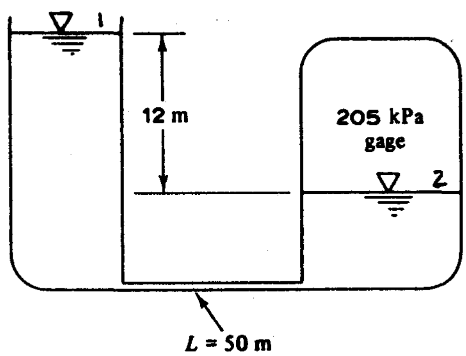


Fig. 9-26

9.100 Compute the diameter of duct in Fig. 9-26 required to maintain a flow rate of $25\ \text{m}^3/\text{h}$.

▮ $$\frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2 = \frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 + h_L \qquad h_L = h_f = \frac{128\mu LQ}{\pi\rho g d^4} = \frac{(128)(0.29)(50)(25/3600)}{(\pi)(917)(9.807)(d)^4} = \frac{4.5621\times 10^{-4}}{d^4}$$

$$(205)(1000)/[(917)(9.807)] + 0 + 0 = 0 + 0 + 12 + 4.5621\times 10^{-4}/d^4 \qquad d = 0.0806\ \text{m}$$

$$v = Q/A = (25/3600)/[(\pi)(0.0806)^2/4] = 1.361\ \text{m/s}$$

$$N_R = \rho dv/\mu = (917)(0.0806)(1.361)/0.29 = 347 \qquad \text{(laminar)}$$

9.101 A 40-ft-long conduit, having an annular cross section with $a = 1$ in and $b = 0.5$ in, connects two reservoirs which differ in surface height by 21 ft. Compute the flow rate if the fluid is water at 20 °C and the conduit is made of commercial steel.

▮ Hydraulic diameter $= D_h = (2)(a - b) = (2)(1 - 0.5) = 1.00$ in $\qquad p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L$

$$0 + 0 + 21 = 0 + 0 + 0 + h_L \qquad h_L = 21\ \text{ft} = h_f = (f)(L/d)(v^2/2g)$$

$$21 = (f)[40/(1.00/12)]\{v^2/[(2)(32.2)]\} \qquad v = 1.679/\sqrt{f}$$

Try $f = 0.02$:

$$v = 1.679/\sqrt{0.02} = 11.87\ \text{ft/s} \qquad N_R = dv/\nu = (1.00/12)(11.87)/(1.05\times 10^{-5}) = 9.42\times 10^4$$

$$\epsilon/d = 0.00015/(1.00/12) = 0.00180$$

From Fig. A-5, $f = 0.0246$. Try $f = 0.0246$:

$$v = 1.679/\sqrt{0.0246} = 10.70\ \text{ft/s} \qquad N_R = (1.00/12)(10.70)/(1.05\times 10^{-5}) = 8.49\times 10^4 \qquad f = 0.0250$$

Try $f = 0.0250$:

$$v = 1.679/\sqrt{0.0250} = 10.62\ \text{ft/s} \qquad N_R = (1.00/12)(10.62)/(1.05\times 10^{-5}) = 8.43\times 10^4$$

$$f = 0.0250 \quad \text{(O.K.)} \qquad Q = Av = \{(\pi)[(1.00/12)^2 - (0.5/12)^2]\}(10.62) = 0.174\ \text{ft}^3/\text{s}$$

9.102 An annular capillary causes a very large pressure drop and is useful in an accurate measurement of viscosity. If a smooth annulus 1 m long with $a = 50$ mm and $b = 49$ mm carries an oil flow at 0.0012 m^3/s, what is the oil viscosity if the pressure drop is 260 kPa?

$$Q = (\pi/8\mu)(\Delta p/L)[a^4 - b^4 - (a^2 - b^2)^2/\ln(a/b)]$$

$$0.0012 = (\pi/8\mu)(260\,000/1)[0.050^4 - 0.049^4 - (0.050^2 - 0.049^2)^2/\ln(0.050/0.049)] \qquad \mu = 0.0738 \text{ Pa}\cdot\text{s}$$

9.103 A metal ventilation duct ($\epsilon = 0.00015$ ft) carries air at 20 °C and 1 atm. The duct section is an equilateral triangle 1 ft on a side, and its length is 100 ft. If a blower can deliver 1 hp to the air, what flow rate can occur?

From Fig. 9-27,

$$A = \tfrac{1}{2}(1)(\tfrac{1}{2}\sqrt{3}) = 0.4330 \text{ ft}^2$$

$$P = Q\gamma h_f \qquad Q = Av = 0.4330v \qquad h_f = (f)(L/d)(v^2/2g) \qquad d = D_h = 4A/p_w$$

$$D_h = (4)(0.4330)/[(12+12+12)/12] = 0.5773 \text{ ft} \qquad h_f = (f)(100/0.5773)\{v^2/[(2)(32.2)]\} = 2.69fv^2$$

$$(1)(550) = (0.4330v)(0.0750)(2.69fv^2) \qquad v = 18.47/f^{1/3}$$

Try $f = 0.02$:

$$v = 18.47/0.02^{1/3} = 68.04 \text{ ft/s} \qquad N_R = dv/\nu = (0.5773)(68.04)/(1.64\times10^{-4}) = 2.40\times10^5$$

$$\epsilon/D_h = 0.00015/0.5773 = 0.000260$$

From Fig. A-5, $f = 0.0176$. Try $f = 0.0176$:

$$v = 18.47/0.0176^{1/3} = 71.00 \text{ ft/s} \qquad N_R = (0.5773)(71.00)/(1.64\times10^{-4}) = 2.60\times10^5$$

$$f = 0.0176 \quad \text{(O.K.)} \qquad Q = (0.4330)(71.00) = 30.7 \text{ ft}^3/\text{s}$$

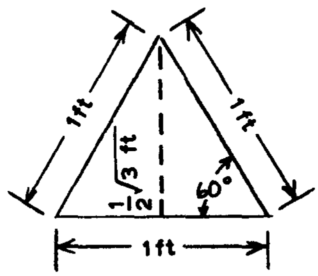


Fig. 9-27

9.104 A fluid with $\rho = 917$ kg/m^3 and $\mu = 0.29$ Pa · s flows between two smooth parallel plates 5 cm apart ($h = 5$ cm) at an average velocity of 3 m/s. Compute the pressure drop, centerline velocity, and head loss for each 100 m of length.

$$d = D_h = 2h = (2)(0.05) = 0.1000 \text{ m} \qquad N_R = \rho dv/\mu = (917)(0.1000)(3)/0.29 = 949 \quad \text{(laminar)}$$

$$v = (h^2/12\mu)(\Delta p/L) \qquad 3 = \{(0.05)^2/[(12)(0.29)]\}(\Delta p/100) \qquad \Delta p = 417\,600 \text{ Pa}$$

$$h_f = \Delta p/\rho g = 417\,600/[(917)(9.807)] = 46.44 \text{ m} \qquad u_{\mathrm{C\!\!\!L}} = 1.5v = (1.5)(3) = 4.50 \text{ m/s}$$

9.105 Air at STP is forced at 25 m/s through a 30-cm-square steel duct 148 m long. Compute the head loss, pressure drop, and power required if the blower efficiency is 60 percent. Use $\epsilon = 0.000046$ m.

$$d = D_h = 4A/p_w = (4)[(0.30)(0.30)]/[(4)(0.30)] = 0.3000 \text{ m}$$

$$h_f = (f)(L/d)(v^2/2g) = (f)(148/0.3000)\{25^2/[(2)(9.807)]\} = 15\,720f$$

$$N_R = \rho dv/\mu = (1.20)(0.3000)(25)/(1.81\times10^{-5}) = 4.97\times10^5 \qquad \epsilon/d = 0.000046/0.3000 = 0.000153$$

From Fig. A-5, $f = 0.0150$.

$$h_f = (15\,720)(0.0150) = 235.8 \text{ m} \qquad \Delta p = \rho g h_f = (1.20)(9.807)(235.8) = 2775 \text{ Pa}$$

$$Q = Av = [(0.30)(0.30)](25) = 2.25 \text{ m}^3/\text{s}$$

$$P = \rho g Q h_f/\eta = (1.20)(9.807)(2.25)(235.8)/0.60 = 10\,406 \text{ W} \quad \text{or} \quad 10.4 \text{ kW}$$

9.106 A 50-m-long wind tunnel has a wooden ($\epsilon = 0.0001$ m) rectangular section 40 cm by 1 m. The average flow velocity is 45 m/s for air at STP. Compute the pressure drop, assuming fully developed conditions, and the power required if the fan has 70 percent efficiency.

▮

$$d = D_h = 4A/p_w = (4)[(\tfrac{40}{100})(1)]/(\tfrac{40}{100} + \tfrac{40}{100} + 1 + 1) = 0.5714 \text{ m}$$

$$h_f = (f)(L/d)(v^2/2g) = (f)(50/0.5714)\{45^2/[(2)(9.807)]\} = 9034f$$

$$N_R = \rho dv/\mu = (1.20)(0.5714)(45)/(1.81 \times 10^{-5}) = 1.70 \times 10^6 \qquad \epsilon/d = 0.0001/0.5714 = 0.000175$$

From Fig. A-5, $f = 0.0140$.

$$h_f = (9034)(0.0140) = 126.5 \text{ m} \qquad \Delta p = \rho g h_f = (1.20)(9.807)(126.5) = 1489 \text{ Pa}$$

$$Q = Av = [(\tfrac{40}{100})(1)](45) = 18.00 \text{ m}^3/\text{s}$$

$$P = \rho g Q h_f/\eta = (1.20)(9.807)(18.00)(126.5)/0.70 = 38\,281 \text{ W} \quad \text{or} \quad 38.3 \text{ kW}$$

9.107 A smooth rectangular duct 82 m long, of aspect ratio 6 : 1, is designed to transport 0.6 m³/s of hydrogen at STP. If the pressure drop is 75 Pa, calculate the dimensions of the cross section. For hydrogen, $\rho = 0.0838$ kg/m³ and $\mu = 9.05\ \mu$Pa · s.

▮ Let h = height of duct; then duct width = $6h$.

$$d = D_h = 4A/p_w = (4)[(6h)(h)]/(6h + 6h + h + h) = 1.714h \qquad v = Q/A = 0.6/[(6h)(h)] = 0.1000/h^2$$

$$h_f = (f)(L/d)(v^2/2g) = (f)(82/1.714h)\{(0.1000/h^2)^2/[(2)(9.807)]\} = 0.02439f/h^5$$

$$\Delta p = \rho g h_f \qquad 75 = (0.0838)(9.807)(0.02439f/h^5) \qquad h = 0.1929f^{1/5}$$

Try $f = 0.02$: $h = (0.1929)(0.02)^{1/5} = 0.08821$ m, $N_R = \rho dv/\mu = 0.0838[(1.714)(0.08821)](0.1000/0.08821^2)/(9.05 \times 10^{-6}) = 1.80 \times 10^4$. From Fig. A-5, $f = 0.0265$. Try $f = 0.0265$: $h = (0.1929)(0.0265)^{1/5} = 0.09332$ m, $N_R = 0.0838[(1.714)(0.09332)](0.1000/0.09332^2)/(9.05 \times 10^{-6}) = 1.70 \times 10^4$, $f = 0.0267$. Try $f = 0.0267$: $h = (0.1929)(0.0267)^{1/5} = 0.09346$ m, $N_R = 0.0838[(1.714)(0.09346)](0.1000/0.09346^2)/(9.05 \times 10^{-6}) = 1.70 \times 10^4$.

$$f = 0.0267 \qquad \text{(O.K.)}$$

Hence, $h = 9.35$ cm and width = $6h$ = 56.1 cm.

9.108 Kerosene at 10 °C flows steadily at 15 L/min through a 150-m-long horizontal length of 5.5-cm-diameter cast iron pipe. Compare the pressure drop of the kerosene flow with that of the same flow rate of benzene at 10 °C through the same pipe. At 10 °C, $\rho = 820$ kg/m³ and $\mu = 0.0025$ N · s/m² for kerosene, and $\rho = 899$ kg/m³ and $\mu = 0.0008$ N · s/m² for benzene.

▮ For kerosene:

$$v = Q/A = [(\tfrac{15}{1000})/60]/[(\pi)(5.5/100)^2/4] = 0.1052 \text{ m/s}$$

$$N_R = \rho dv/\mu = (820)(5.5/100)(0.1052)/0.0025 = 1898 \qquad \text{(laminar)}$$

$$\Delta p = 8\mu v L/r_0^2 = (8)(0.0025)(0.1052)(150)/[(5.5/100)/2]^2 = 417 \text{ Pa}$$

For benzene:

$$N_R = (899)(5.5/100)(0.1052)/0.0008 = 6502 \qquad \text{(turbulent)}$$

$$\Delta p = (f)(L/d)(\rho)(v^2/2) \qquad \epsilon/d = 0.00026/(5.5/100) = 0.00473$$

From Fig. A-5, $f = 0.040$: $\Delta p = (0.040)[150/(5.5/100)](899)(0.1052^2/2) = 543$ Pa. The pressure drop is greater for the benzene than the kerosene, although the benzene has a lower viscosity. If both flows had been laminar or both turbulent, the lower-viscosity fluid, benzene, would have had the lower pressure drop. However, the viscosity of the kerosene is high enough to give laminar flow, while the lower viscosity of the benzene causes a high enough Reynolds number for turbulent flow.

9.109 Water at 60 °F flows through a 250-ft length of horizontal 2-in-diameter drawn tubing. If the pressure drop across the tubing is 10 psi, what is the flow rate?

▮ Assume turbulent flow. $h_f = (f)(L/d)(v^2/2g)$, $h_f = (10)(144)/62.4 = 23.08$ ft, $23.08 = (f)[250/(\tfrac{2}{12})]\{v^2/[(2)(32.2)]\}$, $v = 0.9954/\sqrt{f}$. Try $v = 5$ ft/s: $N_R = \rho dv/\mu = (1.94)(\tfrac{2}{12})(5)/(2.35 \times 10^{-5}) = 6.88 \times 10^4$ (turbulent). $\epsilon/d = 0.000005/(\tfrac{2}{12}) = 0.000030$. From Fig. A-5, $f = 0.0195$. $v = 0.9954/\sqrt{0.0195} = 7.13$ ft/s.

Try $v = 7.13$ ft/s: $N_R = (1.94)(\frac{2}{12})(7.13)/(2.35 \times 10^{-5}) = 9.81 \times 10^4$, $f = 0.0180$, $v = 0.9954/\sqrt{0.0180} = 7.42$ ft/s. Try $v = 7.42$ ft/s: $N_R = (1.94)(\frac{2}{12})(7.42)/(2.35 \times 10^{-5}) = 1.02 \times 10^5$, $f = 0.0180$ (O.K.); $Q = Av = [(\pi)(\frac{2}{12})^2/4](7.42) = 0.162$ ft²/s.

9.110 Air at 200 °F and 15 psig is to be passed through a 150-ft length of new galvanized iron pipe at a rate of 15 ft³/s. If the maximum allowable pressure drop is 5 psi, estimate the minimum pipe diameter.

▮ $$h_f = (f)(L/d)(v^2/2g) = p/\gamma \qquad \gamma = p/RT = (15 + 14.7)(144)/[(53.3)(200 + 460)] = 0.1216 \text{ lb/ft}^3$$

$$h_f = (5)(144)/0.1216 = 5921 \text{ ft} \qquad v = Q/A = 15/(\pi d^2/4) = 19.10/d^2$$

$$5921 = (f)(150/d)\{(19.10/d^2)^2/[(2)(32.2)]\} \qquad d = 0.6782f^{1/5}$$

Try $d = 0.5$ ft:

$$N_R = \rho dv/\mu = (\gamma/g)(dv)/\mu = (0.1216/32.2)(0.5)(19.10/0.5^2)/(4.49 \times 10^{-7}) = 3.21 \times 10^5$$

$$\epsilon/d = 0.0005/0.5 = 0.0010$$

From Fig. A-5, $f = 0.0205$; $d = (0.6782)(0.0205)^{1/5} = 0.3117$ ft.

Try $d = 0.3117$ ft:

$$N_R = (0.1216/32.2)(0.3117)(19.10/0.3117^2)/(4.49 \times 10^{-7}) = 5.15 \times 10^5 \qquad \epsilon/d = 0.0005/0.3117 = 0.0016$$

$$f = 0.0225 \qquad d = (0.6782)(0.0225)^{1/5} = 0.3175 \text{ ft}$$

Try $d = 0.3175$ ft:

$$N_R = (0.1216/32.2)(0.3175)(19.10/0.3175^2)/(4.49 \times 10^{-7}) = 5.06 \times 10^5 \qquad \epsilon/d = 0.0005/0.3175 = 0.0016$$

$$f = 0.0225 \quad \text{(O.K.)}$$

Hence, $d = 0.3175$ ft, or 3.81 in.

9.111 Compute the loss of head and pressure drop in 200 ft of horizontal 6-in-diameter asphalted cast iron pipe carrying water with a mean velocity of 6 ft/s.

▮ $h_f = (f)(L/d)(v^2/2g) \qquad N_R = dv/\nu = (\frac{6}{12})(6)/(1.05 \times 10^{-5}) = 2.86 \times 10^5 \qquad \epsilon/d = 0.0004/(\frac{6}{12}) = 0.00080$

From Fig. A-5, $f = 0.020$:

$$h_f = (0.020)[200/(\tfrac{6}{12})]\{6^2/[(2)(32.2)]\} = 4.47 \text{ ft} \qquad \Delta p = \gamma h_f = (62.4)(4.47) = 279 \text{ lb/ft}^2$$

9.112 Oil with $\rho = 900$ kg/m³ and $\nu = 0.00001$ m²/s flows at 0.2 m³/s through 500 m of 200-mm-diameter cast iron pipe. Determine (*a*) the head loss and (*b*) the pressure drop if the pipe slopes down at 10° in the flow direction.

▮ (*a*) $$h_f = (f)(L/d)(v^2/2g) \qquad v = Q/A = 0.2/[(\pi)(\tfrac{200}{1000})^2/4] = 6.366 \text{ m/s}$$

$$N_R = dv/\nu = (\tfrac{200}{1000})(6.366)/0.00001 = 1.27 \times 10^5 \qquad \epsilon/d = 0.00026/(\tfrac{200}{1000}) = 0.00130$$

From Fig. A-5, $f = 0.0225$, $h_f = 0.0225[500/(\frac{200}{1000})]\{6.366^2/[(2)(9.807)]\} = 116.2$ m.

(*b*) $$h_f = \Delta p/(\rho g) + L \sin 10° \qquad 116.2 = \Delta p/[(900)(9.807)] + (500)(\sin 10°)$$

$$\Delta p = 259\,300 \text{ Pa} \quad \text{or} \quad 259.3 \text{ kPa}$$

9.113 Oil with $\rho = 950$ kg/m³ and $\nu = 0.00002$ m²/s flows through a 30-cm-diameter pipe 100 m long with a head loss of 8 m. The roughness ratio ϵ/d is 0.0002. Find the average velocity and flow rate.

▮ $$h_f = (f)(L/d)(v^2/2g) \qquad 8 = (f)[100/(\tfrac{30}{100})]\{v^2/[(2)(9.807)]\} \qquad v = 0.6861/\sqrt{f}$$

Try $v = 5$ m/s: $N_R = dv/\nu = (\frac{30}{100})(5)/0.00002 = 7.50 \times 10^4$. From Fig. A-5, $f = 0.020$, $v = 0.6861/\sqrt{0.020} = 4.851$ m/s. Try $v = 4.851$ m/s: $N_R = (\frac{30}{100})(4.851)/0.00002 = 7.28 \times 10^4$, $f = 0.020$ (O.K.). $Q = Av = [(\pi)(\frac{30}{100})^2/4](4.851) = 0.343$ m³/s.

9.114 Fluid flows at an average velocity of 6 ft/s between horizontal parallel plates a distance of 2.4 in apart ($h = 2.4$ in). Estimate the head loss and pressure drop for each 100 ft of length for $\rho = 1.9$ slugs/ft³ and $\nu = 0.00002$ ft²/s. Assume smooth walls.

▮ $$h_f = (f)(L/d)(v^2/2g) \qquad d = D_h = 2h \qquad D_h = (2)(2.4/12) = 0.400 \text{ ft}$$

$$N_R = dv/\nu = (0.400)(6)/0.00002 = 1.20 \times 10^5 \quad \text{(turbulent)}$$

From Fig. A-5, $f = 0.0173$.

$$h_f = (0.0173)(100/0.400)\{6^2/[(2)(32.2)]\} = 2.42 \text{ ft} \qquad \Delta p = \rho g h_f = (1.9)(32.2)(2.42) = 148 \text{ lb/ft}^2$$

9.115 Repeat Prob. 9.114 if $\nu = 0.002$ ft²/s.

$$N_R = dv/\nu = (0.400)(6)/0.002 = 1200 \qquad \text{(laminar)} \qquad f = 96/N_R = \tfrac{96}{1200} = 0.0800$$

$$h_f = (f)(L/d)(v^2/2g) = (0.0800)(100/0.400)\{6^2/[(2)(32.2)]\} = 11.18 \text{ ft}$$

$$\Delta p = \rho g h_f = (1.9)(32.2)(11.18) = 684 \text{ lb/ft}^2$$

9.116 Estimate the reservoir level h needed to maintain a flow of 0.01 m³/s through the commercial steel annulus 30 m long shown in Fig. 9-28. Neglect entrance effects and take $\rho = 1000$ kg/m³ and $\nu = 1.02 \times 10^{-6}$ m²/s for water.

$$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_f \qquad v = Q/A = 0.01/\{(\pi)[(\tfrac{5}{100})^2 - (\tfrac{3}{100})^2]\} = 1.989 \text{ m/s}$$

$$h_f = (f)(L/d)(v^2/2g) \qquad d = D_h = (2)(a-b) \qquad D_h = (2)(\tfrac{5}{100} - \tfrac{3}{100}) = 0.0400 \text{ m}$$

$$h_f = (f)(30/0.0400)\{1.989^2/[(2)(9.807)]\} = 151.3f \qquad 0 + 0 + h = 0 + 1.989^2/[(2)(9.807)] + 0 + 151.3f$$

$$h = 0.2017 + 151.3f \qquad N_R = dv/\nu = (0.0400)(1.989)/(1.02 \times 10^{-6}) = 7.80 \times 10^4$$

$$\epsilon/d = 0.000046/0.0400 = 0.00115$$

From Fig. A-5, $f = 0.0232$, $h = 0.2017 + (151.3)(0.0232) = 3.71$ m.

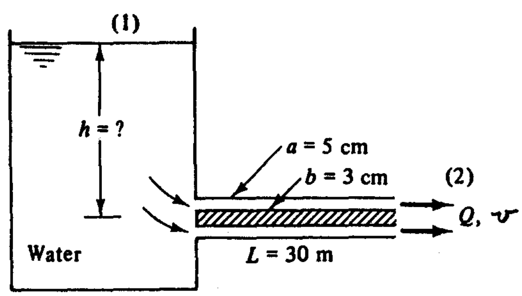


Fig. 9-28

9.117 Air with $\rho = 0.00237$ slug/ft³ and $\nu = 0.000157$ ft²/s is forced through a horizontal square 9-in by 9-in duct 100 ft long at 25 ft³/s. Find the presssure drop if $\epsilon = 0.0003$ ft.

$$h_f = (f)(L/d)(v^2/2g) \qquad N_R = dv/\nu \qquad D_h = 4A/p_w = 4[(\tfrac{9}{12})(\tfrac{9}{12})]/[(4)(\tfrac{9}{12})] = 0.7500 \text{ ft}$$

From Table A-10, for $b/a = \tfrac{9}{9}$, or 1.0, the effective diameter is $d = D_{\text{eff}} = (64/56.91)(D_h) = (64/56.91)(0.7500) = 0.8434$ ft, $v = Q/A = 25/[(\tfrac{9}{12})(\tfrac{9}{12})] = 44.44$ ft/s, $N_R = (0.8434)(44.44)/0.000157 = 2.39 \times 10^5$, $\epsilon/d = 0.0003/0.8434 = 0.000354$. From Fig. A-5, $f = 0.0177$; $h_f = (0.0177)(100/0.7500)\{44.44^2/[(2)(32.2)]\} = 72.37$ ft, $\Delta p = \rho g h_f = (0.00237)(32.2)(72.37) = 5.52$ lb/ft².

9.118 Find the head loss in 1 ft of 6-in-diameter pipe ($\epsilon = 0.042$ in) when turpentine (s.g. = 0.86) having a viscosity of 0.0008 lb · s/ft² flows at a rate of 16 cfs. Also find the shear stress at the wall of the pipe.

$$h_f = (f)(L/d)(v^2/2g) \qquad v = Q/A = 16/[(\pi)(\tfrac{6}{12})^2/4] = 81.49 \text{ ft/s}$$

$$N_R = \rho dv/\mu = [(0.86)(1.94)](\tfrac{6}{12})(81.49)/0.0008 = 8.50 \times 10^4 \qquad \epsilon/d = 0.042/6 = 0.00700$$

From Fig. A-5, $f = 0.0324$.

$$h_f = 0.0324[1/(\tfrac{6}{12})]\{81.49^2/[(2)(32.2)]\} = 6.68\ \text{ft}$$

$$\tau_0 = f\rho v^2/8 = 0.0324[(0.86)(1.94)](81.49)^2/8 = 44.9\ \text{lb/ft}^2$$

9.119 When water at 150 °F flows in a 0.75-in-diameter copper tube at 1.0 gpm, find the head loss per 1000 ft. Also find the centerline velocity.

▮ $Q = (1.0)(0.002228) = 0.002228\ \text{ft}^3/\text{s}$ $v = Q/A = 0.002228/[(\pi)(0.75/12)^2/4] = 0.7262\ \text{ft/s}$

$$h_f = (f)(L/d)(v^2/2g) \qquad N_R = dv/\nu = (0.75/12)(0.7262)/(4.68 \times 10^{-6}) = 9.70 \times 10^3$$

$$\epsilon/d = 0.000005/(0.75/12) = 0.0000800$$

From Fig. A-5, $f = 0.030$.

$$h_f = (0.030)[1000/(0.75/12)]\{0.7262^2/[(2)(32.2)]\} = 3.93\ \text{ft}$$

$$u_{\max} = (v)(1 + 1.33f^{1/2}) = 0.7262[1 + (1.33)(0.030)^{1/2}] = 0.893\ \text{ft/s}$$

$$\delta_1 = 32.8\nu/(vf^{1/2}) = (32.8)(4.68 \times 10^{-6})/[(0.7262)(0.030)^{1/2}] = 0.00122\ \text{ft}$$

9.120 Refer to Prob. 9.119. How thick is the viscous boundary layer for a flow of 20 gpm?

▮ The Reynolds number is now $(20/1.0)(9.70 \times 10^3) = 1.94 \times 10^5$. From Fig. A-5, $f = 0.016$. Then,

$$\delta_1 = 32.8\nu/(vf^{1/2}) = (32.8)(4.68 \times 10^{-6})/[(20)(0.7262)(0.016)^{1/2}] = 8.36 \times 10^{-5}\ \text{ft}$$

9.121 Find the maximum velocity in a 100-mm-diameter pipe ($\epsilon = 0.00085$ m) when oil (s.g. = 0.84), of viscosity 0.0052 Pa · s flows at a rate of 40 L/s.

▮

$$v = Q/A = (40 \times 10^{-3})/[(\pi)(0.100)^2/4] = 5.093\ \text{m/s}$$

$$N_R = \rho dv/\mu = [(0.84)(1000)](0.100)(5.093)/0.0052 = 8.23 \times 10^4 \qquad \epsilon/d = 0.00085/(0.100) = 0.0085$$

From Fig. A-5, $f = 0.0365$. Hence,

$$u_{\max} = (v)(1 + 1.33f^{1/2}) = 5.093[1 + (1.33)(0.0365)^{1/2}] = 6.387\ \text{m/s}$$

9.122 The head loss in 240 ft of 6-in-diameter pipe is known to be 30 ft when oil (s.g. = 0.90) flows at 2.0 ft³/s. Determine the shear stress at the wall of the pipe.

▮

$$v = Q/A = 2.0/[(\pi)(\tfrac{6}{12})^2/4] = 10.19\ \text{ft/s}$$

$$N_R = \rho dv/\mu = [(0.90)(1.94)](\tfrac{6}{12})(10.19)/0.0008 = 1.11 \times 10^4 \qquad \text{(turbulent)}$$

$$h_f = (f)(L/d)(v^2/2g) \qquad 30 = (f)[240/(\tfrac{6}{12})]\{10.19^2/(2)(32.2)]\} \qquad f = 0.0388$$

$$\tau_0 = f\rho v^2/8 = 0.0388[(0.90)(1.94)](10.19)^2/8 = 0.879\ \text{lb/ft}^2$$

9.123 If, in a 1-m-diameter pipe, velocities are 5.03 m/s on the centerline and 4.85 m/s at $r = 100$ mm, what is the flow rate?

▮

$$u = u_{\max} - (5.75)(\tau_0/\rho)^{1/2} \log [r_0/(r_0 - r)]$$

$$4.85 = 5.03 - (5.75)(\tau_0/\rho)^{1/2} \log [500/(500 - 100)] \qquad (\tau_0/\rho)^{1/2} = 0.3230$$

$$v = u_{\max} - (\tfrac{3}{2})(2.5\sqrt{\tau_0/\rho}) = 5.03 - (\tfrac{3}{2})(2.5)(0.3230) = 3.819\ \text{m/s} \qquad Q = Av = [(\pi)(1)^2/4](3.819) = 3.00\ \text{m}^3/\text{s}$$

9.124 The velocities in a 30-in-diameter pipe are measured as 15.0 and 14.5 ft/s at $r = 0$ and $r = 4$ in, respectively. Approximate the flow rate.

▮ $u = u_{max} - (5.75)(\tau_0/\rho)^{1/2} \log [r_0/(r_0 - r)]$ $\quad 14.5 = 15.0 - (5.75)(\tau_0/\rho)^{1/2} \log [15/(15 - 4)]$

$(\tau_0/\rho)^{1/2} = 0.6456$ $\quad \tau_0/\rho = fv^2/8$ $\quad (\tau_0/\rho)^{1/2} = (fv^2/8)^{1/2} = 0.6456$ $\quad f = 3.3344/v^2$

$u_{max} = (v)(1 + 1.33f^{1/2})$ $\quad 15.0 = (v)(1 + 1.33f^{1/2})$ $\quad 15.0 = (v)[1 + (1.33)(3.3344/v^2)^{1/2}]$

$15.0 = v + 2.4286$ $\quad v = 12.57$ ft/s $\quad Q = Av = [(\pi)(\frac{30}{12})^2/4](12.57) = 61.7\ \text{ft}^3/\text{s}$

9.125 With turbulent flow in a circular pipe, prove that the mean velocity occurs at a distance of approximately $0.78r_0$ from the centerline of the pipe.

▮ $u = (1 + 1.33f^{1/2})(v) - (2.04)(f^{1/2})(v) \log [r_0/(r_0 - r)]$ $\quad 0 = 1.33f^{1/2}v - (2.04f^{1/2}v)\{\log [r_0/(r_0 - r)]\}$

$\log [r_0/(r_0 - r)] = 0.65196$ $\quad r_0/(r_0 - r) = \text{antilog}\ 0.65196 = 4.487$ $\quad r = 0.776r_0$

9.126 The flow rate in a 12-in-diameter pipe is 8 cfs. The flow is known to be turbulent, and the centerline velocity is 12.0 fps. Determine the velocity profile, and determine the head loss per foot of pipe.

▮ $v = Q/A = 8/[(\pi)(\frac{12}{12})^2/4] = 10.19$ ft/s $\quad u_{max} = (v)(1 + 1.33f^{1/2})$ $\quad 12.0 = (10.19)(1 + 1.33f^{1/2})$

$f = 0.01784$ $\quad h_f = (f)(L/d)(v^2/2g) = 0.01784[1/(\frac{12}{12})]\{10.19^2/[(2)(32.2)]\} = 0.0288$ ft per foot

$\tau_0 = (f/4)(\rho)(v^2/2) = (0.01784/4)(\rho)(10.19^2/2)$ $\quad (\tau_0/\rho)^{1/2} = 0.4812$

$$u = u_{max} - 5.75(\tau_0/\rho)^{1/2} \log [r_0/(r_0 - r)]$$

For $r = 0$, $u = 12.0 - (5.75)(0.4812) \log [6/(6 - 0)] = 12.0$ ft/s. For $r = 2$, $u = 12.0 - (5.75)(0.4812) \log [6/(6 - 2)] = 11.5$ ft/s. For $r = 4$, $u = 12.0 - (5.75)(0.4812) \log [6/(6 - 4)] = 10.7$ ft/s. For $r = 5$, $u = 12.0 - (5.75)(0.4812) \log [6/(6 - 5)] = 9.85$ ft/s. For $r = 5.5$, $u = 12.0 - (5.75)(0.4812) \log [6/(6 - 5.5)] = 9.01$ ft/s. For $r = 5.9$, $u = 12.0 - (5.75)(0.4812) \log [6/(6 - 5.9)] = 7.08$ ft/s. For $r = 5.99$, $u = 12.0 - (5.75)(0.4812) \log [6/(6 - 5.99)] = 4.31$ ft/s.

9.127 Tung oil (s.g. = 0.82) flows at a temperature of 80 °F ($\nu = 2.21 \times 10^{-5}$ ft²/s) in a 2-in-diameter smooth brass pipeline at 10 gpm. Find the head loss per mile.

▮ $Q = (10)(0.002228) = 0.02228\ \text{ft}^3/\text{s}$ $\quad v = Q/A = 0.02228/[(\pi)(\frac{2}{12})^2/4] = 1.021$ ft/s

$h_f = (f)(L/d)(v^2/2g)$ $\quad N_R = dv/\nu = (\frac{2}{12})(1.021)/(2.21 \times 10^{-5}) = 7.70 \times 10^3$

From Fig. A-5 for smooth pipe, $f = 0.0333$. $h_f = 0.0333[5280/(\frac{2}{12})]\{1.021^2/[(2)(32.2)]\} = 17.1$ ft per mile.

9.128 Water at 40 °C flows in a 20-cm-diameter pipe with $v = 5.1$ m/s. Head loss measurements give $f = 0.022$. Determine the value of ϵ and find the shear stress at the wall of the pipe and at $r = 3$ cm.

▮ $N_R = dv/\nu = (0.20)(5.1)/(6.56 \times 10^{-7})$. From Fig. A-5, $\epsilon/d = 0.0015$, $\epsilon = (20)(0.0015) = 0.0300$ cm; $\tau_0 = f\rho v^2/8 = (0.022)(992)(5.1)^2/8 = 71.0$ Pa; $\tau = (\tau_0)(r/r_0) = (71.0)(0.03) = 21.3$ Pa.

9.129 Water at 15 °C flows through a 200-mm-diameter pipe with $\epsilon = 0.01$ mm. ***(a)*** If the mean velocity is 3.6 m/s, what is the nominal thickness δ_1 of the viscous boundary layer? ***(b)*** What will be the boundary layer thickness if the velocity is increased to 6.0 m/s?

▮ ***(a)*** $\delta_1 = 32.8\nu/(vf^{1/2})$ $\quad N_R = dv/\nu = (0.200)(3.6)/(1.16 \times 10^{-6}) = 6.21 \times 10^5$

$$\epsilon/d = (0.01/10)/20 = 0.0000500$$

From Fig. A-5, $f = 0.0133$. $\delta_1 = (32.8)(1.16 \times 10^{-6})/[(3.6)(0.0133)^{1/2}] = 9.16 \times 10^{-5}$ m, or 91.6 μm.

(b) $N_R = (0.200)(6.0)/(1.16 \times 10^{-6}) = 1.03 \times 10^6$ $\quad f = 0.0126$

$$\delta_1 = (32.8)(1.16 \times 10^{-6})/[(6.0)(0.0126)^{1/2}] = 5.65 \times 10^{-5}\ \text{m} \quad \text{or} \quad 56.5\ \mu\text{m}$$

9.130 When water at 50 °F flows at 3.2 cfs in a 2-ft pipeline, the head loss is 0.0004 ft per foot. What will be the head loss when glycerin at 68 °F flows through the same pipe at the same rate?

▌ $$h_f = (f)(L/d)(v^2/2g) \qquad v = Q/A = 3.2/[(\pi)(\tfrac{24}{12})^2/4] = 1.0186 \text{ ft/s}$$

For water:

$$0.0004 = (f)[1/(2)]\{1.0186^2/[(2)(32.2)]\} \qquad f = 0.04966$$

$$N_R = \rho dv/\mu = (1.94)(2)(1.0186)/(2.72 \times 10^{-5}) = 1.45 \times 10^5 \qquad \text{(turbulent)}$$

From Fig. A-5, $\epsilon/d = 0.021$.

For glycerin:

$$N_R = (2.44)(2)(1.0186)/(3.11 \times 10^{-2}) = 160 \qquad \text{(laminar)}$$

$$f = 64/N_R = 64/160 = 0.4000 \qquad h_f = 0.4000[1/(2)]\{1.0186^2/[(2)(32.2)]\} = 0.00322 \text{ ft per ft}$$

9.131 Air flows at 50 lb/min in a 4-in-diameter welded steel pipe ($\epsilon = 0.0018$ in) at 100 psia and 60 °F. Determine the head loss and pressure drop in 200 ft of this pipe. Assume the air to be of constant density.

▌ $$h_f = (f)(L/d)(v^2/2g) \qquad N_R = \rho dv/\mu \qquad \rho = p/RT = (100)(144)/[(1716)(460+60)] = 0.01614 \text{ slug/ft}^3$$

$$W = \gamma A v = \rho g A v \qquad \tfrac{50}{60} = (0.01614)(32.2)[(\pi)(\tfrac{4}{12})^2/4](v) \qquad v = 18.37 \text{ ft/s}$$

$$N_R = (0.01614)(\tfrac{4}{12})(18.37)/(3.74 \times 10^{-7}) = 2.64 \times 10^5 \qquad \epsilon/d = 0.0018/4 = 0.00045$$

From Fig. A-5, $f = 0.018$. $h_f = 0.018[200/(\tfrac{4}{12})]\{18.37^2/[(2)(32.2)]\} = 56.6$ ft of air, $\Delta p = \rho g h_f = (0.01614)(32.2)(56.6) = 29.4 \text{ lb/ft}^2$, or 0.204 lb/in^2.

9.132 Air flows at an average velocity of 0.5 m/s through a long, 3.2-m-diameter, circular tunnel. Find the head-loss gradient at a point where the air temperature and pressure are 16 °C and 109 kPa abs, respectively. Assume $\epsilon = 2$ mm. Find also the shear stress at the wall and the thickness of the viscous sublayer.

▌ $$h_f/L = (f/d)(v^2/2g) \qquad \rho = p/RT = (109)(1000)/[(287)(273+16)] = 1.314 \text{ kg/m}^3$$

$$N_R = \rho dv/\mu = (1.314)(3.2)(0.5)/(1.79 \times 10^{-5}) = 1.17 \times 10^5 \qquad \epsilon/d = (0.002)/3.2 = 0.000625$$

From Fig. A-5, $f = 0.021$.

$$h_f/L = (0.021/3.2)\{0.5^2/[(2)(9.807)]\} = 8.36 \times 10^{-5} \text{ m/m} \quad \text{or} \quad 83.6\ \mu\text{m/m}$$

$$\tau_0 = f\rho v^2/8 = (0.021)(1.314)(0.5)^2/8 = 0.862 \text{ mPa}$$

$$\delta_1 = 32.8\nu/(vf^{1/2}) = (32.8)(\mu/\rho)/(vf^{1/2}) = (32.8)(1.79 \times 10^{-5}/1.314)/[(0.5)(0.021)^{1/2}] = 0.00620 \text{ m} \quad \text{or} \quad 6.20 \text{ mm}$$

9.133 Repeat Prob. 9.132 for the average velocity 5.0 m/s.

▌ From Fig. A-5, for $N_R = (5.0/0.5)(1.17 \times 10^5) = 1.17 \times 10^6$ and $\epsilon/d = 0.000625$, $f = 0.018$.

$$h_f/L = (f/d)(v^2/2g) = (0.018/3.2)\{5.0^2/[(2)(9.807)]\} = 7.17 \times 10^{-3} \text{ m/m}$$

$$\tau_0 = f\rho v^2/8 = (0.018)(1.314)(5.0)^2/8 = 73.9 \text{ mPa}$$

$$\delta_1 = 32.8\nu/(vf^{1/2}) = (32.8)(\mu/\rho)/(vf^{1/2})$$

$$= (32.8)(1.79 \times 10^{-5}/1.314)/[(5.0)(0.018)^{1/2}] = 6.66 \times 10^{-4} \text{ m} \quad \text{or} \quad 0.660 \text{ mm}$$

9.134 Air at 20 °C and atmospheric pressure flows with a velocity of 6 m/s through a 50-mm-diameter pipe. Find the head loss per meter of pipe if $\epsilon = 0.0025$ mm.

▌ $$h_f/L = (f)(1/d)(v^2/2g) \qquad N_R = dv/\nu = (0.050)(6)/(1.51 \times 10^{-5}) = 1.99 \times 10^4$$

$$\epsilon/d = 0.0025/50 = 0.000050$$

From Fig. A-5, $f = 0.026$. $h_f/L = 0.026[1/0.050]\{6^2/[(2)(9.807)]\} = 0.954$ m/m.

9.135 What is the head loss per foot of pipe when oil (s.g. = 0.88) having a viscosity of 1.9×10^{-4} lb · s/ft² flows in a 2-in-diameter welded steel pipe at 0.15 cfs?

$$h_f/L = (f/d)(v^2/2g) \qquad v = Q/A = 0.15/[(\pi)(\tfrac{2}{12})^2/4] = 6.875 \text{ ft/s}$$

$$N_R = \rho dv/\mu = [(0.88)(1.94)](\tfrac{2}{12})(6.875)/(1.9 \times 10^{-4}) = 1.03 \times 10^4 \qquad \epsilon/d = 0.00015/(\tfrac{2}{12}) = 0.00090$$

From Fig. A-5, $f = 0.033$. $h_f/L = [0.033/(\tfrac{2}{12})]\{6.875^2/[(2)(32.2)]\} = 0.145$ ft/ft.

9.136 Water at 50 °F flows in a 36-in-diameter concrete pipe ($\epsilon = 0.02$ in). For a flow rate of 202 cfs, determine N_R and τ_0.

$$v = Q/A = 202/[(\pi)(\tfrac{36}{12})^2/4] = 28.58 \text{ ft/s} \qquad N_R = dv/\nu = (\tfrac{36}{12})(28.58)/(1.40 \times 10^{-5}) = 6.12 \times 10^6$$

$$\tau_0 = f\rho v^2/8 \qquad \epsilon/d = 0.02/36 = 0.000556$$

From Fig. A-5, $f = 0.0175$.

$$\tau_0 = (0.0175)(1.94)(28.58)^2/8 = 3.47 \text{ lb/ft}^2$$

9.137 What is the flow regime in Prob. 9.136?

$$\delta_1 = 32.8\nu/(vf^{1/2}) = (32.8)(1.40 \times 10^{-5})/[(28.58)(0.0175)^{1/2}] = 0.000121 \text{ ft}$$

Since $[\delta_1 = 0.000121] < [0.3\epsilon = (0.3)(0.02/12) = 0.000500]$, regime is "rough."

9.138 Find the flow rate if water at 60 °F experiences a head loss of $\frac{1}{408}$ ft/ft in 6-in cast iron pipe.

$$h_f/L = (f)(1/d)(v^2/2g) \qquad \tfrac{1}{408} = (f)[1/(\tfrac{6}{12})]\{v^2/[(2)(32.2)]\} \qquad v = 0.2809/\sqrt{f}$$

Try $v = 2$ ft/s: $N_R = dv/\nu = (\tfrac{6}{12})(2)/(1.21 \times 10^{-5}) = 8.26 \times 10^4$, $\epsilon/d = 0.00085/(\tfrac{6}{12}) = 0.00170$. From Fig. A-5, $f = 0.0245$. $v = 0.2809/\sqrt{0.0245} = 1.79$ ft/s. Try $v = 1.79$ ft/s: $N_R = (\tfrac{6}{12})(1.79)/(1.21 \times 10^{-5}) = 7.40 \times 10^4$, $f = 0.025$, $v = 0.2809/\sqrt{0.025} = 1.78$ ft/s. Try $v = 1.78$ ft/s: $N_R = (\tfrac{6}{12})(1.78)/(1.21 \times 10^{-5}) = 7.40 \times 10^4$, $f = 0.025$ (O.K.); $Q = Av = [(\pi)(\tfrac{6}{12})^2/4](1.78) = 0.350$ ft³/s.

9.139 Kerosene with kinematic viscosity 5.1×10^{-7} m²/s flows in a 30-cm-diameter smooth pipe. Find the flow rate when the head loss is 0.4 m per 100 m.

$$h_f = (f)(L/d)(v^2/2g) \qquad 0.4 = (f)[100/(\tfrac{30}{100})]\{v^2/[(2)(9.807)]\} \qquad v = 0.1534/\sqrt{f}$$

Try $v = 1$ m/s: $N_R = dv/\nu = (\tfrac{30}{100})(1)/(5.1 \times 10^{-7}) = 5.88 \times 10^5$. From Fig. A-5, $f = 0.0128$. $v = 0.1534/\sqrt{0.0128} = 1.36$ m/s. Try $v = 1.36$ m/s: $N_R = (\tfrac{30}{100})(1.36)/(5.1 \times 10^{-7}) = 8.00 \times 10^5$, $f = 0.0122$, $v = 0.1534/\sqrt{0.0122} = 1.39$ m/s. Try $v = 1.39$ m/s: $N_R = (\tfrac{30}{100})(1.39)/(5.1 \times 10^{-7}) = 8.18 \times 10^5$, $f = 0.0122$ (O.K.); $Q = Av = [(\pi)(\tfrac{30}{100})^2/4](1.39) = 0.0983$ m³/s.

9.140 A pipe with $\epsilon = 0.00015$ ft is required to carry fluid of kinematic viscosity 0.00021 ft²/s at the rate of 8.0 cfs. If the head loss is to be 0.004 ft/ft, calculate the pipe diameter.

$$h_f/L = (f/d)(v^2/2g) \qquad v = Q/A = 8.0/(\pi d^2/4) = 10.19/d^2 \qquad 0.004 = (f/d)\{(10.19/d^2)^2/[(2)(32.2)]\}$$

$$d = 3.320f^{1/5} \qquad N_R = dv/\nu = (d)(10.19/d^2)/0.00021 = 48\,524/d$$

Try $d = 1$ ft: $N_R = 48\,524/1 = 4.85 \times 10^4$, $\epsilon/d = 0.00015/1 = 0.00015$. From Fig. A-5, $f = 0.0215$. $d = (3.320)(0.0215)^{1/5} = 1.54$ ft. Try $d = 1.54$ ft: $N_R = 48\,524/1.54 = 3.15 \times 10^4$, $\epsilon/d = 0.00015/1.54 = 0.0000974$, $f = 0.0235$, $d = (3.320)(0.0235)^{1/5} = 1.57$ ft. Try $d = 1.57$ ft: $N_R = 50\,950/1.57 = 3.25 \times 10^4$, $\epsilon/d = 0.00015/1.57 = 0.0000955$, $f = 0.0235$ (O.K.). Hence, $d = 1.57$ ft, or 18.8 in.

9.141 (*a*) Find the shear stress on 40-in-diameter asphalted iron pipe if the fluid is water at 72 °F and the average velocity is 10 fps. (*b*) What will be the shear stress if the average velocity is reduced to 5 fps?

▌ $$\tau_0 = f\rho v^2/8 \qquad N_R = dv/\nu$$

(*a*) $$N_R = (\tfrac{40}{12})(10)/(1.02\times10^{-5}) = 3.27\times10^6 \qquad \epsilon/d = 0.0004/(\tfrac{40}{12}) = 0.00012$$

From Fig. A-5, $f = 0.0127$.

$$\tau_0 = (0.0127)(1.93)(10)^2/8 = 0.306\ \text{lb/ft}^2$$

(*b*) $$N_R = (\tfrac{40}{12})(5)/(1.02\times10^{-5}) = 1.63\times10^6 \qquad f = 0.0132$$

$$\tau_0 = (0.0132)(1.93)(5)^2/8 = 0.0796\ \text{lb/ft}^2$$

9.142 A straight steel pipeline ($\epsilon = 0.00015$ ft) slopes downward at a small angle θ, where $\sin\theta \approx \theta = 0.01523$. For gravity flow of oil ($\nu = 0.0006\ \text{ft}^2/\text{s}$) at $10\ \text{ft}^3/\text{s}$, what pipe size is needed?

▌ $$\sin\theta = h_f/L = (f/d)(v^2/2g) \qquad v = Q/A = 10/(\pi d^2/4) = 12.73/d^2$$

$$0.01523 = (f/d)\{(12.73/d^2)^2/[(2)(32.2)]\}$$

$$d = 2.777f^{1/5} \qquad N_R = dv/\nu = (d)(12.73/d^2)/0.0006 = 21\,217/d$$

Try $d = 1$ ft: $N_R = 21\,217/1 = 2.12\times10^4$, $\epsilon/d = 0.00015/1 = 0.00015$. From Fig. A-5, $f = 0.0265$. $d = (2.777)(0.0265)^{1/5} = 1.34$ ft. Try $d = 1.34$ ft: $N_R = 21\,217/1.34 = 1.58\times10^4$, $\epsilon/d = 0.00015/1.34 = 0.00011$, $f = 0.0284$, $d = (2.777)(0.0284)^{1/5} = 1.36$ ft. Try $d = 1.36$ ft: $N_R = 21\,217/1.36 = 1.56\times10^4$, $\epsilon/d = 0.00015/1.36 = 0.00011$, $f = 0.0284$ (O.K.). Hence, $d = 1.36$ ft, or 16.3 in.

9.143 Water at 140 °F flows in a 0.824-in-diameter iron pipe ($\epsilon = 0.00015$ ft) of length 400 ft between points A and B. At point A the elevation of the pipe is 104.0 ft and the pressure is 8.50 psi. At point B the elevation of the pipe is 100.3 ft and the pressure is 9.00 psi. Compute the flow rate.

▌ $$p_A/\gamma + v_A^2/2g + z_A = p_B/\gamma + v_B^2/2g + z_B + h_L$$

$$(8.50)(144)/61.4 + v_A^2/2g + 104.0 = (9.00)(144)/61.4 + v_B^2/2g + 100.3 + h_L \qquad v_A^2/2g = v_B^2/2g$$

$$h_L = 2.52\ \text{ft} \qquad h_f = h_L = (f)(L/d)(v^2/2g) \qquad 2.52 = (f)[400/(0.824/12)]\{v^2/[(2)(32.2)]\} \qquad v = 0.1669/\sqrt{f}$$

Try $v = 1$ ft/s: $N_R = dv/\nu = (0.824/12)(1)/(5.03\times10^{-6}) = 1.37\times10^4$, $\epsilon/d = 0.00015/(0.824/12) = 0.0022$. From Fig. A-5, $f = 0.0325$. $v = 0.1669/\sqrt{0.0325} = 0.926$ ft/s. Try $v = 0.926$ ft/s: $N_R = (0.824/12)(0.926)/(5.03\times10^{-6}) = 1.26\times10^4$, $f = 0.0330$, $v = 0.1669/\sqrt{0.0330} = 0.919$ ft/s. Try $v = 0.919$ ft/s: $N_R = (0.824/12)(0.919)/(5.03\times10^{-6}) = 1.25\times10^4$, $f = 0.0330$ (O.K.); $Q = Av = [(\pi)(0.824/12)^2/4](0.917) = 0.00339\ \text{ft}^3/\text{s}$.

9.144 Air at 50 psia and 160 °F flows in a 1-ft by 1.5-ft by 1000-ft duct at the rate of 1 lb/min. Find the head loss if $\epsilon = 0.0005$ in.

▌ $$h_f = (f)(L/d)(v^2/2g) \qquad d = 4R_h \qquad R_h = A/p_w = (1)(1.5)/(1+1+1.5+1.5) = 0.300\ \text{ft}$$

$$d = (4)(0.300) = 1.20\ \text{ft} \qquad W = \gamma A v \qquad \gamma = p/RT = (50)(144)/[(53.3)(460+160)] = 0.2179\ \text{lb/ft}^3$$

$$1/60 = 0.2179[(1)(1.5)](v) \qquad v = 0.05099\ \text{ft/s} \qquad h_f = (f)(1000/1.20)\{0.05099^2/[(2)(32.2)]\} = 0.03364f$$

$$N_R = dv/\nu = (1.20)(0.05099)/(2.06\times10^{-4}) = 297 \qquad \text{(laminar)}$$

$$f = 64/N_R = 64/297 = 0.2155 \qquad h_f = (0.03364)(0.2155) = 0.00725\ \text{ft}$$

9.145 Find the approximate rate at which 60 °F water will flow in a conduit shaped in the form of an equilateral triangle if the head loss is 2 ft per 100 ft. The cross-sectional area of the duct is $120\ \text{in}^2$, and $\epsilon = 0.0018$ in.

▌ First, find the length of each side (s) of the cross section (see Fig. 9-29):

$$A = bh/2 \qquad 120 = (x+x)(\sqrt{3}x)/2 \qquad x = 8.324\ \text{in} \qquad s = 2x = (2)(8.324) = 16.65\ \text{in}$$

$$h_f = (f)(L/d)(v^2/2g) \qquad R_h = A/p_w = 120/[(3)(16.65)] = 2.402\ \text{in} \quad \text{or} \quad 0.2002\ \text{ft}$$

$$d = 4R_h = (4)(0.2002) = 0.8008\ \text{ft} \qquad 2 = (f)(100/0.8008)\{v^2/[(2)(32.2)]\} \qquad v = 1.016/\sqrt{f}$$

Try $v = 10$ ft/s: $N_R = dv/\nu = (0.8008)(10)/(1.21 \times 10^{-5}) = 6.62 \times 10^5$, $\epsilon/d = (0.0018/12)/0.8008 = 0.000187$. From Fig. A-5, $f = 0.0150$. $v = 1.016/\sqrt{0.0150} = 8.30$ ft/s. Try $v = 8.30$ ft/s: $N_R = (0.8008)(8.30)/(1.21 \times 10^{-5}) = 5.49 \times 10^5$, $f = 0.0155$, $v = 1.016/\sqrt{0.0155} = 8.16$ ft/s. Try $v = 8.16$ ft/s: $N_R = (0.8008)(8.16)/(1.21 \times 10^{-5}) = 5.40 \times 10^5$, $f = 0.0155$ (O.K.); $Q = Av = (\frac{120}{144})(8.16) = 6.80\ \text{ft}^3/\text{s}$.

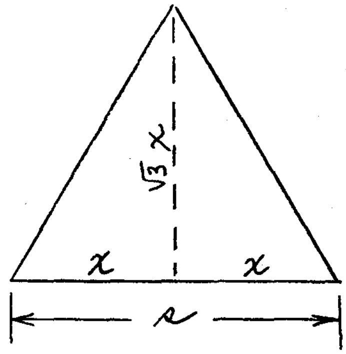


Fig. 9-29

9.146 When fluid of weight density 50 lb/ft³ flows in a 6-in-diameter pipe, the frictional stress between the fluid and the pipe wall is 0.5 lb/ft². Calculate the head loss per mile of pipe. If the flow rate is 2.5 cfs, how much power is lost per mile?

▌ $R_h = d/4 = (\frac{6}{12})/4 = 0.1250$ ft $\qquad h_f = (\tau_0)(L/R_h\gamma) = 0.5\{5280/[(0.1250)(50)]\} = 422.4$ ft per mile

$$P = Q\gamma h_f = (2.5)(50)(422.4) = 52\,800\ \text{ft}\cdot\text{lb/s per mile} = 52\,800/550 = 96.0\ \text{hp per mile}$$

9.147 Prove that for a constant rate of discharge and a constant value of f the friction head loss in a pipe varies inversely as the fifth power of the diameter.

▌ $v = Q/A = 4Q/\pi d^2 \qquad h_f = (f)(L/d)(v^2/2g) = (f)(L/d)[(4Q/\pi d^2)^2/2g] = (f)(L/d^5)(8Q^2/\pi^2 g)$

Thus for constant f and constant Q, $h_f \propto 1/d^5$.

9.148 Two long pipes are used to convey water between two reservoirs whose water surfaces are at different elevations. One pipe has a diameter twice that of the other. If both pipes have the same value of f and if minor losses are neglected, what is the ratio of the flow rates through the two pipes?

▌ $h_f = (f)(L/d)(v^2/2g) = \Delta$ elevation $\qquad h_f \propto Q^2/d^5 \qquad$ (from Prob. 9.147)

$$(h_f)_1 = (h_f)_2$$

Therefore, $Q_1^2/d_1^5 = Q_2^2/d_2^5$, $Q_2/Q_1 = (d_2/d_1)^{5/2} = 2^{5/2} = 5.66$. Thus the flow in the larger pipe will be 5.66 times that in the smaller one.

9.149 Points C and D, at the same elevation, are 500 ft apart in an 8-in pipe and are connected to a differential gage by means of small tubing. When the flow of water is 6.31 cfs, the deflection of mercury in the gage is 6.43 ft. Determine the friction factor f.

▌ $p_C/\gamma + v_C^2/2g + z_C = p_D/\gamma + v_D^2/2g + z_D + h_f$. Since $v_C^2/2g = v_D^2/2g$ and $z_C = z_D$, $P_c/\gamma - p_D/\gamma = h_f = (6.43)(13.6 - 1) = 81.02$ ft, $v = Q/A = 6.31/[(\pi)(\frac{8}{12})^2/4] = 18.08$ ft/s, $h_f = (f)(L/d)(v^2/2g) = (f)[500/(\frac{8}{12})]\{18.08^2/[(2)(32.2)]\} = 3807f$, $81.02 = 3807f$, $f = 0.0213$.

9.150 Oil flows from tank A through 500 ft of 6-in new asphalt-dipped cast iron pipe to point B, as shown in Fig. 9-30. What pressure in pounds per square inch will be needed at A to cause 0.450 cfs of oil to flow? (s.g. = 0.840; $\nu = 2.27 \times 10^{-5}$ ft²/s; $\epsilon = 0.0004$ ft.)

▌ $p_A/\gamma + v_A^2/2g + z_A = p_B/\gamma + v_B^2/2g + z_B + h_L \qquad v_B = Q/A_B = 0.450/[(\pi)(\frac{6}{12})^2/4] = 2.292$ ft/s

$$h_L = h_f = (f)(L/d)(v^2/2g) = (f)[500/(\tfrac{6}{12})]\{2.292^2/[(2)(32.2)]\} = 81.57f$$

$$p_A/[(0.840)(62.4)] + 0 + 80.0 = 0 + 2.292^2/[(2)(32.2)] + 100.0 + 81.57f \qquad p_A = 1053 + 4276f$$

$$N_R = dv/\nu = (\tfrac{6}{12})(2.292)/(2.27 \times 10^{-5}) = 5.05 \times 10^4 \qquad \epsilon/d = 0.0004/(\tfrac{6}{12}) = 0.000800$$

From Fig. A-5, $f = 0.0235$. $p_A = 1053 + (4276)(0.0235) = 1153$ lb/ft², or 8.01 lb/in².

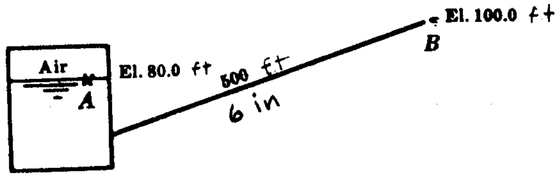


Fig. 9-30

9.151 An old 12-in by 18-in rectangular duct carries air at 15.2 psia and 68 °F through 1500 ft with an average velocity of 9.75 ft/s. Determine the loss of head and the pressure drop, assuming the duct to be horizontal and the size of the surface imperfections is 0.0018 ft.

$$R_h = A/p_w = (\tfrac{12}{12})(\tfrac{18}{12})/(\tfrac{12}{12}+\tfrac{12}{12}+\tfrac{18}{12}+\tfrac{18}{12}) = 0.300 \text{ ft} \qquad d = 4R_h = (4)(0.300) = 1.20 \text{ ft}$$

$$h_f = (f)(L/d)(v^2/2g) = (f)(1500/1.20)\{9.75^2/[(2)(32.2)]\} = 1845f$$

$$N_R = dv/\nu = (1.20)(9.75)/[(14.7/15.2)(1.64\times10^{-4})] = 7.38\times10^4 \text{ (turbulent)} \qquad \epsilon/d = 0.0018/1.20 = 0.00150$$

From Fig. A-5, $f = 0.024$.

$$h_f = (1845)(0.024) = 44.28 \text{ ft of air} \qquad \Delta p = \gamma h_f = [(15.2/14.7)(0.0750)](44.28)/144 = 0.0238 \text{ lb/in}^2$$

9.152 What size of new cast iron pipe, 8000 ft long, will deliver 37.5 cfs of water at 70 °F with a drop in the hydraulic grade line of 215 ft?

$$p_A/\gamma + v_A^2/2g + z_A = p_B/\gamma + v_B^2/2g + z_B + h_L \qquad [(p_A/\gamma + z_A) - (p_B/\gamma + z_B)] = h_L$$

$$[(p_A/\gamma + z_A) - (p_B/\gamma + z_B)] = \text{HGL} = 215 \text{ ft} \qquad v = Q/A = 37.5/(\pi d^2/4) = 47.75/d^2$$

$$h_L = h_f = (f)(L/d)(v^2/2g) = (f)(8000/d)\{(47.75/d^2)^2/[(2)(32.2)]\} = 283\,238f/d^5$$

$$215 = 283\,238f/d^5 \qquad d = 4.207f^{1/5} \qquad N_R = dv/\nu = (d)(47.75/d^2)/(1.05\times10^{-5}) = 4.55\times10^6/d$$

Try $d = 2$ ft: $N_R = (4.55\times10^6)/2 = 2.28\times10^6$, $\epsilon/d = 0.00085/2 = 0.000425$. From Fig. A-5, $f = 0.0164$. $d = (4.207)(0.0164)^{1/5} = 1.85$ ft. Try $d = 1.85$ ft: $N_R = (4.55\times10^6)/1.85 = 2.46\times10^6$, $\epsilon/d = 0.00085/1.85 = 0.000459$, $f = 0.0164$ (O.K.). Hence, $d = 1.85$ ft, or 22.2 in.

9.153 What rate of flow of air at 68 °F will be carried by a new horizontal 2-in-diameter steel pipe at an absolute pressure of 3 atm and with a drop of 0.150 psi in 100 ft of pipe? Use $\epsilon = 0.00025$ ft.

At 68 °F and standard atmospheric pressure, $\gamma = 0.0752$ lb/ft³ and $\nu = 1.60\times10^{-4}$ ft²/s. At a pressure of 3 atm, $\gamma = (0.0752)(3) = 0.2256$ lb/ft³ and $\nu = (1.60\times10^{-4})/3 = 5.333\times10^{-5}$ ft²/s. $h_f = (f)(L/d)(v^2/2g)$, $(0.150)(144)/0.2256 = (f)[100/(\tfrac{2}{12})]\{v^2/[(2)(32.2)]\}$, $v = 3.206/\sqrt{f}$. Try $v = 10$ ft/s: $N_R = dv/\nu = (\tfrac{2}{12})(10)/(5.333\times10^{-5}) = 3.13\times10^4$, $\epsilon/d = 0.00025/(\tfrac{2}{12}) = 0.00150$. From Fig. A-5, $f = 0.027$. $v = 3.206/\sqrt{0.027} = 19.51$ ft/s. Try $v = 19.51$ ft/s: $N_R = (\tfrac{2}{12})(19.51)/(5.333\times10^{-5}) = 6.10\times10^4$, $f = 0.0248$, $v = 3.206/\sqrt{0.0248} = 20.36$ ft/s. Try $v = 20.36$ ft/s: $N_R = (\tfrac{2}{12})(20.36)/(5.333\times10^{-5}) = 6.36\times10^4$, $f = 0.0248$ (O.K.); $Q = Av = [(\pi)(\tfrac{2}{12})^2/4](20.36) = 0.444$ ft³/s.

9.154 Determine the nature of the distribution of shear stress at a cross section in a horizontal, circular pipe under steady flow conditions.

For the free body in Fig. 9-31*a*, since the flow is steady, each particle moves to the right without acceleration. Hence, the summation of the forces in the x direction must equal zero. $(p_1)(\pi r^2) - (p_2)(\pi r^2) - (\tau)(2\pi rL) = 0$ or

$$\tau = (p_1 - p_2)(r)/(2L) \tag{1}$$

When $r = 0$, the shear stress τ is zero; and when $r = r_0$, the stress τ_0 at the wall is a maximum. The variation is linear and is indicated in Fig. 9-31*b*. Equation (*1*) holds for laminar and turbulent flows as no limitations concerning flow were imposed in the derivation. Since $(p_1 - p_2)/\gamma$ represents the drop in the energy line, or the lost head h_L, multiplying Eq. (*1*) by γ/γ yields $\tau = (\gamma r/2L)[(p_1 - p_2)/\gamma]$ or

$$\tau = (\gamma h_L/2L)(r) \tag{2}$$

9.155 Develop the expression for shear stress at a pipe wall.

$h_L = (f)(L/d)(v^2/2g)$. From Prob. 9.154, $h_L = 2\tau_0 L/\gamma r_0 = 4\tau_0 L/\gamma d$, $4\tau_0 L/\gamma d = (f)(L/d)(v^2/2g)$, $\tau_0 = f\gamma v^2/8g = f\rho v^2/8$.

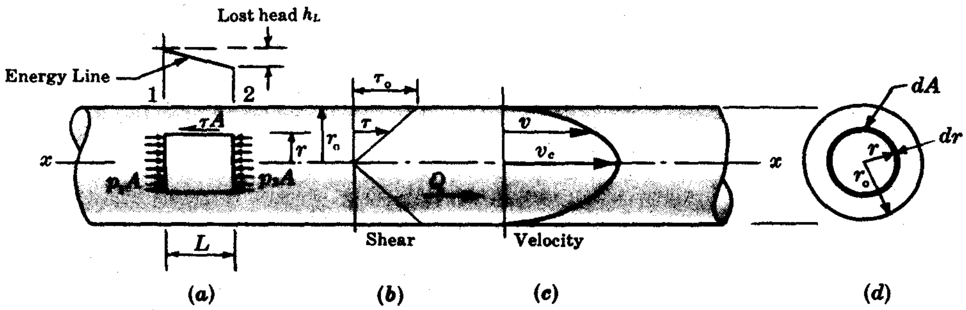


Fig. 9-31

9.156 For steady, laminar flow **(a)** what is the relationship between the velocity at a point in the cross section and the velocity at the center of the pipe, and **(b)** what is the equation for velocity distribution?

▮ **(a)**
$$\tau = -(\mu)(dv/dr) = (p_1 - p_2)(r)/(2L) \qquad \text{(from Prob. 9.154)}$$
$$-(\mu)(dv/dr) = (p_1 - p_2)(r)/(2L)$$

Since $(p_1 - p_2)/L$ is not a function of r,

$$-\int_{v_{\mathbb{C}}}^{v} dv = \frac{p_1 - p_2}{2\mu L}\int_0^r r\,dr \qquad -[v]_{v_{\mathbb{C}}}^{v} = \frac{p_1 - p_2}{4\mu L}[r^2]_0^r \qquad -(v - v_{\mathbb{C}}) = \frac{(p_1 - p_2)r^2}{4\mu L}$$

$$v = v_{\mathbb{C}} - \frac{(p_1 - p_2)r^2}{4\mu L} \tag{1}$$

But the lost head in L feet is $h_L = (p_1 - p_2)/\gamma$; hence,

$$v = v_{\mathbb{C}} - \frac{\gamma h_L r^2}{4\mu L} \tag{2}$$

(b) Since the velocity at the boundary is zero, when $r = r_o$, $v = 0$ in Eq. (*1*), we have

$$v_c = \frac{(p_1 - p_2)r_o^2}{4\mu L} \qquad \text{(at centerline)} \tag{3}$$

Thus, in general,

$$v = \left(\frac{p_1 - p_2}{4\mu L}\right)(r_o^2 - r^2) \tag{4}$$

9.157 Develop the expression for the loss of head in a pipe for steady, laminar flow of an incompressible fluid. Refer to Fig. 9-31*d*.

▮
$$v_{av} = \frac{Q}{A} = \frac{\int v\,dA}{\int dA} = \frac{\int_0^{r_o}(v)(2\pi r\,dr)}{\pi r_o^2} = \frac{(2\pi)(p_1 - p_2)}{(\pi r_o^2)(4\mu L)}\int_0^{r_o}(r_o^2 - r^2)(r\,dr)$$

$$v_{av} = \frac{(p_1 - p_2)(r_o^2)}{8\mu L} \tag{1}$$

Thus for laminar flow, the average velocity is half the maximum velocity v_c in Eq. (*3*) of Prob. 9.156. Rearranging Eq. (*1*), we obtain

$$h_L = \frac{p_1 - p_2}{\gamma} = \frac{8\mu L v_{av}}{\gamma r_o^2} = \frac{32\mu L v_{av}}{\gamma d^2} \tag{2}$$

These expressions apply for laminar flow of all fluids in all pipes and conduits.

9.158 Determine **(a)** the shear stress at the walls of a 12-in-diameter pipe when water flowing causes a measured head loss of 15 ft in 300 ft of pipe length, **(b)** the shear stress 2 in from the centerline of the pipe, **(c)** the shear velocity, **(d)** the average velocity for an f value of 0.50, and **(e)** the ratio v/v_*.

(a) $\tau_o = \gamma h_L r_o/2L = (62.4)(15)[(\frac{12}{12})/2]/[(2)(300)] = 0.780\ \text{lb/ft}^2$, or $0.00542\ \text{lb/in}^2$.
(b) Since τ varies linearly from centerline to wall, $\tau = (0.00542)(\frac{2}{6}) = 0.00181\ \text{lb/in}^2$.
(c) $v_* = \sqrt{\tau_o/\rho} = \sqrt{0.780/1.94} = 0.634\ \text{ft/s}$.
(d) $\tau_o = f\rho v^2/8$, $0.780 = (0.050)(1.94)(v^2)/8$, $v = 8.02\ \text{ft/s}$.
(e) $\tau_o = (\mu)(v/y)$, $\nu = \mu/\rho$, $\tau_0 = \rho\nu(v/y)$, $\tau_o/\rho = (\nu)(v/y) = v_*^2 = (\nu)(v/y)$, $v/v_*^2 = y/\nu$, $v/v_* = v_* y/\nu$.

9.159 If in Prob. 9.158 the water is flowing through a 3-ft by 4-ft rectangular conduit of the same length with the same lost head, what is the shear stress between the water and the pipe wall?

$$R_h = A/p_w = (3)(4)/(3+3+4+4) = 0.8571\ \text{ft}$$

$$\tau = (\gamma h_L/L)(R_h) = [(62.4)(15)/300](0.8571) = 2.67\ \text{lb/ft}^2 \quad \text{or} \quad 0.0186\ \text{lb/in}^2$$

9.160 Medium lubricating oil (s.g. = 0.860) is pumped through 1000 ft of horizontal 2-in pipe at the rate of 0.0436 cfs. If the drop in pressure is 30.0 psi, what is the absolute viscosity of the oil?

Assuming laminar flow,

$$\frac{p_1 - p_2}{\gamma} = \frac{32\mu L v_{av}}{\gamma d^2} \qquad \text{(from Prob. 9.157)}$$

$$v_{av} = Q/A = 0.0436/[(\pi)(\tfrac{2}{12})^2/4] = 1.998\ \text{ft/s}$$

$$(30.0)(144)/[(0.860)(62.4)] = (32)(\mu)(1000)(1.998)/\{[(0.860)(62.4)](\tfrac{2}{12})^2\} \qquad \mu = 0.00188\ \text{lb}\cdot\text{s/ft}^2$$

$$N_R = \rho d v/\mu = [(0.860)(1.94)](\tfrac{2}{12})(1.998)/0.00188 = 296 \qquad \text{(laminar)}$$

9.161 A horizontal wrought iron pipe, 6-in inside diameter and somewhat corroded, is transporting 4.50 lb of air per second from A to B. At A the pressure is 70 psia and at B the pressure must be 65 psia. Flow is isothermal at 68 °F. What is the length of pipe from A to B? Use $\epsilon = 0.0013$ ft.

$$\frac{p_1 - p_2}{\gamma_1} = \frac{2[2\ln(v_2/v_1) + (f)(L/d)](v_1^2/2g)}{1 + p_2/p_1}$$

$$\gamma_1 = (0.0752)(70/14.7) = 0.3581\ \text{lb/ft}^3 \qquad \gamma_2 = (0.0752)(65/14.7) = 0.3325\ \text{lb/ft}^3$$

$$W = \gamma A v \qquad 4.50 = (0.3581)[(\pi)(\tfrac{6}{12})^2/4](v_1) \qquad v_2 = 64.00\ \text{ft/s}$$

$$4.50 = 0.3325[(\pi)(\tfrac{6}{12})^2/4](v_2) \qquad v_1 = 68.93\ \text{ft/s}$$

$$N_R = dv/\nu = (\tfrac{6}{12})(64.00)/[(14.7/70.0)(1.60\times10^{-4})] = 9.52\times10^5 \qquad \epsilon/d = 0.0013/(\tfrac{6}{12}) = 0.0026$$

From Fig. A-5, $f = 0.025$.

$$\frac{(70-65)(144)}{0.3581} = \frac{2\{2\ln(68.93/64.00) + 0.025[L/(\tfrac{6}{12})]\}\{64.00^2/[(2)(32.2)]\}}{1 + \tfrac{65}{70}} \qquad L = 607\ \text{ft}$$

9.162 Heavy fuel oil flows from A to B through 3000 ft of horizontal 6-in steel pipe. The pressure at A is 155 psi and at B is 5.0 psi. The kinematic viscosity is 0.00444 ft²/s and the specific gravity is 0.918. What is the flow rate?

Assuming laminar flow, from Eq. (2) of Prob. 9.157,

$$\frac{p_1 - p_2}{\gamma} = \frac{32\mu L v_{av}}{\gamma d^2} = \frac{(32)(\nu\rho)(L v_{av})}{\gamma d^2} \qquad \frac{(155-5.0)(144)}{(0.918)(62.4)} = \frac{32\{(0.00444)[(0.918)(1.94)]\}(3000)(v_{av})}{[(0.918)(62.4)](\tfrac{6}{12})^2}$$

$$v_{av} = 7.11\ \text{ft/s} \qquad N_R = dv/\nu = (\tfrac{6}{12})(7.11)/0.00444 = 808 \qquad \text{(laminar)}$$

$$Q = Av = [(\pi)(\tfrac{6}{12})^2/4](7.11) = 1.40\ \text{ft}^3/\text{s}$$

9.163 What size pipe should be installed to carry 0.785 cfs of heavy fuel oil ($\nu = 0.00221$ ft²/s, s.g. = 0.912) at 60 °F if the available lost head in the 1000-ft length of horizontal pipe is 22.0 ft?

Assuming laminar flow,

$$h_f = \frac{32\mu L v}{\gamma d^2} \qquad \mu = \rho\nu = [(0.912)(1.94)](0.00221) = 0.003910\ \text{lb}\cdot\text{s/ft}^2$$

$$v = Q/A = 0.785/(\pi d^2/4) = 0.9995/d^2 \qquad 22.0 = (32)(0.003910)(1000)(0.9995/d^2)/\{[(0.912)(62.4)](d^2)\}$$

$$d = 0.562\ \text{ft} \quad \text{or} \quad 6.75\ \text{in} \qquad N_R = dv/\nu = (0.562)(0.9995/0.562^2)/0.00221 = 805 \qquad \text{(laminar)}$$

9.164 Determine the head loss in 1000 ft of new, uncoated 12-in-ID cast iron pipe when water at 60 °F flows at 5.00 ft/s. Use $\epsilon/d = 0.0008$.

$$h_f = (f)(L/d)(v^2/2g) \qquad N_R = dv/\nu = (\tfrac{12}{12})(5.00)/(1.21\times10^{-5}) = 4.13\times10^5$$

From Fig. A-5, $f = 0.0194$. $h_f = 0.0194[1000/(\tfrac{12}{12})]\{5.00^2/[(2)(32.2)]\} = 7.53$ ft.

9.165 Rework Prob. 9.164 if the liquid is medium fuel oil at 60 °F ($\nu = 4.75\times10^{-5}$ ft²/s) flowing at the same velocity.

$$h_f = (f)(L/d)(v^2 2g) \qquad N_R = dv/\nu = (\tfrac{12}{12})(5.00)/(4.75\times10^{-5}) = 1.05\times10^5$$

From Fig. A-5, $f = 0.0213$. $h_f = 0.0213[1000/(\tfrac{12}{12})]\{5.00^2/[(2)(32.2)]\} = 8.27$ ft.

9.166 Points A and B are 4000 ft apart along a new 6-in-ID steel pipe. Point B is 50.5 ft higher than A and the pressures at A and B are 123 psi and 48.6 psi, respectively. How much medium fuel oil at 70 °F will flow from A to B? Use s.g. = 0.854, $\nu = 4.12\times10^{-5}$ ft²/s, $\epsilon = 0.0002$ ft.

$$p_A/\gamma + v_A^2/2g + z_A = p_B/\gamma + v_B^2/2g + z_B + h_L$$

$$(123)(144)/[(0.854)(62.4)] + v_A^2/2g + 0 = (48.6)(144)/[(0.854)(62.4)] + v_B^2/2g + 50.5 + h_L$$

$$v_A^2/2g = v_B^2/2g \qquad h_L = 150.5 \text{ ft} = h_f = (f)(L/d)(v^2/2g)$$

$$150.5 = (f)[4000/(\tfrac{6}{12})]\{v^2/[(2)(32.2)]\} \qquad v = 1.101/\sqrt{f}$$

Try $v = 10$ ft/s: $N_R = dv/\nu = (\tfrac{6}{12})(10)/(4.12\times10^{-5}) = 1.21\times10^5$, $\epsilon/d = 0.0002/(\tfrac{6}{12}) = 0.000400$. From Fig. A-5, $f = 0.0195$. $v = 1.101/\sqrt{0.0195} = 7.884$ ft/s. Try $v = 7.884$ ft/s: $N_R = (\tfrac{6}{12})(7.884)/(4.12\times10^{-5}) = 9.57\times10^4$, $f = 0.0195$ (O.K.); $Q = Av = [(\pi)(\tfrac{6}{12})^2/4](7.884) = 1.55$ ft³/s.

9.167 How much water (60 °F) would flow under the conditions of Prob. 9.166.

$$p_A/\gamma + v_A^2/2g + z_A = p_B/\gamma + v_B^2/2g + z_B + h_L$$

$$(123)(144)/62.4 + v_A^2/2g + 0 = (48.6)(144)/62.4 + v_B^2/2g + 50.5 + h_L$$

$$v_A^2/2g = v_B^2/2g \qquad h_L = 121.2 \text{ ft} = h_f = (f)(L/d)(v^2/2g)$$

$$121.2 = (f)[4000/(\tfrac{6}{12})]\{v^2/[(2)(32.2)]\} \qquad v = 0.9878/\sqrt{f}$$

Try $v = 10$ ft/s: $N_R = dv/\nu = (\tfrac{6}{12})(10)/(1.21\times10^{-5}) = 4.13\times10^5$, $\epsilon/d = 0.0002/(\tfrac{6}{12}) = 0.000400$. From Fig. A-5, $f = 0.0172$. $v = 0.9878/\sqrt{0.0172} = 7.532$ ft/s. Try $v = 7.532$ ft/s: $N_R = (\tfrac{6}{12})(7.532)/(1.21\times10^{-5}) = 3.11\times10^5$, $f = 0.0176$, $v = 0.9878/\sqrt{0.0176} = 7.446$ ft/s. Try $v = 7.446$ ft/s: $N_R = (\tfrac{6}{12})(7.446)/(1.21\times10^{-5}) = 3.08\times10^5$, $f = 0.0176$ (O.K.); $Q = Av = [(\pi)(\tfrac{6}{12})^2/4](7.446) = 1.46$ ft³/s.

9.168 To transport 300 cfs of air, $p = 16$ psia, $T = 68$ °F, with a head loss of 3 in of water per 1000 ft, what size galvanized pipe is needed? ($\epsilon = 0.0005$ ft.)

$$d = 0.66[(\epsilon^{1.25})(LQ^2/gh_f)^{4.75} + (\nu)(Q)^{9.4}(L/gh_f)^{5.2}]^{0.04}$$

$$\gamma = p/RT = (16)(144)/[(53.3)(460+68)] = 0.08187 \text{ lb/ft}^3 \qquad h_f = (\tfrac{3}{12})(62.4/0.08187) = 190.5 \text{ ft of air}$$

$$d = 0.66\left\{(0.0005^{1.25})\left[\frac{(1000)(300)^2}{(32.2)(190.5)}\right]^{4.75} + (1.64\times10^{-4})(300)^{9.4}\left[\frac{1000}{(32.2)(190.5)}\right]^{5.2}\right\}^{0.04} = 2.84 \text{ ft}$$

9.169 Two tanks of a solvent ($\mu = 0.05$ Pa · s, $\gamma = 8$ kN/m³) are connected by 300 m of commercial steel pipe. What size must the pipe be to convey 50 L/s, if one tank is 4 m higher than the other?

$$d = 0.66[(\epsilon^{1.25})(LQ^2/gh_f)^{4.75} + (\nu)(Q)^{9.4}(L/gh_f)^{5.2}]^{0.04}$$

$$\nu = \mu/\rho = \mu g/\gamma = (0.05)(9.807)/[8\times10^3] = 6.129\times10^{-5} \text{ m}^2/\text{s}$$

$$d = 0.66\left\{(0.000046^{1.25})\left[\frac{(300)(\frac{50}{1000})^2}{(9.807)(4)}\right]^{4.75} + (6.129\times10^{-5})(\tfrac{50}{1000})^{9.4}\left[\frac{300}{(9.807)(4)}\right]^{5.2}\right\}^{0.04} = 0.222 \text{ m}$$

9.170 Calculate the diameter of a wooden conduit ($\epsilon = 0.006$ ft) that is to carry 300 ft³/s of water at 60 °F a distance of 1000 ft with a head loss of 1.1 ft.

$$d = 0.66[(\epsilon^{1.25})(LQ^2/gh_f)^{4.75} + (\nu)(Q)^{9.4}(L/gh_f)^{5.2}]^{0.04}$$

$$d = 0.66\left\{(0.0006^{1.25})\left[\frac{(1000)(300)^2}{(32.2)(1.1)}\right]^{4.75} + (1.21\times10^{-5})(300)^{9.4}\left[\frac{1000}{(32.2)(1.1)}\right]^{5.2}\right\}^{0.04} = 7.59 \text{ ft}$$

9.171 An old pipe 2 m in diameter has a roughness of $\epsilon = 30$ mm. A 12-mm-thick lining would reduce the roughness to $\epsilon = 1$ mm. How much would pumping costs be reduced per kilometer of pipe for water at 20 °C with discharge of 6 m³/s? The pumps are 75 percent efficient, and the cost of energy is \$1 per 72 MJ.

$$v_1 = Q/A_1 = 6/[(\pi)(2)^2/4] = 1.910 \text{ m/s} \qquad N_R = dv/\nu$$
$$(N_R)_1 = (2)(1.910)/(1.02\times 10^{-6}) = 3.75\times 10^6 \qquad \epsilon_1/d_1 = (0.030)/2 = 0.015$$

From Fig. A-5, $f_1 = 0.044$.

$$d_2 = [2-(2)(0.012)] = 1.976 \text{ m} \qquad v_2 = Q/A_2 = 6/[(\pi)(1.976)^2/4] = 1.957 \text{ m/s}$$
$$(N_R)_2 = (1.976)(1.957)/(1.02\times 10^{-6}) = 3.79\times 10^6 \qquad \epsilon_2/d_2 = (0.001)/1.976 = 0.000506$$
$$f_2 = 0.017 \qquad h_f = (f)(L/d)(v^2/2g)$$
$$(h_f)_1 = 0.044[1000/2]\{1.910^2/[(2)(9.807)]\} = 4.902 \text{ m}$$
$$(h_f)_2 = 0.017[1000/1.976]\{1.957^2/[(2)(9.807)]\} = 1.680 \text{ m}$$
$$\text{Saving in head} = 4.092 - 1.680 = 2.412 \text{ m} \qquad P = Q\gamma h_f/\eta = (6)(9.79\times 10^3)(2.412)/0.75 = 0.1889 \text{ MJ/s}$$
$$\text{Savings per year} = (0.1889)[(365)(24)(3600)]/72 = \$82\,738$$

9.172 What size of new cast iron pipe is needed to transport 0.5 m³/s of water at 25 °C for 1000 m with head loss of 2 m?

$$d = 0.66[(\epsilon^{1.25})(LQ^2/gh_f)^{4.75} + (\nu)(Q)^{9.4}(L/gh_f)^{5.2}]^{0.04}$$
$$d = 0.66\left\{(0.00026^{1.25})\left[\frac{(1000)(0.5)^2}{(9.807)(2)}\right]^{4.75} + (9.02\times 10^{-7})(0.5)^{9.4}\left[\frac{1000}{(9.807)(2)}\right]^{5.2}\right\}^{0.04} = 0.713 \text{ m}$$

9.173 Pure water at 70 °F runs at 2.778 lb/s through a smooth tube between two reservoirs 30 ft apart and having a difference in elevation of 4.1 ft. What size tubing is needed?

$$Q = 2.778/62.3 = 0.04459 \text{ ft}^3\text{/s} \qquad h_f = (f)(L/d)(v^2/2g)$$
$$v = Q/A = 0.04459/(\pi d^2/4) = 0.05677/d^2 \qquad 4.1 = (f)(30/d)\{(0.05677/d^2)^2/[(2)(32.2)]\} \qquad d = 0.2055f^{1/5}$$
$$N_R = dv/\nu = (d)(0.05677/d^2)/(1.05\times 10^{-5}) = 5.41\times 10^3/d$$

Try $f = 0.020$: $d = (0.2055)(0.020)^{1/5} = 0.09398$ ft, $N_R = 5.41\times 10^3/0.09398 = 5.76\times 10^4$. From Fig. A-5, $f = 0.0205$. Try $f = 0.0205$: $d = (0.2055)(0.0205)^{1/5} = 0.09444$ ft, $N_R = 5.41\times 10^3/0.09444 = 5.73\times 10^4$, $f = 0.0205$ (O.K.). Hence, $d = 0.09444$ ft, or 1.13 in.

9.174 In Fig. 9-10, $H = 20$ m, $L = 150$ m, $d = 50$ mm, s.g. = 0.85, $\mu = 0.400$ N · s/m², and $\epsilon = 1$ mm. Find the newtons per second flowing. Neglect minor losses.

$$h_f = (f)(L/d)(v^2/2g) \qquad 20 = (f)[150/(\tfrac{50}{1000})]\{v^2/[(2)(9.807)]\} \qquad v = 0.03616/\sqrt{f}$$
$$N_R = \rho dv/\mu = [(0.85)(1000)](\tfrac{50}{1000})(v)/0.004 = 10\,625v$$

Try $f = 0.050$: $v = 0.3616/\sqrt{0.050} = 1.617$ m/s, $N_R = (10\,625)(1.617) = 1.72\times 10^4$, $\epsilon/d = (\frac{1}{1000})/(\frac{50}{1000}) = 0.0200$. From Fig. A-5, $f = 0.051$.

$$v = 0.3616/\sqrt{0.051} = 1.601 \text{ m/s} \qquad N_R = (10\,625)(1.601) = 1.70\times 10^4 \qquad f = 0.051 \qquad \text{(O.K.)}$$
$$W = \gamma A v = [(0.85)(9.79)][(\pi)(\tfrac{50}{1000})^2/4](1.601) = 0.0262 \text{ kN/s} \quad \text{or} \quad 26.2 \text{ N/s}$$

9.175 Determine the head loss for flow of 140 L/s of oil, $\nu = 0.00001$ m²/s, through 400 m of 200-mm-diameter cast iron pipe.

$$h_f = (f)(L/d)(v^2/2g) \qquad v = Q/A = (\tfrac{140}{1000})/[(\pi)(\tfrac{200}{1000})^2/4] = 4.456 \text{ m/s}$$
$$N_R = dv/\nu = (\tfrac{200}{1000})(4.456)/0.00001 = 8.91\times 10^4 \qquad \epsilon/d = 0.00026/(\tfrac{200}{1000}) = 0.00130$$

From Fig. A-5, $f = 0.023$. $h_f = 0.023[400/(\frac{200}{1000})]\{4.456^2/[(2)(9.807)]\} = 46.6$ m.

9.176 Water at 15 °C flows through a 300-mm-diameter riveted steel pipe, $\epsilon = 3$ mm, with a head loss of 6 m in 300 m. Determine the flow.

▮ $h_f = (f)(L/d)(v^2/2g)$. Try $f = 0.040$: $6 = 0.040[300/(\frac{300}{1000})]\{v^2/[(2)(9.807)]\}$, $v = 1.715$ m/s; $N_R = dv/\nu = (\frac{300}{1000})(1.715)/(1.16 \times 10^{-6}) = 4.44 \times 10^5$; $\epsilon/d = \frac{3}{300} = 0.0100$. From Fig. A-5, $f = 0.038$. Try $f = 0.038$: $6 = (0.038)[300/(\frac{300}{1000})]\{v^2/[(2)(9.807)]\}$, $v = 1.760$ m/s; $N_R = (\frac{300}{1000})(1.760)/(1.16 \times 10^{-6}) = 4.55 \times 10^5$; $f = 0.038$ (O.K.); $Q = Av = [(\pi)(\frac{300}{1000})^2/4](1.760) = 0.124\ \text{m}^3/\text{s}$.

9.177 Determine the size of clean wrought iron pipe required to convey 4000 gpm of oil, $\nu = 0.0001\ \text{ft}^2/\text{s}$, 10 000 ft with a head loss of 75 ft.

▮
$$Q = (4000)(0.002228) = 8.912\ \text{ft}^3/\text{s} \qquad h_f = (f)(L/d)(v^2/2g)$$
$$v = Q/A = 8.912/(\pi d^2/4) = 11.35/d^2 \qquad 75 = (f)(10\,000/d)\{(11.35/d^2)^2/[(2)(32.2)]\} \qquad d = 3.056 f^{1/5}$$

Try $f = 0.020$: $d = (3.056)(0.020)^{1/5} = 1.398$ ft, $N_R = dv/\nu = (1.398)(11.35/1.398^2)/0.0001 = 8.12 \times 10^4$, $\epsilon/d = 0.00015/1.398 = 0.000107$. From Fig. A-5, $f = 0.019$. Try $f = 0.019$: $d = (3.056)(0.019)^{1/5} = 1.383$ ft, $N_R = (1.383)(11.35/1.383^2)/0.0001 = 8.21 \times 10^4$, $f = 0.019$ (O.K.). Hence, $d = 1.383$ ft, or 16.6 in.

9.178 In Prob. 9.177, for $d = 16.6$ in, if the specific gravity is 0.85, $p_1 = 40$ psi, $z_1 = 200$ ft, and $z_2 = 50$ ft, determine the pressure at point 2.

▮
$$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L$$
$$(40)(144)/[(0.85)(62.4)] + v_1^2/2g + 200 = (p_2)(144)/[(0.85)(62.4)] + v_2^2/2g + 50 + 75$$
$$v_1^2/2g = v_2^2/2g \qquad p_2 = 67.6\ \text{lb/in}^2$$

9.179 What size galvanized iron pipe is needed to be "hydraulically smooth" at $N_R = 3.5 \times 10^5$? (A pipe is said to be hydraulically smooth when it has the same losses as a smoother pipe under the same conditions.)

▮ From Fig. A-5, $\epsilon/d = 0.00001$ is equivalent to smooth pipe. $0.0005/d = 0.00001$, $d = 50.0$ ft.

9.180 Above what Reynolds number is the flow through a 3-m-diameter riveted steel pipe, $\epsilon = 30$ mm, independent of the viscosity of the fluid?

▮ $\epsilon/d = 0.030/3 = 0.010$. From Fig. A-5, $N_R = 1.0 \times 10^5$ at complete turbulence.

9.181 Determine the absolute roughness of a 1-ft-diameter pipe that has a friction factor of 0.032 when $N_R = 1.0 \times 10^6$.

▮ From Fig. A-5, $\epsilon/d = 0.006$. $\epsilon/1 = 0.006$, $\epsilon = 0.006$ ft.

9.182 Galvanized iron pipe of diameter d has the same friction factor for $N_R = 100\,000$ as 400-mm-diameter cast iron pipe. Evaluate d.

▮ **For cast iron:** $\epsilon/d = 0.00026/0.400 = 0.00065$.

For galvanized iron: $\epsilon/d = 0.00015/d = 0.00065$, $d = 0.231$ m, or 231 mm.

9.183 Calculate the friction factor for atmospheric air at 80 °F, flowing at $v = 40$ ft/s in a 2-ft-diameter galvanized pipe.

▮
$$f = 1.325/[\ln(\epsilon/3.7d + 5.74/N_R^{0.9})]^2 \qquad N_R = dv/\nu = (3)(40)/(1.69 \times 10^{-4}) = 710\,059$$
$$f = 1.325/\{\ln[0.0005/(3.7)(2) + 5.74/710\,059^{0.9}]\}^2 = 0.0156$$

9.184 Atmospheric air at 92 °F is conducted through a 4-ft-diameter, 1000-ft-long wrought iron pipe. Find the head loss corresponding to a flow of 266.7 cfs.

▮
$$h_f = (f)(L/d)(v^2/2g) \qquad \rho = p/RT = (14.7)(144)/[(1716)(460 + 92)] = 0.002234\ \text{slug/ft}^3$$
$$v = Q/A = 266.7/[(\pi)(4)^2/4] = 21.22\ \text{ft/s}$$
$$N_R = \rho dv/\mu = (0.002234)(4)(21.22)/(3.90 \times 10^{-7}) = 4.86 \times 10^5 \qquad \epsilon/d = 0.00015/4 = 0.0000375$$

From Fig. A-5, $f = 0.013$. $h_f = (0.013)(1000/4)\{21.22^2/[(2)(32.2)]\} = 22.72$ ft.

9.185 A smooth toroidal wind tunnel is 60 m around at the centerline and has cross-sectional diameter 2 m; the fluid is air at 1 atm and 20 °C. Determine the horsepower rating of a fan that will produce a 500-km/h airstream.

$$h_f = (f)(L/d)(v^2/2g) \qquad v = 5 \times 10^5/3600 = 138.9 \text{ m/s}$$

$$N_R = dv/\nu = (2)(138.9)/(1.46 \times 10^{-5}) = 1.90 \times 10^7$$

From Fig. A-5, $f = 0.0072$ (extrapolated).

$$h_f = (0.0072)(60/2)\{138.9^2/[(2)(9.807)]\} = 212.5 \text{ m} \qquad Q = Av = [(\pi)(2^2/4)](138.9) = 436.4 \text{ m}^3/\text{s}$$

$$P = Q\gamma h_f = (436.4)(11.8)(212.5) = 1.09 \times 10^6\ W \quad \text{or} \quad 1460 \text{ hp}$$

9.186 Assume that 2.0 cfs of oil ($\mu = 0.0003342$ slug/ft · s, $\rho = 1.677$ slug/ft³) is pumped through a 12-in pipeline of cast iron. If each pump produces 100 psi, how far apart can they be placed?

$$h_f = (f)(L/d)(v^2/2g) = p/\rho g = (100 \times 144)/(1.677)(32.2) = 266.7 \text{ft}$$

$$v = Q/A = 2.0/[(\pi)(\tfrac{12}{12})^2/4] = 2.546 \text{ ft/s}$$

$$N_R = \rho dv/\mu = (1.667)(\tfrac{12}{12})(2.546)/0.0003342 = 1.27 \times 10^4 \qquad \epsilon/d = 0.00085/(\tfrac{12}{12}) = 0.00085$$

From Fig. A-5, $f = 0.031$. $266.7 = 0.031[L/(\tfrac{12}{12})]\{2.546^2/[(2)(32.2)]\}$, $L = 85\,473$ ft, or 16.2 miles.

9.187 A 60-mm-diameter smooth pipe 160 m long conveys 36 m³/h of water at 25 °C from a sidewalk hydrant to the top of a building 25 m tall. What pressure can be maintained at the top of the building, if the hydrant pressure is 1.6 MPa?

$$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L \qquad h_L = h_f = (f)(L/d)(v^2/2g)$$

$$v = Q/A = (36/3600)/[(\pi)(0.060)^2/4] = 3.537 \text{ m/s} \qquad N_R = dv/\nu = (0.060)(3.537)/(9.10 \times 10^{-7}) = 2.33 \times 10^5$$

From Fig. A-5, $f = 0.016$.

$$h_L = 0.016[160/0.060]\{3.537^2/[(2)(9.807)]\} = 27.21 \text{ m}$$

$$(1.6)(1000)/9.79 + v_1^2/2g + 0 = (p_2)(1000)/9.79 + v_2^2/2g + 25 + 27.21 \qquad v_1^2/2g = v_2^2/2g \qquad p_2 = 1.09 \text{ MPa}$$

9.188 Calculate the discharge for the pipe of Fig. 9-32; the fluid is water at 150 °F.

$$h_L = h_f = (f)(L/d)(v^2/2g) \qquad 258 = (f)[238/(\tfrac{2}{12})]\{v_2^2/[(2)(32.2)]\} \qquad v = 3.411/\sqrt{f}$$

Try $f = 0.019$: $v = 3.411/\sqrt{0.019} = 24.74$ ft/s, $\epsilon/d = 0.00015/(\tfrac{2}{12}) = 0.000900$, $N_R = dv/\nu = (\tfrac{2}{12})(24.74)/(4.68 \times 10^{-6}) = 8.81 \times 10^5$. From Fig. A-5, $f = 0.019$ (O.K.); $Q = Av = [(\pi)(\tfrac{2}{12})^2/4](24.74) = 0.540$ ft³/s.

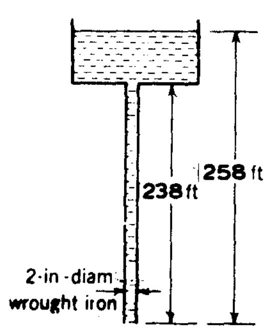


Fig. 9-32

9.189 In Fig. 9-32 how much power would be required to pump 160 gpm of water at 60 °F from a reservoir at the bottom of the pipe to the reservoir shown?

$$Q = (160)(0.002228) = 0.3565 \text{ ft}^3/\text{s} \qquad p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L$$

$$v_1 = Q/A_1 = 0.3565/[(\pi)(\tfrac{2}{12})^2/4] = 16.34 \text{ ft/s} \qquad h_L = h_f = (f)(L/d)(v^2/2g)$$

$$N_R = dv/\nu = (\tfrac{2}{12})(16.34)/(1.21 \times 10^{-5}) = 2.25 \times 10^5 \qquad \epsilon/d = 0.00015/(\tfrac{2}{12}) = 0.000900$$

From Fig. A-5, $f = 0.0205$.

$$h_L = 0.0205[238/(\tfrac{2}{12})]\{16.34^2/[(2)(32.2)]\} = 121.4\ \text{ft} \qquad p_1/\gamma + 16.34^2/[(2)(32.2)] + 0 = 0 + 0 + 260 + 121.4$$

$$p_1/\gamma = 377.3\ \text{ft} \qquad P = Q\gamma(\Delta p/\gamma) = (0.3565)(62.4)(377.3) = 8393\ \text{ft}\cdot\text{lb/s} = 15.3\ \text{hp}$$

9.190 A 12-mm-diameter commercial steel pipe 16 m long is used to drain an oil tank. Determine the discharge when the oil level in the tank is 2 m above the exit end of the pipe. ($\mu = 0.10\ \text{Pa}\cdot\text{s}$, $\gamma = 8\ \text{kN/m}^3$.)

▌ Assuming laminar flow,

$$v = \frac{h_L\gamma d^2}{32\mu L} = \frac{2[(8)(1000)](0.012)^2}{(32)[(0.1)(0.10)](16)} = 0.4500\ \text{m/s}$$

$$N_R = \rho dv/\mu = (\gamma/g)(dv)/\mu = [(8)(1000)/9.807](0.012)(0.4500)/[(0.1)(0.10)] = 440 \qquad \text{(laminar)}$$

$$Q = Av = [(\pi)(0.012)^2/4](0.4500) = 0.0000509\ \text{m}^3/\text{s} \quad \text{or} \quad 0.0509\ \text{L/s}$$

9.191 Two liquid reservoirs are connected by 198 ft of 2-in-diameter smooth tubing. What is the flow rate when the difference in elevation is 50 ft? ($\nu = 0.001\ \text{ft}^2/\text{s}$).

▌ $$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L \qquad h_L = h_f = (f)(L/d)(v^2/2g)$$

Assuming laminar flow,

$$f = 64/N_R \qquad N_R = dv/\nu = (\tfrac{2}{12})(v)/0.001 = 166.7v \qquad h_L = (64/166.7v)[198/(\tfrac{2}{12})]\{v^2/[(2)(32.2)]\} = 7.082v$$

$$0 + 0 + 50 = 0 + 0 + 0 + 7.082v \qquad v = 7.060\ \text{ft/s} \qquad N_R = (\tfrac{2}{12})(7.060)/0.001 = 1177 \qquad \text{(laminar)}$$

$$Q = Av = [(\pi)(\tfrac{2}{12})^2/4](7.060) = 0.154\ \text{ft}^3/\text{s}$$

9.192 Atmospheric air at 16 °C flows for 200 m through a 1.25-m-diameter duct ($\epsilon = 1$ mm). Calculate the flow volume, if the head loss measures 80 mmH$_2$O.

▌ $h_f = (f)(L/d)(v^2/2g) \qquad (\gamma h_f)_{\text{air}} = (\gamma h_f)_{H_2O} \qquad \gamma_{\text{air}} = p/RT = 101.4/[(29.3)(273 + 16)] =$ $0.01197\ \text{kN/m}^3$

$$(0.01197)(h_f)_{\text{air}} = (9.79)(0.080) \qquad (h_f)_{\text{air}} = 65.43\ \text{m} \qquad 65.43 = (f)(200/1.25)\{v^2/[(2)(9.807)]\} \qquad v = 2.832/\sqrt{f}$$

Try $f = 0.020$: $v = 2.832/\sqrt{0.020} = 20.03$ m/s, $N_R = dv/\nu = (1.25)(20.03)/(1.46 \times 10^{-5}) = 1.71 \times 10^6$, $\epsilon/d = 0.001/1.25 = 0.000800$. From Fig. A-5, $f = 0.0205$. Try $f = 0.0205$: $v = 2.832/\sqrt{0.0205} = 19.78$ m/s, $N_R = (1.25)(19.78)/(1.46 \times 10^{-5}) = 1.69 \times 10^6$, $f = 0.0205$ (O.K.); $Q = Av = [(\pi)(1.25^2/4)](19.78) = 24.3\ \text{m}^3/\text{s}$.

9.193 Water at 20 °C is to be pumped through 2 km of 200-mm-diameter wrought iron pipe at the rate of 60 L/s. Compute the head loss and power required.

▌ $$h_f = (f)(L/d)(v^2/2g) \qquad v = Q/A = (60 \times 10^{-3})/[(\pi)(0.200)^2/4] = 1.910\ \text{m/s}$$

$$N_R = dv/\nu = (0.200)(1.910)/(1.02 \times 10^{-6}) = 3.75 \times 10^5 \qquad \epsilon/d = 0.000046/(0.200) = 0.000230$$

From Fig. A-5, $f = 0.016$.

$$h_f = 0.016[2000/0.200]\{1.910^2/[(2)(9.807)]\} = 29.76\ \text{m} \qquad P = Q\gamma h_f = (60 \times 10^{-3})(9.79)(29.76) = 17.48\ \text{kW}$$

9.194 An industrial ventilation system contains 4000 ft of 12-in-diameter galvanized pipe. Neglecting minor losses, what head must a blower produce to furnish 3 ton/h of air at $p = 14$ psia, $T = 90$ °F?

▌ $$h_f = (f)(L/d)(v^2/2g) \qquad N_R = \rho dv/\mu \qquad \rho = p/RT = (14)(144)/[(1716)(460 + 90)] = 0.002136\ \text{slug/ft}^3$$

$$M = \rho Av \qquad 6000/3600 = [(0.002136)(32.2)][(\pi)(\tfrac{12}{12})^2/4](v) \qquad v = 30.85\ \text{ft/s}$$

$$N_R = (0.002136)(\tfrac{12}{12})(30.85)/(3.90 \times 10^{-7}) = 1.69 \times 10^5 \qquad \epsilon/d = 0.0005/(\tfrac{12}{12}) = 0.000500$$

From Fig. A-5, $f = 0.019$.

$$h_f = 0.019[4000/(\tfrac{12}{12})]\{30.85^2/[(2)(32.2)]\} = 1123.2\ \text{ft of air} \quad \text{or} \quad 1.24\ \text{ft of water}$$

9.195 A 2.0-m-diameter pipe of length 1560 m for which $\epsilon = 1.5$ mm conveys water at 12 °C between two reservoirs at a rate of 8.0 m^3/s. What must be the difference in water-surface elevations between the two reservoirs? Neglect minor losses.

$$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L \qquad 0 + 0 + z_1 = 0 + 0 + z_2 + h_f$$

$$z_1 - z_2 = h_f = (f)(L/d)(v^2 2g)$$

$$v = Q/A = 8.0/[(\pi)(2.0)^2/4] = 2.546 \text{ m/s} \qquad N_R = dv/\nu = (2.0)(2.546)/(1.24 \times 10^{-6}) = 4.11 \times 10^6$$

$$\epsilon/d = (1.5/1000)/2.0 = 0.000750$$

From Fig. A-5, $f = 0.018$. $h_f = (0.018)(1560/2.0)\{2.546^2/[(2)(9.807)]\} = 4.64$ m. Hence, the difference in water-surface elevations between the two reservoirs is 4.64 m.

9.196 Water flows from reservoir 1 to reservoir 2 through a 4-in-diameter, 500-ft-length pipe, as shown in Fig. 9-33. Assume an initial friction factor (f) of 0.037 and a roughness (ϵ) of 0.003 ft for the pipe. Find the flow rate.

$$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L \qquad 0 + 0 + 700.6 = 0 + 0 + 655.5 + h_L \qquad h_L = 45.2 \text{ ft} = h_f + h_m$$

(I) **Friction loss:** $h_f = (f)(L/d)(v^2/2g) = 0.037[500/(\frac{4}{12})](v^2/2g) = 55.50v^2/2g$.

(II) **Minor losses:**

(*a*) **Due to entrance:** From Fig. A-7, take $K_1 = 0.45$.

(*b*) **Due to globe valve:** From Table A-11, $K_{open} = 5.7$. From Table A-12, take $K_2/K_{open} = 1.75$. Hence, $K_2 = (5.7)(1.75) = 9.98$.

(*c*) **Due to bend:** $R/D = \frac{12}{4} = 3.0$, $\epsilon/D = 0.003/(\frac{4}{12}) = 0.00900$. From Fig. A-12, $K_3 = 0.45$.

(*d*) **Due to elbow:** From Table A-11, $K_4 = 0.23$.

(*e*) **Due to exit:** From Fig. A-7, $K_5 = 1.0$.

Thus,

$$h_f + h_m = \{v^2/[(2)(32.2)]\}(55.50 + 0.45 + 9.98 + 0.45 + 0.23 + 1.0) = 1.050v^2$$

$$1.050v^2 = 45.2 \qquad v = 6.561 \text{ ft/s}$$

$$N_R = Dv/\nu = (\tfrac{4}{12})(6.561)/(1.90 \times 10^{-5}) = 1.15 \times 10^5 \qquad \epsilon/D = 0.003/(\tfrac{4}{12}) = 0.00900$$

From Fig. A-5, $f = 0.037$. (Assumed value of f O.K.) $Q = Av = [(\pi)(\frac{4}{12})^2/4](6.561) = 0.573$ ft^3/s.

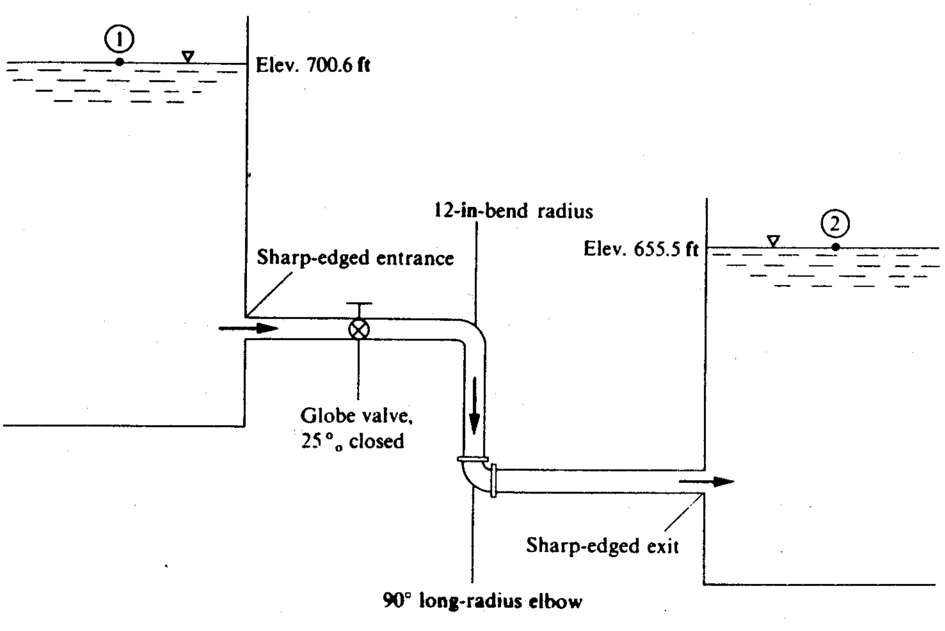


Fig. 9-33