▮ $$N_M = V/\sqrt{kRT} = 540/\sqrt{(1.40)(1716)(100+460)} = 0.466$$

16.103 How do the temperature and the pressure at the stagnation point in isentropic flow compare with reservoir conditions?

▮ They are the same.

16.104 Air flows isentropically from a reservoir at 80 °C, 6 atm. Calculate the temperature, pressure, density, and velocity at a section where $N_M = 0.65$.

▮ $$T_0/T = 1 + [(k-1)/2]N_M^2 \qquad (273+80)/T = 1 + [(1.40-1)/2](0.65)^2 \qquad T = 325\text{ K} \quad \text{or} \quad 52\,°\text{C}$$

$$p_0/p = \{1 + [(k-1)/2]N_M^2\}^{k/(k-1)} \qquad (6)(101.310)/p = \{1 + [(1.40-1)/2](0.65)^2\}^{1.40/(1.40-1)}$$

$$p = 458\text{ kPa} \qquad \rho = p/RT = 458\,000/[(287)(325)] = 4.91\text{ kg/m}^3$$

$$c = \sqrt{kRT} = \sqrt{(1.40)(287)(325)} = 361\text{ m/s} \qquad V = cN_M = (361)(0.65) = 235\text{ m/s}$$

16.105 Oxygen flows from a reservoir where $p_0 = 110$ psia, $T_0 = 100$ °F, to a 6-in-diameter section where the velocity is 500 fps. Calculate the mass rate of flow (isentropic) and the Mach number, pressure, and temperature in the 6-in section.

▮ $$T = T_0 - [(k-1)/2kR]V^2 = (100+460) - [(1.40-1)/(2)(1.40)(1554)](500)^2 = 537\text{ °R} \quad \text{or} \quad 77\text{ °F}$$

$$N_M = V/\sqrt{kRT} = 500/\sqrt{(1.40)(1554)(537)} = 0.463 \qquad \dot{m} = \rho AV$$

$$\rho_0 = p_0/RT_0 = (110)(144)/[(1554)(100+460)] = 0.01820\text{ slug/ft}^3$$

$$\rho_0/\rho = \{1 + [(k-1)/2]N_M^2\}^{1/(k-1)} \qquad 0.01820/\rho = \{1 + [(1.40-1)/2](0.463)^2\}^{1/(1.40-1)} \qquad \rho = 0.01639\text{ slug/ft}^3$$

$$\dot{m} = 0.01639[(\pi)(\tfrac{6}{12})^2/4](500) = 1.61\text{ slugs/s}$$

$$p = \rho RT = (0.01639)(1554)(537) = 13\,677\text{ lb/ft}^2 \quad \text{or} \quad 95.0\text{ lb/in}^2$$

16.106 Air flows adiabatically through a duct. At point 1 the velocity is 800 ft/s, $T_1 = 500$ °R, and $p_1 = 25$ lb/in² abs = 3600 lb/ft². Compute **(a)** T_0; **(b)** p_{01}; **(c)** ρ_0; **(d)** N_M; **(e)** V_{max}; **(f)** V^*. At point 2 further downstream $V_2 = 962$ ft/s, and $p_2 = 2850$ lb/ft². **(g)** What is the stagnation pressure p_{02}?

▮ For air take $k = 1.4$, $c_p = 6010$, and $R = 1717$ (BG units). With V_1 and T_1 known, we can compute T_0 without using ratios.

(a) $$T_{01} = T_1 + [\tfrac{1}{2}(V_1)^2/c_p] = 500 + [\tfrac{1}{2}(800)^2/6010] = 553\text{ °R}$$

Then compute N_M from the known ratio T/T_0, using $(N_M)^2 = 5[(T_0/T) - 1]$:

(d) $$(N_M)_1^2 = 5(\tfrac{553}{500} - 1) \qquad (N_M)_1 = 0.73$$

The stagnation pressure follows:

(b) $$p_{01} = p_1[1 + 0.2(N_M)_1^2]^{3.5} = 3600[1 + 0.2(0.73)^2]^{3.5} = 5130\text{ lb/ft}^2$$

We need the density before we can compute stagnation density: $\rho_1 = p_1/RT_1 = 3600/[1717(500)] = 0.00419$ slug/ft³. Then

(c) $$\rho_0 = \rho_1[1 + 0.2(N_M)_1^2]^{2.5} = 0.00419[1 + 0.2(0.73)^2]^{2.5} = 0.00540\text{ slug/ft}^3$$

However, if we were clever, we could compute ρ_0 directly from p_0 and T_0: $\rho_0 = p_0/RT_0 = 5130/[1717(553)] = 0.00540$ slug/ft³. The maximum velocity follows:

(e) $$V_{max} = (2c_pT_0)^{1/2} = [2(6010)(553)]^{1/2} = 2580\text{ ft/s}$$

and the sonic velocity from $V^* = \{[2k/(k+1)]RT_0\}^{1/2}$:

(f) $$V^* = \{[2(1.4)/(1.4+1)](1717)(553)\}^{1/2} = 1050\text{ ft/s}$$

At point 2, the temperature is not given, but since we know it is adiabatic flow, $T_{02} = T_{01} = 553$ °R from solution **(a)**. Thus we can compute T_2 from $T_2 = T_{02} - (\tfrac{1}{2}V_2^2/c_p) = 553 - [\tfrac{1}{2}(962)^2/6010] = 476$ °R (Trying to find T_2 from the Mach-number relations is a frustratingly laborious procedure.) The speed of sound a_2 thus equals $(kRT_2)^{1/2} \approx 49(476)^{1/2} = 1069$ ft/s, whence the Mach number $(N_M)_2 = V_2/a_2 = \frac{962}{1069} = 0.90$. Finally compute

(g) $$p_{02} = p_2[1 + 0.2(N_M)_2^2]^{3.5} = 28.50[1 + 0.2(0.9)^2]^{3.5} = 4820\text{ lb/ft}^2 \qquad (6\text{ percent less than } p_{01})$$

16.107 Air flows isentropically through a duct. At section 1 the area is 1 ft² and $V_1 = 600$ ft/s, $p_1 = 12\,000$ lb/ft², and

$T_1 = 850\,°\text{R}$. Compute **(a)** T_0, **(b)** $(N_M)_1$, **(c)** p_0, and **(d)** A^*. If $A_2 = 0.75\ \text{ft}^2$, compute $(N_M)_2$ and p_2 if V_2 is **(e)** subsonic or **(f)** supersonic.

▮ With V_1 and T_1 known, the energy equation gives

(a) $$T_0 = T_1 + (V_1^2/2c_p) = 850 + [(600)^2/2(6010)] = 880\,°\text{R}$$

The sound speed $a_1 \approx 49(850)^{1/2} = 1429\ \text{ft/s}$; hence

(b) $$(N_M)_1 = V_1/a_1 = \tfrac{600}{1429} = 0.42$$

With $(N_M)_1$ known $p_0/p_1 = [1 + 0.2(N_M)_1^2]^{3.5} = 1.129$. Hence

(c) $$p_0 = 1.129p_1 = 1.129(12\,000) = 13\,550\ \text{lb/ft}^2$$

Similarly, from $A/A^* = (1/N_M)[(1 + 0.2N_M^2)^3/1.728]$, $A_1/A^* = [1 + 0.2(0.42)^2]^3/[1.728(0.42)] = 1.529$. Hence

(d) $$A^* = 1/1.529 = 0.654\ \text{ft}^2$$

This throat must actually be present in the duct to expand the subsonic $(N_M)_1$ to supersonic flow downstream.

Given $A_2 = 0.75\ \text{ft}^2$, we can compute $A_2/A^* = 0.75/0.654 = 1.147$. For subsonic flow, use $N_M \approx 1 - 0.88[\ln(A/A^*)]^{0.45}$:

(e) $(N_M)_2 \approx 1 - 0.88(\ln 1.147)^{0.45} = 0.64$ whence $p_2 = p_0/[1 + 0.2(0.64)^2]^{3.5} = 13\,550/1.317 = 10\,300\ \text{lb/ft}^2$

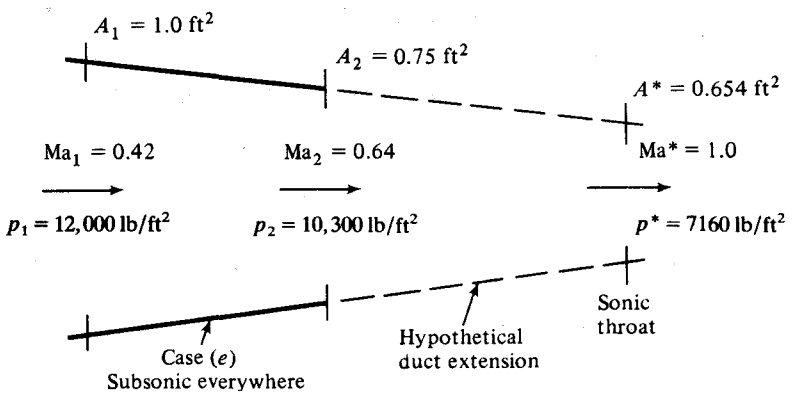


Fig. 16-4

For case **(e)** there is *no* sonic throat; i.e., the duct area has not decreased sufficiently to create supersonic flow. It may possibly do this further downstream, as illustrated in Fig. 16-4. If, on the other hand, the flow at section 2 is supersonic, we use $N_M = 1 + 1.2[(A/A^*) - 1]^{1/2}$:

(f) $(N_M)_2 \approx 1 + 1.2(1.147 - 1)^{1/2} = 1.46$ whence $p_2 = 13\,550/[1 + 0.2(1.46)^2]^{3.5} = 13\,550/3.46 = 3910\ \text{lb/ft}^2$

These answers are accurate within less than 1 percent. Note that the supersonic-flow pressure level is much less than the subsonic-flow condition at section 2 for the same duct area, and a sonic throat ($A^* = 0.654\ \text{ft}^2$) *must* have occurred between sections 1 and 2.

16.108 It is desired to expand air from $p_0 = 200\ \text{kPa}$ and $T_0 = 500\ \text{K}$ through a throat to an exit Mach number of 2.5. If the desired mass flow is 3 kg/s, compute **(a)** the throat area and the exit **(b)** pressure, **(c)** temperature, **(d)** velocity, and **(e)** area, assuming isentropic flow, with $k = 1.4$.

▮ **(a)** The throat area follows from the following equation, because the throat flow must be sonic to produce a supersonic exit: $A^* = [\dot{m}(RT_0)^{1/2}]/(0.6847p_0) = \{3.0[287(500)]^{1/2}\}/[0.6847(200\,000)] = 0.00830\ \text{m}^2 = \frac{1}{4}\pi D^{*2}$ or $D_{\text{throat}} = 10.3\ \text{cm}$. With the exit Mach number known, the isentropic-flow relations give pressure and temperature:

(b) $$p_e = p_0/[1 + 0.2(2.5)^2]^{3.5} = 200/17.08 = 11.70\ \text{kPa}$$

(c) $$T_e = T_0/[1 + 0.2(2.5)^2] = 500/2.25 = 222\ \text{K}$$

(d) The exit velocity follows from the known Mach number and temperature $V_e = (N_M)_e(kRT_e)^{1/2} = 2.5[1.4(287)(222)]^{1/2} = 747\ \text{m/s}$.

(e) The exit area follows from the known throat area and exit Mach number and $A/A^* = (1/N_M)[(1 + 0.2N_M^2)^3/1.728]$: $A_e/A^* = [1 + 0.2(2.5)^2]^3/[1.728(2.5)] = 2.64$ or $A_e = 2.64A^* = 2.64(0.0083) = 0.0219\ \text{m}^2 = \frac{1}{4}\pi D_e^2$: $D_e = 16.7\ \text{cm}$.

16.109 A flow of air ($c_p = 1005$ J/kg · K, $k = 1.40$) has $V = 850$ m/s, $p = 130$ kPa, and $T = 180$ °C. Compute the pressure and temperature of the air if brought isentropically to rest.

▮
$$T_0 = T + V^2/2c_p = (180 + 273) + 850^2/[(2)(1005)] = 812 \text{ K} \quad \text{or} \quad 539\ ^\circ\text{C}$$
$$p_0 = (p)(T_0/T)^{k/(k-1)} = 130[812/(180 + 273)]^{1.40/(1.40-1)} = 1002 \text{ kPa}$$

16.110 Rework Prob. 16.108 for argon ($c_p = 518$ J/kg · K, $k = 1.67$).

▮
$$T_0 = T + V^2/2c_p = (180 + 273) + 850^2/[(2)(518)] = 1150 \text{ K} \quad \text{or} \quad 877\ ^\circ\text{C}$$
$$p_0 = (p)(T_0/T)^{k/(k-1)} = (130)[1150/(180 + 273)]^{1.67/(1.67-1)} = 1326 \text{ kPa}$$

16.111 Rework Prob. 16.108 for steam.

▮ $h_0 = h + V^2/2$. From the steam tables, at 130 kPa and 180 °C, $h = 2834.6$ kJ/kg and $s = 7.6705$ kJ/kg · K: $h_0 = 2835 + (850^2/2)/1000 = 3196$ kJ/kg, $s_0 = 7.695$ kJ/kg · K. From the steam tables, $T_0 = 360$ °C and $p_0 = 480$ kPa.

16.112 Compute the frontal stagnation temperature of an airplane flying at $N_M = 2.0$ at 8000 m altitude. At what Mach number would it fly to have a frontal stagnation temperature of 380 °C?

▮
$$T = 273 - 36.9 = 236.1 \text{ K} \qquad \text{(from Table A-8)}$$
$$T_0 = T\{1 + [(k-1)/2](N_M)^2\} = 236.1\{1 + [(1.40-1)/2](2.0)^2\} = 425 \text{ K} \quad \text{or} \quad 152\ ^\circ\text{C}$$
$$(380 + 273) = 236.1\{1 + [(1.40-1)/2](N_M)^2\} \qquad N_M = 2.97$$

16.113 Air ($c_p = 1005$ J/kg · K) flows at $V = 280$ m/s, $p = 100$ kPa, and $T = 160$ °C. Compute the maximum velocity attainable by adiabatic expansion of this flow.

▮ $T_0 = T + V^2/2c_p = (160 + 273) + 280^2/[(2)(1005)] = 472$ K $\qquad V_{max} = (2c_pT_0)^{1/2} = [(2)(1005)(472)]^{1/2} = 974$ m/s

16.114 Rework Prob. 16.113 for helium ($c_p = 5224$ J/kg · K).

▮ $T_0 = T + V^2/2c_p = (160 + 273) + 280^2/[(2)(5224)] = 441$ K $\qquad V_{max} = (2c_pT_0)^{1/2} = [(2)(5224)(441)]^{1/2} = 2147$ m/s

16.115 Air expands isentropically through a duct from $p_1 = 120$ kPa and $T_1 = 110$ °C to $p_2 = 75$ kPa and $V_2 = 325$ m/s. Compute **(a)** T_2, **(b)** $(N_M)_2$, **(c)** T_0, **(d)** p_0, **(e)** V_1.

▮ **(a)**
$$T_2 = (T_1)(p_2/p_1)^{(k-1)/k} = (110 + 273)(\tfrac{75}{120})^{(1.40-1)/1.40} = 335 \text{ K}$$

(b) $N_M = V/c \qquad c = \sqrt{kRT} \qquad c_2 = \sqrt{(1.40)(287)(335)} = 367$ m/s $\qquad (N_M)_2 = \frac{325}{367} = 0.886$

(c) $(T_0)_1 = (T_0)_2 = (T_2)\{1 + [(k-1)/2](N_M)_2^2\} \qquad T_0 = 335\{1 + [(1.40-1)/2](0.886)^2\} = 388$ K

(d)
$$(p_0)_1 = (p_0)_2 = (p_2)\{1 + [(k-1)/2](N_M)_2^2\}^{k/(k-1)}$$
$$p_0 = 75\{1 + [(1.40-1)/2](0.886)^2\}^{1.40/(1.40-1)} = 125 \text{ kPa}$$

(e) $T_0 = T + V^2/2c_p \qquad 388 = (110 + 273) + V_1^2/[(2)(1005)] \qquad V_1 = 100$ m/s

16.116 Air flows isentropically in a duct. At section 1, $(N_M)_1 = 0.8$, $T_1 = 250$ °C, and $p_1 = 280$ kPa. At section 2, $(N_M)_2 = 3.0$. Compute **(a)** $(p_0)_2$, **(b)** p_2, and **(c)** T_2.

▮ **(a)** $(p_0)_2 = (p_0)_1 = (p_1)\{1 + [(k-1)/2](N_M)_1^2\}^{k/(k-1)} = 280\{1 + [(1.40-1)/2](0.8)^2\}^{1.40/(1.40-1)} = 427$ kPa

(b) $p_2 = (p_0)_2/\{1 + [(k-1)/2](N_M)_2^2\}^{k/(k-1)} = 427/\{1 + [(1.40-1)/2](3.0)^2\}^{1.40/(1.40-1)} = 11.6$ kPa

(c) $(T_0)_2 = (T_0)_1 = (T_1)\{1 + [(k-1)/2](N_M)_1^2\} = (250 + 273)\{1 + [(1.40-1)/2](0.8)^2\} = 590$ K

$$T_2 = (T_0)_2/\{1 + [(k-1)/2](N_M)_2^2\} = 590/\{1 + [(1.40-1)/2](3.0)^2\} = 211 \text{ K}$$

16.117 Show that for isentropic flow of a perfect gas, if a pitot-static probe measures p_0, p, and T_0, the gas velocity can be calculated from $V^2 = 2c_pT_0[1 - (p/p_0)^{(k-1)/k}]$. What would be a source of error if a shock wave is formed in front of the probe?

▮ $T = (T_0)(p/p_0)^{(k-1)/k} = T_0 - V^2/2c_p$. Solving for V^2 gives $V^2 = 2c_pT_0[1 - (p/p_0)^{(k-1)/k}]$. If a shock wave is

formed (i.e., if V is supersonic), the probe measures $(p_0)_2$ inside the shock wave, which is less than stream stagnation pressure $(p_0)_1$.

16.118 At the exit of a nozzle flow, the air temperature is 60 °C and the velocity is 355 m/s. Find the Mach number and stagnation temperature there. If the flow is adiabatic, what is the Mach number upstream, where $T = 120$ °C?

▮ $N_M = V/c \qquad c = \sqrt{kRT} \qquad c_2 = \sqrt{(1.40)(287)(60+273)} = 366$ m/s

$(N_M)_2 = 355/366 = 0.97 \qquad (T_0)_2 = T_2 + V_2^2/2c_p = (60+273) + 355^2/[(2)(1005)] = 396$ K

$T_0/T_1 = 1 + [(k-1)/2](N_M)_1^2 \qquad 396/(120+273) = 1 + [(1.40-1)/2](N_M)_1^2 \qquad (N_M)_1 = 0.195$

16.119 The compressed-air tank in Fig. 16-5 exhausts from a nozzle at 240 m/s. Assuming isentropic flow, compute the pressure in the tank and in the atmosphere. What is the exit Mach number?

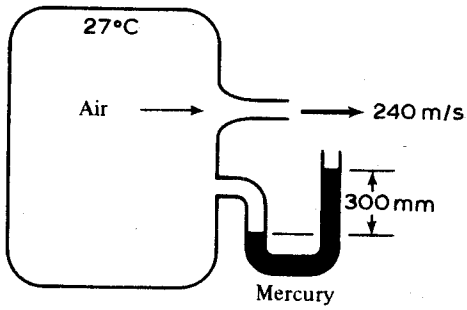


Fig. 16-5

▮ $N_M = V/c \qquad c = \sqrt{kRT} \qquad T_0 = t_{exit} + V_{exit}^2/2c_p \qquad (27+273) = T_{exit} + 240^2/[(2)(1005)] \qquad T_{exit} = 271$ K

$c_{exit} = \sqrt{(1.40)(287)(271)} = 330$ m/s $\qquad (N_M)_{exit} = \frac{240}{330} = 0.727$

$p_{exit} = (p_0)/\{1 + [(k-1)/2](N_M)_{exit}^2\}^{k/(k-1)} \qquad p_0/p_{exit} = \{1 + [(1.40-1)/2](0.727)^2\}^{1.40/(1.40-1)} = 1.421$

Since $\rho_{Hg} \gg \rho_0$, we can simply take $\rho_0 = 1.5$ kg/m^3. $p_0 - p_{exit} = (\rho_{Hg} - \rho_0)gh = (13\,600 - 1.5)(9.807)(0.300) =$ 40 kPa. Then $p_0/p_{exit} - 1 = 1.421 - 1 = (p_0 - p_{exit})/p_{exit} = 40/p_{exit}$, $p_{exit} = 95$ kPa, $p_0 = p_{tank} = 135$ kPa.

16.120 Air flows adiabatically through a duct. At one section, $V_1 = 420$ fps, $T_1 = 210$ °F, and $p_1 = 35$ psia, while farther downstream $V_2 = 1000$ fps and $p_2 = 18$ psia. Compute **(a)** $(N_M)_2$, **(b)** U_{max}, and **(c)** $(p_0)_2/(p_0)_1$.

▮ **(a)** $(N_M)_2 = V_2/c_2 \qquad c_2 = \sqrt{kRT_2} \qquad T_2 = (T_0)_2 - V_2^2/2c_p \qquad (T_0)_2 = (T_0)_1 = T_1 + V_1^2/2c_p$

$(T_0)_2 = (210+460) + 420^2/[(2)(6010)] = 685$ °R $\qquad T_2 = 685 - 1000^2/[(2)(6010)] = 602$ °R

$c_2 = \sqrt{(1.40)(1716)(602)} = 1203$ ft/s $\qquad (N_M)_2 = \frac{1000}{1203} = 0.831$

(b) $U_{max} = \sqrt{2c_pT_0} = \sqrt{(2)(6010)(685)} = 2869$ ft/s

(c) $(p_0) = (p)\{[1 + (k-1)/2](N_M)^2\}^{k/(k-1)} \qquad (N_M)_1 = V_1/\sqrt{kRT_1} = 420/\sqrt{(1.40)(1716)(210+460)} = 0.331$

$(p_0)_1 = 35\{1 + [(1.40-1)/2](0.331)^2\}^{1.40/(1.40-1)} = 37.76$ psia

$(p_0)_2 = 18\{1 + [(1.40-1)/2](0.831)^2\}^{1.40/(1.40-1)} = 28.31$ psia

$(p_0)_2/(p_0)_1 = 28.31/37.76 = 0.75$

16.121 Air flows isentropically from a reservoir, where $p = 320$ kPa and $T = 490$ K, to section 1 in a duct, where $A_1 = 0.2$ m^2 and $V_1 = 600$ m/s. Compute **(a)** T_1, **(b)** $(N_M)_1$, **(c)** p_1, and **(d)** $\dot{m}$.

▮ **(a)** $T_1 = T_0 - V_1^2/2c_p = 490 - 600^2/[(2)(1005)] = 311$ K

(b) $N_M = V/c \qquad c = \sqrt{kRT} \qquad c_1 = \sqrt{(1.40)(287)(311)} = 353$ m/s $\qquad (N_M)_1 = \frac{600}{353} = 1.70$

(c) $p_1 = p_0/\{1 + [(k-1)/2](N_M)_1^2\}^{k/(k-1)} = 320/\{1 + [(1.40-1)/2](1.70)^2\}^{1.40/(1.40-1)} = 64.8$ kPa

(d) $\rho = p/RT \qquad \rho_1 = (64.8)/[(0.287)(311)] = 0.726$ kg/m^3

$\dot{m} = \rho_1 A_1 V_1 = (0.726)(0.2)(600) = 87.1$ kg/s

16.122 Repeat Prob. 16.121 for argon ($k = 1.67$, $R = 0.208$ kJ/kg · K, $c_p = 0.519$ kJ/kg · K).

▮ **(a)** $T_1 = T_0 - V_1^2/2c_p = 490 - 600^2/[(2)(519)] = 143$ K

(b) $N_M = V/c \qquad c = \sqrt{kRT} \qquad c_1 = \sqrt{(1.67)(208)(143)} = 223$ m/s $\qquad (N_M)_1 = \frac{600}{223} = 2.69$

(c) $p_1 = p_0/\{1 + [(k-1)/2](N_M)_1^2\}^{k/(k-1)} = 320/\{1 + [(1.67-1)/2](2.69)^2\}^{1.67/(1.67-1)} = 14.9$ kPa

(d) $\rho = p/RT \qquad \rho_1 = (14.9)/[(0.208)(143)] = 0.501$ kg/m^3

$\dot{m} = \rho_1 A_1 V_1 = (0.501)(0.2)(600) = 60.1$ kg/s

16.123 Sketch a passage which will **(a)** increase the pressure in a subsonic flow isentropically, **(b)** increase the pressure in a supersonic flow isentropically, **(c)** increase the Mach number in a supersonic flow isentropically, **(d)** decrease the Mach number in a subsonic flow isentropically.

▮ See Fig. 16-6.

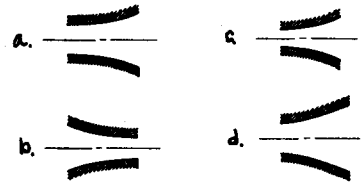


Fig. 16-6

16.124 Air at 28 °C flows from a large tank through a convergent nozzle, which has an exit diameter of 10 mm. Discharge is to the atmosphere where atmospheric pressure is 96.5 kPa. Air pressure inside the tank is 40.0 kPa gage. What is the weight flow rate of air through the nozzle?

▮ $(p_2/p_1)_c = [2/(k+1)]^{k/(k-1)} = [2/(1.40+1)]^{1.40/(1.40-1)} = 0.5283$

$p_2' =$ atmospheric pressure $= 96.5$ kPa (absolute) $\qquad p_1 = 40 + 96.5 = 136.5$ kPa (absolute)

$p_2'/p_1 = 96.5/136.5 = 0.7070$

Since the value of the ratio p_2'/p_1 of 0.7070 is greater than the value of the critical pressure ratio of 0.5283, flow through the nozzle will be subsonic and

$$G = A_2\sqrt{[2gk/(k-1)]p_1\gamma_1[(p_2/p_1)^{2/k} - (p_2/p_1)^{(k+1)/k}]} \qquad A_2 = (\pi)(0.010)^2/4 = 0.00007854 \text{ m}^2$$

$$\gamma = p/RT \qquad \gamma_1 = 136.5/[(29.3)(28+273)] = 0.01548 \text{ kN/m}^3 \qquad p_2 = p_2' = 96.5 \text{ kN/m}^2$$

$$G = (0.00007854)\sqrt{\frac{(2)(9.807)(1.40)}{1.40-1}(136.5)(0.01548)\left[\left(\frac{96.5}{136.5}\right)^{2/1.40} - \left(\frac{96.5}{136.5}\right)^{(1.40+1)/1.40}\right]}$$

$= 0.000227$ kN/s or 0.227 N/s

16.125 Air at 75 °F flows from a large tank through a convergent nozzle, which has an exit diameter of 1.5 in. The discharge is to the atmosphere, where atmospheric pressure is 14.0 psia. Air pressure inside the tank is 25.0 psig. What is the weight flow rate of air through the nozzle?

▮ $(p_2/p_1)_c = 0.5283$ (as determined for air in Prob. 16.124)

$p_2' = 14.0$ psia $\qquad p_1 = 25.0 + 14.0 = 39.0$ psia $\qquad p_2'/p_1 = 14.0/39.0 = 0.3590$

Since the value of the ratio p_2'/p_1 of 0.3590 is less than the value of the critical pressure ratio of 0.5283, flow through the nozzle will be sonic. However, the values substituted into the equation below for p_1 must be the pressure that makes the ratio p_2'/p_1 equal to the critical pressure ratio $(p_2/p_1)_c$.

$$G = (A_2 p_1/\sqrt{T_1})\sqrt{(gk/R)[2/(k+1)]^{(k+1)/(k-1)}} \qquad A_2 = (\pi)(1.5)^2/4 = 1.767 \text{ in}^2 = 0.01227 \text{ ft}^2$$

$$p_1 = p_2'/(p_2/p_1)_c = 14.0/0.5283 = 26.5 \text{ psia} \qquad T_1 = 75° + 460 = 535 \text{ °R}$$

$$G = \{(0.01227)[(26.5)(144)]/\sqrt{535}\}\sqrt{[(32.2)(1.40)/53.3][2/(1.40+1)]^{(1.40+1)/(1.40-1)}} = 1.08 \text{ lb/s}$$

16.126 Air at 308 °C flows from a large tank through a convergent nozzle, which has an exit diameter of 20 mm. Discharge is to the atmosphere where atmospheric pressure is 95.0 kPa. Air pressure inside the tank is 50.0 kPa gage. What is the weight flow of air through the nozzle?

▮ $$(p_2/p_1)_c = [2/(k+1)]^{k/(k-1)} = [2/(1.40+1)]^{1.40/(1.40-1)} = 0.5283 \qquad p_2' = 95.0\ \text{kPa}$$
$$p_1 = 50 + 95.0 = 145.0\ \text{kPa} \qquad p_2'/p_1 = 95.0/145.0 = 0.6522$$

Since $[p_2'/p_1 = 0.6552] > [(p_2/p_1)_c = 0.5283]$, the flow through the nozzle will be subsonic:

$$G = A_2\sqrt{[(2gk)/(k-1)](p_1\gamma_1)[(p_2/p_1)^{2/k} - (p_2/p_1)^{(k+1)/k}]}$$
$$A_2 = (\pi)(0.02)^2/4 = 0.0003142\ \text{m}^2 \qquad \gamma = p/RT$$
$$\gamma_1 = 145.0/[(29.3)(30+273)] = 0.01633\ \text{kN/m}^3 \qquad p_2 = p_2' = 95.0\ \text{kN/m}^2$$
$$G = 0.0003142\sqrt{[(2)(9.807)(1.40)/(1.40-1)](145)(0.01633)(95.0/145.0)^{2/1.40} - (95.0/145.0)^{(1.40+1)/1.40}]}$$
$$= 0.00100\ \text{kN/s} = 1.00\ \text{N/s}$$

16.127 Air at 60 °F flows from a large tank through a convergent nozzle, which has an exit diameter of 1.0 in. The discharge is to the atmosphere, where atmospheric pressure is 14.5 psia. Air pressure inside the tank is 30.0 psig. What is the weight flow rate of air through the nozzle?

▮ From Prob. 16.126, $(p_2/p_1)_c = 0.5283$, $p_2' = 14.5$ psia, $p_1 = 30.0 + 14.5 = 44.5$ psia, $p_2'/p_1 = 14.5/44.5 = 0.3258$. Since $[p_2'/p_1 = 0.3258] < [(p_2/p_1)_c = 0.5283]$, the flow through the nozzle will be sonic.

$$G = [A_2p_1/\sqrt{T_1}]\sqrt{(gk/R)[2/(k+1)]^{(k+1)/(k-1)}} \qquad A = (\pi)(\tfrac{1}{12})^2/4 = 0.005454\ \text{ft}^2$$
$$p_1 = p_2'/(p_2/p_1)_c = 14.5/0.5283 = 27.45\ \text{psia} \qquad T_1 = 60 + 460 = 520\ °\text{R}$$
$$G = [(0.005454)(27.45)(144)/\sqrt{520}]\sqrt{[(32.2)(1.40)/53.3][2/(1.40+1)]^{(1.40+1)/(1.40-1)}} = 0.503\ \text{lb/s}$$

16.128 Air at 80 °F flows from a large tank through a converging nozzle of 2.0-in exit diameter. The discharge is to an atmospheric pressure of 13.5 psia. Determine the flow through the nozzle for pressures within the tank of 5, 10, 15, and 20 psig. Assume isentropic conditions. Plot G as a function of p_1. Assume that the temperature within the tank is 80 °F in all cases.

▮ From Prob. 16.126 the critical pressure ratio for air is $(p_2/p_1)_c = 0.528$. If the flow at the throat is subsonic, $p_2/p_1 > (p_2/p_1)_c$. Thus for subsonic flow at the throat, $p_2/p_1 > 0.528$ and $p_3 = p_2$. So $p_3/p_1 > 0.528$ and $p_1 < p_3/0.528$.

Since $p_3 = 13.5$ psia, the flow at the throat will be subsonic if $p_1 < 25.6$ psia (12.1 psig) and sonic if $p_1 > 25.6$ psia (12.1 psig).

To find the flow rate for conditions where $p_1 < 12.1$ psig (subsonic flow at throat), we use

$$G = A_2\sqrt{(2g)[k/(k-1)]p_1\gamma_1[(p_2/p_1)^{2/k} - (p_2/p_1)^{(k+1)/k}]} \tag{1}$$

Substituting the appropriate value of p_1 into the equation and noting that for this condition $p_2 = p_3 = 13.5$ psia, we get: for $p_1 = 5$ psig (18.5 psia); $G = 1.20$ lb/s; for $p_1 = 10$ psig (23.5 psia), $G = 1.69$ lb/s. To find the

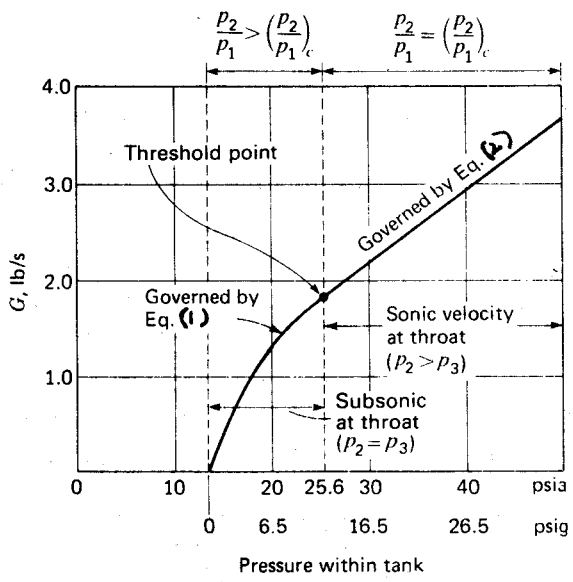


Fig. 16-7

flow rate for conditions where $p_1 > 12.1$ psig (sonic flow at the throat) we use

$$G' = g(A_2 p_1/\sqrt{T_1})\sqrt{(k/R)[2/(k+1)]^{(k+1)/(k-1)}} \tag{2}$$

We get: for $p_1 = 15$ psig (28.5 psia), $G' = 2.04$ lb/s; for $p_1 = 20$ psig (33.5 psia), $G' = 2.40$ lb/s. Substituting $p_1 = 25.6$ psia in Eq. (*1*) for subsonic flow gives $G = 1.84$ lb/s as does Eq. (*2*). This is the threshold point at which the flow in the throat changes from subsonic to sonic. When $p_1 = 25.6$ psia the flow rate as found from Eq. (*2*) is $G'_{p=25.6\,\text{psia}} = [32.2(0.0218)(144)(25.6)]/\sqrt{540}\sqrt{(1.4/1715)(2/2.4)^{2.4/0.4}} = 1.84$ lb/s. As p_1 is increased beyond 12.1 psig, sonic flow prevails at the throat and the flow rate increases linearly with $(p_1)_{\text{abs}}$ as indicated by Eq. (*2*). The variation of the flow rate with p_1 is shown in Fig. 16-7. Other information concerning various aspects of this problem can be found by applying the gas law ($pv = RT$ or $\gamma = gp/RT$), the equation of state (pv^k = constant or p/γ^k = constant), continuity ($G = \gamma AV$), and the energy equation. Applying these, for example, for the case where $p_1 = 5$ psig (18.5 psia) yields $\gamma_1 = 0.093$ lb/ft^3, $\gamma_2 = 0.0743$ lb/ft^3, $V_2 = 740$ fps, $T_2 = 491$ °R, and $p_2 = 13.5$ psia. Note that in this case $p_2 = p_3$.

16.129 Refer to Prob. 16.128. If the pressure in the tank is 5 psig, find G, p_2, and T_2.

▮ Air $p_1 = 5$ psig $\quad p_1 = 5 + p_{\text{atmos}} = 5 + 13.5 = 18.50$ psia

$$\gamma_1 = gp_1/RT_1 = (32.2)(18.50)(144)/[(17715)(540)] = 0.0926 \text{ lb/ft}^3$$

$$p_3/p_1 = 14.5/18.5 = 0.730 > 0.528 = (p_2/p_1)_c \text{ so } G = A_2\sqrt{2g[k/(k-1)]p_1\gamma_1[(p_2/p_1)^{2/k} - (p_2/p_1)^{(k+1)/k}]}$$
$$= (\pi/4)(\tfrac{2}{12})^2\sqrt{64.4(1.4/0.4)[(18.5)(144)(0.0926)][(13.5/18.5)^{2/1.4} - (13.5/18.5)^{2.4/1.4}]}$$
$$= 1.204 \text{ lb/s.}$$

$$V_2^2/2g = (p_2/\gamma_2)[k/(k-1)][(p_1/p_2)^{(k-1)/k} - 1] \tag{1}$$
$$G = \gamma_2 A_2 V_2 \tag{2}$$
$$\gamma_2 = gp_2/RT_2 \tag{3}$$
$$p_1/\gamma_1^k = p_2/\gamma_2^k \tag{4}$$

From (*2*)

$$\gamma_2 = G/A_2V_2 = 1.204/(0.0218V_2) = 55.0/V_2 \tag{5}$$

From (*4*) $p_2 = p_1(\gamma_2^k/\gamma_1^k) = [(18.5)(144)][\gamma_2^k/(0.0926)^{1.4}] = (2660\gamma_2^k)/0.0357 = 74\,500\gamma_2^k$. Substitute for γ_2 from (*5*): $p_2 = 74\,500(55/V_2)^{1.4} = 20.4 \times 10^6/V_2^{1.4}$. From (*4*): $p_1/p_2 = (\gamma_1/\gamma_2)^{1.4} = [0.0926/(55/V_2)]^{1.4} = 0.0001308V_2^{1.4}$. Substitute into (*1*):

$$V_2^2 = (64.4)[(20.4 \times 10^6)(1.4/0.4)/(V_2^{1.4})(55/V_2)][0.0001308V_2^{1.4})^{0.4/1.4} - 1]$$

$V_2^{2.4} = (83.6 \times 10^6)[0.0777V_2^{0.4} - 1]$; by T and E, $V_2 = 741$ fps. From (*2*): $\gamma_2 = G/A_2V_2 = 1.204/(0.0218)(741) = 0.0745$ lb/ft^3. From (*4*): $p_2 = (18.5)[(0.0742)^{1.4}/(0.0926)^{1.4}] = 13.57$ psia. From (*3*): $T_2 = gp_2/R\gamma_2 = (32.2)(13.57)(144)/[(1715)(0.0742)] = 492$ °R.

16.130 Air discharges from a tank through a converging-diverging nozzle with a 2.0-in-diameter throat. Within the tank the pressure is 50 psia and the temperature is 80 °F, while outside the tank the pressure is 13.5 psia. The nozzle is to operate with supersonic flow throughout its diverging section with a 13.5-psia pressure at its outlet. Find the required diameter of the nozzle outlet. Determine the flow rate and the velocities and temperatures at sections 2 and 3. Assume isentropic flow.

▮ The pressure at the throat must be such that sonic velocity will occur there. Hence $p_2 = p_c = 0.528p_1 = 26.4$ psia.

The velocity at the outlet may be found from $V_3^2/2g = (RT_1/g)[k/(k-1)][1 - (p_3/p_1)^{(k-1)/k}] = [(1715)(540)/32.2](1.4/0.4)[1 - (13.5/50)^{0.4/1.4}] = 31\,400$ ft, $V_3 = 1420$ fps. The flow rate is computed from $G = (gA_2p_1/\sqrt{T_1})\sqrt{(k/R)[2/(k+1)]^{(k+1)/(k-1)}}$, $G' = [(32.2)(0.0218)(50)(144)/\sqrt{540}]\sqrt{(1.4/1715)(2/2.4)^{2.4/0.4}} = 3.58$ lb/s. The temperature at 3 may be determined by using $V_3^2 - V_1^2 = 2c_p(T_1 - T_2)$, $(1420)^2 = 2(6000)(540 - T_3)$, $T_3 = 372$ °R $= -88$ °F. From the perfect-gas law $p_3/\gamma_3 = RT_3/g$. $\gamma_3 = (32.2)(13.5)(144)/[(1715)(372)] = 0.098$ lb/ft^3. Isentropic flow between 2 and 3 may be assumed, since the shock wave does not occur within that region. Thus $p_2/\gamma_2^{1.4} = p_3/\gamma_3^{1.4}$. $26.4/\gamma_2^{1.4} = 13.5/(0.098)^{1.4}$, $\gamma_2 = 0.158$ lb/ft^3. The velocity at 2 may now be computed: $V_2 = G/\gamma_2A_2 = 3.58/[(0.158)(0.0218)] = 1040$ fps. The temperature at 2 results from $(1420)^2 - (1040)^2 = 2(6000)(T_2 - 372)$, $T_2 = 450$ °R $= -10$ °F. The area at 3 is computed from $A_3 = G/\gamma_3V_3 = 3.58/[(0.098)(1420)] = 0.0257$ ft^2. Finally, $D_3 = 2.17$ in, the required outlet diameter.

Check for sonic velocity at throat: $c_2 = \sqrt{kRT_2} = \sqrt{(1.4)(1715)(450)} = 1040$ fps $= V_2$. With sonic velocity at the throat, if $D_3 < 2.17$ in, there will be supersonic flow throughout the tube and a shock wave will occur in the flow

field downstream of the nozzle exit. If $D_3 > 2.17$ in, with sonic flow in the throat, in order to satisfy pressure conditions, a shock wave will occur in the tube somewhere between the throat and the nozzle exit.

16.131 Work Prob. 16.130 with all data the same except for the pressure within the tank, which is 100 psia rather than 50 psia.

▮ Pressure at throat: $p_2 = p_c = 0.528(100) = 52.8$ psia. Find the velocity at outlet from $V_3 = (2g)[k/(k-1)](R/g)(T)[1-(p_2/p_1)^{(k-1)/k}] = (64.4)(1.4/0.4)(1715/32.2)(540)[1-(13.5/100)^{0.4/1.4}]$; $V_3 = 1680$ fps. By computation similar to that of Prob. 16.130, $G' = 7.20$ lb/s (exactly twice that of Prob. 16.130), $V_3^2 = [2k/(k-1)](RT_1)[1-(T_3/T_1)]$, $1680^2 = (2.8/0.4)(1715)(540)[1-(T_3/540)]$; $T_3 = 305$ °R $= -155$ °F. $\gamma = pg/RT$; $\gamma_3 = (13.5)(144)(32.2)/[(1715)(305)] = 0.1197$ pcf. $p/\gamma^k =$ constant (isentropic), therefore, $5.28/\gamma_2^{1.4} = 13.5/(0.1197)^{1.4}$; $\gamma_2 = 0.317$ pcf. $V_2 = G/\gamma_2 A_2 = 7.20/[(0.317)(0.0218)] = 1042$ fps, $T_2 = 448$ °R $= -12$ °F, $A_3 = G/\gamma_3 V_3 = 7.20/[(0.1197)(1680)] = 0.0358$ ft^2 and $D = 2.56$ in.

16.132 Air within a tank at 100 °F flows isentropically through a 3-in-diameter convergent nozzle into a 14.2-psia atmosphere. Find the flow rate for a tank pressure of 20 psig.

▮ For air ($k = 1.4$), $(p_2/p_1)_c = 0.528$; $p_2 = 14.2$ psia. If $p_2/p_1 > 0.528$, that is, $p_1 < 14.2/0.528 = 26.9$ psia, find G from

$$G = A_2\sqrt{(2g)[k/(k-1)](p_1\gamma_1)[(p_2/p_1)^{2/k} - (p_2/p_1)^{(k+1)/k}]} \tag{1}$$

If $p_1 > 26.9$ psia, find G' from

$$G' = g(A_2 p_1/\sqrt{T_1})\sqrt{(k/R)[2/(k+1)]^{(k+1)/(k-1)}} \tag{2}$$

Hence, for $p_1 = 20 + 14.2 = 34.2$ psia, $G' = (32.2)[(0.0491)(34.2)(144)/\sqrt{560}]\sqrt{(1.4/1715)(2/2.4)^{2.4/0.4}} = 5.44$ lb/s.

16.133 Air flows at 150 °F from a large tank through a 1.5-in-diameter converging nozzle. Within the tank the pressure is 85 psia. Calculate the flow rate for external pressures of 10, 30, 50, and 70 psia. Assume isentropic conditions. Assume that the temperature within the tank is 150 °F in all cases. Compute also the temperature at the nozzle outlet for each condition.

▮ For air ($k = 1.4$), $(p_2/p_1)_c = 0.528$. If $p_3 > 0.528(85) = 44.9$ psia the flow is subsonic and $p_3 = p_2$. If $p_3 < 0.528(85) = 44.9$ psia the flow is sonic and $p_3 < p_2$. For sonic flows (i.e., $p_3 < 44.9$ psia), G' is given by

$$G' = (g)(A_2 p_1/\sqrt{T_1})\sqrt{(k/R)[2/(k+1)]^{(k+1)/(k-1)}} \tag{1}$$

$$G' = [(32.2\pi)(1.5)^2/4][85/\sqrt{(460+150)}]\sqrt{(1.4/1715)(2/2.4)^{2.4/0.4}} = 3.24 \text{ lb/s}$$

For subsonic flows (i.e., $p_3 > 44.9$ psia), G is given by

$$G = A_2\sqrt{(2g)[k/(k-1)](p_1\gamma_1)[(p_2/p_1)^{2/k} - (p_2/p_1)^{(k+1)/k}]} \tag{2}$$

$$\gamma_1 = gp_1/RT_1 = (32.2)(85)(144)/[(1715)(610)] = 0.378 \text{ pcf}$$

Substitute into Eq. (2): $p_2 = p_3$ (express in psf), $A_2 = 0.01227$ ft^2, $\gamma_1 = 0.378$ pcf, $p_1 = 85$ psia $= 12\,240$ psf, $k = 1.4$. For $p_2 < 44.9$ psia $= 6470$ psf, $G' = 3.35$ lb/s [Eq. (1)]. For $p_2 = p_3 = 50$ psia $= 7200$ psf, $G = 3.22$ lb/s [Eq. (2)]. For $p_2 = p_3 = 70$ psia $= 10\,000$ psf, $G = 2.49$ lb/s [Eq. (2)]. Determination of temperatures: For each case substitute the corresponding values of p_1 and p_2 into $T_2 = T_1 - (p_1 g/R\gamma_1)[1-(p_2/p_1)^{(k-1)/k}]$. Also substitute $T_1 = 150$ °F, $\gamma_1 = 0.378$ pcf. Results are

p_1, psia	T_1, °F	p_3, psia	p_2, psia	G or G', lb/s	T_2, °F
85	150	10	44.9	3.35	49
85	150	30	44.9	3.35	49
85	150	50	50	3.22	64
85	150	70	70	2.49	117
85	150	85	85	0	150

16.134 Air enters a converging-diverging nozzle at a pressure of 100 psia and a temperature of 100 °F. Neglecting the entrance velocity and assuming a frictionless process, find the Mach number at the cross section where the pressure is 30 psia.

▮ $T_2 = T_1(p_2/p_1)^{(k-1)/k}$ (from $pv^k = \text{const}$ and $pv = RT$) $T_2 = 560(\frac{30}{100})^{0.4/1.4} = 397\ °\text{R}$

$V_2^2 = [2k/(k-1)](RT_1)[1-(T_2/T_1)] = 2(1.40/0.40)(1715)(560)[1-(\frac{397}{560})]$ $V_2 = 1399$ fps

$c = (kRT_2)^{1/2} = [1.4(1715)397]^{1/2} = 976$ fps $N_M = V/c = \frac{1399}{976} = 1.433$

16.135 Carbon dioxide within a tank at 42 psia and 90 °F discharges through a convergent nozzle into a 14.2-psia atmosphere. Find the velocity, pressure, and temperature at the nozzle outlet. Assume isentropic conditions.

▮ $(p_2/p_1)_c = [2/(k+1)]^{k/(k-1)}$ $k = 1.28$ $(p_2/p_1)_c = (2/2.28)^{1.28/0.28} = 0.549$

If $p_3 > 0.549p_1 = 0.549(42) = 23.1$ psia, the flow is subsonic. But $p_3 = 14.2$ psia, which is < 23.1 psia; therefore, flow is sonic. The pressure at the nozzle outlet is 23.1 psia. $T_2 = T_1 - (p_1g/R\gamma_1)[1-(p_2/p_1)^{(k-1)/k}]$. Noting that $p_1g/R\gamma_1 = T_1 = (460+90)$ °R: $T_2 = 90 - 550[1-(23.1/42)^{0.28/1.28}] = 22.6$ °F, $V_2^2 = [2k/(k-1)](RT_1)[1-(T_2/T_1)] = 2(1.28/0.28)(1123)(550)[1-(\frac{483}{550})]$; $V_2 = 829$ fps.

16.136 In Prob. 16.135 if the pressure and temperature within the tank had been 18 psia and 100 °F, what would have been the velocity, pressure, and temperature at the nozzle outlet? Assume isentropic conditions.

▮ As for solution to Prob. 16.135: $(p_2/p_1)_c = 0.549$. If $p_3 > 0.549p_1 = 9.88$ psia, the flow is subsonic. $p_3 = 14.2$ psia > 9.88 psia, therefore flow *is* subsonic. With subsonic flow $p_2 = p_3 = 14.2$ psia, $T_2 = T_1 - (p_1g/R\gamma_1)[1-(p_2/p_1)^{(k-1)/k}] = 100 - 560[1-(14.2/18)^{0.219}] = 71.7$ °F, $V_2^2 = [2k/(k-1)](RT_1)[1-(T_2/T_1)] = (2)(1.28/0.28)(1123)(560)[1-(531.7/560)]$; $V_2 = 539$ fps.

16.137 Air discharges from a large tank through a converging-diverging nozzle. The throat diameter is 3.0 in, and the exit diameter is 4.0 in. Within the tank the air pressure and temperature are 40 psia and 150 °F, respectively. Calculate the flow rate for external pressures of 39, 38, 36, and 30 psia. Assume no friction.

▮ $\gamma_1 = gp_1/RT_1 = (32.2)(40)(144)/[(1715)(610)] = 0.1773$ pcf. If sonic flow occurs, $G' = (g)(A_2p_1/\sqrt{T})\sqrt{(k/R)[2/(k+1)]^{(k+1)/(k-1)}} = (32.2)[(0.0491)(40)(144)/\sqrt{460+150}]\sqrt{(1.4/1715)(2.0/2.4)^{2.4/0.4}} = 6.10$ lb/s. For subsonic flow: $G = A_2\sqrt{(2g)[k/(k-1)](p_1\gamma_1)[(p_2/p_1)^{2/k} - (p_2/p_1)^{(k+1)/k}]} = (\pi/4)(\frac{4}{12})^2\sqrt{(2)(32.2)(1.4/0.4)(40)(144)(0.1773)[(p_3/40)^{2/1.4} - (p_3/40)^{2.4/1.4}]}$. By T and E using the above equation, $p_3 = p_2 = 36.75$ psia for $G = 6.10$ lb/s.

p_3, psia	G, lb/s	G', lb/s
40	0	
39	3.49	
38	4.87	
36.75	6.10	6.10
36	—	6.10
30	—	6.10

16.138 Air is to flow through a converging-diverging nozzle at 16 lb/s. At the throat the pressure, temperature, and velocity are to be 21 psia, 120 °F, and 480 fps, respectively. At the outlet the velocity is to be 200 fps. Determine the throat diameter. Assume isentropic flow.

▮ $G = \gamma_2A_2V^2$ $\gamma = p/RT$ $\gamma_2 = (21)(144)/[(53.3)(120+460)] = 0.09782$ lb/ft^3

$16 = (0.09782)(\pi D^2/4)(480)$ $D = 0.6587$ ft or 7.90 in

16.139 Air in a tank under a pressure of 150 psia and 80 °F flows out into the atmosphere through a 2.00-in-diameter converging nozzle. (*a*) Find the flow rate. (*b*) If a diverging section with an outlet diameter of 3.00 in were attached to the converging nozzle, what then would be the flow rate? Neglect friction.

▮ (*a*) $p_3/p_2 = 14.7/150 = 0.098 < 0.528$; so flow is sonic: Use $G' = (g)[A_2p_1/\sqrt{T_1}] \times \sqrt{(k/R)[2/(k+1)]^{(k+1)/(k-1)}}$, $G' = (32.2)[(0.0218)(150)(144)/\sqrt{460+80}]\sqrt{(1.4/1715)(2/2.4)^{2.4/0.4}} = 10.8$ lb/s. (*b*) Attaching a diverging nozzle with sonic flow (and no change in Δp) will not change the flow rate.

16.140 Repeat Prob. 16.139 for the case where the air within the tank is at 18 psia. Assume all other data to be the same.

▮ **(a)** $p_3/p_1 = 14.7/18 = 0.817 > 0.528$; so flow is subsonic. $\gamma_1 = gp_1/RT_1 = (32.2)(18)(144)/[(1715)(540)] = 0.09012$ pcf, $G = A_2\sqrt{(2g)[k/(k-1)](p_1\gamma_1)[(p_2/p_1)^{2/k} - (p_2/p_1)^{(k+1)/k}]} = (\pi/4)(\frac{2}{12})^2\sqrt{2(32.2)(1.4/0.4)(18)(144)(0.09012)(0.817^{2/1.4} - 0.817^{2.4/1.4})} = 1.03$ lb/s.

(b) Attaching a diverging nozzle with subsonic flow (when the air behaves like an incompressible fluid) *will* increase the flow rate. It only changes A_2 in the equation above. Thus $G = 1.03(3.00/2.00)^2 = 2.32$ lb/s *providing* subsonic flow still occurs in the throat. Check this by computing G', which is G_{max} if p_1 is not increased. $G' = (g)(A_2p_1/\sqrt{T_1})\sqrt{(k/R)[2/(k+1)]^{(k+1)/(k-1)}} = (32.2)(\pi/4)(2.0)^2(18/\sqrt{460+80})\sqrt{(1.4/1715)(2/2.4)^{2.4/0.4}} = 1.296$ lb/s. Thus, with the 3.0-in diameter outlet the flow at the throat *has* become sonic, and $G = 1.296$ lb/s.

16.141 Air discharges from a large tank through a converging-diverging nozzle with a 20-mm-diameter throat into the atmosphere. The gage pressure and temperature in the tank are 705 kPa and 42 °C; the barometric pressure is 99.5 kPa. **(a)** Find the nozzle-tip diameter required for p_3 to be equal to the atmospheric pressure. For this case, what are the flow velocity, sonic velocity, and Mach number at the nozzle exit?

▮ $p_1 = 705 + 99.5 = 804.5$ kPa abs. Assume sonic velocity at the throat:

$$V_3^2/2g = (RT/g)[k/(k-1)][1-(p_2/p_1)^{(k-1)/k}] \qquad V_3^2/19.62 = [287(315)/9.81](1.4/0.4)[1-(99.5/804.5)^{0.4/1.4}]$$

$$V_3 = 533 \text{ m/s}$$

$$G' = (g)(A_2p_1/\sqrt{T_1})\sqrt{(k/R)[2/(k+1)]^{(k+1)/(k-1)}}$$
$$= [9.81(\pi)(0.01^2)(804\,500)/\sqrt{273+42}]\sqrt{(1.4/287)(2/2.4)^{2.4/0.4}} = 5.65 \text{ N/s}$$

$$V_3^2 - V_1^2 = 2c_p(T_1 - T_3) \qquad 533^2 - 0 = 2(1003)(315 - T_3) \qquad T_3 = 173.4 \text{ K}$$

$$\gamma_3 = gp_3/RT_3 = (9.81)(99.5)/[(0.287)(173.4)] = 19.61 \text{ N/m}^3$$

$$A_3 = G/\gamma_3V_3 = 5.65/(19.61)(533) = 0.000541 \text{ m}^2 = 541 \text{ mm}^2$$

$$D_3 = 26.3 \text{ mm} \qquad c = [kRT]^{1/2} = [1.4(287)173.4]^{1/2} = 264 \text{ m/s} \qquad M_3 = V_3/c_3 = \tfrac{533}{264} = 2.02$$

Thus sonic velocity occurs at the throat and the shock wave is located downstream of the nozzle tip.

16.142 Helium enters a 100-mm-ID pipe from a converging-diverging nozzle at $N_M = 1.30$, $p = 14$ kPa abs, $T = 225$ K. Determine for isothermal flow **(a)** the maximum length of pipe for no choking, **(b)** the downstream conditions, and **(c)** the length from the exit to the section where $N_M = 1.0$ ($f = 0.016$, $k = 1.66$).

▮ **(a)**

$$(f/D)L_{max} = [(1 - kN_M^2)/kN_M^2] + \ln(kN_M^2)$$
$$0.016L_{max}/0.1 = \{[1 - 1.66(1.3)^2]/(1.66)(1.3)^2\} + \ln[1.66(1.3^2)]$$

from which $L_{max} = 2.425$ m.

(b)

$$p^{*\prime}/p = \sqrt{k}N_M \qquad p^{*\prime} = p\sqrt{k}N_M = 14\sqrt{1.66}(1.30) = 23.45 \text{ kPa abs}$$

The Mach number at the exit is $1/\sqrt{1.66} = 0.776$.

$$\int_V^{V^{*\prime}} \frac{dV}{V} = \frac{1}{2}\int_M^{1/\sqrt{k}} \frac{dN_M^2}{N_M^2} \qquad \text{or} \qquad \frac{V^{*\prime}}{V} = \frac{1}{\sqrt{k}N_M}$$

At the upstream section $V = N_M\sqrt{kRT} = (1.30)\sqrt{(1.66)(2077)(225)} = 1145$ m/s and $V^{*\prime} = V/(\sqrt{k}N_M) = 1145/(\sqrt{1.66})(1.30) = 683.6$ m/s.

(c) Substituting into the equation in **(a)** for $N_M = 1$, $(0.016/0.1)L'_{max} = [(1-1.66)/1.66] + \ln 1.66$ or $L'_{max} = 0.683$ m. $N_M = 1$ occurs 0.683 m from the exit.

16.143 Helium discharges from a $\frac{1}{2}$-in-diameter converging nozzle at its maximum rate for reservoir conditions of $p = 4$ atm, $T = 25$ °C. What restrictions are placed on the downstream pressure? Calculate the mass flow rate and velocity of the gas at the nozzle.

$$\dot{m}_{max} = (A^*p_0/\sqrt{T_0})\sqrt{(k/R)[2/(k+1)]^{(k+1)/(k-1)}} \qquad A^* = (\pi)[(\tfrac{1}{2})(2.54)/100]^2/4 = 0.0001267 \text{ m}^2$$

$$p_0 = (4)(101\,310) = 405\,240 \text{ Pa} \qquad T_0 = 25 + 273 = 298 \text{ K} \qquad k = 1.66 \qquad R = 2077 \text{ N-m/(kg-K)}$$

$$\dot{m}_{max} = [(0.0001267)(405\,240)/\sqrt{298}]\sqrt{(1.66/2077)[2/(1.66+1)]^{(1.66+1)/(1.66-1)}} = 0.0473 \text{ kg/s}$$

$$V^* = \dot{m}_{max}/\rho^* A^* \qquad \rho^* = \rho_0(p^*/p_0)^{1/k} \qquad \rho_0 = p_0/RT_0 = 405\,240/[(2077)(298)] = 0.655\ \text{kg/m}^3$$
$$p^*/p_0 = [2/(k+1)]^{k/(k-1)} = [2/(1.66+1)]^{1.66/(1.66-1)} = 0.488 \qquad p^* = (0.488)(405\,240) = 197\,757\ \text{Pa}$$
$$\rho^* = 0.655(197\,757/405\,240)^{1/1.66} = 0.425\ \text{kg/m}^3$$
$$V^* = 0.0473/(0.425)(0.0001267) = 878\ \text{m/s} \qquad p_{downstream} < p^*$$

16.144 Air in a reservoir at 285 psia, $T = 295$ °F, flows through a 2-in-diameter throat in a converging-diverging nozzle. For $N_M = 1$ at the throat, calculate p, ρ, and T there.

▮ For critical conditions at the throat, $T^* = 0.833$, $p^* = 0.528$, and $\rho^* = 0.632$.

$$T_{throat} = (0.833)(295+460) = 629\ °\text{R} \qquad p_{throat} = (0.528)(285) = 150\ \text{psia}$$
$$\rho = p/RT \qquad \rho_0 = (285)(144)/[(1716)(295+460)] = 0.0317\ \text{slug/ft}^3$$
$$\rho_{throat} = (0.632)(0.0317) = 0.0200\ \text{slug/ft}^3$$

16.145 What must be the velocity, pressure, density, temperature, and diameter at a cross section of the nozzle of Prob. 16.144 where $N_M = 2.5$?

▮
$$T_0/T = 1 + [(k-1)/2]N_M^2 \qquad (295+460)/T = 1 + [(1.40-1)/2](2.5)^2 \qquad T = 336\ °\text{R}$$
$$p_0/p = \{1 + [(k-1)/2]N_M^2\}^{k/(k-1)} \qquad 285/p = \{1 + [(1.40-1)/2](2.5)^2\}^{1.40/(1.40-1)} \qquad p = 16.7\ \text{psia}$$
$$\rho_0/\rho = \{1 + [(k-1)/2]N_M^2\}^{1/(k-1)} \qquad 0.0317/\rho = \{1 + [(1.40-1)/2](2.5)^2\}^{1/(1.40-1)} \qquad \rho = 0.00417\ \text{slug/ft}^3$$
$$N_M = V/\sqrt{kRT} \qquad 2.5 = V/\sqrt{(1.40)(1716)(336)} \qquad V = 2246\ \text{ft/s}$$
$$A/A^* = (1/N_M)[(5+N_M^2)/6]^3 \qquad A/[(\pi)(\tfrac{2}{12})^2/4] = (1/2.5)[(5+2.5^2)/6]^3 \qquad A = 0.0575\ \text{ft}^2$$
$$\pi D^2/4 = 0.0575 \qquad D = 0.271\ \text{ft} \quad \text{or} \quad 3.25\ \text{in}$$

16.146 Nitrogen in sonic flow at a 25-mm-diameter throat section has a pressure of 60 kPa abs, $T = -15$ °C. Determine the mass flow rate.

▮
$$\rho^* = p^*/RT^* = 60/[(0.297)(273-15)] = 0.7830\ \text{kg/m}^3$$
$$V^* = \sqrt{kRT^*} = \sqrt{(1.40)(297)(273-15)} = 327.5\ \text{m/s}$$
$$\dot{m} = \rho^* A^* V^* = 0.7830[(\pi)(\tfrac{25}{1000})^2/4](327.5) = 0.126\ \text{kg/s}$$

16.147 What is the Mach number for Prob. 16.146 at a 40-mm-diameter section in supersonic and in subsonic flow?

▮
$$A/A^* = (1/N_M)[(5+N_M^2)/6]^3 \qquad [(\pi)(40)^2/4]/[(\pi)(25)^2/4] = (1/N_M)[(5+N_M^2)/6]^3$$
$$(N_M)_1 = 0.234 \qquad (N_M)_2 = 2.47$$

16.148 What diameter throat section is needed for critical flow of 0.6 lbm/s of carbon monoxide from a reservoir where $p = 300$ psia and $T = 100$ °F?

▮
$$\dot{m} = \rho^* A^* V^* \qquad \rho_0 = p_0/RT_0 = (300)(144)/[(55.2)(100+460)] = 1.398\ \text{lbm/ft}^3$$
$$\rho^* = 0.634\rho_0 = (0.634)(1.398) = 0.8863\ \text{lbm/ft}^3 \qquad T^* = 0.833T_0 = (0.833)(100+460) = 466\ °\text{R}$$
$$V^* = \sqrt{kRT^*} = \sqrt{(1.40)[(55.2)(32.2)](466)} = 1077\ \text{ft/s} \qquad 0.6 = (0.8863)(A^*)(1077)$$
$$A^* = 0.0006286\ \text{ft}^2 \qquad \pi D^2/4 = 0.0006286 \qquad D = 0.02829\ \text{ft} \quad \text{or} \quad 0.34\ \text{in}$$

16.149 A supersonic nozzle is to be designed for airflow with $N_M = 3.0$ at the exit section, which is 200 mm in diameter and has a pressure of 6 kPa abs and temperature of −80 °C. Calculate throat area and reservoir conditions.

▮
$$A/A^* = (1/N_M)[(5+N_M^2)/6]^3 \qquad A_{exit} = (\pi)(\tfrac{200}{1000})^2/4 = 0.03142\ \text{m}^2$$
$$0.03142/A^* = (1/3.0)[(5+3.0^2)/6]^3 \qquad A^* = 0.007420\ \text{m}^2$$
$$T_0 = (T)\{1 + [(k-1)/2]N_M^2\} = (273-80)\{1 + [(1.40-1)/2](3.0)^2\} = 540\ \text{K}$$
$$p_0 = (p)\{1 + [(k-1)/2]N_M^2\}^{k/(k-1)} = (6)\{1 + [(1.40-1)/2](3.0)^2\}^{1.40/(1.40-1)} = 220.4\ \text{kPa}$$
$$\rho_0 = p_0/RT_0 = 220.4/[(0.287)(540)] = 1.42\ \text{kg/m}^3$$

16.150 In Prob. 16.149, calculate the diameter of cross section for $N_M = 1.5$, 2.0, and 2.5.

▮ Using data from Prob. 16.149,

$$\pi(D^*)^2/4 = 0.007420\ \text{m}^2 \qquad D^* = 0.09720\ \text{m} \qquad D = (D^*)(1/\sqrt{N_M})[(5+N_M^2)/6]^{1.5}$$

$$N_M = 1.5 \qquad D = (0.09720)(1/\sqrt{1.5})[(5+1.5^2)/6]^{1.5} = 0.1054\ \text{m}$$

$$N_M = 2.0 \qquad D = (0.09720)(1/\sqrt{2.0})[(5+2.0^2)/6]^{1.5} = 0.1263\ \text{m}$$

$$N_M = 2.5 \qquad D = (0.09720)(1/\sqrt{2.5})[(5+2.5^2)/6]^{1.5} = 0.158\ \text{m}$$

16.151 Calculate the exit velocity and the mass rate of flow of nitrogen from a reservoir where $p = 3$ atm and $T = 20\ °\text{C}$, through a converging nozzle of 60 mm diameter discharging to the atmosphere.

▮ $T^* = 0.833T_0 = (0.833)(20+273) = 244\ \text{K} \qquad V^* = \sqrt{kRT^*} = \sqrt{(1.40)(297)(244)} = 319\ \text{m/s} \qquad \dot{m} = \rho^* A^* V^*$

$$\rho^* = 0.634\rho_0 \qquad \rho_0 = p_0/RT_0 = (3)(101\,350)/[(297)(20+273)] = 3.494\ \text{kg/m}^3$$

$$\rho^* = (0.634)(3.494) = 2.215\ \text{kg/m}^3 \qquad A^* = (\pi)(\tfrac{60}{1000})^2/4 = 0.002827\ \text{m}^2$$

$$\dot{m} = (2.215)(0.002827)(319) = 2.00\ \text{kg/s}$$

16.152 A converging nozzle has a throat area of 1 in^2 and stagnation air conditions of 120 lb/in^2 abs and 600 °R. Compute the exit pressure and mass flow if the back pressure is (*a*) 90 lb/in^2 abs and (*b*) 45 lb/in^2 abs. Assume $k = 1.4$.

▮ We are given $p_0 = 120$ lb/in^2 abs $= 17\,280$ lb/ft^2, $T_0 = 600$ °R. Hence $\rho_0 = p_0/RT_0 = 17\,280/(1717)(600) = 0.0168$ slug/ft^3, $p^*/p_0 = 0.5283$, or $p^* = 63.4$ lb/in^2 abs $= 9130$ lb/ft^2.

(*a*) Since $p_b = 90$ lb/in^2 abs $> p^*$, the flow is subsonic throughout. The throat Mach number is found from $p_e = p_b$ and

$$N_M^2 = 5[(p_0/p)^{2/7} - 1] \qquad (N_M)_e^2 = 5[(\tfrac{120}{90})^{2/7} - 1] = 0.4283 \qquad (N_M)_e = 0.6545$$

$$T_0/T_e = 1 + 0.2(N_M)_e^2 = 1 + 0.2(0.6545)^2 = 1.0857$$

$$\rho_0/\rho_e = [1 + 0.2(N_M)_e^2]^{2.5} = [1 + (0.2)(0.6545)^2]^{2.5} = 1.2281$$

Hence $T_e = 600/1.0857 = 553$ °R, so that $a_e \approx 49(553)^{1/2} = 1152$ ft/s. $\rho_e = 0.0168/1.2281 = 0.0137$ slug/ft^3. The mass flow is thus given by $\dot{m} = \rho_e A_e V_e = \rho_e A_e[(N_M)a_e] = 0.0137(\frac{1}{144})[0.6545(1152)] = 0.0715$ slug/s. The exit pressure equals the back pressure: $p_e = p_b = 90$ lb/in^2 abs.

(*b*) Since $p_b = 45$ lb/in^2 abs $< p^*$, the throat is choked. The exit pressure is sonic: $p_e = p^* = 63.4$ lb/in^2 abs, $\dot{m} = \dot{m}_{\max} = 0.6847p_0A_t/(RT_0)^{1/2} = 0.6847(17\,280)(\frac{1}{144})/[1717(600)]^{1/2} = 0.0810$ slug/s. Any back pressure less than or equal to $p^* = 63.4$ lb/in^2 abs would cause this same choked condition. Notice that the 50 percent increase in throat Mach number from 0.6545 to 1.0 increases the mass flux only 13 percent, from 0.0715 to 0.0810 slug/s.

16.153 A converging-diverging nozzle has a throat area of 0.002 m^2 and an exit area of 0.008 m^2. Air stagnation conditions are $p_0 = 1000$ kPa and $T_0 = 500$ K. Compute the exit pressure and mass flow for (*a*) design condition and the exit pressure and mass flow if (*b*) $p_b \approx 300$ kPa and (*c*) $p_b \approx 900$ kPa. Assume $\gamma = 1.4$.

▮ (*a*) The design condition corresponds to supersonic isentropic flow at the given area ratio $A_e/A_t = 0.008/0.002 = 4.0$. We can find the design Mach number by $(N_M)_e = [216(A_1/A^*) - 254(A/A^*)^{2/3}]^{1/5}$, $(N_M)_{e,\,\text{design}} \approx [216(4.0) - 254(4.0)^{2/3}]^{1/5} \approx 2.95$, $p_0/p_e = [1 + 0.2(N_M)_e^2]^{3.5}$, $p_0/p_e = [1 + 0.2(2.95)^2]^{3.5} = 34.1$ or $p_{e,\,\text{design}} = 1000/34.1 = 29.3$ kPa. Since the throat is clearly sonic at design conditions, $\dot{m}_{\text{design}} = \dot{m}_{\max} = 0.6847p_0A_t/(RT_0)^{1/2} = 0.6847(10^6)(0.002)/[287(500)]^{1/2} = 3.61$ kg/s.

(*b*) For $p_b = 300$ kPa we are definitely far below the subsonic isentropic condition but we may even be below condition with a normal shock in the exit, where oblique shocks occur outside the exit plane. If it is, $p_e = p_{e,\,\text{design}} = 29.3$ kPa because no shock has yet occurred. To find out, compute condition F by assuming an exit normal shock with $(N_M)_1 = 2.95$, that is, the design Mach number just upstream of the shock. $p_2/p_1 = [1/(k+1)][2k(N_M)_1^2 - (k-1)] = (1/2.4)[2.8(2.95)^2 - 0.4] = 9.99$ or $p_2 = 9.99p_1 = 9.99p_{e,\,\text{design}} = 293$ kPa. Since this is less than the given $p_b = 300$ kPa, there is a normal shock just upstream of the exit plane. The exit flow is subsonic and equals the back pressure $p_e = p_b = 300$ kPa. Also $\dot{m} = \dot{m}_{\max} = 3.61$ kg/s. The throat is still sonic and choked at its maximum mass flow.

(*c*) Finally, for $p_b = 900$ kPa, we compute $(N_M)_e$ and p_e. Again $A_e/A_t = 4.0$ for this condition, with a subsonic $(N_M)_e$: $(N_M)_e = [1 + 0.27(A/A^*)^{-2}]/(1.728)(A/A^*)$, $(N_M)_e(C) \approx [1 + 0.27/(4.0)^2]/(1.728)(4.0) = 0.147$. Then the isentropic exit-pressure ratio for this condition is $p_0/p_e = [1 + 0.2(0.147)^2]^{3.5} = 1.0152$ or $p_e = 1000/1.0152 = 985$ kPa. The given back pressure of 900 kPa is less than this value. Thus for this case there is a normal shock just downstream of the throat and the throat is choked: $p_e = p_b = 900$ kPa, $\dot{m} = \dot{m}_{\max} = 3.61$ kg/s. For this large

exit-area ratio the exit pressure would have to be larger than 985 kPa to cause a subsonic flow in the throat and a mass flow less than maximum.

16.154 Steam in a tank at 450 °F and 100 psia exhausts through a converging nozzle of throat area 0.1 in^2 to a 1-atm environment. Compute the initial mass flow rate. Assume an ideal gas with $k = 1.33$.

▮ It is sure to be choked since $p_0/p_e = 7$; therefore, $A_e = A^*$. $\dot{m} = \dot{m}_{max} = (k^{1/2})[2/(k+1)]^{(k+1)/2(k-1)}(A^* p_0/\sqrt{RT_0})$, $\dot{m} = 1.33^{1/2}[2/(1.33+1)]^{(1.33+1)/2(1.33-1)}\{(0.1/144)[(100)(144)]/\sqrt{(2759)(450+460)}\} = 0.00424$ slug/s.

16.155 Air in a tank at 700 kPa and 20 °C exhausts through a converging nozzle of throat area 65 mm^2 to a 1-atm environment. Compute the initial mass flow rate.

▮ It is sure to be choked since $p_0/p_e = 7$; therefore, $A_e = A^*$. $\dot{m} = \dot{m}_{max} = (k^{1/2})[2/(k+1)]^{(k+1)/2(k-1)}(A^* p_0/\sqrt{RT_0})$, $\dot{m} = 1.40^{1/2}[2/(1.40+1)]^{(1.40+1)/2(1.40-1)}\{(65\times10^{-6})[(700)(1000)]/\sqrt{(287)(20+273)}\} = 0.107$ kg/s.

16.156 At a point upstream of the throat of a converging-diverging nozzle the properties are $V_1 = 190$ m/s, $T_1 = 305$ K, and $p_1 = 120$ kPa. If the exit flow is supersonic, compute, from isentropic theory, $\dot{m}$ and A_1. The throat area is 35 cm^2.

▮ $A_1/A^* = (1+0.2N_M^2)^3/1.728N_M \qquad c = \sqrt{kRT} = \sqrt{(1.40)(287)(305)} = 350$ m/s $\qquad N_M = V/c = \frac{190}{350} = 0.543$

Since the exit flow is supersonic, $A^* = A_{throat} = 35$ cm^2.

$$A_1/35 = [1+(0.2)(0.543)^2]^3/[(1.728)(0.543)] \qquad A_1 = 44.3 \text{ cm}^2$$

$$\rho_1 = p_1/RT = 120/[(0.287)(305)] = 1.371 \text{ kg/m}^3 \qquad \dot{m} = \rho_1 A_1 V_1 = (1.371)(44.3\times10^{-4})(190) = 1.15 \text{ kg/s}$$

16.157 In transonic wind tunnel testing the small area decrease caused by model blockage can be significant. Suppose the test section Mach number is 1.10 and the section area is 1 m^2 for air with $T_0 = 21$ °C. Find, according to one-dimensional theory, the percentage change in test-section velocity caused by a model of cross section 0.005 m^2 (a 0.5 percent blockage).

▮

$$T_1 = T_0/(1+0.2N_M^2) = (21+273)/[1+(0.2)(1.10)^2] = 237 \text{ K} \qquad c = \sqrt{kRT}$$

$$c_1 = \sqrt{(1.40)(287)(237)} = 309 \text{ m/s} \qquad V_1 = c_1(N_M)_1 = (309)(1.1) = 340 \text{ m/s} \qquad \text{(unblocked)}$$

$$A_1/A^* = (1+0.2N_M^2)^3/1.728N_M \qquad 1/A^* = [1+(0.2)(1.1)^2]^3/[(1.728)(1.1)] \qquad A^* = 0.9921 \text{ m}^2$$

If A_1 is blocked by the 0.5 percent model, $A_1 = 0.995$ m^2. $A_1/A^* = 0.995/0.9921 = 1.00292$. For the blocked condition,

$$A_1/A^* = (1+0.2N_M^2)^3/1.728N_M \qquad 1.00292 = (1+0.2N_M^2)^3/1.728N_M \qquad N_M = 1.011$$

$$T_1 = (21+273)/[1+(0.2)(1.011)^2] = 244 \text{ K} \qquad c_1 = \sqrt{(1.40)(287)(244)} = 313 \text{ m/s}$$

$$V_1 = (313)(1.011) = 316 \text{ m/s} \qquad \text{(blocked)}$$

Thus a 0.5 percent decrease in area causes a $(340-316)/340 = 0.071$, or 7.1 percent decrease in velocity for this transonic flow.

16.158 In flow of air in a converging-diverging duct with a supersonic exit, the throat area is 9 cm^2 and the throat pressure is 316 kPa. Find the pressure on either side of the throat where $A = 26$ cm^2.

▮ Supersonic exit means $A^* = A_{throat} = 9$ cm^2 and $p^* = 316$ kPa.

Subsonic side:

$$A_1/A^* = (1+0.2N_M^2)^3/1.728N_M \qquad 26/9 = (1+0.2N_M^2)^3/1.728N_M \qquad N_M = 0.204$$

$$p_1 = p_0/(1+0.2N_M^2)^{k/(k-1)} \qquad p_0 = p^*/[2/(k+1)]^{k/(k-1)} = 316/[2/(1.40+1)]^{1.40/(1.40-1)} = 598 \text{ kPa}$$

$$p_1 = 598/[1+(0.2)(0.204)^2]^{1.40/(1.40-1)} = 581 \text{ kPa}$$

Subsonic side:

$$A_1/A^* = (1+0.2N_M^2)^3/1.728N_M \qquad 26/9 = (1+0.2N_M^2)^3/1.728N_M \qquad N_M = 2.60$$

$$p_1 = 598/[1+(0.2)(2.60)^2]^{1.40/(1.40-1)} = 30.0 \text{ kPa}$$

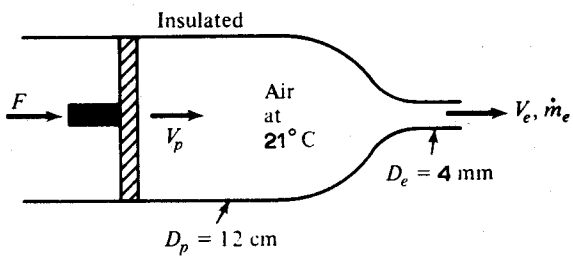


Fig. 16-8

16.159 A force $F = 2.4$ kN pushes a piston through a 12-cm-diameter insulated cylinder containing air at 21 °C, as in Fig. 16-8. The exit diameter is 4 mm and the external pressure is 1 atm. Estimate **(a)** V_e, **(b)** V_p, and **(c)** $\dot{m}_e$.

(a) $$V_e = \{[2k/(k+1)](RT)\}^{1/2} = \{[(2)(1.40)/(1.40+1)](287)(21+273)\}^{1/2} = 314 \text{ m/s}$$

(b) $$V_p = (V_e)(A_e/A_p) = (314)(4/120)^2 = 0.349 \text{ m/s}$$

(c) $$\dot{m}_e = \dot{m}_{\max} = 0.6847 p_0 A^*/(RT_0)^{1/2} \qquad p_0 = F/A_p + p_{\text{atm}} = 2.4/[\pi(0.12)^2/4] + 101.35 = 313.6 \text{ kPa}$$

$$A^* = (\pi)(0.004)^2/4 = 0.0000126 \text{ m}^2$$

$$\dot{m}_e = (0.6847)(313\,600)(0.0000126)/[(287)(21+273)]^{1/2} = 0.00931 \text{ kg/s}$$

16.160 Air flows steadily and subsonically from a tank at 21 °C through a nozzle of exit area 18 cm². Compute **(a)** exit Mach number, **(b)** exit velocity, **(c)** tank pressure (assume 1 atm exit pressure).

(a) Assume an ideal gas, $\rho_e V_e^2 = \gamma p_e (N_M)_e^2$, $\gamma p_e A_e (N_M)_e^2 = 140$, $(N_M)_e = \sqrt{140/[(1.40)(101\,350)(18 \times 10^{-4})]} = 0.740$.

(b) $$V_e = c_e N_M \qquad T_e = T_0/(1 + 0.2N_M^2) = (21+273)/[1 + (0.2)(0.740)^2] = 265 \text{ K}$$

$$c_e = \sqrt{kRT} = \sqrt{(1.40)(287)(265)} = 326 \text{ m/s} \qquad V_e = (326)(0.740) = 241 \text{ m/s}$$

(c) $$p_0 = (p_e)\{1 + [(k-1)/2](N_M)_e^2\}^{k/(k-1)} = (101.350)\{1 + [(1.40-1)/2](0.740)^2\}^{1.40/(1.40-1)} = 146 \text{ kPa abs}$$

16.161 Air flows through a converging-diverging nozzle from a reservoir, where $p_0 = 390$ kPa. The throat area is 8 cm². A normal shock stands in the duct where $A = 12$ cm². Compute the pressure just downstream of this shock.

$$A/A^* = (1 + 0.2N_M^2)^3/1.728N_M = 12/8 = 1.50 \qquad 1.50 = (1 + 0.2N_M^2)^3/1.728N_M \qquad N_M = 1.85$$

$$p_1 = p_0/(1 + 0.2N_M^2)^{k/(k-1)} = 390/[1 + (0.2)(1.85)^2]^{1.40/(1.40-1)} = 62.9 \text{ kPa}$$

$$p_2/p_1 = [1/(k+1)][2k(N_M)_1^2 - (k-1)] = [1/(1.40+1)][(2)(1.40)(1.85)^2 - (1.40-1)] = 3.826$$

$$p_2 = (3.826)(62.9) = 241 \text{ kPa}$$

16.162 Air in a large tank at 102 °C and 152 kPa exhausts to the atmosphere ($p_{\text{atm}} = 101.3$ kPa) through a converging nozzle with a 6-cm² throat area. Compute the exit mass flow.

$$\dot{m} = \rho_e A_e V_e \qquad \rho_e = \rho_0/[1 + (0.2)(N_M)_e^2]^{2.5}$$

$$\rho_0 = p_0/RT_0 = 152/[(0.287)(101.3 + 273)] = 1.415 \text{ kg/m}^3$$

$$p_0/p_a = [1 + (0.2)(N_M)_e^2]^{3.5} \qquad \tfrac{152}{101.3} = [1 + (0.2)(N_M)_e^2]^{3.5} \qquad (N_M)_e = 0.784$$

$$\rho_e = 1.415/[1 + (0.2)(0.784)^2]^{2.5} = 1.059 \text{ kg/m}^3$$

$$T_e = T/[1 + 0.2(N_M)_e^2] = (102 + 273)/[1 + (0.2)(0.784)^2] = 334 \text{ K}$$

$$c_e = \sqrt{kRT_e} = \sqrt{(1.40)(287)(334)} = 366 \text{ m/s} \qquad V_e = (c_e)(N_M)_e = (366)(0.784) = 287 \text{ m/s}$$

$$\dot{m} = (1.059)(6 \times 10^{-4})(287) = 0.182 \text{ kg/s}$$

16.163 Air flows through a converging-diverging nozzle between two reservoirs; a manometer is located at the throat, as shown in Fig. 16-9. Estimate the downstream reservoir pressure.

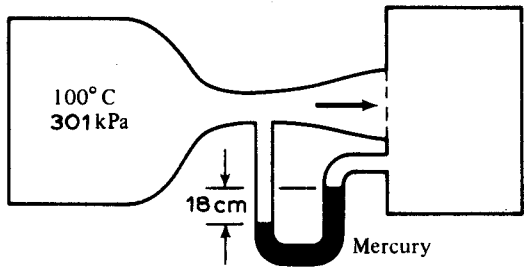


Fig. 16-9

▮ Assume critical flow in the throat: $p_t = p^* = 0.528p_0 = (0.528)(301) = 158.9$ kPa. Guessing $\rho_{exit} = 1.2$ kg/m^3, we have:

$$p^* - p_e = (\rho_M - \rho_e)(gh) \qquad 158\,900 - p_e = (13\,600 - 1.2)(9.807)(0.18)$$
$$p_e = 134\,900 \text{ Pa} = 134.9 \text{ kPa} \qquad \text{(nonisentropic flow)}$$

16.164 Air in a tank at 121 kPa and 301 K exhausts to the atmosphere through a 5-cm^2-throat converging nozzle at a rate of 0.12 kg/s. What is the atmospheric pressure? What is the maximum mass flow possible at low atmospheric pressure?

▮ $\dot{m}_{max} = 0.6847p_0A^*/(RT_0)^{1/2} = (0.6847)[(121)(1000)](5 \times 10^{-4})/[(287)(300)]^{1/2} = 0.141$ kg/s. Therefore, the given $\dot{m} = 0.12$ kg/s is less than critical and we have a subsonic exit.

$$\dot{m} = \rho_e A_e V_e = (\rho_0)(\rho_e/\rho_0)(A_e)(N_M)_e(c_0)(c_e/c_0) = (\rho_0 A_e c_0)(N_M)_e[1 + 0.2(N_M)_e^2]^{-3}$$
$$\rho_0 = p_0/RT_0 = 121/[(0.287)(301)] = 1.401 \text{ kg/m}^3 \qquad c_0 = \sqrt{kRT_0} = \sqrt{(1.40)(287)(301)} = 347.8 \text{ m/s}$$
$$0.12 = [(1.401)(5 \times 10^{-4})(347.8)](N_M)_e[1 + 0.2(N_M)_e^2]^{-3} \qquad (N_M)_e = 0.619 \qquad \text{(by trial and error)}$$
$$p_e = 121/[1 + (0.2)(0.619)^2]^{3.5} = 93.4 \text{ kPa}$$

16.165 A supersonic nozzle with an exit area of 10 cm^2 discharges air at $N_M = 2.5$, $p = 101$ kPa, and $T = 302$ K. Compute **(a)** p_0, **(b)** T_0, and **(c)** $\dot{m}$.

▮ **(a)**
$$p_0 = (p)(1 + 0.2N_M^2)^{3.5} = 101[1 + (0.2)(2.5)^2]^{3.5} = 1726 \text{ kPa}$$

(b)
$$T_0 = (T)(1 + 0.2N_M^2) = 302[1 + (0.2)(2.5)^2] = 680 \text{ K}$$

(c) At $(N_M)_e = 2.5$, $A_e/A^* = 2.637$, $A^* = 10/2.637 = 3.79$ cm^2.

$$\dot{m} = \dot{m}_{max} = 0.6847p_0A^*/(RT_0)^{1/2}$$
$$\dot{m} = 0.6847[(1726)(1000)](3.79 \times 10^{-4})/[(287)(680)]^{1/2} = 1.014 \text{ kg/s}$$

16.166 A perfect gas (not air) at 305 kPa expands isentropically through a supersonic nozzle with an exit area 5 times its throat area. If the exit Mach number is 3.8, what is the exit pressure of the gas?

▮ First find the specific-heat ratio:

$$A_e/A^* = [1/(N_M)_e]\{[2 + (k-1)(N_M)_e^2]/(k+1)\}^{(k+1)/(2)(k-1)}$$
$$5 = (1/3.8)\{[2 + (k-1)(3.8)^2]/(k+1)\}^{(k+1)/(2)(k-1)} \qquad k = 1.67 \qquad \text{(by trial and error)}$$

Then, $p_e = p_0/\{1 + [(k-1)/2](N_M)_e^2\}^{k/(k-1)} = 305/\{1 + [(1.66-1)/2](3.8)^2\}^{1.66/(1.66-1)} = 3.72$ kPa.

16.167 Air with $p_0 = 300$ kPa and $T_0 = 500$ K flows through a converging–diverging nozzle with throat area of 1 cm^2 and exit area of 4 cm^2 into a receiver tank. The mass flow is 195.2 kg/h. For what range of receiver pressure is this mass flow possible?

▮ $\dot{m} = 195.2/3600 = 0.0542$ kg/s, $\dot{m}_{max} = 0.6847p_0A^*/(RT_0)^{1/2} = 0.6847[(3 \times 10^5)(1 \times 10^{-4})/[(287)(500)]^{1/2} = 0.0542$ kg/s. Therefore the throat is choked, and p_b can lie anywhere between $p_b = 0$ and p_e for isentropic subsonic exit: $A_e/A^* = 4.00$, $(N_M)_e = 0.147$, $(p_e)_{max} = p/[1 + (0.2)(N_M)_e^2]^{3.5} = 300/[1 + (0.2)(0.147)^2]^{3.5} = 296$ kPa. Thus the possible exit pressure range is $0 \le p_b \le 296$ kPa.

16.168 A rocket has a thrust of 900 000 lb when operating at its design point ($p_e = p_a$). If the chamber pressure and

temperature are 500 psia and 5000 °R and the gas approximates air with $k = 1.4$, compute the throat diameter of the engine.

▮ $F = \rho_e A_e V_e = k p_e A_e (N_M)_e^2$ $\quad p_0/p_e = [1 + (0.2)(N_M)_e^2]^{3.5}$ $\quad 500/14.7 = [1 + (0.2)(N_M)_e^2]^{3.5}$ $\quad (N_M)_e = 2.95$

$$900\,000 = 1.4[(14.7)(144)](A_e)(2.95)^2 \qquad A_e = 34.90\ \text{ft}^2 \qquad \pi D_e^2/4 = 34.90 \qquad D_e = 6.67\ \text{ft}$$

At $(N_M)_e = 2.95$, $A_e/A^* = [1 + (0.2)(2.95)^2]^3/[(1.728)(2.95)] = 4.038$, $A^* = A_t = 34.90/4.038 = 8.643\ \text{ft}^2$, $\pi D_t^2/4 = 8.643$, $D_t = 3.32$ ft.

16.169 A nozzle for an ideal rocket is to operate at an altitude of 15.25 km, where the pressure is 11.60 kPa, and is to give a 6.67-kN thrust when the chamber pressure is 1345 kPa and the chamber temperature is 2760 °C. Determine the throat and exit areas and the exit velocity and temperature. Take $k = 1.4$ and $R = 0.355$ kJ/kg · K, and assume the exit pressure to be the ambient pressure.

▮ We have p/p_0 for the exit, $p/p_0 = 11.60/1345 = 0.008265$. From Table A-16, we see that $M_{\text{exit}} = 3.8$, and we have an area ratio $A_{\text{exit}}/A_{\text{throat}} = 8.95$. Thus, we get for the exit temperature T_e: $T_e/T_0 = 0.257$, $T_e = (2760 + 273)(0.257) = 779\ \text{K} = 506\ °\text{C}$. We can now determine the exit velocity: $V_e = M_e c = M_e\sqrt{kRT_e} = 3.8\sqrt{(1.4)(355)(779)} = 2364$ m/s. To ascertain the *throat* and *exit* areas, we must consider the thrust. Using a control volume comprising the interior of the combustion chamber and nozzle and considering this control volume to be inertial we have from linear momentum considerations

$$6670 = (\rho_e V_e A_e)V_e = \rho_e A_e (2364^2) \tag{1}$$

By the ideal gas law, $\rho_e = p_e/RT_e = 11\,600/(355)(779) = 0.04195\ \text{kg/m}^3$. Hence, from Eq. (*1*), $A_e = 6670/(2364^2)(0.04195) = 0.02845\ \text{m}^2$ and so $A^* = 0.0285/8.95 = 0.00318\ \text{m}^2$.

16.170 The inlet velocity of an isentropic diffuser is 305 m/s and the undisturbed pressure and temperature are 34.5 kPa abs and 235 °C, respectively. If the pressure is increased by 30 percent at the exit of the diffuser, determine the exit velocity and temperature. Use tables.

▮ Find (N_M) first:

$$(N_M)_1 = V_1/\sqrt{kRT_1} = 305/\sqrt{(1.4)(287)(235 + 273)} = 0.675 \qquad p/p_0 = 0.737$$

$$p_2/p_0 = (p_2/p)(p/p_0) = (1.30)(0.737) = 0.958$$

$$(N_M)_2 = 0.249 \qquad T_2/T_0 = 0.988$$

But $T_1/T_0 = 0.916$, $T_0 = 0.508/0.916 = 555$. Hence, $T_2 = (555)(0.988) = 548$ K. Also $V_2 = (0.249)\sqrt{(1.4)(287)(1)(548)} = 116.8$ m/s.

16.171 Air is kept in a tank at pressure $p_0 = 0.689$ MPa abs and temperature $T_0 = 17$ °C. If one allows the air to issue out in a one-dimensional isentropic flow, what is the greatest possible flow per unit area? What is the flow per unit area at the exit of the nozzle where $p = 0.101325$ MPa?

▮

$$p/p_0 = 0.147 \qquad (N_M)_e = 1.91$$

For the largest G we go to the throat:

$$G^* = \sqrt{k/R}(p_0/\sqrt{T_0})[2/(k+1)]^{(k+1)/[2(k-1)]} = \sqrt{1.4/287}(0.689 \times 10^6/\sqrt{290})(2/2.4)^{2.4/0.8} = 1635\ \text{kg/m}^2 \cdot \text{s}.$$

At the exit, $G = \sqrt{k/R}(p_0/\sqrt{T_0})\left(N_M/\{1 + [(k-1)/2]N_M^2\}^{(k+1)/[2(k-1)]}\right) = \sqrt{1.4/287}(0.689 \times 10^6/\sqrt{290}) \times \left(1.91/\{1 + [(1.4-1)/2](1.91)^2\}^{2.4/0.8}\right) = 1043\ \text{kg/m}^2 \cdot \text{s}$.

16.172 Determine the exit area and the exit velocity for isentropic flow of a perfect gas with $k = 1.4$ in the nozzle shown in Fig. 16-10. How small can we make the exit area and still have isentropic flow with the given conditions entering the nozzle? Use tables.

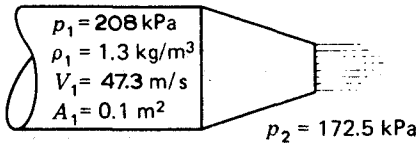


Fig. 16-10

▮ First find $(N_M)_1$: $N_M = V/c$, $c = \sqrt{kp/\rho} = \sqrt{(1.4)(208\,000)/1.3} = 473$ m/s, $(N_M)_1 = 47.3/473 = 0.1$. From Table A-16, $A/A^* = 5.82$, $A^* = 0.1/5.82 = 0.0172$ m^2. Also $p/p_0 = 0.993$, $p_0 = 208/0.993 = 209.47$ kPa. Now go to exit: $p/p_0 = 172.5/209.47 = 0.824$. Hence, $(N_M)_e = 0.53$ and $A_e/A^* = 1.28$, $A_e = (0.0172)(1.28) = 0.0220$ m^2. The smallest possible area is $A^* = 0.0172$ m^2.

At $N_M = 0.1$:

$$\rho_1/\rho_0 = 0.995 \qquad \rho_0 = 1.3/0.995 = 1.31 \text{ kg/m}^3$$

At $(N_M)_e = 0.53$ (exit):

$$\rho_2/\rho_0 = 0.871 \qquad \rho_2 = (0.871)(1.31) = 1.14 \text{ kg/m}^3$$

From continuity:

$$\rho_1 V_1 A_1 = \rho_2 V_2 A_2 \qquad (1.3)(47.3)(0.1) = (1.14)V_2(0.0220) \qquad V_2 = 245 \text{ m/s}$$

16.173 Determine the exit area in Prob. 16.172 for a nozzle efficiency of 90 percent. Take $c_p = 1.005$ kJ/kg · K, $k = 1.4$, and $R = 0.287$ kJ/kg · K.

▮ We shall need $(T_2)_{isen}$. To compute $(T_2)_{isen}$ we use the result that $(N_M)_1 = 0.1$ from Prob. 16.172. Hence, $T_1 = (T_0)(0.998)$, $T_0 = [208/(1.3)(0.287)]0.998 = 556$ K, $T_1 = 555$ K. Since $(N_M)_2 = 0.53$ from Prob. 16.172 we have $T_2/T_0 = 0.947$, $T_2 = 527$ K, $\eta = (V_2^2/2)_{act}/[V_1^2/2 + c_p(T_1 - T_2)]_{isen}$, $0.9 = (V_2^2/2)_{act}/[47.3^2/2 + (1005)(555 - 527)]$, $(V_2)_{act} = 229$ m/s. We need $(\rho_2)_{act}$ now. For this we must evaluate $(T_2)_{act}$. If there were no "reheating" then $V_2^2/2 = 30\,259$ instead of 27 233. Thus we can expect an increase in exit temperature due to reheating of $\Delta T_2 = (30\,259 - 27\,233)/(1005) = 3.01$ K. Hence $(T_2)_{act} = 527 + 3 = 530$ K. Using the equation of state: $\rho_2 = 172\,500/(287)(530) = 1.134$ kg/m^3. From continuity we get $(1.3)(0.1)(47.3) = (1.134)(229)(A_2)$, $A_2 = 0.0237$ m^2.

16.174 A rocket has an area ratio A_{exit}/A^* of 3.5 for the nozzle, and the stagnation pressure is 5200 kPa abs. Fuel burns at the rate of 46 kg/s and the stagnation temperature is 2872 °C. Calculate the throat area and exit area. Take $R = 0.355$ kJ/kg · K and $k = 1.4$.

▮ Using Table A-16 we get for $A_e/A^* = 3.5$, $(N_M)_2 = 2.80$, $T_2/T_0 = 0.389$, $p_2/p_0 = 0.037$. Hence $T_2 = (0.389)(2872 + 273) = 1223$ K, $p_2 = (0.037)(5200) = 192$ kPa. Then

$$\rho_2 = p_2/RT_2 = 192/(0.355)(1223) = 0.442 \text{ kg/m}^3$$
$$V_2 = \sqrt{kRT_2}(N_M)_2 = \sqrt{(1.4)(355)(1223)}(2.80) = 2183 \text{ m/s}$$

By continuity, $A_2 = 46/(0.442)(2183) = 0.0477$ m^2. Finally, $A^* = 0.0477/3.5 = 0.01363$ m^2.

16.175 A nozzle expands air from a pressure $p_0 = 180$ lb/in^2 abs and temperature $T_0 = 100$ °F to a pressure of 18 lb/in^2 abs. If the mass flow w is 50 lbm/s, what is the throat area and the exit area? Take $k = 1.4$ and $R = 53.3$ ft-lb/(lbm-°R).

▮ $p_e/p_0 = \frac{18}{180} = 0.1$. From Table A-16:

$$(N_M)_e = 2.16 \qquad T_e/T_0 = 0.517 \qquad T_e = (560)(0.517) = 289.5 \text{ °R}$$
$$\rho_e/\rho_0 = 0.192 \qquad A_e/A^* = 1.94$$
$$pv = RT \qquad \rho_e = p_e/RT_e = (18)(144)/[(53.3)(32.2)(289.5)] = 5.217 \times 10^{-3} \text{ slug/ft}^3$$
$$V_e = (N_M)_e c_e \qquad c_e = \sqrt{kRT_e} = \sqrt{(1.4)(53.3)(32.2)(289.5)} = 834 \text{ ft/s}$$
$$V_e = (2.16)(834) = 1801 \text{ ft/s}$$

Continuity:

$$(50/g) = \rho_e V_e A_e = (5.217 \times 10^{-3})(1801)(A_e) \qquad A_e = 0.1653 \text{ ft}^2$$
$$A^* = 0.1653/1.94 = 0.0852 \text{ ft}^2$$

16.176 Determine the throat and the exit areas of an ideal rocket motor to give a static thrust of 6.67 kN at 6.1-km altitude standard atmosphere if the chamber pressure is 1.035 MPa abs and chamber temperature is 3588 K. Find the velocity at the throat. Take $k = 1.4$ and $R = 0.355$ kJ/kg · K. Assume that exit pressure is that of surroundings.

▮ From Table A-8, $p_e = 46.583$ kPa and so $p_e/p_0 = 46.583/1035 = 0.045$. Hence, from Table A-16, $(N_M)_e = 2.67$. Also $T_e/T_0 = 0.412$, $T_e = (0.412)(3588) = 1478$ K. Hence, $V_e = (2.67)\sqrt{(1.4)(355)(1478)} = 2290$ m/s, $\rho = p/RT$, $\rho_e = 46.583/(0.355)(1478) = 0.0888$ kg/m^3. Now, by the momentum equation, using gage pressures,

$6670 = \rho_e V_e^2 A_e$, $A_e = 6670/[(0.0888)(2290)^2] = 0.0143\ \text{m}^2$. Also $A_e/A^* = 3.09$, $A^* = 0.0143/3.09 = 0.00463$. Finally we want V^*. For this we need T^*. From Table A-16: $T^*/T_0 = 0.833$, $T^* = (3588)(0.833) = 2988$ K. Hence, $V^* = \sqrt{(1.4)(355)(2988)} = 1220$ m/s.

16.177 Determine the exit area in Prob. 16.176 for a nozzle efficiency of 85 percent. Take $c_p = 1.214$ kJ/kg · K.

▮ $\eta = (V_2^2/2)_{\text{act}}/[c_p(T_1 - T_2)_{\text{isen}}]$ $\quad (V_2^2/2)_{\text{act}} = (0.85)(1214)(3588 - 1478) = 2.177 \times 10^6$ $\quad (V_2)_{\text{act}} = 2087$ m/s

Next we get $(T_2)_{\text{act}}$ from the first law of thermodynamics:

$$(V_2^2/2)_{\text{act}} = c_p[T_0 - (T_2)_{\text{act}}] \qquad 2.177 \times 10^6 = (1214)[3588 - (T_2)_{\text{act}}] \qquad (T_2)_{\text{act}} = 1795\ \text{K}$$

$$\rho = p/RT \qquad (\rho_2)_{\text{act}} = 46.583/[(0.355)(1795)] = 0.07310\ \text{kg/m}^3$$

Finally, from momentum considerations, $6670 = (\rho V_e^2 A_e)_{\text{act}}$, $A_e = 6670/[(0.07310)(2087)^2] = 0.021\ \text{m}^2$.

16.178 Figure 16-11 is a convergent-divergent nozzle attached to a chamber (tank 1) where the pressure is 100 lb/in^2 abs and the temperature is 200 °F. The area of the throat is 3 in^2 and A_1, where we happen to have a normal shock, is 4 in^2. Finally A_e is 6 in^2. What is the Mach number right after the shock wave? What is the Mach number at exit? Compute the stagnation pressure and actual pressure for the jet in tank 2. What is the stagnation temperature at exit? The fluid is air.

▮ **Data:**

$$T_0 = 660\ °\text{R} \qquad p_0 = 100\ \text{psia} \qquad A^* = 3\ \text{in}^2 \qquad A_1 = 4\ \text{in}^2 \qquad A_e = 6\ \text{in}^2$$

Find $(N_M)_1$ to right of shock: Since $A/A^* = \frac{4}{3} = 1.333$, we have from Table A-16, $(N_M)_1 = 1.69$. $(N_M)_2$ from Table A-17 is 0.644. Find N_M at exit: For $N_M = 0.644$ Table A-16 gives $A_1/A^* = 1.16$. Therefore, $A^* = 4/1.16 = 3.45\ \text{m}^2$. Now at exit: $A_e/A^* = 6/3.45 = 1.74$. From Table A-16: $(N_M)_e = 0.360$, $p_1/p_0 = 0.206$, $p_1 = (100)(0.206) = 20.6$ psia. From Table A-17: $p_2/p_1 = 3.165$, $p_2 = (3.165)(20.6) = 65.2$. From Table A-16: $p_2/p_0 = 0.759$, $p_0 = 652/0.759 = 86.0$ psia; $p_e/p_0 = 0.914$, $p_e = (86)(0.914) = 78.5$ psia, $T_0 = 200$ °F.

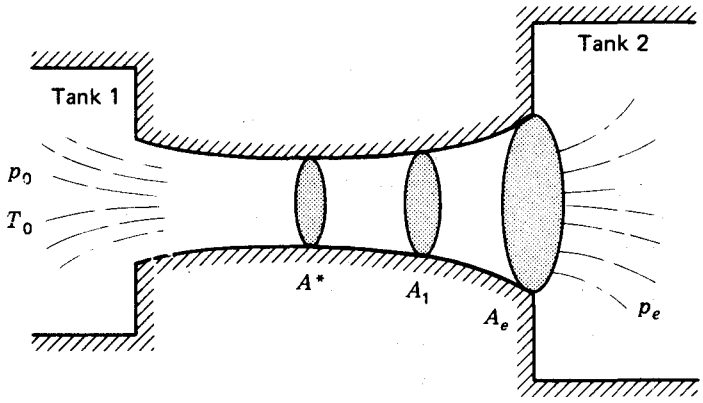


Fig. 16-11

16.179 A convergent nozzle has an exit area of 10 cm^2. It permits flow of air to proceed from a large tank in which the pressure of the air is 138 kPa abs and the temperature is 20 °C. If the ambient pressure outside the tank is 101.325 kPa, what are the velocity of the flow on leaving the nozzle and the mass flow? Neglect friction.

▮ $$p^*/p = 0.528 \qquad p^* = (138)(0.528) = 72.8\ \text{kPa}$$

Hence fluid leaves subsonically and $p_e/p_0 = 101.325/138 = 0.734$. Hence:

$$(N_M)_e = 0.68 \qquad T_e/T_0 = 0.915 \qquad T_e = 268\ \text{K}$$

$$V_e = c(N_M)_e = \sqrt{kRT_e}(N_M)_e = \sqrt{(1.4)(287)(268)}(0.68) = 223\ \text{m/s}$$

$$\rho = p/RT \qquad \rho_e = 101.325/[(0.287)(268)] = 1.317\ \text{kg/m}^3$$

$$\dot{m} = \rho VA = (1.317)(223)(0.0010) = 0.294\ \text{kg/s}$$

16.180 In Prob. 16.179, what is the ambient pressure that will permit the maximum flow through the nozzle? What are the maximum mass flow and temperature of the air leaving the nozzle? Neglect friction.

▮ Maximal pressure is 72.8 kPa, the critical pressure. At $N_M = 1$:

$$T^*/T_0 = 0.833 \qquad T^* = (293)(0.833) = 244\ \text{K}$$
$$V = \sqrt{kRT} \qquad V_e = \sqrt{(1.4)(287)(244)} = 313\ \text{m/s}$$
$$\rho = p/RT \qquad \rho_e = 72.8/[(0.287)(244)] = 1.040\ \text{kg/m}^3$$
$$\dot{m} = \rho VA = (1.040)(313)(1.0 \times 10^{-3}) = 0.326\ \text{kg/s}$$

16.181 A convergent-divergent nozzle with a throat area of 13 cm^2 and an exit area of 20 cm^2 is connected to a tank wherein air is kept at a pressure of 552 kPa abs and a temperature of 15 °C. If the nozzle is operating at design conditions, what should be the ambient pressure outside and the mass flow? What is the critical pressure? Neglect friction.

▮ For design operation we can say $A_e/A^* = 1.46$. From isentropic tables: $(N_M)_e = 1.82$, $p_e/p_0 = 0.169$, $p_e = (0.169)(552) = 93.3$ kPa. Also

$$T_e/T_0 = 0.602 \qquad T_e = (0.602)(288) = 173\ \text{K}$$
$$V_e = c_e(N_M)_e = \sqrt{kRT_e}(N_M)_e = \sqrt{(1.4)(287)(173)}(1.82) = 480\ \text{m/s} \qquad \rho = p/RT$$
$$\rho_e = 93.3/[(0.287)(173)] = 1.879\ \text{kg/m}^3 \qquad \dot{m} = \rho VA = (1.879)(480)(0.0020) = 1.80\ \text{kg/s}$$

At throat $p^*/p_0 = 0.528$, $p^* = 291.5$ kPa.

16.182 In Prob. 16.181, what is the ambient pressure at which a shock will first appear just inside the nozzle? What is the ambient pressure for the completely subsonic flow of maximum mass flow? Neglect friction.

▮ The Mach number just before the shock will be 1.82. From the normal shock tables we have just downstream of the shock $(N_M)_e = 0.612$. Also $p_e/p_1 = 3.698$. Hence $p_e = (3.698)(93.3) = 345$ kPa. For completely subsonic flow we assume isentropic flow and using $A_e/A^* = 1.46$ we have from the isentropic tables $p_e/p_0 = 0.874$. Hence $p_e = (552)(0.874) = 482.45$ kPa.

16.183 How would you determine the approximate position and strength of the shock in Fig. 16-12?

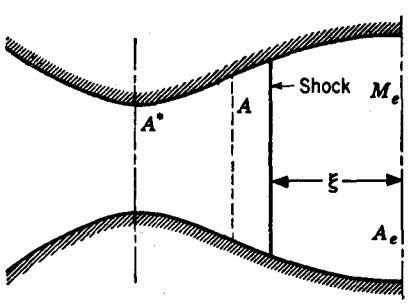


Fig. 16-12

▮ 1. First find $(N_M)_2$ as a function of ξ using isentropic data starting from exit conditions and working in.
2. Find $(N_M)_1$ as a function also of ξ using isentropic data starting from the throat and working downstream.
3. From these curves plot $(N_M)_2$ versus $(N_M)_1$.
4. Also plot $(N_M)_2$ versus $(N_M)_1$ using normal shock relations. Where they intersect gives the proper $(N_M)_1$ and $(N_M)_2$.
5. Now with either $(N_M)_1$ or $(N_M)_2$ go back to the isentropic data of step 2 or 1, respectively, to obtain ξ.

16.184 Air flows through a 2-in constriction in a 3-in diameter pipeline. The pressure and temperature of the air in the pipeline are 108 psig and 105 °F, respectively, and the pressure in the constriction is 81 psig. Barometric pressure is 14.5 psia. Find the weight flow rate of the air in the pipeline.

▮
$$G = [A_2/\sqrt{1-(p_2/p_1)^{2/k}(A_2/A_1)^2}]\sqrt{[2gk/(k-1)](p_1\gamma_1)[(p_2/p_1)^{2/k} - (p_2/p_1)^{(k+1)/k}]}$$
$$A_2 = (\pi)(2)^2/4 = 3.142\ \text{in}^2 = 0.02182\ \text{ft}^2 \qquad p_2 = 81 + 14.5 = 95.5\ \text{psia}$$
$$p_1 = 108 + 14.5 = 122.5\ \text{psia} \qquad A_1 = (\pi)(3)^2/4 = 7.069\ \text{in}^2 = 0.04909\ \text{ft}^2$$
$$\gamma = p/RT \qquad \gamma_1 = (122.5)(144)/[(53.3)(105 + 460)] = 0.5858\ \text{lb/ft}^3$$

$$G = \frac{0.02182}{\sqrt{1-(95.5/122.5)^{2/1.40}(0.02182/0.04909)^2}}\sqrt{\frac{(2)(32.2)(1.40)}{1.40-1}(122.5)(144)(0.5858)}$$
$$\times\sqrt{\left[\left(\frac{95.5}{122.5}\right)^{2/1.40}-\left(\frac{95.5}{122.5}\right)^{(1.40+1)/1.40}\right]} = 7.87\ \text{lb/s}$$

16.185 Air flows through a 1-in constriction in a 2-in-diameter pipeline. The pressure and temperature of the air in the pipeline are 100 psig and 102 °F, respectively, and the pressure in the constriction is 81 psig. Barometric pressure is 14.7 psia. Find the weight flow rate of the air in the pipeline.

▮ $$G = [A_2/\sqrt{1-(p_2/p_1)^{2/k}(A_2/A_1)^2}]\sqrt{[(2gk)/(k-1)](p_1\gamma_1)}\sqrt{[(p_2/p_1)^{2/k}-(p_2/p_1)^{(k+1)/k}]}$$

$$A_2 = (\pi)(\tfrac{1}{12})^2/4 = 0.005454\ \text{ft}^2 \qquad p_2 = 78+14.7 = 92.7\ \text{psia} \qquad p_1 = 100+14.7 = 114.7\ \text{psia}$$

$$A_1 = (\pi)(\tfrac{2}{12})^2/4 = 0.02182\ \text{ft}^2 \qquad \gamma = p/RT \qquad \gamma_1 = (114.7)(144)/[(53.3)(102+460)] = 0.5514\ \text{lb/ft}^3$$

$$G = [(0.005454)/\sqrt{1-(92.7/114.7)^{2/1.4}(0.005454/0.02182)^2}]$$
$$\times\sqrt{[(2)(32.2)(1.40)/(1.40-1)](114.7)(144)(0.5514)}$$
$$\times\sqrt{[(92.7/114.7)^{2/1.40}-(92.7/114.7)^{(1.40+1)/1.40}]} = 1.67\ \text{lb/s}$$

16.186 A normal shock wave occurs in the flow of air where $p_1 = 10$ psia, $T_1 = 40$ °F, and $V_1 = 1400$ ft/s. Find p_2, V_2, and T_2.

▮ $$p_2/p_1 = [2k(N_M)_1^2-(k-1)]/(k+1) \qquad N_M = V/c$$

$$c = \sqrt{kRT} \qquad c_1 = \sqrt{(1.40)(1716)(40+460)} = 1096\ \text{ft/s}$$

$$(N_M)_1 = \tfrac{1400}{1096} = 1.28 \qquad p_2/10 = [(2)(1.40)(1.28)^2-(1.40-1)]/(1.40+1) \qquad p_2 = 17.4\ \text{psia}$$

$$V_2/V_1 = [(k-1)(N_M)_1^2+2]/[(k+1)(N_M)_1^2] \qquad V_2/1400 = [(1.40-1)(1.28)^2+2]/[(1.40+1)(1.28)^2] \qquad V_2 = 945\ \text{ft/s}$$

$$\rho = p/RT \qquad \rho_1V_1 = \rho_2V_2 \qquad \rho_1 = (10)(144)/[(1716)(40+460)] = 0.001678\ \text{slug/ft}^3$$

$$(0.001678)(1400) = (\rho_2)(945) \qquad \rho_2 = 0.00249\ \text{slug/ft}^3$$

$$0.00249 = (17.4)(144)/(1716T_2) \qquad T_2 = 586\ °\text{R} \quad \text{or} \quad 126\ °\text{F}$$

16.187 Rework Prob. 16.186 if $p_1 = 70$ Pa abs, $T_1 = 5$ °C, and $V_1 = 425$ m/s.

▮ $$p_2/p_1 = [2k(N_M)_1^2-(k-1)]/(k+1) \qquad N_M = V/c \qquad c = \sqrt{kRT} \qquad c_1 = \sqrt{(1.40)(287)(5+273)} = 334\ \text{m/s}$$

$$(N_M)_1 = \tfrac{425}{334} = 1.27 \qquad p_2/70 = [(2)(1.40)(1.27)^2-(1.40-1)]/(1.40+1) \qquad p_2 = 120\ \text{Pa}$$

$$V_2/V_1 = [(k-1)(N_M)_1^2+2]/[(k+1)(N_M)_1^2] \qquad V_2/425 = [(1.40-1)(1.27)^2+2]/[(1.40+1)(1.27)^2]$$

$$V_2 = 290\ \text{m/s}$$

$$\rho = p/RT \qquad \rho_1V_1 = \rho_2V_2 \qquad \rho_1 = 70/[(287)(5+273)] = 0.0008773\ \text{kg/m}^3$$

$$(0.0008773)(425) = (\rho_2)(290) \qquad \rho_2 = 0.00129\ \text{kg/m}^3 \qquad 0.00129 = 120/(287T_2) \qquad T_2 = 324\ \text{K} \quad \text{or} \quad 51\ °\text{C}$$

16.188 The pressure, velocity, and temperature just upstream of a normal shock wave in air are 12 psia, 2300 fps, and 24 °F. Determine the pressure, velocity, and temperature just downstream of the wave.

▮ $$p_2 = p_1\{[2k(N_M)_1^2-(k-1)]/(k+1)\} \qquad N_M = V/c \qquad c_1 = [kRT]^{1/2} = [1.4(1715)484]^{1/2} = 1078\ \text{fps}$$

$$(N_M)_1 = \tfrac{2300}{1078} = 2.13 \qquad p_2 = (12)\{[2(1.4)(2.13)^2-0.4]/2.4\} = 61.5\ \text{psia} \qquad p_1-p_2 = \gamma_1V_1/g(V_2-V_1)$$

$$V_2 = V_1+[(p_1-p_2)/\rho_1V_1] = 2300+\{[(12-61.5)144]/[(0.0670/32.2)(2300)]\} = 811\ \text{fps}$$

$$V_2^2-V_1^2 = [2k/(k-1)]\{[p_1/(\gamma_1/g)]-(p_2/\rho_2)\}$$

$$\gamma_1 = gp_1/RT_1 = (32.2)(12)(144)/[(1715)(484)] = 0.0670\ \text{pcf}$$

$$811^2-2300^2 = [2(1.4)/0.4]\{[(12)(144)/(0.0670/32.2)]-[(61.5)(144)/\rho_2]\}$$

$$\rho_2 = 0.00593\ \text{slug/ft}^3 = p_2/RT_2 \qquad T_2 = (61.5)(144)/[(1715)(0.00593)] = 871\ °\text{R} = 411\ °\text{F}$$

16.189 Just downstream of a normal shock wave the pressure, velocity, and temperature are 52 psia, 398 fps, and 115 °F. Compute the Mach number upstream of the shock wave. Consider air and carbon dioxide.

▮ $$(N_M)_2^2 = [2+(k-1)(N_M)_1^2]/[2k(N_M)_1^2-(k-1)]$$

For air:

$$N_M = V/c \qquad c = \sqrt{kRT} \qquad c_2 = \sqrt{(1.40)(1716)(115+460)} = 1175\ \text{ft/s}$$

$$(N_M)_2 = \tfrac{398}{1175} = 0.339 \qquad 0.339^2 = [2 + (1.40-1)(N_M)_1^2]/[(2)(1.40)(N_M)_1^2 - (1.40-1)]$$
$$(N_M)_1^2 = -26.2 \qquad \text{(impossible)}$$

For carbon dioxide:

$$c_2 = \sqrt{(1.28)(1123)(115+460)} = 909 \text{ ft/s} \qquad (N_M)_2 = \tfrac{398}{909} = 0.438$$
$$0.438^2 = [2 + (1.28-1)(N_M)_1^2]/[(2)(1.28)(N_M)_1^2 - (1.28-1)] \qquad (N_M)_1 = 3.12$$

16.190 A schlieren photograph of a bullet shows a Mach angle of 28°. The air is at a pressure of 14 psia and 60 °F. Find the approximate speed of the bullet.

▮ $N_M = V/c = 1/\sin\beta = 1/\sin 28° = 2.13 \qquad c = \sqrt{kRT} = \sqrt{(1.40)(1716)(60+460)} = 1118 \text{ ft/s}$

$$2.13 = V/1118 \qquad V = 2381 \text{ ft/s}$$

16.191 If a normal shock wave occurs in the flow of helium, $p_1 = 1$ psia, $T_1 = 40$ °F, $V_1 = 4500$ ft/s, find p_2, ρ_2, V_2, and T_2.

▮ $p_2 = [1/(k+1)][2\rho_1 V_1^2 - (k-1)p_1] \qquad \rho_1 = p_1/RT_1 = 1(144)/\{386[32.17(460+40)]\} = 0.0000232 \text{ slug/ft}^3$

$$p_2 = [1/(1.66+1)][2(0.0000232)(4500^2) - (1.66-1)(144)(1)] = 317 \text{ lb/ft}^2 \text{ abs} \qquad p_1 + \rho_1 V_1^2 = p_2 + \rho_1 V_1 V_2$$
$$V_2 = V_1 - [(p_2 - p_1)/\rho_1 V_1] = 4500 - [(317-144)(4500)(0.0000232)] = 2843 \text{ ft/s}$$
$$\rho_2 = \rho_1(V_1/V_2) = (0.0000232)(\tfrac{4500}{2843}) = 0.0000367 \text{ slug/ft}^3$$
$$T_2 = p_2/\rho_2 R = 317/\{(0.0000367)[(386)(32.17)]\} = 696 \text{ °R} \quad \text{or} \quad 236 \text{ °F}$$

16.192 A shock wave occurs in a duct carrying air where the upstream Mach number is 2.0 and upstream temperature and pressure are 17 °C and 21 kPa abs. Calculate the Mach number, pressure, temperature, and velocity after the shock wave.

▮ For $(N_M)_1 = 2.0$, from Table A-17, $(N_M)_2 = 0.577$, $p_2/p_1 = 4.500$, and $T_2/T_1 = 1.688$.

$$p_2 = (4.500)(21) = 94.5 \text{ kPa} \qquad T_2 = (1.688)(17+273) = 490 \text{ K} \quad \text{or} \quad 217 \text{ °C}$$
$$c_2 = \sqrt{kRT} = \sqrt{(1.40)(287)(490)} = 444 \text{ m/s} \qquad V_2 = c_2(N_M)_2 = (444)(0.577) = 256 \text{ m/s}$$

16.193 Show that entropy has increased across the shock wave of Prob. 16.192.

▮ $\Delta s = c_v \ln[(T_2/T_1)^k (p_2/p_1)^{1-k}]$, and $[490/(17+273)]^{1.40}(94.5/21)^{1-1.40}$ is slightly larger than 1.

16.194 Conditions immediately before a normal shock wave in airflow are $p_u = 6$ psia, $T_u = 139$ °F, and $V_u = 1860$ ft/s. Find $(N_M)_u$, $(N_M)_d$, p_d, and T_d, where the subscript d refers to conditions just downstream from the shock wave.

▮ $N_M = V/c \qquad c = \sqrt{kRT} = \sqrt{(1.40)(1716)(139+460)} = 1200 \text{ ft/s} \qquad (N_M)_u = \tfrac{1860}{1200} = 1.55$

From Table A-17, $(N_M)_d = 0.683$, $p_d/p_u = 2.65$, and $T_d/T_u = 1.356$. $p_d = (2.65)(6) = 15.9$ psia, $T_d = (1.356)(139+460) = 812$ °R, or 352 °F.

16.195 For $A = 0.20$ ft² in Prob. 16.194, calculate the entropy increase across the shock wave in Btu/s and degrees Rankine.

▮ $\Delta s = (0.171)\ln[(T_2/T_1)^k(p_2/p_1)^{1-k}] = (0.171)\ln\{[812/(139+460)]^{1.40}(15.9/6)^{1-1.40}\} = 0.00617 \text{ Btu/(lbm-°R)}$

$$\Delta s = \dot{m}\,\Delta s$$
$$\dot{m} = \rho_u A V_u = (p_u/RT_u)AV_u = [(6)(144)/(1716)(139+460)](0.20)(1860) = 0.3127 \text{ slug/s}$$
$$\Delta s = (0.3127)(32.17)(0.00617) = 0.0621 \text{ Btu/(s-°R)}$$

16.196 An explosion in air, $\gamma = 1.4$, creates a spherical shock wave propagating radially into still air at standard conditions. At the instant shown in Fig. 16-13, the pressure just inside the shock is 200 lb/in² abs. Estimate **(*a*)** the shock speed C and **(*b*)** the air velocity V just inside the shock.

▮ **(*a*)** In spite of the spherical geometry the flow across the shock moves normal to the spherical wavefront; hence the normal-shock relations apply. Fixing our control volume to the moving shock, we find that the proper conditions to use are $C = V_1$, $p_1 = 14.7$ lb/in² abs, $T_1 = 520$ °R, $V = V_1 - V_2$, $p_2 = 200$ lb/in² abs. The speed of sound outside the shock is $c_1 \approx 49T_1^{1/2} = 1117$ ft/s. We can find $(N_M)_1$ from the known pressure ratio across the shock: $p_2/p_1 = 200/14.7 = 13.61 = [1/(k+1)][2k(N_M)_1^2 - (k-1)]$, $13.61 = (1/2.4)[2.8(N_M)_1^2 - 0.4]$ or $(N_M)_1 = 3.436$. Then, by definition of the Mach number, $C = V_1 = (N_M)_1 c_1 = 3.436(1117) = 3840$ ft/s.

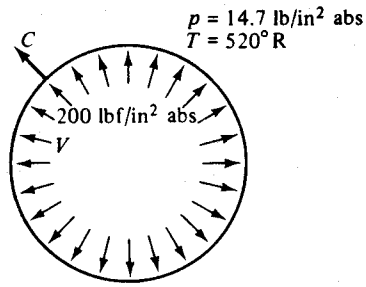


Fig. 16-13

(*b*)
$$T_2/T_1 = [2 + (k-1)(N_M)_1^2]\{[2k(N_M)_1^2 - (k-1)]/(k+1)^2(N_M)_1^2\}$$
$$= [2 + (0.4)(3.436)^2]\{[(2.8)(3.436)^2 - 0.4]/(2.4)^2(3.436)^2\} = 3.228$$
$$T_2 = 3.228T_1 = 3.228(520) = 1679\ °\text{R}$$

At such a high temperature we should account for non-perfect-gas effects or at least use the gas tables, but we won't. Here just estimate from the perfect-gas energy equation that $V_2^2 = 2c_p(T_1 - T_2) + V_1^2 = 2(6010)(520 - 1679) + (3840)^2 = 815\,500$ or $V_2 \approx 903$ ft/s. Notice that we did this without bothering to compute $(N_M)_2$, which equals 0.454, or $c_2 \approx 49T_2^{1/2} = 2000$ ft/s.

Finally, the air velocity behind the shock is $V = V_1 - V_2 = 3840 - 903 \approx 2940$ ft/s. Thus a powerful explosion creates a brief but intense blast wind as it passes.

16.197 Air from a reservoir at 22 °C and 498 kPa flows through a duct and forms a normal shock downstream of a throat of area 12 cm². If the stagnation pressure downstream of this shock exactly equals the throat pressure, what is the area where the shock wave stands?

▮ $A/A^* = (1/N_M)[(1 + 0.2N_M^2)^3/1.728]$ $\quad p_1^* = (p_0)_1/(1 + 0.2N_M^2)^{3.5} = 498/[1 + (0.2)(1)^2]^{3.5} = 263\text{ kPa} = (p_0)_2$

$$(p_0)_2/(p_0)_1 = \tfrac{263}{498} = 0.528$$

From Table A-17, $(N_M)_1 = 2.43$, $A_1/12 = (1/2.43)\{[1 + (0.2)(2.43)^2]^3/1.728\}$, $A_1 = 29.65\text{ cm}^2$.

16.198 Air passes through a normal shock with upstream conditions $V_1 = 790$ m/s, $p_1 = 110$ kPa, and $T_1 = 306$ K. What are the downstream conditions V_2 and p_2?

▮
$$p_2/p_1 = [2k(N_M)_1^2 - (k-1)]/(k+1) \qquad N_M = V/c \qquad c = \sqrt{kRT}$$
$$c_1 = \sqrt{(1.40)(287)(306)} = 351\text{ m/s} \qquad (N_M)_1 = \tfrac{790}{351} = 2.251$$
$$p_2/110 = [(2)(1.40)(2.251)^2 - (1.40-1)]/(1.40+1) \qquad p_2 = 632\text{ kPa}$$
$$V_2/V_1 = [(k-1)(N_M)_1^2 + 2]/(k+1)(N_M)_1^2 \qquad V_2/790 = [(1.40-1)(2.251)^2 + 2]/(1.40+1)(2.251)^2$$
$$V_2 = 262\text{ m/s}$$

16.199 Repeat Prob. 16.197 if the *static* pressure downstream of the shock exactly equals the throat pressure. What is the area where the shock stands?

▮
$$p_2 = p_{\text{throat}} = p_1^* = 263\text{ kPa} \qquad \text{(from Prob. 16.197)}$$
$$p_1 = (p_0)_1/(1 + 0.2N_M^2)^{3.5} = 498/(1 + 0.2N_M^2)^{3.5}$$
$$p_2/p_1 = [(k+1)(N_M)_1^2 - (k-1)]/(k+1) = [2.80(N_M)_1^2 - 0.40]/2.40$$

By trial and error, $(N_M)_1 = 2.15$, $p_1 = 50.3$ kPa, $p_2 = 263$ kPa, $A/A_1^* = 1.919$, $A_1 = (12)(1.919) = 23.0\text{ cm}^2$.

16.200 An atomic explosion propagates into still air at 14.7 lb/in² abs and 515 °R. The pressure just inside the shock is 4950 lb/in² abs. Assuming $k = 1.4$, what is the speed C of the shock and the velocity V just inside the shock?

▮ $C = (N_M)_1(c_1)$ $\quad p_2/p_1 = [2k(N_M)_1^2 - (k-1)]/(k+1)$ $\quad 4950/14.7 = [(2)(1.4)(N_M)_1^2 - (1.4-1)]/(1.4+1)$

$(N_M)_1 = 17.00$ $\quad c = \sqrt{kRT}$ $\quad c_1 = \sqrt{(1.4)(1716)(515)} = 1112$ ft/s $\quad C = (17.00)(1112) = 18\,904\text{ ft/s} = V_1$

$$V_2/V_1 = [(k-1)(N_M)_1^2 + 2]/(k+1)(N_M)_1^2 \qquad V_2/18\,904 = [(1.4-1)(17.00)^2 + 2]/(1.4+1)(17.00)^2 = 0.1696$$
$$V_2 = 3206\text{ ft/s} \qquad V = C - V_2 = 18\,904 - 3206 = 15\,698\text{ ft/s}$$

16.201 The normal-shock wave from an explosion propagates at 1450 m/s into still air at 21 °C and 101 kPa. What are the pressure, velocity, and temperature just inside the shock?

▮ $p_2/p_1 = [2k(N_M)_1^2 - (k-1)]/(k+1)$ $\quad N_M = V/c$ $\quad c = \sqrt{kRT} = \sqrt{(1.4)(287)(21+273)} = 344$ m/s

$(N_M)_1 = \frac{1450}{344} = 4.22$ $\quad p_2/101 = [(2)(1.4)(4.22)^2 - (1.4-1)]/(1.4+1)$ $\quad p_2 = 208.2$ kPa

$$T_2/T_1 = [2 + (k-1)(N_M)_1^2]\{[(2k)(N_M)_1^2 - (k-1)]/(k+1)^2(N_M)_1^2\}$$

$$T_2/(21+273) = [2 + (1.4-1)(4.22)^2]\{[(2)(1.4)(4.22)^2 - (1.4-1)]/(1.4+1)^2(4.22)^2\}$$

$T_2 = 1293$ K $\quad V_2/V_1 = [(k-1)(N_M)_1^2 + 2]/(k+1)(N_M)_1^2$ $\quad V_2/1450 = [(1.4-1)(4.22)^2 + 2]/(1.4+1)(4.22)^2$

$V_2 = 310$ m/s $\quad V_{\text{inside shock}} = C - V_2 = 1450 - 310 = 1140$ m/s

16.202 Air is moving at Mach number 3 in a duct and undergoes a normal shock. If the undisturbed pressure ahead of the shock is 69 kPa abs, what is the increase in pressure after the shock? What is the loss in stagnation pressure across the shock?

▮ From Table A-17, $(N_M)_2 = 0.475$, $p_2/p_1 = 10.333$, $(p_0)_2/(p_0)_1 = 0.328$. Hence $p_2 = (10.333)(69) = 712.98$ kPa. Now go to an isentropic table for $(p_0)_1$: $p_1/(p_0)_1 = 0.027$, $(p_0)_1 = 69/0.027 = 2560$ kPa. Hence $(p_0)_2 = (0.328)(2560) = 838$ kPa, and $\Delta p_0 = 2560 - 838 = 1720$ kPa.

16.203 An airplane having a diffuser designed for subsonic flight has a normal shock attached to the edge of the diffuser when the plane is flying at a certain Mach number. If at the exit of the diffuser the Mach number is 0.3, what must the flight Mach number be for the plane, assuming isentropic diffusion behind the shock? The inlet area is 0.24 m^2 and the exit area is 0.38 m^2.

▮ From Table A-16, $A_e/A^* = 2.04$, $A^* = 0.38/2.04 = 0.186$ m^2. At inlet, $A_1 = 0.24$ m^2. Hence $A_1/A^* = 0.24/0.186 = 1.290$. From isentropic tables, $(N_M)_2 = 0.527$. This is the Mach number behind the inlet shock. From Table A-17, $(N_M)_1 = 2.360$.

16.204 A normal shock forms ahead of the diffuser of a turbojet plane flying at Mach number 1.2. If the plane is flying at 35 000-ft altitude in standard atmosphere, what is the entering Mach number for the diffuser and the stagnation pressure.

▮ $(N_M)_1 = 1.2$. Hence from *normal shock* tables (A-17), $(N_M)_2 = 0.842$.

$$(p_0)_2/(p_0)_1 = 0.993 \qquad (1)$$

Also, from *isentropic* tables (A-16) we have for $(N_M)_1 = 1.2$: $p_1/(p_0)_1 = 0.412$. Hence, $(p_0)_1 = 0.498/0.412 = 1210$ psf, where we have used the standard atmosphere table. Now using Eq. (*1*) we get $(p_0)_2 = (0.993)(1210) = 1200$ psf.

16.205 Consider a supersonic flow of air through a stationary duct wherein a stationary shock is present. The Mach number ahead of the shock is 2.1 and the pressure and temperature are 101.3 kPa abs and 37 °C. What is the velocity of propagation of the shock relative to the air ahead of the shock?

▮ $(N_M)_1 = V_1/c_1 = V_1/\sqrt{kRT_1}$ $\quad 2.1 = V_1/\sqrt{(1.40)(287)(37+273)}$ $\quad V_1 = 741$ m/s

16.206 A jet plane is diving at constant supersonic speed. There is a curved shock wave ahead of it. A static pressure gage near the nose of the plane measures 37.5 kPa abs. The ambient pressure and temperature of the atmosphere are 12 kPa and 254 K. What are the flight Mach number for the plane and its speed if one assumes that in front of the static pressure gage the shock wave is plane?

▮ Consider the reference of observation is from the plane. The ratio of the pressures across the shock wave is $p_2/p_1 = 37.5/12 = 3.125$, $(N_M)_1 = 1.68$. From Tables A-17 the plane is moving at a Mach number of 1.68. The velocity is determined next. The speed of sound in the undisturbed region ahead of the shock is $c = \sqrt{kRT} = \sqrt{(1.4)(287)(254)} = 319.5$ m/s. Therefore, $V = (1.68)(319.5) = 537$ m/s, which is also the velocity of the plane.

16.207 A duct having a square cross section 0.300 m on a side has 25 kg/s of air flowing in it. The air, originally in a chamber where the temperature is 90 °C, has been insulated by the duct walls against heat transfer to the outside. The duct is operating in a choked condition. If the duct has a relative roughness of 0.002, determine the Mach number at a position 6 m from the exit of the duct.

▮ We may solve for N_M at this position by employing $[(1/M^2) - 1] + [(k+1)/2] \ln [(k+1)M^2/2\{1 + [(k-1)/2]M^2\} = (fk/D_H)L$. To do this, we estimate f to be 0.024, from the Moody diagram (Fig. A-5). The hydraulic diameter D_H is $D_H = 4A/p_w = (4)(0.3^2)/(4)(0.3) = 0.3$ m. Using $k = 1.4$, we then have $[(1/N_M^2) - 1] + 1.2 \ln [1.2N_M^2/(1 + 0.2N_M^2)] = [(0.024)(1.4)/0.3](6) = 0.672$. Solving by trial and error, we get $N_M = 0.6$.

To check our friction factor, we must compute other conditions at this section of the pipe. The temperature T is determined from Table A-16 for $N_M = 0.6$. Thus $T/T_0 = 0.933$. Therefore $T = 339$ K $= 66$ °C. We may determine the viscosity of air at that temperature to be 2.15×10^{-5} Pa · s by making use of Fig. A-1, since viscosity does not depend greatly on the pressure. Noting that $G = \rho V = \dot{m}/A = 25/0.300^2 = 277.8$ kg/m² · s, we have $N_R = GD_H/\mu = (277.8)(0.300)/(2.15 \times 10^{-5}) = 3.88 \times 10^6$. Returning to the Moody diagram, we see that our choice of f is close enough not to require further computation and the desired Mach number is 0.6.

16.208 A constant-area duct having a circular cross-sectional area of 0.19 m² is operating in a choked condition. It is highly insulated against heat transfer, and the inside surface has a relative roughness of 0.002. At a distance 9 m from the end of the duct, what is the Mach number of a 36 kg/s airflow? The stagnation temperature is 96 °C. Perform one iteration.

▮
$$[(1/N_M^2) - 1] + [(k+1)/2] \ln\left((k+1)N_M^2/2\{1 + [(k-1)/2]N_M^2\}\right) = (fk/D_H)L$$
$$\pi D^2/4 = 0.19 \text{ m}^2 \qquad D = 0.4918 \text{ m}$$

Try $f = 0.023$:

$$[(1/N_M^2) - 1] + [(1.40+1)/2] \ln\left(1.40+1)N_M^2/(2)\{1 + [(1.40-1)/2]N_M^2\}\right) = [(0.023)(1.40)/0.4918](9)$$
$$N_M = 0.61 \qquad N_R = \rho DV/\mu \qquad \dot{m} = \rho AV \qquad V = cN_M \qquad c = \sqrt{kRT}$$

From Table A-16, $T/T_0 = 0.931$:

$$T = (0.931)(96+273) = 344 \text{ K} \qquad c = \sqrt{(1.40)(287)(344)} = 372 \text{ m/s} \qquad V = (372)(0.61) = 227 \text{ m/s}$$
$$36 = (\rho)(0.19)(227) \qquad \rho = 0.835 \text{ kg/m}^3 \qquad N_R = (0.835)(0.4918)(227)/(2.2 \times 10^{-5}) = 4.24 \times 10^6$$

From Fig. A-5 with $\epsilon/D = 0.002$, $f = 0.023$. Since this value is the same as the guessed value of f, $N_M = 0.61$.

16.209 In Prob. 16.208, determine where Mach number 0.5 is assumed, and ascertain the pressure and temperature of the flow there.

▮ From Fig. A-72, $(fk/H)L = 1.4$, $[(0.023)(1.40)/0.4918]L = 1.4$, $L = 21.4$ m. From Table A-16, $T/T_0 = 0.952$:

$$T = (0.952)(96+273) = 351 \text{ K} \quad \text{or} \quad 77 \text{ °C} \qquad p = (G/N_M)\left(RT_0/k\{1 + [(k-1)/2]N_M^2\}\right)^{1/2}$$
$$p = [(36/0.19)/0.5]\left((287)(96+273)/(1.40)\{1 + [(1.40-1)/2](0.5)^2\}\right)^{1/2} = 101.7 \text{ kPa}$$

16.210 A constant-area duct is operating in the choked condition. The cross section is rectangular, having sides 6 by 4 ft, and the surface has a relative roughness of 0.0001. At 22 ft from the end of the duct, the pressure is 18 lb/in² abs. If there is no heat transferred through the walls, determine the Mach number and Reynolds number at this section of air flow. The exit pressure is that of ambient pressure of the surroundings, which is 14.7 lb/in².

▮ We first compute H.

$$H = 4A/p_w = (4)(24)/20 = 4.8 \text{ ft} \qquad p = (G/N_M)\left(RT_0/k\{1 + [(k-1)/2]N_M^2\}\right)^{1/2}$$
$$(14.7)(144) = (G/1)\{1716T_0/(1.40)[1 + (0.4/2)(1)^2]\}^{1/2}$$
$$GT_0^{1/2} = 66.2 \qquad (1)$$

At position 20 ft from end we get $(18)(144) = (G/N_M)[1716T_0/1.4(1 + 0.2N_M^2)]^{1/2}$:

$$74.0 = (G/N_M)[T_0/(1 + 0.2N_M^2)]^{1/2} \qquad (2)$$

We thus have two unknowns if we consider $GT_0^{1/2}$ and N_M as unknowns. Substituting for $GT_0^{1/2}$ from Eq. (*1*) into Eq. (*2*) we get $74.0 = 66.2/N_M(1 + 0.2N_M^2)^{1/2}$. Solve by trial and error to get $N_M = 0.838$. From Fig. A-72 we get $(fk/H)L = 0.08$, $f = (0.08)(4.8)/(1.4)(22) = 0.0125$. Now using $f = 0.0125$ and $\epsilon/D = 0.0001$, we get from the Moody diagram (Fig. A-5): $N_R = 3.5 \times 10^6$.

16.211 Determine the maximum length of 50-mm-ID pipe, $f = 0.025$ for flow of air, when the Mach number at the entrance to the pipe is 0.35.

▮
$$fL_{\max}/D = \tfrac{5}{7}\{[1/(N_M)_0^2 - 1]\} + \tfrac{6}{7}\ln\{6(N_M)_0^2/[(N_M)_0^2 + 5]\} \qquad k = 1.4$$
$$(0.025/0.05)L_{\max} = \tfrac{5}{7}[(1/0.35^2) - 1] + \tfrac{6}{7}\ln[6(0.35^2)/(0.35^2 + 5)]$$

from which $L_{\max} = 6.90$ m.

16.212 A 4.0-in-ID pipe, $f = 0.020$, has air at 14.7 psia and at $T = 60\,°F$ flowing at the upstream end with Mach number 3.0. Determine L_{max}, p^*, V^*, T^*, and values of p_0', V_0', T_0', and L at $N_M = 2.0$.

▮ $fL_{max}/D = \frac{5}{7}\{[1/(N_M)_0^2] - 1\} + \frac{6}{7}\ln\{6(N_M)_0^2/[(N_M)_0^2 + 5]\}$ $\quad (0.02/0.333)L_{max} = \frac{5}{7}(\frac{1}{9} - 1) + \frac{6}{7}\ln[6(3^2)/(3^2+5)]$

from which $L_{max} = 8.69$ ft. If the flow originated at $N_M = 2$, the length L_{max} is given by the same equation: $(0.02/0.333)L_{max} = \frac{5}{7}(\frac{1}{4} - 1) + \frac{6}{7}\ln[6(2^2)/(2^2+5)]$ from which $L_{max} = 5.08$ ft.

Hence, the length from the upstream section at $N_M = 3$ to the section where $N_M = 2$ is $8.69 - 5.08 = 3.61$ ft. The velocity at the entrance is $V = \sqrt{kRTN_M} = \sqrt{1.4(53.3)[32.17(460+60)](3)} = 3352$ ft/s.

$$p^*/p_1 = (N_M)_0\sqrt{[(k-1)(N_M)_0^2 + 2]/(k+1)} \qquad V^*/V_0 = [1/(N_M)_0]\sqrt{[(k-1)(N_M)_0^2+2]/(k+1)}$$

$$T^*/T_0 = [(k-1)(N_M)_0^2 + 2]/(k+1) \qquad p^*/14.7 = 3\sqrt{[0.4(3^2)+2]/2.4} = 4.583$$

$$V^*/3352 = \tfrac{1}{3}\sqrt{[0.4(3^2)+2]/2.4} = 0.509 \qquad T^*/520 = [0.4(3^2)+2]/2.4 = \tfrac{7}{3}$$

So $p^* = 67.4$ psia, $V^* = 1707$ ft/s, $T^* = 1213\,°R$. For $N_M = 2$ the same equations are now solved for p_0', V_0', and T_0': $67.4/p_0' = 2\sqrt{[0.4(2^2)+2]/2.4} = 2.45$, $1707/V_0' = \frac{1}{2}\sqrt{[0.4(2^2)+2]/2.4} = 0.6124$, $1213/T_0' = [0.4(2^2)+2]/2.4 = \frac{3}{2}$. So $p_0' = 27.5$ psia, $V_0' = 2787$ ft/s, and $T_0' = 809\,°R$.

16.213 What length of 120-mm-diameter insulated duct, $f = 0.020$, is needed when oxygen enters at $N_M = 3.0$ and leaves at $N_M = 2.0$?

▮ $$fL/D = \tfrac{5}{7}\{[1/(N_M)_0^2] - [1/(N_M)^2]\} + \tfrac{6}{7}\ln\left([(N_M)_0/N_M]^2\{[(N_M)^2+5]/[(N_M)_0^2+5]\}\right)$$

$$0.020L/0.120 = \tfrac{5}{7}[(1/3.0^2) - (1/2.0^2)] + \tfrac{6}{7}\ln\{(3.0/2.0)^2[(2.0^2+5)/(3.0^2+5)]\} \qquad L = 1.303\text{ m}$$

16.214 Air enters an insulated pipe at $N_M = 0.3$ and leaves at $N_M = 0.6$. What portion of the duct length is required for the flow to occur at $N_M = 0.5$?

▮ $$\frac{L_1}{L_2} = \frac{5\{[1(N_M)_0^2] - [1/(N_M)_1^2]\} + 6\ln\{[(N_M)_0/(N_M)_1]^2\{[(N_M)_1^2+5]/[(N_M)_0^2+5]\}}{5\{[1/(N_M)_0^2] - [1/(N_M)_2^2]\} + 6\ln\{[(N_M)_0/(N_M)_2]^2\{[(N_M)_2^2+5]/[(N_M)_0^2+5]\}\}}$$

$$= \frac{5[(1/0.3^2) - (1/0.5^2)] + 6\ln\{(0.3/0.5)^2[(0.5^2+5)/(0.3^2+5)]\}}{5[(1/0.3^2) - (1/0.6^2)] + 6\ln\{(0.3/0.6)^2[(0.6^2+5)/(0.3^2+5)]\}} = 0.885$$

Thus 88.5 percent of the duct length is required.

16.215 Determine the maximum length, without choking, for the adiabatic flow of air in a 100-mm-diameter duct, $f = 0.0227$, when upstream conditions are $T = 60\,°C$, $V = 200$ m/s, and $p = 2$ atm.

▮ $$fL_{max}/D = \tfrac{5}{7}\{[1/(N_M)_0^2] - 1\} + \tfrac{6}{7}\ln\{6(N_M)_0^2/[(N_M)_0^2+5]\}$$

$$N_M = V/c = V/\sqrt{kRT} \qquad (N_M)_0 = 200/\sqrt{(1.40)(287)(50+273)} = 0.5552$$

$$0.0227L_{max}/0.100 = \tfrac{5}{7}[(1/0.5552^2) - 1] + \tfrac{6}{7}\ln[(6)(0.5552)^2/(0.5552^2+5)] \qquad L_{max} = 3.077\text{ m}$$

16.216 What minimum size insulated duct is required to transport nitrogen 980 ft? The upstream temperature is 75 °F, and the velocity there is 210 fps ($f = 0.021$).

▮ $$fL_{max}/D = \tfrac{5}{7}\{[1/(N_M)_0^2] - 1\} + \tfrac{6}{7}\ln\{6(N_M)_0^2/[(N_M)_0^2+5]\}$$

$$N_M = V/c = V/\sqrt{kRT} \qquad (N_M)_0 = 210/\sqrt{(1.40)(1776)(75+460)} = 0.1821$$

$$(0.021)(980)/D = \tfrac{5}{7}[(1/0.1821^2) - 1] + \tfrac{6}{7}\ln[(6)(0.1821)^2/(0.1821^2+5)] \qquad D = 1.140\text{ ft}$$

16.217 Find the upstream and downstream pressures in Prob. 16.216 for 3-lbm/s flow.

▮ $$\rho = p/RT \qquad \dot{m} = \rho AV \qquad 3/32.2 = (\rho_u)[(\pi)(1.140)^2/4](210) \qquad \rho_u = 0.0004347\text{ slug/ft}^3$$

$$0.0004347 = p_u/[(1776)(75+460)] \qquad p_u = 413.0\text{ psfa} \quad \text{or} \quad 2.87\text{ psia}$$

$$p^*/p_1 = (N_M)_0\sqrt{[(k-1)(N_M)_0^2+2]/(k+1)} \qquad p^*/413.0 = (0.1821)\sqrt{[(1.40-1)(0.1821)^2+2]/(1.40+1)}$$

$$p^* = p_d = 68.9\text{ psfa} \quad \text{or} \quad 0.478\text{ psia}$$

16.218 What is the maximum mass rate of flow of air from a reservoir, $T = 20\,°C$, through 6 m of insulated 25-mm-diameter pipe, $f = 0.020$, discharging to a standard atmosphere?

▮ $$\dot{m} = \rho_0 AV_0 \qquad \rho = p/RT \qquad p^*/p_0 = (N_M)_0\sqrt{[(k-1)(N_M)_0^2+2]/(k+1)}$$

$$fL_{max}/D = \tfrac{5}{7}\{[1/(N_M)_0^2] - 1\} + \tfrac{6}{7}\ln\{6(N_M)_0^2/[(N_M)_0^2+5]\}$$

$$(0.020)(6)/0.025 = \tfrac{5}{7}\{[1/(N_M)_0^2] - 1\} + \tfrac{6}{7}\ln\{6(N_M)_0^2/[(N_M)_0^2 + 5]\} \qquad (N_M)_0 = 0.311$$

$$101.3/p_0 = 0.311\sqrt{[(1.40)(0.311)^2 + 2]/(1.40 + 1)} \qquad p_0 = 353 \text{ kPa abs}$$

$$\rho_0 = 353/[(0.287)(20 + 273)] = 4.20 \text{ kg/m}^3 \qquad V = (N_M)c = (N_M)\sqrt{kRT}$$

$$V_0 = (0.311)\sqrt{(1.40)(287)(20 + 273)} = 106.7 \text{ m/s} \qquad \dot{m} = (4.20)[(\pi)(0.025)^2/4](106.7) = 0.220 \text{ kg/s}$$

16.219 Air flows subsonically in an adiabatic 1-in-diameter duct. The average friction factor is 0.024. **(*a*)** What length of duct is necessary to accelerate the flow from $(N_M)_1 = 0.1$ to $(N_M)_2 = 0.5$? **(*b*)** What additional length will accelerate it to $(N_M)_3 = 1.0$? Assume $k = 1.4$.

▮ **(*a*)** $f\,(\Delta L/D) = (fL^*/D)_1 - (fL^*/D)_2$. From Table A-24,

$$(fL^*/D)_{N_M=0.1} = 66.9216 \qquad (fL^*/D)_{N_M=0.5} = 1.0691$$

$$(0.024)(\Delta L)/(\tfrac{1}{12}) = 66.9216 - 1.0691 \qquad \Delta L = 229 \text{ ft}$$

(*b*) $$f\,(\Delta L/D) = (fL^*/D)_{N_M=0.5} \qquad (0.024)(\Delta L)/(\tfrac{1}{12}) = 1.0691 \qquad \Delta L = 3.71 \text{ ft}$$

16.220 For the duct flow of Prob. 16.219 assume that at $(N_M)_1 = 0.1$ we have $p_1 = 100$ lb/in² abs and $T_1 = 600$ °R. Compute at section 2 farther downstream $[(N_M)_2 = 0.5]$ **(*a*)** p_2; **(*b*)** T_2; **(*c*)** V_2; and **(*d*)** $(p_0)_2$.

▮ As preliminary information we can compute V_1 and $(p_0)_1$ from the given information: $V_1 = (N_M)_1 c_1 = 0.1[49(600)^{1/2}] = 120$ ft/s, $(p_0)_1 = p_1[1 + \tfrac{1}{2}(k-1)(N_M)_1^2]^{3.5} = 100[1 + 0.2(0.1)^2]^{3.5} = 100.7$ lb/in² abs. Now enter Table A-24, to find the following ratios:

section	N_M	p/p^*	T/T^*	V/V^*	p_0/p_0^*
1	0.1	10.9435	1.1976	0.1094	5.8218
2	0.5	2.1381	1.1429	0.5345	1.3399

Use these ratios to compute all properties downstream:

(*a*) $$p_2 = p_1(p_2/p^*)(p^*/p_1) = 100(2.1381/10.9435) = 19.5 \text{ lb/in}^2 \text{ abs}$$

(*b*) $$T_2 = T_1(T_2/T^*)(T^*/T_1) = 600(1.1429)/1.1976 = 573 \text{ °R}$$

(*c*) $$V_2 = V_1(V_2/V^*)(V^*/V_1) = 120(0.5345)/0.1094 = 586 \text{ ft/s}$$

(*d*) $$(p_0)_2 = (p_0)_1[(p_0)_2/p_0^*][p_0^*/(p_0)_1] = 100.7(1.3399)/5.8218 = 23.2 \text{ lb/in}^2 \text{ abs}$$

16.221 Air enters a duct of $L/D = 38$ at $V_1 = 195$ m/s and $T_1 = 302$ K. The flow at the exit is choked. What is the average friction factor in the duct for adiabatic flow?

▮ $$N_M = V/c = V/\sqrt{kRT} \qquad (N_M)_1 = 195/\sqrt{(1.40)(287)(302)} = 0.560$$

From Table A-24, $fL/D = 0.6736$, $(f)(38) = 0.6736$, $f = 0.0177$.

16.222 Air in a tank at $p_0 = 100$ psia and $T_0 = 518$ °R flows through a converging nozzle into pipe of 1 in diameter. What will be the mass flow through the pipe if its length is **(*a*)** 0 ft, **(*b*)** 1 ft? Assume $f = 0.0409$ and the pressure outside the duct is negligibly small.

▮ **(*a*)** $$\dot{m} = 0.6847 p_0 A^*/(RT_0)^{1/2} \qquad \text{(nozzle is choked)}$$

$$= (0.6847)[(100)(144)][(\pi)(\tfrac{1}{12})^2/4]/[(1716)(518)]^{1/2} = 0.0570 \text{ slug/s}$$

(*b*) $$fL/D = (0.0409)(1)/\tfrac{1}{12} = 0.4908 \qquad \text{(section 2 is choked)}$$

$$\text{From Table A-24} \qquad N_M = 0.6000 \qquad \dot{m} = \rho_1 A_1 V_1$$

$$\rho = p/RT \qquad p_1 = p_0/\{1 + [(k-1)/2](N_M)^2\}^{3.5} = 100/\{1 + [(1.40-1)/2](0.6000)^2\}^{3.5} = 78.4 \text{ psia}$$

$$T_1 = T_0/\{1 + [(k-1)/2](N_M)^2\} = 518/\{1 + [(1.40-1)/2](0.6000)^2\} = 483 \text{ °R}$$

$$\rho = (78.4)(144)/[(1716)(483)] = 0.01362 \text{ slug/ft}^3$$

$$V = (N_M)(c) = (N_M)\sqrt{kRT} = (0.6000)\sqrt{(1.40)(1716)(483)} = 646 \text{ ft/s}$$

$$\dot{m} = (0.01362)[(\pi)(\tfrac{1}{12})^2/4](646) = 0.0480 \text{ slug/s}$$

16.223 Hydrogen ($k = 1.41$ and $R = 4.124$ kJ/kg · K) enters a 6-cm-diameter pipe at $p_1 = 502$ kPa, $V_1 = 298$ m/s, and $T_1 = 21$ °C. The friction factor is 0.025. How long is the duct if the flow is choked? What is the exit pressure?

▮
$$fL/D = \{[1-(N_M)_1^2]/k(N_M)_1^2\} + [(k+1)/2k]\ln\{(k+1)(N_M)_1^2/[2+(k-1)(N_M)_1^2\}$$
$$N_M = V/c = V/\sqrt{kRT} \qquad (N_M)_1 = 298/\sqrt{(1.41)(4124)(21+273)} = 0.227$$
$$0.025L/0.06 = [(1-0.227^2)/(1.41)(0.227)^2]$$
$$+[(1.41+1)/(2)(1.41)]\ln\{(1.41+1)(0.227)^2/[2+(1.41-1)(0.227)^2]\} \qquad L = 25.6\text{ m}$$
$$p_1/p^* = (1/N_M)\{(k+1)/[2+(k-1)(N_M)_2]\}^{1/2}$$
$$502/p^* = (1/0.227)\{(1.41+1)/[2+(1.41-1)(0.227)^2]\}^{1/2} \qquad p^* = p_2 = 104\text{ kPa}$$

16.224 Air enters a 5-cm by 5-cm square duct at $V_1 = 900$ m/s and $T_1 = 300$ K. The friction factor is 0.018. For what length duct will the flow decelerate to exactly $N_M = 1.0$? If the duct length is 2 m, will there be a normal shock in the duct? If so, at what Mach number will it occur?

▮
$$N_M = V/c = V/\sqrt{kRT} \qquad (N_M)_1 = 900/\sqrt{(1.40)(287)(300)} = 2.59 \qquad \text{(supersonic entrance)}$$

From Table A-24, $fL^*/D = 0.451$, $0.018L^*/0.05 = 0.451$, $L^* = 1.25$ m. If $L = 2$ m, which is larger than L^*, there will be a normal shock in the duct. By trial and error, the shock occurs at $N_M = 1.97$ where $fL/D = 0.296$, or $\Delta L_1 = (0.451 - 0.296)(0.05)/0.018 = 0.43$ m. On the downstream side of the shock, $N_M = 0.582$ and $fL^*/D = 0.565$, so $\Delta L_2 = (0.565)(0.05)/0.018 = 1.57$ m; $L_{duct} = \Delta L_1 + \Delta L_2 = 0.43 + 1.57 = 2.00$ m.

16.225 Air enters a 0.5-in-diameter pipe subsonically at $p_1 = 62$ psia and $T_1 = 595$ °R. The pipe length is 25 ft, $f = 0.020$, and the receiver pressure outside the pipe entrance is 18 psia. Compute the mass flow in the pipe, assuming isothermal flow.

▮
$$(\dot{m}/A)^2 = (p_1^2 - p_2^2)/RT[(fL/D) + 2\ln(p_1/p_2)]$$
$$= \{[(62)(144)]^2 - [(18)(144)]^2\}/(1716)(595)\{[(0.020)(25)/(0.5/12)] + 2\ln\tfrac{62}{18}\} = 4.939[\text{slugs/(s-ft}^2)]^2$$
$$\dot{m}/A = 2.222\text{ slugs/(s-ft}^2) \qquad \dot{m} = (2.222)[(\pi)(0.5/12)^2/4] = 0.00303\text{ slug/s}$$

16.226 Oxygen ($R = 0.260$ kJ/kg · K, $k = 1.40$, and $\mu = 2 \times 10^{-5}$ Pa · s) enters a 120-m-long smooth pipe at 250 kPa and 60 °C. The pipe exits into a low-pressure reservoir. The desired mass flow is 0.25 kg/s. Using the Moody chart (Fig. A-5) to compute f, estimate the maximum pipe diameter to transport this flow and the resulting exit pressure and temperature.

▮ Guess $D = 0.05$ m

$$\dot{m} = \rho_1 A_1 V_1 \qquad \rho = p/RT \qquad \rho_1 = 250/[(0.260)(60+273)] = 2.888\text{ kg/m}^3$$
$$0.25 = (2.888)[(\pi)(0.05)^2/4](V_1) \qquad V_1 = 44.09\text{ m/s}$$
$$N_M = V/c = V/\sqrt{kRT} \qquad (N_M)_1 = 44.09/\sqrt{(1.40)(260)(60+273)} = 0.1272$$

From Table A-24, $fL^*/D = 40.39$. $N_R = \rho DV/\mu = (2.888)(0.05)(44.09)/(2 \times 10^{-5}) = 3.18 \times 10^5$. From Fig. A-5, $f = 0.0143$. $(0.0143)(120)/D = 40.39$, $D = 0.0425$ m. Additional iterations are required; they yield a value of $D = 0.0485$ m at $(N_M)_1 = 0.135$.

16.227 Air enters a 1-in-diameter cast iron duct 19 ft long at $V_1 = 200$ fps, $p_1 = 39$ psia, and $T_1 = 521$ °R. Compute the exit pressure and mass flow using the Moody chart (Fig. A-5) to predict the friction factor.

▮
$$\dot{m} = \rho_1 A_1 V_1 \qquad \rho = p/RT \qquad \rho_1 = (39)(144)/[(1716)(521)] = 0.006282\text{ slug/ft}^3$$
$$A_1 = (\pi)(\tfrac{1}{12})^2/4 = 0.005454\text{ ft}^2 \qquad \dot{m} = (0.006282)(0.005454)(200) = 0.006852\text{ slug/s}$$
$$N_M = V/c = V/\sqrt{kRT} \qquad (N_M)_1 = 200/\sqrt{(1.40)(1716)(521)} = 0.1788$$

From Table A-24, $(fL^*/D)_1 = 18.882$.

$$(fL^*/D)_2 = (fL^*/D)_1 - fL/D \qquad N_R = \rho DV/\mu$$
$$(N_R)_1 = (0.006282)(\tfrac{1}{12})(200)/(3.74 \times 10^{-7}) = 2.80 \times 10^5 \qquad \epsilon/D = 0.00085/(\tfrac{1}{12}) = 0.0102$$

From Fig. A-5, $f = 0.0384$.

$$(fL^*/D)_2 = 18.882 - (0.0384)(20)/(\tfrac{1}{12}) = 9.666 \qquad (N_M)_2 = 0.237$$
$$p_1/p^* = 6.112 \quad \text{and} \quad p_2/p^* = 4.601 \qquad p_2 = (4.601/6.112)(39) = 29.4\text{ psia}$$

16.228 Air at 320 K is to be transported through a duct 45 m long. If $f = 0.020$, what is the minimum diameter of duct that can carry the flow without choking, if $V_1 = 50$ m/s? Assume adiabatic flow.

▮ $$N_M = V/c = V/\sqrt{kRT} \qquad (N_M)_1 = 50/\sqrt{(1.40)(287)(320)} = 0.139$$

From Fig. A-24, $(fL^*/D)_1 = 32.80$, $(0.020)(45)/D = 32.80$, $D = 0.0274$ m

16.229 Very cold air, to be used in an air-conditioning system of a test chamber, passes through a rectangular duct of cross-sectional area A ft^2 and of length L ft. It enters the duct at a temperature of T_1 °F at a pressure of p_1 psia. It is estimated that Q Btu per unit length per lbm will be transferred from the surroundings into the flow of air into the duct. If the exit temperature is to be T_2 °F at ambient pressure p_2, how much flow should there be? Set up equations only. Explain how you might go about solving the equations.

▮

equations	unknowns
(1) $QL = c_p[(T_0)_2 - (T_0)_1]$	$(T_0)_1$, $(T_0)_2$
(2) $c_1 = \sqrt{kRT_1}$	c_1
(3) $c_2 = \sqrt{kRT_2}$	c_2
(4) $\dot{m} = \rho_1 V_1 A_1 = (p_1/RT_1)V_1A_1$	$\dot{m}$, V_1
(5) $\dot{m} = (p_2/RT_2)V_2A_2$	V_2
(6) $(N_M)_1 = V_1/c_1$	$(N_M)_1$
(7) $(N_M)_2 = V_2/c_2$	$(N_M)_2$
(8) $T_2/(T_0)_2 = 1/\{1 + [(k-1)/2](N_M)_2^2\}$	
(9) $T_1/(T_0)_1 = 1/\{1 + [(k-1)/2](N_M)_1^2\}$	

We thus have nine unknowns and nine equations. One way of proceeding is as follows. Solve for c_2 and c_3 immediately from Eqs. (2) and (3). Now guess at a "$\dot{m}$." Solve for V_2 from Eq. (5) . Get $(N_M)_2$ from Eq. (7) and then $(T_0)_2$ from Eq. (8). From Eq. (4) get V_1 then go to Eq. (6) for $(N_M)_1$ and finally to Eq. (9) for $(T_0)_1$. Finally go to Eq. (1) and see if the computed $(T_0)_1$ and $(T_0)_2$ satisfy the equation. If not, select another $\dot{m}$, etc.

16.230 Air at $V_1 = 300$ ft/s, $p = 40$ psia, $T = 60$ °F flows into a 4.0-in-diameter duct. How much heat transfer per unit mass is needed for sonic conditions at the exit? Determine pressure, temperature, and velocity at the exit and at the section where $N_M = 0.70$.

▮ $(N_M)_1 = V_1/\sqrt{kRT_1} = 300/\sqrt{1.4(53.3)(32.17)(460+60)} = 0.268$. The isentropic stagnation temperature at the entrance is $T_{01} = T_1\{1 + [(k-1)/2](N_M)_1^2\} = 520[1 + 0.2(0.268^2)] = 527$ °R. The isentropic stagnation temperature at the exit is $T_0^* = T_0(1 + kN_M^2)^2/\{(k+1)N_M^2[2 + (k-1)N_M^2]\} = 527[1 + 1.4(0.268^2)]^2/\{(2.4 \times 0.268^2)[2 + 0.4(0.268^2)]\} = 1827$ °R. The heat transfer per slug of air flowing is $q_H = c_p(T_0^* - T_{01}) = 0.24[32.17(1827 - 527)] = 10\,037$ Btu/slug. The pressure at the exit is $p^* = p[(1 + kN_M^2)/(k+1)] = (40/2.4)[1 + 1.4(0.268^2)] = 18.34$ psia and the temperature is $T^* = T[(1 + kN_M^2)/(k+1)N_M]^2 = 520\{[1 + 1.4(0.268^2)]/2.4(0.268)\}^2 = 1522$ °R. At the exit, $V^* = c^* = \sqrt{kRT^*} = \sqrt{1.4(53.3)(32.17)(1522)} = 1911$ ft/s. At the section where $N_M = 0.7$, $p = p^*[(k+1)/(1 + kN_M^2)] = 18.34(2.4)/[1 + 1.4(0.7^2)] = 26.1$ psia, $T = T^*[(k+1)N_M/(1 + kN_M^2)]^2 = 1522\{2.4(0.7)/[1 + 1.4(0.7^2)]\}^2 = 1511$ °R and $V = N_M\sqrt{kRT} = 0.7\sqrt{1.4(53.3)(32.17)(1511)} = 1333$ ft/s.

16.231 In frictionless oxygen flow through a duct, the following conditions prevail at inlet and outlet: $V_1 = 298$ fps, $T_1 = 82$ °F, $(N_M)_2 = 0.5$. Find the heat added per slug and the pressure ratio p_1/p_2.

▮ $$p_1/p_2 = [1 + k(N_M)_2^2]/[1 + k(N_M)_1^2] \qquad N_M = V/c = V/\sqrt{kRT}$$

$$(N_M)_1 = 298/\sqrt{(1.40)(1554)(82+460)} = 0.274$$

$$p_1/p_2 = [1 + (1.40)(0.5)^2]/[1 + (1.40)(0.274)^2] = 1.22 \qquad q = c_p[(T_0)_2 - (T_0)_1]$$

$$(T_0) = T\{1 + [(k-1)/2]N_M^2\} \qquad (T_0)_1 = (82+460)\{1 + [(1.40-1)/2](0.274)^2\} = 550\ °\text{R}$$

$$T_2 = T_1\big([(N_M)_2/(N_M)_1]\{[1 + k(N_M)_1^2]/[1 + k(N_M)_2^2]\}\big)^2$$

$$= (82+460)\big((0.5/0.274)\{[1 + (1.40)(0.274)^2]/[1 + (1.40)(0.5^2)]\}\big)^2 = 1209\ °\text{R}$$

$$(T_0)_2 = (1209)\{1 + [(1.40-1)/2](0.5)^2\} = 1269\ °\text{R} \qquad q = (0.219)(32.17)(1269 - 550) = 5066\ \text{Btu/slug}$$

16.232 In frictionless air flow through a 100-mm-diameter duct, 0.16 kg/s enters at $T = 0$ °C and $p = 8$ kPa abs. At what rate can heat be added without choking the flow?

▮ $\text{rate} = \dot{m}c_p[T_p^* - (T_0)_1] \qquad T_0^* = T_0(1 + kN_M^2)^2/(N_M)^2(k+1)[2 + (k-1)N_M^2] \qquad T_0 = T\{1 + [(k-1)/2]N_M^2\}$

$$N_M = V/c = V/\sqrt{kRT} \qquad \dot{m} = \rho AV \qquad \rho = p/RT = 8/[(0.287)(0+273)] = 0.1021\ \text{kg/m}^3$$

$$0.16 = (0.1021)[(\pi)(0.100)^2/4](V) \qquad V = 199.5\ \text{m/s} \qquad (N_M)_1 = 199.5/\sqrt{(1.40)(287)(0+273)} = 0.602$$

$$(T_0)_1 = (0+273)\{1 + [(1.40-1)/2](0.602)^2\} = 293\ \text{K}$$

$$T_0^* = (293)[1 + (1.40)(0.602)^2]^2/(0.602)^2(1.40+1)[2 + (1.40-1)(0.602)^2] = 357\ \text{K}$$

$$\text{rate} = (0.16)(1.005)(357 - 293) = 10.3\ \text{kW}$$

16.233 Frictionless flow through a duct with heat transfer causes the Mach number to decrease from 2 to 1.70 ($k = 1.4$). Determine the temperature, velocity, pressure, and density ratios.

▮ $$p_1/p_2 = [1 + k(N_M)_2^2]/[1 + k(N_M)_1^2] = [1 + (1.40)(1.70)^2]/[1 + (1.40)(2)^2] = 0.765$$

$$T_1/T_2 = [(N_M)_1/(N_M)_2]^2(p_1/p_2)^2 = (2/1.70)^2(0.765)^2 = 0.529$$

$$\rho_2/\rho_1 = [(N_M)_1/(N_M)_2]\sqrt{T_1/T_2} = (2/1.70)\sqrt{0.529} = 0.856 \qquad V_1/V_2 = \rho_2/\rho_1 = 0.856$$

16.234 In Prob. 16.233 the duct is 3 in square, $p_1 = 16$ psia, and $V_1 = 2000$ fps. Calculate the mass rate of flow for air flowing.

▮ $$\dot{m} = \rho_1 A_1 V_1 \qquad \rho_1 = p_1/RT_1 \qquad RT_1 = V_1^2/kN_m^2$$

$$\rho_1 = p_1/(V_1^2/kN_M^2) = (16)(144)/\{2000^2/[(1.40)(2)^2]\} = 0.003226\ \text{slug/ft}^3$$

$$\dot{m} = (0.003226)[(\tfrac{3}{12})(\tfrac{3}{12})](2000) = 0.403\ \text{slug/s}$$

16.235 How much heat must be transferred per kilogram to cause the Mach number to increase from 2 to 2.6 in a frictionless duct carrying air? ($V_1 = 500$ m/s)

▮ $$q_H = c_p[(T_0)_2 - (T_0)_1] \qquad T_0 = T\{1 + [(k-1)/2]N_M^2\} \qquad T = V^2/N_M^2 kR$$

$$T_1 = 500^2/(2)^2(1.40)(287) = 156\ \text{K}$$

$$(T_0)_1 = (156)\{1 + [(1.40-1)/2](2)^2\} = 281\ \text{K} \qquad T_1/T_2 = \{[(N_M)_1/(N_M)_2](p_1/p_2)\}^2$$

$$p_1/p_2 = [1 + k(N_M)_2^2]/[1 + k(N_M)_1^2] = [1 + (1.40)(2.6)^2]/[1 + (1.40)(2)^2] = 1.585$$

$$T_1/T_2 = [(2/2.6)(1.585)]^2 = 1.487 \qquad T_2 = 156/1.487 = 105\ \text{K}$$

$$(T_0)_2 = (105)\{1 + [(1.40-1)/2](2.6)^2\} = 247\ \text{K}$$

$$q_H = (1.005)(247 - 281) = -52.3\ \text{kJ/kg} \qquad \text{(i.e., cooling)}$$

16.236 Oxygen at $V_1 = 530$ m/s, $p = 80$ kPa abs, and $T = -10$ °C flows in a 60-mm-diameter frictionless duct. How much heat transfer per kilogram is needed for sonic conditions at the exit?

▮ $$q_H = c_p[(T_0)_2 - (T_0)_1] \qquad T_0 = T\{1 + [(k-1)/2]N_M^2\} \qquad N_M = V/c = V/\sqrt{kRT}$$

$$(N_M)_1 = 530/\sqrt{(1.40)(260)(-10+273)} = 1.71 \qquad (T_0)_1 = (-10+273)\{1 + [(1.40-1)/2](1.71)^2\} = 417\ \text{K}$$

$$T_2 = T_1\{[(N_M)_2/(N_M)_1](p_2/p_1)\}^2 \qquad p_1/p_2 = (1+k)/[1 + k(N_M)_1^2] = (1 + 1.40)/(1 + [(1.40)(1.71)^2] = 0.4712$$

$$T_2 = (-10+273)[(1/1.70)(1/0.4712)]^2 = 410\ \text{K} \qquad (T_0)_2 = (410)\{1 + [(1.40-1)/2](1)^2\} = 492\ \text{K}$$

$$q_H = (0.916)(492 - 417) = 68.6\ \text{kJ/kg}$$

16.237 Apply the first law of thermodynamics, $q_H + p_1/\rho_1 + gz_1 + v_1^2/2 + u_1 = w_s + p_2/\rho_2 + gz_2 + v_2^2/2 + u_2$, to isothermal flow of a perfect gas in a horizontal pipeline, and develop an expression for the heat added per slug flowing.

▮ For isothermal flow in a horizontal pipeline, $w = 0$, $z_1 = z_2$, $T_1 = T_2$, $p_1/\rho_1 = p_2/\rho_2$, $u_1 = u_2$. Hence, $q_H = (v_2^2 - v_1^2)/2$.

16.238 A fuel mixture, assumed equivalent to air, enters a duct combustion chamber at $V_1 = 95$ m/s and $T_1 = 398$ K. Determine the heat input per unit mass that will cause the exit flow to be choked. What will be the exit Mach number and temperature if 1000 kJ/kg is added during combustion?

▮ $$q_{\text{choke}} = c_p[T_0^* - (T_0)_1] \qquad T_0 = T + V^2/2c_p \qquad (T_0)_1 = 398 + [95^2/(2)(1005)] = 402\ \text{K}$$

$$N_M = V/c = V/\sqrt{kRT} \qquad (N_M)_1 = 95/\sqrt{(1.40)(287)(398)} = 0.238$$

From Table A-25, $(T_0)_1/T_0^* = 0.2361$. $T_0^* = 402/0.2361 = 1703$ K, $q_{\text{choke}} = (1.005)(1703 - 402) = 1308$ kJ/kg. For

$q = 1000$ kJ/kg; $(T_0)_2 = (T_0)_1 + (q/c_p) = 402 + (1000/1.005) = 1397$ K, $(T_0)_2/T_0^* = 1397/1703 = 0.8203$. From Table A-25, $(N_M)_2 = 0.601$. $T_2 = (T_0)_2/\{1 + [(k-1)/2](N_M)_2^2\} = (1398)/\{1 + [(1.40-1)/2](0.601)^2\} = 1304$ K.

16.239 What happens to the inlet flow of Prob. 16.238 if the combustion results in 2480 kJ/kg heat addition and $(p_0)_1$ and $(T_0)_1$ remain the same? How much is the mass flow reduced?

▮

$$(T_0)_2 = (T_0)_1 + (q/c_p)$$

Choking:

$$(T_0)_2 = T_0^* = 402 + (2480/1.005) = 2870 \text{ K}$$
$$(T_0)_1/T_0^* = 402/2870 = 0.1400$$

From Table A-25, new $(N_M)_1 = 0.1777$.

$$\dot{m}/A = \rho_1 V_1 = \rho_1 c_1 (N_M)_1 = \rho_0 c_0/\{1 + [(k-1)/2](N_M)_1^2\}^3$$

$$\frac{\dot{m}\text{ (new)}}{\dot{m}\text{ (old)}} = \frac{N_M\text{ (new)}}{N_M\text{ (old)}}\left[\frac{1 + [(1.40-1)/2]N_M^2\text{ (old)}}{1 + [(1.40-1)/2]N_M^2\text{ (new)}}\right]^3 = \frac{0.1777}{0.238}\left[\frac{1 + (0.2)(0.238)^2}{1 + (0.2)(0.1777)^2}\right]^3 = 0.750$$

The mass flow is reduced by 25%.

CHAPTER 17
Flow Measurement

17.1 Water is being discharged through a 3-in-diameter pipe directly into a container that has a volume of 27.0 ft^3. Find the rate and velocity of flow through the pipe if the time required to fill the container is 4 min 19.6 s.

$$Q = V/t = 27.0/[(4)(60) + 19.6] = 0.104\ \text{ft}^3/\text{s} \qquad v = Q/A = 0.104/[(\pi)(\tfrac{3}{12})^2/4] = 2.12\ \text{ft/s}$$

17.2 Water is being discharged through a 6-in-diameter pipe directly into a container that has a volume of 200.0 ft^3. Find the rate and velocity of flow through the pipe if the time required to fill the container is 3 min 21.2 s.

$$Q = V/t = 200.0/[(3)(60) + 21.2] = 0.994\ \text{ft}^3/\text{s} \qquad v = Q/A = 0.994/[(\pi)(\tfrac{6}{12})^2/4] = 5.06\ \text{ft/s}$$

17.3 A Pitot tube being used to determine the velocity of flow of water in a closed conduit indicates a difference between water levels in the Pitot tube and in the piezometer of 48 mm. What is the velocity of flow?

$$v = \sqrt{2gh} = \sqrt{(2)(9.807)(0.048)} = 0.970\ \text{m/s}$$

17.4 A Pitot tube being used to determine the velocity of flow of water in a closed conduit indicates a difference between water levels in the Pitot tube and in the piezometer of 58 mm. What is the velocity of flow?

$$v = \sqrt{2gh} = \sqrt{(2)(9.807)(0.058)} = 1.07\ \text{m/s}$$

17.5 An airplane uses a Pitot measuring device to determine its velocity. The instruments show a stagnation pressure (p_s) of 508 mmHg abs, a static pressure (p_0) of 325 mmHg abs, and a stagnation temperature of 60 °C. What is the plane's velocity?

$$N_M = \sqrt{[2/(k-1)][(p_s/p_0)^{(k-1)/k} - 1]} = \sqrt{[2/(1.40-1)][(\tfrac{508}{325})^{(1.40-1)/1.40} - 1]} = 0.825$$

Since ($N_M = 0.825$) < 1.0, flow is subsonic, and

$$v = \sqrt{[2kRT/(k-1)][(p_s/p_0)^{(k-1)/k} - 1]} \qquad T = 60 + 273 = 333\ \text{K}$$

$$v = \sqrt{[(2)(1.40)(287)(333)/(1.40-1)][(\tfrac{508}{325})^{(1.40-1)/1.40} - 1]} = 302\ \text{m/s}$$

17.6 An airplane uses a Pitot measuring device to determine its velocity. The instruments show a stagnation pressure (p_s) of 498 mmHg abs, a static pressure (p_0) of 318 mmHg abs, and a stagnation temperature of 75 °C. What is the plane's velocity?

$$N_M = \sqrt{[2/(k-1)][(p_s/p_0)^{(k-1)/k} - 1]} = \sqrt{[2/(1.40-1)][(\tfrac{498}{318})^{(1.40-1)/1.40} - 1]} = 0.827$$

Since ($N_M = 0.827$) < 1.0, flow is subsonic, and

$$v = \sqrt{[2kRT/(k-1)][(p_s/p_0)^{(k-1)/k} - 1]} \qquad T = 75 + 273 = 348\ \text{K}$$

$$v = \sqrt{[(2)(1.40)(287)(348)/(1.40-1)][(\tfrac{498}{318})^{(1.40-1)/1.40} - 1]} = 309\ \text{m/s}$$

17.7 Air ($\gamma = 0.075$ pcf) is flowing in Fig. 17-1. If $u = 13.9$ fps and $V_c = 15.7$ fps, determine the manometer reading.

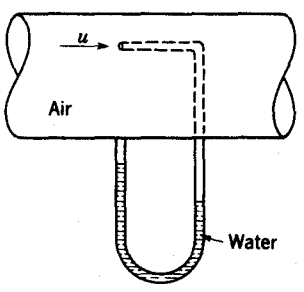


Fig. 17-1

▮ $$u^2/2g = (\gamma_{H_2O}/\gamma_{air} - 1)(x) \qquad 13.9^2/[(2)(32.2)] = (62.4/0.075 - 1)(x)$$
$$x = 0.00361 \text{ ft} \quad \text{or} \quad 0.0433 \text{ in}$$

(Such a reading is too small for practical use.)

17.8 Repeat Prob. 17.7 for Fig. 17-2.

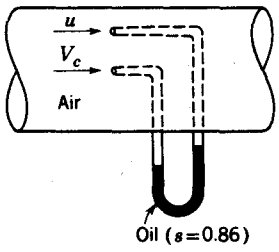


Fig. 17-2

▮ $$V_c/2g - u^2/2g = (\gamma_{oil}/\gamma_{air} - 1)(x)$$
$$15.7^2/[(2)(32.2)] - 13.9^2/[(2)(32.2)] = [(0.86)(62.4)/0.075 - 1](x) \qquad x = 0.00116 \text{ ft} \quad \text{or} \quad 0.0139 \text{ in}$$

(As in Prob. 17.7, such a reading is too small for practical use.)

17.9 In Fig. 17-3, pressure gage A reads 10.8 psi, while pressure gage B reads 11.7 psi. Find the velocity if air at 50 °F is flowing. Atmospheric pressure is 26.9 inHg. Assume $C_I = 1.0$ and neglect compressibility effects.

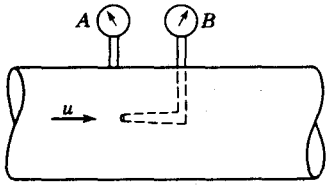


Fig. 17-3

▮ $p = (26.9/29.9)(14.7) + 10.8 = 24.03$ psia $\qquad \gamma = p/RT = (24.03)(144)/[(53.3)(50 + 460)] = 0.1273 \text{ lb/ft}^3$
$$u = C_I\sqrt{2g(p_s/\gamma - p_0/\gamma)} = (1.0)\sqrt{(2)(32.2)[(11.7)(144)/0.1273 - (10.8)(144)/0.1273]} = 256 \text{ ft/s}$$

17.10 In Prob. 17.9, if the two pressure gages were replaced by a differential manometer containing water, what would be the reading on the manometer (y)?

▮ $$\Delta p/\gamma = (\text{s.g.}_M/\text{s.g.}_F - 1)(y) \qquad \Delta p/\gamma = (11.7)(144)/0.1273 - (10.8)(144)/0.1273 = 1018 \text{ ft}$$
$$1018 = (62.4/0.1273 - 1)(y) \qquad y = 2.081 \text{ ft} \quad \text{or} \quad 25.0 \text{ in}$$

17.11 In Fig. 17-3, assume kerosene (s.g. = 0.81) is flowing. The pressure gages at A and B read 74 Pa and 152 Pa, respectively. Find the velocity u, assuming $C_I = 1.0$.

▮ $$\gamma = (0.81)(9.79) = 7.930 \text{ kN/m}^3 \quad \text{or} \quad 7930 \text{ N/m}^3$$
$$u = C_I\sqrt{2g(p_s/\gamma - p_0/\gamma)} = (1.00)\sqrt{(2)(9.807)(\tfrac{152}{7930} - \tfrac{74}{7930})} = 0.439 \text{ m/s}$$

17.12 The piezometer in Fig. 17-4 is connected to a mercury manometer, and the reading is 92 mm. The velocity is 3.5 m/s. If carbon tetrachloride (s.g. = 1.59) is flowing, what is C_I for the measurement?

Fig. 17-4

▮ $$u = C_I\sqrt{2g(p_s/\gamma - p_0/\gamma)} = C_I\sqrt{2g(\Delta p/\gamma)}$$
$$\Delta p/\gamma = (\text{s.g.}_M/\text{s.g.}_F - 1)(y) = (13.6/1.59 - 1)(0.092) = 0.6949 \text{ m}$$
$$3.5 = (C_I)\sqrt{(2)(9.807)(0.6949)} \qquad C_I = 0.948$$

17.13 In Fig. 17-4, suppose air at 90 °F is flowing. The pitometer is attached to a manometer containing a liquid (s.g. = 0.89). Find velocity u versus manometer readings assuming $C_I = 0.93$. Assume the air is at standard atmospheric pressure.

▮ From Table A-4, $\rho_{air} = 0.00224$ slug/ft^3. $\rho_{liquid}/\rho_{air} = (0.89)(1.94)/0.00224 = 770.8$. Thus 1 ft of manometer reading is equivalent to 770.8 ft of air. $u = C_I\sqrt{2g(p_s/\gamma - p_0/\gamma)} = C_I\sqrt{2g(\Delta p/\gamma)}$. Letting y = manometer reading, $u = C_I\sqrt{2g(770.8y)}$. For $y = 1$ in, $u = (0.93)\sqrt{(2)(32.2)[(770.8)(\frac{1}{12})]} = 59.8$ ft/s. For $y = 5$ in, $u = (0.93)\sqrt{(2)(32.2)[(770.8)(\frac{5}{12})]} = 134$ ft/s. For $y = 10$ in, $u = (0.93)\sqrt{(2)(32.2)[(770.8)(\frac{10}{12})]} = 189$ ft/s. For $y = 20$ in, $u = (0.93)\sqrt{(2)(32.2)[(770.8)(\frac{20}{12})]} = 267$ ft/s. (At larger manometer readings, the effect of compressibility must be considered.)

17.14 A Pitot tube is placed in a pipe carrying water at 25 °C. The Pitot tube and a wall piezometer tube are connected to a water-mercury manometer which registers a differential of 66 mm. Assuming $C_I = 0.98$, what is the velocity approaching the tube?

▮
$$\Delta p/\gamma = (\text{s.g.}_M/\text{s.g.}_F - 1)(y) = (13.6/1 - 1)(0.066) = 0.8316 \text{ m}$$
$$u = C_I\sqrt{2g(p_s/\gamma - p_0/\gamma)} = C_I\sqrt{2g(\Delta p/\gamma)} = (0.98)\sqrt{(2)(9.807)(0.8316)} = 3.96 \text{ m/s}$$

17.15 Suppose that the fluids of Prob. 17.14 are reversed so that mercury is flowing in the pipe and water is the gage fluid (with the manometer now inverted). With the same gage differential, what would be the velocity of the mercury?

▮
$$\Delta p/\gamma = (1 - \text{s.g.}_M/\text{s.g.}_F)(y) = (1 - 1/13.6)(0.066) = 0.06115 \text{ m}$$
$$u = C_I\sqrt{2g(p_s/\gamma - p_0/\gamma)} = C_I\sqrt{2g(\Delta p/\gamma)} = (0.98)\sqrt{(2)(9.807)(0.06115)} = 1.07 \text{ m/s}$$

17.16 A Prandtl tube is placed on the centerline of a smooth 10-in-diameter pipe in which water at 80 °F is flowing. The reading on a differential manometer attached to this Prandtl tube is 11 in of carbon tetrachloride (s.g. = 1.59). Find the flow rate.

▮
$$V/u_{max} = 1/(1 + 1.33f^{1/2}) \qquad h = (1.59 - 1)(\tfrac{11}{12}) = 0.5408 \text{ ft}$$
$$u_{max} = \sqrt{2gh} = \sqrt{(2)(32.2)(0.5408)} = 5.901 \text{ ft/s}$$

Try $V = 5$ ft/s: $N_R = DV/\nu = (\frac{10}{12})(5)/(9.15 \times 10^{-6}) = 4.55 \times 10^5$. From Fig. A-5, $f = 0.013$. $V/5.901 = 1/[1 + (1.33)(0.013)^{1/2}]$, $V = 5.12$ ft/s. (f is still 0.013 for $V = 5.12$ ft/s.) $Q = AV = [(\pi)(\frac{10}{12})^2/4](5.12) = 2.79$ ft^3/s.

17.17 A Pitot-static tube for which $C_I = 0.99$ is connected to an inverted U-tube containing oil (s.g. = 0.83). Water is flowing. What is the velocity if the manometer reading is 3.7 in?

▮
$$\Delta p/\gamma = (1 - \text{s.g.}_M/\text{s.g.}_F)(y) = (1 - 0.83/1)(3.7/12) = 0.0524 \text{ ft}$$
$$u = C_I\sqrt{2g(p_s/\gamma - p_0/\gamma)} = C_I\sqrt{2g(\Delta p/\gamma)} = (0.99)(\sqrt{(2)(32.2)(0.0524)} = 1.82 \text{ ft/s}$$

17.18 A Pitot tube having a coefficient of 0.98 is used to measure the velocity of water at the center of a pipe, as shown in Fig. 17-5. What is the velocity?

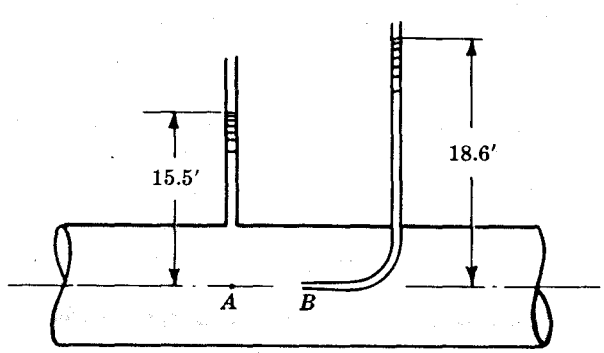


Fig. 17-5

▮
$$u = C_I\sqrt{2g(p_s/\gamma - p_0/\gamma)} = (0.98)\sqrt{(2)(32.2)(18.6 - 15.5)} = 13.8 \text{ ft/s}$$

17.19 Air flows through a duct, and the Pitot-static tube measuring the velocity is attached to a differential gage

containing water. If the deflection of the gage is 4 in, calculate the air velocity, assuming the specific weight of air is constant at 0.0761 lb/ft^3 and that the coefficient of the tube is 0.98.

▮
$$(p_s/\gamma - p_0/\gamma) = (\tfrac{4}{12})(62.4)/0.0761 = 273.3 \text{ ft of air}$$
$$u = C_I\sqrt{2g(p_s/\gamma - p_0/\gamma)} = (0.98)\sqrt{(2)(32.2)(273.3)} = 130 \text{ ft/s}$$

17.20 Carbon tetrachloride (s.g. = 1.60) flows through a pipe. The differential gage attached to the Pitot-static tube shows a 3-in deflection of mercury. Assuming $C = 1.00$, find the velocity.

▮
$$p_s - p_0 = (\tfrac{3}{12})(13.6 - 1.60)(62.4) = 187.2 \text{ lb/ft}^2$$
$$u = C_I\sqrt{2g(p_s/\gamma - p_0/\gamma)} = (1.00)\sqrt{(2)(32.2)\{187.2/[(1.60)(62.4)]\}} = 11.0 \text{ ft/s}$$

17.21 Water flows at a velocity of 4.65 fps. A differential gage ($C_I = 1.0$) which contains a liquid of specific gravity 1.25 is attached to the Pitot-static tube. What is the deflection of the gage fluid?

▮ $u = C_I\sqrt{2g(p_s/\gamma - p_0/\gamma)} = C_I\sqrt{2g(\Delta p/\gamma)}$ $\quad 4.65 = (1.0)\sqrt{(2)(32.2)(\Delta p/\gamma)}$ $\quad \Delta p/\gamma = 0.3358$ ft of water

$\Delta p/\gamma = (\text{s.g.}_M/\text{s.g.}_F - 1)(y)$ $\quad 0.3358 = [(1.25/1) - 1](y)$ $\quad y = 1.34$ ft

17.22 Develop the expression for measuring the flow of a gas with a Pitot tube.

▮ The flow from A to B in the figure of Prob. 17.18 may be considered adiabatic and with negligible loss. Using the Bernoulli equation, A to B, we obtain $\{[k/(k-1)](p_A/\gamma_A) + (V_A^2/2g) + 0\}$ − negligible loss = $[k/(k-1)](p_A/\gamma_A)(p_B/p_A)^{(k-1)/k} + 0 + 0$ or

$$V_A^2/2g = [k/(k-1)](p_A/\gamma_A)[(p_B/p_A)^{(k-1)/k} - 1] \tag{1}$$

The term p_B is the stagnation pressure. This expression (*1*) is usually rearranged, introducing the ratio of the velocity at A to the acoustic velocity c of the undisturbed fluid. The acoustic velocity $c = \sqrt{E/\rho} = \sqrt{kp/\rho} = \sqrt{kpg/\gamma}$. Combining with Eq. (*1*) above,

$$V_A^2 = [c^2/(k-1)][(p_B/p_A)^{(k-1)/k} - 1] \quad \text{or} \quad p_B/p_A = \{1 + [(k-1)/2](V_A/c)^2\}^{k/(k-1)} \tag{2}$$

Expanding by the binomial theorem,

$$p_B/p_A = 1 + (k/2)(V_A/c)^2\{1 + \tfrac{1}{4}(V_A/c)^2 - [(k-2)/24](V_A/c)^4 + \cdots\} \tag{3}$$

In order to compare this expression with the formula in Prob. 17.18, multiply through by p_A and replace kp_A/c^2 by ρ_A, obtaining

$$p_B = p_A + \tfrac{1}{2}\rho_A V_A^2\{1 + \tfrac{1}{4}(V_A/c)^2 - [(k-2)/24](V_A/c)^4 + \cdots\} \tag{4}$$

The above expressions apply to all compressible fluids for ratios of V/c less than unity. For ratios over unity, shock wave and other phenomena occur, the adiabatic assumption is not sufficiently accurate, and the derivation no longer applies. The ratio V/c is called the *Mach number*.

The term in braces in (*4*) is greater than unity and the first two terms provide sufficient accuracy. The effect of compressibility is to increase the stagnation-point pressure over that of an incompressible fluid.

17.23 Air flowing under atmospheric conditions ($\gamma = 0.0763$ lb/ft^3 at 60 °F) at a velocity of 300 fps is measured by a Pitot tube. Calculate the error in the stagnation pressure by assuming the air to be incompressible.

▮ $p_B = p_A + \rho V^2/2 = (14.7)(144) + (0.0763/32.2)(300)^2/2 = 2223$ lb/ft^2 abs

Also:
$$p_B = p_A + \tfrac{1}{2}\rho_A V_A^2\{1 + \tfrac{1}{4}(V_A/c)^2 - [(k-2)/24](V_A/c)^4 + \cdots\}$$
$$c = \sqrt{kgRT} = \sqrt{(1.40)(32.2)(53.3)(60 + 460)} = 1118 \text{ ft/s}$$
$$p_B = (14.7)(144) + [(0.0763/32.2)(300)^2/2]\{1 + (\tfrac{1}{4})(\tfrac{300}{1118})^2 - [(1.40-2)/24](\tfrac{300}{1118})^4 + \cdots\} = 2225 \text{ lb/ft}^2 \text{ abs}$$

The error in the stagnation pressure is $(2225 - 2223)/2225 = 0.0009$, or 0.09 percent.

17.24 The difference between the stagnation pressure and the static pressure measured by a Pitot-static device is 412 lb/ft^2. The static pressure is 14.5 psia and the temperature in the air stream is 60 °F. What is the velocity of the air, assuming the air is (***a***) compressible and (***b***) incompressible.

▮ (*a*) Using Eq. (*2*) of Prob. 17.22,

$$p_B/p_A = \{1 + [(k-1)/2](V_A/c)^2\}^{k/(k-1)}$$
$$c = \sqrt{kgRT} = \sqrt{(1.40)(32.2)(53.3)(60+460)} = 1118 \text{ ft/s}$$
$$[(14.5)(144) + 412]/(14.5)(144) = \{1 + [(1.40-1)/2](V_A/1118)^2]^{1.40/(1.40-1)} \qquad V_A = 574 \text{ ft/s}$$

(*b*)
$$\gamma = p/RT = (14.5)(144)/[(53.3)(60+460)] = 0.07534 \text{ lb/ft}^3$$
$$V = \sqrt{2g(\Delta p/\gamma)} = \sqrt{(2)(32.2)(412/0.07534)} = 593 \text{ ft/s}$$

17.25 Air flows at 800 fps through a duct. At standard barometer, the stagnation pressure is −5.70 ft of water gage. The stagnation temperature is 145 °F. What is the static pressure in the duct?

▮ Using Eq. (*2*) of Prob. 17.22, $p_B/p_A = \{1 + [(k-1)/2](V_A/c)^2\}^{k/(k-1)}$. With two unknowns in the equation, assume a V/c ratio of 0.72. $(-5.70 + 34.0)(62.4)/p_A = \{1 + [(1.40-1)/2](0.72)^2\}^{1.40/(1.40-1)}$, $p_A = 1250$ lb/ft² abs. Check the assumption using the adiabatic relation $T_B/T_A = (p_B/p_A)^{(k-1)/k}$:

$$(145 + 460)/T_A = [(-5.70 + 34.0)(62.4)/1250]^{(1.40-1)/1.40} \qquad T_A = 548 \text{ °R}$$
$$c = \sqrt{kgRT} = \sqrt{(1.40)(32.2)(53.3)(548)} = 1147 \text{ ft/s}$$
$$V/c = \tfrac{800}{1147} = 0.697 \qquad (-5.70 + 34.0)(62.4)/p_A = \{1 + [(1.40-1)/2](0.697)^2\}^{1.40/(1.40-1)}$$
$$p_A = 1277 \text{ lb/ft}^2 \text{ abs}$$

No further refinement is necessary.

17.26 A static tube indicates a static pressure that is 900 Pa too low when liquid is flowing at 2.1 m/s. Calculate the correction to be applied to the indicated pressure for the liquid flowing at 6 m/s.

▮ Since Δh is a function of V^2, $\Delta h_2/\Delta h_1 = V_2^2/V_1^2$, $\Delta p_2/\Delta p_1 = V_2^2/V_1^2$, $\Delta p_2/900 = 6^2/2.1^2$, $\Delta p_2 = 7350$ Pa.

17.27 A simple Pitot tube is inserted into a small stream of flowing oil, $\gamma = 56$ lb/ft³, $\mu = 0.65$ P, $\Delta h = 1.7$ in, $h_0 = 6$ in. What is the velocity at point 1?

▮
$$v = \sqrt{2g\,\Delta h} = \sqrt{(2)(32.2)(1.7/12)} = 3.02 \text{ ft/s}$$

17.28 A stationary body immersed in a river has a maximum pressure of 73 kPa exerted on it at a distance of 5.9 m below the free surface. Calculate the river velocity at this depth.

▮
$$V^2/2g = p_t/\gamma - p_s/\gamma \qquad V^2/[(2)(9.807)] = 73/9.79 - 5.9 \qquad V = 5.53 \text{ m/s}$$

17.29 For Fig. 17-6 derive the equation for velocity at point 1.

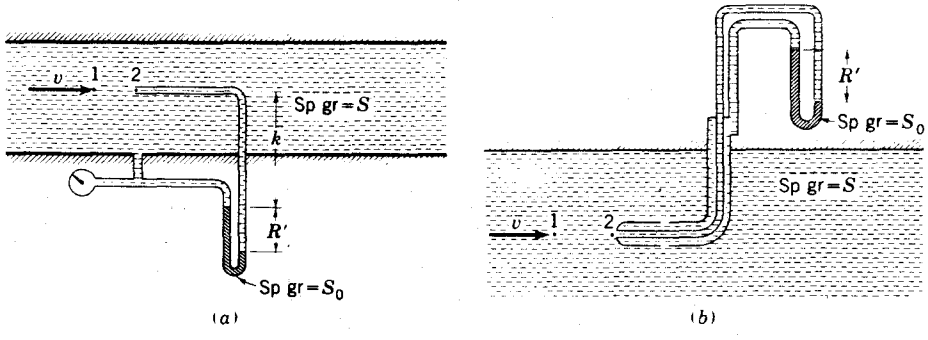


Fig. 17-6

▮ For Fig. 17-6*a* (Pitot tube and piezometer opening), $V_1^2/2g = (p_2 - p_1)/\gamma$.

Manometer equation:

$$p_1(S/\gamma) + kS + R'S_0 - (k + R')S = p_2(S/\gamma) \qquad (p_2 - p_1)/\gamma = R'[(S_0 - S)/S]$$
$$V_1^2/2g = R'[(S_0 - S)/S] \qquad V_1 = \sqrt{2gR'(S_0/S - 1)}$$

For Fig. 17-6*b* (Pitot-static tube), we analyze this system in a manner similar to that for Fig. 17-6*a* and show that the same relations hold, but the uncertainty in the measurement of static pressure requires a corrective coefficient C_I to be applied so that $V_1 = C_I\sqrt{2gR'(S_0/S - 1)}$.

17.30 In Fig. 17-6*a* air is flowing ($p = 20$ psia, $T = 48$ °F) and water is in the manometer. For $R' = 0.9$ in, calculate the velocity of air.

▮ $$V = \sqrt{2gR'(S_0/S - 1)} \qquad \rho = p/RT = (20)(144)/[(53.3)(48 + 460)] = 0.1064 \text{ lbm/ft}^3$$

$$S_0/S = 62.4/0.1064 = 586.5 \qquad V = \sqrt{(2)(32.2)(0.9/12)(586.5 - 1)} = 53.2 \text{ ft/s}$$

17.31 In Fig. 17-6 air is flowing ($p = 102$ kPa abs and $T_1 = 9$ °C) and mercury is in the manometer. For $R' = 188$ mm, calculate the velocity at 1 for air considered incompressible.

▮ $$V = \sqrt{2gR'(S_{Hg}/S_{air} - 1)} \qquad \rho = p/RT \qquad \rho_{air} = 102/[(0.287)(9 + 273)] = 1.260 \text{ kg/m}^3$$

$$S_{Hg}/S_{air} = (13.6)(1000)/1.260 = 10\,794 \qquad V = \sqrt{(2)(9.807)(0.188)(10\,794 - 1)} = 199 \text{ m/s}$$

17.32 Work Prob. 17.31 for isentropic compression of air between 1 and 2.

▮ $$V_1^2/2 = c_p T_1[(p_2/p_1)^{(k-1)/k} - 1] \qquad (p_2 - p_1)/\gamma_{air} = R'[(S_{Hg} - S_{air})/S_{air}]$$

$$p_2 - p_1 = R'\gamma_{Hg} - R'\gamma_{air} \approx R'\gamma_{Hg} \qquad p_2 - 102 = (0.188)[(13.6)(9.79)] \qquad p_2 = 127.0 \text{ kPa}$$

$$V_1^2/2 = (1005)(9 + 273)[(127.0/102)^{(1.40-1)/1.40} - 1] \qquad V_1 = 191 \text{ m/s}$$

17.33 A Pitot-static tube directed into a 3-m/s water stream has a gage difference of 20 mm on a water-mercury differential manometer. Determine the coefficient for the tube.

▮ $$V = C_I\sqrt{2gR'(S_0/S - 1)} \qquad 3 = C_I\sqrt{(2)(9.807)(0.020)[(13.6/1) - 1]} \qquad C_I = 1.35$$

17.34 A Pitot-static tube, $C_I = 1.14$, has a gage difference of 15 mm on a water-mercury manometer when directed into a water stream. Calculate the velocity.

▮ $$V = C_I\sqrt{2gR'(S_0/S - 1)} = (1.14)\sqrt{(2)(9.807)(0.015)[(13.6/1) - 1]} = 2.19 \text{ m/s}$$

17.35 A Pitot-static tube of the Prandtl type has the following values of gage difference R' for the radial distance from the center of a 3-ft-diameter pipe:

r, ft	0.0	0.3	0.6	0.9	1.2	1.48
R', in	4.00	3.89	3.72	3.40	2.94	2.30

Water is flowing, and the manometer fluid has a specific gravity of 2.83. Calculate the discharge.

▮ $V = C_I\sqrt{2gR'(S_0/S - 1)}$. For a Prandtl tube, $C_I = 1.0$

$$V_i = (1.0)\sqrt{(2)(32.2)(R'/12)[(2.83/1) - 1]} = 3.134\sqrt{R'} \qquad (R' \text{ in inches})$$

$$A_1 = (\pi)[(r_2 - r_1)/2]^2 \qquad A_i = (\pi)\{[(r_{i+1} + r_i)/2]^2 - [(r_i + r_{i-1})/2]^2\}$$

$$A_6 = (\pi)\{r_7^2 - [(r_5 + r_6)/2]^2\}$$

i	1	2	3	4	5	6	7	
r_i	0	0.3	0.6	0.9	1.2	1.48	1.5	ft
V_i	6.268	6.181	6.045	5.779	5.374	4.753	—	ft/s
A_i	0.071	0.565	1.131	1.696	2.177	1.428	—	ft²

$$Q = \sum_{i=1}^{6} V_i A_i = (6.268)(0.071) + (6.181)(0.565) + (6.045)(1.131)$$
$$+ (5.779)(1.696) + (5.374)(2.177) + (4.753)(1.428) = 39.1 \text{ ft}^3/\text{s}$$

17.36 Determine the gage difference on a water-nitrogen manometer for flow of nitrogen at 180 m/s, using a Pitot-static tube. The static pressure is 160 kPa abs, and the corresponding temperature is 35 °C. True static pressure is measured by the tube.

$$p_2 = (p_1)(V_1^2/2c_pT_1 + 1)^{k/(k-1)} = (160)\{180^2/[(2)(1040)(35+273)] + 1\}^{1.40/(1.40-1)} = 190.2\text{ kPa}$$

$$R' = (p_t - p_s)/\gamma = (190.2 - 160)/9.79 = 3.08\text{ m}$$

17.37 Measurements in an air stream indicate that the stagnation pressure is 18 psia, the static pressure is 13 psia, and the stagnation temperature is 109 °F. Determine the temperature and velocity of the air stream.

$$V_1^2/2 = c_pT_2[1 - (p_1/p_2)^{(k-1)/k}] = (6000)(109+460)[1 - (\tfrac{13}{18})^{(1.40-1)/1.40}]$$

$$V_1 = 779\text{ ft/s} \qquad T_1 = T_2 - V^2/2c_p = (109+460) - 779^2/[(2)(6000)] = 518\text{ °R} \quad \text{or} \quad 58\text{ °F}$$

17.38 If 0.6 kg/s nitrogen flows through a 60-mm-diameter tube with stagnation temperature of 42 °C and undisturbed temperature of 14 °C, find the velocity and static and stagnation pressures.

$$V_1 = \sqrt{2c_p(T_2 - T_1)} = \sqrt{(2)(1040)[(42+273) - (14+273)]} = 241\text{ m/s}$$

$$p_1 = \rho_1RT_1 = (\dot{m}/AV)RT_1$$

$$A = (\pi)(0.060)^2/4 = 0.002827\text{ m}^2 \qquad p_1 = \{0.6/[(0.002827)(241)]\}(0.297)(14+273) = 75.067\text{ kPa}$$

$$p_2 = (p_1)(T_2/T_1)^{k/(k-1)} = (75.067)[(42+273)/(14+273)]^{1.40/(1.40-1)} = 103.980\text{ kPa}$$

17.39 A Pitot tube is inserted into a flowing air stream at 25 °C and 120 kPa. The differential pressure reads 18 mm on an air-mercury manometer. What is the indicated air speed?

$$\Delta p = (\rho_{Hg} - \rho_{air})gh = \rho_{air}V^2/2 \qquad \rho_{air} = p/RT = 120/[(0.287)(25+273)] = 1.40\text{ kg/m}^3$$

$$(13\,570 - 1.40)(9.807)(0.018) = (1.40)(V^2/2) \qquad V = 58.5\text{ m/s}$$

17.40 For the Pitot-static pressure arrangement of Fig. 17-7, compute the **(*a*)** centerline velocity, **(*b*)** pipe volume flow, and **(*c*)** wall shear stress. The manometer fluid is Meriam red oil (s.g. = 0.827).

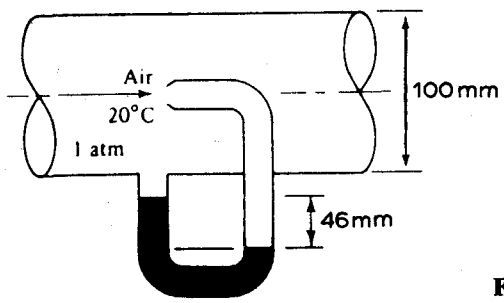


Fig. 17-7

(*a*) $$p_0 - p = (\rho_{oil} - \rho_{air})gh = [(0.827)(998) - 1.20](9.807)(0.046) = 371.8\text{ Pa}$$

$$V_{C_L} = \sqrt{2\Delta p/\rho_{air}} = \sqrt{(2)(371.8)/1.20} = 24.9\text{ m/s}$$

(*b*) $V_{avg} = V_{C_L}/(1 + 1.33\sqrt{f})$. Estimate $V_{avg} = 0.85V_{C_L} = (0.85)(24.9) = 21.2$ m/s. $N_R = \rho DV/\mu = (1.20)(0.100)(21.2)/(1.80 \times 10^{-5}) = 1.41 \times 10^5$. From Fig. A-5, $f = 0.0178$.

$$V_{avg} = 24.9/(1 + 1.33\sqrt{0.0178}) = 21.1\text{ m/s} \quad \text{(O.K.)} \qquad Q = AV = [(\pi)(0.100)^2/4](21.1) = 0.1657\text{ m}^3/\text{s}$$

(*c*) $$\tau = (\tfrac{1}{8})(f\rho V_{avg}^2) = (\tfrac{1}{8})(0.0178)(1.20)(21.1)^2 = 1.19\text{ Pa}$$

17.41 For the water flow at 20 °C of Fig. 17-8, use the manometer measurement to estimate the **(*a*)** centerline velocity and **(*b*)** volume flow in the 8-in-diameter pipe. Assume a smooth wall.

(*a*) $V_{C_L} = \sqrt{2\Delta p/\rho_{H_2O}}$. Assume at first that static pressure at A = static pressure at B.

$$\Delta p = (p_0)_B - p_B = (p_0)_B - p_A = (\rho_{Hg} - \rho_{H_2O})gh = [(13.6-1)(1.94)](32.2)(\tfrac{3}{12}) = 196.8\text{ lb/ft}^2$$

$$V_{C_L} = \sqrt{(2)(196.8)/1.94} = 14.2\text{ ft/s}$$

(*b*) $V_{avg} = V_{C_L}/(1 + 1.33\sqrt{f})$. Estimate $V_{avg} = 0.87V = (0.87)(14.2) = 12.4$ ft/s. $N_R = \rho DV/\mu = (1.94)(\tfrac{8}{12})(12.4)/(2.04 \times 10^{-5}) = 7.86 \times 10^5$. From Fig. A-5, $f = 0.0133$.

$$V_{avg} = 14.2/(1 + 1.33\sqrt{0.0133}) = 12.3\text{ ft/s} \quad \text{(O.K.)}$$

$$Q = AV = [(\pi)(\tfrac{8}{12})^2/4](12.3) = 4.29\text{ ft}^3/\text{s}$$

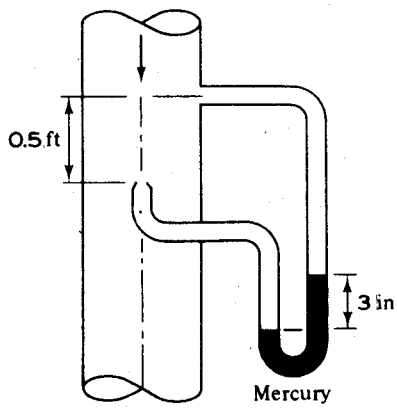


Fig. 17-8

Check:

$$p_A - p_B = \rho g f(L/D)(V^2/2g) = (1.94)(32.2)(0.0133)[0.5/(\tfrac{8}{12})]\{12.3^2/[(2)(32.2)]\} = 1.5 \text{ lb/ft}^2$$

This gives a $1.5/196.8 = 0.008$, or 0.8 percent error in Δp.

17.42 A Pitot tube placed in a flow of methane ($\rho = 0.666$ kg/m^3) at 20 °C and 1 atm shows a methane-water differential manometer reading of 14 mm. What is the methane velocity? What will the manometer reading be if the methane velocity is 35 m/s?

▮
$$\Delta p = (\rho_{H_2O} - \rho_{CH_4})gh = (998 - 0.666)(9.807)(0.014) = 136.9 \text{ Pa}$$
$$V_{\mathrm{Ç}} = \sqrt{2\Delta p/\rho_{CH_4}} = \sqrt{(2)(136.9)/0.666} = 20.3 \text{ m/s}$$

If $V_{\mathrm{Ç}} = 35$ m/s, $\Delta p = (\rho_{H_2O} - \rho_{CH_4})gh = \rho V^2/2$, $(998 - 0.666)(9.807)(h) = (0.666)(35)^2/2$, $h = 0.0417$ m, or 41.7 mm.

17.43 An airplane flying at altitude 7 km uses a Pitot stagnation probe without a static tube. The measured stagnation pressure is 44 kPa. Find the indicated airplane speed and its probable uncertainty.

▮ $\Delta p = p_0 - p = \rho V^2/2$. From Table A-8, p at 7000 m = 41.44 kPa and $\rho = 0.593$ kg/m^3. $(44 - 41.44)(1000) = (0.593)(V)^2/2$, $V = 92.9$ m/s. As for the uncertainty, assume ρ_{air} and p_{air} have ±3 percent error and p_0 has ±1 percent. The worst case would be high ρ and p and low p_0. $\Delta p = [(44)(0.99) - (41.44)(1.03)](1000) = [(0.593)(1.03)](V)^2/2$, $V = 53.6$ m/s. This is $(92.9 - 53.6)/92.9 = 0.42$, or 42 percent less. Thus, small errors in this case make for a large error in the velocity estimate.

17.44 An engineer who took college fluid mechanics on a pass-fail basis has placed the static pressure hole far upstream of the stagnation probe, as in Fig. 17-9, thus contaminating the Pitot measurement ridiculously with pipe friction losses. If the pipe flow is air at 20 °C and 1 atm and the manometer fluid is Meriam red oil (s.g. = 0.827), estimate the air centerline velocity for the given manometer reading. Assume a smooth-walled tube.

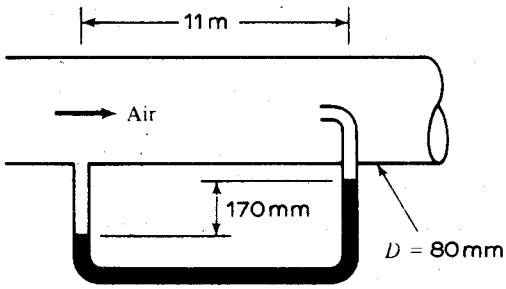


Fig. 17-9

▮
$$(p_0)_B - p_A = (\rho_{oil} - \rho_{air})gh = [(0.827)(998) - 1.20](9.807)(-0.170) = -1374 \text{ Pa}$$

Consider friction loss.

$$p_A - p_B = (f)(L/D)(V_{avg}^2/2)(\rho)$$
$$(p_0)_B - p_B = [(p_0)_B - p_A] + (p_A - p_B) = -1374 + (f)(L/D)(V_{avg}^2/2)(\rho) = (\rho)(V_{\mathrm{Ç}})_B^2/2$$

Guess $f = 0.016$ and $V_{avg} = 0.85V_{℄}$.

$$-1374 + (0.016)[11/0.080](V_{avg}^2/2)(1.20) = (1.20)(V_{avg}/0.85)^2/2 \qquad V_{avg} = 53.0\ \text{m/s}$$

$$N_R = \rho DV/\mu = (1.20)(0.080)(53.0)/(1.81 \times 10^{-5}) = 2.81 \times 10^5$$

From Fig. A-5, $f = 0.016$ (O.K.).

$$V_{℄}/V_{avg} = 1 + 1.33f^{1/2} = 1 + (1.33)(0.016)^{1/2} = 1.168 \qquad V_{avg}/V_{℄} = 1/1.168 = 0.856 \qquad \text{(O.K.)}$$

Hence, $V_{avg} = 53.0$ m/s and $V_{℄} = (53.0)(1.168) = 61.9$ m/s.

17.45 The loss of head due to friction in an orifice nozzle, or tube, may be expressed as $h_L = kV^2/2g$, where V is the actual velocity of the jet. Compute k for the three tubes in Fig. 17-10.

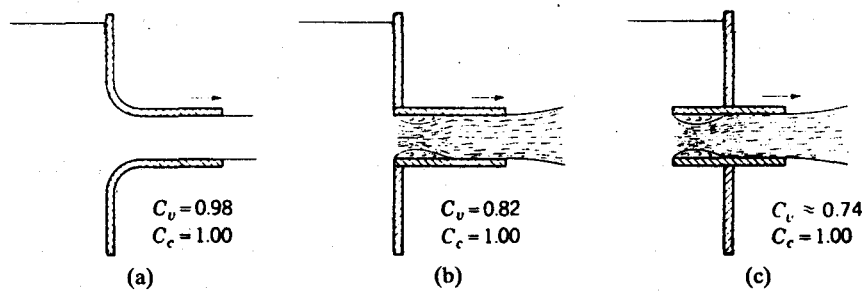


Fig. 17-10

▮ $h_L = kV^2/2g = (1/C_v^2 - 1)(V^2/2g) \qquad k = 1/C_v^2 - 1$

(*a*) $C_v = 0.98 \qquad k = (1/0.98^2) - 1 = 0.0412$

(*b*) $C_v = 0.82 \qquad k = (1/0.82^2) - 1 = 0.487$

(*c*) $C_v = 0.74 \qquad k = (1/0.74^2) - 1 = 0.826$

17.46 If the tubes in Prob. 17.45 and Fig. 17-10 discharge water under a head of 6 ft, compute the loss of head in each case.

▮ $V = C_v\sqrt{2gh} \qquad h_L = kV^2/2g = (1/C_v^2 - 1)(V^2/2g) = (1/C_v^2 - 1)(C_v\sqrt{2gh})^2/2g = (1/C_v^2 - 1)(2hC_v^2/2)$

(*a*) $C_v = 0.98 \qquad h_L = [(1/0.98^2) - 1][(2)(6)(0.98)^2/2)] = 0.238$ ft

(*b*) $C_v = 0.82 \qquad h_L = [(1/0.82^2) - 1][(2)(6)(0.82)^2/2)] = 1.97$ ft

(*c*) $C_v = 0.74 \qquad h_L = [(1/0.74^2) - 1][(2)(6)(0.74)^2/2)] = 2.71$ ft

17.47 The diverging tube shown in Fig. 17-11 discharges water when $h = 4$ ft. The area A is twice area A_0. Neglecting all friction losses, find the (*a*) velocity at the throat and (*b*) pressure head at the throat.

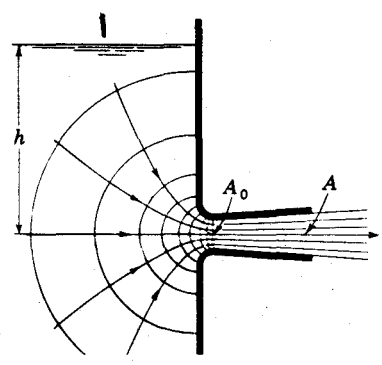


Fig. 17-11

▮ (*a*) $V = \sqrt{2gh} = \sqrt{(2)(32.2)(4)} = 16.05$ ft/s

$$AV = A_0V_0 \qquad (2A_0)(16.05) = A_0V_0 \qquad V_0 = 32.1\ \text{ft/s}$$

(*b*) $p_0/\gamma = V^2/2g - V_0^2/2g = 16.05^2/[(2)(32.2)] - 32.1^2/[(2)(32.2)] = -12.0$ ft

17.48 If the barometric pressure is 14.6 psia and the water temperature is 85 °F, what is the maximum value of h at which the tube will flow full, all other data being the same as in Prob. 17.47? What will happen if the value of h is made greater than this?

▮
$$(p_0)_{min} = p_v = 87.2\ \text{lb/ft}^2 \qquad \text{(from Table A-1)}$$
$$p_0/\gamma + V_0^2/2g = p_a/\gamma + V^2/2g \qquad V_0 = 2V \qquad \text{(from Prob. 17.47)}$$
$$87.2/62.2 + (2V)^2/2g = (14.6)(144)/62.2 + V^2/2g \qquad V^2/2g = 10.80\ \text{ft}$$
$$h_{max} = V^2/2g = 10.80\ \text{ft}$$

If $h > 10.80$ ft, the jet will spring free from the diverging tube, so that $p_0 = p_a$, and discharge will decrease.

17.49 For a rounded entrance and tube flowing full as in the sketch for Prob. 17.47, $C_c = 1.0$ both for the throat and for the exit, and thus $C_v = C_d$ for both sections. For the throat, assume the value of C_v as given in Fig. 17-10*a*, and assume that for the tube as a whole the discharge coefficient applied to the exit end is 0.70. If $h = 4$ ft, find the velocity and the pressure head at the throat, and compare with Prob. 17.47.

▮ $Q = AV = C_cA_0(C_v\sqrt{2g\,\Delta h})$. Since $A = C_cA_0$,
$$V = 0.98\sqrt{(2)(32.2)(4)} = 15.73\ \text{ft/s} \qquad V_0 = 2V \qquad \text{(from Prob. 17.47)}$$
$$V_0 = (2)(15.73) = 31.46\ \text{ft/s}$$
$$p_1/\gamma + v_1^2/2g + z_1 = p_0/\gamma + v_0^2/2g + z_0 + h_L$$
$$0 + 0 + 4 = p_0/\gamma + 31.46^2/[(2)(32.2)] + 0 + (1 - 0.98^2)(4) \qquad p_0/\gamma = -11.5\ \text{ft}$$

17.50 Suppose that the diverging tube shown in the figure for Prob. 17.47 is discharging water when $h = 2.7$ m. The area A is $1.7A_0$. Neglecting all friction losses, find the (***a***) velocity and (***b***) pressure head at the throat.

▮
$$V = \sqrt{2gh} = \sqrt{(2)(9.807)(2.7)} = 7.277\ \text{m/s}$$

(***a***)
$$AV = A_0V_0 \qquad (1.7A_0)(7.277) = A_0V_0 \qquad V_0 = 12.4\ \text{m/s}$$

(***b***)
$$p_0/\gamma = V^2/2g - V_0^2/2g = 7.277^2/[(2)(9.807)] - 12.4^2/[(2)(9.807)] = -5.14\ \text{m}$$

17.51 If the tube of Prob. 17.50 is operating at standard atmospheric conditions at a 2500-m elevation, what would be the maximum value of h at which the tube will flow full?

▮ $p_0/\gamma + V_0^2/2g = p_{atm}/\gamma + V^2/2g$. From Table A-8, at 2500-m elevation, $p_{atm} = 74.66$ kPa and $T = 0.4$ °C. From Table A-2, at 0.4 °C, $p_v = 0.636$ kPa and $\gamma = 9.81$ kN/m³. Hence, $p_v/\gamma = 0.636/9.81 = 0.0648$ m, $V_0 = 1.7V$, $0.0648 + (1.7V)^2/2g = 74.66/9.81 + V^2/2g$, $V^2/2g = 3.99$ m. The tube will not flow full if $h > 3.99$ m.

17.52 Find the maximum theoretical head at which the Borda tube of Fig. 17-12 will flow full if the liquid is water at 90 °F and the barometer reads 28.9 inHg. Assume $C_d = 0.72$ for the tube flowing full.

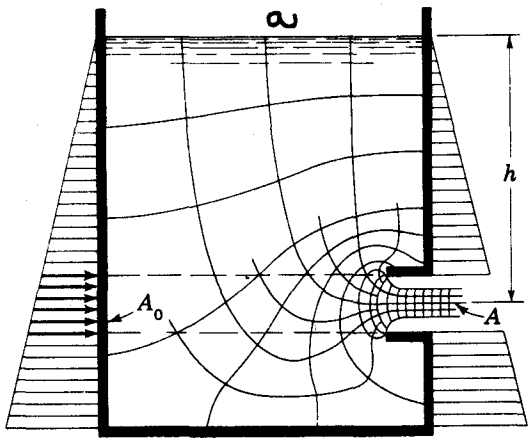


Fig. 17-12

▮ $V = 0.72(2gh)^{1/2}$, $h = (V^2/2g)/0.72^2 = 1.929v^2/2g$. If $C_c = 0.52$ and $C_v = 0.98$ for the contracted throat section, $V_0^2/2g = (V^2/2g)/0.52^2 = (0.72^2/0.52^2)(h) = 1.917h$, $p_a/\gamma + V_a^2/2g + z_a = p_0/\gamma + V_0^2/2g + z_0 + h_L$. From Table A-1, at 80 °F, $p_v = 101$ lb/ft² and $\gamma = 62.1$ lb/ft³. Hence, $(p_0/\gamma)_{min} = p_v/\gamma = 101/62.1 = 1.626$ ft,

$(28.9/12)(13.6) + 0 + h = 1.626 + 1.917h + (1 - 0.98^2)(h + 33.9)$. [$(h + 33.9)$ ft is the total absolute pressure head acting on the orifice.] $h = h_{max} = 31.1$ ft.

17.53 For the 4-in-diameter short tube shown in Fig. 17-13, ***(a)*** what flow of water at 75 °F will occur under a head of 30 ft? ***(b)*** What is the pressure head at section B? Use $C_v = 0.82$.

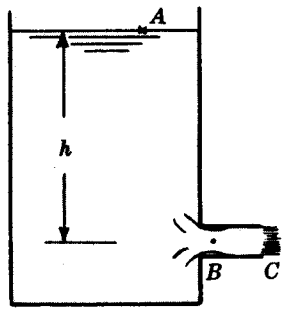


Fig. 17-13

▮ For a standard short tube, the stream contracts at B to about 0.62 of the area of the tube. The lost head from A to B has been measured at about 0.042 times the velocity head at B.

(a)
$$p_A/\gamma + V_A^2/2g + z_A = p_C/\gamma + V_C^2/2g + z_C + h_L$$
$$0 + 0 + 30 = 0 + V_C^2/[(2)(32.2)] + 0 + [(1/0.82^2) - 1]V_C^2/[(2)(32.2)] \qquad V_C = 36.04 \text{ ft/s}$$
$$Q = A_C V_C = [(\pi)(\tfrac{4}{12})^2/4](36.04) = 3.15 \text{ ft}^3/\text{s}$$

(b)
$$p_A/\gamma + V_A^2/2g + z_A = p_B/\gamma + V_B^2/2g + z_B + h_L \qquad V_B = V_C/0.62 = 36.04/0.62 = 58.13 \text{ ft/s}$$
$$h_L = 0.042V_B^2/2g = (0.042)(58.13)^2/[(2)(32.2)] = 2.204 \text{ ft}$$
$$0 + 0 + 30 = p_B/\gamma + 58.13^2/[(2)(32.2)] + 0 + 2.204 \qquad p_B/\gamma = -24.7 \text{ ft of water}$$

17.54 In Prob. 17.53, what maximum head can be used if the tube is to flow full at the exit?

▮ As the head causing flow through the short tube is increased, the pressure head at B will become less and less. For steady flow (and with the tube full at the exit), the pressure head at B must not be less than the vapor pressure head for the liquid at the particular temperature. From Table A-1, for water at 75 °F, $p_v = 63.0$ lb/ft² and $\gamma = 62.3$ lb/ft³; hence, $p_v/\gamma = 63.0/62.3 = 1.0$ ft abs at sea level (-33.0 ft gage). Applying the Bernoulli equation between A and B [see Prob. 17.53(***(b)***)], $C_cAV_B = AV_C = AC_v\sqrt{2gh}$, $V_B(C_v/C_c)\sqrt{2gh}$, $V_B^2/2g = (C_v/C_c)^2(h) = (0.82/0.62)^2(h) = 1.749h$, $0 + 0 + h = -33.0 + 1.749h + 0 + (0.042)(1.749h)$, $h = 40.1$ ft of water. Any head over this value will cause the stream to spring free of the sides of the tube. The tube will then function as an orifice.

17.55 A 4-ft-diameter tank contains oil of specific gravity 0.75. A 3-in-diameter short tube is installed near the bottom of the tank ($C = 0.85$). How long will it take to lower the level of the oil from 6 ft above the tube to 4 ft above the tube?

▮
$$t = t_2 - t_1 = (2A_T/CA_0\sqrt{2g})(h_1^{1/2} - h_2^{1/2}) \qquad A_T = (\pi)(4)^2/4 = 12.57 \text{ ft}^2$$
$$A_0 = (\pi)(\tfrac{3}{12})^2/4 = 0.04909 \text{ ft}^2$$
$$t = [(2)(12.57)/(0.85)(0.04909)\sqrt{(2)(32.2)}](6^{1/2} - 4^{1/2}) = 33.7 \text{ s}$$

17.56 A Borda mouthpiece 60 mm in diameter has a discharge coefficient of 0.52. What is the diameter of the issuing jet?

▮
$$C_c = (D_{jet}/D)^2 \qquad 1 = 2C_dC_v = 2C_v^2C_c \qquad 1 = (2)(0.52)(C_v) \qquad C_v = 0.9615$$
$$1 = (2)(0.9615)^2(C_c) \qquad C_c = 0.5408 \qquad 0.5408 = (D_{jet}/60)^2 \qquad D_{jet} = 44.1 \text{ mm}$$

17.57 A reservoir of variable area is drained by a 150-mm-diameter short pipe with a valve attached. The valve is being adjusted so that the loss (in velocity heads) for the piping system is $K = 1.5 + 0.04t + 0.0001t^2$ with t in seconds. The reservoir area is given by $A = 4 + 0.1y + 0.01y^2$ m² where y is the elevation of the reservoir surface above the centerline of the valve. If $y = 20$ m at $t = 0$, determine y, A, K, and the discharge Q for 300 s. Solve using a computer program.

▮ $Q\,dt = -A\,dy$, $y + 0 + 0 = KV^2/2g = KQ^2/2gA_0^2$, $dy = -HQ/A$, and $Q = \sqrt{C_1 y/K}$ in which $C_1 = 2gA_0^2$. H is the time increment used in Runge–Kutta.

```
10 REM EXAMPLE DRAINAGE OF RESERVOIR—2ND ORDER RUNGE-KUTTA
20 DEF FNQ(YD,TD)=SQR(C1*YD/(K1+K2*TD+K3*TD^2))/(A1+A2*YD+A3*YD^2)
30 DEFINT I: READ A1,A2,A3,K1,K2,K3,Y,TMAX,PI,H,G,D
40 DATA 4.,.1,.01,1.5,.04,.0001,20!,300!,3.1416,1.,9.806001,.15
50 LPRINT: LPRINT"A1,A2,A3,K1,K2,K3=";A1;A2;A3;K1;K2;K3
60 LPRINT"Y,TMAX,PI,H,G,D=";Y;TMAX;PI;H;G;D: LPRINT
70 C1=2!*G*(.25*PI*D^2)^2: T=0!: I=0: A=A1+Y*(A2+A3*Y): Q=FNQ(Y,T)*A: K=K1
80 LPRINT"   T,s     Y,m     Q,m^3/s AREA,m^2    K": LPRINT
90 LPRINT USING"###.### ";T;Y;Q;A;K
100 IF T>= TMAX THEN STOP
110 I=I+1: U1=-H*FNQ(Y,T): U2=-H*FNQ(Y+U1,T+H): Y=Y+.5*(U1+U2): T=T+H
120 A=A1+A2*Y+A3*Y^2: Q=FNQ(Y,T)*A: K=K1+T*(K2+K3*T)
130 IF I MOD 30 = 0 THEN 90 ELSE GOTO 110
140 IF T<TMAX THEN 110 ELSE STOP
```

```
A1,A2,A3,K1,K2,K3= 4   .1   .01   1.5   .04   .0001
Y,TMAX,PI,H,G,D= 20   300   3.1416   1   9.806001   .15

   T,s     Y,m     Q,m^3/s AREA,m^2    K

  0.000  20.000   0.286  10.000   1.500
 30.000  19.265   0.206   9.638   2.790
 60.000  18.687   0.164   9.361   4.260
 90.000  18.202   0.137   9.133   5.910
120.000  17.778   0.119   8.938   7.740
150.000  17.401   0.105   8.768   9.750
180.000  17.060   0.094   8.616  11.940
210.000  16.748   0.085   8.480  14.310
240.000  16.460   0.077   8.355  16.860
270.000  16.191   0.071   8.241  19.590
300.000  15.941   0.066   8.135  22.500
```

17.58 A 2-in ISA flow nozzle is installed in a 3-in pipe carrying water at 72 °F. If a water-air manometer shows a differential of 2 in, find the flow.

▮ This is a trial-and-error type of solution. First assume a reasonable value of K. From Fig. A-20, for $D_2/D_1 = 0.67$, for the level part of the curve, $K = 1.06$. Then $Q = KA_2\sqrt{2g[(p_1/\gamma + z_1) - (p_2/\gamma + z_2)]}$, where $A_2 = (\pi/4)(2^2/144) = 0.0218\ \text{ft}^2$ and $\Delta(p/\gamma + z) = \frac{2}{12} = 0.167$ ft. Thus $Q = (1.06)(0.0218)\sqrt{(64.4)(0.167)} = 0.0757$ cfs. With this first determination of Q, $V_1 = Q/A = 0.0757/0.0492 = 1.54$ fps. Then $D_1'V_1 = (3)(1.54) = 4.62$. From Fig. A-20, $K = 1.04$ and $Q = (1.04/1.06)(0.0757) = 0.0743$ cfs. No further correction is necessary.

17.59 The velocity of water in a 6-in-diameter pipe is 12 ft/s. At the end of the pipe is a nozzle whose velocity coefficient is 0.98. If the pressure in the pipe is 10 psi, what is the velocity in the jet? What is the diameter of the jet? What is the rate of discharge? What is the head loss?

▮ $p_1/\gamma + V_1^2/2g = p_2/\gamma + V_2^2/2g$. Since $p_2 = p_{\text{atm}} = 0$,

$$(V_2)_{\text{ideal}} = [2g(p_1/\gamma + V_1^2/2g)]^{1/2}$$

$$V_2 = C_v(V_2)_{\text{ideal}} = (0.98)\{(2)(32.2)[(10)(144)/62.4 + 12^2/(2)(32.2)]\}^{1/2} = 39.6\ \text{ft/s}$$

$$A_{\text{jet}} = Q/V \qquad Q = AV = [(\pi)(\tfrac{6}{12})^2/4](12) = 2.356\ \text{ft}^3/\text{s} \qquad A_{\text{jet}} = 2.356/39.6 = 0.05949\ \text{ft}^2$$

$$0.05949 = \pi D_{\text{jet}}^2/4 \qquad D_{\text{jet}} = 0.2752\ \text{ft} \quad \text{or} \quad 3.30\ \text{in}$$

$$h_L = h - V^2/2g \qquad h = \text{original head} = p/\gamma + V^2/2g + z$$

$$h = (10)(144)/62.4 + 12^2/[(2)(32.2)] + 0 = 25.31\ \text{ft} \qquad h_L = 25.31 - 39.6^2/[(2)(32.2)] = 0.960\ \text{ft}$$

17.60 A jet of water 4 in in diameter is discharged through a nozzle whose velocity coefficient is 0.96. If the pressure in the pipe is 14 psi and the pipe diameter is 9 in and if it is assumed that there is no contraction of the jet, what is the velocity at the tip of the nozzle? What is the rate of discharge?

▮ $V = \{C_v/[1 - C_v^2(D_2/D_1)^4]^{1/2}\}(2gp/\gamma)^{1/2} = \{0.96/[1 - (0.96)^2(\frac{4}{9})^4]^{1/2}\}\{(2)(32.2)[(14)(144)]/62.4)\}^{1/2} = 44.6$ ft/s

$$Q = AV = [(\pi)(\tfrac{4}{12})^2/4](44.6) = 3.89\ \text{ft}^3/\text{s}$$

17.61 The nozzle in Fig. 17-14 throws a stream of water vertically upward so that the power available in the jet at point 2 is 3.50 hp. If the pressure at the base of the nozzle, point 1, is 36.0 psi, find the ***(a)*** theoretical height to

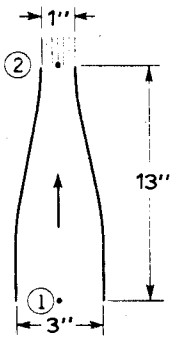


Fig. 17-14

which the jet will rise, (*b*) coefficient of velocity, (*c*) head loss between 1 and 2, and (*d*) theoretical diameter of the jet at a point 18 ft above point 2.

▮

$$P = Q\gamma V_2^2/2g = A_2\gamma V_2^3/2g$$

$$(3.50)(550) = [(\pi)(1.0/12)^2/4](62.4)(V_2^3)/[(2)(32.2)] \qquad V_2 = 71.42 \text{ ft/s}$$

(*a*) $$h_{\text{theor}} = V_2^2/2g = 71.42^2/[(2)(32.2)] = 79.2 \text{ ft}$$

(*b*) $$C_v = V/V_i \qquad V_i = \sqrt{2g(p_1/\gamma + V_1^2/2g + z_1 - z_2)}$$

$$V_1^2/2g = (V_2^2/2g)(D_2/D_1)^4 = \{71.42^2/[(2)(32.2)]\}(1.0/3)^4 = 0.9778 \text{ ft}$$

$$V_i = \sqrt{(2)(32.2)[(36.0)(144)/62.4 + 0.9778 + 0 - \tfrac{13}{12}]} = 73.10 \text{ ft/s}$$

$$C_v = 71.42/73.10 = 0.977$$

(*c*) $$h_L = (1 - C_v^2)h = (1 - 0.977^2)\{73.10^2/[(2)(32.2)]\} = 3.77 \text{ ft}$$

(*d*) $$V_2^2/2g = (V_3^2/2g)(D_3/D_2)^4 \qquad V_3^2/2g = V_2^2/2g - z_3$$

$$D_3 = D_2[(V_2^2/2g)/(V_3^2/2g)]^{1/4} = (1.0)\{[71.42^2/(2)(32.2)]/[71.42^2/(2)(32.2) - 18.0]\}^{1/4} = 1.07 \text{ in}$$

17.62 Suppose a 125-mm ISA flow nozzle is used to measure the flow of water at 50 °C. What would be the reading on a mercury manometer for the flow rate 16 L/s?

▮

$$\Delta(p/\gamma + z) = R'(S_M/S_F - 1) = R'[(13.6/1) - 1] = 12.6R'$$

$$Q = KA_2\sqrt{2g\,\Delta(p/\gamma + z)} = KA_2\sqrt{(2g)(12.6R')}$$

Let $K = 1.0$: $Q = (1.0)[(\pi)(0.125)^2/4]\sqrt{(2)(9.807)(12.6)R'} = 0.1929\sqrt{R'}$. Then

$$16 \times 10^{-3} = 0.1929\sqrt{R'} \qquad R' = 0.00688 \text{ m} = 6.88 \text{ mm}$$

Of course, one should check the Reynolds number to confirm the value of K used.

17.63 A 9-in ISA nozzle is used to measure the flow of crude oil (s.g. = 0.855) at 15 °F. If a mercury manometer shows a reading of 5.5 in, what is the flow? Assume $D_2/D_1 = 0.70$, $\nu = 0.00034 \text{ ft}^2/\text{s}$.

▮

$$\Delta(p/\gamma + z) = R'(S_M/S_F - 1) = R'[(13.6/0.855) - 1] = 14.91R'$$

$$Q = KA_2\sqrt{2g\,\Delta(p/\gamma + z)} = KA_2\sqrt{(2g)(14.91R')}$$

Let $K = 1.08$: $Q = (1.08)[(\pi)(\frac{9}{12})^2/4]\sqrt{(2)(32.2)(5.5/12)(14.91)} = 10.01 \text{ ft}^3/\text{s}$. Check N_R to confirm that the assumed K is applicable: $V = Q/A = 10.01/[(\pi)(\frac{9}{12})^2/4] = 22.66$ ft/s, $N_R = DV/\nu = (\frac{9}{12})(22.66)/0.00034 = 5.00 \times 10^4$. From Fig. A-20, $K = 1.06$ approximately. $Q = (1.06/1.08)(10.01) = 9.82 \text{ ft}^3/\text{s}$.

17.64 Water flows through a 4-in pipe at the rate of 0.952 cfs and thence through a nozzle attached to the end of the pipe. The nozzle tip is 2 in in diameter and the coefficients of velocity and contraction for the nozzle are 0.950 and 0.930, respectively. What pressure head must be maintained at the base of the nozzle if atmospheric pressure surrounds the jet?

▮ Apply the Bernoulli equation between base of nozzle and jet.

$$p_b/\gamma + V_b^2/2g + z_b = p_j/\gamma + V_j^2/2g + z_j + h_L \qquad A_bV_b = A_jV_j = (C_cA_{2\text{-in}})V_j = 0.952$$

$$V_b = 0.952/A_b = 0.952/[(\pi)(\tfrac{4}{12})^2/4] = 10.91 \text{ ft/s}$$

$$V_j = 0.952/(C_cA_{2\text{-in}}) = 0.952/\{(0.930)[(\pi)(\tfrac{2}{12})^2/4]\} = 46.92 \text{ ft/s}$$

$$h_L = (1/C_v^2 - 1)(V_j^2/2g)$$

$$p_b/\gamma + 10.91^2/[(2)(32.2)] + 0 = 0 + 46.92^2/[(2)(32.2)] + 0 + (1/0.950^2 - 1)\{46.92^2/[(2)(32.2)]\}$$

$$p_b/\gamma = 36.0 \text{ ft of water}$$

17.65 A 4-in base diameter by 2-in tip diameter nozzle points downward and the pressure head at the base of the nozzle is 26.0 ft of water. The base of the nozzle is 3.0 ft above the tip and the coefficient of velocity is 0.962. Determine the horsepower in the jet of water.

▮ $P_j = Q\gamma H_j$. For a nozzle, unless C_c is given, it may be taken as unity. Hence, $V_j = V_t$.

$$p_b/\gamma + V_b^2/2g + z_b = p_t/\gamma + V_t^2/2g + z_t + h_L \qquad A_bV_b = A_tV_t$$

$$[(\pi)(4^2/4)](V_b) = [(\pi)(2^2/4)](V_t) \qquad V_b = V_t/4 \qquad h_L = (1/C_v^2 - 1)(V_t^2/2g)$$

$$26.0 + (V_t/4)^2/[(2)(32.2)] + 3.0 = 0 + V_t^2/[(2)(32.2)] + 0 + (1/0.962^2 - 1)\{V_t^2/[(2)(32.2)]\}$$

$$V_t = 42.83 \text{ ft/s} \qquad Q = A_tV_t = [(\pi)(\tfrac{2}{12})^2/4](42.83) = 0.9344 \text{ ft}^3/\text{s}$$

$$H_j = 0 + V_t^2/[(2)(32.2)] + 0 = 42.83^2/[(2)(32.2)] = 28.48 \text{ ft}$$

$$P_j = (0.9344)(62.4)(28.48) = 1661 \text{ ft-lb/s} = \tfrac{1661}{550} = 3.02 \text{ hp}$$

17.66 A nozzle with a 4-in-diameter tip is installed in a 10-in pipe. Medium fuel oil at 80 °F flows through the nozzle at the rate of 3.49 cfs. Assume the calibration of the nozzle is represented by curve $\beta = 0.40$ on Fig. A-21. Calculate the differential gage reading if a liquid of specific gravity 13.6 is the gage liquid.

▮

$$Q = A_4V_4 = A_4C\sqrt{[2g(p_A/\gamma - p_B/\gamma)]/[1 - (A_4/A_{10})^4]}$$
$$= A_4C\sqrt{[2g(p_A/\gamma - p_B/\gamma)]/[1 - (\tfrac{4}{10})^4]} \qquad (1)$$

Figure A-21 indicates that C varies with Reynolds number: $V_4 = Q/A_4 = 3.49/[\frac{1}{4}\pi(\frac{4}{12})^2] = 40.0$ ft/s and $N_R = (40.0)(\frac{4}{12})/(3.65 \times 10^{-5}) = 365\,000$. Curve for $\beta = 0.40$ gives $C = 0.993$. Thus $3.49 = \frac{1}{4}\pi(\frac{4}{12})^2(0.993)\sqrt{[2g(p_A/\gamma - p_B/\gamma)]/[1 - (\frac{4}{10})^4]}$ and $(p_A/\gamma - p_B/\gamma) = 24.6$ ft of fuel oil. Differential gage principles produce, using specific gravity of the oil $= 0.851$, $24.6 = h[(13.6/0.851) - 1]$ and $h = 1.64$ ft (gage reading).

17.67 Derive an expression for the flow of a compressible fluid through a nozzle flowmeter and a Venturi meter.

▮ Since the change in velocity takes place in a very short period of time, little heat can escape and adiabatic conditions will be assumed. The Bernoulli theorem for compressible flow gives $\{[k/(k-1)](p_1/\gamma_1) + (V_1^2/2g) + z_1\} - H_L = \{[k/(k-1)](p_1/\gamma_1)(p_2/p_1)^{(k-1)/k} + (V_2^2/2g) + z_2\}$.

For a nozzle meter and for a horizontal Venturi meter, $z_1 = z_2$ and the lost head will be taken care of by means of the coefficient of discharge. Also, since $C_c = 1.00$, $W = \gamma_1A_1V_1 = \gamma_2A_2V_2$. Then upstream $V_1 = W/\gamma_1A_1$, downstream $V_2 = W/\gamma_2A_2$. Substituting and solving for W, $W^2/\gamma_2^2A_2^2 - W^2/\gamma_1^2A_1^2 = 2g[k/(k-1)](p_1/\gamma_1)[1 - (p_2/p_1)^{(k-1)/k}]$ or (ideal) $W = [\gamma_2A_2/\sqrt{1 - (\gamma_2/\gamma_1)^2(A_2/A_1)^2}\sqrt{[2gk/(k-1)](p_1/\gamma_1)[1 - (p_2/p_1)^{(k-1)/k}]}$. It may be more practical to eliminate γ_2 under the radical. Since $\gamma_2/\gamma_1 = (p_2/p_1)^{1/k}$,

$$\text{(ideal)} \quad W = \gamma_2A_2\sqrt{[2gk/(k-1)](p_1/\gamma_1)[1 - (p_2/p_1)^{(k-1)/k}]/[1 - (A_2/A_1)^2(p_2/p_1)^{2/k}]} \qquad (1)$$

The true value of W in pounds per second is obtained by multiplying the right-hand side of the equation by coefficient C. From Eq. (1) of Prob. 17.66, the following equation may be written: $W = \gamma Q = [\gamma A_2C/\sqrt{1 - (A_2/A_1)^2}]\sqrt{2g(\Delta p/\gamma)}$ or $W = \gamma KA_2\sqrt{2g(\Delta p/\gamma)}$. The above equation can be expressed more generally so that it will apply both to compressible and incompressible fluids. An expansion (adiabatic) factor Y is introduced and the value of γ_1 at inlet is specified. The fundamental relation is then

$$W = \gamma_1KA_2Y\sqrt{2g(\Delta p/\gamma_1)} \qquad (2)$$

For incompressible fluids, $Y = 1$. For compressible fluids, equate expressions (1) and (2) and solve for Y. By so doing,

$$Y = \sqrt{\{[1 - (A_2/A_1)^2]/[1 - (A_2/A_1)^2(p_2/p_1)^{2/k}]\}\{[k/(k-1)][1 - (p_2/p_1)^{(k-1)/k}](p_2/p_1)^{2/k}/(1 - p_2/p_1)\}}$$

This expansion factor Y is a function of three dimensionless ratios. Table A-18 lists some typical values for nozzle flowmeters and for Venturi meters.

17.68 Air at a temperature of 80 °F flows through a 4-in pipe and through a 2-in flow nozzle. The pressure differential is 0.522 ft of oil, s.g. = 0.910. The pressure upstream from the nozzle is 28.3 psi gage. How many pounds per second are flowing for a barometric reading of 14.7 psi, assuming the air has constant density.

▮ $\gamma = p/RT$, $\gamma_1 = [(28.3 + 14.7)144]/[53.3(460 + 80)] = 0.215\ \text{lb/ft}^3$. From differential gage principles, using pressure heads in feet of air, $\Delta p/\gamma_1 = 0.522[(\gamma_{oil}/\gamma_{air}) - 1] = 0.522\{[(0.910)(62.4)/0.215] - 1\} = 137$ ft of air.

Assuming $C = 0.980$ and using equation (*1*) of Prob. 17.66 after multiplying by γ_1, we have $W = \gamma_1 Q = (0.215)(\frac{1}{4}\pi)(\frac{2}{12})^2(0.980)\sqrt{2g(137)/[1 - (\frac{2}{4})^4]} = 0.445$ lb/s.

To check the value of C, find Reynolds number and use the appropriate curve on Fig. A-21 (here $\gamma_1 = \gamma_2$ and $\nu = 16.9 \times 10^{-5}\ \text{ft}^2/\text{s}$ at standard atmosphere).: $V_2 = W/A_2\gamma_2 = W/(\pi d_2^2/4)\gamma_2$. Then $N_R = V_2 d_2/\nu = 4W/\pi d_2 \nu \gamma_2 = 4(0.445)/[\pi(\frac{2}{12})(16.9 \times 14.7/43.0)10^{-5}(0.215)] = 274\,000$.

From Fig. A-21, $C = 0.986$. Recalculating, $W = 0.447$ lb/s.

Further refinement in calculation is not warranted inasmuch as the Reynolds number will not be changed materially, nor will the value of C read from Fig. A-21.

17.69 Find Q for flow of water in a horizontal pipe of inside diameter $D = 100$ mm using a long-radius-type flow nozzle. The value of h for the manometer is 140 mm. The throat diameter of the nozzle is 60 mm. Take $\rho = 999\ \text{kg/m}^3$ and $\nu = 1.12 \times 10^{-3}\ \text{m}^2/\text{s}$.

▮
$$p_1 - p_2 = h(\gamma_{Hg} - \gamma_{H_2O}) = 0.140(13.6 - 1)(999)(9.81) = 17.300\ \text{kPa} \qquad (1)$$
$$C_d = 0.99622 + 0.00059D - (6.36 + 0.13D - 0.24\beta^2)(1/\sqrt{N_R})$$
$$(C_d)_{noz} = [0.99622 + 0.00059(10.0/2.54)] - [6.36 + 0.13(10.0/2.54) - (0.24)(0.6^2)](1/\sqrt{N_R})$$
$$= 0.998 - 6.785(1/\sqrt{N_R}) \qquad (2)$$
$$q_{act} = (C_d)_{noz}A_2\{2[(p_1 - p_2)/\rho]/[1 - (A_2/A_1)^2]\}^{1/2} = (C_d)_{noz}A_2\{2[(p_1 - p_2)/\rho]/(1 - \beta^4)\}^{1/2}$$
$$A_2V_2 = [0.998 - 6.785(1/\sqrt{N_R})](A_2)[2(17\,300/999)/(1 - 0.6^4)]^{1/2}$$

This equation becomes

$$V_2 = (0.998 - 6.785/\sqrt{\rho V_1 d/\mu})(6.31) = \left(0.998 - \{6.785/\sqrt{[(999)(V_1)(0.60)]/(1.12 \times 10^{-3})}\}\right)(6.31) \qquad (3)$$

From *continuity*,

$$V_2 = (\tfrac{100}{60})^2 V_1 \qquad (4)$$

Therefore, substituting for V_2 in Eq. (*3*) using Eq. (*4*), $V_1 = [0.998 - (0.0293/\sqrt{V_1})](2.77)$. Solving by trial and error we get $V_1 = 2.22$ m/s. The Reynolds number is then $N_R = [(999)(0.060)(2.22)/(1.12 \times 10^{-3})] = 1.189 \times 10^5$. We are well within the range of Eq. (*2*), so we get $Q = [(\pi)(0.100^2)/4](2.22) = 0.01744\ \text{m}^3/\text{s}$.

17.70 Determine the flow through a 6-in-diameter water line that contains a 4-in-diameter flow nozzle. The mercury-water differential manometer has a gage difference of 10 in. Water temperature is 60 °F.

▮ $Q = CA_2\sqrt{2gR'(S_0/S_1 - 1)}$. From Fig. A-22, for $A_2/A_1 = (\frac{4}{6})^2 = 0.4444$, assume that the horizontal region of the curves applies. Hence, $C = 1.056$.

$$Q = (1.056)[(\pi)(\tfrac{4}{12})^2/4]\sqrt{(2)(32.2)(\tfrac{10}{12})[(13.6/1) - 1]} = 2.40\ \text{ft}^3/\text{s}$$
$$V = Q/A = 2.40/[(\pi)(\tfrac{6}{12})^2/4] = 12.22\ \text{ft/s}$$
$$N_R = \rho DV/\mu = (1.94)(\tfrac{6}{12})(12.22)/(2.35 \times 10^{-5}) = 5.04 \times 10^5$$

Figure A-22 shows the value of C to be correct; therefore, the discharge is $2.40\ \text{ft}^3/\text{s}$.

17.71 Air flows through a 100-mm-diameter ISA flow nozzle in a 150-mm-diameter pipe. $p_1 = 140$ kPa abs; $T_1 = 10$ °C; and a differential manometer with liquid of s.g. 2.80 has a gage difference of 0.9 m when connected between taps. Calculate the mass rate of flow. [MW of air = 29.]

▮
$$\dot{m} = \rho Q = \rho CYA_2\sqrt{2\,\Delta p/\rho} \qquad \rho = p/RT = 0.140/[(8.312/29)(10 + 273)] = 1.726\ \text{kg/m}^3$$
$$A_2/A_1 = (D_2/D_1)^2 = (\tfrac{100}{150})^2 = 0.4444$$

From Fig. A-22, C is assumed to be 1.055.

$$p_2 = p_1 - \gamma\,\Delta h = 140 - (9.79)(2.80)(0.9) = 115.3\ \text{kPa}$$
$$p_2/p_1 = 115.3/140 = 0.824 \qquad D_2/D_1 = \tfrac{100}{150} = 0.667$$

From Fig. A-26, $Y = 0.875$.

$$A_2 = (\pi)(0.100)^2/4 = 0.007854 \text{ m}^2$$

$$\dot{m} = (1.726)(1.055)(0.875)(0.007854)\sqrt{(2)(140 - 115.3)(1000)/1.726} = 2.12 \text{ kg/s}$$

$$N_R = \rho DV/\mu \qquad V = \dot{m}/\rho A \qquad A_1 = (\pi)(0.150)^2/4 = 0.01767 \text{ m}^2$$

$$V_1 = 2.12/[(1.726)(0.01767)] = 69.51 \text{ m/s}$$

$$N_R = (1.726)(0.150)(69.51)/(1.77 \times 10^{-5}) = 1.02 \times 10^6$$

From Fig. A-22, assumed $C = 1.055$ is O.K.; hence, $\dot{m} = 2.12$ kg/s.

17.72 A 3.5-in-diameter ISA nozzle is used to measure flow of water at 50 °C in a 9-in-diameter pipe. What gage difference on a water-mercury manometer is required for 400 gpm?

▮ $$R' = \Delta p/[(\gamma)(S_0/S_1 - 1)] \qquad \Delta p = (\rho/2)(Q/CA_2)^2 \qquad A_2/A_1 = (3.5/9)^2 = 0.1512$$

$$V_1 = Q/A_1 = (400)(0.002228)/[(\pi)(\tfrac{9}{12})^2/4] = 2.017 \text{ ft/s}$$

$$(N_R)_1 = V_1D_1/\nu_1 = (2.017)(\tfrac{9}{12})/(1.40 \times 10^{-5}) = 1.08 \times 10^5$$

From Fig. A-22, $C = 0.995$.

$$A_2 = (\pi)(3.5/12)^2/4 = 0.06681 \text{ ft}^2$$

$$\Delta p = (1.94/2)\{(400)(0.002228)/[(0.995)(0.06681)]\}^2 = 174.3 \text{ lb/ft}^2$$

$$R' = 174.3/\{(62.4)[(13.6/1) - 1]\} = 0.2217 \text{ ft} \quad \text{or} \quad 2.66 \text{ in}$$

17.73 We want to meter the volume flow of water ($\rho = 1000$ kg/m^3, $\nu = 1.02 \times 10^{-6}$ m^2/s) moving through a 200-mm-diameter pipe at an average velocity of 2.0 m/s. If the differential pressure gage selected reads accurately at $p_1 - p_2 = 50$ kPa, what size meter should be selected for installing a long-radius flow nozzle? What would be the nonrecoverable head loss?

▮ Here the unknown is the β ratio of the meter. Since the discharge coefficient is a complicated function of β, iteration will be necessary. We are given $D = 0.2$ m and $V_1 = 2.0$ m/s. The pipe-approach Reynolds number is thus $N_R = V_1D/\nu = (2.0)(0.2)/(1.02 \times 10^{-6}) = 392\,000$. The generalized formula gives the throat velocity as $V_t = V_1/\beta^2 = \alpha[2(p_1 - p_2)/\rho]^{1/2}$ where everything is known except α and β. Solve for β^2:

$$\beta^2 = (1/\alpha)(\rho V_1^2/2\,\Delta p)^{1/2} \tag{1}$$

With $V_1 = 2.0$ m/s and $\Delta p = 50\,000$ Pa, $\beta^2 = (1/\alpha)\{(1000)(2.0)^2/[2(50\,000)]\}^{1/2} = 0.2/\alpha$ or

$$\beta = 0.447/\alpha^{1/2} \tag{2}$$

The solution depends only on getting the proper flow coefficient. A good guess for flow-nozzle design is $\alpha = 1.0$. Iterate Eq. (2) and list the results:

α	β, Eq. (2)	C_d, Fig. A-23	$E = (1 - \beta^4)^{-1/2}$	$\alpha = EC_d$
1.0	0.447	0.9895	1.0206	1.0099
1.0099	0.445	0.9895	1.0202	1.0095
1.0095	0.445			

Convergence is rapid to $\beta = 0.445$, $d = \beta D = 89$ mm. The throat velocity is $2.0/(0.445)^2 = 10.1$ m/s; the throat head is $(10.1)^2/[2(9.81)] = 5.2$ m. From Fig. A-24 for the nozzle read $K_m \approx 0.7$. Then the nozzle loss is $h_m = 0.7(5.2 \text{ m}) = 3.6$ m.

17.74 Gasoline at 20 °C flows at 80 L/s through a 200-mm-diameter pipe and is metered by a 100-mm-diameter nozzle. What is the expected pressure drop across the nozzle? ($\rho = 680$ kg/m^3; $\nu = 4.29 \times 10^{-7}$ m^2/s)

▮ $$Q = C_dA_t[(2\,\Delta p/\rho)/(1 - \beta^4)]^{1/2} \qquad C_d = 0.9965 - 0.00653\beta^{1/2}[10^6/(N_R)_D]^{1/2}$$

$$\beta = d/D = 100/200 = 0.500 \qquad N_R = DV/\nu \qquad V = Q/A = 80 \times 10^{-3}/[(\pi)(0.200)^2/4] = 2.546 \text{ m/s}$$

$$(N_R)_D = (0.200)(2.546)/(4.29 \times 10^{-7}) = 1.19 \times 10^6$$

$$C_d = 0.9965 - (0.00653)(0.500)^{1/2}[10^6/(1.19 \times 10^6)]^{1/2} = 0.9923$$

$$80 \times 10^{-3} = (0.9923)[(\pi)(0.100)^2/4][(2\,\Delta p/680)/(1 - 0.500^4)]^{1/2} \qquad \Delta p = 33.59 \text{ kPa}$$

17.75 Ethyl alcohol at 20 °C flowing in a 100-mm-diameter pipe is metered through a 60-mm-diameter nozzle. If the measured pressure drop is 55 kPa, calculate the volume flow. ($\rho = 789\ \text{kg/m}^3$; $\nu = 1.52 \times 10^{-6}\ \text{m}^2/\text{s}$)

▮ $Q = C_d A_t[(2\,\Delta p/\rho)/(1-\beta^4)]^{1/2}$. Estimate $C_d = 0.99$.

$$\beta = d/D = 60/100 = 0.600$$

$$Q = (0.99)[(\pi)(0.060)^2/4]\{[(2)(55\,000)/789]/(1-0.600^4)\}^{1/2} = 0.03543\ \text{m}^3/\text{s}$$

$$C_d = 0.9965 - 0.00653\beta^{1/2}[10^6/(N_R)_D]^{1/2} \qquad N_R = DV/\nu$$

$$V = Q/A = 0.03543/[(\pi)(0.100)^2/4] = 4.511\ \text{m/s}$$

$$(N_R)_D = (0.100)(4.511)/(1.52 \times 10^{-6}) = 2.97 \times 10^5$$

$$C_d = 0.9965 - (0.00653)(0.600)^{1/2}[10^6/(2.97 \times 10^5)]^{1/2} = 0.9872$$

$$Q = (0.9872)[(\pi)(0.060)^2/4]\{[(2)(55\,000)/789]/(1-0.600^4)\}^{1/2} = 0.03533\ \text{m}^3/\text{s} = 127\ \text{m}^3/\text{h}$$

17.76 Kerosene at 20 °C flows at 23 m³/h in a 100-mm-diameter pipe. The flow is to be metered by an ISA 1932 flow nozzle so that the pressure drop is 8 kPa. What is the necessary nozzle diameter? ($\rho = 804\ \text{kg/m}^3$; $\nu = 2.39 \times 10^{-6}\ \text{m}^2/\text{s}$)

▮ $Q = C_d A_t(\beta)^2[(2\,\Delta p/\rho)/(1-\beta^4)]^{1/2}$. Estimate $C_d = 0.99$.

$$23/3600 = (0.99)[(\pi)(0.100)^2/4](\beta)^2\{[(2)(8000)/804]/(1-\beta^4)\}^{1/2}$$

$$\beta = 0.426 \qquad C_d = 0.9965 - 0.00653\beta^{1/2}[10^6/(N_R)_D]^{1/2}$$

$$N_R = DV/\nu \qquad V = Q/A = (23/3600)/[(\pi)(0.100)^2/4] = 0.8135\ \text{m/s}$$

$$(N_R)_D = (0.100)(0.8135)/(2.39 \times 10^{-6}) = 3.40 \times 10^4$$

$$C_d = 0.9965 - (0.00653)(0.426)^{1/2}[10^6/(3.40 \times 10^4)]^{1/2} = 0.9734$$

$$23/3600 = (0.9734)[(\pi)(0.100)^2/4](\beta)^2\{[(2)(8000)/804]/(1-\beta^4)\}^{1/2}$$

$$\beta = 0.430 \qquad d_{\text{nozzle}} = \beta D = (0.430)(100) = 43\ \text{mm}$$

17.77 Water flows from a large tank through an orifice and discharges to the atmosphere, as shown in Fig. 17-15. The coefficients of velocity and contraction are 0.96 and 0.62, respectively. Find the diameter and actual velocity in the jet and the discharge from the orifice.

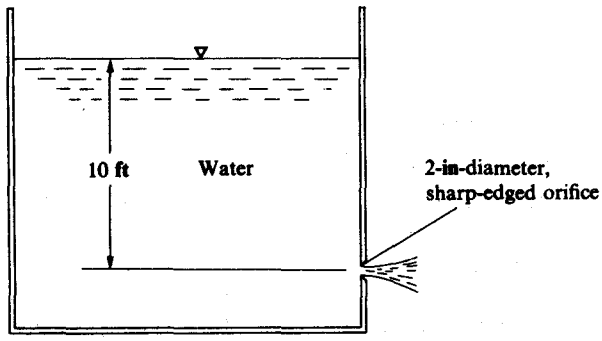


Fig. 17-15

▮

$$C_c = a/A \qquad 0.62 = a/[(\pi)(\tfrac{2}{12})^2/4] \qquad a = 0.01353\ \text{ft}^2$$

$$(\pi)(D_{\text{jet}})^2/4 = 0.01353 \qquad D_{\text{jet}} = 0.1313\ \text{ft} \quad \text{or} \quad 1.58\ \text{in}$$

In order to determine the actual velocity in the jet, the discharge from the orifice will be determined next: $C = C_c C_v = (0.96)(0.62) = 0.595$, $Q = CA\sqrt{2gh} = (0.595)[(\pi)(\tfrac{2}{12})^2/4]\sqrt{(2)(32.2)(10)} = 0.329\ \text{ft}^3/\text{s}$, $v = Q/a = 0.329/0.01353 = 24.3\ \text{ft/s}$.

17.78 Oil discharges from a pipe through a sharp-crested, round orifice, as shown in Fig. 17-16. The coefficients of contraction and velocity are 0.62 and 0.98, respectively. Find the discharge from the orifice and the diameter and actual velocity in the jet.

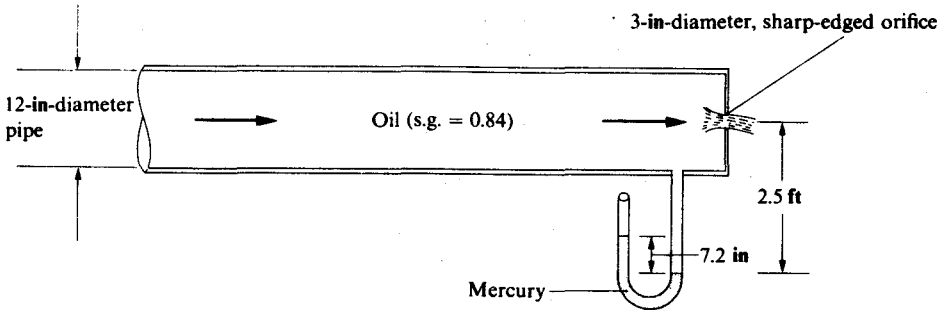


Fig. 17-16

$$Q = CA\sqrt{2g(p_1/\gamma)}/\sqrt{1 - C^2(D/D_p)^4} \qquad C = C_v C_c = (0.98)(0.62) = 0.608$$

$$p_1/\gamma + 2.5 - (13.6/0.84)(7.2/12) = 0 \qquad p_1/\gamma = 7.21 \text{ ft of oil}$$

$$Q = (0.608)[(\pi)(\tfrac{3}{12})^2/4]\sqrt{(2)(32.2)(7.21)}/\sqrt{1 - (0.608)^2(\tfrac{3}{12})^4} = 0.644 \text{ ft}^3/\text{s}$$

$$C_c = a/A \qquad 0.62 = a/[(\pi)(\tfrac{3}{12})^2/4] \qquad a = 0.03043 \text{ ft}^2$$

$$(\pi)(D_{jet})^2/4 = 0.03043 \qquad D_{jet} = 0.1968 \text{ ft} \quad \text{or} \quad 2.36 \text{ in}$$

$$v = Q/a = 0.644/0.03043 = 21.2 \text{ ft/s}$$

17.79 Oil flows through a pipe as shown in Fig. 17-17. The coefficient of discharge for the orifice in the pipe is 0.63. What is the discharge of oil in the pipe?

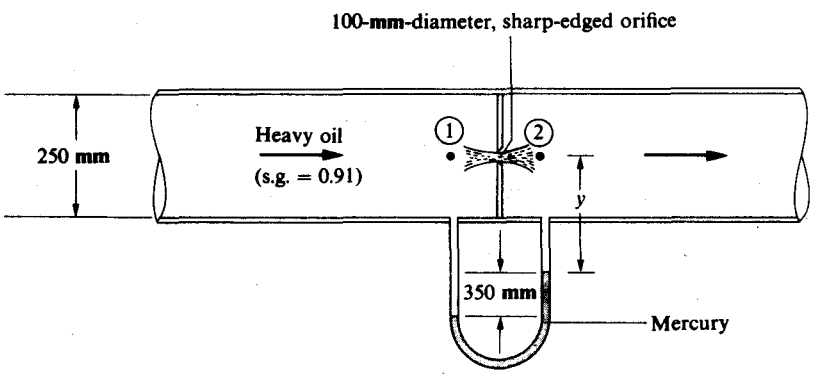


Fig. 17-17

$$Q = CA\sqrt{2g[(p_1 - p_2)/\gamma]}[1 + (C^2/2)(D/D_p)^4] \qquad A = (\pi)(0.100)^2/4 = 0.007854 \text{ m}^2$$

$$p_1/\gamma + y + 0.350 - (13.6/0.91)(0.350) - y = p_2/\gamma \qquad (p_1 - p_2)/\gamma = 4.881 \text{ m of oil}$$

$$Q = (0.63)(0.007854)\sqrt{(2)(9.807)(4.881)}[1 + (0.63^2/2)(\tfrac{100}{250})^4] = 0.0487 \text{ m}^3/\text{s}$$

17.80 The tank shown in Fig. 17-18*a* has the form of a frustum of a cone with dimensions as shown in the figure. The bottom of the tank contains an orifice that has a coefficient of discharge of 0.62. The tank contains water to its depth of 3.5 m. Find the diameter of the orifice needed to empty the tank in 8 min.

$dt = A_s\, dh/CA\sqrt{2gh}$. Rearranging this equation gives

$$CA\sqrt{2gh}\, dt = A_s\, dh \tag{1}$$

In order to solve this problem, A_s must be expressed as a function of the water depth, h. This can be accomplished as follows: By observing Fig. 17-18*b*, it is evident that $y/(y + 3.5) = 1.5/3$, $y = 3.5$; $x/1.5 = (3.5 + h)/(3.5 + 3.5)$,

$$x = 0.7500 + 0.2143h \tag{2}$$

$$A = (\pi)(D)^2/4 = 0.7854D^2$$

$$A_s = \pi x^2 \tag{3}$$

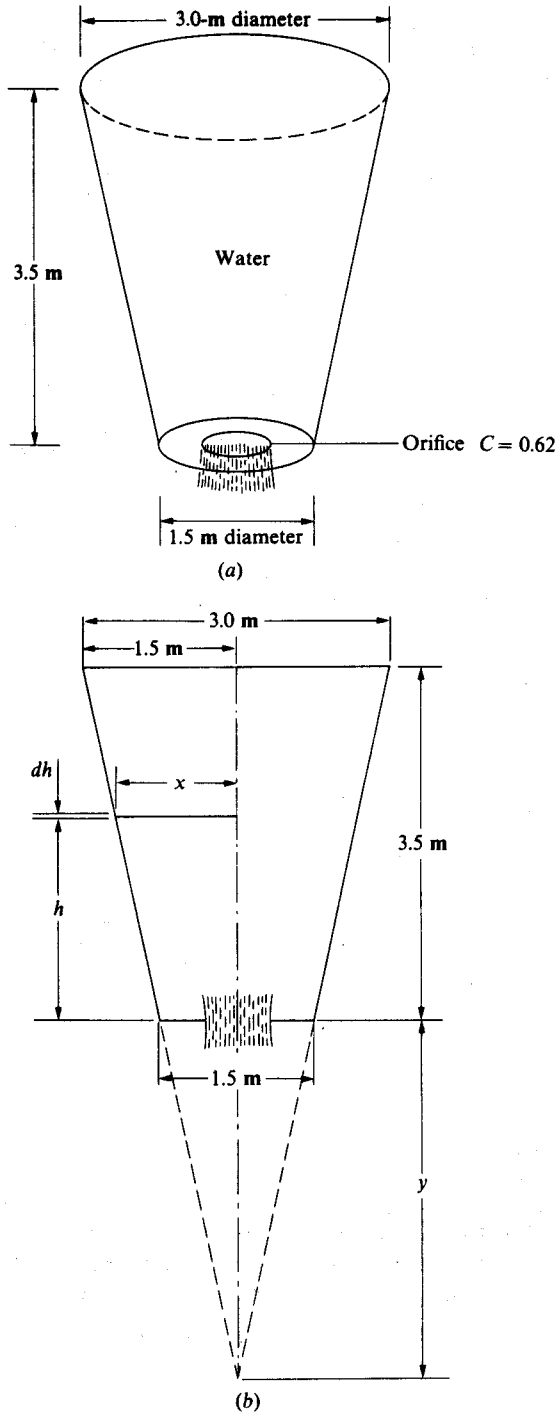


Fig. 17-18

Substituting Eq. (2) into Eq. (3) gives $A_s = (\pi)(0.7500 + 0.2143h)^2$. Therefore, substituting into Eq. (1),

$$(0.62)(0.7854D^2)\sqrt{(2)(9.807)}h^{1/2}\,dt = (\pi)(0.7500 + 0.2143h)^2\,dh$$

$$2.157D^2h^{1/2}\,dt = (1.767 + 1.010h + 0.1443h^2)\,dh$$

$$2.157D^2 = (1.767h^{-1/2} + 1.010h^{1/2} + 0.1443h^{3/2})\,dh/dt$$

$$= \left[\int_0^{3.5} (1.767h^{-1/2} + 1.010h^{1/2} + 0.1443h^{3/2})\,dh\right] \Big/ \left(\int_0^{480} dt\right)$$

$$= [3.534h^{1/2} + 0.6733h^{3/2} + 0.05772h^{5/2}]_0^{3.5} / [t]_0^{480}$$

$$= 12.34/480$$

$$D = 0.109\text{ m} \qquad \text{or} \qquad 109\text{ mm}$$

17.81 A rectangular tank is divided by a partition into two chambers, as shown in Fig. 17-19*a*. A 6-in-diameter orifice, for which C is 0.65, is located near the bottom of the partition. At a certain time, the water level in chamber A is 10.0 ft higher than that in chamber B. Find the time it will take for the water surfaces in the two chambers to be at the same level.

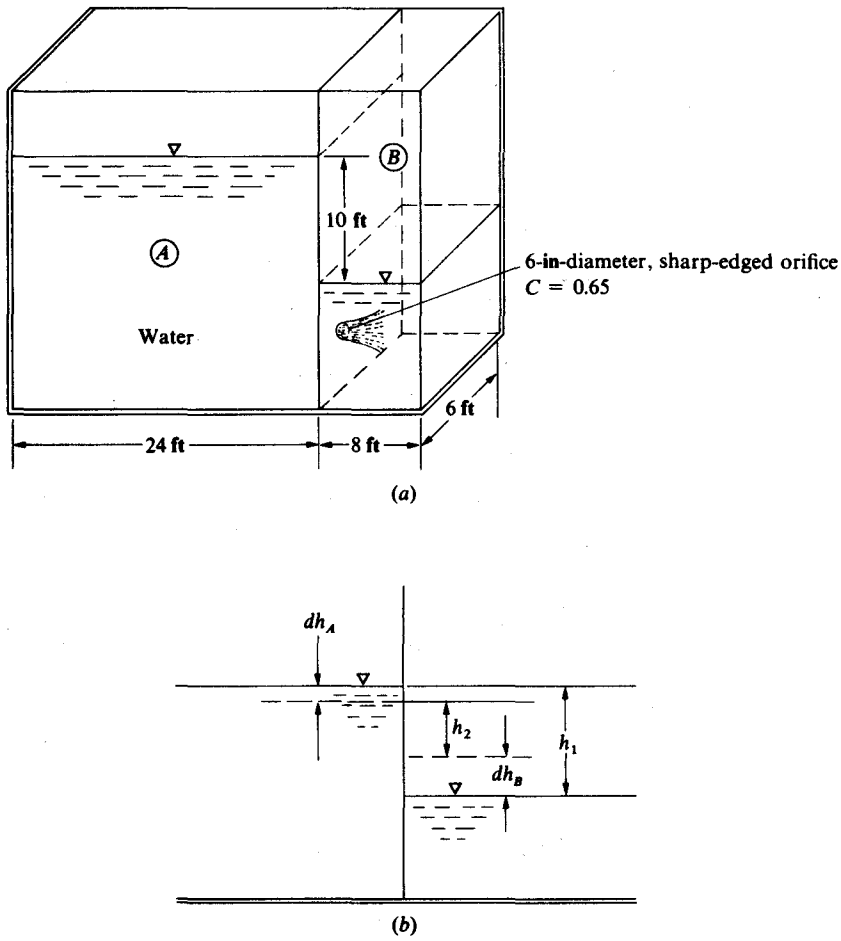


Fig. 17-19

▮ $dV = CA\sqrt{2gh}\,dt$. Let h = difference in elevations of the two water levels at any time, dh = change in h in time dt, and dV = volume of water flowing into chamber B in time dt. As illustrated in Fig. 17-19*b*, during a time dt, the water level in chamber A drops an amount indicated by dh_A while the level in chamber B rises an amount indicated by dh_B. During the same time, the difference in elevations of the two water levels decreases from h_1 to h_2. The volume of water leaving chamber A equals the volume entering chamber B. Hence,

$$(6)(24)(dh_A) = (6)(8)(dh_B) \qquad dh_B = 3.00dh_A \tag{1}$$

Also from Fig. 17-19*b*,

$$dh = dh_A + dh_B \tag{2}$$

Substituting Eq. (*1*) into Eq. (*2*) gives $dh = dh_A + 3.00dh_A = 4.00dh_A$ or $dh_A = 0.250dh$, $dV = (0.250dh)(24)(6) = 36.0dh$, $A = (\pi)(0.5)^2/4 = 0.1963\ \text{ft}^2$, $36.0dh = (0.65)(0.1963)\sqrt{(2)(32.2)}h^{1/2}\,dt$, $35.16h^{-1/2}\,dh = dt$,

$$\int_0^t dt = 35.16\int_0^{10} h^{-1/2}\,dh \qquad [t]_0^t = 35.16[2h^{1/2}]_0^{10} \qquad t = 222\ \text{s}$$

17.82 Water flows from a large tank through an orifice and discharges to the atmosphere, as shown in Fig. 17-15, but the diameter of the orifice is 3 in and the head is 15 ft. The coefficients of velocity and contraction are 0.97 and 0.61, respectively. Find the diameter and actual velocity in the jet and the discharge from the orifice.

$$C_c = a/A \qquad 0.61 = a/[(\pi)(\tfrac{3}{12})^2/4] \qquad a = 0.02994\ \text{ft}^2$$
$$(\pi)(D_{\text{jet}})^2/4 = 0.02994 \qquad D_{\text{jet}} = 0.1952\ \text{ft} \quad \text{or} \quad 2.34\ \text{in}$$

In order to determine the actual velocity in the jet, the discharge from the orifice will be determined next: $C = C_c C_v = (0.97)(0.61) = 0.592$, $Q = CA\sqrt{2gh} = (0.592)[(\pi)(\tfrac{3}{12})^2/4]\sqrt{(2)(32.2)(15)} = 0.903\ \text{ft}^3/\text{s}$, $v = Q/a = 0.903/0.02994 = 30.2\ \text{ft/s}$.

17.83 Oil with a specific gravity of 0.86 discharges from a 250-mm-diameter pipe through a 100-mm-diameter sharp-edged orifice, as shown in Fig. 17-20. Coefficients of velocity and contraction are 0.98 and 0.63, respectively. Determine the discharge of oil in the pipe, diameter of the jet, and velocity of oil in the jet.

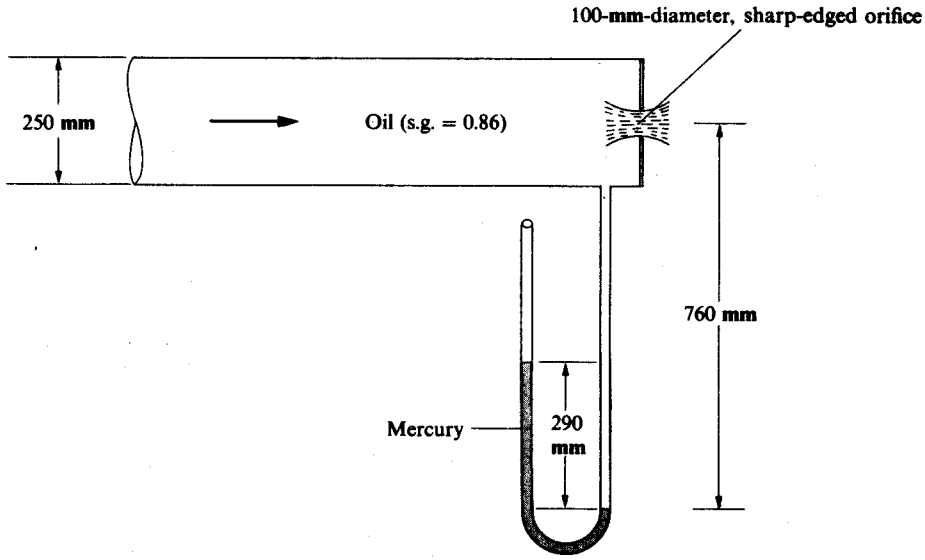


Fig. 17-20

$$Q = CA\sqrt{2g(p_1/\gamma)}/\sqrt{1 - C^2(D/D_p)^4} \qquad C = C_v C_c = (0.98)(0.63) = 0.617$$
$$p_1/\gamma + 0.760 - (13.6/0.86)(0.290) = 0 \qquad p_1/\gamma = 3.826\ \text{m of oil}$$
$$Q = (0.617)[(\pi)(0.100)^2/4]\sqrt{(2)(9.807)(3.826)}/\sqrt{1 - (0.617)^2(0.100)^4} = 0.0420\ \text{m}^3/\text{s}$$
$$C_c = a/A \qquad 0.63 = a/[(\pi)(0.100)^2/4] \qquad a = 0.004948\ \text{m}^2$$
$$(\pi)(D_{\text{jet}})^2/4 = 0.004948 \qquad D_{\text{jet}} = 0.0794\ \text{m} \quad \text{or} \quad 79.4\ \text{mm}$$
$$v = Q/a = 0.0420/0.004948 = 8.49\ \text{m/s}$$

17.84 Oil flows in the pipe shown in Fig. 17-21. The orifice in the pipe has a coefficient of discharge of 0.64. Compute the discharge of the oil in the pipe.

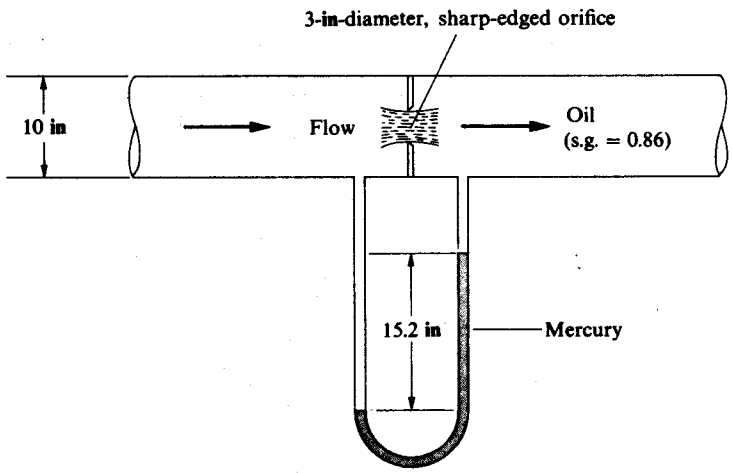


Fig. 17-21

$$Q = CA\sqrt{2g(p_1 - p_2)/\gamma}[1 + (C^2/2)(D/D_p)^4] \qquad A = (\pi)(\tfrac{3}{12})^2/4 = 0.04909\ \text{ft}^2$$

$$p_1/\gamma + y + 15.2/12 - (13.6/0.86)(15.2/12) - y = p_2/\gamma \qquad (p_1 - p_2)/\gamma = 18.76\ \text{ft of oil}$$

$$Q = (0.64)(0.04909)\sqrt{(2)(32.2)(18.76)}[1 + (0.64^2/2)(\tfrac{3}{10})^4] = 1.09\ \text{ft}^3/\text{s}$$

17.85 The water tank shown in Fig. 17-22*a* has the form of the frustum of a cone. The diameter of the top of the tank is 12 ft, while that at the bottom is 8 ft. The bottom of the tank contains a round, sharp-edged orifice, which has a diameter of 4 in. The discharge coefficient of the orifice is 0.60. If the tank is full at a depth of 10.0 ft, as shown in Fig. 17-22*a*, how long will it take to empty the tank?

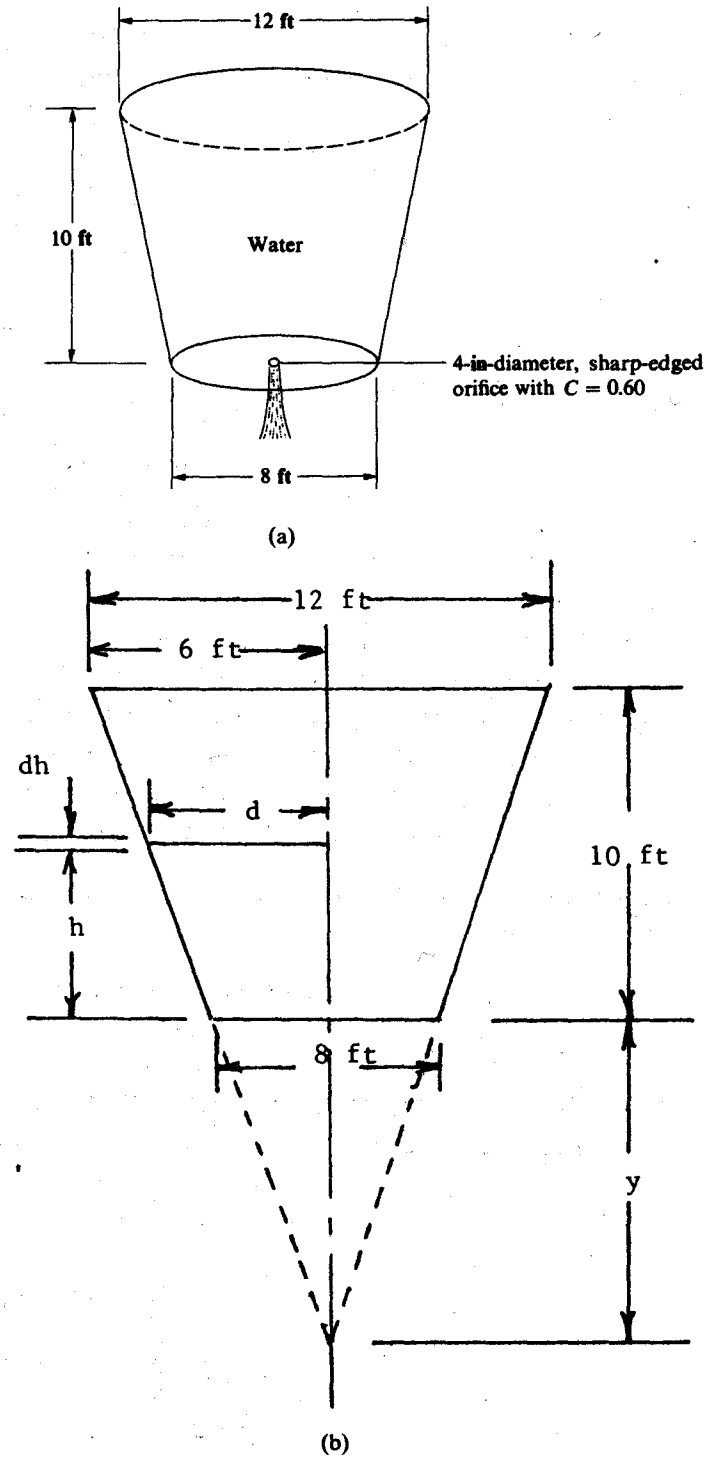


Fig. 17-22

▮ $dt = A_s\, dh/(CA\sqrt{2gh})$. From Fig. 17-22*b*,

$$y/(y+10) = \tfrac{8}{12} \qquad y = 20\text{ ft} \qquad d/12 = (20+h)/30 \qquad d = 8 + 0.4h$$

$$A = (\pi)(\tfrac{4}{12})^2/4 = 0.08727\text{ ft}^2 \qquad A_s = \pi d^2/4 = (\pi)(8+0.4h)^2/4 = 50.27 + 5.027h + 0.1257h^2$$

$$dt = (50.27 + 5.027h + 0.1257h^2)\, dh/[(0.60)(0.08727)\sqrt{(2)(32.2)(h)}] = (119.6h^{-1/2} + 11.96h^{1/2} + 0.2991h^{3/2})\, dh$$

$$\int_0^t dt = \int_0^{10} (119.6h^{-1/2} + 11.96h^{1/2} + 0.2991h^{3/2})\, dh$$

$$[t]_0^t = [239.2h^{1/2} + 7.973h^{3/2} + 0.1196h^{5/2}]_0^{10} \qquad t = 1046.4\text{ s} = 17.44\text{ min}$$

17.86 A rectangular water tank is divided by a partition into two chambers, as shown in Fig. 17-23*a*. In the bottom of the partition is a round, sharp-edged orifice with a diameter of 150 mm. The coefficient of discharge for the orifice is 0.62. At a certain instant, the water level in chamber *B* is 2.5 m higher than it is in chamber *A*. How long will it take for the water surfaces in the two chambers to be at the same level?

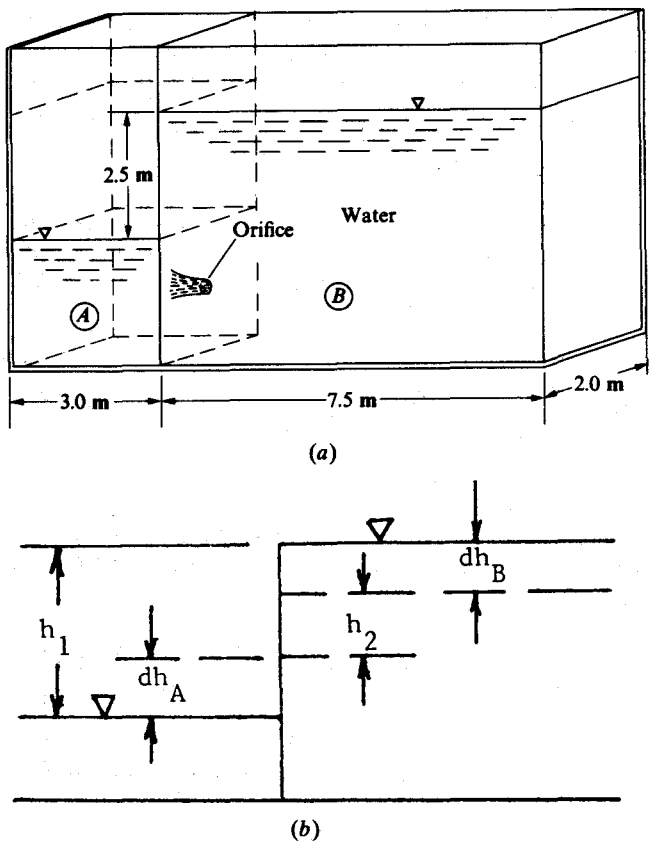


Fig. 17-23

▮ $dV = CA\sqrt{2gh}\, dt$. From Fig. 17-23*b*,

$$(2)(7.5)(dh_B) = (3)(2)(dh_A) \qquad dh_A = 2.5\, dh_B$$

$$dh = dh_A + dh_B = 2.5\, dh_B + dh_B = 3.5\, dh_B \qquad dh_B = 0.2857\, dh$$

$$dV = (dh_B)(7.5)(2.0) = (0.2857\, dh)(15) = 4.286\, dh \qquad A = (\pi)(0.150)^2/4 = 0.01767\text{ m}^2$$

$$4.286\, dh = (0.62)(0.01767)\sqrt{(2)(9.807)(h)}\, dt = 0.04852h\, dt \qquad dt = 88.33h^{-1/2}\, dh$$

$$\int_0^t dt = \int_0^{2.5} 88.33h^{-1/2}\, dh \qquad [t]_0^t = [176.66h^{1/2}]_0^{2.5} \qquad t = 279\text{ s}$$

17.87 A 2-in circular orifice (not standard) at the end of a 3-in-diameter pipe discharges into the atmosphere a measured flow of 0.60 cfs of water when the pressure in the pipe is 10.0 psi, as shown in Fig. 17-24. The jet velocity is determined by a Pitot tube to be 39.2 fps. Find the values of the coefficients C_v, C_c, C_d. Find also the head loss for inlet to throat.

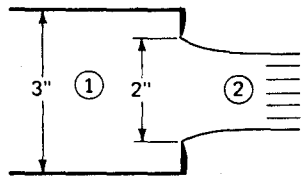


Fig. 17-24

▮ Define the inlet as section 1 and the throat as section 2. $p_1/\gamma = 10(144/62.4) = 23.1$ ft, $V_1 = Q/A_1 = 0.60/0.0491 = 12.22$ fps, $V_1^2/2g = 2.32$ ft. Express the ideal energy equation from 1 to 2 to determine the ideal velocity at 2: $(p_1/\gamma) + (V_1^2/2g) = V_2^2/2g$, $23.1 - 2.3 = V_2^2/2g$, $(V_2)_{\text{ideal}} = 40.4$ fps, $C_v = V/V_i = 39.2/40.4 = 0.97$. Area of jet: $A_2 = Q/V = 0.60/39.2 = 0.0153$ ft^2, $C_c = A_2/A_0 = 0.0153/0.0218 = 0.70$. Hence, $C_d = C_cC_v = 0.68$, $h_{L_{1-2}} = [(1/C_v^2) - 1][1 - (A_2/A_1)^2](V_2^2/2g) = \{[1/(0.97)^2] - 1\}[1 - (\frac{2}{3})^4](V_2^2/2g) = 0.051(V_2^2/2g) = 0.051[(39.2)^2/64.4] = 1.22$ ft. As a check, determine the actual velocity at 2 by expressing the real energy equation from 1 to 2: $(p_1/\gamma) + (V_2^2/2y) - h_{L_{1-2}} = V_2^2/2g$, $23.1 + 2.3 - 1.2 = V_2^2/2g$, $(V_2)_{\text{actual}} = 39.5$ fps, which is a good check. A better check would result if all numbers were carried out to more places.

17.88 Water issues from a circular orifice under a head of 45 ft. The diameter of the orifice is 4.5 in. If the discharge is found to be 740 ft^3 in 3.4 min, what is the coefficient of discharge? If the diameter at the vena contracta is measured to be 3.54 in, what is the coefficient of contraction and what is the coefficient of velocity?

▮
$$Q = C_dA_0(2gh)^{1/2} \qquad 740/[(3.4)(60)] = (C_d)[(\pi)(\tfrac{4.5}{12})^2/4][(2)(32.2)(45)]^{1/2} \qquad C_d = 0.610$$
$$C_c = A/A_0 = [(\pi)(3.54)^2/4]/[(\pi)(4.5)^2/4] = 0.619 \qquad C_v = C_d/C_c = 0.610/0.619 = 0.985$$

17.89 A jet discharges from an orifice in a vertical plane under a head of 16 ft. The diameter of the orifice is 2.0 in, and the measured discharge is 0.42 cfs. The coordinates of the centerline of the jet are 15.39 ft horizontally from the vena contracta and 4.0 ft below the center of the orifice. Find the coefficients of discharge, velocity, and contraction.

▮
$$V = (x)(g/2z)^{1/2} = (15.39)\{32.2/[(2)(4.0)]\}^{1/2} = 30.88 \text{ ft/s} = (C_v)(2gh)^{1/2}$$
$$30.88 = (C_v)[(2)(32.2)(16)]^{1/2} \qquad C_v = 0.962$$
$$Q = C_dA_0(2gh)^{1/2} \qquad 0.42 = (C_d)[(\pi)(2.0/12)^2/4][(2)(32.2)(16)]^{1/2} \qquad C_d = 0.600$$
$$C_v = C_d/C_c \qquad 0.962 = 0.600/C_c \qquad C_c = 0.624$$

17.90 In Fig. 17-25, the Pitot tube in a water jet at elevation 120 ft registers a pressure of 22.7 psi. The orifice at the bottom of the large open tank has a diameter of 1.50 in. Find C_c and C_v of the orifice. Neglect air resistance. The flow rate is 0.39 cfs.

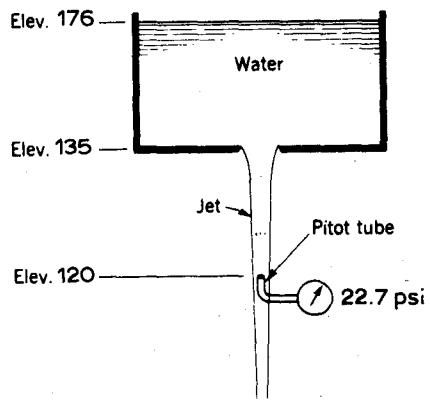


Fig. 17-25

▮
$$V_3^2/2g = p_3/\gamma = (22.7)(144)/62.4 = 52.38 \text{ ft} \qquad V_3^2/[(2)(32.2)] = 52.38 \qquad V_3 = 58.08 \text{ ft/s}$$
$$A_3 = Q/V_3 = 0.39/58.08 = 0.006715 \text{ ft}^2$$
$$V_{\text{jet}}^2/[(2)(32.2)] + 135 = (22.7)(144)/62.4 + 120 \qquad V_{\text{jet}} = 49.07 \text{ ft/s}$$
$$V_{\text{ideal}} = (2gh)^{1/2} = [(2)(32.2)(176 - 135)]^{1/2} = 51.38 \text{ ft/s}$$
$$C_v = V_{\text{jet}}/V_{\text{ideal}} = 49.07/51.38 = 0.955$$
$$A_{\text{jet}}V_{\text{jet}} = C_cA_2V_{\text{jet}} = A_3V_3 \qquad (C_c)[(\pi)(\tfrac{1.5}{12})^2/4](49.07) = (0.006715)(58.08) \qquad C_c = 0.648$$

17.91 Water flows from one tank to an adjacent tank through a 60-mm sharp-edged orifice. The head of water on one side of the orifice is 2.9 m and that on the other is 0.7 m. Assuming $C_c = 0.63$ and $C_v = 0.96$, calculate the flow rate.

▮ $$Q = C_c C_v A(2gh)^{1/2} = (0.63)(0.96)[(\pi)(0.060)^2/4][(2)(9.807)(2.9 - 0.7)]^{1/2} = 0.0112\ \text{m}^3/\text{s}$$

17.92 Repeat Prob. 17.63 for a VDI orifice.

▮ $$Q = KA_2\sqrt{(2g)(14.91R')} \qquad \text{(from Prob. 17.63)}$$

Let $K = 0.70$ (see Fig. A-25): $Q = (0.70)[(\pi)(\frac{9}{12})^2/4]\sqrt{(2)(32.2)[(14.91)(5.5/12)]} = 6.49\ \text{ft}^3/\text{s}$. Check N_R to confirm that the assumed K is applicable.

$$V = Q/A = 6.49/[(\pi)(\tfrac{9}{12})^2/4] = 14.69\ \text{ft/s} \qquad N_R = DV/\nu = (\tfrac{9}{12})(14.69)/0.00034 = 3.24 \times 10^4$$

$$D_0/D_1 = 0.70 \qquad \text{(from Prob. 17.63)}$$

From Fig. A-25, $K = 0.72$, approximately, $Q = (0.72/0.70)(6.49) = 6.68\ \text{ft}^3/\text{s}$.

17.93 Helium, for which $k = 1.66$ and $R = 12\,420$ ft-lb/(slug-°R), is in a tank under a pressure of 60 psia and a temperature of 60 °F. It flows out through an orifice 0.6 in in diameter. For such an orifice, $C_v = 0.98$ and $C_c = 0.62$ for liquids. Find the rate of flow if the pressure into which the gas discharges is 48 psia. Assume $Y = 0.95$.

▮ $$G = CYA_2\sqrt{2g\gamma_1\{(p_1 - p_2)/[1 - (D_2/D_1)^4]\}} \qquad C = C_c C_v = (0.62)(0.98) = 0.608$$

$$Y = 0.95 \qquad \text{(from Fig. A-26)}$$

$$\gamma_1 = p_1/RT = (60)(144)/[(12\,420/32.2)(60 + 460)] = 0.04308\ \text{lb/ft}^3$$

$$G = (0.608)(0.95)[(\pi)(0.6/12)^2/4]\sqrt{(2)(32.2)(0.04308)[(60 - 48)(144)/(1 - 0)]} = 0.0785\ \text{lb/s}$$

17.94 For the data in Prob. 17.93, find the rate of discharge if $p_2 = p_c$.

▮ $$G = CYA_2\sqrt{2g\gamma_1\{(p_1 - p_2)/[1 - (D_2/D_1)^4]\}}$$

$$p_2 = p_c = 0.488p_1 = (0.488)(60) = 29.3\ \text{psia} \qquad Y = 0.85 \qquad \text{(from Fig. A-26)}$$

Using data from Prob. 17.93, for $D_0/D_1 = 1/\infty$,

$$G = (0.608)(0.85)[(\pi)(0.6/12)^2/4]\sqrt{(2)(32.2)(0.04308)[(60 - 29.3)(144)/(1 - 0)]} = 0.112\ \text{lb/s}.$$

17.95 Air is in a tank under a pressure of 1350 kPa abs and a temperature of 90 °C. It flows out through an orifice having an area of 8 cm² into a space where the pressure is 660 kPa abs. Compute the mass flow, assuming $C_d = 0.60$.

▮ $$\dot{m} = CYA_2\sqrt{2\rho_1\{(p_1 - p_2)/[1 - (D_2/D_1)^4]\}} \qquad Y = 0.85 \qquad \text{(from Fig. A-26)}$$

$$\rho_1 = p_1/RT_1 = 1350/[(0.287)(90 + 273)] = 12.9\ \text{kg/m}^3$$

$$\dot{m} = (0.60)(0.85)(8 \times 10^{-4})\sqrt{(2)(12.9)[(1350 - 660)(1000)/(1 - 0)]} = 1.72\ \text{kg/s}$$

17.96 Using the same data as in Prob. 17.95, what would be the flow if the air discharged into a space where the pressure is 125 kPa abs?

▮ With $p_2 = 125$ kPa, $D_0/D_1 = 0$, $p_2/p_1 = 125/1350 = 0.093$, $Y = 0.73$; hence,

$$\dot{m} = (0.60)(0.73)(8 \times 10^{-4})\sqrt{(2)(12.9)[(1350 - 125)(1000)/(1 - 0)]} = 1.97\ \text{kg/s}$$

17.97 Air in a tank at 1600 kPa abs and 70 °C flows out through a 60-mm-diameter orifice into a space where the pressure is 600 kPa abs. Compute the rate of discharge assuming $C_d = 0.60$.

▮ $p_2/p_1 = 600/1600 = 0.375$. Since $p_2/p_1 < 0.528$, sonic velocity occurs at the throat and $\dot{m} = C_d(A_2 p_1/\sqrt{T_1})\sqrt{(k/R)[2/(k + 1)]^{(k+1)/(k-1)}}$.

$$A_2 = (\pi)(0.060)^2/4 = 0.002827\ \text{m}^2$$

$$\dot{m} = (0.60)[(0.002827)(16 \times 10^5)/\sqrt{70 + 273}]\sqrt{(1.40/287)[2/(1.40 + 1)]^{(1.40+1)/(1.40-1)}} = 5.92\ \text{kg/s}$$

17.98 A 4-in-diameter standard orifice discharges water under a 20.0-ft head as shown in Fig. 17-26. What is the flow?

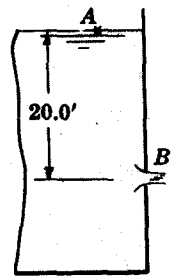


Fig. 17-26

▮ $Q = CA_0\sqrt{2gh}$. From Table A-19, $C = 0.594$, $Q = (0.594)[(\pi)(\frac{4}{12})^2/4]\sqrt{(2)(32.2)(20.0)} = 1.86\ \text{ft}^3/\text{s}$.

17.99 The actual velocity in the contracted section of a jet of liquid flowing from a 2-in-diameter orifice is 28.0 ft/s under a head of 15 ft. **(*a*)** What is the value of the coefficient of velocity? **(*b*)** If the measured discharge is 0.403 cfs, determine the coefficients of contraction and discharge.

▮ **(*a*)** $V_{\text{actual}} = C_v\sqrt{2gH}$ $\quad 28.0 = C_v\sqrt{(2)(32.2)(15)}$ $\quad C_v = 0.901$

(*b*) $Q_{\text{actual}} = CA\sqrt{2gH}$ $\quad 0.403 = (C)[(\pi)(\frac{2}{12})^2/4]\sqrt{(2)(32.2)(15)}$ $\quad C = 0.594$

$C = C_vC_c \quad 0.594 = 0.901C_c \quad C_c = 0.659$

17.100 Oil flows through a standard 1-in-diameter orifice under an 18.0-ft head at the rate of 0.111 cfs. The jet strikes a wall 5.00 ft away and 0.390 ft vertically below the centerline of the contracted section of the jet. Compute the coefficients.

▮ $Q = CA\sqrt{2gH} \quad 0.111 = (C)[(\pi)(\frac{1}{12})^2/4]\sqrt{(2)(32.2)(18.0)} \quad C = 0.598 \quad V = C_v\sqrt{2gH}$

Letting x and y represent the coordinates of the jet as measured, from kinematic mechanics, $x = Vt$, $y = gt^2/2$. Eliminating t gives

$x^2 = (2V^2/g)(y) \quad 5.00^2 = (2V^2/32.2)(0.390) \quad V = V_{\text{actual}} = 32.13\ \text{ft/s}$

$32.13 = C_v\sqrt{(2)(32.2)(18.0)} \quad C_v = 0.944 \quad C = C_cC_v \quad 0.598 = (C_c)(0.944) \quad C_c = 0.633$

17.101 The tank in Prob. 17.98 is closed and the air space above the water is under pressure, causing the flow to increase to 2.65 cfs. Find the pressure in the air space.

▮ $Q = CA_0\sqrt{2gH}$. Table A-19 indicates that C does not change appreciably at the range of head under consideration. Using $C = 0.593$, $2.65 = (0.593)[(\pi)(\frac{4}{12})^2/4]\sqrt{(2)(32.2)(20 + p/\gamma)}$, $p/\gamma = 20.72$ ft of water, $p' = \gamma h = (62.4)(20.72)/144 = 8.98\ \text{lb/in}^2$.

17.102 Oil of specific gravity 0.720 flows through a 3-in-diameter orifice whose coefficients of velocity and contraction are 0.950 and 0.650, respectively. What must be the reading of gage A in Fig. 17-27 in order for the power in the jet C to be 8.00 hp?

▮ $p_B/\gamma + V_B^2/2g + z_B = p_C/\gamma + V_C^2/2g + z_C + h_L \qquad P_{\text{jet}} = \gamma QH_{\text{jet}} = (\gamma)(C_cA_0V_{\text{jet}})(0 + V_{\text{jet}}^2/2g + 0)$

$$(8.00)(550) = [(0.720)(62.4)](0.650)[(\pi)(\tfrac{3}{12})^2/4](V_{\text{jet}})\{V_{\text{jet}}^2/[(2)(32.2)]\}$$

$$V_{\text{jet}} = V_C = 58.25\ \text{ft/s} \qquad h_L = (1/C_v - 1)(V_{\text{jet}}^2/2g)$$

$$p_A/[(0.720)(62.4)] + 0 + 9.0 = 0 + 58.25^2/[(2)(32.2)] + 0 + (1/0.950 - 1)\{58.25^2/[(2)(32.2)]\}$$

$$p_A = 2087\ \text{lb/ft}^2 \quad \text{or} \quad 14.5\ \text{lb/in}^2$$

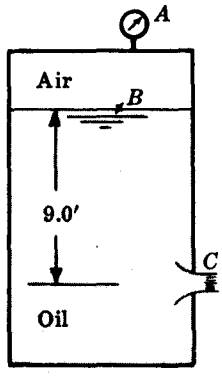


Fig. 17-27

17.103 Water at 100 °F flows at the rate of 0.525 cfs through a 4-in diameter orifice used in an 8-in pipe. What is the difference in pressure head between the upstream section and the contracted section (vena contracta section)?

▮ In Fig. A-27, it is observed that C' varies with the Reynolds number. Note that the Reynolds number must be calculated for the orifice cross section, not for the contracted section of the jet nor for the pipe section. This value is $N_R = V_oD_o/\nu = (4Q/\pi D_o^2)D_o/\nu = 4Q/\nu\pi D_o = 4(0.525)/[\pi(0.00000739)(\frac{4}{12})] = 272\,000$. Figure A-27 for $\beta = 0.500$ gives $C' = 0.604$.

Applying the Bernoulli theorem, pipe section to jet section, produces the general equation for incompressible fluids, as follows: $[(p_8/\gamma) + (V_8^2/2g) + 0] - [(1/C_v^2) - 1](V_{jet}^2/2g) = [(p_{jet}/\gamma) + (V_{jet}^2/2g) + 0]$ and $Q = A_8V_8 = (C_cA_4)V_{jet}$.

Substituting for V_8 in terms of V_{jet} and solving, $V_{jet}^2/2g = C_v^2\{(p_8/\gamma - p_{jet}/\gamma)/[1 - C^2(A_4/A_8)^2]\}$ or $V_{jet} = C_v\sqrt{2g(p_8/\gamma - p_{jet}/\gamma)/[1 - C^2(D_4/D_8)^4]}$. Then $Q = A_{jet}V_{jet} = (C_cA_4)(C_v)\sqrt{2g(p_8/\gamma - p_{jet}/\gamma)/[1 - C^2(D_4/D_8)^4]} = CA_4\sqrt{2g(p_8/\gamma - p_{jet}/\gamma)/[1 - C^2(D_4/D_8)^4]}$.

More conveniently, for an orifice with velocity of approach and a contracted jet, the equation can be written

$$Q = [C'A_4/\sqrt{1 - (D_4/D_8)^4}]\sqrt{2g(\Delta p/\gamma)} \qquad (1)$$

or

$$Q = KA_4\sqrt{2g(\Delta p/\gamma)} \qquad (2)$$

where K is called the flow coefficient. The meter coefficient C' may be determined experimentally for a given ratio of diameter of orifice to diameter of pipe, or the flow coefficient K may be preferred.

Proceeding with the solution by substituting in Eq. (1),

$$0.525 = \frac{(0.604)[(\pi)(\frac{4}{12})^2/4]}{\sqrt{1 - (\frac{1}{2})^4}}\sqrt{(2)(32.2)\left(\frac{\Delta p}{\gamma}\right)} \qquad \frac{\Delta p}{\gamma} = 1.44 \text{ ft of water}$$

17.104 For the pipe orifice in Prob. 17.103, what pressure difference in pounds per square inch would cause the same quantity of turpentine at 68 °F (s.g. = 0.862, $\nu = 0.0000186\ \text{ft}^2/\text{s}$) to flow?

▮

$$Q = [C'A_4/\sqrt{1 - (D_4/D_8)^4}]\sqrt{2g(\Delta p/\gamma)}$$

$$V = Q/A = 0.525/[(\pi)(\tfrac{4}{12})^2/4] = 6.016 \text{ ft/s} \qquad N_R = DV/\nu = (\tfrac{4}{12})(6.016)/0.0000186 = 1.08 \times 10^5$$

From Fig. A-27, for $\beta = 0.500$, $C' = 0.607$.

$$0.525 = \frac{(0.607)[(\pi)(\frac{4}{12})^2/4]}{\sqrt{1 - (\frac{1}{2})^4}}\sqrt{(2)(32.2)\left[\frac{\Delta p}{(0.862)(62.4)}\right]}$$

$$p = 76.9 \text{ lb/ft}^2 \quad \text{or} \quad 0.534 \text{ lb/in}^2$$

17.105 Determine the flow of water at 70 °F through a 6-in orifice installed in a 10-in pipeline if the pressure head differential for vena-contracta taps is 3.62 ft of water.

▮

$$Q = [C'A_6/\sqrt{1 - (D_6/D_{10})^4}]\sqrt{2g(\Delta p/\gamma)}$$

The value of C' cannot be found inasmuch as the Reynolds number cannot be computed. Referring to Fig. A-27, for $\beta = 0.600$, a value of C' will be assumed at 0.610. Using this assumed value,

$$Q = \frac{(0.610)[(\pi)(\frac{6}{12})^2/4]}{\sqrt{1 - (0.600)^4}}\sqrt{(2)(32.2)(3.62)} = 1.96 \text{ ft}^3/\text{s}$$

$$V = Q/A = 1.96/[(\pi)(\tfrac{6}{12})^2/4] = 9.982 \text{ ft/s} \qquad N_R = DV/\nu = (\tfrac{6}{12})(9.982)/0.00001059 = 4.71 \times 10^5$$

From Fig. A-27, for $\beta = 0.600$, $C' = 0.609$.

$$Q = \frac{(0.609)[(\pi)(\frac{6}{12})^2/4]}{\sqrt{1-(0.600)^4}}\sqrt{(2)(32.2)(3.62)} = 1.96\ \text{ft}^3/\text{s}$$

17.106 Carbon dioxide discharges through a $\frac{1}{2}$-in hole in the wall of a tank in which the pressure is 110 psig and the temperature is 68 °F. What is the velocity in the jet (standard atmosphere)?

▮

$$\gamma_1 = p_1/RT_1 = (110+14.7)(144)/[(35.1)(68+460)] = 0.9689\ \text{lb/ft}^3$$

$$(p_2/p_1)_{\text{critical}} = [2/(k+1)]^{k/(k-1)} = [2/(1.30+1)]^{1.30/(1.30-1)} = 0.546$$

$$p_{\text{atm}}/p_{\text{tank}} = 14.7/(110+14.7) = 0.118$$

Since the latter ratio is less than the critical pressure ratio, the pressure of the escaping gas is 0.546 times p_1; hence,

$$p_2 = (0.546)(110+14.7) = 68.1\ \text{psia} \qquad V_2 = c_2 = \sqrt{kgRT_2} \qquad T_2/T_1 = (p_2/p_1)^{(k-1)/k}$$

$$T_2/(68+460) = 0.546^{(1.30-1)/1.30} \qquad T_2 = 459\ °\text{R} \qquad V_2 = \sqrt{(1.30)(32.2)(35.1)(459)} = 821\ \text{ft/s}$$

17.107 Nitrogen flows through a duct in which changes in cross section occur. At a particular cross section the velocity is 1200 ft/s, the pressure is 12.0 psia, and the temperature is 90 °F. Assuming no friction losses and adiabatic conditions, **(*a*)** what is the velocity at a section where the pressure is 18.0 psia and **(*b*)** what is the Mach number at this section?

▮ **(*a*)**

$$V_2^2/2g - V_1^2/2g = [k/(k-1)](p_1/\gamma_1)[1-(p_2/p_1)^{(k-1)/k}]$$

$$\gamma_1 = p_1/RT_1 = (12.0)(144)/[(55.1)(90+460)] = 0.05702\ \text{lb/ft}^3$$

$$\{V_2^2/[(2)(32.2)]\} - \{1200^2/[(2)(32.2)]\} = [1.40/(1.40-1)][(12.0)(144)/0.05702] \times [1-(18.0/12.0)^{(1.40-1)/1.40}] \qquad V_2 = 775\ \text{ft/s}$$

(*b*)

$$N_M = V_2/c_2 = V_2/\sqrt{kgRT} \qquad T_2/T_1 = (p_2/p_1)^{(k-1)/k}$$

$$T_2/(90+460) = (18.0/12.0)^{(1.40-1)/1.40} \qquad T_2 = 618\ °\text{R}$$

$$N_M = 775/\sqrt{(1.40)(32.2)(55.1)(618)} = 0.626$$

17.108 Establish the formula to determine the time to lower the liquid level in a tank of constant cross section by means of an orifice. Refer to Fig. 17-28.

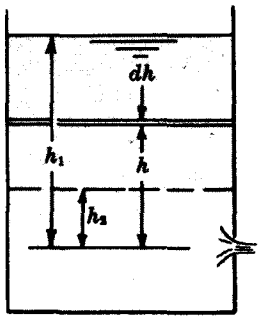


Fig. 17-28

▮ Inasmuch as the head is changing with time, we know that $\partial V/\partial t \neq 0$, i.e., we do not have steady flow. This means that the energy equation should be amended to include an acceleration term, which complicates the solution materially. As long as the head does not change too rapidly, no appreciable error will be introduced by assuming steady flow, thus neglecting the acceleration-head term.

Case A: With *no inflow* taking place, the instantaneous flow will be $Q = CA_0\sqrt{2gh}$ cfs. In time interval dt, the small volume dV discharged will be $Q\,dt$. In the same time interval, the head will decrease dh ft and the volume discharged will be the area of the tank A_T times dh. Equating these values, $(CA_0\sqrt{2gh})\,dt = -A_T\,dh$ where the negative sign signifies that h decreases as t increases. Solving for t yields

$$t = \int_{t_1}^{t_2} dt = (-A_T/CA_0\sqrt{2g})\int_{h_1}^{h_2} h^{-1/2}\,dh$$

or
$$t = t_2 - t_1 = (2A_T/CA_0\sqrt{2g})(h_1^{1/2} - h_2^{1/2}) \tag{1}$$

In using this expression, an average value of coefficient of discharge C may be used without producing significant error in the result. As h_2 approaches zero, a vortex will form and the orifice will cease to flow full. However, using $h_2 = 0$ will not produce serious error in most cases.

Equation (*1*) can be rewritten by multiplying and dividing by $(h_1^{1/2} + h_2^{1/2})$. There results

$$t = t_2 - t_1 = A_T(h_1 - h_2)/[\tfrac{1}{2}(CA_0\sqrt{2gh_1} + CA_0\sqrt{2gh_2})] \tag{2}$$

Noting that the volume discharged in time $(t_2 - t_1)$ is $A_T(h_1 - h_2)$, this equation simplifies to

$$t = t_2 - t_1 = \text{volume discharged}/[\tfrac{1}{2}(Q_1 + Q_2)] = \text{volume discharged/average flow } Q \tag{3}$$

Case B. With a constant rate of inflow less than the flow through the orifice taking place,

$$-A_T\,dh = (Q_{\text{out}} - Q_{\text{in}})\,dt \qquad \text{and} \qquad t = t_2 - t_1 = \int_{h_1}^{h_2} \frac{-A_T\,dh}{Q_{\text{out}} - Q_{\text{in}}}$$

Should Q_{in} exceed Q_{out}, the head would increase, as would be expected.

17.109 The initial head on an orifice was 9 ft and when the flow was terminated the head was measured at 4 ft. Under what constant head H would the same orifice discharge the same volume of water in the same time interval? Assume coefficient C is constant.

▮ Volume under falling head = volume under constant head

$$\tfrac{1}{2}CA_0\sqrt{2g}\,(h_1^{1/2} + h_2^{1/2})(t) = CA_0\sqrt{2gH}\,(t)$$
$$\tfrac{1}{2}CA_0\sqrt{2g}\,(9^{1/2} + 4^{1/2})(t) = CA_0\sqrt{2g}\,\sqrt{H}\,(t) \qquad H = 6.25 \text{ ft}$$

17.110 A 75-mm-diameter orifice under a head of 4.88 m discharges 8900 N water in 32.6 s. The trajectory was determined by measuring $x_0 = 4.76$ m for a drop of 1.22 m. Determine C_v, C_c, C_d, the head loss per unit weight, and the power loss.

▮ The theoretical velocity V_{2t} is $V_{2t} = \sqrt{2gH} = \sqrt{2(9.806)(4.88)} = 9.783$ m/s. The actual velocity is determined from the trajectory. The time to drop 1.22 m is $t = \sqrt{2y_0/g} = \sqrt{2(1.22)/9.806} = 0.499$ s and the velocity is expressed by $x_0 = V_{2a}t$, $V_{2a} = 4.76/0.499 = 9.539$ m/s. Then $C_v = V_{2a}/V_{2t} = 9.539/9.783 = 0.975$. The actual discharge Q_a is $Q_a = (8900/32.6)/9806 = 0.0278$ m³/s. $C_d = Q_a/(A_0\sqrt{2gH}) = 0.0278/[\pi(0.0375^2)\sqrt{2(9.806)(4.88)}] = 0.643$, $C_c = C_d/C_v = 0.643/0.975 = 0.659$. The head loss is loss $= H(1 - C_v^2) = 4.88(1 - 0.975^2) = 0.241$ m. The power loss is $Qy(\text{loss}) = 0.0278(9806)(0.241) = 65.7$ W.

17.111 A tank has a horizontal cross-sectional area of 2 m² at the elevation of the orifice, and the area varies linearly with elevation so that it is 1 m² at a horizontal cross section 3 m above the orifice. For a 100-mm-diameter orifice, $C_d = 0.65$, compute the time to lower the surface from 2.5 m to 1 m above the orifice.

▮
$$t = -\frac{1}{C_dA_0\sqrt{2g}}\int_{y_1}^{y_2} A_r y^{-1/2}\,dy \qquad A_r = 2 - \frac{y}{3}$$
$$t = -\frac{1}{(0.65)[(\pi)(\frac{100}{1000})^2/4]\sqrt{(2)(9.807)}}\int_{2.5}^{1}\left(2 - \frac{y}{3}\right)y^{-1/2}\,dy = -44.23[4y^{1/2} - \tfrac{2}{9}y^{3/2}]_{2.5}^{1} = 73.8 \text{ s}$$

17.112 Determine the equation for trajectory of a jet discharging horizontally from a small orifice with head of 8 m and velocity coefficient of 0.96. Neglect air resistance.

▮
$$x = V_2t \qquad y = gt^2/2 \qquad V_2 = C_v\sqrt{2gH} \qquad t = x/V_2$$
$$y = (g)(x/V_2)^2/2 = (g)[x/(C_v\sqrt{2gH})]^2/2 = (9.807)\{x/[0.96\sqrt{(2)(9.807)(8)}]\}^2/2 = 0.0339x^2$$

17.113 An orifice of area 25 cm² in a vertical plate has a head of 1.13 m of oil, s.g. = 0.89. It discharges 6192 N of oil in 88.8 s. Trajectory measurements yield $x_0 = 2.40$ m, $y_0 = 1.37$ m. Determine C_v, C_c, and C_d.

▮ Letting subscript a denote "actual" and t, "theoretical,"

$$Q_a = W/\gamma = (6192/88.8)/[(0.89)(9790)] = 0.008003 \text{ m}^3/\text{s}$$
$$(V_2)_a = x\sqrt{g/2y} = (2.40)\sqrt{(9.807)/[(2)(1.37)]} = 4.541 \text{ m/s}$$
$$(V_2)_t = \sqrt{2gH} = \sqrt{(2)(9.807)(1.13)} = 4.708 \text{ m/s} \qquad C_v = (V_2)_a/(V_2)_t = 4.541/4.708 = 0.965$$
$$C_d = Q_a/[A(V_2)_t] = 0.008003/[(25 \times 10^{-4})(4.708)] = 0.680 \qquad C_c = C_d/C_v = 0.680/0.965 = 0.705$$

17.114 Calculate Y, the maximum rise of a jet from an inclined plate (Fig. 17-29), in terms of H and α. Neglect losses.

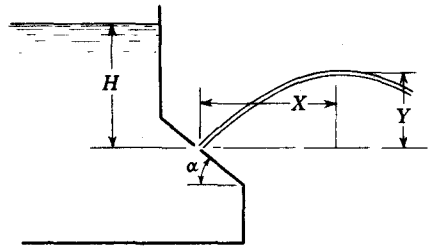


Fig. 17-29

▮ $$y = -gt^2/2 + (V\cos\alpha)(t) \qquad x = Vt\sin\alpha \qquad V = \sqrt{2gH}$$

Substituting for V and t,

$$y = -x^2/(4H\sin^2\alpha) + x\cot\alpha \qquad dy/dx = -x/(2H\sin^2\alpha) + \cot\alpha = 0$$

$$x = 2H\sin\alpha\cos\alpha \qquad y_{\max} = H\cos^2\alpha$$

17.115 In Fig. 17-29, for $\alpha = 45°$, $Y = 0.48H$. Neglecting air resistance of the jet, find C_v for the orifice.

▮ From Prob. 17.114 but with $V = C_v\sqrt{2gH}$, $Y = C_v^2 H\cos^2\alpha$, $C_v = \sqrt{Y/H}/\cos\alpha = \sqrt{0.48H/H}/\cos 45° = 0.980$.

17.116 Show that the locus of maximum points of the jet of Fig. 17-29 is given by $X^2 = 4Y(H - Y)$ when losses are neglected.

▮ From Prob. 17.114, maximum point coordinates are given by $X = 2H\sin\alpha\cos\alpha$ and $Y = H\cos^2\alpha$. Thus, $\cos\alpha = \sqrt{Y/H}$ and $\sin\alpha = \sqrt{1-\cos^2\alpha} = \sqrt{1 - Y/H}$. Hence, $X = 2H\sqrt{Y/H}\sqrt{1 - Y/H}$, $X^2 = 4H^2[Y/H - (Y/H)^2] = 4Y(H - Y)$.

17.117 A 4-in-diameter orifice discharges 135 ft^3 of liquid, s.g. = 1.10, in 95.5 s under a 10-ft head. The velocity at the vena contract is determined by a Pitot-static tube with coefficient 1.0. The manometer liquid is acetylene tetrabromide, s.g. 2.96, and the gage difference is $R' = 3.62$ ft. Determine C_v, C_c, and C_d.

▮ $$V_a = C\sqrt{2gR'(S_0/S - 1)} = (1.0)\sqrt{(2)(32.2)(3.62)[(2.96/1.10) - 1]} = 19.85 \text{ ft/s}$$

$$V_t = \sqrt{2gH} = \sqrt{(2)(32.2)(10)} = 25.38 \text{ ft/s}$$

$$C_v = V_a/V_t = 19.85/25.38 = 0.782 \qquad Q_a = V/t = 135/95.5 = 1.41 \text{ ft}^3/\text{s}$$

$$C_d = Q_a/AV_t = 1.41/\{[(\pi)(\tfrac{4}{12})^2/4](25.38)\} = 0.637 \qquad C_c = C_d/C_v = 0.637/0.782 = 0.815$$

17.118 A 115-mm-diameter orifice discharges 60.0 L/s of water under a head of 2.70 m. A flat plate held normal to the jet downstream from the vena contracta requires a force of 423 N to resist impact of the jet. Find C_d, C_v, and C_c.

▮ $$F = \rho(V_2)_a Q_a \qquad (V_2)_a = F/\rho Q_a = 423/(1000)(60.0\times 10^{-3}) = 7.050 \text{ m/s}$$

$$C_v = (V_2)_a/\sqrt{2gH} = 7.050/\sqrt{(2)(9.807)(2.70)} = 0.969$$

$$C_d = Q_a/(A\sqrt{2gH}) = (60.0\times 10^{-3})/\{[(\pi)(0.115)^2/4]\sqrt{(2)(9.807)(2.70)}\} = 0.794$$

$$C_c = C_d/C_v = 0.794/0.969 = 0.819$$

17.119 Compute the discharge from the tank shown in Fig. 17-30.

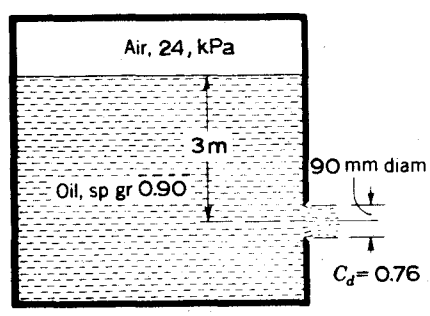


Fig. 17-30

▮ $$Q = C_d A\sqrt{2g(H + p_a/\gamma)} = (0.76)[(\pi)(0.090)^2/4]\sqrt{(2)(9.807)\{3 + 24/[(0.90)(9.79)]\}} = 0.0512 \text{ m}^3/\text{s}$$

17.120 For $C_v = 0.95$ in Fig. 17-30, calculate the head loss.

▮ $$h_L = (H + p_a/\gamma)(1 - C_v^2) = \{3 + 24/[(0.90)(9.79)]\}(1 - 0.95^2) = 0.558\ \text{m}$$

17.121 Calculate the discharge through the orifice of Fig. 17-31.

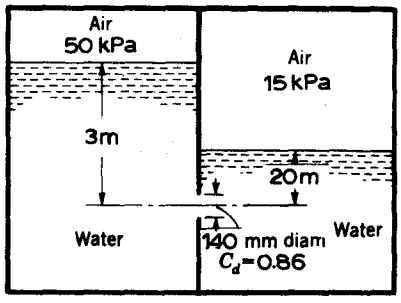


Fig. 17-31

▮ $$H' = (3 + 50/9.79) - (2 + 15/9.79) = 4.575\ \text{m}$$

$$Q = C_d A\sqrt{2gH'} = (0.86)[(\pi)(0.140)^2/4]\sqrt{(2)(9.807)(4.575)} = 0.125\ \text{m}^3/\text{s}$$

17.122 For $C_v = 0.92$ in Fig. 17-31, determine the power loss.

▮ $H' = 4.575$ m (from Prob. 17.121) $\quad h_L = H'(1 - C_v^2) = (4.575)(1 - 0.92^2) = 0.703$ m

$Q = 0.125\ \text{m}^3/\text{s}$ (from Prob. 17.121) $\quad$ power loss $= Q\gamma h_L = (0.125)[(9.79)(1000)](0.703) = 860$ W

17.123 A 3-in-diameter orifice discharges 0.826 cfs of liquid under a head of 9.5 ft. The diameter of the jet at the vena contracta is found by calipering to be 2.65 in. Calculate C_v, C_d, and C_c.

▮ $$V_2 = \sqrt{2gH} = \sqrt{(2)(32.2)(9.5)} = 24.73\ \text{ft/s} \qquad A_0 = (\pi)(\tfrac{3}{12})^2/4 = 0.04909\ \text{ft}^2$$

$$C_d = Q_a/V_2A_0 = 0.826/[(24.73)(0.04909)] = 0.680$$

$$C_c = A_2/A_0 = [(\pi)(2.65/12)^2/4]/[(\pi)(\tfrac{3}{12})^2/4] = 0.780 \qquad C_v = C_d/C_c = 0.680/0.780 = 0.872$$

17.124 A 90-mm-diameter orifice, $C_d = 0.80$, is placed in the bottom of a vertical tank that has a diameter of 2.0 m. How long does it take to draw the surface down from 4 to 3.5 m?

▮ $$t = (2A_r/C_dA_0\sqrt{2g})(y_1^{1/2} - y_2^{1/2}) \qquad A_r = (\pi)(2.0)^2/4 = 3.142\ \text{m}^2$$

$$A_0 = (\pi)(0.090)^2/4 = 0.006362\ \text{m}^2$$

$$t = \{(2)(3.142)/[(0.80)(0.006362)\sqrt{(2)(9.807)}]\}(4^{1/2} - 3.5^{1/2}) = 36.0\ \text{s}$$

17.125 Select the size of orifice that permits a tank of horizontal cross section 2.0 m^2 to have the liquid surface drawn down at the rate of 140 mm/s for 3.50-m head on the orifice ($C_d = 0.65$).

▮ $$Q = (A_r)(\Delta y/\Delta t) = (2.0)(140 \times 10^{-3}) = 0.2800\ \text{m}^3/\text{s}$$

$$A_0 = Q/(C_d\sqrt{2gH}) = 0.2800/[0.65\sqrt{(2)(9.807)(3.50)}] = 0.05199\ \text{m}^2 = 51\,990\ \text{mm}^2$$

$$51\,990 = \pi D^2/4 \qquad D = 257\ \text{mm}$$

17.126 A 3-in-diameter orifice in the side of a 5-ft-diameter tank draws the surface down from 7 ft to 5 ft above the orifice in 53.0 s. Calculate the discharge coefficient.

▮ $$t = [2A_r/(C_dA_0\sqrt{2g})](y_1^{1/2} - y_2^{1/2}) \qquad A_r = (\pi)(5)^2/4 = 19.63\ \text{ft}^2$$

$$A_0 = (\pi)(\tfrac{3}{12})^2/4 = 0.04909\ \text{ft}^2$$

$$53.0 = \{(2)(19.63)/[(C_d)(0.04909)\sqrt{(2)(32.2)}]\}(7^{1/2} - 5^{1/2}) \qquad C_d = 0.770$$

17.127 Select a reservoir of such size and shape that the liquid surface drops 1.1 m/min over a 4-m distance for flow through a 90-mm-diameter orifice ($C_d = 0.75$).

$$dy/dt = -Q/A_r = -(C_d A_0 \sqrt{2gy})/A_r$$

$$A_r = C_d A_0 \sqrt{2g}(\Delta t/\Delta y) y^{1/2} = (0.75)[(\pi)(0.090)^2/4]\sqrt{(2)(9.807)}(60/1.1) y^{1/2} = 1.153 y^{1/2}$$

$$\pi D^2/4 = 1.153 y^{1/2} \qquad D = (1.212\ \mathrm{m}^{3/4}) y^{1/4}$$

17.128 In Fig. 17-32 the truncated cone has an angle $\theta = 60°$. How long does it take to draw the liquid surface down from $y = 5$ m to $y = 2$ m?

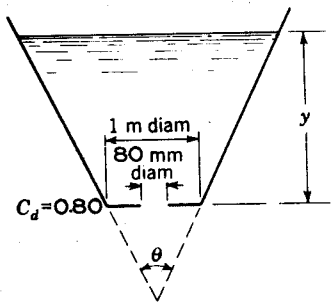


Fig. 17-32

$$t = -\frac{1}{C_d A_0 \sqrt{2g}} \int_{y_1}^{y_2} \frac{A_r}{\sqrt{y}}\, dy \qquad A_r = \pi\left(r_0 + y \tan\frac{\theta}{2}\right)^2$$

$$t = \frac{\pi}{C_d A_0 \sqrt{2g}} \int_{y_2}^{y_1} \frac{[r_0 + y \tan(\theta/2)]^2}{\sqrt{y}}\, dy = \frac{\pi}{C_d A_0 \sqrt{2g}} \left[r_0^2 (2\sqrt{y}) + 2r_0 (\tfrac{2}{3} y^{3/2}) \tan\frac{\theta}{2} + \tfrac{2}{5} y^{5/2} \tan^2\frac{\theta}{2} \right]_{y_2}^{y_1}$$

$$A_0 = (\pi)(0.080)^2/4 = 0.005026\ \mathrm{m}^2$$

$$t = -\frac{\pi}{(0.80)(0.005026)\sqrt{(2)(9.807)}} [(\tfrac{1}{2})^2 (2\sqrt{y}) + (2)(\tfrac{1}{2})(\tfrac{2}{3}) y^{3/2} \tan\tfrac{60}{2} + \tfrac{2}{5} y^{5/2} \tan^2 \tfrac{60}{2}]_5^2 = 1822\ \mathrm{s} \quad \text{or} \quad 30.4\ \mathrm{min}$$

17.129 Calculate the dimensions of a tank such that the surface velocity varies inversely as the distance from the centerline of an orifice draining the tank. When the head is 400 mm, the velocity of fall of the surface is 40 mm/s, orifice diameter is 15 mm, $C_d = 0.64$.

$$dy/dt = -Q/A_r = k/y \qquad k = y(dy/dt) = (0.400)(-0.040) = -0.01600 \qquad -C_d A_0 \sqrt{2gy}/(\pi D^2/4) = k/y$$

$$D = y^{3/4} \sqrt{\frac{C_d A_0 \sqrt{2g}}{\pi/4}} \left(-\frac{1}{k}\right) = y^{3/4} \sqrt{\frac{(0.64)[(\pi)(0.015)^2/4]\sqrt{(2)(9.807)}}{\pi/4}} \left(-\frac{1}{-0.01600}\right)$$

$$= (1.578\ \mathrm{m}^{1/4}) y^{3/4}$$

17.130 Determine the time required to raise the right-hand surface of Fig. 17-33 by 3 ft.

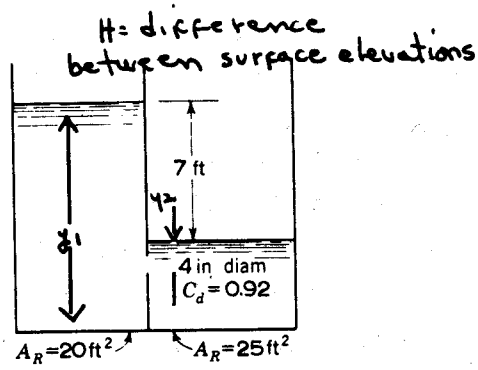


Fig. 17-33

$$A_1 y_1 + A_2 y_2 = C \qquad A_1\, dy_1 + A_2\, dy_2 = 0 \qquad H = y_1 - y_2$$

$$dH = dy_1 - dy_2 = -(1 + A_2/A_1)\, dy_2$$

$$dy_2 = -dH/(1 + A_2/A_1) \qquad Q\, dt = C_d A_0 \sqrt{2gH}\, dt = A_2\, dy_2 = -[A_2\, dH/(1 + A_2/A_1)]$$

Hence,

$$t = \frac{A_2}{C_d A_0 \sqrt{2g}(1 + A_2/A_1)} \int_{H_1}^{H_2} H^{-1/2}\, dH = \frac{2A_2(\sqrt{H_1} - \sqrt{H_2})}{C_d A_0 \sqrt{2g}(1 + A_2/A_1)} \qquad H_1 = 7\ \mathrm{ft}$$

Rise of the right-hand surface by 3 ft will create a fall of the left-hand surface by $(3)(\frac{25}{20}) = 3.75$ ft. Thus, $H_2 = 7 - 3 - 3.75 = 0.25$ ft. $t = (2)(25)(\sqrt{7} - \sqrt{0.25})/\{(0.92)[(\pi)(\frac{4}{12})^2/4]\sqrt{(2)(32.2)(1 + \frac{25}{20})}\} = 74.0$ s.

17.131 How long does it take to raise the water surface of Fig. 17-34 by 1 m? The left-hand surface is a large reservoir of constant water-surface elevation.

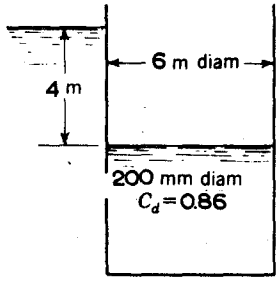


Fig. 17-34

▮ As in Prob. 17.130, $t = [2A_2(\sqrt{H_1} - \sqrt{H_2})]/[C_d A_0 \sqrt{2g}(1 + A_2/A_1)]$ where $A_1 = \infty$, $A_0 = (\pi)(0.200)^2/4 = 0.03142$ m^2, $t = [(2)(\pi 6^2/4)(\sqrt{4} - \sqrt{4-1})]/[(0.86)(0.03142)\sqrt{(2)(9.807)}(1 + 0)] = 126.6$ s.

17.132 Determine the discharge in a 350-mm-diameter line with a 180-mm-diameter VDI orifice for water at 20 °C when the gage difference is 320 mm on an acetylene tetrabromide (s.g. = 2.94, $\nu = 1.01 \times 10^{-6}$ m^2/s)–water differential manometer.

▮
$$Q = CA_0\sqrt{2gR'(S_0/S_1 - 1)} \qquad A_0/A_1 = (180/350)^2 = 0.264$$

From Fig. A-28, assume $C = 0.632$ (for high N_R).

$$Q = (0.632)[(\pi)(0.180)^2/4]\sqrt{(2)(9.807)(0.320)[(2.94/1) - 1]} = 0.0561 \text{ m}^3/\text{s}$$
$$N_R = DV/\nu = 4Q/\pi D\nu = (4)(0.0561)/[(\pi)(0.350)(1.01 \times 10^{-6})] = 2.02 \times 10^5$$

From Fig. A-28, $C = 0.632$ as assumed is O.K.

17.133 A 15-mm-diameter VDI orifice is installed in a 30-mm-diameter pipe carrying nitrogen at $p_1 = 10$ atm, $T_1 = 30$ °C. For a pressure drop of 120 kPa across the orifice, calculate the mass flow rate.

▮
$$\dot{m} = CYA_0\sqrt{2\rho_1 \Delta p} \qquad A_0/A_1 = (15/30)^2 = 0.2500$$

From Fig. A-28, assume $C = 0.63$.

$$k = 1.4 \qquad D_0/D_1 = 0.5 \qquad p_2/p_1 = [(10)(101.3) - 120]/[(10)(101.3)] = 0.882$$

From Fig. A-26, $Y = 0.97$.

$$\rho = p/RT = (10)(101.3)/[(0.297)(30 + 273)] = 11.26 \text{ kg/m}^3$$
$$\dot{m} = (0.63)(0.97)[(\pi)(0.015)^2/4]\sqrt{(2)(11.26)[(120)(1000)]} = 0.1775 \text{ kg/s} \qquad N_R = \rho DV/\mu$$
$$V = \dot{m}/\rho A = 0.1775/\{(11.26)[(\pi)(0.030)^2/4]\} = 22.30 \text{ m/s}$$
$$N_R = (11.26)(0.030)(22.30)/(2.15 \times 10^{-5}) = 3.50 \times 10^5$$

From Fig. A-28, $C = 0.63$; hence, $\dot{m} = 0.1775$ kg/s.

17.134 Air at 1 atm, $T = 29$ °C, flows through a 1.1-m-square duct that contains a 400-mm-diameter square-edged orifice. With a head loss of 55 mmH_2O across the orifice, compute the mass flow.

▮
$$\dot{m} = CYA_0\sqrt{2\rho\,\Delta p} \qquad A_0/A_1 = [(\pi)(0.400)^2/4]/[(1.1)(1.1)] = 0.104$$

From Fig. A-28, assume $C = 0.605$ (for high N_R).

$$p_2 = p_1 - \gamma h = 101\,300 - (9790)(0.055) = 100\,762 \text{ Pa} \qquad p_2/p_1 = 100\,762/101\,300 = 0.995$$
$$D_0/D_1 = (0.400)/[(1.1)(1.1)] = 0.331 \qquad k = 1.4$$

From Fig. A-26, $Y = 0.995$.

$$\rho = p/RT = 101.3/[(0.287)(29 + 273)] = 1.170 \text{ kg/m}^3$$

$$\dot{m} = (0.605)(0.995)[(\pi)(0.400)^2/4]\sqrt{(2)(1.170)(9790)(0.055)} = 2.685 \text{ kg/s}$$

$$N_R = \rho D_h V/\mu = (1.170)[(4)(1.1)]\{2.29/[(1.1)(1.1)]\}/(1.915 \times 10^{-5}) = 5.09 \times 10^5$$

From Fig. A-28, $C = 0.605$, as assumed.

17.135 A 4-in-diameter VDI orifice is installed in an 8-in-diameter oil line, $\mu = 6$ cP, $\gamma = 54$ lb/ft^3. An oil-air differential manometer is used. For a gage difference of 18 in determine the flow rate in gallons per minute.

▮ $$Q = CA_0\sqrt{2gR'} \qquad A_0/A_1 = (\tfrac{4}{8})^2 = 0.250$$

From Fig. A-28, assume $C = 0.625$ (for high N_R).

$$Q = (0.625)[(\pi)(\tfrac{4}{12})^2/4]\sqrt{(2)(32.2)(\tfrac{18}{12})} = 0.536 \text{ ft}^3/\text{s}$$

$$N_R = \rho DV/\mu = 4Q\rho/\pi D\mu = (4)(0.536)(54/32.2)/\{(\pi)(\tfrac{8}{12})[(\tfrac{6}{100})/479]\} = 1.37 \times 10^4$$

From Fig. A-28, $C = 0.63$ (O.K.) $Q = 0.536/0.002228 = 241$ gpm.

17.136 Solve Prob. 17.73 for an orifice with $D:\frac{1}{2}D$ taps (instead of a long-radius flow nozzle).

▮ A good initial guess for an orifice is $\alpha \approx 0.62$. From Eq. (2) of Prob. 17.73 compute $\beta \approx 0.447/(0.62)^{1/2} = 0.568$. From Fig. A-29 compute the discharge coefficient $C_d = 0.6064$ for $\beta = 0.568$ and $N_R = 392\,000$. Then $E = [1 - (0.568)^4]^{-1/2} = 1.0565$ and $\alpha = C_dE = 0.6407$. Iterate Eq. (2) again: $\beta = 0.447/(0.6407)^{1/2} = 0.558$, $C_d = 0.6061$, and $\alpha = 0.6378$. Stop. We have converged satisfactorily to a design value: $\beta \approx 0.56$, $d = \beta D = 112$ mm. The throat velocity is $V_t = V_1/\beta^2 = 2.0/(0.56)^2 = 6.38$ m/s. The throat head is $V_t^2/2g = (6.38)^2/[2(9.81)] = 2.07$ m. From Fig. A-24 for the orifice at $\beta = 0.56$, estimate $K_m \approx 1.7$. Then the nonrecoverable loss of the orifice will be $h_m = K_m(V_t^2/2g) \approx 1.7(2.07) = 3.5$ m.

17.137 Water at 20 °C in a 15-cm-diameter pipe flows through a 7-cm-diameter thin-plate orifice with $D:\frac{1}{2}D$ taps ($\beta = 0.5$). If the measured pressure drop is 73 kPa, find the volume flow.

▮ $Q = C_dA_t\sqrt{2\,\Delta p/[(\rho)(1-\beta^4)]}$. Guess $C_d = 0.61$:

$$Q = (0.61)[(\pi)(0.07)^2/4]\sqrt{(2)(73)(1000)/[(998)(1-0.5^4)]} = 0.0293 \text{ m}^3/\text{s}.$$

$$V = Q/A = 0.0293/[(\pi)(0.15)^2/4] = 1.658 \text{ m/s}$$

$$N_R = \rho DV/\mu = (998)(0.15)(1.658)/(1.02 \times 10^{-3}) = 2.43 \times 10^5$$

From Fig. A-29, $C_d = 0.605$. $Q = (0.605)[(\pi)(0.07)^2/4]\sqrt{(2)(73)(1000)/[(998)(1-0.5^4)]} = 0.0291$ m^3/s, or 29 L/s.

17.138 Gasoline at 20 °C flows in a 60-mm-diameter pipe at 30 m^3/h. If a 30-mm-diameter thin-plate orifice with corner taps is installed in the pipe, what will the measured pressure drop be? $\beta = 0.4$. Use $\rho = 680$ kg/m^3 and $\mu = 0.000292$ Pa · s.

▮ $$Q = C_dA_t\sqrt{2\,\Delta p/[(\rho)(1-\beta^4)]} \qquad V = Q/A = (30/3600)/[(\pi)(0.060)^2/4] = 2.947 \text{ m/s}$$

$$N_R = \rho DV/\mu = (680)(0.060)(2.947)/0.000292 = 4.12 \times 10^5$$

For corner taps,

$$f(\beta) = 0.5959 + 0.0312\beta^{2.1} - 0.184\beta^8 = 0.5959 + (0.0312)(0.4)^{2.1} - (0.184)(0.4)^8 = 0.6003$$

$$C_d = f(\beta) + 91.71\beta^{2.5}N_R^{-0.75} = 0.6003 + (91.71)(0.4)^{2.5}(4.12 \times 10^5)^{-0.75} = 0.6009$$

$$30/3600 = (0.6009)[(\pi)(0.030)^2/4]\sqrt{2\,\Delta p/[(680)(1-0.4^4)]} \qquad \Delta p = 128\,000 \text{ Pa} = 128 \text{ kPa}$$

17.139 A 2-m-diameter cylindrical tank in Fig. 17-35 is initially filled with kerosene at 20 °C to a depth of 3 m. There is a 40-mm-diameter thin-plate orifice in the bottom. When the orifice is opened, what is the initial flow rate?

▮ $Q = C_dA_t\sqrt{2\,\Delta p/[(\rho)(1-\beta^4)]}$, Δp across the orifice is $\rho gh(t)$. The orifice simulates "corner taps" as $\beta \to 0$, so $C_d \approx 0.596$. $Q_0 = C_dA_t\sqrt{2gh_0/(1-\beta^4)} = (0.596)[(\pi)(0.040)^2/4]\sqrt{(2)(9.807)(3)/(1-0^4)} = 0.005745$ m^3/s, or 5.745 L/s.

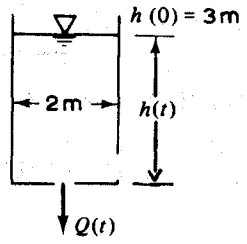


Fig. 17-35

17.140 For the conditions given in Prob. 17.139, how long will it take $h(t)$ to drop from 3 m to 1.9 m?

▮ $Q(t) = -dV_{\text{tank}}/dt = -(A_0)\,(dh/dt)$. On the other hand, from Prob. 17.139,

$$Q(t) = (0.596)[(\pi)(0.040)^2/4]\sqrt{(2)(9.807)(h)/(1-0^4)} = 0.003317h^{1/2}$$

Then, since $A_0 = (\pi)(2)^2/4 = 3.142\ \text{m}^2$,

$$(-3.142)(dh/dt) = 0.003317h^{1/2} \qquad \int_3^{1.9} \frac{dh}{h^{1/2}} = -\left(\frac{0.003317}{3.142}\right)\int_0^t dt \qquad [2h^{1/2}]_3^{1.9} = -0.001056[t]_0^t$$

$$-0.001056t = (2)(1.9^{1/2} - 3.0^{1/2}) \qquad t = 670\ \text{s} = 11.2\ \text{min}$$

17.141 Water flowing in a 12-in-diameter pipe passes through a Venturi meter with a 6-in throat diameter. The measured pressure head differential is 150.5 in of water. Assuming a discharge coefficient of 0.98, find the flow rate of water through the pipe.

▮

$$Q = CA_2\sqrt{[2g(p_1 - p_2)/\gamma]/[1-(A_2/A_1)^2]} \qquad A_2 = (\pi)(\tfrac{6}{12})^2/4 = 0.1963\ \text{ft}^2$$

$$A_1 = (\pi)(\tfrac{12}{12})^2/4 = 0.7854\ \text{ft}^2$$

$$Q = (0.98)(0.1963)\sqrt{[(2)(32.2)(150.5/12)]/[1-(0.1963/0.7854)^2]} = 5.65\ \text{ft}^3/\text{s}$$

17.142 Oil is flowing upward through a Venturi meter as shown in Fig. 17-36. Assume a discharge coefficient of 0.984. What is the rate of flow of the oil?

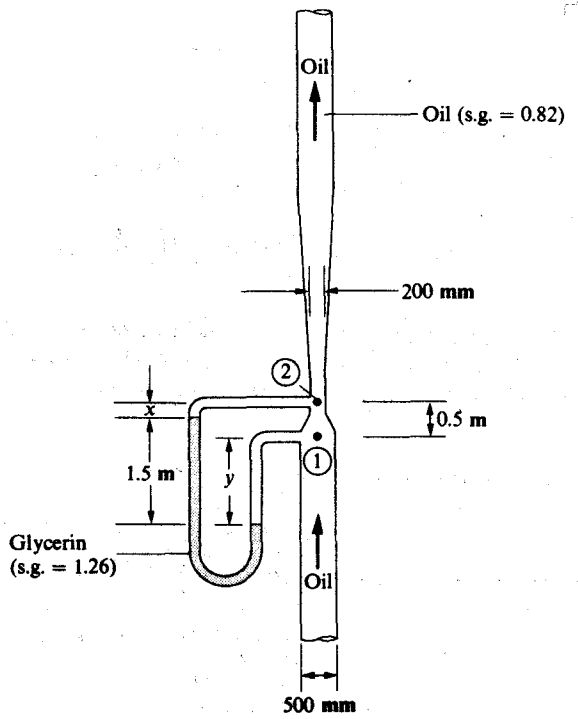


Fig. 17-36

$$Q = CA_2\sqrt{[2g(p_1 - p_2)/\gamma]/[1 - (A_2/A_1)^2]} \qquad A_2 = (\pi)(0.200)^2/4 = 0.03142\ \text{m}^2$$
$$A_1 = (\pi)(0.500)^2/4 = 0.1963\ \text{m}^2$$
$$p_1/\gamma + y - (1.26/0.82)(1.5) - x = p_2/\gamma \qquad x + 1.5 = 0.5 + y \qquad x = y - 1.0$$
$$p_1/\gamma + y - (1.26/0.82)(1.5) - (y - 1.0) = p_2/\gamma \qquad (p_1 - p_2)/\gamma = 1.305\ \text{m of oil}$$
$$Q = (0.984)(0.03142)\sqrt{(2)(9.807)(1.305)/[1 - (0.03142/0.1963)^2]} = 0.158\ \text{m}^3/\text{s}$$

17.143 Carbon dioxide at 20 °C flowing in a 200-mm-diameter pipe passes through a Venturi meter with a 100-mm throat diameter. Measured (gage) pressures are $p_1 = 22.0$ kPa and $p_2 = 5.2$ kPa. Atmospheric pressure is 762 mmHg, and the discharge coefficient is 0.98. Find the weight flow rate of carbon dioxide through the pipe.

$$G = C\gamma_2 A_2\sqrt{[2gk/(k-1)](p_1/\gamma_1)[1 - (p_2/p_1)^{(k-1)/k}]/[1 - (A_2/A_1)^2(p_2/p_1)^{2/k}]} \qquad \gamma = p/RT$$
$$p_2 = 5.2 + (0.762)(9.79)(13.6) = 106.7\ \text{kPa} \qquad \gamma_2 = 106.7/[(19.3)(20 + 273)] = 0.01887\ \text{kN/m}^3$$
$$p_1 = 22.0 + (0.762)(9.79)(13.6) = 123.5\ \text{kPa} \qquad \gamma_1 = 123.5/[(19.3)(20 + 273)] = 0.02184\ \text{kN/m}^3$$
$$A_2 = (\pi)(0.100)^2/4 = 0.007854\ \text{m}^2 \qquad A_1 = (\pi)(0.200)^2/4 = 0.03142\ \text{m}^2$$
$$G = (0.98)(0.01887)(0.007854)$$
$$\times\sqrt{\frac{[(2)(9.807)(1.30)/(1.30 - 1)](123.5/0.02184)[1 - (106.7/123.5)^{(1.30-1)/1.30}]}{1 - (0.007854/0.03142)^2(106.7/123.5)^{2/1.30}}} = 18.8\ \text{N/s}$$

17.144 Water flows through a Venturi meter, as shown in Fig. 17-37. Determine the discharge coefficient of the Venturi meter if the discharge is determined to be 2.12 cfs.

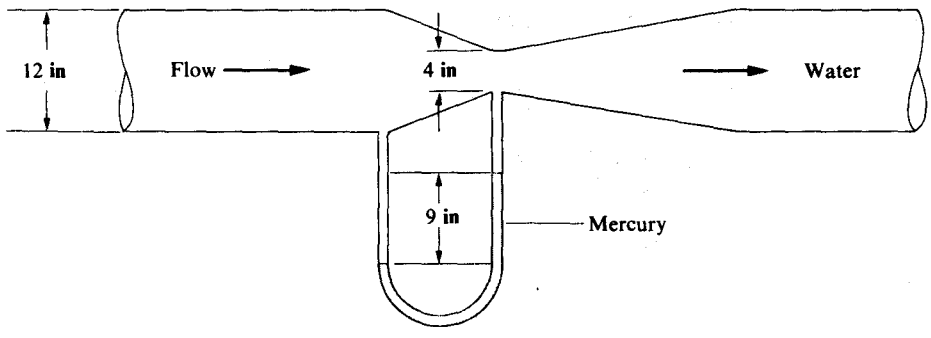


Fig. 17-37

$$Q = CA_2\sqrt{[2g(p_1 - p_2)/\gamma]/[1 - (A_2/A_1)^2]} \qquad A_2 = (\pi)(\tfrac{4}{12})^2/4 = 0.08727\ \text{ft}^2$$
$$A_1 = (\pi)(\tfrac{12}{12})^2/4 = 0.7854\ \text{ft}^2$$
$$p_1/\gamma + y + \tfrac{9}{12} - (13.6)(\tfrac{9}{12}) - y = p_2/\gamma \qquad (p_1 - p_2)/\gamma = 9.45\ \text{ft of water}$$
$$2.12 = (C)(0.08727)\sqrt{[(2)(32.2)(9.45)]/[1 - (0.08727/0.7854)^2]} \qquad C = 0.979$$

17.145 A Venturi meter having a throat diameter of 150 mm is installed in a horizontal 300-mm-diameter water main, as shown in Fig. 17-38. The coefficient of discharge is 0.982. Determine the difference in level of the mercury columns of the differential manometer attached to the Venturi meter if the discharge is 0.142 m^3/s.

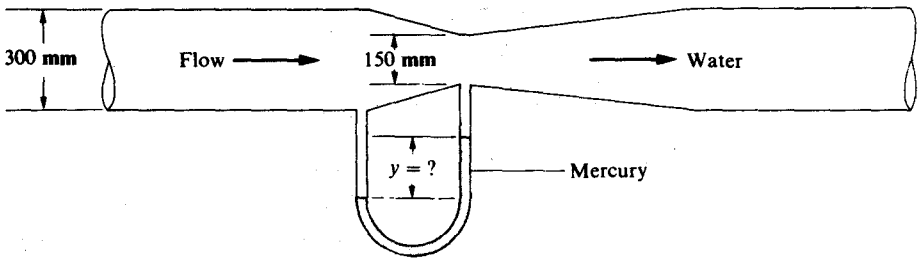


Fig. 17-38

$$Q = CA_2\sqrt{[2g(p_1 - p_2)/\gamma]/[1 - (A_2/A_1)^2]} \qquad A_2 = (\pi)(0.150)^2/4 = 0.01767\ \text{m}^2$$

$$A_1 = (\pi)(0.300)^2/4 = 0.07069\ \text{m}^2$$

$$0.142 = (0.982)(0.01767)\sqrt{[(2)(9.807)(p_1 - p_2)/\gamma]/[1 - (0.01767/0.07069)^2]}$$

$$(p_1 - p_2)/\gamma = 3.201\ \text{m}$$

$$p_1/\gamma + x + y - 13.6y - x = p_2/\gamma \qquad (p_1 - p_2)/\gamma = 12.6y = 3.201 \qquad y = 0.254\ \text{m} = 254\ \text{mm}$$

17.146 Determine the mass flow rate when air at 40 °C and 755 kPa abs flows through a Venturi meter, if the pressure at the throat of the meter is 455 kPa abs. The diameters at inlet and throat are 350 mm and 175 mm, respectively. Assume that $C = 0.985$.

$$\dot{m} = CYA_2\sqrt{2\rho_1\{(p_1 - p_2)/[1 - (D_2/D_1)^4]\}}$$

$$p_2/p_1 = 455/755 = 0.60 \qquad D_2/D_1 = 1/2 \qquad Y = 0.74 \qquad \text{(from Fig. A-26)}$$

$$\rho = p/RT \qquad \rho_1 = 755/[(0.287)(40 + 273)] = 8.40\ \text{kg/m}^3$$

$$\dot{m} = (0.985)(0.74)[(\pi)(0.175)^2/4]\sqrt{2(8.40)\{(3 \times 10^5)/[1 - (1/2)^4]\}} = 40.7\ \text{kg/s}$$

17.147 Find the flow rate of water at 72 °F for the Venturi tube of Fig. 17-39 if the mercury manometer reads $y = 5$ in for the case where $D_1 = 10$ in, $D_2 = 5$ in, and $\Delta z = 2.0$ ft. Assume the discharge coefficients of Fig. A-30 are applicable. In addition, find the head loss from inlet to throat. Also, determine the total head loss across the meter. Assume a diverging cone angle of 10°.

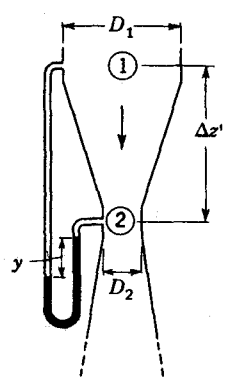


Fig. 17-39

$$Q = \{CA_2/[1 - (D_2/D_1)^4]^{1/2}\}[2g\,\Delta(z + p/\gamma)]^{1/2}$$

$$(z_1 + p_1/\gamma) - (z_2 + p_2/\gamma) = (R')(S_M/S_F - 1) = (\tfrac{5}{12})[(13.6/1) - 1] = 5.25\ \text{ft}$$

$$A_2 = (\pi)(\tfrac{5}{12})^2/4 = 0.1364\ \text{ft}^2$$

From Fig. A-30, assume $C = 0.95$.

$$Q = \{(0.95)(0.1364)/[1 - (\tfrac{5}{10})^4]^{1/2}\}[(2)(32.2)(5.25)]^{1/2} = 2.46\ \text{ft}^3/\text{s}$$

$$V_2 = Q/A_2 = 2.46/0.1364 = 18.0\ \text{ft/s} \qquad D''V_2 = (5)(18.0) = 90.0$$

From Fig. A-30, $C = 0.985$. Since the curve is flat in this area, $C = 0.985$ is O.K.

$$Q = (0.985/0.95)(2.46) = 2.55\ \text{ft}^3/\text{s} \qquad V_2 = 2.55/0.1364 = 18.7\ \text{ft/s}$$

$$V_3 = V_2/4 = 18.7/4 = 4.68\ \text{ft/s}$$

$$(h_L)_{1\text{-}2} = (1/C^2 - 1)[1 - (D_2/D_1)^4](V_2^2/2g) = [(1/0.985^2) - 1][1 - (\tfrac{5}{10})^4]\{18.7^2/[(2)(32.2)]\} = 0.156\ \text{ft}$$

$$(h_L)_{2\text{-}3} = k'(V_2 - V_3)^2/2g$$

From Fig. A-31, $k' = 0.175$.

$$(h_L)_{2\text{-}3} = (0.175)(18.7 - 4.68)^2/[(2)(32.2)] = 0.534\ \text{ft} \qquad (h_L)_{\text{total}} = 0.156 + 0.534 = 0.690\ \text{ft}$$

17.148 Repeat Prob. 17.147 for the case of a horizontal Venturi tube (i.e., $\Delta z = 0$) with all other data the same.

The answers are the same as for Prob. 17.147, since $R'(S_M/S_F - 1) = \Delta(p/\gamma + z)$.

17.149 Refer to Prob. 17.147. What would be the flow rate if the manometer fluid was carbon tetrachloride (s.g. = 1.59) with all other data remaining the same?

▮ $$Q = \{CA_2/[1-(D_2/D_1)^4]^{1/2}\}[2g\,\Delta(z+p/\gamma)]^{1/2}$$

Assume $C = 0.95$ (see Fig. A-30).

$$A_2 = 0.1364\ \text{ft}^2 \quad \text{(from Prob. 17.147)} \qquad \Delta(p/\gamma + z) = (\tfrac{5}{12})[(1.59/1)-1] = 0.2458\ \text{ft}$$

$$Q = \{(0.95)(0.1364)/[1-(\tfrac{5}{10})^4]^{1/2}\}[(2)(32.2)(0.2458)]^{1/2} = 0.532\ \text{ft}^3/\text{s}$$

$$V_2 = Q/A_2 = 0.532/0.1364 = 3.90\ \text{ft/s} \qquad D''V_2 = (5)(3.90) = 19.50$$

From Fig. A-30, $C = 0.985$.

$$Q = (0.985/0.95)(0.532) = 0.552\ \text{ft}^3/\text{s}$$

17.150 Find the flow rate of water for the Venturi tube shown in Fig. 17-39 if $D_1 = 70$ cm, $D_2 = 35$ cm, $\Delta z = 186$ cm, and $y = 13$ cm of mercury.

▮ $Q = \{CA_2/[1-(D_2/D_1)^4]^{1/2}\}\sqrt{2gy(S_0/S_1 - 1)}$. Try $C = 0.985$.

$$Q = \{(0.985)[(\pi)(0.35)^2/4]/[1-(\tfrac{35}{70})^4]^{1/2}\}\sqrt{(2)(9.807)(0.13)[(13.6/1)-1]} = 0.555\ \text{m}^3/\text{s}$$

$$V_2 = Q/A_2 = 0.555/[(\pi)(0.35)^2/4] = 5.77\ \text{m/s} \qquad N_R = DV/\nu = (0.35)(5.77)/10^{-6} = 2.02\times 10^6$$

From Fig. A-30, $C = 0.985$. Hence, $Q = 0.555\ \text{m}^3/\text{s}$.

17.151 In Fig. 17-39 suppose $D_1 = 4$ in, $D_2 = 2$ in, and $\Delta z = 8$ in. Oil (s.g. = 0.89) with a kinematic viscosity of $0.0005\ \text{ft}^2/\text{s}$ is flowing. Determine the manometer reading y if mercury is used as the manometer fluid. The rate of flow is 0.12 cfs.

▮ $Q = \{CA_2/[1-(D_2/D_1)^4]^{1/2}\}\sqrt{2gy(S_0/S - 1)}$. Try $C = 0.96$:

$$0.12 = \{(0.96)[(\pi)(\tfrac{2}{12})^2/4]/[1-(\tfrac{2}{4})^4]^{1/2}\}\sqrt{(2)(32.2)(y)[(13.6/0.89)-1]}$$

$$y = 0.03346\ \text{ft} \quad \text{or} \quad 0.40\ \text{in} \qquad V = Q/A = 0.12/[(\pi)(\tfrac{2}{12})^2/4] = 5.50\ \text{ft/s}$$

$$N_R = DV/\nu = (\tfrac{2}{12})(5.50)/0.0005 = 1833$$

From Fig. A-30, $C = 0.905$. $y = (0.905/0.96)^2(0.40) = 0.36$ in.

17.152 Assume that air at 50 °F and 90 psia flows through a Venturi tube and that the pressure at the throat is 54 psia. The inlet area is $0.55\ \text{ft}^2$, and the throat area is $0.10\ \text{ft}^2$. The tube coefficient is 0.98, and k is 1.4. Find the rate of discharge.

▮ $$G = CA_t\sqrt{2g[k/(k-1)]p_1\gamma_1(p_2/p_1)^{2/k}\{[1-(p_2/p_1)^{(k-1)/k}]/[1-(A_t/A)^2(p_2/p_1)^{2/k}]\}}$$

$$p_1 = (90)(144) = 12\,960\ \text{lb/ft}^2 \qquad \gamma = p/RT$$

$$\gamma_1 = 12\,960/[(53.3)(50+460)] = 0.4768\ \text{lb/ft}^3$$

$$G = (0.98)(0.10)\sqrt{(2)(32.2)\left(\frac{1.4}{1.4-1}\right)(12\,960)(0.4768)(\tfrac{54}{90})^{2/1.4}\left[\frac{1-(\frac{54}{90})^{(1.4-1)/1.4}}{1-(0.10/0.55)^2(\frac{54}{90})^{2/1.4}}\right]} = 29.8\ \text{lb/s}$$

17.153 Solve Prob. 17.152 by evaluating Y from Fig. A-26 and using it to find the rate of discharge.

▮ $$G = CYA_t\sqrt{2g\gamma_1\{(p_1-p_2)/[1-(D_2/D_1)^4]\}}$$

For $D_2/D_1 = \sqrt{\frac{0.10}{0.55}} = 0.426$ and $p_2/p_1 = \frac{54}{90} = 0.6$, from Fig. A-26, $Y = 0.745$.

$$G = (0.98)(0.745)(0.10)\sqrt{(2)(32.2)(0.4768)\{[(90-54)(144)]/(1-0.426^4)\}} = 29.6\ \text{lb/s}$$

17.154 What would be the value of Y and the rate of discharge for a square-edged orifice for the same data as in Prob. 17.152, assuming $C = 0.60$?

▮ $$G = CYA_t\sqrt{2g\gamma_1\{(p_1-p_2)/[1-(D_0/D_1)^4]\}}$$

With $D_0/D_1 = 0.426$ and $p_2/p_1 = 0.6$, $Y = 0.875$ (from Fig. A-26).

$$G = (0.60)(0.875)(0.10)\sqrt{(2)(32.2)(0.4768)[(90-54)(144)/(1-0.426^4)]} = 21.3\ \text{lb/s}$$

17.155 What is the value of the throat velocity in Prob. 17.152?

▮ $$V^2/2 = C_pT_1[1-(p_2/p_1)^{(k-1)/k}] = (6000)(50+460)[1-(\tfrac{54}{90})^{(1.40-1)/1.40}] \qquad V = 912\text{ ft/s}$$

17.156 Air flows through a 20-cm by 10-cm Venturi meter. At the inlet the air temperature is 30 °C and the pressure is 150 kPa. Determine the mass flow if a mercury manometer reads 27 cm. Assume an atmospheric pressure of 101.3 kPa abs.

▮ $\dot{m} = CYA_t\sqrt{2\rho_1\{(p_1-p_2)/[1-(D_0/D_1)^4]\}}$. From Fig. A-30, $C = 0.985$.

$$\rho = p/RT \qquad \rho_1 = (150+101.3)/[(0.287)(30+273)] = 2.89\text{ kg/m}^3$$

$$p_1 - p_2 = \gamma h = [(13.6)(9.79)](0.27) = 35.9\text{ kPa}$$

$$p_1 = 150+101.3 = 251.3\text{ kPa} \qquad p_2 = 251.3 - 35.9 = 215.4\text{ kPa}$$

For $D_2/D_1 = 0.5$ and $p_2/p_1 = 215.4/251.3 = 0.857$, from Fig. A-26, $Y = 0.92$. Thus

$$\dot{m} = (0.985)(0.92)[(\pi)(0.10/2)^2]\sqrt{2(2.89)[(35.9)(1000)/(1-0.5^4)]} = 3.34\text{ kg/s}.$$

17.157 Natural gas, for which $k = 1.3$ and $R = 3100$ ft-lb/(slug-°R), flows through a Venturi tube with pipe and throat diameters of 10 in and 5 in, respectively. The initial pressure of the gas is 159 psia, and its temperature is 40 °F. If the meter coefficient is 0.98, find the rate of flow for a throat pressure of 109 psia.

▮ $$G = CA_t\sqrt{2g[k/(k-1)]p_1\gamma_1(p_2/p_1)^{2/k}\{[1-(p_2/p_1)^{(k-1)/k}]/[1-(A_2/A_1)^2(p_2/p_1)^{2/k}]\}}$$

$$A_t = (\pi)(\tfrac{5}{12})^2/4 = 0.1364\text{ ft}^2 \qquad \gamma = p/RT$$

$$\gamma_1 = (159)(144)/[(3100/32.2)(40+460)] = 0.4756\text{ lb/ft}^3$$

$$G = (0.98)(0.1364)$$
$$\times\sqrt{(2)(32.2)[1.3/(1.3-1)][(159)(144)](0.4756)(\tfrac{109}{159})^{2/1.3}\{[1-(\tfrac{109}{159})^{(1.3-1)/1.3}]/[1-(5^2/10^2)^2(\tfrac{109}{159})^{2/1.3}]\}}$$
$$= 51.2\text{ lb/s}$$

17.158 Referring to Fig. 17-40, assume that liquid flows from A to C at the rate of 222 L/s and that the friction loss between A and B is negligible but that between B and C it is $0.1V_B^2/2g$. Find the pressure heads at A and C.

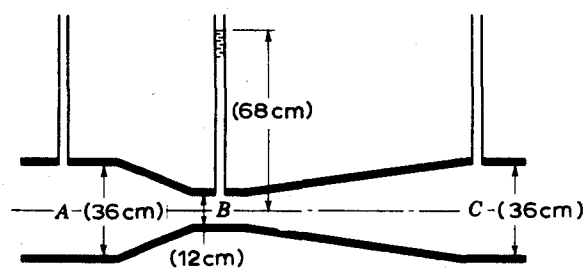


Fig. 17-40

▮ $$p_A/\gamma + V_A^2/2g + z_A = p_B/\gamma + V_B^2/2g + z_B + h_L$$

$$V_A = Q/A_A = 0.222/[(\pi)(0.36)^2/4] = 2.18\text{ m/s} \qquad V_B = 0.222/[(\pi)(0.12)^2/4] = 19.6\text{ m/s}$$

$$p_A/\gamma + 2.18^2/[(2)(9.807)] + 0 = 0.68 + 19.6^2/[(2)(9.807)] + 0 + 0 \qquad p_A/\gamma = 20.0\text{ m}$$

$$p_B/\gamma + V_B^2/2g + z_B = p_C/\gamma + V_C^2/2g + z_C + h_L$$

$$0.68 + 19.6^2/[(2)(9.807)] + 0 = p_C/\gamma + 2.18^2/[(2)(9.807)] + 0 + 0 + (0.1)\{19.6^2/[(2)(9.807)]\}$$

$$p_C/\gamma = 18.1\text{ m}$$

17.159 Water flows through a 12-in by 6-in Venturi meter at the rate of 1.49 cfs and the differential gage is deflected 3.50 ft, as shown in Fig. 17-41. The specific gravity of the gage liquid is 1.25. Determine the coefficient of the meter.

▮ Applying the Bernoulli equation, A to B, ideal case, yields $(p_A/\gamma + V_{12}^2/2g + 0)$ – no lost head $= (p_B/\gamma + V_6^2/2g + 0)$ and $V_{12}^2 = (A_6/A_{12})^2V_6^2$. Solving, $V_6 = \sqrt{2g(p_A/\gamma - p_B/\gamma)/[1-(A_6/A_{12})^2]}$ (no lost head).

The true velocity (and hence the true value of flow Q) will be obtained by multiplying the ideal value by the coefficient C of the meter. Thus

$$Q = A_6V_6 = A_6C\sqrt{2g(p_A/\gamma - p_B/\gamma)/[1-(A_6/A_{12})^2]} \qquad (1)$$

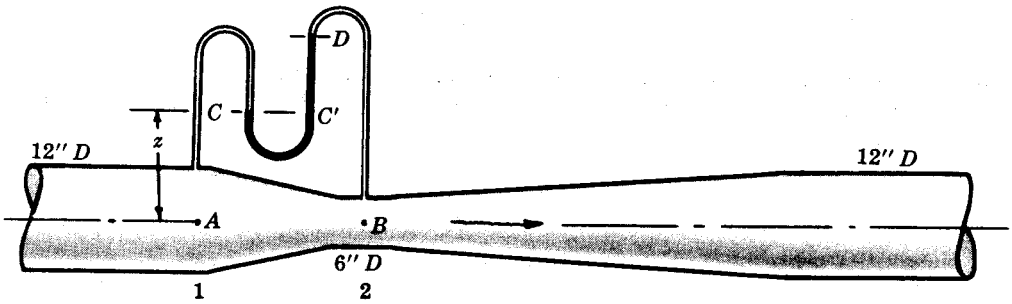


Fig. 17-41

To obtain the differential pressure head above, the principles of the differential gage must be used. $p_C = p_{C'}$, $(p_A/\gamma - z) = p_B/\gamma - (z + 3.50) + 1.25(3.50)$ or $p_A/\gamma - p_B/\gamma = 0.875$ ft.

Substituting in (*1*), $1.49 = \frac{1}{4}\pi(\frac{1}{2})^2 C\sqrt{2g(0.875)/(1 - \frac{1}{16})}$ and $C = 0.978$ (use 0.98).

17.160 Water flows upward through a vertical 12-in by 6-in Venturi meter whose coefficient is 0.980. The differential gage deflection is 3.88 ft of liquid of specific gravity 1.25, as shown in Fig. 17-42. Determine the flow in cfs.

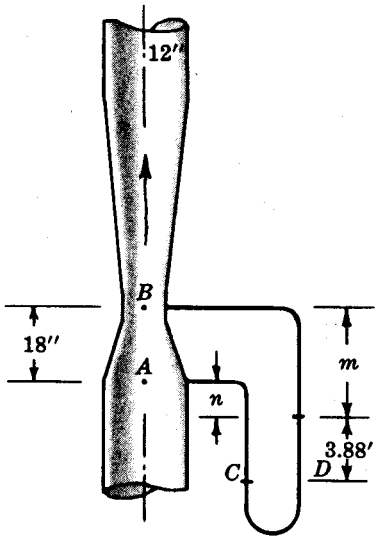


Fig. 17-42

▮ Reference to the Bernoulli equation in Prob. 17.159 indicates that, for this problem, $z_A = 0$ and $z_B = 1.50$ ft. Then $Q = CA_6\sqrt{2g[(p_A/\gamma - p_B/\gamma) - 1.50]/[1 - (\frac{1}{2})^4]}$.

Using the principles of the differential gage to obtain $\Delta p/\gamma$,

$$p_C/\gamma = p_D/\gamma \quad \text{(ft of water units)}$$

$$p_A/\gamma + (n + 3.88) = p_B/\gamma + m + 1.25(3.88)$$

$$(p_A/\gamma - p_B/\gamma) - (m - n) = 3.88(1.25 - 1.00)$$

$$(p_A/\gamma - p_B/\gamma) - 1.50 = 0.97 \text{ ft of water}$$

Substituting in the equation for flow, $Q = 0.980(\frac{1}{4}\pi)(\frac{1}{2})^2\sqrt{2g(0.97)/(1 - \frac{1}{16})} = 1.57$ cfs.

17.161 An 8-in by 4-in Venturi meter is used to measure the flow of carbon dioxide at 68 °F. The deflection of the water column in the differential gage is 71.8 in and the barometer reads 30.0 in of mercury. For a pressure at entrance of 18.0 psi abs, calculate the weight flow.

▮ The absolute pressure at entrance $= p_1 = (18.0)(144) = 2590$ psf abs and the specific weight γ_1 of the carbon dioxide is $\gamma_1 = 2590/[34.9(460 + 68)] = 0.1405$ lb/ft³. The pressure difference $= (71.8/12)(62.4 - 0.141) = 372$ psf and hence the absolute pressure at the throat $= p_2 = 2590 - 372 = 2220$ psf abs.

To obtain the specific weight γ_2, we use $p_2/p_1 = \frac{2220}{2590} = 0.858$ and $\gamma_2/\gamma_1 = (0.858)^{1/k}$. Thus $\gamma_2 = (0.1405)(0.858)^{1/1.30} = 0.1250$ lb/ft³. $W = \gamma_1 K A_2 Y\sqrt{2g(\Delta p/\gamma_1)}$ in lb/s. Using $k = 1.30$, $d_2/d_1 = 0.50$ and $p_2/p_1 =$

0.858, Y (Table A-18) = 0.909 by interpolation. Assuming $C = 0.985$, from Fig. A-32, and noting that $K = 1.032$, we have $W = (0.1405)[(1.032)(0.985)][\frac{1}{4}\pi(\frac{4}{12})^2](0.909)\sqrt{2g(372/0.1405)} = 4.67$ lb/s.

To check the assumed value of C, determine the Reynolds number and use the appropriate curve on Fig. A-32. $N_R = 4W/\pi d_2 \nu\gamma_2 = 4(4.67)/\{\pi(\frac{4}{12})[(9.1)(14.7)/18.0](10^{-5})](0.1250)\} = 1.92 \times 10^6$.

From Fig. A-32, $C = 0.984$. Recalculating, $W = 4.66$ lb/s.

17.162 A 100-mm-diameter pipe has a 50-mm-diameter, long constriction. If the mercury manometer indicates a column-height difference of 200 mm, estimate the flow rate of water passed by the pipe.

▮ We assume that the streamlines are parallel at locations 1, 2, and 3 (Fig. 17-43). Applying the Bernoulli equation with mechanical-energy loss for the sudden contraction between points 1 and 2 gives $p_1/\rho g + \bar{V}_1^2/2g - K_{SC}(\bar{V}_2^2/2g) = p_2/\rho g + \bar{V}_2^2/2g$. The Bernoulli equation with mechanical-energy loss for the sudden enlargement between points 2 and 3 gives $p_2/\rho g + \bar{V}_2^2/2g - K_{SE}(\bar{V}_2^2/2g) = p_3/\rho g + \bar{V}_3^2/2g$. The steady incompressible continuity equation gives $\bar{V}_1 = \bar{V}_3 = (d/D)^2\bar{V}_2$. The pressure change indicated by the manometer is $p_1 - p_3 = (p_1 - p_2) + (p_2 - p_3)$. Using the Bernoulli equation to eliminate the pressure differences $p_1 - p_2$ and $p_2 - p_3$ and continuity to eliminate $\bar{V}_1$ and $\bar{V}_3$, we get $p_1 - p_3 = (K_{SC} + K_{SE})\frac{1}{2}\rho\bar{V}_2^2$. But $\dot{Q} = (\pi d^2/4)\bar{V}_2$ and $p_1 - p_3 = \rho g(\text{s.g.}_{\text{Hg}} - 1)h$. Then $\dot{Q} = (\pi d^2/4)\sqrt{2gh(\text{s.g.}_{\text{Hg}} - 1)/(K_{SC} + K_{SE})}$. From Fig. A-33, $K_{SC} = 0.478$ and from $K_{SE} = (1 - A_1/A_2)^2$, $K_{SE} = (1 - 0.25)^2 = 0.5625$. Then $\dot{Q} = (\pi/4)(0.050)^2\sqrt{2(9.80)(0.200)(13.6 - 1)/(0.478 + 0.5625)}$ $0.0135\ \text{m}^3/\text{s}$.

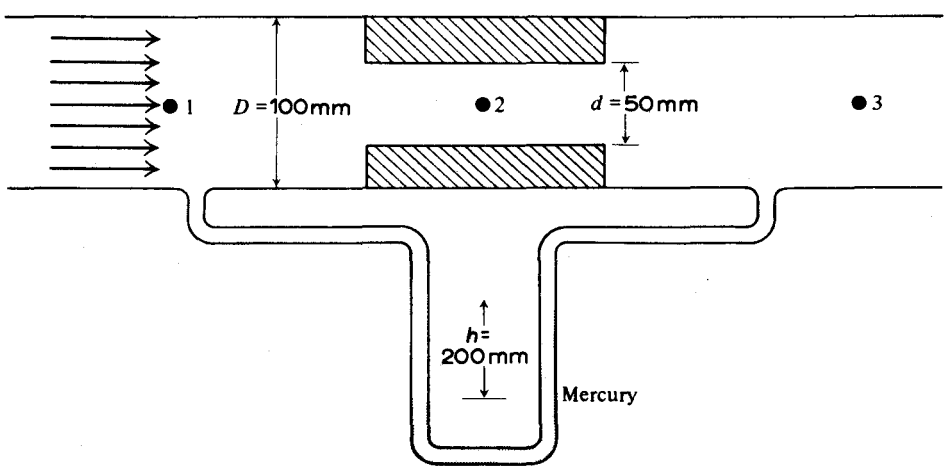


Fig. 17-43

17.163 Find q for flow of water in a pipe of inside diameter D of 100 mm using a Venturi meter with a machined convergent section. The throat diameter d of the Venturi meter is 60 mm. The Venturi meter is in a pipe section having an inclination θ (see Fig. 17-44) of 45°. The distance from 1 to 2 in the meter is 120 mm and the value of h for the manometer is 140 mm. Take $\rho = 999\ \text{kg/m}^3$ and $\mu = 1.12 \times 10^{-3}$ Pa · s.

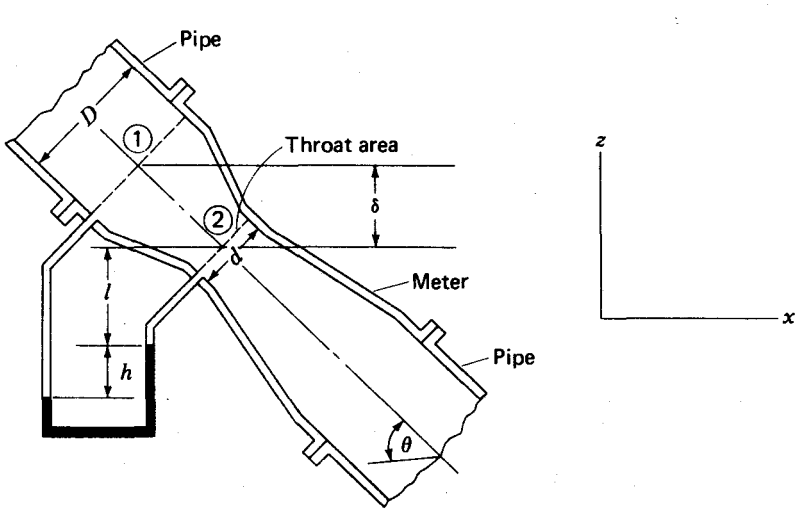


Fig. 17-44

▮ We note that $(C_d)_{\text{ven}}$ for this case is 0.995. We need next the value of $p_1 - p_2$. This is easily determined as follows from manometry: $p_1 + \gamma_{H_2O}(\delta + l + h) = p_2 + l\gamma_{H_2O} + h\gamma_{Hg}$, $p_1 - p_2 = h(\gamma_{Hg} - \gamma_{H_2O}) - \gamma_{H_2O}\delta = (0.140)(13.6 - 1)(999)(9.81) - (999)(9.81)(0.120)(0.707) = 16\,456$ Pa. We now compute V_2 in $q_{act} = (C_d)_{\text{ven}}A_2(2\{[(p_1 - p_2)/\rho] + (z_1 - z_2)g\}/[1 - (A_2/A_1)^2])^{1/2}$ by deleting A_2 in the formula. Thus, $V_2 = 0.995(2\{[16\,456/999] + (0.120)(0.707)(9.81)\}/[1 - (\frac{60}{100})^4]\})^{1/2}$ where we used diameters squared in place of areas. We get from the last equation and continuity $V_2 = 6.27$ m/s, $V_1 = (0.6^2)(6.27) = 2.26$ m/s. We must next compute N_R for the pipe: $N_R = \rho V_1 D/\mu = [(999)(2.26)(0.100)/(1.12 \times 10^{-3})] = 2.02 \times 10^5$. We see that the Reynolds number is within the proper range for the $(C_d)_{\text{ven}}$ we have used, and so $q = (V_2)(A_2) = (6.27)[(\pi)(0.060^2)/4] = 0.01773\ \text{m}^3/\text{s}$.

17.164 Consider in Prob. 17.163 that we have a flow of air at a temperature of 40 °C and a pressure of 200 kPa abs. The pipe as before is 100 mm in inside diameter, but the diameter in the Venturi is now 20 mm. The height h is 140 mmHg. What is the mass flow rate?

▮ We can use the same C_d as in incompressible flow, which is 0.995. We next determine Y, so we can use $\dot{m} = \rho_1 C_d A_3\{2[(p_1 - p_2)/\rho_1]/[1 - (A_2/A_1)^2]\}^{1/2}Y$. We need p_2 for this purpose. Thus, $p_2 = p_1 - \Delta p = 200\,000 - (0.140)(9806)(13.6) = 181.3$ kPa. The ratio p_2/p_1 is then $p_2/p_1 = 181.3/200 = 0.907$. Noting that $A_2/A_1 = (d/D)^2 = (0.020/0.100)^2 = 0.04$, we go to Fig. A-34 to get Y: $Y = 0.95$. Finally, we need ρ_1. Using the equation of state of a perfect gas, $\rho_1 = p_1/RT_1 = 200\,000/[(287)(313)] = 2.2264\ \text{kg/m}^3$, $\dot{m} = \rho_1 C_d A_2\{2[(p_1 - p_2)/\rho_1]/[1 - (A_2/A_1)^2]\}^{1/2}Y = (2.2264)(0.995)[(\pi)(0.020^2)/4]\{2[(200\,000 - 181\,300)/2.2264]/[1 - (0.04^2)]\}^{1/2}(0.95) = 0.0858$ kg/s.

17.165 A 3-m by 1.5-m Venturi meter carries water at 25 °C. A water-air differential manometer has a gage difference of 72 mm. What is the discharge?

▮ $Q = C_v A_2\sqrt{2g(\Delta p/\gamma)/[1 - (D_2/D_1)^4]}$. Assume $N_R > 3 \times 10^5$. From Fig. A-35, $C_v = 0.983$.

$$Q = (0.983)[(\pi)(1.5)^2/4]\sqrt{(2)(9.807)(0.072)/[1 - (\tfrac{1.5}{3})^4]} = 2.13\ \text{m}^3/\text{s}$$

$$N_R = D_1V_1/\nu \qquad V_2 = Q/A_2 = 2.13/[(\pi)(1.5)^2/4] = 1.21\ \text{m/s}$$

$$A_1V_1 = A_2V_2 \qquad [(\pi)(3)^2/4](V_1) = [(\pi)(1.5)^2/4](1.21) \qquad V_1 = 0.302\ \text{m/s}$$

$$N_R = (3)(0.302)/(9.02 \times 10^{-7}) = 1.00 \times 10^6$$

From Fig. A-35, $C_v = 0.983$. Hence, $Q = 2.13\ \text{m}^3/\text{s}$.

17.166 What is the pressure difference between the upstream section and throat of a 140-mm by 70-mm horizontal Venturi meter carrying 46 L/s of water at 58 °C?

▮

$$Q = C_v A_2\sqrt{2g(\Delta p/\gamma)/[1 - (D_2/D_1)^4]} \qquad V_1 = Q/A_1 = (46 \times 10^{-3})/[(\pi)(0.140)^2/4] = 2.988\ \text{m/s}$$

$$N_R = V_1D_1/\nu = (2.988)(0.140)/(5.70 \times 10^{-7}) = 7.34 \times 10^5$$

From Fig. A-35, $C_v = 0.984$.

$$46 \times 10^{-3} = (0.984)[(\pi)(0.070)^2/4]\sqrt{(2)(9.807)(\Delta p/9.79)/[1 - (\tfrac{70}{140})^4]} \qquad \Delta p = 69.0\ \text{kPa}$$

17.167 A 14-in by 7-in Venturi meter is mounted in a vertical pipe with the flow upward. 2900 gpm of oil, s.g. = 0.82, $\mu = 0.1$ P, flows through the pipe. The throat section is 7 in above the upstream section. What is $p_1 - p_2$?

▮

$$Q = C_v A_2\sqrt{2g[-h + (p_1 - p_2)/\gamma]/[1 - (D_2/D_1)^4]}$$

$$V_1 = Q/A_1 = (2900)(0.002228)/[(\pi)(\tfrac{14}{12})^2/4] = 6.044\ \text{ft/s}$$

$$N_R = \rho D_1V_1/\mu = [(0.82)(1.94)](\tfrac{14}{12})(6.044)/(0.1/479) = 5.37 \times 10^4$$

From Fig. A-35, $C_v = 0.962$.

$$(2900)(0.002228) = (0.962)[(\pi)(\tfrac{7}{12})^2/4]\sqrt{(2)(32.2)\{-\tfrac{7}{12} + (p_1 - p_2)/[(0.82)(62.4)]\}/[1 - (\tfrac{7}{14})^4]}$$

$$p_1 - p_2 = 500.3\ \text{lb/ft}^2 \quad \text{or} \quad 3.47\ \text{lb/in}^2$$

17.168 Air flows through a Venturi meter in a 50-mm-diameter pipe having a throat diameter of 25 mm, $C_v = 0.97$. For $p_1 = 800$ kPa abs, $T_1 = 35$ °C, $p_2 = 520$ kPa abs, calculate the mass per second flowing.

$$\dot{m} = C_v YA_2\sqrt{2\rho_1\,\Delta p/[1 - (D_2/D_1)^4]} \qquad p_2/p_1 = 520/800 = 0.650 \qquad D_2/D_1 = 25/50 = 1/2$$

From Fig. A-26, $Y = 0.78$.

$$\rho = p/RT \qquad \rho_1 = 800/[(0.287)(35+273)] = 9.050\ \text{kg/m}^3$$
$$\dot{m} = (0.97)(0.78)[(\pi)(0.025)^2/4]\sqrt{(2)(9.050)[(800-520)(1000)]/[1-(\tfrac{1}{2})^4]} = 0.864\ \text{kg/s}$$

17.169 Oxygen, $p_1 = 37$ psia, $T_1 = 100\ °\text{F}$, flows through a 2-in by 1-in Venturi meter with a pressure drop of 5 psi. Find the mass per second flowing.

▮ $\dot{m} = C_v Y A_2 \sqrt{(2\rho_1\,\Delta p)/[1-(D_2/D_1)^4]}$. Assume $C_v = 0.984$.

$$p_2/p_1 = (37-5)/37 = 0.865 \qquad D_2/D_1 = 1/2 = 0.500$$

From Fig. A-26, $Y = 0.91$.

$$\rho = p/RT \qquad \rho_1 = (37)(144)/[(1551)(100+460)] = 0.006134\ \text{slug/ft}^3$$
$$\dot{m} = (0.984)(0.91)[(\pi)(1/12)^2/4]\sqrt{(2)(0.006134)[(5)(144)]/[1-(1/2)^4]} = 0.01499\ \text{slug/s}$$
$$V_1 = \dot{m}/\rho_1 A_1 = 0.01499/\{(0.006134)[(\pi)(\tfrac{2}{12})^2/4]\} = 112\ \text{ft/s}$$
$$N_R = \rho_1 D_1 V_1/\mu = (0.006134)(\tfrac{2}{12})(112)/(4.18\times10^{-7}) = 2.74\times10^5$$

From Fig. A-35, $C_v = 0.984$ is O.K.

17.170 Solve Prob. 17.73 for a Venturi nozzle.

▮ For the Venturi guess $\alpha \approx 1.0$ and iterate Eq. (2) of Prob. 17.73: $\alpha = 1.0$, $\beta = 0.447$, $C_d = 0.9806$. This rapidly converges to give $\beta = 0.4468$, $d = \beta D = 89$ mm. The throat velocity is $2.0/(0.4468)^2 = 10.0$ m/s, and the throat head is $(10.0)^2/[2(9.81)] = 5.12$ m. From Fig. A-24 for the Venturi estimate $K_m \approx 0.15$; hence $h_m = 0.15(5.12\ \text{m}) = 0.8$ m.

17.171 It is planned to equip a 20-cm-diameter pipe carrying water at 20 °C with a modern Venturi nozzle. In order for international standards to be met, what is the permissible range of flow rates and nozzle diameters?

▮ **Flow range:**

$$1.5\times10^5 < N_R < 2\times10^6 \qquad 1.5\times10^5 < 4Q\rho/\pi\mu D < 2\times10^6$$
$$1.5\times10^5 < (4)(Q)(998)/[(\pi)(1.02\times10^{-3})(0.20)] < 2\times10^6 \qquad 0.0241\ \text{m}^3/\text{s} < Q < 0.321\ \text{m}^3/\text{s}$$

Size range:

$$0.316 < [\beta = d/D] < 0.775 \qquad 0.316 < d/20 < 0.775 \qquad 6.32\ \text{cm} < d < 15.50\ \text{cm}$$

17.172 For Prob. 17.171, what is the permissible range of pressure drops? Would compressibility be a problem for the highest pressure-drop condition?

▮ From Fig. A-36, $0.9236 < C_d < 0.9847$.

$$Q = (C_d \pi d^2/4)\sqrt{(2\,\Delta p)/[\rho(1-\beta^4)]} \qquad d = \beta D = 0.20\beta$$
$$\Delta p = (1.598\times10^6)(1-\beta^4)(Q/C_d\beta^2)^2$$

Smallest Δp occurs at largest β and smallest Q.

$$(\Delta p)_{\min} = (1.598\times10^6)(1-0.775^4)\{0.0241/[(0.9847)(0.775)^2]\}^2 = 1.696\ \text{kPa}$$

Largest Δp occurs at smallest β and largest Q.

$$(\Delta p)_{\max} = (1.598\times10^6)(1-0.316^4)\{0.321/[(0.9236)(0.316)^2]\}^2 = 19.2\ \text{MPa}$$

Since $(\Delta p)_{\max}$ is much smaller than the bulk modulus of water (2190 MPa), compressibility is not important.

17.173 Light oil (s.g. = 0.92, $\nu = 10^{-5}\ \text{ft}^2/\text{s}$) flows down an 8-in vertical pipe through a 4-in Venturi nozzle as in Fig. 17-45. If the mercury manometer reads a deflection of 5 in, what is the estimated flow rate?

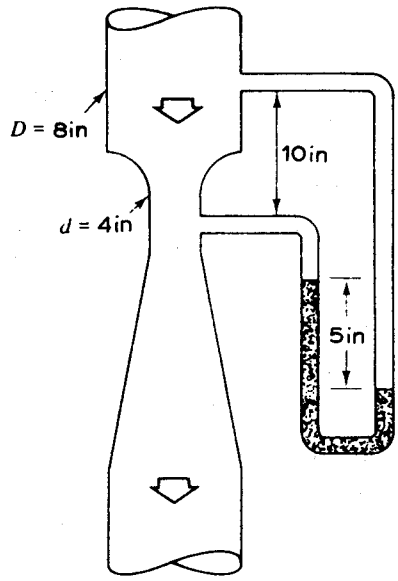


Fig. 17-45

$$Q = C_d A_t \sqrt{(2\,\Delta p)/[\rho(1-\beta^4)]} \qquad C_d = 0.9858 - 0.196\beta^{4.5} \qquad \beta = d/D = \tfrac{4}{8} = 0.500$$

$$C_d = 0.9858 - (0.196)(0.500)^{4.5} = 0.9771$$

$$\Delta p = (\rho_M - \rho_0)gh = (\text{s.g.}_M - \text{s.g.}_0)\gamma h = (13.6 - 0.92)(62.4)(\tfrac{5}{12}) = 329.7\ \text{lb/ft}^2$$

$$Q = (0.9771)[(\pi)(\tfrac{4}{12})^2/4]\sqrt{(2)(329.7)/\{[(0.92)(1.94)](1-0.500^4)\}} = 1.69\ \text{ft}^3/\text{s}$$

$$N_R = 4Q/\pi\nu D = (4)(1.69)/(\pi)(10^{-5})(\tfrac{8}{12}) = 3.23 \times 10^5 \quad \text{(O.K.)}$$

17.174 A Venturi meter (Fig. 17-46) is a carefully designed constriction whose pressure difference is a measure of the flow rate in a pipe. Using Bernoulli's equation for steady incompressible flow with no losses, show that flow rate Q is related to manometer reading h by $Q \approx [A_2/\sqrt{1-(D_2/D_1)^4}]\sqrt{[2gh(\rho_M - \rho)/\rho]}$ where ρ_M is the density of the manometer fluid.

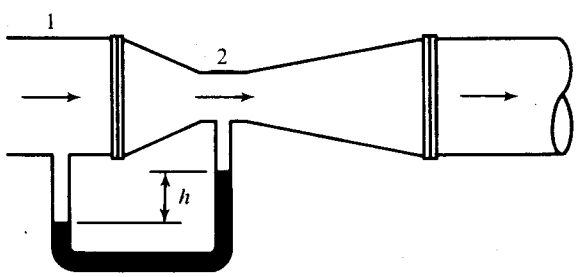


Fig. 17-46

Continuity: $\rho_1 A_1 V_1 = \rho_2 A_2 V_2$. Since $\rho_1 = \rho_2$, $V_2 = V_1(D_1/D_2)^2$.

Bernoulli (no losses): $p_1/\rho + V_1^2/2 + gz_1 = p_2/\rho + V_2^2/2 + gz_2$. Since $gz_1 = gz_2$, $V_2 = \sqrt{2(p_1 - p_2)/\{\rho[1-(D_2/D_1)^4]\}}$, $Q = A_2 V_2$.

Manometer: $p_1 - p_2 = (\rho_M - \rho)gh$.
Combining these, $Q \approx [A_2/\sqrt{1-(D_2/D_1)^4}]\sqrt{2gh(\rho_M - \rho)/\rho}$.

17.175 Water flows from a reservoir over a dam that is essentially a broad-crested weir. The dam is 20 ft long. How much higher will the water surface in the reservoir be above the top of the dam when the discharge over the dam is 1000 cfs?

$$Q = CLH^{1.5} \qquad 1000 = (3.09)(120)(H^{1.5}) \qquad H = 1.94\ \text{ft}$$

17.176 A sharp-crested, suppressed weir is under a head of 1.65 m. The weir is 2.5 m long and has a height of 4.50 m. What is the flow rate of water over the weir?

▮ $$C = 0.611 + 0.075(H/H_w) = 0.611 + 0.075(1.65/4.50) = 0.6385$$
$$Q = C(\tfrac{2}{3})L\sqrt{2g}\,H^{1.5} = (0.6385)(\tfrac{2}{3})(2.50)\sqrt{(2)(9.807)}(1.65)^{1.5} = 9.99\ \text{m}^3/\text{s}$$

17.177 A sharp-crested, contracted, rectangular weir is to carry water at a flow rate of 12.75 cfs. What must the weir width be in order for the head of flow to be 1.50 ft?

▮ $$Q = CL^{1.02}H^{1.47} \qquad 12.75 = (3.10)(L^{1.02})(1.50)^{1.47} \qquad L = 2.23\ \text{ft}$$

17.178 A sharp-crested, contracted, triangular weir with a weir angle of 80° carries water under a head of 0.955 m. Find the flow of water over the weir.

▮ $$Q = C(\tfrac{8}{15})\sqrt{2g}\,[\tan(\theta/2)]H^{2.5} = (0.60)(\tfrac{8}{15})\sqrt{(2)(9.807)}\,[\tan(80°/2)](0.955)^{2.5} = 1.06\ \text{m}^3/\text{s}$$

17.179 Water flows over a Cipolletti weir 8.0 ft long under a head of 1.2 ft. What is the flow rate of water over the weir?

▮ $$Q = CLH^{1.5} = (3.367)(8.0)(1.2)^{1.5} = 35.4\ \text{ft}^3/\text{s}$$

17.180 For the same conditions as given in Prob. 17.178 except that the weir is submerged and the downstream water surface is 0.450 m above the top of the weir, find the flow rate over the submerged weir.

▮ $$Q = Q_1[1 - (d/h)^n]^{0.385} \qquad Q_1 = 1.06\ \text{m}^3/\text{s} \qquad \text{(from Prob. 17.178)}$$
$$Q = (1.06)[1 - (0.450/0.955)^{2.5}]^{0.385} = 0.995\ \text{m}^3/\text{s}$$

17.181 A sharp-crested, suppressed weir is under a head of 2.2 ft. The weir's length is 10.0 ft, and its height is 4.0 ft. Determine the flow rate of water over the weir.

▮ $$C = 0.611 + 0.075(H/H_w) = 0.611 + 0.075(2.2/4.0) = 0.6522$$
$$Q = C(\tfrac{2}{3})L\sqrt{2g}\,H^{1.5} = (0.6522)(\tfrac{2}{3})(10)\sqrt{(2)(32.2)}\,(2.2)^{1.5} = 114\ \text{ft}^3/\text{s}$$

17.182 A sharp-crested, contracted, rectangular weir carries water at a flow rate of 1.81 m³/s. What must the length of the weir be in order for the head of flow to be 0.8 m?

▮ $$Q = CL^{1.02}H^{1.47} \qquad 181 = (1.69)(L^{1.02})(0.8)^{1.47} \qquad L = 1.475\ \text{m}$$

17.183 A sharp-crested, contracted, triangular weir with a weir angle of 60° carries water under a head of 1.23 ft. Determine the flow rate of water over the weir.

▮ $$Q = C(\tfrac{8}{15})\sqrt{2g}\,[\tan(\theta/2)]H^{2.5} = (0.60)(\tfrac{8}{15})\sqrt{(2)(32.2)}\,[\tan(60°/2)](1.23)^{2.5} = 2.49\ \text{ft}^3/\text{s}$$

17.184 The rate of flow of water over a sharp-crested, contracted, 45° triangular weir is 0.823 cfs. Determine the head of flow.

▮ $$Q = C(\tfrac{8}{15})\sqrt{2g}\,[\tan(\theta/2)]H^{2.5} \qquad 0.823 = (0.60)(\tfrac{8}{15})\sqrt{(2)(32.2)}\,[\tan(45°/2)](H)^{2.5} \qquad H = 0.902\ \text{ft}$$

17.185 What length of a Cipolletti weir should be constructed in order that the head of flow will be 0.91 m when the rate of flow is 3.45 m³/s?

▮ $$Q = CLH^{1.5} \qquad 3.45 = (1.859)(L)(0.910)^{1.5} \qquad L = 2.14\ \text{m}$$

17.186 Compute the rate of flow of water over a 4.5-ft-long Cipolletti weir under a head of 1.15 ft.

▮ $$Q = CLH^{1.5} = (3.367)(4.5)(1.15)^{1.5} = 18.7\ \text{ft}^3/\text{s}$$

17.187 Water flows from a reservoir over a dam that is essentially a broad-crested weir. The dam is 45 m long, and the head of flow is 0.83 m. Compute the discharge of water over the dam.

▮ $$Q = CLH^{1.5} = (1.71)(45)(0.83)^{1.5} = 58.2\ \text{m}^3/\text{s}$$

17.188 Given the same conditions as Prob. 17.181 except that the weir is submerged and the downstream water surface is 0.96 ft above the top of the weir, determine the flow rate of water over the weir.

▮ $$Q = Q_1[1 - (d/h)^n]^{0.385} \qquad Q_1 = 114\ \text{ft}^3/\text{s} \qquad \text{(from Prob. 17.181)}$$
$$Q = (114)[1 - (0.96/2.2)^{1.5}]^{0.385} = 100\ \text{ft}^3/\text{s}$$

17.189 Flow is occurring in a rectangular channel at a velocity of 3 fps and depth of 1.0 ft. Neglecting the effect of velocity of approach, determine the height of a sharp-crested, suppressed weir that must be installed to raise the water depth upstream of the weir to 4 ft.

▮ $$Q = 3.33LH^{3/2} = Av = (1.0L)(3) = 3.00L \qquad 3.00L = (3.33L)(H)^{3/2}$$
$$H = 0.93 \text{ ft} \qquad \text{Height of weir} = 4 - 0.93 = 3.07 \text{ ft}$$

17.190 A rectangular, sharp-crested weir 1.0 m high extends across a rectangular channel which is 3.3 m wide. When the head is 0.49 m, find the rate of discharge by neglecting the velocity of approach.

▮ $$Q = 1.84LH^{3/2} = (1.84)(3.3)(0.49)^{3/2} = 2.08 \text{ m}^3/\text{s}$$

17.191 Suppose the rectangular weir of Prob. 17.190 is contracted at both ends. Find the rate of discharge for a head of 0.49 m by the Francis formula.

▮ $$Q = 1.84LH^{3/2} = (1.84)[3.3 - (2)(0.1)(0.49)](0.49)^{3/2} = 2.02 \text{ m}^3/\text{s}$$

17.192 Develop a table of C_d versus P/H with H as a parameter. Use the Rehbock formula.

▮ $C_d = 0.605 + 1/305H + 0.08H/P$. For $P/H = 0.5$ and $H = 0.2$, $C_d = 0.605 + 1/[(305)(0.2)] + 0.08/0.5 = 0.781$. Similarly, for additional values of P/H and H:

	C_d		
P/H	$H = 0.2$ ft	1.0 ft	5.0 ft
0.5	0.781	0.768	0.765
1.0	0.701	0.688	0.685
2.0	0.661	0.648	0.645
5.0	0.637	0.624	0.621
10.0	0.629	0.616	0.613

17.193 What is the rate of discharge of water over a 60° triangular weir when the head is 0.9 ft?

▮ $Q = C_d(\frac{8}{15})\sqrt{2g}\,[\tan(\theta/2)](H^{5/2})$. From Fig. A-37, $C_d = 0.582$. $Q = (0.582)(\frac{8}{15})\sqrt{(2)(32.2)}\,[\tan(60°/2)](0.9)^{5/2} = 1.11 \text{ ft}^3/\text{s}$.

17.194 With the same head, what would be the increase in discharge obtained by doubling the notch angle in Prob. 17.193?

▮ $Q = C_d(\frac{8}{15})\sqrt{2g}\,[\tan(\theta/2)](H^{5/2})$. From Fig. A-37, $C_d = 0.581$. $Q = (0.581)(\frac{8}{15})\sqrt{(2)(32.2)}\,[\tan(120°/2)](0.9)^{5/2} = 3.31 \text{ ft}^3/\text{s}$, increase $= (3.31 - 1.11)/1.11 = 1.98$, or 198 percent.

17.195 What would be the head for a discharge of 2.6 cfs of water over a 90° triangular weir? Use a value of C_d of 0.58.

▮ $$Q = C_d(\tfrac{8}{15})\sqrt{2g}\,[\tan(\theta/2)](H^{5/2}) \qquad 2.6 = (0.58)(\tfrac{8}{15})\sqrt{(2)(32.2)}\,[\tan(90°/2)](H^{5/2}) \qquad H = 1.02 \text{ ft}$$

17.196 For the Cipolletti weir, derive the slope ($\frac{1}{4}$:1) of the sides of the trapezoid by setting the reduction in discharge due to contraction equal to the increase in discharge due to the triangular area added.

▮ By the Francis formula, the decrease in flow due to two contractions is $Q = C_d(\frac{2}{3})\sqrt{2g}\,LH^{3/2}$, $\Delta Q = C_d(\frac{2}{3})\sqrt{2g}\,[(2)(0.1h)]H^{3/2}$. This deficiency is made up by the equivalent triangular weir of angle θ, for which $Q = C_d(\frac{8}{15})\sqrt{2g}\,[\tan(\theta/2)](H^{5/2})$. Equating these, $C_d(\frac{2}{3})\sqrt{2g}\,[(2)(0.1H)]H^{3/2} = C_d(\frac{8}{15})\sqrt{2g}\,[\tan(\theta/2)](H^{5/2})$. Assuming C_d is the same for both weirs, $\tan(\theta/2) = 0.2500$, or side slope is $\frac{1}{4}$:1.

17.197 Develop in general terms an expression for the percent of error in Q over a triangular weir if there is a small error in the measurement of the vertex angle. Assume there is no error in the weir coefficient. Compute the percent error in Q if there is a 3° error in the measurement of the total vertex angle of a triangular weir having a total vertex angle of 60°.

▮ $Q = C_d(\frac{8}{15})(2g)^{1/2}[\tan(\theta/2)]H^{5/2}$ $\qquad \partial Q/\partial\theta = C_d(\frac{8}{15})(2g)^{1/2}H^{5/2}\{(\partial/\partial\theta)[\tan(\theta/2)]\}$

$$\partial Q/Q = (\partial/\partial\theta)\{[\tan(\theta/2)]\,\partial\theta\}/\tan(\theta/2) = \sec^2(\theta/2)(\tfrac{1}{2})\,\partial\theta/[\tan(\theta/2)] = \partial\theta/[2\sin(\theta/2)\cos(\theta/2)]$$

For the case where $\partial\theta = 3°$ and $\theta = 60°$: $\theta/2 = 30°$, so that $\partial Q/Q = 3(\pi/180)/[2(0.500)0.866] = 0.0453 = 4.53$ percent.

17.198 A broad-crested weir rises 0.4 m above the bottom of a horizontal channel. With a measured head of 0.7 m above the crest, what is the rate of discharge per unit width?

▮
$$Q = L\sqrt{g}\,(\tfrac{2}{3})^{3/2}E^{3/2} = L\sqrt{9.807}\,(\tfrac{2}{3})^{3/2}(0.7 + V_1^2/2g)^{3/2}$$
$$Q/L = q = (1.705)\{0.7 + V_1^2/[(2)(9.807)]\}^{3/2}$$

But $q = (0.7 + 0.4)V_1$: $V_1 = q/1.1$, $q = (1.705)\{0.7 + (q/1.1)^2/[(2)(9.807)]\}^{3/2}$, $q = 1.11\ \text{m}^3/\text{s/m}$ (by trial and error).

17.199 A broad-crested weir of height 2.50 ft in a channel 6.00 ft wide has a flow over it of 11.50 cfs. What is the water depth just upstream of the weir?

▮ $q = 11.5/6 = 1.92$ cfs/ft $\qquad y_2 = y_c = (q^2/g)^{1/3} = (1.92^2/32.2)^{1/3} = 0.4856$ ft

$V_2 = q/y_2 = 1.92/0.4856 = 3.954$ ft/s $\qquad y_1 + V_1^2/2g = 2.5 + 0.4856 + 3.954^2/[(2)(32.2)]$

But $V_1 = 1.92/y_1$: $y_1 + (1.92/y_1)^2/[(2)(32.2)] = 3.228$, $y_1 = 3.22$ ft (by trial and error).

17.200 A 90° V-notch weir is used to measure a flow rate of 0.29 cfs. Assuming C_d is known precisely, compute the percentage of error in Q that would result from an error of 0.01 ft in head measurement.

▮ $Q = C_d(\frac{8}{15})(2g)^{1/2}[\tan(\theta/2)]H^{5/2}$. For $\theta = 90°$, $C_d = 0.587$, and $Q = 0.29$ cfs: $H = 0.422$ ft. $\partial Q/\partial H = C_d(\frac{8}{15})(2g)^{1/2}[\tan(\theta/2)](\frac{5}{2})H^{3/2}$, $\partial Q/Q = \frac{5}{2}H^{3/2}(\partial H)/H^{5/2} = 5(\partial H)/2H = 5(0.01)/[2(0.422)] = 0.0592 = 5.92$ percent.

17.201 Work Prob. 17.200 for a rectangular weir with end contractions having a crest length of 3 ft.

▮ $Q = 3.33LH^{3/2}$ $\quad L = 3$ ft $\quad$ and $\quad Q = 0.29$ cfs $\quad H = 0.0945$ ft

$\partial Q/\partial H = C_w L(\frac{3}{2})H^{1/2}$ $\qquad \partial Q/Q = \frac{3}{2}H^{1/2}(\partial H)/H^{3/2} = 3(\partial H)/2H = 3(0.01)/[2(0.0945)] = 0.159 = 15.9$ percent

17.202 Derive the theoretical formula for flow over a rectangular weir. Refer to Fig. 17-47.

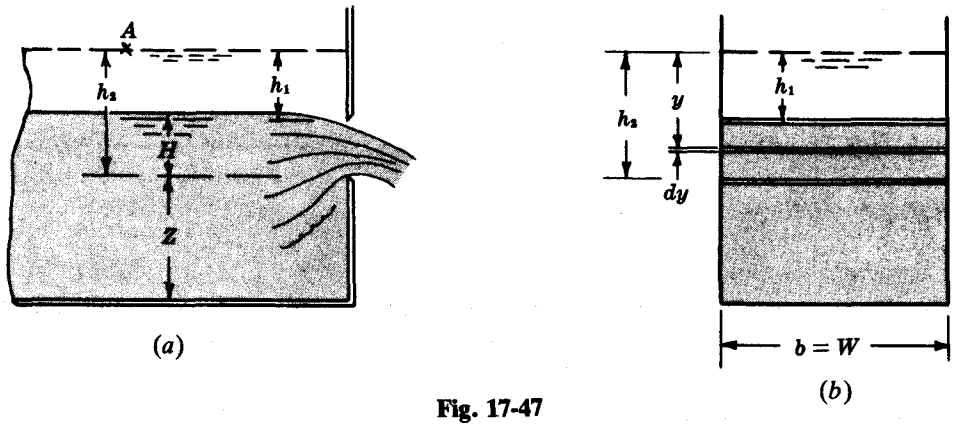


Fig. 17-47

▮ Consider the rectangular opening in Fig. 17-47 to extend the full width W of the channel ($b = W$). With the liquid surface in the dashed position, application of the Bernoulli theorem between A and an elemental strip of height dy in the jet produces, for ideal conditions, $(0 + V_A^2/2g + y) - \text{no losses} = (0 + V_{jet}^2/2g + 0)$ where V_A represents the average velocity of the particles approaching the opening.

Thus the ideal $V_{jet} = \sqrt{2g(y + V_A^2/2g)}$ and

$$\text{Ideal } dQ = dAV_{jet} = (b\,dy)V_{jet} = b\sqrt{2g}\left(y + \frac{V_A^2}{2g}\right)^{1/2} dy = b\sqrt{2g}\int_{h_1}^{h_2}\left(y + \frac{V_A^2}{2g}\right)^{1/2} dy$$

A weir exists when $h_1 = 0$. Let H replace h_2 and introduce a coefficient of discharge C to obtain the actual

flow. Then

$$Q = Cb\sqrt{2g}\int_0^H \left(y + \frac{V_A^2}{2g}\right)^{1/2} dy = \tfrac{2}{3}Cb\sqrt{2g}\left[\left(H + \frac{V_A^2}{2g}\right)^{3/2} - \left(\frac{V_A^2}{2}\right)^{3/2}\right] = mb\left[\left(H + \frac{V_A^2}{2g}\right)^{3/2} - \left(\frac{V_A^2}{2g}\right)^{3/2}\right] \quad (1)$$

Notes:

(1) For a fully contracted rectangular weir the end contractions cause a reduction in flow. Length b is corrected to recognize this condition, and the formula becomes

$$Q = m(b - \tfrac{2}{10})[H + V_A^2/2g)^{3/2} - (V_A^2/2g)^{3/2}] \quad (2)$$

(2) For high weirs and most contracted weirs the velocity head of approach is negligible and

$$Q = m(b - \tfrac{2}{10}H)H^{3/2} \quad \text{for contracted weirs} \quad (3)$$

or

$$Q = mbH^{3/2} \quad \text{for suppressed weirs} \quad (4)$$

(3) Coefficient of discharge C is not constant. It embraces the many complexities not included in the derivation, such as surface tension, viscosity, density, nonuniform velocity distribution, secondary flows, and possibly others.

17.203 Derive the theoretical formula for flow through a triangular-notched weir. Refer to Fig. 17-48.

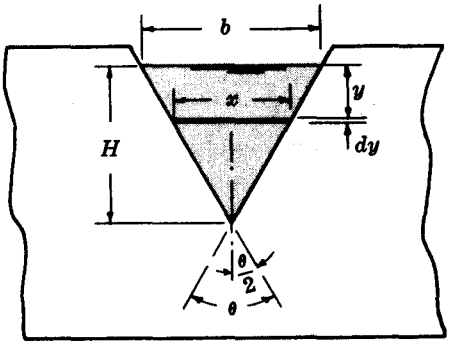


Fig. 17-48

▮ From Prob. 17.202, $V_{jet} = \sqrt{2g(y + \text{negligible } V^2/2g)}$ and ideal $dQ = dAV_{jet} = x\,dy\sqrt{2gy}$. By similar triangles, $x/b = (H - y)/H$ and $b = 2H\tan(\theta/2)$. Then

$$\text{Actual } Q = \left(\frac{b}{H}\right)C\sqrt{2g}\int_0^H (H - y)y^{1/2}\,dy$$

Integrating and substituting,

$$Q = \tfrac{8}{15}C\sqrt{2g}\,H^{5/2}\tan(\tfrac{1}{2}\theta) \quad (1)$$

A common V-notch is the 90° opening. Expression (*1*) then becomes $Q = 4.28CH^{5/2}$ where an *average* value of C is about 0.60 for heads above 1.0 ft.

17.204 During a test on an 8-ft suppressed weir which was 3 ft high, the head was maintained constant at 1.000 ft. In 38.0 s, 7600 gal of water were collected. Find weir factor m in Eqs. (*1*) and (*4*) of Prob. 17.202.

▮ (*a*) Change the measured flow to cfs. $Q = 7600/[(7.48)(38)] = 26.7$ cfs.

(*b*) Check the velocity of approach. $V = Q/A = 26.7/[(8)(4)] = 0.834$ ft/s. Then $V^2/2g = (0.834)^2/2g = 0.018$ ft.

(*c*) Using (*1*), $Q = mb[(H + V^2/2g)^{3/2} - (V^2/2g)^{3/2}]$ or $26.7 = (m)(8)[(1.000 + 0.011)^{3/2} - (0.0108)^{3/2}]$ and $m = 3.29$. Using (*4*), $Q = 26.7 = mbH^{3/2} = (m)(8)(1.000)^{3/2}$ and $m = 3.34$ (about 1.8 percent higher neglecting the velocity of approach terms).

17.205 Determine the flow over a suppressed weir 10.0 ft long and 4.00 ft high under a head of 3.000 ft. The value of m is 3.46.

▮ Since the velocity head term cannot be calculated, an approximate flow is $Q = mbH^{3/2} = 3.46(10)(3.000)^{3/2} = 178.8$ cfs. For this flow, $V = 178.8/[(10)(7)] = 2.554$ ft/s and $V^2/2g = 0.101$ ft. Using Eq. (*1*) of Prob. 17.202, $Q = 3.46(10)[(3.000 + 0.101)^{3/2} - (0.101)^{3/2}] = 188$ cfs.

This second calculation shows an increase of 9 cfs or about 5.0 percent over the first calculation. Further calculation will generally produce an unwarranted refinement, i.e., beyond the accuracy of the formula itself. However, to illustrate, the revised velocity of approach would be $V = 188/[(10)(7)] = 2.69$ ft/s, $V^2/2g = 0.112$ ft and $Q = 3.46(10)[(3.000 + 0.112)^{3/2} - (0.112)^{3/2}] = 189$ cfs.

17.206 A suppressed weir, 25.0 ft long, is to discharge 375.0 cfs into a channel. The weir factor $m = 3.42$. To what height Z (nearest $\frac{1}{100}$ ft) may the weir be built, if the water behind the weir must not exceed 6 ft in depth?

▮ Velocity of approach $V = Q/A = 375.0/[(25)(6)] = 2.50$ ft/s. Then $375.0 = (3.42)(25.0)\{[H + (2.50)^2/2g]^{3/2} - [(2.50)^2/2g]^{3/2}\}$ and $H = 2.59$ ft. Height of weir is $Z = 6.00 - 2.59 = 3.41$ ft.

17.207 A contracted weir, 4.00 ft high, is to be installed in a channel 8 ft wide. The maximum flow over the weir is 60.0 cfs when the total depth back of the weir is 7.00 ft. What length of weir should be installed if $m = 3.40$?

▮ Velocity of approach $V = Q/A = 60.0/[(8)(7)] = 1.071$ ft/s. It appears that the velocity head is negligible in this case. $Q = m(b - \frac{2}{10}H)(H)^{3/2}$, $60.0 = 3.40[b - \frac{2}{10}(3.00)](3.00)^{3/2}$, $b = 4.00$ ft long.

17.208 The discharge from a 6-in-diameter orifice, under a 10.0-ft head, $C = 0.600$, flows into a rectangular weir channel and over a contracted weir. The channel is 6 ft wide and, for the weir, $Z = 5.00$ ft and $b = 1.00$ ft. Determine the depth of water in the channel if $m = 3.35$.

▮ The discharge through the orifice is $Q = CA\sqrt{2gh} = (0.600)[\frac{1}{4}\pi(\frac{1}{2})^2]\sqrt{2g(10.0)} = 2.99$ cfs. For the weir, $Q = m(b - \frac{2}{10}H)H^{3/2}$ (velocity head neglected) or $2.99 = 3.35(1.00 - 0.20H)H^{3/2}$ and $H^{3/2} - 0.20H^{5/2} = 0.893$. By successive trials, $H = 1.09$ ft; and the depth $= Z + H = 5.00 + 1.09 = 6.09$ ft.

17.209 The discharge of water over a 45° triangular weir is 0.750 cfs. For $C = 0.580$, determine the head on the weir.

▮
$$Q = C(\tfrac{8}{15})\sqrt{2g}\,[\tan(\theta/2)]H^{5/2}$$
$$0.750 = (0.580)(\tfrac{8}{15})\sqrt{(2)(32.2)}\,[\tan(45°/2)]H^{5/2} \qquad H = 0.881 \text{ ft}$$

17.210 What length of Cipolletti weir should be constructed so that the head will be 1.54 ft when the discharge is 122 cfs?

▮
$$Q = 3.367bH^{3/2} \qquad 122 = (3.367)(b)(1.54)^{3/2} \qquad b = 19.0 \text{ ft}$$

17.211 Derive the expression for the time to lower the liquid level in a tank, lock, or canal, by means of a suppressed weir.

▮
$$Q\,dt = -A_T\,dH \qquad (mLH^{3/2})\,dt = -A_T\,dH$$
$$t = \int_{t_1}^{t_2} dt = \left(\frac{-A_T}{mL}\right)\int_{H_1}^{H_2} H^{-3/2}\,dH \qquad t = t_2 - t_1 = \left(\frac{2A_T}{mL}\right)(H_2^{-1/2} - H_1^{-1/2})$$

17.212 A rectangular flume, 50 ft long and 10 ft wide, feeds a suppressed weir under a head of 1.000 ft. If the supply to the flume is cut off, how long will it take for the head on the weir to decrease to 4 in? Use $m = 3.33$.

▮
$$t = (2A_T/mL)(H_2^{-1/2} - H_1^{-1/2}) \qquad \text{(from Prob. 17.211)}$$
$$= (2)[(50)(10)]/[(3.33)(10)][(\tfrac{4}{12})^{-1/2} - 1.000^{-1/2}] = 22.0 \text{ s}$$

17.213 The head on a sharp-crested, rectangular weir of height 2.0 ft and crest length 4.0 ft was incorrectly observed to be 0.38 ft when it was actually 0.40 ft. Determine the percentage error in the computed value of flow rate.

▮
$$Q = CLH^{3/2} \qquad \partial Q/\partial H = (\tfrac{3}{2})CLH^{1/2} \approx \Delta Q/\Delta H \qquad \Delta Q \approx \Delta H(\tfrac{3}{2})CLH^{1/2}$$
$$\Delta Q/Q \approx (\tfrac{3}{2})(\Delta H/H) = (\tfrac{3}{2})(0.02/0.40) = 0.075 \quad \text{or} \quad 7.5 \text{ percent}$$

17.214 It is desired to measure a discharge which may vary from 0.6 to 2.4 cfs with a relative accuracy of at least 0.4 percent through the entire range. A stage recorder which is accurate to the nearest 0.001 ft is available. What is the maximum width of rectangular sharp-crested weir that will satisfy these conditions? Assume the weir has end contractions and $C_w = 3.5$. Neglect velocity of approach.

▮
$$Q = 3.5Lh^{3/2} \qquad \partial Q/\partial h = 5.25Lh^{1/2} \approx \Delta Q/\Delta h$$

Thus $\Delta Q = 5.25Lh^{1/2}(\Delta h)$. $\Delta Q/Q = (5.25/3.50)(\Delta h/h) = 1.5(\Delta h/h)$, $0.004 = 1.5(0.001/h)$, $h = 0.0015/0.004 = 0.375$ ft. h must be ≥ 0.375 ft to acquire desired accuracy: $Q = 0.6 = 3.5L(0.375)^{3/2}$, $L \leq 0.747$ ft.

17.215 What is the maximum permissible vertex angle of a V-notch weir that will satisfy the conditions of Prob. 17.214?

▮
$$Q = 4.28(0.58)h^{5/2}[\tan(\theta/2)] = 2.48h^{5/2}[\tan(\theta/2)]$$
$$\partial Q/\partial h = \tfrac{5}{2}(2.48)h^{3/2}[\tan(\theta/2)] = 6.2h^{3/2}[\tan(\theta/2)] \approx \Delta Q/\Delta h$$

Thus $\Delta Q = 6.2h^{3/2}[\tan(\theta/2)]\,\Delta h$; $\Delta Q/Q = 2.5(\Delta h/h)$, $0.004 = 2.5(0.001/h)$, $h \geq 0.625$ ft; $Q = 0.6 = 2.48h^{5/2}[\tan(\theta/2)] = 2.48(0.625)^{5/2}[\tan(\theta/2)]$, $\tan(\theta/2) = 0.6/[2.48(0.309)] = 0.783$, $\theta/2 \leq 38°04'$, $\theta < 76°08'$.

17.216 With a head of 0.12 m on a 60° V-notch weir, what error in the measured head will produce the same percentage error in the computed flow rate as an error of 1° in the vertex angle?

▮
$$Q = 2.48h^{5/2}[\tan(\theta/2)] \qquad \partial Q/\partial h = 6.2h^{3/2}[\tan(\theta/2)] \approx \partial Q/\partial h$$

Thus $\Delta Q = 6.2h^{3/2}[\tan(\theta/2)]\,\Delta h$; $\Delta Q/Q = 2.5(\Delta h/h)$, $\partial Q/\partial\theta = 2.48h^{5/2}[\sec^2(\theta/2)]\frac{1}{2} = 1.24h^{5/2}[\sec^2(\theta/2)] = \Delta Q/\Delta\theta$. Thus $\Delta Q = 1.24h^{5/2}[\sec^2(\theta/2)]\,\Delta\theta$; $\Delta Q/Q = [(\Delta\theta)\sec^2(\theta/2)]/(2\tan\theta)$. For $\theta = 30°$, $\sec^2(\theta/2)/\tan(\theta/2) = 2.31$, $\Delta Q = 2.5(\Delta h/0.12) = 2.31(1°)/2 = (2.31/2)(1/57.3)]$, $\Delta h = 0.00097$ m $= 0.10$ cm.

17.217 Tests on a 60°-notch weir yield the following values of head H on the weir and discharge Q:

H, ft	0.345	0.356	0.456	0.537	0.568	0.594	0.619	0.635	0.654	0.665
Q, cfs	0.107	0.110	0.205	0.303	0.350	0.400	0.435	0.460	0.490	0.520

Use the theory of least squares to determine the constants in $Q = CH^m$ for this weir.

▮ Taking the logarithm of each side of the equation, $\ln Q = \ln C + m \ln H$ or $y = B + mx$, it is noted that the best values of B and m are needed for a straight line through the data when plotted on log-log paper.

By the theory of least squares, the best straight line through the data points is the one yielding a minimum value of the sums of the squares of vertical displacements of each point from the line; or, from Fig. 17-49,

$$F = \sum_{i=1}^{i=n} s_i^2 = \sum [y_i - (B + mx_i)]^2$$

where n is the number of experimental points. To minimize F, $\partial F/\partial B$ and $\partial F/\partial m$ are taken and set equal to zero, yielding two equations in the two unknowns B and m: $\partial F/\partial B = 0 = 2\sum[y_i - (B + mx_i)](-1)$ from which

$$\sum y_i - nB - m\sum x_i = 0 \tag{1}$$

and $\partial F/\partial m = 0 = 2\sum[y_i - (B + mx_i)](-x_i)$ or

$$\sum x_i y_i - B\sum x_i - m\sum x_i^2 = 0 \tag{2}$$

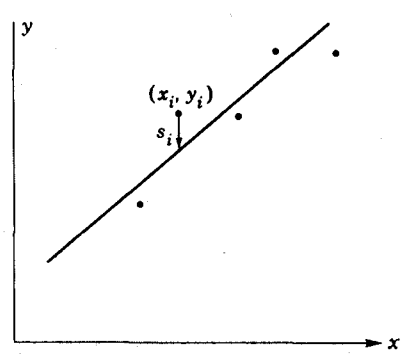


Fig. 17-49

Solving Eqs. (1) and (2) for m gives $m = (\sum x_i y_i/\sum x_i - \sum y_i/n)/(\sum x_i^2/\sum x_i - \sum x_i/n)$, $B = (\sum y_i - m \sum x_i)/n$. These equations are readily solved by hand calculator having the $\sum$ key, or a simple program can be written for the digital computer. The answer for the data of this problem is $m = 2.437$, $C = 1.395$.

17.218 A rectangular, sharp-crested weir 15 m long with end contractions suppressed is 1.6 m high. Determine the discharge when the head is 300 mm.

▮ Since the height of the weir is large compared to H, the upstream velocity head can be omitted; hence, $Q = 1.83LH^{1.5} = (1.83)(15)(0.300)^{1.5} = 4.51\ \text{m}^3/\text{s}$.

17.219 In Fig. 17-50, $L = 9$ ft, $P = 1.9$ ft, and $H = 0.90$ ft. Estimate the discharge over the weir ($C = 3.33$).

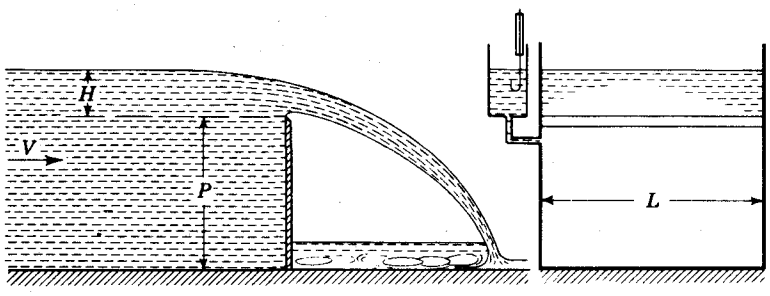


Fig. 17-50

▮ $Q = CL(H + \alpha V^2/2g)^{1.5}$ $\quad V = Q/[(L)(P + H)]$ $\quad Q = CL\{H + \alpha Q^2/[(2gL^2)(P + H)^2]\}^{1.5}$

$Q = (3.33)(9)\{0.90 + 1.4Q^2/[(2)(32.2)(9)^2(1.9 + 0.90)^2]\}^{1.5} = 26.7\ \text{ft}^3/\text{s}$ (by trial and error)

17.220 A rectangular, sharp-crested weir with end contractions is 1.4 m long. How high should it be placed in a channel to maintain an upstream depth of 2.35 m for 0.40 m^3/s flow?

▮ $Q = 1.83L'H^{3/2}$ where $L' = L - 0.2H$ (due to end contractions).

$0.40 = (1.83)(1.4 - 0.2H)(H)^{3/2}$ $\quad H = 0.30$ m $\quad P = 2.35 - 0.30 = 2.05$ m (height of weir)

17.221 Determine the head on a 45° V-notch weir for a discharge of 225 L/s.

▮ $Q = C(\frac{8}{15})\sqrt{2g}\,[\tan(\theta/2)]H^{5/2}$ $\quad \frac{225}{1000} = (0.58)(\frac{8}{15})\sqrt{(2)(9.807)}\,[\tan(45°/2)]H^{5/2}$ $\quad H = 0.691$ m

17.222 Tests on a 90° V-notch weir gave the following results: $H = 200$ mm, $Q = 21.3$ L/s, $H = 430$ mm, $Q = 165$ L/s. Determine the formula for the weir.

▮ Let $Q = AH^n$: $\frac{165}{1000} = A(\frac{430}{1000})^n$, $21.3/1000 = A(\frac{200}{1000})^n$. Dividing one equation by the other, $7.746 = 2.150^n$, $n = 2.67$. $\frac{165}{1000} = A(\frac{430}{1000})^{2.67}$, $A = 1.57$; $Q = 1.57H^{2.67}$.

17.223 A broad-crested weir 1.8 m high and 9 m long has a well-rounded upstream corner. What head is required for a flow of 8.80 m^3/s?

▮ $Q = 1.67LH^{3/2}$ $\quad 8.80 = (1.67)(9)H^{3/2}$ $\quad H = 0.700$ m

17.224 A weir in a horizontal channel is 12 ft wide and 4 ft high. The upstream water depth is 5.2 ft. Estimate the discharge if the weir is sharp-crested.

▮ $C_w = 0.611 + 0.075H/Y = 0.611 + (0.075)(5.2 - 4)/4 = 0.6335$

$q = (\frac{2}{3})(C_w)(2g)^{1/2}H^{3/2} = (\frac{2}{3})(0.6335)[(2)(32.2)]^{1/2}(5.2 - 4)^{3/2} = 4.455\ \text{ft}^3/\text{s/ft}$ $\quad Q = (12)(4.455) = 53.5\ \text{ft}^3/\text{s}$

17.225 Solve Prob. 17.224 if the weir is broad-crested.

▮ $C_w = 0.65/(1 + H/Y)^{1/2} = 0.65/[1 + (5.2 - 4)/4]^{1/2} = 0.5701$. Since everything else is the same as in Prob. 17.224, we simply scale down the sharp-crested result: $Q = (0.5701/0.6335)(53.5) = 48.1\ \text{ft}^3/\text{s}$.

17.226 A weir in a horizontal channel is 10 m wide and 1.4 m high. If the upstream depth is 3 m, estimate the channel discharge for a sharp-crested weir.

▮ $$C_w = 0.611 + 0.075H/Y = 0.611 + (0.075)(3 - 1.4)/1.4 = 0.6967$$

$$q = (\tfrac{2}{3})(C_w)(2g)^{1/2}H^{3/2} = (\tfrac{2}{3})(0.6967)[(2)(9.807)]^{1/2}(3 - 1.4)^{3/2} = 4.163\ (\text{m}^3/\text{s})/\text{m} \qquad Q = (10)(4.163) = 41.6\ \text{m}^3/\text{s}$$

17.227 Solve Prob. 17.226 if the weir is broad-crested.

▮ $C_w = 0.65/(1 + H/Y)^{1/2} = 0.65/[1 + (3 - 1.4)/1.4]^{1/2} = 0.4440$. Since everything else is the same as in Prob. 17.226, we simply scale down the sharp-crested result: $Q = (0.4440/0.6967)(41.6) = 26.5\ \text{m}^3/\text{s}$.

17.228 Show that the discharge of the V-notch weir in Fig. 17-51 is given by $Q = C_w(\frac{8}{15})(2g)^{1/2}(\tan\alpha)H^{5/2}$. Experiments show that C_w varies from 0.65 at $\alpha = 5°$ to 0.58 at $\alpha = 50°$.

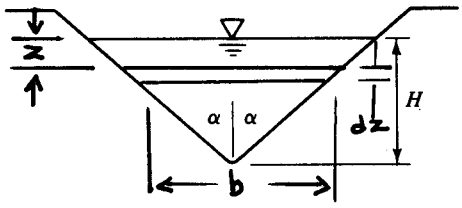


Fig. 17-51

▮ Assume V at any strip $= \sqrt{2gz}$, where z is measured down from the top.

$$Q = \int V\,dA = \int_0^H (2gz)^{1/2} b\,dz \qquad \text{where } b = b_0\left(1 - \frac{z}{H}\right)$$

$$= \int_0^H (2gz)^{1/2}\left[b_0\left(1 - \frac{z}{H}\right)\right] dz = (\tfrac{4}{15})(2g)^{1/2}\left(\frac{b_0}{H}\right)(H^{5/2})$$

$$b_0/H = 2\tan\alpha \qquad Q = (\tfrac{8}{15})(2g)^{1/2}(\tan\alpha)H^{5/2}$$

A correction factor C_w is inserted to account for end and edge effects and upstream kinetic energy; hence, $Q = C_w(\frac{8}{15})(2g)^{1/2}(\tan\alpha)H^{5/2}$.

17.229 Wastewater flows through a Parshall flume with a throat width of 3.0 ft at a depth of flow of 2.5 ft. Find the flow rate.

▮ $$Q = 4.0Bh^{1.522B^{0.026}} = (4.0)(3.0)(2.5)^{(1.522)(3.0)^{0.026}} = 50.4\ \text{ft}^3/\text{s}$$

17.230 Water is to flow through a proposed Parshall flume at a maximum flow rate of 30 m^3/s. The width of the Parshall flume's throat is to be 10.0 m. What should be the channel depth entering the flume?

▮ $$Q = (2.293B + 0.474)h^{1.6} \qquad 30 = [(2.293)(10.0) + 0.474]h^{1.6} \qquad h = 1.17\ \text{m}$$

17.231 Wastewater flows through a Parshall flume with a throat width of 5.0 ft at a depth of flow of 3.2 ft. Determine the rate of flow of the wastewater.

▮ $$Q = 4.0Bh^{1.522B^{0.026}} = (4.0)(5.0)(3.2)^{(1.522)(5.0)^{0.026}} = 127\ \text{ft}^3/\text{s}$$

17.232 Water is to flow through a Parshall flume at a maximum flow rate of 25 m^3/s. The width of the throat of the Parshall flume is 8.0 m. What will be the depth of flow at maximum flow rate?

▮ $$Q = (2.293B + 0.474)h^{1.6} \qquad 25 = [(2.293)(8.0) + 0.474]h^{1.6} \qquad h = 1.19\ \text{m}$$

17.233 The stilling-well depth-measurement scale of a Parshall flume with a 3-ft throat gives a head reading that is 0.04 ft too large. Compute the percentage errors in flow rate when the observed head readings are 0.6 ft and 2.2 ft.

▮ $$Q = 4Bh^{1.522B^{0.026}} \qquad B = 3\ \text{ft} \qquad \Delta h = 0.04\ \text{ft} \qquad e = 0.04\ \text{ft}$$

$$\%\ \text{Error} = \{[(h + e)/h]^{1.522B^{0.026}} - 1\}(100) \qquad 3^{0.026} = 1.0290$$

$$\%\ \text{Error} = [(0.64/0.60)^{1.566} - 1] = 0.106 \quad \text{or} \quad 10.6\ \text{percent}$$

If $h = 2.2$ ft, % Error $= [(2.24/2.20)^{1.566} - 1] = 0.0286$, or 2.86 percent.

17.234 It is desired to measure the flow in a canal which may carry between 30 cfs and 90 cfs. The flow is to be measured to an accuracy of 2 percent, and the available water-level recorder is accurate to 0.01 ft. The canal is

on a very flat slope and the head loss in the measuring device should be as small as possible. What type and size of flow-measuring device would you recommend?

▮ Try an 8-ft-throat Parshall flume: $Q = 4Bh^{1.552B^{0.026}} = (4)(8)h^{1.522(8)^{0.026}} = 32h^{1.607}$, $dQ = 51.4h^{0.607}\,dh$. For $Q = 30$ cfs, $h = 0.961$ ft, $dQ/Q = (51.4)(0.961)^{0.607}(0.01)/30 = 0.0167 < 0.02$. For $Q = 90$ cfs, $h = 1.90$ ft, $dQ/Q = (51.4)(1.90)^{0.607}(0.01)/90 = 0.0084 < 0.02$. So a Parshall flume with an 8-ft throat provides the required accuracy.

17.235 A rectangular channel 2.0 m wide contains a sluice gate which extends across the width of the channel. If the gate produces free flow when it is open 0.15 m with an upstream depth of 1.15 m, find the rate of discharge, assuming $C_d = 0.60$ and $C_c = 0.62$.

▮ $$Q = C_d Ba[2g(y_1 - y_2 + V_1^2/2g)]^{1/2} \qquad y_2 = C_c a = (0.62)(0.15) = 0.0930 \text{ m}$$

Find a trial Q by neglecting the $V_1^2/2g$ term in the equation above.

$$Q = (0.60)(2.0)(0.15)[(2)(9.807)(1.15 - 0.0930 + 0)]^{1/2} = 0.820 \text{ m}^3/\text{s}$$

$$V_1 = Q/A_1 = 0.820/[(2.0)(1.15)] = 0.357 \text{ m/s} \qquad V_1^2/2g = 0.357^2/[(2)(9.807)] = 0.00650 \text{ m}$$

$$Q = (0.60)(2.0)(0.15)[(2)(9.807)(1.15 - 0.0930 + 0.00650)]^{1/2} = 0.822 \text{ m}^3/\text{s}$$

17.236 For the sluice gate shown in Fig. 17-52, if $C_v = 0.98$, what is the flow rate? If $C_c = 0.62$, what is the height of the opening?

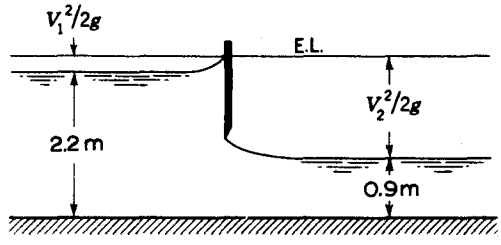


Fig. 17-52

▮ $V_2 = C_v[2g(y_1 - y_2 + V_1^2/2g)]^{1/2}$. Find a trial V_2 by neglecting the $V_1^2/2g$ term in the equation above.

$$V_2 = (0.98)[(2)(9.807)(2.2 - 0.9 + 0)]^{1/2} = 4.949 \text{ m/s} \qquad V_2 y_2 = V_1 y_1$$

$$(4.949)(0.9) = (V_1)(2.2) \qquad V_1 = 2.025 \text{ m/s}$$

$$V_1^2/2g = 2.025^2/[(2)(9.807)] = 0.209 \text{ m} \qquad V_2 = (0.98)[(2)(9.807)(2.2 - 0.9 + 0.209)]^{1/2} = 5.332 \text{ m/s}$$

Second trial:

$$(5.332)(0.9) = (V_1)(2.2) \qquad V_1 = 2.181 \text{ m/s}$$

$$V_1^2/2g = 2.181^2/[(2)(9.807)] = 0.243 \text{ m} \qquad V_2 = (0.98)[(2)(9.807)(2.2 - 0.9 + 0.243)]^{1/2} = 5.391 \text{ m/s}$$

Third trial:

$$(5.391)(0.9) = (V_1)(2.2) \qquad V_1 = 2.205 \text{ m/s}$$

$$V_1^2/2g = 2.205^2/[(2)(9.807)] = 0.248 \text{ m} \qquad V_2 = (0.98)[(2)(9.807)(2.2 - 0.9 + 0.248)]^{1/2} = 5.400 \text{ m/s}$$

Fourth trial:

$$(5.400)(0.9) = (V_1)(2.2) \qquad V_1 = 2.209 \text{ m/s} \qquad V_1^2/2g = 2.209^2/[(2)(9.807)] = 0.249 \text{ m}$$

$$V_2 = (0.98)[(2)(9.807)(2.2 - 0.9 + 0.249)]^{1/2} = 5.402 \text{ m/s} \qquad \text{(O.K.)}$$

$$q = y_2 V_2 = (0.9)(5.402) = 4.86 \text{ (m}^3/\text{s)/m} \qquad a = y_2 C_c = 0.9/0.62 = 1.45 \text{ m}$$

17.237 A disk meter has a volumetric displacement of 30 mL for one complete oscillation. Calculate the flow for 76.5 oscillations per minute.

$$Q = \text{(displacement per oscillation)(number of oscillations per unit time)}$$
$$= (30)(76.5) = 2295 \text{ mL/min} = 2.295 \text{ L/min}$$

17.238 A disk water meter with volumetric displacement of 35 mL per oscillation requires 450 oscillations per minute to pass 0.30 L/s and 3740 oscillations per minute to pass 2.19 L/s. Calculate the percent error, or slip, in the meter.

▮

$$\text{Slip} = (Q_{\text{actual}} - Q_{\text{indicated}})/Q_{\text{actual}}$$

$$Q = (\text{displacement per oscillation})(\text{number of oscillations per unit time})$$

$$Q_1 = (0.035)(450)/60 = 0.2625 \text{ L/s} \qquad \text{Slip}_1 = (0.30 - 0.2625)/0.30 = 0.0278 \quad \text{or} \quad 2.78 \text{ percent}$$

$$Q_2 = (0.035)(3740)/60 = 2.182 \text{ L/s} \qquad \text{Slip}_2 = (2.19 - 2.182)/2.19 = 0.0037 \quad \text{or} \quad 0.37 \text{ percent}$$

17.239 A volumetric tank 3 ft in diameter and 6 ft high was filled with oil in 17 min 22.4 s. What is the average discharge in gallons per minute?

▮

$$Q = V/t = [(\pi)(3)^2/4](6)/[17 + 22.4/60)] = 2.441 \text{ ft}^3/\text{min} = (2.441)(7.48) = 18.3 \text{ gpm}$$

17.240 A weigh tank receives 79 N of liquid, s.g. = 0.83, in 13.9 s. What is the volume flow rate?

▮

$$V = W/\gamma = 79/[(0.83)(9.79)(1000)] = 0.009722 \text{ m}^3$$

$$Q = V/t = 0.009722/(13.9) = 0.0007 \text{ m}^3/\text{s} = 0.7 \text{ L/s}$$

17.241 A critical-depth meter 15 ft wide has a rise in the bottom of 1.8 ft. For an upstream depth of 3.45 ft, determine the flow through the meter.

▮

$$q = 0.517g^{1/2}(y_1 - z + 0.55q^2/gy_1^2)^{3/2}$$

$$= (0.517)(32.2)^{1/2}\{3.45 - 1.8 + 0.55q^2/[(32.2)(3.45)^2]\}^{3/2}$$

$$= (2.934)(1.65 + 0.001435q^2)^{3/2}$$

$$q = 6.57 \text{ cfs/ft} \qquad \text{(by trial and error)}$$

$$Q = (6.57)(15) = 98.6 \text{ cfs}$$

CHAPTER 18
Dimensional Analysis and Similitude

18.1 The *Brinkman number* N_B, often used in analysis of organic–liquid flows, is the ratio of viscous dissipation to heat conduction in a fluid. It is a dimensionless combination of viscosity μ, flow velocity V, thermal conductivity k, and fluid temperature T. Derive the Brinkman number, using the fact that it is proportional to viscosity.

▮ $N_B = f(\mu, V, k, T)$ and is proportional to μ.

$$N_B = \{1\} = \mu^1 V^a k^b T^c = \{M/LT\}^1\{L/T\}^a\{ML/T^3\theta\}^b\{\theta\}^c = M^0L^0T^0\theta^0$$

$$L:\ -1+a+b=0 \qquad M:\ 1+b=0 \qquad T:\ -1-a-3b=0 \qquad \theta:\ -b+c=0$$

Hence, $a = 2$ and $b = c = -1$. $N_B = \mu V^2/kT$.

18.2 The velocity of sound c of a gas varies with pressure p and density ρ. Show by dimensional reasoning that the proper form must be $c = (\text{constant})(p/\rho)^{1/2}$.

▮

$$c = f(p, \rho) = (\text{constant})p^a\rho^b \qquad \{L/T\} = \{M/LT^2\}^a\{M/L^3\}^b$$

$$M:\ 0 = a + b \qquad L:\ 1 = -a - 3b \qquad T:\ -1 = -2a$$

Hence, $a = \frac{1}{2}$ and $b = -\frac{1}{2}$. $c = (\text{constant})(p/\rho)^{1/2}$.

18.3 The speed of propagation C of a capillary wave in deep water is known to be a function only of density ρ, wavelength λ, and surface tension σ. Find the proper functional relationship, completing it with a dimensionless constant. For a given density and wavelength, how does the propagation speed change if surface tension is doubled?

▮

$$C = f(\rho, \lambda, \sigma) = (\text{constant})\rho^a\lambda^b\psi^c \qquad \{L/T\} = \{M/L^3\}^a\{L\}^b\{M/T^2\}^c$$

$$M:\ 0 = a + c \qquad L:\ 1 = -3a + b \qquad T:\ -1 = -2c$$

Hence, $a = -\frac{1}{2}$, $b = -\frac{1}{2}$, $c = \frac{1}{2}$. $C = (\text{constant})(\sigma/\rho\lambda)^{1/2}$. If σ is doubled, C increases as $\sqrt{2}$, or 41 percent.

18.4 The excess pressure Δp inside a bubble is known to be a function of the surface tension and the radius. By dimensional reasoning determine how the excess pressure will vary if we double **(*a*)** the surface tension and **(*b*)** the radius.

▮

$$\Delta p = f(\sigma, r) = (\text{constant})\sigma^a r^b \qquad \{F/L^2\} = \{F/L\}^a\{L\}^b$$

$$F:\ 1 = a \qquad L:\ -2 = -a + b$$

Hence, $a = 1$ and $b = -1$. $\Delta p = (\text{constant})(\sigma/r)$. **(*a*)** If σ is doubled, Δp is doubled. **(*b*)** If r is doubled, Δp is halved.

18.5 The pressure drop in a Venturi meter varies only with fluid density, velocity of approach, and the diameter ratio of the meter. Venturi meter 1 in water at 20 °C shows a 6-kPa drop when the approach velocity is 5 m/s. Meter 2, geometrically similar to 1, is used in a $\frac{1}{6}$-m³/s flow of benzene ($\rho = 680$ kg/m³). Find the upstream pipe diameter that will yield a 16-kPa drop in meter 2.

▮

$$\Delta p = f(\rho, V, d/D) \qquad \Delta p/\rho V^2 = g(d/D) \qquad (\Delta p/\rho V^2)_1 = (\Delta p/\rho V^2)_2$$

$$(\Delta p/\rho V^2)_1 = (6)(1000)/[(998)(5)^2] = 0.2405$$

$$(\Delta p/\rho V^2)_2 = (16)(1000)/(680V^2) = 23.53/V^2 \qquad 0.2405 = 23.53/V^2 \qquad V = 9.891 \text{ m/s}$$

$$Q = AV \qquad \tfrac{1}{6} = (\pi D_p^2/4)(9.891) \qquad D_p = 0.146 \text{ m} \quad \text{or} \quad 146 \text{ mm}$$

18.6 For two hydraulic machines to be homologous, they must **(*a*)** be geometrically similar, **(*b*)** have the same discharge coefficient when viewed as an orifice, $Q_1/(A_1\sqrt{2gH_1}) = Q_2/(A_2\sqrt{2gH_2})$, and **(*c*)** have the same ratio of peripheral speed to fluid velocity, $\omega D/(Q/A)$. Show that the scaling ratios can be expressed as $Q/ND^3 = \text{constant}$ and $H/(ND)^2 = \text{constant}$. N is the rotational speed.

▮ (1) From condition (b),

$$Q_1/(A_1\sqrt{H_1}) = Q_2/(A_2\sqrt{H_2}) \tag{1}$$

From condition (c),

$$\omega_1 D_1 A_1/Q_1 = \omega_2 D_2 A_2/Q_2 \tag{2}$$

$$(Q_1/\omega_1 D_1)(\omega_2 D_2/Q_2) = A_1/A_2$$

Geometrical similarity yields $A_1/A_2 = D_1^2/D_2^2 = (Q_1/\omega_1 D_1)(\omega_2 D_2/Q_2)$. Hence, $Q_1/\omega_1 D_1^3 = Q_2/\omega_2 D_2^3$.

(2) From Eq. (1), $Q_1/A_1 = \sqrt{H_1/H_2}Q_2/A_2$. From Eq. (2), $Q_1/A_1 = (Q_2/A_2)(\omega_1 D_1/\omega_2 D_2)$. Hence, $\omega_1 D_1/\omega_2 D_2 = \sqrt{H_1/H_2}$, $H_1/(\omega_1 D_1)^2 = H_2/(\omega_2 D_2)^2$. ($N$ is rotational speed in revolutions per minute; ω is in radians per second. They are equivalent for modeling purposes.)

18.7 Use the scaling ratios of Prob. 18.6 to determine the head and discharge of a 1:5 model of a centrifugal pump that discharges 42 m^3/min at a 33 m head when turning at 200 rpm. The model operates at 1000 rpm.

▮ From Prob. 18.6, $H_1/(\omega_1 D_1)^2 = H_2/(\omega_2 D_2)^2$ or $H_m = H_p[(\omega_m/\omega_p)(D_m/D_p)]^2 = (33)[(\frac{1000}{200})(\frac{1}{5})]^2 = 33.00$ m; $Q_1/\omega_1 D_1 = Q_2/\omega_2 D_2$ or $Q_m = Q_p(\omega_m/\omega_p)(D_m/D_p)^3 = (42)(\frac{1000}{200})(\frac{1}{5})^3 = 1.68$ m^3/min.

18.8 For an ideal liquid, express the flow Q through an orifice in terms of the density of the liquid, the diameter of the orifice, and the pressure difference.

▮ $Q = f(\rho, p, d)$ or $Q = K\rho^a p^b d^c$. Then, dimensionally, $F^0L^3T^{-1} = (F^aT^{2a}L^{-4a})(F^bL^{-2b})(L^c)$ and $0 = a + b$, $3 = -4a - 2b + c$, $-1 = 2a$, from which $a = -\frac{1}{2}$, $b = \frac{1}{2}$, $c = 2$. Substituting, $Q = K\rho^{-1/2}p^{1/2}d^2$ or ideal $Q = Kd^2\sqrt{p/\rho}$. Factor K must be obtained by physical analysis and/or experimentation.

For an orifice in the side of a tank under head h, $p = \gamma h$. To obtain the familiar orifice formula, let $K = \sqrt{2}(\pi/4)$. Then ideal $Q = \sqrt{2}(\pi/4)d^2\sqrt{\gamma h/\rho}$. But $g = \gamma/\rho$; hence ideal $Q = \frac{1}{4}\pi d^2\sqrt{2gh}$.

18.9 Determine the dynamic pressure exerted by a flowing incompressible fluid on an immersed object, assuming the pressure is a function of the density and the velocity.

▮ $p = f(\rho, V)$ or $p = K\rho^a V^b$. Then, dimensionally, $F^1L^{-2}T^0 = (F^aT^{2a}L^{-4a})(L^bT^{-b})$ and $1 = a$, $-2 = -4a + b$, $0 = 2a - b$, from which $a = 1$, $b = 2$. Substituting, $p = K_p V^2$.

18.10 Develop the expression for lost head in a horizontal pipe for turbulent incompressible flow.

▮ For any fluid, the lost head is represented by the drop in the pressure gradient and is a measure of the resistance to flow through the pipe. The resistance is a function of the diameter of the pipe, the viscosity and density of the fluid, the length of the pipe, the velocity of the fluid, and the roughness K of the pipe. We may write $(p_1 - p_2) = f(d, \mu, \rho, L, V, K)$ or

$$(p_1 - p_2) = Cd^a\mu^b\rho^c L^d V^e(\epsilon/d)^f \tag{1}$$

From experiment and observation, the exponent of the length L is unity. The value of K is usually expressed as a ratio of the size of the surface protuberances ϵ to the diameter d of the pipe, a dimensionless number. We may now write $F^1L^{-2}T^0 = (L^a)(F^bT^bL^{-2b})(F^cT^{2c}L^{-4c})(L^eT^{-e})(L^f/L^f)$ and $1 = b + c$, $-2 = a - 2b - 4c + 1 + e + f - f$, $0 = b + 2c - e$ from which the values of a, b, and c may be determined in terms of e, or $c = e - 1$, $b = 2 - e$, $a = e - 3$. Substituting in (1), $(p_1 - p_2) = Cd^{e-3}\mu^{2-e}\rho^{e-1}L^1V^e(\epsilon/d)^f$. Dividing the left side of the equation by w and the right side by its equivalent ρg, $(p_1 - p_2)/w$ = lost head = $C(\epsilon/d)^f L(d^{e-3}V^e\rho^{e-1}\mu^{2-e})/(\rho g)$, which becomes (introducing 2 in numerator and in denominator): lost head = $2C(\epsilon/d)^f(L/d)(V^2/2g)(d^{e-2}V^{e-2}\rho^{e-2}/\mu^{e-2}) = K'(R_E^{e-2})(L/d)(V^2/2g) = f(L/d)(V^2/2g)$ (Darcy formula).

18.11 To find the drag on a 1-mm-diameter sphere in slowly streaming water, a 100-mm sphere is tested in glycerin at $V = 300$ mm/s. The measured drag on the model is 1.3 N. Under similar conditions, what are the water velocity and the drag on the 1-mm sphere?

▮ Water (prototype): $\mu_p = 0.001$ Pa · s $\quad \rho_p = 999$ kg/m^3

Glycerin (model): $\mu_m = 1.5$ Pa · s $\quad \rho_m = 1263$ kg/m^3

$$(N_R)_m = \rho_m V_m L_m/\mu_m = (1263)(0.300)(0.100)/1.5 = 25.3$$

$$(C_F)_m = F_m/\rho_m V_m^2 L_m^2 = 1.3/[(1263)(0.300)^2(0.100)^2] = 1.14$$

$$(N_R)_p = (N_R)_m = 25.3 = 999V_p(0.001)/0.001 \quad \text{or} \quad V_p = 0.0253 \text{ m/s}$$

$$(C_F)_p = (C_F)_m = 1.14 = F_p/[999(0.0253)^2(0.001)^2] \quad \text{or} \quad F_p = 0.731\ \mu\text{N}$$

18.12 A 0.1-ft-diameter steel sphere ($\rho_s = 15.2$ slugs/ft^3) is dropped in water [$\rho = 1.94$ slugs/ft^3, $\mu = 0.000021$ slug/(ft-s)] until it reaches terminal velocity or zero acceleration. From the sphere data in Fig. A-38 compute the terminal velocity of the falling sphere in feet per second.

▮ At terminal velocity, the net weight of the sphere equals the drag; hence the drag is known in this problem: $D = W_{net} = (\rho_s - \rho)g(\pi/6)d^3 = (15.2 - 1.94)(32.2)(\pi/6)(0.1)^3 = 0.224$ lb. We can compute that portion of C_D and N_R which excludes the unknown velocity: $C_D = D/[\frac{1}{2}\rho(\pi/4)d^2V^2] = 0.224/[1.94(\pi/8)(0.1)^2V^2] = 29.4/V^2$, $N_R = \rho Vd/\mu = (1.94)(V)(0.1)/(2.1 \times 10^{-5}) = 9240V$. Now we will just have to guess an initial velocity V to get started on the iteration.

Guess $V = 1.0$ ft/s; then $N_R = 9240(1.0) = 9240$. From Fig. A-38 read $C_D \approx 0.38$; then $V \approx (29.4/C_D)^{1/2} = 8.8$ ft/s. Now try again with this new guess.

Guess $V = 8.8$ ft/s, $N_R = 9240(8.8) = 81\,000$. From Fig. A-38 read $C_D \approx 0.52$, $V \approx (29.4/0.52)^{1/2} = 7.5$ ft/s. One more try will give pretty good convergence.

Guess $V = 7.5$ ft/s, $N_R = 9240(7.5) = 69\,000$. From Fig. A-38 read $C_D \approx 0.51$, $V \approx (29.4/0.51)^{1/2} = 7.6$ ft/s. To the accuracy of the figure, $V_{term} \approx 7.6$ ft/s.

18.13 Repeat Prob. 18.12, using the regrouped chart, Fig. A-39.

▮ We must repeat the calculation of the net weight to establish that $D = W_{net} = 0.224$ lb. But now we can go directly to the new drag coefficient: $C_F' = D\rho/\mu^2 = (0.224)(1.194)/(2.1 \times 10^{-5})^2 = 9.85 \times 10^8$. Now enter Fig. A-39 and read $N_R = 70\,000$. Then the desired velocity is $V = \mu N_R/\rho d = (2.1 \times 10^{-5})(70\,000)/[(1.94)(0.1)] = 7.58$ ft/s.

18.14 A 7-cm-diameter sphere is tested in water at 20 °C and a velocity of 4 m/s and has a measured drag of 8 N. What will be the velocity and drag force of a 3-m-diameter weather balloon moving in air at 20 °C and 1 atm under similar conditions?

▮ $(N_R)_m = (N_R)_p \qquad N_R = \rho DV/\mu \qquad (998)(0.07)(4)/(1.02 \times 10^{-3}) = (1.20)(3)(V_b)/(1.81 \times 10^{-5})$

$$V_b = 1.38 \text{ m/s}$$

$(C_F)_m = (C_F)_p \qquad C_F = F/\rho V^2D^2 \qquad 8/[(998)(4)^2(0.07)^2] = F_b/[(1.20)(1.38)^2(3)^2] \qquad F_b = 2.10 \text{ N}$

18.15 To find the drag on an airplane whose velocity is 275 mph, could one test a 1:25 scale model in a subsonic wind tunnel at the same pressure and temperature?

▮ $(N_R)_m = (N_R)_p \qquad N_R = LV/\nu \qquad L_m = L_p/25$

Since $p_m = p_p$ and $T_m = T_p$, $\nu_m = \nu_p$. Therefore, $V_m = V_p(L_p/L_m) = (275)(25) = 6875$ mph. As this velocity is hypersonic, a subsonic test is impossible.

18.16 A 1:25 scale model of a submarine is tested at 180 ft/s in a wind tunnel using sea-level standard air. What is the prototype speed in sea water at 20 °C for dynamic similarity? If the model drag is 1.6 lb, what is the prototype drag?

▮ $(N_R)_m = (N_R)_p \qquad N_R = \rho LV/\mu \qquad L_m = L_p/25$

$$(0.00234)(L_p/25)(180)/(3.78 \times 10^{-7}) = (2.00)(L_p)(V_p)/(2.23 \times 10^{-5}) \qquad V_p = 0.497 \text{ ft/s}$$

$$(C_F)_m = (C_F)_p \qquad C_F = F/\rho V^2L^2$$

$$1.6/[(0.00234)(180)^2(L_p/25)^2] = F_p/[(2.00)(0.497)^2(L_p)^2] \qquad F_p = 6.52 \text{ lb}$$

18.17 A blimp is designed to move in air at 20 °C at 7 m/s. If a 1:20 scale model is tested in water at 20 °C, what should the water velocity be? If the measured water drag on the model is 3.09 kN, calculate the drag on the prototype blimp and the power required to propel it.

▮ $(N_R)_m = (N_R)_p \qquad N_R = \rho LV/\mu \qquad L_m = L_p/20$

$$(998)(L_p/20)(V_m)/(1.02 \times 10^{-3}) = (1.20)(L_p)(7)/(1.81 \times 10^{-5}) \qquad V_m = 9.49 \text{ m/s}$$

$(C_F)_m = (C_F)_p \qquad C_F = F/\rho V^2L^2 \qquad 3.09/[(998)(9.49)^2(L_p/20)^2] = F_p/[(1.20)(7)^2(L_p)^2]$

$$F_p = 0.809 \text{ kN} \qquad P_p = F_pV_p = (0.809)(7) = 5.663 \text{ kW}$$

18.18 A 1:6 scale model automobile is tested in a wind tunnel in the same air properties as the prototype. The

prototype velocity is 48 km/h. For dynamically similar conditions, the model drag is 320 N. What are the drag of the prototype automobile and the power required to overcome this drag?

▮ $(N_R)_m = (N_R)_p \qquad N_R = \rho LV/\mu \qquad L_m = L_p/6$

$$(\rho)(L_p/6)(V_m)/\mu = (\rho)(L_p)[(48)(1000)/3600]/\mu \qquad V_m = 80.00 \text{ m/s}$$

$$(C_F)_m = (C_F)_p \qquad C_F = F/\rho V^2 L^2 \qquad 320/[(\rho)(80.00)^2(L_p/6)^2] = F_p/[(\rho)[(48)(1000)/3600]^2(L_p)^2]$$

$$F_p = 320 \text{ N} \qquad P_p = F_p V_p = (320)[(48)(1000)/3600] = 4267 \text{ W}$$

18.19 A model airplane is built to the scale 1:20. If the prototype is to fly at 425 mph, what must be the air velocity in the wind tunnel to achieve the same Reynolds number at the same air temperature and pressure?

▮ $(N_R)_m = (N_R)_p \qquad N_R = \rho LV/\mu \qquad L_m = L_p/20 \qquad (\rho)(L_p/20)(V_m)/\mu = (\rho)(L_p)(425)/\mu$

$$V_m = 8500 \text{ mph} \quad \text{[impractically high]}$$

18.20 A model airplane has dimensions that are 1/18 those of its prototype. It is desired to test it in a pressure wind tunnel at a speed the same as that of the prototype. If the air temperature is the same and the Reynolds number is the same, what must be the pressure in the wind tunnel relative to the atmospheric pressure?

▮ $(N_R)_m = (N_R)_p \qquad N_R = LV/\nu \qquad L_m = L_p/18 \qquad (L_p/18)(V)\nu_m = (L_p)(V)\nu_p$

$$\nu_m/\nu_p = \tfrac{1}{18} = (\mu_m/\rho_m)/(\mu_p/\rho_p) = (\mu_m/\mu_p)(\rho_p/\rho_m)$$

For $T_m = T_p$, assume $\mu_m = \mu_p$; hence, $\rho_p/\rho_m = \gamma_p/\gamma_m = \frac{1}{18}$, $\gamma_p/\gamma_m = (p_p/RT)/(p_m/RT) = \frac{1}{18}$. Therefore, $p_m = 18p_p = 18$ atm.

18.21 What weight flow rate of air at 70 °F ($\mu = 3.82 \times 10^{-7}$ lb-s/ft^2) at 58 psi in a 2-in-diameter pipe will give dynamic similarity to a 275-gpm flow of water at 60 °F in a 6-in-diameter pipe?

▮ $(N_R)_{\text{air}} = (N_R)_{H_2O} \qquad N_R = DV/\nu \qquad \nu = \mu/\rho$

$$\rho_{\text{air}} = p/RT = (58)(144)/[(1716)(70 + 460)] = 0.009183 \text{ slug/ft}^3$$

$$\nu_{\text{air}} = (3.82 \times 10^{-7})/0.009183 = 4.160 \times 10^{-5} \text{ ft}^2/\text{s}$$

$$V_{\text{air}} = Q/[(\pi)(\tfrac{2}{12})^2/4] = 45.84Q \qquad V_{H_2O} = Q/A = (275)(0.002228)/[(\pi)(\tfrac{6}{12})^2/4] = 3.120 \text{ ft/s}$$

$$(\tfrac{2}{12})(45.84Q)/(4.160 \times 10^{-5}) = (\tfrac{6}{12})(3.120)/(1.21 \times 10^{-5}) \qquad Q = 0.7020 \text{ ft}^3/\text{s}$$

$$\gamma = \rho g \qquad \gamma_{\text{air}} = (0.009183)(32.2) = 0.2957 \text{ lb/ft}^3 \qquad G = \gamma Q = (0.2957)(0.7020) = 0.208 \text{ lb/s}$$

18.22 A 1:40 scale model of a submarine is tested in a wind tunnel. It is desired to know the drag on the submarine when it is operating at 9 knots (15.2 fps) in ocean water at 40 °F. At what velocity should the object be tested in a wind tunnel containing air at 70 °F at atmospheric pressure?

▮ $(N_R)_{\text{air}} = (N_R)_{H_2O} \qquad N_R = LV/\nu \qquad L_{\text{air}} = L_{H_2O}/40$

$$(L_{H_2O}/40)(V_{\text{air}})/(1.64 \times 10^{-4}) = (L_{H_2O})(15.2)/(1.66 \times 10^{-5}) \qquad V_{\text{air}} = 6007 \text{ ft/s}$$

Since this velocity exceeds sonic velocity, the model will not operate properly to indicate prototype behavior.

18.23 A ship 650 ft long is to operate at a speed of 22 mph in ocean water whose kinematic viscosity is 0.00001261 ft^2/s. What should be the kinematic viscosity of a fluid used with an 11-ft model so that both the Reynolds number and the Froude number would be the same?

▮ $(N_R)_m = (N_R)_p \qquad N_R = LV/\nu$

Let subscript r denote ratio of prototype to model.

$$L_r V_r/\nu_r = 1 \qquad (N_F)_m = (N_F)_p \qquad N_F = V/(gL)^{1/2} \qquad V_r/(g_r L_r)^{1/2} = 1 \qquad L_r V_r/\nu_r = V_r/(g_r L_r)^{1/2}$$

$$g_r = 1.0 \qquad L_r = \tfrac{650}{11} = 59.1 \qquad (59.1)(V_r)/\nu_r = V_r/[(1.0)(59.1)]^{1/2} \qquad \nu_r = 454.3$$

Hence, $\nu_m = 0.00001261/454.3 = 0.0000000278$ ft^2/s [impossibly small].

18.24 The valve coefficients $K = \Delta p/(\rho V^2/2)$ for a 600-mm-diameter valve are to be determined from tests on a geometrically similar 300-mm-diameter valve using atmospheric air at 80 °F ($\nu = 1.57 \times 10^{-5}$ m²/s). What range of air flows is required to model flows of water at 70 °F ($\nu = 9.96 \times 10^{-7}$ m²/s) between 1 m/s and 2.5 m/s?

▮ $$N_R = DV/\nu \qquad (N_R)_{min} = (0.600)(1)/(9.96 \times 10^{-7}) = 6.02 \times 10^5$$

$$(N_R)_{max} = (0.600)(2.5)/(9.6 \times 10^{-7}) = 1.51 \times 10^6$$

For air:

$$6.02 \times 10^5 = (0.300)(V_{min})/(1.57 \times 10^{-5}) \qquad V_{min} = 31.5 \text{ m/s}$$

$$1.51 \times 10^6 = (0.300)(V_{max})/(1.57 \times 10^{-5}) \qquad V_{max} = 79.0 \text{ m/s}$$

$$Q_{min} = AV_{min} = [(\pi)(0.300)^2/4](31.5) = 2.23 \text{ m}^3/\text{s} \qquad Q_{max} = AV_{max} = [(\pi)(0.300)^2/4](79.0) = 5.58 \text{ m}^3/\text{s}$$

18.25 A model of a Venturi meter has linear dimensions 1/6 those of the prototype. The prototype operates on water at 10 °C, and the model on water at 100 °C. For a throat diameter of 700 mm and a velocity at the throat of 7 m/s, what discharge is needed through the model for similitude?

▮ $$(N_R)_m = (N_R)_p \qquad N_R = LV/\nu \qquad (L_p/6)(V_m)/(2.94 \times 10^{-7}) = (L_p)(7)/(1.30 \times 10^{-6}) \qquad V_m = 9.498 \text{ m/s}$$

$$Q_m = A_m V_m \qquad D_m = D_p/6 = \tfrac{700}{6} = 117 \text{ mm} \qquad Q_m = [(\pi)(0.117)^2/4](9.498) = 0.102 \text{ m}^3/\text{s}$$

18.26 The losses in a Y in a 1.3-m-diameter pipe system carrying gas ($\rho = 40$ kg/m³, $\mu = 2 \times 10^{-4}$ Pa · s, $V = 28$ m/s) are to be determined by testing a model with water at 40 °C. The laboratory has a water capacity of 80 L/s. What should be the diameter of the model?

▮ $$(N_R)_m = (N_R)_p \qquad N_R = \rho DV/\mu \qquad V_m = Q_m/A_m = (80 \times 10^{-3})/(\pi D_m^2/4) = 0.1019/D_m^2$$

$$(998)(D_m)(0.1019/D_m^2)/(6.51 \times 10^{-4}) = (40)(1.3)(28)/(2 \times 10^{-4})$$

$$D_m = 0.02146 \text{ m} \quad \text{or} \quad 21 \text{ mm}$$

18.27 A 1:4 scale model of a water pumping station pumping system is to be tested to determine overall head losses. Air at 30 °C, 1 atm, is available. For a prototype velocity of 600 mm/s in a 3-m-diameter section with water at 20 °C, determine the air velocity and quantity needed. How may losses determined from the model be converted into prototype losses?

▮ $$(N_R)_m = (N_R)_p \qquad N_R = DV/\nu \qquad [(3)(\tfrac{1}{4})](V_m)/(1.60 \times 10^{-5}) = (3)(0.600)/(1.02 \times 10^{-6})$$

$$V_m = 37.65 \text{ m/s} \qquad Q_m = A_m V_m = \{(\pi)[(3)(\tfrac{1}{4})]^2/4\}(37.65) = 16.6 \text{ m}^3/\text{s}$$

Losses are the same when expressed in velocity heads.

18.28 Water at 60 °F flows at 12.0 fps in a 6-in pipe. At what velocity must medium fuel oil at 90 °F ($\nu = 3.19 \times 10^{-5}$ ft²/s) flow in a 3-in pipe for the two flows to be dynamically similar?

▮ $$(N_R)_{H_2O} = (N_R)_{oil} \qquad N_R = DV/\nu \qquad (\tfrac{6}{12})(12.0)/(1.21 \times 10^{-5}) = (\tfrac{3}{12})(V_{oil})/(3.19 \times 10^{-5}) \qquad V_{oil} = 63.3 \text{ ft/s}$$

18.29 Air at 68 °F is to flow through a 24-in pipe at an average velocity of 6.00 fps. For dynamic similarity, what size pipe carrying water at 60 °F at 3.65 fps should be used?

▮ $$(N_R)_{air} = (N_R)_{H_2O} \qquad N_R = DV/\nu \qquad (\tfrac{24}{12})(6.00)/(1.63 \times 10^{-4}) = (D/12)(3.65)/(1.21 \times 10^{-5}) \qquad D = 2.93 \text{ in}$$

(Would probably use a 3-in pipe.)

18.30 A 1:5 model of a submarine is to be tested in a towing tank containing salt water. If the submarine moves at 12.0 mph, at what velocity should the model be towed for dynamic similarity?

▮ $(N_R)_m = (N_R)_p \qquad N_R = LV/\nu \qquad (L/15)(V_m)/\nu = (L)(12.0)/\nu \qquad V_m = 180 \text{ mph}$

18.31 A model of a torpedo is tested in a towing tank at a velocity of 80.0 fps. The prototype is expected to attain a velocity of 20.0 fps in water at 60 °F. (***a***) What model scale has been used? (***b***) What would be the model speed if tested in a wind tunnel under a pressure of 20 atm and at constant temperature 80 °F?

▮ $(N_R)_m = (N_R)_p \qquad N_R = DV/\nu$

(***a***) $(L/x)(80.0)/\nu = (L)(20.0)/\nu$, $x = 4.00$. Hence, the scale model is 1:4.

(***b***) $\nu = \mu/\rho \qquad \rho_{air} = p/RT = (20)(14.7)(144)/[(1716)(80+460)] = 0.04569 \text{ slug/ft}^3$

$$\nu_{air} = (3.85 \times 10^{-7})/0.04569 = 8.426 \times 10^{-6} \text{ ft}^2/\text{s}$$

$$(L/4)(V_m)/(8.426 \times 10^{-6}) = (L)(20.0)/(1.21 \times 10^{-5}) \qquad V_m = 55.7 \text{ ft/s}$$

18.32 A centrifugal pump pumps medium lubricating oil at 60 °F ($\nu = 188 \times 10^{-5}$ ft²/s) while rotating at 1200 rpm. A model pump, using air at 68 °F, is to be tested. If the diameter of the model is 3 times the diameter of the prototype, at what speed should the model run?

▮ $(N_R)_m = (N_R)_p$, $N_R = DV/\nu$. Using the peripheral speeds (which equal radius times angular velocity in radians per second) as the velocities in Reynolds number, we obtain $(3D)[(3D/2)(\omega_m)]/(1.63 \times 10^{-4}) = (D)[(D/2)(\omega_p)]/(188 \times 10^{-5})$, $\omega_p = 103.8\omega_m$. Hence, model speed = 1200/103.8 = 11.6 rpm.

18.33 An airplane wing of 3-ft chord is to move at 90 mph in air. A model of 3-in chord is to be tested in a wind tunnel with air velocity at 108 mph. For air temperature of 68 °F in each case, what should be the pressure in the wind tunnel?

▮ $(N_R)_m = (N_R)_p \qquad N_R = LV/\nu \qquad (\tfrac{3}{12})(108)/\nu_m = (3)(90)/(1.63 \times 10^{-4}) \qquad \nu_m = 1.63 \times 10^{-5} \text{ ft}^2/\text{s}$

The pressure that produces this kinematic viscosity of air at 68 °F can be found by remembering that the absolute viscosity is not affected by pressure changes. The kinematic viscosity equals absolute viscosity divided by density. But density increases with pressure (temperature constant). $\nu = \mu/\rho$, $\nu_p/\nu_m = (1.63 \times 10^{-4})/(1.63 \times 10^{-5}) = 10$. Thus the density of air in the tunnel must be ten times standard (68 °F) air and the resulting pressure in the tunnel must be 10 atm.

18.34 A dam spillway is to be tested using Froude scaling with a 1:20 model. The model flow has an average velocity of 0.7 m/s and a volume flow of 0.06 m³/s. What are the velocity and flow of the prototype?

▮ For Froude scaling,

$$V_p/V_m = 1/\sqrt{\alpha} \qquad \alpha = L_m/L_p = \tfrac{1}{20} \qquad V_p/0.7 = 1/\sqrt{\tfrac{1}{20}} \qquad V_p = 3.13 \text{ m/s}$$

$$Q_p/Q_m = (V_p/V_m)(L_p/L_m)^2 = (1/\sqrt{\alpha})(1/\alpha)^2 = (1/\alpha)^{5/2} \qquad Q_p/0.06 = [1/(\tfrac{1}{20})]^{5/2} \qquad Q_p = 107 \text{ m}^3/\text{s}$$

18.35 A prototype ship is 40 m long and designed to cruise at 10 m/s (about 19 knots). Its drag is to be simulated by a 0.8-m-long model pulled in a tow tank. For Froude scaling find (***a***) the tow speed, (***b***) the ratio of prototype to model drag, and (***c***) the ratio of prototype to model power.

▮ For Froude scaling,

(***a***) $V_p/V_m = 1/\sqrt{\alpha} \qquad \alpha = L_m/L_p = \tfrac{0.8}{40} = \tfrac{1}{50} \qquad 10/V_m = 1/\sqrt{\tfrac{1}{50}} \qquad V_m = 1.41 \text{ m/s}$

(***b***) $F_m/F_p = (\rho_m/\rho_p)(V_m/V_p)^2(L_m/L_p)^2 = (1)(\sqrt{\alpha})^2(\alpha)^2 = \alpha^3 = (\tfrac{1}{50})^3 = 1/125\,000$

(***c***) $P_m/P_p = (F_m/F_p)(V_m/V_p) = (\alpha)^3(\sqrt{\alpha}) = \alpha^{3.5} = (\tfrac{1}{50})^{3.5} = 1/883\,883$

18.36 A ship 555 ft long is to operate at a speed of 22 mph. If a model is 8 ft long, what should be its speed to give the same Froude number? What is the value of the Froude number?

▮ $(N_F)_m = (N_F)_p \qquad N_F = V/(gL)^{1/2} \qquad V_m/[(g)(8)]^{1/2} = 22/[(g)(555)]^{1/2}$

$$V_m = 2.64 \text{ mph} \quad \text{or} \quad 3.87 \text{ ft/s} \qquad N_F = 3.87/[(32.2)(8)]^{1/2} = 0.241$$

18.37 In a 1:30 model of the flow in a spillway, the velocity at point A is 0.4 m/s and the force exerted on a small area about A is 0.10 N. What would be the force on the corresponding area in the prototype?

▮ $(N_F)_m = (N_F)_p \qquad N_F = V/(gL)^{1/2} \qquad 0.4/[(g)(L_p/30)]^{1/2} = V_p/(gL_p)^{1/2} \qquad V_p = 2.19\ \text{m/s}$

$(C_F)_m = (C_F)_p \qquad C_F = F/\rho V^2 L^2 \qquad 0.10/[(\rho)(0.4)^2(L_p/30)^2] = F_p/[(\rho)(2.19)^2(L_p)^2] \qquad F_p = 2698\ \text{N}$

18.38 A 1:400 model is constructed to study tides. What length of time in the model corresponds to a day in the prototype? Suppose the model could be transported to the moon and tested there. What then would be the time relationship between the model and prototype? g of earth equals six times g of moon.

▮ $(N_F)_m = (N_F)_p \qquad N_F = V/(gL)^{1/2} \qquad [V/(gL)^{1/2}]_m = [V/(gL)^{1/2}]_p$

$V_r = V_p/V_m = (gL)_r^{1/2} \qquad T_r = L_r/V_r = L_r(gL)_r^{1/2} \qquad T_p/T_m = [L/(gL)^{1/2}]_p/[L/(gL)^{1/2}]_m$

On earth:

$$1/T_m = \{L_p/[(g)(L_p)]^{1/2}\}/\{(L_p/400)/[(g)(L_p/400)]^{1/2}\} \qquad T_m = 0.05000\ \text{day} \quad \text{or} \quad 1.20\ \text{h}$$

On the moon:

$$1/T_m = \{L_p/[(g_p)(L_p)]^{1/2}\}/\{L_p/400)/[(g_p/6)(L_p/400)]^{1/2}\} \qquad T_m = 0.1225\ \text{day} \quad \text{or} \quad 2.94\ \text{h}$$

18.39 A sectional model of a spillway 2 ft high is placed in a laboratory flume of 9-in width. Under a head of 0.345 ft the flow is 0.62 cfs. What flow does this represent in the prototype if the scale model is 1:20 and the spillway is 700 ft long?

▮ $(N_F)_m = (N_F)_p \qquad N_F = V/(gL)^{1/2} \qquad (N_F)_r = (N_F)_p/(N_F)_m = [V/(gL)^{1/2}]_p/[V/(gL)^{1/2}]_m = V_r/L_r^{1/2} = 1.00$

$V_r/20^{1/2} = 1.00 \qquad V_r = 4.47 \qquad Q_r = A_r V_r \qquad A_r = A_p/A_m = (700)[(0.345)(20)]/[(\frac{9}{12})(0.345)] = 18\,667$

$$Q_r = A_r V_r = (18\,667)(4.47) = 83\,441 \qquad Q = (83\,441)(0.62) = 51\,733\ \text{ft}^3/\text{s}$$

18.40 The flow over a model spillway is 98 L/s per meter of width. What flow does this represent in the prototype spillway if the model scale is 1:20?

▮ $q = Q/b = [(by)(V)]/b = yV \qquad q_r = y_r V_r \qquad y_r = L_r$

$V_r/L_r^{1/2} = 1.00 \quad \text{or} \quad V_r = L_r^{1/2} \quad$ (from Prob. 18.39)

$q_r = L_r L_r^{1/2} = L_r^{3/2} = 20^{3/2} = 89.44 \qquad q_p = (89.44)(98) = 8770\ (\text{L/s})/\text{m} = 8.77\ (\text{m}^3/\text{s})/\text{m}$

18.41 The velocity at a point in a model of a spillway for a dam is 2 m/s. For a geometric ratio of prototype to model of 12:1, what is the velocity at the corresponding point in the prototype under similar conditions?

▮ $(N_F)_m = (N_F)_p \qquad N_F = V/(gL)^{1/2} \qquad 2/[(g)(L_p/12)]^{1/2} = V_p/(gL_p)^{1/2} \qquad V_p = 6.93\ \text{m/s}$

18.42 The wave drag on a model of a ship is 25 N at a speed of 4 m/s. For a prototype 12 times as long, what will the corresponding speed and wave drag be if the liquid is the same in each case?

▮ $(N_F)_m = (N_F)_p \qquad N_F = V/(gL)^{1/2} \qquad 4/[(g)(L_p/12)]^{1/2} = V_p/(gL_p)^{1/2} \qquad V_p = 13.86\ \text{m/s}$

$$C_D = F/(A\rho V^2/2)$$

At the same Froude number, C_D is equal for model and prototype; hence, $F_m/(A_m V_m^2/2) = F_p/(A_p V_p^2/2)$, $F_p = (F_m)(A_p/A_m)(V_p/V_m)^2 = (F_m)(L_p/L_m)^3 = (25)(12)^3 = 43\,200$ N, or 43.2 kN.

18.43 Oil of kinematic viscosity 50×10^{-5} ft²/s is to be used in a prototype in which both viscous and gravity forces dominate. A model scale of 1:5 is also desired. What viscosity of model liquid is ncessary to make both Froude number and Reynolds number the same in model and prototype?

▮ $(N_F)_m = (N_F)_p \qquad N_F = V/(gL)^{1/2} \qquad V_r = (g_r L_r)^{1/2} = L_r^{1/2} \qquad (N_R)_m = (N_R)_p \qquad N_R = LV/\nu$

$V_r = \nu_r/L_r \qquad L_r^{1/2} = \nu_r/L_r \qquad \nu_r = L_r^{3/2} = (\frac{1}{5})^{3/2} = 0.08944$

$$\nu_m = (0.08944)(50 \times 10^{-5}) = 4.47 \times 10^{-5}\ \text{ft}^2/\text{s}$$

18.44 A ship whose hull length is 460 ft is to travel at 25.0 fps. (***a***) Compute the Froude number. (***b***) For dynamic similarity, at what velocity should a 1:30 model be towed through water?

(a) $N_F = V/(gL)^{1/2} = 25.0/[(32.2)(469)]^{1/2} = 0.205$

(b) $(N_F)_m = (N_F)_p \qquad V_m/[(32.2)(\frac{460}{30})]^{1/2} = 25.0/[(32.2)(460)]^{1/2} \qquad V_m = 4.56\ \text{ft/s}$

18.45 An airplane is designed to fly at 240 m/s at 8 km altitude. If a 1:12 model is tested in a wind tunnel at 25 °C, find the tunnel pressure that will reproduce both the Reynolds number and the Mach number.

At 8000 meters, $\rho_p = 0.526\ \text{kg/m}^3$, $T_p = -36.9\ °\text{C}$, and $\mu_p = 1.53 \times 10^{-5}\ \text{Pa} \cdot \text{s}$ (from Table A-8).

$$(N_M)_m = (N_M)_p \qquad N_M = V/c \qquad V_m/c_m = V_p/c_p \qquad c \approx (20\ \text{m/s} \cdot \text{K}^{1/2})T^{1/2}$$

$$c_m \approx (20)(25 + 273)^{1/2} = 345.3\ \text{m/s} \qquad c_p \approx (20)(-36.9 + 273)^{1/2} = 307.3\ \text{m/s}$$

$$V_m/345.3 = 240/307.3 \qquad V_m = 269.7\ \text{m/s} \qquad (N_R)_m = (N_R)_p \qquad N_R = \rho LV/\mu$$

$$(\rho_m)(L_p/12)(269.7)/(1.84 \times 10^{-5}) = (0.526)(L_p)(240)/(1.53 \times 10^{-5}) \qquad \rho_m = 6.75\ \text{kg/m}^3$$

$$p_m = \rho_m R T_m = (6.75)(287)(25 + 273) = 577\,300\ \text{Pa} \quad \text{or} \quad 577\ \text{kPa}$$

18.46 One wishes to model the flow about a missile that travels at 975 mph through the atmosphere at elevation 9000 ft. The model is to be tested in a wind tunnel at standard atmospheric conditions with air at 60 °F. What air speed in the wind tunnel is required for dynamic similarity?

$$(N_M)_m = (N_M)_p \qquad N_M = V/c = V/(p/\rho)^{1/2}$$

$$V_m/(14.7/0.00227)^{1/2} = 975/(10.54/0.001814)^{1/2} \qquad V_m = 1029\ \text{mph} \quad \text{or} \quad 1510\ \text{ft/s}$$

18.47 A model of a supersonic aircraft is tested in a variable-density wind tunnel at 1290 fps. The air is at 90 °F with a pressure of 20 psia. At what velocity should this model be tested to maintain dynamic similarity if the air temperature is raised to 130 °F and the pressure increased to 27 psia?

$N_M = V/(p/\rho)^{1/2}$, $\rho = p/RT$. At 90 °F, $\rho = (20)(144)/[(53.3)(90 + 460)] = 0.09824\ \text{slug/ft}^3$, $N_M = 1290/[(20)(144)/0.09824]^{1/2} = 7.534$. At 130 °F, $\rho = (27)(144)/[(53.3)(130 + 460)] = 0.1236\ \text{slug/ft}^3$, $N_M = V/[(27)(144)/0.1236]^{1/2} = 7.534$, $V = 1336\ \text{ft/s}$.

18.48 The flow about a ballistic missile which travels at 1400 fps through air at 80 °F and 14.7 psia is to be modeled in a high-speed wind tunnel with a 1:10 model. If the air in the wind tunnel test section has a temperature of 10 °F at a pressure of 12 psia, what velocity is required in the model test section?

$$(N_M)_m = (N_M)_p \qquad N_M = V/(p/\rho)^{1/2}$$

$$\rho = p/RT \qquad \rho_m = (12)(144)/[(53.3)(10 + 460)] = 0.06898\ \text{slug/ft}^3$$

$$\rho_p = (14.7)(144)/[(53.3)(80 + 460)] = 0.07355\ \text{slug/ft}^3$$

$$V_m/[(12)(144)/0.06898]^{1/2} = 1400/[(14.7)(144)/0.07355]^{1/2} \qquad V_m = 1306\ \text{ft/s}$$

18.49 A prototype spillway has a characteristic velocity of 2 m/s and a characteristic length of 9 m. A small model is constructed using Froude scaling. What is the minimum scale ratio of the model that will ensure that its minimum Weber number is 100? Both flows use water at 30 °C.

$$N_W = \rho V^2 L/\sigma \qquad (N_W)_p = (996)(2)^2(9)/0.0712 = 503\,596$$

Froude scaling:

$$(N_W)_m/(N_W)_p = (\rho_m/\rho_p)(V_m/V_p)^2(L_m/L_p)/(\sigma_m/\sigma_p) = (1)(\sqrt{\alpha})^2(\alpha)(1) = \alpha^2 \geq 100/503\,596$$

$$\alpha = 0.01409 \qquad L_p/L_m \leq 1/\alpha \leq 1/0.01409 = 71:1$$

18.50 At low velocities (laminar flow), the volume flow Q through a small-bore tube is a function only of the tube radius R, the fluid viscosity μ, and the pressure drop per unit tube length dp/dx. Using the Buckingham pi theorem, find an appropriate dimensionless relationship.

Write the given relation and count variables: $Q = f(R, \mu, dp/dx)$ four variables ($n = 4$). Make a list of the dimensions of these variables:

Q	R	μ	dp/dx
$\{L^3T^{-1}\}$	$\{L\}$	$\{ML^{-1}T^{-1}\}$	$\{ML^{-2}T^{-2}\}$

There are three primary dimensions (M, L, T), hence $j \leq 3$. By trial and error we determine that R, μ, and dp/dx cannot be combined into a pi group. Then $j = 3$, and $n - j = 4 - 3 = 1$. There is only *one* pi group, which we find by combining Q in a power product with the other three: $\Pi_1 = R^a\mu^b(dp/dx)^cQ^1 = (L)^a(ML^{-1}T^{-1})^b(ML^{-2}T^{-2})^c(L^3T^{-1}) = M^0L^0T^0$. Equate exponents:

Mass:	$b + c = 0$
Length:	$a - b - 2c + 3 = 0$
Time:	$-b - 2c - 1 = 0$

Solving simultaneously, we obtain $a = -4$, $b = 1$, $c = -1$. Then $\Pi_1 = R^{-4}\mu^1(dp/dx)^{-1}Q$ or $\Pi_1 = Q\mu/[R^4(dp/dx)] = \text{const}$. Since there is only one pi group, it must equal a dimensionless constant.

18.51 The capillary rise h of a liquid in a tube varies with tube diameter d, gravity g, fluid density ρ, surface tension σ, and the contact angle θ. **(*a*)** Find a dimensionless statement of this relation. **(*b*)** If $h = 3$ cm in a given experiment, what will h be in a similar case if diameter and surface tension are half as much, density is twice as much, and the contact angle is the same?

▮ **(*a*)** *Step 1* Write down the function and count variables $h = f(d, g, \rho, \sigma, \theta)$, $n = 6$ variables.
Step 2 List the dimensions (FLT):

h	d	g	ρ	Υ	θ
$\{L\}$	$\{L\}$	$\{LT^{-1}\}$	$\{FT^2L^{-4}\}$	$\{FL^{-1}\}$	None

Step 3 Find j. Several groups of three form no pi: σ, ρ, and g or ρ, g, and d. Therefore $j = 3$, and we expect $n - j = 6 - 3 = 3$ dimensionless groups. One of these is obviously θ, which is already dimensionless: $\Pi_3 = \theta$. If we chose carelessly to search for it using steps 4 and 5, we would still find $\Pi_3 = \theta$.
Step 4 Select j variables which do not form a pi group: ρ, g, d.
Step 5 Add one additional variable in sequence to form the pi's:

Add h: $$\Pi_1 = \rho^a g^b d^c h = (FT^2L^{-4})^a(LT^{-2})^b(L)^c(L) = F^0L^0T^0$$

Solve for $a = b = 0$, $c = -1$. Therefore $\Pi_1 = \rho^0g^0d^{-1}h = h/d$. Finally add Υ, again selecting its exponent to be 1: $\Pi_2 = \rho^a g^b d^c \sigma = (FT^2L^{-4})^a(LT^{-2})^b(L)^c(FL^{-1}) = F^0L^0T^0$. Solve for $a = b = -1$, $c = -2$. Therefore $\Pi_2 = \rho^{-1}g^{-1}d^{-2}\sigma = \sigma/\rho g d^2$.

Step 6 The complete dimensionless relation for this problem is thus

$$h/d = F(\sigma/\rho g d^2, \theta) \qquad (1)$$

This is as far as dimensional analysis goes. Theory, however, establishes that h is proportional to Υ. Since Υ occurs only in the second parameter, we can slip it outside $(h/d)_{\text{actual}} = (\sigma/\rho g d^2)F_1(\theta)$ or $h\rho g d/\sigma = F_1(\theta)$.

(*b*) We are given h_1 for certain conditions d_1, σ_1, ρ_1, and θ_1. If $h_1 = 3$ cm, what is h_2 for $d_2 = \frac{1}{2}d$, $\sigma_2 = \frac{1}{2}\sigma$, $\rho_2 = 2\rho_1$, and $\theta_2 = \theta_1$? We know the functional relation, Eq. (*1*), must still hold at condition 2: $h_2/d_2 = F(\sigma_2/\rho_2 g d_2^2, \theta_2)$. But $\sigma_2/\rho_2 g d_2^2 = \frac{1}{2}\sigma_1/2\rho_1 g(\frac{1}{2}d_1)^2 = \sigma_1/\rho_1 g d_1^2$. Therefore, functionally, $h_2/d_2 = F(\sigma_1/\rho_1 g d_1^2, \theta_1) = h_1/d_1$. We are given a condition 2 which is exactly similar to condition 1, and therefore a scaling law holds: $h_2 = h_1(d_2/d_1) = (3)(\frac{1}{2}d_1/d_1) = 1.5$ cm. If the pi groups had not been exactly the same for both conditions, we would have to know more about the functional relation F to calculate h_2.

18.52 Under laminar conditions, the volume flow Q through a small triangular-section pore of side length b and length L is a function of viscosity μ, pressure drop per unit length $\Delta p/L$, and b. Using the pi theorem, rewrite this relation in dimensionless form. How does the volume flow change if the pore size b is tripled?

▮ $$Q = f(\Delta p/L, \mu, b) \qquad \{L^3/T\} = \{M/L^2T^2\}\{M/LT\}\{L\}$$

$n = 4$, $j = 3$ (Q, μ, b do not make a Π), $n - j = 1$ Π expected. $\Pi_1 = Q^1(\Delta p/L)^a(\mu)^b(b)^c = \{L^3/T\}\{M/L^2T^2\}^a\{M/LT\}^b\{L\}^c = M^0L^0T^0$.

M: $a+b=0$

L: $3-2a-b+c=0$

T: $-1-2a-b=0$

$a=-1$, $b=+1$, $c=-4$; $\Pi_1 = Q\mu/(\Delta p/L)b^4 =$ constant. If b is tripled, Q increases 3^4, or 81 times.

18.53 The power input P to a centrifugal pump is assumed to be a function of volume flow Q, impeller diameter D, rotational rate Ω, and the density ρ and viscosity μ of the fluid. Rewrite this as a dimensionless relationship.

▮ $P_{\text{pump}} = f(Q, D, \Omega, \rho, \mu)$. $n=6$, $j=3$, $n-j=6-3=3$ Π's expected. $P/(\rho\Omega^3D^5) = f(Q/\Omega D^3, \rho\Omega D^2/\mu)$.

18.54 The resistance force F of a surface ship is a function of its length L, velocity V, gravity g, and the density ρ and viscosity μ of the water. Rewrite in dimensionless form.

▮ $F_{\text{ship}} = f(L, V, g, \rho, \mu)$. $m=6$, $j=3$, $n-j=6-3=3$ Π's expected. $F/(\rho V^2L^2) = f(V^2/gL, \rho VL/\mu)$.

18.55 The torque M on an axial-flow turbine is a function of fluid density ρ, rotor diameter D, angular rotation rate Ω, and volume flow Q. Rewrite in dimensionless form. If it is known that M is proportional to Q for a particular turbine, how would M vary with Ω and D for that turbine?

▮ $M_{\text{turbine}} = f(\rho, D, \Omega, Q)$. $n=5$, $j=3$, $n-j=5-3=2$ Π's expected. $M/(\rho\Omega^2D^5) = f(Q/\Omega D^3)$. If $M \approx Q$, then $M \approx \Omega$ and $M \approx D^2$.

18.56 A *weir* is an obstruction in a channel flow that can be calibrated to measure flow rate. The volume flow Q varies with gravity g, weir width b, and upstream water height H above the weir crest. If it is known that Q is proportional to b, use the pi theorem to find a unique functional relationship $Q(g, b, H)$.

▮ $Q/b = f(g, H)$. $n=3$, $j=2(L, T)$, $n-j=3-2=1$ Π expected. $(Q/b)/(g^{1/2}H^{3/2}) =$ constant.

18.57 The size of droplets produced by a liquid spray nozzle is thought to depend upon the nozzle diameter D, jet velocity U, and the properties of the liquid ρ, μ, and σ. Rewrite this relation in dimensionless form.

▮ $d = f(D, U, \rho, \mu, \sigma)$. $n=6$, $j=3$, $n-j=6-3=3$ Π's expected. $d/D = f(\rho UD/\mu, \sigma/\rho U^2D)$.

18.58 Perfume (specific gravity 0.92) in a tube of 25 mm diameter has a capillary rise of 3 mm. Determine its capillary rise in a 40-mm-diameter tube.

▮ $h = f(\rho g, \sigma, D)$. $n=4$, $j=3$, $n-j=4-3=1$ Π expected. $\rho ghD/\sigma =$ constant. For a given fluid (i.e., given ρg and σ), $hD =$ constant, or $h_mD_m = h_pD_p$. Thus, $(3)(2.5) = (h_p)(4)$ or $h_p = 1.88$ mm.

18.59 An estuary has a lunar tide of period 12.02 h and a tidal current of approximately 2.7 km/h. In a 1:400 scale model, what should the period and current speed be?

▮ $T_m/T_p = \sqrt{\alpha}$ $\quad \alpha = L_m/L_p = \frac{1}{400}$ $\quad T_m/12.02 = \sqrt{\frac{1}{400}}$ $\quad T_m = 0.601$ h or 36.1 min

$V_m/V_p = \sqrt{\alpha}$ $\quad V_m/2.7 = \sqrt{\frac{1}{400}}$ $\quad V_m = 0.135$ km/h $= 2.25$ m/min

18.60 Derive the expression for the drag on a submerged torpedo. The parameters are the size of the torpedo L, the velocity of the torpedo V, the viscosity of the water μ, and the density of the water ρ. The size of a torpedo may be represented by its diameter or its length.

▮ $F_D = f(L, V, \rho, \mu) = (\text{constant})L^aV^b\rho^c\mu^d$ $\quad \{ML/T^2\} = \{L\}^a\{L/T\}^b\{M/L^3\}^c\{M/LT\}^d$

M: $1 = c+d$

L: $1 = a+b-3c-d$

T: $-2 = -b-d$

$c = 1-d$ $\quad b = 2-d$ $\quad a = 2-d$

$$F_D = (\text{constant})L^{2-d}V^{2-d}\rho^{1-d}\mu^d = (\text{constant})C\rho L^2V^2(\mu/LV\rho)^d = (\text{constant})C\rho L^2V^2(LV\rho/\mu)^{-d}$$

Since $LV\rho/\mu = N_R$, $F_D = \phi(N_R)\rho L^2V^2$.

18.61 Derive an expression for the drag on a surface vessel. Use the same parameters as in Prob. 18.60, and add the acceleration due to gravity g to account for the effect of wave action.

▮ $F_D = f(L, V, \rho, \mu, g)$, $f'(F_D, L, V, \rho, \mu, g) = 0$, $n = 6$, $j = 3$, $n - j = 6 - 3 = 3$ Π's expected.

$$\Pi = \rho^a L^b V^c F_D^d = F_D/L^2V^2 \qquad \Pi_2 = \rho^a L^b V^c \mu^d = LV\rho/\mu = N_R \qquad \Pi_3 = \rho^a L^b V^c g^d = Lg/V^2 = 1/N_F^2$$

$$F_D = \phi(N_F, N_R)\rho L^2 V^2$$

18.62 The discharge through a horizontal capillary tube is thought to depend upon the pressure drop per unit length, the diameter, and the viscosity. Find the form of the equation.

▮ The quantities are listed with their dimensions:

quantity	symbol	dimensions
Discharge	Q	L^3T^{-1}
Pressure drop per length	$\Delta p/l$	$ML^{-2}T^{-2}$
Diameter	D	L
Viscosity	μ	$ML^{-1}T^{-1}$

Then $F(Q, \Delta p/l, D, \mu) = 0$. Three dimensions are used, and with four quantities there will be one Π parameter: $\Pi = Q^{x_1}(\Delta p/l)^{y_1}D^{z_1}\mu$. Substituting in the dimensions gives $\Pi = (L^3T^{-1})^{x_1}(ML^{-2}T^{-2})^{y_1}L^{z_1}ML^{-1}T^{-1} = M^0L^0T^0$. The exponents of each dimension must be the same on both sides of the equation. With L first, $3x_1 - 2y_1 + z_1 - 1 = 0$ and similarly for M and T: $y_1 + 1 = 0$, $-x_1 - 2y_1 - 1 = 0$; from which $x_1 = 1$, $y_1 = -1$, $z_1 = -4$, and $\Pi = Q\mu/(D^4\Delta p/l)$. After solving for Q, $Q = C(\Delta p/l)(D^4/\mu)$, from which dimensional analysis yields no information about the numerical value of the dimensionless constant C; experiment (or analysis) shows that it is $\pi/128$.

18.63 The discharge Q of a V-notch weir is some function of the elevation H of upstream liquid surface above the bottom of the notch. In addition, the discharge depends upon gravity and upon the velocity of approach V_0 to the weir. Determine the form of discharge equation.

▮ A functional relation $F(Q, H, g, V_0, \phi) = 0$ is to be grouped into dimensionless parameters. ϕ is dimensionless; hence, it is one Π parameter. Only two dimensions are used, L and T. If g and H are the repeating variables, $\Pi_1 = H^{x_1}g^{y_1}Q = L^{x_1}(LT^{-2})^{y_1}L^3T^{-1}$, $\Pi_2 = H^{x_2}g^{y_2}V_0 = L^{x_2}(LT^{-2})^{y_2}LT^{-1}$. Then

$$x_1 + y_1 + 3 = 0 \qquad x_2 + y_2 + 1 = 0$$
$$-2y_1 - 1 = 0 \qquad -2y_2 - 1 = 0$$

from which $x_1 = -\frac{5}{2}$, $y_1 = -\frac{1}{2}$, $x_2 = -\frac{1}{2}$, $y_2 = -\frac{1}{2}$, and $\Pi_1 = Q/(\sqrt{g}H^{5/2})$, $\Pi_2 = V_0/\sqrt{gH}$, $\Pi_3 = \phi$, or $f(Q/\sqrt{g}H^{5/2}), V_0/\sqrt{gH}, \phi) = 0$. This can be written $Q/(\sqrt{g}H^{5/2}) = f_1(V_0/\sqrt{gH}, \phi)$ in which both f and f_1 are unknown functions. After solving for Q, $Q = \sqrt{g}H^{5/2}f_1(V_0/\sqrt{gH}, \phi)$. Either experiment or analysis is required to yield additional information about the function f_1. If H and V_0 were selected as repeating variables in place of g and H, $\Pi_1 = H^{x_1}V_0^{y_1}Q = L^{x_1}(LT^{-1})^{y_1}L^3T^{-1}$, $\Pi_2 = H^{x_2}V_0^{y_2}g = L^{x_2}(LT^{-1})^{y_2}LT^{-2}$. Then

$$x_1 + y_1 + 3 = 0 \qquad x_2 + y_2 + 1 = 0$$
$$-y_1 - 1 = 0 \qquad -y_2 - 2 = 0$$

from which $x_1 = -2$, $y_1 = -1$, $x_2 = 1$, $y_2 = -2$, and $\Pi_1 = Q/H^2V_0$, $\Pi_2 = gH/V_0^2$, $\Pi_3 = \phi$ or $f(Q/H^2V_0, gH/V_0^2, \phi) = 0$. Since any of the Π parameters can be inverted or raised to any power without affecting their dimensionless status, $Q = V_0H^2f_2(V_0/\sqrt{gH}, \phi)$. The unknown function f_2 has the same parameters as f_1, but it could not be the same function. The last form is not very useful, in general, because frequently V_0 may be neglected with V-notch weirs. This shows that a term of minor importance should not be selected as a repeating variable.

18.64 The losses $\Delta p/l$ in turbulent flow through a smooth horizontal pipe depend upon velocity V, diameter D, dynamic viscosity μ, and density ρ. Use dimensional analysis to determine the general form of the equation $F(\Delta p/l, V, D, \rho, \mu) = 0$.

▮ If V, D, and ρ are repeating variables, $\Pi_1 = V^{x_1}D^{y_1}\rho^{z_1}\mu = (LT^{-1})^{x_1}L^{y_1}(ML^{-3})^{z_1}ML^{-1}T^{-1}$

$$\begin{aligned} x_1 + y_1 - 3z_1 - 1 &= 0 \\ -x_1 \qquad\quad -1 &= 0 \\ z_1 + 1 &= 0 \end{aligned}$$

from which $x_1 = -1$, $y_1 = -1$, $z_1 = -1$, and $\Pi_2 = V^{x_2}D^{y_2}\rho^{z_2}(\Delta p/l) = (LT^{-1})^{x_2}L^{y_2}(ML^{-3})^{z_2}ML^{-2}T^{-2}$

$$\begin{aligned} x_2 + y_2 - 3z_2 - 2 &= 0 \\ -x_2 \qquad\quad -2 &= 0 \\ z_2 + 1 &= 0 \end{aligned}$$

From which $x_2 = -2$, $y_2 = 1$, and $z_2 = -1$. Then $\Pi_1 = \mu/VD\rho$, $\Pi_2 = (\Delta p/l)/(\rho V^2/D)$, $F[VD\rho/\mu, (\Delta p/l)/(\rho V^2/D)] = 0$, since the Π quantities can be inverted if desired. The first parameter. $VD\rho/\mu$, is the *Reynolds number* N_R. After solving for $\Delta p/l$ we have $\Delta p/l = f_1(N_R, \rho V^2/D)$. The usual formula is $\Delta p/l = f(N_R)(\rho V^2/2D)$ or, in terms of head loss, $\Delta h/l = f(N_R)(1/D)(V^2/2g)$.

18.65 A fluid-flow situation depends on the velocity V, the density ρ, several linear dimensions, l, l_1, l_2, pressure drop Δp, gravity g, viscosity μ, surface tension σ, and bulk modulus of elasticity K. Apply dimensional analysis to these variables to find a set of Π parameters: $F(V, \rho, l, l_1, l_2, \Delta p, g, \mu, \sigma, K) = 0$.

▮ As three dimensions are involved, three repeating variables are selected. For complete situations, V, ρ, and l are generally helpful. There are seven Π parameters: $\Pi_1 = V^{x_1}\rho^{y_1}l^{z_1}\Delta p$, $\Pi_2 = V^{x_2}\rho^{y_2}l^{z_2}g$, $\Pi_3 = V^{x_3}\rho^{y_3}l^{z_3}\mu$, $\Pi_4 = V^{x_4}\rho^{y_4}l^{z_4}\sigma$, $\Pi_5 = V^{x_5}\rho^{y_5}l^{z_5}K$, $\Pi_6 = l/l_1$, $\Pi = l/l_2$. By expanding the Π quantities into dimensions, $\Pi_1 = (LT^{-1})^{x_1}(ML^{-3})^{y_1}L^{x_1}ML^{-1}T^{-2}$

$$\begin{aligned} x_1 - 3y_1 + z_1 - 1 &= 0 \\ -x_1 \qquad\quad -2 &= 0 \\ y_1 \qquad +1 &= 0 \end{aligned}$$

from which $x_1 = -2$, $y_1 = -1$, $z_1 = 0$. $\Pi_2 = (LT^{-1})^{x_2}(ML^{-3})^{y_2}L^{z_2}LT^{-2}$

$$\begin{aligned} x_2 - 3y_2 + z_2 + 1 &= 0 \\ -x_2 \qquad\quad -2 &= 0 \\ y_2 \qquad\quad &= 0 \end{aligned}$$

from which $x_2 = -2$, $y_2 = 0$, $z_2 = 1$. $\Pi_3 = (LT^{-1})^{x_3}(ML^{-3})^{y_3}L^{z_3}ML^{-1}T^{-1}$

$$\begin{aligned} x_3 - 3y_3 + z_3 - 1 &= 0 \\ -x_3 \qquad\quad -1 &= 0 \\ y_3 \qquad +1 &= 0 \end{aligned}$$

from which $x_3 = -1$, $y_3 = -1$, $z_3 = 1$. $\Pi_4 = (LT^{-1})^{x_4}(ML^{-3})^{y_4}L^{z_4}MT^{-2}$

$$\begin{aligned} x_4 - 3y_4 + z_4 \qquad &= 0 \\ -x_4 \qquad\quad -2 &= 0 \\ y_4 \qquad +1 &= 0 \end{aligned}$$

from which $x_4 = -2$, $y_4 = -1$, $z_4 = -1$. $\Pi_5 = (LT^{-1})^{x_5}(ML^{-3})^{y_5}L^{z_5}ML^{-1}T^{-2}$

$$\begin{aligned} x_5 - 3y_5 + z_5 - 1 &= 0 \\ -x_5 \qquad\quad -2 &= 0 \\ y_5 \qquad +1 &= 0 \end{aligned}$$

from which $x_5 = -2$, $y_5 = -1$, $z_5 = 0$. $\Pi_1 = \Delta p/\rho V^2$, $\Pi_2 = gl/V^2$, $\Pi_3 = \mu/Vl\rho$, $\Pi_4 = \sigma/V^2\rho l$, $\Pi_5 = K/\rho V^2$, $\Pi_6 = l/l_1$, $\Pi_7 = l/l_2$, and $f(\Delta p/\rho V^2, gl/V^2, \mu/Vl\rho, \sigma/V^2\rho l, K/\rho V^2, l/l_1, l/l_2) = 0$. It is convenient to invert some of the parameters and to take some square roots. $f_1(\Delta p/\rho V^2, V/\sqrt{gl}, Vl\rho/\mu, V^2l\rho/\sigma, V/\sqrt{K/\rho}, l/l_1, l_1/l_2) = 0$. The first parameter, usually written $\Delta\rho/(\rho V^2/2)$, is the *pressure coefficient*; the second parameter is the *Froude* number N_F; the third is the *Reynolds* number N_R; the fourth is the *Weber* number N_W; and the fifth is the *Mach* number N_M. Hence, $f_1(\Delta p/\rho V^2, N_F, N_R, N_W, N_M, l/l_1, l/l_2) = 0$. After solving for pressure drop, $\Delta p = \rho V^2 f_2(N_F, N_R, N_W, N_M, l/l_1, l/l_2)$ in which f_1 and f_2 must be determined from analysis or experiment. By selecting other repeating variables, a different set of Π parameters could be obtained.

18.66 The thrust due to any one of a family of geometrically similar airplane propellers is to be determined experimentally from a wind-tunnel test on a model. Use dimensional analysis to find suitable parameters for plotting test results.

▮ The thrust F_T depends upon speed of rotation ω, speed of advance V_0, diameter D, air viscosity μ, density ρ, and speed of sound c. The function $F(F_T, V_0, D, \omega, \mu, \rho, c) = 0$ is to be arranged into four dimensionless parameters, since there are seven quantities and three dimensions. Starting first by selecting ρ, ω, and D as repeating variables, $\Pi_1 = \rho^{x_1}\omega^{y_1}D^{z_1}F_T = (ML^{-3})^{x_1}(T^{-1})^{y_1}L^{z_1}MLT^{-2}$, $\Pi_2 = \rho^{x_2}\omega^{y_2}D^{z_2}V_0 = (ML^{-3})^{x_2}(T^{-1})^{y_2}L^{z_2}LT^{-1}$, $\Pi_3 = \rho^{x_3}\omega^{y_3}D^{z_3}\mu = (ML^{-3})^{x_3}(T^{-1})^{y_3}L^{z_3}ML^{-1}T^{-1}$, $\Pi_4 = \rho^{x_4}\omega^{y_4}D^{z_4}c = (ML^{-3})^{x_4}(T^{-1})^{y_4}L^{z_4}LT^{-1}$. Writing the simultaneous equations in x_1, y_1, z_1, etc., as before and solving them gives $\Pi_1 = F_T/\rho\omega^2D^2$, $\Pi_2 = V_0/\omega D$, $\Pi_3 = \mu/\rho\omega D^2$, $\Pi_4 = c/\omega D$. Solving for the thrust parameter leads to $F_T/\rho\omega^2D^4 = f_1(V_0/\omega D, \rho\omega D^2/\mu, c/\omega D)$. Since the parameters can be recombined to obtain other forms, the second term is replaced by the product of the first and second terms, $VD\rho/\mu$, and the third term is replaced by the first term divided by the third term, V_0/c; thus $F_T/\rho\omega^2D^4 = f_2(V_0/\omega D, V_0D\rho/\mu, V_0/c)$. Of the dimensionless parameters, the first is probably of the most importance since it relates speed of advance to speed of rotation. The second parameter is a Reynolds number and accounts for viscous effects. The last parameter, speed of advance divided by speed of sound, is a Mach number, which would be important for speeds near or higher than the speed of sound. Reynolds effects are usually small, so that a plot of $F_T/\rho\omega^4$ against $V_0/\omega D$ should be most informative.

18.67 The variation Δp of pressure in static liquids is known to depend upon specific weight γ and elevation difference Δz. By dimensional reasoning determine the form of the hydrostatic law of variation of pressure.

▮ $p = f(\gamma, \Delta z) = (\text{constant})\gamma^a(\Delta z)^b \qquad \{M/LT^2\} = \{M/L^2T^2\}^a\{L\}^b$

M: $a = 1$

L: $-2a + b = -1$

T: $-2a = -2$

$a = 1 \qquad b = 1 \qquad \Delta p = (\text{constant})\gamma\Delta z$

18.68 When viscous and surface-tension effects are neglected, the velocity V of efflux of liquid from a reservoir is thought to depend upon the pressure drop Δp of the liquid and its density ρ. Determine the form of expression for V.

▮ $V = f(\Delta p, \rho) \qquad (\Delta p)^a\rho^b = (\text{constant})V \qquad \{M/LT^2\}^a\{M/L^3\}^b = L/T$

M: $a + b = 1$

L: $-a - 3b = 1$

T: $-2a = -1$

$a = \frac{1}{2} \qquad b = -\frac{1}{2} \qquad V = (\text{constant})\sqrt{\Delta p/\rho}$

18.69 The buoyant force F_b on a body is thought to depend upon its volume submerged V and the gravitational body force acting on the fluid. Determine the form of the buoyant-force equation.

▮ $F_b = f(V, \rho g) \qquad V^a(\rho g)^b = (\text{constant})F_b \qquad \{L^3\}^a\{(M/L^3)(L/T^2)\}^b = \{ML/T^2\}$

M: $b = 1$

L: $3a - 2b = 1$

T: $-2b = -2$

$a = 1 \qquad b = 1 \qquad F_b = (\text{constant})V\rho g$

18.70 In a fluid rotated as a solid about a vertical axis with angular velocity ω, the pressure rise p in a radial direction depends upon speed ω, radius r, and fluid density ρ. Obtain the form of equation for p.

▮ $\Delta p = f(\omega, r, \rho) \qquad \omega^a r^b \rho^c = (\text{constant})\Delta p \qquad \{1/T\}^a\{L\}^b\{M/L^3\}^c = \{M/LT^2\}$

M: $c = 1$

L: $b - 3c = -1$

T: $-a = -2$

$a = 2 \qquad b = 2 \qquad c = 1 \qquad \Delta p = (\text{constant})\omega^2r^2\rho$

18.71 The Mach number N_M for flow of a perfect gas in a pipe depends upon the specific-heat ratio k (dimensionless), the pressure p, the density ρ, and the velocity V. Obtain by dimensional reasoning the form of the Mach number expression.

▮ $N_M = f(k, \rho, p, V)$, $f'(N_M, k, \rho, p, V) = 0$; $n = 5$, $j = 3$, $n - j = 5 - 3 = 2\ \Pi$'s expected.

$$\Pi_1 = M \qquad \Pi_2 = k \qquad p^a\rho^b V = \Pi_3 \qquad \{M^a/L^a\}\{M^b/T^{2a}L^{3b}\}\{L/T\} = 1$$

M: $\quad a + b = 0$

L: $\quad -a - 3b + 1 = 0$

T: $\quad -2a - 1 = 0$

$$a = -\tfrac{1}{2} \qquad b = \tfrac{1}{2} \qquad N_M = f(V/\sqrt{p/\rho}, k)$$

18.72 The moment exerted on a submarine by its rudder is to be studied with a 1:25 scale model in a water tunnel. If the torque measured on the water model is 7 N · m for a tunnel velocity of 17 m/s, what are the corresponding torque and speed for the prototype?

▮ $V_p/V_m = L_m/L_p \qquad V_p/17 = \frac{1}{25} \qquad V_p = 0.68$ m/s $\qquad T = (\text{force})(\text{arm}) \propto (\rho V^2 A)(L)$

$T_r = \rho_r V_r^2 A_r L_r = (1)(1/L_r)^2 L_r^2 L_r = L_r \qquad T_p/T_m = L_p/L_m \qquad T_p/7 = \frac{25}{1} \qquad T_p = 175$ N · m

18.73 A rotary machine is to mix paint [$\rho = 789$ kg/m^3, $\mu = 1.20 \times 10^{-3}$ Pa · s]. Tests with a 1:5 scale model in SAE30 oil [$\rho = 917$ kg/m^2, $\mu = 0.29$ Pa · s] indicate best mixing at $\omega = 1850$ rpm. What should the rotation speed of the prototype be?

▮ $(N_R)_m = (N_R)_p$. Use a characteristic velocity ωD, so that $N_R = (\rho)(\omega D)(D)/\mu = \rho\omega D^2/\mu$. Then

$$(917)(1850)(D_p/5)^2/0.29 = (789)(\omega_p)(D_p)^2/(1.20 \times 10^{-3}), \quad \omega_p = 0.356 \text{ rpm}.$$

18.74 A one-fourteenth-scale model of a parachute has a drag of 400 lb when tested at 23 ft/s in a water tunnel. If Reynolds number effects are negligible, estimate the terminal fall velocity at 5000 ft standard altitude of a parachutist using the prototype if chute and chutist together weight 175 lb. Neglect the drag coefficient of the woman.

▮ $(C_D)_m = (C_D)_p$ (if Reynolds number effects are negligible), $C_D = F/\rho V^2 D^2$.

$$\rho_p = 0.002048 \text{ slug/ft}^3 \quad \text{(from Table A-7)} \qquad D_m = D_p/14$$

$$400/[(1.94)(23)^2(D_p/14)^2] = 175/[(0.002048)(V_p)^2(D_p)^2] \qquad V_p = 33.4 \text{ ft/s}$$

18.75 A one-tenth-scale model of a weir has a measured flow rate of 2.5 cfs when the upstream water height is $H = 6.9$ in. Use the results of Prob. 18.56 to predict the prototype flow rate when $H = 3.1$ ft.

▮ $(Q/b)/(g^{1/2}H^{3/2}) = \text{constant}$ (from Prob. 18.56) $\qquad [(Q/b)/(g^{1/2}H^{3/2})]_m = [(Q/b)/(g^{1/2}H^{3/2})]_p$

$$b_m = b_p/10 \qquad [2.5/(b_p/10)]/[(32.2)^{1/2}(6.9/12)^{3/2}] = (Q_p/b_p)/[(32.2)^{1/2}(3.1)^{3/2}] \qquad Q_p = 313 \text{ ft}^3/\text{s}$$

18.76 For model and prototype, show that, when gravity and inertia are the only influences, the ratio of flows Q is equal to the ratio of the length dimension to the five-halves power.

▮ $Q_m/Q_p = (L_m^3/T_m)/(L_p^3/T_p) = L_r^3/T_r$. The time ratio must be established for the conditions influencing the flow. Expressions can be written for the gravitation and inertia forces, as follows:

Gravity: $\qquad F_m/F_p = W_m/W_p = (\gamma_m/\gamma_p)(L_m^3/L_p^3) = \gamma_r L_r^3$

Inertia $\qquad F_m/F_p = M_m a_m/M_p a_p = (\rho_m/\rho_p)(L_m^3/L_p^3)(L_r/T_r^2) = \rho_r L_r^3(L_r/T_r^2)$

Equating the force ratios, $\gamma_r L_r^3 = \rho_r L_r^3(L_r/T_r^2)$, which, when solved for the time ratio, yields

$$T_r^2 = L_r(\rho_r/\gamma_r) = L_r/g_r \tag{1}$$

Recognizing that the value of g_r is unity, substitution in the flow ratio expression gives

$$Q_r = Q_m/Q_p = L_r^3/L_r^{1/2} = L_r^{5/2} \tag{2}$$

18.77 A spillway model is to be built to a scale of 1:25 across a flume which is 2 ft wide. The prototype is 37.5 ft high and the maximum head expected is 5.0 ft. What height of model and what head on the model should be used? If

the flow over the model at 0.20 ft head is 0.70 cfs, what flow per foot of prototype may be expected;

▮ $L_m/L_p = \frac{1}{25}$ Height of model $= (37.5)(\frac{1}{25}) = 1.50$ ft

Head on model $= (5.0)(\frac{1}{25}) = 0.20$ ft $Q_r = L_r^{5/2}$ (from Prob. 18.76)

$Q_m/Q_p = (L_m/L_p)^{5/2}$ $0.070/Q_p = (\frac{1}{25})^{5/2}$ $Q_p = 2188\ \text{ft}^3/\text{s}$

$b_m/b_p = \frac{1}{25}$ $2/b_p = \frac{1}{25}$ $b_p = 50.0$ ft $q_p = Q_p/b_p = 2188/50.0 = 43.8\ \text{ft}^3/\text{s}$

18.78 For the model described in Prob. 18.77, if the model shows a measured hydraulic jump of 1.0 in, how high is the jump in the prototype? If the energy dissipated in the model at the hydraulic jump is 0.15 hp, what would be the energy dissipation in the prototype?

▮ Since $L_m/L_p = \frac{1}{25}$, height of jump $= (1.0)(25) = 25.0$ in, or 2.08 ft, $P_r = F_rL_r/T_r = (\gamma_rL_r^3)(L_r)/\sqrt{L_r/g_r}$. Since $g_r = \gamma_r = 1$, $P_r = L_r^{7/2}$, $0.15/P_p = (\frac{1}{25})^{7/2}$, $P_r = 11\,720$ hp.

18.79 A model of a reservoir is drained in 4 min by opening the sluice gate. The model scale is 1:225. How long should it take to empty the prototype?

▮ Since gravity is the dominant force, $Q_r = L_r^{5/2}$ (from Prob. 18-76), $Q_r = Q_m/Q_p = (L_m^3/t_m)/(L_p^3/t_p)$, $L_r^{5/2} = (L_r^3)(t_p/t_m)$, $t_p = t_m/L_r^{1/2} = (4)/(\frac{1}{225})^{1/2} = 60.0$ min.

18.80 A rectangular pier in a river is 4 ft wide by 12 ft long and the average depth of water is 9.0 ft. A model is built to a scale of 1:16. The velocity of flow of 2.50 fps is maintained in the model and the force acting on the model is 0.90 lb. What are the values of velocity in and force on the prototype?

▮ Since the gravity forces predominate, $V_m/V_p = \sqrt{L_r}$, $2.50/V_p = \sqrt{\frac{1}{16}}$, $V_p = 10.0$ ft/s. $F_m/F_p = \gamma_rL_r^3$, $0.90/F_p = (1.0)(\frac{1}{16})^3$, $F_p = 3686$ lb.

18.81 If a standing wave in the model of Prob. 18.80 is 0.16 ft high, what height of wave should be expected at the nose of the pier? What is the coefficient of drag resistance?

▮ $V_m/V_p = \sqrt{L_m}/\sqrt{L_p} = \sqrt{h_m}/\sqrt{h_p}$ $2.50/10.0 = \sqrt{0.16}/\sqrt{h_p}$ $h_p = 2.56$ ft

$F_D = C_D\rho AV^2/2$ $3686 = (C_D)(1.94)[(4)(9)](10.0)^2/2$ $C_D = 1.06$

18.82 The measured resistance in fresh water of an 8-ft ship model moving at 6.50 fps was 9.60 lb. **(*a*)** What would be the velocity of the 128-ft prototype? **(*b*)** What force would be required to drive the prototype at this speed in salt water?

▮ **(*a*)** Since gravity forces predominate, $V_m/V_p = \sqrt{L_r}$, $6.50/V_p = \sqrt{\frac{8}{128}}$, $V_p = 26.0$ ft/s.

(*b*) $F_m/F_p = \gamma_rL_r^3$ $9.60/F_p = (62.4/64.2)(\frac{1}{16})^3$ $F_p = 40\,460$ lb

CHAPTER 19
Unsteady Flow

19.1 The surface of a frictionless fluid column 2.18 m long is dropping at 2 m/s when $z = 0.5$ m (measured upwards from the reference level). Find **(a)** the maximum value of z, **(b)** the maximum speed.

(a) $z = Z\cos\omega t$, $dz/dt = -\omega Z\sin\omega t$, where $\omega = \sqrt{2g/L} = \sqrt{2(9.807)/2.18} = 3.00$ rad/s. Then, if t_1 is the time when $z = 0.5$ and $dz/dt = 2$, $0.5 = Z\cos 3.00t_1$, $-2 = -3.00Z\sin 3.00t_1$. From these,

$$\sin^2 3.00t_1 + \cos^2 3.00t_1 = 1 = \left(\frac{2}{3Z}\right)^2 + \left(\frac{1}{2Z}\right)^2$$

whence $Z^2 = 25/36$, or $Z = 5/6 = 0.833$ m.
(b) $\omega Z = (3.00)(0.833) = 2.50$ m/s.

19.2 A 1.0-in-diameter U-tube contains oil, $\nu = 1\times10^{-4}$ ft²/s, with a total column length of 120 in. Applying air pressure to one of the tubes makes the gage difference 16 in. By quickly releasing the air pressure, the oil column is free to oscillate. Find the maximum velocity, the maximum Reynolds number, and the equation for position of one meniscus z, in terms of time.

The assumption is made that the flow is laminar, and the Reynolds number will be computed on this basis. The constants m and n are $m = 16\nu/D^2 = (16\times10^{-4})/(\frac{1}{12})^2 = 0.2302$, $n = \sqrt{(16\nu/D^2)^2 - (2g/L)} = \sqrt{0.2302^2 - [2(32.2)/10]} = \sqrt{-6.387} = i2.527$, or $n' = 2.527$. The liquid will oscillate above and below $z = 0$. The oscillation starts from the maximum position, that is, $Z = 0.667$ ft. The velocity (fictitious) when $z = 0$ at time t_0 before the maximum is determined to be $V_0 = Z\sqrt{2g/L}\exp[(m/n')\tan^{-1}(n'/m)] = 0.667\sqrt{64.4/10}\exp[(0.2302/2.527)\tan^{-1}(2.527/0.2302)] = 1.935$ ft/s and $\tan n't_0 = n'/m$, $t_0 = (1/2.527)\tan^{-1}(2.527/0.2302) = 0.586$ s; $z = (V_0/n')e^{-mt}\sin(n't) = 0.766\exp[-0.2302(t+0.586)]\sin 2.527(t+0.586)$ in which $z = Z$ at $t = 0$. The maximum velocity (actual) occurs for $t > 0$. Differentiating with respect to t to obtain the expression for velocity, $V = dz/dt = -0.1763\exp[-0.2302(t+0.586)]\sin 2.527(t+0.586) + 1.935\exp[-0.2302(t+0.586)]\cos 2.527(t+0.586)$. Differentiating again with respect to t and equating to zero to obtain maximum V produces $\tan 2.527(t+0.586) = -0.1837$. The solution in the second quadrant should produce the desired maximum, $t = 0.584$ s. Substituting this time into the expression for V produces $V = -1.48$ ft/s. The corresponding Reynolds number is $VD/\nu = 1.48(\frac{1}{12}\times10^4) = 1234$; hence the assumption of laminar resistance is justified.

19.3 A U-tube consisting of 500-mm-diameter pipe with $f = 0.03$ has a maximum oscillation (Fig. 19-1) of $z_m = 6$ m. Find the minimum position of the surface and the following maximum.

$[1 + (fz_m/D)]e^{-fz_m/D} = [1 + (fz_{m+1}/D)]e^{-fz_{m+1}/D}$, $\{1 + [0.03(6)/0.5]\}e^{0.03(6)0.5} = (1 + 0.06z_{m+1})e^{-0.06z_{m+1}}$ or $(1 + 0.06z_{m+1})e^{-0.06z_{m+1}} = 0.9488$, which is satisfied by $z_{m+1} = -4.84$ m. Using $z_m = 4.84$ m in the previous equation, $(1 + 0.06z_{m+1})e^{-0.06z_{m+1}} = [1 + 0.06(4.84)]e^{-0.06(4.84)} = 0.9651$, which is satisfied by $z_{m+1} = -4.05$ m. Hence, the minimum water surface is $z = -4.84$ m and the next maximum is $z = 4.05$ m.

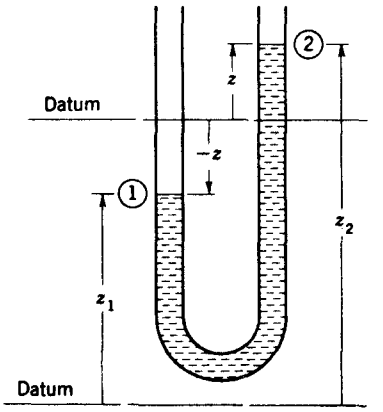


Fig. 19-1

19.4 The pipe of Prob. 19.3 is 1000 m long. By numerical integration of the equation

$$\frac{d^2z}{dt^2} + \frac{f}{2D}\frac{dz}{dt}\left|\frac{dz}{dt}\right| + \frac{2g}{L}z = 0$$

using the third-order Runge–Kutta method [see Prob. 19.22], determine by a computer program the time to the first minimum z and the next maximum z and check the results of Prob. 19.3.

```
10 REM          EXAMPLE          TURBULENT OSCILLATION OF A U-TUBE.
20 DEFINT I:     DEF FNZ(ZZ,YY)=C1*YY*ABS(YY)+C2*ZZ
30 READ D,F,L,ZM,G,H,IPR:     DATA .5,.03,1000.,6.,9.806,.5,4
40 LPRINT:     LPRINT"D,F,L,ZM,G,H,IPR=";D;F;L;ZM;G;H;IPR
50 C1=-.5*F*H/D: C2=-2!*G*H/L: Y=0!: Z=6!                          ' Y IS DZ/DT
60 OZ=6!:     I=0:     EPS=.001
70 LPRINT:     LPRINT"  T,sec     Z,m       V,m/s":     LPRINT
80 I=I+1:    OOZ=OZ:    OZ=Z        ' OOZ and OZ used to determine max and min Z
90  U11=FNZ(Z,Y):                          U12=H*Y
100  U21=FNZ(Z+.3333*U12,Y+.3333*U11):     U22=H*(Y+.3333*U11)
110 U31=FNZ(Z+.6667*U22,Y+.6667*U21):     U32=H*(Y+.6667*U21)
120 Y=Y+.25*U11+.75*U31:     Z=Z+.25*U12+.75*U32:     T=T+H
130 IF (OZ<OOZ-EPS) AND (OZ<Z-EPS) THEN LPRINT"ZMIN=";OZ;"TIME=";T-H
140 IF I MOD IPR=0 THEN LPRINT USING"###.###  ";T;Z;Y
150 IF (OZ>OOZ+EPS) AND (OZ>Z+EPS) THEN LPRINT"ZM=";OZ;"TIME=";T-H: STOP
    ELSE GOTO 80
```

```
D,F,L,ZM,G,H,IPR= .5  .03  1000  6  9.806001  .5  4

  T,sec     Z,m       V,m/s

  2.000    5.767    -0.231
  4.000    5.091    -0.438
  6.000    4.042    -0.602
  8.000    2.721    -0.709
 10.000    1.247    -0.755
 12.000   -0.257    -0.740
 14.000   -1.677    -0.671
 16.000   -2.912    -0.558
 18.000   -3.884    -0.409
 20.000   -4.533    -0.236
 22.000   -4.821    -0.050
ZMIN=-4.834092 TIME= 22.5
 24.000   -4.732     0.138
 26.000   -4.278     0.313
 28.000   -3.502     0.457
 30.000   -2.478     0.559
 32.000   -1.298     0.613
 34.000   -0.061     0.616
 36.000    1.136     0.574
 38.000    2.206     0.490
 40.000    3.075     0.374
 42.000    3.688     0.235
 44.000    4.006     0.082
ZM= 4.048564 TIME= 45
```

19.5 In Fig. 19-2 a valve is opened suddenly in the pipeline when $z_1 = 40$ ft. $L = 2000$ ft, $A_1 = 200$ ft^2, $A_2 = 300$ ft^2, $D = 3.0$ ft, $f = 0.024$, and minor losses are $3.50V^2/2g$. Determine the subsequent maximum negative and positive surges in the reservoir A_1.

▮ The equivalent length of minor losses is $KD/f = 3.5(3)/0.024 = 438$ ft. Then $L_e = 2000 + 438 = 2438$ and $z_m = z_1A_1/A = 40(200/(2.25\pi) = 1132$ ft. The corresponding ϕ is $\phi = f(L_e/L)(z_m/D) = 0.024(\frac{2438}{2000})(\frac{1132}{3}) = 11.04$ and $F(\phi) = (1+\phi)e^{-\phi} = (1+11.04)e^{-11.04} = 0.000193$, which is satisfied by $\phi \approx -1.0$. Then $F(\phi) = (1+1)e^{-1} = 0.736 = (1+\phi)e^{-\phi}$ which is satisfied by $\phi = -0.593$. The values of z_m are, for $\phi = -1$. $z_m = \phi LD/fL_e = -1(2000)(3)/[0.024(2438)] = -102.6$ and, for $\phi = 0.593$, $z_m = 0.593(2000)(3)/[0.024(2438)] = 60.9$. The corresponding values of z_1 are $z_1 = z_m(A/A_1) = -102.6(2.25\pi/200) = -3.63$ ft and $z_1 = 60.9(2.25\pi/200) = 2.15$ ft.

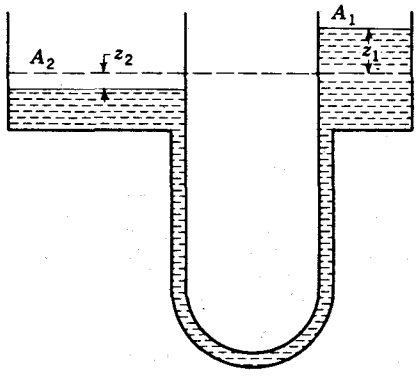


Fig. 19-2

19.6 In Fig. 19-3 the minor losses are $16V^2/2g$, $f = 0.030$, $L = 3000$ m, $D = 2.4$ m, and $H = 20$ m. Determine the time, after the sudden opening of a valve, for velocity to attain nine-tenths the final velocity.

▮
$$H = f(L_e/D)(V_0^2/2g) \qquad L_e = 3000 + [16(2.4)/0.03] = 4280 \text{ m}$$
$$V_0 = \sqrt{2gHD/fL_e} = \sqrt{19.612(20)(2.4)/[0.030(4280)]} = 2.708 \text{ m/s}$$

Substituting $V = 0.9V_0$ into $t = (LV_0/2gH)\ln[(V_0 + V)/(V_0 - V)]$, $t = [3000(2.708)/19.612(20)]\ln(1.90/0.10) = 60.98$ s.

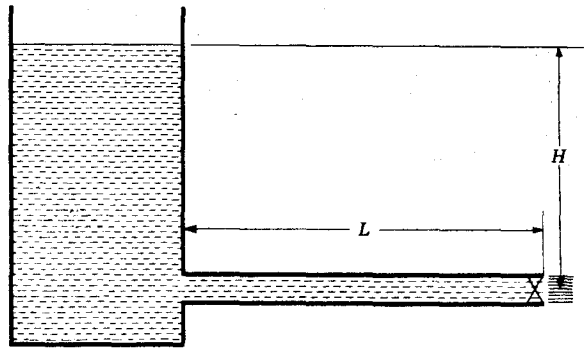


Fig. 19-3

19.7 In Fig. 19-4 the water surface changes according to the equation $H_{P_A} = H_0 + \Delta H \sin \omega t$ while the right end of the pipe contains a small orifice. The frequency of the waves is set at the natural period of the pipe, $4L/a$, which yields an ω of $2\pi/(4L/a)$. Determine the resulting motion of fluid in the pipe and the head fluctuations.

▮ One reach is used. The upstream section is solved by $H_{P_i} = C_M + B_M Q_{P_i}$ together with the head, which is a known function of time. The downstream boundary condition solves the orifice equation together with $Q_{P_B}^2 = C_1 H_{P_A}$, $H_{P_B} = C_P - B_P Q_{P_B}$. The program and results are shown below.

```
10 REM                          TRANSIENT IN SINGLE-REACH PIPE.
20 DEFINT I:     READ F,L,D,A,DH,HO,IPR,PI,G,TM,CD,DORIF
30 DATA .018,600.,.5,1200.,3.,40.,1,3.1416,9.806,12.,.6,.03
40 AO=.25*PI*DORIF^2:     LPRINT"CD,DORIF,AO=";CD;DORIF;AO:   C1=(CD*AO)^2*2!*G
50 LPRINT"F,L,D,A,DH=";F;L;D;A;DH:      LPRINT"HO,IPR,PI,G,TM=";HO;IPR;PI;G;TM
60 PER=4!*L/A:     OM=2!*PI/PER:     AR=.25*PI*D^2
70 B=A/(G*AR):     R=.5*F*L/(G*D*AR^2):     QO=SQR(HO/(R+1!/C1))
80 T=0!:     DT=L/A:     I=0:     HA=HO:     HB=HO-R*QO^2:     QA=QO:     QB=QO
90 LPRINT"     T,s     HA,m     QA,l/s     HB,m     QB,l/s"
100 LPRINT USING"####.#### ";T;HA;1000!*QA;HB;1000!*QB
110 T=T+DT:     I=I+1:     IF T>TM THEN STOP
120 HPA=HO+DH*SIN(OM*T):     CM=HB-B*QB:     BM=B+R*ABS(QB):     QPA=(HPA-CM)/BM
130 CP=HA+B*QA:     BP=B+R*ABS(QA):     QPB=-.5*BP*C1+SQR((.5*BP*C1)^2+CP*C1)
140 HPB=CP-BP*QPB:     HA=HPA:     HB=HPB:     QA=QPA:     QB=QPB
150 IF I MOD IPR=0 THEN 100 ELSE GOTO 110
CD,DORIF,AO= .6  .03  7.0686E-04
F,L,D,A,DH= .018  600  .5  1200  3
HO,IPR,PI,G,TM= 40  1  3.1416  9.806001  12
     T,s      HA,m      QA,l/s     HB,m      QB,l/s
   0.0000   40.0000   11.8783   39.9960   11.8783
   0.5000   43.0000   16.6892   39.9960   11.8783
   1.0000   40.0000   11.8782   45.4994   12.6692
   1.5000   37.0000   -0.9676   39.9959   11.8783
   2.0000   40.0000   11.8784   29.9865   10.2851
   2.5000   43.0000   31.1506   39.9961   11.8783
   3.0000   39.9999   11.8780   53.8149   13.7783
   3.5000   37.0000  -13.1930   39.9958   11.8783
   4.0000   40.0001   11.8787   23.1428    9.0355
   4.5000   43.0000   40.8796   39.9963   11.8783
   5.0000   39.9999   11.8776   59.4364   14.4801
   5.5000   37.0000  -21.5050   39.9955   11.8782
   6.0000   40.0001   11.8792   18.5504    8.0895
   6.5000   43.0000   47.3015   39.9966   11.8784
   7.0000   39.9998   11.8771   63.1574   14.9265
   7.5000   37.0000  -27.0247   39.9952   11.8782
   8.0000   40.0002   11.8798   15.5371    7.4034
   8.5000   43.0000   51.4503   39.9970   11.8784
   9.0000   39.9998   11.8765   65.5652   15.2084
   9.5000   37.0000  -30.6034   39.9948   11.8781
  10.0000   40.0002   11.8804   13.6031    6.9273
  10.5000   43.0000   54.0776   39.9974   11.8785
  11.0000   39.9998   11.8759   67.0916   15.3844
  11.5000   37.0000  -32.8746   39.9944   11.8781
  12.0000   40.0003   11.8810   12.3853    6.6099
```

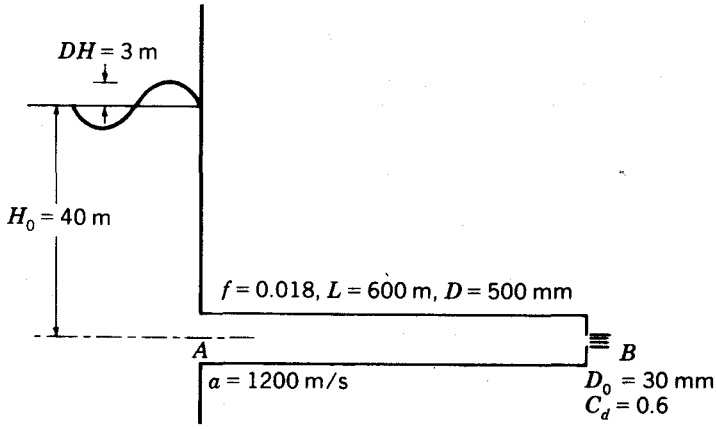


Fig. 19-4

19.8 Develop the necessary boundary-condition equations for the pump in Fig. 19-5. The pump is to be started with a linear speed rise to N_R in t_0 s. A check valve exists in the discharge pipe. The initial no-flow steady-state head on the downstream side of the check valve is H_C. For a steady flow of Q_0 there is a loss of ΔH_0 across the open check valve. Assume that the check valve opens when the pump has developed enough head to exceed H_C.

▮ The equation for the hydraulic grade line downstream from the pump and check valve (after the check valve is open) is $H_P = C_1N^2 + C_2NQ_P + C_3Q_P^2 - Q_P^2(\Delta H_0/Q_0^2)$, $Q_P = \{(B_M - C_2N)/[2(C_3 - \Delta H_0/Q_0^2)]\}\{1 - [1 + 4(C_3 - \Delta H_0/Q_0^2)^2(C_M - C_1N^2)/(B_M - C_2N)^2]^{1/2}\}$. The equations for the boundary condition are $B_M = (a/gA) + R\,|Q_2|$, $C_M = H_2 - BQ_2$,

$$N = \begin{cases} N_R(t/t_0) & t \le t_0 \\ N_R & t > t_0 \end{cases}$$

If $C_1N^2 > H_C$, Q_P is defined by the above equation for the quadratic, and $H_P = C_M + B_MQ_P$. If $C_1N^2 < H_C$, $Q_P = 0$ and $H_P = H_C$.

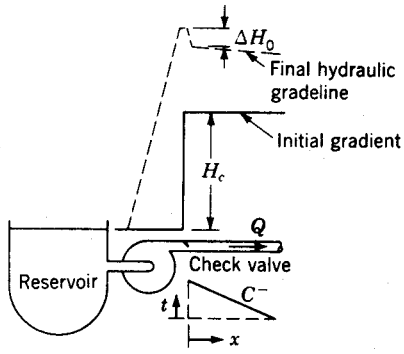


Fig. 19-5

19.9 The system of Fig. 19-6 initially has a valve opening $C_DA = 0.06\ \text{m}^2$. At intervals of 5 s, C_DA takes on the values 0.03, 0.01, 0.003, 0.001, 0.0005, 0.0002, 0.0, and remains closed. Using a computer program, calculate the transients of the system for 40 s after the valve starts to close.

▮ The data for this problem and some of the output are given below. The number of reaches was selected as $N = 4$. Hydraulic-grade-line elevations are given in meters and discharges in cubic meters per second.

```
10 REM B:WH   WATERHAMMER PROGRAM IN IBMPC BASICA FOR UPSTREAM RESERVOIR,
20 ' SINGLE PIPE, AND DOWNSTREAM VALVE. DARCY-WEISBACH FRICTION. F GIVEN.
30 ' A=WAVE SPEED, CVA=PRODUCT OF DISCHARGE COEF. AND AREA OF VALVE
40 ' OPENING, WITH VALUES GIVEN FOR INTERVALS DCV.
50 LPRINT "      BASIC WATERHAMMER PROGRAM"
60 LPRINT "      DATE=";DATE$;"    TIME=";TIME$
65 CLEAR:   DIM CVA(11):   DEFINT I,J,K
70 READ F,L,A,N,D,HRES,G,JPR,DCV,TMAX
80 DATA .022,4800.,1200.,4,2.,100.,9.806,1,5.,40.
90 LPRINT:   LPRINT"F,L,A,N,D=";F;L;A;N;D
100 LPRINT"HRES,G,JPR,DCV,TMAX=";HRES;G;JPR;DCV;TMAX
105 LPRINT"CVA=";
110 FOR I=1 TO 11:   READ CVA(I):   NEXT I:   LPRINT
115 FOR I=1 TO 11:   LPRINT CVA(I);:   NEXT I:   LPRINT
120 DATA .06,.03,.01,.003,.001,.0005,.0002,.0,.0,.0,.0
```

```
130 AR=.7854*D^2:   B=A/(G*AR):   NS=N+1:   DT=L/(A*N):   CV=CVA(1)
140 HP(1)=HRES:  J=0:  T=0!:  Q0=SQR(HRES/(F*L/(2!*G*D*AR^2)+1!/(2!*G*CV^2)))
150 H0=(Q0/CV)^2/(2!*G):   R=(HRES-H0)/(Q0^2*N)
160 FOR I=1 TO NS:   Q(I)=Q0:   H(I)=HRES-(I-1)*R*Q0^2:   NEXT I
170 LPRINT"     PIEZOMETER HEADS AND FLOWS ALONG THE PIPE"
180 LPRINT"   TIME    CV   X/L=     .0     .25     .5     .75     1."
190 LPRINT USING"###.###";T;:   LPRINT USING" #.#####";CV;:   LPRINT"  H=";
200 LPRINT USING"  ###.##";H(1);H(2);H(3);H(4);H(5)
210 LPRINT SPC(17);"Q=";:   LPRINT USING"  ###.##";Q(1);Q(2);Q(3);Q(4);Q(5)
220 T=T+DT:   J=J+1:   K=FIX(T/DCV)+1:   IF T>TMAX THEN STOP
230 CV=CVA(K)+(T-(K-1)*DCV)*(CVA(K+1)-CVA(K))/DCV
240 REM DOWNSTREAM BOUNDARY CONDITION
250 CP=H(N)+B*Q(N):   BP=B+R*ABS(Q(N))
260 QP(NS)=-G*BP*CV^2+SQR((G*BP*CV^2)^2+2!*G*CV^2*CP):   HP(NS)=CP-BP*QP(NS)
270 REM UPSTREAM BOUNDARY CONDITION
280 QP(1)=(HP(1)-H(2)+B*Q(2))/(B+R*ABS(Q(2)))
290 REM INTERIOR SECTIONS
300 FOR I=2 TO N:   CP=H(I-1)+B*Q(I-1):   BP=B+R*ABS(Q(I-1))
310 CM=H(I+1)-B*Q(I+1):   BM=B+R*ABS(Q(I+1))
320 QP(I)=(CP-CM)/(BP+BM):   HP(I)=CP-BP*QP(I):   NEXT I
330 FOR I=1 TO NS:   H(I)=HP(I):   Q(I)=QP(I):   NEXT I
340 IF J MOD JPR=0 THEN 190 ELSE 220
```

```
F,L,A,N,D= .022  4800  1200  4  2
HRES,B,JPR,DCV,TMAX= 100  9.806001  1  5  36
CVA=
 .06  .03  .01  .003  .001  .0005  .0002  0  0  0  0
    PIEZOMETER HEADS AND FLOWS ALONG THE PIPE
  TIME     CV    X/L=      .0      .25      .5      .75      1.
 0.000 0.06000  H=  100.00   99.53   99.06   98.58   98.11
                Q=    2.63    2.63    2.63    2.63    2.63
 1.000 0.05400  H=  100.00   99.53   99.06   98.58  105.14
                Q=    2.63    2.63    2.63    2.63    2.45
 2.000 0.04800  H=  100.00   99.53   99.06  105.59  112.77
                Q=    2.63    2.63    2.63    2.45    2.26
 3.000 0.04200  H=  100.00   99.53  106.03  113.18  121.07
                Q=    2.63    2.63    2.45    2.26    2.05
 4.000 0.03600  H=  100.00  106.47  113.59  121.45  130.07
                Q=    2.63    2.45    2.26    2.05    1.82
 5.000 0.03000  H=  100.00  114.00  121.83  130.41  139.85
                Q=    2.28    2.26    2.05    1.82    1.57
 6.000 0.02600  H=  100.00  115.33  130.76  140.17  146.84
                Q=    1.90    1.88    1.82    1.57    1.40
 7.000 0.02200  H=  100.00  116.76  133.63  147.14  154.25
                Q=    1.48    1.46    1.40    1.40    1.21
 8.000 0.01800  H=  100.00  118.29  133.13  147.70  162.04
                Q=    1.03    1.01    1.04    1.04    1.01
 9.000 0.01400  H=  100.00  116.37  132.36  148.02  157.80
                Q=    0.54    0.61    0.64    0.65    0.78
10.000 0.01000  H=  100.00  114.08  131.27  142.47  152.18
                Q=    0.19    0.18    0.22    0.38    0.55
11.000 0.00860  H=  100.00  114.90  124.19  135.44  139.90
                Q=   -0.19   -0.20   -0.08    0.12    0.45
12.000 0.00720  H=  100.00  110.11  119.07  121.62  126.02
                Q=   -0.58   -0.45   -0.30   -0.02    0.36
13.000 0.00580  H=  100.00  104.18  107.56  109.66  110.38
                Q=   -0.70   -0.69   -0.38   -0.06    0.27
14.000 0.00440  H=  100.00   97.46   94.78   96.32   99.66
                Q=   -0.79   -0.64   -0.44   -0.09    0.19
15.000 0.00300  H=  100.00   90.62   86.23   84.79   87.91
                Q=   -0.57   -0.55   -0.35   -0.19    0.12
16.000 0.00260  H=  100.00   88.78   80.63   77.83   73.65
                Q=   -0.31   -0.28   -0.29   -0.13    0.10
17.000 0.00220  H=  100.00   90.01   80.38   69.49   69.43
                Q=    0.00   -0.05   -0.07   -0.01    0.08
18.000 0.00180  H=  100.00   91.60   78.88   71.98   66.65
                Q=    0.20    0.22    0.23    0.15    0.07
30.000 0.00020  H=  100.00   96.69   93.19   93.90   94.64
                Q=   -0.87   -0.72   -0.53   -0.21    0.01
31.000 0.00016  H=  100.00   90.08   85.27   83.34   85.51
                Q=   -0.64   -0.61   -0.43   -0.28    0.01
32.000 0.00012  H=  100.00   88.59   80.24   76.88   72.20
                Q=   -0.36   -0.34   -0.36   -0.21    0.00
33.000 0.00008  H=  100.00   90.16   80.20   69.10   68.40
                Q=   -0.05   -0.11   -0.13   -0.08    0.00
34.000 0.00004  H=  100.00   91.61   79.03   71.72   66.12
                Q=    0.15    0.16    0.18    0.09    0.00
35.000 0.00000  H=  100.00   88.87   83.13   76.04   75.15
                Q=    0.38    0.43    0.38    0.26    0.00
36.000 0.00000  H=  100.00   91.52   85.88   86.56   86.02
                Q=    0.72    0.60    0.51    0.29    0.00
```

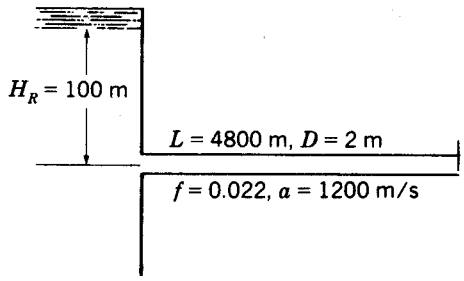


Fig. 19-6

19.10 Determine the period of oscillation of a U-tube containing 333 mL of water. The cross-sectional area is 270 mm^2. Neglect friction.

▮
$$L = V/A = (333 \times 10^{-6})/(270 \times 10^{-6}) = 1.235 \text{ m}$$
$$T = 2\pi\sqrt{L/2g} = (2)(\pi)\sqrt{1.235/[(2)(9.807)]} = 1.58 \text{ s}$$

19.11 A fluid-filled U-tube is oscillating with maximum displacement from an equilibrium position of 0.1 m. The total column length is 0.9 m. Determine the maximum fluid velocity and the period of oscillation. Neglect friction.

▮ $z = Z\cos\sqrt{2g/L}\,t = (0.1)\cos\sqrt{2(9.807)/0.9}\,t = 0.100\cos 4.668t \qquad dz/dt = -0.4668\sin 4.668t$

Maximum velocity = 0.4668 m/s; $T = 2\pi/4.668 = 1.35$ s.

19.12 A U-tube contains liquid oscillating with a velocity of 1.8 m/s at the instant the menisci are at the same elevation. Find the time to the instant the menisci are next at the same elevation and determine the velocity then. Take $\nu = 1 \times 10^{-5}$ m^2/s, $D = 8$ mm, and $L = 800$ mm.

▮
$$N_R = DV/\nu = (0.008)(1.8)/(1 \times 10^{-5}) = 1440 \qquad \text{(laminar flow)}$$
$$m = 16\nu/D^2 = (16)(1 \times 10^{-5})/(0.008)^2 = 2.500$$
$$n = \sqrt{(16\nu/D^2)^2 - 2g/L} = \sqrt{2.500^2 - (2)(9.807)/(0.800)} = 4.274i$$
$$n' = 4.274 \qquad z = (V_0/n')(e^{-mt})\sin(n't)$$

At next $z = 0$, $4.274t = \pi$. $t = 0.735$ s, $V = dz/dt = V_0e^{-mt}[\cos(n't) - (m/n')\sin(n't)]$. When $n't = \pi$, $V = (1.8)[e^{-(2.500)(0.735)}][\cos\pi - (m/n')\sin\pi] = 0.287$ m/s.

19.13 A 9-ft-diameter horizontal tunnel has 9-ft-diameter vertical shafts spaced 1.1 mi apart. When valves are closed isolating this reach of tunnel, the water surges to a depth of 48 ft in one shaft when it is 18 ft in the other shaft. For $f = 0.020$, find the height of the next two surges.

▮
$$z_m = (48 - 18)/2 = 15.00 \text{ ft} \qquad [1 + (fz_m/D)]e^{-fz_m/D} = [1 + (fz_{m+1}/D)]e^{-fz_{m+1}/D}$$
$$fz_m/D = (0.020)(15.00)/9 = 0.0333$$
$$(1 + 0.0333)e^{-0.0333} = [1 + (0.020z_{m+1}/9)]e^{-0.020z_{m+1}/9} \qquad z_{m+1} = 14.99 \text{ ft}$$

Repeat, using $z_m = 14.99$ ft.

$$fz_m/D = (0.020)(14.99)/9 = 0.0333$$
$$(1 + 0.0333)e^{-0.0333} = [1 + (0.020z_{m+1}/9)]e^{-0.020z_{m+1}/9} \qquad z_{m+1} = 14.98 \text{ ft}$$

Hence, $z_1 = 14.99$ ft and $z_2 = 14.98$ ft.

19.14 Two standpipes 5 m in diameter are connected by 965 m of 2.0-m-diameter pipe; $f = 0.020$ and minor losses are 4.2 velocity heads. One reservoir level is 9.6 m above the other one when a valve is rapidly opened in the pipeline. Find the maximum fluctuation in water level in the standpipe.

▮ $L_e = L + KD/f = 965 + (4.2)(2.0)/0.020 = 1385$ m $\qquad z_m = z_1(A_1/A) = (\frac{9.6}{2})(5/2.0)^2 = 30.00$ m
$$\phi = (f)(L_e/L)(z_m/D) = (0.020)(\tfrac{1385}{965})(30.00/2.0) = 0.4306$$

$F(\phi) = (1 + \phi)e^{-\phi} = 0.954$, which is satisfied by $\phi = -0.275$.

$$z_m = \phi LD/fL_e = (-0.275)(965)(2.0)/[(0.020)(1385)] = -19.16 \text{ m}$$
$$z_1 = z_m(A/A_1) = (-19.16)(2.0/5)^2 = -3.07 \text{ m}$$
$$\text{Maximum fluctuation} = \tfrac{9.6}{2} - (-3.07) = 7.87 \text{ m}$$

19.15 A valve is quickly opened in a pipe 1270 m long, $D = 0.7$ m, with a 0.4-m-diameter nozzle on the downstream end. Minor losses are $3V^2/2g$, with V the velocity in the pipe, $f = 0.024$, and $H = 8$ m. Find the time to attain 95 percent of the steady-state discharge.

▮ $t = (LV/2gH)\ln[(V_0 + V)/(V_0 - V)]$ $\quad H = (fL_e/D)(V^2/2g) + V_1^2/2g$ $\quad (V_1 = \text{nozzle velocity})$

$$L_e = L + KD/f = 1270 + (3)(0.7)/0.024 = 1358 \text{ m} \qquad V_1/V = (0.7/0.4)^2 \qquad V_1 = 3.062V$$

$$8 = [(0.024)(1358)/0.7]\{V^2/[(2)(9.807)]\} + (3.062V)^2/[(2)(9.807)] \qquad V = 1.675 \text{ m/s}$$

$$t = \{(1270)(1.675)/[(2)(9.807)(8)]\}\ln(1.95/0.05) = 49.7 \text{ s}$$

19.16 A globe valve ($K = 10$) at the end of a pipe 2400 ft long is rapidly opened. $D = 3.5$ ft, $f = 0.019$, minor losses are $2V^2/2g$, and $H = 85$ ft. How long does it take for the discharge to attain 85 percent of its steady-state value?

$$t = (LV/2gH)\ln[(V_0 + V)/(V_0 - V)] \qquad V = \sqrt{2gHD/fL_e}$$

$$L_e = L + KD/f = 2400 + (10 + 2)(3.5)/0.019 = 4611 \text{ ft}$$

$$V = \sqrt{(2)(32.2)(85)(3.5)/[(0.019)(4611)]} = 14.79 \text{ ft/s}$$

$$t = \{(2400)(14.79)/[(2)(32.2)(85)]\}\ln(1.85/0.15) = 16.3 \text{ s}$$

19.17 Benzene ($K = 150\,000$ psi, s.g. $= 0.88$) flows through 1-in-ID steel ($E = 30\,000\,000$ psi) tubing with $\frac{3}{16}$-in wall thickness. Determine the speed of a pressure wave.

▮ $$a = \sqrt{(K/\rho)/[1 + (K/E)(D/e)]}$$

$$= \sqrt{\{[(150\,000)(144)]/[(0.88)(1.94)]\}/\{1 + (150\,000/30\,000\,000)[(1)/(\tfrac{3}{16})]\}} = 3511 \text{ ft/s}$$

19.18 A valve is closed in 4.5 s at the downstream end of a 3200-m pipeline carrying water at 2.7 m/s ($a = 1000$ m/s). What is the peak pressure developed by the closure?

▮ $2L/a = (2)(3200)/1000 = 6.40$ s. Since $[2L/a = 6.40 \text{ s}] > 4$ s, this is rapid closure. Then, $h = aV_0/g = (1000)(2.7)/9.807 = 275.3$ m, $p = \gamma h = (9.79)(275.3) = 2695$ kPa.

19.19 Determine the length of pipe in Prob. 19.18 subject to the peak pressure.

▮ $$x = L - at_c/2 = 3200 - (1000)(4.5)/2 = 950 \text{ m}$$

19.20 A valve is closed at the downstream end of a pipeline in such a manner that only one-fourth of the line is subjected to maximum pressure. During what proportion of the time $2L/a$ is it closed?

▮ $$x = L - at_c/2 \qquad L/4 = L - at_c/2 \qquad 3L/4 = at_c/2 \qquad t_c/(2L/a) = \tfrac{3}{4}$$

19.21 A 10-mm-diameter U-tube contains oil, $\nu = 5$ m²/s, with a total column length of 2 m. If the initial half-amplitude of displacement is 250 mm, prepare a computer program to find the first 10 maximum and minimum displacements and the times they occur.

▮
```
10 REM                          LAMINAR OSCILLATION OF A U-TUBE.
20 '  TO=ATN(N'/M)/N'
30 '  VO=Z*N'/(EXP(-M*TO)*SIN(N'*TO))
40 ' The angle THO is THO=N'*TO, and the
50 ' general angle is TH=THO+I*PI, with T=TH/N'-TO.
60 '  Z=VO/N'*EXP(-M*TH/N')*SIN(TH)
70 DEFINT I: DEF FNZ(DT,DTH)=C1*EXP(-M*DT)*SIN(DTH)
80 READ D,NU,L,G,DZ,PI: DATA .01,5E-6,2.,9.806,.25,3.1416
90 LPRINT: LPRINT"D,NU,L,G,DZ,PI=";D;NU;L;G;DZ;PI: LPRINT
100 M=16!*NU/D^2:CC=2!*G/L-M^2:IF CC>0! THEN NP=SQR(CC) ELSE
    LPRINT"NON-OSCILLATING CASE":STOP
110 TO=ATN(NP/M)/NP:VO=NP*DZ*EXP(M*TO)/SIN(NP*TO):THO=NP*TO
120 LPRINT"M,NP,TO,VO,THO=";M;NP;TO;VO;THO: LPRINT
130 C1=VO/NP: FOR I=0 TO 10:TH=THO+I*PI:T=TH/NP:Z=FNZ(T,TH)
140 LPRINT"T,Z,TH=";:LPRINT USING " ####.##### ";T-TO;Z;TH
150 NEXT I
160 LPRINT: LPRINT"MAXIMUM REYNOLD'S NUMBER,V*D/NU=";VO*D/NU
```

```
D,NU,L,G,DZ,PI= .01   .000005   2   9.806001   .25   3.1416

M,NP,TO,VO,THO= .8   3.02754   .4335071   1.107393   1.31246
```

```
T,Z,TH=     0.00000     0.25000     1.31246
T,Z,TH=     1.03767    -0.10900     4.45406
T,Z,TH=     2.07535     0.04752     7.59566
T,Z,TH=     3.11302    -0.02072    10.73726
T,Z,TH=     4.15070     0.00903    13.87886
T,Z,TH=     5.18837    -0.00394    17.02046
T,Z,TH=     6.22604     0.00172    20.16206
T,Z,TH=     7.26372    -0.00075    23.30366
T,Z,TH=     8.30139     0.00033    26.44526
T,Z,TH=     9.33907    -0.00014    29.58686
T,Z,TH=    10.37674     0.00006    32.72846

MAXIMUM REYNOLD'S NUMBER,V*D/NU= 2214.786
```

19.22 An equation for dealing with turbulent resistance is given by

$$\frac{d^2z}{dt^2}+\frac{f}{2D}\frac{dz}{dt}\left|\frac{dz}{dt}\right|+\frac{2g}{L}=0$$

Put this equation in suitable form for solution by the third-order Runge-Kutta method.

▮ Let $y = dz/dt$, then $dy/dt = d^2z/dt^2$, and $dy/dt = -(f/2D)y\,|y| - (2g/L)z = F_1(y, z, t)$, $dz/dt = y = F_2(y, z, t)$. The two equations are solved simultaneously, from known initial conditions y_n, z_n, t_n: $u_{11} = hF_1(y_n, z_n, t_n) = h[-(f/2D)y_n\,|y_n| - (2g/L)z_n]$, $u_{12} = hF_2(y_n, z_n, t_n) = hy_n$, $u_{21} = hF_1[y_n + (u_{11}/3), z_n + (u_{12}/3), t_n + (h/3)] = h\{-(f/2D)[y_n + (u_{11}/3)]\,|y_n + (u_{11}/3)| - (2g/L)[z_n + (u_{12}/3)]\}$, $u_{22} = hF_2[y_n + (u_{11}/3), z_n + (u_{12}/3), t_n + (h/3)] = h[y_n + (u_{11}/3)]$, $u_{31} = hF_1(y_n + \frac{2}{3}u_{21}, z_n + \frac{2}{3}u_{22}, t_n + \frac{2}{3}h) = h[-(f/2D)(y_n + \frac{2}{3}u_{21})\,|y_n + \frac{2}{3}u_{21}| - (2g/L)(z_n + \frac{2}{3}u_{22})]$, $u_{32} = hF_2(y_n + \frac{2}{3}u_{21}, z_n + \frac{2}{3}u_{22}, t_n + \frac{2}{3}h) = h(y_n + \frac{2}{3}u_{21})$, $y_{n+1} = y_n + (u_{11}/4) + \frac{3}{4}u_{31}$, $z_{n+1} = z_n + (u_{12}/4) + \frac{3}{4}u_{32}$, $t_{n+1} = t_n + h$. The equations for simultaneous solution have been written for a general case as well as for the specific case of solution of the given equation.

19.23 Prepare a program to carry out the solution of Prob. 19.22 and apply it to the following case: $t = 0$, $z = 12$ ft, $V_0 = 0$, $L = 400$ ft, $f = 0.017$, $d = 2$ ft, $dt = 0.1$ s, and $t_{max} = 30$ s.

▮

```
10 REM                          TURBULENT OSCILLATION OF A U-TUBE
20 DEFINT I: DEF FNN(DY,DZ)=H*(-C1*DY*ABS(DY)-C2*DZ): DEF FNM(DY)=H*DY
30 READ Z,V,L,F,D,DT,TMAX,IPR,G: DATA 12.,.0,400.,.017,2.5,.1,30.,20,32.2
40 LPRINT: LPRINT"Z,V,L,F,D=";Z;V;L;F;D: LPRINT"DT,TMAX,IPR,G=";DT,TMAX,IPR,G
50 II=FIX(TMAX/DT)+1: H=DT: C1=.5*F/D: C2=2!*G/L: Y=0!: T=0!
60 LPRINT: LPRINT"        T           V           Z"
70 FOR I=0 TO II: IF I MOD IPR=0 THEN LPRINT USING "  ###.### ";T;Y;Z
80 U11=FNN(Y,Z):U12=FNM(Y)
90 U21=FNN(Y+U11/3!,Z+U12/3!):U22=FNM(Y+U11/3!)
100 U31=FNN(Y+2!*U21/3,Z+2!*U22/3!):U32=FNM(Y+2!*U21/3!)
110 Y=Y+.25*U11+.75*U31:Z=Z+.25*U12+.75*U32:T=T+H
120 NEXT I

Z,V,L,F,D= 12   0   400   .017   2.5
DT,TMAX,IPR,G= .1              30              20              32.2

      T           V           Z
   0.000       0.000      12.000
   2.000      -3.434       8.354
   4.000      -4.681      -0.240
   6.000      -3.083      -8.433
   8.000       0.308     -11.355
  10.000       3.459      -7.356
  12.000       4.418       0.982
  14.000       2.702       8.495
  16.000      -0.585      10.723
  18.000      -3.466       6.425
  20.000      -4.162      -1.647
  22.000      -2.345      -8.513
  24.000       0.834     -10.104
  26.000       3.458      -5.552
  28.000       3.911       2.244
  30.000       2.011       8.492
```

19.24 A surge tank of area AR is to be used with a hydroelectric power plant. The turbines may change from full flow to zero flow in TC seconds, and similarly accept full flow in TC seconds. Assuming linear changes in flow to the

turbines, determine the elevation of maximum and minimum water surfaces in the surge tank. The maximum flow is QM; elevation of reservoir is Z. Prepare a program using the Runge–Kutta second-order integration of the equations. Specifically, let $Z = 100$ m, $QM = 20$ m^3/s, $AR = 75$ m^2, and a pressure pipe $L = 10$ km, $D = 3$ m, $F = 0.016$, with $TC = 4$ s and $DT = H = 1$ s.

```
10 REM                          SIMPLE SURGE TANK.
20 ' Continuity equation: DY/DT=(QIN-QOUT)/AR
30 ' Eq.Motion: GAM*AR*(Z-Y-R*Q*ABS(Q))=GAM*AR*(L/G)*DQ/DT
40 ' or DQ/DT=G*AP*(Z-Y-R*Q*ABS(Q))/L: in which Y=elev of water surface in
50 ' surge tank; AP=area of pressure pipe; R=head loss in pressure pipe
60 ' per unit discharge.
70 DEFINT I: DEF FNQ(DY,DQ)=C1*(Z-DY-R*DQ*ABS(DQ))
80 READ F,L,D,G,Z,AR,QM,TC,H,TM,IPR,ICLOSE
90 DATA .016,10000.,3.,9.806,100.,75.,20.,4.,1.,900.,60,0
100 LPRINT: LPRINT"F,L,D,G,Z=";F;L;D;G;Z: LPRINT
110 LPRINT"AR,QM,TC,H,TM,IPR,ICLOSE=";AR;QM;TC;H;TM;IPR;ICLOSE: LPRINT
120 AP=.7854*D^2: R=F*L/(2!*G*.7854^2*D^5): C1=G*AP*H/L: C2=H/AR: II=FIX(TM/H)+1

130 T=0!: IF ICLOSE=1 THEN Y=Z-R*QM^2: QT=QM: Q=QM ELSE Y=Z: QT=0!: Q=0!
140 LPRINT"     TIME      DISCHARGE       Y          QT"
150 YMAX=Z: YMIN=Z: LPRINT USING"  ###.###   ";T;Q;Y;QT: IF ICLOSE=0 THEN 270
160 FOR I=0 TO II:T=I*H:IF T<=TC THEN QT=QM*(1!-T/TC) ELSE QT=0!
170 U11=FNQ(Y,Q):U12=C2*(Q-QT):TT=T+H
180 IF TT<=TC THEN QT=QM*(1!-TT/TC) ELSE QT=0!
190 U21=FNQ(Y+U12,Q+U11):U22=C2*(Q+U11-QT)
200 Q=Q+.5*(U11+U21):Y=Y+.5*(U12+U22):T=T+H
210 IF Y>YMAX THEN YMAX=Y:TMAX=T
220 IF Y<YMIN THEN YMIN=Y:TMIN=T
230 IF (I+1) MOD IPR=0 THEN LPRINT USING" ####.###   ";T;Q;Y;QT
240 NEXT I
250 GOTO 360
260 END
270 FOR I=1 TO II:T=I*H:IF T<=TC THEN QT=QM*T/TC ELSE QT=QM
280 U11=FNQ(Y,Q):U12=C2*(Q-QT):TT=T+H
290 IF TT<=TC THEN QT=QM*TT/TC ELSE QT=QM
300 U21=FNQ(Y+U12,Q+U11):U22=C2*(Q+U11-QT)
310 Q=Q+.5*(U11+U21):Y=Y+.5*(U12+U22):T=T+H
320 IF Y>YMAX THEN YMAX=Y:TMAX=T
330 IF Y<YMIN THEN YMIN=Y:TMIN=T
340 IF (I+1) MOD IPR=0 THEN LPRINT USING" ####.###   ";T;Q;Y;QT
350 NEXT I
360 LPRINT"YMAX,TMAX=";:LPRINT USING"  ####.##   ";YMAX;TMAX
370 LPRINT"YMIN,TMIN=";:LPRINT USING"  ####.##   ";YMIN;TMIN
```

```
F,L,D,G,Z= .016  10000  3  9.806001  100

AR,QM,TC,H,TM,IPR,ICLOSE= 75  20  4  1  900  60  0

     TIME      DISCHARGE       Y          QT
    0.000        0.000     100.000      0.000
   60.000        2.978      85.346     20.000
  120.000       10.451      74.568     20.000
  180.000       17.524      69.934     20.000
  240.000       21.355      69.706     20.000
  300.000       22.561      71.399     20.000
  360.000       22.479      73.466     20.000
  420.000       21.944      75.248     20.000
  480.000       21.346      76.561     20.000
  540.000       20.839      77.427     20.000
  600.000       20.466      77.940     20.000
  660.000       20.220      78.206     20.000
  720.000       20.075      78.318     20.000
  780.000       20.000      78.345     20.000
  840.000       19.970      78.331     20.000
  900.000       19.964      78.303     20.000
YMAX,TMAX=     100.00        0.00
YMIN,TMIN=      69.44      212.00
```

19.25 The open wedge-shaped tank in Fig. 19-7 has a length of 15 ft perpendicular to the sketch. It is drained with a 3-in-diameter pipe of length 10 ft whose discharge end is at elevation zero. The coefficient of loss at the pipe

entrance is 0.50, the total of the bend loss coefficient is 0.20, and f for the pipe is 0.018. Find the time required to lower the water surface in the tank from elevation 8 ft to 5 ft. Neglect the possible change of f with N_R, and assume that the acceleration effects in the pipe are negligible.

▮ Energy equation from water surface to jet at discharge: $z - [0.5 + 0.2 + 0.018(10/0.25)](V^2/2g) = V^2/2g$, $z - 1.42(V^2/2g) = V^2/2g$, $V = 5.16z^{1/2}$; $Q_0 = AV = (\pi/4)(0.25)^2 5.16z^{1/2} = 0.253z^{1/2}$. The area of the water surface may be expressed as $A_s = 15b = 15Kz$. At the top of the tank, $A_s = 15(6) = 15K(10)$, $K = 0.6$. Thus $A_s = 15(0.6)z = 9z$.

$$t = \int_{z_1}^{z_2} \frac{A_s\,dz}{Q_i - Q_0} = \int_8^5 \frac{9z\,dz}{0 - 0.253z^{1/2}} = -\frac{9}{0.253}\int_8^5 z^{1/2}\,dz = -35.5[\tfrac{2}{3}z^{3/2}]_8^5 = 271\text{ s}$$

Note that if the pipe had discharged at an elevation other than zero, the integral would have been different, because the head on the pipe would then have been $z + h$, where h is the vertical distance of the discharge end of the pipe below (h positive) or above (h negative) point A of the figure.

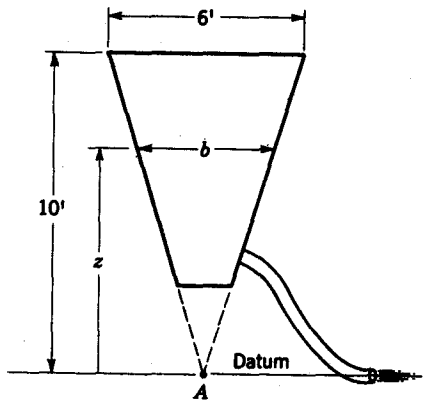


Fig. 19-7

19.26 Two large water reservoirs are connected to one another with a 10-cm-diameter pipe ($f = 0.02$) of length 15 m. The water-surface elevation difference between the reservoirs is 2.0 m. A valve in the pipe, initially closed, is suddenly opened. Determine the times required for the flow to reach $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$ of the steady-state flow rate. Assume the water-surface elevations remain constant. Repeat for pipe lengths of 150 m and 1500 m with all other data remaining the same. In the first case $L/D = 15/0.10 = 150$, hence minor losses are significant. Assume square-edged entrance.

▮ Square-edge entrance: $k_L = 0.5$, $k = k_L + f(L/D) = 0.5 + 0.02(15/0.10) = 3.5$. For steady flow: $V_0 = \sqrt{2gH/(1+k)} = \sqrt{2(9.81)_2/(1+3.5)} = 2.95$ m/s. For unsteady flow use $t = \{L/[(1+k)V_0]\} \ln[(V_0 + V)/(V_0 - V)] = [15/4.5(2.95)] \ln[(2.95 + V)/(2.95 - V)] = 1.129 \ln[(2.95 + V)/(2.95 - V)]$. For $Q = \frac{1}{4}Q_0$ substitute $V = \frac{1}{4}V_0$, etc.:

Q	V, m/s	$\dfrac{2.95 + V}{2.95 - V}$	ln	t, s
$0.25Q_0$	0.74	1.667	0.511	0.577
$0.50Q_0$	1.48	3.00	1.099	1.240
$0.75Q_0$	2.21	7.00	1.946	2.197

For the other two lengths the results are as follows:

Q	$L = 150$ m	$L = 1500$ m
$0.25Q_0$	2.18 s	7.04 s
$0.50Q_0$	4.69 s	15.15 s
$0.75Q_0$	8.30 s	26.84 s

19.27 In Fig. 19-8 the elasticity and dimensions of the pipe are such that the celerity of the pressure wave is 3200 fps. Suppose the pipe has a length of 2000 ft and a diameter of 4 ft. The flow rate is initially 30 cfs. Water is flowing.

Find (*a*) the water-hammer pressure for instantaneous valve closure; (*b*) the approximate water-hammer pressure at the valve if it is closed in 4.0 s; (*c*) the water-hammer pressure at the valve if it is manipulated so that the flow rate drops almost instantly from 30 to 10 cfs; (*d*) the maximum water-hammer pressure at a point in the pipe 300 ft from the reservoir if a 1.0-s valve closure reduces the flow rate from 10 cfs to zero.

▮ $$V = Q/A = 30/(\pi 2^2) = 2.39 \text{ fps}$$

(*a*) $$p_h = \rho c_P V = 1.94(3200)(2.39) = 14\,840 \text{ lb/ft}^2 = 103.0 \text{ psi}$$

(*b*) $$p_h' \approx (\tfrac{4000}{3200}/4.0)p_h = (1.25/4.00)(103.0) = 32.2 \text{ psi}$$

(*c*) For this case of partial closure $\Delta p_h = -\rho c_P(\Delta V)$. $\Delta V = (10-30)/\pi 2^2 = -1.592$ fps, $\Delta p_h = -1.94(3200)(-1.592) = 9880 \text{ lb/ft}^2 = 68.6$ psi.

(*d*) If $2x_0/c_P = 1.0$ s, $x_0 = 1600$ ft, so that full water-hammer pressure will be developed in the pipe only in the region that is farther than 1600 ft from the reservoir.

For this cases, at valve, $p_h = 1.94(3200)(2.39/3) = 4940 \text{ lb/ft}^2 = 34.3$ psi. At point 300 ft from reservoir: $p_h = 34.3(\tfrac{300}{1600}) = 6.43$ psi.

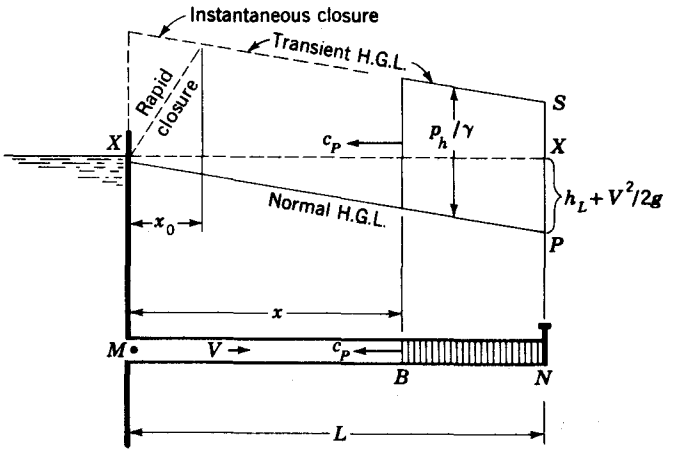


Fig. 19-8

19.28 Suppose a ship lock has vertical sides and that water enters or discharges through a conduit area A such that $Q = C_d A\sqrt{2gz}$, where z is the variable difference in level between the water surface in the lock and that outside. Prove that for the water level in the lock to change from z_1 to z_2 the time is $t = [2A_s/(C_d A\sqrt{2g})](z_1^{1/2} - z_2^{1/2})$. (*Note:* If the lock is being filled, the signs must be reversed.)

▮ $$t = \int_{z_1}^{z_2} \frac{A_s\,dz}{Q_i - Q_o} \qquad Q_i = 0 \qquad Q_o = C_d A(2gz)^{1/2}$$

thus

$$t = \int_{z_1}^{z_2} \frac{A_s\,dz}{-C_d A(2gz)^{1/2}} = \frac{A_s}{C_d A(2g)^{1/2}} \int_{z_1}^{z_2} \frac{dz}{-z^{1/2}} = \frac{A_s}{C_d A(2g)^{1/2}}[-2z^{1/2}]_{z_1}^{z_2} = \frac{2A_s}{C_d A(2g)^{1/2}}(z_1^{1/2} - z_2^{1/2})$$

19.29 Suppose the lock of Prob. 19.28 is 285 ft long by 100 ft wide, and water enters through a conduit for which the discharge coefficient is 0.52. If the water surface in the lock is initially 29 ft below the level of the surface of the water upstream, how large must the conduit be if the lock is to be filled in 6 min?

▮ $t = [2A_s/(C_d A\sqrt{2g})](z_1^{1/2} - z_2^{1/2})$ $\qquad (6)(60) = \{(2)[(285)(100)]/[(0.52)(A)\sqrt{(2)(32.2)}]\}(29^{1/2} - 0^{1/2})$

$$A = 204 \text{ ft}^2$$

19.30 (*a*) Suppose a reservoir has vertical sides and that initially there is a steady flow into it such that the height of the surface above the level of a spillway ($Q = C_W L H^{3/2}$) is z_1. If the inflow is suddenly cut off, prove that the time required for the water level to fall from z_1 to z_2 is $t = (2A_s/C_W L)(1/\sqrt{z_2} - 1/\sqrt{z_1})$. (*Note:* $z = H$.) (*b*) How long will it take theoretically for the outflow to cease entirely? What factors make this theoretical answer unrealistic?

▮ (*a*) $$t = \int_{z_1}^{z_2} \frac{A_s\,dz}{Q_i - Q_o} \qquad Q_i = 0 \qquad Q_o = C_W L z^{3/2}$$

$$= \int_{z_1}^{z_2} \frac{A_s\,dz}{-C_W L z^{3/2}} = \frac{A_s}{C_W L}\left[\frac{2}{z^{1/2}}\right]_{z_1}^{z_2} = \frac{2A_s}{C_W L}\left[\frac{1}{z_2^{1/2}} - \frac{1}{z_1^{1/2}}\right]$$

(*b*) As $z_2 \to 0$, $t \to \infty$, so theoretically the outflow would never cease. Factors which make this theoretical answer unrealistic include surface tension, surface ripples, evaporation, and leakage/seepage losses.

19.31 The crest of the overflow of a reservoir is 82 ft long, and the value of C_W is 3.45. For the range of levels considered here the area of the water surface is constant and is 600 000 ft^2. Initially, there is a flow into the reservoir at such a rate that the height of the water surface above that of the spillway crest is stabilized at 4 ft, and then the inflow is suddenly diverted. Find the length of time for the water surface to fall to a height of 2 ft above the level of the spillway.

▮

$$t = \int_{z_1}^{z_2} \frac{A_s\,dz}{Q_i - Q_o} \qquad Q_i = 0 \qquad Q_o = C_W L z^{3/2}$$

$$= \int_{z_1}^{z_2} \frac{A_s\,dz}{-C_W L z^{3/2}} = \frac{2A_s}{C_W L}\left[\frac{1}{z_2^{1/2}} - \frac{1}{z_1^{1/2}}\right]$$

$$= \frac{2(600\,000)}{3.45(82)}\left[\frac{1}{2^{1/2}} - \frac{1}{4^{1/2}}\right] = 879\text{ s} = 14.6\text{ min}$$

19.32 The crest of the overflow spillway of a reservoir is 46 ft long, and the value of C_W is 3.50. The area of the water surface is assumed constant at 700 000 ft^2 for the range of heights considered here. Initially, the water surface is 4 ft below the level of the spillway crest. If suddenly there is turned into this reservoir a flow of 560 cfs, what will be the height of water in the reservoir for equilibrium? How long a time will be required for this height to be reached? How long a time will be required for the water surface to reach a height of 1 ft above the level of the spillway? (*Note:* This last can be solved by integration after substituting x^3 for $z^{3/2}$ and consulting integral tables. However, it will be easier to solve it graphically either by plotting and actually measuring the area under the curve or by computing the latter by some method, such as Simpson's rule.)

▮ $Q = C_W L z^{3/2}$, $Q_o = 3.50(46)z^{3/2} = 161z^{3/2}$. For equilibrium $Q_i = Q_o = 560$ cfs, from which $z = (\frac{560}{161})^{2/3} = 2.30$ ft. The time required to reach this equilibrium height is theoretically infinite, but surface ripples and minor irregularities in flow cause practical equilibrium to be reached in a finite time. However, this does point out that true equilibrium is not quickly obtained. This should be borne in mind in experimental work. The time for the water level to reach the spillway crest = 4(700 000)/560 = 5000 s. The time for water level to rise from crest to 2 ft above it is

$$t = \int_{z_1}^{z_2} \frac{A_s\,dz}{Q_i - Q_o} = 700\,000\int_0^1 \frac{dz}{560 - 161z^{3/2}}$$

Letting $z^{3/2} = x^3$, $(\frac{3}{2})z^{1/2}\,dz = 3x^2\,dx$ and $dz = (3x^2\,dx)/[(\frac{3}{2})z^{1/2}] = (3x^2\,dx)/[(\frac{3}{2})x] = 2x\,dx$.

$$t = 2(700\,000)\int_0^{1^{1/2}} \frac{x\,dx}{560 - 161x^3} = 1\,400\,000\,\frac{(-\frac{560}{161})^{1/3}}{3(560)}\left[\frac{1}{2}\ln\frac{[x + (-\frac{560}{161})^{1/3}]^2}{(-\frac{560}{161})^{2/3} - (-\frac{560}{161})^{1/3}x + x^2} + 3^{1/2}\tan^{-1}\frac{2x - (-\frac{560}{161})^{1/3}}{(-\frac{560}{161})^{1/3}3^{1/2}}\right]_0^{1^{1/2}}$$

$$= 1\,400\,000\left(\frac{-1.515}{1680}\right)\left[\frac{1}{2}\ln\frac{(x - 1.515)^2}{2.30 + 1.515x + x^2} + 3^{1/2}\tan^{-1}\frac{2x + 1.515}{-1.515(3)^{1/2}}\right]_0^{1^{1/2}} = 3863\text{ s}$$

$$\text{Total time} = 5000 + 3863 = 8863\text{ s}$$

19.33 Water enters a reservoir at such a rate that the height of water above the level of the spillway crest is 2.5 ft. The spillway ($Q = C_W L H^{3/2}$) is 80 ft long, and the value of C_W is 3.45. The area of the water surface for various water levels is as follows:

z, ft	A_s, ft^2
2.50	730 000
2.00	620 000
1.50	490 000
1.25	435 000
1.00	380 000

If the inflow is suddenly reduced to 150 cfs, what will be the height of water for equilibrium? How long will it take, theoretically, for equilibrium to be attained? How long will it take for the level to drop from 2.5 ft to 1 ft above that of the spillway?

▮ $Q_o = 276z^{3/2}$; when $Q_o = Q_i = 150$ cfs, $z = 0.666$ ft and the time for equilibrium is theoretically infinite.

$$t = \int_{z_1}^{z_2} \frac{A_s\,dz}{Q_i - Q_o} = \int \frac{A_s\,dz}{276z^{3/2} - 150} \qquad (z \text{ gets smaller})$$

z, ft	Δz, ft	A_s, ft^2	$\bar{A}_s$, ft^2	$\bar{A}_s(\Delta z)$, ft^3	$Q_o = 276z^{3/2}$, cfs	$\bar{Q}_o$, cfs	$\bar{Q}_o - 150$, cfs	$\dfrac{\bar{A}_s(\Delta z)}{\bar{Q}_o - 150}$
2.5		730 000			1091			
	0.5		675 000	337 500		936	786	429
2.0		620 000			781			
	0.5		555 000	277 500		644	494	562
1.5		490 000			507			
	0.25		463 000	115 750		446	296	391
1.25		435 000			386			
	0.25		408 000	102 000		331	181	564
1.00		380 000			276			
								$\Sigma = 1946$ s

Total time for z to change from 2.5 ft to 1.0 ft is approximately 1946 s = 32.4 min.

19.34 Work Prob. 19.33 using the same numbers but changing feet to meters, square feet to square meters, and cubic feet per second to cubic meters per second.

▮ This problem is solved in the same manner as Prob. 19.33. $Q_o = 1.84(3.45/3.33)LH^{3/2} = 1.906(80)z^{3/2} = 152.5z^{3/2}$. When $Q_o = Q_i = 150$ m^3/s, $z = (150/152.5)^{2/3} = 0.989$ m and the time for equilibrium is theoretically infinite. Columns 2 to 5 are omitted from the following solution table since they are numerically the same as those in Prob. 19.33.

z, m	$Q_o = 152.5z^{3/2}$, m^3/s	$\bar{Q}_o$, m^3/s	$\bar{Q}_o - 150$, m^3/s	$\dfrac{A_s(\Delta z)}{\bar{Q}_o - 150}$
2.5	603			
		517	367	920
2.0	431			
		356	206	1347
1.5	280			
		246	96	1206
1.25	213			
		182	32	3188
1.00	152			
				$\Sigma = 6661$ s

Total time for z to change from 2.5 m to 1.0 m is approximately 6661 s = 111.0 min.

19.35 Figure 19-9 shows a tank whose shape is the frustum of a cone with a 1.5-ft^2 orifice in the bottom. Assume $C_d = 0.62$. If the water level outside the tank is constant at section 2, how long will it take the water level in the tank to drop from section 1 to section 2? (*Note:* Diameter of tank = Ky, and $y = z + h_2$, where z is the variable distance between surface levels.)

▮ $D = ky$; $k = (39.6 - 18)/34 = 0.635$ $A_s = (\pi/9.92)y^2$; $y = z + h_2 = z + 28.35$; $Q_o = C_dA(2gz)^{1/2} = 0.62(1.5)(2gz)^{1/2}$.

$$t = \int_{z_1}^{z_2} \frac{A_s\,dz}{Q_i - Q_o} = \int_{34}^{0} \frac{(\pi/9.92)(z + 28.35)^2\,dz}{-(0.62)1.5(2gz)^{1/2}} = 830 \text{ s}$$

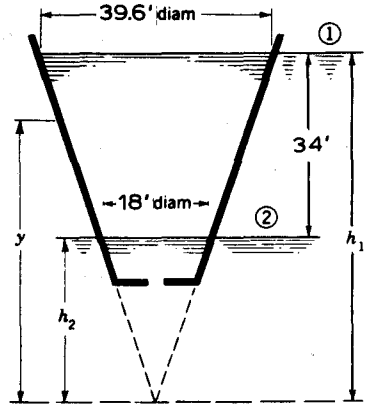


Fig. 19-9

19.36 If in Fig. 19-9 the water surface outside the tank is constant at section 1 and the tank is initially empty, how long will it take for the water level in the tank to rise from section 2 to section 1? Assume a 1.5-ft^2 orifice with $C_d = 0.62$.

▮ $z = h_1 = y$; $y = h_1 - z = 62.35 - z$; $Q_o = C_d A(2gz)^{1/2} = 0.62(1.5)(2gz)^{1/2}$.

$$t = \int_{z_1}^{z_2} \frac{A_s\,dz}{Q_i - Q_o} = \int_{34}^{0} \frac{(\pi/9.92)(62.35 - z)^2(-dz)}{(0.62)1.5(2gz)^{1/2}} = 1339 \text{ s}$$

19.37 A 1-in-diameter smooth brass pipe 900 ft long drains an open cylindrical tank which contains oil having $\rho = 1.8$ slugs/ft^3, $\mu = 0.0006$ lb-s/ft^2. The pipe discharges at elevation 80 ft. Find the time required for the oil level to drop from elevation 100 ft to elevation 88 ft if the tank is 5 ft in diameter.

▮ Check if flow is always laminar. When $V = V_{max}$, the elevation of oil in the tank is at 100 ft, then $100 - f(L/D)(V^2/2g) = 80$; i.e., $20 = f(L/D)(V^2/2g)$. Assuming laminar flow: $f = 64/N_R$, $N_R = DV\rho/\mu$. Substituting: $20 = (64\mu/DV\rho)(L/D)(V^2/2g)$, $20 = \{64(0.0006)/[(\frac{1}{12})V(1.8)]\}(900/\frac{1}{12})\{V^2/[2(32.2)]\} = 42.9V$, $V = V_{max} = 0.466$ fps; $N_{Rmax} = (\frac{1}{12})(0.466)(1.8)/0.0006 = 116.5$, $N_{Rmax} < 2000$, so the flow is always laminar. In general, from the above: elev $- 80 = 42.9V$, $V = (\text{elev} - 80)/42.9 = h/42.9$ (defining $h = \text{elev} - 80$); $Q = AV = 0.00545(h/42.9)$; $A_s\,dz = Q_i\,dt - Q_o\,dt$, $Q\,dt = -A\,dh$, $dt = -6.25\pi\,dh/[0.00545(h/42.9)] = -154\,558\,dh/h$.

$$\int_0^t dt = -154\,558\int_{20}^{8} \frac{dh}{h} = -154\,558[\ln h]_{20}^{8} \qquad t = -154\,558(2.08 - 3.00) = 141\,620 \text{ s} = 39.3 \text{ h}$$

19.38 A large reservoir is being drained with a pipe system as shown in Fig. 19-10. Initially, when the pump is rotating at 190 rpm, the flow rate is 6.0 cfs. If the pump speed is increased instantaneously to 240 rpm, determine the flow rate as a function of time. Assume that the head h_p developed by the pump is proportional to the square of the rotative speed; that is, $h_p \propto n^2$.

▮ Initial condition neglecting minor losses: $z - h_{L_f} + h_p = V^2/2g$, $37 - 0.022(\frac{2900}{1})V^2/2g + h_p = V^2/2g$, where $V = Q/A = 6.0/0.785 = 7.64$ fps, from which $h_p = 21.7$ ft; $h_p' = 21.7(\frac{240}{190})^2 = 34.6$ ft.

New conditions: $37 - 63.8(V^2/2g) + 34.6 = V^2/2g + (L/g)(dV/dt)$. Eventually $dV/dt = 0$, $V^2/2g = 71.6/64.8 = 1.105$ ft, $V = 8.44$ fps when steady flow is eventually achieved.

In general: $71.6 - 64.8(V^2/2g) = (2900/32.2)(dV/dt)$, $dt = 89.6\,dV/(71.2 - V^2)$. Integrate the left side from zero to t; integrate the right side from 7.64 to V_t. Select various V_t's and find the corresponding t's:

$$\int dt = \frac{89.6}{16.88}\left[\ln\frac{8.44 + V}{8.44 - V}\right]_{7.64}^{V_t} = 5.31 \ln\frac{8.44 + V_t}{8.44 - V_t} - 15.93$$

V_t, fps	t, s	Q, cfs
7.64	0	6.00
7.90	2.176	6.20
8.10	4.70	6.36
8.25	7.84	6.48
8.40	16.16	6.60
8.42	19.84	6.61
8.44	∞	6.63

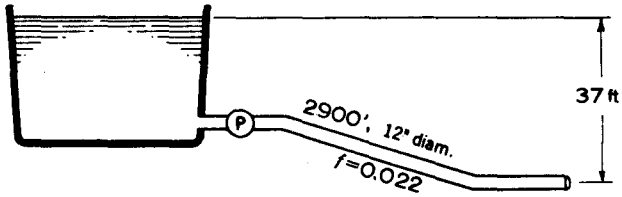


Fig. 19-10

19.39 (*a*) Repeat Prob. 19.38 with all data the same except use a 24-in-diameter pipe rather than a 12-in pipe. (*b*) Repeat also for the case of a 10-inch-diameter pipe.

▮ (*a*) **24-in pipe:** Initial conditions: $37 - 0.022(\frac{2900}{2})(V^2/2g) + h_p = V^2/2g$, $V = 6.0/3.142 = 1.91$ fps; $h_p = 32.9(V^2/2g) - 37 = -35.1$ ft. Thus a pump is not required; actually a head of 35.1 ft is available to drive a turbine when the flow rate is 6.0 cfs.

(*b*) **10-in pipe:** Initial conditions: $37 - 0.022(2900/0.833)(V^2/2g) + h_p = V^2/2g$ where $V = Q/A = 6.0/[\pi(\frac{5}{12})^2] = 11.00$ fps, $h_p = 77.6(V^2/2g) - 37 = 108.8$ ft; $h_p' = 108.8(\frac{240}{190})^2 = 173.6$ ft.

New conditions: $37 - 76.6(V^2/2g) + 173.6 = V^2/2g + (L/g)(dV/dt)$. Eventually $dV/dt = 0$, $V^2/2g = \frac{210.6}{77.6} = 2.71$ ft, $V = 13.21$ fps when steady flow is eventually achieved.

In general: $210.6 - 77.6(V^2/2g) = (2900/32.2)(dV/dt)$, $dt = 74.7\,dV/(174.8 - V^2)$.

$$\int_0^t dt = t = \int_{11.00}^{V_t} \frac{74.7\,dV}{(174.8 - V^2)} = \frac{74.7}{26.42}\left[\ln\frac{13.21 + V}{13.21 - V}\right]_{11.00}^{V_t} = 2.83 \ln\frac{13.21 + V_t}{13.21 - V_t} - 6.77$$

V_t, fps	t, s	Q, cfs
11.00	0	6.00
12.00	1.82	6.54
13.00	6.90	7.09
13.20	15.53	7.20
13.21	∞	7.20

19.40 Attached to the tank in Fig. 19-11 is a flexible 30-mm-diameter hose ($f = 0.016$) of length 72 m. The tank is hoisted in such a manner that $h = 6 + 0.8t$, where h is the head in meters and t is the time in seconds. Estimate the flow at $t = 9$ s.

▮ Find steady-state velocity for $t = 9$ s; then $h = 6 + 0.8(9) = 13.2$ m, $13.2 - 0.016(72/0.030)(V^2/2g) = V^2/2g$; $V = 2.56$ m/s at steady state. Now find the effect of changing head.

Given $h = 6 + 0.8t$, therefore $\partial h/\partial t = 0.8$ m/s, $h - 38.4(V^2/2g) = V^2/2g + (L/g)(dV/dt)$, $\partial h/\partial t = 39.4(2V/2g)(\partial h/\partial t) + (L/g)(\partial^2 V/\partial t^2)$. Neglect the last term (of higher order): $0.8 = 39.4(V/g)(\partial V/\partial t)$; $\partial V/\partial t = 0.8g/(39.4V) = 0.0778$ m/s per second. Finally, $13.2 = 39.4[V^2/2(9.81)] + (72/9.81)(0.0778)$; $39.4V = 2.51$ m/s; $Q = 0.001774$ m³/s, or 1.77 L/s.

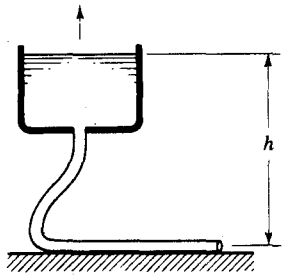


Fig. 19-11

19.41 Repeat Prob. 19.40 for a 100-mm-diameter hose, all other data remaining the same.

▮ Find steady-state velocity for $t = 9$ s: $13.2 - 0.016(72/0.100)(V^2/2g) = V^2/2g$; $V = 4.55$ m/s. Now find effect of changing head: $h = 12.52(V^2/2g) + (L/g)(dV/dt)$. Differentiating with respect to t and neglecting the higher-order term: $\partial h/\partial t = 0.8 = 12.52(2V/2g)(dV/dt)$; $\partial V/\partial t = 0.8g/(12.52V) = 0.1377$ m/s per second.

Hence,

$$13.2 = 12.52[V^2/2(9.81)] + (72/9.81)(0.1377) \qquad V = 4.37 \text{ m/s} \qquad Q = 0.0343 \text{ m}^3/\text{s} = 34.3 \text{ L/s}$$

19.42 In Prob. 19.40, suppose h was changed instantaneously from 6 m to 13.2 m. Under these conditions, find the flow rate at $t = 9$ s.

▮ Initial condition where $h = 6$ m: $6 - 0.016(72/0.030)(V^2/2g) = V^2/2g$; $V = 1.728$ m/s (initially). After head is changed to 13.2 m: $13.2 = 39.4(V^2/2g) + (L/g)(dV/dt)$. Eventually $dV/dt = 0$, $V = 2.56$ m/s, $13.2 = [39.4/(2)(9.81)]V^2 + (72/9.81)(dV/dt)$; $13.2 = 2.008V^2 + 7.34(dV/dt)$; $6.57 = V^2 + 3.66(dV/dt)$; $dt = 3.66\,dV/(2.56^2 - V^2)$.

$$\int_0^t dt = \int_{1.728}^{V} \frac{3.66\,dV}{2.56^2 - V^2} = \frac{3.66}{5.12}\left[\ln\frac{2.56+V}{2.56-V}\right]_{1.728}^{V} \qquad t = 0.715 \ln\frac{2.56+V}{2.56-V} - 1.172$$

V, m/s	t, s
1.728	0
2.1	0.484
2.4	1.283
2.5	2.00
2.56	∞

For all practical purposes $V = 2.00$ m/s at $t = 9$ s when $Q = (0.000707)2.00 = 0.00141$ m³/s = 1.41 L/s.

19.43 A 6-in-diameter pipe of length 3600 ft drains a reservoir. The elevation difference between the reservoir water surface and the pipe outlet is 95 ft. The pipe entrance is flush. Initially there is no flow since there is a plug at the outlet. The plug is then removed. Tabulate Q versus t, assuming $f = 0.022$.

▮ $L/D = 3600/0.50 > 1000$, neglect minor losses, i.e., $k_L = 0$, $k = k_L + fL/D = 0 + 0.022(3600)/(\frac{6}{12}) = 158$, $V_0^2 = 2gH/(1+k) = 2(32.2)95/(1+158)$; steady $V_0 = 6.20$ fps.

$$t = [L/(1+k)V_0] \ln[(V_0+V)/(V_0-V)] = [3600/159(6.20)] \ln[(V_0+V)/(V_0-V)]$$
$$= 3.65 \ln[(6.20+V)/(6.20-V)]$$

V, fps	t, s	Q, cfs
0	0	0
1	1.19	0.196
3	3.85	0.589
5	8.15	0.982
5.6	10.87	1.100
6.20	∞	1.217

19.44 Repeat Prob. 19.43 for the case where the pipe length is 360 ft rather than 3600 ft.

▮ $L/D = 360/0.50 = 900 < 1000$, minor losses are *not* negligible; $k_L = 0.5$ (flush entrance). $k = k_L + fL/D = 0.5 + 0.022(360)/(\frac{6}{12}) = 16.34$, $V_0^2 = 2gH/(1+k) = 2(32.2)95/(1+16.34)$; steady $V_0 = 18.78$ fps.

$$t = [L/(1+k)V_0] \ln[(V_0+V)/(V_0-V)] = [360/17.34(18.78)] \ln[(V_0+V)/(V_0-V)]$$
$$= 1.105 \ln[(18.78+V)/(18.78-V)]$$

V, fps	t, s	Q, cfs
0	0	0
5	0.603	0.982
10	1.312	1.963
15	2.420	2.945
18	4.258	3.534
18.78	∞	3.687

19.45 A 10-cm-diameter pipe of length 400 m drains a reservoir. The elevation difference between the reservoir water surface and the pipe outlet is 50 m. Initially there is no flow because a valve at the pipe outlet is closed. The valve is suddenly opened; tabulate Q versus t, given $f = 0.028$.

▮ $L/D = 400/0.10 > 1000$, neglect minor losses, i.e., $k_L = 0$, $k = k_L + fL/D = 0 + 0.028(400)/0.10 = 112$, $V_p^2 = 2gH/(1+k) = 2(9.81)50/(1+112)$; steady-flow velocity $V_0 = 2.95$ m/s.

$$t = [L/(1+k)V_0] \ln [(V_0 + V)/(V_0 - V)] = [400/112(2.95)] \ln [(V_0 + V)/(V_0 - V)]$$
$$= 1.211 \ln [(2.95 + V)/(2.95 - V)]$$

V, m/s	t, s	Q, L/s
0	0	0
1.0	0.855	7.85
2.0	1.999	15.7
2.3	2.53	18.1
2.6	3.35	20.4
2.8	4.42	22.0
2.9	5.77	22.8
2.95	∞	23.2

19.46 A large water reservoir is drained by a pipeline that consists of 225 ft of 4-in-diameter pipe ($f = 0.028$) followed by 490 ft of 12-in-diameter pipe ($f = 0.022$). The point of discharge is 125 ft below the elevation of the reservoir water surface. A valve at the discharge end of the pipe is initially closed. It is then quickly opened. Derive an equation applicable to this situation, and tabulate flow rate versus time. Neglect minor losses.

▮ $h = f_1(L_1/D_1)(V_1^2/2g) - f_2(L_2/D_2)(V_2^2/2g) = V_2^2/2g + (L_1/g)(dV_1/dt) + (L_2/g)(dV_2/dt)$. By continuity: $V_1 = V_2(D_2/D_1)^2$; $dV_1 = dV_2(D_2/D_1)^2$. Eliminate V_1: $h - [f_1(L_1/D_1)(D_2/D_1)^4 + f_2(L_2/D_2) + 1](V_2^2/2g) = [L_1(D_2/D_1)^2\, dV_2]/(g\, dt) + (L_2/g)(dV_2/dt)$. For steady-state condition when $dV_2/dt = 0$ and $V_2 = V_{20}$. Substituting known values yields steady-flow $V_{20} = 2.28$ fps. In general, rearranging: $dt = \{2[L_1(D_2/D_1)^2 + L_2]/[1 + f_1(L_1/D_1)(D_2/D_1)^4 + f_2(L_2/D_2)]\}[dV/(V_{20}^2 - V^2)]$; integrating

$$t = \{[L_1(D_2/D_1)^2 + L_2]/[1 + f_1(L_1/D_1)(D_2/D_1)^4 + f_2(L_2/D_2)]\}(1/V_{20}) \ln [(V_{20} + V_2)/(V_{20} - V_2)]$$
$$= 0.7150 \ln [(2.28 + V_2)/(2.28 - V_2)]$$

V_1, fps	V_2, fps	t, s	Q, cfs
0	0	0	0
11.25	1.25	0.88	0.98
13.50	1.5	1.13	1.18
15.75	1.75	1.45	1.37
18.00	2.0	1.95	1.57
19.80	2.20	2.88	1.73
20.52	2.28	∞	1.79

19.47 A 12-in-diameter pipe ($f = 0.022$) of length 360 ft is connected to a reservoir. Entrance losses are negligible. At the discharge end of the pipe is a nozzle that produces a 4.8-in-diameter jet. The elevation difference between the jet and the water surface in the reservoir is 52 ft. The nozzle has a coefficient of velocity of 0.95. Initially, there is a tight-fitting plug in the nozzle, which is then removed. For this situation derive an equation and tabulate flow rate versus time. Assume that the liquid level in the reservoir does not drop.

▮ $h - k(V^2/2g) - [(1/C_V^2) - 1][1 - (D_j/D)^4](V_j^2/2g) = (V_j^2/2g) + (L/g)(dV/dt)$. But $V_j = V(A/A_j) = V(D/D_j)^2$, so $h = C(V^2/2g) + (L/g)(dV/dt)$, where $C = k + (D/D_j)^4\{1 + [(1/C_V^2) - 1][1 - (D_j/D)^4]\}$. For steady-state conditions when $dV/dt = 0$ and $V = V_0$: $h = CV_0^2/2g$, or $V_0 = (2gh/C)^{1/2}$. Eliminating h between the above two equations and rearranging: $dt = (2L/C)[dV/(V_0^2 - V^2)]$ and integrating $t = (L/CV_0) \ln [(V_0 + V)/(V_0 - V)]$. Given $k_L = 0$: $k = 0 + 0.022(360)/(\frac{12}{12}) = 7.92$. Given $C_V = 0.95$ and $D/D_j = \frac{12}{4.8}$: $C = 50.4$, $V_0 = [2(32.2)52/50.4]^{1/2} = 8.15$ fps, $t = [360/50.4(8.15)] \ln [(V_0 + V)/(V_0 - V)] = 0.876 \ln [(8.15 + V)/(8.15 - V)]$.

V_j, fps	V, fps	t, s	Q, cfs
0	0	0	0
12.5	2	0.439	1.57
25.0	4	0.941	3.14
37.5	6	1.65	4.71
50.0	8	4.10	6.28
50.6	8.1	5.07	6.36
50.9	8.15	∞	6.40

19.48 An open tank containing oil ($s = 0.85$, $\mu = 0.0005$ lb-s/ft^2) is connected to a 3-in-diameter smooth pipe of length 3800 ft. The elevation drop from the liquid surface in the tank to the point of discharge is 19 ft. A valve on the discharge end of the pipe, initially closed, is then opened. Tabulate the ensuing flow rate versus time.

▮ $L/D = 3800/(\frac{3}{12}) = 15\,200 > 1000$, neglect minor losses. Assume laminar flow, neglect velocity head: $h_L = 19 = 32(\mu/\gamma)(L/D^2)V$, $V = 19(0.85)(62.4)(0.25)^2/[32(0.0005)(3800)] = 1.036$ fps, $N_R = DV\rho/\mu = 0.25(1.036)(0.85)(1.94)/0.0005 = 854$. $N_R < 2000$, therefore flow *is* laminar, assumption was O.K. Assuming the velocity head is negligible: $h - 32(\mu/\gamma)(L/D^2)V = (L/g)(dV/dt)$, or $h - \alpha V = \beta(dV/dt)$, where $\alpha = 32(\mu/\gamma)(L/D^2) = 32(0.0005)3800/[(0.85)(62.4)(0.25)^2] = 18.3$ and $\beta = L/g = 3800/32.2 = 118$.

$$dt = \frac{\beta\, dV}{h - \alpha V} = \frac{dV}{(h/\beta) - (\alpha/\beta)V}$$

$$t = \frac{-1}{\alpha/\beta}\ln\left[\left(\frac{h}{\beta}\right) - \left(\frac{\alpha}{\beta}\right)V\right]_0^V = -6.45\ln[0.1610 - 0.155V]_0^V = -6.45\ln\left(\frac{0.1610 - 0.155V}{0.1610}\right) = -6.45\ln(1 - 0.963V)$$

V, fps	t, s	Q, cfs
0	0	0
0.20	1.38	0.0098
0.40	3.14	0.0196
0.60	5.56	0.0295
0.80	9.49	0.0393
1.00	21.26	0.0491
1.036	∞	0.0509

Check neglect of velocity head: $(V^2/2g)/h_L = (V^2/2g)/\alpha V = V/(2g\alpha) = V/1179$. As V varies from 0 to 1.036, $(V^2/2g)/h_L$ varies from 0 to 0.000877; therefore $V^2/2g < 0.022$ percent of h_L, and velocity head *is* negligible.

19.49 An open tank containing oil ($s = 0.82$, $\nu = 0.002$ m^2/s) is connected to a 125-mm-diameter pipe of length 470 m. The oil surface in the tank is 3.5 m above the pipe outlet. A valve at the end of the pipe, initially closed, is suddenly opened. Plot the ensuing flow rate as a function of time.

▮ $L/D = 470/0.125 = 3760 > 1000$, neglect minor losses. Assuming laminar flow for h_L, $H - 32\nu(L/gD^2)V = (L/g)(dV/dt)$; $3.5 - 32(0.002)[470/9.81(0.125)^2]V = (470/9.81)(dV/dt)$, $3.5 - 196V = 47.9\, dV/dt$; $dt = (47.9\, dV)/(3.5 - 196V) = (13.69\, dV)/(1 - 56.00V)$. For steady-state conditions when $dV/dt = 0$ and $V = V_0$: $3.5 - 196V_0 = 0$; steady-flow velocity $V_0 = 0.01786$ m/s. Integrating:

$$t = -(13.69/56.00)[\ln(1 - 56.00V)]_0^V = -0.2445\ln(1 - 56.00V)$$

V, m/s	t, s	Q, L/s
0.001	0.014	0.0123
0.002	0.029	0.0245
0.005	0.080	0.0614
0.010	0.201	0.123
0.015	0.448	0.184
0.017	0.742	0.209
0.01786	∞	0.219

$(N_R)_{max} = (0.125)0.01786/0.002 = 1.12$. $N_R < 2000$, so flow *is* laminar (as assumed) and answers are valid.

19.50 Find the celerity of a pressure wave in benzene (s.g. = 0.90, E_v = 150 000 psi) contained in a 9-in-diameter steel pipe having a wall thickness of 0.300 in.

▮
$$c = (E_v/\rho)^{1/2} = [(150\,000)(144)/(0.9)(1.94)]^{1/2} = 3520 \text{ fps}$$
$$c_p = c/\sqrt{1 + (D/t)(E_v/E)} = 3520/\{1 + (9/0.300)[150\,000/(30 \times 10^6)]\}^{1/2} = 3282 \text{ fps}$$

19.51 (***a***) What is the celerity of a pressure wave in a 4.5-ft-diameter water pipe with 0.4-in steel walls? (***b***) If the pipe is 3750 ft long, what is the time required for a pressure wave to make the round trip from the valve?

▮ (***a***) $c = 4720$ fps in water
$$c_p = c/\sqrt{1 + (D/t)(E_v/E)} = 4720/\{1 + [(4.5)(12)/0.4][300\,000/(30 \times 10^6)]\}^{1/2} = 3079 \text{ fps}$$
(***b***)
$$T_r = 2L/c_p = 2(3750)/3079 = 2.44 \text{ s}$$

19.52 In Prob. 19.51, (***a***) if the initial water velocity is 7 fps, what will be the rise in pressure at the valve if the time of closure is less than the time of a round trip? (***b***) If the valve is closed at such a rate that the velocity in the pipe decreases uniformly with respect to time and closure is completed in a time $t_c = 5L/c_P$, approximately what will be the pressure head at the valve when the first pressure unloading wave reaches the valve?

▮ (***a***)
$$p = (\gamma/g)c_pV = (62.4/32.2)(3079)7 = 41\,767 \text{ psf} = 669 \text{ ft}$$
(***b***) If velocity changes uniformly from 7.0 fps to 0 in time $5L/c_p$, the change in V in $t' = T_r = 2L/c_p$ is $(\frac{2}{5})7.0 = 2.80$ fps. $\Delta p'/\gamma = c_p\,\Delta V/g = 3079(2.80)/32.2 = 268$ ft.

19.53 For the situation depicted in Prob. 19.27, find the water-hammer pressure at the valve if a flow of 75 cfs is reduced to 20 cfs in 3.2 s. Under these conditions what would be the maximum water-hammer pressures at points 600 ft and 1600 ft from the reservoir?

▮ $\Delta p'_h = (T_r/t_c)(\Delta p_h)$, $V_1 = 75/4\pi = 5.97$ fps, $V_2 = 20/4\pi = 1.592$ fps, $\Delta V = 4.38$ fps over 3.2 s; $\Delta p_h = \rho c_p(\Delta V) = 1.94(3200)(4.38) = 27\,200$ psf for instantaneous drop in flow rate. $T_r = 2L/c_p = \frac{4000}{3200} = 1.250$ s. At valve: $\Delta p'_h \simeq (1.25/3.2)(27\,200) = 10\,625$ psf = 73.8 psi.
600 ft from reservoir: $\Delta p'_h = (\frac{600}{2000})73.8 = 22.1$ psi.
1600 ft from reservoir: $\Delta p'_h = (\frac{1600}{2000})73.8 = 59.0$ psi.

19.54 Water is flowing through a 35-cm-diameter welded steel pipe ($f = 0.014$) of length 2300 m that drains a reservoir under a head of 50 m. The pipe has a thickness of 9 mm. If a valve at the end of the pipe is closed in 9 s, approximately what water-hammer pressure will be developed?

▮
$$h_f = (f)(L/D)(V^2/2g) \qquad 50 = 0.014(2300/0.35)V^2/2g \qquad V = 3.26 \text{ m/s}$$
$$p'_h = 2LV\rho/t_c = [2(2300)3.26(1000)/9] = 1.667 \text{ MPa}$$

19.55 In Fig. 19-12, the total length of pipe is 9800 ft, its diameter is 30 in, and its thickness is $\frac{5}{8}$ in. Assume E = 30 000 000 psi and E_v = 300 000 psi. If the initial velocity for steady flow is 8 fps and the valve at G is partially closed so as to reduce the flow to half of the initial velocity in 3 s, find (***a***) the maximum pressure rise from the water hammer; (***b***) the location of the point of maximum total pressure.

▮ (***a***) $c = 4720$ ft/s
$$c_p = c/\sqrt{1 + (D/t)(E_V/E)} = 4720/[1 + (30/\tfrac{5}{8})(300\,000/30\,000\,000)]^{1/2} = 3880 \text{ fps}$$
$$\Delta p = (\gamma/g)c_p(\Delta V) = (62.4/32.2)(\tfrac{3880}{144})4 = 209 \text{ psi} \quad \text{or} \quad 482 \text{ ft}$$
(***b***) $x_0 = c_pt_c/2 = 3880(3)/2 = 5820$ ft. Thus water-hammer pressure rise, at B: $\Delta p = 209(\frac{1800}{5820}) = 64.6$ psi; at E: $\Delta p = 209(100 \text{ percent}) = 209$ psi. Total pressure = static pressure plus water hammer. Total pressure at B: $p/\gamma = 700 + 64.6(144/62.4) = 849$ ft; at E: $p/\gamma = 100 + 209(144/62.4) = 582$ ft. Maximum total pressure occurs at point B.

19.56 Refer to Fig. 19-12, but take all the dimensions given in feet to be in meters instead. This 9.8-km-long pipe has a diameter of 1.4 m and a wall thickness of 30 mm. Assume E = 200 GPa and E_v = 2 GPa. The initial steady-flow velocity is 5 m/s. The valve at G is then partially closed so as to reduce the velocity to 1 m/s in 9 s. Find (***a***) the maximum pressure rise due to water hammer, and (***b***) the location of the point of maximum total pressure.

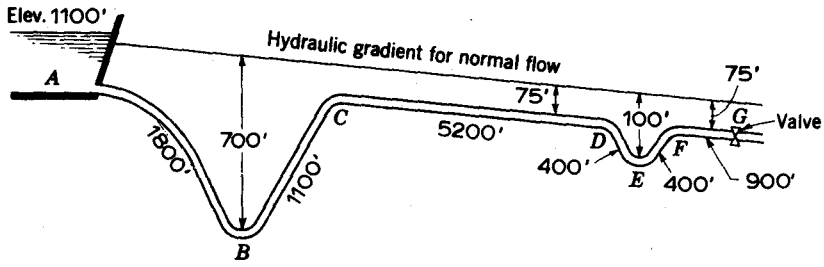


Fig. 19-12

▮ (a) $c_p = 1440$ m/s $\quad c_p = c/\sqrt{1+(D/t)(E_v/E)} = 1440/[1+(\frac{1400}{30})(\frac{2}{200})]^{1/2} = 1189$ m/s

$\Delta p = \rho c_p \Delta V = 1000(1189)4 = 4.756$ MPa

(b) $x_0 = c_p t_c/2 = 1189(9)/2 = 5350$ m. Thus water-hammer pressure rise, at B: $\Delta p = 4.756(\frac{1800}{5350}) = 1.600$ MPa, or 163.4 m; at E: $\Delta p = 4.756$ MPa or 486 m. Total pressure = static pressure plus water-hammer pressure. Total pressure at B: $p/\gamma = 700$ m + 163.4 m = 863.4 m; at E: $p/\gamma = 100$ m + 486 m = 586 m. Maximum total pressure occurs at point B.

19.57 Derive the equation $V^2 = (2gAD^2/LA_sf^2)[1-(fA_s/AD)z] - Ce^{-(fA_s/AD)z}$, which expresses the relationship between velocity in a pipe and water-surface level in a tank over an interval from valve closure to the top of the first surge.

▮ Write dV/dt as $(dV/dz)(dz/dt)$ in $0 - f(L/D)(V^2/2g) = z + (L/g)(dV/dt)$ and substitute for dz/dt from $AV = A_s(dz/dt)$; $0 - (fL/D)(V^2/2g) = z + (L/g)(dV/dz)(AV/A_s)$ or $z + (fL/2gd)V^2 + (LA/2gA_s)[(2V\,dV)/dz] = 0$. Define $K = fL/2gD$; $M = (L/2g)(A/A_s)$; and $V^2 = x$ so that $2V\,dV = dx$. This gives $z + kx + (M\,dx)/dz = 0$ or $dx/dz + (K/M)x + (1/M)z = 0$. Now define $\alpha = K/M$, $\beta = 1/M$; then $dx/dz + \alpha Z + \beta z = 0$. Multiply by $e^{\alpha z}$: $(dx/dz)e^{\alpha z} + \alpha x e^{\alpha z} + \beta z e^{\alpha} = 0$, i.e., $d/dz(xe^{\alpha z}) + e^{\alpha z}\beta z = 0$ or $d(xe^{\alpha z}) + e^{\alpha z}\beta z\,dz = 0$. Integrate: $\int d(xe^{\alpha z}) + \int e^{\alpha z}\beta z\,dz = 0$, $xe^{\alpha z} + \beta(e^{\alpha z}/\alpha^2)(\alpha z - 1) + C = 0$ or $x = (\beta/\alpha^2)(1-\alpha z) - Ce^{-\alpha z}$. But $\beta/\alpha^2 = M/K^2 = 2gAD^2/LA_sf^2$, $\alpha = K/M = fA_s/AD$, and $x = V^2$; therefore $V^2 = (2gAD^2/LA_sf^2)[1-(fA_s/AD)z] - Ce^{-(fA_s/AD)z}$.

19.58 A 36-in steel pipe 3300 ft long supplies water to a small power plant. What height would be required for a simple surge tank 7 ft in diameter situated 59 ft upstream from the valve at a point where the centerline of the pipe is 100 ft below the water surface in the reservoir if the tank is to protect against instantaneous closure of a valve at the plant? The valve is 130 ft below reservoir level, and the discharge is 140 cfs. Take $f = 0.014$. The surge tank is not to overflow. Neglect all velocity heads and minor losses; in the surge tank (only) neglect fluid friction and inertial effects.

▮ $$V^2 = (2gAD^2/LA_sf^2)[1-(fA_s/AD)z] - Ce^{-(fA_s/AD)z}$$
$$= [2(32.2)/3241](\tfrac{3}{7})^2(3/0.014)^2[1-(0.014/3)(\tfrac{7}{3})^2z] - Ce^{-(0.014/3)(7/3)^2z} = 168(1-0.025z) - Ce^{-0.025z}$$

First find the constant of integration C, from initial (steady) conditions, when $Q_1 = 140$ cfs, $V_1 = Q_1/A = 140/[(\pi/4)3^2] = 19.8$ fps, and $-z_1 = (fL/D)(V_1^2/2g) = (0.014)(\frac{3241}{3})19.8^2/[(2)(32.2)] = 92.1$ ft. Substituting these values $19.8^2 = 168[1-0.025(-92.1)] - Ce^{-0.025(-92.1)}$, from which $C = 16.3$. For the first surge, $z = z_{max}$ when $V = 0$, so $168(1-0.025z_{max}) - 16.3e^{-0.025z_{max}}$. [*Note:* The last (3rd) term is always negative. For a positive middle (2nd) term we need $0.025z_{max} < 1$, i.e., $z_{max} < 1/0.025 = 40$.] By trial and error: $z_{max} = 37.5$ ft. Required height of surge tank = (100 + 37.5) = 137.5 ft.

19.59 Repeat Prob. 19.58 for the case where the surge tank is to have a diameter of 11 ft.

▮ Following the procedure of Prob. 19.58: $V^2 = 67.87(1-0.0627z) - Ce^{-0.0627z}$. From initial (steady) conditions: $z_1 = -92.1$ ft, $C = 0.211$. (Note the large change in C from Prob. 19.58.) For the first surge, we need $0.0627z_{max} < 1$, i.e., $z_{max} < 15.95$ ft. By trial and error: $z_{max} = 15.93$ ft. Required height of surge tank = (100 + 15.93) = 115.9 ft.

19.60 Using the data of Prob. 19.58, find the diameter of a surge tank that will produce a surge requiring a tank height of 145 ft.

▮ As in Prob. 19.58 for initial (steady) conditions: $V_1 = 19.8$ fps, $z_1 = -92.1$ ft. Substituting these and other known values into $V^2 = (2gAD^2/LA_sf^2)[1-(fA_s/AD)z] - Ce^{-(fA_s/AD)z}$, while writing A/A_s as $(D/D_s)^2$: $19.8^2 = 8212/D_s^2 + 392 - Ce^{0.0478D_s^2}$, that is,

$$0 = 8212/D_s^2 - Ce^{0.0478D_s^2} \qquad (1)$$

When $z = z_{max} = 145 - 100 = 40$ ft, $V = 0$; substituting these and other known values

$$0 = 8212/D_s^2 - 168 - Ce^{-0.0207D_s^2} \qquad (2)$$

Eliminating the unknown C between Eqs. (*1*) and (*2*): $1 - 0.0205D_s^2 = e^{-0.0695D_s^2}$. By trial and error: $D_s = 6.85$ ft.

19.61 A 0.95-m-diameter steel pipe ($f = 0.015$) of length 1100 m carries drinking water from a reservoir. What height would be required for a simple surge tank 3.2 m in diameter situated 11 m upstream from the valve at a point where the centerline of the pipe is 38 m below the water surface in the reservoir? Assume instantaneous closure of the valve. The valve is 48 m below reservoir level and the flow is 3.4 m³/s. The surge tank is not to overflow. Neglect all velocity heads and minor losses; in the surge tank (only) neglect fluid friction and inertial effects.

▮
$$\begin{aligned} V^2 &= (2gAD^2/LA_sf^2)[1 - (fA_s/AD)z] - Ce^{-(fA_s/AD)z} \\ &= [2(9.81)/1089](0.95/3.2)^2(0.95/0.015)^2[1 - (0.015/0.95)(3.2/0.95)^2z] - Ce^{-(0.015/0.95)(3.2/0.95)^2z} \\ &= 6.37(1 - 0.1792z) - Ce^{-0.1792z} \end{aligned}$$

First, find the constant of integration C from initial (steady) conditions when $Q_1 = 3.4$ m³/s, $V_1 = Q_1/A = 3.4/[(\pi/4)0.95^2] = 4.80$ m/s, and $-z_1 = (fL/D)(V_1^2/2g) = (0.015)(1089/0.95)4.80^2/[(2)(9.81)] = 20.19$ m. Substituting these values, $4.80^2 = 6.37[1 - 0.1792(-20.19)] - Ce^{-0.1792(-20.19)}$, from which $C = 0.1712$. For the first surge, $z = z_{max}$ when $V = 0$, so $0 = 6.37(1 - 0.1792z_{max}) - 0.1712e^{-0.1792z_{max}}$. [*Note:* The last (3rd) term is always negative. For a positive middle (2nd) term we need $0.1792z_{max} < 1$, i.e., $z_{max} < 1./0.1792 = 5.58$ m.] By trial and error: $z_{max} = 5.52$ m. Required height of surge tank $= 38 + 5.52 = 43.52$ m.

19.62 A 1.5-m-diameter sluice ($f = 0.018$) 980 m long carries water from a dam. When the outlet valve is closed instantaneously, water rises in a 2-m-diameter surge tank immediately adjacent to the outlet valve. Determine the maximum allowable initial discharge Q so that the resulting surge will not rise more than 9 m above the reservoir water surface. Neglect all velocity heads and minor losses; in the surge tank (only) neglect fluid friction and inertial effects.

▮
$$\begin{aligned} V^2 &= (2gAD^2/LA_sf^2)[1 - (fA_s/AD)z] - Ce^{-(fA_s/AD)z} \\ &= [2(9.81)/980](1.5/2)^2(1.5/0.018)^2[1 - (0.018/1.5)(2/1.5)^2] - Ce^{-(0.018/1.5)(2/1.5)^2z} \\ &= 78.2(1 - 0.0213z) - Ce^{-0.0213z} \end{aligned}$$

For the first surge, $z = z_{max} = 9$ m when $V = 0$, so $0 = 78.2[1 - 0.0213(9)] - Ce^{-0.0213(9)}$; from which $C = 76.6$. For the initial conditions, when V = steady-flow velocity $= V_1$, $z_1 = -h_L = -(fL/D)(V_1^2/2g)$. Substitute this expression for z_1 for V_1^2, and simplify: $V_1^2 = B[1 + (V_1^2/B)] - Ce^{V_1^2/B}$, where $B = 78.2$, $B = Ce^{V_1^2/B}$ or $\ln(B/C) = V_1^2/B$, $V_1 = [B \ln(B/C)]^{1/2} = [78.2 \ln(78.2/76.6)]^{1/2} = 1.271$ m/s. So $Q = AV = (\pi/4)(1.5)^2(1.271) = 2.25$ m³/s.

CHAPTER 20
Pumps and Fans

20.1 A pump delivers 0.300 m^3/s against a head of 200 m with a rotative speed of 2000 rpm. Find the specific speed.

$$N_s = 51.64NQ^{0.5}/H^{0.75} = (51.64)(2000)(0.300)^{0.5}/200^{0.75} = 1064$$

20.2 A pump delivers 0.019 m^3/s against a head of 16.76 m with a rotative speed of 1750 rpm. Find the specific speed.

$$N_s = 51.64NQ^{0.5}/H^{0.75} = (51.64)(1750)(0.019)^{0.5}/16.76^{0.75} = 1504$$

20.3 A radial-flow pump must deliver 2000 gpm against a head of 950 ft. Find the minimum practical rotative speed.

$N_s = NQ^{0.5}/H^{0.75}$. From Fig. A-40, it is apparent that the minimum practical specific speed for a radial-flow pump is about 500. Hence, $500 = (N)(2000)^{0.5}/950^{0.75}$, $N = 1913$ rpm.

20.4 A radial-flow pump must deliver 300 gpm against a head of 30 ft. Find the operating rotative speed.

$N_s = NQ^{0.5}/H^{0.75}$. From Fig. A-40, $N_s = 2500$ for maximum efficiency of 92.5 percent for a radial-flow pump. Hence, $2500 = (N)(300)^{0.5}/30^{0.75}$, $N = 1850$ rpm.

20.5 An axial-flow pump is to be operated at a rotative speed of 2500 rpm against a head of 400 m. What flow rate will be delivered by the most efficient pump?

From Fig. A-40, it is apparent that the most efficient axial-flow pump has a specific speed of around 12 500. $N_s = 51.64NQ^{0.5}/H^{0.75}$, $12\,500 = (51.64)(2500)(Q)^{0.5}/400^{0.75}$, $Q = 75.0\ m^3/s$.

20.6 A mixed-flow pump is to be operated at a rotative speed of 1500 rpm against a head of 15 m at maximum pump efficiency. Determine the flow rate the pump will deliver.

From Fig. A-40, it is apparent that the most efficient mixed-flow pump has a specific speed of around 6500. $N_s = 51.64NQ^{0.5}/H^{0.75}$, $6500 = (51.64)(1500)(Q)^{0.5}/15^{0.75}$, $Q = 0.409\ m^3/s$.

20.7 A radial-flow pump operating at maximum efficiency is to deliver 260 gpm against a head of 129 ft at a rotative speed of 2100 rpm. Find the required number of stages (i.e., impellers).

From Fig. A-40, it is apparent that maximum efficiency for a radial-flow pump is about 93 percent at a specific speed of 2500. $N_s = NQ^{0.5}/H^{0.75}$, $2500 = (2100)(260)^{0.5}/H^{0.75}$, $H = 32.29$ ft. Since the given head is 129 ft, a total of 129/32.29, or 4 stages will be needed.

20.8 A radial-flow pump operating at maximum efficiency is to deliver 400 gpm against a head of 191 ft at a rotative speed of 1920 rpm. Find the required number of stages (i.e., impellers).

From Fig. A-40, it is apparent that maximum efficiency for a radial-flow pump is about 93 percent at a specific speed of 2500. $N_s = NQ^{0.5}/H^{0.75}$, $2500 = (1920)(400)^{0.5}/H^{0.75}$, $H = 38.18$ ft; number of stages = 191/38.18 = 5.

20.9 The value of $(NPSH)_{min}$ for a pump is given by the manufacturer as 20 ft. Water is being pumped from a reservoir at a rate of 25 cfs. The water level in the reservoir is 6.0 ft below the pump. Atmospheric pressure is 14.7 psia and water temperature is 40 °F. If the total head loss in the suction pipe is 4.0 ft, is the pump safe from cavitation effects?

$$\text{NPSH} = p_A/\gamma - z_s - h_L - p_v/\gamma = (14.7)(144)/62.4 - 6.0 - 4.0 - 18.5/62.4 = 23.6\ \text{ft}$$

Since [NPSH = 23.6] > [$(NPSH)_{min}$ = 20], cavitation should not be a problem.

20.10 The $(NPSH)_{min}$ for a pump is given by the manufacturer as 7.0 m. This pump is being used to pump water from a reservoir at a rate of 0.2832 m^3/s. The water level in the reservoir is 1.280 m below the pump. Atmospheric pressure is 98.62 kN/m^2 and water temperature is 20 °C. Assume total head loss in the suction pipe is 1.158 m of water. Determine whether or not the pump is safe from cavitation effects.

▮ $$\text{NPSH} = p_A/\gamma - z_s - h_L - p_v/\gamma = 98.62/9.79 - 1.280 - 1.158 - 2.34/9.79 = 7.40\text{ m}$$

Since [NPSH = 7.40 m] > [$(\text{NPSH})_{\min}$ = 7.0 m], cavitation should not be a problem.

20.11 A commercial pump is operating at 2150 rpm and delivers 1800 gpm against a head of 340 ft. Find the approximate efficiency of the pump.

▮ $$N_s = NQ^{0.5}/H^{0.75} = (2150)(1800)^{0.5}/340^{0.75} = 1152$$

From Fig. A-41, with $N_s = 1152$ and $Q = 1800$ gpm, the approximate efficiency of the pump is determined to be 82 percent.

20.12 A pump operating at 1600 rpm delivers 0.189 m^3/s against a head of 47.03 m. Determine the approximate efficiency of the pump.

▮ $$N_s = 51.64NQ^{0.5}/H^{0.75} = (51.64)(1600)(0.189)^{0.5}/47.03^{0.75} = 2000$$

From Fig. A-41, with $N_s = 2000$ and $Q = 0.189\ \text{m}^3/\text{s}$, the approximate efficiency of the pump is determined to be 87 percent.

20.13 A centrifugal pump with a 700-mm-diameter impeller runs at 1800 rpm. The water enters without whirl, and $\alpha_2 = 60°$. The actual head produced by the pump is 17 m. Find its hydraulic efficiency when $V_2 = 6$ m/s.

▮ The theoretical head is $H = (u_2V_2\cos\alpha_2)/g = [(\frac{1800}{60})(2\pi)(\frac{700}{1000})/2](6)(\cos 60°)/9.807 = 20.18$ m, $e_h = 17/20.18 = 0.842$, or 84.2 percent.

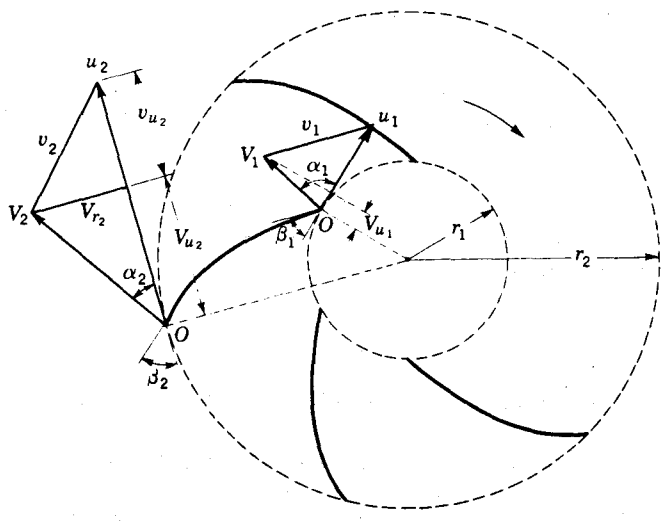
20.14 A centrifugal water pump has an impeller (Fig. 20-1*a*) with $r_2 = 12$ in, $r_1 = 4$ in, $\beta_1 = 20°$, $\beta_2 = 10°$. The impeller is 2 in wide at $r = r_1$ and $\frac{3}{4}$ in wide at $r = r_2$. For 1800 rpm, neglecting losses and vane thickness, determine **(*a*)** the discharge for shockless entrance when $\alpha_1 = 90°$, **(*b*)** α_2 and the theoretical head H, **(*c*)** the horsepower required, and **(*d*)** the pressure rise through the impeller.

▮ **(*a*)** The peripheral speeds are $u_1 = \frac{1800}{60}(2\pi)(\frac{1}{3}) = 62.8$ ft/s, $u_2 = 3u_1 = 188.5$ ft/s. The vector diagrams are shown in Fig. 20-1*b*. With u_1 and the angles α_1, β_1 known, the entrance diagram is determined, $V_1 = u_1\tan 20° = 22.85$ ft/s; hence, $Q = 22.85(\pi)(\frac{2}{3})(\frac{2}{12}) = 7.97$ cfs.

(*b*) At the exit the radial velocity V_{r2} is $V_{r2} = 7.97(12)/[2\pi(0.75)] = 20.3$ ft/s. By drawing u_2 (Fig. 20-1*b*) and a parallel line distance V_{r2} from it, the vector triangle is determined when β_2 is laid off. Thus, $v_{u2} = 20.3\cot 10° = 115$ ft/s, $V_{u2} = 188.5 - 115 = 73.5$ ft/s, $\alpha_2 = \tan^{-1}(20.3/73.5) = 15°26'$, $V_2 = 20.3\csc 15°26' = 76.2$ ft/s; $H = (u_2V_2\cos\alpha_2)/g = u_2V_{u2}/g = 188.5(73.5)/32.2 = 430$ ft.

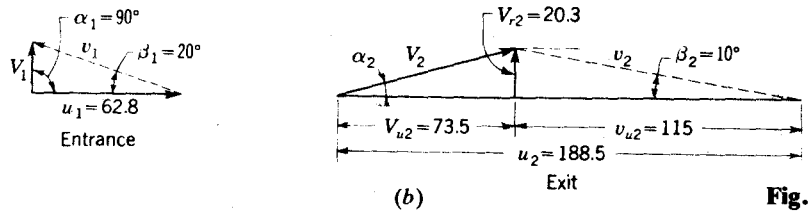
(*c*) $$\text{Power} = Q\gamma H/550 = [7.97(62.4)(430)]/550 = 338\text{ hp}$$

(*d*) By applying the energy equation from entrance to exit of the impeller, including the energy H added (elevation change across impeller is neglected), $H + (V_1^2/2g) + (p_1/\gamma) = (V_2^2/2g) + (p_2/\gamma)$ and $(p_2 - p_1)/\gamma = 430 + (22.85^2/64.4) - (76.2^2/64.4) = 348$ ft or $p_2 - p_1 = 348(0.433) = 151$ psi.



(*a*)

Fig. 20-1



(*b*) **Fig. 20-1** (*continued*)

20.15 Tests of a pump model indicate a σ_c of 0.10. A homologous unit to be installed at a location where $p_a = 90$ kPa and $p_v = 3.5$ kPa is to pump water against a head of 25 m. The head loss from suction reservoir to pump impeller is 0.35 N-m/N. What is the maximum permissible suction head?

▮
$$\sigma' = V_e^2/2gH = (p_a - p_v - \gamma z_s + h_l)/\gamma H$$
$$z_s = [(p_a - p_v)/\gamma] - \alpha' H + h_l = [(90\,000 - 3500)/9806] - 0.10(25) + 0.35 = 6.67 \text{ m}$$

20.16 A prototype test of a mixed-flow pump with a 72-in-diameter discharge opening, operating at 225 rpm, resulted in the following characteristics:

H, ft	Q, cfs	e, %	H, ft	Q, cfs	e, %	H, ft	Q, cfs	e, %
60	200	69	47.5	330	87.3	35	411	82
57.5	228	75	45	345	88	32.5	425	79
55	256	80	42.5	362	87.4	30	438	75
52.5	280	83.7	40	382	86.3	27.5	449	71
50	303	86	37.5	396	84.4	25	459	66.5

What size and synchronous speed (60 Hz) of homologous pump should be used to produce 200 cfs at 60-ft head when operating at highest efficiency? Find the characteristic curves for this case.

▮ Subscript 1 refers to the 72-in pump. For best efficiency $H_1 = 45$, $Q_1 = 345$, $e = 88$ percent. $H/N^2D^2 = H_1/N_1^2D_1^2$, $Q/ND^3 = Q/N_1D_1^3$, or $60/N^2D^2 = 45/[225^2(72^2)]$, $200/ND^3 = 345/[225(72^3)]$. After solving for N and D, $N = 366.7$ rpm, $D = 51.0$ in. The nearest synchronous speed (3600 divided by number of pairs of poles) is 360 rpm. To maintain the desired head of 60 ft, a new D is necessary. When its size is computed, $D = \sqrt{\frac{60}{45}}(\frac{225}{360})(72) = 52$ in, the discharge at best efficiency is $Q = Q_1ND^3/N_1D_1^3 = 345(\frac{360}{225})(\frac{52}{72})^3 = 208$ cfs, which is slightly more capacity than required. With $N = 360$ and $D = 52$, equations for transforming the corresponding values of H and Q for any efficiency can be obtained: $H = H_1(ND/N_1D_1)^2 = H_1[(\frac{360}{225})(\frac{52}{72})]^2 = 1.335H_1$ and $Q = Q_1(ND^3/N_1D_1^3) = Q_1(\frac{360}{225})(\frac{52}{72})^3 = 0.603Q_1$. The characteristics of the new pump are

H, ft	Q, cfs	e, %	H, ft	Q, cfs	e, %	H, ft	Q, cfs	e, %
80	121	69	63.5	200	87.3	46.7	248	82
76.7	138	75	60	208	88	43.4	257	79
73.4	155	80	56.7	219	87.4	40	264	75
70	169	83.7	53.5	231	86.3	36.7	271	71
66.7	183	86	50	239	84.4	33.4	277	66.5

The efficiency of the 52-in pump might be a fraction of a percent less than that of the 72-in pump, as the hydraulic radii of flow passages are smaller, so that the Reynolds number would be less.

20.17 Develop a program for calculating homologous pump characteristics and apply it to Prob. 20.16.

▮
```
10 REM                          HOMOLOGOUS CHARACTERISTICS
20 DEFINT I: DIM H1(20),Q1(20),E(20),H(20),Q(20)
30 FOR I=1 TO 15: READ H1(I): NEXT I
40 DATA 60.,57.5,55.,52.5,50.,47.5,45.,42.5,40.,37.5,35.,32.5,30.,27.5,25.
50 LPRINT:LPRINT"H1=";: FOR I=1 TO 15:LPRINT USING"##.# ";H1(I);: NEXT I:LPRINT
60 FOR I=1 TO 15: READ Q1(I): NEXT I
70 DATA 200.,228.,256.,280.,303.,330.,345.,362.,382.,396.,411.,425.,438.,449.,45
9.
80 LPRINT:LPRINT"Q=";: FOR I=1 TO 15: LPRINT USING"###.# ";Q1(I);:NEXT I:LPRINT
90 FOR I=1 TO 15: READ E(I): NEXT I
100 DATA 69.,75.,80.,83.7,86.,87.3,88.,87.4,86.3,84.4,82.,79.,75.,71.,66.5
110 LPRINT:LPRINT"E=";: FOR I=1 TO 15: LPRINT USING"##.# ";E(I);: NEXT I:LPRINT
120 EE=0!: FOR I=1 TO 15: IF E(I)>EE THEN II=I: EE=E(I): NEXT I
```

```
130 LPRINT"II,E(I)=";II;EE
140 READ HH,QQ,NSYN,D1,N1:DATA 60.,200.,3600.,72.,225.
150 LPRINT"HH,QQ,NSYN,D1,N1=";HH;QQ;NSYN;D1;N1
160 D=((QQ/Q1(II))^2*H1(II)/HH)^.25*D1: N=N1*SQR(HH/H1(II))*D1/D
170 LPRINT"D,N=";D;N: I=FIX(NSYN/N)
180 NN1=FIX(NSYN/I): NN2=FIX(NSYN/(I+1)): LPRINT"N,NN1,NN2=";N;NN1;NN2
190 IF (NN1-N)<(N-NN2) THEN N=NN1 ELSE N=NN2
200 D=D1*N1*SQR(HH/H1(II))/N: QQ=(D/D1)^3*N*Q1(II)/N1: LPRINT"D,N,QQ=";D;N;QQ
210 C1=(N*D/(N1*D1))^2: C2=(D/D1)^3*N/N1
220 FOR I=1 TO 15: H(I)=C1*H1(I): Q(I)=C2*Q1(I): NEXT I
230 LPRINT: LPRINT"H=";: FOR I=1 TO 15:LPRINT USING" ###.#";H(I);:NEXT I:LPRINT
240 LPRINT"Q=";: FOR I=1 TO 15: LPRINT USING" ###.#";Q(I);: NEXT I: LPRINT

H1=60.0 57.5 55.0 52.5 50.0 47.5 45.0 42.5 40.0 37.5 35.0 32.5 30.0 27.5 25.0

Q=200.0 228.0 256.0 280.0 303.0 330.0 345.0 362.0 382.0 396.0 411.0 425.0 438.0
449.0 459.0

E=69.0 75.0 80.0 83.7 86.0 87.3 88.0 87.4 86.3 84.4 82.0 79.0 75.0 71.0 66.5
II,E(I)= 7  88
HH,QQ,NSYN,D1,N1= 60  200  3600  72  225
D,N= 51.01563  366.6749
N,NN1,NN2= 366.6749  400  360
D,N,QQ= 51.96153  360  207.4853

H=  80.0  76.7  73.3  70.0  66.7  63.3  60.0  56.7  53.3  50.0  46.7  43.3  40.0
  36.7  33.3
Q= 120.3 137.1 154.0 168.4 182.2 198.5 207.5 217.7 229.7 238.2 247.2 255.6 263.4
 270.0 276.0
```

20.18 Develop the characteristic curve for a homologous pump of the series of Prob. 20.16 for 21-in-diameter discharge and 1600 rpm.

▮

$$Q = Q_1(N/N_1)(D/D_1)^3 = Q_1\tfrac{1600}{225}(1.75/6)^3 = 0.176Q_1$$

$$H = H_1(ND/N_1D_1)^2 = H_1[(1600)(1.75)/(225)(6)]^2 = 4.30H_1$$

H_1	H	Q_1	Q
60	258	200	35.2
55	236	256	45.1
50	215	303	53.3
45	194	345	60.7
40	172	382	67.2
35	150	411	72.3
30	129	438	77.1
25	108	459	80.8

See curve plotted in Fig. 20-2.

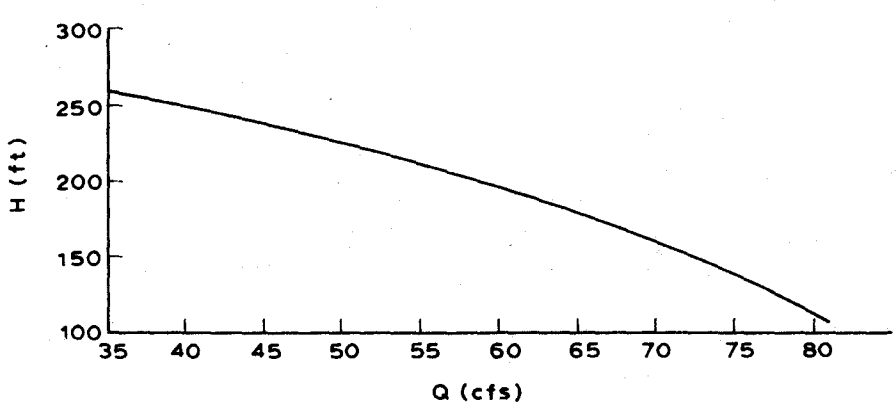


Fig. 20-2

20.19 Determine the size and synchronous speed of a pump homologous to the 72-in pump of Prob. 20.16 that will produce 2.5 m^3/s at 90-m head when operating at highest efficiency.

▮

$$Q/ND^3 = Q_1/N_1D_1^3 \qquad Q_1 = 2.5/0.3048^3 = 88.3\text{ cfs} \qquad H_1 = 90/0.3048 = 295.3\text{ ft}$$

$$N_1D_1^3 = (Q_1/Q)ND^3 = (88.3/345)(225)(\tfrac{72}{12})^3 = 12\,439 \qquad H/N^2D^2 = H_1/N_1^2D_1^2$$

$$N_1^2D_1^2 = (H_1/H)N^2D^2 = (295.3/45)(225)^2(\tfrac{72}{12})^2 = 11\,959\,650$$

$D_1 = 1.897$ ft, or 22.76 in, and $N_1 = 1822$ rpm. Use $N = 1800$ rpm (one pair of poles). Then, $D_1 = \sqrt{H_1/H}(N/N_1)D = \sqrt{295.3/45}(\frac{225}{1800})(6) = 1.92$ ft.

20.20 Sketch the head versus discharge curve for a centrifugal pump (Fig. 20-1*a*) having $r_1 = 55$ mm, $r_2 = 110$ mm, $b_1 = 27.5$ mm, $b_2 = 22$ mm, $N = 1300$ rpm, and $\beta_2 = 33°$.

▮ $H = u_2^2/g - [(u_2 Q \cot \beta_2)/(2\pi r_2 b_2 g)]$ where $u_2 = N(2\pi/60)r_2 = 1300(2\pi/60)(0.110) = 14.975$ m/s, $H = (14.975^2/9.807) - \{[(14.975 \cot 33°)Q]/[2\pi(0.110)(0.022)(9.807)]\} = 22.87 - 173.94Q$. See Fig. 20-3.

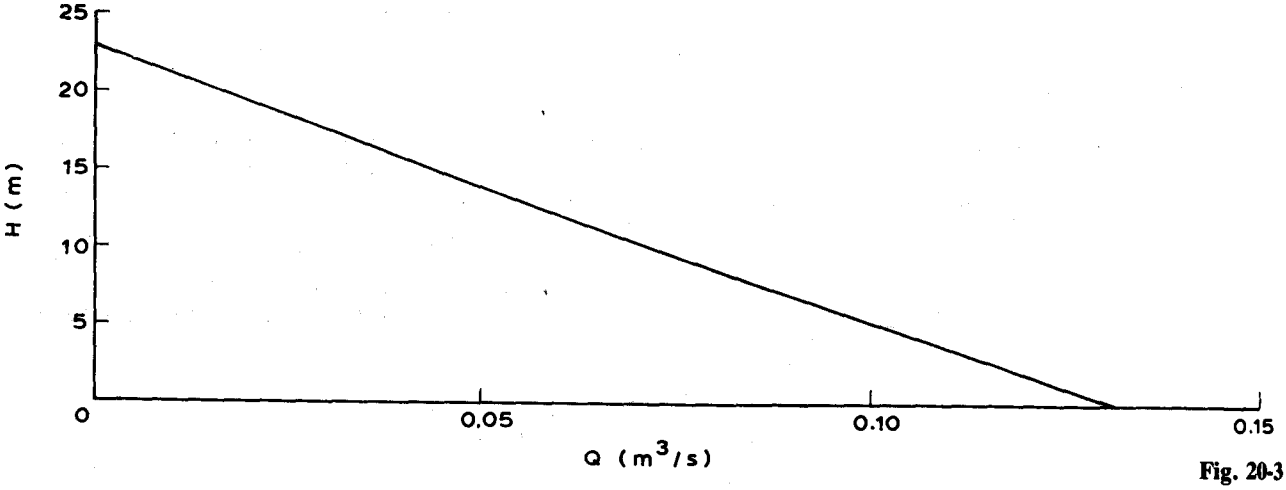


Fig. 20-3

20.21 A centrifugal pump (see Fig. 20-1*a*) has an impeller with dimensions $r_1 = 75$ mm, $r_2 = 150$ mm, $b_1 = 50$ mm, $b_2 = 30$ mm, $\beta_1 = \beta_2 = 30°$. For a discharge of 70 L/s and shockless entry to vanes, compute (***a***) the speed, (***b***) the head, (***c***) the torque, and (***d***) the power. Neglect losses ($\alpha_1 = 90°$).

▮
$$V_1 = V_{r1} = Q/2\pi r_1 b_1 = (70 \times 10^{-3})/[(2)(\pi)(0.075)(0.050)] = 2.971 \text{ m/s}$$
$$u_1 = V_1/(\tan \beta_1) = 2.971/(\tan 30°) = 5.146 \text{ m/s}$$

(***a***) $$N = (u_1/r_1)(60/2\pi) = [5.146/(0.075)](60/2\pi) = 655.2 \text{ rpm}$$

(***b***) $V_{r2} = Q/2\pi r_2 b_2 = (r_1 b_1/r_2 b_2)V_1 = [(75)(50)/(150)(30)](2.971) = 2.476$ m/s $\quad u_2/u_1 = r_2/r_1 = 150/75 = 2$

$u_2 = 2u_1 = (2)(5.146) = 10.292$ m/s $\quad V_{u2} = u_2 - V_{r2} \cot \beta_2 = 10.292 - 2.476 \cot 30° = 6.003$ m/s

$$H = u_2 V_{u2}/g = (10.292)(6.003)/9.807 = 6.30 \text{ m}$$

(***c***) $$T = \rho Q(r_2 V_{u2}) = (1000)(70 \times 10^{-3})(0.150)(6.003) = 63.0 \text{ N} \cdot \text{m}$$

(***d***) $$P = T\omega = T(u_1/r_1) = (63.0)[5.146/(0.075)] = 4323 \text{ W}$$

20.22 A centrifugal pump with impeller dimensions $r_1 = 2$ in, $r_2 = 5$ in, $b_1 = 3$ in, $b_2 = 1.5$ in, $\beta_2 = 60°$ is to pump 6 cfs at a 64-ft head. Determine (***a***) β_1, (***b***) the speed, (***c***) the pressure rise across the impeller. Neglect losses and assume no shock at the entrance ($\alpha_1 = 90°$).

▮
$$V_{r1} = Q/2\pi r_1 b_1 = 6/[(2)(\pi)(\tfrac{2}{12})(\tfrac{3}{12})] = 22.92 \text{ ft/s}$$
$$V_{2r} = Q/2\pi r_2 b_2 = (r_1 b_1/r_2 b_2)V_{r1} = [(2)(3)/(5)(1.5)](22.92) = 18.34 \text{ ft/s}$$
$$H = u_2 V_{u2}/g = u_2(u_1 - v_{r2} \cot \beta_2) \qquad u_2^2 - u_2 V_{r2} \cot \beta_2 - gH = 0$$
$$u_2 = (V_{r2} \cot \beta_2 + \sqrt{V_{r2}^2 \cot^2 \beta_2 + 4gH})/2 = [18.34 \cot 60° + \sqrt{18.34^2 \cot^2 60° + (4)(32.2)(64)}]/2 = 51.0 \text{ ft/s}$$
$$u_1 = (r_1/r_2)u_2 = (\tfrac{2}{5})(51.0) = 20.4 \text{ ft/s}$$

(***a***) $$\tan \beta_1 = V_{r1}/u_1 = 22.92/20.4 = 1.124 \qquad \beta_1 = 48.33°$$

(***b***) $$N = u_2/r_2 = [51.0/(\tfrac{5}{12})](60/2\pi) = 1169 \text{ rpm}$$

(***c***) $V_1 = V_{r1}/(\sin \beta_1) = 22.92/(\sin 48.33°) = 30.68$ ft/s $\quad V_2 = V_{r2}/(\sin \beta_2) = 18.34/(\sin 60°) = 21.18$ ft/s

$\Delta p = (\rho/2)(u_2^2 - u_1^2 - V_2^2 + V_1^2) = (1.94/2)(51.0^2 - 20.4^2 - 21.18^2 + 30.68^2) = 2597$ lb/ft² or 18.0 lb/in²

20.23 Evaluate r_1, r_2, β_1, β_2, b_1, and b_2 of a centrifugal impeller that will take 36 L/s of water from a 100-mm-diameter suction line and increase its energy head by 18 m ($N = 1440$ rpm; $\alpha_1 = 90°$). Neglect losses.

▮ Select sample values $r_1 = 50$ mm, $r_2 = 100$ mm, $b_1 = 40$ mm, $b_2 = 25$ mm. Then find β_1 and β_2.

$$V_1 = Q/2\pi r_1 b_1 = (36 \times 10^{-3})/[(2)(\pi)(0.050)(0.040)] = 2.865 \text{ m/s}$$

$$u_1 = \omega r_1 = [(2)(\pi)(1440)/60](0.050) = 7.540 \text{ m/s} \qquad u_2 = (r_2/r_1)u_1 = (\tfrac{100}{50})(7.540) = 15.08 \text{ m/s}$$

Since $\alpha_1 = 90°$, $\tan\beta_1 = V_1/u_1 = 2.865/7.540 = 0.37997$, $\beta_1 = 20.8°$.

$$V_{u2} = Hg/u_2 = (18)(9.807)/15.08 = 11.71 \text{ m/s} \qquad V_{r2} = V_{r1}(r_1 b_1/r_2 b_2) = (2.865)[(50)(40)/(100)(25)] = 2.292 \text{ m/s}$$

$$\tan\beta_2 = V_{r2}/(u_2 - V_{u2}) = 2.292/(15.08 - 14.04) = 2.20385 \qquad \beta_2 = 65.6°$$

Note: It is best to choose $r_1 = 50$ mm (100-mm inlet diameter). Then r_2, b_2, and b_1 can be arbitrarily selected within reasonable limits.

20.24 A mercury–water differential manometer, $R' = 740$ mm, is connected from the 100-mm-diameter suction pipe to the 80-mm-diameter discharge pipe of a pump, as shown in Fig. 20-4. The centerline of the suction pipe is 330 mm below the discharge pipe. For $Q = 3.42$ m³/min of water, calculate the head developed by the pump.

▮ $V_1 = Q/A_1 = (3.42/60)/[(\pi)(0.100)^2/4] = 7.257$ m/s $\qquad V_2 = V_1(D_1^2/D_2^2) = (7.257)(100^2/80^2) = 11.34$ m/s

The energy equation yields $p_1/\gamma + V_1^2/2g + H_p = p_2/\gamma + V_2^2/2g + h$, $H_p = \Delta p/\gamma + (V_2^2 - V_1^2)/2g + h$. The manometer equation yields $\Delta p/\gamma = R'(\text{s.g.} - 1) - h = (0.740)(13.6 - 1) - 0.330 = 8.99$ m of water, $H_p = 8.99 + (11.34^2 - 7.257^2)/[(2)(9.807)] + 0.330 = 13.19$ m.

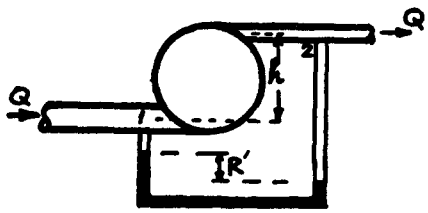


Fig. 20-4

20.25 An air blower is to be designed to produce pressure of 90 mmH₂O when operating at 3200 rpm; $r_2 = 1.1r_1$; $\beta_2 = \beta_1$; width of impeller is 100 mm; $\alpha_1 = 90°$. Find r_1. Assume a temperature of 30 °C.

▮ $Q = 2\pi r_1 b_1 V_1 = 2\pi r_2 b_2 V_2$. Since $r_2 = 1.1r_1$ and $b_1 = b_2$,

$$V_{r2} = V_{r1}/1.1 = V_1/1.1 \qquad u_2 = (r_2/r_1)u_1 = 1.1u_1$$

$$h_{\text{air}} = H_{\text{H}_2\text{O}}(\gamma_{\text{H}_2\text{O}}/\gamma_{\text{air}}) = (0.090)[(9.77)(1000)/11.4] = 77.13 \text{ m of air}$$

$$V_{u2} = gh_{\text{air}}/u_2 = (9.807)(77.13)/1.1u_1 = 687.6/u_1 \qquad u_2 = V_{u2} + V_{r2}\cot\beta$$

Since $\beta_1 = \beta_2 = \beta$ and $u_1 = V_1 \cot\beta$, $1.1u_1 = 687.6/u_1 + (V_1\cot\beta)/1.1 = 687.6/u_1 + u_1/1.1$, $u_1 = 60.0$ m/s; $r_1 = u_1/\omega = 60.0/[(3200)(2\pi)/60] = 0.179$ m, or 179 mm.

20.26 In Prob. 20.25, calculate the discharge for $\beta_1 = 32°$.

▮
$$V_1 = u_1 \tan\beta_1 = 60.0 \tan 32° = 37.5 \text{ m/s}$$

$$Q = 2\pi r_1 b_1 V_1 = (2)(\pi)(0.179)(0.100)(37.5) = 4.218 \text{ m}^3/\text{s} \quad \text{or} \quad 253 \text{ m}^3/\text{min}$$

20.27 What is the cavitation index at a point in flowing water where $T = 20$ °C, $p = 16$ kPa, and the velocity is 13 m/s.

▮
$$\sigma' = (p - p_v)/(\rho V^2/2) = (16 - 2.34)(1000)/[(998)(13)^2/2] = 0.162$$

20.28 Two reservoirs A and B are connected with a long pipe which has characteristics such that the head loss through the pipe is expressible as $h_L = 20Q^2$, where h_L is in feet and Q is the flow rate in hundreds of gpm. The water-surface elevation in reservoir B is 35 ft above that in reservoir A. Two identical pumps are available for use to pump the water from A to B. The characteristic curve of the pump when operating at 1800 rpm is given in the table on page 616.

At the optimum point of operation the pump delivers 200 gpm at a head of 75 ft. Determine the specific speed N_s of the pump and find the rate of flow under the following conditions: (***a***) a single pump operating at 1800 rpm; (***b***) two pumps in series, each operating at 1800 rpm; (***c***) two pumps in parallel, each operating at 1800 rpm.

▮ The head-capacity curves for the pumping alternatives are plotted in Fig. 20-5 and so is the h_L versus Q curve for the pipe system. In this case $h = \Delta z + h_L = 35 + 20Q^2$. The answers are found at the points of

operation at 1800 rpm	
head, ft	**flow rate, gpm**
100	0
90	110
80	180
60	250
40	300
20	340

intersection of the curves. They are as follows: (*a*) single pump, 156 gpm; (*b*) two pumps in series, 224 gpm; (*c*) two pumps in parallel, 170 gpm.

If Δz had been greater than 100 ft, neither the single pump nor the two pumps in parallel would have delivered any water. If Δz had been -20 ft (i.e., with the water-surface elevation in reservoir B 20 ft below that in A), the flows would have been (*a*) 212 gpm; (*b*) 258 gpm; and (*c*) 232 gpm.

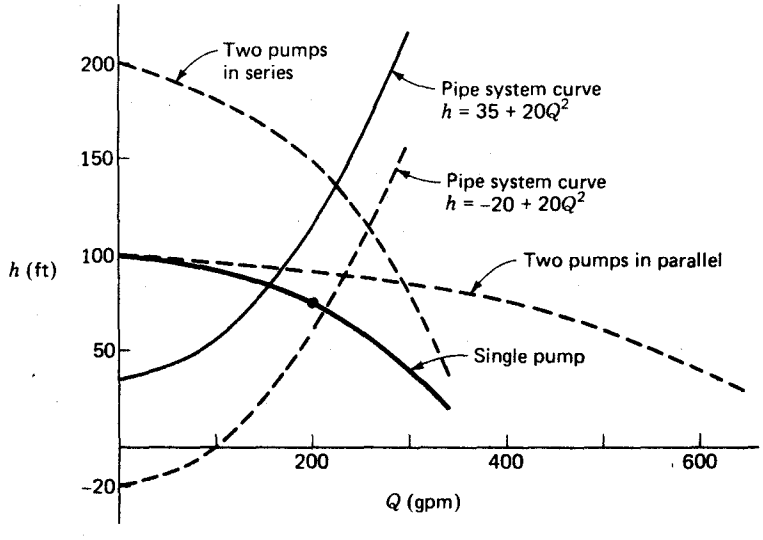


Fig. 20-5

20.29 The diameter of the discharge pipe of a pump is 8 in, and that of the intake pipe is 10 in. The pressure gage at discharge reads 32 psi, and the vacuum gage at intake reads 12 inHg. If $Q = 4.0\ \text{ft}^3/\text{s}$ of water and the brake horsepower is 49.0, find the efficiency. The intake and the discharge are at the same elevation.

▮ $$V_s = Q/A_s = 4.0/[(\pi)(\tfrac{10}{12})^2/4] = 7.334\ \text{ft/s} \qquad V_p = 4.0/[(\pi)(\tfrac{8}{12})^2/4] = 11.46\ \text{ft/s}$$

$$\text{vacuum pressure} = -(12/29.9)(14.7) = -5.900\ \text{lb/ft}^2$$

$$H_p = p_d/\gamma + V_d^2/2g + z_d - (p_s/\gamma + V_s^2/2g + z_s) = (32)(144)/62.4 + 11.46^2/[(2)(32.2)] + 0$$
$$- \{(-5.900)(144)/62.4 + 7.334^2/[(2)(32.2)] + 0\} = 88.67\ \text{ft}$$

$$P = Q\gamma H_p = (4.0)(62.4)(88.67)/550 = 40.24\ \text{hp} \qquad \eta = 40.24/49.0 = 0.821 \quad \text{or} \quad 82.1\%$$

20.30 A centrifugal pump with an impeller diameter of 2.94 in delivers 260 gpm of water at a head of 820 ft with an efficiency of 62 percent at 21 000 rpm. Compute the peripheral velocity u, specific speed N_s, and peripheral-velocity factor ϕ.

▮ $$u = \omega r = [(21\,000)(2\pi)/60][(2.94/2)/12] = 269\ \text{ft/s} \qquad N_s = NQ^{0.5}/H^{0.75} = (21\,000)(260)^{0.5}/820^{0.75} = 2210$$

$$u = \phi\sqrt{2gh} \qquad 269 = (\phi)\sqrt{(2)(32.2)(820)} \qquad \phi = 1.171$$

20.31 Select the specific speed of the pump or pumps required to lift 5835 gpm of water 350 ft through 9400 ft of 2.5-ft-diameter pipe ($f = 0.022$). The pump rotative speed is to be 1700 rpm. Consider the following cases: single pump, two pumps in series, two pumps in parallel.

$$Q = (5835)(0.002228) = 13\ \text{ft}^3/\text{s} \qquad V = Q/A = 13/[(\pi)(2.5)^2/4] = 2.648\ \text{ft/s}$$

$$h_f = (f)(L/D)(V^2/2g) = (0.022)(9400/2.5)\{2.648^2/[(2)(32.2)]\} = 9.0\ \text{ft}$$

$$h_p = 350 + 9.0 = 359.0\ \text{ft} \qquad N_s = NQ^{0.5}/H^{0.75}$$

	gpm per pump	h_p per pump	N_s
Single pump	5835	359.0	1575
Two pumps in series	5835	179.5	2648
Two pumps in parallel	2918	359.0	1113

20.32 The pump of Fig. 20-6*a* is used to lift water from one reservoir to another through a 9-in-diameter pipe ($f = 0.022$), 692 ft long. The difference Δz in surface elevation between the reservoirs fluctuates from 20 ft to 110 ft. Plot delivery rate Q versus pump head h_p. Plot also the corresponding efficiencies. The pump is operated at a constant speed of 1450 rpm. Neglect minor losses.

With Q in gpm,

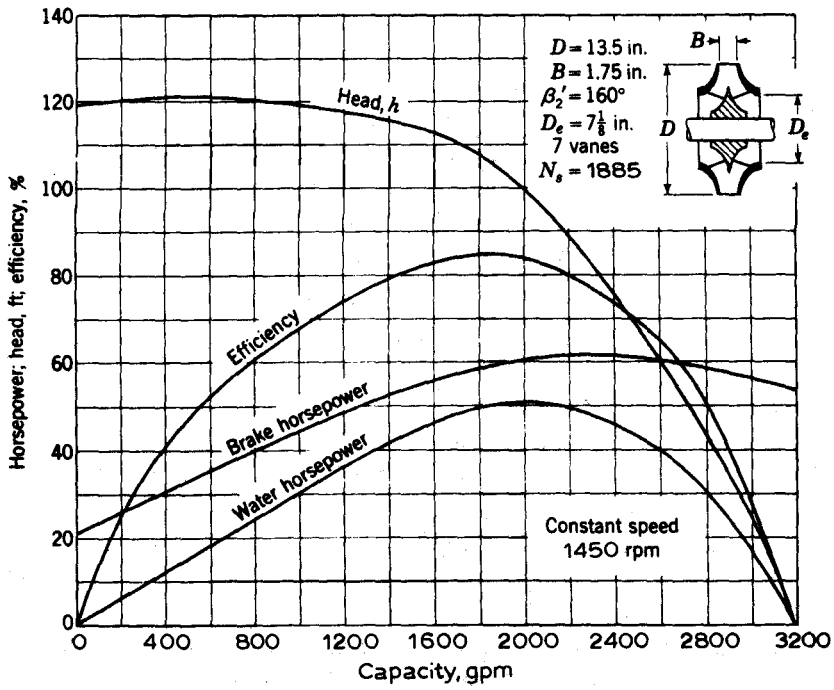
$$V = 0.002228Q/A = 0.002228Q/[(\pi)(\tfrac{9}{12})^2/4] = 0.0050Q$$

$$h_p = \Delta z + (f)(L/D)(V^2/2g) = \Delta z + (0.022)[692/(\tfrac{9}{12})]\{(0.0050Q)^2/[(2)(32.2)]\} = \Delta z + (8.0 \times 10^{-6})Q^2$$

Plot the pump characteristic curve and pipe system curves for a variety of Δz's.

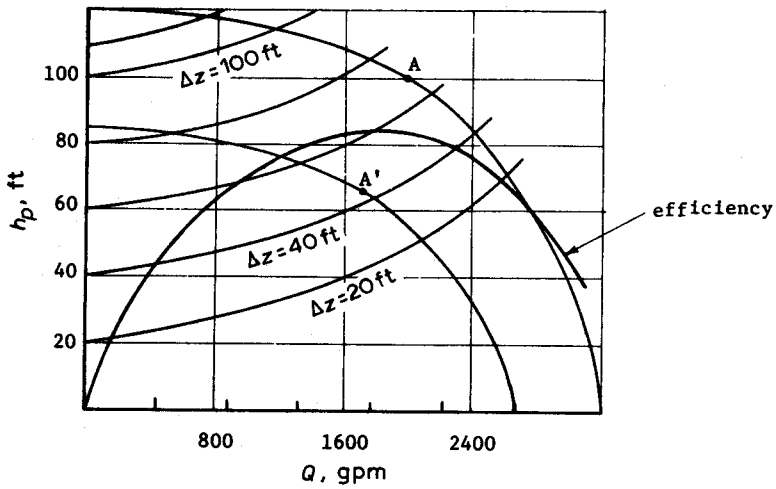
Δz, ft	Q, gpm	efficiency, %
20	2500	70
40	2320	76
60	2100	81
80	1860	83
100	1350	77
110	940	66

See plotted curves in Fig. 20-6*b*.



(*a*)

Fig. 20-6



(*b*)

Fig. 20-6 (*continued*)

20.33 Repeat Prob. 20.32 for the same pump operating at 1250 rpm. Assume efficiency pattern and values remain the same.

▮ The pump characteristic curve must be transformed using $Q \propto N$ and $h \propto N^2$. Thus point A in Fig. 20-6*b* transforms to A', where $Q' = (2000)(\frac{1250}{1450}) = 1724$ gpm and $h_p' = (100)(\frac{1250}{1450})^2 = 74.3$ ft, etc.

Δz, ft	Q, gpm	efficiency, %
20	2000	73
40	1750	82
60	1350	84
80	550	58
82.3	(shutoff)	0

The transformed pump characteristic curve is shown in Fig. 20-6*b*. (The transformed efficiency curve is not shown in the figure.)

20.34 Repeat Prob. 20.32 for a homologous pump whose diameter is 21% smaller. Assume efficiency pattern and values remain the same.

▮ In this case the pump characteristic curve is transformed using $Q \propto D^3$ and $h \propto D^2$. Thus point A in Fig. 20-6*b* transforms as follows: $Q = (2000)(0.79)^3 = 986$ gpm, $h = (100)(0.79)^2 = 62.4$ ft.

Δz, ft	Q, gpm	efficiency, %
20	1400	60
40	1200	76
60	800	81
76.8	(shutoff)	0

The transformed pump characteristic curve is shown in Fig. 20-7.

20.35 Repeat Prob. 20.32 if the pump diameter and rotation speed are reduced by 21% and 13.8%, respectively. Assume efficiency pattern and values remain the same.

▮ In this case the pump characteristic curve is transformed using $Q \propto ND^3$ and $h \propto N^2D^2$. Thus point A in Fig. 20-6*b* for the solution of Prob. 20.32 transforms as follows: $Q = (2000)(0.862)(0.79)^3 = 850$ gpm, $h = (100)(0.862)^2(0.79)^2 = 46.4$ ft. Transforming the pump characteristic curve according to these rules and following

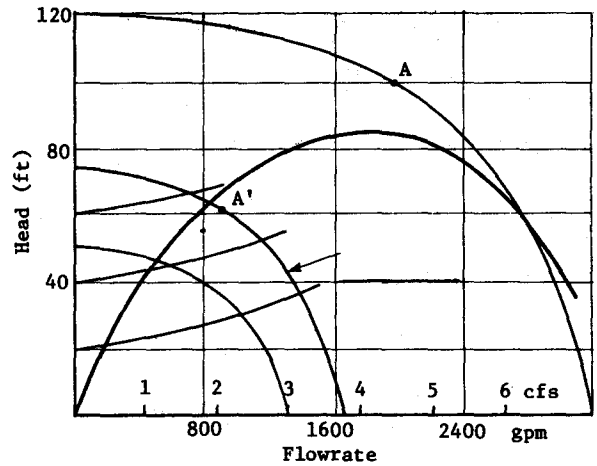


Fig. 20-7

the procedure as outlined in Prob. 20.34 gives

Δz, ft	Q, gpm	efficiency, %
20	1080	69
40	700	83
52.6	Shutoff	0

20.36 A pump with a critical value of σ_c of 0.10 is to pump against a head of 450 ft. The barometric pressure is 14.4 psia, and the vapor pressure of the water is 0.6 psia. Assume the friction losses in the intake piping are 6 ft. Find the maximum allowable height of the pump relative to the water surface at intake.

▮ $$(z_s)_{max} = (p_0)_{abs}/\gamma - p_v/\gamma - \sigma_c h - h_L \qquad p_0/\gamma = (14.4)(144)/62.4 = 33.2 \text{ ft}$$

$$p_v/\gamma = (0.6)(144)/62.4 = 1.385 \text{ ft} \qquad (z_s)_{max} = 33.2 - 1.385 - (0.10)(450) - 6 = -19.2 \text{ ft} \qquad \text{(submerged)}$$

20.37 A pump draws water at 212 °F from a boiler with a friction loss of 3 ft in the intake pipe. Barometric pressure is 29 in of mercury, σ_c for the pump is 0.10, and the total pumping head is 300 ft. Locate the pump intake relative to the water surface in the boiler.

▮ $(z_s)_{max} = (p_0)_{abs}/\gamma - p_v/\gamma - \sigma_c h - h_L \qquad p_0 = (\frac{29}{12})(847.3/144) = 14.2 \text{ psia} \qquad p_0/\gamma = (14.2)(144)/59.8 = 34.2 \text{ ft}$

$$p_v/\gamma = (14.7)(144)/59.8 = 35.4 \text{ ft} \qquad (z_s)_{max} = 34.2 - 35.4 - (0.10)(300) - 3 = -34.2 \text{ ft}$$

Thus the water surface must be 34.2 ft above the pump suction.

20.38 Water is to be pumped at a head of 142 ft, the water temperature being 100 °F and the barometric pressure being 14.3 psia. At intake, the pressure is a vacuum of 15 inHg and the velocity is 13 fps. What are the values of NPSH and σ?

▮ $\text{NPSH} = (p_s)_{abs}/\gamma + V_s^2/2g - p_v/\gamma \qquad (p_s)_{abs} = p_0/\gamma + p_s/\gamma = (14.3)(144)/62.0 + (-\frac{15}{12})(847.3/62.0) = 16.13 \text{ ft}$

$$V_s^2/2g = 13^2/[(2)(32.2)] = 2.62 \text{ ft} \qquad p_v/\gamma = 135/62.0 = 2.18 \text{ ft}$$

$$\text{NPSH} = 16.13 + 2.62 - 2.18 = 16.57 \text{ ft} \qquad \sigma = 16.57/142 = 0.117$$

20.39 A pump discharges 7700 gpm of water at 140 °F at a head of 250 ft, the barometric pressure being 13.8 psia. Determine the reading on a pressure gage at the suction flange when cavitation is just starting. Assume the suction pipe diameter equals 2.1 ft and neglect the effects of prerotation. Take $\sigma_c = 0.085$.

▮ $\sigma = [(p_s)_{abs}/\gamma + V_s^2/2g - p_v/\gamma]/h$. Let p = gage pressure at suction flange.

$$(p_s)_{abs} = p_{atm} + p = 13.8 + p \qquad V_s = Q/A_s = (7700)(0.002228)/[(\pi)(2.1)^2/4] = 4.953 \text{ ft/s}$$

$$p_v/\gamma = 416/61.4 = 6.775 \text{ ft} \qquad 0.085 = \{(13.8 + p)(144)/61.4 + 4.953^2/[(2)(32.2)] - 6.775\}/250$$

$$p = -2.013 \text{ psia} = +2.013 \text{ psia vacuum}$$

20.40 If the maximum efficiency of the pump of Prob. 20.32 is 81%, approximately what would be the efficiency of the pump of Prob. 20.34?

▮ $(1-\eta_1)/(1-\eta_2) = (D_2/D_1)^{1/5}$ $\quad (1-0.81)/(1-\eta_2) = (0.79/1.0)^{1/5}$ $\quad \eta_2 = 0.801$ or 80.1%

20.41 Figure 20-8 shows the dimensions and angles of the diffuser vanes of a centrifugal pump. The vane passages are 0.75 in wide perpendicular to the plane of the figure. If the impeller delivers water at the rate of 3 cfs under ideal and frictionless conditions, what is the rise in pressure through the diffuser?

▮ $V_1^2/2g = V_2^2/2g + \Delta p/\gamma \qquad V = Q/A \qquad A_1 = (2\pi)(\frac{6.5}{12})(\sin 15°)(0.75/12) = 0.05505\ \text{ft}^2$

$$V_1 = 3/0.05505 = 54.50\ \text{ft/s} \qquad A_2 = (2\pi)(9/12)(\sin 28°)(0.75/12) = 0.1382\ \text{ft}^2$$

$$V_2 = 3/0.1382 = 21.71\ \text{ft/s}$$

$$54.50^2/[(2)(32.2)] = 21.71^2/[(2)(32.2)] + \Delta p/\gamma \qquad \Delta p/\gamma = 38.80\ \text{ft} \qquad \Delta p = (62.4)(38.80)/144 = 16.8\ \text{lb/in}^2$$

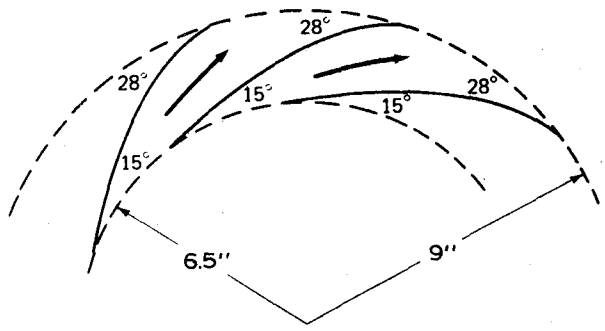


Fig. 20-8

20.42 Two pumps whose characteristics are given in Prob. 20.28 are to be used in parallel. They must develop a head $h = 35 + 20Q^2$ as in Prob. 20.28. One pump is to be operated at 1800 rpm. The speed of the other pump is to be gradually reduced until it no longer delivers water. At approximately what speed will this happen?

▮ From Prob. 20.28, with a single pump operating at 1800 rpm, the flow rate is 156 gpm and the head is 83.8 ft. If we add a pump in parallel to this pump with a shutoff head of 83.8 ft, the second pump will convey no flow. Thus, $h/h' = (n/n')^2$, $83.8/100 = (n/1800)^2$, $n = 1648$ rpm.

20.43 A pump is installed to deliver water from a reservoir of surface elevation zero to another of elevation 290 ft. The 10-in-diameter suction pipe ($f = 0.022$) is 90 ft long, and the 8-in-diameter discharge pipe ($f = 0.028$) is 4900 ft long. The pump characteristic at 1100 rpm is defined by $h_p = 365 - 20Q^2$ where h_p, the pump head, is in feet and Q is in cubic feet per second. Compute the rate at which this pump will deliver water under these conditions assuming the setting is low enough to avoid cavitation.

▮ $h_p = 365 - 20Q^2$. Also, since $h_f = (f)(L/D)(V^2/2g)$,

$$h_p = 290 + (0.022)[90/(\tfrac{10}{12})]V_{10}^2/2g + (0.028)(4900)(\tfrac{8}{12})V_8^2/2g \qquad V = Q/A$$

$$V_{10} = Q/[(\pi)(\tfrac{10}{12})^2/4] = 1.833Q \qquad V_8 = Q/[(\pi)(\tfrac{8}{12})^2/4] = 2.865Q$$

$$h_p = 290 + (0.022)[90/(\tfrac{10}{12})]\{(1.833Q)^2/[(2)(32.2)]\}$$
$$+ (0.028)[4900/(\tfrac{8}{12})]\{(2.865Q)^2/[(2)(32.2)]\} = 290 + 26.35Q^2$$

Equating the two expressions for h_p gives $365 - 20Q^2 = 290 + 26.35Q^2$, $Q = 1.27\ \text{ft}^3/\text{s}$.

20.44 Repeat Prob. 20.43 for the case of two such pumps in parallel.

▮ From Prob. 20.43, h_p per pump $= 290 + 26.35Q^2 = 365 - 20(Q/2)^2$, or $Q = 1.55\ \text{ft}^3/\text{s}$.

20.45 A centrifugal pump with a 14-in-diameter impeller is rated at 690 gpm against a head of 91 ft when rotating at 1800 rpm. What would be the rating of a pump of identical geometry with a 9-in impeller? Assume pump efficiencies and rotative speeds are identical.

▮ $h \propto D^2 \qquad h = (91)(\frac{9}{12})^2 = 51.2\text{ ft} \qquad Q \propto D^3 \qquad Q = (690)(\frac{9}{12})^3 = 291\text{ gpm}$

20.46 The following are data for a centrifugal water pump: $r_1 = 4$ in, $r_2 = 7$ in, $\beta = 30°$, $\beta_2 = 20°$, speed = 1440 rpm. If $b_1 = b_2 = 1.75$ in, estimate **(*a*)** design-point discharge; **(*b*)** water horsepower; **(*c*)** head.

▮ **(*a*)** The angular velocity is $\omega = 2\pi(\frac{1440}{60}) = 150.8$ rad/s. Thus the tip speeds are $u_1 = \omega r_1 = 150.8(\frac{4}{12}) = 50.3$ ft/s, and $u_2 = \omega r_2 = 150.8(\frac{7}{12}) = 88.0$ ft/s. From the inlet-velocity diagram, Fig. 20-9*a*, with $\alpha_1 = 90°$ for design point, we compute $V_{n1} = u_1 \tan 30° = 29.0$ ft/s, whence the discharge is $Q = 2\pi r_1 b_1 V_{n1} = 2\pi(\frac{4}{12})(1.75/12)(29.0) = 8.86\text{ ft}^3/\text{s} = 3980$ gal/min.

(*b*) The outlet radial velocity follows from Q: $V_{n2} = Q/(2\pi r_2 b_2) = 8.86/[2\pi(\frac{7}{12})(1.75/12)] = 16.6$ ft/s. This enables us to construct the outlet-velocity diagram as in Fig. 20-9*b*, given $\beta_2 = 20°$. The tangential component is $V_{t2} = u_2 - V_{n2}\cot\beta_2 = 88.0 - 16.6\cot 20° = 42.4$ ft/s, $\alpha_2 = \tan^{-1}(16.6/42.4) = 21.4°$. The power is then computed with $V_{t1} = 0$ at the design point: $P_w = pQu_2V_{t2} = [(1.94)(8.8)(88.0)(42.4)]/550 = 117$ hp.

(*c*) The head is estimated as $H \approx P_w/\rho g Q = (117)(550)/[(62.4)(8.86)] = 116$ ft.

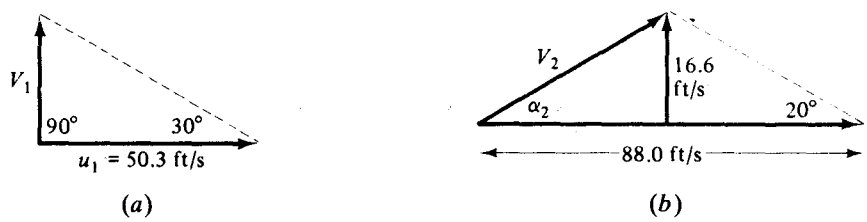
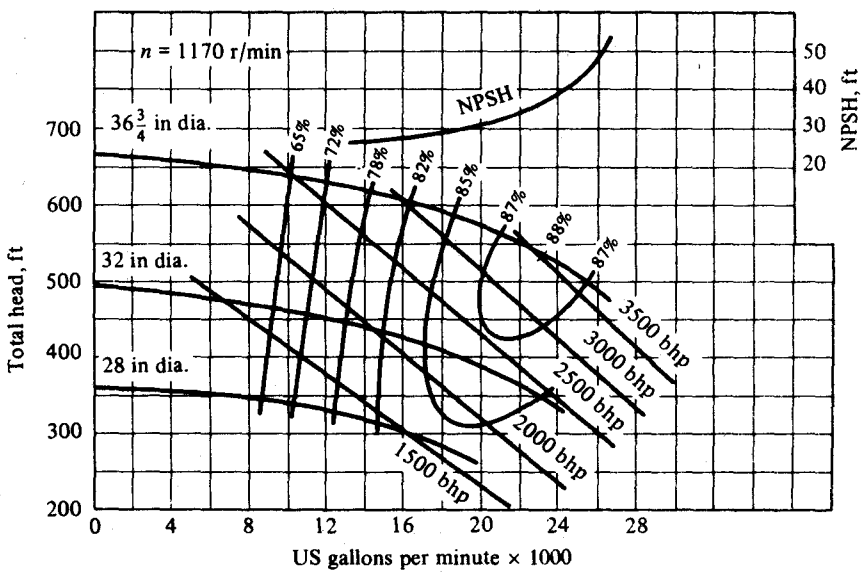


Fig. 20-9

20.47 The 32-in pump of Fig. 20-10 is to pump 24 000 gal/min of water at 1170 rpm from a reservoir whose surface is at 14.7 lb/in² abs. If head loss from reservoir to pump inlet is 6 ft, where should the pump inlet be placed to avoid cavitation for water at **(*a*)** 60 °F, $p_v = 0.26$ lb/in² abs, s.g. = 1.0 and **(*b*)** 200 °F, $p_v = 11.52$ lb/in² abs, s.g. = 0.9635?

▮ For either case read from Fig. 20-10 at 24 000 gal/min that the required NPSH is 40 ft.

(*a*) For this case $\rho g = 62.4\text{ lb/ft}^3$. NPSH $\le [(p_a - p_v)/\rho g] - Z_i - h_{fi}$ or 40 ft $\le [(14.7 - 0.26)(144)/62.4] - Z_i - 6.0$ or $Z_i \le 27.3 - 40 = -12.7$ ft. The pump must be placed at least 12.7 ft *below* the reservoir surface to avoid cavitation.



(From Ingersoll-Rand Corporation, Cameron Pump Division)

Fig. 20-10

(*b*) For this case $\rho g = 62.4(0.9635) = 60.1\ \text{lb/ft}^3$. $40\ \text{ft} \le [(14.7 - 11.52)(144)/60.1] - Z_i - 6.0$ or $Z_i \le 1.6 - 40 = 38.4$ ft. The pump must now be placed at least 38.4 ft below the reservoir surface. These are unusually stringent conditions because a large, high-discharge pump requires a large NPSH.

20.48 A pump from the family of Fig. 20-11 has $D = 21$ in and $n = 1500$ rpm. Estimate (*a*) discharge, (*b*) head, (*c*) pressure rise, and (*d*) brake horsepower of this pump for water at 60 °F and best efficiency.

▮ (*a*) In BG units take $D = \frac{21}{12} = 1.75$ ft and $n = \frac{1500}{60} = 25$ r/s. At 60 °F, ρ of water is 1.94 slugs/ft^3. The BEP parameters are known from Fig. 20-11. The BEP discharge is thus $Q^* = C_Q \cdot nD^3 = 0.115(25)(1.75)^3 = 15.4\ \text{ft}^3/\text{s} = 6920$ gal/min.

(*b*) Similarly, the BEP head is $H^* = C_H \cdot n^2D^2/g = 5.0(25)^2(1.75)^2/32.2 = 297$ ft water.

(*c*) Since we are not given elevation or velocity-head changes across the pump, we neglect them and estimate $\Delta p \approx \rho g H = 1.94(32.2)(297) = 18\,600\ \text{lb/ft}^2 = 129\ \text{lb/in}^2$.

(*d*) Finally, the BEP power is $P^* = C_P \cdot \rho n^3 D^5 = 0.65(1.94)(25)^3(1.75)^5/550 = 588$ hp.

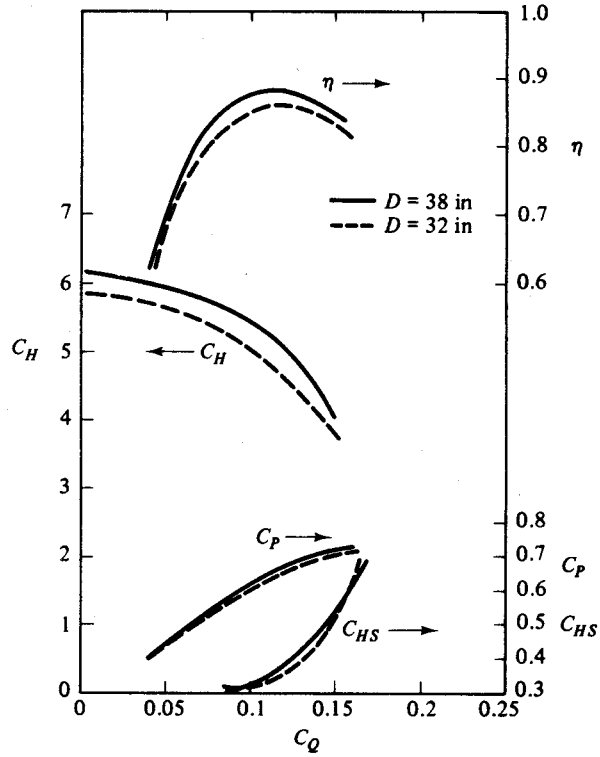


Fig. 20-11

20.49 We want to build a pump from the family of Fig. 20-11, which delivers 3000 gal/min water at 1200 rpm at best efficiency. Estimate (*a*) the impeller diameter, (*b*) the maximum discharge, (*c*) the shutoff head, and (*d*) the NPSH at best efficiency.

▮ (*a*) 3000 gal/min = 6.68 ft^3/s and 1200 rpm = 20 r/s. At BEP we have $Q^* = C_Q \cdot nD^3 = 6.68\ \text{ft}^3/\text{s} = (0.115)(20)D^3$ or $D = [6.68/0.115(20)]^{1/3} = 1.43$ ft = 17.1 in.

(*b*) The max Q is related to Q^* by a ratio of capacity coefficients: $Q_{max} = Q^* C_{Q\cdot max}/C_{Q^*} \approx 3000(0.23)/0.115 = 6000$ gal/min.

(*c*) From Fig. 20-11 we estimated the shutoff-head coefficient to be 6.0. Thus $H(0) \approx C_H(0)n^2D^2/g = 6.0(20)^2(1.43)^2/32.2 = 152$ ft.

(*d*) Finally, the NPSH at BEP is approximately $\text{NPSH}^* = C_{HS} \cdot n^2D^2/g = 0.37(20)^2(1.43)^2/32.2 = 9.4$ ft. Since this is a small pump, it will be less efficient than the pumps in Fig. 20-11, probably about 82 percent maximum.

20.50 We want to use a centrifugal pump from the family of Fig. 20-11 to deliver 100 000 gal/min of water at 60 °F with a head of 25 ft. What should be (*a*) the pump size and speed and (*b*) brake horsepower assuming operation at best efficiency?

▮ (a) Enter the known head and discharge into the BEP parameters: $H^* = 25\ \text{ft} = C_H \cdot n^2D^2/g = 5.0n^2D^2/32.2$, $Q^* = 100\,000\ \text{gal/min} = 222.8\ \text{ft}^3/\text{s} = C_Q \cdot nD^3 = 0.115nD^3$. The two unknowns are n and D. Solve simultaneously for $D = 12.4$ ft, $n = 1.03$ r/s = 62 rpm.

(*b*) The most efficient horsepower is, then, bhp* $\approx C_p \cdot \rho n^3 D^5 = 0.65(1.94)(1.03)^3(12.4)^5/550 = 734$ hp.

The solution to this problem is mathematically correct but results in a grotesque pump: an impeller more than 12 ft in diameter, rotating so slowly one can visualize oxen walking in a circle turning the shaft.

20.51 We want to use the 32-in pump of Fig. 20-10 at 1170 rpm to pump water at 60 °F from one reservoir to another 120 ft higher through 1500 ft of 16-in-ID pipe with friction factor $f = 0.030$. (*a*) What will the operating point and efficiency be? (*b*) To what speed should the pump be changed to operate the BEP?

▮ (*a*) For reservoirs the initial and final velocities are zero; thus the system head is $H_s = z_2 - z_1 + (V^2/2g)(fL/D) = 120 + (V^2/2g)[0.030(1500)/\frac{16}{12}]$. From continuity in the pipe, $V = Q/A = Q/[\frac{1}{4}\pi(\frac{16}{12})^2]$, and so we substitute for V above to get

$$H_s = 120 + 0.269Q^2 \qquad Q \text{ in ft}^3/\text{s} \tag{1}$$

Since Fig. 20-10 uses thousands of gallons per minute for the abscissa, we convert Q in Eq. (*1*) to this unit:

$$H_s = 120 + 1.335Q^2 \qquad Q \text{ in } 10^3 \text{ gal/min} \tag{2}$$

We can plot Eq. (*2*) on Fig. 20-10 and see where it intersects the 32-in pump-head curve, as in Fig. 20-12. A graphical solution gives approximately $H \approx 430$ ft, $Q \approx 15\,000$ gal/min. The efficiency is about 82 percent, slightly off design.

An analytic solution is possible if we fit the pump-head curve to a parabola, which is very accurate:

$$H_{\text{pump}} \approx 490 - 0.26Q^2 \qquad Q \text{ in } 10^3 \text{ gal/min} \tag{3}$$

Equations (*2*) and (*3*) must match at the operating point: $490 - 0.26Q^2 = 120 + 1.335Q^2$ or $Q^2 = (490 - 120)/(0.26 + 1.335) = 232$, $Q = 15.2 \times 10^3$ gal/min $= 15\,200$ gal/min; $H = 490 - 0.26(15.2)^2 = 430$ ft.

(*b*) To move the operating point to BEP, we change n, which changes both $Q \propto n$ and $H \propto n^2$. From Fig. 20-10, at BEP, $H^* \approx 386$ ft; thus for any n, $H^* = 386(n/1170)^2$. Also read $Q^* \approx 20 \times 10^3$ gal/min; thus for any n, $Q^* = 20(n/1170)$. Match H^* to the system characteristics, Eq. (*2*), $H^* = 386(n/1170)^2 \approx 120 + 1.335[20(n/1170)]^2$, which gives $n^2 < 0$. Thus it is impossible to operate at maximum efficiency with this particular system and pump.

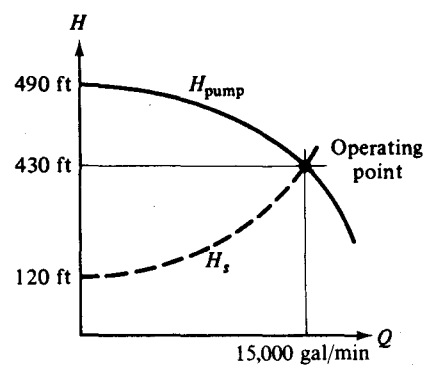


Fig. 20-12

20.52 Investigate extending Prob. 20.51 by using two 32-in pumps in parallel to deliver more flow. Is this efficient?

▮ Since the pumps are identical, each delivers $\frac{1}{2}Q$ at the same 1170 rpm speed. The system curve is the same, and the balance-of-head relation becomes $H = 490 - 0.26(\frac{1}{2}Q)^2 = 120 + 1.335Q^2$ or $Q^2 = (490 - 120)/(1.335 + 0.065)$, $Q = 16\,300$ gal/min. This is only 7 percent more than a single pump. Each pump delivers $\frac{1}{2}Q = 8150$ gal/min, for which the efficiency is only 60 percent. The total brake horsepower required is 3200, whereas a single pump used only 2000 bhp.

20.53 Suppose the elevation change in Prob. 20.51 is raised from 120 to 500 ft, greater than a single 32-in pump can supply. Investigate using 32-in pumps in series at 1170 rpm.

▮ Since the pumps are identical, the total head is twice as much and the constant 120 in the system-head curve is replaced by 500. The balance of heads becomes $H = 2(490 - 0.26Q^2) = 500 + 1.335Q^2$ or $Q^2 = (980 - 500)/(1.335 + 0.52)$, $Q = 16.1 \times 10^3$ gal/min. The operating head is $500 + 1.335(16.1)^2 = 846$ ft, or 97 percent more than a single pump in Prob. 20.50. Each pump is operating at 16.1×10^3 gal/min, which from Fig. 20-10 is 83 percent efficient, a pretty good match to the system. To pump at this operating point requires 4100 bhp, or about 2050 bhp for each pump.

20.54 A piston positive-displacement pump (PDP) has a 6-in diameter and a 2.5-in stroke. Its crankshaft rotates at 300 rpm. Calculate its output at 100 percent volumetric efficiency.

▮ $V_{\text{displaced}} = [(\pi)(\frac{6}{12})^2/4](\frac{2.5}{12}) = 0.04091\ \text{ft}^3 \qquad Q = (0.04091)(300) = 12.27\ \text{ft}^3/\text{min}$

20.55 If the PDP of Prob. 20.54 delivers water against a total head of 24 ft, what horsepower is required at 72% efficiency?

▮ $P = Q\gamma E_p = (12.27/60)(62.4)(24) = 306.3\ \text{ft-lb/s} \qquad P_{\text{required}} = (306.3/550)/0.72 = 0.77\ \text{hp}$

20.56 A pump delivers 1.6 m^3/min of water at 20 °C against a pressure rise of 287 kPa. Kinetic- and potential-energy changes are negligible. If the driving motor supplies 10 kW, what is the overall efficiency?

▮ $P = Q\gamma E_p = \Delta p\, Q = (287)(1.6/60) = 7.65\ \text{kN}\cdot\text{m/s} = 7.65\ \text{kW} \qquad \eta = 7.65/10 = 0.765$

20.57 A 26-hp pump delivers 475 gpm of gasoline ($\gamma = 42.5\ \text{lb/ft}^3$) at 20 °C with 78% efficiency. What head and pressure rise result across the pump?

▮ $P_{\text{required}} = Q\gamma E_p/\eta \qquad (26)(550) = [(475)(0.002228)](42.5)(E_p)/0.78 \qquad E_p = 248\ \text{ft}$

$$\Delta p = \gamma E_p = (42.5)(248) = 10\,540\ \text{lb/ft}^2 \quad \text{or} \quad 73.2\ \text{lb/in}^2$$

20.58 A sump pump turning at 1600 rpm provides a discharge of 9 gpm against a head of 18 ft. Compute the minimum power input.

▮ $N_s = NQ^{0.5}/H^{0.75} = (1600)(9)^{0.5}/18^{0.75} = 549$. From Fig. A-41, $\eta_{\max} = 0.32$. $P_{\min} = Q\gamma H/\eta_{\max} = [(9)(0.002228)](62.4)(18)/0.32 = 70.4$ ft-lb/s, or 0.128 hp.

20.59 A model pump delivering water at 180 °F at 900 gpm and 2500 rpm begins to cavitate when the inlet pressure and velocity are 13 psia and 22 fps. Find the required NPSH of a prototype which is 4 times larger and runs at 1100 rpm.

▮ $\text{NPSH} = (p_i - p_v)/\gamma + V_i^2/2g \qquad \text{NPSH}_{\text{model}} = [(13)(144) - 1086]/60.6 + 22^2/[(2)(32.2)] = 20.49\ \text{ft}$

$$\text{NPSH}_{\text{proto}} = (\text{NPSH}_{\text{model}})(N_p/N_m)^2(D_p/D_m)^2 = (20.49)(\tfrac{1100}{2500})^2(4)^2 = 63.5\ \text{ft}.$$

20.60 Determine the specific speed of the pump whose operating characteristics are shown in Fig. 20–13. If this pump were operated at 1200 rpm, what head and discharge would be developed at rated capacity, and what power would be required?

▮ $N_s = NQ^{0.5}/H^{0.75}$. At its rated capacity of 700 gpm, this pump develops 120 ft of head when operating at 1450 rpm. Thus $N_s = (1450)(700)^{0.5}/120^{0.75} = 1058$. If the efficiency remains constant with the speed change, at 1200 rpm, $Q = (\frac{1200}{1450})(700) = 579$ gpm, $H = (\frac{1200}{1450})^2(120) = 82.2$ ft, $P = (\frac{1200}{1450})^3(26) = 14.7$ hp.

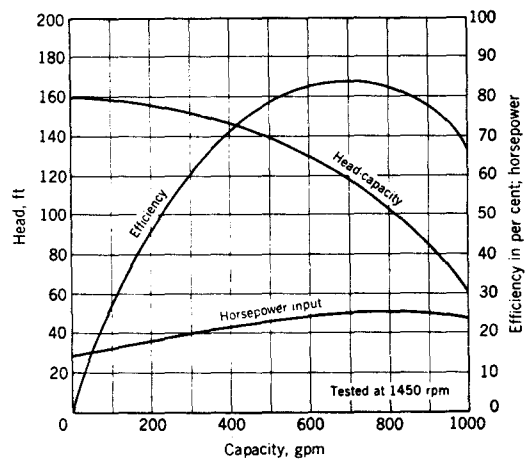


Fig. 20-13

20.61 A mixed-flow centrifugal pump ($N_s = 400$, $\sigma_c = 0.30$) is to develop 50 ft of head. Find the maximum permissible suction lift on the pump if the pump is at sea level and the water is at 80 °F.

▮ Neglecting head loss between reservoir and pump, $z_s \approx (p_{atm}/\gamma - p_v/\gamma) - \text{NPSH}$, $\text{NPSH} = \sigma_c h = (0.30)(50) = 15.0$ ft, $z_s \approx [(14.7)(144)/62.4 - 73.5/62.4] - 15.0 = 17.7$ ft.

20.62 A centrifugal pump (efficiency 79%) driven by an electric motor (efficiency 88%) lifts water through a total height of 135 ft from reservoir to discharge. Pumping is through 977 ft of 3-in-diameter pipe at the rate of 290 gpm. If $f = 0.025$ and electricity costs \$7 per megawatt-hour, find the cost of pumping one million gallons.

▮
$$Q = (290)(0.002228) = 0.6461 \text{ ft}^3/\text{s} \qquad V = Q/A = 0.6461/[(\pi)(\tfrac{3}{12})^2/4] = 13.16 \text{ ft/s}$$

$$h_f = (f)(L/D)(V^2/2g) = (0.025)[977/(\tfrac{3}{12})]\{13.16^2/[(2)(32.2)]\} = 262.7 \text{ ft} \qquad E_p = 135 + 262.7 = 397.7 \text{ ft}$$

$$P_{H_2O} = Q\gamma E_p = (0.6461)(62.4)(397.7) = 16\,034 \text{ ft-lb/s} = 21.74 \text{ kW}$$

$$P_{motor} = 21.74/[(0.79)(0.88)] = 31.27 \text{ kW} = 0.03127 \text{ MW}$$

$$\text{Cost to pump 1 000 000 gallons} = \left(\frac{10^6 \text{ gal}}{290 \times 60 \text{ gal/h}}\right)(0.03127 \text{ MW})\left(\frac{\$7}{1 \text{ MWh}}\right) = \$12.59$$

20.63 A model of a mixed-flow water pump has been tested in the laboratory to give performance curves as is shown in Fig. 20-14. What would be the total head ΔH delivered by a prototype pump with an impeller size of 1.2 m operating at a speed of 1750 rpm and delivering 1.300 m³/s flow? What is its mechanical efficiency? Consider that dynamic similarity can be achieved between model flow and prototype flow.

▮ We first compute the flow coefficient for the prototype flow: $(Q/ND^3)_p = 1.300/\{[(1750)(2\pi)/60](1.2^3)\} = 0.00411$. Examining Fig. 20-14 we see that the efficiency for this operating point is about 75 percent for the prototype. Also note that the head coefficient is about 0.17. Therefore we have for the head ΔH of the prototype $(g\,\Delta H)/N^2D^2 = 0.17$, $\Delta H = (0.17/9.81)[1750(2\pi/60)]^2(1.2^2) = 838$ m.

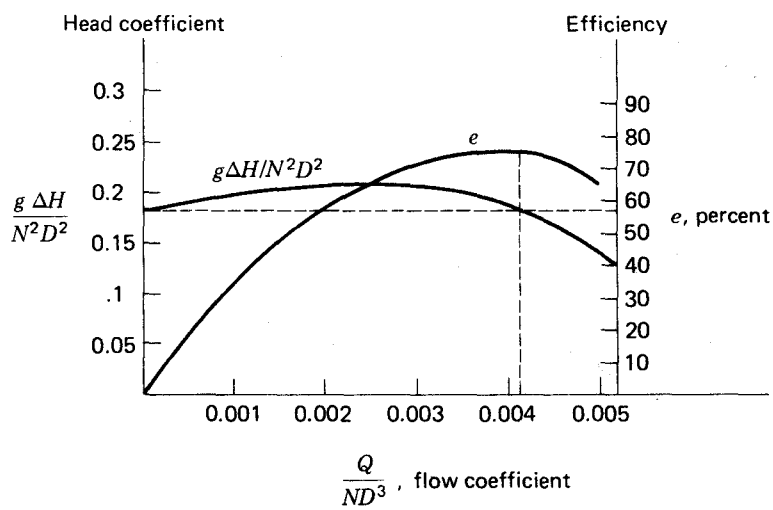


Fig. 20-14

20.64 A centrifugal blower compresses 1.4 lbm/s of air at a temperature of 60 °F and a pressure of 13.00 psia to a pressure of 14.50 psia. The impeller runs at 3500 rpm. The exit area of the blower is 0.14 ft² and the inlet area is 0.4 ft². How much power is needed to run the blower at 80% efficiency?

▮
$$(dW_p/dt)_{theo} = \{[(V_2)^2_{theo} - V_1^2]/2 + c_pT_1[(p_2/p_1)^{(k-1)/k} - 1]\}\dot{m}$$

Evaluate the two velocities:

$$p_1 = \rho_1 RT_1 \qquad \rho_1 = (13)(144)/[(53.3)(g)(520)] = 0.002098 \text{ slug/ft}^3 \qquad \rho_1/\rho_2 = (p_1/p_2)^{1/k}$$

$$\rho_2 = (0.002098)(14.5/13)^{1/1.4} = 0.002268 \text{ slug/ft}^3$$

$$\dot{m} = (1.4/g) = \rho_2(V_2)_{theo}A_2 \qquad 1.4/g = (0.002268)(V_2)_{theo}(0.14) \qquad (V_2)_{theo} = 136.93 \text{ ft/s}$$

Also $\dot{m} = 1.4/g = \rho_1V_1A_1 = (0.002098)(V_1)(0.4)$, $V_1 = 51.81$ ft/s. Consequently,

$$(dW_p/dt)_{theo} = \{[(136.93^2 - 51.81^2)/2] + (0.24)(778)(g)(520)[(14.5/13)^{0.4/1.4} - 1]\}(1.4/g)/550 = 8.47 \text{ hp}$$

$$(dW_p/dt)_{act} = 8.47/0.80 = 10.59 \text{ hp}$$

20.65 In Prob. 20.64, if the impeller has an outside radius of 9 in and is 1.6 in wide, what should the blade angle β_2 be at exit?

▮
$$(dW_p/dt)_{\text{theo}} = (\omega r_2 \dot{m})[(U_t)_2 + (V_r)_2 \cot \beta_2]$$

$$(8.47)(550) = [(3500)(2\pi)/(60)](\tfrac{9}{12})(1.4/g)\{[(3500)(2\pi)/60](\tfrac{9}{12}) + (V_r)_2 \cot \beta_2\} \qquad (1)$$

Writing the continuity equation as $\dot{m} = \rho_2(V_r)_2(2\pi r_2)(b)$ and taking ρ_2 from Prob. 20.64, we find

$$1.4/g = (0.002268)(V_r)_2(2\pi)(\tfrac{9}{12})(1.6/12) \qquad (V_r)_2 = 30.51 \text{ ft/s}$$

Now Eq. (*1*) is solved to give $\beta_2 = 14.86°$.

20.66 An air compressor has a mass flow of 0.7 kg/s, taking the fluid from a pressure of 100 kPa at the inlet to 101.67 kPa at the outlet. The inlet temperature is 15 °C. The exit angle β_2 is 160 °. The inlet area is 8000 mm^2 and the exit area is 7000 mm^2. Determine the speed of the compressor, if the diameter of the rotor is 175 mm, the width of the impeller is 80 mm, and the compressor efficiency is $\eta = 0.80$. Base your calculation on actual, not theoretical, torque.

▮
$$dW_p/dt = \{[(V_2)^2_{\text{theo}} - V_1^2]/2 + c_p T_1[(p_2/p_1)^{(k-1)/k} - 1]\}\dot{m}/\eta$$

From the continuity equation, $\rho_2(V_2)_{\text{theo}}A_2 = 0.7$ kg/s, $V_2 = 0.7/[(1.2244)(7000 \times 10^{-6})] = 81.7$ m/s; hence,

$$dW_p/dt = \{[(81.7^2 - 71.5^2)/2] + (1003)(288)[(101.67/100)^{0.4/1.4} - 1]\}(0.7)/0.80 = 1883 \text{ N} \cdot \text{m/s}$$

Now, $dW_p/dt = T_s\omega = r_2\dot{m}[(U_t)_2 + (V_r)_2 \cot \beta_2]\omega$, or

$$1883 = (0.175/2)(0.7)[(\omega)(0.175/2) + (V_r)_2 \cot 160°]\omega \qquad (1)$$

By the continuity equation, $(1.2244)(V_{r_2})[(\pi)(0.175)](0.08) = 0.7$, or $(V_r)_2 = 13.00$ m/s. Then (*1*) gives: $\omega^2 - 408.2\omega - 3.513 \times 10^5 = 0$, $\omega = [408.2 \pm \sqrt{408.2^2 + (4)(3.518 \times 10^5)(1)}]/(2)(1) = 831$ rad/s $= 7935$ rpm.

20.67 A performance chart is shown in Fig. 20-15 for a 3000-rpm blower from a bulletin of the Buffalo Forge Co. For a 21-in wheel having a flow of 3000 ft^3/min, what is the efficiency and the required torque? The exit diameter is 8 in and the entrance diameter is 16 in. At the entrance $p = 14.7$ lb/in^2 and $T = 70$ °F. The capacity is given at the exit pressure. *Note:* The upper curves apply to the left ordinate and the lower curves apply to the right ordinate.

▮ From the chart:

$$\begin{cases} hp = 21 \\ (\Delta p)_{\text{static}} = 24.4 \text{ inH}_2\text{O} = (24.4)(62.4)/12 = 126.9 \text{ psf} \end{cases}$$

$$p_2 = (14.7)(144) + 126.9 = 2244 \text{ psf} \qquad \dot{m} = \rho_2 Q = [(0.002378)(\tfrac{2216}{2117})^{1/k}](\tfrac{3000}{60}) = 0.1228 \text{ slug/s}$$

$$V_2 = Q/A_2 = (\tfrac{3000}{60})/[\pi(\tfrac{8}{12})^2/4] = 143.2 \text{ ft/s}$$

Now get V_1.

$$\rho_1 V_1 A_1 = \dot{m} = 0.1228 \text{ slug/s} \qquad (0.002378)(V_1)(\pi)(\tfrac{16}{12})^2/4 = 0.1228 \qquad V_1 = 36.98 \text{ ft/s}$$

$$dW_p/dm = [(V_2^2/2) - (V_1^2/2)] + c_p T_1[(p_2/p_1)^{(k-1)/k} - 1]$$

$$dW_p/dt = (dW_p/dm)\dot{m} = (0.1228)\{[(143.2^2/2) - (36.98^2/2)] + (0.24)(778)(g)(460 + 70)[(\tfrac{2216}{2117})^{(1.4-1)/1.4} - 1]\}/550 = 11.49 \text{ hp}$$

$$e = [(21 - 11.49)/21](100) = 45.3\% \qquad T = (11.49)(550)/[3000(2\pi/60)] = 20.12 \text{ ft-lb}$$

20.68 A centrifugal pump in a horizontal pipe drives 213 gal/min of water from an entrance pressure $p_1 = 13$ psia. Pump data are: inside radius of impeller is 3 in; outside radius of impeller is 0.8 ft; pump outputs 25 hp running at 1650 rpm; pump efficiency is 80%; width b of the vanes is 2 in. Determine the exit pressure p_2. Take $\rho = 1.938$ slug/ft^3 for water.

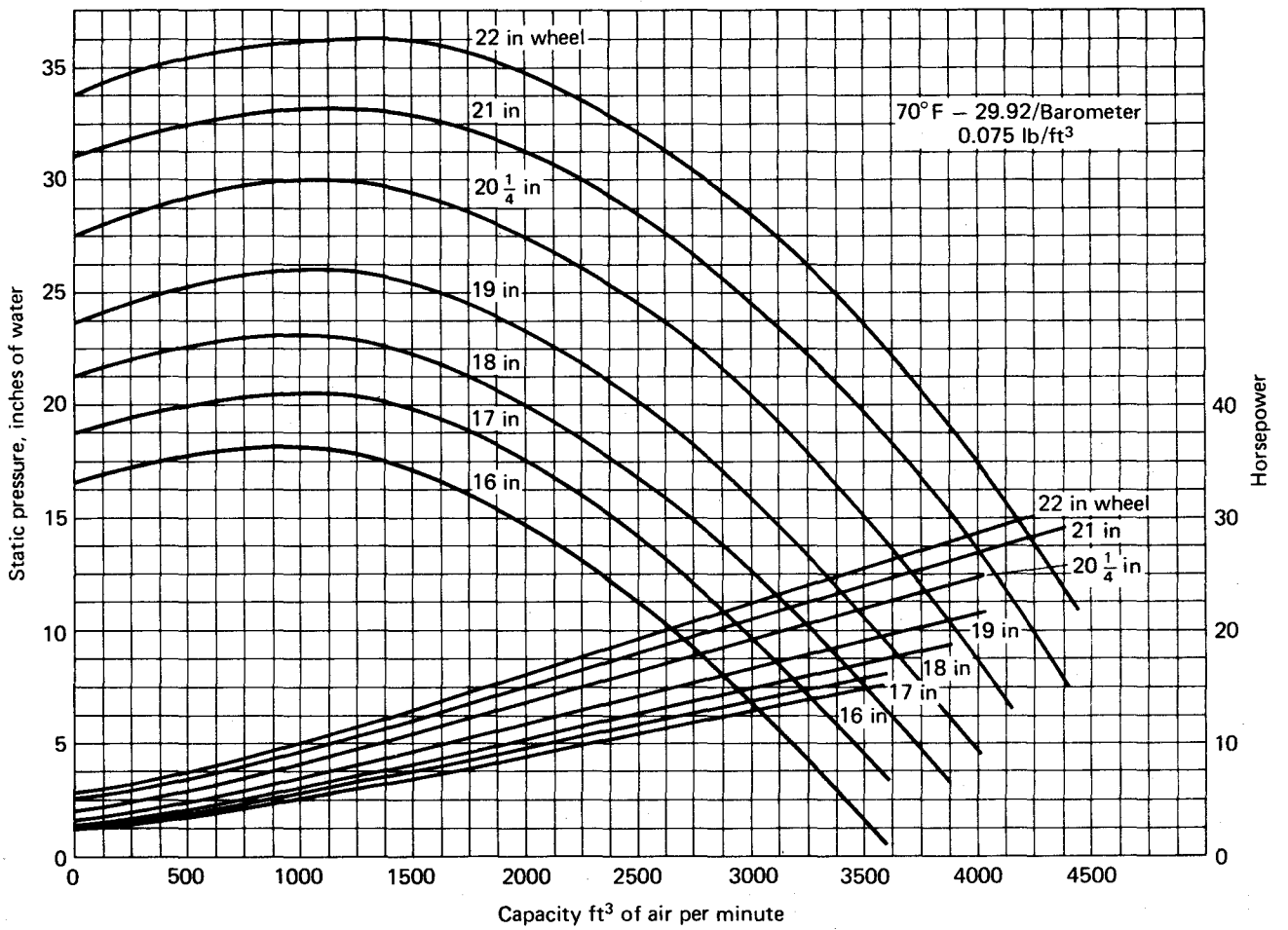


Fig. 20-15 (*Courtesy Buffalo Forge Company. Reprinted with permission.*)

First, find the blade angle from T_s = (theoretical power)/$\omega = r_2\dot{m}[(U_t)_2 + (V_r)_2 \cot \beta_2]$:

$$(25/0.80)(550)/[(1650)(2\pi)/60]\} = (0.8)[(213)(231)/(1728)(60)](1.938)([(1650)(2\pi)/60](0.8) + \{[(213)(231)/(1728)(60)]/[(2\pi)(0.8)(\tfrac{2}{12})]\} \cot \beta_2)$$

$$\beta_2 = 169.4°$$

Next compute the exit velocity:

$$(V_2)_{rel} \sin(\pi - \beta_2) = (V_r)_2 \qquad (V_2)_{rel} = \{[(213)(231)/(1728)(60)]/[2\pi(0.8)(\tfrac{2}{12})]\}/[\sin(180° - 169.4°)] = 3.079 \text{ ft/s}$$

and, from the law of cosines,

$$V_2^2 = (U_t)_2^2 + (V_{rel})_2^2 - 2(U_t)(V_{rel})_2 \cos(\pi - \beta_2)$$
$$= \{[(1650)(2\pi)/60](0.8)\}^2 + 3.079^2 - (2)\{[(1650)(2\pi)/60](0.8)\}(3.079 \cos 10.6°) = 18\,280 \text{ ft}^2/\text{s}^2$$

Since the entrance velocity is $V_1 = [(213)(231)/(1728)(60)]/[(2\pi)(\tfrac{3}{12})(\tfrac{2}{12})] = 1.813$ ft/s, we can now write

$$\text{theoretical power} = \left(\frac{25}{0.80}\right)(550) = \dot{m}\left[\frac{\Delta(V^2)}{2} + \frac{\Delta p}{\rho}\right] = (\tfrac{213}{60})(\tfrac{231}{1728})(1.938)\left[\frac{18\,280 - 1.813^2}{2} + \frac{\Delta p}{1.938}\right]$$

Solving, $\Delta p = 18\,507$ psf = 129 psi and $p_2 = 13 + 129 = 142$ psi.

20.69 A centrifugal pump moves 9.0 L/s of water from a pressure of 90 kPa to a pressure of 520 kPa. The section inlet has a diameter of 50 mm and the discharge has a diameter of 40 mm. What power is required if there is an efficiency of 65 percent and the discharge pipe is 0.3 m above the inlet pipe?

▮ $(dW_p/dt)_{\text{theo}} = \dot{m}(\{[(V_2^2)_{\text{theo}} - V_1^2]/2\} + (p_2 - p_1)/\rho + g(z_2 - z_1))$ $\qquad \dot{m} = (0.0090)(1000) = 9$ kg/s

$$V_1 = Q/A_1 = 0.0090/[(\pi)(0.050)^2/4] = 4.584 \text{ m/s} \qquad V_2 = Q/A_2 = 0.0090/[(\pi)(0.040)^2/4] = 7.162 \text{ m/s}$$

$$dW_p/dt = (9)\{[(7.162^2 - 4.584^2)/2] + [(520 - 90)/1000] \times 10^3 + 9.81(0.3)\} = 4033 \text{ W}$$

$$P = 4033/0.65 = 6.20 \text{ kW}$$

20.70 Shown in Fig. 20-16 are performance curves as found in a Buffalo Forge Co. bulletin for multistage centrifugal pumps. The number designations 2 × 3, 4 × 4, etc., give the exit and the inlet diameters, respectively. For a 2 × 3 (four-stage) pump operating at 25 hp with a flow Q of 80 gal/min of water, compute the efficiency. What is the total required torque on the four rotors?

▮ $\Delta H(\text{output}) = 540$ ft $\qquad \Delta H(\text{input}) = [(\text{hp})(550)/Q\gamma] = (25)(550)/\{[(80)(231)/(1728)(60)]62.4\} = 1236$ ft

$$\eta = \tfrac{540}{1236}(100) = 44 \text{ percent} \qquad \text{Torque} = (25)(550)/[(3500)(2\pi)/60] = 37.5 \text{ ft-lb}$$

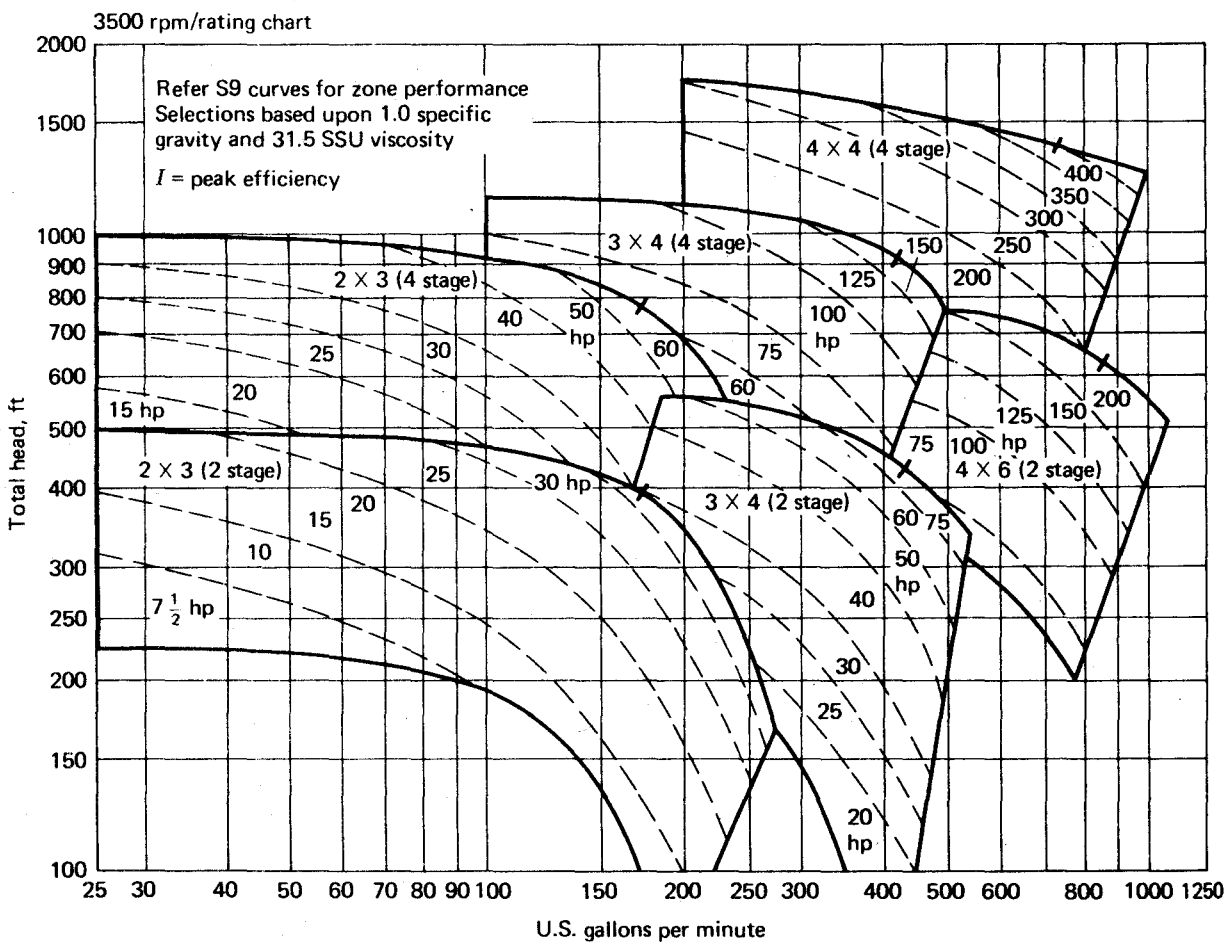


Fig. 20-16 (*Courtesy Buffalo Forge Company. Reprinted with permission.*)

20.71 Using Fig. 20-16, select a centrifugal pump to move 120 gal/min of water from tank A to tank B in Fig. 20-17. The inlet and outlet pipes are steel. Neglect head loss for the short inlet piping. Water is at 60 °F.

▮ Use the first law for pump and 2-in outlet pipe.

$$(1/g)(dW_p/dm) + (V_1^2/2g) + (p_1/\gamma) + 0 = (V_2^2/2g) + (p_2/\gamma) + 390 + [(h_L)/g] \qquad (1)$$

For p_1 (gage pressure) apply Bernoulli's equation between free surface of tank A and pump inlet:

$$(V_{fs}^2/2g) + (p_{atm}/\gamma) + 1 = (V_1^2/2g) + (p_1/\gamma) \qquad (2)$$

$$V_1 = [(120)(231)/(1728)(60)]/[(\pi)(3^2)/(4)(144)] = 5.447 \text{ ft/s} \qquad p_1/\gamma = 1 - (5.447^2/2g) = 0.539 \text{ ft}$$

$$V_2 = [(120)(231)/(1728)(60)]/[(\pi)(2^2)/(4)(144)] = 12.25 \text{ ft/s} \qquad N_R = (12.25)(\tfrac{2}{12})/(1.217 \times 10^{-5}) = 1.398 \times 10^5$$

$$\epsilon/D = 0.00015/(\tfrac{2}{12}) = 0.0009 \qquad f = 0.021 \qquad h_L = f(L/d)(V^2/2) = (0.021)(390/\tfrac{1}{6})(12.25^2/2) = 3687 \text{ ft}$$

Now substitute in Eq. (*1*):

$$(1/g)(dW_p/dm) + (5.447^2/2g) + 0.539 = (12.25^2/2g) + [(\gamma)(20)/\gamma] + 390 + (3687/g) = 527 \text{ ft}$$

Use 2 × 3(four-stage) 30-hp pump.

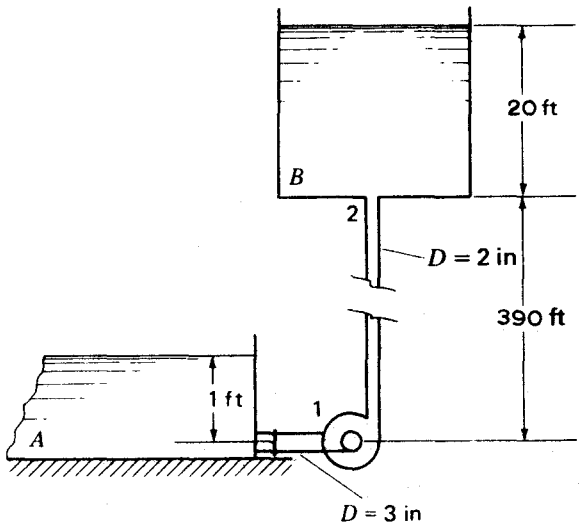


Fig. 20-17

20.72 A pump impeller, 12 in in diameter, discharges 5.25 cfs when running at 1200 rpm. The blade angle β_2 is 160° and the exit area A_2 is 0.25 ft². Assuming losses of $2.8(v_2^2/2g)$ and $0.38(V_2^2/2g)$, compute the efficiency of the pump (exit area A_2 is measured normal to v_2).

▮ The absolute and relative velocities at exit must be calculated first. Velocities u_2 and v_2 are $u_2 = r_2\omega = (\frac{6}{12})(2\pi)(\frac{1200}{60}) = 62.8$ ft/s, $v_2 = Q/A_2 = 5.25/0.25 = 21.0$ ft/s.

From the vector diagram shown in Fig. 20-18, the value of the absolute velocity at exit is $V_2 = 43.7$ ft/s. Head furnished by impeller, $H' = (u_2^2/2g) - (v_2^2/2g) + (V_2^2/2g) = [(62.8)^2/2g] - [(21.0)^2/2g] + [(43.7)^2/2g] = 84.0$ ft. Head delivered to water, $H = H' - \text{losses} = 84.0 - \{2.8[(21.0)^2/2g] + 0.38[(43.7)^2/2g]\} = 53.6$ ft.

Efficiency $e = H/H' = 53.6/84.0 = 63.8$ percent. The value of H' might have been calculated by means of the commonly used expression $H' = (u_2/g)(u_2 + v_2 \cos \beta_2) = (62.8/g)[62.8 + 21.0(-0.940)] = 84.0$ ft.

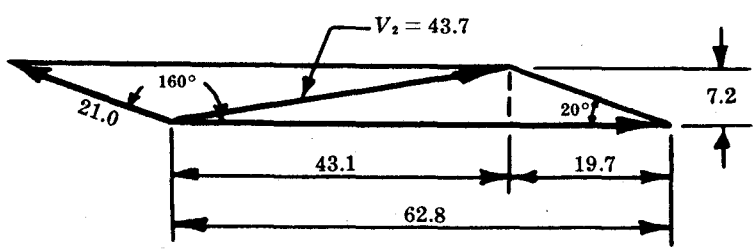


Fig. 20-18

20.73 A 6-in pump delivers 1300 gpm against a head of 90 ft when rotating at 1750 rpm. The head-discharge and efficiency curves are shown in Fig. 20-19. For a geometrically similar 8-in pump running at 1450 rpm and delivering 1800 gpm, determine (*a*) the probable head developed by the 8-in pump. (*b*) Assuming a similar efficiency curve for the 8-in pump, what power would be required to drive the pump at the 1800-gpm rate?

▮ (*a*) The homologous pumps will have identical characteristics at corresponding flows. Choose several rates of flow for the 6-in pump and read off the corresponding heads. Calculate the values of Q and H so that

a curve for the 8-in pump can be plotted. One such calculation is detailed below and a table of values established by similar determinations.

Using the given 1300 gpm and the 90-ft head, we obtain from the speed relation $H_8 = (D_8/D_6)^2(N_8/N_6)^2H_6 = (\frac{8}{6})^2(\frac{1450}{1750})^2H_6 = 1.22H_6 = 1.22(90) = 109.8$ ft. From the flow relation, $Q/D^3N =$ constant, we obtain $Q_8 = (D_8/D_6)^3(N_8/N_6)Q_6 = (\frac{8}{6})^3(\frac{1450}{1750})Q_6 = 1.964Q_6 = 1.964(1300) = 2550$ gpm.

Additional values, which have been plotted as the dashed line in Fig. 20-19, are as follows:

For 6-in pump at 1750 rpm			For 8-in pump at 1450 rpm		
Q, gpm	H, ft	Efficiency, %	Q, gpm	H, ft	Efficiency, %
0	124	0	0	151.6	0
500	119	54	980	145.5	54
800	112	64	1570	134.5	64
1000	104	68	1960	127.0	68
1300	90	70	2550	110.0	70
1600	66	67	3140	80.6	67

From the head-discharge curve, for $Q = 1800$ gpm the head is 130 ft.

(*b*) The efficiency of the 8-in pump would probably be somewhat higher than that of the 6-in pump at comparable rates of flow. For this case, the assumption is that the efficiency curves are the same at comparable rates of flow. The table above lists the values for the flows indicated. The figure gives the efficiency curve for the 8-in pump and, for the 1800 gpm flow, the value is 67 percent. Then $P = \gamma QH/550e = 62.4[1800/(60)(7.48)](130)/[550(0.67)] = 88.3$ horsepower required.

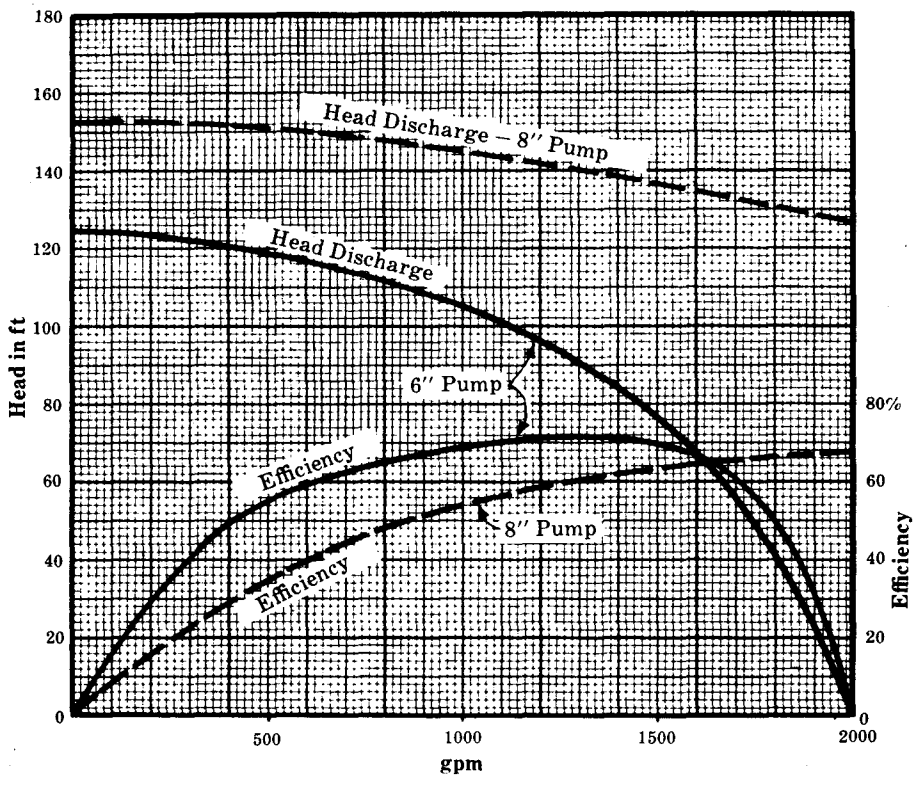


Fig. 20-19

20.74 In order to predict the behavior of a small oil pump, tests are to be made on a model using air. The oil pump is to be driven by a $\frac{1}{20}$-hp motor at 1800 rpm and a $\frac{1}{4}$-hp motor is available to drive the air pump at 600 rpm. Using the s.g. of oil at 0.912 and the density of air constant at 0.00238 slug/ft^3, what size model should be built?

▮ Using the power relation, we obtain prototype $[P/(\rho D^5N^3)] =$ model$[P/(\rho D^5N^3)]$. Then $\frac{1}{20}/[0.912(62.4/32.2)D_p^5(1800)^3] = \frac{1}{4}/[0.00238D_m^5(600)^3]$ and $D_m/D_p = \frac{10}{1}$. The model should be 10 times as large as the oil pump.

20.75 What is the power ratio of a pump and its $\frac{1}{5}$ scale model if the ratio of the heads is 4 to 1?

▮ For geometrically similar pumps, $P/(D^2H^{3/2})$ for pump $= P/(D^2H^{3/2})$ for model. Then $P_p/[(5D)^2(4H)^{3/2}] = P_m/(D^2H^{3/2})$ and $P_p = 25(4)^{3/2}P_m = 200P_m$.

20.76 Determine the thrust coefficient of a propeller which is 4 in in diameter, revolves at 1800 rpm, and develops a thrust of 2.50 lb in fresh water.

▮ Thrust coefficient $= F/(\rho N^2D^4) = 2.50/[1.94(\frac{1800}{60})^2(\frac{4}{12})^4] = 0.116$. The coefficient is dimensionless when F is in pounds, N in revolutions/per second, and D in feet.

20.77 The power and thrust coefficients of an 8-ft-diameter propeller moving forward at 100 ft/s at a rotational speed of 2400 rpm are 0.068 and 0.095, respectively. **(*a*)** Determine the power requirement and thrust in air ($\rho = 0.00237$ slug/ft^3). **(*b*)** If the advance-diameter ratio for maximum efficiency is 0.70, what is the air speed for the maximum efficiency?

▮ **(*a*)** Power $P = C_P\rho N^3D^5$ in ft-lb/s $= [0.068(0.00237)(\frac{2400}{60})^3(8)^5]/550 = 615$ horsepower.

Thrust $F = C_F\rho N^2D^4$ in lb $= 0.095(0.00237)(\frac{2400}{60})^2(8)^4 = 1476$ lb.

(*b*) Since $V/ND = 0.70$, $V = 0.70(\frac{2400}{60})(8) = 224$ ft/s.

20.78 An airplane flies at 180 mph in still air, $\gamma = 0.0750$ lb/ft^3. The propeller is 5.5 ft in diameter and the velocity of the air through the propeller is 320 ft/s. Determine **(*a*)** the slipstream velocity, **(*b*)** the thrust, **(*c*)** the horsepower input, **(*d*)** the horsepower output, **(*e*)** the efficiency, and **(*f*)** the pressure difference across the propeller.

▮ **(*a*)** $V = \frac{1}{2}(V_1 + V_4)$ $\quad 320 = \frac{1}{2}[180(\frac{5280}{3600}) + V_4]$ $\quad V_4 = 376$ ft/s (relative to fuselage)

(*b*) Thrust $F = (\gamma/g)Q(V_4 - V_1) = (0.0750/32.2)[\frac{1}{4}\pi(5.5)^2(320)](376 - 264) = 1983$ lb

(*c*) Power input $P_i = FV/550 = 1983(320)/550 = 1154$ hp

(*d*) Power output $P_0 = FV_1/550 = 1983(264)/550 = 952$ hp

(*e*) Efficiency $e = \frac{952}{1154} = 82.5$ percent or $e = 2V_1/(V_4 + V_1) = 2(264)/(376 + 264) = 82.5$ percent.

(*f*) Pressure difference $=$ (thrust F)/[area $(\frac{1}{4}\pi D^2)$] $= 1983/[\frac{1}{4}\pi(5.5)^2] = 83.5$ psf or pressure difference $= (0.0750/32.2)\{[(376)^2 - (264)^2]/2\} = 83.5$ psf.

20.79 Water is being pumped from the lower reservoir to the upper reservoir, as shown in Fig. 20-20. Find the pump horsepower required, assuming 75 percent pump efficiency. Neglect minor losses.

▮ $p_1/\gamma + V_1^2/2g + z_1 + E_p = p_2/\gamma + V_2^2/2g + z_2 + h_L$. From Fig. A-13, with $Q = 23$ cfs and $D = 24$ in, $h_1 = 0.0076$ ft/ft.

$$h_L = h_f = (0.0076)(3600) = 27.36 \text{ ft} \qquad 0 + 0 + 97.5 + E_p = 0 + 0 + 132.0 + 27.36$$

$$E_p = 61.86 \text{ ft} \qquad P = Q\gamma E_p/550 = (23)(62.4)(61.86)/550 = 161 \text{ hp}$$

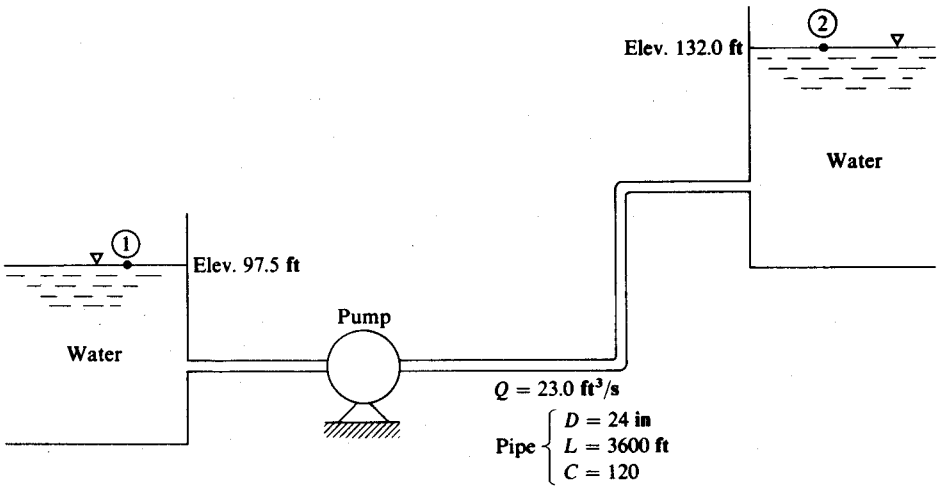


Fig. 20-20

20.80 Water is being pumped from a reservoir to the top of a hill, where it is discharged, as shown in Fig. 20-21. The pump, which is 70 percent efficient, is rated at 150 kW. Find the flow rate at which water is being discharged from the pipe. Neglect minor losses.

▮ $p_1/\gamma + V_1^2/2g + z_1 + E_p = p_2/\gamma + V_2^2/2g + z_2 + h_L \qquad E_p = P/Q\gamma = (150)(0.70)/[(Q)(9.79)] = 10.725/Q$

$$V = Q/A \qquad V_2 = Q/[(\pi)(0.500)^2/4] = 5.093Q \qquad h_L = h_f = (f)(L/D)(V^2/2g)$$

Assume $f = 0.018$. $h_L = (0.018)[975/0.500]\{(5.093Q)^2/[(2)(9.807)]\} = 46.42Q^2$, $0 + 0 + 111.0 + 10.725/Q = 0 + (5.093Q)^2/[(2)(9.807)] + 150.2 + 46.42Q^2$, $Q = 0.254\ \text{m}^3/\text{s}$ (by trial and error). This solution was based on the assumed value of f of 0.018. However, the value of f is dependent on the Reynolds number and relative roughness. Therefore a new value of f should be determined based on the computed value of Q of $0.254\ \text{m}^3/\text{s}$.

$$\epsilon/D = 0.00030/0.500 = 0.00060 \qquad V = Q/A = 0.254/[(\pi)(0.500)^2/4] = 1.294\ \text{m/s}$$

$$N_R = DV/\nu = (0.500)(1.294)/(1.02 \times 10^{-6}) = 6.34 \times 10^5$$

From Fig. A-5, $f = 0.018$. Hence, the computed flow rate of $0.254\ \text{m}^3/\text{s}$ is correct.

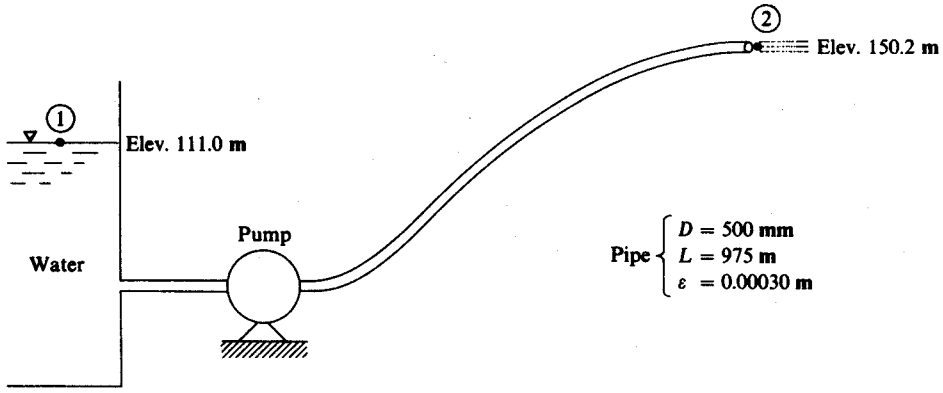


Fig. 20-21

20.81 Oil with a specific gravity of 0.87 is being pumped from a lower reservoir to an elevated tank as shown in Fig. 20-22. The pump in the system is 78 percent efficient and is rated at 185 kW. Determine the flow rate of the oil in the pipe if the total head loss from point 1 to point 2 is 12 m of oil.

▮ $p_1/\gamma + v_1^2/2g + z_1 + E_p = p_2/\gamma + v_2^2/2g + z_2 + h_L \qquad p_1/\gamma = p_2/\gamma = v_1^2/2g = 0 \qquad z_1 = 150\ \text{m}$

$$p = Q\gamma E_p \qquad (0.78)(185) = (Q)[(0.87)(9.79)](E_p) \qquad E_p = 16.94/Q$$

$$v_2^2/2g = (Q/A)^2/2g = \{Q/[(\pi)(0.160)^2/4]\}^2/[(2)(9.807)] = 126.12Q^2$$

$$z_2 = 200\ \text{m} \qquad h_L = 12\ \text{m} \qquad 0 + 0 + 150 + 16.94/Q = 0 + 126.12Q^2 + 200 + 12 \qquad 126.12Q^2 - 16.94/Q + 62 = 0$$

By trial and error solution, $Q = 0.244\ \text{m}^3/\text{s}$.

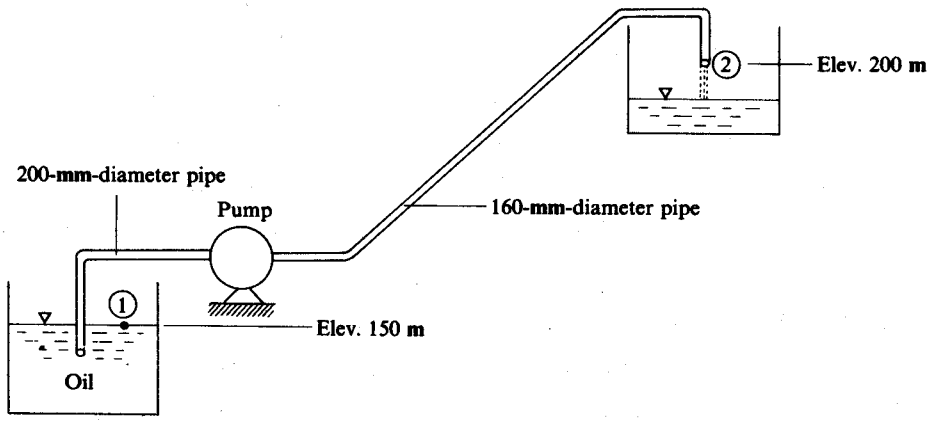


Fig. 20-22

20.82 For the conditions given in Prob. 20-80, find the flow rate at which water is being discharged from the pipe using the computer program written for Prob. 9.295.

INPUT

```
1SAMPLE ANALYSIS ØF INCØMPRESSIBLE FLØW
                                1.0        111.0      150.2      105.0
        500.0      975.0      .00000102 .00030      9.79
WATER                      (RØUGHNESS = 0.300 MM)

SAMPLE ANALYSIS OF INCOMPRESSIBLE FLOW
```

OUTPUT

```
GIVEN DATA FOR A CIRCULAR CLOSED CONDUIT CARRYING INCOMPRESSIBLE FLOW

    PRESSURE AT POINT 1 =     0.0 KPA

    PRESSURE AT POINT 2 =     0.0 KPA

    ELEVATION AT POINT 1 =  111.0 M

    ELEVATION AT POINT 2 =  150.2 M

    ACTUAL ENERGY ADDED BETWEEN POINTS 1 AND 2 =105.0 KW

    ACTUAL ENERGY REMOVED BETWEEN POINTS 1 AND 2 =  0.0 KW

    MINOR HEAD LOSSES BETWEEN POINTS 1 AND 2 =  0.0 M

    DIAMETER OF CONDUIT =  500.0 MM

    LENGTH OF CONDUIT =   975.0 M

    FLUID FLOWING IS WATER

    CONDUIT MATERIAL IS (ROUGHNESS = 0.300 MM)


THE FLOW RATE WILL BE  0.254 CU M/S

    VELOCITY AT POINT 1 =  0.00 M/S

    VELOCITY AT POINT 2 =  1.29 M/S
```

20.83 A pump draws water from a reservoir and then discharges it to an elevated tank, as shown in Fig. 20-23. The pipe's ends are squared-cornered (i.e., sharp-edged). There are two 90° bends as shown. The pipe roughness (ϵ) is 0.0084 ft and C is 120. If the rate of flow is 8.0 ft^3/s and the efficiency of the pump is 75 percent, determine the required horsepower of the pump. Consider both friction and minor losses.

$$p_1/\gamma + v_1^2/2g + z_1 + E_p = p_2/\gamma + v_2^2/2g + z_2 + h_L \qquad p_1/\gamma = p_2/\gamma = v_1^2/2g = v_2^2/2g = 0$$

$$z_1 = 10\text{ ft} \qquad z_2 = 120\text{ ft} \qquad h_L = h_f + h_m$$

For the 12-in pipe with $Q = 8$ cfs, $(h_1)_{12} = 0.032$ ft/ft. For the 10-in pipe with $Q = 8$ cfs, $(h_1)_{10} = 0.074$ ft/ft.

$$h_f = (0.032)(40) + (0.074)(200) = 16.08\text{ ft} \qquad h_m = h_c + 2h_b + h_e \qquad h_c = K_c v_{10}^2/2g \qquad K_c = 0.45$$

$$v_{12} = Q/A = 8.0/[(\pi)(1.0)^2/4] = 10.19\text{ ft/s} \qquad h_c = (0.45)(10.19)^2/[(2)(32.2)] = 0.73\text{ ft}$$

$$h_b = K_b v_{10}^2/2g \qquad v_{10} = Q/A = 8.0/[(\pi)(0.833)^2/4] = 14.68\text{ ft/s}$$

With $\epsilon/D = 0.0084/(\frac{10}{12}) = 0.101$ and $R/D = \frac{20}{10} = 2.0$, $K_b = 0.6$.

$$h_b = (0.6)(14.68)^2/[(2)(32.2)] = 2.01\text{ ft} \qquad h_e = K_e v_{10}^2/2g \qquad K_e = 1.0$$

$$h_e = (1.0)(14.68)^2/[(2)(32.2)] = 3.35\text{ ft} \qquad h_m = 0.73 + (2)(2.01) + 3.35 = 8.10\text{ ft}$$

$$h_L = h_f + h_m = 16.08 + 8.10 = 24.18\text{ ft} \qquad 0 + 0 + 10 + E_p = 0 + 0 + 120 + 24.18 \qquad E_p = 134.18\text{ ft}$$

$$P = Q\gamma E_p = (8.0)(62.4)(134.18) = 66\,983\text{ ft-lb/s} \qquad \text{Required horsepower} = 66\,983/[(550)(0.75)] = 162\text{ hp}$$

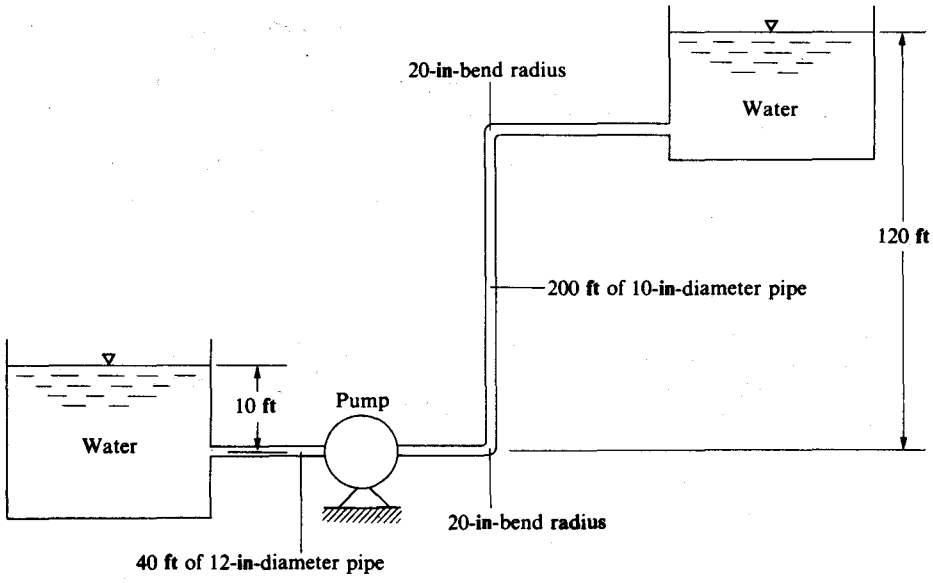


Fig. 20-23

20.84 Oil with a specific gravity of 0.86 is being pumped from a reservoir as shown in Fig. 20-24. The pressures at points 1 and 2 are −4.0 psi and 43.0 psi, respectively. The rate of flow in the pipe is 0.50 ft³/s. The pump is rated at 8 hp. Determine the efficiency of the pump. Neglect energy losses in the system.

▮ $p_1/\gamma + v_1^2/2g + z_1 + E_p = p_2/\gamma + v_2^2/2g + z_2 + h_L \qquad p_1/\gamma = (-4)(144)/[(0.86)(62.4)] = -10.73\text{ ft}$

$$v_1^2/2g = v_2^2/2g \qquad z_1 = h_L = 0 \qquad p_2/\gamma = (43)(144)/[(0.86)(62.4)] = 115.38\text{ ft}$$

$$z_2 = 3\text{ ft} \qquad -10.73 + E_p = 115.38 + 3 \qquad E_p = 129.11\text{ ft}$$

$$P = Q\gamma E_p = (0.50)[(0.86)(62.4)](129.11) = 3464\text{ ft-lb/s} = \tfrac{3464}{550} = 6.30\text{ hp}$$

$$\text{Efficiency of pump} = (6.30/8)(100) = 78.8\text{ percent}$$

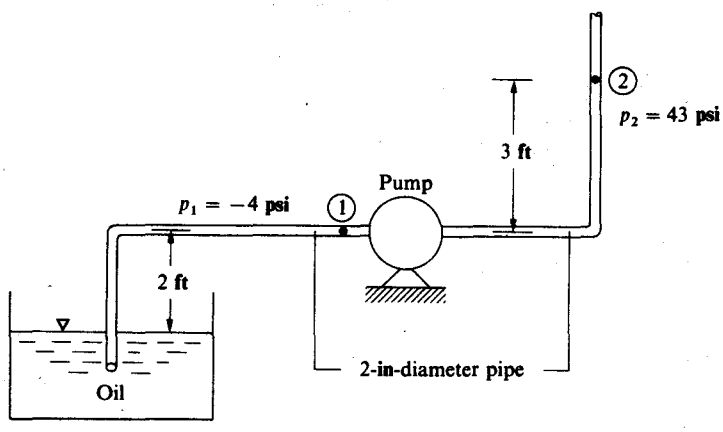


Fig. 20-24

20.85 A pump circulates water at the rate of 1900 gpm in a closed circuit holding 9000 gal. The total pump head is 290 ft and the pump efficiency is 0.89. Assuming the bearing friction to be negligible and the system to be adiabatic, find the rate of temperature rise in the water. [For water, c = 25 000 ft-lb/(slug-F°).]

▮ By the first law $mc(\Delta T/\Delta t) = Q\gamma H/\eta$, or

$$\frac{\Delta T}{\Delta t} = \frac{Q\gamma H}{mc\eta} = \frac{[(1900)(0.002228](62.4)(290)}{[(9000/7.48)(1.94)](25\,000)(0.89)} = 0.00147\text{ F°/s}$$

or 5.3 F°/h.

20.86 The pump in Fig. 20-25 delivers water at 3 ft³/s to a machine at section 2, which is 20 ft higher than the reservoir surface. The losses between 1 and 2 are given by $h_f = KV_2^2/2g$, where $K = 7.5$ is a dimensionless loss coefficient. Find the power required to drive the pump if it is 80 percent efficient.

▮ The flow is steady except for the slow decrease in reservoir depth, which we neglect, taking $V_1 \approx 0$. We can compute V_2 from the given flow rate and diameter: $V_2 = Q/A_2 = 3/[(\pi/4)(\frac{3}{12})^2] = 61.1$ ft/s. Because of the solid walls and one-dimensional ports, the viscous work is zero. $(p_1/\rho g) + (V_1^2/2g) + z_1 = (p_2/\rho g) + (V_2^2/2g) + z_2 + h_s + h_f$. With $V_1 = z_1 = 0$ and $h_f = KV_2^2/2g$, we can solve for

$$h_s = [(p_1 - p_2)/\rho g] - z_2 - (1 + K)(V_2^2/2g) \tag{1}$$

The pressures must be in pounds per square foot for consistent units, with $\rho g = 62.4$ lb/ft^3 for water. Introducing numerical values, we have

$$h_s = [(14.7 - 10)(144)/62.4] - 20 - [(1 + 7.5)(61.1)^2/2(32.2)] = -502 \text{ ft} \tag{2}$$

The pump head is negative, indicating work done on the fluid. The power delivered is $P = \dot{m}w_s = \rho Q g h_s = (1.94)(3)(32.2)(-502) = -94\,100$ ft-lb/s, hp $= 94\,100/550 = 171$ hp. The input power thus required to drive the 80 percent efficient pump is $P_{\text{input}} = P/\text{efficiency} = 171/0.8 = 214$ hp.

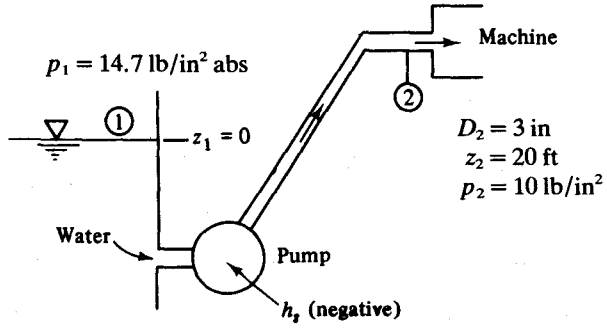


Fig. 20-25

20.87 For the pump of Fig. 20-26, total friction head loss is 6 m. If the pump delivers 40 kW of power to the water, determine the exit velocity V_e and the flow rate Q.

▮ $$p_1/\gamma + V_1^2/2g + z_1 + E_p = p_2/\gamma + V_2^2/2g + z_2 + h_L \qquad 0 + 0 + 0 + E_p = 0 + V_e^2/[(2)(9.807)] + 3 + 6$$

$$E_p = 0.05098V_e^2 + 9$$

Also, $P = Q\gamma E_p$, $Q = AV = [(\pi)(0.050)^2/4](V_e) = 0.001964V_e$, $(40)(1000) = (0.001964V_e)[(9.79)(1000)](E_p)$, $E_p = 2080/V_e$. Equating expressions for E_p, $0.05098V_e^2 + 9 = 2080/V_e$, $V_e = 32.7$ m/s (by trial and error); $Q = (0.001964)(32.7) = 0.0642$ m^3/s, or 64.2 L/s.

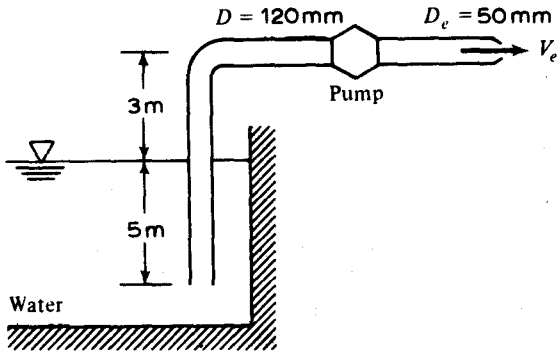


Fig. 20-26

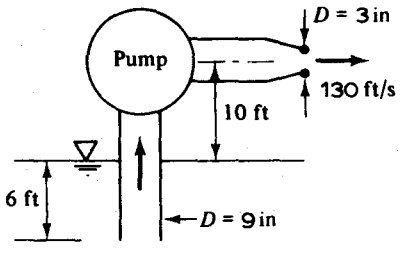


Fig. 20-27

20.88 A fireboat pump draws sea water (s.g. = 1.025) from a 9-in submerged pipe and discharges it as 130 fps through a 3-in nozzle, as in Fig. 20-27. Total head loss is 9 ft. If the pump is 72 percent efficient, how much horsepower is required to drive it?

▮ $$P = Q\gamma E_p/\eta \qquad Q = AV = [(\pi)(\tfrac{3}{12})^2/4](130) = 6.381 \text{ ft}^3/\text{s}$$

$$p_1/\gamma + V_1^2/2g + z_1 + E_p = p_2/\gamma + V_2^2/2g + z_2 + h_L \qquad 0 + 0 + 0 + E_p = 0 + 130^2/[(2)(32.2)] + 10 + 9$$

$$E_p = 281.4 \text{ ft} \qquad P = \{(6.381)[(1.025)(62.4)](281.4)/550\}/0.72 = 290 \text{ hp}$$

20.89 The pump in Fig. 20-28 discharges water at 30 L/s. Neglecting losses and elevation changes, what power is delivered to the water by the pump?

▮ $p_1/\gamma + V_1^2/2g + z_1 + E_p = p_2/\gamma + V_2^2/2g + z_2 + h_L \qquad V_1 = Q/A_1 = (30 \times 10^{-3})/[(\pi)(0.10)^2/4] = 3.820 \text{ m/s}$

$$V_2 = 0.03/[(\pi)(0.04)^2/4] = 23.87 \text{ m/s}$$

$$125/9.79 + 3.820^2/[(2)(9.807)] + 0 + E_p = 409/9.79 + 23.87^2/[(2)(9.807)] + 0 + 0$$

$$E_p = 57.31 \text{ m} \qquad P = Q\gamma E_p = (30 \times 10^{-3})(9.79)(57.31) = 16.8 \text{ kW}$$

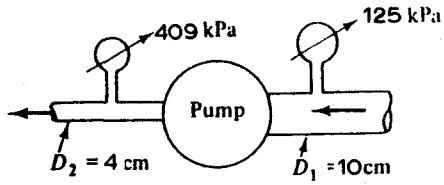


Fig. 20-28

20.90 What is the flow for the system shown in Fig. 20-29*a*? The pump has the characteristics shown in Fig. 20-29*b*. What is the power required?

▮ Express the first law for thermodynamics for pipe interior as shown:

$$(p_1/\rho) + (V_1^2/2) + gz_1 + gH = (p_2/\rho) + (V_2^2/2) + gz_2 + h \tag{1}$$

Bernoulii in left tank: Use gage pressures, $[(35)(144)/1.94] + 36g = (p_1/\rho) + (V^2/2) + 0$:

$$p_1/\rho = 3757 - V^2/2 \tag{2}$$

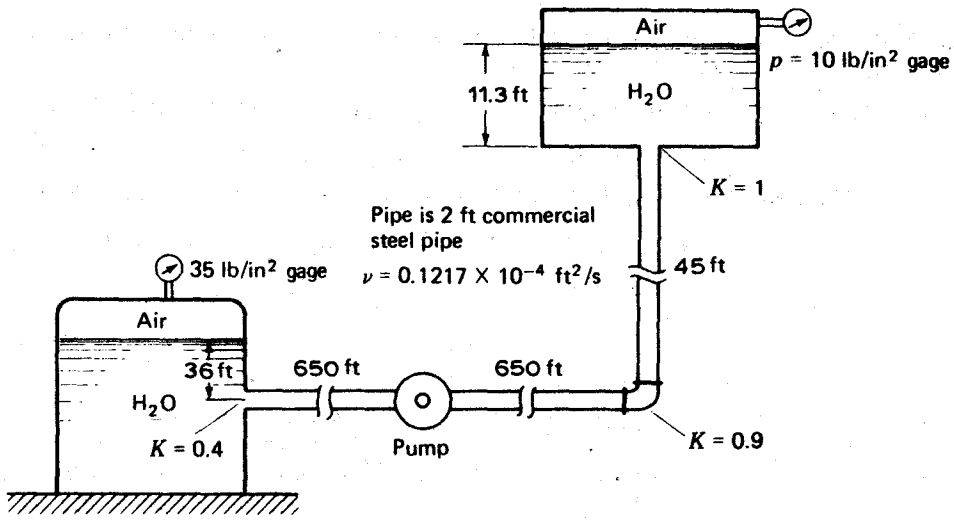
Hydrostatics in right-hand tank:

$$p_2/\rho = (1/\rho)[(10)(144) + 11.3(62.4)] = 1106 \tag{3}$$

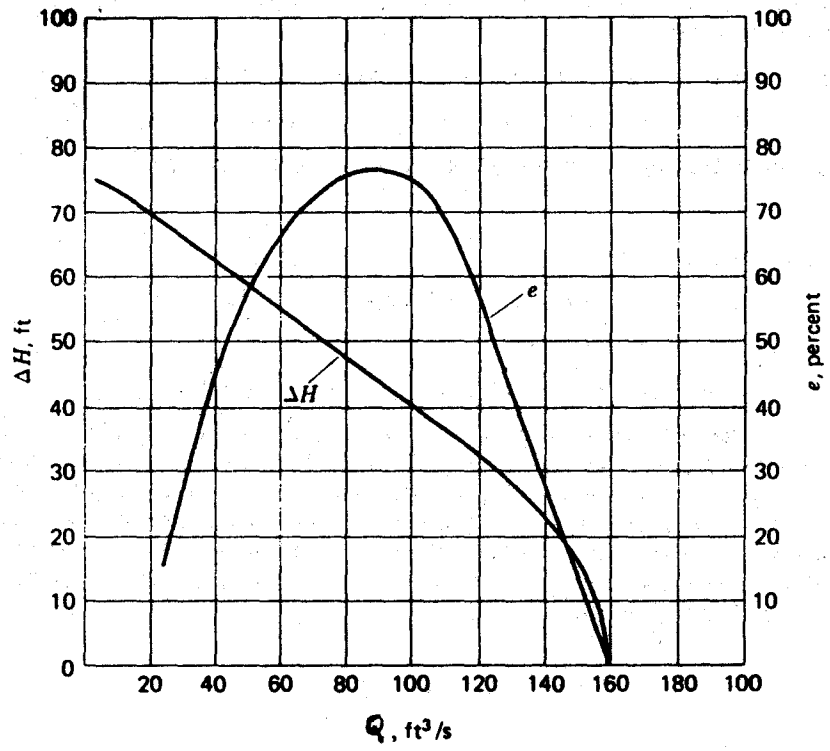
Substitute Eqs. (*2*) and (3) into Eq. (*1*):

$$[3757 - (V^2/2)] + gH = 1106 + (g)(45) + f(1345/2.00)(V^2/2) + 2.3(V^2/2) \tag{4}$$

Assume $Q_1 = 80$ cfs. Then $V_1 = 80/[(\pi/4)(2.00)^2] = 25.5$ ft/s, $(N_R)_1 = [(25.5)(2.00)/(0.1217 \times 10^{-4})] = 4.19 \times 10^6$, $f_1 = 0.0118$. Solve for H_1 in Eq. (*4*): $H_1 = 76.1$ ft. The point (1) for H_1 and Q_1 is above Q curve. We take as a second estimate $Q_2 = 70$, $V_2 = 70/[(\pi/4)(2)^2] = 22.28$ ft/s, $(N_R)_2 = (22.28)(2)/(0.1217 \times 10^{-4}) = 3.66 \times 10^6$, $f_2 = 0.0118$, $H_2 = 49.3$ ft. New point is just below Q line. Third estimate can now be easily made: $Q_3 = 71$ cfs, $V_3 = 71/[(\pi/4)(2^2)] = 22.60$ ft/s, $(N_R)_3 = (22.60)(2)/(0.1217 \times 10^{-4}) = 3.71 \times 10^6$, $f_3 = 0.0118$, $H_3 = 51.8$ ft. We are close enough to the intersection so that we can say: $Q = 71$ cfs. The power needed for this operation is then Power $= g(51.8)(1.938)(71)/[(0.73)(550)] = 572$ hp.


Air
p = 10 lb/in² gage
11.3 ft
H₂O
K = 1
Pipe is 2 ft commercial steel pipe
ν = 0.1217 × 10⁻⁴ ft²/s
45 ft
35 lb/in² gage
Air
36 ft
650 ft
650 ft
H₂O
K = 0.4
Pump
K = 0.9

(*a*)


ΔH, ft
e, percent
Q, ft³/s
e
ΔH
0
10
20
30
40
50
60
70
80
90
100
20
40
60
80
100
120
140
160
180
100

(*b*)

Fig. 20-29

CHAPTER 21
Turbines

21.1 A radial-flow turbine has the following dimensions: $r_1 = 1.6$ ft, $r_2 = 1.0$ ft, and $\beta_1 = 80°$. The width of the flow passage between the two sides of the turbine is 0.8 ft. At 300 rpm the flow rate through the turbine is 120 cfs. Find **(a)** the blade angle β_2 such that the water exits from the turbine in the radial direction, **(b)** the torque exerted by the water on the runner and the horsepower thus developed, **(c)** the head utilized by the runner and the power resulting therefrom. Assume that water enters and leaves the blades smoothly. Assume the blades are so thin that they do not occupy any of the available flow area.

(a) At the outer periphery ($r_1 = 1.6$ ft): $u_1 = \omega r_1 = (2\pi/60)(300)(1.6) = 50.3$ fps. From continuity $Q = 120 = 2\pi r_1(z)V_{r_1} = 2\pi(1.6)(0.8)(V_{r_1})$, $V_{r_1} = 120/8.04 = 14.92$ fps $= v_1 \sin\beta_1 = v_1 \sin 80°$, $v_1 = 14.92/(\sin 80°) = 15.15$ fps, $v_1 \cos 80° = (15.15)(0.1736) = 2.63$ fps, $V_1 \cos\alpha_1 = u_1 + v_2 \cos\beta_1 = 50.3 + 2.6 = 52.9$ fps. (See Fig. 21-1*a*.) At the inner periphery ($r_2 = 1.0$ ft): $u_2 = \omega r_2 = (2\pi/60)(300)(1.0) = 31.4$ fps, $V_{r_2} = V_{r_1}(r_1/r_2) = 14.92(1.6/1.0) = 23.9$ fps. Because the water exits from the turbine in the radial direction, $\alpha_2 = 90°$ and $V_2 = V_{r_2}$. $\tan(180° - \beta_2) = V_2/u_2 = 23.9/31.4 = 0.760$. Thus $\beta_2 = 180° - 37.2° = 142.8°$, the required blade angle. (See Fig. 21-1*b*.)
(b) $T = \rho Q(r_1 V_1 \cos\alpha_1 - r_2 V_2 \cos\alpha_2) = 1.94(120)[(1.6)(52.9 - 0)] = 19\,700$ ft-lb and power $= T\omega = 19700(31.4) = 619\,000$ ft-lb/s $= 1125$ hp.
(c) $h'' = (u_1 V_1 \cos\alpha_1 - u_2 V_2 \cos\alpha_2)/g = [50.3(52.9) - 0]/32.2 = 82.6$ ft and power $= \gamma Q h'' = (62.4)(120)(82.6) = 619\,000$ ft-lb/s $= 1125$ hp.

Further calculations indicate that the absolute velocity of the water changed from $V_1 = 55.0$ fps at entry to $V_2 = 23.9$ fps at exit while the velocity of the water relative to the blades changed from $v_1 = 15.15$ fps at entry to $v_2 = 39.5$ fps at exit.

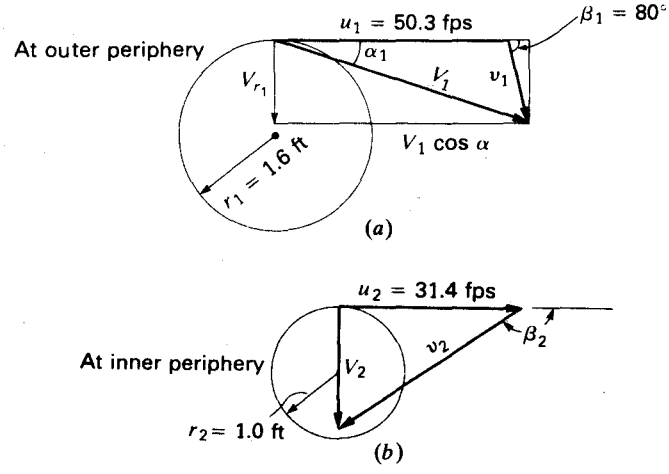


Fig. 21-1

21.2 A 6-in-diameter pipe ($f = 0.020$) of length 1000 ft delivers water from a reservoir with a water-surface elevation of 500 ft to a nozzle at elevation 300 ft. The jet from the nozzle is used to drive a small impulse turbine. If the head loss through the nozzle can be expressed as $0.04\, V_j^2/2g$, find the jet diameter that will result in maximum power in the jet. Neglect the head loss at entrance to the pipe from the reservoir. Evaluate the power in the jet.

Energy equation: $500 \times 0.02(1000/0.5)(V_p^2/2g) - 0.04(V_j^2/2g) = 300 + (V_j^2/2g)$. If we define the pipe diameter and velocity as D_p and V_p and the jet diameter and velocity as D_j and V_j, from continuity we get $A_p V_p = A_j V_j$, $D_p^2 V_p = D_j^2 V_j$.

Since the pipe diameter $D_p = 0.50$ ft, $0.25 V_p = D_j^2 V_j$ and $V_p = 4 D_j^2 V_j$. Substituting this expression for V_p in the energy equation gives $200 - (V_j^2/2g)(1.04 + 640 D_j^4)$.

Assuming different values for D_j, we can compute corresponding values of V_j and Q, and then the jet power can be computed using $p_{jet} = [\gamma Q(V_j^2/2g)]/550$. The results are shown in the table on page 639.

Thus a 2-in-diameter jet is the optimum; it will have about 30 hp.

An alternative procedure for solving this problem is to set up an algebraic expression for the power of the jet, P_{jet}, as a function of the jet diameter, D_j, and differentiate P_{jet} with respect to D_j and equate to zero to find the value of D_j for which P_{jet} is a maximum.

D_j, in	D_j, ft	V_j, fps	A_j, ft^2	$Q = A_jV_j$, cfs	P_{jet}, hp
1.0	0.083	111	0.0054	0.60	12.8
1.5	0.125	105	0.0122	1.28	24.2
2.0	0.167	91	0.0218	2.00	29.8
2.5	0.208	76	0.0338	2.57	26.2
3.0	0.250	60	0.0491	2.94	18.8
4.0	0.333	38	0.0873	3.29	8.4
6.0	0.500	18	0.197	3.49	1.9

21.3 A turbine is to operate at 400 rpm under a net head of 1320 ft. If a single 6-in-diameter water jet is used, find the specific speed of this machine assuming $C_v = 0.98$, $\phi = 0.45$, and $\eta = 0.85$. Find also the required pitch diameter of the wheel.

▮ $$V = C_v\sqrt{2gh} = 0.98\sqrt{(64.4)(1320)} = 286 \text{ fps} \qquad Q = AV = (0.196)(286) = 56.0 \text{ cfs}$$

$$\text{bhp} = \eta(\gamma Qh/550) = 0.85[(62.4)(56)(1320)/550] = 7130$$

$$N_s = N\sqrt{\text{bhp}}/h^{5/4} = 400\sqrt{7130}/(1320)^{5/4} = 4.25 \qquad u = \phi\sqrt{2gh} = 0.45\sqrt{(64.4)(1320)} = 131.2 \text{ fps}$$

$$N = 400 \text{ rpm} = u(60)/\pi D = (131.2)(60)/\pi D \qquad D = 6.26 \text{ ft} = 75.2 \text{ in}$$

21.4 In lieu of the single impulse wheel of Prob. 21.3, suppose that three identical single-nozzle wheels are to be used, operating under the same head of 1320 ft. The total flow rate is to be 56.0 cfs. Determine the required specific speed of these turbines, their pitch diameter, the jet diameter, and the operating speed. Once again, assume $C_v = 0.98$, $\phi = 0.45$, and $\eta = 0.85$.

▮ $$Q = \tfrac{56}{3} = 18.7 \text{ cfs} \qquad \text{bhp} = \tfrac{7130}{3} = 2377 \qquad N_s = N\sqrt{2377}/(1320^{5/4}) = N(0.00613)$$

$$N = 131.2(60)/\pi D = 2056/D$$

From the two preceding expressions it is apparent that the required N_s depends on the operating speed, as does D. Hence there are a number of possible answers. If we let the operating speed be 400 rpm (18-pole generator for 60-cycle electricity), $N_s = 400(0.00613) = 2.45$, $D = \frac{2506}{400} = 6.26$ ft $= 75.2$ in. Thus three $N_s = 2.45$ wheels of pitch diameter 75 in operating at 400 rpm would suffice. These wheels would have relatively small buckets. At such a low specific speed the optimum efficiency of impulse wheels is usually less than 0.85. An alternative solution, for example, would be to use an operating speed of 600 rpm (12-pole generator for 60-cycle electricity). For this case, $N_s = (600)(0.00613) = 3.68$, $D = \frac{2506}{600} = 4.18$ ft $= 50.1$ in. Thus three $N_s = 3.68$ wheels, of pitch diameter 50.1 in, operating at 600 rpm would suffice.

Let us now determine the required jet diameter for these 50.1-in-diameter wheels operating at 600 rpm: $Q = AV = (\pi D_j^2/4)(286) = 18.7$ cfs from which $D_j = 0.289$ ft $= 3.46$ in.

21.5 A two-nozzle single-wheel turbine, with C_v, ϕ, and η as in Prob. 21.3, is designed to operate under a head of 1300 ft with a total flow of 56 cfs. Express the specific speed N_s and pitch diameter D as functions of the rotation speed N.

▮ $$V_j = C_v(2gh)^{1/2} = (0.98)[(2)(32.2)(1300)]^{1/2} = 283.6 \text{ fps} \qquad Q_j = A_jV_j = (A_j)(283.6) = \tfrac{56}{2}$$

$$A_j = 0.09873 \text{ ft}^2 \qquad \pi D_j^2/4 = 0.09800 \qquad D_j = 0.3546 \text{ ft} \quad \text{or} \quad 4.25 \text{ in}$$

$$bp_j = (\eta)(\gamma Q_jh)/550 = (0.85)(62.4)(28.0)(1300)/550 = 3510 \text{ bhp per jet}$$

$$N_s = N(\text{bhp})^{0.5}/h^{5/4} = (N)(3510)^{0.5}/1300^{5/4} = 0.007590N$$

$$D = 60\phi(2gh)^{1/2}/\pi N = (60)(0.45)[(2)(32.2)(1300)]^{1/2}/(\pi N) = 2487/N$$

21.6 A series of vanes is acted on by a 4-in water jet having a velocity of 102 fps, $\alpha_1 = \beta_1 = 0°$. In terms of the vane velocity v, find the blade angle β_2 that makes the force acting on the vanes in the direction of the jet equal 210 lb. Neglect friction.

▮ From the velocity vector diagram of Fig. 21-2,

$$\Delta V_x = (u + v_2\cos\beta_2) - V_1 \qquad Q = AV_1 = [(\pi)(\tfrac{4}{12})^2/4](102) = 8.901 \text{ ft}^3/\text{s}$$

$$F_x = \rho Q(-\Delta V_x) = -\rho Q[(u + v_2\cos\beta_2) - V_1] \qquad 210 = -(1.94)(8.901)[(u + v_2\cos\beta_2) - 102]$$

$$u + v_2\cos\beta_2 = 89.8 \qquad v_2 = v_1 = 102 - u \qquad u + (102 - u)\cos\beta_2 = 89.8$$

$$\beta_2 = \arccos\frac{89.8 - u}{102 - u}$$

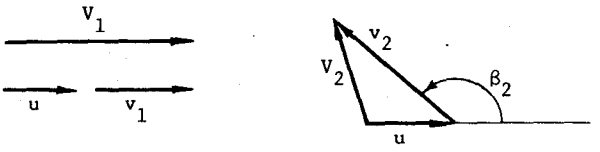


Fig. 21-2

21.7 Under a net head of 1 ft, the Pelton wheel for Fig. 21-3*a* discharges 0.288 cfs at full nozzle opening and the maximum power is 0.026 bhp for a value of $\phi = 0.465$. The corresponding brake torque is 3.70 ft-lb as shown in Fig. 21-3*b*. Assuming that the similarity laws apply precisely, determine the discharge, torque, power, and rotative speed of this wheel when it operates under a head of 1600 ft.

▮ On Fig. 21-3*a*, $h_1 = 1.0$ ft, $\phi = 0.465$, $Q_1 = 0.288$ cfs, $T_1 = 3.70$ ft-lb, $\text{bhp}_1 = 0.026$, and $r = 1$ ft.

$$u_1 = (\phi)(2gh_1)^{1/2} = (0.465)[(2)(32.2)(1.0)]^{1/2} = 3.732 \text{ fps} \qquad \omega_1 = u_1/r = 3.732/1 = 3.732 \text{ rad/s}$$

From the similarity laws, $Q \propto h^{1/2}$, $T \propto h$, $\omega \propto h^{1/2}$, and $\text{bhp} \propto h^{3/2}$. At $h_2 = 1600$ ft,

$$Q_2 = (0.288)(1600)^{1/2} = 11.5 \text{ ft}^3/\text{s} \qquad T_2 = (3.70)(1600) = 5920 \text{ ft-lb}$$

$$\omega_2 = (3.73)(1600)^{1/2} = 149.2 \text{ rad/s} \quad \text{or} \quad 1425 \text{ rpm} \qquad \text{bhp}_2 = (0.026)(1600)^{3/2} = 1664$$

21.8 A wheel and nozzle similar to that of Fig. 21-3*a* and *b* with a pitch diameter of 12 ft is used under a net head of 1500 ft. What are the torque, power, and rotative speed at point of best efficiency for full nozzle opening?

▮ In Fig. 21-3*a*, $\phi_e = 0.46$. In Fig. 21-3*b*, for $\phi = 0.46$, nozzle fully open, $T = 3.76$ ft-lb.

$$u = (\phi)(2gh)^{1/2} = (0.46)[(2)(32.2)(1)]^{1/2} = 3.691 \text{ fps} \qquad \omega = u/r = 3.691/1 = 3.691 \text{ rad/s}$$

$$\text{bhp} = Tu/550 = (3.76)(3.691)/550 = 0.02523 \qquad N = 60u/\pi D = (60)(3.691)/[(\pi)(2)] = 35.25 \text{ rpm}$$

For a 12-ft-diameter wheel and 1500 ft head, $T = (3.76)(\frac{12}{2})^3(1500) = 1.22 \times 10^6$ ft-lb, $P = (0.02523)(\frac{12}{2})^2(1500)^{3/2} = 52\,770$ hp, $N = (35.25)(1500)^{1/2}/6 = 228$ rpm.

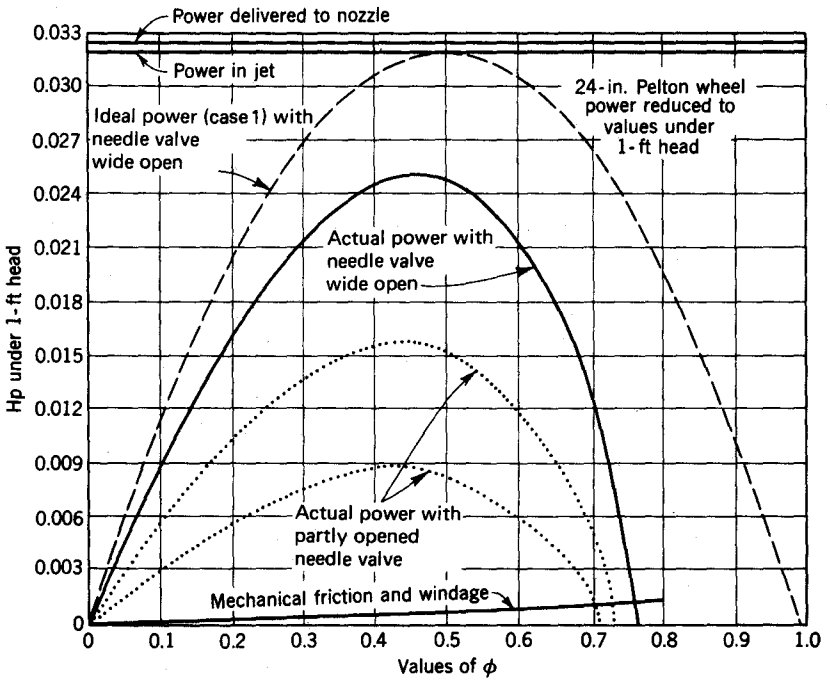


Fig. 21-3(*a*)

Relation between power and speed at constant head with maximum nozzle opening. (From tests made by F. G. Switzer and R. L. Daugherty.)

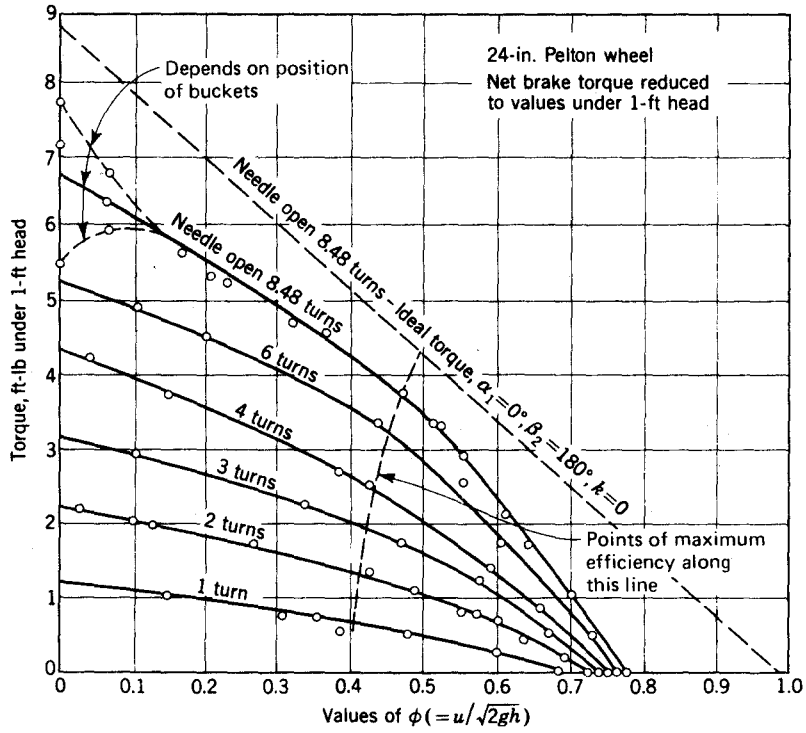


Fig. 21-3(*b*)

Relation between torque and speed at constant head. (From tests made by F. G. Switzer and R. L. Daugherty.)

21.9 Find the torque and power transferred to the buckets of an impulse wheel with $\alpha_1 = 0°$, $\beta_2 = 160°$, $k = 0.44$, $\phi = 0.46$, $C_v = 0.98$, a jet diameter of 10 in, and a pitch diameter of 10 ft. Find also, the hydraulic efficiency, and, expressed as a percentage of the total head, find the head loss in bucket friction, the energy head loss at discharge, and the head loss in the nozzle.

▮ For operation under the purely artificial value of 1-ft net head, $u = \phi\sqrt{2g} = 8.02\phi = 3.68$ fps, $V_1 = 0.98\sqrt{2g} = 7.86$ fps, and $\gamma Q = \gamma AV = (62.4)(0.545)(7.86) = 267$ lb/s. Hence, the expression for the torque exerted on the wheel by the water is $T = (r)(F_u) = 5(267/32.2)[1 + (0.940/1.2)](8.02)(C_v - \phi)$. With $C_v = 0.98$ and $\phi = 0.46$, $T = 309$ ft-lb. The power transferred from the water to the buckets is $F_u u = T\omega = T(u/r) = 309[(0.46)(8.02)/5] = 228$ ft-lb/s. The power input is $\gamma Qh = 267(1) = 267$ ft-lb/s. The hydraulic efficiency is $\frac{228}{267} = 0.85$. Note that the answers given here are for $h = 1$ ft. For other values of h, the flow rate, torque, and power can be found by adjusting the values for $h = 1$ ft according to similarity laws.

Under 1-ft head, $v_1 = V_1 - u = 7.86 - 3.68 = 4.18$ fps and $v_2 = v_1/\sqrt{1+k} = 4.18/\sqrt{1.44} = 3.48$ fps. Hence the head loss in bucket friction is $0.44(3.48)^2/2g = 0.083$ ft, or 8.3 percent. $V_2 \cos\alpha_2 = u + v_2 \cos\beta_2 = 3.68 + 3.48 \cos 160° = 3.68 - 3.27 = 0.41$ fps, $V_2 \sin\alpha_2 = v_2 \sin\beta_2 = 3.48 \sin 160° = 1.19$ fps. Hence $\cot\alpha_2 = 0.41/1.19 = 0.344$ or $\alpha_2 = 71°$, and $V_2 = 1.19/0.945 = 1.26$ fps from which the energy head loss at discharge is $1.26^2/2g = 0.025$ ft, or 2.5 percent. The head loss in the nozzle is approximately $(1/C_v^2 - 1)V_1^2/2g$, about 4.0 percent; so the total hydraulic loss is $8.3 + 2.5 + 4.0 \approx 14.8$ percent, which gives a close check on the computed hydraulic efficiency of 85 percent.

21.10 A nozzle having a velocity coefficient of 0.98 discharges a jet 6 in in diameter under a head of 900 ft. The wheel diameter is 8 ft, $\beta_2 = 170°$, $\alpha_1 = 0°$, $k = 0.5$, and the mechanical efficiency of the wheel is 96%. Calculate the hydraulic efficiency and the gross efficiency, assuming $\phi = 0.45$.

▮ Nozzle efficiency $= 0.98^2 = 0.9604$, $\eta_h = (2)[1 - (\cos\beta_2/\sqrt{1+k})](C_v - \phi)(\phi) = (2)[1 - (\cos 170°/\sqrt{1+0.5})](0.98 - 0.45)(0.45) = 0.861$. As C_v is included, η_h is here the hydraulic efficiency of both the wheel and the nozzle. $\eta = \eta_h\eta_m = (0.861)(0.96) = 0.830$.

21.11 A 24-in Pelton wheel is tested under a head of 65.5 ft. For a certain nozzle setting, the net brake load at 276 rpm

is 40 lb at a brake arm of 5.0 ft, and the discharge is 1.90 cfs. What are the brake horsepower and efficiency under these conditions?

▮ $T_2 = dW = (5.0)(40) = 200$ ft-lb $\qquad \omega = 2\pi n/60 = (2)(\pi)(276)/60 = 28.9$ rad/s

$$\text{bp} = T_2\omega = (200)(28.9) = 5780 \text{ ft-lb/s} \quad \text{or} \quad 10.5 \text{ hp}$$

$$P_w = Q\gamma H = (1.90)(62.4)(65.5) = 7766 \text{ ft-lb/s} \quad \text{or} \quad 14.1 \text{ hp} \qquad \eta = 10.5/14.1 = 74.5\%$$

21.12 Supposing the bearing-friction and windage losses in Prob. 21.11 to be 0.2 hp, calculate the mechanical efficiency and the hydraulic efficiency.

▮ Bearing-friction and windage loss = 0.2/10.5 = 0.019, or 1.9 percent.

$$\eta_m = 100 - 1.9 = 98.1 \text{ percent} \qquad \eta_h = \eta/(\eta_v\eta_m) \qquad \eta_v = 1$$

$$\eta_h = 0.745/[(1)(0.981)] = 0.759 \quad \text{or} \quad 75.9 \text{ percent}$$

21.13 Find the approximate hydraulic efficiency of an impulse wheel for which the nozzle velocity coefficient is 0.96 and the bucket angle is 165°, if $\phi = 0.46$ and $k = 0.1$.

▮ $$\eta_h = (2)[1 - (\cos\beta_2/\sqrt{1+k})](C_v - \phi)(\phi) = (2)[1 - (\cos 165°/\sqrt{1+0.1})](0.96 - 0.46)(0.46)$$
$$= 0.884 \quad \text{or} \quad 88.4 \text{ percent}$$

21.14 A double-overhung impulse-turbine installation is to develop 20 000 hp at 275 rpm under a net head of 1100 ft. Determine N_s, wheel-pitch diameter, and approximate jet diameter.

▮ $$N_s = NP^{0.5}/H^{5/4} = (275)(20\,000/2)^{0.5}/1100^{5/4} = 4.34$$
$$D = 153.2\phi\sqrt{h}/N = (153.2)(0.45)(\sqrt{1100})/275 = 8.31 \text{ ft}$$
$$D_j = D/12 = 8.31/12 = 0.6925 \text{ ft} \quad \text{or} \quad 8.31 \text{ in}$$

21.15 Repeat Prob. 21.14 for a single wheel with a single nozzle.

▮ $N_s = (275)(20\,000/1)^{0.5}/1100^{5/4} = 6.14$. Noting the computation of D in Prob. 21.14, D is unchanged; hence, $D_j = 8.31$ in.

21.16 Repeat Prob. 21.14 for a single wheel with four nozzles.

▮ $N_s = (275)(20\,000/4)^{0.5}/1100^{5/4} = 3.07$. Noting the computation of D in Prob. 21.14, D is unchanged; hence, $D_j = 8.98$ in.

21.17 An impulse turbine ($N_s = 5$) develops 105 800 hp under a head of 2050 ft. For 60-Hz electricity calculate the turbine speed, wheel diameter, and number of poles in the generator. Assume $\phi = 0.45$.

▮ $$N_s = NP^{0.5}/H^{5/4} \qquad 5 = (N)(105\,800)^{0.5}/2050^{5/4} \qquad N = 212 \text{ rpm} = 7200/N_{\text{poles}}$$
$$N_{\text{poles}} = 7200/212 \approx 34 \qquad D = 153.2\phi\sqrt{h}/N = (153.2)(0.45)(\sqrt{2050})/212 = 14.72 \text{ ft}$$

21.18 Rework Prob. 21.17 for a six-nozzle unit using the same N_s, bhp, and head.

▮ $$N_s = NP^{0.5}/H^{5/4} \qquad 5 = (N)(105\,800/6)^{0.5}/2050^{5/4} \qquad N = 519 \text{ rpm} = 7200/N_{\text{poles}}$$
$$N_{\text{poles}} = 7200/519 \approx 14 \qquad D = 153.2\phi\sqrt{h}/N = (153)(0.45)(\sqrt{2050})/514 = 6.06 \text{ ft}$$

21.19 Making suitable assumptions, estimate the wheel diameter and the rotative speed of a turbine that will develop 14 900 bhp under a head of 995 ft.

▮ $N_s = NP^{0.5}/H^{5/4}$. Best efficiency occurs at $N_s < 4.5$. Assume $N_s = 4$: $4 = (N)(14\,900)^{0.5}/995^{5/4}$, $N = 183$ rpm, $D = 153.2\phi\sqrt{h}/N$. Assume synchronous speed = 180 rpm and $\phi = 0.46$: $D = (153.2)(0.46)(\sqrt{995})/180 = 12.35$ ft.

21.20 An 18-in-diameter model turbine is operated at 600 rpm under a total head of 55.0 ft; the brake horsepower and flow are measured as 330 and 61 cfs. Calculate **(*a*)** the specific speed, **(*b*)** the value of ϕ, **(*c*)** the efficiency, and **(*d*)** the shaft torque.

▮ **(*a*)** $$N_s = NP^{0.5}/H^{5/4} = (600)(330)^{0.5}/55.0^{5/4} = 72.8$$

(*b*) $$\phi = u/(2gh)^{1/2} = (2\pi N/60)r/(2gH)^{1/2} = [(2)(\pi)(600)/60][9/12]/[(2)(32.2)(55.0)]^{1/2} = 0.792$$

(c) $\eta = P/\gamma QH = (330)(550)/[(62.4)(61)(55.0)] = 0.867$ or 86.7 percent

(d) $T = P/\omega = (330)(550)/[(2\pi)(600)/60] = 2889$ ft-lb

21.21 Refer to Prob. 21.20. (*a*) What would be the efficiency of a 12-ft-diameter prototype turbine? (*b*) If the prototype is to operate at 140 rpm under a net head of 202 ft, find its horsepower output and flow rate.

▮ (*a*) $(1-\eta_2)/(1-\eta_1) = (D_1/D_2)^{1/5}$ $\quad (1-\eta_2)/(1-0.867) = (1.5/12)^{1/5}$ $\quad \eta_2 = 0.912$ or 91.2 percent

(*b*) $N_s = NP^{0.5}/H^{5/4}$ $\quad 72.8 = 140P^{0.5}/202^{5/4}$ $\quad P = 156\,800$ bhp

$P = Q\gamma H/550$ $\quad 156\,800 = (Q)(62.4)(202)(0.912)/550$ $\quad Q = 7502$ ft^3/s

21.22 A single turbine with $N_s \approx 25$ is to develop 4180 hp under a head of 245 ft. What rotative speed would occur with 50-Hz electricity? How many poles should the generator have?

▮ $N_s = NP^{0.5}/H^{5/4}$ $\quad 25 = (N)(4180)^{0.5}/245^{5/4}$ $\quad N = 375$ rpm

For 50 cycles, $N_{\text{poles}} = 6000/N = \frac{6000}{375} = 16$.

21.23 The Francis turbine for which the test curves are shown in Figs. 21-4*a* and *b* has a 27-in-diameter runner and a maximum efficiency of 86% when discharging 39 cfs and developing 560 bhp at 600 rpm under a net head of 142.0 ft. Compute N_s, ϕ_e, and C_r, assuming $B/D = 0.15$.

▮ $N_s = NP^{0.5}/H^{5/4} = (600)(560)^{0.5}/142.0^{5/4} = 29.0$ $\quad ND = 153.2\phi\sqrt{H}$

$(600)(\frac{27}{12}) = (153.2)(\phi_e)\sqrt{142}$ $\quad \phi_e = 0.740$

$C_r = N_s^2/[(63\,800)(\phi_e^2)(B/D)(\eta)] = (29.0)^2/[(63\,800)(0.740)^2(0.15)(0.86)] = 0.1866$ $\quad V_r = Q/A_c$

$A_c = 0.95\pi(B/D)D^2 = (0.95)(\pi)(0.15)(\frac{27}{12})^2 = 2.266$ ft^2 $\quad V_r = 39/2.266 = 17.21$ fps

$C_r = V_r/(2gH)^{1/2} = 17.21/[(2)(32.2)(142.0)]^{1/2} = 0.1800$ (a good check)

21.24 If a turbine homologous to that in Prob. 21.23 were made with a runner diameter of 130 in, what would be its probable efficiency under the same head?

▮ $(1-\eta_2)/(1-\eta_1) = (D_1/D_2)^{1/5}$ $\quad (1-\eta_2)/(1-0.86) = (27/130)^{1/5}$ $\quad \eta = 0.898 = 89.8\%$

21.25 The turbine of Prob. 21.23 has a horizontal shaft, and at the time of the test the centerline of the shaft was 12.70 ft above the surface of the water in the tailrace. The discharge edge of the runner at its highest point is 0.85 ft above the centerline of the shaft. Evaluate the cavitation factor σ, assuming a barometric pressure of 14.6 psia and a vapor pressure of 0.5 psia.

▮ $\sigma = (p_{\text{atm}}/\gamma - p_v/\gamma - z_{\max})/h$ $\quad p_{\text{atm}}/\gamma = (14.6)(144)/62.4 = 33.69$ ft

$p_v/\gamma = (0.5)(144)/62.4 = 1.15$ ft $\quad z_{\max} = 12.70 + 0.85 = 13.55$ ft

$\sigma = (33.69 - 1.15 - 13.55)/142 = 0.134$

21.26 If the critical value of the cavitation factor for the turbine of Prob. 21.25 is 0.06, compute the maximum allowable height of the centerline of the shaft above the tailwater surface.

▮ $z_{\max} = p_{\text{atm}}/\gamma - p_v/\gamma - \sigma_c h = 33.69 - 1.15 - (0.06)(142.0) = 24.02$ ft

Height to centerline of shaft $= 24.02 - 0.85 = 23.17$ ft

21.27 Rework Prob. 21.26, if the total head on the turbine is 420 ft.

▮ $z_{\max} = p_{\text{atm}}/\gamma - p_v/\gamma - \sigma_c h = 33.69 - 1.15 - (0.06)(420) = 7.34$ ft

Height to centerline of shaft $= 7.34 - 0.85 = 6.49$ ft

21.28 Find the maximum permissible head under which an axial-flow turbine ($N_s = 160$) can operate if it is set 6 ft below the tailwater. The installation is at elevation 3150 ft where $p_{\text{atm}} = 14.15$ psia, and the water temperature is 65 °F. Assume $\sigma_c = 0.90$.

▮ $z_{\max} = p_{\text{atm}}/\gamma - p_v/\gamma - \sigma_c h$ $\quad -6 = (14.15)(144)/62.4 - 44.4/62.4 - 0.90h$ $\quad h = 42.2$ ft

21.29 In Prob. 21.23, the Francis turbine has frictional losses of 2.8 hp. Assuming that the leakage is 1% of the measured discharge and that $\alpha_2 = 90°$, find the values of η_h, α_1, and β_1.

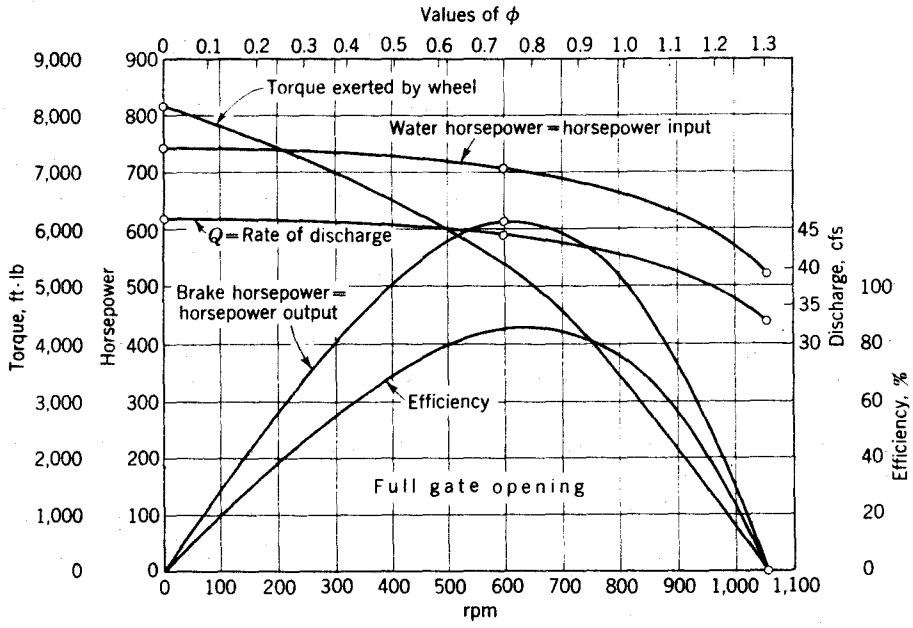


Fig. 21-4(*a*). Francis turbine operating at variable speed under constant head with constant gate opening. Runner diameter = 27 in, head = 140.5 ft.

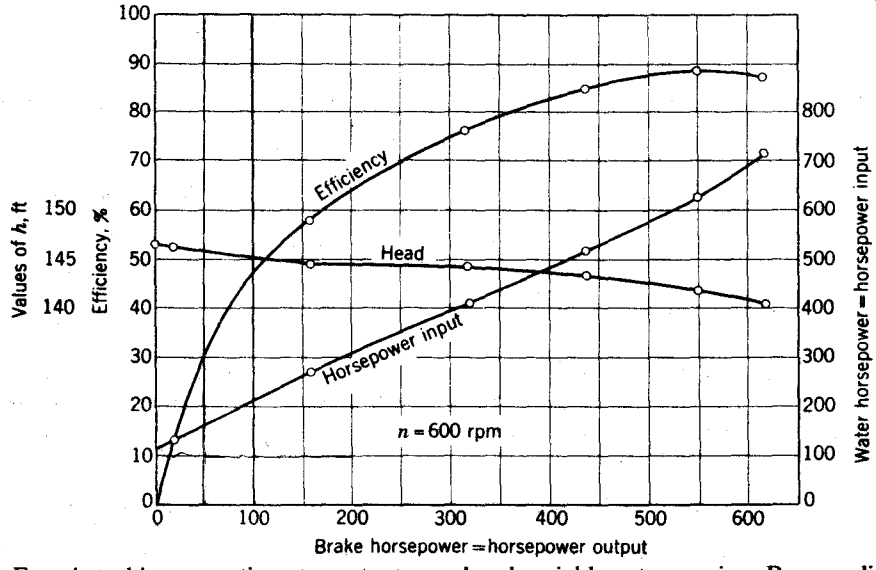


Fig. 21-4(*b*). Francis turbine operating at constant speed and variable gate opening. Runner diameter = 27 in.

▮ $\eta_v = 99\% = 0.99 \qquad \eta_m = bp/(bp + fp) = 560/(560 + 2.8) = 0.995$

$$\eta_h = 0.86/[(0.99)(0.995)] = 0.873 \qquad C_1 = \cos\alpha_1 = 0.873/[(2)(0.74)] = 0.5899 \qquad \eta_h = 2\phi_e C_1 \cos\alpha_1$$

$$\tan\alpha_1 = (C_1 \sin\alpha_1)/(C_1 \cos\alpha_1) = C_r/(C_1 \cos\alpha_1) = 0.1866/0.5899 = 0.3163 \qquad \alpha_1 = 17.55°$$

$$\cot\beta_1 = (C_1 \cos\alpha_1 - \phi)/(C_1 \sin\alpha_1) = (C_1 \cos\alpha_1 - \phi)/C_r = (0.5899 - 0.74)/0.1866 = -0.80439 \qquad \beta_1 = 141.2°$$

21.30 A dam turbine has runner diameter 16′ 6″. The height of the guide vanes is 34.4 in. The turbine is rated at 150 000 hp under a head of 331 ft at 120 rpm. At this power the efficiency is 88 percent and the absolute velocity of the water entering the runner is 77.2 fps. Compute the rated specific speed, ϕ, and C_1 for this full-gate opening.

▮ $N_s = NP^{0.5}/H^{5/4} = (120)(150\,000)^{0.5}/331^{5/4} = 32.9 \qquad \phi = u_1/(2gH)^{1/2}$

$$u_1 = r\omega = r(2\pi N/60) = [16.5/2][(2)(\pi)(120)/60] = 103.7 \text{ ft/s}$$

$$\phi = 103.7/[(2)(32.2)(331)]^{1/2} = 0.710 \qquad C_1 = V_1/(2gh)^{1/2} = 77.2/[(2)(32.2)(331)]^{1/2} = 0.529$$

21.31 For the data of Prob. 21.30, find C_r, α_1, and β_1.

▮ $C_r = V_r/(2gh)^{1/2}$ $\quad V_r = Q/A_c$ $\quad A_c = 0.95\pi BD = (0.95)(\pi)(34.4/12)(16.5) = 141.2\ \text{ft}^2$

$$Q = \frac{\text{bhp}}{\gamma H\eta} = \frac{(150\,000)(550)}{(62.4)(331)(0.88)} = 4539\ \text{cfs} \qquad V_r = 4539/141.2 = 32.15\ \text{fps}$$

$$C_r = 32.15/[(2)(32.2)(331)]^{1/2} = 0.220 \qquad \sin\alpha_1 = C_r/C_1 = 0.220/0.529 = 0.415879 \qquad \alpha_1 = 24.6°$$

$$\tan\beta_1 = (C_1\sin\alpha_1)/(\phi - C_1\cos\alpha_1) = (0.529)(\sin 24.6°)/[0.710 - (0.529)(\cos 24.6°)] = 0.96157 \qquad \beta_1 = 43.9°$$

21.32 How many identical turbines, operating at 139.0 rpm and 91% efficiency, are needed to exploit a head of 1200 ft and a flow of 1660 ft^3/s?

▮ bhp $= \eta Q\gamma H_p/550 = (0.91)(1660)(62.4)(1200)/550 = 206\,000$. From Fig. A-42, $N_s \le 5.4$. Let x = number of identical turbines. $N_s \ge NP^{0.5}/H_p^{5/4}$, $5.4 = (139.0)(206\,000/x)^{0.5}/1200^{5/4}$, $x = 2.7$; use three turbines.

21.33 A Francis turbine, working a total head of 102 ft, is to produce 25 000 hp at maximum efficiency. What speed and diameter should this turbine have?

▮ For a Francis turbine, $N_s = 20$ to 80; try $N_s = 50$: $N_s = NP^{0.5}/H^{5/4}$, $50 = N(25\,000)^{0.5}/102^{5/4}$, $N = 103$ rpm; $D = 153.2\phi\sqrt{H}/N$. From Fig. A-43 for $N_s = 50$, $\phi_e = 0.78$. $D = (153.2)(0.78)\sqrt{102}/103 = 11.70$ ft.

21.34 Solve Prob. 21.33 for a propeller turbine.

▮ For a propeller turbine, $N_s = 100$ to 250; try $N_s = 120$: $N_s = NP^{0.5}/H^{5/4}$, $120 = N(25\,000)^{0.5}/102^{5/4}$, $N = 246$ rpm; $D = 153.2\phi\sqrt{H}/N$. From Fig. A-43 for $N_s = 120$, $\phi_e = 1.63$. $D = (153.2)(1.63)\sqrt{102}/246 = 10.25$ ft.

21.35 How may poles should a 50-Hz generator have, if it is connected to a turbine operating under a design head of 3000 ft with a flow of 82 cfs? Assume turbine efficiencies as given in Fig. A-44 and avoid cavitation.

▮ $N_s = NP^{0.5}/H^{5/4}$. From Fig. A-42, we must use $N_s \le 3$; use $N_s = 3$.

$$P = \eta Q\gamma H/550 = (0.84)(82)(62.4)(3000)/550 = 23\,444\ \text{hp}$$

$$3 = (N)(23\,444)^{0.5}/3000^{5/4} \qquad N = 435\ \text{rpm} \qquad N_{\text{poles}} = \frac{(50)(120)}{435} = 13.8$$

Use a 12- or 14-pole generator.

21.36 A turbine is to be installed where the net available head is 186 ft, and the available flow will average 885 cfs. Specify type, operating speed, and number of generator poles for 60-Hz electricity, if the turbine is to have the highest tolerable specific speed without cavitation. Assume the turbine is set 5 ft above tailwater and is 91% efficient. Approximately what size of runner is required?

▮ $N_s = NP^{0.5}/H^{5/4}$ $\quad$ bhp $= \eta Q\gamma H/550 = (0.91)(885)(62.4)(186)/550 = 16\,995$

From Fig. A-42 for $H = 186$ ft and draft head = +5 ft, highest tolerable $N_s = 50$. $50 = (N)(16\,995)^{0.5}/186^{5/4}$, $N = 263$ rpm. With 60-cycle electricity, a 28-pole generator will operate at $\frac{7200}{28} = 257$ rpm. So use a 28-pole generator operating at 257 rpm. $N_s = (257)(16\,995)^{0.5}/186^{5/4} = 48.8$. Use a Francis turbine with $N_s = 48.8$. $D = 153.3\phi\sqrt{H}/N = (153.2)(0.78)(\sqrt{186})/257 = 6.34$ ft.

21.37 Repeat Prob. 21.36 for the following data: head, 176 ft; flow, 1000 cfs; 60-Hz power; draft head, 10 ft; efficiency, 90%.

▮ $N_s = NP^{0.5}/H^{5/4}$. From Fig. A-42 for $H = 176$ ft and draft head = +10 ft, $N_s \le 48$.

$$\text{bhp} = \eta Q\gamma H/550 = (0.90)(1000)(62.4)(176)/550 = 17\,971 \qquad 48 = (N)(17\,971)^{0.5}/176^{5/4} \qquad N = 230\ \text{rpm}$$

$N_{\text{poles}} = 7200/N = \frac{7200}{230} = 31$; use 32 poles. $32 = 7200/N$, $N = 225$ rpm; $N_s = (225)(17\,971)^{0.5}/176^{5/4} = 47.1$; use a Francis turbine. From Fig. A-43 for $N_s = 47.3$, $\phi_e = 0.77$. $D = 153.2\phi\sqrt{H}/N = (153.2)(0.77)(\sqrt{176})/225 = 6.96$ ft.

21.38 For the data of Prob. 21.37, select a set of identical turbines to be operated in parallel. Specify the speed and size of the units.

▮ $$N_s = NP^{0.5}/H^{5/4} \qquad N_s \le 48 \qquad \text{bhp} = 17\,971/2 = 8986$$
$$48 = (N)(8986)^{0.5}/176^{5/4} \qquad N = 325 \qquad N_{\text{poles}} = 7200/N$$

$N_{\text{poles}} = \frac{7200}{325} = 22.2$; use 22 poles. $22 = 7200/N$, $N = 327$; $N_s = (327)(8986)^{0.5}/176^{5/4} = 48.4$. From Fig. A-43 for $N_s = 48.4$, $\phi_e = 0.75$. $D = 153.2\phi\sqrt{h}/N = (153.2)(0.75)(\sqrt{176})/327 = 4.66$ ft.

21.39 Water enters a rotating wheel with a relative velocity of 198 fps; $r_1 = 4.0$ ft, $r_2 = 3.0$ ft, and $N = 425$ rpm. There is no pressure drop in flow over the vanes. Assume $k = 0.2$. Find the relative velocity at discharge.

▮ With no pressure drop, $(V_1^2 - u_1^2)/2g - (V_2^2 - u_2^2)/2g = kV_2^2/2g$, $u = \pi DN/60$.

$$u_1 = (\pi)[(2)(4)](425)/60 = 178.0 \text{ ft} \qquad u_2 = (\pi)[(2)(3)](425)/60 = 133.5 \text{ ft}$$
$$(198^2 - 178.0^2)/[(2)(32.2)] - (V_2^2 - 133.5^2)/[(2)(32.2)] = 0.2V_2^2/[(2)(32.2)] \qquad V_2 = 145 \text{ fps}$$

21.40 Water enters a rotating wheel in such manner that the passages are completely filled. If $Q = 406$ cfs, $a_1 = 10$ ft^2, $a_2 = 8$ ft^2, $r_1 = 1.5$ ft, $r_2 = 1.0$ ft, $N = 538$ rpm, and $k = 0.2$, find the drop in pressure head between entrance and exit.

▮ $$(p_1/\gamma - p_2/\gamma) + (V_1^2 - u_1^2)/2g - (V_2^2 - u_2^2)/2g = kV_2^2/2g \qquad V_1 = Q/a_1 = \tfrac{400}{10} = 40.6 \text{ fps}$$
$$V_2 = \tfrac{406}{8} = 50.8 \text{ fps} \qquad u = \pi DN/60$$
$$u_1 = (\pi)[(2)(1.5)](538)/60 = 84.51 \text{ fps} \qquad u_2 = (\pi)[(2)(1.0)](538)/60 = 56.34 \text{ fps}$$
$$(p_1/\gamma - p_2/\gamma) + (40.6^2 - 84.51^2)/[(2)(32.2)] - (50.8^2 - 56.34^2)/[(2)(32.2)] = (0.2)(50.8^2)/[(2)(32.2)]$$
$$p_1/\gamma - p_2/\gamma = 103 \text{ ft}$$

21.41 It is desired to develop 300 000 hp under a head of 48 ft and to operate at 60 rpm. How many Francis turbines with a specific speed of 80 are required?

▮ $$N_s = NP^{0.5}/H^{5/4} \qquad 80 = 60P^{0.5}/49^{5/4} \qquad P = 29\,879 \text{ bhp}$$

Number of units required = 300 000/29 879 = 10.0.

21.42 Air flows steadily, as shown in Fig. 21-5, through a turbine which produces 700 hp. For the inlet and exit conditions shown, estimate **(*a*)** the exit velocity V_2 and **(*b*)** the heat transferred $\dot{Q}$.

▮ **(*a*)** The inlet and exit densities can be computed from the perfect-gas law: $\rho_1 = p_1/RT_1 = 150(144)/[1715(460 + 300)] = 0.0166$ slug/ft^3, $\rho_2 = p_2/RT_2 = 40(144)/[1715(460 + 35)] = 0.00679$ slug/ft^3. The mass flow is determined by the inlet conditions $\dot{m} = \rho_1 A_1 V_1 = (0.0166)(\pi/4)(\frac{6}{12})^2(100) = 0.326$ slug/s. Knowing mass flow, we compute the exit velocity

$$\dot{m} = 0.326 = \rho_2 A_2 V_2 = (0.00679)(\pi/4)(\tfrac{6}{12})^2 V_2 \quad \text{or} \quad V_2 = 245 \text{ fps}$$

(*b*) The steady-flow energy equation applies with $\dot{W}_v = 0$, $z_1 = z_2$, and $\hat{h} = c_p T$:

$$\dot{Q} = \dot{W}_s + \dot{m}(c_p T_2 + \tfrac{1}{2}V_2^2 - c_p T_1 - \tfrac{1}{2}V_1^2)$$
$$\dot{Q} = 700(550) + 0.326[6003(495) + \tfrac{1}{2}(245)^2 - 6003(760) - \tfrac{1}{2}(100)^2] = -125\,000 \text{ ft-lb/s}$$

The negative sign indicates that this heat transfer is a *loss* from the control volume.

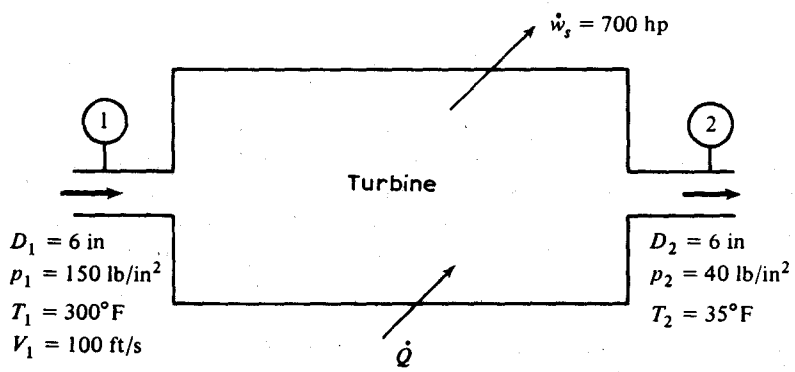


Fig. 21-5

21.43 A hydroelectric power plant (Fig. 21-6) takes in 30 m^3/s of water through its turbine and discharges it at $V_2 = 2$ m/s at atmospheric pressure. The head loss in the turbine and penstock system is $h_f = 20$ m. Estimate the power extracted by the turbine.

▮ $(p_1/\rho g) + (V_1^2/2g) + z_1 = (p_2/\rho g) + (V_2^2/2g) + z_2 + h_s$. We neglect viscous work and heat transfer and take section 1 at the reservoir surface, where $V_1 \approx 0$ and $p_1 = p_a$. Section 2 is at the turbine outlet.

$$(p_a/\rho g) + \tfrac{1}{2}(0)^2 + 100 = (p_a/\rho g) + [\tfrac{1}{2}(2)^2/9.81] + 0 + h_f + h_s \qquad (1)$$

With h_f given as 20 m, we solve for $h_s = 100 - 20 - 0.2 = 79.8$ m; then

$$w_s = h_s g = (79.8)(9.81) = 783 \text{ m}^2/\text{s}^2 = 783 \text{ J/kg} \qquad (2)$$

The result is positive, as expected from our sign convention that work done by the fluid on a turbine is positive. The power extracted is $P = \dot{m}w_s = \rho Q w_s = (1000)(30)(783) = 23.5 \times 10^6$ J/s = 23.5 MW.

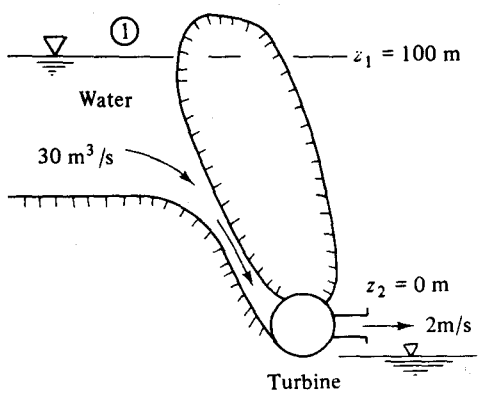


Fig. 21-6

21.44 It is proposed to build a dam in a river where the flow rate is 10 m^3/s and a 32-m drop in elevation can be achieved for flow through a turbine. If the turbine is 82 percent efficient, what is the maximum power that can be achieved?

▮
$$P = \eta\gamma QH = (0.82)(9.79)(10)(32) = 2570 \text{ kW}$$

21.45 Steam enters a turbine at 400 psia, 600 °F, and 10 fps and is discharged at 100 fps and 20 psia saturated conditions. The mass flux is 2.0 lb/s, and the heat loss from the turbine is 6 Btu per pound of steam. Head losses and elevation changes are negligible. How much horsepower does the turbine develop?

▮ $Q - W_s - W_v = \dot{m}(h_2 + V_2^2/2 - h_1 - V_1^2/2)$. 1 Btu/lbm = 25 000 ft-lb/slug; hence, $Q = (-6)(25\,000)(2.0/32.2) = -9332$ ft-lb/s. From the steam tables:

At 400 psia, 600 °F: $h_1 = 1306.6$ Btu/lbm

At 20 psia saturated: $h_2 = 1156.4$ Btu/lbm

$$-9332 - W_s - 0 = (2.0/32.2)[(1156.4)(25\,040) + 100^2/2 - (1306.6)(25\,000) - 10^2/2]$$

$$W_s = 224\,337 \text{ ft-lb/s} = 224\,337/550 = 408 \text{ hp}$$

21.46 Investigate the possibility of using a Pelton wheel similar to Fig. A-45 to deliver 30 000 bhp from a net head of 1200 ft.

▮ From Fig. A-46, the most efficient Pelton wheel occurs at about $N_s \approx 4.5 = (N)(30\,000)^{1/2}/(1200)^{1.25}$ or $N = 183$ rpm. From Fig. A-45 the best operating point is $\phi \approx 0.47 = \pi D(3.06)/[2(32.2)(1200)]^{1/2}$ or $D = 13.6$ ft. This Pelton wheel is perhaps a little slow and a trifle large. You could reduce D and increase N by increasing N_s to, say, 6 or 7 and accepting the slight reduction in efficiency. Or you could use a double-hung, two-wheel configuration, each delivering 15 000 bhp, which changes D and N by the factor $2^{1/2}$.

Double wheel: $N = (183)2^{1/2} = 259$ rpm $\qquad D = 136/2^{1/2} = 9.6$ ft

21.47 For the data given in Prob. 21.46, investigate the possibility of using the Francis-turbine family of Fig. A-47.

▮ The Francis wheel of Fig. A-47 must have $N_s = 29 = (N)(30\,000)^{1/2}/(1200)^{1.25}$ or $N = 1183$ rpm. Then the optimum power coefficient is $C_{P*} = 2.70 = P/\rho N^3 D^5 = 30\,000(550)/[(1.94)(\frac{1183}{60})^3 D^5]$ or $D = 3.33$ ft = 40 in. This is a faster speed than normal practice, and the casing would have to withstand 1200 ft of water or about 520 lb/in^2 internal pressure, but the 40-in size is extremely attractive. Francis turbines are now being operated at heads up to 1500 ft.

21.48 A lawn sprinkler can be used as a simple turbine. As shown in Fig. 21-7*a*, the flow enters normal to the paper in the center and splits evenly into $Q/2$ and V_{rel} leaving each nozzle. The arms rotate at angular velocity ω and do work on a shaft. Draw the velocity diagram for this turbine. Neglecting friction, find an expression for the power delivered to the shaft. Find the rotation rate for which the power is a maximum.

▮ The velocity diagram for this turbine is shown in Fig. 21-7*b*.

$$P = \omega T = \rho Q(u_2 V_{t2} - u_1 V_{t1}) = \rho Q u(W - u) = \rho Q \omega R(V_{rel} - \omega R) \qquad dP/du = \rho Q(W - 2u) = 0$$

$$u = W/2 \quad \text{or} \quad \omega R = V_{rel}/2 \qquad P_{max} = \rho Q(\omega R)^2 = \rho Q u^2$$

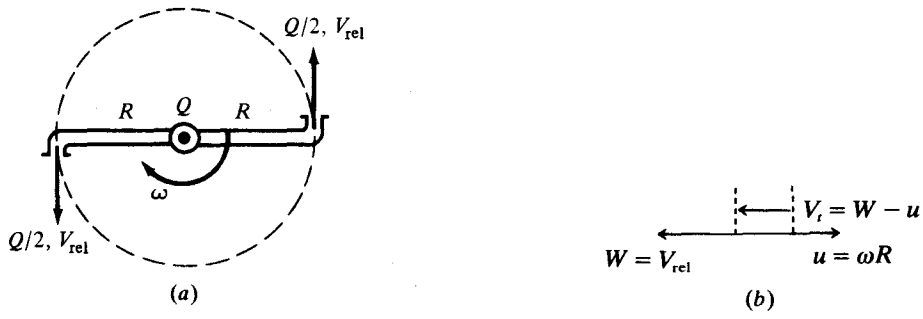


Fig. 21-7

21.49 Apply the result of Prob. 21.48 to a nozzle flow of water, $Q = 5$ L/s, where the exit diameter is 1 cm.

▮ $$P_{max} = \rho Q u^2 \qquad u = V_{rel}/2 \qquad V_{rel} = (Q/2)/A_{exit} = [(5 \times 10^{-3})/2]/[(\pi)(0.01)^2/4] = 31.83 \text{ m/s}$$

$$u = 31.83/2 = 15.92 \text{ m/s} \qquad P_{max} = (998)(5 \times 10^{-3})(15.92)^2 = 1265 \text{ W}$$

21.50 Show that if the net head H is varied for a turbine operating at a given valve opening and efficiency, the speed will vary as $H^{1/2}$ and the output power as $H^{3/2}$.

▮ At constant efficiency, gH/N^2D^2 is constant; and, since g and D are constant, $H \propto N^2$. Hence, $N \propto H^{1/2}$. Also at constant efficiency, $P/\rho N^3 D^5$ is constant; and, since ρ and D are constant, $P \propto N^3 \propto (H^{1/2})^3$. Hence, $P \propto H^{3/2}$.

21.51 Turbines are to be installed where the net head is 398 ft and the flow rate is 250 000 gal/min. Discuss the type, number, and size of turbine which might be selected if the generator selected is **(*a*)** 48-pole, 60 cycle ($n = 150$ rpm); and **(*b*)** 8-pole ($n = 900$ rpm). Why are at least two turbines desirable from a planning point of view?

▮ Assume $\eta = 90$ percent. Select two turbines so that one will still be available for power if the other is shut down.

(*a*) $$P = \eta\gamma QH = (0.90)(62.4)[(250\,000)(0.002228)](398)/550 = 22\,640 \text{ bhp}$$

$$N_s = NP^{0.5}/H^{5/4} = (150)(22\,640/2)^{0.5}/398^{5/4} = 9.0$$

Select two impulse turbines. $\phi = \pi ND/(2gH)^{1/2}$. Estimate $\phi = 0.47$. $0.47 = (\pi)(\frac{150}{60})(D)/[(2)(32.2)(398)]^{1/2}$, $D = 9.60$ ft.

(*b*) $N_s = (900)(22\,640/2)^{0.5}/398^{5/4} = 53.9$. Select two Francis turbines. $C_p^* = P/\rho N^3 D^5$. From Fig. A-47, $C_p^* = 2.6$. $2.6 = (22\,640/2)(550)/[(1.94)(\frac{900}{60})^3(D^5)]$, $D = 3.26$ ft.

21.52 What type of turbine develops 54 000 bhp at 80 rpm under a head of 90 ft? Estimate the flow rate.

▮ $$N_s = NP^{0.5}/H^{5/4} = (80)(54\,000)^{0.5}/90^{5/4} = 67.1 \qquad \text{(Francis turbine)} \qquad P = \eta\gamma QH$$

From Fig. A-48, $\eta = 93$ percent: $(54\,000)(550) = (0.93)(62.4)(Q)(90)$, $Q = 5687$ ft^3/s.

21.53 What type of turbine delivers 25 000 bhp at 500 rpm under a net head of 5340 ft? Estimate the impeller diameter.

▮ $N_s = NP^{0.5}/H^{5/4} = (500)(25\,000)^{0.5}/5340^{5/4} = 1.73$ (impulse turbine) $\phi = \pi ND/(2gH)^{1/2}$.

Estimate $\phi = 0.47$. $0.47 = (\pi)(\frac{500}{60})(D)/[(2)(32.2)(5340)]^{1/2}$, $D = 10.5$ ft.

21.54 A Pelton wheel of 12-ft pitch diameter operates under a net head of 1990 ft. Estimate the speed, power output, and flow rate for best efficiency if the nozzle exit diameter is 4 in.

▮ $V_{jet} = C_v\sqrt{2gH}$. Take $C_v = 0.94$: $V_{jet} = (0.94)\sqrt{(2)(32.2)(1990)} = 337$ ft/s. $\phi = \pi ND/(2gH)^{1/2}$. Estimate $\phi = 0.47$: $0.47 = (\pi)(N/60)(12)/[(2)(32.2)(1990)]^{1/2}$, $N = 268$ rpm.

$$Q = AV = [(\pi)(\tfrac{4}{12})^2/4](337) = 29.4\ \text{ft}^3/\text{s} \qquad P_w = \rho Qu(V_{jet} - u)(1 - \cos\beta)$$

Assume $\beta = 165°$.

$$u = V_{jet}/2 = \tfrac{337}{2} = 168.5\ \text{fps} \qquad P_w = (1.94)(29.4)(168.5)(337 - 168.5)(1 - \cos 165°)/550 = 5788\ \text{hp}$$

$$N_s = NP^{0.5}/H^{5/4} = (268)(5788)^{0.5}/1990^{5/4} = 1.53$$

Hence, $\eta = 0.75$. $P = (0.75)(5788) = 4341$ hp.

21.55 It is planned to use the Francis-turbine family of Fig. A-47 at an installation with a head of 602 ft and a flow rate of 198 ft^3/s. What are the proper impeller diameter and the optimum speed and power produced?

▮ $N_s = NP^{0.5}/H^{5/4}$, $P^* = \eta\gamma QH$. Estimate $\eta = 87$ percent.

$$P^* = (0.87)(62.4)(198)(602)/550 = 11\,765\ \text{hp} \qquad N_s = 29 \qquad 29 = (N)(11\,765)^{0.5}/602^{5/4} \qquad N = 797\ \text{rpm}$$

$$C_H^* = gH/N^2D^2 = 9.03 \qquad 9.03 = (32.2)(602)/[(\tfrac{797}{60})^2(D)^2] \qquad D = 3.49\ \text{ft}$$

21.56 A giant wind-tunnel fan has rotor blades 125 ft in diameter. For a 20 mph wind (the optimum output), compute **(*a*)** the power generated, **(*b*)** the rotor speed, and **(*c*)** the velocity V_2 behind the rotor.

▮ **(*a*)** $(C_p)_{max} = P/(\rho AV_1^3/2)$. From Fig. A-49, $(C_p)_{max} = 0.46$.

$$A = [(\pi)(125)^2/4] = 12\,272\ \text{ft}^2 \qquad V_1 = (20)(5280)/3600 = 29.33\ \text{fps}$$

$$0.46 = P/[(0.00233)(12\,272)(29.33)^3/2] \qquad P = 165\,934\ \text{ft-lb/s} = (165\,934)(0.3048)(4.448)/1000 = 225\ \text{kW}$$

(*b*) At P_{max}, $\omega r/V_1 = 5.5$. $(\omega)(\frac{125}{2})/29.33 = 5.5$, $\omega = 2.581$ rad/s, or 24.6 rpm.

(*c*) From ideal theory, $V_2 = V_1/3$, $V_2 = 29.33/3 = 9.78$ fps.

21.57 A hydroelectric plant (Fig. 21-8) has a difference in elevation from head water to tail water of $H = 50$ m and a flow $Q = 5$ m^3/s of water through the turbine. The turbine shaft rotates at 180 rpm, and the torque in the shaft is measured to be $T = 1.16 \times 10^5$ N · m. Output of the generator is 2100 kW. Determine **(*a*)** the reversible power for the system, **(*b*)** the irreversibility, or losses, in the system, and **(*c*)** the losses and the efficiency in the turbine and in the generator.

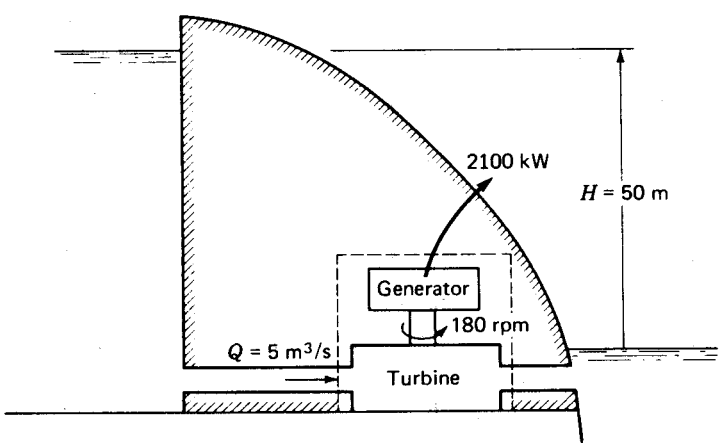


Fig. 21-8

▮ (*a*) The water's potential energy per unit weight is $H = 50$ J/N. Hence, for perfect conversion the reversible power is $\gamma QH = (9806)(5)(50) = 2\,451\,500$ J/s $= 2451.5$ kW.

(*b*) The irreversibility, or lost power, in the system is the difference between the power into and out of the system, or $2451.5 - 2100 = 351.5$ kW.

(*c*) The rate of work by the turbine is the product of the shaft torque and the rotational speed: $T\omega = (1.16 \times 10^5)[180(2\pi)/60] = 2186.5$ kW. The irreversibility through the turbine is then $2451.5 - 2186.5 = 265.0$ kW, or, when expressed as lost work per unit weight of fluid flowing, $(265.0)(\frac{1000}{1})(\frac{1}{9806})(\frac{1}{5}) = 5.4$ J/N. The generator power loss is $2186.5 - 2100 = 86.5$ kW, or $86.5(1000)/[9806(5)] = 1.76$ J/N. Efficiency of the turbine η_t is $\eta_t = 100[(50 - 5.4)/50] = 89.2$ percent and efficiency of the generator η_g is $\eta_g = 100[(50 - 5.4 - 1.76)/(50 - 5.4)] = 96.1$ percent.

21.58 A turbine has a velocity of 6 m/s at the entrance to the draft tube and a velocity of 1.2 m/s at the exit. For friction losses of 0.1 m and a tailwater 5 m below the entrance to the draft tube, find the pressure head at the entrance.

▮ $$p_1/\gamma = -z_s - V_1^2/2g + \text{losses} = -5 - 6^2/[(2)(9.807)] + 1.2^2/[(2)(9.807)] + 0.1 = -6.66 \text{ m}$$

21.59 The wicket gates of the propeller turbine of Fig. 21-9 are turned so that the flow makes an angle of 45° with a radial line at section 1, where the speed is 4.005 m/s. Determine the magnitude of tangential velocity component V_u over section 2.

▮ Since no torque is exerted on the flow between sections 1 and 2, the moment of momentum is constant and the motion follows the free-vortex law $V_u r =$ constant. At section 1, $V_{u1} = 4.005 \cos 45° = 2.832$ m/s, $V_{u1} r_1 = (2.832)(1.5/2) = 2.124$ m²/s. Across section 2, $V_{u2} = 2.124/r$. At the hub, $V_u = 2.124/(0.45/2) = 9.44$ m/s. At the outer edge, $V_u = 2.124/(1.2/2) = 3.54$ m/s.

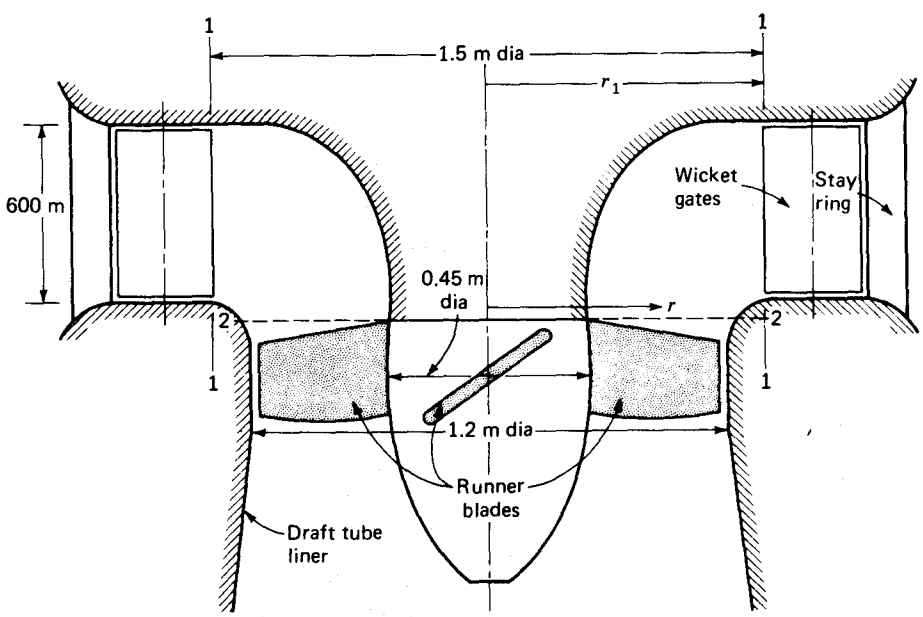


Fig. 21-9

21.60 Assuming uniform axial velocity over section 2 of Fig. 21-9 using the data of Prob. 21.59, determine the angle of the leading edge of the propeller at $r = 0.225$, 0.45, and 0.6 m for a propeller speed of 240 rpm.

▮ At $r = 0.225$ m, $u = \frac{240}{60}(2\pi)(0.225) = 5.66$ m/s, $V_u = 9.44$ m/s. At $r = 0.45$ m, $u = \frac{240}{60}(2\pi)(0.45) = 11.3$ m/s, $V_u = 4.72$ m/s. At $r = 0.6$ m, $u = \frac{240}{60}(2\pi)(0.6) = 15.06$ m/s, $V_u = 3.54$ m/s. The discharge through the turbine is, from section 1, $Q = (0.6)(1.5)(\pi)(4.005)(\cos 45°) = 8.01$ m³/s. Hence, the axial velocity at section 2 is $V_a = 8.01/[\pi(0.6^2 - 0.225^2)] = 8.24$ m/s. Figure 21-10 shows the initial vane angle for the three positions.

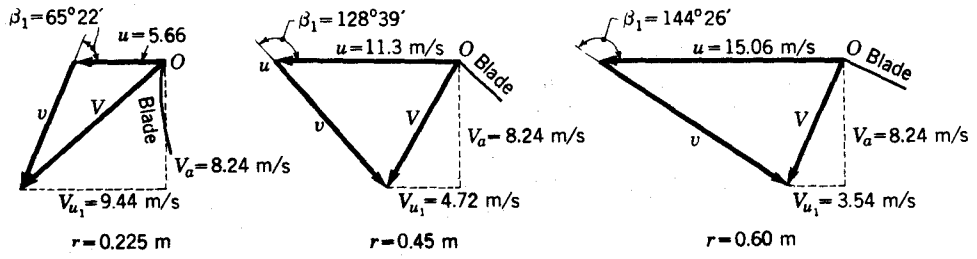


Fig. 21-10

21.61 A Pelton wheel is to be selected to drive a generator at 600 rpm. The water jet is 75 mm in diameter and has a velocity of 100 m/s. With the blade angle at 170°, the ratio of vane speed to initial jet speed at 0.47, and neglecting losses, determine **(*a*)** diameter of wheel to centerline of buckets (vanes), **(*b*)** power developed, and **(*c*)** kinetic energy per newton remaining in the fluid.

▮ **(*a*)** The peripheral speed of the wheel is $u = 0.47(100) = 47$ m/s. Then $\frac{600}{60}[2\pi(D/2)] = 47$ m/s or $D = 1.496$ m.

(*b*) The power is $P = \rho Q u V_r(1 - \cos\theta) = (1000)(\pi/4)(0.075)^2(100)(47)(100 - 47)[1 - (-0.9848)] = 2.184$ MW.

(*c*) From Fig. 21.11, the absolute-velocity components leaving the vane are $V_x = (100 - 47)(-0.9848) + 47 = -5.2$ m/s, $V_y = (100 - 47)(0.1736) = 9.2$ m/s. The kinetic energy remaining in the jet is $(5.2^2 + 9.2^2)/[2(9.807)] = 5.69$ J/N.

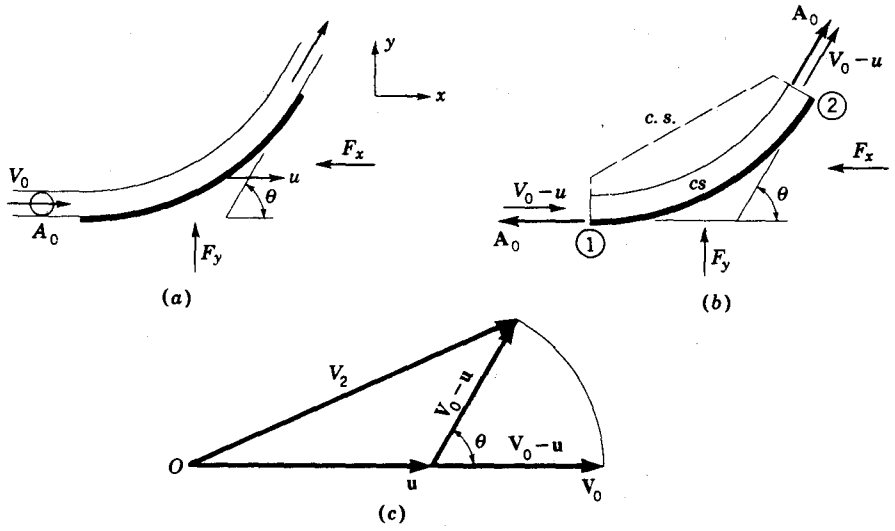


Fig. 21-11

21.62 A small impulse wheel is to be used to drive a generator for 60-Hz power. The head is 100 m, and the discharge is 40 L/s. Determine the diameter of the wheel at the centerline of the buckets and the speed of the wheel ($C_v = 0.98$). Assume efficiency of 80 percent.

▮ The power is $P = \gamma Q H_a \eta = 9806(0.040)(100)(0.80) = 31.38$ kW. By taking a trial value of N_s of 15, $N = N_s H_a^{5/4}/\sqrt{P} = 15(100^{5/4})/\sqrt{31.38} = 847$ rpm. For 60-Hz power the speed must be 3600 divided by the number of pairs of poles in the generator. For five pairs of poles the speed would be $\frac{3600}{5} = 720$ rpm, and for four pairs of poles it would be $\frac{3600}{4} = 900$ rpm. The closer speed 900 is selected. Then $N_s = N\sqrt{P}/H_a^{5/4} = 900\sqrt{31.38}/100^{5/4} = 15.94$. For $N_s = 15.94$, take $\phi = 0.448$, $u = \phi\sqrt{2gH_a} = 0.448\sqrt{2(9.807)(100)} = 19.84$ m/s and $\omega = \frac{900}{60}(2\pi) = 94.25$ rad/s. The peripheral speed u and D and ω are related: $u = \omega D/2$, $D = 2u/\omega = 2(19.84)/94.25 = 421$ mm. The diameter d of the jet is obtained from the jet velocity V_2; thus $V_2 = C_v\sqrt{2gH_a} = 0.98\sqrt{2(9.807)(100)} = 43.4$ m/s, $a = Q/V_2 = (40 \times 10^{-3})/43.4 = 0.000922$ m^2 = 922 mm^2, $d = \sqrt{4a/\pi} = \sqrt{922/0.7854} = 34.3$ mm, where a is the area of jet. Hence, the diameter ratio D/d is $D/d = 421/34.3 = 12.27$. The desired diameter ratio for the best efficiency is $D/d = 206/N_s = 206/15.94 = 12.92$ so the ratio D/d is satisfactory. The wheel diameter is 421 mm, and the speed is 900 rpm.

21.63 If 24 m^3/s of water flowing through the fixed vanes of a turbine has a tangential component of 2 m/s at a radius of 1.25 m and the impeller, turning at 240 rpm, discharges in an axial direction, what torque is exerted on the impeller? Neglecting losses, what is the head on the turbine?

▮ $$T = \rho Q[(rV_t)_{in} - (rV_t)_{out}] = \rho Q(r_1V_{u1} - r_2V_{u2}) = (1000)(24)[(1.25)(2) - 0] = 60\ \text{kN}\cdot\text{m}$$

$$T\omega = Q\gamma H \qquad (60\,000)[(\tfrac{240}{60})(2\pi)] = (24)[(9.79)(1000)](H) \qquad H = 6.42\ \text{m}$$

21.64 A turbine model test with 250-mm-diameter impeller showed an efficiency of 90 percent. What efficiency could be expected from a 1.5-m-diameter impeller?

▮ $$\eta_2 = 1 - (1 - \eta_1)(D_1/D_2)^{1/4} = 1 - (1 - 0.90)[0.250/1.5]^{1/4} = 0.936 \quad \text{or} \quad 93.6\%$$

21.65 A tangential turbine develops 7200 hp at 200 rpm under a head of 790 ft at an efficiency of 82 percent. (*a*) If the speed factor is 0.46 compute the wheel diameter, the flow, the unit speed, unit power, unit flow, and specific speed. (*b*) For this turbine, what would be the speed, power, and flow under a head of 529 ft? (*c*) For a turbine having the same design, what size of wheel should be used to develop 3800 hp under a 600-ft head and what would be its speed and rate of discharge? Assume no change in efficiency.

▮ (*a*) Since $\phi = D_1N/(1840\sqrt{H})$, $D_1 = 1840\sqrt{790}(0.46)/200 = 119$ in. From horsepower output $= \gamma QH/550$, $Q = (7200)(550)/[(62.4)(790)(0.82)] = 98.0$ cfs.

$$N_u = ND_1/\sqrt{H} = (200)(119)/\sqrt{790} = 847\ \text{rpm} \qquad P_u = P/D_1^2H^{3/2} = 7200/[(119)^2(790)^{3/2}] = 0.0000229\ \text{hp}$$

$$Q_u = Q/D_1^2\sqrt{H} = 98.0/[(119)^2\sqrt{790}] = 0.000246\ \text{cfs} \qquad N_s = N\sqrt{P}/H^{5/4} = 200\sqrt{7200}/(790)^{5/4} = 4.05\ \text{rpm}$$

(*b*)
$$\text{Speed } N = Nu\sqrt{H}/D_1 = 847\sqrt{529}/119 = 164\ \text{rpm}$$
$$\text{Power } P = P_uD_1^2H^{3/2} = 0.0000229(119)^2(529)^{3/2} = 3946\ \text{hp}$$
$$\text{Flow } Q = Q_uD_1^2\sqrt{H} = 0.000246(119)^2\sqrt{529} = 80.1\ \text{cfs}$$

The above three quantities might have been obtained by noting that, for the same turbine (D_1 unchanged), the speed varies as $H^{1/2}$, the power varies as $H^{3/2}$, and Q varies as $H^{1/2}$. Thus $N = 200\sqrt{\frac{529}{790}} = 164$ rpm, $P = 7200(\frac{529}{790})^{3/2} = 3945$ hp, $Q = 98.0\sqrt{\frac{529}{790}} = 80.2$ cfs.

(*c*) From $P = P_uD_1^2H^{3/2}$ we obtain $3800 = 0.0000229(D_1)^2(600)^{3/2}$ from which $D_1^2 = 11\,290$ and $D_1 = 106$ in. $N = N_u\sqrt{H}/D_1 = 847\sqrt{600}/106 = 196$ rpm, $Q = Q_uD_1^2\sqrt{H} = 0.000246(11\,290)\sqrt{600} = 68.0$ cfs.

21.66 An impulse wheel at best produces 125 hp under a head of 210 ft. By what percent should the speed be increased for a 290-ft head? Assuming equal efficiencies, what power would result?

▮ For the same wheel, the speed is proportional to the square root of the head. Thus $N_1/\sqrt{H_1} = N_2/\sqrt{H_2}$, $N_2 = N_1\sqrt{H_2/H_1} = N_1\sqrt{\frac{290}{210}} = 1.175N_1$. Thus, the speed should be increased 17.5 percent.

$$N_s = NP^{0.5}/H^{5/4} \qquad N_1P_1^{0.5}/H_1^{5/4} = N_2P_2^{0.5}/H_2^{5/4} \qquad (N_1)(125)^{0.5}/210^{5/4} = (1.175N_1)(P_2)^{0.5}/290^{5/4} \qquad P_2 = 203\ \text{hp}$$

21.67 The reaction turbines at the Hoover Dam installation have a rated capacity of 115 000 hp at 180 rpm under a head of 487 ft. The diameter of each turbine is 11 ft and the discharge is 2350 cfs. Evaluate the speed factor, the unit speed, unit discharge and unit power, and the specific speed.

▮ $$\phi = D_1N/1840\sqrt{H} = (11)(12)(180)/(1840\sqrt{487}) = 0.585 \qquad N_u = D_1N/\sqrt{H} = (11)(12)(180)/\sqrt{487} = 1077\ \text{rpm}$$

$$Q_u = Q/D_1^2\sqrt{H} = 2350/[(132)^2\sqrt{487}] = 0.00611\ \text{cfs}$$

$$P_u = \text{hp}/D_1^2H^{3/2} = 115\,000/[(132)^2(487)^{3/2}] = 0.000614\ \text{hp} \qquad N_s = N_u\sqrt{P_u} = 1077\sqrt{0.000614} = 26.7$$

21.68 A single-stage impulse turbine receives from the nozzles 6 lbm/ft^3 of combustion products moving at 6000 ft/s. The radius R to the nozzles is 2 ft. Find the most efficient speed ω of the turbine, given $\alpha_1 = 20°$? If the turbine runs at 0.72 of this speed, how much power does it develop?

▮ $$U_t = R\omega = \tfrac{1}{2}V_1\cos\alpha_1 \qquad (2)(\omega) = \tfrac{1}{2}(6000)\cos 20° \qquad \omega = 1410\ \text{rad/s} = 13\,460\ \text{rpm}$$

$$\text{Actual speed} = (0.72)(1410) = 1015\ \text{rad/s}$$

$$dW_s/dt = 2\rho QU_t(V_1\cos\alpha_1 - U_t) = (2)(6/g)(2)(1015)[6000\cos 20° - (2)(1015)]/550 = 4963\ \text{hp}$$

21.69 Three nozzles expand combustion products to a speed of 1800 m/s; the nozzles are at a distance $R = 0.6$ m from the axis of rotation. The density of the fluid as it reaches the turbine blades is 0.5 kg/m^3. If the turbine is to rotate at its most efficient speed, 12 000 rpm, and if the power desired is 5900 hp, determine the exit areas of the nozzles.

▮ $$U_t = \tfrac{1}{2}V_1 \cos\alpha \qquad (0.6)[(12\,000)(2\pi)/60] = \tfrac{1}{2}(1800)\cos\alpha_1 \qquad \alpha_1 = 33.1°$$

$$dW_s/dt = 2\rho Q U_t(V_1 \cos\alpha_1 - U_t)$$

$$(5900)(745.7) = (2)(0.5)(Q)(0.6)[(12\,000)(2\pi)/60]\{1800\cos 33.1° - [(0.6)(12\,000)(2\pi)/60]\}$$

$$Q = 7.74\ \text{m}^3/\text{s} \qquad 3(A)(V_a) = 7.74 \qquad 3(A)(1800\sin 33.1°) = 7.74$$

$$A = 2.625 \times 10^{-3}\ \text{m}^2 = 2625\ \text{mm}^2$$

21.70 The effective head H for an axial-flow hydraulic turbine is 92 m and the volumetric flow Q is 21 m^3/s. The radius R_o of the guide vanes is 1.6 m and the height b is 1 m. The exit flow is axial only and the guide vanes are at an angle α of 11.0°. Calculate the rotor angular velocity ω.

▮ $g(\Delta H) = (U_tV_t)_2 - (U_tV_t)_1$. At any position r:

$$(9.81)(-92) = -(\omega)(r)(V_t)_1 \qquad (1)$$

From continuity: $Q = (2)(\pi)(R_0)(b)(V_0)\sin\alpha$, $21 = (2\pi)(1.6)(1)(V_0)\sin 11°$, $V_0 = 10.95$ m/s. At any distance r:

$$V_t = (V_oR_o\cos\alpha)/r = [(10.95)(1.6)\cos 11°]/r = 17.20/r \qquad (2)$$

Replace $(V_t)_1$ in Eq. (*1*) using Eq. (*2*): $(9.81)(92) = \omega r(17.20/r)$, $\omega = 52.5$ rad/s = 501 rpm.

21.71 The first stage of an axial-flow gas turbine has a hub-to-tip diameter ratio of 0.85 with the tip having a diameter of 4 ft. Air [$R = 53.3$ ft-lbf/slug-°R] flows through the machine at 52 lbm/s. At the first stage, the pressure is 4 atm and the temperature is 1800 °R. If $\alpha_1 = 38°$ and $\beta_2 = 55°$ for the first stage and it is to develop 25 hp, at what speed should the rotor turn?

▮ Find V_a.

$$Q = V_a(\pi/4)\{4^2 - [(0.85)(4)]^2\} \qquad (1)$$

Also we can say $Q = (52/g)/\rho$, $\rho = \rho/RT = (14.7)(144)(4)/[(53.3)(32.2)(1800)] = 0.002741$ slug/ft^3, $Q = (50/g)/0.002741 = 566.5$ cfs. Go back to Eq. (*1*): $566.5 = V_a(\pi/4)\{4^2 - [(0.85)(4)]^2\}$, $V_a = 162.45$ ft/s. Compute ΔH for the first stage.

$$-\Delta H(Q\rho g) = 25(550) \qquad \Delta H = -(25)(550)/[(566.2)(0.002741)(32.2)] = -275\ \text{ft}$$

$$\Delta Hg/U_t^2 = [1 - (V_a/U_t)(\cot\alpha_1 + \cot\beta_2)] \qquad U_t^2 - (162.45)(\cot 38° + \cot 55°)U_t + (275)(32.2) = 0$$

$$U_t^2 - 322U_t + 8855 = 0 \qquad U_t = [322 \pm \sqrt{322^2 - (4)(8855)(1)}]/(2)(1) \qquad (U_t)_1 = 292\ \text{ft/s} \qquad (U_t)_2 = 30.5\ \text{fps}$$

$$U_t = R_{av}\omega \qquad 292 = (2 - 0.15)(\omega_1) \qquad \omega_1 = 157.8\ \text{rad/s} = 1507\ \text{rpm} \qquad 30.5 = (2 - 0.15)(\omega_2)$$

$$\omega_2 = 16.49\ \text{rad/s} = 157\ \text{rpm}$$

21.72 Water flows from an upper reservoir to a lower one while passing through a turbine, as shown in Fig. 21-12. Find the power generated by the turbine. Neglect minor losses.

▮ $p_1/\gamma + V_1^2/2g + z_1 - E_t = p_2/\gamma + V_2^2/2g + z_2 + h_L$. From Fig. A-14, with $Q = 0.15$ m^3/s and $D = 250$ mm, $h_1 = 0.037$ m/m.

$$h_L = h_f = (0.037)(100) = 3.70\ \text{m} \qquad 0 + 0 + 197.3 - E_t = 0 + 0 + 50.0 + 3.70$$

$$E_t = 143.6\ \text{m} \qquad P = Q\gamma E_t = (0.15)(9.79)(143.6) = 211\ \text{kW}$$

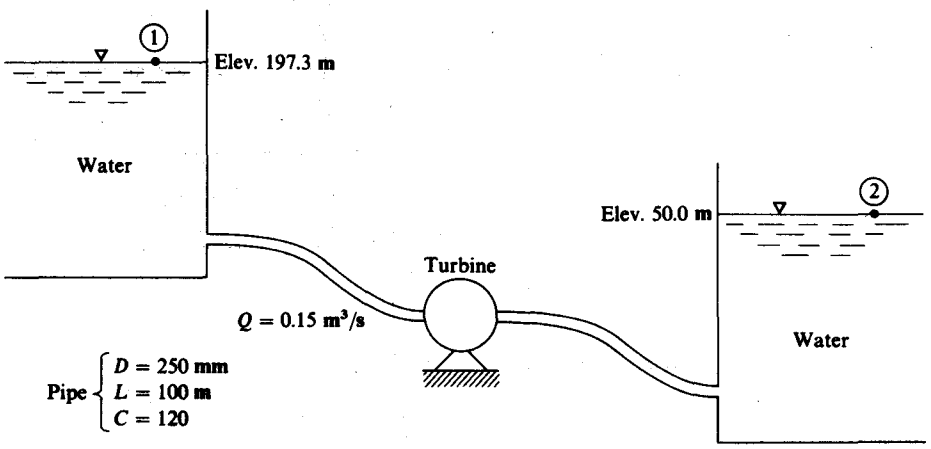


Fig. 21-12

21.73 Water is being discharged from a reservoir through a turbine, as shown in Fig. 21-13. What water-surface elevation is required in the reservoir in order for the turbine to generate 100 hp? Neglect minor losses.

▮ $p_1/\gamma + V_1^2/2g + z_1 - E_t = p_2/\gamma + V_2^2/2g + z_2 + h_L$ $\quad P = Q\gamma E_t \quad 100 = (20)(62.4)(E_t)/550 \quad E_t = 44.07\text{ ft}$

$$V_2 = Q/A_2 = 20/[(\pi)(\tfrac{36}{12})^2/4] = 2.829\text{ ft/s}$$

From Fig. A-13, with $Q = 20\text{ ft}^3/\text{s}$ and $D = 36$ in, $h_1 = 0.00084$ ft/ft.

$$h_L = h_f = (0.00084)(270) = 0.23\text{ ft} \qquad 0 + 0 + z_1 - 44.07 = 0 + 2.829^2/[(2)(32.2)] + 100 + 0.23$$

$$z_1 = 144.4\text{ ft}$$

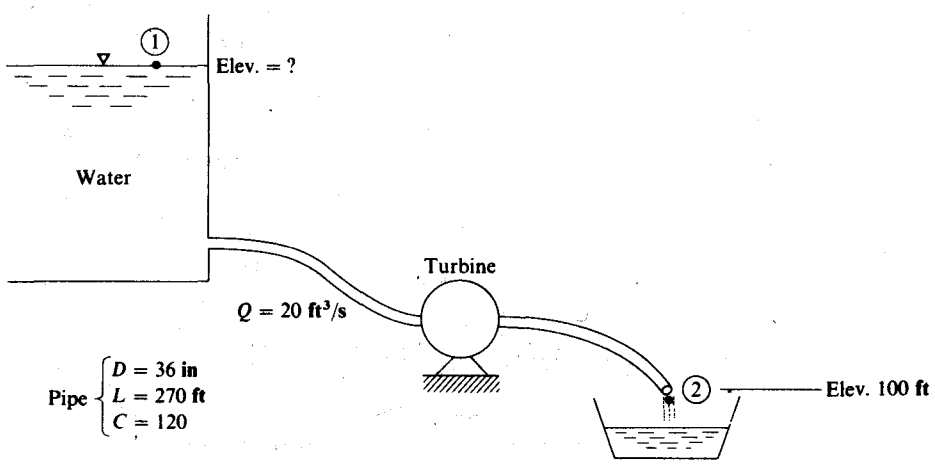


Fig. 21-13

21.74 The flow rate of water through the turbine shown in Fig. 21-14 is $0.20\text{ m}^3/\text{s}$, and the pressures at points 1 and 2 are 150 and −35.0 kPa, respectively. Determine the power generated by the turbine.

▮ $$p_1/\gamma + v_1^2/2g + z_1 - E_t = p_2/\gamma + v_2^2/2g + z_2 + h_L \qquad p_1/\gamma = 150/9.79 = 15.32\text{ m}$$

$$v_1 = Q/A_1 = 0.20/[(\pi)(0.25)^2/4] = 4.07\text{ m/s}$$

$$v_1^2/2g = (4.07)^2/[(2)(9.807)] = 0.84\text{ m} \qquad z_1 = 1.00\text{ m} \qquad p_2/\gamma = -35/9.79 = -3.58\text{ m}$$

$$v_2 = Q/A_2 = 0.20/[(\pi)(0.50)^2/4] = 1.02\text{ m/s} \qquad v_2^2/2g = (1.02)^2/[(2)(9.807)] = 0.05\text{ m} \qquad z_2 = 0 \qquad h_L = 0$$

$$15.32 + 0.84 + 1.0 - E_t = -3.58 + 0.05 + 0 + 0 \qquad E_t = 20.69\text{ m} \qquad P = Q\gamma E_t = (0.20)(9.79)(20.69) = 40.5\text{ kW}$$

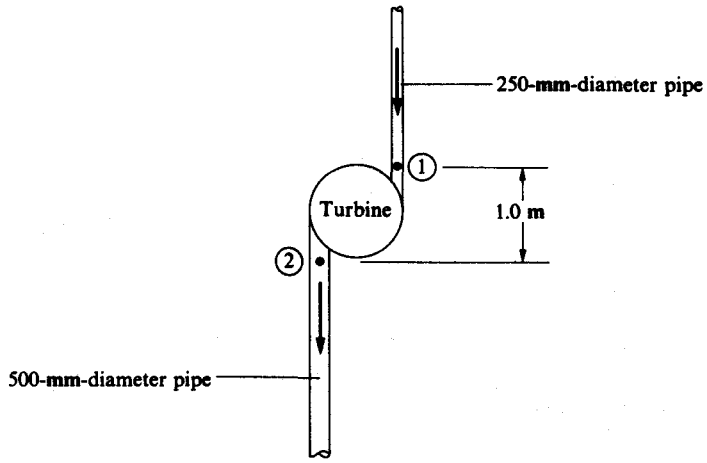


Fig. 21-14

21.75 Water flows from reservoir A to a turbine and then discharges into tailrace B, as shown in Fig. 21-15. The loss of head from A to point 1 is six times the velocity head in the pipe, and the loss of head from point 2 to B is 0.5 times the velocity head in the pipe. If the flow rate in the pipe is 30.0 ft^3/s and the diameter of the pipe is 24 in, calculate the horsepower being generated by the turbine.

▮ $$p_A/\gamma + v_A^2/2g + z_A - E_t = p_B/\gamma + v_B^2/2g + z_B + h_L \qquad p_A/\gamma = p_B/\gamma = v_A^2/2g = v_B^2/2g = 0$$

$$v = Q/A = 30/[(\pi)(2.0)^2/4] = 9.55 \text{ ft/s}$$

$$v^2/2g = (9.55)^2/[(2)(32.2)] = 1.416 \text{ ft} \qquad h_L \text{ from } A \text{ to } 1 = (6)(1.416) = 8.50 \text{ ft}$$

$$h_L \text{ from } 2 \text{ to } B = (0.5)(1.416) = 0.71 \text{ ft}$$

$$0 + 0 + 210 - E_t = 0 + 0 + 0 + 8.50 + 0.71 \qquad E_t = 200.79 \text{ ft}$$

$$P = Q\gamma E_t = (30)(62.4)(200.79) = 375\,879 \text{ ft-lb/s} = 375\,879/550 = 683 \text{ hp}$$

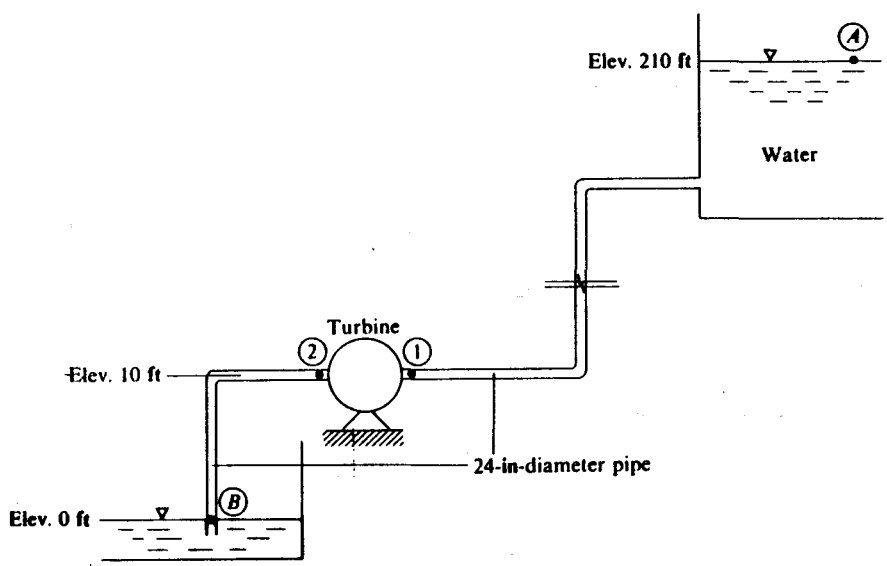


Fig. 21-15

21.76 Water is being discharged from a reservoir through a turbine, as shown in Fig. 21-16. Determine the elevation of the water surface in the reservoir that would be required in order for the turbine to generate 56 kW of power. Neglect minor losses. Assume $C = 120$ for all pipes.

▮ $$p_1/\gamma + v_1^2/2g + z_1 - E_t = p_2/\gamma + v_2^2/2g + z_2 + h_L \qquad p_1/\gamma = p_2/\gamma = v_1^2/2g = 0 \qquad p = Q\gamma E_t$$

$$56 = (0.28)(9.79)(E_t) \qquad E_t = 20.43 \text{ m} \qquad v_2^2/2g = (Q/A)^2/2g = \{0.28/[(\pi)(0.60)^2/4]\}^2/[(2)(9.807)] = 0.05 \text{ m}$$

$$z_2 = 50 \text{ m}$$

With $Q = 0.28$ m^3/s and $D = 300$ m, $(h_1)_{300} = 0.050$ m/m. With $Q = 0.28$ m^3/s and $D = 600$ mm, $(h_1)_{600} = 0.0017$ m/m.

$h_L = h_f = (0.050)(50) + (0.0017)(20) = 2.53\ \text{m}$ $\quad 0 + 0 + z_1 - 20.43 = 0 + 0.05 + 50 + 2.53 \quad z_1 = 73.0\ \text{m}$

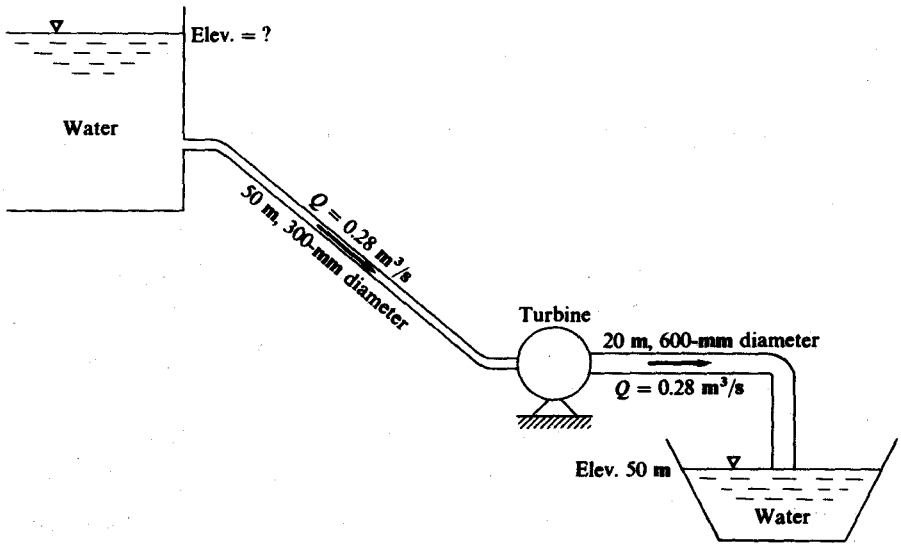


Fig. 21-16

CHAPTER 22
Hydraulic and Energy Grade Lines

22.1 Plot the hydraulic grade line and the energy grade line for the pipe shown in Fig. 22-1*a*. The turbine develops 45 kW; the water is at 5 °C.

First compute a.

$$(V_1^2/2) + (p_1/\rho) + 0 - (dW_s/dm) = (V_2^2/2) + (p_2/\rho) + 0 + h_L \qquad (1)$$

$$dW_s/dm = (dW_s/dt)(dt/dm) = (45\,000)/[V_2(\pi/4)(0.3^2)(1000)] = 637/V_2 \qquad (2)$$

$$h = f_1(90/0.5)(V_1^2/2) + f_2(90/0.3)(V_2^2/2) + 0.05(V_1^2/2) + K(V_1^2/2) \qquad (3)$$

Let $f_1 = 0.023$, $f_2 = 0.0258$: $K = 0.5[1-(0.3/0.5)^2]/(0.3/0.5)^4 = 2.47$. Substitute the above results into Eq. (*1*) noting that $V_1 = (0.3/0.5)^2 V_2 = 0.36V_2$,

$$[(0.36V_2)^2/2] + (p_1/\rho) - (637/V_2) = (V_2^2/2) + (0.023)(90/0.5)[(0.36V_2)^2/2] + (0.0258)(90/0.3)(V_2^2/2) + [(0.05)(0.36V_2)^2/2] + [2.47(0.36V_2)^2/2] \qquad (4)$$

But, by Bernoulli's equation, $90g = (p_1/\rho) + [(0.36V_2)^2/2] + 0$. Substitute this expression for (p_1/ρ) in Eq. (*4*) to find: $4.80V_2^3 - 90gV_2 + 637 = 0$. Solve by trial and error: $V_2 = 13.18$ m/s, $V_1 = 4.74$ m/s.

At *A*:

$$(H_{\text{Hyd}})_a = 90 - (0.05)(4.74^2/2g) - (0.023)(90/0.5)(4.74^2/2g) \qquad (H_{\text{Hyd}})_a = 85.2 \text{ m}$$

After contraction:

$$(H_{\text{Hyd}})_{a'} = -(2.47)(4.74^2/2g) + 85.2 = 82.4 \text{ m}$$

At *B*:

$$(H_{\text{Hyd}})_b = 82.4 - (0.0258)(45/0.3)(13.18^2/2g) \qquad (H_{\text{Hyd}})_b = 48.1 \text{ m}$$

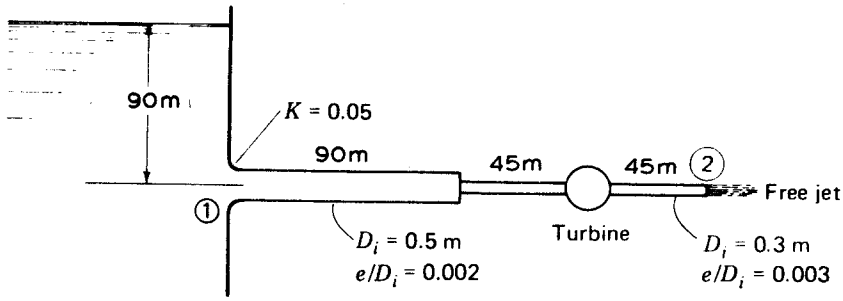
After turbine (b'):

$$(H_{\text{Hyd}})_{b'} = 48.1 - [637/(13.18)(g)] = 43.2 \text{ m}$$

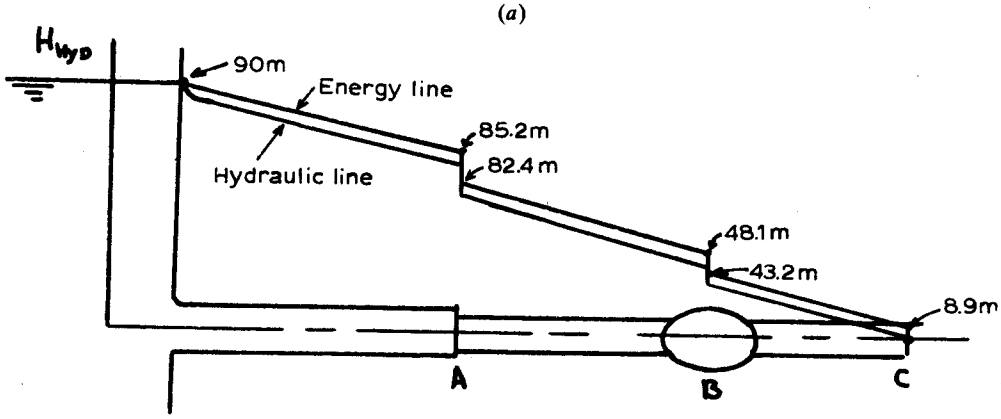
At *C*:

$$(H_{\text{Hyd}})_c = 43.2 - (0.0258)(45/0.3)(13.18^2/2g) = 8.9 \text{ m}$$

This just equals $V_2^2/2g$, the energy head at exit, as it must. The graphs are shown in Fig. 22-1*b*.



(*a*)



(*b*)

Fig. 22-1

22.2 At a point A in a horizontal 12-in pipe ($f = 0.020$) the pressure head is 200 ft. At a distance of 200 ft from A the 12-in pipe reduces suddenly to a 6-in pipe. At a distance of 100 ft from this sudden reduction the 6-in pipe ($f = 0.015$) suddenly enlarges to a 12-in pipe and point F is 100 ft beyond this change in size. For a velocity of 8.025 ft/s in the 12-in pipes, draw the energy and hydraulic grade lines. Refer to Fig. 22-2.

▮ The velocity heads are $V_{12}^2/2g = (8.025)^2/2g = 1.00$ ft and $V_6^2/2g = 16.0$ ft. The energy line drops in the direction of flow by the amount of the lost head. The hydraulic grade line (gradient) is below the energy line by the amount of the velocity head at any cross section. Note (in Fig. 22-2) that the *hydraulic grade line* can rise where a change (enlargement) in size occurs.

Tabulating the results to the nearest 0.1 ft,

lost head in feet			elevation energy line	$V^2/2g$	elevation of hyd. gradient
at	from	calculated			
A	(Elevation 0.0)		201.0	1.0	200.0
B	A to B	$(0.020)(\frac{200}{1})(1) = 4.0$	197.0	1.0	196.0
C	B to C	$(K_c)(16) = (0.37)(16) = 5.9$	191.1	16.0	175.1
D	C to D	$(0.015)(100/\frac{1}{2})(16) = 48.0$	143.1	16.0	127.1
E	D to E	$\frac{(V_6 - V_{12})^2}{2g} = \frac{(32.1 - 8.0)^2}{64.4} = 9.0$	134.1	1.0	133.1
F	E to F	$(0.020)(\frac{100}{1})(1) = 2.0$	132.1	1.0	131.1

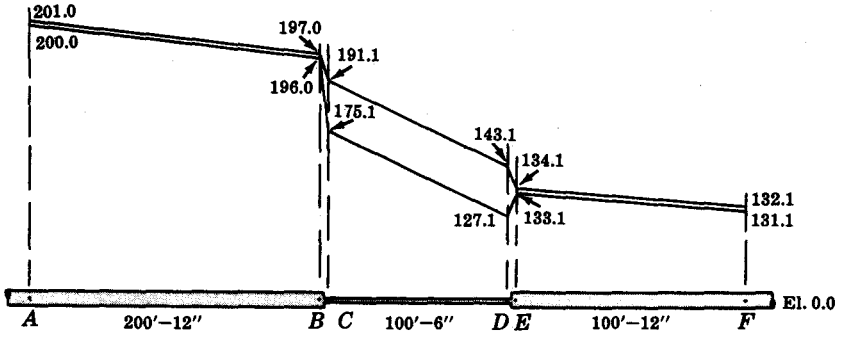


Fig. 22-2

22.3 The elevations of the energy line and hydraulic grade line at point G are 44.0 and 42.0 ft, respectively. For the system shown in Fig. 22-3, calculate **(*a*)** the power extracted between G and H if the energy line at H is at elevation 4.0 and **(*b*)** the pressure heads at E and F which are at elevation 20.0. **(*c*)** Draw, to the nearest 0.1 ft, the energy and hydraulic grade lines, assuming K for valve CD is 0.40 and $f = 0.010$ for the 6-in pipes.

▮ The flow must be from the reservoir since the energy line at G is below the reservoir level. GH is a turbine. Before the power extracted can be calculated, flow Q and the head extracted must be obtained.

(*a*) At G, $V_{12}^2/2g = 2.0$ (the difference between the elevations of the energy and hydraulic grade lines). Also $V_6^2/2g = (16)(2.0) = 32.0$ and $V_{24}^2/2g = \frac{1}{16}(2.0) = 0.13$ ft. To obtain Q, $V_{12} = 11.34$ ft/s and $Q = \frac{1}{4}\pi(1)^2(11.34) = 8.91$ cfs. h.p. $= \gamma Q H_T/550 = 62.4(8.91)(44.0 - 4.0)/550 = 40.4$ extracted.

(*b*) F to G, datum zero

$$(\text{Energy at } F) - 0.030(\tfrac{100}{1})(2.0) = (\text{energy at } G = 44.0) \qquad \text{Energy at } F = 44.0 + 6.0 = 50.0 \text{ ft}$$

E to F, datum zero:

$$(\text{Energy at } E) - (45.4 - 11.3)^2/2g = (\text{energy at } F = 50.0) \qquad \text{Energy at } E = 50.0 + 18.0 = 68.0 \text{ ft}$$

$$z + V^2/2g$$

The pressure head at $E = 68.0 - (20 + 32) = 16.0$ ft water. The pressure head at $F = 50.0 - (20 + 2) = 28.0$ ft water.

(*c*) Working back from E:

Drop in energy line $DE = 0.010(25/\frac{1}{2})(32.0) = 16.0$ ft Drop in energy line $CD = 0.40(32.0) = 12.8$ ft

Drop in energy line BC = same as $DE = 16.0$ ft Drop in energy line $AB = 0.50(32.0) = 16.0$ ft

(Elevation at $D - 16.0$) = elevation at E of 68.0 Elev. $D = 84.0$

(Elevation at $C - 12.8$) = elevation at D of 84.0 Elev. $C = 96.8$

(Elevation at $B - 16.0$) = elevation at C of 96.8 Elev. $B = 112.8$

(Elevation at $A - 16.0$) = elevation at B of 112.8 Elev. $A = 128.8$

The hydraulic grade line is $V^2/2g$ below the energy line: 32.0 ft in the 6-in, 2.0 ft in the 12-in and 0.13 ft in the 24-in. The values are shown in Fig. 22-3.

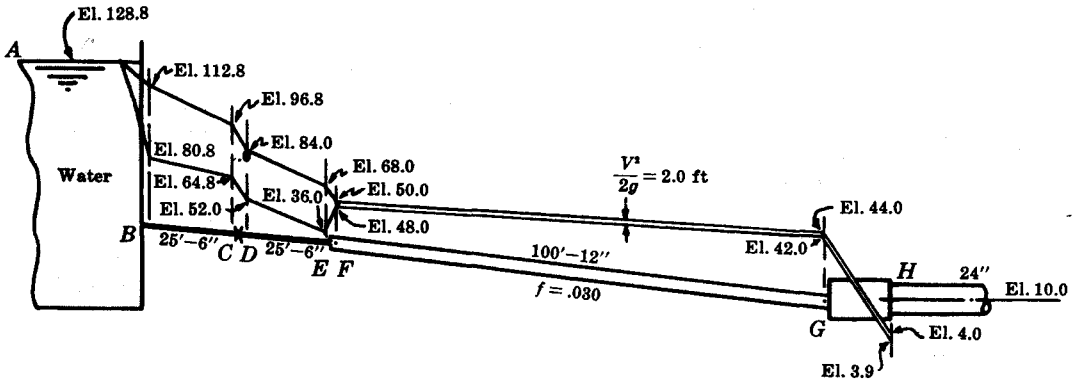


Fig. 22-3

22.4 The head extracted by turbine CR in Fig. 22-4 is 200 ft and the pressure at T is 72.7 psi. For losses of $2.0(V_{24}^2/2g)$ between W and R and $3.0(V_{12}^2/2g)$ between C and T, determine **(*a*)** how much water is flowing and **(*b*)** the pressure head at R. Draw the energy line.

▮ Because the energy line at T is at elevation $\{250.0 + [(72.7)(44)/62.4] + (V_{12}^2/2g)\}$ and well above the elevation at W, the water flows into the reservoir W.

(*a*) Using T to W, datum zero,

$$\underbrace{\left[\frac{(72.7)(144)}{62.4} + \frac{V_{12}^2}{2g} + 250\right]}_{\text{at } T} - \underbrace{\left(3.0\frac{V_{12}^2}{2g}\right.}_{T \text{ to } C} + \underbrace{\left.2.0\frac{V_{24}^2}{2g}\right)}_{R \text{ to } W} - \underbrace{200}_{H_T} = \underbrace{(0 + \text{negl} + 150)}_{\text{at } W}$$

Substituting $V_{24}^2 = \frac{1}{16}V_{12}^2$ and solving, $V_{12}^2/2g = 32.0$ ft or $V_{12} = 45.4$ ft/s. Then $Q = \frac{1}{4}\pi(1)^2(45.4) = 35.7$ cfs.

(*b*) Using R to W, datum R, $[(p_R/\gamma) + \frac{1}{16}(32.0) + 0] - 2(\frac{1}{16})(32.0) = (0 + \text{negl} + 50)$ and $p_R/\gamma = 52.0$ ft. The reader may check this pressure head by applying the Bernoulli equation between T and R.

To plot the energy line in the figure, evaluate the energy at the four sections indicated:

Elevation of energy line at $T = 168.0 + 32.0 + 250.0 = 45.0$

Elevation of energy line at $C = 450.0 - 3(32.0) = 354.0$

Elevation of energy line at $R = 354.0 - 200.0 = 154.0$

Elevation of energy line at $W = 154.0 - 2(\frac{1}{16})(32) = 150.0$

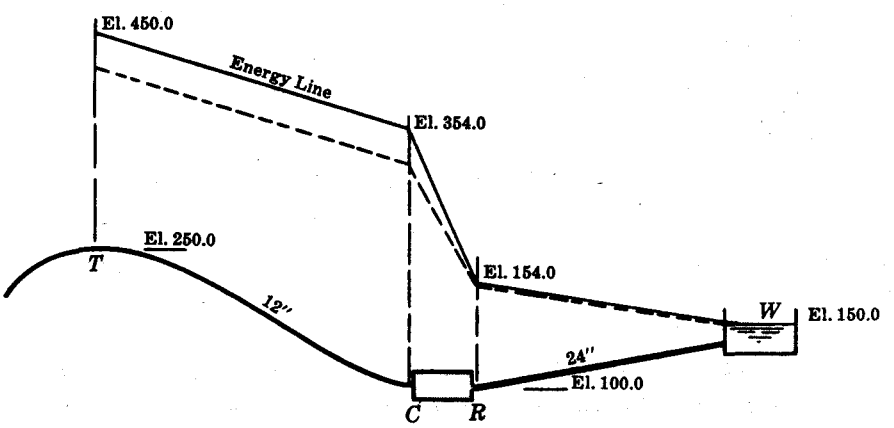


Fig. 22-4

22.5 A pipeline with a pump leads to a nozzle as shown in Fig. 22-5. Find the flow rate when the pump develops a head of 80 ft. Assume that the head loss in the 6-in-diameter pipe may be expressed by $h_L = 5V_6^2/2g$, while the head loss in the 4-in-diameter pipe is $h_L = 12V_4^2/2g$. Sketch the energy line and hydraulic grade line, and find the pressure head at the suction of the pump.

▮ Select the datum as the elevation of the water surface in the reservoir. Note from continuity that $V_6 = (\frac{3}{6})^2 V_3 = 0.25V_3$ and $V_4 = (\frac{3}{4})^2 V_3 = 0.563V_3$, where V_3 is the jet velocity. Writing an energy equation from the surface of the reservoir to the jet, $[z_1 + (p_1/\gamma) + (V_1^2/2g)] - h_{L_6} + h_p - h_{L_4} = z_3 + (p_3/\gamma) + (V_3^2/2g)$, $0 + 0 + 0 - 5(V_6^2/2g) + 80 - 12(V_4^2/2g) = 10 + 0 + (V_3^2/2g)$. Express all velocities in terms of V_3: $-[5(0.25V_3)^2/2g] + 80 - 12[(0.563V_3)^2/2g] = 10 + (V_3^2/2g)$, $V_3 = 29.7$ fps; $Q = A_3V_3 = (\pi/4)(\frac{3}{12})^2(29.7) = 1.45$ cfs. Head loss in suction pipe: $h_L = 5(V_6^2/2g) = [5(0.25V_3)^2/2g] = 0.312V_3^2/2g = 4.3$ ft. Head loss in discharge pipe: $h_L = 12(V_4^2/2g) = 12(0.563V_3)^2/2g = 52.1$ ft, $V_3^2/2g = 13.7$ ft, $V_4^2/2g = 4.3$ ft, $V_6^2/2g = 0.86$ ft ≈ 0.9 ft. The energy line and hydraulic grade line are drawn on the figure to scale. Inspection of the figure shows that the pressure head on the suction side of the pump is $p_B/\gamma = 14.8$ ft. Likewise, the pressure head at any point in the pipe may be found if the figure is to scale.

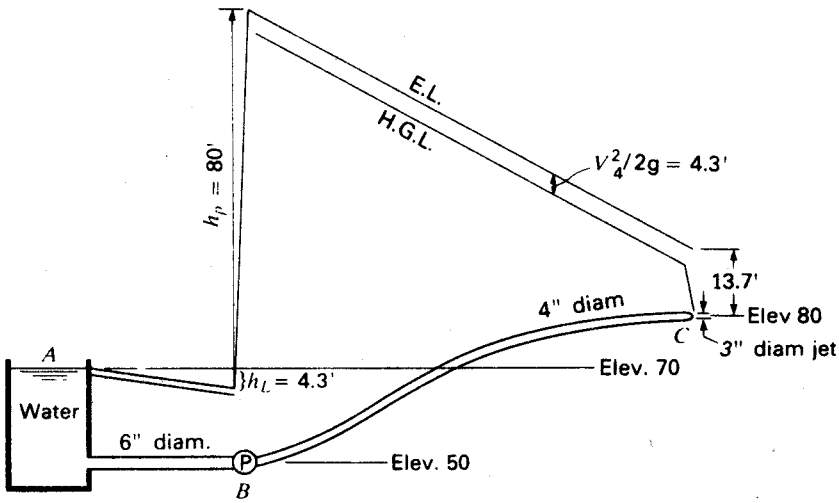


Fig. 22-5

22.6 A pump lifts water at the rate of 180 cfs to a height of 375 ft, at a pipe friction loss of 27 ft. Find the power required if the pump efficiency is 88%. Sketch the energy grade line and the hydraulic grade line.

▮ Water hp $= Q\gamma(\Delta z + h_L)/550 = (180)(62.4)(375 + 27)/550 = 8210$ Input power $= 8210/0.88 = 9330$

The energy and hydraulic grade lines are sketched in Fig. 22-6.

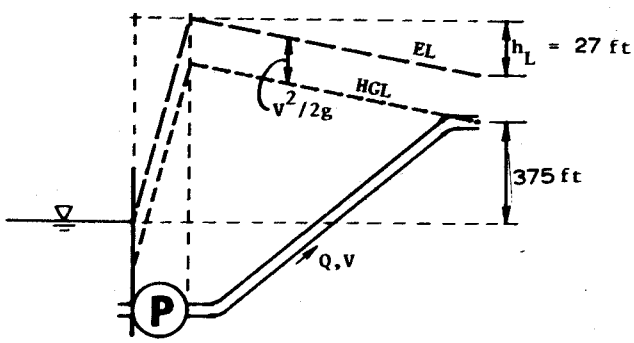


Fig. 22-6

22.7 The diameters of the suction and discharge pipes of a pump are 6 in and 4 in, respectively. The discharge pressure is read by a gage at a point 5 ft above the centerline of the pump, and the suction pressure is read by a gage 2 ft below the centerline. If the pressure gage reads 20 psi and the suction gage reads a vacuum of 10 inHg when a fluid of specific gravity 0.79 is pumped at the rate of 2.0 cfs, find the power delivered to the fluid. Sketch the energy grade line and the hydraulic grade line.

▮ $$p_1/\gamma + z_1 + V_1^2/2g + E_p = p_2/\gamma + z_2 + V_2^2/2g + h_L \qquad V_1 = Q/A_1 = 2.0/[(\pi)(\tfrac{6}{12})^2/4] = 10.19 \text{ fps}$$

$$V_2 = 2.0/[(\pi)(\tfrac{4}{12})^2/4] = 22.92 \text{ fps} \qquad h_L = 0$$

$$-(\tfrac{10}{12})(13.6/0.79) - 2 + 10.19^2/[(2)(32.2)] + E_p = (20)(144)/[(0.79)(62.4)] + 5 + 22.92^2/[(2)(32.2)] + 0$$

$$E_p = 86.31 \text{ ft}$$

$$P = Q\gamma E_p = (2.0)[(0.79)(62.4)](86.31)/550 = 15.5 \text{ hp}$$

The energy and hydraulic grade lines are sketched in Fig. 22-7.

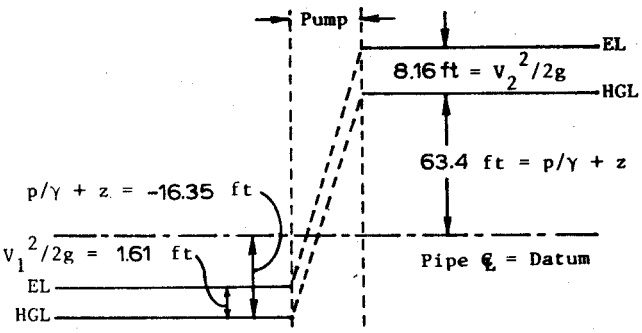


Fig. 22-7

22.8 Determine the elevation of hydraulic and energy grade lines at points A, B, C, D, and E of Fig. 22-8 ($z = 10$ ft).

▮ Solving for the velocity head is accomplished by applying the energy equation from the reservoir to E, $10 + 60 + 0 + 0 = (V_E^2/2g) + 10 + 0 + \frac{1}{2}(V^2/2g) + 0.020(200/0.50)(V^2/2g) + 10(V^2/2g) + 0.10(V_E^2/2g)$. From the continuity equation, $V_E = 4V$. After simplifying, $60 = (V^2/2g)[16 + \frac{1}{2} + 8 + 10 + 16(0.1)] = 36.1(V^2/2g)$ and $V^2/2g = 1.66$ ft. Applying the energy equation for the portion from the reservoir to A gives $70 + 0 + 0 = (V^2/2g) + (p/\gamma) + z + 0.5(V^2/2g)$. Hence, the hydraulic grade line at A is

$$\left.\frac{p}{\gamma} + z\right|_A = 70 - 1.5\frac{V^2}{2g} = 70 - 1.5(1.66) = 67.51 \text{ ft}$$

The energy grade line for A is $(V^2/2g) + z + (p/\gamma) = 67.51 + 1.66 = 69.17$ ft. For B, $70 + 0 + 0 = (V^2/2g) + (p/\gamma) + z + 0.5(V^2/2g) + 0.02(80/0.5)(V^2/2g)$ and

$$\left.\frac{p}{\gamma} + z\right|_B = 70 - (1.5 + 3.2)(1.66) = 62.19 \text{ ft}$$

The energy grade line is at $62.19 + 1.66 = 63.85$ ft.

Across the valve the hydraulic grade line drops by $10V^2/2g$, or 16.6 ft. Hence, at C the energy and hydraulic grade lines are at 47.25 ft and 45.59 ft, respectively.

At point D, $70 = (V^2/2g) + (p/\gamma) + z + [10.5 + 0.02(200/0.50)](V^2/2g)$ and

$$\left.\frac{p}{\gamma} + z\right|_D = 70 - 19.5(1.66) = 37.6 \text{ ft}$$

with the energy grade line at $37.6 + 1.66 = 39.26$ ft.

At point E the hydraulic grade line is 10 ft, and the energy grade line is $z + (V_E^2/2g) = 10 + 16(V^2/2g) = 10 + 16(1.66) = 36.6$ ft.

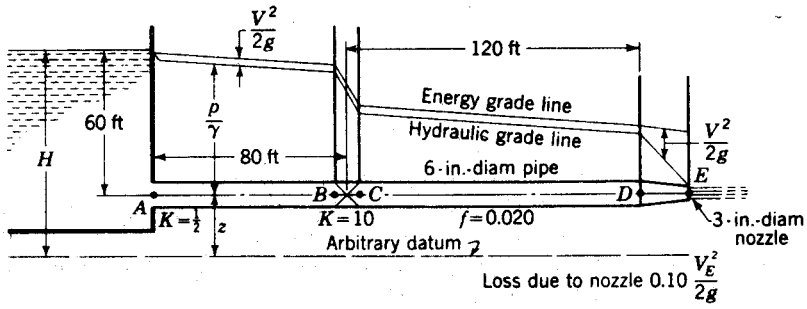


Fig. 22-8

22.9 A pump with a shaft input of 7.5 kW and an efficiency of 70 percent is connected in a water line carrying 0.1 m^3/s. The pump has a 150-mm-diameter suction line and a 120-mm-diameter discharge line. The suction line enters the pump 1 m below the discharge line. For a suction pressure of 70 kN/m^2, calculate the pressure at the discharge flange and the rise in the hydraulic grade line across the pump.

▮ The energy added per unit weight of water, E, is given by $Q\gamma E = 7500(0.70)$, or $E = 7500(0.7)/[0.1(9806)] = 5.354$ m. Applying the energy equation from suction flange to discharge flange gives $(V_s^2/2g) + (p_s/\gamma) + 0 + 5.354 = (V_d^2/2g) + (p_d/\gamma) + 1$ in which the subscripts s and d refer to the suction and discharge conditions, respectively. From the continuity equation, $V_s = 0.1(4)/(0.15^2\pi) = 5.66$ m/s, $V_d = 0.1(4)/(0.12^2\pi) = 8.84$ m/s. Solving for p_d gives $p_d/\gamma = [5.66^2/2(9.806)] + (70\,000/9806) + 5.354 - [8.84^2/2(9.806)] - 1 = 9.141$ m and $p_d = 89.6$ kPa. The rise in hydraulic grade line is $[(p_d/\gamma) + 1] - (p_s/\gamma) = 9.141 + 1 - (70\,000/9806) = 3.002$ m.

22.10 Sketch the hydraulic and energy grade lines for Fig. 22-9*a* ($H = 10$ m).

▮ $$p_1/\gamma + V_1^2/2g + z_1 = p_2/\gamma + V_2^2/2g + z_2 + h_L \qquad h_L = h_f + h_m \qquad h_f = (f)(L/D)(V^2/2g)$$

$(\epsilon/D)_1 = 0.000046/0.150 = 0.000307$; try $f_1 = 0.015$. $(\epsilon/D)_2 = 0.000046/0.300 = 0.000153$; try $f_2 = 0.015$.

$$h_f = (0.015)[25/0.150]\{V_1^2/[(2)(9.807)]\} + (0.015)[(20+8)/0.300]\{(V_1/4)^2/[(2)(9.807)]\} = 0.1319V_1^2$$

$$h_m = KV^2/2g = [1 + (1 - \tfrac{1}{4})^2 + 3.5/16 + \tfrac{1}{16}]\{V_1^2/[(2)(9.807)]\} = 0.09400V_1^2$$

$$h_L = 0.1319V_1^2 + 0.09400V_1^2 = 0.2259V_1^2 \qquad 0 + 0 + 10 = 0 + 0 + 0 + 0.2259V_1^2 \qquad V_1 = 6.653 \text{ m/s}$$

$$V_1^2/2g = 6.653^2/[(2)(9.807)] = 2.257 \text{ m} \qquad V_2 = 6.653/4 = 1.663 \text{ m/s} \qquad V_2^2/2g = 1.663^2/[(2)(9.807)] = 0.141 \text{ m}$$

Friction losses are $(f_1)(L/D)(v_1^2/2g) = 5.642$ m, $(f_2)(L/D)(v_2^2/2g) = 0.197$ m. Minor losses are $(V_1 - V_2)^2/2g = 1.270$ m, $3.5V_2^2/2g = 0.494$ m. The hydraulic and energy grade lines are sketched in Fig. 22-9*b*. [Further checking (not shown) of the Reynolds number indicates that the assumed values of f_1 and f_2 are acceptable.]

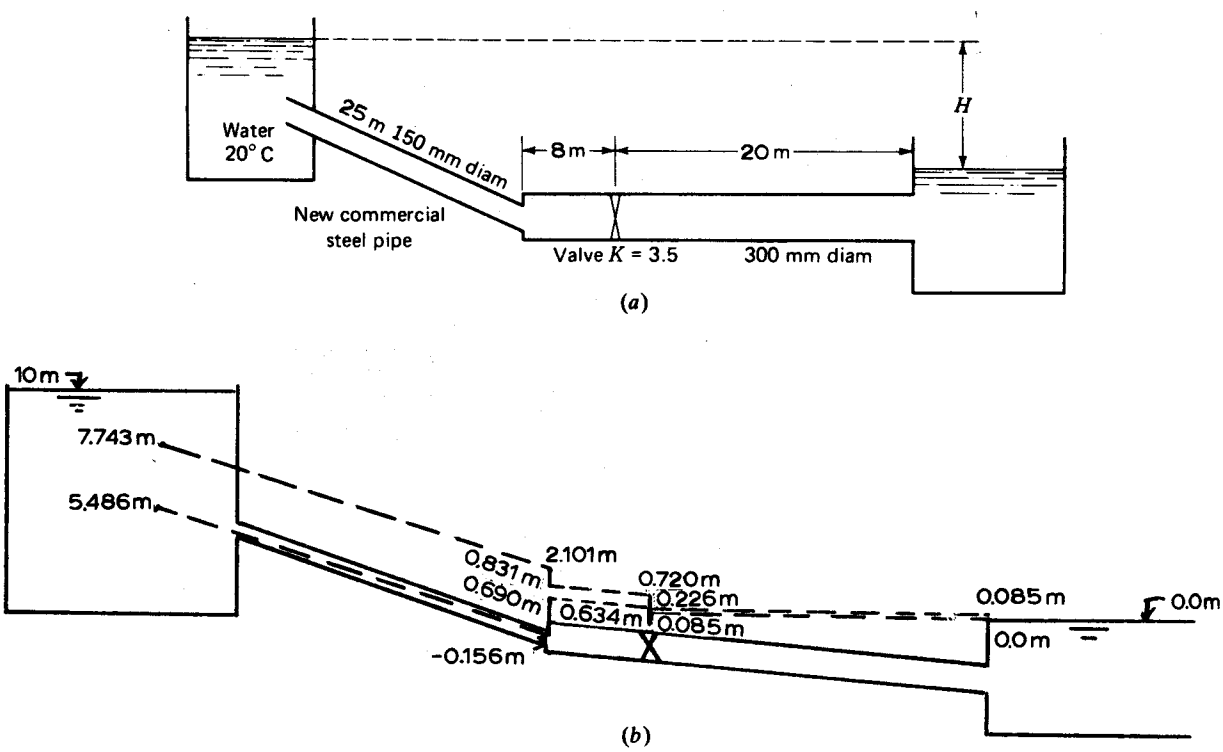


Fig. 22-9

22.11 Determine the slope of the hydraulic grade line for flow of atmospheric air through a rectangular 21-in by 7-in galavanized iron conduit ($V = 25$ fps).

▮ $h_f = (f)(L/D)(V^2/2g)$. Slope of the hydraulic grade line $= h_f/L = (f/D)(V^2/2g)$.

$$\epsilon = 0.0005 \text{ ft} \qquad R_h = (\tfrac{21}{12})(\tfrac{7}{12})/4 = 0.2552 \text{ ft} \qquad D = 4R_h = (4)(0.2552) = 1.021 \text{ ft}$$

$$\epsilon/D = 0.0005/1.021 = 0.000490 \qquad N_R = DV/\nu = (1.021)(25)/(1.69 \times 10^{-4}) = 1.51 \times 10^5$$

From Fig. A-5, $f = 0.0195$. Slope of the hydraulic grade line $= (0.0195/1.021)\{25^2/[(2)(32.2)]\} = 0.185$ ft/ft.

22.12 What size square conduit is needed to convey 375 L/s of water at 15 °C with a slope of the hydraulic grade line of 0.002? ($\epsilon = 1$ mm)

▮ Let d = side of conduit.

$$R_h = d^2/4d = d/4 \qquad D = 4R_h = 4d/4 = d \qquad V = Q/A = (375 \times 10^{-3})/d^2$$

$$N_R = DV/\nu = (d)(0.375/d^2)/(1.16 \times 10^{-6}) = 323\,276/d \qquad \epsilon/D = 0.001/d \qquad h_f = (f)(L/D)(V^2/2g)$$

$$h_f/L = 0.002 = (f/d)\{(0.375/d^2)^2/[(2)(9.807)]\} \qquad d^5 = 3.585f$$

Assume $f = 0.022$.

$$d^5 = (3.585)(0.022) \qquad d = 0.602 \text{ m} \qquad N_R = 323\,276/0.602 = 537\,000$$

From Fig. A-5, $f = 0.023$. $d^5 = (3.585)(0.023)$, $d = 0.607$ m.

22.13 Neglecting minor losses other than the valve, sketch the hydraulic grade line for Fig. 9-54. The globe valve has a loss coefficient of $K = 4.5$.

▮ $$p_1/\gamma + V_1^2/2g + z_1 = p_2/\gamma + V_2^2/2g + z_2 + h_L \qquad h_L = (f)(L/D)(V^2/2g) + KV^2/2g$$

Try $f = 0.013$.

$$h_L = (0.013)[(100 + 70 + 105)/(\tfrac{8}{12})]\{V^2/[(2)(32.2)]\} + (4.5)\{V^2/[(2)(32.2)]\} = 0.1531V^2$$

$$0 + 0 + 12 = 0 + 0 + 0 + 0.1531V^2 \qquad V = 8.853 \text{ fps}$$

$$12 - (0.013)[(100 + 70)/(\tfrac{8}{12})]\{8.853^2/[(2)(32.2)]\} = 7.97 \text{ ft} \qquad 4.5V^2/2g = (4.5)\{8.853^2/[(2)(32.2)]\} = 5.48 \text{ ft}$$

The hydraulic grade line is sketched in Fig. 22-10. [Further checking (not shown) of the Reynolds number indicates that the assumed value of f is acceptable.]

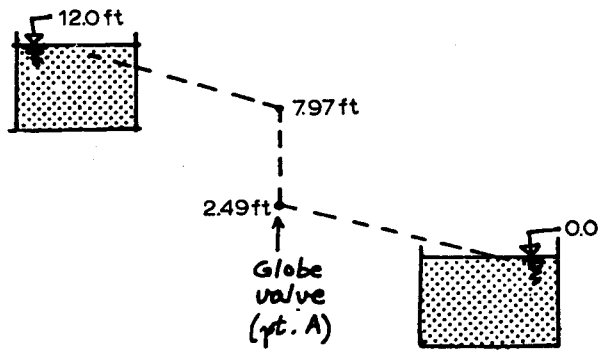


Fig. 22-10

CHAPTER 23
Forces Developed by Fluids in Motion

23.1 A hose and nozzle discharge a horizontal water jet against a nearby vertical plate, as shown in Fig. 23-1. The flow rate of water is 0.025 m^3/s, and the diameter of the nozzle tip is 30 mm. Find the horizontal force necessary to hold the plate in place.

▮ $F = \rho Q(v_2 - v_1)$. The net external force acting on the fluid (F in the equation) is the horizontal force necessary to hold the plate in place (i.e., R in Fig. 23-1). Assume it acts toward the left, as shown in the figure, and this direction is taken to be positive. $v_1 = Q/A_1 = 0.025/[(\pi)(0.030)^2/4] = 35.37$ m/s, $v_2 = 0$, $R = (1000)(0.025)[0 - (-35.37)] = 884$ N (leftward).

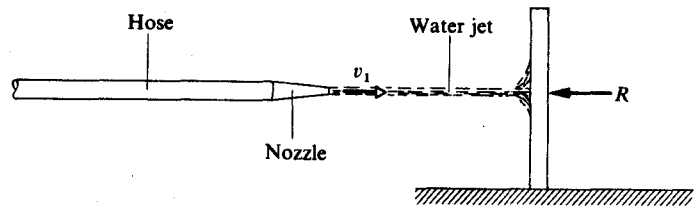


Fig. 23-1

23.2 Water flows from a large tank through an orifice of 3 in diameter and against a block, as shown in Fig. 23-2. The water jet strikes the block at the vena contracta. The block weighs 50 lb, and the coefficient of the friction between block and floor is 0.57. The orifice's coefficient of discharge (C) is 0.60, and its coefficient of contraction (C_c) is 0.62. What is the minimum height to which water must rise in the tank (y in Fig. 23-2) in order to start the block moving to the right?

▮ $F = \rho Q(v_2 - v_1)$. The force caused by the water striking the block must equal (or slightly exceed) the friction force between block and floor (F_f in Fig. 23-2). In other words, the net external force acting on the fluid (F in the equation above) is F_f when the block begins to move. Hence, $F = F_f = (0.57)(50) = 28.5$ lb, $-28.5 = (1.94)(Q)(0 - v_1)$, $Qv_1 = 14.69$. But $V_1 = Q/a$, where a is the area of the jet at its vena contracta.

$$a = (0.62)[(\pi)(\tfrac{3}{12})^2/4] = 0.03043 \text{ ft}^2 \qquad v_1 = Q/0.03043 = 32.86Q \qquad (Q)(32.86Q) = 14.69$$

$$Q = 0.6686 \text{ ft}^3/\text{s} = CA\sqrt{2gh}$$

$$0.6686 = (0.60)[(\pi)(\tfrac{3}{12})^2/4]\sqrt{(2)(32.2)(y - 1.0)} \qquad y = 9.00 \text{ ft}$$

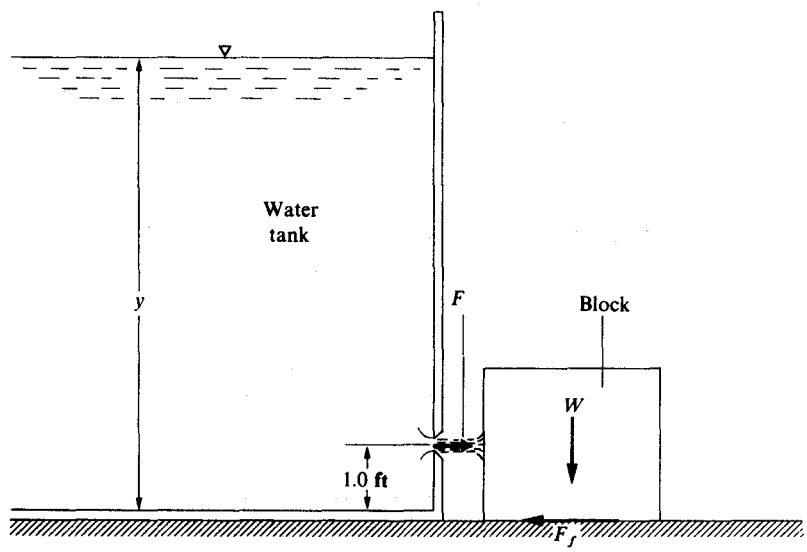


Fig. 23-2

23.3 A hose and nozzle discharge a horizontal water jet against a nearby vertical plate, as shown in Fig. 23-1. The flow rate of water is 0.043 m^3/s, and the diameter of the nozzle tip is 50 mm. Find the horizontal force necessary to hold the plate in place.

▮ $F = \rho Q(v_2 - v_1) \qquad v_1 = Q/A_1 = 0.042/[(\pi)(0.050)^2/4] = 21.39 \text{ m/s}$

$v_2 = 0 \qquad R = (1000)(0.042)[0 - (-21.39)] = 898 \text{ N} \qquad \text{(leftward)}$

23.4 In Fig. 23-3 a small ingot and platform rest on a steady water jet. If the total weight supported is 825 N, what is the jet velocity?

▮ $F = \rho Q(v_2 - v_1) \qquad Q = Av_1 = [(\pi)(0.060)^2/4](v_1) = 0.002827v_1$

$825 = (1000)(0.002827v_1)(v_1) \qquad v_1 = 17.1 \text{ m/s}$

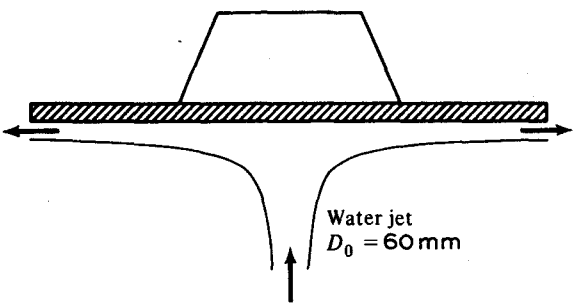


Fig. 23-3

23.5 The horizontal nozzle in Fig. 23-4 has $D_1 = 10$ in and $D_2 = 6$ in. The inlet pressure $p_1 = 60$ psia, and the exit velocity $V_2 = 85$ fps. Compute the tensile force in the flange bolts. Assume incompressible steady flow.

▮ $\Sigma F = \rho Q(V_2 - V_1) \qquad Q = AV = [(\pi)(\frac{6}{12})^2/4](85) = 16.69 \text{ ft}^3/\text{s} \qquad V_1 = (\frac{6}{10})^2(V_2) = (\frac{6}{10})^2(85) = 30.60 \text{ fps}$

$(60 - 15)[(\pi)(10)^2/4] - F_{\text{bolts}} = (1.94)(16.69)(85 - 30.60) \qquad F_{\text{bolts}} = 1773 \text{ lb}$

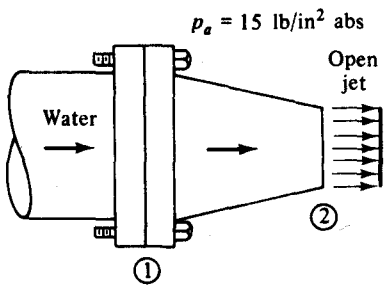


Fig. 23-4

23.6 The tank in Fig. 23-5 weighs 920 N empty and contains 1.1 m^3 of water at 20 °C. The entrance and exit pipes are identical, $D_1 = D_2 = 60$ mm, and both carry 80 L/s. Determine the scale reading W.

▮ $\Sigma F_y = W - W_{H_2O} - W_{\text{tank}} = -\rho Q[(v_y)_2 - (v_y)_1] \qquad (v_y)_1 = Q/A = 80 \times 10^{-3}/[(\pi)(0.060)^2/4] = 28.29 \text{ m/s}$

$W - (1.1)[(9.79)(1000)] - 920 = -(1000)(80 \times 10^{-3})(0 - 28.29) \qquad W = 13.952 \text{ kN}$

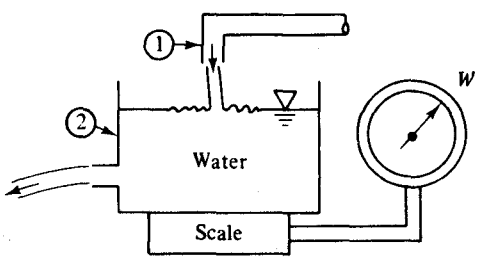


Fig. 23-5

23.7 Garbage is discharged onto a scow, as in Fig. 23-6; the discharge velocity is 4 fps and the weight flux is 700 lb/s. Calculate the tension in the mooring line.

▮ The x-equilibrium of the scow is expressed by

$$T = F_{x\,\text{scow}} = -F_{x\,\text{garbage}} = -\dot{m}(v_2 - v_1) = -(700/g)(0 - 4\cos 35°) = +71.3\text{ lb}$$

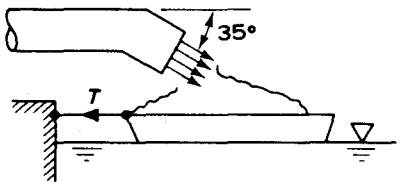


Fig. 23-6

23.8 The water jet in Fig. 23-7, moving at 45 ft/s, is split so that one-third of the water moves toward A. Calculate the magnitude and direction of the force on the stationary splitter. Assume ideal flow in a horizontal plane.

▮
$$F = \rho Q(v_2 - v_1) \qquad -F_x = \rho Q_A[(v_x)_2 - (v_x)_1] + \rho Q_B[(v_x)_2 - (v_x)_1]$$
$$F_y = \rho Q_A[(v_y)_2 - (v_y)_1] + \rho Q_B[(v_y)_2 - (v_y)_1]$$
$$Q = AV = (0.15)(45) = 6.75\text{ ft}^3/\text{s} \qquad Q_A = 6.75/3 = 2.25\text{ ft}^3/\text{s} \qquad Q_B = 6.75 - 2.25 = 4.50\text{ ft}^3/\text{s}$$
$$-F_x = (1.94)(2.25)(-45\cos 60° - 45) + (1.94)(4.50)(45\cos 60° - 45) = -491\text{ lb}$$
$$F_y = (1.94)(2.25)(-45\cos 30° - 0) + (1.94)(4.50)(45\cos 30° - 0) = 170\text{ lb}$$
$$F_r = \sqrt{491^2 + 170^2} = 520\text{ lb} \qquad \tan\alpha = \tfrac{170}{491} = 0.34623 \qquad \alpha = 19.1°$$

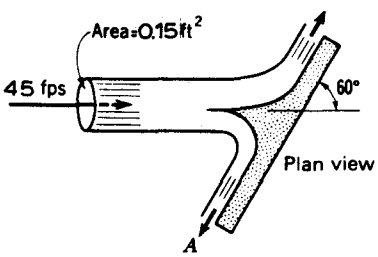


Fig. 23-7

23.9 Rework Prob. 23.8, if the splitter is moving to the right at 8 fps.

▮
$$F = \rho Q(v_2 - v_1) \qquad -F_x = \rho Q_A[(v_x)_2 - (v_x)_1] + \rho Q_B[(v_x)_2 - (v_x)_1]$$
$$F_y = \rho Q_A[(v_y)_2 - (v_y)_1] + \rho Q_B[(v_y)_2 - (v_y)_1]$$
$$V = 45 - 8 = 37\text{ ft/s} \qquad Q = AV = (0.15)(37) = 5.55\text{ ft}^3/\text{s} \qquad Q_A = 5.55/3 = 1.85\text{ ft}^3/\text{s}$$
$$Q_B = 5.55 - 1.85 = 3.70\text{ ft}^3/\text{s}$$
$$-F_x = (1.94)(1.85)(-37\cos 60° - 37) + (1.94)(3.70)(37\cos 60° - 37) = -332\text{ lb}$$
$$F_y = (1.94)(1.85)(-37\cos 30° - 0) + (1.94)(3.70)(37\cos 30° - 0) = 115\text{ lb}$$
$$F_r = \sqrt{332^2 + 115^2} = 351\text{ lb} \qquad \tan\alpha = \tfrac{115}{332} = 0.34639 \qquad \alpha = 19.1°$$

23.10 A locomotive tender running at 18 mph scoops up water from a trough between the rails, as shown in Fig. 23-8. The scoop delivers water at a point 9 ft above its original level and in the direction of motion. The area of the stream of water at entrance is 45 in². The water is everywhere under atmospheric pressure. (*a*) Find the ground velocity of the water as it leaves the scoop and the reaction force acting on the tender. How slow can the train move and still deliver water to the same point?

▮
$$F_u = \rho A_1 V_1(\Delta V_u) \qquad u_1 = u_2 = -(18)(5280)/3600 = -26.40\text{ ft/s} \qquad v_1 = 26.40\text{ fps}$$

Relative to the moving car,

$$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 \qquad 0 + 26.40^2/[(2)(32.2)] + 0 = 0 + v_2^2/[(2)(32.2)] + 9 \qquad v_2 = -10.83\text{ fps}$$
$$V_2 = -26.40 - 10.83 = -37.23\text{ fps} \qquad F_u = (1.94)(\tfrac{45}{144})(26.40)(37.23) = 596\text{ lb}$$

At minimum speed, $V^2/2g = 9$ ft, since $v_2 = 0$.

$$V^2/[(2)(32.2)] = 9 \qquad V = 24.07 \text{ fps} = 16.4 \text{ mph}$$

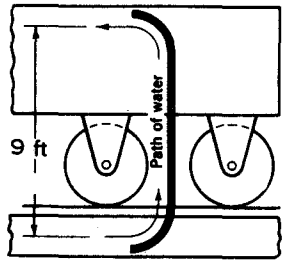


Fig. 23-8

23.11 Referring to Fig. 23-9, a 2-in-diameter stream of water strikes a 4-ft-square door which is at an angle of 30° with the stream's direction. The velocity of the water in the stream is 60.0 ft/s and the jet strikes the door at its center of gravity. Neglecting friction, what normal force applied at the edge of the door will maintain equilibrium?

▮ The force exerted by the door on the water will be normal to the door (no friction). Hence, since no forces act in the W direction in the figure, there will be no change in momentum in that direction. Thus, using W components, initial momentum ± 0 = final momentum, $+M(V\cos 30°) = +M_1V_1 - M_2V_2$, $(\gamma/g)(A_{jet}V)(V\cos 30°) = (\gamma/g)(A_1V_1)V_1 - (\gamma/g)(A_2V_2)V_2$. But $V = V_1 = V_2$ (friction neglected). Then $A_{jet}\cos 30° = A_1 - A_2$ and, from the equation of continuity, $A_{jet} = A_1 + A_2$. Solving, $A_1 = A_{jet}(1 + \cos 30°)/2 = A_{jet}(0.933)$ and $A_2 = A_{jet}(1 - \cos 30°)/2 = A_{jet}(0.067)$.

The stream divides as indicated and the momentum equation produces, for the X direction, $[(62.4/32.2)(\frac{1}{4}\pi)(\frac{1}{6})^2 60](60) - F_x(1) = [(62.4/32.2)(\frac{1}{4}\pi)(\frac{1}{6})^2(0.933)(60)](52.0) + [(62.4/32.2)(\frac{1}{4}\pi)(\frac{1}{6})^2(0.067)(60)](-52.0)$ and $F_x = 38.0$ lb.

Similarly, in the Y direction, $M(0) + F_y(1) = [(62.4/32.2)(0.0218)(0.933)(60)](30.0) + [(62.4/32.3)(0.0218)(0.067)(60)](-30.0)$ and $F_y = 65.9$ lb.

For the door as the free body, $\Sigma M_{hinge} = 0$ and $+38.0(1) + 65.9(2)(0.866) - P(4) = 0$ or $P = 38.0$ lb.

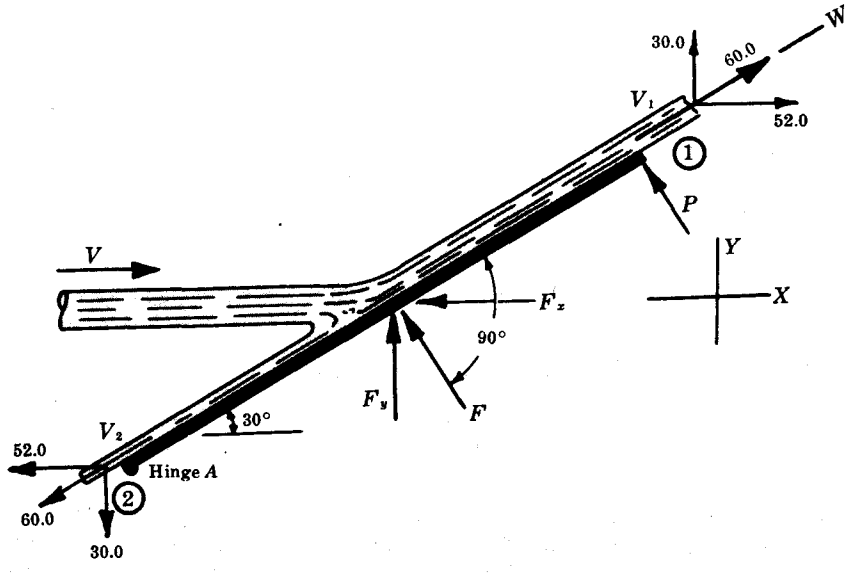


Fig. 23-9

23.12 The force exerted by a 1-in-diameter stream of water against a flat plate held normal to the stream's axis is 145 lb. What is the flow of water?

▮ $F = \rho Q(v_2 - v_1) \qquad v_1 = Q/A = Q/[(\pi)(\frac{1}{12})^2/4] = 183.3Q \qquad -145 = (1.94)(Q)(0 - 183.3Q) \qquad Q = 0.639 \text{ ft}^3/\text{s}$

23.13 Water flow in open channels can be controlled and measured with the sluice gate in Fig. 23-10. At a moderate distance upstream and downstream of the gate, sections 1 and 2, the flow is uniform and the pressure is hydrostatic. Derive an expression for the force F required to hold the gate as a function of ρ, V_1, g, h_1, and h_2. Neglect bottom friction in your analysis.

▮ $\Sigma F_x = \dot{m}(V_2 - V_1)$ $\quad -F + \gamma h_1(h_1 b)/2 - \gamma h_2(h_2 b)/2 = \rho h_1 b V_1[(h_1/h_2)(V_1) - V_1]$

$$F = \gamma b(h_1^2 - h_2^2)/2 - \rho h_1 b V_1^2(h_1/h_2 - 1)$$

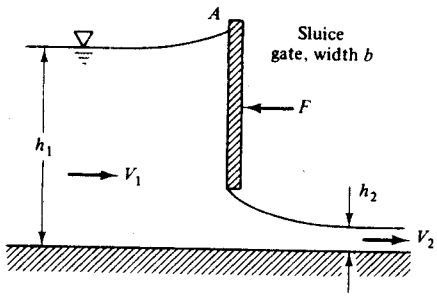


Fig. 23-10

23.14 Compute the force per unit width required to hold the gate in Fig. 23-10 if $h_1 = 4$ m, $h_2 = 0.7$ m, and $V_1 = 1.25$ m/s.

▮ $$F/b = \gamma(h_1^2 - h_2^2)/2 - \rho h_1 V_1^2(h_1/h_2 - 1)$$
$$= (9.79)(4^2 - 0.7^2)/2 - (9.98)(4)(1.25)^2(4/0.7 - 1) = 46.5 \text{ kN/m}$$

23.15 Find the thrust developed when water is pumped through a 300-mm-diameter pipe in the bow of a boat at $V = 2.0$ m/s and emitted through a 200-mm-diameter pipe in the stern.

▮ $Q = A_1 V_1 = [(\pi)(0.300)^2/4](2.0) = 0.1414 \text{ m}^3/\text{s}$ $\quad V_2 = (2.0)(\frac{300}{200})^2 = 4.50$ m/s

$$F = \rho Q(V_2 - V_1) = (988)(0.1414)(4.50 - 2.0) = 349 \text{ N}$$

23.16 A jet of water flowing freely in the atmosphere is deflected by a curved vane, as shown in Fig. 23-11*a*. If the water jet has a diameter of 1.5 in and a velocity of 25.5 fps, what is the force required to hold the vane in place?

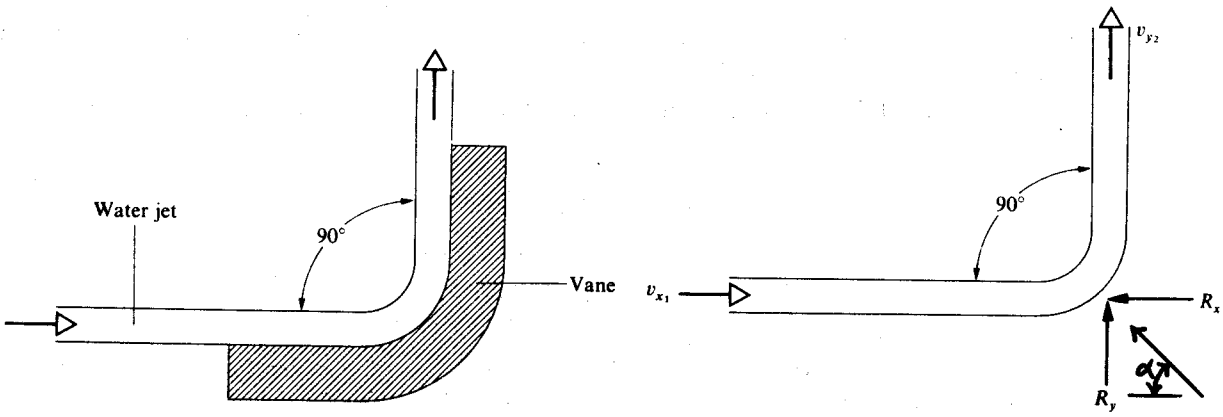
▮ Forces acting on the water jet are shown in Fig. 23-11*b*.

$F_x = \rho Q[(v_x)_2 - (v_x)_1]$ $\quad Q = AV = [(\pi)(1.5/12)^2/4](25.5) = 0.3129 \text{ ft}^3/\text{s}$

$R_x = (1.94)(0.3129)[0 - (-25.5)] = 15.48$ lb (leftward) $\quad F_y = \rho Q[(v_y)_2 - (v_y)_1]$

$R_y = (1.94)(0.3129)(25.5 - 0) = 15.48$ lb (upward) $\quad R = \sqrt{15.48^2 + 15.48^2} = 21.49$ lb

With equal x and y components, the direction of R is at a 45° angle (i.e., $\alpha = 45°$ in Fig. 23-11*b*).



(*a*) **Fig. 23-11** (*b*)

23.17 A jet of water having a velocity of 10 m/s and a flow rate of 60 L/s is deflected through a right angle, as shown in Fig. 23-12. Determine the reaction on the chute. Neglect the friction of the chute.

▮ $F_x = \rho Q[(v_x)_2 - (v_x)_1]$ $\quad R_x = (1000)(60 \times 10^{-3})[-10 \sin 30° - (-10 \cos 30°)] = 219.6$ N (leftward)

$F_y = \rho Q[(v_y)_2 - (v_y)_1]$ $\quad R_y = (1000)(60 \times 10^{-3})[10 \sin 60° - (-10 \cos 60°)] = 819.6$ N (upward)

$$R = \sqrt{219.6^2 + 819.6^2} = 849 \text{ N} \qquad \alpha = \arctan(819.6/219.6) = 75.0°$$

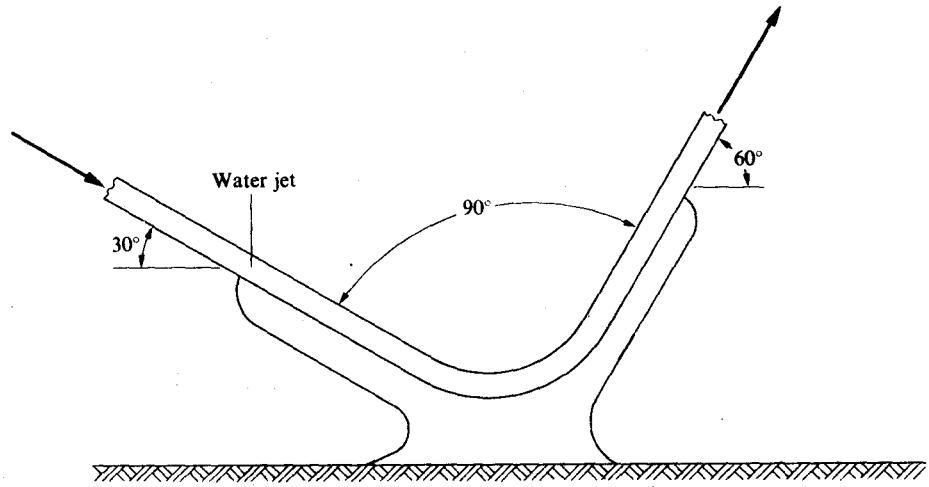


Fig. 23-12

23.18 For the cart of Fig. 23-13, compute the force on the wheels caused by deflecting the jet and the compression of the spring, if its stiffness is 1.6 kN/m.

▮ The force on the jet is upward and to the left. The upward component is supplied by the ground, acting on the wheels:

$$F_y = \rho Q[(v_y)_2 - (V_y)_1] \qquad Q = Av = [(\pi)(0.040)^2/4](20) = 0.02513\ \text{m}^3/\text{s}$$

$$F_y = (998)(0.02513)(20 \sin 45° - 0) = 355\ \text{N}$$

The leftward component is supplied by the compressed spring:

$$F_x = \rho Q[(v_x)_2 - (V_x)_1] = (998)(0.02513)(20 \cos 45° - 20) = -147\ \text{N}$$

Then, by Hooke's law, $F_x = -k\ \Delta x$, $-147 = (1600)(\Delta x)$, $\Delta x = 92$ mm.

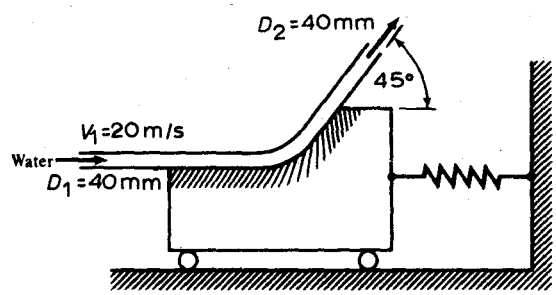


Fig. 23-13

23.19 In Fig. 23–14 the vane turns the water jet completely around. Find an expression for the maximum jet velocity V_0 if the maximum possible support force is F_0.

▮ $F = \rho Q(u_2 - u_1) \qquad F_0 = (\rho_0)(A_0 V_0)(-V_0 - V_0) = (\rho_0)[(\pi D_0^2/4)(V_0)](-V_0 - V_0) \qquad V_0 = \sqrt{2F_0/(\rho_0 D_0^2 \pi)}$

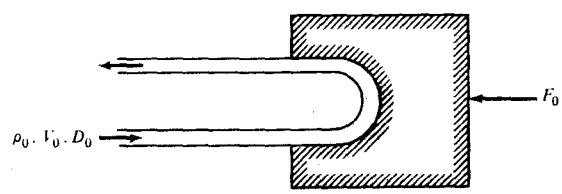


Fig. 23-14

23.20 The water tank in Fig. 23-15 stands on a frictionless cart and feeds a jet of diameter 3 cm and velocity 10 m/s, which is deflected 50° by a vane. Compute the tension in the supporting cable.

▮ Contrast Prob. 23.18; here the force on the water is upward and *rightward*.

$$F_x = \rho Q[(v_x)_2 - (v_x)_1] \qquad Q = Av = [(\pi)(0.03)^2/4](10) = 0.007069\ \text{m}^3/\text{s}$$

$$T = (998)(0.007069)(10 \cos 50° - 0) = 45.3\ \text{N}$$

23.21 Water exits to the atmosphere ($p_{atm} = 101$ kPa) through a split nozzle as shown in Fig. 23-16. Duct areas are $A_1 = 200\ \text{cm}^2$ and $A_2 = A_3 = 90\ \text{cm}^2$. The flow rate is $Q_2 = Q_3 = 200\ \text{m}^3/\text{h}$, and inlet pressure $p_1 = 165$ kPa abs. Compute the force on the flange bolts at section 1.

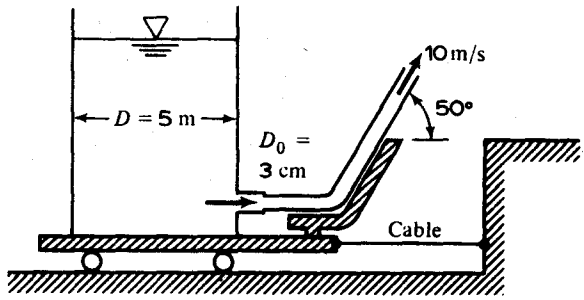


Fig. 23-15

▮ $$\Sigma F_x = \rho Q[(v_x)_2 - (v_x)_1] \qquad -F_{\text{bolts}} + p_1 A_1 = 2\rho Q_2(-v_2 \cos 30°) - \rho Q_1 v_1$$

$$v_2 = Q_2/A_2 = (\tfrac{200}{3600})/(90 \times 10^{-4}) = 6.173 \text{ m/s} \qquad v_1 = (2)(\tfrac{200}{3600})/(200 \times 10^{-4}) = 5.556 \text{ m/s}$$

$$-F_{\text{bolts}} + [(165 - 101)(1000)](200 \times 10^{-4}) = (2)(998)(\tfrac{200}{3600})(-5.556 \cos 30°) - (998)[(2)(\tfrac{200}{3600})](5.556)$$

$$F_{\text{bolts}} = 2430 \text{ N}$$

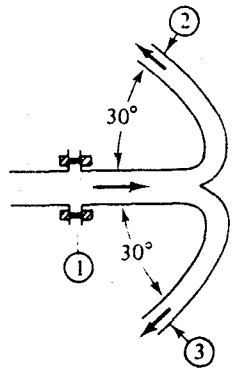


Fig. 23-16

23.22 A pump in a water tank directs a water jet at 25 ft/s and 0.3 ft³/s against a vane, as in Fig. 23-17. Compute the force F to hold the cart stationary if the jet follows path A.

▮ If the water follows path A, no momentum flux is created; thus, $F = 0$.

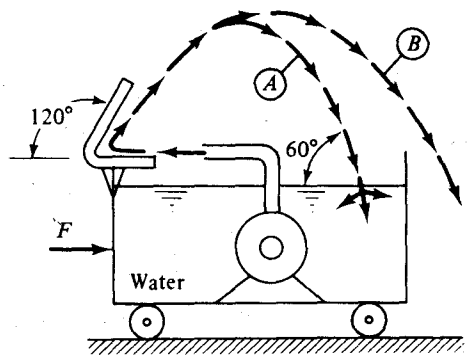


Fig. 23-17

23.23 Compute the force F in Prob. 23.22 if the fluid jet follows path B. The tank holds 50 ft³ of water at this instant.

▮ $F = \rho Q(v_2 - v_1)$. From "frictionless particle" concept, jet B will have constant horizontal velocity $v = v_{\text{jet}} \cos(180 - 120)°$. $F = (1.94)(0.3)[25 \cos(180 - 120)° - 0] = 7.28$ lb.

23.24 The cart in Fig. 23-18 moves at constant velocity $V_0 = 25$ m/s and takes on water with a scoop 1.7 m wide which dips $h = 25$ mm into a pond. Find the mass flow into the cart and the force needed to propel the cart.

▮ $$\dot{m} = \rho A V_0 = (998)[(1.7)(0.025)](25) = 1060 \text{ kg/s}$$

The cart is gaining momentum at the rate (1060 kg/s)(25 m/s) = 26 500 (kg · m/s)/s = 26.5 kN, which, by Newton's law, must be the force on the cart.

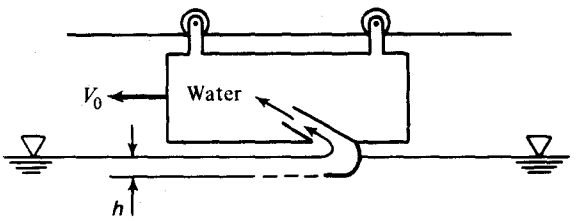


Fig. 23-18

23.25 A racing car, coasting, is to be decelerated by a scoop, as shown in Fig. 23-19. If the car and scoop weigh 2000 lb and $h = 3$ in, find the deceleration when $V = 540$ ft/s. Assume that the scoop has a width $b = 1$ ft into the paper.

▮ Force on car $= \rho bhV(1 - \cos 60°) = Ma$, whence ($\rho = 1.94$ slug/ft^3)

$$a = \frac{(1.94)(1)(3/12)(540)(1 - 0.5)}{(2000/32.2)} = 1138 \text{ ft/s}^2$$

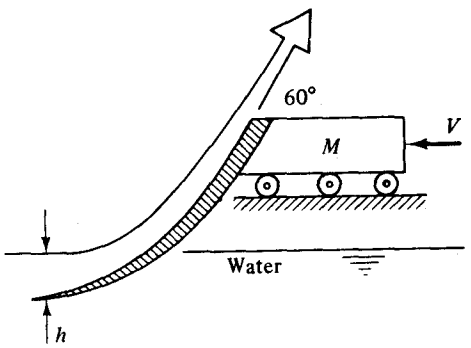


Fig. 23-19

23.26 A vane of turning angle $(180° - \theta)$ is mounted on a water tank, as shown in Fig. 23-20. It is struck by a 3-in-diameter, 40-fps water jet, which turns and falls off into the water tank without spilling. What is the force F required to hold the water tank stationary if $\theta = 90°$?

▮ $\Sigma F_x = \rho Q(v_2 - v_1)$ $\quad Q = Av = [(\pi)(\frac{3}{12})^2/4](40) = 1.963 \text{ ft}^3/\text{s}$ $\quad F = (1.94)(1.963)[0 - (-40)] = 152$ lb

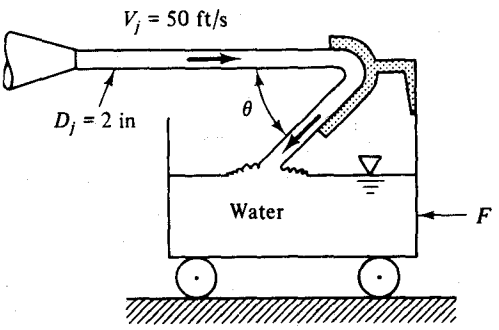


Fig. 23-20

23.27 In Fig. 23-21, air of density 1.2 kg/m^3 flows in the 30-cm duct at 12 m/s; it is choked by a 90° cone in the exit. Estimate the force of the airflow on the cone.

▮ $$\Sigma F_x = \dot{m}_2 u_2 - \dot{m}_1 u_1 \qquad \dot{m}_1 = \rho A_1 V_1 = (1.2)[(\pi)(0.30)^2/4](12) = 1.018 \text{ kg/s}$$

$$A_2 = (\pi)[0.40 + (0.01)(\cos 45°)](0.01) = 0.01279 \text{ m}^2 \qquad \dot{m}_2 = \rho A_2 V_2 = (1.2)(0.01279)(V_2) = 1.018$$

$$V_2 = 66.33 \text{ m/s} \qquad F = (1.018)(66.33 \cos 45°) - (1.018)(12) = 35.5 \text{ N} \qquad \text{(rightward)}$$

23.28 For the same situation and conditions as given in Prob. 23.16 except that the vane itself is moving to the right with a velocity of 10.0 ft/s, find the force exerted by the water on the vane.

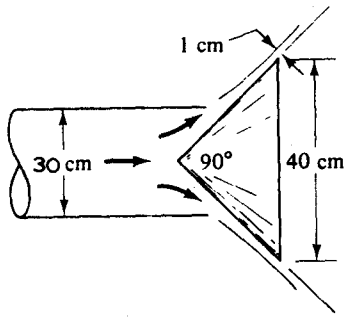


Fig. 23-21

▮ $F_x = \rho Q_r V_r(\cos\beta + 1)$ $\quad V_r = 25.5 - 10.0 = 15.5$ ft/s $\quad Q_r = A_r V_r = [(\pi)(1.5/12)^2/4](15.5) = 0.1902$ ft^3/s

$$R_x = (1.94)(0.1902)(15.5)(\cos 90° + 1) = 5.719 \text{ lb} \qquad F_y = \rho Q_r V_r(\sin\beta)$$

$$R_y = (1.94)(0.1092)(15.5)(\sin 90°) = 5.719 \text{ lb} \qquad R = \sqrt{5.719^2 + 5.719^2} = 8.09 \text{ lb}$$

As in Prob. 23.16, R acts at an angle β of 45°.

23.29 Solve Prob. 23.28 if the "angle of the vane" is 30° (i.e., the 90° angle in Fig. 23-11*b* is 30° instead).

▮ $F_x = \rho Q_r V_r(\cos\beta + 1)$. Using data from Prob. 23.28, $R_x = (1.94)(0.1902)(15.5)(\cos 30° + 1) = 10.67$ lb, $F_y = Q_r V_r(\sin\beta)$, $R_y = (1.94)(0.1902)(15.5)(\sin 30°) = 2.86$ lb, $R = \sqrt{10.67^2 + 2.86^2} = 11.0$ lb. The direction can be determined by referring to Fig. 23.22: $\alpha = \arctan(2.86/10.67) = 15.0°$.

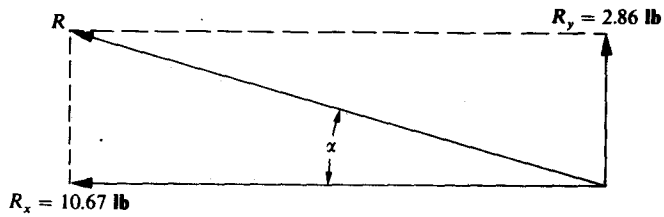


Fig. 23-22

23.30 A jet of water flowing freely in the atmosphere is deflected by a curved, 90° vane (see Fig. 23-11*a*). The vane is part of a turbine and is moving to the right with a velocity of 20 ft/s. The water jet has a diameter of 2 in and a velocity of 60 ft/s. Determine the force exerted by the water on the vane.

▮ $F_x = \rho Q_r V_r(\cos\beta + 1)$ $\quad V_r = 60 - 20 = 40$ ft/s $\quad Q_r = A_r V_r = [(\pi)(\frac{2}{12})^2/4](40) = 0.8727$ ft^3/s

$$R_x = (1.94)(0.8728)(40)(\cos 90° + 1) = 67.72 \text{ lb} \qquad F_y = Q_r V_r(\sin\beta)$$

$$R_y = (1.94)(0.8728)(40)(\sin 90°) = 67.72 \text{ lb} \qquad R = \sqrt{67.72^2 + 67.72^2} = 95.8 \text{ lb}$$

Since R_x and R_y are equal, R acts at an angle of 45° with the horizontal.

23.31 Solve Prob. 23.30 if the "angle of the vane" is 60° (i.e., the 90° angle in Fig. 23-11*b* is 30° instead).

▮ $F_x = \rho Q_r V_r(\cos\beta + 1)$. Using data from Prob. 23.30, $R_x = (1.94)(0.8727)(40)(\cos 60° + 1) = 101.6$ lb, $F_y = \rho Q_r V_r(\sin\beta)$, $R_y = (1.94)(0.8728)(40)(\sin 60°) = 58.65$ lb, $R = \sqrt{101.6^2 + 58.65^2} = 117.3$ lb. The direction can be determined by referring to Fig. 23-22: $\alpha = \arctan(58.65/101.6) = 30.0°$.

23.32 As shown in Fig. 23-23, a vane is curved so that the water jet is entirely reversed in direction. The vane is moving to the right at a velocity of 9.0 m/s, and the velocity of the water jet is 21.0 m/s. If the diameter of the water jet is 50 mm, find the force exerted by the vane on the water.

▮ $F_x = \rho Q_r V_r(\cos\beta + 1)$ $\quad V_r = 21.0 - 9.0 = 12.0$ m/s $\quad Q_r = A_r V_r = [(\pi)(0.050)^2/4](12.0) = 0.0236$ m^3/s

$$R_x = (1000)(0.0236)(12.0)(\cos 0° + 1) = 566 \text{ N} \qquad F_y = \rho Q_r V_r \sin\beta \qquad R_y = (1000)(0.0236)(12.0)(\sin 0°) = 0$$

Hence, $R = 566$ N (acting horizontally).

23.33 A 2-in-diameter water jet with a velocity of 100 fps impinges on a single vane moving in the same direction (thus $F_x = F_u$) at a velocity of 60 fps. If $\beta_2 = 150°$ and friction losses over the vane are such that $v_2 = 0.9v_1$, compute the force exerted by the water on the vane. See Fig. 23-24*a*.

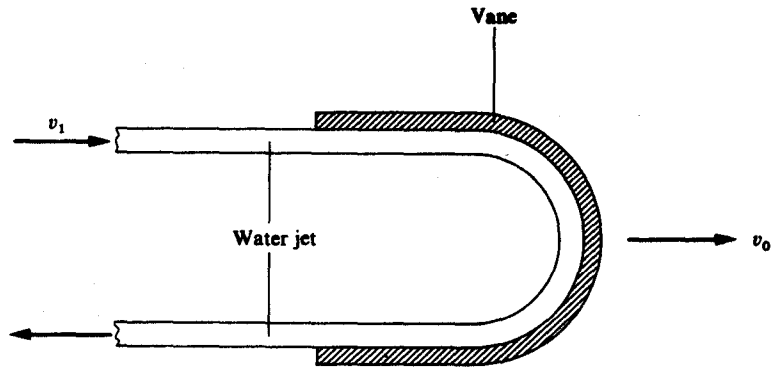


Fig. 23-23

▮ The velocity vector diagrams at entrance and exit to the vane are shown in Fig. 23-24*b*. Since $v_2 = (0.9)(40) = 36$ fps;

$$V_2 \sin \alpha_2 = v_2 \sin \beta_2 = (36)(0.5) = 18 \text{ fps} \qquad (1)$$

$$V_2 \cos \alpha_2 = u + v_2 \cos \beta_2 = 60 + 36(-0.866) = 28.8 \text{ fps} \qquad (2)$$

Solving (*1*) and (*2*) simultaneously yields $V_2 = 34$ fps, $\alpha_2 = 32°$. Hence $-F_x = \rho Q'(V_2 \cos \alpha_2 - V_1) = 1.94(0.0218)(100 - 60)(28.8 - 100) = -120.4$ lb. So $F_x = 120.4$ lb. The force of vane on water is to the left as assumed; hence force of water on vane is 120.4 lb to the right. $-F_y = \rho Q'(V_2 \sin \alpha_2 - 0) = 1.94(0.0218)(40)(-18) = -30.5$ lb. Thus $F_y = 30.5$ lb in the direction shown. The force of water on the vane is equal and opposite and thus 30.5 lb upward. If the blade were one of a series of blades, $-F_x = \rho Q(V_2 \cos \alpha_2 - V_1) = 1.94(0.0218)(100)(28.8 - 100) = -301$ lb.

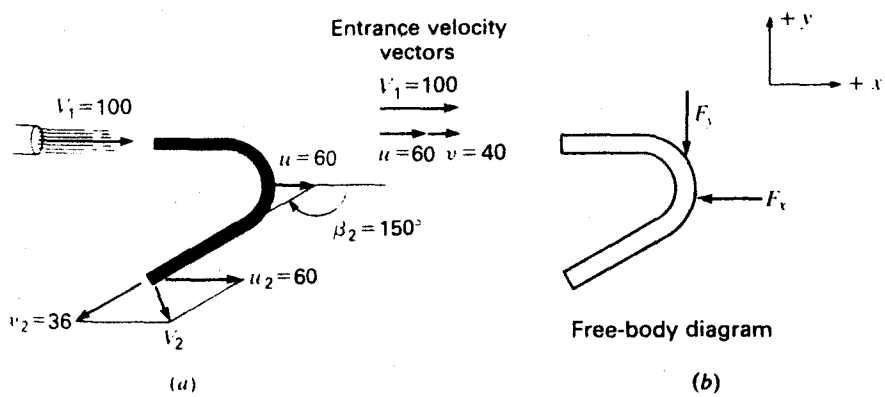


Fig. 23-24

23.34 A jet of water with an area of 4 in^2 and a velocity of 175 fps strikes a single vane which reverses it through 180° without friction loss. Find the force exerted if the vane moves (***a***) in the same direction as the jet with a velocity of 65 fps; (***b***) in a direction opposite to that of the jet with a velocity of 65 fps.

▮ $$F = \rho Q(v_2 - v_1)$$

(***a***) $v_1 = 175 - 65 = 110$ ft/s $\quad Q = Av = (\frac{4}{144})(110) = 3.056$ ft^3/s $\quad F = (1.94)(3.056)[110 - (-110)] = 1304$ lb

(***b***) $v_1 = 175 - (-65) = 240$ ft/s $\quad Q = (\frac{4}{144})(240) = 6.667$ ft^3/s $\quad F = (1.94)(6.667)[240 - (-240)] = 6208$ lb

23.35 The pipe bend shown in Fig. 23-25 is in a horizontal plane. A fluid of weight density 8.615 kN/m^3 enters the bend with a velocity of 3.50 m/s and a pressure of 280 kPa. Neglecting any energy losses in the bend, find the force required to hold the bend in place.

▮ $$\Sigma F_x = \rho Q[(v_x)_2 - (v_x)_1] \qquad p_1 A_1 + p_2 A_2 \cos 65° - R_x = \rho Q(-v_2 \cos 65° - v_1)$$

$$A_1 = (\pi)(0.200)^2/4 = 0.03142 \text{ m}^2$$

p_2 can be determined using the Bernoulli equation.

$$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 \qquad A_2 v_2 = A_1 v_1$$

$$A_2 = (\pi)(0.080)^2/4 = 0.005027 \text{ m}^2 \qquad 0.005027 v_2 = (0.03142)(3.50) \qquad v_2 = 21.88 \text{ m/s}$$

$$280/8.615 + 3.50^2/[(2)(9.807)] + 0 = p_2/8.615 + 21.88^2/[(2)(9.807)] + 0$$

$$p_2 = 75.10\ \text{kPa} \qquad Q = A_1 v_1 = (0.03142)(3.50) = 0.1100\ \text{m}^3/\text{s}$$

$$[(280)(1000)](0.03142) + [(75.10)(1000)](0.005027)(\cos 65°) - R_x$$
$$= [8615/9.807](0.1100)[(-21.88)(\cos 65°) - 3.50]$$

$$R_x = 10\,190\ \text{N} \quad \text{or} \quad 10.19\ \text{kN} \qquad \Sigma F_y = \rho Q[(v_y)_2 - (v_y)_1] \qquad -p_2 A_2 \sin 65° + R_y = \rho Q(v_2 \sin 65° - 0)$$

$$-[(75.10)(1000)](0.005027)(\sin 65°) + R_y = [8615/9.807](0.1100)[(21.88)(\sin 65°) - 0]$$

$$R_y = 2262\ \text{N} \quad \text{or} \quad 2.26\ \text{kN} \qquad R = \sqrt{10.19^2 + 2.26^2} = 10.4\ \text{kN}$$

$$\alpha = \arctan(2.26/10.19) = 12.5° \qquad \text{(see Fig. 23-25}b\text{)}$$

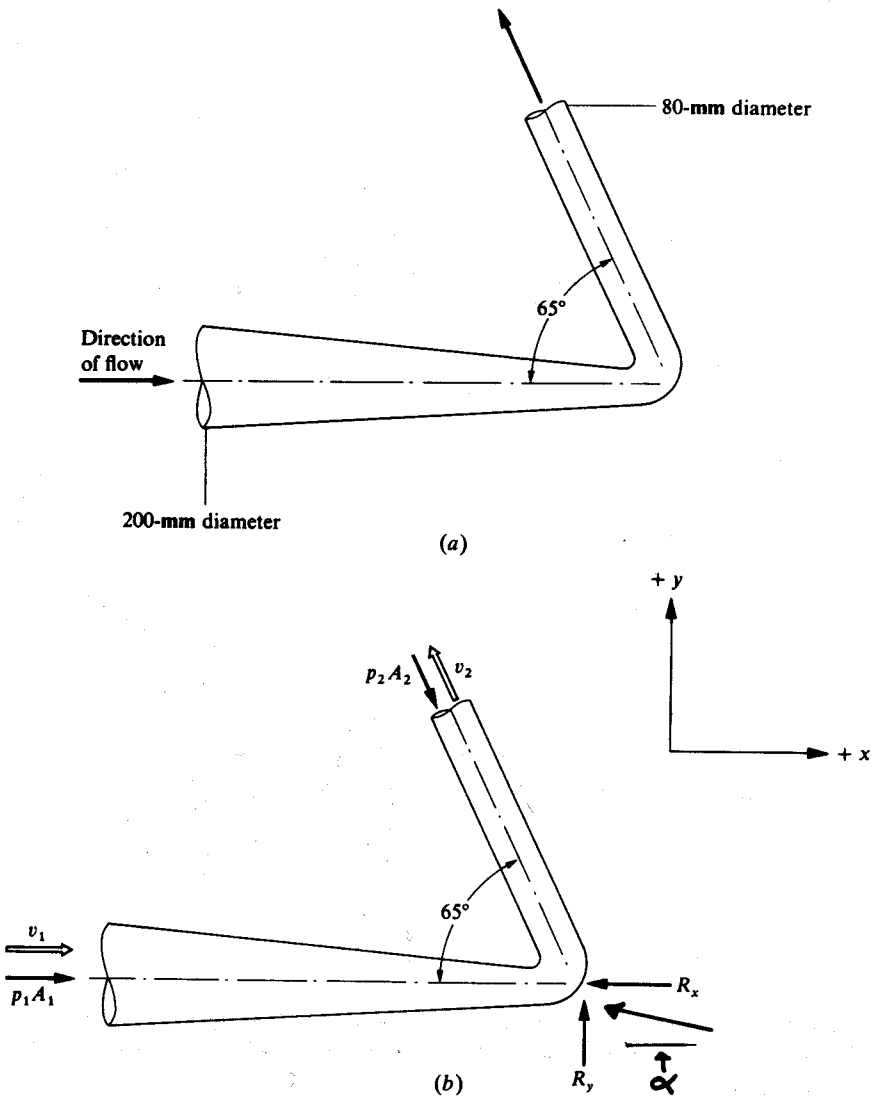


Fig. 23-25

23.36 The pipe bend shown in Fig. 23-26*a* is in a horizontal plane. Oil with a specific gravity of 0.86 enters the reducing bend at section *A* with a velocity of 3.2 m/s and a pressure of 150 kPa. Determine the force required to hold the bend in place. Neglect any energy loss in the bend.

▮ External forces acting on the oil within the bend are shown in Fig. 23-26*b*.

$$A_1 v_1 = A_2 v_2 \qquad A_1 = \pi(0.150)^2/4 = 0.01767\ \text{m}^2 \qquad A_2 = \pi(0.100)^2/4 = 0.007854\ \text{m}^2$$

$$(0.01767)(3.2) = 0.007854 v_2 \qquad v_2 = 7.199\ \text{m/s} \qquad p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2$$

$$150/[(0.86)(9.79)] + (3.2)^2/[(2)(9.807)] + 0 = p_2/[(0.86)(9.79)] + (7.199)^2/[(2)(9.807)] + 0 \qquad p_2 = 132.1\ \text{kPa}$$

$$F_x = \rho Q(v_{x2} - v_{x1}) \qquad p_1A_1 - p_2A_2\cos 30° - R_x = \rho Q(v_{x2}\cos 30° - v_{x1})$$

$$Q = Av = (0.01767)(3.2) = 0.05654\ \text{m}^3/\text{s}$$

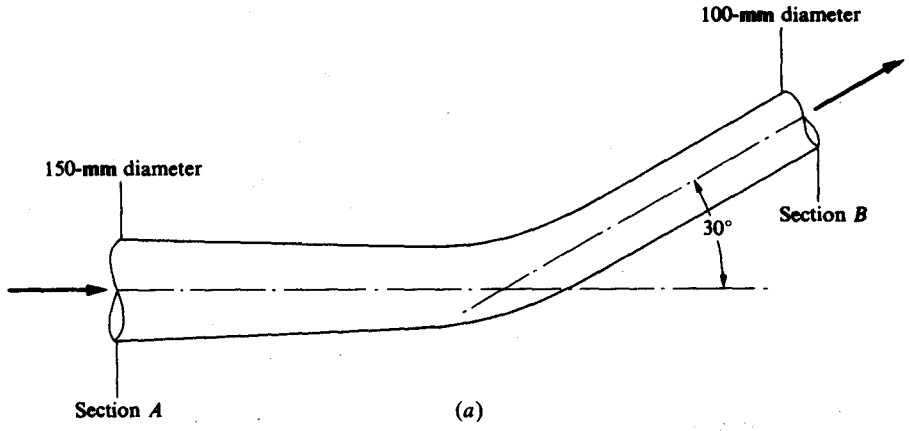
$$(150\,000)(0.01767) - (132\,100)(0.007854)\cos 30° - R_x = (0.86)(1000)(0.05654)(7.199\cos 30° - 3.2)$$

$$R_x = 1604\ \text{N}$$

$$F_y = \rho Q(v_{y2} - v_{y1}) \qquad -p_2A_2\sin 30° + R_y = \rho Q(v_{y2}\sin 30° - 0)$$

$$(-132\,100)(0.007854)\sin 30° + R_y = (0.86)(1000)(0.05654)(7.199\sin 30°) \qquad R_y = 694\ \text{N}$$

$$R = \sqrt{(1604)^2 + (694)^2} = 1748\ \text{N} \qquad \alpha = \arctan\tfrac{694}{1604} = 23.4° \qquad \text{(see Fig. 23-26}b\text{)}$$



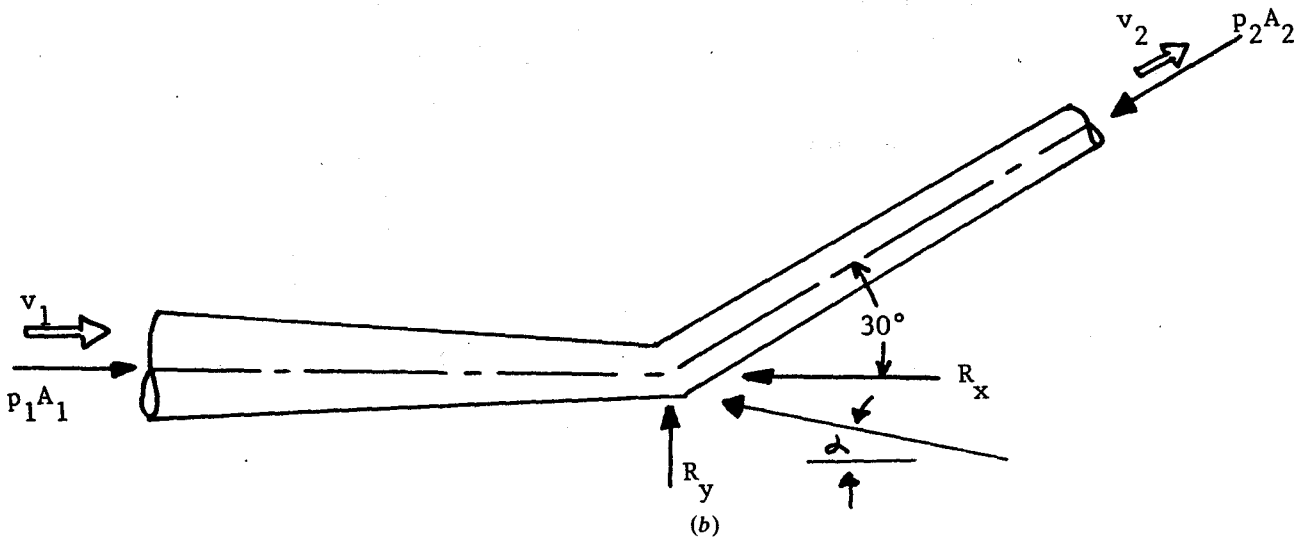


Fig. 23-26

23.37 Find the magnitude and direction of the resultant force on the compressor shown in Fig. 23-27. Air ($\gamma = 0.075\ \text{lb/ft}^3$) enters at A through a 4-ft^2 area at a velocity of 12 fps. Air is discharged at B through a 3-ft^2 area at a velocity of 14 fps.

▮

$$F_x = \rho Q[(V_2)_x - (V_1)_x] \qquad Q = AV = (4)(12) = 48.0\ \text{ft}^3/\text{s}$$

$$F_x = (0.075/32.2)(48.0)(14 - 12\cos 50°) = 0.703\ \text{lb} \qquad \text{(rightward)}$$

$$F_y = \rho Q[(V_2)_y - (V_1)_y] = (0.075/32.2)(48.0)[0 - (-12\sin 50°)] = 1.028\ \text{lb} \qquad \text{(upward)}$$

Force of fluid on compressor is 0.703 lb leftward and 1.028 lb downward. $F = \sqrt{0.703^2 + 1.028^2} = 1.245$ lb, $\alpha = \arctan(1.028/0.703) = 55.6°$ to the horizontal.

23.38 Repeat Prob. 23.37 for the case where a gas ($\gamma = 12.1\ \text{N/m}^3$) enters at A through a 75-cm-diameter pipe at 3.5 m/s and leaves at B through a 60-cm-diameter pipe at 4.5 m/s.

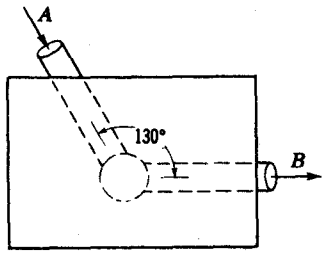


Fig. 23-27

▮ $F_x = \rho Q[(V_2)_x - (V_1)_x]$ $\quad Q = AV = [(\pi)(0.75)^2/4](3.5) = 1.546\ \text{m}^3/\text{s}$

$$F_x = (12.1/9.807)(1.546)(4.5 - 3.5\cos 50°) = 4.29\ \text{N} \quad \text{(rightward)}$$

$$F_y = \rho Q[(V_2)_y - (V_1)_y] = (12.1/9.807)(1.546)[0 - (-3.5\sin 50°)] = 5.11\ \text{N} \quad \text{(upward)}$$

Force of fluid on compressor is 4.29 N leftward and 5.11 N downward. $F = \sqrt{4.29^2 + 5.11^2} = 6.67$ N, $\alpha = \arctan(5.11/4.29) = 50.0°$ to the horizontal.

23.39 In Fig. 23-28*a* is shown a curved pipe section of length 40 ft that is attached to the straight pipe section as shown. Determine the resultant force on the curved pipe, and find the horizontal component of the jet reaction. All significant data are given in the figure. Assume an ideal liquid with $\gamma = 55\ \text{lb/ft}^3$.

▮ The energy equation between sections 1 and 3 gives $[(30)(144)/55] + 35 = 20 + (V_3^2/2g)$ and $V_3 = 77.6$ fps (jet velocity); $Q = A_3V_3 = 3.81$ cfs, $V_2 = Q/A_2 = 43.6$ fps. The energy equation between section 2 and 3 gives $10 + [p_2(144)/55] + [(43.6)^2/64.4] = 20 + [(77.6)^2/2g]$, and $p_2 = 28.3$ psi. The free-body diagram of the forces acting on the liquid contained in the curved pipe is shown in Fig. 23-28*b*. $p_2A_2 - p_3A_3\cos 20° - (F_{N/L})_x = \rho Q(V_3\cos 20° - V_2)$ where $(F_{N/L})_x$ represents the force of the curved pipe on the liquid in the x direction. Since section 3 is a jet in contact with the atmosphere, $p_3 = 0$. Thus $28.3[(\pi/4)(4^2)] - (F_{N/L})_x = [(1.94)(55/62.4)](3.81)[(77.6)(0.94) - 43.6]$, $356 - (F_{N/L})_x = 191$, $(F_{N/L})_x = +165$ lb where the plus sign indicates that the assumed direction is correct. In the y direction the p_2A_2 force has no component. Estimating the weight of liquid W as 150 lb, $(F_{N/L})_y = \rho Q[(77.6)(0.342) - 0] + 150 = +323$ lb. The resultant force of liquid on the curved pipe is equal and opposite to that of the curved pipe on liquid. The resultant force of liquid on the curved pipe is $[(165)^2 + (323)^2]^{1/2} = 363$ lb downward and to the right at an angle of 62°56′ with the horizontal.

The horizontal jet reaction is best found by taking a free-body diagram of the liquid in the system as shown in Fig. 23-28*c*: $(F_{S/L})_x = \rho Q(V_3\cos 20° - 0) = 475$ lb where $(F_{S/L})_x$ respresents the force of the system on the liquid in the x direction. $(F_{S/L})_x$ is equivalent to the integrated effect of the x components of the pressure vectors shown in Fig. 23-28*c*. Equal and opposite to $(F_{S/L})_x$ is the force of the liquid on the system, i.e., the jet reaction. Hence the horizontal jet reaction is a 475-lb force to the left. Thus there is a 165 lb force to the right tending to separate the curved pipe section from the straight pipe section, while at the same time there is a 475-lb force tending to move the entire system to the left.

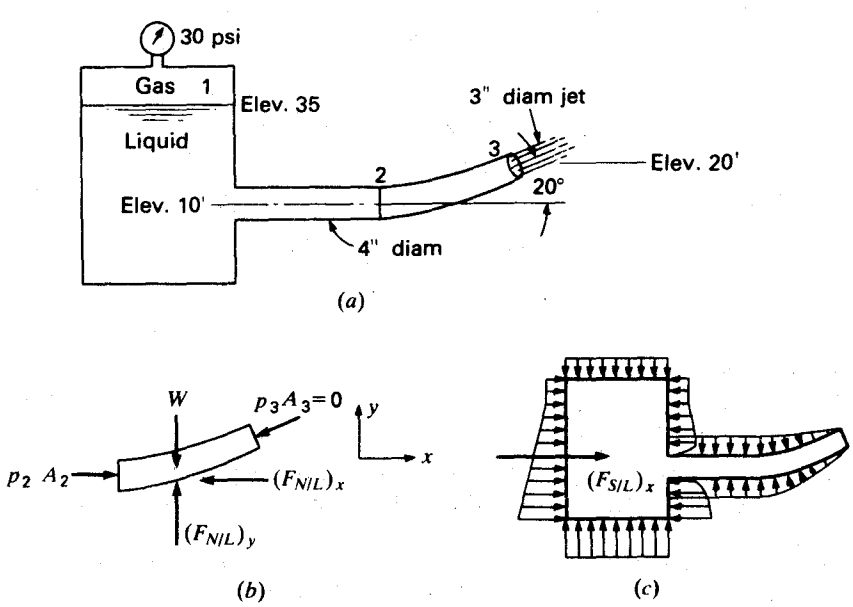


Fig. 23-28

23.40 A 24-in pipe carrying 31.4 cfs of oil (s.g. = 0.85) has a 90° bend in a horizontal plane. The loss of head in the bend is 3.50 ft of oil and the pressure at entrance is 42.5 psi. Determine the resultant force exerted by the oil on the bend.

▮ Referring to Fig. 23-29, the free-body diagram indicates the static and dynamic forces acting on the mass of oil in the bend. These forces are calculated as follows:

1. $$P_1 = p_1 A = (42.5)(\tfrac{1}{4}\pi)(24)^2 = 19\,200 \text{ lb}$$
2. $P_2 = p_2 A$, where $p_2 = p_1 -$ loss in psi, from the Bernoulli equation since $z_1 = z_2$ and $V_1 = V_2$. Then $P_2 = [42.5 - (0.85)(62.4)(3.50/144)](\tfrac{1}{4}\pi)(24)^2 = 18\,600$ lb.
3. Using the impulse-momentum principle, and knowing that $V_1 = V_2 = Q/A = 10.0$ ft/s, $MV_{x_1} + \Sigma$ (forces in X direction)(1) $= MV_{x_2}$, $19\,200 - F_x = [(0.85)(62.4)(31.4/32.2)](0 - 10.0) = -517$ lb, and $F_x = +19\,720$ lb, to the left on the oil.
4. Similarly, for $t = 1$ s, $MV_{y_1} + \Sigma$ (forces in Y direction)(1) $= MV_{y_2}$, $F_y - 18\,600 = [(0.85)(62.4)(31.4/32.2)](10.0 - 0) = +517$ lb, and $F_y = +19\,120$ lb, upward on the oil.

On the pipe bend, the resultant force R acts to the right and downward and is $R = \sqrt{(19\,720)^2 + (19\,120)^2} = 27\,470$ lb at $\theta_x = \tan^{-1}(19\,120/19\,720) = 44.1°$.

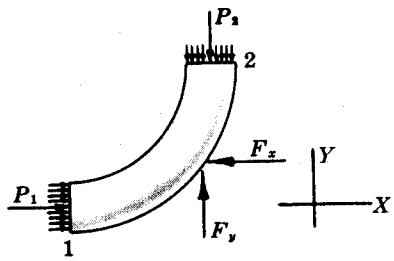


Fig. 23-29

23.41 The 24-in pipe of Prob. 23.40 is connected to a 12-in pipe by a standard *reducer* fitting. For the same flow of 31.4 cfs of oil, and a pressure of 40.0 psi, what force is exerted by the oil on the reducer, neglecting any lost head?

▮ Since $V_1 = 10.0$ fps, $V_2 = (\frac{2}{1})^2(10.0) = 40.0$ fps. Also, the Bernoulli equation between section 1 at entrance and section 2 at exit yields $\{(p_1/\gamma) + [(10)^2/2g] + 0\}$ − negligible lost head $= \{(p_2/\gamma) + [(40)^2/2g] + 0\}$. Solving, $p_2/\gamma = [(40.0)(144)/(0.85)(62.4)] + (100/2g) - (1600/2g) = 85.3$ ft of oil and $p_2' = 31.4$ psi.

Figure 23-30 represents the forces acting on the mass of oil in the reducer: $P_1 = p_1 A_1 = (40.0)(\tfrac{1}{4}\pi)(24)^2 = 18\,100$ lb (to the right), $P_2 = p_2 A_2 = (31.4)(\tfrac{1}{4}\pi)(12)^2 = 3550$ lb (to the left).

In the X direction the momentum of the oil is changed. Then $MV_{x_1} + \Sigma$ (forces in X direction)(1) $= MV_{x_2}$, $(18\,100 - 3550 - F_x)1 = [(0.85)(62.4)(31.4/32.2)](40.0 - 10.0)$, and $F_x = 13\,000$ lb acting to the left on the oil.

The forces in the Y direction will balance each other and $F_y = 0$. Hence the force exerted by the oil on the reducer is 13 000 lb to the right.

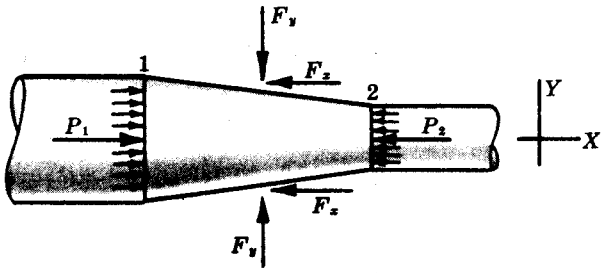


Fig. 23-30

23.42 A 45° reducing bend, 24-in diameter upstream, 12-in diameter downstream, has water flowing through it at the rate of 15.7 cfs under a pressure of 21.0 psi. Neglecting any loss in the bend, calculate the force exerted by the water on the reducing bend.

▮ $V_1 = 15.7/A_1 = 5.00$ ft/s and $V_2 = 20.0$ ft/s. The Bernoulli equation, section 1 to section 2, produces $\{[(21.0)(144)/62.4] + (25/2g) + 0\}$ − negligible lost head $= [(p_2/\gamma) + (400/2g) + 0]$ from which $p_2/\gamma = 42.6$ ft and $p_2' = 18.5$ psi.

In Fig. 23-31 is shown the mass of water acted upon by static and dynamic forces: $P_1 = p_1 A_1 = (21.0)(\tfrac{1}{4}\pi)(24)^2 = 9500$ lb, $P_2 = p_2 A_2 = (18.5)(\tfrac{1}{4}\pi)(12)^2 = 2090$ lb, $P_{2_x} = P_{2_y} = (2090)(0.707) = 1478$ lb.

In the X direction, $MV_{x_1} + \Sigma$ (forces in X direction)(1) $= MV_{x_2}$, $(9500 - 1478 - F_x)1 = [(62.4)(15.7/32.2)][(20.0)(0.707) - 5.00]$ and $F_x = 7740$ lb to the left.

In the Y direction, $(+F_y - 1478)1 = [(62.4)(15.7/32.2)][(20.0)(0.707) - 0]$ and $F_y = 1910$ lb upward.

The force exerted by the water on the reducing bend is $F = \sqrt{(7740)^2 + (1910)^2} = 7970$ lb to the right and downward, at an angle $\theta_x = \tan^{-1}\frac{1910}{7740} = 13°52'$.

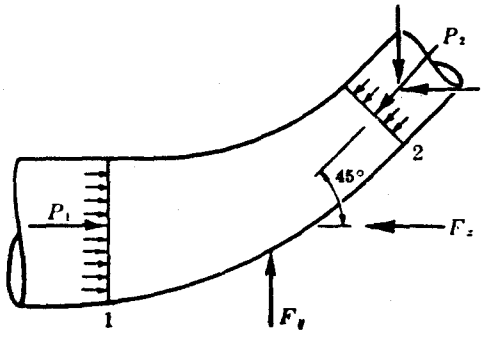


Fig. 23-31

23.43 Determine the reaction of a jet flowing through an orifice on the containing tank.

▮ In Fig. 23-32 a mass of liquid $ABCD$ is taken as a free body. The only horizontal forces acting are F_1 and F_2, which change the momentum of the water. $(F_1 - F_2)(1) = M(V_2 - V_1)$, where V_1 can be considered negligible. Reaction $F = F_1 - F_2 = (\gamma Q/g)V_2 = (\gamma A_2 V_2/g)V_2$. But $A_2 = c_c A_0$ and $V_2 = c_v\sqrt{2gh}$. Hence $F = [\gamma(c_c A_0)/g]c_v^2(2gh) = (cc_v)\gamma A_0(2h)$ (to the right on the liquid).

1. For average values of $c = 0.60$ and $c_v = 0.98$, the reaction $F = 1.176\gamma h A_0$. Hence the force acting to the left on the tank is about 18 percent more than the static force on a plug which would just fill the orifice.
2. For ideal flow (no friction, no contraction), $F = 2(\gamma h A_0)$. This force is equal to twice the force on a plug which would just fill the orifice.
3. For a nozzle ($c_c = 1.00$), the reaction $F = c_v^2\gamma A(2h)$ where h would be the effective head causing the flow.

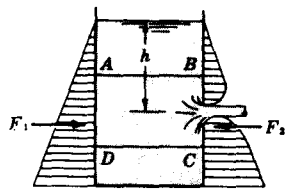


Fig. 23-32

23.44 The jets from a garden sprinkler are 1-in in diameter and are normal to the 2-ft radius. If the pressure at the base of the nozzles is 50 psi, what force must be applied on each sprinkler pipe, 1 ft from the center of rotation, to maintain equilibrium? (Use $c_v = 0.80$ and $c_c = 1.00$.)

▮ The reaction of the sprinkler jet may be calculated from the momentum principle. Inasmuch as the force which causes a change in momentum in the X direction acts along the X axis, no torque is exerted. We are interested, therefore, in the change in momentum in the Y direction. But the initial momentum in the Y direction is zero. The jet velocity $V_Y = c_v\sqrt{2gh} = 0.80\sqrt{2g}[(50)(2.31) + \text{negligible velocity head}] = 69.0$ ft/s. Thus $F_Y\,dt = M(V_Y) = [(62.4/32.2)(\frac{1}{4}\pi)(\frac{1}{12})^2(69.0)\,dt](-69.0)$ or $F_Y = -50.3$ lb downward on the water. Hence the force of the jet on the sprinkler is +50.3 lb upward. Then $\Sigma M_O = 0$, $F(1) - 2(50.3) = 0$, $F = 100.6$ lb for equilibrium.

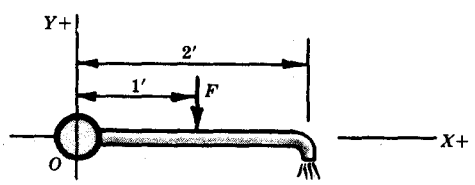


Fig. 23-33

23.45 A rocket engine burns 5.0 kg of fuel and oxidizer per second, and combustion gases exit the rocket at a velocity of 550 m/s relative to the rocket, at a pressure approximately equal to the ambient air pressure. Find the thrust produced by the rocket engine.

▮

$$F = \rho Q v_2 = Mv = (5.0)(550) = 2750 \text{ N}$$

23.46 A jet craft traveling at a velocity of 590 fps takes in air at a rate of 1.55 slugs/s. The air : fuel ratio is 25 : 1, and the exhaust velocity relative to the jet is 1950 fps. Find the thrust produced by the jet engine.

▮ $$F = \rho_a Q_a(v_2 - v_1) + \rho_f Q_f v_2 \qquad \rho_a Q_a = M_a = 1.55 \text{ slugs/s}$$

Since the air : fuel ratio is 25 : 1, $\rho_f Q_f = 1.55/25 = 0.0620$ slug/s, $F = (1.55)(1950 - 590) + (0.0620)(1950) = 2229$ slug-ft/s^2. Since slug = lb-s^2/ft, $F = 2229$ lb.

23.47 A rocket engine burns 8.5 kg of fuel and oxidizer per second, and combustion gases exit the rocket at a velocity of 565 m/s relative to the rocket, at a pressure approximately equal to the surrounding atmospheric pressure. Find the thrust produced by the rocket engine.

▮ $$F = \rho Q v_2 = Mv = (8.5)(565) = 4802 \text{ N}$$

23.48 A jet craft traveling at a velocity of 700 fps takes in air at a rate of 1.68 slugs/s. The air : fuel ratio is 20 : 1, and the exhaust velocity relative to the jet is 2105 fps. Find the thrust produced by the jet engine.

▮ $$F = \rho_a Q_a(v_2 - v_1) + \rho_f Q_f v_2 \qquad \rho_a Q_a = M_a = 1.68 \text{ slugs/s}$$

Since the air : fuel ratio is 20 : 1, $\rho_f Q_f = 1.68/20 = 0.0840$ slug/s, $F = (1.68)(2105 - 700) + (0.0840)(2105) = 2537$ slug-ft/s^2. Since slug = lb-s^2/ft, $F = 2537$ lb.

23.49 A jet engine is being tested in the laboratory. The engine consumes 50 lb of air per second and 0.5 lb of fuel per second. If the exit velocity of the gases is 1500 fps, what is the thrust?

▮ $$F = W_{exit}V_2/g - W_1V_1/g = (50 + 0.5)(1500)/32.2 - (50)(0)/32.2 = 2352 \text{ lb}$$

23.50 A jet engine operates at 600 fps and consumes air at the rate of 50.0 lb/s. At what velocity should the air be discharged in order to develop a thrust of 1500 lb?

▮ $$F = 1500 = (50.0/32.2)(V_{exit} - 600) \qquad V_{exit} = 1566 \text{ fps}$$

23.51 A turbojet engine is tested in the laboratory under conditions simulating an altitude where atmospheric pressure is 785.3 lb/ft^2 abs, temperature is 429.5 °R, and specific weight is 0.0343 lb/ft^3. If the exit area of the engine is 1.50 ft^2 and the exit pressure is atmospheric, what is the Mach number if the gross thrust is 1470 lb? Use $k = 1.33$.

▮ $$F = W_{exit}V_{exit}/g = (\gamma A_{exit}V_{exit})(V_{exit})/g \qquad 1470 = (0.0343)(1.50)(V_{exit})^2/32.2 \qquad V_{exit} = 959 \text{ fps}$$
$$N_M = V_{exit}/\sqrt{kgRT} = 959/\sqrt{(1.33)(32.2)(53.3)(429.5)} = 0.969$$

23.52 A rocket device burns its propellant at a rate of 15.2 lb/s. The exhaust gases leave the rocket at a relative velocity of 3220 fps and at atmospheric pressure. The exhaust nozzle has an area of 50.0 in^2 and the gross weight of the rocket is 500 lb. At the given instant, 2500 hp is developed by the rocket engine. What is the rocket velocity?

▮ $$P = FV_{rocket} \qquad F = W_{exit}V_{exit}/g = (15.2)(3220)/32.2 = 1520 \text{ lb}$$
$$(2500)(550) = 1520V_{rocket} \qquad V_{rocket} = 905 \text{ fps}$$

23.53 A rocket motor is operating steadily, as shown in Fig. 23-34. The exhaust products approximate a perfect gas of molecular weight 26. Calculate the exit velocity.

▮ $$\dot{m}_{exit} = \rho_{exit}A_{exit}V_{exit} = 0.6 + 0.12 = 0.72 \text{ slug/s} \qquad \rho = p/RT \qquad R = 49\,750/26 = 1913 \text{ ft-lb/slug-°R}$$
$$\rho_{exit} = (15)(144)/[(1913)(1000 + 460)] = 0.0007734 \text{ slug/ft}^3 \qquad A_{exit} = (\pi)(6/12)^2/4 = 0.1964 \text{ ft}^2$$
$$0.72 = (0.0007734)(0.1964)(V_{exit}) \qquad V_{exit} = 4740 \text{ fps}$$

23.54 In contrast to the liquid rocket in Fig. 23-34, the solid-propellant rocket in Fig. 23-35 is self-contained and has no entrance ducts. Using a control-volume analysis for the conditions shown in Fig. 23-35, compute the rate of mass loss of the propellant, assuming that the exit gas has molecular weight 31.

▮ $$\dot{m}_{propellant} = \rho_{exit}A_{exit}V_{exit} \qquad \rho = p/RT \qquad R = \tfrac{8313}{31} = 268 \text{ J/kg}\cdot\text{K}$$
$$\rho_{exit} = (80)(1000)/[(268)(725)] = 0.4117 \text{ kg/m}^3 \qquad A_{exit} = (\pi)(0.200)^2/4 = 0.03142 \text{ m}^2$$
$$m_{propellant} = (0.4117)(0.03142)(1100) = 14.2 \text{ kg/s}$$

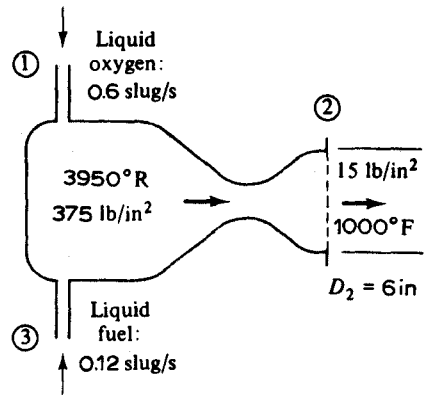


Fig. 23-34

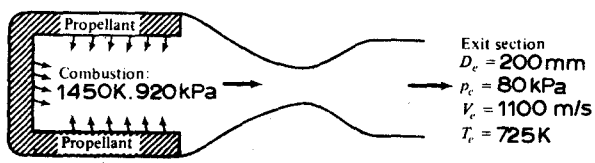


Fig. 23-35

23.55 For the rocket engine of Prob. 23.53, compute the thrust if atmospheric pressure is 15 psia.

▮ $$F = \dot{m}_{exit}V_{exit} = (0.72)(4740) = 3413 \text{ lb.}$$

23.56 A rocket ship is moving at 8.5 km/s in outer space. It is desired to slow the rocket down to 8.4 km/s by firing a retrorocket forward. The retrorocket burns 8 kg/s of fuel and oxidizer at an exhaust velocity of 1.3 km/s relative to the rocket. If the initial mass of the rocket ship is 1550 kg, how long should the retrorocket burn, and how much fuel will be burned?

▮ $$\int_{V_0}^{V_{final}} dV = \dot{m}V_{exit}\int_0^{t_{burn}} \frac{dt}{m_0 - \dot{m}t} \qquad V_{final} - V_0 = V_{exit}\ln\left(1 - \frac{\dot{m}t_{burn}}{m_0}\right) \qquad 8.4 - 8.5 = 1.3\ln\left(1 - \frac{8t_{burn}}{1550}\right)$$

$$t_{burn} = 14.3 \text{ s} \qquad m_{burned} = (8)(14.3) = 114 \text{ kg}$$

23.57 Determine the magnitude and direction of the resultant force exerted on the double nozzle of Fig. 23-36. The axes of the pipes and both nozzles lie in a horizontal plane. Both nozzle jets have a water velocity of 12 m/s.

▮ **Continuity:**

$$A_1V_1 = A_2V_2 + A_3V_3 \qquad [(\pi)(0.150)^2/4](V_1) = [(\pi)(0.100)^2/4](12) + [(\pi)(0.075)^2/4](12)$$

$$V_1 = 8.33 \text{ m/s} \qquad Q_1 = A_1V_1 = [(\pi)(0.150)^2/4](8.33) = 0.147 \text{ m}^3/\text{s}$$

$$Q_2 = [(\pi)(0.100)^2/4](12) = 0.094 \text{ m}^3/\text{s} \qquad Q_3 = [(\pi)(0.075)^2/4](12) = 0.053 \text{ m}^3/\text{s}$$

Energy equation:

$$(p_1/\gamma) + [8.33^2/2(9.81)] = 0 + [12^2/2(9.81)] \qquad p_1/\gamma = 3.80 \text{ m} \qquad p_1 = 37.3 \text{ kPa} \qquad p_1A_1 = 0.659 \text{ kN}$$

$$\sum F_x = p_1A_1 - (F_{N/L})_x = (\rho Q_2V_{2_x} + \rho Q_3V_{3_x}) - \rho Q_1V_{1_x} \qquad \rho = 10^3 \text{ kg/m}^3$$

$$V_{2_x} = V_2\cos 15° = 12(0.966) = 11.6 \text{ m/s} \qquad V_{3_x} = V_3\cos 30° = 12(0.866) = 10.4 \text{ m/s} \qquad V_{1_x} = V_1 = 8.33 \text{ m/s}$$

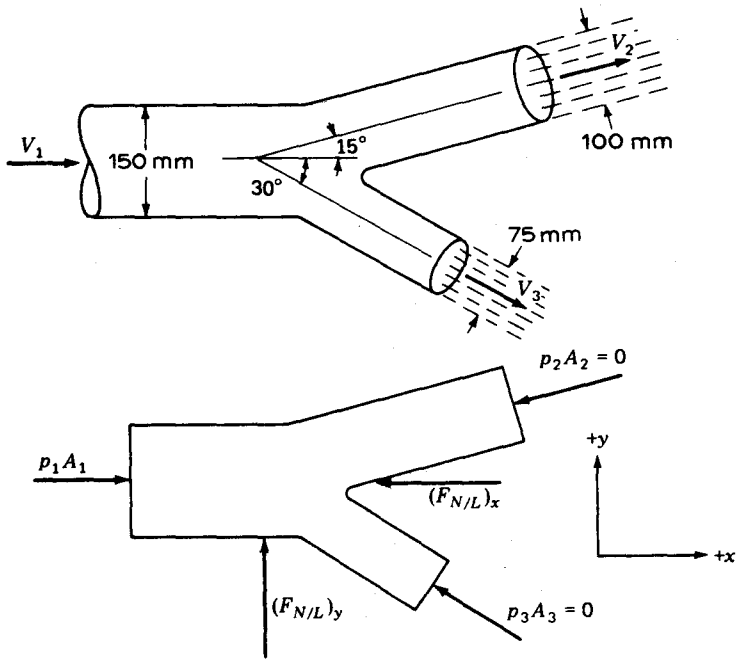
$$0.659 - (F_{N/F})_x = 10^3(0.094)(11.6) + 10^3(0.053)(10.4) - 10^3(0.147)8.33 = 0.417 \text{ kN}$$

$$(F_{N/L})_x = 0.659 - 0.417 = 0.242 \text{ kN}$$

$$\sum F_y = (F_{N/L})_y = (\rho Q_2V_{2_y} + \rho Q_3V_{3_y}) - \rho Q_1V_{1_y} \qquad V_{2_y} = V_2\sin 15° = 12(0.259) = 3.1 \text{ m/s}$$

$$V_{3_y} = -V_3\sin 30° = -12(0.50) = -6.0 \text{ m/s} \qquad V_{1_y} = 0$$

$$(F_{N/L})_y = 10^3(0.094)(3.1) + 10^3(0.053)(-6.0) - 10^3(1.47)(0) = -0.027 \text{ kN}$$



Free-body diagram

Fig. 23-36

The minus sign indicates that the assumed direction of $(F_{N/L})_y$ was wrong. Therefore $(F_{N/L})_y$ acts in the negative y direction. Equal and opposite to $F_{N/L}$ is $F_{L/N}$: $(F_{L/N})_x = 0.242$ kN (in positive x direction), $(F_{L/N})_y = 0.027$ kN (in positive y direction).

23.58 In Fig. 23-37 suppose that $\theta = 30°$, $V_1 = 100$ fps, and the stream is a jet of water with an initial diameter of 2 in. Assume friction losses such that $V_2 = 95$ fps. Find the resultant force on the blade. Assume that flow occurs in a horizontal plane.

▮ This problem is best solved by taking a free-body diagram of the element of fluid in contact with the blade. The forces acting on this element are as shown in the sketch. The forces $(F_{B/W})_x$ and $(F_{B/W})_y$ represent the components of force of blade on water in the x and y directions. These forces include shear stresses tangential to the blade as well as pressure forces normal to the blade. $A = 0.0218\ \text{ft}^2$, $-(F_{B/W})_x = \rho Q(V_{2_x} - V_{1_x}) = 1.94(0.0218)(100)[(0.866)(95) - 100] = 4.23(-17.7) = -74.9$ lb. Hence, $(F_{B/W})_x = +74.9$ lb. The plus sign indicates that the assumed direction of $(F_{B/W})_x$ was correct. $+(F_{B/W})_y = \rho Q(V_{2_y} - V_{1_y}) = 4.23[(0.50)(95) - 0] = 201$ lb. Thus the force of the blade on the fluid is the resultant of a 74.9 lb component to the left and a 201-lb component upward in the y direction. Equal and opposite to this is the force of the fluid on the blade (downward and to the right). The resultant force is 215 lb at an angle of 69.6° below the horizontal.

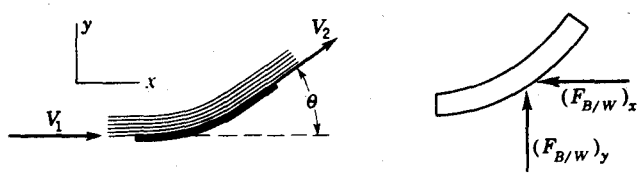


Fig. 23-37

23.59 On the right end of a 9-in diameter pipe is a nozzle which discharges a 3-in-diameter water jet into the air. The pressure in the pipe is 60 psi, and the pipe velocity is 9.9 fps. Compute the axial force on the nozzle and the head loss in the nozzle.

▮

$$\sum F = \rho Q(v_2 - v_1) \qquad Q = A_1 v_1 = [(\pi)(\tfrac{9}{12})^2/4](9.9) = 4.374\ \text{ft}^3/\text{s} \qquad v_2 = (\tfrac{9}{3})^2(9.9) = 89.1\ \text{fps}$$

$$-F + (60)(144)[(\pi)(\tfrac{9}{12})^2/4] = (1.94)(4.374)(89.1 - 9.9) \qquad F = 3145\ \text{lb}$$

$$h_L = p_1/\gamma + v_1^2/2g - v_2^2/2g = (60)(144)/62.4 + 9.9^2/[(2)(32.2)] - 89.1^2/[(2)(32.2)] = 16.7\ \text{ft}$$

23.60 A jet of steam hits the blades of a steam turbine at 3700 fps (absolute speed); the exit speed is 2500 fps. $\alpha_1 = 20°$, $\alpha_2 = 150°$, $u_1 = u_2 = 475$ fps, and $r_1 = r_2 = 0.6$ ft. Find the torque exerted on the rotor and the power delivered to it, if $G = 0.5$ lb/s.

▮ $T = \rho Q(r_1 V_1 \cos\alpha_1 - r_2 V_2 \cos\alpha_2) = (0.5/32.2)[(0.6)(3700)(\cos 20°) - (0.6)(2500)(\cos 150°)] = 52.6$ ft-lb

$$P = FV = T\omega = (52.6)(475/0.6)/550 = 75.7 \text{ hp}$$

23.61 When a turbine runner is held fixed, the discharge under a head of 44 ft is 30 cfs. $\alpha_1 = 35°$, $\beta_2 = 155°$, $r_1 = 0.75$ ft, $r_2 = 0.45$ ft, $A_1 = 0.850$ ft^2, $A_2 = 0.900$ ft^2. Find the zero-speed torque? Neglect shock loss.

▮ $T = \rho Q(r_1 V_1 \cos\alpha_1 - r_2 V_2 \cos\beta_2)$ $\quad \rho Q = (30)(62.4)/32.2 = 58.14$ slugs/s

$$V_1 = Q/A_1 = 30/0.850 = 35.29 \text{ fps} \qquad v_2 = 30/0.900 = 33.33 \text{ fps}$$

For stopped runner, $V_2 = v_2$ and $\alpha_2 = \beta_2 = 155°$. $T = (58.14)[(0.75)(35.29)(\cos 35°) - (0.45)(33.33)(\cos 155°)] = 1274$ ft-lb.

23.62 A 24-in-diameter fan drives air ($\gamma = 0.076$ lb/ft^3) at a rate of 1.90 lb/s. Find the horsepower required to drive the fan. Neglect losses.

▮ $\gamma Q = \gamma A(\Delta V/2) \qquad 1.90 = (0.076)[(\pi)(\frac{24}{12})^2/4](\Delta V/2) \qquad \Delta V = 15.92$ fps

$$F = \rho Q(\Delta V) = (1.90/32.2)(15.92) = 0.939 \text{ lb} \qquad P = FV = F(\Delta V/2) = (0.939)(15.92/2)/550 = 0.0136 \text{ hp}$$

23.63 A fan sucks air ($\rho = 0.0022$ slug/ft^3) into a building through a 20-in-diameter duct. If the pressure difference across the two sides of the fan is 3.9 inH_2O, determine the flow rate of the air. What thrust must the fan support be designed to withstand?

▮ $F = A(\Delta p) = [(\pi)(\frac{20}{12})^2/4](62.4)(3.9/12) = 44.2$ lb $\qquad Q = A(\Delta V/2)$

$$F = \rho Q(\Delta V) = \rho[A(\Delta V/2)](\Delta V) \qquad 44.2 = (0.0022)\{[(\pi)(\tfrac{20}{12})^2/4]/2\}(\Delta V)^2$$

$$\Delta V = 135.7 \text{ fps/s} \qquad Q = [(\pi)(\tfrac{20}{12})^2/4](135.7/2) = 148 \text{ ft}^3/\text{s}$$

23.64 Find the thrust and efficiency of two 6.5-ft-diameter propellers through which flows a total of 20 000 cfs of air (0.072 lb/ft^3). The propellers are attached to an airplane moving at 150 mph through still air. Find also the pressure rise across the propellers and the horsepower input to each propeller. Neglect eddy losses.

▮ Velocity of air relative to airplane is $V_1 = 150$ mph $= (150)(44)/30 = 220$ fps. Velocity of air through the actuating disk is $V = V_1 + (\Delta V/2) = Q/A = (20\,000/2/[(\pi/4)(6.5)^2] = 301$ fps. Thus $\Delta V = 2(301 - 220) = 162$ fps, $F_T = \rho Q\,\Delta V = (0.072/32.2)(20\,000)(162) = 7240$ lb (total thrust of both propellers), $\eta = 1/[1 + \Delta V/2V_1] = 1/(1 + \frac{162}{440}) = 0.73 = 73$ percent. F_T on one propeller $= \frac{7240}{2} = 3620$ lb. But $F_T = (\Delta p)(A)$, thus $3620 = \Delta p(\pi/4)(6.5)^2$, $\Delta p = 109$ psf $= 0.758$ psi, hp/propeller $= \gamma Q(\Delta p/\gamma)/550 = Q(\Delta p)/550 = 10\,000(109)/550 = 1980$. Check: hp/propeller $= F_T(V_1 + \Delta V/2)/550 = 3620(301)/550 = 1980$.

23.65 This water passage (Fig. 23-38) is 10 ft wide normal to the figure. Determine the horizontal force acting on the shaded structure. Assume ideal flow.

▮ In free-surface flow such as this where the streamlines are parallel, the water surface is coincident with the hydraulic grade line. Writing an energy equation from the upstream section to the downstream section,

$$6 + (V_1^2/2g) = 3 + (V_2^2/2g) \tag{1}$$

From continuity,

$$6(10)V_1 = 3(10)V_2 \tag{2}$$

Substituting Eq. (*2*) into Eq. (*1*) yields $V_1 = 8.02$ fps, $V_2 = 16.04$ fps, $Q = A_1V_1 = A_2V_2 = 481$ cfs. Next take a free-body diagram of the element of water shown in the figure $F_1 - F_2 - F_x = \rho Q(V_2 - V_1)$, where F_x represents the force of the structure on the water in the horizontal direction. $F_1 = \gamma h_{c_1} A_1$ and $F_2 = \gamma h_{c_2} A_2$. Hence $62.4(3)(10)(6) - 62.4(1.5)(10)(3) - F_x = 1.94(481)(16.04 - 8.02)$ and $F_x = 940$ lb. The positive sign means that the assumed direction is correct. Hence the force of the water on the structure is equal and opposite, namely, 940 lb to the right.

The momentum principle will not permit one to obtain the vertical component of the force of the water on the shaded structure because the pressure distribution along the bottom of the channel is unknown. The pressure distribution along the boundary of the structure and along the bottom of the channel can be estimated by sketching a flow net and applying Bernoulli's principle. The horizontal and vertical components of the force can be found by computing the integrated effect of the pressure-distribution diagram.

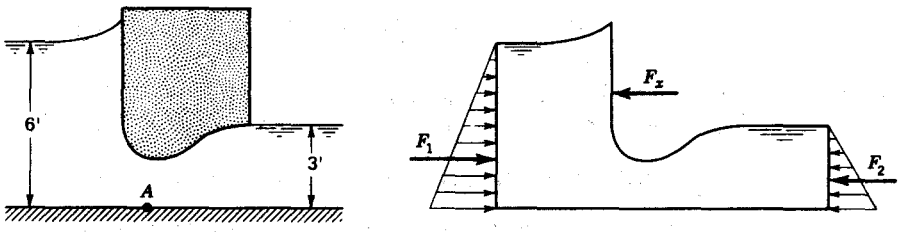


Fig. 23-38

23.66 Flow occurs over a spillway, as shown in Fig. 23-39. Determine the horizontal force on the spillway per foot of width. Assume ideal flow.

▮ $\sum F = \rho Q(v_2 - v_1)$. Energy consideration: $y_1 + v_1^2/2g = y_2 + v_2^2/2g$, $5 + v_1^2/[(2)(32.2)] = 0.9 + v_2^2/[(2)(32.2)]$. Continuity consideration: $y_1 v_1 = y_2 v_2$, $5v_1 = 0.9v_2$, $v_1 = 0.1800v_2$.

$$5 + (0.1800v_2)^2/[(2)(32.2)] = 0.9 + v_2^2/[(2)(32.2)] \qquad v_2 = 16.52 \text{ fps} \qquad v_1 = (0.1800)(16.52) = 2.973 \text{ fps}$$

$$(62.4)(5)(\tfrac{5}{2}) - (62.4)(0.9)(0.9/2) - F_x = (1.94)[(5)(2.973)](16.52 - 2.973) \qquad F_x = 364 \text{ lb/ft} \qquad \text{(rightward)}$$

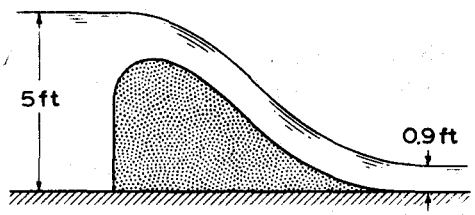


Fig. 23-39

CHAPTER 24
Dynamic Drag and Lift

24.1 A pitcher throws a baseball through air at 50 °F at a velocity of 92 mph. The diameter of the baseball is 2.82 in. Find the drag force on the baseball.

▮ $F_D = C_D\rho(v^2/2)A$ $\quad v = (92)(\frac{5280}{3600}) = 134.9$ ft/s $\quad N_R = Dv/\nu = (2.82/12)(134.9)/(1.52 \times 10^{-4}) = 2.09 \times 10^5$

From Fig. A-50, $C_D = 0.42$. $F_D = (0.42)(0.00242)(134.9^2/2)[(\pi)(2.82/12)^2/4] = 0.401$ lb.

24.2 The pole of Fig. 24-1 is a cylinder 100 mm in diameter. Wind is blowing against it with a velocity of 15 m/s, and air temperature is 30 °C. Find the bending moment about the base (at ground level). Neglect end effects.

▮ $F_D = C_D\rho(v^2/2)A \qquad N_R = Dv/\nu = (0.100)(15)/(1.60 \times 10^{-5}) = 9.38 \times 10^4$

From Fig. A-51, $C_D = 1.3$. $F_D = (1.3)(1.16)(15^2/2)[(17)(0.100)] = 288$ N. Wind acts uniformly along the pole; hence, the resultant drag force acts at the pole's midpoint as shown in Fig. 24-1. The bending moment about the base is, therefore, $M_{base} = (288)(8.5) = 2448$ N · m.

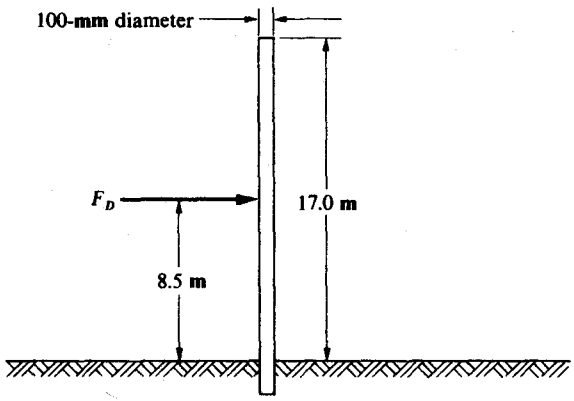


Fig. 24-1

24.3 A flat plate, 0.1 m by 0.1 m, moves at a velocity of 5.0 m/s perpendicular to the plate. Find the drag force on the plate if it is moving through (*a*) still air ($\rho = 1.200$ kg/m^3) and (*b*) still water ($\rho = 1000$ kg/m^3).

▮ $F_D = C_D\rho(v^2/2)A$. From Fig. A-52 with $x/y = 0.1/0.1 = 1$, $C_D = 1.1$.

(*a*) $\qquad F_D = (1.1)(1.200)(5.0^2/2)[(0.1)(0.1)] = 0.165$ N

(*b*) $\qquad F_D = (1.1)(1000)(5.0^2/2)[(0.1)(0.1)] = 138$ N

24.4 A smooth, flat plate 8.0 ft wide and 80.0 ft long is being towed lengthwise (parallel to the plate) through still water at 70 °F at a velocity of 17.0 fps. What is the drag force on one side of the plate (i.e., the skin-friction drag)?

▮ $F_D = C_D\rho(v^2/2)A \qquad N_R = Dv/\nu = (80.0)(17.0)/(1.05 \times 10^{-5}) = 1.30 \times 10^8$

From Fig. A-53, $C_D = 0.0020$. $F_D = (0.0020)(1.93)(17.0^2/2)[(80.0)(8.0)] = 357$ lb.

24.5 A basketball is thrown through air at 50 °F at a velocity of 27 mph. The diameter of the basketball is 9.3 in. Determine the drag force on the basketball.

▮ $F_D = C_D\rho(v^2/2)A$ $\quad v = (27)(\frac{5280}{3600}) = 39.60$ ft/s $\quad N_R = Dv/\nu = (9.3/12)(39.60)/(1.52 \times 10^{-4}) = 2.02 \times 10^5$

From Fig. A-50, $C_D = 0.41$. $F_D = (0.41)(0.00242)(39.60^2/2)[(\pi)(9.3/12)^2/4] = 0.367$ lb.

24.6 A silo with a height of 10 m above ground has the shape of a cylinder 3.5 m in diameter. Wind is blowing against the silo at a velocity of 8.3 km/h when air temperature is 30 °C. Determine the bending moment about the base of the silo.

▮ $F_D = C_D\rho(v^2/2)A$ $\quad v = (8.3)(\frac{1000}{3600}) = 2.306$ m/s $\quad N_R = Dv/\nu = (3.5)(2.306)/(1.60 \times 10^{-5}) = 5.04 \times 10^5$

From Fig. A-51, $C_D = 0.35$. $F_D = (0.35)(1.16)(2.306^2/2)[(3.5)(10)] = 37.8$ N, $M_{base} = (37.8)(\frac{10}{2}) = 189$ N-m.

24.7 A flat plate, 0.5 m by 2.0 m, moves at a velocity of 3.5 m/s perpendicular to the plate. Find the drag force on the plate if it is moving through **(*a*)** still air ($\rho = 1.200$ kg/m^3) and **(*b*)** still water ($\rho = 1000$ kg/m^3).

▮ $F_D = C_D\rho(v^2/2)A$. From Fig. A-52 with $x/y = 2.0/0.5 = 4.0$, $C_D = 1.2$.

(*a*) $F_D = (1.2)(1.200)(3.5^2/2)[(0.5)(2.0)] = 8.82$ N

(*b*) $F_D = (1.2)(1000)(3.5^2/2)[(0.5)(2.0)] = 7350$ N

24.8 Wind with a velocity of 35 mph is blowing perpendicularly against a 14-ft by 10-ft highway sign. If the mass density of the air is 0.00234 slug/ft^3, compute the total drag force on the sign.

▮ $F_D = C_D\rho(v^2/2)A$. From Fig. A-52 with $x/y = 14/10 = 1.4$, $C_D = 1.12$. $v = (35)(\frac{5280}{3600}) = 51.33$ ft/s, $F_D = (1.12)(0.00234)(51.33^2/2)[(14)(10)] = 483$ lb.

24.9 A highway sign is illustrated in Fig. 24-2. Assuming the sign can be analyzed as a flat plate, determine the wind velocity required to overturn the sign if it was designed to resist a total moment of 6000 lb-ft at its base. Use a mass density of air of 0.00234 slug/ft^3.

▮ $F_D = C_D\rho(v^2/2)A$. From Fig. A-52 with $x/y = \frac{15}{9} = 1.67$, $C_D = 1.16$.

$$F_D = (1.16)(0.00234)(v^2/2)[(15)(9)] = 0.1832v^2 \qquad \sum M_{base} = 0$$

$$6000 - (6 + \tfrac{9}{2})(0.1832v^2) = 0 \qquad v = 55.85 \text{ ft/s} \quad \text{or} \quad 38.1 \text{ mph}$$

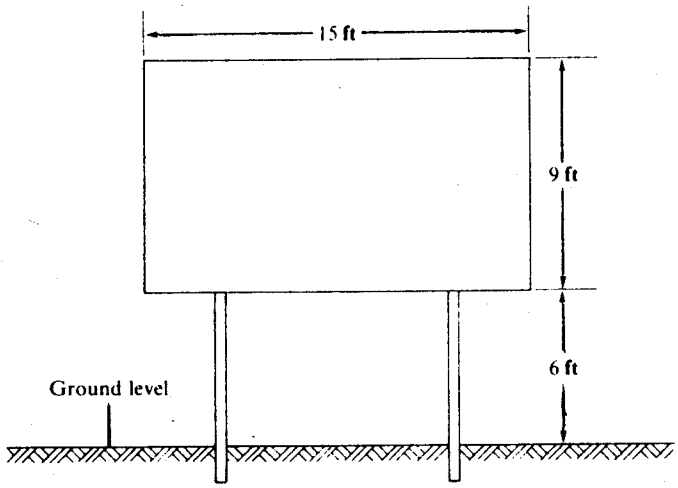


Fig. 24-2

24.10 A smooth, flat plate 2.0 m wide and 10.0 m long is being towed lengthwise through still water at 20 °C at a velocity of 5.0 m/s. What is the drag force on one side of the plate (i.e., the skin-friction drag)?

▮ $F_D = C_D\rho(v^2/2)A$ $\quad N_R = Dv/\nu = (10.0)(5.0)/(1.02 \times 10^{-6}) = 4.90 \times 10^7$

From Fig. A-53, $C_D = 0.0023$. $F_D = (0.0023)(998)(5.0^2/2)[(10.0)(2.0)] = 574$ N.

24.11 Find the friction drag on one side of a smooth flat plate 6 in wide and 18 in long, placed longitudinally in a stream of crude oil (s.g. = 0.925) at 60 °F flowing with undistributed velocity of 2 fps.

▮ From Fig. A-2, $\nu = 0.0010$ ft^2/s. Then, at $x = L$, $N_R = LU/\nu = (1.5)(2)/0.0010 = 3000$, well within the laminar range; that is, $N_R < 500\,000$. $C_f = 1.328/\sqrt{N_R} = 1.328\sqrt{3000} = 0.0242$, $F_f = C_f\rho(V^2/2)BL = (0.0242)(0.925)(1.94)(2^2)(6)(18)/[(2)144] = 0.065$ lb. Find the thickness of the boundary layer and the shear stress at the trailing edge of the plate. $\delta/x = 4.91/\sqrt{N_R} = 4.91/\sqrt{3000} = 0.0896$, $\delta = (0.0896)(1.5) = 0.1344$ ft = 1.61 in. At $x = L$, $\tau_0 = 0.332(\nu\rho U/L)\sqrt{N_R} = (0.332)[(0.0010)(0.925)(1.94)(2)/1.5]\sqrt{3000} = 0.0435$ lb/ft^2.

24.12 (*a*) Find the frictional drag on the top and sides of a box-shaped moving van 8 ft wide, 10 ft high, and 35 ft long, traveling at 60 mph through air ($\gamma = 0.0725$ lb/ft^3) at 50 °F. Assume that the vehicle has a rounded nose so that the flow does not separate from the top and sides (see Fig. A-54*b*). Assume also that even though the top and sides of the van are relatively smooth there is enough roughness so that for all practical purposes a turbulent boundary layer starts immediately at the leading edge. (*b*) Find the thickness of the boundary layer and the shear stress of the trailing edge.

▮ (*a*) From Fig. A-2, for air at 50 °F, $\nu = 0.00015$ ft^2/s. Then $N_R = LU/\nu = (35)(88)/0.00015 = 20\,530\,000$. As $N_R > 10^7$, $C_f = 0.455/(\log N_R)^{2.58} = 0.455/(7.31)^{2.58} = 0.00269$, $F_f = C_f\rho(V^2/2)BL = (0.00269)(0.0725/32.2)[(88)^2/2](10 + 8 + 10)35 = 23.0$ lb.

(*b*) $\delta/x = 0.377/N_R^{1/5}$ $\quad \delta_{35} = (35)(0.377)/[(205.3)^{1/5} \times 10] = 0.455$ ft

$\tau_0 = 0.0587\rho(U^2/2)(\nu/Ux)^{1/5}$ $\quad (\tau_0)_{35} = (0.0587)(0.0725/32.2)(88^2/2)[0.00015/(88)(35)]^{1/5} = 0.0176$ lb/ft^2

24.13 A small submarine, which may be supposed to approximate a cylinder 10 ft in diameter and 50 ft long, travels submerged at 3 knots (5.06 fps) in sea water at 40 °F. Find the friction drag assuming no separation from the sides.

▮ $$F_f = C_f\rho(V^2/2)A \qquad N_F = DV/\nu$$

Kinematic viscosity of sea water ≈ kinematic viscosity of fresh water = 0.0000166 ft^2/s. Then $N_F = (50)(5.06)/0.0000166 = 1.52 \times 10^7$. From Fig. A-55, $C_f = 0.0028$ and $F_f = (0.0028)(64/32.2)[(5.06)^2/2](\pi)(10)(50) = 112$ lb. Find the value of the critical roughness for a point 1 ft from the nose of the submarine.

At $x = 1$ ft, $(N_F) = (5.06)(1)/0.0000166 = 305\,000$, $e_c = 26(\nu/V)(N_R)_x^{1/4} = [(26)(0.0000166)/5.06](305\,000)^{1/4} = 0.0020$ ft. Find the height of roughness at the midsection of the submarine which would class the surface as truly rough.

At $x = 25$ ft, $(N_R)_x = (5.05)(25)/0.0000166 = 7.61 \times 10^6$, $\tau_0/\rho = 0.0587(V^2/2)(\nu/Vx)^{1/5} = (0.0587)[(5.06)^2/2][1/(76.2 \times 10^5)^{1/5}] = 0.0316$ ft^2/s^2, $\delta_t = 60\nu/\sqrt{\tau_0/\rho} = (60)(0.0000166)/\sqrt{0.0316} = 0.0056$ ft.

24.14 Using the data of Prob. 24.12 determine the total drag exerted by the air on the van. Assume that $C_D \approx 0.45$ (see Fig. A-54).

▮ $$F_D = C_D\rho(V^2/2)A = 0.45(0.0725/32.2)[(88)^2/2](8)(10) = 314 \text{ lb}$$

Thus the pressure drag = 314 − 23 = 291 lb; in this case the pressure drag is responsible for about 93 percent of the total drag while the friction drag comprises only 7 percent of the total.

24.15 Find the "free-fall" velocity of an 8.5-in-diameter sphere (bowling ball) weighing 16 lb when falling through the following fluids under the action of gravity: through the standard atmosphere at sea level; through the standard atmosphere at 10 000-ft elevation; through water at 60 °F; through crude oil (s.g. = 0.925) at 60 °F.

▮ When first released the sphere will accelerate (Fig. 24-3*a*) because the forces acting on it are out of balance. This acceleration results in a buildup of velocity which causes an increase in the drag force. After a while the drag force will increase to the point where the forces acting on the sphere are in balance, as indicated in Fig. 24-3*b*. When that point is reached the sphere will attain a constant or terminal (free-fall) velocity. Thus for free-fall conditions, $\sum F_x = W - F_B - F_D = (\text{mass})(\text{acceleration}) = 0$ where W is the weight, F_B the buoyant force, and F_D the drag force. The buoyant force is equal to the unit weight of the fluid multiplied by the volume ($\pi D^3/6 = 0.186$ ft^3) of the sphere. The given data are approximately as follows:

fluid	γ, lb/ft^3	ρ, slugs/ft^3	$\nu = \mu/\rho$ ft^2/s	F_B, lb
Air (sea level)	0.0765	0.00238	1.57×10^{-4}	0.0142
Air (10 000 ft)	0.0564	0.00176	2.01×10^{-4}	0.0105
Water, 60 °F	62.4	1.94	1.22×10^{-5}	11.6
Oil, 60 °F	57.7	1.79	0.001	10.7

The detailed analysis for the sphere falling through the standard sea-level atmosphere is as follows: $16 - 0.0142 - C_D\rho(V^2/2)A = 0$ where $\rho = 0.00238$ slug/ft^3 and $A = \pi(8.5/12)^2/4 = 0.394$ ft^2 or $15.986 = C_D(0.00238)(V^2/2)(0.394) = 0.00047C_DV^2$. A trial-and-error solution is required. Let $C_D = 0.2$, then $V = 412$ fps. $N_R = DV/\nu = (8.5/12)412/(1.57 \times 10^{-4}) = 1.86 \times 10^6$. The values of C_D and N_R check, hence $C_D = 0.2$ and $V = 412$ fps.

Following a similar procedure for the other three fluids gives the following free-fall velocities:

fluid	C_D	N_R	V_{fall}
Standard atmosphere at 10 000 ft	0.20	1.69×10^6	480 fps
Water at 60 °F	0.19	453 000	7.4 fps*
Crude oil (s.g. = 0.925) at 60 °F	0.39	4 390	6.2 fps

* In this instance the Reynolds number is 453 000 which, for the case of a sphere, generally indicates a turbulent boundary layer (Fig. A-56). This is very close to the point where the boundary layer changes from laminar to turbulent. If the water had been at a somewhat lower temperature and, hence, more viscous, a laminar boundary layer might have been present, in which case the free-fall velocity would have been only about 5.2 fps.

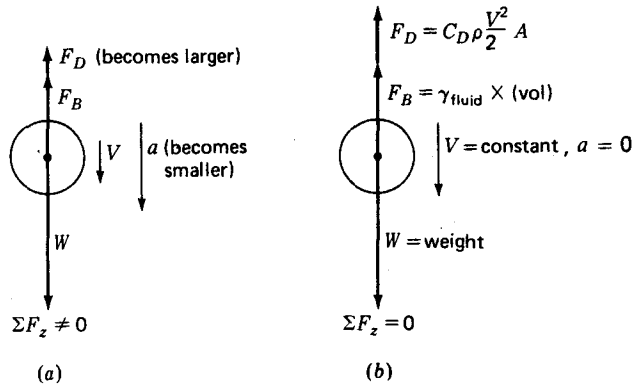


Fig. 24-3

24.16 Find the critical Reynolds number (indicating transition from laminar to turbulent boundary layer) for a circular pipe, given the value 480 000 for a flat plate.

▮ For the plate, $\delta/x = 4.91/\sqrt{(N_R)_x} = 4.91/\sqrt{480\,000} = 0.00709$; therefore, $N_R = 480\,000 = Ux/\nu = U\delta/0.00709\nu$. Imagine the plate rolled up to form a circular pipe of radius R: the critical conditions become $U \rightarrow U_{centerline} = 2V$ (V = cross-sectional average velocity) and $\delta = R = D/2$. Hence, for unchanged ν, $480\,000 = (2V)(D/2)/0.00709\nu$ or $DV/\nu = (N_R)_{pipe} = (480\,000)(0.00709) = 3403$.

24.17 Refer to the data of Prob. 24.12. Find the shear stress on the sides of the van at 4, 12, and 20 ft back from the front edge of the sides.

▮ $\tau_0 = 0.0587\rho(U^2/2)(\nu/Ux)^{1/5}$. At $x = 4$ ft, $\tau_0 = (0.0587)(0.0725/32.2)(88^2/2)\{0.00015/[(88)(4)]\}^{1/5} = 0.0272$ lb/ft^2. At $x = 12$ ft, $\tau_0 = (0.0587)(0.0725/32.2)(88^2/2)\{0.00015/[(88)(12)]\}^{1/5} = 0.0219$ lb/ft^2. At $x = 20$ ft, $\tau_0 = (0.0587)(0.0725/32.2)(88^2/2)\{0.00015/[(88)(20)]\}^{1/5} = 0.0197$ lb/ft^2.

24.18 If the wind velocity 6 ft above a soccer field is 18 fps, approximately what are the wind velocities 0.4 ft and 0.8 ft above the field?

▮ $u/U = (y/\delta)^{1/7}$. At 0.4 ft, $u/18 = (0.4/6)^{1/7}$, $u = 12.2$ fps. At 0.8 ft, $u/18 = (0.8/6)^{1/7}$, $u = 13.5$ fps.

24.19 A metal ball of diameter 1.2 ft and weight 99 lb is dropped into the ocean. Determine the maximum velocity the ball will achieve if, for seawater, $\rho = 2.0$ slugs/ft^3 and $\mu = 3.3 \times 10^{-5}$ lb-s/ft^2.

▮
$$F_D = W - F_b = C_D\rho(V^2/2)A \qquad W - (\gamma)(\pi D^3/6) = C_D\rho(V^2/2)(\pi D^2/4)$$
$$99 - [(62.4)(2.0/1.94)](\pi)(1.2)^3/6 = C_D(2.0)(V^2/2)(\pi)(1.2)^2/4 \qquad 99 - 58.20 = 1.131C_DV^2 \qquad C_DV^2 = 36.07$$
$$N_R = DV\rho/\mu = (1.2)(V)(2.0)/(3.3 \times 10^{-5}) = 72\,727V$$

Try $C_D = 0.4$; $0.4V^2 = 36.07$, $V = 9.496$ ft/s, $N_R = (72\,727)(9.496) = 6.91 \times 10^5$. From Fig. A-56, $C_D = 0.20$. Try $C_D = 0.20$: $0.20V^2 = 36.07$, $V = 13.43$ ft/s, $N_R = (72\,727)(13.43) = 9.77 \times 10^5$, $C_D = 0.20$ (O.K.). Hence, $V = 13.43$ fps.

24.20 Two 5-ft-diameter bodies—the airship hull of Fig. A-56 and a 4:3 oblate ellipsoid—move at 80.7 ft/s through air at STP. For each body, find the power required to overcome air resistance.

▮ $P = F_D V \qquad F_D = C_D \rho (V^2/2)A \qquad N_R = DV/\nu = (5)(80.7)/0.0001564 = 2.58 \times 10^6$

Assuming from Fig. A-56 that C_D remains constant for $N_R > 10^6$, C_D for airship hull = 0.04 and C_D for 1:0.75 ellipsoid = 0.20. Thus, $(F_D)_{\text{hull}} = (0.04)(0.002378)(80.7^2/2)[(\pi)(5)^2/4] = 6.08$ lb, $P_{\text{hull}} = (6.08)(80.7)/550 = 0.892$ hp, and $(F_D)_{\text{ellip.}} = (0.20)(0.002378)(80.7^2/2)[(\pi)(5)^2/4] = 30.4$ lb, $(P)_{\text{ellip.}} = (30.4)(80.7)/550 = 4.46$ hp.

24.21 The drag coefficient for a hemispherical shell with the concave side upstream is approximately 1.33 if $N_R > 1000$. Design a parachute that will land a 100-kg Marine as though he had jumped (parachuteless) off a 3.0-m wall. Assume standard air at sea level and neglect the parachute's weight.

▮ In jump from 3.0-m wall,

$$V = \sqrt{2gH} = \sqrt{(2)(9.80)(3.0)} = 7.67 \text{ m/s} \qquad F_D = C_D \rho (V^2/2)A$$

$$(100)(9.80) = (1.33)(1.225)(7.67^2/2)(\pi D^2/4) \qquad D = 5.10 \text{ m}$$

$$N_R = DV/\nu = (5.10)(7.67)/(1.455 \times 10^{-5}) = 2.69 \times 10^6 \qquad \text{(O.K.)}$$

24.22 A football, 6.78 in in diameter and 14.5 oz in weight, can be modeled by the prolate ellipsoid of Fig. A-56. Find the resistance when the ball is passed through still air (14.7 psia and 40 °F) at a velocity of 35 fps. Neglect spin.

▮ $F_D = C_D \rho (V^2/2)A \qquad N_R = DV/\nu = (6.78/12)(35)/(1.46 \times 10^{-4}) = 1.35 \times 10^5$

From Fig. A-56, $C_D = 0.065$, $F_D = (0.065)(0.00247)(35^2/2)[(\pi)(6.78/12)^2/4] = 0.0247$ lb.

24.23 Repeat Prob. 24.22, if the air temperature is 100 °F.

▮ $F_D = C_D \rho (V^2/2)A$. At $T = 100$ °F, $\rho = 0.00220$ slug/ft^3 (from Table A-4). $F_D = (0.065)(0.00220)(35^2/2)[(\pi)(6.78/12)^2/4] = 0.0220$ lb. [The air is rarer at the higher temperature.]

24.24 At sea level, a 3.2-m-diameter spherical balloon travels at 22 m/s. At a 3 km altitude, what speed will give the same drag force? Let C_D be the same at both altitudes.

▮ $F_D = C_D \rho (V^2/2)A$, whereby $V \propto 1/\sqrt{\rho}$. Hence

$$V_3 = \sqrt{\frac{\rho_0}{\rho_3}}\, V_0 = \sqrt{\frac{1.225}{0.958}}\,(22) = 24.9 \text{ m/s}$$

24.25 Find the drag force on a radio antenna 0.33 in in diameter extended to 5 ft in length on an automobile traveling through 80 °F air at 90 mph.

▮ $F_D = C_D \rho (V^2/2)A \qquad N_R = DV/\nu = (0.33/12)[\frac{90}{60}(88)]/(1.69 \times 10^{-4}) = 2.15 \times 10^4$

From Fig. A-57, $C_D = 1.1$.

$$F_D = (1.1)(0.00228)\{[\tfrac{90}{60}(88)]^2/2\}[(0.33/12)(5)] = 3.00 \text{ lb}$$

24.26 A 75-mph (110-fps) wind blows across a 0.100-in-diameter wire at standard sea level ($\nu = 1.57 \times 10^{-4}$ ft^2/s). What note (approximately) does the wire "sing"?

▮ $f \approx 0.20(V/D)(1 - 20/N_R) \qquad N_R = DV/\nu$

$$N_R = (0.100/12)(110)/(1.57 \times 10^{-4}) = 5839 \qquad f = (0.20)[110/(0.100/12)](1 - \tfrac{20}{5839}) = 2631 \text{ Hz}$$

On the piano, this would be an E (third octave above middle C).

24.27 Determine the deceleration experienced by the blunt-nosed projectile of Fig. A-58 when it is moving horizontally at 950 mph (1393 fps). Assume standard sea-level atmosphere. The projectile has a diameter of 20 in and it weighs 550 lb.

▮ $F_D = C_D\rho(V^2/2)A$. At sea level, $c = 1116$ fps. $V/c = \frac{1393}{1116} = 1.25$. From Fig. A-58, $C_D = 1.22$.

$$F_D = (1.22)(0.002377)(1393^2/2)[(\pi)(\tfrac{20}{12})^2/4] = 6138 \text{ lb}$$

$$F_D = -a(W/g) \qquad 6138 = -a(550/32.2) \qquad a = -359 \text{ ft/s}^2$$

24.28 Solve Prob. 24.27 if the projectile is moving upward at an angle of 35° with the horizontal.

▮ In this case, $F_D - W\sin\theta = -a(W/g)$, $6138 - 550\sin 35° = -a(550/32.2)$, $a = -341$ ft/s^2.

24.29 A sharp flat plate with $L = 1$ m and $b = 3$ m is immersed parallel to a stream of velocity 2 m/s. Find the drag on one side of the plate, and at the trailing edge find the thicknesses δ, δ^*, and θ for air ($\rho = 1.23$ kg/m^3 and $\nu = 1.46 \times 10^{-5}$ m^2/s).

▮ The airflow Reynolds number is $VL/\nu = (2.0)(1.0)/(1.46 \times 10^{-5}) = 137\,000$. Since this is less than 3×10^6, we assume that the boundary layer is laminar and the drag coefficient is $C_D = 1.328/N_R^{1/2} = 1.328/(137\,000)^{1/2} = 0.00359$. Thus the drag on one side in the airflow is $D = C_D\frac{1}{2}\rho U^2 bL = 0.00359(\frac{1}{2})(1.23)(2.0)^2(3.0)(1.0) = 0.0265$ N. The boundary-layer thickness at the end of the plate is $\delta/L = 5.0/N_R^{1/2} = 5.0/(137\,000)^{1/2} = 0.0135$ or $\delta = 0.0135(1.0) = 0.0135$ m $= 13.5$ mm. We find the other two thicknesses by proportion: $\delta^* = (1.721/5.0)\delta = 4.65$ mm, $\theta = \delta^*/2.59 = 1.80$ mm.

24.30 Solve Prob. 24.29 for water ($\rho = 1000$ kg/m^3 and $\nu = 1.02 \times 10^{-6}$ m^2/s).

▮ The water Reynolds number is $N_R = 2.0(1.0)/(1.02 \times 10^{-6}) = 1.96 \times 10^6$. This is rather close to the critical value of 3×10^6, so that a rough surface or noisy free stream might trigger transition to turbulence; but let us assume that the flow is laminar. The water drag coefficient is $C_D = 1.328/(1.96 \times 10^6)^{1/2} = 0.000949$ and $D = 0.000949(\frac{1}{2})(1000)(2.0)^2(3.0)(1.0) = 5.69$ N. The drag is 215 times more for water in spite of the higher Reynolds number and lower drag coefficient because water is 57 times more viscous and 813 times denser than air.

The boundary-layer thickness is given by $\delta/L = 5.0/(1.96 \times 10^6)^{1/2} = 0.00357$ or $\delta = 0.00357(1000 \text{ mm}) = 3.57$ mm. By ratioing down we have $\delta^* = (1.721/5.0)\delta = 1.23$ mm, $\theta = \delta^*/2.59 = 0.47$ mm. The water layer is 3.8 times thinner than the air layer, which reflects the square root of the 14.3 ratio of air to water kinematic viscosity.

24.31 A hydrofoil 1.2 ft long and 6 ft wide is placed in a water flow of 40 fps, with $\rho = 1.99$ slugs/ft^3 and $\nu = 0.000011$ ft^2/s. Estimate the boundary-layer thickness at the end of the plate.

▮ The Reynolds number is $N_R = UL/\nu = (40)(1.2)/0.000011 = 4.36 \times 10^6$. Thus the trailing-edge flow is certainly turbulent. The maximum boundary-layer thickness would occur for turbulent flow starting at the leading edge. $\delta/L = 0.16/N_F^{1/7} = 0.16/(4.36 \times 10^6)^{1/7} = 0.018$ or $\delta = 0.018(1.2 \text{ ft}) = 0.0216$ ft.

24.32 For the data in Prob. 24.31, estimate the friction drag for turbulent smooth-wall flow from the leading edge.

▮ This is 7.5 times thicker than a fully laminar boundary layer at the same Reynolds number. For fully turbulent smooth-wall flow, the drag coefficient on one side of the plate is $C_D = 0.031/N_F^{1/7} = 0.031/(4.36 \times 10^6)^{1/7} = 0.00349$. Then the drag on both sides of the foil is approximately $D = 2C_D(\frac{1}{2}\rho U^2)bL = 2(0.00349)(\frac{1}{2})(1.99)(40)^2(6.0)(1.2) = 80.0$ lb.

24.33 Solve Prob. 24.32 for laminar-turbulent flow with $(N_R)_{\text{trans}} = 5 \times 10^5$.

▮ With a laminar leading edge and $(N_R)_{\text{trans}} = 5 \times 10^5$, $C_D = (0.031/N_R^{1/7}) - (1440/N_R) = 0.00349 - [1440/(4.36 \times 10^6)] = 0.00316$. The drag can be recomputed for this lower drag coefficient: $D = 2C_D(\frac{1}{2}\rho U^2)bL = 72.4$ lb.

24.34 Solve Prob. 24.32 for turbulent rough-wall flow with $\epsilon = 0.0004$ ft.

▮ For the rough wall, we calculate $L/\epsilon = 1.2/0.0004 = 3000$. From Fig. A-59 at $N_R = 4.36 \times 10^6$, this condition is just inside the fully rough regime. $C_D \approx [1.89 + 1.62\log(L/\epsilon)]^{-2.5} = (1.89 + 1.62\log 3000)^{-2.5} = 0.00644$ and the drag estimate is $D = 2C_D(\frac{1}{2}\rho U^2)bL = 148$ lb. This small roughness nearly doubles the drag. It is probable that the total hydrofoil drag is still another factor of 2 larger because of trailing-edge flow-separation effects.

24.35 A square 6-in piling is acted on by a water flow of 5 ft/s 20 ft deep, as shown in Fig. 24-4. Estimate the maximum bending exerted by the flow on the bottom of the piling.

▮ Assume sea water with $\rho = 1.99$ slugs/ft^3 and kinematic viscosity $\nu = 0.000011$ ft^2/s. With piling width of 0.5 ft, we have $N_R = VD/\nu = (5)(0.5)/0.000011 = 2.3 \times 10^5$. This is the range where Table A-20 applies. The worst case occurs when the flow strikes the flat side of the piling, $C_D \approx 2.1$. The frontal area is $A = Lh = (20)(0.5) = 10$ ft^2. The drag is estimated by $F = C_D(\frac{1}{2}\rho V^2 A) \approx 2.1(\frac{1}{2})(1.99)(5)^2(10) = 522$ lb. If the flow is uniform, the center of this force should be at approximately middepth. Therefore the bottom bending moment is $M_0 \approx FL/2 = 522(\frac{20}{2}) = 5220$ ft-lb. According to the flexure formula from strength of materials, the bending stress at the bottom would be $S = M_0 y/I = (5220)(0.25)/[\frac{1}{12}(0.5)^4] = 251\,000$ lb/ft^2 = 1740 lb/in^2 to be multiplied, of course, by the stress-concentration factor due to the built-in end conditions.

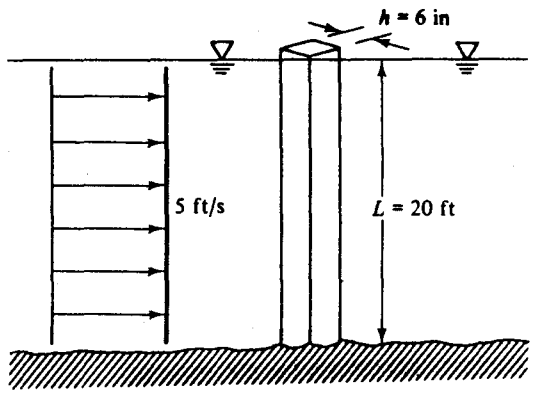


Fig. 24-4

24.36 A high-speed car with $m = 2000$ kg, $C_D = 0.3$, and $A = 1$ m^2, deploys a 2-m parachute to slow down from an initial velocity of 100 m/s (Fig. 24-5). Assuming constant C_D, brakes free, and no rolling resistance, calculate the distance and velocity of the car after 1, 10, 100, and 1000 s. For air, assume $\rho = 1.2$ kg/m^3 and neglect interference between the wake of the car and the parachute.

▮ Newton's law applied in the direction of motion gives $F_x = m(dV/dt) = -F_c - F_p = -\frac{1}{2}\rho V^2(C_{Dc}A_c + C_{Dp}A_p)$ where subscript c is the car and subscript p the parachute. This is of the form $dV/dt = -(K/m)V^2$, $K = \sum C_D A(\rho/2)$. Separate the variables and integrate

$$\int_{V_0}^{V} \frac{dV}{V^2} = -\frac{K}{m}\int_0^t dt$$

or $V_0^{-1} - V^{-1} = -(K/m)t$. Rearrange and solve for the velocity V: $V = V_0/[1 + (K/m)V_0 t]$, $K = (C_{Dc}A_c + C_{Dp}A_p)\rho/2$. We can integrate this to find the distance traveled; $S = (V_0/\alpha)\ln(1 + \alpha t)$, $\alpha = (K/m)V_0$. Now work out some numbers. From Table A-21, $C_{Dp} \approx 1.2$; hence $C_{Dp}A_c + C_{Dp}A_p = 0.3(1) + 1.2(\pi/4)(2)^2 = 4.07$ m^2. Then $(K/m)V_0 = \frac{1}{2}(4.07)(1.2)(100)/2000 = 0.122$ s$^{-1} = \alpha$. Now make a table of the results for V and S

t, s	1	10	100	1000
V, m/s	89	45	7.6	0.8
S, m	94	654	2100	3940

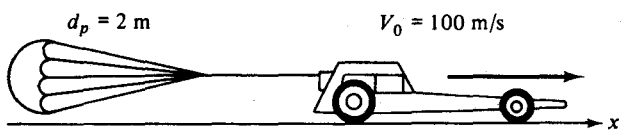


Fig. 24-5

24.37 A thin flat plate, 0.5 m by 1.0 m, is immersed in a stream of glycerin at 20 °C and a velocity of 6 m/s. Compute the viscous drag if the stream is parallel to the short side.

▮ $F_D = C_D\rho(V^2/2)A$ $\quad N_R = \rho DV/\mu = (1258)(0.5)(6)/1.49 = 2533$ (laminar)

$C_D = 1.328/\sqrt{N_R} = 1.328/\sqrt{2533} = 0.0264$ $\quad (F_D)_{\text{total}} = (2)(0.0264)(1258)(6^2/2)[(1.0)(0.5)] = 598$ N

24.38 Solve Prob. 24.37 if the stream is parallel to the long side.

▮ $$F_D = C_D\rho(V^2/2)A \qquad N_R = \rho DV/\mu = (1258)(1.0)(6)/1.49 = 5066$$
$$C_D = 1.328/\sqrt{N_R} = 1.328/\sqrt{5066} = 0.0187 \qquad (F_D)_{\text{total}} = (2)(0.0187)(1258)(6^2/2)[(1.0)(0.5)] = 423\text{ N}$$

24.39 A ship is 180 m long and has a wetted area of 8000 m^2. Estimate the power expended against drag when the ship moves at 3.86 m/s in sea water at 20 °C.

▮ $$P = F_DV \qquad F_D = C_D\rho(V^2/2)A \qquad C_D = 0.031/N_R^{1/7}$$
$$N_R = \rho DV/\mu = (1028)(180)(3.86)/(1.07\times10^{-3}) = 6.68\times10^8$$
$$C_D = 0.031/(6.68\times10^8)^{1/7} = 0.00170 \qquad F_D = (0.00170)(1028)(3.86^2/2)(8000) = 104\,154\text{ N}$$
$$P = (104\,154)(3.86) = 0.402\text{ MW}$$

24.40 A blimp has the form of an ellipsoid of revolution, $L = 225$ ft long and $D = 45$ ft in diameter. Estimate its skin-friction drag when the blimp moves at 55 mph through still air at 68 °F and 12 psia ($\rho = 0.001906$ slug/ft^3; $\mu = 3.76\times10^{-7}$ slug/ft-s).

▮ $$F_D = C_D\rho(V^2/2)A \qquad C_D = 0.031/N_R^{1/7}$$
$$N_R = \rho LV/\mu = (0.001906)(225)(\tfrac{55}{60}\times88)/(3.76\times10^{-7}) = 9.20\times10^7 \qquad C_D = 0.031/(9.20\times10^7)^{1/7} = 0.00226$$

By calculus, $A = (\pi LD/2)[\lambda + (\sin^{-1}\sqrt{1-\lambda^2})/\sqrt{1-\lambda^2}]$, where $\lambda = D/L = 1/5$. For an estimate, neglect λ^2:

$$A \approx \frac{\pi(225)(45)}{2}\left[\frac{1}{5}+\frac{\pi}{2}\right] \approx 28\,000\text{ ft}^2 \qquad F_D \approx (0.00226)(0.001906)\frac{(\frac{55}{60}\times88)^2}{2}(28\,000) = 392\text{ lb}$$

24.41 A hydrofoil 750 mm long and 3.2 m wide skims at 15 m/s. Using flat-plate theory, estimate the drag if $(N_R)_{\text{trans}} = 5\times10^5$ for a smooth bottom.

▮ $$F_D = C_D\rho(V^2/2)A \qquad C_D = 0.031/N_R^{1/7} - 1440/N_R$$
$$N_R = \rho DV/\mu = (998)(0.750)(15)/(1.02\times10^{-3}) = 1.10\times10^7 \qquad \text{(turbulent)}$$
$$C_D = 0.031/(1.10\times10^7)^{1/7} - 1440/(1.10\times10^7) = 0.00293$$
$$(F_D)_{\text{total}} = (2)(0.00293)(998)(15^2/2)[(0.750)(3.2)] = 1579\text{ N}$$

24.42 Solve Prob. 24.41 for a rough bottom ($\epsilon = 0.1$ mm).

▮ $$F_D = C_D\rho(V^2/2)A \qquad L/\epsilon = 750/0.1 = 7500 \qquad \text{(fully rough)}$$
$$C_D = [1.89 + 1.62\log(L/\epsilon)]^{-2.5} = (1.89 + 1.62\log 7500)^{-2.5} = 0.00525$$
$$(F_D)_{\text{total}} = (2)(0.00525)(998)(15^2/2)[(0.750)(3.2)] = 2829\text{ N}$$

24.43 An airplane travels at 230 m/s at an altitude of 9 km. It has a smooth wing measuring 6.5 m by 50 m (transverse dimension). Estimate the power required to overcome friction drag.

▮ $$P = F_DV \qquad F_D = C_D\rho(V^2/2)A \qquad C_D = 0.031/N_R^{1/7}$$
$$N_R = \rho LV/\mu = (0.470)(6.5)(230)/(1.50\times10^{-5}) = 4.68\times10^7 \qquad C_D = 0.031/(4.68\times10^7)^{1/7} = 0.00249$$
$$(F_D)_{\text{total}} = (2)(0.00249)(0.470)(230^2/2)[(6.5)(50)] = 0.02012\text{ MN}$$
$$P = (0.02012)(230) = 4.628\text{ MW}$$

24.44 If, in Prob. 24.43, the wing requires 14 MW to overcome friction, estimate the wing roughness ϵ.

▮ $$P = F_DV \qquad 14\times10^6 = (F_D)(230) \qquad F_D = 60\,870\text{ N} = C_D\rho(V^2/2)A$$
$$60\,870/2 = (C_D)(0.470)(230^2/2)[(6.5)(50)] \qquad C_D = 0.00753$$

From Fig. A-59, $L/\epsilon = 1750$. $\epsilon = \frac{6.5}{1750} = 0.00371$ m, or 3.71 mm.

24.45 A flat barge 12 m wide and 35 m long moves at 2 knots (1.029 m/s) in sea water at 20 °C. Estimate the friction drag if the bottom of the barge has roughness height 4 mm.

▮ $$F_D = C_D\rho(V^2/2)A \qquad L/\epsilon = 35/0.004 = 8750.$$

From Fig. A-59, $C_D = 0.0052$; $F_D = (0.0052)(1028)(1.029^2/2)[(12)(35)] = 1189$ N.

24.46 A shark 60 cm in diameter and 5.5 m long swims at 24.69 m/s in sea water at 20 °C. Estimate the power required to overcome friction drag if $(N_R)_{crit} = 5 \times 10^5$ and $\epsilon = 0.5$ mm.

▮ $F_D = C_D\rho(V^2/2)A$ $\quad N_R = \rho DV/\mu = (1028)(5.5)(24.69)/(1.07 \times 10^{-3}) = 1.30 \times 10^8$ (roughness prevails)

$$L/\epsilon = 5.5/5 \times 10^{-4} = 11\,000$$

From Fig. A-59, $C_D = 0.00490$. $F_D = (0.00490)(1028)(24.69^2/2)[(\pi)(0.60)(5.5)] = 15.92$ kN.

24.47 A smokestack 2.2 m in diameter and 42 m high is subjected to a wind of 22 m/s. Estimate the drag force and the bending moment about the base ($\rho_{air} = 1.2255$ kg/m^3 and $\mu_{air} = 1.78 \times 10^{-5}$ Pa · s).

▮ $F_D = C_D\rho(V^2/2)A$ $\quad N_R = \rho DV/\mu = (1.2255)(2.2)(22)/(1.78 \times 10^{-5}) = 3.33 \times 10^6$

Turbulent: $C_D \approx 0.3$. $F_D = (0.3)(1.2255)(22^2/2)[(2.2)(42)] = 8221$ N, $M_{base} = (8221)(\frac{42}{2}) = 173$ kN · m.

24.48 A log 2.2 m in diameter and 22 m long is towed at a constant speed of 3 m/s in fresh water at 20 °C. Find the horsepower required if the log is towed lengthwise.

▮ $F_D = C_D\rho(V^2/2)A$ $\quad N_R = \rho DV/\mu = (998)(22)(3)/(1.02 \times 10^{-3}) = 6.46 \times 10^7$

From Table A-21, $C_D \approx 1.05$ at $L/D \approx 10$. $F_D = (1.05)(998)(3^2/2)[(\pi)(2.2)^2/4] = 17.925$ kN. $P = (17.925)(3) = 53.78$ kW $= 72$ hp.

24.49 A seine consists of 1-mm-diameter cord knotted into 20-mm squares. Estimate the drag of 1 m^2 of such a net when towed normal to its plane at 3.5 m/s in sea water.

▮ $F_D = C_D\rho(V^2/2)A$ $\quad N_R = \rho DV/\mu = (1028)(0.001)(3.5)/(1.07 \times 10^{-3}) = 3.36 \times 10^3$

From Fig. A-60*a*, $C_D \approx 1.0$. For a single 20-mm side: $F_D = (1.0)(1028)(3.5^2/2)[(0.020)(0.001)] = 0.126$ N. Since 1 m^2 of net contains 5000 sides, $F_D = (0.126)(5000) = 630$ N.

24.50 A filter may be idealized as an array of cylindrical fibers normal to the flow, as in Fig. 24-6*a*. Assuming that the fibers are uniformly distributed and have drag coefficients given by Fig. A-61*a* derive an approximate expression for the pressure drop Δp through a filter of thickness L.

▮ Let N = number of fibers per unit area of filter, D = fiber diameter, b = filter width, H = filter height, and L = filter length (see Fig. 24-6*b*). Each fiber has drag of $F_D = C_D\rho(V^2/2)Db$. Force balance on filter: $\Delta pHb = \sum F_{fibers} = NHLC_D\rho(V^2/2)Db$, $\Delta p = NLC_D\rho(V^2/2)D$.

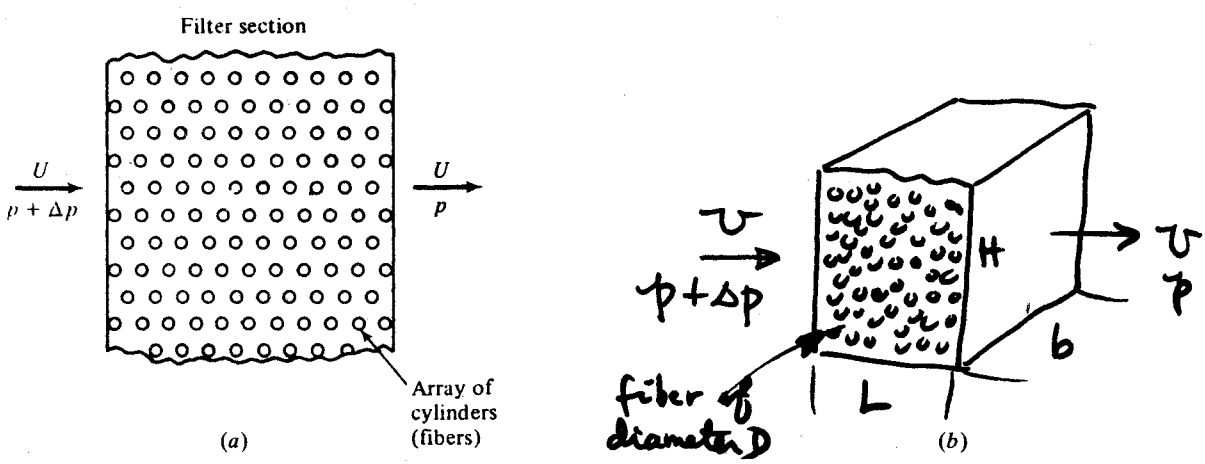


Fig. 24-6

24.51 Apply Prob. 24.50 to a filter consisting of 0.25-mm-diameter fibers packed 9/mm^2 section in the plane of Fig. 24-6*a*. For sea-level standard air flowing at 1.5 m/s, estimate the pressure drop through a filter 35 mm thick.

▮ $\Delta p = NLC_D\rho(V^2/2)D$ $\quad N = 9 \times 10^6/\text{m}^2$

$$N_R = \rho DV/\mu = (1.2255)(0.25 \times 10^{-3})(1.5)/(1.78 \times 10^{-5}) = 25.8$$

From Fig. A-61*a*, $C_D \approx 1.0$. $\Delta p = (9 \times 10^6)(0.035)(1.0)(1.2255)(1.5^2/2)(0.25 \times 10^{-3}) = 109$ Pa.

24.52 A 1-μm-diameter volcanic dust particle has a specific weight of about 26.2 kN/m^3. Approximately how long will it take the particle to reach the earth from an altitude of 8 km?

▮ By *Stokes' law*, the velocity of slow fall, v, satisfies $F_D = W_{net} = 3\pi\mu Dv$. Assume average values of $\gamma_{air} = 8.6$ N/m^3 and $\mu_{air} = 1.7 \times 10^{-5}$ Pa · s.

$$W = \gamma V \qquad W_{net} = (26\,200 - 8.6)(\pi D^3/6) = 13\,714D^3$$

$$13\,714D^3 = (3)(\pi)(1.7 \times 10^{-5})(D)(u) \qquad u = 8.56 \times 10^7 D^2 = 8.56 \times 10^{-5}\ \text{m/s}$$

$$t = (8 \times 10^3)/(8.56 \times 10^{-5}) = 9.35 \times 10^7\ \text{s} \quad \text{or} \quad 1082\ \text{days}$$

24.53 A radar installation is approximated by a 16-m-diameter sphere atop a 1.25-m-diameter, 22-m-tall pole. Estimate the bending moment at the base during hurricane winds of 100 mph. Use $\rho_{air} = 1.2255$ kg/m^3.

▮ $F = C_D\rho(V^2/2)A$. Estimate $C_D = 0.2$ for sphere and 0.3 for rod; 100 mph = 45 m/s.

$$F_{sphere} = (0.2)(1.2255)(45^2/2)[(\pi)(16^2)/4] = 49.90\ \text{kN} \qquad F_{rod} = (0.3)(1.2255)(45^2/2)[(1.25)(22)] = 10.24\ \text{kN}$$

$$M_{base} = (49.90)(22 + \tfrac{16}{2}) + (10.24)(\tfrac{22}{2}) = 1600\ \text{kN} \cdot \text{m}$$

24.54 A boxcar measures 10 ft high, 40 ft long, and 6 ft wide; it stands 3 ft above tracks 4.8 ft apart and has gross weight 45 000 lb. What hurricane wind would topple the boxcar?

▮ $F_D = C_D\rho(V^2/2)A$. From Table A-21, $C_D = 1.2$; and the area is that of one side of the boxcar; hence, $F_D = (1.2)(0.00237)(V^2/2)[(10)(40)] = 0.5688V^2$, $M_{overturning} = (0.5688V^2)(3 + \frac{10}{2}) = 4.550V^2$, $M_{righting} = (45\,000)(4.8/2) = 108\,000$ lb-ft, $4.550V^2 = 108\,000$, $V = 154.1$ ft/s, or 105 mph.

24.55 A long copper wire, 0.5 in in diameter, is stretched taut and is exposed to a wind of velocity 90.0 fps normal to the wire. Compute the drag force per foot of length.

▮ $$F_D = C_D\rho(V^2/2)A \qquad N_R = DV/\nu = (0.5/12)(90.0)/(1.6 \times 10^{-4}) = 2.34 \times 10^4$$

From Fig. A-62, $C_D = 1.30$. $F_D = (1.30)(0.00233)(90.0^2/2)[(0.5/12)(1)] = 0.511$ lb per foot.

24.56 Consider the area on one side of a moving van to be 600 ft^2. Determine the resultant force acting on the side of the van when the wind is blowing at 10 mph normal to the area **(*a*)** when the van is at rest and **(*b*)** when the van is moving at 30 mph normal to the direction of the wind. In **(*a*)** use $C_D = 1.30$, and in **(*b*)** use $C_D = 0.25$ and $C_L = 0.60$. ($\rho = 0.00237$ slug/ft^3)

▮ **(*a*)** The force acting normal to the area $= C_D(\rho/2)AV^2$. Then Resultant force $= 1.30(0.00237/2)(600)[(10)(\frac{5280}{3600})]^2 = 199$ lb normal to area.

(*b*) It will be necessary to calculate the relative velocity of the wind with respect to the van. From kinetic mechanics, $V_{wind} = V_{wind/van} \leftrightarrow V_{van}$.

Figure 24-7 indicates this vector relationship, i.e., $OB = OA \leftrightarrow AB = 30.0 \leftrightarrow V_{w/v}$. Thus the relative velocity $= \sqrt{(30)^2 + (10)^2} = 31.6$ mph to the right and upward at an angle $\theta = \tan^{-1}\frac{10}{30} = 18.4°$.

The component of the resultant force normal to the relative velocity of wind with respect to van is Lift force $= C_L(\rho/2)AV^2 = 0.60(0.00237/2)(600)[(31.6)(\frac{5280}{3600})]^2 = 916$ lb normal to the relative velocity. The component of the resultant force parallel to the relative motion of wind to van is Drag force $= C_D(\rho/2)AV^2 = 0.25(0.00237/2)(600)[(31.6)(\frac{5280}{3600})]^2 = 382$ lb parallel to the relative velocity.

Referring to Fig. 24-7*b*, the resultant force $= \sqrt{(916)^2 + (382)^2} = 992$ lb at an angle $\alpha = \tan^{-1}\frac{916}{382} = 67.4°$. Hence the angle with the longitudinal axis (X axis) is $18.4° + 67.4° = 85.8°$.

24.57 A man weighing 170 lb is descending from an airplane using an 18-ft-diameter parachute. Assuming a drag coefficient of 1.00 and neglecting the weight of the parachute, what maximum terminal velocity will be attained?

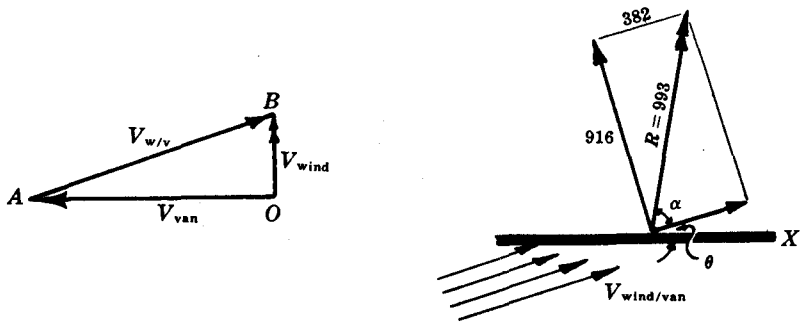
▮ The forces on the parachute are the weight down and the drag force up. $F_D = C_D\rho(V^2/2)A$, $170 = (1.00)(0.00237)(V^2/2)[(\pi)(18)^2/4]$, $V = 23.7$ ft/s.

24.58 A 1-in-diameter sphere of lead weighing 710 lb/ft^3 is moving downward in an oil at a constant velocity of 1.17 fps. Calculate the absolute viscosity of the oil if its specific gravity is 0.93.

▮ $$F_D = (\gamma_{lead} - \gamma_{oil})(V_{lead}) = C_D\rho(V^2/2)A$$

$$[710 - (0.93)(62.4)][(\pi)(\tfrac{1}{12})^3/6] = C_D[(1.94)(0.93)](1.17^2/2)[(\pi)(\tfrac{1}{12})^2/4] \qquad C_D = 29.3$$

From Fig. A-62, $N_R = 0.85$ for $C_D = 29.3$. $N_R = \rho DV/\mu$, $0.85 = [(1.94)(0.93)](\frac{1}{12})(1.17)/\mu$, $\mu = 0.207$ lb-s/ft^2.



(a) (b) **Fig. 24-7**

24.59 A sphere, 0.5 in in diameter, rises in oil at the maximum velocity of 0.12 fps. What is the specific weight of the sphere if the density of the oil is 1.78 slugs/ft^3 and the absolute viscosity is 0.000710 lb-s/ft^2?

▮ $F_D = F_b - W = C_D\rho(V^2/2)(A)$ $\quad N_R = \rho DV/\mu = (1.78)(0.5/12)(0.12)/0.000710 = 12.5$

From Fig. A-62, $C_D = 3.9$ for $N_R = 12.5$. $[(\pi)(0.5/12)^3/6][(1.78/1.94)(62.4)] - W = (3.9)(1.78)(0.12^2/2)[(\pi)(0.5/12)^2/4]$, $W = 0.002100$ lb; $\gamma = 0.002100/[(\pi)(0.5/12)^3/6] = 55.4$ lb/ft^3.

24.60 Measurements on a smooth sphere, 6 in in diameter, in an air stream (68 °F) gave a force for equilibrium equal to 0.250 lb. At what velocity was the air moving?

▮ Total drag $= C_D\rho AV^2/2$, where C_D = overall drag coefficient. Since neither Reynolds number nor C_D can be found directly, assume $C_D = 1.00$. Then $0.250 = C_D(0.00233)\frac{1}{4}\pi(\frac{1}{2})^2(V^2/2)$, $V^2 = 1093/C_D$, $V = 33.1$ ft/s. Calculate $N_R = Vd/\nu = 33.1(\frac{1}{2})/(16.0 \times 10^{-5}) = 103\,000$. From Fig. A-62, $C_D = 0.59$ (for spheres). Then $V^2 = 1093/0.59 = 1853$, $V = 43.0$ ft/s. Anticipating result, use $V = 44.0$ ft. Recalculate $N_R = Vd/\nu = 44.0(\frac{1}{2})/(16.0 \times 10^{-5}) = 137\,500$. From Fig. A-62, $C_D = 0.56$. Then $V^2 = 1093/0.56 = 1952$, $V = 44.2$ ft/s (satisfactory accuracy).

24.61 What should be the diameter of a sphere (s.g. = 2.50) in order that its freely falling velocity at 60 °F attains the acoustic velocity?

▮ $F_D = W = C_D\rho(V^2/2)A$. From Fig. A-63, $C_D = 0.80$. $V = c = \sqrt{kgRT}$. For air at 60 °F, $c = \sqrt{(1.40)(32.2)(53.3)(460 + 60)} = 1118$ ft/s, $[(2.50)(62.4)](\pi d^3/6) = (0.80)(0.00237)(1118^2/2)(\pi d^2/4)$, $d = 11.4$ ft.

24.62 The fixed keel of a Columbia 22 sailboat is about 38 in long (see Fig. 24-8). Moving in Lake Ontario at a speed of 3 knots, what is the skin drag from the keel? The water is at 40 °F. Solve this problem using rectangular plate of length 38 in and width 24.5 in, which is the average width of the keel. Transition takes place at a Reynolds number of 10^6.

▮ Compute the plate Reynolds number for the rectangular model of the keel.

$$N_R = Vl/\nu = [(3)(1.689)](24.5/12)/(1.664 \times 10^{-5}) = 6.22 \times 10^5 \qquad (1)$$

We thus have a laminar boundary layer. We get the plate coefficient of drag using

$$C_f = 1.328/\sqrt{N_R} = 1.328/\sqrt{6.22 \times 10^5} = 1.684 \times 10^{-3} \qquad (2)$$

Next we get the skin drag, realizing that there are two sides to be considered. Thus,

$$D = 2[(C_f)(\tfrac{1}{2}\rho V_0^2)(A)] = 2\{(1.684 \times 10^{-3})(\tfrac{1}{2})(1.940)[(3)(1.689)]^2[(38)(24.5)/144]\} = 0.542 \text{ lb} \qquad (3)$$

24.63 Solve Prob. 24.62 using the actual dimensions of the keel as shown in Fig. 24-8. Compare answers and comment on the result.

▮ For this purpose, consult Fig. 24-9 in which a keel with an infinitesimal slice dz is depicted. The length $l(z)$ of the slice is

$$l = \tfrac{15}{12} + (z/38)(19) = 1.250 + 0.5z \quad \text{ft} \qquad (4)$$

with z in feet. Now let us first see if we have transition anywhere on the keel. Looking at the uppermost

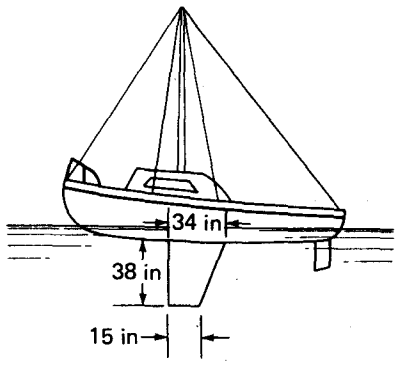


Fig. 24-8

portion, we have for $(N_R)_{max}$

$$(N_R)_{max} = Vl_{max}/\nu = [(3)(1.689)](\tfrac{34}{12})/(1.664 \times 10^{-5}) = 8.63 \times 10^5 \tag{5}$$

We have only a laminar boundary layer. For an infinitesimal plate of length $l = 1.250 + 0.5z$ ft, $C_f = 1.328/(Vl/\nu)^{1/2} = 1.328\{[(3)(1.689)/(1.664 \times 10^{-5})](1.250 + 0.5z)\}^{-1/2}$. Now for the drag D,

$$D = 2\int_0^{38/12} C_f(\tfrac{1}{2}\rho V_0^2)(1.250 + 0.5z)(dz)$$

$$= 2\int_0^{38/12} 1.238\left[\frac{(3)(1.689)}{1.664 \times 10^{-5}}(1.250 + 0.5z)\right]^{-1/2} (\tfrac{1}{2})(1.940)[(3)(1.689)]^2(1.250 + 0.5z)dz$$

$$= 0.1199\int_0^{3.17} (1.250 + 0.5z)^{1/2}\, dz$$

Let $1.250 + 0.5z = \eta$; therefore $0.5dz = d\eta$, $dz = 2d\eta$. Hence,

$$D = \frac{0.1199}{0.5}\int_{1.250}^{2.833} \eta^{1/2}\, d\eta = \frac{0.1199}{0.5}\eta^{3/2}(\tfrac{2}{3})\Big|_{1.250}^{2.833} = 0.539 \text{ lb}$$

The averaging process in Prob. 24.62 gave a very good result when compared with the result above.

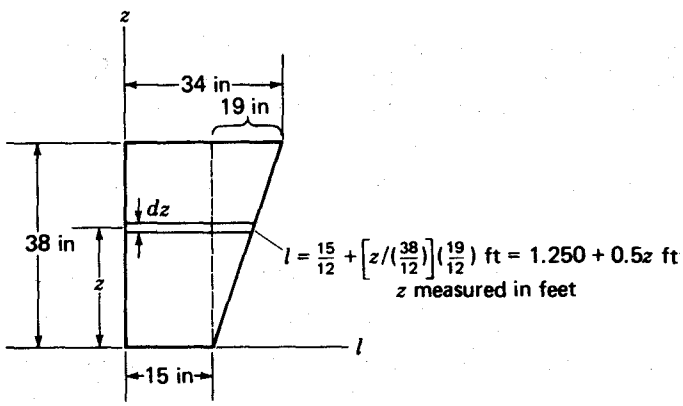


Fig. 24-9

24.64 The United States at one time in the thirties had three large dirigibles—the Los Angeles, the Graf Zeppelin, and the Akron. Two of them were destroyed by accidents. The largest was the Akron, having a length of 785 ft and a maximum diameter of 132 ft. Its maximum speed was 84 mph. The useful lift was 182 000 lb.

Moving at top speed, estimate the power needed to overcome skin friction, which is a significant part of the drag. Disregard effects of protrusion from engine cowlings, cabin region, etc. Assume that the surface is smooth. Take the critical Reynolds number to be 500 000. Consider the Akron at 10 000-ft standard atmosphere.

▮ We can make a reasonable estimate of the skin drag by "unwrapping" the outer surface of the Akron to form a flat plate. As a first step, we wish to calculate the plate Reynolds number. For this we find from the standard atmosphere table that $\rho = (0.7385)(0.002378) = 0.001756$ slug/ft^3, $T = 23.3$ °F. From the viscosity curves in Fig. A-1, we then find that $\mu = 3.7 \times 10^{-7}$ lb-s/ft^2. We can now compute $N_R = \rho dV/\mu$:

$$N_R = (0.001756)[(84)(\tfrac{5280}{3600})](785)/(3.7 \times 10^{-7}) = 4.59 \times 10^8 \qquad (1)$$

We will use the Prandtl–Schlichting skin-friction formula with $A = 1700$:

$$C_f = [0.455/(\log N_R)^{2.58}] - (A/N_R) = \{0.455/[\log(4.59 \times 10^8)]^{2.58}\} - [1700/(4.59 \times 10^8)] = 1.730 \times 10^{-3} \qquad (2)$$

Now going to $D = C_f(\frac{1}{2}\rho V_0^2)(bL)$, we have for the drag D owing to skin friction using the maximum diameter of 132 ft:

$$D = (1.730 \times 10^{-3})(\tfrac{1}{2})(0.001756)[84(\tfrac{5280}{3600})]^2(785)[(\pi)(132)] = 7505 \text{ lb} \qquad (3)$$

The power needed then is

$$\text{Power} = (7505)(84)(\tfrac{5280}{3600})/550 = 1681 \text{ hp} \qquad (4)$$

24.65 In Prob. 24.64, we computed the skin drag for the dirigible Akron using smooth flat-plate theory. Let us now evaluate the admissible roughness ϵ_{adm} for the results Prob. 24.64 to be valid. Then using a roughness of 0.05 in, recompute the skin-friction drag. What is the power needed to overcome this drag at the top speed of 84 mph?

▮ We can immediately solve for the admissible roughness and hence the largest roughness that will still give hydraulically smooth flow. Using the plate Reynolds number from Prob. 24.64, we have

$$\epsilon_{\text{adm}} = L(100/N_R) = (785)[100/(4.59 \times 10^8)] = 1.710 \times 10^{-4} \text{ ft} = 0.00205 \text{ in} \qquad (1)$$

For the dirigible, the actual roughness coefficient is 0.05 in. Now L/ϵ is $785/(0.05/12) = 1.884 \times 10^5$, so that on consulting Fig. A-64 we are clearly in the rough zone. We accordingly use

$$C_f = [1.89 + 1.62 \log(L/\epsilon)]^{-2.5} = \{1.89 + 1.62 \log[785/(0.05/12)]\}^{-2.5} = 0.002843 \qquad (2)$$

For the skin drag, we have, using 0.001756 slug/ft^3 for ρ, $D = (0.002843)(\frac{1}{2})(0.001756)[(84)(\frac{5280}{3600})]^2(785)(\pi)(132) = 12\,333$ lb. The power needed to overcome skin friction is Power $= (12\,333)(84)(\frac{5280}{3600})/550 = 2763$ hp.

24.66 In Prob. 24.62, consider that the surface is rough with $\epsilon = 0.009$ in. Also, consider that transition takes place at $(N_R)_{\text{crit}} = 3.2 \times 10^6$. Calculate the skin drag on the keel as accurately as you can.

▮ From Prob. 24.62, we see that the plate Reynolds number ranges from 3.81×10^5 at the bottom of the keel to 8.63×10^5 at the top of the keel. The ratio L/ϵ ranges from 1.667×10^3 at the bottom to 3.778×10^3 at the top. Considering Fig. A-64, we see that at both extremes we are in the transition zone of flow. We accordingly will use $C_f = [0.031/(N_R)^{1/7}] - (A/N_R)$ for C_f. We will furthermore use strips of width dz (see Fig. 24-9) and of length $(1.250 + 0.5z)$ ft, with z measured in feet. We have for C_f for such a strip

$$C_f = \frac{0.031}{\{[(3)(1.689)(1.250 + 0.5z)]/(1.664 \times 10^{-5})\}^{1/7}} - \frac{1050}{\{[(3)(1.689)(1.250 + 0.5z])/(1.664 \times 10^{-5})\}}$$
$$= 0.00510(1.250 + 0.5z)^{-1/7} - 0.00345(1.250 + 0.5z)^{-1}$$

The drag D then becomes

$$D = 2\int_0^{38/12} C_f(\tfrac{1}{2}\rho V_0^2)(1.250 + 0.5z)\,dz$$
$$= 2\int_0^{38/12} [(0.00510)(1.250 + 0.5z)^{-1/7} - (0.00345)(1.250 + 0.5z)^{-1}](\tfrac{1}{2})(1.490)[(3)(1.689)]^2(1.250 + 0.5z)\,dz$$
$$= \int_0^{3.17} [0.254(1.250 + 0.5z)^{6/7} - (0.1718)\,dz$$

Integrating, we get

$$D = (0.254)(1.250 + 0.5z)^{13/7}\left(\frac{7}{13}\right)\left(\frac{1}{0.5}\right)\Bigg|_0^{3.17} - (0.1718)(3.17) = 0.936 \text{ lb}$$

We see that we get almost a doubling of the drag. In racing sailboats, every ounce of drag counts, so sailors carefully smooth the wetted surfaces of their boats.

24.67 We will consider the dirigible Akron once again for drag (see Prob. 24.64). This time we will use an ellipsoidal body of revolution from Table A-22 to represent the dirigible. Estimate the pressure drag.

▮ The coefficient of drag should correspond to an ellipsoid whose $L/D_{max} = \frac{785}{132} = 5.95$. For turbulent flow, we estimate C_D using simple interpolation from the 4:1 to the 8:1 ellipsoids:

$$C_D = 0.06 + [(5.95 - 4)/4](0.13 - 0.06) = 0.094 \tag{1}$$

Now the total drag is next computed using

$$D = C_D A(\rho V_0^2/2) = (0.094)[(\pi)(132^2)/4](\tfrac{1}{2})(0.001756)[84(\tfrac{5280}{3600})]^2 = 17\,143 \text{ lb} \tag{2}$$

We can now estimate the pressure drag on the Akron using the drag from Prob. 24.64. Thus considering hydraulically smooth flow, $D_{press} = 17\,143 - 7505 = 9638$ lb. The total power needed to move the Akron should be $P = (17\,143)[(84)(\frac{5280}{3600})]/550 = 3840$ hp. The Akron actually had 8450-hp diesel engines.

24.68 A well-streamlined car can have a drag coefficient as low as 0.45 compared with old-fashioned cars whose drag coefficient could be as high as 0.9 (see Fig. A-65). Consider a car with a drag coefficient of 0.45 moving at the speed of 100 km/h. The frontal area of the car is 2 m^2 and its mass with driver is 1300 kg. How far must it move to halve its speed if the engine is disengaged and the vehicle rolls freely along a straight flat road with negligible wind present. Neglect the rotational effects of the wheels but consider that the coefficient of rolling resistance, a, of the tires is 0.50 mm. The air temperature is 20 °C. The tire diameter is 450 mm.

▮ You will recall from your statics course that the resistance to rolling F_R is given as

$$F_R = Pa/r \tag{1}$$

where P = normal force on wheel, a = coefficient of rolling resistance, r = radius of wheel.

For this problem, we have for the total effect of the four wheels $F_R = [(1300)(9.81)](0.50 \times 10^{-3})/0.225 = 28.34$ N. The drag force from the air is next computed:

$$D = C_D(\tfrac{1}{2}\rho V^2)(A) = (0.45)(\tfrac{1}{2})(\rho)(V^2)(2) \tag{2}$$

For ρ we have $\rho = p/RT = 101\,325/[(287)(293)] = 1.205$ kg/m^3. Hence we have for D

$$D = 0.542V^2 \tag{3}$$

We next express Newton's law for the car: $M(dV/dt) = F$.

$$(1300)(dV/dt) = -0.542V^2 - 28.34 \tag{4}$$

In order to separate variables with the distance x appearing, we may say, using the chain rule,

$$dV/dt = (dV/dx)(dx/dt) = V(dV/dx) \tag{5}$$

Now Eq. (4) becomes $1300V(dV/dx) = -(0.542V^2 + 28.34)$. Separating variables, we get

$$(1300V\,dV)/(0.542V^2 + 28.34) = -dx \tag{6}$$

Let $\eta = 0.542V^2 + 28.34$:

$$d\eta = 1.084V\,dV \tag{7}$$

Making the above substitution of variable, we get $(1300/1.084)(d\eta/\eta) = -dx$. Integrating, $1.199 \times 10^3 \ln \eta = -x + C$:

$$1.199 \times 10^3 \ln (0.542V^2 + 28.34) = -x + C \tag{8}$$

When $x = 0$, $V = 100(\frac{1000}{3600}) = 27.78$ m/s. We can determine C: $1.199 \times 10^3 \ln [(0.542)(27.78)^2 + 28.34] = C$, $C = 1.199 \times 10^3 \ln 446.6$. Equation (8) can now be written as

$$\ln [(0.542V^2 + 28.34)/446.6] = -x/(1.199 \times 10^3) \tag{9}$$

Let $V = 50(\frac{1000}{3600}) = 13.89$ m/s. We get for x

$$\ln \{[(0.542)(13.89)^2 + 28.34]/446.6\} = -x/(1.199 \times 10^3) \qquad x = 1453 \text{ m} \tag{10}$$

The vehicle moves a distance of 1453 m. Actually, by including the rotational energy of the wheels, the vehicle would move even farther.

24.69 A jet aircraft discharges solid particles of matter 10 μm in diameter, s.g. = 2.5, at an altitude of 11 km. Assume the viscosity μ of air, in Pa · s, to be expressed by $\mu = 1.78 \times 10^{-5} - 3.06 \times 10^{-10}y$, where y is altitude, in m.

Estimate the time for these particles to reach sea level. Neglect air currents and wind effects.

▮ By Stokes' law (Prob. 24.52), $u = (D^2/18\mu)(\gamma_s - \gamma)$. Writing $u = -dy/dt$ and recognizing the unit weight of air to be much smaller than the unit weight of the solid particles, one has $-dy/dt \approx (D^2/18)(\gamma_s/\mu)$:

$$\int_0^T dt = \int_{11\,000}^0 18(1.78 \times 10^{-5} - 3.06 \times 10^{-10}y)\left[\frac{1}{(10 \times 10^{-6})^2}\right]\left[\frac{1}{2.5(9806)}\right] dy$$

$$T = (1/86\,400)[1.78y - (3.06 \times 10^{-5}y^2)/2]_0^{11\,000}(73.45) = 15.07 \text{ days}$$

24.70 How many 32-m-diameter parachutes ($C_D = 1.2$) should be attached to a 100-kN load to ensure a terminal speed of 10 m/s, if air pressure and temperature are 100 kPa and 10 °C?

▮ For one parachute,

$$F_D = C_D\rho(V^2/2)A \qquad \rho = p/RT = 100\,000/[(287)(10 + 273)] = 1.231 \text{ kg/m}^3$$

$$F_D = (1.2)(1.231)(10^2/2)[(\pi)(32)^2/4] = 59.4 \text{ kN}$$

Thus, two parachutes should be sufficient.

24.71 An 85-cm cubical box is tied to the roof of a car. Estimate the additional power needed to travel at 75 km/h ($C_D = 1.1$ and $\rho_{air} = 1.2$ kg/m^3).

▮

$$P = F_D V \qquad F_D = C_D\rho(V^2/2)A$$

$$V = (75)(1000)/3600 = 20.83 \text{ m/s} \qquad F_D = (1.1)(1.2)(20.83^2/2)[(0.85)(0.85)] = 206.9 \text{ N}$$

$$P = (206.9)(20.83) = 4310 \text{ W} = 5.78 \text{ hp}$$

24.72 A semitubular cylinder of 8-in radius with concave side upstream is submerged in water flowing 2 fps. Calculate the drag for a cylinder 20 ft long.

▮ $F_D = C_D\rho(V^2/2)A$. From Table A-23, $C_D = 2.3$. $F_D = (2.3)(1.94)(2^2/2)[(20)(8 + 8)/12] = 238$ lb.

24.73 A fully loaded, small aircraft weighing 5000 lb with a wing area (projected chord area) of 350 ft^2 is to take off at a horizontal velocity of 100 mph (146.7 fps). What is the necessary angle of attack (i.e., angle the wings make with the horizontal)? Assume the wings have the characteristics of the airfoil of Fig. A-66. Use $\rho_{air} = 0.00234$ slug/ft^3.

▮

$$F_L = C_L\rho(v^2/2)A \qquad 5000 = C_L(0.00234)(146.7^2/2)(350) \qquad C_L = 0.567$$

From Fig. A-66, $\alpha = 6.0°$.

24.74 An aircraft weighing 1000.0 kN when empty has a wing area of 226 m^2. It is to take off at a velocity of 300 km/h and a 20° angle of attack. Assume $\rho_{air} = 1.20$ kg/m^3. Also, assume the wing has the characteristics of the airfoil in Fig. A-66. What is the allowable weight of cargo?

▮ $F_L = C_L\rho(v^2/2)A$. From Fig. A-66, $C_L = 1.42$. $v = (300)(1000)/3600 = 83.33$ m/s, $F_L = (1.42)(1.20)(83.33^2/2)(226) = 1\,337\,000$ N, or 1337 kN. This value (1337 kN) represents the total weight that can be lifted. Since the aircraft weighs 1000.0 kN when empty, it can carry, in theory, $1337 - 1000 = 337$ kN of cargo.

24.75 A fully loaded aircraft weighing 10 000 lb with a wing area of 450 ft^2 is to take off at a horizontal velocity of 125 mph (183.3 fps). What is the necessary angle of attack? Assume the wings have the characteristics of the airfoil of Fig. A-66. Use $\rho_{air} = 0.00234$ slug/ft^3.

▮

$$F_L = C_L\rho(v^2/2)A \qquad 10\,000 = C_L(0.00234)(183.3^2/2)(450) \qquad C_L = 0.565$$

From Fig. A-66, $\alpha = 6.0°$.

24.76 An aircraft weighing 65.2 kN when empty has a wing area of 62.3 m^2. It is to take off at a velocity of 250 km/h and a 5.0° angle of attack. Assume $\rho_{air} = 1.20$ kg/m^3. Also, assume the wing has the characteristics of the airfoil in Fig. A-66. What is the allowable weight of cargo?

▮ $F_L = C_L\rho(v^2/2)A$. From, Fig. A-66, $C_L = 0.50$. $v = (250)(1000)/3600 = 69.44$ m/s, $F_L = (0.50)(1.20)(69.44^2/2)(62.3) = 90\,000$ N, or 90.1 kN; $W_{cargo} = 90.1 - 65.2 = 24.9$ kN.

24.77 A 3-ft by 4-ft plate moves at 44 ft/s in still air at an angle of 12° with the horizontal. Using a coefficient of drag $C_D = 0.17$ and a coefficient of lift $C_L = 0.72$, determine (*a*) the resultant force exerted by the air on the plate, (*b*) the frictional force and (*c*) the horsepower required to keep the plate moving. (Use $\gamma = 0.0752\ \text{lb/ft}^3$.)

▮ (*a*) Drag force $= C_D(\gamma/g)A(V^2/2)$ $\quad F_D = 0.17(0.752/32.2)(12)[(44)^2/2] = 4.61$ lb

Lift force $= C_L(\gamma/g)A(V^2/2)$ $\quad F_L = 0.72(0.0752/32.2)(12)[(44)^2/2] = 19.5$ lb

Referring to Fig. 24-10, the resultant of the drag and lift components is $R = \sqrt{(4.61)^2 + (19.5)^2} = 20.0$ lb acting on the plate at $\theta_x = \tan^{-1}(19.5/4.61) = 76°42'$.

(*b*) The resultant force might also have been resolved into a normal component and a frictional component (shown dotted in the figure). From the vector triangle, frictional component $= R\cos(\theta_x + 12°) = 20.0(0.0227) = 0.45$ lb.

(*c*) Horsepower = (force in direction of motion)(velocity)/550 = (4.61)(44)/550 = 0.369

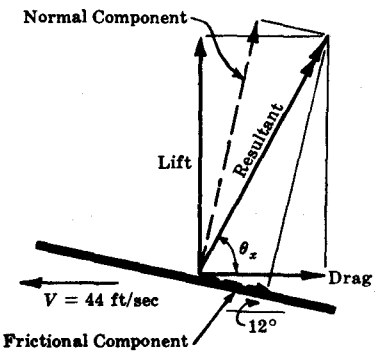


Fig. 24-10

24.78 If an airplane weighs 4000 lb and has a wing area of 300 ft², what *angle of attack* must the wings make with the horizontal at a speed of 100 mph? Assume the coefficient of lift varies linearly from 0.35 at 0° to 0.80 at 6° and use $\gamma = 0.0752\ \text{lb/ft}^3$ for air.

▮ For equilibrium in the vertical direction, $\sum Y = 0$. Hence, lift − weight = 0, or weight $= C_L\gamma A(V^2/2g)$, $4000 = C_L(0.0752)(300)\{[(100)(\frac{5280}{3600})]^2/2g\}$, $C_L = 0.53$. By interpolation between 0° and 6°, angle of attack = 2.4°.

24.79 What wing area is required to support a 5000-lb plane when flying at an angle of attack of 5° at 88 ft/s? Use coefficients given in Prob. 24.78.

▮ From given data, $C_L = 0.725$ for 5° angle by interpolation. As in Prob. 24.78, weight = lift, $5000 = 0.725(0.0752/32.2)A(88)^2/2$, $A = 763$ ft².

24.80 A kite weighs 2.50 lb and has an area of 8.00 ft². The tension in the kite string is 6.60 lb when the string makes an angle of 45° with the horizontal. For a wind of 20 mph, what are the coefficients of lift and drag if the kite assumes an angle of 8° with the horizontal? Consider the kite essentially a flat plate and $\gamma_{air} = 0.0752\ \text{lb/ft}^3$.

▮ Figure 24-11 indicates the forces acting on the kite taken as a free body. The components of the tension are 4.66 lb each.

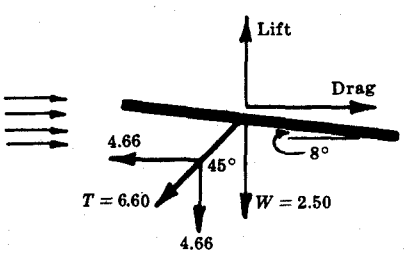


Fig. 24-11

From $\sum X = 0$, drag = 4.66 lb. From $\sum Y = 0$, lift = 4.66 + 2.50 = 7.16 lb. Drag force = $C_D \rho A V^2/2$, $4.66 = C_D(0.0752/32.2)(8.00)[(20)(\frac{5280}{3600})]^2/2$, $C_D = 0.58$. Lift force = $C_L \rho A V^2/2$, $7.16 = C_L(0.0752/32.2)(8.00)[(20)(\frac{5280}{3600})]^2/2$, $C_L = 0.89$.

24.81 If the minimum landing speed (V_{min}) of the Mustang racer is 1.3 times the stall speed, what is the minimum landing speed for this plane with flaps at 40°? The Mustang has a wing area of 233 ft² and a weight of 9500 lb. Take ρ to be 0.002378 slug/ft³ and $(C_L)_{max}$ as 1.65.

▮ $$V_{stall} = \sqrt{2W/[(\rho)(C_L)_{max}A]} = \sqrt{(2)(9500)/[(0.002378)(1.65)(233)]} = 144.2 \text{ ft/s} \quad \text{or} \quad 98.3 \text{ mph}$$

$$V_{min} = (1.3)(98.3) = 128 \text{ mph}$$

24.82 When a wing of 2-m chord moves through still air ($\gamma = 11.3$ N/m³) at 138 km/h, the mean velocity along the top surface is 160 km/h (44.45 m/s) and that along the bottom surface is 130 km/h (36.11 m/s). Estimate the lift per unit length of span.

▮ $$\text{Lift} = (\Delta p)(c) \qquad p_{top}/\gamma + v_{top}^2/2g = p_{bottom}/\gamma + v_{bottom}^2/2g$$

$$p_{top}/11.3 + 44.45^2/[(2)(9.807)] = p_{bottom}/11.3 + 36.11^2/[(2)(9.807)]$$

$$p_{bottom} - p_{top} = \Delta p = 387.1 \text{ N/m}^2 \qquad \text{Lift} = (387.1)(2) = 774 \text{ N/m}$$

24.83 For a rectangular Clark Y airfoil of 6-ft chord by 36-ft span, find the value of the friction coefficient η if the angle of attack $\alpha = 5.4°$ when the wing is moving at 300 fps through standard atmosphere at altitude 10 000 ft. Find the weight which the wing will carry and the horsepower required to drive it.

▮ From Fig. A-67, with $\alpha = 5.4°$, $C_L = 0.8$, $C_D = 0.047$. From Fig. A-68, for $B/c = 6$, $\tau = 0.175$. α_i(radians) = $[C_L/\pi(B/c)](1+\tau) = [0.8/\pi(\frac{36}{6})](1+0.175) = 0.0499$ rad = 2.86°. From Fig. A-69, $\alpha_0 = \alpha - \alpha_i = 5.40 - 2.86 = 2.54°$ and since the angle of zero lift is −5.6°, $\alpha_0' = 2.54 + 5.6 = 8.14° = 0.1421$ rad, $\eta = C_L/2\pi\alpha_0' = 0.8/[(2\pi)(0.1421)] = 0.896$. The wing will support a weight equal to the lift force, $F_L = C_L\rho(V^2/2)(B)(c)$.

From Table A-7, at 10 000 ft, $\rho = 0.001756$ slug/ft³. $F_L = (0.8)(0.001756)[(300)^2/2](36)(6) = 13\,650$ lb while $F_D = (0.047/0.8)(13\,650) = 802$ lb, horsepower required = (802)(300)/550 = 437 hp.

24.84 A cylinder 4 ft in diameter and 25 ft long rotates at 90 rpm with its axis perpendicular to an air stream with a wind velocity of 120 fps. The specific weight of the air is 0.0765 lb/ft³. Assuming no slip between the cylinder and the circulatory flow, find **(a)** the value of the circulation; **(b)** the transverse or lift force; and **(c)** the position of the stagnation points.

▮ **(a)** Peripheral velocity $v_t = 2\pi Rn/60 = (2\pi)(2)(\frac{90}{60}) = 18.85$ fps, $\Gamma = 2\pi R v_t = (2\pi)(2)(18.85) = 237$ ft²/s.

(b) $$F_L = \rho B U \Gamma = (0.0765/32.2)(25)(120)(237) = 1690 \text{ lb}$$

(c) $$\sin\theta_s = -\Gamma/4\pi RU = -237/(4\pi 2)(120) = -0.0786$$

Therefore $\theta_x = 184.5°, 355.5°$. Actually, the real circulation produced by surface drag of the rotating cylinder would be only about one-half of that obtained for the no-slip assumption.

24.85 A wing with a 22-m span and 64-m² planform area moves horizontally through the standard atmosphere at 8 km, with a velocity of 760 km/h. If the wing supports 280 kN, find **(a)** the required value of the lift coefficient, **(b)** the downwash velocity, assuming semielliptical distribution of lift over the span, and **(c)** the induced drag.

▮ From Table A-8, $\rho = 0.526$ kg/m³; $V = (760)(1000)/3600 = 211.1$ m/s.

(a) $$F_L = C_L\rho(V^2/2)A \qquad 280\,000 = C_L(0.526)(211.1^2/2)(64) \qquad C_L = 0.373$$

(b) $$V_i/V = C_L/[\pi(B^2/A)] \qquad V_i/211.1 = 0.373/[(\pi)(22^2/64)] \qquad V_i = 3.31 \text{ m/s}$$

(c) $$(C_D)_i = C_L^2/[\pi(B^2/A)] = 0.373^2/[(\pi)(22^2/64)] = 0.00586$$

$$(F_D)_i = (C_D)_i(F_L/C_L) = (0.00586)(280\,000/0.373) = 4399 \text{ N}$$

24.86 A glider, of gross weight 350 lb, has a 3-ft-chord by 18-ft-span wing of the Clark Y section. Assuming that its characteristics are the same as those for the larger wing of the same aspect ratio shown in Fig. A-67, find the angle of glide through standard air at 2000 ft which will produce the greatest horizontal range.

▮ From Fig. A-67 for maximum C_L/C_D: $\alpha = -0.1°$, $C_L = 0.4$, and $C_D = 0.019$. For maximum range, glide angle β is minimum. $\beta = \arctan(F_D/F_L) = \arctan(0.019/0.4) = 2.7°$.

24.87 A boat fitted with a hydrofoil weighs 4000 lb. At a velocity of 60 fps, what size hydrofoil is needed to support the boat? Use the lift characteristics in Fig. A-70 at an angle of attack of 2°.

▮ $F_L = C_L\rho(V^2/2)A$. From Fig. A-70, $C_L = 0.35$. $4000 = (0.35)(1.94)(60^2/2)A$, $A = 3.27\ \text{ft}^2$.

24.88 At what angle of attack should a 128-kN airplane be flown at a speed of 225 km/h, if the planform area is 75 m²? What power is required to overcome wing drag? Assume a temperature of 30 °C and flaps at zero degrees.

▮

$$F_L = C_L(\tfrac{1}{2})\rho V^2 A \qquad 128\,000 = (C_L)(\tfrac{1}{2})(\rho)(225/3.6)^2(75) \qquad \rho = p/RT$$

$$\rho = 101\,325/[(287)(303)] = 1.165\ \text{kg/m}^3 \qquad C_L = 0.751 \qquad \alpha = 8° \qquad C_D = 0.04$$

$$F_D = (0.04)(\tfrac{1}{2})(1.165)(225/3.6)^2(75) = 6.825\ \text{kN} \qquad P = (6.825)(225/3.6) = 426\ \text{kW} = 570\ \text{hp}$$

24.89 The takeoff speed of the aircraft of Prob. 24.88 is about 1.3 times the stall speed. Calculate the takeoff distance for a constant thrust of 24 kN and a rolling resistance of 1.8 kN. Assume that the flaps are at 40°, the air is at 30 °C, the overall coefficient of drag for the plane is 0.20, and the frontal area is 45 m².

▮ The stall speed occurs at the condition of $(C_L)_{\max}$ which here is 1.64.

$$\rho = 101\,325/[(287)(303)] = 1.165\ \text{kg/m}^3$$

$$V_{\text{stall}} = \sqrt{(2)W/[(\rho)(C_L)_{\max}A]} = \sqrt{(2)(128\,000)/[(1.165)(1.64)(75)]} = 42.3\ \text{m/s}$$

V at takeoff is $(1.3)(42.3) = 55.0$ m/s. By Newton's second law,

$$24\,000 - 1800 - (0.2)(\rho V^2/2)(45) = (128\,000/g)V(dV/dx)$$

$$7400 - 1.748V^2 = 4354V(dV/dx) \qquad (4354)(V\,dV)/(7400 - 1.748V^2) = dx$$

Let $7400 - 1.748V^2 = \eta$, $-3.50V\,dV = d\eta$, $V\,dV = (d\eta/3.50)$.

$$\int_{7400}^{2112}(4354)\frac{(-d\eta/3.50)}{\eta} = \int_0^{L_1} dx \qquad -\frac{4354}{3.50}\ln\eta\Big]_{7400}^{2112} = L_1 \qquad L_1 = 1560\ \text{m}$$

24.90 An airplane with a planform wing area of 233 ft² is flying at an angle of attack of 3° at a speed of 200 mph. The air is at 40 °F, and the flaps are at zero degrees. Compute the weight of the plane.

▮

$$C_L = 0.35 \qquad L = C_L(\tfrac{1}{2})\rho V^2 A = W = (0.35)(\tfrac{1}{2})(\rho)(V_0^2)(A)$$

$$\rho = (14.7)(144)/[(53.3)(g)(500)] = 0.00247\ \text{slug/ft}^3$$

$$W = (0.35)(\tfrac{1}{2})(0.00247)[(200)\tfrac{5280}{3600}]^2(233) = 8666\ \text{lb} = 4.33\ \text{ton}$$

24.91 A boat is fitted with hydrofoils having a total planform area of 1.2 m². At 11 knots, the stall speed, the coefficient of lift is 1.6 and the coefficient of drag is 0.6. Find the maximum weight and the minimum power corresponding to this stall speed.

▮

$$W = F_L = C_L(\tfrac{1}{2})\rho V^2 A = (1.6)(\tfrac{1}{2})(1000)[(11)(0.5144)]^2(1.2) = 30.737\ \text{kN}$$

$$F_D = C_D(\tfrac{1}{2})\rho V^2 A = (0.6)(\tfrac{1}{2})(1000)[(11)(0.5144)]^2(1.2) = 11.526\ \text{kN}$$

$$P = F_D V = (11.526)[(11)(0.5144)] = 65.2\ \text{kW}$$

24.92 The 9-in-wide, 4.5-ft-long flat blades of a ceiling fan are attached to an 8-in-diameter core; the blades' angle of attack is 15°. At what speed of rotation will the core experience zero vertical force? Each blade weighs 1.6 lb, and the room temperature is 40 °F.

From Fig. A-71, $C_L = 0.79$. Determine ω by equating the lift on a single blade to its weight.

$$F_L = C_L \int (\tfrac{1}{2}\rho V^2)\, dA = (0.79)\int_{0.333}^{4.833} (\tfrac{1}{2}\rho V_0^2)\left(\frac{9\, dr}{12}\right) \qquad \rho = \frac{(14.7)(144)}{(53.3)(g)(500)} = 0.002467 \text{ slug/ft}^3$$

$$F_L = 1.6 = (0.79)(\tfrac{1}{2})(0.002467)(\tfrac{9}{12})\int_{0.333}^{4.833} (r\omega)^2\, dr \qquad 1.6 = (0.000731)\omega^2 \frac{r^3}{3}\bigg|_{0.333}^{4.833} = 0.000244\omega^2(4.833^3 - 0.333^3)$$

$$\omega = 7.623 \text{ rad/s} = (7.623/2\pi)(60) = 72.8 \text{ rpm}$$

CHAPTER 25
Basic Hydrodynamics

25.1 Assuming ρ to be constant, do these flows satisfy continuity? **(a)** $u = -2y$, $v = 3x$; **(b)** $u = 0$, $v = 3xy$; **(c)** $u = 2x$, $v = -2y$. Continuity for incompressible fluids is satisfied if $\partial u/\partial y + \partial v/\partial y = 0$.

(a) $[\partial(-2y)/\partial x] + [\partial(3x)/\partial y] = 0 + 0 = 0$ (continuity is satisfied)

(b) $[\partial(0)/\partial x] + [\partial(3xy)/\partial y] = 0 + 3x \neq 0$ (continuity is not satisfied)

(c) $[\partial(2x)/\partial x] + [\partial(-2y)/\partial y] = 2 - 2 = 0$ (continuity is satisfied)

Note: If **(b)** did indeed describe a flow field, the fluid must be compressible.

25.2 Determine whether these flows are rotational or irrotational. **(a)** $u = -2y$, $v = 3x$; **(b)** $u = 0$, $v = 3xy$; **(c)** $u = 2x$, $v = -2y$.

If $(\partial v/\partial x) - (\partial u/\partial y) = 0$ (flow is irrotational):

(a) $[\partial(3x)/\partial x] - [\partial(-2y)/\partial y] = 3 + 2 \neq 0$ (flow is rotational)

(b) $[\partial(3xy)/\partial x] - [\partial(0)/\partial y] = 3y - 0 \neq 0$ (flow is rotational)

(c) $[\partial(-2y)/\partial x] - [\partial(2x)/\partial y] = 0 - 0 = 0$ (flow is irrotational)

25.3 Check these flows for continuity and determine the vorticity of each: **(a)** $v_t = 6r$, $v_r = 0$; **(b)** $v_t = 0$, $v_r = -5/r$.

Applying the equations $(v_r/r) + (\partial v_r/\partial r) + (\partial v_t/r\,\partial\theta) = 0$, $\xi = (\partial v_t/\partial r) + (v_t/r) - (\partial v_r/r\,\partial\theta)$.

(a) $(0/r) + [\partial(0)/\partial r] + [\partial(6r)/r\,\partial\theta] = 0$ (continuity is satisfied)

$\xi = [\partial(6r)/\partial r] + (6r/r) - [\partial(0)/r\,\partial\theta] = 6 + 6 - 0 = 12$ (flow is rotational)

(b) $-[(5/r)/r] + [\partial(-5r^{-1})/\partial r] + [\partial(0)/r\,\partial\theta] = -(5/r^2) + (5/r^2) + 0 = 0$ (continuity is satisfied)

$$\xi = \frac{\partial(0)}{\partial r} + \frac{0}{r} - \frac{\partial(-5/r)}{r\,\partial\theta} = 0$$ (flow is irrotational)

25.4 A flow is defined by $u = 2x$ and $v = -2y$. Find the stream function and potential function for this flow and plot the flow net.

Check continuity: $(\partial u/\partial x) + (\partial v/\partial y) = 2 - 2 = 0$. Hence continuity is satisfied and it is possible for a stream function to exist: $d\psi = -v\,dx + u\,dy = 2y\,dx + 2x\,dy$, $\psi = 2xy + C_1$. Check to see if the flow is irrotational: $(\partial v/\partial x) - (\partial u/\partial y) = 0 - 0 = 0$. Hence the flow is irrotational and a potential function exists: $d\phi = -u\,dx - v\,dy = -2x\,dx + 2y\,dy$, $\phi = -(x^2 - y^2) + C_2$. The location of lines of equal ψ can be found by substituting values of ψ into the expression $\psi = 2xy$. Thus for $\psi = 60$, $x = 30/y$. This line is plotted (in the upper right-hand quadrant) in Fig. 25-1. In a similar fashion lines of equal potential can be plotted. For example, for $\phi = 60$ we have $-(x^2 - y^2) = 60$ and $x = \pm\sqrt{y^2 - 60}$. This line is also plotted on the figure. The flow net depicts *flow in a corner*. Mathematically the net will plot symmetrically in all four quadrants.

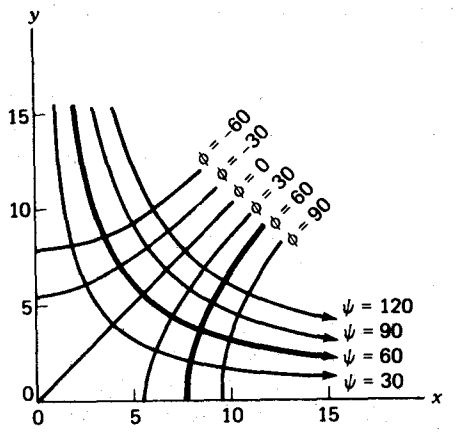
25.5 Given a two-dimensional flow defined by $u = 4 + 3x$, $v = 0$. If this flow satisfies continuity, what can be said about the density of the fluid?

$\partial(\rho u)/\partial x = 0$, or $\rho v = c(y)$, or $\rho = c(y)/(4 + 3x)$. Therefore, for each fixed y, the density ρ must decrease as x increases.

25.6 The flow of an incompressible fluid is defined by $u = 3$, $v = 9x$. Does a stream function exist for this flow? If so, determine the expression for the stream function.

To find if a stream function exists, check for continuity. $\partial u/\partial x = 0$, $\partial v/\partial y = 0$, $w = 0$. Thus, $(\partial u/\partial x) + (\partial v/\partial y) + (\partial w/\partial z) = 0$ (continuity) is satisfied and a stream function exists.

$$d\psi = -v\,dx + u\,dy = -9x\,dx + 3\,dy \qquad \psi = -4.5x^2 + 3y$$



Flow net

Fig. 25-1

25.7 Plot the streamlines in the upper right-hand quadrant for the flow defined by $\psi = 1.6x^2 + y^2$ and determine the value of the velocity at $x = 3$, $y = 4$.

▮ $u = \partial\psi/\partial y = 2y$, $v = -\partial\psi/\partial x = -3.2x$. At $(3, 4)$ $u = 8$, $v = -9.6$, $V = \sqrt{8^2 + 9.6^2} = 12.50$. To plot streamlines, rewrite as $y = (\psi - 1.6x^2)^{1/2}$. Assume ψ, compute y's for different x's. See Fig. 25-2.

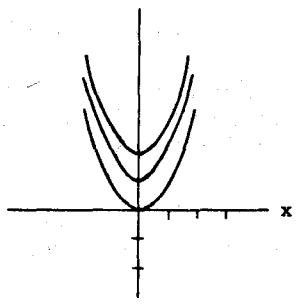


Fig. 25-2

25.8 The components of the velocities of a certain flow system are $u = -(Q/2\pi)[x/(x^2 + y^2)] + By + C$, $v = -A[y/(x^2 + y^2)] + Dx + E$. **(a)** Calculate a value of A consistent with continuous flow. **(b)** Sketch the streamlines for this flow system, assuming $B = C = D = E = 0$.

▮ **(a)** To satisy continuity 2-D flow: $(\partial u/\partial x) + (\partial v/\partial y) = 0$.

$$\partial u/\partial x = -(Q/2\pi)\{[(x^2 + y^2) - x(2x)]/(x^2 + y^2)^2\} = -(Q/2\pi)[(y^2 - x^2)/(x^2 + y^2)^2]$$

$$\partial v/\partial y = -A\{[(x^2 + y^2) - y(2y)]/(x^2 + y^2)^2\} = -A[(x^2 - y^2)/(x^2 + y^2)^2]$$

Substituting, $-Q/2\pi(y^2 - x^2) - A(x^2 - y^2) = 0$; thus $A = Q/2\pi$.

(b)

$$u = -(Q/2\pi)[x/(x^2 + y^2)] \qquad v = -(Q/2\pi)[y/(x^2 + y^2)]$$

$$d\psi = -v\,dx + u\,dy = (Q/2\pi)[(y\,dx)/(x^2 + y^2)] - (Q/2\pi)[(x\,dy)/(x^2 + y^2)]$$

Integrating, $\psi = (Q/2\pi)[\tan^{-1}(x/y) - \tan^{-1}(y/x)]$. See Fig. 25-3.

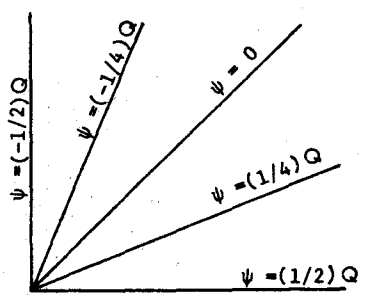


Fig. 25-3

25.9 A flow field is described by $\psi = x^2 - 2y$. Describe the streamlines for $\psi = 0$, 1, and 2. Derive an expression for the velocity at any point in the flow field and determine the vorticity of the flow.

▮ From the equation for ψ, the flow field is a family of parabolas symmetric about the y axis. The streamline $\psi = 0$ passes through the origin. $u = \partial u/\partial y = -2$, $v = -\partial u/\partial x = -2x$. Thus $V = (u^2 + v^2)^{1/2} = (4 + 4x^2)^{1/2}$. Vorticity: $\xi = (\partial v/\partial x) - (\partial u/\partial y) = -2$. (Since $\xi \neq 0$, the flow is rotational.)

25.10 Locate the stagnation point of the incompressible flow due to a source of strength 9π located at (3, 0) and a source of strength 18π located at (−4, 0).

▮ It is obvious that this flow has a single stagnation point, located between the sources on the x-axis (where v is automatically zero). $\psi = (q_1\theta_1/2\pi) + (q_2\theta_2/2\pi) = (9\pi/2\pi)\arctan[y/(x-3)] + (18\pi/2\pi)\arctan[y/(x+4)]$, $u = \partial\psi/\partial y = 4.5\{(x-3)/[(x-3)^2 + y^2]\} + 9\{(x+4)/[(x+4)^2 + y^2]\}$. If $y = 0$, $u = [4.5/(x-3)] + [9/(x+4)]$. At stagnation point, $u = 0 = [4.5/(x-3)] + [9/(x+4)]$; thus $x = \frac{2}{3}$.

25.11 A source discharging 18 m³/s per m is located at the origin and a uniform flow at 2 m/s in the $+x$-direction is superimposed on the source flow. Write the overall stream function in polar and rectangular coordinates

▮
$$\psi_1 = 18(\theta/2\pi) = (18/2\pi)\arctan(y/x) \qquad \psi_2 = 2y = 2r\sin\theta$$

$$\psi_{\text{polar}} = (9\theta/\pi) + 2r\sin\theta \qquad \psi_{\text{rect}} = (9/\pi)\arctan(y/x) + 2y$$

25.12 Refer to Prob. 25.11. Find the difference in pressure head between point $A(-9\text{ m}, 0)$ and point $B(0, 1.33\text{ m})$.

▮ $u = \partial\psi_{\text{rect}}/\partial y = (9/\pi)[x/(x^2 + y^2)] + 2$; $v = -\partial\psi_{\text{rect}}/\partial x = (9/\pi)[y/(x^2 + y^2)]$. At point A (−9 m, 0): $u = 1.68$ m/s, $v = 0$; $V_A = 1.68$ m/s. At point B (0, 1.33 m): $u = 2.0$ m/s, $v = 2.15$ m/s; $V_B = \sqrt{2.0^2 + 2.15^2} = 2.94$ m/s. From Bernoulli's theorem: $p_A/\gamma - p_B/\gamma = V_B^2/2g - V_A^2/2g = (2.94^2 - 1.68^2)/[2(9.81)] = 0.297$ m.

25.13 (*a*) Does the flow $u = x^2 + 3x - 4y$, $v = -2xy - 3y$ satisfy continuity? (*b*) Compute the vorticity. (*c*) Locate all stagnation points. (*d*) Find the equation of the stream function and graph it in the first quadrant.

▮ (*a*) $(\partial u/\partial x) + (\partial v/\partial y) = 2x + 3 - 2x - 3 = 0$; continuity is satisfied.
(*b*) $\xi = (\partial v/\partial x) - (\partial u/\partial y) = -2y - 4 \neq 0$; flow is rotational.
(*c*) Stagnation points occur where both $u = 0$ and $v = 0$. $v = 0 = -2xy - 3y = -2y(x + \frac{3}{2})$; true when $y = 0$ or $x = -\frac{3}{2}$, $u = 0 = x^2 + 3x - 4y$. If $x = -1$, $y = -\frac{1}{2}$. If $y = 0$, $x = 0$ or −3. Thus there are three stagnation points: (0, 0), (−3, 0), and $(-\frac{3}{2}, -\frac{1}{2})$.

(*d*)
$$u = \partial\psi/\partial y \qquad \psi = \int u\,\partial y = \int (x^2 + 3x - 4y)\,\partial y = x^2y + 3xy - 2y^2 + C(x)$$

$$v = -2xy - 3y = -\partial\psi/\partial x = -2xy - 3y - C'(x) \qquad C(x) = \text{constant} = 0$$

So $\psi = x^2y + 3xy - 2y^2$. Notice in the graph, Fig. 25-4, the branching of $\psi = 0$ at the origin (a stagnation point).

25.14 Is the function $\psi = 4x - 3y$ the stream function of a two-dimensional potential flow?

▮ Yes: $\partial^2\psi/\partial x^2 + \partial^2\psi/\partial y^2 = 0 + 0 = 0$.

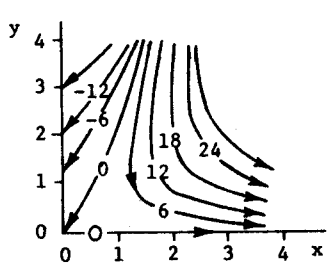


Fig. 25-4

25.15 An ideal fluid flows in a two-dimensional 90° bend. The inner and outer radii of the bend are 0.5 and 1.5 ft. Sketch the flow net and estimate the velocity at the inner and outer walls of the bend if the velocity in the 1.0-ft-wide straight section is 9 fps. Develop an analytic expression for the stream function, noting that $v_t = -\partial\psi/\partial r$ and $v_r = \partial\psi/r\,\partial\theta$. Determine the inner and outer velocities accurately.

▮ Velocity estimates: $V = -\partial\phi/\partial s$, $-\Delta\phi \approx V\,\Delta s \approx 9\,\Delta s_0$, $V \approx 9\,\Delta s_0/\Delta s$. From Fig. 25-5, $V_{out} \approx 9(\frac{5}{8}) = 5.6$ fps, $V_{in} \approx 9(5/2.5) = 18$ fps.

Accurate velocity determinations: $v_t(\Delta r) = \text{constant}$, Δr is proportional to r. $v_t(kr) = \text{constant}$, therefore $v_t = C/r$. Also $v_r = 0$. $\psi = \psi(r, \theta)$, so by chain rule of differentiation $d\psi = (\partial\psi/r\,\partial\theta)r\,d\theta + (\partial\psi/\partial r)\,dr = v_r r\,d\theta - v_t\,dr = 0 - v_t\,dr$. If $\psi_{in} = 0$, $\psi_{out} = \psi_{in} + dq = 0 + (1)(9) = 9$ cfs/ft. Integrating

$$\int_0^{10} d\psi = -\int_{0.5}^{1.5} v_t\,dr = -C\int_{0.5}^{1.5}\frac{dr}{r} \qquad \psi = 9 = -C[\ln r]_{0.5}^{1.5} = -C\ln\frac{1.5}{0.5} = -C\ln 3.0$$

and $C = -8.19$. Thus the analytic expression is $\psi = 8.19\ln r$; $v_t = -8.19/r$, $(v_t)_{r=0.5} = 16.4$ fps, $(v_t)_{r=1.5} = 5.5$ fps.

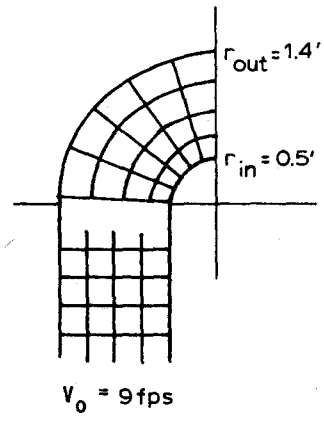
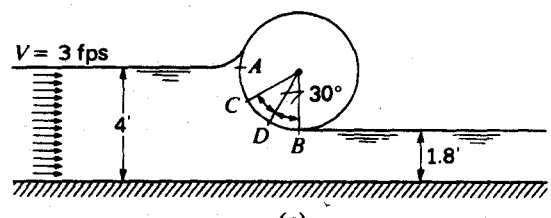
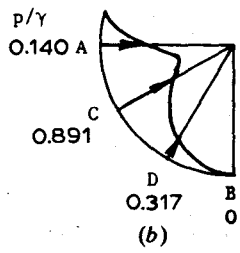


Fig. 25-5

25.16 A 4-ft-diameter cylindrical drum is fixed in an open channel 5 ft wide. Water flows at the rate of 60 cfs beneath the drum, as shown in Fig. 25-6*a*. Sketch the flow net, and infer values of the pressure at points *A*, *C*, *D*, and *B*. Sketch the pressure distribution, and by numerical integration estimate the horizontal thrust on the cylinder.



(*a*)



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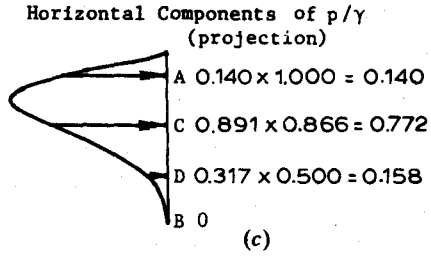


(*c*)

Fig. 25-6

▮ Sketch streamlines by eye and use continuity to determine velocities. High degree of accuracy is not to be expected. See Fig. 25-6*b*. At A, $p/\gamma = p_s/\gamma = V^2/2g = 3^2/2g = 0.140$ ft. At B, in contact with atmosphere ($p_{atm} = 0$ psig): $p/\gamma = 0$. At C, from flow net, $V \approx 4$ fps: $4 + 3^2/2g = (4 - 1.0) + p/\gamma + 4^2/2g$; $p/\gamma = 0.891$ ft. At D, from flow net, $V \approx 10$ fps: $4 + 3^2/2g = (4 - 1.73) + p/\gamma + 10^2/2g$; $p/\gamma = 0.317$ ft. See Fig. 25-6*c*. Area under projected p/γ diagram $\approx$ avg horizontal component = area/2.2 = 0.36 ft = 0.16 psi, horizontal thrust $\approx$ 0.16(2.2)(12)(5)(12) = 253 lb.

25.17 For the two-dimensional flow of a frictionless incompressible fluid against a flat plate normal to the initial velocity, the stream function is given by $\psi = -2axy$, while its *conjugate function*, the velocity potential, is $\phi = a(x^2 - y^2)$ where a is a constant and the flow is symmetrical about the yz plane. By direct differentiation demonstrate that these functions satisfy the equation $(\partial^2\phi/\partial x^2) + (\partial^2\phi/\partial y^2) = 0$. Using a scale of 1 in = 1 unit of distance, plot the streamlines given by $\psi = \pm 2a, \pm 4a, \pm 6a, \pm 8a$, and the equipotential lines given by $\phi = 0, \pm 2a, \pm 4a, \pm 6a, \pm 8a$. Observe that this flow net also gives the ideal flow around an inside square corner.

▮ Given: $\psi = -2axy$; $\phi = a(x^2 - y^2)$. Differentiating: $\partial\psi/\partial y = -2ax$; $\partial\phi/\partial x = 2ax$; $-\partial\psi/\partial x = 2ay$; $\partial\phi/\partial y = a(-2y)$; $\partial^2\phi/\partial x^2 = 2a$; $\partial^2\phi/\partial y^2 = -2a$; thus the equation indicated is satisfied. See Fig. 25-7 for required plot.

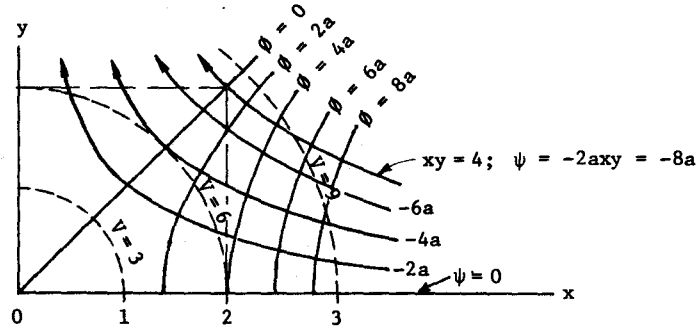


Fig. 25-7

25.18 (*Equivalent Source Method*). The flow around the body of Fig. 25-8*a* may be considered as that due to the sum of two velocity potentials, $\phi_1 = -Ux$, representing an undisturbed flow of velocity U in the x direction, and $\phi_2 = -S \ln r$, representing the radial flow from a source located inside the body behind the stagnation point. To relate U and S, it is observed that the total flow $2\pi S$ from the source (which is hydrodynamically equivalent to the body itself) must be equal to the flow of the main stream which is not passing through the body of width b, or $2\pi S = Ub$. This gives $\phi_2 = -(Ub/2\pi) \ln r$. The velocity at any point of the field is given by the vector sum of the components U and $v_r = -\partial\phi_2/\partial r$. (***a***) Prove that the distance from source to stagnation point is as given in Fig. 25-8*a*.

There follows an ingenious method of plotting the boundary of such a streamlined body, as shown in Fig. 25-8*b*. Suppose that the streamlines in the undisturbed flow are spaced a distance a apart, where $b/2a = n$, an integer. Next divide the upper half of the source into n radial sectors, each of angle α, that is, $n\alpha = \pi$. Then the undisturbed flow between the x axis and the first streamline is associated with the source flow in the first sector from the stagnation point. Thus the intersection of the first streamline with the first line must be a point on the boundary of the body, through which there can be no flow. Similarly, the intersection of the horizontal line at $2a$ with the radial line at 2α forms another point, and so on. Further streamlines can be plotted by connecting successive intersections of the original horizonal lines with the radial lines, recognizing that the same flow must exist between any adjacent pair of streamlines. Thus the intersection of a horizontal line ea above the axis with a radial line at $f\alpha$ from the stagnation point must lie on a streamline which is $(e - f)a$ distant from the axis in the undisturbed region, where e and f are integers. (***b***) Given $U = 18$ fps and $b = 34$ ft, determine the velocity vector at point E of Fig. 25-8*b*.

▮ (***a***) Given: source potential, $\phi_2 = -(Ub/2\pi) \ln r$; $v_r = -\partial\phi_2/\partial r = Ub/2\pi r$. Setting $v_r = U$ for stagnation condition (on tip of nose) and solving for r, the desired distance behind the nose, $r = b/2\pi$.

(***b***) For $b = 34$ ft, distance to stagnation point $= -34/2\pi = -5.41$ ft. Since $b/2a = n = 6 = 34/2a =$ (the number of radial sectors from half of source), streamline spacing $= a = \frac{34}{12} = 2.83$ ft. From Fig. 25-8*b*: At E, $r = 3a = 3(2.83) = 8.49$ ft and $v_r = Ub/2\pi r = (18)(34)/[2\pi(8.49)] = 11.47$ fps. By superposition, $V = (U^2 + v_r^2)^{1/2} = (18^2 + 11.47^2)^{1/2} = 21.3$ fps. Direction relative to axis is $\tan^{-1}(11.47/18) = 32.5°$.

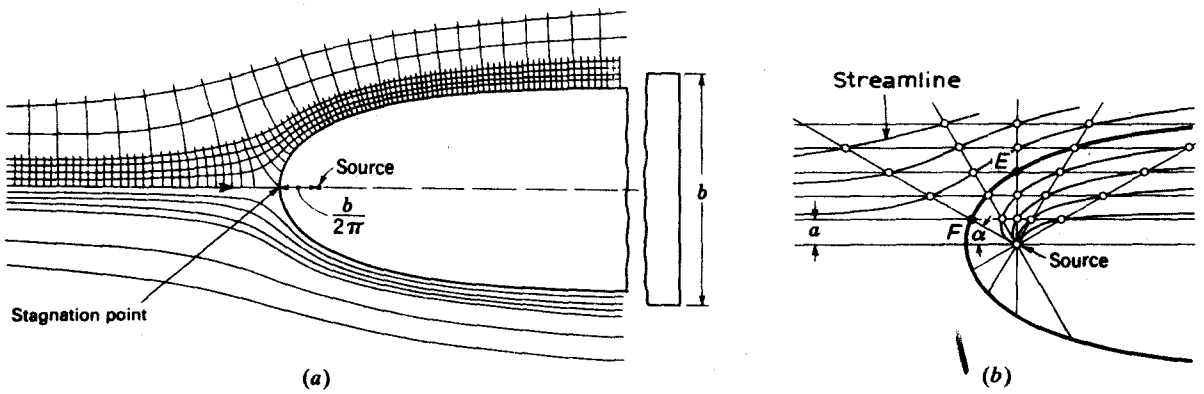


Fig. 25-8

25.19 Find the flow velocity at point F of Fig. 25-8b.

▮ At F ($\alpha = 30°$), $r = a/\sin 30° = 2.83/0.5 = 5.66$ ft.

$$v_r = \partial\phi_2/\partial r = Ub/2\pi r = 18(34)/[2\pi(5.66)] = 17.21 \text{ fps}$$

$$V^2 = U^2 + v_r^2 - 2Uv_r \cos\alpha = 18^2 + 17.21^2 - 2(18)17.21\cos 30° \qquad V = 9.14 \text{ fps}$$

25.20 Superimpose a point source ($Q = 95$ cfs/ft) on a rectilinear flow field ($U = 18$ fps). Plot the body contour at $\theta = 30, 60, 90, 120, 150, 180°$. Compute the velocities along the body contour at these points, as well as the pressures, assuming $\rho = 1.94$ slug/ft^3 and zero pressure in the undisturbed rectilinear flow field.

▮ Given: $U = 18$ fps, $v_r = 95/2\pi r$ fps. Distance from source to stagnation point: $U = v_r$, $r = 5.28/2\pi = 0.840$ ft. Refer to Prob. 25.18; for upper half let $n = 6$, thus $\theta = 30°$. $q = Ua = 95/2n$; thus $a = 95/(2nU) = 0.440$ ft. See Fig. 25-9. Scale off radial distances to points a, b, c, d, e, and f; $v_x = U - v_r\cos\theta$, $v_y = v_r \sin\alpha$. Vector addition: $V = (v_x^2 + v_y^2)^{1/2}$.

point	r, ft	θ	v_r, fps	v_x, fps	v_y, fps	V, fps
S	0.84	0	18.0	0	0	0
a	0.85	30°	17.8	2.6	8.9	9.3
b	1.00	60°	15.1	10.4	13.1	16.7
c	1.28	90°	11.8	18.0	11.8	21.5
d	1.94	120°	7.8	21.9	6.8	22.9
e	4.20	150°	3.6	21.1	1.8	21.2

$$p_0 + \rho V_0^2/2 = p + \rho V^2/2 \qquad 1.94(18^2/2) = 314 \text{ lb/ft}^2 = p + \rho V^2/2$$

Therefore, $p = 314 - \rho V^2/2$. Hence desired pressures are 314, 230, 43, −134, −195, −122 psf.

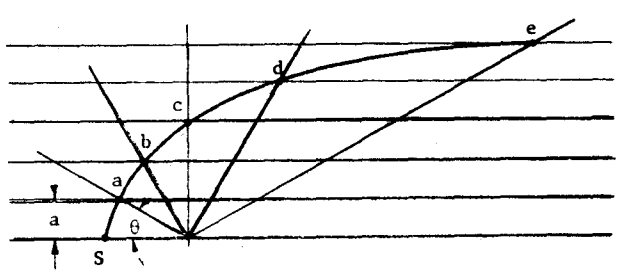


Fig. 25-9

Appendixes

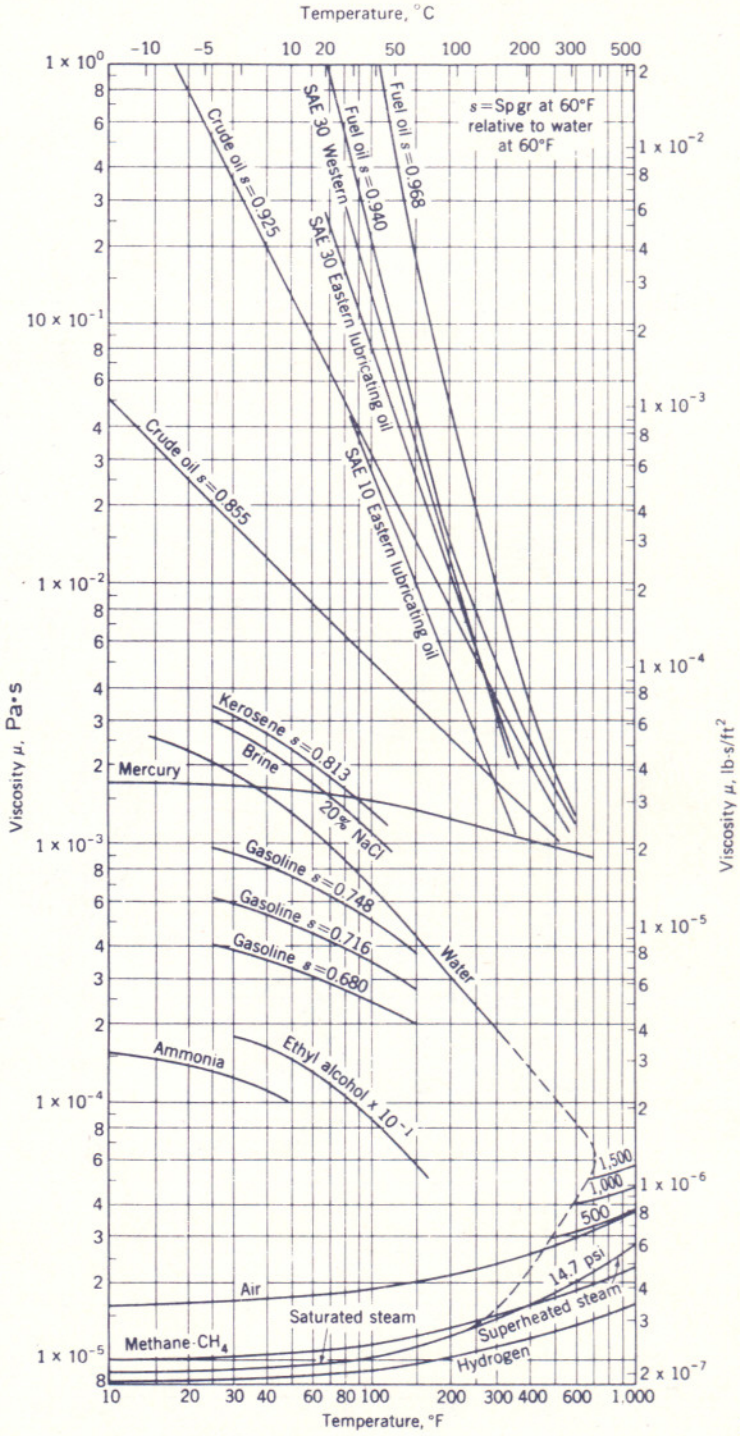


Fig. A-1

Absolute viscosity μ of fluids.

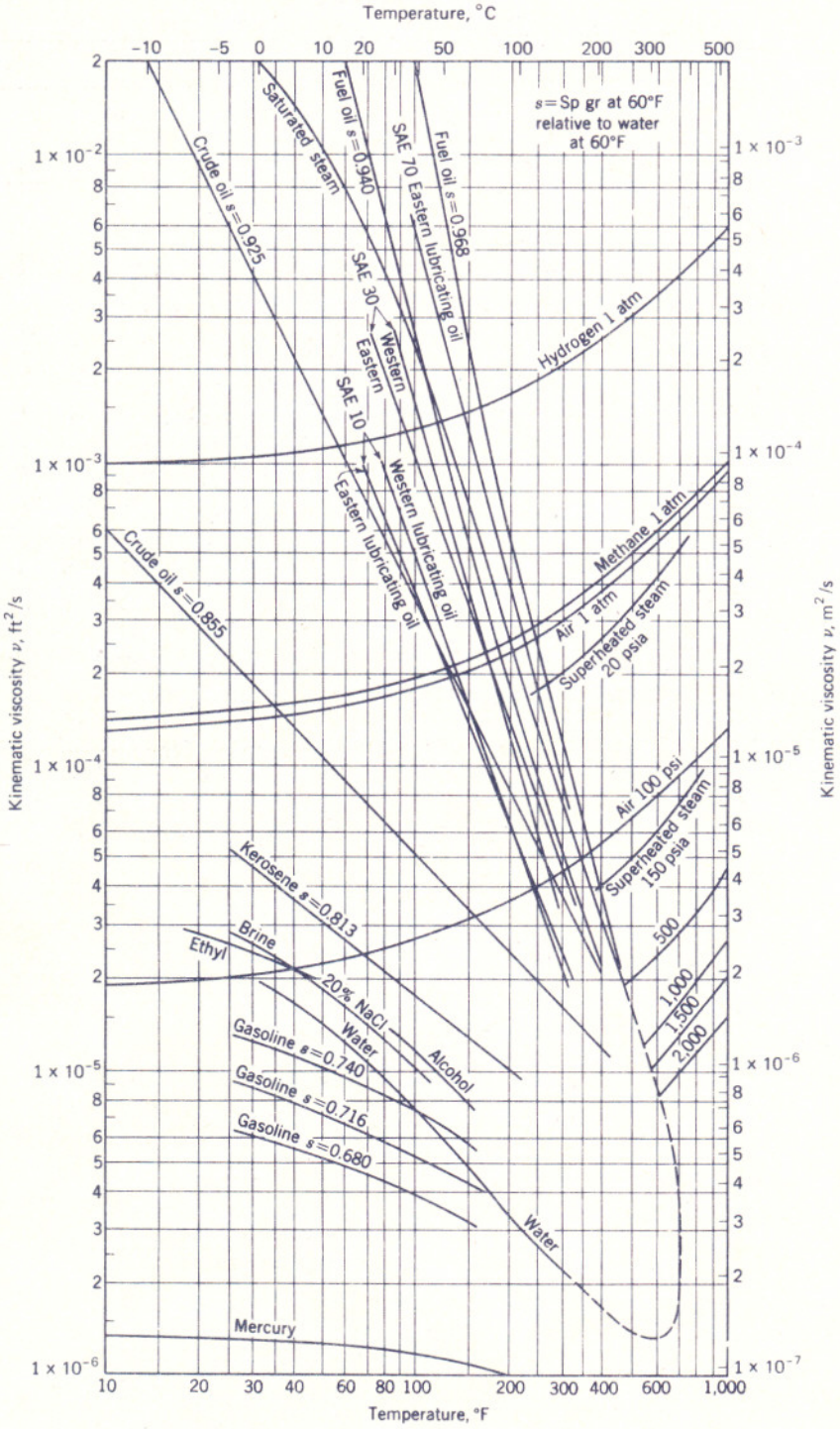


Fig. A-2

Kinematic viscosity ν of fluids.

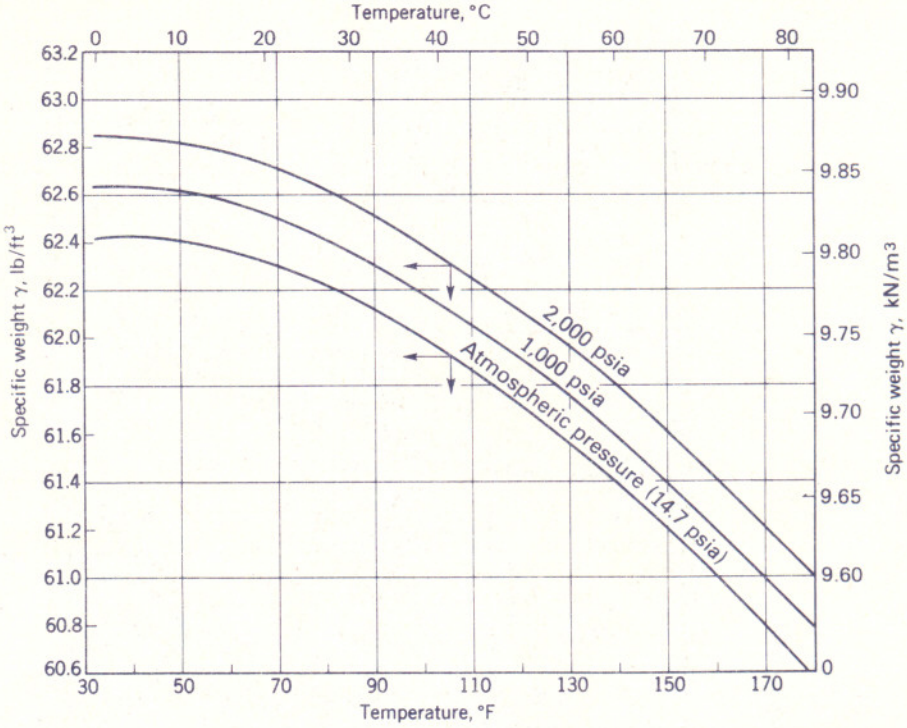


Fig. A-3

Specific weight γ of pure water as a function of temperature and pressure for condition where $g = 32.2\ \text{ft/s}^2$ ($9.81\ \text{m/s}^2$).

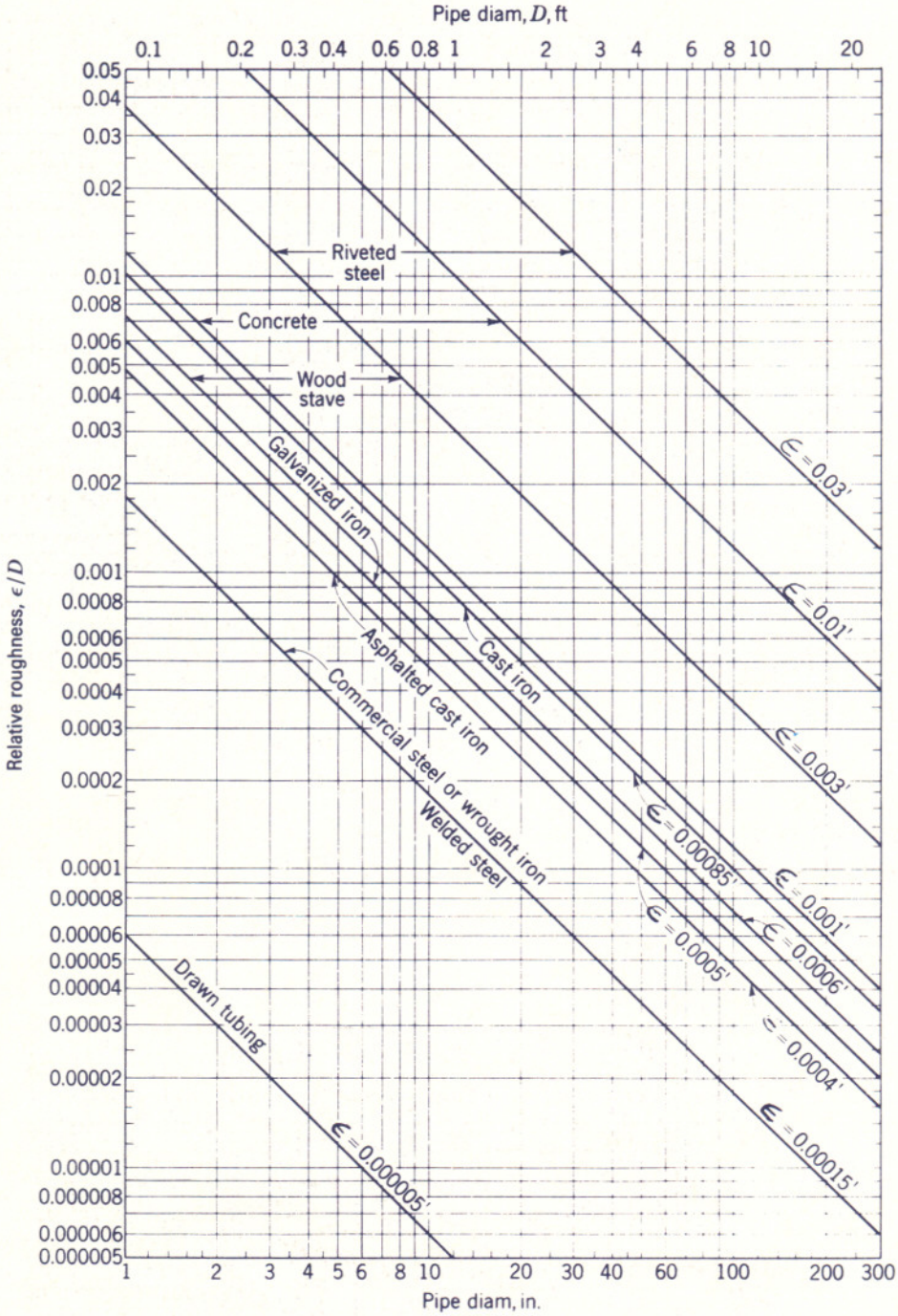


Fig. A-4

Roughness factors (ϵ expressed in feet) for commercial pipes.

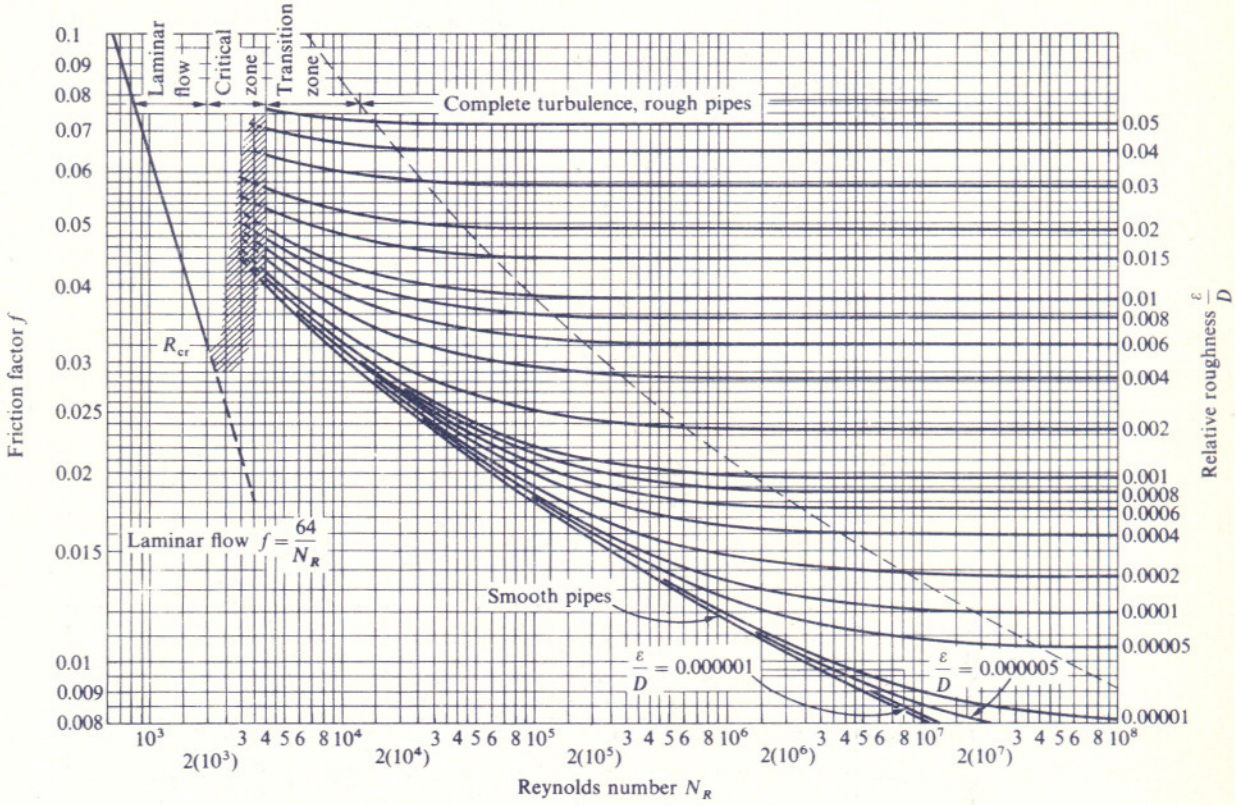


Fig. A-5

Moody diagram.

From Lewis F. Moody, "Friction Factors for Pipe Flows," *ASME Trans.*, vol. 66, pp. 671–684, 1944.

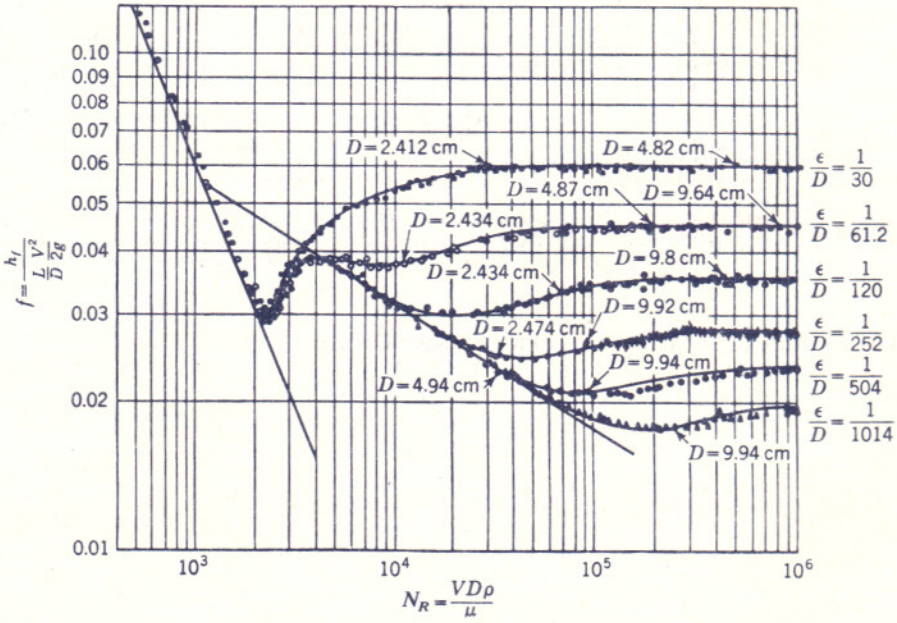


Fig. A-6

Nikuradse's sand-roughened-pipe tests.

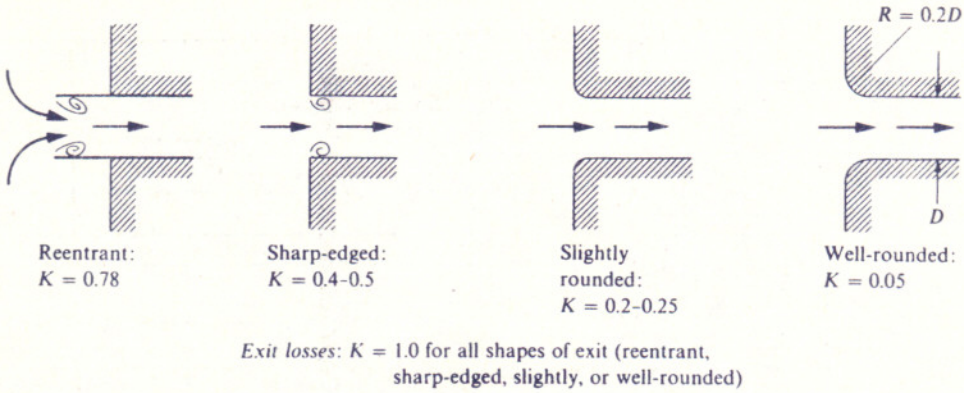


Fig. A-7

Entrance and exit loss coefficients.

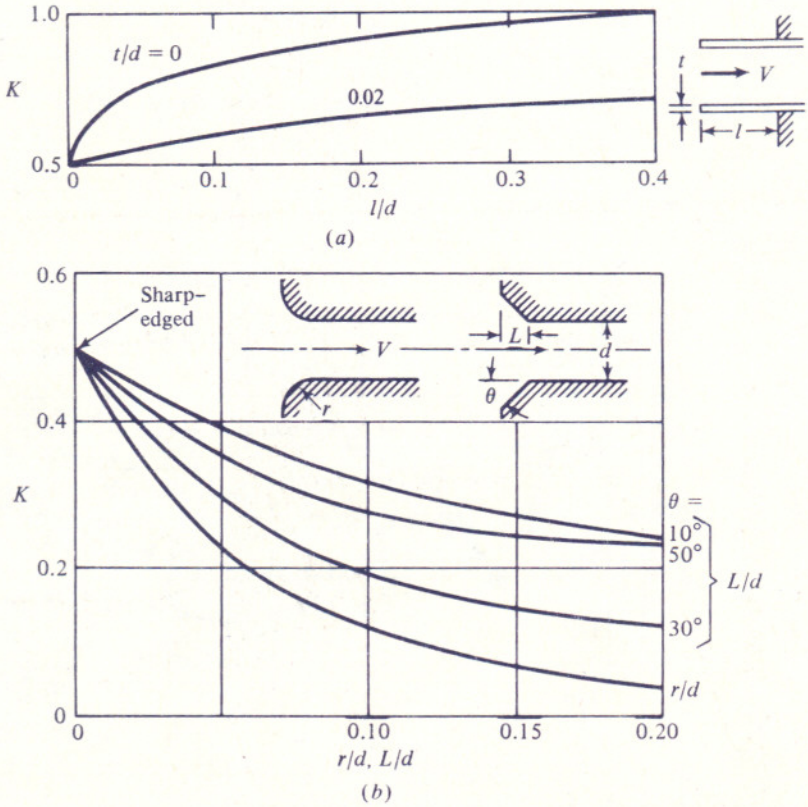


Fig. A-8

Entrance and exit loss coefficients: (*a*) reentrant inlets; (*b*) rounded and beveled inlets. Exit losses are $K \approx 1.0$ for all shapes of exit (reentrant, sharp, beveled, or rounded). (Adapted by permission from *ASHRAE Handbook of Fundamentals*, Atlanta, 1985.)

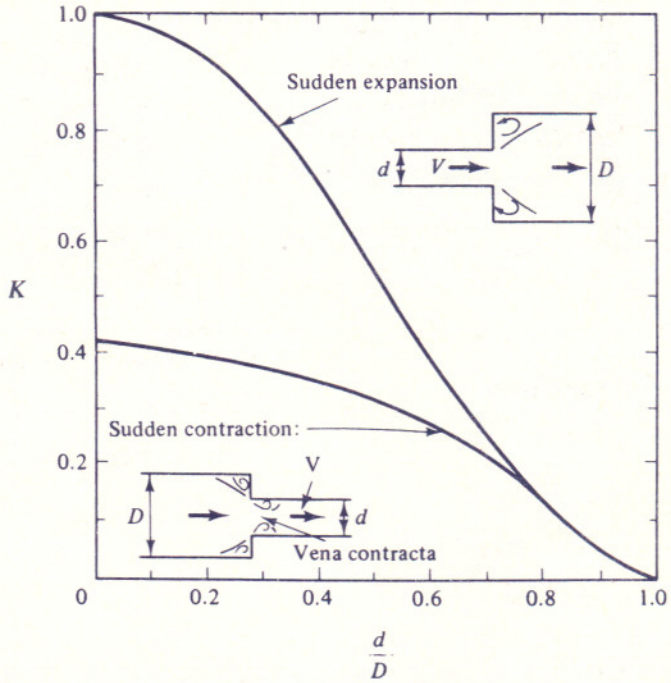


Fig. A-9

Sudden expansion and contraction losses. Note that the loss is based on velocity head in the small pipe.

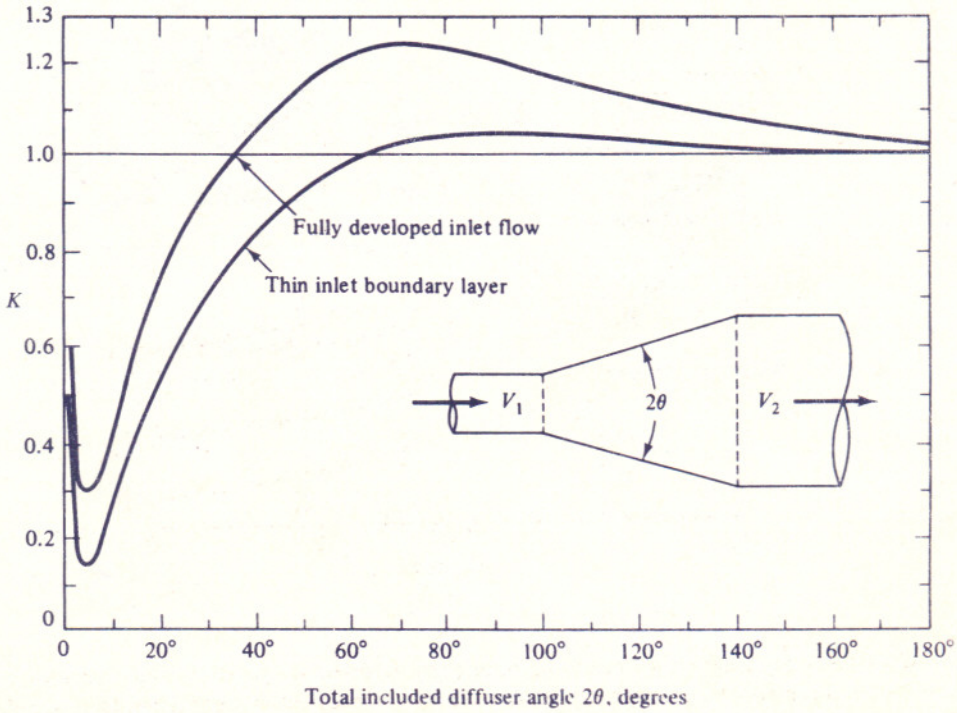
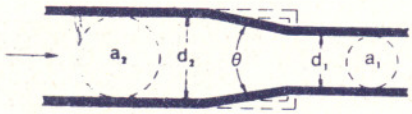


Fig. A-10

Flow losses in a gradual conical expansion region.

SUDDEN AND GRADUAL CONTRACTION



If: $\theta \leq 45°$ . . . . . K_2 = Formula 1

$45° < \theta \leq 180°$ . . . K_2 = Formula 2

• **Formula 1**

$$K_2 = \frac{0.8\left(\sin\frac{\theta}{2}\right)(1-\beta^2)}{\beta^4} = \frac{K_1}{\beta^4}$$

• **Formula 2**

$$K_2 = \frac{0.5\,(1-\beta^2)\sqrt{\sin\frac{\theta}{2}}}{\beta^4} = \frac{K_1}{\beta^4}$$

$$\beta = \frac{d_1}{d_2}$$

$$\beta^2 = \left(\frac{d_1}{d_2}\right)^2 = \frac{a_1}{a_2}$$

Subscript 1 defines dimensions and coefficients with reference to the smaller diameter.

Subscript 2 refers to the larger diameter.

Fig. A-11

From "Flow of Fluids through Valves, Fittings, and Pipe," *Crane Co. Tech. Pap. 410,* New York, 1985.

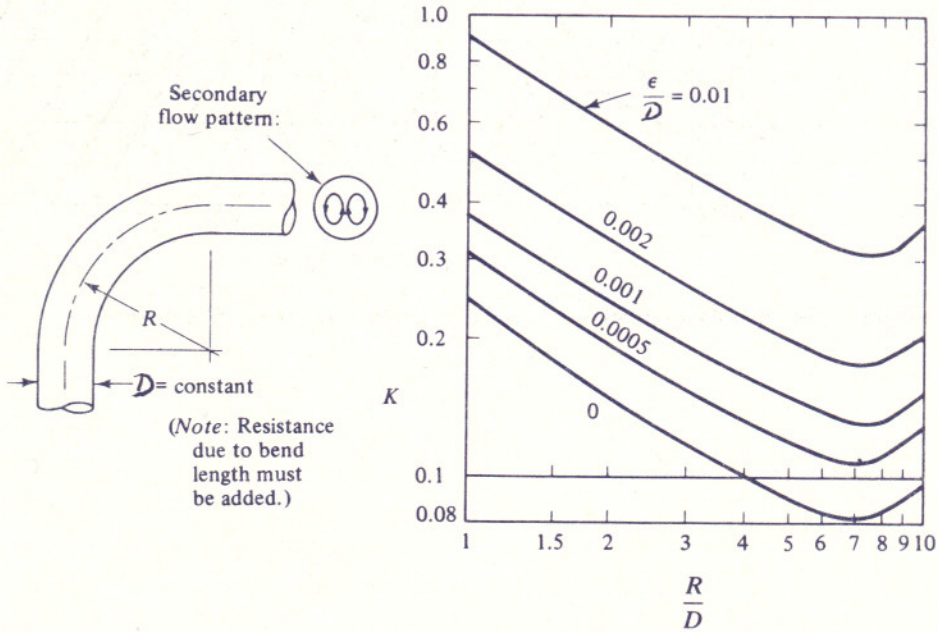


Fig. A-12

Resistance coefficients for 90° bends.

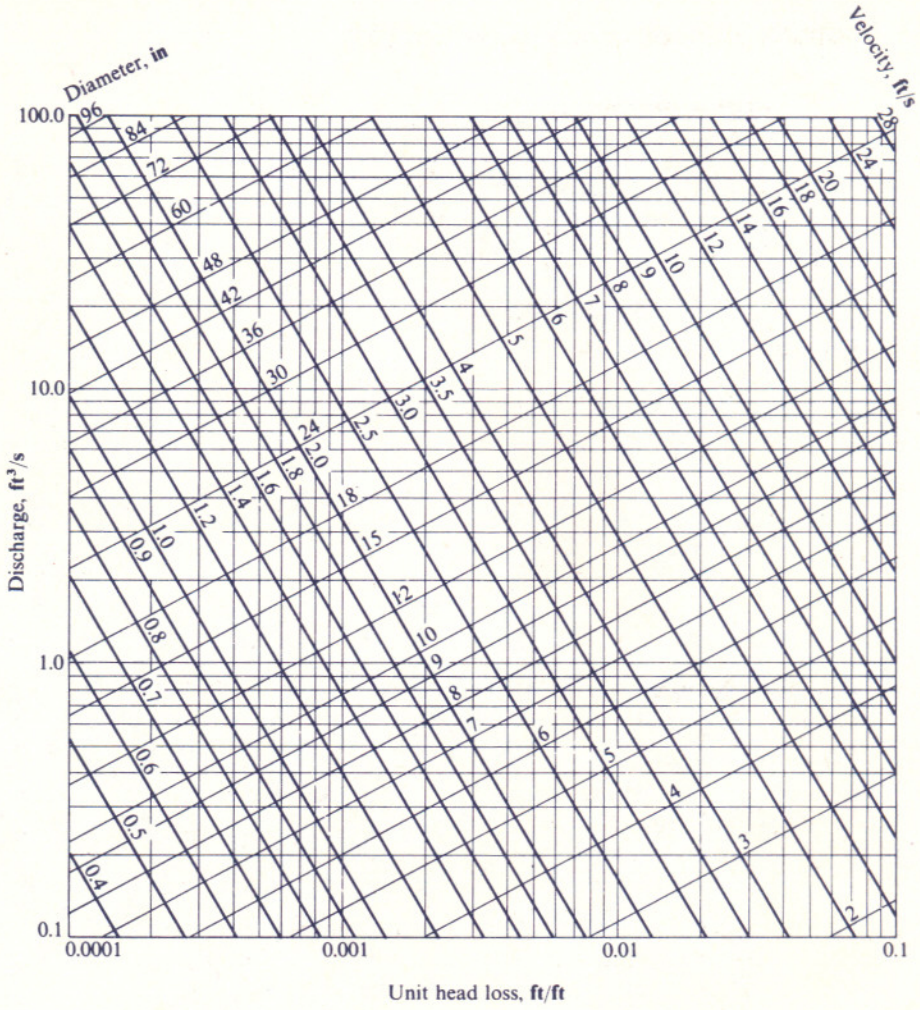


Fig. A-13

Pipe diagram: Hazen-Williams equation ($C = 120$); English Gravitational System.

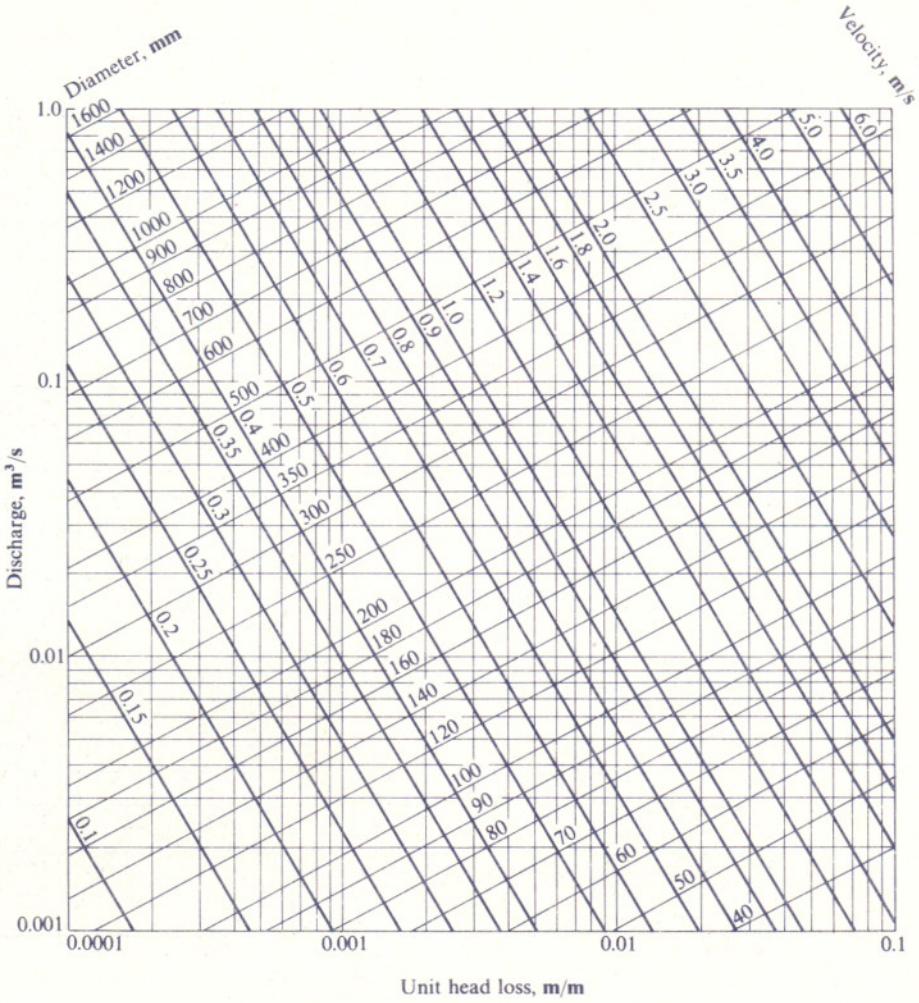


Fig. A-14

Pipe diagram: Hazen-Williams equation ($C = 120$); International System.

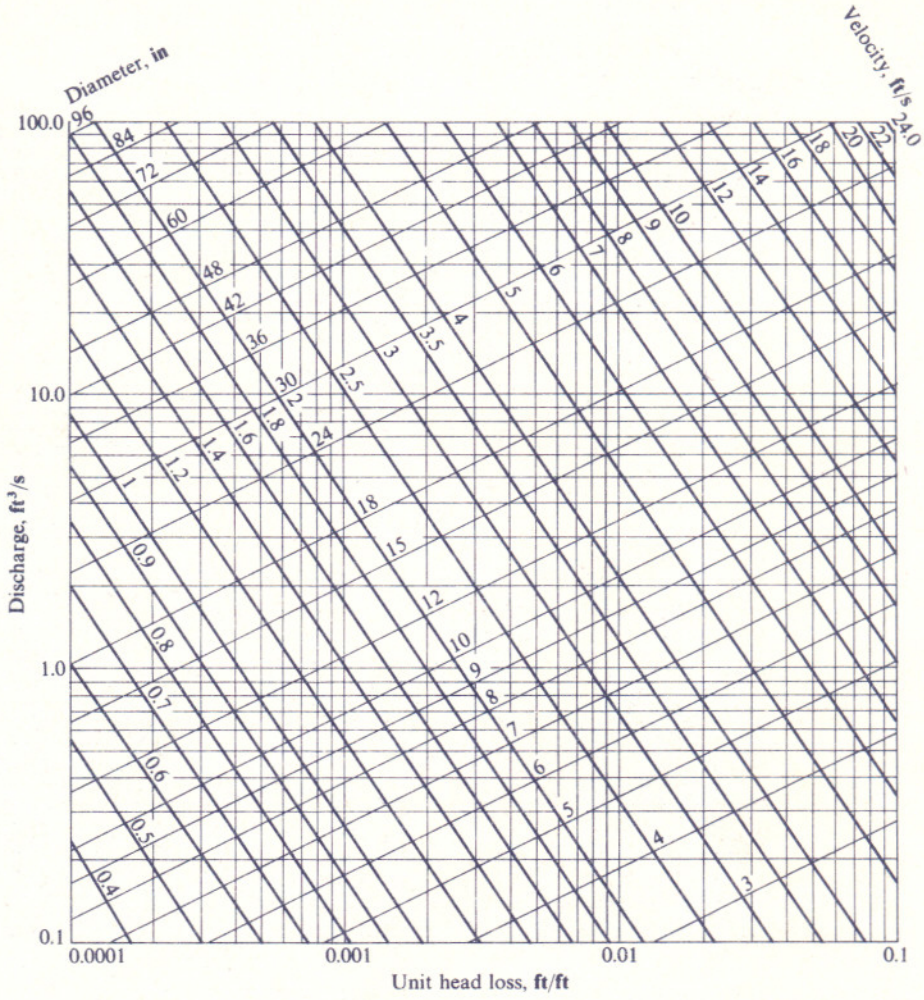


Fig. A-15

Pipe diagram: Manning equation ($n = 0.013$); English Gravitational System.

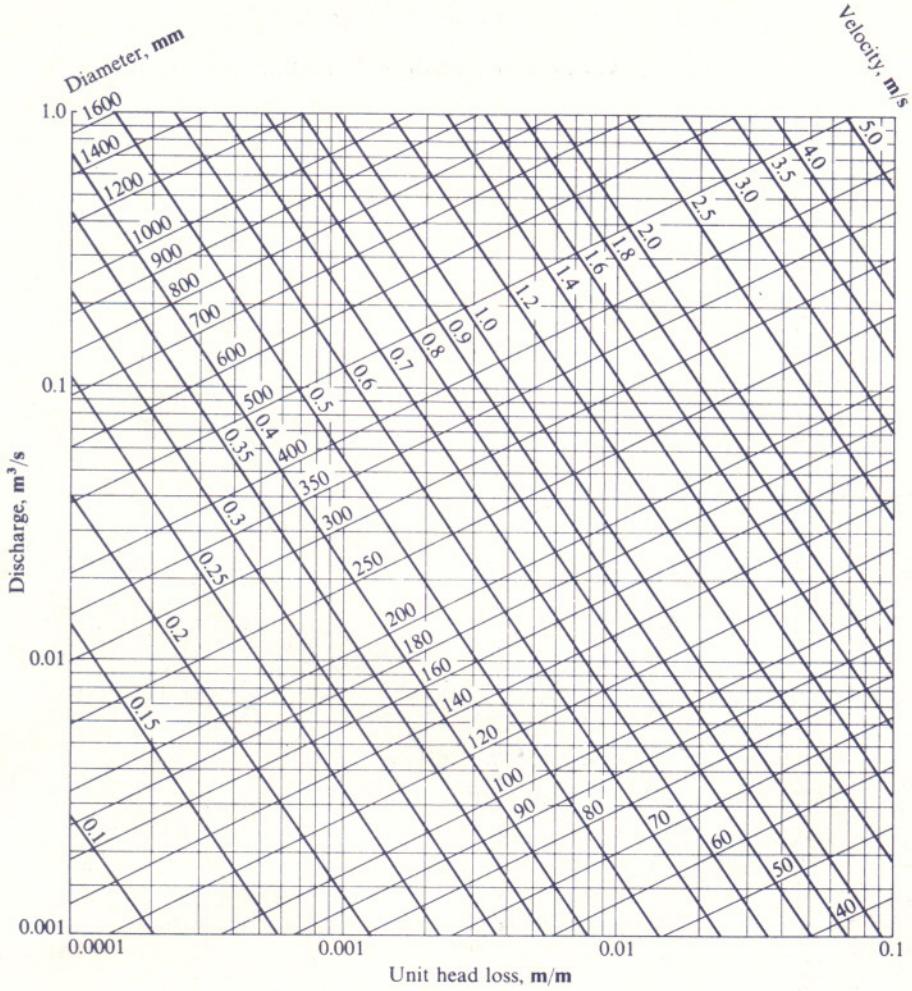
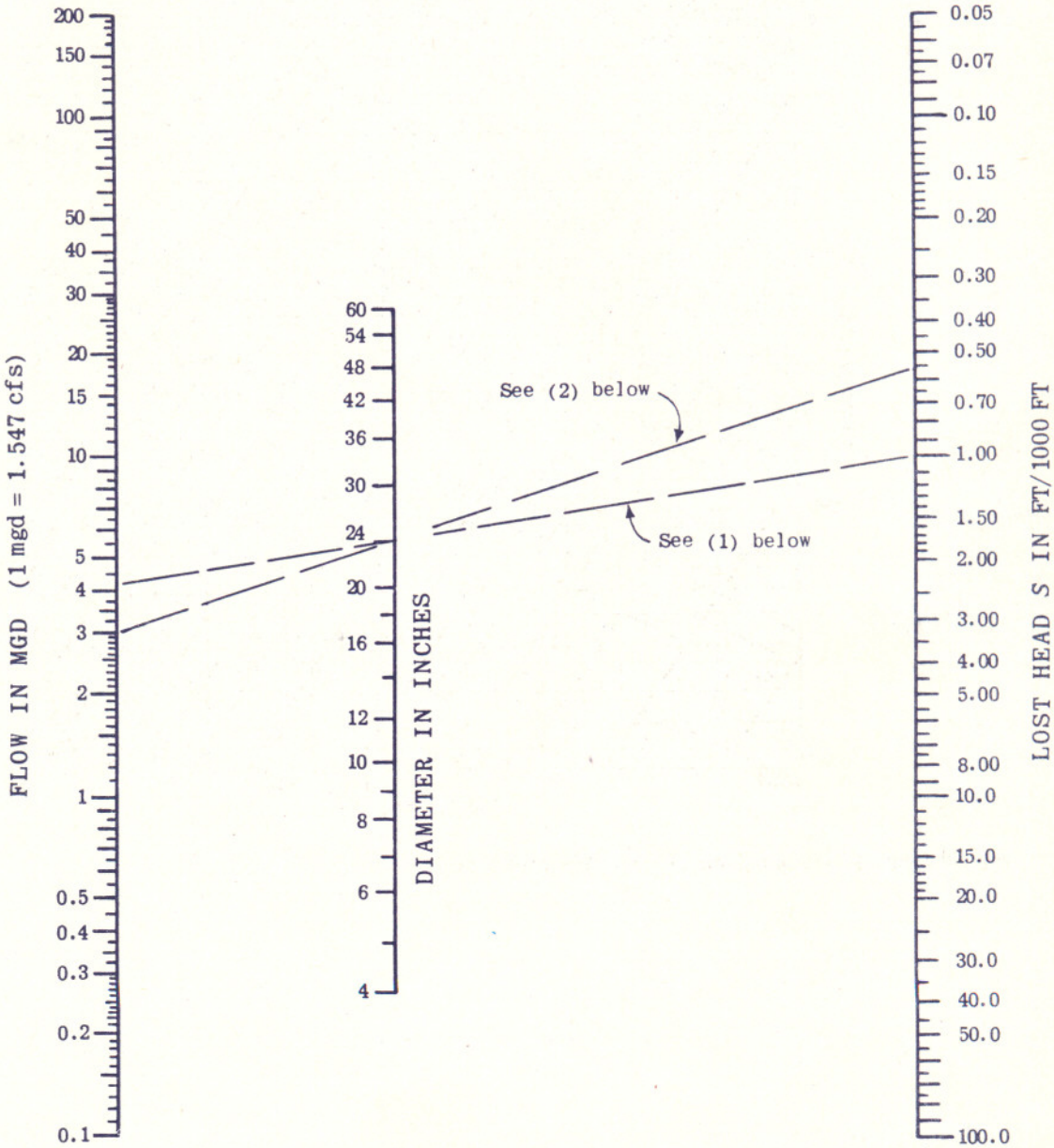


Fig. A-16

Pipe diagram: Manning equation ($n = 0.013$); International System.

FLOW CHART

HAZEN-WILLIAMS FORMULA, $C_1 = 100$



USE OF CHART

(1) Given $D = 24''$, $S = 1.0$ ft/1000 ft, $C_1 = 120$; find flow Q.
Chart gives $Q_{100} = 4.2$ mgd.
For $C_1 = 120$, $Q = (120/100)(4.2) = 5.0$ mgd.

(2) Given $Q = 3.6$ mgd, $D = 24''$, $C_1 = 120$; find Lost Head.
Change Q_{120} to Q_{100}: $Q_{100} = (100/120)(3.6) = 3.0$ mgd.
Chart gives $S = 0.55$ ft/1000 ft.

Fig. A-17

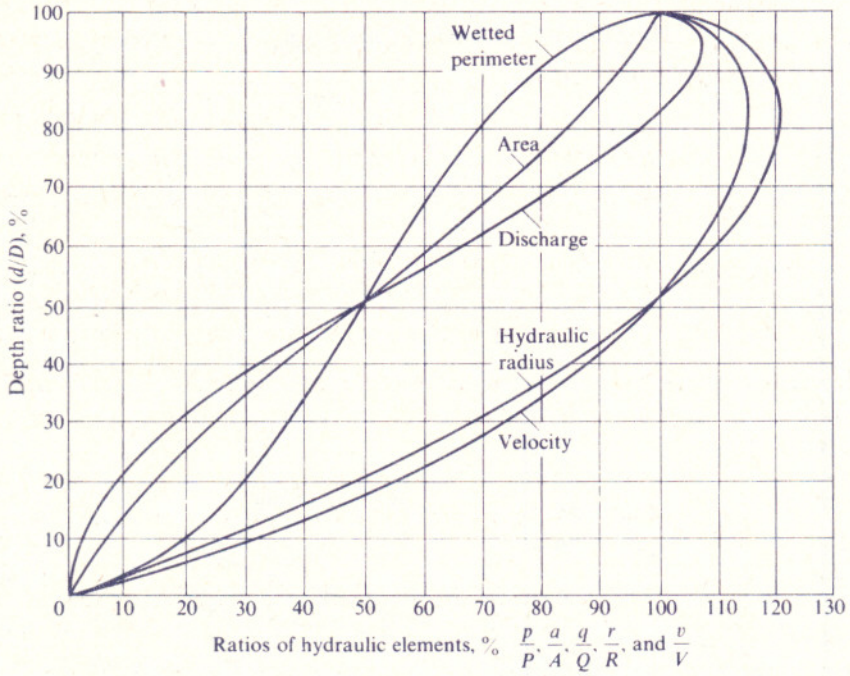


Fig. A-18

Hydraulic elements of a circular section.

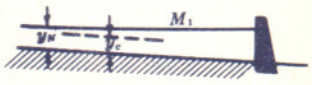
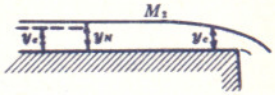
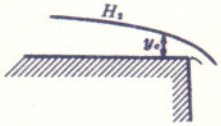
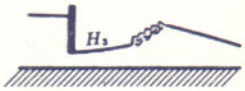
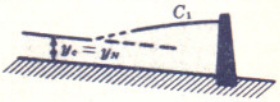
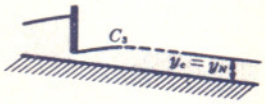
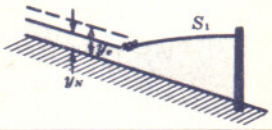
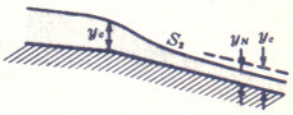
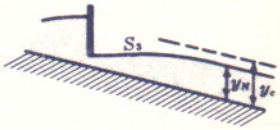
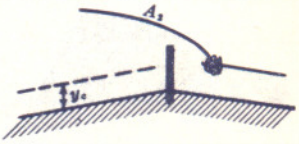
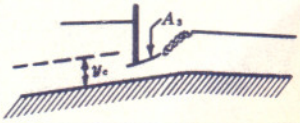
Channel Slope	Depth Relations	$\left(\frac{dy}{dL}\right)$	Type of Profile	Symbol	Type of Flow	Form of Profile
Mild $0 < S < S_c$	$y > y_N > y_c$	+	Backwater	M_1	Subcritical	
	$y_N > y > y_c$	−	Dropdown	M_2	Subcritical	
	$y_N > y_c > y$	+	Backwater	M_3	Supercritical	
Horizontal $S = 0$ $y_N = \infty$	$y > y_c$	−	Dropdown	H_2	Subcritical	
	$y_c > y$	+	Backwater	H_3	Supercritical	
Critical $S_N = S_c$ $y_N = y_c$	$y > y_c = y_N$	+	Backwater	C_1	Subcritical	
	$y_c = y = y_N$		Parallel to bed	C_2	Uniform, Critical	
	$y_c = y_N > y$	+	Backwater	C_3	Supercritical	
Steep $S > S_c > 0$	$y > y_c > y_N$	+	Backwater	S_1	Subcritical	
	$y_c > y > y_N$	−	Dropdown	S_2	Supercritical	
	$y_c > y_N > y$	+	Backwater	S_3	Supercritical	
Adverse $S < 0$ $y_N = \infty$	$y > y_c$	−	Dropdown	A_2	Subcritical	
	$y_c > y$	+	Backwater	A_3	Supercritical	

Fig. A-19

Types of profiles in open-channel flow.

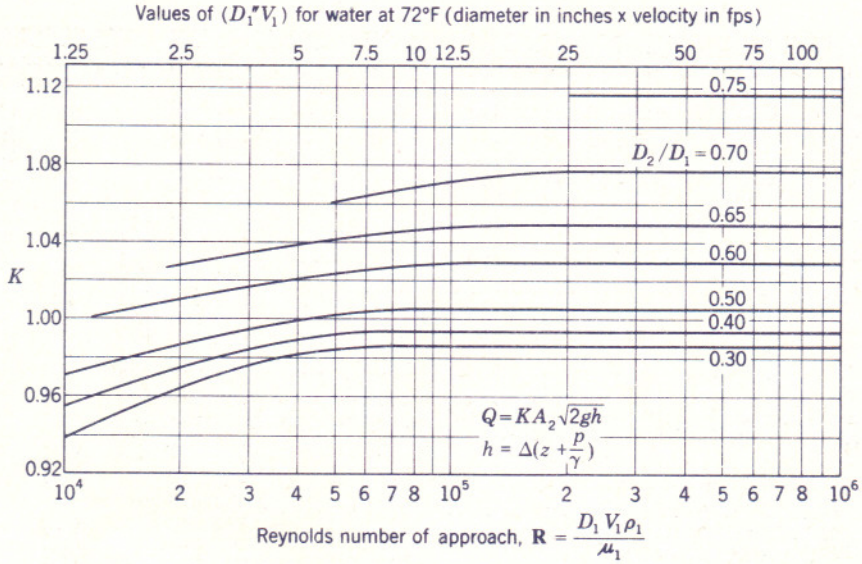


Fig. A-20

Flow coefficients for ISA nozzle. (Adapted from *ASME Flow Measurement*, 1959.)

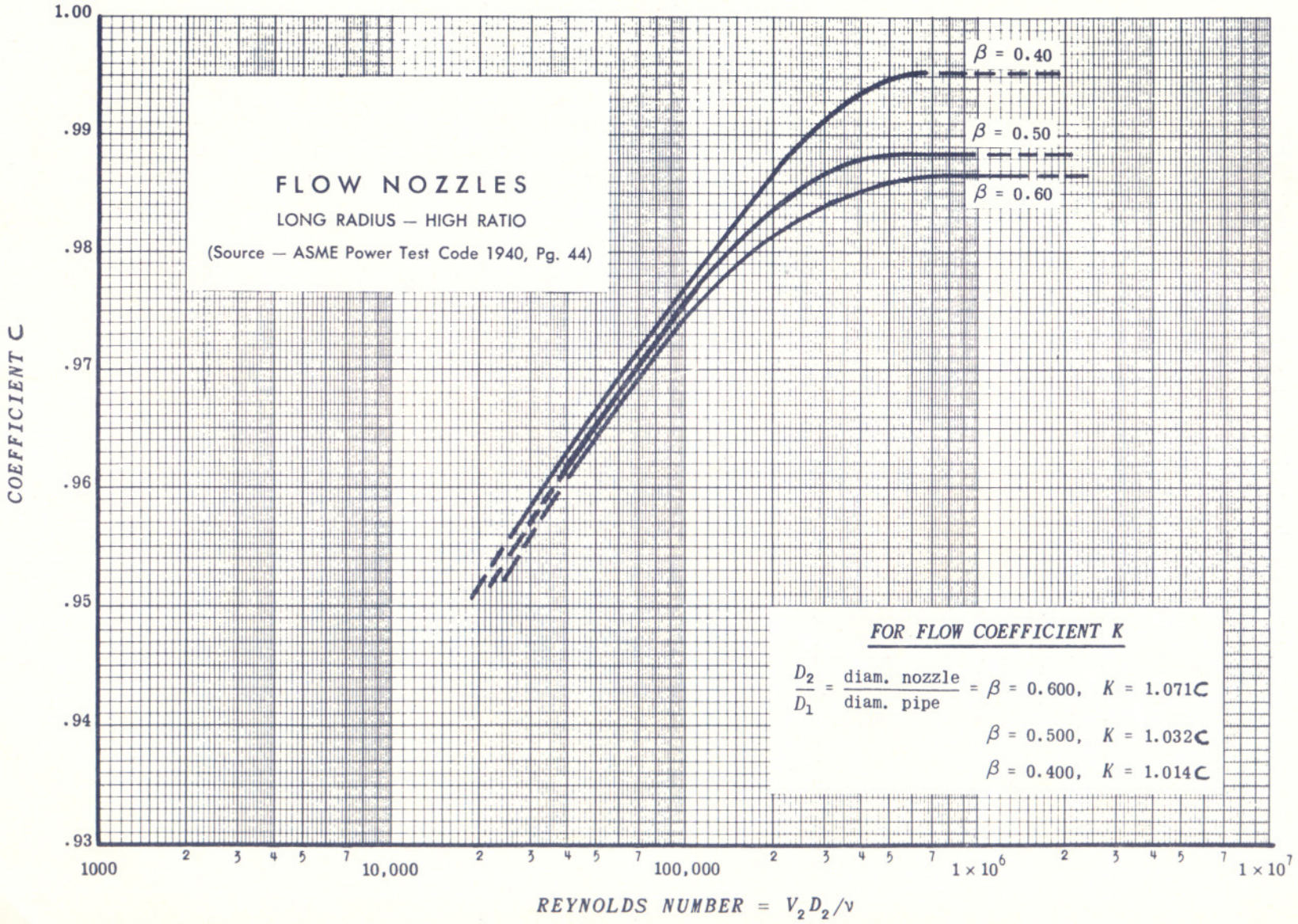

FLOW NOZZLES
LONG RADIUS — HIGH RATIO
(Source — ASME Power Test Code 1940, Pg. 44)
β = 0.40
β = 0.50
β = 0.60
FOR FLOW COEFFICIENT K
D2/D1 = diam. nozzle / diam. pipe = β = 0.600, K = 1.071C
β = 0.500, K = 1.032C
β = 0.400, K = 1.014C
COEFFICIENT C
1.00
.99
.98
.97
.96
.95
.94
.93
1000
10,000
100,000
1 × 10^6
1 × 10^7
REYNOLDS NUMBER = V2D2/ν

Fig. A-21

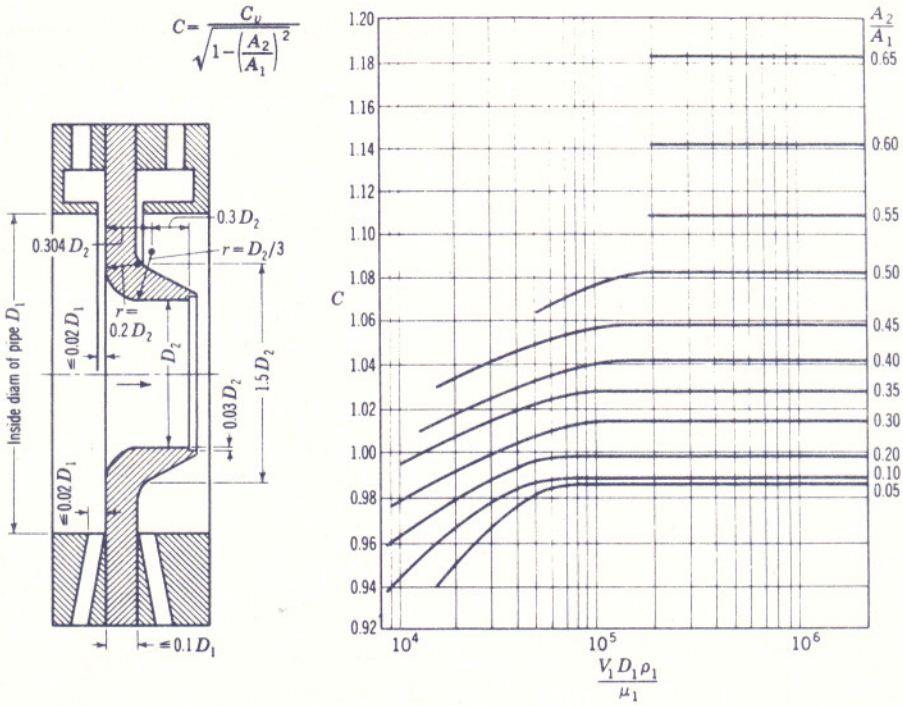


Fig. A-22

ISA (VDI) flow nozzle and discharge coefficients. (Ref. 11 in *NACA Tech. Mem. 952.*)

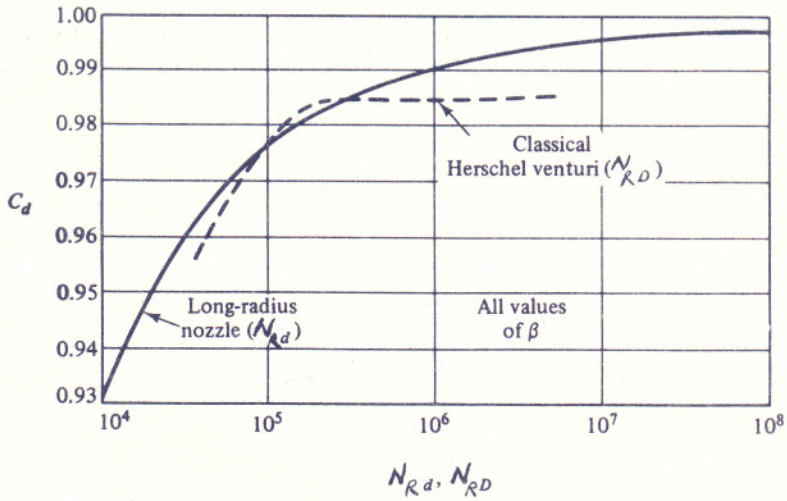


Fig. A-23

Discharge coefficient for long-radius nozzle and classical Herschel-type venturi.

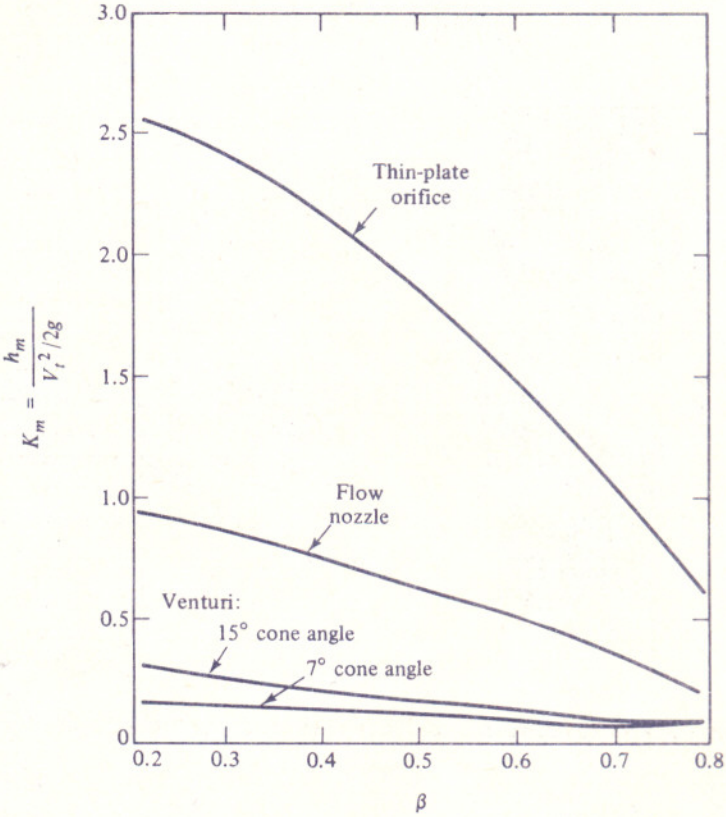


Fig. A-24

Nonrecoverable head loss in Bernoulli obstruction meters. [H. S. Bean (ed.), '*Fluid Meters: Their Theory and Application*,' 6th ed., American Society of Mechanical Engineers, New York, 1971.]

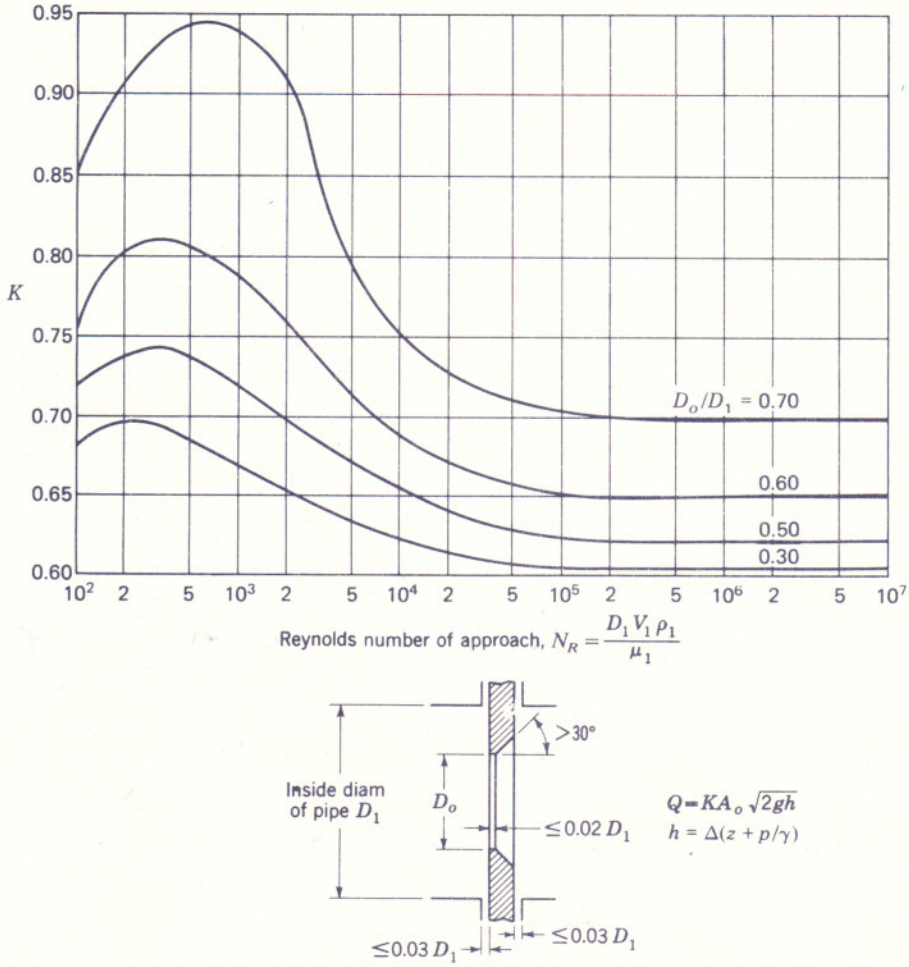


Fig. A-25

VDI orifice meter and flow coefficients for flange taps. (Adapted from *NACA Tech. Mem. 952.*)

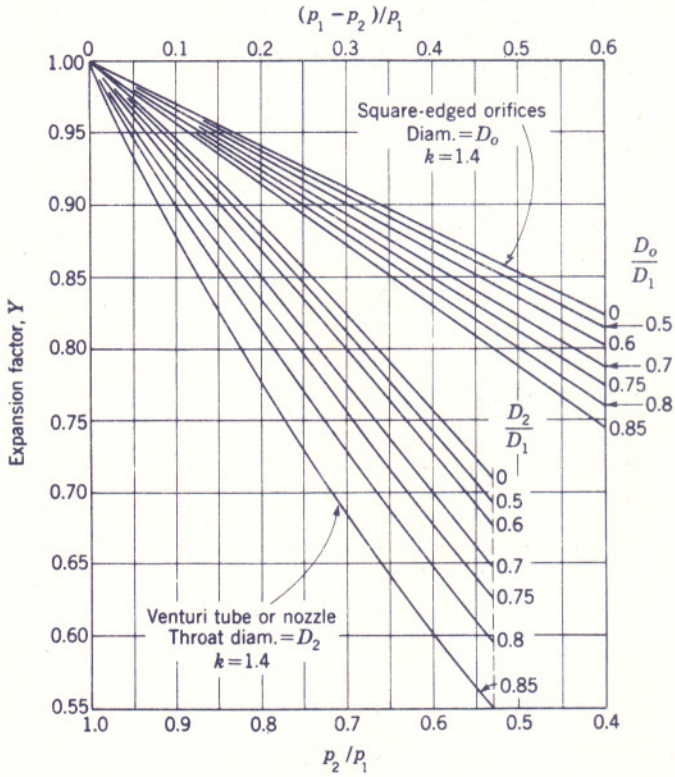


Fig. A-26

Expansion factors.

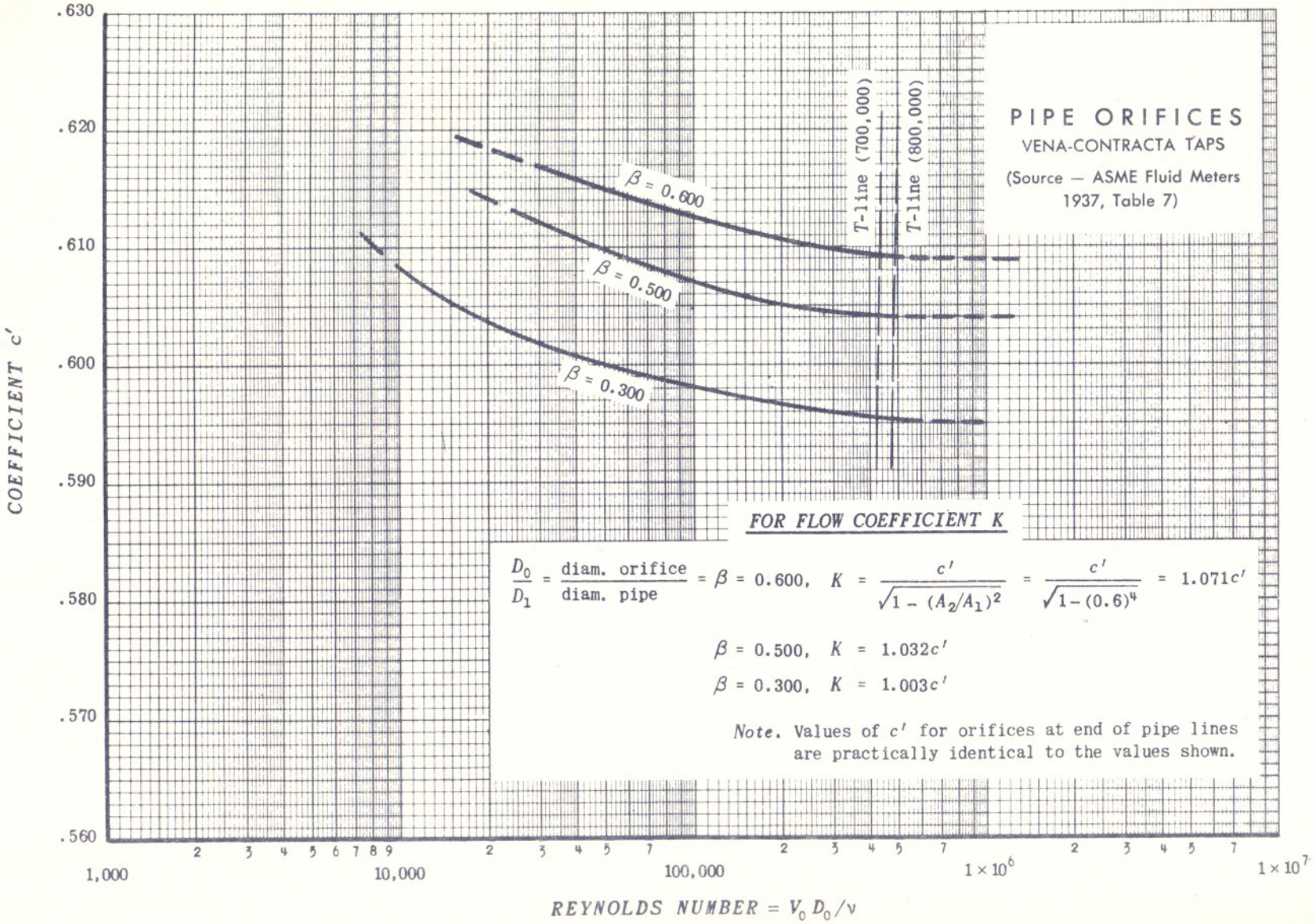

PIPE ORIFICES
VENA-CONTRACTA TAPS
(Source — ASME Fluid Meters 1937, Table 7)
T-line (700,000)
T-line (800,000)
β = 0.600
β = 0.500
β = 0.300
FOR FLOW COEFFICIENT K
$\frac{D_0}{D_1} = \frac{\text{diam. orifice}}{\text{diam. pipe}} = \beta = 0.600, \quad K = \frac{c'}{\sqrt{1-(A_2/A_1)^2}} = \frac{c'}{\sqrt{1-(0.6)^4}} = 1.071c'$
$\beta = 0.500, \quad K = 1.032c'$
$\beta = 0.300, \quad K = 1.003c'$
Note. Values of c′ for orifices at end of pipe lines are practically identical to the values shown.
COEFFICIENT c′
.630
.620
.610
.600
.590
.580
.570
.560
1,000
10,000
100,000
1 × 10^6
1 × 10^7
REYNOLDS NUMBER = V₀ D₀/ν

Fig. A-27

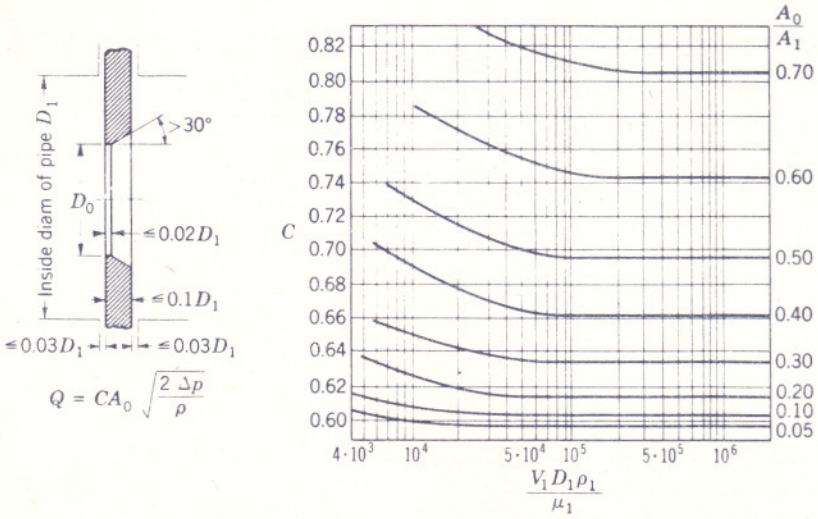


Fig. A-28

VDI orifice and discharge coefficients. (Ref. 11 in *NACA Tech. Mem. 952.*)

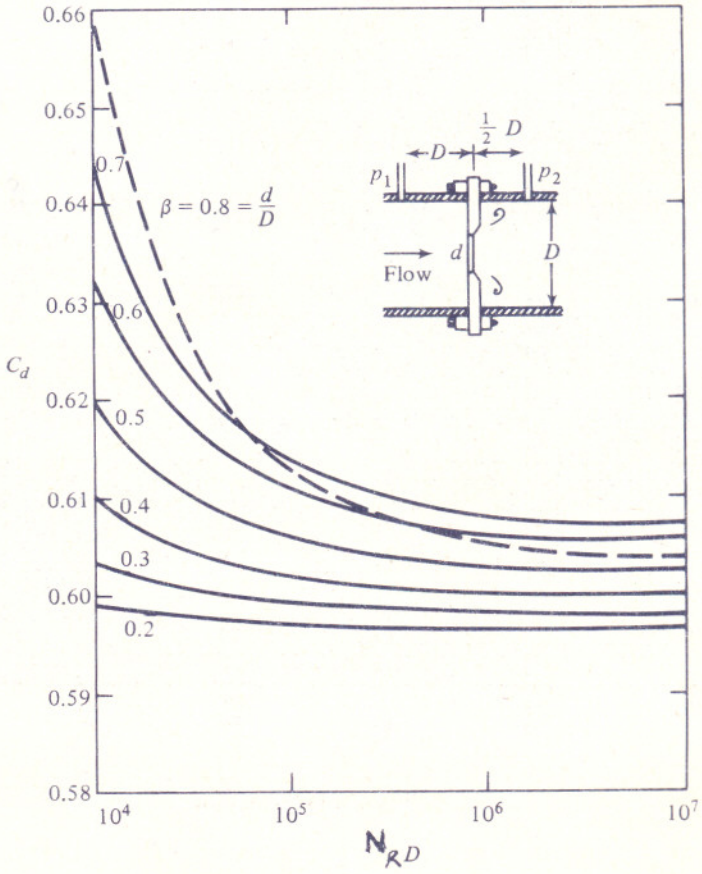
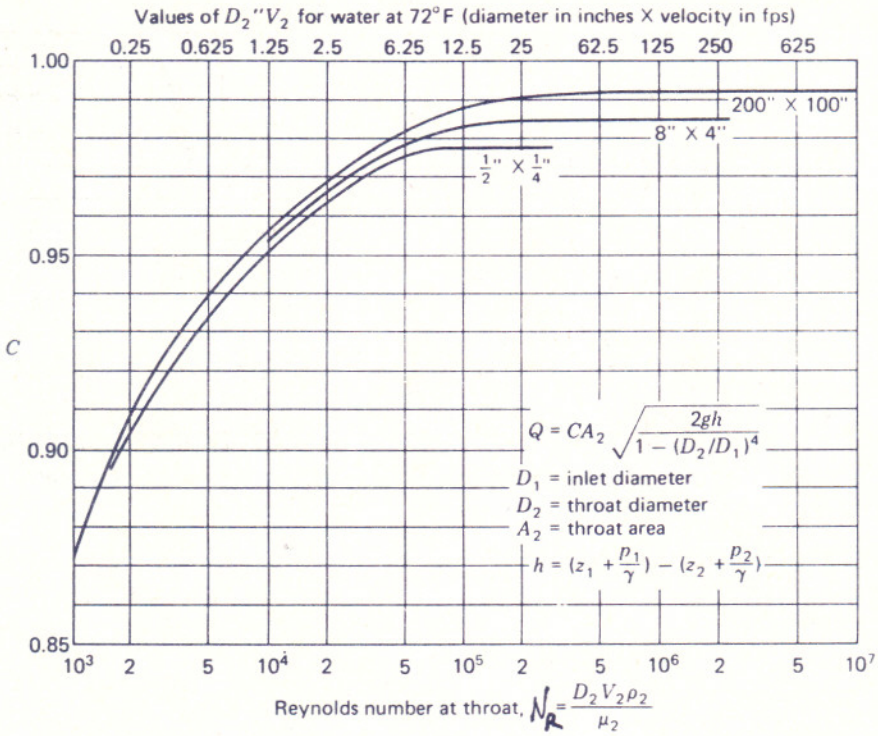


Fig. A-29

Discharge coefficient for a thin-plate orifice with D: $\frac{1}{2}D$.



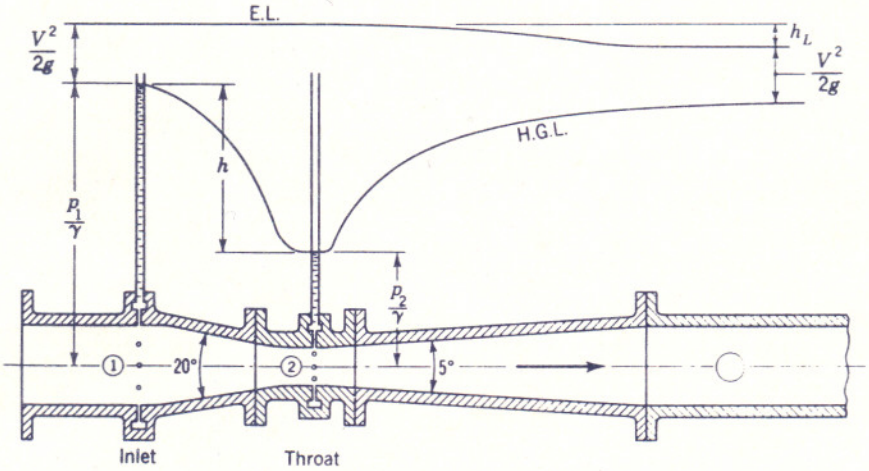


Fig. A-30

Venturi meter with conical entrance and flow coefficients for $D_2/D_1 = 0.5$.

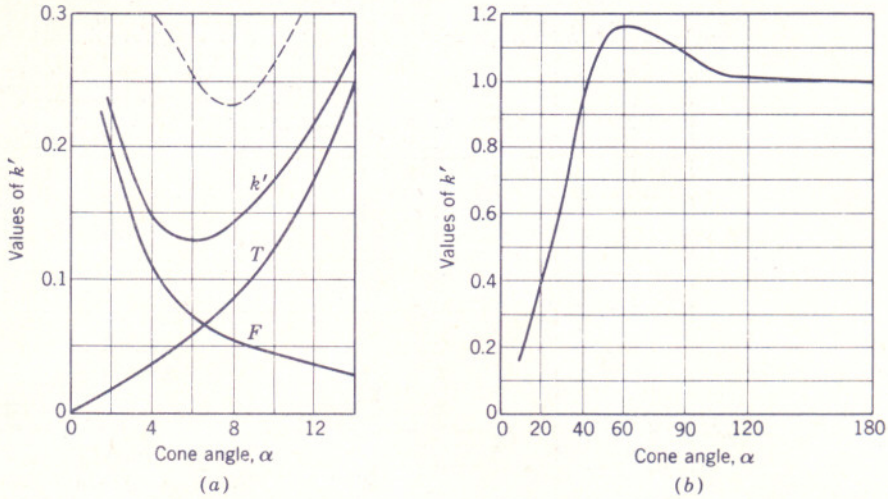


Fig. A-31

Loss coefficient for conical diffusers.

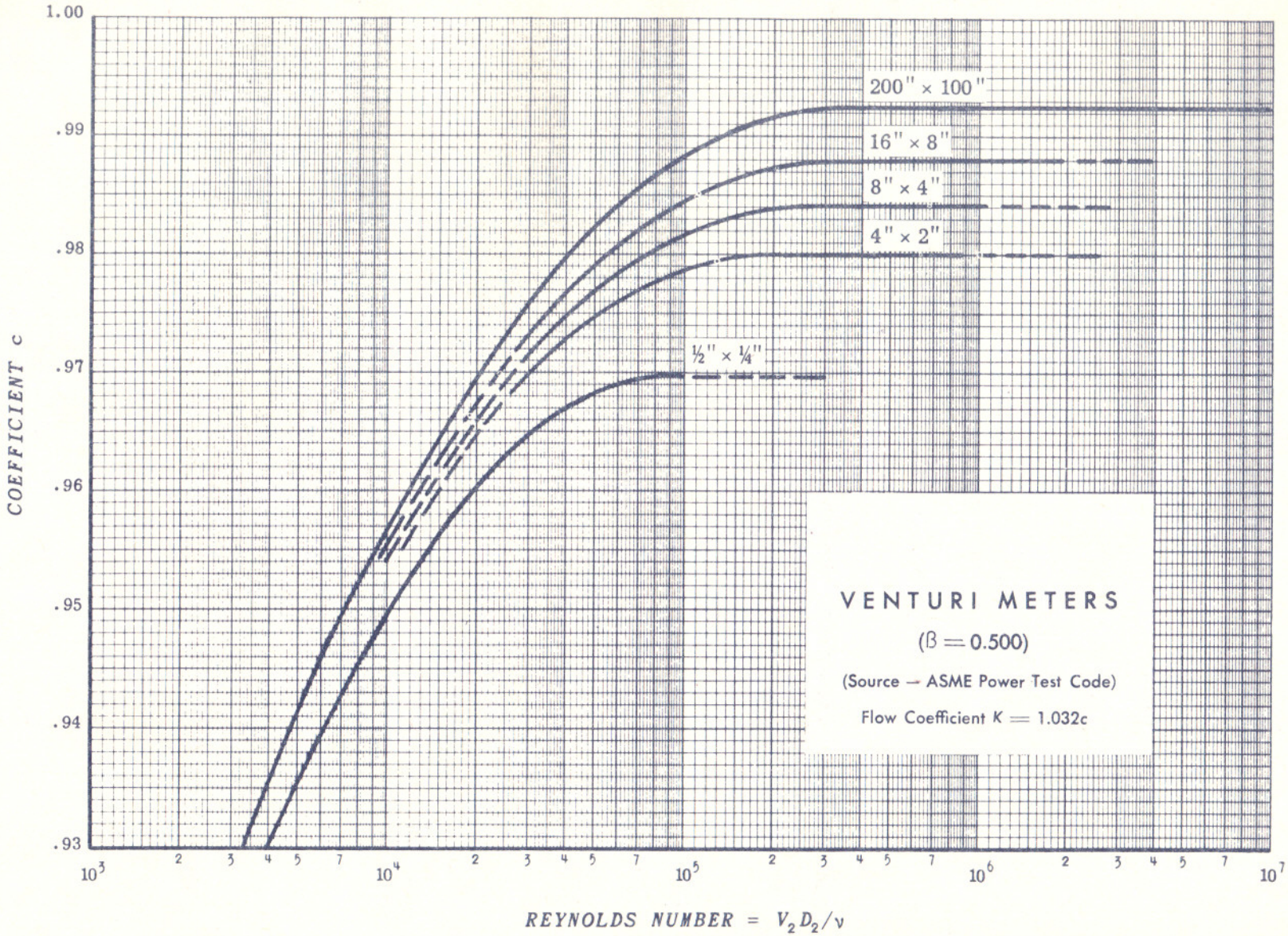

1.00
.99
.98
.97
.96
.95
.94
.93
COEFFICIENT c
200" × 100"
16" × 8"
8" × 4"
4" × 2"
½" × ¼"
VENTURI METERS
(β = 0.500)
(Source — ASME Power Test Code)
Flow Coefficient K = 1.032c
10³
10⁴
10⁵
10⁶
10⁷
REYNOLDS NUMBER = V₂D₂/ν

Fig. A-32

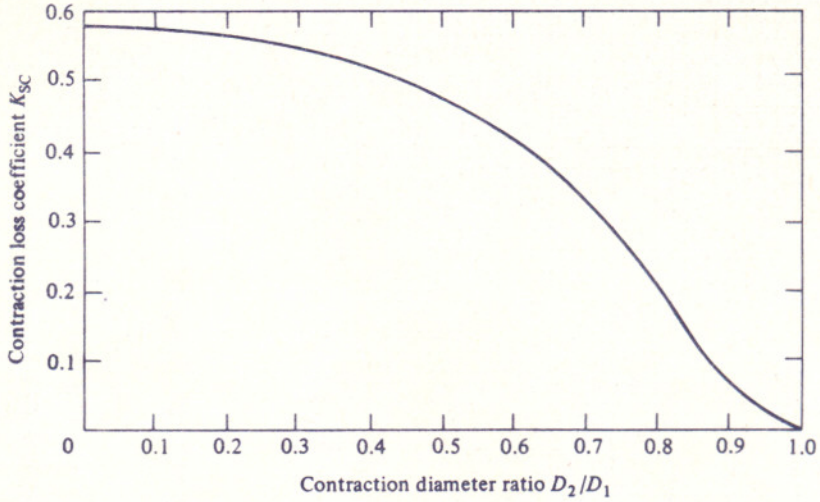


Fig. A-33

Sudden-contraction loss coefficient. (Adapted from R. P. Benedict, N. A. Carlucci, and S. D. Swetz, "Flow Losses in Abrupt Enlargements and Contractions," *J. Eng. Power,* vol. 48, January 1966.)

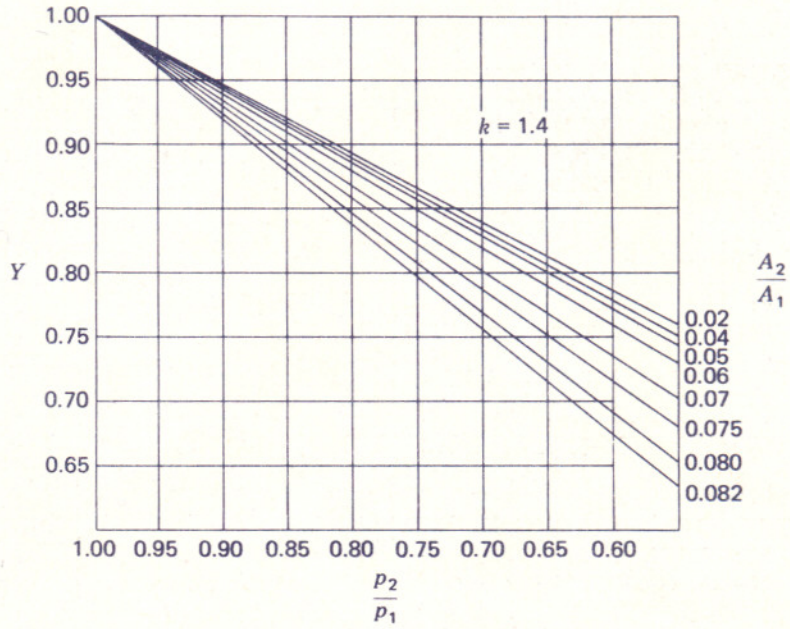


Fig. A-34

Compressibility factor Y for flow nozzles and Venturi meters. $k = 1.4$.

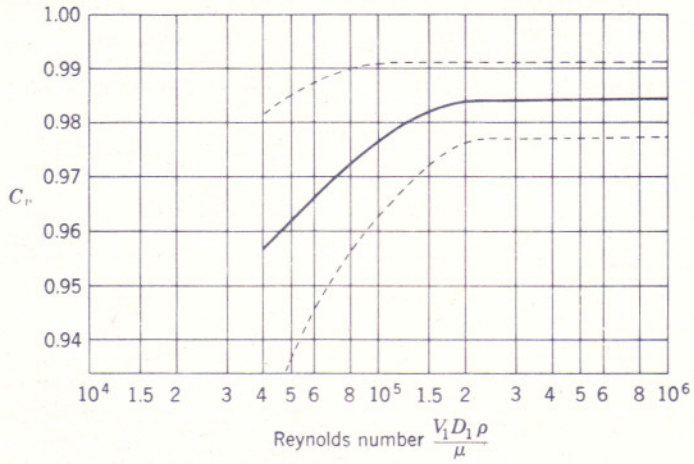


Fig. A-35

Coefficient C_v for venturi meters (*Fluid Meters: Their Theory and Application,* 5th ed., American Society of Mechanical Engineers, 1959.)

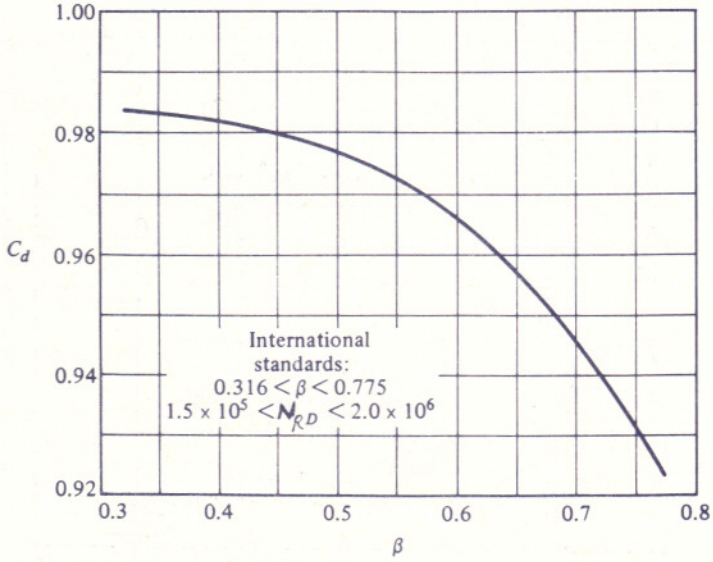


Fig. A-36

Discharge coefficient for a venturi nozzle.

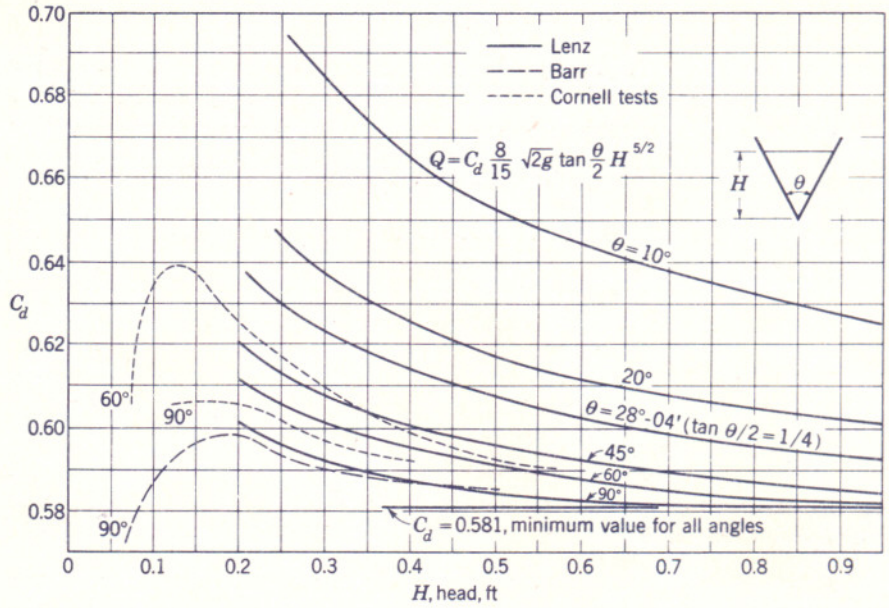
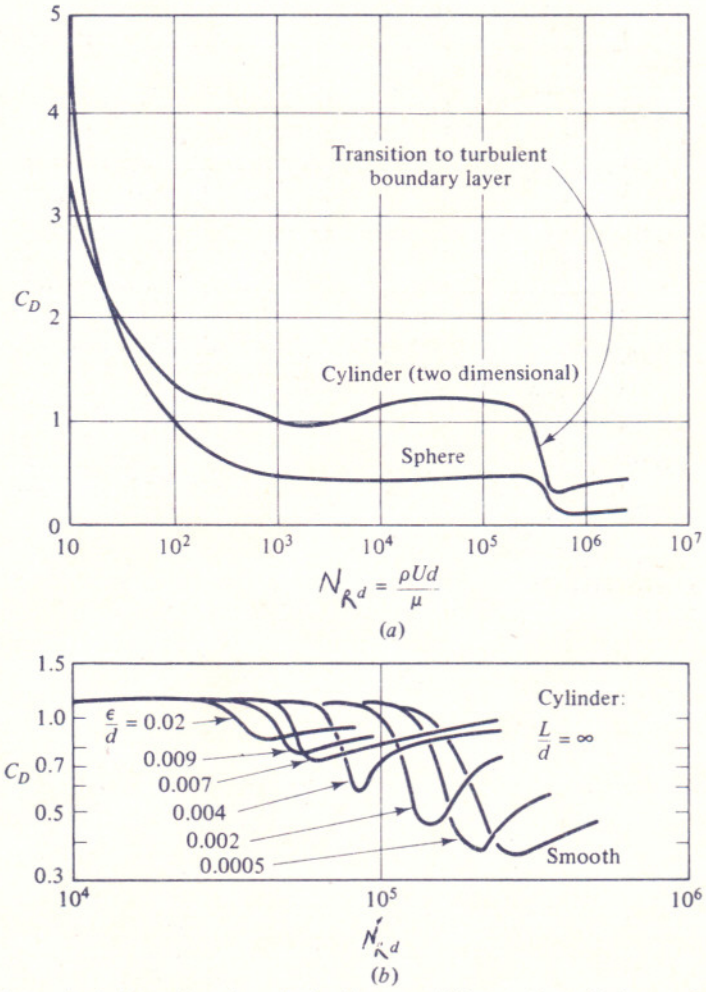


Fig. A-37

Coefficients for triangular weirs.



Cylinder length effect ($10^4 < N_R < 10^5$)	
L/d	C_D
∞	1.20
40	0.98
20	0.91
10	0.82
5	0.74
3	0.72
2	0.68
1	0.64

Fig. A-38

The proof of practical dimensional analysis: drag coefficients of a cylinder and sphere: (*a*) drag coefficient of a smooth cylinder and sphere (data from many sources); (*b*) increased roughness causes earlier transition to a turbulent boundary layer.

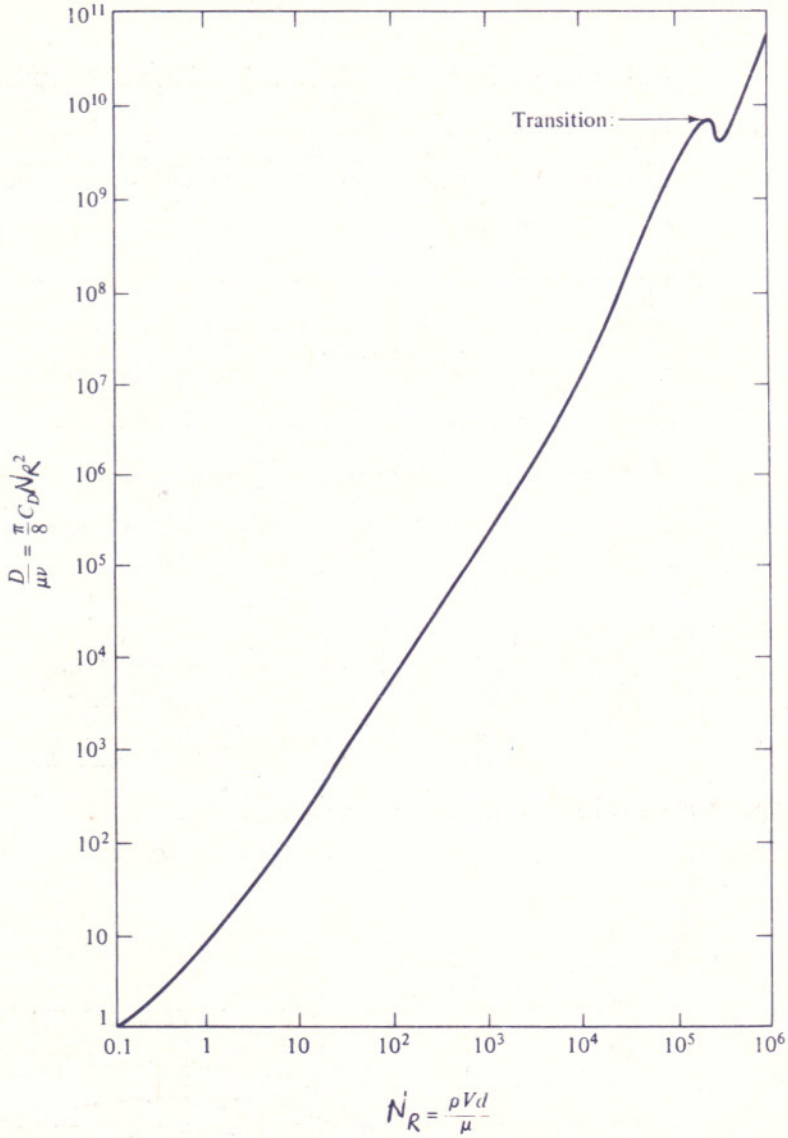


Fig. A-39

Crossplot of sphere-drag data from Fig. A-38*a* to isolate diameter and velocity.