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and at the base. Use both the seismic coefficient method as well as response spectrum method.

Solution

(a) Seismic coefficient method

For zone IV, $\alpha_0 = 0.05$

$\therefore \quad \alpha_h = 2\,\alpha_0 = 2 \times 0.05 = 0.1$

Hence vertical acceleration at top is

$$\alpha_v^T = 0.75\,\alpha_h = 0.75 \times 0.1 = 0.075$$

It varies linearly from a value of 0.75 at the top to zero at the base, as shown in Fig. 8.14.

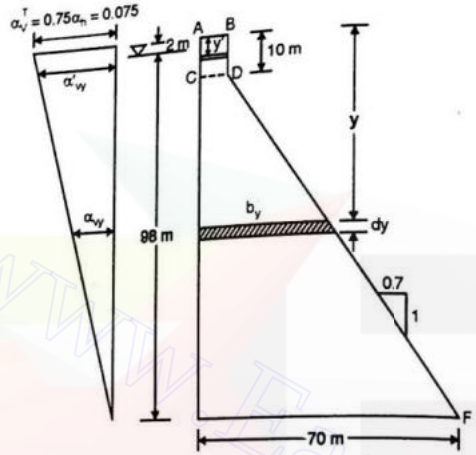

FIG. 8.12

At 10 m below top

Consider an elementary strip of thickness dy' at y' below the top.

Weight of strip $W_y' = (1 \times 7\,dy')\,24 = 168\,dy'$

$\alpha_{vy}' = 0.075\,(1 - y'/100) = 0.075\,(1 - 0.01y')$

$\therefore$ Increase or decrease in weight

$$= \int_0^{10} \alpha_{vy'}\,W_y'\,dy' = \int_0^{10} 0.075(1 - 0.01\,y')168\,dy'$$

$$= 12.6 \int_0^{10} (1 - 0.01\,y')\,dy' = 12.6\left[y' - \frac{0.01y'^2}{2}\right]_0^{10} = \mathbf{119.7\ kN}$$

At the base (100 m below top) : Consider an elementary strip of thickness dy at y below the top.

$$W_y = \left\{7 + 0.7\,(y - 10)\right\} dy \times 24 = 16.8y\,.\,dy$$

$$\alpha_{vy} = 0.075\,(1 - 0.01y)$$

$\therefore$ Increase or decrease in weight $= 119.7 + \int_{10}^{100} (16.8\,y\,dy)\,0.075\,(1 - 0.01y)$

$$= 119.7 + 1.26\left[\frac{y^2}{2} - \frac{0.01\,y^3}{3}\right]_{10}^{100} = \mathbf{2160.9\ kN}$$

(b) Response spectrum method

For zone IV, $F_0 = 0.25$. Also $S_a/g = 0.19$ (from Example 8.2)

$$\therefore \quad \alpha_h = 2\,F_0\,\frac{S_a}{g} = 2 \times 0.25 \times 0.19 = 0.095$$

Hence vertical acceleration at top $= 0.75\,\alpha_h = 0.75 \times 0.095 = 0.07125$. It varies linearly from a value of 0.07125 at the top to zero at the base.

At 10 m below top

Weight of strip, $W_y' = (1 \times 7\,dy')\,24 = 168\,dy'$

$\alpha_{vy'} = 0.07125\,(1 - y'/100) = 0.07125\,(1 - 0.01\,y')$

$\therefore$ Increase or decrease in weight

$$= \int_0^{10} \alpha_{vy'}\,W_y'\,dy' = \int_0^{10} 0.07125\,(1 - 0.01y')\,168\,dy'$$

$$= 11.97 \int_0^{10} (1 - 0.01y')\,dy' = 11.97\left[y' - \frac{0.01y'^2}{2}\right]_0^{10} = \mathbf{113.715\ kN}$$

At the base (i.e. 100 m from top)

$$W_y = \left\{7 + 0.7\,(y - 10)\right\} dy \times 24 = 16.8\,y\,dy$$

$$\alpha_{vy} = 0.07125\,(1 - 0.01y)$$

$\therefore$ Increase or decrease in weight $= 113.715 + \int_{10}^{100} (16.8\,y dy))\,0.07125\,(1 - 0.01\,y)$

$$= 113.715 + 1.197\left[\frac{y^2}{2} - \frac{0.01y^3}{3}\right]_{10}^{100} = \mathbf{2052.86\,kN}$$

Example 8.4. ***Hydrodynamic pressure on inclined u/s face***

A 100 m high concrete dam shown in Fig. 8.13 is located in zone IV and has to resist pressure from 98 m depth of water. Find the hydrodynamic pressure at the following locations : (a) 10 m from the top (b) 40 m from the top, and (c) at the base. Take the value of $\alpha_h = 0.1$ determined on the basis of response spectrum method.

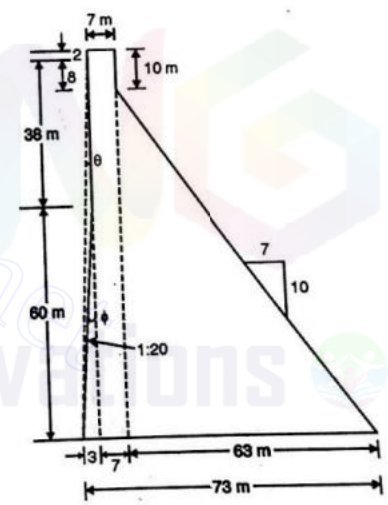

FIG. 8.13

Solution

(a) Hydrodynamic Pressure intensities

Hydrodynamic pressure at any depth y below the reservoir surface is given by

$$p_{ey} = C_y\,\alpha_h\,wh$$

where $\alpha_h = 0.1$; $w = 9.81\ \text{kN/m}^2$; $h = 98$ m.

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Since the height of sloping face (= 60 m) is more than $h/2$ (= 98/2), the modified slope for the design will be obtained by joining the point of intersection of the u/s face of the dam and the reservoir surface with the point of intersection of u/s face of the dam and the foundation, as shown in Fig. 8.15. Thus $\theta = \tan^{-1} 3/98 = 1.75°$. Hence from Fig. 8.9, we get $C \triangleq 0.72$.

Alternatively, from Eq. 8.18, $C_m \triangleq 0.735\,(1 - 1.75/90) = 0.72$

$$\therefore \quad C_y = \frac{0.72}{2}\left[\frac{y}{98}\left(2-\frac{y}{98}\right)+\sqrt{\frac{y}{98}\left(2-\frac{y}{98}\right)}\right]$$

and
$$p_{ey} = \frac{0.72}{2}\left[\frac{y}{98}\left(2-\frac{y}{98}\right)+\sqrt{\frac{y}{98}\left(2-\frac{y}{98}\right)}\right]\times 0.1\times 9.81\times 98$$

or
$$p_{ey} = 34.61\left[\frac{y}{98}\left(2-\frac{y}{98}\right)+\sqrt{\frac{y}{98}\left(2-\frac{y}{98}\right)}\right]$$

At depth of 8 m below reservoir level or 10 m below top of dam,

$$p_{e10} = 19.116 \text{ kN/m}^2$$

At depth of 38 m below reservoir level or 40 m below top of dam,

$$p_{e40} = 49.002 \text{ kN/m}^2$$

At a depth of 98 m below the reservoir level or 100 m below the top of dam

$$p_{e100} = 69.22 \text{ kN/m}^2$$

(b) *Horizontal shear*

Horizontal shear at any depth of y from the reservoir level is given by

$$P_{ey} = 0.726\, p_{ey} \cdot y \qquad ...(8.19)$$

Hence
$$P_{e10} = 0.726 \times 19.116 \times 8 = 111.026 \text{ N kN/m}$$
$$p_{e40} = 0.726 \times 49.002 \times 38 = 1351.867 \text{ kN/m}$$
$$P_{e100} = 0.726 \times 69.22 \times 98 = 4924.864 \text{ kN/m}$$

(c) *Moment*

$$M_{ey} = 0.299\, p_{ey} \cdot y^2 \qquad ..(8.20)$$

$$\therefore \quad M_{e10} = 0.299 \times 19.116\,(8)^2 = 365.8 \text{ kN-m/m}$$
$$M_{e40} = 0.299 \times 49.002\,(38)^2 = 21156.9 \text{ kN-m/m}$$
$$M_{e100} = 0.299 \times 69.22\,(98)^2 = 198771.9 \text{ kN-m/m}$$

(d) Vertical component of reservoir water on horizontal section

Since the hydrodynamic pressure acts normal to the face of the dam, there will be a vertical component of this force, if the face of the dam against which it is acting is sloping. The magnitude of this load (W_h) on any horizontal section is given by

$$W_h = (P_{e2} - P_{e1}) \tan \varphi$$

where φ = the actual slope of the u/s face. Here $\tan \varphi = 3/60 = 1/20 = 0.05$

P_{e2} = total shear due to horizontal component of hydrodynamic force at the elevation of the section being considered.

P_{e1} = total shear due to horizontal component of hydrodynamic force at the elevation at which the slope of the dam face commences,

Hence at the base of the dam, $= P_{e2} = P_{e100} = 4924.864$ and $P_{e1} = P_{e40} = 1351.867$

$$\therefore \quad W_{100} = (4924.864 - 1351.867)\,0.05 = 178.65 \text{ kN/m}$$

8.12. MODES OF FAILURE : STABILITY REQUIREMENTS

Following are the modes of failure of a gravity dam :

1. Overturning
2. Sliding
3. Compression or Crushing
4. Tension.

1. **Overturning.** The overturning of the dam section takes place when the resultant force at any section cuts the base of the dam downstream of the toe. In that case the resultant moment at the toe becomes clockwise (or – ve). On the other hand, if the resultant cuts the base within the body of the dam, there will be no overturning.

For stability requirements, the dam must be safe against overturning. The factor of safety against overturning is defined as the ratio of the righting moment (+ ve) to the overturning moments, *i.e.*

$$\text{F.S.} = \frac{\Sigma \text{ Righting moments}}{\Sigma \text{ Overturning moments}} = \frac{\Sigma M_R}{\Sigma M_0} \qquad ...(8.25)$$

Since the magnitude and direction of each and every force considered in the previous articles are known, their moments about the toe can very easily be found and F.S. computed. The factor of safety against overturning should not be less than 1.5.

2. **Sliding.** A dam will fail in sliding at its base, or at any other level, if the horizontal forces causing sliding are more than the resistance available to it at that level. The resistance against sliding may be due to friction alone, or due to friction and shear strength of the joint. Shear strength develops at the base if benched foundations are provided and at other joints if the joints are carefully laid so that a good bond develops. Shear strength also comes into play because of the interlocking of stone in masonry dams.

If the shear strength is not taken into account, the factor of safety is known as *factor of safety against sliding*. The factor of safety against sliding is defined as the ratio of *actual coefficient of static friction* (μ) on the horizontal joint to the *sliding friction*. The sliding factor is the minimum coefficient of friction required to prevent sliding. If ΣH = sum of the horizontal forces causing the sliding and ΣV is the net vertical forces, the *sliding factor* ($\tan \theta$) is given by

$$\text{S.F.} = \tan \theta = \frac{\Sigma H}{\Sigma V} \qquad ...(8.26)$$

and factor of safety against sliding (F.S.S) is

$$\text{F.S.S.} = \frac{\mu}{\tan \theta} = \frac{\mu\, \Sigma V}{\Sigma H} \qquad ...(8.27)$$

The coefficient of friction μ varies from 0.65 to 0.75. The factor of safety against sliding should be greater than the value given in Table 8.4 (IS 6512-1972)

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It is considered that a low gravity dam should be safe against sliding considering friction alone. However, in large dams, shear strength of the joint should also be considered for an economical design. The factor of safety in that case is commonly known as the *shear friction factor* S.F.F. and is defined by the equation

$$\text{S.F.F.} = \frac{\mu\,\Sigma\,(V) + A.c}{\Sigma\,H} = \frac{\mu\,\Sigma\,(V) + b\,.\,c}{\Sigma\,H} \qquad ...(8.28)$$

where c = average cohesion or shear strength of the joint the value of which varies, from 1300 kN/m² to 4500 kN/m², for good rock to 650 to 1300 kN/m² for concrete.

b = width of the joint or section

A = Area of joint $= b \times 1$ for unit length of the dam.

Table 8.4 gives permissible values of factor of the safety against sliding and the S.F.F.

TABLE 8.4. FACTOR OF SAFETY AGAINST SLIDING AND SHEAR FRICTION FACTOR (IS : 6512 - 1972)

S.No.	Loading condition	F.S. against sliding (F.S.S.)	S.F.F.
1.	A, B, C	2.0	4.0
2.	D, E	1.5	3.0
3.	F, G	1.2	1.5

Recommendations of IS 6512-1984 : The above criterion (i.e. Eq. 8.28 and Table 8.4) given in IS 6512-1972 was completely changed in IS 6512-1984. According to the revised version, the factor of safety against sliding is calculated on the basis of partial factor of safety in respect of friction (F_φ) and partial factor of safety in respect of cohesion or shear (F_c) by using the following equation :

$$F = \frac{\dfrac{\mu\,\Sigma\,(V)}{F_\varphi} + \dfrac{cA}{F_c}}{\Sigma\,H} = \frac{\dfrac{\mu\,\Sigma\,V}{F_\varphi} + \dfrac{cb}{F_c}}{\Sigma\,H} \qquad ...(8.29)$$

where F = factor of safety against sliding

and A = Area of the joint $= b \times 1$ for unit length of the dam.

The values of F_φ and F_c are given in Table 8.5.

TABLE 8.5. VALUES OF PARTIAL SAFETY FACTORS F_φ AND F_c (IS 6512-1984)

S.N.	Load combination	F_φ	F_c		
			For dams and the contact plane with foundation	For foundation	
				Thoroughly investigated	others
(i)	A,B,C	1.5	3.6	4.0	4.5
(ii)	D,E	1.2	2.4	2.7	3.0
(iii)	F,G	1.0	1.2	1.35	1.5

The factor of safety against sliding, computed from Eq. 8.29, should not be less than 1.

3. Compression or Crushing. In order to calculate the normal stress distribution at the base, or at any section, let H be the total horizontal force, V be the total vertical force and R be the resultant forces cutting the base at an eccentricity e from the centre of the base of width b (Fig. 8.14).

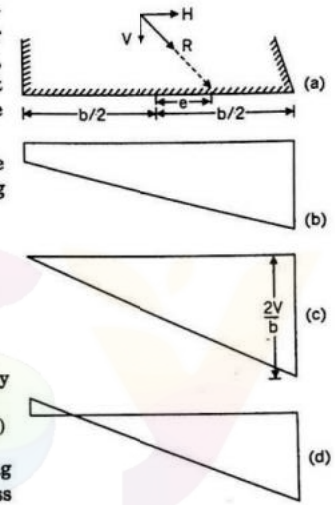

FIG. 8.14. NORMAL STRESS DISTRIBUTION AT BASE.

The normal stress at any point on the base will be the sum of the direct stress and the bending stress.

Thus, direct stress $= \dfrac{V}{b \times 1}$

$$\text{Bending stress} = \pm\frac{M.y}{I} = \pm\frac{V\,.\,e}{\frac{1}{6}b^2} = \pm\frac{6\,V\,.\,e}{b^2}$$

Hence the *total normal stress* p_n is given by

$$p_n = \frac{V}{b}\left(1 \pm \frac{6\,e}{b}\right) \qquad ...(8.30)$$

The positive sign will be used for calculating normal stress at the toe, since the bending stress will be compressive there, and negative sign will be used for calculating normal stress at the heel.

Thus, the normal stress at the toe is

$$(p_n)_{toe} = \frac{V}{b}\left(1 + \frac{6\,e}{b}\right) \qquad ...[8.30(a)]$$

The normal stress at the heel is

$$(p_n)_{heel} = \frac{V}{b}\left(1 - \frac{6\,e}{b}\right) \qquad ...[8.30(b)]$$

Fig. 8.14(*b*) shows the normal stress distribution for a general case when the pressure at both toe and heel are compressive. Evidently, the maximum compressive stress occurs at the toe and for safety, this should not be greater than the allowable compressive stress f for the foundation material. Hence, from strength point of view

$$\frac{V}{b}\left(1 + \frac{6\,e}{b}\right) \le f \qquad ...(8.31)$$

When the eccentricity e is equal to $\dfrac{b}{6}$, we get

$$(p_n)_{toe} = \frac{V}{b}\left(1 + \frac{6}{b} \times \frac{b}{6}\right) = \frac{2V}{b} \qquad ...(8.32)$$

The corresponding stress at the heel in that circumstance will evidently be zero. Fig. 8.14(*c*) shows the pressure distribution for this case.

4. Tension. From equation 8.30(*b*) the normal stress at the heel is

$$(p_n)_{heel} = \frac{V}{b}\left(1 - \frac{6\,e}{b}\right)$$

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It is evident that if $e > \frac{b}{6}$, the normal stress at the heel will be – ve or tensile. No tension should be permitted at any point of the dam under any circumstance for moderately high dams. For no tension to develop, the eccentricity should be less than $b/6$. In other words, the resultant should always lie within the *middle third*.

However, in case of extra high dams, 230 to 260 m, small tension within the permissible limit is generally permitted for comparatively small periods of loading such as heavy flood or earthquake.

Effect of Tension Cracks. Since concrete cannot resist the tension, a crack develops at the heel, which modifies the uplift pressure diagram, as illustrated in Fig. 8.15. Due to tension crack, the uplift pressure increases in magnitude and net downward vertical force or the stabilising force reduces. The resultant force thereby gets further shifted towards the toe and this leads to further lengthening of the crack. The base width thus goes on reducing and the compressive stresses on toe goes on increasing, till the toe fails in compression.

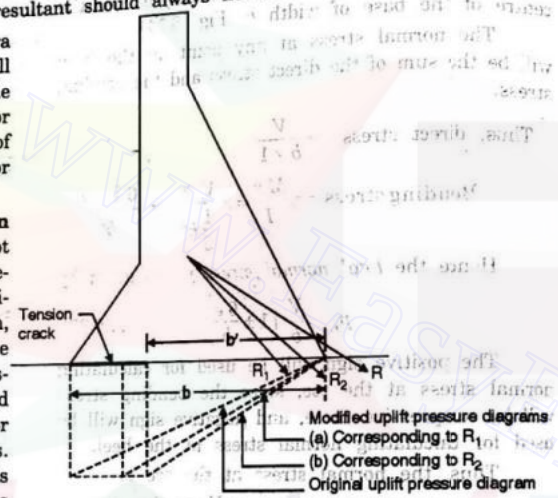

FIG. 8.15. EFFECT OF TENSION CRACKS.

8.13. PRINCIPAL AND SHEAR STRESSES

Principal Stress. Consider an elementary triangular section at either the heel or the toe of the dam section such that stress intensites may be assumed to be uniform on its faces. The face of the dam will be a principal plane as water pressure acts on it in the perpendicular dirction, with no accompanying shear stress. Since the principal planes are mutually at right angles, the plane *AB*, considered at right angles to the face *AC*, will also have only a normal stress on it, and will be the other principal

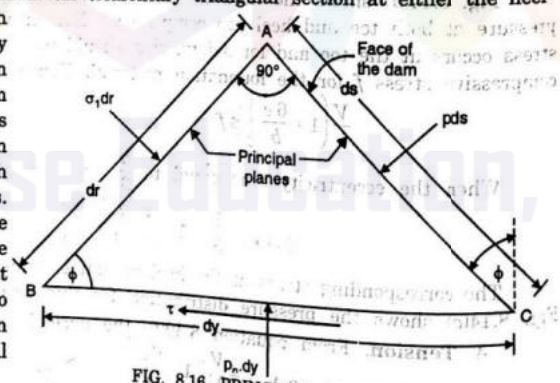

FIG. 8.16. PRINCIPAL STRESSES.

plane. The forces acting on the elementary section are shown in Fig. 8.16.

Let ds, dr and dy be the lengths of *AC*, *AB* and *BC*

p = intensity of water pressure acting normal to the face of the dam

σ_1 = principal stress on plane *AB*

τ = shear stress on plane *BC*

p_n = normal stress on plane *BC*, known from Eq. 8.30

Considering unit length of the dam, the normal forces on the planes *AB*, *BC* and *CA* are respectively $\sigma_1 dr$, $p_n . dy$ and $p . ds$.

Resolving all the forces in the vertical direction, we get

$$p_n . dy = p.ds \sin\phi + \sigma_1 . dr \cos\phi$$

But $dr = dy \cos\phi$ and $ds = dy \sin\phi$

$$\therefore \quad p_n dy = p.dy \sin^2\phi + \sigma_1 .dy \cos^2\phi$$

or

$$p_n = p \sin^2\phi + \sigma_1 . \cos^2\phi$$

Hence

$$\sigma_1 = \frac{(p_n - p \sin^2\phi)}{\cos^2\phi}$$

or

$$\sigma_1 = p_n \sec^2\phi - p \tan^2\phi \quad ...(8.33)$$

Equation 8.33 is known as the principal stress relationship, and is applicable to both upstream and downstream faces. However, for the downstream side, the worst condition will be when there is no tailwater, and hence p will be zero. In that circumstance, the principal stress σ_1 is given by

$$\sigma_1 = p_n \sec^2\phi \quad ...(8.34)$$

If p_e' is the intensity of hydrodynamic pressure of tailwater due to an earthquake the principal stress at the downstream is given by

$$\sigma_1 = p_n \sec^2\phi - (p - p_e') \tan^2\phi \quad ...(8.35)$$

Similarly, considering the hydrodynamic pressure (p_e) to the upstream side, principal stress is given by

$$\sigma_1 = p_n \sec^2\phi - (p + p_e) \tan^2\phi \quad ...(8.36)$$

It should be noted, however, that for the upstream face σ_1 will always be less than p. Hence σ_1 is the minor principal stress and p is the major principal stress for the upstream face. Also, if u/s face is vertical, $\varphi = 0$ and hence $\sigma_1 = p_n$

Shear Stress : Resolving all the forces in the horizontal direction, we get

$$\tau dy = \sigma_1 dr \sin\phi - pds \cos\phi$$

$$\therefore \quad \tau = \sigma_1 \sin\phi \frac{dr}{dy} - p \frac{ds}{dy} \cos\phi$$

or

$$\tau = (\sigma_1 - p) \cos\phi \sin\phi.$$

Substituting the value of σ_1 from Eq. 8.33, we get

$$\tau = (p_n \sec^2\phi - p \sec^2\phi) \cos\phi \sin\phi$$

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or $$\tau = (p_n - p) \tan \phi \quad ...(8.37)$$

The above equation is applicable for downstream side only. For the upstream side, the magnitude of τ will be the same as that given by Eq. 8.37, but its direction will be reversed. Hence, for upstream

$$\tau = -(p_n - p) \tan \phi \quad ...(8.38)$$

If tailwater is neglected, the shear stress at the downstream side is given by

$$\tau = p_n \tan \phi \quad ...[8.37(a)]$$

Considering the hydrodynamic pressure due to earthquake, the shear stress at downstream is given by

$$\tau = [p_n - (p - p_e')] \tan \phi \quad ...[8.39.]$$

Similarly, the shear stress for the upstream side is given by

$$\tau = -[p_n - (p + p_e')] \tan \phi \quad ...(8.40)$$

However, if u/s face is vertical, $\varphi = 0$ and $\tau = 0$ from Eq. 8.40

8.14. QUALITY AND STRENGTH OF CONCRETE / MASONRY (IS : 6512-1984)

According to the Indian Standard, the strength of concrete / masonry shall exceed the stresses anticipated in the structure by a safe margin. The maximum compressive stresses occur at the heel or toe and on planes normal to the faces of the dam. The strength of concrete and masonry varies with age, the kind of cement and other ingredients and their proportions in the work can be determined only by experiment.

(a) Compressive Strength : The compressive strength of concrete and masonry shall conform to the following requirements :

(i) ***Concrete.*** Concrete strength is determined by compressing to failure 15 cm × 30 cm cylinders. The strength of concrete should satisfy early load and construction requirements and at the age of the one year it should be four times the maximum computed stress in the dam or 14000 kN/m^2 (14 N/mm^2), whichever is more. The allowable working stress in any part of the structure shall not also exceed 7000 kN/m^2 (7 N/mm^2).

(ii) ***Masonry.*** The compressive strength of masonry is determined by compressing to failure 75 cm cubes of masonry fabricated and cured at temperatures approximating to those expected in the structures or 45 × 90 cm cylinders cored out of the structures or blocks made (for the purpose). This strength should satisfy early load and construction requirements and at one year it should be five times the maximum computed stress on the dam or 17500 kN/m^2 (17.5 N/mm^2), whichever is more.

[***Note.*** For the purpose of quality control, correlation between the strength of mortar and that of masonry may be established in suitable small size specimens].

Recommendations of IS 6512-1984 against over stressing : Provision of fillets

If the compressive stress at toe and heel are excessive, they can be brought to the permissible limits by provision of fillets with slopes one horizontal to two vertical (1:2) at the heel and two horizontal to one vertical (i.e. 2:1) at the toe. The heights of these fillets above heel and toe are determined from the following expressions.

(i) For heel : $$h_{fu} = 6\,H_0 - 0.658\,H_0^2 - 1.07 \quad ...(8.41\ a)$$

(ii) For toe : $$h_{fd} = 6.5\,H_0 - 1.1\,H_0^2 - 0.9 \quad ...(8.41\ b)$$

where h_{fu} = height of fillet at heel (i.e. u/s side)

h_{fd} = height of fillet at toe (i.e. d/s side)

H_0 = height of dam in 100 metres

For example, if height of dam = 150 m, $H_0 = 1.5$

$\therefore$ $$h_{fu} = 6 \times 1.5 - 0.658\,(1.5)^2 - 1.07 \triangleq 6.45 \text{ m}$$

and $$h_{fd} = 6.5 \times 1.5 - 1.1\,(1.5)^2 - 0.9 \triangleq 6.4 \text{ m}$$

(b) Tensile Strength

(i) Although concrete and masonry are known to possess some tensile strength, quantitative evaluations are uncertain and difficult. Moreover, construction joints can be planes of weakness against tensile strength. Tensile failures in the form of cracking may, in some cases, have significant adverse effects on the structural behaviour of the dam.

(ii) Tensile stresses are not permitted at the u/s face of the dam for load combination *B* (normal operation condition). However, normal tensile stresses may be permitted in case of other load combinations but their values should not exceed the permissible values given in Table 8.6.

TABLE 8.6. PERMISSIBLE TENSILE STRESS IN CONCRETE AND MASONRY

Load combination	*Permissible tensile stress*	
	Concrete	*Masonry*
C	0.01 f_c	0.005 f_c
E	0.02 f_c	0.01 f_c
F	0.02 f_c	0.01 f_c
G	0.04 f_c	0.02 f_c

(iii) Small values of tension on the d/s face may be permitted since it is very improbable that a fully constructed dam is kept empty and downstream cracks which are not extensive and for limited depths from the surface may not be detrimental to the safety of the structure.

Structural Competency of the Foundation

The true conditions at the dam site and reservoir area shall be determined and the design of the dam made to suit the conditions imposed by nature. The ground water level may cause softening of the rocks or the dam may slide on a slippery bedding plane or the impounded water may pass through fissures and caverns around and under the dam. The structural competency of the foundation is not usually amenable to precise and complete evaluation ; data from explorations and tests should be supplemented by engineering and geological judgement. When the quality of the foundation materials approaches or falls below that of the concrete/masonry of the structure on it, the foundation competency becomes a controlling factor and should be appraised in full detail. It is to be noted that field and laboratory tests respectively evaluate relatively small areas and small samples only and the overall characteristics of the foundation including geological structures or other inhomogeneities are interpreted from these limited data.

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8.15. STABILITY ANALYSIS

The stabilty analysis of a given gravity dam section may be carried out by the following methods :

1. Gravity method or two-dimensional method :
   (*a*) Graphical method.
   (*b*) Analytical method.
2. Trial load twist method.
3. Slab analogy method.
4. Lattice analogy method.
5. Experimental methods :
   (*a*) Direct method.
   (*b*) Indirect method.

1. Gravity method. This is an approximate method in which the dam is considered to be composed of parallel sided vertical cantilevers, each of which is free to act without supporting or interfering with the adjoining cantilevers. The loads or forces are resisted entirely by the weight of the individual cantilevers of unit length. Other methods of design and analysis are complicated and time consuming, and hence preliminary calculations are always done with the help of this method.

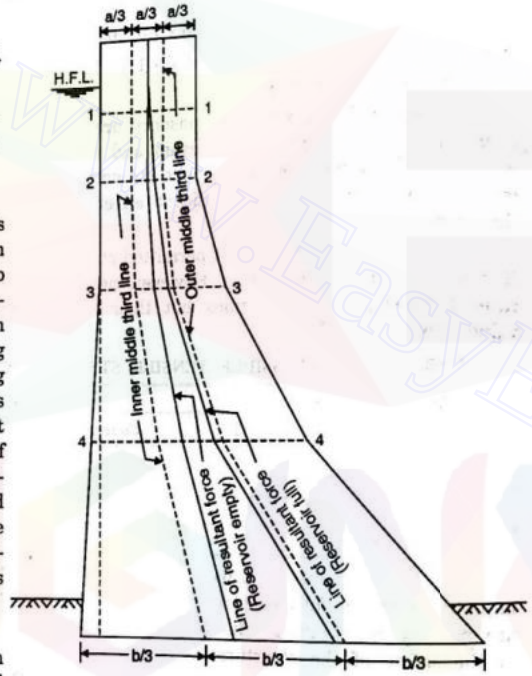

FIG. 8.17. GRAPHICAL METHOD.

(a) Graphical method. In the graphical method, the dam section is divided into horizontal section 1 – 1 , 2 – 2 , 3 – 3 etc. at some suitable interval, or at the places where slope changes (Fig. 8.17). For each section, the sum of all horizontal forces (ΣH) and the sum of all vertical forces (ΣV), *acting above that section* are calculated and their line of action are graphically located. The resultant force R is then found, and its line of action is located graphically. This is done for each section, and finally, a line is drawn joining the point at which the resultants cut the various sections. The line of resultant force so obtained should evidently lie within the middle third, for no tension to develop. This procedure is adopted both for the reservoir full condition as well as reservoir empty condition. Both the lines of resultant pressure so obtained should lie in the middle third portion, as shown in Fig. 8.17.

(b) Analytical method. The stability analysis by analytical method is done in the following steps :

1. Consider unit length of the dam. Calculate all the vertical loads acting. They include the weight of the dam, weight of water acting on the inclined faces, uplift pressure and inertia forces due to vertical acceleration. Find their algebraic sum (ΣV).

2. Find out the sum of horizontal forces ΣH, and the horizontal pressure due to hydrodynamic pressure.

3. Find out the sum of the overturning moments (ΣM_0) and the sum of righting moments (ΣM_R) at the toe. Also find the algebraic sum of all the moments as under :

$$\Sigma M = \Sigma M_R - \Sigma M_0.$$

4. Find out the location (*i.e.* distance $\bar{x}$) of the resultant force R from toe, by relation

$$\bar{x} = \frac{\Sigma M}{\Sigma V}.$$

5. Find out the eccentricity e of the resultant R, from the centre by the relation,

$$e = \frac{b}{2} - \bar{x}, \text{ where } b = \text{base width of the section.}$$

6. Find the normal stress at the toe of dam :

$$p_n = \frac{\Sigma V}{b}\left(1 + \frac{6e}{b}\right)$$

Also, find normal stress at the heel by the expression :

$$p_n = \frac{\Sigma V}{b}\left(1 - \frac{6e}{b}\right)$$

Sometimes, the normal compressive stresses are found by ignoring the uplift.

7. Find out the principal and shear stresses at the toe and the heel by Eqs. 8.35 to 8.40.

8. Find out the factor of safety against overturning by the expression :

$$\text{F.S.} = \frac{\Sigma M_R}{\Sigma M_0}$$

9. Find out the factor of safety against sliding by the following expressions :

(*i*) Sliding factor : $\text{S.F.} = \dfrac{\mu \Sigma V}{\Sigma H}$

(*ii*) Shear friction factor : $\text{S.F.F.} = \dfrac{\mu \Sigma V + b\,c}{\Sigma H}$

(*iii*) Factor of safety against sliding : $F = \dfrac{\dfrac{\mu \Sigma V}{F_\varphi} + \dfrac{c\,b}{F_c}}{\Sigma H}$

where ΣV is the net vertical load, inclusive of the uplift.

The method is illustrated in a tabular form in Examples 8.8 and 8.9.

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2. Trial Load Twist Method. Gravity method of analysis described above is based on the assumption that the dam behaves as vertical cantilevers independent of each other. The assumption is very nearly true if the dam is constructed in a wide U–shaped valley, and the transverse joints are not keyed or grouted. If the transverse joints are keyed or grouted, the dam will tend to behave as a monolithic structure and the two dimensional analysis or gravity method will not be true. Trial load twist method is based on the assumption that the horizontal elements of the dam will behave as a beam and will transfer the water pressure acting upon it to the abutments by beam action. Further, the height of the dam reduces as it proceeds from the centre towards the side as the valley on both sides rises steeply. The successive cantilever elements, therefore, go on reducing in the height, and their deflection also goes on reducing. Each cantilever element is thus dragged on its one face to greater deflections by deeper adjacent cantilevers on the valley side and is held on by the shorter cantilever on hill side. This develops a twist effect on each cantilever. In the trial load twist method therefore the entire dam is divided into a number of vertical cantilevers and horizontal beam elements. These elements are then supposed to form three structural systems namely :

(*i*) *Cantilever structural system* to resist the external load by gravity action through bending and shear along horizontal planes.

(*ii*) *Beam structural system* to resist bending.

(*iii*) *Twist structural system* to resist twisting moments caused by shearing of horizontal elements and angular rotation of vertical elements.

The portion of the pressures taken by each element is obtained by dividing out the load distribution between them so that the deflection of a common point considered on any element, is the same.

3. Slab Analogy Method. According to this method, the dam is considered as a slab and the analysis carried out is similar to the calculation of stresses in bridge slabs. It is a time consuming and laborious method.

4. Lattice Analogy Method. According to this method, the dam is considered to be composed of diagonally braced square frames. The analysis, though simple than the slab analysis method, is quite cumbersome.

5. Experimental Methods. (*a*) ***Direct method*** (*Three dimensional model analysis*). In this method, geometrically similar three dimensional models, which are exact replica of the prototype, are made of elastic materials. The models are located in similar manner as the prototype. The model is then acted upon by various pressures proportional to those of the prototype. In order to simulate the high water pressure acting on the prototype, a liquid of high specific gravity such as mercury may be used or the effect may be obtained by the jetting action of water. The deflections occurring at various points of the model are thereafter measured. Then from the correlation developed between the model and prototype for the stress and deflection, the stresses for the prctotype can be obtained. The model analysis has been found to be quite useful for the analysis of high dams, particularly for the Cupola dam of Italy.

(*b*) ***Indirect method*** (***Photoelastic model analysis***). Photoelastic method and methods of magnetic and electrical analogy are usually considered under the indirect methods of determining the stresses in the model and hence in the prototype dam. In case of photoelastic method, the stresses in the photoelastic model are determined by an optical instrument known as photoelastic polariscope, wherein a polarised light passes through the model while it is under load. Studies employing photoelastic models are limited. Usually the study is carried out to supplement an analysis by determining regions of stress concentration. The material used for the photoelastic models must be elastic, isotropic and free from initial or residual stresses.

8.16. ELEMENTARY PROFILE OF A GRAVITY DAM

In the absence of any force other than the forces due to water, an elementary profile will be triangular in section, having zero width at the water level at the top, where water pressure is zero, and a maximum base width *b*, where the maximum water pressure acts. Thus, the section of the elementary profile is of the same shape as the hydrostatic pressure distribution diagram. For reservoir empty condition, a right angled triangular profile as shown in Fig. 8.18, will provide the maximum possible stabilising force against overturning, without causing tension in the base. This is so because the weight of the dam acts at distance *b*/3 from the upstream face and is closer to it. If any triangular profile, other than the right angled one, is provided, its weight will act still closer to the upstream face to provide a higher stabilising force, but tension will be developed at the toe when the dam is empty.

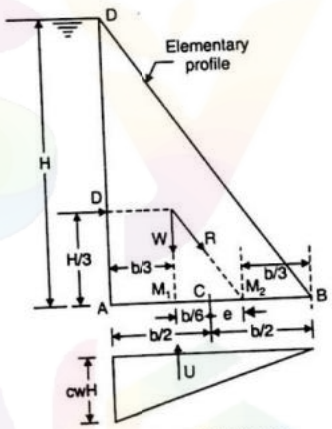

FIG. 8.18. ELEMENTARY PROFILE.

We shall consider the following forces acting on the elementary profile of a gravity dam :

(1) *Weight of the dam* (*W*)

$$W = \frac{1}{2} bH.\rho w$$

where ρ = specific gravity of dam material

w = unit weight of water.

(2) Water pressure (*P*)

$$P = \frac{1}{2} w H^2, \text{ acting at } \frac{1}{3} H \text{ from the base.}$$

(3) *Uplift pressure* (*U*)

$$U = \frac{1}{2} c.w.b.H$$

where c = uplift pressure intensity coefficient.

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Base width of elementary profile. The base width of the elementary profile is to be found under two criteria :

(1) Stress criterion. (2) Stability or sliding criterion

(1) ***Stress Criterion.*** We have already seen that when reservoir is empty, for no tension to develop, the resultant should act at the inner 'third point' (M_1) For the 'reservoir full' condition, for no tension to develop, the resultant R must pass through the 'outer third' point (M_2) shown in Fig. 8.18.

Taking the moment of all forces about M_2 and equating it to zero (since the moment of R about M_2 is zero), we get,

$$\frac{1}{2} wH^2 \cdot \frac{H}{3} + \frac{1}{2} cwb\, H \cdot \frac{b}{3} - \frac{1}{2} b\, H\rho w \cdot \frac{b}{3} = 0.$$

Multiplying all the terms by $\frac{6}{wH}$, we get

$$H^2 + cb^2 - b^2 \rho = 0$$

or

$$b^2 (\rho - c) = H^2$$

$$\therefore \quad b = \frac{H}{\sqrt{\rho - c}} \qquad ...(8.42)$$

The above equation can also be alternatively derived by considering the force triangle in Fig. 8.20. Thus

$$\frac{W-U}{P} = \frac{H/3}{b/3} = \frac{H}{b} \quad \text{or} \quad \frac{\frac{1}{2} bH\rho w - \frac{1}{2} wcb\, H}{\frac{1}{2} wH^2} = \frac{H}{b}$$

or $\quad b^2 (\rho - c) = H^2 \quad$ or $\quad b = \frac{H}{\sqrt{\rho - c}}$

If uplift is not considered, $c = 0$

$$b = \frac{H}{\sqrt{\rho}} \qquad ...[8.42(a)]$$

(2) ***Stability or Sliding criterion*** . For no sliding to occur, horizontal force causing sliding should be balanced by the frictional forces opposing the same.

Hence $\quad P = \mu (W - U)$

$$\therefore \quad \frac{1}{2} wH^2 = \mu \left(\frac{1}{2} bH\, \rho w - \frac{1}{2} cbw\, H\right)$$

From which $\quad b = \frac{H}{\mu (\rho - c)} \qquad ...(8.43)$

If uplift is neglected, $\quad b = \frac{H}{\mu \cdot \rho} \qquad ...[8.43(a)]$

The width provided for the elementary profile should be greater of the width given by Eqs. 8.42 and 8.43.

Stresses developed in the elementary profile

The normal stress, in general, is given by Eq. 8.30, *i.e.*

$$p_n = \frac{V}{b}\left(1 \pm \frac{6e}{b}\right)$$

where $\quad V = (W - U)$ in this case and $e = \frac{b}{6}$.

Hence, for full reservoir case, the normal stresses at the toe is

$$p_n = \frac{(W-U)}{b}(1+1) = \frac{2(W-U)}{b}$$

or $\quad p_n = \frac{2}{b}\left(\frac{1}{2} b\, H\rho w - \frac{1}{2} cbwH\right)$

or $\quad p_n = wH(\rho - c) \qquad ...(8.44)$

The corresponding stress at the heel will be

$$p_n = \frac{(W-U)}{b}(1-1) = 0.$$

Principal stress at the toe

$$\sigma_1 = p_n \sec^2 \phi = wH(\rho - c)\left[\left(\frac{b}{H}\right)^2 + 1\right]$$

But $\quad \left(\frac{b}{H}\right)^2 = \frac{1}{\rho - c}$, from Eq. 8.42

Hence $\quad \sigma_1 = wH(\rho - c)\left[\frac{1}{\rho - c} + 1\right]$

or $\quad \sigma_1 = wH(\rho - c + 1) \qquad ...(8.45)$

Shear stress at the toe

$$\tau = p_n \tan \phi$$

or $\quad \tau = wH(\rho - c) \times \frac{b}{H}$

or $\quad \tau = wH(\rho - c) \times \frac{1}{\sqrt{\rho - c}}$

or $\quad \tau = wH\sqrt{\rho - c} \qquad ...(8.46)$

Since the normal stress at the heel is zero, the principal stress and shear stress will be zero there.

Reservoir empty condition. When the reservoir is empty, the only force acting on the elementary profile will be its weight, acting through the first third point M_1. Hence, the maximum compressive stress at the heel will be $= \frac{W}{b}(1+1) = \frac{2W}{b}$, and the corresponding normal stress at the toe will be $\frac{W}{b}(1-1) =$ zero.

Maurice Levy's criterion of elementary profile

Maurice Levy laid down the criterion for fixing the base width of elementary profile of gravity dam, according to which the vertical stress (or the normal stress)

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at the heel of the dam under full reservoir condition but without uplift, should be compressive and should be equal to the hydrostatic pressure.

The, according to the criterion, $\frac{\Sigma V}{b}\left(1-\frac{6e}{b}\right) \geq wH$ ...(a)

Substituting $\Sigma V = W = \frac{1}{2} bH\rho w$, we get

$$\frac{1}{2}\frac{b\,H\rho\,w}{b}\left(1-\frac{6e}{b}\right) \geq wH$$

or

$$\left(\frac{\rho}{2}-\frac{3\rho e}{b}\right) > 1$$

which gives $e = \frac{b}{6} - \frac{b}{3\rho}$, in the limiting case ...(b)

The weight W of the elementary profile dam will pass through M_1 point, which is at a distance of $b/6$ from the centre of the lamina. Let the resultant of weight W and hydrostatic pressure P strike the base at any point A, with an eccentricity e from the centre of the lamina. Then the point of application of weight W will be at a distance of $\left(\frac{b}{6}+e\right)$ from point A. Taking the moments of P and W about point A, we get.

$$\frac{wH^2}{2}\times\frac{H}{3} = \frac{1}{2}bH\rho w\left(e+\frac{b}{6}\right)$$

which gives $e = \frac{H^2}{3b\rho} - \frac{b}{6}$ ...(c)

Equating (b) and (c), $\frac{b}{6}-\frac{b}{3\rho} = \frac{H^2}{3b\rho}-\frac{b}{6}$ or $\frac{b}{3}\left(1-\frac{1}{\rho}\right) = \frac{H^2}{3b\rho}$

From which $b = \frac{H}{\sqrt{\rho-1}}$ which is the same as Eq. 8.42 with $c = 1$

Hence the base width required according to Maurice-Levy's criterion is the same as that required for no tension basis with uplift intensity coefficient (c) equal to unity.

8.17. PRACTICAL PROFILE OF A GRAVITY DAM

The elementary profile of the gravity dam is only a theorectical profile. However, such a profile is not possible in practice because of the provision of (*i*) roadway at the top, (*ii*) additional loads due to the roadway, and (*iii*) freeboard. Due to these provisions, the resultant force of the weight of the dam and the water pressure falls outside the middle third of the base of the dam when the reservoir is full. To eliminate tension, some masonry is to be provided to the upstream side. Fig. 8.19 shows the dimensions of the practical profile of a gravity dam.

Freeboard.

Freeboard is the margin provided between the top of dam and H.F.L. in the reservoir to prevent the splashing of the waves over the non-overflow section. It incidently also takes care of any unforeseen floods in the reservoir. The freeboard usually provided

is $\frac{3}{2}h_w$, where h_w can be found from Eq. 8.22. The top width equal to about 14% of h has been found to be economical

Indian Standard Recommendations for Free board

According to IS 6512-1972, the freeboard adopted shall be one and a half times the corresponding wave height above full reservoir level (FRL) or for the maximum water level condition (MWL) whichever gives the higher crest elevation for the non over flow section of the dam.

The freeboard above maximum water level shall, however, be in no case less than 0.9 m. Wind velocities of 120 km/h over water in the case of full reservoir condition and 80 km/h over water in case of maximum reservoir condition are generally assumed for calculation of wave heights.

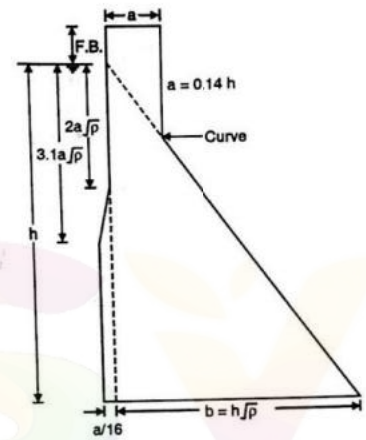

FIG. 8.19. PRACTICAL PROFILE OF A GRAVITY DAM.

However, according to the revised Code recommendations (IS 6512-1984) the free board should be equal to $\frac{4}{3}h_w$ plus *wind set-up*, where h_w is the wave height. The wave height and wind set-up should be calculated according to the method given by T. Saville

Thus, Free board $= \frac{4}{3}h_w +$ wind set-up

The *wind set-up* is the piling up of the water at one end of the reservoir and is determined by the following formula by Zuider Zee.

$$S = \frac{V^2 F \cos\beta}{62000\,D} \quad ...(8.47)$$

where S = wind set-up in metres,

V = wind velocity (km/hr)

F = fetch (km)

β = angle between wind direction and fetch

D = average depth of water, in metres, over the fetch.

The wind velocity is taken as 120 km/hr for normal reservoir level condition and 80 km/hr. for maximum water level condition. In no case shall the F.B. be less than 1 m above the maximum water level.

Modern practice is to provide a maximum free board equal to 3 to 4% of the height, though free board equal to 5% or more might prove economical.

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8.18. LIMITING HEIGHT OF A GRAVITY DAM: HIGH AND LOW GRAVITY DAMS

From Eq. 8.45, the principal stress at the toe is given by

$$\sigma_1 = wH(\rho - c + 1).$$

In this expression the only variable, changing the value of σ_1, is H. The maximum value of this principal stress should not exceed the allowable stress (f) for the material. In the limiting case,

$$f = \sigma_1 = wH(\rho - c + 1).$$

From which, the height H is given by

$$H = \frac{f}{w(\rho - c + 1)} \quad ...(8.48a)$$

For finding the limiting height H, it is usual not to consider the uplift. Hence, putting $c = 0$, we get,

$$H = \frac{f}{w(\rho + 1)} \quad ...(8.48)$$

If the height of the dam is more than that given by Eq. 8.48, the maximum compressive stress will exceed the permissible stress. This condition is undesirable.

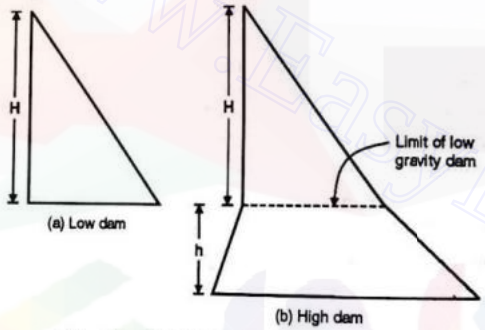

FIG. 8.20. HIGH AND LOW GRAVITY DAM.

Eq. 8.48 for the limiting height defines the distinction between a low and a high gravity dam. A *low gravity dam* is the one in which the height H is less than that given by Eq. 8.48, so that maximum compressive stress is not greater than the allowable stress. For a general case, taking $w = 9.81$ kN/m^3, and $\rho = 2.40$, the limiting height, in metres is given by $H \simeq 0.03\,f$, where f is the allowable stress in kN/m^2. If a common value of f is taken equal to be 2940 kN/m^2 (or 30 kg/cm^2) for concrete dam, the limiting height of low gravity dam comes out to be

$$H = \frac{2940}{9.81\,(2.4 + 1)} \simeq 88 \text{ m}.$$

If the height of the dam to be constructed is more than that given by Eq. 8.31, the dam is known as *high gravity dam*. For such a dam, the section will have to be given extra slopes to the upstream and downstream sides, below the limiting height, to bring the compressive stress within the limits, as illustrated in Fig. 8.20(*b*).

8.19. PROFILE OF HIGH MASONRY GRAVITY DAM

G. Molesworth gave the following expressions for profile of a high gravity dam of masonry (Fig. 8.21) :

$$x = \sqrt{\frac{0.05\,y^3}{f + (0.03\,y)}}$$

subject to a minimum of $\frac{y}{\sqrt{\rho}}$

(F. P. S. units) ...(8.49)

and $$z = \left(\frac{0.09\,y}{f}\right)^4 \quad ...(8.50)$$

where x = downstream offset from the vertical line (axis of the dam) at a depth y below the maximum reservoir level (feet)

z = upstream offset from the vertical line at a depth y (feet)

b_1 = value of x at $h/4$ (feet)

a = thickness of the dam at the full reservoir level = $0.4b_1$ (feet)

f = allowable pressure (compressive stress) in masonry, in tons per sq. ft. which may vary between 7 to 10 tons/ft^2.

The above formula is applicable for masonry dams only.

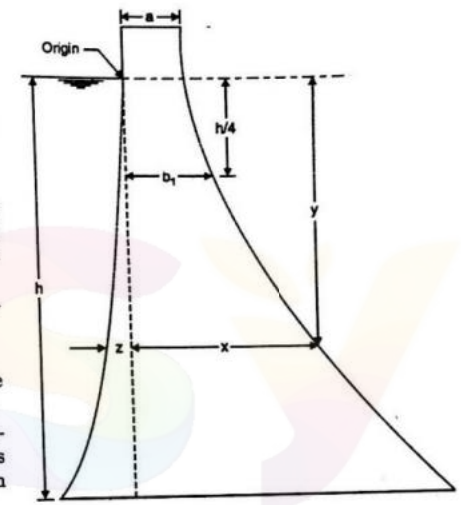

FIG. 8.21

8.20. DESIGN OF GRAVITY DAMS

The preliminary design is always done by the two dimensional method or *gravity method* in which the dam is thought to be composed of a number of cantilevers of unit length, acting independent of adjacent cantilevers. We have already seen in § 8.16 that the elementary profile of a gravity dam is triangular in shape, having zero width at the top. However, a truly triangular section is not practical nor is it necessarily the most economical section.

Top width.

Before discussing the design procedure, let us first study the effect of top width added at the

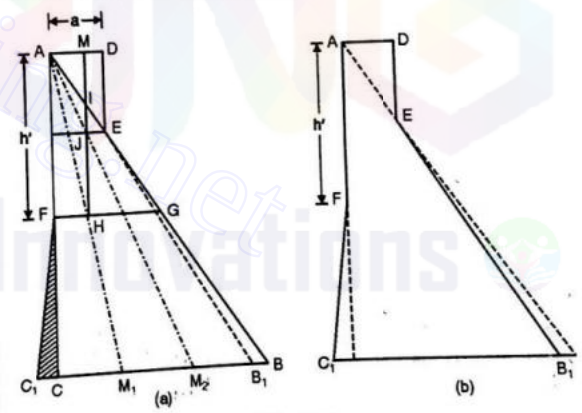

FIG. 8.22.

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apex of the triangular profile. Fig. 8.22(*a*) shows the elementary section *ABC*. If some top width $a = AD$ is provided, the resultant of the dam section will be shifted to the u/s when the reservoir is empty. AM_1 is the inner third point line, and *MI* is the line passing through the centroid of the added triangle *ADE*. Both these lines intersect at point *H*. For all sections below plane *FHG*, the resultant will, therefore, be shifted to the left of line AM_1, causing tension at the down stream face when the reservoir is empty. This will, therefore, require the provision of u/s batter FC_1 below the plane *FHG*. In order to find the depth *h'* of the plane *FHG* below which u/s batter is required, we have :

$$FH = AM = \frac{2}{3}a$$

$$FG = 3\,FH = 3 \times \frac{2}{3}a = 2a$$

Hence $h' = AF = FG\sqrt{\rho - c}$ (Eq. 8.42)

(where *c* is the uplift pressure intensity co-efficient)

Hence $h' = 2a\sqrt{\rho - c}$ ...(8.51)

Thus for heights greater than *h'*, u/s batter will have to be provided. The centroidal line *MJ* intersects with the outer-third -point line AM_2 at *J*. Hence, when the reservoir is full, the resultant of all sections below the plane *JE* is shifted to the u/s side. In order to bring the resultant back to the outer third point line, for the sake of economy, the slope of d/s face may be flattened, bringing it from *EB* to position EB_1. Thus, due to the provision of some top width *a*, the net economical section will be $ADEB_1C_1FA$ as shown in Fig. 8.22(*b*).

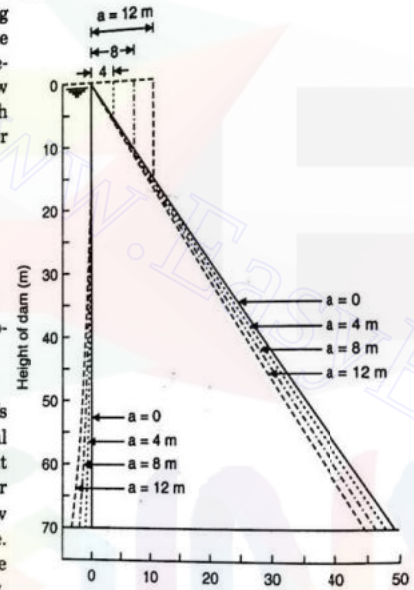

FIG. 8.23. EFFECT OF TOP WIDTH.

Fig. 8.23 shows the effect of top width of various magnitudes. We see that as the top width is increased, the u/s batter is *increased* while the d/s slope is *decreased*. It can be shown, that within limits, *the masonry added for the provision of top width decreases, rather than increases, the total masonry volume in the dam.*

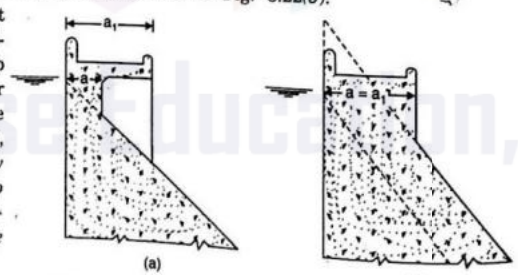

FIG. 8.24. METHODS OF PROVIDING TOP WIDTH.

The increase in masonry volume due to provision of top width is counter-balanced by the reduction in the d/s slope at lower levels. However, the most economical top width is the function of height of dam. Creager (1916) has shown that the most economical top width, without considering earthquake effects, is found to be about 14% of the height of dam. However, for dams of low height, the top width provided on the basis of economy (*i.e.* 14 % of height) may have to be increased from other practical considerations, such as provision of roadway on the top etc. Fig. 8.24 shows two alternative methods of providing the top width. Method shown in Fig. 8.24(*a*) is applicable for low dams where economical top width *a* falls short of the top width a_1 required from practical consideration. Method of Fig. 8.24(*b*) is applicable for high dams.

Free board.

Free board is provided on the requirements of height of waves, as discussed earlier in § 8.17. However, modern practice is to provide a maximum free board equal to 3 to 4 % of the height, though free board equal to 5 % or more might prove economical.

METHODS OF DESIGN OF GRAVITY DAMS :

Gravity dams can be designed by two methods :

(*a*) Multible step method (*b*) Single step method

(A) MULTIPLE STEP METHOD OF DESIGN OF GRAVITY DAM

For the economic design of gravity dam, the section of the dam is divided into various zones. Each zone is designed in such a way that all the requirements of stability are satisfied. Fig. 8.25 shows typical seven zones in a non-overflow gravity dam.

Zone I. This is the portion aa_1b_1b of the dam situated above the maximum reservoir level (or bottom of ice sheet where ice sheet exists). When no ice exists, the height of zone I is controlled by free board requirements and the width is determined by practical considerations or economy for the whole section. In the case of ice sheet the height of block I is fixed on the consideration of sliding of the zone due to ice pressure.

Zone II. This is the portion of the dam in which both the u/s and d/s faces remain vertical. The

FIG. 8.25. ZONING FOR NON-OVERFLOW GRAVITY DAMS.

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position of the bottom plane cc_1 of zone II is such that the resultant forces, when the reservoir is full, passes through the outer third point of the plane cc_1. However, when reservoir is empty, the line of resultant forces lies well within the middle third, throughout the height of zone II.

Zone III. In this Zone, the u/s face continues to be vertical while the d/s face is inclined. The line of resultant continues to conicide with the d/s extremity of middle third when the reservoir is full. The position of bottom plane dd_1 of zone III is located such that when the reservoir is empty the line of resultant passes through the u/s middle third point.

Zone IV. In this zone, the upstream face also begins to batter so that the lines of resultants lie along the corresponding extremities of middle third. The position of lower limit plane ee_1 of this zone is governed by the criterion that the maximum inclined pressure at d/s toe, for reservoir full condition is just equal to the allowable limit. The design of zone IV, specially the height as well as both u/s and d/s slopes, are determined by trial, by dividing zone IV into number of convenient blocks, till the bottom of zone IV is reached. It should be noted that *low dams* lie within the limits of zone IV. *Zones V, VI and VII are applicable only for high dams.*

Zone V. In this zone, the d/s slope is *flattened* so that the maximum inclined pressure at d/s toe, under reservoir full condition remains within the working stress. Thus the resultant for reservoir full condition remains well within middle third. For the reservoir empty condition, the resultant continues to intersect the u/s extremity of the kern. The lower limit of zone V is marked by plane ff_1, where the inclined compressive stress on the u/s heel reaches the permissible limit, for the reservoir empty condition.

Zone VI. In this zone, the conditions of the design are determined by the maximum pressures at both u/s and d/s faces under reservoir empty and reservoir full conditions, respectively. The line of resultants under both the conditions lie well within the middle third. The position of the bottom plane gg_1 is reached when the inclined pressure at d/s toe just reaches its maximum value.

Zone VII. This is the zone situated below zone VI, in which maximum compression at the downstream toe exceeds the working limit. This zone is usually eliminated by the revision of the entire design. That is, if the height of dam is so large that it is more than the position of plane gg_1 of zone VI, various changes are made in the upper zones so that the height of the dam lies within the zone VI. If this is not possible, then the height of the dam is reduced or superior material is utilised so that the height is accommodated within VIth zone.

(B) SINGLE STEP METHOD OF DESIGN OF HIGH DAMS

For high dams, going beyond zone IV, it is found that the shape of u/s and d/s slopes are sometimes obtained of unusual shape. Fig. 8.26(*a*) shows a typical section of high dam obtained by multiple step method. The u/s face has steep slope while the d/s slope has convex shape outwards. A convexed face under compression, whether smoothy curved or polygonal, may be subject to tensile stresses on surface parallel to the face. Such shape is usually not desirable since the outer layer of such a section

tends to buckle out-ward. Hence, the whole section may have to be revised in such a way that such a reversed curvature is avoided. The simple method would be to start the redesign work from the top, converting the whole dam into a single block controlled by the rules of zone VI.

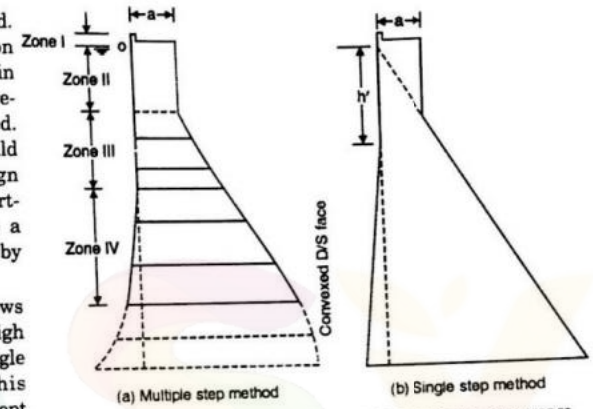

FIG. 8.26. SINGLE STEP METHOD FOR DESIGN OF HIGH

Fig. 8.26(*b*) shows the section of the same high dam designed by the single step method. In this method, the u/s slope is kept vertical for some depth, to be determined by trial. It may preliminarily be fixed by Eq. 8.51 such that the plane *FG* [Fig. 8.24(*a*)] passes through the intersection of the middle third line *AH* and the centroidal line *MH* of the top width triangle. After this height, both the u/s and d/s faces are given such slopes that the maximum inclined pressure at both u/s and d/s face under the conditions of reservoir empty and reservoir full conditions, reach their maximum values simultaneously. This is to be accomplished by trial and error. When this is accomplished, check computations will show that all the stability requirements for the reservoir full or empty conditions are satisfied at all points above the base.

Fig. 8.27 shows the sections of a high dam, designed both by the single step method and multiple step method. It is quite clear from the two sections that the multiple-step design is more economical for the upper portion of the dam. *The single-step-method section is under-stressed at all the points* except at the base. *Hence if the height of the dam is less, multiple-step-design method would give substantial saving in material.*

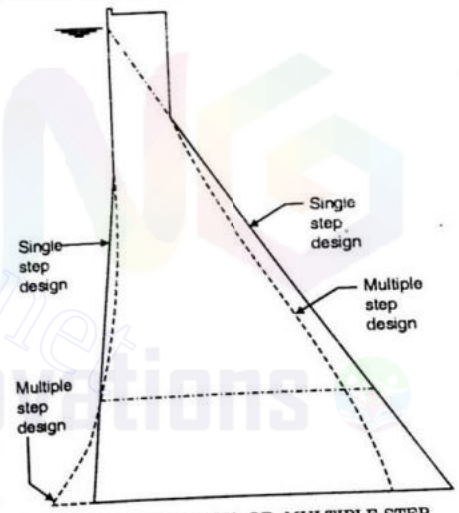

FIG. 8.27. COMPARISON OF MULTIPLE-STEP METHOD AND SINGLE STEP METHOD FOR THE DESIGN OF HIGH DAMS.

From the above discussions we draw the following conclusions :

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(*i*) Dams of lesser heights can be designed economically only by multiple step-design method.

(*ii*) It may be economical to increase the masonry strength through the use of more expensive materials, thus keeping out of zones V and VI.

(*iii*) High dams beyond zones VI are designed by single step method so that convex-curvature of d/s face is avoided.

Alternative method for the design of high gravity dam.

We have seen earlier that if the height of the dam is greater than that given by equation $h = \frac{f}{w(\rho+1)}$, the dam is known as a high dam. The design of such a dam, *below the limiting height*, is done by dividing the height into a number of suitable strips as shown in Fig. 8.28.

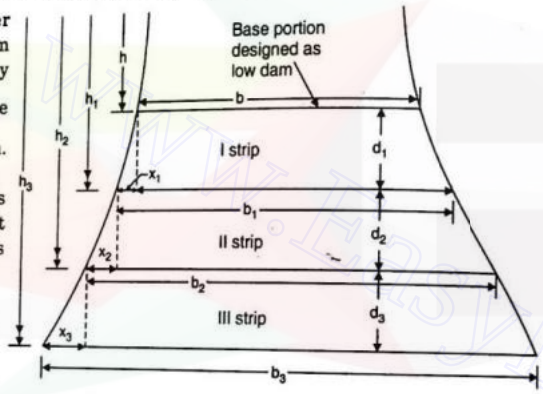

FIG. 8.28. DESIGN OF HIGH GRAVITY DAM.

Design of first strip

Let b_1 = base width of the first strip.

b = base width of the low dam.

h_1 = height of the dam portion from H.F.L. to the bottom of the first strip.

W_1 = total vertical load of dam and water, above the base of the first strip.

d_1 = height of the first strip.

W = total weight of dam portion and water, on the top of the first strip.

Then the base width b_1, sufficient to keep the maximum compressive stress within the safe value f given by

$$b_1 = \sqrt{\frac{wh_1^3}{f}\left(1+\frac{w^2 h_1^4}{4W_1^2}\right)} \quad ...(8.52)$$

The projection x_1 on the u/s of the heel is given by the following expression:

$$\frac{\rho w d_1}{24}\left\{3b^2 - b_1^2 + 6x_1(b+b_1) + 2bb_1\right\} - \frac{wx_1}{12}(h+h_1)(2b_1 - 3x_1) - W\left(\frac{b_1 - b}{3} - x_1\right) = 0 \quad ...(8.53)$$

Design for second strip

Using suffix 2 for the second strip, the corresponding expressions for b_2 and x_2 are as follows :

$$b_2 = \sqrt{\frac{wh_2^3}{f}\left(1+\frac{w^2 h_2^4}{4W_2^2}\right)} \quad ...[8.52(a)]$$

$$\frac{\rho w d_2}{24}\left\{3b_1^2 - b_2^2 + 6x_2(b_1+b_2) + 2b_1b_2\right\} - \frac{wx_2}{12}(h_1+h_2)(2b_2 - 3x_2) - W_1\left(\frac{b_2 - b_1}{3} - x_2\right) = 0 \quad ...[8.53(a)]$$

Similarly, the remaining strips are designed, till the base of the dam is reached.

8.21. GALLERIES

A gallery is a formed opening left in a dam. This may run in transverse or longitudinal direction and may run horizontally or on a slope. The shape and size varies from dam to dam and is generally governed by the functions it has to perform. Following are the purposes for which a gallery is formed in the dams.

1. To provide drainage of the dam section. Some amount of water constantly seeps through the upstream face of the dam which is drained off through galleries.
2. To provide facilities for drilling and grouting operations for foundations etc. Drillings for drain is generally resorted to clean them if they are clogged. High pressure grouting and required drilling for it is generally carried out after the completion of dam. This can be best done through galleries.
3. To provide space for header and return pipes for post cooling of concrete and grouting the longitudinal joints after completion of dam.
4. To provide access to observe and measure the behaviour of the structure after its completion by fixing thermo-couples and examining development of cracks etc.
5. To provide an access of mechanical contrivances needed for the operation of outlet gates and spillway gates.

Fig. 8.29 shows two typical shapes of galleries. In the rectangular gallery [Fig. 8.29(*a*)], all the corners should be rounded so that stress concentration is minimum. Where drainage trough is required, a rectangular trough such as at *c* is necessary, though shape *d* is preferred. Fig. 8.29(*b*) shows another form. To provide walkway, the semi-circular bottom is filled in with an unbounded slab.

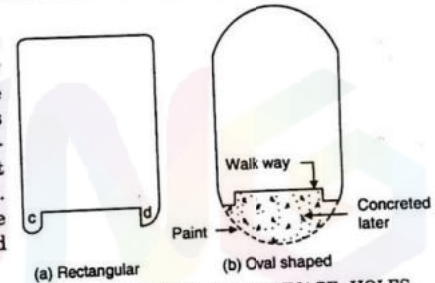

FIG. 8.29. SHAPES OF DRAINAGE HOLES.

Shafts. Vertical openings in the dam are called shafts. Shafts are provided to connect galleries at various levels. *Plumber shaft* is provided to measure the deflections of dam by suspending a plumb bob in it.

8.22. JOINTS, KEYS AND WATER SEALS

Joints. Various joints provided in dams may be classified under two heads : (*a*) construction joints, (*b*) contraction joints.

(*a*) **Construction Joints**. Construction joints, usually called *horizontal joints* are necessary since the entire work of concreting the whole dam cannot be completed in one stretch. The concreting is, therefore, done in various stages. In soild gravity dams, the height between horizontal joints is usually limited to about 1.5 m. This

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height between two successive construction joints or horizontal joints is known as *lift*. Evidently, this height is limited to the necessity of providing sufficient cooling between pours. For the first layer immediately on the rock, half this amount, *i.e.* 0.75 m height is adopted as the lift. Modern treatment of the surface and good concrete create an automatic water-tight horizontal joints, and as such no provisions, such as water stops or keyways etc., are made in the horizontal joints. The contraction joints, though provided for different purpose, also serve as construction joints

(*b*) **Contraction Joints.** Contraction joints are mainly provided to avoid cracks caused due to shrinkage of concrete due to temperature changes. The number of cracks

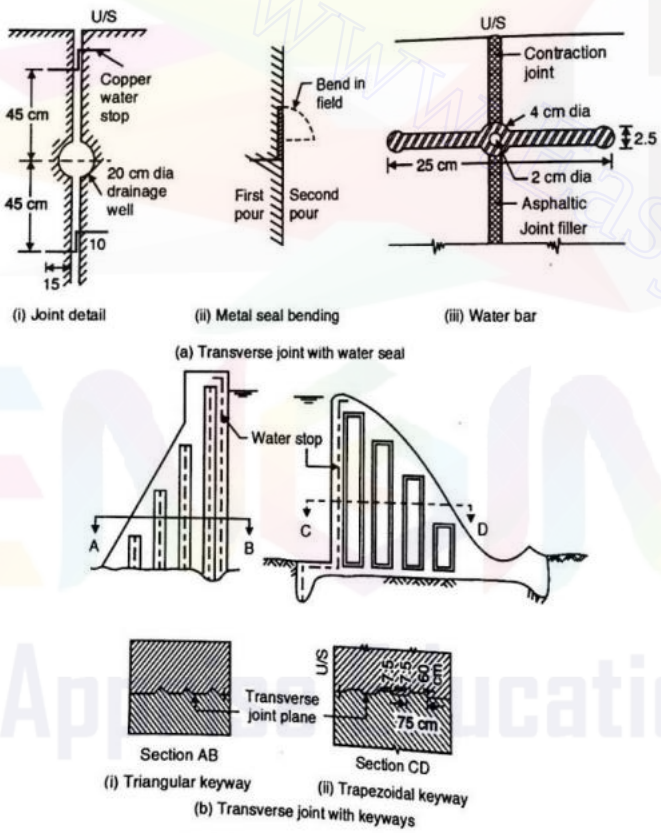

FIG. 8.30. TRANSVERSE JOINTS.

and their size can also be controlled to some extent, by controlled temperature of concrete while setting and curing. Contraction joints are of two types : (*i*) transverse joints, and (*ii*) longitudinal joints, depending upon the plane in which they lie.

(*i*) ***Transverse Joints.*** Transverse joints are provided normal to the axis of the dam. They are provided to prevent haphazard transverse cracks due to contraction of the concrete. The spacing of such joints depends upon a number of factors, including the position of the structure and the topographic features of dam site. The general practice is to restrict the spacing of the joints to about 15 m or the height of the dam, whichever is less. These joints extend entirely through the structure. In order to ensure intimate contact for transmission of stresses, transverse joints are consolidated either by grouting or by leaving a slot which is filled later when the monoliths have ceased shrinking. However, the transverse joints of solid gravity dams are not generally grouted. In order to make the joints water-tight, keyways or water stops are invariably provided in the transverse joints.

Fig. 8.30(*a*) shows a transverse joint with two ordinary copper water stops near the u/s side of the joint which confines a 20 cm drainage well which may be filled later with melted asphalt or grouted, Fig. 8.30(*a*)(*ii*) shows the method of bending the water stop in the field. When the foundations of the dam are of yielding type, the metal seals are damaged or broken. In that case, *water bars* of natural or synthetic rubbers and plastics, such as polyvinyl chloride etc. may be used, as shown in Fig. 8.30 (*a*) (*iii*). The joint is filled with asphaltic joint-filler or any other joint sealing compound which may provide a water-tight seal by adhesion to the concrete throughout the joint movement.

(*ii*) **Longitudinal Joint.** These are provided parallel to the axis of the dam to prevent longitudinal cracks. Since longitudinal cracks may coincide closely with the planes of maximum shear, these cracks are objectionable from the stand point of safety. Longitudinal joints run between two adjacent transverse joints and are thus not continuous all along the length of the dam. However, they are continuous in vertical plane, dividing the dam into a number of blocks. Fig. 8.31 shows the positions of transverse and longitudinal joints.

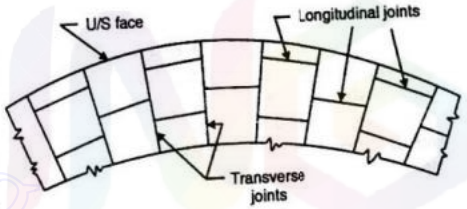

FIG. 8.31. THE SPACING OF LONGITUDINAL JOINTS.

The spacing of longitudinal joints is also limited to 15 m (or 50 ft). Keyways are invariably provided in the vertical longitudinal joints. These keyways are designed to transmit the principal stresses, and are spaced at 1.5 m (or 5 ft), *i.e.* one key in each lift. Fig. 8.32 shows a typical section through vertical longitudinal joints at Shasta Dam.

Keys. Keys are invariably provided in longitudinal joints to permit the transfer of shearing stresses from one block to the other. However, their provision in the transverse

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joints is optional. The adjoining surfaces of each side of the joint are given such a shape as to be interlocked together for the transfer of stresses. Keyways give a measure of shearing strength required at times when the dam is only partly full and the lines of first principal stress are not parallel to the joints. Keyways may be of several shapes, such a triangular [Fig. 8.30(*b*)], trapezoidal [Fig .8.30(*c*)] or trough shaped (Fig. 8.32).

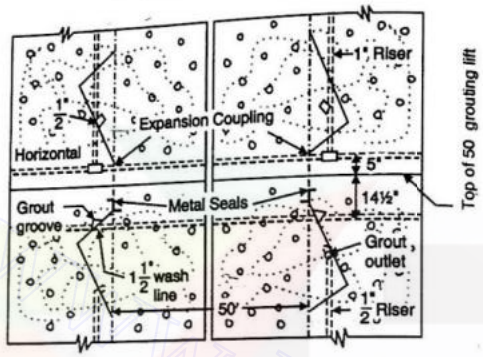

FIG. 8.32. SECTION THROUGH VERTICAL LONGITUDINAL JOINT : SHASTA DAM.

Water stops.

Water stops are provided in both the types of joints, to prevent leakage of water. Water stops are also sometimes known as *water bars*. Water stops may be either of metal, such as annealed copper, steel, monel, metal or sheet lead, or of natural or synthetic rubbers and plastics such as polyvinyl chloride. Metal water stops are provided only in the case of non-yielding foundations. For other cases of yielding foundation, rubber water bars are preferred. Sometimes, along with water bars, drainage well is also used which is filled with asphalt. Water stop, when accompanied by drainge wells are used in pairs [Fig. 8.30(*a*)].

8.23. CONTROL OF CRACKING IN CONCRETE DAMS

If proper temperature control is not exercised, the large concrete block between the joints may crack due to high temperature gradient between the interior and the surface. These large blocks of concrete are subject to deep as well as surface cracking. The cracks in the interior of the blocks are produced due to heat of hydration liberated by cement thus giving rise to high temperature gradient. The surface cracks may appear due to daily variations of the temperature at surface. The surface cracks are more harmful, since the disintegration starts through them by wedge action. Water enters these surface cracks, accumulates there and then solidifies. The ice so formed begins to expand at 4°C, resulting in widening and deepening of cracks.

Following are some of the methods employed to check or minimise the development of the cracks in mass concrete :

1. ***Pre-cooling of concrete.*** The concrete is pre-cooled before it is placed in the dam. This is accomplished by cooling the aggregate by refrigerated water, blowing air through them, cooling of sand and using refrigerated water for the manufacture of concrete. All this involves high cost of preparing the concrete.

2. ***Post cooling of concrete.*** The post cooling is achieved by circulating refrigerated water through pipes embedded in concrete in each lift. The cooling is begun immediately after a block is laid and is continued till the mass temperature falls to the mean annual temperature of the locality. For the purposes of circulating cool water, thin

walled tubing 2.5 cm outside diameter and 250 m long joined with expansion coupling are made into coils and are placed in the middle of each lift. The horizontal spacing of these tubings may be between 0.5 to 2 m and the velocity of flow is kept more than 60 cm per second. Resistance thermometrs are embeded in the concrete to ascertain the temperature.

3. Using ***low heat cement*** for the concrete.
4. Using ***lower percentage of cement*** in the concrete for the interior of the blocks, say about 80 % of that for the exterior.
5. ***Restricting the height of lift***, to say 1.5 m.
6. ***Allowing considerable time*** between laying of two successive vertical lifts. The usual time is about 4 days.
7. ***Providing contraction joints*** at suitable spacings.

Example 8.5. *Considering earthquake forces due to uniform horizontal acceleration $\alpha_h = \alpha$ and uniform vertical acceleration $\alpha_v = \alpha$ in addition to the hydrostatic pressure and uplift pressure, determine the base width of the elementary profile of gravity dam so that resultant passes through the outer third point.*

Solution

Let b = base width of elementary profile dam *ABC*.

The various forces acting on the dam are shown in Fig. 8.33.

(a) Vertical forces

1. Force due to self-weight of dam $= W = \frac{1}{2} bHw\,\rho\,(\downarrow)$
2. Force due to vertical acceleration of earthquake

$$= \alpha W = \frac{\alpha\, bHw\, \rho}{2} \uparrow$$

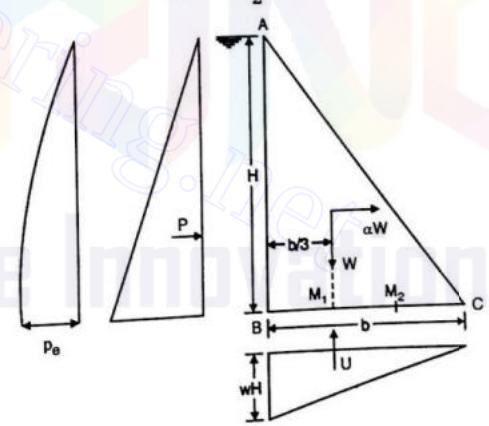

FIG. 8.33.

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3. Force due to uplift $= U = \frac{1}{2} b \,.\, wH\ (\uparrow)$

Hence $\Sigma V = W - \alpha W - U = \frac{1}{2} bwH\,[(1-\alpha)\rho - 1]$

where w = unit weight of water, ρ = specific weight of concrete

and α = coefficient of earthquake acceleration (both vertical as well as horizontal).

(b) *Horizontal forces*

1. Force due to water pressure $P = \frac{1}{2} wH^2$

2. Force due to hydrodynamic pressure of water at base :

$$C_m = 0.735\left(1 - \frac{\theta}{90°}\right) = 0.735 \text{ (since } \theta = 90°)$$

$$\therefore \quad p_e = C_m\, \alpha\, wH = 0.735\ \alpha\, wH$$

$$P_e = 0.726\ p_e \,.\, H$$

$$M_e = 0.299\, p_e\ H^2 = 0.299 \times 0.735\, \alpha\, wH^3 = 0.2198\ \alpha\, w\, H^3 .$$

3. ***Inertia force* (horizontal)** $= \alpha W = \frac{1}{2}\alpha\, b\, H\, w\, \rho$

If the resultant of all forces has to pass through the outer third point M_2, moment of all these forces at this point must be zero.

$$\therefore \quad \Sigma V \times \frac{b}{3} - \left[\frac{wH^2}{2} \times \frac{H}{3} + 0.2198\ \alpha\, w\, H^3 + \frac{\alpha\, w\, b\, H\, \rho}{2} \times \frac{H}{3}\right] = 0$$

or $$\frac{b^2 H w}{6}\left[(1-\alpha)\rho - 1\right] - \frac{bH^2 w \rho \alpha}{6} - \frac{w H^3}{6}\left[1 + \alpha\,(1.3186)\right] = 0$$

or $$b^2\,[(1-\alpha)\rho - 1] - bH\rho\alpha - H^2\,[1 + 1.3186\,\alpha] = 0$$

or $$b = H\,\frac{\rho\alpha \pm \sqrt{\rho^2\alpha^2 + 4(1 + 1.3186\alpha)\left[(1-\alpha)\rho - 1\right]}}{2\,[(1-\alpha) - 1]} \qquad ...(8.54)$$

which is the required expression.

Putting $\alpha = 0$ when no earthquake acts, the value of b reduces to

$$b = \frac{H}{\sqrt{\rho - 1}}$$

which is the same as Eq. 8.42 when $c = 1$.

Example 8.6. *Analysis of Rectangular profile dam*

A rectangular dam has constant width b and height H with reservoir full upto its top. Analyse the dam completely.

Solution

Consider a rectangular dam of width b and height H. (Fig. 8.34)

$$W = bH\rho w;\ P = \frac{wH}{2}\ ;\ U = \frac{1}{2} cwbH$$

Let the resultant pass through a point M, distant $\bar{x}$ from the toe B. Taking moments about the toe, we have.

$$\Sigma M = W \,.\, \frac{b}{2} - \frac{PH}{3} - U \,.\, \frac{2b}{3}$$

Also $$\Sigma W = W - U.$$

$$\therefore \quad \bar{x} = \frac{\Sigma M}{\Sigma W} = \frac{W\frac{b}{2} - \frac{PH}{3} - \frac{U\,.\,2b}{3}}{W - U}$$

Substituting the values of W, P and U, we get.

$$\bar{x} = \frac{bH\rho w \,.\, \frac{b}{2} - \frac{wH^3}{6} - \frac{2}{3} b\,(\frac{1}{2} cwbH)}{bH\rho w - \frac{1}{2} cwb\,H}$$

or $$\bar{x} = \frac{\frac{b}{2}(bH\rho w - \frac{1}{2} cwbH) - \frac{wH^3}{6} - (\frac{1}{2} cwbH)\frac{b}{6}}{bH\rho w - \frac{1}{2} cwbH}$$

$$= \frac{b}{2} - \frac{\frac{wH^3}{6} + \frac{1}{2} cwbH.\frac{b}{6}}{bH\rho w - \frac{1}{2} cwbH}$$

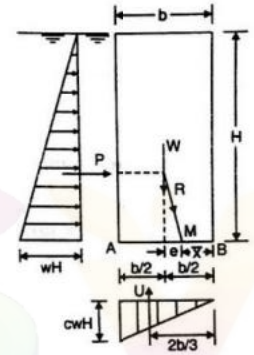

FIG. 8.34. RECTANGULAR PROFILE

Hence $$e = \frac{b}{2} - \bar{x} = \frac{\frac{wH^3}{6} + \frac{1}{2} cwbH \,.\, \frac{b}{6}}{bH\rho w - \frac{1}{2} cwbH} = \frac{\frac{H^2}{6} + \frac{cb^2}{12}}{b\rho - \frac{1}{2} cb}$$

For no tension to develop

$$e = \frac{b}{6} = \frac{\frac{H^2}{6} + \frac{cb^2}{12}}{b\rho - \frac{1}{2} cb}$$

or $$\frac{b^2}{6}\left(\rho - \frac{c}{2}\right) = \frac{H^2}{6} + \frac{cb^2}{12} \quad \text{or} \quad b^2(\rho - c) = H^2$$

From which $b = \frac{H}{\sqrt{\rho - c}}$ If $c = 0$ (no uplift), $b = \frac{H}{\sqrt{\rho}}$

Also, the limiting height is given by, $H_{\lim} = b\sqrt{\rho - c}$

From stability considerations, providing a F.S. of 1.5 against sliding, we have

$$\mu\,(W - U) = 1.5\,P$$

or $$\mu\,(b\,H\rho w - \frac{1}{2} cwbH) = \frac{3}{2} \times \frac{wH^2}{2} \quad \text{or} \quad \mu b\left(\rho - \frac{c}{2}\right) = \frac{3}{4} H$$

From which $$b = \frac{3}{4}\,\frac{H}{\mu\left(\rho - \frac{c}{2}\right)}$$

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If $c = 0$ (i.e. no uplift) $b = \frac{3}{4}\frac{H}{\mu\rho}$

Also, limiting height is, $H_{lim} = \frac{4}{3} b\mu\left(\rho - \frac{c}{2}\right)$

Neglecting uplift, $H_{lim} = \frac{4}{3} b\mu\rho$

From *crushing point of view*, when $e = b/6$, we get

$$p_n = \frac{2(W-U)}{b} = \frac{2}{b}\left(bH\rho w - \frac{1}{2}cwbH\right)$$

$$\sigma_c = p_n = 2wH\left(\rho - \frac{1}{2}c\right)$$

$$H_{lim} = \frac{\sigma_c}{2w\left(\rho - \frac{1}{2}c\right)}$$

If uplift is neglected, $H_{lim.} = \frac{\sigma_c}{2w\rho}$

Example 8.7. Analysis of trapezoidal dam

A trapezoidal dam with u/s face vertical, has top width a and bottom width b. Analyse the dam completely.

Solution :

Consider a trapezoidal profile with top width a and bottom width b, with upstream face vertical, as shown in Fig. 8.35.

W_1 = weight of rectangular portion $= aH\rho w$

W_2 = weight of triangular portion $= \frac{1}{2}bH \cdot \rho w$

P = Water pressure $= \frac{wH^2}{2}$

C = uplift pressure $= \frac{1}{2}cwbH$

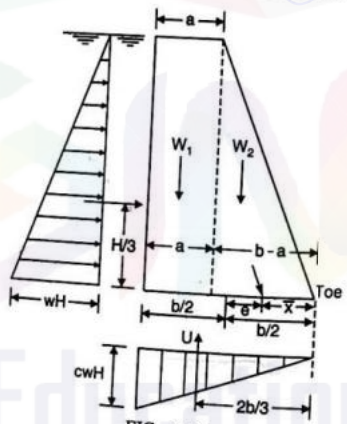

FIG. 8.35

Let the resultant fall at $\bar{x}$ from the toe.

If ΣM is the sum of moments of all the forces about the toe, we have

$$\Sigma M = \bar{x} \cdot \Sigma W \quad \text{or} \quad \bar{x} = \frac{\Sigma M}{\Sigma W}$$

$$\therefore \quad \bar{x} = \frac{aH\rho w\left(b - \frac{a}{2}\right) + \frac{1}{2}bH\rho w \times \frac{2}{3}(b-a) - \frac{wH^3}{6} - \frac{1}{2}cwb\,H \times \frac{2}{3}b}{aH\rho w + \frac{1}{2}bH\rho\, w - \frac{1}{2}cwbH}$$

or

$$\bar{x} = \frac{a\rho\left(b - \frac{a}{2}\right) + \frac{1}{3}b\rho(b-a) - \frac{H^2}{6} - \frac{1}{3}cb^2}{a\rho + \frac{1}{2}\rho b - \frac{1}{2}cb}$$

Eccentricity $e = \frac{b}{2} - \bar{x}$ and $p_n = \frac{\Sigma W}{b}\left(1 \pm \frac{6e}{b}\right)$.

Thus, stress at toe and heel can be found.

Example 8.8. *Rectangular masonry dam*

A rectangular masonry dam is 3 m at the base. Compute the maximum permissible height H (a) when no tension is permissible, and (b) when the factor of safety against sliding is 1.5. Given the following : (i) $\mu = 0.5$, (ii) density of masonry = 24 times the density of water, and (iii) $c = 1$. What will be corresponding values of H if uplift is neglected ?

Solution

Given : $\rho = 2.4$; $w = 9.81$ kN/m³

(*i*) ***Considering uplift***

$$W = bH\rho w = 3 \times 2.4 \times 9.81\,H = 70.632\,H \text{ kN}$$

$$U = \frac{1}{2}cwbH = \frac{1}{2} \times 1 \times 9.81 \times 3\,H = 14.715\,H \text{ kN}$$

$$P = \frac{wH^2}{2} = \frac{9.81\,H^2}{2} = 4.905\,H^2 \text{ kN}$$

$$\Sigma W = 70.632\,H - 14.715\,H = 55.917\,H$$

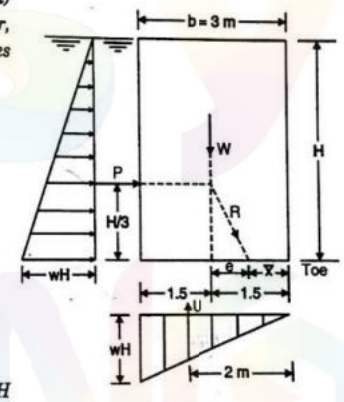

FIG. 8.36

Taking moments about the toe,

$$\Sigma M = (70.632\,H \times 1.5) - (14.715\,H \times 2) - 4.905\,H^3/3$$

$$= 76.518\,H - 1.635\,H^3$$

$$\therefore \quad \bar{x} = \frac{\Sigma M}{\Sigma W} = \frac{76.518\,H - 1.635\,H^3}{55.917\,H} = 1.368 - 0.02924\,H^2$$

(*a*) For no tension to develop, $e = b/6 = 3/6 = 0.5$ m

Hence, $e = (1.5 - \bar{x}) = 0.132 + 0.02924\,H^2 = 0.5$

From which, $H_{lim.} \approx$ **3.55 m**

Alternatively, from Eq. 18.45 (*b*), $H_{lim.} = b\sqrt{\rho - c} = 3\sqrt{2.4 - 1} = 3.55$ m

(*b*) From stability consideration, providing a F.S. of 1.5 against sliding, we have

$$\mu(W - U) = 1.5\,P$$

$$\therefore \quad 0.5\,(70.632\,H - 14.715\,H) = 1.5 \times 4.905\,H^2 \quad \text{or} \quad 27.9585 = 7.3575\,H$$

From which $H = H_{lim} =$ **3.8 m**

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Alternatively, from Eq. 18.47, $H_{lim.} = \frac{4}{3} b\,\mu \left(\rho - \frac{c}{2}\right) = \frac{4}{3} \times 3 \times 0.5\,(2.4 - 0.5) = 3.8$ m.

(*ii*) ***Neglecting uplift***

$W = 70.632\,H$ and $P = 4.905\,H^2$, as before,

$\Sigma M = 70.632\,H \times 1.5 - 4.905\,H^3/3 = 105.948\,H - 1.635\,H^3$.

$\Sigma W = W = 70.632\,H$

$$\bar{x} = \frac{\Sigma M}{\Sigma W} = \frac{105.948\,H - 1.635\,H^3}{70.632\,H} = 1.5 - 0.02315\,H^2$$

$$e = \frac{b}{2} - \bar{x} = 1.5 - (1.5 - 0.02315\,H^2) = 0.02315 H^2$$

(*a*) For no tension to develop, $e = b/6 = 3/6 = 0.5$ m

$0.02315\,H^2 = 0.5$, from which $H = H_{lim.} \approx$ **4.65 m**

(*b*) From stability consideration, providing a F.S. of 1.5 against sliding,

$\mu W = 1.5\,P$ or $0.5 \times 70.632\,H = 1.5 \times 4.905\,H^2$. From which $H =$ **4.8 m**

Example 8.9. *Trapezoidal masonry dam*

A masonry dam 6 m high is 1.5 m wide at top and 4.5 m wide at the bottom, with vertical water face. Determine the normal stresses at the toe and heel for reservoir empty and reservoir full conditions. Take ρ = 2.4 and c = 1.

Solution (Fig. 8.37)

Consider 1 m length of the dam.

(*a*) ***Reservoir empty***

Weight $W_1 = 1.5 \times 6 \times 1(2.4 \times 9.81) = 211.9$ kN

Weight $W_2 = \frac{1}{2} \times 3 \times 6\,(2.4 \times 9.81) = 211.9$ kN

$\Sigma W = 211.9 + 211.9 = 423.8$ kN

Taking moments about the toe (*B*) we have

$$\bar{x} = \frac{\Sigma M}{\Sigma W} = \frac{211.9 \times 3.75 + 211.9 \times 2}{423.8}$$

$= 2.875$ m

$$\therefore \quad e = \frac{b}{2} - \bar{x} = 2.25 - 2.875 = -0.625 \text{ m}$$

(i.e. resultant falls to the left of middle point).

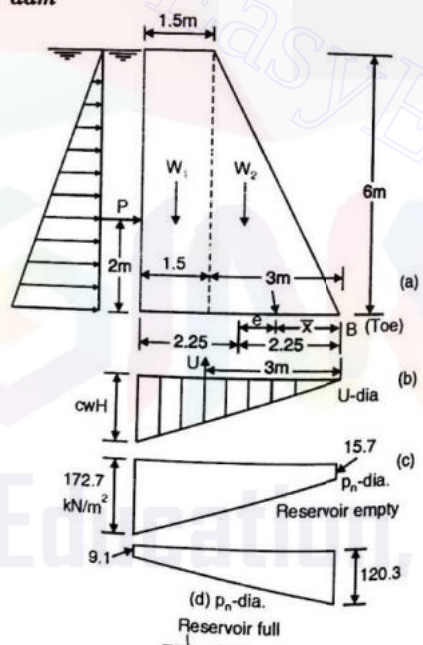

FIG. 8.37

$$\therefore \quad (p_n)_{toe} = \frac{\Sigma W}{b}\left(1 - \frac{6e}{b}\right) = \frac{423.8}{4.5}\left(1 - \frac{6 \times 0.625}{4.5}\right) = \mathbf{15.7\ kN/m^2} \text{ (comp.)}$$

$$\therefore \quad (p_n)_{heel} = \frac{\Sigma W}{b}\left(1 + \frac{6e}{b}\right) = \frac{423.8}{4.5}\left(1 + \frac{6 \times 0.625}{4.5}\right) = \mathbf{172.7\ kN/m^2} \text{ (comp.)}$$

The p_n distribution below the base is shown in Fig. 18.24 (*c*)

(*b*) ***Reservoir full*** : $W_1 = 211.9$ kN and $W_2 = 211.9$ kN, as before.

$$P = \frac{wH^2}{2} = \frac{9.81\,(6)^2}{2} = 176.58 \text{ kN}$$

$$U = \frac{1}{2} cwH.\,b = \frac{1}{2} \times 1 \times 9.81 \times 6 \times 4.5 = 132.435 \text{ kN.}$$

$\Sigma W = 211.9 + 211.9 - 132.435 = 291.365$ kN

Taking moments about the toe, we have

$$\bar{x} = \frac{\Sigma M}{\Sigma W} = \frac{211.9 \times 3.75 + 211.9 \times 2 - 132.435 \times 3 - 176.58 \times 2}{291.365} = 1.606 \text{ m}$$

$$\therefore \quad e = \frac{b}{2} - \bar{x} = 2.25 - 1.606 = 0.644 \text{ m}$$

$$\therefore \quad (p_n)_{toe} = \frac{\Sigma W}{b}\left(1 + \frac{6e}{b}\right) = \frac{291.365}{4.5}\left(1 + \frac{6 \times 0.644}{4.5}\right) = \mathbf{120.3\ kN/m^2} \text{ (comp.)}$$

$$\therefore \quad (p_n)_{heel} = \frac{\Sigma W}{b}\left(1 - \frac{6e}{b}\right) = \frac{291.365}{4.5}\left(1 - \frac{6 \times 0.644}{4.5}\right) = \mathbf{9.1\ kN/m^2} \text{ (comp.)}$$

The p_n distribution below the base is shown in Fig. 18.37 (*d*).

Example 8.10. *Trapezoidal masonry dam*

A dam 6 m high and 1.5 m wide at the top has vertical water face. Find the base width of the dam if no tension is to develop. Take unit weight of masonry as 20 kN/m³ and c = 1.

Investigate the stability of the above dam if the coefficient of friction is 0.6 and maximum allowable compressive stress is 1800 kN/m².

Solution

Let the bottom width be *b* m, as shown in Fig. 8.38.

$W_1 = 1.5 \times 6 \times 20 = 180$ kN

$W_2 = \frac{1}{2}(b - 1.5)\,6 \times 20 = 60\,(b - 1.5)$

$$P = \frac{wH^2}{2} = \frac{9.81\,(6)^2}{2} = 176.58 \text{ kN}$$

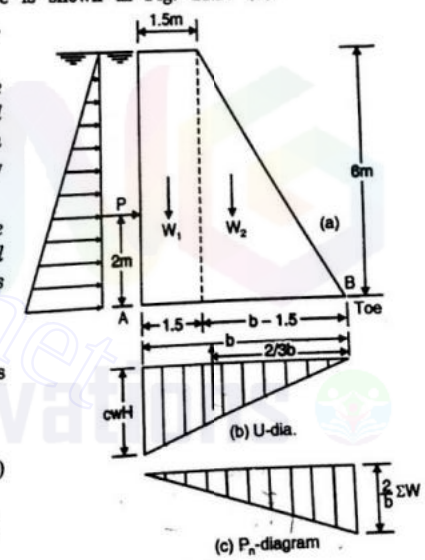

FIG. 8.38

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$$U = \frac{1}{2} cwH \,.\, b = \frac{1}{2} \times 1 \times 9.81 \times 6\, b = 29.43\, b.$$

$$\Sigma W = 180 + 60\,(b - 1.5) - 29.43\, b = 90 + 30.57\, b$$

Taking moments about the toe, we get

$$\bar{x} = \frac{\Sigma M}{\Sigma W} = \frac{180\,(b - 1.5 + 0.75) + 60\,(b - 1.5)\,\frac{2}{3}\,(b - 1.5) - 176.58 \times 2 - 29.43\, b\,\frac{2}{3}\, b}{90 + 30.57\, b}$$

For no tension to develop, e should be equal to $b/6$.

$$\therefore \qquad \frac{b}{2} - \bar{x} = \frac{b}{6} \quad \text{or} \quad \bar{x} = \frac{b}{3}$$

Hence,
$$\frac{b}{3} = \frac{180\,(b - 0.75) + 40\,(b - 1.5)^2 - 353.16 - 19.62\, b^2}{90 + 30.57\, b}$$

From which, we get $\quad b^2 - 2.944\, b - 39.074 = 0$

Hence $\quad b \simeq$ **4.95 m**

Check for crushing

$$\Sigma W = 90 + 30.57\, b = 90 + 30.57 \times 4.95 = 241.322 \text{ kN}$$

$$(p_n)_{max} = \frac{2\,\Sigma W}{b} = \frac{2\,(241.322)}{4.95} = 97.5 \text{ kN/m}^2 < 1800 \text{ OK.}$$

Check for sliding :

$$\text{F.S.} = \frac{\mu\,\Sigma W}{P} = \frac{0.6 \times 241.322}{176.58} = 0.82$$

Hence the dam is *unsafe* against sliding.

If a minimum factor of safety of 1 is required under the conditions of uplift, we have

$$\mu\,\Sigma W = P$$

$$\therefore \qquad 0.6\,(90 + 30.57\, b) = 176.58 \text{ from which } b = \mathbf{6.683\ m}$$

Example 8.11. *What will be the base width in the dam of Example 8.10 if uplift is neglected ? Investigate the corresponding stability conditions.*

Solution

$$W_1 = 180 \text{ kN} \;;\; W_2 = 60\,(b - 1.5) \;;\; P = 176.58 \text{ kN}$$

$$\Sigma W = 180 + 60\,(b - 1.5) = 90 + 60\, b$$

$$\bar{x} = \frac{\Sigma M}{\Sigma W} = \frac{180\,(b - 1.5 + 0.75) + 60\,(b - 1.5)\,\frac{2}{3}\,(b - 1.5) - 176.58 \times 2}{90 + 60\, b}$$

But for no tension to develop, $\bar{x} = b/3$

$$\frac{b}{3} = \frac{180\,(b - 0.75) + 40\,(b - 1.5)^2 - 353.16}{90 + 60\, b}$$

From which, we get $b^2 + 1.5\, b - 19.908 = 0$. Hence $b =$ **3.774 m.**

Check for crushing : $\Sigma W = 90 + 60\, b = 90 + 60 \times 3.774 = 316.44$ kN

$$(p_n)_{max} = \frac{2\,\Sigma W}{b} = \frac{2 \times 316.44}{3.774} = 167.7 \text{ kN/m}^2 < 1800. \text{ Hence safe.}$$

Check for sliding

$$\text{F.S.} = \frac{\mu\,\Sigma W}{P} = \frac{0.6 \times 316.44}{176.58} = 1.076 > 1. \text{ Hence safe.}$$

Example 8.12 : *Trapezoidal masonry dam with top shutter*

A trapezoidal dam section of 12 m height has vertical upstream face, with 3 m top width and 9 m bottom width. In order to increase the storage, it is proposed to raise the water height by erecting steel shutter at the top of the dam, in line with the vertical face of the dam. Determine the maximum permissible height of shutter so that no tension is developed at the upstream end of the base, under the reservoir full condition. Neglect foundation seepage. Take unit weight of masonry as 21 kN/m³.

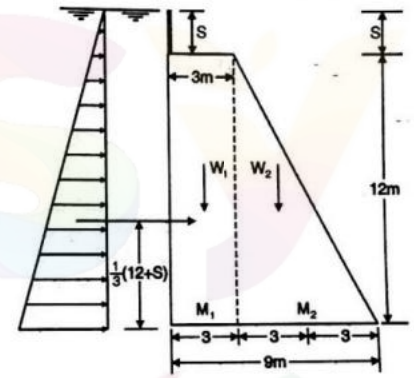

FIG. 8.39

Solution (Fig. 8.39)

Let the height of shutter be s. In order that no tension is developed, the resultant must pass through the outer third point M_2, distant 3 m from the toe. Hence ΣM at point M_2 must be zero.

Now $\quad W_1 = 3 \times 12 \times 21 = 756$ kN acting at 4.5 m from M_2

$$W_2 = \frac{1}{2} \times 6 \times 12 \times 21 = 756 \text{ kN acting at 1 m from } M_2.$$

$$P = \frac{wH^2}{2} = \frac{9.81\,(12 + s)^2}{2} = 4.905\,(12 + s)^2 \text{ kN}$$

acting at $\frac{1}{3}(12 + s)$ m above the base.

Taking moments about M_2,

$$\Sigma M = 756 \times 4.5 + 756 \times 1 - 4.905\,(12 + s)^2 \times \frac{1}{3}\,(12 + s) = 0$$

or $\quad (12 + s)^3 = 2543 \quad$ or $\quad (12 + s) = 2543^{1/3} = 13.65$

$$s = \mathbf{1.65\ m}$$

Hence height of shutter should not exceed 1.65 m.

Example 8.13. *A gravity dam has the following dimensions :*

Height of dam $\quad = 100$ *m*

Free board $\quad = 1$ *m*

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Slope of u/s face = *0.15 : 1.*

Taking uniform value of $\alpha_h = 0.1$, determine (i) hydrodynamic earthquake pressure and (ii) its moment at a joint situated 50 m below maximum water suface.

Solution. If θ is the angle that u/s slope makes with vertical, we have

$$\theta = \tan^{-1}\left(\frac{0.15}{1}\right) \triangleq 8.5°$$

$$C_m = 0.735\left(1 - \frac{8.5°}{90°}\right) = 0.666$$

Here $h = 100$ m and $y = 50$ m,

$$\therefore \quad C_y = \frac{C_m}{2}\left[\frac{y}{h}\left(2 - \frac{y}{h}\right) + \sqrt{\frac{y}{h}\left(2 - \frac{y}{h}\right)}\right] = \frac{0.666}{2}\left[0.5\,(2 - 0.5) + \sqrt{0.5\,(2 - 0.5)}\right] = 0.538$$

Now $p_{ey} = C_y\, \alpha_h\, w\, h = 0.538 \times 0.1 \times 9.81 \times 100 = 52.778$ kN/m².

Hence $P_{ey} = 0.726\, p_{ey} \,.\, y = 0.726 \times 52.778 \times 50 =$ **1915.8 kN/m**

$M_{ey} = 0.229\, p_{ey} \,.\, y^2 = 0.299 \times 52.778\,(50)^2 =$ **39451.4 kN-m/m**

Example 8.14. ***Trapezoidal dam*** *: A masonry dam 10 m high is trapezoidal in section with a top width of 1 m and bottom width of 8.25 m. The face exposed to water has a batter of 1 : 10. Test the stability of the dam. Find out the principal stresses at the toe and heel of the dam. Assume unit weight of masonry as 22.4 kN/m³, w for water = 9.81 kN/m³ and permissible shear stress of joint =1400 kN/m².*

Solution. Assuming no free board, stability is tested by considering only three forces. Consider 1 m length of dam.

FIG. 8.40.

1. Vertical forces

(*a*) Self-weight of the dam

$$= \frac{8.25 + 1}{2} \times 10 \times 1 \times 22.4 = 1036 \text{ kN.}$$

(*b*) Weight of water in column *AA′B*

$$= \frac{10 \times 1}{2} \times 9.81 = 49.05 \text{ kN.}$$

(*c*) Uplift force

$$= \tfrac{1}{2} \times 8.25 \times 9.81 \times 10 = 404.66 \text{ kN}$$

$$\therefore \quad \Sigma V = 1036 + 49.05 - 404.66 = 680.39 \text{ kN.}$$

2. Horizontal water pressure

$$\Sigma H = \frac{wh^2}{2} = \frac{9.81 \times 100}{2} = 490.05 \text{ kN.}$$

Moment due to various forces at toe

Consider oveturning moment as –ve and restoring moment is + ve.

(*a*) Due to self-weight of dam :

Moment $= \left\{\left(\tfrac{1}{2} \times 1 \times 10 \times 22.4\right)(7.25 + \tfrac{1}{3})\right\} + \left\{(1 \times 10 \times 22.4)\,(6.25 + 0.5)\right\}$

$+ \left\{\left(\tfrac{1}{2} \times 6.25 \times 10 \times 22.4\right)\left(\frac{2}{3} \times 6.25\right)\right\} = 5278$ kN-m (+ ve)

(*b*) Due to column of water in *AA′B* :

Moment $= \tfrac{1}{2}(10 \times 1 \times 9.81)\,(8.25 - \tfrac{1}{3}) = 388.31$ kN-m (+ ve)

(*c*) Due to uplift force :

Moment $= 404.66 \times 2/3 \times 8.25 = 2225.63$ kN-m (-ve)

(*d*) Due to horizontal water pressure

$= 490.5 \times 10/3 = 1635$ kN-m (–ve)

$\therefore \quad \Sigma M = 5278 + 388.31 - 2225.63 - 1635 = +5666.31 - 3860.63$

$= +1805.68$ kN-m

Calculation of factor of safety

Factor of safety against overturning

$$= \frac{M_R}{M_O} = \frac{(+)\,M}{(-)\,M} = \frac{5666.31}{3860.63} = \mathbf{1.47 < 1.5} \quad \text{Unsafe.}$$

Factor of safety against sliding

$$= \frac{\mu\,\Sigma V}{\Sigma H} \quad (\text{Assuming } \mu = 0.75) = \frac{0.75 \times 680.39}{490.5} = \mathbf{1.04 > 1}$$

Shear friction factor $= \dfrac{\mu\,\Sigma V + bs}{\Sigma H} = \dfrac{0.75 \times 680.39 + 8.25 \times 1400}{490.5} = \mathbf{24.6.}$

Calculation of stresses

The resultant acts at a distance $\bar{x}$ from toe,

where $\bar{x} = \dfrac{\Sigma M}{\Sigma V} = \dfrac{1805.68}{680.39} = 2.65$ m.

Its distance from centre is $e = \dfrac{b}{2} - \bar{x} = \dfrac{8.25}{2} - 2.65 = 1.475$ m

Compressive stress at toe is

$$p_n = \frac{\Sigma V}{b}\left[1 + \frac{6e}{b}\right] = \frac{680.39}{8.25}\left[1 + \frac{6(1.475)}{8.55}\right] = 167.8 \text{ kN/m}^2 \text{ (comp.)}$$

Compressive stress at heel

$$= \frac{\Sigma V}{b}\left[1 - \frac{6e}{b}\right] = \frac{680.39}{8.25}\left[1 - \frac{6\,(1.475)}{8.25}\right] = -2.9 \text{ kN/m}^2. \text{ (i.e. tension)}$$

From Fig. 8.36, $\tan\alpha = \dfrac{1}{10}$, $\tan\beta = \dfrac{6.25}{10}$

$\sec\alpha = \sqrt{1.01}$, $\sec\beta = \sqrt{1.391}$

Principal stress at toe of dam $= p_n \sec^2\beta = 167.8 \times 1.391 =$ **233.4 kN/m².**

Principal stress at heel $= p_n \sec^2\alpha - p \tan^2\alpha = -2.91 \times 1.01 - p\,(0.01)$

$= -2.91 \times 1.01 - 9.81 \times 10 \times 0.01 =$ **– 3.9 kN/m².**

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Shear stress at toe, $\tau = p_n \tan\beta = \dfrac{167.8 \times 6.25}{10} = \mathbf{104.9}$ **kN/m²**

Shear stress at heel $= -[p_n - p]\tan\alpha = -[-2.91 - 98.1] \times \dfrac{1}{10} = \mathbf{10.1}$ **kN/m².**

Example 8.15. *A concrete dam section is shown in Fig. 8.41. Check the stability of the section and determine the value of sliding factor and shear factor. Find out the magnitude and direction of principal stresses, normal stress and shear stress at toe and heel. Analysis of dam section it to be carried out under the following conditions:*

(i) Effect of only horizontal earthquake is to be considered by taking uniform value of $a_h = 0.1$ and by taking $C_m = 0.735$.

(ii) Reservoir full condition.

Take unit weight of concrete = 23.5 kN/m³ and unit shear for concrete = 1400 kN/m².

Uplift pressure at the base of any horizontal section varies from full reservoir level pressure at u/s to zero or tail water pressure at d/s and is considered to act over 2/3 area of section.

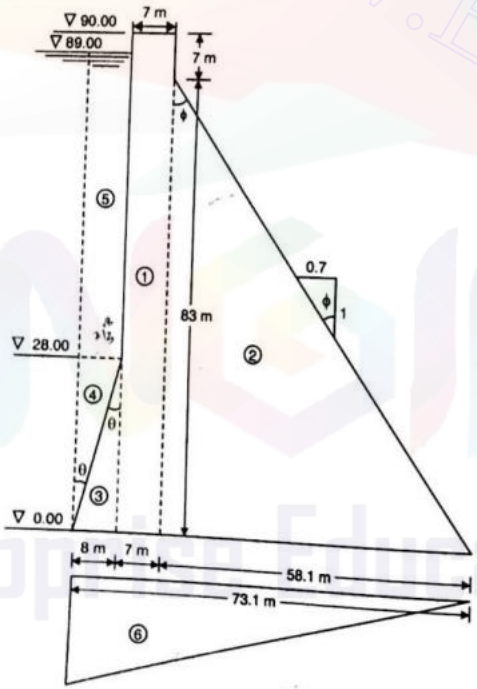

FIG. 8.41.

Solution. The analysis for stability is carried out for 1 m length of dam. The forces considered for the stability analysis are self-weight, uplift water pressure and horizontal earthquake. The analysis can be carried out as detailed in the tabular form (Table 8.7). Note the consideration of uplift since it has been multiplied by 2/3.

TABLE 8.7.

S.N.	Dimensions	Forces (kg.) Vertical	Forces (kg.) Horizontal	Lever Arm (m)	Moment at toe (kN-m) + ve	Moment at toe (kN-m) − ve
1.	***Self-weight of the dam*** $7 \times 90 \times 23.5$	14805		61.60	911988	
2.	$83 \times \frac{58.1}{2} \times 1 \times 23.5$	56662		38.73	2194709	
3.	$8 \times \frac{28}{2} \times 1 \times 23.5$	2632		67.77	178362	
	Weight of super-imposed column of water					
4.	$8 \times \frac{28}{2} \times 9.81$	1098.7		70.43	77387	
5.	$61 \times 8 \times 9.81$	4787.3		69.10	330801	
	Uplift Pressure					
6.	$89 \times \frac{73.1}{2} \times \frac{2}{3} \times 9.81$	− 21274.3		48.73		1036767
	Water pressure					
7.	$\frac{89^2}{2} \times 9.81$		38852.5	29.67		1152624
	Hydrodynamic pressure due to water					
8.	$0.726 \times 0.735 \times 0.1 \times 9.81 \times 89^2$		4146.4			
	Moment due to hydrodynamic pressure of water					
9.	$0.299 \times 0.735 \times 0.1 \times 9.81 \times 89^3$					151984
	Inertial load because of horizontal acceleration					
10.	Due to 1st rectangle		1480.5	45		66623
11.	Due to 2nd triangle		5666.2	83/3		156765
12.	Due to 3rd triangle		263.2	28/3		2457
	Sum	58710.7	50408.8		3693247	2567220
				$\Sigma M = 1126027$		

Calculation of stresses

From Fig. 8.41, $\tan\phi = 0.7$, $\sec\varphi = (\sqrt{1+0.49})/1 = \sqrt{1.49} = 1.2207$

and $\tan\theta = 8/28 = 2/7$

Distance where resultant acts from toe is $\bar{x} = \dfrac{\Sigma M}{\Sigma V} = \dfrac{1126027}{58710.7} = 19.18$ m.

Its distance e from the centre of the base is

$$e = \frac{b}{2} - \bar{x} = 36.55 - 19.18 = 17.37 \text{ m.}$$

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Normal stress at toe $p_n = \frac{\Sigma V}{b}\left[1 + \frac{6e}{b}\right] = \frac{58710.7}{73.1}\left[1 + \frac{6 \times 17.37}{73.1}\right]$

$= 1948 \text{ kN/m}^2 \ (\triangleq 1.95 \text{ N/mm}^2)$

Normal stress at heel $p_n = \frac{\Sigma V}{b}\left(1 - \frac{6e}{b}\right) = \frac{58710.7}{73.1}\left(1 - \frac{6 \times 17.37}{73.1}\right)$

$= -342 \text{ kN/m}^2$, *i.e.* tensile.

Principal stress at toe (there is no tail water) :

$$\sigma_1 = p_n \sec^2 \phi = 1948 \times 1.49 = 2903 \text{ kN/m}^2 \ (2.9 \text{ N/mm}^2)$$

Its direction is parallel to downstream face.

Principal stress of heel is : $\sigma_1 = p_n \sec^2 \theta - (p + p_e) \tan^2 \theta$

where p = water pressure at base $= 89 \times 9.81 = 873.1 \text{ kN/m}^2$

p_e = hydrodynamic pressure at base $= C_m \alpha_h w h$

$C_m = 0.735$

$\alpha_h = 0.1$

$\therefore \quad p_e = 0.735 \times 0.1 \times 9.81 \times 89 = 64.2 \text{ kN/m}^2$

$\sigma_1 = -342\,[1 + 4/49] - [873.1 + 64.2] \times 4/49 = -446 \text{ kN/m}^2$.

Hence, the dam becomes unsafe as the concrete cannot withstand tensile stress more than $0.02 f_c = 0.02 \times 15 = 0.3 \text{ N/mm}^2 = 300 \text{ kN/m}^2$ for M15 concrete for load combination E (Table 8.6). Its direction is perpendicular to upstream face.

Shear stress at toe $\tau = p_n \tan\phi = 1948 \times 0.7 = 1364 \text{ kN/m}^2$

Shear stress at heel

$$\tau = -[p_n - (p + p_e)] \tan\theta] = -[342 - (873.1 + 64.2)] \times 2/7 = 170 \text{ kN/m}^2.$$

Calculation of factor of safety

(*a*) Factor of safety against sliding as per IS 6512-1972 : Taking $\mu = 0.75$,

$$\text{F.S.S.} = \frac{\mu \Sigma V}{\Sigma H} = \frac{0.75 \times 58710.7}{50408.8} = 0.874 < 1.5 \text{ Unsafe.}$$

(*b*) Shear friction factor as per IS 6512-1972 :

$$\text{S.F.F.} = \frac{\mu \Sigma V + bc}{\Sigma H} = \frac{0.75 \times 58710.7 + 73.1 \times 1 \times 1400}{50408.8}$$

$= 2.91 < 6$; hence unsafe.

(*c*) Factor of safety againsty sliding as per IS 6512-1984 :

For load condition E, taking $F_\varphi = 1.2$ and $F_c = 2.4$ (Table 8.5), we get

$$F = \frac{\frac{\mu \Sigma V}{F_\varphi} + \frac{cb}{F_c}}{\Sigma H} = \frac{\frac{0.75 \times 58710.7}{1.2} + \frac{1400 \times 73.1}{2.4}}{50408.8} = 1.57 > 1 \text{ (Safe)}$$

(*d*) Factor of safety against overturning

$$= \frac{\Sigma M_R}{\Sigma M_O} = \frac{(+)M}{(-)M} = \frac{3693247}{2567220} = 1.44 < 1.5$$

The dam section is therefore, ***unsafe*** for the present loading conditions.

Example 8.16. (*a*) *Check the stability of the gravity dam section shown in Fig. 8.42 for reservoir empty and reservoir full conditions. Neglect seismic forces, assume reasonable values for uplift and a line of drain holes 6.5 m downstream of the upstream face. For the purpose of this check, assume water level at the top of the dam and no tail water. Also find principal and shear stresses at the toe and heel of the dam. Take unit weight of concrete as 24 kN/m³, shear strength of concrete as 1400 kN/m² and* $\mu = 0.7$.

(*b*) *Check the stability of the dam section by considering seismic forces also, assuming a value of horizontal accelertion coefficient* $\alpha_h = 0.1$ *and vertical acceleration coefficient* $\alpha_v = 0.08$, *both uniform over the depth.*

Solution

(*a*) **Stability analysis neglecting seismic forces :**

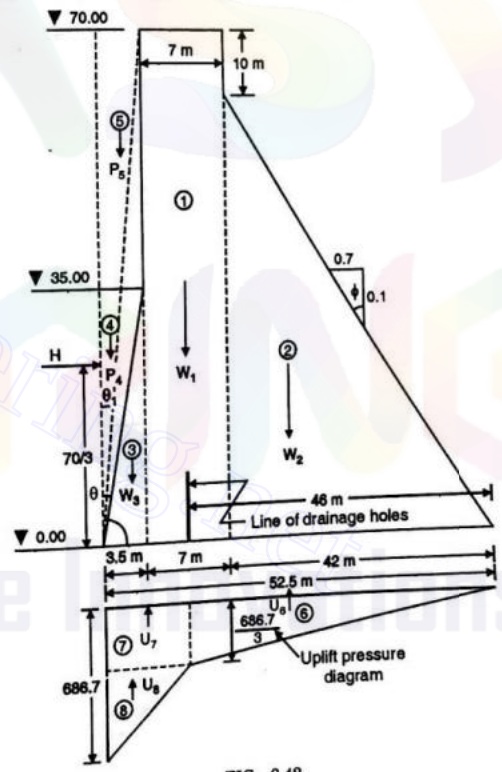

FIG. 8.42

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Case I. Reservoir Empty Condition

When reservoir is empty, only self weight of the dam will be acting as force. Other forces namely water pressure and uplift will be zero. The resulting force ΣV_1 and resulting moment ΣM_1 for this case has been worked out in Table 8.8.

Position of resultant from toe :

$$\bar{x} = \frac{\Sigma M_1}{\Sigma V_1} = \frac{1455545}{43470} = 33.48 \text{ m}$$

Its distance from centre is

$$e = b/2 - \bar{x} = 52.5/2 - 33.48 = -7.23 \text{ m}$$

(*i.e.*, the resultant falls to the left of the centre)

Normal compressive stress at toe :

$$p_n = \frac{\Sigma V_1}{b}\left[1 + \frac{6e}{b}\right] = \frac{43470}{52.5}\left[1 + \frac{6(-7.23)}{52.5}\right]$$

$$= 143.8 \text{ kN/m}^2 \text{ compressive}$$

Normal compressive stress at heel :

$$p_n = \frac{\Sigma V}{b}\left[1 - \frac{6e}{b}\right] = \frac{43470}{52.5}\left[1 - \frac{6(-7.23)}{52.5}\right] = 1512.2 \text{ kN/m}^2.$$

TABLE 8.8. STABILITY CALCULATIONS NEGLECTING SEISMIC FORCES

S. No.	Item	Description	Forces		Lever arm (m)	Movement at toe		Remarks
			Vertical	Horizontal		+ ve	− ve (kN-m)	
		Self weight of dam						
1.	W_1	$70 \times 7 \times 1 \times 24$	11760		45.5	535080		Reservoir Empty Condition
2.	W_2	$60 \times 42 \times \frac{1}{2} \times 24$	30240		28	846720		
3.	W_3	$35 \times 3.5 \times \frac{1}{2} \times 24$	1470		50.17	73745		
		Sum	$\Sigma V_1 = 43470$		Sum	+1455545	0.00	
						$\Sigma M_1 = +1455545$		
		Water Pressure						
4.	P_4	$3.5 \times 35 \times \frac{1}{2} \times 9.81$	600.9		51.33	30844		Reservoir full with no uplift
5.	P_5	$3.5 \times 35 \times 9.81$	1201.7		50.75	60988		
	H	$\frac{1}{2} \times 70^2 \times 9.81$		24034.5	70/3		560805	
		Sum	$\Sigma V_2 = 45272.6$	240345	Sum	+1547377	− 560805	
						$\Sigma M_2 = +986572$		
		Uplift forces						
6.	U_6	$686.7/3 \times 46 \times \frac{1}{2} \times 1$	− 5264.7		30.67		161451	Reservoir full with uplift
7.	U_7	$686.7/3 \times 6.5 \times 1$	− 1487.9		49.25		73277	
8.	U_8	$2 \times 686.7/3 \times 6.5 \times \frac{1}{2} \times 1$	− 1487.9		50.33		74888	
		Sum	$\Sigma V_3 = 37032.1$	24034.5	Sum	+1547377	− 870421	
						$\Sigma M_3 = +676956$		

Principal stress at toe :

$$\sigma_1 = p_n \sec^2\phi \quad \text{where } \tan\phi = 0.7, \text{ and } \sec^2\phi = 1.49$$

$$\therefore \quad \sigma_1 = 143.8 \times 1.49 = 214.3 \text{ kN/m}^2.$$

Principal stress at heel :

$$\sigma = p_n \sec^2\theta \quad \text{where } \tan\theta = 3.5/35 = 0.1 \text{ and } \sec^2\theta = 1.01$$

$$\therefore \quad \sigma = 1512.2 \times 1.01 = 1527.3 \text{ kN/m}^2.$$

Shear stress at toe :

$$\tau = p_n \tan\varphi = 143.8 \times 0.7 = 100.7 \text{ kN/m}^2$$

Shear stress at heel :

$$\tau = p_n \tan\theta = 1512.2 \times 0.1 = 151.2 \text{ kN/m}^2.$$

Note that there cannot be any sliding or overturning when reservoir is empty.

II. *Reservoir full with no uplift* : Sometimes values of stresses at toe and heel are worked out without considering uplift as the vertical forces are maximum in this case.

Calculation of stresses : $\tan\theta = 0.1$; $\sec^2\theta = 1.01$; $\tan\phi = 0.7$; $\sec^2\phi = 1.49$

The values of vertical forces ΣV_2 and moments ΣM_2 have been worked out in Table 8.8 where the. ΣV_2 and ΣM_2 represent the sum of vertical forces and sum of moments of all forces when the reservoir is full but when uplift is not acting.

Position of the resultant from the toe is

$$\bar{x} = \frac{\Sigma M_2}{\Sigma V_2} = \frac{986572}{45272.6} = 21.79 \text{ m}$$

Position of the resultant from the centre of the base is

$$e = b/2 - \bar{x} = 26.25 - 21.79 = 4.46 \text{ m.}$$

Normal compressive stress at toe

$$p_n = \frac{\Sigma V_2}{b}\left(1 + \frac{6e}{b}\right) = \frac{45272.6}{52.5}\left(1 + \frac{6 \times 4.46}{52.5}\right) = 1301.9 \text{ kN/m}^2 (\triangleq 1.3 \text{ N/mm}^2)$$

Normal compressive stress at heel

$$p_n = \frac{\Sigma V_2}{b}\left[1 - \frac{6e}{b}\right] = \frac{45272.6}{52.5}\left[1 - \frac{6 \times 4.46}{52.5}\right] = 422.8 \text{ kN/m}^2 (\triangleq 0.42 \text{ N/mm}^2)$$

Principal stress at toe is

$$\sigma_1 = p_n \sec^2\phi = 1301.9 \times 1.49 = 1939.8 \text{ kN/m}^2 (\triangleq 1.94 \text{ N/mm}^2)$$

Principal stess at heel is

$$\sigma = p_n \sec^2\theta - p \tan^2\theta = 422.8 \times 1.01 - 9.81 \times 70 \times 0.01$$

$$= 420.2 \text{ kN/m}^2 (\triangleq 0.42 \text{ N/mm}^2)$$

Note. This is the value of minor principal stress at the heel. The value of major principal stress at heel is equal to $9.81 \times 70 = 686.7 \text{ kN/m}^2$ ($\triangleq 0.69 \text{ N/mm}^2$).

Shear stress at toe, $\tau = p_n \tan\phi = 1301.9 \times 0.7 = 911.3 \text{ kN/m}^2$ ($\triangleq 0.91 \text{ N/mm}^2$).

Shear stress at heel is $\tau = -(p_n - p)\tan\theta = -(422.8 - 686.7) \times 0.1 = 26.4 \text{ kN/m}^2$.

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The factor of safety against sliding and overturning should be found out only when reservoir is full and full uplift acts, since the condition of overturning and sliding will be more critical in that case.

Case III. Reservoir full with uplift : Values of vertical forces ΣV_3 and moment ΣM_3 have been worked out in Table 8.8.

Position of resultant from the toe is

$$\bar{x} = \frac{\Sigma M_3}{\Sigma V_3} = \frac{676956}{37032.1} = 18.27 \text{ m}$$

Position of resultant from the centre of the base is

$$e = \frac{b}{2} - \bar{x} = 26.25 - 18.27 \text{ m} = 7.98 \text{ m.}$$

Normal compressive stress at toe is

$$p_n = \frac{\Sigma V_3}{b}\left[1 + \frac{6e}{b}\right] = \frac{37032.1}{52.5}\left[1 + \frac{6 \times 7.98}{52.5}\right] = 1348.7 \text{ kN/m}^2 \ (\triangleq 1.35 \text{ N/mm}^2).$$

This compressive stress is greater than the one obtained for condition II ; hence this is the worst case.

Normal compressive stress at heel

$$p_n = \frac{37032.1}{52.5}\left[1 - \frac{6 \times 7.98}{52.5}\right] = 62.1 \text{ kN/m}^2 \text{ (Comp.).}$$

Principal stress at toe

$$\sigma_1 = p_n \sec^2 \phi = 1348.7 \times 1.49 = 2010 \text{ kN/m}^2 (\triangleq 2.01 \text{ N/mm}^2).$$

Shear stress at heel

$$\sigma_1 = p_n \sec^2 \theta - p \tan^2 \theta = 62.1 \times 1.01 - 9.81 \times 70 \times 0.01 = 56 \text{ kN/m}^2.$$

Shear stress at toe

$$\tau = p_n \tan \phi = 1348.7 \times 0.7 = 944 \text{ kN/m}^2 \ (\triangleq 0.94 \text{ N/mm}^2).$$

Shear stress at heel

$$\tau = -(p_n - p) \tan \theta = -(62.1 - 9.81 \times 70) \times 0.1 = 62 \text{ kN/m}^2.$$

Calculation of factor of safety

Factor of safety against overtuning $= \dfrac{\Sigma M(+)}{\Sigma M(-)} = \dfrac{1547377}{870421} = 1.78 > 1.5$

Factor of safety against sliding $= \dfrac{\mu \Sigma V_3}{\Sigma H} = \dfrac{0.70 \times 37032.1}{24034.5} = 1.079 < 2.$

Shear friction factor S.F.F. $= \dfrac{\mu \Sigma V_3 + b.c}{\Sigma H} = \dfrac{0.70 \times 37032.1 + 52.5 \times 1400}{24034.5} = 4.14 > 4$

Safety against sliding according to IS 6512-1984 :

Taking $F_\varphi = 1.5$ and $F_c = 3.6$ for load combination B,

$$F \cong \frac{\dfrac{\mu \Sigma V}{F_\varphi} + \dfrac{cb}{F_c}}{\Sigma H} = \frac{\dfrac{0.7 \times 37032.1}{1.5} + \dfrac{1400 \times 52.5}{3.6}}{24034.5} = 1.568 > 1 \qquad \text{Hence safe.}$$

(*b*) **Stability check of dam by considering seismic forces**

For worst condition consider that :

(*a*) Horizontal earthquake acceleration acts upstream.

(*b*) Vertical earthquake acceleration acts downwards.

Hydrodynamic pressure due to water caused by earthquake can be found out from Zanger's formula. Since the slope is upto middle depth, approximate value of θ can be found out by joining heel to the upstream edge,

$$\tan \theta_1 = \frac{3.5}{70} = 0.05 \quad \text{or} \quad \theta_1 = 2.862°$$

$$\therefore \quad C_m = 0.735 \times \left(1 - \frac{2.862°}{90°}\right) = 0.712$$

At base $\quad C = C_m$

Therefore, $\quad p_e = C_m \, \alpha_h \, w \, h = 0.712 \times 0.1 \times 9.81 \times 70 = 48.89 \text{ kN/m}^2$

Total pressure $P_e = 0.726 \, p_e \, h = 0.726 \times 48.89 \times 70 = 2485$ kN

Moment due to this force at base,

$$M_e = 0.299 \, p_e \, h^2 = 0.299 \times 48.89 \times 4900 = 71629 \text{ kN-m.}$$

Calculation of forces and moments due to inertial earthquake force is done in the tabular form (Table 8.9). This Table is to be prepared in continuation of the previous Table (Table 8.8) made without considering earthquake forces.

Values of vertical forces ΣV_4 and moment ΣM_4 have been worked out in the Table 8.9.

TABLE 8.9. ADDITIONAL FORCES AND THEIR MOMENTS DUE TO EARTHQUAKE

S. No.	Item	Description	Forces (kN)		Lever arm (m)	Moment of toe (kN-m)		Remarks
			Vertical	Horizontal		+ ve	- ve	
		Inertial force due to earthquake on weight of dam					116444	Inertial force acting at C.G. is considered to be acting upwards
1.	ΣV_1	$\Sigma V_1 \times \alpha_v$	- 3477.6		35		41160	
2.	W_1	$W_1 \times \alpha_h$		1176	20		60480	
3.	W_2	$W_2 \times \alpha_h$		3024	35/3		1715	
4.	W_3	$W_3 \times \alpha_h$		147			71629	
5.	P_e	*Hydrodynamic pressure*		2485			$\Sigma M = -291428$	
		Total	$\Sigma V = -3477.6$	$\Sigma H = 6832$		+1547377	- 870421	
		Sum of forces and moments from previous table	$\Sigma V_3 = 37032.1$	$H = 24034.5$				
						+1547377	- 1161849	
		Sum	$\Sigma V_4 = 33554.5$	$\Sigma H = 30866.5$		$\Sigma M_4 = 385528$		

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Position of resultant from toe is

$$\bar{x} = \frac{\Sigma M_4}{\Sigma V_4} = \frac{385528}{33554.5} = 11.48 \text{ m}$$

Hence the resultant does not lie within the middle third. Its distance from centre is

$$e = \frac{b}{2} - \bar{x} = 26.25 - 11.48 = 14.77 \text{ m}$$

i.e., the resultent falls to the right of centre.

Normal compressive stress at toe

$$p_n = \frac{\Sigma V_4}{b}\left[1 + \frac{6e}{b}\right] = \frac{33554.5}{52.5}\left[1 + \frac{6 \times 14.77}{52.5}\right]$$

$$= 1718 \text{ kN/m}^2 \ (\triangleq 1.72 \text{ N/mm}^2) \text{ compressive.}$$

Normal compressive stress at heel

$$p_n = \frac{\Sigma V_4}{b}\left[1 - \frac{6e}{b}\right] = \frac{33554.5}{52.5}\left[1 - \frac{6 \times 14.77}{52.5}\right]$$

$$= -440 \text{ kN/m}^2 \ (\triangleq 0.44 \text{ N/mm}^2) \text{ tensile.}$$

Since the resultant does not fall within middle third and the value of tensile stress is substantial (> 0.2 f_c for M15 concrete) the dam is ***unsafe*** under seismic condition.

Principal stress at toe is

$$p_n \sec^2\phi = 1718 \times 1.49 = 2560 \text{ kN/m}^2 \ (\triangleq 2.56 \text{ N/mm}^2).$$

Principal stress at heel is

$$\sigma_1 = p_n \sec^2\theta - (p + p_e)\tan^2\theta = -440 \times 1.01 - (9.81 \times 70 + 48.89) \times 0.01$$

$$= -452 \text{ kN/m}^2 \quad \text{Hence unsafe.}$$

Shear stress at toe

$$\tau = p_n \tan\phi = 1718 \times 0.7 = 1203 \text{ kN/m}^2.$$

Shear stress at heel

$$\tau = -[p_n - (p + p_e)]\tan\theta = -[-440 - (9.81 \times 70 + 48.89)] \times 0.1$$

$$= 1176 \text{ kN/m}^2.$$

Calculations for factor of safety

Factor of safety against overturning

$$= \frac{\Sigma M(+)}{\Sigma M(-)} = \frac{1547377}{1161849} = 1.33 < 1.5. \text{ Hence unsafe.}$$

Factor of safety against sliding

$$= \frac{\mu \Sigma V_4}{\Sigma H} = \frac{0.70 \times 33554.5}{30866.5}$$

$$= 0.761 < 1.5 \quad \text{Hence unsafe.}$$

Shear friction factor

$$\text{S.F.F.} = \frac{\mu \Sigma V_4 + b.c}{\Sigma H} = \frac{0.70 \times 33554.5 + 52.5 \times 1400}{30866.5} = 3.14 > 3.0.$$

Safety against sliding as per IS 6512-1984 :

Taking $F_\varphi = 1.2$ and $F_c = 2.7$ for loading combination E,

$$F = \frac{\frac{\mu \Sigma V}{F_\varphi} + \frac{cb}{F_c}}{\Sigma H} = \frac{\frac{0.7 \times 33554.5}{1.2} + \frac{1400 \times 52.5}{2.7}}{30866.5} = 1.52 > 1 \text{ Hence Safe.}$$

Example 8.17. *Design the practical profile of a gravity dam of stone masonry, given the following data :*

R.L. of base of dam =1450 m.

R.L. of F.R.L. = 1480.5 m

Specific gravity of the masonry = 2.4

Safe compressive stress for masonry = 1200 kN/m².

Height of waves = 1 m.

Solution. $h_w = 1$ m

Free board $= 1.5\ h_w = 1.5$ m

$\therefore$ R.L. of top of dam = 1480.5 + 1.5 = 1482 m

Hence height of dam = 1482 − 1450 = 32 m

$$\text{Limiting height of dam} = \frac{f}{w(\rho + 1)} = \frac{1200}{9.81(2.4 + 1)} = 35.98 \text{ m}$$

Hence the dam is a low gravity dam.

The pratical profile of the dam can be designed with respect to recommendations of § 8.17 (Fig. 8.19).

Depth of water h = 1480.5 − 1450 = 30.5 m.

Top width a can be determined on the following criteria :

(*i*) a = 14% of H for economy = 0.14 × 32 $\triangleq$ 4.5 m

(*ii*) a = width of roadway, if any.

However, keep a = 4.5 m.

Base width of elementary profile

$$= \frac{h}{\sqrt{\rho}} = \frac{30.5}{\sqrt{2.4}} = 19.7 \text{ m.}$$

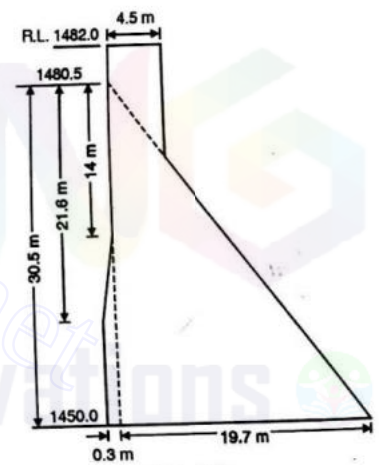

FIG. 8.43

Upstream offset $= \frac{a}{16} = \frac{4.5}{16} \triangleq 0.3$ m..

Total base width = 19.7 + 0.3 = 20 m.

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Distance upto which u/s slope is vertical from the u/s water level

$$= 2a\sqrt{\rho} = 2 \times 4.5\sqrt{2.4} \simeq 14 \text{ m.}$$

Fig. 8.43 shows the practical profile of the dam. The stability of this section can now be tested.

8.24. EXAMPLES FROM COMPETETIVE EXAMINATIONS

Example 8.18. *The following Fig. (Fig. 8.44 a) gives the profile of a gravity dam with reservoir level as shown. If the coefficient of friction is 0.75, is the dam safe against sliding ? Take weight density of concrete = 2.4 tonnes/m³.*

(Engg. Services Exam. 1997)

Solution : Let us neglect the uplift pressure. The forces due horizontal water pressure (*P*) and the self weight of the dam are shown marked in Fig. 8.44 (*b*)

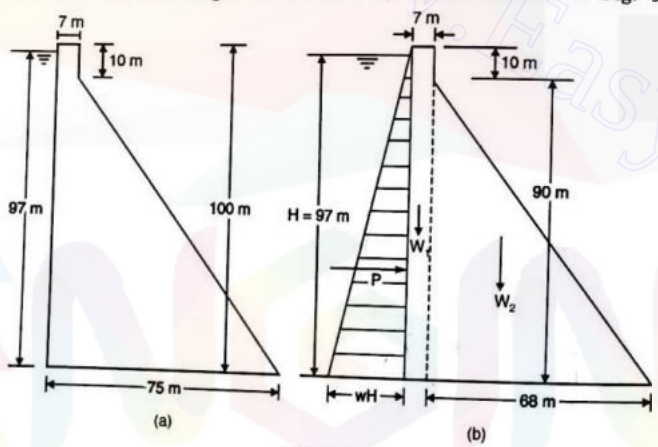

FIG. 8.44

$$P = \frac{wH^2}{2} = \frac{1000}{2}(97)^2 \times 10^{-3} = 4704.5 \text{ tonnes/m}$$

Self weight $W_1 = 7 \times 100 \times 2.4 = 1680$ tonnes/m

Self weight $W_2 = \frac{1}{2} \times 68 \times 90 \times 2.4 = 7344$ tonnes/m

Total self weight $= W = W_1 + W_2 = 1680 + 7344 = 9024$ tonnes/m

$$\therefore \quad \text{F.S. against sliding} = \frac{\mu W}{P} = \frac{0.75 \times 9024}{4704.5} = 1.439 > 1$$

Hence the dam is safe against sliding.

Example 8.19. *A masonry dam 10 m high is trapezoidal in section with a top width of 1 m and bottom width of 8.25 m. The face exposed to water has a batter of 1 : 10. Calculate*

(*i*) *Factor of safety against overturning*

(*ii*) *Factor of safety against sliding.*

(*iii*) *Shear friction factor.*

Assume coefficient of friction as 0.75, unit weight of masonry as 2240 kg/m³, and permissible stress of joint = 14 kg/cm². Based on the results, give your remark.

(Engg. Services Exam. 1995)

Solution : ***This problem was set from Example 8.14 of this book. For solution, see example 8.14.***

Example 8.20. *Fig. 8.45 (a) shows the section of a gravity dam (non-overflow portion) built of concrete. Calculate the maximum vertical stress at the heel and toe of the dam. Assume weight of concrete as 23.5 kN/m³. Neglect earthquake effects.*

(Engg. Services Exam. 2000)

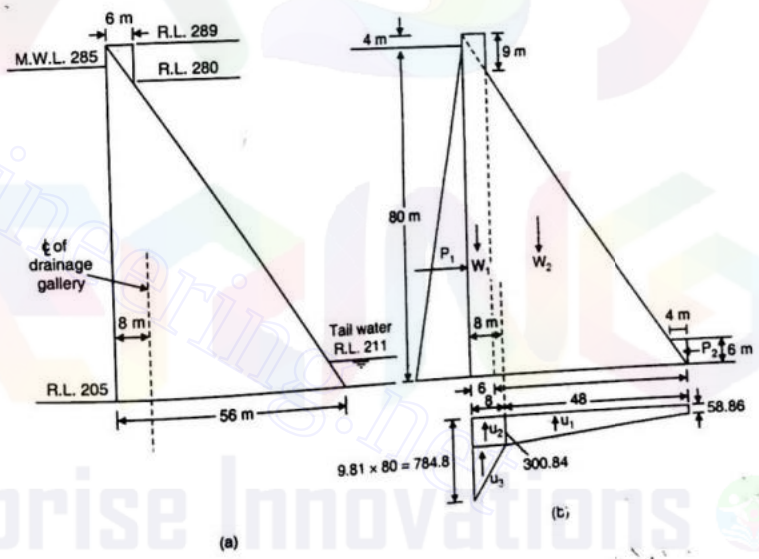

FIG. 8.45

Solution : Various forces acting on the dam are shown marked in Fig. 8.45 (*a*). The computations are arranged in Tabular form below (Table 8.10).

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TABLE 8.10

Designation	Dimensions	Force (kN) Vertical	Force (kN) Horizontal	L.A. (m)	Moment about toe (kN-m) RL +ve	Moment about toe (kN-m) RNA -ve
	Self weight of dam					
W_1	$6 \times 84 \times 1 \times 23.5$	11844		53.0	627732	
W_2	$\frac{1}{2} \times 50 \times 75 \times 23.5$	44062.5		33.33	1468750	
	Wt. of water on d/s face					
W_3	$\frac{1}{2} \times 4 \times 6 \times 9.81$	117.7		1.33	157	
	Uplift pressure					
U_1	$\frac{1}{2}(58.86 + 300.84) \times 48$	– 8632.8		29.38		253647
U_2	300.84×8	– 2406.7		52		125149
U_3	$\frac{1}{2}(784.8 - 300.84) \times 8$	– 1935.8		53.33		103245
	Horizontal water pressure					
P_1	$9.81 \times 80^2/2$		31392 (→)	26.67		837120
P_2	$9.81 \times 6^2/2$		176.6 (←)	2	353	
	Sum	43048.9	31215.4 (→)		2096992	131961
					777831	

$$\bar{x} = \frac{\Sigma M}{\Sigma V} = \frac{777831}{43048.9} = 18.07 \text{ m} \quad ; \quad e = \frac{b}{2} - \bar{x} = \frac{56}{2} - 18.07 = 9.93 \text{ m}$$

$$\therefore \quad p_{n,\max} = \frac{\Sigma V}{b}\left[1 \pm \frac{6e}{b}\right] = \frac{43048.9}{56}\left[1 \pm \frac{6 \times 9.93}{56}\right] = 768.73\left[1 \pm 1.064\right]$$

$\therefore$ $p_{n,\max}$ at toe = **1586.6 kN/m²**

$p_{n,\min}$ at heel = **– 49.2 kN/m²** (i.e. tension)

PROBLEMS

1. What do you understand by gravity dam ? Explain various forces that act on a gravity dam.

2. Discuss in brief the U.S.B.R. recommendations for determining uplift pressure under the base of a dam, provided with a drainage gallery.

3. Explain how do you account for earthquake effects in the design of a gravity dam.

4. Discuss in brief various modes of failure of a gravity dam.

5. Explain the method of determining principal and shear stresses in a gravity dam.

6. What do you understand by the elementary profile of a gravity dam ? Derive expressions for determining base width of such a dam based on (*i*) stress criterion, (*ii*) sliding criterion. Also derive expression for normal, principal and shear stresses at the base of such a dam.

7. Give a practical profile of a low gravity dam.

8. Distinguish clearly between a low gravity dam and high gravity dam. Derive the expression used for such a distinction.

Determine the critical height of a low gravity dam of concrete, taking the specific gravity of concrete as 2.40 and allowable compressive stress as 3340 kN/m². [Ans. 100 m]

9. Give a practical profile for a high gravity dam as suggested by G. Molesworth.

10. Explain the step by step method of designing a high gravity dam.

11. What do you understand by galleries and shafts and why are they provided in gravity dams ?

12. Explain with the help of diagrams various joints and water seals provided in gravity dams.

13. How do you control the cracking in a concrete gravity dam ?

14. Design the pratical profile of a gravity dam of stone masonry, given the following data :

R.L .of base of dam = 1250 m

R.L .of H.F.L. = 1280 m

Specific gravity of masonry = 2.4

Safe compressive stress for masonry of dam =1200 kN/m².

15. Check the stability (at section *AB*) of the concrete gravity dam section shown in Fig. 8.46. Assume any other data not given. Neglect gallery in calculations.

16. The cross-section of low gravity dam is showing Fig 8.47. Assuming the reservoir to be full, determine : (1) normal stress, principal stress and shear stress at the toe, (2) Factor of safety against sliding and (3) shear friction factor at level *A–B*. Full uplift pressure is to be accounted according to the U.S.B.R. recommendations. Neglect earthquake pressure, wave pressure and silt pressure.

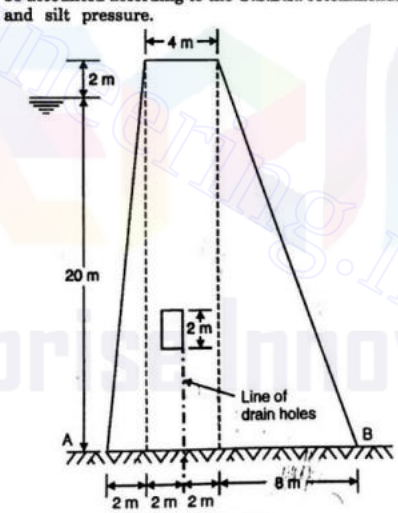

FIG. 8.46

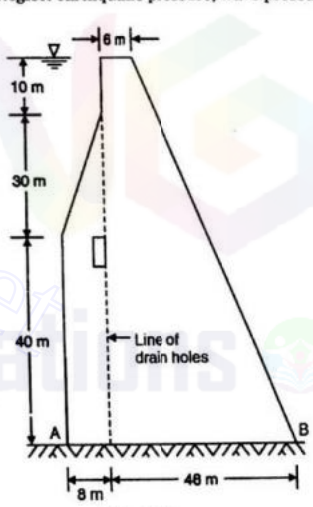

FIG. 8.47

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9

Dams - III : Arch and Buttress Dams

9.1. TYPES OF ARCH DAMS

An arch dam is a dam curved in plan and carries a major part of its water load horizontally to the abutments by arch action. The part of the water load depends primarily upon the amount of curvature. The balance of the water load is transferred to the foundation by cantilever action. The thrust, developed by the water load carried by arch action, essentially requires strong side walls of the canyon to resist the arch forces. Many early arch dams were built of rubble, ashlar etc. However, practically all arch dams constructed during recent years have been built of concrete. The famous Hoover dam of U.S.A., constructed in 1936, is an arch dam of constant radius type having the following dimensions : Height 222 m (726 ft); Length 380 m (1244 ft); Top width 13.7 m (45 ft) and Base thickness 201 m (660 ft).

The arch dams may be divided into the following three classes:

1. Constant radius dam. 2. Variable radius dam, and
3. Constant angle dam.

1. **Constant radius arch dam** (Fig. 9.1). *A constant radius arch dam is the one in which the radius to some prominent feature* (such as upstream face, or the

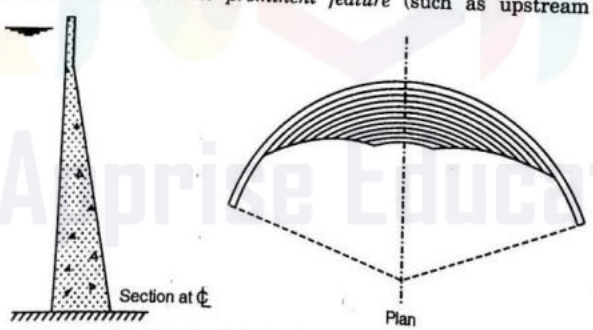

FIG. 9.1. CONSTANT RADIUS ARCH DAM.

downstream face, or the arch axis) *is constant*. In most cases, however, the constant radius refers to the upstream side. In a constant radius arch dam, the arch centres for upstream face, downstream face and the centre line of the horizontal arch rings at various elevations lie on a straight vertical line that passes through the centre of the horizontal arch ring of the upstream face at the crest. Hence this type of dam is also sometimes known as the constant centre arch dam. In most of the simple types, the upstream face is vertical (*i.e.*, its radius is constant at all elevations), and the thickness is proportional to the height. This makes the central angles of the arch rings of the introdos vary at various elevations, the maximum being at the top of the dam and a certain minimum at the bottom of the dam, as shown in Fig. 9.1.

A constant radius arch dam is particlarly adopted to U-shaped canyons [Fig. 9.2 (*a*)] as cantilever action will carry a large proportion of the load at lower levels. The constant radius arch dam is less economical than the constant angle arch dam, and the latter is constructed in preference to constant radius arch dam. However, the form work for the constant radius arch dam is much simpler to construct.

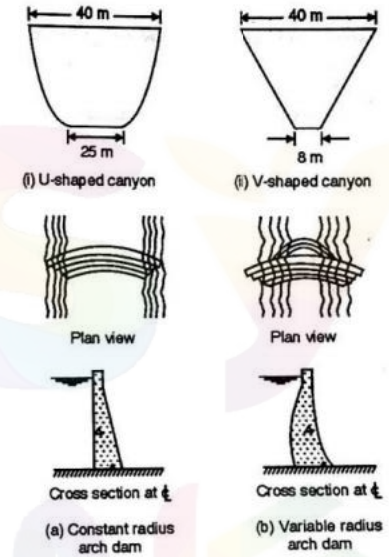

FIG. 9.2 CONSTANT AND VARIABLE RADIUS ARCH DAMS.

2. **Variable radius arch dam** (Fig. 9.3). A variable radius arch dam is the one in which the radii of the extrodos curves (*i.e.*, arch rings corresponding to upstream face) and introdos curves (*i.e.*, arch rings corresponding to downstream face) vary at various elevations being maximum at the top and certain minimum at its bottom. This makes the central angle as large as possible so that maximum arch efficiency may be obtained at all elevations. The variable

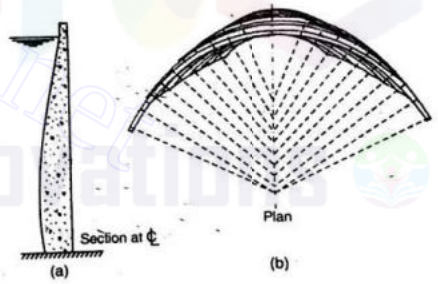

FIG. 9.3 VARIABLE RADIUS ARCH DAM.

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radius arch dams have often vertical or even overhanging faces at the upstream side near the abutments and the downstream side near the crown, as shown in Fig. 9.3(*b*). The centres of the horizontal arch rings at various elevation do not lie on the single vertical line. Hence it is also known as variable centre arch dam.

A variable radius arch dam is often adapted to narrow V-shaped canyons [Fig. 9.2(*b*)] since arch action can be depended upon at the lower levels. A variable radius arch dam has greater arch efficiency, resulting in saving of concrete. Hence it is preferred in comparision to the constant radius arch dam.

3. Constant angle arch dam (Fig. 9.4). The constant angle arch dam is a special type of variable radius arch dam in which the central angle of the horizontal arch rings is of the same magnitude at all elevations, as shown in Fig. 9.4. It has been shown in § 9.3 that the volume of concrete is minimum when the central angle is 133° 34′.

A variable radius arch uses about 58 % of the concrete required in constant radius arch dam. A constant angle arch dam requires about 42.6 % of the concrete required in constant radius arch dam. Hence a constant angle arch dam is the most economical.

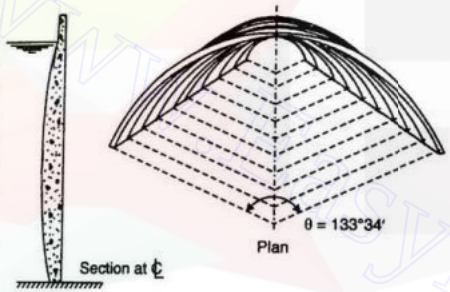

FIG. 9.4 CONSTANT ANGLE ARCH DAM.

9.2. FORCES ON AN ARCH DAM

As in a gravity dam, the various forces acting on an arch dam are :

(*i*) Water pressure (*ii*) Self-weight of the dam
(*iii*) Silt pressure (*iv*) Uplift pressure (negligibly small)
(*v*) Ice pressure (*vi*) Earthquake pressure.

However, the relative importance of the above forces is not the same as in gravity dam. Due to the narrow base width of an arch dam, the uplift pressure is less important than for gravity dam. The internal stresses caused by ice pressure, temperature changes and yielding of supports (abutments) are very important in the arch dam. Ice pressure causes a continuous concentrated load along the arch element at the elevation of the ice. The internal forces caused by temperature changes move the dam upstream during the summer and downstream during the winter. Thus, the weather condition is more important in the stress analysis since it acts with the reservoir loads. The slight yielding of abutments may also cause high internal stresses in the arch.

9.3. DESIGN METHODS FOR ARCH DAMS

The following are some of the theories for the design of arch dams :

1. The thin cylinder theory 2. The elastic theory
3. The trial load analysis.

THE THIN CYLINDER THEORY

In the thin cylinder theory, the stresses in the arch are assumed to be approximately the same as in a thin cylinder of equal outside radius.

Fig. 9.5 shows the section of a thin arch, having average radius r, angle subtended at the centre of the arch equal to θ, and thickness t.

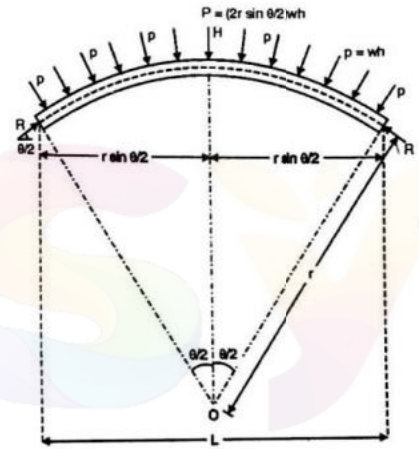

FIG. 9.5 THIN CYLINDER THEORY.

Intensity of hydrostatic pressure, p, at any depth h is given by

$$p = wh \qquad ...(i)$$

where w = unit weight of water.

This horizontal pressure (p) acts in the radial direction. The total downstream component of the horizontal force (P_H) acting along the axis of the river is given by

$$P_H = \text{(Intensity of pressure)} \times \text{(Projected area)}$$

or

$$P_H = wh \,.\, 2r \sin \theta/2 = 2\, whr \sin \theta/2 \qquad ...(ii)$$

If R is the reaction at each abutment, its upward component is equal to $2\,R \sin \theta/2$. Equating this to P_H, for equilibrium, we get

$$2\,R \sin \theta/2 = 2\, whr \sin \theta/2$$

or

$$R = whr \qquad ....(iii)$$

This gives an expression for the abutment reaction and is equal to the maximum compressive stress induced in the arch. If σ is the compressive stress introduced in the arch, at the abutments, we get

$$\sigma = \frac{R}{t} = \frac{whr}{t} \qquad ...(iv)$$

If f is the allowable compressive stress for the material, we get

$$t = \frac{whr}{f} \qquad ..(9.1)$$

This is the expression for the thickness of the arch.

Central angle for minimum concrete. It has been stated earlier that the constant angle arch requires minimum concrete for a fixed combination of span, loading and the permissible cylinder theory stress. Let us now find the numerical value of the central angle θ for minimum concrete.

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From Eq. 9.1, it is clear that

$$t \propto \text{ radius of curvature } r$$

and

$$t \propto \text{ water depth } h.$$

Now volume (V) of the concrete, per unit height is given by

$$V = r \,.\, \theta \,.\, A; \text{ where } A = \text{cross-sectional area of arch}$$

Now $A \propto t \propto r$

or $A = k.r$, where k is a constant

$$\therefore \quad V = r\,\theta \,.\, k\,r = k \,.\, \theta\, r^2 \qquad ...(i)$$

From Fig. 9.5, we observe that

$$\sin\frac{\theta}{2} = \frac{L}{2r} \quad \text{or} \quad r = \frac{L}{2\sin\theta/2} \qquad ...(ii)$$

Substituting this in (i), we get

$$V = k\,\theta\left(\frac{L}{2\sin\theta/2}\right)^2 = \frac{k\,L^2}{4}\cdot\frac{\theta}{\sin^2\theta/2}$$

For minimum volume, $\dfrac{dV}{d\theta} = 0$

r

$$\frac{dV}{d\theta} = \frac{kL^2}{4}\,\frac{\left\{\sin^2\frac{\theta}{2} - \theta\sin\frac{\theta}{2}\,.\,\cos\frac{\theta}{2}\right\}}{\sin^4\frac{\theta}{2}} = 0$$

$$\sin^2\frac{\theta}{2} = \theta\,\sin\frac{\theta}{2}\,.\,\cos\frac{\theta}{2} \quad \text{or} \quad \tan\frac{\theta}{2} = \theta$$

which gives $\theta = 133°\ 34'$

This is the most economical angle. In actual practice, θ varies between 100° to 140°.

Limitations of cylinder theory

Following are the limitations of the cylinder theory :

1. The arch sections are not thin cylinders. They are also not free at abutments.
2. There is no provision for considering shearing as well as bending stress in the arch.
3. The analysis is based on only the water pressure. Temperatre stress and ice thrust have not been considered.
4. Plastic flow of concrete and shrinkage in concrete have also not been accounted for.
5. Stress due to rib shortening and that due to yielding of the abutments have also not been taken into account.

9.4. THE ELASTIC THEORY

The elastic theory of design of an arch dam takes into accout the additional stresses developed because of the following effects :

1. Rib shortening
2. Temperature changes
3. Shrinkage
4. Yielding of abutments.

In the case of arch, the ends are fixed. The distance between them, *i.e.* the span, does not change due to the water pressure acting radially from upstream side. This restriction of the change of span causes additional stresses known as *rib shortening stresses*. Similarly the change in the temperature tends to cause the change in the span and due to fixidity at the end, temperature stresses are introduced. Similarly, additional stresses are introduced if the abutments yield. The concrete on setting shrinks giving rise to further stresses.

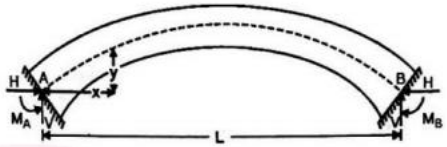

FIG 9.6. THE ELASTIC ARCH.

The cylinder formulae are based on only uniform radial pressure However, the elastic theory can take into consideration variable loads along the length of arch, including the ice thrust etc.

The following are the formulae of elastic arches (Fig. 9.6):

$$M = \mu + M_A + (M_B - M_A)\,\frac{x}{L} + Hy \qquad ...(9.2)$$

$$\int\frac{M\,.\,y}{EI}\,ds = 0 = \int\frac{\mu y}{EI}\,ds + M_A\int\frac{y\,ds}{EI} + \frac{M_B - M_A}{L}\int\frac{xy\,.\,ds}{EI} + H\int\frac{y^2}{EI}\,ds = 0 \qquad ...(9.3)$$

$$\int\frac{M\,.\,x}{EI}\,ds = 0 = \int\frac{\mu x}{EI}\,ds + M_A\int\frac{x\,ds}{EI} + \frac{M_B - M_A}{L}\int\frac{x^2\,.\,ds}{EI} + H\int\frac{x\,y}{EI}\,ds = 0 \qquad ...(9.4)$$

and

$$\int\frac{M}{EI}\,ds = 0 = \int\frac{\mu}{EI}\,ds + M_A\int\frac{ds}{EI} + \frac{M_B - M_A}{L}\int\frac{x\,ds}{EI} + H\int\frac{y\,ds}{EI} \qquad ...(9.5)$$

where M = bending moment at any point on the arch

M_A and M_B = fixed end bending moments at both the ends

μ = bending moment on a straight horizontal freely supported beam

H = horizontal thrust at the abutments.

If temperature effect is taken into account, equations 9.3, 9.4 and 9.5 are modified as below :

$$\alpha\,TL - \int\frac{M\,.\,y}{EI}\,ds = 0 \qquad ...[9.3(a)]$$

$$\int\frac{M\,.\,x}{EI}\,ds = 0 \qquad ...(9.4)$$

and

$$\int\frac{M\,ds}{EI} = 0 \qquad ...(9.5)$$

where α is the coefficient of thermal expansion and T is the temperature variation.

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9.5. THE TRIAL LOAD ANALYSIS

Both the cylinder theory as well as the elastic theory are based on the assumption that any horizontal arch ring is independent of the arch ring below and above it. This is not true. The arch rings are not independent. Due to this restriction, the actual analysis is extremely complicated. The U.S.B.R. devised the 'trial load analysis', which gives a reasonably satifactory approximation.

In the trial load method, the dam is assumed to be made up of two system of elements : (*a*) horizontal arches and (*b*) vertical cantilevers, as shown in Fig. 9.7. Each system occupies the whole dam. The horizontal component of the water load is resisted jointly by the arch and cantilever action, and the distribution of the load between the arches and the cantilevers is determined by 'trial and error'. The structure and the loading is assumed to be divided between them in such a manner that the computed deflection at any point in the dam, considered as a point in the arch system will be identical with its computed deflections, considered as a point in the cantilever system. If computed deflection are not equal, new loads are assumed until a distribution is found which produces equal arch and cantilever deflections at all points.

The various forces on the arch dam can be supposed to be divided into primary load because of water pressure and secondary load because of other forces. The arches will carry part of primary load and total of secondary load. The cantilever will take the remaining of the primary load and total secondary load. The stresses in the dam can then be computed on the basis of the final load distribution.

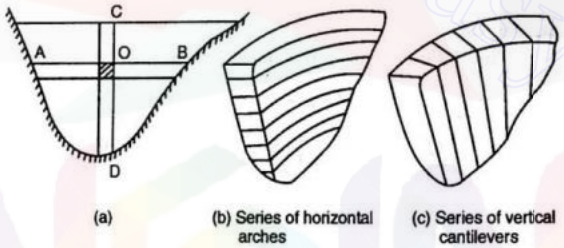

FIG. 9.7 TRIAL LOAD ANALYSIS

9.6. BUTTRESS DAMS

A buttress dam consists of the following component parts:

(1) ***Sloping membrane.*** The sloping membrane or upstream deck supports the water and transmits the water load to a series of buttresses at right angles to the axis of the dam.

(2) ***Buttresses.*** The buttresses are constructed at right angles to the axis of the dam, at certain intervals. They support the sloping membrane and transfer the load to the foundation. The function of the buttress is similar to that of a counterfort of a retaining wall.

(3) ***Mat foundation footing.*** The footings distribute the pressure to the soil.

(4) ***Lateral Braces.*** They are constructed at right angles to the buttress. They reinforce the buttresses.

(5) ***Corbels.*** Corbels help in transmitting the load from the upstream deck to the buttresses.

(6) ***Concrete Cutoffs.*** The cutoff walls are constructed to reduce or prevent seepage.

Fig. 9.8 illustrates a flat-slab type buttress dam showing all the above component parts.

Forces on buttress dam. A buttress dam is also subjected to the same forces as gravity arch dams. However, the ice pressure in not important as the ice tends to slide up the sloping membrane. The total uplift pressure is usually of smaller magnitude since the uplift pressure is relieved by the gaps between the buttresses.

9.7. TYPES OF BUTTRESS DAMS

Buttress dams may be broadly classified under three heads :

(*i*) Rigid. (*ii*) Articulated.

(*iii*) Intermediate or semi-rigid.

Rigid type buttress dams have their upstream deck or sloping membrane constructed monolithically and unyielding with their buttresses. No provision is made for unequal foundation settlement or for expansion and contraction. Under this category fall : (*a*) the multiple arch dam, and (*b*) multiple dome dams.

Articulated buttress dams are quite flexible, and are not constructed monolithically with the buttresses. The flat-slab buttress dam is an example of this type.

Intermediate or semi-rigid buttress dams fall between rigid type and the articulated type. The design of such dams eliminate the undesirable rigidity of the multiple arch and multiple dome designs, although not securing the high degree of flexibility or articulation incidental to the flat slab design. The round-head buttress dam and the diamond head buttress dam fall under this category.

The various types of buttress dams falling under the above three categories are as follows :

(1) Flat slab or Ambursen type
(2) Multiple-arch type
(3) Multiple dome type
(4) Cylinder or massive head type
(5) Columnar buttress type
(6) Truss buttress type.

(1) **Flat slab or Ambursen type.** The flat slab buttress dams has borne the name of its inventor, Nils F. Ambursen in 1903. The upstream deck or sloping membrane consists of reinforced concrete slab, separated by buttress tongues, and supported by reinforced haunches which are constructed monolithically with the buttress, such as shown in Fig. 9.8.

The most typical feature of the Ambursen type is its *articulation* or flexibilty. The slab is not rigidly attached to the buttresses ; the joint between the slab and the buttresses is filled with some flexible joint compound or asphaltic putty. Such provision permits each slab to act independently.

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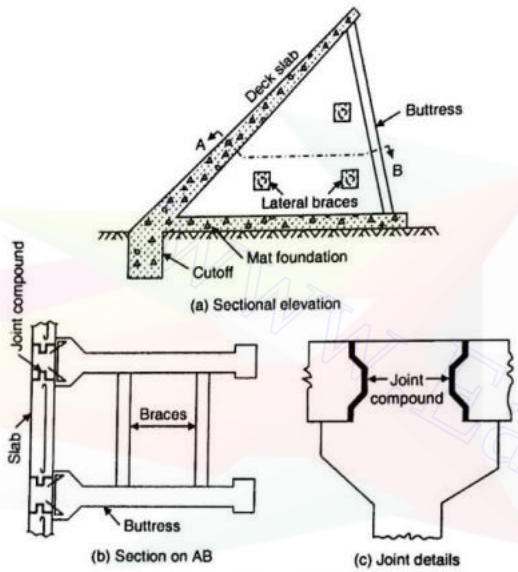

FIG. 9.8. FLAT SLAB BUTTRESS DAM.

The inclination of the deck may be kept between 35 to 45 degrees with horizontal. For proper sliding resistance, too steep slope increases the tendency to sliding , while too flat a slope will cause a very high inclined stress at the heel. For constant buttress spacing, a variation of few degrees in the upstream slope results in an appreciable change in the quantity of concrete. Hence the determination of most economic upstream angle from point of view of saving in the cost is very important. Concrete quantities are worked out for various angles of upstream slope and different buttress spacing and plotted to get a curve for each angle. A *master curve* is then drawn through the junctions of the curves and vertical line corresponding to each angle. The master curve shows the absolute economic value of buttress spacing and the angle of upstream slope for the given height of the dam. Buttresses spacing varies with the height of the dam from about 5 m for dams under 16 m to 16 m for dams more than 50 m high. The most economical spacing is that which gives the minimum thickness for slab and buttress so that the concrete is fully stressed at all sections.

The other types of flat-slab buttress dams are (*i*) *fixed slab type* and (*ii*) *cantilever type*, as shown in Fig. 9.9.

In the *fixed slab type*, the deck slab is cast monolithically with the buttress, with provision of expansion joints at convenient places. Such dams are not very much in use, except for very long heights. The stresses are there both at the upstream as well as downstream face of the slab. The reinforcement placed at the u/s face may

corrode. Also, it is difficult to assess the stresses due to temperature variations, shrinkage of decks and buttresses, and due to settlement of foundations.

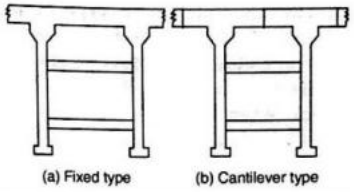

FIG. 9.9. OTHER TYPES OF FLAT-SLAB BUTTRESS DAMS

In the *cantilever type*, the slab is laid monolithically with the buttresses, with overhangs at both the sides. Thus a joint is left at the centre of each span between two buttresses. Due to cantilever action, steel has to be placed towards the upstream face. It is also subjected to high stresses due to temperature, shrinkage and settlement.

The *Ambursen dam* is called the *free deck type* dam. In this, steel is placed at the d/s face, and hence there is no danger due to rusting. Also the difficulties due to influence of temperature, shrinkage and settlement of foundations are eliminated. This type of dam is particularly adapted to wide valleys where a long dam is required and foundation materials are of interior strength.

(2) Multiple Arch Type Buttress Dam. This type of dam consists of a series of arches transmitting its water load to buttresses. Meer Alam dam in India, built in about 1800 A.D. is the earliest recorded example of such a type. This type is used for high heights. The stability of any unit depends upon the adjacent units so that any settlement in one may affect other units. Hence it requires better foundation. For short span (10 to 15 m), circular arches of uniform thickness is economical while for greater spans, arches of variable thickness are provided. If the central angle of each arch is kept between 150° to 180° with semicircular arches (central angle 180°), the horizontal thrust between adjoining arches is eliminated. The arches may be supported on either the single wall stiffened buttresses or double wall hollow buttresses.

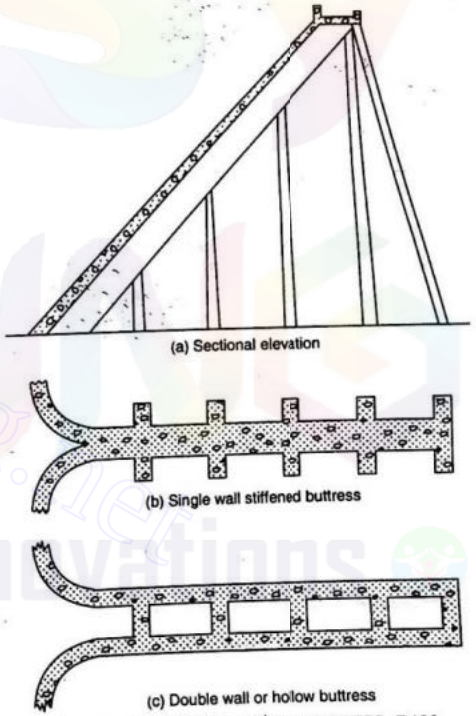

FIG. 9.10. MULTIPLE-ARCH BUTTRESS DAM.

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(3) **Multiple Dome Type of Buttress Dam.** This is a modification of multiple arch dam and consists of a dome shaped deck supported on buttresses.

(4) **Cylinder or Massive Head Type.** In this type, the water supporting member is formed by enlarging the upstream end of the buttress until it meets the adjacent surface. When the enlarged head is circular, it is known as *round head deck* [Fig. 9.11 (*b*)]. When the head converges to a point, it is known as *diamond head deck* [Fig. 9.11(*c*)]. The pressure acts radially to the surface of the cylindrical type, or perpendicularly to the diamond type and hence all pressures are compressive. Reinforcement is therefore, not required for the deck and there is no danger of corrosion.

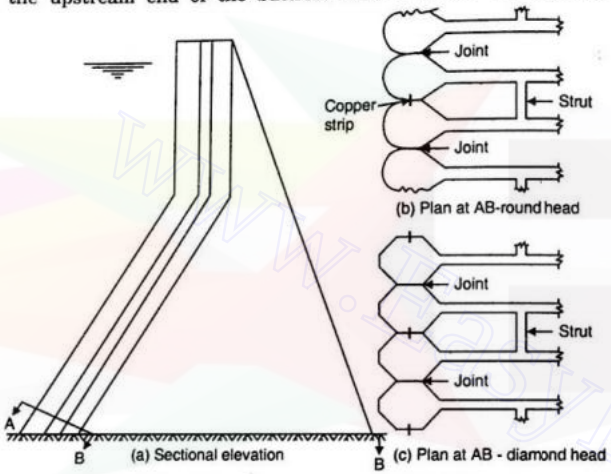

FIG. 9.11. MASSIVE HEAD BUTTERESS DAMS

(5) **Columnar Buttress Type.** This is a modication of flat slab type . In this, the deck-slab is suppported on a number of inclined columns, instead of buttresses.

(6) **Truss Buttress Type.** This is another modification of flat slab type. This deck slab is supported on reinforced concrete trusses.

PROBLEMS

1. Discuss in brief various types of arch dams.
2. Prove that the most economical central angle for a constant radius arch dam is 133° 34′.
3. Discuss various forces that act on an arch dam.
4. What is a buttress dam ? Describe the functions of its component parts.
5. Describe with the help of sketches flat-slab buttress dam (Ambursen type.)
6. Sketch various types of buttress dams in common use.
7. Discuss advantages and disadvantages of buttress dams.

10

Dams-IV : Earth and Rockfill Dams

10.1. INTRODUCTION : TYPES OF EARTH DAMS

Earth dams have been built since the early days of civilisation. Today, as in the past, the earth dam constitutes to be the most common type of dams, since it is generally built of locally available materials in their natural state with a minimum of proccessing. Earth dams are composed of fragmental materials which maintain their individual identity. These fragments have spaces and voids between them, and derive their strength from their position, friction and cohesion. Some of the earth dams built in antiquity were of considerable size. One earth dam, 17.6 km (11 miles) long, 23.33 m (70 ft) height, and containing about 13 million cubic metres (17 millon cubic yards) of embankment was completed in Ceylon in the year 504 B.C.

Depending upon the method of construction, earth dams can be divided into two categories :

(*i*) Rolled fill dam (*ii*) Hydraulic fill dam.

In the *rolled fill dam,* the embankment is constructed in successive, mechanically compacted layers. The suitable materials are transported from the borrow pits to the construction site by earth moving machinery. It is then spread by bulldozers, and sprinkled to form layers of limited thickness having proper water content. They are then thoroughly compacted and bonded with the preceding layer by means of power-operated rollers of proper design and weight.

In the case of *hydraulic fill dam*, the materials are excavated, transported and placed by hydraulic methods. Flumes are laid at a suitable falling gradient along the outer edge of the embankment. The material, mixed with water at the borrow pits, is pumped and washed into these flumes. The slush is discharged through the out-lets in the flumes, at suitable interval along their length. The slush thus flows towards the centre of the bank.The coarse materials of the slush settle at the outer edge while the fine materials settle at the centre. No compaction is done.

Rolled-fill earth dams can further be sub-divided into following types :

1. Homogeneous embankment type,
2. Zoned embankment type, and

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3. Diaphragm embankment type.

(1) **Homogeneous Embankment Type.** A *purely homogeneous* type earth dam is composed of single kind of material (exclusive of the slope protection). Homogeneous dams have been built since the earliest times and are used today whenever only one type of material is economically available. However, they are used only for low to moderate heights. A purely homogeneous section has been replaced by a *modified homogeneous section* in which internal drainage system in the form of horizontal filter drain or rock toe, is provided. This controls the action of seepage so as to permit much steeper slopes. The drainage system also keeps the pheratic line well within the body of the dam. For the purpose of further discussions, however, we shall designate the 'homogeneous section' as the one composed of one type of material, with or without internal drain.

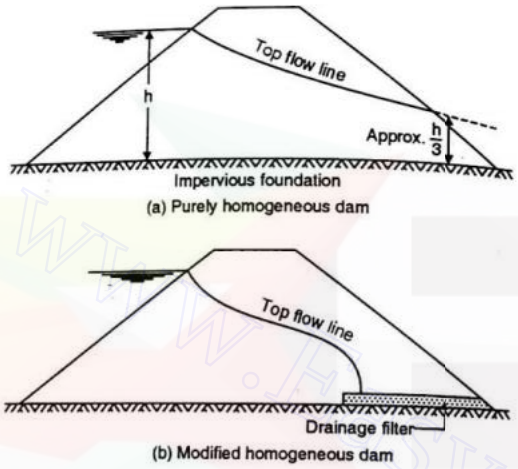

FIG. 10.1. HOMOGENEOUS DAM.

Homogeneous dams are usually composed of impervious or semi-impervious soils to provide an adequate water barrier. However, the upstream slope has to be flatter, to make it safe during the 'sudden drawdown condition'. Many successful embankments have been built to relatively pervious sands and sand-gravel mixtures.

(2) Zoned Embankment Type. Zoned embankment type earth dam is the one in which the dam is made up of more than one material. The most common type of a rolled earth dam section is that in which a central impervious core is flanked by zones of material considerably more pervious. A suitable drainage system, in the form of a horizontal drain or a rock toe is also provided at the d/s side. If a variety of soils are readily available, the choice of type of earth-fill dam should always be zoned embankment type because its inherent advantages will lead to economy in the cost of construction.

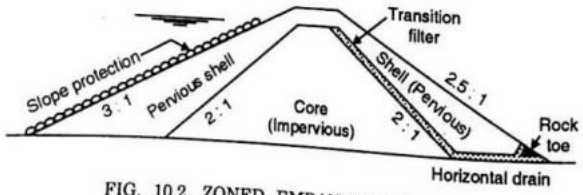

FIG. 10.2. ZONED EMBANKMENT TYPE.

In the zoned embankement type, outer shells are made of pervious, freely draining materials. The shells give stability to the central impervious fill, and at the same time distribute the load over a larger area in the foundation. Sometimes, a semi pervious zone is provided to serve as a transition between the outer shell and the central core. The upstream pervious zone affords stability against rapid drawdown while the downstream pervious zone acts as a drain to control the line of seepage. The central impervious core checks the seepage.

(3) **Diaphragm Embankment Type.** This is a modification over the homogeneous embankment type, in which the bulk of the embankment is constructed of pervious material and a *thin* diaphragm of impermeable material is provided to check the seepage. The diaphragm may be of impervious soil, cement concrete, bituminous concrete, or any other material, and may be placed either at the centre of the section as a central vertical core, or at the upstream face as a blanket. However, the distinction between a diaphragm type and zoned type must be clearly known. If the horizontal thickness of the diaphragm (made of impervious soil) at any elevation is less than 10 m or less than the height of embankment above any corresponding elevation in the dam, the dam is of the diaphragm type. If the impervious earth diaphragm equals or exceeds these thicknesses, the dam is considered to be 'zoned embankment' type.

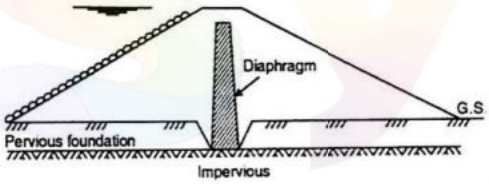

FIG. 10.3. DIAPHRAGM TYPE EMBANKMENT.

10.2. CAUSES OF FAILURES OF EARTH DAMS

Inspite of taking great care in construction of earth dams, some failures have occurred in the past. However, a knowledge of the principal lessons learned from failures and damages in the past is an essential part of the training of earth dam designer.

On the basis of investigation reports on most of the past failures, it has now been possible to categorise the types of failures into threee main classes :

1. Hydraulic failures : 40%
2. Seepage failures : 30%
3. Structural failures : 30%

HYDRAULIC FAILURES. Hydraulic failures include the following :

(*i*) Overtopping (*ii*) Wave erosion
(*iii*) Toe erosion (*iv*) Gullying

(*i*) **Overtopping.** The earth dam may get overtopped if the design flood is underestimated, or if the spillway is of insufficient capacity. Faulty operation of spillway gates, etc. may also sometimes lead to overtopping. Insufficient free board or settlement of foundation and embankment may also lead to overtopping.

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(*ii*) **Wave Erosion.** Fig. 10.4 shows failures due to wave erosion. The effects of wave is to notch out earth from the upstream slope in absence of proper slope protection in the form of riprap. Rollers are developed in the waves which try to scoop out the earth. Waves can also cause upstream slips [Fig. 10.4(*b*)].

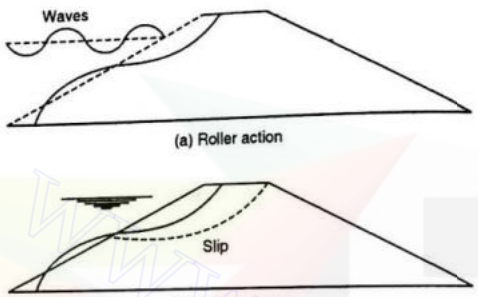

FIG. 10.4. WAVE EROSION.

(*iii*) **Toe Erosion.** Toe erosion may occur due to two reasons: (*a*) erosion due to tail water, and (*b*) erosion due to cross currents that may come from spillway buckets or from exit areas of outlets. The toe erosion can be avoided by providing thick riprap on the d/s slope, upto a height slightly above the tail water level. Diaphragm walls of sufficient lengh and height should be provided to check the cross flow towards the earth dam.

(*iv*) **Gullying.** Downstream slope may fail due to the formation of gullies by heavy downpour. To eliminate failure due to gullying, proper berms, turfing and good drainage system should be provided to the downstream side.

2. SEEPAGE FAILURES. Seepage failures may be due to (*a*) piping and (*b*) sloughing.

(*a*) **Piping**. The seepage of water through the body and foundation of the earth dam may lead to *piping* or progressive erosion of concentrated leaks, causing a large number of catastrophic failures. Water seeping through the earth dam may have four bad effects : (*i*) seeping water generates erosive forces which dislodge particles from the soil structure and causes rearraggement or migration of the fines to voids between larger grains, (*ii*) the flow with its associated differential pore

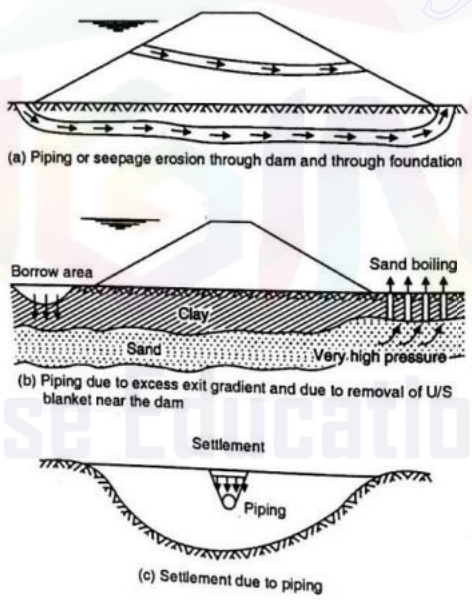

FIG. 10.5. PIPING FAILURES.

pressure can lift portion of soil mass causing sand boiling, (*iii*) internal erosion of the soil mass, progressive backwards from the point of exit leads to the formation of an open conduit through the soil (*i.e.* piping), and (*iv*) the internal pressures in the soil water can reduce that part of the soil strength that is developed by internal friction and thereby lead to weakening of the soil mass and even failure by shear. Most of the serious trouble from piping has resulted from progressive backward erosion of concentrated leaks which develop through or under the dam.

Leaks in the embankment may also lead to piping failure. Probably the most common cause of embankment leaks has been poor construction control which can result in inadequately compacted or pervious layers in the embankment, inferior compaction adjacent to concrete outlet pipes or other structures, or poor compaction and bond between the embankment and the foundation or abutments. Embankment leaks through differential settlement cracks have also been a major source of trouble.

(*b*) **Sloughing**. Failure due to progressive sloughing or ravelling is closely related to piping. Under the full reservoir condition, the downstream toe remains saturated and may erode, producing a small slump or miniature slide. This miniature slide leaves a relatively steep face, which becomes saturated by seepage from the reservoir and slumps again, forming a slightly higher and more unstable face. This ravelling process can continue till the remaining portion of the dam is too thin to withstand the water pressure and complete failure occurs suddenly as the reservoir breaks through. The failure of the Sinker Creek dam in Idaho in 1943 is good illustration of the mechanics of progressive backward sloughing.

3. STRUCTURAL FAILURES. Structural failures may be due to the following reasons :

(*i*) Upstream and downstream slope failures due to construction pore pressures.
(*ii*) Upstream slope failure due to sudden drawdown.
(*iii*) Downstream slope failure during full reservoir condition.
(*iv*) Foundation slide : spontaneous liquefaction.
(*v*) Failure by spreading.
(*vi*) Failure due to earthquake.
(*vii*) Slope protection failures.
(*viii*) Failure due to damage caused by burrowing animals.
(*ix*) Damage caused by water soluble materials.

(*i*) **Upstrem and Downstream Slope Failures due to Construction Pore Pressure.** When a dam is built of relatively impervious compressible soil, the drainage is extremely slow and excess pore pressures develop during and immediately after construction. When the permeabiltiy is low (less than 10^{-6} cm/sec), there may be no substantial drop in pore pressure in the central zone of the dam by the end of the construction if this lies within the usual range of 2 – 4 years. An initial pore pressure upto almost 100% of total weight of soil above the point considered has been measured in the field. Under these circumstances, the construction stage in many cases, may be more critical from the point of view of stability and the slopes may fail. Experience indicates that there are two types of construction slides, which differ primarily in speed and

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magnitude of the movement. The first type of slide occurs slowly and continues at a uniform rate for a period usually of one to two weeks. The failure of North Ridge Dam in Canada (Sept. 1953), is the example of such slow slide. The second type of slide occurs rapidly and suddenly. The major part of the sliding (magnitude of movement) usually equal to one half or more of the total height of the dam is over in few minutes. The failure at Marshall Creek Dam in Kansas (1937) is the typical example of this type, where the major movement covered a period or 15 to 20 minutes and was accompanied by rumbling noise and dense clouds of dust.

(*ii*) **Upstream Slope Slide During Sudden Drawdown.** For the upstream slope, the critical condition is when the reservoir is suddenly emptied without allowing any appreciable change in water level within the saturated soil mass. This stage is known as sudden draw-down. With complete draw-down, the hydrostatic force acting along the upstream slope at the time of full reservoir is removed without the hydrostatic pressure on the slope to counteract it. Up-stream slides have not caused complete failures or loss of water from the reservoir, although they have occasionally blocked the entrances to outlet conduits and made these useless for further lowering of the reservoir. When the upstream slide occurs due to sudden drawdown, the pore pressure along the surface of slide are dissipated to a large extent. Hence there is lesser tendency for the continued sloughing and sliding, and there is small likelihood of catastrophic failure even though a large earth movement has taken place.

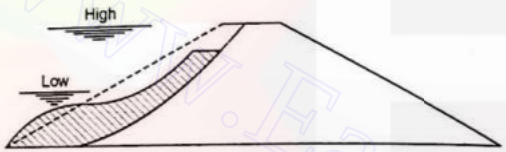

FIG. 10.6. U/S SLOPE FAILURE DURING SUDDEN DRAWDOWN.

(*iii*) **Downstream Slope Slide during Full Reservoir Condition.** Critical condition for downstream slope occurs when the reservoir is full and percolation is at its maximum rate. The direction of seepage forces tend to decrease stability. Or, in other words, the pore water pressure acting on the soil mass below the saturation line reduces the effective stress responsible for mobilising shearing resistance. There are two types of d/s slides : (*a*) deep slides, and (*b*) shallow slides. The deep slides generally pass through the clay foundation and frequently reduce the free board by extending further upstream than the upstream edge of the crest. When such slide occurs, there is no relief of pore pressure and the unstable vertical slide scarp left standing often sloughs or slides again until it breaches the dam and releases the reservoir water in one great flood wave. Shallow slides do not as a rule extend in the embankment in a direction normal to the slope more than 1 to 2 m.

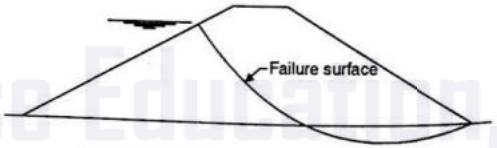

FIG. 10.7. DEEP D/S SLIDES DURING STEADY SEEPAGE.

(*iv*) **Foundation Slides : Spontaneous Liquefaction.** When the earth dam has foundation of fine silt or soft soil it can slide wholly. Sometimes a soft and week clayey seam exists under the foundation and the dam can slide over it causing the failure. Excess water pressure on confined sand and silt seams in the foundation may also cause unbalanced conditions causing foundation failure. Expansion of soils on saturation may cause lifting of the slopes and thus may cause failure of foundation. One of the most difficult problems facing the earth dam designer is the analysis of the stability of loose sand foundations against the possibility of liquefaction or flow slides. The failure of the Sheffield Dam in Santa Barbara in 1925 following an earthquake shock probably was caused by liquefaction of lower portion of the dam or the foundation.

(*v*) **Failure by Spreading.** Failures by spreading have been observed only in connection with fills located above stratified deposits that contain layers of soft clay. Outstanding failures due to spreading are those of Lafayets Dam in California (1928), Marshall Creek Dam in Kansas (1927), and the Hartford flood protection dyke in Connecticut (1941).

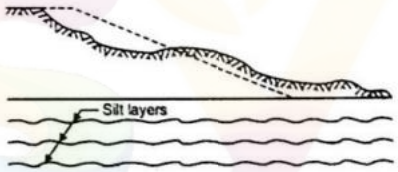

FIG. 10.8. FAILURE BY SPREADING.

(*vi*) **Failure due to Earthquake.** Most of the failures due to earthquake have occurred only with respect to those dams which were constructed before 1920 with inferior design details and construction methods. The accumulation of experience concerning the behavior or performance of dams when shaken by earthquakes is small. However, we can speculate with some degree of assurance that most serious damages and failures may be due to the following effects due to earthquake : (*i*) Cracks in the core of dam leading to leakage and piping failure, (2) Settlement of the crest due to compression of foundation and/or embankment, thereby reducing the free board and increasing the chances of overtopping, (3) Shaking of reservoir bottom causing slow waves (seiches), and hence the failure due to overtopping (4) Sliding of natural hill side causing overtopping and damaging appurtenant structures, (5) Liquefaction of sand below foundation, (6) Acceleration forces on embankment may cause shear slide of appreciable point of the slope of the dam and (7) Fault movement causing reduction in the reservoir capacity and consequent overtopping.

(*vii*) **Slope Protection Failure.** Slopes are generally protected by riprap (either hand placed or dumped) over a layer of gravel or filter blanket. During a heavy storm, the waves on the surface of the reservoir beat repeatedly against the slope just above the reservoir level. The wave energy is dissipated in turbulent action on and within the rocks of the riprap layer. This action may have two effects : (1) the waves may pass through the voids of riprap and

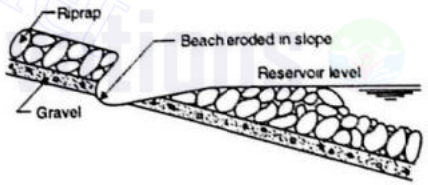

FIG. 10.9. TYPICAL FAILURE OF RIPRAP.

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may wash away the filter layer, setting the riprap layer and exposing the embankment to wave eroson, and (2) if the average size of rock comprising riprap is not heavy, it may be washed out of the layer by the hydraulic forces generated by the waves. However, failure of filter layer is more common.

(*viii*) **Failure due to Damage Caused by Burrowing Animals.** Burrowing animals may cause piping failure of small dams only. Animals like muskrats burrow into embankment either to make homes or to dig passage from one pond to another as shown in Fig. 10.10(*a*) and (*b*). If many muskrats are involved, their holes may dangerously honeycomb a small earth dam, making it extremely weak. Ground squirrels normally dig only in dry soil and stop at the point where seepage is encountered. However, if the water level in the dam is very low for a number of years, the squirrels may completely dig dam.

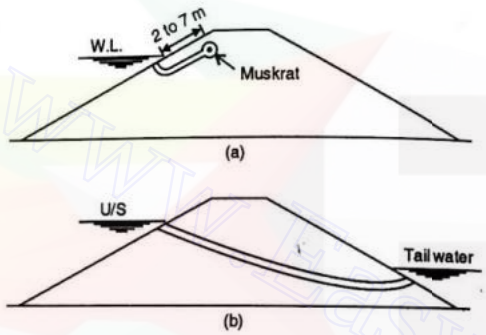

FIG. 10.10. HOLES MADE BY MUSKRATS.

(*ix*) **Damage caused by Water Soluble Materials.** The leaching of natural deposits of water soluble materials, such as gypsum, has created trouble at many dams. Where considerable foundation leakage occurs, leaching of gypsum of the foundation may cause excessive settlement. The disposition of soluble material previously leached from the natural soil may tend to plug specially designed filters. The failure of the Apishapa Dam in Southern Colorado in 1923 was attributed, at least in part, by some engineers to dissolution of salts from the embankment material.

10.3. CRITERIA FOR SAFE DESIGN OF EARTH DAM

An earth dam must be safe and stable during phases of construction and operation of the reservoir. The practical criteria for the design of earth dams may be stated briefly as follows :

(1) The embankment must be safe against overtopping during occurrence of the inflow design flood by the provision of sufficient spillway and outlet works capacity.

(2) The dam must have sufficient free board so that it is not overtopped by wave action.

(3) The seepage line should be well within the d/s face so that no sloughing of the slope takes place.

(4) Seepage flow through the embankment, foundation and abutments must be controlled by suitable design provisions so that no internal erosion takes place. The amount of water lost through seepage must be controlled so that it does not interfere with planned project functions.

(5) There should be no opportunity for the free passage of water from upstream to the downstream either through the dam or through the foundation.

(6) The portion of the downstream of the impervious core should be properly drained.

(7) The upstream and downstream slopes should be so designed that they are safe during and immediately after the construction.

(8) The downstream slope should be so designed that it is safe during steady seepage case under full reservoir condition.

(9) The upstream slope should be stable during rapid drawdown condition.

(10) The upstream and downstream slopes of the dam should be flat enough so that shear stress induced in the foundation is enough less than the shear strength of the material in the foundation to ensure a suitable factor of safety.

(11) The dam as a whole should be earthquake resistant.

(12) The upstream slope must be protected against erosion by wave action, and the crest and down stream slope must be protected against erosion due to wind and rain.

The above criteria of design have been covered at length in the subsequent articles.

10.4. SECTION OF AN EARTH DAM

The preliminary design of an earth dam is done on the basis of past experience and on the basis of the performance of the dams built in the past. We shall discuss here the preliminary selection of the following terms :

(1) Top width. (2) Free board.
(3) Casing or outer shells. (4) Central impervious core.
(5) Cut-off trench. (6) Downstream drainge system.

(1) Top Width.

The crest width of an earth dam depends on the following considerations :

(*i*) Nature of the embankment materials and minimum allowable percolation distance through the embankment at the normal reservoir level.
(*ii*) Height of the structure.
(*iii*) Importance of the structure.
(*iv*) Width of highway on the top of the dam.
(*v*) Practicability of construction.
(*vi*) Protection against earthquake forces.

Following are some of the empirical expressions for the top width b of the earth dam, in terms of the height Z of the dam :

$$b = \frac{Z}{5} + 3 \quad ...(10.1)$$

$$b = 0.55\, Z^{1/2} + 0.2\, Z \quad ...(10.2)$$

$$b = 1.65\, (Z + 1.5)^{1/3} \quad ...(10.3)$$

Eq. 10.1 is applicable for very low dams. Eq. 10.2 is applicable for dams lower than 30 m. Eq. 10.3 is given by U.S.B.R., and is applicable for dams higher than 30 m.

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According to the Indian Standard, no dam should have crest width of less than 6 m.

(2) **Free board.**

Free board is the vertical distance between the horizontal crest of the embankment and the reservoir level. *Normal free board* is the difference in the level between the crest or top of the embankment and normal reservoir level. *Minimum free board* is the difference in the elevation between the crest of the dam and the maximum reservoir water surface that would result should the inflow design flood occur and should the outlet works and spillway function as planned. Sufficient free board must be provided so that there is no possibility whatsoever of the embankment being overtopped. The U.S.B.R. suggests the following free boards :

TABLE 10.1 U.S.B.R. PRACTICE FOR FREE BOARD

Nature of spillway	*Height of Dam*	*Free Board*
Free	Any	Minimum 2 m and maximum 3 m over the maximum flood level
Controlled	Less than 60 m	2.5 m above the top of gates.
Controlled	Over 60 m	3 m above the top of gates.

(3) **Casing or outer shells.**

The function of *casing or outer shells* is to impart stability and protect the core. The relatively pervious materials, which are not subject to cracking on direct exposure to atmosphere, are suitable for casing. Table 10.2 (*a*) gives recommendations for suitability of soils used for earth dams as per IS: 8826–1978.

TABLE 10.2 (*a*). SUITABILITY OF SOILS FOR CONSTRUCTION OF EARTH DAMS

Relative Suitability	*Homogeneous Sections*	*Zoned Earth Dam*	
		Pervious Casing	*Impervious Core*
1. Very Suitable	GC	SW, GW	GC
2. Suitable	CL, CI	GM	CL, CI
3. Fairly suitable	SP, SM, CH	SP, GP	CM, GC, SM SC, CH
4. Poor	-	-	ML, MI, MH
5. Not suitable	-	-	OL, NI, OH, Pt

The design slopes of the upstream and downstream embankments may vary widely, depending on the character of the materials available, foundation conditions and the height of the dam. The slopes also depend upon the type of the dam (*i.e.* homogeneous, zoned or diaphragm).

The upstream slope may vary from 2 : 1 to as flat as 4 :1 for stability. A storage dam subjected to rapid drawdown of the reservoir should have an upstream zone with permeability sufficient to dissipate pore water pressures exerted outwardly in the upstream part of the dam. If only materials of low permeability are available, it is necessary to provide flat slope for the rapid drawdown requirement. However, a steep slope may be provided if free draining sand and gravel are available to provide a superimposed weight for holding down the fine material of low permeability. The

usual downstream slopes are 2 : 1, where a downstream pervious zone is provided, to $2\frac{1}{2}$: 1, where embankment is impervious. Table 10.2(*b*) gives the side slopes for preliminary design of earth dams according to Terzaghi. Table 10.2(*c*) gives preliminary dimensions of earth dams according to Strange.

TABLE 10.2(*b*). SIDE SLOPES FOR EARTH DAMS ACCORDING TO TERZAGHI

	Type of material	*Upstream slope*	*Downstream slope*
(*i*)	Homogeneous well graded material	$2\frac{1}{2}$: 1	2 : 1
(*ii*)	Homogeneous coarse silt	3 : 1	$2\frac{1}{2}$: 1
(*iii*)	Homogeneous silty clay, or clay Ht. less than 15 m Ht. more than 15 m	$2\frac{1}{2}$: 1 3 : 1	2 : 1 $2\frac{1}{2}$: 1
(*iv*)	Sand or sand and gravel with clay core	3 : 1	$2\frac{1}{2}$: 1
(*v*)	Sand or sand and gravel with R.C. core wall	$2\frac{1}{2}$: 1	2 : 1

TABLE 10.2(*c*). PRELIMINARY DIMENSIONS OF EARTH DAMS ACCORDING TO STRANGE

Height of dam above foundation level (m)	*Height of dam above H.F.L. (m)*	*Top width (m)*	*U/S slope*	*D/S slope*
Upto 4.5	1.2 to 1.5	1.8	2 : 1	$1\frac{1}{2}$: 1
4.5 to 7.5	1.5 to 1.8	1.85	$2\frac{1}{2}$: 1	$1\frac{3}{4}$: 1
7.5 to 15	1.85	2.5	3 : 1	2 : 1
15 to 22.5	2.1	3.0	3 : 1	2 : 1

(4) **Central Impervious Core.**

The minimum safe thickness of the central impervious core depends the following factors:

(*i*) Tolerable seepage loss.

(*ii*) Maximum width that will permit proper construction.

(*iii*) Types of the materials available for the core and the shells.

(*iv*) Design of the proposed filter layers.

(*v*) Precedent on similar projects.

It should be noted that the shear strength of core materials is always lesser than the rest of the embankment. Hence a thinner shell is preferable from the point of view of stability. However, a thick core has more resistance to piping which may develop in differential settlement cracks. The governing criterion is that the thickness of the core at any elevation is not less than the height of the embankment at that elevation so that the average hydraulic gradient is less than unity. The width of the core at the crest of the dam should be a minimum of 3 m to permit economical placement and compaction of impervious embankment material by construction equipment, which

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may include trucks, dozers and tamping rollers. The top level of the core should be at least 1 metre above the maximum water level to prevent seepage by capillary syphoning.

Fig. 10.11 shows the size range of impervious core in zoned embankments. The minimum core for a dam on pervious foundation is based on consideration of seepage pressures in the foundation.

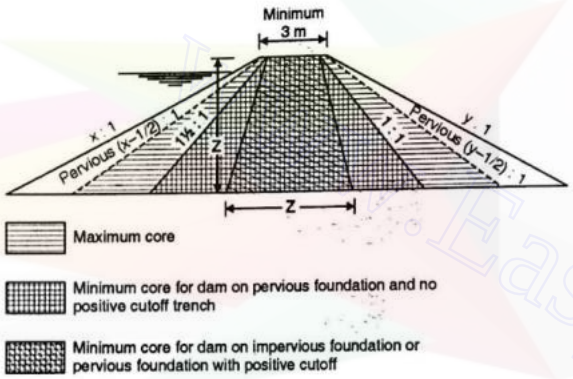

FIG. 10.11. SIZE RANGE OF IMPERVIOUS CORES IN ZONED EMBANKMENT.

Sloping Cores in Earth Dam[1]

The impervious core may be placed with axis either vertical (Fig. 10.11), or inclined (Fig. 10.12). The advantages of the vertical core are : (*i*) the higher pressure exists on the contact between the core and the foundations, reducing the possibility of leakage along the contact, and (*ii*) thickness of vertical core is greater than the thickness of a sloping core, for a given quantity of impervious soil. However, the advantages of the sloping cores are : (*i*) the downstream portion of the embankment can be constructed first and the core placed later, (*ii*) foundation grouting can be carried out while the embankment is being placed and (*iii*) filter layers between the core and the upstream and downstream pervious zones can be made thinner with less difficulty in construction than the first layers with vertical core dam. The vertical core dam has a steeper upstream

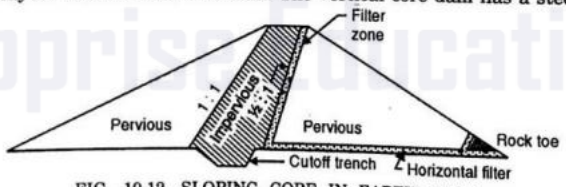

FIG. 10.12. SLOPING CORE IN EARTH DAMS.

[1] Lewis. J.G.,"The use of a Sloping Core in Earth and Rockfill Dams". Fifth Congress on Large Dams, Paris, (1955), Vol. I, p. 532.

slope than the sloping core dam, so that the total volumes are about the same. The theoretical minimum volume for a thin core dam is obtained when the core is located in such a way that the upstream and downstream slopes have approximately the same safety factors against shear failure. When the core is sloped upstream, this condition is achieved with an average inclination[1] of about $\frac{1}{2}$: 1.

Suitability of Material for Core

The soils having high compressibility and liquid limit are not suitable as they are prone to swelling and formation of cracks. Soils having organic contents are also not suitable. Table 10.2(*a*) gives the suitability of soils for impervious core of zoned earth dams. IS : 1498 – 1970 may further be referred for suitability of soils for core. Table 10.3 gives the recommendation based on IS : 8826– 1978 regarding suitability of soils for construction of core for earth dams in earthquake zones.

TABLE 10.3. SUITABILITY OF SOILS FOR CONSTRUCTION OF CORE OF EARTH DAM IN EARTHQUAKE ZONES

Relative Suitability	*Type of Soil*
1. Very good	Very well graded coarse mixtures of sand, gravel and fines, D_{85} coarser than 50 mm. D_{50} coarser than 6 mm.
2. Good	(*a*) Well graded mixture of sand, gravel and clayely fines. D_{85} coarser than 25 mm ; Fines consisting of inorganic clay (CL with plasticity index > 12). (*b*) Highly plastic tough clay (CH with plasticity index > 20).
3. Fair	(*a*) Fairly well graded, gravelly, medium to coarse sand with cohesionless fines, D_{85} coarser than 19 mm. D_{50} between 0.5 mm to 3.0 mm. Not more than 25% finer then 75 μ. (*b*) Clay of medium plasticity (CL with plasticity index > 12).
4. Poor	(*a*) Clay of low plasticity (CL and CL–ML) with little coarse fraction Plasticity index between 5 and 8. Liquid limit > 25. (*b*) Silts of medium to high plasticity (ML or MH) with little coarse fraction. Plasticity index > 10. (*c*) Medium sand with cohesionless fines.
5. Very poor	(*a*) Fine, uniform cohesionless silty sand, D_{85} finer than 0.3 mm. (*b*) Silt from medium plasticity to cohesionless (ML). Plasticity index less than 10.

(5) Cutoff Trench.

Cutoff is required to (*a*) reduce loss of stored water through foundation and abutments, and (*b*) prevent sub-surface erosion by piping. Following are the Indian Standard Recommendations (IS : 8826 – 1978).

(*i*) The alignment of the cutoff trench should be fixed in such a way that its central line should be within the u/s base of the impervious core and it should be keyed into rock or continuous impervious strata.

(*ii*) The bottom width of cutoff trench may be fixed taking following factors into consideration :

(*a*) Provide sufficient working space for compaction equipment,

(*b*) Provide sufficient working space to carry out curtain grouting, and

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(c) Provide safety against piping.

A minimum width of 4 m is recommended. A bottom width of 10 % of hydraulic head may be provided to safety requirements of piping. This may be suitably increased to satisfy other requirements of mechanical equipments and curtain grouting. The side slopes depend upon sub-strata. Side slopes of atleast 1 : 1 or flatter may be provided in case of overburden, while 1/2 and 1/4 : 1 may be provided in soft rock and hard rock respectively.

(*iii*) The positive cutoff should be taken atleast one metre into continuous impervious substratum.

(*iv*) The partial cutoff is specially suited for horizontally stratified foundations with relatively more pervious layer near top. The depth of the partial cutoff in deep pervious alluvium will be governed by :

(*a*) Permeability of substrata, and

(*b*) Relative economics of depth of excavation governed usually by cost of dewatering versus length of u/s impervious blanket.

(*v*) The backfill material for cutoff trench shall have same properties as those prescribed for impervious core.

(*vi*) The cutoff in the flanks on either side should normally extend upto the top of the impervious core, particularly in case of steep abutments.

10.5. DOWNSTREAM DRAINAGE SYSTEM

Filter zones are invariably provided in all earth dams. They are constructed of material appreciably more pervious than the embankment soil. Provision of filter zones at the d/s side serves two purposes :

(1) It reduces the pore water pressure in the downstream portion of the dam, and hence increases its stability.

(2) It checks the piping by checking the migration of the particles.

Generally, three types of drains are provided in the earth dams :

(*i*) Toe drains [Fig. 10.13(*a*)]

(*ii*) Horizontal blanket drains [Fig. 10.13(*b*)]

(*iii*) 'Chimney' drains [Fig. 10.13(*c*)]

Toe drains were installed in some of the oldest homogeneous dams in an effect to prevent softening which was observed at the downstream toe. Horizontal drainage blankets are widely used for dams of moderate height. The greatest disadvantage in the provision of horizontal drain results from the fact of stratification of the embankment material having permeability in the horizontal direction more than the vertical.

In such a circumstance, inspite of horizontal drain, the seepage water may flow horizontally at top of a relatively impervious layer and discharge on the downstream face, causing surface sloughing. In an effort to prevent trouble with stratification and to intercept seepage water before it reaches the downstream slope, many of the higher homogeneous dams have been constructed with inclined or vertical 'chimney' drains. In several recent major dams, the chimney drain has been inclined at a considerable slopes, sometimes upstream and sometimes downstream.

Filter Criteria. The dimensions and permeability of filter drains must be chosen in such a way that the drainage system can carry away the anticipated flow with an ample margion of safety. The rational approach to the design of filter is generally credited to Terzaghi. Considerable experimentation has been performed by the U.S. Corps of Engineers and the U.S.B.R. Generally, a *multilayered filter* is provided, in which each subsequent layer becomes increasingly coarser than the previous one. Such a filter is sometimes known as *inverted filter* or *reverse filter*. The soil to be protected (*i.e.* the embankment and foundation material surrounding the filter) is known as the *base material*.

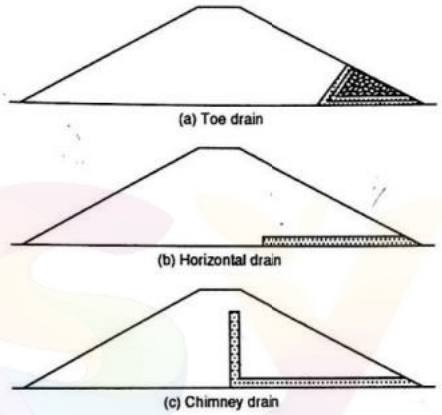

FIG. 10.13. DRAINS IN EARTH DAMS.

The four main requirements to be satisfied by filter material are as follows[1] :

(1) The filter material should be sufficiently fine and properly graded so that the voids of the filter are small enough to prevent base material particles from penetrating and clogging the filter.

(2) The filter material should be sufficiently coarse and pervious compared to the base material so that the incoming water is rapidly removed without any appreciable build up of seepage forces within the filter.

(3) The filter material should be coarse enough not to be carried away through the drainage pipe openings. The drainage pipes should be provided with sufficiently small openings or perforations, or additional coarser layer should be used, if necessary.

(4) The filter layer should be sufficiently thick to provide a good distribution of all particle size throughout the filter and to be able to carry the seepage discharge. The filter thickness should ensure an adequate safety against piping and proper insulation for frost susceptible base material, as the case may be.

According to Terzaghi, the filter material should fulfil the following two criteria:

(1) The D_{15} size of the filter material must not be more than 4 to 5 times D_{85} size of the base material. This prevents the foundation material from passing through the pores of the filter material.

(2) The D_{15} size of the filter material must be at least 4 to 5 times the D_{15} size of base material. This keeps seepage forces within the filter to permissible small magnitude.

[1]Earth Manual, U.S. Bureau of Reclamation, Washington, 1960.

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The above two criteria may be expressed as

$$\frac{D_{15} \text{ of filter}}{D_{85} \text{ base material}} < 4 \text{ to } 5 < \frac{D_{15} \text{ of filter}}{D_{15} \text{ of base material}}$$

The requirements cited above must be satisfied between any two adjacent layers of the filter. The criteria given by Terzaghi have been further modified as follows[1] :

(*a*) $\dfrac{D_{15} \text{ of filter}}{D_{15} \text{ of base material}} = 5 \text{ to } 40$, provided that the filter does not contain more than 5 percent of material finer than 0.074 mm (no. 200 sieve).

(*b*) $\dfrac{D_{15} \text{ of filter}}{D_{85} \text{ of base material}} = 5$ or less.

(*c*) $\dfrac{D_{85} \text{ of filter}}{\text{Maximum opening of pipe of drain}} = 2$ or more.

(*d*) The grain size curve of the filter should be roughly parallel to that of the base material.

Drainage of earth dams. The filter or the rock toe discharges the water into a network of drains. The drains may consist of open trenches or filled with rough stones. It may also enclose tile drains in addition and protected by an inverted filter. The number of drains and their interval depends upon (*i*) nature of soil on which the dam is founded, (*ii*) nature of the ground, and (*iii*) type of material used for dams. Fig. 10.14 shows the arrangement of the network of drains.

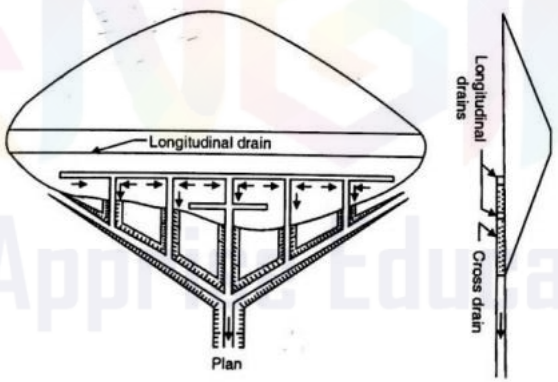

FIG. 10.14. DRAINAGE SYSTEM.

[1]"Design of small Dams", U.S. Bureau of Reclamation, pp. 174.

10.6. SEEPAGE ANALYSIS

The quantity of water passing through the body and foundation of the earth dam, as well as the distribution of water pressures can be estimated by the theory of flow of fluids through porous media. While computing these quantities with the help of theoretical analysis that follow, the following assumptions must be kept in mind :

(1) The rolled embankment and the natural soil foundation of the earth dam are *incompressible* porous media. The size of the pore spaces do not change with time, regardless of water pressure.

(2) The seeping water flows under a hydraulic gradient which is due only to gravity head loss, or Darcy's law for flow through porous medium is valid.

(3) There is no change in the degree of saturation in the zone of soil through which the water seeps and the quantity flowing into any element of volume is equal to quantity which flows out in the same length of time.

(4) The hydraulic boundary conditions at entry and exit are known.

(5) Water is incompressible.

1. LAPLACE EQUATION FOR TWO DIMENSIONAL FLOW.

In earth dams, the flow is essentially two dimensional. Hence we shall consider only two dimensional flows.

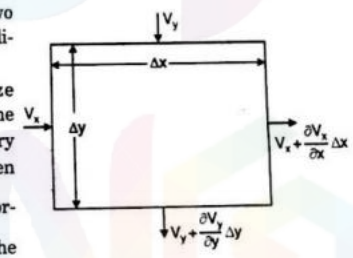

FIG. 10.15.

Consider an element of soil of size $\Delta x, \Delta y$ and of unit thickness perpendicular to the plane of the paper. Let v_x and v_y be the entry velocity components in x and y directions. Then $\left(v_x + \dfrac{\partial v_x}{\partial x}\Delta x\right)$ and $\left(v_y + \dfrac{\partial v_y}{\partial y}\Delta y\right)$ will be the corresponding velocity components at the exit of the element. According to assumption 3 stated above, the quantity of water entering the element is equal to the quantity of water leaving it. Hence, we get

$$v_x(\Delta y.\ 1) + v_y(\Delta x.1) = \left(v_x + \frac{\partial v_x}{\partial x}\Delta x\right)(\Delta y.\ 1) + \left(v_y + \frac{\partial v_y}{\partial y}.\ \Delta y\right)(\Delta x.\ 1)$$

From which

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0 \qquad ...(i)$$

This is the *continuity equation.*

According to assumption 2 :

$$v_x = k_x\, i_x = k_x \frac{\partial h}{\partial x} \qquad ...(ii)$$

and

$$v_y = k_y.\ i_y = k_y \frac{\partial h}{\partial y} \qquad ...(iii)$$

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where h = hydraulic head under which water flows.

k_x and k_y = coefficients of permeability in x and y directions.

Substituting in (*ii*) and (*iii*), we get

$$\frac{\partial^2 (k_x h)}{\partial x^2} + \frac{\partial^2 (k_y . h)}{\partial y^2} = 0 \qquad ...(10.4)$$

For an isotropic soil,

$$k_y = k_x = k$$

Hence we get from Eq. 10.4

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = 0$$

Substituting $\Phi = kh$ = velocity potential, we get

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} = 0 \qquad ...(10.5)$$

This is the Laplace equation of flow in two dimensions. *The velocity potential Φ may be defined as a scalar function of space and time such that its derivative with respect to any direction gives the fluid velocity in that direction.*

This is evident, since we have $\Phi = kh$

$$\therefore \quad \frac{\partial \Phi}{\partial x} = k \frac{\partial h}{\partial x} = k . i_x = v_x$$

Similarly, $$\frac{\partial \Phi}{\partial y} = k \frac{\partial h}{\partial y} = k . i_y = v_y$$

The solution of Eq. 10.5 can be obtained by

(*i*) Analytical methods, (*ii*) Graphical method, (*iii*) Experimental methods.

The solution gives two sets of curves, known as *equipotential lines* and *stream lines* (or flow lines), mutually orthogonal to each other, as shown in Fig. 10.16. The equipotential lines represent contours of equal head (potential). The direction of seepage is always perpendicular to the equipotential lines. The path along which the individual particles of water seep through the soil are called stream lines or flow lines.

2. COMPUTATION OF RATE OF SEEPAGE FROM FLOW NET.

A network of equipotential lines and flow lines is known as a *flow net*. Fig. 10.16 shows a portion of such a flow net. The portion between any two successive flow lines is known as *flow channel*. The portion enclosed between two successive equipotential lines and successive flow lines is known *as field* such as that shown hatched in Fig. 10.16.

Let b and l be the width and length of the field.

Δh = head drop through the field.

Δq = discharge passing through the flow channel.

h = total head causing flow

= difference between upstream and downstream heads.

Then, from Darcy's law of flow through soils :

$$\Delta q = k . \frac{\Delta h}{l} (b \times 1)$$

(considering unit thickness). ...(*i*)

If N_d = total number of potential drops in the complete flow net,

Then $\Delta h = \frac{h}{N_d}$

and $$\Delta q = k \frac{h}{N_d} \left(\frac{b}{l}\right) \qquad ...(ii)$$

Hence the total discharge through the complete flow net is given by

$$q = \Sigma \Delta q = k . \frac{h}{N_d} \left(\frac{b}{l}\right) . N_f$$

or $$q = kh \frac{N_f}{N_d} . \frac{b}{l}$$

where N_f = total number of flow channels in the net.

The field is square, and hence $b = l$.

Thus $$q = kh \frac{N_f}{N_d} \qquad ...(10.6)$$

This is the required expression for the discharge passing through a flow net, and is valid only for isotropic soils in which

$$k_x = k_y = k.$$

FIG. 10.16. PORTION OF A FLOW NET.

3. SEEPAGE DISCHARGE FOR ANISOTROPIC SOIL.

Let us now consider the case of an anisotropic flow medium in which $k_x \neq k_y$.

For such a case, the flow equation 10.4 becomes

$$k_x \frac{\partial^2 h}{\partial x^2} + k_y \frac{\partial^2 h}{\partial y^2} = 0$$

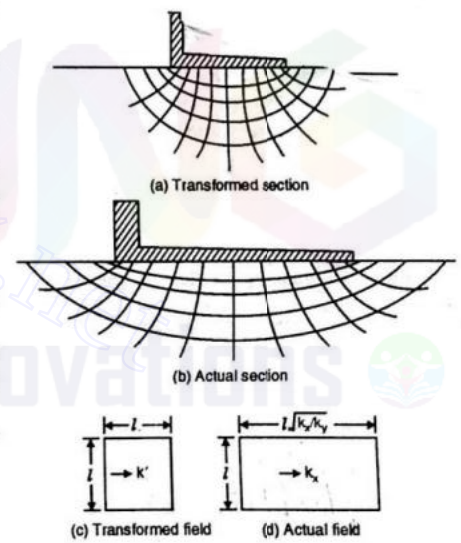

FIG. 10.17. ANISOTROPIC FLOW.

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which is not a Laplacian equation. Hence flow net cannot be directly drawn. Rewriting it, we get

$$\frac{k_x}{k_y} \cdot \frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = 0.$$

Let us put $x_n = x\sqrt{k_y/k_x}$

where x_n is the new co-ordinate variable in the x-direction.

Then the above equation becomes,

$$\frac{\partial^2 h}{\partial x_n^2} + \frac{\partial^2 h}{\partial y^2} = 0 \qquad ....(10.7)$$

This is in Laplacian form.

To plot the flow for such a case, the cross-section through anisotropic soil is plotted to a natural scale in the y-direction, but to a transformed scale in the x-direction, all dimensions parallel to x-axis being reduced by multiplying by the factor $\sqrt{k_y/k_x}$. The flow net obtained for this *transformed section* will now be constructed in the normal manner as if the soil were isotropic. The *actual flow net is* then obtained by re-transforming the cross-section including the flow net, back to the natural scale by multiplying the x-coordinates by a factor $\sqrt{k_x/k_y}$. The actual flow net, thus will not have orthogonal set of curves.

As shown in Fig. 10.17, field of the transformed section will be a square one, while the field of the actual section (re-transformed) will be rectangular one having its length in x-direction equal to $\sqrt{k_x/k_y}$ times the width in the y-direction.

Let k_x = permeability coefficient in x-direction, of the actul anisotropic soil field.

k' = equivalent permeability of the transformed field.

Then, for the transformed section.

$$\Delta q = k' \frac{\Delta h}{l}(l \times 1) \qquad ....(1)$$

For the actual field, $$\Delta q = k_x \frac{\Delta h}{(l)\sqrt{k_x/k_y}}(l \times 1) \qquad ...(2)$$

Since the quantity of flow is the same,

$$k' \frac{\Delta h}{l}(l) = k_x \frac{\Delta h}{\sqrt{k_x/k_y}}(l)$$

Hence $$k' = k_x\sqrt{k_y/k_x} = \sqrt{k_x \cdot k_y} \qquad ....(10.8)$$

Hence the discharge is given by

$$q = k' h \frac{N_f}{N_d} = \sqrt{k_x \cdot k_y}\; h \frac{N_f}{N_d} \qquad ...(10.9)$$

4. PHREATIC LINE IN EARTH DAM.

Let us now consider the case of a homogeneous earth dam with a horizontal filter, as shown in Fig.10.18. In order to draw the flow net, it is first essential to

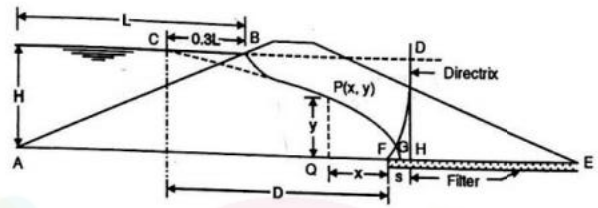

FIG. 10.18. CASAGRANDE'S METHOD OF DETERMINING PHREATIC LINE IN A DAM WITH HORIZONTAL DRAINAGE FILTER

find the location and shape of the *phreatic line* or the *top flow line* separating the saturated and unsaturated zones. The phreatic line or seepage line is defined as the line within a dam section below which there are positive hydrostatic pressures in the dam. The hydrostatic pressure on the phreatic line itself is atmospheric or zero. The phreatic line can be located by (*i*) analytical method, (*ii*) graphical method, and (*iii*) experimental method. We shall first discuss the graphical method of the determination of phreatic line by *Casagrande method.*

Casagrande assumed the phreatic line to be a *base parabola* with its focus at F, the starting point of the filter FE. The following is the procedure for locating the phreatic line graphically :

1. AB is the upstream face. Let its horizontal projection be L. On the water surface, measure the distance $BC = 0.3\ L$. Then the point C is the starting point of the base parabola.

2. To locate the position of the directrix of the parabola, we utilise the principle that any point on the parabola is equidistant from the focus as well as directrix. Hence with the point C as the centre, and CF as the radius, draw an arc to cut the horizontal line through CB in D. Draw a vertical tangent to the curve FD at D. Evidently, $CD=CF$. Hence the vertical line DH is the directrix.

3. The last point G on the parabola will evidently lie midway between F and H.

4. In order to locate the intermediate points on the parabola we use the principle that its distances from focus and directrix must be equal. For example, to locate any point P, draw vertical line QP at any distance x from F. Measure the distance QH. With F as the centre and QH as the radius, draw an arc to cut the vertical line through Q in point P.

5. Join all these points to get the base parabola. However, correction is to be made at the entry point. The phreatic line must start from B, and not from C. Also the phreatic line is a flow line, and must start perpendicularly to the u/s face AB which is a 100 % equipotential line. Hence the portion of the phreatic line at B is sketched free hand in such a way that it starts perpendicularly to AB, and meets the rest of the parabola tangentially without any kink. The base parabola should also meet the d/s filter perpendicularly (*i.e*, vertically) at G.

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In order to find the equation of the base parabola, consider any point P on it, with co-ordinate (x, y) with respect to focus F as origin.

From the property of the parabola, we have $PF = QH$

$$\therefore \quad \sqrt{(x^2+y^2)} = QF + FH = x + s \quad \text{where } s = FH = \text{focal distance.} \quad ...(i)$$

From (i)
$$x^2 + y^2 = x^2 + s^2 + 2xs$$

$$\therefore \quad x = \frac{y^2 - s^2}{2s} \quad ...[10.10(a)]$$

or
$$y^2 = 2xs + s^2 \quad ...(ii) \quad ....[10.10(b)]$$

This is the equation of the base parabola.

In order to get an expression for the discharge q through the body of the dam for the present case of horizontal filter, we observe that, through the vertical section PQ,

$$q = kiA = k.\frac{\partial y}{\partial x}(y \times 1) \quad ...(iii)$$

But from Eq.(ii),
$$y = (2xs + s^2)^{1/2} \quad \text{and} \quad \frac{\partial y}{\partial x} = \frac{s}{(2xs + s^2)^{1/2}} \quad ...(iv)$$

Substituting in (iii), we get

$$q = k\frac{s}{(2xs + s^2)^{1/2}} \times (2xs + s^2)^{1/2} \quad \text{or} \quad q = ks \quad ...(10.11)$$

This is very simple expression for discharge q in terms of focal distance s. The distance s can either be determined graphically (Fig. 10.18), or can be calculated analytically by considering coordinates of point C, as follows :

From (i)
$$s = \sqrt{x^2 + y^2} - x$$

At C, $x = D$ and $y = H$

$$\therefore \quad s = \sqrt{D^2 + H^2} - D$$

Hence
$$q = ks = k(\sqrt{D^2 + H^2} - D) \quad ...[10.11(a)]$$

5. PHREATIC LINE FOR A DAM WITH NO FILTER

(a) General Solution by Casagrande. Fig. 10.19 shows a homogeneous dam with no horizontal drainage filter at the d/s side. The focus in this case will be the lowest point F of the d/s slope, and the base parabola BJG will evidently cut the d/s slope at J and extend beyond the limits of the dam, as shown by dotted line. However, according to exit conditions (Fig. 10.20 a), the phreatic

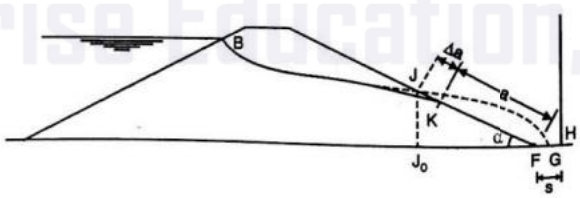

FIG. 10.19. DAM WITH NO DRAINAGE FILTER

line must emerge out at some point K, meeting the d/s face tangentially there. The portion KF is then known as *discharge face* and always remains wet. The correction Δa, by which the parabola is to be shifted downwards, is found by the value of $\frac{\Delta a}{a + \Delta a}$ given by Casagrande for various values of the slope α of the discharge face. The slope α can even exceed the value of 90°, specially in case of rockfill toe, as shown in Fig. 10.20 (a).

Thus we observe that

$$\frac{\Delta a}{a + \Delta a} = \text{value found from Fig. 10.20}(b) \text{ for given values of } \alpha \quad ...(i)$$

$$a + \Delta a = JF = \text{value measured from Fig. 10.19} \quad ...(ii)$$

Alternatively, from Fig. 10.19, we have

$$JF = (a + \Delta a) = J_0H = J_0F + FH = JF\cos\alpha + s$$

Hence
$$JF = \frac{s}{1 - \cos\alpha}$$

From the above expression, JF (and hence $a + \Delta a$) can be computed. Solving (i) and (ii), the value a and Δa can be found.

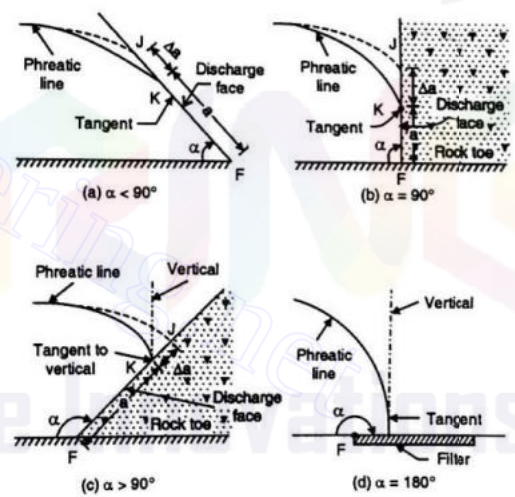

FIG. 10.20(a). EXIT CONDITIONS FOR VARIOUS SLOPES OF THE DISCHARGE FACE.

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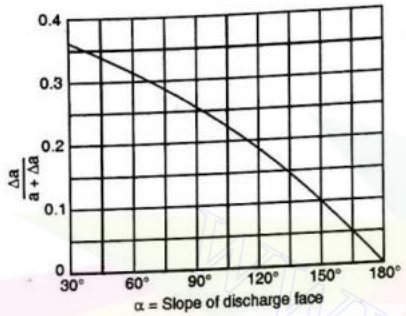

α	$\frac{\Delta a}{a+\Delta a}$
30°	0.36
60°	0.32
90°	0.26
120°	0.18
135°	0.14
150°	0.1
180°	0

FIG. 10.20(*b*). RELATION BETWEEN a AND $\frac{\Delta a}{a+\Delta a}$.

(*b*) Analytical Solution of Schaffernak and Van Iterson for α < 30°. In order to find the value of a analytically, Schaffernak and Van Iterson assumed that the energy gradient $i = \tan\alpha = \frac{dy}{dx}$. This means that the gradient is equal to the slope of the line of seepage, which is approximately true so long as the slope is gentle (*i.e.* < 30°).

For the vertical section KK_1,

$$q = k\frac{dy}{dx}y \qquad ....(i)$$

But $\frac{dy}{dx} = i = \tan\alpha.$

and $y = KK_1 = a\sin\alpha$

Substituting in (*i*), we get

$$q = ka\sin\alpha\tan\alpha \qquad ...(10.12)$$

This is the expression for discharge.

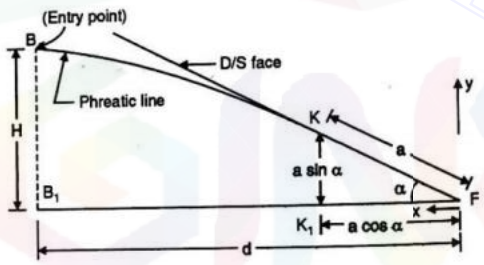

FIG. 10.21. SCHAFFERNAK'S METHOD.

Again $q = k\frac{dy}{dx}y = ka\sin\alpha\tan\alpha$

$\therefore \quad a\sin\alpha\tan\alpha\, dx = y\,dy$

Integrating between the limits:

$x = a\cos\alpha$ to $x = d$

$y = a\sin\alpha$ to $y = H$,

and

we get

$$a\sin\alpha\tan\alpha\int_{a\cos\alpha}^{d} dx = \int_{a\sin\alpha}^{H} y\,dy$$

$$\therefore \quad a\sin\alpha\tan\alpha\,(d - a\cos\alpha) = \frac{H^2 - a^2\sin^2\alpha}{2}$$

From which, we obtain, after simplification,

$$a = \frac{d}{\cos\alpha} - \sqrt{\frac{d^2}{\cos^2\alpha} - \frac{H^2}{\sin^2\alpha}} \qquad ....(10.13)$$

(*c*) Analytical Solution by Casagrande for 30° < α < 60°.

It will be observed that the previous solution gives satisfactory results for slope < 30°. For steeper slopes, the deviation from correct values increases rapidly beyond tolerable limits. Casagrande suggested the use of sin α instead of tan α.

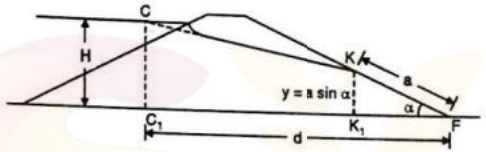

FIG. 10.22. CASAGRANDE'S METHOD.

Thus $$q = kiA = k\frac{dy}{ds}A \qquad ....(1)$$

At K, $s = a$, and $y = a\sin\alpha = A$; hence $\frac{dy}{ds} = \sin\alpha$

where s = distance measured along the curve.

Substituting in (1), we get

$$q = k\,.\,\sin\alpha\,.\,a\sin\alpha = ka\sin^2\alpha \qquad ...(10.14)$$

which is the expression for the discharge.

Again, $$q = k\frac{dy}{ds}y = ka\sin^2\alpha.$$

$\therefore \quad a\sin^2\alpha\, ds = y\,dy$

Integrating between the following limits

$s = a$ to $s = S$ (where S = total length of the parabola)

and $y = a\sin\alpha$ to $y = H$, we get

$$a\sin^2\alpha\int_a^S ds = \int_{a\sin\alpha}^{H} y\,dy$$

$$\therefore \quad a\sin^2\alpha\,.\,(S - a) = \frac{H^2 - a^2\sin^2\alpha}{2}$$

or $$a^2 - 2aS + \frac{H^2}{\sin^2\alpha} = 0$$

from which $$a = S - \sqrt{S^2 - \frac{H^2}{\sin^2\alpha}} \qquad ....(10.15)$$

Taking $S \triangleq (H^2 + d^2)^{1/2}$ for α upto 60°, we get

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$$a = \sqrt{H^2 + d^2} - \sqrt{H^2 + d^2 - \frac{H^2}{\sin^2 \alpha}}$$

or

$$a = \sqrt{d^2 + H^2} - \sqrt{d^2 - H^2 \cot^2 \alpha} \quad ...[10.15(a)]$$

For $\alpha = 90°$, Eq. 10.15 (*a*) becomes

$$a = \sqrt{d^2 + H^2} - d \quad ...(10.15\ b)$$

which provides value of *a* upto 25% deviations from the actual values.

6. CHARACTERISTICS OF PHREATIC LINE (SEEPAGE LINE).

Based on the above discussions, the characteristics of the phreatic line may be summarised below:

1. At the entry point, the phreatic line must be normal to the upstream face since the upstream face is a 100 % equipotential line. For other entry conditions (Fig. 10.23), the phreatic line starts tangentially with the water surface.

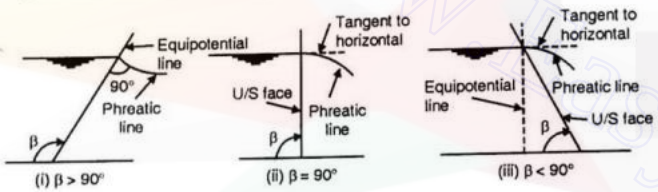

FIG. 10.23. ENTRY CONDITIONS OF PHREATIC LINE.

2. The pressure along the phreatic line is atmospheric. Hence the only change in the head along it is due to drop in the elevation of various points on it. Due to this, the successive equipotential lines will meet it at equal vertical intervals.

3. The focus of the base parabola lies at the break out point of the bottom flow line, where the flow emerges out from relatively impervious medium to a highly pervious medium.

4. When horizontal filter or drainage toe is provided, the phreatic would tend to emerge vertically.

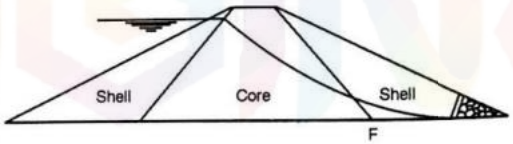

FIG. 10.24. INFLUENCE OF CENTRAL CORE ON PHREATIC LINE.

5. In the absence of any filter, the seepage line will cut the downstream slope at some point above the base. The location of this point, and the phreatic line itself, is not dependent on the permeability or any other property, so long as the dam is homogeneous. The geometry of the dam alone decides these.

6. The presence of pervious foundation below the dam does not influence the position of phreatic line.

The top surface of a freely moving ground water reservoir is an imaginary surface called phreatic line.

7. In the case of a zoned dam with central impervious core, the effect of outer shells can be neglected altogether. The focus of the base parabola will be located at the downstream toe of the core (Fig. 10.24).

7. GRAPHICAL DETERMINATION OF FLOW NET.

After having located phreatic line in an earth dam the flow net can be plotted by trial and error by observing the following properties of flow net (Fig. 10.25), and by following the practical suggestions given by A. Casagrande[1].

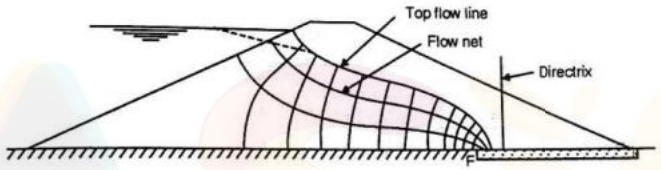

FIG. 10.25. FLOW NET BY GRAPHICAL METHOD.

Properties of flow net. 1. The flow lines and equipotential lines meet at right angles to each other.

2. The fields are approximately squares, so that a circle can be drawn touching all the four sides of square.

3. The quantity flowing through each flow channel is the same. Similarly, the same potential drop occurs between two successive equipotential lines.

4. Smaller the dimensions of the field, greater will be the hydraulic gradient and velocity of flow through it.

5. In a homogeneous soil, every transition in the shape of curves is smooth, being either elliptical or parabolic in shape.

Arthur Casagrande[1] gives the following excellent hints for the beginner in flow net sketching :

1. Use every opportunity to study the appearance of well constructed flow nets. When the picture is sufficiently absorbed in your mind, try to draw the same flow net without looking at the available solution ; repeat this until you are able to sketch this flow net in a satisfactory manner.

2. Four or five flow channels are usually sufficient for the first attempt ; the use of too many flow channels may distract the attention from the essential features.

3. Always watch the apperance of the entire flow net. Do not try to adjust details before the entire flow net is approximately correct.

4. The beginner usually makes the mistake of drawing too sharp transitions between straight and curved sections of flow lines or equipotential lines. Keep in mind that all transitions are smooth; of elliptical or parabolic shape. The size of the squares in each channel will change gradually.

[1]A. Casagrande "Seepage Through Dams", J. New Eng. Water Works Association, June, 1937, pp. 136-137.

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8. FLOW NET BY ELECTRICAL ANALOGY.

The Darcy's law governing the flow of water through soil is analogous to Ohm's law governing the flow of electric current through conductor. The corresponding analogous quantities are shown in Table 10.4.

TABLE 10.4

Darcy's law for water seepage	*Ohm's law for electric flow*
$Q = k \frac{H}{l} A$	$I = C. \frac{E}{l} A$
Q = quantity of seepage	I = rate of flow of electricity
k = coefficient of permeability	C = electric conductivity coefficient
A = cross-sectional area	A = cross-sectional area
H = pressure head	E = electric potential
l = length of seepage path	l = length of path of electric current

Thus, solutions to seepage problems can be obtained with electric models which have the same geometric shape as the soil through which the water flows. The seepage medium is replaced by an electric conductor consisting of water with some salt or dilute hydrochloric acid. The boundary equipotentials (i.e. 100 % equipotential line at u/s face and 0 % equipotential line at the d/s filter) are made of copper strips. The boundary flow lines (i.e. the phreatic line and the bottom flow line) are simulated by non-conducting strips such as ebonite or perspex etc. An alternating voltage (generally of value 5 to 20 volts) is applied across the boundary equipotential strips. A potential divider is connected in parallel with the alternating current source. Fig. 10.26 shows the electrical analogy tray, the model and the complete circuit diagram.

To determine a line of contour of equal potentials, the potentiometer is adjusted to percentage of the total voltage drop and the probe of a galvanometer is used to find the corresponding balance points (null points) on the model. Changes in the coefficient of permeability of soil zones in the seepage analogue are simulated by changes in the electrical conductivity coefficient in the model. When once the equipotential lines are obtained, orthogonal flow lines conforming to the boundary conditions are then drawn as in the graphical method. Fig. 10.27 shows a typical flow net for steady seepage case for a earth dam having the same foundation material as that of the body

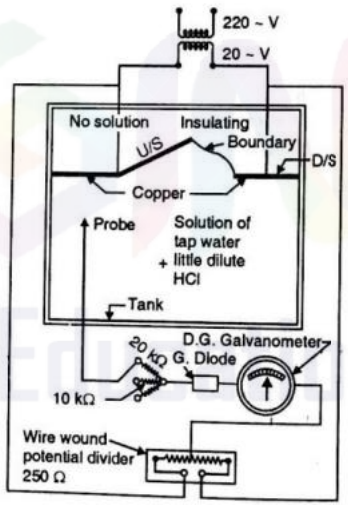

FIG. 10.26. CIRCUIT DIAGRAM FOR ELECTRICAL ANALOGY TRAY (AFTER ALAM SINGH & B.C. PUNMIA).

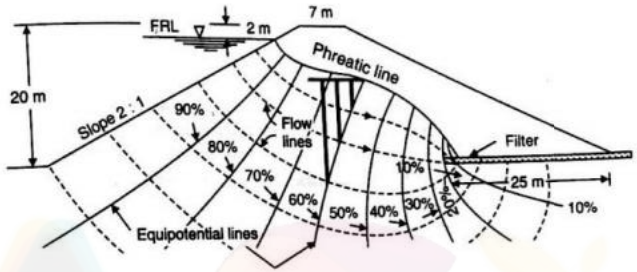

FIG. 10.27 FLOW NET FOR STEADY SEEPAGE CASE

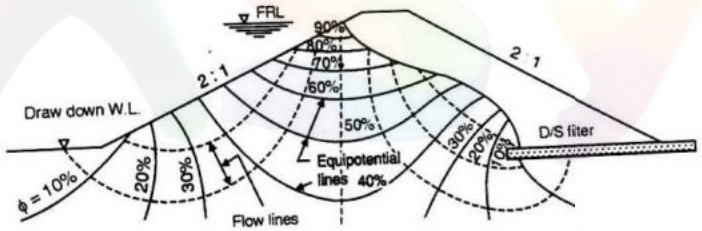

FIG. 10.28 FLOW NET FOR SUDDEN DRAWDOWN CASE (AFTER ALAM SINGH AND B.C. PUNMIA[1]).

of the dam. Fig. 10.28 shows the flow net for the sudden drawdown case for the same dam.

10.7. STABLITY ANALYSIS

We shall be discussing the stability analysis under the following four heads :

1. Stability of downstream slope during steady seepage.
2. Stability of upstream slope during sudden dradown.
3. Stability of upstream and downstream slopes during and immediately after construction.
4. Stability of foundation against shear.

SWEDISH CIRCLE METHOD OF SLOPE STABILITY.

Of the numerous methods of slope stability which have been devised over the last 40 years, the Swedish Slip Circle Method of stability analysis is the one most generally accepted. In this method the potential failure surface is assumed to be cylindrical, and the factor of safety against sliding is defined as the ratio of the average shearing

[1]Alam Singh and B.C. Punmia, 'Critical slope stability of an earthen dam', 2nd Annual No. Institution of Engineers (India), Rajasthan Centre, Jan. 1962.

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strength, as determined by the Coulomb's equation $s = c + \sigma \tan\varphi$ to the average shearing stress determined by statics on the potential sliding surface.

In order to test the stability of the slope, a trial slip circle is drawn, and the soil material above assumed slip surface is divided into a convenient number of vertical strips or slices. The forces between the slices are neglectd and each slice is assumed to act independently as a column of soil of unit thickness and width b. The weight W of each slice is assumed to act at its centre. If this weight of each slice is resolved in normal (N) and tangential (T) components, then the normal component will pass through the centre of rotation(O), and hence does not cause any driving moment on the slice. However, the tangential component T cause a driving moment $= (T \times r)$, where r is the radius of the slip circle. The tangential components of the few slices at the base may cause resisting moment; in that case T is considered negative.

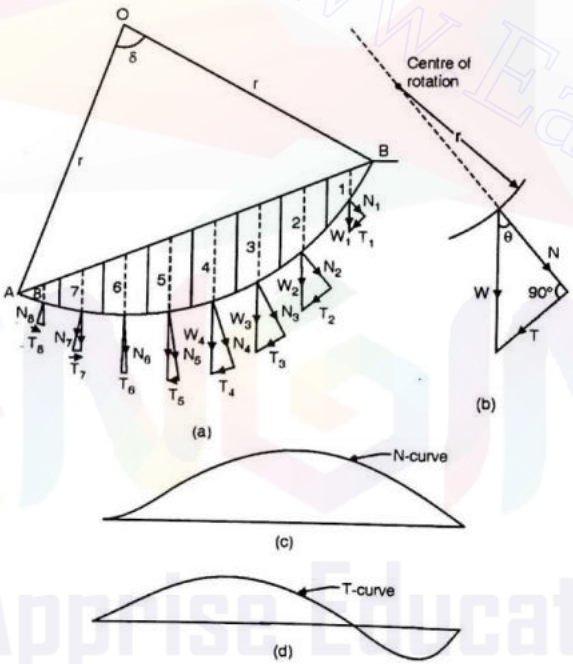

FIG. 10.29 SLIP CIRCLE METHOD

If c is the unit cohesion and ΔL is the curved length of each slice, then the resisting force, from Coulomb's equation is

$$= c \,.\, \Delta L + N \tan\phi$$

For the entire slip surface AB, we have

Driving moment $\quad M_d = r\,\Sigma T$

Resisting moment $\quad M_r = r.[c.\,\Sigma\,\Delta L + \tan\phi\,\Sigma N]$

where ΣT = sum of all tangential components

ΣN = sum of all normal components

$$\Sigma\,\Delta L = \hat{L} = \frac{2\pi r\,\delta}{360^\circ} = \text{length } AB \text{ of slip circle.}$$

Hence factor of safety against sliding is

$$F = \frac{M_r}{M_d} = \frac{c\,\hat{L} + \tan\phi\,\Sigma N}{\Sigma T} \qquad \text{....(10.16)}$$

In order to find ΣN and ΣT, N-curves and T-curves are drawn [Fig. 10.29(c), (d)] by making the ordinates of these diagrams equal to N and T values for different strips and joining them by smooth curves. The area of these diagrams can then be measured by a planimeter and ΣN and ΣT can be computed.

A simplified *Rectangular Plot Method* has been suggested by Prof. Alam Singh.[1] Referring to Fig. 10.30, let z_1, z_2..... be the boundary ordinates of the slices. Let the width of each slice be b, and the width of the last slice be mb. The total weight ΣW of the sliding sector may be written as

$$\Sigma W = b\left\{z_1 + z_2 + \ldots\ldots \left(\frac{1+m}{2}\right) z_n\right\}$$

(in terms of area)

This shows that the weight can be taken corresponding to the end ordinates of each slice. The weight can be resolved into corresponding various N-values (i.e. N_1, N_2 ..... etc.) as abscissa and the constant width b as the ordinate. Similarly, T-rectangle can be constructed. Then ΣN = area of N-rectangle and ΣT = area of T-rectangle.

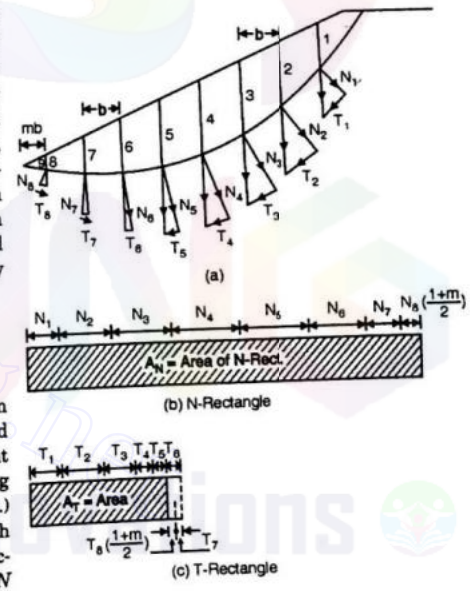

FIG. 10.30. RECTANGULAR PLOT METHOD.

[1]Alam Singh : "A Rectangular Plot Method for Slope Stability Analysis", Journal, Indian National Society of Soil, and Mech. and Found. Engg., Vol 1, No. 2, pp. 18-21, April 1962.

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It should be clearly noted that in the rectangular plot method, if there are n-slices, the total number of ordinates will be $(n-1)$, the ordinate at the end of the last strip (9) being zero. If A_N = area of N-rectangle, in sq.cm, γ = unit weight of the soil, and the scale of drawing is 1 cm = x metres, then the area in sq. metres $= A_N . x^2$ and volume $V = A_N . x^2 \times 1 \text{ m}^3$.

Hence $\Sigma N = A_N . x^2 \gamma$

Similarly $\Sigma T = A_T . x^2 \gamma$

The Third method of computing ΣN and ΣT is with the help of Table shown below :

TABLE 10.5

Slice No.	W	$N = W \cos \alpha$	$T = W \sin \alpha$
1	W_1	N_1	T_1
2	W_2	N_2	T_2
3	W_3	N_3	T_3
.	.	.	.
.	.	.	.
.	.	.	.
n	W_n	N_n	$-T_n$
	sum	$\Sigma N =$	$\Sigma T =$

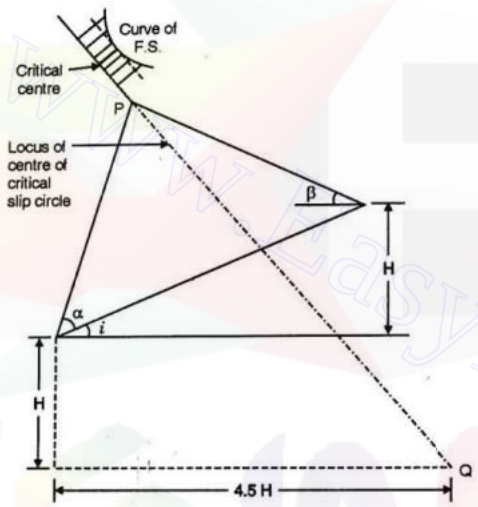

FIG. 10.31. FELLENIOUS METHOD.

Method of locating centre of critical slip circle. In order to reduce the number of trials to find the centre of critical slip circle, Fellenious has given a method of locating the locus on which the probable centre may lie. For homogenous c-ϕ soil, centre of critical slip circle lies on a line PQ, in which the point Q has its coordinates H downwards from toe and 4.5 H horizontally away, as shown in Fig. 10.31.

The other point P is obtained with the help of directional angles and given in the Table below :

TABLE 10.6

Slope	Slope angle (degrees)	Directional angles (degrees)	
		α	β
0.58 : 1	60	29	40
1 : 1	45	28	37
1.5 : 1	33.8	26	35
2 : 1	26.6	25	35
3 : 1	18.4	25	35
5 : 1	11.3	25	35

When the line PQ is obtained, trial centres are selected on it. According to Fellenous, the centre of critical slip surface is located at the point P for purely cohesive soil ($\varphi = 0$). However for $c - \varphi$ soil, the centre of critical slip surface-lies above P, on the line QP produced. The factor of safety corresponding to each centre is calculated from Eq. 10.16. These various factors of safety so obtained are plotted as ordinates (Fig. 10.31) on the corresponding centres, and a smooth curve is obtained. The centre corresponding to the lowest factor of safety is then the critical centre.

Centre of critical circle for zoned dams

The above method of locating the centre of critical slip circle, given by Felleneous, is applicable only to simple finite slopes of homogeneous soils. The method is therefore not directly applicable to zoned earth dams consisting of outer shells and inner core, having different properties. However, the number of trials can be considerably reduced if the position of the centre of the first trial slip surface is taken at point P (Fig 10.31) and the centres of other slip circles are taken in its neighbourhood. In these trials, usually a (3×3) matrix of nine trial centres, including point P in the centre is considered. All the points in this nine-point matrix are taken equidistant in the near vicinity around point P, and the factor of safety for each such trial surface is computed. The point which gives the minimum factor of safety is selected out of these nine points. With this selected point in the centre, a new matrix of 9 points is selected, including this point and the factor of safety for each trial slip circle is found. The point which gives a minimum factor of safety is then selected and the process is repeated. Usually two or three such trials are sufficient for locating the most critical centre.

Critical circle passing below the toe

For most of earth dams in practice, the critical slip circle passes through the toe of the dam. However, in the following mentioned circumstances, the slip surface may pass through the foundations, below the toe of the dam :

(*i*) For small values of φ (usually below 10° to 15°) the critical circle may pass below the toe of the slope if the inclination of the slope is less than 53°.

(*ii*) If the soils of the dam and the foundation are homogeneous, the critical slip surface may pass below the toe.

(*iii*) If the foundation soil has the *same* or *lesser* shear strength than that of the dam, the critical slip surface may pass below the toe of the slope.

For such cases, the centre P of critical slip surface is located on a vertical line through the mid-point of the slope. Felleneous found that the critical slip surface for such cases subtends an angle of $133\frac{1}{2}°$ at the centre.

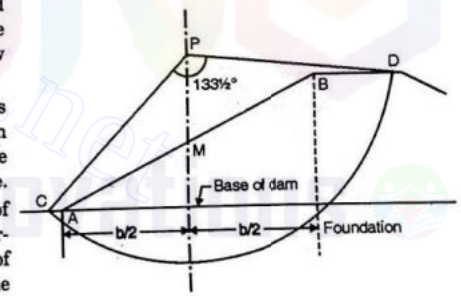

FIG. 10.32. CENTRE OF CRITICAL SLIP CIRCLE PASSING BELOW THE TOE.

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Procedure

(*i*) Draw vertical line through the mid-point (M) of the side slope.

(*ii*) Select a trial centre P on the vertical line such that when a possible slip surface is drawn with P as the centre, the slips surface subtends an angle of $133\frac{1}{2}°$ at P. Compute the factor of safety.

(*iii*) Move the next trial centre slightly to the left of P, and draw a new slip surface with reduced radius such that the circle subtends an angle of $133\frac{1}{2}°$ at the centre of the circle. Compute the factor of safety

(*iv*) Repeat the procedure with a new centre, till minimum factor of safety is obtained.

10.8. STABILITY OF DOWNSTREAM SLOPE DURING STEADY SEEPAGE

Critical condition for d/s slope occurs when the reservoir is full and percolation is at its maximum rate. The direction of seepage forces tend to decrease stability. In other words, the pore water pressure (U) acting on the soil mass below the saturated line reduces the effective stress responsible for mobilising shearing resistance. The factor of safety in this case is given by

$$\text{F . S.} = \frac{c\hat{L} + \tan\phi\,\Sigma(N-U)}{\Sigma T} \qquad ...(10.17)$$

when ΣU is the total pore pressure on the slip surface.

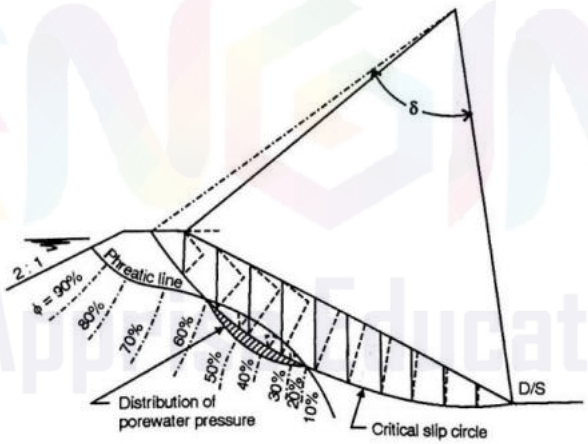

FIG. 10.33. STABILITY OF D/S SLOPE DURING STEADY SEEPAGE.

The pore-water pressure at any point is represented by the piezometric head (h_w) at that point. Thus the variations of pore water pressure along a likely slip surface

is obtained by measuring at each of its intersections with an equipotential line, the vertical height from that intersection to the level at which the equipotential line cuts the phreatic line. The pore pressure represented by vertical height so obtained is plotted to scale in a direction normal to the sliding surface at the respective point of intersection. The distribution of pore-water pressure on the critical slip surface during steady seepage is shown hatched in Fig. 10.33. The area of U-diagram can be measured with the help of a planimeter, or else the rectangular plot method may be utilised. If A_U is the area of U-rectangle, then $\Sigma U = A_U \times x^2 \gamma_w$ where x is the scale of the drawing (*i.e.* 1 cm $= x$ metres).

In the absence of a flow net, the F.S. of the d/s slope can approximately be calculated from the equation

$$\text{F . S.} = \frac{c\hat{L} + \tan\phi\,\Sigma N'}{\Sigma T} \qquad ...(10.18)$$

where the normal components N' are to be calculated on the buoyant unit weight (γ') of the dam, while T components are to be calculated on the basis of saturated weight.

10.9. STABILITY OF UPSTREAM SLOPE DURING SUDDEN DRAWDOWN

(A) Solution with the Help of Flow Net and Pressure Net.

For the u/s slope, steady seepage does not represent the critical state, because the seepage pressure then acts inwards from this slope and tends to increase the stability on the upstream side. For the u/s slope, the critical condition is when the reservoir is suddenly emptied without allowing any appreciable change in the water level within the saturated mass of soil. This state is known as sudden drawdown. With complete drawdown, the hydrostatic force acting along the u/s slope at the time of full reservoir is removed. The weight of water within the soil now tends to help a sliding failure, without the hydrostatic pressure on the slope to counteract it. Or, by another interpretation, while the disturbing force is the tangential component of the saturated weight, the shear resistance is considerably reduced on account of development of pore water pressure

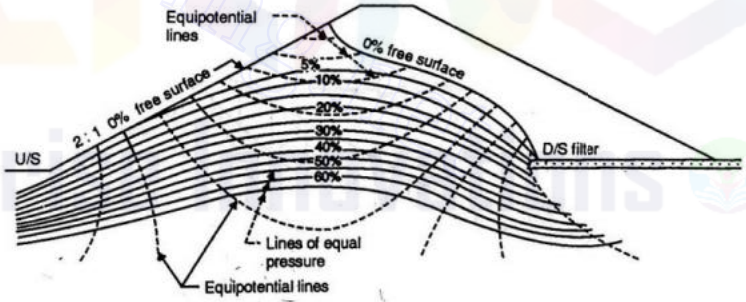

FIG. 10.34. PRESSURE NET FOR SUDDEN DRAWDOWN (AFTER ALAM SINGH & B.C. PUNMIA)

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on a likely slip surface. The effect of drawdown on slope stability varies appreciably with opportunity for drainage at the base. If the base is quite impervious, flow lines tend to be horizontal and in outward direction towards the slope, which is quite unfavourable to stability. On the other hand, if the base material is pervious, flow pattern tends to be downwards and this is more favourable with respect to stability.

The magnitude and distribution of pore water pressure on a likely slip surface is estimated from a *pressure-net* which is developed from the flow net. Fig. 10.34 represents the pressure- net for sudden drawdown case. The pressure net gives lines of equal piezometric heads (h_w) expressed in terms of percentage of total hydraulic heads H when the reservoir is full. At any point, we have

$$\Phi = h_w + y \qquad ...(1)$$

where Φ = hydraulic potential at the point, expressed as percentage of total head H.

h_w = piezometric head, expressed as percentage of total head H.

y = elevation head, expressed as percentage of total head H.

If we want to plot the 'line of equal pressure' corresponding to $h_w = 20$ % (say), we have

$$h_w = 20 = \Phi - y$$

To find a point corresponding to $h_w = 20$ % on $\Phi = 30$ %, we get from Eq. (2), $y = 30 - 20 = 10\%$ of H. Thus the point is located. Similarly, to get another point of $h_w = 20$ % on $\Phi = 40\%$, $y = 40 - 20 = 20\%$ of H and so on. The various points so obtained

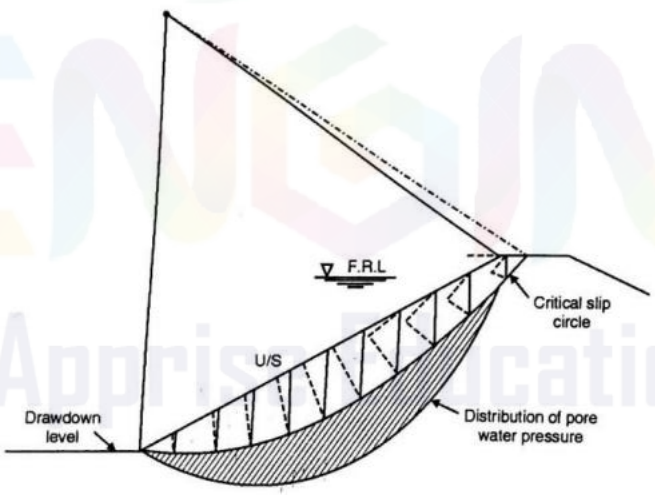

FIG. 10.35. STABILITY ANALYSIS DURING DRAWDOWN.

may be joined by a smooth curve to get a contour for $h_w = 20$ %. In this way, a pressure net may be prepared (Fig. 10.34).

In order to plot the distribution of pore water pressure on the assumed slip surface, the pressure net of Fig. 10.34 is superimposed on the diagram showing the location of slip surface. At the intersection points of the slip surface with the contours of pressure net, radial ordinates of the values of h_w's of corresponding contours are erected. The curve joining the ends of these ordinates give the distribution of pore water pressure, as illustrated in Fig. 10.35. The F.S. is then calculated by the expression:

$$\text{F.S.} = \frac{c\hat{L} + \tan\phi\,\Sigma(N - U)}{\Sigma T}$$

(B) Approximate Analysis. In the absence of the flow net the factor of safety can be calculated from the expression.

$$\text{F.S.} = \frac{c\hat{L} + \tan\phi\,\Sigma N'}{\Sigma T}$$

in which the N' components are computed with respect to the submerged density (γ') of the u/s slope, while T components are calculated with respect to the saturated density.

(C) Analysis For Sloughing of Upstream Slope During Sudden Drawdown. This is another approximate method for finding the factor of safety against sliding or sloughing of u/s slope during sudden drawdown. Refer Fig. 10.36.

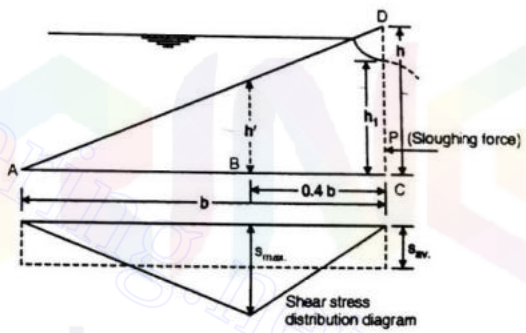

FIG. 10.36. SLOUGHING OF U/S SLOPE DURING SUDDEN DRAWDOWN.

Let γ' = submerged unit weight of the material in u/s portion of dam.

γ_s = saturated unit weight of the material in u/s portion of dam

b = horizontal length of the slope AD

P = total horizontal shear or sloughing force on the upstream portion of dam

s_{av} = average unit shear over upstream part of dam

s_{max} = maximum unit shear over upstream part of dam

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h, h_1, h' = heights as marked in Fig. 10.36.

Then the horizontal shear force is given by

$$P = \frac{\gamma_s h^2}{2} \tan^2\left(45° - \frac{\phi}{2}\right) + \frac{\gamma_w h_1^2}{2} \quad ...(10.19)$$

$$s_{av} = \frac{P}{b} \quad ...[10.20(a)]$$

and

$$s_{max} = 2\, s_{av} = \frac{2P}{b} \quad ...[10.20(b)]$$

The maximum shear occurs at a distance of 0.4 b from the u/s point, as shown.

Let R = shearing resistance $= N_e \tan\phi + c \,.\, b$ ...(10.21)

where $N_e = \gamma' A$

and A = area of u/s slope $= \frac{1}{2} bh$

$$\therefore \quad \text{Factor of safety} = \frac{R}{P} \quad ...(10.22)$$

This should be greater than 2.

(**Note**. If the analysis is done by slip circle method, factor of safety may be taken as 1.5.)

The above factor of safety is with respect to average shear. In order to find the factor of safety with respect to maximum shear, we have

$$s_{max} = 2\, s_{av} = \frac{2P}{b} \text{ (at point B)}$$

$$s_r = \text{shearing resistance at } B = h'\gamma' \tan\phi + c = 0.6\, h\, \gamma' \tan\phi + c \quad ....(10.23)$$

$$\therefore \text{ F.S. at the point of maximum shear} = \frac{s_r}{s_{max}} \quad ...(10.24)$$

This should be greater than 1.

10.10. STABILITY OF U/S AND D/S SLOPES DURING CONSTRUCTION

When a dam is built of relatively impervious compressible soil, excess pore pressures develop in air and water entrapped in the pore space. This is because the soil mass undergoes a change in the volume due to compaction of the dam during construction and also due to its own weight. In due course of time, this initial excess pore pressure gets gradually dissipated. As the pore-pressure greatly affects the shear strength of soil, it is essential to know its magnitude for stability analysis. The estimation of the construction pore pressure is made with the help of Hilf's equation[1] :

$$u = \frac{p_a \Delta}{V_a + h_c V_w - \Delta} \quad ....(10.25)$$

where u = induced pore pressure

[1]Hilf, J.W. "Estimating Construction Pore-pressure in Rolled Earth Dam", Proc. 2nd Int. Conf. Soil Mech., Vol. 3, p.3, p .234, Rotherdam, July 1948.

p_a = air pressure in the voids of soil mass after initial compaction (absolute; considered to be atmospheric; should be corrected to site elevation)

Δ = embankment compression, in percent of original total embankment volume

V_a = volume of free air voids, in percent of original total embankment volume

V_w = volume of pore water, in percent of original total embankment volume

h_c = Henry's constant of solubility of air in water by volume(= 0.02 at 68° F).

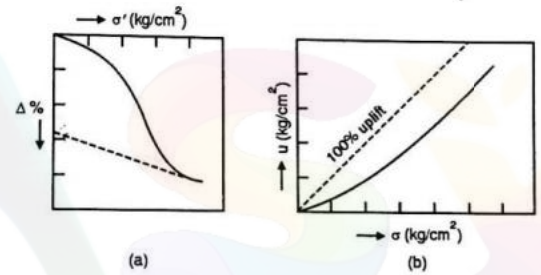

FIG. 10.37. CONSOLIDATION TEST DATA FOR ESTIMATING CONSTRUCTION PORE PRESSURE.

For use in this method a graph is plotted between effective stress σ' and percent consolidation (Δ), as obtained from consolidation test. Fig. 10.37 (*a*) gives a sample consolidation/load curve. The value of pore pressure u, for various values of Δ are calculated from Eq. 10.25. Knowing effective stress σ' and pore pressure u, the total stress σ is obtained by the relation $\sigma = \sigma' + u$. The results are tabulated as shown in Table 10.7. A plot is then made between pore water pressure (u) and total stresses (σ) as shown in Fig. 10.37 (*b*).

TABLE 10.7

Consolidation Δ (%)	*Effective Stress* σ' (*kg/cm²*)	*Pore-pressure* *u* (*kg/cm²*)	*Total stress* σ (*kg/cm²*)
1	0.7	0.065	0.765
2	1.3	0.139	1.439
4	2.3	0.322	2.622
6	3.0	0.576	3.576
8	3.6	0.948	4.548
10	4.1	1.550	5.650
12	4.9	2.680	7.580
14	6.0	6.160	12.160
16	7.9	38.20	46.10

In order to find the construction pore-pressure at the bottom of various slices (Fig. 10.29), the mid-height, z_1, z_2 ...., z_n of each strip is measured and the total stress

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σ (*i.e.*, σ_1, σ_2 ....., σ_n) is calculated. Then, from Fig. 10.37(*a*), the values of u corresponding to various values of σ are found. The results are tabulated as shown Table 10.8

TABLE 10.8

Slice No.	*Height z*	*Total stress* $\sigma = \gamma z$	u
		Sum	Σu

Hence $\Sigma U = (\Sigma u) \times$ (width of strip). Alternatively, using ordinates of u from Table 10.8, pore pressure distribution diagram can be plotted on the slip circle and ΣU can be calculated from its area A_U.

Knowing ΣU, the factor of safety of the slope is calculated by the equation :

$$\text{F. S.} = \frac{c\hat{L} + \tan\phi\,(\Sigma N - \Sigma U)}{\Sigma T}.$$

10.11. STABILITY OF FOUNDATION AGAINST SHEAR

Foundations consisting of fine, loose, cohesionless meterials or of unconsolidated clays and silts may be very weak in shear and may require thorough investigation. The present method of finding factor of safety of foundation against horizontal shear is an approximate one, and is based on the assumption that earth material has an equivalent liquid unit weight which would produce the same shear stress as the material itself.

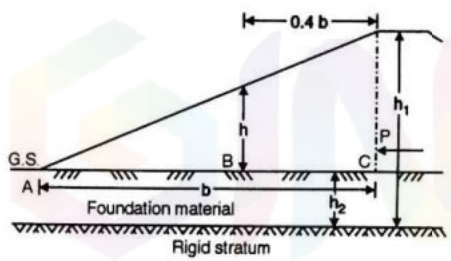

FIG. 10.38. FOUNDATION SHEAR.

If P = total horizontal shear down to rigid boundary, then

$$P = \frac{h_1^2 - h_2^2}{2}\gamma_m \tan^2\left(45° - \frac{\phi_1}{2}\right) \quad \text{.... (10.26)}$$

where $\gamma_m \tan^2\left(45° - \frac{\phi_1}{2}\right)$ = equivalent liquid unit weight

ϕ_1 = equivalent angle of friction, given by

$$\tan\phi_1 = \frac{\gamma_m h_1 \tan\phi + c}{\gamma_m h_1} \quad \text{.....(10.27)}$$

where ϕ and c are the shear parameters for the foundation material.

γ_m = mean unit weight of the dam and foundation weighted in proportion to the depth of each.

i.e. $$\gamma_m = \frac{\gamma_D(h_1 - h_2) + \gamma_F h_2}{h_1}$$

where γ_D = unit weight of dam material

γ_F = unit weight of foundation material.

Now average unit shear $= s_{av.} = \frac{P}{b}$

Maximum unit shear $= s_{max} = 2\,s_{av}$ and occurs at 0.4 b from point C.

Let s_1 = unit *shear strength* below toe (at A) $= c + \gamma_F . h_2 \tan\phi$

s_2 = unit shear strength at point $C = c + \gamma_m h_1 \tan\phi$

$\therefore$ Average shear strength $s = \frac{s_1 + s_2}{2}$

Hence overall factor of safety against shear $= \frac{s}{s_{av}}$.

This should be greater than 1.5.

In order to get factor of safety at maximum shear, let

s = unit shear strength at point of max shear $(B) = c + \gamma_{av}\, h \tan\phi$

where γ_{av} = mean effective unit weight at $B = \frac{\gamma_D h + \gamma_F h_2}{h + h_2}$

$\therefore$ $\text{F.S.} = \frac{s}{s_{max}}$. This should be greater than 1.

10.12. DESIGN TO SUIT AVAILABLE MATERIALS

The section of a zoned earth dam should be so selected that the available materials are utilised to the maximum and limited quantity of other required materials are imported to the site. The available materials may be classified into three heads :

1. Gravel or coarse sand is available along with clayey silt
2. Only fine gravel or coarse sand is available
3. Only silty clay is available.

In addition to these, three general foundation conditions may be there :

(*a*) Foundations impervious to a large depth

(*b*) Foundations pervious to moderate depth, after which impervious strata is available.

(*c*) Foundations pervious to large depth.

We shall treat each one of these conditions separately.

CASE 1. Gravel or Coarse Sand Available along with Clayey Silt.

(*a*) ***Foundations impervious to a large depth***. Figs.10.39(*a*) and (*b*) show the typical sections when the impervious or hard stratum is exposed to the surface, and the foundations are impervious to a large depth. The outer shells are made of the

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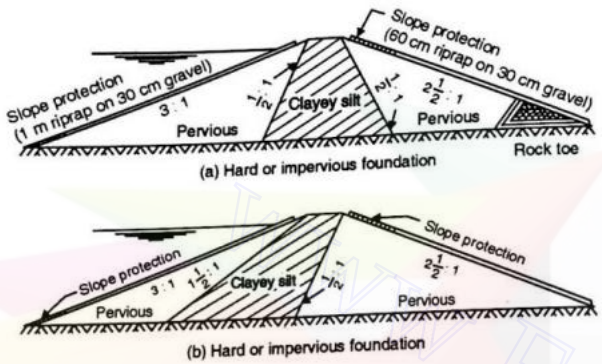

FIG. 10.39

pervious material available at the nearby site. The central core of clayey silt may have its axis either vertical or inclined.

(b) Foundation pervious to a moderate depth, after which impervious stratum is available. The construction is similar to the previous case, except that the central core is taken upto impervious foundation level, as shown in Fig. 10.40.

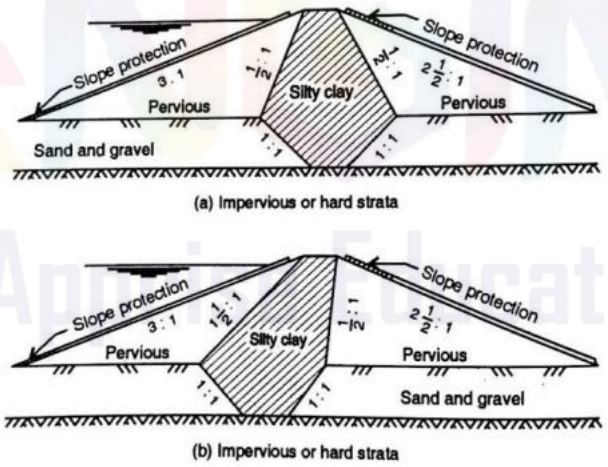

FIG. 10.40.

(c) Foundation pervious to a great depth. In this case there is a possibility of large seepage through foundations. Hence some form of cutoff is to be provided. If sufficient impervious material is available, an upstream clay blanket, such as shown in Fig. 10.41(*a*) is suggested. Otherwise, a sheet pile cutoff may be provided to some reasonable depth. Complete cutoff through foundation is not possible.

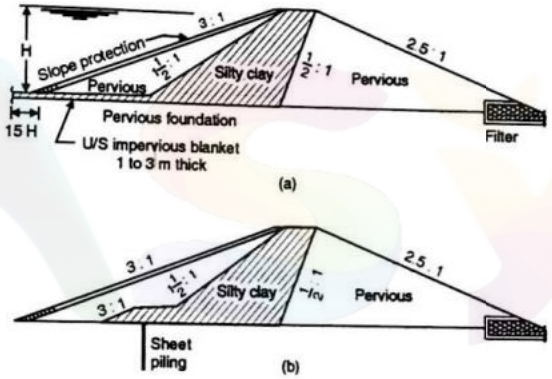

FIG. 10.41

CASE 2. Only Pervious Material Available

(a) Foundation impervious to a great depth. Since the impervious material is to be imported, thin central core is provided as shown in Fig. 10.42.

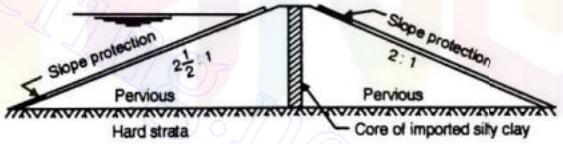

FIG. 10.42

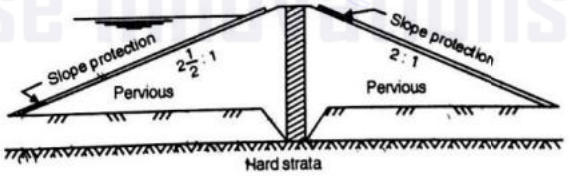

FIG. 10.43

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(b) *Foundation pervious to moderate depth, after which impervious stratum is available*. The central thin core, is taken up to the impervious stratum as shown in Fig. 10.43.

(c) *Foundation pervious to a great depth.* In this case, a thin central core is provided along with an upstream blanket of clayey silt (Fig. 10.44).

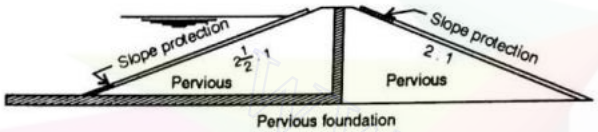

FIG. 10.44.

CASE 3. Only Silty Clay is Available

(a) *Foundation impervious to a great depth*. The whole dam will be homogeneous, made of silty clay.

However, a horizontal drainage filter will be provided upto a great distance inside the body of the dam (Fig. 10.45).

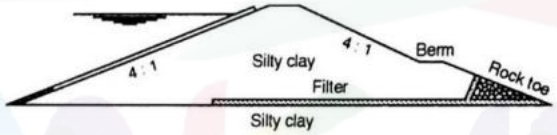

FIG. 10.45.

(b) *Pervious Foundation*. If the foundation is pervious and impervious material is available the whole dam may be made homogeneous, consisting of silty clay with a provision of sheet pile cutoff and a long horizontal filter (Fig. 10.46).

FIG. 10.46.

10.13. SLOPE PROTECTION

The upstream slope should be protected against the wave action and the downstream slope should be protected against rain. In some instances, provision must be made against burrowing animals. Following are the various materials used for slope protection:

1. *Rock riprap* (*a*) Dry dumped (*b*) Hand packed

2. Concrete pavement
3. Bituminous pavement
4. Precast concrete blocks.

One of the most comprehensive and reliable collection of data on slope protection performance was gathered by the U.S. Corps of Engineers during 1946–47 in a survey of approximately 100 dams in United States. The survey found that :

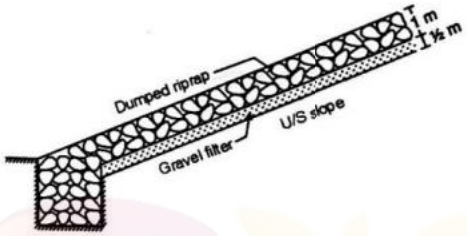

FIG. 10.47. UPSTREAM SLOPE PROTECTION

1. Dumped riprap failed in 5 % of the cases where used, failures being attributed to improper size of stones.

2. Hand placed riprap failed in 30 % of the cases where used, due to the usual method of single course construction.

3. Concrete pavements failed in 30 % of the cases where used, due generally to the inherent deficiencies of this type of construction.

This shows that the layers of dumped rock riprap are more successful than equivalent thickness of carefully hand-packed rock. The efficiency of dumped rock riprap depends upon the following characteristics :

1. Quality of the rock
2. Weight or size of the individual pieces
3. Thickness of riprap
4. Shape of the stones or rock fragments
5. Slope of the embankments
6. Stability and effectiveness of the filter (*i.e.*, gravel layer) on which riprap is placed.

The U.S. Bureau of Reclamation has found a thickness of about 1 m of dumped riprap, placed on 30 to 50 cm of gravel filter to be quite satisfactory for major dams.

A layer or blanket of graded gravel should be provided underneath the riprap when the compacted material of underlying earth-fill is of such gradation that there is danger that fines may be washed out through the voids in the riprap by wave action.

10.14. SEEPAGE CONTROL MEASURES

The seepage control measures are necessitated to prevent adverse effects of water percolating through embankment and its fondation. The following devices are used for seepage control through earth dam :

(A) *Embankment seepage control*

1. Toe filter
2. Horizontal drainage filter

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3. Protective filter d/s of the toe
4. D/s coarse section (embankment zoning)
5. Chimney drain extending upward into embankments.

(B) Foundation seepage control

6. Impervious cutoff
7. Upstream impervious blanket
8. D/s seepage berms
9. Drainage trenches
10. Relief wells.

A brief description of the these measures will now follow.

1. Toe filter. Rock toe keeps the phreatic line well within the section and also facilitates drainage. Its height is generally kept equal to 30 to 40 % of reservoir head. The gradation of various materials and layers should satisfy filter criteria.

FIG. 10.48. DRAINAGE FILTERS.

2. Horizontal drainage filter. The horizontal drainage filter may extend from 25 to 100 % of the distance from downstream toe to the centre line of the dam. The horizontal filter serves the following purposes : (*i*) It keeps the phreatic line well within the embankment, (*ii*) It gives greater leakage because of shorter seepage path, (*iii*) It also provides drainage for foundation, (*iv*) It accelerates consolidation.

Where a high degree of stratification of the embankment may occur, the rock-fill toe has an advantage over drainage blanket in that it intercepts more layers of the embankment.

3. Filter downstream of the toe. The provision of such filter [Fig. 10.49 (*a*)] provides additional weight and thus makes the upward flow more safe.

4. Downstream coarse section. This also intercepts the flow through the embankment, and makes the d/s slope safe againt piping. It is also an earthquake resistant measure.

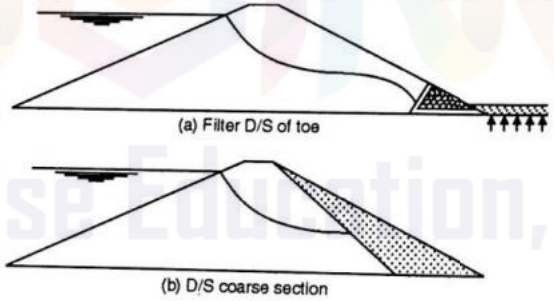

FIG. 10.49

5. Chimney drain. When there is high degree of embankment stratification, the horizontal permeability is greater than the vertical, resulting in greater horizontal spread of seepage. A correctly built vertical drain can completely intercept embankment seepage, regardless of the degree of stratification of the embankment. It is also earthquake resistant.

6. Impervious cutoffs. Whenever economically possible, the seepage should be cutoff. However, no cutoff is absolutely impervious and the reduction of seepage is a relative matter. The various cutoffs may consist of : (1) Cutoff trenches, (2) Grout curtains, (3) Sheet pile cutoffs. It may be impossible or extremely expensive to continue the cutoff to an impervious stratum. In such a case, only *partial cutoff* is provided. For homogeneous foundation material, the effectiveness of partial cutoff is limited, since 90 % depth of cutoff reduces the discharge only by about 64 % and 50 % depth by 25 %.

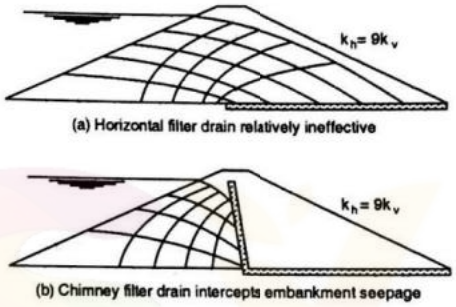

FIG.10.50. EFFECTIVENESS OF CHIMNEY DRAIN.

7. Downstream Seepage berms (Fig. 10.51). Berms can be used to control seepage efficiently where the d/s top stratum is relatively thin and uniform, or where no d/s top stratum is present.

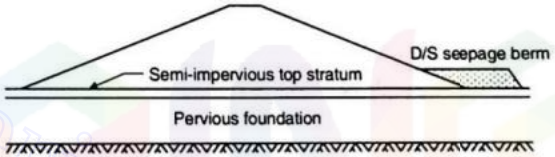

FIG. 10.51. D/S SEEPAGE BERMS.

They serve two purposes :

(*i*) They provide additional weight to resist uplift pressure beneath the top stratum, and

(*ii*) They afford some protection against possible sloughing of the d/s slope of the dam as a result of seepage.

8. Drainage trenches (Fig. 10.52). They are provided when top stratum is thin and pervious stratum is also shallow.

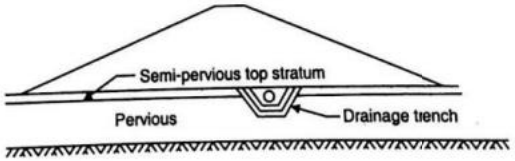

FIG. 10.52 DRAINAGE TRENCH

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9. Relief wells (Fig. 10.53). The preliminary purpose of relief well is to reduce the sub-stratum uplift pressure d/s of the dam which otherwise would cause formation of sand boils and possibly sub-surface piping. They intercept the seepage and control the outlet for seepage. They were first used by U.S. Corps of Engineers.

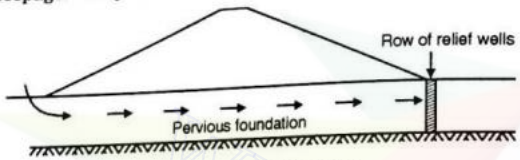

FIG. 10.53. RELIEF WELL.

10. Upstream impervious blanket. Impervious upstream or riverside blankets overlaying a pervious foundation are effective in reducing the quantity of seepage. They also, to some extent, reduce uplift pressure and escape gradients downstream of land side. A part of reservoir head is dissipated through the blanket. The length of the blanket can be determined by the Benett's analysis.

Benett's analysis

Benett gave the mathematical solution for the performance of the u/s impervious blanket, shown in Fig. 10.54.

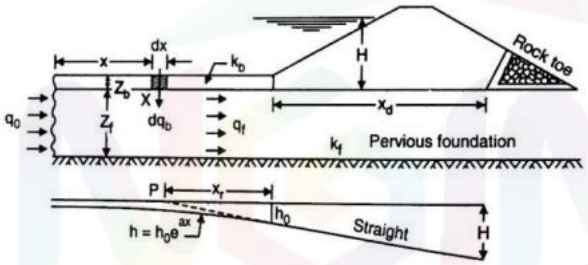

FIG. 10.54. BENETT'S ANALYSIS.

At any point X, under the blanket, the horizontal flow through the foundation (q_f) is equal to the flow through the blanket itself (q_b) upstream from the point plus the inflow q_o under the upstream end of the blanket.

Thus $$q_f = q_0 + \int_0^x dq_b \qquad ....(1)$$

where dq_b is the vertical flow through a small element of the blanket of width dx and thickness Z_b. The value of dq_b varies directly with the head loss h at X, and inversely with the blanket thickness Z_b. If k_b is the permeability coefficient of the blanket, we have

$$dq_b = k.\,i.\,A = k_b \frac{h}{Z_b} dx \qquad ....(2)$$

Substituting in (1), and differentiating with respect to x, we get

$$\frac{dq_f}{dx} = \frac{dq_0}{dx} + \frac{d}{dx}\left[\int_0^x k_b \frac{h}{Z_b} dx\right]$$

But $\frac{dq_0}{dx} = 0$, since q_0 is independent of x.

Hence $$\frac{dq_f}{dx} = \frac{k_b h}{Z_b} \qquad ...(3)$$

Again $q_f = k_f \frac{dh}{dx} Z_f$ for the foundation

$$\therefore \quad \frac{dq_f}{dx} = k_f Z_f \frac{d^2h}{dx^2} \qquad ....(4)$$

Equating (3) and (4), we get

$$\frac{d^2h}{dx^2} = \frac{k_b h}{k_f Z_b Z_f} = a^2 h \qquad ...(10.28)$$

where $$a = \sqrt{\frac{k_b}{k_f Z_b Z_f}} = \text{constant} \qquad ...(10.29)$$

Eq. 10.28 is the differential equation for the pressure dissipation. Its solution is to be obtained in two cases : (*i*) Infinite length of the blanket, (*ii*) Finite length of the blanket.

Also, we will consider two cases of blanket thickness :

(a) Blanket of uniform thickness, and (*b*) blanket on variable thickness.

(a) BLANKET OF UNIFORM THICKNESS

For blanket of uniform thickness, Z_b is constant and hence $a = \sqrt{k_b/(k_f Z_b Z_f)}$ is constant.

(*i*) Solution for Infinite length of blanket

The solution of Eq. 10.28 for infinite length is :

$$h = h_0 e^{ax} \qquad ....(10.30)$$

where h_0 = total head lost through blanket at $x = 0$ (which is taken for convenience in this case at downstream end of the blanket)

Equivalent resistance X_r of the foundation. The resistance X_r is a measure of the efficiency of the blanket of any length x. It may be defined as the length of a prism of the foundation material, of thickness Z_f and permeability k_f, which under head loss h_f would carry flow equivalent to the flow which passes the blanket system.

Thus $$k_f Z_f \frac{dh}{dx} = k_f Z_f \frac{h}{X_r}$$

or $$\frac{dh}{dx} = \frac{h}{X_r} \qquad ...(5)$$

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But $\frac{dh}{dx} = h_o . ae^{ax}$ (from Eq. 10.30)

or $\frac{dh}{dx} = a\, h_o\, e^{ax} = a\,h$

Substituting in (5), we get $ah = \frac{h}{X_r}$

$$\therefore \quad X_r = \frac{h}{ah} = \frac{1}{a} = \sqrt{\frac{k_f Z_b Z_f}{k_b}} \qquad ...(10.31)$$

Discharge reduction

If there were no blanket,

$$q_f = k_f \frac{H}{X_d} . Z_f$$

Due to the provision of blanket

$$q_f' = k_f \frac{H}{X_r + X_d} Z_f$$

$$\therefore \quad \% \text{ reduction} = \frac{q_f - q_f'}{q_f} \times 100 = \frac{X_r}{X_r + X_d} \times 100 \qquad ...(10.32)$$

(*ii*) **Solution for finite length of blanket**

The solution of Eq. 10.28 for finite length is

$$h = h_n (e^{ax} - e^{-ax}) \qquad ...(10.33)$$

where h_n = constant = total head loss through the blanket at the d/s edge of the blanket, and is proportional to the ratio of the blanket resistance to the remainder of the system.

Differentiating Eq 10.33,

$$\frac{dh}{dx} = a\,h_n (e^{ax} + e^{-ax}) \ .$$

Now, if X_r is the equivalent resistance of the foundation, we have

$$\frac{dh}{dx} = \frac{h}{X_r}$$

$$X_r = \frac{h}{dh/dx} = \frac{h_n (e^{ax} - e^{-ax})}{ah_n (e^{ax} + e^{-ax})}$$

or

$$X_r = \frac{(e^{2ax} - 1)}{a(e^{2ax} + 1)} \qquad ....(10.34)$$

If there were no blanket, $q_f = k_f \frac{H}{X_d} Z_f$

Due to the provision of blanket

$$q_f' = k_f \frac{H}{X_r + X_d} . Z_f$$

$$\therefore \quad \% \text{ reduction is discharge} = \frac{q_f - q_f'}{q_f} \times 100 = \frac{X_r}{X_r + X_d} \times 100$$

It can be shown that for any value of a, very little benefit will be gained by extending the blanket beyound a length X_0, known as the *optimum length of blanket* given by the expression.

$$X_0 = \sqrt{2}/a. \qquad ...(10.34\,a)$$

(*ii*) **Steps for the design of blanket of uniform thickness**

(*i*) Find a by the expression $a = \sqrt{\frac{k_b}{k_f Z_b Z_f}}$

(*ii*) Find the optimum length X_o by $X_0 = \sqrt{2}/a$

(*iii*) Find X_r by the relation $X_r = \frac{1}{a}$ for infinite length

or $X_r = \frac{(e^{2ax} - 1)}{a(e^{2ax} + 1)}$ for finite length

(*iv*) Find the hydraulic gradient $i = \frac{H}{X_r + X_d}$

(*v*) Find the loss of head (or head dissipated) through the blanket by the expression

$$h_0 = \frac{X_r}{X_r + X_d} . H \qquad ...(10.35)$$

(*v*) Find percentage reduction in discharge.

The actual design procedure will, therefore, be to select various values of x and to find % reduction in discharge, and also to find the head dissipated through the blanket.

The recommended length will be the one beyond which there will be no appreciable reduction in the discharge. However, the *optimum length* of the blanket is given by $X_0 = (\sqrt{2}/a)$.

(*b*) BLANKET OF VARIABLE THICKNESS

It is seen that the blanket is more effective towards the downstream end. Hence for more *efficient* use of the blanket, we should provide increasing thickness in the down stream direction, thus making Z_b variable. Hence Eq. 10.28 can be expressed as follows :

$$\frac{d^2 h}{dx^2} = \frac{\alpha h}{Z_b} \qquad ...(10.36)$$

where $\alpha = \frac{k_b}{k_f Z_f}$ = constant ...(10.36 *a*)

while Z_b varies with x.

Considering blanket thickness to vary triangularly, we have $Z_b = S . x$ where S is the slope of blanket thickness. However the solution of Eq. 10.36 for this case is obtained in terms of a series which can not be computed conveniently. On the contrary, if we assume the blanket to be of parabolic profile with its thickness represented by the following equation :

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$$Z_b = \frac{\alpha x^2}{n(n-1)} \quad ...(10.37)$$

Then the solution of Eq. 10.36 is given by :

$$h = h_n x^n \quad ...(10.38)$$

For such a case, $$X_r = \frac{h}{(dh/dx)} = \frac{x}{n} \quad ...(10.39)$$

The head dissipated through the blanket is given by

$$h_0 = \frac{X_r}{X_r + X_d} H \quad ...(10.35)$$

The percentage reduction in discharge is given by Eq. 10.32.

Design steps

1. Compute the value of α from Eq. 10.36 (a), corresponding to the known values of k_b, k_f and Z_f.

2. Assume a *suitable value of length* of the blanket, and compute Z_b (from Eq. 10.37) and X_r (From Eq. 10.39) by taking different values of $n = 1.25, 1.5, 1.75, 2.0$ etc.

However, the length of the blanket may be assumed close to the one given by $X = \sqrt{2}/a$, by assuming *average uniform thickness* (Z_b) and taking a equal to the one given by Eq. 10.29.

3. Compute head dissipated through the blanket and reduction in discharge.

4. Repeat the above procedure for several other lengths of the blanket.

5. Select the length of the blanket which causes considerable reduction in the discharge and is also economical.

10.15. SLOPE STABILITY ANALYSIS UNDER EARTH QUAKE EFFECTS

It is recognized that an earth dam vibrates when subjected to ground motion during an earthquake requiring thereby a *dynamic analysis* of the structure for its design. Nevertheless, currently accepted design procedure is based on assumption that the portion of the dam above the rupture surface is rigid. Therefore the method given below in IS 1893-1984, which assumes additional horizontal and vertical loads on the soil mass within the rupture surface, is adopted. It is desirable, however, to carry out dynamic analysis for final design of important dams in order to estimate deformations in dams in probable future earthquakes.

Design seismic coefficient (α_h)

As per IS : 1893-1984, the value of design seismic coefficient (α_h) in the horizontal direction is given by Eq. 8.4 reproduced here.

$$\alpha_h = \beta I F_0 \frac{S_a}{g} = 2 F_0 \frac{S_a}{g} \quad ...(10.40)$$

where β = soil foundation system factor, the value of which is taken as 1.0 for dams.

I = importance factor, the value of which is taken as 2 for dams.

F_0 = seismic zone factor, the value of which for each of the four zones is given in Table 8.2.

$\frac{S_a}{g}$ = average acceleration spectra, as read from Fig. 8.4 for appropriate natural period (T) and 10% damping.

The natural (or fundamental) period (T) of vibration of an earth dam is determined from the following expression.

$$T = 2.9 H \sqrt{\rho/G} \quad ...(10.41)$$

where T = fundamental period of earth dam, is seconds

H = height of the dam above toe of the slope

ρ = mass density of the shell material

G = modulus of rigidity of the shell material.

Note : The quantity $\sqrt{G/\rho}$ is the *shear wave velocity* through the material of the dam and may be used if known instead of ρ and G.

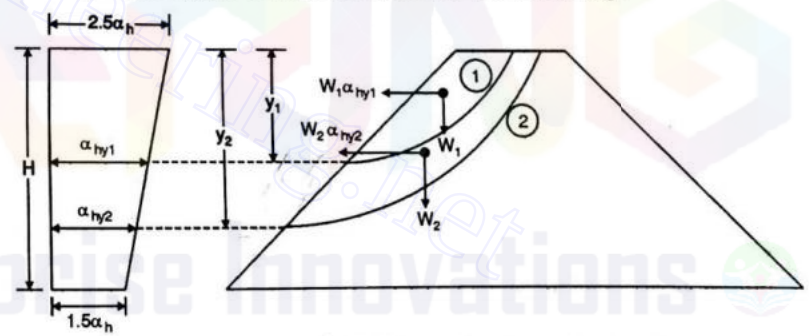
Equivalent uniform horizontal seismic coefficient and horizontal inertial force

For checking slope failure with the lowest point of the rupture surface at any depth y below the top of the dam, the value of *equivalent uniform horizontal seismic coefficient* is given by

$$\alpha_{hy} = \left(2.5 - 1.5\frac{y}{H}\right)\alpha_h \quad ...(10.42)$$

where y = depth of the lowest point of the rupture surface

H = total height of the dam above the toe of the slope



(a) Variation of equivalent uniform horizontal seismic coefficient with depth

(b) Earth dam section and assumed rupture surface

FIG. 10.55. EQUIVALENT UNIFORM HORIZONTAL SEISMIC COEFFICIENT

Fig. 10.55 (a) shows the variation of horizontal seismic coefficient with depth, with value at the top equal to $2.5\,\alpha_h$ and value at bottom as $1.5\,\alpha_h$. Thus, for assumed

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rupture surface 1 (Fig. 10.55 *b*), if y_1 is the depth of the lowest point of the rupture surface, the value of equivalent uniform horizontal seismic coefficient will be equal to α_{hy1}. Similarly, for rupture surface 2, with y_2 as the depth of the lower point, the value of uniform horizontal seismic coefficient will be α_{hy2}.

The *horizontal inertia force* acting at the centre of gravity of the *rupture wedge* will be equal to $W_1 . \alpha_{hy1}$ for rupture surface 1 and $W_2 . \alpha_{hy2}$ for rupture surface 2, as shown in Fig. 10.55 (*b*), where W_1 and W_2 are the weight of the rupture wedges acting at their centre of gravity.

Equivalent uniform vertical seismic coefficient and vertical inertia force:

The value of *equivalent uniform vertical seismic coefficient* (α_{vy}) is taken equal to half the corresponding value of equivalent uniform horizontal seismic coefficient. Thus,

$$\alpha_{vy} = 0.5\,\alpha_{hy} = \frac{1}{2}\left(2.5 - 1.5\,\frac{y}{H}\right)\alpha_h \qquad ...(10.43)$$

where y = depth of the lowest point of the rupture surface under consideration.

The vertical inertia force, acting *upwards* through the centre of gravity of the rupture wedge will be equal to $W . \alpha_{vy}$ where W is the weight of the rupture wedge.

Choice of rupture surfaces

From the elastic analysis of a typical dam section, it can be found that the crest of the dam is subjected to much larger forces than the toe, the increase in acceleration from the toe to the crest being almost parabolic in nature. This behaviour has been borne out even by actual measurements in different dams in Japan. Therefore, design of upper portions of the dam to resist larger forces become essential. This can be achieved by assuming probable slip surfaces entirely in the upper ranges only, say extending to the upper 7/8 to 3/4 range from the top. Because of larger horizontal forces, the stability will be least for such slip surfaces, specially in the absence of cohesion.

Stability of upstream slope

The stability of the upstream slope of an earth or rockfill dam shall be tested with full reservoir level with horizontal forces due to earthquake acting in the upstream direction and vertical forces due to earthquake (taken as one half of the horizontal) acting *upwards*. For preliminary design, a factor of safety of unity shall be accepted as being adequate for ensuring stability of upstream slope. The factor of safety need be tested only for failure surface which passes through the lower half of the dam.

Stability of downstream slope

The provision mentioned above for u/s slope shall also apply in determining the stability of the d/s slope except that the horizontal forces due to earthquake should be considered acting in the downstream direction.

Stability during construction stage or for reservoir empty condition

Earthquake forces shall not be normally included in stability analysis for construction stage or for reservoir empty condition. However, where construction or operating schedule requires the reservoir empty condition to exist for prolonged periods, earthquake forces may be included and may be calculated on 50 percent of the value obtained above.

Note **:** Junction between spillways and abutments shall be constructed with great care in view of the damage that may be caused by differential vibration of the dam and the spillway.

10.16. DESIGN CONSIDERATIONS IN EARTHQUAKE REGIONS

The failures of earth dams due to earthquake have already been discussed in § 10.2. In most of the cases, the designer should anticipate that the dam will crack and settle. Hence he must provide means to control leakage through the cracks and sufficient free board to prevent overtopping in spite of the settlement. The following *design details* should be provided to protect the dam against failure due to earthquake.

(1) Cohesionless graded filter downstream. If a graded filter is provided just to the d/s of the impervious core, it will protect the dam against excessive leakage and progressive piping through the cracks developed by the earthquake. When water flows through the cracks of the core and discharges into d/s graded filter, the core material will either swell and seal the crack, or the particles of the core materials in the crack will be eroded and carried to the d/s filter. The graded d/s filter, in turn, will stop the migrating particles and the crack will thus be sealed (Fig. 10.56).

(2) Choesionless graded filter upstream. When the central core, made of tough clay, develops cracks due to earthquake, an upstream graded filter with appreciable content of fine sand sizes, will prove to be very helpful. The fine sand from the filter will just flow into the cracks of the core and will partially seal them. This was one of the main design provisions for Coyote Dam constructed on the Hayward Fault (Fig. 10.56).

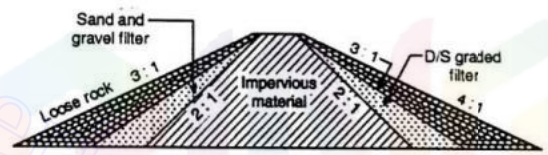

FIG. 10.56. CROSS-SECTION OF COYOTE DAM LOCATED ON THE HYWARD FAULT.

(3) Very pervious downstream zone. Provision should be made so that the discharge through the cracks developed, if any, in the core is discharged to the d/s as quickly as possible so that critically high water pressure may not develop in the d/s portion of the embankment. D/s zones of large quarried rock offers best combination of permeability and stability. Such a zone also provides the greatest security against erosive action of large quantities of water passing over the crest, if overtopped.

(4) Provision of flatter slopes near the top of the dam. During an earthquake, the top of the dam vibrates with considerably greater amplitude than the base and may be subjected to worst damage. It is, therefore, desirable to make the top of the dam thicker either by increasing the crest width or by using flatter slopes at the top. The thicker the top is made, the longer is the path of seepage through cracks which may develop, and safer will be the dam against earthquake damage.

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(5) Core treatment. The core material should be most resistant to piping. A thicker core provides greater piping resistance, and makes the dam safe when the earthquake tend to twist the dam alignment. Inferior concrete or masonry wall should be avoided.

(6) Foundation treatment. All the loose or soft soil should be excavated because a dam on soft foundation is more severely shaken.

(7) Provision of extra free board. In most of the cases, the worst earthquake damage that may be anticipated is overtopping of the dam, due to (*i*) the settement of the crest of the dam, or (*ii*) rise in the water level due to land slide into reservoir basin, or (*iii*) upward movement of reservoir bottom due to fault movement. Hence extra free board should be provided.

(8) Slope stability analysis using horizontal acceleration forces. While testing the stability of slope, it has become common practice to use horizontal acceleration forces in addition to the other usual forces. Depending upon the location of the area with respect to various seismic zone, the horizontal force is assigned a value of 1/2 to 1/5 of the weight of the sliding mass..

10.17. COMPACTION OF ROLLED FILL DAM

The rapid increase in knowledge of soil mechanics since the year 1925 has resulted in substantial progress towards understanding the factors involved in transforming loose earth into a structural material. The development of huge economical earth-moving machines has increased the placing rate of the earth fill many times thereby intensifying the problem of quality control. Construction control is obtained by inspection, testing and reports. Proper control of earthwork requires the use of laboratory facilities.

After having decided the quarry sites for various materials to be used in a dam, the next step it to determine, by laboratory tests, the relationship between the water content and dry density obtained on compaction. *The maximum dry density* and the *optimum water content* for each soil can be determined in the laboratory either by Proctor's test or by 'Jodhpur-mini-compactor test'. While compacting these soils in the field, the same dry density should be achieved.

The various earth-moving machinery used in embakment construction are listed below :

1. Excavating equipment, such as power shovels, draglines, scrapers etc.
2. Hauling equipment, such as delivery trucks, scrapers, trucks and belt conveyors.
3. Spreading equipment such as bulldozers, graders etc.
4. Rollers and special compacting equipment.
5. Rock seperation equipment.
6. Discs, harrows and ploughs.
7. Watering equipment.

The various compaction equipment used in the field can be grouped under three classes :

(*i*) Rollers,

(*ii*) Rammers, and

(*iii*) Vibrators.

The rolling equipment (rollers) can further be classified into five types :

(*i*) Smooth wheel rollers,

(*ii*) Sheep's foot rollers,

(*iii*) Pneumatic tyred rollers,

(*iv*) Lorries, and

(*v*) Track laying tractors.

For a detailed description of these, the reader is referred to and standard text on 'Soil Mechanics' or 'Earthwork'.

10.18. ROCKFILL DAMS

A rock fill dam is an embankment which uses variable sizes of rock to provide stability, and an impervious membrane to provide water-tightness. Various materials used for the membrane are : earth, concrete, steel, asphalt and wood.

The principal parts of a rockfill dam are as follows (Fig. 10.57):

1. Upstream impervious membrane resting on the upstream rock cushion.
2. Upstream rock-cushion constructed on carefully laid stone bonding with main rockfill on the downstream.
3. Main rockfill on the downstream.
4. Upstream cutoff.

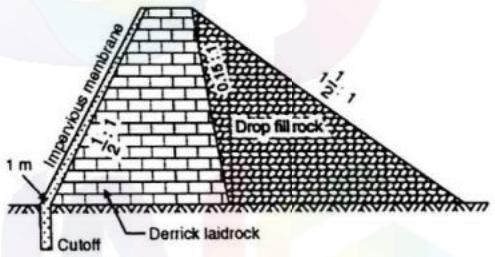

FIG. 10.57 ROCKFILL DAM.

Sometimes, a horizontal pervious zone is provided, instead of rock cushioning, at the u/s to support the impervious membrane (Fig. 10.58).

The watertight membrane should be constructed on the u/s slope so that it can be inspected when the reservoir is drawn down, and repairs made as necessary. A cutoff wall should be provided along the contact of the impervious membrane with the foundation and abutments at the u/s toe of the dam to prevent seepage under the dam. The placement of the rockfill zone is one of the most important operations in the construction of a rockfill dam, as it is essential to minimise total settlement and the possibility of damage to the impervious membrane. The rock cushioning provided at the u/s of the main rockfill acts as a cushioning in stabilising the u/s face and equalising the settlement.

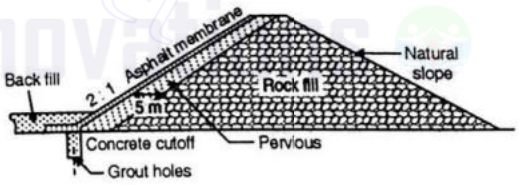

FIG. 10.58. ROCKFILL DAM.

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ILLUSTRATIVE EXAMPLES

Example 10.1. *For the earth dam of homogeneous section with a horizontal filter as shown in Fig. 10.59, draw the top flow line. If the coefficient of permeability of the soil material used in the dam is 5×10^{-4} cm/sec, find the seepage flow per unit length of the dam.*

Solution : The top flow line which is a parabola can either be drawn graphically or by calculation of its ordinates. The procedure has been explained in § 10.6.

Taking the focus F as the origin, the equation of parabola is given by

$$\sqrt{x^2+y^2} = x + s$$

where s = distance of directrix from the origin and x, y are the co-ordinates of any point on the parabola.

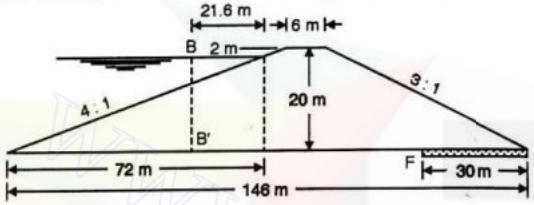

FIG. 10.59.

The parabola cuts the reservoir water surface at point B, such that

$$FB' = 146 - 30 - 72 + 72 \times 0.3 = 65.6 \text{ m} = x$$

and $$BB' = 18 \text{ m} = y$$

Then $$s = \sqrt{(65.6)^2 + 18^2} - 65.6 = 68.0247 - 65.60 = 2.4247 \triangleq \mathbf{2.425 \text{ m}}$$

Equation of parabola, therefore, is

$$y = \sqrt{2sx + s^2} = \sqrt{4.849\,x + 5.879}$$

The co-ordinates of parabola are tabulated below :

x	0	10	20	30	40	50	60	65.6
y	2.425	7.374	10.132	12.303	14.137	15.759	17.229	18

The top flow line can, therefore, be drawn by the above table. A correction as explanied in § 10.6 at entry and exit should be made.

Seepage flow per unit length of dam= $k.s.$

$$= 5 \times 10^{-4} \times 2.425 \times 100 \text{ cm}^3/\text{sec/cm} = 12.125 \times 10^{-2} \text{ cm}^3/\text{sec/cm}$$

$$= \mathbf{12.125 \times 10^{-6}} \text{ cumecs per metre length of the dam.}$$

Example 10.2. *For a homogeneous earth dam 52 m high, and 2 m free board, a flow net was constructed and following results were obtained.*

Number of potential drops = 25

Number of flow channels = 4

The dam has a horizontal filter of 40 m length at its downstream end. Calculate the discharge per metre length of the dam if the coefficient of permeability of the dam is 3×10^{-3} cm/sec.

Solution. The discharge per unit length is given by

$$q = kh\frac{N_f}{N_d}$$

Here h = water depth = 52 − 2 = 50 m ; $k = 3 \times 10^{-3}$ cm/sec = 3×10^{-5} m/sec

$N_f = 4$; $N_d = 25$

$$q = 3 \times 10^{-5} \times 50 \times 4/25 = 24 \times 10^{-5} \text{ m}^3/\text{sec/m}$$

$$= \mathbf{0.00024} \quad \textbf{cumec/metre length.}$$

Example 10.3. *In order to find the factor of safety of d/s slope during steady seepage, the section of the dam was drawn to scale of 1 cm = 4 m, and the following results obtained on a critical slip circle :*

Area of N-rectangle = 14.4 sq.cm

Area of T rectangle = 6.4 sq. cm

Area of U-rectangle = 4.9 sq. cm

Length of arc = 12.6 cm.

Laboratory tests have furnished values of 26° for effective angle and 19.5 kN/m² for cohesion. Unit weight of soil = 19 kN/m³. Determine the factor of safety of the slope.

Solution. Let the scale be 1 cm = x m. (Here x = 4)

Consider 1 m length of the dam.

$$\Sigma N = A_N (x)^2 \gamma = A_N (16) \times 19 \text{ kN} = 14.4 \times 16 \times 19 = 4377.6 \text{ kN}$$

$$\Sigma T = A_T \times 16 \times \gamma = 6.4 \times 16 \times 19 = 1945.6 \text{ kN}$$

$$\Sigma U = A_U \times 16 \times \gamma_w = A_U \times 16 \times 9.81 = 4.9 \times 16 \times 9.81 = 769.1 \text{ kN}$$

Length of arc $= \hat{L} = 12.6 \times 4 = 50.4$ m

$$\therefore \quad F = \frac{c'\hat{L} + \Sigma(N-U)\tan\phi'}{\Sigma T}$$

or $$F = \frac{(19.5 \times 50.4) + (4377.6 - 769.1)\tan 26°}{1945.6} = \mathbf{1.41}$$

Example 10.4. *A homogeneous earth dam has the following data :*

(a) Dimensions

(i) *Height of the dam* = 20 m

(ii) *Top width of dam* = 6 m

(iii) *U/S slope* = 4 : 1

(iv) *D/S slope* = 3 : 1

(v) *Length of d/s horizontal filter* = *Height of dam*

(vi) *Free board* = 2 m.

(b) Properties of the material of the dam

(i) *Dry density* = 17.27 kN/m³

(ii) *Optimum water content* = 15 %

(iii) *Saturated density* = 21.19 kN/m³

(iv) *Average unit weight under steady seepage* = 19.62 kN/m³

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(v) *Average angle of internal friction* = 26°

(vi) *Average cohesion* = 19.13 kN/m^2

(vii) *Coefficient of permeability* = 5×10^{-4} cm/sec.

(c) Properties of the foundation material

(i) *Average unit weight* = 17.27 kN/m^3

(ii) *Average cohesion* = 47.87 kN/m^2

(iii) *Average angle of internal friction* = 8°

(iv) *Coefficient of permeability : negligible.*

The foundation material consists of 6 m thick layer of clay.

Check the section for

(i) *Stability of d/s slope against steady seepage*

(ii) *Sloughing of u/s slope against sudden drawdown*

(iii) *Stability of the foundation against shear*

(iv) *Seepage through the body of dam.*

Permissible factor of safety of slope = 1.5

Permissible seepage = 8×10^{-6} cumecs.

Solution. Fig. 10.60 shows the section of the dam with phreatic line drawn by the method explained earlier.

(a) Stability of d/s Slope During Steady Seepage

In order to check stability of the d/s slope, Fellinious line is first drawn, and a trial centre is taken on it. Fig. 10.60 shows one such centre, with a slip circle passing through the toe. The sliding mass has been divided into strips each of 6 m width The pore-pressure diagram has been drawn by the method explained in § 10.8. From Fig. 10.60, we obtain the following :

Area of N-diagram = 302.4 m^2

$$\therefore \quad \Sigma N = 302.4 \times 21.19 = 6407.86 \text{ kN}$$

Area of T-diagram = 91.2 m^2

$$\therefore \quad \Sigma T = 91.2 \times 21.19 = 1932.53 \text{ kN}$$

Area of U-diagram = 98.4 m^2

$$\Sigma U = 98.4 \times 9.81 = 965.3 \text{ kN}$$

Length of the arc = 60 m

Hence
$$\text{F.S.} = \frac{c\hat{L} + \Sigma(N - U)\tan\phi}{\Sigma T}$$

$$= \frac{(19.13 \times 60) + (6407.86 - 965.3) \times \tan 26°}{1932.53} = 1.97 > 1.5$$

Hence Safe.

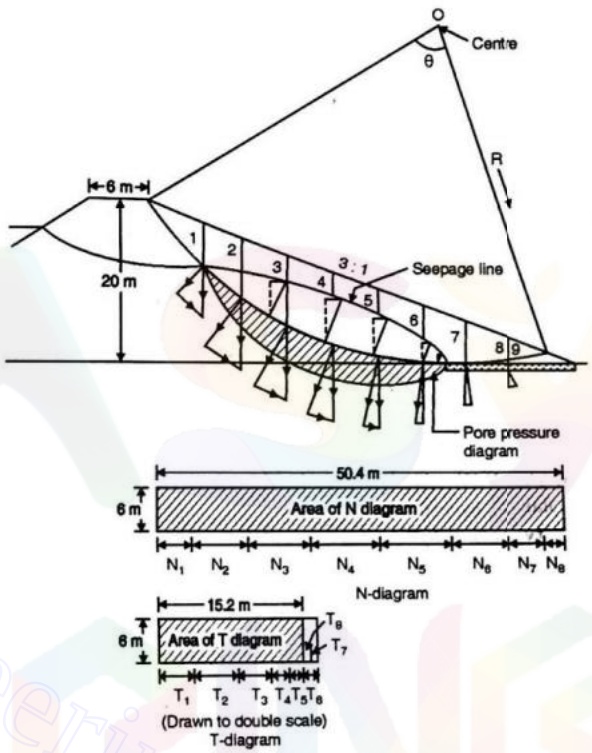

FIG. 10.60

(b) Sloughing of u/s slope against sudden drawdown

The horizontal shear force or sloughing force is given by Eq. 10.19 :

$$P = \frac{\gamma_s h^2}{2} \tan^2\left(45° - \frac{\phi}{2}\right) + \frac{\gamma_w h_1^2}{2}$$

Here $h = 20$ m and $h_1 = 15$ m (measured from Fig. 10.60).

$$\therefore \quad P = \frac{1}{2} \times 21.19 \times (20)^2 \tan^2\left(45° - \frac{26}{2}\right) + \frac{9.81 \times (15)^2}{2} = 2758.4 \text{ kN}$$

$$s_{av} = \frac{P}{b} = \frac{2758.4}{80} = 34.48 \text{ kN/m}^2.$$

$$s_{max} = 2s_{av} = 68.96 \text{ kN/m}^2$$

Shearing resistance R is given by $R = N_e \tan\phi + cb$

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Hence $N_e = \gamma' A = (21.19 - 9.81) \times \frac{1}{2} \times 80 \times 20 = 9104$ kN/m

$\therefore$ $R = 9104 \tan 26° + 19.13 \times 80 = 5970.72$ kN

$\therefore$ $\text{F.S.} = \frac{R}{P} = \frac{5970.72}{2758.4} = 2.16 > 2.$ Hence safe.

The above factor of safety is with respect to average shear.
In order to find the F.S. with respect to maximum shear, we have

$$s_{max} = 2\, s_{av} = 68.96 \text{ kN/m}^2$$

If s_r = shearing resistance at point of maximum shear (*i.e.* at B in Fig. 10.34)

$s_r = h'\gamma' \tan\phi + c = 0.6\, h\, \gamma' \tan\phi + c = 0.6\,(20)\,(21.19 - 9.81) \tan 26° + 19.13$

$= 85.73$ kN/m^2

$\therefore$ $\text{F.S.} = \frac{s_r}{s_{max}} = \frac{85.73}{68.96} = 1.24 > 1.$ Hence safe.

(*c*) **Stability Against Foundation Shear** : Refer Fig. 10.38.

The horizontal shear P is given by Eq. 10.26,

$$P = \frac{h_1^{\,2} - h_2^{\,2}}{2} \gamma_m \tan^2\left(45 - \frac{\phi_1}{2}\right)$$

where $\tan\phi_1 = \frac{\gamma_m h_1 \tan\phi + c}{\gamma_m h_1}$ where ϕ and c are the shear parameters for the foundation material.

γ_m = mean unit weight of the dam and foundation weighed in proportion to the depth of each.

Here $\phi = 8°$; $c = 47.87$ kN/m^2 ; $\gamma_m = \frac{\gamma_D (h_1 - h_2) + \gamma_F h_2}{h_1}$

where γ_D = Average unit weight of dam material = 19.62 kN/m^3

γ_F = Unit weight of foundation material = 17.27 kN/m^3.

$\therefore$ $\gamma_m = \frac{19.62\,(26 - 6) + 17.27 \times 6}{26} = 19.08$ kN/m^3

$\therefore$ $\tan\phi_1 = \frac{19.08\,(26) \tan 8° + 47.87}{19.08 \times 26} = 0.237$ or $\phi_1 = 13.335°$

$\therefore$ $P = \frac{26^2 - 6^2}{2} \times 19.08 \tan^2\left(45° - \frac{13.335°}{2}\right) = 3817$ kN

Average unit shear $s_{av} = \frac{P}{b} = \frac{3817}{80} = 47.71$ kN/m^2

Maximum unit shear $= s_{max} = 2\, s_{av} = 95.42$ kN/m^2

which occurs at $0.4 \times 80 = 32$ m from point C.

Let s_1 = unit shear strength below toe (at A) $= c + \gamma_F h_2 \tan\phi$

$= 47.87 + 17.27 \times 6 \tan 8° = 62.43$ kN/m^2.

s_2 = unit shear strength at point C $= c + \gamma_m h_1 \tan\phi$

$= 47.87 + 19.08 \times 26 \times \tan 8° = 117.59$ kN/m^2

$\therefore$ Average shear strength $= \frac{62.43 + 117.59}{2} = 90$ kN/m^2

$\therefore$ Overall factor of safety against shear $= \frac{90}{47.71} = 1.89 > 1.5$ (Safe).

Factor of safety at maximum shear :

s = unit shear strength at point of maximum shear (point B)

$= c + \gamma_{av} h \tan\phi$

where γ_{av} = mean effective unit weight at $B = \frac{\gamma_D h + \gamma_F h_2}{h + h_2}$

$= \frac{19.62\,(0.6 \times 20) + (17.27 \times 6)}{(0.6 \times 20) + 6} = 18.84$ kN/m^2

$\therefore$ $s = 47.87 + 18.84\,(12) \tan 8° = 79.64$ kN/m^2

$\therefore$ $\text{F.S.} = \frac{s}{s_{max}} = \frac{79.64}{95.42} = 0.83 < 1.$ Hence unsafe.

(*d*) **Seepage Through the Body of the Dam**

From the construction of parabola (Fig. 10.60).

$s = 2$ m (measured)

$q = k.s = (5 \times 10^{-4} \times 10^{-2}) \times 2 = 1 \times 10^{-5}$ cumecs/m length of the dam.

Example 10.5. ***Design of U/S impervious blanket*** : *For a homogeneous earth dam section shown in Fig. 10.61, design an upstream impervious blanket of 1.2 m uniform thickness. Take the coefficient of permeability of blanket material as 0.06 m/day and that of foundation soil as 72 m/day.*

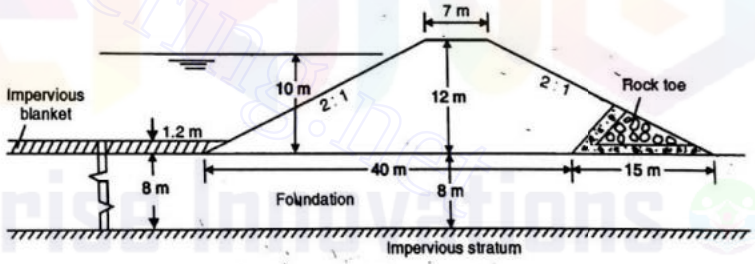

FIG. 10.61.

Solution : Given $Z_b = 1.2$ m ; $Z_f = 8$ m ; $k_b = 0.06$ m/day ; $k_f = 72$ m/day

Hence $a = \sqrt{\frac{k_b}{k_f Z_b Z_f}} = \sqrt{\frac{0.06}{72 \times 1.2 \times 8}} = \frac{1}{107.33}$

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Optimum length of blanket, $X_0 = \sqrt{2}/a = 107.33\sqrt{2} \triangleq 151.8$ m

Let us vary the length of blanket as 25 m, 50 m, 75 m, 100 m, 125 m, 151.8 m and ∞ and calculate the values of X_r, h_0 and % reduction in the discharge. The results are tabulated in Table below. Sample computations are given below for blanket length $x = 50$ m.

$$e^{2ax} = e^{(2 \times 50)/107.33} = 2.5388$$

Hence
$$X_r = \frac{e^{2ax} - 1}{a\,(e^{2ax} + 1)} = \frac{(2.5388 - 1)\,107.33}{(2.5388 + 1)} = 46.67 \text{ m}$$

$$h_0 = \frac{X_r}{X_r + X_d} H = \frac{46.67}{46.67 + 40} \times 10 = 5.38 \text{ m}$$

$$\%\text{ reduction in } q = \frac{X_r}{X_r + X_d} \times 100 = 53.8\%$$

Length of blanket; x (m)	X_r *(m)*	h_0 *(m)*	*Reduction in q (%)*
25	24.56	3.80	38.04
50	46.67	5.38	53.80
75	64.78	6.18	61.83
100	78.50	6.62	66.24
125	88.28	6.88	68.82
151.8	99.35	7.04	70.45
∞	107.33	7.29	72.85

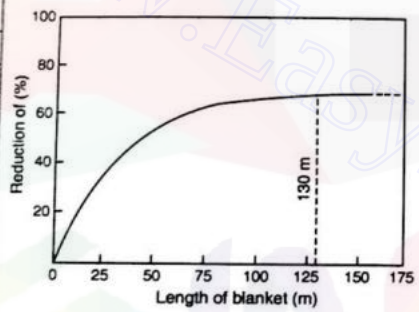

FIG. 10.62

Fig. 10.62 shows the variation of discharge reduction with length of blanket. From the above table as well as graph, we find that by providing the length of blanket beyond 130 m length (say), there would be only slight increase in the head dissipated (h_0) through the blanket as well as in the reduction of seepage discharge. ***Hence provide 1.2 m thick blanket of 130 m length***, though the optimum length of the blanket is 151.8 m.

PROBLEMS

1. Explain, with the help of a sketch, the components of a zoned embankment dam, with their functions.
2. Discuss in brief the causes of failure of earth dams.
3. What are the criteria for safe design of earth dam ?
4. Discuss the recommendations for the section of an earth dam.
5. Write a note on 'filter criteria' for earth dam.

6. Derive Laplace equation for seepage through the homogeneous mass of an earth dam. What are its methods of solution ?
7. Explain the method of plotting the flownet for seepage through anisotropic soil.
8. Explain the method of plotting phreatic line for an earth dam with horizontal filter at down stream. Also, sketch the phreatic line if there is no filter at the d/s end.
9. Derive the equation for the phreatic surface for an earth dam.
10. Derive an expression for discharge through the body of an earth dam with horizontal filter at the downstream. Also, derive an alternative expression if the flownet for the dam is known.
11. Explain the method of stability analysis of d/s slope during steady seepage.
12. Explain the method of stability analysis of u/s slope during sudden drawdown.
13. Explain the method of checking the stability of earth dam foundation against shear failure.
14. What do you understand by construction pore pressure in earth dams, and how are they determined ?
15. Sketch out sections of earth dam to suit various conditions of available material and existing foundation conditions.
16. The cross-section of an earth dam is shown in Fig. 10.63. Check the stability of foundation against shear.

Given : (*i*) Angle of internal friction for foundation clay : 6°
(*ii*) Unit cohesion of foundation clay : 45 kg/m^2
(*iii*) Average unit weight for foundation clay : 1700 kN/m^3
(*iv*) Average unit weight for dam : 20 kN/m^3.

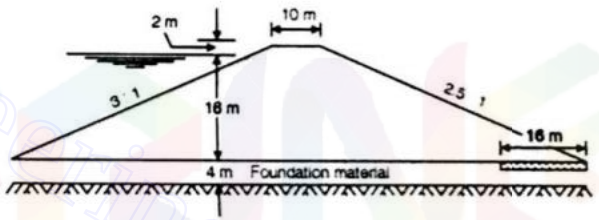

FIG. 10.63.

17. Fig. 10.64 shows a homogeneous earth dam. Calculate the seepage, per metre length, through the body of the dam. Coefficient of permeability of the dam material may be taken as 8×10^{-3} cm/sec. [Ans 28.8×10^{-5} cumec/m]

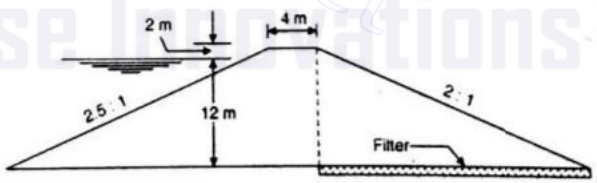

FIG. 10.64

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18. In order to find the factor of safety of d/s slope during steady seepage, the section of a homogeneous earth dam was drawn to a scale of 1 cm = 10 m and the following results obtained on a trial circle :

Area of N-rectangle = 12.15 sq.cm
Area of T-rectangle = 6.50 sq.cm
Area of U-rectangle = 4.82 sq.cm
Length of arc = 11.60 cm. The dam material has the following properties :
Effective angle of friction = 26°
Unit cohesion = 20 kN/m²
Unit weight of soil = 20 kN/m³
Determine the factor of safety of the slope. [Ans. 1.21]

19. A homogeneous earth dam section shown in Fig. 10.65 has the following data:
(a) Properties of the material of the dam ;
(i) Dry density = 18 kN/m³
(ii) Saturated density = 22 kN/m³
(iii) Average angle of friction = 30°
(iv) Average cohesion = 16 kN/m²
(b) Properties of foundation material :
(i) Average unit weight = 17 kg/m³
(ii) Average cohesion = 54 kN/m²
(iii) Average angle of internal friction = 7°
Check the section for the following :
(i) Sloughing of u/s slope during sudden drawdown.
(ii) Stability of foundation against shear.

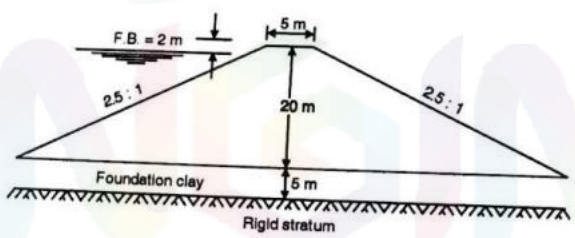

FIG. 10.65.

20. Explain, with a sketch, the component parts and their function, of a rockfill dam.

21. Explain the method of locating centre of the critical slip circle of stability analysis of the slope of an earth dam.

11

Dams-V : Spillways

11.1. INTRODUCTION

A spillway is the overflow portion of dam, over which surplus discharge flows from the reservoir to the downstream. A spillway is, therefore, called a *surplussing work*, designed to carry this flood water not required to be stored in the reservoir, safely to the river lower down. Spillways are invariably provided for all storage and detention dams. Ordinarily, this surplus water is drawn from the top of the pool (or reservoir) created by the dam. Spillways are very important structures ; many failures of the dams have been caused by improperly designed spillways or by spillways of insufficient capacity. A spillway is thus the *safety valve* for a dam.

Sufficient capacity of spillway is of paramount importance, specially in earth and rockfill dams where overtopping may be very dangerous. A spillway must have the capacity to discharge major floods without damage to the dam or any appurtenant structures, at the same time keeping the reservoir level below some predetermined maximum level. The design flood discharge required to be passed over the spillway can be determined by flood routing (Chapter 6). In addition to providing sufficient capacity, the spillway must be hydraulically and structurally adequate and must be located so that spillway does not erode or undermine the downstream of the dam.

Types of Spillways

Based on their *utility*, spillways can be of two types :

(i) Main spillway (ii) Subsidiary or emergency spillway.

The *main spillway* is the one which is called upon to work under the design floods. A subsidiary or *emergency spillway* is provided for additional safety should emergencies not contemplated by normal design assumptions arise. Under normal reservoir operation, emergency spillways are never required to function. Their functioning is required under the situations caused by an enforced shutdown of the outlet works or malfunctioing of spillway gates or when a flood greater than the design flood occurs. *The crest of an emergency spillway is therefore, placed at or above the designed maximum reservoir water surface.* Emergency spillways are provided for earth and rockfill dams only, to avoid the overtopping of the main dam embankment because of an emergency condition.

Fig. 11.1 shows the main and emergency spillways for an earth dam. Emergency spillways are often formed by lowering the crest of a dyke section below that of

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the main embankement or by utilizing saddles or depressions along the reservoir rim. The emergency spillways are thus constructed of low height earth embankments which, when overtopped during heavy floods, get washed away leaving the main dam having greater free board entirely safe. The embankment of the emergency spillway is therefore, called the *breaching section*, the top width of which is kept less than 9 m.

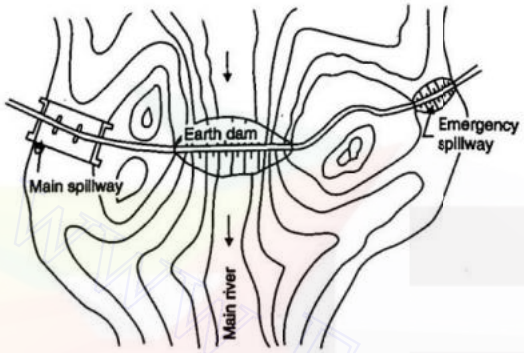

FIG. 11.1. MAIN AND EMERGENCY SPILLWAYS.

Based on their most prominent features, pertaining to either its control or to the discharge channel or to some other component, following are the *main types* of spillways :

1. Free overfall or straight drop spillway. 2. Ogee or overflow spillway.
3. Side channel spillway.
4. Chute or open channel or trough spillway. 5. Conduit or tunnel spillway
6. Drop inlet or shaft or morning glory spillway.
7. Siphon spillway.

Spillways are often referred to as *controlled or uncontrolled* depending upon whether they are gated or ungated.

11.2. STRAIGHT DROP SPILLWAY

This is the simplest type of spillway which is constructed in the form of a low height weir having downstream face either vertical or nearly vertical. Water drops *freely* from the crest, and the underside of the falling nappe is ventilated sufficiently to prevent a pulsating, fluctuating, jet. Occasionally, the crest is extended in the form of an overhanging lip (similar to that provided in notch falls) to direct the small discharge away from the face of the overfall section [Fig.11.2(*b*)].

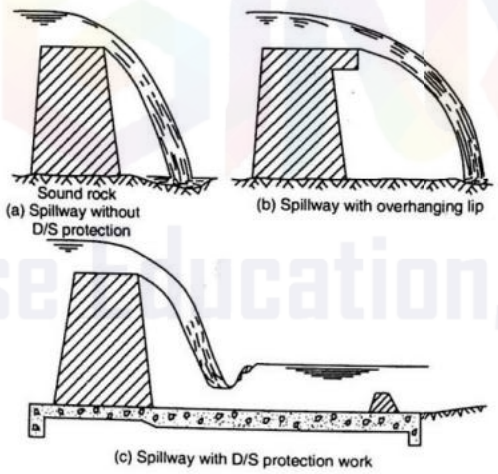

FIG. 11.2. STRAIGHT DROP SPILLWAY.

Fig. 11.2(*a*) shows a straight drop spillway constucted on sound rock. However, after sometime, the falling jet will form a deep plunge pool. Where erosion is not permissible, a low secondary dam may be constructed to create an artificial pool, or a concrete apron may be provided. If tailwater depths are sufficient, a hydraulic jump will form when the free jet falls on the flat apron. This type of spillway is not recommended for high head since the vibrations caused by the falling jet might crack or displace the structure, with danger from failure by piping or undermining.

Fig. 11.2(*c*) shows an *impact block basin* useful for low heads. The block gives reasonably good dissipation of energy for a wide range of tailwater depth. The energy is dissipated by means of turbulence caused by the impingement of the flow upon the impact block.

11.3. OGEE OR OVERFLOW SPILLWAY

This is the most common type of spillway provided on gravity dams. The profile of the spillway is *Ogee* or 'S' shaped. The oveflowing water is guided smoothly over the crest and profile of the spillway so that the overflow water does not break contact with the spillway surface. If this is not assured, a vacuum may form at the point of separation and cavitation may cocur. In addition to cavitation, vibration from the alternate making and breaking of contact between the water and face of the dam may result in serious structural damage. Hence the upper profile of the ogee is made to conform with the lower nappe of a freely falling jet of water over a sharp crested weir, when the flow rate corresponds to the maximum designed capacity of the spillway. Here, the essential difference between the straight drop spillway and the ogee or overflow spillway should be clearly noted. In the former type, the jet falls clearly away from the face of the spillway and the gap between the jet and face is kept ventilated. In the ogee or overflow spillway, the falling water is made to glide over the curved profile of the spillway. A smooth gradual reverse curvature on the downstream face of the spillway is provided. This reverse curve turns the flow on the apron of a stilling basin or into the spillway discharge channel.

High overflow spillway : Overflow or ogee spillways are classified as ***high*** and *low* depending on whether the ratio of the height (P) of spillway crest measured from the river bed to the design head (H_d) is greater than, and equal to or less than 1.33 respectively. In case of *high overflow spillways*, the velocity of approach head (H_a) which is generally of the order of 0.04 H or less, may be considered negligible.

Head : The head (H) is the distance measured vertically from the water surface (upstream of the commencement of drawdown) to the crest axis.

Design head (H_d) : The design head is that value of head for which the ogee profile is designed.

Head due to velocity of approach (H_a) : It is the velocity head given by

$$H_a = V_a^2/2g, \text{ where } V_a \text{ is the velocity of approach.}$$

Total energy head (H_e) : It is equal to the actual head plus the head due velocity of approach. Thus.

$$H_e = H + H_a$$

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If $H = H_d$, $H_{ed} = H_d + H_a$

Ogee profile : The ogee profile to be acceptable should provide maximum possible hydraulic efficiency, structural stability, and economy and also avoid the formation of objectionable sub-atmospheric pressures at the surface.

Fig. 11.3(*a*) shows the flow over a sharp crested weir. The ideal spillway [Fig.11.3(*b*)] should have the form of the underside nappe of a sharp crested weir [Fig. 11.3(*a*)] when the flow rate corresponds to the maximum design capacity of spillway. However, the head during an unanticipated flood may rise higher than the design head [Fig.11.3(*c*)] and the lower nappe of the falling jet may leave the ogee profile, thereby causing negative pressure and cavitation. On the contrary, when the head on the spillway is less than the design head, the falling jet would *adhere* to the profile of ogee, causing a positive hydrostatic pressure and reducing the discharging capacity.

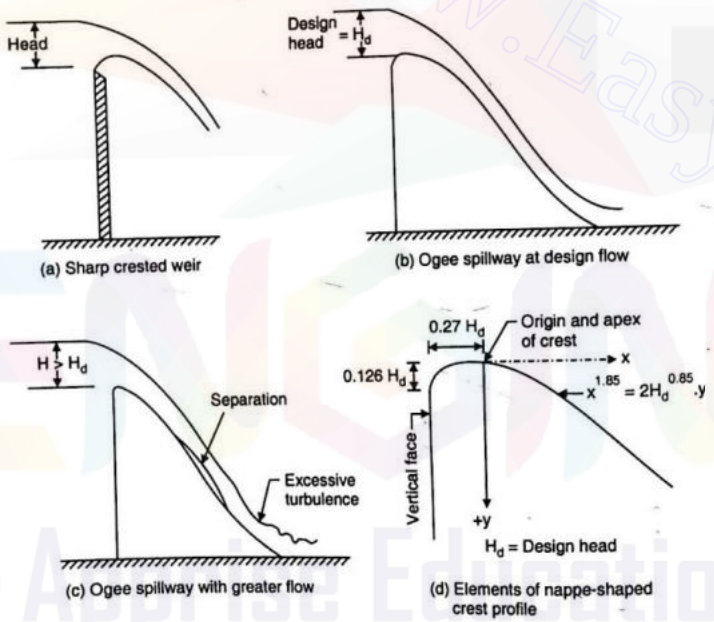

FIG. 11.3. OGEE SPILLWAY.

(*a*) Crest profile for vertical u/s Face

(*i*) *Design criteria of downstram profile*

The details of the dowstream crest profile for vertical u/s face are shown in [Fig.11.3(*d*)]. The downstream curve of the ogee has the following equation:

$$x^{1.85} = 2\,H_d^{0.85}\,.\,y \qquad ...(11.1)$$

where x and y are the co-ordinates of the crest profile measured from the apex of the crest, and H_d is the design head excluding head due to velocity of approach.

Table 11.1 gives the values of various parameters used in Eq. 11.1

TABLE 11.1. VALUES OF $x^{1.85}$ AND $H_d^{0.85}$

x	$x^{1.85}$	x	$x^{1.85}$	H_d	$2H_d^{0.85}$	H_d	$2H_d^{0.85}$	H_d	$2H_d^{0.85}$
0.10	0.014	3.00	7.633	0.5	1.110	10.5	14.758	20.5	26.063
0.15	0.030	3.50	10.151	1.0	2.000	11.0	15.354	21.0	26.602
0.20	0.051	4.00	12.996	1.5	2.823	11.5	15.945	21.5	27.140
0.25	0.077	4.50	16.160	2.0	3.605	12.0	16.532	22.0	27.674
0.30	0.108	5.00	19.638	2.5	4.358	12.5	17.116	22.5	28.209
0.35	0.143	6.00	27.515	3.0	5.088	13.0	17.696	23.0	28.741
0.40	0.184	7.00	36.596	3.5	5.801	13.5	18.273	23.5	29.271
0.45	0.228	8.00	46.851	4.0	6.498	14.0	18.847	24.0	29.799
0.50	0.277	9.00	58.257	4.5	7.182	14.5	19.417	24.5	30.327
0.60	0.389	10.00	70.795	5.0	7.855	15.0	19.985	25.0	30.352
0.70	0.520	12.00	99.194	5.5	8.518	15.5	20.580		
0.80	0.662	14.00	131.928	6.0	9.172	16.0	21.112		
0.90	0.823	16.00	168.897	6.5	9.818	16.5	21.671		
1.00	1.00	18.00	210.017	7.0	10.460	17.0	22.229		
1.20	1.401	20.00	255,215	7.5	11.087	17.5	22.783		
1.40	1.864	25.00	385.646	8.0	11.713	18.0	23.335		
1.60	2.386	30.00	540.349	8.5	12.332	18.5	23.885		
1.80	2.967	35.00	718.664	9.0	12.946	19.0	24.433		
2.00	3.605	40.00	920.049	9.5	13.555	19.5	24.978		
2.50	5.447			10.0	14.159	20.0	25.521		

The profile is designed, following Eq. 11.1, for one value of design head (H_d) only which is generally chosen to give the maximum practicable hydraulic efficiency in keeping with the operational requirements, stability and economy. Generally, operational needs dictate that the spillway be operated under other than the design head also. For lower heads of flow, the pressure on the crest will be above atmospheric and for higher heads of flow, the pressure will be less than atmospheric and may, at times, become so low as to develop cavitation. The lower the design head of the crest profile, the greater will be the discharge coefficients for full range of heads. Model tests show that the design head may be exceeded by 25 percent upto which limit no harmful cavitation is seen to develop.

(*ii*) *Design criteria of upstream crest profile*

The u/s profile should be tangential to the vertical face and should have zero slope at the crest axis to ensure that there is no discontinuity along the surface of flow. The recommended curve, given below, incorporates the co-ordinates computed in the region close to the sharp-crested weir where accurate experimental data measurements are difficult and fits the experimental data over the remaining portion.

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According to the latest analytical studies of U.S. Army, the upstream curve of the ogee shape has the following equation.

$$y = \frac{0.724(x + 0.270\,H_d)^{1.85}}{H_d^{0.85}} + 0.126\,H_d - 0.4315\,H_d^{0.375}(x + 0.27\,H_d)^{0.625} \qquad ...(11.2)$$

It should be noted here that the upstream curve at the crest should neither be made too sharp nor too broad. A broader curve at crest *supports* the lower nappe, produces positive hydrostatic pressure and thus reduces the discharging capacity. If the curve is sharp, the nappe will leave the ogee profile, thereby causing negative pressure and consequent cavitation, and increasing the coefficent of discharge. Table 11.2 gives the dimensionless co-ordinates of the u/s profile.

TABLE 11.2. DIMENSIONLESS COORDINATES OF U/S PROFILE

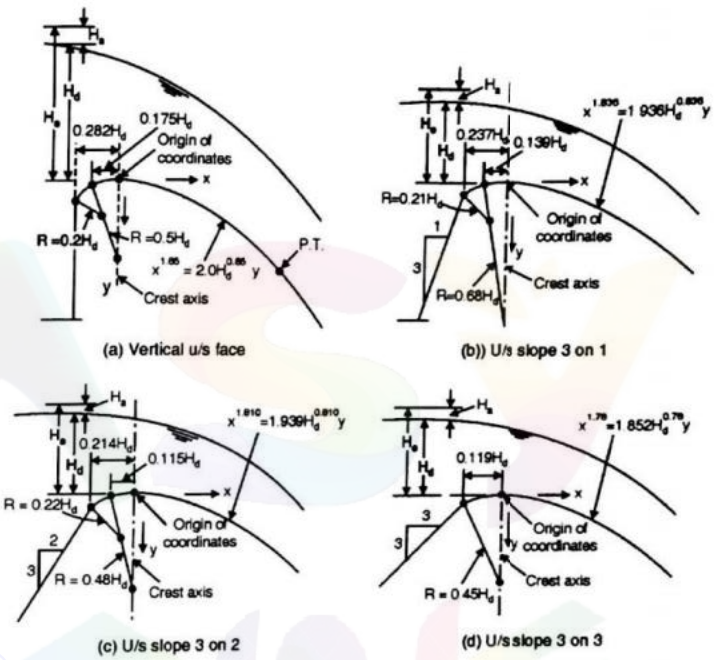
$\frac{x}{H_d}$	$\frac{y}{H_d}$	$\frac{x}{H_d}$	$\frac{y}{H_d}$
0.0000	0.0000	– 0.2000	0.0494
– 0.0200	0.0004	– 0.2100	0.0556
– 0.0400	0.0016	– 0.2200	0.0624
– 0.0600	0.0038	– 0.2300	0.0701
– 0.0800	0.0068	– 0.2400	0.0787
– 0.1000	0.0108	– 0.2450	0.0836
– 0.1200	0.0158	– 0.2500	0.0889
– 0.1400	0.0221	– 0.2550	0.0948
– 0.1600	0.0296	– 0.2600	0.1016
– 0.1700	0.0339	– 0.2650	0.1099
– 0.1800	0.0386	– 0.2680	0.1165
– 0.1900	0.0437	– 0.2700	0.1260

(b) Crest profiles for inclined u/s face

In general, the equation for the nappe-shaped profile depends upon (*i*) head over the crest, (*ii*) inclination of the upstream face of the spillway and (*iii*) height of the spillway crest above the stream bed or above the bed of approach/entrance channel, on which the approach velocity depends. Based on extensive experiments conducted by U.S.B.R., U.S. Army Corps of Engineers have developed several standard crests of overflow spillways at its waterways experiment station (W.E.S.) at Vickesburg. These shapes, given in Fig. 11.4 for both vertical as well as inclined face, are known as *WES-standard spillway crests*. IS 6934-1973 has also adopted and recommended these shapes. These shapes can be represented by the following general equation :

$$x^n = K\,H_d^{n-1}\,.\,y \qquad ...(11.3)$$

where x and y are the coordinates of the profile of the crest of the spillway with origin at the highest point of the crest, H_d is the design head (excluding the head due to velocity of approach) and K and n are constants the values of which, given in Table 11.3, depends on the slope of the upstream face of the spillway. For vertical face, $K = 2.000$ and $n = 1.850$, and hence Eq. 11.3 reduces to Eq. 11.1. For intermediate slopes, approximate values of K and n may be obtained from Fig. 11.5.



(a) Vertical u/s face (b)) U/s slope 3 on 1 (c) U/s slope 3 on 2 (d) U/s slope 3 on 3

FIG. 11.4. WES-STANDARD SPILLWAY CRESTS

TABLE. 11.3. VALUES OF *K* AND *n*

S.N.	Slope of u/s face	K	n
1.	Vertical	2.000	1.850
2.	3(V) to 1(H)	1.936	1.836
3.	3(V) to 2(H)	1.939	1.810
4.	3(V) to 3(H)	1.852	1.780

Thus, the equations for spillways shapes for the three standard slopes of u/s face are as under :

(i) For 3(V) to 1 (H) : Slope 3 on 1

$$x^{1.836} = 1.936\,H_d^{0.836}\,.\,y \qquad ...(11.3\ a)$$

(ii) For 3(V) to 2(H) : Slope 3 on 2

$$x^{1.810} = 1.939\,H_d^{0.810}\,.\,y \qquad ...(11.3\ b)$$

(iii) For 3 (V) to 3(H) : Slope 3 on 3

$$x^{1.78} = 1.852\,H_d^{0.78}\,.\,y \qquad ...(11.3\ c)$$

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In all these equations, x is taken positive towards downstream and y is taken positive in downward direction. Eq. 11.3 is applicable only for the positive values of x and y.

Tables 11.4, 11.5, and 11.6 give downstream profile data (as per Eq. 11.3) in cases of upstream slopes of 3 on 1, 3 on 2 and 3 on 3 respectively, given by IS : 6934-1973. Similarly, Table 11.7 gives the coordinates of u/s quadrants for the above three slopes.

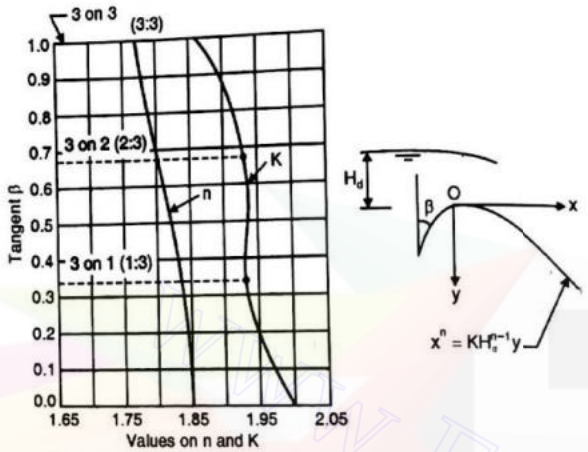

FIG. 11.5. CREST SHAPE, DOWNSTREAM PROFILE, n AND K VALUES (IS : 6934-1973)

TABLE 11.4. DOWNSTREAM PROFILE DATA FOR U/S SLOPE 3 ON 1

x	$x^{1.836}$	x	$x^{1.836}$	x	$x^{1.836}$	x	$x^{1.836}$	H_d	$1.936\ H_d^{0.836}$	H_d	$1.936\ H_d^{0.836}$
0.05	0.0041	1.90	3.2493	4.70	17.1385	17.50	191.5216	0.50	1.0845	13.50	17.0550
0.10	0.0146	2.00	3.5702	4.80	17.8139	18.00	201.6881	1.00	1.9360	14.00	17.5820
0.15	0.0307	2.10	3.9043	4.90	18.5012	18.50	212.0934	1.50	2.7172	14.50	18.1954
0.20	0.0521	2.20	4.2529	5.00	19.2003	19.00	222.7366	2.00	3.4559	15.00	18.6259
0.25	0.0785	2.30	4.6146	5.50	22.8721	19.50	233.6165	2.50	4.1647	15.50	19.1436
0.30	0.1097	2.40	4.9896	6.00	26.8340	20.00	244.7322	3.00	4.8510	16.00	19.6585
0.35	0.1455	2.50	5.3780	6.50	31.0820	20.50	256.0826	3.50	5.5176	16.50	20.1707
0.40	0.1859	2.60	5.7795	7.00	35.6123	21.00	267.6669	4.00	6.1692	17.00	20.6805
0.45	0.2308	2.70	6.1942	7.50	40.4215	21.50	279.4841	4.50	6.8076	17.50	21.1878
0.50	0.2801	2.80	6.6219	8.00	45.5065	22.00	291.5332	5.00	7.4344	18.00	21.6927
0.55	0.3337	2.90	7.0625	8.50	50.8643	22.50	316.8135	5.50	8.0510	18.50	22.1953
0.60	0.3915	3.00	7.5170	9.00	56.4923	23.00	316.3240	6.00	8.6590	19.00	22.6957
0.65	0.4534	3.10	7.9825	9.50	62.3879	23.50	329.0642	6.50	9.2577	19.50	23.1939
0.70	0.5195	3.20	8.4617	10.00	68.5489	24.00	342.0329	7.00	9.8493	20.00	23.6901
0.75	0.5897	3.30	8.9535	10.50	74.9727	24.50	355.2395	7.50	10.4341	20.50	24.1841
0.80	0.6639	3.40	9.4579	11.00	81.6577	25.00	368.6531	8.00	11.0126	21.00	24.6763
0.85	0.7420	3.50	9.9749	11.50	88.6015	25.50	382.3031	8.50	11.5851	21.50	25.1666
0.90	0.8241	3.60	10.5044	12.00	95.8025	26.00	396.1786	9.00	12.1521	22.00	25.6549
0.95	0.9010	3.70	11.0463	12.50	103.2587	26.50	410.2792	9.50	12.7140	22.50	26.1415
1.00	1.0000	3.80	11.6007	13.00	110.9686	27.00	424.6038	10.00	13.2711	23.00	26.6262
1.10	1.1912	3.90	12.1673	13.50	118.9304	27.50	439.1519	10.50	13.8235	23.50	27.1093
1.20	1.3976	4.00	12.7460	14.00	127.1425	28.00	453.9229	11.00	14.3717	24.00	27.5907
1.30	1.6188	4.10	13.3374	14.50	135.6038	28.50	468.9161	11.50	14.9159	24.50	28.0704
1.40	1.8547	4.20	13.9407	15.00	144.3124	29.00	484.1308	12.00	15.4561	25.00	28.5485
1.50	2.1053	4.30	14.5563	15.50	153.2671	29.50	499.5663	12.50	15.9927		
1.60	2.3701	4.40	15.1837	16.00	162.4667	30.00	515.224	13.00	16.5258		
1.70	2.6491	4.50	15.8233	16.50	171.9097						
1.80	2.9423	4.60	16.4749	17.00	181.5951						

TABLE 11.5. DOWNSTREAM PROFILE DATA FOR U/S SLOPE 3 ON 2

x	$x^{1.81}$	x	$x^{1.81}$	x	$x^{1.81}$	x	$x^{1.81}$	H_d	$1.939\ H_d^{0.81}$	H_d	$1.939\ H_d^{0.81}$
0.05	0.0044	1.90	3.1955	4.70	16.4626	17.50	177.7865	0.50	1.1060	13.50	15.9644
0.10	0.0155	2.00	3.5064	4.80	17.1020	18.00	187.0868	1.00	1.9390	14.00	16.4415
0.15	0.0323	2.10	3.8302	4.90	17.7524	18.50	196.5988	1.50	2.6929	14.50	16.9154
0.20	0.0543	2.20	4.1666	5.00	18.4135	19.00	206.3213	2.00	3.3995	15.00	17.3865
0.25	0.0813	2.30	4.5157	5.50	21.8805	19.50	216.2633	2.50	4.0720	15.50	17.8545
0.30	0.1131	2.40	4.8773	6.00	25.6126	20.00	226.3937	3.00	4.7211	16.00	18.3196
0.35	0.1495	2.50	5.2514	6.50	29.6056	20.50	236.7417	3.50	5.3490	16.50	18.7819
0.40	0.1904	2.60	5.6377	7.00	33.8554	21.00	247.2959	4.00	5.9600	17.00	19.2416
0.45	0.2357	2.70	6.0363	7.50	38.3584	21.50	258.0058	4.50	6.5566	17.50	19.6987
0.50	0.2852	2.80	6.4470	8.00	43.1155	22.00	269.0203	5.00	7.1407	18.00	20.1539
0.55	0.3389	2.90	6.8697	8.50	48.1114	22.50	280.1885	5.50	7.7139	18.50	20.6057
0.60	0.3967	3.00	7.3049	9.00	53.3555	23.00	291.5596	6.00	8.2771	19.00	21.0556
0.65	0.4585	3.10	7.7511	9.50	58.8401	23.50	303.1328	6.50	8.8316	19.50	21.5033
0.70	0.5244	3.20	8.2096	10.00	64.5655	24.00	314.9069	7.00	9.3779	20.00	21.9489
0.75	0.5941	3.30	8.6798	10.50	70.5265	24.50	326.8817	7.50	9.9169	20.50	22.3922
0.80	0.6677	3.40	9.1617	11.00	76.7222	25.00	339.0560	8.00	10.4491	21.00	22.8337
0.85	0.7452	3.50	9.6552	11.50	83.1501	25.50	351.4291	8.50	10.9751	21.50	23.2730
0.90	0.8264	3.60	10.1603	12.00	89.8086	26.00	364.0003	9.00	11.4951	22.00	23.7105
0.95	0.9113	3.70	10.6769	12.50	96.6957	26.50	376.7689	9.50	12.0097	22.50	24.1460
1.00	1.0000	3.80	11.2040	13.00	103.8096	27.00	389.7342	10.00	12.5192	23.00	24.5797
1.10	1.1883	3.90	11.7443	13.50	111.1486	27.50	402.8953	10.50	13.0239	23.50	25.0117
1.20	1.3910	4.00	12.2950	14.00	118.7112	28.00	416.2517	11.00	13.5240	24.00	25.4418
1.30	1.6078	4.10	12.8570	14.50	126.4958	28.50	429.8029	11.50	14.0197	24.50	25.8705
1.40	1.8386	4.20	13.4302	15.00	134.5009	29.00	443.5477	12.00	14.5116	25.00	26.2972
1.50	2.0832	4.30	14.0145	15.50	142.7251	29.50	457.4860	12.50	14.9993		
1.60	2.3413	4.40	14.6100	16.00	151.1670	30.00	471.6172	13.00	15.4836		
1.70	2.6128	4.50	15.2165	16.50	159.8254						
1.80	2.8980	4.60	15.8340	17.00	168.6990						

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TABLE 11.6. DOWNSTREAM PROFILE DATA FOR U/S SLOPE 3 ON 3

x	$x^{1.78}$	x	$x^{1.78}$	x	$x^{1.78}$	x	$x^{1.78}$	H_d	$1.852\,H_d^{0.78}$	H_d	$1.852\,H_d^{0.78}$
0.05	0.0048	1.90	3.1345	4.70	15.716	16.50	146.934	0.50	1.0785	14.50	14.9110
0.10	0.0166	2.00	3.4342	4.80	16.316	17.00	154.953	1.00	1.8520	15.00	15.3106
0.15	0.0342	2.10	3.7458	4.90	16.926	17.50	163.158	1.50	2.5409	15.50	15.7072
0.20	0.0570	2.20	4.0692	5.00	17.546	18.00	171.548	2.00	3.1810	16.00	16.1011
0.25	0.0848	2.30	4.4043	5.50	20.789	18.50	180.122	2.50	2.7847	16.50	16.4922
0.30	0.1173	2.40	4.7508	6.00	24.272	19.00	188.878	3.00	4.3631	17.00	16.8808
0.35	0.1543	2.50	5.1090	6.50	27.989	19.50	197.816	3.50	4.9205	17.50	17.2667
0.40	0.1957	2.60	5.4782	7.00	31.935	20.00	206.935	4.00	5.4606	18.00	17.6503
0.45	0.2413	2.70	5.8590	7.50	36.109	20.50	216.233	4.50	5.9860	18.50	18.0316
0.50	0.2912	2.80	6.2509	8.00	40.504	21.00	225.710	5.00	6.4988	19.00	18.4105
0.55	0.3450	2.90	6.6538	8.50	42.120	21.50	235.364	5.50	7.0004	19.50	18.7874
0.60	0.2490	3.00	7.0676	9.00	49.952	22.00	245.195	6.00	7.4921	20.00	19.1621
0.65	0.4645	3.10	7.4924	9.50	54.997	22.50	255.202	6.50	7.9747	20.50	19.5349
0.70	0.5300	3.20	7.9279	10.00	60.256	23.00	265.384	7.00	8.4492	21.00	19.9051
0.75	0.5992	3.30	8.3745	10.50	65.723	23.50	275.741	7.50	8.9164	21.50	20.2735
0.80	0.6722	3.40	8.8314	11.00	71.396	24.00	286.270	8.00	9.3766	22.00	20.6411
0.85	0.7488	3.50	9.2991	11.50	77.097	24.50	296.972	8.50	9.8308	22.50	21.0059
0.90	0.8290	3.60	9.7771	12.00	83.358	25.00	307.846	9.00	10.2789	23.00	21.3693
0.95	0.9127	3.70	10.266	12.50	89.640	25.50	318.890	9.50	10.7218	23.50	21.7308
1.00	1.0000	3.80	10.765	13.00	96.120	26.00	330.105	10.00	11.1594	24.00	22.0905
1.10	1.1849	3.90	11.274	13.50	102.800	26.50	341.490	10.50	11.15922	24.50	22.4386
1.20	1.3334	4.00	11.794	14.00	109.675	27.00	353.043	11.00	12.0206	25.00	22.8051
1.30	1.5952	4.10	12.324	14.50	116.744	27.50	364.764	11.50	12.4447		
1.40	1.8202	4.20	12.864	15.00	124.006	28.00	376.653	12.00	12.8647		
1.50	2.0580	4.30	13.415	15.50	131.459	28.50	388.708	12.50	13.2810		
1.60	2.3085	4.40	13.975	16.00	139.102	29.00	400.930	13.00	13.6935		
1.70	2.5716	4.50	14.545			29.50	413.317	13.50	14.1026		
1.80	2.8470	4.60	15.125			30.00	425.869	14.00	14.5084		

TABLE 11.7. COORDINATES OF UPSTREAM QUADRANT FOR U/S SLOPES 3 ON 1, 3 ON 2 AND 3 ON 3.

u/s slope 3 on 1		*u/s slope 3 on 2*		*u/s slope 3 on 3*	
$\frac{x}{H_d}$	$\frac{y}{H_d}$	$\frac{x}{H_d}$	$\frac{y}{H_d}$	$\frac{x}{H_d}$	$\frac{y}{H_d}$
0.000	0.0000	0.0000	0.0000	0.000	0.0000
– 0.020	0.0004	– 0.0200	0.0004	– 0.020	0.0004
– 0.040	0.0016	– 0.0400	0.0016	– 0.010	0.0016
– 0.060	0.0037	– 0.0600	0.0036	0.060	0.0036
– 0.080	0.0067	– 0.0800	0.0066	– 0.080	0.0065
– 0.100	0.0106	– 0.1000	0.0104	– 0.100	0.0103
– 0.120	0.0156	– 0.1200	0.0153	– 0.120	0.0150
– 0.160	0.0291	– 0.1600	0.0283	– 0.160	0.0275
– 0.170	0.0330	– 0.1700	0.0365	– 0.170	0.0313
– 0.180	0.0376	– 0.1900	0.0412	– 0.180	0.0354
– 0.190	0.0425	– 0.2000	0.0464	– 0.190	0.0399
– 0.200	0.0480	– 0.2100	0.0554	– 0.200	0.0450
– 0.210	0.0550				
– 0.220	0.0650				
– 0.230	0.0800				

Further details of downstream profile

Since Eq. 11.3 is applicable only for the positive values of x and y, it gives the crest shape downstream from the vertex (or the origin of coordinates. The curved profile of the crest is continued till it meets tangentially the straight sloping surface of the downstream face of the overflow dam. The location of the *point of tangency* depends upon the slope of the d/s face, which usually varies from 0.7:1 to 0.8:1, based on stability requirements. At the end of the straight sloping face of the spillway, a curved circular surface, called *bucket* is provided to create a smooth transition of flow from the spillway surface to the outlet channel or the river on the downstream side. The bucket also serves the purpose of dissipating the energy as well as prevention of scour at the downstream side, as discussed in §11.10. The radius R (in m) of the bucket is obtained from the following *approximate* expression :

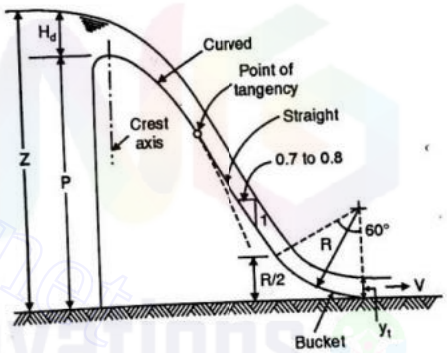

FIG. 11.6. PROVISION OF BUCKET AT D/S TOE

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$$R = (10)^a \quad ...(11.4)$$

where

$$a = \frac{V + 6.4\,H_d + 4.88}{3.6\,H_d + 19.52} \quad ...(11.4\ a)$$

and V = velocity of flow at the toe of the spillway (m/s), given by

$$V = \sqrt{2g\,(Z + H_a - y_t)} \quad ...(11.4\ a)$$

Z = fall or vertical distance from u/s reservoir level to the floor level at the d/s toe (Fig. 11.6)

y_t = depth of water at the toe.

H_d = design head, exclusive of velocity of approach.

Generally, $R = P/4$ is found to be satisfactory, where P is the height of spillway crest above approach channel or river bed.

Practical profile of spillway

When the ogee profile, given by Eq. 11.1 or 11.3 is plotted over the triangular profile of the non-overflow section of the gravity dam, it is found (Fig. 11.7 *a*) that the d/s curved profile of the spillway goes beyond the straight downstream face of the dam, thus requiring the thickening of the d/s face to accommodate the ogee profile. However, this thickening (or extra concrete) can be saved by shifting the curve of the nappe towards the upstream direction till the ogee curve becomes tangential to the down stream face of the dam, as shown in Fig. 11.7 (*b*).

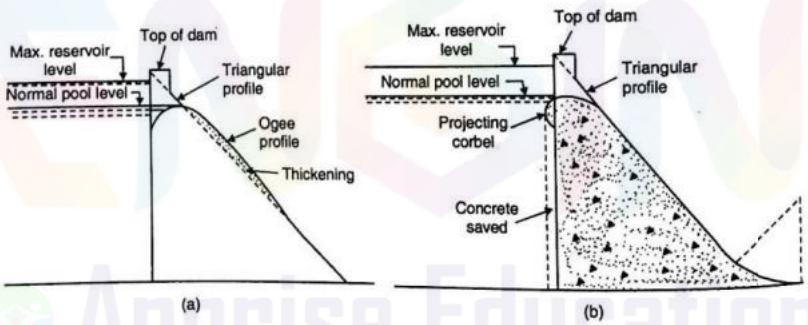

FIG. 11.7. PRACTICAL PROFILE OF SPILLWAY.

Such shifting of the ogee profile towards u/s side will give rise to a projecting portion known as *corbel* at the u/s side of the dam. The portion below the corbel need not be concreted, thus giving rise to saving in concrete, as shown in Fig. 11.7 (*b*). However, the provision of corbel will interfere with the gate operation.

Overflow dams with crest offsets and risers

Whenever structural requirements permit, removal of some mass from the upstream face, leading to *offsets* and *risers* results in economy (Fig. 11.8). The maximum permitted projection (or offset S) from the crest line is equal to $0.315\,H_d$ while the maximum depth of bulging is equal to $0.25\,H_d$. Fig. 11.8 (*a*) shows vertical faced overhang; the vertical depth M (called *riser* of the crest) should be equal to or greater than $0.5\,H_d$. Flow conditions would be extremely unstable for M/H_d ratio less than 0.53; hence crests with M/H_d ratio less than 0.5 are not recommended. The crest shape given by Eq. 11.2 is applicable to over hanging crest having M/H_d ratio greater than 0.5. Also, the ratio M/N should not be less than 0.5, though it can have a zero value. For M/N ratio between 0 to 0.5, the flow conditions become extremely unstable.

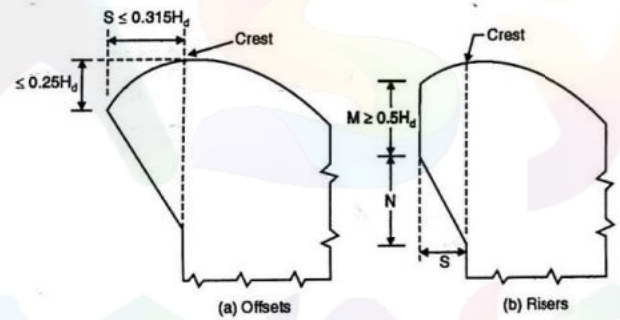

FIG. 11.8. PROVISION OF OFFSETS AND RISERS

Discharge equation. The discharge over an ogee shaped spillway is given by

$$Q = C_d\,L_e\,H_e^{3/2} \quad ...(11.5)$$

where Q = discharge.

C_d = a variable coefficient of discharge which is influenced by several factors such as (*i*) ratio of actual total head with design total head (*ii*) depth of approach (*iii*) relation of actual crest shape to the ideal nappe shape (*iv*) upstream face slope, (*v*) downstream apron interference and (*vi*) down stream submergence. Its value varies from 1.6 to 2.28.

L_e = effective length of crest.

H_e = total head on the crest, including velocity of approach head = $H + H_a$

Factors affecting coefficient of discharge (C_d)

Various factors affecting the coefficient of discharge (C_d) have already been mentioned above. Let us elaborate some of these factors.

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(i) Ratio of actual total head to the design total head

Fig. 11.9 gives the coefficient of discharge C_d, as a function of H/H_d for the crest profile given for vertical upstream face, as given by IS : 6934-1973. When $H = H_d$, the value of $C_d = 2.2$ for high overflow spillway. Prototype tests for the low-head ranges and model tests for values of H/H_d above 0.4 form the basis. The coefficient of discharge for inclined upstream face spillways will not differ significantly from those given in Fig. 11.9.

(ii) Depth of approach channel below spillway crest

The height (P) of the spillway crest above stream bed or bed of approach channel affects the velocity of approach (V_a), which in turn affects the coefficient of discharge. Fig. 11.10 shows the variation of C_d with P/H_{de} ratio. When P *increases*, the velocity of approach (V_a) decreases, but the coefficient of discharge *increases*. From model tests on spillways, it is known that when $P \geq 1.33\, H_d$, the effect of velocity of approach is negligible.

(iii) Downstream submergence (Fig. 11.11 a)

The coefficient of discharge reduces due to down stream submergence. Fig. 11.11 (a) shows the variation of C_s/C_d ratio with the degree of downstream submergence in terms of h/H ratio, where C_s is the coefficient of submerged discharge and h is the depth of submergence. The discharge in such case is given by

$$Q = C_s L_e H_e^{3/2}. \qquad ...(11.5\ a)$$

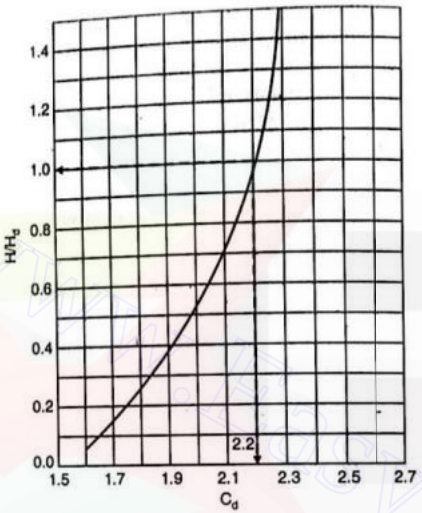

FIG. 11.9. VARIATION OF C_d WITH H/H_d

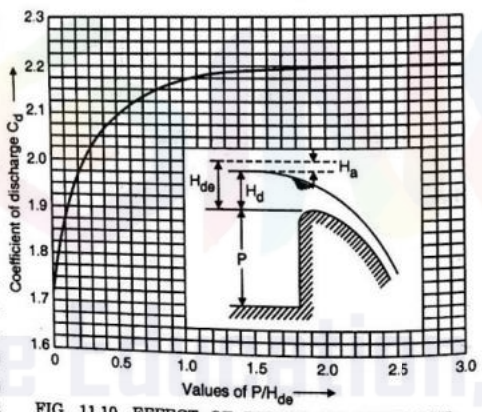

FIG. 11.10. EFFECT OF DEPTH OF APPROACH CHANNEL BELOW SPILLWAY CREST

It would be noticed that even for a degree of submergence of as much as 60%, the reduction of C_d is only 5 percent.

(iv) D/S apron interference :

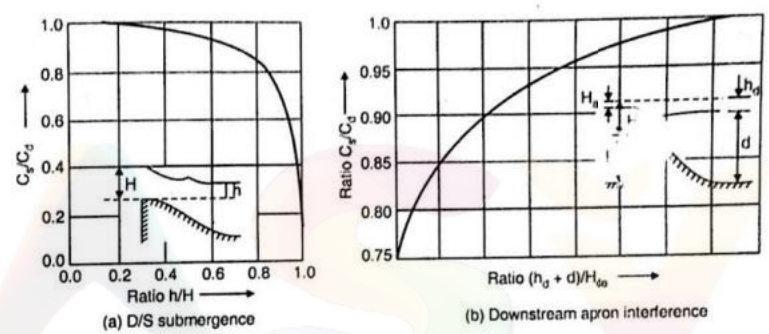

FIG. 11.11. EFFECTS OF D/S SUNERGENCE AND APRON INTERFERENCE

Fig. 11.11 (b) shows the effect of d/s apron interference on coefficient of discharge (C_d). It is observed that when the value of ratio $(h_d + d)/H_{de}$ exceeds 1.7, the downstream apron has little effect on coefficient of discharge. However, for lower values of this ratio, the actual coefficient of discharge (C_a) is lower. From the geometry of Fig. 11.11 (b) we have $h_d + d = P + H_{de}$.

(v) Slope of upstream face of spillway

For *small* values of P/H_{de} ratio, a spillway with sloping upstream face is slightly higher value of coefficient of discharge than for the spillway with vertical u/s face. However, for large values of P/H_{de} ratio, the value of C_d for spillways with sloping u/s face tends to decrease.

Effective length (L_e) of the crest

The effective length of the crest is given by

$$L_e = L - 2\,(NK_p + K_a)\,H_e \qquad ...(11.6)$$

where L_e = effective length of the crest

L = net length of the crest

N = number of piers

K_p = pier contraction coefficient

K_a = abutment contraction coefficient

The *pier contraction coefficient* depends upon many factors such as (i) shape and location of pier nose, (ii) thickness of pier, (iii) approach velocity and (iv) actual head

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of flow in relation to the design head. For the flow under the design head, the values of K_p can be taken from Table 11.8.

TABLE 11.8. PIER CONTRACTION COEFFICIENT (K_p)

Pier Condition	K_p
1. Square nosed piers with corners rounded on a radius equal to 0.1 of pier thickness.	0.02
2. Round-nose piers.	0.01
3. Pointed-nose piers.	0.0

Similarly, the *abutment contraction coefficient* depends upon several factors such as (*i*) shape of abutment, (*ii*) angle between upstream approach wall and axis of flow (*iii*) approach velocity and (*iv*) actual head of flow in relation to the design head, For the flow under the design head, the abutment contraction coefficient may be taken from Table 11.9.

TABLE 11.9. ABUTMENT CONTRACTION COEFFICIENT (K_a)

Abutment Condition	K_a
1. Square abutment with head wall at 90° to the direction of flow.	0.2
2. Rounded abutment with head wall at 90° to the direction of flow when $0.5\ H_d \geq r \geq 0.15\ H_d$	0.10
3. Rounded abutment where $r > 0.5\ H_d$ and wall is placed not more than 45° to direction of flow (where H_d = design head and r = radius of abutment rounding).	0.0

Orifice flow : Discharge formula at partial gate opening

Orifice flow is the flow that occurs at partial gate opening in a gated spillway. Sub-atmospheric pressures develop on the crest immediately below the gate if the crest profile is steeper than the one conforming to the trajectory. Where a thicker crest is needed for stability and the discharge efficiency is unimportant, the trajectory profile is an obvious choice and avoids sub-atmospheric pressures. Where the crest is to be shaped to the nappe profile, the sub-atmospheric pressure zone can be reduced by placing the gate sill downstream of the crest. In such a case the trajectory is steeper and conforms more nearly to the free overflow lower nappe profile. Model tests have shown that even when the gate is located at the crest axis and the head exceeds the design head (H_d) by $33\frac{1}{3}$%, negative pressure on the crest axis observed will be less than $0.15\ H_d$. While deciding on the design and crest profile, a certain extent of sub-atmospheric

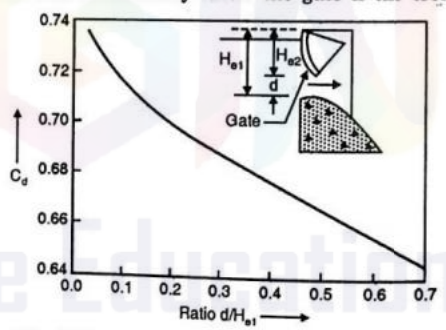

FIG. 11.12. COEFFICIENT OF DISCHARGE (C_{d0}) FOR ORIGINCE FLOW

pressure may be considered permissible taking into account the rareness and shortness of duration of occurrence of sub-atmospheric pressure.

The discharge for a gated spillway at partial gate opening is given by the following low-head orifice formula :

$$Q = \frac{2}{3} C_{do} \sqrt{2g}\ L_e\ (H_{e1}^{3/2} - H_{e2}^{3/2}) \qquad ...(11.7)$$

where C_{do} = coefficient of discharge of the office

L_e = effective length of the spillway crest

H_{e1} = total head above the bottom of the opening

H_{e2} = total head above the top of the opening

Fig. 11.12 shows the variation of coefficient of discharge (C_{d0}) with different values of d/H_{e1} where d is the height of opening. However, C_{d0} is also influenced by approach condition as well as downstream conditions.

11.4. SIDE CHANNEL SPILLWAY

A side channel spillway is the one in which the flow, after passing over a weir or ogee crest, is carried away by channel running essentially parallel to the crest. Discharge characteristics of a side channel spillway are similar to those of an ordinary overflow spillway and are dependent on the selected profile of the weir crest. However, this flow may differ from that of overflow spillway in that the flow in the trough may partly submerge the flow over the crest. Side channel spillway is suitable for earth or rockfill dams in narrow canyons and for other situations where direct over-flow is not permissible. Side channel spillway is also the best choice where a long overflow crest is desired in order to limit the surcharge head and the abutments are steep. This type of spillway is also desirable where the spillway discharge is to be connected to narrow discharge channel or

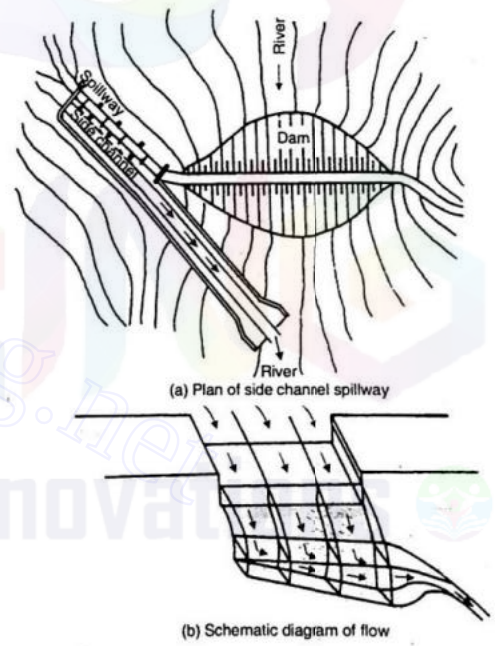

FIG. 11.13. SIDE CHANNEL SPILLWAY.

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a tunnel. Fig. 11.13(*a*) shows the plan of a side channel spillway, while Fig. 11.13(*b*) shows the schematic diagram of flow through such a spillway. The hydraulics and design of a side channel spillway is beyond the scope of this book and hence has not been included.

11.5. CHUTE OR TROUGH SPILLWAY

A chute spillway is the one which passes the surplus discharge through a steep sloped open channel, called a chute or trough, placed either along a dam abutment or through a saddle. Generally, this type of spillway is provided on earth or rockfill dam, and is isolated from the main dam. Its crest is kept normal to its centre line. It consist of a discharge channel to the river in an excavated trench which is usually paved with concrete in whole or in part. The crest or the spillway proper is usually of insignificant height or actually flat, as shown in Fig. 11.14(*c*). The chute is sometimes of constant width, but usually narrowed for economy and then widened near the end to reduce discharge velocity. Factors influencing the selection of chute spillways are the simplicity of their design and construction, their adaptability to almost any foundation condition and the overall economy often obtained by the use of large amounts of spillway excavation in the dam embankment.

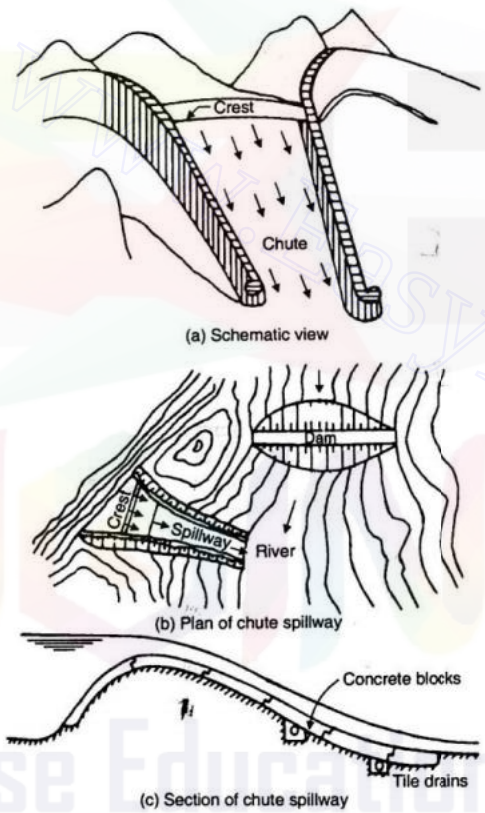

FIG. 11.14 CHUTE OR TROUGH SPILLWAY.

11.6. CONDUIT OR TUNNEL SPILLWAY

Conduit spillway or tunnel spillway is the one in which a *closed channel* is used to convey the discharge around or under a dam. The closed channel may be in the form of a vertical or inclined shaft, a horizontal tunnel through earth dam or a conduit constructed with open cut and backfilled with earth materials. These spillways are designed to flow partly full. Full flow is not allowed in the tunnel or conduit, lest the siphonic action may develop and negative pressures may be created in the conduit. To ensure free flow in the tunnel, the ratio

of flow area to the total tunnel area is often limited to 75% and air vents are provided at critical points along the tunnel or conduit to ensure an adequate air supply which will avoid unsteady flow through the spillway.

11.7. SHAFT SPILLWAY

Shaft spillway, drop inlet spillway or morning glory spillway is the one which has horizontally positioned lip through which water enters and then drops through a vertical or sloping shaft, and then to a horizontal conduit which convey the water past the dam. A shaft spillway can often be used where there is inadequate space for other types of spillways. It is generally not desirable to use a spillway over or through an earth dam. Thus, on an earth dam location, if there is no enough space or if the topography prevents the use of a chute or side channel spillway, the best alternative would be to use shaft spillway.

A shaft spillway normally consists of three elements : (*i*) overflow control weir (called a flared inlet or *morning glory*, (*ii*) vertical control, and (*iii*) closed discharge channel. Small shaft spillway may be constructed entirely of metal or concrete pipe or clay tile. The vertical shaft of large structures is usually made of reinforced concrete and the horizontal conduit is tunnelled in rock. Sometimes the diversion tunnel, used for diverting river water during the construction may be plugged at the upstream end, and then connected to the vertical shaft of shaft spillway as shown in Fig 11.15(*a*) and (*b*).

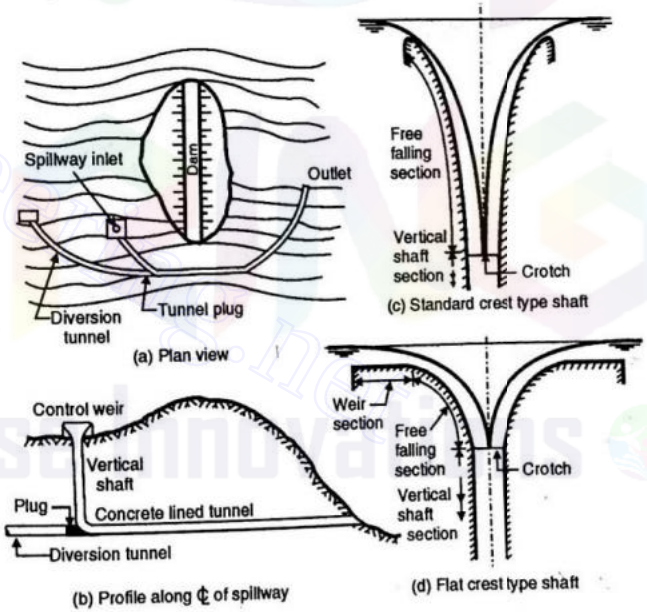

FIG. 11.15 SHAFT SPILLWAY.

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There are two general types of shaft spillways : (*i*) *Standard crest type* and (*ii*) *Flat crest type*. In the flat crest type [Fig 11.15(*d*)], the weir section preceeds free fall section, while in the standard crest type [Fig. 11.15(*c*)], there is no weir section and the water begins its free fall immediately upon leaving the crest. The standard crest type has the advantage of having smaller crest diameter since its coefficient of discharge is greater than that of flat crest. However, flat crest design has smaller funnel diameter, and is therefore, preferred in cases where the shaft has to be excavated in rock.

11.8. SIPHON SPILLWAYS

Instead of allowing water to spill over the crest of a dam or weir, the surplus water may be discharged downstream through a siphon spillway consisting of one or more siphon units. A siphon spillway is the one which utilizes the siphonic action to discharge the surplus water. Generally, a siphon spillway consists of a closed conduit system formed in the shape of an inverted U positioned so that the inside of the bend of the upper passageway is at normal resevoir level. When the water level in the reservoir rises above its normal level, water flows over the crest, and then siphonic action takes place. Continuous flow is then maintained by suction effect due to the gravity pull of the water in the lower leg of the siphon.

There are generally two types of siphon spillways :

(*i*) Saddle siphon spillway (*ii*) Volute siphon spillway.

SADDLE SIPHON SPILLWAY

Fig. 11.16 shows two types of saddle siphons, the former being more common. It essentially consists of reinforced concrete hood constructed over an overflow section of gravity dam. The *inlet* or *mouth* of the main hood is kept submerged in water so that floating debris etc. do not enter the siphon. A small deprimer hood is kept above the main hood, and both these hoods are connected through an air vent. The

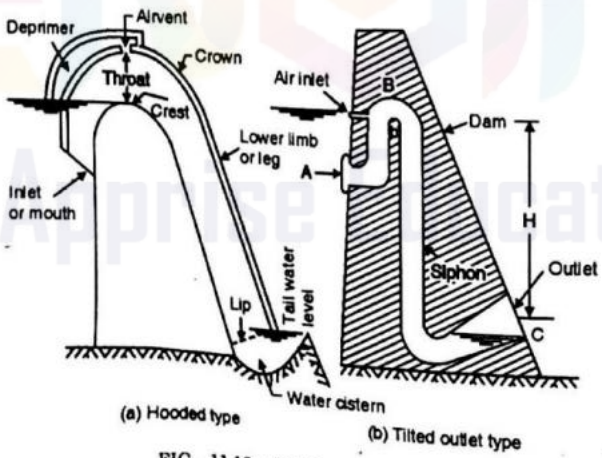

FIG. 11.16. SADDLE SIPHONS.

inlet of the deprimer hood is kept just at the reservoir level. The lower limb or leg of the main hood is generally kept submerged in the tailwater so that there is no air entry from the down stream end.

When the water level in the reservoir increases, it seals the air entry though the mouth of the deprimer dome. Water spills over the crest of the spillway. Since air entry is sealed both from u/s as well as d/s end, the spilling water sucks all the remaining air from the hood within minutes. Siphonic action is established after the air in the bend over the crest has been exhausted. This action is known as *priming*. The siphon runs full and water is discharged downstream under a greater effective (siphonic) head. During receding flood, when water level has gone down just to the reservoir level, air enters through the mouth of deprimer dome and the siphonic action is broken. This action is called *depriming* of the siphon and is achieved through the deprimer dome.

Priming devices. Priming is the action of the siphon spillway from the moment the water first spills over the crest to the instant when the whole siphon is filled with water. Priming is achieved by sealing the down-stream outlets. The following are the common methods employed for early priming in spillways :

1. ***Provision of cup type basin.*** This is shown in Fig. 11.16(*a*). The lip of the lower limb is kept immersed in the water stored in the cup. This will prevent the entry of air.

2. ***Provision of tilted outlet.*** This is another type of saddle siphon in which the vertical shaft has a tilted outlet which maintains a *water seal* [Fig. 11.16(*b*)]. The air inlet is maintained just at the reservoir level.

3. ***Provision of auxiliary or baby siphon*** (Fig.11.17). A small auxiliary or baby siphon is formed just below the crest. When the water level reaches the crest, the baby siphon will run full. The sheet of water issuing from it is arranged to shoot

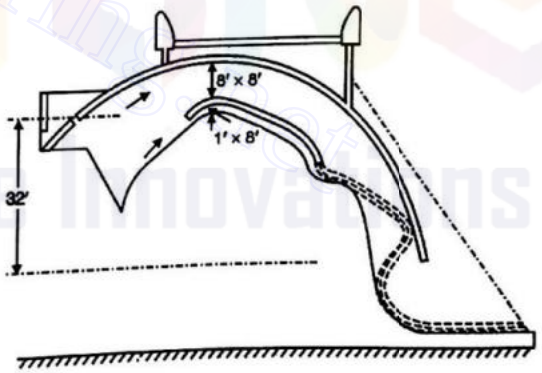

FIG. 11.17 MARAMSILI SIPHON INSTALLATION.

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across the lower limb of the siphon so as to seal it and prevent air entering from below. The priming thus takes place. One such installation was made at Maramsili siphon (India) in 1921. The baby siphon is 1 ft. (0.3 m) high and 8 ft. (2.4 m) wide at throat while the main siphon is 8 ft. (2.4 m) high and 8 ft. (2.4 m) wide. The priming is followed just in 15 to 30 seconds. Though priming is good, but the effective top water level for the consideration of storage is not the crest of the main siphon but the crest of the auxiliary siphon. Hence from the point of view of maximum discharge, the auxiliary siphon may be looked upon as an obstruction. As the flow over the main siphon increases, the jet through the baby siphon is depressed down and care must be taken that the combined jet does not strike below the lip of the lower leg. The baby siphon is expensive and difficult to construct.

4. *Provision of tudol or step* [Fig. 11.9(*a*)]. A horizontal tudol or step, provided near the lip, deflects the jet to strike the lower limb thus sealing the air entry.

5. *Provision of step just after crest* [Fig. 11.18(*b*)]. A step is provided just after the crest to deflect the water which strikes to the opposite side, thus providing a water seal.

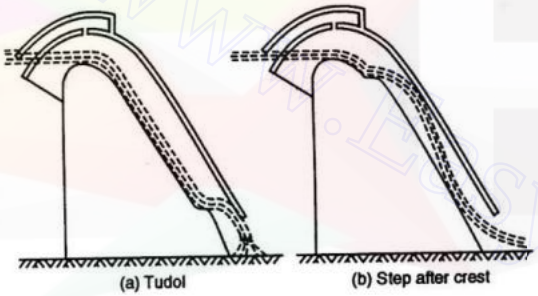

FIG. 11.18

6. *Provision of hinged adjustable plate.* Fig. 11.19(*a*) shows a system suggested by Heyn in which a flexible steel tongue directs the over-flow across the siphon until priming begins. When weight of water presses down the steel tongue against the inner wall, an unobstructed flow is ensured.

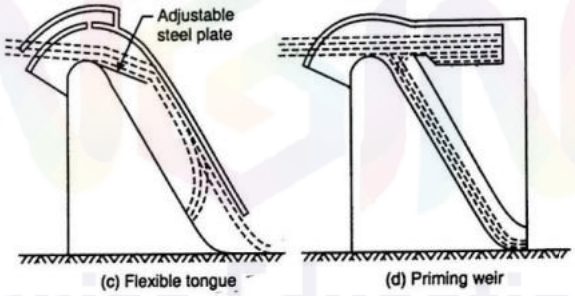

FIG. 11.19

7. *Provision of priming weir.* A priming weir, shown in Fig. 11.19(*d*) connected with the main reservoir is made. When the overflow commences, the falling nappe completely seals the air in the crown of the siphon and priming is ensured.

Depriming. Depriming is the reverse process of priming. It is the action of the siphon from the time air starts entering the siphon through the vents until the siphonic action is completely stopped. Depriming is carried out by providing *deprimers*. A deprimer is a siphon breaker air vent and is provided to break the siphonic action when the

reservoir water surface is drawn below it. If an air vent is not provided, siphonic action once initiated would continue till the reservoir level is brought down to the level at inlet which is provided much below normal reservoir level. The air vent consists of a vent pipe connected to the siphon with its open end in the plane of normal reservoir level. It can be provided either on the side or the top of the crown. The open end should be provided in a horizontal plane to facilitate instantaneous sealing and opening of the pipe. This prevents the development of unsteady flow leading to *knocking* of the siphon, which is undesirable.

Discharge Through Saddle Siphon

The usual siphon discharge formula is :

$$Q = CA\sqrt{2gH} \qquad ...(11.8)$$

where A = Area of the cross-section at crown = $L \times b$, where L is the length and b is the height of throat.

H = Operating head.

= Reservoir level – Centre of outlet, if outlet is discharging freely.

= Reservoir level – Downstream tail water level if the outlet is submerged.

C = Coefficient of discharge, the average value of which may be taken as 0.65.

The discharge equation can also be expressed in terms of radii of the crest and crown, as shown in Fig. 11.20.

Let R_1 = Radius of crest and R_2 = Radius of crown.

Consider any point P on the vertical section at the throat, and let its radius be r. The L-section of the throat has been shown in Fig. 11.20(*b*). The section of the throat is rectangular, having length L and height b.

Let V_1, V_2, and V be the velocities of flow at crest, crown and point P respectively. The flow through the throat is governed by free vortex flow for which we have :

$$V \times r = \text{Constant.}$$

$$\therefore \quad V_1 R_1 = V_2 R_2 = V \cdot r.$$

Let h_0 be the *net operative head* at the crest. This head is calculated after making due allowances of all losses occurring in the siphon, that is

$h_0 = H$ – Losses through the siphon.

Hence $V_1 = \sqrt{2gh_0}$

$$\therefore \quad V = V_1 \cdot \frac{R_1}{r} = \frac{R_1}{r}\sqrt{2gh_0}.$$

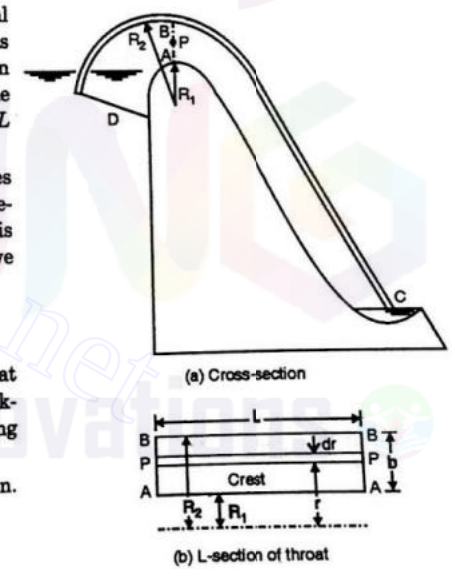

FIG. 11.20.

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Consider a thin rectangular strip of thickness dr at P, and let δq be the discharge through this strip of *unit length*.

$$\therefore \quad \delta q = V \,.\, dr \,.\, 1 = \frac{R_1}{r}\sqrt{2gh_0}\, dr.$$

Hence discharge q through the throat per unit length is given by

$$q = \int_{R_1}^{R_2} \delta q = R_1 \sqrt{2gh_0} \int_{R_1}^{R_2} \frac{dr}{r}$$

or
$$q = R_1 \sqrt{2gh_0} \log_e \frac{R_2}{R_1} \qquad ...[11.9(a)]$$

The total discharge through the siphon unit having throat length L is therefore:

$$Q = q \,.\, L = R_1 \,.\, L \sqrt{2gh_0} \log_e \frac{R_2}{R_1} \qquad ...[11.9(b)]$$

The maximum value of V occurs corresponding to atmospheric pressure head of 10.35 m (34 ft). But to prevent cavitation, the pressure at any point cannot be allowed to approach zero value. Hence safe maximum head is assumed to be 7.92 m (26 ft.).

Substituting the values of $h_0 = 7.92$ m, we get

$$Q = R_1 L \sqrt{2g \times 7.92} \log_e \frac{R_2}{R_1} \simeq 12.47\, R_1 L \log_e \frac{R_2}{R_1}$$

or
$$Q = 28.71\, R_1 L \log_{10} \frac{R_2}{R_1} \qquad ...(11.9)$$

Pressure Conditions in the Siphon Spillway

(Refer Fig. 11.20)

Let H = total head at any instant
= difference between the u/s water level and d/s water level if the outlet end is submerged.

or = difference between u/s water level and centre of the outlet if the outlet discharges freely.

h = difference in elevation in the throat and outlet end (*i.e.* between points A and C).

P_a = atmospheric pressure.

H_l = head loss in the whole siphon.

h_l = head loss between throat and the outlet end.

σ = absolute pressure at which the air dissolved in water starts escaping and cavitation begins (average value of σ may be taken as 2.43 m).

Applying Bernoulli's theorem to the points D and C, we have

$$p_a + H = p_a + \frac{v_c^2}{2g} + H_l \qquad ...(1)$$

Hence
$$\frac{v_c^2}{2g} = H - H_l$$

or
$$v_c = \sqrt{2g\,(H - H_l)} \qquad ...(2)$$

Hence the discharge through the spillway is

$$Q = A\sqrt{2g\,(H - H_l)} \qquad ...(3)$$

where A is the area of cross-section of the bore.

Equating this to Eq. 11.7, we get

$$A\sqrt{2g\,(H - H_l)} = CA\sqrt{2gH}$$

or
$$H - H_l = C^2 H$$

Hence
$$H_l = (1 - C^2)\, H \qquad ...(11.10\ a)$$

Let p_T = pressure head at the throat.

v_T = velocity of flow at the throat.

Applying Bernoulli's theorem between the throat and the exit end, we get

$$p_T + \frac{v_T^2}{2g} + h = p_a + \frac{v_c^2}{2g} + h_l$$

Since the siphon is to run full bore and the area of section is constant between the throat and outlet end, $v_T = v_c$.

Hence
$$p_T + h = p_a + h_l$$

or
$$h = (p_a - p_T) + h_l \qquad ...(4)$$

It should be noted that the pressure (p_T) at the throat cannot be allowed to approach zero value. Since air dissolved in water begins to separate much earlier than the zero value, the value p_T may be assumed to be equal to σ.

Hence
$$h = (p_a - \sigma) + h_l$$

Putting $p_a = 10.35$ m and $\sigma = 2.43$ m, we get

$$h \le (10.35 - 2.43) + h_l$$

$$h \le 7.92 + h_l \qquad ...(11.10)$$

Eq. 11.10 limits the maximum difference in elevation between the throat and the outlet end. This limit can be slightly increased by reducing the area of outlet end, in which case v_c will be greater than v_T and Eq. 11.10 will be modified to:

$$h < 7.92 + h_l + \left(\frac{v_c^2}{2g} - \frac{v_T^2}{2g}\right) \qquad ...[11.10\ (b)]$$

The quantity $(p_a - p_T)$ or $(p_a - \sigma)$ in Eq. (4) above is known as the *sub-atmospheric pressure head* (p_{aT}) at the crest of the throat. The maximum allowable sub-atmospheric pressure head is equal to probable minimum atmospheric head at the location under consideration minus the minimum tolerable residual pressure head (σ) i.e. $p_{aT} = p_a - \sigma$.

It should be noted that h is always less than H. The total head loss H_l consists of (*i*) entry head loss, (*ii*) velocity head loss, (*iii*) conversion head loss, (*iv*) friction head loss, (*v*) head loss in bends etc. and its maximum value seldom exceeds 20 % of the operating head H.

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Again, applying the Bernoulli's theorem between the inlet and the throat, with crest as the datum, we get

$$p_a = p_T + \frac{v_T^2}{2g} + h_{lu}$$

where h_{lu} = head loss from inlet to throat.

Hence, we get $$\frac{v_T^2}{2g} = (p_a - p_T) - h_{lu} = p_{aT} - h_{lu} \quad ...(11.11)$$

Taking $p_T = \sigma = 2.43$ m and $h_a = 10.35$ m, we get

$$\frac{v_T^2}{2g} \le 7.92 - h_{lu} \quad ...(11.12)$$

Neglecting h_{lu}, we get $v_T = \sqrt{2g \times 7.92} = 12.46$ m/s. Thus, the maximum possible velocity at the throat is 12.46 m/s.

Classification of Siphon

Siphon spillway may be classified according to head as *high head, medium head, or low head siphons.*

1. *Medium head siphon*. We have seen in Eq. 11.10 that the difference in level between siphon crest and the exit end is given by

$$h \le 7.92 + h_l.$$

This is true only when the area of the siphon at the throat and at the exit end are equal. Hence a medium head siphon spillway is the one in which the outlet area is the same as the area at the throat.

2. *High head siphon.* In Eq. [11.10 (*b*)] we have seen that the limit of the value of h can be increased if the velocity at the exit end is greater than the velocity at the throat, the increase being equal to $\left(\frac{v_e^2}{2g} - \frac{v_T^2}{2g}\right)$. This can be achieved by reducing the outlet area by a nozzle or by tapering the lower part of the siphon, or in some other manner increasing the resistance to flow. Such convergent siphons are known as *high head siphons.*

3. *Low head siphon.* A low head siphon is the one in which the outlet tube is made divergent. This gives velocity v_e lesser than v_T thus further decreasing the value of h given by Eq. 11.10.

Qualities of Good Siphon

A good siphon should have the following qualities :

1. It should have quick priming.
2. It should have low depth of priming so that the siphon becomes effective even with a small rise in the water level.
3. There should be no knocking at the times of priming and depriming. The flow through the siphon should be steady and continuous.
4. It should have automatic working.
5. It should be self-cleaning so that no silt or debris deposits in the siphon.
6. Air should neither get access to the siphon nor be entrained.

The dissolved air in water should not be separated. The flow should be smooth and free of vibration.

7. It should have high efficiency.
8. It should have low initial and maintenance cost. It should have easy access to all parts for repairs.
9. It should be simple in design and construction.

VOLUTE SIPHON SPILLWAY

Volute siphon spillway is a special type of siphon spillway designed in India by Ganesh Iyer. It consists of a vertical barrel or shaft bent at the discharge end and opened out in the form of a funnel at the top. The top or lip of the funnel is kept at the reservoir level and a number of volutes (like blades of a centrifugal pump or turbine) are fixed in the funnel to induce a spiral motion to the water passing along them. A dome, supported on number of pillars, is placed over the funnel. Over the main dome is attached a deprimer dome. The entrance end of the deprimer dome is kept just at reservoir level.

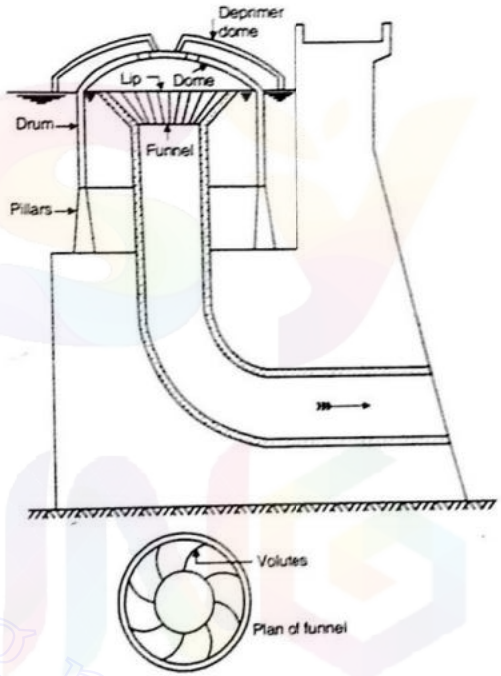

FIG. 11.21. VOLUTE SIPHON SPILLWAY

When water level in the reservoir rises, it seals the air entry and sheet of water enters to the funnel from all sides. The spiral flow of water through the volute causes suction which sucks all the remaining air and priming is thus achieved very quickly. The siphon then runs full. When the reservoir level falls, air enters through the deprimer dome, thus breaking the siphonic action.

The discharge through the volute siphon is given by

$$Q = A\sqrt{2g(H - H_L)} = CA \cdot \sqrt{2gH} \quad ...(11.13)$$

where A = area of cross-section of pipe.
C = coefficient of discharge.
H = maximum operating head.
H_L = head loss through the siphon.

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11.9. DYNAMIC FORCE ON SPILLWAY

When water flows over the curved surface of ogee spillway there is continuous change of velocity, and hence there is change in momentum from section to section. According to Newton's second law of motion, this change in momentum causes a force on the spillways structure. This force is known as the *dynamic force.*

Consider an element of water between two sections A and B on a curved surface [Fig.11.22(*b*)]. The resultant of the forces on the element of water is given by

$$\Sigma F = \rho Q \Delta V = \rho Q (V_2 - V_1) \quad ...(11.14)$$

where ρ = mass density of water $= w/g$

Q = discharge.

Rewriting Eq. 11.14 in horizontal and vertical directions separately, we get

$$\Sigma F_H = \rho Q (V_{2H} - V_{1H}) \quad ...[11.14(a)]$$

and

$$\Sigma F_V = \rho Q (V_{2V} - V_{1V}) \quad ...[11.14(b)]$$

The force F_H and F_V are those acting on a significant free body of fluid and include gravity forces, hydrostatic pressure and the reaction of any object in contact with water. Note that $\rho = w/g = 9.81/9.81 = 1$ in SI units.

See example 11.1 for illustration.

Example 11.1. *Fig. 11.13(a) shows an ogee spillway which discharges water with a head of 1.2 m over the crest. Taking the coefficient of discharge as 2.2, compute the dynamic force on the curved section AB which has a constant radius of 3 m.*

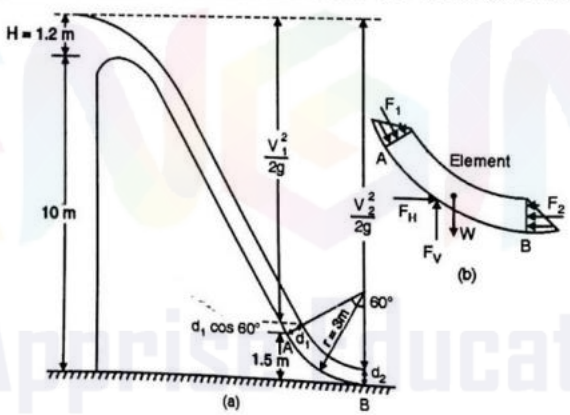

FIG. 11.22.

Solution. The discharge over the spillway is given by

$$Q = CLH^{3/2}$$

or

$$q = \frac{Q}{L} = CH^{3/2} = 2.2\,(1.2)^{3/2} = 2.9 \text{ cumecs/m.}$$

Let d_1 and d_2 be the depth of sheet of water at A and B, and V_1 and V_2 be the velocities. Assuming that there is no loss of energy and neglecting approach velocity, we may apply Bernoulli's theorem at u/s water surface, and at sections A and B.

Thus we get $$11.2 = 1.5 + d_1 \cos 60° + \frac{V_1^2}{2g} = d_2 + \frac{V_2^2}{2g} \quad ...(1)$$

But $$V_1 d_1 = q = 2.9 = V_2 d_2$$

Hence $$V_1 = \frac{2.9}{d_1} \text{ and } V_2 = \frac{2.9}{d_2}$$

Substituting these in (1) and solving these by trial and error, we get

$$d_1 = 0.212 \text{ m and } d_2 = 0.197 \text{ m}$$

Hence $$V_1 = 13.7 \text{ m/sec and } V_2 = 14.7 \text{ m/sec.}$$

Fig. 11.22(*b*) shows the free body diagram of the curved element between sections A and B in which F_x and F_y represent components of force on the water by the curved section AB. If F_1 and F_2 are the resultant hydrostatic forces at section A and B, we have

$$F_1 = \frac{1}{2} w d_1^2 \cos 60° = \frac{1}{2} \times 9.81\,(0.212)^2 \times \frac{1}{2} = 0.11 \text{ kN.}$$

$$F_2 = \frac{1}{2} w d_2^2 = \frac{1}{2} \times 9.81\,(0.197)^2 = 0.19 \text{ kN.}$$

The weight W of the water body in the curved portion between sections A and B is

$$W = 9.81 \times \frac{60°}{360°} \times 2\pi \left(3 \times \frac{0.212 + 0.197}{2}\right) = 6.3 \text{ kN.}$$

Applying Eq. 11.14(*a*) for 1 m length of spillway, we get

$$\Sigma F_H = F_1 \cos 60° - F_2 + F_x = \rho Q (V_2 - V_1 \cos 60°)$$

or $$0.11 \times \frac{1}{2} - 0.19 + F_x = (1 \times 2.9 \times 1)(14.7 - 13.7 \times \frac{1}{2})$$

From which $$F_x = \mathbf{22.9 \text{ kN/m.}}$$

Similarly, from Eq. 11.14(*b*), we have

$$\Sigma F_V = F_y - F_1 \sin 60° - W = \rho Q\,[0 + 13.7 \sin 60°)].$$

or $$F_y - 0.11 \times 0.866 - 6.3 = (1 \times 2.9 \times 1)(13.7 \times 0.866)$$

From which $$F_y = \mathbf{40.80 \text{ kN/m}}$$

Resultant force $$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(22.9)^2 + (40.8)^2} = \mathbf{46.79 \text{ kN/m}}$$

Example 11.2. *Compute the discharge over an ogee weir with coefficient of discharge equal to 2.4 at a head of 2 m. The length of the spillway is 100 m. The weir crest is 8 m above the bottom of the approach channel having the same width as that of the spillway.*

Solution. Neglecting the approach velocity and end contractions,

$$Q = C\,L\,H^{3/2} = 2.4 \times 100(2)^{3/2} = 678.82 \text{ cumecs}$$

The approach velocity is given by

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$$v_a = \frac{Q}{\text{Height} \times \text{Width of channel}} = \frac{Q}{(8+2)\times 100} = \frac{678.82}{10 \times 100} = 0.679 \text{ m/sec}$$

$$\therefore \quad h_a = \frac{v_a^2}{2g} = \frac{(0.679)^2}{2 \times 9.81} = 0.023 \text{ m}$$

and $$H_e = H + h_a = 2 + 0.023 = 2.023 \text{ m}$$

Hence $$Q = 2.4 \times 100\,(2.023)^{3/2} = \mathbf{690.6 \text{ cumecs.}}$$

Example 11.3. *A dam has siphon spillway whose cross-section is 1 m high and 4 m wide. The tail water elevation at design flow is 6 m below the summit of the siphon and the head water elevation is 1.5 m above the summit. Assuming a coefficient of discharge of 0.6, what is the capacity of the siphon ? What head would be required on an ogee spillway (C = 2.25) 4 m long to discharge this flow ? What length of ogee weir would be required to discharge the same flow as the siphon with a head of 1.5 m on the weir crest ?*

Solution. (*a*) *For the siphon spillway*

Net operative head $= 6 + 1.5 = 7.5$ m

$$Q = CA\sqrt{2gH} = 0.6 \times 1 \times 4\sqrt{2 \times 9.81 \times 7.5} = \mathbf{29.1 \text{ cumecs.}}$$

(*b*) *For the ogee spillway 4 m long.*

$$Q = CLh^{3/2}$$

$$\therefore \quad h = \left(\frac{29.1}{2.25 \times 4}\right)^{2/3} = \mathbf{2.19} \text{ m}$$

(*c*) *For the ogee spillway with* $h = 1.5$ m.

$$L = \frac{Q}{Ch^{3/2}} = \frac{29.1}{2.25 \times (1.5)^{3/2}} = \mathbf{7.04 \text{ m.}}$$

It is thus seen that for the same discharge, a siphon spillway works with lesser head over the crest (thereby reducing the afflux), or with lesser length than an ogee spillway.

Example 11.4. *A saddle siphon has the following data :*

Full reservoir level	= 435.00 m.
Level of centre of siphon outlet	= 429.60 m.
High flood level	= 435.85 m.
High flood discharge	= 600 cumecs.

If the dimensions of the throat of the siphon are : width = 4 m and height = 2 m, determine the number of siphon units required to pass the flood safely. The siphon discharges freely in air.

Solution. Maximum operative head H = H.F.L. – R.L. of outlet centre

$$= 435.85 - 429.60 = 6.25 \text{ m.}$$

The discharge is given by $Q = CA \cdot \sqrt{2gH}$

Assume $C = 0.65$, Also, $A = 4 \times 2 = 8 \text{ m}^2$

$$\therefore \quad Q = 0.65 \times 8\sqrt{2 \times 9.81 \times 6.25} = 57.58 \text{ cumecs.}$$

$\therefore$ No. of units required $= \dfrac{600}{57.58} = 10.42$. Hence provide 11 units.

11.5. Example 11.5. *Design an ogee spillway for concrete gravity dam, for the following data :*

1.	*Average river bed level*	= 250.00 m
2.	*R.L. of spillway crest*	= 350.00 m
3.	*Slope of d/s face of gravity dam*	= 0.75 : 1
4.	*Design discharge*	= 6500 cumecs
5.	*Length of spillway*	= 5 spans with a clear length of 9 m each
6.	*Thickness of each pier*	= 2 m

Solution

Step 1 : Computation of design head

$$Q = C_d \cdot L_e H_e^{3/2} \text{ where } L_e = L - 2\,[NK_p + K_a]\,H_e$$

To start with, let us assume $C_d = 2.2$, applicable for high over flow spillway

Also, let $L_e \triangleq L =$ clear water way $= 5 \times 9 = 45$ m

$$\therefore \quad 6500 = 2.2 \times 45\, H_e^{3/2} \text{ from which } H_e = 16.27 \text{ m}$$

The height (P) of spillway crest above river bed $= 350 - 250 = 100$ m

Since $\dfrac{P}{H_d} \triangleq \dfrac{P}{H_e} = \dfrac{100}{16.27} = 6.15$ is greater than 1.33, it is a high overflow spillway, for which the effect of velocity of approach is negligible.

Also, with reference to Fig. 11.11 (*b*), we have

$$\frac{h_d + d}{H_{de}} = \frac{H_{de} + P}{H_{de}} = \frac{16.27 + 100}{16.27} = 7.15 > 1.7$$

Hence the discharge coefficient is not affected by down stream apron interference as well as on the tail water conditions.

The above value of H_e has been computed by taking $L_e \triangleq L$. Actual value of L_e is given by $L_e = L - 2\,[NK_p + K_a]\,H_e$

Let us assume rounded nose piers and rounded abutments.

Thus we have $K_p = 0.01$ and $K_a = 0.1$. Also $N = 4$. Assuming $H_e \triangleq 17.5$ m (i.e. slightly more than the one computed above), we get

$$\therefore \quad L_e = 45 - 2\,[4 \times 0.01 + 0.1] \times 17.5 = 40.1 \text{ m}$$

Substituting this value of L_e in discharge equation, we get

$$6500 = 2.2 \times 40.1 \times H_e^{3/2} \text{ from which } H_e = 17.58 \text{ m}$$

which is quite near to the assumed value of $H_e = 17.5$ m.

Hence the crest profile will be designed for $H_d = 17.58$ m, thus neglecting the head due to velocity of approach.

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However, velocity of approach, $V_a = \frac{Q}{Area} = \frac{6500}{(45 + 4 \times 2)(100 + 17.58)} = 1.043$ m/s

$\therefore$ Velocity head, $H_a = \frac{V_a^2}{2g} = \frac{(1.043)^2}{2 \times 9.81} \simeq 0.055$ m

Hence the velocity head is quite small (being only 0.3% of the total head) and it is quite justified in neglecting this.

Step 2. Determination of d/s profile

Let us keep u/s face vertical. The d/s profile suggested by WES, for vertical face is given by Eq. 11.1 :

$$x^{1.85} = 2 H_d^{0.85} . y$$

Hence
$$y = \frac{x^{1.85}}{2 H_d^{0.85}} = \frac{x^{1.85}}{2 (17.58)^{0.85}} = 0.04372 x^{1.85} \quad ...(1)$$

Let us now determine the tangent point (T.P.), for a downstream slope of 0.75 (*H*) : 1 (*V*)

$$\therefore \quad \frac{dy}{dx} = \frac{1}{0.75} \quad ...(2)$$

However, differentiating Eq. (1), w.r. to x, we get

$$\frac{dy}{dx} = 1.85 \times 0.04372 x^{1.85 - 1} = 0.0809 x^{0.85} \quad ...(3)$$

Equating (2) and (3), $0.0809 x^{0.85} = \frac{1}{0.75}$

or
$$x^{0.85} = 16.484, \text{ from which } x = 27.03 \text{ m}$$

Hence from (1), $y = 19.48$ m

Hence the coordinates of the tangent points are (27.03 m , 19.48 m).

The co-ordinates of the curve for $x = 0$ to $x = 27.03$ m are given in the table below.

x (m)	$y = 0.04372 x^{1.85}$	x (m)	y (m)	x (m)	y (m)
1	0.044	7	1.600	18	9.182
2	0.158	8	2.048	20	11.158
3	0.334	10	3.095	22	13.310
4	0.568	12	4.337	24	15.634
5	0.859	14	5.768	26	18.129
6	1.203	16	7.384	27.03	19.480

Step 3. Determination of u/s profile

For vertical u/s face, the u/s profile is given by Eq. 11.2. The u/s curve extends upto $x = -0.27 H_d$. Thus

$$y = \frac{0.724 (x + 0.27 H_d)^{1.85}}{H_d^{0.85}} + 0.126 H_d - 0.4315 H_d^{0.375} (x + 0.27 H_d)^{0.625}$$

Taking $H_d = 17.58$ m, we get

$$y = \frac{0.724 (x + 0.27 \times 17.58)^{1.85}}{(17.58)^{0.85}} + 0.126 \times 17.58 - 0.4315 (17.58)^{0.375} (x + 0.27 \times 17.58)^{0.625}$$

or
$$y = 0.0633 (x + 4.7466)^{1.85} + 2.2151 - 1.2643 (x + 4.7466)^{0.625} \quad ...(4)$$

The curve will extend upto $x = -0.27 \times 17.58 \simeq -4.75$ m

Hence the value of y coordinates are calculated for the values of $x = -0.5$ m , -1.0 m, -1.5 m , -2 m , -3 m , -4 m and -4.75 m and tabulated as under.

x (m)	y (m)
– 0.5	0.012
– 1.0	0.057
– 1.5	0.135
– 2.0	0.248
– 3.0	0.601
– 4.0	1.199
– 4.75	2.215

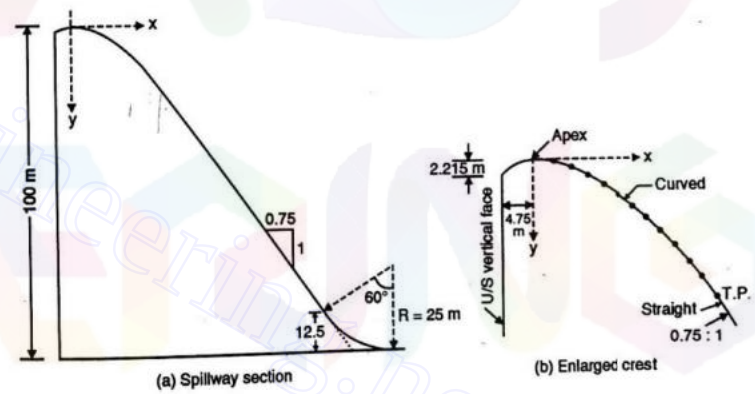

FIG. 11.23

4. Step 4: Design of d/s bucket

The profile of the spillway is shown in Fig. 11.23 (*a*). A reverse curve at the toe is provided to form a bucket. The radius of the bucket is generally kept at $R = P/4 = 100/4 = 25$ m. The bucket will subtend an angle of 60° at the centre, as shown.

11.10. ENERGY DISSIPATION BELOW SPILLWAYS

When flood discharge passes over the spillway crest, it has high potential energy which gets converted into kinetic energy as it glides along it. This high energy has to be dissipated otherwise it would cause erosion at the downstream toe. There is

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a varied practice regarding the extent of protection against erosion below a spillway to be provided. In general, the protection can be rendered by two ways :

(*i*) By dissipating the energy by means of hydraulic jump.

(*ii*) By directing the jet of water so as to fall away from the structure by a deflector bucket or lip, and dissipating the energy by impact.

Where the energy of flow is to be dissipated before the discharge is returned to the main river channel, the hydraulic jump basin is an effective device for reducing the exit velocity to a tranquil state. However, where the spillway discharge may be safely delivered directly to the river channel, without providing a dissipating or stilling device, the jet is often projected beyond the structure by a *deflector bucket*.

Hydraulic jump computations. The hydraulic jump that occurs at the stilling basin has some distinctive characteristics and assumes a definite form, depending upon the energy of flow which must be dissipated in relation to the depth of flow. The jump form and flow characteristics depends upon the following factors :

(*i*) Discharge entering the basin,

(*ii*) Critical depth of flow, d_c

(*iii*) Froude number parameter $\frac{V}{\sqrt{gd}}$

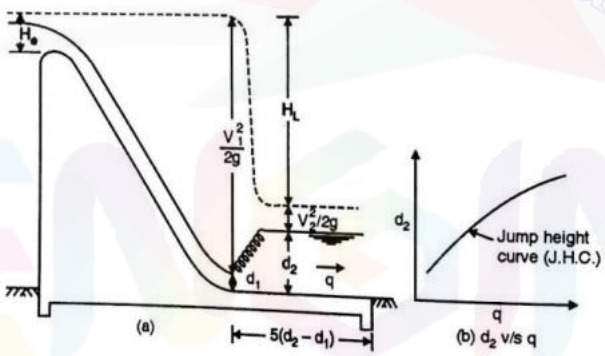

FIG. 11.24.

Fig. 11.24 shows the elements of hydraulic jump. The post jump depth d_2 can be computed for a given discharge as under :

1. For the given discharge q per metre length of the spillway, calculate the head H_e over the crest to the total energy level :

$$H_e = \left(\frac{q}{C}\right)^{2/3}$$

2. Find total energy level at u/s :

$$\text{u/s T.E.L.} = \text{crest level} + H_e$$

3. Assuming no losses, the specific energy E_1 at the toe of the spillway will be equal to the T.E.L. u/s.

$$E_1 = \text{T.E.L. u/s.}$$

4. Knowing E_1 and q, find the prejump depth d_1 by trial and error from the relation:

$$E_1 = d_1 + \frac{V_1^2}{2g} = d_1 + \frac{q^2}{2\,g\,d_1^2}$$

5. Calculate Froude Number F_1 :

$$F_1 = \frac{V_1}{\sqrt{g\,d_1}} = \frac{q}{\sqrt{g\,d_1^3}} \qquad ...(11.15)$$

6. Compute the post jump depth d_2 from the relations :

$$d_2 = \frac{d_1}{2}\left[\sqrt{1+8F_1^2} - 1\right] \qquad ...(11.16)$$

or by

$$d_2 = \frac{d_1}{2}\left[\sqrt{1+\frac{8q^2}{gd_1^3}} - 1\right] \qquad ...(11.17)$$

Plotting jump height curve. Eq. 11.16 or 11.17 can be used for calculating post jump depths d_2 for various values of flow discharge q. Fig. 11.24(*b*) shows such a curve, known as *jump height curve* (J.H.C.) showing the variation of post jump depth d_2 with the discharge q.

Plotting tails water curve (T.W.C.). The efficiency of the hydraulic jump in dissipating the energy, and the corresponding protection works in the stilling basin will depend upon the position or height of tail water in relation to the post jump depth d_2 for that discharge. Tail water curve [Fig. 11.25(*a*)] gives the relation between the discharge q and the tail water depth D. This can be prepared by actual gauge readings and measurements may be conveniently made at some control point available downstream. However, the effect of channel retrogression must be taken into account.

Relative position of jump height curve and tail water curve. There may be *five conditions* that govern the relationship between the jump height curve (J.H.C.) and tail water curve (T.W.C.) :

1. Both the curves coincide [Fig. 11.25(*b*)].
2. J.H.C. lies lower than T.W.C. at all discharges [Fig. 11.25(*c*)].
3. J.H.C. lies above T.W.C. at all discharges [Fig. 11.25(*d*)].
4. J.H.C. lies lower than T.W.C. at small discharges and higher than T.W.C. at large discharges [Fig. 11.25(*e*)].
5. J.H.C. lies above T.W.C. at small discharges and lower at higher discharges [Fig.11.25(*f*)].

1. Protection works for condition I. In this condition, both the curves coincide, as shown in Fig. 11.25(*b*). This is an ideal condition, since the post jump depth required for the formation of hydraulic jump is just available at the channel. The jump formation

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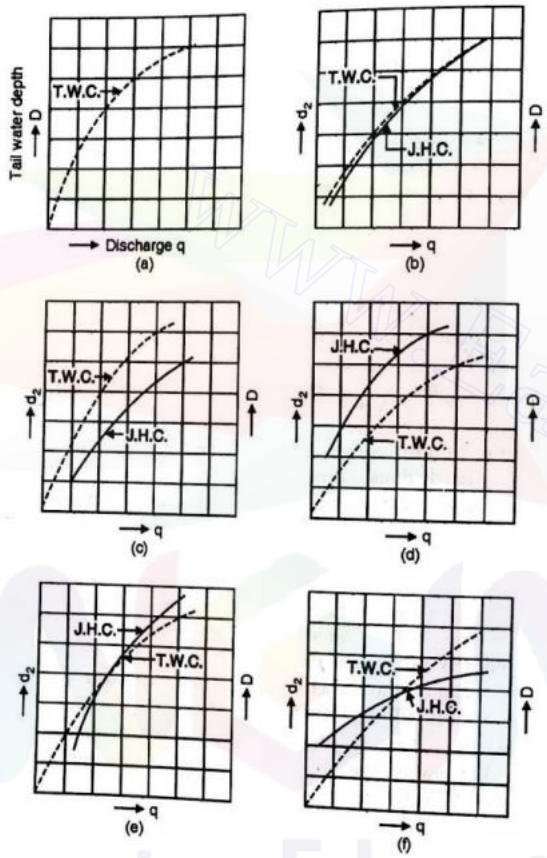

FIG. 11.25

will thus be perfect at all discharges and hydraulic jump will be formed at the toe of the spillway.

Protective measures. In such a case a simple horizontal apron of a length $5\,(d_2 - d_1)$ will be sufficient, as shown in Fig. 11.24(*a*).

2. Protection works for condition II. In this condition, shown in Fig. 11.25(*c*), the jump height curve lies lower than the tail water curve at all discharges. In other

words, the available tail water depth is greater than required for the formation of hydraulic jump. The jump in such a case will be completely submerged and no visible standing wave would occur. Thus, very little energy will be dissipated unless arrangements are made to reduce the tail water depth at the point of formation of hydraulic jump.

Protective measures. The following protection measures may be provided :

(*i*) Using a sloping apron above the bed level as shown in Fig. 11.26(*a*). The jump will be formed on the sloping glacis where depth d_2 is available. This was done at Madden dam on Panama canal.

(*ii*) Using a low level bucket sharply upturned at the end, as shown in Fig. 11.26(*b*). This serves the purpose of a deflector.

(*iii*) Providing end sills with baffles [Fig. 11.26(*c*)] so that energy is dissipated by impact and friction.

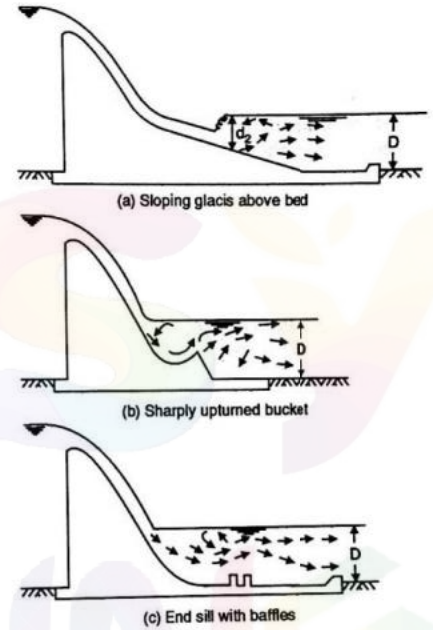

FIG. 11.26. PROTECTION WORKS FOR HIGH TAIL WATER.

3. Protection works for condition III. In this condition the jump height curve is higher than the tail water curve at all discharges, as shown in Fig. 11.25(*d*). Thus the available tail water depth is lesser than the depth required for the formation of hydraulic jump.

Protective measures. The protection works depend upon the nature of foundations. The following measures are provided.

(*i*) Providing cistern depressed below bed with sloping glacis upstream [Fig. 11.27(*a*)].

(*ii*) Providing a cistern excavated in the rock to a depth required for the formation of jump. Baffles may also be provided [Fig. 11.27(*b*)].

(*iii*) Providing a low secondary weir and baffles [Fig. 11.27(*c*)].

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(*iv*) Providing a sharply upturned bucket or *ski jump bucket* to deflect the jet of water away from the main structure [Fig. 11.27(*d*)], if sound rock is available at ground surface.

4. Protection works for condition IV. In this condition the jump height curve lies lower than the tail water curve at low discharges, and higher at high discharges, as shown in Fig. 11.25(*e*). Thus, at the low discharges, the jump will be drowned, while at high discharges, *insufficient* tail water depth will be there.

Protection measures. In such a case, a sloping apron partly above and partly below the bed is provided, with a sufficient length of horizontal apron and end sill. The jump will form higher up on the apron at low discharges and lower down at higher discharges, as shown in Fig. 11.28.

Protection works for condition V. This is just the reverse of condition IV. As illustrated in Fig. 11.25(*f*), the jump height curve lies above the T.W.C. at low discharge, and lower at high discharges. Thus at low discharges insufficient tail water will be there while at higher discharges the jumps will be drowned. The *protection measure* similar to that shown in Fig. 11.28 is provided. At low discharges, the jump will be formed lower down, while at high discharges, the jump will be formed higher up.

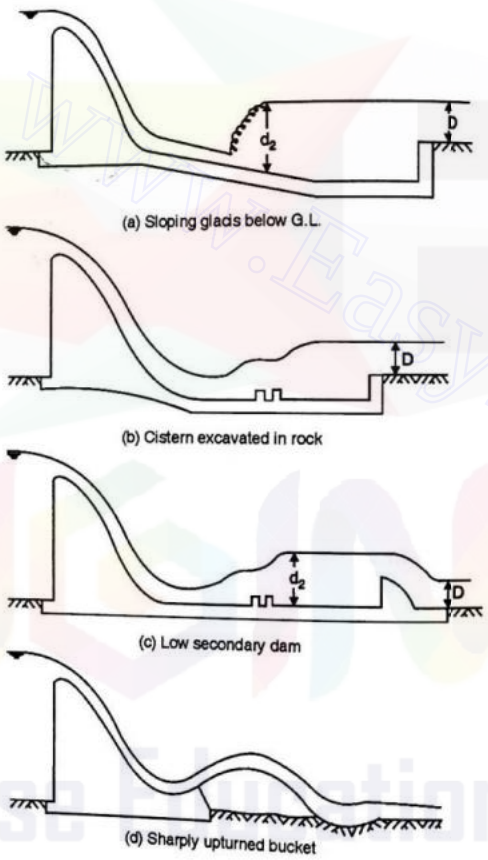

FIG. 11.27. PROTECTION WORKS FOR LOW TAIL WATER DEPTH.

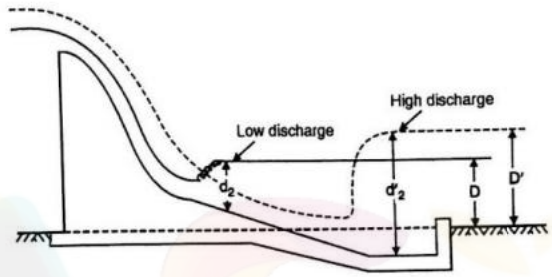

FIG. 11.28. SLOPING APRON PARTLY BELOW AND PARTLY ABOVE THE BED.

11.11. INDIAN STANDARD ON CRITERIA FOR DESIGN OF HYDRAULIC JUMP TYPE STILLING BASINS WITH HORIZONTAL AND SLOPING APRONS

1. Introduction. The Indian Standard IS : 4997 – 1968 lays down the criteria for the design of hydraulic jump type stilling basins of rectangular cross section with horizontal and sloping apron utilising various energy dissipators such as chute blocks, basin or floor blocks and end sill. *A stilling basin* is a structure in which all or part of the energy dissipating action is confined. In a stilling basin the kinetic energy first causes turbulence and is ultimately lost as heat energy. A *hydraulic jump type stilling basin* is basin in which dissipation of energy is accomplished basically by hydraulic jump which may be stabilised using chute blocks, basin blocks, end sill, etc. A hydraulic jump type stilling basin may be either with *horizontal apron* (Fig. 11.29 and 11.31) or with *sloping apron* (Fig. 11.37).

2. Hydraulic Jump Type Stilling Basin with Horizontal Apron.

When the tail water rating curve approximately follows the hydraulic jump curve, or is only slightly above or below it, then hydraulic jump type stilling basin with horizontal apron provides the best solution for energy dissipation. In this case the requisite depth may be obtained on a proper apron near or at the ground level so that it is quite economical. For spillways on weak bed conditions, end weirs, and barrages on sand and loose gravel, hydraulic jump type stilling basins are recommended.

Conjugate depths : Water depths at the beginning and the end of the hydraulic jump are related by the following formula, for horizontal aprons :

$$\frac{D_2}{D_1} = \frac{1}{2}\left[\left(\sqrt{1+8F_1^2} - 1\right)\right] \quad ...(11.18)$$

where F_1 is the Froude number of the flow at the beginning of the jump. In general, Froude number is given by the following formula :

$$F = \frac{V}{\sqrt{gD}} \quad ...(11.19)$$

Classification : Hydraulic jump type stilling basin with horizontal apron may be classified into the following two categories :

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(*a*) ***Basin type I*** : Stilling basin in which the Froude number of the incoming flow in less than 4.5. This case is generally encountered on weirs and barrages.

(*b*) ***Stilling basin type II*** : Stilling basin in which Froude number of the incoming flow is greater than 4.5. This case is general feature for dams.

Stilling basin type I. Fig. 11.29(*b*) shows the stilling basin type I. The factors involved in the design of stilling basin are : (*i*) determination of the elevation of basin floor, (*ii*) basin length , and (*iii*) basin appurtenances.

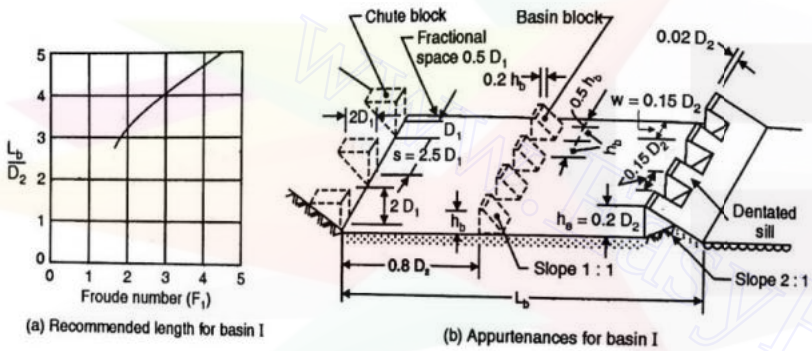

FIG. 11.29. STILLING BASIN TYPE I.

(*i*) ***Elevation of basin floor :***

Let H_L be the head loss in hydraulic jump and q be the discharge intensity.

To calculate H_L from the known upstream and downstream total energy lines, the following procedure may be adopted :

(*a*) Where the basin is directly downstream from the crest or where the chute is not longer than the hydraulic head, H_L may be assumed equal to the difference in the upstream and downstream total energy levels.

(*b*) Where the chute length exceeds the hydraulic head, D_1 should be first determined by taking H_L equal to the difference between the upstream and downstream total energy levels. Next calculate friction losses on the chute by Manning's formula and determine the more accurate value of H_L and consquently that of D_1 . The process may be repeated till the desired accuracy is achieved.

Knowing H_L and q, we can calculate D_c , D_1 and D_2 either from the following formulae or from Fig. 11.30.

$$H_L = \frac{(D_2 - D_1)^2}{4\,D_1\,D_2} \qquad ...(11.20)$$

$$D_2 = \frac{D_1}{2} + \sqrt{\frac{2q^2}{D_1 \cdot q} + \frac{D_1^2}{4}} \quad ; \quad D_c = \left(\frac{q^2}{g}\right)^{1/3}$$

Having obtained D_1 and D_2 the elevation of the basin floor may be calculated by either deducting the specific energy at section 1–1 (prejump) from the energy line at the section or that at section 2–2 (end of jump) from the downstream total energy line.

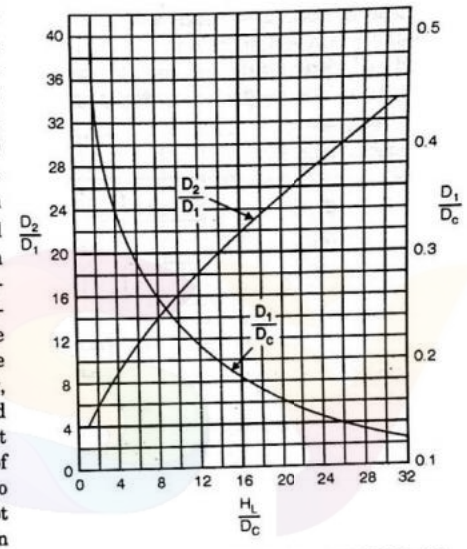

FIG. 11.30. CURVE FOR DETERMINATION OF SEQUENT DEPTH (HORIZONTAL APRON).

(ii) Basin length and depth

The length of the basin (L_b) will be determined from the curve given in Fig.11.29(*a*). The basin will be provided with an end sill, preferably dentated one. In the boulder reach the sloping face of the end sill will be kept on the upstream side. Generally, the basin floor should not be raised above the level required from sequent depth consideration. If the rising of the floor becomes obligatory due to site conditions, the same should not exceed 15 percent of D_2 and the basin in that case should be further supplemented by chute blocks and basin blocks. The basin blocks should not be used if the velocity of flow at the location of basin blocks exceeds 15 m/s and in that case the floor or the basin should be kept at a depth equal to D_2 below the tail-water level. The tail water depth should not generally exceed 10 percent of D_2 .

(*iii*) ***Basin Appurtenances***

The requirements of basin appurtenances, such as chute blocks, basin blocks, end sill are shown in Fig. 11.29(*b*).

Chute blocks : The chute blocks should be kept at a height equal to $2D_1$ on the glacis slope. Their top length should also be equal to $2D_1$. The width of the chute blocks should be kept equal to D_1 and their spacing as $2.5\,D_1$. A space equal to $\frac{D_1}{2}$ should be left along each wall.

Basin blocks : The height of the basin blocks in terms of D_1 may be obtained from Fig. 11.31(*c*). The width and spacing of the basin block should be equal to their height. The upstream face of all the basin blocks should be vertical and in one plane. A half space is recommended adjacent to the wall. The upstream face of the basin blocks should be set at a distance of $0.8D_2$ from the down stream face of the chute blocks.

End sill. The height of the dentated end sill is recommended as $0.2D_2$. The maximum width and spacing of dents shall be according to Fig. 11.29(*b*). A dent is

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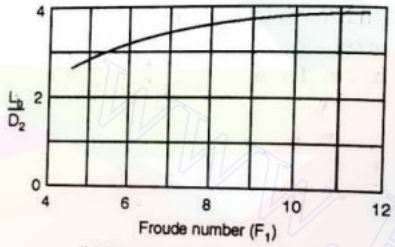
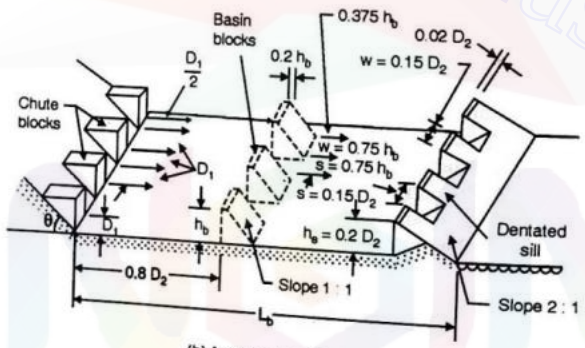
recommended adjacent to each side wall. In case of narrow basin, it is advisable to reduce the width and spacing but in the same proportion. It is not necessary to stagger the end sill dents with reference to chute blocks.

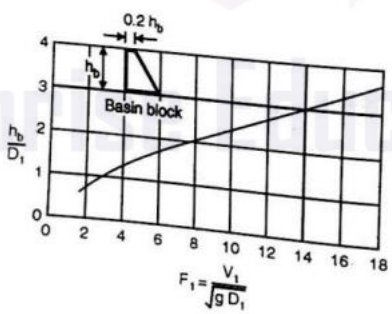
Stilling Basin Type II. Fig. 11.31(*b*) shows the stilling basin type II.



(a) Recommended length for basin II



(b) Appurtenances for basin



(c)

FIG. 11.31. STILLING BASIN TYPE II.

(*i*) ***Elevation of basin floor.*** Same procedure is followed as for basin I.

(*ii*) ***Basin length and depth.*** The length of the basin is determined from Fig. 11.31(*a*). The basin should be provided with chute blocks and end sill. The maximum raising of the basin floor shall not exceed 15 percent of D_2 and the basin in that case will be further supplemented by basin blocks. However, when the flow velocity at the location of basin blocks exceeds 15 m/s, no basin blocks are recommended and in that case the floor of the basin should be kept at depth equal to D_2 below the tail-water level. The tail water depth should not generally exceed 10 percent of D_2.

(*iii*) ***Basin appurtenances.*** Requirements of basin appurtenances, such as chute blocks, basin blocks and end sill are shown in Fig. 11.31(*b*).

Chute blocks. The height, width and spacing of the chute blocks should be kept equal to D_1. The width and spacing may be varied to eliminate frictional blocks. A space equal to $D_1/2$ is preferable along each wall.

Basin blocks. The height of basin blocks in terms of D_1 can be obtained from Fig. 11.31(*c*). The width and spacing should be kept three-fourth of the height. These should be placed at distance of $0.8D_2$, downstream from the chute blocks.

End sill. Same requirements as that for basin I.

3. Hydraulic Jump Type Stilling Basin with Sloping Apron

General. When the tail water is too deep as compared to the sequent depth D_2, the jet left at the natural ground level would continue to go as a strong current

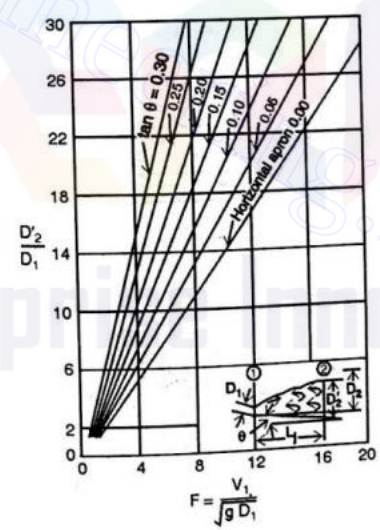


FIG. 11.32 DETERMINATION OF D_2' FOR BASIN III.

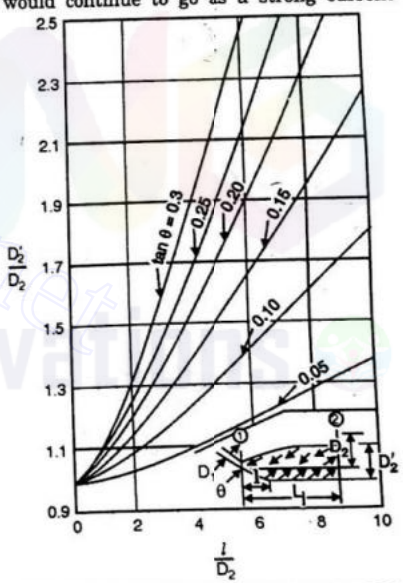


FIG. 11.33. DETERMINATION OF D_2' FOR BASIN IV.

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near the bed forming a drowned jump which is harmful to the river bed. In such a case, a hydraulic jump type stilling basin with sloping apron should be preferred as it would allow an efficient jump to be formed at suitable level on the sloping apron.

Conjugate depths. Water depths at the beginning and the end of the hydraulic jump are related by the following formula for a fully sloping apron:

$$\frac{D_2}{D_1} = \frac{1}{2\cos\theta}\left[\left(\frac{8F_1^2\cos^3\theta}{1-2K\tan\theta}+1\right)^{1/2} - 1\right] \quad ...(11.21)$$

where D_2' = conjugate depth sequent to D_1 for sloping apron, and may be determined from Fig. 11.32 (for basin III) or Fig. 11.33 (for basin IV).

K = Shape factor. It is defined as a dimensionless parameter which varies with Froude number and the slope of the apron. This has been plotted against slope in Fig. 11.34 on the assumption that it is independent of Froude number.

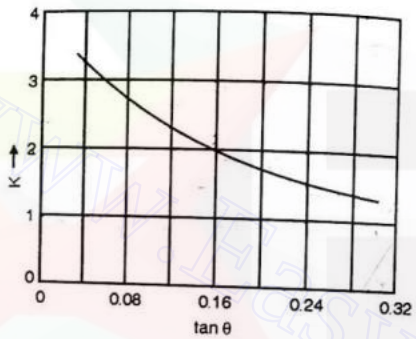

FIG. 11.34. DETERMINATION OF SHAPE FACTOR K.

Classification. The hydraulic jump on a sloping apron may occur in four different forms depending on the tail water conditions, as shown in Fig. 11.35. The action in cases (*c*) and (*d*) is same if it is assumed that horizontal floor begins at the end of jump of case (*d*). Case (*b*) is virtually case (*a*) operating with excessive tail-water

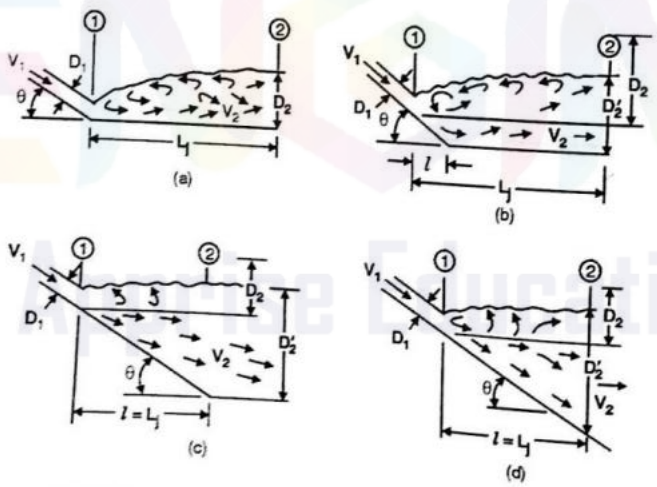

FIG. 11.35. DIFFERENT FORMS OF HYDRAULIC JUMPS ON SLOPING APRON.

depth. Case (*a*) has been dealt with under 'horizontal aprons'. Cases (*d*) and (*b*) hereafter will be known as basin III and IV respectively.

Basin III [Fig. 11.37(*a*)] is recommended for the case where tail water curve is higher than the D_2 curve at all discharges.

Basin IV [Fig. 11.37(*b*)] is suitable for the case where the tail water depth at maximum discharge exceeds D_2 considerably but is equal to or slightly greater than D_2 at lower discharges.

Design criteria. It is not possible to standardise design criteria for sloping aprons to the same extent as in case of horizontal apron. In this case, greater individual judgement is required. The slope and overall shape of the apron are determined from economic consideration, the length being judged by the type and soundness of the river bed downstream. The following design criteria should serve only as a guide in proportioning the sloping apron designs.

Basin type III. In the design of Basin III, the following procedure may be adopted (Fig. 11.37) :

(1) Assume a certain level at which the front of jump will form for the maximum tail-water depth and discharge.

(2) Determine D_1 from the known upstream total energy line by applying Bernoulli's theorem and calculate F_1. Then find out the conjugate depth D_2 from Eq. 11.15.

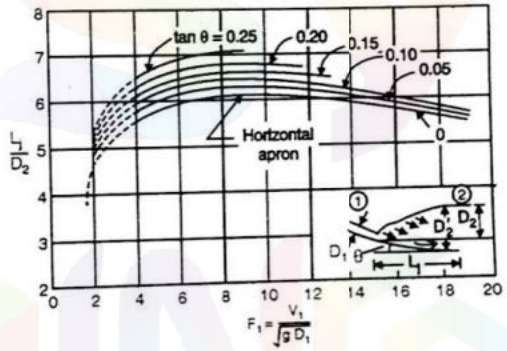

FIG. 11.36. LENGTH OF JUMP IN TERMS OF CONJUGATE DEPTH D_2 FOR BASIN III.

(3) Assume a certain slope and determine the conjugate depth D_2' and the length of the jump for the above Froude number from Fig. 11.32 and 11.36 respectively. The length of the apron should be kept equal to 60 per cent of the jump length.

(4) Test whether the available tail-water depth at the end of the apron matches the conjugate depth D_2'. If not, change the slope or the level of upstream end of the apron or both. Several trials may be required before the slope and the location of the apron are compatible with the hydraulic requirement.

(5) The apron designed for maximum discharge may then be tested at lower discharges, say $\frac{1}{4}$, $\frac{1}{2}$ and $\frac{3}{4}$. If the tail-water depth is sufficient, or in excess of the conjugate depth for the intermediate discharges, the design is acceptable. If not, a flatter slope at lower apron level should be tried or Basin IV may be adopted.

(6) The basin should be supplemented by a solid or dentated end sill of height (h_3) equal to 0.05 to 0.2 D_2 with an upstream slope to 2 : 1 to 3 : 1.

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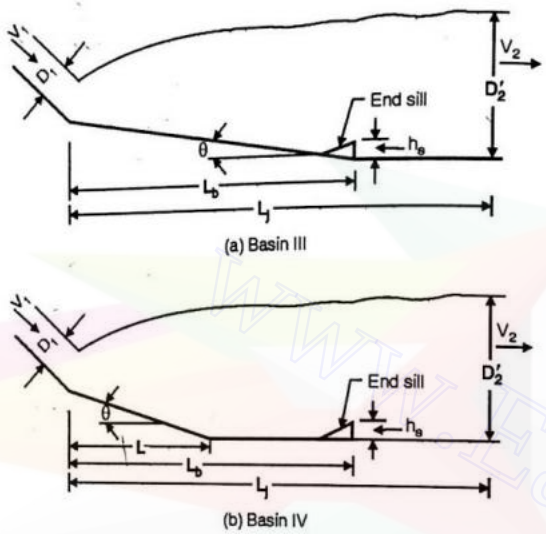

FIG. 11.37. STILLING BASIN TYPES III AND IV.

Basin type IV. In the design of basin IV, the following procedure may be adopted.

1. Determine the discharge at which the tail-water depth is most deficient.
2. For the above discharge, determine the level and length of the apron on the basis of criteria given for horizontal apron.
3. Assume a certain level at which the front of jump will form for the maximum tail-water depth and discharge.
4. Determine D_1 from the known upstream total energy line by applying Bernoulli's theorem and calculate F_1. Then find out conjugate depth D_2 from Eq. 11.15.
5. Determine a suitable slope (by trial and error) so that the available tail-water depth matches the required conjugate depth D_2' determind from Fig. 11.33.
6. Determine the length of jump for the above slope from Fig. 11.26. If the sum of lengths of inclined portion and horizontal portion is equal to about 60 per cent of jump length, the design is acceptable. If not, fresh trials may be done by changing the level of the upstream end of the jump formation.
7. The basin should be supplemented by a solid or dentated end sill of height 0.05 to 0.2 D_2 and upstream slope of 2 : 1 to 3 : 1.

11.12. SPILLWAY CREST GATES

As stated earlier, spillways are of two types : *controlled* and *uncontrolled*. Controlled spillway is the one the flow over which is controlled by using gates over its crest. By installing gates over spillways, additional storage can be made available, specially

during low water flow. However, during floods, these gates are removed or lifted so that full use of spillway capacity can be made. However, crest gates over spillways for earth dams should be used with caution since faulty operation may cause the rise in the reservoir water level and subsequent overtopping of the earth dam.

The following are some of the common types of gates used for spillways :

1. Flash boards, stop logs and needles.
2. Radial gates.
3. Drum gates.
4. Vertical lift gates.
5. Bear trap gates.
6. Rolling gates.

1. Flash Boards, Stop Logs and Needles

Flash boards : These are the temporary gates used only for small spillway of minor importance. Fig.11.38(*a*) shows two types of *flash boards*. They consist of wooden panels supported by pins on the edges. The pins are supported on pipe sockets along the crest of the dam. Flash board may be either *braced or hinged or automatic*. Automatic flash boards are designed in such a way that they fall automatically at a certain head.

Stop logs . They consist of horizontal timber planks spanning across piers having grooves. These planks are kept over one another as shown in Fig. 11.38(*b*), and can be removed either by hand or with the help of any hoisting equipment.

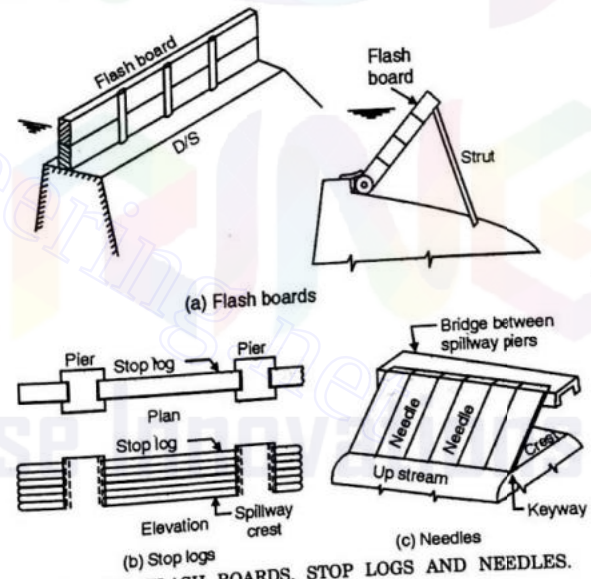

FIG. 11.38. FLASH BOARDS, STOP LOGS AND NEEDLES.

Needles : Needles consists of wooden planks kept in inclined position, with lower ends resting in a keyway on the spillway crest and upper ends at the top of a bridge girder, as shown in Fig. 11.38 (*c*).

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2. Radial Gates

A radial gate, also known as a *tainter gate* has its water supporting face, made of steel plates, in the shape of sector of a circle, properly braced and hinged at the pivot, as shown in Fig. 11.39. The gate can thus be made to rotate about fixed horizontal axis. The load of the gate and water etc. is carried on bearings, mounted on piers. The gate can be lifted by means of ropes and chains acting simultaneously at both ends or with the help of power driven winches.

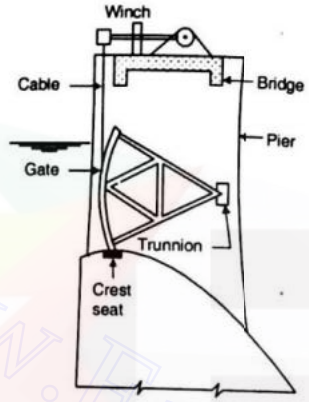

FIG. 11.39. TAINTER GATE.

3. Drum Gates

Drum gates are normally used for long span. Fig. 11.40 shows the U.S.B.R. type drum gate which consists of a circular sector in cross-section formed by skin plates attached to internal bearings. It is hinged at the centre of curvature in such a way that the entire sector may be raised above the crest or may be lowered so that upper surface becomes coincident with the crest line. The buoyant forces due to head water pressure underneath aid in its lifting. It is enclosed on all the three faces and at ends to form a water-tight vessel.

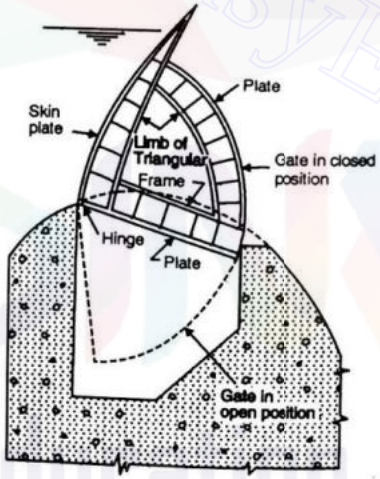

FIG. 11.40. U.S.B.R. DRUM GATE.

4. Vertical Lift Gates

A vertical gate consists of a frame work of skin plate at the u/s face along with beams and girder suitably placed. Vertical gates are rectangular in shape, and move vertically in their own plane. Because of the hydrostatic pressure, high frictional forces are developed in the guides. These frictional forces can be reduced by placing cylindrical rollers between the bearing surfaces of the gate and guides. There are several types of vertical lifts gates such as (*i*) Sliding gates, (*ii*) Fixed wheel gates, (*iii*) Stoney gates. In the case of *sliding gates*, the frame of the gate bears directly on the downstream guide members, the seal being formed in contact between the two. In the *fixed wheel type* the gates have fixed wheels mounted on each side to carry the water load to a vertical track on down-stream side of the gate groove. This reduces the frictional force and hence fixed wheel lift gates are the one most commonly adopted, for heights upto 15 m and spans of 15 m or more. The gate is hoisted by horizontal lifting beam, the ends of which travel in the gate guides.

Fig. 11.41 shows the plan and elevation of stoney gate. The moving train of rollers is substituted for fixed wheel. These rollers are independent of gates and guides, thus eliminating axle friction. Only rolling friction is developed. The rollers transmit the entire load from a bearing strip on the gate to a roller path on the guide.

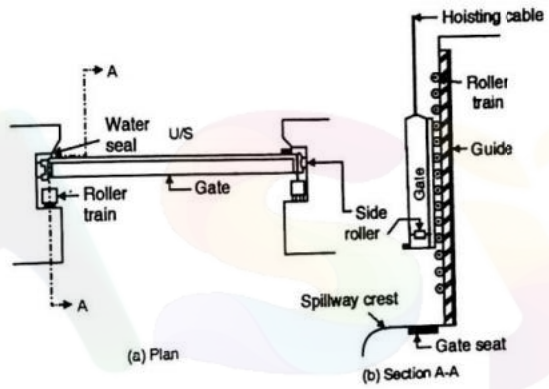

FIG. 11.41. STONEY GATES.

5. Bear Trap Gates

Fig. 11.42 (*a*) shows a bear trap gate consisting of two leaves of either timber or steel hinged to the dam. These gates are lifted up by admitting water to the space under the leaves. The downward or downstream leaf is often made hollow so that its buoyancy aids in the lifting operation. These gates are often used for low navigation dams.

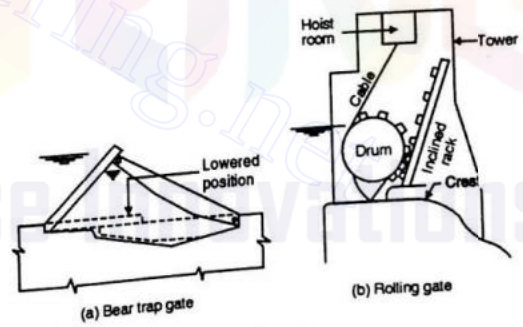

FIG. 11.42.

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6. Rolling Gate

Fig. 11.42(*b*) shows a rolling gate. It essentially consists of steel cylinder as large in diameter as the height of opening and spanning between piers. A heavy annular rim having gear teeth at its periphery encircles each end of the cylinder. Each pier has an inclined rack which engages the gear teeth. The gate is rolled up the inclined rack by means of pull from the hoisting cable operated from the hoist room. A cylindrical segment, attached to the lower portion of the gate, makes contact with spillway.

11.13. OUTLET WORKS

An outlet work serves to regulate to release water impounded by a dam. It provides a controlled release of water from the reservoir in such quantities and at such time as may be required. Outlet may be required for several purposes such as feeding water to power turbines, discharging heavily laden silty water, supplying water to irrigation channels, or for a combination of multipurpose requirements. In certain instances, the outlet works of a dam may be used in lieu of a service spillway in conjunction with an auxiliary or secondary spillway. An outlet works may also act as a flood control regulator, to release water temporarily stored in flood control storage space or to evacuate storage in anticipation of flood inflows.

An outlet works may consist of the following components :

1. Water way.
2. Control device, such as gates, valves etc.
3. Intake structure.

1. Water way. The water way for outlet structure may be either in the form of a pipe or tunnel that passes directly through the dam. The water way is also called *sluice way*. In the case of masonry or concrete dam, sluice way may pass through the body of the dam while in the case of earth dams, sluice way is generally placed outside the limits of the embankment. However in case it is essential to place the sluice way through the body of the earth dam, it should have projecting collars at regular intervals. These collars increase the path of water seeping along the boundary of the sluice way. The length of the seepage path should be more so that no damage is done to the earth embankment, since many failures of earth dams have been reported due to the seepage of water around sluice way. If there are n number of collars, each having x projection, then in order that the collars may increase the seepage path atleast by 25%, we have

$$2\,n\,x = 0.25L. \qquad \text{where } L \text{ is the length of the sluice way.}$$

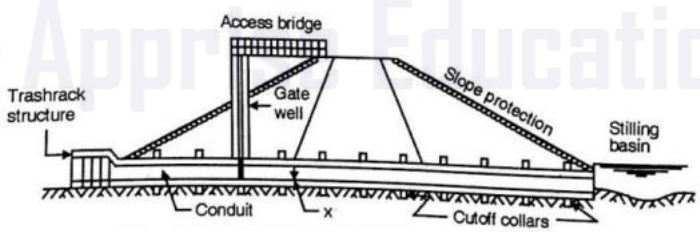

FIG. 11.43. OUTLET STRUCTURE THROUGH DAM.

The inlet of the sluice way must be at minimum reservoir level. However, in big dams outlets may be at different levels. The sluice way may be either circular or rectangular in shape. They must have bellmouth entry.

2. Control devices.

The sluice ways should have simple gates or valves at its entry. Cast iron slide gates, which may be used for control and guard gates are commercially available both for rectangular and circular openings and for design heads upto 15 m. However, simple radial gates may also be used for ordinary surface installation where the sluice way is in the form of an open channel. These gates may be operated either through as gate-well in the case of an earth dam (Fig. 11.43) or through a separate hoisting arrangement kept at top of a gravity dam, or sometimes through the gallery installed in case of gravity dam, as shown in Fig. 11.44.

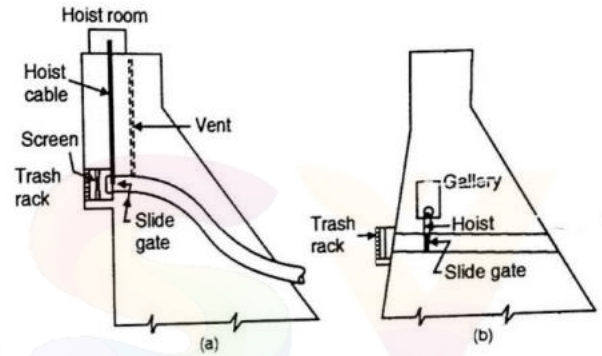

FIG. 11.44. CONTROL OF OUTLET GATES.

3. Intake structure.

An intake structure forms the entrance into the outlet works. It supports trash racks, fish screens and byepass devices, and accommodates the control device (gates and valves). An intake structure may either be submerged or extended as a tower to some height above the maximum reservoir water surface. Submerged intake structure is used when trash cleaning is ordinarily not required. In that case, it only serves as entrance to the outlet conduit. A tower intake is provided if an operating platform is needed for trash racking, maintaining and cleaning of fish screens, or installing stop logs.

Trash racks.

Trash rack is a structure which is provided at the entrance to the intakes and sluice ways, to prevent entrance to debris. These are made in the form of semi-polygonal grid of iron or steel bars, as shown in Fig. 11.45. Steel flats or bars are spaced from 5 to 15 cm centres, depending upon the maximum size of debris which can be permitted in the sluice way. A velocity of 0.5 m/sec is generally allowed when trash racks are inaccessible for cleaning, but it may be increased to 15 m/sec if it is accessible for cleaning purposes. The debris which is collected on the trash rack may be removed either manually or with the help of automatic power driven racks.

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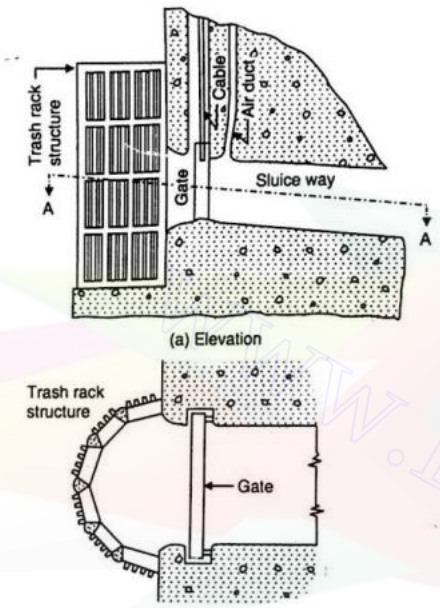

FIG. 11.45. TRASH RACK.

11.14. EXAMPLES FROM COMPETETIVE EXAMINATIONS

Example 11.6. *A round crested spillway passes a design discharge of 1 m^3/sec per metre length. The coefficient of discharge may be taken as $C_d = 0.7$. If height of the crest above the downstream stilling basin floor level is 10 m, design the (i) depth and (ii) length of the stilling basin. Depth of flow in the stream on the down stream of spillway is 1 m at the design discharge of 1 m^3/sec. Enquire if the bed of the stilling basin has to be depressed.* ***(Engg. Services Exam. 1987)***

Solutions : Given $q = 1$ cumec/m and $C_d = 0.7$

The discharge (q) over the round crested weir, per metre length is given by

$$q = \frac{2}{3} C_d \sqrt{2g}\, h^{3/2}$$

or

$$1 = \frac{2}{3} \times 0.7 \times \sqrt{2 \times 9.81}\, h^{3/2} \quad \text{from which } h = 0.62 \text{ m.}$$

$$\text{Average fall of water} = 10 + \frac{0.62}{2} = 10.31 \text{ m.}$$

Theoretical velocity of flow at the foot of the spillway

$$= \sqrt{2gH} = \sqrt{2 \times 9.81 \times 10.31} = 14.22 \text{ m/sec.}$$

Assuming coefficient of velocity $C_v = 0.9$,

Velocity of approach, $V_1 = 0.9 \times 14.22 = 12.80$ m/sec.

Depth of flow at the foot of spillway is given by

$$y_1 = \frac{q}{V_1} = \frac{1}{12.8} = 0.078 \text{ m}$$

Froude's Number at u/s, $F_1 = \dfrac{V_1}{\sqrt{g y_1}} = \dfrac{12.8}{\sqrt{9.81 \times 0.078}} = 14.62$

Since $F_1 > 1$, the flow is super-critical. Depth at the downstream side, $y_2 = 1$ m.

$$\therefore \quad V_2 = \frac{q}{y_2} = \frac{1}{1} = 1 \text{ m/sec.}$$

$\therefore$ Froude's number at d/s, $F_2 = \dfrac{V_2}{\sqrt{g y_2}} = \dfrac{1}{\sqrt{9.81 \times 1}} = 0.319 < 1$

Hence the downstream flow is sub-critical and hydraulic jump will be formed

$$\therefore \quad y_2 = \frac{y_1}{2}\left[\sqrt{1 + 8F_1^2} - 1\right] = \frac{0.078}{2}\sqrt{1 + 8(14.62)^2 - 1}\,] = 1.574 \text{ m.}$$

Depth available in the stream at d/s = 1.0 m only.

Hence the stilling basin is to be *depressed* by value $= 1.574 - 1.0 = \mathbf{0.574}$ m.

Length of stilling basin = 5 times $(y_2 - y_1)$.

Keep length of stilling basin $= 5(y_2 - y_1) = 5(1.574 - 0.078) \simeq \mathbf{7.5}$ **m.**

Example 11.7. *An overfall spillway shown in Fig. 11.46 passes a discharge of 7.83 m^3/sec/metre width with a fall of 12.5 m. Depth of water available on the down stream is 2.0 m. Calculate the leading dimensions of hydraulic jump stilling basin.* ***(Gate, 1988)***

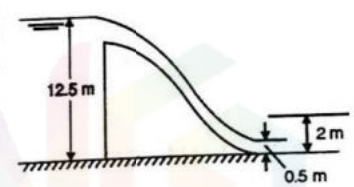

FIG. 11.46

Solution

Depth of flow at the foot of spillway $= y_1 = 0.5$ m

$$\therefore \text{ Velocity of flow, } \quad V_1 = \frac{q}{y_1} = \frac{7.83}{0.5} = 15.66 \text{ m/s}$$

$$\text{Froude number, } \quad F_1 = \frac{V_1}{\sqrt{g y_1}} = \frac{15.66}{\sqrt{9.81 \times 0.5}} = 7.07 \text{ m}$$

Since $F_1 > 1$, the flow is super-critical

At the downstream, $d = 2$ m and $V_2 = q/d = \dfrac{7.83}{2} = 3.915$ m/s

$$\therefore \text{ Froude number, } \quad F_2 = \frac{V_2}{\sqrt{gd}} = \frac{3.915}{\sqrt{9.81 \times 2}} = 0.884 \text{ m/s.}$$

Since $F_2 < 1$, the flow at the downstream is sub-critical, and hence hydraulic jump will be formed.

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Taking prejump depth $y_1 = 0.5$ m, the post jump depth y_2 is given by

$$y_2 = \frac{y_1}{2}\left[\sqrt{1+8F_1^2}-1\right] = \frac{0.5}{2}\left[\sqrt{1+8(7.07)^2}-1\right] = 4.76 \text{ m.}$$

Depth available at d/s = 2 m

Hence depression of the stilling basin below G.L. $= 4.76 - 2 = \mathbf{2.76\ m}$

Length of stilling basin = 5 times height of jump.

Keep length $= 5(y_2 - y_1) = 5(4.76 - 0.5) \triangleq \mathbf{21.5\ m}$

Example 11.8. *A vertical lift gate 5 m × 2.5 m size weighing 0.5 tonnes slides along guides (coefficient of friction 0.25) fitted on the side walls of an overflow spillway and its crest. What force will have to be exerted at the hoisting mechanism to lift the gate when the head of water over crest is 2 m.* ***(Engg. Services Exam. 1998)***

Solution : Mass of gate = 500 kg = 500 × 9.81 N

Total hydrostatic force exterted by water on the gate is

$F = wg\,A\,\bar{h}$

where A = immersed area of gate $= 2 \times 2.5 = 5\ \text{m}^2$

$\bar{h}$ = depth of c.g. of area below the free surface $= 2/2 = 1$

$\therefore \quad F = 1000 \times 9.81 \times 5 \times 1 = 49.05 \times 10^3$ N = 49.05 kN

Coefficient of friction between gate and guide = 0.25

$\therefore$ Downward frictional force

$= \mu F = 0.25 \times 49.05 = 12.2625$ kN

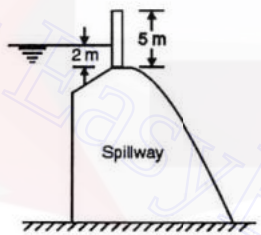

FIG. 11.47

$\therefore$ Total downward force

= force due to self wt. of gate + downward frictional force

$= (500 \times 9.81 \times 10^{-3}) + 12.2625 = \mathbf{17.1675\ kN}$

Example 11.9. *At an energy dissipater structure below a low spillway, the discharge is 19 m³/s and the energy loss is 1 m at hydraulic jump forming therein. Determine the depths of flow at both ends of the jumps.* ***(Engg. Services Exam. 2002)***

Solution : Given $q = 19\ \text{m}^3/\text{s}$ and $\Delta E = 1$ m.

We have $\quad \Delta E = \dfrac{(y_2 - y_1)^3}{4\,y_1\,y_2}$

Hence $\quad \dfrac{\Delta E}{y_1} = \dfrac{\left[\dfrac{y_2}{y_1} - 1\right]^3}{4\,(y_2/y_1)} = \dfrac{(x-1)^3}{4x}$ where $x = y_2/y_1$

or $$\frac{1}{y_1} = \frac{(x-1)^3}{4x} \qquad ...(1)$$

Also, $$\frac{y_2}{y_1} = \frac{1}{2}\left[-1 + \sqrt{1 + \frac{8q^2}{g\,y_1^3}}\right]$$

or $$x = \frac{1}{2}\left[-1 + \sqrt{1 + \frac{8 \times 19^2}{9.81\,y_1^3}}\right] = \frac{1}{2}\left[-1 + \sqrt{1 + \frac{294.39}{y_1^3}}\right] \qquad ...(2)$$

Substituting the value of $\dfrac{1}{y_1}$ from (1) into (2), we get

$$x = \frac{1}{2}\left[-1 + \sqrt{1 + \frac{294.39\,(x-1)^9}{64\,x^3}}\right] = \frac{1}{2}\left[-1 + \sqrt{1 + \frac{4.6\,(x-1)^9}{x^3}}\right] \qquad ...(3)$$

Solving Eq. (3) by trial and error we get $x = 2.806$

Hence $$y_1 = \frac{4x}{(x-1)^3} = \frac{4 \times 2.806}{(2.806-1)^3} = \mathbf{1.905\ m}$$

and $$y_2 = x y_1 = 2.806 \times 1.905 = \mathbf{5.347\ m}$$

PROBLEMS

1. What is a spillway ? What are its functions ? Enumerate various types of spillways.
2. Write a note on ogee-shaped spillway.
3. What is a siphon spillway ? Sketch a saddle siphon spillway and explain the functions of its various component parts.
4. Derive expression for discharge through a saddle siphon spillway. Also, find maximum negative pressure developed at the centre of the throat of saddle siphon spillway.
5. What do you understand by 'priming' and 'depriming' ? Describe various devices used for early priming in saddle siphon spillway.
6. Explain the qualities of a good siphon.
7. Describe, with the help of sketch, the working of a volute siphon.
8. Compute the discharge over an ogee spillway with a coefficient of discharge $C = 2.5$ at a head of 4 m. The effective length of the spillway is 100 m. Neglect the velocity of approach. [**Ans** 200 cumecs]
9. Discuss various methods used for energy dissipation below spillways.
10. A siphon spillway has the following cross-section at its throat : height of throat = 1.5 m and width = 4 m. At the design flow, the tail water is 2 m above the summit. Taking a coefficient of discharge as 0.6, determine the capacity of the siphon. (*a*) Determine the head that would be required on an ogee spillway 4 m long to discharge this flow if coefficient of discharge is 2.25. (*b*) What length of this ogee weir would be required to discharge the same flow with a head of 2 m on the crest ? [Ans. (*a*) 48 cumecs, (*b*) $h = 3.05$ m, (*c*) 7.75 m]
11. Explain the design procedure for the standard stilling basin type *I*.
12. Explain with the help of a sketch, the components and working of a tainter gate.
13. Write a note on U.S.B.R. drum gate.

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12

Diversion Headworks

12.1. INTRODUCTION

Any hydraulic structure which supplies water to the off-taking canal is called a *headwork*. Headworks may be divided into two classes :

1. Storage headwork.
2. Diversion headwork.

A *storage headwork* comprises the construction of a dam across the river. It stores water during the period of excess supplies in the river and releases it when demand overtakes available supplies.

A *diversion headwork* serves to divert the required supply into the canal from the river. A diversion headwork serves the following *purposes*.

1. It raises the water level in the river so that the commanded area can be increased.
2. It regulates the intake of water into the canal.
3. It controls the silt entry into the canal.
4. It reduces fluctuations in the level of supply in the river.
5. It stores water for tiding over small periods of short supplies.

A diversion headwork can further be sub-divided into two principal classes :

1. Temporary spurs or bunds.
2. Permanent weirs and barrages.

Temporary spurs or bunds are those which are temporary and are constructed every year after the floods. However, for important works, weirs or barrages are constructed since they are of permanent nature if designed properly.

Weir. The weir is a solid obstruction put across the river to raise its water level and divert the water into the canal (Fig. 12.1). If a weir also stores water for tiding over small periods of short supplies, it is called a *storage weir*. The main difference between a storage weir and a dam is only in height and the duration for which the supply is stored. A dam stores the supply for a comparatively longer duration.

Barrage. The function of a barrage is similar to that of weir, but the heading up of water is effected by the gates alone (Fig. 12.2). No solid obstruction is put across the river. The crest level in the barrage is kept at a low level. During the floods,

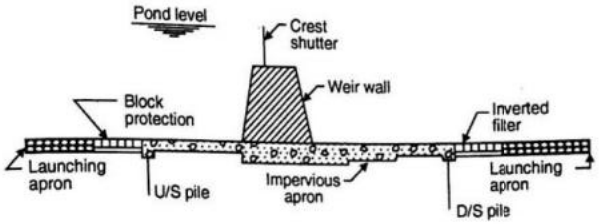

FIG. 12.1. VERTICAL DROP WEIR.

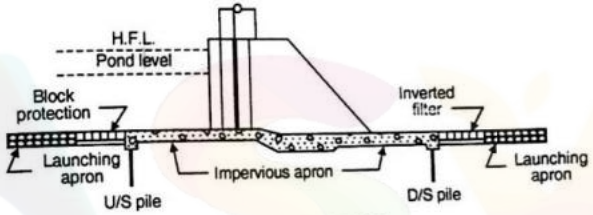

FIG. 12.2. BARRAGE.

the gates are raised to clear off the high flood level, enabling the high flood to pass downstream with minimum afflux. When the flood recedes, the gates are lowered and the flow is obstructed, thus raising the water level to the upstream of the barrage. Due to this, there is less silting and better control over the levels. However, barrages are much more costlier than the weirs.

12.2. COMPONENT PARTS OF A DIVERSION HEADWORK

A diversion headwork consists of the following component parts (Fig. 12.3) :

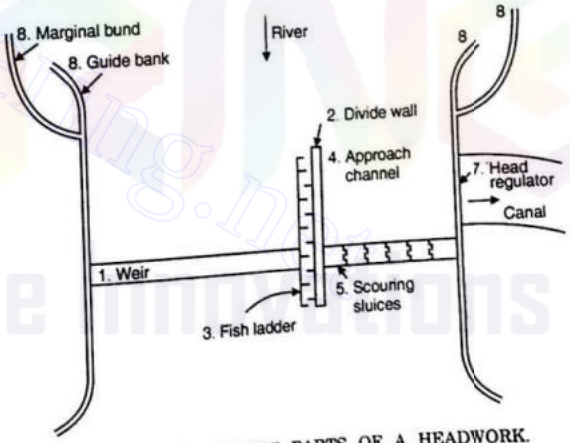

FIG. 12.3. COMPONENT PARTS OF A HEADWORK.

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1. Weir or barrage
2. Divide wall or divide groyne
3. Fish ladder
4. Pocket or approach channel
5. Scouring sluices
6. Silt prevention devices
7. Canal head regulator
8. River training works (Marginal bunds and guide banks)

The description and design details of these parts are discussed in the following articles.

12.3. THE WEIR

Weirs are classified into two heads, depending upon the criterion of the design of their floors.

1. Gravity weirs 2. Non-gravity weirs

A gravity weir is the one in which the uplift pressure due to the seepage of water below the floor is resisted entirely by the weight of floor. In the non-gravity type, the floor thickness is kept relatively less, and the uplift pressure is largely resisted by the bending action of the reinforced concrete floor.

Depending upon the material and certain design features, gravity weir (or simply weirs) can further be sub-divided into the following types :

1. Vertical drop weir.
2. Sloping weir :
   (*a*) Masonry or concrete slope weir
   (*b*) Dry stone slope weir.
3. Parabolic weir.

1. **Vertical drop weir** (Fig. 12.1). A vertical drop weir consists of a vertical drop wall or crest wall, with or without crest gates. At the upstream and downstream ends of the impervious floor, cutoff piles are provided. To safeguard against scouring action, launching aprons are provided both at upstream and downstream end of the floor. A graded inverted filter is provided immediately at the downstream end of the impervious floor to relieve the uplift pressure. Vertical drop weirs are suitable for any type of foundation.

2. **Masonry or Concrete Sloping Weir** [Fig.12.4(*a*)]. Weirs of this type are of recent origin. They are suitable for soft sandy foundations, and are generally used where the difference in weir crest and downstream river bed is limited to 3 metres, When water passes over such a weir, hydraulic jump is formed on the sloping glacis.

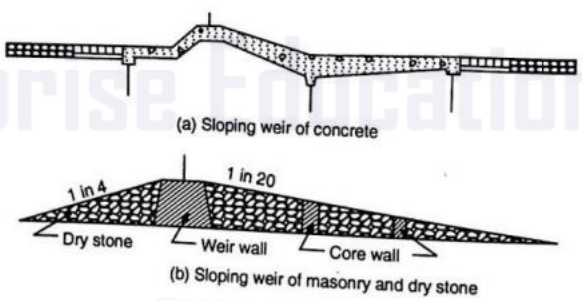

FIG. 12.4. SLOPING WEIRS.

3. **Dry stone slope weir** [Fig. 12.4(*b*)]. A dry stone weir or a rockfill weir consists of a body wall (or weir wall) and upstream and downstream rockfills laid in the form of glacis, with few intervening core walls. Okhla weir on Yamuna river, near Delhi, is the example of such weir.

4. **Parabolic Weir** (Fig. 12.5). A parabolic weir is similar to the spillway section of a dam. The body wall for such a weir is designed as a low dam. A cistern is provided at the downstream side to dissipate the energy. The upstream and down-stream protection works are similar to that of a vertical drop or sloping glacis weir.

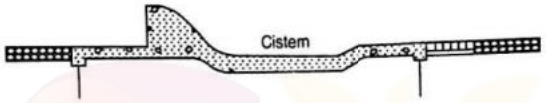

FIG. 12.5. PARABOLIC WEIR.

12.4. LOCATION OF HEADWORKS

A river, in its course, is divided into four distinct regions or stages :

(*i*) *The torrential, rocky or mountaineous stage* : The river in this stage has very steep bed slope and high velocity.

(*ii*) *The sub-mountaineous or boulder stage* : The sides and bed of the river are composed of boulder and gravel. There is strong subsoil flow in this region.

(*iii*) *Trough stage or alluvial plain* : The cross-section of the river is made of alluvial sand and silt. The bed slopes are small and velocities are gentle.

(*iv*) *Delta stage* : The first and last stages are unsuitable for construction of weirs. The choice of site selection lies between the boulder stage and the trough stage. The advantages and disadvantages of the first three stages are given below :

(*a*) **Rocky Stage**

(*i*) ***Disadvantages of Rocky Stage***

1. The soil, suitable for irrigation, must have good depth. Such a soil is not available in the rocky stage. This results in a *long idle length of the canal* from the headworks to the plains where good soil is available.
2. A canal taking off from a headwork situated in rocky stage will have to cross more discharge lines due to uneven mountaineous ground. Hence, *more cross drainage works* are required.
3. The ground in this region has steep gradients. Hence, either the canal bed will have to be lined to permit high velocities or *more falls are necessary* to dissipate the energy.
4. A shingle-excluding regulator is necessary if the headwork is situated on the rocky stage. This requires a *costly head regulator.*
5. The shingle crossing over the weir may damage it. This requires *frequent repairs* of the weir.

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6. The high velocity associated with the rocky stage requires *use of better material,* increasing the cost of construction.
7. The water in this region does not contain silt and is thus *devoid of fertilising materials.*
8. The rivers in mountaineous regions are more flashy, resulting in floods appearing and disappearing suddenly. This causes *frequent failures.*

(ii) *Advantages of Rocky Stage :*

1. Good rocky foundations are available at surface or shallow depths. Hence, cost of construction of weir is less.
2. High heads are available for hydroelectric work.
3. There are no chances of the supply channel getting silted, since the steep slopes give rise to high velocities.
4. Comparative silt free water is fed to the turbines.

(d) Sub-mountaineous or Boulder stage

(i) *Disadvantages*

1. There is *strong subsoil flow.* This decreases the storage and may cause damage to the floor downstream.
2. As in rocky stage, long *idle length of canal* is required.
3. There will be *more percolation losses* from the canal taking off in boulder tracts. This will increase the cost per cumec per net supply.
4. *More cross-drainage works* are required.
5. In the boulder stage, *rivers are more flashy* than in trough stage.
6. There is less demand of water at head reaches.

(ii) *Advantages of Boulder Stage :*

1. Cash crops like transplanted rice, tobacco, sugar, tea etc., can be grown in the sub-mountaineous regions, where there was no irrigation earlier.
2. Cost of headworks is less due to availability of local material.
3. Less river training works are required.
4. Falls can be utilized for power generation.

(e) Trough stage

(i) *Disadvantages of trough reach :*

1. Cost of headworks more due to poor foundation.
2. More river training works are required.
3. There is problem of silt in the canal.

(ii) *Advantages of trough reach :*

1. Subsoil flow is comparatively less.
2. The head works constructed in this region serve large area than a weir in boulder region can. There is no idle length of canal.

3. The tracts lying at the foot of hills do not require water with the same urgency as those lower down. Consequently headworks in trough regions are more remunerative as the cultivators will be willing to pay higher assessment.
4. The cross-drainage works across the canals are less.
5. The water contains silt and other fertilising material.

12.5. EFFECTS OF CONSTRUCTION OF A WEIR ON THE REGIME OF RIVER

The weir is an obstruction thrown in the path of water. Due to its construction, therefore, the regime of river is affected in the following ways :

1. The silt supporting power of a river or channel mainly depends upon the hydraulic slope. When the weir is constructed, the heading up of water leads to flattening of the surface slope on the upstream side.

2. Due to decrease in the water surface slope, the silt carrying capacity is decreased, and the bulk of silt charge of the river water deposits in the pond, leading to the formation of irregular shoals at u/s of the weir.

3. Also due to silt excluding devices provided at the head regulator, the canal takes less silt. This results in further deposition of silt in the pond.

4. The water passing over the weir and through the scouring sluices now contains a deficient silt charge because much of it has been deposited u/s. In order to maintain a constant silt charge, the flowing water at the d/s scours the bed. This results in a progressive degradation or retrogression of bed levels downstream. The retrogression may undermine the stability of a work by an increase in exit gradient beyond safe limits. During high floods, the retrogression of bed may be from 0.3 to 0.5 m, while at low water levels, it may be as high as 1.2 to 2 metres.

5. As the silting and consequent shoals formation at u/s increases, the resistance to flow of river is increased due to tortuous route the water has to take about shoals. To overcome this resistance, increased head is required. The river starts regaining its original slope, and the afflux is extended more and more to the u/s. A stage is then reached when the upstream section of river cannot take up any more silt, and the normal silt charge is passed on the downstream side. The silt excluding device will also discharge more silt downstream.

6. Due to this, river below the weir will carry an excessive silt charge with a lower discharge. This will result in progressive silting up downstream, an increase in tortuosity and, therefore, a recovery of bed levels downstream. The process of recovery of downstream bed levels after the initial retrogression is slow and steady, and it may take 20 to 30 years to regain the original bed slope. The recovery of levels to the d/s may lead loss of control on the silt regulation. Hence, sufficient margin should be provided between canal F.S.L. and the pond level so that the crest level of the head regulator can be increased in the event of necesssity.

12.6. CAUSES OF FAILURE OF WEIRS AND THEIR REMEDIES

A weir may fail due to the following reasons :

(i) Piping

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(*ii*) Rupture of floor due to uplift

(*iii*) Rupture of floor due to suction caused by standing wave

and (*iv*) Scour at the upstream and downstream side of the weir floor.

1. Piping. Water seeps under the base of the weirs founded on permeable soils. When the flow lines emerge out at the d/s end of the impervious floor of the weir, the hydraulic gradient or the *exit gradient* may exceed a certain critical value for the soil. In that case, the surface soil starts *boiling* and is washed away by percolating water. With the removal of the surface soil, there is further concentration of flow lines into the resulting depression and still more soil is removed. This process of erosion thus progressively works backwards towards the upstream and results in the formation of a channel or a pipe underneath the floor of the weir, causing its failure.

Remedies. Piping failures can be prevented by :

(*i*) Providing sufficient length of the impervious floor so that path of percolation is increased and the exit gradient is decreased.

(*ii*) Providing pile at downstream end.

2. Rupture of Floor Due to Uplift. If the weight of floor is insufficient to resist the uplift pressure, the floor may burst and effective length of impervious floor is thereby reduced. The final failure, however, is due to the reduction of the effective length with the consequent increase in the exit gradient. Examples of such failures are Khanki weir on Chenab and Narora weir on the Ganga.

Remedies. Failures due to rupture of floor may be prevented by :

(*i*) Providing impervious floor of sufficient length.

(*ii*) Providing impervious floor of appropriate thickness at various points, and

(*iii*) Providing pile at the upstream end so that the uplift pressure to the down stream (d/s) is reduced.

3. Rupture of Floor Due to Suction Caused by Standing Wave.

The standing wave or hydraulic jump formed at the d/s of the weir causes suction which also acts in the direction of uplift pressure. If the floor thickness is insufficient, it may fail by rupture. Examples of such failures are Marala weir on the Chenab and Rasul weir.

Remedies. Failures can be prevented by :

(*i*) Providing additional thickness of floor to counterbalance the extra pressure due to the standing wave.

(*ii*) Constructing the floor thickness in one concrete mass instead of in masonry layers.

4. Scour on the Upstream and Downstream of the Weir. When the natural waterway of a river is contracted, the water may scour the bed both at upstream and downstream of the structure. The scour holes so formed may progress towards the structure, causing its failure. Examples of such failures are Islam weir and Deoha weir.

Remedies. Such failures can be prevented by :

(*i*) Taking the piles at upstream and downstream ends of the impervious floor, much below the calculated scour level.

(*ii*) Providing suitable length and thickness of launching aprons at u/s and d/s sides, so that stones of the aprons may settle in the scour holes.

12.7. DESIGN OF IMPERVIOUS FLOOR FOR SUB-SURFACE FLOW

We have already seen that the sub-surface flow or the foundation seepage may cause harm in two ways (*i*) piping, and (*ii*) uplift. Following the damage to Khanki weir in 1895, a group of Anglo Indian Engineers under the leadership of Colonel J. Clibborn, Dean of the Roorkee College and J.S. Beresfored, Inspector General of Irrigation, India, carried out experiments and confirmed the Darcy's law for seepage through granular soils. Later, in 1912, Bligh advanced a theory for the sub-surface flow and published it in his book, *'The practical design of Irrigation Works'*. In 1932, after analysing about 200 dams all over the World, Lane evolved his *weighted creep theory*. Scientific study of the sub-surface flow was, however, made by Pavlovsky (1922), and Khosla and his associates (1936).

BLIGH'S CREEP THEORY. The design of the impervious floor, or the *apron* is directly dependent on the possibilities of percolation in the porous soil on which the apron is built. Bligh assumes as an approximation that the hydraulic slope or gradient is *constant* throughout the impervious length of the apron. He further assumed the percolating water to *creep* along the contact of the base profile of the apron with the sub-soil, losing head enroute, proportional to the length of its travel. He designated the length of the travel as the 'creep length', which is the sum of horizontal as well as vertical length of creep. Bligh asserted that no amount of sheet piling or another cut-off could ever stop the percolation unless the cut-off extends upto the impermeable sub-soil strata. Thus, according to Bligh's theory, the total *creep length L* for the case of Fig. 12.6(*a*) is

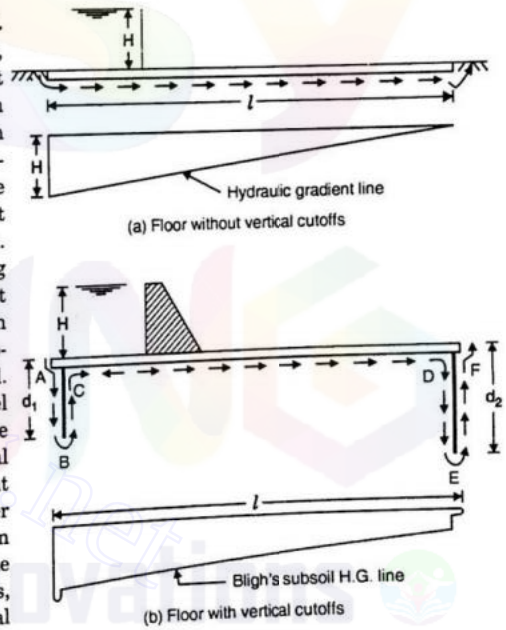

FIG. 12.6. BLIGH'S CREEP THEORY.

$$L = l$$

For the case of Fig. 12.6(*b*), the total creep length is

$$L = 2\,d_1 + l + 2\,d_2$$

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This means that in calculating the length of creep, the depth of every cutoff (*i.e.* vertical creep) is multiplied by the coefficient 2.

If H is the total loss of head, the *'loss of head per unit length of the creep'* (c) would be

$$c = \frac{H}{2d_1 + l + 2d_2} = \frac{H}{L} \quad ...(12.1)$$

He called the ' loss of head per unit length of creep' as *percolation coefficient*. The reciprocal of this $(i.e.\ L/H)$ is called the *coefficient of creep (C)* and Bligh assigned its safe values for different soils, as given in Table 12.1.

TABLE 12.1. COEFFICIENT OF CREEP

Type of soil	*Value of* $C = 1/c$
1. Light sand and mud (as in Nile)	18
2. Fine micaceous sand (as in Northern India rivers)	15
3. Coarse grained sand (as is Central and South India)	12
4. Boulders or shingle, gravel and sand mixed	5 to 9

Design Criteria : Bligh gave two criteria for design :

(i) *Safety against piping* . The length of creep should be sufficient to provide a safe hydraulic gradient according to the type of the soil.

Thus, the safe creep length is given by

$$L = CH \quad ...(12.2)$$

where C = coefficient of creep = $1/c$.

(ii) *Safety against uplift pressure* . Let h' be uplift pressure head at any point of the apron.

The uplift pressure $= wh'$ ...(i)

If t = thickness of the floor at the point

and ρ = specific gravity of the floor material

Then, downward force (resisting force) per unit area $= t\,w\,\rho$ ...(ii)

Equating the two, we get

$$wh' = tw\rho \quad ...(iii)$$

or

$$h' = t\rho$$

$$\therefore \quad h' - t = t\rho - t = t(\rho - 1)$$

From which

$$t = \frac{h' - t}{\rho - 1} = \frac{h}{\rho - 1} \quad ...[12.3(a)]$$

where h = ordinate of the hydraulic gradient line measured above the top of the floor.

Providing a factor of safety of 4/3, we have

$$t = \frac{4}{3}\frac{h}{\rho - 1} \quad ...(12.3)$$

Eq. 12.3 gives the floor thickness t at point of the apron where the uplift pressure ordinate, measured above the top of the floor is h.

For the portion of the floor to the upstream of the barrier, only nominal thickness need be provided since the weight of the water will counterbalance the uplift pressure. It is clear from Fig. 12.7 that out of the total impervious length (l) of the floor, if greater length is provided to the upstream of the barrier, the design will be comparatively economical. However, as we shall see later, a certain minimum length of the impervious floor is always necessary to the d/s of the barrier.

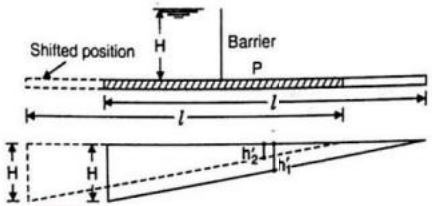

FIG. 12.7 EFFECT OF SHIFTING THE FLOOR RELATIVE TO THE BARRIER.

Limitations of Bligh's Theory

1. Bligh made no distinction between horizontal and vertical creep.
2. Bligh's method holds good so long as the horizontal distance between the pile lines is greater than twice their depth.
3. Bligh did not explain the idea of exit gradient. The safety against undermining cannot simply be obtained by considering a flat average gradient but by keeping this gradient well below critical.
4. Bligh makes no distinction between outer and inner faces of sheet piles or the intermediate sheet pile, whereas according to investigations, the outer faces of the end sheet piles are much more effective than inner ones. Also, intermediate sheet piles of shorter length than the outer ones are ineffective except for local redistribution of pressure.
5. Loss of head does not take place in the same proportion as creep length. Also, the uplift pressure distribution is not linear but follows a sine curve.
6. Bligh does not specify the absolute necessity of providing a sheet pile at d/s end whereas it is absolutely essential to have a deep vertical cut off at d/s end to prevent undermining.

LANE'S WEIGHTED CREEP THEORY

Based on statistical investigations of as many as 278 dams, weirs and barrages all over the World, Lane observed that vertical creep is more effective than the horizontal creep. He therefore modified Bligh's creep theory by evolving *'Lanes Weighted creep theory'*. According to this theory, the *weighted creep length* (L_w) is given by

$$L_w = \frac{1}{3}l + V \quad ...(12.4)$$

where l = the sum of all horizontal contacts and all the sloping contacts having slope less than 45°

V = sum of all the vertical contacts and all the sloping contacts steeper than 45°

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Thus, in Fig. 12.6 (*b*)

$$L_w = \frac{1}{3} l + 2 d_1 + 2 d_2$$

To ensure safety against piping, lane suggested that the weighted creep length must not be less than the following :

$$L_w = C_w \sqrt{H} \qquad \text{...(12.4 a)}$$

where L_w = weighted creep length given by Eq. 12.4

C_w = Lane's creep coefficient, the value of which depends on the type of soil (Table 12.2).

TABLE 12.2 LANE'S CREEP COEFFICIENT

S.N.	Type of soil	Lane's creep coefficient C_w	Lane's safe Hydraulic gradient $(1/C_w)$
1.	Very fine sand or silt	8.5	1/8.5
2.	Fine sand	7.0	1/7.0
3.	Medium sand	6.0	1/6.0
4.	Coarse sand	5.0	1/5.0
5.	Fine Gravel	4.0	1/4.0
6.	Medium Gravel	3.5	1/3.5
7.	Coarse gravel including cobbles	3.0	1/3.0
8.	Boulders with same cobbles and gravel	2.5	1/2.5
9.	Soft clay	3.0	1/3.0
10.	Medium clay	2.0	1/2.0
11.	Hard clay	1.8	1/1.8
12.	Very hard clay or hard pan	1.6	1/1.6

However, his theory, though an improvement over the Bligh's creep theory, is empirical and lacked the background for a rational basis of design.

Example 12.1. *Fig. 12.8 shows the section of a hydraulic structure founded on sand. Calculate the average hydraulic gradient. Also, find the uplift pressures at points 6, 12 and 18 m from the u/s end of the floor and find the thickness of the floor at these points taking* $\rho = 2.24$.

Solution

Total length of creep $= (2 \times 6) + 22 + (2 \times 8) = 50$

Hydraulic gradient $= \frac{4}{50} = \frac{1}{12.5}$

(i) Uplift pressure at a point A, 6 m from u/s

Length of creep upto $A = (6 \times 2) + 6 = 18$ m

$\therefore$ Unbalanced head $= h_1 = 4\left(1 - \frac{18}{50}\right) = \mathbf{2.56}$ **m**

$\therefore$ Uplift pressure $= wh_1 = 9.81 \times 2.56 = \mathbf{25.11}$ **kN/m²**

Thickness $t = \frac{4}{3} \frac{h_1}{\rho - 1}$

$= \frac{4}{3} \times \frac{2.56}{2.24 - 1} = \mathbf{2.76}$ **m**

(ii) Uplift pressure at point B, 12 m from u/s.

Length of creep upto B

$= (6 \times 2) + 12 = 24$ m

Unbalanced head

$h_2 = 4\left(1 - \frac{24}{50}\right) = \mathbf{2.08}$ **m**

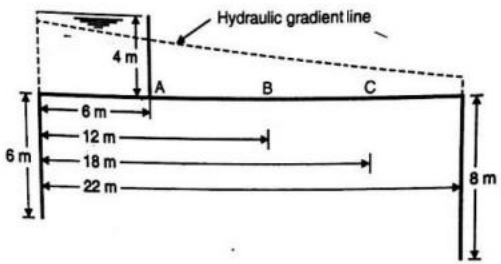

FIG. 12.8. BLIGH'S METHOD.

$\therefore$ Uplift pressure $= wh_2 = 9.81 \times 2.08 = \mathbf{20.4}$ **kN/m²**

Thickness $t = \frac{4}{3} \frac{h_2}{\rho - 1} = \frac{4}{3} \frac{2.08}{2.24 - 1} = \mathbf{2.23}$ **m**

(iii) Uplift pressure at point C, 18 m from u/s.

Length of creep upto $C = (6 \times 2) + 18 = 30$ m

$\therefore$ Unbalanced head $h_3 = 4\left(1 - \frac{30}{50}\right) = 1.6$ m

$\therefore$ Uplift pressure $= wh_3 = 9.81 \times 1.6 = \mathbf{15.7}$ **kN/m²**

Thickness $t = \frac{4}{3} \frac{h_3}{2.24 - 1} = \frac{4}{3} \times \frac{1.6}{2.24 - 1} = \mathbf{1.72}$ **m**

12.8. KHOSLA'S THEORY

In 1926-27, some siphons on Upper Chenab canal, designed on Bligh's theory, gave trouble. Actual pressure measurements made with the help of pipes inserted in the floors of two of these siphons did not show any relationship with the pressure calculated on the basis of Bligh's theory. This led to the following provisional conclusions by Khosla :

1. The outer faces of the end sheet piles were much more effective than the inner ones and the horizontal length of the floor.

2. The intermediate piles if smaller in length than the outer ones were ineffective except for the local redistribution of pressure.

3. Undermining of the floor started from the tail end. If the hydraulic gradient at exit was more than the critical gradient for the particular soil, the particles would move with the flow of water, thus causing progressive degradation of the sub-soil, resulting in cavities and ultimate failure.

4. It was absolutely essential to have a reasonably deep vertical cutoff at the downstream end to prevent undermining.

Khosla and his associates took into account the flow pattern below the impermeable base of hydraulic structures, to calculate the uplift pressure and exit gradient. Various cases were analysed mathematically. In order to appreciate the essential difference between

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Khosla's theory and Bligh's theory. we will start with a simple case of horizontal floor with negligibly small thickness, as shown in Fig. 12.9 analysed by Weaver (1932).

WEAVER'S SOLUTION :

For the case of two dimensional flow occurring under a straight floor, it can be shown that the streamlines are confocal ellipses given by the equation

$$\frac{x^2}{\left(\frac{b}{2}\cosh u\right)^2}+\frac{y^2}{\left(\frac{b}{2}\sinh u\right)^2}=1 \quad ...(1)$$

where u = stream line function given by

$$\cosh u=\frac{1}{2}(e^u+e^{-u})$$

and $\sinh u=\frac{1}{2}(e^u-e^{-u})$ ...(2)

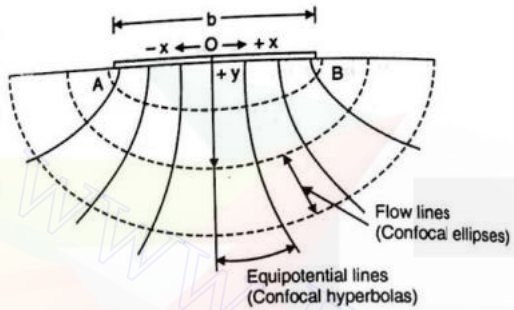

FIG. 12.9. HORIZONTAL FLOOR WITH NEGLIGIBLE THICKNESS.

From Eq. 1, any particular streamline can be determined by giving a suitable value to u. To calculate the gradient diagram, we must consider the first streamline AB which 'hugs' to the outline of the floor. Since, the discharge between this line and the solid floor is zero, the stream function $u = 0$.

Hence $\cosh u = \cosh 0 = 1$; $\sinh u = \sinh 0 = 0$.

Substituting in Eq. (1), we get

$$\frac{x^2}{b^2/4}+\frac{y^2}{0}=1$$

which is true only when $y = 0$. This gives the equation of the straight line AB.

Again, the equipotential lines consists of a set of confocal hyperbolas given by

$$\frac{x^2}{\left(\frac{b}{2}\cos v\right)^2}-\frac{y^2}{\left(\frac{b}{2}\sin v\right)^2}=1 \quad ...(3)$$

where v = pressure function, given by

$$v=\pi\Phi=\pi\frac{P}{H} \quad ...(4)$$

where P = pressure head at any point

= height of heading up against the structure.

In order to solve equations (1) and (3) for x and y, let the complex co-ordinate z of any point be represented by

$$z=\frac{b}{2}\cosh w$$

where $z = x + iy$ and $w = u + iv$

$$\therefore \quad x+iy=\frac{b}{2}\cosh(u+iv)$$

or

$$x+iy=\frac{b}{2}(\cosh u\cos v+i\sinh u\sin v)$$

Separating real and imaginary parts and equating them, we get

$$x=\frac{b}{2}\cosh u\cos v \quad \text{and} \quad y=\frac{b}{2}\sinh u\sin v \quad ...(5)$$

These values of x and y will be acceptable only if they satisfy both Eqs. 1 and 3. Thus, substituting these values of x and y in (1), we get

$$\cos^2 v+\sin^2 v=1 \text{ (satisfied)}$$

Similarly, from (3),

$$\cosh^2 u-\sinh^2 u=1 \text{ (satisfied)}$$

Thus the values of x and y are given by Eq. 5.

Hence $$x=\frac{b}{2}\cosh u\cos v$$

In the limiting case, for AB, $u = 0$

$$x=\frac{b}{2}\cosh 0\cos v=\frac{b}{2}\cos v$$

or $$v=\cos^{-1}\frac{2x}{b} \quad ...(12.5)$$

But $$v=\pi\Phi=\pi\frac{P}{H}$$

$$\therefore \quad \Phi=\frac{1}{\pi}\cos^{-1}\frac{2x}{b} \quad ...[12.5(a)]$$

or $$P=\frac{H}{\pi}\cos^{-1}\frac{2x}{b} \quad ...[12.5(b)]$$

Thus, for the first flow line AB, the pressures can be obtained by giving x different values in Eq. 12.5. Fig. 12.10 shows the pressure distribution diagrams both by Eq. 12.5 as well as by Bligh's method.

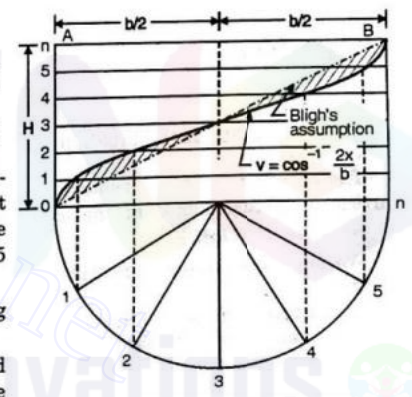

FIG. 12.10. ACTUAL PRESSURE DISTRIBUTION.

From the study of Fig. 12.10, following two conclusions can be drawn :

1. Slope of pressure diagram at A and B is infinite. Hence, the force at A will be theoretically infinite acting downwards, and that at B will also be infinite acting upwards. This will cause sand boiling, and hence either the floor should be depressed or a cutoff should be provided at the d/s end.

2. D/s half of the floor has more uplift pressure than that calculated by Bligh's method.

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