sq. km. Columns 3, 4, 5, 6, 7, 8 show the ordinates of successive unit hydrographs, each shifted by the unit time interval = T_0 = 6 hours. Column 9 shows the horizontal summation of the ordinates of the original unit hydrograph (column 2) and various successive unit hydrographs. Evidently, column 9 gives the ordinates of S-hydrograph. The summation is continued upto the time equal to the base of the unit hydrograph minus one unit duration (T_0 = 6 hrs.) to reach the equilibrium rate of discharge. It should be noted that the ordinates of S-hydrograph approach the rate corresponding to the effective rainfall (R_0) at the time of equilibrium. For the present case, the equilibrium discharge is given by

$$Q_0 = 2.778\, AR_0 \qquad \text{...(4.63)}$$

where R_0 = constant rate of continuous effective rainfall $= \frac{1}{T_0}$ cm/hour $= \frac{1}{6}$ cm/hour

A = catchment area in sq. km = 311 sq. km

$$\therefore \quad Q_0 = \frac{2.778 \times 311}{6} = 144 \text{ cumecs.}$$

This rate is reached in Table 4.20 at 36 hours (= base period − T_0 hours).

Alternative Method

An *alternative simple method* to compute ordinates of S-hydrograph is illustrated in Table 4.21 for the same data. Column 1 shows Line No., Column 2 shows time, while column 3 shows the ordinates of unit hydrograph. Column 5 shows the ordinates of S-hydrograph (to be computed) in which the values in the first two lines (upto T_0 = 6 hours) are the same as the corresponding values of unit hydrograph in column (3). These two values of Lines 1 and 2 of column (5) are entered in column 4 (offset ordinates) at an offset = T_0 = 6 hours against line 3 and 4 respectively.

TABLE 4.21. COMPUTATIONS FOR S-HYDROGRAPH (ALTERNATIVE METHOD)

Line No. (1)	*Time (hours)* (2)	*Ordinates of 6-hours unit hydrograph* (3)	*Off-set ordinates* (4)	*Ordinates of S-hydrograph (R_0=1/6 cm/hr.)* (5)
1	00	0	–	0
2	03	9	–	9
3	06	20	0	20
4	09	35	9	44
5	12	49	20	69
6	15	43	44	87
7	18	35	69	104
8	21	28	87	115
9	24	22	104	126
10	27	17	115	132
11	30	12	126	138
12	33	9	132	141
13	36	6	138	144
14	39	3	141	144
15	42	0	144	144

Now lines 3 and 4 of columns (3) and (4) are respectively added horizontally, and entered in lines 3 and 4 of column (5), thus giving values of ordinates as 20 + 0 = 20 and 35 + 9 = 44 cumecs. These two values of lines 3 and 4 of column (5) are shifted to lines 5 and 6 respectively of column 4. Now lines 5 and 6 of columns (3) and (4) are respectively added horizontally and entered in lines 5 and 6 respectively, of column 5 thus getting ordinates of 69 and 87. This process of shifting or offsetting the ordinates of column (5) by time interval T_0, entering into column (4) and adding with corresponding ordinates of column (3) to get new ordinates for column (5) is continued, till the base of the unit hydrograph is reached, or till the equilibrium discharge is reached. Sometimes, the computed ordintes of S-curve may not fall along a smooth curve. In that case, smoothening is carried out and the corresponding new ordinates along smoothened curves are found.

It should be noted that the S-hydrograph so derived from a 6 hours unit hydrograph represents contant $R_0 = 1/T_0 = 1/6$ cm/hour. The technique of constructing a S-hydrograph is very useful in deriving a unit hydrograph of shorter or longer unit duration from a given unit hydrograph of same unit duration. This is explained in the next article.

4.24. CONSTRUCTION OF UNIT HYDROGRAPH OF DIFFERENT UNIT DURATION FROM A UNIT HYDROGRAPH OF SOME GIVEN UNIT DURATION

(a) Construction of Longer Period Unit hydrograph from a Given Unit Hydrograph of Shorter Unit period

Let it be required to derive a unit hydrograph of unit duration t_0 hours, from a unit hydrograph of unit duration T_0 where $t_0 > T_0$. If t_0 is an integral multiple of T_0, this can be done by the simple principle of superposition.

Let it be required to derive a unit hydrograph of 6 hour unit duration (t_0 = 6 hours) from a given unit hydrograph of 2 hour duration (T_0 = 2 hours). This can be obtained by taking the sum of the ordinates of the *three* unit hydrographs of 2 hours duration, each lagging from the other by 2-hours, and dividing the sum by 3. This is illustrated in Fig. 4.57, in which it is clear that the peak discharge (Q_m') of the new hydrograph is less than Q_m of the original hydrograph, and also that the peak occurs later. If, however, t_0 is not an integral multiple of T_0, the method of S-hydrograph is used, as described below.

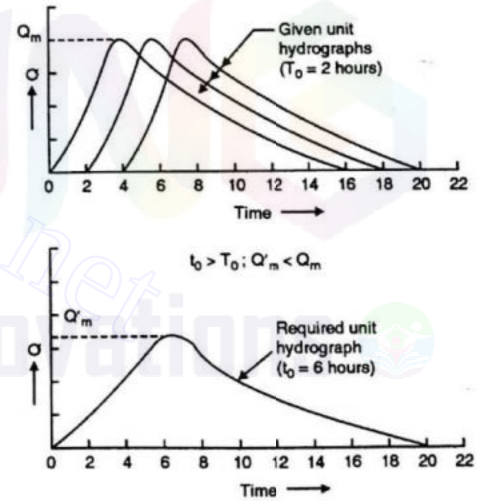

FIG. 4.57

(b) Construction of Shorter or Longer Period Unit Hydrograph from a Given Unit Hydrograph

Let it be required to obtain a unit hydrograph of unit period t_0 from a given unit hydrograph of unit period. T_0, where t_0 can be either greater or smaller than T_0. For this, the S-hydrograph method described below may be used.

From the given unit hydrograph of unit period T_0, S-curve can be derived, as discussed in § 4.23. This S-curve will represent a constant effective rainfall of $R_0 = 1 / T_0$ cm/hour. Fig. 4.58 (*a*) shows the S-curve so obtained. An *offset curve* is then drawn by advancing or offsetting the position of original S-curve for a period equal to the desired unit period t_0 hours. The difference between the ordinates of original S-curve and offset S-curve, divided by $R_0\, t_0$ will give the ordinate of the desired unit hydrogrph [Fig. 4.58 (*b*)].

Thus, at any time period *t*, if the difference between the ordinates of the two S-curves is Δy, then the ordinate of the desired unit hydrograph of t_0 unit period

$$= \frac{\Delta y}{R_0\, t_0} = \frac{\Delta y}{(1/T_0)\, t_0} = \Delta y \,.\, \frac{T_0}{t_0}.$$

Thus, ordinates at various time intervals can be computed and the desired unit hydrograph may be obtained.

For numerical illustration, let it be required to obtain unit hydrograph of 3 hours unit duration from the hydrograph of 6 hours unit duration given in Table 4.21. Column (2) of Table 4.22 gives the ordinates of the given hydrograph of 6 hours unit duration. Column (4) gives the ordinates of S-hydrograph, derived from the given unit hydrograph. This S-hydrograph represents $R_0 = 1/T_0 = 1/6$ cm/hour.

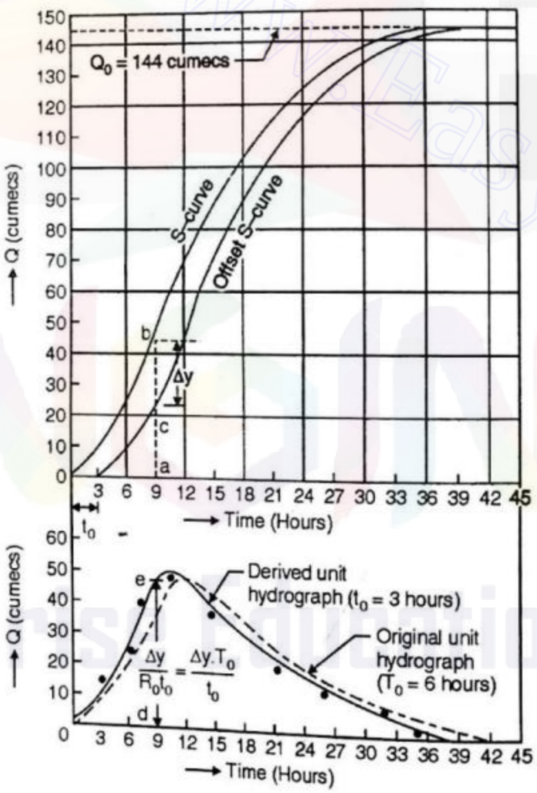

FIG. 4.58

TABLE 4.22. DERIVATION OF UNIT HYDROGRAPH OF t_0 (= 3 HOURS) UNIT DURATION FROM THE ONE OF T_0 (= 6 HOURS) UNIT DURATION

Time (1)	Ordinate of unit hydrograph (2)	Offset ordinate (3)	Ordinate of S-curve (4)	Ordinate of offset S-curve (5)	Δy (cumecs) (6)	$O = \Delta y \frac{T_0}{t_0} = 2\,\Delta y$ (7)	Ordinate of 3 hr. hydrograph smoothened (8)
0	0	–	0	–	0	0	0
03	9	–	9	0	9	18	18
06	20	0	20	9	11	22	23
09	35	9	44	20	24	48	47
12	49	20	69	44	25	50	49
15	43	44	87	69	18	36	40
18	35	69	104	87	17	34	32
21	28	87	115	104	11	22	25
24	22	104	126	115	11	22	19
27	17	115	132	126	6	12	14
30	12	126	138	132	6	12	10
33	9	132	141	138	3	6	7
36	6	138	144	141	3	6	4
39	3	141	144	144	0	0	0
42	0	144	144	144	0		
					$\Sigma 0$	288	288

Column (5) gives the ordinates of the *offset S-curve*, shifted by t_0 = 3 hours. Column (6) gives the difference Δy of the columns (4) and (5) of the two S-curves. Column (7) gives the ordinates of the required unit graph of 3 hours duration. In general, the ordinates of the unit hydrograph so obtained does not fall on a smooth curve. In that case smoothening has to be carried out [Fig. 4.58(*b*)]. Column 8 shows the new values of ordinates along the smoothened curve. In each case $\Sigma\, 0$ should be the same. As a check, $\Sigma\, 0 = 288$.

$$R = 0.36\, \frac{\Sigma\, 0}{A} \times t = \frac{0.36 \times 288}{311} \times 3 = 1 \text{ cm.}$$

4.25. DERIVATION OF UNIT HYDROGRAPH FROM COMPLEX STORMS

The simple approach for the determination of unit hydrograph discussed in § 4.20 is not always applicable, as an ideal storm assumed in the derivation is seldom available in practice. For the case of complex storms, the following procedure is adopted to derive the unit hydrograph.

Procedure (Fig. 4.59)

(1) Let the direct run-off ordinates of the given complex hydrograph be represented by $Q_1, Q_2, Q_3, \ldots\ldots Q_n$ in successive periods during the storm.

(2) Let the ordinates of the unit hydrograph be designated by $U_1, U_2, U_3 \ldots\ldots U_m$ at the successive time intervals.

(3) For the purpose of illustration, we shall assume that the whole period of complex storm has three storms, each of 'effective duration' equal to 'unit duration' for which unit hydrograph is to be drawn. On to the hyetographs of rainfall, superimpose the infiltration rate curves and find the 'rainfall excess' rates. Let R_1, R_2, and R_3 be the respective rainfall-excesses for the three storms. Thus, for the whole duration,

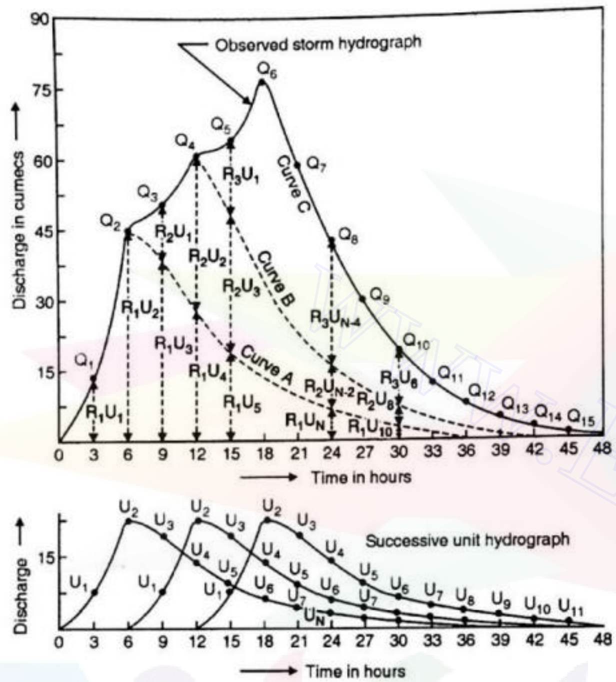

FIG. 4.59

there will be three similar unit hydrographs, each operating at a time lag equal to the 'unit duration'.

(4) In Fig. 4.59, the curve with thick line is the observed storm hydrograph. We observe that :

First ordinate $Q_1 = R_1 U_1$ ...(1)

Knowing R_1 and Q_1, U_1 is known.

Second ordinate $Q_2 = R_1 U_2$ ...(2)

Knowing R_1 and Q_2, U_2 is known.

Third ordinate $Q_3 = R_1 U_3 + R_2 U_1$ ...(3)

Knowing R_1, R_2, Q_3, U_1 and U_2, U_3 is known.

Fourth ordinate $Q_4 = R_1 U_4 + R_2 U_2$ ...(4)

Hence U_4 is known.

Fifth ordinate $Q_5 = R_1 U_5 + R_2 U_3 + R_3 U_1$ ...(5)

Hence U_5 is known.

N th ordinate $Q_N = R_1 U_N + R_2 U_{N-2} + R_3 U_{N-4}$ ...(N)

where N is less than m

Thus the ordinates $U_1, U_2, U_3, \ldots, U_m$ of the unit hydrograph are known.

In Fig. 4.59, ordinates of curve A are equal to $R_1 \times$ successive ordinates of the first unit hydrograph. Similarly, ordinates of curve B = (successive ordinates of curve A) + ($R_2 \times$ successive ordinates of second unit hydrograph).

And, ordinates of curve C = (successive ordinates of curves $A + B$) + $R_3 \times$ successive ordinates of Third Unit hydrograph). See example 4.29 for illustration.

Example 4.26. *The following table gives the observed total runoff hydrograph for a drainage basin of 92 km². The ordinates of the rainfall mass curve, which produced the above runoff are also given. Derive the unit hydrograph.*

Date and time	*Runoff (m^3/s)*	*Date and time*	*Runoff (m^3/s)*
20-7-60		*21-7-60*	
06	10.6	02	58.2
08	9.7	04	48.0
10	107.8	06	36.2
12	175.6	08	28.4
14	193.9	10	20.2
16	150.3	12	14.0
18	126.2	14	10.2
20	106.9	16	10.4
22	90.0	18	
24	72.8	20	

Date	*Time (h)*	*Cumulative rainfall (cm)*
20-7-60		
	06	0
	08	2
	10	4.6
	12	10.8
	14	18.8
	16	21.4
	18	21.4

Solution : Fig. 4.60 (a) shows the storm hydrograph while Fig. 4.60 (b) shows the rainfall hyetograph, each constructed adopting a time interval $\Delta t = 2$ hours. The point of rise (A) is easily located at the end of 08 hours of 20th July. The time distance

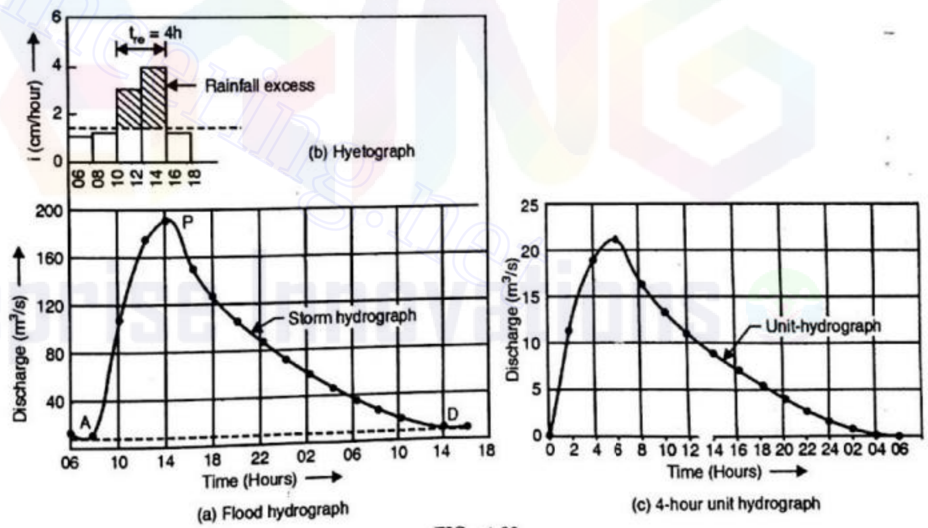

FIG. 4.60

N for the location of point D is given by

$$N = 0.827\,(A)^{0.2} = 0.827\,(92)^{0.2} = 2.043 \text{ days} \simeq 49 \text{ hours}$$

From the plotted hydrograph, we observe that this value of N is too large. Hence the end point (D) of direct runoff is arbitrarily fixed at 14 hours of 21st July. The base line AD is thus obtained by joining points A and D.

The ordinates of direct-runoff are now computed by subtracting the base flow from total runoff, as shown in Table below.

Date	Time (hours)	Total runoff (m^3/s)	Base flow (m^3/s)	Direct runoff ordinate (O) m^3/s)	Ordinate of unit hydro-graph (m^3/s)	Time of start of direct-runoff (h)
(1)	(2)	(3)	(4)	(5)= (3) – (4)	(6)= (5)/n	(7)
20-7-60	06	10.6	10.6	0		
	08	9.7	9.7	0	0.00	00
	10	107.8	9.73	98.07	11.51	02
	12	175.6	9.77	165.83	19.46	04
	14	193.9	9.80	184.10	21.61	06
	16	150.3	9.83	140.47	16.49	08
	18	126.2	9.87	116.33	13.65	10
	20	106.9	9.90	97.00	11.38	12
	22	90.0	9.93	80.07	9.40	14
	24	72.8	9.97	62.83	7.37	16
21-7-70	02	58.2	10.00	48.20	5.66	18
	04	48.0	10.03	37.97	4.46	20
	06	36.2	10.07	26.13	3.07	22
	08	28.4	10.10	18.30	2.15	24
	10	20.2	10.13	10.07	1.18	02
	12	14.0	10.16	3.84	0.45	04
	14	10.2	10.20	0.00	0.00	06
	16	10.4	10.40	0.00		
			Σ	1089.21		

$$n = \frac{0.36\,\Delta t\,\Sigma O}{A} = \frac{0.36 \times 2\,(1089.21)}{92} = 8.52 \text{ cm}$$

Hence the ordinates of unit hydrograph (column 6) are obtained by dividing the ordinates of direct runoff (column 5) by $n = 8.52$.

In order to determine unit duration, let us first determine the value of infiltration index Φ_i. The rainfall intensities during the successive 2-hr periods are $\frac{2-0}{2} = 1.0$ cm/hr, $\frac{4.6-2.0}{2} = 1.3$ cm/hr, $\frac{10.8-4.6}{2} = 3.1$ cm/hr, $\frac{18.8-10.8}{2} = 4.0$ cm/hr, $\frac{21.4-18.8}{2} = 1.3$ cm/hr and $\frac{21.4-21.4}{2} = 0$ cm/hr. To start with, let us assume that $\Phi_i > 1.3$ cm/hr. The total rainfall excess or effective rainfall (P_e) is then given by

$$P_e = [(3.1 - \Phi_i) + (4.0 - \Phi_i)]\,2$$

This should be equal to depth (n) of direct runoff. Hence we have

$$[(3.1 - \Phi_i) + (4.0 - \Phi_i)]\,2 = n = 8.52 \text{ cm which gives } \Phi_i = 1.42 \text{ cm/hr.}$$

Since this greater than 1.3 cm/hr, no further trials are required. The line $\Phi_i = 1.42$ cm/hr is drawn or in Fig. 4.60 (*b*), to get the effective rainfall hyetograph, from which we get effective rainfall duration (t_{re}) equal to 4 hours. Hence the unit time duration is 4 hours and the resulting unit hydrograph is a ***4-hr unit hydrograph***.

Example 4.27. *Given below are the observed flows from a storm of 6 hours duration on a stream with a drainage area of 316 sq. km.*

Time	*Flow (in cumecs)*	*Time*	*Flow (in cumecs)*
June 1, 0.0 A.M.	*17.0*	June 3, 0.0 A.M.	*53.8*
6.0 A.M.	*113.2*	6.0 A.M.	*42.5*
Noon	*254.5*	Noon	*31.1*
6.0 A.M.	*198.0*	6.0 P.M.	*22.6*
June 2, 0.0 A.M	*150.0*	June 4, 0.0 A.M	*17.0*
6.0 A.M.	*113.2*		
Noon	*87.7*		
6.0 P.M.	*67.9*		

Assuming constant base flow of 17.0 cumecs, derive and plot a 6 hours unit hydrograph. How many cm of rainfall excess does the above storm hydrograph represent?

Solution : The example has been solved in the following tabular form below

Time (1)	Ordinates of storm hydrograph (cumecs) (2)	Base flow (cumecs) (3)	Ordinates of direct run-off (4) = (2) – (3)	Ordinates of unit hydrograph (5) = (4)/6.477
June 1, 0.0 AM.	17.0	17	0	0
6.0 A.M.	113.2	17	96.2	14.9
Noon	254.5	17	237.5	36.6
6.0 A.M.	198.0	17	181.0	28.0
June 2, 0.0 A.M.	150.0	17	133.0	20.5
6.0 A.M.	113.2	17	96.2	14.9
Noon	87.7	17	70.7	10.9
6.0 P.M.	67.9	17	50.9	7.86
June 3, 0.0 A.M.	53.8	17	36.8	5.69
6.0 A.M.	42.5	17	25.5	3.94
Noon	31.1	17	14.1	2.18
6.0 P.M.	22.6	17	5.6	0.87
June 4, 0.0 A.M.	17.0	17	0	0
		Sum	947.5	

$$\text{Rainfall excess} = \frac{947.5 \times 60 \times 60 \times 6 \times 10^6}{316 \times 10^4 \times 10^6} = \mathbf{6.477 \text{ cm.}}$$

Alternatively from Eq. 4.61 :

$$\text{Direct run-off} = 0.36\,\frac{\Sigma O \times t}{A} = 0.36\,\frac{947.5 \times 6}{316} = 6.477 \text{ cm.}$$

Example 4.28. *Find the ordinates of a storm hydrograph resulting from a 3 hour storm with rainfall of 2, 6.75 and 3.75 cm during subsequent 3 hours intervals. The ordinates of unit 3-hour hydrograph are given in the following table :*

Hours	03	06	09	12	15	18	21	24	03	06	09	12	15	18	21	24
Ordinates of unit hydrograph (cumecs)	0	110	365	500	390	310	250	235	175	130	95	65	40	22	10	0

Assume an initial loss of 5 mm, infiltration index of 2.5 mm/hour and base flow of 10 cumecs.

Solution

(*i*) Rainfall excess during the first three hours

$= 20 - (2.5 \times 3) - 5 = 7.5$ mm $= 0.75$ cm.

(*ii*) Rainfall excess during the second three hours

$= 67.5 - (3 \times 2.5) = 60$ mm $= 6$ cm.

(*iii*) Rainfall excess during the last three hours

$= 37.5 - (3 \times 2.5) = 30$ mm $= 3$ cm

Rainfall excess as ratio of unit rainfall of 1 cm during the subsequent 3 hours intervals are 0.75, 6 and 3. The example has been solved in a tabular form below. The computations of run-off due to 0.75 cm rainfall excess will start from 03 hours. The computations of run-off due to 6 cm rainfall excess will start from 06 hours. Lastly, the computations of run-off due to 3 cm rainfall excess will start from 09 hours.

Time in hours	*Ordinates of 3-hour unit hydrograph (cumecs)*	*Rainfall excess cm/2 hours*	*Surface run-off from rain fall excess during sucessive unit periods (cumecs)*				*Base flow (cumecs)*	*Total discharge (cumecs)*
			0.75	*6.0*	*3.0*	*Sub total*		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
03	0	0.75	0			0	10	10.0
06	110	6.00	82.5	0		82.5	10	92.5
09	365	3.0	274.0	660	0	934.0	10	944.0
12	500		375.0	2190	330	2895.0	10	2905.0
15	390		295.5	3000	1045	4337.5	10	4347.5
18	310		232.5	2340	1500	4072.5	10	4082.5
21	250		187.5	1860	1170	3217.5	10	3227.5
24	235		176.0	1500	930	2606.0	10	2616.0
03	175		131.5	1410	750	2291.5	10	2301.5
06	130		97.5	1050	705	1852.5	10	1862.5
09	95		71.3	780	525	1376.3	10	1386.0
12	65		48.6	570	390	1008.6	10	1018.6
15	40		30.0	390	285	705.0	10	715.0
18	22		16.5	240	195	451.5	10	461.5
21	10		7.5	132	120	259.5	10	269.5
24	0		0	60	66	126.0	10	136.0
30				0	30	30.0	10	40.0
06					0	0	10	100.0

Example 4.29. *The following direct run-off hydrograph resulted from three successive 6 hours periods of rainfall having run-off estimated as 2, 4 and 3 cm respectively :*

Time (hr) 0 3 6 9 12 15 18 21 24 27 30 33 36

Flow (cumecs) 0 21 80 82 189 123 184 87 55.5 25.25 9 6 0

Using the method of complex hydrographs, derive and plot a 6 hour unit hydrograph for the basin having area = 103.4 sq. km.

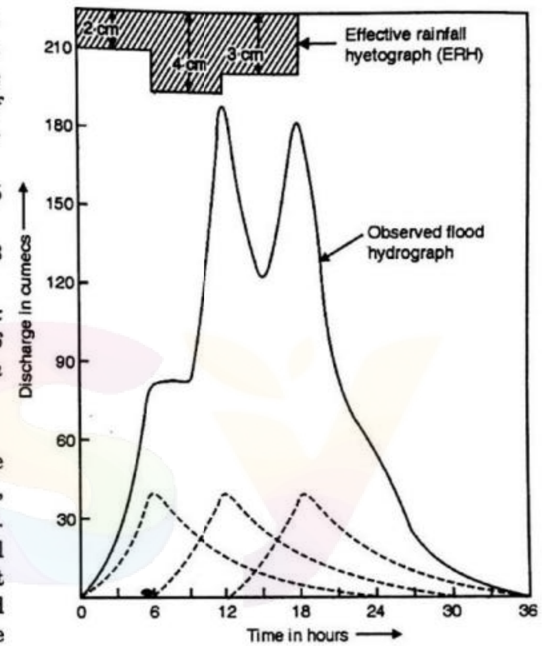

FIG. 4.61

Solution

Let the unit hydrograph have ordinates of U_0, U_3, U_6, U_9, U_{12}, U_{15}, U_{18}, U_{24}, at 3 hours intervals. The direct run-off hydrograph will be obtained by multiplying the unit hydrograph values with 2, 4 and 3 cm respectively with successive hydrographs having six hours lag.

Then $U_0 = 0$	or $U_0 = 0$	
$2U_3 = 21$	or $U_3 = 10.5$	cumecs
$2U_6 + 4U_0 = 80$	or $U_6 = 40$	cumecs
$2U_9 + 4U_3 = 82$	or $U_9 = 20$	cumecs
$2U_{12} + 4U_6 + 3U_0 = 189$	or $U_{12} = 14.5$	cumecs
$2U_{15} + 4U_9 + 3U_3 = 123$	or $U_{15} = 5.75$	cumecs
$2U_{18} + 4U_{12} + 3U_6 = 184$	or $U_{18} = 3.0$	cumecs
$2U_{21} + 4U_{15} + 3U_9 = 87$	or $U_{21} = 2.0$	cumecs
$2U_{24} + 4U_{18} + 3U_{12} = 55.5$	or $U_{24} = 0.0$	cumecs

Therefore, ordinates of unit hydrograph are as follows :

Time hour	0	3	6	9	12	15	18	21	24	Total
Flow cumecs	0	10.5	40	20	14.5	5.75	3.0	2.0	0	95. 75

The ordinates of unit hydrograph will be correct if it gives a direct run-off of 1 cm.

$$\text{Direct run-off} = \frac{95.75 \times 3 \times 3600}{103.4 \times 10^4} \simeq 1 \text{ cm.}$$

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The successive unit hydrographs will have the same ordinates but will be shifted laterally by 6 hous as shown in Fig. 4.61.

Example 4.30. *The hourly ordinates of a two-hour unit hydrograph are given below. Derive a 6-hours unit hydrograph for the same catchment.*

Time (hours)	*00*	*01*	*02*	*03*	*04*	*05*	*06*	*07*	*08*	*09*	*10*	*11*	*12*	*13*	*14*	*15*
Discharge (cumecs)	*0.0*	*1.0*	*2.7*	*5.0*	*8.0*	*9.8*	*9.0*	*7.5*	*6.3*	*5.0*	*4.0*	*2.9*	*2.1*	*1.3*	*0.5*	*0.0*

Solution Given $T_0 = 2$ hours ; $t_0 = 6$ hours.

Since t_0 is an integral multiple of T_0 (*i.e.* $t_0 = 3T_0$), the desired hydrograph can be obtained by taking the sum of the ordinates of three unit hydrographs of 2-hours duration, each lagging from the other by 2-hours, and dividing the sum by 3. The computations are arranged in table below.

Time	**Given 2 hours unit hydrograph**	**Offset unit hydrograph**	**Offset unit hydrograph**	**Sum**	**Ordinates of 6 hr unit hydrograph**	**Ordinate of 6 hr moderated hydrograph**
00	0.0	–	–	0.0	0.00	0.0
01	1.0	–	–	1.0	0.33	0.3
02	2.7	0	–	2.7	0.90	0.9
03	5.0	1.0	–	6.0	2.00	2.0
04	8.0	2.7	0	10.7	3.57	3.6
05	9.8	5.0	1.0	15.8	5.27	5.3
06	9.0	8.0	2.7	19.7	6.57	6.6
07	7.5	9.8	5.0	22.3	7.43	7.4
08	6.3	9.0	8.0	23.3	7.77	7.8
09	5.0	7.5	9.8	22.3	7.43	7.4
10	4.0	6.3	9.0	19.3	6.43	6.4
11	2.9	5.0	7.5	15.4	5.13	5.1
12	2.1	4.0	6.3	12.4	4.13	4.1
13	1.3	2.9	5.0	9.2	3.07	3.1
14	0.5	2.1	4.0	6.6	2.20	2.2
15	0.0	1.3	2.9	4.2	1.40	1.4
16		0.5	2.1	2.6	0.87	0.9
17		0.0	1.3	1.3	0.48	0.4
18			0.5	0.5	0.17	0.2
19			0.0	0.0	0.00	0.0
$\Sigma 0$	65.1				65.10	65.1

It should be clearly noted that $\Sigma 0$ of the 6-hours unit hydrograph so obtained should be the same as $\Sigma 0$ of the original 2-hours unit hydrograph, so that each of the unit hydrographs represent direct run off 1 cm. Also, we note that while the peak discharge of the 2-hours unit hydrograph was 9.8 cumecs occuring at 05 hours, the peak discharge of the 6-hours unit hydrograph is 7.8 cumecs, occurring at 08 hours.

Example 4.31. *Given the 6 hour unit hydrograph of Table 4.22 (column 2). Derive a 9 hour unit hydrograph for the same catchment.*

Solution: The method has already been illustrated in § 4.24, using S-hydrograph. Column 2 of Table below gives the given 6 hours unit hydrograph while colume (4) gives the ordinates of S-curve derived from it. Column (5) gives the ordinates of the offset S-curve, shifted by $t_0 = 9$ hours. Column (6) gives the difference Δy between the ordinates of the two S-curves. The ordinates of the 9 hours unit hydrograph are given by

$$O = \Delta y \frac{T_0}{t_0} = \Delta y \frac{6}{9} = \frac{2}{3} \Delta y$$

Column (8) gives the ordinates of the moderated 9 hour unit hydrograph.

Time (hours) (1)	**Ordinates of 6 hour unit hydrograph (cumecs)** (2)	**Offset ordinates** (3)	**Ordinates of S-hydrograph (cumecs)** (4)	**Ordinates of offset S-curve** (5)	**Δy (cumecs)** (6)	**$O = \Delta y \frac{T_0}{t_0} = \frac{2}{3}\Delta y$ (cumecs)** (7)	**Ordinates of moderated 9 hour unit hydrograph** (8)
00	0	–	0	–	0	0.0	0.0
03	9	–	9	–	9	6.0	5.5
06	20	0	20	–	20	13.33	15.0
09	35	9	44	0	44	29.33	28.5
12	49	20	69	9	60	40.0	40.0
15	43	44	87	20	67	44.67	45.0
18	35	69	104	44	60	40.0	40.0
21	28	87	115	69	46	30.67	31.5
24	22	104	126	87	39	26.0	25.0
27	17	115	132	104	28	18.67	19.0
30	12	126	138	115	23	15.33	14.5
33	9	132	141	126	15	10.0	10.5
36	6	138	144	132	12	8.0	7.0
39	3	141	144	138	6	4.0	4.3
42	0	144	144	141	3	2.0	2.2
45			144	144	0	0.0	0.0
$\Sigma 0$	288					288.0	288.0

4.26. DISTRIBUTION GRAPH

Distribution graph was introduced by Bernard (1935), and hence it is also known as '*Bernard's distribution graph*'. It represents the unit hydrogaph in the form of *percentage of total runoff* occurring in successive periods of equal intervals of unit period. The unit period or unit interval is taken equal to the duration of rainfall excess. The ordinates of the distribution graph are indicated at successive such unit intervals. Since the discharge represented by a unit hydrograph is directly proportional to net rain, the percentage in unit time will remain constant whatever be the net rain. The percentage are drawn as rectangular bars or steps against successive unit times (Fig. 4.62).

To draw a distribution graph (Fig 4.62*b*) corresponding to a given unit hydrograph (Fig 4.62 *a*), the base period of unit hydrograph is divided into a number of unit time periods, each equal to the unit duration (T_0) of unit hydrograph. The volume of runoff in each period of unit hydrograph is expressed as a percentage of the total volume of runoff represented by the entire unit bydrograph. Thus, in Fig. 4.62 (*a*) we observe that

$$\frac{\text{Shaded area of U.H.}}{\text{Total area of U.H.}} = 0.29 = 29\%$$

Hence the ordinate of distribution graph for the unit time period (T_0) will be 29%. The use of the distribution graph to generate a direct runoff hydrograph is exactly the same as that of a unit hydrograph. The area under the curve and under the step lines are the same, and hence in deriving unit hydrograph from distribution percentages, a smooth line must be drawn through steps to give equal areas. The distribution graph is therefore less precise than the unit hydrograph. However, it is much better suited for Collin's iterative process of deriving unit hydrograph from complex storms. The distribution graph is also useful in comparing runoff characteristics of different catchments.

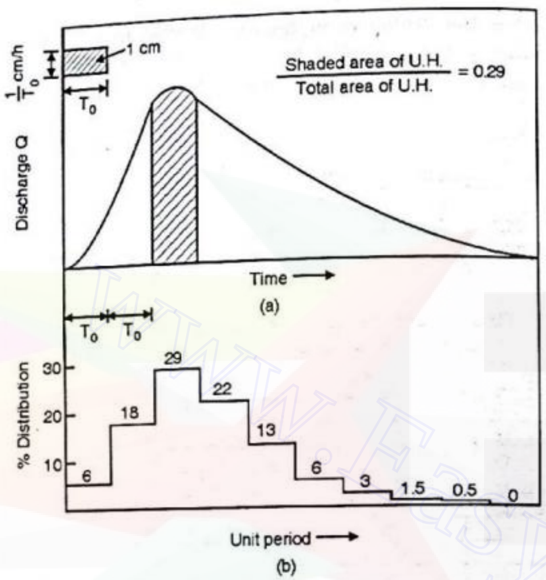

FIG. 4.62. DERIVATION OF DISTRIBUTION ORAPH

4.27. INSTANTANEOUS UNIT HYDROGRAPH

We have seen earlier that each unit hydrograph representing 1 cm of direct runoff, is for some unit duration T_0. For a catchment, there can be a number of unit hydrographs corresponding to various values of unit duration T_0. To obtain the runoff hydrograph resulting from a storm of *varying duration* and *varying intensities*, it is preferable to have a unit hydrograph of very *short unit duration*. Theoretically, the shortest unit duration in zero. If the duration of rainfall excess becomes infinitesimally small, the resulting unit hydrograph is called *instantaneous unit hydrograph*. Thus IUH is a fictitious, conceptual unit hydrograph which represents surface runoff from the catchment due to an

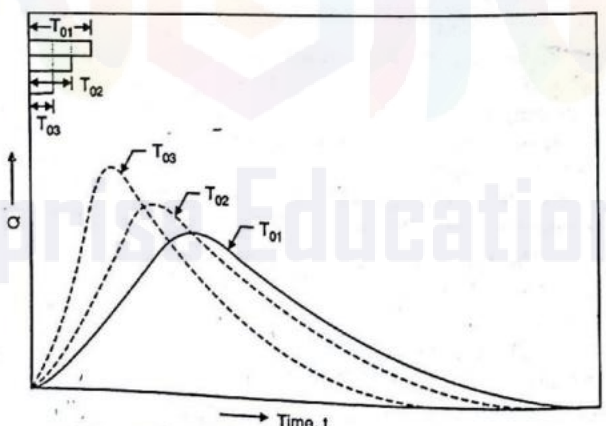

FIG. 4.63. UNIT HYDROGRAPHS OF DIFFERENT UNIT DURATIONS (T_0)

instantaneous precipitation of rainfall excess volume of 1 cm. The IUH is designated as *u(t,0)* or simply as *u(t)*.

IUH can be easily derived from S-curve hydrograph obtained from an available unit hydrograph of duration T_0. If two S-curves are drawn at a time lag of t_0, the ordinate of unit hydrograph of t_0 hour unit duration at any time t is given by

$$u(t, t_0) = \frac{T_0}{t_0}\left[S_t - S_{t-t_0}\right] \qquad ...(4.64)$$

where $u(t, t_0)$ = ordinate of unit hydrograph of unit duration t_0

T_0 = unit duration of unit hydrograph from which S-curve has been obtained

S_t = ordinate of S-curve at any time t

S_{t-t_0} = ordinate of shift S–curve, shifted by t_0

If t_0 is taken as Δt, the ordinates of resulting unit hydrograph of Δt unit duration is given by

$$u(t, \Delta t) = \frac{T_0}{\Delta t}\left[S_t^{T_0} - S_{t-\Delta t}^{T_0}\right] \quad \text{or} \quad u(t, \Delta t) = T_0 \frac{\Delta S_t^{T_0}}{\Delta t} \qquad ...(4.65)$$

where $S_t^{T_0}$ is the S-curve ordinate derived from unit hydrograph of T_0 unit duration. In the limit $\Delta t \to 0$, we get the IUH, given by

$$\underset{\Delta t \to 0}{\text{Lt}} \; u(t, \Delta t) = T_0 \frac{d S_t^{T_0}}{dt}$$

or
$$u(t) = T_0 \frac{d S_t^{T_0}}{dt} = \frac{1}{R_0} \frac{d S_t^{T_0}}{dt} \qquad ...(4.66)$$

Hence $\begin{Bmatrix}\textbf{The ordinate of}\\ \textbf{IUH at any time } t\end{Bmatrix} = T_0 \begin{Bmatrix}\textbf{The slope of S-curve derived}\\ \textbf{from } T_0 \textbf{ hour unit hydrograph at time } t\end{Bmatrix}$

In the above expression, R_0 is the intensity of rainfall excess, given by $R_0 = 1/T_0$. If $R_0 = 1$ cm, we get

$$u(t) = \frac{d S_t}{dt} \qquad ...(4.67)$$

where S_t is the ordinate of S-curve of intensity 1 cm/hour. *Thus, the ordinate of IUH at any time t is the slope of S-curve of intensity 1 cm/hour.*

An IUH, *designated by u(t) is a single peaked hydrograph with a finite base width*. It has the following properties :

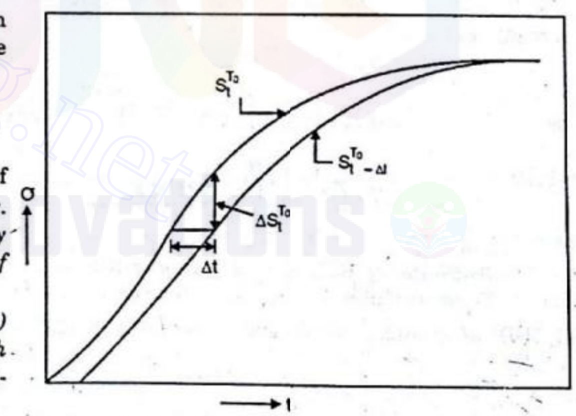

FIG. 4.64. THE INSTANTANEOUS UNIT HYDROGRAPH AS S-CURVE DERIVATIVE

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1. $0 \le u(t) <=$ a positive value at $t > 0$
2. $u(t) = 0$ at $t \le 0$
3. $u(t) \rightarrow 0$ as $t \rightarrow \infty$
4. $\int_0^\infty u(t)\,dt$ = unit depth over catchment

and 5. (Time to peak) < (time to centroid of curve)

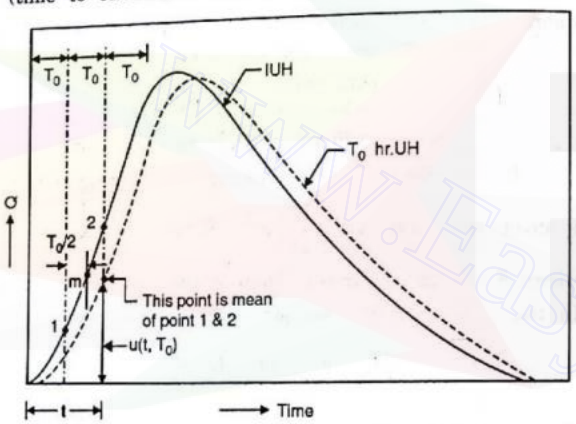

FIG. 4.65. DERIVATION OF T_0 HR. UNIT HYDROGRAPH FROM IUH

It is interesting to note that IUH is a unique demonstration of a particular catchment's response to rain, independant of duration, just as unit hydrograph is its response to rain of a particular unit duration. *IUH is not time dependant. It is a graphical expression of the integration of all the parameters of the catchment, such as length, shape, slope etc that control such a response.*

The IUH can be developed either directly from the observed data or by adopting conceptual models. When once IUH is available for a catchment, unit hydrographs of various unit durations can be easily derived. Let us derive unit hydrograph $u(t, T_0)$ of T_0 unit duration. We already know that

$$u(t, T_0) = S_t - S_{t-T_0}$$

or
$$u(t, T_0) = \frac{1}{T_0}\left[\int_0^t u(z)\,dz - \int_0^{t-T_0} u(z)\,dz\right]$$

or
$$u(t, T_0) = \frac{1}{T_0}\int_{t-T_0}^{t} u(z)\,dz \qquad ...(4.68)$$

Hence
$$\left[\begin{array}{l}\textit{Ordinate of U.H. of}\\ T_0 \textit{ at any time } t\end{array}\right] = \frac{1}{T_0}\left[\begin{array}{l}\textit{area of IUH in the limits}\\ \textit{between } (t-T_0) \textit{ and } t\end{array}\right]$$

If IUH is assumed to be linear between $(t - T_0)$ and t, the Eq. 4.68 reduces to

$$u(t, T_0) = u\left(t - \frac{T_0}{2}\right) \qquad ...(4.69)$$

Thus, the ordinate of T_0 unit hydrograph at any time t is the average of IUH for $(t - T_0)$ hours and that of t hour.

From Fig. 4.65, it is clear that if IUH is divided into T_0 hour time interval, and if the averages of the ordinates at the beginning and the end of each interval are plotted at the end of the interval, we get ordinate of T_0 unit hydrograph.

Thus,
$$u(t, T_0) = \frac{1}{2}\left[u(t) + u(t - T_0)\right] \qquad ...(4.70)$$

4.28. SYNTHETIC UNIT HYDROGRAPH

We have seen that if the rainfall and runoff records are available, unit hydrographs can be derived. However, in the case of *ungauged rivers*, these data are not available. In some other cases, the data available may be scanty. For such catchments, unit hydrographs are derived by relating the selected basin characteristics to the unit hydrograph shape. The resulting hydrograph, derived from basin characteristic relationships is known as a *synthetic unit hydrograph*. There are several methods to derive synthetic unit hydrograph, but the one given by Synder (1938), is widely used, and will therefore be discussed here.

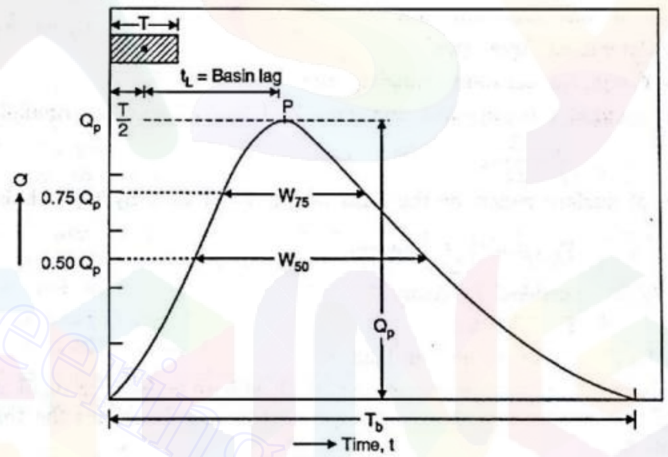

FIG. 4.66. PARAMETERS OF SYNTHETIC UNIT HYDROGRAPH

Synder's Method

Synder studied the data of catchments of Appalachian Highlands of Eastern United States, and then developed empirical equations for synthetic hydrograph.

Synder selected three parameters of unit hydrograph : (*i*) base width T_b (*ii*) peak discharge Q_p and (*iii*) basin lag t_L, as marked in Fig 4.66. The equations given by him take into account catchment area, shape of basin, topography, channel slopes, stream density and channel storage. He eliminated all these parameters except the first two, by including them in a single coefficient C_t. He dealt with the size and shape of the catchment by measuring the length of the main stream channel, by proposing the following equation for basin lag :

$$t_L = C_t (L_{ca} \cdot L)^{0.3} \qquad \text{...(4.71)}$$

where t_L = *basin lag* in hours, (basin lag is the time between mass of centre of unit rainfall of T hour duration and runoff peak flow)

C_t = a coefficient depending upon units and drainage basin characteristics. The value of C_t varies between 1.35 and 1.65 for the Appalachian High-lands Catchments studied by Synder.

L = distance from station to catchment boundary measured along the main-stream, in km.

L_{ca} = distance along the mainstream from gauging station to a point opposite the centroid of the watershed, in km.

The peak discharge of Q_p unit hydrograph of standard duration T is given by

$$Q_p = \frac{2.778\, C_p \cdot A}{t_L} \qquad \text{...(4.72)}$$

where Q_p = peak discharge (m³/sec)

A = Catchment area (km²)

C_p = a regional constant, ranging from 0.56 to 0.69

Synder adopted the standard duration (T) hours of effective rainfall given by

$$T = \frac{2}{11} t_L \qquad \text{...(4.73)}$$

The duration of surface runoff or the base length T_b of unit hydrograph is given by

$$T_b = 3 + 3\left(\frac{t_L}{24}\right) \text{ (days)} \qquad \text{...(4.74a)}$$

When T_b is expressed in hours,

$$T_b = 72 + 3\, t_L \qquad \text{...(4.74)}$$

(where both T_b and t_L are in hours)

It is found that Eq. 4.74 gives unreasonably long base periods for small catchments. Some investigators recommend that a base period equal to five times the time to peak should be taken :

$$T_b = 5\left(\frac{T}{2} + t_L\right) = 5\left(\frac{t_L}{11} + t_L\right)$$

or

$$T_b = 5.455\, t_L \qquad \text{...(4.75)}$$

Alternatively, assuming a triangular shape of unit hydrograph,

$$T_b = \frac{5.556}{q_p} \qquad \text{...(4.75a)}$$

Sketching of unit hydrographs becomes easier by adopting the following recommendations given for U.S. catchments, by US Army Corps of Engineers, for widths of unit hydrographs at 50 and 75% of the peak discharge :

$$W_{50} = \frac{5.87}{q_p^{1.08}} \qquad \text{...[4.76(a)]}$$

and
$$W_{75} = \frac{W_{50}}{1.75} \qquad \text{...[4.76(b)]}$$

where W_{50} = width of unit hydrograph in hours, at 50% peak discharge

W_{75} = width of unit hydrograph, in hours, at 75% peak discharge

$q_p = \frac{Q_p}{A}$ = peak discharge per unit area.

Usually, one third of this width is kept before the unit hydrograph peak and two thirds after the peak.

If synthetic unit hydrograph of any other duration T' is required, then the modified basin lag is

$$T_L' = T_L + \frac{T' - T}{4} \qquad \text{...[4.77(a)]}$$

Also,
$$Q_p' = \frac{2.778\, A\, C_p}{t_L'} \qquad \text{...[4.77(b)]}$$

4.29. PEAK FLOW DETERMINATION

The extreme hydrologic events are *floods* and *droughts*. In general qualitative terms, these refer to periods of unusually high and low water supplies. A flood is an unusually high stage of a river due to runoff from rainfall and/or melting of snow in quantities too great to be confined in the normal water surface elevations of the river or stream. In the peak flow determination, reference is made to the following *three types of* floods :

***(i)* Standard Project Flood (SPF)** : It is the flood that would result from the most severe combination of meteorological and hydrological factors that are reasonably applicable to the basin. However, extreme rare combinations are excluded.

***(ii)* Maximum Probable Flood (MPF)** : This includes the extreme rare combination also, excluded in SPF. Thus, it is defined as extreme flood that is physically possible in a region as a result of severemost combination including extreme rare combination of meteorological and hydrological factors. It is observed that SPF is about 80% of MPF.

***(iii)* Design Flood** : A design flood is the flood discharge adopted for design of a hydraulic structure after careful considerations of hydrologic and economic factors. A design flood in most of the cases may be less than MPF. There may be several design flood values adopted for various components of a project. For example, design flood used for spillway will be more than the design flood used for cofferdam of the same project.

The peak flood discharge in a stream may be determined by the following methods.

1. By physical indication of past floods
2. By empirical flood formulae
3. By unit hydrograph
4. By flood frequency analysis
5. By enveloping curves
6. By rational formula

PHYSICAL INDICATION OF PAST FLOODS

Ancient monuments, etc., situated on river banks always bear past floods marks. Old persons in the villages situated on the bank of the river may be contacted to know the maximum water level attained in the past 35 years. The cross-section of the river may be plotted and the water line corresponding to the highest flood can be drawn on it. From such a cross-section, the water-flow-area, wetted perimeter and hydraulic mean depth can be calculated. By longitudinal sectioning with the help of levelling slightly to the upstream and downstream of the site where cross-section has been plotted, the longitudinal slope of the bed of the river can be determined. Assuming this to be the same as the hydraulic slope during the past flood, the mean velocity of flow can be computed by Chezy's or any other suitable hydraulic formula. This velocity can be multiplied with the probable area of water section at the time of past flood to calculate the flood discharge. This procedure should be repeated at several villages or water marks, to get consistent results.

4.30. EMPIRICAL FORMULAE FOR FLOOD DISCHARGE

Some of the empirical formulae for estimating the flood discharge are given below. Most of these are in the form :

$$Q = CA^n$$

where : Q = flood discharge
A = catchment area
n = flood index
C = flood coefficient.

Both C and n depend upon various factors, such as (*i*) size, shape and location of catchment, (*ii*) topography of the catchment, and (*iii*) intensity and duration of rainfall, and distribution pattern of the storm over the basin.

1. **Dicken's Formula**

$$Q = CA^{3/4} \qquad \text{...(4.78)}$$

where Q = discharge in cumecs
A = area of basin in sq.km

The constants C depend upon the catchment and may be obtained from Table 4.23.

TABLE 4.23

Region	C
Northern India	11.4
Central India	13.9 — 19.5
Western India	22.2 — 25

2. **Ryve's Formula**

For Madras catchments

$$Q = CA^{2/3} \qquad \text{...(4.79)}$$

(where Q is in cumecs and A in sq. km)

Values of C may be obtained form Table 4.24.

TABLE 4.24

Location of the catchment	C
1. Areas within 24 km from the coast.	6.75
2. Areas within 24 km to 161 km from the coast.	8.45
3. Limited areas near hills.	10.1

3. **Inglis formula**

Inglis formula is applicable for catchments of former Bombay Presidency.

$$Q = \frac{123\,A}{\sqrt{A + 10.4}} \simeq 123\,A^{1/2} \qquad \text{...(4.80)}$$

4. **Nawab Jang Bahadur Formula**

For catchments of Old Hyderabad state :

$$Q = CA^{\left(0.993 - \frac{1}{14}\log A\right)} \qquad \text{...(4.81)}$$

where C varies from 48 to 60.

5. **Fanning's Formula**

For American catchments :

$$Q = CA^{5/6} \qquad \text{...(4.82)}$$

where average value of C may be taken equal to 2.54.

6. **Creager's Formula**

Applicable for American catchments : Expressed in F.P.S. units,

$$Q_1 = 46\,C_1\,A_1^{(0.8904\,A_1^{-0.048})} \qquad \text{...(4.83)}$$

The constant varies from 30 to 100.

7. **Fuller's Formula**

The formula takes into account the flood frequency also; expressed in metric units,

$$Q_{max} = CA^{0.8}\,(1 + 0.8 \log T)\,(1 + 2.67\,A^{-0.3}) \qquad \text{...(4.84)}$$

where T = Number of years after which such a flood is to reccur.
Q = Maximum flood (in cumecs) during any part of the day that could occur in T-years.
A = Area of drainage basin, in sq. km
C = Constant, varying from 0.185 to 1.3.

4.31. FLOOD FREQUENCY STUDIES

Since the exact sequence of stream flow for future years cannot be predicted, probability concepts must be used to study the probable variations in flow so that the design can be completed on the basis of a calculated risk.

Flood Frequency. Flood frequency denotes the likelyhood of flood being equalled or exceeded. A 10% frequency means that the flood has 10 out of 100 chances of being equalled or exceeded.

Recurrence Interval. Recurrence interval denotes the number of years in which a flood can be expected once. It is the period of time between the equalling or exceeding of a specific flood. This is usually denoted by a symbol T

Return period : It is the average recurrence interval for a certain event or flood.

Probability of occurrence (P) : The probability of an event being equalled or exceeded in any one year is the *probability of its occurrence*.

The probability (P) of occurrence of a flood having a recurrence interval of T years in any year or the probability of exceedance is

$$P = \frac{1}{T}$$

The probability that it will not occur in a year, is known as *probability of non-exceedance* (q) and is given by

$$q = 1 - P$$

Frequency (f) : The probability of occurrence of an event expressed as a percent is known as frequency (f). Thus $f = 100\,P = \dfrac{100}{T}$

Frequency studies for flood

Frequency studies interpret a past record of events to predict the future probabilities of occurrence. It is based on the assumption that combination of the numerous factors which produce floods are a matter of pure chance and therefore are subject to analysis according to mathematical theory of probability. There are two methods of compiling flood peak data :

(*i*) Annual duration series (*ii*) Partial duration series

In the *annual series*, the largest flood observed in each water year only is taken. It ignores the second and lower order events of each year which may sometimes exceed many of the annual maximum. In the *partial duration series* all flood events above a selected base value are included. The base is usually so chosen that not more than three or four events are included for each year. If the extreme flood are of primary concern, wherein the flood magnitude with exceedance probability of 0.5 or less are estimated, the annual series is used. When estimates of very frequent events with return periods of less than 5 years are required, (such as in design of coffer dams, urban drainage etc.) the partial series is preferable to annual series. However, for spillway design flood, the annual series is preferable since the flood should not be exceeded in the dams life time, say 100 years.

Annual flood series

Annual flood series consist of the values of annual maximum flood from a given catchment area, for large number of successive years. The data of the series are arranged in the decreasing order of magnitude. The probability (P) of each event being equalled to or exceeded (known as *plotting position*) is computed from one of the following *plotting position formulae*:

1. *California method* (1923) :

$$P = \frac{m}{N} \;;\; T = \frac{N}{m}$$

2. *Allen Hazen method* (1939) :

$$P = \frac{2m - 1}{2N} \;;\; T = \frac{2N}{2m - 1}$$

3. *Weibull method* (1939) :

$$P = \frac{m}{N + 1} \quad \text{or} \quad T = \frac{N + 1}{m}$$

4. *Gumbel's Method*

$$P = \frac{m + C - 1}{N} \quad \text{or} \quad T = \frac{N}{m + C - 1}$$

where C is known as Gumbel's correction. The correction depends upon $\frac{m}{N}$ ratio and can be found from Table 4.25.

TABLE 4.25. GUMBEL'S CORRECTION

$\frac{m}{N}$	1	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.08	0.04
C	1	0.95	0.88	0.845	0.78	0.73	0.66	0.59	0.52	0.4	0.38	0.28

The exceedance probability of the event (such as flood discharge etc.) obtained by the use of an empirical formula (such as Weibull formula) is called the *plotting position*. Weibull equation given above is the most popular *plotting position formula*.

Binomial Distribution

The *binomial distribution* given below can be used to find the probability of occurrence of the event r times in n successive years.

$$P_{r,n} = {}^nC_r\, P^r q^{n-r} = \frac{n!}{(n-r)!\,r!} P^r q^{n-r} \qquad \ldots(4.85)$$

where $P_{r,n}$ = probability of flood of given magnitude and exceedance probability P, occurring r times in n successive years,

and $q = 1 - P$

Thus, the probability of flood of given magnitude occurring 3 times in n successive years is

$$P_{3,n} = \frac{n!}{(n-3)!\,3!} P^3 q^{n-3}$$

Also, $$P_{2,n} = \frac{n!}{(n-2)!\,2!} P^2 q^{n-2}$$

and $$P_{1,n} = \frac{n!}{(n-1)!\,1} p^1 q^{n-1} = nPq^{n-1}$$

The probability of the flood of given magnitude not occurring at all (i.e. $r = 0$) in n successive years is

$$P_{0,n} = q^n = (1 - P)^n$$

The probability of the flood of given magnitude occurring *at least once* in n successive years is called risk (R_{sk}) and is given by

$$R_{sk} = 1 - q^n = 1 - (1 - P)^n$$

Probability Plotting

The annual flood peak can be plotted against the return period either on (*i*) ordinary probability plot, or on (*ii*) logarithmic probability plot.

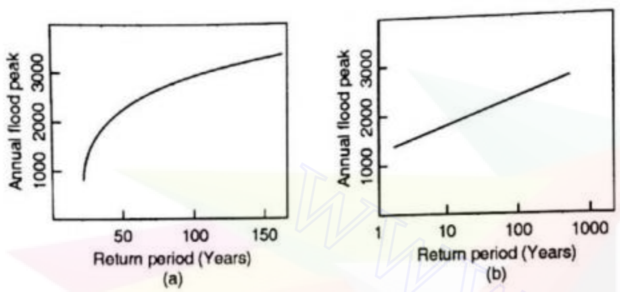

FIG. 4.67. PROBABILITY PLOTS

In the ordinary probability plot (Fig. 4,67 *a*) both the annual peak flood as well as return periods are plotted on an algebraic scale. In the logarithmic probability plot, (Fig. 4.67 *b*) annual flood peak is plotted on arithmetic scale while return period is plotted on log scale, giving rise to a straight line plotting.

Frequency distribution functions

According to Chow (1951), most *frequency distribution functions* can be expressed by the following general equation :

$$x_T = \bar{x} + K\,\sigma_x \qquad \text{...(4.86)}$$

where x_T = Value of the variate X (such as flood discharge, max. rainfall depth etc.) of a random hydrologic series with return period T

$\bar{x}$ = mean of the variate

σ_x = standard deviation of the variate

K = *Frequency factor* depending upon the return period (T) and the assumed frequency distribution.

Following are some of the commonly used frequency distribution functions :

(*a*) Gumbel's extreme value distribution

(*b*) Log-Pearson Type III distribution, and

(*c*) Log-normal distribution.

We shall, however, discuss only the first type of distribution in this Chapter.

Gumbel's extreme value distribution

This method is useful for obtaining values of flood discharges for a high recurrence interval. According to Gumbel (1941), the probability of occurrence of an event equal to or larger than a value x_0 is given by

$$P(X \geq x_0) = 1 - e^{-e^{-y}} \qquad \text{...(4.87)}$$

In the above equation, y is a dimensionless variable, given by the expression

$$y = \alpha\,(x - a)$$

where $\alpha = 1.2825/\sigma_x$ and $a = \bar{x} - 0.45005\,\sigma_x$

Hence
$$y = \frac{1.2825\,(x - \bar{x})}{\sigma_x} + 0.577 \qquad \text{...(4.88)}$$

Transposing Eq. 4.87, we get

$$y_P = -\ln[-\ln(1 - P)] \qquad \text{...(4.87 a)}$$

If y_T = value of y for a given T = *reduced variate* and noting that $T = 1/P$,

we have
$$y_T = -\left[\ln . \ln \frac{T}{T-1}\right] \qquad \text{...(4.89)}$$

or
$$y_T = -\left[0.834 + 2.303 \log\log \frac{T}{T-1}\right] \qquad \text{..(4.89 a)}$$

Again, re-arranging Eq. 4.88, the value x_T for the variate X with a return period T is given by

$$x_T = \bar{x} + K\,\sigma_x \qquad \text{...(4.90)}$$

where
$$K = \frac{y_T - 0.577}{1.2825} \qquad \text{...(4.90 a)}$$

Eqs. 4.90 are the basic Gumbel's equations which are applicable to an infinite series (i.e. where $N \to \infty$). In actual practice, N is finite and hence Eq. 4.90 is modified as under :

$$x_T = \bar{x} + K_T\,\sigma_{n-1} \qquad \text{...(4.91)}$$

where σ_{n-1} = standard deviation of the sample of size $N = \sqrt{\dfrac{\Sigma (x - \bar{x})^2}{N - 1}}$

$$K_T = \text{Modified frequency factor} = \frac{y_T - \bar{y}_n}{S_n} \qquad \text{...(4.91 a)}$$

y_T = *reduced variate* given by Eq. 4.89.

$\bar{y}_n$ = *reduced mean*, a function of sample size N, the values of which are given in Table 4.21 (*a*). Note that when $N \to \infty$, $\bar{y}_n \to 0.577$

S_n = *reduced standard deviation*, a function of sample size N, the values of which are given in Table 4.21 (*b*). Note that when $N \to \infty$, $S_n \to 1.2825$

Procedure

Step 1 : For the given annual flood discharge data of size N, find mean ($\bar{x}$) and standard deviation (σ_{n-1})

Step 2 : From Table 4.21 (*a*) find reduced mean $\bar{y}_n$ for the given value of N. For example if $N = 33$, we get $\bar{y}_n = 0.5388$ from Table 4.21 (*a*).

Step 3 : From Table 4.21 (*b*), find reduced standard deviation S_n for the given value of N. For example, if $N = 33$, we get $S_n = 1.1226$.

TABLE 4.26 (a) VALUES OF GUMBEL'S REDUCED MEAN $\bar{y}_n$ FOR SAMPLE SIZE N

$N \rightarrow$	0	1	2	3	4	5	6	7	8	9
10	0.4952	0.4996	0.5035	0.5070	0.5100	0.5128	0.5157	0.5181	0.5202	0.5220
20	0.5236	0.5252	0.5268	0.5283	0.5296	0.5309	0.5320	0.5332	0.5343	0.5353
30	0.5362	0.5371	0.5380	0.5388	0.5396	0.5402	0.5410	0.5418	0.5424	0.5430
40	0.5436	0.5442	0.5448	0.5453	0.5458	0.5463	0.5468	0.5473	0.5477	0.5481
50	0.5485	0.5489	0.5493	0.5497	0.5501	0.5504	0.5508	0.5511	0.5515	0.5518
60	0.5521	0.5524	0.5527	0.5530	0.5533	0.5535	0.5538	0.5540	0.5543	0.5545
70	0.5548	0.5550	0.5552	0.5555	0.5557	0.5559	0.5561	0.5563	0.5565	0.5567
80	0.5569	0.5570	0.5572	0.5574	0.5576	0.5578	0.5580	0.5581	0.5583	0.5585
90	0.5586	0.5587	0.5589	0.5591	0.5592	0.5593	0.5595	0.5596	0.5598	0.5599
100	0.5600									
∞	0.577									

TABLE 4.26 (b) VALUES OF GUMBEL'S REDUCED STANDARD DEVIATION S_n FOR SAMPLE SIZE N

N	0	1	2	3	4	5	6	7	8	9
10	0.9496	0.9676	0.9833	0.9971	1.0095	1.0206	1.0316	1.0411	1.0493	1.0565
20	1.0628	1.0696	1.0754	1.0811	1.0864	1.0915	1.0961	1.1004	1.1047	1.1086
30	1.1124	1.1159	1.1193	1.1226	1.1255	1.1285	1.1313	1.1339	1.1363	1.1388
40	1.1413	1.1436	1.1458	1.1480	1.1499	1.1519	1.1538	1.1557	1.1574	1.1590
50	1.1607	1.1623	1.1638	1.1658	1.1667	1.1681	1.1696	1.1708	1.1721	1.1734
60	1.1747	1.1759	1.1770	1.1782	1.1793	1.1803	1.1814	1.1824	1.1834	1.1844
70	1.1854	1.1863	1.1873	1.1881	1.1890	1.1898	1.1906	1.1915	1.1923	1.1930
80	1.1938	1.1945	1.1953	1.1959	1.1967	1.1973	1.1980	1.1987	1.1994	1.2001
90	1.2007	1.2013	1.2020	1.2026	1.2032	1.2038	1.2044	1.2049	1.2055	1.2061
100	1.2065									
∞	1.2825									

Step 4 : For the given value of recurrence interval T, determine y_T from Eq. 4.89.

Step 5 : Knowing $y_T, \bar{y}_n$ and S_n, calculate the value of K_T from Eq. 4.91 (*a*).

Step 6 : Lastly, compute the value of x_T (i.e. the desired flood discharge at recurrence interval T) from Eq. 4.91 in which $\bar{x}, K_T$ and σ_{n-1} are known.

See example 4.34 for illustration.

Note 1 : Gumbel's distribution has the property which gives $T = 2.33$ years for the average of the annual series, when N is very large. Hence the value of the flood with $T = 2.33$ years is called *mean annual flood.*

Note 2 : In the above mentioned procedure, a series of steps are required to compute the value of K_T for given sample size (N) and currence interval or frequency (T). However, Table 4.27 gives the computed values of K_T for given set of values of N and T.

TABLE 4.27. VALUES OF FREQUENCY FACTOR K_T FOR GUMBEL'S METHOD

Sample size (N) in years	*Frequency (T) in years*									
	5	*10*	*15*	*20*	*25*	*30*	*50*	*60*	*75*	*100*
15	0.967	1.703	2.117	2.410	2.632	2.823	3.321	3.501	3.721	4.005
20	0.919	1.625	2.023	2.302	2.517	2.690	3.179	3.352	3.563	3.836
25	0.888	1.575	1.963	2.235	2.444	2.614	3.088	3.257	3.463	3.729
30	0.866	1.541	1.922	2.188	2.393	2.560	3.026	3.191	3.393	3.653
35	0.851	1.516	1.891	2.152	2.354	2.520	2.979	3.142	3.341	3.598
40	0.838	1.495	1.866	2.126	2.326	2.489	2.943	3.104	3.301	3.554
45	0.829	1.478	1.847	2.104	2.303	2.464	2.913	3.078	3.268	3.520
50	0.820	1.466	1.831	2.086	2.283	2.443	2.889	3.048	3.241	3.491
55	0.813	1.455	1.818	2.071	2.267	2.426	2.869	3.027	3.219	3.467
60	0.807	1.446	1.806	2.059	2.253	2.411	2.852	3.008	3.200	3.446
65	0.801	1.437	1.796	2.048	2.243	2.398	2.837	2.992	3.183	3.429
70	0.797	1.430	1.788	2.038	2.230	2.387	2.824	2.979	3.169	3.413
75	0.792	1.423	1.780	2.029	2.220	2.377	2.812	2.967	3.155	3.400
80	0.788	1.417	1.773	2.020	2.212	2.368	2.802	2.956	3.145	3.387
85	0.785	1.413	1.767	2.013	2.205	2.361	2.793	2.946	3.135	3.376
90	0.782	1.409	1.762	2.007	2.198	2.353	2.785	2.938	3.125	3.367
95	0.780	1.405	1.757	2.002	2.193	2.347	2.777	2.930	3.116	3.357
100	0.779	1.401	1.752	1.993	2.187	2.341	2.770	2.922	3.109	3.349

Plot of x_T versus T

In order to obtain the variation of x_T with T, the values of x_T are computed for some return periods $T(<N)$ using Gumbel's formula. These pairs of values of x_T and T are plotted on semi-log, log-log or Gumbel's probability paper. If these are plotted on Gumbel's probability paper, it will give rise to a straight line. This straight line will pass through the *mandatory point* corresponding to the pair of values of $T = 2.33$ years and $x_T = \bar{x}$. (Fig. 4.68).

Construction of Extreme value probability paper (Gumbel's probability paper)

The above mentioned *probability paper*, which plots Gumbel's extreme value distribution as a straight line can be easily constructed as follows :

Step 1 : Construct the arithmetic scale of y_T values, say from -2 to $+6$ on the abscissa

Step 2 : Compute the value of y_T for some selected values of T (say, 2, 5, 10, 50, 100, 500 etc.), using Eqs. 4.89 (*a*) or 4.89 as under :

$$y_T = -\left[\ln . \ln \frac{T}{T-1}\right] = -\left[0.834 + 2.303 \log \log \frac{T}{T-1}\right]$$

Step 3 : Mark off these positions on the abscissa, thus giving rise to the T-scale or probability scale for recurrence interval T

It is to be noted that the variate x_T (i.e. flood discharge for the present case) is generally plotted on arithmetic scale on the Gumbel's paper.

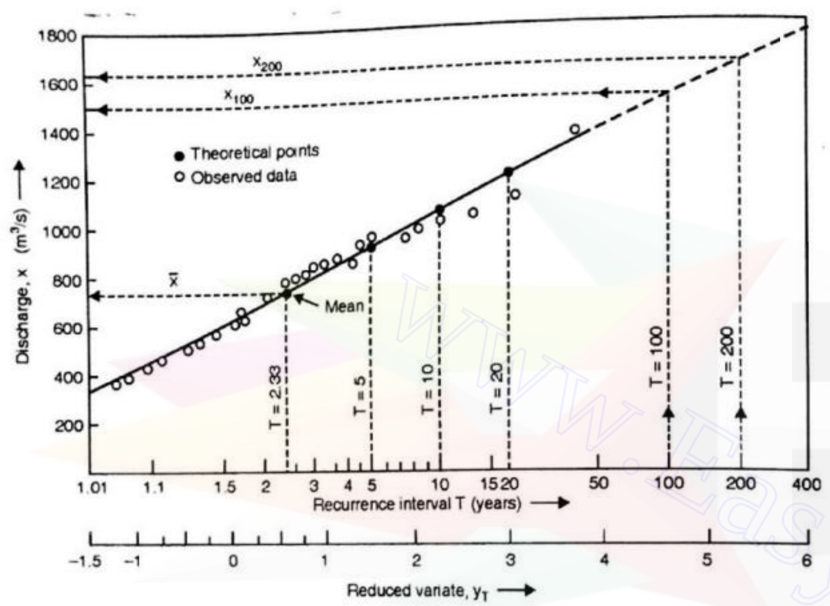

FIG. 4.68. FLOOD PROBABILITY PLOT ON GUMBEL'S PAPER

Test for observed flood data

The Gumbel's straight line mentioned above may be obtained by plotting two or three pairs of values of x_T and T, though the line will pass through the mandatory point $T = 2.33$ years and $x_T = \bar{x}$. In order to confirm whether the observed data follows Gumbel's distribution or not, the following procedure is followed.

***Step* 1** : Arrange the observed data (about flood) in descending order of magnitude and assign each the order m, the total number of observations being N.

***Step* 2** : Find the return periods (plotting positions) for each discharge, using Weibull formula : $T = (N + 1)/m$

***Step* 3** : Plot these pairs of values of discharge and the corresponding return periods, on the Gumbel's paper.

***Step* 4** : If a good fit of the observed data with the theoretical variation line is seen, it will be concluded that Gumbel's distribution is applicable to the given series.

The values of x_T (i.e. flood discharge) for T greater than N can be determined by extrapolating Gumbel's straight line (x_T *vs* T).

Selection of design return period : Risk and Reliability

In the terminology of probability theory, $1/T$ indicates the *probability* with which T year flood may be equalled or exceeded in any one year. Hence if it is desired to select a design flood which is not likely to occur during the life period of the structure, it becomes necessary to use a return period greater than the estimated useful life.

Let us take the case of a weir which is designed for T years flood and its useful life is n years. The probability that the design flood is equalled or exceeded, and hence the probability that the weir may fail in any year is $1/T$. In other words, the probability that the structure does not fail in any year is $\left(1 - \frac{1}{T}\right)$. Assuming that the annual flood peaks are independent events, the probability that the weir does not fail in the next n years is $\left(1 - \frac{1}{T}\right)^n$.

Hence the risk R_{sk} in the design, which is the probability that the weir may fail in any one of the next n years, is given by

$$R_{sk} = 1 - \left(1 - \frac{1}{T}\right)^n = 1 - (1 - P)^n \quad \text{...(4.92)}$$

The *reliability* R_{el} is defined as

$$R_{el} = 1 - R_{sk} = \left(1 - \frac{1}{T}\right)^n = (1 - P)^n \quad \text{..(4.92 a)}$$

The above equations can also be used to determine the required return period of the design flood for a given risk and given life period n. For example, if the weir with life period of 50 years is designed for a 50 years flood, the risk of failure is

$$R_{sk} = 1 - \left(1 - \frac{1}{50}\right)^{50} = 0.636 \text{ or } 63.6\%$$

If the weir is designed for a 100 year flood,

$$R_{sk} = 1 - \left(1 - \frac{1}{100}\right)^{50} = 0.605 \text{ or } 60.5\%$$

However, if we want to reduce the risk to 0.10 (or 10%) we have

$$0.10 = 1 - \left(1 - \frac{1}{T}\right)^{50} \text{ which givens } T = 475 \text{ years.}$$

4.32. FLOOD DISCHARGE BY RATIONAL FORMULA

Amongst various types of empirical relations, *rational formula* is the most rational method of calculating peak discharge for *small catchments*. In this method, it is assumed that the maximum flood flow is produced by a certain rainfall intensity which lasts for a time equal to or greater than the period of *concentration time* (t_c). The maximum rate of runoff from the watershed appears when the entire area contributes at the basin outlet. The runoff gradually increases from zero to peak when the rainfall duration reaches the time of concentration (t_c). If the rainfall continues beyond t_c, the runoff will be constant and at the peak value. The peak value of runoff is given by

$$Q_p = F_u \cdot C\, i\, A \quad \text{...(4.93)}$$

where C = runoff coefficient representing a ratio of runoff to rainfall

A = Catchment area.

i = rainfall intensity

F_u = a factor which permits the expression of terms Q_p, A and i in consistent units

Let $\quad Q_p$ = discharge in cubic metres per second (cumecs)

A = catchment area in km^2

i = intensity of rainfall in cm/hour

In order to find corresponding value of factor F_u, we have

$$Q_p = (10^6 A)(C)\left(\frac{i}{100 \times 3600}\right) = \frac{C\,i\,A}{0.36} = 2.778\,C\,i\,A \qquad ...(4.94)$$

Hence factor F_u = **2.778**

If, however, i is expressed in mm/hour, we have

$$Q_p = (10^6 A)(C)\left(\frac{i}{1000 \times 3600}\right) = \frac{C\,i\,A}{3.6} = 0.2778\,C\,i\,A \qquad ...(4.95)$$

Runoff coefficient C

Runoff coefficient is a highly critical element that serves the purpose of converting the average rainfall rate of a particular recurrence interval to the peak runoff intensity of the same frequency. Its magnitude depends upon the following factors (*i*) antecedent moisture conditions (*ii*) ground slope (*iii*) ground cover (*iv*) depression storage (*v*) soil moisture, (*vi*) shape of drainage area (*vii*) overland flow velocity, (*viii*) intensity of rainfall (*ix*) geology of catchment, etc. In spite of this, its value is generally considered fixed for any drainage area, depending only on the surface type. This simplistic approach is a major cause of criticism for the rational method. Some typical values of C are indicated in Table 4.28.

TABLE 4.28. TYPICAL VALUES OF RUNOFF COEFFICIENT C

Type of area	*Value of C*	*Type of area*	*Value of C*
(a Urban area			
(a) Lawns :		*(d) Industrial area*	
(*i*) Sandy soil, flat 2%	0.05 - 0.10	(*i*) Unimproved	0.1 - 0.3
(*ii*) Sandy-soil, average 2.7%	0.1 - 0.15	(*ii*) Light areas	0.5 - 0.8
(*iii*) Sandy-soil, steep < 7%	0.15 - 0.2	(*iii*) Heavy areas	0.6 - 0.9
(*iv* Heavy soil, flat	0.13 - 0.18	(*iv*) Rail yards	0.2 - 0.4
(*v*) Heavy soil, average	0.18 - 0.22	*(e) Streets*	0.7 - 0.95
(*vi*) Heavy soil, steep	0.25 - 0.35	***(B) Agricultural area***	
(b) Residential area		*(a) Flat*	
(*i*) Single House area	0.3 - 0.5	(*i*) Light clay; cultivated	0.50
(*ii*) Multi-units attached	0.6 - 0.75	(*ii*) Light clay; wood land	0.40
(*iii* Suburban	0.25 - 0.4	(*iii*) Sandy loam : cultivated	0.7
(c) Business area	0.5 - 0.95	(iv) Sand loam : wood land	0.1
		(b) Hilly	
		(*i*) Light clay ; cultivated	0.7
		(*ii*) Light clay ; wood land	0.6
		(*iii*) Sandy loam ; Cultivated	0.4
		(*iv*) Sandy loam ; wood land	0.3

If a watershed of total area A is non-homogeneous, having component sub-areas having different values of C, a weighted runoff coefficient (C_w) is computed from the following equation :

$$C_w = \frac{C_1 A_1 + C_2 A_2 + ...C_n A_n}{A_1 + A_2 + ...A_n} = \frac{\sum_{j=1}^{j=n} C_j A_j}{A} \qquad ....(4.96)$$

where $\quad A_1 ; A_2 ; A_3 ...... A_n$ = areas of sub-zones

$C_1, C_2, C_3, ...C_n$ = runoff coefficients for the corresponding sub-zones.

Rainfall intensity (*i*)

The rainfall intensity (i) corresponding to a duration t_c and the desired probability of exceedance P (or return period T) is given by Eq. 4.19 indicating the relationship between intensity, duration and return period.

$$i = \frac{K\,T^x}{(t+b)^n} \qquad ...(4.19)$$

where $\quad i$ = intensity of rainfall (cm/h).

Typical values of constants K, x, b and n are given in Table 4.8.

In the above equation, t is the *time of concentration* (in hours) which can be found from the following formula by Kirpich (1940).

$$t = t_c = 0.000323\,L^{0.77}\,S^{-0.385} \qquad ...(4.58)$$

4.33. METHOD OF ENVELOPING CURVES

In this method, areas having similar topographical features and climatic conditions are grouped together, the available data regarding discharges are compiled along with

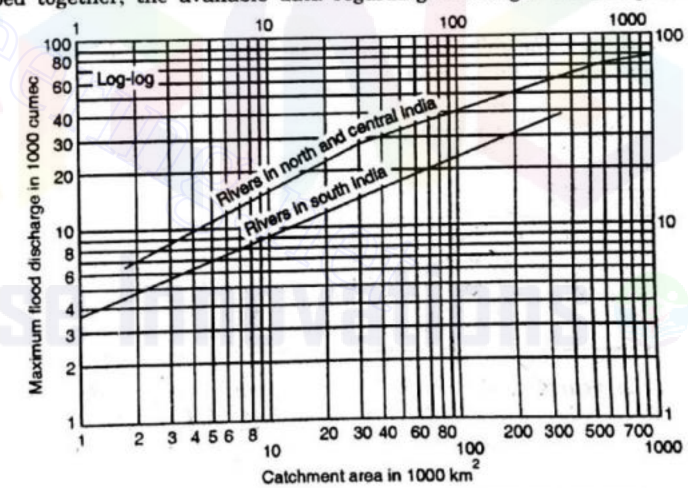

FIG. 4.69. ENVELOPING CURVES FOR INDIAN RIVERS

their catchment areas, the peak flood discharges are plotted against the drainage areas, and a curve is then drawn to cover or envelope the highest plotted points. Kanwar Sain and Karpov (1967) have presented two enveloping curves, one for Northern and Central Indian rivers and the other for the Southern Indian rivers, as shown in Fig. 4.69.

Mcilluraith (1951) gave the following equation for the enveloping curve of maximum floods (Q_{mp}), based on maximum recorded flood throughout the World :

$$Q_{mp} = \frac{3025\,A}{(278 + A)^{0.78}} \qquad \text{...(4.97)}$$

where Q_{mp} is in m^3/s and A is in km^2.

Limitations of Rational Formula

1. The formula gives good results only for small catchments, having area upto 50 km^2.

2. It is applicable only if the duration of rainfall is equal to or more than time of concentration (t_c).

3. Rainfall intensity (i) should be constant over the entire catchment, during the time of concentration.

4. It assumes constant value of C for a given area, for all storms, which is not reasonable.

5. If a plot is made between Q_p and i, a straight line is obtained with zero intercept. Nature does not follow such a linear relationship.

Due to the above limitations, the rational formula is generally used to estimate the peak flood (Q_p) in the design of urban drainage system, storm severs, design of small culverts, and bridges etc.

Example 4.32. *The following Table gives flood data for 16 years recorded at the Bhakra Dam site on Sutlej river :*

Year	*Discharge (cumecs)*	*year*	*Discharge (cumecs)*
1937	3110	1944	2290
1938	5800	1945	2380
1939	3090	1946	3810
1940	1723	1947	7800
1941	3630	1948	4525
1942	6600	1949	3254
1943	5260	1950	4980
		1951	9200

Find out the recurrence interval for the flood of various magnitudes by the following methods : (i) California method, (ii) Hazen's method, and (iii) Gumbel's method.

Solution : The recurrence interval has been calculated by the three methods in the tabular form below. The value of C for Gumbel's method has been taken from Table 4.20. Here N=15.

Order No. (m)	*Peak yearly discharges arranged in decreasing order (cumecs)*	*Recurrence Interval*		
		California Method $T = \frac{N}{m}$	*Hazen's method* $T = \frac{2N}{2m-1}$	*Gumbel's method* $T = \frac{N}{m+C-1}$
1	9200	15.0	30	50
2	7800	7.50	10	10.41
3	6600	5.00	6	5.95
4	5800	3.75	4.28	4.21
5	5260	3.00	3.33	3.25
6	4980	2.50	2.72	2.65
7	4525	2.14	2.30	2.24
8	3810	1.87	2.00	1.94
9	3630	1.66	1.82	1.71
10	3250	1.50	1.56	1.53
11	3110	1.36	1.42	1.38
12	3090	1.25	1.30	1.26
13	2380	1.15	1.20	1.17
14	2390	1.07	1.11	1.08
15	1723	1.00	1.03	1.00

Example 4.33. *For a river valley project, the following results were obtained from flood frequency analysis using Gumbel's method :*

Return period T (years)	*Peak flood (m^3/s)*
40	27000
80	31000

Estimate the flood magnitude with a return period of 240 years.

Solution

From Gumbel's equation (Eq. 4.91)

$$x_T = \bar{x} + K_T\,\sigma_{n-1}$$

Hence $\quad x_{80} = \bar{x} + K_{80}\,\sigma_{n-1}$ and $x_{40} = \bar{x} + K_{40}\,\sigma_{n-1}$

$$\therefore \quad (K_{80} - K_{40})\,\sigma_{n-1} = x_{80} - x_{40} = 31000 - 27000 = 4000 \qquad \text{...(1)}$$

But $K_T = \frac{y_T - \bar{y}_n}{S_n} = \frac{y_T}{S_n} - \frac{\bar{y}_n}{S_n}$, where $\bar{y}_n$ and S_n are constants for given series

$$\therefore \quad (y_{80} - y_{40})\,\frac{\sigma_{n-1}}{S_n} = 4000 \qquad \text{...(2)}$$

From Eq. 4.89, $\quad y_T = -\left[\ln . \ln \frac{T}{T-1}\right]$

$$\therefore \quad y_{80} = -\,[\ln . \ln (80/79)] = 4.37574$$

and $\quad y_{40} = -\,[\ln . \ln (40/39)] = 3.67625$

Hence from (2), $\quad \frac{\sigma_{n-1}}{S_n} = \frac{4000}{4.37574 - 3.67625} = 5718.5 \qquad \text{...(3)}$

For $T = 240$ years, $\quad y_{240} = -\,[\ln . \ln (240/239)] = 5.47855$

Now $(y_{240} - y_{80}) \dfrac{\sigma_{n-1}}{S_n} = x_{240} - x_{80}$ or $(5.47855 - 4.37574) \times 5718.5 = x_{240} - 31000$

From which $x_{240} \simeq 37300 \text{ m}^3/\text{s}$

Example 4.34. *The annual maximum recorded floods for a river, for a period of 40 years from 1961 to 2000 are given below. Verify whether the Gumbel extreme-value distribution fits the recorded values. Estimate the flood discharge with a recurrence interval of 100 years, and 200 years by graphical interpolation as well as theoretical computations.*

Year	*1961*	*1962*	*1963*	*1964*	*1965*	*1966*	*1967*	*1968*	*1969*	*1970*
Max. flood (m^3/s)	530	835	1030	570	380	1330	950	710	700	585
Year	*1971*	*1972*	*1973*	*1974*	*1975*	*1976*	*1977*	*1978*	*1979*	*1980*
Max. flood (m^3/s)	940	840	670	795	810	485	925	980	708	420
Year	*1981*	*1982*	*1983*	*1984*	*1985*	*1986*	*1987*	*1988*	*1989*	*1990*
Max. flood (m^3/s)	550	975	1095	620	595	853	465	855	625	505
Year	*1991*	*1992*	*1993*	*1994*	*1995*	*1996*	*1997*	*1998*	*1999*	*2000*
Max. flood (m^3/s)	390	500	705	460	605	495	945	756	610	825

Solution : The given flood data is entered in column (2) of the Table below, in descending order, while column (1) shows the rank of each discharge. The plotting position recurrence interval (T_p) for each discharge is computed from Weibull equation: $T_p = (N + 1)/m$ where $N = 40$ in the present case.

Order No. (m)	*Discharge* x (m^3/s)	T_p *(years)*	$(x - \bar{x})^2$	*Order No. (m)*	*Discharge* x (m^3/s)	T_p *(years)*	$(x - \bar{x})^2$
1	1330	41	377549	21	700	1.95	242
2	1095	20.5	143982	22	670	1.86	2075
3	1030	13.67	98879	23	625	1.78	8199
4	980	10.25	69934	24	620	1.71	9130
5	975	8.2	67314	25	610	1.64	11140
6	950	6.83	54967	26	605	1.58	12221
7	945	5.86	52647	27	595	1.52	14532
8	940	5.13	50378	28	585	1.46	17043
9	925	4.56	43869	29	570	1.41	21185
10	855	4.10	19446	30	550	1.37	27407
11	853	3.73	18893	31	530	1.32	34429
12	840	3.42	15488	32	505	1.28	44331
13	835	3.15	14268	33	500	1.24	46462
14	825	2.93	11979	34	495	1.21	48642
15	810	2.73	8921	35	485	1.17	53153
16	795	2.56	6312	36	465	1.14	62775
17	756	2.41	1636	37	460	1.11	65306
18	710	2.28	31	38	420	1.08	87350
19	708	2.16	57	39	390	1.05	105983
20	705	2.05	111	40	380	1.03	112594
				Σ	28622	Σ	1840860

$$\Sigma x = 28622 \text{ and hence } \bar{x} = 28622/40 \simeq 715.6 \text{ m}^3/\text{s}$$

$$\Sigma (x - \bar{x})^2 = 1840860 \quad ; \text{ hence } \sigma_{n-1} = \sqrt{1840860/39} = 217.3$$

Here $N = 40$ years. Hence from Tables 4.21 (*a*) and (*b*), we get $\bar{y}_n = 0.5436$ and $S_n = 1.1413$

Choosing $N = 10$, years, we get

$$y_T = y_{10} = -[\ln . \ln (10/9)] = 2.25037$$

$$K_T = K_{10} = \frac{y_T - \bar{y}_n}{S_n} = \frac{2.25037 - 0.5436}{1.1413} = 1.4955$$

$$x_T = x_{10} = \bar{x} + K_T \, \sigma_{n-1} = 715.6 + 1.4955 \times 217.3 \simeq 1041 \text{ m}^3/\text{s}$$

Choosing $N = 20$ years, we get

$$y_T = y_{20} = -[\ln . \ln (20/19)] = 2.9702$$

$$K_T = K_{20} = \frac{y_T - \bar{y}_n}{S_n} = \frac{2.9702 - 0.5436}{1.1413} = 2.1262$$

$$\therefore \quad x_T = x_{20} = \bar{x} + K_T \, \sigma_{n-1} = 715.6 + 2.1262 \times 217.3 \simeq 1178 \text{ m}^3/\text{s}$$

Finally, choosing $N = 5$ years, we have

$$y_T = y_5 = -[\ln . \ln (5/4)] = 1.4999$$

$$K_T = K_5 = \frac{y_T - \bar{y}_n}{S_n} = \frac{1.4999 - 0.5436}{1.1413} = 0.8379$$

$$\therefore \quad x_T = x_5 = \bar{x} + K_T \, \sigma_{n-1} = 715.6 + 0.8379 \times 217.3 \simeq 898 \text{ m}^3/\text{s}$$

Thus we have following (theoretical) three sets of values

T (years)	*Theoretical values of* x_T
5	898
10	1041
20	1178

These values are plotted on Gumbel's probability paper, as shown in Fig. 4.68. It is seen that this theoretical line will also pass through mandatory point corresponding to $x_T = \bar{x} = 715.6$ and $T = 2.33$ years.

All these four points are shown by solid dots. On the same paper, the observed sets of values (x_T, T) are plotted, as shown by hollow dots in Fig. 4.68. It is seen that the data of the given series fit well with Gumbel's extreme value distribution function.

From the same graph, we get following values by extrapolation :

When $T = 100$, $x_{100} \simeq 1500 \text{ m}^3/\text{s}$

When $T = 200$, $x_{200} \simeq 1620 \text{ m}^3/\text{s}$

However, these values can also be computed *theoretically* as under :

When $T = 100$ ***years***

$$y_T = y_{100} = -[\ln . \ln (100/99)] = 4.6001$$

$$K_T = K_{100} = \frac{y_T - \bar{y}_n}{S_n} = \frac{4.6001 - 0.5436}{1.1413} = 3.5543$$

$\therefore$ $$x_T = x_{100} = \bar{x} + K_T\,\sigma_{n-1} = 715.6 + 3.5543 \times 217.3 \doteq \mathbf{1488\ m^3/s}$$

When $T = 200$ ***years***

$$y_T = y_{200} = -[\ln . \ln(200/199)] = 5.2958$$

$$K_T = (5.2958 - 0.5436)/1.1413 = 4.1639$$

$$x_T = x_{200} = 715.6 + 4.1639 \times 217.3 \doteq \mathbf{1620\ m^3/s}$$

Example 4.35. *From the analysis of available data on annual flood peaks of a stream for a period of 40 years, the 50 year and 100 year floods have been estimated to be 878 m^3/s and 970 m^3/s. Using Gumbel's method, estimate the 200 year flood for the stream.*

Solution

For N = 40 years, we get the following values from Table 4.21 (*a*) and (*b*).

$$\bar{y}_n = 0.5436 \text{ and } S_n = 1.1413$$

These are thus constants for the series.

Now $$y_{50} = -[\ln . \ln(50/49)] = 3.90194$$

and $$K_{50} = \frac{y_{50} - \bar{y}_n}{S_n} = \frac{3.90194 - 0.5436}{1.1413} = 2.9426$$

Similarly, $$y_{100} = -[\ln . \ln(100/99)] = 4.60015$$

and $$K_{100} = \frac{y_{100} - \bar{y}_n}{S_n} = \frac{4.60015 - 0.5436}{1.1413} = 3.5543$$

From the general equation $x_T = \bar{x} + K_T\,\sigma_{n-1}$, we have

$$x_{50} = 878 = \bar{x} + 2.9426\ \sigma_{n-1} \quad \text{...(1)}$$

and $$x_{100} = 970 = \bar{x} + 3.5543\,\sigma_{n-1} \quad \text{...(2)}$$

Solving the two simultaneous equations (1) and (2), we get

$$\bar{x} = 435.4\ m^3/s \text{ and } \sigma_{n-1} = 150.4\ m^3/s$$

Hence, we have $$x_T = 435.4 + K_T\,(150.4) \quad \text{...(3)}$$

Again, $$y_{200} = -[\ln . \ln(200/199)] = 5.29581$$

$$K_{200} = \frac{y_{200} - \bar{y}_n}{S_n} = \frac{5.29581 - 0.5436}{1.1413} = 4.16386$$

Hence $$x_{200} = 435.4 + 4.16386\,(150.4) \doteq \mathbf{1062\ m^3/s}$$

Example 4.36. *A coffer dam is designed for a 30 year flood and constructed. If it takes 6 years to complete the construction of main dam, what is the risk that the coffer dam may fail before the end of the construction period ? What return period in the design of coffer dam would have reduced the risk to 10% ?*

Solution The risk of failure is given by Eq. 4.92.

$$R_{sk} = 1 - \left(1 - \frac{1}{T}\right)^n, \text{ where } T = 30 \text{ years and } n = 6 \text{ years.}$$

$\therefore$ $$R_{sk} = 1 - \left(1 - \frac{1}{30}\right)^6 = 0.184 \text{ or } \mathbf{18.4\%}$$

If risk is reduced to 10%, we have

$$0.1 = 1 - \left(1 - \frac{1}{T}\right)^6$$

which gives $T = 57.45 \doteq$ **58 years** (say)

Example 4.37. *A flood of a certain magnitude has a return period of 40 years. Determine : (a) the probability of exceedance, (b) probability of the flood of magnitude equal to or greater than the given magnitude occurring*

(*i*) *at least once in 10 successive years.*
(*ii*) *two times in 10 successive years, and*
(*iii*) *once in 10 successive years*

Solution : Given : $T = 40$ years.

(*a*) Exceedance probability $= P = \frac{1}{T} = \frac{1}{40} = \mathbf{0.025}$

(*b*) Probability of equaling or exceeding *at least* once in n successive years in given by Eq. 4.92.

$$R_{sk} = 1 - (1 - P)^n \text{ where } n = 10 \text{ here}$$

$\therefore$ $$R_{sk} = 1 - (1 - 0.025)^{10} = \mathbf{0.224}$$

Probability of occurrence twice in n years is given by Eq. 4.85

$$P_{2,n} = \frac{n\,!}{(n-2)\,!\,2\,!}\,p^2 q^{n-2} \text{ where } q = 1 - P = 1 - 0.025 = 0.975 \text{ and } n = 10$$

$\therefore$ $$P_{2,10} = \frac{10\,!}{8\,!\,2\,!}(0.025)^2\,(0.975)^8 = \frac{8\,! \times 9 \times 10}{8\,!\,2 \times 1}(0.025)^2\,(0.975)^8 = \mathbf{0.0229}$$

Probability of exceedance once in $n\,(=10)$ years is

$$P_{1,n} = n\,.\,P\,.\,q^{n-1} \text{ where } n = 10 \text{ years}$$

$\therefore$ $$P_{1,10} = 10\,(0.025)\,(0.975)^{10-1} = \mathbf{0.199}$$

Example 4.38. *A small water shed consists of 3.2 km^2 of cultivated area with $C = 0.22$, 4.8 km^2 under forest with $C = 0.12$ and 1.8 km^2 under grass cover with $C = 0.32$. The water course, 2.4 km in length has a fall of 30 m. The intensity-frequency-duration relation for the area may be expressed by the following relation :*

$$i = \frac{78\,T^{0.22}}{(t+12)^{0.45}}$$

where i is in cm/h, T is in years and t is in minutes. Estimate the peak rate of runoff for a 30 years frequency, using rational formula.

Solution : Slope of water course, $S = \Delta H/L = 30/2400 = 1/80$

$$t = t_c = 0.000323\,L^{0.77}\,S^{-0.385} = 0.000323\,(2400)^{0.77} \times \left(\frac{1}{80}\right)^{-0.385}$$

$$= 0.6993 \text{ hours} \simeq 42 \text{ minutes}$$

$$i = \frac{78\,T^{0.22}}{(t+12)^{0.45}} = \frac{78\,(30)^{0.22}}{(42+12)^{0.45}} = 27.38 \text{ cm/h}$$

$$\therefore \quad Q_p = 2.778\,C\,i\,A = 2.778\,i\,\Sigma\,C \times A$$

$$= 2.778 \times 27.38\,(3.2 \times 0.22 + 4.8 \times 0.12 + 1.8 \times 0.32) \simeq \mathbf{141.2\ m^3/s}$$

Example 4.39. *A small watershed has an area of 2.4 km². The slope of the catchment is 1/200 and the maximum length of travel of water is 1.8 km. The maximum depth of rainfall with a 30-year return period is given in the following Table*

Duration (min)	5	10	15	20	25	30	40	50	60
Depth of rainfall (cm)	2.5	3.8	4.8	5.9	6.7	7.4	8.4	8.7	9.2

Determine the peak flow rate for a return period of 30 years if the average runoff coefficient for the watershed is 0.25.

Solution :

The time of concentration is given by Kirpich formula :

$$t_c = 0.000323\,L^{0.77}\,S^{-0.385}$$

$$= 0.000323\,(1800)^{0.77}\left(\frac{1}{200}\right)^{-0.385} = 0.7974 \text{ hours} = 47.84 \text{ min.}$$

For a duration ($t = t_c$) of 47.84 min., the maximum depth of rainfall can be determined from the above table by linear interpolation and is equal to

$$8.4 + \frac{8.7 - 8.4}{10} \times 7.84 \simeq 8.64 \text{ cm}$$

Hence average intensity $\quad i_{av} = \dfrac{8.64}{47.84} \times 60 = 10.83$ cm/h

Hence $\quad Q_p = 2.778\,C\,i\,A = 2.778 \times 0.25\,(10.83) \times 2.4 = \mathbf{18.05\ m^3/s}$

4.34. EXAMPLES FROM COMPETITIVE EXAMINATIONS

Example 4.40. *A precipitation station X was inoperative for some time during which a storm occurred. At three stations A, B and C surrounding X, the total precipitation recorded during this storm are 75, 58 and 47 mm respectively. The normal annual precipitation amounts at stations X, A, B and C are 757, 826, 618 and 482 mm respectively. Estimate the storm precipitation for station X..*

(Engg. services Exam. 1993)

Solution : ***This problem is similar to Example 4.3.***

$$P_x = \frac{N_x}{3}\left[\frac{P_A}{N_A} + \frac{P_B}{N_B} + \frac{P_C}{N_C}\right] = \frac{757}{3}\left[\frac{75}{826} + \frac{58}{618} + \frac{47}{482}\right] \simeq \mathbf{71.2\ mm}$$

Example 4.41. *In a basin, six rain gauge stations are reporting rainfall in a year. The annual rainfall recorded by the gauges are :*

Station	A	B	C	D	E	F
Rainfall (cm)	41	51	32	55	50	68

For 80% error in estimation of mean rainfall, calculate the optimum no. of rain gauge stations in the basin. ***(Civil Services Exam. 1995)***

Solution : ***This problem is similar to Example 4.1.***

From Eq. 4.3, $\quad N = \left[\dfrac{C_v}{p}\right]^2$

where $\quad C_v = \dfrac{s_x}{\bar{x}} \times 100$; $\bar{x} = \dfrac{\Sigma\,x_i}{n}$ and $s_x = \sqrt{\dfrac{\Sigma\,(x_i - \bar{x})^2}{n-1}}$; $n = 6$

Here $\quad \bar{x} = \dfrac{41 + 51 + 32 + 55 + 50 + 68}{6} = 49.5$ cm

$$s_x = \left[\left\{(41-49.5)^2 + (51-49.5)^2 + (32-49.5)^2 + (55-49.5)^2 + (50-49.5)^2 + (68-49.5)^2\right\}/5\right]^{1/2}$$

$$= 12.28$$

$$\therefore \quad C_v = \frac{s_x}{\bar{x}} \times 100 = \frac{12.28}{49.5} \times 100 = 24.80$$

$$p = 8\%$$

$$\therefore \quad N = \left(\frac{C_v}{p}\right)^2 = \left(\frac{24.80}{8}\right)^2 = 9.61 \simeq \mathbf{10}$$

Example 4.42. *Find the mean precipitation for the area sketched in Fig. 4.70 by Thiessen's method. The area is composed of a square plus an equilateral triangular plot of side 4 kilometers. Rainfall readings at the various station are also given in centimeters in the Fig.* ***(Engg Services Exam. 1968)***

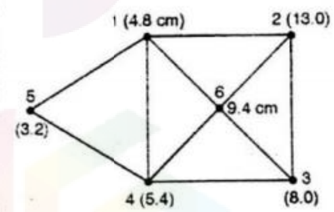

FIG. 4.70

Solution

Let the stations 1,2,3,4,5,6 be named as station A, B, C, D, E and F respectively, for convenience. Let the length of the sides of square $ABCD$ be a (= 4 km). Then the length of each side of the equilateral triangular plot will be also a. Now for the triangular plot, draw perpendicular bisectors Aa, Dd and Ee, so that they meet in point g. Similarly, draw the perpendicular bisectors eb, bc, cf and fe of the lines FA, FB, FC and FD respectively, shown in Fig. 4.71.

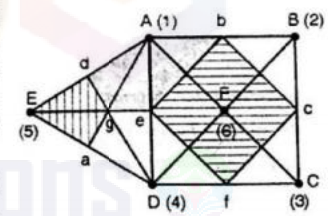

FIG. 4.71

Evidently, station F (or station 6) will be fed by the rectangular area, $bcfe$, where length of its side, say bc will be equal to $\frac{1}{2}AC = \frac{1}{2}\sqrt{2}\,a = a/\sqrt{2}$.

$$\text{Hence area} \quad bcfe = A_6 = \frac{a}{\sqrt{2}} \times \frac{a}{\sqrt{2}} = \frac{a^2}{2} = \frac{(4)^2}{2} = 8 \text{ km}^2 \qquad \ldots(i)$$

Then each of the corner stations, say station A, will be fed by triangular area Abe and sectorial area $Adge$.

Triangular area $\quad Abe = \frac{1}{2} \cdot \frac{a}{\sqrt{2}} \cdot \left(\frac{\sqrt{2}\,a}{4}\right) = \frac{a^2}{8}$

Sectorial area $\quad Adge = \frac{1}{3}$ area of triangle $ADE = \frac{1}{3} \times \frac{1}{2} a \times \frac{\sqrt{3}}{2} a = \frac{a^3}{4\sqrt{3}}$

Hence station A will be fed by area = area Abe + area $Adge$

$$= \frac{a^2}{8} + \frac{a^2}{4\sqrt{3}} = \frac{(4)^2}{8} + \frac{(4)^2}{4\sqrt{3}} = 4.3094 \text{ km}^2$$

Hence, $\quad A_1 = A_4 = 4.3094 \text{ km}^2$ ...(ii)

Also station E will be fed by sectorial area $Edga$ = area $Adge = \frac{a^2}{4\sqrt{3}}$

Hence $\quad A_5 = \frac{a^2}{4\sqrt{3}} = \frac{(4)^2}{4\sqrt{3}} = 2.3094 \text{ km}^2$ ...(iii)

Station B will be fed by area bcB = area $Abe = \frac{a^2}{8} = \frac{(4)^2}{8} = 2 \text{ km}^2$

$\therefore \quad A_2 = A_3 = 2 \text{ km}^2$ ...(iv)

Lastly, station F will be fed by area $ebcf = \left(\frac{a}{\sqrt{2}}\right)^2$

$\therefore \quad A_6 = \frac{(4)^2}{2} = 8 \text{ km}^2$

Hence $\quad P_{av} = \frac{P_1 A_1 + P_2 A_2 + \ldots\ldots P_6 A_6}{A_1 + A_2 + \ldots\ldots A_6}$

$$= \frac{4.8 \times 4.3094 + 13.0 \times 2 + 8.0 \times 2 + 5.4 \times 4.3094 + 3.2 \times 2.3094 + 9.4 \times 8}{4.3094 + 2 + 2 + 4.3094 + 2.3094 + 8}$$

= 7.35 cm

Example 4.43. *For a drainage basin of 640 km², isohyetals based on a storm event yield the following data :*

Isohyetal interval (cm)	*14 – 12*	*12 – 10*	*10 – 8*	*8 – 6*	*6 – 4*	*4 – 2*	*2 – 0*
Inter isohyetal area (km²)	90	140	125	140	85	40	20

Estimate the average depth of precipitation over the basin.

(Engg. Services Exam. 2002)

Solution : $\quad P_{av} = \dfrac{\Sigma A \times \left[\dfrac{P_1 + P_2}{2}\right]}{\Sigma A}$ ...(4)

$\therefore \quad P_{av} = [(90 \times 13) + (140 \times 11) + (125 + 9) + (140 \times 7) + (85 \times 5) + (40 \times 3) + (20 \times 1)]/640$

$\simeq$ **8.41 cm**

Example 4.44. *The information available from a Isohyetal map of 1100 sq. km basin is as follows :*

Zone	*Area (km²)*	*Rain gauge station*	*Normal annual rainfall (cm)*
I	*85*	*A*	*120*
II	*290*	*B*	*95*
		C	*96*
III	*395*	*D*	*60*
		E	*65*
		F	*70*
IV	*230*	*G*	*45*
V	*65*	*H*	*21*

How many additional rain gauge stations will be required if the desired limit of error in the mean value of rainfall is not to exceed 10 cm ? Suggest how you propose to distribute these stations. What factors will you consider in locating the additional rain gauge stations between different isohyetals. ***(Engg Services Exam 1994)***

Solution : ***This problem is similar to Example 4.1.***

Optimum No. of rain gauge stations is given by Eq. 4.4.

$$N = \left[\frac{C_v}{p}\right]^2 \quad \text{where} \quad C_v = \frac{s_x}{\bar{x}} \times 100$$

where $\quad s_x$ = standard deviation $\sqrt{\frac{\Sigma (x_i - \bar{x})^2}{n - 1}}$, where $n = 8$ here.

$$\bar{x} = \frac{1}{8}[120 + 95 + 96 + 60 + 65 + 70 + 45 + 21] = 71.5 \text{ cm.}$$

$$s_x = \left[\left((120 - 71.5)^2 + (95 - 71.5)^2 + (96 - 71.5)^2 + (60 - 71.5)^2 + (65 - 71.5)^2 + (70 - 71.5)^2 + (45 - 71.5)^2 + (21 - 71.5)^2\right)/7\right]^{1/2} = 31.47$$

$$\therefore \quad C_v = \frac{s_x}{\bar{x}} \times 100 = \frac{31.47}{71.5} \times 100 = 44.02$$

$$p = \frac{10}{\bar{x}} \times 100 = \frac{10}{71.5} \times 100 = 13.99\%$$

$$N = \left(\frac{C_v}{p}\right)^2 = \left(\frac{44.02}{13.99}\right)^2 = 9.91 \simeq 10$$

Hence two more rain gauge station are required. One of these should be in zone IV and the other should be located between zones III and IV.

Example 4.45. *The mass curve of precipitation resulted from the storm of 14th Aug 1983 gave the following values.*

Hour-min	*Accumulated depth at the end of periods (in mm)*	*Hour- min*	*Accumulated depth at the end of periods (in mm)*
22.00	*0.0 (Beginning of storm)*	**22.30**	*64.0*
22.05	*10.2*	**22.35**	*71.6*
22.10	*20.8*	**22.40**	*78.8*
22.15	*33.0*	**22.45**	*85.4*
22.20	*47.2*	**22.50**	*91.4 (End of storm)*
22.25	*55.8*		

From the above storm, construct hyetograph and draw the maximum intensity-duration curve. ***(Engg. Services Exam. 1984)***

Solution : ***This problem is similar to Example 4.8.***

First of all, let us calculate the rainfall intensities, using uniform time interval of 5 minutes, in a tabular form below, which is self explanatory.

Time (hr. min)	*Accumulated rainfall (cm)*	*Rainfall in successive 5 min. interval (cm)*	*Rainfall intensity cm/hr.*
(1)	(2)	(3)	(4) = (3) × 60/5
22.00	0.00		
22.05	1.02	1.02	12.24
22.10	2.08	1.06	12.72
22.15	3.30	1.22	14.64
22.20	4.72	1.42	17.04
22.25	5.58	0.86	10.32
22.30	6.40	0.82	9.84
22.35	7.16	0.76	9.12
22.40	7.88	0.72	8.64
22.45	8.54	0.66	7.92
22.50	9.14	0.60	7.20

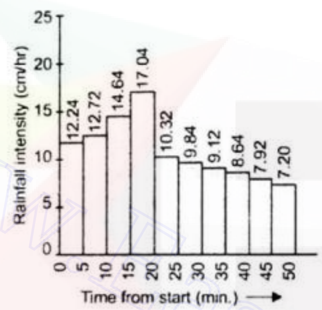

FIG. 4.72

Fig 4.72 shows the rainfall hyetograph.

Next let us find the maximum rainfall values of rainfall durations of 5 min, 10 min, 15 min, 20 min. .....50 min. For this, the computations are done in tabular form below. In this table, the rainfall values of 5 min time interval of rain will be the same as computed in column (3) of the table above. However, for interval of 10 min. the rainfall values will be obtained by adding the rainfall values of two successive durations of 5 minutes each. Similarly, for interval of 15 min. the rainfall values will be obtained by adding the rainfall values of three successive durations of 5 minutes each, and so on. Thus, the rainfall values of various times intervals of 5 min, 10 min, 15 min .....50 min will be obtained, as tabulated in columns (3), (4), (5) ....(12) respectively, of the table. For each of these time interval, the maximum rainfall value will be selected, as shown in bold letters in the Table.

Time (hr. min.)	*Accumulated rainfall (cm)*	*Rainfall in any possible time interval equal to*									
		5 min	*10 min*	*15 min*	*20 min*	*25 min*	*30 min*	*35 min*	*40 min*	*45 min*	*50 min*
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
22.00	0.00										
22.05	1.02	1.02									
22.10	2.08	1.06	2.08								
22.15	3.30	1.22	2.28	3.30							
22.20	4.72	**1.42**	**2.64**	**3.70**	**4.72**						
22.25	5.58	0.86	2.28	3.50	4.56	**5.58**					
22.30	6.40	0.82	1.68	3.10	4.32	5.38	**6.40**				
22.35	7.16	0.76	1.58	2.44	3.86	5.08	6.14	**7.16**			
22.40	7.88	0.72	1.48	2.30	3.16	4.58	5.80	6.86	**7.88**		
22.45	8.54	0.66	1.38	2.14	2.96	3.82	5.24	6.46	7.52	**8.54**	
22.50	9.14	0.60	1.26	1.98	2.74	3.56	4.42	5.84	7.06	8.12	**9.14**

After having found the maximum rainfall during each of the possible rainfall durations, we compute the max. intensity of rainfall for each of these durations, in Tabular form below. Finally, we plot a graph between rainfall intensity (col.3) and the rainfall durations (col. 1), as shown in Fig. 4.73.

Duration (min.)	*Max. rainfall (cm)*	*Max. rainfall intensity (cm/hr).*
(1)	(2)	(3) = (2) × 60/(1)
05	1.42	17.04
10	2.64	15.84
15	3.70	14.80
20	4.72	14.16
25	5.58	13.39
30	6.40	12.80
35	7.16	12.27
40	7.88	11.82
45	8.54	11.39
50	9.14	10.97

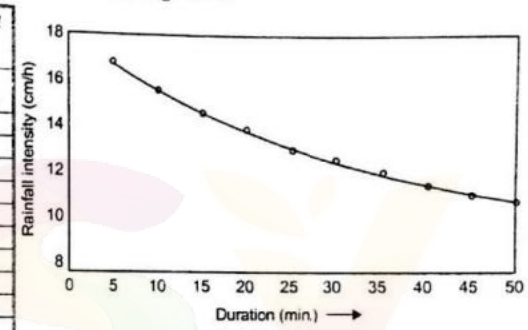

FIG. 4.73

Example 4.46. *Compute and draw the storm hyetograph and the intensity duration curve for the following storm (of a given frequency) on a drainage basin :*

Duration (minute)	*Accumulated Precipitation (cm)*
0	-
30	5.0
60	7.5
90	8.5
120	9.0

(Engg. Services Exam. 1998)

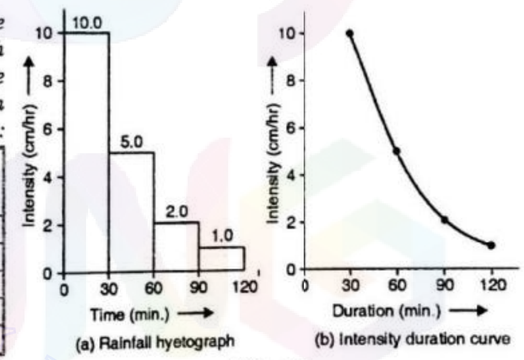

FIG. 4.74

Solution

The computations are done in a tabular form below :

Time (min.)	*Accumulated Rainfall (cm)*	*Rainfall in successive 30 min. interval (cm)*	*Rainfall intensity (cm/hr)*
(1)	(2)	(3)	(4) = (3) × 60/30
0	0.0		
30	5.0	5.0	10.0
60	7.5	2.5	5.0
90	8.5	1.0	2.0
120	9.0	0.5	1.0

The rainfall hyetograph is shown in Fig. 4.74 (*a*) while the intensity-duration curve is shown in Fig. 4.74 (*b*).

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Example 4.47. *Estimate the average depth of precipitation, from depth area curve, that may be expected over an area of 2400 sq. km, due to the storm of 27th Sept. 1978, lasting for 24 hours, assuming the storm centre to be located at the centre of the area. The Isohyctal map for the storm gave the areas enclosed between different isohyets as follows :*

Isohyet in mm	*21*	*20*	*19*	*18*	*17*	*16*	*15*	*14*	*13*	*12*
Enclosed area in sq. km	*543*	*1345*	*2030*	*2545*	*2955*	*3280*	*3535*	*3710*	*3880*	*3915*

Hence, determine the depth of rainfall due to the storm, that may be expected to be recorded by a rain gauge placed at the storm centre.

(Engg. Services Exam, 1986)

Solution : ***This problem is exactly similar to Example 4.10.*** The computations for equivalent uniform depth (EUD) are done in a tabular from below :

Isohyet (mm)	*Area enclosed between Isohyet and boundary of basin (km²)*	*Net incremental area between isohyets (km²)*	*Average rainfall on incremental area (mm)*	*Rainfall volume on incremental area (mm. km²)*	*Cumulative volume (mm km²)*	*EUD (mm)*
(1)	*(2)*	*(3)*	*(4)*	*(5) = (3) × (4)*	*(6) = Σ (5)*	*(7)=(5)/(2)*
21	543	543	21.5 (say)	11675	11675	21.50
20	1345	802	20.5	16441	28116	20.90
19	2030	685	19.5	13357	41473	20.43
18	2545	515	18.5	9528	51001	20.04
17	2955	410	17.5	7175	58176	19.69
16	3280	325	16.5	5362	63538	19.37
15	3535	255	15.5	3953	67491	19.09
14	3710	175	14.5	2537	70028	18.88
13	3880	170	13.5	2295	72323	18.64
12	3915	35	12.5	438	72761	18.59

Fig. 4.75 shows the plot between mean depth of rainfall (EUD) and area enclosed between the isohyets and the boundary of the basin. From the above depth-area curve, we obtain average depth of precipitation = **20.15** mm for an area of 2400 sq. km.

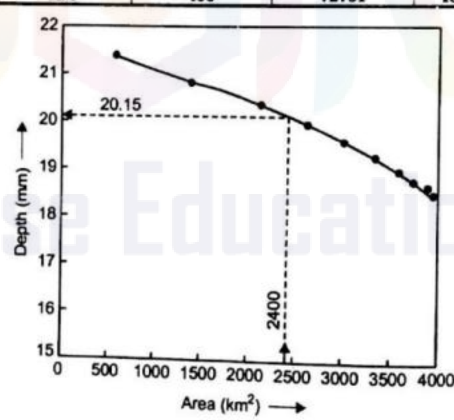

FIG. 4.75

Example 4.48. *At the beginning of a certain week, the depth of water in an evaporation pan, 1.2 metres diameter, was 7.75 cm. During the week, the rainfall was 3.8 cm, and 2.5 cm of water was removed from the pan to keep the depth of water in it within a fixed*

range. At the end of the week, the gauge indicated a depth of 8.32 cm of water in the pan. Using a suitable evaporation pan coefficient, estimate the evaporation during the week from the surface of reservoir under similar atmospheric conditions.

(Engg. Services Exam. 1970)

Solution : Given :

Initial depth of water in the pan = 7.75 cm

Rainfall during the week = 3.80 cm

Depth of water removed = 2.50 cm

∴ Net addition of water in the pan = 3.8 − 2.50 = 1.30 cm

If there were no loss, depth at the end of week = 7.75 + 1.30 = 9.05 cm

Actual depth at the end of week = 8.32 cm

∴ Evaporation loss from the pan = 9.05 − 8.32 = 0.73 cm

Let the Pan coefficient = 0.7

∴ Evaporation loss from reservoir = 0.7 × 0.73 ≏ **0.51 cm** during the week.

Example 4.49. *A 12 hour storm rainfall has the following depth in cm for each hour occurring over a basin :*

1.8, 2.6, 7.8, 3.9, 10.6, 5.4, 7.8, 9.2, 6.5, 4.4, 1.8 and 1.6

The surface runoff resulting from the above storm is found to be 24.4 cm depth over the basin. Determine the average infiltration index for the basin

(Gate : 1992)

Solution :

This problem is similar to Example 4.19.

Total $P = 1.8 + 2.6 + 7.8 + 3.9 + 10.6 + 5.4 + 7.8 + 9.2 + 6.5 + 4.4 + 1.8 + 1.6$

$= 63.4$ cm

$t_r = 12$ hours

Total surface runoff = 24.4 cm (given)

$$\therefore \quad W_i = \frac{P - R}{t_r} = \frac{63.4 - 24.4}{12} = 3.25 \text{ cm/hour.}$$

Now Φ_i will be greater than W_i, i.e. greater than 3.25 cm/hour

Let us assume period of effective rainfall, t_{re} to be equal to 7 hours (i.e. $\Phi_i \geq 3.9$), we get

$$\Phi_i = \frac{(63.4 - 24.4) - (1.8 + 2.6 + 3.9 + 1.8 + 1.6)}{7} = \mathbf{3.9 \text{ cm/hour}}$$

Check : This value of Φ_i is satisfactory since it gives $t_{re} = 7$ hours as assumed.

Let us verify the rainfall excess (i.e. direct runoff) for these values

Time from start (h)	1	2	3	4	5	6	7	8	9	10	11	12	sum↓
Rainfall excess = $P - \Phi_i$	0	0	3.9	0	6.7	1.5	3.9	5.3	2.6	0.5	0	0	24.4

From the above Table, we get R = rainfall excess = 24.4 cm.

Hence Φ_i = **3.9 cm/hour**

Example 4.50. *A storm with 15.0 cm precipitation produced a direct runoff of 8.7 cms. The time distribution of the storm is as follows :*

Time from start (hours)	1	2	3	4	5	6	7	8
Incremental rainfall in each hour (cm)	0.6	1.35	2.25	3.45	2.70	2.40	1.50	0.75

(Civil Services Exam. 1987)

Solution : ***This problem is similar to Example 4.23.***

The given data about rainfall in each hour also represent rainfall intensity in cm/hr. Fig. 4.76 shows the rainfall hyetograph for total rainfall duration $t_r = 8$ h.

Total $P = 15$ cm

Total direct runoff, $R = 8.7$ cm

$$\therefore \quad W_i = \frac{P - R}{t_r} = \frac{15 - 8.7}{8} = 0.7875 \text{ cm/h}$$

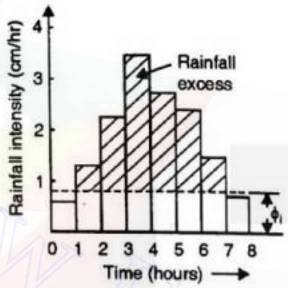

FIG. 4.76

Since Φ_i will be more than W_i and hence more than 0.7875 cm/h, we find that the first hour and the last hour of the rainfall will be ineffective in producing rainfall excess. Hence effective rainfall duration, t_{re} will be $= 8 - 2 = 6$ hours.

$$\therefore \quad \Phi_i = \frac{(15 - 8.7) - 0.6 - 0.75}{6} = \mathbf{0.825 \text{ cm/hr.}}$$

Example 4.51. *Assuming the initial infiltration rate of 10 mm/h, final infiltration rate of 5 mm/h and the constant value (describing the rate of decay of the difference between the initial and final infiltration rates) as 0.95 h^{-1}, calculate the total infiltration depth for a storm lasting 6 h.* ***(Civil Services Exam. 2004)***

Solution

The equation of the infiltration curve is

$$f_t = f_c + (f_0 - f_c)\, e^{-kt} \qquad \text{...(4.36)}$$

Given : $f_c = 5$ mm/h, $f_0 = 10$ mm/h and $k = 0.95$ /h

Hence substituting the values in Eq. 4.36, we get

$$f_t = 5 + (10 - 5)\, e^{-0.95t} = 5 + 5\, e^{-0.95t} \qquad \text{...(1)}$$

Integrating the above equation for t between 0 hour to 6 hours, we get

$$\text{Total infiltration} = \sum_0^6 f_t = \int_0^6 (5 + 5e^{-0.95t}) = 5\left[\,t\,\right]_0^6 + 5\left[\frac{e^{-0.95t}}{-0.95}\right]_0^6$$

$$= 5\,[6 - 0] - \frac{5}{0.95}\,[e^{-0.95 \times 6} - e^0]$$

$$= 30 - \frac{5}{0.95}\,[3.346 \times 10^{-3} - 1] = \mathbf{35.25 \text{ mm}}$$

Example 4.52. *The infiltration capacities of an area at different intervals of time are indicated below. Find an equation for the infiltration capacity in the exponential form.*

Time in hours	0	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
Infiltration capacity (f) in cm/hr.	10.4	5.6	3.2	2.1	1.5	1.2	1.1	1.0	1.0

(Engg Services Exam. 1970)

Solution

Let the equation of the infiltration curve be :

$$f_t = f_c + (f_0 - f_c)\, e^{-kt} \qquad \text{...(4.36)}$$

where f_c = constant infiltration rate ≈ 1.0 cm/hr

f_0 = infiltration rate at the beginning $= 10.4$ cm/hr.

k = constant to be determined.

Hence $$f_t = 1.0 + (10.4 - 1.0)\, e^{-kt}$$

or $$f_t = 1.0 + 9.4\, e^{-kt} \qquad \text{...(1)}$$

In order to get the value of k, let us make a plot between t on simple algebraic scale on the ordinate and $\log_{10}(f_t - f_c)$ on the abscissa.

t (hours)	0	0.25	0.50	0.75	1.0	1.25	1.50	1.75	2.00
f_t (cm/hr)	10.4	5.6	3.2	2.1	1.5	1.2	1.1	1.0	1.0
$(f_t - f_c)$	9.4	4.6	2.2	1.1	0.5	0.2	0.1	0.0	0.0
$\log_{10}(f_t - f_c)$	+ 0.973	+ 0.663	+ 0.342	+ 0.041	− 0.301	− 0.699	− 1.00		

Fig. 4.77 shows the straight line plot between t and $\log(f_t - f_c)$.

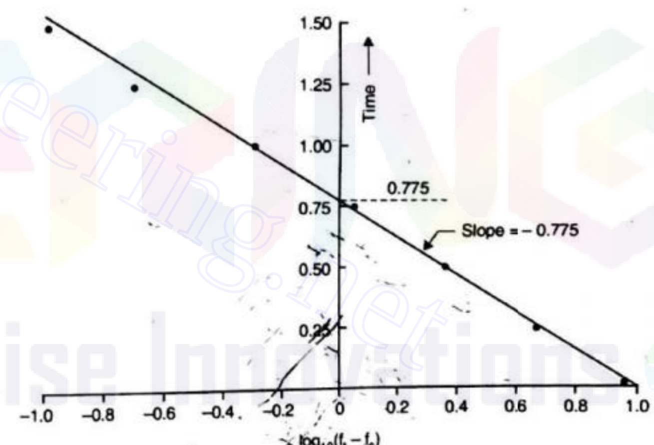

FIG. 4.77

$$\text{Slope of the line} = \frac{0.775}{1} = -\,0.775$$

Hence $$-\frac{1}{k \log_{10} e} = -\,0.775$$

or $\quad k \log_{10} 2.7183 = \dfrac{1}{0.775} = 1.2903$

From which $\quad k = \dfrac{1.2903}{0.43429} \simeq 2.97$

Hence the desired equation is

$$f_t = 1.0 + 9.4\, e^{-2.97\, t}$$

Example 4.53. *The following is the set of observed data for successive 15 minutes period of 105 minutes storm in a catchment.*

Duration (min.)	*Rainfall (cm/hour)*
15	*2.0*
30	*2.0*
45	*8.0*
60	*7.0*
70	*1.25*
90	*1.25*
105	*4.5*

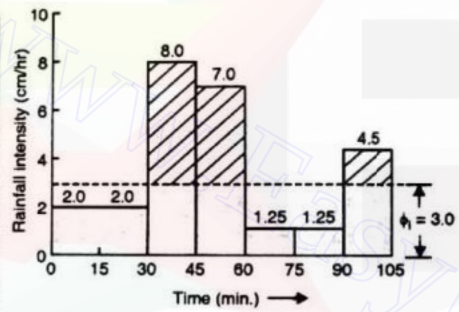

FIG. 4.78

If the value of Φ-index is 3.0 cm/hr, estimate the net runoff, the total rainfall and the value of W-index.

(*Engg. Services Exam. 1996*)

Solution

Fig. 4.78 shows the rainfall hyetograph, Φ-index and rainfall excess hyetograph situated above the Φ – line.

Total rainfall $P = (2 \times 30 + 8 \times 15 + 7 \times 15 + 1.25 \times 30 + 4.5 \times 15)/60 = 6.5$ cm

Net runoff R = rainfall excess (shown hatched in Fig. 4.78)

$= [(8.0 - 3.0) \times 15 + (7.0 - 3.0) \times 15 + (4.5 - 3)\, 15]/60 = 2.625$ cm

$$\therefore \quad W_i = \frac{P - R}{t_r} = \frac{6.5 - 2.625}{(105/60)} = \mathbf{2.214} \ \textbf{cm / hour}$$

Example 4.54. *An isolated 3 hour storm occurred over an area of 120 ha. as below :*

Partial area of catchment (ha)	*Φ-index (cm/hr)*	*Rainfall (cms)*		
		1st hour	*2nd hour*	*3rd hour*
36	*0.90*	*0.6*	*2.4*	*1.3*
18	*1.10*	*0.9*	*2.1*	*1.5*
60	*0.50*	*1.0*	*2.0*	*0.9*

What is the total rainfall on the catchment in this storm ? Estimate the runoff from the catchment. If the Φ-index were to remain at the same value, what runoff would be produced by a uniform rainfall of 3.3 cm in 3 hours uniformly spread over the catchment ?.

(*Civil Services Exam. 1989*)

Solution

Total rainfall on 36 hectares = 0.6 + 2.4 + 1.3 = 4.3 cm

Total rainfall on 18 hectares = 0.9 + 2.1 + 1.5 = 4.5 cm

Total rainfall on 66 hectares = 1.0 + 2.0 + 0.9 = 3.9 cm

$$\therefore \text{ Total rainfall on the entire catchment} = \frac{\Sigma P \times A}{\Sigma A} = \frac{4.3 \times 36 + 4.5 \times 18 + 3.9 \times 66}{120}$$

$$= \mathbf{4.11 \ cm}$$

Runoff from the catchment is computed in a tabular form below

Area (ha)	*Runoff = Rainfall – Φ_i (cm)*			*Total (cm)*
	(1)	*(2)*	*(3)*	
36	0	1.5	0.4	1.9
18	-	1.0	0.4	1.4
66	0.5	1.5	0.4	2.4

$$\therefore \text{ Runoff from the entire catchment } = \frac{36 \times 1.9 + 18 \times 1.4 + 66 \times 2.4}{120} = \mathbf{2.1 \ cm}$$

$$\text{Average } \Phi_i \text{ for the entire catchment } = \frac{0.9 \times 36 + 1.10 \times 18 + 0.5 \times 60}{120} = 0.71 \text{ cm/hr}$$

Total uniform rainfall = 3.3 cm in 3 hours = 1.1 cm/hr.

$\therefore$ Total runoff = (1.1 – 0.71) 3 = **1.17 cm**

Example 4.55. *Given below are the monthly rainfall, P and the corresponding runoff values R, for a period of 10 months for a catchment. Develop a correlation between R and P.* **(*Engg. Services Exam. 2003*)**

Month	*P (cms)*	*R (cms)*
1	*4*	*0.2*
2	*22*	*7.1*
3	*28*	*10.9*
4	*15*	*4.0*
5	*12*	*3.0*
6	*8*	*1.3*
7	*4*	*0.4*
8	*15*	*4.1*
9	*10*	*2.0*
10	*5*	*0.3*

Solution : ***This problem is similar to Example 4.20.***

Let the correlation equation be

$$R = aP + b \qquad \ldots(4.43)$$

where constants a and b are given by Eqs. 4.43 (a) and (b) respectively. The computations are done in a Tabular form below :

S.N. of month	P	P^2	R	R^2	PR
1	4	16	0.2	0.04	0.8
2	22	484	7.1	50.41	156.2
3	28	784	10.9	118.81	305.2
4	15	225	4.0	16.00	60
5	12	144	3.0	9.00	36
6	8	64	1.3	1.69	10.4
7	4	16	0.4	0.16	1.6
8	16	225	4.1	16.81	61.5
9	10	100	2.0	4.00	20
10	5	25	0.3	0.09	1.5
Σ	123	2083	33.3	217.01	653.2

From the Table, $\Sigma P = 123; (\Sigma P)^2 = 15129 \;;\; \Sigma P^2 = 2083 \;; N = 10$

$\Sigma R = 33.3 \;, (\Sigma R)^2 = 1108.9 \;; \Sigma R^2 = 217.01 \;; \Sigma PR = 653.2$

$$\therefore \quad a = \frac{N(\Sigma PR) - (\Sigma P)(\Sigma R)}{N(\Sigma P^2) - (\Sigma P)^2} = \frac{10 \times 653.2 - 123 \times 33.3}{10 \times 2083 - 15129} = 0.4273$$

and
$$b = \frac{\Sigma R - a\,\Sigma P}{N} = \frac{33.3 - 0.4273 \times 123}{10} = -1.9259$$

Hence the correlation equation is

$$\boldsymbol{R = 0.4273\,P - 1.9259}$$

Example 4.56. *A peak of a flood hydrograph due to a six-hour storm is 470 m³/sec. The average depth of rainfall is 8.0 cms. Assume an infiltration loss of 0.25 cm/ hour and a constant base flow of 15 m³/sec. Estimate the peak discharge of 6 hour unit hydrograph for this catchment.* ***(Civil Services Exam. 1996)***

Solution

Peak discharge of flood hydrograph $= 470 \text{ m}^3/\text{sec}$

Base flow $= 15 \text{ m}^3/\text{sec}$

$\therefore$ Peak discharge of direct runoff hydrograph $= 470 - 15 = 455 \text{ m}^3/\text{sec}$

Average depth of rainfall $= 8.0$ cm.

$\therefore$ Rainfall excess $= 8.0 - 0.25 \times 6 = 6.5$ cm

$\therefore$ Peak discharge of unit hydrograph $= \dfrac{455}{6.5} = \mathbf{70\ m^3/sec}$

Example 4.57. *The ordinates of a 4-hour unit hydrograph of catchment are given below :*

Time (hours)	*0*	*4*	*8*	*12*	*16*	*20*	*24*	*28*	*32*	*36*	*40*	*44*
Ordinate of 4h UH (m³/sec)	*0*	*20*	*60*	*150*	*120*	*90*	*70*	*50*	*30*	*20*	*10*	*0*

Derive the flood hydrograph in the catchment due to the storm given below :

Time from start of storm (h)	*0*	*4*	*8*	*12*
Accumulated rainfall (cm)	*0*	*5.0*	*5.8*	*8.8*

The Φ – index for the catchment can be assumed to be 0.25 cm/hour and a constant base flow of 20 cm³/sec. is appropriate. ***(Civil services Exam 1991)***

Solution : ***This problem is similar to Example 4.28.***

		(1st 4 hr)		*(2nd 4 hour)*		*(3rd 4 hour)*	
Time from start of storm	0		4		8		12
Accumulated rainfall (cm)	0		5.0		5.8		8.8
Rainfall in each period (cm)		5.0		0.8		3.0	
Loss in each period = 4 × Φ_i		1.0		1.0		1.0	
Effective rainfall		4.0		0		2.0	

The computation for the ordinates of flood hydrograph are in a Tabular form below which is self explanatory

Time (hours)	*Ordinate of 4-h UH (m³/sec)*	*Surface runoff from rainfall excess during successive unit periods*				*Base flow*	*Ordinate of flood hydrograph*
		$n_1 = 4.0$	$n_2 = 0.0$	$n_3 = 2.0$	*Sub-total*		
(1)	*(2)*	*(3) = (2) × 4.0*	*(4) = (2) × 0.0*	*(5) = (2) × 2.0*	*(6)*	*(7)*	*(8) = (6) + (7)*
0	0	0			0	20	20
4	20	80	0		80	20	100
8	60	240	0	0	240	20	260
12	150	600	0	40	640	20	660
16	120	480	0	120	600	20	620
20	90	360	0	300	660	20	680
24	70	280	0	240	520	20	540
28	50	200	0	180	380	20	400
32	30	120	0	140	260	20	280
36	20	80	0	100	180	20	200
40	10	40	0	60	100	20	120
44	0	0	0	40	40	20	60
48			0	20	20	20	40
52				0	0	20	20

Example 4.58. *On a catchment area of 200 sq. km, rainfalls of 7.5 cm 2.0 cm and 5.0 cm occurred on three consecutive days. The Φ-index was 2.5 cm/day. The distribution graph percentages of surface runoff which extended over six days for every rainfall of such magnitudes are 5, 15, 40, 25, 10 and 5. Determine the ordinates of the discharge hydrograph and determine the peak discharge. Neglect base flow.* ***(Engg Services Exam. 1982)***

Solution:

The runoff, measured as depth on three consecutive days will be equal to rainfall excess on these three days, with the values as follows :

Runoff on first day $= 7.5 - 2.5 = 5.0$ cm

Runoff on second day $= 2.0 - 2.5 = 0.0$ cm

Runoff on third day $= 5.0 - 2.5 = 2.5$ cm

Runoff, measured as depth of water per day, spread over the whole catchment area can be converted into discharge as follows :

1 cm runoff per day over 200 km^2 area $= \frac{1}{100} (200 \times 10^6)$ m^3/day

$$= \frac{(2 \times 10^6)}{24 \times 3600} \text{ m}^3/\text{sec} = 23.148 \text{ m}^3/\text{sec}.$$

However, the discharge pattern is governed by distribution graph percentages, contributed by effective rain of each day, spread over 6 days from its occurrence. Since there is effective rainfall on three consecutive days, these discharge contributions by each day rainfall excess are added to give total discharge. The computations are arranged in Tabular form below which is self explanatory.

Time interval in days	Rainfall excess (cm)	Distribution graph %	Distributed runoff for rainfall excess of			Total runoff	
			5 cm	0 cm	2.5 cm	As depth (cm)	As discharge (cumecs)
(1)	(2)	(3)	(4)	(5)	(6)	(7) = (4) + (5) + (6)	(8) = 23.148 × (7)
0-1	5.0	5	0.25			0.25	5.787
1-2	0.0	15	0.75	0		0.75	17.361
2-3	2.5	40	2.00	0	0.125	2.125	49.190
3-4		25	1.25	0	0.375	1.625	37.616
4-5		10	0.50	0	1.00	1.500	34.722
5-6		5	0.25	0	0.625	0.875	20.255
6-7			0	0	0.25	0.25	5.787
7-8				0	0.125	0.125	2.894
8-9					0	0	0
Total	7.5	100	5.0	0	2.5	7.5	173.612

Check : Total runoff$(R) = \frac{0.36 \, \Sigma \, O \times t}{A} = \frac{0.36 \times 173.612 \times 24}{200}$ = **7.5 cm**

Example 4.59. *The ordinates of a flood hydrograph, resulting from two successive storms each of 1 cm rainfall excess and 6 hour duration, are tabulated below. Find a 6 hour unit hydrograph.*

Time (hours)	*Ordinate of 6 hr flood hydrograph (m^3/s)*	*Time (hours)*	*Ordinate of 6 hr flood hydrograph (m^3/s)*
0	10	42	126
6	30	48	92
12	90	54	62
18	220	60	40
24	280	66	20
30	220	72	10
36	166		

(Engg. Services Exam. 1997)

Solution :

If R_1 and R_2 are the rainfall excesses, we have $R_1 = 1$ cm and $R_2 = 1$ cm. Assuming a base flow of 10 m^3/s, the ordinates $Q_1, Q_2 ...... Q_{13}$ of the direct runoff hydrograph will be as given in Table below.

Let $U_1\, U_2, U_3 ...... U_{13}$ be the ordinates of the 6 hours unit hydrograph. The computations are done in tabular form below.

Time (hours)	Ordinate of 6 hr. flood hydrograph (m^3/s)	Base flow (m^3/s)	Ordinate of direct runoff	Equations for direct runoff hydrograph	Values put in the equation	Ordinate of 6 hr. unit hydrograph
0	10	10	0 (Q_1)	$Q_1 = R_1 U_1$	$0 = 1 \times U_1$	$U_1 = 0$
6	30	10	20 (Q_2)	$Q_2 = R_1 U_2 + R_2 U_1$	$20 = 1\, U_2 + 1 \times 0$	$U_2 = 20$
12	90	10	80 (Q_3)	$Q_3 = R_1 U_3 + R_2 U_2$	$80 = 1\, U_3 + 1 \times 20$	$U_3 = 60$
18	220	10	210 (Q_4)	$Q_4 = R_1 U_4 + R_2 U_3$	$210 = 1\, U_4 + 1 \times 60$	$U_4 = 150$
24	280	10	270 (Q_5)	$Q_5 = R_1 U_5 + R_2 U_4$	$270 = 1\, U_5 + 1 \times 150$	$U_5 = 120$
30	220	10	210 (Q_6)	$Q_6 = R_1 U_6 + R_2 U_5$	$210 = 1\, U_6 + 1 \times 120$	$U_6 = 90$
36	166	10	156 (Q_7)	$Q_7 = R_1 U_7 + R_2 U_6$	$156 = 1\, U_7 + 1 \times 90$	$U_7 = 60$
42	126	10	116 (Q_8)	$Q_8 = R_1 U_8 + R_2 U_7$	$116 = 1\, U_8 + 1 \times 60$	$U_8 = 50$
48	92	10	82 (Q_9)	$Q_9 = R_1 U_9 + R_2 U_8$	$82 = 1\, U_9 + 1 \times 50$	$U_9 = 32$
54	62	10	52 (Q_{10})	$Q_{10} = R_1 U_{10} + R_2 U_9$	$52 = 1\, U_{10} + 1 \times 32$	$U_{10} = 20$
60	40	10	30 (Q_{11})	$Q_{11} = R_1 U_{11} + R_2 U_{10}$	$30 = 1\, U_{11} + 1 \times 20$	$U_{11} = 10$
66	20	10	10 (Q_{12})	$Q_{12} = R_1 U_{12} + R_2 U_{11}$	$10 = 1\, U_{12} + 1 \times 10$	$U_{12} = 0$
72	10	10	10 Q_{13})	$Q_{13} = R_1 U_{13} + R_2 U_{12}$	$10 = 1\, U_3 + 1 \times 10$	$U_{13} = 0$

Example 4.60. *The following are the ordinates for a flood hydrograph resulting from an isolated storm of 6 hours duration.*

Time (hr)	0	12	24	36	48	60	72	84	96
Ordinate of flood hydrograph (cumec)	5	15	40	80	60	50	25	15	5

Determine the ordinates of 1 cm–6 hour unit hydrograph if catchment area is 450 sq. km.

(Engg. Services Exam. 2001)

Solution

Let us assume a constant base flow of 5 cumecs. The ordinates of *direct runoff hydrograph* are obtained by subtracting base flow (= 5 cumecs) from each ordinate of the given flood hydrograph, as shown in the Table below.

Time (hours)	0	12	24	36	48	60	72	84	96
Ordinates of flood hydrograph	5	15	40	80	60	50	25	15	5
Ordinate of direct runoff	0	10	35	75	55	45	20	10	0
Ordinate of unit hydrograph	0	4.17	14.58	31.25	22.92	18.75	8.33	4.17	0

The direct runoff is given by

Direct Runoff $n = 0.36 \dfrac{(\Sigma\, 0) \times t}{A}$ cm

$\Sigma\, 0 = 0 + 10 + 35 + 75 + 55 + 45 + 20 + 10 + 0 = 250$ cumecs

t = time interval between ordinates, in hours, = 12 hours

A = catchment area in sq. km = 450 sq. km

$\therefore$ Direct runoff $n = \dfrac{0.36 \times 250 \times 12}{450} = 2.4$ cm

Hence the ordinates of 6-hour unit hydrograph are obtained by dividing the ordinates of direct runoff by $n = 2.4$, as shown in the Table above.

Example 4.61. *The ordinates of 8-hour unit hydrograph for a drainage basin are given below.*

Time in hours	*Ordinates of 8-h unit hydrograph*	*Time in hours*	*Ordinate of 8-h unit hydrograph*	*Time in hours*	*Ordinate of 8-h unit hydrograph*
0	0.0	32	231.0	64	14.0
4	5.5	36	165.0	68	9.5
8	13.5	40	112.0	72	6.6
12	26.5	44	79.0	76	4.0
16	45.0	48	57.0	80	2.0
20	82.0	52	42.0	84	1.0
24	162.0	56	31.0	88	0
28	240.0	60	22.0		

Obtain 24-hour unit hydrograph by tabulation method

(Engg. Services Exam. 1988)

Solution

When 3 unit hydrographs, each of 8 hour duration, are added together placed at 8 hour lag successively from one another, then we will get the ordinates of a 24

hr. surface runoff hydrograph, containing 3 cm of direct runoff. Hence the ordinates of a 24-hour unit hydrograph are obtained by dividing each of the above ordinates by 3. The computations are done a tabular form below.

Time (hours)	***1st 8-hr unit hydrograph***	***2nd 8-hr unit hydrograph***	***3rd 8-hr unit hydrograph***	***Total 24 hr hydrograph of 3 cm runoff***	***Ordinates of 24-h unit hydrograph***
(1)	(2)	(3)	(4)	(5) = (2) + (3) + (4)	(6) = (5)/3
0	0.0			0	0
4	5.5			5.5	1.8
8	13.5	0.0		13.5	4.5
12	26.5	5.5		32.0	10.7
16	45.0	13.5	0	58.5	19.5
20	82.0	26.5	5.5	114.0	38.0
24	162.0	45.0	13.5	220.5	73.5
28	240.0	82.0	26.5	348.5	116.2
32	231.0	162.0	45.0	438	146.0
36	165.0	240.0	82.0	487	162.3
40	112.0	231.0	162.0	505	168.3
44	79.0	165.0	240.0	484	161.3
48	57.0	112.0	231.0	400	133.3
52	42.0	79.0	165.0	286	95.3
56	31.0	57.0	112.0	200	66.7
60	22.0	42.0	79.0	143	47.7
64	14.0	31.0	57.0	102	34.0
68	9.5	22.0	42.0	73.5	24.5
72	6.6	14.0	31.0	51.6	17.2
76	4.0	9.5	22.0	35.5	11.8
80	2.0	6.6	14.0	22.6	7.5
84	1.0	4.0	9.5	14.5	4.8
88	0.0	2.0	6.6	8.6	2.9
92		1.0	4.0	5.0	1.7
96		0.0	2.0	2.0	0.7
100			1.0	1.0	0.3
104			0.0	0.0	0.0

Example 4.62. *A 2 hr. unit hydrograph in a rather steep catchment is given below :*

Time hr	0	2	4	6	8	10	12
Discharge 100 m^3/s	0	0.54	1.75	1.27	0.58	0.25	0

Compute the 1 hr. unit hydrograph for the catchment

(Engg. services Exam. 1989)

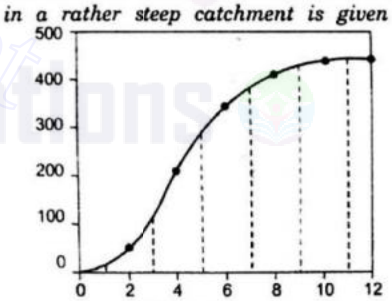

FIG. 4.79

Solution : For the derivation of 1 hr ($= t_0$) unit hydrograph from the unit hydrograph of T_0 ($= 2\ hr$) unit duration, we will follow the method of S- curve discussed in §4.24 Note that ordinates of S-curve, obtained in col.(4) are at 2 hours ($=T_0$) interval. In order to obtain the ordinate of curve at 1 hours ($=t_0$) interval,

S-curves is plotted and the its offsets are read off at 1 hours interval and entered in column (4). The S-curve is shown plotted in Fig. 4.79.

Time (in hr)	*Ordinate of 2 hr unit hydrograph (cumecs)*	*Offset ordinate (cumecs)*	*Ordinate of S-curve (cumes)*	*Ordinate of offset S-curve (cumes)*	*Δy (cumecs)*	*Ordinate of 1 hr unit hydrograph* $0=\Delta y \times \frac{T_0}{t_0} = 2\,\Delta y$
(1)	(2)	(3)	(4)	(5)	(6) = (4) − (5)	(7)
0	0		0	-	0	0
1			25	0	25	50
2	54	0	54	25	29	58
3			120	54	66	132
4	175	54	229	120	109	218 (peak)
5			300	229	71	142
6	127	229	356	300	56	112
7			390	356	34	68
8	58	356	414	390	24	48
9			430	414	16	32
10	25	414	439	430	9	18
11			439	439	0	0
12	0	439	439	0	0	0

Example 4.63. *The regression analysis of a 30-year flood data at a point on a river yielded sample mean $\bar{x}=1200\ m^3/s$ and standard deviation $s_x = 650\ m^3/sec$. For what discharge would you design the structure at this point to 95% assurance that the structure would not fail in the next 50 years ? Use Gumbel's method. The value of mean and standard deviation of the reduced variate for $n=30$ are 0.53622 and 1.11238 respectively.* ***(Engg. Services Exam. 2000)***

Solution

Given : Assurance = 95%, are hence risk $R_{sk} = 100 - 95 = 5\% = 0.05$

$$\sigma_{n-1} = s_x = 650\ \text{m}^3/\text{sec}$$

But from Eq. 4.92, $R_{sk} = 1 - \left(1 - \frac{1}{T}\right)^n$, where $n = 50$ years

$$\therefore \quad 0.05 = 1 - \left(1 - \frac{1}{T}\right)^{50} \quad \text{or} \quad 1 - \frac{1}{T} = (0.95)^{1/50}$$

From which we get $T \simeq 975.3$ years

For $T = 975.3$ years, the reduced variate y_T is given by

$$y_T = -[\ln . \ln (975.3/974.3)] = 6.88184$$

$$\therefore \quad K_T = \frac{y_T - \bar{y}_n}{S_n} = \frac{6.88184 - 0.53622}{1.11238} = 5.70454$$

Hence the design flood $x_T = \bar{x} + K_T\,\sigma_{n-1}$

or $x_T = 1200 + 5.70454 \times 650 \simeq$ **4908 m³/sec**

Example 4.64. *Flood frequency computations for a flash river at a point 50 km upstream of a bund site indicated the following:*

Return period; T (years)	*50*	*100*
Peak flood; m³/sec	*20600*	*22150*

Estimate the flood magnitude in the river with a return period of 500 years through use of Gumbel's method. ***(Engg. Services Exam. 2003)***

Solution :

This problem is similar to the one solved at Example 4.33.

From Gumbel's equation (Eq. 4.91).

$$(K_{100} - K_{50})\,\sigma_{n-1} = x_{100} - x_{50} = 22150 - 20600 = 1550\ \text{m}^3/\text{s} \quad ...(1)$$

Now $K_T = \frac{y_T - \bar{y}_n}{S_n} = \frac{y_T}{S_n} - \frac{\bar{y}_n}{S_n}$ where $\bar{y}_n$ and S_n are constants for a given series.

$$\therefore \quad \frac{y_{100} - y_{50}}{S_n} = (K_{100} - K_{50}) = \frac{1550}{\sigma_{n-1}}$$

or $$(y_{100} - y_{50})\frac{\sigma_{n-1}}{S_n} = 1550 \quad ...(2)$$

But $y_{100} = -[\ln . \ln (100/99)] = 4.60015$ and $y_{50} = -[\ln . \ln (50/49)] = 3.90194$

$$\therefore \quad \frac{\sigma_{n-1}}{S_n} = \frac{1550}{4.60015 - 3.90194} \simeq 2220 \quad ...(3)$$

For $T = 500$ years, $y_{500} = -[\ln . \ln (500/499)] = 6.21361$

Now $$(y_{500} - y_{100})\frac{\sigma_{n-1}}{S_n} = x_{500} - x_{100}$$

or $$(6.21361 - 4.60015) \times 2220 = x_{500} - 22150$$

From which $x_{500} \simeq$ **25732 m³/s**

Example 4.65. *In a frequency analysis of rainfall, based on 15 years of data of 10 minutes storm, the following values were obtained*

Arithmetic mean of data = *1.65 cm*

Standard deviation = *0.45 cm*

Find, using Gumbel's extreme distribution, the recurrence interval of a storm of 10 minutes duration and a depth equal to 3 cm. ***(Civil Services Exam. 1996)***

Solution Given : $\bar{x} = 1.65$ cm ; $\sigma_x = 0.45$ cm and $x = 3$ cm

From Eq. 4.88, $$y = \frac{1.2825\,(x - \bar{x})}{\sigma_x} + 0.577$$

or $$y = \frac{1.2825\,(3 - 1.65)}{0.45} + 0.577 = 4.4245$$

From Eq. 4.89,

$$\ln . \ln \frac{T}{T-1} = -y_T = -4.4245$$

$$\therefore \quad \ln \frac{T}{T-1} = 0.01198 \quad \text{or} \quad \frac{T}{T-1} = 1.01205$$

From which $\quad T = \mathbf{83.8\ years}$

Example 4.66. *A large sample of peak floods was available for a river. Flood frequency computations using Gumbel's method yield the following results :*

Return Period (years)	*Peak flood m^3/sec)*
50	30800
100	36300

Estimate the magnitude of a flood for this river with a return period of 200 years.

(Engg. Services Exam 1997)

Solution : ***This problem is similar to the one solved in Example 4.33.*** From Gumbel's equation (Eq. 4.91) :

$$(K_{100} - K_{50})\ \sigma_{n-1} = x_{100} - x_{50} = 36300 - 30800 = 5500 \quad ...(1)$$

Now $\quad K_T = \dfrac{y_T - \bar{y}_n}{S_n} = \dfrac{y_T}{S_n} - \dfrac{\bar{y}_n}{S_n}$ where $\bar{y}_n$ and S_n are constants for a series.

$$\therefore \quad \frac{(y_{100} - y_{50})}{S_n} = (K_{100} - K_{50}) = \frac{5500}{\sigma_{n-1}} \quad ...(2)$$

or $\quad (y_{100} - y_{50}) \dfrac{\sigma_{n-1}}{S_n} = 5500$

But $\quad y_{100} = -[\ln . \ln (100/99)] = 4.60015$

and $\quad y_{50} = -[\ln . \ln [50/49)] = 3.90194$

$$\therefore \quad \frac{\sigma_{n-1}}{S_n} = \frac{5500}{y_{100} - y_{50}} = \frac{5500}{4.60015 - 3.90194} = 7877.3 \quad ...(3)$$

For $\quad T = 200$ years, $y_{200} = -[\ln . \ln (200/199)] = 5.29581$

Now $\quad (y_{200} - y_{100}) \dfrac{\sigma_{-1}}{S_n} = x_{200} - x_{100}$

or $\quad (5.29581 - 4.60015) \times 7877.3 = x_{200} - 36300$

From which $\quad x_{200} \triangleq \mathbf{41780\ m^3/sec}$

Example 4.67. *For a data of maximum recorded flood of a river, the mean and standard deviations are 4200 m^3/s and 1705 m^3/s respectively. Using Gumbel's extreme value distribution, estimate the return period of a design flood of 9950 m^3/s. Assume an infinite sample size.*

(Engg. Services Exam. 1996)

Solution : For an infinite sample size (i.e. infinite series)

$$x_T = \bar{x} + K\sigma_x \quad ...(4.90)$$

For the present case, $\quad \bar{x} = 4200\ m^3/s$; $\sigma_x = 1705\ m^3/s$ and $x_T = 9550\ m^3/s$

$$\therefore \quad 9550 = 4200 + K \times 1705$$

From which $\quad K = 3.13783$

Hence from Eq. 4.90 (*a*)

$$y_T = 1.2825\,K + 0.577 = 1.2825 \times 3.13783 + 0.577 = 4.6013$$

But $\quad y_T = -\left[\ln . \ln \dfrac{T}{T-1}\right]$ from Eq. 4.89.

$$\therefore \quad \ln . \ln \frac{T}{T-1} = -y_T = -4.6013$$

or $\quad \ln \dfrac{T}{T-1} = e^{-4.6013} = 0.01004 \quad$ or $\quad \dfrac{T}{T-1} = 1.01009$

which gives $\quad T \triangleq \mathbf{101\ years}$

Example 4.68. *For a river, the estimated flood peaks for two return periods by the use of Gumbel's method are given below:*

Return period (years)	*Peak flood (m^3/s)*
100	485
50	445

What flood discharge in this river will have a return period of 1000 years.

(Civil Services Exam. 1990)

Solution :

This problem is similar to the one solved at Example 4.33.

From Gumbel's Equation (Eq. 4.91).

$$(K_{100} - K_{50})\ \sigma_{n-1} = x_{100} - x_{50} = 485 - 445 = 40\ m^3/s \quad ...(1)$$

Now $\quad K_T = \dfrac{y_T - \bar{y}_n}{S_n} = \dfrac{y_T}{S_n} - \dfrac{\bar{y}_n}{S_n}$

where $\bar{y}_n$ and S_n are constant for a given series

$$\therefore \quad \frac{y_{100} - y_{50}}{S_n} = (k_{100} - k_{50}) = \frac{40}{\sigma_{n-1}} \quad ...(2)$$

or $\quad (y_{100} - y_{50}) \dfrac{\sigma_{n-1}}{S_n} = 40$

But $\quad y_{100} = -[\ln . \ln (100/99)] = 4.60015$ and $y_{50} = -[\ln . \ln (50/49)] = 3.90194$

$$\therefore \quad \frac{\sigma_{n-1}}{S_n} = \frac{40}{4.60015 - 3.90194} = 57.29 \quad ...(3)$$

For $\quad T = 1000$ years, $y_{1000} = -[\ln . \ln (1000/999)] = 6.90726$

Now $\quad (y_{1000} - y_{100}) \dfrac{\sigma_{n-1}}{S_n} = x_{1000} - x_{100}$

or $\quad (6.90726 - 4.60015) \times 57.29 = x_{1000} - 485$

From which $\quad x_{1000} \triangleq \mathbf{617.2\ m^3/s}$

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Example 4.69. *A flood of a certain magnitude has a return period of 25 years. (a) what is its probability of exceedance ? (b) what is the probability that this flood may occur in the next 12 years.* ***(Engg. Services Exam. 1983)***

Solution : Given : $T = 25$ years.

(a) Probability of exceedance, $P = \frac{1}{T} = \frac{1}{25} = \mathbf{0.04}$

(b) Probability of non-exceedance in next n successive years is

$$y_{0,n} = q^n = (1 - P)^n = (1 - 0.04)^{12} = 0.6127$$

∴ Probability of occurrence at least once in next 12 years is

$$R_{sk} = 1 - (1 - P)^n = 1 - 0.6127 = \mathbf{0.3873}$$

PROBLEMS

1. Explain with the help of a diagram the hydrologic cycle.
2. What do you understand by precipitation? Explain various types of precipitation.
3. Explain any one type of automatic rain gauge.
4. Describe various methods of computing average rainfall over a basin.
5. What is run-off ? What are the factors that affect the run-off from a catchment area?
6. What are the methods of computing run-off from a catchment area? Give various formulae stating clearly the area for which each is applicable.
7. Explain various methods of determining flood discharge in a stream.
8. Give various flood discharge formulae applicable for Indian catchments.
9. What is a hydrograph ? Draw a single peaked hydrograph and explain its components.
10. Explain the method of determining direct run-off from a given storm hydrograph.
11. What do you understand by unit hydrograph ? How is it derived ? Explain its use in construction of flood hydrograph resulting from two or more periods of rainfall.
12. Find out the ordinates of a storm hydrograph resulting from a 3 hour storm with rainfall of 3, 4.5 and 1.5 cm during subsequent 3 hours intervals. The ordinates of unit hydrograph are given in the Table below :

Hours	0	03	06	09	12	15	18	21	24	03	6	09	12
Ordinates of unit hydrograph (cumecs).	0	90	200	350	450	350	260	190	130	80	45	20	0

Assume an initial loss of 5 mm, infiltration index of 5mm/hour and base flow of 20 cumecs.

Ans. 20, 110, 490, 970, 1520, 1720, 1330, 990, 720, 490, 305, 175, 80 and 20.

13. What do you understand by infiltration index ? How do you determine it ?

5

Ground Water : Well Irrigation

5.1. INTRODUCTION

Ground water hydrology is the science of occurrence, distribution and movement of water below the surface of earth. The largest available source of fresh water lies underground. The total ground water potential is estimated to be one third the capacity of oceans.

The main source of ground water is precipitation. A portion of rain falling on the earth's surface infiltrates into ground, travels down and, when checked by impervious layer to travel further down, forms ground water. The ground water reservoir consists of water held in voids within a geologic stratum. Other sources of ground water include water from deep in the earth which is carried upward in intrusive rocks and water which is trapped in sedimentary rocks during their formation. The quantities of such water are small and they are often so highly mineralised as to be unsuited for use. Water bearing formations of the earth's crust act as conduits for transmission and as reservoirs of storage of ground water.

The discharge from ground water occurs in two ways : (1) natural way, (2) artificial way. The natural discharge occurs as flow in lakes, reservoirs, rivers, oceans and springs. Pumpage from wells constitutes the major artifical discharge from ground water.

5.2. SOME DEFINITIONS

Aquifer. Aquifers are the permeable formations having structures which permit appreciable quantity of water to move through them under ordinary field conditions. Thus these are the geologic formations in which ground water occurs (*i.e.*, sands and gravels).

Aquiclude. Aquicludes are the impermeable formations which contain water but are not capable of transmitting and supplying a significant quantity (*e.g*, clays).

Aquifuge. Aquifuge is an impermeable formation which neither contains water nor transmits any water.

Probably, 90 percent of all developed aquifers consist of unconsolidated rocks, chiefly gravel and sand. *Sands* are composed of macroscopic particles that are rounded (bulky) or angular in shape. They drain easily, do not swell, possess insignificant capillary potential and when dry exhibit no shrinkage. *Clays* on the other hand, are composed of microscopic particles of plate like shape. They are highly impervious, exhibit considerable

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(*a*) Probability of exceedance, $P = \frac{1}{T} = \frac{1}{25} = \mathbf{0.04}$

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5.2. SOME DEFINITIONS

Aquifer. Aquifers are the permeable formations having structures which permit appreciable quantity of water to move through them under ordinary field conditions. Thus these are the geologic formations in which ground water occurs (*i.e.*, sands and gravels).

Aquiclude. Aquicludes are the impermeable formations which contain water but are not capable of transmitting and supplying a significant quantity (*e.g*, clays).

Aquifuge. Aquifuge is an impermeable formation which neither contains water nor transmits any water.

Probably, 90 percent of all developed aquifers consist of unconsolidated rocks, chiefly gravel and sand. *Sands* are composed of macroscopic particles that are rounded (bulky) or angular in shape. They drain easily, do not swell, possess insignificant capillary potential and when dry exhibit no shrinkage. *Clays* on the other hand, are composed of microscopic particles of plate like shape. They are highly impervious, exhibit considerable

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swelling, possess a high capillary potential and demonstrate considerable volume reduction upon drying.

Porosity. Porosity (n) is defined as the ratio of the volume of openings or pores (or voids) V_v in the material to its total volume V and is expressed as percentage :

$$n = \frac{V_v}{V} \times 100$$

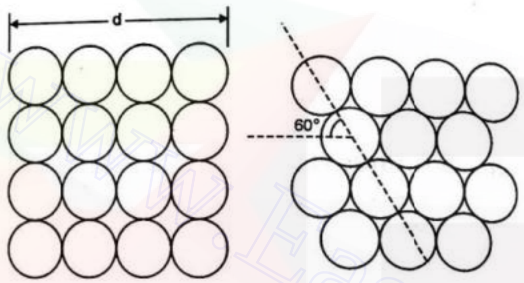
For a cubical array of spheres [Fig. 5.1 (*a*)]

$$V = d^3, \; V_s = \frac{\pi}{6} d^3$$

$$V_v = V - V_s = d^3\left(1 - \frac{\pi}{6}\right)$$

$$\therefore \; n = \frac{d^3\left(1 - \frac{\pi}{6}\right)}{d^3} \times 100 = 47.6\% \quad ...(11)$$

For a rhombohedral packing [Fig. 5.1 (*b*)], it can be shown that



(a) Cubical (b) Rhombohedral

FIG. 5.1.

$$n = \left(1 - \frac{\sqrt{2}}{6}\pi\right) 100 = 26\%$$

Values of the porosity of some soils in their natural form are given below : (Terzaghi and Pack) :

Soil	*Porosity (%)*
Uniform sand, loose	46
Uniform sand, dense	34
Glacial till, very mixed-grained	20
Soft glacial clay	55
Stiff glacial clay	37
Soft very organic clay	75
Soft bentonite	84

Specific Yield. The capacity of a formation to contain water is measured by porosity. However, a high porosity does not indicate that an aquifer will yield large volumes of water to a well. The only water which can be obtained from the aquifer is that which will flow by gravity. *The specific yield of an aquifer is defined as the ratio expressed as a percentage, of the volume of water which after being saturated, can be drained by gravity to its own volume.*

Thus, $\quad$ Specific yield $= \dfrac{\text{Volume of water drained by gravity}}{\text{Total volume}}$

or $$S_y = \frac{w_y}{V} \times 100$$

where w_y = volume of water drained by gravity.

Specific yield is an indication of water yielding capacity of an unconfined aquifer.

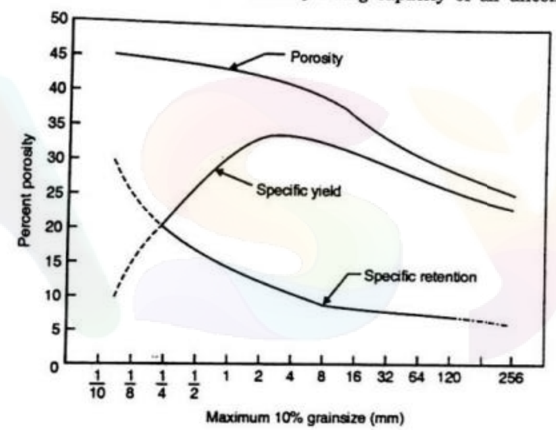


FIG. 5.2 POROSITY, SPECIFIC YIELD AND SPECIFIC RETENTION VARIATION WITH GRAIN SIZE (AFTER ECKIS).

Specific Retention. Specific yield is always less than porosity since some water will be retained in the aquifer by molecular and surface tension forces. *The specific retention (S_r) of an aquifer is the ratio, expressed as a percentage, of the volume of water it will retain after saturation against the force of gravity to its own volume.* Thus,

Specific retention $\quad S_r = \dfrac{w_r}{V} \times 100$

where w_r = volume of water retained.

Porosity $\quad n = \dfrac{V_v}{V} \times 100 = \dfrac{w}{V} \times 100$

where w = volume of water $= V_v$ in a saturated aquifer $= w_y + w_r$

Hence $\quad n = S_y + S_r$.

The value of specific yield depends upon gain size, shape and distribution of pores, and compaction of stratum. Fig. 5.2 shows the variations of porosity, specific yield and specific retention of soil with grain size.

5.3. DIVISIONS OF SUB-SURFACE WATER

The sub-surface water, as indicated in Fig. 5.3 can be divided into following zones:

The zone in which below the water table relatively all pores and fractures are saturated with water is called zone of saturation

(a) Zone of Aeration

Consisting of :

(i) Soil water zone : Soil water.

(ii) Intermediate zone : Pellicular and gravitational water.

(iii) Capillary zone : Capillary water.

(b) Zone of Saturation

Ground water fills all the interstices in the saturated zone.

Water Table

The saturated zone is bounded at the top by either a limiting surface of saturation or overlying (confining) impermeable strata, and extends down to underlying impermeable strata. In the absence of the confining impermeable layer, the static level of water in wells penetrating the zone of saturation is called the *water table*. The water table is the surface of a water body which is constantly adjusting itself towards an equilibrium condition. If there were no recharge to or outflow from the ground water in basin, the water table would eventually become horizontal.

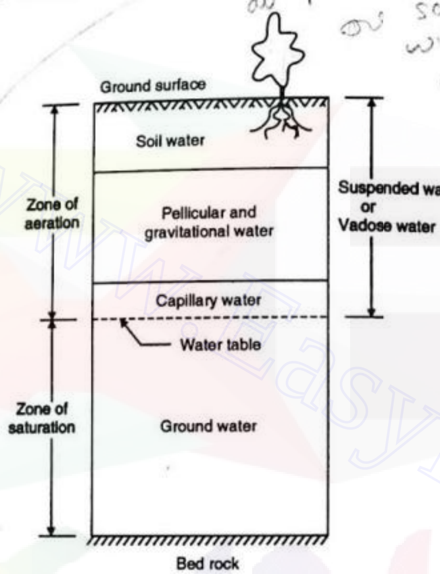


FIG. 5.3 DIVISIONS OF SUB-SURFACE WATER.

5.4. TYPES OF AQUIFERS

Aquifers are mainly of two types:

1. Unconfined aquifer.
2. Confined aquifer (artesian aquifer).

The aquifer is said to be unconfined when the upper surface is open to the atmosphere through permeable material

Unconfined Aquifer

Unconfined aquifer, or water table aquifer is the one in which a water table serves as the upper surface of the zone of saturation. It is also sometimes known as the *free*, *phreatic* or *non-artesian aquifer*. In such an aquifer, the water table varies in undulating form and in slope. Rises and falls in the water table corresponds to changes in the volume of water in storage within unconfined aquifer.

Confined Aquifer or Artesian Aquifer

Confined aquifer or artesian aquifer is the one in which ground water is confined under pressure greater than atmospheric by over-lying, relatively impermeable strata. Artesian aquifers are analogous to pipelines. The static pressure at a point within the artesian aquifer is equivalent to the elevation of the water table in the recharge

[illegible] aquifer in which impermeable clay or rock layer exist [illegible] prevents the water from seeping [illegible] [illegible] ground surface located directly above.

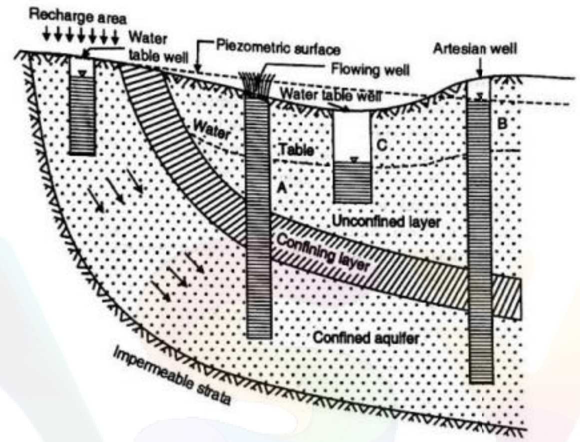


FIG. 5.4 CONFINED AND UNCONFINED AQUIFERS.

area less the loss in head through the aquifer to the point under consideration. In a well penetrating such an aquifer, the water level will rise to the level of the local static pressure or artesian head. Water enters a confined aquifer in an area where the confining bed rises to the surface or ends underground and the aquifer becomes unconfined. Artesian aquifers usually have relatively small recharge areas as compared with unconfined aquifers. When water is withdrawn from an artesian well, a local depression of the piezometric surface results. This decrease in pressure permits a silight expansion of the water and in some cases a compaction of the aquifer.

Flowing Well and Artesian Well

When a well penetrates a confined aquifer, water rises in the well to the level of local static pressure or artesian head. If this artesian pressure is sufficient to raise the water above the ground level, a *flowing well* occurs, such as well *A* in Fig. 5.4. If, however, the water level in such a well is below the ground level, but is above the local water table it is known as the *artesian well* such as well *B* in Fig. 5.4.

Perched Aquifer

Perched aquifer (Fig. 5.5) is a special type of unconfined aquifer, and occurs where a ground water body is separated from the main ground water by a relatively imper-

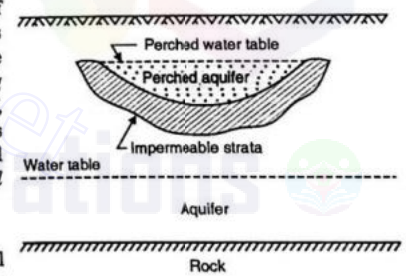


FIG. 5.5 PERCHED AQUIFER.

meable stratum of small aerial extent and by the zone of aeration above the main body of ground water.

5.5. STORAGE COEFFICIENT

The water yielding capacity of a confined aquier can be expressed in terms of its *storage coefficient*. Storage coefficient is defined as the volume of water that an aquifer releases from or takes into storage per unit surface area of aquifer per unit change in the component of head normal to that surface.

Let us consider a vertical column of unit area (one metre × one metre) extending through a confined aquifer (Fig. 5.6). Then, the storage coefficient, S is the volume of water, in cubic metres, released from the aquifer when the piezometric surface declines by one metre. In most of the confined aquifers, the value of storage coefficient ranges between 0.00005 to 0.005. Its value can be determined from pumping out tests on wells penetrating fully into confined aquifer.

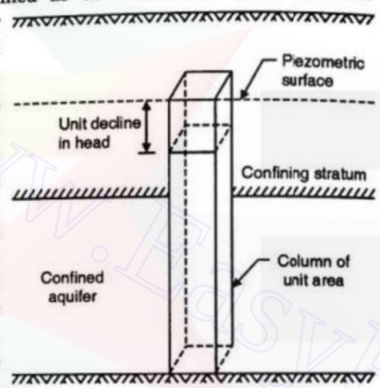


FIG. 5.6 STORAGE COEFFICIENT.

In an unconfined aquifer, when the water table is lowered by one metre, the water from one metre height of the vertical column of unit area drains freely under gravity. Thus, storage coefficient for an unconfined aquifer corresponds to its specific yield.

Coefficient of Permeability (k)

The coefficient of permeability is defined as the velocity of flow which will occur through the total cross-sectional area of the soil (or aquifer) under a unit hydraulic gradient. Some typical values of the coefficient of permeability are given in Table 5.1.

TABLE 5.1

Soil type	*Coefficient of permeability (cm/sec)*
Clean gravel	1.0 and greater
Clean sand (coarse)	1.0–0.01
Sand (mixture)	0.01–0.005
Fine sand	0.05–0.001
Silty sand	0.002–0.0001
Silt	0.0005–0.00001
Clay	0.000001 and smaller

Coefficient of Transmissibility (T)

Coefficient of transmissibility is defined as the rate of flow of water (in m^3/day or gallons/day) through vertical strip of aquifer of unit width (1 m or 1 ft) and extending the full saturation height under unit hydraulic gradient, at a temperature of 60° F.

Thus, the coefficient of transmissibility T equals to the field coefficient of permeability multiplied by the aquifer thickness (B) :

$$T = B\,k.$$

5.6. WELL HYDRAULICS

Darcy's Law. The percolation of water through soil was first studied by Darcy (1856) who demonstrated experimentally that for laminar flow conditions in a saturated soil, the rate of flow, or the discharge per unit time is proportional to the hydraulic gradient, and it could be expressed as follows :

$$Q = kiA \quad \text{...(5.1)}$$

or

$$v = \frac{Q}{A} = ki \quad \text{...(5.2)}$$

where Q = rate of flow

i = hydraulic gradient

k = Darcy's coefficient of permeability

A = total cross-sectional area of soil mass perpendicular to the direction of flow

v = flow velocity.

Eq. 5.2 demonstrates the linear dependency between the hydraulic gradient and the discharge velocity. However, it in no way describes the state of affairs within an individual pore. Strictly speaking, Darcy's law represents the statistical macroscopic equivalent of the Navier-Stokes equation of motion for the viscous flow of ground water.

Darcy's law is vaild only for laminar flow. Because of very small pore dimensions in fine grained soils, a laminar flow should exist, but in course grained soils turbulent flow may be expected under certain conditions. It has been borne out by experiments that the limits of validity of Darcy's law may be fixed with respect to particle size, velocity of flow and hydraulic gradient. Fancher, Lewis and Barnes demonstrated that flow through sands remains laminar and the Darcy's law valid so long as the Reynold's number expressed in the form below is equal to or less than unity :

$$\frac{\rho v d}{\mu} \le 1 \quad \text{...(5.3)}$$

where ρ = mass density

μ = dynamic viscosity

d = diameter or particle size

v = velocity of flow

For the ground water flow occuring in nature, the law is generally within its validity limits. But in rock aquifers, in unconsolidated aquifers with steep hydraulic gradients, or in those containing large diameter solution openings, Darcy's law may not be applicable. Also, the flow in the immediate vicinity of wells have steep hydraulic gradient and the Darcy's law is not applicable in the immediate vicinity of the well.

STEADY RADIAL FLOW TO A WELL : DUPUIT'S THEORY

When a well is penetrated into an extensive homogeneous aquifer, the water table initially remains horizontal in the well. When the well is pumped, water is removed

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from the aquifer and the water table or piezometric surface, depending upon the type of the aquifer, is lowered resulting in a circular depression in the water table or the piezometric surface. This depression is called the *cone of depression* or the *drawdown curve*. At any point, away from the well, the *drawdown* is the vertical distance by which the water table or the piezometric surface is lowered. The analysis of such radial flow towards a well was originally proposed by Dupuit in 1863 and later modified by Thiem (1906). For the sake of analysis, we shall take two cases : (1) well in unconfined aquifer, and (2) well fully penetrating a confined aquifer.

1. **Unconfined Aquifer.**

Fig. 5.7 shows a well penetrating an unconfined or free aquifer to its full depth.

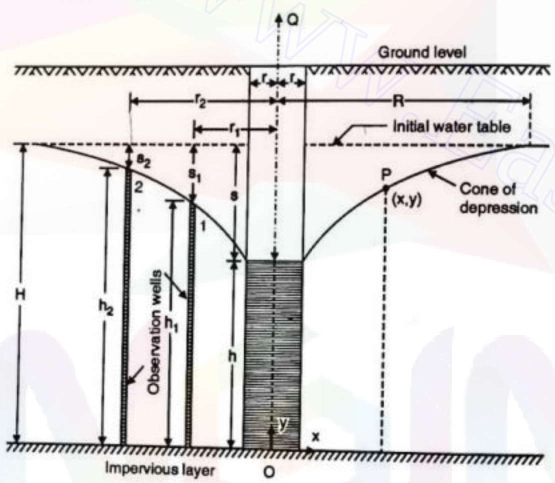

FIG. 5.7 UNCONFINED AQUIFER

Let r = radius of the well

H = thickness of the aquifer, measured from the impermeable layer to the initial level of water table.

s = drawdown at the well

h = depth of water in the well measured above impermeable layer.

Considering the origin of co-ordinates at a point O at the centre of the well at its bottom, let the co-ordinates of any point P on the drawdown curve be (x, y). Then, from Darcy's law

$$Q = k A_x i_x$$

where A_x = area of cross-section of the saturated part of aquifer at P
$= (2\pi x) \times (y) = 2\pi x y$

i_x = hydraulic gradient at $P = \frac{dy}{dx}$

Hence
$$Q = k(2\pi x y)\frac{dy}{dx}$$

$$\therefore \quad Q\frac{dx}{x} = 2\pi k y\, dy.$$

Integrating between the limits (R, r) for x and (H, h) for y we get

$$Q\int_r^R \frac{dx}{x} = 2\pi k \int_h^H y\, dy$$

$$\therefore \quad Q\left[\log_e x\right]_r^R = 2\pi k\left[\frac{y^2}{2}\right]_h^H$$

From which
$$Q = \frac{\pi k (H^2 - h^2)}{\log_e \frac{R}{r}} = \frac{1.36\, k (H^2 - h^2)}{\log_{10} \frac{R}{r}} \qquad ...(5.4)$$

If k (coefficient of permeability) is expressed in cubic metre per day per square metre ($m^3/day/m^2$) of the area of sub-soil, the above expression for discarge will directly be in cubic metre per day (m^3/day) units. If, however, k is in gallons per day per sq. foot of area of sub-soil, the discharge will be in gallons per day.

In the above expression, R, commonly known as *radius of zero drawdown*, is the radius measured from the centre of the well to a point where the drawdown curve meets the original water table tangentially. *In practice, the selection of the radius of influence R is approximate and arbitrary but the variation in Q is small for a wide range of R.* Suggested values of R fall in the range of 150 to 300 metres.

Alternatively, R may be computed from the following approximate expression given by Sichardt :

$$R = 3000\, s\sqrt{k}$$

where R and s are in metres, and k is in m/sec.

If there are two observation wells at radial distance r_1 and and r_2 $(r_2 > r_1)$ and if the depths of water in them are h_1 and h_2 respectively, Eq. 5.4 can also be expressed in the form :

$$Q = \frac{\pi k (h_2^2 - h_1^2)}{\log_e \frac{r_2}{r_1}} \qquad ....[5.4(a)]$$

or
$$Q = \frac{1.36\, k (h_2^2 - h_1^2)}{\log_{10} \frac{r_2}{r_1}} \qquad ....[5.4(b)]$$

If the drawdown (s) is measured at the well face, we have

$$s = H - h$$

and $$H = s + h \quad or \quad H + h = (s + 2h)$$

Then, from Eq. 5.4, $$Q = \frac{\pi k (H-h)(H+h)}{\log_e \frac{R}{r}} = \frac{\pi k s (s + 2h)}{\log_e \frac{R}{r}}$$

$$Q = \frac{\pi k s (s + 2L)}{\log_e \frac{R}{r}} = \frac{2 \pi k s (L + s/2)}{\log_e \frac{R}{r}}$$

where $h = L$ = Length of the strainer

$$\therefore \quad Q = \frac{2.72\, ks (L + s/2)}{\log_{10} \frac{R}{r}} \qquad \ldots(5.5)$$

Assumptions and Limitations of Dupuit's Theroy

Dupuit's theory of flow for unconfined aquifer is based on the following assumptions :

1. The velocity of flow is proportional to the tangent of the hydraulic gradient instead of sine.
2. The flow is horizontal and uniform everywhere in the vertical section.
3. Aquifer is homogeneous, isotropic and of infinite aerial extent.
4. The well penetrates and receives water from the entire thickess of the aquifer.
5. The coefficient of transmissibility is constant at all places and at all times.
6. Natural ground water regime affecting an aquifer remains constant with time.
7. Flow is laminar and Darcy's law is applicable.

Out of these, assumptions (1), (2) and (7) are of particular importance. The flow is not horizontal, especially near the well. Also, the piezometric surface attains greater slope as it approaches the well boundary, with the result that assumption 1 is an approximation. Due to these reasons, the parabolic form of piezometric surface computed from the Dupuit's theory deviates from the observed surface. This deviation is large at the well face, resulting in the formation of seepage face. In addition to these, the velocity near the well increases and the flow no longer remains laminar. Thus, Darcy's law equation is not valid near the well face.

2. **Confined Aquifer.**

Fig. 5.8 shows a well fully penetrating a confined or artesian aquifer. Let (x, y) be the co-ordinates, of any point P on the drawdown curve, measured with respect to the origin O. Then, from Darcy's law, flow crossing a vertical plane through P is given by

$$Q = k\, i_x A_x$$

where A_x = cross-sectional area of flow, measured at $P = 2\pi x b$

b = thickness of confined aquifer

i_x = hydraulic gradient at $P = \frac{dy}{dx}$

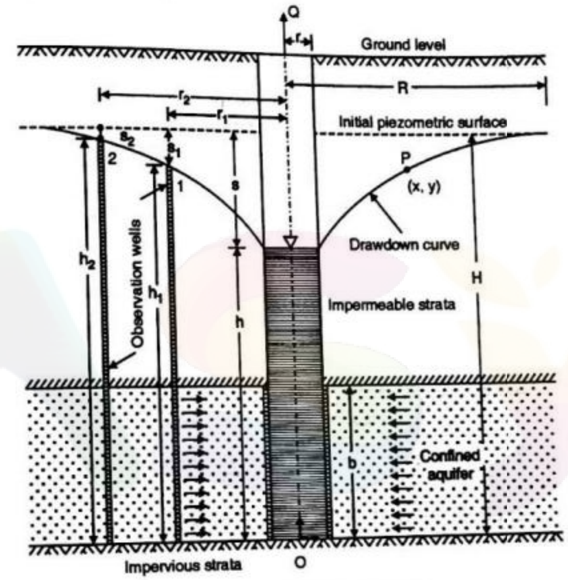

FIG. 5.8 CONFINED AQUIFER.

$$\therefore \quad Q = k \left(\frac{dy}{dx}\right)(2\pi x b)$$

$$\therefore \quad Q \frac{dx}{x} = 2\pi k b\, dy$$

Integrating between the limits (R, r) for x and (H, h) for y, we get

$$Q \int_r^R \frac{dx}{x} = 2\pi k b \int_h^H dy$$

$$\therefore \quad Q \left[\log_e x\right]_r^R = 2\pi k b \left[y\right]_h^H$$

From which $$Q = \frac{2\pi k b (H-h)}{\log_e \frac{R}{r}} = \frac{2.72\, bk (H-h)}{\log_{10} \frac{R}{r}} = \frac{2\pi b k s}{\log_e \frac{R}{r}} = \frac{2.72\, b k s}{\log_{10} \frac{R}{r}} \qquad \ldots[5.6(a)]$$

or $$Q = \frac{2.72\, T . s}{\log_{10} \frac{R}{r}} \qquad \ldots[5.6(b)]$$

where T = coefficient of transmissibility = bk

s = drawdown at the well

Eq. 5.6(a) is known as the *equilibrium equation* or *Thiem equation.*

If h_1 and h_2 are the measured depth of water in two observation wells situated radially at distances r_1 and r_2 respectively, the above equations can also be expressed as

$$Q = \frac{2.72\, bk\,(h_2 - h_1)}{\log_{10} \frac{r_2}{r_1}} = \frac{2.72\, T(h_2 - h_1)}{\log_{10} \frac{r_2}{r_1}} \quad ...[5.6(d)]$$

If h_x is the depth at any radial distance x, the discharge can be written as

$$Q = \frac{2\,\pi kb\,(h_x - h)}{\log_e \frac{r_x}{r}}$$

Equating this to Eq. 5.6(a), we get

$$h_x - h = (H - h)\,\frac{\log_e \frac{r_x}{r}}{\log_e \frac{R}{r}} \quad ...(5.7)$$

This shows that head varies linearly with the logarithm of distance regardless of rate of discharge.

5.7. DETERMINATION OF AQUIFER CONSTANT T

(a) **Confined aquifer**. The aquifer constant T (coefficient of transmissibility) can be determined by a pumping out test, and by observing drawdown at various observation wells. Pumping must continue at a unifrom rate for a sufficient time to approach a steady state condition for which equilibrium Eq. 5.6 is applicable. Steady state condition is the one in which the drawdown changes negligibly with time. Refer Fig. 5.8.

Let s_1 = drawdown in well 1 = $(H - h_1)$

s_2 = drawdown in well 2 = $(H - h_2)$.

$\therefore \quad h_2 - h_1 = (H - s_2) - (H - s_1) = s_1 - s_2$

Then from Eq. 5.6 (d),

$$Q = \frac{2.72\, T\,(h_2 - h_1)}{\log_{10} \frac{r_2}{r_1}} = \frac{2.72\, T\,(s_1 - s_2)}{\log_{10} \frac{r_2}{r_1}}$$

$$\therefore \quad T = \frac{Q}{2.72\,(s_1 - s_2)} \log_{10} \frac{r_2}{r_1} \quad ...(5.8)$$

Choosing $r_2 = 10\, r_1$, we find $\log_{10} \frac{r_2}{r_1} = 1$.

Hence $$T = \frac{Q}{2.72\,(s_1 - s_2)} = \frac{Q}{2.72\,\Delta s} \quad ...(5.9)$$

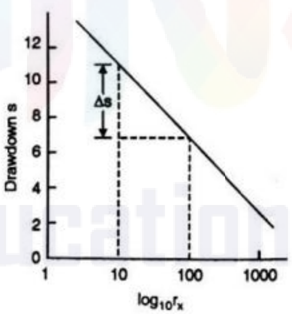

FIG. 5.9

where Δs = difference in drawdowns at the two wells so selected that $r_2 = 10\, r_1$.

The method, therefore, consists in observing drawdowns s_1, s_2 ...., s_x at certain observation wells distant r_1, r_2,....r_x etc., and plotting a graph between s_x as ordinate

and $\log_{10} r_x$ as abscissa, thus getting a straight line as shown in Fig. 5.9. From the graph, Δs can be obtained for one log cycle of distance and can be substituted in Eq. 5.9 to get T.

(b) **Unconfined Aquifer.** The above method of determination of T can also be extended to unconfined aquifer. Refer Fig. 5.7.

$$h_1 = H - s_1 \; ; \qquad h_2 = H - s_2$$

$$\therefore \qquad h_2^2 - h_1^2 = (H - s_2)^2 - (H - s_1)^2$$

$$= 2H\left[\left(s_1 - \frac{s_1^2}{2H}\right) - \left(s_2 - \frac{s_2^2}{2H}\right)\right] = 2\,H\,(s_1' - s_2')$$

where s_1' and s_2' are *modified drawdowns* given by

$$s_1' = s_1 - \frac{s_1^2}{2H}$$

and $$s_2' = s_2 - \frac{s_2^2}{2H}$$

Now from Eq. [5.4(a)],

$$Q = \frac{\pi\, k\,(h_2^2 - h_1^2)}{\log_e \frac{r_2}{r_1}} = \frac{1.36\, k\,(h_2^2 - h_1^2)}{\log_{10} \frac{r_2}{r_1}}$$

Substituting the value of $(h_2^2 - h_1^2)$ from above, we get

$$Q = \frac{1.36\, k\,[2\,H\,(s_1' - s_2')]}{\log_{10} \frac{r_2}{r_1}} = \frac{2.72\, kH(s_1' - s_2')}{\log_{10} \frac{r_2}{r_1}}$$

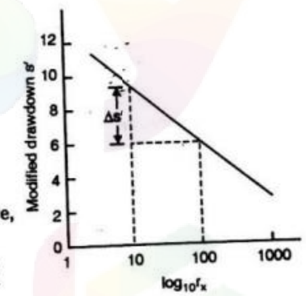

FIG. 5.10

Since H = aquifer thickness, we have $kH = T$

$$\therefore \qquad Q = \frac{2.72\, T\,(s_1' - s_2')}{\log_{10} \frac{r_2}{r_1}} \quad ...(5.10)$$

From which $$T = \frac{Q}{2.72\,(s_1' - s_2')} \log_{10} \frac{r_2}{r_1}$$

It should be noted that Eq. 5.8 and 5.10 are identical. Choosing $r_2 = 10\, r_1$ as before, we get

$$T = \frac{Q}{2.72\,\Delta s'} \quad ...(5.11)$$

Thus the observed drawdowns are corrected or modified, and a graph is plotted between the modified drawdowns (s') and logarithm of distance of observation wells from the discharge well. Measuring $\Delta s'$ for one log cycle of distance, we get T from Eq. 5.11.

5.8. CHARACTERISTIC WELL LOSSES : SPECIFIC CAPACITY OF WELL

Well Loss. When water is pumped out of a well, the total drawdown caused includes not only that of the logarithmic drawdown curve at the well face, but also drawdown caused by flow through well screen and axial movement within the well. The latter drawdown is called *well loss.* Since turbulent flow generally occur near the well face, this loss may be taken to be proportional to Q^n.

Rewriting Eq. 5.4(a) and adding well loss CQ^n to it, we get

$$s = (H - h) = \frac{Q}{2\pi kb} \log_e \frac{R}{r} + CQ^n$$

where the constant C is governed by several factors such as well radius, construction and condition of the well. Rewriting the above, we get

$$s = BQ + CQ^n \qquad \ldots(5.12)$$

where $B = \dfrac{\log_e \dfrac{R}{r}}{2\pi kb}$; BQ = aquifer loss and CQ^n = well loss.

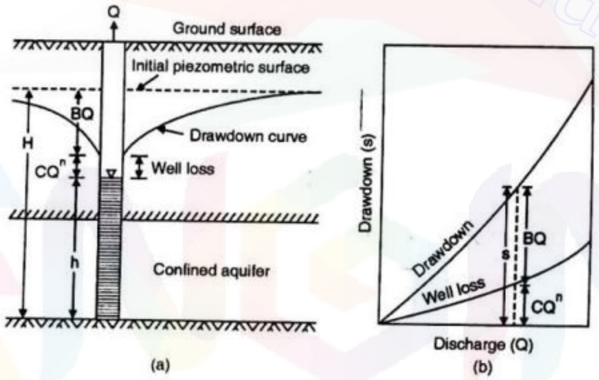

FIG. 5.11. WELL LOSS IN CONFINED AQUIFER.

Fig. 5.11(a) illustrates that the total drawdown consists of the sum of the aquifer loss (BQ) and the well loss (CQ^n). Fig. 5.11(b) shows the variations of drawdown and well loss with the discharge. For a screen which is not clogged or encrusted and whose size is compatible to the surrounding porous media, the portion of the well loss caused by water entering the well is small in comparison with the portion resulting from axial movement inside the well to the pump intake.

Specific Capacity.

The specific capacity of a well is the measure of the effectiveness of the well, and is defined as the yield of the well per unit drawdown.

Thus, if s = drawdown and Q = well discharge or the yield.

Then, specific capacity $= \dfrac{Q}{s}$

For a confined aquifer, from Eq. 5.12, we have $s = BQ + CQ^n$

Hence specific capacity

$$= \frac{Q}{BQ + CQ^n} = \frac{1}{B + CQ^{n-1}} \qquad \ldots(5.3)$$

This shows that the specific capacity of a well is not constant, but decreases as discharge increases.

5.9. INTERFERENCE AMONG WELLS

When two wells, situated near to each other, are discharging, their drawdown curves intersect within their radius of zero drawdown. Thus, though the total discharge is increased, the discharge in individual well is decreased due to interference.

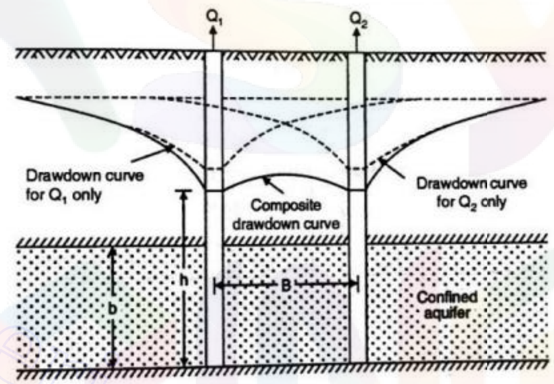

FIG. 5.12 INTERFERENCE BETWEEN TWO WELLS

Fig. 5.12 shows interference between two wells. If the two wells are a distance B apart, and have the same diameter and drawdown and discharge over the same period of time, it can be shown with the help of method of complex variables, that the discharge through each well is given by

$$Q_1 = Q_2 = \frac{2\pi kb\,(H - h)}{\log_e \dfrac{R^2}{rB}} \qquad \ldots(5.14)$$

where R is the radius of area of influence ($R >> B$).

If there were only one well, then the discharge, under the same drawdown, would have been, from Eq. [5.6(a)],

$$Q = \frac{2\pi kb\,(H - h)}{\log_e \dfrac{R}{r}} \qquad \ldots[5.6(a)]$$

Since $\quad R >> B, \ \frac{R^2}{rB} > \frac{R}{r}$

Hence $\quad Q > Q_1$.

Thus, discharge in each well decreases due to the interference will decrease.

Similarly, if there are three wells forming an equilateral triangle a distance B on a side, and if all the three wells have the same characteristics,

$$Q_1 = Q_2 = Q_3 = \frac{2\pi kb\,(H-h)}{\log_e \frac{R^3}{rB^2}} \qquad \text{...(5.15)}$$

5.10. FULLY PENETRATING ARTESIAN GRAVITY WELL

Sometimes, in an artesian well (*i.e.*, a well in a confined aquifer) high pumping rates may lower the water at the well face to a level below the top confined aquifer, as shown in Fig. 5.13. In such a case, the flow pattern close to the well is similar to that for a gravity well (*i.e.*, a well in unconfined aquifer) whereas at distances farther from the well the flow is artesian. This type of well is known as a *combined artesian gravity well.*

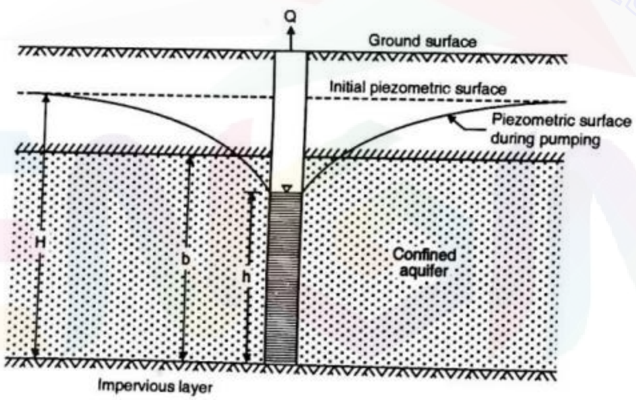

FIG. 5.13 ARTESIAN GRAVITY WELL.

Fig. 5.13 shows an artesian gravity well. The flow from such a well can be computed from the following expression developed by Muskat :

$$Q = \frac{\pi k\,(2bH - b^2 - h^2)}{\log_e \frac{R}{r}} \qquad \text{...[5.16(a)]}$$

5.11. PARTIALLY PENETRATING ARTESIAN WELL

A partially penetrating artesian well is the one in which the well screen does not penetrate to the full depth of the confined aquifer. The pattern of flow in the

aquifer in the vicinity of such a well deviates from that for a fully penetrating well. In practice, we often encounter such wells that extend only part way through the water bearing strata.

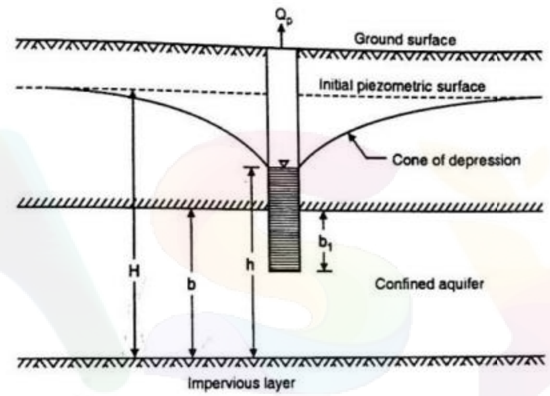

FIG. 5.14. PARTIALLY PENETRATING ARTESIAN WELL.

Fig. 5.14 shows a partially penetrating artesian well in which the strainer length b_1 is less than the aquifer thickness b. The discharge Q_p from such a well can be computed from the following equation :

$$Q_p = \frac{2\,\pi kb\,(H-h)}{\log_e \frac{R}{r}}\, G = Q.G \qquad \text{...[5.16(b)]}$$

where $\quad Q_p$ = discharge for partially penetrating well.

Q = discharge for a fully penetrating well for the same drawdown $(H-h)$.

G = correction factor for partial penetration $= \frac{Q_p}{Q}$.

A reasonable estimation of the correction factor G can be obtained from the following expression developed by Kozeny :

$$G = \frac{b_1}{b}\left(1 + 7\sqrt{\frac{r}{2b_1}}\cos\frac{\pi b_1}{2b}\right) \qquad \text{...[5.17.]}$$

5.12. SPHERICAL FLOW IN A WELL

Fig. 5.15 shows a special case of partially penetrating well where the well just penetrates the top surface of a semi-infinite porous medium. Here $b_1 = 0$ and equation 5.16 does not apply because the flow towards the well is purely spherical. Thus discharge Q_s from such a well can be computed from the expression

$$Q_s = 2\,\pi\, k\, r\,(H-h) \qquad \text{...(5.18)}$$

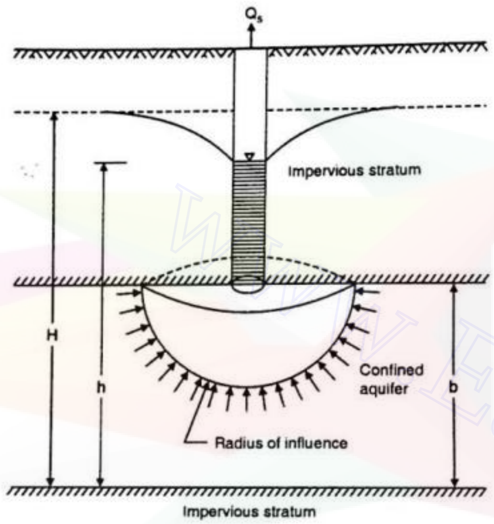

FIG. 5.15 SPHERICAL FLOW IN A WELL.

For the case of simple radial flow is a fully penetrating well the discharge Q is given by equation 5.6(*a*)

$$Q = \frac{2 \pi k b (H-h)}{\log_e \frac{R}{r}} \qquad \ldots[5.6(a)]$$

$$\therefore \quad \frac{Q_s}{Q} = \frac{r}{b} \log_e \frac{R}{r} = 2.303 \frac{r}{b} \log_{10} \frac{R}{r} \qquad \ldots(5.19)$$

As a numerical example,

Let r = 8 cm = 0.8 m. Hence $\frac{R}{r} = 1000$

b = thickness of aquifer =16 m.

$$\therefore \quad \frac{Q_s}{Q} = 2.303 \times \frac{0.08}{10} \log_{10} 1000 \approx \frac{1}{30}$$

This shows that the spherical flow is very much less efficient than the radial flow.

5.13. TUBE WELLS

A *water well* is a hole or shaft, usually vertical, excavated in the ground for bringing ground water to the surface. Well can be mainly divided into two classes:

1. Dug wells or open wells.
2. Bored or drilled wells or tube wells.

An *open well* is comparatively of bigger diameter and is suitable for discharge upto 0.005 cumec. This is because the cross-sectional area of flow is less in the open well, and the water can be withdrawn safely only at the critical velocity for the soil. A *tube well* is a long pipe sunk into the ground with a strainer which allows water to pass through but prevents sand from coming in. Because of the strainer, high velocity of flow can be permitted without danger of soil particles being carried away with water. Also because of the radial flow towards the well, the cross-sectional area of flow is more. Due to the increased velocity and more cross-sectional area of flow a tube well, though much less in diameter than an open well, gives discharge many times more than the open well.

Types of Tube Wells. The tube well may be of three types :

1. Strainer well. 2. Cavity well. 3. Slotted well.

(1) Strainer Type Tube Well

The strainer well is the most common and widely used well. In common term, the word "tube well" refers to the strainer type of tube well. In this type of well, a strainer, which is a special type of wire mesh, is wrapped round the main tube of the well. The main pipe contains bigger holes or slots than the openings of the strainer. The total area of the openings of the tube is kept equal to the openings of the strainer so that the velocity of flow does not change. Due to fineness of the openings of the strainer, a higher operational velocity of water can be permitted. Little annular space is left between the strainer and the pipe so that the open area of pipe perforations is not reduced. Thus mesh size of the strainer is generally kept equal to D_{60} to D_{70} of the surrounding soil.

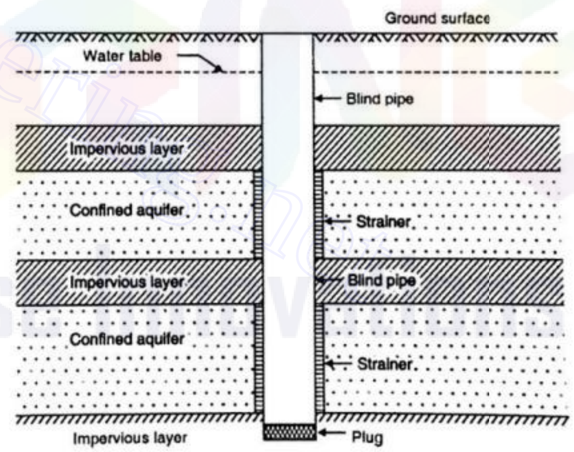

FIG. 5.16. STRAINER TYPE TUBE WELL.

A strainer well may draw water either from an unconfined aquifer of unlimited extent, or from one or more confined aquifer layers. The strainers are provided only in that length of the pipe where it crosses the aquifer. The pipe in the aquifer portion is kept perforated. In the rest of the portion, plain or blind pipe is provided. At the bottom, a short pipe is provided to permit settlement of any sand if passed through the strainer. The well is generally plugged at the bottom.

Abyssinian tube well is a special type of strainer tube well, in which the diameter of pipe is kept 38 mm and the strainer is provided only for a length of about 1.2 m to 1.5 m.

Design Criteria for Strainer Type Tube Well

The follwing points should be noted for the design of a strainer type of tube well :

(1) The wire screen should not be in contact with the slotted tubes otherwise a large part of the opening will be covered by the wire screen.

(2) The area of the opening in the wire screen should be equal to the area of waterway in the perforated or slotted tube to permit no change in the velocity of incoming water particles. If there is any change, sand will be deposited in the annular space.

(3) The design should permit less velocity through soil than the exit critical velocity which is 1.25 cm/sec.

(4) The discharging velocity should vary from 1 to 2 m/sec.

(5) The total surface area of tube well should be more than three times the area of perforations.

(6) The material of the strainer (wire screen) should be such that it can withstand the strain of sinking and at the same time can give maximum amount of water way consistent with fineness of openings necessary to prevent entry of sand.

(7) The material should preferably be of one metal. A bimetal construction poses a danger of electrolytic action in water, which may ultimately lead to deposition of salt and chocking of openings.

(8) The material should be sturdy and withstand rough handling.

Following metals and alloys have been found useful for strainers. They are corrosion-resistive :

(*i*) Zinc-free-brass or cupro-nickel alloy.

(*ii*) Stainless steel.

(*iii*) Low carbon steel.

(*iv*) High copper-alloy.

Type of Strainers. Following are some of the common types of strainers used in tube well :

(*i*) Cook strainer.

(*ii*) Tej strainer.

(*iii*) Brownlie strainer.

(*iv*) Ashford strainer.

(*v*) Leggett strainer.

(*vi*) Phoenix strainer.

(*vii*) Layne and Bowler strainer.

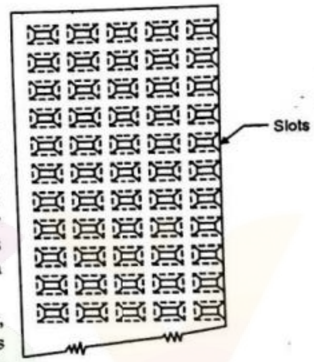

FIG. 5.17. COOK STRAINER.

(*i*) ***Cook strainer.*** This is a very costly strainer of American patent. It is made up of solid drawn brass tube slotted with wedge-shaped horizontal slots. The slots are made with a slot cutting machine from inside the tube. The slots are wide inside and narrower outside, as shown in Fig. 5.17. The gauge of slots depend on the coarseness of sand and varies from 0.15 to 0.4 mm.

(*ii*) ***Tej strainer.*** It is similar to Cook strainer, but is manufactured in India. It consists of brass tube constructed of a brass sheet bent round to form the tube, the vertical joint being brazed. The slots are cut in the sheet before it is bent. The strainer is generally manufactured from 7.5 cm diameter upwards, and is made in $2\frac{1}{2}$ meter lengths. The individual lengths of the strainer are then joined together by means of screwed collars of brass.

(*iii*) ***Brownlie strainer.*** The Brownlie strainer is made of a polygonal convoluted steel plate having perforations. A wire mesh surrounds the steel tube, as shown in Fig. 5.18. The mesh consists of heavy parallel copper wires woven with copper ribbons. Since the wire mesh is slightly away from the perforated tube, it is known as the best type of strainer.

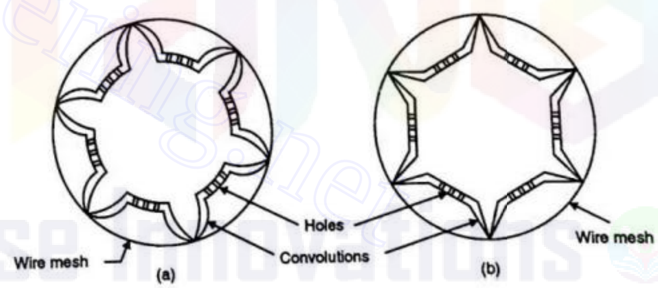

FIG. 5.18 BROWNLIE STRAINER.

(*iv*) ***Ashford strainer.*** This is very delicate strainer and consists of perforated tube with a wire round it over which a wire mesh is soldered. The wire keeps the mesh away from the tube. The wire mesh is protected and strengthened by wire net around it, as shown in Fig. 5.19.

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(v) ***Leggett strainer.*** It is an expensive type of strainer in which a cleaning device is provided. The cleaning device is in the shape of cutters which can be turned in the slits. The cutters are operated from the top (ground surface). These cutters clean the strainer clogged by the solid matter.

(vi) ***Phoenix strainer.*** It is a mild steel tube in which the openings are made by cutting slits from inside. The tube is cadmium plated to keep it free from danger of chocking and corrosion caused by chemical action.

(vii) ***Layne and Bowler strainer.*** It is a robust type strainer manufactured in America. It consists of wedge-shaped steel wire wound to a suitable pitch round a slotted or perforated steel or wrought iron pipe. The joints of the strainer pipes are made by screwed collars.

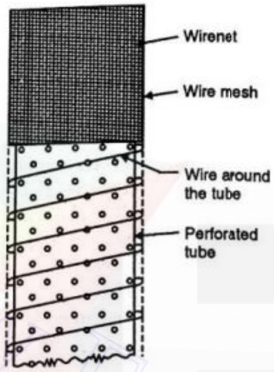

FIG. 5.19 ASHFORD STRAINER.

Chocking of Strainers. The strainer of a tube well may get chocked due to two actions : (a) Mechanical action, (b) Chemical action.

(a) ***Mechanical chocking.*** Mechanical chocking may result from the chocking of slits with sand and other particles. This may, however, be prevented by providing such slits which expand inwards. The pulsating action of the centrifugal pump may also remove the chocking. To safeguard against chocking, proper screening or shrouding should be provided. Another method of eliminating chocking is to permit inflow velocity lesser than the critical.

(b) ***Chemical chocking.*** The strainer may be chocked due to chemical action of salts present in water. The chemical action may also deteriorate a strainer by corrosion. If calcium bicarbonate present in water exceeds by an amount of 15 parts per million part of water, carbon dioxide is released when pressure is reduced due to pumping and calcium carbonate is precipitated on the strainer. The cumulative action of such precipitation reduces the yield. The chemical chocking by deposition of carbonates is reduced by providing large slit area and having low inflow velocity. Similarly sodium salts may attack mild steel and cast iron strainer pipes causing chocking. Sodium bicarbonate may attack copper to certain extent, though brass is not readily attacked.

(2) Cavity Type Tube Well

This is a special tupe of tube well in which water is not drawn through the strainer, but it is drawn through the bottom of the well where a cavity is formed. The tube well pipe penetrates a strong clay layer which acts as a strong roof. Thus, a cavity tube well is similar to a deep well. However, a deep well draws from the first aquifer below the *mota* while a cavity well need not do so. The essential condition for a cavity tube well to function efficiently is to have confined aquifer of good specific yield, and the aquifer should have a strong impervious material above it. In the inital stage of pumping with the help of a centrifugal pump or an air lift pump, fine sand comes with water and consequently a hollow or cavity is formed. As the spherical

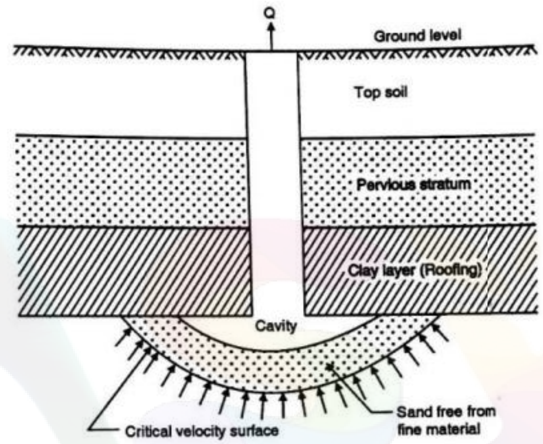

FIG 5.20. CAVITY TUBE WELL.

surface area of cavity increases outwards, the radial critical velocity decreases, and the sand particles stop entering the well. At this stage, an equilibrium in the cavity formation is established and clean water continues to enter the well on further pumping at the same constant discharge. After the formation of the cavity, the velocity of entry of water at the bottom of the pipe is lesser than the critical.

The essential difference between a strainer tube well and a cavity tube well lies in the fact that in the former case, due to the provision of a strainer, the flow is *radial* , while in the later case the flow is spherical. In the strainer tube well, the area of flow is increased by increasing the length of the strainer pipe (in an unconfined aquifer of infinite extent) while in the case of cavity type, the area of flow is increased by enlarging the size of the cavity. The cavity, formed due to a particular discharge Q_1, increases if an increased discharge Q_2 is pumped out.

(3) Slotted Type Tube Well

A slotted tube well is resorted to under two circumstances : (1) *sufficient* depth of water bearing startum is not available even upto a depth of 75 to 100 m, so that strainer type tube well cannot be used and (2) Suitable strong roof is not available so that a cavity well cannot be formed. In such a circumstance, a slotted well is used and it is made to penetrate to some depth in the water bearing strata. A slotted tube well essentially consists of a slotted tube penetrating the confined aquifer. The size of slots may be 25 mm × 3 mm at 10 to 12 mm spacings. In order to prevent the fine particles entering the pipe, it is shrouded with a mixture of gravel and bajri (coarse sand).

First of all, a casing pipe of 36 cm diameter is lowered, and soil is excavated out, and the water bearing strata is penetrated by a depth of about 5 m length. The perforated pipe, sometimes known as the education pipe of 15 cm diameter is then lowered, the slotted portion being only 5 m long and the rest of the length being of plain pipe. Gravel is then poured from the top, upto about 3 to 4 m higher than the top level of perforated portion of the pipe. The casing pipe is then withdrawn 5 cm at a time and the well is developed with the help of compressed air pumped into the education pipe. Finally, when the casing pipe is fully withdrawn, the annular space between the casing pipe and the education pipe is suitably plugged. By developing the well with the help of compressed air, the sand surrounding the gravel filter is freed of finer particles, and the chances of getting the filter chocked are reduced. Due to the provision of gravel shrouding, a larger area of radial flow is obtained.

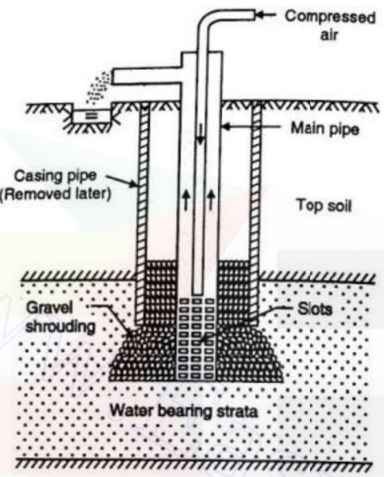

FIG. 5.21. SLOTTED TYPE TUBE WELL.

There are two essential differences between a strainer tube well and a slotted tube well : (1) in the strainer tube well, the strainer pipes are surrounded by wire mesh to prevent the fine particles from entering the well, while in the slotted tube well the gravel shrouding serves this purpose, (2) a strainer tube well the may have several alternative lengths of strainer pipes and plain pipes, while a slotted tube well has the slotted pipe length only at its bottom. Thus a strainer tube well draws water from several aquifers sandwiched between impervious layers, while a slotted tube well draws water only from one pervious stratum which has sufficient water bearing capacity.

5.14. METHODS FOR DRILLING TUBE WELLS

For installing a tube well in the ground, so as to penetrate the required stratum, a hole, slightly of larger diameter than the diameter of the strainer pipe, is bored. The most common methods for boring a well are :

1. Wash boring or water jet boring method.
2. Cable tool method (also known as percussion or standard method).
3. Hydraulic rotary method.
4. Reverse rotary method.

1. Wash Boring or Water-jet Boring Method

This method is suitable at places where the well is to be sunk into formations consisting of gravel, sand, clay or other soft deposits. The boring is done by cutting action of a downward directed stream of water. The outer casing in first erected in position in a suitable pit dug at the surface. A jet pipe with a nozzle is then lowered

in the casing tube, and water under pressure is forced through it. The dislodged soil particles and broken pieces form slurry with water and are lifted up through the annular space between the casing and jet pipe by the returning water in the upward direction. The casing pipe having shoes at the bottom is kept rotated slowly, and is thus lowered. In penetrating clays and hard pans, various types of jetting drill bits are fitted to the drill pipe which is raised and lowered sharply, causing the bit to shatter the formation. When the casing pipe has penetrated to a sufficient depth into the aquifer, the well pipe attached with screen etc. is lowered in the casing pipe. The outer casing is then pulled. Sometimes a *self jetting well point* is used. In this, the casing pipe is not used but, instead a tube of brass screen ending into a jetting nozzle to the main well pipe (Fig. 5.22). As the jetting action progresses, the well pipe goes on sinking. An annular space round the well pipe is automatically created due to the upward motion of water carrying the dislodged particles. When the well pipe has been sunk to the desired depth, jetting is stopped, and the annular space is packed with gravel.

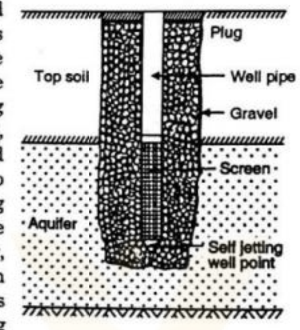

FIG. 5.22. SELF-JETTING WELL POINT.

2. Cable Tool Method (Percussion Method)

Cable tool method, also sometimes known as the percussion method or Standard method, is used for drilling deep wells through consolidated rock materials. In this method, a standard well drilling rig consists of a multiline hoist, a walking beam and an engine — all assembled and mounted on a truck for easy portability. A string of percussion tools consists of (Fig. 5.23) a rope socket, a set of jars, a drill stem and a drilling bit — the total weight of these amounting to several thousand kilograms.

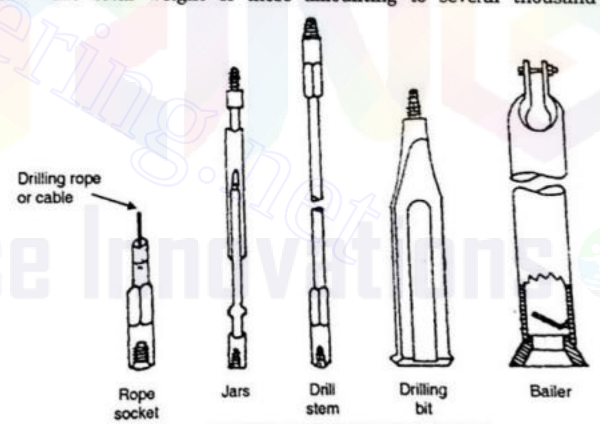

FIG. 5.23. PERCUSSION DRILLING TOOLS.

The drilling bits are manufactured in 1 to 3 meter lengths, and may weigh upto 1500 kg. A pit is dug at the site where the well is to be drilled. A casing pipe, with a drive shoe is inserted in the pit. The string of drilling tools is inserted in the first length of the casing pipe. Drilling is then accomplished by regular lifting and dropping of the string of tools mechanically. During drilling, the tools make 40 to 60 strokes per minute, ranging from 40 cm to 1 m in length. The Drilling line is kept continuously rotated so that the drilling bit will form a round hole. After the bit has cut 1 to $1\frac{1}{2}$ metre through the formation, the string of tools are taken out and a bailer (Fig. 5.23) is inserted in the hole to remove the drill cutting. The bailer essentially consists of a pipe like section with a valve at the bottom. When the bailer is inserted in the hole, the valve is automatically opened by the upward movement of the cuttings. The valve, however, prevents the cuttings from moving in the downward direction and thus escaping during lifting. When the bailer is full, it is lifted up to the surface and emptied. The length of the bailer varies with its diameter, and may range from 3 to 13 metres. When the cuttings have been taken out, the string of tools are again inserted and blows given to break the formation by impact. If no water is encountered in the hole, water is added from the surface to form a paste with the cuttings. The casing is driven down by motion of the tools striking the top of the casing, protected by a drive head, sinks the casing. The individual subsequent lengths of casing are joined by threaded or welded joints.

In soft and fissured rock formations, manual labour may be used (Fig. 5.24). In this case, the boring is done with the help of 'sludger' or 'sand pump'. The sludger, almost similar to the bailer, is a steel pipe 2 to 4 metres long, having a cutting shoe of hard steel riveted to its bottom. A flap valve at its lower end permits the entry of the cut material. The sludger is inserted in the casing pipe and is worked up and down by means of a rope the other end of which passes over a pulley fixed centrally to a tripod. A platform is attached to the upper end of the casing pipe, and the weight placed on the platform drives the casing pipe slowly into the hole. When the sludger is full with the cutting paste, it is taken out and emptied. The process goes on till the required level is reached. However. if the rock formations are hard, the sludger is unable to cut it and a string of drilling bits, shown in Fig. 5.23, is inserted and operated by an engine.

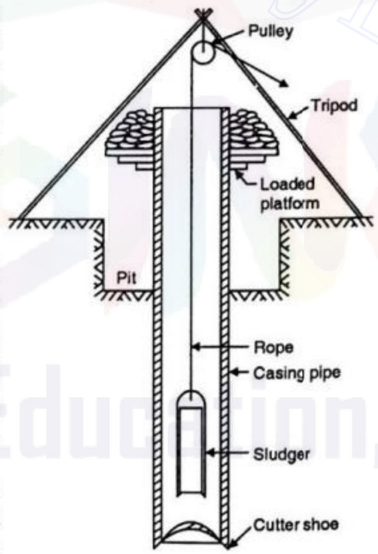

FIG. 5.24. PERCUSSION BORING BY MANUAL LABOUR.

In both the methods, the record of the material collected by bailer or sludger is kept. A bore log is then plotted to know the depths of various formations. The well pipe with strainers at determined levels of aquifer is then lowered to the desired depth. The well is then shrouded and the casing pipe is taken out when the shrouding is done in steps. Wells by cable tool method have been drilled in diameters up to 30 cm and to depths as great as 600 metres.

3. Hydraulic Rotary Method

Hydraulic rotary method, sometimes known as rotary boring method is used for drilling large bores in unconsolidated strata. This is fastest method and has been used for wells upto 45 cm diameter (upto 150 cm with a reamer), and for depth over 163 m. Oil wells over 7000 metre deep have been drilled by this method.

In this method, the boring is done with the help of a drilling bit attached at the end of a string of hollow pipe (Fig. 5.25). A mixture of clay and water, known as *drilling mud*, is continuously circulated through the drill shaft in the hole. Material loosened by the bit is carried upward in the hole by the rising mud. Ordinarily no casing is required since the drilling mud forms a clay lining and supports the walls of the hole.

The drill bits have hollow shanks and one or more centrally located orifices for jetting the mud into the bottom of the hole. The drill rod made of heavy pipe, carries the drill bit at one end and is screwed to a square section known as *kelly.* A rotating table, which fits closely the kelly, rotates the drill rod which slides downward as the hole deepens. The rising drilling mud carrying rock fragments is taken to the settling basin where the cuttings settle. The mud is recirculated to the hole. To maintain the required consistency, clay and water is added to the circulating mud from time to time. A complete boring record is maintained to know the type of formations at various depths. When desired level is reached, the drill rod, etc. are taken out and well pipe containing strainers pipes at appropriate locations (opposite aquifers) is lowered. Since the well walls are coated with clay, it should be washed to get more discharge. Back washing is done by lowering the drill pipe and bit in the well-pipe, and forcing water containing calgon (sodiumhexa- meta phosphate). Calgon has the property of dispersing clay colloids. A collar of the size of well pipe, is attached to the drill rod just above the bit. This forces the water through the strainer causing washing action on the clay wall. At the same time, the drill rod is plunged up and down causing surging action. When washing in the bottom is done, the bit is raised through some distance and the operation is repeated.

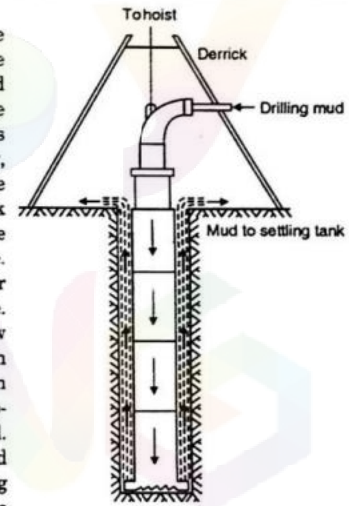

FIG. 5.25. HYDRAULIC ROTARY METHOD.

4. Reverse Rotary Method

Reverse rotary method, similar to the hydraulic rotary method, is very much used in Europe. In this method, the cutting are removed by a suction pipe. A large capacity centrifugal pump is used for this purpose. A mixture of water and fine grained material is circulated in the hole. The procedure is essentially a suction dredging method. The walls of the hole during drilling are supported by hydrostatic pressure acting against the film of fine-grained material deposited on the walls by the drilling water. The method of recirculation of drilling water containing fine grained particles and cleaning the well after inserting the well-pipe is similar to that of hydraulic rotary method.

5.15. WELL SHROUDING AND WELL DEVELOPMENT

(a) Well Shrouding

Well shrouding is a process of interposing coarse material such as gravel and coarse sand between the well-pipe (strainer pipe) and the aquifer soil to prevent finer particles of soil coming in contact with the strainer and chocking it. This is essential in sandy and unconsolidated formations of aquifer. This is also essential in slotted type tube well where a strainer is not used. Such a tube well is also sometimes known as a *gravel-packed well*. The shrouding increases the effective well diameter, acts as a strainer to keep fine material out of the well, and protects the well-pipe from caving of surrounding formations. A gravel packed well has a greater specific capacity than one of the same diameter not shrouded by gravel. A minimum thickness of 40 cm gravel pack is necessary to make it effective. The proper grain size distribution of the shrouding material depends upon the mechanical analysis of the aquifer and upon the perforation or screen slot size.

The amount of shrouding material per 30 cm length of the casing pipe can be calculated accurately before hand. In the beginning, the shrouding material for 60 cm lengths is shovelled in from the top betweeen the tube-pipe and casing. The casing is raised by 30 cm with the help of jack. Then the quantity for each 30 cm is added and the casing pipe is withdrawn 30 cm at a time till the strainer is covered. The shrouding material is sometimes placed through small pipes or pilot holes around the tube-pipe which feed the material down into position. A further refinement is the use of bladeless pumps for pumping the shrouding material in place.

(b) Well Development

Well development is the process of removing fine material from the aquifer formation surrounding the strainer pipe, and is aimed at (*i*) increasing the specific capacity of the well, (*ii*) preventing sand flowing in, and (*iii*) obtaining maximum economic well life. The actual yield of the well can be known only after well development. Thus, it also helps in determining the requiring characteristics of the pump and power unit to be installed. Depending upon formation characteristics of the aquifer, a well may be developed by one of the following methods :

1. Development by pumping.
2. Development by surging.
3. Development by compressed air.
4. Development by back washing.
5. Development by dry ice.

1. Development by Pumping

In this method a variable speed pump is used. The method is based on the principle that irregular and non-continuous pumping agitates the fine material surrounding the well so that it can be carried into the well and pumped out. Initially, the pump is started with a very low discharge. The fine particles start coming. This low speed is maintained till clear water comes. The discharge is then increased in steps until maximum discharge or well capacity is reached. The pump is then stopped and levels permitted to increase till it comes to normal. The pump is then again started and the procedure repeated, till no fine particles come.

2. Development by Surging

In this method, surging effect is created by up and down movement of a hollow surge block or a bailer. Calgon (sodium hexametaphosphate) is added to water so that it acts as dispersing agent for fine grained particles. When the surge block is moved up it sucks water in. When it is moved down, it forces water-calgon solution back in the formation. Further upward motion brings with it fine material. The surge block is connected to a string of hollow pipe from which the water charged with fine particles is pumped out continuously. The procedure is repeated by increasing the speed of surging till clear water comes out.

3. Development by Compressed Air

In this method, the development is done with the help of an air compressor, a discharge pipe and air pipe. The air pipe is put into the discharge pipe and is lowered into the well tube, till the assembly reaches near the bottom of the strainer-pipe section. The lower end of the air pipe is kept emerging out of the discharge pipe by a small length. The air entry to the air pipe is first closed and, the compressor is then started till a pressure of 6 to 10 kg/cm^2 is built up. The air is then suddenly made to enter the pipe, at this pressure, with the help of suitable quick-opening valve.This sudden entry of air into well creates a powerful surge within the well causing loosening of fine material surrounding the perforations. When ice pressure decreases, water enters the well bringing the loosened particles with it. The continuous air injection creates an air lift pump, and the water carrying fine particles is pumped out. The process is repeated till clear water comes. The pipe assembly is then lifted up, and the surging is again created. This operation is repeated at interval along the screen section till the well is fully developed.

4. Development by Back Washing

In this method, in addition to the compressor, a discharge pipe and an air pipe, and additional small air pipe is used. The well is sealed at its top so that it becomes air tight. The discharge pipe and air pipe assembly is lowered in the well, as in the previous method, but the end of the air pipe is kept inside the discharge pipe. A small air pipe is fitted at the top of the air-tight cover, and is provided with a three-way cock. With the help of the three way cock, air can be admitted to the well either through the long air pipe (but inside the discharge pipe) or through the small air pipe fitted at the top. Air is first made to enter the long air pipe. This forces air and water out of the well through the discharge pipe. When clear water comes, the

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valve is closed, and water level is allowed to increase in the well. The valve is then turned to the other-side so that air enters through the small air pipe . This *back washes* the water from the well through the discharge pipe and at the same time agitates the fine particles surrounding the well. Calgon is often added to water. When air starts escaping from the discharge pipe, the valve is turned so that air enters the long air pipe, so that assembly works at an air-lift pump and the water is pumped out. The procedure is repeated till clear water comes and the well is fully developed.

5 Development by Dry Ice (Solid Sodium Dioxide)

In this method, well is developed with the help of two chemicals : hydrochloric acid and solid sodium dioxide (know as dry ice). First of all hydrochloric acid is poured into the well. The well is capped at the top and compressed air is forced into the well. The pressure of the compressed air forces the chemical into the formation The cap is then removed and blocks of dry ice are dropped into the well. The sublimation releases gaseous carbon dioxide, and a high pressure of this gas is built up in the well. On releasing the pressure the muddy water is forced up in the form of a jet and is automatically thrown out of the well. Explosion of mud and water extending 40 metres into the air form a well in Utah (U.S.A.) was observed when the well was developed with dry ice.

5.16. OPEN WELL

As stated earlier, an open well is essentially of a bigger diameter than that of tube well, and derives its water only from one pervious startum. Since a tube well, in general may derive water from more than one aquifer formation, it has greater depth than an open well. The economically feasible depth of an open well is limited to 30 metres below the ground surface. In a lined open well, the entry of water is from the bottom and *not from* the sides.

An open well is classsified as :

(*i*) Shallow well. (*ii*) Deep well.

The nomenclature of shallow and deep well has nothing to do with the actual depth of the well. A *deep well* is a well which is supported on a *mota* layer and

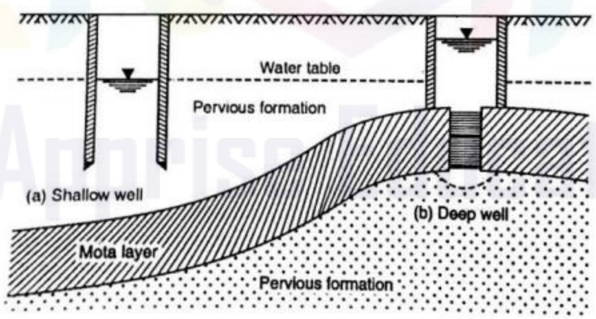


FIG. 5.26. SHALLOW AND DEEP WELLS.

draws its water supply through a hole bored in it from the pervious formation below the *mota* layer. A *shallow well*, on the other hand, penetrates the pervious startum only and draws its water supply through it. The term *mota* layer also sometimes known as *matbarwa* or *nagasan*, refers to a layer of clay, cemented sand, kankar or any other hard material. The *mota* layer gives structural support to the open well, and is found throughout the Indo-Gangetic plain. The mota layer may either be continuous or may be localised sand and may be found in different thicknesses and depths at different places.

Fig. 5.26(*a*) shows a shallow well which derives water from the pervious startum, and does not rest on a mota layer. Fig. 5.26(*b*) shows a deep well resting on a continuous mota layer. Fig. 5.27 shows a deep well resting on a localised mota layer and deriving its water from the second pervious stratum. Actually, a shallow well can be deeper than deep well. However, since a shallow well draws water from the first pervious stratum (*i.e.*, the top formation), the water in it is liable to be contaminated by rain water percolating in the vicinity, and may take with it mineral organic matter such as decomposing animals and plants. The water in a deep well is not liable to get such impurities and infection. Also, the pervious formation, below a mota layer, normally has greater water content and specific yield. Hence discharge from a deep well is generally more than a shallow well.

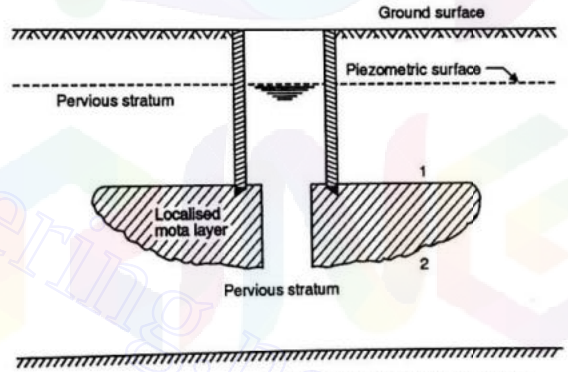


FIG. 5.27. DEEP WELL ON LOCALISED MOTA LAYER.

The open well may further be classified as :

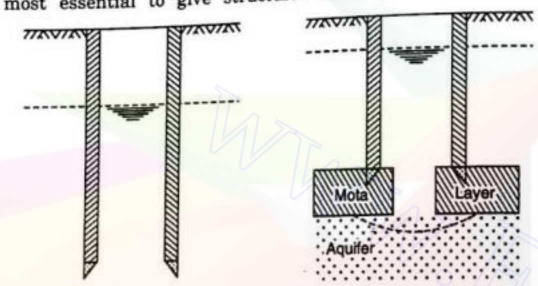
(*i*) Kachha well or unlined well.

(*ii*) Well with impervious lining.

(*iii*) Well with pervious lining.

Kachha Well . A kachha well is a temporary well of a very shallow depth. It is suitable only in hard formations the walls of which can stand vertically. They are suitable only when the water table is very near the ground surface. Such well often collapse after some time and are dangerous.

Well with Impervious or Pucca Lining. This is the most common type of open well, and is suitable for all types of formations. Once constructed, it becomes a permanent source of water supply. Impervious lining for an open well in sandy formations is most essential to give structural stability to the well.



(a) Shallow well with pucca lining (b) Deep well with pucca lining

FIG. 5.28. WELLS WITH THE PUCCA LINING

The thickness of impervious lining (steining) varies from 30 to 60 cm and may be either in brick masonry or in stone masonry. The linings carry well curbs under them. Well curbs may be constructed of either wood, iron or reinforced concrete. In a pucca well, the flow is not radial. Water enters only from the bottom and, after a virtual cavity has been formed at the bottom, the flow is spherical.

Well with Pervious Lining. Such types of wells are suitable in coarse formations. The lining consists of dry bricks or stones with no mortar or binding material. Due to this, water enters from the sides, and the flow is, therfore, radial. Such wells are generally plugged at the bottom. If there is no plug at the bottom the flow is a combination of radial and spherical pattern.

If a well with pervious lining is constructed in a sandy formation, brick ballast upto 20 mm size and gravel is placed behind the lining to prevent sand coming into the well along with the flowing water.

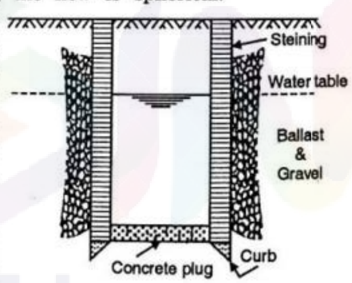


FIG. 5.29. WELL WITH PERVIOUS LINING.

5.17. YIELD OF AN OPEN WELL

The hydraulics of a lined open well is essentially different from that of a tube well. The yield from an open well can be found by the following two tests :

(1) Constant level pumping test. (2) Recuperation test.

(1) CONSTANT LEVEL PUMPING TEST

In this test, a pump with suitable regulating arrangement is used. The water level is depressed by an amount h (say) known as the *depression head*. The speed

of the pump is so adjusted that whatever water enters the well under this depression head is pumped out and a constant water level is maintained in the well. The amount of water pumped out is measured with the help of a V-notch or any other arrangement, in a given amount of time for which the pump speed was regulated to a constant value. The quantity pumped out in one hour gives the yield of the well per hour.

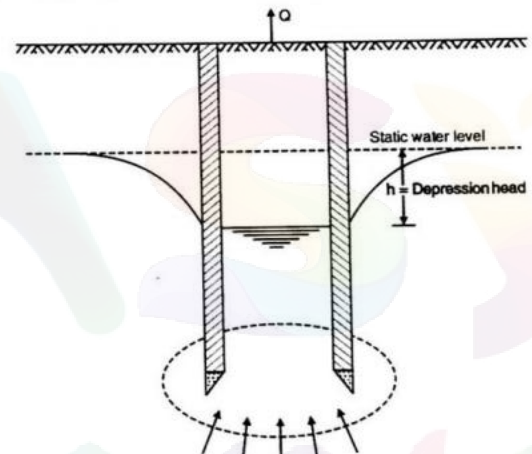


FIG. 5.30 CONSTANT LEVEL PUMPING TEST.

The formula for discharge, in cumecs from an open well with impervious lining may be written as :

$$Q = A \times v$$

or

$$Q = A \times C \times h \quad \text{cumecs} \qquad \ldots(5.20)$$

where Q = discharge, in cubic metres per second

A = cross-sectional area of flow into the well at its base, in m^2

v = mean velocity of water percolating into the well, in metres/sec

h = depression head in metres

C = percolation intensity coefficient. This is a constant of the formation around the well. Its value is greater for coarser soil and smaller for finer soils (m/sec under unit head).

The above formula can also be derived from Darcy's law as under :

$$Q = k.A.i = k.A.\frac{h}{L} = \frac{k}{L}.Ah$$

or

$$Q = C.Ah \qquad \ldots(5.20)$$

Due to cavity formation, the area A is taken to be equal to 4/3 times the actual cross-sectional area of the bottom of the well.

From the above expression, it is clear that the discharge increases with the percolation head h. However, the percolation head cannot be increased beyond a certain critical value because otherwise the percolation velocity will be exceeded and the soil particles will be disturbed and dislodged. The critical value of h at which the velocity is critical is known as the *critical depression head*. Normally, the depression head is kept equal to 1/3 of the critical head ; such a head is known as the *working head*.

Maximum yield or critical yield, therefore, will be obtained corresponding to the critical depression head. The yield under the working head is known as the *maximum safe yield*. From a pumping test, we can find the maximum safe yield.

(2) RECUPERATION TEST

Though the constant level pumping test gives an accurate value of the safe yield of an open well, it is sometimes very difficult to regulate the pump in such a way that constant level is maintained in the well . In such a circumstance, a recuperation test is resorted to. In the recuperation test, water level is depressed to any level below the normal and the pumping is stopped. The time taken for the water to recuperate to the normal level is noted. From the data, the discharge from the well can be calculated as under (Fig. 5.31).

Let aa = static water level in the well, before the pumping started.
bb = water level in the well when the pumping stopped
h_1 = depression head in the well when the pumping stopped (metres)
cc = water level in the well at a time T after the pumping stopped
h_2 = depression head in the well at the time T after the pumping stopped (metres)
h = depression head in the well at a time t after the pumping stopped (metres)
dh = decrease in depression head in a time dt
t, T = time in hours.

Thus, in a time t, reckoned from the instant of stopping the pump, the water level recuperates by $(h_1 - h)$ metres. In a time dt after this, the head recuperates by a value dh metres.

$\therefore$ Volume of water entering the well, when the head recuperates by dh is

$$dV = A\ dh \qquad \text{...(1)}$$

where A = cross-sectional area of well at its bottom

Again, if Q is the rate of discharge in the well at the time t, under the depression head h, the volume of water entering the well in time t hours is given by

$$dV = Q\,dt$$

But $\qquad Q \propto h$

or $\qquad Q = Kh \qquad \text{...(2)}$

$\therefore \qquad dV = K\,h\,dt \qquad \text{...(3)}$

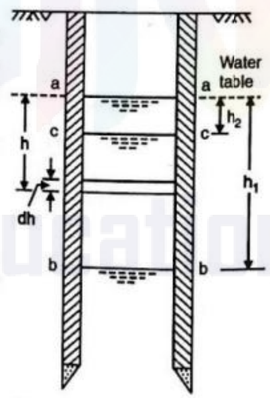


FIG. 5.31. RECUPERATION TEST.

where K is a constant depending upon the soil at the base of the well through which water enters.

Equating (1) and (3), we get

$$K\,h\,dt = -A\,dh \qquad \text{...(4)}$$

The minus sign indicates that h decreases as time t increases. Integrating the above between the limits : $t = 0$ when $h = h_1$; $t = T$ when $h = h_2$.

we get $$\frac{K}{A}\int_0^T dt = -\int_{h_1}^{h_2} \frac{dh}{h}$$

or $$\frac{K}{A}\int_0^T dt = -\int_{h_2}^{h_1} \frac{dh}{h}$$

From which $$\frac{K}{A}T = \Big[\log_e h\Big]_{h_2}^{h_1}$$

$$\therefore \qquad \frac{K}{A} = \frac{1}{T}\log_e\frac{h_1}{h_2} = \frac{2.303}{T}\log_{10}\frac{h_1}{h_2} \qquad \text{...(5.21)}$$

Thus knowing the value of h_1 , h_2 and T from recuperation test, the quantity K/A can be calculated. K/A is known as the *specific yield or specific capacity* of an open well, in cubic meter per hour per sq meter of the area through which water percolates under one metre depression head. In the absence of the recuperation test, the following rough values of K/A specified by Marriot can be adopted:

Type of soil	*K/A Cubic metres per hour, per sq metre of area under unit depression load*
Clay	0.25
Fine sand	0.5
Coarse sand	1.00

Knowing the value of K/A by observation, the discharge from a well under a constant depression head H can be calculated as under :

$$Q = K\,H \qquad \text{(from 2)}$$

or $$Q = \left(\frac{K}{A}\right)A.H. \qquad \text{...(5.22)}$$

or $$Q = \frac{2.303}{T}\left(\log_{10}\frac{h_1}{h_2}\right)AH \text{ m}^3\text{/hour} \qquad \text{...(5.23)}$$

It should be noted that the time T in above expression is in hours. If T is substituted in seconds, Q will be in cumecs. The discharge Q will be the maximum yield of the well if H corresponds to the maximum depression head. If H is the average depression head, Q well be the average yield.

5.18. METHODS OF LIFTING WATER

The various devices for lifting water can be divided into two heads :

(*a*) Indigenous methods.
(*b*) Mechanical methods.

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(a) **Indigenous Methods.** They include :

1. Basket
2. Doon
3. Archemedian screw
4. Denki
5. Rati or Girni
6. Windlass
7. Mote or Churus
8. Persian wheel.

(b) **Mechanical Methods.** They include:

9. Windmill
10. Hydraulic Ram
11. Pumps.

It is assumed that the reader is familiar with the methods (1) to (10).

Pumps. The discharge from a shallow open well is usually less, and hence hand operated pitcher pumps, wind mill operated plunger pumps, turbine pumps, gear pumps or centrifugal pumps may be used. Though atmospheric pressure is equal to 13.6 m (34 ft) head of water, dissociation takes place much earlier. Hence, the suction lift in these pumps should not exceed 7.62 m (25 ft). For deep wells and tube wells, where the lift is more than 8 m, large capacity pumps are used. The factors to be considered for the selection of pump for such wells are : diameter and depth of well, depth of water table, drawdown, seasonal variability of ground water table, duration of pumping, capacity, initial and maintenance costs, Power required and water quality. The following common types of pumps are suitable for deep well and tube well operation:

1. Deep well turbine pump.
2. Deep well jet pump.
3. Air lift pump.

1. Deep Well Turbine Pump

This is the most common type of pump used in tube wells. Deep well turbines are available for diameters varying from 10 cm to 35 cm, and their usual lengths vary from 20 to 60 cm. For each 5 to 8 m lift, small diameter impellers or bowls are used in series. The power impellers are of centrifugal type and the upper impellers are of mixed flow type. The driving shaft is enclosed in a pipe and the space between the two pipes serves as an outlet for water.

2. Deep Well Jet Pump

Jet pumps are used only for small discharges and are suitable for 10 to 20 cm diameter wells and for lifts between 6 to 50 metres. In a jet pump, a rotor pumps water into a draft tube which has a valve at the lower end. Water rises up in the tube due to suction created by the jet. Some quantity of water is again utilised by the jet.

3. Air lift pump

In this type, a compressor is used to force air into a small diameter pipe, called the *air pipe*. The air pipe is placed in a bigger diameter discharge pipe, sometimes known as the *education pipe*. When air is forced into water, air-water mixture is formed whose specific gravity is below that of water. Due to this, water rises in the education pipe and is discharged out. Due to this, water rise in arranging air and education pipes are shown in Fig. 5.32.

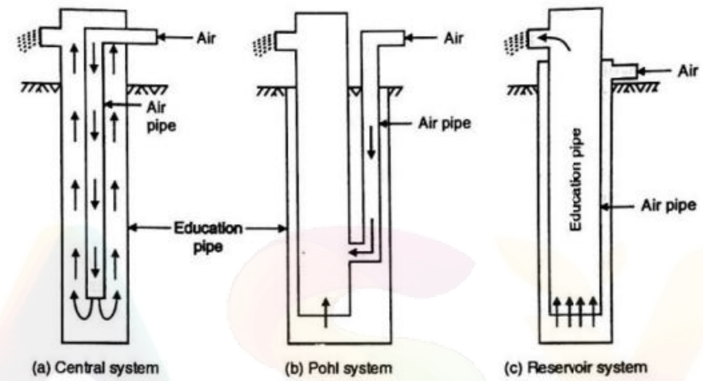

FIG. 5.32. SCHEMATIC DIAGRAMS OF AIR LIFT PUMP SYSTEMS.

5.19. ADVANTAGES AND DISADVANTAGES OF WELL IRRIGATION OVER CANAL IRRIGATION

Advantages

(1) The well is under the direct control of the owner. Hence wells may be sunk and equipped as required.

(2) Isolated area can be irrigated by a well. Wells may be sited to command any desired land.

(3) The supply from a well can be maintained fairly constant. Water can be turned off at any moment, taking advantage of the rainfall.

(4) Since a well is generally centrally located, the loss in transit is very much reduced. The duty of water in well irrigation is generally higher.

(5) Volumetric assessment is possible.

(6) Well irrigation is helpful in lowering the sub-soil water level and in draining off irrigated land which might become water logged. In canal irrigation, the chances of water logging in much more than in the well irrigation.

(7) Unless drought continues for several years, well irrigation does not fail in drought seasons, while a canal may fail.

(8) With the help of well irrigation, more than one crop in a year can be grown.

(9) The well water, which is warmer in cold weather, and cooler in hot weather, is more agreeable to crops.

(10) The cost of construction of a well is low and irrigation in a locality can be introduced in stages.

Disadvantages

(1) Since water has to be lifted from the well, the working expenses are very high in comparison to the canal irrigation.

(2) Due to mechanical defects in the pump or due to interruption in electric supply, water may not be made available to the crop in right time.

(3) The well water is clear and free from silt. Clear water does not have the manuring value which silt in canal water provides.

(4) The tube well strainer is subject to progressive determination due to mechanical and chemical action. Thus, replacements are necessary after frequent interval of time.

(5) The maintenance of mechanical and electrical machinery also requires great care and more funds.

5.20. SELECTION OF SUITABLE SITE FOR A TUBE WELL

The site where a tube well is to be located must be carefully selected. Improper or careless selection of the site have resulted in tube wells having very low yield or without any yield during summer season. Examples of such tube wells are present in Mahi area where some tube wells did not discharge any water during summers and had to be shifted.

The following points should be considered for the selection of the site :

(1) Tube well should be selected at a site where large underground reservoir exists. A judgement of such a place can be made on the following considerations:

(a) The place will be usually dark and green in summer also indicating a high water table.

(b) Certain types of worms and insects such as gnats hover in a column round the spot where there is likelihood of sub-soil water.

(c) By a study of geological strata by actual boring.

(2) If the geological explorations indicate ridges and depressions of impermeable strata inside the ground, the tube well should be located where there is the valley (Fig. 5.33).

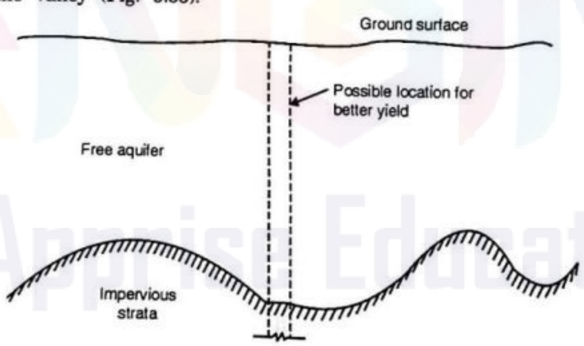

FIG. 5.33.

(3) The area should have an access for the availability of a cheap electric supply so that motor driven pumps can be fitted with tube wells without an exhorbitant cost. Diesel driven sets are not as efficient as those by electric power.

(4) The area should have a well distributed and uniform demand for irrigation throughout the year. If the demand is not uniform, tube well will be idling for some time or has to work inefficiently and thus the cost for overall irrigation will increase.

(5) The area around the tube well should have an intensive cultivation and the tube well be located centrally so as to reduce and the length of guls and thereby transient losses. In any case the tube well should have a command.

(6) The water available from the tube well should be tested to find out the irrigation quality of water. If the water is found to contain harmful salts, the site may have to be changed in spite of various advantages.

Section of a Tube Well. Fig. 5.34 shows the section of tube well with a centrifugal pump.

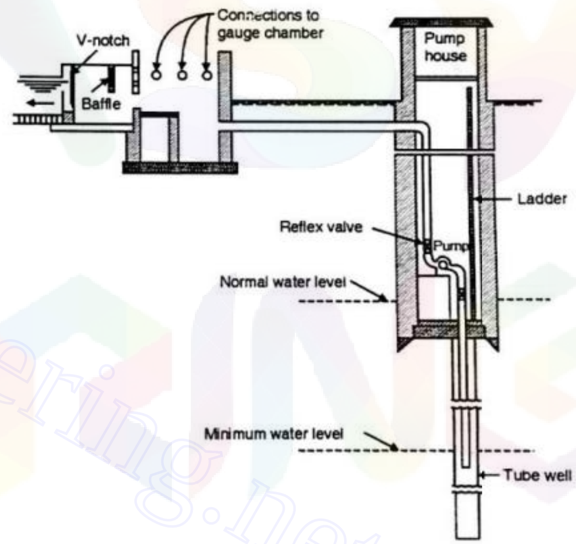

FIG. 5.34. SECTION OF A TUBE WELL.

ILLUSTRATIVE EXAMPLES

Example 5.1. *design an open well in fine sand to give a discharge of 0.003 cumec when worked under a depression head of 2.5 metres.*

Solution

Required discharge $Q = 0.003$ cumec $= 0.003 \times 3600 = 10.8\ m^3/\text{hour}$

From Eq. 5.22