34. Surface area $= \left(\frac{34398}{13}\right) = 2646 \text{ cm}^2$.

$\therefore \quad 6a^2 = 2646 \Rightarrow a^2 = 441 \Rightarrow a = 21.$

So, Volume $= (21 \times 21 \times 21) \text{ cm}^3 = 9261 \text{ cm}^3$.

35. $a^3 = 729 \Rightarrow a = 9.$

$\therefore$ Surface area $= (6 \times 9 \times 9) \text{ cm}^2 = 486 \text{ cm}^2$.

36. Required length = Diagonal $= \sqrt{3}\,a = (\sqrt{3} \times \sqrt{3}) \text{ m} = 3 \text{ m}$.

37. $\sqrt{3}\,a = 4\sqrt{3} \Rightarrow a = 4.$

$\therefore$ Volume $= (4 \times 4 \times 4) \text{ cm}^3 = 64 \text{ cm}^3$.

38. $6a^2 = 600 \Rightarrow a^2 = 100 \Rightarrow a = 10.$

$\therefore$ Diagonal $= \sqrt{3}\,a = 10\sqrt{3}$ cm.

39. $a^3 = 6a^2 \Rightarrow a = 6.$

40. Number of cubes $= \left(\frac{100 \times 100 \times 100}{10 \times 10 \times 10}\right) = 1000.$

41. Number of blocks $= \left(\frac{160 \times 100 \times 60}{20 \times 20 \times 20}\right) = 120.$

42. Number of cubes $= \left(\frac{18 \times 18 \times 18}{3 \times 3 \times 3}\right) = 216.$

43. Volume of block $= (6 \times 9 \times 12) \text{ cm}^3 = 648 \text{ cm}^3$.

Side of largest cube = H.C.F. of 6 cm, 9 cm, 12 cm = 3 cm.

Volume of this cube $= (3 \times 3 \times 3) = 27 \text{ cm}^3$.

$\therefore$ Number of cubes $= \left(\frac{648}{27}\right) = 24.$

44. Side of smallest cube = L.C.M. of 5 cm, 10 cm, 20 cm = 20 cm.

Volume of the cube $= (20 \times 20 \times 20) \text{ cm}^3 = 8000 \text{ cm}^3$.

Volume of the block $= (5 \times 10 \times 20) \text{ cm}^3 = 1000 \text{ cm}^3$.

$\therefore$ Number of blocks $= \left(\frac{8000}{1000}\right) = 8.$

45. Let the sides of the sheet be x and $5x$. Then,

Volume of the sheet = Volume of the cube

$\Rightarrow \quad x \times 5x \times \frac{1}{2} = 10 \times 10 \times 10 \Rightarrow 5x^2 = 2000 \Rightarrow x^2 = 400 \Rightarrow x = 20.$

$\therefore$ The sides are 20 cm and 100 cm.

46. Volume of the new cube $= (6^3 + 8^3 + 10^3) \text{ cm}^3 = 1728 \text{ cm}^3$.

Let the edge of the new cube be a cm.

$\therefore \quad a^3 = 1728 \Rightarrow a = 12.$

47. The new solid formed is a cuboid of length 25 cm, breadth 5 cm and height 5 cm.

$\therefore$ Volume $= (25 \times 5 \times 5) \text{ cm}^3 = 625 \text{ cm}^3$.

48. Required ratio $= \frac{6 \times 1 \times 1}{6 \times 5 \times 5} = \frac{1}{25} = 1:25.$

49. Volume of the large cube $= (3^3 + 4^3 + 5^3) \text{ cm}^3 = 216 \text{ cm}^3$.

Let the edge of the large cube be a.

So, $a^3 = 216 \Rightarrow a = 6$ cm.

$\therefore$ Required ratio $= \frac{6 \times (3^2 + 4^2 + 5^2)}{6 \times 6^2} = \frac{50}{36} = 25:18.$

50. Let the sides of the three cubes be $3x$, $4x$ and $5x$.
Then, Volume of the new cube = $[(3x)^3 + (4x)^3 + (5x)^3] = 216x^3$.
Edge of the new cube = $(216x^3)^{1/3} = 6x$.
Diagonal of the new cube = $6\sqrt{3}\,x$.
$\therefore \quad 6\sqrt{3}\,x = 12\sqrt{3} \Rightarrow x = 2$.
So, the sides of the cubes are 6 cm, 8 cm and 10 cm.

51. Let their edges be a and b. Then,

$$\frac{a^3}{b^3} = \frac{27}{1} \Leftrightarrow \left(\frac{a}{b}\right)^3 = \left(\frac{3}{1}\right)^3 \Leftrightarrow \frac{a}{b} = \frac{3}{1} \Leftrightarrow a : b = 3 : 1.$$

52. Let their edges be a and b. Then,

$$\frac{a^3}{b^3} = \frac{8}{27} \Leftrightarrow \left(\frac{a}{b}\right)^3 = \left(\frac{2}{3}\right)^3 \Leftrightarrow \frac{a}{b} = \frac{2}{3} \Leftrightarrow \frac{a^2}{b^2} = \frac{4}{9} \Leftrightarrow \frac{6a^2}{6b^2} = \frac{4}{9}.$$

53. Let their edges be a and b. Then,

$$\frac{a^3}{b^3} = \frac{1}{27} \Leftrightarrow \left(\frac{a}{b}\right)^3 = \left(\frac{1}{3}\right)^3 \Leftrightarrow \frac{a}{b} = \frac{1}{3} \Leftrightarrow \frac{a^2}{b^2} = \frac{1}{9}.$$

54. Let original edge = a. Then, volume = a^3.
New edge = $2a$. So, new volume = $(2a)^3 = 8a^3$.
$\therefore$ Volume becomes 8 times.

55. Let original edge = a. Then, surface area = $6a^2$.

New edge $= \frac{125}{100}a = \frac{5a}{4}$.

New surface area $= 6 \times \left(\frac{5a}{4}\right)^2 = \frac{75a^2}{8}$.

Increase in surface area $= \left(\frac{75a^2}{8} - 6a^2\right) = \frac{27a^2}{8}$.

$\therefore$ Increase % $= \left(\frac{27a^2}{8} \times \frac{1}{6a^2} \times 100\right)\% = 56.25\%$.

56. Volume $= \pi r^2 h = \left(\frac{22}{7} \times 1 \times 1 \times 14\right) \text{m}^3 = 44 \text{ m}^3$.

57. Volume of the tank = 246.4 litres = 246400 cm^3.
Let the radius of the base be r cm. Then,

$$\left(\frac{22}{7} \times r^2 \times 400\right) = 246400 \Leftrightarrow r^2 = \left(\frac{246400 \times 7}{22 \times 400}\right) = 196 \Leftrightarrow r = 14.$$

$\therefore$ Diameter of the base = $2r$ = 28 cm.

58. $2\pi r = 66 \Rightarrow r = \left(66 \times \frac{1}{2} \times \frac{7}{22}\right) = \frac{21}{2}$ cm.

$\frac{2\pi rh}{2\pi r} = \left(\frac{2640}{66}\right) \Rightarrow h = 40$ cm.

$\therefore$ Volume $= \left(\frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times 40\right) \text{cm}^3 = 13860 \text{ cm}^3$.

59. Let the radius and height be r cm each.

Then, $\frac{22}{7} \times r^2 \times r = \frac{176}{7} \Rightarrow r^3 = \left(\frac{176}{7} \times \frac{7}{22}\right) = 8 \Rightarrow r = 2.$

60. $\frac{2\pi rh}{h} = \frac{704}{14} \Rightarrow 2\pi r = \frac{704}{14}.$

$\therefore \quad r = \left(\frac{704}{14} \times \frac{1}{2} \times \frac{7}{22}\right) = 8$ cm.

$\therefore$ Volume $= \left(\frac{22}{7} \times 8 \times 8 \times 14\right) \text{cm}^3 = 2816 \text{ cm}^3.$

61. Total surface area $= 2\pi r(h + r) = \left[2 \times \frac{22}{7} \times \frac{35}{100} \times (1.25 + 0.35)\right] \text{m}^2$

$= \left(2 \times \frac{22}{7} \times \frac{35}{100} \times \frac{16}{10}\right) \text{m}^2 = 3.52 \text{ m}^2.$

$\therefore$ Cost of the material = Rs. (3.52 × 80) = Rs. 281.60.

62. Curved surface area $= 2\pi rh = (\pi r^2 h) \cdot \frac{2}{r} = \left(\text{Volume} \times \frac{2}{r}\right).$

63. $\frac{\text{Total surface area}}{\text{Lateral surface area}} = \frac{2\pi rh + 2\pi r^2}{2\pi rh} = \frac{(h + r)}{h} = \frac{80}{60} = \frac{4}{3}.$

64. Difference in capacities $= \left(8 \times 8 \times 14 - \frac{22}{7} \times 4 \times 4 \times 14\right) \text{cm}^3 = 192 \text{ cm}^3.$

65. Let radius = $2x$ and height = $3x$. Then,

$\frac{22}{7} \times (2x)^2 \times 3x = 12936 \Leftrightarrow x^3 = \left(12936 \times \frac{7}{22} \times \frac{1}{12}\right) = 343 = 7^3$

$\therefore$ $x = 7$. So, radius = 14 cm and height = 21 cm.

$\therefore$ Total surface area $= 2 \times \frac{22}{7} \times 14 \times (21 + 14) = \left(2 \times \frac{22}{7} \times 14 \times 35\right) \text{cm}^2 = 3080 \text{ cm}^2.$

66. It is given that $r = \frac{1}{2}h$ and $2\pi rh + \pi r^2 = 616 \text{ m}^2$

$\therefore \quad 2\pi \times \frac{1}{2}h \times h + \pi \times \frac{1}{4}h^2 = 616$

$\Rightarrow \quad \frac{5}{4} \times \frac{22}{7} \times h^2 = 616 \Rightarrow h^2 = \left(616 \times \frac{28}{110}\right) = \frac{28 \times 28}{5}.$

$\therefore$ Volume $= \pi r^2 h = \frac{22}{7} \times \frac{1}{4}h^2 \times h = \frac{22}{7} \times \frac{1}{4} \times \frac{28 \times 28}{5} \times \frac{28}{\sqrt{5}} \text{ cm}^3$

$= \left(\frac{22 \times 28 \times 28}{25} \times \sqrt{5}\right) \text{cm}^3 = \left(\frac{22 \times 28 \times 28 \times 2.23}{25 \times 1000}\right)$ litres = 1.53 litre.

67. $(h + r) = 37$ and $2\pi r(h + r) = 1628.$

$\therefore \quad 2\pi r \times 37 = 1628$ or $r = \left(\frac{1628}{2 \times 37} \times \frac{7}{22}\right) = 7.$

So, $r = 7$ m and $h = 30$ m.

$\therefore$ Volume $= \left(\frac{22}{7} \times 7 \times 7 \times 30\right) \text{m}^3 = 4620 \text{ m}^3.$

68. $\frac{\pi r^2 h}{2\pi r h} = \frac{924}{264} \Rightarrow r = \left(\frac{924}{264} \times 2\right) = 7$ m.

And, $2\pi r h = 264 \Rightarrow h = \left(264 \times \frac{7}{22} \times \frac{1}{2} \times \frac{1}{7}\right) = 6$ m.

$\therefore$ Required ratio $= \frac{2r}{h} = \frac{14}{6} = 7:3$.

69. $V = \pi r^2 h$ and $S = 2\pi r h + 2\pi r^2$

$\Rightarrow S = 2\pi r(h + r)$, where $h = \frac{V}{\pi r^2}$

$\Rightarrow S = 2\pi r\left(\frac{V}{\pi r^2} + r\right) = \frac{2V}{r} + 2\pi r^2 \Rightarrow \frac{dS}{dr} = \frac{-2V}{r^2} + 4\pi r$ and $\frac{d^2S}{dr^2} = \left(\frac{4V}{r^3} + 4\pi\right) > 0$

$\therefore$ S is minimum when $\frac{dS}{dr} = 0$

$\Leftrightarrow \frac{-2V}{r^2} + 4\pi r = 0 \Leftrightarrow V = 2\pi r^3 \Leftrightarrow \pi r^2 h = 2\pi r^3 \Leftrightarrow h = 2r$.

70. Let original radius = R. Then, new radius $= \frac{R}{2}$.

$$\frac{\text{Volume of reduced cylinder}}{\text{Volume of original cylinder}} = \frac{\pi \times \left(\frac{R}{2}\right)^2 \times h}{\pi \times R^2 \times h} = \frac{1}{4}.$$

71. Let their radii be $2x$, $3x$ and heights be $5y$, $3y$.

Ratio of their volumes $= \frac{\pi \times (2x)^2 \times 5y}{\pi \times (3x)^2 \times 3y} = \frac{20}{27}$.

72. Let their heights be h and $2h$ and radii be r and R respectively. Then,

$\pi r^2 h = \pi R^2 (2h) \Rightarrow \frac{r^2}{R^2} = \frac{2h}{h} = \frac{2}{1} \Rightarrow \frac{r}{R} = \frac{\sqrt{2}}{1}$ *i.e.* $\sqrt{2}:1$.

73. Let the height of X and Y be h, and their radii be r and $2r$ respectively. Then,
Volume of X $= \pi r^2 h$ and Volume of Y $= \pi (2r)^2 h = 4\pi r^2 h$.
New height of X $= 2h$.
So, new volume of X $= \pi r^2 (2h) = 2\pi r^2 h = \frac{1}{2}(4\pi r^2 h) = \frac{1}{2} \times$ (Volume of Y).

74. Let original radius $= r$ and original length $= h$.

New radius $= \frac{r}{3}$ and let new length = H.

Then, $\pi r^2 h = \pi\left(\frac{r}{3}\right)^2 \times H$ or $H = 9h$.

75. Let the drop in the water level be h cm. Then,

$\frac{22}{7} \times \frac{35}{2} \times \frac{35}{2} \times h = 11000 \Leftrightarrow h = \left(\frac{11000 \times 7 \times 4}{22 \times 35 \times 35}\right)$ cm $= \frac{80}{7}$ cm $= 11\frac{3}{7}$ cm.

76. Volume of earth dug out = $\left(\frac{22}{7} \times 7 \times 7 \times 10\right) m^3 = 1540\ m^3$.

Area of embankment = $\frac{22}{7} \times \left[(28)^2 - (7)^2\right] = \left(\frac{22}{7} \times 35 \times 21\right) m^2 = 2310\ m^2$.

Height of embankment = $\left(\frac{\text{Volume}}{\text{Area}}\right) = \left(\frac{1540}{2310}\right) m = \frac{2}{3}$ m.

77. Volume of water flown in 1 sec. = $\left(\frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 200\right) cm^3 = 7700\ cm^3$.

Volume of water flown in 10 min. = $(7700 \times 60 \times 10)\ cm^3$

$= \left(\frac{7700 \times 60 \times 10}{1000}\right)$ litres = 4620 litres.

78. Volume of one coin = $\left(\frac{22}{7} \times \frac{75}{100} \times \frac{75}{100} \times \frac{2}{10}\right) cm^3 = \frac{99}{280}\ cm^3$.

Volume of larger cylinder = $\left(\frac{22}{7} \times 3 \times 3 \times 8\right) cm^3$.

$\therefore$ Number of coins = $\left(\frac{22 \times 9 \times 8}{7} \times \frac{280}{99}\right) = 640$.

79. Let the radius of the vessel be R. Then,

$\pi R^2 \times 15 = \pi \times (15)^2 \times 35 + \pi \times (10)^2 \times 15$

$\Leftrightarrow \pi R^2 \times 15 = 9375\pi \Leftrightarrow R^2 = 625 \Leftrightarrow R = 25$ cm.

80. Let the length of the wire be h.

Radius = $\frac{1}{2}$ mm = $\frac{1}{20}$ cm. Then,

$\frac{22}{7} \times \frac{1}{20} \times \frac{1}{20} \times h = 66 \Leftrightarrow h = \left(\frac{66 \times 20 \times 20 \times 7}{22}\right) = 8400$ cm = 84 m.

81. Circumference of the girth = 440 cm.

$\therefore\ 2\pi R = 440 \Rightarrow R = \left(440 \times \frac{1}{2} \times \frac{7}{22}\right) = 70$ cm.

So, Outer radius = 70 cm. Inner radius = (70 − 4) cm = 66 cm.

Volume of iron = $\pi\ [(70)^2 - (66)^2] \times 63 = \left(\frac{22}{7} \times 136 \times 4 \times 63\right) cm^3 = 58752\ cm^3$.

82. Internal radius = $\left(\frac{11.2}{2}\right)$ cm = 5.6 cm, External radius = (5.6 + 0.4) cm = 6 cm.

Volume of metal = $\left\{\frac{22}{7} \times [(6)^2 - (5.6)^2] \times 21\right\} cm^3 = (66 \times 11.6 \times 0.4)\ cm^3 = 306.24\ cm^3$.

83. External radius = 6 cm, Internal radius = (6 − 0.25) cm = 5.75 cm.

Volume of material in hollow cylinder

$= \left\{\frac{22}{7} \times [(6)^2 - (5.75)^2] \times 15\right\} cm^3 = \left(\frac{22}{7} \times 11.75 \times 0.25 \times 15\right) cm^3$

$= \left(\frac{22}{7} \times \frac{1175}{100} \times \frac{25}{100} \times 15\right) cm^3 = \left(\frac{11 \times 705}{56}\right) cm^3$.

Let the length of solid cylinder be h. Then,

$\frac{22}{7} \times 1 \times 1 \times h = \left(\frac{11 \times 705}{56}\right) \Leftrightarrow h = \left(\frac{11 \times 705}{56} \times \frac{7}{22}\right)$ cm = 44.0625 cm.

84. External radius = 4 cm, Internal radius = 3 cm.

Volume of iron = $\left\{\frac{22}{7} \times [(4)^2 - (3)^2] \times 21\right\}$ cm^3 = $\left(\frac{22}{7} \times 7 \times 1 \times 21\right)$ cm^3 = 462 cm^3.

$\therefore$ Weight of iron = (462 × 8) gm = 3696 gm = 3.696 kg.

85. Let the internal radius of the cylinder be x. Then,

$$\frac{22}{7} \times r^2 \times 40 = \frac{616}{10} \Leftrightarrow r^2 = \left(\frac{616 \times 7}{10 \times 22 \times 40}\right) = 0.49 \Leftrightarrow r = 0.7.$$

So, internal radius = 0.7 cm = 7 mm.

$\therefore$ Thickness = (8 – 7) mm = 1 mm.

86. $$\frac{\text{Volume of cone}}{\text{Volume of cylinder}} = \frac{\frac{1}{3} \times \pi \times (3)^2 \times 5}{\pi \times (2)^2 \times 4} = \frac{45}{48} = \frac{15}{16}.$$

87. h = 15 cm, r = 8 cm. So, $l = \sqrt{r^2 + h^2} = \sqrt{8^2 + (15)^2} = 17$ cm.

$\therefore$ Curved surface area = πrl = (π × 8 × 17) cm^2 = 136π cm^2.

88. h = 14 cm, r = 7 cm. So, $l = \sqrt{(7)^2 + (14)^2} = \sqrt{245} = 7\sqrt{5}$ cm.

$\therefore$ Total surface area = $\pi rl + \pi r^2 = \left(\frac{22}{7} \times 7 \times 7\sqrt{5} + \frac{22}{7} \times 7 \times 7\right)$ cm^2

= [154 ($\sqrt{5}$ + 1)] cm^2 = (154 × 3.236) cm^2 = 498.35 cm^2.

89. Clearly, we have r = 3 cm and h = 4 cm.

$\therefore$ Volume = $\frac{1}{3}\pi r^2 h = \left(\frac{1}{3} \times \pi \times 3^2 \times 4\right)$ cm^3 = 12π cm^3.

90. l = 10 m, h = 8 m. So, $r = \sqrt{l^2 - h^2} = \sqrt{(10)^2 - 8^2} = 6$ m.

$\therefore$ Curved surface area = πrl = (π × 6 × 10) m^2 = 60π m^2.

91. $$\frac{1}{3} \times \frac{22}{7} \times r^2 \times 24 = 1232 \Leftrightarrow r^2 = \left(\frac{1232 \times 7 \times 3}{22 \times 24}\right) = 49 \Leftrightarrow r = 7.$$

Now, r = 7 cm, h = 24 cm. So, $l = \sqrt{(7)^2 + (24)^2} = 25$ cm.

$\therefore$ Curved surface area = $\left(\frac{22}{7} \times 7 \times 25\right)$ cm^2 = 550 cm^2.

92. Let the radius of the base be r km. Then,

$$\pi r^2 = 1.54 \Rightarrow r^2 = \left(\frac{1.54 \times 7}{22}\right) = 0.49 \Rightarrow r = 0.7 \text{ km}.$$

Now, l = 2.5 km, r = 0.7 km.

$\therefore$ $h = \sqrt{(2.5)^2 - (0.7)^2}$ km = $\sqrt{6.25 - 0.49}$ km = $\sqrt{5.76}$ km = 2.4 km.

So, height of the mountain = 2.4 km.

93. $$\pi r^2 = 3850 \Rightarrow r^2 = \left(\frac{3850 \times 7}{22}\right) = 1225 \Rightarrow r = 35.$$

Now, r = 35 cm, h = 84 cm.

So, $l = \sqrt{(35)^2 + (84)^2} = \sqrt{1225 + 7056} = \sqrt{8281} = 91$ cm.

$\therefore$ Curved surface area = $\left(\frac{22}{7} \times 35 \times 91\right)$ cm^2 = 10010 cm^2.

94. $\frac{22}{7} \times 70 \times l = 40040 \Rightarrow l = \left(\frac{40040 \times 7}{22 \times 70}\right) = 182.$

Now, $l = 182$ cm, $r = 70$ cm.

So, $h = \sqrt{(182)^2 - (70)^2} = \sqrt{252 \times 112} = 168$ cm.

$\therefore$ Volume $= \left(\frac{1}{3} \times \frac{22}{7} \times 70 \times 70 \times 168\right) \text{cm}^3 = 862400 \text{ cm}^3.$

95. Let the radius and the height of the cone be $3x$ and $4x$ respectively. Then,

$\frac{1}{3} \times \pi \times (3x)^2 \times 4x = 96\pi \Leftrightarrow 36x^3 = (96 \times 3) \Leftrightarrow x^3 = \left(\frac{96 \times 3}{36}\right) = 8 \Leftrightarrow x = 2.$

$\therefore$ Radius = 6 cm, Height = 8 cm.

Slant height $= \sqrt{6^2 + 8^2}$ cm $= \sqrt{100}$ cm $= 10$ cm.

96. $\pi r^2 = 346.5 \Rightarrow r^2 = \left(346.5 \times \frac{7}{22}\right) = \frac{441}{4} \Rightarrow r = \frac{21}{2}.$

$\therefore l = \sqrt{r^2 + h^2} = \sqrt{\frac{441}{4} + (14)^2} = \sqrt{\frac{1225}{4}} = \frac{35}{2}.$

So, area of canvas needed $= \pi r l = \left(\frac{22}{7} \times \frac{21}{2} \times \frac{35}{2}\right) \text{m}^2 = \left(\frac{33 \times 35}{2}\right) \text{m}^2.$

$\therefore$ Length of canvas $= \left(\frac{33 \times 35}{2 \times 1.1}\right)$ m $= 525$ m.

97. Let the original radius and height of the cone be r and h respectively.

Then, new radius $= 2r$. New height $= 2h$.

$\therefore \frac{\text{New Volume}}{\text{Original Volume}} = \frac{\frac{1}{3} \times \pi \times (2r)^2 \times 2h}{\frac{1}{3} \times \pi \times r^2 \times h} = \frac{8}{1}.$

98. Let the original radius and height of the cone be r and h respectively.

Then, Original volume $= \frac{1}{3} \pi r^2 h.$

New radius $= \frac{120}{100} r = \frac{6}{5} r$, New height $= \frac{6}{5} h.$

New volume $= \frac{1}{3} \pi \times \left(\frac{6}{5} r\right)^2 \times \left(\frac{6}{5} h\right) = \frac{216}{125} \times \frac{1}{3} \pi r^2 h.$

Increase in volume $= \frac{91}{125} \times \frac{1}{3} \pi r^2 h.$

$\therefore$ Increase % $= \left(\frac{\frac{91}{125} \times \frac{1}{3} \pi r^2 h}{\frac{1}{3} \pi r^2 h} \times 100\right)\% = 72.8\%.$

99. Let the original radius and height of the cone be r and h respectively.

Then, original volume $= \frac{1}{3} \pi r^2 h.$

New radius $= \frac{r}{2}$ and new height $= 3h.$

New volume $= \frac{1}{3} \times \pi \times \left(\frac{r}{2}\right)^2 \times 3h = \frac{3}{4} \times \frac{1}{3} \pi r^2 h.$

$\therefore$ Decrease % $= \left(\frac{\frac{1}{4} \times \frac{1}{3} \pi r^2 h}{\frac{1}{3} \pi r^2 h} \times 100\right)\% = 25\%.$

100. Required ratio $= \frac{\frac{1}{3} \pi r^2 h}{\frac{1}{3} \pi r^2 \times (2h)} = \frac{1}{2}.$

101. Let their heights be x, $3x$ and their radii be $3y$, y.

Then, Ratio of volumes $= \frac{\frac{1}{3} \times \pi \times (3y)^2 \times x}{\frac{1}{3} \times \pi \times y^2 \times (3x)} = \frac{9}{3} = 3 : 1.$

102. Let their radii be $2x$, x and their heights be h and H respectively. Then,

$\frac{1}{3} \times \pi \times (2x)^2 \times h = \frac{1}{3} \times \pi \times x^2 \times \text{H}$ or $\frac{h}{\text{H}} = \frac{1}{4}.$

103. Let their radii be $4x$ and $5x$, and their heights be h and H respectively. Then,

$\frac{\frac{1}{3} \times \pi \times (4x)^2 \times h}{\frac{1}{3} \times \pi \times (5x)^2 \times \text{H}} = \frac{1}{4}$ or $\frac{h}{\text{H}} = \frac{1}{4} \times \frac{25}{16} = \frac{25}{64}.$

104. Volume of the largest cone

= Volume of the cone with diameter of base 7 cm and height 7 cm

$= \left(\frac{1}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times 7\right) \text{cm}^3 = \left(\frac{269.5}{3}\right) \text{cm}^3 = 89.8 \text{ cm}^3.$

105. Volume of the block $= (10 \times 5 \times 2) \text{ cm}^3 = 100 \text{ cm}^3.$

Volume of the cone carved out $= \left(\frac{1}{3} \times \frac{22}{7} \times 3 \times 3 \times 7\right) \text{cm}^3 = 66 \text{ cm}^3.$

$\therefore$ Wood wasted $= (100 - 66)\% = 34\%.$

106. Let their radius and height be $5x$ and $12x$ respectively.

Slant height of the cone, $l = \sqrt{(5x)^2 + (12x)^2} = 13x.$

$\frac{\text{Total surface area of cylinder}}{\text{Total surface area of cone}} = \frac{2\pi r\,(h + r)}{\pi r\,(l + r)} = \frac{2\,(h + r)}{(l + r)} = \frac{2 \times (12x + 5x)}{(13x + 5x)} = \frac{34x}{18x} = \frac{17}{9}.$

107. Let the radius of the cone be r cm.

Then, $\frac{1}{3} \pi \times r^2 \times 6 = \pi \times 8 \times 8 \times 2 \Leftrightarrow r^2 = \left(\frac{8 \times 8 \times 2 \times 3}{6}\right) = 64 \Leftrightarrow r = 8 \text{ cm}.$

108. Let radius of each be r and height of each be h.

Then, number of cones needed $= \frac{\text{Volume of cylinder}}{\text{Volume of 1 cone}} = \frac{\pi r^2 h}{\frac{1}{3} \pi r^2 h} = 3.$

109. Volume of cylinder $= (\pi \times 3 \times 3 \times 5) \text{ cm}^3 = 45\pi \text{ cm}^3.$

Volume of 1 cone $= \left(\frac{1}{3} \pi \times \frac{1}{10} \times \frac{1}{10} \times 1\right) \text{cm}^3 = \frac{\pi}{300} \text{ cm}^3.$

$\therefore$ Number of cones $= \left(45\pi \times \frac{300}{\pi}\right) = 13500.$

110. Volume flown in conical vessel $= \frac{1}{3}\pi \times (20)^2 \times 24 = 3200\pi$.

Volume flown in 1 min. $= \left(\pi \times \frac{2.5}{10} \times \frac{2.5}{10} \times 1000\right) = 62.5\pi$.

$\therefore$ Time taken $= \left(\frac{3200\pi}{62.5\pi}\right) = 51$ min. 12 sec.

111. Slant height of the cone, $l = \sqrt{(12)^2 + (5)^2} = 13$ cm.

Lateral surface of the solid = Curved surface of cone + Curved surface of cylinder + Surface area of bottom

$= \pi rl + 2\pi rh + \pi r^2$, where h is the height of the cylinder

$= \pi r\,(l + h + r) = \left[\frac{22}{7} \times 12 \times (13 + 18 + 12)\right] \text{cm}^2$

$= \left(\frac{22}{7} \times 12 \times 43\right) \text{cm}^2 = \left(\frac{11352}{7}\right) \text{cm}^2 = 1621\frac{5}{7} \text{cm}^2$.

112. Volume of parallelopiped $= (5 \times 3 \times 4)\ \text{cm}^3 = 60\ \text{cm}^3$.

Volume of cube $= (4)^3\ \text{cm}^3 = 64\ \text{cm}^3$.

Volume of cylinder $= \left(\frac{22}{7} \times 3 \times 3 \times 3\right) \text{cm}^3 = 84.86\ \text{cm}^3$.

Volume of sphere $= \left(\frac{4}{3} \times \frac{22}{7} \times 3 \times 3 \times 3\right) = 113.14\ \text{cm}^3$.

113. $\frac{4}{3} \times \frac{22}{7} \times R^3 = 4851 \Rightarrow R^3 = \left(4851 \times \frac{3}{4} \times \frac{7}{22}\right) = \left(\frac{21}{2}\right)^3 \Rightarrow R = \frac{21}{2}$.

$\therefore$ Curved surface area $= \left(4 \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2}\right) \text{cm}^2 = 1386\ \text{cm}^2$.

114. $4\pi R^2 = 5544 \Rightarrow R^2 = \left(5544 \times \frac{1}{4} \times \frac{7}{22}\right) = 441 \Rightarrow R = 21$.

$\therefore$ Volume $= \left(\frac{4}{3} \times \frac{22}{7} \times 21 \times 21 \times 21\right) \text{cm}^3 = 38808\ \text{cm}^3$.

115. Volume $= \frac{4}{3}\pi r^3 = \frac{r}{3}(4\pi r^2) = \frac{r}{3} \times$ Surface area.

116. $\frac{\frac{4}{3}\pi R^3}{4\pi R^2} = 27 \Rightarrow R = 81$ cm.

117. Let the radii of A and B be r and R respectively.

$\therefore$ Required ratio $= \frac{4\pi r^2}{4\pi R^2} = \frac{r^2}{R^2} = \left(\frac{r}{R}\right)^2 = \left(\frac{40}{10}\right)^2 = 16 : 1$.

118. Let the original radius be r.

Then, original surface area $= 4\pi r^2 = 2464\ \text{cm}^2$ (given).

New radius $= 2r$.

$\therefore$ New surface area $= 4\pi\,(2r)^2 = 4 \times 4\pi r^2 = (4 \times 2464)\ \text{cm}^2 = 9856\ \text{cm}^2$.

119. Let the original radius be r. Then, original volume $= \frac{4}{3}\pi r^3$.

New radius $= 2r$.

$\therefore$ New volume $= \frac{4}{3}\pi\,(2r)^3 = 8 \times \frac{4}{3}\pi r^3 = 8 \times$ original volume.

120. $4\pi(r+2)^2 - 4\pi r^2 = 352 \Leftrightarrow (r+2)^2 - r^2 = \left(352 \times \frac{7}{22} \times \frac{1}{4}\right) = 28.$

$\Leftrightarrow (r+2+r)(r+2-r) = 28 \Leftrightarrow 2r+2 = 14 \Rightarrow r = \left(\frac{14}{2} - 1\right) = 6$ cm.

121. Let the correct radius be 100 cm. Then, measured radius = 101.5 cm.

$\therefore$ Error in volume $= \frac{4}{3}\pi[(101.5)^3 - (100)^3]$ cm^3

$= \frac{4}{3}\pi(1045678.375 - 1000000)$ cm^3 $= \left(\frac{4}{3} \times \pi \times 45678.375\right)$ cm^3.

$\therefore$ Error % $= \left\{\frac{\frac{4}{3}\pi(45678.375)}{\frac{4}{3}\pi(100 \times 100 \times 100)} \times 100\right\}$ % = 4.56% = 4.6% (app.).

122. Let their radii be R and r. Then,

$$\frac{\frac{4}{3}\pi R^3}{\frac{4}{3}\pi r^3} = \frac{64}{27} \Rightarrow \left(\frac{R}{r}\right)^3 = \frac{64}{27} = \left(\frac{4}{3}\right)^3 \Rightarrow \frac{R}{r} = \frac{4}{3}.$$

Ratio of surface areas $= \frac{4\pi R^2}{4\pi r^2} = \left(\frac{R}{r}\right)^2 = \left(\frac{4}{3}\right)^2 = \frac{16}{9}.$

123. Let their radii be R and r. Then,

$$\frac{4\pi R^2}{4\pi r^2} = \frac{4}{25} \Rightarrow \left(\frac{R}{r}\right)^2 = \left(\frac{2}{5}\right)^2 \Rightarrow \frac{R}{r} = \frac{2}{5}.$$

$\therefore$ Ratio of volumes $= \frac{\frac{4}{3}\pi R^3}{\frac{4}{3}\pi r^3} = \left(\frac{R}{r}\right)^3 = \left(\frac{2}{5}\right)^3 = \frac{8}{125}.$

124. Volume of new sphere $= \left[\frac{4}{3}\pi \times (6)^3 + \frac{4}{3}\pi \times (8)^3 + \frac{4}{3}\pi \times (10)^3\right]$ cm^3

$= \left\{\frac{4}{3}\pi[(6)^3 + (8)^3 + (10)^3]\right\}$ cm^3

$= \left(\frac{4}{3}\pi \times 1728\right)$ cm^3 $= \left[\frac{4}{3}\pi \times (12)^3\right]$ cm^3.

Let the radius of the new sphere be R. Then,

$\frac{4}{3}\pi R^3 = \frac{4}{3}\pi \times (12)^3 \Rightarrow R = 12$ cm.

$\therefore$ Diameter = 2R = 24 cm.

125. Volume of bigger sphere $= \left[\frac{4}{3}\pi \times (8)^3\right]$ cm^3 $= \left(\frac{4}{3}\pi \times 512\right)$ cm^3.

Volume of 1 ball $= \left[\frac{4}{3}\pi \times (2)^3\right]$ cm^3 $= \left(\frac{4}{3}\pi \times 8\right)$ cm^3.

$\therefore$ Number of balls $= \left(\frac{\frac{4}{3}\pi \times 512}{\frac{4}{3}\pi \times 8}\right) = \frac{512}{8} = 64.$

126. Let the radius of the third ball be R cm. Then,

$$\frac{4}{3}\pi\times\left(\frac{3}{4}\right)^3+\frac{4}{3}\pi\times(1)^3+\frac{4}{3}\pi\times R^3=\frac{4}{3}\pi\times\left(\frac{3}{2}\right)^3$$

$$\Rightarrow\ \frac{27}{64}+1+R^3=\frac{27}{8}\ \Rightarrow\ R^3=\frac{125}{64}=\left(\frac{5}{4}\right)^3\ \Rightarrow\ R=\frac{5}{4}.$$

$\therefore$ Diameter of the third ball $= 2R = \frac{5}{2}$ cm $= 2.5$ cm.

127. Volume of each ball $=\frac{1}{8}\times\left(\frac{4}{3}\pi\times10\times10\times10\right)\text{cm}^3.$

$$\therefore\ \frac{4}{3}\pi R^3=\frac{1}{8}\times\frac{4}{3}\pi\times10\times10\times10\ \Rightarrow\ R^3=\left(\frac{10}{2}\right)^3=5^3\ \Rightarrow\ R=5.$$

128. External radius = 3 cm, Internal radius = (3 − 0.5) cm = 2.5 cm.

Volume of the metal $=\left[\frac{4}{3}\times\frac{22}{7}\times\{(3)^3-(2.5)^3\}\right]\text{cm}^3$

$$=\left(\frac{4}{3}\times\frac{22}{7}\times\frac{91}{8}\right)\text{cm}^3=\left(\frac{143}{3}\right)\text{cm}^3=47\frac{2}{3}\text{cm}^3.$$

129. Volume of the solid = $(49\times33\times24)$ cm^3.

Let the radius of the sphere be r.

Then, $\frac{4}{3}\pi r^3=(49\times33\times24)\ \Leftrightarrow\ r^3=\left(\frac{49\times33\times24\times3\times7}{4\times22}\right)=(21)^3\ \Leftrightarrow\ r=21.$

130. Number of bullets $=\frac{\text{Volume of the cube}}{\text{Volume of 1 bullet}}=\left(\frac{22\times22\times22}{\frac{4}{3}\times\frac{22}{7}\times1\times1\times1}\right)=2541.$

131. Volume of each lead shot $=\left[\frac{4}{3}\pi\times\left(\frac{0.3}{2}\right)^3\right]\text{cm}^3=\left(\frac{4}{3}\times\frac{22}{7}\times\frac{27}{8000}\right)\text{cm}^3=\frac{99}{7000}\text{cm}^3.$

$\therefore$ Number of lead shots $=\left(9\times11\times12\times\frac{7000}{99}\right)=84000.$

132. $4\pi R^2=6a^2\ \Rightarrow\ \frac{R^2}{a^2}=\frac{3}{2\pi}\ \Rightarrow\ \frac{R}{a}=\frac{\sqrt{3}}{\sqrt{2\pi}}.$

$$\frac{\text{Volume of sphere}}{\text{Volume of cube}}=\frac{\frac{4}{3}\pi R^3}{a^3}=\frac{4}{3}\pi\cdot\left(\frac{R}{a}\right)^3=\frac{4}{3}\pi\cdot\frac{3\sqrt{3}}{2\pi\sqrt{2\pi}}=\frac{2\sqrt{3}}{\sqrt{2\pi}}=\frac{\sqrt{12}}{\sqrt{2\pi}}=\frac{\sqrt{6}}{\sqrt{\pi}}.$$

133. Let the edge of the cube be a. Then, volume of the cube $= a^3$.

Radius of the sphere $= (a/2)$.

Volume of the sphere $=\frac{4}{3}\pi\left(\frac{a}{2}\right)^3=\frac{\pi a^3}{6}.$

$\therefore$ Required ratio $= a^3:\frac{\pi a^3}{6}=6:\pi.$

134. $4\pi R^2=2\pi\times6\times12\ \Rightarrow\ R^2=\left(\frac{6\times12}{2}\right)=36\ \Rightarrow\ R=6$ cm.

135. Let the radius of the cylinder be R.

Then, $\pi \times R^2 \times \frac{7}{3} = \frac{4}{3}\pi \times 7 \times 7 \times 7$

$\Rightarrow \quad R^2 = \left(\frac{4 \times 7 \times 7 \times 7}{3} \times \frac{3}{7}\right) = 196 = (14)^2 \Rightarrow R = 14$ cm.

$\therefore$ Diameter = 2R = 28 cm.

136. Required volume = Volume of a sphere of radius 1 cm

$= \left(\frac{4}{3}\pi \times 1 \times 1 \times 1\right) \text{cm}^3 = \frac{4}{3}\pi \text{ cm}^3.$

137. Volume of cylinder = $\pi \times (3)^2 \times 15 = 135\pi$ cm^3.

Radius of 1 bullet = $\frac{5}{2}$ mm = $\frac{5}{20}$ cm = $\frac{1}{4}$ cm.

Volume of 1 bullet = $\left(\frac{4}{3}\pi \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4}\right) \text{cm}^3 = \frac{\pi}{48} \text{ cm}^3.$

$\therefore$ Number of bullets = $\left(135\pi \times \frac{48}{\pi}\right) = 6480.$

138. Let the radius of the cylindrical rod be r.

Then, height of the rod = $8r$ and radius of one ball = $\frac{r}{2}$.

$\therefore$ Number of balls = $\dfrac{\pi \times r^2 \times 8r}{\frac{4}{3}\pi \times \left(\frac{r}{2}\right)^3} = \left(\frac{8 \times 8 \times 3}{4}\right) = 48.$

139. Let the length of the wire be h.

Then, $\pi \times \frac{3}{20} \times \frac{3}{20} \times h = \frac{4}{3}\pi \times 4 \times 4 \times 4$

$\Leftrightarrow \quad h = \left(\frac{4 \times 4 \times 4 \times 4 \times 20 \times 20}{3 \times 3 \times 3}\right) \text{cm} = \left(\frac{102400}{27}\right) \text{cm} = 3792.5 \text{ cm} = 37.9 \text{ m}.$

140. Let the rise in the water level be h cm.

Then, $\pi \times 4 \times 4 \times h = \frac{4}{3}\pi \times 3 \times 3 \times 3 \Rightarrow h = \left(\frac{3 \times 3}{4}\right) = \frac{9}{4}$ cm.

141. Let the radius of each sphere be r cm.

Then, Volume of 12 spheres = Volume of cylinder

$\Rightarrow \quad 12 \times \frac{4}{3}\pi \times r^3 = \pi \times 8 \times 8 \times 2 \Rightarrow r^3 = \left(\frac{8 \times 8 \times 2 \times 3}{12 \times 4}\right) = 8 \Rightarrow r = 2$ cm.

$\therefore$ Diameter of each sphere = $2r$ = 4 cm.

142. Let the radius of the ball be r cm.

Volume of ball = Volume of water displaced by it

$\therefore \quad \frac{4}{3}\pi r^3 = \pi \times 12 \times 12 \times 6.75 \Rightarrow r^3 = 9 \times 9 \times 9 \Rightarrow r = 9$ cm.

143. Let the height of the cone be h cm. Then,

$\frac{1}{3}\pi \times 6 \times 6 \times h = \frac{4}{3}\pi \times 3 \times 3 \times 3 \Rightarrow h = \left(\frac{36 \times 3}{36}\right) = 3$ cm.

144. Volume of sphere = $\left(\frac{4}{3}\pi \times 9 \times 9 \times 9\right)$ cm^3.

Volume of cone = $\left(\frac{1}{3}\pi \times 9 \times 9 \times 9\right)$ cm^3.

Volume of wood wasted = $\left[\left(\frac{4}{3}\pi \times 9 \times 9 \times 9\right) - \left(\frac{1}{3}\pi \times 9 \times 9 \times 9\right)\right]$ cm^3

$= (\pi \times 9 \times 9 \times 9)$ cm^3.

$\therefore$ Required percentage = $\left(\frac{\pi \times 9 \times 9 \times 9}{\frac{4}{3}\pi \times 9 \times 9 \times 9} \times 100\right)\% = \left(\frac{3}{4} \times 100\right)\% = 75\%$.

145. Number of spheres = $\frac{\text{Volume of cone}}{\text{Volume of 1 sphere}} = \frac{\frac{1}{3}\pi \times 12 \times 12 \times 24}{\frac{4}{3}\pi \times 2 \times 2 \times 2} = 108$.

146. Volume of material in the sphere = $\left[\frac{4}{3}\pi \times \{(4)^3 - (2)^3\}\right]$ cm^3 = $\left(\frac{4}{3}\pi \times 56\right)$ cm^3.

Let the height of the cone be h cm.

Then, $\frac{1}{3}\pi \times 4 \times 4 \times h = \left(\frac{4}{3}\pi \times 56\right) \Leftrightarrow h = \left(\frac{4 \times 56}{4 \times 4}\right) = 14$ cm.

147. Let radius = R and height = H. Then,

Ratio of their volumes = $\pi R^2 H : \frac{1}{3}\pi R^2 H : \frac{4}{3}\pi R^3 = H : \frac{1}{3}H : \frac{4}{3}R$

$= H : \frac{1}{3}H : \frac{4}{3} \times \frac{H}{2}$ $\left[\text{In sphere, } H = 2R \text{ or } R = \frac{H}{2}\right]$

$= 3 : 1 : 2$.

148. Total surface area = $3\pi R^2 = \left(3 \times \frac{22}{7} \times 7 \times 7\right)$ cm^2 = 462 cm^2.

149. Let the radius be R cm. Then,

$\frac{2}{3} \times \frac{22}{7} \times R^3 = 19404 \Leftrightarrow R^3 = \left(19404 \times \frac{21}{44}\right) = (21)^3 \Leftrightarrow R = 21$ cm.

150. Let their radii be R and r. Then,

$\frac{\frac{2}{3}\pi R^3}{\frac{2}{3}\pi r^3} = \frac{6.4}{21.6} \Leftrightarrow \left(\frac{R}{r}\right)^3 = \frac{8}{27} = \left(\frac{2}{3}\right)^3 \Leftrightarrow \frac{R}{r} = \frac{2}{3}$.

$\therefore$ Ratio of curved surface areas = $\frac{2\pi R^2}{2\pi r^2} = \left(\frac{R}{r}\right)^2 = \frac{4}{9}$.

151. Let the height of the vessel be x. Then, radius of the bowl = radius of the vessel = $\frac{x}{2}$.

Volume of the bowl, $V_1 = \frac{2}{3}\pi\left(\frac{x}{2}\right)^3 = \frac{1}{12}\pi x^3$.

Volume of the vessel, $V_2 = \pi\left(\frac{x}{2}\right)^2 x = \frac{1}{4}\pi x^3$.

Since $V_2 > V_1$, so the vessel can contain 100% of the beverage filled in the bowl.

152. $\frac{2}{3}\pi R^3 = \frac{1}{3}\pi R^2 H \Rightarrow H = 2R.$

153. Let the radius of the cone be R cm. Then,

$$\frac{1}{3}\pi \times R^2 \times 75 = \frac{2}{3}\pi \times 6 \times 6 \times 6$$

$$\Leftrightarrow R^2 = \left(\frac{2 \times 6 \times 6 \times 6}{75}\right) = \left(\frac{144}{25}\right) = \left(\frac{12}{5}\right)^2 \Leftrightarrow R = \frac{12}{5}\text{cm} = 2.4 \text{ cm}.$$

154. Let the radius of each be R. Height of hemisphere, H = R.
So, height of cone = height of hemisphere = R.

Slant height of cone $= \sqrt{R^2 + R^2} = \sqrt{2}\,R.$

$$\frac{\text{Curved surface area of hemisphere}}{\text{Curved surface area of cone}} = \frac{2\pi R^2}{\pi R \times \sqrt{2}\,R} = \sqrt{2} : 1.$$

155. Volume of hemisphere $= \frac{2}{3}\pi r^3.$

Volume of biggest sphere = Volume of sphere with diameter $r = \frac{4}{3}\pi\left(\frac{r}{2}\right)^3 = \frac{1}{6}\pi r^3$

$$\therefore \text{ Required ratio} = \frac{\frac{2}{3}\pi r^3}{\frac{1}{6}\pi r^3} = \frac{4}{1} \text{ i.e. } 4:1.$$

EXERCISE 25B

(DATA SUFFICIENCY TYPE QUESTIONS)

Directions (*Questions 1 to 10*) : ***Each of the questions given below consists of a statement and/or a question and two statements numbered I and II given below it. You have to decide whether the data provided in the statement(s) is/are sufficient to answer the given question. Read both the statements and***

Give answer (a) if the data in Statement I alone are sufficient to answer the question, while the data in Statement II alone are not sufficient to answer the question;

Give answer (b) if the data in Statement II alone are sufficient to answer the question, while the data in Statement I alone are not sufficient to answer the question;

Give answer (c) if the data either in Statement I or in Statement II alone are sufficient to answer the question;

Give answer (d) if the data even in both Statements I and II together are not sufficient to answer the question;

Give answer (e) if the data in both Statements I and II together are necessary to answer the question.

1. What is the weight of the iron beam ?
   I. The beam is 9 m long, 40 cm wide and 20 cm high.
   II. Iron weighs 50 kg per cubic metre.
2. What is the volume of 32 metre high cylindrical tank ? **(Bank P.O. 2003)**
   I. The area of its base is 154 m^2.
   II. The diameter of the base is 14 m.
3. What is the volume of a cube ? **(Bank P.O. 2003)**
   I. The area of each face of the cube is 64 square metres.
   II. The length of one side of the cube is 8 metres.

4. What is the total cost of painting the inner surface of an open box at the rate of 50 paise per 100 sq. cm ?
   I. The box is made of wood 3 cm thick.
   II. The external dimensions of the box are 50 cm, 40 cm and 23 cm.
5. What is the capacity of a cylindrical tank ? **(I.B.P.S. 2002)**
   I. Radius of the base is half of its height which is 28 metres.
   II. Area of the base is 616 sq. metres and its height is 28 metres.
6. What is the volume of the cylinder ? **(Bank P.O. 2003)**
   I. Height is equal to the diameter.
   II. Perimeter of the base is 352 cm.
7. What will be the total cost of whitewashing the conical tomb at the rate of 80 paise per square metre ?
   I. The diameter and the slant height of the tomb are 28 m and 50 m.
   II. The height of the tomb is 48 m and the area of its base is 616 sq. m.
8. What is the height of a circular cone ? **(Bank P.O. 1999)**
   I. The area of that cone is equal to the area of a rectangle whose length is 33 cm.
   II. The area of the base of that cone is 154 sq. cm.
9. Is a given rectangular block, a cube ? **(M.A.T. 1999)**
   I. At least 2 faces of the rectangular block are squares.
   II. The volume of the block is 64.
10. A spherical ball of given radius x cm is melted and made into a right circular cylinder. What is the height of the cylinder ? **(S.B.I.P.O. 2003)**
   I. The volume of the cylinder is equal to the volume of the ball.
   II. The area of the base of the cylinder is given.

Directions (*Questions 11-13*) : *Each of the questions given below consists of a question followed by three statements. You have to study the question and the statements and decide which of the statement(s) is/are necessary to answer the question.*

11. What is the capacity of the cylindrical tank ? **(R.B.I. 2003)**
   I. The area of the base is 61,600 sq. cm.
   II. The height of the tank is 1.5 times the radius.
   III. The circumference of base is 880 cm.
   (*a*) Only I and II (*b*) Only II and III (*c*) Only I and III
   (*d*) Any two of the three (*e*) Only II and either I or III
12. A solid metallic cone is melted and recast into a sphere. What is the radius of the sphere ?
   I. The radius of the base of the cone is 2.1 cm.
   II. The height of the cone is four times the radius of its base.
   III. The height of the cone is 8.4 cm.
   (*a*) Only I and II (*b*) Only II and III (*c*) Only I and III
   (*d*) Any two of the three (*e*) All I, II and III
13. What is the total surface area of the cone ?
   I. The area of the base of the cone is 154 cm^2.
   II. The curved surface area of the cone is 550 cm^2.
   III. The volume of the cone is 1232 cm^3.
   (*a*) I, and either II or III (*b*) II, and either I or III
   (*c*) III, and either I or II (*d*) Any two of the three
   (*e*) None of these

ANSWERS

1. (*e*)	2. (*c*)	3. (*c*)	4. (*e*)	5. (*c*)	6. (*e*)	7. (*c*)	8. (*d*)
9. (*d*)	10. (*b*)	11. (*e*)	12. (*d*)	13. (*a*)			

SOLUTIONS

1. I gives, $l = 9$ m, $b = \frac{40}{100}$ m $= \frac{2}{5}$ m and $h = \frac{20}{100}$ m $= \frac{1}{5}$ m.

This gives, volume $= (l \times b \times h) = \left(9 \times \frac{2}{5} \times \frac{1}{5}\right)$ m^3 $= \frac{18}{25}$ m^3.

II gives, weight of iron is 50 kg / m^3.

$\therefore$ Weight $= \left(\frac{18}{25} \times 50\right)$ kg $= 36$ kg.

Thus, both I and II are needed to get the answer.

$\therefore$ Correct answer is (*e*).

2. Given, height = 32 m.

I gives, area of the base = 154 m^2.

$\therefore$ Volume = (area of the base × height) = (154 × 32) m^3 = 4928 m^3.

Thus, I alone gives the answer.

II gives, radius of the base = 7 m.

$\therefore$ Volume $= \pi r^2 h = \left(\frac{22}{7} \times 7 \times 7 \times 32\right)$ m^3 = 4928 m^3.

Thus, II alone gives the answer.

$\therefore$ Correct answer is (*c*).

3. Let each edge be *a* metres. Then,

I. $a^2 = 64 \Rightarrow a = 8$ m $\Rightarrow$ Volume = (8 × 8 × 8) m^3 = 512 m^3.

Thus, I alone gives the answer.

II. $a = 8$ m $\Rightarrow$ Volume = (8 × 8 × 8) m^3 = 512 m^3.

Thus, II alone gives the answer.

$\therefore$ Correct answer is (*c*).

4. I gives, thickness of the wall of the box = 3 cm.

II gives, Internal length = (50 − 6) cm = 44 cm, Internal breadth = (40 − 6) = 34 cm,

Internal height = (23 − 3) cm = 20 cm.

Area to be painted = (area of 4 walls + area of floor) = $[2\,(l + b) \times h + (l \times b)]$

$= [2\,(44 + 34) \times 20 + (44 \times 34)]$ cm^2 = 4616 cm^2.

Cost of painting = Rs. $\left(\frac{1}{2 \times 100} \times 4616\right)$ = Rs. 23.08.

Thus, both I and II are needed to get the answer.

$\therefore$ Correct answer is (*e*).

5. I gives, $h = 28$ m and $r = 14$ cm.

$\therefore$ Capacity $= \pi r^2 h$, which can be obtained.

Thus, I alone gives the answer.

II gives, $\pi r^2 = 616$ m^2 and $h = 28$ m.

$\therefore$ Capacity $= (\pi r^2 \times h) = (616 \times 28)$ m^3.

Thus, II alone gives the answer.

$\therefore$ Correct answer is (*c*).

6. I gives, $h = 2r$.

II gives, $2\pi r = 352 \Rightarrow r = \left(\frac{352}{2} \times \frac{7}{22}\right)$ cm $= 56$ cm.

From I and II, we have $r = 56$ cm, $h = (2 \times 56)$ cm $= 112$ cm.

Thus, we can find the volume.

$\therefore$ Correct answer is (*e*).

7. I gives, $r = 14$ m, $l = 50$ m.

$\therefore$ Curved surface $= \pi rl = \left(\frac{22}{7} \times 14 \times 50\right)$ m^2 $= 2200$ m^2.

Cost of whitewashing = Rs. $\left(2200 \times \frac{80}{100}\right)$ = Rs. 1760.

Thus, I alone gives the answer.

II gives, $h = 48$ m, $\pi r^2 = 616$ m^2.

These results give r and h and so l can be found out.

$\therefore$ Curved surface $= \pi rl$.

Thus, II alone gives the answer.

$\therefore$ Correct answer is (*c*).

8. II gives the value of r.

But, in I, the breadth of rectangle is not given.

So, we cannot find the surface area of the cone.

Hence, the height of the cone cannot be determined.

$\therefore$ Correct answer is (*d*).

9. I gives, any two of l, b, h are equal.

II gives, $lbh = 64$.

From I and II, the values of l, b, h may be (1, 1, 64), (2, 2, 16), (4, 4, 4).

Thus, the block may be a cube or cuboid.

$\therefore$ Correct answer is (*d*).

10. Clearly, I is not needed, since it is evident from the given question.

From II, we get radius of the base of the cylinder.

Now, $\frac{4}{3}\pi x^3 = \pi r^2 h$ in which x and r are known.

$\therefore$ h can be determined.

$\therefore$ Correct answer is (*b*).

11. Capacity $= \pi r^2 h$.

I gives, $\pi r^2 = 61600$. This gives r.

II gives, $h = 1.5\ r$.

Thus, I and II give the answer.

Again, III gives $2\pi r = 880$. This gives r.

So, II and III also give the answer.

$\therefore$ Correct answer is (*e*).

12. $\frac{4}{3}\pi R^3 = \frac{1}{3}\pi r^2 h.$

Now r and h can be determined from any two of I, II and III.

Thus, R can be calculated.

$\therefore$ Correct answer is (*d*).

13 Total surface area of the cone = $(\pi rl + \pi r^2)$ cm^2.

I gives, $\pi r^2 = 154$. Thus, we can find r.

II gives, $\pi rl = 550$.

From I and II we get the answer.

III gives, $\frac{1}{3}\pi r^2 h = 1232$.

From I and III, we can find h and therefore, l.

Hence the surface area can be determined.

$\therefore$ Correct answer is (*a*).

26. RACES AND GAMES OF SKILL

IMPORTANT FACTS

Races : *A contest of speed in running, riding, driving, sailing or rowing is called a race.*

Race Course : *The ground or path on which contests are made is called a race course.*

Starting Point : *The point from which a race begins is known as a starting point.*

Winning Point or Goal : *The point set to bound a race is called a winning point or a goal.*

Winner : *The person who first reaches the winning point is called a winner.*

Dead Heat Race : *If all the persons contesting a race reach the goal exactly at the same time, then the race is said to be a dead heat race.*

Start : Suppose A and B are two contestants in a race. If before the start of the race, A is at the starting point and B is ahead of A by 12 metres, then we say that '*A gives B, a start of 12 metres*'.

To cover a race of 100 metres in this case, A will have to cover 100 metres while B will have to cover only (100 – 12) = 88 metres.

In a 100 m race, '*A can give B 12 m*' or '*A can give B a start of 12 m*' or '*A beats B by 12 m*' means that while A runs 100 m, B runs (100 – 12) = 88 m.

Games : '*A game of 100, means that the person among the contestants who scores 100 points first is the winner*'.

If A scores 100 points while B scores only 80 points, then we say that '*A can give B 20 points*'.

SOLVED EXAMPLES

Ex. 1. ***In a km race, A beats B by 28 metres or 7 seconds. Find A's time over the course.***

Sol. Clearly, B covers 28 m in 7 seconds.

$\therefore$ B's time over the course $= \left(\frac{7}{28} \times 1000\right)$ sec = 250 seconds.

$\therefore$ A's time over the course = (250 – 7) sec = 243 sec = 4 min. 3 sec.

Ex. 2. ***A runs $1\frac{3}{4}$ times as fast as B. If A gives B a start of 84 m, how far must the winning post be so that A and B might reach it at the same time ?***

Sol. Ratio of the rates of A and B $= \frac{7}{4} : 1 = 7 : 4$.

So, in a race of 7 m, A gains 3 m over B.

$\therefore$ 3 m are gained by A in a race of 7 m.

$\therefore$ 84 m are gained by A in a race of $\left(\frac{7}{3} \times 84\right)$ m = 196 m.

$\therefore$ Winning post must be 196 m away from the starting point.

Ex. 3. ***A can run 1 km in 3 min. 10 sec. and B can cover the same distance in 3 min. 20 sec. By what distance can A beat B ?***

Sol. Clearly, A beats B by 10 sec.

Distance covered by B in 10 sec. $= \left(\frac{1000}{200} \times 10\right)$ m = 50 m.

$\therefore$ A beats B by 50 metres.

Ex. 4. ***In a 100 m race, A runs at 8 km per hour. If A gives B a start of 4 m and still beats him by 15 seconds, what is the speed of B ?***

Sol. Time taken by A to cover 100 m $= \left(\frac{60 \times 60}{8000} \times 100\right)$ sec = 45 sec.

$\therefore$ B covers (100 – 4) m = 96 m in (45 + 15) sec = 60 sec.

$\therefore$ B's speed $= \left(\frac{96 \times 60 \times 60}{60 \times 1000}\right)$ km/hr = 5.76 km/hr.

Ex. 5. ***A, B and C are three contestants in a km race. If A can give B a start of 40 m and A can give C a start of 64 m, how many metre's start can B give C ?***

Sol. While A covers 1000 m, B covers (1000 – 40) m = 960 m and C covers (1000 – 64) m or 936 m.

When B covers 960 m, C covers 936 m.

When B covers 1000 m, C covers $\left(\frac{936}{960} \times 1000\right)$ m = 975 m.

$\therefore$ B can give C a start of (1000 – 975) or 25 m.

Ex. 6. ***In a game of 80 points, A can give B 5 points and C 15 points. Then how many points B can give C in a game of 60 ?***

Sol. A : B = 80 : 75, A : C = 80 : 65.

$$\frac{B}{C} = \left(\frac{B}{A} \times \frac{A}{C}\right) = \left(\frac{75}{80} \times \frac{80}{65}\right) = \frac{15}{13} = \frac{60}{52} = 60 : 52.$$

$\therefore$ In a game of 60, B can give C 8 points.

EXERCISE 26

(OBJECTIVE TYPE QUESTIONS)

Directions : ***Mark (✓) against the correct answer :***

1. In a 100 m race, A covers the distance in 36 seconds and B in 45 seconds. In this race A beats B by :
   (*a*) 20 m (*b*) 25 m (*c*) 22.5 m (*d*) 9 m
2. In a 200 metres race A beats B by 35 m or 7 seconds. A's time over the course is :
   (*a*) 40 sec (*b*) 47 sec (*c*) 33 sec (*d*) None of these
3. In a 300 m race A beats B by 22.5 m or 6 seconds. B's time over the course is :
   (*a*) 86 sec (*b*) 80 sec (*c*) 76 sec (*d*) None of these
4. A can run 22.5 m while B runs 25 m. In a kilometre race B beats A by :
   (*a*) 100 m (*b*) $111\frac{1}{9}$ m (*c*) 25 m (*d*) 50 m
5. In a 500 m race, the ratio of the speeds of two contestants A and B is 3 : 4. A has a start of 140 m. Then, A wins by :
   (*a*) 60 m (*b*) 40 m (*c*) 20 m (*d*) 10 m

6. A runs $1\frac{2}{3}$ times as fast as B. If A gives B a start of 80 m, how far must the winning post be so that A and B might reach it at the same time ?

(*a*) 200 m (*b*) 300 m (*c*) 270 m (*d*) 160 m

7. In a 100 m race, A can beat B by 25 m and B can beat C by 4 m. In the same race, A can beat C by :

(*a*) 21 m (*b*) 26 m (*c*) 28 m (*d*) 29 m

8. In a 100 m race, A can give B 10 m and C 28 m. In the same race B can give C :

(*a*) 18 m (*b*) 20 m (*c*) 27 m (*d*) 9 m

9. In a 100 m race, A beats B by 10 m and C by 13 m. In a race of 180 m, B will beat C by :

(*a*) 5.4 m (*b*) 4.5 m (*c*) 5 m (*d*) 6 m

10. In a race of 200 m, A can beat B by 31 m and C by 18 m. In a race of 350 m, C will beat B by :

(*a*) 22.75 m (*b*) 25 m (*c*) 19.5 m (*d*) $7\frac{4}{7}$ m

11. A and B take part in a 100 m race. A runs at 5 km per hour. A gives B a start of 8 m and still beats him by 8 seconds. The speed of B is :

(*a*) 5.15 kmph (*b*) 4.14 kmph (*c*) 4.25 kmph (*d*) 4.4 kmph

12. In a game of 100 points, A can give B 20 points and C 28 points. Then, B can give C :

(*a*) 8 points (*b*) 10 points (*c*) 14 points (*d*) 40 points

13. At a game of billiards, A can give B 15 points in 60 and A can give C 20 points in 60. How many points can B give C in a game of 90 ?

(*a*) 30 points (*b*) 20 points (*c*) 10 points (*d*) 12 points

ANSWERS

1. (*a*) 2. (*c*) 3. (*b*) 4. (*a*) 5. (*c*) 6. (*a*) 7. (*c*) 8. (*b*)
9. (*d*) 10. (*b*) 11. (*b*) 12. (*b*) 13. (*c*)

SOLUTIONS

1. Distance covered by B in 9 sec. $= \left(\frac{100}{45} \times 9\right)$ m = 20 m.

$\therefore$ A beats B by 20 metres.

2. B runs 35 m in 7 sec.

$\therefore$ B covers 200 m in $\left(\frac{7}{35} \times 200\right) = 40$ sec.

B's time over the course = 40 sec.

$\therefore$ A's time over the course = (40 – 7) sec = 33 sec.

3. B runs $\frac{45}{2}$ m in 6 sec.

$\therefore$ B covers 300 m in $\left(6 \times \frac{2}{45} \times 300\right)$ sec = 80 sec.

Time taken by A to cover 100 m = $\left(100 \times \frac{18}{25}\right)$ sec = 72 sec.

$\therefore$ Time taken by B to cover 92 m = (72 + 8) sec = 80 sec.

$\therefore$ B's speed = $\left(\frac{92}{80} \times \frac{18}{5}\right)$ kmph = 4.14 kmph.

12. A : B = 100 : 80 and A : C = 100 : 72.

$\therefore \frac{B}{C} = \left(\frac{B}{A} \times \frac{A}{C}\right) = \left(\frac{80}{100} \times \frac{100}{72}\right) = \frac{10}{9} = \frac{100}{90} = 100 : 90.$

$\therefore$ B can give C 10 points.

13. A : B = 60 : 45 and A : C = 60 : 40.

$\therefore \frac{B}{C} = \left(\frac{B}{A} \times \frac{A}{C}\right) = \left(\frac{45}{60} \times \frac{60}{40}\right) = \frac{45}{40} = \frac{90}{80} = 90 : 80.$

$\therefore$ B can give C 10 points in a game of 90.

27. CALENDAR

IMPORTANT FACTS AND FORMULAE

Under this heading we mainly deal with finding the day of the week on a particular given date. The process of finding it lies on obtaining the number of odd days.

I. **Odd Days** : Number of days more than the complete number of weeks in a given period is the number of odd days during that period.

II. **Leap Year** : Every year which is divisible by 4 is called a leap year.

Thus, each one of the years 1992, 1996, 2004, 2008, 2012, etc. is a leap year.

Every 4th century is a leap year but no other century is a leap year.

Thus, each one of 400, 800, 1200, 1600, 2000, etc. is a leap year.

None of 1900, 2010, 2020, 2100, etc. is a leap year.

An year which is not a leap year is called an ordinary year.

III. (*i*) *An ordinary year has 365 days.* (*ii*) *A leap year has 366 days.*

IV. **Counting of Odd Days :**

(*i*) 1 ordinary year = 365 days = (52 weeks + 1 day).

∴ *An ordinary year has 1 odd day.*

(*ii*) 1 leap year = 366 days = (52 weeks + 2 days).

∴ *A leap year has 2 odd days.*

(*iii*) 100 years = 76 ordinary years + 24 leap years

= [(76 × 52) weeks + 76 days] + [(24 × 52) weeks + 48 days]

= 5200 weeks + 124 days = (5217 weeks + 5 days).

∴ *100 years contain 5 odd days.*

200 years contain 10 and therefore 3 odd days.

300 years contain 15 and therefore 1 odd day.

400 years contain (20 + 1) and therefore 0 odd day.

Similarly, each one of 800, 1200, 1600, 2000, etc. contains 0 odd days.

Remark : $(7n + m)$ odd days, where $m < 7$ is equivalent to m odd days.

Thus, 8 odd days ≡ 1 odd day etc.

V.

No. of odd days	0	1	2	3	4	5	6
Day	Sun.	Mon.	Tues.	Wed.	Thur.	Fri.	Sat.

SOLVED EXAMPLES

Ex. 1. ***What was the day of the week on 16th July, 1776 ?***

Sol. 16th July, 1776 = (1775 years + Period from 1st Jan., 1776 to 16th July, 1776)

Counting of odd days :

1600 years have 0 odd day. 100 years have 5 odd days.

75 years = (18 leap years + 57 ordinary years)

= [(18 × 2) + (57 × 1)] odd days = 93 odd days

= (13 weeks + 2 days) = 2 odd days.

∴ 1775 years have (0 + 5 + 2) odd days = 7 odd days = 0 odd day.

Jan.		Feb.		March		April		May		June		July	
31	+	29	+	31	+	30	+	31	+	30	+	16	= 198 days

= (28 weeks + 2 days) = 2 odd days.

∴ Total number of odd days = (0 + 2) = 2. Required day was **'Tuesday'**.

Ex. 2. ***What was the day of the week on 15th August, 1947 ?***

Sol. 15th August, 1947 = (1946 years + Period from 1st Jan., 1947 to 15th Aug., 1947)

Counting of odd days :

1600 years have 0 odd day. 300 years have 1 odd day.

47 years = (11 leap years + 36 ordinary years)

= [(11 × 2) + (36 × 1)] odd days = 58 odd days = 2 odd days.

Jan.		Feb.		March		April		May		June		July		Aug.
31	+	28	+	31	+	30	+	31	+	30	+	31	+	15

= 227 days = (32 weeks + 3 days) = 3 odd days.

Total number of odd days = (0 + 1 + 2 + 3) odd days = 6 odd days.

Hence, the required day was **'Saturday'**.

Ex. 3. ***What was the day of the week on 16th April, 2000 ?***

Sol. 16th April, 2000 = (1999 years + Period from 1st Jan., 2000 to 16th April, 2000)

Counting of odd days :

1600 years have 0 odd day. 300 years have 1 odd day.

99 years = (24 leap years + 75 ordinary years)

= [(24 × 2) + (75 × 1)] odd days = 123 odd days

= (17 weeks + 4 days) = 4 odd days.

Jan.		Feb.		March		April	
31	+	29	+	31	+	16	= 107 days = (15 weeks + 2 days) = 2 odd days.

Total number of odd days = (0 + 1 + 4 + 2) odd days = 7 odd days = 0 odd day.

Hence, the required day was **'Sunday'**.

Ex. 4. ***On what dates of July 2004 did Monday fall ?***

Sol. Let us find the day on 1st July, 2004.

2000 years have 0 odd day. 3 ordinary years have 3 odd days.

Jan.		Feb.		March		April		May		June		July
31	+	29	+	31	+	30	+	31	+	30	+	1

= 183 days = (26 weeks + 1 day) = 1 odd day.

Total number of odd days = (0 + 3 + 1) odd days = 4 odd days.

∴ 1st July 2004 was 'Thursday'.

Thus, 1st Monday in July 2004 was on 5th July.

Hence, during July 2004, Monday fell on 5th, 12th, 19th and 26th.

Ex. 5. ***Prove that the calendar for the year 2003 will serve for the year 2014.***

Sol. In order that the calendar for the year 2003 and 2014 be the same, 1st January of both the years must be on the same day of the week.

For this, the number of odd days between 31st Dec., 2002 and 31st Dec., 2013 must be the same.

We know that an ordinary year has 1 odd day and a leap year has 2 odd days.

During this period, there are 3 leap years, namely 2004, 2008 and 2012 and 8 ordinary years.

Total number of odd days = (6 + 8) days = 0 odd day.

Hence, the calendar for 2003 will serve for the year 2014.

Ex. 6. ***Prove that any date in March of a year is the same day of the week as the corresponding date in November that year.***

Sol. We will show that the number of odd days between last day of February and last day of October is zero.

March		April		May		June		July		Aug.		Sept.		Oct.
31	+	30	+	31	+	30	+	31	+	31	+	30	+	31

= 241 days = 35 weeks = 0 odd day.

∴ Number of odd days during this period = 0.

Thus, 1st March of an year will be the same day as 1st November of that year. Hence, the result follows.

EXERCISE 27

(OBJECTIVE TYPE QUESTIONS)

Directions : *Mark (✓) against the correct answer* :

1. January 1, 2004 was a Thursday. What day of the week lies on Jan. 1, 2005 ?
   (*a*) Thursday (*b*) Friday (*c*) Saturday (*e*) Sunday
2. On 8th March, 2005, Wednesday falls. What day of the week was it on 8th March, 2004 ?
   (*a*) Sunday (*b*) Monday (*c*) Tuesday (*d*) Wednesday
3. The calendar for the year 2005 is the same as for the year :
   (*a*) 2010 (*b*) 2011 (*c*) 2012 (*d*) 2013
4. On what dates of April 2001 did Sunday fall ?
   (*a*) 1st, 8th, 15th, 22nd, 29th (*b*) 2nd, 9th, 16th, 23rd, 30th
   (*c*) 4th, 11th, 18th, 25th (*d*) 6th, 13th, 20th, 27th
5. What will be the day of the week on 1st January, 2010 ?
   (*a*) Friday (*b*) Saturday (*c*) Sunday (*d*) Monday
6. What was the day of the week on 17th June, 1998 ?
   (*a*) Monday (*b*) Tuesday (*c*) Wednesday (*d*) Thursday
7. What was the day of the week on 28th May, 2003 ?
   (*a*) Friday (*b*) Saturday (*c*) Sunday (*d*) Monday
8. Today is Friday. After 62 days, it will be :
   (*a*) Saturday (*b*) Monday (*c*) Tuesday (*d*) Thursday
9. The last day of a century cannot be :
   (*a*) Monday (*b*) Wednesday (*c*) Friday (*d*) Tuesday
10. The first Republic Day of India was celebrated on 26th January, 1950. It was :
    (*a*) Tuesday (*b*) Wednesday (*c*) Thursday (*d*) Friday

SOLUTIONS

1. The year 2004 being a leap year, it has 2 odd days. So, first day of 2005 will be 2 days beyond Thursday and so it will be Saturday.
2. The year 2004 being a leap year, it has 2 odd days.
   So, the day on 8th March, 2005 will be two days beyond the day on 8th March, 2004.
   But, 8th March, 2005 is Wednesday. So, 8th March, 2004 is Monday.
3. Count the number of days from 2005 onwards to get 0 odd day.

Year	2005	2006	2007	2008	2010	2011
Odd days	1	1	1	2	1	1 = 7 or 0 odd day.

∴ Calendar for the year 2005 is the same as that for the year 2012.

4. Find the day on 1st April, 2001. 2000 years contain 2 odd days.

Jan.		Feb.		March		April
31	+	28	+	31	+	1 = 91 days = 13 weeks 0 day = 0 odd day.

Sunday fell on 1st, 8th, 15th, 22nd and 29th of April 2001.

5. 2000 years have 2 odd days.

Year	2001	2002	2003	2004	2005	2006	2007	2008	2009
Odd days	1	1	1	2	1	1	1	2	1

= 11 odd days = 4 odd days.

1st January, 2010 has 1 odd day. Total number of odd days = (2 + 4 + 1) = 7 = 0.

∴ 1st January, 2010 will be a Sunday.

6. 1600 years have 0 odd day. 300 years have 1 odd day.

97 years = 24 leap years + 73 ordinary years

= $[(24 \times 2) + (73 \times 1)]$ odd days = 121 odd days

= (17 weeks + 2 days) odd days = 2 odd days.

Jan.		Feb.		March		April		May		June
31	+	28	+	31	+	30	+	31	+	17 = 168 days = 0 odd day.

Total number of odd days = (0 + 1 + 2 + 0) = 3 odd days.

Hence, the required day was **'Wednesday'**.

7. 2000 years have 2 odd days.

The years 2001 and 2002 have (1 + 1) = 2 odd days.

Jan.		Feb.		March		April		May
31	+	28	+	31	+	30	+	28

= 148 days = 21 weeks + 1 day = 1 odd day.

Total number of odd days = (2 + 2 + 1) = 5.

∴ The required day was **'Friday'**.

8. Each day of the week is repeated after 7 days. So, after 63 days, it will be Friday. Hence, after 62 days, it will be Thursday.

9. 100 years contain 5 odd days. So, last day of 1st century is 'Friday'.

200 years contain (5 × 2) = 10 odd days = 3 odd days.

So, last day of 2nd century is 'Wednesday'.

300 years contain (5 × 3) = 15 odd days = 1 odd day.

∴ Last day of 3rd century is 'Monday'.

400 years contain 0 odd day.

∴ Last day of 4th century is 'Sunday'.

Since the order is continually kept in successive cycles, we see that the last day of a century cannot be Tuesday, Thursday or Saturday.

10. 26th Jan., 1950 = (1949 years + Period from 1st Jan., 1950 to 26th Jan., 1950)

1600 years have 0 odd day. 300 years have 1 odd day.

49 years = (12 leap years + 37 ordinary years)

= $[(12 \times 2) + (37 \times 1)]$ odd days = 61 odd days = 5 odd days.

Number of days from 1st Jan. to 26th Jan. = 26 = 5 odd days.

Total number of odd days = (0 + 1 + 5 + 5) = 11 = 4 odd days.

∴ The required day was **'Thursday'**.

28. CLOCKS

IMPORTANT FACTS

The face or dial of a watch is a circle whose circumference is divided into 60 equal parts, called minute spaces.

A clock has two hands, the smaller one is called the *hour hand* or *short hand* while the larger one is called the *minute hand* or *long hand*.

(*i*) In 60 minutes, the minute hand gains 55 minutes on the hour hand.

(*ii*) In every hour, both the hands coincide once.

(*iii*) The hands are in the same straight line when they are coincident or opposite to each other.

(*iv*) When the two hands are at right angles, they are 15 minute spaces apart.

(*v*) When the hands are in opposite directions, they are 30 minute spaces apart.

(*vi*) Angle traced by hour hand in 12 hrs = 360°.

(*vii*) Angle traced by minute hand in 60 min. = 360°.

Too Fast and Too Slow : *If a watch or a clock indicates 8.15, when the correct time is 8, it is said to be 15 minutes too fast.*

On the other hand, if it indicates 7.45, when the correct time is 8, it is said to be 15 minutes too slow.

SOLVED EXAMPLES

Ex. 1. ***Find the angle between the hour hand and the minute hand of a clock when the time is 3.25.***

Sol. Angle traced by the hour hand in 12 hours = 360°.

Angle traced by it in 3 hrs 25 min. *i.e.* $\frac{41}{12}$ hrs = $\left(\frac{360}{12} \times \frac{41}{12}\right)^\circ = 102\frac{1}{2}^\circ$.

Angle traced by minute hand in 60 min. = 360°.

Angle traced by it in 25 min. = $\left(\frac{360}{60} \times 25\right)^\circ = 150^\circ$.

$\therefore$ Required angle = $\left(150^\circ - 102\frac{1}{2}^\circ\right) = 47\frac{1}{2}^\circ$.

Ex. 2. ***At what time between 2 and 3 o'clock will the hands of a clock be together?***

Sol. At 2 o'clock, the hour hand is at 2 and the minute hand is at 12, *i.e.* they are 10 min. spaces apart.

To be together, the minute hand must gain 10 minutes over the hour hand.

Now, 55 minutes are gained by it in 60 min.

$\therefore$ 10 minutes will be gained in $\left(\frac{60}{55} \times 10\right)$ min. = $10\frac{10}{11}$ min.

$\therefore$ The hands will coincide at $10\frac{10}{11}$ min. past 2.

Ex. 3. ***At what time between 4 and 5 o'clock will the hands of a clock be at right angle ?***

Sol. At 4 o'clock, the minute hand will be 20 min. spaces behind the hour hand.
Now, when the two hands are at right angles, they are 15 min. spaces apart.
So, they are at right angles in following two cases.

Case I. *When minute hand is 15 min. spaces behind the hour hand* :
In this case min. hand will have to gain (20 – 15) = 5 minute spaces.
55 min. spaces are gained by it in 60 min.

5 min. spaces will be gained by it in $\left(\frac{60}{55} \times 5\right)$ min. = $5\frac{5}{11}$ min.

∴ They are at right angles at $5\frac{5}{11}$ min. past 4.

Case II. *When the minute hand is 15 min. spaces ahead of the hour hand* :
To be in this position, the minute hand will have to gain (20 + 15) = 35 minute spaces.
55 min. spaces are gained in 60 min.

35 min. spaces are gained in $\left(\frac{60}{55} \times 35\right)$ min. = $38\frac{2}{11}$ min.

∴ They are at right angles at $38\frac{2}{11}$ min. past 4.

Ex. 4. ***Find at what time between 8 and 9 o'clock will the hands of a clock be in the same straight line but not together.***

Sol. At 8 o'clock, the hour hand is at 8 and the minute hand is at 12, *i.e.* the two hands are 20 min. spaces apart.
To be in the same straight line but not together they will be 30 minute spaces apart.
So, the minute hand will have to gain (30 – 20) = 10 minute spaces over the hour hand.
55 minute spaces are gained in 60 min.

10 minute spaces will be gained in $\left(\frac{60}{55} \times 10\right)$ min. = $10\frac{10}{11}$ min.

∴ The hands will be in the same straight line but not together at $10\frac{10}{11}$ min. past 8.

Ex. 5. ***At what time between 5 and 6 o'clock are the hands of a clock 3 minutes apart ?***

Sol. At 5 o'clock, the minute hand is 25 min. spaces behind the hour hand.

Case I. *Minute hand is 3 min. spaces behind the hour hand.*
In this case, the minute hand has to gain (25 – 3) = 22 minute spaces.
55 min. are gained in 60 min.

22 min. are gained in $\left(\frac{60}{55} \times 22\right)$ min. = 24 min.

∴ The hands will be 3 min. apart at 24 min. past 5.

Case II. *Minute hand is 3 min. spaces ahead of the hour hand.*
In this case, the minute hand has to gain (25 + 3) = 28 minute spaces.
55 min. are gained in 60 min.

28 min. are gained in $\left(\frac{60}{55} \times 28\right)$ = $31\frac{5}{11}$ min.

∴ The hands will be 3 min. apart at $31\frac{5}{11}$ min. past 5.

Ex. 6. ***The minute hand of a clock overtakes the hour hand at intervals of 65 minutes of the correct time. How much a day does the clock gain or lose?***

Sol. In a correct clock, the minute hand gains 55 min. spaces over the hour hand in 60 minutes.

To be together again, the minute hand must gain 60 minutes over the hour hand.

55 min. are gained in 60 min.

60 min. are gained in $\left(\frac{60}{55} \times 60\right)$ min. $= 65\frac{5}{11}$ min.

But, they are together after 65 min.

$\therefore$ Gain in 65 min. $= \left(65\frac{5}{11} - 65\right) = \frac{5}{11}$ min.

Gain in 24 hours $= \left(\frac{5}{11} \times \frac{60 \times 24}{65}\right)$ min. $= 10\frac{10}{43}$ min.

$\therefore$ The clock gains $10\frac{10}{43}$ minutes in 24 hours.

Ex. 7. ***A watch which gains uniformly, is 5 min. slow at 8 o'clock in the morning on Sunday and it is 5 min. 48 sec. fast at 8 p.m. on following Sunday. When was it correct?***

Sol. Time from 8 a.m. on Sunday to 8 p.m. on following Sunday = 7 days 12 hours
= 180 hours

$\therefore$ The watch gains $\left(5 + 5\frac{4}{5}\right)$ min. or $\frac{54}{5}$ min. in 180 hrs.

Now $\frac{54}{5}$ min. are gained in 180 hrs.

$\therefore$ 5 min. are gained in $\left(180 \times \frac{5}{54} \times 5\right)$ hrs. = 83 hrs 20 min. = 3 days 11 hrs 20 min.

$\therefore$ Watch is correct 3 days 11 hrs 20 min. after 8 a.m. of Sunday.

$\therefore$ It will be correct at 20 min. past 7 p.m. on Wednesday.

Ex. 8. ***A clock is set right at 5 a.m. The clock loses 16 minutes in 24 hours. What will be the true time when the clock indicates 10 p.m. on 4th day?***

Sol. Time from 5 a.m. on a day to 10 p.m. on 4th day = 89 hours.

Now 23 hrs 44 min. of this clock = 24 hours of correct clock.

$\therefore$ $\frac{356}{15}$ hrs of this clock = 24 hours of correct clock.

89 hrs of this clock $= \left(24 \times \frac{15}{356} \times 89\right)$ hrs of correct clock.

= 90 hrs of correct clock.

So, the correct time is 11 p.m.

Ex. 9. ***A clock is set right at 8 a.m. The clock gains 10 minutes in 24 hours. What will be the true time when the clock indicates 1 p.m. on the following day?***

Sol. Time from 8 a.m. on a day to 1 p.m. on the following day = 29 hours.

24 hours 10 min. of this clock = 24 hours of the correct clock.

$\frac{145}{6}$ hrs of this clock = 24 hrs of the correct clock

29 hrs of this clock = $\left(24 \times \frac{6}{145} \times 29\right)$ hrs of the correct clock

= 28 hrs 48 min. of correct clock

∴ The correct time is 28 hrs 48 min. after 8 a.m.

This is 48 min. past 12.

EXERCISE 28

(OBJECTIVE TYPE QUESTIONS)

Directions : *Mark (✓) against the correct answer* :

1. A clock is started at noon. By 10 minutes past 5, the hour hand has turned through :
   (*a*) 145° (*b*) 150° (*c*) 155° (*d*) 160°
2. An accurate clock shows 8 o'clock in the morning. Through how many degrees will the hour hand rotate when the clock shows 2 o'clock in the afternoon ? (I.A.S. 2000)
   (*a*) 144° (*b*) 150° (*c*) 168° (*d*) 180°
3. At 3.40, the hour hand and the minute hand of a clock form an angle of :
   (*a*) 120° (*b*) 125° (*c*) 130° (*d*) 135°
4. The angle between the minute hand and the hour hand of a clock when the time is 8.30, is :
   (*a*) 80° (*b*) 75° (*c*) 60° (*d*) 105°
5. The angle between the minute hand and the hour hand of a clock when the time is 4.20, is :
   (*a*) 0° (*b*) 10° (*c*) 5° (*d*) 20°
6. At what angle the hands of a clock are inclined at 15 minutes past 5 ?
   (*a*) $58\frac{1}{2}°$ (*b*) 64° (*c*) $67\frac{1}{2}°$ (*d*) $72\frac{1}{2}°$
   (L.I.C.A.A.O. 2003)
7. The reflex angle between the hands of a clock at 10.25 is : (S.C.R.A. 1996)
   (*a*) 180° (*b*) $192\frac{1}{2}°$ (*c*) 195° (*d*) $197\frac{1}{2}°$
8. How many times do the hands of a clock coincide in a day ?
   (*a*) 20 (*b*) 21 (*c*) 22 (*d*) 24
9. How many times in a day, the hands of a clock are straight ?
   (*a*) 22 (*b*) 24 (*c*) 44 (*d*) 48
10. How many times are the hands of a clock at right angle in a day ? (I.A.S. 1997)
    (*a*) 22 (*b*) 24 (*c*) 44 (*d*) 48
11. How many times in a day, are the hands of a clock in straight line but opposite in direction ? (R.R.B. 2003)
    (*a*) 20 (*b*) 22 (*c*) 24 (*d*) 48
12. How much does a watch lose per day, if its hands coincide every 64 minutes ?
    (*a*) $32\frac{8}{11}$ min. (*b*) $36\frac{5}{11}$ min. (*c*) 90 min. (*d*) 96 min.
13. At what time, in minutes, between 3 o'clock and 4 o'clock, both the needles will coincide each other ? (R.R.B. 2002)
    (*a*) $5\frac{1}{11}''$ (*b*) $12\frac{4}{11}''$ (*c*) $13\frac{4}{11}''$ (*d*) $16\frac{4}{11}''$

14. At what time between 9 and 10 o'clock will the hands of a watch be together ?
(*a*) 45 min. past 9 (*b*) 50 min. past 9
(*c*) $49\frac{1}{11}$ min. past 9 (*d*) $48\frac{2}{11}$ min. past 9

15. At what time between 7 and 8 o'clock will the hands of a clock be in the same straight line but, not together ? **(A.A.O. Exam. 2003)**
(*a*) 5 min. past 7 (*b*) $5\frac{2}{11}$ min. past 7
(*c*) $5\frac{3}{11}$ min. past 7 (*d*) $5\frac{5}{11}$ min. past 7

16. At what time between 4 and 5 o'clock will the hands of a watch point in opposite directions ?
(*a*) 45 min. past 4 (*b*) 40 min. past 4
(*c*) $50\frac{4}{11}$ min. past 4 (*d*) $54\frac{6}{11}$ min. past 4

17. At what time between 5.30 and 6 will the hands of a clock be at right angles ?
(*a*) $43\frac{5}{11}$ min. past 5 (*b*) $43\frac{7}{11}$ min. past 5
(*c*) 40 min. past 5 (*d*) 45 min. past 5

18. A watch which gains uniformly is 2 minutes low at noon on Monday and is 4 min. 48 sec fast at 2 p.m. on the following Monday. When was it correct ? **(R.R.B. 2001)**
(*a*) 2 p.m. on Tuesday (*b*) 2 p.m. on Wednesday
(*c*) 3 p.m. on Thursday (*d*) 1 p.m. on Friday

19. A watch which gains 5 seconds in 3 minutes was set right at 7 a.m. In the afternoon of the same day, when the watch indicated quarter past 4 o'clock, the true time is :
(*a*) $59\frac{7}{12}$ min. past 3 (*b*) 4 p.m.
(*c*) $58\frac{7}{11}$ min. past 3 (*d*) $2\frac{3}{11}$ min. past 4

ANSWERS

1. (*c*) **2.** (*d*) **3.** (*c*) **4.** (*b*) **5.** (*b*) **6.** (*c*) **7.** (*d*) **8.** (*c*) **9.** (*c*) **10.** (*c*)
11. (*b*) **12.** (*a*) **13.** (*d*) **14.** (*c*) **15.** (*d*) **16.** (*d*) **17.** (*b*) **18.** (*b*) **19.** (*b*)

SOLUTIONS

1. Angle traced by hour hand in 12 hrs = 360°.

Angle traced by hour hand in 5 hrs 10 min. *i.e.* $\frac{31}{6}$ hrs. $= \left(\frac{360}{12} \times \frac{31}{6}\right)^\circ = 155^\circ$.

2. Angle traced by the hour hand in 6 hours $= \left(\frac{360}{12} \times 6\right)^\circ = 180^\circ$.

3. Angle traced by hour hand in 12 hrs. = 360°.

Angle traced by it in $\frac{11}{3}$ hrs $= \left(\frac{360}{12} \times \frac{11}{3}\right)^\circ = 110^\circ$.

Angle traced by minute hand in 60 min. = 360°.

Angle traced by it in 40 min. $= \left(\frac{360}{60} \times 40\right)^\circ = 240^\circ$.

$\therefore$ Required angle $(240 - 110)^\circ = 130^\circ$.

4. Angle traced by hour hand in $\frac{17}{2}$ hrs $= \left(\frac{360}{12} \times \frac{17}{2}\right)^\circ = 255$.

Angle traced by min. hand in 30 min. $= \left(\frac{360}{60} \times 30\right)^\circ = 180$.

$\therefore$ Required angle $= (255 - 180)^\circ = 75^\circ$.

5. Angle traced by hour hand in $\frac{13}{3}$ hrs $= \left(\frac{360}{12} \times \frac{13}{3}\right)^\circ = 130^\circ$.

Angle traced by min. hand in 20 min. $= \left(\frac{360}{60} \times 20\right)^\circ = 120^\circ$.

$\therefore$ Required angle $= (130 - 120)^\circ = 10^\circ$.

6. Angle traced by hour hand in $\frac{21}{4}$ hrs $= \left(\frac{360}{12} \times \frac{21}{4}\right)^\circ = 157\frac{1}{2}^\circ$.

Angle traced by min. hand in 15 min. $= \left(\frac{360}{12} \times 15\right)^\circ = 90^\circ$.

$\therefore$ Required angle $= \left(157\frac{1}{2}\right)^\circ - 90^\circ = 67\frac{1}{2}^\circ$.

7. Angle traced by hour hand in $\frac{125}{12}$ hrs $= \left(\frac{360}{12} \times \frac{125}{12}\right)^\circ = 312\frac{1}{2}^\circ$.

Angle traced by minute hand in 25 min $= \left(\frac{360}{60} \times 25^\circ\right) = 150^\circ$.

$\therefore$ Reflex angle $= 360^\circ - \left(312\frac{1}{2} - 150\right)^\circ = 360^\circ - 162\frac{1}{2}^\circ = 197\frac{1}{2}^\circ$.

8. The hands of a clock coincide 11 times in every 12 hours (Since between 11 and 1, they coincide only once, *i.e.* at 12 o'clock).

$\therefore$ The hands coincide 22 times in a day.

9. In 12 hours, the hands coincide or are in opposite direction 22 times.

$\therefore$ In 24 hours, the hands coincide or are in opposite direction 44 times a day.

10. In 12 hours, they are at right angles 22 times.

$\therefore$ In 24 hours, they are at right angles 44 times.

11. The hands of a clock point in opposite directions (in the same straight line) 11 times in every 12 hours (Because between 5 and 7 they point in opposite directions at 6 o'clock only). So, in a day, the hands point in the opposite directions 22 times.

12. 55 min. spaces are covered in 60 min.

60 min. spaces are covered in $\left(\frac{60}{55} \times 60\right)$ min. $= 65\frac{5}{11}$ min.

Loss in 64 min. $= \left(65\frac{5}{11} - 64\right) = \frac{16}{11}$ min.

Loss in 24 hrs $= \left(\frac{16}{11} \times \frac{1}{64} \times 24 \times 60\right)$ min. $= 32\frac{8}{11}$ min.

13. At 3 o'clock, the minute hand is 15 min. spaces apart from the hour hand.
To be coincident, it must gain 15 min. spaces.
55 min. are gained in 60 min.

15 min. are gained in $\left(\frac{60}{55} \times 15\right)$ min. = $16\frac{4}{11}$ min.

$\therefore$ The hands are coincident at $16\frac{4}{11}$ min. past 3.

14. To be together between 9 and 10 o'clock, the minute hand has to gain 45 min. spaces.
55 min. spaces gained in 60 min.

45 min. spaces are gained in $\left(\frac{60}{55} \times 45\right)$ min. or $49\frac{1}{11}$ min.

$\therefore$ The hands are together at $49\frac{1}{11}$ min. past 9.

15. When the hands of the clock are in the same straight line but not together, they are 30 minute spaces apart.
At 7 o'clock, they are 25 min. spaces apart.
$\therefore$ Minute hand will have to gain only 5 min. spaces.
55 min. spaces are gained in 60 min.

5 min. spaces are gained in $\left(\frac{60}{55} \times 5\right)$ min = $5\frac{5}{11}$ min.

$\therefore$ Required time = $5\frac{5}{11}$ min. past 7.

16. At 4 o'clock, the hands of the watch are 20 min. spaces apart.
To be in opposite directions, they must be 30 min. spaces apart.
$\therefore$ Minute hand will have to gain 50 min. spaces.
55 min. spaces are gained in 60 min.

50 min. spaces are gained in $\left(\frac{60}{55} \times 50\right)$ min. or $54\frac{6}{11}$ min.

$\therefore$ Required time = $54\frac{6}{11}$ min. past 4.

17. At 5 o'clock, the hands are 25 min. spaces apart.
To be at right angles and that too between 5.30 and 6, the minute hand has to gain (25 + 15) = 40 min. spaces.
55 min. spaces are gained in 60 min.

40 min. spaces are gained in $\left(\frac{60}{55} \times 40\right)$ min. = $43\frac{7}{11}$ min.

$\therefore$ Required time = $43\frac{7}{11}$ min. past 5.

18. Time from 12 p.m. on Monday to 2 p.m. on the following Monday = 7 days 2 hours
= 170 hours.

$\therefore$ The watch gains $\left(2 + 4\frac{4}{5}\right)$ min. or $\frac{34}{5}$ min. in 170 hrs.

Now, $\frac{34}{5}$ min. are gained in 170 hrs.

$\therefore$ 2 min. are gained in $\left(170 \times \frac{5}{34} \times 2\right)$ hrs = 50 hrs.

$\therefore$ Watch is correct 2 days 2 hrs. after 12 p.m. on Monday *i.e.* it will be correct at 2 p.m. on Wednesday.

19. Time from 7 a.m. to 4.15 p.m. = 9 hrs 15 min. = $\frac{37}{4}$ hrs.

3 min. 5 sec. of this clock = 3 min. of the correct clock.

$\Rightarrow$ $\frac{37}{720}$ hrs of this clock = $\frac{1}{20}$ hrs of the correct clock

$\Rightarrow$ $\frac{37}{4}$ hrs of this clock = $\left(\frac{1}{20} \times \frac{720}{37} \times \frac{37}{4}\right)$ hrs of the correct clock

= 9 hrs of the correct clock

$\therefore$ The correct time is 9 hrs after 7 a.m. *i.e.* 4 p.m.

29. STOCK AND SHARES

To start a big business or an industry, a large amount of money is needed. It is beyond the capacity of one or two persons to arrange such a huge amount. However, some persons associate together to form a company. They, then, draft a proposal, issue a prospectus (in the name of the company), explaining the plan of the project and invite the public to invest money in this project. They, thus, pool up the funds from the public, by assigning them ***shares*** of the company.

IMPORTANT FACTS AND FORMULAE

1. **Stock-capital :** *The total amount of money needed to run the company is called the* ***stock-capital.***
2. **Shares or Stock :** *The whole capital is divided into small units, called* ***shares*** *or* ***stock.***

   For each investment, the company issues a *share-certificate*, showing the value of each share and the number of shares held by a person.

   The person who subscribes in shares or stock is called a ***share holder*** *or* ***stock holder.***
3. **Dividend :** *The annual profit distributed among share holders is called* ***dividend.***

   Dividend is paid annually as per share or as a percentage.
4. **Face Value :** *The value of a share or stock printed on the share-certificate is called its* ***Face Value*** *or* ***Nominal Value*** *or* ***Par Value.***
5. **Market Value :** The stocks of different companies are sold and bought in the open market through brokers at stock-exchanges. A share (or stock) is said to be :

   (*i*) ***At premium*** or ***Above par,*** if its market value is more than its face value.

   (*ii*) ***At par,*** if its market value is the same as its face value.

   (*iii*) ***At discount*** or ***Below par,*** if its market value is less than its face value.

   Thus, if a Rs. 100 stock is quoted at a premium of 16, then market value of the stock = Rs. (100 + 16) = Rs. 116.

   Likewise, if a Rs. 100 stock is quoted at a discount of 7, then market value of the stock = Rs. (100 – 7) = Rs. 93.
6. **Brokerage :** *The broker's charge is called* ***brokerage.***

   (*i*) When stock is purchased, brokerage is added to the cost price.

   (*ii*) When stock is sold, brokerage is subtracted from the selling price.

   Remember :

   (*i*) The face value of a share always remains the same.

   (*ii*) The market value of a share changes from time to time.

   (*iii*) Dividend is always paid on the face value of a share.

   (*iv*) Number of shares held by a person

$$= \frac{\text{Total Investment}}{\text{Investment in 1 share}} = \frac{\text{Total Income}}{\text{Income from 1 share}} = \frac{\text{Total Face Value}}{\text{Face value of 1 share}}.$$

Thus, by a Rs. 100, 9% stock at 120, we mean that :

(*i*) Face Value (N.V.) of stock = Rs. 100.

(*ii*) Market Value (M.V.) of stock = Rs. 120.

(*iii*) Annual dividend on 1 share = 9% of face value = 9% of Rs. 100 = Rs. 9.

(*iv*) An investment of Rs. 120 gives an annual income of Rs. 9.

(*v*) Rate of interest p.a. = Annual income from an investment of Rs. 100

$$= \left(\frac{9}{120} \times 100\right)\% = 7\frac{1}{2}\%.$$

SOLVED EXAMPLES

Ex. 1. ***Find the cost of :***

(*i*) ***Rs. 7200, 8% stock at 90;***

(*ii*) ***Rs. 4500, 8.5% stock at 4 premium;***

(*iii*) ***Rs. 6400, 10% stock at 15 discount.***

Sol. (*i*) Cost of Rs. 100 stock = Rs. 90.

Cost of Rs. 7200 stock = Rs. $\left(\frac{90}{100} \times 7200\right)$ = Rs. 6480.

(*ii*) Cost of Rs. 100 stock = Rs. (100 + 4) = Rs. 104.

Cost of Rs. 4500 stock = Rs. $\left(\frac{104}{100} \times 4500\right)$ = Rs. 4680.

(*iii*) Cost of Rs. 100 stock = Rs. (100 – 15) = Rs. 85.

Cost of Rs. 6400 stock = Rs. $\left(\frac{85}{100} \times 6400\right)$ = Rs. 5440.

Ex. 2. ***Find the cash required to purchase Rs. 3200, $7\frac{1}{2}$% stock at 107 $\left(\text{brokerage } \frac{1}{2}\%\right)$.***

Sol. Cash required to purchase Rs. 100 stock = Rs. $\left(107 + \frac{1}{2}\right)$ = Rs. $\frac{215}{2}$.

Cash required to purchase Rs. 3200 stock = Rs. $\left(\frac{215}{2} \times \frac{1}{100} \times 3200\right)$ = Rs. 3440.

Ex. 3. ***Find the cash realised by selling Rs. 2440, 9.5% stock at 4 discount $\left(\text{brokerage } \frac{1}{4}\%\right)$.***

Sol. By selling Rs. 100 stock, cash realised = Rs. $\left[(100 - 4) - \frac{1}{4}\right]$ = Rs. $\frac{383}{4}$.

By selling Rs. 2400 stock, cash realised = Rs. $\left(\frac{383}{4} \times \frac{1}{100} \times 2400\right)$ = Rs. 2298.

Ex. 4. ***Find the annual income derived from Rs. 2500, 8% stock at 106.***

Sol. Income from Rs. 100 stock = Rs. 8.

Income from Rs. 2500 stock = Rs. $\left(\frac{8}{100} \times 2500\right)$ = Rs. 200.

Ex. 5. ***Find the annual income derived by investing Rs. 6800 in 10% stock at 136.***

Sol. By investing Rs. 136, income obtained = Rs. 10.

By investing Rs. 6800, income obtained = Rs. $\left(\frac{10}{136} \times 6800\right)$ = Rs. 500.

Ex. 6. ***Which is better investment ? $7\frac{1}{2}$% stock at 105 or $6\frac{1}{2}$% stock at 94.***

Sol. Let the investment in each case be Rs. (105 × 94).

Case I : $7\frac{1}{2}$% stock at 105 :

On investing Rs. 105, income = Rs. $\frac{15}{2}$.

On investing Rs. (105 × 94), income = Rs. $\left(\frac{15}{2} \times \frac{1}{105} \times 105 \times 94\right)$ = Rs. 705.

Case II : $6\frac{1}{2}$% stock at 94 :

On investing Rs. 94, income = Rs. $\frac{13}{2}$.

On investing Rs. (105 × 94), income = Rs. $\left(\frac{13}{2} \times \frac{1}{94} \times 105 \times 94\right)$ = Rs. 682.50.

Clearly, the income from $7\frac{1}{2}$% stock at 105 is more.

Hence, the investment in $7\frac{1}{2}$% stock at 105 is better.

Ex. 7. ***Find the cost of 96 shares of Rs. 10 each at $\frac{3}{4}$ discount, brokerage being $\frac{1}{4}$ per share.*** **(L.I.C. 2003)**

Sol. Cost of 1 share = Rs. $\left[\left(10 - \frac{3}{4}\right) + \frac{1}{4}\right]$ = Rs. $\frac{19}{2}$.

Cost of 96 shares = Rs. $\left(\frac{19}{2} \times 96\right)$ = Rs. 912.

Ex. 8. ***Find the income derived from 88 shares of Rs. 25 each at 5 premium, brokerage being $\frac{1}{4}$ per share and the rate of dividend being $7\frac{1}{2}$% per annum. Also, find the rate of interest on the investment.***

Sol. Cost of 1 share = Rs. $\left(25 + 5 + \frac{1}{4}\right)$ = Rs. $\frac{121}{4}$.

Cost of 88 shares = Rs. $\left(\frac{121}{4} \times 88\right)$ = Rs. 2662.

∴ Investment made = Rs. 2662.

Face value of 88 shares = Rs. (88 × 25) = Rs. 2200.

Dividend on Rs. 100 = $\frac{15}{2}$.

Dividend on Rs. 2200 = Rs. $\left(\frac{15}{2} \times \frac{1}{100} \times 2200\right)$ = Rs. 165.

∴ Income derived = Rs. 165.

Rate of interest on investment = $\left(\frac{165}{2662} \times 100\right)$ = 6.2%.

Ex. 9. ***A man buys Rs. 25 shares in a company which pays 9% dividend. The money invested is such that it gives 10% on investment. At what price did he buy the shares?***

Sol. Suppose he buys each share for Rs. x.

Then, $\left(25 \times \frac{9}{100}\right) = \left(x \times \frac{10}{100}\right)$ or $x = 22.50$.

$\therefore$ Cost of each share = Rs. 22.50.

Ex. 10. ***A man sells Rs. 5000, 12% stock at 156 and invests the proceeds partly in 8% stock at 90 and 9% stock at 108. He thereby increases his income by Rs. 70. How much of the proceeds were invested in each stock?***

Sol. S.P. of Rs. 5000 stock = Rs. $\left(\frac{156}{100} \times 5000\right)$ = Rs. 7800.

Income from this stock = Rs. $\left(\frac{12}{100} \times 5000\right)$ = Rs. 600.

Let investment in 8% stock be x and that in 9% stock = $(7800 - x)$.

$\therefore \quad \left(x \times \frac{8}{90}\right) + (7800 - x) \times \frac{9}{108} = (600 + 70)$

$\Leftrightarrow \quad \frac{4x}{45} + \frac{7800 - x}{12} = 670 \quad \Leftrightarrow \quad 16x + 117000 - 15x = (670 \times 180) \quad \Leftrightarrow \quad x = 3600.$

$\therefore$ Money invested in 8% stock at 90 = Rs. 3600.

Money invested in 9% at 108 = Rs. (7800 – 3600) = Rs. 4200.

EXERCISE 29

(OBJECTIVE TYPE QUESTIONS)

Directions : ***Mark (✓) against the correct answer :***

1. The cost price of a Rs. 100 stock at 4 discount, when brokerage is $\frac{1}{4}$% is :
   (*a*) Rs. 95.75 (*b*) Rs. 96 (*c*) Rs. 96.25 (*d*) Rs. 104.25
2. The cash realised on selling a 14% stock at Rs. 106.25, brokerage being $\frac{1}{4}$%, is :
   (*a*) Rs. 105.50 (*b*) Rs. 106 (*c*) Rs. 106.50 (*d*) Rs. 113.75
3. How many shares of market value Rs. 25 each can be purchased for Rs. 12750, brokerage being 2% ? **(M.A.T. 2002)**
   (*a*) 450 (*b*) 500 (*c*) 550 (*d*) 600
4. A man invests in a 16% stock at 128. The interest obtained by him is :
   (*a*) 8% (*b*) 12% (*c*) 12.5% (*d*) 16%
5. The income derived from a Rs. 100, 13% stock at Rs. 105, is :
   (*a*) Rs. 5 (*b*) Rs. 8 (*c*) Rs. 13 (*d*) Rs. 18
6. A man invested Rs. 4455 in Rs. 10 shares quoted at Rs. 8.25. If the rate of dividend be 12%, his annual income is :
   (*a*) Rs. 207.40 (*b*) Rs. 534.60 (*c*) Rs. 648 (*d*) Rs. 655.60
7. A man invested Rs. 14,400 in Rs. 100 shares of a company at 20% premium. If the company declares 5% dividend at the end of the year, then how much does he get ?
   (*a*) Rs. 500 (*b*) Rs. 600 (*c*) Rs. 650 (*d*) Rs. 720
   (Hotel Management, 2003)
8. A 6% stock yields 8%. The market value of the stock is :
   (*a*) Rs. 48 (*b*) Rs. 75 (*c*) Rs. 96 (*d*) Rs. 133.33

9. A 9% stock yields 8%. The market value of the stock is :
(a) Rs. 72 (b) Rs. 92 (c) Rs. 112.50 (d) Rs. 116.50

10. A 12% stock yielding 10% is quoted at :
(a) Rs. 83.33 (b) Rs. 110 (c) Rs. 112 (d) Rs. 120

11. By investing Rs. 1620 in 8% stock, Michael earns Rs. 135. The stock is then quoted at :
(a) Rs. 80 (b) Rs. 96 (c) Rs. 106 (d) Rs. 108

12. To produce an annual income of Rs. 1200 from a 12% stock at 90, the amount of stock needed is :
(a) Rs. 10,000 (b) Rs. 10,800 (c) Rs. 14,400 (d) Rs. 16,000

13. In order to obtain an income of Rs. 650 from 10% stock at Rs. 96, one must make an investment of :
(a) Rs. 3100 (b) Rs. 6240 (c) Rs. 6500 (d) Rs. 9600

14. By investing in $16\frac{2}{3}\%$ stock at 64, one earns Rs. 1500. The investment made is :
(a) Rs. 5640 (b) Rs. 5760 (c) Rs. 7500 (d) Rs. 9600

15. A man invested Rs. 1552 in a stock at 97 to obtain an income of Rs. 128. The dividend from the stock is :
(a) 7.5% (b) 8% (c) 9.7% (d) None of these.

16. A man bought 20 shares of Rs. 50 at 5 discount, the rate of dividend being $13\frac{1}{2}\%$. The rate of interest obtained is :
(a) $12\frac{1}{2}\%$ (b) $13\frac{1}{2}\%$ (c) 15% (d) $16\frac{2}{3}\%$

17. A man buys Rs. 20 shares paying 9% dividend. The man wants to have an interest of 12% on his money. The market value of each share is :
(a) Rs. 12 (b) Rs. 15 (c) Rs. 18 (d) Rs. 21

18. A man buys Rs. 50 shares in a company which pays 10% dividend. If the man gets 12.5% on his investment, at what price did he buy the shares ? **(L.I.C.A.A.O. 2003)**
(a) Rs. 37.50 (b) Rs. 40 (c) Rs. 48 (d) Rs. 52

19. The market value of a 10.5% stock, in which an income of Rs. 756 is derived by investing Rs. 9000, brokerage being $\frac{1}{4}\%$, is :
(a) Rs. 108.25 (b) Rs. 112.20 (c) Rs. 124.75 (d) Rs. 125.25

20. Sakshi invests a part of Rs. 12,000 in 12% stock at Rs. 120 and the remainder in 15% stock at Rs. 125. If his total dividend per annum is Rs. 1360, how much does he invest in 12% stock at Rs. 120 ?
(a) Rs. 4000 (b) Rs. 4500 (c) Rs. 5500 (d) Rs. 6000

21. Rs. 9800 are invested partly in 9% stock at 75 and 10% stock at 80 to have equal amount of incomes. The investment in 9% stock is :
(a) Rs. 4800 (b) Rs. 5000 (c) Rs. 5400 (d) Rs. 5600

22. A man invests some money partly in 9% stock at 96 and partly in 12% stock at 120. To obtain equal dividends from both, he must invest the money in the ratio :
(a) 3 : 4 (b) 3 : 5 (c) 4 : 5 (d) 16 : 15

23. Which is better investment — 11% stock at 143 or $9\frac{3}{4}\%$ stock at 117 ?
(a) 11% stock at 143 (b) $9\frac{3}{4}\%$ stock at 117 (c) Both are equally good
(d) Cannot be compared, as the total amount of investment is not given

24. Which is better investment, 12% stock at par with an income tax at the rate of 5 paise per rupee or $14\frac{2}{7}\%$ stock at 120 free from income tax ?

(*a*) 12% stock (*b*) $14\frac{2}{7}\%$ stock (*c*) Both are equally good

(*d*) Cannot be compared

25. A invested some money in 10% stock at 96. If B wants to invest in an equally good 12% stock, he must purchase a stock worth of :

(*a*) Rs. 80 (*b*) Rs. 115.20 (*c*) Rs. 120 (*d*) Rs. 125.40

ANSWERS

1. (*c*) **2.** (*b*) **3.** (*b*) **4.** (*c*) **5.** (*c*) **6.** (*c*) **7.** (*b*) **8.** (*b*) **9.** (*c*) **10.** (*d*)
11. (*b*) **12.** (*a*) **13.** (*b*) **14.** (*b*) **15.** (*b*) **16.** (*c*) **17.** (*b*) **18.** (*b*) **19.** (*c*) **20.** (*a*)
21. (*b*) **22.** (*d*) **23.** (*b*) **24.** (*b*) **25.** (*b*)

SOLUTIONS

1. C.P. = Rs. $\left(100 - 4 + \frac{1}{4}\right)$ = Rs. 96.25.

2. Cash realised = Rs. (106.25 – 0.25) = Rs. 106.

3. C.P. of each share = Rs. (25 + 2% of 25) = Rs. 25.50.

$\therefore$ Number of shares = $\left(\frac{12750}{25.50}\right)$ = 500.

4. By investing Rs. 128, income derived = Rs. 16.

By investing Rs. 100, income derived = Rs. $\left(\frac{16}{128} \times 100\right)$ = Rs. 12.5.

$\therefore$ Interest obtained = 12.5%.

5. Income on Rs. 100 stock = Rs. 13.

6. Number of shares = $\left(\frac{4455}{8.25}\right)$ = 540.

Face value = Rs. (540 × 10) = Rs. 5400.

Annual income = Rs. $\left(\frac{12}{100} \times 5400\right)$ = Rs. 648.

7. Number of shares = $\left(\frac{14400}{120}\right)$ = 120.

Face value = Rs. (100 × 120) = Rs. 12000.

Annual income = Rs. $\left(\frac{5}{100} \times 12000\right)$ = Rs. 600.

8. For an income of Rs. 8, investment = Rs. 100.

For an income of Rs. 6, investment = Rs. $\left(\frac{100}{8} \times 6\right)$ = Rs. 75.

$\therefore$ Market value of Rs. 100 stock = Rs. 75.

9. To obtain Rs. 8, investment = Rs. 100.

To obtain Rs. 9, investment = Rs. $\left(\frac{100}{8} \times 9\right)$ = Rs. 112.50.

∴ Market value of Rs. 100 stock = Rs. 112.50.

10. To earn Rs. 10, money invested = Rs. 100.

To earn Rs. 12, money invested = Rs. $\left(\frac{100}{10} \times 12\right)$ = Rs. 120.

∴ Market value of Rs. 100 stock = Rs. 120.

11. To earn Rs. 135, investment = Rs. 1620.

To earn Rs. 8, investment = Rs. $\left(\frac{1620}{135} \times 8\right)$ = Rs. 96.

∴ Market value of Rs. 100 stock = Rs. 96.

12. For an income of Rs. 12, stock needed = Rs. 100.

For an income of Rs. 1200, stock needed = Rs. $\left(\frac{100}{12} \times 1200\right)$ = Rs. 10,000.

13. To obtain Rs. 10, investment = Rs. 96.

To obtain Rs. 650, investment = Rs. $\left(\frac{96}{10} \times 650\right)$ = Rs. 6240.

14. To earn Rs. $\frac{50}{3}$, investment = Rs. 64.

To earn Rs. 1500, investment = Rs. $\left(64 \times \frac{3}{50} \times 1500\right)$ = Rs. 5760.

15. By investing Rs. 1552, income = Rs. 128.

By investing Rs. 97, income = Rs. $\left(\frac{128}{1552} \times 97\right)$ = Rs. 8.

∴ Dividend = 8%.

16. Investment = Rs. [20 × (50 − 5)] = Rs. 900.

Face value = Rs. (50 × 20) = Rs. 1000.

Dividend = Rs. $\left(\frac{27}{2} \times \frac{1000}{100}\right)$ = Rs. 135.

Interest obtained = $\left(\frac{135}{900} \times 100\right)$% = 15%.

17. Dividend on Rs. 20 = Rs. $\left(\frac{9}{100} \times 20\right)$ = Rs. $\frac{9}{5}$.

Rs. 12 is an income on Rs. 100.

∴ Rs. $\frac{9}{5}$ is an income on Rs. $\left(\frac{100}{12} \times \frac{9}{5}\right)$ = Rs. 15.

18. Dividend on 1 share = Rs. $\left(\frac{10}{100} \times 50\right)$ = Rs. 5.

Rs. 12.50 is an income on an investment of Rs. 100.

Rs. 5 is an income on an investment of Rs. $\left(100 \times \frac{2}{25} \times 5\right)$ = Rs. 40.

∴ Cost of 1 share = Rs. 40.

19. For an income of Rs. 756, investment = Rs. 9000.

For an income of Rs. $\frac{21}{2}$, investment = Rs. $\left(\frac{9000}{756} \times \frac{21}{2}\right)$ = Rs. 125.

$\therefore$ For a Rs. 100 stock, investment = Rs. 125.

Market value of Rs. 100 stock = Rs. $\left(125 - \frac{1}{4}\right)$ = Rs. 124.75.

20. Let investment in 12% stock be Rs. x.

Then, investment in 15% stock = Rs. $(12000 - x)$.

$$\frac{12}{120} \times x + \frac{15}{125} \times (12000 - x) = 1360$$

$$\Leftrightarrow \quad \frac{x}{10} + \frac{3}{25}(12000 - x) = 1360$$

$$\Leftrightarrow \quad 5x + 72000 - 6x = 1360 \times 50 \quad \Leftrightarrow \quad x = 4000.$$

21. Let the investment in 9% stock be Rs. x.

Then, investment in 10% stock = Rs. $(9800 - x)$.

$$\frac{9}{75} \times x = \frac{10}{80} \times (9800 - x) \quad \Leftrightarrow \quad \frac{3x}{25} = \frac{9800 - x}{8}$$

$$\Leftrightarrow \quad 24x = 9800 \times 25 - 25x \quad \Leftrightarrow \quad 49x = 9800 \times 25 \quad \Leftrightarrow \quad x = 5000.$$

22. For an income of Re. 1 in 9% stock at 96, investment = Rs. $\left(\frac{96}{9}\right)$ = Rs. $\frac{32}{3}$.

For an income of Re. 1 in 12% stock at 120, investment = Rs. $\left(\frac{120}{12}\right)$ = Rs. 10.

$\therefore$ Ratio of investments = $\frac{32}{3} : 10 = 32 : 30 = 16 : 15$.

23. Let investment in each case be Rs. (143×117).

Income in 1st case = Rs. $\left(\frac{11}{143} \times 143 \times 117\right)$ = Rs. 1287.

Income in 2nd case = Rs. $\left(\frac{39}{4 \times 117} \times 143 \times 117\right)$ = Rs. 1394.25.

Clearly, $9\frac{3}{4}\%$ stock at 117 is better.

24. Let investment in each case = Rs. (100×120).

Income from 12% stock = Rs. $\left(\frac{12}{100} \times 100 \times 120\right)$ = Rs. 1440.

Net income = Rs. $\left(1440 - \frac{5}{100} \times 1440\right)$ = Rs. 1368.

Income from $14\frac{2}{7}\%$ stock = Rs. $\left(\frac{100}{7 \times 20} \times 100 \times 120\right)$ = Rs. 1428.57.

Clearly, $14\frac{2}{7}\%$ stock is better.

25. For an income of Rs. 10, investment = Rs. 96.

For an income of Rs. 12, investment = Rs. $\left(\frac{96}{10} \times 12\right)$ = Rs. 115.20.

30. PERMUTATIONS AND COMBINATIONS

IMPORTANT FACTS AND FORMULAE

Factorial Notation : Let n be a positive integer. Then, factorial n, denoted by $\lfloor n$ or $n\,!$ is defined as :

$$n\,! = n\,(n-1)\,(n-2)\,.....\,3.2.1.$$

Examples : (*i*) $5\,! = (5 \times 4 \times 3 \times 2 \times 1) = 120$; (*ii*) $4\,! = (4 \times 3 \times 2 \times 1) = 24$ etc.

We define, $0\,! = 1$.

Permutations : *The different arrangements of a given number of things by taking some or all at a time, are called permutations.*

Ex. 1. All permutations (or arrangements) made with the letters a, b, c by taking two at a time are (***ab, ba, ac, ca, bc, cb***).

Ex. 2. All permutations made with the letters a, b, c, taking all at a time are : (***abc, acb, bac, bca, cab, cba***).

Number of Permutations : Number of all permutations of n things, taken r at a time, is given by :

$$^{n}P_{r} = n\,(n-1)\,(n-2)\,.....\,(n-r+1) = \frac{n\,!}{(n-r)\,!}$$

Examples : (*i*) $^{6}P_{2} = (6 \times 5) = 30$. (*ii*) $^{7}P_{3} = (7 \times 6 \times 5) = 210$.

Cor. ***Number of all permutations of n things, taken all at a time = n !***

An Important Result : *If there are n objects of which p_1 are alike of one kind; p_2 are alike of another kind; p_3 are alike of third kind and so on and p_r are alike of rth kind, such that $(p_1 + p_2 + ... + p_r) = n$.*

Then, ***number of permutations of these n objects is :***

$$\frac{n\,!}{(p_1\,!)\cdot(p_2\,!)\,.....\,(p_r\,!)}.$$

Combinations : *Each of the different groups or selections which can be formed by taking some or all of a number of objects, is called a combination.*

Ex. 1. Suppose we want to select two out of three boys A, B, C. Then, possible selections are AB, BC and CA.

Note that AB and BA represent the same selection.

Ex. 2. All the combinations formed by a, b, c, taking two at a time are ***ab, bc, ca.***

Ex. 3. The only combination that can be formed of three letters a, b, c taken all at a time is ***abc.***

Ex. 4. Various groups of 2 out of four persons A, B, C, D are :

AB, AC, AD, BC, BD, CD.

Ex. 5. Note that ab and ba are two different permutations but they represent the same combination.

Number of Combinations : The number of all combinations of n things, taken r at a time is :

$$^nC_r = \frac{n!}{(r!)(n-r)!} = \frac{n(n-1)(n-2)\ldots\text{ to } r \text{ factors}}{r!}.$$

Note that : $^nC_n = 1$ *and* $^nC_0 = 1$.

An Important Result : $^nC_r = {}^nC_{(n-r)}$.

Example : (*i*) $^{11}C_4 = \frac{(11\times10\times9\times8)}{(4\times3\times2\times1)} = 330.$

(*ii*) $^{16}C_{13} = {}^{16}C_{(16-13)} = {}^{16}C_3 = \frac{16\times15\times14}{3!} = \frac{16\times15\times14}{3\times2\times1} = 560.$

SOLVED EXAMPLES

Ex. 1. *Evaluate :* $\frac{30!}{28!}$

Sol. We have, $\frac{30!}{28!} = \frac{30\times29\times(28!)}{28!} = (30\times29) = 870.$

Ex. 2. *Find the value of* (*i*) $^{60}P_3$ (*ii*) $^4P_4$

Sol. (*i*) $^{60}P_3 = \frac{60!}{(60-3)!} = \frac{60!}{57!} = \frac{60\times59\times58\times(57!)}{57!} = (60\times59\times58) = 205320.$

(*ii*) $^4P_4 = 4! = (4\times3\times2\times1) = 24.$

Ex. 3. *Find the value of* (*i*) $^{10}C_3$ (*ii*) $^{100}C_{98}$ (*iii*) $^{50}C_{50}$

Sol. (*i*) $^{10}C_3 = \frac{10\times9\times8}{3!} = \frac{10\times9\times8}{3\times2\times1} = 120.$

(*ii*) $^{100}C_{98} = {}^{100}C_{(100-98)} = {}^{100}C_2 = \left(\frac{100\times99}{2\times1}\right) = 4950.$

(*iii*) $^{50}C_{50} = 1.$ [$\because {}^nC_n = 1$]

Ex. 4. ***How many words can be formed by using all letters of the word 'BIHAR' ?***

Sol. The word BIHAR contains 5 different letters.

$\therefore$ Required number of words $= {}^5P_5 = 5! = (5\times4\times3\times2\times1) = 120.$

Ex. 5. ***How many words can be formed by using all the letters of the word 'DAUGHTER' so that the vowels always come together ?***

Sol. Given word contains 8 different letters. When the vowels AUE are always together, we may suppose them to form an entity, treated as one letter.

Then, the letters to be arranged are DGHTR (AUE).

These 6 letters can be arranged in $^6P_6 = 6! = 720$ ways.

The vowels in the group (AUE) may be arranged in $3! = 6$ ways.

$\therefore$ Required number of words $= (720\times6) = 4320.$

Ex. 6. ***How many words can be formed from the letters of the word 'EXTRA', so that the vowels are never together ?***

Sol. The given word contains 5 different letters.

Taking the vowels EA together, we treat them as one letter.

Then, the letters to be arranged are XTR (EA).

These letters can be arranged in 4 ! = 24 ways.

The vowels EA may be arranged amongst themselves in 2 ! = 2 ways.

Number of words, each having vowels together = (24 × 2) = 48.

Total number of words formed by using all the letters of the given words

$= 5\,! = (5 \times 4 \times 3 \times 2 \times 1) = 120.$

Number of words, each having vowels never together = (120 − 48) = 72.

Ex. 7. ***How many words can be formed from the letters of the word 'DIRECTOR' so that the vowels are always together ?***

Sol. In the given word, we treat the vowels IEO as one letter.

Thus, we have DRCTR (IEO).

This group has 6 letters of which R occurs 2 times and others are different.

Number of ways of arranging these letters $= \frac{6\,!}{2\,!} = 360.$

Now 3 vowels can be arranged among themselves in 3 ! = 6 ways.

∴ Required number of ways = (360 × 6) = 2160.

Ex. 8. ***In how many ways can a cricket eleven be chosen out of a batch of 15 players?***

Sol. Required number of ways $= {}^{15}C_{11} = {}^{15}C_{(15-11)} = {}^{15}C_4$

$$= \frac{15 \times 14 \times 13 \times 12}{4 \times 3 \times 2 \times 1} = 1365.$$

Ex. 9. ***In how many ways, a committee of 5 members can be selected from 6 men and 5 ladies, consisting of 3 men and 2 ladies ?***

Sol. (3 men out 6) and (2 ladies out of 5) are to be chosen.

∴ Required number of ways $= ({}^6C_3 \times {}^5C_2) = \left(\frac{6 \times 5 \times 4}{3 \times 2 \times 1} \times \frac{5 \times 4}{2 \times 1}\right) = 200.$

EXERCISE 30

(OBJECTIVE TYPE QUESTIONS)

Directions : ***Mark (✓) against the correct answer :***

1. The value of ${}^{75}P_2$ is :
   (*a*) 2775 (*b*) 150 (*c*) 5550 (*d*) None of these
2. How many 4-letter words with or without meaning, can be formed out of the letters of the word, 'LOGARITHMS', if repetition of letters is not allowed ?
   (*a*) 40 (*b*) 400 (*c*) 5040 (*d*) 2520
3. How many words with or without meaning, can be formed by using all the letters of the word, 'DELHI', using each letter exactly once ?
   (*a*) 10 (*b*) 25 (*c*) 60 (*d*) 120
4. In how many ways can the letters of the word 'APPLE' be arranged ?
   (*a*) 720 (*b*) 120 (*c*) 60 (*d*) 180 (*e*) None of these
5. In how many ways can the letters of the word 'LEADER' be arranged ?
   (*a*) 72 (*b*) 144 (*c*) 360 (*d*) 720 (*e*) None of these
   (Bank P.O. 2003)

6. In how many different ways can the letters of the word 'RUMOUR' be arranged ?
(a) 180 (b) 90 (c) 30 (d) 720 (e) None of these
(Bank P.O. 2003)

7. How many words can be formed by using all the letters of the word, 'ALLAHABAD' ?
(a) 3780 (b) 1890 (c) 7560 (d) 2520 (e) None of these

8. How many arrangements can be made out of the letters of the word 'ENGINEERING' ?
(a) 277200 (b) 92400 (c) 69300 (d) 23100 (e) None of these

9. How many words can be formed from the letters of the word 'SIGNATURE' so that the vowels always come together ? **(Bank P.O. 2003)**
(a) 720 (b) 1440 (c) 2880 (d) 3600 (e) 17280

10. In how many different ways can the letters of the word 'OPTICAL' be arranged so that the vowels always come together ? **(M.B.A. 2002)**
(a) 120 (b) 720 (c) 4320 (d) 2160 (e) None of these

11. In how many different ways can the letters of the word 'SOFTWARE' be arranged in such a way that the vowels always come together ? **(Bank P.O. 2003)**
(a) 120 (b) 360 (c) 1440 (d) 13440 (e) 720

12. In how many different ways can the letters of the word 'LEADING' be arranged in such a way that the vowels always come together ? **(Bank P.O. 2002)**
(a) 360 (b) 480 (c) 720 (d) 5040 (e) None of these

13. In how many different ways can the letters of the word 'JUDGE' be arranged in such a way that the vowels always come together ? **(S.B.I.P.O. 2001)**
(a) 48 (b) 120 (c) 124 (d) 160 (e) None of these

14. In how many different ways can the letters of the word 'AUCTION' be arranged in such a way that the vowels always come together ? **(S.B.I.P.O. 2000)**
(a) 30 (b) 48 (c) 144 (d) 576 (e) None of these

15. In how many different ways can the letters of the word 'BANKING' be arranged so that the vowels always come together ? **(Bank P.O. 2003)**
(a) 120 (b) 240 (c) 360 (d) 540 (e) 720

16. In how many different ways can the letters of the word 'CORPORATION' be arranged so that the vowels always come together ? **(S.B.I.P.O. 2003)**
(a) 810 (b) 1440 (c) 2880 (d) 50400 (e) 5760

17. In how many different ways can the letters of the word 'MATHEMATICS' be arranged so that the vowels always come together ?
(a) 10080 (b) 4989600 (c) 120960 (d) None of these

18. In how many different ways can the letters of the word 'DETAIL' be arranged in such a way that the vowels occupy only the odd positions ? **(Bank P.O. 2002)**
(a) 32 (b) 48 (c) 36 (d) 60 (e) 120

19. In how many different ways can the letters of the word 'MACHINE' be arranged so that the vowels may occupy only the odd positions ?
(a) 210 (b) 576 (c) 144 (d) 1728 (e) 3456

20. In how many ways can a group of 5 men and 2 women be made out of a total of 7 men and 3 women ? **(Bank P.O. 2003)**
(a) 63 (b) 90 (c) 126 (d) 45 (e) 135

21. In how many ways a committee, consisting of 5 men and 6 women can be formed from 8 men and 10 women ? **(Bank P.O. 2003)**
(a) 266 (b) 5040 (c) 11760 (d) 86400 (e) None of these

22. From a group of 7 men and 6 women, five persons are to be selected to form a committee so that at least 3 men are there on the committee. In how many ways can it be done ? **(M.B.A. 2002)**
(a) 564 (b) 645 (c) 735 (d) 756 (e) None of these

23. In a group of 6 boys and 4 girls, four children are to be selected. In how many different ways can they be selected such that at least one boy should be there ?
(*a*) 159 (*b*) 194 (*c*) 205 (*d*) 209 (*e*) None of these
(S.B.I.P.O. 2000)

24. A box contains 2 white balls, 3 black balls and 4 red balls. In how many ways can 3 balls be drawn from the box, if at least one black ball is to be included in the draw ?
(*a*) 32 (*b*) 48 (*c*) 64 (*d*) 96 (*e*) None of these
(Bank P.O. 1998)

25. How many 3-digit numbers can be formed from the digits 2, 3, 5, 6, 7 and 9, which are divisible by 5 and none of the digits is repeated ? **(S.S.C. 2000)**
(*a*) 5 (*b*) 10 (*c*) 15 (*d*) 20

26. In how many ways can 21 books on English and 19 books on Hindi be placed in a row on a shelf so that two books on Hindi may not be together ?
(*a*) 3990 (*b*) 1540 (*c*) 1995 (*d*) 3672 (*e*) None of these

27. Out of 7 consonants and 4 vowels, how many words of 3 consonants and 2 vowels can be formed ?
(*a*) 210 (*b*) 1050 (*c*) 25200 (*d*) 21400 (*e*) None of these

ANSWERS

1. (*c*)	2. (*c*)	3. (*d*)	4. (*c*)	5. (*c*)	6. (*a*)
7. (*c*)	8. (*a*)	9. (*e*)	10. (*c*)	11. (*e*)	12. (*c*)
13. (*a*)	14. (*d*)	15. (*e*)	16. (*d*)	17. (*c*)	18. (*c*)
19. (*b*)	20. (*a*)	21. (*c*)	22. (*d*)	23. (*d*)	24. (*c*)
25. (*d*)	26. (*b*)	27. (*c*)			

SOLUTIONS

1. $^{75}P_2 = \frac{75\,!}{(75-2)\,!} = \frac{75\,!}{73\,!} = \frac{75 \times 74 \times (73\,!)}{73\,!} = (75 \times 74) = 5550.$

2. 'LOGARITHM' contains 10 different letters.
Required number of words = Number of arrangements of 10 letters, taking 4 at a time
$= {}^{10}P_4 = (10 \times 9 \times 8 \times 7) = 5040.$

3. The word 'DELHI' contains 5 different letters.
Required number of words = Number of arrangements of 5 letters, taken all at a time
$= {}^{5}P_5 = 5\,! = (5 \times 4 \times 3 \times 2 \times 1) = 120.$

4. The word 'APPLE' contains 5 letters, 1A, 2P, 1L and 1E.
$\therefore$ Required number of ways $= \frac{5\,!}{(1\,!)(2\,!)(1\,!)(1\,!)} = 60.$

5. The word 'LEADER' contains 6 letters, namely 1L, 2E, 1A, 1D and 1R.
$\therefore$ Required number of ways $= \frac{6\,!}{(1\,!)(2\,!)(1\,!)(1\,!)(2\,!)} = 360.$

6. The word 'RUMOUR' contains 6 letters, namely 2R, 2U, 1M and 1U.
$\therefore$ Required number of ways $= \frac{6\,!}{(2\,!)(2\,!)(1\,!)(1\,!)} = 180.$

7. The word 'ALLAHABAD' contains 9 letters, namely 4A, 2L, 1H, 1B and 1D.

$\therefore$ Requisite number of words $= \dfrac{9\,!}{(4\,!)(2\,!)(1\,!)(1\,!)(1\,!)} = 7560.$

8. The word 'ENGINEERING' contains 11 letters, namely 3E, 3N, 2G, 2I and 1R.

$\therefore$ Required number of arrangements $= \dfrac{11\,!}{(3\,!)(3\,!)(2\,!)(2\,!)(1\,!)} = 277200.$

9. The word 'SIGNATURE' contains 9 different letters.
When the vowels IAUE are taken together, they can be supposed to form an entity, treated as one letter.
Then, the letters to be arranged are SGNTR (IAUE).
These 6 letters can be arranged in ${}^6P_6 = 6\,! = 720$ ways.
The vowels in the group (IAUE) can be arranged amongst themselves in ${}^4P_4 = 4\,! = 24$ ways.
$\therefore$ Required number of words $= (720 \times 24) = 17280.$

10. The word 'OPTICAL' contains 7 different letters.
When the vowels OIA are always together, they can be supposed to form one letter.
Then, we have to arrange the letters PTCL (OIA).
Now, 5 letters can be arranged in $5\,! = 120$ ways.
The vowels (OIA) can be arranged among themselves in $3\,! = 6$ ways.
$\therefore$ Required number of ways $= (120 \times 6) = 720.$

11. The word 'SOFTWARE' contains 8 different letters.
When the vowels OAE are always together, they can be supposed to form one letter.
Thus, we have to arrange the letters SFTWR (OAE).
Now, 5 letters can be arranged in $6\,! = 720$ ways.
The vowels (OAE) can be arranged among themselves in $3\,! = 6$ ways.
$\therefore$ Required number of ways $= (720 \times 6) = 4320.$

12. The word 'LEADING' has 7 different letters.
When the vowels EAI are always together, they can be supposed to form one letter.
Then, we have to arrange the letters LDNG (EAI).
Now, 5 letters can be arranged in $5\,! = 120$ ways.
The vowels (EAI) can be arranged among themselves in $3\,! = 6$ ways.
$\therefore$ Required number of ways $= (120 \times 6) = 720.$

13. The word 'JUDGE' has 5 different letters.
When the vowels UE are always together, they can be supposed to form one letter.
Then, we have to arrange the letters JDG (UE).
Now, 4 letters can be arranged in $4\,! = 24$ ways.
The vowels (UE) can be arranged among themselves in $2\,! = 2$ ways.
$\therefore$ Required number of ways $= (24 \times 2) = 48.$

14. The word 'AUCTION' has 7 different letters.
When the vowels AUIO are always together, they can be supposed to form one letter.
Then, we have to arrange the letters CTN (AUIO).
Now, 4 letters can be arranged in $4\,! = 24$ ways.
The vowels (AUIO) can be arranged among themselves in $4\,! = 24$ ways.
$\therefore$ Required number of ways $= (24 \times 24) = 576.$

15. In the word 'BANKING', we treat the two vowels AI as one letter. Thus, we have BNKNG (AI).
This has 6 letters of which N occurs 2 times and the rest are different.

Number of ways of arranging these letters $= \dfrac{6\,!}{(2\,!)(1\,!)(1\,!)(1\,!)(1\,!)} = 360.$

Now, 2 vowels AI can be arranged in $2\,! = 2$ ways.

$\therefore$ Required number of ways $= (360 \times 2) = 720.$

16. In the word 'CORPORATION', we treat the vowels OOAIO as one letter. Thus, we have CRPRTN (OOAIO).

This has 7 letters of which R occurs 2 times and the rest are different.

Number of ways of arranging these letters $= \dfrac{7\,!}{2\,!} = 2520.$

Now, 5 vowels in which O occurs 3 times and the rest are different, can be arranged in $\dfrac{5\,!}{3\,!} = 20$ ways.

$\therefore$ Required number of ways $= (2520 \times 20) = 50400.$

17. In the word 'MATHEMATICS' we treat the vowels AEAI as one letter. Thus, we have MTHMTCS (AEAI).

Now, we have to arrange 8 letters, out of which M occurs twice, T occurs twice and the rest are different.

$\therefore$ Number of ways of arranging these letters $= \dfrac{8\,!}{(2\,!)(2\,!)} = 10080.$

Now, AEAI has 4 letters in which A occurs 2 times and the rest are different.

Number of ways of arranging these letters $= \dfrac{4\,!}{2\,!} = 12.$

$\therefore$ Required number of words $= (10080 \times 12) = 120960.$

18. There are 6 letters in the given word, out of which there are 3 vowels and 3 consonants. Let us mark these positions as under :

$$\begin{pmatrix}1\end{pmatrix}\begin{pmatrix}2\end{pmatrix}\begin{pmatrix}3\end{pmatrix}\begin{pmatrix}4\end{pmatrix}\begin{pmatrix}5\end{pmatrix}\begin{pmatrix}6\end{pmatrix}$$

Now, 3 vowels can be placed at any of the three places out of 4, marked 1, 3, 5.

Number of ways of arranging the vowels $= {}^3P_3 = 3\,! = 6.$

Also, the 3 consonants can be arranged at the remaining 3 positions.

Number of ways of these arrangements $= {}^3P_3 = 3\,! = 6.$

Total number of ways $= (6 \times 6) = 36.$

19. There are 7 letters in the given word, out of which there are 3 vowels and 4 consonants. Let us mark the positions to be filled up as follows :

$$\begin{pmatrix}1\end{pmatrix}\begin{pmatrix}2\end{pmatrix}\begin{pmatrix}3\end{pmatrix}\begin{pmatrix}4\end{pmatrix}\begin{pmatrix}5\end{pmatrix}\begin{pmatrix}6\end{pmatrix}\begin{pmatrix}7\end{pmatrix}$$

Now, 3 vowels can be placed at any of the three places, out of the four marked 1, 3, 5, 7.

$\therefore$ Number of ways of arranging the vowels $= {}^4P_3 = (4 \times 3 \times 2) = 24.$

Also, the 4 consonants at the remaining 4 positions may be arranged in

$= {}^4P_4 = 4\,! = 24$ ways.

$\therefore$ Required number of ways $= (24 \times 24) = 576.$

20. Required number of ways $= ({}^7C_5 \times {}^3C_2) = ({}^7C_2 \times {}^3C_1) = \left(\dfrac{7 \times 6}{2 \times 1} \times 3\right) = 63.$