21. Required number of ways = $({}^8C_5 \times {}^{10}C_6)$

$$= ({}^8C_3 \times {}^{10}C_4) = \left(\frac{8\times7\times6}{3\times2\times1} \times \frac{10\times9\times8\times7}{4\times3\times2\times1}\right) = 11760.$$

22. We may have (3 men and 2 women) or (4 men and 1 woman) or (5 men only)

$\therefore$ Required number of ways $= ({}^7C_3 \times {}^6C_2) + ({}^7C_4 \times {}^6C_1) + ({}^7C_5)$

$$= \left(\frac{7\times6\times5}{3\times2\times1} \times \frac{6\times5}{2\times1}\right) + ({}^7C_3 \times {}^6C_1) + ({}^7C_2)$$

$$= 525 + \left(\frac{7\times6\times5}{3\times2\times1} \times 6\right) + \left(\frac{7\times6}{2\times1}\right)$$

$$= (525 + 210 + 21) = 756.$$

23. We may have (1 boy and 3 girls) or (2 boys and 2 girls) or (3 boys and 1 girl) or (4 boys).

$\therefore$ Required number of ways $= ({}^6C_1 \times {}^4C_3) + ({}^6C_2 \times {}^4C_2) + ({}^6C_3 \times {}^4C_1) + ({}^6C_4)$

$$= ({}^6C_1 \times {}^4C_1) + ({}^6C_2 \times {}^4C_2) + ({}^6C_3 \times {}^4C_1) + ({}^6C_2)$$

$$= (6 \times 4) + \left(\frac{6\times5}{2\times1} \times \frac{4\times3}{2\times1}\right) + \left(\frac{6\times5\times4}{3\times2\times1} \times 4\right) + \left(\frac{6\times5}{2\times1}\right)$$

$$= (24 + 90 + 80 + 15) = 209.$$

24. We may have (1 black and 2 non-black) or (2 black and 1 non-black) or (3 black).

$\therefore$ Required number of ways $= ({}^3C_1 \times {}^6C_2) + ({}^3C_2 \times {}^6C_1) + ({}^3C_3)$

$$= \left(3 \times \frac{6\times5}{2\times1}\right) + \left(\frac{3\times2}{2\times1} \times 6\right) + 1 = (45 + 18 + 1) = 64.$$

25. Since each desired number is divisible by 5, so we must have 5 at the unit place. So, there is 1 way of doing it.

Tens place can be filled by any of the remaining 5 numbers.

So, there are 5 ways of filling the tens place.

The hundreds place can now be filled by any of the remaining 4 digits. So, there are 4 ways of filling it.

$\therefore$ Required number of numbers $= (1 \times 5 \times 4) = 20$.

26. In order that two books on Hindi are never together, we must place all these books as under :

X E X E X E X ..... X E X

where E denotes the position of an English book and X that of a Hindi book.

Since there are 21 books on English, the number of places marked X are therefore, 22.

Now, 19 places out of 22 can be chosen in ${}^{22}C_{19} = {}^{22}C_3 = \dfrac{22\times21\times20}{3\times2\times1} = 1540$ ways.

Hence, the required number of ways = 1540.

27. Number of ways of selecting (3 consonants out of 7) and (2 vowels out of 4)

$$= ({}^7C_3 \times {}^4C_2) = \left(\frac{7\times6\times5}{3\times2\times1} \times \frac{4\times3}{2\times1}\right) = 210.$$

Number of groups, each having 3 consonants and 2 vowels = 210.

Each group contains 5 letters.

Number of ways of arranging 5 letters among themselves

$= 5\,! = (5 \times 4 \times 3 \times 2 \times 1) = 120.$

$\therefore$ Required number of words $= (210 \times 120) = 25200$.

31. PROBABILITY

IMPORTANT FACTS AND FORMULAE

1. **Experiment** : An operation which can produce some well-defined outcomes is called an experiment.
2. **Random Experiment** : An experiment in which all possible outcomes are known and the exact output cannot be predicted in advance, is called a random experiment.

   Examples of Performing a Random Experiment :

   (*i*) Rolling an unbiased dice.

   (*ii*) Tossing a fair coin.

   (*iii*) Drawing a card from a pack of well-shuffled cards.

   (*iv*) Picking up a ball of certain colour from a bag containing balls of different colours.

   Details :

   (*i*) When we throw a coin. Then either a Head (H) or a Tail (T) appears.

   (*ii*) A dice is a solid cube, having 6 faces, marked 1, 2, 3, 4, 5, 6 respectively. When we throw a die, the outcome is the number that appears on its upper face.

   (*iii*) A pack of cards has 52 cards.

   It has 13 cards of each suit, namely **Spades, Clubs, Hearts** and **Diamonds.**

   Cards of spades and clubs are **black cards.**

   Cards of hearts and diamonds are **red cards.**

   There are 4 honours of each suit.

   These are **Aces, Kings, Queens** and **Jacks.**

   These are called **face cards.**
3. **Sample Space** : When we perform an experiment, then the set S of all possible outcomes is called the **Sample Space.**

   Examples of Sample Spaces :

   (*i*) In tossing a coin, S = {H, T}.

   (*ii*) If two coins are tossed, then S = {HH, HT, TH, T T}.

   (*iii*) In rolling a dice, we have, S = {1, 2, 3, 4, 5, 6}.
4. **Event** : Any subset of a sample space is called an event.
5. **Probability of Occurrence of an Event :**

   Let S be the sample space and let E be an event.

   Then, $E \subseteq S$.

   $$\therefore \quad P(E) = \frac{n(E)}{n(S)}.$$
6. **Results on Probability :**

   (*i*) $P(S) = 1$ (*ii*) $0 \le P(E) \le 1$ (*iii*) $P(\phi) = 0$

   (*iv*) For any events A and B, we have :

   $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

   (*v*) If $\overline{A}$ denotes (not-A), then $P(\overline{A}) = 1 - P(A)$.

SOLVED EXAMPLES

Ex. 1. ***In a throw of a coin, find the probability of getting a head.***

Sol. Here S = {H, T} and E = {H}.

$\therefore \quad P(E) = \dfrac{n(E)}{n(S)} = \dfrac{1}{2}.$

Ex. 2. ***Two unbiased coins are tossed. What is the probability of getting at most one head ?***

Sol. Here S = {HH, HT, TH, T T}.

Let E = event of getting at most one head.

$\therefore$ E = {T T, HT, TH}.

$\therefore \quad P(E) = \dfrac{n(E)}{n(S)} = \dfrac{3}{4}.$

Ex. 3. ***An unbiased die is tossed. Find the probability of getting a multiple of 3.***

Sol. Here S = {1, 2, 3, 4, 5, 6}.

Let E be the event of getting a multiple of 3.

Then, E = {3, 6}.

$\therefore \quad P(E) = \dfrac{n(E)}{n(S)} = \dfrac{2}{6} = \dfrac{1}{3}.$

Ex. 4. ***In a simultaneous throw of a pair of dice, find the probability of getting a total more than 7.***

Sol. Here, $n(S) = (6 \times 6) = 36$.

Let E = Event of getting a total more than 7

= {(2, 6), (3, 5), (3, 6), (4, 4), (4, 5), (4, 6), (5, 3), (5, 4), (5, 5), (5, 6), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}.

$\therefore \quad P(E) = \dfrac{n(E)}{n(S)} = \dfrac{15}{36} = \dfrac{5}{12}.$

Ex. 5. ***A bag contains 6 white and 4 black balls. Two balls are drawn at random. Find the probability that they are of the same colour.***

Sol. Let S be the sample space. Then,

$n(S)$ = Number of ways of drawing 2 balls out of $(6+4) = {}^{10}C_2 = \dfrac{(10 \times 9)}{(2 \times 1)} = 45.$

Let E = Event of getting both balls of the same colour. Then,

$n(E)$ = Number of ways of drawing (2 balls out of 6) or (2 balls out of 4)

$= ({}^6C_2 + {}^4C_2) = \dfrac{(6 \times 5)}{(2 \times 1)} + \dfrac{(4 \times 3)}{(2 \times 1)} = (15 + 6) = 21.$

$\therefore \quad P(E) = \dfrac{n(E)}{n(S)} = \dfrac{21}{45} = \dfrac{7}{15}.$

Ex. 6. ***Two dice are thrown together. What is the probability that the sum of the numbers on the two faces is divisible by 4 or 6 ?***

Sol. Clearly, $n(S) = 6 \times 6 = 36$.

Let E be the event that the sum of the numbers on the two faces is divisible by 4 or 6. Then

E = {(1, 3), (1, 5), (2, 2), (2, 4), (2, 6), (3, 1), (3, 3), (3, 5), (4, 2), (4, 4), (5, 1), (5, 3), (6, 2), (6, 6)}

$\therefore \quad n(E) = 14.$

Hence, $P(E) = \dfrac{n(E)}{n(S)} = \dfrac{14}{36} = \dfrac{7}{18}.$

Ex. 7. ***Two cards are drawn at random from a pack of 52 cards. What is the probability that either both are black or both are queens ?***

Sol. We have $n(S) = {}^{52}C_2 = \frac{(52 \times 51)}{(2 \times 1)} = 1326.$

Let A = event of getting both black cards;
B = event of getting both queens.

$\therefore$ $A \cap B$ = event of getting queens of black cards.

$\therefore$ $n(A) = {}^{26}C_2 = \frac{(26 \times 25)}{(2 \times 1)} = 325,\ n(B) = {}^{4}C_2 = \frac{(4 \times 3)}{(2 \times 1)} = 6$ and $n(A \cap B) = {}^{2}C_2 = 1.$

$\therefore$ $P(A) = \frac{n(A)}{n(S)} = \frac{325}{1326};\ P(B) = \frac{n(B)}{n(S)} = \frac{6}{1326}$ and $P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{1}{1326}.$

$\therefore$ $P(A \cup B) = P(A) + P(B) - P(A \cap B) = \left(\frac{325}{1326} + \frac{6}{1326} - \frac{1}{1326}\right) = \frac{330}{1326} = \frac{55}{221}.$

EXERCISE 31

(OBJECTIVE TYPE QUESTIONS)

Directions : ***Mark (✓) against the correct answer :***

1. In a simultaneous throw of two coins, the probability of getting at least one head is :
   (*a*) $\frac{1}{2}$ (*b*) $\frac{1}{3}$ (*c*) $\frac{2}{3}$ (*d*) $\frac{3}{4}$
2. Three unbiased coins are tossed. What is the probability of getting at least 2 heads ?
   (*a*) $\frac{1}{4}$ (*b*) $\frac{1}{2}$ (*c*) $\frac{1}{3}$ (*d*) $\frac{1}{8}$
3. Three unbiased coins are tossed. What is the probability of getting at most two heads ?
   (*a*) $\frac{3}{4}$ (*b*) $\frac{1}{4}$ (*c*) $\frac{3}{8}$ (*d*) $\frac{7}{8}$
4. In a single throw of a die, what is the probability of getting a number greater than 4 ?
   (*a*) $\frac{1}{2}$ (*b*) $\frac{1}{3}$ (*c*) $\frac{2}{3}$ (*d*) $\frac{1}{4}$
5. In a simultaneous throw of two dice, what is the probability of getting a total of 7 ?
   (*a*) $\frac{1}{6}$ (*b*) $\frac{1}{4}$ (*c*) $\frac{2}{3}$ (*d*) $\frac{3}{4}$
6. What is the probability of getting a sum 9 from two throws of a dice ?
   (*a*) $\frac{1}{6}$ (*b*) $\frac{1}{8}$ (*c*) $\frac{1}{9}$ (*d*) $\frac{1}{12}$
   (M.B.A. 2002)
7. In a simultaneous throw of two dice, what is the probability of getting a doublet ?
   (*a*) $\frac{1}{6}$ (*b*) $\frac{1}{4}$ (*c*) $\frac{2}{3}$ (*d*) $\frac{3}{7}$
8. In a simultaneous throw of two dice, what is the probability of getting a total of 10 or 11 ?
   (*a*) $\frac{1}{4}$ (*b*) $\frac{1}{6}$ (*c*) $\frac{7}{12}$ (*d*) $\frac{5}{36}$

9. Two dice are thrown simultaneously. What is the probability of getting two numbers whose product is even ? **(Asstt. PF Commissioner's Exam, 2002)**

(*a*) $\frac{1}{2}$ (*b*) $\frac{3}{4}$ (*c*) $\frac{3}{8}$ (*d*) $\frac{5}{16}$

10. Tickets numbered 1 to 20 are mixed up and then a ticket is drawn at random. What is the probability that the ticket drawn bears a number which is a multiple of 3 ?

(*a*) $\frac{3}{10}$ (*b*) $\frac{3}{20}$ (*c*) $\frac{2}{5}$ (*d*) $\frac{1}{2}$

11. Tickets numbered 1 to 20 are mixed up and then a ticket is drawn at random. What is the probability that the ticket drawn has a number which is a multiple of 3 or 5 ?

(*a*) $\frac{1}{2}$ (*b*) $\frac{2}{5}$ (*c*) $\frac{8}{15}$ (*d*) $\frac{9}{20}$

12. In a lottery, there are 10 prizes and 25 blanks. A lottery is drawn at random. What is the probability of getting a prize ?

(*a*) $\frac{1}{10}$ (*b*) $\frac{2}{5}$ (*c*) $\frac{2}{7}$ (*d*) $\frac{5}{7}$

13. One card is drawn at random from a pack of 52 cards. What is the probability that the card drawn is a face card ?

(*a*) $\frac{1}{13}$ (*b*) $\frac{4}{13}$ (*c*) $\frac{1}{4}$ (*d*) $\frac{9}{52}$

14. A card is drawn from a pack of 52 cards. The probability of getting a queen of club or a king of heart is :

(*a*) $\frac{1}{13}$ (*b*) $\frac{2}{13}$ (*c*) $\frac{1}{26}$ (*d*) $\frac{1}{52}$

15. One card is drawn from a pack of 52 cards. What is the probability that the card drawn is either a red card or a king ?

(*a*) $\frac{1}{2}$ (*b*) $\frac{6}{13}$ (*c*) $\frac{7}{13}$ (*d*) $\frac{27}{52}$

16. From a pack of 52 cards, one card is drawn at random. What is the probability that the card drawn is a ten or a spade ?

(*a*) $\frac{4}{13}$ (*b*) $\frac{1}{4}$ (*c*) $\frac{1}{13}$ (*d*) $\frac{1}{26}$

17. The probability that a card drawn from a pack of 52 cards will be a diamond or a king, is :

(*a*) $\frac{2}{13}$ (*b*) $\frac{4}{13}$ (*c*) $\frac{1}{13}$ (*d*) $\frac{1}{52}$

18. From a pack of 52 cards, two cards are drawn together at random. What is the probability of both the cards being kings ? **(M.B.A. 2002; Railways, 2002)**

(*a*) $\frac{1}{15}$ (*b*) $\frac{25}{57}$ (*c*) $\frac{35}{256}$ (*d*) $\frac{1}{221}$

19. Two cards are drawn together from a pack of 52 cards. The probability that one is a spade and one is a heart, is : **(M.B.A. 2000)**

(*a*) $\frac{3}{20}$ (*b*) $\frac{29}{34}$ (*c*) $\frac{47}{100}$ (*d*) $\frac{13}{102}$

20. Two cards are drawn from a pack of 52 cards. The probability that either both are red or both are kings, is :

(*a*) $\frac{7}{13}$ (*b*) $\frac{3}{26}$ (*c*) $\frac{63}{221}$ (*d*) $\frac{55}{221}$

9. Two dice are thrown simultaneously. What is the probability of getting two numbers whose product is even ? **(Asstt. PF Commissioner's Exam, 2002)**

(*a*) $\frac{1}{2}$ (*b*) $\frac{3}{4}$ (*c*) $\frac{3}{8}$ (*d*) $\frac{5}{16}$

10. Tickets numbered 1 to 20 are mixed up and then a ticket is drawn at random. What is the probability that the ticket drawn bears a number which is a multiple of 3 ?

(*a*) $\frac{3}{10}$ (*b*) $\frac{3}{20}$ (*c*) $\frac{2}{5}$ (*d*) $\frac{1}{2}$

11. Tickets numbered 1 to 20 are mixed up and then a ticket is drawn at random. What is the probability that the ticket drawn has a number which is a multiple of 3 or 5 ?

(*a*) $\frac{1}{2}$ (*b*) $\frac{2}{5}$ (*c*) $\frac{8}{15}$ (*d*) $\frac{9}{20}$

12. In a lottery, there are 10 prizes and 25 blanks. A lottery is drawn at random. What is the probability of getting a prize ?

(*a*) $\frac{1}{10}$ (*b*) $\frac{2}{5}$ (*c*) $\frac{2}{7}$ (*d*) $\frac{5}{7}$

13. One card is drawn at random from a pack of 52 cards. What is the probability that the card drawn is a face card ?

(*a*) $\frac{1}{13}$ (*b*) $\frac{4}{13}$ (*c*) $\frac{1}{4}$ (*d*) $\frac{9}{52}$

14. A card is drawn from a pack of 52 cards. The probability of getting a queen of club or a king of heart is :

(*a*) $\frac{1}{13}$ (*b*) $\frac{2}{13}$ (*c*) $\frac{1}{26}$ (*d*) $\frac{1}{52}$

15. One card is drawn from a pack of 52 cards. What is the probability that the card drawn is either a red card or a king ?

(*a*) $\frac{1}{2}$ (*b*) $\frac{6}{13}$ (*c*) $\frac{7}{13}$ (*d*) $\frac{27}{52}$

16. From a pack of 52 cards, one card is drawn at random. What is the probability that the card drawn is a ten or a spade ?

(*a*) $\frac{4}{13}$ (*b*) $\frac{1}{4}$ (*c*) $\frac{1}{13}$ (*d*) $\frac{1}{26}$

17. The probability that a card drawn from a pack of 52 cards will be a diamond or a king, is :

(*a*) $\frac{2}{13}$ (*b*) $\frac{4}{13}$ (*c*) $\frac{1}{13}$ (*d*) $\frac{1}{52}$

18. From a pack of 52 cards, two cards are drawn together at random. What is the probability of both the cards being kings ? **(M.B.A. 2002; Railways, 2002)**

(*a*) $\frac{1}{15}$ (*b*) $\frac{25}{57}$ (*c*) $\frac{35}{256}$ (*d*) $\frac{1}{221}$

19. Two cards are drawn together from a pack of 52 cards. The probability that one is a spade and one is a heart, is : **(M.B.A. 2000)**

(*a*) $\frac{3}{20}$ (*b*) $\frac{29}{34}$ (*c*) $\frac{47}{100}$ (*d*) $\frac{13}{102}$

20. Two cards are drawn from a pack of 52 cards. The probability that either both are red or both are kings, is :

(*a*) $\frac{7}{13}$ (*b*) $\frac{3}{26}$ (*c*) $\frac{63}{221}$ (*d*) $\frac{55}{221}$

33. Two dice are tossed. The probability that the total score is a prime number is :

(*a*) $\frac{1}{6}$ (*b*) $\frac{5}{12}$ (*c*) $\frac{1}{2}$ (*d*) $\frac{7}{9}$

34. A speaks truth in 75% cases and B in 80% of the cases. In what percentage of cases are they likely to contradict each other, narrating the same incident ?

(*a*) 5% (*b*) 15% (*c*) 35% (*d*) 45%

(Bank P.O. 2000)

35. A man and his wife appear in an interview for two vacancies in the same post. The probability of husband's selection is (1 / 7) and the probability of wife's selection is (1 / 5). What is the probability that only one of them is selected ?

(*a*) $\frac{4}{5}$ (*b*) $\frac{2}{7}$ (*c*) $\frac{8}{15}$ (*d*) $\frac{4}{7}$

ANSWERS

1. (*d*)	**2.** (*b*)	**3.** (*d*)	**4.** (*b*)	**5.** (*a*)	**6.** (*c*)	**7.** (*a*)	**8.** (*d*)	**9.** (*b*)
10. (*a*)	**11.** (*d*)	**12.** (*c*)	**13.** (*b*)	**14.** (*c*)	**15.** (*c*)	**16.** (*a*)	**17.** (*b*)	**18.** (*d*)
19. (*d*)	**20.** (*d*)	**21.** (*b*)	**22.** (*d*)	**23.** (*c*)	**24.** (*a*)	**25.** (*a*)	**26.** (*d*)	**27.** (*a*)
28. (*b*)	**29.** (*a*)	**30.** (*d*)	**31.** (*b*)	**32.** (*a*)	**33.** (*b*)	**34.** (*c*)	**35.** (*b*)	

SOLUTIONS

1. Here S = {HH, HT, TH, T T}.

Let E = event of getting at least one head = {HT, TH, HH}.

$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{3}{4}.$

2. Here S = {TTT, TTH, THT, HTT, THH, HTH, HHT, HHH}.

Let E = event of getting at least two heads = {THH, HTH, HHT, HHH}.

$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{4}{8} = \frac{1}{2}.$

3. Here S = {TTT, TTH, THT, HTT, THH, HTH, HHT, HHH}.

Let E = event of getting at most two heads.

Then, E = {TTT, TTH, THT, HTT, THH, HTH, HHT}.

$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{7}{8}.$

4. When a die is thrown, we have S = {1, 2, 3, 4, 5, 6}.

Let E = event of getting a number greater than 4 = {5, 6}.

$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{2}{6} = \frac{1}{3}.$

5. We know that in a simultaneous throw of two dice, $n(S) = 6 \times 6 = 36$.

Let E = event of getting a total of 7 = {(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)}.

$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}.$

6. In two throws of a die, $n(S) = (6 \times 6) = 36$.

Let E = event of getting a sum 9 = {(3, 6), (4, 5), (5, 4), (6, 3)}.

$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{4}{36} = \frac{1}{9}.$

7. In a simultaneous throw of two dice, n (S) = (6 × 6) = 36.
Let E = event of getting a doublet = {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)}.

$$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}.$$

8. In a simultaneous throw of two dice, we have n (S) = (6 × 6) = 36.
Let E = event of getting a total of 10 or 11 = {(4, 6), (5, 5), (6, 4), (5, 6), (6, 5)}.

$$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{5}{36}.$$

9. In a simultaneous throw of two dice, we have n (S) = (6 × 6) = 36.
Let E = event of getting two numbers whose product is even.
Then, E = {(1, 2), (1, 4), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 2), (3, 4), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 2), (5, 4), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5),(6, 6)}.

$\therefore \quad n(E) = 27.$

$$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{27}{36} = \frac{3}{4}.$$

10. Here, S = {1, 2, 3, 4, ..., 19, 20}.
Let E = event of getting a multiple of 3 = {3, 6, 9, 12, 15, 18}.

$$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{6}{20} = \frac{3}{10}.$$

11. Here, S = {1, 2, 3, 4, ..., 19, 20}.
Let E = event of getting a multiple of 3 or 5 = {3, 6, 9, 12, 15, 18, 5, 10, 20}.

$$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{9}{20}.$$

12. P (getting a prize) $= \dfrac{10}{(10+25)} = \dfrac{10}{35} = \dfrac{2}{7}.$

13. Clearly, there are 52 cards, out of which there are 16 face cards.

$$\therefore \quad P(\text{getting a face card}) = \frac{16}{52} = \frac{4}{13}.$$

14. Here, n (S) = 52.
Let E = event of getting a queen of club or a king of heart.
Then, n (E) = 2.

$$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{2}{52} = \frac{1}{26}.$$

15. Here, n (S) = 52.
There are 26 red cards (including 2 kings) and there are 2 more kings.
Let E = event of getting a red card or a king.
Then, n (E) = 28.

$$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{28}{52} = \frac{7}{13}.$$

16. Here, n (S) = 52.
There are 13 spades (including one ten) and there are 3 more tens.
Let E = event of getting a ten or a spade.
Then, n (E) = (13 + 3) = 16.

$$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{16}{52} = \frac{4}{13}.$$

17. Here, $n(S) = 52$.

There are 13 cards of diamond (including one king) and there are 3 more kings.

Let E = event of getting a diamond or a king.

Then, $n(E) = (13 + 3) = 16$.

$$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{16}{52} = \frac{4}{13}.$$

18. Let S be the sample space. Then,

$$n(S) = {}^{52}C_2 = \frac{(52 \times 51)}{(2 \times 1)} = 1326.$$

Let E = event of getting 2 kings out of 4.

$$\therefore \quad n(E) = {}^{4}C_2 = \frac{(4 \times 3)}{(2 \times 1)} = 6.$$

$$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{6}{1326} = \frac{1}{221}.$$

19. Let S be the sample space. Then,

$$n(S) = {}^{52}C_2 = \frac{(52 \times 51)}{(2 \times 1)} = 1326.$$

Let E = event of getting 1 spade and 1 heart.

$\therefore$ $n(E)$ = number of ways of choosing 1 spade out of 13 and 1 heart out of 13

$$= ({}^{13}C_1 \times {}^{13}C_1) = (13 \times 13) = 169.$$

$$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{169}{1326} = \frac{13}{102}.$$

20. Clearly, $n(S) = {}^{52}C_2 = \dfrac{(52 \times 51)}{2} = 1326.$

Let E_1 = event of getting both red cards,

E_2 = event of getting both kings.

Then, $E_1 \cap E_2$ = event of getting 2 kings of red cards.

$$\therefore \quad n(E_1) = {}^{26}C_2 = \frac{(26 \times 25)}{(2 \times 1)} = 325; \; n(E_2) = {}^{4}C_2 = \frac{(4 \times 3)}{(2 \times 1)} = 6;$$

$$n(E_1 \cap E_2) = {}^{2}C_2 = 1.$$

$$\therefore \quad P(E_1) = \frac{n(E_1)}{n(S)} = \frac{325}{1326}; \; P(E_2) = \frac{n(E_2)}{n(S)} = \frac{6}{1326}; \; P(E_1 \cap E_2) = \frac{1}{1326}.$$

$\therefore$ P (both red or both kings) $= P(E_1 \cup E_2)$

$$= P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

$$= \left(\frac{325}{1326} + \frac{6}{1326} - \frac{1}{1326}\right) = \frac{330}{1326} = \frac{55}{221}.$$

21. Total number of balls = (6 + 8) = 14.

Number of white balls = 8.

$$P(\text{drawing a white ball}) = \frac{8}{14} = \frac{4}{7}.$$

22. Let S be the sample space. Then,

$n(S)$ = number of ways of drawing 3 marbles out of 12

$$= {}^{12}C_3 = \frac{(12 \times 11 \times 10)}{(3 \times 2 \times 1)} = 220.$$

Let E be the event of drawing 3 balls of the same colour.

Then, E = event of drawing (3 balls out of 5) or (3 balls out of 4) or (3 balls out of 3)

$\Rightarrow \quad n(E) = ({}^5C_3 + {}^4C_3 + {}^3C_3) = ({}^5C_2 + {}^4C_1 + 1) = \dfrac{(5 \times 4)}{(2 \times 1)} + 4 + 1 = 15.$

$\Rightarrow \quad P(E) = \dfrac{n(E)}{n(S)} = \dfrac{15}{220} = \dfrac{3}{44}.$

$\therefore \quad$ Required probability $= \left(1 - \dfrac{3}{44}\right) = \dfrac{41}{44}.$

23. Let S be the sample space. Then,

$n(S)$ = number of ways of drawing 3 balls out of 15 $= {}^{15}C_3 = \dfrac{(15 \times 14 \times 13)}{(3 \times 2 \times 1)} = 455.$

Let E = event of getting all the 3 red balls.

$\therefore \quad n(E) = {}^5C_3 = {}^5C_2 = \dfrac{(5 \times 4)}{(2 \times 1)} = 10.$

$\therefore \quad P(E) = \dfrac{n(E)}{n(S)} = \dfrac{10}{455} = \dfrac{2}{91}.$

24. Let S be the sample space. Then,

$n(S)$ = number of ways of drawing 3 balls out of 10

$= {}^{10}C_3 = \dfrac{(10 \times 9 \times 8)}{(3 \times 2 \times 1)} = 120.$

Let E = event of drawing 1 red and 2 white balls

$\therefore \quad n(E)$ = Number of ways of drawing 1 red ball out of 4 and 2 white balls out of 6

$= ({}^4C_1 \times {}^6C_2) = \left(4 \times \dfrac{6 \times 5}{2 \times 1}\right) = 60.$

$\therefore \quad P(E) = \dfrac{n(E)}{n(S)} = \dfrac{60}{120} = \dfrac{1}{2}.$

25. Total number of balls = (2 + 3 + 2) = 7.

Let S be the sample space. Then,

$n(S)$ = Number of ways of drawing 2 balls out of 7 $= {}^7C_2 = \dfrac{(7 \times 6)}{(2 \times 1)} = 21.$

Let E = Event of drawing 2 balls, none of which is blue.

$\therefore \quad n(E)$ = Number of ways of drawing 2 balls out of (2 + 3) balls

$= {}^5C_2 = \dfrac{(5 \times 4)}{(2 \times 1)} = 10.$

$\therefore \quad P(E) = \dfrac{n(E)}{n(S)} = \dfrac{10}{21}.$

26. Total number of balls = (8 + 7 + 6) = 21.

Let E = event that the ball drawn is neither red nor green

= event that the ball drawn is red.

$\therefore \quad n(E) = 8$

$\therefore \quad P(E) = \dfrac{8}{21}.$

27. Total number of balls = 20.

Let S be the sample space. Then,

$n(S)$ = Number of ways of drawing 2 balls out of 20 $= {}^{20}C_2 = \dfrac{(20 \times 19)}{(2 \times 1)} = 190.$

Let E = event of drawing 2 balls of the same colour

$$n(E) = ({}^{10}C_2 + {}^{10}C_2) = 2 \times \left(\frac{10 \times 9}{2 \times 1}\right) = 90.$$

$$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{90}{190} = \frac{9}{19}.$$

28. Total number of balls = (4 + 5 + 6) = 15.

$\therefore \quad n(S) = 15.$

Let E_1 = event of drawing a red ball

and E_2 = event of drawing a green ball.

Then, $E_1 \cap E_2 = \phi$.

$$P(E_1 \text{ or } E_2) = P(E_1) + P(E_2) = \left(\frac{4}{15} + \frac{5}{15}\right) = \frac{9}{15} = \frac{3}{5}.$$

29. Let S be the sample space and E be the event of selecting 1 girl and 2 boys. Then,

$n(S)$ = Number of ways of selecting 3 students out of 25

$$= {}^{25}C_3 = \frac{(25 \times 24 \times 23)}{(3 \times 2 \times 1)} = 2300.$$

$$n(E) = ({}^{10}C_1 \times {}^{15}C_2) = \left\{10 \times \frac{(15 \times 14)}{(2 \times 1)}\right\} = 1050.$$

$$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{1050}{2300} = \frac{21}{46}.$$

30. Let S be the sample space and E be the event of choosing four persons such that 2 of them are children. Then,

$n(S)$ = Number of ways of choosing 4 persons out of 9

$$= {}^{9}C_4 = \frac{(9 \times 8 \times 7 \times 6)}{(4 \times 3 \times 2 \times 1)} = 126.$$

$n(E)$ = Number of ways of choosing 2 children out of 4 and 2 persons out of (3 + 2) persons

$$= ({}^{4}C_2 \times {}^{5}C_2) = \frac{(4 \times 3)}{(2 \times 1)} \times \frac{(5 \times 4)}{(2 \times 1)} = 60.$$

$$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{60}{126} = \frac{10}{21}.$$

31. $$P(\text{None is defective}) = \frac{{}^{16}C_2}{{}^{20}C_2} = \left(\frac{16 \times 15}{2 \times 1} \times \frac{2 \times 1}{20 \times 19}\right) = \frac{12}{19}.$$

$$P(\text{at least one is defective}) = \left(1 - \frac{12}{19}\right) = \frac{7}{19}.$$

32. $P(E) = \frac{30}{100} = \frac{3}{10}$, $P(H) = \frac{20}{100} = \frac{1}{5}$ and $P(E \cap H) = \frac{10}{100} = \frac{1}{10}$.

$$P(E \text{ or } H) = P(E \cup H)$$
$$= P(E) + P(H) - P(E \cap H)$$
$$= \left(\frac{3}{10} + \frac{1}{5} - \frac{1}{10}\right) = \frac{4}{10} = \frac{2}{5}.$$

33. Clearly, $n(S) = (6 \times 6) = 36$.

Let E = Event that the sum is a prime number.

Then, E = {(1, 1), (1, 2), (1, 4), (1, 6), (2, 1), (2, 3), (2, 5), (3, 2), (3, 4), (4, 1), (4, 3), (5, 2), (5, 6), (6, 1), (6, 5)}

$\therefore \quad n(E) = 15$

$\therefore \quad P(E) = \dfrac{n(E)}{n(S)} = \dfrac{15}{36} = \dfrac{5}{12}$.

34. Let A = Event that A speaks the truth

and B = Event that B speaks the truth.

Then, $P(A) = \dfrac{75}{100} = \dfrac{3}{4}$, $P(B) = \dfrac{80}{100} = \dfrac{4}{5}$.

$\therefore \quad P(\overline{A}) = \left(1 - \dfrac{3}{4}\right) = \dfrac{1}{4}$ and $P(\overline{B}) = \left(1 - \dfrac{4}{5}\right) = \dfrac{1}{5}$.

P (A and B contradict each other)

= P [(A speaks the truth and B tells a lie) or (A tells a lie and B speaks the truth)]

$= P[(A \text{ and } \overline{B}) \text{ or } (\overline{A} \text{ and } B)]$

$= P(A \text{ and } \overline{B}) + P(\overline{A} \text{ and } B)$

$= P(A) \cdot P(\overline{B}) + P(\overline{A}) \cdot P(B)$

$= \left(\dfrac{3}{4} \times \dfrac{1}{5}\right) + \left(\dfrac{1}{4} \times \dfrac{4}{5}\right) = \left(\dfrac{3}{20} + \dfrac{1}{5}\right) = \dfrac{7}{20} = \left(\dfrac{7}{20} \times 100\right)\% = 35\%$.

$\therefore$ A and B contradict each other in 35% of the cases.

35. Let A = Event that the husband is selected

and B = Event that the wife is selected.

Then, $P(A) = \dfrac{1}{7}$ and $P(B) = \dfrac{1}{5}$.

$\therefore \quad P(\overline{A}) = \left(1 - \dfrac{1}{7}\right) = \dfrac{6}{7}$ and $P(\overline{B}) = \left(1 - \dfrac{1}{5}\right) = \dfrac{4}{5}$.

$\therefore$ Required probability = P [(A and not B) or (B and not A)]

$= P[(A \text{ and } \overline{B}) \text{ or } (B \text{ and } \overline{A})]$

$= P(A \text{ and } \overline{B}) + P(B \text{ and } \overline{A})$

$= P(A) \cdot P(\overline{B}) + P(B) \cdot P(\overline{A}) = \left(\dfrac{1}{7} \times \dfrac{4}{5}\right) + \left(\dfrac{1}{5} \times \dfrac{6}{7}\right) = \dfrac{10}{35} = \dfrac{2}{7}$.

32. TRUE DISCOUNT

IMPORTANT CONCEPTS

Suppose a man has to pay Rs. 156 after 4 years and the rate of interest is 14% per annum. Clearly, Rs. 100 at 14% will amount to Rs. 156 in 4 years. So, the payment of Rs. 100 now will clear off the debt of Rs. 156 due 4 years hence. We say that :

Sum due = Rs. 156 due 4 years hence;

Present Worth (P.W.) = Rs. 100;

True Discount (T.D.) = Rs. (156 – 100) = Rs. 56 = (Sum due) – (P.W.).

We define : ***T.D. = Interest on P.W.***

Amount = (P.W.) + (T.D.).

Interest is reckoned on P.W. and true discount is reckoned on the amount.

IMPORTANT FORMULAE

Let rate = R% per annum and Time = T years. Then,

1. $\text{P.W.} = \dfrac{100 \times \text{Amount}}{100 + (\text{R} \times \text{T})} = \dfrac{100 \times \text{T.D.}}{\text{R} \times \text{T}}$.

2. $\text{T.D.} = \dfrac{(\text{P.W.}) \times \text{R} \times \text{T}}{100} = \dfrac{\text{Amount} \times \text{R} \times \text{T}}{100 + (\text{R} \times \text{T})}$.

3. $\text{Sum} = \dfrac{(\text{S.I.}) \times (\text{T.D.})}{(\text{S.I.}) - (\text{T.D.})}$.

4. (S.I.) – (T.D.) = S.I. on T.D.

5. When the sum is put at compound interest, then $\text{P.W.} = \dfrac{\text{Amount}}{\left(1 + \dfrac{\text{R}}{100}\right)^{\text{T}}}$.

SOLVED EXAMPLES

Ex. 1. ***Find the present worth of Rs. 930 due 3 years hence at 8% per annum. Also find the discount.***

Sol. $\text{P.W.} = \dfrac{100 \times \text{Amount}}{100 + (\text{R} \times \text{T})} = \text{Rs.}\left[\dfrac{100 \times 930}{100 + (8 \times 3)}\right] = \text{Rs.}\left(\dfrac{100 \times 930}{124}\right) = \text{Rs. } 750.$

T.D. = (Amount) – (P.W.) = Rs. (930 – 750) = Rs. 180.

Ex. 2. ***The true discount on a bill due 9 months hence at 12% per annum is Rs. 540. Find the amount of the bill and its present worth.***

Sol. Let amount be Rs. x. Then,

$$\frac{x \times \text{R} \times \text{T}}{100 + (\text{R} \times \text{T})} = \text{T.D.} \Rightarrow \frac{x \times 12 \times \frac{3}{4}}{100 + \left(12 \times \frac{3}{4}\right)} = 540 \Rightarrow x = \left(\frac{540 \times 109}{9}\right) = \text{Rs. } 6540.$$

$\therefore$ Amount = Rs. 6540.

P.W. = Rs. (6540 – 540) = Rs. 6000.

Ex. 3. ***The true discount on a certain sum of money due 3 years hence is Rs. 250 and the simple interest on the same sum for the same time and at the same rate is Rs. 375. Find the sum and the rate percent.***

Sol. T.D. = Rs. 250 and S.I. = Rs. 375.

$$\therefore \quad \text{Sum due} = \frac{\text{S.I.} \times \text{T.D.}}{(\text{S.I.}) - (\text{T.D.})} = \text{Rs.}\left(\frac{375 \times 250}{375 - 250}\right) = \text{Rs. } 750.$$

$$\text{Rate} = \left(\frac{100 \times 375}{750 \times 3}\right)\% = 16\frac{2}{3}\%.$$

Ex. 4. ***The difference between the simple interest and true discount on a certain sum of money for 6 months at $12\frac{1}{2}\%$ per annum is Rs. 25. Find the sum.***

Sol. Let the sum be Rs. x. Then,

$$\text{T.D.} = \frac{x \times \frac{25}{2} \times \frac{1}{2}}{100 + \left(\frac{25}{2} \times \frac{1}{2}\right)} = \left(x \times \frac{25}{4} \times \frac{4}{425}\right) = \frac{x}{17}.$$

$$\text{S.I.} = \left(x \times \frac{25}{2} \times \frac{1}{2} \times \frac{1}{100}\right) = \frac{x}{16}.$$

$$\therefore \quad \frac{x}{16} - \frac{x}{17} = 25 \Rightarrow 17x - 16x = 25 \times 16 \times 17 \Rightarrow x = 6800.$$

Hence, sum due = Rs. 6800.

Ex. 5. ***A bill falls due in 1 year. The creditor agrees to accept immediate payment of the half and to defer the payment of the other half for 2 years. By this arrangement he gains Rs. 40. What is the amount of the bill, if the money be worth $12\frac{1}{2}\%$?***

Sol. Let the sum be Rs. x. Then,

$$\left[\frac{x}{2} + \frac{\frac{x}{2} \times 100}{100 + \left(\frac{25}{2} \times 2\right)}\right] - \frac{x \times 100}{100 + \left(\frac{25}{2} \times 1\right)} = 40 \Rightarrow \frac{x}{2} + \frac{2x}{5} - \frac{8x}{9} = 40 \Rightarrow x = 3600.$$

$\therefore$ Amount of the bill = Rs. 3600.

EXERCISE 32

(OBJECTIVE TYPE QUESTIONS)

Directions : ***Mark (✓) against the correct answer :***

1. The present worth of Rs. 2310 due $2\frac{1}{2}$ years hence, the rate of interest being 15% per annum, is :
   (*a*) Rs. 1750 (*b*) Rs. 1680 (*c*) Rs. 1840 (*d*) Rs. 1443.75
2. If the true discount on a sum due 2 years hence at 14% per annum be Rs. 168, the sum due is :
   (*a*) Rs. 768 (*b*) Rs. 968 (*c*) Rs. 1960 (*d*) Rs. 2400
3. The true discount on Rs. 2562 due 4 months hence is Rs. 122. The rate percent is :
   (*a*) 12% (*b*) $13\frac{1}{3}\%$ (*c*) 15% (*d*) 14%

4. The true discount on Rs. 1760 due after a certain time at 12% per annum is Rs. 160. The time after which it is due is :
(a) 6 months (b) 8 months (c) 9 months (d) 10 months

5. The true discount on a bill due 9 months hence at 16% per annum is Rs. 189. The amount of the bill is :
(a) Rs. 1386 (b) Rs. 1764 (c) Rs. 1575 (d) Rs. 2268

6. The interest on Rs. 750 for 2 years is the same as the true discount on Rs. 960 due 2 years hence. If the rate of interest is the same in both cases, it is :
(a) 12% (b) 14% (c) 15% (d) $16\frac{2}{3}\%$

7. The simple interest and the true discount on a certain sum for a given time and at a given rate are Rs. 85 and Rs. 80 respectively. The sum is :
(a) Rs. 1800 (b) Rs. 1450 (c) Rs. 1360 (d) Rs. 6800

8. If Rs. 10 be allowed as true discount on a bill of Rs. 110 due at the end of a certain time, then the discount allowed on the same sum due at the end of double the time is :
(a) Rs. 20 (b) Rs. 21.81 (c) Rs. 22 (d) Rs. 18.33

9. A man wants to sell his scooter. There are two offers, one at Rs. 12,000 cash and the other at a credit of Rs. 12,880 to be paid after 8 months, money being at 18% per annum. Which is the better offer ?
(a) Rs. 12,000 in cash (b) Rs. 12,880 at credit (c) Both are equally good

10. Goods were bought for Rs. 600 and sold the same day for Rs. 688.50 at a credit of 9 months and thus gaining 2%. The rate of interest per annum is :
(a) $16\frac{2}{3}\%$ (b) $14\frac{1}{2}\%$ (c) $13\frac{1}{3}\%$ (d) 15%

11. The present worth of Rs. 1404 due in two equal half-yearly instalments at 8% per annum simple interest is :
(a) Rs. 1325 (b) Rs. 1300 (c) Rs. 1350 (d) Rs. 1500

12. A trader owes a merchant Rs. 10,028 due 1 year hence. The trader wants to settle the account after 3 months. If the rate of interest is 12% per annum, how much cash should he pay ?
(a) Rs. 9025.20 (b) Rs. 9200 (c) Rs. 9600 (d) Rs. 9560

13. A man buys a watch for Rs. 1950 in cash and sells it for Rs. 2200 at a credit of 1 year. If the rate of interest is 10% per annum, the man :
(a) gains Rs. 55 (b) gains Rs. 50 (c) loses Rs. 30 (d) gains Rs. 30

14. A man purchased a cow for Rs. 3000 and sold it the same day for Rs. 3600, allowing the buyer a credit of 2 years. If the rate of interest be 10% per annum, then the man has a gain of :
(a) 0% (b) 5% (c) 7.5% (d) 10%

15. A owes B, Rs. 1573 payable $1\frac{1}{2}$ years hence. Also B owes A, Rs. 1444.50 payable 6 months hence. If they want to settle the account forthwith, keeping 14% as the rate of interest, then who should pay and how much ?
(a) A, Rs. 28.50 (b) B, Rs. 37.50 (c) A, Rs. 50 (d) B, Rs. 50

16. A has to pay Rs. 220 to B after 1 year. B asks A to pay Rs. 110 in cash and defer the payment of Rs. 110 for 2 years. A agrees to it. If the rate of interest be 10% per annum, in this mode of payment :
(a) There is no gain or loss to any one (b) A gains Rs. 7.34
(c) A loses Rs. 7.34 (d) A gains Rs. 11

17. Rs. 20 is the true discount on Rs. 260 due after a certain time. What will be the true discount on the same sum due after half of the former time, the rate of interest being the same ?
(a) Rs. 10 (b) Rs. 10.40 (c) Rs. 15.20 (d) Rs. 13

ANSWERS

1. (*b*) 2. (*a*) 3. (*c*) 4. (*d*) 5. (*b*) 6. (*b*) 7. (*c*) 8. (*d*) 9. (*a*)
10. (*a*) 11. (*a*) 12. (*b*) 13. (*b*) 14. (*a*) 15. (*d*) 16. (*b*) 17. (*b*)

SOLUTIONS

1. P.W. = Rs. $\left[\dfrac{100 \times 2310}{100 + \left(15 \times \dfrac{5}{2}\right)}\right]$ = Rs. 1680.

2. P.W. = $\dfrac{100 \times \text{T.D.}}{\text{R} \times \text{T}} = \dfrac{100 \times 168}{14 \times 2} = 600.$

   ∴ Sum = (P.W. + T.D.) = Rs. (600 + 168) = Rs. 768.

3. P.W. = Rs. (2562 – 122) = Rs. 2440.

   ∴ S.I. on Rs. 2440 for 4 months is Rs. 122.

   ∴ Rate = $\left(\dfrac{100 \times 122}{2440 \times \dfrac{1}{3}}\right)\%$ = 15%.

4. P.W. = Rs. (1760 – 160) = Rs. 1600.

   ∴ S.I. on Rs. 1600 at 12% is Rs. 160.

   ∴ Time = $\left(\dfrac{100 \times 160}{1600 \times 12}\right) = \dfrac{5}{6}$ years = $\left(\dfrac{5}{6} \times 12\right)$ months = 10 months.

5. Let P.W. be Rs. x. Then, S.I. on Rs. x at 16% for 9 months = Rs. 189.

   ∴ $x \times 16 \times \dfrac{9}{12} \times \dfrac{1}{100} = 189$ or $x = 1575$.

   ∴ P.W. = Rs. 1575.

   ∴ Sum due = P.W. + T.D. = Rs. (1575 + 189) = Rs. 1764.

6. S.I. on Rs. 750 = T.D. on Rs. 960.

   This means P.W. of Rs. 960 due 2 years hence is Rs. 750.

   ∴ T.D. = Rs. (960 – 750) = Rs. 210.

   Thus, S.I. on Rs. 750 for 2 years is Rs. 210.

   ∴ Rate = $\left(\dfrac{100 \times 210}{750 \times 2}\right)\%$ = 14%.

7. Sum = $\dfrac{\text{S.I.} \times \text{T.D.}}{(\text{S.I.}) - (\text{T.D.})} = \dfrac{85 \times 80}{(85 - 80)}$ = Rs. 1360.

8. S.I. on Rs. (110 – 10) for a certain time = Rs. 10.

   S.I. on Rs. 100 for double the time = Rs. 20.

   T.D. on Rs. 120 = Rs. (120 – 100) = Rs. 20.

   T.D. on Rs. 110 = Rs. $\left(\dfrac{20}{120} \times 110\right)$ = Rs 18.33.

9. P.W. of Rs. 12,880 due 8 months hence

   = Rs. $\left[\dfrac{12880 \times 100}{100 + \left(18 \times \dfrac{8}{12}\right)}\right]$ = Rs. $\left(\dfrac{12880 \times 100}{112}\right)$ = Rs. 11500.

   Clearly, Rs. 12,000 in cash is a better offer.

10. S.P. = 102% of Rs. 600 = Rs. $\left(\frac{102}{100} \times 600\right)$ = Rs. 612.

Now, P.W. = Rs. 612 and sum = Rs. 688.50.

$\therefore$ T.D. = Rs. (688.50 − 612) = Rs. 76.50.

Thus, S.I. on Rs. 612 for 9 months is Rs. 76.50.

$$\therefore \quad \text{Rate} = \left(\frac{100 \times 76.50}{612 \times \frac{3}{4}}\right)\% = 16\frac{2}{3}\%.$$

11. Required sum = P.W. of Rs. 702 due 6 months hence + P.W. of Rs. 702 due 1 year hence

$$= \text{Rs.}\left[\left(\frac{100 \times 702}{100 + 8 \times \frac{1}{2}}\right) + \left(\frac{100 \times 702}{100 + (8 \times 1)}\right)\right] = \text{Rs. } (675 + 650) = \text{Rs. } 1325.$$

12. Required money = P.W. of Rs. 10028 due 9 months hence

$$= \text{Rs.}\left[\frac{10028 \times 100}{100 + \left(12 \times \frac{9}{12}\right)}\right] = \text{Rs. } 9200.$$

13. S.P. = P.W. of Rs. 2200 due 1 year hence = Rs. $\left[\frac{2200 \times 100}{100 + (10 \times 1)}\right]$ = Rs. 2000.

$\therefore$ Gain = Rs. (2000 − 1950) = Rs. 50.

14. C.P. = Rs. 3000. S.P. = Rs. $\left[\frac{3600 \times 100}{100 + (10 \times 2)}\right]$ = Rs. 3000.

Gain = 0%.

15. A owes = P.W. of Rs. 1573 due $\frac{3}{2}$ years hence

$$= \text{Rs.}\left[\frac{1573 \times 100}{100 + \left(14 \times \frac{3}{2}\right)}\right] = \text{Rs.}\left(\frac{1573 \times 100}{121}\right) = \text{Rs. } 1300.$$

B owes = P.W. of Rs. 1444.50 due 6 months hence

$$= \text{Rs.}\left[\frac{1444.50 \times 100}{100 + \left(14 \times \frac{1}{2}\right)}\right] = \text{Rs.}\left(\frac{1444.50 \times 100}{107}\right) = \text{Rs. } 1350.$$

$\therefore$ B must pay Rs. 50 to A.

16. A has to pay = P.W. of Rs. 220 due 1 year hence = Rs. $\left[\frac{220 \times 100}{100 + (10 \times 1)}\right]$ = Rs. 200.

A actually pays = Rs. 110 + P.W. of Rs. 110 due 2 years hence

$$= \left[110 + \frac{110 \times 100}{100 + (10 \times 2)}\right] = \text{Rs. } 192.66.$$

$\therefore$ A gains = Rs. (200 − 192.66) = Rs. 7.34.

17. S.I. on Rs. (260 − 20) for a given time = Rs. 20.

S.I. on Rs. 240 for half the time = Rs. 10.

T.D. on Rs. 250 = Rs. 10.

$\therefore$ T.D. on Rs. 260 = Rs. $\left(\frac{10}{250} \times 260\right)$ = Rs. 10.40.

33. BANKER'S DISCOUNT

IMPORTANT CONCEPTS

Banker's Discount : Suppose a merchant A buys goods worth, say Rs. 10,000 from another merchant B at a credit of say 5 months. Then, B prepares a bill, called the bill of exchange. A signs this bill and allows B to withdraw the amount from his bank account after exactly 5 months.

The date exactly after 5 months is called *nominally due date*. Three days (known as *grace days*) are added to it to get a date, known as *legally due date*.

Suppose B wants to have the money before the legally due date. Then he can have the money from the banker or a broker, who deducts S.I. on the face value (*i.e.*, Rs. 10,000 in this case) for the period from the date on which the bill was discounted (*i.e.*, paid by the banker) and the legally due date. This amount is known as ***Banker's Discount (B.D.)***

Thus, **B.D.** *is the S.I. on the face value for the period from the date on which the bill was discounted and the legally due date.*

Banker's Gain (B.G.) = (B.D.) – (T.D.) *for the unexpired time.*

Note : When the date of the bill is not given, grace days are not to be added.

IMPORTANT FORMULAE

1. B.D. = S.I. on bill for unexpired time.

2. B.G. = (B.D.) – (T.D.) = S.I. on T.D. = $\frac{(\text{T.D.})^2}{\text{P.W.}}$.

3. T.D. = $\sqrt{\text{P.W.} \times \text{B.G.}}$

4. B.D. = $\left(\frac{\text{Amount} \times \text{Rate} \times \text{Time}}{100}\right)$.

5. T.D. = $\left[\frac{\text{Amount} \times \text{Rate} \times \text{Time}}{100 + (\text{Rate} \times \text{Time})}\right]$.

6. Amount = $\left(\frac{\text{B.D.} \times \text{T.D.}}{\text{B.D.} - \text{T.D.}}\right)$.

7. T.D. = $\left(\frac{\text{B.G.} \times 100}{\text{Rate} \times \text{Time}}\right)$

SOLVED EXAMPLES

Ex. 1. ***A bill for Rs. 6000 is drawn on July 14 at 5 months. It is discounted on 5th October at 10%. Find the banker's discount, true discount, banker's gain and the money that the holder of the bill receives.***

Sol. Face value of the bill = Rs. 6000.

Date on which the bill was drawn = July 14 at 5 months.

Nominally due date = December 14. Legally due date = December 17.

Date on which the bill was discounted = October 5.

Unexpired time : Oct. Nov. Dec.

26 + 30 + 17 = 73 days = $\frac{1}{5}$ year.

$\therefore$ B.D. = S.I. on Rs. 6000 for $\frac{1}{5}$ year = Rs. $\left(6000 \times 10 \times \frac{1}{5} \times \frac{1}{100}\right)$ = Rs. 120.

$$\text{T.D.} = \text{Rs.}\left[\frac{6000 \times 10 \times \frac{1}{5}}{100 + \left(10 \times \frac{1}{5}\right)}\right] = \text{Rs.}\left(\frac{12000}{102}\right) = \text{Rs. } 117.64.$$

$\therefore$ B.G. = (B.D.) – (T.D.) = Rs. (120 – 117.64) = Rs. 2.36.

Money received by the holder of the bill = Rs. (6000 – 120) = Rs. 5880.

Ex. 2. ***If the true discount on a certain sum due 6 months hence at 15% is Rs. 120, what is the banker's discount on the same sum for the same time and at the same rate?***

Sol. B.G. = S.I. on T.D. = Rs. $\left(120 \times 15 \times \frac{1}{2} \times \frac{1}{100}\right)$ = Rs. 9.

$\therefore$ (B.D.) – (T.D.) = Rs. 9.

$\therefore$ B.D. = Rs. (120 + 9) = Rs. 129.

Ex. 3. ***The banker's discount on Rs. 1800 at 12% per annum is equal to the true discount on Rs. 1872 for the same time at the same rate. Find the time.***

Sol. S.I. on Rs. 1800 = T.D. on Rs. 1872.

$\therefore$ P.W. of Rs. 1872 is Rs. 1800.

$\therefore$ Rs. 72 is S.I. on Rs. 1800 at 12%.

$\therefore$ Time = $\left(\frac{100 \times 72}{12 \times 1800}\right)$ year = $\frac{1}{3}$ year = 4 months.

Ex. 4. ***The banker's discount and the true discount on a sum of money due 8 months hence are Rs. 120 and Rs. 110 respectively. Find the sum and the rate percent.***

Sol. Sum = $\left(\frac{\text{B.D.} \times \text{T.D.}}{\text{B.D.} - \text{T.D.}}\right)$ = Rs. $\left(\frac{120 \times 110}{120 - 110}\right)$ = Rs. 1320.

Since B.D. is S.I. on sum due, so S.I. on Rs. 1320 for 8 months is Rs. 120.

$\therefore$ Rate = $\left(\frac{100 \times 120}{1320 \times \frac{2}{3}}\right)\% = 13\frac{7}{11}\%$.

Ex. 5. ***The present worth of a bill due sometime hence is Rs. 1100 and the true discount on the bill is Rs. 110. Find the banker's discount and the banker's gain.***

Sol. T.D. = $\sqrt{\text{P.W.} \times \text{B.G.}}$

$\therefore$ B.G. = $\frac{(\text{T.D.})^2}{\text{P.W.}}$ = Rs. $\left(\frac{110 \times 110}{1100}\right)$ = Rs. 11.

$\therefore$ B.D. = (T.D. + B.G.) = Rs. (110 + 11) = Rs. 121.

Ex. 6. ***The banker's discount on Rs. 1650 due a certain time hence is Rs. 165. Find the true discount and the banker's gain.***

Sol. Sum = $\frac{\text{B.D.} \times \text{T.D.}}{\text{B.D.} - \text{T.D.}} = \frac{\text{B.D.} \times \text{T.D.}}{\text{B.G.}}$.

$\therefore$ $\frac{\text{T.D.}}{\text{B.G.}} = \frac{\text{Sum}}{\text{B.D.}} = \frac{1650}{165} = \frac{10}{1}$.

Thus, if B.G. is Re 1, T.D. = Rs. 10.

If B.D. is Rs. 11, T.D. = Rs. 10. If B.D. is Rs. 165, T.D. = Rs. $\left(\frac{10}{11} \times 165\right)$ = Rs. 150.

And, B.G. = Rs. (165 – 150) = Rs. 15.

Ex. 7. ***What rate percent does a man get for his money when in discounting a bill due 10 months hence, he deducts 10% of the amount of the bill?***

Sol. Let, amount of the bill = Rs. 100. Money deducted = Rs. 10.

Money received by the holder of the bill = Rs. (100 − 10) = Rs. 90.

∴ S.I. on Rs. 90 for 10 months = Rs. 10.

∴ $\text{Rate} = \left(\dfrac{100 \times 10}{90 \times \dfrac{10}{12}}\right)\% = 13\frac{1}{3}\%.$

EXERCISE 33

(OBJECTIVE TYPE QUESTIONS)

Directions : ***Mark (✓) against the correct answer :***

1. The true discount on a bill of Rs. 540 is Rs. 90. The banker's discount is :
   (*a*) Rs. 60 (*b*) Rs. 108 (*c*) Rs. 110 (*d*) Rs. 112
2. The present worth of a certain bill due sometime hence is Rs. 800 and the true discount is Rs. 36. The banker's discount is :
   (*a*) Rs. 37 (*b*) Rs. 37.62 (*c*) Rs. 34.38 (*d*) Rs. 38.98
3. The present worth of a certain sum due sometime hence is Rs. 1600 and the true discount is Rs. 160. The banker's gain is :
   (*a*) Rs. 20 (*b*) Rs. 24 (*c*) Rs. 16 (*d*) Rs. 12
4. The banker's gain of a certain sum due 2 years hence at 10% per annum is Rs. 24. The present worth is :
   (*a*) Rs. 480 (*b*) Rs. 520 (*c*) Rs. 600 (*d*) Rs. 960
5. The banker's gain on a bill due 1 year hence at 12% per annum is Rs. 6. The true discount is :
   (*a*) Rs. 72 (*b*) Rs. 36 (*c*) Rs. 54 (*d*) Rs. 50
6. The banker's discount on a bill due 4 months hence at 15% is Rs. 420. The true discount is :
   (*a*) Rs. 400 (*b*) Rs. 360 (*c*) Rs. 480 (*d*) Rs. 320
7. The banker's gain on a sum due 3 years hence at 12% per annum is Rs. 270. The banker's discount is :
   (*a*) Rs. 960 (*b*) Rs. 840 (*c*) Rs. 1020 (*d*) Rs. 760
8. The present worth of a sum due sometime hence is Rs. 576 and the banker's gain is Rs. 16. The true discount is :
   (*a*) Rs. 36 (*b*) Rs. 72 (*c*) Rs. 48 (*d*) Rs. 96
9. The banker's discount on Rs. 1600 at 15% per annum is the same as true discount on Rs. 1680 for the same time and at the same rate. The time is :
   (*a*) 3 months (*b*) 4 months (*c*) 6 months (*d*) 8 months
10. The banker's discount on a sum of money for $1\frac{1}{2}$ years is Rs. 558 and the true discount on the same sum for 2 years is Rs. 600. The rate percent is :
    (*a*) 10% (*b*) 13% (*c*) 12% (*d*) 15%
11. The banker's discount of a certain sum of money is Rs. 72 and the true discount on the same sum for the same time is Rs. 60. The sum due is :
    (*a*) Rs. 360 (*b*) Rs. 432 (*c*) Rs. 540 (*d*) Rs. 1080

12. The banker's discount on a certain sum due 2 years hence is $\frac{11}{10}$ of the true discount. The rate percent is :

(a) 11% (b) 10% (c) 5% (d) 5.5%

13. The banker's gain on a certain sum due $1\frac{1}{2}$ years hence is $\frac{3}{25}$ of the banker's discount. The rate percent is :

(a) $5\frac{1}{5}\%$ (b) $9\frac{1}{9}\%$ (c) $8\frac{1}{8}\%$ (d) $6\frac{1}{6}\%$

ANSWERS

1. (b)	2. (b)	3. (c)	4. (c)	5. (d)	6. (a)	7. (c)
8. (d)	9. (b)	10. (c)	11. (a)	12. (c)	13. (b)	

SOLUTIONS

1. P.W. = Rs. (540 – 90) = Rs. 450.

∴ S.I. on Rs. 450 = Rs. 90

S.I. on Rs. 540 = Rs. $\left(\frac{90}{450} \times 540\right)$ = Rs. 108.

∴ B.D. = Rs. 108.

2. B.G. = $\frac{(\text{T.D.})^2}{\text{P.W.}}$ = Rs. $\left(\frac{36 \times 36}{800}\right)$ = Rs. 1.62.

∴ B.D. = (T.D. + B.G.) = Rs. (36 + 1.62) = Rs. 37.62.

3. B.G. = $\frac{(\text{T.D.})^2}{\text{P.W.}}$ = Rs. $\left(\frac{160 \times 160}{1600}\right)$ = Rs. 16.

4. T.D. = $\left(\frac{\text{B.G.} \times 100}{\text{Rate} \times \text{Time}}\right)$ = Rs. $\left(\frac{24 \times 100}{10 \times 2}\right)$ = Rs. 120.

∴ P.W. = $\frac{100 \times \text{T.D.}}{\text{Rate} \times \text{Time}}$ = Rs. $\left(\frac{100 \times 120}{10 \times 2}\right)$ = Rs. 600.

5. T.D. = $\frac{\text{B.G.} \times 100}{\text{R} \times \text{T}}$ = Rs. $\left(\frac{6 \times 100}{12 \times 1}\right)$ = Rs. 50.

6. T.D. = $\frac{\text{B.D.} \times 100}{100 + (\text{R} \times \text{T})}$ = Rs. $\left[\frac{420 \times 100}{100 + \left(15 \times \frac{1}{3}\right)}\right]$ = Rs. $\left(\frac{420 \times 100}{105}\right)$ = Rs. 400.

7. T.D. = $\left(\frac{\text{B.G.} \times 100}{\text{R} \times \text{T}}\right)$ = Rs. $\left(\frac{270 \times 100}{12 \times 3}\right)$ = Rs. 750.

∴ B.D. = Rs. (750 + 270) = Rs. 1020.

8. T.D. = $\sqrt{\text{P.W.} \times \text{B.G.}}$ = $\sqrt{576 \times 16}$ = 96.

9. S.I. on Rs. 1600 = T.D. on Rs. 1680.

∴ Rs. 1600 is the P.W. of Rs. 1680, *i.e.*, Rs. 80 is S.I. on Rs. 1600 at 15%.

∴ Time = $\left(\frac{100 \times 80}{1600 \times 15}\right)$ year = $\frac{1}{3}$ year = 4 months.

10. B.D. for $\frac{3}{2}$ years = Rs. 558. B.D. for 2 years = Rs. $\left(558 \times \frac{2}{3} \times 2\right)$ = Rs. 744.

T.D. for 2 years = Rs. 600.

$\therefore$ Sum = $\frac{\text{B.D.} \times \text{T.D.}}{\text{B.D.} - \text{T.D.}}$ = Rs. $\left(\frac{744 \times 600}{144}\right)$ = Rs. 3100.

Thus, Rs. 744 is S.I. on Rs. 3100 for 2 years.

$\therefore$ Rate = $\left(\frac{100 \times 744}{3100 \times 2}\right)\%$ = 12%.

11. Sum = $\frac{\text{B.D.} \times \text{T.D.}}{\text{B.D.} - \text{T.D.}}$ = Rs. $\left(\frac{72 \times 60}{72 - 60}\right)$ = Rs. $\left(\frac{72 \times 60}{12}\right)$ = Rs. 360.

12. Let T.D. be Re 1. Then, B.D. = Rs. $\frac{11}{10}$ = Rs. 1.10.

$\therefore$ Sum = Rs. $\left(\frac{1.10 \times 1}{1.10 - 1}\right)$ = Rs. $\left(\frac{110}{10}\right)$ = Rs. 11.

$\therefore$ S.I. on Rs. 11 for 2 years is Rs. 1.10.

$\therefore$ Rate = $\left(\frac{100 \times 1.10}{11 \times 2}\right)\%$ = 5%.

13. Let, B.D. = Re 1. Then, B.G. = Re $\frac{3}{25}$.

$\therefore$ T.D. = (B.D. – B.G.) = Re $\left(1 - \frac{3}{25}\right)$ = Re $\frac{22}{25}$.

Sum = $\left(\frac{1 \times \frac{22}{25}}{1 - \frac{22}{25}}\right)$ = Rs. $\frac{22}{3}$.

S.I. on Rs. $\frac{22}{3}$ for $1\frac{1}{2}$ years is Re 1.

$\therefore$ Rate = $\left(\frac{100 \times 1}{\frac{22}{3} \times \frac{3}{2}}\right)\%$ = $9\frac{1}{9}\%$.

34. HEIGHTS AND DISTANCES

IMPORTANT FACTS AND FORMULAE

1. We already know that :

   In a rt. angled Δ OAB, where $\angle BOA = \theta$,

   (*i*) $\sin\theta = \dfrac{\text{Perpendicular}}{\text{Hypotenuse}} = \dfrac{AB}{OB}$;

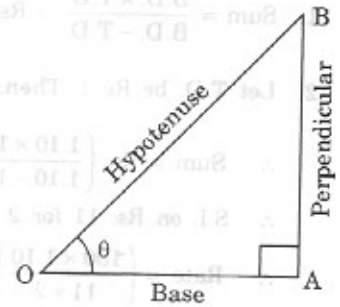
   (*ii*) $\cos\theta = \dfrac{\text{Base}}{\text{Hypotenuse}} = \dfrac{OA}{OB}$;

   (*iii*) $\tan\theta = \dfrac{\text{Perpendicular}}{\text{Base}} = \dfrac{AB}{OA}$;

   (*iv*) $\text{cosec}\,\theta = \dfrac{1}{\sin\theta} = \dfrac{OB}{AB}$;

   (*v*) $\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{OB}{OA}$;

   (*vi*) $\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{OA}{AB}$.



2. **Trigonometrical Identities :**

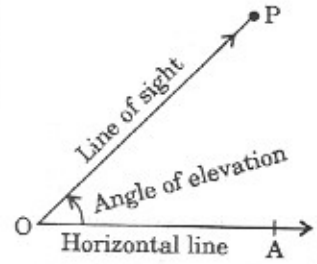
   (*i*) $\sin^2\theta + \cos^2\theta = 1$. (*ii*) $1 + \tan^2\theta = \sec^2\theta$. (*iii*) $1 + \cot^2\theta = \text{cosec}^2\theta$.

3. **Values of T-ratios :**

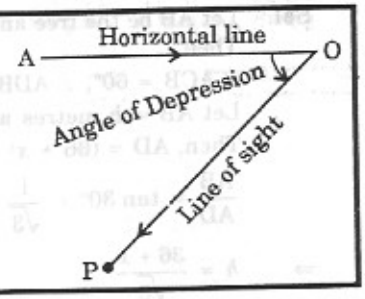
θ	$0°$	$(\pi/6)$ $30°$	$(\pi/4)$ $45°$	$(\pi/3)$ $60°$	$(\pi/2)$ $90°$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	not defined

4. **Angle of Elevation :** Suppose a man from a point O looks up at an object P, placed above the level of his eye. Then, the angle which the line of sight makes with the horizontal through O, is called the angle of elevation of P as seen from O.

   $\therefore$ Angle of elevation of P from O = $\angle AOP$.



5. **Angle of Depression :** Suppose a man from a point O looks down at an object P, placed below the level of his eye, then the angle which the line of sight makes with the horizontal through O, is called the angle of depression of P as seen from O.



SOLVED EXAMPLES

Ex. 1. ***If the height of a pole is $2\sqrt{3}$ metres and the length of its shadow is 2 metres, find the angle of elevation of the sun.***

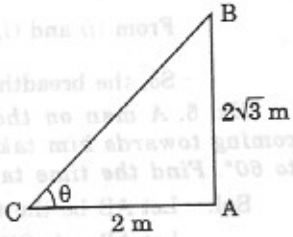
Sol. Let AB be the pole and AC be its shadow.

Let angle of elevation, $\angle ACB = \theta$.

Then, $AB = 2\sqrt{3}$ m, $AC = 2$ m.

$$\tan\theta = \frac{AB}{AC} = \frac{2\sqrt{3}}{2} = \sqrt{3} \Rightarrow \theta = 60°.$$

So, the angle of elevation is 60°.



Ex. 2. ***A ladder leaning against a wall makes an angle of 60° with the ground. If the length of the ladder is 19 m, find the distance of the foot of the ladder from the wall.***

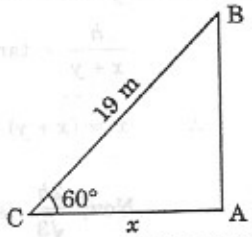
Sol. Let AB be the wall and BC be the ladder.

Then, $\angle ACB = 60°$ and $BC = 19$ m.

Let $AC = x$ metres

$$\frac{AC}{BC} = \cos 60° \Rightarrow \frac{x}{19} = \frac{1}{2} \Rightarrow x = \frac{19}{2} = 9.5.$$

$\therefore$ Distance of the foot of the ladder from the wall = 9.5 m.



Ex. 3. ***The angle of elevation of the top of a tower at a point on the ground is 30°. On walking 24 m towards the tower, the angle of elevation becomes 60°. Find the height of the tower.***

Sol. Let AB be the tower and C and D be the points of observation. Then,

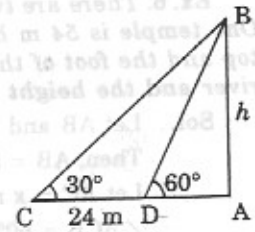
$$\frac{AB}{AD} = \tan 60° = \sqrt{3} \Rightarrow AD = \frac{AB}{\sqrt{3}} = \frac{h}{\sqrt{3}}.$$

$$\frac{AB}{AC} = \tan 30° = \frac{1}{\sqrt{3}} \Rightarrow AC = AB \times \sqrt{3} = h\sqrt{3}.$$

$$CD = (AC - AD) = \left(h\sqrt{3} - \frac{h}{\sqrt{3}}\right).$$

$$\therefore \quad h\sqrt{3} - \frac{h}{\sqrt{3}} = 24 \Rightarrow h = 12\sqrt{3} = (12 \times 1.73) = 20.76.$$

Hence, the height of the tower is 20.76 m.



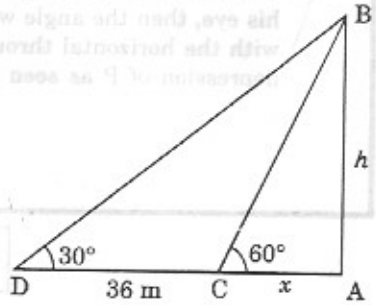
Ex. 4. ***A man standing on the bank of a river observes that the angle subtended by a tree on the opposite bank is 60°. When he retires 36 m from the bank, he finds the angle to be 30°. Find the breadth of the river.***

Sol. Let AB be the tree and AC be the river. Let C and D be the two positions of the man. Then,

$\angle ACB = 60°$, $\angle ADB = 30°$ and CD = 36 m.

Let AB = h metres and AC = x metres.

Then, AD = $(36 + x)$ metres.



$$\frac{AB}{AD} = \tan 30° = \frac{1}{\sqrt{3}} \Rightarrow \frac{h}{36+x} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad h = \frac{36+x}{\sqrt{3}} \quad ...(i)$$

$$\frac{AB}{AC} = \tan 60° = \sqrt{3} \Rightarrow \frac{h}{x} = \sqrt{3}$$

$$\Rightarrow \quad h = \sqrt{3}\,x \quad ...(ii)$$

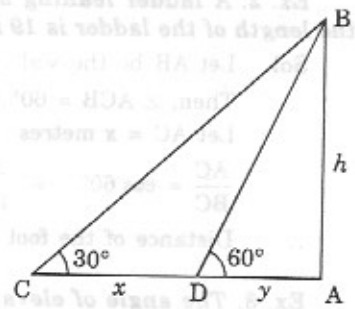
From (i) and (ii), we get : $\frac{36+x}{\sqrt{3}} = \sqrt{3}\,x \Rightarrow x = 18$ m.

So, the breadth of the river = 18 m.

Ex. 5. ***A man on the top of a tower, standing on the seashore finds that a boat coming towards him takes 10 minutes for the angle of depression to change from 30° to 60°. Find the time taken by the boat to reach the shore from this position.***

Sol. Let AB be the tower and C and D be the two positions of the boat.

Let AB = h, CD = x and AD = y.



$$\frac{h}{y} = \tan 60° = \sqrt{3} \Rightarrow y = \frac{h}{\sqrt{3}}.$$

$$\frac{h}{x+y} = \tan 30° = \frac{1}{\sqrt{3}} \Rightarrow x + y = \sqrt{3}\,h.$$

$$\therefore \quad x = (x+y) - y = \left(\sqrt{3}\,h - \frac{h}{\sqrt{3}}\right) = \frac{2h}{\sqrt{3}}.$$

Now, $\frac{2h}{\sqrt{3}}$ is covered in 10 min.

$$\therefore \quad \frac{h}{\sqrt{3}} \text{ will be covered in } \left(10 \times \frac{\sqrt{3}}{2h} \times \frac{h}{\sqrt{3}}\right) = 5 \text{ min.}$$

Hence, required time = 5 minutes.

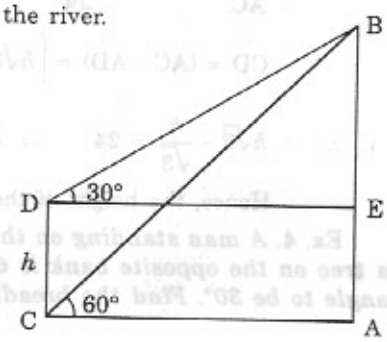
Ex. 6. ***There are two temples, one on each bank of a river, just opposite to each other. One temple is 54 m high. From the top of this temple, the angles of depression of the top and the foot of the other temple are 30° and 60° respectively. Find the width of the river and the height of the other temple.***

Sol. Let AB and CD be the two temples and AC be the river.

Then, AB = 54 m.

Let AC = x metres and CD = h metres.

$\angle ACB = 60°$, $\angle EDB = 30°$.



$$\frac{AB}{AC} = \tan 60° = \sqrt{3}$$

$$\Rightarrow \quad AC = \frac{AB}{\sqrt{3}} = \frac{54}{\sqrt{3}} = \left(\frac{54}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}\right) = 18\sqrt{3} \text{ m.}$$

DE = AC = $18\sqrt{3}$ m.

$$\frac{BE}{DE} = \tan 30° = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad BE = \left(18\sqrt{3} \times \frac{1}{\sqrt{3}}\right) = 18 \text{ m.}$$

$\therefore$ $CD = AE = AB - BE = (54 - 18)$ m $= 36$ m.

So, Width of the river $= AC = 18\sqrt{3}$ m $= (18 \times 1.73)$ m $= 31.14$ m.

Height of the other temple $= CD = 18$ m.

EXERCISE 34

(OBJECTIVE TYPE QUESTIONS)

Directions : *Mark (✓) against the correct answer* :

1. The angle of elevation of the sun, when the length of the shadow of a tree is $\sqrt{3}$ times the height of the tree, is : **(R.R.B. 2003)**
   (*a*) 30° (*b*) 45° (*c*) 60° (*d*) 90°
2. From a point P on a level ground, the angle of elevation of the top of a tower is 30°. If the tower is 100 m high, the distance of point P from the foot of the tower is :
   (*a*) 149 m (*b*) 156 m (*c*) 173 m (*d*) 200 m
   (R.R.B. 2002)
3. The angle of elevation of a ladder leaning against a wall is 60° and the foot of the ladder is 4.6 m away from the wall. The length of the ladder is :
   (*a*) 2.3 m (*b*) 4.6 m (*c*) 7.8 m (*d*) 9.2 m
4. An observer 1.6 m tall is $20\sqrt{3}$ m away from a tower. The angle of elevation from his eye to the top of the tower is 30°. The height of the tower is :
   (*a*) 21.6 m (*b*) 23.2 m (*c*) 24.72 m (*d*) None of these
5. Two ships are sailing in the sea on the two sides of a lighthouse. The angles of elevation of the top of the lighthouse as observed from the two ships are 30° and 45° respectively. If the lighthouse is 100 m high, the distance between the two ships is :
   (*a*) 173 m (*b*) 200 m (*c*) 273 m (*d*) 300 m
6. A man standing at a point P is watching the top of a tower, which makes an angle of elevation of 30° with the man's eye. The man walks some distance towards the tower to watch its top and the angle of elevation becomes 60°. What is the distance between the base of the tower and the point P ? **(Bank P.O. 1999)**
   (*a*) $4\sqrt{3}$ units (*b*) 8 units (*c*) 12 units
   (*d*) Data inadequate (*e*) None of these
7. The angle of elevation of the top of a tower from a certain point is 30°. If the observer moves 20 m towards the tower, the angle of elevation of the top of the tower increases by 15°. The height of the tower is :
   (*a*) 17.3 m (*b*) 21.9 m (*c*) 27.3 m (*d*) 30 m
8. A man is watching from the top of a tower a boat speeding away from the tower. The boat makes an angle of depression of 45° with the man's eye when at a distance of 60 metres from the tower. After 5 seconds, the angle of depression becomes 30°. What is the approximate speed of the boat, assuming that it is running in still water ?
   (*a*) 32 kmph (*b*) 36 kmph (*c*) 38 kmph
   (*d*) 40 kmph (*e*) 42 kmph **(S.B.I.P.O. 1999)**
9. On the same side of a tower, two objects are located. Observed from the top of the tower, their angles of depression are 45° and 60°. If the height of the tower is 150 m, the distance between the objects is :
   (*a*) 63.5 m (*b*) 76.9 m (*c*) 86.7 m (*d*) 90 m

10. A man on the top of a vertical observation tower observes a car moving at a uniform speed coming directly towards it. If it takes 12 minutes for the angle of depression to change from 30° to 45°, how soon after this will the car reach the observation tower ?

(*a*) 14 min. 35 sec. (*b*) 15 min. 49 sec. (*c*) 16 min. 23 sec. (*d*) 18 min. 5 sec.

(R.R.B. 2002)

11. The top of a 15 metre high tower makes an angle of elevation of 60° with the bottom of an electric pole and angle of elevation of 30° with the top of the pole. What is the height of the electric pole ?

(*a*) 5 metres (*b*) 8 metres (*c*) 10 metres
(*d*) 12 metres (*e*) None of these

ANSWERS

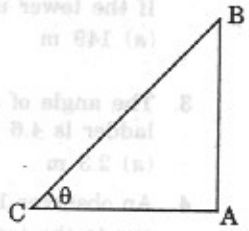
1. (*a*)	2. (*c*)	3. (*d*)	4. (*a*)	5. (*c*)	6. (*d*)
7. (*c*)	8. (*a*)	9. (*a*)	10. (*c*)	11. (*c*)	

SOLUTIONS

1. Let AB be the tree and AC be its shadow.

Let $\angle ACB = \theta$.

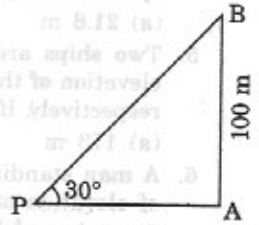
Then, $\frac{AC}{AB} = \sqrt{3} \Rightarrow \cot\theta = \sqrt{3} \Rightarrow \theta = 30°$.



2. Let AB be the tower. Then, $\angle APB = 30°$ and AB = 100 m.

$\frac{AB}{AP} = \tan 30° = \frac{1}{\sqrt{3}} \Rightarrow AP = (AB \times \sqrt{3}) = 100\sqrt{3}$ m.

$= (100 \times 1.73)$ m $= 173$ m.

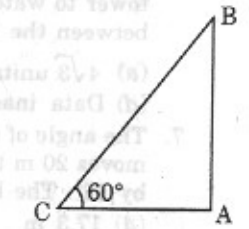


3. Let AB be the wall and BC be the ladder.

Then, $\angle ACB = 60°$ and AC = 4.6 m.

$\frac{AC}{BC} = \cos 60° = \frac{1}{2}$

$\Rightarrow BC = 2 \times AC = (2 \times 4.6)$ m $= 9.2$ m.



4. Let AB be the observer and CD be the tower.

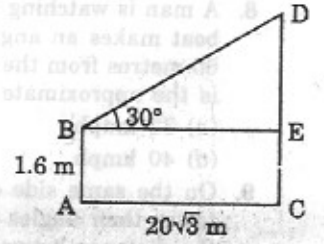
Draw BE ⊥ CD.

Then, CE = AB = 1.6 m, BE = AC = $20\sqrt{3}$ m.

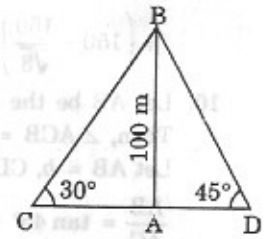
$\frac{DE}{BE} = \tan 30° = \frac{1}{\sqrt{3}}$

$\Rightarrow DE = \frac{20\sqrt{3}}{\sqrt{3}}$ m $= 20$ m.

$\therefore CD = CE + DE = (1.6 + 20)$ m $= 21.6$ m.



5. Let AB be the lighthouse and C and D be the positions of the ships. Then,
AB = 100 m, $\angle ACB = 30°$ and $\angle ADB = 45°$.



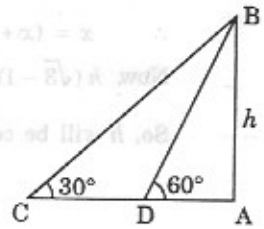
$\frac{AB}{AC} = \tan 30° = \frac{1}{\sqrt{3}} \Rightarrow AC = AB \times \sqrt{3} = 100\sqrt{3}$ m.

$\frac{AB}{AD} = \tan 45° = 1 \Rightarrow AD = AB = 100$ m.

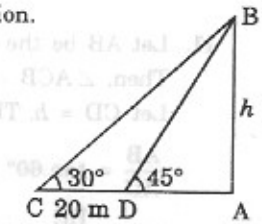
$\therefore$ CD = (AC + AD) = $(100\sqrt{3} + 100)$ m

$= 100(\sqrt{3} + 1)$ m $= (100 \times 2.73)$ m $= 273$ m.

6. One of AB, AD and CD must have been given.
So, the data is inadequate.



7. Let AB be the tower and C and D be the points of observation.
Then, $\angle ACB = 30°$, $\angle ADB = 45°$ and CD = 20 m.
Let AB = h.



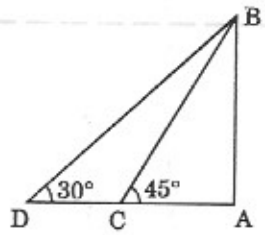
Then, $\frac{AB}{AC} = \tan 30° = \frac{1}{\sqrt{3}} \Rightarrow AC = AB \times \sqrt{3} = h\sqrt{3}$.

And, $\frac{AB}{AD} = \tan 45° = 1 \Rightarrow AD = AB = h$.

CD = 20 $\Rightarrow$ (AC − AD) = 20 $\Rightarrow h\sqrt{3} - h = 20$.

$\therefore \quad h = \frac{20}{(\sqrt{3}-1)} \times \frac{(\sqrt{3}+1)}{(\sqrt{3}+1)} = 10(\sqrt{3}+1)$ m $= (10 \times 2.73)$ m $= 27.3$ m.

8. Let AB be the tower and C and D be the two positions of the boats.
Then, $\angle ACB = 45°$, $\angle ADB = 30°$ and AC = 60 m.
Let AB = h.



Then, $\frac{AB}{AC} = \tan 45° = 1 \Rightarrow AB = AC \Rightarrow h = 60$ m.

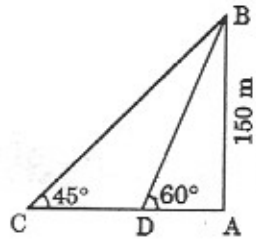
And, $\frac{AB}{AD} = \tan 30° = \frac{1}{\sqrt{3}} \Rightarrow AD = (AB \times \sqrt{3}) = 60\sqrt{3}$ m.

$\therefore$ CD = (AD − AC) = $60(\sqrt{3} - 1)$ m.

Hence, required speed $= \left[\frac{60(\sqrt{3}-1)}{5}\right]$ m/s $= (12 \times 0.73)$ m/s

$= \left(12 \times 0.73 \times \frac{18}{5}\right)$ km/hr = 31.5 km/hr ≈ 32 km/hr.

9. Let AB be the tower and C and D be the objects.
Then, AB = 150 m, $\angle ACB = 45°$ and $\angle ADB = 60°$.



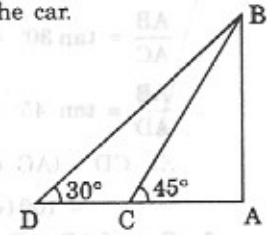
$\frac{AB}{AD} = \tan 60° = \sqrt{3} \Rightarrow AD = \frac{AB}{\sqrt{3}} = \frac{150}{\sqrt{3}}$ m.

$\frac{AB}{AC} = \tan 45° = 1 \Rightarrow AC = AB = 150$ m.

$\therefore$ CD = (AC − AD)

$$= \left(150 - \frac{150}{\sqrt{3}}\right) \text{ m} = \left[\frac{150(\sqrt{3}-1)}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}\right] \text{ m} = 50(3-\sqrt{3}) \text{ m} = (50 \times 1.27) \text{ m} = 63.5 \text{ m.}$$

10. Let AB be the tower and C and D be the two positions of the car.



Then, $\angle ACB = 45°$, $\angle ADB = 30°$.

Let AB = h, CD = x and AC = y.

$$\frac{AB}{AC} = \tan 45° = 1 \Rightarrow \frac{h}{y} = 1 \Rightarrow y = h.$$

$$\frac{AB}{AD} = \tan 30° = \frac{1}{\sqrt{3}} \Rightarrow \frac{h}{x+y} = \frac{1}{\sqrt{3}} \Rightarrow x + y = \sqrt{3}\,h.$$

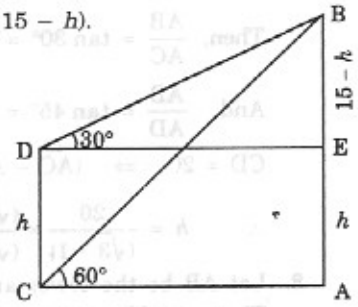
$$\therefore \quad x = (x+y) - y = \sqrt{3}\,h - h = h(\sqrt{3}-1).$$

Now, $h(\sqrt{3}-1)$ is covered in 12 min.

So, h will be covered in $\left[\frac{12}{h(\sqrt{3}-1)} \times h\right] = \frac{12}{(\sqrt{3}-1)}$ min.

$$= \left(\frac{1200}{73}\right) \text{ min.} \approx 16 \text{ min. } 23 \text{ sec.}$$

11. Let AB be the tower and CD be the electric pole.



Then, $\angle ACB = 60°$, $\angle EDB = 30°$ and AB = 15 m.

Let CD = h. Then, BE = (AB – AE) = (AB – CD) = $(15 - h)$.

$$\frac{AB}{AC} = \tan 60° = \sqrt{3} \Rightarrow AC = \frac{AB}{\sqrt{3}} = \frac{15}{\sqrt{3}}.$$

And, $\frac{BE}{DE} = \tan 30° = \frac{1}{\sqrt{3}} \Rightarrow DE = (BE \times \sqrt{3}) = \sqrt{3}(15-h).$

$$AC = DE \Rightarrow \frac{15}{\sqrt{3}} = \sqrt{3}(15-h)$$

$$\Rightarrow 3h = (45 - 15) \Rightarrow h = 10 \text{ m.}$$

35. ODD MAN OUT AND SERIES

EXERCISE 35

Directions : ***Find the odd man out :***

1. 3, 5, 7, 12, 17, 19
   (*a*) 19 (*b*) 17 (*c*) 13 (*d*) 12
2. 10, 14, 16, 18, 21, 24, 26
   (*a*) 26 (*b*) 24 (*c*) 21 (*d*) 18
3. 3, 5, 9, 11, 14, 17, 21
   (*a*) 21 (*b*) 17 (*c*) 14 (*d*) 9
4. 1, 4, 9, 16, 23, 25, 36
   (*a*) 9 (*b*) 23 (*c*) 25 (*d*) 36
5. 6, 9, 15, 21, 24, 28, 30
   (*a*) 28 (*b*) 21 (*c*) 24 (*d*) 30
6. 41, 43, 47, 53, 61, 71, 73, 81
   (*a*) 61 (*b*) 71 (*c*) 73 (*d*) 81
7. 16, 25, 36, 72, 144, 196, 225
   (*a*) 36 (*b*) 72 (*c*) 196 (*d*) 225
8. 10, 25, 45, 54, 60, 75, 80
   (*a*) 10 (*b*) 45 (*c*) 54 (*d*) 75
9. 1, 4, 9, 16, 20, 36, 49
   (*a*) 1 (*b*) 9 (*c*) 20 (*d*) 49
10. 8, 27, 64, 100, 125, 216, 343
    (*a*) 27 (*b*) 100 (*c*) 125 (*d*) 343
11. 1, 5, 14, 30, 50, 55, 91
    (*a*) 5 (*b*) 50 (*c*) 55 (*d*) 91
12. 385, 462, 572, 396, 427, 671, 264
    (*a*) 385 (*b*) 427 (*c*) 671 (*d*) 264
13. 835, 734, 642, 751, 853, 981, 532
    (*a*) 751 (*b*) 853 (*c*) 981 (*d*) 532
14. 331, 482, 551, 263, 383, 242, 111
    (*a*) 263 (*b*) 383 (*c*) 242 (*d*) 111
15. 2, 5, 10, 17, 26, 37, 50, 64
    (*a*) 50 (*b*) 26 (*c*) 37 (*d*) 64
16. 19, 28, 39, 52, 67, 84, 102
    (*a*) 52 (*b*) 102 (*c*) 84 (*d*) 67
17. 253, 136, 352, 460, 324, 631, 244
    (*a*) 136 (*b*) 324 (*c*) 352 (*d*) 631
18. 2, 5, 10, 50, 500, 5000
    (*a*) 0 (*b*) 5 (*c*) 10 (*d*) 5000
19. 4, 5, 7, 10, 14, 18, 25, 32
    (*a*) 7 (*b*) 14 (*c*) 18 (*d*) 33

Directions : ***Find out the wrong number in each sequence :***

20. 22, 33, 66, 99, 121, 279, 594
(*a*) 33 (*b*) 121 (*c*) 279 (*d*) 594

21. 36, 54, 18, 27, 9,18.5, 4.5
(*a*) 4.5 (*b*) 18.5 (*c*) 54 (*d*) 18

22. 582, 605, 588, 611, 634, 617, 600
(*a*) 634 (*b*) 611 (*c*) 605 (*d*) 600

23. 46080, 3840, 384, 48, 24, 2, 1
(*a*) 1 (*b*) 2 (*c*) 24 (*d*) 384

24. 1, 8, 27, 64, 124, 216, 343
(*a*) 8 (*b*) 27 (*c*) 64 (*d*) 124

25. 5, 16, 6, 16, 7, 16, 9
(*a*) 9 (*b*) 7 (*c*) 6 (*d*) None of these

26. 6, 13, 18, 25, 30, 37, 40
(*a*) 25 (*b*) 30 (*c*) 37 (*d*) 40

27. 56, 72, 90, 110, 132, 150
(*a*) 72 (*b*) 110 (*c*) 132 (*d*) 150

28. 8, 13, 21, 32, 47, 63, 83
(*a*) 47 (*b*) 63 (*c*) 32 (*d*) 83

29. 25, 36, 49, 81, 121, 169, 225
(*a*) 36 (*b*) 49 (*c*) 121 (*d*) 169

30. 1, 2, 6, 15, 31, 56, 91
(*a*) 31 (*b*) 91 (*c*) 56 (*d*) 15

31. 52, 51, 48, 43, 34, 27, 16
(*a*) 27 (*b*) 34 (*c*) 43 (*d*) 48

32. 105, 85, 60, 30, 0, – 45, – 90
(*a*) 0 (*b*) 85 (*c*) – 45 (*d*) 60

33. 4, 6, 8, 9, 10, 11, 12
(*a*) 10 (*b*) 11 (*c*) 12 (*d*) 9

34. 125, 127, 130, 135, 142, 153, 165
(*a*) 130 (*b*) 142 (*c*) 153 (*d*) 165

35. 16, 36, 64, 81, 100, 144, 190
(*a*) 81 (*b*) 100 (*c*) 190 (*d*) 36

36. 125, 123, 120, 115, 108, 100, 84
(*a*) 123 (*b*) 115 (*c*) 100 (*d*) 84

37. 3, 10, 21, 36, 55, 70, 105
(*a*) 105 (*b*) 70 (*c*) 36 (*d*) 55

38. 4, 9, 19, 39, 79, 160, 319
(*a*) 319 (*b*) 160 (*c*) 79 (*d*) 39

39. 10, 14, 28, 32, 64, 68, 132
(*a*) 32 (*b*) 68 (*c*) 132 (*d*) 28

40. 8, 27, 125, 343, 1331
(*a*) 1331 (*b*) 343 (*c*) 125 (*d*) None of these

Directions : *Insert the missing number* :

41. 4, – 8, 16, – 32, 64, (.....)
(*a*) 128 (*b*) – 128 (*c*) 192 (*d*) – 192

42. 5, 10, 13, 26, 29, 58, 61, (.....)
(*a*) 122 (*b*) 64 (*c*) 125 (*d*) 128

43. 1, 4, 9, 16, 25, 36, 49, (.....)
(*a*) 54 (*b*) 56 (*c*) 64 (*d*) 81

44. 1, 8, 27, 64, 125, 216, (.....)
(*a*) 354 (*b*) 343 (*c*) 392 (*d*) 245

45. 11, 13, 17, 19, 23, 29, 31, 37, 41, (.....)
(*a*) 43 (*b*) 47 (*c*) 53 (*d*) 51

46. 16, 33, 65, 131, 261, (.....)
(*a*) 523 (*b*) 521 (*c*) 613 (*d*) 721

47. 3, 7, 6, 5, 9, 3, 12, 1, 15, (.....)
(*a*) 18 (*b*) 13 (*c*) – 1 (*d*) 3

48. 15, 31, 63, 127, 255, (.....)
(*a*) 513 (*b*) 511 (*c*) 517 (*d*) 523

49. 2, 6, 12, 20, 30, 42, 56, (.....)
(*a*) 60 (*b*) 64 (*c*) 72 (*d*) 70

50. 8, 24, 12, 36, 18, 54, (.....)
(*a*) 27 (*b*) 108 (*c*) 68 (*d*) 72

51. 165, 195, 255,285, 345, (.....)
(*a*) 375 (*b*) 420 (*c*) 435 (*d*) 390

52. 7, 26, 63, 124, 215, 342, (.....)
(*a*) 481 (*b*) 511 (*c*) 391 (*d*) 421

53. 2, 4, 12, 48, 240, (.....)
(*a*) 960 (*b*) 1440 (*c*) 1080 (*d*) 1920

54. 8, 7, 11, 12, 14, 17, 17, 22, (.....)
(*a*) 27 (*b*) 20 (*c*) 22 (*d*) 24

55. 10, 5, 13, 10, 16, 20, 19, (.....)
(*a*) 22 (*b*) 40 (*c*) 38 (*d*) 23

56. 1, 2, 4, 8, 16, 32, 64, (.....), 256
(*a*) 148 (*b*) 128 (*c*) 154 (*d*) 164

57. 71, 76, 69, 74, 67, 72, (.....)
(*a*) 77 (*b*) 65 (*c*) 80 (*d*) 76

58. 9, 12, 11, 14, 13, (.....), 15
(*a*) 12 (*b*) 16 (*c*) 10 (*d*) 17

59. Complete the series : 2, 5, 9, 19, 37, ...
(*a*) 76 (*b*) 74 (*c*) 75 (*d*) None of these

60. Find the wrong number in the series : 3, 8, 15, 24, 34, 48, 63
(*a*) 15 (*b*) 24 (*c*) 34 (*d*) 48 (*e*) 63

61. Find the wrong number in the series : 2, 9, 28, 65, 126, 216, 344
(*a*) 2 (*b*) 28 (*c*) 65 (*d*) 126 (*e*) 216

62. Find out the wrong number in the series : 5, 15, 30, 135, 405, 1215, 3645
(*a*) 3645 (*b*) 1215 (*c*) 405 (*d*) 30 (*e*) 15

63. Find out the wrong number in the series : 125, 106, 88, 76, 65, 58, 53
(*a*) 125 (*b*) 106 (*c*) 88 (*d*) 76 (*e*) 65

Directions : ***Find out the wrong number in the series :***

64. 190, 166, 145, 128, 112, 100, 91
(*a*) 100 (*b*) 166 (*c*) 145 (*d*) 128 (*e*) 112

65. 1, 1, 2, 6, 24, 96, 720
(*a*) 720 (*b*) 96 (*c*) 24 (*d*) 6 (*e*) 2

66. 40960, 10240, 2560, 640, 200, 40, 10
(*a*) 640 (*b*) 40 (*c*) 200 (*d*) 2560 (*e*) 10240

67. 64, 71, 80, 91, 104, 119, 135, 155
(*a*) 71 (*b*) 80 (*c*) 104 (*d*) 119 (*e*) 135

68. 7, 8, 18, 57, 228, 1165, 6996
(*a*) 8 (*b*) 18 (*c*) 57 (*d*) 228 (*e*) 1165

69. 3, 7, 15, 27, 63, 127, 255
(*a*) 7 (*b*) 15 (*c*) 27 (*d*) 63 (*e*) 127

70. 19, 26, 33, 46, 59, 74, 91
(*a*) 26 (*b*) 33 (*c*) 46 (*d*) 59 (*e*) 74

71. 2880, 480, 92, 24, 8, 4, 4
(*a*) 480 (*b*) 92 (*c*) 24 (*d*) 8 (*e*) 4

72. 445, 221, 109, 46, 25, 11, 4
(*a*) 221 (*b*) 109 (*c*) 46 (*d*) 25 (*e*) 11

73. 3, 7, 15, 39, 63, 127, 255, 511
(*a*) 7 (*b*) 15 (*c*) 39 (*d*) 63 (*e*) 127

74. 1, 3, 10, 21, 64, 129, 356, 777
(*a*) 10 (*b*) 21 (*c*) 64 (*d*) 129 (*e*) 356

75. 196, 169, 144, 121, 100, 80, 64
(*a*) 169 (*b*) 144 (*c*) 121 (*d*) 100 (*e*) 80

76. 6, 12, 48, 100, 384, 768, 3072
(*a*) 768 (*b*) 384 (*c*) 100 (*d*) 48 (*e*) 12

77. 10, 26, 74, 218, 654, 1946, 5834
(*a*) 26 (*b*) 74 (*c*) 218 (*d*) 654 (*e*) 1946

78. 15, 16, 34, 105, 424, 2124, 12576
(*a*) 16 (*b*) 34 (*c*) 105 (*d*) 424 (*e*) 2124

79. 2807, 1400, 697, 347, 171, 84, 41, 20
(*a*) 697 (*b*) 347 (*c*) 171 (*d*) 84 (*e*) 41

80. 32, 36, 41, 61, 86, 122, 171, 235
(*a*) 41 (*b*) 61 (*c*) 86 (*d*) 122 (*e*) 171

81. 3, 4, 9, 22.5, 67.5, 202.5, 810
(*a*) 4 (*b*) 9 (*c*) 22.5 (*d*) 67.5 (*e*) 202.5

82. 1, 2, 8, 33, 148, 760, 4626
(*a*) 2 (*b*) 8 (*c*) 33 (*d*) 148 (*e*) 760

83. 3, 8, 18, 46, 100, 210, 432
(*a*) 8 (*b*) 18 (*c*) 46 (*d*) 100 (*e*) 210

84. 789, 645, 545, 481, 440, 429, 425
(*a*) 645 (*b*) 545 (*c*) 481 (*d*) 440 (*e*) 429

85. 1050, 510, 242, 106, 46, 16, 3
(*a*) 510 (*b*) 242 (*c*) 106 (*d*) 46 (*e*) 16

86. 5, 8, 20, 42, 124, 246, 736
(*a*) 8 (*b*) 20 (*c*) 42 (*d*) 124 (*e*) 246

87. 2, 3, 6, 15, 52.5, 157.5, 630
(*a*) 3 (*b*) 6 (*c*) 15 (*d*) 52.5 (*e*) 157.5

88. 888, 440, 216, 104, 48, 22, 6
(*a*) 440 (*b*) 216 (*c*) 104 (*d*) 48 (*e*) 22

89. 4, 5, 15, 49, 201, 1011, 6073
(*a*) 5 (*b*) 15 (*c*) 49 (*d*) 201 (*e*) 1011

ANSWERS

1. (*d*)	**2.** (*c*)	**3.** (*c*)	**4.** (*b*)	**5.** (*a*)	**6.** (*d*)	**7.** (*b*)	**8.** (*c*)	**9.** (*c*)
10. (*b*)	**11.** (*b*)	**12.** (*b*)	**13.** (*a*)	**14.** (*b*)	**15.** (*d*)	**16.** (*b*)	**17.** (*b*)	**18.** (*d*)
19. (*c*)	**20.** (*c*)	**21.** (*b*)	**22.** (*a*)	**23.** (*c*)	**24.** (*d*)	**25.** (*a*)	**26.** (*d*)	**27.** (*d*)
28. (*a*)	**29.** (*a*)	**30.** (*b*)	**31.** (*b*)	**32.** (*a*)	**33.** (*b*)	**34.** (*d*)	**35.** (*c*)	**36.** (*c*)
37. (*b*)	**38.** (*b*)	**39.** (*c*)	**40.** (*d*)	**41.** (*b*)	**42.** (*a*)	**43.** (*c*)	**44.** (*b*)	**45.** (*a*)
46. (*a*)	**47.** (*c*)	**48.** (*b*)	**49.** (*c*)	**50.** (*a*)	**51.** (*c*)	**52.** (*b*)	**53.** (*b*)	**54.** (*b*)
55. (*b*)	**56.** (*b*)	**57.** (*b*)	**58.** (*b*)	**59.** (*c*)	**60.** (*c*)	**61.** (*e*)	**62.** (*d*)	**63.** (*c*)
64. (*d*)	**65.** (*b*)	**66.** (*c*)	**67.** (*e*)	**68.** (*d*)	**69.** (*c*)	**70.** (*b*)	**71.** (*b*)	**72.** (*c*)
73. (*c*)	**74.** (*e*)	**75.** (*e*)	**76.** (*c*)	**77.** (*d*)	**78.** (*e*)	**79.** (*b*)	**80.** (*a*)	**81.** (*a*)
82. (*e*)	**83.** (*b*)	**84.** (*d*)	**85.** (*c*)	**86.** (*b*)	**87.** (*d*)	**88.** (*e*)	**89.** (*a*)	

SOLUTIONS

1. Each of the numbers except 12, is a prime number.
2. Each of the numbers except 21, is an even number.
3. Each of the numbers except 14, is an odd number.
4. Each of the given numbers except 23, is a perfect square.
5. Each of the numbers except 28, is a multiple of 3.
6. Each of the numbers except 81, is a prime number.
7. Each of the numbers except 72, is a perfect square.
8. Each of the numbers except 54, is a multiple of 5.
9. The pattern is $1^2, 2^2, 3^2, 4^2, 5^2, 6^2, 7^2$. But, instead of 5^2, it is 20, which is to be turned out.
10. The pattern is $2^3, 3^3, 4^3, 5^3, 6^3, 7^3$. But, 100 is not a perfect cube.
11. The pattern is 1^2, $1^2 + 2^2$, $1^2 + 2^2 + 3^2$, $1^2 + 2^2 + 3^2 + 4^2$, $1^2 + 2^2 + 3^2 + 4^2 + 5^2$, $1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2$. But, 50 is not of this pattern.
12. In each number except 427, the middle digit is the sum of the other two.
13. In each number except 751, the difference of third and first digit is the middle one.
14. In each number except 383, the product of first and third digits is the middle one.
15. The pattern is $x^2 + 1$, where $x = 1, 2, 3, 4, 5, 6, 7, 8$ etc. But, 64 is out of pattern.
16. The pattern is $x^2 + 3$, where $x = 4, 5, 6, 7, 8, 9$ etc. But, 102 is out of pattern.
17. Sum of the digits in each number, except 324 is 10.
18. Pattern is 1st × 2nd = 3rd; 2nd × 3rd = 4th; 3rd × 4th = 5th.
But, 4th × 5th = 50 × 500 = 25000 ≠ 5000 = 6th.

19. 2nd = (1st + 1); 3rd = (2nd + 2); 4th = (3rd + 3); 5th = (4th + 4).
But, 18 = 6th ≠ 5th + 5 = 14 + 5 = 19.
20. Each number except 279 is a multiple of 11.
21. The terms are alternately multiplied by 1.5 and divided by 3. However, 18.5 does not satisfy it.
22. Alternately 23 is added and 17 is subtracted from the terms. So, 634 is wrong.
23. The terms are successively divided by 12, 10, 8, 6, ... etc. So, 24 is wrong.
24. The numbers are 1^3, 2^3, 3^3, 4^3 etc. So, 124 is wrong; it must have been 5^3 *i.e.*, 125.
25. Terms at odd places are 5, 6, 7, 8 etc. and each term at even place is 16.
So, 9 is wrong.
26. The difference between two successive terms from the beginning are 7, 5, 7, 5, 7, 5.
So, 40 is wrong.
27. The numbers are 7 × 8, 8 × 9, 9 × 10, 10 × 11, 11 × 12, 12 × 13. So, 150 is wrong.
28. Go on adding 5, 8, 11, 14, 17, 20.
So, the number 47 is wrong and must be replaced by 46.
29. The numbers are squares of odd natural numbers, starting from 5 upto 15.
So, 36 is wrong.
30. Add 1^2, 2^2, 3^2, 4^2, 5^2, 6^2. So, 91 is wrong.
31. Subtract 1, 3, 5, 7, 9, 11 from successive numbers. So, 34 is wrong.
32. Subtract 20, 25, 30, 35, 40, 45 from successive numbers. So, 0 is wrong.
33. Each number is a composite number except 11.
34. Prime numbers 2, 3, 5, 7, 11, 13 are to be added successively. So, 165 is wrong.
35. Each number is the square of a composite number except 190.
36. Prime numbers 2, 3, 5, 7, 11, 13 have successively been subtracted.
So, 100 is wrong. It must be (108 − 11) *i.e.*, 97.
37. The pattern is 1 × 3, 2 × 5, 3 × 7, 4 × 9, 5 × 11, 6 × 13, 7 × 15 etc.
38. Double the number and add 1 to it, to get the next number. So, 160 is wrong.
39. Alternately, we add 4 and double the next.
So, 132 is wrong. It must be (68 × 2) *i.e.*, 136.
40. The numbers are cubes of primes *i.e.*, 2^3, 3^3, 5^3, 7^3, 11^3. Clearly, none is wrong.
41. Each number is the preceding number multiplied by − 2.
So, the required number is − 128.
42. Numbers are alternately multiplied by 2 and increased by 3.
So, the missing number = 61 × 2 = 122.
43. Numbers are 1^2, 2^2, 3^2, 4^2, 5^2, 6^2, 7^2. So, the next number is 8^2 = 64.
44. Numbers are 1^3, 2^3, 3^3, 4^3, 5^3, 6^3. So, the missing number is 7^3 = 343.
45. Numbers are all primes. The next prime is 43.
46. Each number is twice the preceding one with 1 added or subtracted alternately.
So, the next number is (2 × 261 + 1) = 523.
47. There are two series, beginning respectively with 3 and 7. In one 3 is added and in another 2 is subtracted. The next number is 1 − 2 = − 1.
48. Each number is double the preceding one plus 1.
So, the next number is (255 × 2) + 1 = 511.
49. The pattern is 1 × 2, 2 × 3, 3 × 4, 4 × 5, 5 × 6, 6 × 7, 7 × 8.
So, the next number is 8 × 9 = 72.
50. Numbers are alternately multiplied by 3 and divided by 2.
So, the next number = 54 ÷ 2 = 27.

51. Each number is 15 multiplied by a prime number *i.e.*, 15×11, 15×13, 15×17, 15×19, 15×23. So, the next number is $15 \times 29 = 435$.

52. Numbers are $(2^3 - 1)$, $(3^3 - 1)$, $(4^3 - 1)$, $(5^3 - 1)$, $(6^3 - 1)$, $(7^3 - 1)$ etc.
So, the next number is $(8^3 - 1) = (512 - 1) = 511$.

53. Go on multiplying the given numbers by 2, 3, 4, 5, 6. So, the correct next number is 1440.

54. There are two series (8, 11, 14, 17, 20) and (7, 12, 17, 22) increasing by 3 and 5 respectively.

55. There are two series (10, 13, 16, 19) and (5, 10, 20, 40), one increasing by 3 and the other multiplied by 2.

56. Each previous number is multiplied by 2.

57. Alternately, we add 5 and subtract 7.

58. Alternately, we add 3 and subtract 1.

59. Second number is one more than twice the first; third number is one less than twice the second; fourth number is one more than twice the third; fifth number is one less than the fourth. Therefore, the sixth number is one more than twice the fifth.
So, the missing number is 75.

60. The difference between consecutive terms are respectively 5, 7, 9, 11 and 13.
So, 34 is a wrong number.

61. $2 = (1^3 + 1)$; $9 = (2^3 + 1)$; $28 = (3^3 + 1)$; $65 = (4^3 + 1)$; $125 = (5^3 + 1)$; $216 \neq (6^3 + 1)$ and $344 = (7^3 + 1)$. So, 216 is a wrong number.

62. Multiply each term by 3 to obtain the next term. Hence, 30 is a wrong number.

63. Go on subtracting prime numbers, 19, 17, 13, 11, 7, 5 from the numbers to get the next number. So, 88 is wrong.

64. Go on subtracting 24, 21, 18, 15, 12, 9 from the numbers to get the next number.
Clearly, 128 is wrong.

65. Go on multiplying with 1, 2, 3, 4, 5, 6 to get the next number. So, 96 is wrong.

66. Go on dividing by 4 to get the next number. So, 200 is wrong.

67. Go on adding 7, 9, 11, 13, 15, 17, 19 respectively to obtain the next number.
So, 135 is wrong.

68. Let the given numbers be A, B, C, D, E, F, G. Then,
$A, A \times 1, B \times 2 + 2, C \times 3 + 3, D \times 4 + 4, E \times 5 + 5, F \times 6 + 6$ are the required numbers.
Clearly, 228 is wrong.

69. Go on multiplying the number by 2 and adding 1 to it to get the next number.
So, 27 is wrong.

70. Go on adding 7, 9, 11, 13, 15, 17 respectively to obtain the next number.
So, 33 is wrong.

71. Go on dividing by 6, 5, 4, 3, 2, 1 respectively to obtain the next number.
Clearly, 92 is wrong.

72. Go on subtracting 3 and dividing the result by 2 to obtain the next number.
Clearly, 46 is wrong.

73. Go on multiplying 2 and adding 1 to get the next number. So, 39 is wrong.

74. $A \times 2 + 1$, $B \times 3 + 1$, $C \times 2 + 1$, $D \times 3 + 1$ and so on. So, 356 is wrong.

75. Numbers must be $(14)^2$, $(13)^2$, $(11)^2$, $(10)^2$, $(9)^2$, $(8)^2$. So, 80 is wrong.

76. Each even term of the series is obtained by multiplying the previous term by 2.
2nd term = (1st term) $\times 2 = 6 \times 2 = 12$; 4th term = (3rd term) $\times 2 = 48 \times 2 = 96$;
6th term = (5th term) $\times 2 = 384 \times 2 = 768$.
$\therefore$ 4th term should be 96 instead of 100.

77. 2nd term = (1st term) × 3 − 4 = 10 × 3 − 4 = 26;
3rd term = (2nd term) × 3 − 4 = 26 × 3 − 4 = 74;
4th term = (3rd term) × 3 − 4 = 74 × 3 − 4 = 218;
5th term = (4th term) × 3 − 4 = 218 × 3 − 4 = 650.
∴ 5th term must be 650 instead of 654.

78. 2nd term = (1st term) × 1 + 1 = 15 × 1 + 1 = 16;
3rd term = (2nd term) × 2 + 2 = 16 × 2 + 2 = 34;
4th term = (3rd term) × 3 + 3 = 34 × 3 + 3 = 105;
5th term = (4th term) × 4 + 4 = 105 × 4 + 4 = 424;
6th term = (5th term) × 5 + 5 = 425 × 5 + 5 = 2125.
∴ 6th term should be 2125 instead of 2124.

79. 7th term = (8th term) × 2 + 1 = 20 × 2 + 1 = 41;
6th term = (7th term) × 2 + 2 = 41 × 2 + 2 = 84;
5th term = (6th term) × 2 + 3 = 84 × 2 + 3 = 171;
4th term = (5th term) × 2 + 4 = 171 × 2 + 4 = 346.
∴ 4th term should be 346 instead of 347.

80. 2nd term = (1st term) + 2^2 = 32 + 4 = 36; 3rd term = (2nd term) + 3^2 = 36 + 9 = 45;
4th term = (3rd term) + 4^2 = 45 + 16 = 61; 5th term = (4th term) + 5^2 = 61 + 25 = 86.
∴ 3rd term should be 45 instead of 41.

81. There are two sequences (3, 9, 67.5, 810) and (4, 22.5, 202.5).
Pattern is : (1st term × 3), (2nd term × 7.5), (3rd term × 12) for the first sequence and (1st term × 5), (2nd term × 9) and so on for the second sequence.

82. 2nd term = (1st term × 1 + 1^2) = 1 × 1 + 1^2 = 2;
3rd term = (2nd term × 2 + 2^2) = 2 × 2 + 2^2 = 8;
4th term = (3rd term × 3 + 3^2) = 8 × 3 + 3^2 = 33;
5th term = (4th term × 4 + 4^2) = 33 × 4 + 4^2 = 148;
6th term = (5th term × 5 + 5^2) = 148 × 5 + 5^2 = 765.
∴ 760 is wrong.

83. 2nd term = (1st term × 2 + 2) = 3 × 2 + 2 = 8;
3rd term = (2nd term × 2 + 4) = 8 × 2 + 4 = 20;
4th term = (3rd term × 2 + 6) = 20 × 2 + 6 = 46;
5th term = (4th term × 2 + 8) = 46 × 2 + 8 = 100 and so on.
∴ 18 is wrong.

84. 2nd term = 1st term − $(12)^2$ = 789 − 144 = 645;
3rd term = (2nd term) − $(10)^2$ = 645 − 100 = 545;
4th term = (3rd term) − $(8)^2$ = 545 − 64 = 481;
5th term = (4th term) − $(6)^2$ = 481 − 36 = 445.
∴ 440 is wrong.

85. 2nd term = (1st term − 30) ÷ 2 = $\left(\frac{1050-30}{2}\right)$ = 510;
3rd term = (2nd term − 26) ÷ 2 = $\left(\frac{510-26}{2}\right)$ = 242;
4th term = (3rd term − 22) ÷ 2 = $\left(\frac{242-22}{2}\right)$ = 110.
∴ 106 is wrong.

86. 2nd term = (1st term × 2 − 2) = (5 × 2 − 2) = 8;
3rd term = (2nd term × 3 − 2) = (8 × 3 − 2) = 22;
4th term = (3rd term × 2 − 2) = (22 × 2 − 2) = 42;
5th term = (4th term × 3 − 2) = (42 × 3 − 2) = 124 and so on.
∴ 20 is wrong.

87. 2nd term = (1st term × 1.5) = 2 × 1.5 = 3; 3rd term = (2nd term × 2) = 3 × 2 = 6;
4th term = (3rd term × 2.5) = 6 × 2.5 = 15; 5th term = (4th term × 3) = 15 × 3 = 45.
∴ 52.5 is wrong.

88. 2nd term $= \left(\frac{\text{1st term} - 8}{2}\right) = \left(\frac{888 - 8}{2}\right) = 440;$

3rd term $= \left(\frac{\text{2nd term} - 8}{2}\right) = \left(\frac{440 - 8}{2}\right) = 216;$

4th term $= \left(\frac{\text{3rd term} - 8}{2}\right) = \left(\frac{216 - 8}{2}\right) = 104;$

5th term $= \left(\frac{\text{4th term} - 8}{2}\right) = \left(\frac{104 - 8}{2}\right) = 48;$

6th term $= \left(\frac{\text{5th term} - 8}{2}\right) = \left(\frac{48 - 8}{2}\right) = 20.$

∴ 22 is wrong.

89. 2nd term = (1st term × 1 + 2) = (4 × 1 + 2) = 6;
3rd term = (2nd term × 2 + 3) = (6 × 2 + 3) = 15;
4th term = (3rd term × 3 + 4) = (15 × 3 + 4) = 49;
5th term = (4th term × 4 + 5) = (49 × 4 + 5) = 210 and so on.
∴ 5 is wrong.

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36. TABULATION

This section comprises of questions in which certain data regarding common disciplines as production over a period of a few years : imports, exports, incomes of employees in a factory, students applying for and qualifying a certain field of study etc. are given in the form of a table. The candidate is required to understand the given information and thereafter answer the given questions on the basis of comparative analysis of the data.

Thus, here the data collected by the investigator are arranged in a systematic form in a table called the *tabular form*. In order to avoid some heads again and again, tables are made consisting of horizontal lines called *rows* and vertical lines called *columns* with distinctive heads, known as *captions*. Units of measurements are given with the captions.

SOLVED EXAMPLES

Ex. 1. ***The following table gives the sales of batteries manufactured by a company over the years. Study the table and answer the questions that follow :***

(S.B.I.P.O. 1998)

NUMBER OF DIFFERENT TYPES OF BATTERIES SOLD BY A COMPANY OVER THE YEARS (NUMBERS IN THOUSANDS)

Year	TYPES OF BATTERIES					
	4AH	7AH	32AH	35AH	55AH	Total
1992	75	144	114	102	108	543
1993	90	126	102	84	126	528
1994	96	114	75	105	135	525
1995	105	90	150	90	75	510
1996	90	75	135	75	90	465
1997	105	60	165	45	120	495
1998	115	85	160	100	145	605

1. The total sales of all the seven years is the maximum for which battery ?
   (*a*) 4AH (*b*) 7AH (*c*) 32AH (*d*) 35AH (*e*) 55AH
2. What is the difference in the number of 35AH batteries sold in 1993 and 1997 ?
   (*a*) 24000 (*b*) 28000 (*c*) 35000 (*d*) 39000 (*e*) 42000
3. The percentage of 4AH batteries sold to the total number of batteries sold was maximum in the year :
   (*a*) 1994 (*b*) 1995 (*c*) 1996 (*d*) 1997 (*e*) 1998
4. In the case of which battery there was a continuous decrease in sales from 1992 to 1997 ?
   (*a*) 4AH (*b*) 7AH (*c*) 32AH (*d*) 35AH (*e*) 55AH
5. What was the approximate percentage increase in the sales of 55AH batteries in 1998 compared to that in 1992 ?
   (*a*) 28% (*b*) 31% (*c*) 33% (*d*) 34% (*e*) 37%

Sol. 1. (*c*) : The total sales (in thousands) of all the seven years for various batteries are :

For 4AH = 75 + 90 + 96 + 105 + 90 + 105 + 115 = 676

For 7AH = 144 + 126 + 114 + 90 + 75 + 60 + 85 = 694

For 32AH = 114 + 102 + 75 + 150 + 135 + 165 + 160 = 901

For 35 AH = 102 + 84 + 105 + 90 + 75 + 45 + 100 = 601

For 55 AH = 108 + 126 + 135 + 75 + 90 + 120 + 145 = 799.

Clearly, sales are maximum in case of 32AH batteries.

2. (*d*) : Required difference = [(84 – 45) × 1000] = 39000.

3. (*d*) : The percentages of sales of 4AH batteries to the total sales in different years are :

For 1992 $= \left(\frac{75}{543} \times 100\right)\% = 13.81\%$; **For 1993** $= \left(\frac{90}{528} \times 100\right)\% = 17.05\%$;

For 1994 $= \left(\frac{96}{525} \times 100\right)\% = 18.29\%$; **For 1995** $= \left(\frac{105}{510} \times 100\right)\% = 20.59\%$;

For 1996 $= \left(\frac{96}{465} \times 100\right)\% = 19.35\%$; **For 1997** $= \left(\frac{105}{495} \times 100\right)\% = 21.21\%$;

For 1998 $= \left(\frac{115}{605} \times 100\right)\% = 19.01\%$.

Clearly, the percentage is maximum in 1997.

4. (*b*) : From the table it is clear that the sales of 7AH batteries have been decreasing continuously from 1992 to 1997.

5. (*d*) : Required Percentage $= \left[\frac{(145-108)}{108} \times 100\right]\% = 34.26\% \approx 34\%$.

Ex. 2. ***Study the following table carefully and answer these questions :*** **(S.B.I.P.O. 2002)**

NUMBER OF CANDIDATES APPEARED AND QUALIFIED IN A COMPETITIVE EXAMINATION FROM DIFFERENT STATES OVER THE YEARS

Year / State	1997		1998		1999		2000		2001	
	App.	Qual.	App.	Qual.	App.	Qual.	App.	Qual.	App.	Qual.
M	5200	720	8500	980	7400	850	6800	775	9500	1125
N	7500	840	9200	1050	8450	920	9200	980	8800	1020
P	6400	780	8800	1020	7800	890	8750	1010	9750	1250
Q	8100	950	9500	1240	8700	980	9700	1200	8950	995
R	7800	870	7600	940	9800	1350	7600	945	7990	885

1. Combining the states P and Q together in 1998, what is the percentage of the candidates qualified to that of the candidates appeared ?
(*a*) 10.87% (*b*) 11.49% (*c*) 12.35% (*d*) 12.54% (*e*) 13.05%

2. The percentage of the total number of qualified candidates to the total number of appeared candidates among all the five states in 1999 is :
(*a*) 11.49% (*b*) 11.84% (*c*) 12.21% (*d*) 12.57% (*e*) 12.73%

3. What is the percentage of candidates qualified from State N for all the years together, over the candidates appeared from State N during all the years together ?
(*a*) 12.36% (*b*) 12.16% (*c*) 11.47% (*d*) 11.15% (*e*) None of these

4. What is the average of candidates who appeared from State Q during the given yeas ?

(a) 8700 (b) 8760 (c) 8810 (d) 8920 (e) 8990

5. In which of the given years the number of candidates appeared from State P has maximum percentage of qualified candidates ?

(a) 1997 (b) 1998 (c) 1999 (d) 2000 (e) 2001

6. Total number of candidates qualified from all the states together in 1997 is approximately what percentage of the total number of candidates qualified from all the states together in 1998 ?

(a) 72% (b) 77% (c) 80% (d) 83% (e) 86%

Sol. 1. (c) : Required Percentage $= \left[\frac{(1020+1240)}{(8800+9500)} \times 100\right]\% = \left(\frac{2260}{18300} \times 100\right)\%$

$= 12.35\%.$

2. (b) : Required Percentage $= \left[\frac{(850+920+890+980+1350)}{(7400+8450+7800+8700+9800)} \times 100\right]\%$

$= \left(\frac{4990}{42150} \times 100\right)\% = 11.84\%.$

3. (d) : Required Percentage $= \left[\frac{(840+1050+920+980+1020)}{(7500+9200+8450+9200+8800)} \times 100\right]\%$

$= \left(\frac{4810}{43150} \times 100\right)\% = 11.15\%.$

4. (e) : Required average $= \frac{8100+9500+8700+9700+8950}{5} = \frac{44950}{5} = 8990.$

5. (e) : The percentages of candidates qualified to candidates appeared from State P during different years are :

For 1997 $= \left(\frac{780}{6400} \times 100\right)\% = 12.19\%$; **For 1998** $= \left(\frac{1020}{8800} \times 100\right)\% = 11.59\%$;

For 1999 $= \left(\frac{890}{7800} \times 100\right)\% = 11.41\%$; **For 2000** $= \left(\frac{1010}{8750} \times 100\right)\% = 11.54\%$;

For 2001 $= \left(\frac{1250}{9750} \times 100\right)\% = 12.82\%.$

$\therefore$ Maximum percentage is for the year 2001.

6. (c) : Required Percentage $= \left[\frac{(720+840+780+950+870)}{(980+1050+1020+1240+940)} \times 100\right]\%$

$= \left(\frac{4160}{5230} \times 100\right)\% = 79.54\% = 80\%.$

Ex. 3. ***The following table gives the percentage of marks obtained by seven students in six different subjects in an examination. Study the table and answer the questions based on it. The numbers in the brackets give the maximum marks in each subject.*** **(Bank P.O. 2003)**

Subjects (Max. Marks) / Student	Maths (150)	Chemistry (130)	Physics (120)	Geography (100)	History (60)	Computer Science (40)
Ayush	90	50	90	60	70	80
Aman	100	80	80	40	80	70
Sajal	90	60	70	70	90	70
Rohit	80	65	80	80	60	60
Muskan	80	65	85	95	50	90
Tanvi	70	75	65	85	40	60
Tarun	65	35	50	77	80	80

1. What was the aggregate of marks obtained by Sajal in all the six subjects ?
(*a*) 409 (*b*) 419 (*c*) 429 (*d*) 439 (*e*) 449
2. What is the overall percentage of Tarun ?
(*a*) 52.5% (*b*) 55% (*c*) 60% (*d*) 63% (*e*) 64.5%
3. What are the average marks obtained by all the seven students in Physics ? (rounded off to two digits after decimal)
(*a*) 77.26 (*b*) 89.14 (*c*) 91.37 (*d*) 96.11 (*e*) 103.21
4. The number of students who obtained 60% and above marks in all the subjects is :
(*a*) 1 (*b*) 2 (*c*) 3 (*d*) None (*e*) None of these
5. In which subject is the overall percentage the best ?
(*a*) History (*b*) Maths (*c*) Physics (*d*) Chemistry (*e*) Geography

Sol. 1. (*e*) : Aggregate marks obtained by Sajal
= [(90% of 150) + (60% of 130) + (70% of 120) + (70% of 100) + (90% of 60) + (70% of 40)] = 135 + 78 + 84 + 70 + 54 + 28 = 449.

2. (*c*) : Aggregate marks obtained by Tarun
= [(65% of 150) + (35% of 130) + (50% of 120) + (77% of 100) + (80% of 60) + (80% of 40)] = 97.5 + 45.5 + 60 + 77 + 48 + 32 = 360.
Total maximum marks (of all the six subjects)
= (150 + 130 + 120 + 100 + 60 + 40) = 600.

Overall percentage of Tarun $= \left(\frac{360}{600} \times 100\right)\% = 60\%.$

3. (*b*) : Average marks obtained in Physics by all the seven students

$$= \frac{1}{7} \times [(90\% \text{ of } 120) + (80\% \text{ of } 120) + (70\% \text{ of } 120) + (80\% \text{ of } 120) + (85\% \text{ of } 120) + (65\% \text{ of } 120) + (50\% \text{ of } 120)]$$

$$= \frac{1}{7} \times [(90 + 80 + 70 + 80 + 85 + 65 + 50)\% \text{ of } 120]$$

$$= \frac{1}{7} \times [520\% \text{ of } 120] = \frac{624}{7} = 89.14.$$

4. (*b*) : From the table it is clear that Sajal and Rohit have 60% or more marks in each of the six subjects.

5. (*b*) : We shall find the overall percentage (for all the seven students) with respect to each subject.
The overall percentage for any subject is equal to the average of percentages obtained by all the seven students since the maximum marks for any subject is the same for all the students.

Therefore, overall percentage for :

(*i*) Maths $= \left[\frac{1}{7} \times (90 + 100 + 90 + 80 + 80 + 70 + 65)\right]\%$

$= \left[\frac{1}{7} \times (575)\right]\% = 82.14\%.$

(*ii*) Chemistry $= \left[\frac{1}{7} \times (50 + 80 + 60 + 65 + 65 + 75 + 35)\right]\%$

$= \left[\frac{1}{7} \times (430)\right]\% = 61.43\%.$

(*iii*) Physics $= \left[\frac{1}{7} \times (90 + 80 + 70 + 80 + 85 + 65 + 50)\right]\%$

$= \left[\frac{1}{7} \times (520)\right]\% = 74.29\%.$

(*iv*) Geography $= \left[\frac{1}{7} \times (60 + 40 + 70 + 80 + 95 + 85 + 77)\right]\%$

$= \left[\frac{1}{7} \times (507)\right]\% = 72.43\%.$

(*v*) History $= \left[\frac{1}{7} \times (70 + 80 + 90 + 60 + 50 + 40 + 80)\right]\%$

$= \left[\frac{1}{7} \times (470)\right]\% = 67.14\%.$

(*vi*) Computer Science $= \left[\frac{1}{7} \times (80 + 70 + 70 + 60 + 90 + 60 + 80)\right]\%$

$= \left[\frac{1}{7} \times (510)\right]\% = 72.86\%.$

Clearly, this percentage is highest for Maths.

Ex. 4. ***Study the following table carefully and answer the questions given below :*** **(Bank P.O. 2001)**

CLASSIFICATION OF 100 STUDENTS BASED ON THE MARKS OBTAINED BY THEM IN PHYSICS AND CHEMISTRY IN AN EXAMINATION

Marks out of 50 / Subject	40 and above	30 and above	20 and above	10 and above	0 and above
Physics	9	32	80	92	100
Chemistry	4	21	66	81	100
(Aggregate) Average	7	27	73	87	100

1. The number of students scoring less than 40% marks in aggregate is :
   (*a*) 13 (*b*) 19 (*c*) 20 (*d*) 27 (*e*) 34
2. If at least 60% marks in Physics are required for pursuing higher studies in Physics, how many students will be eligible to pursue higher studies in Physics ?
   (*a*) 27 (*b*) 32 (*c*) 34 (*d*) 41 (*e*) 68
3. What is the difference between the number of students passed with 30 as cut-off marks in Chemistry and those passed with 30 as cut-off marks in aggregate ?
   (*a*) 3 (*b*) 4 (*c*) 5 (*d*) 6 (*e*) 7

4. The percentage of the number of students getting at least 60% marks in Chemistry over those getting at least 40% marks in aggregate, is approximately :
(a) 21% (b) 27% (c) 29% (d) 31% (e) 34%

5. If it is known that at least 23 students were eligible for a Symposium on Chemistry, the minimum qualifying marks in Chemistry for eligibility to Symposium would lie in the range :
(a) 40-50 (b) 30-40 (c) 20-30 (d) Below 20 (e) Cannot be determined

Sol. 1. (d) : We have 40% of 50 = $\left(\frac{40}{100} \times 50\right) = 20$.

∴ Required number = Number of students scoring less than 20 marks in aggregate
= 100 – number of students scoring 20 and above marks in aggregate = 100 – 73 = 27.

2. (b) : We have 60% of 50 = $\left(\frac{60}{100} \times 50\right) = 30$.

∴ Required number = Number of students scoring 30 and above marks in Physics = 32.

3. (d) : Required difference = (Number of students scoring 30 and above marks in Chemistry) – (Number of students scoring 30 and above marks in aggregate) = 27 – 21 = 6.

4. (c) : Number of students getting at least 60% marks in Chemistry
= Number of students getting 30 and above marks in Chemistry = 21.
Number of students getting at least 40% marks in aggregate
= Number of students getting 20 and above marks in aggregate = 73.

∴ Required Percentage = $\left(\frac{21}{73} \times 100\right)\% = 28.77\% \approx 29\%$.

5. (c) : Since 66 students get 20 and above marks in Chemistry and out of these 21 students get 30 and above marks, therefore to select top 35 students in Chemistry, the qualifying marks should lie in the range 20-30.

EXERCISE 36

Directions (Questions 1 to 6) : ***Study the following table and answer the questions based on it.*** **(Bank P.O. 2003)**

NUMBER OF CANDIDATES APPEARED, QUALIFIED AND SELECTED IN A COMPETITIVE EXAMINATION FROM FIVE STATES DELHI, H.P., U.P., PUNJAB AND HARYANA OVER THE YEARS 1994 TO 1998

Year	Delhi			H.P.			U.P.			Punjab			Haryana		
	App.	Qual.	Sel.	App.	Qual.	Sel.	App.	Qual.	Sel.	App.	Qual.	Sel.	App.	Qual.	Sel.
1997	8000	850	94	7800	810	82	7500	720	78	8200	680	85	6400	700	75
1998	4800	500	48	7500	800	65	5600	620	85	6800	600	70	7100	650	75
1999	7500	640	82	7400	560	70	4800	400	48	6500	525	65	5200	350	55
2000	9500	850	90	8800	920	86	7000	650	70	7800	720	84	6400	540	60
2001	9000	800	70	7200	850	75	8500	950	80	5700	485	60	4500	600	75

1. In the year 1997, which state had the lowest percentage of candidates selected over the candidates appeared ?
(a) Delhi (b) H.P. (c) U.P. (d) Punjab (e) Haryana

2. The percentage of candidates qualified from Punjab over those appeared from Punjab is highest in the year :
(a) 1997 (b) 1998 (c) 1999 (d) 2000 (e) 2001

3. The percentage of candidates selected from U.P. over those qualified from U.P. is highest in the year :
(a) 1997 (b) 1998 (c) 1999 (d) 2000 (e) 2001

4. The number of candidates selected from Haryana during the period under review is approximately what percent of the number selected from Delhi during this period ?
(a) 79.5% (b) 81% (c) 84.5% (d) 88.5% (e) 92.5%

5. For which state the average number of candidates selected over the years is the maximum ?
(a) Delhi (b) H.P. (c) U.P. (d) Punjab (e) Haryana

6. What is the approximate percentage of total number of candidates selected to the total number of candidates qualified for all the five states together during the year 1999 ?
(a) 10% (b) 11% (c) 12% (d) 13% (e) 14%

Directions (Questions 7 to 11) : ***Study the following table to answer the questions that are given below it.*** **(R.B.I. 2003)**

EXPENDITURES OF A COMPANY (IN LAKH RUPEES) PER ANNUM OVER THE GIVEN YEARS

Item of Expenditure / Year	Salary	Fuel and Transport	Bonus	Interest on Loans	Taxes
1998	288	98	3.00	23.4	83
1999	342	112	2.52	32.5	108
2000	324	101	3.84	41.6	74
2001	336	133	3.68	36.4	88
2002	420	142	3.96	49.4	98

7. The ratio between the total expenditure on Taxes for all the years and the total expenditure on Fuel and Transport for all the years respectively is approximately :
(a) 4 : 7 (b) 10 : 13 (c) 15 : 18 (d) 5 : 8 (e) 2 : 3

8. The total expenditure of the Company over these items during the year 2000 is :
(a) Rs. 544.44 lakhs (b) Rs. 501.11 lakhs (c) Rs. 446.46 lakhs
(d) Rs. 478.87 lakhs (e) Rs. 612.13 lakhs

9. What is the average amount of interest per year which the Company had to pay during this period ?
(a) Rs. 32.43 lakhs (b) Rs. 33.72 lakhs (c) Rs. 34.18 lakhs
(d) Rs. 35.69 lakhs (e) Rs. 36.66 lakhs

10. Total expenditure on all these items in 1998 was approximately what percent of the total expenditure in 2002 ?
(a) 62% (b) 66% (c) 69% (d) 71% (e) 73%

11. The total amount of bonus paid by the Company during the given period is approximately what percent of the total amount of salary paid during this period ?
(a) 0.1% (b) 0.5% (c) 1% (d) 1.25% (e) 1.11%

Directions (Questions 12 to 16) : ***A school has four sections A, B, C, D of Class IX students. The results of half-yearly and annual examinations are shown in the table given below. Answer the questions based on this table.*** **(Bank P.O. 2000)**

Result	Number of Students			
	Section A	Section B	Section C	Section D
Students failed in both Exams	28	23	17	27
Students failed in half-yearly but passed in Annual Exams	14	12	8	13
Students passed in half-yearly but failed in Annual Exams	6	17	9	15
Students passed in both Exams	64	55	46	76

12. How many students are there in Class IX in the school ?
(*a*) 336 (*b*) 189 (*c*) 335 (*d*) 286 (*e*) 430

13. Which section has the minimum failure rate in half-yearly examination ?
(*a*) A (*b*) B (*c*) C
(*d*) D (*e*) Cannot be determined

14. Which section has the maximum success rate in annual examination ?
(*a*) A (*b*) B (*c*) C
(*d*) D (*e*) Cannot be determined

15. Which section has the maximum pass percentage in at least one of the two examinations ?
(*a*) A (*b*) B (*c*) C
(*d*) D (*e*) Cannot be determined

16. If the number of students passing an examination be considered a criteria for comparison of difficulty level of two examinations, which of the following statements is true in this context ?
(*a*) Half-yearly examinations were more difficult.
(*b*) Annual examinations were more difficult.
(*c*) Both the examinations had almost the same difficulty level.
(*d*) The two examinations cannot be compared for difficulty level.
(*e*) For students of Sections A and B, the annual examinations seem to be more difficult as compared to the half-yearly examinations.

Directions (Questions 17 to 21) : ***The following table shows the number of new employees added to different categories of employees in a Company and also the number of employees from these categories who left the company every year since the foundation of the Company in 1995.*** **(Bank P.O. 2001)**

Year	Managers		Technicians		Operators		Accountants		Peons	
	New	Left	New	Left	New	Left	New	Left	New	Left
1995	760	—	1200	—	880	—	1160	—	820	—
1996	280	120	272	120	256	104	200	100	184	96
1997	179	92	240	128	240	120	224	104	152	88
1998	148	88	236	96	208	100	248	96	196	80
1999	160	72	256	100	192	112	272	88	224	120
2000	193	96	288	112	248	144	260	92	200	104

17. During the period between 1995 and 2000, the total number of Operators who left the Company is what percent of the total number of Operators who joined the Company ?
(a) 19% (b) 21% (c) 27% (d) 29% (e) 32%

18. For which of the following categories the percentage increase in the number of employees working in the Company from 1995 to 2000 was the maximum ?
(a) Managers (b) Technicians (c) Operators (d) Accountants (e) Peons

19. What is the difference between the total number of Technicians added to the Company and the total number of Accountants added to the Company during the years 1996 to 2000 ?
(a) 128 (b) 112 (c) 96 (d) 88 (e) 72

20. What was the total number of Peons working in the Company in the year 1999 ?
(a) 1312 (b) 1192 (c) 1088 (d) 968 (e) 908

21. What is the pooled average of the total number of employees of all categories in the year 1997 ?
(a) 1325 (b) 1285 (c) 1265 (d) 1235 (e) 1195

Directions (Questions 22 to 25) : ***The following table gives the percentage distribution of population of five states, P, Q, R, S and T on the basis of poverty line and also on the basis of sex. Study the table and answer the questions based on it.***

(Bank P.O. 2000)

State	Percentage of Population below Poverty Line	Proportion of Males and Females	
		Below Poverty Line	Above Poverty Line
		M : F	M : F
P	35	5 : 6	6 : 7
Q	25	3 : 5	4 : 5
R	24	1 : 2	2 : 3
S	19	3 : 2	4 : 3
T	15	5 : 3	3 : 2

22. What will be the number of females above poverty line in the State S if it is known that the population of State S is 7 million ?
(a) 3 million (b) 2.43 million (c) 1.33 million
(d) 5.7 million (e) 1.61 million

23. If the male population above poverty line for State R is 1.9 million, then the total population of State R is :
(a) 4.5 million (b) 4.85 million (c) 5.35 million
(d) 6.25 million (e) 7.6 million

24. What will be the male population above poverty line for State P if the female population below poverty line for State P is 2.1 million ?
(a) 2.1 million (b) 2.3 million (c) 2.7 million
(d) 3.3 million (e) 3.4 million

25. If the population of males below poverty line for State Q is 2.4 million and that for State T is 6 million, then the total populations of states Q and T are in the ratio :
(a) 1 : 3 (b) 2 : 5 (c) 3 : 7 (d) 4 : 9 (e) 5 : 12

ANSWERS

1. (*d*)	2. (*d*)	3. (*b*)	4. (*d*)	5. (*a*)	6. (*d*)	7. (*b*)	8. (*a*)	9. (*e*)
10. (*c*)	11. (*c*)	12. (*e*)	13. (*d*)	14. (*a*)	15. (*d*)	16. (*c*)	17. (*d*)	18. (*a*)
19. (*d*)	20. (*b*)	21. (*e*)	22. (*b*)	23. (*d*)	24. (*d*)	25. (*b*)		

SOLUTIONS

1. The percentages of candidates selected over the candidates appeared in 1997, for various states are :

(*i*) For Delhi $= \left(\frac{94}{8000} \times 100\right)\% = 1.175\%$; (*ii*) For H.P. $= \left(\frac{82}{7800} \times 100\right)\% = 1.051\%$;

(*iii*) For U.P. $= \left(\frac{78}{7500} \times 100\right)\% = 1.040\%$; (*iv*) For Punjab $= \left(\frac{85}{8200} \times 100\right)\% = 1.037\%$;

(*v*) For Haryana $= \left(\frac{75}{6400} \times 100\right)\% = 1.172\%$.

Clearly, this percentage is lowest for Punjab.

2. The percentages of candidates qualified from Punjab over those appeared from Punjab during different years are :

For 1997 $= \left(\frac{680}{8200} \times 100\right)\% = 8.29\%$; **For 1998** $= \left(\frac{600}{6800} \times 100\right)\% = 8.82\%$;

For 1999 $= \left(\frac{525}{6500} \times 100\right)\% = 8.08\%$; **For 2000** $= \left(\frac{720}{7800} \times 100\right)\% = 9.23\%$;

For 2001 $= \left(\frac{485}{5700} \times 100\right)\% = 8.51\%$.

Clearly, this percentage is highest for the year 2000.

3. The percentages of candidates selected from U.P. over those qualified from U.P. during different years are :

For 1997 $= \left(\frac{78}{720} \times 100\right)\% = 10.83\%$; **For 1998** $= \left(\frac{85}{620} \times 100\right)\% = 13.71\%$;

For 1999 $= \left(\frac{48}{400} \times 100\right)\% = 12\%$; **For 2000** $= \left(\frac{70}{650} \times 100\right)\% = 10.77\%$;

For 2001 $= \left(\frac{80}{950} \times 100\right)\% = 8.42\%$.

Clearly, this percentage is highest for the year 1998.

4. Required Percentage $= \left[\frac{(75 + 75 + 55 + 60 + 75)}{(94 + 48 + 82 + 90 + 70)} \times 100\right]\%$

$= \left(\frac{340}{384} \times 100\right)\% = 88.54\% \approx 88.5\%$.

5. The average number of candidates selected over the given period for various states are :

For Delhi $= \frac{94 + 48 + 82 + 90 + 70}{5} = \frac{384}{5} = 76.8$

For H.P. $= \frac{82 + 65 + 70 + 86 + 75}{5} = \frac{378}{5} = 75.6$

For U.P. $= \dfrac{78+85+48+70+80}{5} = \dfrac{361}{5} = 72.2$

For Punjab $= \dfrac{85+70+65+84+60}{5} = \dfrac{364}{4} = 72.8$

For Haryana $= \dfrac{75+75+55+60+75}{5} = \dfrac{340}{5} = 68.$

Clearly, this average is maximum for Delhi.

6. Required Percentage $= \left[\dfrac{(82+70+48+65+55)}{(640+560+400+525+350)} \times 100\right]\%$

$= \left(\dfrac{320}{2475} \times 100\right)\% = 12.93\% \approx 13\%.$

7. Required Ratio $= \dfrac{(83+108+74+88+98)}{(98+112+101+133+142)} = \dfrac{451}{586} = \dfrac{1}{1.3} = \dfrac{10}{13}.$

8. Total expenditure of the Company during 2000

$=$ Rs. $(324 + 101 + 3.84 + 41.6 + 74)$ lakhs $=$ Rs. 544.44 lakhs.

9. Average amount of interest paid by the Company during the given period

$=$ Rs. $\left(\dfrac{23.4+32.5+41.6+36.4+49.4}{5}\right)$ lakhs $=$ Rs. $\left(\dfrac{183.3}{5}\right)$ lakhs

$=$ Rs. 36.66 lakhs.

10. Required Percentage $= \left[\dfrac{(288+98+3.00+23.4+83)}{(420+142+3.96+49.4+98)} \times 100\right]\%$

$= \left(\dfrac{495.4}{713.36} \times 100\right)\% = 69.45\%.$

11. Required Percentage $= \left[\dfrac{(3.00+2.52+3.84+3.68+3.96)}{(288+342+324+336+420)} \times 100\right]\%$

$= \left(\dfrac{17}{1710} \times 100\right)\% \approx 1\%.$

12. Since the classification of the students on the basis of their results and sections form independent groups, so the total number of students in the class :

$= (28 + 23 + 17 + 27 + 14 + 12 + 8 + 13 + 6 + 17 + 9 + 15 + 64 + 55 + 46 + 76) = 430.$

13. Total number of failures in half-yearly exams in a section

$=$ [(Number of students failed in both exams) + (Number of students failed in half-yearly but passed in Annual exams)] in that section

$\therefore$ Failure rate in half-yearly exams in **Section A**

$= \left[\dfrac{\text{Number of students of Section A failed in half-yearly}}{\text{Total number of students in Section A}} \times 100\right]\%$

$= \left[\dfrac{(28+14)}{(28+14+6+64)} \times 100\right]\% = \left(\dfrac{42}{112} \times 100\right)\% = 37.5\%$

Similarly, failure rate in half-yearly exams in :

Section B $= \left[\dfrac{(23+12)}{(23+12+17+55)} \times 100\right]\% = \left(\dfrac{35}{107} \times 100\right)\% = 32.71\%$

Section C $= \left[\dfrac{(17+8)}{(17+8+9+46)} \times 100\right]\% = \left(\dfrac{25}{80} \times 100\right)\% = 31.25\%$