Short Notes for Soil Mechanics & Foundation Engineering

Properties of Soils

Water content

- $w = \frac{W_W}{W_S} \times 100$

W_W = Weight of power
W_S = Weight of solids

Void ratio

- $e = \frac{V_v}{V_s}$

  V_v = Volume of voids
  V = Total volume of soil

Degree of Saturation

- $S = \frac{V_w}{V_v} \times 100$

V_w = Volume of water
V_v = Volume of voids
$0 \le S \le 100$
for perfectly dry soil : S = O
for Fully saturated soil : S = 100%

Air Content

- $a_c = \frac{V_a}{V_v} = 1 - s$ V_a = Volume of air

$S_r + a_c = 1$

% Air Void

- $\% n_a = \frac{\text{Volume of air}}{\text{Total volume}} \times 100 = \frac{V_a}{V} \times 100$

Unit Weight

- **Bulk unit weight**

$$\gamma = \frac{W}{V} = \frac{W_s + W_w}{V_s + V_w + V_a}$$

- **Dry Unit Weight**

$$\gamma_d = \frac{W_s}{V}$$

  - Dry unit weight is used as a **measure of denseness** of soil
- **Saturated unit weight:** It is the ratio of total weight of fully saturated soil sample to its total volume.

$$\gamma_{sat} = \frac{W_{sat}}{V}$$

- **Submerged unit weight or Buoyant unit weight**

$$\gamma' = \gamma_{sat} - \gamma_w$$

γ_{sat} = unit wt. of saturated soil

γ = unit wt. of water

- **Unit wt. of solids:**

$$\gamma_s = \frac{W_s}{V_s}$$

Specific Gravity

True/Absolute Special Gravity, G

- Specific gravity of soil solids (G) is the ratio of the weight of a given volume of solids to the weight of an equivalent volume of water at 4°C.

$$G = \frac{W_s}{V_s.\gamma_w} = \frac{\gamma_s}{\gamma_w}$$

- Apparent or mass specific gravity (G_m):

$$G_m = \frac{W}{V.\gamma_w} = \frac{\gamma \text{ or } \gamma_d \text{ or } \gamma_{sat}}{\gamma_w}$$

where, γ is bulk unit wt. of soil

γ = γ_{sat} for saturated soil mass

γ = γ_d for dry soil mass

$G_m < G$

Relative density (I_D)

- **To compare degree of denseness of two soils.**

$$I_D \infty Shear\ strength \alpha \frac{1}{Compressibility}$$

$$\%I_D = \frac{e_{\max} - e}{e_{\max} - e_{\min}} \times 100$$

$$\%I_D = \frac{\frac{1}{\gamma_{d\min}} - \frac{1}{\gamma_d}}{\frac{1}{\gamma_{d\min}} - \frac{1}{\gamma_{d\max}}} \times 100$$

Relative Compaction

- **Indicate:** Degree of denseness of cohesive + cohesionless soil

$$R_c = \frac{\gamma_D}{\gamma_{D_{\max}}}$$

Relative Density

- **Indicate:** Degree of denseness of natural cohesionless soil

Some Important Relationships

- **Relation between** γ_d, γ

$$\gamma_d = \frac{\gamma}{1+w}$$

(ii) $V_s = \dfrac{V}{1+e}$ (iii) $W_s = \dfrac{W}{1+w}$

- **Relation between e and n**

$$n = \frac{e}{1+e} \quad \text{or} \quad e = \frac{n}{1-n}$$

- **Relation between e, w, G and S**:

Se = w. G

- **Bulk unit weight** (γ) **in terms of G, e, w and** γ_w γ **, G, e, S$_r$,** γ_w

$$\gamma = \frac{(G+eS_r)\gamma_w}{1+e}$$

$$\gamma = \frac{G\gamma_w(1+w)}{(1+e)} \quad \textbf{\{Srxe = wG\}}$$

- **Saturated unit weight** $(\gamma sat.)$ **in terms of G, e &** γ_w

S$_r$ = 1 $\gamma_{sat} = \left[\frac{G+e}{1+e}\right].\gamma_w$

- **Dry unit weight** (γ_d) **in terms of G, e and** γ_w

S$_r$ = 0 $\gamma_d = \frac{G\gamma_w}{1+e} = \frac{G\gamma_w}{1+\frac{wG}{S}} = \frac{(1-\eta_a)G\gamma_w}{1+wG}$

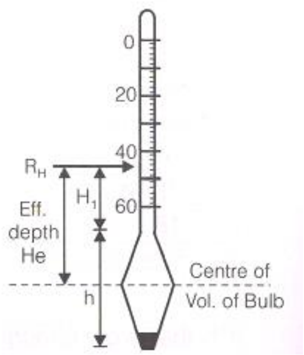
- **Submerged unit weight** (γ') **in terms of G, e and** γ_w

$$\gamma = \gamma_{sat} - \gamma_w = \gamma' = \left(\frac{G-1}{1+e}\right).\gamma_w$$

- **Relation between degree of saturation (s) w and G**

$$S = \frac{W}{\frac{\gamma_w}{\gamma}(1+W) - \frac{1}{G}}$$

- **Calibration of Hydrometer**



  - Effective depth is calculated as

$$H_e = H_1 + \frac{1}{2}\left(h - \frac{V_H}{A_j}\right)$$

where, H_1 = distance (cm) between any hydrometer reading and neck.
h = length of hydrometer bulb
V_H = volume of hydrometer bulb

Plasticity Index (I_p):

- It is the range of moisture content over which a soil exhibits plasticity.

$$I_p = W_L - W_p$$

W_L = water content at LL

W_p = water content at PL

I_p (%)	Soil Description
0	Non plastic
1 to 5	Slight plastic
5 to 10	Low plastic
10 to 20	Medium plastic
20 to 40	Highly plastic
> 40	Very highly plastic

Relative Consistency or Consistency – index (I_c):

$$I_C = \frac{W_L - W_N}{I_p}$$

$$\left.\begin{aligned} \therefore For\ W_N = W_L \Rightarrow I_C = 0 \\ For\ W_N = W_P \Rightarrow I_C = 1 \end{aligned}\right\}$$

Liquidity Index (I_L)

$$I_L = \frac{W_N - W_P}{I_P}$$

For a soil in plastic state I_L varies from 0 to 1.

Consist.	Description	I_C	I_L
Liquid	Liquid	<0	>1
Plastic	Very soft	0-0.25	0.75-1.00
	soft	0.25-0.5	0.50-0.75
	medium	0.50-0.75	0.25-0.50
	stiff	0.75-1.00	0.0-0.25
	stiff		
Semi-solid	Very stiff OR Hard	>1	< 0
Solid	Hard OR very hard	>1	< 0

Flow Index (I_f)

$$I_f = \frac{W_1 - W_2}{\log 10(N_2 / N_1)}$$

Toughness Index (I_t)

$$I_T = \frac{I_P}{I_F}$$

- For most of the soils: $0 < I_T < 3$

- When $I_T < 1$, the soil is friable (easily crushed) at the plastic limit.

- **Shrinkage Ratio (SR)**

$$SR = \frac{\frac{V_1 - V_2}{V_d} \times 100}{w_1 - w_2}$$

V_1 = Volume of soil mass at water content w_1%.
V_2 = volume of soil mass at water content w_2%.
V_d = volume of dry soil mass

$$\therefore\ SR = \frac{\left(\frac{V_1 - V_d}{V_d} \times 100\right)}{(W_1 - W_s)}$$

If w_1 & w_2 are expressed as ratio,

$$SR = \frac{(V_1 - V_2)/V_d}{W_1 - W_2}\ But, w_1 - w_2 = \frac{(V_1 - V_2)/\gamma_w}{W_s}$$

$$\therefore SR\ = \frac{W_s}{V_d}.\frac{1}{\gamma_w} = \frac{\gamma_d}{\gamma_w}$$

Properties	**Relationship**	**Governing Parameters**
Plasticity	$\propto$	Plasticity Index
Better Foundation Material upon Remoulding	$\propto$	Consistency Index
Compressibility	$\propto$	Liquid Limit
Rate of loss in shear strength with increase in water content	$\propto$	Flow Index
Strength of Plastic Limit	$\propto$	Toughness Index

Compaction of Soil

Optimum moisture content

$$(\delta_d)_{\max imum} = \frac{\delta}{1 + w_{optimum}}$$

$(\delta_d)_{\max imum}$ = Maximum dry density
δ = Density of soil
$w_{optimum}$ = Optimum moisture content

Comparison of Standard & Modified Proctor Test
Inference

- $\gamma_d = \dfrac{G\gamma_w}{1+\dfrac{wG}{S}}$ for, $r_{d\ max'}$ $S = 1$, $h_a = 0$ correspond to 100% saturation or zero air void line.
- $\gamma_d = \dfrac{(a-n_a)G\gamma_w}{1+wG}$
- Ratio of total energy given in heavy compaction test to that given in light compaction test

$$= \frac{4.9\times g\times(5\times 25)\times 450}{2.6\times g\times(3\times 25)\times 310} = 4.5$$

Compaction Equipments

	Type of Equipment	Suitability for soil type	Nature of project
1.	Rammers or Tampers	All soils	In confined areas such as fills behind retaining walls, basement walls etc. Trench fills.
2.	Smooth wheeled rollers	Crushed rocks, gravels sands	Road construction
3.	Pneumatic tyred rollers	Sand, gravels silts, clayey soils	Base, sub-base and embankment compaction for highways, air fields etc. Earth dams.
			Core of earth dams.
		Clayey soils	Embankment for oil storage tanks etc.

4. 5.	Sheep foot Rollers Vibratory Rollers	Sands	

Compaction Tests

Standard proctor test (Light compaction test)	Modified proctor test (Heavy compaction test)
• Volume of mould 942cc	• Volume of mould 942 cc
• No. of layers -3	• No. of layers -5
• No. of blows per layer - 25	• No. of blows per layer -25
• Height of free fall - 304.8 mm (12 inches)	• Height of free fall - 457.2 mm (18 inches)
• Wt. of hammer - 2.495 kg (5.5 /b)	• Wt. of hammer - 4.54 kg (10 /b)

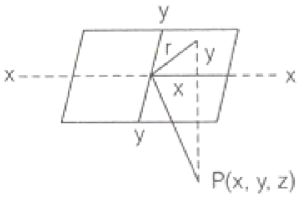
Indian standard light compaction	Indian standard heavy compaction
V – Volume of mould 1000 cc	• Volume of mould 1000 cc
H – Height of free fall 310 mm	• Height of free fall 450 mm
W – Wt. of hammer 2.6 kg	• Wt. of hammer 4.9 kg
N – No. of layers 3	• No. of layers 5
N – Blows per layer 25	• Blows per layer 25

Stress Distribution in The Soil

Boussinesq's Theory

Vertical stress at point 'P'. (σ_z)

- $$\sigma_Z = \frac{3Q}{2\pi z^2}\left[\frac{1}{1+\frac{r^2}{z^2}}\right]^{5/2}$$ where, Q = Point load in newton



- $$\sigma_Z = k_B.\frac{Q}{z^2}$$

$$k_B = = \frac{3}{2\pi}\left[\frac{1}{1+\frac{r^2}{z^2}}\right]^{5/2}$$

$$kB|_{\max} = \frac{3}{2\pi} = 0.4775$$

- 'σ_Z' below the point load at depth z,

$$\sigma_z = 0.4775.\frac{Q}{z^2}$$

Westergaard's Theory

- $$\sigma_z = \frac{Q}{\pi z^2}\left[\frac{1}{1+\frac{2r^2}{z^2}}\right]^{3/2}$$
- $$\sigma_z = k_W.\frac{Q}{z^2}$$
- $$k_w|_{\max} = 0.3183$$

Boussinesq's Result

- $$\sigma_z|_{\max} = 0.0888\frac{Q}{r^2}$$
- $$\sigma_z|_{\max} = 0.1332\frac{Q^2}{2^2}$$

Westergaard's Results

- Vertical Stresss due to Live Loads

$$\sigma_z = \frac{2q'}{\pi z}\left[\frac{1}{1+\frac{X^2}{z^2}}\right]^2$$

where, σ_z = Vertical stress of any point having coordinate (x, z)

- Vertical Stress due to Strip Loading

$$\sigma_z = \frac{2q}{\pi}\left(\frac{X}{B}\alpha - \frac{\sin 2\beta}{2}\right)$$

where, σ_z = Vertical stress at point 'p'

- $\sigma_Z = \frac{q}{\pi}[\beta + \sin\beta]$

- Vertical stress below uniform load acting on a circular area.

$$\sigma_Z = q(1-\cos^3\theta)$$

where, $\cos\theta = \frac{z}{\sqrt{r^2+z^2}}$

Newmark's Chart Method

- Influence of each area

$$= \frac{1}{\text{Total no. of sectoral area}} = 0.005$$

$\sigma_Z = 0.005qN_A$ where, N_A = Total number of sectorial area of Newmark's chart.

Equivalent Load Method

- $\sigma_Z = \sigma_{Z_1} + \sigma_{Z_2} + \sigma_{Z_3} + \ldots$

where, σ_{Z_1} = $k_{B_1}\frac{Q_1}{z^2}$ σ_{Z_2} = $k_{B_2}.\frac{Q_2^2}{z^2}\ldots$

Trapezoidal Method

- σ_z at depth $'z' = \frac{q(B\times L)}{(B+2\eta z)(L+2\eta z)}$
- $\sigma_z = \frac{q(B\times L)}{(B+2z)(L+2z)}$
- $\sigma_z = \frac{q(B\times L)}{(B+4z)(L+4z)}$

Shear Strength of Soil

Shear Strength

- $\theta_c = \frac{\pi}{4} + \frac{\beta_{maximum}}{2}$ where, $\beta_{max.}$ = Angle between resultant stress and normal stress on critical plane.

= Friction angle of soil = Ø

$$\theta_c = \frac{\pi}{4} + \frac{\phi}{2}$$

↓

for clay Ø = 0

$$\theta_c = \frac{\pi}{4}$$

- $\tau = \sigma_\eta \tan\phi$ (iii) $\tau = C + \sigma_\eta \tan\phi$, for C-Ø soil.

- $\tau = C$, for C-soil (clays).
- $\sigma_1 = \sigma_3 \tan^2(45° + \frac{\phi}{2}) + 2C \tan(45° + \frac{\phi}{2})$, for C-Ø soil.
- $\sigma_1 = \sigma_3 \tan^2(45° + \frac{\phi}{2})$, for ϕ-soil.
- $\sigma_1 = 2C$, for C-soil.

Mohr Coulomb's Theory

- $\tau = s = C' + \overline{\sigma_n} \tan\phi'$

C' = Effective cohesion

$\overline{\sigma_n}$ = Effective normal stress

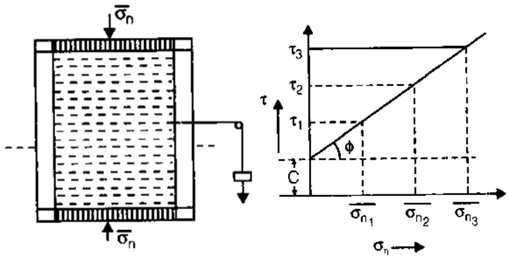
and Ø' = Effective friction angle

Drained condition	Effective stress analysis and post construction stability is checked.
Undrained condition with positive pole water pressure	Total stress analysis and stability should be checked immediately after construction.
Undrained condition with	Effective stress analysis and long term

negative pore water pressure	stability should be checked.

Direct Shear Test

- $\tau = s = C' + \overline{\sigma_n} \tan\phi'$



Results of Direct Shear Test

- $\sigma_1 = \sigma_3 + \sigma_d$
- $(\sigma_d)_{failure} = (\sigma_1 + \sigma_3)_{failure} \dfrac{P}{A}$
- $\tau = S = C + \overline{\sigma_n} \tan\phi$

- σ_3 = Cell pressure or all-round confining pressure
- σ_d = Deviator stress   A = Area of failure

$A = \dfrac{A_0(1 \pm \in_v)}{(1 - \in_L)}$ where, A_0 = Area of beginning

$\in_v$ = Volumetric strain

$\in_v = 0 \; for U - U - test$

where, ΔV = Volume of water escaped out

$$\in_v = \frac{\Delta V}{V} \; for C - D test$$

$V = \dfrac{\pi}{4} D^2 L$ = Initial Volume

∈ = Axial strain

Unconfined Compression Test

- $q_u = (\sigma_1)_f$ where, q_u = unconfined compressive strength.

Here, σ_3 =0

- $(\sigma_1)_f = 2C \tan\left(45° \dfrac{\phi}{2}\right), \; for C - \phi soil$
- $(\sigma_1)_f = 2C, \; for C - soil.$
- $\tau = S = C = \dfrac{q_u}{2}$, for clay's or c-soil.

- For clays as sand/coarse grained soil/can't sland in equipment with no lateral pressure.
- Used to rapidly assess clay consistency in field.
- To get sensitivity values of clay.

Vane Shear Test

	Lab Size	Field Size
Height of vane (H)	20 mm	10 to 20 cm
Dia of vane (D)	12 mm	5 to 10 cm
Thickness of vane (t)	0.5 to 0.1 mm	2 to 3 cm

Shear Strength

$$S = \tau = \frac{T}{\pi D^2\left(\frac{H}{2}+\frac{D}{6}\right)}$$

When top and bottom of vanes both take part in shearing.

- $S = \tau = \dfrac{T}{\pi D^2\left(\frac{H}{2}+\frac{D}{12}\right)}$

When only bottom of vanes take part in shearing.

- $S_t = \dfrac{(q_u)_{undisturbed}}{(q_u)_{remolded}}$
  where s_f = Sensitivity

Pore Pressure Parameter

- $B = \dfrac{\Delta U_c}{\Delta \sigma_c} = \dfrac{\Delta U_c}{\Delta \sigma_3}$
  - 0 ≤ B ≤ 1
  - B = 0, for dry soil.
  - B = 1, for saturated soil.

$\overline{A} = A.B$ where A = Pore pressure parameter

- $\overline{A} = \dfrac{\Delta U_d}{\Delta \sigma_d}$

ΔU_d = Change in pore pressure due to deviator stress.

$\Delta \sigma_d$ = Change in deviator stress

ΔU = Change in pore pressure

$$\Delta U = \Delta U_c + \Delta U_d$$
$$\Delta U = B[\Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)]$$

Deep Foundation

Bearing capacity of piles

- **Analytical Method**

$$Q_{up} = Q_{eb} + Q_{sf}$$
$$Q_{up} = q_b A_b + q_s A_s$$

Q_{up} = Ultimate load on pile
Q_{eb} = End bearing capacity
Q_{sf} = Skin friction
q_b = End bearing resistance of unit area.
q_s = Skin friction resistance of unit area.
A_b = Braking area
A_s = Surface area

- $q_b \approx 9C$,C = Unit Cohesion at base of pile for clays
- $q_S = \alpha\overline{C}$, α = Adhesion factor
- $\alpha\overline{C}$ = C_a = Unit adhesion between pile and soil.
- $\overline{C}$ = Average Cohesion over depth of pile.
- $Q_{safe} = \dfrac{Q_{up}}{F_s}$ where, F_s = Factor of safety.
- $Q_{safe} = \dfrac{Q_{eb}}{F_1} + \dfrac{Q_{sf}}{F_2}$

Dynamic Approach

- **Engineering News Records Formula**

$$Q_{up} = \frac{WH}{S+C}$$
$$Q_{ap} = \frac{Q_{up}}{6} = \frac{WH}{(S+C)}$$

Q_{up} = Ultimate load on pile
Q_{ap} = Allowable load on pile
W = Weight of hammer in kg.
H = Height of fall of hammer in cm.
S = Final set (Average penetration of pile per blow of hammer for last five blows in cm)
C = Constant
= 2.5 cm → for drop hammer
= 0.25 cm → for steam hammer (single acting or double acting)

- **Hiley Formula (I.S. Formula)**

$$Q_{ap} = \frac{\eta_h . \eta_b . WH}{S + \dfrac{C}{2}}$$

$$Q_{ap} = \frac{Q_{up}}{F_s}$$

F_s = Factor of safety = 3

η_h = Efficiency of hammer

η_b = Efficiency of blow.

$\eta_h = 0.75 to 0.85$ for single acting steam hammer

$\eta_h = 0.75 to 0.80$ for double acting steam hammer

$\eta_h = 1$ for drop hammer.

$$\eta_b = \frac{\text{Energy of hammer after impact}}{\text{Energy of hammer just before impact}}$$

$$\eta_b = \frac{W + e^2 P}{W + P} \text{ when } \mathbf{w > e.p}$$

$$\eta_b = \left(\frac{W + e^2 P}{W + P}\right) - \left(\frac{W - e^2 P}{W + P}\right)^2 \text{ .. when } \mathbf{w < e.p}$$

w = Weight of hammer in kg.

p = Weight of pile + pile cap

e = Coefficient of restitutions

= 0.25 for wooden pile and cast iron hammer

= 0.4 for concrete pile and cast iron hammer

= 0.55 for steel piles and cast iron hammer

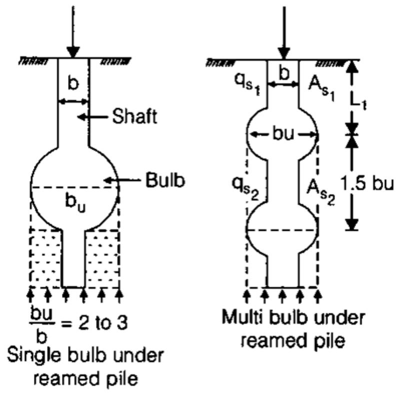
S = Final set or penetrations per blow

C = Total elastic compression of pile, pile cap and soil

H = Height of fall of hammer.

Under-Reamed Pile

An 'under-reamed' pile is one with an enlarged base or a bulb;



- $A_{s_1} = \pi b L_1 \quad q_{s_1} = \alpha C \quad \alpha < 1.$
- $A_{s_2} = \pi b_u L_2 \quad q_{s_2} = \alpha C \quad \alpha = 1$

.

$$Q_{up} = q_b A_b + q_{s_1} A_{s_1} + q_{s_2} A_{s_2}$$

- **For Cohesive soil**

$$Q_{nf} = Perimeter.L_1 \alpha C \text{ for Cohesive soil.}$$

Q_{nf} = Total negative skin frictions

$$F_s = \frac{Q_{up} - Q_{nf}}{Applied\ load} \quad \text{where, } F_s = \text{Factor of safety.}$$

- **For cohesion less soils**

Q_{nf} = P x force per unit surface length of pile $= P \times \frac{1}{2} K \gamma D_n^2 . \tan \delta$

$$Q_{nf} \ \frac{1}{2} P D_n^2 K . \tan \delta . \gamma \quad \text{(friction force = μH)}$$

where γ = unit weight of soil.

Group Action of Pile

- **Group Efficiency (η_g)**

$$\eta_g = \frac{Q_{ug}}{n.Q_{up}}$$

  - For sandy soil → η_g >1
  - For clay soil → η_g <1 and η_g >1
  - Minimum number of pile for group = 3.
  - $Q_{ug} = q_b A_b + q_s A_s$ where q_b = 9C for clays
  - $A_b = B^2$ $q_s = \overline{C}$ A_s =4 B.L
- **For Square Group**
- $Q_{ug} = \eta . Q_{up}$
- $Q_{ug} = \frac{Q_{ug}}{FOS}$ where, Q_{ug} = Allowable load on pile group.
- $S_r = \frac{S_g}{S_i}$
- **When Piles are Embended on a Uniform Clay**

$$S_g = \Delta H = \frac{C_c H_0}{1 + e_0} \log_{10} \left(\frac{\overline{\sigma_0} + \overline{\Delta \sigma}}{\overline{\sigma_0}} \right) \text{and}$$

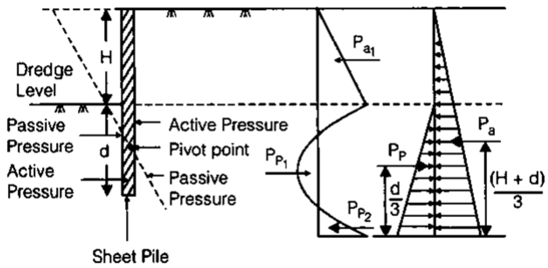
$$\overline{\sigma_0} = \frac{Q}{(B+z)^2}$$

- **In case of Sand**

$$S_r = \frac{S_g}{S_i} = \left(\frac{4B + 2.7}{B + 3.6} \right)^2 \text{ where, B = Size of pile group in meter.}$$

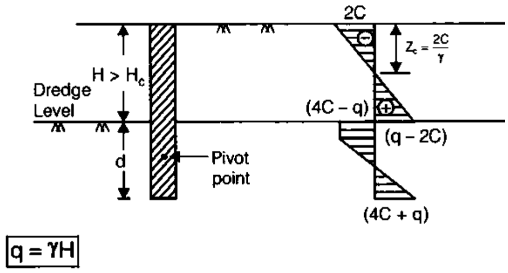
Sheet Pile Walls

Sheet Pile Walls Embedded in Sands



- $P_p.\dfrac{d}{3} = P_a\dfrac{(H+d)}{3}$ … without factor of safety.
- $\dfrac{P_p}{Fos}\dfrac{d}{3} = P_a\dfrac{(H+d)}{3}$ …. with factor of safety.
- $P_p\dfrac{1}{2} = k_p\gamma d^2 \qquad P_p\dfrac{1}{2} = k_p\gamma(H+d)^2$

Sheet Pile Walls Embedded in Clays



- Active earth pressure at depth H.

$$P_p = q - 2C$$

- Passive earth pressure at depth 'H'.

$$P_p = 2C$$

- Resultant earth pressure of depth H. is

(P_p – P_a)

$$P_p - P_a = 4c - q$$

- Resultant earth pressure at base i.e. at depth (H + d) is (P_p – P_a)

$$P_p - P_a = (4c - q)$$

- Resultant earth pressure of base i.e. of depth (H + d) is (P_p – P_a)

$$P_p - P_a = (4c + q)$$

Shallow Foundation & Bearing Capacity

Bearing Capacity

- The load carrying capacity of foundation soil or rock which enables it to bear and transmit loads from a structure.

Gross Pressure Intensity

- It is the total pressure at the base of the footing due to the weight of the super structure, self weight of the footing and weight of the earth fill.

Net safe bearing capacity

- $q_{ns} = \dfrac{q_{nu}}{F_s}$ where q_{ns} = Net safe bearing capacity

F_s = Factor of safety

Safe bearing capacity

$q_s = q_{ns} + \overline{\sigma}$ where, q_s = Safe bearing capacity.

Method to determine bearing capacity

- **Rankines Method (Ø - soil)**
  - $q_u = \gamma D_f \tan^4\left(45° + \dfrac{\phi}{2}\right)$ or $q_u = \gamma D_f \left(\dfrac{1+\sin\phi}{1-\sin\phi}\right)^2$
  - **Bells Theory (C - Ø)**
  - $q_u = CN_c + \gamma D_f N_q$ where, N_c and N_q are bearing capacity factors.

Fellinious Method: (C-soil)

$$q_{ult} = \frac{W.I_r + CR}{b.I_0} \quad q_{ult} = 5.5C$$

Prandtl Method: (C - Ø)

$$q_u = CN_c + \gamma D_f N_q + \frac{1}{2}\gamma BN_\gamma \rightarrow \text{For strip footing}$$

For C-soil $N_c = 5.14, \quad N_q = 1, \quad N_\gamma = 0$

Terzaghi Method (C - Ø)

- **For strip footing**

$$q_u = CN_c + \gamma D_f N_q + \frac{1}{2}\gamma BN_\gamma$$

- **For square footing**

$$q_u = 1.3CN_c + \gamma D_f N_q + 0.4\gamma BN_\gamma$$

- **For rectangular footing**

$$q_u = \left(1 + 0.3\frac{B}{L}\right)CN_C +$$

$$\gamma D_f N_q + \frac{1}{2}\left(1 - \frac{0.2B}{L}\right)\gamma BN_\gamma$$

- **For circular footing**

$$q_u = 1.3CN_c + \gamma D_f N_q + 0.3\gamma DN_\gamma$$

Skemptons Method (c-soil)

$$q_{nu} = CN_c$$

- If $\frac{D_f}{B} = 0 i.e.$ of the surface.

Then N_C = 5 For strip footing
N_C = 6.0 For square and circular footing.
where D_f = Depth of foundation.

- If $0 \le \frac{D_f}{B} \le 2.5$

$$N_C = 5\left[1 + 0.2\frac{D_f}{B}\right], \text{for strip footing}$$

$$N_C = 6\left[1 + 0.2\frac{D_f}{B}\right], \text{ For square and circular footing.}$$

$$N_C = 5\left[1 + 0.2\frac{B}{L}\right]\left[1 + 0.2\frac{D_f}{B}\right] \text{for rectangular footing}$$

- if $\frac{D_f}{B} \le 2.5$ N_C =7.5

for strip footing
N_C =9.0 for circular, square and rectangular footing.

Meyorhoff's Method → (C - Ø soil)

$$q_u = CN_c.s_c.d_c.i_c + \gamma D_f N_q.s_q.d_q.i_q + \frac{1}{2}\gamma BN_\gamma s_\gamma.d_\gamma.i_\gamma$$

Plate Load Test

$$\frac{q_{uf}}{q_{up}} = \frac{B_f}{B_p}$$

$$q_{uf} = q_{up}$$

- If plate load test carried at foundation level then

$$\frac{S_f}{S_p} = \left[\frac{B_f(B_p + 0.3)}{B_p(B_f + 0.3)}\right]^2$$

$$S_{f\,corrected} = S_f \times \left[\frac{1}{1 + \frac{D_2}{B_f}}\right]^{0.5}$$

$$\frac{S_f}{S_p} = \left[\frac{B_f(B_p + 0.3)}{B_p(B_f + 0.3)}\right]^2$$

$$\frac{S_f}{S_p} = \frac{B_f}{B_p}$$

$$\frac{S_f}{S_p} = \left(\frac{B_f}{B_p}\right)^{n+1}$$

Housels Approach

$$Q_P = mA_p + nP_p$$

$$Q_f = mA_f + nP_f$$

Q_P = Allowable load on plate m and n are constant

P = Perimeter A_p = Area of plate

A_f = Area of foundation

Standard Penetration Test

$$N_1 = N_0 \frac{350}{(\overline{\sigma}+70)} \text{ and } \overline{\sigma} \not> 280$$

N_1 = Overburden pressure correction

N_0 = Observed value of S.P.T. number.

$\overline{\sigma}$ = Effective overburden pressure at the level of test in kM/m^2.

- **For Saturated** $\overline{\sigma}$ fine sand and silt, when $N_1 > 15$

$$N_2 = \frac{1}{2}(N_1 - 15) + 15$$

N_2 = Dilatancy correction or water table correction.

$N_q + N_\gamma$ related to N value using peck Henson curve or (code method)

Pecks Equation

$$q_{a\ net} = 0.44NS = C_w kN/m^2$$

$$C_w = 0.5\left(1 + \frac{D_w}{D_f + B}\right)$$

Teng's Equations

$$q_{ns} = 1.4(N-3)\left(\frac{B+0.3}{2B}\right)^2 SC_w C_D kN/m^2$$

$$C_W = 0.5\left(1 + \frac{D_w}{B}\right)$$

$$C_D = \left(1 + \frac{D_f}{B}\right) \le 2$$

C_w =Water table correction factor

D_w = Depth of water table below foundation level

B = Width of foundation

C_d =Depth correction factor

S = Permissible settlement in 'mm'.

I.S Code Method

$$q_{ns} = 1.38(N-3)\left(\frac{B+0.3}{2B}\right)^2 SC_w$$

q_{ns} =Net safe bearing pressure in kN/m^2
B = Width in meter.
S = Settlement in 'mm'.

I.S. Code Formula for Reft:

$$q_{ns} = 0.88NSC_w$$

C_w : Same as of peck Henson.

Meyer-hoffs Equation

- $q_{ns} = 0.49NSC_wC_d$ where, q_{ns} = Net safe bearing capacity in kN/m^2.

B < 1.2 m

$$C_d = \left(1+\frac{D_f}{B}\right) \le 2 \quad C_w = \frac{1}{2}\left(1+\frac{D_w}{B}\right)$$

$$q_{ns} = 0.32N\left(\frac{B+0.3}{2B}\right)^2 .S.C_d.C_w$$

B ≥ 1.2 m (where q_{ns} is in kN/m^2.

Cone Penetrations Test

$$C = 1.5\left[\frac{q_c}{\overline{\sigma_0}}\right]$$

q_c = Static cone resistance in kg/cm^2
c = Compressibility coefficient
$\overline{\sigma_0}$ = Initial effective over burden pressure in kg/cm^2.

$$S = 2.3\frac{H_0}{C}\log_{10}\left[\frac{\overline{\sigma_0}+\overline{\Delta\sigma}}{\overline{\sigma_0}}\right]$$

where, 'S' = Settlement.

$$q_{ns} = 3.6q_sR_w \text{ B > 1.2 m.}$$

where, q_{ns} = Net safe bearing pressure in kN/m^2.

$$q_{ns} = 2.7q_c.R_w \text{ B < 1.2 m.}$$

where, R_w = Water table correction factor.

Retaining Wall/Earth Pressure Theories

Earth Pressure at Rest

$$\sigma_h = K_0.\gamma.z, \quad K_0 = \frac{\sigma_h}{\sigma_v}, \quad K_0 = \frac{\mu}{1-\mu},$$

σ_h = Earth pressure at rest

K_0 = Coefficient of earth pressure at rest
μ = Poissons ratio of soil 0.4
K_0 = 1 – sin Ø → for Ø soil.
where, Ø = Angle of internal friction.
(K_0) over consolidation = (K_0) normally consolidation $\sqrt{OCR}$
where, OCR = Over Consolidation Ratio.

Active Earth Pressure

- Length of Failure block = $\mathrm{Hcot}\left(45^\circ + \frac{\phi}{2}\right)$
- $\Delta H = 0.2\%$ of H for dense sand
  $\Delta H = 0.5\%$ of H for loose sand
  $\Delta H = 0.4\%$ of H for clay's
- $k_a = \frac{1-\sin\phi}{1+\sin\phi}$ $k_a = \tan^2\left(45^\circ - \frac{\phi}{2}\right)$

where k_a = Coefficient of active earth pressure.

Passive Earth Pressure

- Length of Failure block = $\mathrm{Hcot}\left(45^\circ - \frac{\phi}{2}\right)$
- $\Delta H = 0.2\%$ of H for dense sand
  $\Delta H = 15\%$ of H for loss sand
- $k_P = \frac{1+\sin\phi}{1-\sin\phi}$ or $k_a = \tan^2\left(45^\circ + \frac{\phi}{2}\right)$

k_P = Coefficient of passive earth pressure.

- $K_a.K_P = 1$
- $P_a < P_0 < P_P$

P_a = Active earth pressure.
P_0 = Earth pressure at rest.
P_P = Passive earth pressure.

Active Earth pressure by Rankine Theory

- $P_a = \frac{1}{2}K_a\gamma H^2$ acts at $\frac{H}{3}$ from base.

where, P_a = Active earth pressure force on unit length of wall.

- $P_a = \frac{1}{2}K_a\gamma' H^2 + \frac{1}{2}\gamma_w H^2$ acts at $\frac{H}{3}$ from base

where γ = Submerged unit weight of soil.

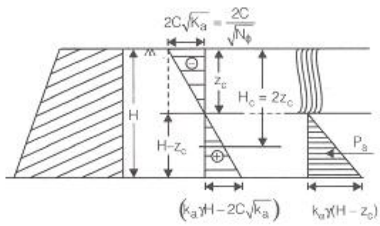
$$P_{a_1} = \frac{1}{2}K_a H_1^2 \text{ --- acts of } \left(H_2 + \frac{H_1}{3}\right) \text{from base } = \overline{H_1}$$

$$P_{a_2} = K_a\gamma_1 H_1 H_2 \text{ --- acts of} \left(\frac{H_2}{2}\right) \text{from base } = \overline{H_2}$$

$$P_{a_3} = \frac{1}{2}K_a\gamma' H_2^2 \text{ --- acts at } \left(\frac{H_2}{3}\right) \text{ from base } = \overline{H_3}$$

$$P_{a_4} = \frac{1}{2}\gamma_w H_2^2 \text{ --- acts of } \left(\frac{H_2}{3}\right) \text{ from base } = \overline{H_4}$$

Active Earth Pressure for Cohesive Soil



- $K_a = \tan^2\left(45° - \frac{\phi}{2}\right) = \dfrac{1}{\tan^2\left(45° + \frac{\phi}{2}\right)} = \dfrac{1}{N_\phi}$ where N_ϕ = Influence Factor.
  - Active Earth Pressure of Any Depth z
    $P_a = k_a\gamma z - 2c\sqrt{k_a}$
  - Active Earth Pressure of Surface. i.e., at z = 0 $P_a = -2c\sqrt{k_a}$
  - At $z = z_c \rightarrow P_a = O$
  - $Z_c = \frac{2c}{\gamma}\tan\left(45° + \frac{\phi}{2}\right)$
    $H_c = \frac{4c}{\gamma}\tan\left(45° + \frac{\phi}{2}\right)$
  - When Tension Cracks are not Developed

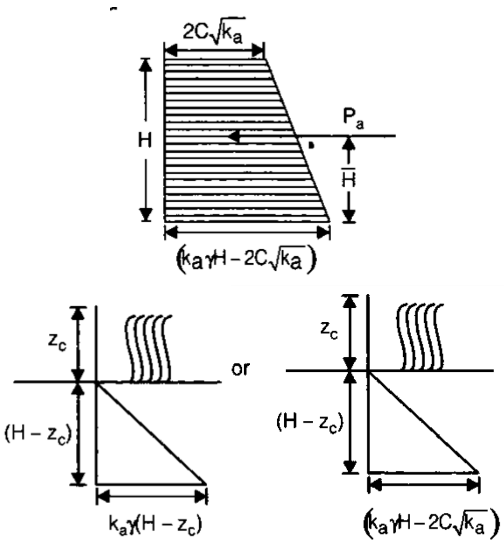
$$P_a = \frac{1}{2}k_a\gamma H^2 - 2CH\sqrt{k_a}$$

  - When Tension Cracks are Developed

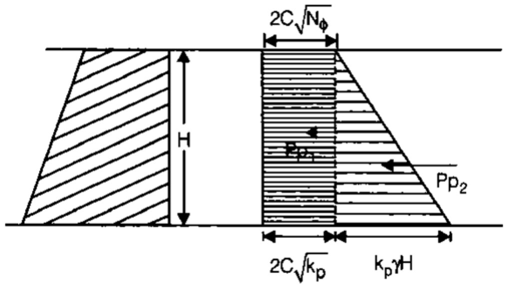
$$P_a = \frac{1}{2}(k_a\gamma H - 2C\sqrt{k_a})(H - Z_c)$$

$$P_a = \frac{1}{2}k_a\gamma H^2 - 2CH\sqrt{k_a} + \frac{2C^2}{\gamma}$$

$$\text{or } P_a = \frac{1}{2}(k_a\gamma(H - Z_c)^2 \text{ acts at } \left(\frac{H - Z_c}{3}\right)$$



Passive Earth Pressure for Cohesive Soil



- Passive Earth Pressure at any depth 'z',

$$P_p = \frac{1}{2} k_p \gamma H z + 2C\sqrt{k_p}$$

  - Total P_p on Unit Length

$$P_p = \frac{1}{2} k_p \gamma H^2 + 2C\sqrt{k_p H}$$

Coulombs Wedge Theory

$$k_a = \left[\frac{\dfrac{\sin(\alpha+\phi)}{\sin\alpha}}{\sqrt{\sin(\alpha+\delta)} + \sqrt{\dfrac{\sin(\phi+\delta).\sin(\phi+\beta)}{\sin(\alpha+\beta)}}} \right]^2$$

$$k_p = \left[\frac{\dfrac{\sin(\alpha-\phi)}{\sin\alpha}}{\sqrt{\sin(\alpha+\delta)} + \sqrt{\dfrac{\sin(\phi+\delta)\sin(\phi+\beta)}{\sin(\alpha+\beta)}}} \right]^2$$

Special points:

- Retaining wall are designed for active earth P.
- Ranking theory
- Overstimate → Active earth pressure
  Underestimates → Passive earth pressure

Stability Analysis of Slopes

Factor of safety w.r.t. shear strength (F_s)

- $F_s = \dfrac{C + \overline{\sigma}\tan\phi}{\tau}$

τ = Developed shear strength.

$(C + \overline{\sigma}\tan\phi)$ = Developed or mobilized shear stress

C = Effective cohesion

∅ = Effective friction

$\overline{\sigma}$ = Effective normal stress

- $\sigma = C_m + \overline{\sigma}\tan\phi_m$

C_m = Mobilized Cohesion

ϕ_m = Mobilized Friction Angle

$$C_m = \frac{C}{F_s} \text{ and } \tan\phi_m = \frac{\tan\phi}{F_s}$$

Factor of Safety w.r.t. Cohesion (f_C)

$$F_c = \frac{H_c}{H} \text{ and } F_c = \frac{C}{C_m}$$

H_c = Critical depth

H = Actual depth

$$H_c = \frac{4C}{\gamma}\tan\left(45° + \frac{\phi}{2}\right)$$

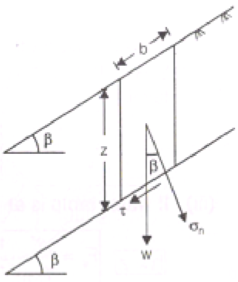
Stability Analysis of Infinite Slopes

- **Cohesionless dry soil/dry sand**

$$W = \gamma z \cos\beta$$

$$\tau = \frac{W\sin\beta}{(b\times 1)} \Rightarrow \tau = \lambda Z \sin\beta\cos\beta$$

$$\sigma_n = \frac{W\cos\beta}{(b\times 1)} \Rightarrow \sigma_n = \lambda Z \cos^2\beta$$



τ = Developed shear stress or mobilized shear stress

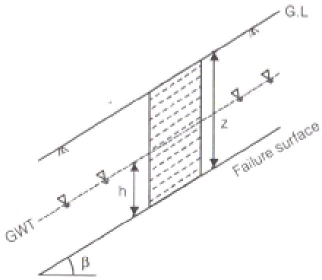
σ_n = Normal stress.

$$F_s \frac{\tan\phi}{\tan\beta}$$ where, F_s = Factor of safety against sliding $= \frac{S}{\tau} = \frac{C + \overline{\sigma_n}\tan\phi}{\tau}$

- **For safety of Slopes**

$$\beta < \phi$$
$$\downarrow$$
$$F_s > 1$$

- **Seepage taking place and water table is parallel to the slope in Cohesionless soil**



  - h = Height of water table above the failure surface.

$$F_s = \left[1 - \left(\frac{\gamma_w}{\gamma}\right)\left(\frac{h}{z}\right)\right]\frac{\tan\phi'}{\tan\beta}$$

ϕ' is effective friction angle

γ – avg. total unit weight of soil above the slip surface upto ground level.

$$\gamma = \frac{\gamma_1 h_1 + \gamma_2 h_2}{h_1 + h_2}$$

- **If water table is at ground level: i.e.,**

h = z $F_s = \frac{\gamma'}{\gamma_{Sat}} \cdot \frac{\tan\phi}{\tan\beta}$ $\quad F_s \square \frac{1}{2} \cdot \frac{\tan\phi}{\tan\beta}$

- **Infinite Slope of Purely Cohesive Soil**

$$F_s = F_c \frac{C}{\gamma z \sin\beta . \cos\beta} \quad F_c = \frac{H_c}{H}$$

$$S_\eta = \frac{C}{\gamma H_c} = \sin\beta . \cos\beta = \frac{C}{\gamma F_c H} = \frac{C}{\gamma F_c z}$$

S_η =Stability Number.

- **C-Ø Soil in Infinite Slope**

$$F_s \frac{C}{\gamma H \sin\beta . \cos\beta} + \frac{\tan\phi}{\tan\beta}$$

- **Taylor's stability no.**

$$S_\eta = \frac{C}{\gamma . H_c} = \sin\beta . \cos\beta$$ **(for cohesive soil)**

$$S_\eta = [\tan\beta - \tan\phi]\cos^2\beta$$ (for C-Ø soils)

Stability Analysis of Finite Slopes

- **Fellinious Method**

- $F = \frac{Cr^2\theta}{we}$ where, F = Factor of safety

- $F = \frac{Cr^2\theta^1}{we}$

- **Swedish Circle Method**

$$F = \frac{Cr\theta + \sum w\cos\alpha.\tan\phi}{\sum w\sin\alpha}$$

- **Friction Circle Method**

$$F_C = \frac{C}{C_m} \qquad F_\phi = \frac{\tan\phi}{\tan\beta} = \frac{\tan\phi}{\tan\phi_m}$$

- **Taylor's Stability Method (C-Ø soil)**

$$S_\eta = \frac{C}{\gamma H_c} = \frac{C}{\gamma F_C H}$$

$\phi_w = \frac{\gamma'}{\gamma_{Sat}}.\phi$ where $Ø_w$ = weight friction angle.