2019-1 Computer Algorithms Homework #1

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(Deadline : March 19)

1. Watching assignments : watch the following lecture(s).
   (a) Fall 2005 MIT Algorithm Lecture 1: Administrivia; Introduction; Analysis of Algorithms, Insertion Sort, Mergesort (https://bit.ly/2srWNNd)
   (b) Fall 2005 MIT Algorithm Lecture 2: Asymptotic Notation; Recurrences; Substitution, Master Method (https://bit.ly/2tRc1Nu)
2. Prove the following:
   (a) $nlogn + 5n \in O(n^2)$.
   (b) $3n \in O(n^2)$.
   (c) $3n^2 + 2n \in O(n^2)$.
   (d) $3n^2 + 2n \in \Theta(n^2)$.
3. Show the time complexity of an algorithm that calculates π upto 100 digits, in terms of big O notation($O(fn)$).
4. Calculate how many times instructions in the following algorithm are executed. And express the time complexity of the algorithm in terms of big O notation($O(fn)$).

```
function algo1(int n)
{
  for (int i = 1 to n)
  {
    for (int j = i to n)
    {
      for (int k = j to n)
      {
        A[i][j][k] = i*j*k;
      }
    }
  }
}
```

5. Calculate running time $f(n)$ of the following code fragments, and their asymptotic complexities in big-O notation $O(.)$ respectively. Calculate the bounds as tight as possible. For each bound calculation, show c and n_0.
   (a)
   ```
   for(int i = 0; i < n; i++)
         sum++;
   ```
   (b)
   ```
   for(int i = 0; i < n; i+=2)
         sum++;
   ```

(c)
```
for(int i = 0; i < n; i++)
    for( int j = 0; j < n; j++)
        sum++;
```

(d)
```
for(int i = 0; i < n; i+=2)
    sum++;
for(int j = 0; j < n; j++)
    sum++;
```

(e)
```
for(int i = 0; i < n; i++)
    for( int j = 0; j < n * n; j++)
        sum++;
```

(f)
```
for(int i = 0; i < n; i++)
    for( int j = 0; j < i; j++)
        sum++;
```

(g)
```
for(int i = 0; i < n; i++)
    for( int j = 0; j < n * n; j++)
        for(int k = 0; k < j; k++)
            sum++;
```

(h)
```
for(int i = 1; i < n; i = i * 2)
    sum++;
```

6. Problem 0.4 of DPV in Page 18 of Chapter 0.

7. Using mathematical induction, prove $\sum_{k=1}^{n} k^3 = \{\sum_{k=1}^{n} k\}^2$.

8. Show that
$$log(n!) = \Theta(n \times log(n)) \tag{1}$$
(Hint: To show an upper bound, compare $n!$ with n^n. To show a lower bound, compare it with $(n/2)^{n/2}$)

9. A d-ary tree is a rooted tree in which each node has at most d children. Show that any d-ary tree with n nodes must have a depth of $\Omega(log(n)/log(d))$.

10. Mina, Sana, Momo, and Tzuyu have had 11 cookies. Everyone has eaten at least one cookies, and everyone knows it. But no one knows how many cookies each of the four has had.

    Mina: Sana, did you eat more cookies than I?
    Sana: I don't know. Momo, did you eat more cookies than I?
    Momo: I don't know.

    After hearing this conversation, Tzuyu has found out how many cookies that each of Mina, Sana, Momo and herself has had.
    How many cookies each of the four women has had? (Mina:?, Sana:?, Momo:?, Tzuyu:?)

11. Using your favorite computer programming language (but C/C++, Java, Python, C# recommended), write a program that calculate the following:

    A group of zombies walk on a very narrow bridge. The length of the bridge is k meters. The moving speed of each zombie is one meter per second. When a zombie reaches an end of the bridge, he leaves the bridge. When two zombies meet, they turn back and walk in opposite directions.

    You are given the original positions of zombies in the bridge, but you don't know the initial moving directions of the zombies. You have to compute the earliest time and the latest time for all zombies to leave the bridge.

    Input:
    The first line of input, you are given an integer number for the number of zombies (let it n). The second line gives you the length of the bridge (k in meters). The third (the last) line has n integers denoting the initial positions of zombies in the bridge.
    All the input integers are not bigger than 1,000,000, separated by space.

    Output:
    You print two numbers, the earliest time and the latest time (in seconds).

    Example:

    ```
    Input
    4
    10
    2 3 6 7
    Output
    4 8
    3
    ```

    How do zombie walk? Answer of a question from one student

    ```
    123456789  (time=1)
       ><<

    123456789  (time=2)
       <>
        <

    123456789  (time=3)
      << >
    ```