Este es el link de la clase de hoy 18-4 por si no pueden ingresar a pedco:

https://us02web.zoom.us/j/89193885302?pwd=MjdCUlhuZnpxK2VnUURDZnJveWJydz09

ID de la reunión 891 9388 5302

Ejemplos: Resolver las siguientes ecuaciones:

(1) $(x-2)^2 + x^2 - 4 = 0$

$$(x-2)^2 + x^2 - 4 = 0 \iff (x^2 - 2.2x + 4) + x^2 - 4 = 0 \iff x^2 - 4x + \cancel{4} + x^2 - \cancel{4} = 0 \iff$$

$$\iff 2x^2 - 4x = 0 \iff 2x(x-2) = 0 \iff 2x = 0 \lor x - 2 = 0 \iff \frac{1}{2}.2x = \frac{1}{2}.0 \lor x - 2 + 2 = 0 + 2$$

$$\iff \boxed{x = 0 \lor x = 2}$$

$$\boxed{Sol: \ x \in \{0, 2\}}$$

$$Sol = \{0, 2\}$$

$$(2)\quad \frac{1}{x}-\frac{1}{x^2}=0. \qquad \boxed{x\neq 0}$$

$$\frac{1}{x}-\frac{1}{x^2}=0 \iff \frac{x-1}{x^2}=0 \iff x-1=0 \iff \boxed{x=1}$$

$$\boxed{Sol:\ x\in\{1\}}$$

$$\frac{2}{5}-\frac{k}{2}=\frac{?}{?}$$

$$\frac{1}{2}+\frac{1}{4}=$$

$$\frac{\cancel{x}(x-1)}{\cancel{x}}$$

$$\frac{\cancel{x}\cdot(x-1)}{\cancel{x}}=\frac{x-1}{1}=x-1$$

$$\frac{\cancel{x}(x-1)}{2\cancel{x}}=\frac{x-1}{2}$$

$$(2')\quad \frac{1}{x^2-x}+\frac{2}{x^2}=0. \qquad \boxed{x\neq 0,\ x\neq 1}$$

$$\frac{1}{x^2-x}+\frac{2}{x^2}=0 \iff \frac{1}{x(x-1)}+\frac{2}{x^2}=0 \iff \frac{x+2(x-1)}{x^2(x-1)}=0 \iff \frac{x+2x-2}{x^2(x-1)}=0 \iff \frac{3x-2}{x^2(x-1)}=0$$

$$\boxed{\frac{x^{\cancel{2}}\cancel{(x-1)}}{\cancel{x}\cancel{(x-1)}}=x \ // \ \frac{\cancel{x^2}(x-1)}{\cancel{x^2}}=x-1}$$

$$\iff 3x-2=0 \iff 3x=2 \iff \boxed{x=\frac{2}{3}} \qquad Sol=\{\tfrac{2}{3}\}$$

$(3)\ \underbrace{(x-1)}\underbrace{(x+2)} \neq 0.$

Buscamos los valores de $x \in \mathbb{R}$ que hacen que: $\overbrace{(x-1)}\overbrace{(x+2)}=0$

$(x-1)(x+2)=0 \iff x-1=0$ ó $x+2=0 \iff x-1+1=0+1$ ó $x+2+(-2)=-2 \iff x=1$ ó $x=-2$

1 y -2 hacen que $(x-1)(x+2)=0$.

Luego la solución de $(x-1)(x+2)\neq 0$ es $\boxed{\text{sol: } x \in \mathbb{R}-\{1,-2\}}$

solución

(4) $2x^2 + 2x - 4 = 0.$

$$a x^2 + b x + c = 0$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \begin{cases} \to x_1 \\ \to x_2 \end{cases}$$

Por Baskhara $\dfrac{-2 \pm \sqrt{4 - 4.2.(-4)}}{2.2}$

$$\to \frac{-2 + \sqrt{36}}{4} = \frac{-2+6}{4} = \boxed{1}$$

$$\to \frac{-2 - \sqrt{36}}{4} = \frac{-2-6}{4} = \boxed{-2}$$

Otra forma de resolv.

$$2x^2 + 2x - 4 = 2\left(x - 1\right)\left(x - (-2)\right) = 0 \iff 2(x-1)(x+2) = 0$$

$$\iff \frac{1}{2}.2(x-1)(x+2) = \frac{1}{2}.0 \iff (x-1)(x+2) = 0$$

$$\iff x - 1 = 0 \text{ ó } x + 2 = 0$$

$$\iff \boxed{x = 1 \text{ ó } x = -2}$$

¿Cómo resuelvo $\underbrace{2x^2 + 2x - 4}_{} > 0$?

$$2(x-1)(x+2) > 0$$

$$\overset{+}{a} . \overset{+}{b} > 0$$

$$\underset{-}{a} . \underset{-}{b}$$

Ej. $2.4 = 8 > 0$

$(-2).(-4) = 8 > 0$

$$2 < 4 \Rightarrow 2(-2) \quad 4(-2)$$

$$-4 > -8$$

$-8 \quad -7 \quad -6 \quad -5 \quad -4 \quad -3$

Ejemplo. Resolver la siguiente inecuación: $\dfrac{-2}{x+1} > 0.$

$-8+1$

$\dfrac{B}{A} > 0 \iff \begin{cases} B > 0 \wedge A > 0 \\ \text{ó } B < 0 \wedge A < 0 \end{cases}$

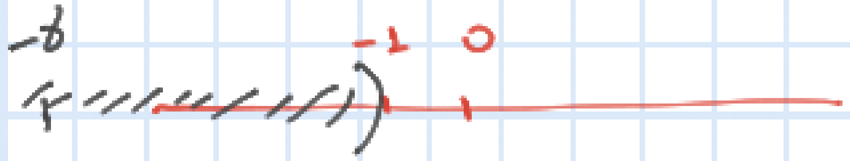
$-2 < 0$

Observamos: $x+1 \neq 0 \iff \boxed{x \neq -1}$

$\dfrac{-2}{x+1} > 0$

Como $-2 < 0$ debe ser $x+1 < 0 \Rightarrow x+1(-1) < 0+(-1) \Rightarrow \boxed{x < -1}$

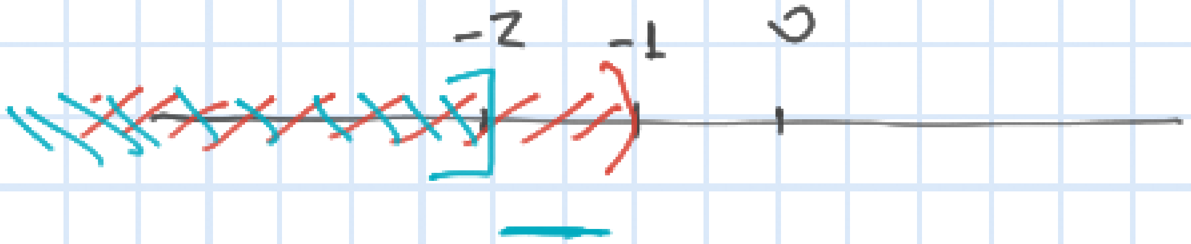
$\boxed{\text{Sol: } x \in (-\infty, -1)}$



0

$x < -1 \iff x \leq -2$ $\boxed{\text{No}}$

Cuidado!



Ejemplo. Hallar los valores reales de x que verifiquen:

1) $-5 < -x + 1$ y $-2x < 4$.

$a < b \Rightarrow a + c < b + c$

$a < b$, $c > 0 \Rightarrow a.c < b.c$

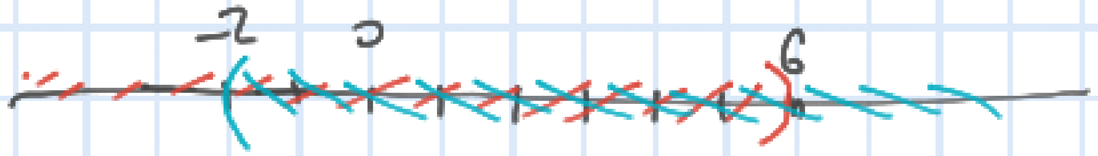
$a < b$, $c < 0 \Rightarrow a.c > b.c$

$$-5 < -x+1 \;\wedge\; -2x < 4 \iff -5-1 < -x+1-1 \;\wedge\; \left(-\tfrac{1}{2}\right)(-2x) > \left(-\tfrac{1}{2}\right)\cdot 4$$

$$\iff -6 < -x \;\wedge\; x > -2$$

$$\overset{-1<0}{\iff} (-6)(-1) > (-x)(-1) \;\wedge\; x > -2$$

$$\iff 6 > x \;\wedge\; x > -2$$



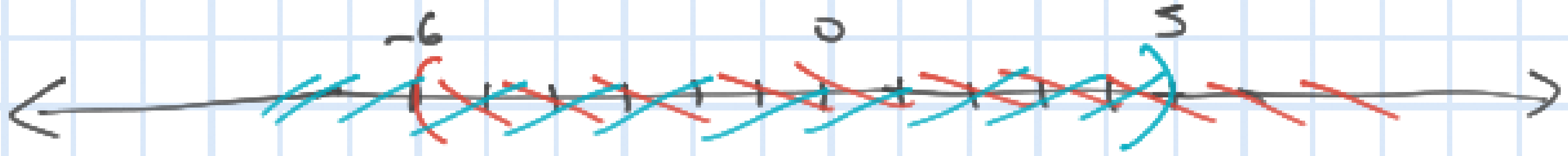
Sol: $\boxed{x \in (-2, 6)} = \{x \in \mathbb{R} : -2 < x < 6\}$

2) $-5 < -x < 6$.

$-x \neq x$

$$-5 < -x < 6 \overset{-1<0}{\iff} (-5).(-1) > (-x)(-1) > 6(-1)$$

$$\iff 5 > x > -6$$



$\boxed{\text{Sol: } x \in (-6, 5)}$

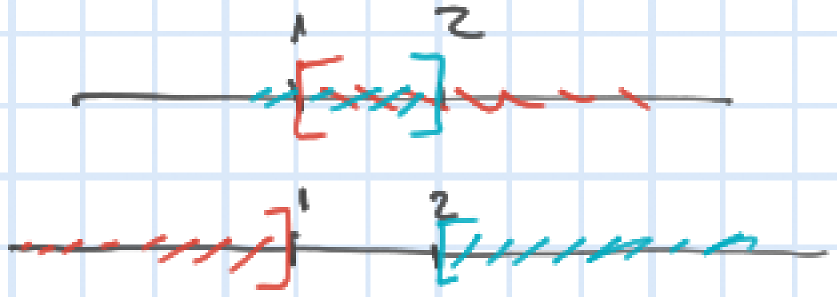
Ej: $\boxed{x = -2}$

$-5 < -(-2) < 6$ ✓

$-5 < 2 < 6$

3) $\underbrace{(x-1)}\underbrace{(-3x+6)} \geq 0.$

$$(x-1)(-3x+6) \geq 0 \iff_{\text{ó}} \begin{cases} \text{I)}\ x-1 \geq 0 \ \wedge\ -3x+6 \geq 0 \\ \text{II)}\ x-1 \leq 0 \ \wedge\ -3x+6 \leq 0 \end{cases} \iff_{\text{ó}} \begin{cases} x \geq 1 \ \wedge\ -3x \geq -6 \\ x \leq 1 \ \wedge\ -3x \leq -6 \end{cases} \iff \begin{cases} x \geq 1 \ \wedge\ x \leq \frac{-6}{-3} \\ x \leq 1 \ \wedge\ x \geq \frac{-6}{-3} \end{cases}$$



$$\iff \begin{cases} x \geq 1 \ \wedge\ x \leq 2 \\ x \leq 1 \ \wedge\ x \geq 2 \end{cases} \iff \begin{cases} x \in [1,2] \\ \emptyset \end{cases}$$

$$\boxed{\text{Sol: Caso I} \cup \text{Caso II} = [1,2] \cup \emptyset = \underline{[1,2]}}$$

4) $-(x-1)(-3x+6) > 0.$

$\iff (x-1)(-3x+6) < 0 \cdot (-1) = 0$

Por ej anterior Sol: $x \notin [1,2] \iff x \in [1,2]' \iff x \in (-\infty,1) \cup (2,+\infty)$

5) $\frac{2}{x-1} \le 4.$ $\boxed{x \neq 1}$ $\frac{\square}{\triangle} \le 0$

$$\frac{2}{x-1} \le 4 \iff \frac{2}{x-1} - \frac{4}{1} \le 0 \iff \frac{2-4(x-1)}{x-1} \le 0$$

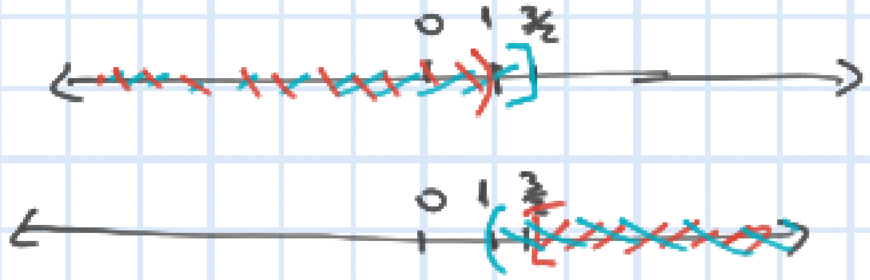
$$\iff \frac{2-4x+4}{x-1} \le 0 \iff \frac{-4x+6}{x-1} \le 0 \iff \begin{cases} \mathrm{I}) & -4x+6 \ge 0 \ \wedge \ x-1<0 \\ & \text{o} \\ \mathrm{II} & -4x+6 \le 0 \ \wedge \ x-1>0 \end{cases} \iff \begin{cases} -4x \ge -6 \ \wedge \ x<1 \\ -4x \le -6 \ \wedge \ x>1 \end{cases}$$

$$\iff \begin{cases} x \le \frac{-6}{-4} \ \wedge \ x<1 \\ x \ge \frac{-6}{-4} \ \wedge \ x>1 \end{cases} \iff \begin{cases} x \le \frac{3}{2} \ \wedge \ x<1 \\ x \ge \frac{3}{2} \ \wedge \ x>1 \end{cases} \iff \begin{cases} x \in (-\infty, 1) \\ x \in \left[\frac{3}{2}, +\infty\right) \end{cases}$$

$$\boxed{Sol = Sol\,\mathrm{I} \cup Sol\,\mathrm{II} = (-\infty, 1) \cup \left[\tfrac{3}{2}, +\infty\right)}$$

I)

II



6) $\dfrac{2}{x-1} \leq 0.$

$x-1 \neq 0$

$\boxed{x \neq 1}$

$\dfrac{■}{▲} \leq 0 \iff \begin{cases} ■ \geq 0 \wedge ▲ < 0 \\ ■ \leq 0 \wedge ▲ > 0 \end{cases}$

Como $2 \geq 0$ debe ser $x-1<0 \Rightarrow \boxed{x<1}$

$\boxed{\text{Sol}: \; x \in (-\infty, 1)}$

Es $\dfrac{2}{x-1} \leq 0 \iff 2 \leq 0.(x-1)$? NO!!! pues no sé el signo de $x-1$.

$\dfrac{2}{x-1} \leq 0 \iff \begin{cases} \text{I)} \; 2 \leq 0.(x-1) & \text{si supongo } x-1>0 \\ \text{II)} \; 2 \geq 0.(x-1) & \text{si sup. } x-1<0 \end{cases} \iff \begin{cases} \overbrace{2 \leq 0}^{\text{Abs!}} & \text{si sup. } x>1 \\ \underbrace{2 \geq 0}_{\text{pasa siempre } \forall x \in \mathbb{R}} & \text{si } x<1 \end{cases} \iff \begin{cases} \emptyset \\ x \in (-\infty, 1) \end{cases}$

$\text{Sol} = \text{Sol I} \cup \text{Sol II} = \emptyset \cup (-\infty,1) = \boxed{(-\infty,1)}$

ojo

Ejemplo: Hallar los $x \in \mathbb{R}$ que hacen real el resultado:

1) $\dfrac{1}{-x^2+9}$.

Condición: $-x^2+9 \neq 0$

Averiguemos cuando $-x^2+9=0 \iff -x^2=-9 \iff x^2=9 \iff \boxed{x=3 \vee x=-3}$

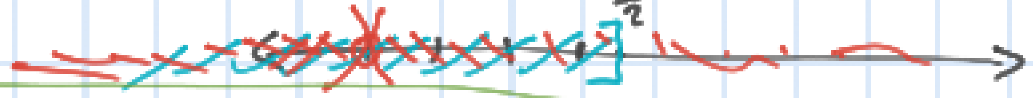
$\boxed{\text{Rta: } x \in \mathbb{R}-\{-3,3\}}$

2) $\dfrac{\sqrt{-2x+1}}{\sqrt[3]{x+3}}$.

Condición: $\sqrt[3]{x+3} \neq 0 \wedge -2x+1 \geq 0$

$\sqrt[3]{x+3} \neq 0 \wedge -2x+1 \geq 0 \iff x+3 \neq 0 \wedge -2x \geq -1 \overset{-2<0}{\iff} x \neq -3 \wedge x \leq \frac{-1}{-2}$

$\iff x \neq -3 \wedge x \leq \frac{1}{2}$

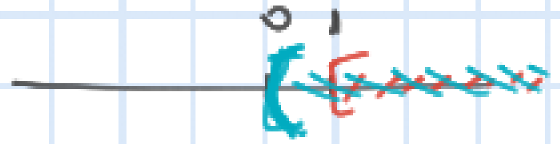


$\boxed{\text{Sol: } x \in (-\infty,-3) \cup (-3,\tfrac{1}{2}]}$

3) $\sqrt[6]{1-\frac{1}{x}}$.

Condición: $\underbrace{1-\frac{1}{x}\geqslant 0}_{\text{I}} \quad \wedge \quad x\neq 0$

I) $1-\frac{1}{x}\geqslant 0 \iff \frac{x-1}{x}\geqslant 0 \iff \begin{cases} \text{I)} & x-1\geqslant 0 \ \wedge \ x>0 \\ \text{ó} & \\ \text{II)} & x-1\leqslant 0 \ \wedge \ x<0 \end{cases} \iff \begin{cases} x\geqslant 1 \ \wedge \ x>0 \\ x\leqslant 1 \ \wedge \ x<0 \end{cases} \iff \begin{cases} x\in[1,+\infty) \\ x\in(-\infty,0) \end{cases}$

I) 

I) $\boxed{\text{Sol: } x\in(-\infty,0)\cup[1,+\infty)}$

$\boxed{\text{Rta: } x\in(-\infty,0)\cup[1,+\infty)}$

Demostrar:

1) $|x| \geq 0$

$$|x| = \begin{cases} x & \text{si } x \geq 0, \\ -x & \text{si } x \leq 0, \end{cases}$$

D/ Sea $x \in \mathbb{R}$

- Si $x \geq 0 \overset{\text{Df v.A}}{\Rightarrow} |x| = x \wedge \overbrace{x \geq 0}^{\checkmark} \Rightarrow |x| \geq 0$ ✓
- Si $x < 0 \overset{\text{df v.A}}{\Rightarrow} |x| = -x \wedge -x > (-1).0 = 0 \Rightarrow |x| > 0$ ✓

$\therefore |x| \geq 0 \quad \forall x \in \mathbb{R}.$

Ejemplo: Resolver:

1) $-|-x+1|-2<4$.

2) $|x-1|<-2$.

1) $-|-x+1|-2<4 \iff -|-x+1| \overset{(-1)}{<} \overset{6}{4+2} \iff \underbrace{|-x+1|}_{\geq 0} > -6$ $-1<0$

Por prop. $|-x+1| \geq 0$, $0>-6$ $\Rightarrow |-x+1|>-6$ $\forall x \in \mathbb{R}$

$\boxed{\text{Sol: } x \in \mathbb{R}}$

2) $\underbrace{\overbrace{|x-1|}^{\geq 0} < -2}_{F}$

Como $|x-1| \geq 0$ $\forall x \in \mathbb{R}$ $\Rightarrow |x-1|<-2$ nunca es verd.

$\boxed{\text{Sol: } \phi}$

¿ $|x-2|<4$?

Demostrar: (2) $|x| < a \Leftrightarrow -a < x < a.$

Ejemplo: Hallar los $x \in \mathbb{R}$ que verifican:

$$(1)\ |2x-5| = 2.$$

Si $a > 0$ y $x, y \in \mathbb{R}$ entonces valen:

(1) $|x| = a \iff x = a \vee x = -a$

(2) $|x| < a \iff -a < x < a$. (También vale para $\leq$)

(3) $|x| > a \iff x > a \vee x < -a$. (También vale para $\geq$)

(4) $|x| = |y| \iff x = y \vee x = -y$.

(5) $\sqrt{x^2} = |x|$.

$|2x-5| = 2 \overset{1)}{\iff} 2x-5 = 2$ ó $2x-5=-2 \iff 2x = 7$ ó $2x = 3 \iff x = \frac{7}{2}$ ó $x = \frac{3}{2}$

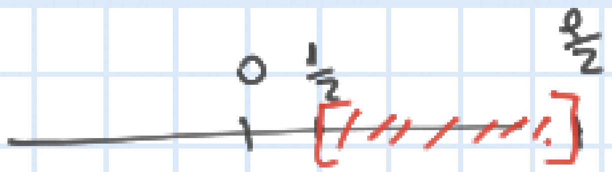
Sol: $x \in \{\frac{3}{2}, \frac{7}{2}\}$

Ejercicio: chequearlo.

$$(2)\ |2x-5| \leq 4.$$

$|2x-5| \leq 4 \overset{2)}{\iff} -4 \leq 2x-5 \leq 4 \iff \frac{1}{2} \leq \frac{2x}{2} \leq \frac{9}{2} \overset{2>0}{\iff} \frac{1}{2} \leq x \leq \frac{9}{2}$

$\boxed{\text{Sol: } x \in [\frac{1}{2}, \frac{9}{2}]}$



$(3)\ (2x-4)^2 \geq 9.$

Recordar $\boxed{x^2 \geq 0}$

Si $a > 0$ y $x, y \in \mathbb{R}$ entonces valen:

(1) $|x| = a \quad \Leftrightarrow \quad x = a \vee x = -a$

(2) $|x| < a \quad \Leftrightarrow \quad a < x < a.$ (También vale para $\leq$)

(3) $|x| > a \quad \Leftrightarrow \quad x > a \vee x < -a.$ (También vale para $\geq$)

(4) $|x| = |y| \quad \Leftrightarrow \quad x = y \vee x = -y.$

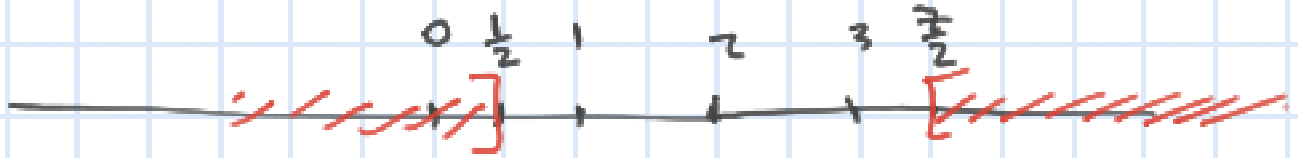
(5) $\sqrt{x^2} = |x|.$

$$(2x-4)^2 \geq 9 \iff \sqrt{(2x-4)^2} \geq \sqrt{9}$$

$$\overset{5)}{\iff} |2x-4| \geq 3 \overset{3)}{\iff} 2x-4 \geq 3 \ \vee\ 2x-4 \leq -3 \iff$$

$$\iff 2x \geq 7 \ \vee\ 2x \leq 1$$

$$\overset{2>0}{\iff} x \geq \frac{7}{2} \ \vee\ x \leq \frac{1}{2}$$



$$\boxed{\text{Sol: } x \in \left(-\infty, \tfrac{1}{2}\right] \cup \left[\tfrac{7}{2}, +\infty\right)}$$

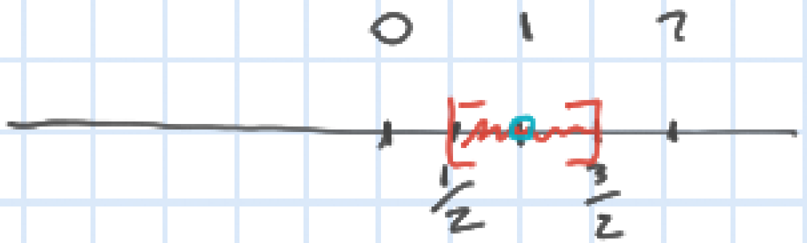
$(4)\ \left|\dfrac{-4}{x-1}\right| \geq 8.$

$x - 1 \neq 0$

$\boxed{x \neq 1}$

$$\left|\frac{-4}{x-1}\right| \geq 8 \iff \frac{|-4|}{|x-1|} \geq 8 \iff \frac{4}{|x-1|} \geq 8 \overset{|x-1|>0}{\iff} 4 \geq 8.|x-1| \overset{8>0}{\iff} \frac{4}{8} \geq |x-1| \iff \frac{1}{2} \geq |x-1| \overset{2)}{\iff}$$

$$-\frac{1}{2} \leq x-1 \leq \frac{1}{2} \iff -\frac{1}{2}+1 \leq x \leq \frac{1}{2}+1 \iff \frac{1}{2} \leq x \leq \frac{3}{2}$$



$$\text{Sol: } x \in \left[\tfrac{1}{2}, 1\right) \cup \left(1, \tfrac{3}{2}\right] = \left[\tfrac{1}{2}, \tfrac{3}{2}\right] - \{1\}$$

$$(5)\ |x| = |2x-1|.$$

Si $a > 0$ y $x, y \in \mathbb{R}$ entonces valen:

(1) $|x| = a \iff x = a \vee x = -a$

(2) $|x| < a \iff -a < x < a$. (También vale para $\leq$)

(3) $|x| > a \iff x > a \vee x < -a$. (También vale para $\geq$)

(4) $|x| = |y| \iff x = y \vee x = -y$.

(5) $\sqrt{x^2} = |x|$.