23/10/21 Saturday

Two-Valued Boolean algebra:-

$B = \{0, 1\}$

OR

x	y	$x+y$
0	0	0
0	1	1
1	0	1
1	1	1

AND

x	y	$x \cdot y$
0	0	0
0	1	0
1	0	0
1	1	1

NOT

Complement

x	x'
0	1
1	0

$x+y+z$

x	y	z	$x+y+z$
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

2^n

n - no. of Variables

$2^3 = 8$

$8 \div 2 = 4$

$4 \div 2 = 2$

$2 \div 2 = 1$

$x \cdot (y+z) = (x \cdot y) + (x \cdot z)$

$x + (y \cdot z) = (x+y) \cdot (x+z)$

✓

x	y	z	$y+z$	$x \cdot (y+z)$	$x \cdot y$	$x \cdot z$	$(x \cdot y)+(x \cdot z)$
0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	0	0	0	0
1	0	0	0	0	0	0	0
1	0	1	1	1	0	1	1
1	1	0	1	1	1	0	1
1	1	1	1	1	1	1	1

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1) Closure 2) Identity 3) Commutative 4) dis

5) Complement 6) $x \neq y$ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ ×

I Associative → $a+(b+c) = (a+b)+c$

$a \cdot (b \cdot c) = (a \cdot b) \cdot c$

II Idempotent → $a+a=a$; $a \cdot a = a$

III ABSORPTION → $a \cdot (a+b) = a$; $a + (a \cdot b) = a$

Duality Principle :-

Every algebraic expression deducible from the postulates of Boolean algebra remains valid if the operators and the identity elements are interchanged.

$x + y = y + x$ $\qquad$ $x + (y + z) = (x + y) + z$

$x \cdot y = y \cdot x$ $\qquad$ $x \cdot (y \cdot z) = (x \cdot y) \cdot z$

$x \cdot (y + z) = (x \cdot y) + (x \cdot z)$

$x + (y \cdot z) = (x + y) \cdot (x + z)$

$x \cdot x' = 0$

$x + x' = 1$

Theorem 1 :-

a) $x + x = x$

b) $x \cdot x = x$

$x + 0 = x$

$x \cdot 1 = x$

$x \cdot x' = 0$

$x + x' = 1$

Proof :- a) $x = x + 0$ [By identity element property]

$= x + (x \cdot x')$ [$\because x \cdot x' = 0$]

$= (x + x) \cdot (x + x')$ [By distributive law]

$= (x + x) \cdot 1$ [$\because x + x' = 1$]

$= x + x$ [$\because x \cdot 1 = x$]

$\therefore x + x = x$

b) $x = x \cdot 1$ [$\because x = x \cdot 1$]

$= x \cdot (x + x')$ [$\because x + x' = 1$]

$= (x \cdot x) + (x \cdot x')$ [By distributive property]

$= x \cdot x + 0$ [$\because x \cdot x' = 0$]

$= x \cdot x$ [$\because x + 0 = x$]

$\therefore x \cdot x = x$