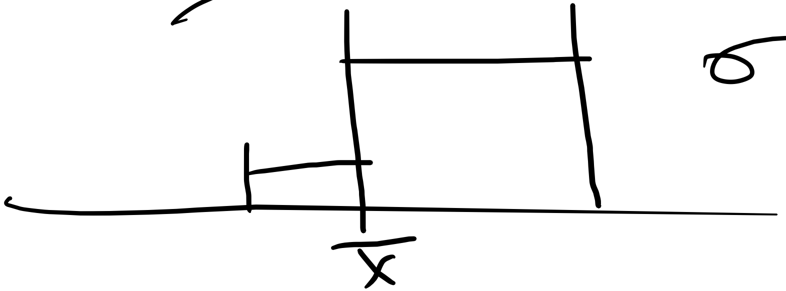
# Measures of dispersion.

1) Range $\longrightarrow L - S$
2) Quartile deviation $\longrightarrow \dfrac{Q_3 - Q_1}{2}$
3) Mean deviation $\longrightarrow \dfrac{\sum f\,|m - \bar{x}|}{\sum f}$ (MD about mean)
4) Standard deviation. $\longrightarrow \sigma = \sqrt{\frac{1}{n}\sum x^2 - \left(\frac{\sum x}{n}\right)^2}$
5) Moments.




$\sigma^2 = $ Variance

# Coefficient of dispersion.

Coeff of range $= \dfrac{L-S}{L+S}$.

(independent of units)

Coeff of QD $= \dfrac{Q_3 - Q_1}{Q_3 + Q_1}$.

Coeff of MD $= \dfrac{M.D\ (\text{Mean})}{\text{Mean}}$

Coeff of Variation $= \dfrac{\sigma}{\bar{X}} \times 100$.

Combined S.D.

$$\text{Combined S.D} = \frac{n_1(\sigma_1^2 + d_1^2) + n_2(\sigma_2^2 + d_2^2) + \cdots + n_k(\sigma_k^2 + d_k^2)}{n_1 + n_2 + \cdots + n_k}$$

$$d_i = \bar{x}_i - \bar{x}$$

$\bar{x}$ → Combined mean

$$\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2 + \cdots + n_k\bar{x}_k}{n_1 + n_2 + \cdots + n_k}$$

Calculate Q.D & coeff of Q D for the following:

| CI: | 20-30 | 30-40 | 40-50 | 50-60 | 60-70 | 70-80 | 80-90 |
|---|---|---|---|---|---|---|---|
| f: | 3 | 61 | 132 | 153 | 140 | 51 | 2 |

Soln

| | CI | f | cf |
|---|---|---|---|
| | 20-30 | 3 | 3 |
| | 30-40 | 61 | 64 |
| $Q_1$ | 40-50 | 132 | 196 |
| | 50-60 | 153 | 349 |
| $Q_3$ | 60-70 | 140 | 489 |
| | 70-80 | 51 | 540 |
| | 80-90 | 2 | 542 |
| | | 542 | |

$$Q.D = \frac{Q_3 - Q_1}{2}$$

$$\frac{3N}{4} = \frac{3(542)}{4} = 406.5$$

$$Q_3 = L + \frac{\frac{3N}{4} - cf}{f} \times i$$

$$\frac{N}{4} = \frac{542}{4} = 135.5$$

$$= 60 + \frac{406.5 - 349}{140} \times 10 = 64.107$$

$$Q_1 = L + \frac{\frac{N}{4} - cf}{f} \times i$$

$$= 40 + \frac{135.5 - 64}{132} \times 10 = 45.416.$$

$$QD = \frac{Q_3 - Q_1}{2}$$

$$= \frac{64.107 - 45.416}{2} = 9.345$$

$$\text{Coeff of } QD = \frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{64.107 - 45.416}{64.107 + 45.416} = \cancel{0.085}\ 0.17068$$

→ Find the SD for the following:

| | | | | | | |
|---|---|---|---|---|---|---|
| Marks : | 0-4 | 4-8 | 8-12 | 12-16 | 16-20 | 20-24 |
| No of students : | 10 | 12 | 18 | 7 | 5 | 3 |

Soln

| CI | m | f | $d = \frac{m-A}{4}$ | fd | $fd^2$ |
|---|---|---|---|---|---|
| 0-4 | 2 | 10 | -3 | -30 | 90 |
| 4-8 | 6 | 12 | -2 | -24 | 48 |
| 8-12 | 10 | 18 | -1 | -18 | 18 |
| 12-16 | 14 A | 7 | 0 | 0 | 0 |
| 16-20 | 18 | 5 | 1 | 5 | 5 |
| 20-24 | 22 | 3 | 2 | 6 | 12 |
| | | 55 | | -61 | 173 |

$$\sigma = \sqrt{\frac{\Sigma fd^2}{\Sigma f} - \left(\frac{\Sigma fd}{\Sigma f}\right)^2} \times i$$

$$= \sqrt{\frac{173}{55} - \left(\frac{-61}{55}\right)^2} \times 4$$

$$= \sqrt{3.145 - 1.23} \times 4$$

$$= 1.3838 \times 4$$

$$= 5.535$$

Find the Mean deviation about Mean & coeff of M.D for the following:

| CI : | 20-25 | 25-30 | 30-35 | 35-40 | 40-45 | 45-50 | 50-55 | 55-60. |
|---|---|---|---|---|---|---|---|---|
| f : | 35 | 45 | 70 | 105 | 90 | 74 | 51 | 30. |

Soln

| CI | m | f | $d = \frac{m-A}{5}$ | fd | $\lvert m-39.96\rvert$ | $f\lvert m-39.96\rvert$ |
|---|---|---|---|---|---|---|
| 20-25 | 22.5 | 35 | -3 | -105 | 17.46 | 611.1 |
| 25-30 | 27.5 | 45 | -2 | ~~-90~~ | 12.46 | 560.7 |
| 30-35 | 32.5 | 70 | -1 | -70 | 7.46 | 522.2 |
| 35-40 A | 37.5 | 105 | 0 | 0 | 2.46 | 258.3 |
| 40-45 | 42.5 | 90 | 1 | ~~90~~ | 2.54 | 228.6 |
| 45-50 | 47.5 | 74 | 2 | 148 | 7.54 | 557.96 |
| 50-55 | 52.5 | 51 | 3 | 153 | 12.54 | 639.54 |
| 55-60 | 57.5 | 30 | 4 | 120 | 17.54 | 526.2 |
| | | 500 | | 246 | | 3904.6 |

148
98
246

$$\bar{X} = A + \frac{\sum fd}{\sum f} \times i$$

$$\begin{array}{r} 37.5 \\ 2.46 \\ \hline 39.96 \\ \hline \end{array}$$

$$= 37.5 + \left(\frac{\overset{2.46}{\cancel{246}}}{\underset{100}{\cancel{500}}}\right) \times \cancel{5}$$

$$= \underline{\underline{39.96.}}$$

$$M.D = \frac{\sum f|m - 39.96|}{\sum f} = \frac{3904.6}{500} = \underline{\underline{7.809}}$$

about Mean

$$\text{Coeff of MD} = \frac{7.809}{39.96} = \underline{\underline{0.1954}}$$

→ An analysis of monthly wages paid to the workers of two firms A & B belonging to the same industry gives the following:

| | firm A | firm B |
|---|---|---|
| No of workers | $500 \rightarrow n_1$ | $600 \rightarrow n_2$ |
| Average daily wage | $186 \rightarrow \bar{x}_1$ | $175 \rightarrow \bar{x}_2$ |
| Variance of wage | $81 \rightarrow \sigma_1^2$ | $100 \rightarrow \sigma_2^2$ |

(i) which firm, A or B has large wage bill.

C.V ← (ii) which firm, A or B, is there greater variability in individual wages?

Combined mean & combined SD (iii) Calculate the avg daily wage & variance of the distribution of the wages of all the workers in firm A & B together.

(ii) $CV(A) = \frac{\sigma_1}{\overline{X}_1} \times 100 = \frac{9}{186} \times 100 = \frac{900}{186} = 4.838$

$CV(B) = \frac{\sigma_2}{\overline{X}_2} \times 100 = \frac{10}{175} \times 100 = 5.714.$

$CV(B) > CV(A)$. $\Rightarrow$ B has got greater variability when compared with A.

(i) $n_1 = 500$; $\bar{x}_1 = 186$; $n_2 = 600$; $\bar{x}_2 = 175$.

Total wages in firm A $= n_1 \times \bar{x}_1 = 500 \times 186 = 93{,}000$

Total wages in firm B $= n_2 \times \bar{x}_2 = 600 \times 175 = 1{,}05{,}000$

More wage is paid by firm B.

(iii)

$$\text{Combined mean} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1+n_2}$$

$d_1^2 = (186-180)^2 = 6^2 = 36$

$d_2^2 = (175-180)^2 = 25$

$$= \frac{500(186)+600(175)}{500+600} = 180.$$

$$\text{Combined SD} = \frac{n_1(\sigma_1^2+d_1^2)+n_2(\sigma_2^2+d_2^2)}{n_1+n_2}$$

$$= \frac{500(81+36)+600(100+25)}{500+600}$$

$$= \frac{500(117)+600(125)}{1100} = \frac{133500}{1100} = 121.36$$