SASTRA DEEMED TO BE UNIVERSITY	School of Arts, Sciences and Humanities First CIA Test – April 2022 I-B.Tech Computer Science and Business Systems Course Code: MAT134 Course Name: **Linear Algebra** Duration: 90 minutes Max Marks: 50

PART A

10 x 2 = 20 Marks

1.	Find the value of a for which the system of equations $x+y+z=1, x-y+2z=0, 2x+3z=a$, has infinitely many solutions.
2.	(a) If A be an 100×100 matrix and $rank(Adj(A))=1$ then what is the $rank(A)$? (b) If A be an 100×100 matrix and $rank(Adj(A))=0$ then what are the possible values of $rank(A)$? Justify your answer
3.	State whether the matrix AB is always singular matrix or BA is always singular matrix if A_{mxn} and B_{nxm} be a matrix over real numbers with $m<n$. and verify it.
4.	Let A be an 2022×2022 matrix. Then (i) If $rank(A)=1011$ then A has an invertible matrix. (ii) If $Ax=0$ such that $rank(A)=2021$ then A has an non-trivial solution. (iii) All the eigen values are non-zero of A then $rank(A)$ is 2022. (iv) If A are linearly independent column vector $\mathbb{R}^{2022}$ Then $Ax=0$ has trivial solution. (v) If A are linearly independent column vector $\mathbb{R}^{2022}$ Then $Ax=b$ has unique solution. (vi) If $rank(A)=2022$ and any $B_{2022\times 2022}$ matrix then $rank(AB)=rank(B)$ **Pick the correct Option** (a) (i),(ii) and (v) are correct always statements (b) (ii),(iii) and (iv) are correct always statements (c) (ii),(v) and (vi) are correct always statements (d) (iii),(iv) and (vi) are correct always statements
5.	Find the trace of the matrix $\begin{bmatrix}2&5&7\\0&3&1\\0&0&4\end{bmatrix}^{50}$?

6.	Find the determinant of the matrix $\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & 2 & 0 \\ 0 & 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 & 1 & 0 \\ 2 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$
7.	Given that the matrix $\begin{pmatrix} \alpha & 1 \\ 2 & 3 \end{pmatrix}$ has 1 as an eigenvalue, compute its trace and its determinant.
8.	Discuss the following statements are true (or) false and explain it (a) If A and B be two arbitrary $n \times n$ matrices. Then $(A+B)^2 = A^2 + 2AB + B^2$. (b) There exist $n \times n$ matrices A and B such that $AB - BA = I$.
9.	Let A be a 3×3 upper triangular matrix whose diagonal entries are $1, 2$ and -3. Express A^{-1} as a linear combination of I, A and A^2.
10.	If $A_{9\times 9}$ matrix and p be the characteristic polynomial of A. Then (a) What is $e^{p(A)}$ matrix? (b) What is $\sin(P(A))$ matrix?

PART B **3 x 10 = 30 Marks**

11.	Find the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{bmatrix}$
12.	(a) Test for the consistency of the following system of equation $3x_1 + 4x_2 + 5x_3 + 6x_4 = 7, 4x_1 + 5x_2 + 6x_3 + 7x_4 = 8, 5x_1 + 6x_2 + 7x_3 + 8x_4 = 9, 10x_1 + 11x_2 + 12x_3 + 13x_4 = 14,\ 15x_1 + 16x_2 + 17x_3 + 18x_4 = 19.$ (5-mark)
	(b) Test for the consistency of the following systems of equations and solve it $x - 3y - 8z = -10, 3x + y = 4z, 2x + 5y + 6z = 13.$ (5-mark)
13.	(a) Find the condition on $a, b, c,$ so that the equations $x + y + z = a, x + 2y + 3z = b, 3x + 5y + 7z = c$ may have a one-parameter family of solutions. (5-mark)
	(b) Find the eigenvalues of $A^2,\ A^{-1},\ 5A$ and $adj(A)$, when $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ (5-mark)

SASTRA DEEMED TO BE UNIVERSITY	School of Arts, Sciences and Humanities Second CIA Test – May 2022 Course Code: MAT134 Course Name: Linear Algebra Duration: 90 minutes Max Marks: 50

PART A 10 x 2 = 20 Marks

1. If A & B are positive definite matrix of order n. Then prove that $A+B$ is positive definite.

2. Discuss the nature of quadratic form corresponding
$$x_1^2+x_2^2+x_3^2+x_4^2+x_5^2+x_6^2+4x_1^2x_6^2+4x_2^2x_5^2+4x_3^2x_4^2=0.$$

3. List the following sets are subspace over $\mathbb{R}$?
   (a) $V=\{A\in M_3(\mathbb{R}): \text{Trace}(A)=0\}$
   (b) $V=\{A\in M_{2022}(\mathbb{R}): \text{Trace}(A)=2022\}$
   (c) $V=\{A\in M_{2022}(\mathbb{R}): \text{Det}(A)=0\}$
   (d) $V=\{A\in M_{2022}(\mathbb{R}): \text{Det}(A)=2022\}$
   (e) $V=\{A\in M_{2022}(\mathbb{R}): A=A^T\}$
   (f) $V=\{A\in M_{2022}(\mathbb{R}): A=-A^T\}$
   (g) $V=\{A\in M_{2022}(\mathbb{R}): AA^T=I\}$

4. Let W_1 & W_2 are subspace of V such that $\dim W_1=5$ & $\dim W_2=8$ of $\dim V=10$. What are the possible $\dim(W_1\cap W_2)$?

5. Find the singular value of the matrix $\begin{bmatrix}1&0&2\\0&1&-1\end{bmatrix}$?

6. Find the dimension following sets
   (a) $V=\{A\in M_3(\mathbb{R}): A=A^T\}$,
   (b) $V_1=\{A\in M_3(\mathbb{R}): A=-A^T\}$,

7. Derive the basis vector for the following vector space?
   (a) $V=\{(x_1,x_2,x_3,x_4)\in\mathbb{R}^4 : x_1+2x_2+3x_3+4x_4=0\}$
   (b) $V=\{A\in M_3(\mathbb{R}): \text{Trace}(A)=0\}$

8. Discuss the following statements are true (or) false and explain it
   (a) $V=\{a_0+a_1x+a_2x^2=0 :\ a_i\in\mathbb{R}\}$ is a vector space of $\mathbb{R}$.
   (b) $V=\{A\in M_n(\mathbb{R}): \det A=0\}$ is a vector space of $\mathbb{R}$.

9. If A is similar to B then prove that eigenvalue of A same as eigenvalue of B.

10. Construct an example such that $W_1\oplus W_2=V(Direct\ sum)$ and verify it.

PART B(Answer any three question) 3 x 10 = 30 Marks

11. Reduce a matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ -2 & 4 & 3 \end{bmatrix}$ to diagonal form by similar transformation.

12. (a) State the Cayley-Ham ton theorem (2-mark)

(b) Use Cayley Hamilton theorem to find the value of the matrix given by $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I = ?$

If $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$. (8-mark)

13. Reduce to $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ a singular value decomposition.

14. If W_1 and W_2 are subspaces of a finite dimensional vector space, then $\dim(W_1 + W_2) = \dim W_1 + \dim W_2 - \dim(W_1 \cap W_2)$.

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School of Arts, Sciences and Humanities
III CIA Test – June 2022
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PART A **5 x 2 = 10 Marks**

1.	If A & B are invertible matrix such that $AB = -BA$, show that $Trace(A) = 0$.
2.	Prove that $V = \{A \in M_2(\mathbb{R}) : A = A^T\}$ subspace of $M_2(\mathbb{R})$.
3.	Let V be the vector space of any real polynomial of degree ≤ 3. Let $Tp(x) = p'(x) for\ p \in V$ be a linear transformation from V to V with respect to basis $\{1, x, x^2, x^3\}$. Then find A matrix of T with respect to this basis.
4.	What are different types of Machine learning algorithm?
5.	Discuss the consistency of the system of equations $x + y + z = 1, x - y + 2z = 0, 2x + 3z = 1$.

PART B(Answer any four question) **4 x 10 = 40 Marks**

6.	Find the value of k such that the following system of equation has unique solution, infinitely many solutions and no solution $$kx + y + z = 1, x + ky + z = 1, x + y + kz = 1.$$
7.	If $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ Find QR decomposition using Gram-Schmidt method.
8.	State and prove Rank-Nullity theorem.
9.	Given data {2,3,4,5,7 : 1,5,3,6,7,8 }, Compute the principal component using PCA algorithm.
10.	(a) If A is an invertible then prove that A^2 is positive definite matrix (3-mark) (b) If A and B are positive definite matrix then prove that A+B is positive definite matrix. (3-mark) (c) If $\{(1,-1,1), (1,0,1), (1,1,2)\}$ basis of $\mathbb{R}^3$ using Gram-Schmidt algorithm. To find Orthogonal basis (4-mark)