Geometric mean

$$GM = (x_1 x_2 \cdots\cdots x_n)^{1/n} = G.$$

(Individual observations)

$$G = \text{Antilog}\left\{\frac{1}{n}\sum_i \log x_i\right\}.$$

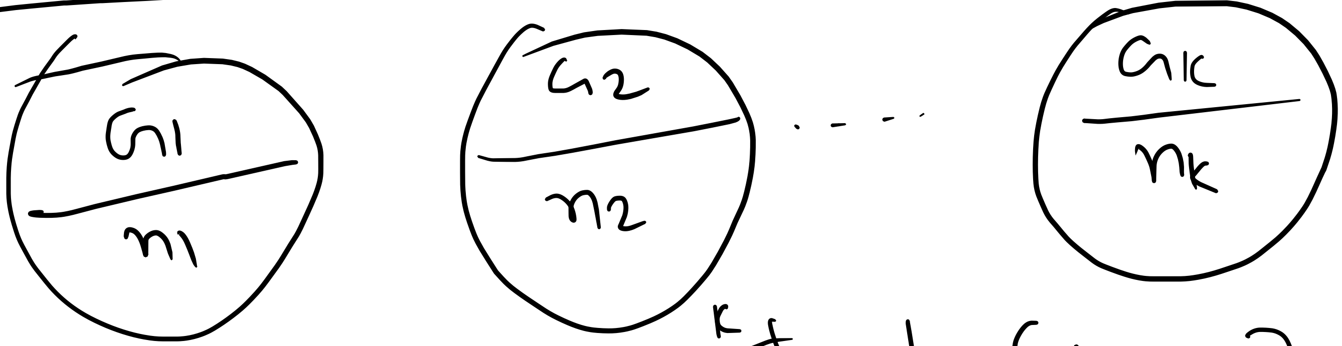
discrete distributions.

$$G = \text{antilog}\left\{\frac{1}{N}\sum_i f_i \log x_i\right\}; \quad N = \sum_i f_i$$

↓

continuous distribution

Combined data



$$G = \text{Antilog}\left\{ \frac{\sum_{i=1}^{k} n_i \log G_i}{\sum_{i=1}^{k} n_i} \right\}.$$

* $A.M > G.M.$

* The G.M of 10 observations on a certain variable was calculated as 16.2. It was later discovered that one of the observation was wrongly recorded as 12.9 instead of 21.9. Calculate the correct G.M.

soln

$$G' = (x_1 x_2 \cdots x_n)^{1/n} = 16.2$$

$\boxed{\text{Let } x_1 \to 12.9 \ \&}$

$x_1' = 21.9.$

$$(12.9 \cdot x_2 \cdot x_3 \cdots x_{10})^{1/10} = 16.2$$

$$(x_2 x_3 \cdots x_{10})^{1/10} = \frac{16.2}{(12.9)^{1/10}}.$$

$$\text{Corrected G.M} = (x_1' x_2 x_3 \cdots x_{10})^{\frac{1}{10}} = (x_1')^{1/10} (x_2 x_3 \cdots x_{10})^{1/10}$$

$$= (21.9)^{1/10} \times \frac{16.2}{(12.9)^{1/10}}.$$

$$G.M = 16.2 \left(\frac{21.9}{12.9}\right)^{+\frac{1}{10}} \quad //$$

* S.T in finding the A.M of a set of readings on a thermometer it does not matter whether we measure temperature in centigrade or Fahrenheit, but that in finding the GM it does matter which scale we use.

$$\left\{\frac{F-32}{180}=\frac{C}{100}\right\}.$$

$$\downarrow$$

$$F-32=\frac{\overset{9}{\cancel{180}}}{\underset{5}{\cancel{100}}}\times C$$

$$\boxed{F=32+\frac{9}{5}C}$$

Soln suppose $c_1, c_2, \ldots c_n$ are in centigrade.

$$A\cdot M \quad \bar{C}=\frac{1}{n}\left\{c_1+c_2+\cdots\cdots+c_n\right\}.$$

$$GM=G_c=\left(c_1c_2\cdots\cdots c_n\right)^{1/n}.$$

The observations corresponding to Fahrenheit

$$32+\frac{9}{5}c_1,\ 32+\frac{9}{5}c_2,\ \ldots,\ 32+\frac{9}{5}c_n.$$

$$\bar{C}_f = \frac{1}{n}\left\{\left(32+\frac{9}{5}C_1\right)+\left(32+\frac{9}{5}C_2\right)+\cdots+\left(32+\frac{9}{5}C_n\right)\right\}$$

$$= 32+\frac{9}{5}\left\{\frac{C_1+C_2+\cdots+C_n}{n}\right\} = 32+\frac{9}{5}\bar{C}$$

$$G_{cf} = \frac{1}{n}\left\{\left(32+\frac{9}{5}C_1\right)\left(32+\frac{9}{5}C_2\right)\cdots\left(32+\frac{9}{5}C_n\right)\right\}^{1/n}$$

$$\neq \frac{9}{5}\left(C_1C_2\cdots C_n\right)^{1/n}+32.$$

→ In a frequency table, the upper bound of each CI has a const ratio to the lower bound. S.T GM is given by

$$\log G = \boxed{x_0} + \frac{c}{N}\sum_{i} f_i\,(i-1)$$

x_0 → log of mid value of first CI ; c → log of ratio of upper bound & lower bound.

$\overset{\text{lower}}{10} - \overset{\text{upper}}{20}$

20 − 40

40 − 80

Soln Let the i^{th} class interval is denoted as $I_i - I_{i+1}$ & the frequency is f_i

$i = 1, 2, \ldots n$

$I_1 - I_2 \quad f_1$

$I_2 - I_3 \quad f_2$

$I_3 - I_4 \quad f_3$

$\vdots$

Given $\dfrac{I_2}{I_1} = \dfrac{I_3}{I_2} = \cdots\cdots = \dfrac{I_i}{I_{i-1}} \cdots\cdots = \lambda$ (const)

$\rightarrow I_2 = \lambda I_1$

$I_3 = \lambda I_2$

$= \lambda^2 I_1$

$$I_i = \lambda I_{i-1}$$

$$= \lambda(\lambda I_{i-2}) = \cdots = \lambda^{i-1} I_1 .$$

$$\boxed{\{ I_i = \lambda^{i-1} I_1 \}}$$

$$I_2 = \lambda I_1$$

Let x_i be the mid pt of i^{th} class.

$$x_1 = \frac{1}{2}(I_1 + I_2) = \frac{1}{2}(I_1 + \lambda I_1)$$

$$= \frac{I_1}{2}(1+\lambda).$$

$$x_i = \frac{1}{2}(I_i + I_{i+1}) = \frac{1}{2}\{\lambda^{i-1} I_1 + \lambda^{i} I_1\}$$

$$= \frac{1}{2}\lambda^{i-1} I_1 (1+\lambda).$$

$x_1 = \frac{(1+\lambda)}{2} I_1$ & $x_i = \frac{1}{2}\lambda^{i-1} I_1 (1+\lambda)$

$$x_i = \lambda^{i-1} x_1$$

Let G be the G.M.

$$\log G = \frac{\sum f_i \log x_i}{\sum f_i} \quad N$$

$$= \frac{1}{N} \sum f_i \log(\lambda^{i-1} x_1)$$

$$\log G = \frac{1}{N} \sum_i f_i \log(x_1 \lambda^{i-1})$$

$$= \frac{1}{N} \sum_i f_i \{\log x_1 + \log \lambda^{i-1}\}$$

$$= \frac{1}{N} \sum_i f_i \log x_1 + \frac{1}{N} \sum_i f_i \log \lambda^{(i-1)}$$

$$= \frac{1}{N} \sum_i f_i \log x_1 + \frac{1}{N} \sum_i f_i (i-1) \log \lambda .$$

$$= \frac{\log x_1}{\cancel{N}} \cancel{\sum_i f_i} + \frac{\log \lambda}{N} \sum_i f_i (i-1)$$

$$= x_0 + c \sum_i f_i (i-1)$$

Harmonic mean

Harmonic mean of a number of observations, none are zero, is the reciprocal of A.M of the reciprocal of the given values.

ie $x_1, x_2, \dots x_n$ is the given data.

$$HM = \frac{1}{\frac{1}{n}\sum_{i=1}^{n}\left(\frac{1}{x_i}\right)}.$$

Discrete | continuous

$$HM = \frac{1}{\frac{1}{N}\sum_{i=1}^{n}\left(\frac{f_i}{x_i}\right)} \quad ; \; N = \sum_i f_i$$

→ A cyclist pedals from his house to his college at a speed of <u>10 kmph</u> & back from college to his house at 15 kmph. Find the average speed.

<u>Soln</u> Let x be the distance b/w house & college.

House to college → The distance travelled in $\frac{x}{10}$ hours.

College to house → The distance travelled in $\frac{x}{15}$ hrs.

The total distance $2x$ is covered in $\left(\frac{x}{10}+\frac{x}{15}\right)$ hrs.

$$\text{average speed} = \frac{\text{Total distance}}{\text{Total time take}}$$

$$= \frac{2x}{\cancel{x}\left(\frac{1}{10}+\frac{1}{15}\right)}$$

$$= \frac{2\times\cancel{10}_2\times\cancel{15}^3}{\cancel{25}\ \cancel{5}} = \underline{\underline{12\text{hrs}}}$$