Dispersion

Dispersion is scatteredness.

To have an idea about the homogeneity & heterogenity of the data.

Defn:- The degree to which numerical data tend to spread about an average Value is called Variation or dispersion of data

$\bar{X}$ | Median | Mode

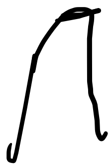
Measures of dispersion.

1) Range
2) Quartile deviation
3) Mean deviation
4) Standard deviation

Moments.

Skewness — size of the data

Kurtosis — flatness of the data



Suppose ~~Cb~~ we wish to compare dispersion of two samples. We use coefficient of dispersion.

* It is independent of units.

- Coefficient of range
- Coefficient of Q.D
- Coeff of Mean deviation
- Coefficient of Variation.

* Range = $L - S$

$L \rightarrow$ Longest value

$S \rightarrow$ Smallest value

Coeff of range = $\dfrac{L-S}{L+S}$ ✓

* Quartile deviation = $\dfrac{Q_3 - Q_1}{2}$

$Q_3 \rightarrow$ III Quartile

$Q_1 \rightarrow$ I Quartile

Coeff of QD = $\dfrac{Q_3 - Q_1}{Q_3 + Q_1}$

Mean deviation.

MD = Mean deviation from $\boxed{\alpha}$ = $\frac{1}{N}\sum_i f_i |x_i - \alpha|$

Coeff of MD

$\alpha \rightarrow$ mean $\rightarrow$ MD about Mean

$\alpha \rightarrow$ median $\rightarrow$ M.D about Median.

$\alpha \rightarrow$ mode $\rightarrow$ M.D about Mode

$\frac{MD}{Mean}$

$\frac{MD}{median}$

$\frac{MD}{mode}$.

Standard deviation (σ)

* $\sigma = \sqrt{\frac{1}{N}\sum_i f_i (x_i - \bar{x})^2}$; $\sum_i f_i = N$.

called as Root mean square value.

* σ^2 = Variance of the distribution. ✓

Equivalent formula for SD

$$\sigma = \sqrt{\frac{\sum fd^2}{\sum f} - \left(\frac{\sum fd}{\sum f}\right)^2} \times i \quad ; \quad d = \frac{m-A}{i}$$

Coefficient of Variation:-

$$CV(x) = \frac{\sigma}{\bar{x}} \times 100.$$

→ For a group of 200 students, the mean & SD of scores are found to be $\underline{40}$ & 15 respectively. Later it was discovered that the scores 43 & 35 were misread as $\underline{34}$ and $\underline{53}$ respectively. Find the correct mean & SD.

	correct value
34	43
53	35

$\underline{\text{soln}}$

$$n = 200; \quad \bar{X} = 40; \quad \sigma = 15.$$

To calculate corrected mean & corrected SD.

$$\bar{X} = \frac{\sum x}{n} \Rightarrow \sum x = n \cdot \bar{X} = 200(40) = 8000.$$

-87
$+78$

$$\text{Corrected } \sum x = \sum x - 34 - 53 + 43 + 35$$

$$= 8000 - 9 = 7991.$$

$$\text{Corrected mean} = \frac{7991}{200} = \_\_\_\_$$

$$\sigma^2 = \frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2.$$ {Individual observation}

$$15^2 = \frac{\sum x^2}{200} - (40^2)$$

$$225 = \frac{\sum x^2}{200} - 1600.$$

$$\frac{\sum x^2}{200} = 1825$$

$$\underline{\underline{\sum x^2}} = 1825 \times 200 = 3{,}65{,}000$$

$$\text{Corrected } \sum x^2 = \sum x^2 - 34^2 - 53^2 + 43^2 + 35^2$$

$$= 365000 - 1156 - 2809 + 1849 + 1225$$

$$= 364109.$$

$$\text{Corrected } \sigma^2 = \frac{364109}{200} - \left(\frac{7991}{200}\right)^2$$

$$= 224 \cdot 1429$$

$$\sigma = \sqrt{224 \cdot 1429} = 14 \cdot 97$$

→ The first of the two samples has 100 items with mean 15 and S.D 3. If the whole group has 250 items with mean 15.6 & S.D $\sqrt{13.44}$. Find the SD of the second group.

Combined ~~SD~~ Variance is given by

$$\left\{ \sigma^2 = \frac{n_1(\sigma_1^2+d_1^2)+n_2(\sigma_2^2+d_2^2)+\cdots+n_k(\sigma_k^2+d_k^2)}{n_1+n_2+\cdots+n_k} \right.$$

$$\left. \text{where } d_i = \bar{x}_i - \bar{x} \quad (i=1,2,\ldots k) \right\}$$

$\bar{x} \rightarrow$ Combined mean.

Soln

$n_1 = 100$; $n_2 = 150$

$\bar{x}_1 = 15$ $\bar{x} = 15.6 \longrightarrow$ (Combined mean)

$\sigma_1 = 3$ $\sigma^2 = 13.44$ (Combined Variance)

$$\bar{x} = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2} \Rightarrow 15.6 = \frac{100(15) + 150(\bar{x}_2)}{250}$$

$$\bar{x}_2 = \frac{15.6 \times 250 - 1500}{150}$$

$$\bar{x}_2 = \frac{2400}{150} = 16.$$

$$\sigma^2 = \frac{n_1(\sigma_1^2 + d_1^2) + n_2(\sigma_2^2 + d_2^2)}{n_1 + n_2}$$

$$d_1 = \bar{x}_1 - \bar{x}$$
$$= 15 - 15.6$$
$$= -0.6$$

$$d_2 = \bar{x}_2 - \bar{x}$$
$$= 16 - 15.6$$
$$= 0.4$$

$$13.44 = \frac{100(9 + 0.36) + 150(\sigma_2^2 + 0.16)}{250}$$

$$13.44 \times 250 = 936 + 150(\sigma_2^2 + 0.16)$$

$$3360 - 936 = 150(\sigma_2^2 + 0.16)$$

$$2424 = 150(\sigma_2^2 + 0.16)$$

$$\sigma_2^2 = 16 \Rightarrow \sigma_2 = 4$$