Unit - II
2.4 Mesh Analysis Problems and Supermesh

Dr.Santhosh.T.K.

V

I

Toolchest

ohm's law
Division
Reduction
KVL, KCL
Mesh.

Syllabus

UNIT – II **14 Periods**

DC Circuit Analysis: Voltage source and current sources, ideal and practical, Kirchhoff's laws and applications to network solutions using mesh analysis, - Simplifications of networks using series- parallel, Star/Delta transformation, DC circuits-Current-voltage relations of electric network by mathematical equations to analyse the network (Superposition theorem, Thevenin's theorem, Maximum Power Transfer theorem), Transient analysis of R-L, R-C and R-L-C Circuits.

AC Steady-state Analysis: AC waveform definitions - Form factor - Peak factor - study of R-L - R-C -RLC series circuit - R-L-C parallel circuit - phasor representation in polar and rectangular form - concept of impedance - admittance - active - reactive - apparent and complex power - power factor, Resonance in R-L-C circuits - 3 phase balanced AC Circuits

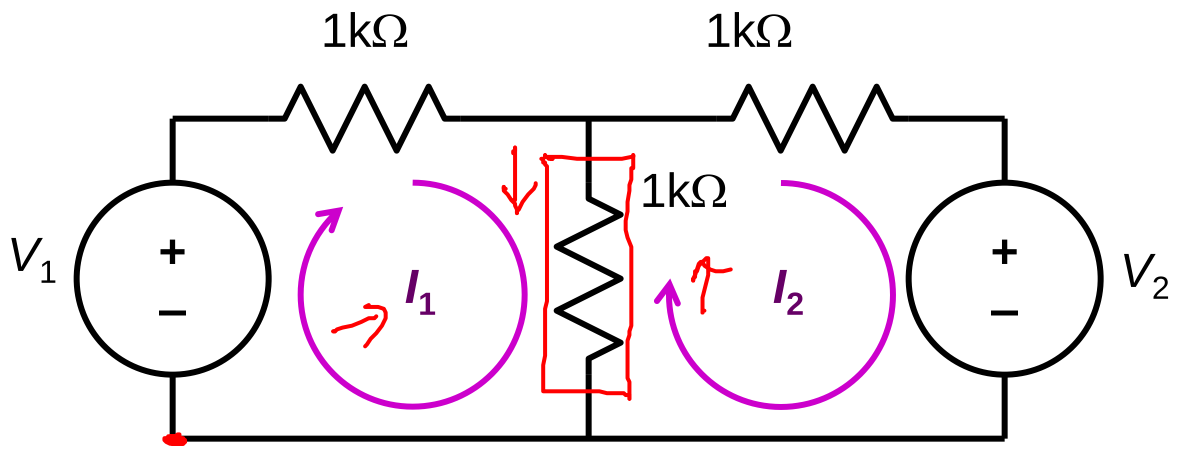
Steps of Mesh Analysis

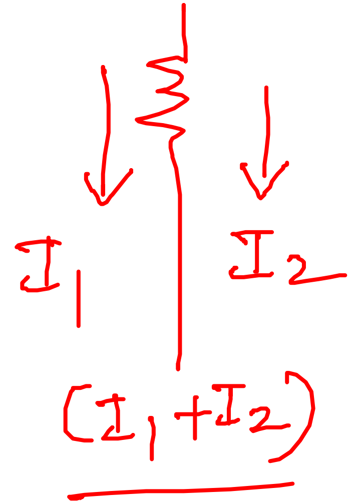
1. Identify mesh (loops).
2. Assign a current to each mesh. → clockwise
3. Apply KVL around each loop to get an equation in terms of the loop currents.

   V = IR → Resistors → drop
4. **Solve the resulting system of linear equations for the mesh/loop currents.**

3. KVL Around Mesh 1

$+V_1 - I_1(1k) - (I_1 - I_2)\,1k = 0$





$$+V_1 - I_1\,1k\Omega - (I_1 - I_2)\,1k\Omega = 0$$
$$I_1\,1k\Omega + (I_1 - I_2)\,1k\Omega = V_1$$
$$I_1\,2k\Omega - I_2\,1k\Omega = V_1$$

3. KVL Around Mesh 2

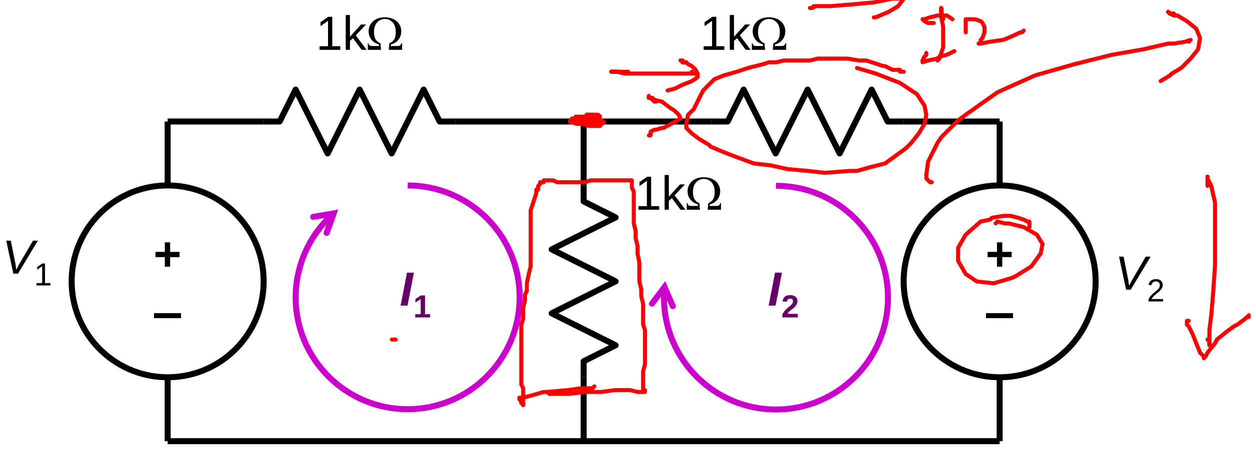
$V = IR$

$-I_2(1k) - V_2 - 1k(I_2 - I_1) = 0$

$V_1 = 7V$

$V_2 = 4V$

I_1 & I_2



$$-(I_2 - I_1)\,1k\Omega - I_2\,1k\Omega - V_2 = 0$$

$$(I_2 - I_1)\,1k\Omega + I_2\,1k\Omega = -V_2$$

$$-I_1\,1k\Omega + I_2\,2k\Omega = -V_2 \quad (2)$$

$$I_1(2k) - I_2(1k) = V_1 \quad (1)$$

4. Solving the Equations

Let: $V_1 = 7V$ and $V_2 = 4V$

Results:

$$I_1 = 3.33 \text{ mA}$$

$$I_2 = -0.33 \text{ mA}$$

Finally

$$V_{out} = (I_1 - I_2)\ 1k\Omega = 3.66V$$

- The two equations can be combined into a single matrix/vector equation

$I_2 - I_1$

$I_1 - I_2$

$I_1\, 2\text{k}\Omega \;-\; I_2\, 1\text{k}\Omega = V_1$

$-I_1\, 1\text{k}\Omega + I_2\, 2\text{k}\Omega = -V_2$

Resistance matrix

+ve

$$\begin{bmatrix} 1\text{k}\Omega + 1\text{k}\Omega & -1\text{k}\Omega \\ -1\text{k}\Omega & 1\text{k}\Omega + 1\text{k}\Omega \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ -V_2 \end{bmatrix}$$

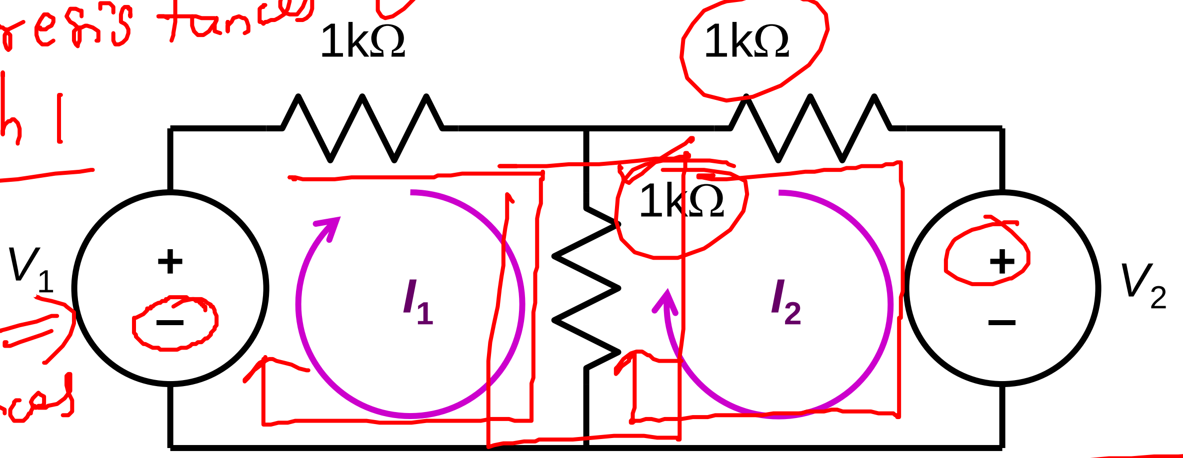
-ve

R I = V

Inspection Method

$a_{11} \rightarrow$ Sum of resistances in mesh 1

$a_{22} \rightarrow$ Sum of resistances in mesh 2.



$a_{12} \rightarrow -1k = a_{21}$

$$\begin{bmatrix} \overset{a_{11}}{1k\Omega + 1k\Omega} & \overset{a_{12}}{-1k\Omega} \\ \overset{a_{21}}{-1k\Omega} & \overset{a_{22}}{1k\Omega + 1k\Omega} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} +V_1 \\ -V_2 \end{bmatrix}$$

Practice Problem

Determine the power dissipation in the 4Ω resistor of the given network.

1. 3 mesh
2. I_1, I_2, I_3
3. KVL → Inspection.

$$\begin{bmatrix} 8 & -3 & 0 \\ -3 & 9 & -4 \\ 0 & -4 & 10 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} +60 \\ 0 \\ -20 \end{bmatrix}$$

R I V

$a_{11} = 5+3 = 8$

$a_{22} = 3+4+2 = 9$

$a_{23} = a_{32} =$

$a_{33} = 6+4 = 10$

$a_{12} = a_{21} = -3$

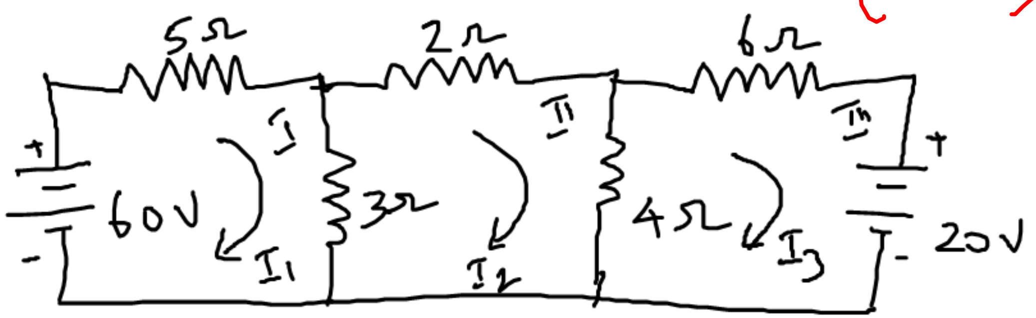
$a_{13} = a_{31} = 0$

Problem

Cramer's

999MS

Determine the power dissipation in the 4Ω resistor of the given network.



(Thrice) Mode

↓

EQN(1)

↓

3

↓

$$\begin{matrix} a_{11} & a_{12} & a_{13} & d_1 \\ a_{21} & a_{22} & a_{23} & d_2 \\ a_{31} & a_{32} & a_{33} & d_3 \end{matrix}$$

↓

= →

→ Cramer's rule

→ Calculator

→ Simultaneous eqn

$$\boxed{ax + by + cz = d}$$

Mode

↓

Matrix

↓

Dimension (3x3)

↓

Enter Values

MAT A

↓

dim - 3

↓

a_{11} → All Values

Det(MAT A)

Cramer's rule

$$\Delta = |R| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$\Delta I_1 = \begin{vmatrix} V_1 & a_{12} & a_{13} \\ V_2 & a_{22} & a_{23} \\ V_3 & a_{32} & a_{33} \end{vmatrix} \rightarrow I_1 = \frac{\Delta I_1}{\Delta}$$

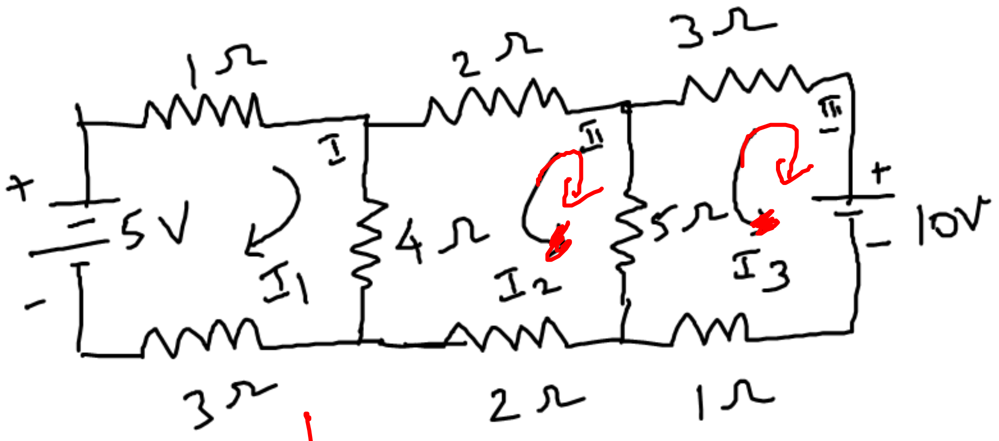
$\Delta I_2 =$

$\Delta I_3 =$

$$\boxed{\begin{aligned} I_1 &= 8.36\ A \\ I_2 &= 2.31\ A \\ I_3 &= -1.07\ A \end{aligned}}$$

Problem

- Find the mesh current and determine the power supplied by each of the voltage source in the given circuit.

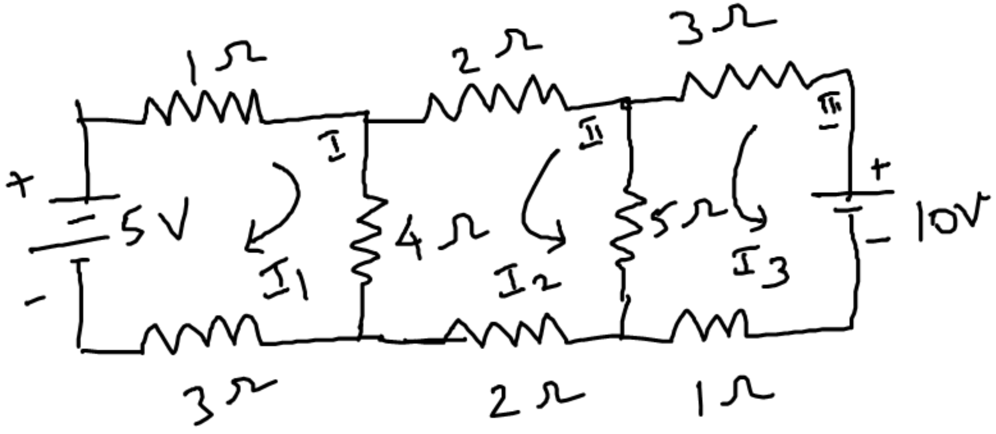


$$\begin{bmatrix} 8 & -4 & 0 \\ -4 & 13 & -5 \\ 0 & -5 & 9 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} +5 \\ 0 \\ -10 \end{bmatrix}$$

$$\boxed{\begin{aligned} I_1 &= 1.1\,A \\ I_2 &= 0.97\,A \\ I_3 &= 1.65\,A \end{aligned}}$$

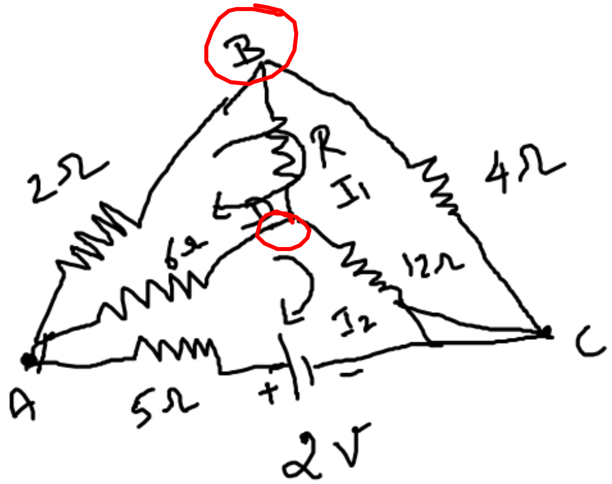
Problem

- Find the mesh current and determine the power supplied by each of the voltage source in the given circuit.

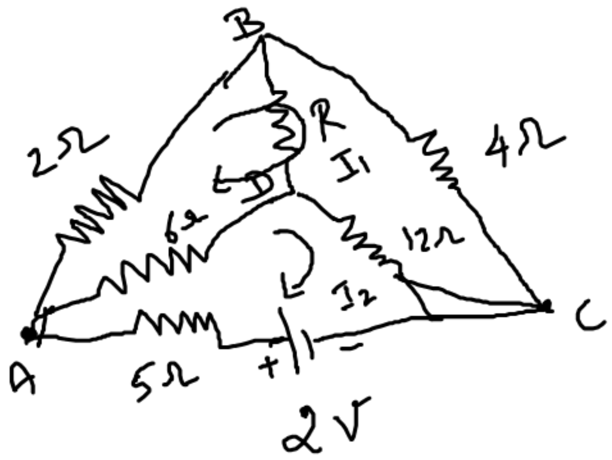


Exercise

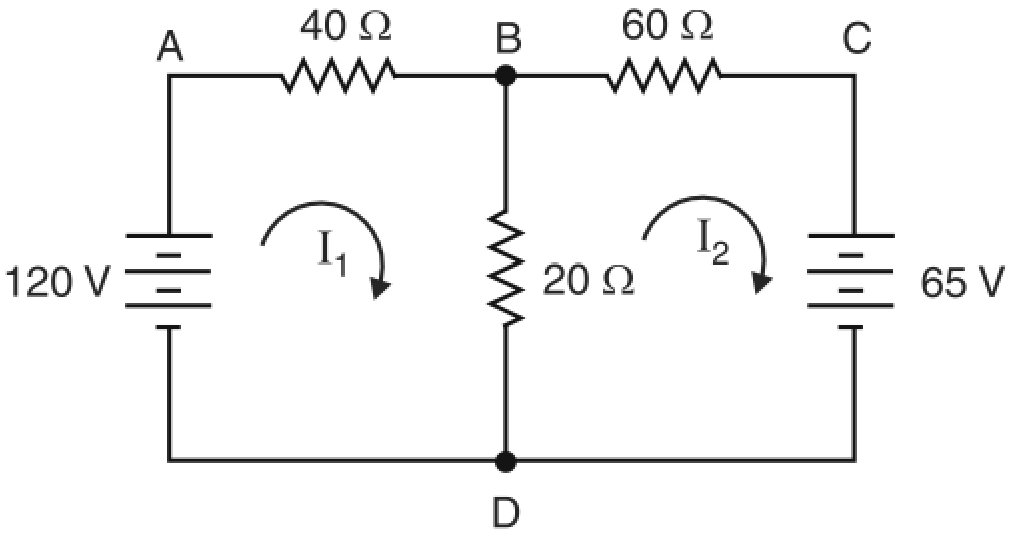
- Find the mesh currents in the given network when the current through the branch BD is zero.



- Find the mesh currents in the given network when the current through the branch BD is zero.



Cramer’s Rule



$$60I_1 - 20I_2 = 120$$
$$-20I_1 + 80I_2 = -65$$

$$I_1 = \frac{\begin{vmatrix} 120 & -20 \\ -65 & 80 \end{vmatrix}}{\begin{vmatrix} 60 & -20 \\ -20 & 80 \end{vmatrix}} = \frac{(120 \times 80) - (-65 \times -20)}{(60 \times 80) - (-20 \times -20)} = \frac{8300}{4400} = 1{\cdot}886 \text{ A}$$

$$I_2 = \frac{\begin{vmatrix} 60 & 120 \\ -20 & -65 \end{vmatrix}}{\text{Denominator}} = \frac{(60 \times -65) - (-20 \times 120)}{4400} = \frac{-1500}{4400} = -0{\cdot}341 \text{ A}$$

Advantages of Loop Analysis

- Solves directly for some currents
- Voltage sources are easy
- Current sources are either very easy or somewhat difficult
- Works best for circuits with few loops

Disadvantages of Loop Analysis

- Some currents must be computed from loop currents

- Does not work with non-planar circuits

- Choosing the supermesh may be difficult.

Summary

Circuit Analysis
- V
- I
- Toolchest → Mesh
  - steps
  - Inspection
    - Linear eq.
    - Cramer's ✓
    - Calc.

Exercise ✓