04/12/21 Saturday

K+1 or more objects are placed into K boxes then at least one box containing 2 or more objects.

GPHP: If N objects are placed into K boxes then there is atleast one box containing $\lceil N/K \rceil$ objects.

What is the least number of area codes needed to guarantee that the 25 million phones in a state can be assigned distinct 10-digit telephone numbers? (Assume that telephone numbers are of the form NXX-NXX-XXXX, where the first three digits form the area code, N represents a digit from 2 to 9 inclusive, and X represents any digit.)

0-9

2 to 9

$\boxed{NXX}$ - $\boxed{NXX - XXXX}$

Area code

8	10	10	10	10	10	10
N	X	X	X	X	X	X

$= 8{,}000{,}000$

$\boxed{25 \text{ million Phones}}$

8 million Phone numbers.

25m objects among 8 m boxes

$\left\lceil \frac{25000000}{8000000} \right\rceil = \lceil 3.025 \rceil = 4$

We need 4 area codes to guarantee that 25 m Phones in a state can be assigned different telephone numbers.

A1	A2	A3	A4
8m	8m	8m	

25m 1m

During a month with 30 days, a baseball team plays at least one game a day, but no more than 45 games. Show that there must be a period of some number of consecutive days during which the team must play exactly 14 games.

Sol:- Let a_j be the number of games played on or before jth day of the month.

Then $a_1, a_2, \ldots, a_{30}$ is an increasing sequence of '+'ve integers with $1 \le a_j \le 45$.

$1 \to 3$
$a_1 = 3$
$a_2 = 4$
$a_3 =$

$\Rightarrow a_1+14, a_2+14, \ldots, a_{30}+14$ is also an increasing sequence of '+'ve integers with $15 \le a_j+14 \le 59$

The 60 positive integers $a_1, a_2, \ldots a_{30}, a_1+14, \ldots a_{30}+14$ are all less than or equal to 59

$\Rightarrow$ By Pigeonhole Principle two of the integers are equal

$\Rightarrow$ There must be indices i, j s.t. $\boxed{a_i = a_j + 14}$

$\Rightarrow$ There is exactly 14 games are played b/w $j+1$ th day to ith day

Rough

60

1 2 3 4 --- 59

$\boxed{a_1, a_2, \ldots a_{30}+4}$

$\overbrace{a_1 \cdots a_{30}}^{30}$ $\overbrace{a_1+14, \cdots}^{30}$ $\boxed{a_{30}+14}$

$\boxed{60}$

59

$i = 15$
$j = 4$

No. of games played till 15th day = No. of games played till 4th day + 14