19/10/21 Tuesday

$72 = 8 \times 9 = 2^3 \cdot 3^2$

(S_{72}, D) $S_{72} = \{1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72\}$

Let $X = S_{72}$

$A = \{4, 6, 8, 12\}$

LUB

$4 \leq 4, 8, 12, 24, 36, 72$

$6 \leq 6, 12, 18, 24, 36, 72$

$8 \leq 8, 24, 72$

$12 \leq 12, 24, 36, 72$

$LUB(A) = 24$

GLB

$4 \geq 1, 2, 4$

$6 \geq 1, 2, 3, 6$

$8 \geq 1, 2, 4, 8$

$12 \geq 1, 2, 3, 4, 6, 12$

$GLB(A) = 2$

JOIN $\leftarrow$ LUB $\rightarrow$ $\vee, \sqcup, +, \oplus$

MEET $\leftarrow$ GLB $\rightarrow$ $\wedge, \sqcap, \cdot, \otimes$

LATTICE :-

A Poset $(X, \leq)$ is said to be a lattice if both $a \vee b$ and $a \wedge b$ exist for all $a, b \in X$.

$X = \{1, 2, 3, 5, 6, 10, 15\}$

$R = \{(a, b) : a / b\}$

(X, R) is a poset

$a = 2; b = 3$

$2 \leq 2, 6, 10$

$3 \leq 3, 6, 15$

$2 \vee 3 = 6$

$2 \geq 1, 2$

$3 \geq 1, 3$

$2 \wedge 3 = 1$

$GLB\{3, 10\} = 1$

$LUB\{3, 10\}$ does not exist

$\therefore (X, R)$ is not a lattice

Bounded Lattice

A lattice $(L, \leq)$ is said to be a bounded lattice if it has both least element and greatest element.

$(\mathbb{Z}, \leq)$

$1 \leq 1, 2, 3, 4, 5$

$2 \leq 2, 3, 4, 5$

$3 \leq 3, 4, 5$

$4 \leq 4, 5$

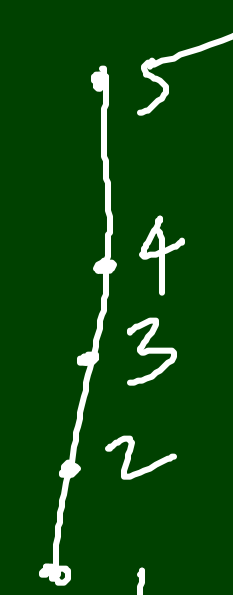
$5 \leq 5$

$(\{1, 2, 3, 4, 5\}, \leq)$

chain

a, b

$a \leq b$ or $b \leq a$



usual LESS THAN OR EQUAL TO RELATION

$(\mathbb{Z}^+, \leq)$ usual less than or equal to

least element $= 1$

$(\mathbb{Z}^+ \cup \{0\}, \leq)$

least element $= 0$

$(\mathbb{Z}^-, \leq)$

Greatest element $= -1$

Complement:-

Let $(L, \leq)$ be a bounded lattice. Then an element $a \in L$ is said to have complement if $\exists\, b \in L$ s.t. $a \wedge b = 0$ and $a \vee b = 1$

Ex:- (S_{24}, D) $\quad S_{24} = \{1, 2, 3, 4, 6, 8, 12, 24\}$

$1 \to 0$, $24 \to 1$

$1' \to 24$

$2' \to$ does not have any complement

$3' \to 8$

$4' \to -$

$6' \to -$

$8' \to 3$

$12' \to -$

$24' \to 1$

A'

$2 \wedge 3 = 1$ (0)

$2 \vee 3 = 6$

$0' = 1$

$1' = 0$

$x \wedge y$ (and) $\to 0$ $\quad$ $x \vee y$ (&) $\to 1$

$x' = y$

$y' = x$

Complemented Lattice:-

A Bounded lattice $(L, \leq)$ is said to be a complemented lattice if each and every element of L has atleast one complement.

Boolean algebra :-

A complemented distributive lattice is known as Boolean algebra

$a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$

$$a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$$

<u>Note</u> :- In Boolean algebra the complement is unique

Algebraic System or Algebraic Structure or Algebra

$X \to$ one or more n-ary operations

<u>H.W</u> :- Find the complements of all the elements of the following lattices.

(1) (S_{12}, D) (2) (S_{30}, D) (3) (S_{48}, D)

(4) (S_{18}, D) (5) (S_{36}, D)
