CSE211 – Formal Languages and Automata Theory

Unit 1-L5: Grammars and Dérivations
Problems Part 2

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Agenda

- Recap-Grammar and Derivation
- Formal definition of grammar
- Production rule –def.
- Formal definition of language
- How to find Language generated by grammar?
- How to find Grammar to generate language?

Formal Definition: Grammar

- A grammar G is defined as a quadruple

$$G = (V, T, S, P),$$

where V is a finite set of objects called variables,

T is a finite set of objects called terminal symbols,

$S \in V$ is a special symbol called the start variable,

P is a finite set of productions.

- It will be assumed that the sets V and T are nonempty and disjoint.

Production Rule

- The heart of a grammar
- All production rules are of the form

$$\boxed{x \to y}$$

  - where x is an element of $(V \cup T)^+$ and
  - y is in $(V \cup T)^*$
- Based on the restriction of variables and terminals the on the left or right side of the production it can be
  - Regular
  - Context-free,
  - Context-sensitive or
  - Unrestricted grammar

Formal Def.: Language

- Let G = (V, T, S, P) be a grammar. Then the set,

$$L(G) = \{w : S \overset{*}{\Rightarrow} w, \quad w \in T^*\}$$

is the language generated by G.

$\boxed{S \to \varepsilon}$

- If w ∈ L (G), then the sequence

$$S \Rightarrow \overset{VT}{w_1} \Rightarrow \overset{VT}{w_2} \Rightarrow w_3 \Rightarrow w_4 \ldots\ldots\ldots \Rightarrow \boxed{w}$$

is a derivation of the sentence w. The strings

S, w1, w2,…, wn,

which contain variables as well as terminals, are called sentential forms of the derivation.

Find the grammar

$L = \{a^n : n \geq 0\}$

soln: $\Sigma = \{a\}$. $n = 0, 1, 2 \ldots\ldots$

$L = \{\varepsilon, a, aa, aaa \ldots\ldots\}$

$\varepsilon \to a \cup aa$

$a \cap aa$

Let the starting symbol is S

(i) Minimum string is ε, so

$S \to \varepsilon$

(ii) strings are multiples of 'a'

$S \to aS$

∴ The production set is

$P = \{ S \to \varepsilon$
$S \to aS \}$

The Grammar is

$G = \{V, T, P, S\}$

$= \{\{S\}, \{a\}, S, \{S \to \varepsilon, S \to aS\}\}$

Find the Grammar-Problems

aaa aa ε

$L = \{ a^n : n \geq 0 \} = \{ \varepsilon, a, aa, aaa, \dots \}$

$\Sigma = \{ a \}$

$G = (V, T, S, P)$

P

$S \to \varepsilon$
$S \to aS$

$T = \{ a \}$

$V = \{ S \}$

$S \Rightarrow aS$
$\Rightarrow a\varepsilon$
$\Rightarrow a$

$S \Rightarrow aS$
$\Rightarrow aaS$
$\Rightarrow aa\varepsilon$
$\Rightarrow aa$

$S \Rightarrow aS$
$\Rightarrow aaS$
$\Rightarrow aaaS$
$\Rightarrow aaa\varepsilon$
$\Rightarrow aaa$

$L = \{ a^n b^m : n \geq 1, n \geq 0 \}$

$S \to a$
$S \to aSb$

a
ab
aaab

aSb
aaSbb

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Find the grammar for

$L = \{ a^n : n \text{ is even} \}$

soln: $\Sigma = \{a\}$, $n = 0, 2, 4, 6 \ldots$ a^n

$L = \{ \epsilon, aa, aaaa, aaaaaa \ldots \} = (aa)^*$

we have grammar rule for a^* as $S \rightarrow aS/\epsilon$

∴ The productions for $(aa)^*$ is $S \rightarrow aaS/\epsilon$

∴ The grammar for the given language is

$G = \{ \{S\}, \{a\}, S, \{ S \rightarrow aaS, S \rightarrow \epsilon \} \}$

Find the grammar for

$L = \{ a^n : n \text{ is odd} \}$

Solution

$\Sigma = \{a\}$, $n = 1, 3, 5, 7, 9, 11 \ldots$

$L = \{a, aaa, aaaaa, \ldots\} = a(aa)^*$

For $(aa)^*$ we have written $P = \{ S \to aaS / \epsilon \}$

$\therefore$ The productions for the given language are

$S \to aA$

$A \to aaA / \epsilon$

$\boxed{S \to aaS / a}$

$\therefore$ The grammar for the given language is

$G = \{ \{S, A\}, \{a\}, S, \{ S \to aA, A \to aaA / \epsilon \} \}$

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Find the grammar for

$L = \{ a^n b^m : n \geq 1, m \geq 0 \}$

Solution

$\Sigma = \{a, b\}$, $n = 1, 2, 3 \ldots$ $m = 0, 1, 2, 3 \ldots$

$L = \{ a, ab, aa, aab, aabb, \ldots \}$

a^n | b^m $S \to AB$

with min one a — with min zero b

to be concatenated

Let starting symbol is S

Each string has two parts

so, $S \to AB$ → for b's

A → for a's

$A \to$ no of a's with min 1

$\therefore A \to aA / a$

$B \to$ no of b's with min 0

$\therefore B \to bB / \varepsilon$

$\therefore$ The total production set is

$P = \{ S \to AB$

$A \to aA / a$

$B \to bB \mid \varepsilon \}$

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Find the grammar for

$L = \{ a^n b^m : (n+m) \text{ is even} \}$

Solution

$\underline{a^n} \quad \underline{b^m}$

To be concatenated and total should even

$S \to AB$

$L = \{ \varepsilon, ab, abbb, aabb, aaab, aa, bb, aaaa, bbbb, \ldots \}$

From the list we can understand that no of a's and b's are either even or odd, but total should be even

ie: $a^n \quad b^n$

even + even = even

(or) odd + odd = even

Let the starting symbol is 'S'

$\therefore S \to AB \mid aAbB$

$A \to aaA \mid \varepsilon$

$B \to bbB \mid \varepsilon$

$\therefore P = \{ S \to AB \mid aAbB$

$A \to aaA \mid \varepsilon$

$B \to bbB \mid \varepsilon \}$

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① $L = \{ a^n b^m : (n+m) \text{ is odd} \}$

$P = \{ S \to AbB \mid aAB$

$A \to aaA \mid \varepsilon$

$B \to bbB \mid \varepsilon \}$

$S \to AbB \to \varepsilon b \varepsilon = b$

$S \to aAB \to a\varepsilon B \to a\varepsilon\varepsilon = a$

$L = \{ a, b,$

$S \Rightarrow AbB \Rightarrow aaAbB \Rightarrow aa\varepsilon bB = aab\varepsilon = aab$

abb

aabbb

aaabb

② $L = \{ a^n b^m : n \geq 3, m \geq 2 \}$

$S \to aaaAbbB$

$A \to aA \mid \varepsilon$

$B \to bB \mid \varepsilon$

aaabb

$S \to AB$

$A \to aA \mid aaa$

$B \to bB \mid bb$

aaabb

aaaabb

aaaaabbb

Find the G

$L = \{ \underbrace{a^n b^n}_{A} \underbrace{c^m d^m}_{B} : n, m \geq 0 \}$

$a^n b^n$

$S \to aSb / \varepsilon$

$S \to AB$

$A \to aAb / \varepsilon$

$B \to cBd / \varepsilon$

$L = \{ \varepsilon, ab, cd, abcd, aabbcd, aabbccdd \ldots \}$

$S \to abS / \varepsilon$

$\Sigma = \{a, b, c, d\}$, $n = 0, 1, 2 \ldots$, $m = 0, 1, 2 \ldots$

The grammar for the given language

$G = \{ V, T, S, P \}$

$V = \{ S, A, B \}$

$T = \{ a, b, c, d \}$

$S = S$

$P = \{ S \to AB,$
$A \to aAb \mid \varepsilon$
$B \to cBd / \varepsilon$

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$L = \{ \overbrace{a^n \underbrace{b^m c^m}_{A}\, d^n}^{B} : m, n \geq 1 \}$

$a^n b^n : n \geq 1$

$S \to aSb / ab$

$L = \{ abcd, aabcdd, abbccd, \ldots \}$

$G = \{ S \to aAd / aSd$
$A \to bAc / bc$

$S \Rightarrow aAd$
$\Rightarrow abcd$

$S \Rightarrow aSd$
$\Rightarrow aaAdd$
$\Rightarrow aabcdd$

$S \Rightarrow aAd$
$\Rightarrow abAcd$
$\Rightarrow abbccd$

$S \to aSd / \cancel{aAd}$

$S \to aAd$

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Find the grammar that generates the strings of a's & b's starts and ends with same letter

$L = \{a, b, aa, bb, aaa, bbb, aba, bab, baabaab, aabbba, \ldots\}$

General form $a(a/b)^*a \mid b(a/b)^*b \mid a \mid b$

$S \rightarrow aAa \,/\, bAb \,/\, a \,/\, b$

$A \rightarrow aA \,/\, bA \,/\, \epsilon$

$S \Rightarrow aaa$

$S \Rightarrow aAa$
$\Rightarrow aaAa$
$\Rightarrow aa\epsilon a$
$\Rightarrow aaa$

aba

$S \Rightarrow aAa$
$\Rightarrow abAa$
$\Rightarrow ab\epsilon a$
$\Rightarrow aba$

Find the Grammar

- Find the grammar that generates

$$L = \{0^n 1^n : n \geq 0\}$$

$$S \to 0S1$$

$$S \to \lambda$$

Find the Grammar

- Find the grammar that generates

$$L = \{a^n b^{n+1} : n \geq 0\}$$

S -> Ab
A -> aAb
A -> ε

Find the Grammar

- Let $\sum = \{a, b\}$, and let $n_a(w)$ and $n_b(w)$ denote the number of a's and b's in the string w, respectively. Find the grammar G which generates

$$L = \{w : n_a(w) = n_b(w)\}$$

S -> SS
S -> aSb
S -> bSa
S -> ε

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Summary

- Why Grammars for a language
- What is a grammar?
- What is derivation?
- What is sentential form?
- Language generated by grammar
- Grammar to generate language

References

- John E. Hopcroft, Rajeev Motwani and Jeffrey D. Ullman, *Introduction to Automata Theory*, Languages, and Computation, Pearson, 3rd Edition, 2011.
- Peter Linz, An Introduction to Formal Languages and Automata, Jones and Bartle Learning International, United Kingdom, 6th Edition, 2016.

Next Class:

Types and Chomsky hierarchy of Grammar

THANK YOU.