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Autonomous Mobile Robots
MCT 308

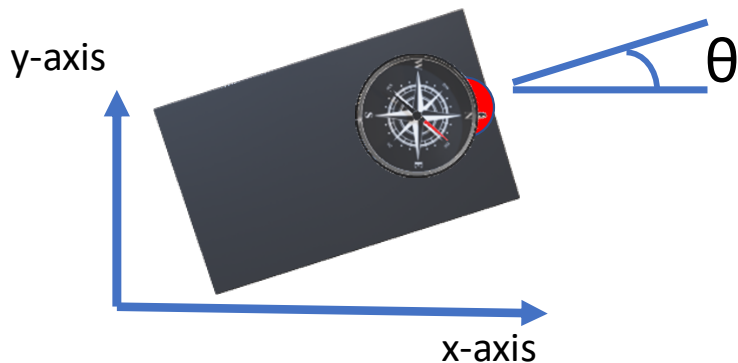
Autonomous Mobile Robots

Module 12: Heading Sensors

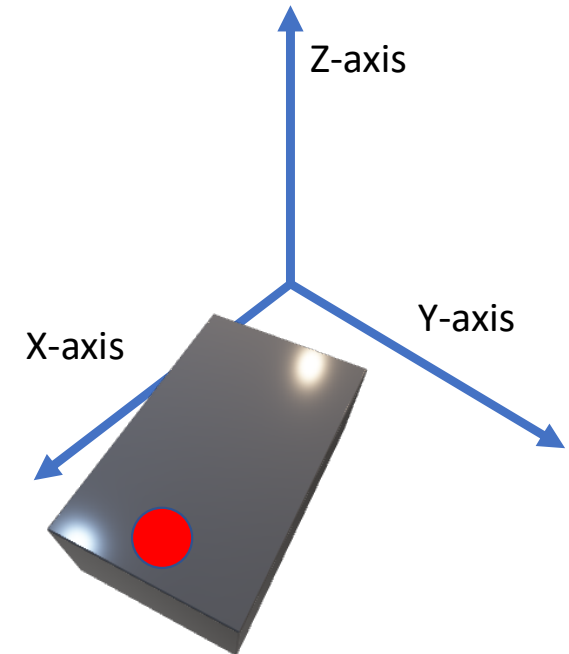
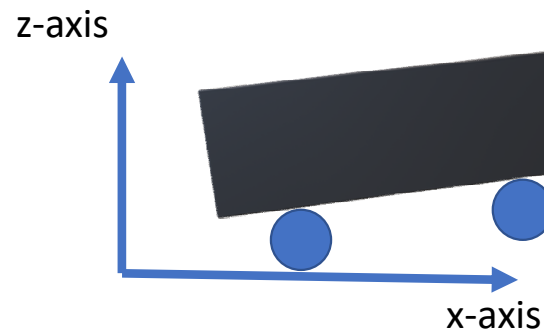
Heading Sensors

- Heading sensors are used to determine the robot's orientation (in which direction is the robot heading) and inclination
- This can be treated as a proprioceptive or exteroceptive sensor
- Heading sensors can be of two types: Compasses and Gyroscopes
- Robot can have rotation about x-axis, y-axis or z-axis

Top-view: rotation about z-axis



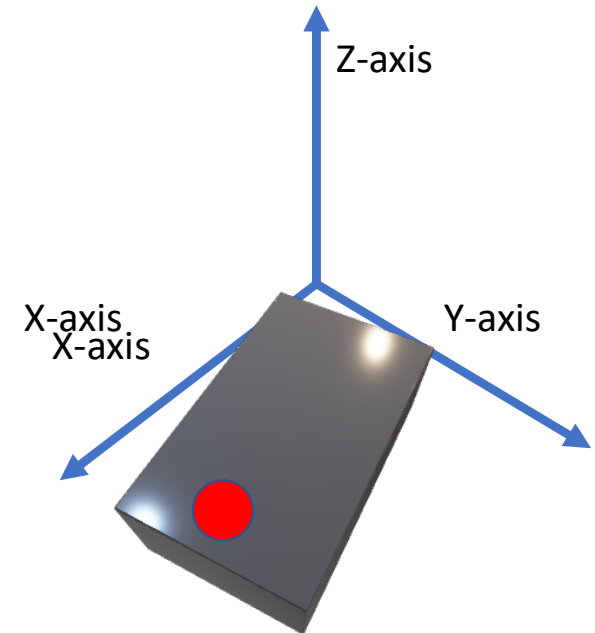
Side-view: rotation about x-axis





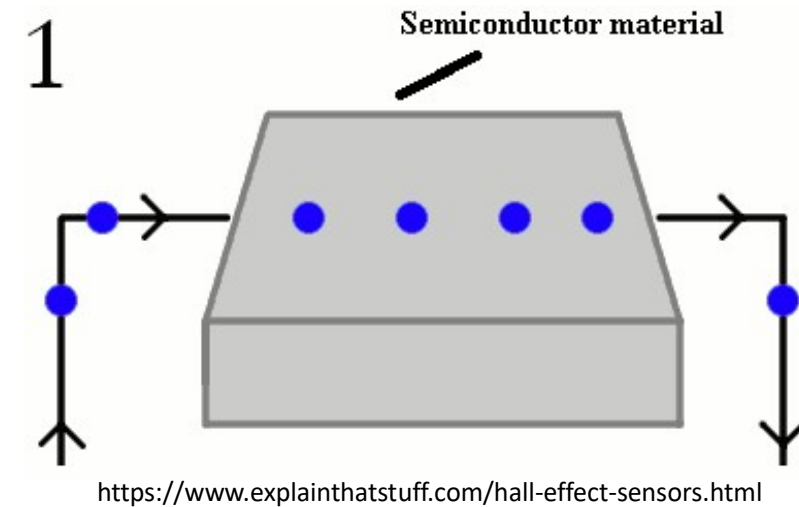
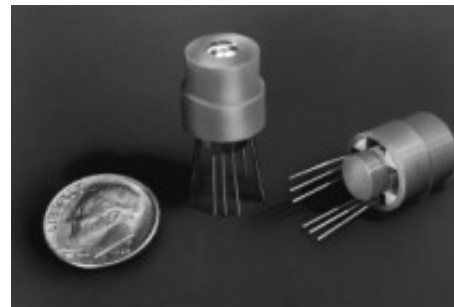
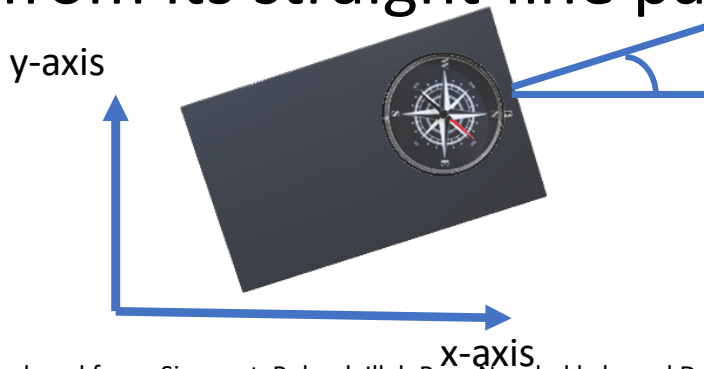
Heading Sensors - Compass

- A compass measures the direction of the magnetic field and uses this measurement to determine the orientation of the robot
- Two common sensors for measuring direction of magnetic field are
 - Hall-effect compasses
 - Flux-gate compasses

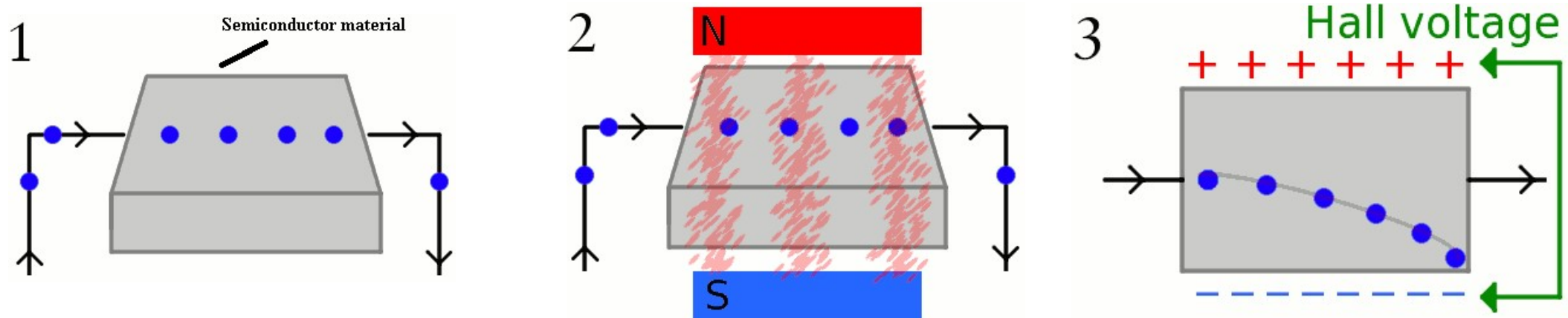


Heading Sensors – Compass – Hall Effect

- Hall effect describes the behavior of electric potential in a semiconductor when it is placed in a magnetic field
- When a beam of charge particles passes through a magnetic field, forces act on the particles and the current beam is deflected from its straight-line path.



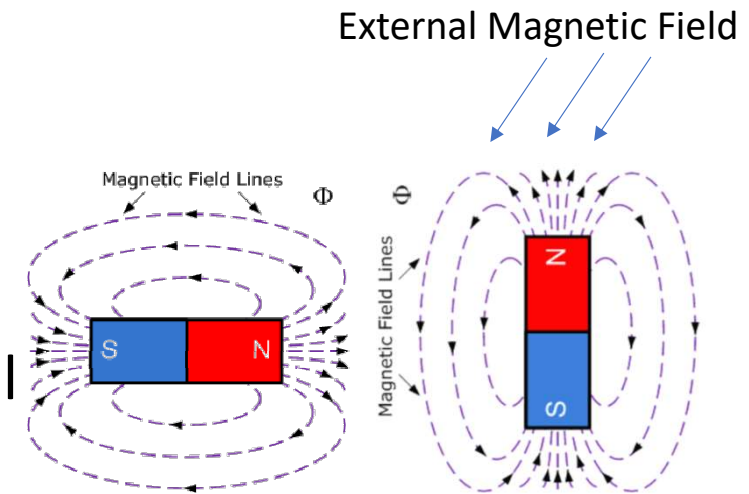
Heading Sensors – Compass – Hall Effect



- Thus one side of the material will become negatively charged and the other side will be of positive charge. This charge separation generates a potential difference.
- The voltage potential depends on the relative orientation of the semiconductor to magnetic flux lines
- Sensor is compact, light
- Resolution can be poor, filtering may be necessary

Heading Sensors – Fluxgate Compass

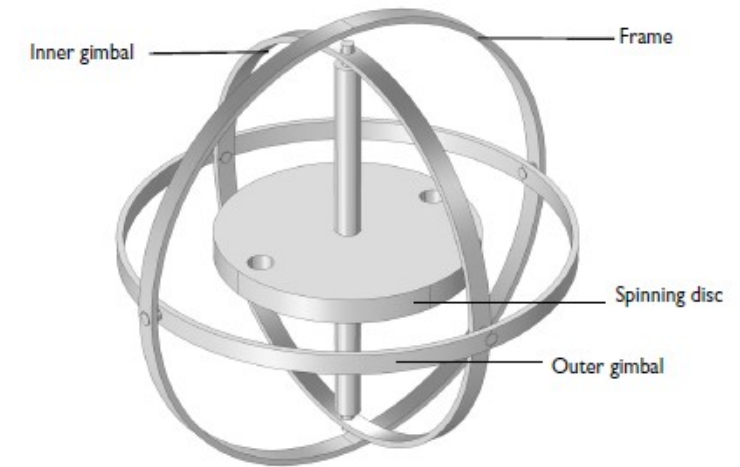
- Two small coils are wound on ferrite cores and fixed perpendicular to each other
- When AC current is passed through both coils, external magnetic field causes shifts in the phase depending on the relative alignment with each coil
- By measuring both phase shifts, direction of the magnetic field in two dimensions can be computed
- FGC has improved accuracy and resolution compared to Hall Effect compass
- FGC is larger and more expensive, however





Heading Sensors - Gyroscopes

- Gyroscopes are heading sensors that preserve their orientation in relation to a fixed reference frame
- It provides an absolute measure for heading of a mobile robot
- It consists of a fast-spinning rotor, which will try to maintain its orientation to conserve angular momentum
- The spinning axis of gyroscope is to be selected based on the desired axis of orientation
- By arranging the spinning wheel along desired axis of rotation, no torque will be transmitted from the outer pivot to wheel axis
- Using the relative displacement between the fixed wheel axis and rotating outer pivot, the orientation angle of the robot can be determined.





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Thanks!



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Autonomous Mobile Robots

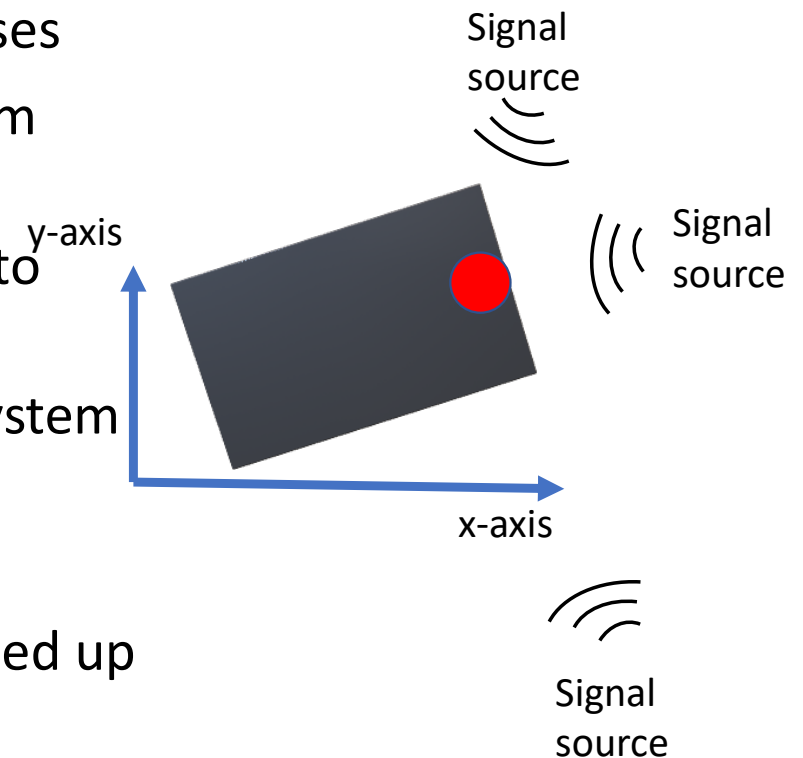
Module 13: Ground-based Beacons



Ground-based beacon

- Basic idea: inspired from stars, mountains, lighthouses
- Requires robot to carry a sensor to detect signal from beacon
- Uses interaction of the on-board sensor & beacons to enable the robot to identify its position precisely
- One such beacon is the GPS or Global Positioning System
- GPS is good only for outdoors though, indoors is a challenge
- Developed for military purposes initially, later opened up for civilian use
- 24 operational GPS satellites orbit at a height of 20.19 km.
- Emit EM waves which are received by the GPS receiver

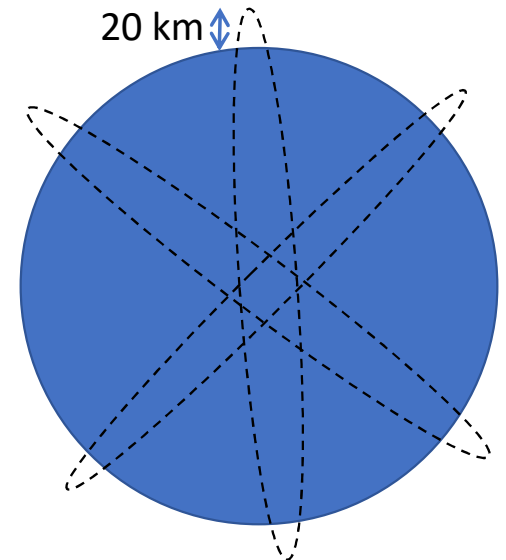
Top-view: rotation about z-axis





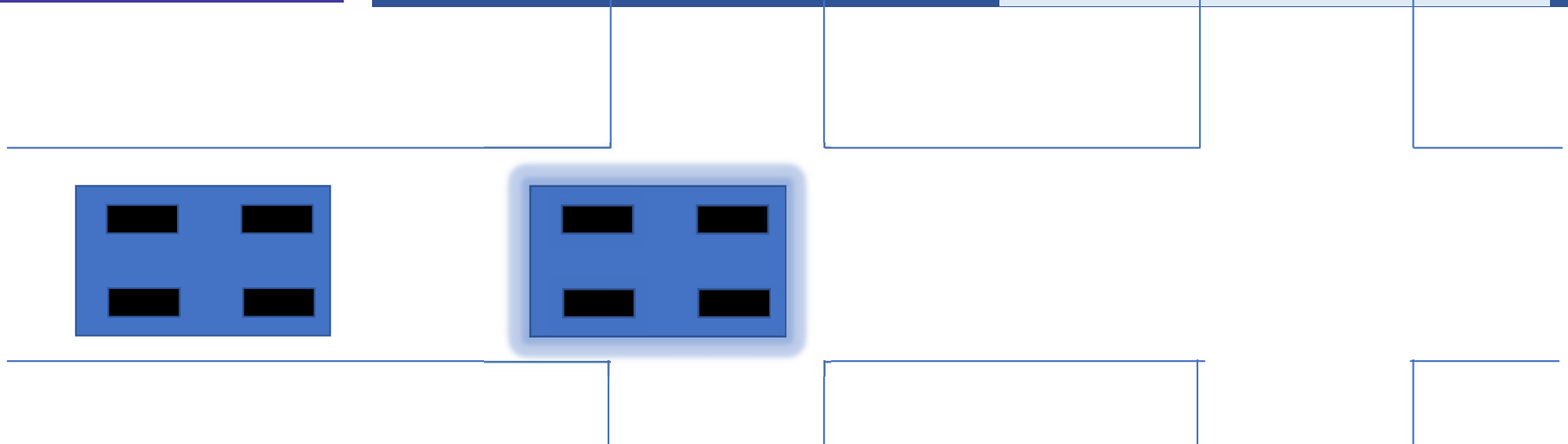
GPS

- 6 orbits that are spaced at 55 degrees from each other
- 4 satellites are present in each orbit
- Orbiting happens at around 20 km from the surface of the earth
- Satellite continuously transmits its location and current time
- GPS receiver receives signals from multiple satellites simultaneously
- The GPS is a passive sensor since it only receives signals from satellite
- The sensor combines information regarding arrival time and instantaneous location of 4 satellites
- Receiver infers its position from the above data using trilateration or triangulation
- GPS can give updates only every 200-300 ms
- On fast moving robots, faster updates are needed





GPS

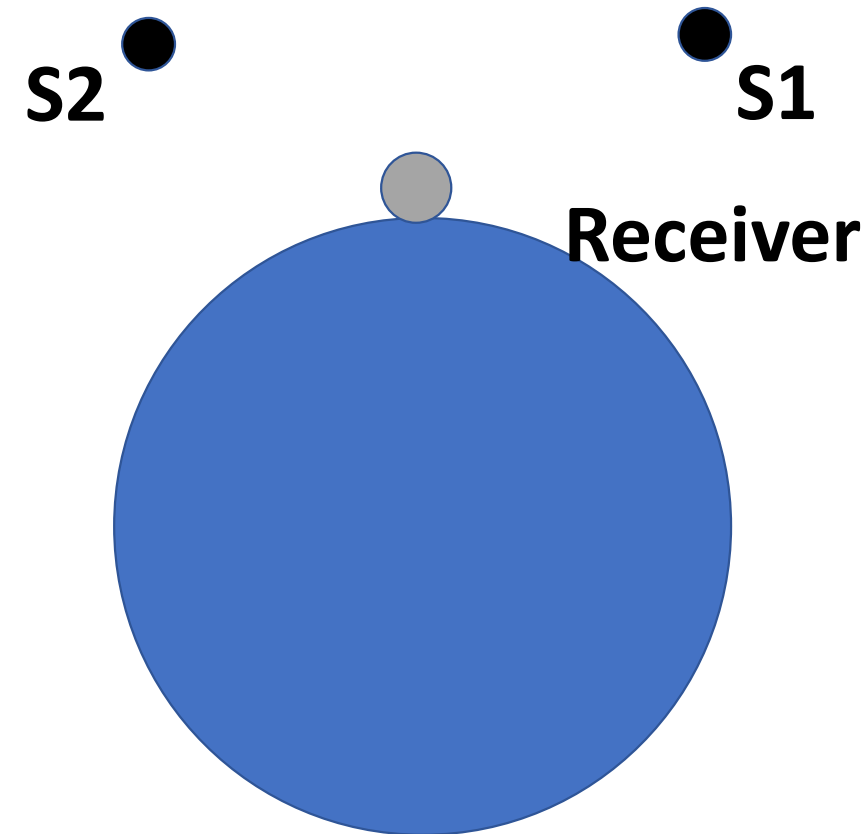


- Fast moving robots require fast updates, GPS may not be suitable
- Pedestrians, road blocks, buildings are all obstacles with respect to the robot
- If it takes 300 ms for update, and if robot moves at 1 m/s, it would have moved 30 cms without update
- If robot moves at 15 m/s (54 kmph), it would have moved 450 cms or 4.5 m (around 15 ft) without update



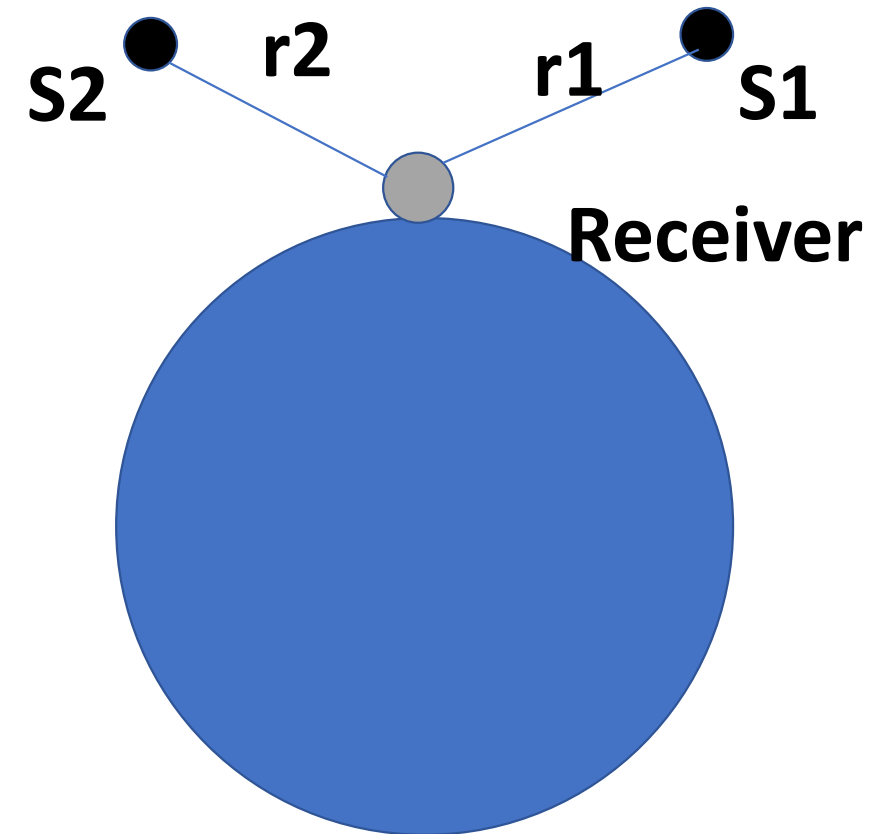
GPS Operation - Planar Explanation

- Consider the earth as a 2D circle, S1 and S2 are locations of two satellites
- Receiver is located on the circumference of the circle
- Both S1 and S2 emit signals which are read by the Receiver
- Let velocity of transmission of the signal be 'C'
- Let time of receiving signals be marked as t_{recv}
- Time of transmission of the signal is t_{r1} and t_{r2}
- The data sent by the satellite is this time information along with its location
- Time taken for signal from S1 to be received by sensor is $t_{\text{recv1}} - t_{r1}$



GPS Operation - Planar Explanation

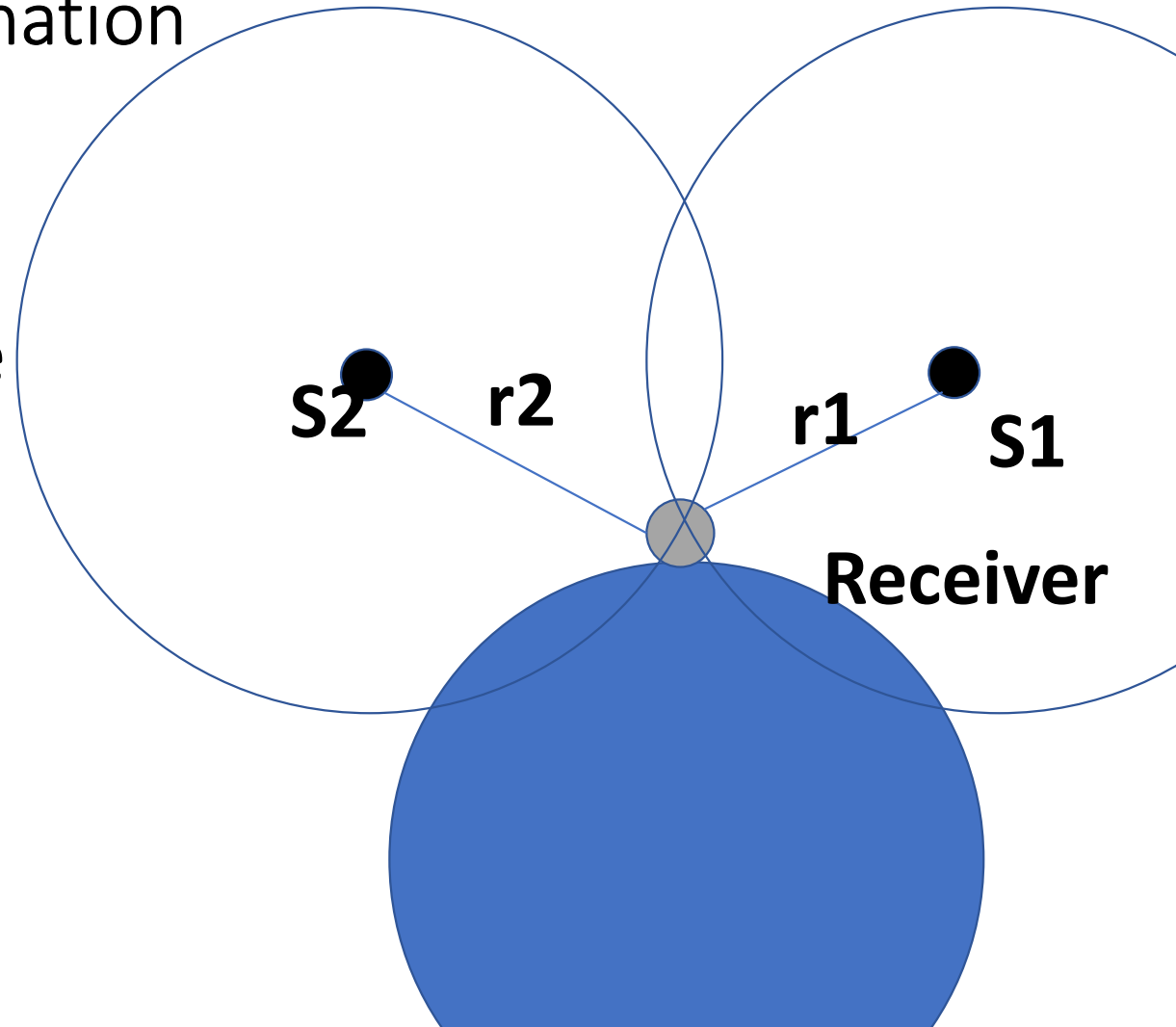
- Time taken for signal to be received by sensor is $t_{\text{recv1}} - t_{r1}$
- Above time difference is the time taken to travel the distance $r1$ with speed C
- Similarly for satellite $S2$, $t_{\text{recv2}} - t_{r2}$ is the time take for traveling a distance $r2$ with speed C
- Distance to $S1$: $C * (t_{\text{recv1}} - t_{r1})$
- Distance to $S2$: $C * (t_{\text{recv2}} - t_{r2})$





GPS Operation - Planar Explanation

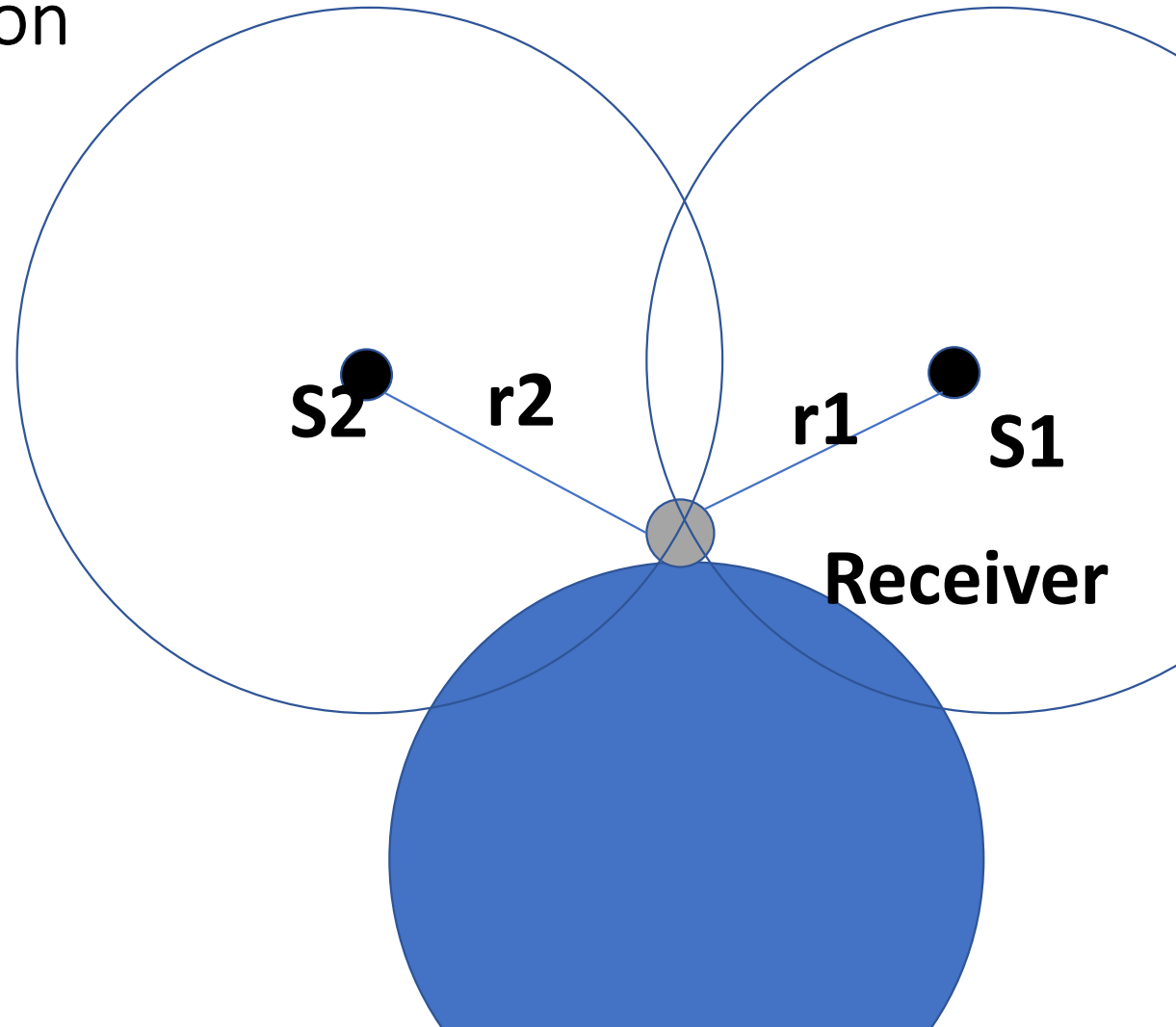
- Based on the location information of the Satellite and the distance estimate r_1 and r_2 , two circles can be drawn keeping the location of S_1 and S_2 as centers
- The points of intersection are found and based on the condition that the intersecting point has to lie on the earth, the correct location is determined



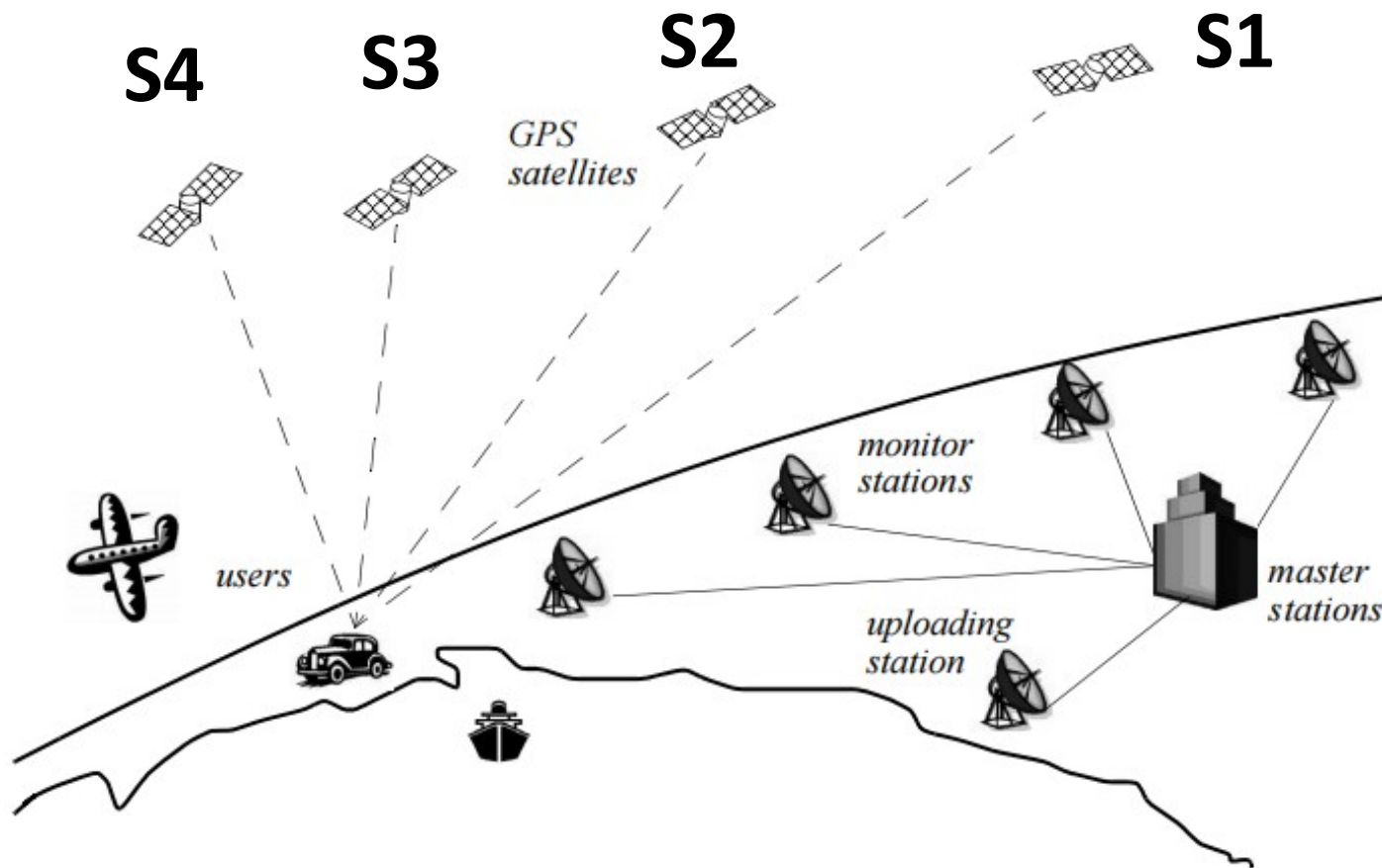


GPS Operation - 3D Explanation

- Need to consider at least 3 satellites
- A sphere is drawn from each satellite and the point of intersection of all 3 spheres along with the surface of the earth will be the location of the receiver



GPS Operation - Applications



- Autonomous cars make use of this GPS information for planning paths and tracking
- Drones make use of GPS to reach specific waypoints
- Package delivery robots operating on the streets similarly make use of GPS for path planning and tracking



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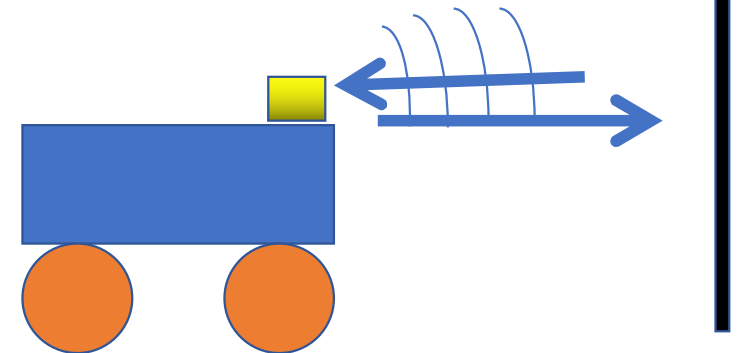
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Module 14: Active Ranging Sensors



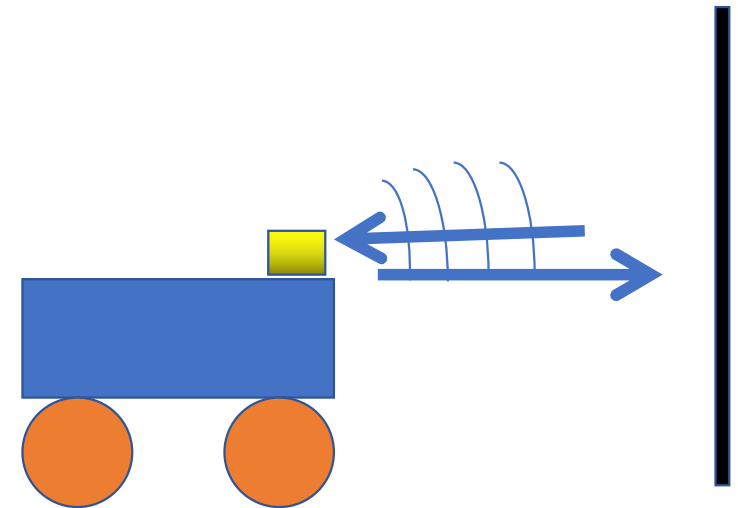
Active Ranging Sensors

- Ranging implies measuring distance, say to an object
- Active ranging implies a signal is emitted from the sensor and reflected signal is used to measure distance
- Used for obstacle detection and avoidance
- This is also known as Time of flight Active Ranging (ToFAR) since time between transmission and reception is made use of to determine range
- A sensor operating on this principle can be classified as either Ultrasonic sensor or Laser rangefinder



Active Ranging Sensors

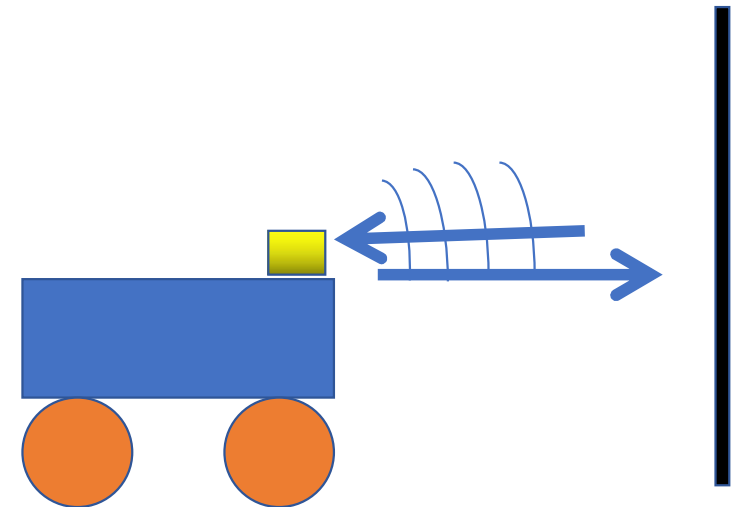
- Geometric active ranging is another type of active ranging based on triangulation
- ToFAR makes use of propagation speed of sound of EM wave
- To estimate the distance to the object, we need to determine the distance travelled by the wave
- If the object is at a distance 'd' from the sensor, the wave travels a distance '2d'.
- Then, $2d = c \cdot t$, where 'c' is speed of the wave and 't' is the time of flight
- Speed of sound wave is $\sim 340 \text{ ms}^{-1}$
- Speed of EM wave is $3 \times 10^8 \text{ ms}^{-1}$





Active Ranging Sensors

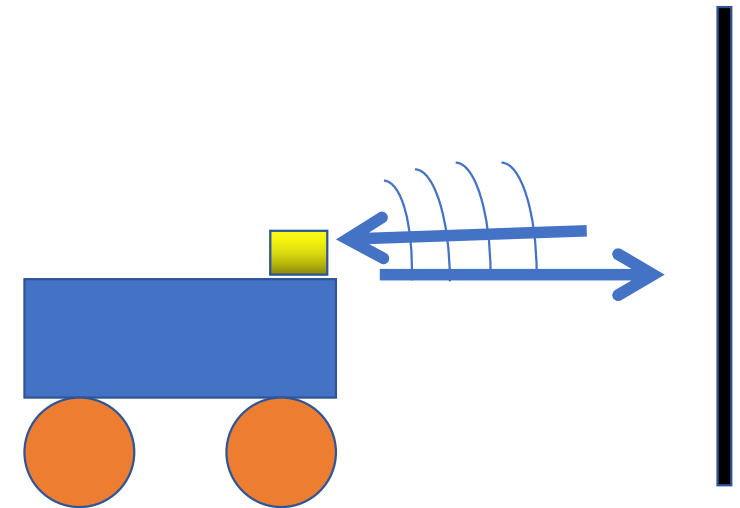
- Ex: For an object at a distance of 1.5 m from the robot, total distance travelled by the wave is 3m
- Ultrasonic sensor will take 10 ms
- Laser rangefinder will take 10 ns
- Laser rangefinder (LRF) is capable of providing very fast updates, but can be more expensive compared to ultrasonic sensors





Ultrasonic sensors

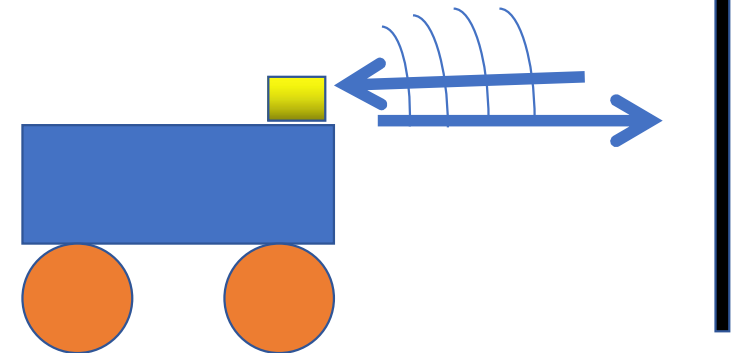
- An ultrasonic sensor is a Time of Flight, sound-based active sensor
- It transmits an ultrasonic pressure wave
- It measures the time take for the wave to reflect and return to the receiver
- The total distance travelled by the wave = $2d = c * t$
- Distance to the object is $d = c * t / 2$
- Time of flight = $2d / c$
- To transmit sound, it is necessary to create a vibration using a transducer
- There is a need to wait for a duration for the transducer to settle down before making a measurement





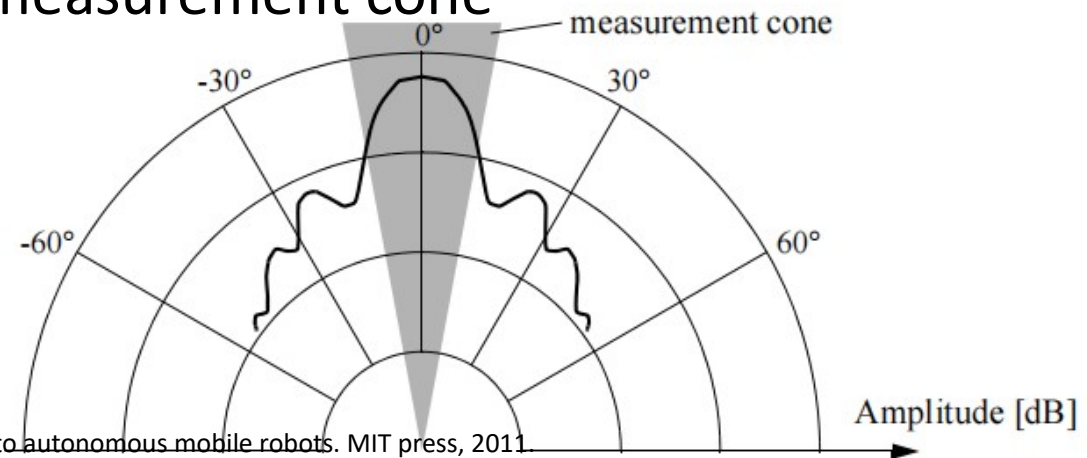
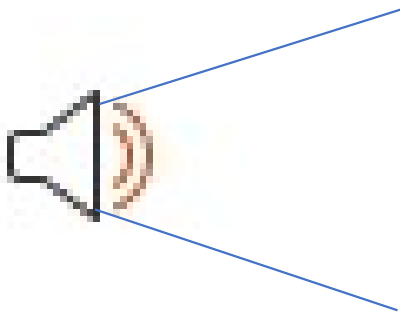
Ultrasonic sensors

- If an object is very near to the transducer, transmitted sound reflects and returns to sensor before transducer settles down, therefore making the reading unusable
- Most ultrasonic sensors have a range roughly between 12 cm to 5 m
- The resolution of these sensors is around 2 cm
- Accuracy of these sensors is $> 90\%$
- The operating frequency of this type of sensor is 40 kHz - 180 kHz
- The speed of operation of an ultrasonic sensor is a disadvantage and slower compared to LRF
- For a distance of 3 m, time taken by an ultrasonic sensor to make a measurement is 10 ms
- If the object or the robot is moving fast, the actual distance will change before the reading the updated



Ultrasonic sensors

- In a manner similar to a flashlight, an ultrasonic sensor produces beams which occupy an angle
- With a narrower beam, more precise information may be obtained regarding objects
- The opening angle for an ultrasonic sensor is around 20-40 degrees
- Because of this, the sensor is only able to tell us that there is an object at a certain distance within the area of the measurement cone





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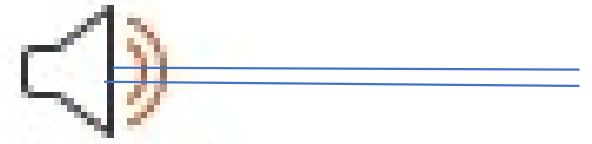


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Laser Rangefinder

- The laser rangefinder (LRF) provides significantly better performance in comparison to an ultrasonic sensor
- The transmitter sends out an illuminated beam (laser)
- The receiver needs to be capable of detecting reflected light
- The sensor is usually referred to as LiDAR (Light detection and ranging)
- The beam produced by a laser is highly focussed and narrow in nature





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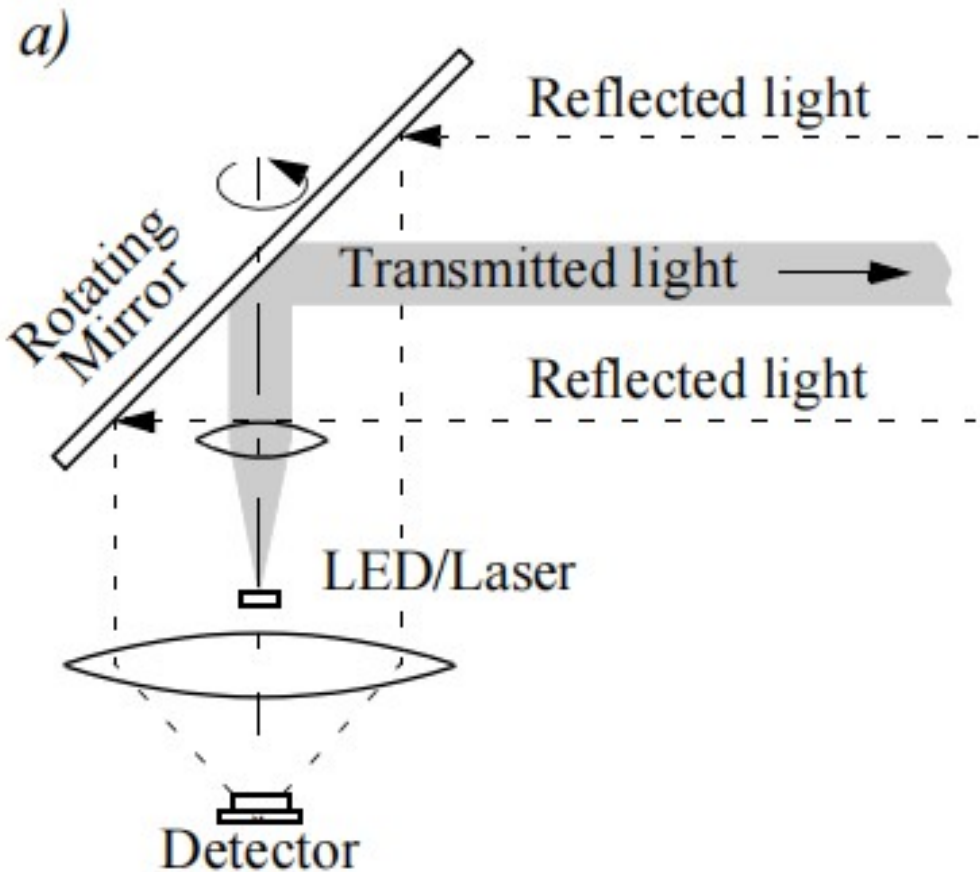
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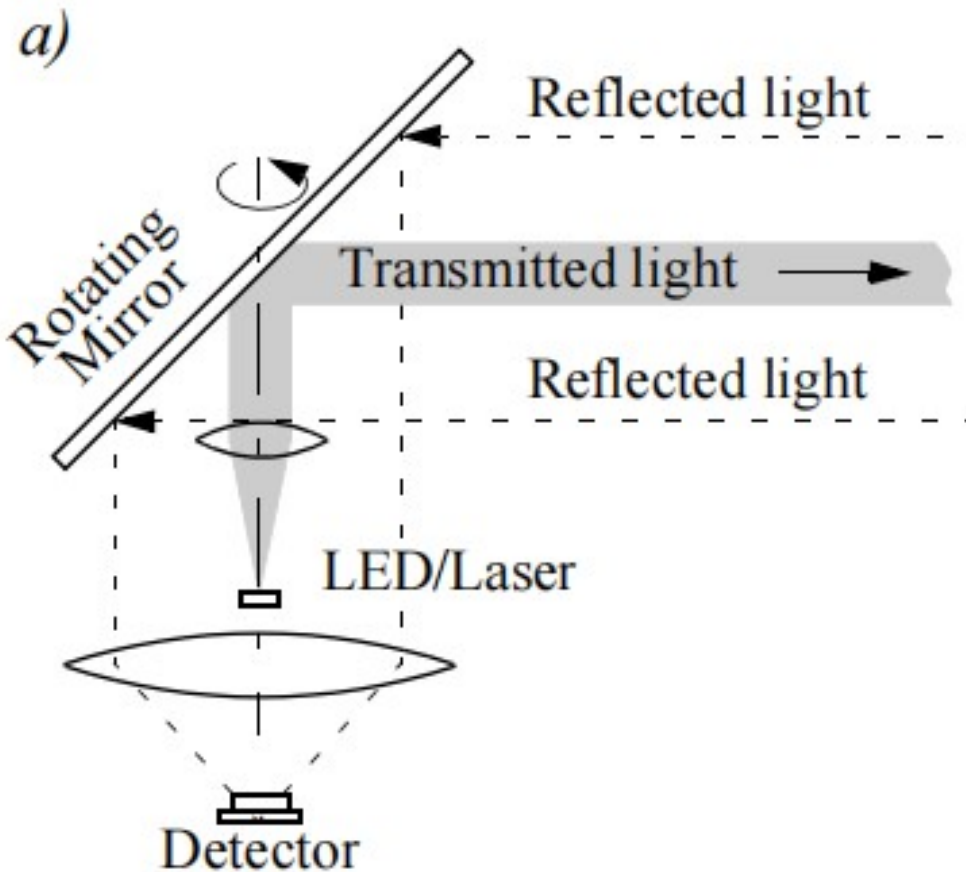
Laser Rangefinder

- The light direction can be modified by using a mirror which can be rotated about a suitable axis
- Because of the high speed of the laser light wave, the electronics must be capable of resolving picosecond level information and this leads to an increase in cost of the sensor
- The angular resolution of a LiDAR is much better than the ultrasonic sensors and can be lower than 0.5 degree
- The range of an LRF is around 5 cm to 10 m
- The linear resolution can be around 1 cm
- The update rate can be greater than 200 Hz
- The sensor may not work with optically transparent objects such as glass

Scanning Laser Rangefinder



Scanning Laser Rangefinder



- A mechanism with a mirror sweeps the light beam to cover the required scene in a plane or even in three dimensions, using a rotating, nodding mirror.
- Example specs of Sick LMS 200
- Angular resolution : 0.25 degree
- Depth resolution: ~ 15 mm
- Accuracy : 35 mm
- Range : 5 cm to 20+ m
- Sampling rate: Seventy five 180-degrees scans per second



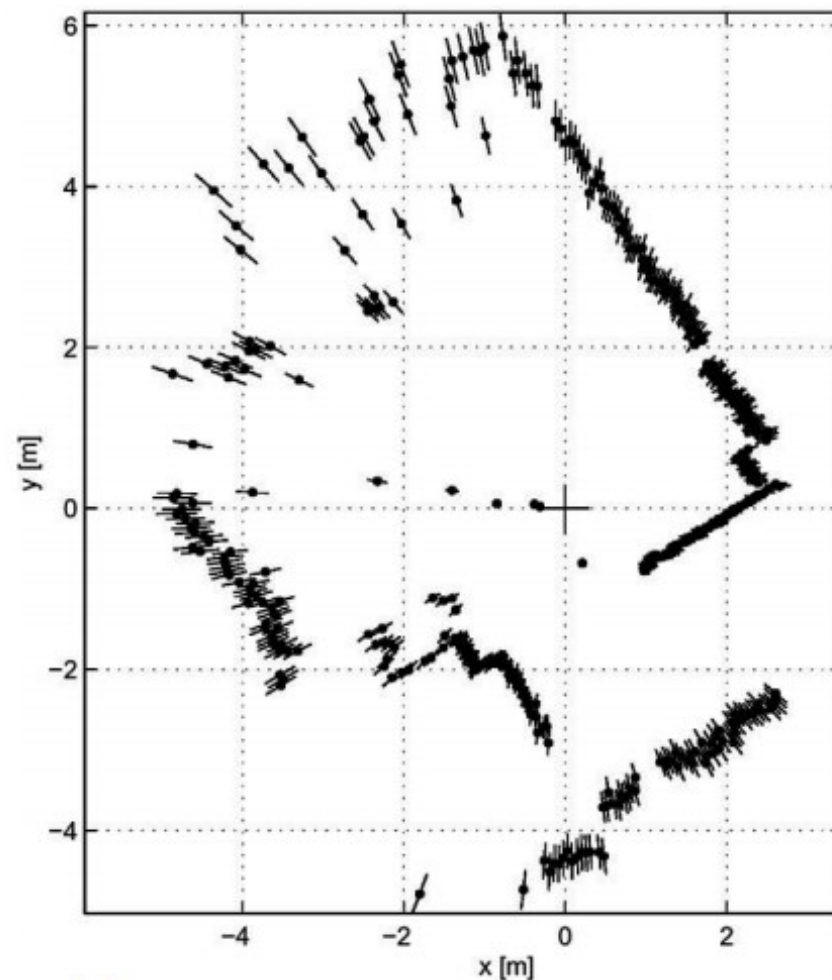
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Scanning Laser Rangefinder - Range image of a 2D Laser rangefinder



Reproduced from: Siegwart, Roland, Illah Reza Nourbakhsh, and Davide Scaramuzza. Introduction to autonomous mobile robots. MIT press, 2011.



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Module 15: Motion/speed Sensors



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Motion/Speed Sensors

- These sensor measure directly the relative motion between the robot and its environment.
- It requires that the object is moving with respect to the robot's reference frame
- Different types of sensors can be made use of for detecting relative motion/speed
- Doppler effect based sensors can be used
- The change in frequency of a signal that is observed when a moving object approaches is called as the Doppler effect
- Ex: Change in siren pitch that occurs when an approaching fire engine passes by and recedes



Motion/Speed Sensors - Doppler effect

- A transmitter emits an electromagnetic or sound wave with a frequency f_t .
- It is either received by a receiver or reflected from an object.
- If the transmitter is moving, the measured frequency f_r at the receiver is a function of the relative speed v between transmitter and receiver according to

$$f_r = f_t \frac{1}{1 + v/c}$$

- If the receiver is moving, the measured frequency f_r is obtained as

$$f_r = f_t (1 + v/c)$$



Motion/Speed Sensors - Doppler effect

- If the transmitter and receiver are both moving at a speed of V_t and V_r respectively, the measured frequency f_r is obtained as

$$f_r = \frac{c+v_r}{c+v_t} f_t$$

- In general, the expression is given as

$$f_r = \frac{c \pm v_r}{c \pm v_t} f_t$$



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Motion/Speed Sensors - Doppler effect

- When applied to sound waves, we call the sensor as a Doppler sensor
- When applied to EM waves, it is called as a Radar system
- **Application areas: Autonomous vehicles, manned vehicles**
- An example of a radar system used on vehicles is VORAD (Vehicle On-board RADAR)
- It makes use of a microwave radar and has a beam of approximately 4 degrees
- Range : 150 m
- Accuracy : 97%
- Range : 0 - 160 km/hr
- Resolution : 1 km/r
- Sample rate : 2 Hz



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Module 16: Vision-based Sensors



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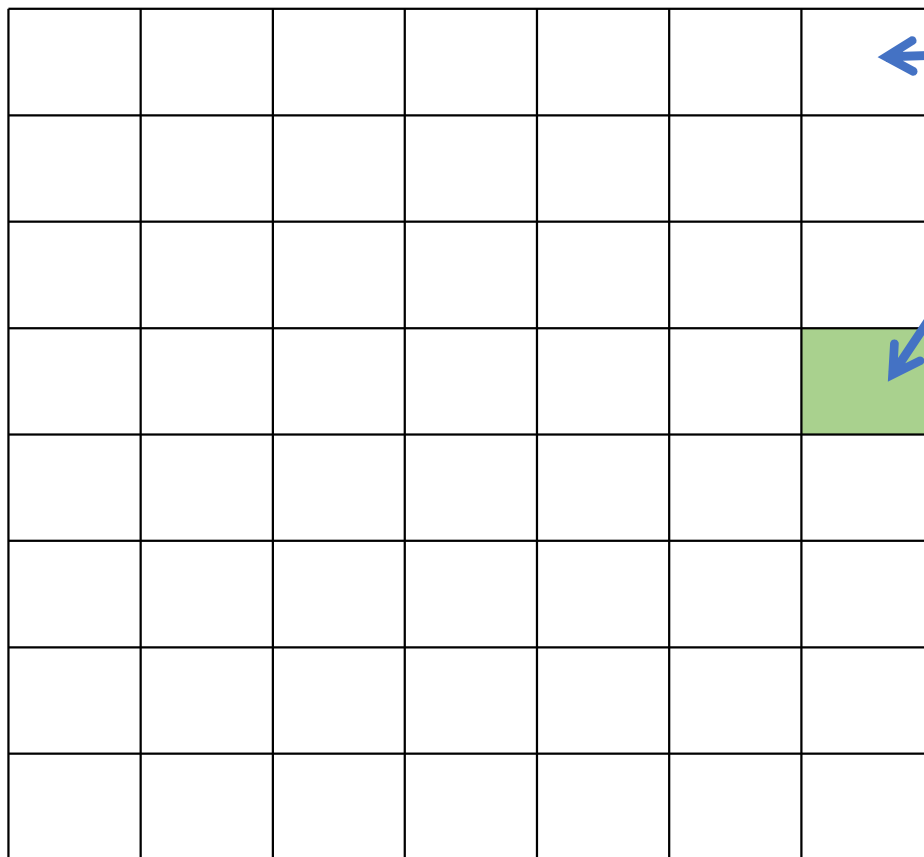
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Vision-based sensors

- Vision is a very powerful sense and can provide very rich information about the environment
- It can also allow for complex interactions with the environment
- Human vision serves as the reference for these sensors
- Cameras can be based on two technologies
 - Charge coupled devices (CCD)
 - Complementary Metal Oxide Semiconductor (CMOS)
- Imaging done by cameras similar to human eye



Camera Image



Pixel or picture element

Colored Pixel

Red $R = 5$

Green $G = 195$

Blue $B = 10$

If $R=G=B=0$, black color results

If $R=G=B=255$, white color results



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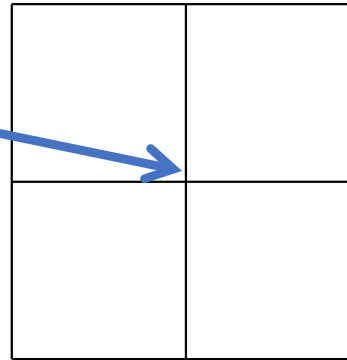
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Vision-based sensors

- When light falls on the camera, only photons impinge on the sensor
- It does not automatically indicate the energy in the Red, Green and Blue regions
- Red, Green and Blue filters are necessary to extract the energy in respective portion of the impinging light

Consider one pixel and light impinging on it

- Break up the pixel into 4 parts
- Place filters for each color on the parts





Vision-based sensors

- R represents region with Red filter
- G represents region with Green filter
- B represents region with Blue filter
- Since there are 4 parts and only 3 colors, Green is repeated twice
- A filter which is constructed as shown is called Bayer filter, which is a composite arrangement of RGB filter
- Pixel size is a square with a side that is twice the filter size

R	G
G	B



Charge coupled devices (CCD)

- A CCD chip is an array of light sensitive pixels
- A pixel is a light sensitive, discharging capacitor that is a 5 to 25 μm in size
- Incident light photons strike each pixel
- Charge that is present at each pixel corresponds to the incident photon energy
- Dark regions will result in low energy incident photons, Bright light will result in high energy incident photons



5 to 25 μm



Reading of a CCD Array

5	7	21	.	.	18	29
9	11	14	.	.	71	33
.
.
.
.
19	15	75	.	.	51	33
61	49	87	.	.	36	43

61	49	87	.	.	36	43
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- Consider a CCD Array, say of size 2048x2048 pixels
- Assume that the incident photon energy on the array results in a set of pixel values shown
- The reading of the CCD array is done all at one edge of the array
- A special row known as the reading row receives the bottom row of pixel values (charge values)
- The charges stored in the reading row is then serially transferred into a reading circuit at the corner of the array which enables storage of all the pixel values from the array into memory
- Once one row of pixels is read, the next row is brought into the reading row
- The process of shifting and reading is repeated
- This type of sensor requires specialized control circuitry and custom fabrication techniques to stably transport charges

← Reading Row



Reading of a CCD Array

5	7	21	.	.	18	29
9	11	14	.	.	71	33
.
.
.
.
19	15	75	.	.	51	33
61	49	87	.	.	36	43

Reading Row

61	49	87	.	.	36	43
19	15	75	.	.	51	33

Reading Circuit

61,49,87,...,36,43,19,15,75,...,51,33

- CCD cameras can be more expensive than CMOS cameras
- Use more power
- Less noisy
- Better in low light conditions



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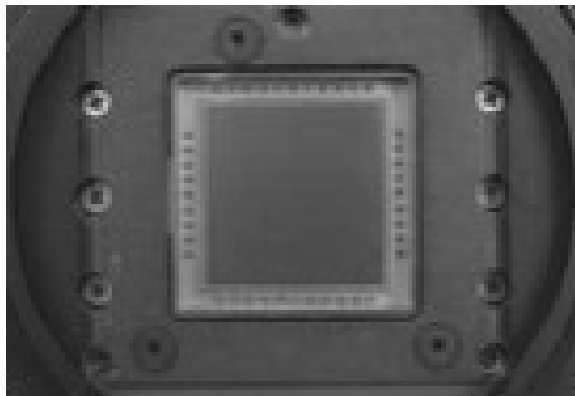
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Complementary Metal Oxide Semiconductor (CMOS) Camera

- A CMOS camera consists of pixels each of which have several transistors associated
- These transistors enable reading the pixels individually
- There is no requirement as in the case of a CCD camera, to transfer the reading to a different location, since it can be read at each pixel with the associated transistors
- All signals can be amplified in parallel
- No complex circuitry needed for transporting charge
- Circuitry next to the pixel occupies real estate
- Some photons will hit the transistor instead of the photodiode
- CMOS camera becomes less sensitive and more noisy as a result
- Resolution may be lower than a CCD chip

Examples of commercial CCD and CMOS cameras



2048 x 2048 CCD array



Orangemicro iBOT Firewire



Sony DFW-X700



Canon IXUS 300



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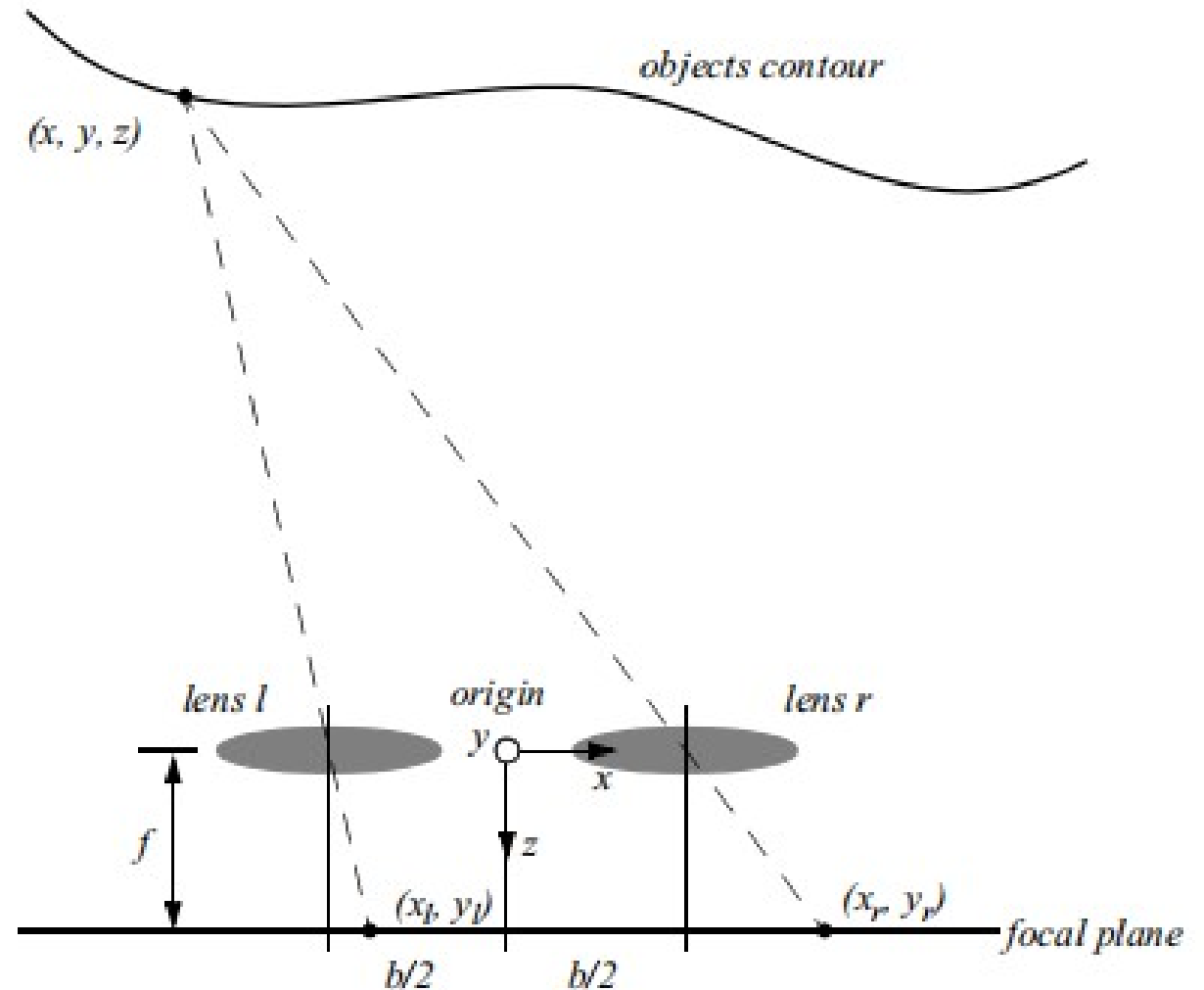
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Vision-based ranging

- Cameras are typically 2D representations of a 3D world
- Depth information is lost and makes the information ambiguous.
- For instance, a small object which is closer and a big object which is farther, both create similar 2D images
- Rangefinders can provide depth information, but they cannot provide other important feature information which we can extract from cameras e.g. edge, corner etc
- Humans have two eyes which allow for estimation of depth based on a camera-like sensor
- Depth information can be recovered by modification to sensor setup
- Stereo vision recovers depth from 2 images that depict scene from different perspective

Vision-based ranging

2 cameras are placed with their optic axis parallel



Reproduced from: Siegwart, Roland, Illah Reza Nourbakhsh, and Davide Scaramuzza. Introduction to autonomous mobile robots. MIT press, 2011.



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Autonomous Mobile Robots

Module 17: Representing uncertainty, Statistical representation, Error propagation



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Representing uncertainty

- In an ideal world, sensors will give perfect readings
- But in reality, when sensors measure the same quantity multiple times, there is variation in readings
- Sensors are imperfect with both systematic and random errors
- Suppose a laser rangefinder is placed in front of an obstacle at 10 m distance.
- When continuously read, the sensor may produce readings such as 10.1, 10.08, 10.12 and so on
- This variation is not because of any movement between the sensor and the object, but due to randomness
- Similar variations can be expected in all types of sensors
- Systematic errors can be corrected for, but random errors cannot be



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Representing uncertainty

- Uncertainty exists in all sensors and need to be combined and when combined, it will result in overall uncertainty for the robot system
- First, we will deal with uncertainty due to a single sensor
- Based on a set of uncertain measurements, we compute conclusions with uncertainty, which is known as error propagation law
- Statistical representation of uncertainty is used for such scenarios
- Starting point: **error = measured value - true value**
- Measured value is the sensor measurement
- Ex. Say, $d_{\text{true}} = 3 \text{ m}$, and measurements are showing 2.99, 3.01, 3.05, 2.95, 2.9, 3.1
- Sensor makes n measurements: $\rho_1, \rho_2, \rho_3, \dots, \rho_n$
- **Goal:** estimate true value given these measurements



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Statistical representation

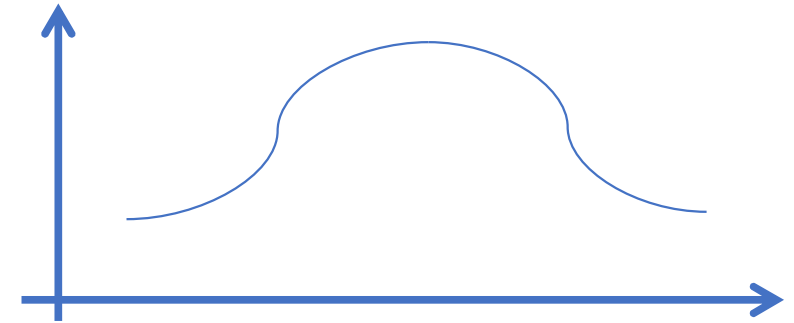
- Ex. Say, $d_{\text{true}} = 3$ m, and measurements are showing 2.99, 3.01, 3.05, 2.95, 2.9, 3.1
- How can we estimate the true value, when d_{true} is not known?
- We are used to taking the average of measurements as an *estimate* of the true value
- There is a statistical reasoning for why we are choosing the average
- True value is represented by a **random variable X**
- Random variables have probability density functions to characterize its statistical properties instead of fixed values



Statistical representation

- Ex. Probability Density function:
- Can ask questions such as:
- What is the probability that the random variable will take a value between $-\infty$ to ∞ ?
- What is the probability that the random variable will take a value between -2 to 1?
- Probability that value of X falls between 2 limits a & b is given as

$$\int_a^b f(x)dx$$



$$\int_{-\infty}^{\infty} f(x)dx = 1$$

$$\int_{-2}^1 f(x)dx$$



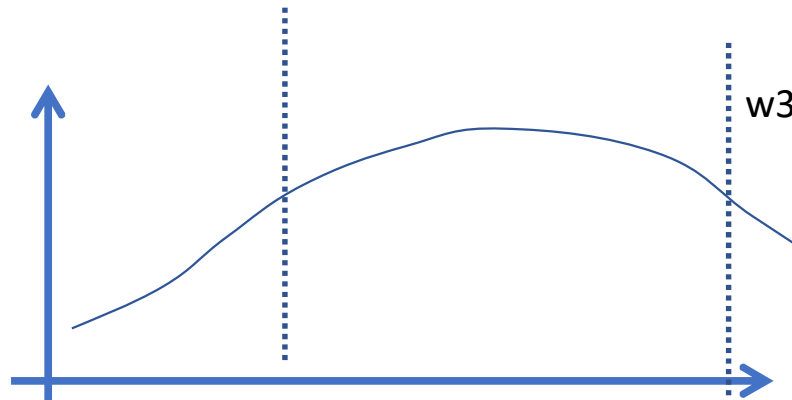
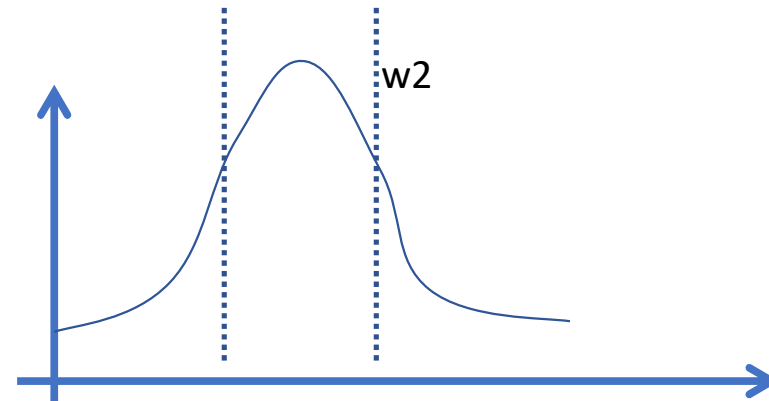
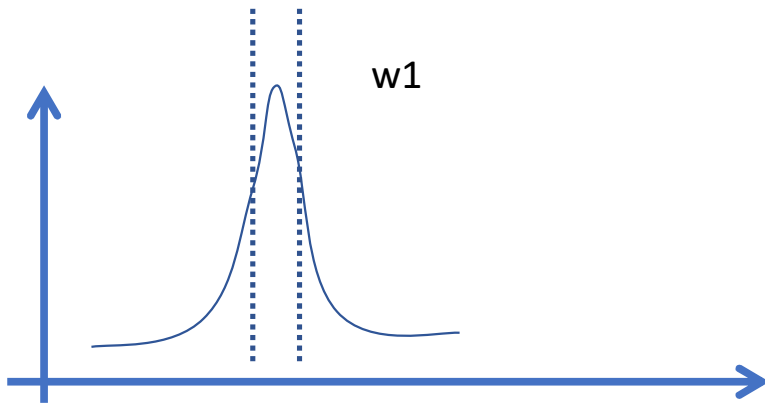
Statistical representation

- X : random variable
- - Characterize the possible values of X using properties of the probability distribution function
- - We can define mean, variance and standard deviation
- - Mean value μ : equivalent to making an infinite number of measurements and averaging all of them
- - $\mu = E[X] = \int_{-\infty}^{\infty} x \cdot f(x) dx$
- Mean square error : $E[X^2] = \int_{-\infty}^{\infty} x^2 \cdot f(x) dx$
- $Var[X] = \sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 \cdot f(x) dx$



Statistical representation

- Variance captures the width of the distribution





Independence of random variables

- 2 random variables X_1 and X_2 are considered independent if the particular value of one has no bearing on the particular value of the other
- The following properties hold in this case

$$E[X_1 X_2] = E[X_1] E[X_2]$$

$$Var[X_1 + X_2] = Var[X_1] + Var[X_2]$$

- Gaussian Distribution: Also called as the normal distribution

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x - \mu)^2}{2\sigma^2}\right)$$



Error Propagation: combining uncertain measurements

- Probability mechanisms may be used to describe errors associated with single sensor's measurement of real-world value
- But in general, multiple uncertain measurements may be used to extract a single environmental measure
- Error propagation in a multi-input, multi-output system (MIMO)





Error Propagation: combining uncertain measurements

X_i are input signals, $i = 1, 2, \dots, n$

Y_i are output signals, $i = 1, 2, \dots, m$

$$Y_1 = f_1(X_1, X_2, \dots, X_n)$$

$$Y_2 = f_2(X_1, X_2, \dots, X_n)$$

\vdots

$$Y_m = f_m(X_1, X_2, \dots, X_n)$$



Error Propagation: combining uncertain measurements

Covariance propagation

$$C_Y = F_X C_X F_X^T$$

$$F_X = \begin{bmatrix} \frac{\partial f_1}{\partial X_1} & \frac{\partial f_1}{\partial X_2} \cdots \frac{\partial f_1}{\partial X_n} \\ \frac{\partial f_2}{\partial X_1} & \frac{\partial f_2}{\partial X_2} \cdots \frac{\partial f_2}{\partial X_n} \\ \vdots & \vdots \\ \frac{\partial f_m}{\partial X_1} & \frac{\partial f_m}{\partial X_2} \cdots \frac{\partial f_m}{\partial X_n} \end{bmatrix}$$

Examine size of F_X , C_X , C_Y