CSE211-Formal Languages and Automata Theory

U3L3 – Designing Turing Machines

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Agenda

- Recap of Previous class
- Turing Machine def.
- Instantaneous Descriptions for Turing Machine
- Moves of a TM
- Designing a TM with example
- Languages of a TM
- Halting of TM

Turing-Machine Formal def.

- A Turing machine (TM) is a 7-tuple $M = (Q, \Sigma, \Gamma\ \delta, q_0, B, F)$ where
  1. Q: A finite set of *states*
  2. Σ: An *input alphabet*
  3. Γ: A *tape alphabet* (with Σ being a *subset* of it).
  4. δ: A *transition function,* $\delta(q, X) = (p, Y, D)$
  5. q_0 : A *start state*
  6. B: A *blank symbol* (B, in Γ− Σ, typically).
     1. All tape except for the input is blank initially.
  7. F: A set of *final states* ($F \subseteq Q$, typically).

The Transition Function

- δ: a transition function $\delta\,(q, X) = (p, Y, D)$ where
- Takes two arguments:
  1. A state q, in Q.
  2. A tape symbol X in Γ.
- $\delta(q, Z)$ is either undefined or a triple of the form (p, Y, D).
  - p is a state.
  - Y is the new tape symbol.
  - D is a *direction*, L or R.

Instantaneous Descriptions for Turing Machine

- **The *instantaneous description* (ID) of a TM ---**

The ID of a TM is represented by $X_1X_2\ldots X_{i-1}qX_iX_{i+1}\ldots X_n$ in which

- q is the current state;
- the tape head is scanning the ith symbol X_i from the left;
- $X_1X_2\ldots X_n$ is the portion of the tape between the leftmost and the rightmost nonblank symbols.

Moves of a TM

- The moves of a TM *M* are denoted by $\vdash_M$ or $\vdash$.
- If $\delta(q, X_i) = (p, Y, L)$ (a leftward move), then we write the following to describe the left move:

$$X_1X_2 \ldots X_{i-1}qX_iX_{i+1} \ldots X_n \vdash X_1X_2 \ldots X_{i-2}pX_{i-1}YX_{i+1} \ldots X_n.$$

- Right moves are defined similarly.

Design a TM: Example 1

Design a TM to accept the language $L = \{0^n1^n \mid n \geq 1\}$.

- The machine may be designed by the following steps.
- Starting at the left end of the input.
  - Change 0 to an *X*.
  - Move to the right over 0's and *Y*'s until a 1.
  - Change 1 to *Y*.
- Move left over *Y*'s and 0's until an *X*.
  - Look for a 0 immediately to the right.
  - If a 0 is found, change it to *X* and repeat the above process.

Design a TM: Example 1

- An example illustrating the above steps is as follows (the Green character indicates the position of the reading head).
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- 0011 → X011 → X0Y1 → XXY1 → ... → XXYY → XXYYB
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- The TM is defined formally as follows:
- $M = (\{q0\sim q4\}, \{0, 1\}, \{0, 1, X, Y, B\}, \delta, q0, B, \{q4\})$

Design a TM: Example 1

- Transition table for δ is as shown in Table

	symbol				
State	0	1	X	Y	B
q_0	$(q_1, X, R)_1$	-	-	$(q_3, Y, R)_8$	-
q_1	$(q_1, 0, R)_2$	$(q_2, Y, L)_4$	-	$(q_1, Y, R)_3$	-
q_2	$(q_2, 0, L)_5$	-	$(q_0, X, R)_7$	$(q_2, Y, L)_6$	-
q_3	-	-	-	$(q_3, Y, R)_9$	$(q_4, B, R)_{10}$
q_4	-	-	-	-	-

Red numbers are used to distinguish the transitions.

Design a TM: Example 1

- The moves to accept the input string w = 0011 are as follows (use $\Rightarrow$ instead of):
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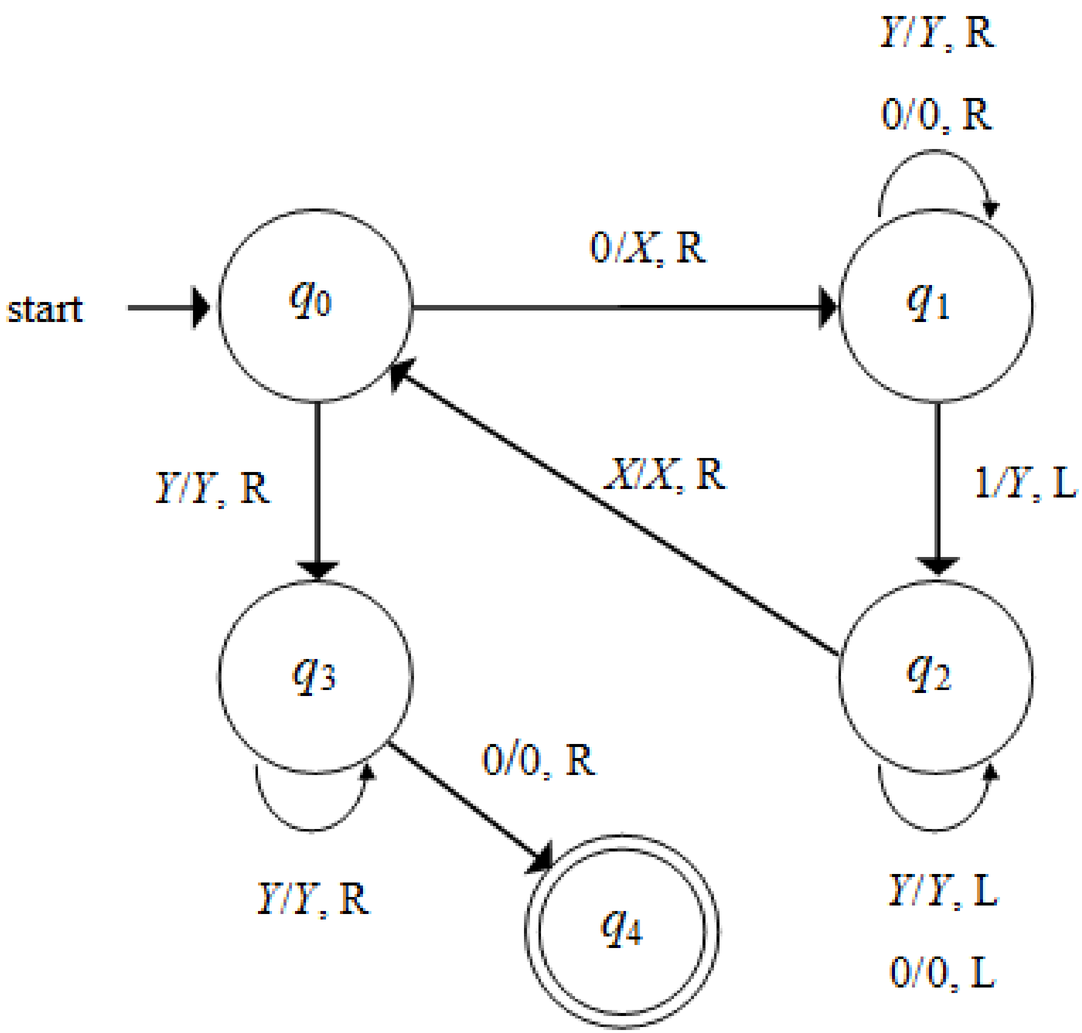
- $q00011 \Rightarrow_1 Xq1011 \Rightarrow_2 X0q111 \Rightarrow_4 Xq20Y1 \Rightarrow_5 q2X0Y1$
  $\Rightarrow_7 Xq00Y1 \Rightarrow_1 XXq1Y1 \Rightarrow_3 XXYq11$
  $\Rightarrow_4 XXq2YY \Rightarrow_6 Xq2XYY \Rightarrow_7 XXq0YY$
  $\Rightarrow_8 XXYq3Y \Rightarrow_9 XXYYq3B \Rightarrow_{10} XXYYBq4B.$

Transition Diagrams for TM's



TM to accept the language
$L = \{0^n1^n \mid n \geq 1\}$.

Example 2

- Design a TM that accepts the language denoted by the RE 00*

Summary

- Recap of Previous class
- Turing Machine def.
- Instantaneous Descriptions for Turing Machine
- Moves of a TM
- Designing a TM with example

References

- John E. Hopcroft, Rajeev Motwani and Jeffrey D. Ullman, *Introduction to Automata Theory*, Languages, and Computation, Pearson, 3rd Edition, 2011.
- Peter Linz, An Introduction to Formal Languages and Automata, Jones and Bartle Learning International, United Kingdom, 6th Edition, 2016.

Next Class: **Unit III**

Designing Turing Machines Part 2

Thank you.