# Measures of dispersion

1) Range $= L - S$

2) Quartile deviation $= \frac{Q_3 - Q_1}{2}$

3) Mean deviation about mean $= \frac{\sum f|x - \bar{x}|}{\sum f}$

4) Standard deviation $\sigma = \sqrt{\frac{\sum fd^2}{\sum f} - \left(\frac{\sum fd}{\sum f}\right)^2} \times i$ $\quad \left(d = \frac{m - A}{i}\right)$

Variance $= \sigma^2$.

# Coefficient of dispersion.

Coeff of range $= \dfrac{L-S}{L+S}$

Coeff of Mean deviation $= \dfrac{\text{M.D about Mean}}{\text{Mean}}$

Coeff of QD $= \dfrac{Q_3 - Q_1}{Q_3 + Q_1}$.

Coeff of Variation $= \dfrac{\sigma}{\bar{X}} \times 100$. $\longrightarrow$ Variability of the data.

# Moments

The $r^{th}$ moment of a variable X about any point $x = A$, denoted by $\mu_r'$ is given by

(raw moment)

$$\mu_r' = \frac{1}{\sum_i' f_i} \sum_i' f_i (x_i - A)^r \rightarrow \mu_1' = \frac{\sum_i' f_i \overset{d_i}{\cancel{(x_i - A)}}}{\sum_i' f_i}$$

The $r^{th}$ moment of a variable X about the mean $\bar{x}$, denoted by $\mu_r$ is defined as

$$\mu_r = \frac{1}{\sum_i' f_i} \sum_i' f_i (x_i - \bar{x})^r .$$

Note:-

$$\mu_r = \frac{\sum_i f_i (x - \bar{x})^r}{\sum f_i}$$

r=0 $\quad \mu_0 = \frac{\sum f_i}{\sum f_i} = 1.$

r=1 $\quad \mu_1 = \frac{\sum f_i (x_i - \bar{x})}{\sum f_i} = \frac{\sum f_i x_i}{\sum f_i} - \bar{x}\frac{\sum f_i}{\sum f_i}$

$= \bar{x} - \bar{x}.1 = 0.$

r=2 $\quad \mu_2 = \frac{\sum f_i (x - \bar{x})^2}{\sum f_i} = \sigma^2.$

Relationship b/w $\mu_r$ & $\mu_r'$.

Let $d_i = x_i - A$. & $\bar{x} = A + \frac{\sum' f_i d_i}{\sum' f_i} = A + \mu_1'$

$N = \sum' f_i$

$\Rightarrow \boxed{\bar{x} - A = \mu_1'}$

$\bar{x} = A + \mu_1'$

$$\mu_r = \frac{1}{N} \sum' f_i (x_i - \bar{x})^r$$

$$= \frac{1}{N} \sum' f_i \{ \overbrace{x_i - A} + \overbrace{A - \bar{x}} \}^r$$

$$= \frac{1}{N} \sum' f_i \{ d_i - \mu_1' \}^r.$$

$$\mu_r = \frac{1}{N} \sum_i f_i (d_i - \mu_1')^r$$

$$= \frac{1}{N} \sum_i f_i \left\{ d_i^r - {}^rC_1 d_i^{r-1} \mu_1' + {}^rC_2 d_i^{r-2} (\mu_1')^2 + \cdots + (-1)^r (\mu_1')^r \right\}$$

$$= \frac{1}{N} \sum f_i d_i^r - {}^rC_1 \mu_1' \frac{\sum f d_i^{r-1}}{N} + {}^rC_2 (\mu_1')^2 \frac{\sum f d_i^{r-2}}{N} + \cdots + (-1)^r (\mu_1')^r$$

$$\mu_r = \mu_r' - {}^rC_1 \mu_{r-1}' \mu_1' + {}^rC_2 \mu_{r-2}' (\mu_1')^2 + \cdots + (-1)^r (\mu_1')^r.$$

$$\mu_r = \mu'_r - {}^rc_1\, \mu'_{r-1}\, \mu'_1 + {}^rc_2\, \mu'_{r-2} (\mu'_1)^2 + \cdots + (-1)^r (\mu'_1)^r.$$

$r=2$ $$\mu_2 = \mu'_2 - {}^2c_1\, \mu'_1\, \mu'_1 + (-1)^2 (\mu'_1)^2 = \mu'_2 - (\mu'_1)^2$$ (Variance)

$r=3$ $$\mu_3 = \mu'_3 - {}^3c_1\, \mu'_2\, \mu'_1 + {}^3c_2\, \mu'_1 (\mu'_1)^2 - (\mu'_1)^3$$

$$\mu_3 = \mu'_3 - 3\mu'_2\, \mu'_1 + 2(\mu'_1)^3.$$

$r=4$ $$\mu_4 = \mu'_4 - {}^4c_1\, \mu'_3\, \mu'_1 + {}^4c_2\, \mu'_2\, \mu'^2_1 - {}^4c_3\, \mu'_1 (\mu'_1)^3 + (-1)^4 (\mu'_1)^4$$

$$\mu_4 = \mu'_4 - 4\mu'_3\, \mu'_1 + 6\mu'_2\, \mu'^2_1 - 3(\mu'_1)^4.$$

$$\mu_1 = \frac{\sum f(x-\bar{x})}{\sum f} = \frac{\sum fx}{\sum f} - \bar{x}\frac{\sum f}{\sum f} = \bar{x} - \bar{x} = 0.$$

$$\mu_1 = \mu_1' - \mu_1' = 0$$

$$\text{Skewness} = \beta_1 = \frac{\mu_3^2}{\mu_2^3}$$

{ bulkness of the data }

$$\text{Kurtosis} = \beta_2 = \frac{\mu_4}{\mu_2^2}$$

{ flatness of the curve }

for a symmetrical data $\beta_2 = 3$

1) Raw moments

2) Central moments

3) Relationship b/w raw & central moments

4) Particular cases in central moments

$\mu_2$ = Variance $= \mu_2' - (\mu_1')^2$.

$\mu_3$ & $\mu_4$.

5) $\beta_1 = \dfrac{\mu_3^2}{\mu_2^3}$ (Skewness) & $\beta_2 = \dfrac{\mu_4}{\mu_2^2}$ (Kurtosis).

→ The first four moments about the value $x = 4$ are $-1.5, 17, -30$ & $108$. Find skewness & kurtosis.

Soln: Given $\mu_1' = -1.5$; $\mu_2' = 17$; $\mu_3' = -30$; $\mu_4' = 108$.

To find $\beta_1 = \dfrac{\mu_3^2}{\mu_2^3}$ & $\beta_2 = \dfrac{\mu_4}{\mu_2^2}$

$$\mu_2 = \mu_2' - (\mu_1')^2 = 17 - (-1.5)^2 = 17 - 2.25 = 14.75$$

$$\mu_3 = \mu_3' - 3\mu_2'(\mu_1') + 2(\mu_1')^3 = -30 - 3(17)(-1.5) + 2(-1.5)^3$$

$$= -30 + 51(+1.5) - 2(1.5)^3$$

$$= -30 + 76.5 - 6.75 = 39.75$$

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3} = \frac{39.75^2}{(14.75)^3} =$$

$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'(\mu_1')^2 - 3(\mu_1')^4$$

$$= 108 - 4(-30)(-1.5) + 6(17)(-1.5)^2 - 3(-1.5)^4$$

$$= 108 - 120(1.5) + 6(17)(2.25) - 3(2.25)(2.25)$$

$$= 142.3125$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{142.3125}{14.75^2} = 0.6541.$$

**Note:-** $\bar{x} = A + \mu_1' = 4 + (-1.5) = 2.5.$

$$\boxed{\bar{x} = 2.5}$$

How will you find moments about $x = 0$?

$\bar{x} = A + \mu_1'$. Here assume $A = 0$

$\Rightarrow \bar{x} = \mu_1' = 2.5$

$\mu_2 = \mu_2' - (\mu_1')^2 \Rightarrow \mu_2' = \mu_2 + (\mu_1')^2 = 14.75 + (2.5)^2$

$= 14.75 + 6.25$

$= \underline{\underline{21}}$

→ find the first four moments for the following:

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $x$: | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| $f$: | 1 | 8 | 28 | 56 | 70 | 56 | 28 | 8 | 1 |

Soln

| $x$ | $f$ | $d = x - A$ | $fd$ | $fd^2$ | $fd^3$ | $fd^4$ |
|---|---|---|---|---|---|---|
| 0 | 1 | −4 | −4 | 16 | −64 | 256 |
| 1 | 8 | −3 | −24 | 72 | −216 | 648 |
| 2 | 28 | −2 | −56 | 112 | −224 | 448 |
| 3 | 56 | −1 | −56 | 56 | −56 | 56 |
| A 4 | 70 | 0 | 0 | 0 | 0 | 0 |
| 5 | 56 | 1 | 56 | 56 | 56 | 56 |
| 6 | 28 | 2 | 56 | 112 | 224 | 448 |
| 7 | 8 | 3 | 24 | 72 | 216 | 648 |
| 8 | 1 | 4 | 4 | 16 | 64 | 256 |

$\boxed{\sum fd = 0.}$

$\sum fd^2 = 512$

$\sum fd^3 = 0$

$\sum fd^4 = 2816.$

$$\bar{x} = A + \frac{\sum fd}{\sum f} = 4 + \frac{0}{\sum f} = 4.$$

256
648
448
56
1408

168
56
32
256

$$\mu_1 = 0.$$

$$\mu_2 = \frac{\sum fd^2}{\sum f} = \frac{512}{256} = 2$$

$$\mu_3 = \frac{\sum fd^3}{\sum f} = \frac{0}{256} = 0$$

$$\mu_4 = \frac{\sum fd^4}{\sum f} = \frac{2816}{256} = 11$$

$$\begin{array}{r} 2816 \\ 256 \\ \hline 256 \end{array}$$