SASTRA DEEMED UNIVERSITY

(A University under section 3 of the UGC Act, 1956)

End Semester Examinations

July 2022

Course Code: **MAT134**

Course: **LINEAR ALGEBRA**

Question Paper No. :**U0881**

Duration: **3** hours

Max. Marks:**100**

PART - A

Answer all the questions **10 x 2 = 20 Marks**

1. Find the value of a for which the system of equations $x + y + z = 1, x - y + 2z = 0, 2x + 3z = a$, has infinitely many solutions.

2. Find the determinant of the matrix $\begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & -4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix}$

3. If A & B are invertible matrix such that $AB = -BA$, show that $Trace(A) = 0$.

4. Find the singular value of the matrix $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \end{bmatrix}$.

5. If A be an 2 by 2 matrix and eigenvalue of A are 2,3 such that $A^{-1} = \alpha A + \beta$, Find α and β .

6. Let A be any symmetric matrix of order n then, which of the following statements are true?

(a) If A is an invertible then A^2 is positive definite matrix
(b) If A and B are positive definite matrix then A+B is positive definite matrix.

7. Let W_1 & W_2 are subspace of V such that $\dim W_1 = 5$ & $\dim W_2 = 8$ of $\dim V = 10$. What are the possible $\dim(W_1 \cap W_2)$?

8. Which of the following are subspace of the vector space R^3?
   a. $\{(x, y, z): 2022x + y = 0\}$
   b. $\{(x, y, z): x - 2022y = 0\}$
   c. $\{(x, y, z): x - 2y = 1\}$
   d. $\{(x, y, z): x + y = 1\}$

9. Let V be the vector space of any real polynomial of degree ≤ 3. Let $Tp(x) = p'(x) for\ p \in V$ be a linear transformation from V to V with respect to basis $\{1, x, x^2, x^3\}$. Then find A matrix of T with respect to this basis.

10. What are different types of Machine learning algorithm?

PART – B

Answer all the questions **4 x 15 = 60 Marks**

11. Find the value of k such that the following system of equation has unique solution, infinitely many solutions and no solution
$kx + y + z = 1, x + ky + z = 1, x + y + kz = 1.$

(OR)

12. (a) Test for the consistency of the following system of equation $x_1 + 2x_2 + 3x_3 + 4x_4 = 5, 6x_1 + 7x_2 + 8x_3 + 9x_4 = 10, 11x_1 + 12x_2 + 13x_3 + 14x_4 = 15, 16x_1 + 17x_2 + 18x_3 + 19x_4 = 20, 21x_1 + 22x_2 + 23x_3 + 24x_4 = 25.$ (7)
(b) Test for the consistency of the following systems of equations and solve it $x - 3y - 8z = -10, 3x + y = 4z, 2x + 5y + 6z = 13.$ (8)

13. (a) If W_1 and W_2 are subspaces of a finite dimensional vector space, then $\dim(W_1 + W_2) = \dim W_1 + \dim W_2 - \dim(W_1 \cap W_2)$. (10)

(b) Prove that $V = \{A \in M_n(\mathbb{R}) : A = A^T\}$ and $V_1 = \{A \in M_n(\mathbb{R}) : A = -A^T\}$ are subspace of $M_n(\mathbb{R})$. (5)

(OR)

14. (a) If $A = \begin{bmatrix} 1 & -1 & 4 \\ 1 & 4 & -2 \\ 1 & 4 & 2 \\ 1 & -1 & 0 \end{bmatrix}$. Find QR decomposition using Gram-Schmidt method. (10)

(b) If $\{(1,-1,1), (1,0,1), (1,1,2)\}$ basis of $\mathbb{R}^3$ using Gram-Schmidt algorithm. To find Orthogonal basis. (5)

15. Reduce a matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ -4 & 4 & 3 \end{bmatrix}$ to diagonal form by similar transformation.

(OR)

16. State and prove Rank-Nullity theorem.

17. Reduce to $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ a singular value decomposition.

(OR)

18. Given data $\{2,3,4,5,7 : 1,5,3,6,7,8\}$, Compute the principal component using PCA algorithm.

PART – C

Answer the following $1 \times 20 = 20$ Marks

19. If $x + 2y + 3z = 3, 2x + 4y + 5z = -1, 3x + 5y + 2z = 2$ system of non homogeneous equations.

(a) Test for the consistency of the above systems of equations. (5)

(b) If $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 2 \end{bmatrix}$, Discuss the nature of the definite matrix and rank. (5)

(c) Find the null space of A.($x + 2y + 3z = 0, 2x + 4y + 5z = 0, 3x + 5y + 2z = 0$). (5)

(d) Find the singular value of $A = \begin{bmatrix} 0 & 1 & 1 \\ \sqrt{2} & 2 & 0 \\ 0 & 1 & 1 \end{bmatrix}$. (5)

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