SASTRA DEEMED UNIVERSITY

(A University under section 3 of the UGC Act, 1956)

End Semester Examinations

JULY 2023

Course Code: **MAT134**

Course: **LINEAR ALGEBRA**

QP No. :**UF026-2**

Duration: **3 hours**

Max. Marks:**100**

PART-A

Answer all the questions **10 x 2 = 20 Marks**

1. Write down the general form LU-Decomposition.

2. What is the difference between the Gauss elimination and Rank method for solving system of equations?

3. If RankA $< n-1$ for $n \times n$ real matrix A. Then Rank(adjA) =?. Justify your answer.

4. Find the basis and dimension for $V(R) = \left\{\begin{pmatrix} a & b \\ -b & c \end{pmatrix}; a, b, c \in R\right\}$.

5. Find $W^{\perp}$ for $W = \{(a, b, 0); a, b \in R\}$ in R^3. with standard inner product in R^3.

6. If A is negative definite matrix, then A^3 is? Justify your answer.

7. Examine $T: C \to C$ is defined by$T(z) = \bar{z}$, whether T is a linear transformation, where C is the vector space of complex numbers over reals.

8. Find the singular values of the matrix $A = \begin{pmatrix} 1 & 0 \\ 0 & -3 \end{pmatrix}$

9. Define first and second principal component of the data.

10. Define projection.

PART-B

Answer all the questions **4 x 15 = 60 Marks**

11. a) Solve the equations $3x + y + 2z = 3, 2x - 3y - z = -3,\ x + 2y + z = 4$ by cramers rule.

    b) Solve the equations $x - 3y = -5, y + 3z = -1,$
    $2x - 10y + 2z = -20$ by LU-Decomposition. (7+8)

(OR)

12. a) Find the values of λ for which the equations

$$(\lambda - 1)x + (3\lambda + 1)y + 2\lambda z = 0$$
$$(\lambda - 1)x + (4\lambda - 2)y + (\lambda + 3)z = 0$$
$$2x + (3\lambda + 1)y + 3(\lambda - 1)z = 0$$

    Are consistent, and find the ratios of $x: y: z$ when λ has the smallest and greatest of these values?

    b) Investigate for what values of λ and μ the simultaneous $x + y + z = 6, x + 2y + 3z = 0, x + 2y + \lambda z = \mu$ have i)No solution (ii) a unique solution (iii) infinitely many solution. . (7+8)

13. Prove that $\{A_{nxn}(R) / A = A^t\}$ is vector spaces over R-Real numbers also find the basis and dimension.

(OR)

14. Find QR-Decomposition of the matrix $A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$ and also check the definiteness of the matrix A.

15. Diagonalise the matrix $A = \begin{pmatrix} 3 & 1 & 1 \\ 1 & 3 & -1 \\ 1 & -1 & 3 \end{pmatrix}$

(OR)

16. a) State and prove Rank Nullity Theorem
b) Verify the above theorem $T: R^2 \to R^2$ is defined is rotation

(12+3)

17. A Landsat image with three spectral components was made of Homestead Air Force Base in Florida (after the base was hit by Hurricane Andrew in 1992). The covariance matrix of the data is shown below. Find the first principal component of the data, and compute the percentage of the total variance that is contained in this component

$$S = \begin{pmatrix} 164.12 & 32.73 & 81.04 \\ 32.73 & 539.44 & 249.13 \\ 81.04 & 249.13 & 189.11 \end{pmatrix}$$

(OR)

18. Find the singular value decomposition of the matrix

$$A = \begin{pmatrix} 1 & -1 \\ -2 & 2 \\ 2 & -2 \end{pmatrix}$$

PART-C

Answer the following **1 x 20 = 20 Marks**

19. a) Obtain an orthonormal basis for V=The space of all real polynomials of degree atmost 2, the inner product is defined by $< f, g >= \int_0^1 f(x)g(x)dx$

b) The following table list the weights and height of 5 boys

Boy	1	2	3	4	5
Weight(lb)	120	125	125	135	145
Height(in)	61	60	64	68	72

i) Find covariance Matrix.

ii) Make a principal component analysis of the data to find a single size index that explains most of the variation in the data. (10 +10)

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