PART A

Answer all the questions **35 marks**

1. Derive the covariance of regression coefficient of linear regression model. (7)

Ans: \

Vriance

$$Cov(\hat{\beta}) = E\left[\{\hat{\beta} - E[\hat{\beta}]\}\{\hat{\beta} - E[\hat{\beta}]\}^T\right]$$

$$\hat{\beta}_{(p+1)\times 1} ; \quad Cov(\hat{\beta})_{(p+1)\times(p+1)}$$

$$Cov(\hat{\beta}) = E\left[(\hat{\beta} - \beta)\cdot(\hat{\beta} - \beta)^T\right]$$

Consider we know $\hat{\beta} = (x^T x)^{-1} x^T y$

$\hat{\beta} - \beta$

$$\therefore \hat{\beta} - \beta = (x^T x)^{-1} x^T y - \beta$$

$$= (x^T x)^{-1} x^T (x\beta + \varepsilon) - \beta$$

$$= (x^T x)^{-1} x^T x\beta + (x^T x)^{-1} x^T \varepsilon - \beta$$

$$\hat{\beta} - \beta = (\cancel{\beta} + (x^T x)^{-1} x^T \varepsilon) - \cancel{\beta}$$

$$= (x^T x)^{-1} x^T \varepsilon$$

$$(\hat{\beta} - \beta)^T = [(x^T x)^{-1} x^T \varepsilon]^T$$

$$= \varepsilon^T x (x^T x)^{-1}$$

$$E\left[(\hat{\beta} - \beta)(\hat{\beta} - \beta)^T\right] = E\left[\left((x^T x)^{-1} x^T \varepsilon\right) \cdot \left(\varepsilon^T x (x^T x)^{-1}\right)\right]$$

$$= (x^T x)^{-1} x^T E[\varepsilon \varepsilon^T] x (x^T x)^{-1}$$

$$= (x^T x)^{-1} x^T (\sigma^2 I) x (x^T x)^{-1}$$

$$= \sigma^2 (x^T x)^{-1} x^T x (x^T x)^{-1}$$

$$\boxed{\begin{aligned} &= \sigma^2 (x^T x)^{-1} = \mathrm{Cov}(\hat{\beta}) \\ &= s_e^2 (x^T x)^{-1} \end{aligned}}$$

$$\sigma^2 = s_e^2 = \frac{SSE}{n-(p+1)}$$

2. Find out the regression coefficient of advertisement and predict sales value for the given data. (10)

   $\text{Sales}^T = [5,6,7,8,9]$; $\text{Advertisement}^T=[0.5,0.6,0.7,0.8,0.9]$

   Ans: X is independent and y is dependent. So Assumption: Advertisement is X, Sales is Y.

   $$\hat{\beta} = (x^T x)^{-1} x^T y$$

   If the assumption is wrong and still answer is correct- 8 marks.

   If assumption is correct then :

$$X^T x = \begin{bmatrix} 5 & 3.5 \\ 3.5 & 2.55 \end{bmatrix} \quad -2 \text{ marks}$$

$$(X^T X)^{-1} = \begin{bmatrix} 5.1 & -7 \\ -7 & 10 \end{bmatrix} \quad -2 \text{ marks}$$

$$X^T y = \begin{bmatrix} 35 \\ 25.5 \end{bmatrix} -2 \text{ marks} \qquad \beta = \begin{bmatrix} 0 \\ 10 \end{bmatrix} -2 \text{ marks} \qquad \hat{y} -2 \text{ marks}$$

$$\varepsilon = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix} -2 \text{ marks}$$

3.Sales (in lakhs) of two products P1 and P2 for many branches where the amount follow a bivariate normal distribution with parameters:

- μ_x= 80 and μ_y=90. Are the marginal means
- σ_x= 20 and σ_y=25 are the marginal standard deviation
- ρ=0.70 Is the correlation co-efficient

Suppose we select branch at random, what is the probability that

a) A branch sales over 95 for P2?

b) The sum of P1 and P2 over 180? (7)

   Ans: a): p(z>(95-90)/25; 1-φ(0.2) ; 1-0.58=0.42 --- 3 marks

b) p(z>(180-170)/sqrt(1725); 1-φ(0.24)=1-0.59=0.41. ---4 marks

3. Calculate eigen values and eigen vector for the given matrix

$$\begin{bmatrix} 4 & 8 \\ 10 & 6 \end{bmatrix} \qquad (8)$$

X^2-10x-56=0

A=14, b=-4 --4marks

For a=14, (4,5); For b=-4, (1,-1). ---4marks

PART B

Answer all the questions **15 marks**

4. Derive the equation of independent multivariate normal distribution (15)

   Ans: Derivation of component term and exponent term by considering covariance as 0.

   Notes pages :8 to 11