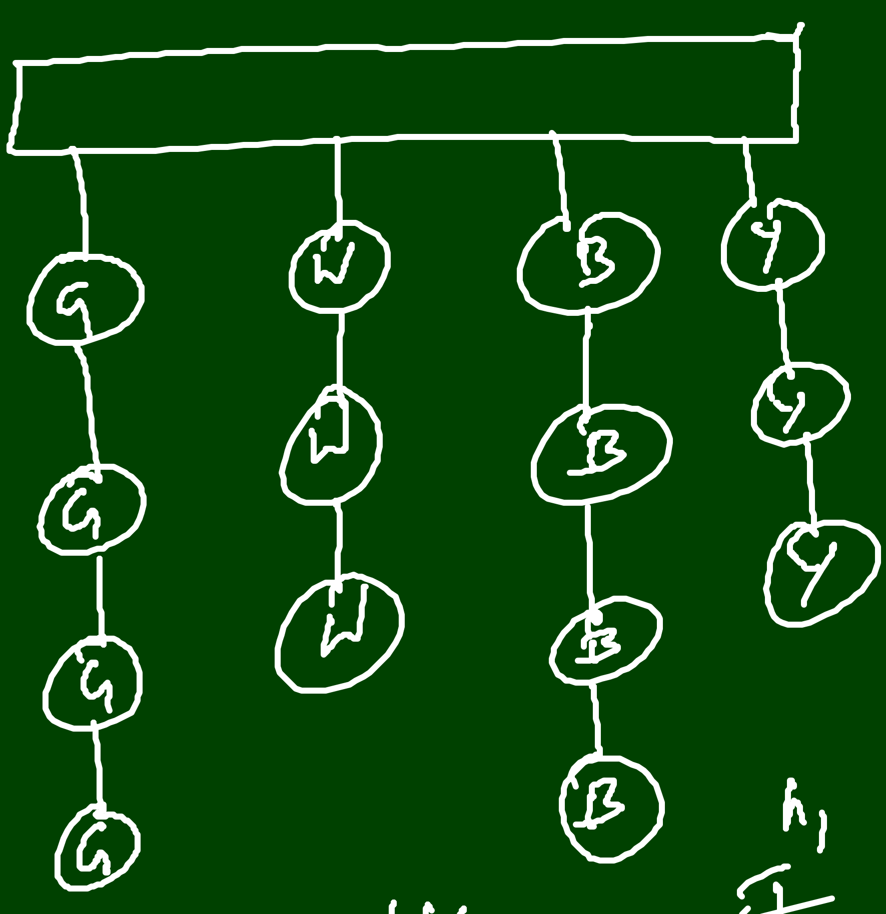
23/11/21 Tuesday

$14P_4$ ✗



GGWWBB YYY GWBBG
GGGG WWW BBBB YYY
GGGG YYYY WWBBBB W
GWBY GWBY GWBY GW

I^{n_1} II^{n_2} III^{n_3} IV^{n_4}

GGGG WWW BBBB YYY

$$\frac{\lfloor 14}{\lfloor 4 \lfloor 3 \lfloor 4 \lfloor 3} \quad (n_1, n_2, n_3, n_4)$$

$$\frac{n!}{14} = \frac{\cancel{1\times2\times3\times4}\times5\times6\times7\times8\times9\times10\times11\times12\times13\times14}{\cancel{1\times2\times3\times4}\times1\times2\times3\times1\times2\times3\times4\times1\times2\times3}$$

$= 5\times7\times3\times10\times11\times4\times13\times7$

$= 105\times10\times44\times91$

$= 105\times4004\times10$

$= 420420\times10$

$= 4204200$

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(2) In how many ways can we select 6 numbers from the set {1,2,3,4,5,6,7,8,9}?

$n=9$ $r=6$ $^9C_6$ ways

$$^9C_6 = {}^9C_3 = \frac{9\times8\times7}{1\times2\times3} = 84$$

(ii) select both / select none

$$^7C_4 + {}^7C_6$$

$= 35\times7$

$= 245$

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(3) Find the number of arrangements of all the letters of the word MISSISSIPPI if
(i) all I's must come together
(ii) no two I's are adjacent.

Sol:
(i) Treat all I's as a single object

M I SSSS PP

$n=8$, $n_1=1$, $n_2=1$, $n_3=4$, $n_4=2$

$$\frac{\lfloor 8}{\lfloor 1 \lfloor 1 \lfloor 4 \lfloor 2}$$

$n=11$, $n_1=1$, $n_2=4$, $n_3=4$, $n_4=2$

bal

ball

$$\frac{\lfloor 4}{\lfloor 1 \lfloor 1 \lfloor 2} = 12$$

llab rbllal
llba rlabl
rlalb alrbal
rlblla
xalbl

ball
abll
allb
blla

(ii) ↑M↑S↑S↑S↑P↑P↑ $\quad n=7 \quad n_1=1,\ n_2=4,\ n_3=2$

$$\frac{\lfloor 7}{\lfloor 1 \lfloor 4 \lfloor 2} \times {}^8C_4$$

$$= \frac{1\times 2\times 3\times 4\times 5\times 6\times 7}{(1\times 2\times 3\times 4)\times 1\times 2} \times \frac{8\times 7\times 6\times 5}{1\times 2\times 3\times 4}$$

$$= 105\times 70 = 7350$$

In how many ways can we arrange all the letters of the word PERMUTATIONS if the difference in position between P and S is 4?

$$\frac{\lfloor 10}{\lfloor 1 \lfloor 1 \lfloor 1 \lfloor 1 \lfloor 1 \lfloor 1 \lfloor 1 \lfloor 1 \lfloor 1 \lfloor 1 \lfloor 2} \times {}^8C_1 \times 2$$

(1,5) (6,10)
(2,6) (7,11)
(3,7) (8,12)
(4,8)
(5,9)

$$= 10\times 8$$