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* For two variables X and Y the equations of the regression lines are $9Y - X - 288 = 0$ & $X - 4Y + 38 = 0$. Find

(i) mean values of X & Y.

(ii) coeff of correlation

(iii) the ratio of the SD of Y to that of X.

(iv) Most probable value of Y when $X = 145$

(v) Most probable value of X when $Y = 35$.

$$x - \bar{x} = b_{xy}(y - \bar{y})$$
$$y - \bar{y} = b_{yx}(x - \bar{x})$$

Mean passes thro' two regression lines

(i)

$$-x + 9y - 288 = 0$$
$$x - 4y + 38 = 0$$

Add

$$5y - 250 = 0$$
$$y = \frac{250}{5} = 50$$
$$\Rightarrow \bar{y} = 50.$$

$$-x + 9y - 288 = 0$$
$$-x = 288 - 9y$$
$$x = -288 + 9y$$
$$= -288 + 450$$
$$= 172$$
$$\bar{x} = 172$$

Let $9y - x - 288 = 0$ represent regression line of x on y.

$$x = 9y - 288$$
$$= 9\left(y - \frac{288}{9}\right) \Rightarrow b_{xy} = 9.$$

The line $x - 4y + 38 = 0$ is regression line of y on x.

$$4y = x + 38 \Rightarrow y = \frac{1}{4}(x + 38) \Rightarrow b_{yx} = \frac{1}{4}.$$

$$r^2 = b_{xy} b_{yx} = 9 \times \frac{1}{4} \Rightarrow r = \frac{3}{2} > 1 \text{ (impossible)}$$

$\therefore$ $9y - x - 288 = 0$ is regression line of y on x

& $x - 4y + 38 = 0$ is regression line of x on y

$$9y = x + 288 \Rightarrow y = \frac{1}{9}(x + 288) \Rightarrow b_{yx} = \frac{1}{9}.$$

$$x = 4y - 38 \Rightarrow x = 4\left(y - \frac{38}{4}\right) \Rightarrow b_{xy} = 4.$$

$$r^2 = b_{yx} b_{xy} = \frac{4}{9}$$
$$r = 2/3$$

(iii) $\frac{\sigma_y}{\sigma_x} = ?$

$b_{yx} = \frac{1}{9}$

$r\frac{\sigma_y}{\sigma_x} = \frac{1}{9}$

$\frac{\sigma_y}{\sigma_x} = \frac{1}{9r} = \frac{1}{9\times\frac{2}{3}} = \frac{1}{6}.$

$\begin{array}{r} 288 \\ 145 \\ \hline 433 \end{array}$

Regression line of y on x is $9y - x - 288 = 0$

when $x = 145$,

$9y - 145 - 288 = 0$

$9y = 433 \Rightarrow y = \frac{433}{9}$

Regression line x on y is $x - 4y + 38 = 0$

$x = 4y - 38$

when $y = 35$

$x = 4(35) - 38 = 140 - 38 = 102$

A panel of judges A & B graded seven debators and independently awarded the marks:

(x) Judge A:	40	34	28	30	44	38	31	36
(y) Judge B:	32	39	26	30	38	34	28	?

Eight debator was awarded 36 marks by Judge A while Judge B was not present. If Judge B was also present, how many marks would he allot for the 8 debator.

Soln

Judge A = X & Judge B = Y.

To find regression line of Y on X.

Then substitute X = 36 in the above to find Y.

X	y	$dx = x - 35$	$dy = y - 32$	$dxdy$	dx^2	dy^2
40	32	$\cancel{5}$	0	0	25	0
34	39	-1	7	-7	1	49
28	26	-7	$\cancel{-6}$	42	49	36
30	30	$\cancel{-5}$	$\cancel{-2}$	10	25	4
44	38	9	$\cancel{6}$	54	81	36
38	34	3	$\cancel{2}$	6	9	4
31	28	-4	-4	16	16	16
		0	3	121	206	145

$$\bar{y} = A + \frac{\sum dy}{n}$$

$$= 32 + \frac{3}{7}$$

$$= 32 + 0.4$$

$$= 32.4.$$

$$\bar{X} = 35$$

$$r_{yx} = \frac{N \sum dxdy - \cancel{\sum dx \sum dy}^{\,0}}{N \sum dx^2 - \cancel{(\sum dx)^2}_{\,0}}$$

$$= \frac{\cancel{7}(121)}{\cancel{7}(206)} = 0.5873$$

Regression line of Y on X is

$$y - \bar{y} = b_{yx}(x - \bar{x})$$

$\bar{x} = 35$

$$y - 32.4 = 0.5873x - 0.5873(35)$$

$$y = 0.5873x + 32.4 - 0.5873(35)$$

$$y = 0.5873x + 32.4 - 20.55$$

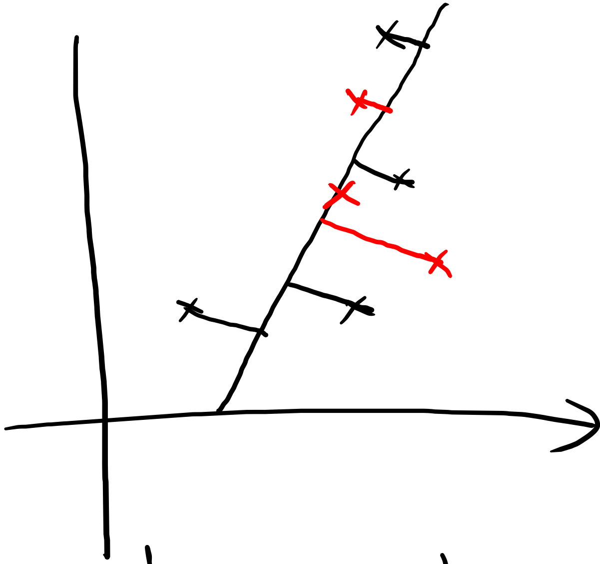
Given $x = 36 \Rightarrow y = 0.5873(36) + 32.4 - 20.55$

$$= 53.54 - 20.55 = 32.99$$

Method of least squares.

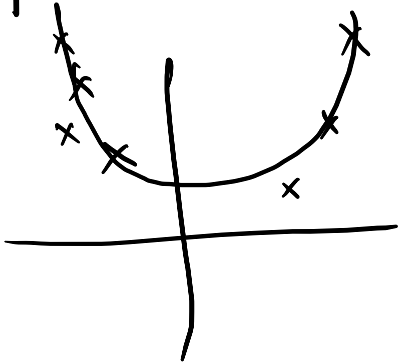
Fit $y = a + bx$

Normal eqns
$$\begin{cases} \sum y = na + b\sum x \\ \sum xy = a\sum x + b\sum x^2 \end{cases}$$



Fit $y = a + bx + cx^2$

Normal eqns
$$\begin{cases} \sum y = na + b\sum x + c\sum x^2 \\ \sum xy = a\sum x + b\sum x^2 + c\sum x^3 \\ \sum x^2 y = a\sum x^2 + b\sum x^3 + c\sum x^4 \end{cases}$$



Fit $y = ae^{bx}$ (exponential curve)

$$y = ae^{bx}$$

$$\log_e y = \log_e (ae^{bx})$$

$$= \log_e a + \log_e e^{bx}$$

α $\cancel{\log_e y} = A + bX$

($A = \log_e a$

$\alpha = \log_e y$)

$$\alpha = A + bX$$

Normal eqns are
$$\begin{cases} \sum_1 \alpha = nA + b\sum_1 X \\ \sum_1 \alpha X = A\sum_1 X + b\sum_1 X^2 \end{cases}$$