30/11/21 Tuesday

Pigeonhole principle :-

If K is a positive integer and K+1 or more objects are placed into K boxes, then there is atleast one box containing two or more objects.

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5 balls

B_1	B_2	B_3
5	0	0
4	1	0
4	0	1
3	2	0
3	0	2
3	1	1
2	3	0
2	0	3
2	2	1
2	1	2
1	4	0
1	0	4
1	3	1
1	1	3
1	2	2
0	5	0
0	0	5

Ex 1 :-

Among 367 people there must be atleast two with the same birth day.

Jan 1 - - - Dec 31 → 365 or 366

Ex 2 :- In any group of 27 English words there must be atleast two that begin with the same letter.

A, B, C - - - Z → Pigeonholes (26)

27 words (Pigeons)

Problem 1 :- How many students must be in a class to guarantee that at least two students will receive the same score in the final exam if the exam is graded on a scale from 0 to 100 points?

Ans :- No. of Pigeonholes = 101

No. of Pigeons = No. of students = 102

Generalized Pigeonhole Principle :-

If N objects are placed into k boxes then there is atleast one box containing at least $\lceil N/k \rceil$ objects.

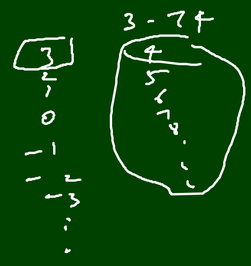
Note :-

Ceiling function — $\lceil x \rceil$ = The least integer greater than or equal to x.

Floor function — $\lfloor x \rfloor$ = The greatest integer less than or equal to x

$x = 3.74$

$\lfloor x \rfloor = 3$ $\quad \lceil x \rceil = 4$



Problem 2 :- What is the minimum number of students required in a discrete Mathematics class to be sure that at least six will receive the same grade, if there are five possible grades? (A, B, C, D, E)

Sol :- Here $N = ?$; $k = 5$; $\lceil N/k \rceil = 6$

(i.e) $\lceil N/5 \rceil = 6$

$5 < \frac{N}{5} \leq 6$

$N = 5 \cdot 5 + 1$

$N = 26$

$\lceil N/5 \rceil = 6$

$\lceil x \rceil = 6$

$5 < x \leq 6$

$\lceil N/5 \rceil = 6$

$\frac{N}{5} \neq 5$

$\lceil N/5 \rceil \neq 5$

$\frac{N}{5} > 5$

$N > 25$

$N = 26$

∴ There are atleast 26 students in discrete maths class to guarantee that atleast six will receive the same grade.

A B C D E

A B C D E
1 2 3 4 5
6 7 8 9 10
11 12 13 14 15
16 17 18 19 20
21 22 23 24 25