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CSE211-Formal Languages and Automata Theory

U2L12 – Greibach Normal Form (GNF)

Dr. P. Saravanan

School of Computing

SASTRA Deemed University

Agenda

- Recap of previous class
 - Chomsky Normal Form
- Greibach Normal Form
 - Definition
 - Steps to convert CFG to a GNF
- Example problems to convert CFG to GNF

Chomsky Normal Form (CNF)

■ Definition:

A grammar G is said to be in *Chomsky Normal form (CNF)*, if the following two conditions hold:

- all its productions are in one of the following two simple forms:
 - $A \rightarrow BC$
 - $A \rightarrow a$

where A , B and C are nonterminals and a is a terminal;
and

- G has no useless symbol.

Ex 1: Converting to CNF

(2) transformation of $E \rightarrow E + T \mid T * F$
 $\mid (E) \mid a \mid b \mid Ia \mid Ib \mid IO \mid I1$

- $\Rightarrow E \rightarrow EPT \mid TMF \mid LER \mid a \mid b \mid IA \mid IB \mid IZ \mid IO$
- $T \rightarrow TMF \mid LER \mid a \mid b \mid IA \mid IB \mid IZ \mid IO$
- $F \rightarrow LER \mid a \mid b \mid IA \mid IB \mid IZ \mid IO$
- $I \rightarrow a \mid b \mid IA \mid IB \mid IZ \mid IO$
- $\Rightarrow E \rightarrow EC_1, C_1 \rightarrow PT,$
 $E \rightarrow TC_2, C_2 \rightarrow MF,$
 $E \rightarrow LC_3, C_3 \rightarrow ER,$
- $\Rightarrow T \rightarrow TC_2, C_2 \rightarrow MF,$
 $T \rightarrow LC_3, C_3 \rightarrow ER,$
- $\Rightarrow F \rightarrow LC_3, C_3 \rightarrow ER,$

$E \rightarrow T \mid E + T$
 $T \rightarrow F \mid T * F$
 $F \rightarrow I \mid (E)$
 $I \rightarrow a \mid b \mid Ia \mid Ib \mid IO \mid I1$

The grammar in CNF

- $\Rightarrow E \rightarrow EC_1 \mid TC_2 \mid LC_3 \mid a \mid b \mid IA \mid IB \mid IZ \mid IO$
- $T \rightarrow TC_2 \mid LC_3 \mid a \mid b \mid IA \mid IB \mid IZ \mid IO$
- $F \rightarrow LC_3 \mid a \mid b \mid IA \mid IB \mid IZ \mid IO$
- $I \rightarrow a \mid b \mid IA \mid IB \mid IZ \mid IO$
- $C_1 \rightarrow PT,$
- $C_2 \rightarrow MF,$
- $C_3 \rightarrow ER,$
- $A \rightarrow a$
 $Z \rightarrow 0$
 $P \rightarrow +$
 $L \rightarrow ($
- $B \rightarrow b$
 $O \rightarrow 1$
 $M \rightarrow *$
 $R \rightarrow)$

Greibach Normal Form

- A production is said to be of the **Greibach normal form** (GNF) if it is of the form

- $A \rightarrow a\alpha$

where a is a **terminal** and α is a string of zero or more **nonterminals**

(OR)

i. $A \rightarrow aX_1...X_n$

ii. $A \rightarrow a$

iii. $S \rightarrow \lambda$

where $X_1...X_n \in V$

Steps to GNF

- **Step 1.** Convert the grammar into CNF.
If the given grammar is not in CNF, convert it to GNF.
- **Step 2.** Eliminate left recursion from grammar if it exists.

$$A \rightarrow A \alpha \mid \beta \quad \xrightarrow{\text{Can be written as}} \quad \begin{aligned} A &\rightarrow \beta A' \\ A' &\rightarrow \alpha A' / \varepsilon \end{aligned}$$

- **Step 3.** Convert the production rules into GNF form
 - Rename the variables like A_1, A_2, \dots, A_n starting with $S = A_1$
 - Modify the rules such that

$A_i \rightarrow A_j X_k \quad j > i$
 - Replace A_j by its terminals

Greibach Normal Form

Example 1:

Step 1

$$\begin{aligned} S &\rightarrow XA \mid BB \\ B &\rightarrow b \mid SB \\ X &\rightarrow b \\ A &\rightarrow a \end{aligned}$$

CNF

Step 2

$$\begin{aligned} S &= A_1 \\ X &= A_2 \\ A &= A_3 \\ B &= A_4 \end{aligned}$$

New
Labels

$$\begin{aligned} A_1 &\rightarrow A_2A_3 \mid A_4A_4 \\ A_4 &\rightarrow b \mid A_1A_4 \\ A_2 &\rightarrow b \\ A_3 &\rightarrow a \end{aligned}$$

Updated CNF

Greibach Normal Form

Example:

Step 3

$$A_1 \rightarrow A_2A_3 \mid A_4A_4$$

$$A_4 \rightarrow b \mid A_1A_4$$

$$A_2 \rightarrow b$$

$$A_3 \rightarrow a$$

$$A_i \rightarrow A_j X_k \quad j > i$$

X_k is a string of zero
or more variables

✗ $A_4 \rightarrow A_1A_4$

Greibach Normal Form

Example:

$$A_i \rightarrow A_j X_k \quad j > i$$

$$A_4 \rightarrow \underline{A_1} A_4 \mid b$$

$$A_4 \rightarrow \underline{A_2} A_3 A_4 \mid A_4 A_4 A_4 \mid b$$

$$A_4 \rightarrow b A_3 A_4 \mid A_4 A_4 A_4 \mid b$$

$$A_1 \rightarrow A_2 A_3 \mid A_4 A_4$$

$$A_4 \rightarrow b \mid A_1 A_4$$

$$A_2 \rightarrow b$$

$$A_3 \rightarrow a$$

Greibach Normal Form

Example:

$$A_1 \rightarrow A_2 A_3 \mid A_4 A_4$$

$$A_4 \rightarrow b A_3 A_4 \mid A_4 A_4 A_4 \mid b$$

$$A_2 \rightarrow b$$

$$A_3 \rightarrow a$$

Eliminate Left
Recursions

$$A \rightarrow A \alpha \mid \beta$$

Can be written as

$$A \rightarrow \beta A'$$

$$A' \rightarrow \alpha A' / \epsilon$$

$$\times \quad A_4 \rightarrow A_4 A_4 A_4$$

Greibach Normal Form

Example:

Second Step

Eliminate Left
Recursions

$$A \rightarrow A \alpha \mid \beta$$

Can be written as

$$A \rightarrow \beta A' \\ A' \rightarrow \alpha A' / \epsilon$$

$$A_4 \rightarrow bA_3A_4 \mid b \mid bA_3A_4Z \mid bZ$$

$$Z \rightarrow A_4A_4 \mid A_4A_4Z$$

$$A_1 \rightarrow A_2A_3 \mid A_4A_4$$

$$A_4 \rightarrow bA_3A_4 \mid A_4A_4A_4 \mid b$$

$$A_2 \rightarrow b$$

$$A_3 \rightarrow a$$

Greibach Normal Form

Example:

$$A_1 \rightarrow A_2 A_3 \mid A_4 A_4$$

$$A_4 \rightarrow b A_3 A_4 \mid b \mid b A_3 A_4 Z \mid b Z$$

$$Z \rightarrow A_4 A_4 \mid A_4 A_4 Z$$

$$A_2 \rightarrow b$$

$$A_3 \rightarrow a$$

$$A \rightarrow aX$$

GNF

Greibach Normal Form

Example:

$$\begin{aligned} A_1 &\rightarrow \underline{A_2}A_3 \mid \underline{A_4}A_4 \\ A_4 &\rightarrow \textcircled{bA_3A_4} \mid b \mid bA_3A_4Z \mid bZ \\ Z &\rightarrow A_4A_4 \mid A_4A_4Z \\ A_2 &\rightarrow b \\ A_3 &\rightarrow a \end{aligned}$$

$$A_1 \rightarrow bA_3 \mid bA_3A_4A_4 \mid bA_4 \mid bA_3A_4ZA_4 \mid bZA_4$$

$$Z \rightarrow bA_3A_4A_4 \mid bA_4 \mid bA_3A_4ZA_4 \mid bZA_4 \mid bA_3A_4A_4 \mid bA_4 \mid bA_3A_4ZA_4 \mid bZA_4$$

Greibach Normal Form

Example:

$$A_1 \rightarrow bA_3 \mid bA_3A_4A_4 \mid bA_4 \mid bA_3A_4ZA_4 \mid bZA_4$$

$$A_4 \rightarrow bA_3A_4 \mid b \mid bA_3A_4Z \mid bZ$$

$$Z \rightarrow bA_3A_4A_4 \mid bA_4 \mid bA_3A_4ZA_4 \mid bZA_4 \mid \\ bA_3A_4A_4 \mid bA_4 \mid bA_3A_4ZA_4 \mid bZA_4$$

$$A_2 \rightarrow b$$

$$A_3 \rightarrow a$$

Grammar in Greibach Normal Form

Ex 2: Convert to GNF

- Find the GNF of the following Grammar

$$S \rightarrow AB$$
$$A \rightarrow BS \mid b$$
$$B \rightarrow SA \mid a$$

Summary

- Greibach Normal Form
 - Definition
 - Steps to convert CFG to a GNF
- Example problems to convert CFG to GNF

References

- John E. Hopcroft, Rajeev Motwani and Jeffrey D. Ullman, *Introduction to Automata Theory, Languages, and Computation*, Pearson, 3rd Edition, 2011.
- Peter Linz, *An Introduction to Formal Languages and Automata*, Jones and Bartle Learning International, United Kingdom, 6th Edition, 2016.

Next Class

Greibach Normal Form

Thank you.