ASSIGNMENT

I. 2 Marks:

1) What is LPP?

It is a mathematical method used to optimize complex system where resources are limited

It involves maximizing and minimizing a linear objective function.

2) What is feasible solution?

It satisfies all of the constraints of the LPP but may no necessarily optimize the objective funct

3) What is optimal solution?

It is not only satisfies all the constraints but also max or min the objective functi representing the best possible outcome.

4) What is unrestricted varible?

i) It can be either positive, negative or zero

ii) It is expressed as the difference between two non-negative variable

$$x = y_1 - y_2 \quad , \quad y_1, y_2 \geq 0$$

5) What is Optimization:

It refers to the process of finding the best solution from a set of feasible alternatives, typically to max or min an objective function.

II 10 Marks:

1) Graphical Method:

Maximize $Z = 5x_1 + 4x_2$

S.T

$6x_1 + 4x_2 \leq 24$

$x_1 + 2x_2 \leq 6$

$-x_1 + x_2 \leq 1$

$x_2 \leq 2$

$x_1, x_2 \geq 0$

Sol:

$Z = 5x_1 + 4x_2$

put $x_1 = 0$

$6x_1 + 4x_2 = 24$

$0 + 4x_2 = 24$

$x_2 = 6$

$(0, 6)$

put $x_2 = 0$

$6x_1 + 4x_2 = 24$

$6x_1 = 24$

$x_1 = 4$

$(4, 0)$

$x_1 + 2x_2 = 6$

put $x_1 = 0$

$2x_2 = 6$

$x_2 = 3$

$(0, 3)$

put $x_2 = 0$

$x_1 = 6$

$(6, 0)$

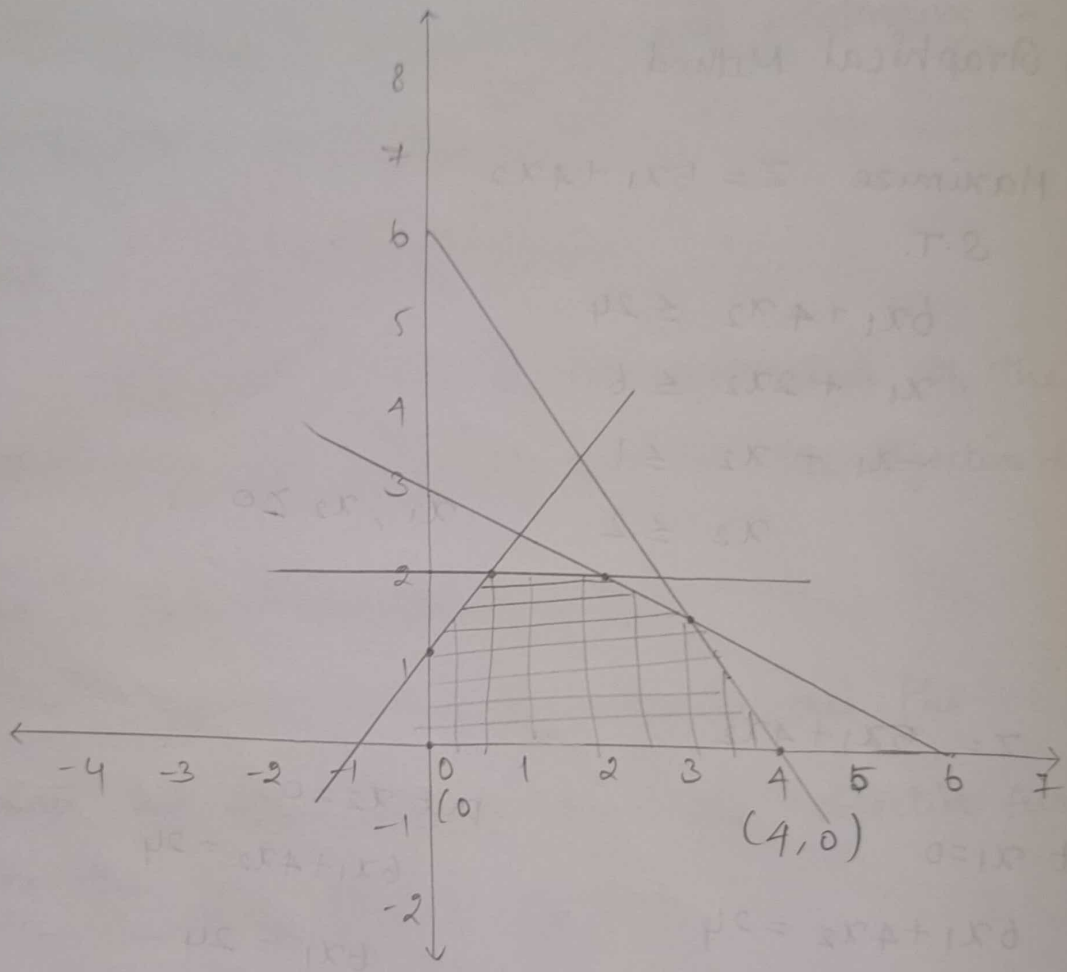
$-x_1 + x_2 = 1$

$x_1 = 0$ $\quad$ $x_2 = 0$

$x_2 = 1$ $\quad$ $x_1 = -1$

$(0, 1)$ $\quad$ $(-1, 0)$

$x_2 = 2 \Rightarrow (0, 2)$



$6x_1 + 4x_2 = 24$ —①

$x_1 + 2x_2 = 6$ —②

① $\Rightarrow$ $6x_1 + 4x_2 = 24$

② $\times 2 \Rightarrow$ $2x_1 + 4x_2 = 12$

(−) (−) (−)

$4x_1 = 12$

$x_1 = 3$

$x_1 = 4$ sub in ①

$6(3) + 4x_2 = 24$

$18 + 4x_2 = 24$

$4x_2 = 24 - 18$

$4x_2 = 6$

$x_2 = \frac{6}{4} = 1.5$

$(3, 1.5)$

$x_1 + 2x_2 = 6$ — (5)

$x_2 = 2$ — (4)

$x_1 + 2(2) = 6$

$x_1 = 6 - 4$

$x_1 = 2$ $\quad (2, 2)$

$-x_1 + x_2 = 1$ — (3)

$x_2 = 2$ — (4)

$-x_1 + 2 = 1$

$-x_1 = -1$

$x_1 = 1$ $\quad (1, 2)$

$Z = 5x_1 + 4x_2$

Corner values		Values of Z
(0,0)	$\Rightarrow 5(0) + 4(0) \Rightarrow$	$Z = 0$
(4,0)	$\Rightarrow 5(4) + 4(0) \Rightarrow$	$Z = 20$
(3,1.5)	$\Rightarrow 5(3) + 4(1.5) \Rightarrow$	$Z = 21$ ←
(2,2)	$\Rightarrow 5(2) + 4(2) \Rightarrow$	$Z = 18$
(1,2)	$\Rightarrow 5(1) + 4(2) \Rightarrow$	$Z = 13$
(0,1)	$\Rightarrow 5(0) + 4(1) \Rightarrow$	$Z = 4$

Max $Z = 21$

3) Simple Method:

Max $Z = 5x_1 + 7x_2$

S.T

$x_1 + x_2 \leq 4$

$3x_1 + 8x_2 \leq 24$

$10x_1 + 7x_2 \leq 35$ $\quad x_1, x_2 \geq 0$

Sol:

Max $Z = 5x_1 + 7x_2 + 0S_1 + 0S_2 + 0S_3$

$x_1 + x_2 + s_1 = 4$

$3x_1 + 8x_2 + s_2 = 24$

$10x_1 + 7x_2 + s_3 = 35$

		c_j	[5	7	0	0	0]	
CB	Y_B	X_B	x_1	x_2	s_1	s_2	s_3	ratio
0	s_1	4	1	1	1	0	0	4
0	s_2	24	3	[8] pivot	0	1	0	3 ←
0	s_3	35	10	7	0	0	1	5
	$Z_j - c_j$		-5	-7 ↑	0	0	0	

		c_j	[5	7	0	0	0]	
CB	Y_B	X_B	x_1	x_2	s_1	s_2	s_3	ratio
0	s_1	1	[5/8] pivot	0	1	-1/8	0	8/5 = 1.64
7	x_2	3	3/8	1	0	1/8	0	8
0	s_3	14	59/8	0	0	-7/8	1	112/59 = 1.89
	$Z_j - c_j$		$\frac{-19}{8}$ ↑	0	0	$\frac{7}{8}$	0	

	C_j	5	7	0	0	0
C_B	Y_B / X_B	x_1	x_2	s_1	s_2	s_3
5	x_1	8/5	1	0	8/5	-1/5	0
7	x_2	12/5	0	1	-3/5	1/5	0
0	s_3	11/5	0	0	-59/5	6/10	1
	$Z_j - C_j$	0	0	$\frac{19}{5}$	$\frac{6}{5}$	0

$Z = \frac{124}{5}$

3) Big M Method:

Minimize $Z = 4x_1 + 3x_2$

S.T

$$2x_1 + x_2 \geq 10$$
$$-3x_1 + 2x_2 \leq 6$$
$$x_1 + x_2 \geq 6 \quad \text{and} \quad x_1, x_2 \geq 0$$

Max $Z^* = -4x_1 - 3x_2$

S.T

$$2x_1 + x_2 \geq 10$$
$$-3x_1 + 2x_2 \leq 6$$
$$x_1 + x_2 \geq 6 \quad \text{and} \quad x_1, x_2 \geq 0$$

$$\text{Max } Z^* = -4x_1 - 3x_2 + 0s_1 + 0s_2 + 0s_3 - MA_1 - MA_2$$

S.T

$$2x_1 + x_2 - s_1 + 0s_2 + 0s_3 + A_1 = 10$$

$$-3x_1 + 2x_2 + 0s_1 + s_2 + 0s_3 = 6$$

$$x_1 + x_2 + 0s_1 + 0s_2 - s_3 + A_2 = 6$$

			C_j	[-4	-3	0	0	0	-M	-M]	
C_B	Y_B	X_B	x_1	x_2	s_1	s_2	s_3	A_1	A_2	Ratio
-M	A_1	10	[2]	1	-1	0	0	1	0	5 ←
0	s_2	6	-3	2	0	1	0	0	0	-
-M	A_2	6	1	1	0	0	-1	0	1	6
	$Z_j^* - C_j$		-3M+4	-2M+3	M	0	M	0	0	
			↑							

			C_j	[-4	-3	0	0	0	-M]	
C_B	Y_B	X_B	x_1	x_2	s_1	s_2	s_3	A_2	Ratio
-4	x_1	5	1	1/2	$\frac{-1}{2}$	0	0	0	10
0	s_2	21	0	7/2	-3/2	1	0	0	42/7
-M	A_2	1	0	[1/2]	1/2	0	-1	1	2 ←
	$Z_j^* - C_j$		0	$\frac{-M+2}{2}$	$\frac{-M+4}{2}$	0	M	0	
				↑					

		c_j	[-4	-3	0	0	0]
CB	Y_B	X_B	x_1	x_2	s_1	s_2	s_3
-4	x_1	4	1	0	-1	0	1
0	s_2	14	0	0	-5	1	7
-3	x_2	2	0	1	1	0	-2
$Z_j^* = -22$		$z_j^* - c_j$	0	0	1	0	2

Max $z^* = -22$

Min $Z = -(-22) \Rightarrow 22$.

4) Big M method:

Minimize $Z = 4x_1 + x_2$

S.T

$3x_1 + x_2 = 3$

$4x_1 + 3x_2 \geq 6$

$x_1 + 2x_2 \leq 4$ and $x_1, x_2 \geq 0$.

Max $z^* = -4x_1 - x_2 + 0S_1 + 0S_2 - MA_1 - MA_2$

$3x_1 + x_2 + A_1 = 3$

$4x_1 + 3x_2 - S_1 + A_2 = 6$

$x_1 + 2x_2 + S_2 = 4$

~~The objective function is~~

			c_j	[-4	-1	0	0	-M	-M]	
C_B	Y_B	X_B	x_1	x_2	s_1	s_2	A_1	A_2	Ratio
-M	A_1	3	3 (pivot)	1	0	0	1	0	1 ←
-M	A_2	6	4	3	-1	0	0	1	1.5
0	s_2	4	1	2	0	1	0	0	4
	$z_j^*-c_j$		-7M-4	-4M-1	M	0	0	0	

↑

			c_j	[-4	-1	0	0	-M	-M]	
C_B	Y_B	X_B	x_1	x_2	s_1	s_2	A_1	A_2	ratio
-4	x_1	1	1	1/3	0	0	1/3	0	3
-M	A_2	2	0	5/3	-1	0	-4/3	1	$\frac{6}{5}=1.2$ ←
0	s_2	3	0	5/3	0	1	-1/3	0	$\frac{9}{5}=1.8$
	$z_j^*-c_j$		~~-4M~~ -1-5M	0	$\frac{-1}{3}-\frac{5M}{3}$	M	0	$\frac{-4}{3}+\frac{7M}{3}$	0

↑

			c_j	[-4	-1	0	0	-M	-M]	
C_B	Y_B	X_B	x_1	x_2	s_1	s_2	A_1	A_2	ratio
-4	x_1	3/5	1	0	1/5	0	3/5	-1/5	3
-1	x_2	6/5	0	1	-3/5	0	-4/5	3/5	-2
0	s_2	1	0	0	1 (pivot)	1	1	-1	1 ←
	$z_j^*-c_j$		0	0	$\frac{-1}{5}$	0	$\frac{-8}{5}+M$	$\frac{1}{5}+M$	

↑

CB	YB	XB	x_1	x_2	S_1	S_2	A_1	A_2	θ
-4	x_1	2/5	1	0	0	-1/5	2/5	0	
-1	x_2	9/5	0	1	0	3/5	-1/5	0	
0	S_1	1	0	0	1	1	1	-1	
	$Z_j^* - c_j$		0	0	0	1/5	$\frac{-7}{5}+M$	M	

$$Z^* = -\frac{17}{5}$$

$$\text{Max } Z^* = -\frac{17}{5}$$

$$\text{Min } Z = -\left(-\frac{17}{5}\right)$$

$$\text{Min } Z = 17/5$$

5) Graphical Method:

$$\text{Max } Z = 10x_1 + 6x_2$$

S.T

$$5x_1 + 3x_2 \leq 30$$

$$x_1 + 2x_2 \leq 18 \quad \text{and} \quad x_1, x_2 \geq 0$$

$$\text{Max } Z = 2(5x_1 + 3x_2)$$

S.T

$$5x_1 + 3x_2 = 30$$

$$x_1 + 2x_2 = 18$$

$$5x_1 + 3x_2 = 30$$

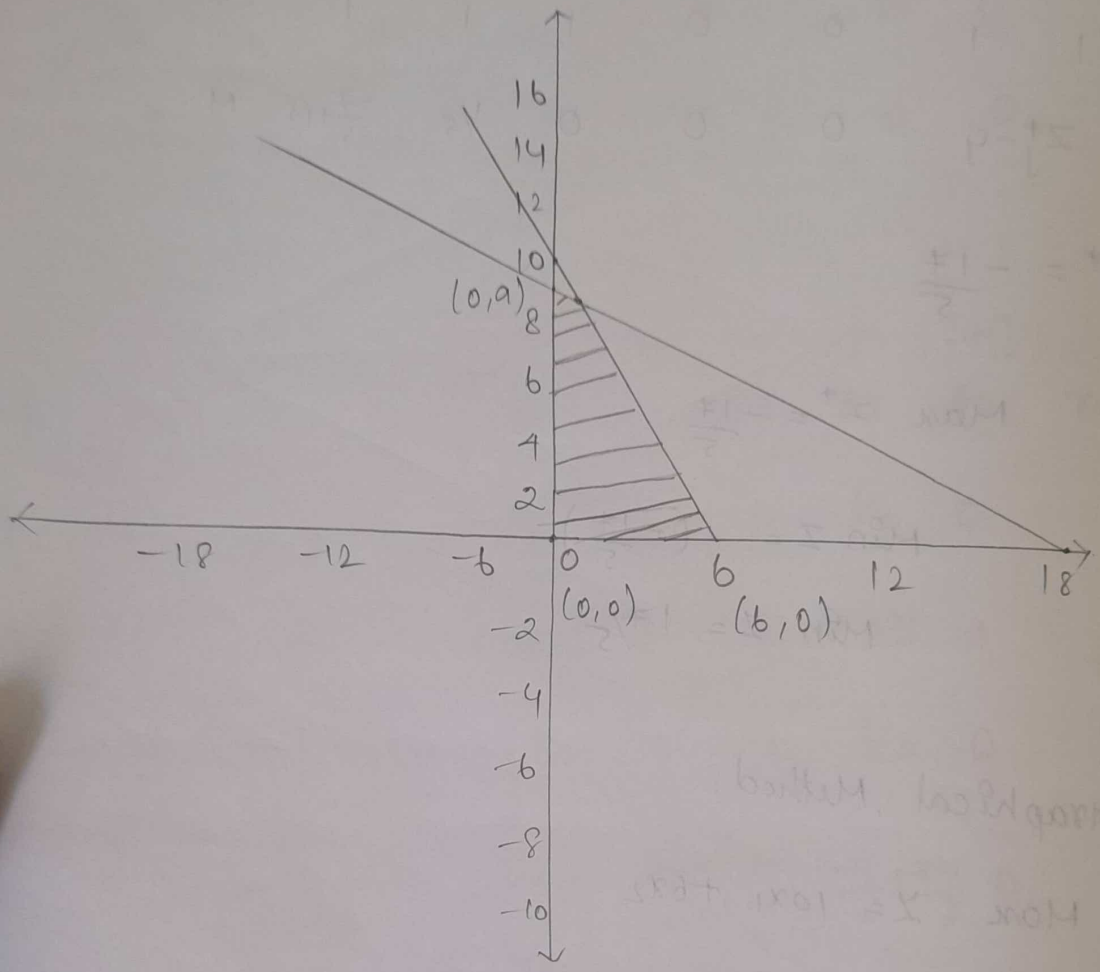
$x_1 = 0 \Rightarrow x_2 = 10$ (0, 10)

$x_2 = 0 \Rightarrow x_1 = 6$ (6, 0)

$x_1 + 2x_2 = 18$

$x_1 = 0 \Rightarrow x_2 = 9 \quad (0, 9)$

$x_2 = 0 \Rightarrow x_1 = 18 \quad (18, 0)$



Equating ① & ②

$$5x_1 + 3x_2 = 30$$
$$-5x_1 - 10x_2 = -90$$

$$-7x_2 = -60$$

$$x_2 = 60/7$$

$$x_1 + 2\left(\frac{60}{7}\right) = 18$$

$$x_1 = \frac{6}{7} \qquad \left(\frac{6}{7}, \frac{60}{7}\right)$$

$$Z = 2(5x_1 + 3x_2)$$

Corner value		Z value
$(0,0)$	$\Rightarrow Z = 2(0+0) \Rightarrow$	0
$(6,0)$	$\Rightarrow Z = 2(5(6)+0) \Rightarrow$	60 ←
$(0,9)$	$\Rightarrow Z = 2(0+3(9)) \Rightarrow$	54
$\left(\frac{6}{7}, \frac{60}{7}\right)$	$\Rightarrow Z = 2\left(5\left(\frac{6}{7}\right)+3\left(\frac{-60}{7}\right)\right) \Rightarrow$	42.85

Max $Z = 60$.

$x_1 = 6 \quad x_2 = 0$.