

CSE211 - Formal Languages and Automata Theory

Unit 1-L5: Grammars and Dérivations Problems Part 2

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Agenda

- Recap-Grammar and Derivation
- Formal definition of grammar
- Production rule –def.
- Formal definition of language
- How to find Language generated by grammar?
- How to find Grammar to generate language?



Formal Definition: Grammar

A grammar G is defined as a quadruple

$$G = (V, T, S, P),$$

where V is a finite set of objects called variables,

T is a finite set of objects called terminal symbols,

S ∈ V is a special symbol called the start variable,

P is a finite set of productions.

It will be assumed that the sets V and T are nonempty and disjoint.



Production Rule

- The heart of a grammar
- All production rules are of the form

$$x \rightarrow y$$

- where x is an element of $(\underline{V} \cup \underline{T})^{\pm}$ and
- y is in (<u>V</u> ∪ <u>T</u>)*
- Based on the restriction of variables and terminals the on the left or right side of the production it can be
 - Regular
 - Context-free,
 - Context-sensitive or
- Dr.PS Unrestricted grammar



Formal Def.: Language

Let G = (V, T, S, P) be a grammar. Then the set,

$$L(G) = \{w \colon \underbrace{S \Rightarrow w}, \quad w \in T^*\}$$
 is the language generated by G.

■ If w ∈ L (G), then the sequence/

$$\underline{S} \Longrightarrow \underline{\bar{w}_1} \Longrightarrow \underline{w_2} \Longrightarrow \underline{w_3} \Longrightarrow \underline{w_4} \Longrightarrow \underline{w}$$

is a derivation of the sentence w. The strings

which contain variables as well as terminals, are called sentential forms of the derivation.



Find the grammar

$$L = \{a^n : n > 0\}$$

$$\frac{1}{2} = \left(\frac{1}{2} \right) \cdot \pi = 0, 1, 2 \dots$$

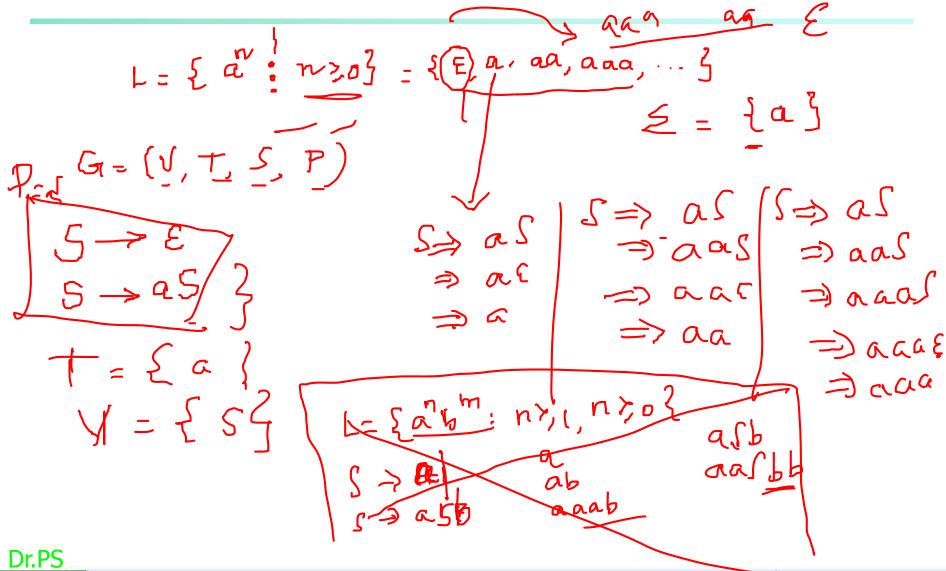
The production Set is

$$P = \{S \rightarrow E\}$$
 $\{S \rightarrow S \in S \rightarrow S \in S \rightarrow S \in S \rightarrow S \}$

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Find the Grammar-Problems





Find the grammar for L= { on : n is even } $5d9.5 = {a3} n = 0, 2, 4, 6...$ We have grammar for at as $S \rightarrow aS/E$: The productions for (aa)* is S-> aas/E .. The grammar for the given language is $G = \{ \{ s \}, \{ a \}, \{ s \rightarrow aa \}, s \rightarrow \epsilon \} \}$



Find the grownnar for L= & an: nie odd?

Santon Z = { a } n = 1,3,5,7,9,11... L= {a, aaa, aaaaa,....} = a(aa) For $(aa)^{*}$ we have written $\beta = \frac{5}{5}$ on $as/\frac{5}{5}$.. The productions for the given language are $S \rightarrow aA = \frac{5}{4}$ $A \rightarrow aaA/E$.. The grannmar for the given I Enguerye 15 $G = \{ S, A^2, A^2, S, S > aA, A > acA/E \}$

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tind the grammar for $L = \{ a^{n} b^{m} : n > 1, m > 0 \}$ $Solution \leq = \{ a_{1} b^{3} \mid n = 1, 2, 3, ... \mid m = 0, 1, 2, 3 ... \}$ L={a, ab, aa, aab, aabb, } A -> no of all with min 1 an bon sy AB .. A saAla 8 - s min of bir min o with min with min : B > 6B/E Zera b :- The total production set is to be concatenated P= { 5 -> AB A -> aA/a B -> bB/E } Let starting symbol is s Ench string has two parts SO, SO AB -> for b's







S = I

$$L = \left\{ \frac{\alpha^{h}b^{n}}{A} c^{m}d^{m} : n, m > 0 \right\}$$

$$5 \rightarrow AB$$

$$A \rightarrow aAb/E$$

$$B \rightarrow cBd/E$$

$$7 = \{a, b, c, d\}$$
, $n = 0, 1, 2 - \cdots$, $m = 0, 1, z - \cdots$

The grammar for the given lamper of
$$G_1 = \{ V_1, T_2, S_1, P_2 \}$$

$$V = \{ S_1, A_2, B_3 \}$$

$$T = \{ a_1, b_2, c_2, d_3 \}$$







Find the grammar that generates the strings of a's & b's
starts and ends with some letter L={a,b,aa,bb,aa,bbb,aba,bab,babab,aabbba,...} Creveral Form a (a)b) to b (a/b) b (a)b S-> aAa/bAb/a/b s> aaa S=) aAa

DabAa

DabAa

DabAa

DabAa

DabAa

DabAa A -> aA/bA/E



Find the Grammar

Find the grammar that generates

$$L = \{0^n 1^n : n \ge 0\}$$

$$S \rightarrow 0S1$$

$$S \to \lambda$$

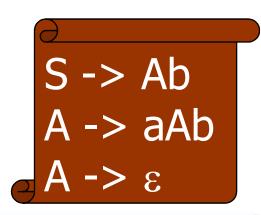




Find the Grammar

Find the grammar that generates

$$L = \{a^n b^{n+1} : n \ge 0\}$$

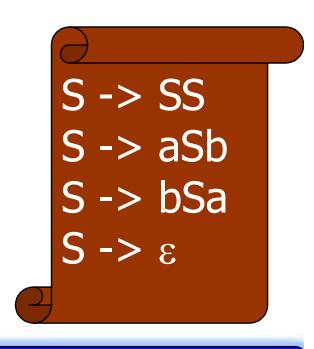




Find the Grammar

Let ∑ = {a, b}, and let n_a (w) and n_b (w) denote the number of a's and b's in the string w, respectively. Find the grammar G which generates

$$L = \{w : n_a(w) = n_b(w)\}$$





Summary

- Why Grammars for a language
- What is a grammar?
- What is derivation?
- What is sentential form?
- Language generated by grammar
- Grammar to generate language



References

- John E. Hopcroft, Rajeev Motwani and Jeffrey D.
 Ullman, Introduction to Automata Theory, Languages, and Computation, Pearson, 3rd Edition, 2011.
- Peter Linz, An Introduction to Formal Languages and Automata, Jones and Bartle Learning International, United Kingdom, 6th Edition, 2016.



Next Class:

Types and Chomsky hierarchy of Grammar

THANK YOU.