CSE211 – Formal Languages and Automata Theory

U1L9 – Non-Deterministic Finite Automata

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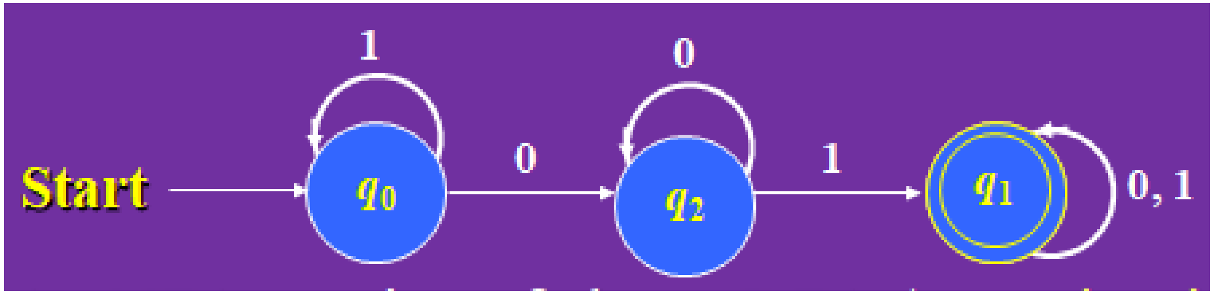
Agenda

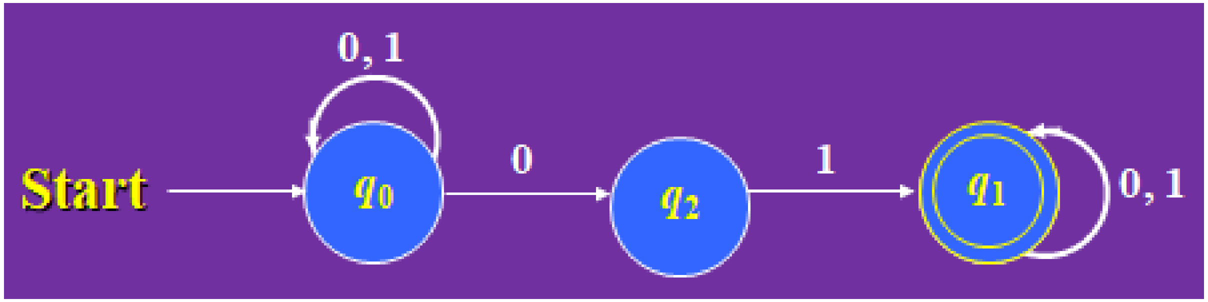
- Recap of previous class
- What is Non-deterministic Finite Automata(NFA)?
- Examples for NFA
- Definition of NFA
- Epsilon-NFA
- E-closure of a state in e-NFA
- Example for identifying e-closure

Nondeterministic Finite Automata (NFA)

- DFA for $L = \{x01y \mid x \text{ and } y \text{ are any strings of 0's and 1's}\}$



- NFA for the above DFA



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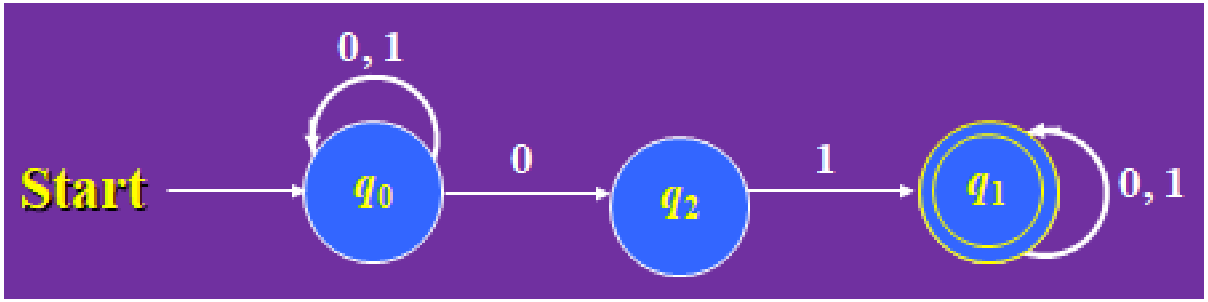
Nondeterministic Finite Automata (NFA)

- Some properties of NFA's



  - Some transitions may "die," like $\delta(q_2, 0)$.
  - Some transitions have multiple choices, like

    $\delta(q_0, 0) = q_0$ and q_2.

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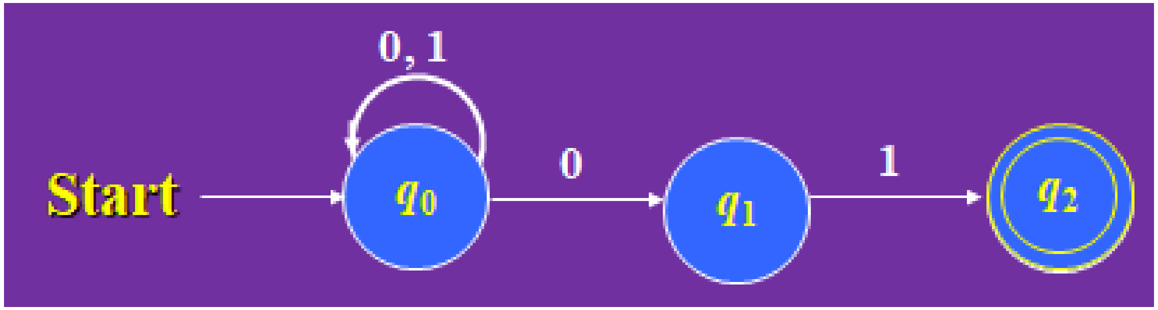
NFA - Example

- Design an NFA accepting the following language

  L = {w | w ∈ {0, 1}* and ends in 01}.



- Non-determinism creates many transition paths
- But if there is one path leading to a final state, then the input is accepted
- Check the input x = 1000101

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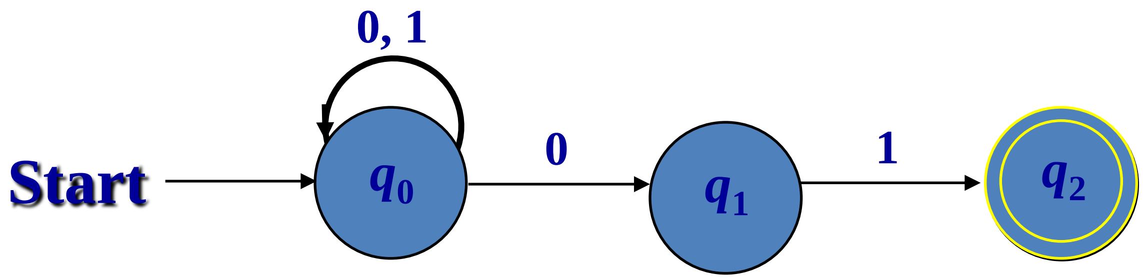
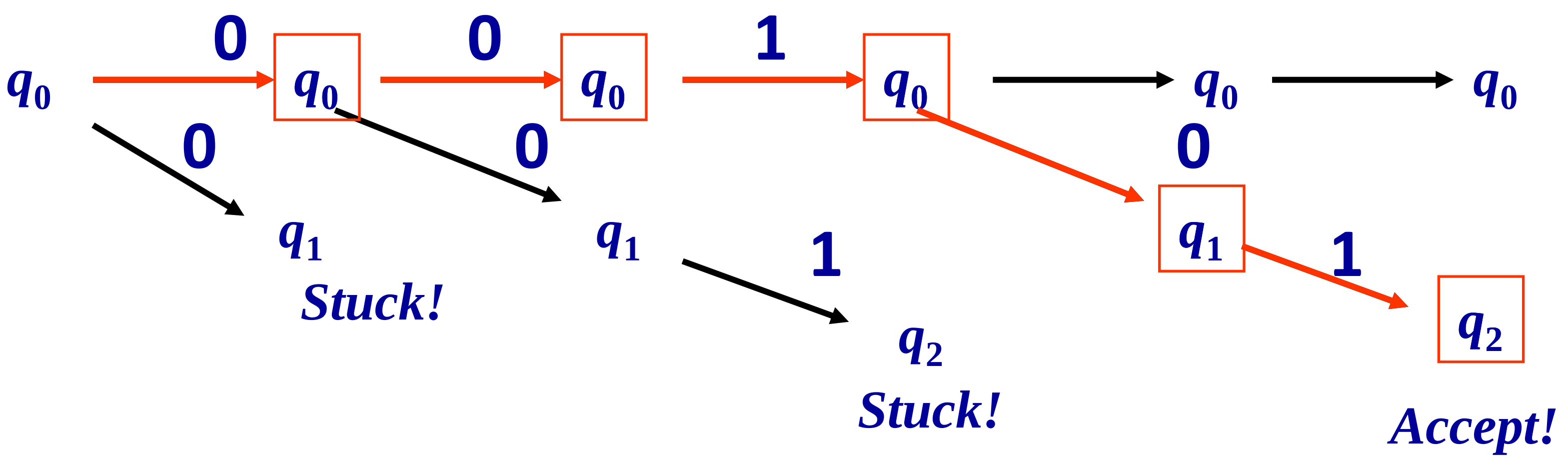
NFA Example

- Example



- When input x = 00101, the NFA processes x in the following way:



Nondeterministic Finite Automata

Definition of NFA

- An NFA A is a 5-tuple $A = (Q, \Sigma, \delta, q_0, F)$ where
  - Q = a finite (nonempty) set of states;
  - Σ = a finite (nonempty) set of input symbols;
  - q_0 = a start state;
  - F = a set of (nonempty) final or accepting states;
  - $\delta: Q \times \Sigma \rightarrow 2^Q$ is a transition function

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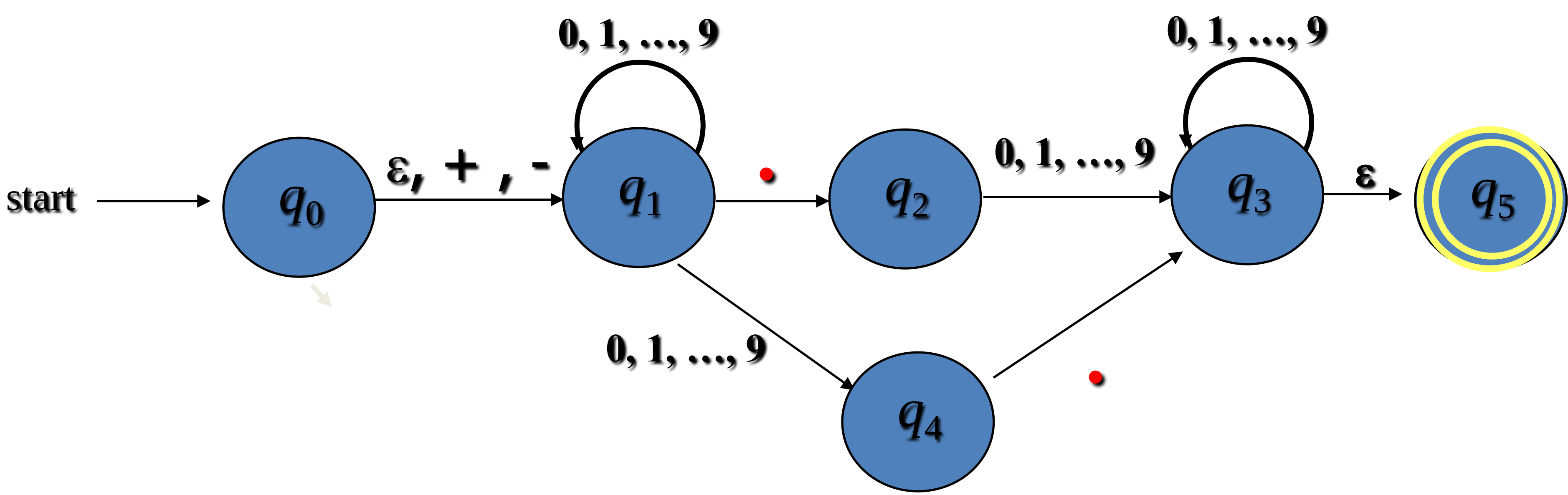
Finite Automata with Epsilon-Transitions

- **Use of ε-transitions**
  - We allow the automaton to accept the empty string ε.
  - This means that a transition is allowed to occur without reading in a symbol.
  - The resulting NFA is called ε-NFA.
  - It adds "programming (design) convenience" (more intuitive for use in designing FA's)
  - But,
    - ε cannot be used as an input symbol, but can be accepted to yield a transition!
    - $\delta(q_i, \varepsilon)$ is defined for every state q_i

Finite Automata with Epsilon-Transitions

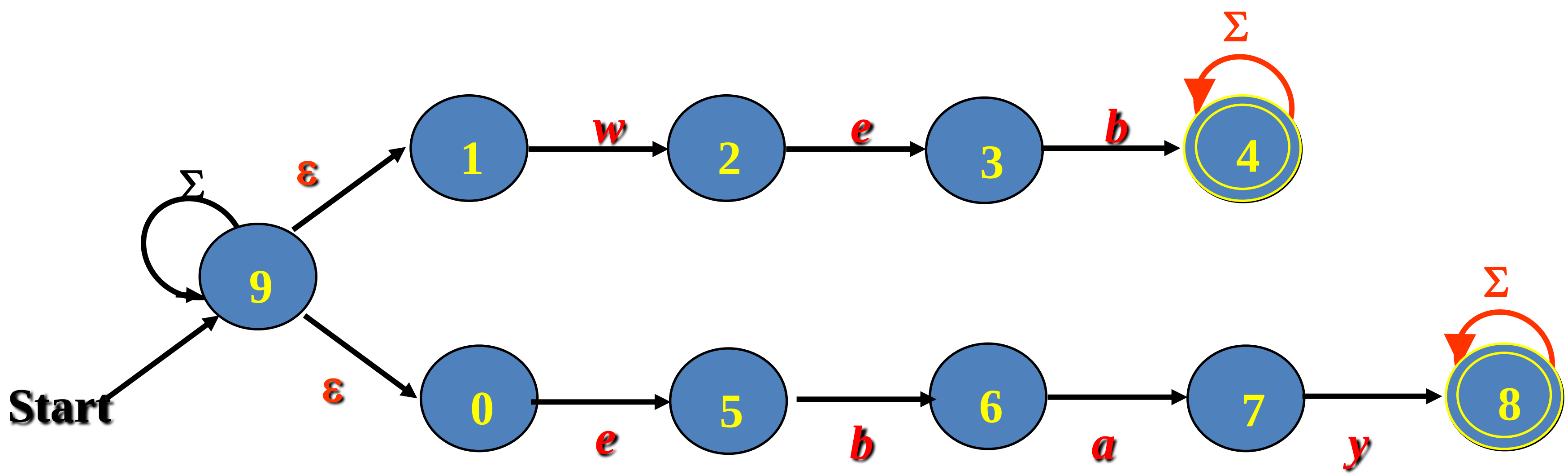
- Example : an ε-NFA accepting decimal numbers like 2.15, .125, +1.4, -0.501…



- To accept a number like "+5." (nothing after the decimal point), we have to add q_4.

Finite Automata with Epsilon-Transitions

- Example : a more intuitive ε-NFA for Example 2.14



Design an NFA with ε-transitions.

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Finite Automata with Epsilon-Transitions

Formal Notation for an ε-NFA

- Definition: an ε-NFA A is denoted by $A = (Q, \Sigma, \delta, q_0, F)$
  - Q = a finite (nonempty) set of states;
  - Σ = a finite (nonempty) set of input symbols;
  - q_0 = a start state;
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  - $\delta: Q \times \Sigma \cup \{\varepsilon\} \rightarrow 2^Q$ is a transition function

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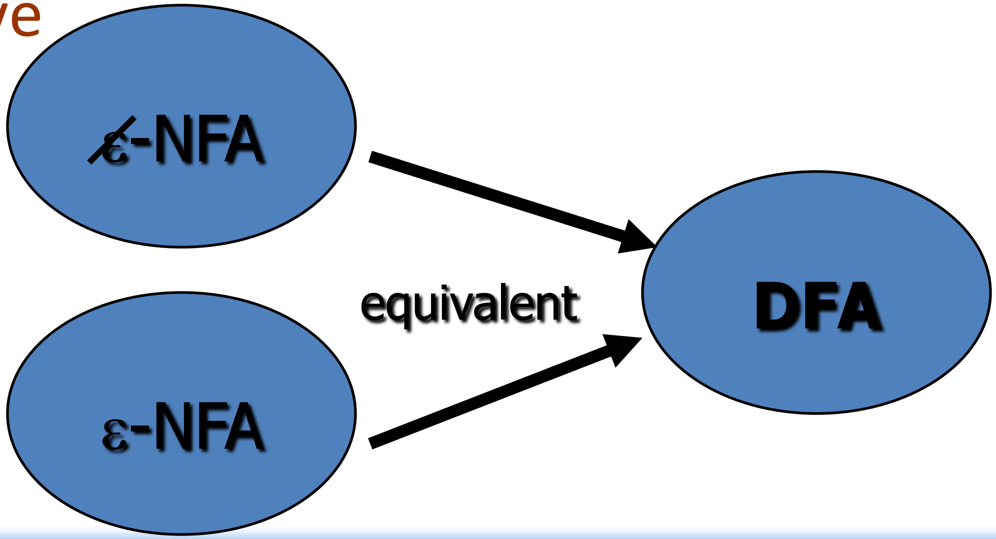
Review

3 Types of Automata

- DFA ---good for soft/hardware implementation
  - $\delta: \mathcal{Q} \times \Sigma \rightarrow \mathcal{Q}$ is the transition function
- NFA ---intermediately intuitive
  - $\delta: \mathcal{Q} \times \Sigma \rightarrow 2^{\mathcal{Q}}$ is the transition function
- ε-NFA ---most intuitive
  - $\delta: \mathcal{Q} \times \Sigma \cup \{\varepsilon\} \rightarrow 2^{\mathcal{Q}}$ is the transition function



Summary

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- Examples for NFA
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- E-closure of a state in e-NFA
- Example for identifying e-closure

References

- John E. Hopcroft, Rajeev Motwani and Jeffrey D. Ullman, *Introduction to Automata Theory*, Languages, and Computation, Pearson, 3rd Edition, 2011.
- Peter Linz, An Introduction to Formal Languages and Automata, Jones and Bartle Learning International, United Kingdom, 6th Edition, 2016.

Next Class:

Converting NFA or e-NFA to DFA

THANK YOU.