

04/12/21 Saturday

$k+1$  or more objects are placed into  $k$  boxes then at least one box containing 2 or more objects.

QHP: If  $N$  objects are placed into  $k$  boxes then there is at least one box containing  $\lceil N/k \rceil$  objects.

What is the least number of area codes needed to guarantee that the 25 million phones in a state can be assigned distinct 10-digit telephone numbers? (Assume that telephone numbers are of the form XXX-XXX-XXXX, where the first three digits form the area code,  $N$  represents a digit from 2 to 9 inclusive, and  $X$  represents any digit.)

Example

$$\boxed{NXX} - \boxed{NXX-XXXX} \quad (2 \text{ to } 9)$$

8	10	10	10	10	10	10
N	X	X	X	X	X	X

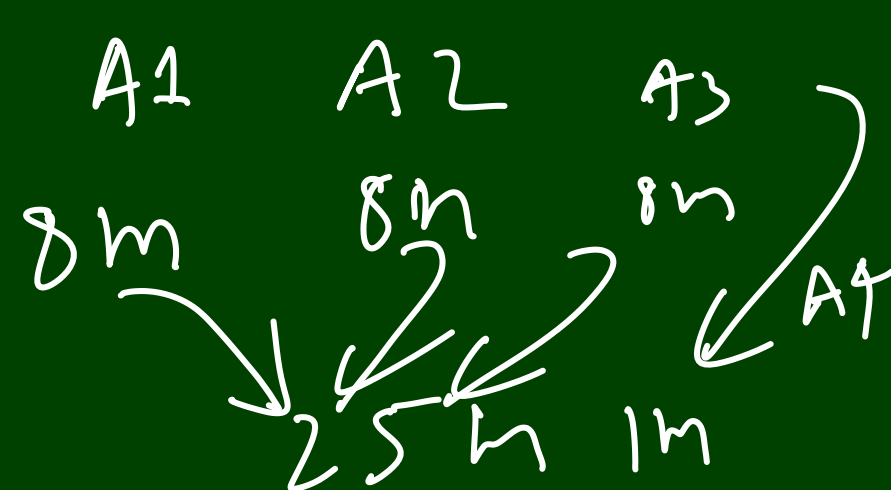
$= 8,000,000$

25 million phones  
8 million phone numbers.

25m objects among 8m boxes

$$\left\lceil \frac{25,000,000}{8,000,000} \right\rceil = \lceil 3.025 \rceil = 4$$

We need 4 area codes to guarantee that 25m phones in a state can be assigned different telephone numbers.



During a month with 30 days, a baseball team plays at least one game a day, but no more than 45 games. Show that there must be a period of some number of consecutive days during which the team must play exactly 14 games.

Sol: Let  $a_j$  be the number of games played on or before  $j^{\text{th}}$  day of the month.

Then  $a_1, a_2, \dots, a_{30}$  is an increasing sequence of  $i^{\text{th}}$  integers with  $1 \leq a_i \leq 45$ .

$$\begin{aligned} 1 &\rightarrow 3 \\ a_1 &= 3 \\ a_2 &= 4 \\ a_3 &= \end{aligned}$$

$\Rightarrow a_1+14, a_2+14, \dots, a_{30}+14$  is also an increasing sequence of  $i^{\text{th}}$  integers with  $15 \leq a_i+14 \leq 59$

The 60 positive integers  $a_1, a_2, \dots, a_{30}, a_1+14, \dots, a_{30}+14$  are all less than or equal to 59

$\Rightarrow$  By Pigeonhole Principle two of the integers are equal

$\Rightarrow$  There must be indices  $i < j$

s.t.  $a_i = a_j + 14$

$\Rightarrow$  There is exactly 14 games are played b/w  $j+1^{\text{st}}$  day to  $i^{\text{th}}$  day

