

FUNDAMENTALS

OF PHYSICS

- * Concepts like mechanics, electrical, electronics, magnetics, thermodynamics, semiconductors and optics play a role of great importance in the process of innovation & development.
- * Everything around us uses energy in one way or other. Search for new tech, to enhance by modifying properties like internal & external parameters like ext force, temp, chem, struct, etc.
- * Physics - natural science that involves study of matter and its motion and behavior through space and time with related compounds like Energy and force.

UNIT - 1

WAVES AND OSCILLATION

Depending on the system, force value changes

$F = ma$
$F = m \frac{d^2x}{dt^2}$

$$\int m \frac{d^2x}{dt^2} = \int \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} = \int -kx$$
$$= \int \frac{GmM}{R^2} = \int$$

PERIODIC MOTION:

Periodic motion can cause disturbances that move through a medium in the form of a wave

WAVE MOTION:

- Wave is a motion of a disturbance
- Mechanical waves require:
 - some source of disturbance
 - medium that can be disturbed
 - phy connection or mechanism through which adjacent portions of the medium influence each other
 - waves carry energy & momentum.

Waves \rightarrow Transverse wave
 \rightarrow Longitudinal wave

- Transverse waves - each element ~~to the wave~~ that is disturbed moves in a direction \perp to wave motion.
- Longitudinal aka compression waves - the elements of medium undergo displacements parallel to the motion of the wave.

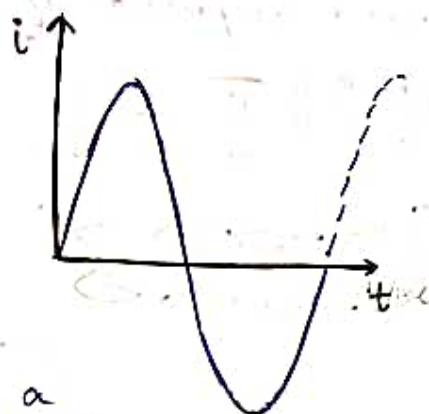
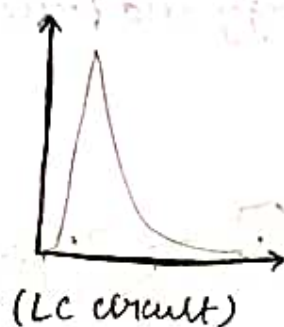
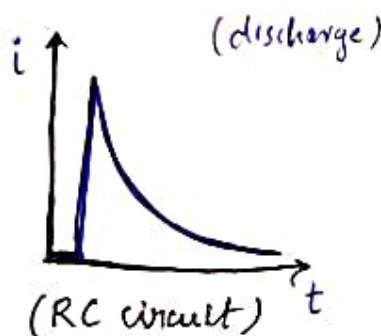
(Particular type of periodic motion)

★ SIMPLE HARMONIC MOTION:



LC oscillator

$$V_L = -L \frac{di}{dt}$$



→ In a clock when current flows through a quartz crystal which is given a force and vibrates. Vibration rate is controlled by a microchip which is connected to a motor that converts it to circulatory motion.

- Periodic motion is a motion that regularly returns to a given position after a fixed time interval.
- SHM occurs when the net force acting along obeys "Hookes law".
i.e. $F = -kx$
- In SHM, force acting on the object is proportional to the position of the object about some equilibrium position.

NOTE: [Force is always directed towards equilibrium position]

Ex: Spring mass system.

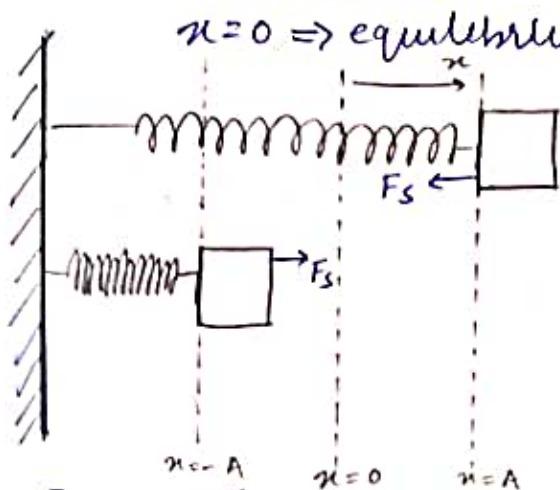
- Clock - piezoelectric principle

(when F is applied - gives rise to V difference)

[voltage → quartz crystal → vibrates → microchip controls the vib]
oscillate with same \leftarrow stepper motor \leftarrow

[law] $F_s = -kx$

k - spring constant
defines spring stiffness



→ k affects the distance it moves given the force F
→ F is always directed opp to the displacement i.e. towards the eq. position aka "Restoring force"

$$\Rightarrow m \frac{d^2x}{dt^2} = F = -kx$$

$$\boxed{\omega^2 = \frac{k}{m}}$$

$$\frac{d^2x}{dt^2} = -\omega^2 x \quad \text{--- (1)}$$

$$\boxed{\frac{d^2x}{dt^2} + \omega^2 x = 0} \quad \text{[Equation of motion for simple harmonic motion]}$$

$$x = ce^{\alpha t} \quad \text{(solution) --- (2)}$$

$$\begin{cases} D^2 = -\omega^2 \\ D = \pm i\omega \end{cases}$$

$$\frac{dx}{dt} = ce^{\alpha t} \cdot \alpha$$

$$\frac{d^2x}{dt^2} = ce^{\alpha t} \cdot \alpha^2 \quad \text{--- (3)}$$

$$ce^{\alpha t} \alpha^2 + \omega^2 ce^{\alpha t} = 0$$

$$ce^{\alpha t} (\alpha^2 + \omega^2) = 0$$

$$\alpha^2 + \omega^2 = 0$$

$$\boxed{\alpha = \pm i\omega}$$

$$\boxed{x = ce^{\pm i\omega t}} \quad \text{(or)} \quad \boxed{x = C_1 e^{i\omega t} + C_2 e^{-i\omega t}}$$

$$x = C_1 (\cos \omega t + i \sin \omega t) + C_2 (\cos \omega t - i \sin \omega t)$$

$$= (C_1 + C_2) \cos \omega t + i(C_1 - C_2) \sin \omega t$$

$$= a \sin \phi \cos \omega t + a \cos \phi \sin \omega t$$

$$\begin{cases} C_1 + C_2 = a \sin \phi \\ C_1 - C_2 = a \cos \phi \end{cases}$$

$$\boxed{x = A \sin(\omega t + \phi)}$$

→ Phase const.
→ Frequency
→ Amplitude

$$x = a \sin(\omega t + \phi)$$

$$v = \frac{dx}{dt} = a\omega \cos(\omega t + \phi)$$

$$v = A\omega \cos(\omega t + \phi)$$

$$\begin{cases} \sin(\omega t + \phi) = \frac{x}{a} \\ \cos(\omega t + \phi) = \sqrt{1 - \frac{x^2}{a^2}} \end{cases}$$

$$[\cos \theta = \sqrt{1 - \sin^2 \theta}]$$

$$v = a\omega \sqrt{\frac{a^2 - x^2}{a^2}}$$

$$v = \omega \sqrt{a^2 - x^2}$$

$$v = \omega \sqrt{A^2 - x^2}$$

$$a = -\omega^2 A \sin(\omega t + \phi)$$

(To calculate time period)

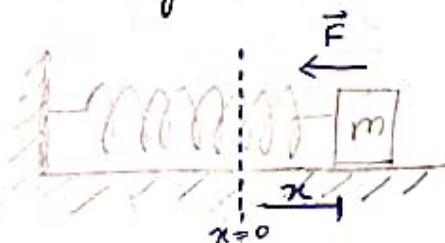
$$T = \frac{1}{\nu} = \frac{2\pi}{\omega}$$

$$\omega = \sqrt{\frac{k}{m}}$$

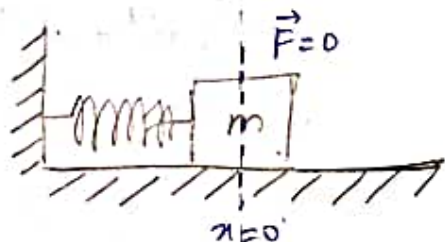
$$T = \frac{2\pi}{\sqrt{k/m}} \Rightarrow T = 2\pi \sqrt{\frac{m}{k}}$$

★ Restoring force and Spring mass system.

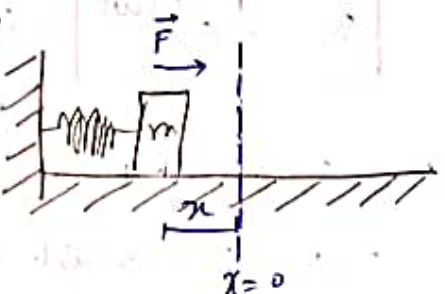
- Here the block is displaced to the right of $x=0$. The restoring force exerted by the spring is directed to the left (negative)



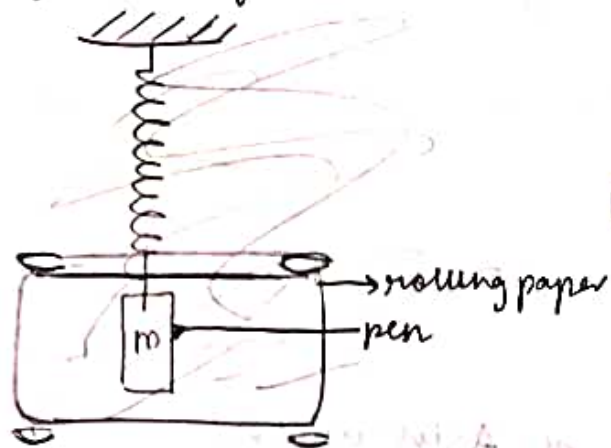
- Block is at eq. position ($x=0$). Thus the spring is neither stretched nor compressed. $\vec{F}_s = 0$



- Block is displaced to the left of $x=0$. (position is -ve) The restoring force is directed to the right (positive)



* Verification of Sinusoidal Nature:



BLAH
BLAH

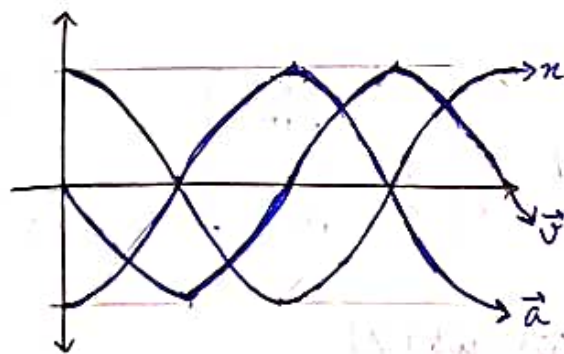
* Amplitude:

- Amplitude is the maximum position of the object from its equilibrium position.
- [Ideal SHM involves oscillation of m between $x = \pm A$]

* Period:

- The period is the time taken by object to complete one complete cycle of motion.
[$x=A$ to $x=-A$ to $x=A$]
- Represented by 'T' (seconds)
- The frequency is the no of complete cycles per unit time

$$f = 1/T \quad (\text{Hertz})$$



- velocity is 90° out of phase with displacement
- acceleration is 180° out of phase with displacement

$$x = A \cos \omega t$$

- when $x = \text{max}$; $v = \text{Zero}$
- when $x = 0$; $v = \text{max}$ and in negative direction
- when $x = +\text{max}$; $a = \text{max}$ and in -ve direction

★ Kinetic and Potential energies w.r.t Force.

$$F = -kx$$

$$F = -\frac{dU}{dx}$$

$$\int \frac{dU}{dx} = \int kx$$

$$U = \frac{1}{2} kx^2 + C$$

$$\boxed{U = \frac{1}{2} kx^2}$$

$$x = A \sin(\omega t + \phi)$$

$$U = \frac{1}{2} k A^2 \sin^2(\omega t + \phi)$$

$$\boxed{U = \frac{1}{2} k A^2 \sin^2(\omega t + \phi)}$$

$$\boxed{U_{\max} = \frac{1}{2} k A^2}$$

$$K.E = \frac{1}{2} mv^2 = \frac{1}{2} m \left(\frac{dx}{dt} \right)^2$$

$$\boxed{K.E = \frac{1}{2} mv^2}$$

$$= \frac{1}{2} m [A \omega \cos(\omega t + \phi)]^2$$

$$\boxed{K.E = \frac{1}{2} m \omega^2 A^2 \cos^2(\omega t + \phi)}$$

$$KE = \frac{1}{2} m \omega^2 A^2 \cos^2(\omega t + \phi)$$

$$K.E_{\max} = \frac{1}{2} m \omega^2 A^2$$

$$\boxed{K.E_{\max} = \frac{1}{2} k A^2}$$

$$\left[\begin{array}{l} \omega = \sqrt{\frac{k}{m}} \\ \omega^2 = \frac{k}{m} \end{array} \right]$$

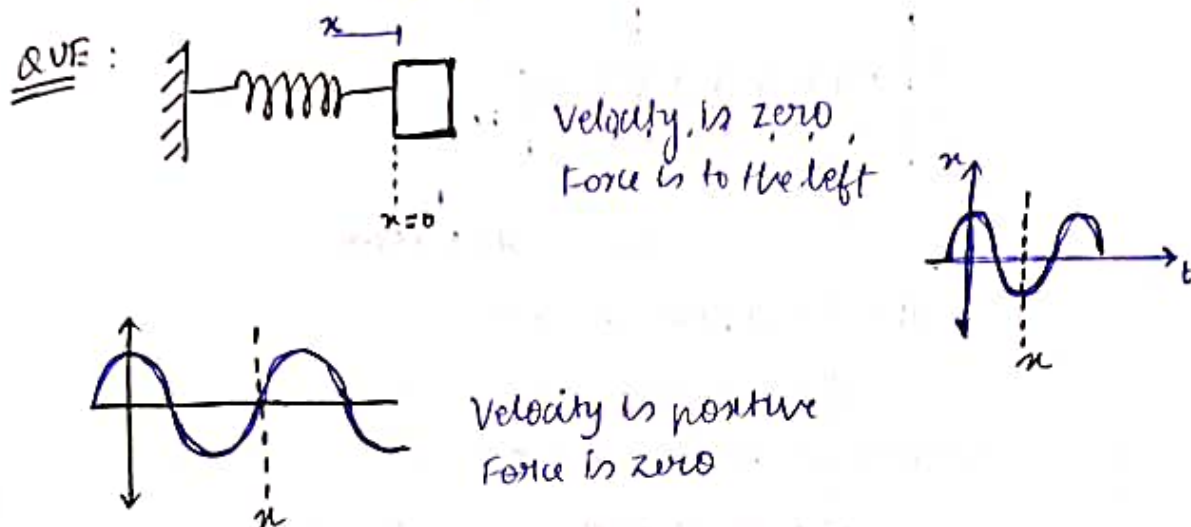
$$T.E = \frac{1}{2} k A^2 \sin^2(\omega t + \phi) + \frac{1}{2} k A^2 \cos^2(\omega t + \phi)$$

$$= \frac{1}{2} k A^2$$

$$\boxed{T.E = \frac{1}{2} k A^2}$$

* Transferring of Energy of SHM:

- Total energy is always constant. $E = \frac{1}{2} k A^2$
- Energy is continuously being transferred from P.E in the spring to K.E in the block



QVE: (i) Total distance travelled by m in SHM in T $\Rightarrow 4A$

(ii) displacement after T $\Rightarrow 0$

(iii) At what point is $v = 0$ and $a = 0$ simultaneously
not possible. [v and a cannot be zero at the same time]

QVE: In a SHM, when the m is doubled and A is unchanged, T.E = ?

$$T.E \propto A^2$$

Total energy does not change
wz it does not depend on mass

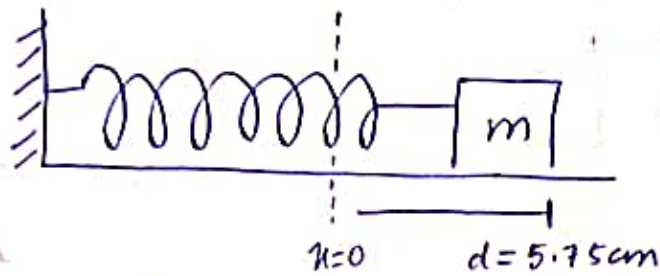
$$T.E = \frac{1}{2} k A^2$$

QVE: mass oscillating on a vertical spring with T is taken to the moon.

Time period will not change

$$T = 2\pi \sqrt{\frac{m}{k}} \quad (\text{does not depend on } g')$$

Q.1: 1.55 Kg block sliding on a $\mu = 0$ plane is connected to a horizontal spring of $K = 2.55 \text{ N/cm}$. Block is pulled to the right by $d = 5.75 \text{ cm}$ and released from rest. v after 1.5 s?



$$x = d = A \sin(\omega t + \phi)$$

$$d = A \sin \phi \quad \text{--- (1) } (t=0)$$

$$v = \omega_0 A \cos(\omega t + \phi)$$

$$v = \omega_0 A \cos \phi \quad (t=0)$$

$$[v = 0 \text{ at } t = 0] \text{ (extreme end)}$$

$$\therefore \cos \phi = 0$$

$$\boxed{\phi = \pi/2}$$

$$d_{\max} = A$$

$$[\cos(\omega t + \pi/2) = -\sin \omega t]$$

$$\Rightarrow \therefore v = -\omega_0 d \sin \omega t$$

$$= -\omega_0 d \sin(\sqrt{K/m} t)$$

$$= -\sqrt{\frac{2.55}{1.55}} (0.0575) \sin[\omega_0 (1.5)] \quad \omega = \sqrt{\frac{K}{m}}$$

$$= \dots$$

$$\underline{\text{Ans:}} \quad \boxed{v = 6.92 \text{ cm/s}} \quad //$$

QVE: If displacement of a moving particle at any time is $x = a \cos \omega t + b \sin \omega t$, show that the motion is SHM. If $a=3$, $b=4$, $\omega=2$, find period, v_{\max} , a_{\max}

$$x = a \cos \omega t + b \sin \omega t$$

$$\frac{dx}{dt} = -a \sin \omega t (\omega) + b \omega \cos \omega t$$

$$\frac{d^2x}{dt^2} = -a \omega^2 \cos \omega t - b \omega^2 \sin \omega t$$

$$= -\omega^2 (a \cos \omega t + b \sin \omega t)$$

$$= -\omega^2 x \quad \therefore \text{SHM}$$

$$A_{\max} = \sqrt{a^2 + b^2} = \sqrt{9 + 16} = 5 \text{ cm}$$

$$\boxed{A = 5 \text{ cm}}$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi$$

$$\boxed{T = \pi \text{ s}}$$

$$v_{\max} = \omega A = 2 \times 5$$

$$\boxed{v_{\max} = 10 \text{ cm/s}}$$

$$a_{\max} = \omega^2 A = 4 \times 5$$

$$\boxed{a_{\max} = 20 \text{ cm/s}^2}$$

QVE:

$$A = 5 \text{ cm} \quad \nu = 1 \text{ Hz}$$

$$x = A \cos(\omega t + \alpha)$$

$$\Rightarrow x = 5 \cos(\omega t + \pi/2)$$

$$0 = 5 \cos \alpha$$

$$(\text{at } t = 8/3 \text{ s})$$

$$\cos \alpha = 0$$

$$\boxed{\alpha = \pi/2}$$

$$\frac{dx}{dt} = 5 \omega \sin(\omega t + \pi/2)$$

$$v = -5 \omega \sin(8\omega/3 + \pi/2)$$

$$T = \frac{2\pi}{\omega} = 1$$

$$v = -10\pi \sin\left(\frac{16\pi}{3} + \frac{\pi}{2}\right)$$

$$\boxed{\omega = 2\pi}$$

$$= -10\pi \sin\left(\frac{32\pi}{6}\right) = -10\pi \sin\left(6\pi - \frac{\pi}{6}\right)$$

$$= -10\pi \sin\left(5.83\pi\right) = -10\pi \sin\left(-\frac{\pi}{6}\right)$$

$$= 10\pi \left[\sin \frac{\pi}{6}\right]$$

$$= 10\pi \left(\frac{1}{2}\right)$$

$$\Rightarrow 5\pi \text{ cm/s}$$

$$\Rightarrow 15.7 \text{ cm/s}$$

QVE: $m = 10g$ is placed in a potential field.

$$V = (50x^2 + 100) \text{ ergs/gm} \cdot v = ?$$

$$U = mV = 10 \times 10^{-3} \times (50x^2 + 100)$$

$$\boxed{U = 0.5x^2 + 1}$$

$$F = -\frac{dU}{dx} = -\frac{d}{dx}(0.5x^2 + 1)$$

$$U = -Fdx$$

$$F = -(0.5 \times 2)x$$

$$\boxed{F = -x}$$

$$F = kx = m\omega^2 x$$

$$m\omega^2 = 1$$

$$10^{-2} \times \omega^2 = 1$$

$$\omega^2 = 100$$

$$\boxed{\omega = 10}$$

$$T = \frac{\omega}{2\pi} = \frac{10}{2\pi}$$

$$\boxed{T = \frac{5}{\pi} \text{ s}}$$

QVE: Write eqn of SHM

$$(i) \phi_i = 0 \quad (ii) \phi_i = \pi/2 \quad (iii) A = 5 \text{ cm} \quad T = 8 \text{ s.}$$

$$(i) x = A \sin(\omega t + \phi)$$

$$\omega = \frac{2\pi}{T} = \frac{\pi}{6}$$

$$x = 5 \sin(0.785 t) //$$

$$\boxed{\omega = 0.785 \text{ rad/s}}$$

$$(ii) x = 5 \sin(0.785 t + \pi/2) //$$

$$\underline{\underline{QVE}}: x = 2 \sin\left(\frac{\pi t}{2} + \frac{\pi}{4}\right) \text{ cm.}$$

$$T = ? \quad v_{\max} = ?$$

$$\omega = \frac{\pi}{2} = \frac{2\pi}{T}$$

$$v = \pi \sin(\omega t + \pi/4)$$

$$v = \pi \cos(\pi/2 t + \pi/4)$$

$$\boxed{T = 4 \text{ s}} //$$

$$\boxed{v_{\max} = \pi \text{ cm/s}} //$$

$$\boxed{v_{\max} = \omega A = \frac{\pi}{2} \times 2 = \pi \text{ cm/s}} //$$

QVE $T = 31.4 \text{ s}$; $A = 5 \text{ cm}$ $v_{\text{max}} = ?$

$T = \frac{2\pi}{\omega}$ $\omega = 0.2$

$v_{\text{max}} = \omega A = 1 \text{ cm/s}$

$v_{\text{max}} = \omega A$
 $a_{\text{max}} = -\omega^2 A$

$a_{\text{max}} = -\omega^2 A = 0.2 \text{ cm/s}^2$

QVE : $T = 10 \text{ s}$; $A = 0.1 \text{ m}$

Write the equation. What are phase & displacement at $t = 5 \text{ s}$ after a passage of the particle through its extreme positive elongation. $v_{\text{max}} = ?$

$\omega = \pi/5 \text{ rad/s}$

$v_{\text{max}} = \omega A = 2\pi \text{ m/s}$

$x = A \sin(\omega t + \phi)$

$x = 0.1 \sin(\pi/5 t + \phi)$

$t=0$

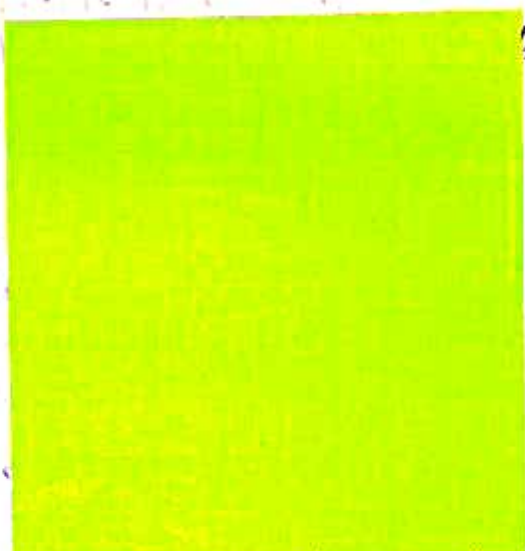
[starts at extrem]

$A = 0.1 \sin \phi$

$0.1 \sin \phi = 0.1$

$\sin \phi = 1$

$\phi = \pi/2$



* SUPER-POSITION OF WAVES

* HUYGEN'S PRINCIPLE:

- A wave is continuously repeating change or oscillation in matter or in a physical field. Light is also a wave.
- Huygen's believed that light was made of waves vibrating up & down i.e. to direction of motion (i.e) transverse waves

$$\boxed{v = \frac{1}{\lambda}}$$

* Wave characteristics:

Wavelength (λ):

distance between 2 crests or 2 troughs is λ (lambda).

Frequency (ν):

no. of waves that pass a pt in one second.

Amplitude (A):

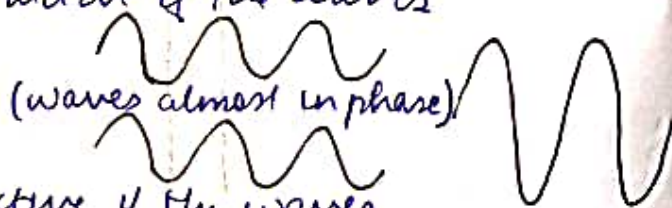
the vertical distance from midline of waves to the top of peak or bottom of trough.

* Combination of waves:

- Composite wave = 2 waves combined
is the algebraic sum of the 2 original waves
point by point - (superposition principle)

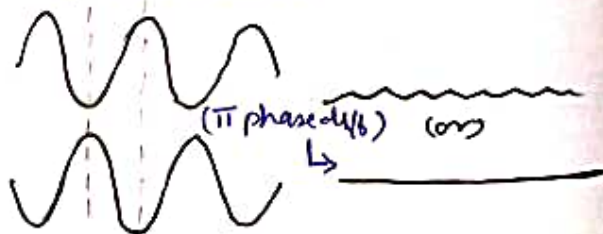
- direction, Amplitude & Phase need to be taken into account while adding
- phase difference should not be there.

- The interference is constructive if the waves reinforce each other



- The interference is destructive if the waves tend to cancel each other.

- If the phases are exactly opp (i.e) π , then the result is nothing since the waves cancel each other completely



- The resultant Amplitude of the new wave depends on the phase differences of the 2 original waves.

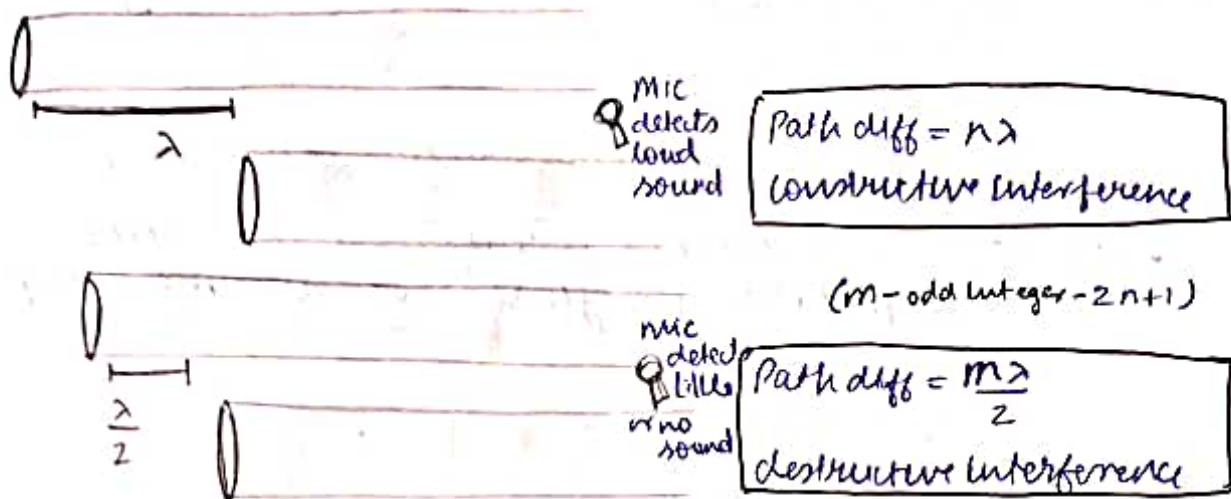
* Coherence:

Coherent sources \rightarrow same frequency

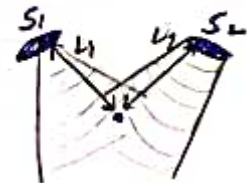
\rightarrow same phase values

[To keep the phase diff 0 along the length along the direction of propagation, the frequency must be same]

- In phase - peaks line up with peaks
valleys line up with valleys } waves add up and
 $A_R = A_1 + A_2$
- Ex: sound amplifiers; surround sound



Noise cancellation headphones use destructive interference.



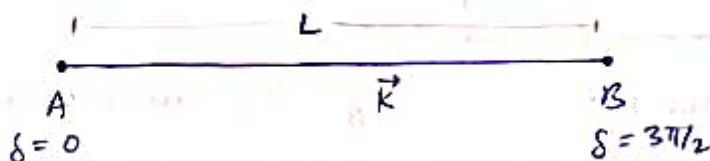
* Path difference (ΔL):

- Path difference ΔL is the absolute value of the difference between in the distances from each source to a point being considered.

$$\Delta L = |L_1 - L_2|$$

- We express the path difference in terms of number of wavelengths. Ex: $3\frac{1}{2}\lambda$, 5λ , etc

* Theory of Super-position:



- Let light travel from A to B and travels a distance L , its phase changes from 0 to $3\pi/2$.

$$\delta = nKL$$

$$\delta = \frac{n 2\pi L}{\lambda}$$

$$K = \frac{2\pi}{\lambda}$$

K - wave vector
 λ - wavelength

$$\delta \propto n$$

$$\delta \propto \frac{1}{\lambda}$$

$$\delta \propto L$$

- Consider a constructive interference situation where Amplitudes of original waves are A & A resp.

$$\Rightarrow A_R = A + A = 2A$$

$$\boxed{A_R = A_1 + A_2}$$

$$I \propto |A|^2$$

$$\Rightarrow I_R = (2A)^2 = 4I$$

$$\boxed{I_R > I_1 + I_2} \rightarrow \text{constructive interference}$$

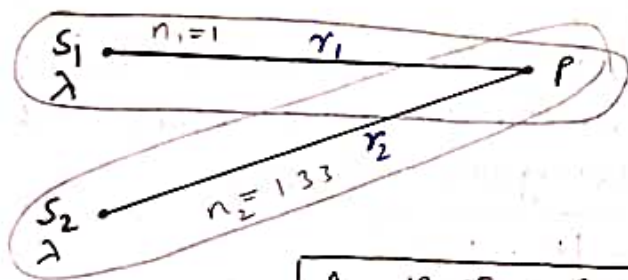
- Consider a destructive interference situation where amplitudes of original waves are A & A resp.

$$\Rightarrow A_R = A - A = 0$$

$$\Rightarrow I_R = (0)^2 = 0$$

$$\boxed{A_R = A_1 + A_2}$$

$$\boxed{I_R < I_1 + I_2} \rightarrow \text{destructive interference}$$



- Consider 2 coherent sources but in 2 different media of refractive indexes n_1 & n_2 resp.

$$\boxed{\Delta = n_2 r_2 - n_1 r_1}$$

$$\Delta = n\lambda$$

→ constructive

$$\Delta = \frac{(2n+1)\lambda}{2}$$

→ Destructive

$$\bullet E_A = E_1 \sin \omega t$$

$$E_B = E_2 \sin(\omega t + \delta)$$

$$E_R = E_1 + E_2 = E_1 \sin \omega t + E_2 \sin(\omega t + \delta)$$

$$= E_1 \sin \omega t + E_2 (\sin \omega t \cos \delta + \cos \omega t \sin \delta)$$

$$= (E_1 + E_2 \cos \delta) \sin \omega t + (E_2 \sin \delta) \cos \omega t$$

$\leftarrow E \cos \phi$

$$\left[\begin{array}{l} \text{Let } E_1 + E_2 \cos \delta = E \cos \phi \quad \text{--- ①} \\ E_2 \sin \delta = E \sin \phi \quad \text{--- ②} \end{array} \right.$$

$$= E \cos \phi \sin \omega t + E \sin \phi \cos \omega t$$

$$\textcircled{1}^2 + \textcircled{2}^2 \Rightarrow (E_1 + E_2 \cos \delta)^2 + (E_2 \sin \delta)^2 = E^2 \cos^2 \phi + E^2 \sin^2 \phi$$

$$= E_1^2 + E_2^2 \cos^2 \delta + 2E_1 E_2 \cos \delta + E_2^2 \sin^2 \delta = E^2$$

$$\Rightarrow E_1^2 + E_2^2 + 2E_1 E_2 \cos \delta = E^2$$

$$\boxed{I_R = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \delta}$$

$$\boxed{E^2 = E_1^2 + E_2^2 + 2E_1 E_2 \cos \delta}$$

2 waves of same freq but diff phase

$$\delta = \frac{2\pi nL}{\lambda}$$

$$K = \frac{2\pi}{\lambda}$$

Constructive:

$$\Delta = n\lambda$$

$$A_R = A_1 + A_2$$

$$I_R > I_1 + I_2$$

Destructive:

$$\Delta = \frac{(2n+1)\lambda}{2}$$

$$A_R = A_1 + A_2$$

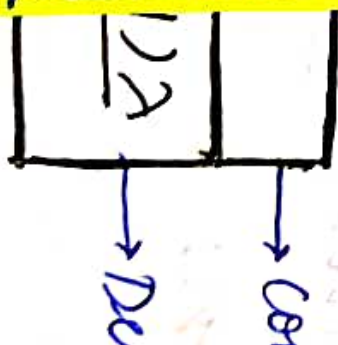
$$I_R < I_1 + I_2$$

$$E_R^2 = E_1^2 + E_2^2 + 2E_1E_2 \cos \delta$$

$$I_R = I_1 + I_2 + 2\sqrt{I_1I_2} \cos \delta$$

$$I_{\max} = 4I$$

$$I_{\min} = 0$$



$n_2 r_2 - 1$

$$I_R < I_1 + I_2 \rightarrow \text{dark}$$

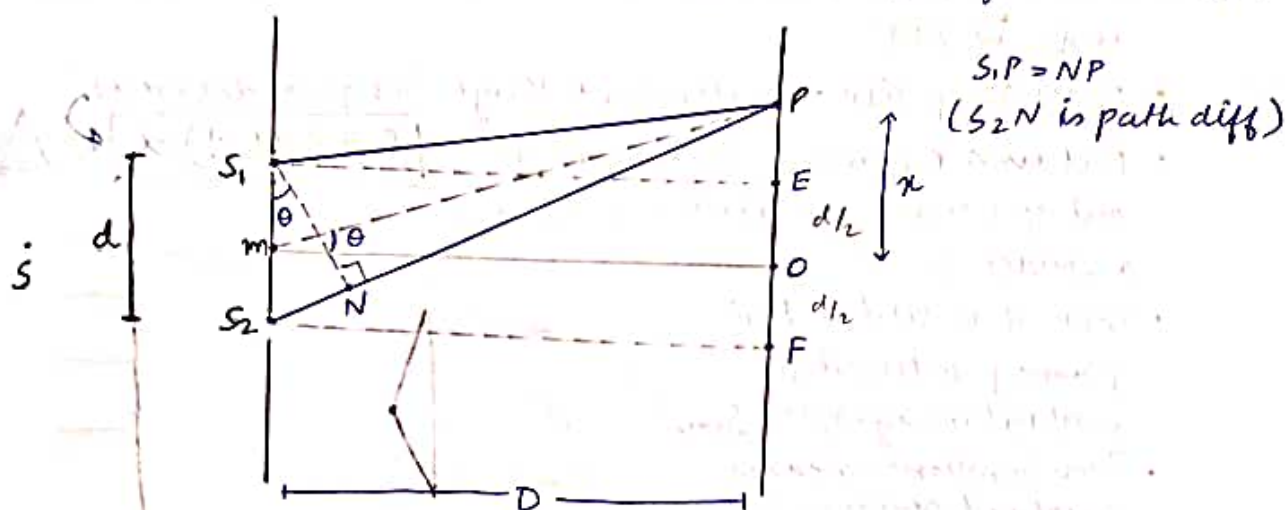
$E_A = E_1 \sin \omega t$

E

★ YOUNG'S DOUBLE SLIT EXPERIMENT:

Describes the wave nature of light and demonstrates the interference of light.

The results can be proved only by taking light as wave.



In $\Delta S_1 P E$;

$$P E = x - d/2$$

$$(S_1 P)^2 = D^2 + (x - d/2)^2$$

In $\Delta S_2 P F$;

$$P F = x + d/2$$

$$(S_2 P)^2 = D^2 + (x + d/2)^2$$

$$(S_2 P)^2 - (S_1 P)^2 = D^2 - D^2 + x^2 + \left(\frac{d}{2}\right)^2 + x d - x^2 - \left(\frac{d}{2}\right)^2 + x d$$

$$(S_2 P)^2 - (S_1 P)^2 = 2 x d$$

$$(S_2 P - S_1 P)(S_2 P + S_1 P) = 2 x d$$

$$S_2 P - S_1 P = \frac{2 x d}{S_2 P + S_1 P}$$

$$S_2 P - S_1 P = \frac{2 x d}{D + D} = \frac{x d}{D}$$

$$S_2 P - S_1 P = \frac{x d}{D}$$

$$x_m = \frac{m \lambda D}{d}$$

$$x_{m+1} = \frac{(m+1) \lambda D}{d}$$

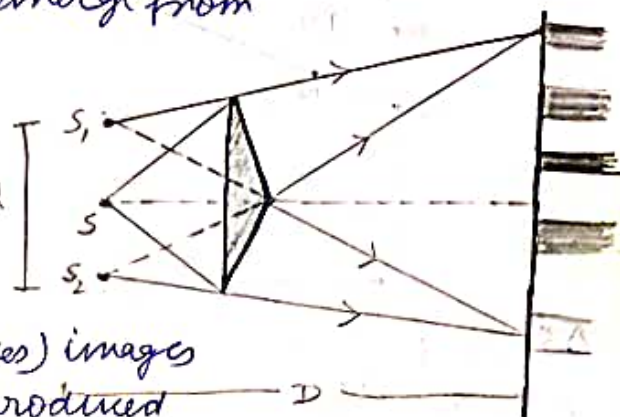
(Fringe width) $\boxed{\beta = \frac{\lambda D}{d}}$

$d \sin \theta = m \lambda$	→ constructive (maxima)
$d \sin \theta = \frac{2m+1}{2} \lambda$	→ Destructive (minima)

Light can hold coherent only for a limit and then intensity gradually fades to zero.

★ FRESNEL'S BIPRISM:

- Biprism consists of 2 prisms of very small refractive angles joined base to base
- Usually 2 side angles are $30'$ (0.5°) and other angle is 179°
- ordinary prism - ray is bent through 'angle of deviation'.
- But in a Biprism; the ray coming out of prism, appears to emerge from a source S
- Then it is said that prism produced a virtual image of the source d
- Then biprism creates 2 virtual sources S_1, S_2
- S_1 & S_2 are (virtual sources) images of the same source S produced by 'Refraction'. They are coherent



$$\boxed{\frac{x d}{D} = m \lambda}$$

(Bright fringe)

$$\boxed{\frac{x d}{D} = (2m+1) \frac{\lambda}{2}}$$

Dark fringe

$$x_m = \frac{m \lambda D}{d}$$

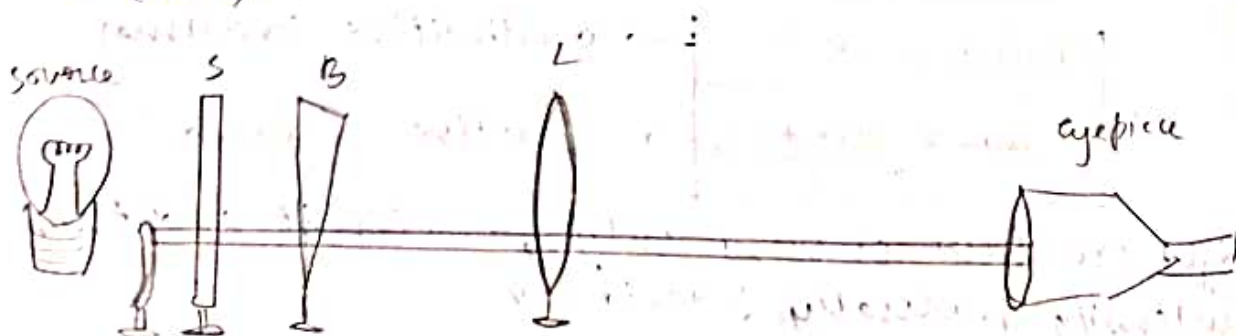
$$x_{m+1} = \frac{(m+1) \lambda D}{d}$$

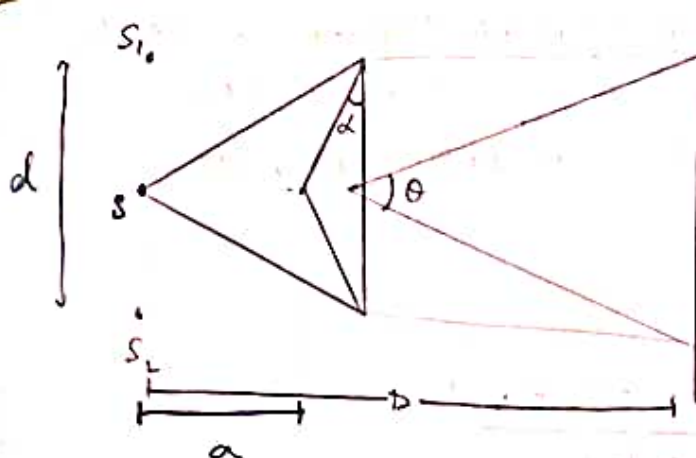
$$\beta = x_m - x_{m+1} = \frac{\lambda D}{d}$$

$$\boxed{\beta = \frac{\lambda D}{d}} \Rightarrow \text{Fringe width}$$

Expt (Experimental arrangement)

- Biprism mounted on an optical bench - (2 horizontal 11th rods)
- (Blah)





$$\delta = \frac{\theta}{2}$$

$$\theta = \frac{d}{a}$$

$$\delta = (\mu - 1)d$$

$$\theta/2 = (\mu - 1)d = \frac{d}{2a} = (\mu - 1)d$$

$$d = 2a(\mu - 1)d$$

$$d = 2a(\mu - 1)d$$

(a → source to prism)

$$t = \frac{\mu d}{D(\mu - 1)}$$

(thickness of mica sheet)

$$d = \sqrt{d_1 d_2} = 2a(\mu - 1)d$$

QUR: Viewing screen is put separation from a double slit by 1.2 m. $d = 0.030$ nm. Second order bright fringe ($m=2$) is 4.5 cm from center line.

(a) $\lambda = ?$ (b)

$$x = \frac{m \lambda D}{d}$$

$$\lambda = \frac{x d}{m D} = \frac{0.045 \times 0.030 \times 10^{-9}}{2 \times 1.2}$$

$$= \frac{4.5 \times 3 \times 10^{-13}}{2 \times 1.2}$$

$$= 5.625 \times 10^{-13}$$

\Rightarrow



QUR: Screen placed 13.7 m apart. 3rd order fringe is seen 2.50 cm from central. $d = 0.960$ cm. $\lambda = ?$

$$D = 13.7 \text{ m}$$

$$x = 2.50 \text{ cm} = 0.025 \text{ m}$$

$$d = 0.0096 \text{ m}$$

$$x = \frac{m \lambda D}{d} \Rightarrow \lambda = \frac{x d}{m D} = \frac{0.025 \times 0.0096}{3 \times 13.7} = 5.84 \times 10^{-7} \text{ m}$$

$$x = \frac{3 \lambda D}{d} = \frac{3 \lambda (13.7)}{96 \times 10^{-4}} = 25 \times 10^{-3}$$

$$\lambda = \frac{25 \times 96 \times 10^{-7}}{13.7 \times 3}$$

$$\lambda = 5.84 \times 10^{-7} \text{ m}$$

$$\boxed{\lambda = 584 \text{ nm}}$$

QUR: How far from the central fringe the first order violet ($\lambda = 350$ nm) & Red ($\lambda = 700$ nm)

$$D = 10 \text{ m}$$

$$d = 0.50 \text{ cm}$$

$$x_v = \frac{\lambda m D}{d} = \frac{350 \times 10^{-9} \times 1 \times 10}{0.05 \times 10^{-2}} \Rightarrow 0.007 \text{ m}$$

$$x_r = \frac{\lambda m D}{d} = \frac{700 \times 10^{-9} \times 1 \times 10}{0.05 \times 10^{-2}} \Rightarrow 0.014 \text{ m}$$

QUE: If yellow light with $\lambda = 540 \text{ nm}$ shines on a double slit; $d = 0.01 \text{ mm}$ $\theta = ?$ $n = 2$

$$\left[\sin \theta = \frac{x}{D} \right]$$

$$x = \frac{\lambda m D}{d} = \frac{540 \times 2 \times 10^{-9}}{1 \times 10^{-5}}$$

$$\lambda = \frac{d}{m} \times \frac{x}{D} = \frac{d}{m} \sin \theta$$

$$\sin \theta = \frac{2 \times 540 \times 10^{-9}}{10^{-5}} = 1080 \times 10^{-4}$$

$$\sin \theta \approx \frac{1}{10}$$

$$\left[\theta = \sin^{-1}(1/10) \right]$$

$$\left[\theta \approx 6.20^\circ \right]$$

QUE: Distance between adjacent dark spots from a double slit; $\lambda = 500 \text{ nm}$; $d = 1 \text{ mm}$; $D = 2 \text{ m}$

$$\theta = ? \quad \text{distance} = \frac{2\lambda D}{d} - \frac{\lambda D}{d} = \frac{\lambda D}{d}$$

$$\left[\beta = \frac{\lambda D}{d} \right] \Rightarrow$$

$$= \frac{500 \times 2 \times 10^{-9}}{10^{-3}}$$

$$= 1000 \times 10^{-6}$$

$$\left[y = 10^{-3} \text{ m} \right]$$

$$\sin \theta_1 = \frac{x}{D} = \frac{\lambda D}{d D} = \frac{\lambda}{d} = \frac{500}{10^{-3}} \times 10^{-9} = 500 \times 10^{-6}$$

$$\sin \theta_1 = 5 \times 10^{-4}$$

$$\text{Ans: } \left[\theta_1 = \sin^{-1}(5 \times 10^{-4}) \Rightarrow 0.0286^\circ \right] //$$

$$m\lambda = d \sin \theta$$

$$\theta = \sin^{-1} \left(\frac{m\lambda}{d} \right)$$

$$\sin \theta = \frac{1 \times 500 \times 10^{-9}}{10^{-3}}$$

$$\sin \theta = 5 \times 10^{-4}$$

$$\left[\theta = 0.0286^\circ \right] //$$

QVE: Biprism exp; refracting angles $\Rightarrow 1.5$; $\mu = 1.5$
 with single slit of 5cm from biprism; $\lambda = 580 \text{ nm}$;
 fringes were formed 1m from the slit. fringe width = ?

$$\mu = 1.5$$

$$a = 5 \text{ cm}$$

$$d = \sqrt{d_1 d_2}$$

$$\alpha = 1.5 \times \frac{\pi}{180}$$

$$d = 2 \times 5 \times 10^{-2} (1.5 - 1) \left(\frac{1.5 \times \pi}{180} \right)$$

$$d = 0.13 \times 10^{-2} \text{ m} //$$

$$\beta = \frac{\lambda D}{d}$$

$$\beta = \frac{\lambda D}{d} = \frac{580 \times 1}{0.13 \times 10^{-2}}$$

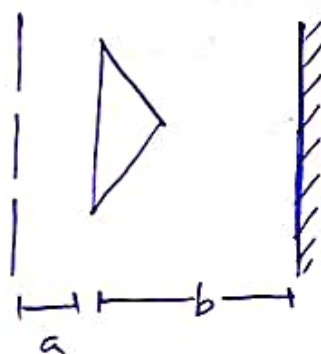
$$\beta = 4461.5 \times 10^{-7} \text{ m} //$$

$$d = 2a(\mu - 1)\alpha$$

QVE: $a = 25 \text{ cm}$ $b = 100 \text{ cm}$
 (biprism to slit) (slit to screen)

$$\alpha = 20'$$

$$\beta = 0.55 \text{ mm}$$



$$\rightarrow \Delta x = \frac{l\lambda}{d} \quad [l = a + b = 125 \text{ cm}]$$

$$\Delta x = \frac{125 \times \lambda}{d}$$

$$\rightarrow d = 2a(\mu - 1)\alpha$$

$$= 2 \times 0.25 (1.5 - 1) 20 \times \pi / 180$$

$$= 0.25 \times \pi / 9$$

$$\Delta x = \beta = \frac{D\lambda}{d}$$

$$\rightarrow \Delta x = \frac{l\lambda}{d} = \frac{1.25 \times \lambda}{0.25 \times \pi / 9} = \beta$$

$$\lambda = 0.25 \times \frac{\pi}{9} \times \beta$$

$$1.25$$

$$\lambda = \frac{25 \times \pi \times 0.55 \times 10^{-3}}{9 \times 125}$$

$$9 \times 125$$

$$\underline{\underline{\lambda = 0.64 \times 10^{-6} \text{ m} //}}$$

* NEWTON'S RING :

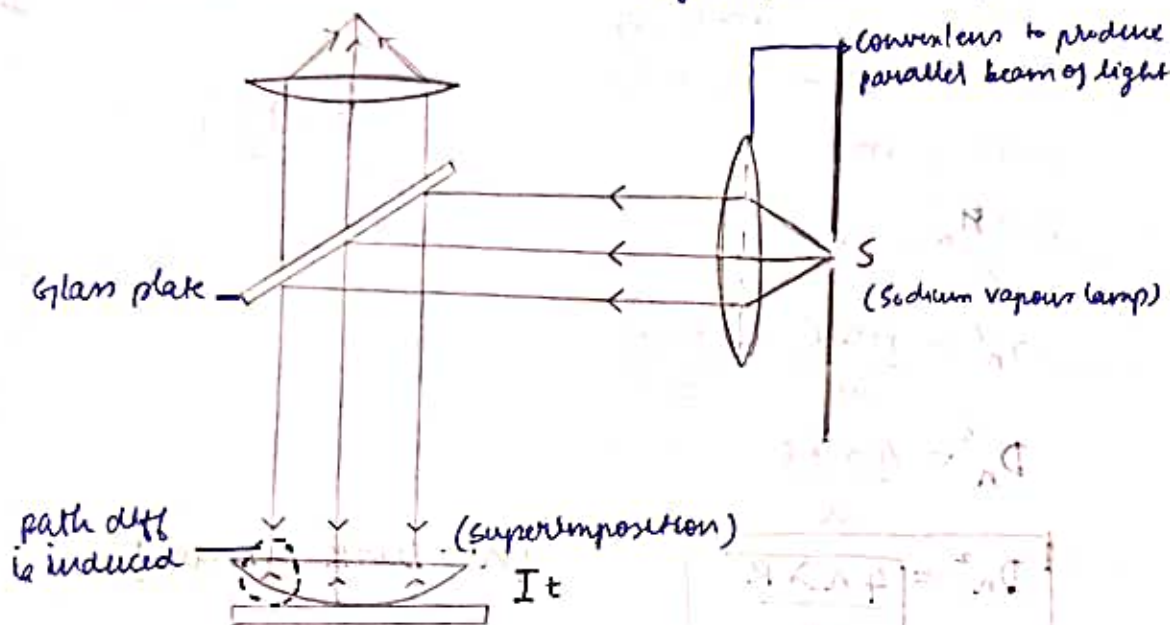
- Used to calculate the refractive index of the given material.

- Works based on superposition principle of wave



[partially the light will pass through the lens and partially get reflected back by the mirror]

2000
copy



$$\Delta = 2\mu t \cos(\gamma + \theta) - \lambda/2$$

$$\Delta = 2\mu t - \lambda/2$$

$$\text{Constructive} \Rightarrow 2\mu t - \lambda/2 = m\lambda$$

$$\text{Destructive} \Rightarrow 2\mu t = (2m+1)\lambda/2$$

Conclusion: The central fringe will be dark for Newton's ring

* For constructive interference;

$$r \times r = t(2R - t)$$

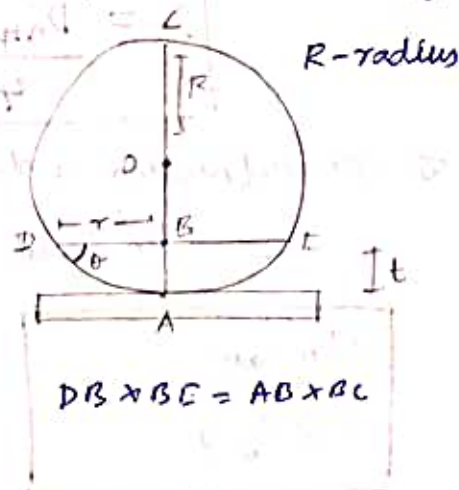
$$r^2 = 2Rt - t^2$$

$$r^2 = 2Rt$$

$$t = \frac{r^2}{2R} \quad \text{--- (1)}$$

$$\Delta = \frac{2\mu r_n^2}{2R} = \frac{(2n+1)\lambda}{2}$$

$$r_n^2 = \frac{(2n+1)\lambda R}{2\mu} = \frac{D_n^2}{4}$$



$$D_n^2 = \frac{4(2n+1)\lambda R}{2\mu} = 2 \frac{(2n+1)\lambda R}{\mu}$$

$$D_n = \sqrt{\frac{2(2n+1)\lambda R}{\mu}} \quad \text{--- (2) } [n^{\text{th}} \text{ Bright fringe}]$$

★ For destructive interference;

$$2\mu t + \lambda/2 = (2n+1)\lambda/2$$

$$2\mu t = n\lambda + \lambda/2 \pm \lambda/2$$

$$2\mu t = n\lambda$$

$$\frac{2\mu r_n^2}{2R} = n\lambda$$

$$r_n^2 = \frac{n\lambda R}{\mu} = \left(\frac{D_n}{2}\right)^2$$

$$D_n^2 = \frac{4n\lambda R}{\mu}$$

$$D_n = \sqrt{\frac{4n\lambda R}{\mu}} \quad [n^{\text{th}} \text{ Dark fringe}]$$

★ For wavelength;

$$D_n^2 = \frac{4n\lambda R}{\mu}$$

$$; D_{n+p} = \frac{4(n+p)\lambda R}{\mu}$$

$$D_{n+p}^2 - D_n^2 = \frac{4p\lambda R}{\mu}$$

$$\lambda = \frac{D_{n+p}^2 - D_n^2}{4pR}$$

$$[\mu=1]$$

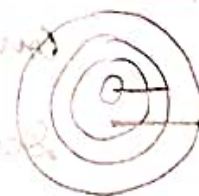
$$\lambda = \frac{D_{n+p}^2 - D_n^2}{4pR}$$

★ For refractive index;

$$(D_n^2)_{\text{med}} = \frac{4n\lambda R}{\mu}$$

$$; (D_n^2)_{\text{air}} = 4n\lambda R$$

$$\frac{(D_n^2)_{\text{air}}}{(D_n^2)_{\text{med}}} = \mu$$



* DIFFRACTION :

(Bending of light)

- For a single slit diffraction, when the slit width decreases the number of observable fringes
Narrower width of slit \rightarrow more fast drop on intensities

[single slit] $\Rightarrow I = I_0 \left(\frac{\sin \alpha}{\alpha} \right)^2$

$\left[\alpha = \pi \frac{a \sin \theta}{\lambda} \right]$

mth minima when $\Rightarrow \sin \theta = \frac{m\lambda}{a}$

[a - dist b/w 2 slits]

[multi slit] $I = I_0 \left(\frac{\sin \beta}{\beta} \right)^2 \cos^2 \gamma$

$\left[\beta = \frac{\pi}{\lambda} b \sin \theta \right]$

$\left[\gamma = \frac{\pi}{\lambda} d \sin \theta \right] \left(\frac{N > 1}{\downarrow \text{no. slit}} \right)$

phase diff $\Delta \phi = 2\pi \frac{\text{path diff}}{\lambda}$

- When no. of slits is increased, the θ - diffraction angle also increases.

$\frac{\Delta \theta}{\Delta \lambda} = \frac{m}{d \cos \theta}$

- Diffraction is a wave effect. (explained in terms of Huygen's principle)
- "Diffraction" is the bending of waves around obstacles or the edges of an opening. (spreading of wavefronts as they pass the edge of an object).
- Single slit fringe pattern - alternate B & D fringes and fringes fade away from 'centre bright'. (series of narrower less intense secondary bands - 'secondary maxima')
- In terms of Huygen's principle, each point on a wavefront can be considered as a source of a new wave. (wavelets)

For single slit diffraction;

$a \sin \theta = m\lambda$ [dark fringes - minima]

$I(\theta) = I_m \left(\frac{\sin(\phi/2)}{\phi/2} \right)^2$

$\left[\frac{\phi}{2} = \frac{\pi}{\lambda} a \sin \theta \Rightarrow m\lambda \right]$

$$\text{maxima} \Rightarrow \boxed{a \sin \theta = \frac{(2m+1)\lambda}{2}} \quad m \in \mathbb{N}$$

$$\text{minima} \Rightarrow \boxed{a \sin \theta = m\lambda} \quad m \in \mathbb{N}$$

QVE: Single slit; width $1.50 \mu\text{m}$; $\lambda = 500 \text{ nm}$

Find angular width of central maxima

spread of the maximum is from 1st order minima from either sides.

$$\sin \theta = \frac{\lambda}{b} = \frac{500 \times 10^{-9}}{1.5 \times 10^{-6}} = 0.333$$

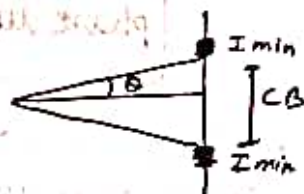
$$\theta = \sin^{-1}(0.333)$$

$$\boxed{\theta = 19.5^\circ} \quad (\text{or } 0.34 \text{ rad}) //$$

$$b \sin \theta = m\lambda \quad (\text{for one minima})$$

$$\therefore \text{Angle difference} = \theta + \theta = 2\theta$$

$$\underline{\text{Ans}}: \text{Angular width} \Rightarrow 39^\circ //$$



QVE: 580 nm light incident on slit 0.30 mm
 $D = 2 \text{ m}$. Position of first minima

$$b \sin \theta = (1)\lambda$$

$$\sin \theta = \frac{\lambda}{b} = \frac{580 \times 10^{-9}}{0.3 \times 10^{-3}} = 1.9 \times 10^{-3}$$

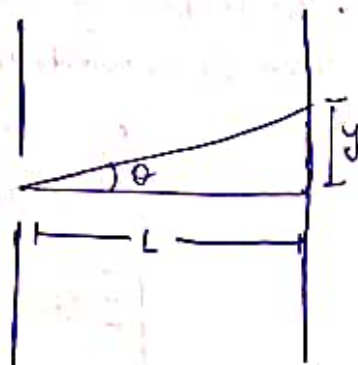
$$\sin \theta \approx \theta \Rightarrow 1.9 \times 10^{-3}$$

For small angles; $\sin \theta \approx \tan \theta$

$$\therefore \tan \theta = \frac{y}{L} \quad [L = D = 2 \text{ m}]$$

$$y = L \tan \theta = 2 \times 1.9 \times 10^{-3}$$

$$\underline{\text{Ans}}: \boxed{y = 3.8 \times 10^{-3} \text{ m}} //$$



* Diffraction and Interference by a double slit:

$$I = I(\text{double slit interference}) \times I(\text{diffraction})$$

$$I(\theta) = I_m \cos^2(\beta) \times \frac{\sin^2 \alpha}{\alpha^2}$$

$$\beta = \frac{\pi d \sin \theta}{\lambda}$$

$$d \sin \theta = m \lambda$$

$m = 0, 1, 2, \dots$
[constructive] [maxima]

$$\alpha = \frac{\pi a \sin \theta}{\lambda}$$

$$a \sin \theta = m \lambda$$

$m = 1, 2, 3, \dots$
[destructive] [minima]

[a - slit width
 d - distance between the 2 slits]

QVE: Screen is sep from a double slit source by 1.2 m
 $d = 0.030 \text{ mm}$. 2nd order maxima is 4.5 cm from C.B.
 $\lambda = ?$ $L = 1.2 \text{ m}$; $d = 3 \times 10^{-5} \text{ m}$; $y = 0.045 \text{ m}$
 $\tan \theta = y/L$

$$\theta = \tan^{-1}\left(\frac{y}{L}\right) = \tan^{-1}\left(\frac{0.045}{1.2}\right) = 2.15^\circ //$$

$$d \sin \theta = m \lambda$$

$$3 \times 10^{-5} \times \sin(2.15^\circ) = 2 \lambda$$

$$\underline{\text{Ans}}: \lambda = 5.62 \times 10^{-7} \text{ m} //$$

NOTE:

As the number of slits grow, the peaks become narrower & more intense.

QVE: A grating consists of 4000 slits per cm. Produces 2nd order bright line at 30° . $\lambda = ?$

No. slits per cm = 4000

$$d = \text{dist between slits} = \frac{1}{4000} = 0.00025 \text{ cm} = 2.5 \times 10^{-6} \text{ m}$$

order $(n) = 2$

$$d \sin \theta = n \lambda$$

$$2.5 \times 10^{-6} \times \sin(30^\circ) = 2 \lambda$$

$$\lambda = 2.5 \times \frac{1}{2} \times \frac{1}{2} \times 10^{-6} = 0.625 \times 10^{-6}$$

$$\underline{\text{Ans}}: \lambda = 6250 \text{ \AA} //$$

QUE: $\lambda = 2.5 \times 10^{-7} \text{ m}$; 15000 slits/cm ; 2nd bright θ_2 ?

$$d = \frac{1}{10k} = 0.0001 \text{ cm} = 10^{-6} \text{ m} \quad \left. \begin{array}{l} n\lambda = d \sin \theta \\ 2\lambda = 10^{-6} \sin \theta \end{array} \right\}$$

$$\sin \theta = \frac{2 \times 2.5 \times 10^{-7}}{10^{-6}} = 0.5 = \frac{1}{2} \quad \text{Ans } \boxed{\theta = 30^\circ} //$$

QUE: $\lambda = 5 \times 10^{-7} \text{ m}$; $L = 2 \text{ m}$; $y_3 = 1.5 \text{ m}$; $d = ?$

$$\sin \theta \approx \tan \theta = \frac{y}{L} = \frac{1.5}{2} = 0.75 = 75 \times 10^{-2}$$

$$d \sin \theta = n\lambda$$

$$d = \frac{3 \times 5 \times 10^{-7}}{75 \times 10^{-2}} \quad \left. \begin{array}{l} \\ \end{array} \right\} \quad d = 2 \times 10^{-6} \text{ m}$$

$$\boxed{d = 2 \mu \text{ m}} //$$

QUE: $\lambda = 500 \text{ nm}$; $\theta_{4B} = 30^\circ$; no. slits/cm = ?

$$(i) d \sin \theta = n\lambda$$

$$d \times \frac{1}{2} = 4 \times 500 \times 10^{-9} = 40 \times 100 \times 10^{-9} \Rightarrow \boxed{d = 4 \times 10^{-6} \text{ m}}$$

$$\boxed{d = 4 \times 10^{-4} \text{ cm}} \quad \text{no. slits per cm} \Rightarrow \frac{1}{d} = \frac{1}{4 \times 10^{-4}}$$

$$\text{Ans: No. slits per cm} \Rightarrow 2500 //$$

$$(ii) \theta_2 = 30^\circ \quad \frac{d}{2} = 2 \times 500 \times 10^{-9} = 2000 \times 10^{-9} =$$

$$\boxed{d = 2 \times 10^{-4} \text{ m}} \Rightarrow \text{no. slits per cm} \Rightarrow 5000 //$$

QUE: $\lambda = 450 \text{ nm}$; $L = 1.8 \text{ m}$; Dist between dark

(i) $d = ?$ (ii) no. slits/m = ?

$$\theta = \tan^{-1} (y/L) = \left(\frac{4.2}{2} \times 10^{-3} \right) \times \frac{1}{1.8}$$

$$\Rightarrow \tan^{-1} \left(\frac{0.0021}{1.8} \right) = 0.067^\circ //$$

$$d \sin \theta = \left(\frac{2m+1}{2} \right) \lambda$$

$$d = \frac{1}{2} \times 450 \times 10^{-9} \times \sin (0.067^\circ)$$

$$d = 0.0001924 \text{ m}$$

$$\Rightarrow \boxed{d = 1.924 \times 10^{-4} \text{ m}} //$$

$$d = \frac{\text{metres}}{\text{line}}$$

$$\frac{\text{lines}}{\text{metre}} = d^{-1} = \frac{1}{0.0001924}$$

$$\Rightarrow \boxed{\text{no. lines/m} \Rightarrow 5197.2} //$$

superimposed, form an interference pattern.

• 2 independent coherent waves superimpose.

• Fringes are of uniform I
All bright fringes equally bright

• Good contrast between the maxima & minima

• Constructive $\Rightarrow \boxed{n\lambda = d \sin \theta}$
 $\theta = 0, 1, 2, 3, \dots$

Destructive $\Rightarrow \boxed{\frac{(2n+1)\lambda}{2} = d \sin \theta}$

• Wavelets result in diffraction

• C.B is max width and subsequent maxima have diminishing I.

• comparatively poor contrast between maxima & minima.

• Constructive $\Rightarrow \boxed{\frac{(2n+1)\lambda}{2} = d \sin \theta}$
 $(n = 1, 2, 3, \dots)$

Destructive $\Rightarrow \boxed{n\lambda = d \sin \theta}$
 $(1, 2, 3, \dots)$

QVE: $\lambda = 2.5 \times 10^{-7} \text{ m}$; 10,000 slits/cm ; 2nd bright 0.2?

$$d = \frac{1}{10^4} = 0.0001 \text{ cm} = 10^{-6} \text{ m} \quad \left. \begin{array}{l} n\lambda = d \sin \theta \\ 2\lambda = 10^{-6} \sin \theta \end{array} \right\}$$

$$\sin \theta = \frac{2 \times 2.5 \times 10^{-7}}{10^{-6}} = 0.5 = \frac{1}{2} \quad \text{Ans } \boxed{\theta = 30^\circ} //$$

QVE: $\lambda = 5 \times 10^{-7} \text{ m}$; $L = 2 \text{ m}$; $y_3 = 1.5 \text{ m}$; $d = ?$

$$\sin \theta \approx \tan \theta = \frac{y}{L} = \frac{1.5}{2} = 0.75 = 75 \times 10^{-2}$$

$$d \sin \theta = n\lambda$$

$$d = \frac{3 \times 5 \times 10^{-7}}{75 \times 10^{-2}} \quad \left. \begin{array}{l} \\ \end{array} \right\} \quad d = 2 \times 10^{-6} \text{ m} \\ \boxed{d = 2 \mu \text{ m}} //$$

QVE: $\lambda = 500 \text{ nm}$; $\theta_{4B} = 30^\circ$; no. slits per cm = ?

$$(i) d \sin \theta = n\lambda$$

$$d \times \frac{1}{2} = 4 \times 500 \times 10^{-9} = 40 \times 100 \times 10^{-9} \Rightarrow \boxed{d = 4 \times 10^{-6} \text{ m}}$$

- Diffraction observed at small distances is called "near field diffraction" or "Fresnel diffraction". (Zone plates)
- Diffraction observed at larger distances are called "far-field diffractions" or "Fraunhofer diffraction".



Interference

- Two or more waves travelling in forward direction when superimposed, form an interference pattern.
- 2 independent coherent waves superimpose.
- Fringes are of uniform I
- All bright fringes equally bright
- Good contrast between the maxima & minima

$$\text{Constructive} \Rightarrow \boxed{n\lambda = d \sin \theta} \quad (n=1, 2, 3, \dots)$$

$$\text{Destructive} \Rightarrow \boxed{\frac{(2n+1)\lambda}{2} = d \sin \theta}$$

Diffraction

- Light bends around the corners of sharp edge and enter shadow region.
- Wavelets result in diffraction
- C.B is max width and subsequent maxima have diminishing I.
- comparatively poor contrast between maxima & minima.

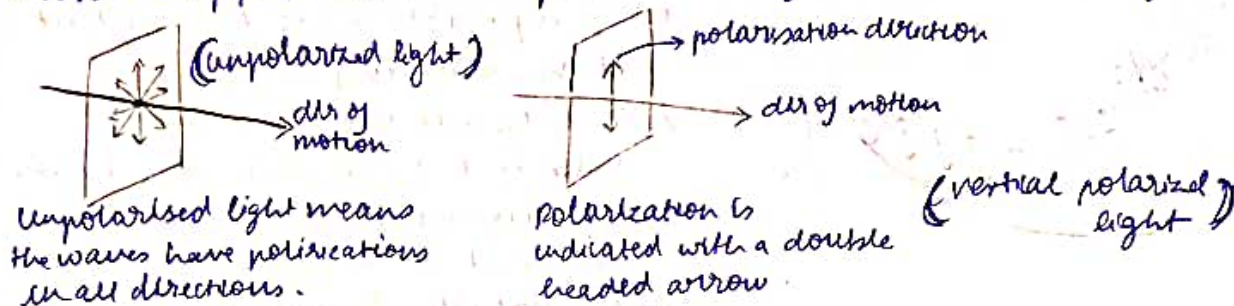
$$\text{Constructive} \Rightarrow \boxed{\frac{(2n+1)\lambda}{2} = d \sin \theta} \quad (n=1, 2, 3, \dots)$$

$$\text{Destructive} \Rightarrow \boxed{n\lambda = d \sin \theta} \quad (1, 2, 3, \dots)$$

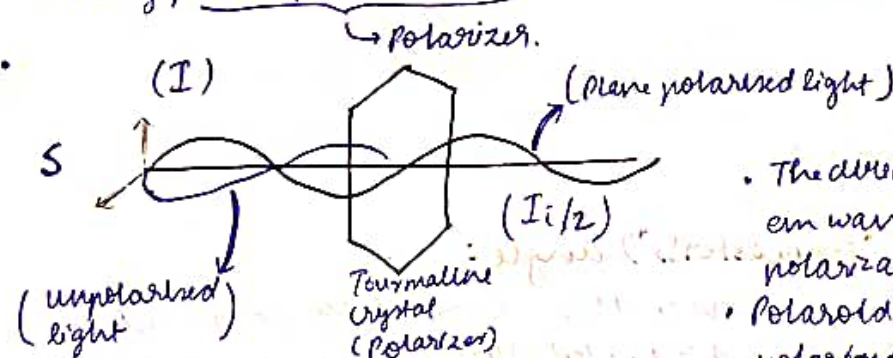
UNIT - II

BASIC IDEA OF ELECTROMAGNETISM

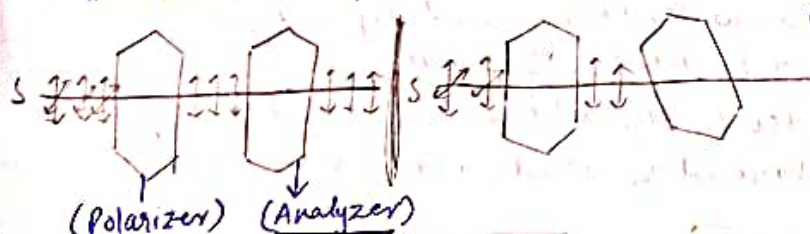
- Electromagnetic waves have both electric and magnetic fields that oscillate perpendicularly. (Transverse wave)
- Light should be considered as an electromagnetic wave
- What happens when unpolarised light encounters a filter?



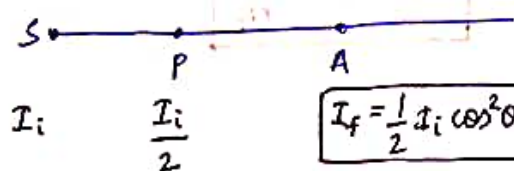
- Direction of propagation $\Rightarrow \vec{c}$
 $\vec{E} \perp \vec{B} \perp \vec{c}$
- Controlling a light wave or any em wave to vibrate in one specific dir by making it pass through a filter made out of polaroid material is called "Polarization"



- How to check if polarized?



- The direction of \vec{E} in the em wave is called the polarization direction.
- Polaroid filter absorbs light polarized along one axis & transmits light polarized along the other.



$$I = I_0 \cos^2 \theta$$

$$I_f = \frac{1}{2} I_i \cos^2 \theta$$

(Law of Malus)

[Applicable only of Analyzer]

* Polarization by reflection:

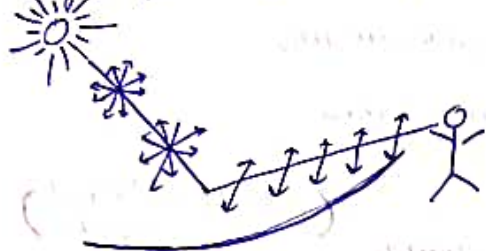
- When unpolarized light beam is reflected from a surface, the reflected light can be completely polarized or partially polarized or unpolarized.
- Depends on angle of incidence θ :

If $\theta = 0^\circ$; reflected beam is unpolarized.

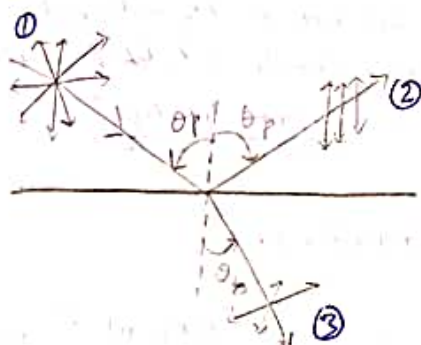
$0 < \theta < 90^\circ$; partially polarized.

one particular angle; completely polarized [polarizing angle]

Ex: Light reflected from a lake is partially horizontally polarized



- If unpolarized light is incident at the polarizing angle
- Then the reflected light is 100% polarized \perp to plane of incidence
- The transmitted light is partly polarized \parallel to plane of incidence
- Alternatively, if unpolarized light is incident on the reflecting surface at an angle other than θ_p , the reflected light is partly polarized.



* Polarizing ("Brewster's") angle:

- Light in a reflection will be completely polarized if the reflected and refracted rays make a 90° angle
- The incident angle required to achieve this is called the polarizing or Brewster angle, and depends on the ref indices of refraction of the media.
- The angle of incidence for which the reflected ray is completely polarized is called polarizing angle (θ_p)

$$\frac{\sin \theta_p}{\sin \theta_2} = \frac{n_2}{n_1}$$

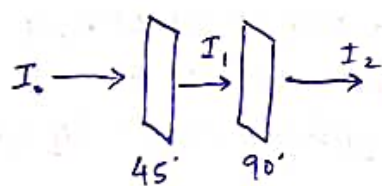
$$\tan \theta_p = \frac{n_2}{n_1}$$

$$[\theta_2 = 90^\circ - \theta_p]$$

* Selective absorption:

- A material that polarizes light through selective absorption is called a "Polaroid".
- The molecules readily absorb light whose \vec{E} is parallel to their lengths and transmit light whose \vec{E} is \perp to their lengths.
- When unpolarised light passes through polaroid filter;
 - half the intensity is absorbed by the filter
 - The transmitted light becomes linearly polarized in the same direction as the filter's polarizing axis.
- When linearly polarized light passes through the filter;
 - The intensity is reduced depending on ϕ , the angle between the polarization direction and polarizing axis. $I = I_0 \cos^2 \phi$
 - The transmitted light becomes linearly polarized in the dir of the filter's polarizing axis.
- If light (any) is passed through 2 ideal polaroid filters with \perp polarization axes, it will be completely absorbed. (Principle behind sunglasses)

QUB: 3 polarizing filters are stacked with p'ing axis of 2nd and 3rd filters at 45° & 90° resp.



$$I_1 = I_0 \cos^2 45^\circ = I_0 / 2$$

$$I_2 = I_1 \cos^2 90^\circ = I_1 \times 0 = 0 //$$

$$\therefore I_2 = 0 //$$

$$I_1 = I_0 \cos^2 45^\circ = I_0 / 2$$

$$I_2 = I_1 \cos^2 45^\circ = I_0 / 2 \times 1/2 = I_0 / 4$$

$$I_0 = I / 2$$

$$\therefore I_2 = I / 8 //$$

$$E_x = E_0 \sin(kz - \omega t)$$

$$B_y = B_0 \sin(kz - \omega t)$$

$$\omega = kc$$

$$c = \frac{E_0}{B_0}$$

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

\vec{E} determines the polarization

$$E = \hat{e} E_0 \sin(kz - \omega t + \phi)$$

$$E_x = E_0 \cos \theta \sin(kz - \omega t + \phi)$$

$$E_y = E_0 \sin \theta \sin(kz - \omega t + \phi)$$

$$\omega = kc$$

$$c = \frac{E_0}{B_0} = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

* Polarization —
 → Linear polarization
 → Circular polarization
 → Elliptical polarization

• For linearly polarized light the \vec{E} and \vec{B} are in phase and the resultant vector oscillates linearly.

• When we shift one of the component by quarter cycle; i.e. [phase difference = $\pi/2 = 90^\circ$]

Here, the resultant vector appears to be rotating either in clockwise or anti-clockwise direction. This light is called "Circular polarized light."

• In circularly polarized light, if the amplitude of the waves \vec{E} and \vec{B} are different, then the resultant vector moves (or rotates) like an ellipse and this is called "elliptically polarized light" [phase diff = $\pi/2 = 90^\circ$]

* Wave plate:

- Optical device — birefringent crystal with chosen thickness
- Light polarized along the extraordinary / fast axis propagates faster than the ordinary / slow axis.

Phase shift

$$\phi = \frac{2\pi \Delta n L}{\lambda}$$

$$\phi = n K L$$

↑ color of light
↓ n_K index

Making ϕ_x diff from ϕ causes circular polarization

$$\underline{\underline{Ex}}: \quad \phi_x - \phi_y = \pi/2 \quad \boxed{\theta = 45^\circ}$$

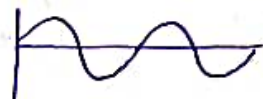
$$E_x = \frac{E_0}{\sqrt{2}} \cos(kz - \omega t)$$

$$E_y = \frac{E_0}{\sqrt{2}} \sin(kz - \omega t)$$

$$y = a_1 \sin(\omega t - \alpha_1)$$

$$z = a_2 \sin(\omega t - \alpha_2)$$

$$[\alpha_1 - \alpha_2 = \phi]$$



$$\Rightarrow \frac{y}{a_1} = \sin \omega t \cos \alpha_1 - \cos \omega t \sin \alpha_1 \quad \text{--- (1)}$$

$$\Rightarrow \frac{z}{a_2} = \sin \omega t \cos \alpha_2 - \cos \omega t \sin \alpha_2 \quad \text{--- (2)}$$

$$\frac{y}{a_1} \sin \alpha_2 = (\sin \alpha_2) (1) \quad \text{--- (3)}$$

$$= \frac{z}{a_2} \sin \alpha_1 = (\sin \alpha_1) (1) \quad \text{--- (4)}$$

$$\frac{z}{a_2} \sin \alpha_1 - \frac{y}{a_1} \sin \alpha_2 = \sin \omega t (\cos \alpha_2 \sin \alpha_1 - \sin \alpha_2 \cos \alpha_1) \quad \text{--- (5)}$$

$$= \frac{y}{a_1} \cos \alpha_2 = (\cos \alpha_2) (1) \quad \text{--- (6)}$$

$$= \frac{z}{a_2} \cos \alpha_1 = (\cos \alpha_1) (1) \quad \text{--- (7)}$$

$$\frac{y}{a_1} \cos \alpha_2 - \frac{z}{a_2} \cos \alpha_1 = -\cos \omega t (\cos \alpha_2 \sin \alpha_1 - \cos \alpha_1 \sin \alpha_2) \quad \text{--- (8)}$$

$$\left[\frac{y}{a_1} \cos \alpha_2 - \frac{z}{a_2} \cos \alpha_1 \right]^2 = \cos^2 \omega t (\cos \alpha_2 \sin \alpha_1 - \cos \alpha_1 \sin \alpha_2)^2$$

$$\left[\frac{z}{a_2} \sin \alpha_2 - \frac{y}{a_1} \sin \alpha_1 \right]^2 = \sin^2 \omega t (\cos \alpha_2 \sin \alpha_1 - \sin \alpha_2 \cos \alpha_1)^2$$

$$\left. \begin{aligned} & \frac{y^2}{a_1^2} \cos^2 \alpha_2 + \frac{z^2}{a_2^2} \cos^2 \alpha_1 + \frac{z^2}{a_2^2} \sin^2 \alpha_1 + \frac{y^2}{a_1^2} \sin^2 \alpha_2 \\ & - \frac{2yz}{a_1 a_2} \cos \alpha_1 \cos \alpha_2 - \frac{2yz}{a_1 a_2} \sin \alpha_1 \sin \alpha_2 \end{aligned} \right\} = 1$$

$$= \frac{y^2}{a_1^2} + \frac{z^2}{a_2^2} - \frac{2yz}{a_1 a_2} (\cos \alpha_1 \cos \alpha_2 + \sin \alpha_1 \sin \alpha_2) = 1$$

$$\Rightarrow \boxed{\frac{y^2}{a_1^2} + \frac{z^2}{a_2^2} - \frac{2yz}{a_1 a_2} \cos(\alpha_1 - \alpha_2) = 1}$$

Relation when the 2 waves are superimposed

NOTE: When $\alpha_1 - \alpha_2 = \delta = \pi/2$;

$$\Rightarrow \boxed{\frac{y^2}{a_1^2} + \frac{z^2}{a_2^2} = 1}$$

[Equation of ellipse]

NOTE: When $a_1 = a_2$ & $\delta = \pi/2$;

$$\Rightarrow \boxed{y^2 + z^2 = a^2}$$

[Equation of circle]

When $\delta = \alpha_1 - \alpha_2 = 0$;

NOTE: $\frac{y^2}{a_1^2} + \frac{z^2}{a_2^2} - \frac{2yz}{a_1 a_2} = 1 = \left(\frac{y}{a_1} - \frac{z}{a_2}\right)^2$

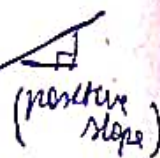
$$\frac{y}{a_1} - \frac{z}{a_2} = 1$$

$$\frac{y}{a_1} = \frac{z}{a_2}$$

$$\Rightarrow y = z \left(\frac{a_1}{a_2}\right) m$$

$$y = mx + c$$

[Equation of straight line]



NOTE $\delta = \alpha_1 - \alpha_2 = \pi$; $(\cos \pi = -1)$

$$\therefore \left(\frac{y}{a_1} + \frac{z}{a_2}\right)^2 = 1 = \left(\frac{y}{a_1} + \frac{z}{a_2}\right)$$

$$\Rightarrow y = -z \left(\frac{a_1}{a_2}\right) m$$

[Equation of straight line]

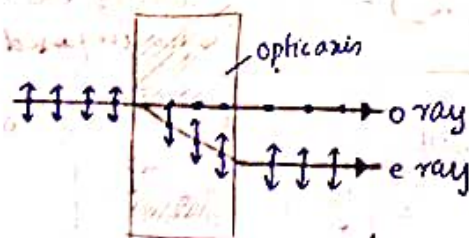


- If the phase difference can be induced 180° , half wave plate is used to retard 180° and rotates polarization direction of linear polarized light
- Quarter-wave plate - retards 90° , changed linearly polarized light to circular. Intensity due to quarter wave plate does not change. I - constant.

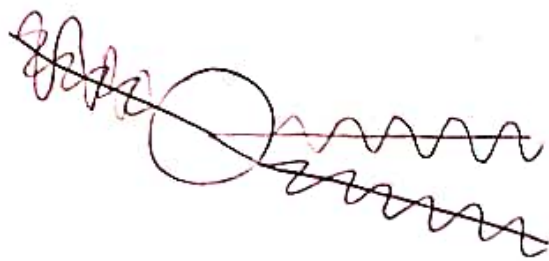
★ DOUBLE REFRACTION:

A light ray incident on a birefringent material is split into two beams namely ordinary ray and extraordinary ray that have mutually \perp polarizations.

Example for a birefringent material \Rightarrow Calcite, Iceland spar, etc.



This splitting of a ray into two and two refractions is called double refraction.



Optical path travelled = $n \cdot d$

" travelled by o ray = $n_o \cdot d$

" travelled by e ray = $n_e \cdot d$

Since optical path travelled

by o & e are different,

the refractive indexes for

o and e are different

Ordinary ray	Extra-ordinary ray.
<ul style="list-style-type: none"> • This ray obeys the laws of refraction • Plane of vibration lies \perp to the direction of propagation • The vibration of particles are perpendicular to the direction of ray. • plane of polarization lies in the principal plane • Refractive index is constant along optic axis. • It travels in constant speed in all directions. 	<ul style="list-style-type: none"> • This ray does not obey the law of refraction. • plane of vibration is parallel to direction of propagation. • The vibration of particle is parallel to the direction of the ray. • plane of polarization lies perpendicular to the principal axis. • Refractive index varies along optic axis. • It travels with different speed in different directions but travel with equal speed along optic axis

* Polarization by double refraction:

$$E = E_0 \cos(kx - \omega t) \quad \text{or} \quad E = E_0 \cos(kx - \omega t) \quad \text{or} \quad E = E_0 \cos(kx - \omega t)$$

$$B = B_0 \cos(kx - \omega t) \quad \text{or} \quad B = B_0 \cos(kx - \omega t) \quad \text{or} \quad B = B_0 \cos(kx - \omega t)$$

$$E = E_0 \cos(kx - \omega t) \quad \text{or} \quad E = E_0 \cos(kx - \omega t) \quad \text{or} \quad E = E_0 \cos(kx - \omega t)$$

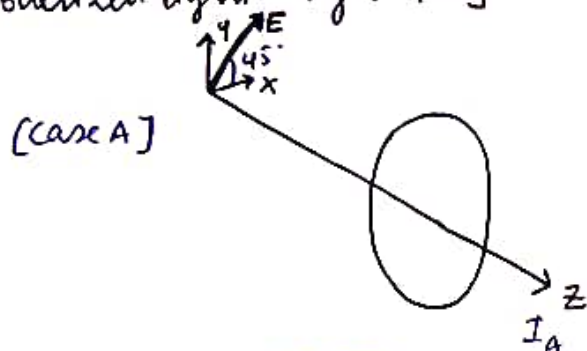
$$B = B_0 \cos(kx - \omega t) \quad \text{or} \quad B = B_0 \cos(kx - \omega t) \quad \text{or} \quad B = B_0 \cos(kx - \omega t)$$

QVE: What happens when circularly polarized light is put through a polarizer along x or y axis?

$$E_x = E_0 \cos(kz) \quad E_y = E_0 \sin(kz) \quad \left. \vphantom{E_x = E_0 \cos(kz)} \right\} \therefore \boxed{I = \frac{1}{2} I_0} //$$

Intensity is reduced by half

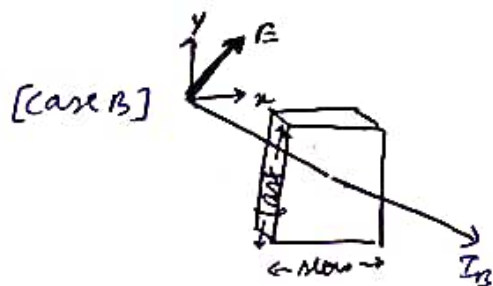
QVE: Linearly polarized light from the y axis is passed through A - linear polarizer & B - quarter wave plate. [polarized light angle 45°] [fast axis = y axis]



$$\boxed{I_A = I_0 / 2}$$

E_x is absorbed and thus;

$$I_A = I_0 \cos^2(45^\circ) \Rightarrow \boxed{I_A < I_B} //$$



$$\boxed{I_B = I_0}$$

(E_x, E_y) phase changed

QVE: Unpolarized light falls on 2 polarizing sheets, one on top of the other. What must be the angle between the characteristic directions of the sheets if the I of the transmitted light is one-third of the incident beam?

$$I = I_0 \cos^2 \theta$$

$$I_2 = I_0 / 3 = I_0 / 2 \cos^2 \theta$$

$$\cos^2 \theta = 2/3$$

$$\cos \theta = \sqrt{2/3}$$

$$\boxed{\theta = \cos^{-1}(\sqrt{2/3})} //$$

$$\xrightarrow{I} \left[\frac{I_0}{2} \right] \left[\frac{I_0}{2} \cos^2 \theta \right]$$

QVE: Natural light falls on 3 identical polaroids, the principle dir of the mid one forming an angle $\theta = 60^\circ$ with those of the other 2. Max transmission coeff of each $P \Rightarrow T = 0.81$ when plane pol light falls on them. $I = ?$

$$\left[\begin{array}{c} \rightarrow I_1 \\ \rightarrow I_2 \\ \rightarrow I_3 \end{array} \right]$$

$$I_1 = I_0 / 2 T; \quad I_2 = \frac{I_0}{2} \cos^2 \theta \times T^2; \quad I_3 = I_2 \times T \times \cos^2 \theta$$

$$\left[\begin{array}{c} 100W \\ \rightarrow \\ 81W \\ T = 0.81 \end{array} \right]$$

$$= I_1 T^2 \cos^4 \theta$$

$$= \frac{I_0}{2} T^3 \cos^4 \theta$$

$$= \frac{I_0}{2} \times (0.81)^2 \times \left(\frac{1}{2}\right)^4 //$$

$$\therefore \text{dec} = \frac{I_0}{I_3} = \frac{I_0 \times 2 \times 16}{I_0 (0.81)^3}$$

$$\boxed{\therefore \text{dec} = 60.2 \text{ times}} //$$

QVE: Light beam travelling in water enters glass plane
 $\theta_i = ?$ for which reflected light is completely polarized.

$$\tan \theta_2 = \frac{1.55}{1.33} = 1.127$$

$$\Rightarrow \theta_i = \tan^{-1}(1.127) \Rightarrow$$

QVE: $\lambda = 5400 \text{ \AA}$ $\phi = 60^\circ \rightarrow$ phase diff b/w e & o
 (Plane polarized light) (Incident on quartz)

$$n_e = 1.553$$

$$n_o = 1.544$$

[L = thickness of crystal = ?]

$$\phi = \frac{2\pi \Delta n L}{\lambda}$$

$$60 = \frac{\pi}{2} = \frac{2\pi \times 0.009 \times L}{5400 \times 10^{-10}}$$

$$\frac{1}{2} = \frac{2 \times 9 \times 10^{-3} \times L}{54 \times 10^{-8}}$$

$$L = \frac{10^{-8}}{10^{-3}}$$

$$\Rightarrow \boxed{L = 10^{-5} \text{ m}} //$$

$$\boxed{L = 10 \mu\text{m}} //$$

QVE: $\lambda = 6000 \text{ \AA}$ Incid on calcite $D = 0.04 \text{ mm}$
 $\mu_o = 1.642$ $\mu_e = 1.478$

$$\phi = \frac{2\pi \Delta n L}{\lambda} = \frac{2\pi \times 0.164 \times 4 \times 10^{-5}}{6000 \times 10^{-10}}$$

$$= \frac{\pi \times 0.164 \times 4 \times 10^{-5} \times 10^7}{3}$$

$$= \pi \times 0.2186 \times 10^2$$

$$= 218\pi \quad 21.86 \times 3.14$$

$$\boxed{\phi \Rightarrow 68.64 \text{ rad}} //$$

QVE: Left circularly polarized beam ($\lambda = 5893 \text{ \AA}$) is incident on calcite (optic axis \parallel to surface) $L = 0.005141 \text{ mm}$
State of polarization of emergent beam?

$$E_x = E_0 \sin \omega t \quad ; \quad E_y = E_0 (\sin \omega t + \pi/2) = E_0 \cos \omega t$$

emergent beam $\Rightarrow E_y = \frac{E_0}{\sqrt{2}} \sin \omega t \quad E_z = \frac{E_0}{\sqrt{2}} \cos \omega t$

$[\Delta n = 0.17195]$

$$\theta = \frac{\Delta n \times L \times 2\pi}{\lambda_0}$$

$$= \frac{0.17195 \times 0.05141 \times 2\pi}{5893 \times 10^{-7}}$$

$$\boxed{\theta = 3\pi} \quad \therefore //$$

Thus, emergent wave will be;

$$E_y = \frac{E_0}{\sqrt{2}} \sin(\omega t - 3\pi) = -\frac{E_0}{\sqrt{2}} \sin(\omega t)$$

$$\boxed{E_y = -\frac{E_0}{\sqrt{2}} \sin \omega t} \quad //$$

$$\boxed{E_z = \frac{E_0}{\sqrt{2}} \cos \omega t} \quad //$$

Ans \Rightarrow which represents a right circularly polarized beam

QVE: A half wave plate is fabricated for a $\lambda = 3800 \text{ \AA}$
For what λ it works as quarter wave plate?

$$\frac{\pi}{2} = \frac{\Delta n L 2\pi}{\lambda_0} = \frac{1}{3800} = \frac{\pi}{2}$$

$$\frac{\pi}{4} = \frac{2\pi \Delta n L}{\lambda_1}$$

$$\left[\pi = \frac{2}{3800} \right]$$

$$\frac{\pi}{4} \times \frac{2}{\pi} = \frac{1}{\lambda_1} \times 3800$$

$$\frac{1}{2} = \frac{3800}{\lambda_1}$$

$$\Rightarrow \boxed{\lambda_1 = 7600 \text{ \AA}} \quad //$$