SASTRA DEEMED UNIVERSITY

(A University under section 3 of the UGC Act, 1956)

End Semester Examinations

February 2022

Course Code: **MAT133**

Course: **DISCRETE MATHEMATICS**

Question Paper No. : **UGF035** Max. Marks:**100**

PART - A

Answer all the questions **10 x 2 = 20 Marks**

1. Find the complement of the Boolean function F(w,x,y,z)=wx+x'z+wxy.

2. Construct a circuit that produces the output x'(y+z')'.

3. Sketch the region of integration $\int_{y=-2}^{2}\int_{x=0}^{y}(x^2+y^2)dxdy$.

4. Evaluate $\int_0^{\frac{\pi}{2}}\int_0^2 r\cos^7\theta\, dr\, d\theta$.

5. Obtain $\frac{d^2y}{dx^2}$, if $y = \tan x + 3^x$.

6. How many reflexive relations can be defined on a set having 10 elements?

7. Is the converse of Lagrange's theorem true? Justify.
8. If n^{th} term of a sequence {s(n)} is the sum of two times of $(n-1)^{th}$ term and three times $(n-2)^{th}$ term such that s(0)=1 and s(1)=3,then find the recurrence relation whose solution is {s(n)}. Also, find S(6).

9. How many different five letter words with or without meaning can be formed using the letters a,b,c,d,e,i,o,u,v,x if the words start with a consonant?

10. State the pigeonhole principle.

PART - B

Answer all the questions **4 x 15 = 60 Marks**

11. A committee of three individuals decides issues for an organization. Each individual vote either yes or no for each proposal that arises. A proposal is passed if it receives at least two yes votes. Design a circuit that determines whether a proposal passes. Also, obtain the sum of product and product of sum canonical forms for the output of the above circuit.

(OR)

12. (a) Prove the following laws for Boolean algebra. (12)
    (i) Absorption law
    (ii) De Morgan's law.
    (b) Simplify the Boolean function.
    F(a,b,c,d)=a'bcd + ab'cd + a'b'cd +a'b'c'd. (3)

13. (a) Obtain $\frac{dy}{dx}$, if $x = 2a\, sin^{-1}\left(\sqrt{\frac{y}{2a}}\right) - \sqrt{2ay - y^2}$. (7)
    (b) Change the order of integration in $\int_0^2 \int_x^{1-x} xy\, dy\, dx$ and hence evaluate it. (8)

(OR)

14. (a) Find the volume of the rectangular box which is bounded by x=0, x=2, y=0, y=4, z=0 and z=6 using triple integrals. (6)
    (b) Evaluate $\int_0^{log2} \int_0^{x} \int_0^{x+y} e^{x+y+z}\, dx\, dy\, dz$. (5)
    (c) Obtain $\frac{du}{dv}$, $where\ u = e^{sin^{-1}x}; v = e^{cos^{-1}x}$. (4)

15. (a) State and prove Lagrange's theorem. (12)
    (b) Is a group of prime order cyclic? Justify. (3)

(OR)

16. (a) If A, B and C are sets then prove that $A X (B \cap C) = (A X B) \cap (A X C)$. (6)
(b) Show that the relation $R=\{(a,b) / a \equiv b \bmod n\}$ on the set of integers is an equivalence relation. (7)
(c) Draw Hasse diagram for (S_{48}, D). (2)

17. (a) Prove that $\sum_{j=0}^{n}\left(-\frac{1}{2}\right)^{j} = \frac{2^{n+1}+(-1)^{n}}{3.2^{n}}$, whenever n is a non-negative integer. (9)
(b) How many license plates can be made either using 2 uppercase English letters, followed by 4 digits or 2 digits, followed by 4 uppercase English letters? (3)
(c) What is the minimum number of students required in a Mathematics class to be sure that at least six will receive the same grade, if there are five possible grades, A, B, C, D and F? (3)

(OR)

18. (a) Find the number of integer solutions to the equation $u + v + w + x = 10$, if $u, x \geq 0$, $v \geq 1$, $0 \leq w \leq 4$. (8)
(b) How many words with or without meaning can be formed using all the letters in the word "RATIONALIZATION"? (2)
(c) Prove that $\frac{1}{1.4} + \frac{1}{4.7} + \frac{1}{7.11} + \cdots + \frac{1}{(3n-2)(3n+1)} = \frac{n}{3n+1}$. (5)

PART - C

Answer the following **1 x 20 = 20 Marks**

19. (a) Solve $S(k+1) - 8S(k) + 16S(k-1) = 4^{k}$, $k \geq 1$, given that $S(0)=1$ and $S(1)=8$ using generating function. (10)
(b) Prove that the union of two subgroups of a group is a subgroup if and only if one is contained in the other. (10)

****