Central tendency

1) Arithmetic mean
2) Mode
3) Median
4) Geometric mean
5) Harmonic mean.

A.M

1) Individual observations $\rightarrow \bar{X} = \dfrac{\sum x}{n}$.
2) Discrete Series $\rightarrow \bar{X} = \dfrac{\sum fx}{\sum f}$
3) Continuous series $\rightarrow \bar{X} = \underbrace{A}_{\text{assumed mean}} + \dfrac{\sum fd}{\sum f} \times i$

$d = \dfrac{m - A}{i}$

m → mid pt of CI

i → Length of the CI.

Properties of A.M.

1) Algebraic sum of the deviations of a set of values from their arithmetic mean is zero.

{ Discrete case }

T.P.T $\sum_i f_i \underbrace{(x_i - \bar{x})}_{\text{deviations from A.M.}} = 0$

Proof:- Consider $\sum_i f_i (x_i - \bar{x})$

$$= \sum_i f_i x_i - \sum_i f_i \bar{x}$$

$$\bar{x} = \frac{\sum_i f_i x_i}{\sum_i f_i}$$

$$= \bar{x} \sum_i f_i - \bar{x} \sum_i f_i = 0.$$

Property 2:- The sum of the squares of the deviations of a set of values is minimum when taken about mean.

Soln

$$X: x_1 \quad x_2 \quad \cdots\cdots \quad x_n$$
$$f: f_1 \quad f_2 \quad \cdots\cdots \quad f_n$$

$$Z = \sum_{i=1}^{n} f_i (x_i - A)^2 .$$

To prove, Minimum Value of Z is obtained when $A = \bar{x}$.

Diff w.r.t A,
$$\frac{dZ}{dA} = \sum_{i=1}^{n} 2 f_i (x_i - A)(-1)$$

$$\frac{d^2Z}{dA^2} = \sum_{i=1}^{n} 2 f_i (-1)(-1) = \sum 2 f_i > 0 .$$

$\Rightarrow$ Min Value is obtained when
$$-2\sum f_i (x_i - A) = 0 .$$

$$-2\sum f_i(x_i - A) = 0$$

$$\Rightarrow \sum f_i x_i - A\sum f_i = 0.$$

$$A = \frac{\sum f_i x_i}{\sum f_i} = \bar{x}.$$

Dispersion.

$$\downarrow$$

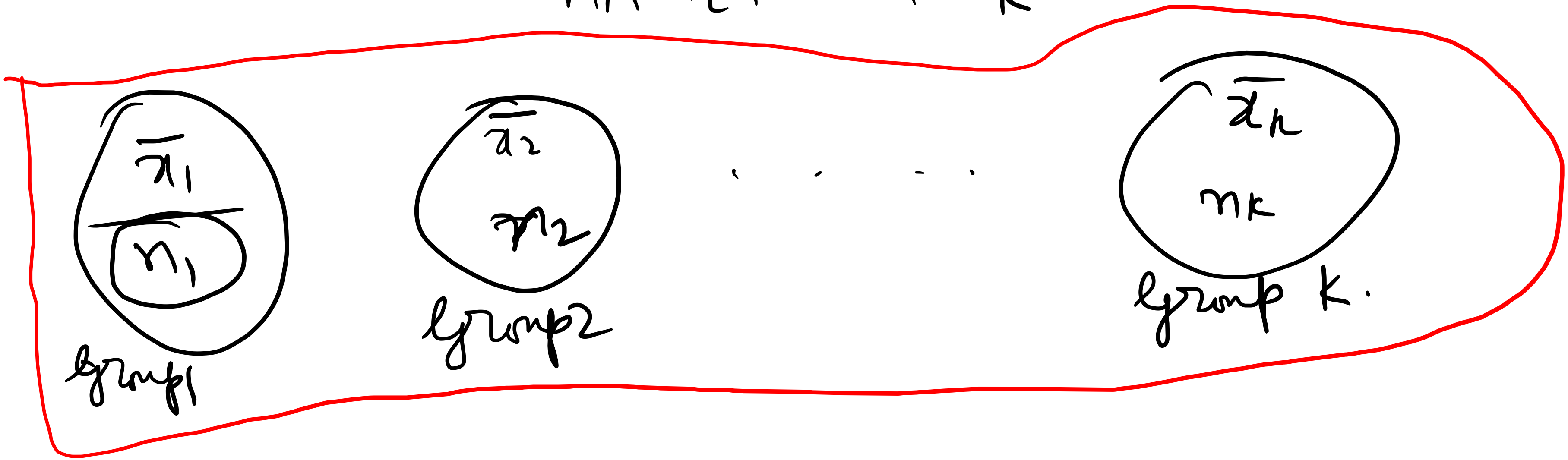
$\left\{ \begin{array}{l} S.D \\ M.D \end{array} \right.$

about Mean.

Property 3:- If $\bar{x}_i$ $(i=1,2,\dots k)$ are the means of k component series of size n_i $(i=1,2,\dots k)$ respectively, then the mean $\bar{x}$ of the composite series obtained by combining the component series is given by

$$\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2 + \dots + n_k\bar{x}_k}{n_1 + n_2 + \dots + n_k}$$

{ Combined Mean }



Pf:-

$$\bar{x}_1 = \frac{(x_{11}+x_{12}+\cdots+x_{1n_1})}{n_1} \Rightarrow \sum_i x_{1i} = n_1\bar{x}_1$$

$\vdots$

$$\bar{x}_k = \frac{x_{k1}+x_{k2}+\cdots+x_{kn_k}}{n_k}.$$

$$\bar{x} = \frac{(x_{11}+x_{12}+x_{13}+\cdots+x_{1n_1})+\cdots+(x_{k1}+x_{k2}+\cdots+x_{kn_k})}{n_1+n_2+\cdots+n_k}.$$

$$= \frac{n_1\bar{x}_1+n_2\bar{x}_2+\cdots+n_k\bar{x}_k}{n_1+n_2+\cdots+n_k}.$$

1) Find the mean of 32, 45, 28, 13, 0, 16, 32, 27.

Soln

$$\bar{X} = \frac{\sum x_i}{n} = \frac{32+45+28+13+0+16+32+27}{8}$$

$$= \frac{193}{8} = \underline{\underline{24 \cdot 125}}.$$

2) The algebraic sum of the deviations of 20 observations measured from 30 is 2. Find the mean.

Soln:

$$\sum_{i=1}^{20}(x_i - \cancel{A}^{\,30}) = 2$$

$$\sum_{i=1}^{20} x_i - 20 \times 30 = 2$$

$$\sum_{i=1}^{20} x_i = 602.$$

$$\overline{X} = \frac{\sum_{i=1}^{20} x_i}{20} = \frac{602}{20} = 30.1$$

3) Calculate the mean for the following

x:	0	10	20	30	40	50	60	
f:	8	10	11	16	20	25	15	$\boxed{105}$

Soln

x	f	fx
0	8	0
10	10	100
20	11	220
30	16	480
40	20	800
50	25	1250
60	15	900
Total		$\boxed{3750}$

$$\bar{X} = \frac{\sum fx}{\sum f} = \frac{3750}{105} = 35.71$$

(Correct to 3 decimal places)

4) Calculate the mean for the following frequency distribution:

C.I :	0-8	8-16	16-24	24-32	32-40	40-48
f :	4	12	20	28	36	44.

Soln

$$\boxed{\bar{X} = A + \frac{\sum fd}{\sum f} \times i \; ; \quad d = \frac{m-A}{i}}$$

CI	m	f	$d = \frac{m-28}{8}$	fd	fm
0-8	4	4	−3	−12	16
8-16	12	12	−2	−24	144
16-24	20	20	−1	−20	400
24-32	28 A	28	0	0	784
32-40	36	36	1	36	1296
40-48	44	44	2	88	1936

$\sum f = 144.$

$\sum fd = 68$

$$\boxed{\sum fm = 4576.}$$

$$\bar{X} = A + \frac{\sum fd}{\sum f} \times i$$

$$= 28 + \frac{68}{144} \times 8$$

$$= 31.778$$

$$\bar{X} = \frac{\sum fm}{\sum f}$$

$$= \frac{4576}{144}$$

$$= 31.778$$

5) find the mean of the following:

CI	0-7	8-14	15-22	23-30	31-38	39-46
f	8	7	16	24	15	7

Soln

CI	f	m	d=m-A	fd
-0.5-7.5	8	3.5	-23	-184
7.5-14.5	7	11	-15.5	-108.5
14.5-22.5	16	18.5	-8	-128
22.5-30.5	24	$\boxed{26.5}$	0	0
30.5-38.5	15	34.5	8	120
38.5-46.5	7	42.5	16	112

$\sum f = 77$

$\sum fd = -188.5$

$$\boxed{\bar{X} = A + \frac{\sum fd}{\sum f}}$$

$$= 26.5 + \left(\frac{-188.5}{77}\right)$$

$$= 24.05$$