15/11/21 Monday

Unit IV COMBINATORICS

1) Basic Counting Principles (Perm./Comb.)
2) Pigeon hole principle
3) Recurrence relations & Difference eqns.
4) Generating functions
5) Mathematical Induction

$\lfloor n$

$0! = 1$

$F_n = \lfloor n$

$\lfloor n = n \lfloor n-1$

$F_n = n \cdot F_{n-1}$, $F_0 = 1$

$\frac{1\times2\times3\times4\times5 = 120}{\lfloor 4 \times 5}$

System

Cont. → Differential eqns → ODE, PDE

$\frac{dy}{dx} = e^x$

$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = \sin x$

C.F. + P.I

$m^2 + 5m + 6 = 0$

$-2, -3$

C.F + P.I

Disc Crete → Recurrence relation (or) Difference equation

$S(n) - 5S(n-1) + 6S(n-2) = 2^n$

$S(n) = H.S + P.S$

Generating functions

Z-transform

$5C_3 = \frac{\lfloor 5}{\lfloor 3 \lfloor 2}$

$nC_r = \frac{\lfloor n}{\lfloor r \lfloor n-r}$

$5C_3 = \frac{5\times4\times3}{1\times2\times3} = 10$

$10C_2 = \frac{10\times9}{1\times2}$

$nC_r = \frac{n(n-1)(n-2)\cdots(n-(r-1))}{\lfloor r}$

PASCAL'S TRIANGLE

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1

$nC_0 = 1 = nC_n$

$nC_1 = n$

$nC_2 = nC_{n-2}$

$10C_7$ ← Pascal's triangle

$\frac{10!}{7!3!}$

$= 10C_{10-7} = 10C_3 = \frac{10 \times 9 \times 8}{1 \times 2 \times 3} = 120$

$$nC_0 + nC_1 + nC_2 + \cdots + nC_n = 2^n$$

$(a+b)^4 = a^4b^0 + 4a^3b + 6a^2b^2 + 4ab^3 + a^0b^4$

$= a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

COMBINATORICS

Sum Rule ← → Product Rule

5 Red balls

3 White balls

$5C_1 + 3C_1$ ✓

1

$5C_2 \times 3C_1$ ✓

2R 1W

$5C_2 + 3C_1$ ✗

Sum rule :- Let the disjoint events $E_1, E_2, \ldots, E_K$ be happened in $n_1, n_2, \ldots n_K$ ways respectively. Then any one of the K events can happen in $n_1 + n_2 + \ldots + n_K$ ways.

Product rule :- suppose that a procedure can be broken down into a sequence of two tasks. If there are n_1 ways to do the first task and for each of these ways of doing the first task there are n_2 ways to do the

second task. Then there are $n_1 . n_2$ ways to do the procedure.

Permutation :- An ordered arrangement of r elements from a set containing n elements is called r-permutation of n elements and it is denoted by nP_r or $P(n, r)$ where $r \leq n$, where $nP_r = \frac{\lfloor n}{\lfloor n-r}$

A B C B C A
A C B C A B
B A C C B A

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(1) How many 6 digit number can be formed using the digits $0, 1, 2, \ldots, 9$
(i) without repetition
(ii) with repetition

$0, 1, 2, 3, 4, 5, 6, 7, 8, 9$

Sol :-
(i)

9	9	8	7	6	5

$9 \times 9 \times 8 \times 7 \times 6 \times 5 = 81 \times 56 \times 30$
$= 136080$

56×40
81
4536

(ii)

9	10	10	10	10	10

$9 \times 10 \times 10 \times 10 \times 10 \times 10$
$= 900000$