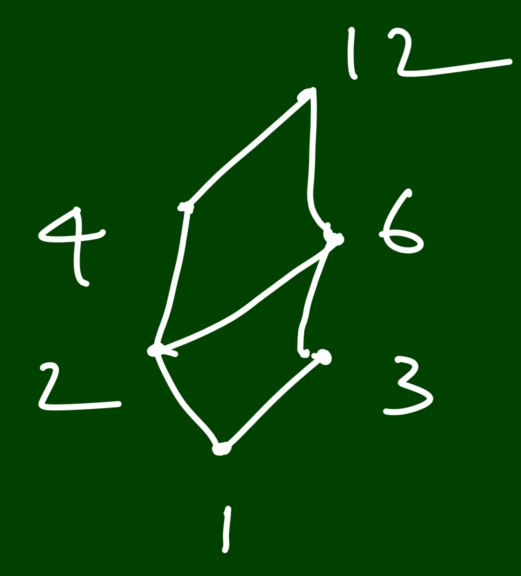
22/10/21

Solution for Homework Problems (19/10/21)

① (S_{12}, D)

$S_{12} = \{1, 2, 3, 4, 6, 12\}$

$1' = 12$
$2' = -$
$3' = 4$
$4' = 3$
$6' = -$
$12' = 1$

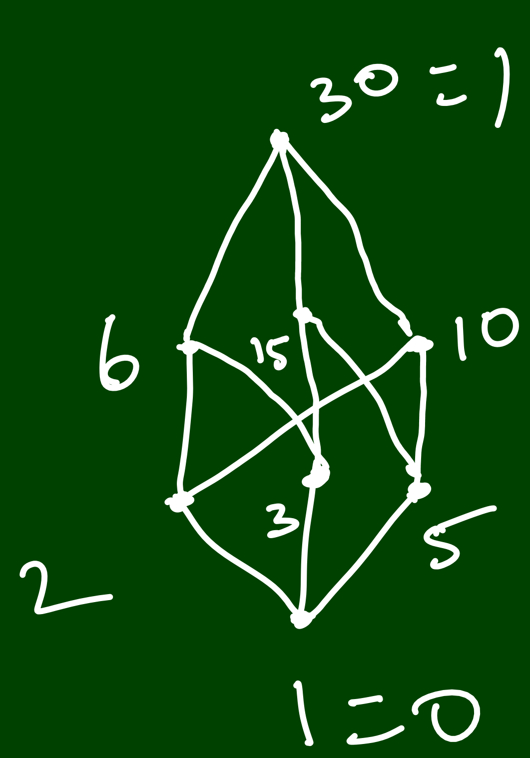


$\boxed{\begin{array}{l} a' = b \\ b' = a \end{array}}$

$2 \wedge 3 = 0$
$2 \vee 3 \neq 1$
$glb(3, 4)$
$= 3 \wedge 4 = 1$
$lub(3, 4) = 3 \vee 3 = 12$

② $S_{30} = \{1, 2, 3, 5, 6, 10, 15, 30\}$

$1' = 30$
$2' = 15$
$3' = 10$
$5' = 6$
$6' = 5$
$10' = 3$
$15' = 2$
$30' = 1$



$1 \to \cancel{1}, 2, 3, 5, \cancel{6}, \cancel{10}, \cancel{15}, \cancel{30}$
$2 \to \cancel{2}, 6, 10, \cancel{30}$
$3 \to \cancel{3}, 6, 15, \cancel{30}$
$5 \to \cancel{5}, 10, 15, \cancel{30}$
$6 \to \cancel{6}, 30$
$10 \to \cancel{10}, 30$
$15 \to \cancel{15}, 30$
$30 \to \cancel{30}$

$2 \vee 0 = 2$
$3 \vee 0 = 3$
$5 \vee 0 = 5$ $\quad 6 \vee 0 = 6 \quad 10 \vee 0 = 10$

Boolean algebra :-

A Complemented distributive lattice is known as Boolean algebra.

$\vee(\wedge)$

$$a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$$
$$a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$$

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n-ary operation :-

A^n

Any function from $\underline{A \times A \times A \times \cdots n \text{ times}}$ to A is called as n-ary operation

$A = \{0, 1\}$ $\quad \underline{A \times A} = \{(0,0), (0,1), (1,0), (1,1)\}$

$A \times A \times \cdots n \text{ times} = \{(0, 0, \ldots 0), (0, 0 \ldots 1), (0, 0, \ldots 1, 0), \ldots (1, 1, 1, \ldots 1)\}$

n-tuples

$n = 1 \rightarrow$ unary operation

$n = 2 \rightarrow$ binary operation

$n = 3 \rightarrow$ ternary operation

Algebra & Algebraic System & Algebraic Structure.

$(X, *, \oplus, \odot, \ldots)$

$(B, \underbrace{+_{\text{lub}}, \cdot_{\text{glb}}}_{\text{binary operations}}, '_{\text{complement}})$ unary operation

Boolean algebra :-

Boolean algebra is an algebraic structure defined on a set of elements B together with two binary operators + and · provided the following postulates are satisfied.

(1) a) Closure w.r.t the operator +
b) closure w.r.t. the operator ·

(2) a) An identity element w.r.t. + designated by 0;

$$x + 0 = 0 + x = x$$

b) An identity element w.r.t. · designated by 1;

$$x \cdot 1 = 1 \cdot x = x$$

(3) a) Commutative w.r.t. + ; $x + y = y + x$
b) commutative w.r.t. · ; $x \cdot y = y \cdot x$

(4) a) · is distributive over +

$$x \cdot (y + z) = (x \cdot y) + (x \cdot z)$$

b) + is distributive over ·

$$x + (y \cdot z) = (x + y) \cdot (x + z)$$

(5) For every element $x \in B$ ∃ an element $x' \in B$ such that

$$x + x' = 1 \text{ and } x \cdot x' = 0$$

(x' is called the complement of x)

(6) There exists atleast two elements $x, y \in B$ such that $x \neq y$

Note:-

(*) Huntington postulates

B

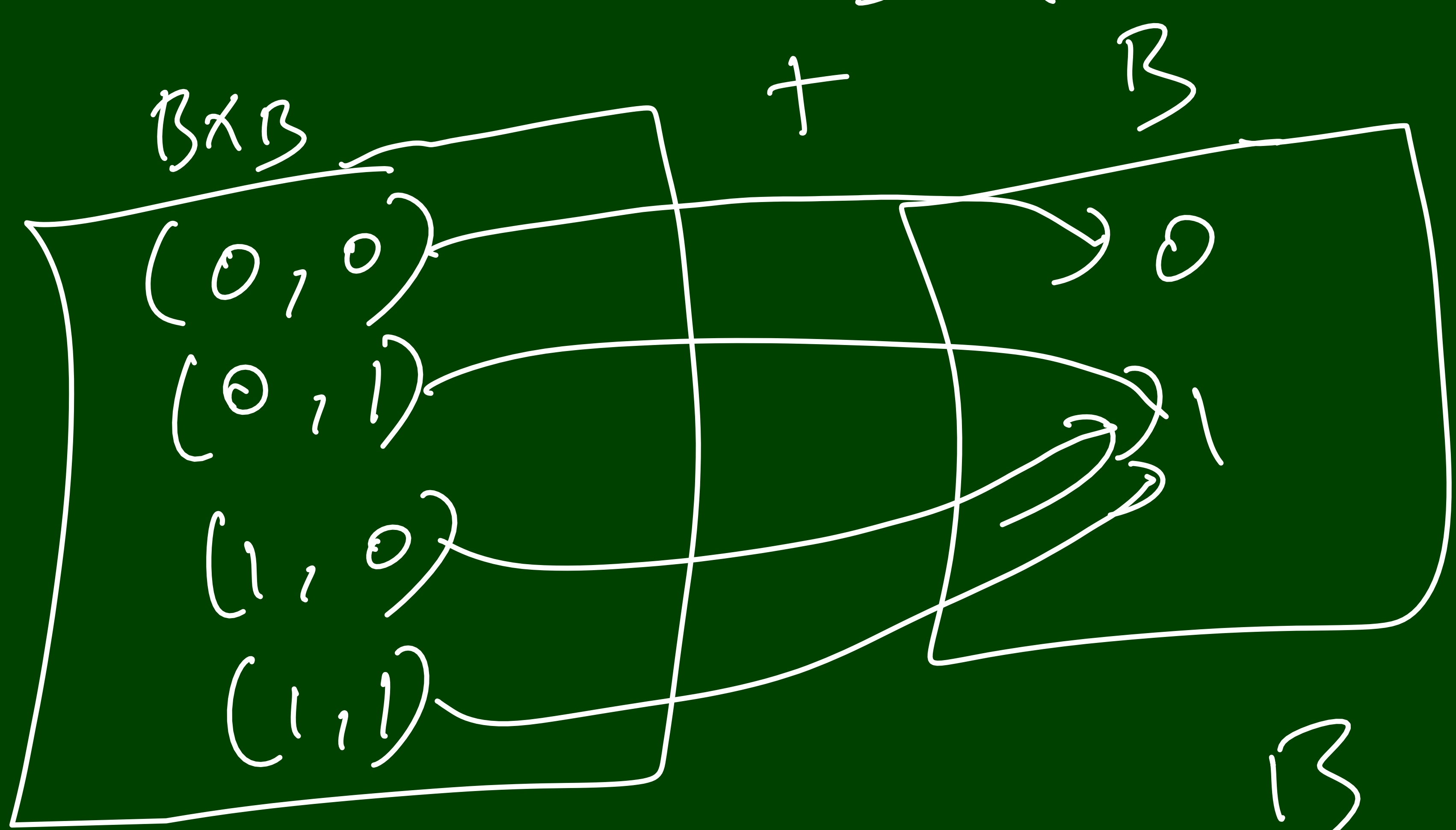
$x \quad y$

$x + y = z \in B$

$x \cdot y = z_1 \in B$

$+ : B \times B \longrightarrow B$

$\cdot \; : B \times B \longrightarrow B$



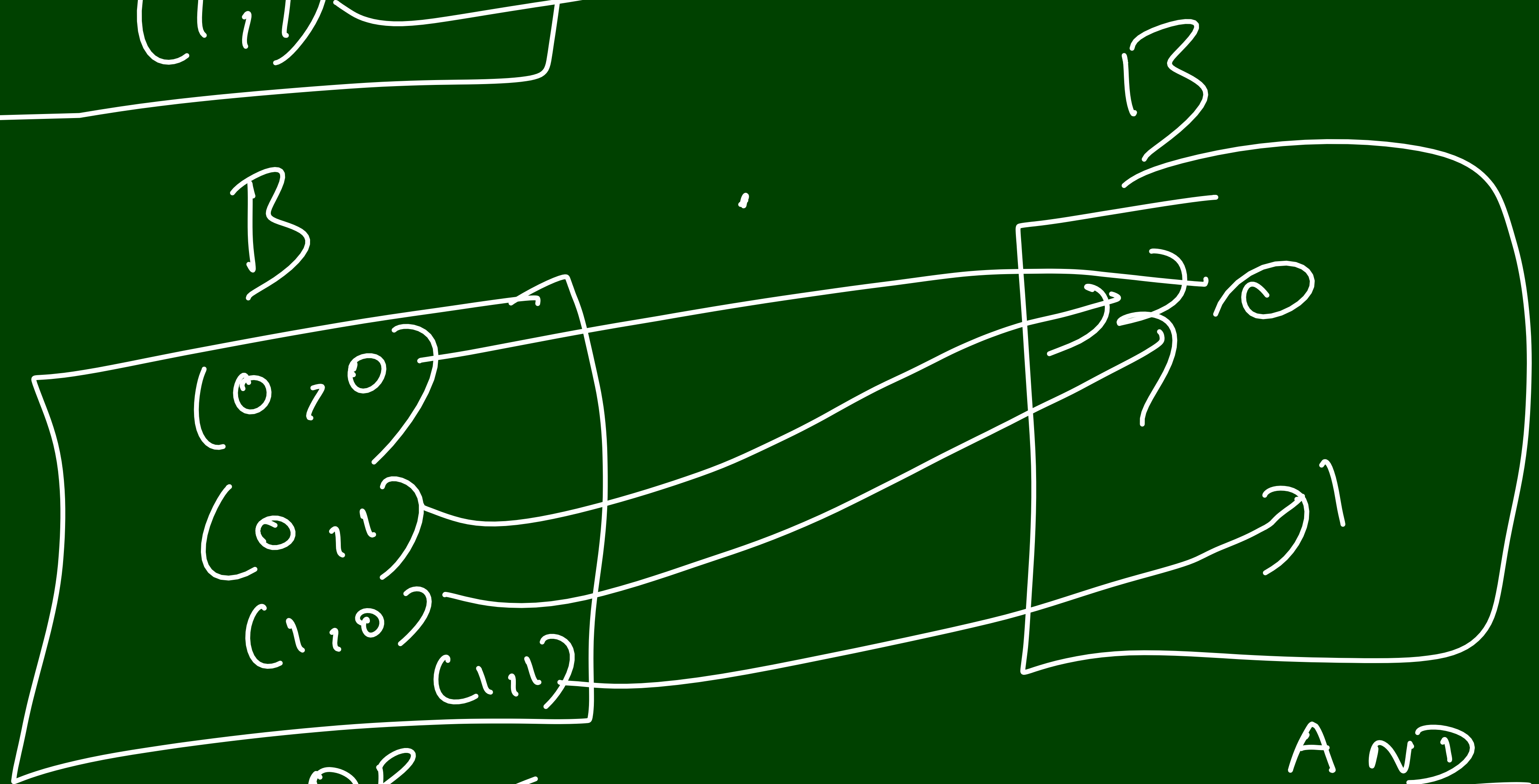
$B = \{0, 1\}$

$0 \vee 0 = 0$

$0 \vee 1 = 1$

$1 \vee 0 = 1$

$1 \vee 1 = 1$



$0 \wedge 0 = 0$

$0 \wedge 1 = 0$

$1 \wedge 0 = 0$

$1 \wedge 1 = 1$

OR

x	y	$x+y$
0	0	0
0	1	1
1	0	1
1	1	1

AND

x	y	$x \cdot y$
0	0	0
0	1	0
1	0	0
1	1	1

$\{0, 1\} \quad \{1, 0\}$

LUB

$0 \leq 0, 1$

$1 \leq 1$

GLB

$0 \geq 0$

$1 \geq 0, 1$