

20/11/21 Saturday

66810

$\boxed{1}$	$\boxed{2}$	$\boxed{3}$
6	0	0
0	6	0
0	0	6
1	5	0
1	4	1
1	3	2
1	2	3
1	1	4
1	0	5
2	4	0
2	3	1
2	2	2
2	1	3
2	0	4
⋮		

$$n=6$$

$$r=3$$

$$(n+r-1)C_r = (3+6-1)C_6$$

$$= 8C_6 = 8C_2$$

$$= \frac{8 \times 7}{1 \times 2} = 28$$

$$\boxed{1} \boxed{2} \rightarrow 4$$

$$n=2 \quad r=4$$

$$(n+r-1)C_r = 5C_4 = 5C_1 = 5$$

Combinations with Repetition:- There are

$(n+r-1)C_r$ ways to select r objects from a set having n elements when repetition is permitted.

(1) How many integer solns does the equation $x_1 + x_2 + x_3 + x_4 = 7$ have?
if $x_i \geq 0, i=1, 2, 3, 4$

Sol:- $x_1 + x_2 + x_3 + x_4 = 7$ } \rightarrow (1)
where $x_i \geq 0, i=1, 2, 3, 4$

Here $n=4; r=7$

\therefore There are $(n+r-1)C_r$ integer solutions to the eqn. (1)

$$(4+7-1)C_7 = 10C_7 = 10C_3$$

$$= \frac{10 \times 9 \times 8}{1 \times 2 \times 3}$$

$$= 120$$

(2) How many integer solns does the eqn. $x_1 + x_2 + x_3 + x_4 = 7$ have?
if $x_1 > 0$ and $x_2, x_3, x_4 \geq 0$.

Sol:- $x_1 + x_2 + x_3 + x_4 = 7$ } \rightarrow (1)
 $x_1 > 0$ and $x_2, x_3, x_4 \geq 0$

Put $x_1 = u + 1, u \geq 0$

$$u + 1 + x_2 + x_3 + x_4 = 7$$

$$u + x_2 + x_3 + x_4 = 6$$

Now (1) becomes $u + x_2 + x_3 + x_4 = 6$

where $u, x_2, x_3, x_4 \geq 0$

Here $n = 4; \lambda = 6$

$$\begin{aligned} (n + \lambda - 1) C_1 &= (4 + 6 - 1) C_6 \\ &= 9 C_6 = 9 C_3 = \frac{9 \times 8 \times 7}{1 \times 2 \times 3} \\ &= 84 \end{aligned}$$

(3) How many integer solns does the eqn $x_1 + x_2 + x_3 + x_4 = 7$ have? if $x_1, x_2, x_3, x_4 \geq 0$.

Sol:- Let $x_i = u_i + 1, u_i \geq 0, i = 1, 2, 3, 4$

$$\text{Then } u_1 + u_2 + u_3 + u_4 = 3$$

Here $n = 4; \lambda = 3$

$$(n + \lambda - 1) C_1 = 7 C_3 = \frac{7 \times 5 \times 4}{1 \times 2 \times 3} = 20$$

(4) How many integer solns. does the eqn. $x_1 + x_2 + x_3 + x_4 = 7$ have? if $x_1 \geq 2$ and $x_2, x_3, x_4 \geq 0$

Sol:- Put $x_1 = u + 2$, where $u \geq 0$

$$u + x_2 + x_3 + x_4 = 5$$

$n = 4; \lambda = 5$

$$(5 + 4 - 1) C_5 = 8 C_5 = 8 C_3 = \frac{8 \times 7 \times 6}{1 \times 2 \times 3} = 56$$

(5) How many integer solns does the equation $x_1 + x_2 + x_3 + x_4 = 7$ have? if $0 \leq x_1 \leq 2, x_2, x_3, x_4 \geq 0$

Sol:- $x_1 + x_2 + x_3 + x_4 = 7$, where $0 \leq x_1 \leq 2, x_2, x_3, x_4 \geq 0 \rightarrow$ (1)

Put $x_1 = u + 3$, where $u \geq 0$

Then (1) becomes

$$u + x_2 + x_3 + x_4 = 7 - 3$$

$$(u) \quad u + x_2 + x_3 + x_4 = 4$$

Here $n = 4; \lambda = 4$

$$(n + \lambda - 1) C_1 = (4 + 4 - 1) C_4 = 7 C_4 = 7 C_3$$

$$= \frac{7 \times 6 \times 5}{1 \times 2 \times 3}$$

$$= 35$$

The no. of intgy solns. for the given
eqn $x_1 + x_2 + x_3 + x_4 = 7$ if $0 \leq x_1 \leq 2$,
 $x_2, x_3, x_4 \geq 0$

= The no. of intgy solns for the given
eqn $x_1 + x_3 + x_3 + x_4 = 7$ if $x_i \geq 0$
($1 \leq i \leq 4$)

- The no. of intgy solns for the
given eqn $x_1 + x_2 + x_3 + x_4 = 7$ if
 $x_1, x_2 \leq 1, x_3, x_4 \geq 0$

$$= 120 - 35$$

$$= 85$$