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| SASTRA<br>DEEMED TO BE UNIVERSITY | **School of Arts Science &Humanities, Education**<br>**First CIA – Mar 2023**<br>Course Code: **MAT 134**<br>Course Name: **Linear Algebra**<br>Duration: 90 Minutes Max Marks: 50 |

**Answer all the questions.** **PART A** **5 x 2 = 10 Marks**

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| 1 | Write down the formulas for Cramers rule –Solving system of equations |
| 2 | Find the rank of a matrix $A = \begin{pmatrix} 1 & -1 & 1 & -1 \\ 1 & -3 & 4 & -6 \\ 4 & 3 & -2 & -3 \\ 7 & -4 & 7 & -16 \end{pmatrix}$ $2R_2 + R_1 + R_3 = R_4$ |
| 3 | For what values of $k$ is the following an inner product on $R^2(R)$. $< u, v >= x_1y_1 - 3x_1y_2 - 3x_3y_1 + kx_2y_2$ where $u = (x_1, x_2)$ , $v = (y_1, y_2)$ |
| 4 | Prove that if two vectors are linearly dependent, one of them is a scalar multiple of other. |
| 5 | Examine the set $B = \{(1,2,1), (3,1,5), (3,-4,7)\}$ is LI or LD |
| | **Answer the following questions. PART B 4 x 10 = 40marks** |
| 6 | Investigate for what values of $\lambda$ and $\mu$ the simultaneous $x + y + z = 6, x + 2y + 3z = 0, x + 2y + \lambda z = \mu$ have i)No solution (ii) a unique solution (iii) infinitely many solution |
| 7 | Find the values of $\lambda$ for which the equations $(\lambda - 1)x + (3\lambda + 1)y + 2\lambda z = 0,$ $(\lambda - 1)x + (4\lambda - 2)y + (\lambda + 3)z = 0$ $2x + (3\lambda + 1)y + 3(\lambda - 1)z = 0$ Are consistent, and find the ratios of $x: y: z$ when $\lambda$ has the smallest and greatest of these values? |
| 8 | (a) Solve the system of equation by LU –Decomposition $x + y - z = 2, 2x + 3y + 5z = -3, 3x + 2y - 3z = 6$<br>(b) Convert the set $S = \{(1,0,2,0), (1,2,3,1)\}$ is an orthonormal set. Use standard inner product in $R^2(R)$. **(6+4)** |
| 9 | Prove that $V = \{A_{2\times2}(R)/Trace(A) = 0\}$ is a vector space over R-Set of real numbers, also Find Basis and dimension. |

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| SASTRA DEEMED TO BE UNIVERSITY | **School of Arts Science &Humanities, Education**<br>**2nd CIA – May2023**<br>Course Code: **MAT 134**<br>Course Name: **Linear Algebra**<br>Duration: 90 Minutes Max Marks: 50 |

**Answer all the questions.** **PART A** **5 x 2 = 10 Marks**

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| 1 | Verify the following is linear transformation $T: R^3 \rightarrow R^3$, $T(x, y, z) = (z, y + z, x + y + z)$. |
| 2 | Let $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 1 & 2 & 0 \end{pmatrix}$ and I be the $3 \times 3$ identity matrix. If $6A^{-1} = aA^2 + bA + cI$ for $a, b, c$ *in* $R$. Then $(a, b, c)$ is equals to? |
| 3 | For every $4 \times 4$ real symmetric invertible matrix A, there exist a positive integer $p$ such that which of the following is / are true Justify your answers (a)$pI + A$ is positive definite (b) $A^p$ is negative definite (c) $A^{-p}$ is positive definite |
| 4 | If $A = \begin{pmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{pmatrix}$ then find the eigen values of $A^{99} - 2I$. |
| 5 | Let A and B be $n \times n$ matrices. Then show that AB and BA have same eigen values. |
| | **Answer any four questions. PART B 4 x 10 = 40marks** |
| 6 | State and prove Rank nullity theorem |
| 7 | Find the QR-Decomposition of $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 1 & 2 & 0 \end{pmatrix}$ |
| 8 | Diagonalise the matrix $A = \begin{pmatrix} 3 & 1 & 1 \\ 1 & 3 & -1 \\ 1 & -1 & 3 \end{pmatrix}$ |
| 9 | Verify T is linear and also find Range,rank,null space and nullity of T. $T(a, b) = (a + b, a - b, b)$ where $T: R^2 \rightarrow R^3$. |
| 10 | Verify that $A = \begin{pmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$ satisfies its characteristic equation and hence find $A^4$. |

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| **SASTRA** DEEMED TO BE UNIVERSITY | **School of Arts Science &Humanities, Education** <br> **3rd CIA – June 2023** <br> Course Code: **MAT 134** <br> Course Name: **Linear Algebra** <br> Duration: 90 Minutes Max Marks: 50 |

**Answer all the questions.** **PART A 5 x 2 = 10 Marks**

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| 1 | What is the difference between the Gauss elimination and Rank method for solving system of equations? |
| 2 | Find the singular values of $\begin{pmatrix} 4 & 11 & 14 \\ 8 & 7 & -2 \end{pmatrix}$ |
| 3 | Find the basis and dimension for $V(R) = \left\{ \begin{pmatrix} a & b \\ 2b & c \end{pmatrix}; a, b, c \in R \right\}$. |
| 4 | Let A and B be $n \times n$ matrices are positive definite Then what can you say about *A-B?* |
| 5 | Define projection with Example |

**Answer any two questions. PART B 15 x 2= 30marks**

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| 6 | Find the singular value decomposition of $\begin{pmatrix} 4 & 11 & 14 \\ 8 & 7 & -2 \end{pmatrix}$ **(OR)** |
| 7 | A Landsat image with three spectral components was made of Homestead Air Force Base in Florida (after the base was hit by Hurricane Andrew in 1992). The covariance matrix of the data is shown below. Find the first principal component of the data, and compute the percentage of the total variance that is contained in this component $S = \begin{pmatrix} 164.12 & 32.73 & 81.04 \\ 32.73 & 539.44 & 249.13 \\ 81.04 & 249.13 & 189.11 \end{pmatrix}$ |
| 8 | Obtain the orthonormal basis for V= the space of all polynomials of degree atmost 2, the inner product is defined by $< f, g >= \int_0^1 f(x)g(x)dx$. **(OR)** |
| 9 | Verify T is linear and also find Range,rank,null space and nullity of T. $T(a,b) = (a,a,b)$ where $T: R^2 \to R^3$. |

**Part C Answer the following 1x 10= 10marks**

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| 10 | Solve the system of equations by rank method $x + y + z = 3, x + 2y + 3z = 4, x + 4y + 9z = 6$ |