16/10/21 Saturday

Relation

Any Subset of $A \times B$ is a relation from A to B.

Any Subset of $A \times A$ is a relation on A.

Void : $\{\}$ Universal : $A \times A$

Properties :- Let R be a relation on A. Then R is said to be

(i) Reflexive : if $(a, a) \in R$, $\forall a \in A$

(ii) Symmetric : if $(a, b) \in R \Rightarrow (b, a) \in R$ $\forall a, b \in A$

(iii) Transitive : if $(a, b) \in R$ and $(b, c) \in R$ then $(a, c) \in R$, $\forall a, b, c \in R$

(iv) Anti-Symmetic : if $(a, b) \in R$ and $(b, a) \in R$ whenever $a = b$

(v) Asymmetric : if $(a, b) \in R$ then $(b, a) \notin R$, $\forall a, b \in A$

(vi) Irreflexive : if $(a, a) \notin R$ $\forall a \in A$

A = Set of all real numbers

$R = \{(a,b) / a \leq b, a,b \in A\}$

$2.45 \leq 2.45$ $\quad 5 \leq 5$

$6 \leq 8$ $\quad 8 \not\leq 6$ $\quad \leq$

$a \leq b \quad b \leq c \Rightarrow a \leq c$

$a \leq b$ and $b \leq a \Rightarrow \boxed{a = b}$

'=' $R = \{(a,b) / a = b\}$

$\boxed{a = a}$ $\quad \underline{a R b}$ $\quad a = b \Rightarrow b = a \Rightarrow bRa$

$a R b \quad b R c \Rightarrow a = b$ and $b = c$

$\Rightarrow a = c \quad \boxed{a R c}$

$\boxed{a R b \text{ and } bRa \quad a = b}$

Z '/' $R = \{(a,b) : a/b\}$ $\quad \not a/b$ $\quad a/b$ $\quad a \not/ b$

Relation

Equivalence

Partial order relation

Equivalence relation : Reflexive =
Symmetric $a \equiv b$
Transitive

Partial order relation : Reflexive
Anti-Symmetric
Transitive

$A = \{1, 2, 3, \dots, 10\}$

$R = \{(a,b) / a \equiv b \pmod 4\}$

$4 / a-b$

$a \equiv a \pmod 4$ $\quad \frac{0}{4}$

$a \equiv b \pmod 4 \Rightarrow 4 / a-b$

$\Rightarrow 4 / -(a-b)$

$\Rightarrow 4 / b-a$

$\Rightarrow b \equiv a \pmod 4$

$a \equiv b \pmod 4$ and $b \equiv c \pmod 4$

$\Rightarrow 4 / a-b$ and $4 / b-c$

$\Rightarrow a-b = 4k_1$ and $b-c = 4k_2$

$a - c = a - b + b - c$

$= 4k_1 + 4k_2 = 4(k_1 + k_2)$

$\Rightarrow 4 \mid a - c$

$\Rightarrow a \equiv c \pmod 4$

$A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$[1]_R = \{1, 5, 9\}$ ✓ equivalence class of 1

$[2]_R = \{2, 6, 10\}$ ✗

$[3]_R = \{3, 7\}$ •

$[4]_R = \{4, 8\}$ ••

$[5]_R = \{1, 5, 9\}$ ✓

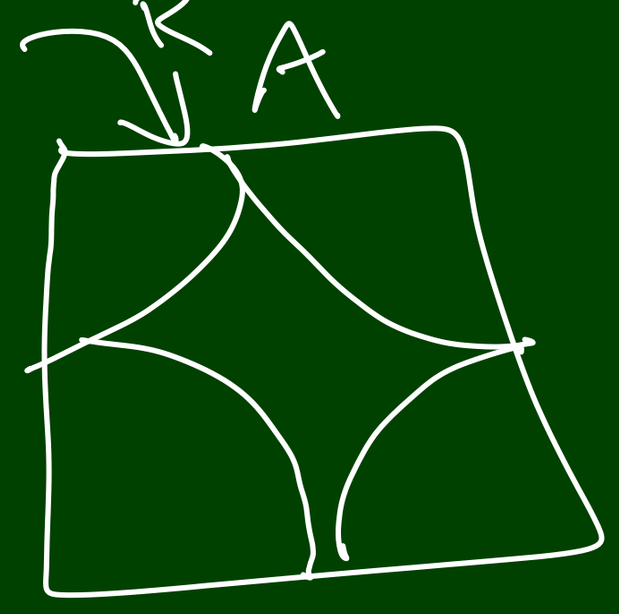
$[6]_R = \{2, 6, 10\}$ ✗ $1 \equiv 2 \pmod 4$? $1 - 2 = -1$

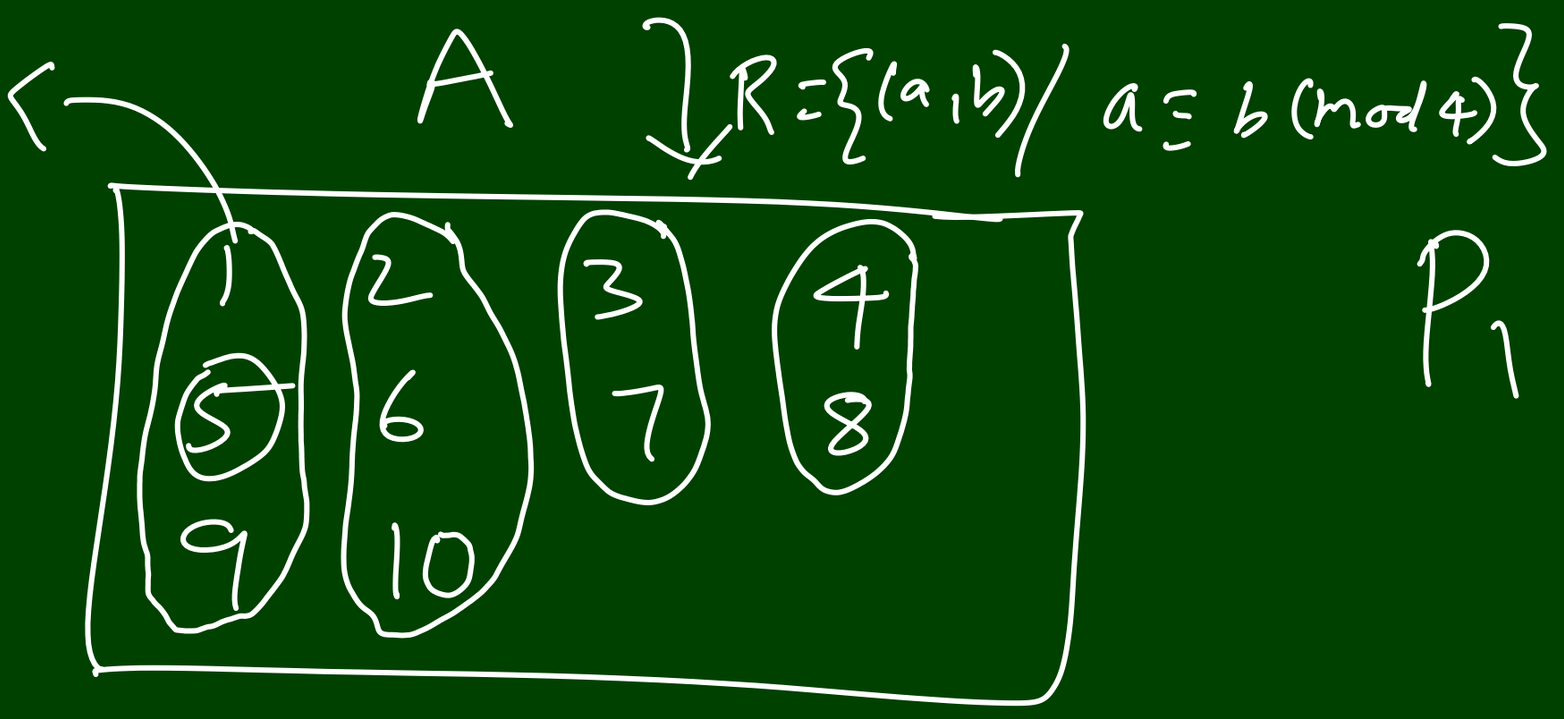
$[7]_R = \{3, 7\}$ •

$[8]_R = \{4, 8\}$ ••

$[9]_R = \{1, 5, 9\}$ ✓

$[10]_R = \{2, 6, 10\}$ ✗





Boolean algebra → Lattice

Lattice → Poset or partially ordered set

Poset → A non-empty set together with a partial order relation

$A = \{1, 2, 3, \ldots, 10\}$

$R = \{(a, b) : a \mid b\}$

precedence

$a \leq b$

$(A, \leq)$ – Poset

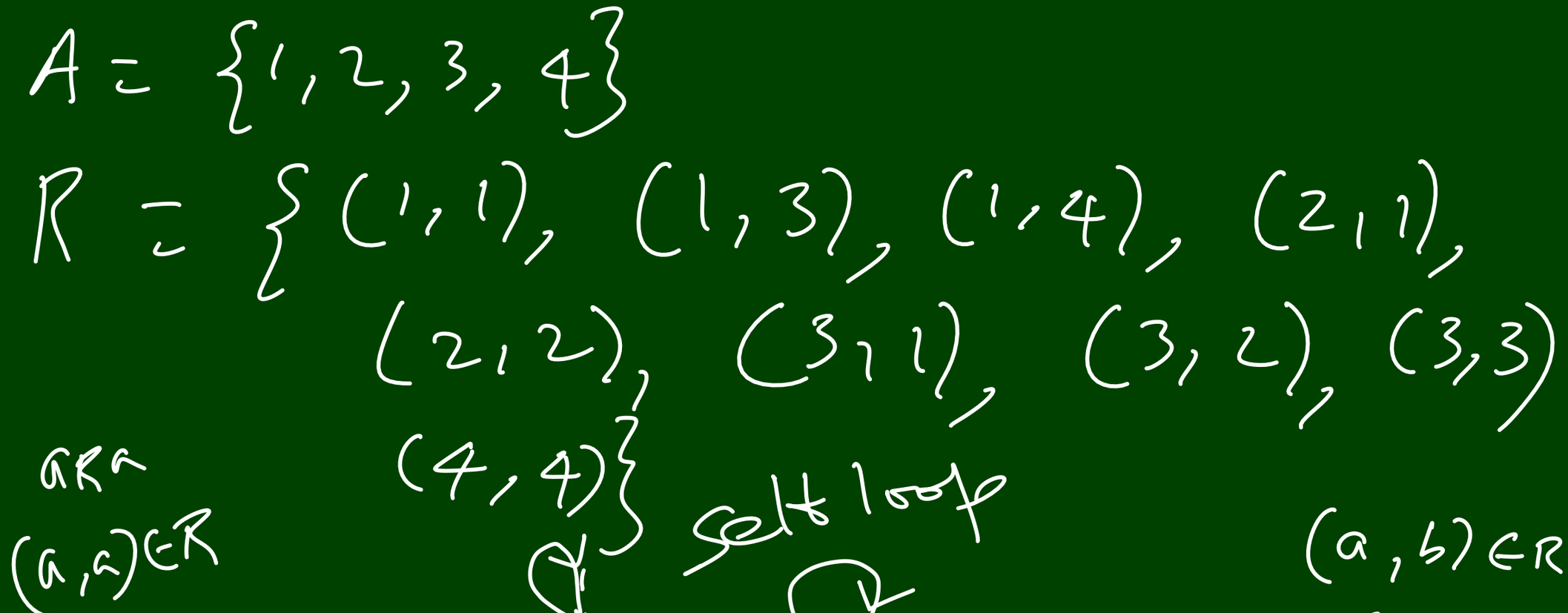
lub

glb

$R = \{(1,1), (1,2), (1,3), \ldots (1,10)$
$(2,2), (2,4), (2,6), (2,8), (2,10)$
$(3,3), (3,6), (3,9)$
$(4,4), (4,8), (5,5), (5,10),$
$(6,6), (7,7), (8,8), (9,9), (10,10)\}$

$A = \{1, 2, 3, 4\}$

$R = \{(1,1), (1,3), (1,4), (2,1),$
$(2,2), (3,1), (3,2), (3,3)$
$(4,4)\}$



Hasse diagram

$a \leq b$ immediate

$a \leq c$

$c \leq b$

$S_{24} = \{1, 2, 3, 4, 6, 8, 12, 24\}$
D_{24}

$24 = 12 \times 2$
$= 3 \times 4 \times 2$
$= 3 \times 2^3$

$(D_{24}, /)$ is a Poset

$a \leq b$

$1 \leq 2$
$2 \leq 4$

$a \leq c \quad c \leq b$

$2 \leq c \quad c \leq 6$

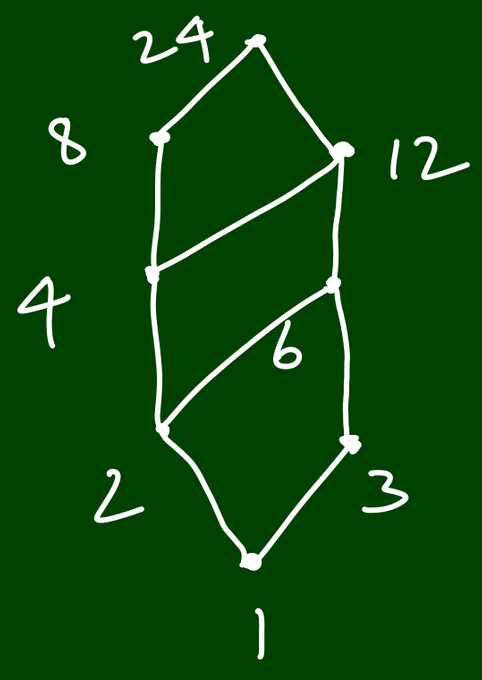
$1 \to 1, 2, 3, 4, 6, 8, 12, 24$
$2 \to 2, 4, 6, 8, 12, 24$
$3 \to 3, 6, 12, 24$
$4 \to 4, 8, 12, 24$
$6 \to 6, 12, 24$
$8 \to 8, 24$
$12 \to 12, 24$
$24 \to 24$

$(1+1) \times (3+1)$
$2 \times 4 = 8$

$20 = 5 \times 2^2$

$(1+1) \times (2+1)$

$a \leq b$



$a \leq b$

Join $l.u.b$

meet $g.l.b$

H.W:- Draw Hasse diagram for the following Posets.

(1) $(S_{36}, /)$ (2) $(S_{32}, /)$ (3) $(S_{48}, /)$