18|10|21 Monday

Solution of home work problems (16|10|21)

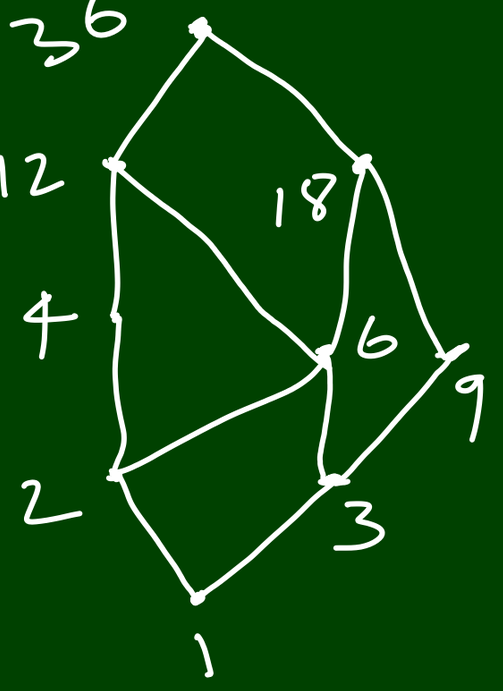
(1) $(S_{36}, /)$

$S_{36} = \{1, 2, 3, 4, 6, 9, 12, 18, 36\}$

$36 = 9 \times 4$
$= 3^2 \times 2^2$

(S_{36}, D)

Division 36

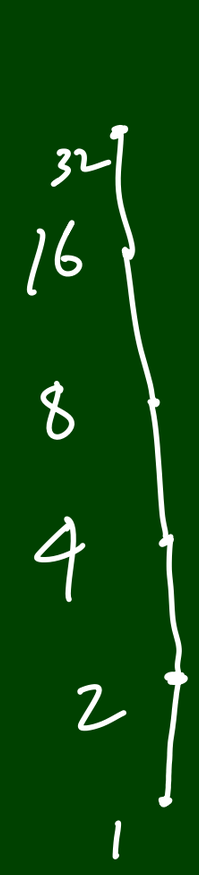


1 → ~~1, 2, 3, 4, 6, 9, 12, 18, 36~~
2 → ~~2, 4, 6, 12, 18, 36~~
3 → ~~3, 6, 9, 12, 18, 36~~
4 → ~~4, 12, 36~~
6 → ~~6, 12, 18, 36~~
9 → ~~9, 18, 36~~
12 → ~~12, 36~~
18 → ~~18, 36~~
36 → ~~36~~

(2) $(S_{32}, /)$

$S_{32} = \{1, 2, 4, 8, 16, 32\}$

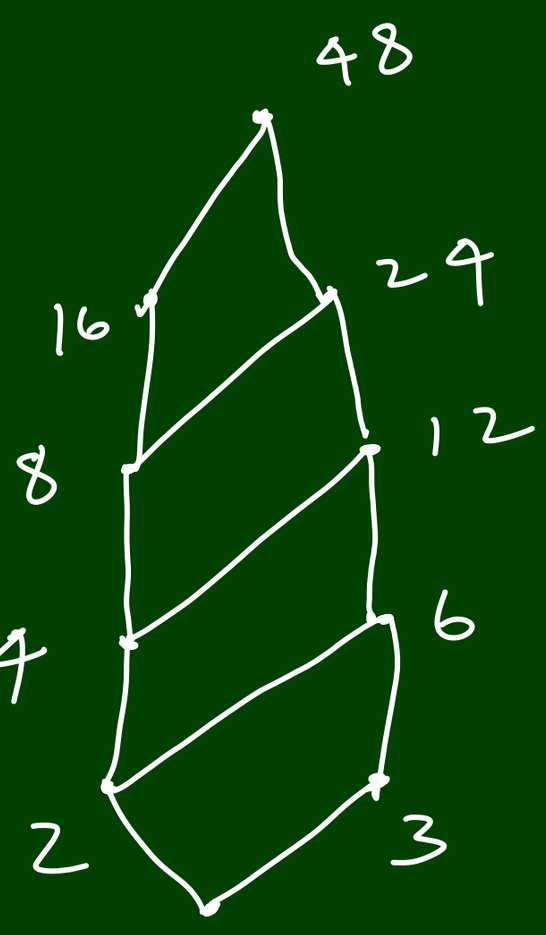
1 → ~~1, 2, 4, 8, 16, 32~~
2 → ~~2, 4, 8, 16, 32~~
4 → ~~4, 8, 16, 32~~
8 → ~~8, 16, 32~~
16 → ~~16, 32~~
32 → ~~32~~

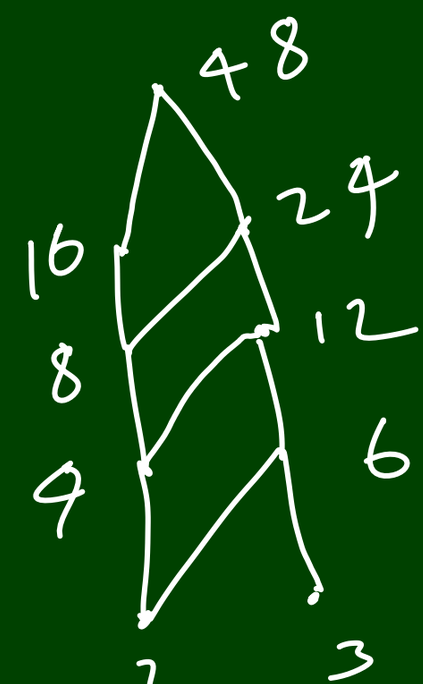


(3) $(S_{48}, /)$

$$S_{48} = \{1, 2, 3, 4, 6, 8, 12, 16, 24, 48\}$$

1 → ~~1, 2, 3, 4, 6, 8, 12, 16, 24, 48~~
2 → ~~2, 4, 6, 8, 12, 16, 24, 48~~
3 → ~~3, 6, 12, 24, 48~~
4 → ~~4, 8, 12, 16, 24, 48~~
6 → ~~6, 12, 24, 48~~
8 → ~~8, 16, 24, 48~~
12 → ~~12, 24, 48~~
16 → ~~16, 48~~
24 → ~~24, 48~~
48 → ~~48~~





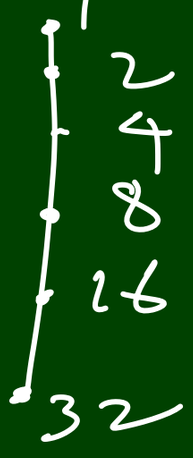
$(S_n, /)$ & (S_n, D) & $(D_n, /)$

Least element :-

Let $(X, \leq)$ be a poset. Then an element $a \in X$ is said to be a least element if $a \leq x$, $\forall x \in X$

$\{1, 2, 4, 8, 16, 32\}$ $\geq$

$R = \{(a, b) : a \geq b\}$



Greatest element :-

Let $(X, \leq)$ be a poset. Then an element $a \in X$ is said to be a greatest element if $x \leq a$, $\forall x \in X$

Note :- 1) Least element and Greatest element are unique

2) Least element is denoted by '0'

3) Greatest element is denoted by '1'

Lower bound :-

Lower bound :-

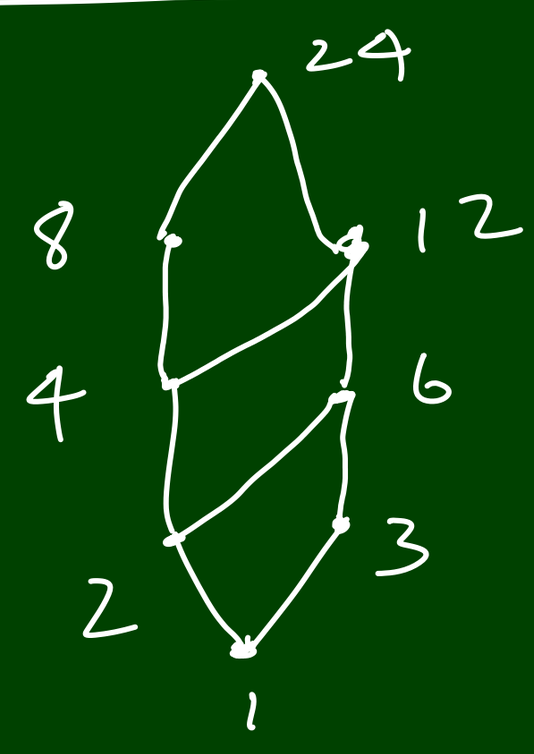
Let $(X \leq)$ be a poset and $A \subseteq X$. Then an element $x \in X$ is said to be a lower bound of A if $x \leq a$, $\forall a \in A$

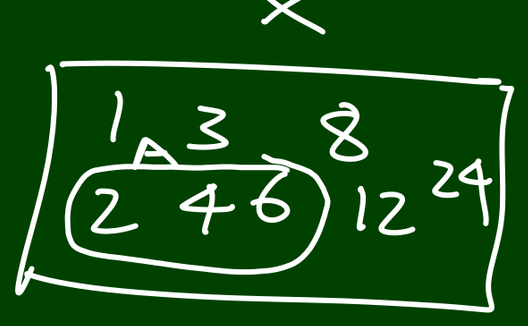
Upper bound :- Let $(X, \leq)$ be a poset and $A \subseteq X$. Then an element $x \in X$ is said to be an upper bound for A if $a \leq x$, $\forall a \in A$

(S_{24}, D)

$X = S_{24}$

$A = \{2, 4, 6\}$





Lower bounds

$2 \geq 1, 2$

$4 \geq 1, 2, 4$

$6 \geq 1, 2, 3, 6$

Lower bounds for A are $\{1, 2\}$

Upper bounds

$2 \leq 2, 4, 6, 8, 12, 24$

$4 \leq 4, 8, 12, 24$

$6 \leq 6, 12, 24$

Upper bounds of $A = \{12, 24\}$

Least upper bound (lub)

Let $(X, \leq)$ be a poset and $A \subseteq X$. Then an element $x \in X$ is said to be a least upper bound of A if

(i) x is an upper bound of A

(ii) $x \leq y$, for all upper bounds y of A

Greatest lower bound (glb)

Let $(X, \leq)$ be a poset and $A \subseteq X$. Then an element $x \in X$ is said to be a glb of A if (i) x is a lower bound for A

(ii) $y \leq x$, for all lower bounds y of A

Ex- (S_{30}, D)

let $X = S_{30} = \{1, 2, 3, 5, 6, 10, 15, 30\}$

$A = \{3, 5, 6\}$

LUB

$3 \leq 3, 6, 15, 30$

$5 \leq 5, 10, 15, 30$

$6 \leq 6, 30$

$Ub = \{30\}$

$lub = 30$

GLB

$3 \geq 1, 3$

$5 \geq 1, 5$

$6 \geq 1, 2, 3, 6$

$LB = \{1\}$

$glb = 1$

$A = \{2, 5, 10\}$

LUB

$2 \leq 2, 6, 10, 30$

$5 \leq 5, 10, 15, 30$

$10 \leq 10, 30$

$UB = \{10, 30\}$

$LUB = 10$

GLB

$2 \geq 1, 2$

$5 \geq 1, 5$

$10 \geq 1, 2, 5$

$LB = \{1\}$

$glb = 1$