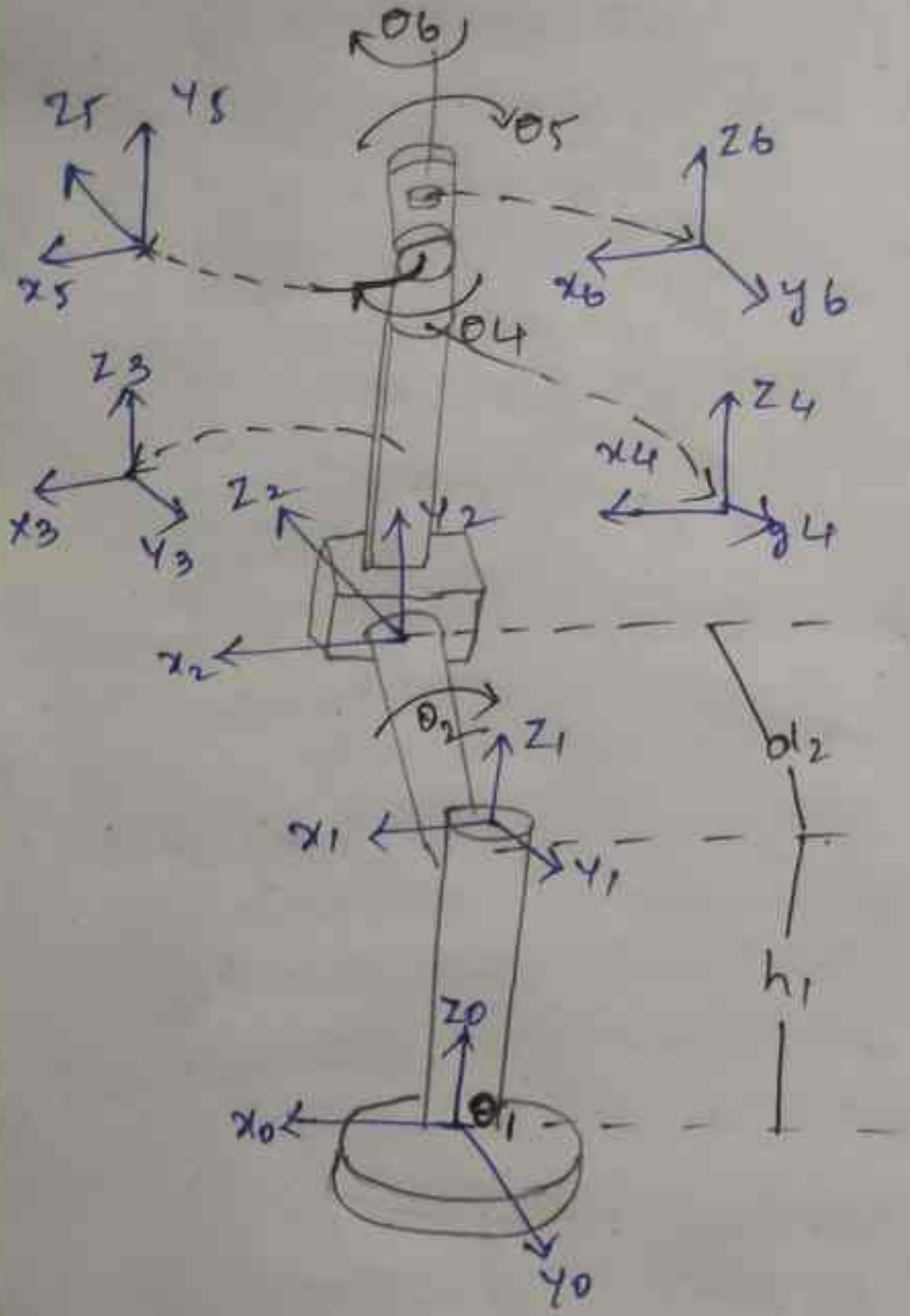

θ6
θ5
θ4
θ3
θ2
θ1
Z5
Y5
X5
Z6
X6
Y6
Z3
X3
Y3
Z4
X4
Y4
Z2
Y2
X2
Z1
X1
Y1
dl2
h1
Z0
X0
Y0

Forward kinematics

Step 1: Locate and label the joint axes $z_0, \ldots, z_{n-1}$

Step 2: Establish the base frame. Set the origin anywhere on the z_0-axis. The x_0 and y_0 axes are chosen conveniently to form a right-hand frame.

For $i = 1, \ldots, n-1$, perform steps 3 to 5

Step 3: Locate the origin O_i where the common normal to z_i and z_{i-1} intersects z_i. If z_i intersects z_{i-1} locate O_i at this intersection. If z_i and z_{i-1} are parallel, locate O_i in any convenient position along z_i.

Step 4: Establish x_i along the common normal between z_{i-1} and z_i through O_i, or in the direction normal to the $z_{i-1} - z_i$ plane if z_{i-1} and z_i intersect.

Step 5: Establish y_i to complete a right-hand frame.

Step 6: Establish the end-effector frame $O_n x_n y_n z_n$. Assuming the nth joint is revolute, set $z_n = a$ along the direction z_{n-1}. Establish the origin O_n conveniently along z_n, preferably at the center of the gripper or at the tip of any tool that the manipulator may be carrying. Set $y_n = s$ in the direction of the gripper closure and set $x_n = n \times a$. If the tool is not a simple gripper set x_n & y_n conveniently to form a right-hand frame.

Step 7: Create a table of link parameters $a_i, d_i, \alpha_i, \theta_i$.

a_i = distance along x_i from O_i to the intersection of the x_i and Z_{i-1} axes.

d_i = distance along Z_{i-1} from O_{i-1} to the intersection of the x_i and Z_{i-1} axes. d_i is variable if joint i is prismatic.

α_i = the angle angle b/w Z_{i-1} and Z_i measured about x_i.

θ_i = the angle b/w x_{i-1} and x_i measured about Z_{i-1}.
θ_i is variable if joint i is revolute.

Step 8: Form the homogeneous transformation matrices A_i by substituting the above parameters into.

Step 9: Form $T_n^0 = A_1 \ldots A_n$. This then gives the position and orientation of the tool frame expressed in base coordinates.

Link parameters for 2-link planar manipulator

Link	a_i	α_i	d_i	θ_i
1	a_1	0	0	θ_1*
2	a_2	0	0	θ_2*

*variable

$$A_1 = \begin{bmatrix} c_1 & -s_1 & 0 & a_1 c_1 \\ s_1 & c_1 & 0 & a_1 s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} c_2 & -s_2 & 0 & a_2 c_2 \\ s_2 & c_2 & 0 & a_2 s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The T- matrices are then, given by

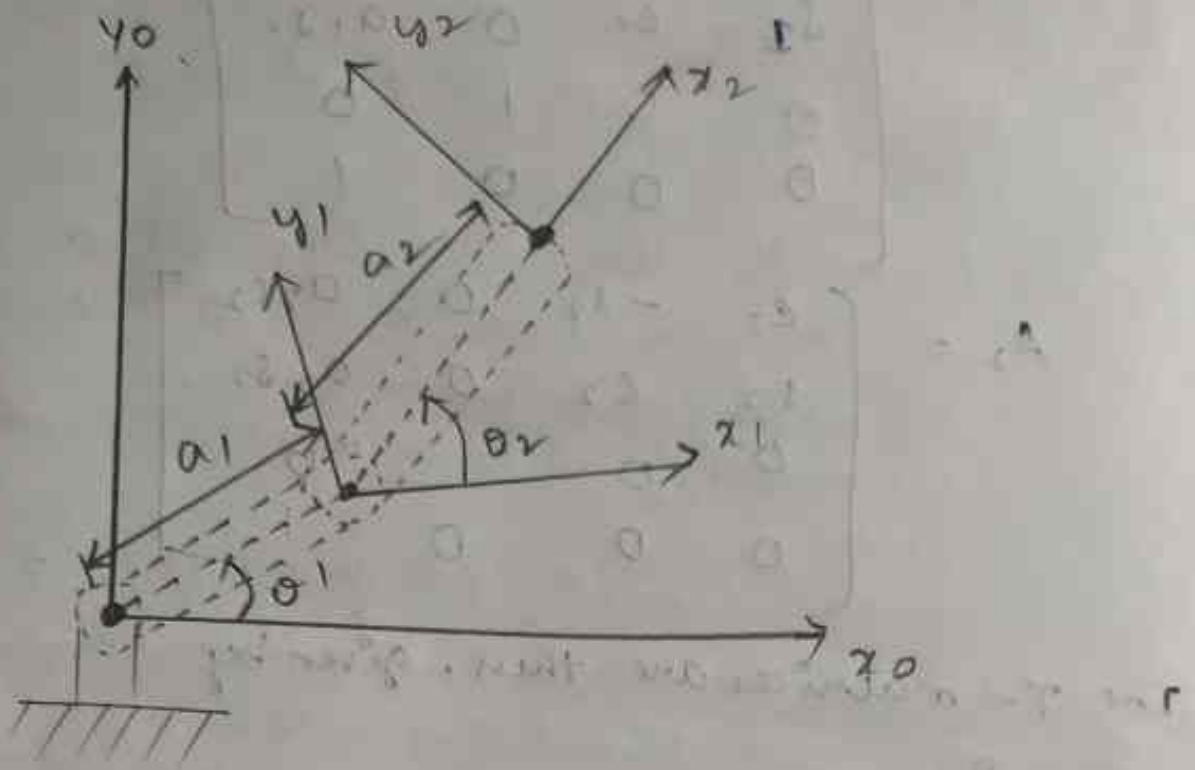
$$T_1^0 = A_1$$

$$T_2^0 = A_1 A_2 = \begin{bmatrix} c_{12} & -s_{12} & 0 & a_1 c_1 + a_2 c_{12} \\ s_{12} & c_{12} & 0 & a_1 s_1 + a_2 s_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Notice, that the first two entries of the last column of T_2^0 are the x and y components of the origin O_2 in the base frame; that is,

$$x = a_1 c_1 + a_2 c_{12}$$
$$y = a_1 s_1 + a_2 s_{12}$$

are the coordinates of the end effector in the base frame. The rotational part of T_2^0 gives the orientation of the frame $O_2 x_2 y_2 z_2$ relative to the base frame.



Inverse kinematics

To derive the inverse kinematic solution, we want to find the joint angles (θ_1 & θ_2) that achieve a desired end-effector position (x, y)

The forward kinematic eqns for this manipulator are as follows:-

For the end effectors position:

$x = a_1 \cos(\theta_1) + a_2 \cos(\theta_1 + \theta_2)$

$y = a_1 \sin(\theta_1) + a_2 \sin(\theta_1 + \theta_2)$

We want to solve the eqns for θ_1 & θ_2

Let's denote

$r = \sqrt{x^2 + y^2}$

$\phi = \text{atan2}(y, x)$

Then, from the equations above

$$\cos(\theta_1+\theta_2) = \frac{x - a_1\cos(\theta_1)}{a_2} = \frac{r^2 - a_1^2 - a_2^2}{2a_1a_2}$$

$$\sin(\theta_1+\theta_2) = \frac{y - a_1(\sin\theta_1)}{a_2} = \frac{\sqrt{1-(r^2-a_1^2-a_2^2)^2}}{2a_1a_2}$$

Now, we can solve for θ_1 and θ_2 using inverse trigonometric functions:

$$\theta_1 = \text{atan2}(y,x) - \text{atan2}(a_2\sin(\theta_2), a_1 + a_2\cos(\theta_2))$$

$$\theta_2 = \text{atan2}\left(\sqrt{1-\left(\frac{r^2-a_1^2-a_2^2}{2a_1a_2}\right)^2}, \frac{r^2-a_1^2-a_2^2}{2a_1a_2}\right)$$

These equations give the joint angles ~~(theta1 & the~~ (θ_1 & θ_2) for a given end-effector position (x, y).