CSE211 – Formal Languages and Automata Theory

Unit 1-L3: Grammars and Dérivations

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Agenda

- Why Grammars for a language?
- What is a grammar?
- What is derivation?
- What is sentential form?
- How to find Language generated by grammar?
- How to find Grammar to generate language?

Grammar

- A formal grammar is a set of rules for rewriting strings, along with a "start symbol" from which rewriting starts
- It is a mechanism to describe languages
- A grammar for the English language tells us whether a particular sentence is well-formed or not
- A typical rule of English grammar is “a sentence can consist of a noun phrase followed by a predicate.”
- More concisely we write this as

$$\langle sentence \rangle \rightarrow \langle noun\_phrase \rangle \ \langle predicate \rangle$$

Example: Grammar for Sentence

- The set of grammar rules for the sentence formation in English language

$$\langle sentence \rangle \rightarrow \langle noun\_phrase \rangle \ \langle predicate \rangle$$

$$\langle noun\_phrase \rangle \rightarrow \langle article \rangle \ \langle noun \rangle$$

$$\langle predicate \rangle \rightarrow \langle verb \rangle$$

$$\langle article \rangle \rightarrow a \qquad \langle noun \rangle \rightarrow cat \qquad \langle verb \rangle \rightarrow runs$$

$$\langle article \rangle \rightarrow the \qquad \langle noun \rangle \rightarrow dog \qquad \langle verb \rangle \rightarrow sleeps$$

Derivation of String in English

- Consider “the dog sleeps”

$$\langle sentence \rangle \Rightarrow \langle noun\_phrase \rangle \ \langle predicate \rangle$$
$$\Rightarrow \langle noun\_phrase \rangle \ \langle verb \rangle$$
$$\Rightarrow \langle article \rangle \ \langle noun \rangle \ \langle verb \rangle$$
$$\Rightarrow the \ \langle noun \rangle \ \langle verb \rangle$$
$$\Rightarrow the \ dog \ \langle verb \rangle$$
$$\Rightarrow the \ dog \ sleeps$$

Derivation of String in English

- Consider “the cat eats”

$$\langle sentence \rangle \Rightarrow \langle noun\_phrase \rangle \ \langle predicate \rangle$$
$$\Rightarrow \langle noun\_phrase \rangle \ \langle verb \rangle$$
$$\Rightarrow \langle article \rangle \ \langle noun \rangle \ \langle verb \rangle$$
$$\Rightarrow the \ \langle noun \rangle \ \langle verb \rangle$$
$$\Rightarrow the \ cat \ \langle verb \rangle$$
$$\Rightarrow the \ cat \ eats$$

Language of the grammar

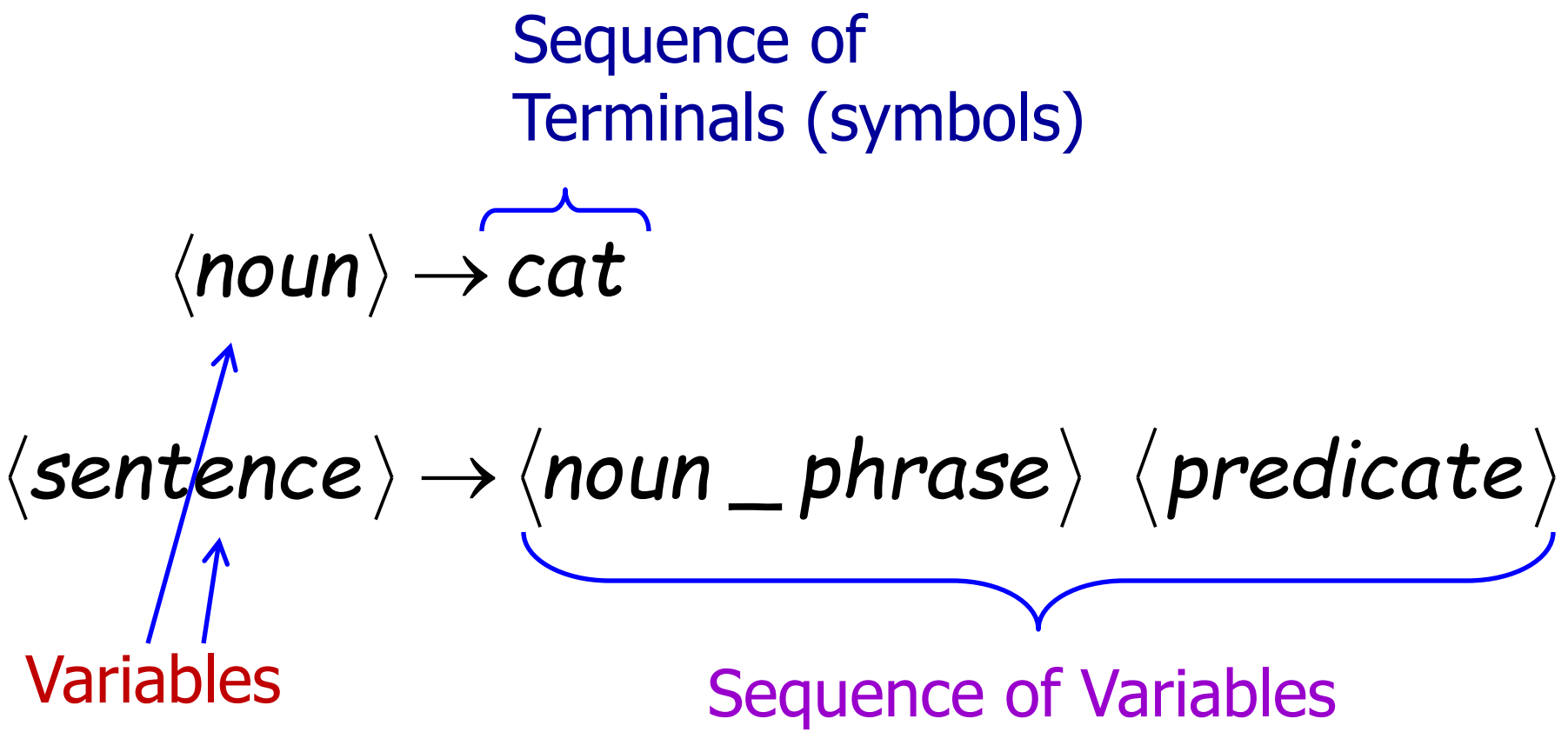
$\langle sentence \rangle \rightarrow \langle noun\_phrase \rangle \ \langle predicate \rangle$

$\langle noun\_phrase \rangle \rightarrow \langle article \rangle \ \langle noun \rangle$

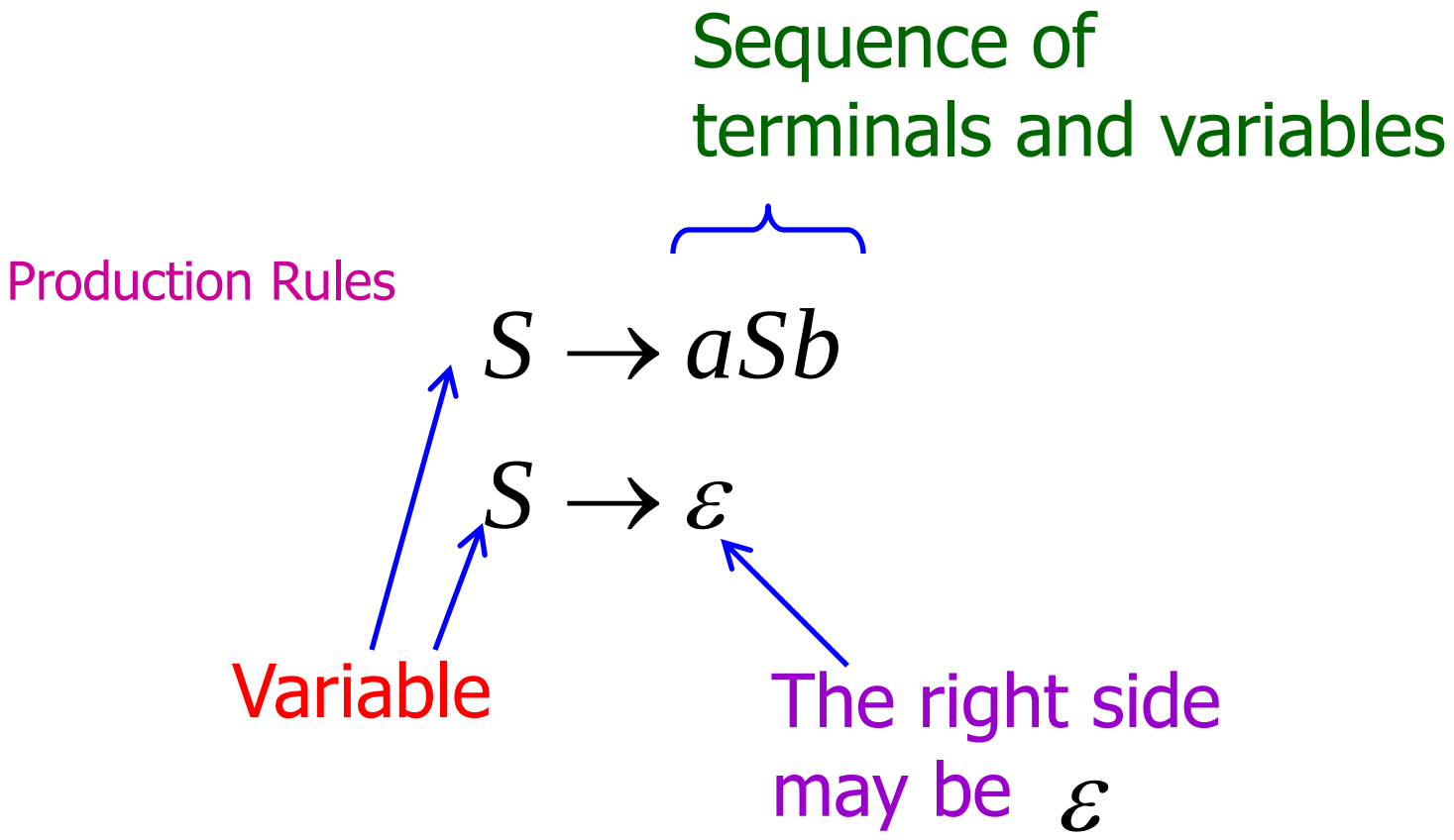
$\langle predicate \rangle \rightarrow \langle verb \rangle$

L = { “a cat runs”, “a cat sleeps”,
“the cat runs”, “the cat sleeps”,
“a dog runs”, “a dog sleeps”,
“the dog runs”, “the dog sleeps” }

Production of Grammar



Production of Grammar ...



Derivation of String

- Production rules:

$$S \rightarrow aSb$$

$$S \rightarrow \lambda$$

- Derivation of string : ab

$$S \Rightarrow aSb \Rightarrow ab$$

$S \rightarrow aSb$ $\qquad$ $S \rightarrow \lambda$

Derivation of String...

- Production: $S \rightarrow aSb$

  $S \rightarrow \lambda$

- Derive the string:

$aaabbb$

String is a part of the language generated by the grammar

$$S \Rightarrow aSb \qquad [\because S \rightarrow aSb]$$

$$\Rightarrow aaSbb \qquad [\because S \rightarrow aSb]$$

$$\Rightarrow aaaSbbb \qquad [\because S \rightarrow aSb]$$

$$\Rightarrow aaabbb \qquad [\because S \rightarrow \lambda]$$

Derivation of String...

- Production rules:

$$S \rightarrow aSb$$

$$S \rightarrow \lambda$$

- Derivation of string : $aabb$

$$S \Rightarrow aSb \Rightarrow aaSbb \Rightarrow aabb$$

$S \rightarrow aSb$ $\qquad$ $S \rightarrow \lambda$

Derivation of String...

- Productions: $S \rightarrow aSb$

  $S \rightarrow \lambda$

- Derive the string:

$aaabb$

String is not a part of the language generated by the grammar

$$S \Rightarrow aSb \qquad [\because S \rightarrow aSb]$$

$$\Rightarrow aaSbb \qquad [\because S \rightarrow aSb]$$

$$\Rightarrow incomplete$$

Language Accepted by Grammar

Productions: $S \rightarrow aSb$

$S \rightarrow \lambda$

Language accepted by the grammar:

$$L = \{\lambda, ab, aabb, aaabbb, aaaabbbb, \ldots\}$$

Language accepted by the grammar in general:

$$L = \{a^n b^n : n \geq 0\}$$

Formal Definition: Grammar

- A grammar G is defined as a quadruple

$$G = (V, T, S, P),$$

where V is a finite set of objects called variables,

T is a finite set of objects called terminal symbols,

$S \in V$ is a special symbol called the start variable,

P is a finite set of productions.

- It will be assumed that the sets V and T are nonempty and disjoint.

Summary

- Why Grammars for a language
- What is a grammar?
- What is derivation?
- What is sentential form?
- Language generated by grammar
- Grammar to generate language

References

- John E. Hopcroft, Rajeev Motwani and Jeffrey D. Ullman, *Introduction to Automata Theory*, Languages, and Computation, Pearson, 3rd Edition, 2011.
- Peter Linz, An Introduction to Formal Languages and Automata, Jones and Bartle Learning International, United Kingdom, 6th Edition, 2016.

Next Class:

Examples for Grammar and Derivation

THANK YOU.