25/10/21 Monday

Theorem 2 :- a) $x+1=1$

b) $x\cdot 0 = 0$

$x+\textcircled{1}$ — $x+x'$

$x+x$

Proof :-

$x+1 = 1\cdot(x+1)$ [∵ $x\cdot 1 = x$]

$= (x \underline{+} x')\cdot(x \underline{+} 1)$ [$x+x'=1$]

$= x+(x'\cdot 1)$ [By distributive property]

$= x+x'$ [By Identity law]

$=1$ [By Complement law]

Rough

$a = x$

$b = x'$

$c = 1$

Rough $a\cdot(b+c) = a\cdot b + a\cdot c$

$\boxed{a+(b\cdot c) = (a+b)\cdot(a+c)}$

∴ $\boxed{x+1=1}$

b) $x\cdot 0 = 0+(x\cdot 0)$ [By $x\cdot 0=0$]

$= (x\cdot x')+(x\cdot 0)$ [By Complement law]

$= x\cdot(x'+0)$ [By distributive law]

$= x\cdot x'$ [By Identity law]

$= 0$ [By Complement law]

Rough

$a = x$

$b = x'$

$c = 0$

∴ $\boxed{x\cdot 0 = 0}$

Theorem 3 :- $(x')' = x$ [Involution]

Proof :-

We know that $x+x'=1$ and $x\cdot x'=0$ by Complement law.

$\Rightarrow (x')' = x$

∴ $(x')' = x$

Rough

a b

$a\cdot b = 0$

$a+b=1$

$a'=b$ a/ $b'=a$

Theorem 4 :- [Associative]

a) $x+(y+z) = (x+y)+z$

b) $x\cdot(y\cdot z) = (x\cdot y)\cdot z$

Theorem 5 :- De Morgan's law

$\overline{A \cup B} = \overline{A} \cap \overline{B}$

$\overline{A \cap B} = \overline{A} \cup \overline{B}$

$\cup \to +$

$\cap \to \cdot$

$\overline{} \to '$

$A \quad B$

$x \quad y$

a) $(x+y)' = x' \cdot y'$

b) $(x \cdot y)' = x' + y'$

Proof :-

$\boxed{(x+y)} + (2 \cdot 3)$

$1 + (2 \cdot 3)$

a) $\overset{I}{(x\underset{1}{+}y)} \underset{2}{+} \overset{II}{(x' \underset{3}{\cdot} y')}$

$= ((x+y)+x') \cdot ((x+y)+y')$ [By dis.]

$= ((y+x)+x') \cdot ((x+y)+y')$ [By Comm]

$= (y+(x+x')) \cdot (x+(y+y'))$ [By Asso.]

$= (y+1) \cdot (x+1)$ [By Complement law]

$= 1 \cdot 1$ [$\because x+1 = x$]

$= 1$ [$\because 1 \cdot 1 = 1$]

$a \to b$

$a = x+y$

$b = x' \cdot y'$

$+ \downarrow 1 \quad \cdot \downarrow 0$

$\overset{I}{(1)} 2 \overset{II}{(3)}$

$1 = 2 \to I$

$2 = 3 \to II$

$1 = 3 \to I$ & II

$\therefore \underset{I}{(x+y)} + \underset{II}{(x' \cdot y')} = 1 \longrightarrow ①$

Now, $(x\underset{1}{+}y) \underset{2}{\cdot} (x' \underset{3}{\cdot} y') = (x \cdot (x' \cdot y')) + (y \cdot (x' \cdot y'))$

[By distributive law]

$= [(x \cdot x') \cdot y') + [y \cdot (x' \cdot y'))$

[By Associative laws]

$= (0 \cdot y') + (y \cdot (y' \cdot x'))$

[By Commutative law and Complement laws]

$= 0 + ((y \cdot y') \cdot x')$ [By Associative law and $x \cdot 0 = 0$]

$= (0 \cdot x')$ [By Identity law and Complement law]

$= 0$ [$\because x \cdot 0 = 0$]

$\therefore (x+y) \cdot (x' \cdot y') = 0 \longrightarrow ②$

From ① and ②, we infer that

$(x+y)' = x' \cdot y'$

b) H.W.

Theorem 6 :- Absorption law

a) $x \cdot (x+y) = x$

b) $x + (x \cdot y) = x$

$x \cdot x' = 0$

Proof :- a) $x \cdot (x+y) = x \cdot x + x \cdot y$ [By distributive law]

$= x + x \cdot y$ [By Idempotent law]

$= x \cdot 1 + x \cdot y$ [By Identity law]

$= x \cdot (1+y)$ [By distributive law]

$= x \cdot 1$ [∵ $x+1=x$]

$= x$ [By Identity law]

$\therefore x \cdot (x+y) = x$

b) $x + x \cdot y = x \cdot 1 + x \cdot y$ [By Identity law]

$= x \cdot (1+y)$ [By distributive law]

$= x \cdot 1$ [∵ $x+1=1$]

$= x$ [By Identity law]

$\therefore x + x \cdot y = x$
