CSE211 – Formal Languages and Automata Theory

U1L18 – Properties of RL – Pumping Lemma for RL

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Agenda

- Recap of previous class
- Properties of Regular Languages
- Proving Languages Not to Be Regular
- Pumping Lemma for Regular Languages
- Examples for Non-Regular Languages

Pumping Lemma for Regular Languages

- It is first noted that *not* every language is regular.
  - For example, $L_{01} = \{0^n 1^n \mid n \geq 1\}$ is not a regular language.
- How to prove that a language is not regular?
  - Answer: use the *pumping lemma*.

Pumping lemma for RL's

- Let L be an RL. Then, there exists an integer constant n (depending on L) such that for every string w in L with $|w| \geq n$, we can break w into three substrings, $w = xyz$, such that:
  - $y \neq \varepsilon$ or $|y| \geq 1$ (i.e., y has at least one symbol);
  - $|xy| \leq n$; and
  - for all $k \geq 0$, the "pumped" string xy^kz is also in L.

Applications of Pumping Lemma

- The pumping lemma may be used for proving "a given language is *not* an RL," instead of proving "*is* an RL."

Example 1:

- Prove that the language $L_{eq} = \{w \mid w$ has equal numbers of 0's and 1's$\}$ is *not* an RL. (OR)
- $L_{eq} = \{ 0^n 1^n \mid n \geq 0 \}$ is non-regular
- *Proof* (by contradiction):
  - Assume that L_{eq} is an RL.
  - The *pumping lemma* says that there exists an integer n such that for every string w in L with length $|w| \geq n$, w can be broken into 3 pieces, $w = xyz$, such that the three conditions mentioned in Theorem .
  - In particular, pick the string $w = 0^n1^n$ whose length is $2n$.

(A)

To prove Non Regular

- We know that w is in L_{eq}.
- Because $|w| = 2n > n$, by Theorem , *string* w can be broken into 3 pieces, $w = xyz$, so that
  - $y \neq \varepsilon$ or $|y| \geq 1$;
  - $|xy| \leq n$;
  - for all $k \geq 0$, $xy^kz \in L$.
- Also, the inequality $|xy| \leq n$ says that xy consists of all 0's because $w = 0^n1^n$.
- Furthermore, $y \neq \varepsilon$ says y has at least one 0.

Contd…

- Now, take *k* to be 0 and the *pumping* in the 3rd condition says that the following is true:
- $$xy^0z = x\varepsilon z = xz \in L. \quad \text{(B)}$$
- However, by (A) at least one 0 disappears when *y* was "pumped" out.
- This means that the resulting string *xz cannot have equal numbers of 0's and 1's, i.e., xz* $\notin$ *L. Contradictive to* (B) *above!*
- So, the original assumption "L_{eq} is an RL" is false (according to principle of "proof by contradiction."). Done!

Example 1: Non-Regular

- $L_{eq} = \{ 0^n 1^n \mid n \geq 0 \}$ is non-regular by example

Example 2: Non-Regular

- Prove that L={ww|w Є{0,1}*} is not a regular language

Example 3: Non-Regular

- Prove that the Language $L=\{w \in \{0,1\}^* | w=w^R\}$ or palindrome is not a regular language.

Summary

- Properties of Regular Languages
- Proving Languages Not to Be Regular
- Pumping Lemma for Regular Languages
- Examples for Non-Regular Languages

References

- John E. Hopcroft, Rajeev Motwani and Jeffrey D. Ullman, *Introduction to Automata Theory*, Languages, and Computation, Pearson, 3rd Edition, 2011.
- Peter Linz, An Introduction to Formal Languages and Automata, Jones and Bartle Learning International, United Kingdom, 6th Edition, 2016.

Next Class:

Closure properties of RL

THANK YOU.