Probability of an Event :-

1) Sample space
2) Event
3) Probability.
4) Random Variables.

All possible outcomes of an Expt

Subset of a Sample space

Roll a dice.

$S = \{1,2,3,4,5,6\}$.

Event :- getting an odd number.

$\{1,3,5\}$

$$P(E) = \frac{n(E)}{n(S)}$$

$$\boxed{0 \le P(E) \le 1}$$

$$y = a e^{bx}$$
$$\log y = \log a + bx$$
$$Y = A + bx$$
$\downarrow$
fitting a line

$$y = \underline{\underline{a}}x^2 + \underline{\underline{b}}x + \underline{\underline{c}}$$
$$\sum_1 y = a\sum_1 x^2 + b\sum_1 x + nc$$
$$\sum_1 xy = a\sum_1 x^3 + b\sum_1 x^2 + c\sum_1 x$$
$$\sum_1 x^2 y = a\sum_1 x^4 + b\sum_1 x^3 + c\sum_1 x^2$$
fitting a parabola

$$y = \underline{\underline{a}} + \underline{\underline{b}}x$$
$$\begin{cases} \sum_1 y = na + b\sum_1 x \\ \sum_1 xy = a\sum_1 x + b\sum_1 x^2 \end{cases}$$
fit a line

Random Variable:-

define a function $f: S \to \mathbb{R}$

S → sample space

$\mathbb{R}$ → real number.

random variable → $(X, Y, \ldots)$ → (Random Variable)

Event ~~Sample space~~	prob
E_1	a_1
E_2	a_2
$\vdots$	
E_n	a_n

$a_1, a_2, \ldots, a_n$ → real number b/w 0 & 1.

$f: S \to \mathbb{R}$ → Random Variable.

Single dimension r.v.

Probability density function. / Mass function. $\begin{cases} \text{Discrete type} \\ \text{Continuous type.} \end{cases}$

$$P(X=x) = \begin{cases} \cancel{f(x)}\ p_i & x \in S = [a,b]. \\ 0 & \text{otherwise.} \end{cases}$$

$$f : S \to \mathbb{R}. \hookrightarrow [0,1]$$

$$\sum_{x \in S} P(X=x) = 1$$

$$\int_a^b f(x)\,dx = 1.$$

$$P(X=x) = {}^nC_x\, p^x q^{n-x}$$

$n, x = 0, 1, 2, \dots$

$$P(X=x) = e^{-\lambda} \lambda^x / \underline{|x}.$$

Distribution function:-

$$F(x) = P(X \le x) \rightarrow \text{r.v}$$

$$\begin{cases} x: & 1 & 2 & 3 & 4 \\ P(x): & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{cases}$$

$$F(3) = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}.$$

$$\sum_{x \le a} P(X = x) \qquad \int_{-\infty}^{a} f(x)\,dx.$$

→ Cumulative frequency

$$F'(x) = f(x)$$

Two Dimensional Random Variable.

(X, Y).

Joint Probability density function:-

$$P(x, y)$$

$$\sum_{y} \sum_{x} P(x,y) = 1.$$

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y)\, dx\, dy = 1.$$

$$\boxed{\begin{gathered} \sum p_i = 1 \\ \int_a^b f(x)\, dx = 1 \end{gathered}}$$

$Y \backslash X$	x_1	x_2	$\cdots$	x_m
y_1	p_{ij}	p_{12}		p_{1m}
y_2	- - -	- - -	- - -	- -
$\vdots$	- - -	- - -	- - -	p_{nm}
y_n				

$$f(x,y) = \begin{cases} \dfrac{xy}{2} & 0 \le x \le 1,\ 0 \le y \le 1 \\ 0 & \text{otherwise} \end{cases}$$

Ex:- In the random placement of three marbles in three cells, describe the possible outcomes of the expt. Let X_i denote the number of marbles in cell $i = 1, 2, 3$ & N, the no. of cells occupied. Obtain the Joint distribution of (X_1, N) & (X_1, X_2).

Soln Let a, b, c be the marbles.

$X_1 \to$ one marble in a cell

$X_2 \to$ Two marbles in a cell.

$N \to$ no of cells occupied.

1) a-b-c
2) a-c-b
3) b-a-c
4) b-c-a
5) c-a-b
6) c-b-a
7) ab-c-* ✓
8) ab-*-c ✓
9) *-ab-c ✓
10) ac-b-* ✓
11) ac-*-b ✓
12) *-ac-b ✓
13) bc-a-* ✓
14) bc-*-a ✓
15) *-bc-a ✓
16) a-bc-* ✓
17) a-*-bc ✓
18) *-a-bc ✓
19) b-ca-* ✓
20) b-*-ca ✓
21) *-b-ca ✓
22) c-ab-* ✓
23) c-*-ab ✓
24) *-c-ab ✓
25) abc-*-*
26) *-abc-*
27) *-*-abc.

9, 12, 15, 18, 21, 24,
26, 27

$P(N=1) = \frac{3}{27}$; $P(N=2) = \frac{18}{27}$; $P(N=3) = \frac{6}{27}$

$X_1 \rightarrow$ no of marbles placed in first cell.

$P(X_1=0) = \frac{8}{27}$; $P(X_1=1) = \frac{12}{27}$; $P(X_1=2) = \frac{6}{27}$

$P(X_1=3) = \frac{1}{27}$