Module II

Topics: Knowledge representation issues, Predicate logic, Representation knowledge using rules. Concept Learning: Concept learning task, Concept learning as search, Find-S algorithm, Candidate Elimination Algorithm, Inductive bias of Candidate Elimination Algorithm

CHAPTER 4-CONCEPT LEARNING

- Inducing general functions from specific training examples is a main issue of machine learning.
- Concept learning - a learning task in which a human or machine learner is trained to classify objects by being shown a set of example objects along with their class labels. The learner simplifies what has been observed by condensing it in the form of an example.
- Concept learning - also known as category learning, concept attainment, and concept formation.
- Concept Learning: Acquiring the definition of a general category from given sample of positive and negative training examples of the category.

4.1 A Formal Definition for Concept Learning:

- Inferring a Boolean-valued function from training examples of its input and output. Learn the definition of a concept from examples.
- An example for concept-learning is the learning ofbird-concept from the given examples of birds(positive examples) and non-birds (negative examples).
- The concept of a bird is the subset of all objects (i.e.,the set of all things or all animals) that belong to the category of bird.
- Each concept is a Boolean-valued function defined over this larger set. [Example: a function defined over all animals whose value is true for birds and false for every other animal.

Every concept has two components:

- **Attributes:** These are features of a stimulus that one must look for to decide if that stimulus is a positive instance of the concept.
- **A rule:** This statement that specifies which attributes must be present or absent for a stimulus to qualify as a positive instance of the concept.

  The simplest rules refer to the presence or absence of a single attribute. For example, a "vertebrate" animal is defined as an animal with a backbone. Which of these stimuli are positive instances?

- This rule is called affirmation. It says at a stimulus must possess a single specifiedattribute to qualify as a positive instance of a concept.
- The opposite or “complement” of affirmation is negation. To qualify as a positive instance, a stimulus must lack a single specified attribute.
- An invertebrate animal is one that lacks a backbone. These are the positive and negative instances when the negation rule is applied.

4.2 A Concept Learning Task – EnjoySport Training

Table 1: Positive and Negative training examples for the target concept EnjoySport

Example	Sky	AirTemp	Humidity	Wind	Water	Forecast	EnjoySport
1	Sunny	Warm	Normal	Strong	Warm	Same	Yes
2	Sunny	Warm	High	Strong	Warm	Same	Yes
3	Rainy	Cold	High	Strong	Warm	Change	No
4	Sunny	Warm	High	Strong	Cool	Change	Yes

A set of example days, and each is described by six attributes.

- The task is to learn to predict the value of Enjoy Sport for arbitrary day, based onthe values of its attribute values -Target concept.
- Goal: To infer the “best” concept-description from the set of all possiblehypotheses.
- Each hypothesis consists of a conjunction of constraints on the instance attributes.
- Each hypothesis will be a vector of six constraints, specifying the values of the sixattributes
- (Sky, AirTemp, Humidity, Wind, Water, and Forecast).
- Each attribute will be:
- Indicating any value is acceptable for the attribute (don’t care)
- single value – specifying a single required value (ex. Warm) (specific)
- Indicating no value is acceptable for the attribute (no value)
- hypothesis:
- Sky AirTemp Humidity Wind Water Forecast
- < Sunny, ?, ?, Strong , ? , Same >Most General Hypothesis: Every day is a good day for water sports <?, ?, ?, ?, ?, ?> (Positive example)
- Most Specific Hypothesis: No day is a good day for water sports <0, 0, 0, 0, 0, 0> No day is

Positive example)

- Enjoy Sport concept learning task requires learning the sets of days for which Enjoy Sport = yes, describing this set by a conjunction of constraints over the instance attributes.

Notation

Given:

Instances X: Set of all Possible days, each described by the attributes

- Sky (Sunny, Cloudy, and Rainy)
- Temp (Warm and Cold)
- Humidity (Normal and High)
- Wind (Strong and Weak)
- Water (Warm and Cool)
- Forecast (Same and Change)

Hypotheses *H:* Each hypothesis is described by a conjunction of constraints on theattributes ***Sky, AirTemp, Humidity, Wind, Water,*** and ***Forecast.*** The constraints may be "?" (any value is acceptable), *"**0*** (no value is acceptable), or a specific value.

- Target concept ***c: EnjoySport*** : ***X*** → {0,l}
- Training Examples D : positive and negative examples of the target function alongwith their target concept value c(x).

Determine: A hypothesis h in H such that h(x) = c(x) for all x in X.

When learning the target concept, the learner is presented a set of ***training examples,*** each consisting of an instance ***x*** from X, along with its target concept value ***c(x)*** (e.g., the training examples in Table 1.1). Instances for which ***c(x)*** = 1 are called ***positive examples,*** or members ofthe target concept. Instances for which ***C(X)*** = 0 are called ***negative examples,*** or non members of the target concept.
We will often write the ordered pair ***(x, c(x))*** to describe the training example consisting of the instance **x** and its target concept value ***c(x).*** We use the symbol ***D*** to denote the set of available training examples.

Given a set of training examples of the target concept ***c,*** the problem faced by the learner is to hypothesize, or estimate, ***c.***
We use the symbol H to denote the set of ***all possible hypotheses*** that the learner may consider regarding the identity of the target concept. Usually H is determined by the human designer's choice of hypothesis

representation. In general, each hypothesis ***h*** in H represents a boolean-valued function defined over X; that is, ***h*** : X → {O, 1).

The goal of the learner is to find a hypothesis ***h*** such that ***h(x)*** = ***c(x)*** for a" ***x*** in X.

4.3 The Inductive Learning Hypothesis

- Inductive learning algorithms can at best guarantee that the output hypothesis fits the target concept over the training data. Lacking any further information, our assumption is that the best hypothesis regarding unseen instances is the hypothesis that best fits the observed training data.
- **The inductive learning hypothesis**. Any hypothesis found to approximate the target function well over a sufficiently large set of training examples will also approximate the target function well over other unobserved examples.

4.4 Concept Learning as Search

- Concept Learning can be viewed as the task of searching through a large space of hypotheses implicitly defined by the hypothesis representation.
- The goal of this search is to find the hypothesis that best fits the training examples.
- The hypothesis space has a general-to-specific ordering of hypotheses, and the search can be efficiently organized by taking advantage of a naturally occurring structure over the hypothesis space.
- By selecting a hypothesis representation, the designer of the learning algorithm implicitly defines the space of all hypotheses that the program can ever represent and therefore can ever learn.

Example: EnjoySport Hypothesis Space

✓ Sky has 3 possible values, and other 5 attributes ave 2 possible values.

✓ There are 96 (= 3.2.2.2.2.2) distinct instances in X.

✓ There are 5120 (=5.4.4.4.4.4) syntactically distinct hypotheses in H.

– Two more values for attributes: ? and 0

✓ Every hypothesis containing one or more 0 symbols represents the empty set ofinstances. that is, it classifies every instance as negative.

- ✓ There are 973 (= 1 + 4.3.3.3.3.3) semantically distinct hypotheses in H.
  – Only one more value for attributes: ?, and
- ✓ one hypothesis representing empty set of
- ✓ instances.
- ✓ Although EnjoySport has small, finite hypothesis
- ✓ space, most learning tasks have much larger(even infinite) hypothesis spaces.
  – We need efficient search algorithms on the hypothesis spaces.

4.5 General-to-Specific Ordering of Hypotheses

- ✓ The hypothesis space has a general-to-specific ordering of hypotheses, and the search can be efficiently organized.

Now consider the sets of instances that are classified positive by h_l and by h_2.Because h_2 imposes fewer constraints on the instance, it classifies more instances as positive. In fact, any instance classified positive by h_l will also be classified positive by h_2. Therefore, we say that h_2 is more general than h_l.

This intuitive "more general than" relationship between hypotheses can be defined more precisely as follows. First, for any instance x in X and hypothesis h in H, we say that x satisfies h if and only if h(x) = 1.

We now define the ***more_general_than_or equa_to*** *relat*ion in terms of the sets of instances that satisfy the two hypotheses: Given hypotheses h_j and h_k, h_j is more-general-than m- equal do h_k if and only if any instance that satisfies hk also satisfies ***hi.***

Definition: Let $\boldsymbol{h_j}$ and $\boldsymbol{h_k}$ ***be*** boolean-valued functions defined over X. Then ***hj*** is **more general-than-or-equal-to** $\boldsymbol{h_k}$(written ***hj*** $\boldsymbol{>=_g h_k}$**)** if and only if

$$(\forall x \in X)[(h_k(x) = 1) \rightarrow (h_j(x) = 1)]$$

Example:

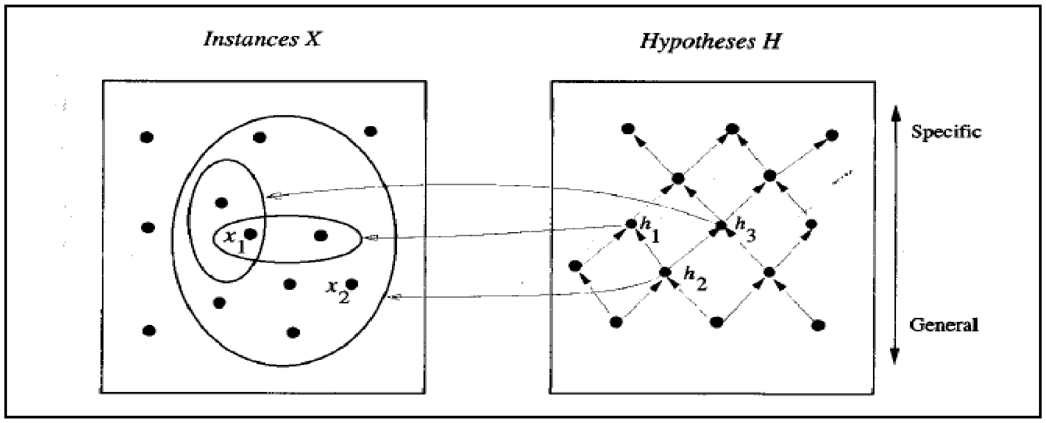
h1 = <Sunny, ?, ?, Strong, ?, ?>

h2 = <Sunny, ?, ?, ?, ?, ?>

- Every instance that are classified as positive by h1 will also be classified as positiveby h2 in our example data set. Therefore, h2 is more general than h1.

- We also use the ideas of “strictly” -more-general-than, and more-specific-than [Mitchell].

More general than relation



x_1 = <Sunny, Warm, High, Strong, Cool, Same>
x_2 = <Sunny, Warm, High, Light, Warm, Same>

h_1 = <Sunny, ?, ?, Strong, ?, ?>
h_2 = <Sunny, ?, ?, ?, ?, ?>
h_3 = <Sunny, ?, ?, ?, Cool, ?>

Figure 1: Instances, hypotheses, and the *m o r e - g e n e r a l - t h a n* relation.

The box on the left represents the set X of all instances, the box on the right the set *H* of all hypotheses. Each hypothesis corresponds to some subset of X-the subset of instances that it classifies positive. The arrows connecting hypotheses represent the *m o r e - g e n e r a l - t h a n* relation, with the arrow pointing toward the less general hypothesis. Note the subset of instances characterized by *h2* subsumes the subset characterized by h_l, hence h_2 is *m o r e - g e n e r a l - t h a n* h_l.

To illustrate these definitions, consider the three hypotheses ***hl, h2,*** and ***h3*** from our ***Enjoysport*** example, shown in Figure. How are these three hypotheses related by the ***p,*** relation? As noted earlier, hypothesis ***h2*** is more general than ***hl*** because every instance that satisfies ***hl*** also satisfies ***h2.***

Similarly,***h2*** is more general than ***h3.*** Note that neither ***hl*** nor ***h3*** is more general than the other; although the instances satisfied by these two hypotheses intersect,neither set subsumes the other. Notice also that the ***p,*** and >, relations are defined independent of the target concept. They depend only on which instances satisfy the two hypotheses and not on the classification of those instances according to the target concept.

Formally, the ***p,*** relation defines a partial order over the hypothesis space H (the relation is reflexive, antisymmetric, and transitive).

Informally, when we say the structure is a partial (as opposed to total) order, we mean there may be pairs of hypotheses such as ***hl*** and ***h3,*** such that ***hl not greater than equal to h3*** and *h3 not greater than equal to* ***hl.***

The $\geq_g$ relation is important because it provides a useful structure over the hypothesis space H for ***any*** concept learning problem. The following sections present concept learning algorithms that take advantage of this partial order to efficiently organize the search for hypotheses that fit the training data**.**

4.6 Find-S: Finding a Maximally Specific Hypothesis

```
FIND-S Algorithm.
1. Initialize h to the most specific hypothesis in H
2. For each positive training instance x
        For each attribute constraint a, in h
            If the constraint a, is satisfied by
            xThen do nothing
            Else replace a, in h by the next more general constraint that is satisfied by x
3. Output hypothesis h
```

The first step of FIND- S is to initialize h to the most specific hypothesis in H

$$h \leftarrow \langle \emptyset, \emptyset, \emptyset, \emptyset, \emptyset, \emptyset \rangle$$

none of the "0" constraints in h are satisfied by this example, so each is replaced by the next more general constraint {hat fits the example; namely, the attribute values for this training example.

$$h \leftarrow \langle Sunny, Warm, Normal, Strong, Warm, Same \rangle$$

Next, the second training example (also positive in this case) forces the algorithm to further generalizeh, this time substituting a "?' in place of any attribute value in h that is not satisfied bythe new example. The refined hypothesis in this case is

$$h \leftarrow \langle Sunny, Warm, ?, Strong, Warm, Same \rangle$$

Upon encountering the third training example-in this case a negative example-the algorithm makes no change to h. In fact, the FIND-S algorithm simply ignores every negative example!. To complete our trace of FIND-S, the fourth (positive) example leads to a further generalization of h.

$$h \leftarrow \langle Sunny, Warm, ?, Strong, ?, ? \rangle$$

The FIND-S algorithm illustrates one way in which the more-general-than partial ordering can be used to organize the search for an acceptable hypothesis. The search moves from hypothesis to hypothesis, searching from the most specific to progressively more general hypotheses along one chain of the partial ordering.

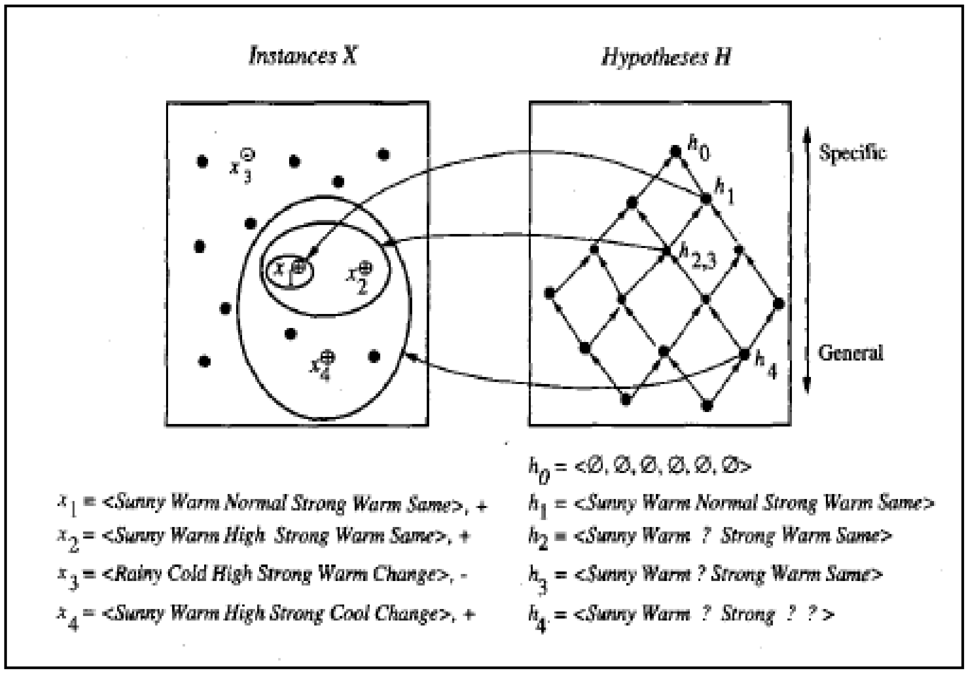

Instances X
Hypotheses H
$x_3^{\ominus}$
$x_1^{\oplus}$
$x_2^{\oplus}$
$x_4^{\oplus}$
h_0
h_1
$h_{2,3}$
h_4
Specific
General
h_0 = <∅, ∅, ∅, ∅, ∅, ∅>
x_1 = <Sunny Warm Normal Strong Warm Same>, +
h_1 = <Sunny Warm Normal Strong Warm Same>
x_2 = <Sunny Warm High Strong Warm Same>, +
h_2 = <Sunny Warm ? Strong Warm Same>
x_3 = <Rainy Cold High Strong Warm Change>, -
h_3 = <Sunny Warm ? Strong Warm Same>
x_4 = <Sunny Warm High Strong Cool Change>, +
h_4 = <Sunny Warm ? Strong ? ? >

Figure 2 : illustrates this search in terms of the instance and hypothesis spaces.

At each step, thehypothesis is generalized only as far as necessary to cover the new positive example. Therefore, ateach stage the hypothesis is the most specific hypothesis consistent with the training examples observed up to this point (hence the name FIND-S).

x1=<Sunny Warm Normal Strong Warm Same>, +x2
= <Sunny Warm High Strong Warm Same>, + x3=
<Rainy Cold High Strong Warm Change>, -
x4 =<Sunny Warm High Strong Cool Change>, +

h1=<Sunny Warm Normal Strong Warm same>

h2 = <Sunny Warm ? Strong Warm same>

h3= <Sunny Warm ? Strong Warm same >

h4= <Sunny Warm ? Strong ? ? >

The hypothesis space search performed by FIND-S. The search begins (ho) with the most specific hypothesis in H, then considers increasingly general hypotheses (hl through h4) as mandated bythe training examples. In the instance space diagram, positive training examples are denoted by "+," negative by "-," and instances that have not been presented as training examples are denoted by a solid circle.

The key property of the find-s Algorithm is that for hypothesis spaces described by conjunctions of attribute constraints. FIND-S is guaranteed to output the most specific hypothesis within H that is consistent with the positive training examples. Its final hypothesis will also be consistent with the negative examples provided the correct target concept is contained in H, and provided the training examples are correct.

Issues(unanswered questions) in Find-S algorithm:

- **Has FIND-S converged to the correct target concept?**
  – Although FIND-S will find a hypothesis consistent with the training data, it has no way to determine whether it has found the only hypothesis in H consistent with the data (i.e., the correct target concept), or whether there are many other consistent hypotheses as well.
  – We would prefer a learning algorithm that could determine whether it had converged and, if not, at least characterize its uncertainty regarding the true identity of the target concept.
- **Why prefer the most specific hypothesis?**
  – In case there are multiple hypotheses consistent with the training examples, FIND-S will find the most specific.
  – It is unclear whether we should prefer this hypothesis over, say, the most general, or some other hypothesis of intermediate generality.
- **Are the training examples consistent?**
  – In most practical learning problems there is some chance that the training examples will contain at least some errors or noise. – Such inconsistent sets of training examples can severely mislead FIND-S, given the fact that it ignores negative examples.
  – We would prefer an algorithm that could at least detect when the training data is inconsistent and, preferably, accommodate such errors.
- **What if there are several maximally specific consistent hypotheses?**
  – In the hypothesis language H for the EnjoySport task, there is always a unique, most specific hypothesis consistent with any set of positive examples.

– However, for other hypothesis spaces there can be several maximally specific hypotheses consistent with the data.
– In this case, FIND-S must be extended to allow it to backtrack on its choices of how to generalize the hypothesis, to accommodate the possibility that the target concept lies along a different branch of the partial ordering than the branch it has selected.

4.7 Version Spaces and the Candidate Elimination Algorithm

Representation

The CANDIDATE-ELIMINATION algorithm finds all describable hypotheses that areconsistent with the observed training examples. In order to define this algorithm precisely, we begin with a few basic definitions. First, let us say that a hypothesis is consistent with the training examples if it correctly classifies these examples.

Definition: A hypothesis h is consistent with a set of training examples D if and only ifh(x) = c(x) for each example (x, c(x)) in D.

$$Consistent(h, D) \equiv (\forall \langle x, c(x) \rangle \in D)\ h(x) = c(x)$$

An example x is said to satisfy hypothesis h when h(x) = 1,regardless of whether x is a positive or negative example of the target concept. However, whether such an example is consistent with h depends on the target concept, and in particular, whether h(x) = c(x).

The CANDIDATE-ELIMINATION algorithm represents the set of all hypotheses consistent with the observed training examples. This subset of all hypotheses is called the version space with respect to the hypothesis space H and the training examples D, because it contains all plausible versions of the target concept.

Definition: The version space, denoted VSHVD, with respect to hypothesis space H and training examples D, is the subset of hypotheses from H consistent with the training examples in D.

The LIST-THEN-ELIMINATE Algorithm

$$VS_{H,D} \equiv \{h \in H | Consistent(h, D)\}$$

The LIST-THEN-ELIMINATE algorithm first initializes the version space to contain all hypotheses in H, then eliminates any hypothesis found inconsistent with any training example. The version space of candidate hypotheses thus shrinks as more examples are observed, until ideally just one hypothesis remains that is consistent with all the observed examples. The LIST-THEN-ELIMINATE algorithm can be applied whenever the hypothesis space H is finite. It has many advantages, including the fact that it is guaranteed to output all hypotheses consistent with the training data.

More Compact Representation for Version Spaces

The version space is represented by its most general and least general members.

The LIST-THEN-ELIMINATE Algorithm

```
1. VersionSpace c a list containing every hypothesis in H
2. For each training example, (x, c(x))
        remove from VersionSpace any hypothesis h for which h(x) # c(x)
3. Output the list of hypotheses in VersionSpace
```

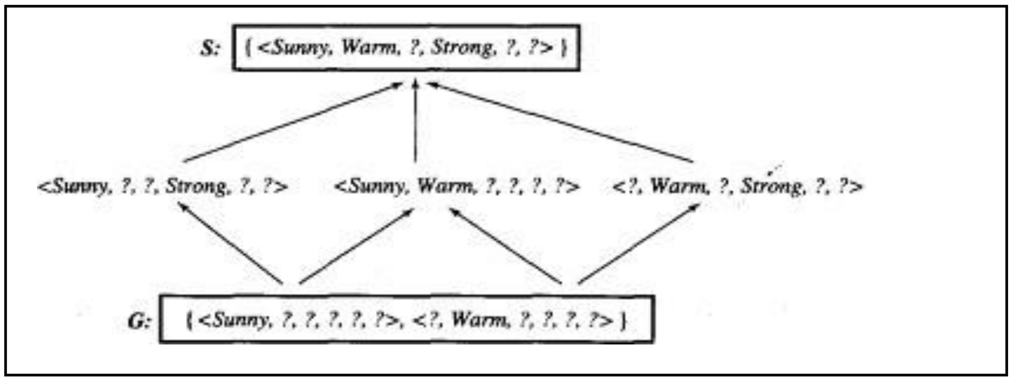


Figure3 : A version space with its general and specific boundary sets.

The version space includes all six hypotheses shown here, but can be represented more simply by S and G. Arrows indicate instances of the more-general-than relation. This is the version space for the Enjoy sport concept learning problem and training examples.

To illustrate this representation for version spaces, consider again the En*joysport* concept learning problem described in Table 1. Recall that given the four training examples from Table 1,
FIND-S outputs the hypothesis

***h* = (*Sunny, Warm,* ?, *Strong,* ?, ?)**

In fact, this is just one of six different hypotheses from H that are consistent with these training examples. All six hypotheses are shown in Figure 3. They constitute the version space relative to this set of data and this hypothesis representation.
The arrows among these six hypotheses in Figure 3 indicate instances of the *more-general~han*r elation. The CANDIDATE-ELIMINATlON Algorithm represents the version space by storing only its most

general members and its most specific (labeled *S* in the figure). Given only these two sets *S* and G, it is possible to enumerate all members of the version space as needed by generating the hypotheses that lie between these two sets in the general-to-specific partial ordering over hypotheses.

Definition: The general boundary G, with respect to hypothesis space H and trainingdata D, is the set of maximally general members of H consistent with D.

$$G \equiv \{g \in H | Consistent(g, D) \wedge (\neg \exists g' \in H)[(g' >_g g) \wedge Consistent(g', D)]\}$$

Definition: The specific boundary S, with respect to hypothesis space H and training data D, is the set of

$$S \equiv \{s \in H | Consistent(s, D) \wedge (\neg \exists s' \in H)[(s >_g s') \wedge Consistent(s', D)]\}$$

minimally general (i.e., maximally specific) members of H consistent withD.

Theorem : Version space representation theorem.

Let X be an arbitrary set of instances and let H be a set of boolean-valued hypotheses defined over X. Let c : X → {O, 1) be an arbitrary target concept defined over X, and let D be an arbitrary set of training examples

{(x, c(x))). For all X, H, c, and D such that S and G are well defined,

$$VS_{H,D} = \{h \in H | (\exists s \in S)(\exists g \in G)(g \geq_g h \geq_g s)\}$$

Proof.

- To prove the theorem it suffices to show that (1) every h satisfying the right- hand side of the above expression is in $VS_{H,D.}$
- (2) every member of VS_{HD}, satisfies the right-hand side of the expression. To show (1) letg be an arbitrary member of G, s be an arbitrary member of S, and h be an arbitrarymember of H, such that $g \geq_g h \geq_g s.$ 1
- Then by the definition of S, s must be satisfied by all positive examples in D. Because $h \geq_g s$ h must also be satisfied by all positive examples in D.
- Similarly, by the definition of G, g cannot be satisfied by any negative example in D, and because $g \geq_g h$, h cannot be satisfied by any negative example in D.
- Because h is satisfied by all positive examples in D and by no negative examples in D, h is consistent with D, and therefore h is a member of $VS_{H,D}$. This proves step (1). The argument for (2) is a bit more complex. It can be proven by assuming some h in $VS_{H,D}$ that does not satisfy the

right-hand side of the expression, then showing that this leads to an inconsistency.

4.8 CANDIDATE-ELIMINATION Learning Algorithm

- The CANDIDATE-ELIMINATION algorithm computes the version space containingall hypotheses from H that are consistent with an observed sequence of training examples.
- It begins by initializing the version space to the set of all hypothesesin H; that is, by initializing the G boundary set to contain the most generalhypothesis inH.

$$G_0 \leftarrow \{\langle ?, ?, ?, ?, ?, ? \rangle\}$$

- And initializing the S boundary set to contain the most specific (least general)hypothesis.

$$S_0 \leftarrow \{\langle \emptyset, \emptyset, \emptyset, \emptyset, \emptyset, \emptyset \rangle\}$$

- These two boundary sets delimit the entire hypothesis space, because every otherhypothesisinH is both more general than So and more specific than Go.

```
Initialize G to the set of maximally general hypotheses in H
Initialize S to the set of maximally specific hypotheses in H
For each training example d, do
  • If d is a positive example
      o Remove from G any hypothesis inconsistent with d , For each hypothesis s in S that is
        not consistent with d-Remove s from S
          Add to S all minimal generalizations h of s such that
              h is consistent with d, and some member of G is more general than h
          Remove from S any hypothesis that is more general than another hypothesis in S
  • If d is a negative example
      o Remove from S any hypothesis inconsistent with d For each hypothesis g in G that is not
        consistent with d
          Remove g from G
          Add to G all minimal specializations h of g such that
              h is consistent with d, and some member of S is more specific than h
          Remove from G any hypothesis that is less general than another hypothesis in G
```

An Illustrative Example.

The CANDIDATE-ELIMINATION algorithm applied to the first two training examples fromTable 1.

Figure 4 traces the CANDIDATE- ELIMINATION algorithm applied to the first two training examples from Table 1. As described above, the boundary sets are first initialized to Go and So, the most general and most specific hypotheses in H, respectively.

When the first training example is presented (a positive example in this case), the CANDIDATE-ELIMINATION Algorithm checks the *S* boundary and finds that it is overly specific-it fails to cover the positive example. The boundary is therefore revised by moving it to the least more general hypothesis that covers this new example. This revised boundary is shown as S1 in Figure 4. No update of the G boundary is needed in response to this training example because Go correctly covers this example. When the second training example (also positive) is observed, it hasa similar effect of generalizing ***S*** further to ***S2,*** leaving G again unchanged (i.e., G2 = **GI** = GO).

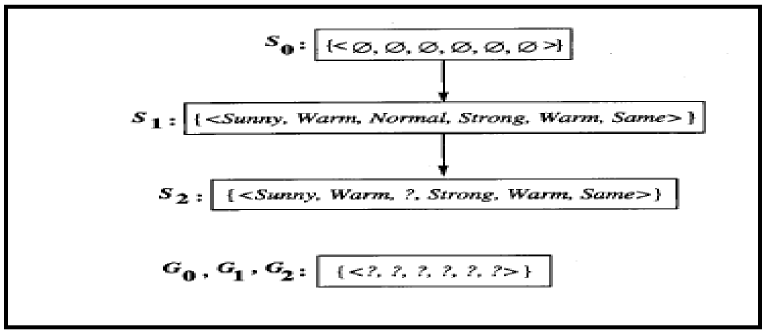

S_0: {<∅, ∅, ∅, ∅, ∅, ∅>}
S_1: {<Sunny, Warm, Normal, Strong, Warm, Same>}
S_2: {<Sunny, Warm, ?, Strong, Warm, Same>}
G_0, G_1, G_2: {<?, ?, ?, ?, ?, ?>}

Figure 4: CANDIDATE-ELIMINATION Algorithm Trace1.

S_o and G_o are the initial boundary sets corresponding to the most specific and most general hypotheses. Training examples 1 and 2 force the *S* boundary to become more general, as in the FIND-S algorithm. They have no effect on the *G* boundary.

Notice the processing of these first two positive examples is very similar to the processing performed by the FIND-S algorithm.

As illustrated by these first two steps, positive training examples may force the **S** boundary of the version space to become increasingly general. Negative training examples play the complimentary role of forcing the **G** boundary to become increasingly specific. Consider the third training example, shown in Figure 7.

This negative example reveals that the **G** boundary of the version space is overly general; that is, the hypothesis in **G** incorrectly predicts that this new example is a positive example. The hypothesis in the **G** boundary must therefore be specialized until it correctly classifies this new negative example. As shown in Figure 5, there are several alternative minimally more specific hypotheses. All of these become members of the new **G3** boundary set.

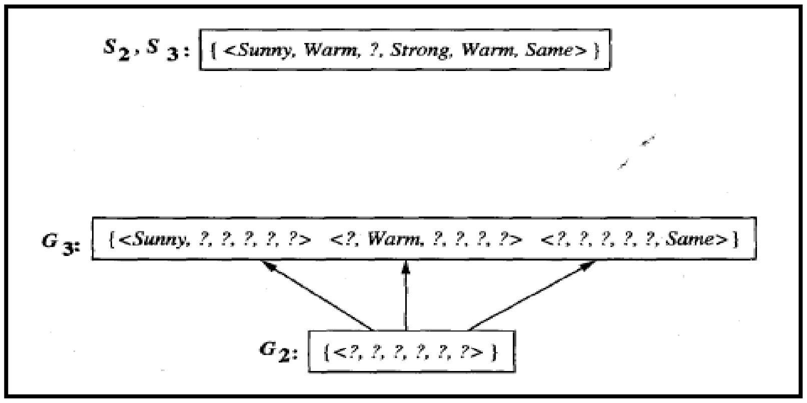


Figure 5: CANDIDATE-ELIMINATION Trace 2.

Training example 3 is a negative example that forces the G_2 boundary to be specialized to G_3. Note several alternative maximally general hypotheses are included in G_3.

Given that there are six attributes that could be specified to specialize $\mathbf{G_2}$**,**why are there only three new hypotheses in $\mathbf{G_3}$**?** For example, the hypothesis ***h*** = (?, ?, ***Normal,*** ?, ?, ?) is a minimal specialization of $\mathbf{G_2}$ that correctly labels the new example as a negative example, but it is not included in $\mathbf{G_3}$**.** The reason this hypothesis is excluded is that it is inconsistent with the previously encountered positive examples. The algorithm determines this simply by noting that ***h***is not more general than the current specific boundary, $\mathbf{S_2}$**.** In fact, the **S** boundary of the version space forms a summary of the previously encountered positive examples that can be used to determine whether any given hypothesis is consistent with these examples.

Any hypothesis more general than ***S*** will, by definition, cover any example that ***S*** covers and thus will cover any past positive example. In a dual fashion, the G boundary summarizes the information from previously encountered negative examples. Any hypothesis more specific than G is assured to be consistent with past negative examples. This is true because any such hypothesis, by definition, cannot cover examples that G does not cover.

The fourth training example, as shown in Figure 2.0, further generalizes the S boundary of the version space. It also results in removing one member of the G boundary, because this member fails to cover the new positive example. This last action results from the first step under the condition "If d is a positive example" shown in candidate elimination algorithm. To understand the rationale for this step, it is useful to consider why the offending hypothesis must be removed from

G. Notice it cannot be specialized, because specializing it would not make it cover the new example. It also cannot be generalized, because by the definition of G, any more general hypothesis will cover at least one negative training example. Therefore, the hypothesis must be dropped from the G boundary, thereby removing an entire branch of the partial ordering from the version space of hypotheses remaining under consideration.

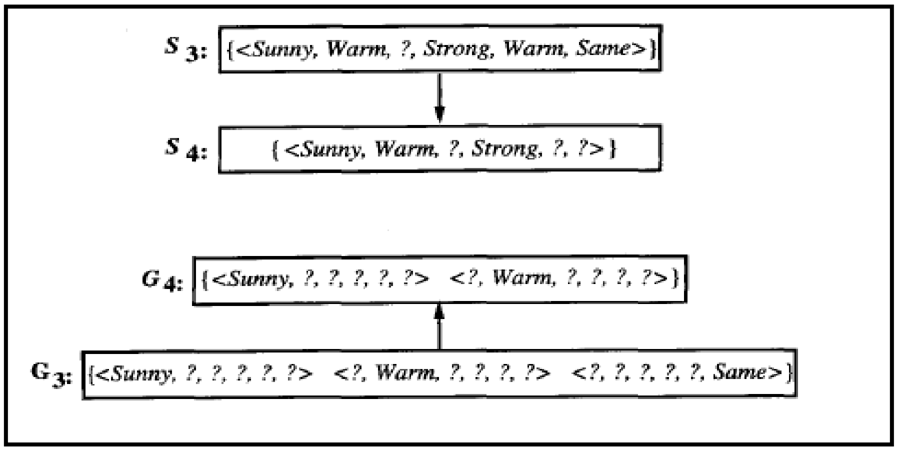


Figure 7: CANDIDATE-ELIMINATION Trace 3 .

The positive training example generalizes the *S* boundary, from *S3* to *S4.* One member of *Gg* must also be deleted, because it is no longer more general than the *S4* boundary.

- No up-date of the G boundary is needed in response to this training example because Go correctly covers this example.
- When the second training example (also positive) is observed, it has a similar effect of generalizing S further to S2, leaving G again unchanged (i.e., G2 = GI = GO). Notice the processing of these first two positive examples is very similar to the processing performed by the FIND-S algorithm.
- Negative training examples play the complimentary role of forcing the G boundary to become increasingly specific. Consider the third training example,
- The fourth training example, as shown in Figure 1.9, further generalizes the S boundary of the

version space. It also results in removing one member of the G boundary, because this member fails to cover the new positive example. This last action results from the first step under the condition.

- After processing these four examples, the boundary setsS4 and G4 delimit the version space of all hypotheses consistent with the set of incrementally observed training examples. The entire version space, including those hypotheses bounded by S4 and G4, is shown in Figure 8
- This learned version space is independent of the sequence in which the training examples are presented (be-cause in the end it contains all hypotheses consistent with the set of examples).

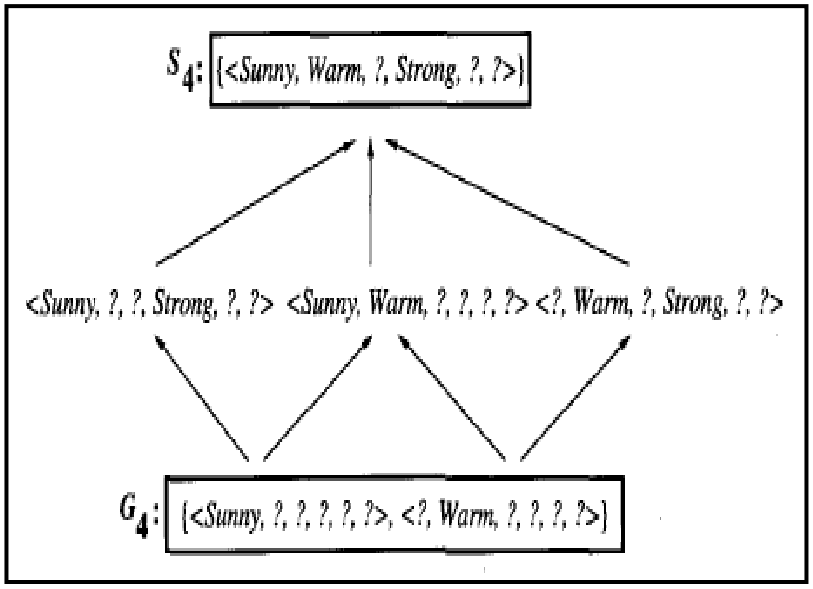


Figure 8 : The final version space for the *EnjoySport* concept learning problem and training examples described earlier.

- As further training data is encountered, the S and G boundaries will move monotonically closer to each other, delimiting a smaller and smaller version space of candidate hypotheses.

4.9 REMARKS ON VERSION SPACES AND CANDIDATE-ELIMINATION

Will the CANDIDATE-ELIMINATION Algorithm Converge to theCorrect Hypothesis?

The version space learned by the **CANDIDATE-ELIMINATION Algorithm** will convergetoward the hypothesis that correctly describes the target concept, provided

(1) There are no errors in the training examples

(2) There is some hypothesis in H that correctly describes the target concept.

In fact, as new training examples are observed, the version space can be monitored to determine the remaining ambiguity regarding the true target concept and to determine when sufficient training examples have been observed to unambiguously identify the target concept. The target concept is exactly learned when the S and ***G*** boundary sets converge to a single, identical, hypothesis.

What Training Example Should the Learner Request Next?

Consider again the version space learned from the four training examples of the **Enjoysport** ***concept***. Clearly, the learner should attempt to discriminate among the alternative competing hypotheses in its current version space. Therefore, it should choose an instance that would be classified positive by some of these hypotheses, but negative by others.

One such instance is

(Sunny, Warm, Normal, Light, Warm, Same)

Note that this instance satisfies three of the six hypotheses in the current version space.

If the trainer classifies this instance as a positive example,the ***S*** boundary of the version space can then be generalized.

Alternatively, if the trainer indicates that this is a negative example, the G boundary can then be specialized. Either way, the learner will succeed in learning more about the true identity of the target concept, shrinking the version space from six hypotheses to half this number.

How Can Partially Learned Concepts Be Used?

Instance A was not among the training examples, it is classified as a positive instance by ***every*** hypothesis in the current version space .Because the hypotheses in the version space unanimously agree that this is a positive instance, the learner can classify instance A as positive with the same confidence it would have if it had already converged to the single ,correct target concept. Regardless of which hypothesis in the version space is eventually found to be the correct target concept, it is already clear that it will classify instance A as a positive example. Notice furthermore that we need not enumerate every hypothesis in the version space in order to test whether each classifies the instance as positive. This condition will be met if and only if the instance satisfies every member of S (why?). The reason is that every other hypothesis in theversion space is at least as general as some member of S. By our definition of ***more-general_thani***f the new instance satisfies **all** members of S it must also satisfy each of these more general hypotheses.

- Similarly, instance B is classified as a negative instance by every hypothesis in the version space. This instance can therefore be safely classified as negative, given the partially learned concept. An efficient test for this condition is that the instance satisfies none of the members of G.

Table 2: New Instances to be classified

Instance	Sky	AirTemp	Humidity	Wind	Water	Forecast	Enjoy Sport
A	Sunny	Warm	Normal	Strong	Cool	Change	?
B	Rainy	Cold	Normal	Light	Warm	Same	?
C	Sunny	Warm	Normal	Light	Warm	Same	?
D	Sunny	Cold	Normal	Strong	Warm	Same	?

- Instance C presents a different situation. Half of the version space hypotheses classify it as positive and half classify it as negative. Thus, the learner cannot classify this example with confidence until further training examples are available. Notice that instance C is the same instance presented in the previous section as an optimal experimental query for the learner.
- Finally, instance D is classified as positive by two of the version space hypotheses and negative by the other four hypotheses. In this case we have less confidence in the classification than in the unambiguous cases of instances A and B. Still, the vote is in favor of a negative classification, and one approach we could take would be to output the majority vote, perhaps with a confidence rating indicating how close the vote was.

INDUCTIVE BIAS : A Biased Hypothesis Space

Suppose we wish to assure that the hypothesis space contains the unknown target concept. The obvious solution is to enrich the hypothesis space to include ***every possible*** hypothesis. To illustrate, consider again the ***EnjoySport*** example in which we restricted the hypothesis space to include only conjunctions of attribute values.

Because of this restriction, the hypothesis space is unable to represent even simple disjunctive target concepts such as ***"Sky*** = ***Sunny*** or ***Sky*** = ***Cloudy."***

In fact, given the following three training examples of this disjunctive hypothesis, our algorithm would find that there are zero hypotheses in the version space To see why there are no hypotheses consistent with these three examples,note that the most specific hypothesis consistent with the first two examples *and representable in the given hypothesis space H* is **S2 : (?, Warm, Normal, Strong, Cool, Change)**

Example	**Sky**	**AirTemp**	**Humidity**	**Wind**	**Water**	**Forecast**	**Enjoy Sport**
1	Sunny	Warm	Normal	Strong	Cool	Change	Yes
2	Cloudy	Warm	Normal	Strong	Cool	Change	Yes
3	Rainy	Warm	Normal	Strong	Cool	Change	Yes

This hypothesis, although it is the maximally specific hypothesis from H that is consistent with the first two examples, is already overly general: it incorrectly covers the third (negative) training example. The problem is that we have biased the learner to consider only conjunctive hypotheses. In this case we require a more expressive hypothesis space.

An Unbiased Learner

In the ***EnjoySport*** learning task, for example, the size of the instance space X of days described by the six available attributes is 96. How many possible concepts can be defined overthis set of instances? In other words, how large is the power set of X? In general, the number of distinct subsets that can be defined over a set X containing 1x1 elements (i.e., the size of the power set of X) is 21'1.

Thus, there are **296,** or approximately distinct target concepts that could be defined over this instance space and that our learner might be called upon to learn.

Let us reformulate the ***Enjoysport*** learning task in an unbiased way by defining a new hypothesis space ***H'*** that can represent every subset of instances; that is, let H' correspond to the power set of X. One way to define such an H' is to allow arbitrary disjunctions, conjunctions, and negations of our earlier hypotheses.

For instance, the target concept ***"Sky*** = ***Sunny*** or ***Sky*** = ***Cloudy"*** could then be described as ***(Sunny,*** ?, ?, ?, ?, ?) v ***(Cloudy,*** ?, ?, ?, ?, ?)

Given this hypothesis space, we can safely use the CANDIDATE-ELIMINATION algorithm without worrying that the target concept might not be expressible. However, while this hypothesis space eliminates any problems of expressibility, it unfortunately raises a new, equally difficult problem: our concept learning algorithm is now completely unable to generalize beyondthe observed examples! To see why, suppose we present three positive examples **(xl , x2, x3)** and two negative examples **(x4,** x5) to the learner. At this point, the ***S*** boundary of the version space will contain the hypothesis which is just the disjunction of the positive examples because this is the most specific possible hypothesis.

$$\mathbf{S:\{(x_1 \vee x_2 \vee x_3)\}}$$

because this is the most specific possible hypothesis that covers these three examples. Similarly, the G boundary will consist of the hypothesis that rules out only the observed negative examples

$$\mathbf{G : \{ \neg (x_4 \vee x_5)\}}$$

It might at first seem that we could avoid this difficulty by simply using the partially learned version space and by taking a vote among the members of the version space

To see the reason, note that when H is the power set of X and **x** is some previously unobserved instance, then for any hypothesis h in the version space that covers **x,** there will be another hypothesis *h'* in the power set that is identical to h except for its classification of x. And of courseif *h* is in the version space, then h' will be as well, because it agrees with *h* on all the observed training examples.

The Futility of Bias-Free Learning

Because inductive learning requires some form of prior assumptions, or inductive bias, we will find it useful to characterize different learning approaches by the inductive bias they employ. Let us define this notion of inductive bias more precisely. The key idea we wish to capture here is the policy by which the learner generalizes beyond the observed training data, to infer the classification of new instances. Therefore, consider the general setting in which an arbitrary learning algorithm L is provided an arbitrary set of training data D, = {(x, c(x))} of some arbitrary target concept c. After training, L is asked to classify a new instance xi. Let L(xi, D,) denote the classification (e.g., positive or negative) that L assigns to xi after learning from the training data D,. We can describe this inductive inference step performed by L as follows

$$\mathbf{(D_c \wedge x_i) > L(x_i, Dc)}$$

where the notation *y* > z indicates that z is inductively inferred from *y*. For example, if we take Lto be the CANDIDATE-ELIMINATION Algorithm, D, to be the training data, and x_i to be the first instance from Table 2.6,then the inductive inference performed in this case concludes that

L(x_i, Dc) =*(EnjoySport = yes)*

Because L is an inductive learning algorithm, the result L(xi, D,) that it infers will not in general be provably correct; that is, the classification L(xi, D,) need not follow deductively from the training data D, and the description of the new instance xi. However, it is interesting to ask what additional assumptions could be added to D, r\xi so that L(xi, D,) would follow deductively. We define the inductive bias of ***L*** as this set of additional assumptions. More precisely, we define the inductive bias of ***L*** to be the set of assumptions ***B*** such that for all new instances

xi (B A D, A xi) ├ L(xi, D,)

where the notation ***y*** ├ *z* indicates that *z* follows deductively from y (i.e., that z is provable from ***y).*** Thus, we define the inductive bias of a learner as the set of additional assumptions B sufficient to justify its inductive inferences as deductive inferences.

Definition: Consider a concept learning algorithm ***L*** for the set of instances X. Let c be an arbitrary concept defined over X, and let ***D,*** = ***((x , c(x))}*** be an arbitrary set of training examplesof c. Let ***L(xi, Dc)*** denote the classification assigned to the instance ***xi*** by ***L*** after training on the data ***Dc.*** The **inductive bias** of ***L*** is any minimal set of assertions ***B*** such that for any target concept c and corresponding training examples ***Dc***

$$(\forall x_i \in X)[(B \wedge D_c \wedge x_i) \vdash L(x_i, D_c)]$$

What, then, is the inductive bias of the CANDIDATE-ELIMINATION algorithm? To answer this, let us specify L(xi, D,) exactly for this algorithm: given a set of data D, the CANDIDATE-ELIMINATION algorithm will first compute the version space VSH,D, then classify the new instance xi by a vote among hypotheses in this version space. Here let us assume that it will output a classification for xi only if this vote among version space hypotheses is unanimously positive or negative and that it will not output a classification otherwise. Given this definition of L(xi, D,) for the CANDIDATE-ELIMINATION algorithm, what is its inductive bias? It is simply the assumption c E H. Given this assumption, each inductive inference performed by the CANDIDATE-ELIMINATION algorithm can be justified deductively.

To see why the classification L(xi, D,) follows deductively from B = {c ϵ H), together with the data D, and description of the instance xi, consider the fol lowing argument. First, notice that if we assume c E H then it follows deductively that c ϵ VSH,Dc. This follows from c E H, from the definition of the version space VSH,D, as the set of all hypotheses in H that are consistent with D,, and from our definition of D, = {(x, c(x))} as training data consistent with the target concept c. Second, recall that we defined the classification L(xi, D,) to be the unanimous vote of all hypotheses in the version space. Thus, if L outputs the classification L(x,, D,), it must be the case the every hypothesis in VSH,~, also produces this classification, including the hypothesis c ϵ VSHYDc. Therefore c(xi) = L(xi, D,). To summarize, the

CANDIDATE-ELIMINATION algorithm defined in this fashion can be characterized by the following bias Inductive bias of CANDIDATE-ELIMINATION algorithm. The target concept c is contained in the given hypothesis space H.

Figure summarizes the situation schematically. The inductive CANDIDATE ELIMINATION algorithm at the top of the figure takes two inputs: the training examples and a new instance to be classified. At the bottom of the figure, a deductive theorem prover is given these same two inputs plus the assertion "H contains the target concept." These two systems will in principle produce identical outputs for every possible input set of training examples and every possible new instance in X. Of course the inductive bias that is explicitly input to the theorem prover is only implicit in the code of the CANDIDATE-ELIMINATION algorithm. In a sense, it exists only in the eye of us beholders. Nevertheless, it is a perfectly well-defined set of assertions. One advantage of viewing inductive inference systems in terms of their inductive bias is that it provides a nonprocedural means of characterizing their policy for generalizing beyond the observed data. A second advantage is that it allows comparison of different learners according to the strength of the inductive bias they employ.

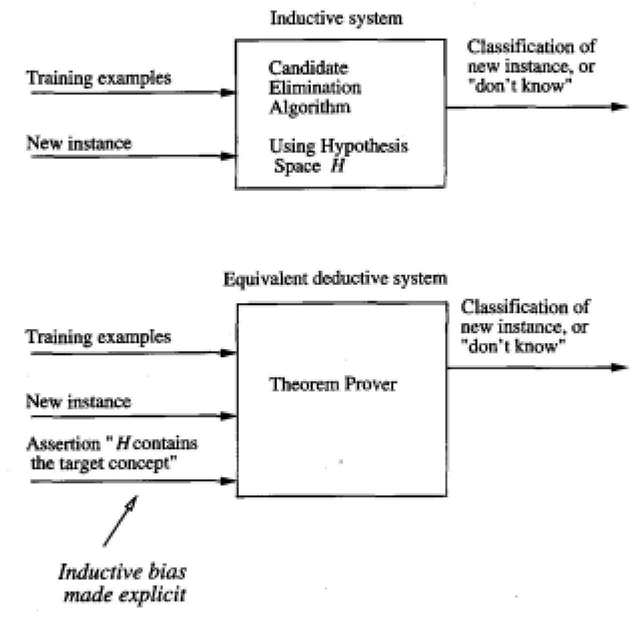

Inductive system
Training examples
New instance
Candidate Elimination Algorithm
Using Hypothesis Space H
Classification of new instance, or "don't know"
Equivalent deductive system
Training examples
New instance
Assertion "H contains the target concept"
Theorem Prover
Classification of new instance, or "don't know"
Inductive bias made explicit

Figure 8 : Modeling inductive systems by equivalent deductive systems. The input-output behavior of the CANDIDATE-ELIMINATION algorithm using a hypothesis space H is identical to that of a deduc tive theorem prover utilizing the assertion "H contains the target concept." This assertion is therefore called the inductive bias of the CANDIDATE-ELIMINATION algorithm. Characterizing inductive systems by their inductive bias allows modeling them by their equivalent deductive systems. This provides a way to compare inductive systems according to their policies for generalizing beyond the observed training data.

Inductive Bias – Three Learning Algorithms

- **ROTE-LEARNER:** Learning corresponds simply to storing each observed training example in memory. Subsequent instances are classified by looking them up in memory. If the instance is found in memory, the stored classification is returned. Otherwise, the system refuses to classify the new instance. Inductive Bias: No inductive bias
- **CANDIDATE-ELIMINATION:** New instances are classified only in the case where all members of the current version space agree on the classification. Otherwise, the system refuses to classify the new instance. Inductive Bias: the target concept can be represented in its hypothesis space.
- **FIND-S:** This algorithm, described earlier, finds the most specific hypothesis consistent with the training examples. It then uses this hypothesis to classify all subsequent instances. Inductive Bias: the target concept can be represented in its hypothesis space, and all instances are negative instances unless the opposite is entailed by its other know1edge.

Concept Learning – Summary

• Concept learning can be seen as a problem of searching through a large predefined space of potential hypotheses.

• The general-to-specific partial ordering of hypotheses provides a useful structure for organizing the search through the hypothesis space.

• The FIND-S algorithm utilizes this general-to-specific ordering, performing a specific-to-general search through the hypothesis space along one branch of the partial ordering, to find the most specific hypothesis consistent with the training examples.

• The CANDIDATE-ELIMINATION algorithm utilizes this general-tospecific ordering to compute the version space (the set of all hypotheses consistent with the training data) by incrementally computing the sets of maximally specific (S) and maximally general (G) hypotheses. Because the S and G sets delimit the entire set of hypotheses consistent with the data, they provide the learner with a description of its uncertainty regarding the exact identity of the target concept.

This version space of alternative hypotheses can be examined

– to determine whether the learner has converged to the target concept,

– to determine when the training data are inconsistent, – to generate informative queries to further refine the version space, and

– to determine which unseen instances can be unambiguously classified based on the partially learned concept.

• The CANDIDATE-ELIMINATION algorithm is not robust to noisy data or to situations in which the unknown target concept is not expressible in the provided hypothesis space. Inductive learning algorithms are able to classify unseen examples only because of their implicit inductive bias for selecting one consistent hypothesis over another.

• If the hypothesis space is enriched to the point where there is a hypothesis corresponding to every possible subset of instances (the power set of the instances), this will remove any inductive bias from the CANDIDATE-ELIMINATION algorithm.

– Unfortunately, this also removes the ability to classify any instance beyond the observed training examples.

– An unbiased learner cannot make inductive leaps to classify unseen examples.