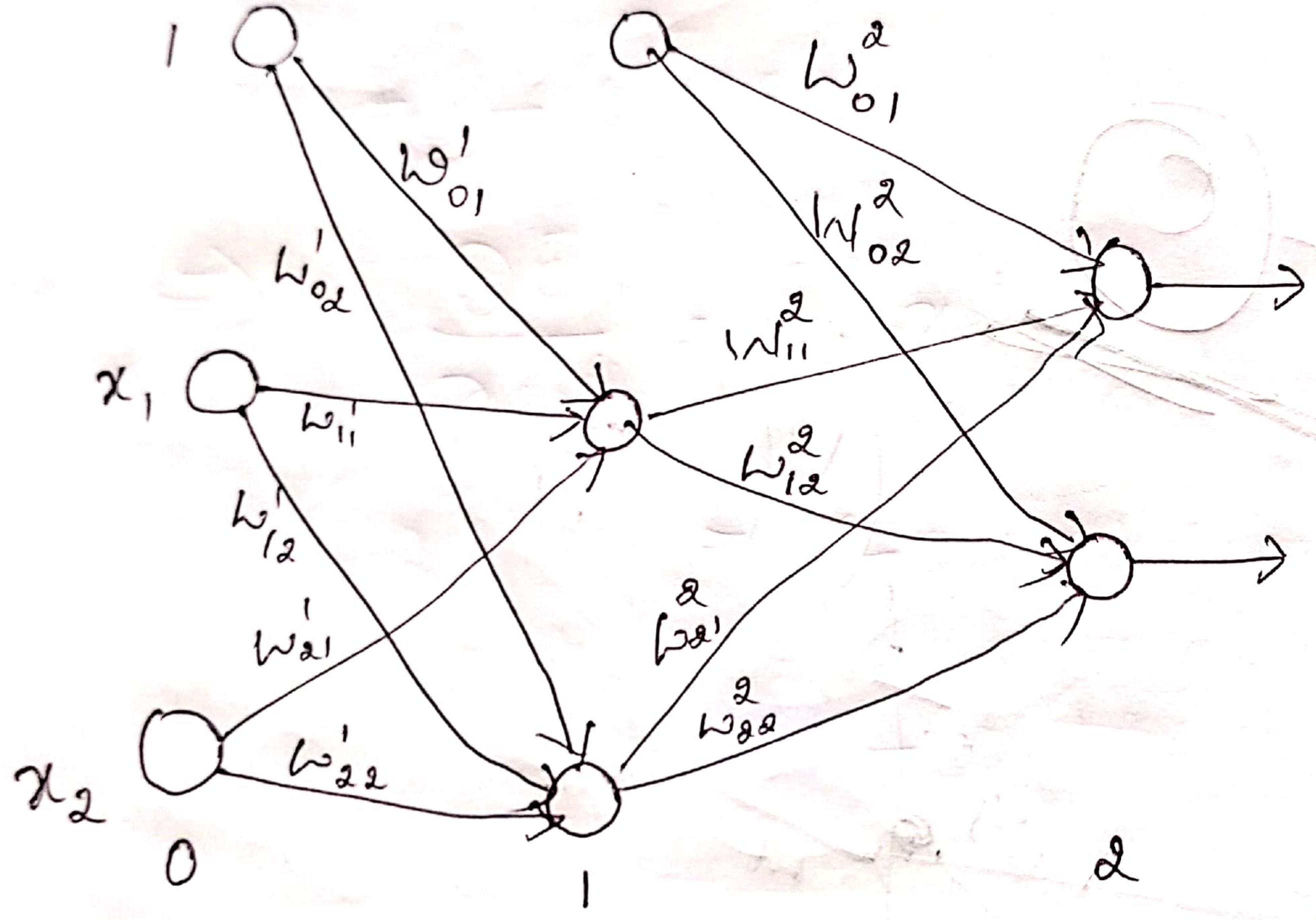
Backpropagation Learning Example-II

Feed Forward Pass



$$W^1 \qquad x_i^0 \qquad O_j^1 = \sum w_{ij}^1 x_i^0 \qquad x_j^1 = \frac{1}{1+e^{-O_j^1}}$$

$$\begin{bmatrix} 0.5 & 1.5 & 0.8 \\ 0.8 & 0.2 & -1.6 \end{bmatrix} \begin{bmatrix} 1 \\ 0.7 \\ 1.2 \end{bmatrix} = \begin{bmatrix} 2.31 \\ -9.8 \end{bmatrix} \Rightarrow \begin{bmatrix} 0.92 \\ 0.27 \end{bmatrix}$$

$$x_i^1 \qquad O_j^2 = \sum_i w_{ij}^2 x_i^1 \qquad x_j^2 = \frac{1}{1+e^{-O_j^2}}$$

$$W^2$$

$$\begin{bmatrix} 0.9 & -1.7 & 1.6 \\ 1.2 & 2.1 & -0.20 \end{bmatrix} \begin{bmatrix} 1 \\ 0.92 \\ 0.27 \end{bmatrix} = \begin{bmatrix} -0.232 \\ 3.057 \end{bmatrix} \Rightarrow \begin{bmatrix} 0.44 \\ 0.95 \end{bmatrix}$$

$$t = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Backpropagation Learning: Output layer

$$E = \frac{1}{2} \sum_{j=1}^{2} (x_j^2 - t_j)^2 \qquad x_j^2 = \frac{1}{1+e^{-O_j^2}} \qquad O_j^2 = \sum_{i=0}^{2} w_{ij}^2 x_i^1$$

$$\frac{\partial E}{\partial w_{ij}^2} = \frac{\partial E}{\partial x_j^2} \cdot \frac{\partial x_j^2}{\partial O_j^2} \cdot \frac{\partial O_j^2}{\partial w_{ij}^2} = (x_j^2 - t_j)\, x_j^2 (1 - x_j^2)\, x_i^1$$

We set $\delta_j^2 = x_j^2 (1 - x_j^2)(x_j^2 - t_j)$

$$\Rightarrow \frac{\partial E}{\partial w_{ij}^2} = \delta_j^2 x_i^1$$

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$$\delta_j^2 = x_j^2 (1 - x_j^2)(x_j^2 - t_j)$$

$$\delta_1^2 = x_1^2 (1 - x_1^2)(x_1^2 - t_1)$$

$$= 0.44(1 - 0.44) * (0.44 - 1)$$

$$= -0.138$$

$$\Rightarrow \frac{\partial E}{\partial w_{11}^2} = \delta_1^2 x_1^1$$

$$= (-0.138)(0.92)$$

$$= -0.126$$

$$\boxed{w_{11}^2 \leftarrow w_{11}^2 - \eta(-0.126)}$$

increased

$$\delta_2^2 = x_2^2(1 - x_2^2)(x_2^2 - t_2)$$

$$= 0.95(1-0.95) * (0.95-0)$$

$$= \underline{0.045}$$

$$\Rightarrow \frac{\partial E}{\partial w_{12}^2} = \delta_2^2 x_1^1 = 0.04$$

$$\boxed{W_{12}^2 \leftarrow w_{12}^2 - \eta * 0.04}$$

Decreased

III^{ly} $\frac{\partial E}{\partial w_{21}^1} = \delta_1^2 x_2^1 = \underline{0.037}$

$$\frac{\partial E}{\partial w_{22}^1} = \delta_2^2 x_2^1 = \underline{0.012}$$

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$$\frac{\partial E}{\partial w_{01}^{2}} = \delta_1^2 x_0^1 = -1.38$$

$$\frac{\partial E}{\partial w_{02}^{2}} = \delta_2^2 x_0^1 = 0.045$$

Hidden Layer

We set

Backpropagated Error term

$$\delta_i^{k} = x_i^{k}(1-x_i^{k}) \sum_{j=1}^{M_{k+1}} \delta_j^{k+1} w_{ij}^{k+1}$$

$$\Downarrow$$

$$\delta_i^1 = x_i^1(1-x_i^1) \sum_{j=1}^{2} \delta_j^2 w_{ij}^2$$

$$\boxed{\frac{\partial E}{\partial w_{ij}^{k}} = \delta_j^{k} x_i^{k-1}}$$

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$$\delta_1^1 = x_1^1 (1 - x_1^1) [\delta_1^2 * w_{11}^2 + \delta_2^2 w_{12}^2]$$

$$= 0.92 * (1 - 0.92) [(-0.137 * (-1.7)) + 0.045 * (2.1)]$$

$$= 0.024$$

$$\delta_2^1 = x_2^1 (1 - x_2^1) [\delta_1^2 * w_{21}^2 + \delta_2^2 w_{22}^2]$$

$$= 0.27 * (1 - 0.27) [(-0.137) * 1.6 + (0.045) * (-0.2)]$$

$$= -0.02$$

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$$\frac{\partial E}{\partial w_{11}^1} = \delta_1^1 * x_1^0$$

$$= 0.024 \times 0.7$$

$$= 0.017$$

$$\frac{\partial E}{\partial w_{12}^1} = \delta_2^1 * x_1^0$$

$$= -0.02 * 0.7$$

$$= -0.014$$

$$\frac{\partial E}{\partial w_{21}^1} = \delta_1^1 * x_2^0$$

$$= (0.024) \times 1.2$$

$$= 0.0288$$

$$\frac{\partial E}{\partial w_{22}^{1}} = \delta_2^{1} \times x_2^{0}$$

$$= (-0.02) \times (1.2)$$

$$= -0.024$$

$$\frac{\partial E}{\partial w_{01}^{1}} = \delta_1^{1} \times x_0^{0}$$

$$= 0.024 \times 1$$

$$= 0.024$$

$$\frac{\partial E}{\partial w_{02}^{1}} = \delta_2^{1} \times x_0^{0}$$

$$= -0.02 \times 1$$

$$= -0.02$$

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$$\boxed{w_{ij}^{1} \leftarrow w_{ij}^{1} - \eta \frac{\partial E}{\partial w_{ij}^{1}}}$$

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