Module – 2

Decision Trees

Prof. Mahesh G Huddar

Dept. of Computer Science and Engineering

3.1 Give decision trees to represent the following boolean functions:

(a) $A \wedge \neg B$

(b) $A \vee [B \wedge C]$

(c) A XOR B

(d) $[A \wedge B] \vee [C \wedge D]$

Decision Tree for Boolean Functions

(a) $A \wedge \neg B$

(b) $A \vee [B \wedge C]$

(c) A XOR B

(d) $[A \wedge B] \vee [C \wedge D]$

Decision Tree for Boolean Functions

- Every Variable in Boolean function such as A, B, C etc. has two possibilities that is True and False
- Every Boolean function is either True or False
- If the Boolean function is true we write YES (Y)
- If the Boolean function is False we write NO (N)

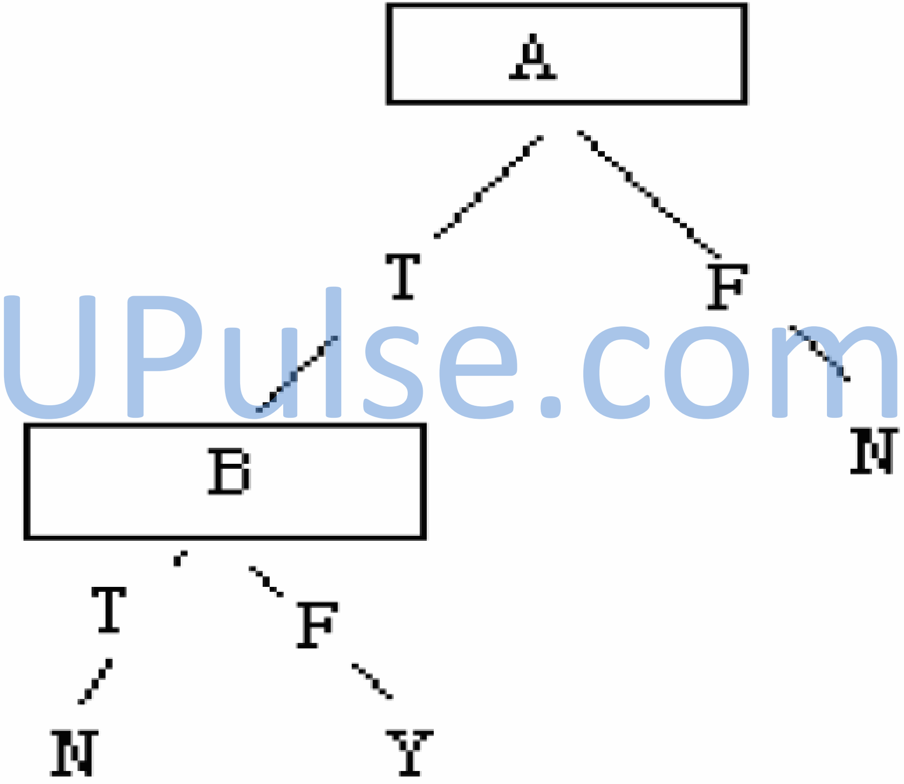
Decision Tree for Boolean Functions

(c) A XOR B

Decision Tree for Boolean Functions

(d) $[A \wedge B] \vee [C \wedge D]$

(a) $A \wedge \neg B$



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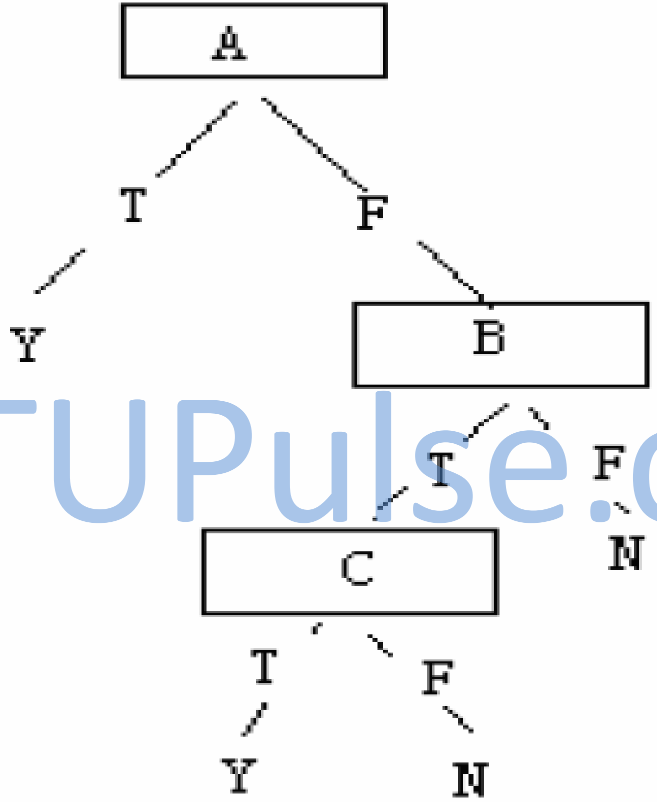

A
T
F
Y
B
T
F
N
C
T
F
Y
N

3.1 Give decision trees to represent the following boolean functions:

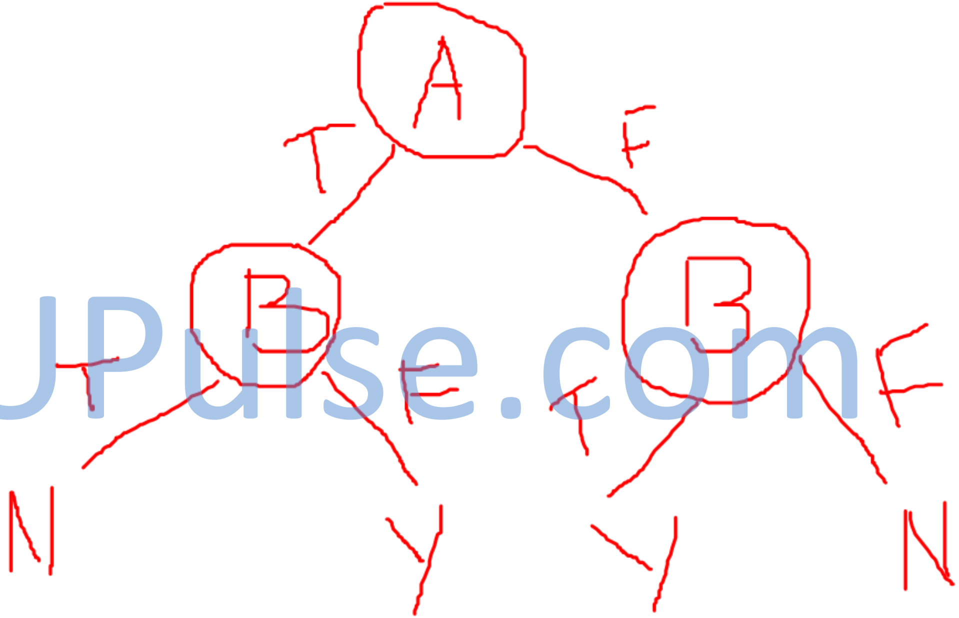
(a) $A \wedge \neg B$

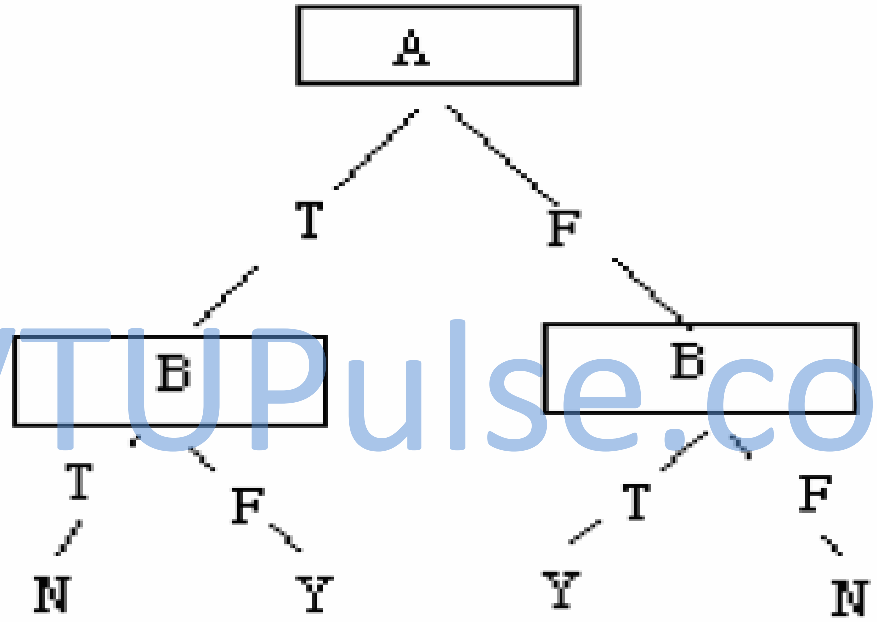
(b) $A \vee [B \wedge C]$

(c) A XOR B

(d) $[A \wedge B] \vee [C \wedge D]$

$(A \wedge \neg B) \vee (\neg A \wedge B)$




A
T
F
B
B
T
F
T
F
N
Y
Y
N

3.1 Give decision trees to represent the following boolean functions:

(a) $A \wedge \neg B$

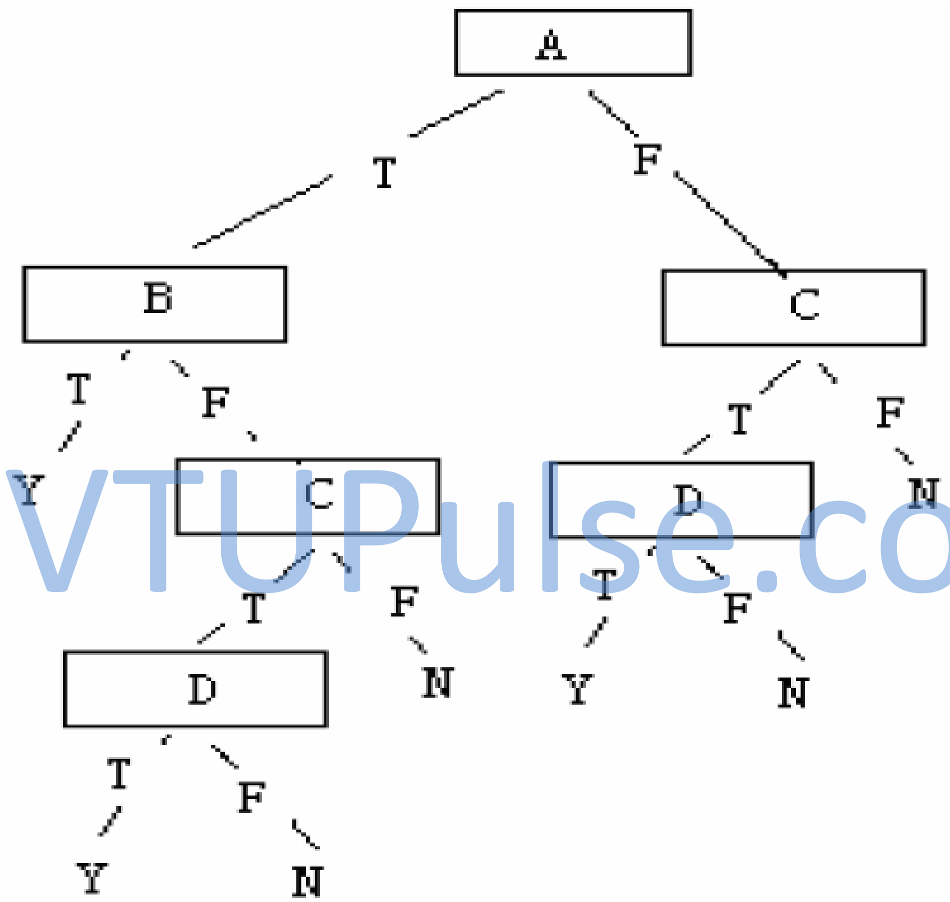
(b) $A \vee [B \wedge C]$

(c) $A \text{ XOR } B$

(d) $[A \wedge B] \vee [C \wedge D]$

$(A=T \wedge B=T)$

$\Rightarrow T$


A
T
F
B
C
T
F
T
F
Y
C
D
N
T
F
T
F
D
N
Y
N
T
F
Y
N

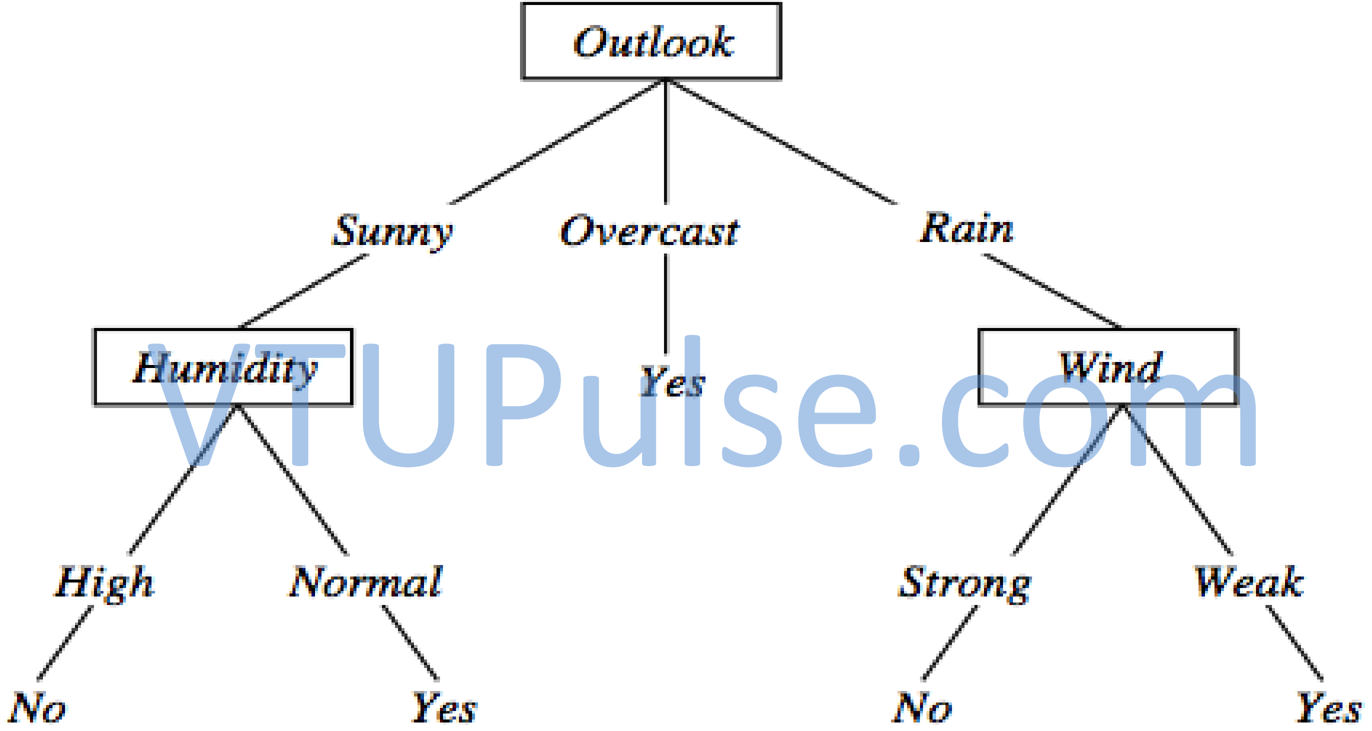
Decision Trees

- *Decision Trees* is one of the most widely used Classification Algorithm
- **Features**
  - Method for approximating *discrete-valued* functions (including boolean)
  - Learned functions are represented as *decision trees* (or *if-then-else* rules*)*
  - Expressive hypotheses space, including disjunction
  - Robust to noisy data

Example

Day	*Outlook*	*Temperature*	*Humidity*	*Wind*	*PlayTennis*
D1	Sunny	Hot	High	Weak	No
D2	Sunny	Hot	High	Strong	No
D3	Overcast	Hot	High	Weak	Yes
D4	Rain	Mild	High	Weak	Yes
D5	Rain	Cool	Normal	Weak	Yes
D6	Rain	Cool	Normal	Strong	No
D7	Overcast	Cool	Normal	Strong	Yes
D8	Sunny	Mild	High	Weak	No
D9	Sunny	Cool	Normal	Weak	Yes
D10	Rain	Mild	Normal	Weak	Yes
D11	Sunny	Mild	Normal	Strong	Yes
D12	Overcast	Mild	High	Strong	Yes
D13	Overcast	Hot	Normal	Weak	Yes
D14	Rain	Mild	High	Strong	No

Decision Tree Representation (PlayTennis)

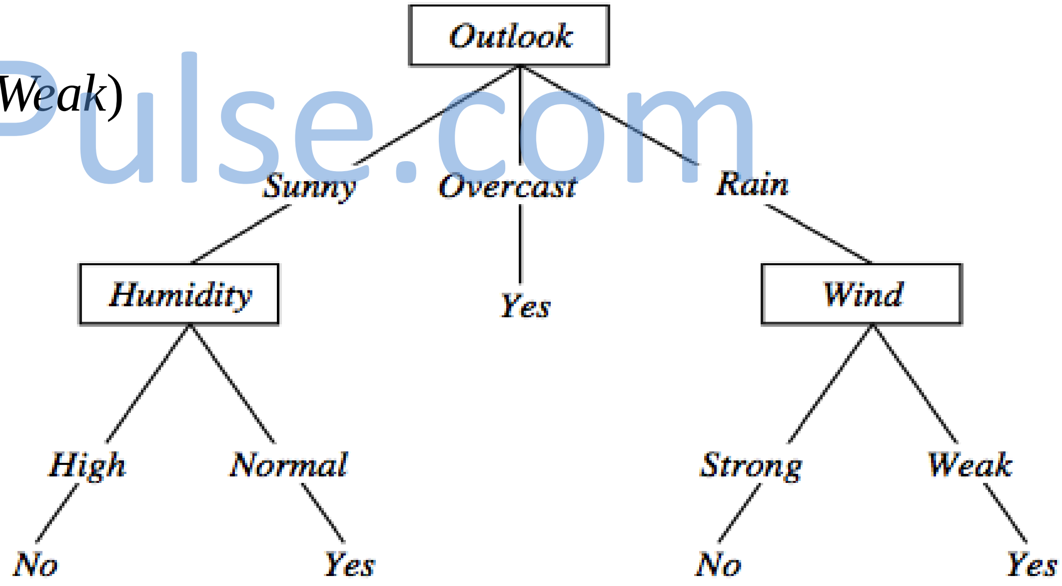


⟨*Outlook=Sunny, Temp=Hot, Humidity=High, Wind=Strong*⟩ No

Decision trees expressivity

- Decision trees represent a disjunction of conjunctions on constraints on the value of attributes:

$$(Outlook = Sunny \wedge Humidity = Normal) \vee$$
$$(Outlook = Overcast) \vee$$
$$(Outlook = Rain \wedge Wind = Weak)$$



Decision tree representation (PlayTennis)

- Decision trees classify instances by sorting them down the tree from the root to some leaf node, which provides the classification of the instance.
- Each node in the tree specifies a test of some attribute of the instance, and each branch descending from that node corresponds to one of the possible values for this attribute.
- An instance is classified by starting at the root node of the tree, testing the attribute specified by this node, then moving down the tree branch corresponding to the value of the attribute in the given example.
- This process is then repeated for the subtree rooted at the new node.

Decision tree representation (PlayTennis)

- In general, decision trees represent a disjunction of conjunctions of constraints on the attribute values of instances.
- Each path from the tree root to a leaf corresponds to a conjunction of attribute tests, and the tree itself to a disjunction of these conjunctions.

$$
\begin{aligned}
& (Outlook = Sunny \ \wedge \ Humidity = Normal) \\
\vee \quad & (Outlook = Overcast) \\
\vee \quad & (Outlook = Rain \ \wedge \ Wind = Weak)
\end{aligned}
$$

Example

Day	*Outlook*	*Temperature*	*Humidity*	*Wind*	*PlayTennis*
D1	Sunny	Hot	High	Weak	No
D2	Sunny	Hot	High	Strong	No
D3	Overcast	Hot	High	Weak	Yes
D4	Rain	Mild	High	Weak	Yes
D5	Rain	Cool	Normal	Weak	Yes
D6	Rain	Cool	Normal	Strong	No
D7	Overcast	Cool	Normal	Strong	Yes
D8	Sunny	Mild	High	Weak	No
D9	Sunny	Cool	Normal	Weak	Yes
D10	Rain	Mild	Normal	Weak	Yes
D11	Sunny	Mild	Normal	Strong	Yes
D12	Overcast	Mild	High	Strong	Yes
D13	Overcast	Hot	Normal	Weak	Yes
D14	Rain	Mild	High	Strong	No

APPROPRIATE PROBLEMS FOR DECISION TREE LEARNING

Although a variety of decision tree learning methods have been developed with somewhat differing capabilities and requirements, decision tree learning is generally best suited to problems with the following characteristics:

1. ***Instances are represented by attribute-value pairs.*** Instances are described by a fixed set of attributes (e.g., ***Temperature***) and their values (e.g., ***Hot***). The easiest situation for decision tree learning is when each attribute takes on a small number of disjoint possible values (e.g., ***Hot, Mild, Cold***). However, extensions to the basic algorithm allow handling real-valued attributes as well (e.g., representing ***Temperature*** numerically).

APPROPRIATE PROBLEMS FOR DECISION TREE LEARNING

2. ***The target function has discrete output values.*** The decision tree is usually used for Boolean classification (e.g., ***yes*** or ***no)*** kind of example. Decision tree methods easily extend to learning functions with more than two possible output values. A more substantial extension allows learning target functions with real-valued outputs, though the application of decision trees in this setting is less common.

3. ***Disjunctive descriptions may be required.*** Decision trees naturally represent disjunctive expressions.

APPROPRIATE PROBLEMS FOR DECISION TREE LEARNING

4. ***The training data may contain errors.*** Decision tree learning methods are robust to errors, both errors in classifications of the training examples and errors in the attribute values that describe these examples.

5. ***The training data may contain missing attribute values.*** Decision tree methods can be used even when some training examples have unknown values (e.g., if the ***Humidity*** of the day is known for only some of the training examples).

APPROPRIATE PROBLEMS FOR DECISION TREE LEARNING

- Many practical problems have been found to fit these characteristics.
- Decision tree learning has therefore been applied to problems such as learning to classify medical patients by their disease, equipment malfunctions by their cause, and loan applicants by their likelihood of defaulting on payments.
- Such problems, in which the task is to classify examples into one of a discrete set of possible categories, are often referred to as ***classification problems.***

THE BASIC DECISION TREE LEARNING ALGORITHM

- Most algorithms that have been developed for learning decision trees are variations on a core algorithm that employs a top-down, greedy search through the space of possible decision trees.
- This approach is exemplified by the ID3 algorithm (Quinlan 1986) and its successor C4.5 (Quinlan 1993), which form the primary focus of our discussion here.
- The basic algorithm for decision tree learning, corresponding approximately to the ID3 algorithm.
- Next, we consider a number of extensions to this basic algorithm, including extensions incorporated into C4.5 and other more recent algorithms for decision tree learning.

Example

Day	*Outlook*	*Temperature*	*Humidity*	*Wind*	*PlayTennis*
D1	Sunny	Hot	High	Weak	No
D2	Sunny	Hot	High	Strong	No
D3	Overcast	Hot	High	Weak	Yes
D4	Rain	Mild	High	Weak	Yes
D5	Rain	Cool	Normal	Weak	Yes
D6	Rain	Cool	Normal	Strong	No
D7	Overcast	Cool	Normal	Strong	Yes
D8	Sunny	Mild	High	Weak	No
D9	Sunny	Cool	Normal	Weak	Yes
D10	Rain	Mild	Normal	Weak	Yes
D11	Sunny	Mild	Normal	Strong	Yes
D12	Overcast	Mild	High	Strong	Yes
D13	Overcast	Hot	Normal	Weak	Yes
D14	Rain	Mild	High	Strong	No

CONSTRUCTING DECISION TREE – ID3 ALGORITHM

Which Attribute Is the Best Classifier?

- The central choice in the ID3 algorithm is selecting which attribute to test at each node in the tree.
- We would like to select the attribute that is most useful for classifying examples.
- What is a good quantitative measure of the worth of an attribute? We will define a statistical property, called ***information gain,*** that measures how well a given attribute separates the training examples according to their target classification.
- ID3 uses this information gain measure to select among the candidate attributes at each step while growing the tree.

CONSTRUCTING DECISION TREE – ID3 ALGORITHM

ENTROPY MEASURES HOMOGENEITY OF EXAMPLES

- ***Entropy,*** characterizes the (im)purity of an arbitrary collection of examples.
- Given a collection S, containing positive and negative examples of some target concept, the entropy of S relative to this boolean classification is

$$Entropy(S) \equiv -p_{\oplus} \log_2 p_{\oplus} - p_{\ominus} \log_2 p_{\ominus}$$

- where p+, is the proportion of positive examples in S and p-, is the proportion of negative examples in S.
- In all calculations involving entropy we define 0 log 0 to be 0.

CONSTRUCTING DECISION TREE – ID3 ALGORITHM

ENTROPY MEASURES HOMOGENEITY OF EXAMPLES

- Entropy measures the *(im)purity* of a collection of examples. It depends from the distribution of the random variable *p*.
  - S is a collection of training examples
  - p_+ the proportion of positive examples in S
  - p_- the proportion of negative examples in S

$$Entropy(S) \equiv \sum_{i=1}^{c} -p_i \log_2 p_i$$

Examples

$Entropy\ (S) \equiv -p_+ \log_2 p_+ - p_- \log_2 p_- \quad [0 \log_2 0 = 0]$

$Entropy\ ([14+, 0-]) = -14/14 \log_2 (14/14) - 0 \log_2 (0) = 0$

$Entropy\ ([9+, 5-]) = -9/14 \log_2 (9/14) - 5/14 \log_2 (5/14) = 0{,}94$

$Entropy\ ([7+, 7-]) = -7/14 \log_2 (7/14) - 7/14 \log_2 (7/14) =$

$= 1/2 + 1/2 = 1$

Example

Day	*Outlook*	*Temperature*	*Humidity*	*Wind*	*PlayTennis*
D1	Sunny	Hot	High	Weak	No
D2	Sunny	Hot	High	Strong	No
D3	Overcast	Hot	High	Weak	Yes
D4	Rain	Mild	High	Weak	Yes
D5	Rain	Cool	Normal	Weak	Yes
D6	Rain	Cool	Normal	Strong	No
D7	Overcast	Cool	Normal	Strong	Yes
D8	Sunny	Mild	High	Weak	No
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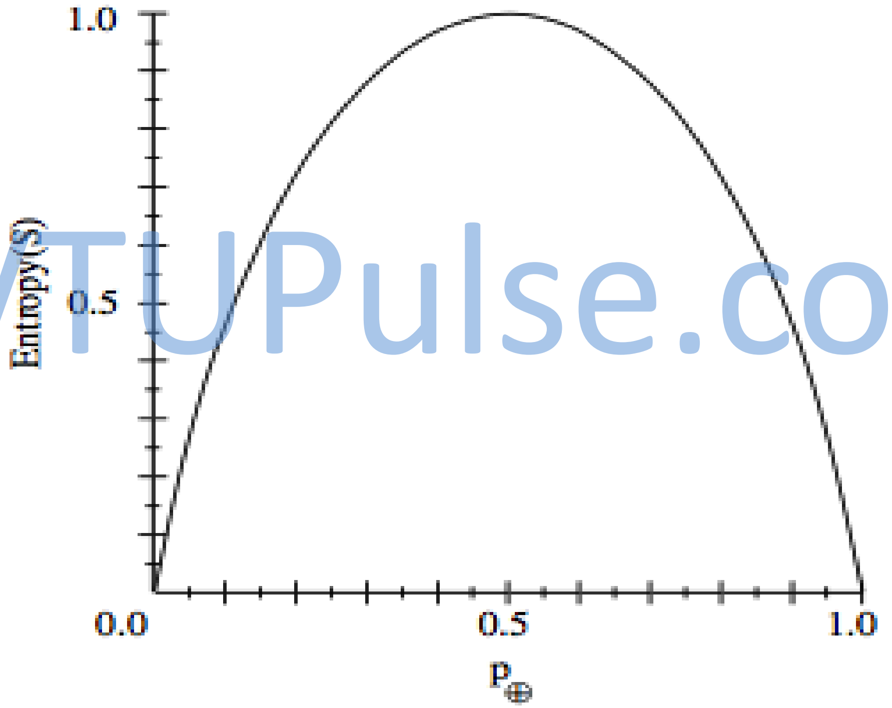
D10	Rain	Mild	Normal	Weak	Yes
D11	Sunny	Mild	Normal	Strong	Yes
D12	Overcast	Mild	High	Strong	Yes
D13	Overcast	Hot	Normal	Weak	Yes
D14	Rain	Mild	High	Strong	No

CONSTRUCTING DECISION TREE – ID3 ALGORITHM

Entropy



CONSTRUCTING DECISION TREE – ID3 ALGORITHM

INFORMATION GAIN MEASURES THE EXPECTED REDUCTION IN ENTROPY

- Given entropy as a measure of the impurity in a collection of training examples, we can now define a measure of the effectiveness of an attribute in classifying the training data.
- Now, the ***information gain,*** is simply the expected reduction in entropy caused by partitioning the examples according to this attribute.
- More precisely, the information gain, ***Gain(S, A)*** of ***an*** attribute **A,** relative to a collection of examples ***S,*** is defined as,

$$Gain(S, A) \equiv Entropy(S) - \sum_{v \in Values(A)} \frac{|S_v|}{|S|} Entropy(S_v)$$

- where ***Values(A)*** is the set of all possible values for attribute A, and ***S,*** is the subset of S for which attribute A has value v (i.e., $S_v = \{\boldsymbol{s} \in \boldsymbol{S} | \boldsymbol{A}(\boldsymbol{s}) = \boldsymbol{v}\}$)

CONSTRUCTING DECISION TREE – ID3 ALGORITHM

- For example, suppose ***S*** is a collection of training-example days described by attributes including ***Wind,*** which can have the values ***Weak*** or ***Strong.***

$$Values(Wind) = Weak, Strong$$

$$S = [9+, 5-]$$

$$S_{Weak} \leftarrow [6+, 2-]$$

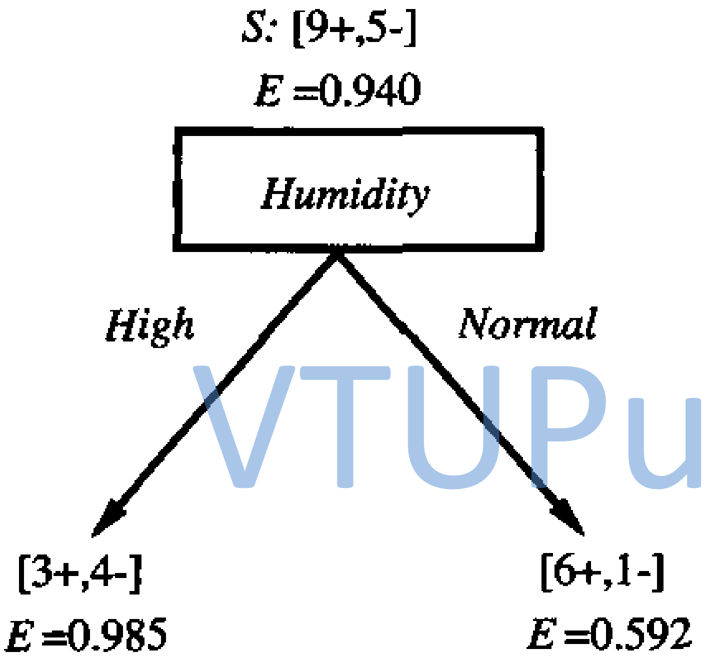
$$S_{Strong} \leftarrow [3+, 3-]$$

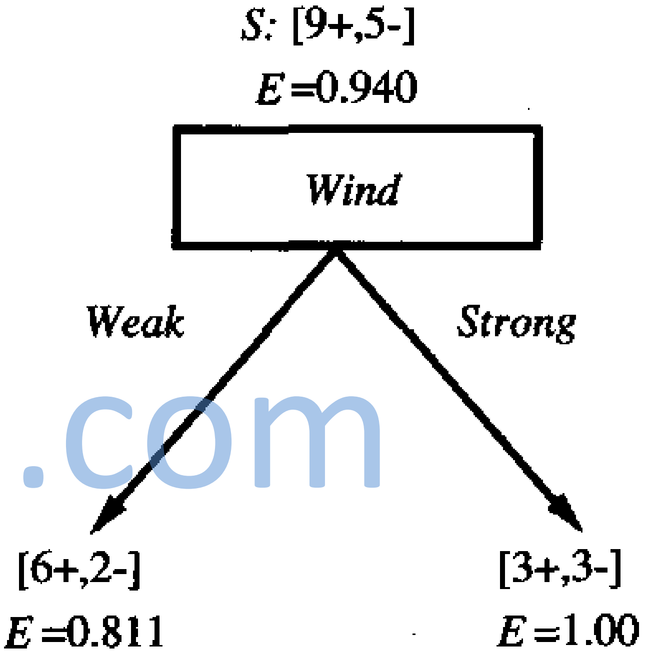
$$\begin{aligned} Gain(S, Wind) &= Entropy(S) - \sum_{v \in \{Weak, Strong\}} \frac{|S_v|}{|S|} Entropy(S_v) \\ &= Entropy(S) - (8/14) Entropy(S_{Weak}) \\ &\quad - (6/14) Entropy(S_{Strong}) \\ &= 0.940 - (8/14)0.811 - (6/14)1.00 \\ &= 0.048 \end{aligned}$$

CONSTRUCTING DECISION TREE – ID3 ALGORITHM

- Information gain is precisely the measure used by ID3 to select the best attribute at
- each step in growing the tree.
- The use of information gain to evaluate the relevance of attributes**.**
- Here the information gain of two different attributes, ***Humidity*** and ***Wind,*** is computed in order to determine which is the better attribute for classifying the training examples**.**

CONSTRUCTING DECISION TREE – ID3 ALGORITHM





Gain (S, Humidity)

$$= .940 - (7/14).985 - (7/14).592$$
$$= .151$$

Gain (S, Wind)

$$= .940 - (8/14).811 - (6/14)1.0$$
$$= .048$$

ID3(*Examples*, *Target_attribute*, *Attributes*)

Examples are the training examples. Target_attribute is the attribute whose value is to be predicted by the tree. Attributes is a list of other attributes that may be tested by the learned decision tree. Returns a decision tree that correctly classifies the given Examples.

- Create a *Root* node for the tree
- If all *Examples* are positive, Return the single-node tree *Root*, with label = +
- If all *Examples* are negative, Return the single-node tree *Root*, with label = −
- If *Attributes* is empty, Return the single-node tree *Root*, with label = most common value of *Target_attribute* in *Examples*
- Otherwise Begin
    - $A \leftarrow$ the attribute from *Attributes* that best* classifies *Examples*
    - The decision attribute for *Root* $\leftarrow A$
    - For each possible value, v_i, of A,
        - Add a new tree branch below *Root*, corresponding to the test $A = v_i$
        - Let $Examples_{v_i}$ be the subset of *Examples* that have value v_i for A
        - If $Examples_{v_i}$ is empty
            - Then below this new branch add a leaf node with label = most common value of *Target_attribute* in *Examples*
            - Else below this new branch add the subtree
              ID3($Examples_{v_i}$, *Target_attribute*, *Attributes* − $\{A\}$))
- End
- Return *Root*

DECISION TREE – ID3 ALGORITHM NUMERICAL EXAMPLE

Day	Outlook	Temp	Humidity	Wind	PlayTennis
D1	Sunny	Hot	High	Weak	No
D2	Sunny	Hot	High	Strong	No
D3	Overcast	Hot	High	Weak	Yes
D4	Rain	Mild	High	Weak	Yes
D5	Rain	Cool	Normal	Weak	Yes
D6	Rain	Cool	Normal	Strong	No
D7	Overcast	Cool	Normal	Strong	Yes
D8	Sunny	Mild	High	Weak	No
D9	Sunny	Cool	Normal	Weak	Yes
D10	Rain	Mild	Normal	Weak	Yes
D11	Sunny	Mild	Normal	Strong	Yes
D12	Overcast	Mild	High	Strong	Yes
D13	Overcast	Hot	Normal	Weak	Yes
D14	Rain	Mild	High	Strong	No

Day	Outlook	Temp	Humidity	Wind	Play Tennis
D1	Sunny	Hot	High	Weak	No
D2	Sunny	Hot	High	Strong	No
D3	Overcast	Hot	High	Weak	Yes
D4	Rain	Mild	High	Weak	Yes
D5	Rain	Cool	Normal	Weak	Yes
D6	Rain	Cool	Normal	Strong	No
D7	Overcast	Cool	Normal	Strong	Yes
D8	Sunny	Mild	High	Weak	No
D9	Sunny	Cool	Normal	Weak	Yes
D10	Rain	Mild	Normal	Weak	Yes
D11	Sunny	Mild	Normal	Strong	Yes
D12	Overcast	Mild	High	Strong	Yes
D13	Overcast	Hot	Normal	Weak	Yes
D14	Rain	Mild	High	Strong	No

Attribute: Outlook

$$Values\ (Outlook) = Sunny, Overcast, Rain$$

$$S = [9+, 5-] \qquad Entropy(S) = -\frac{9}{14}log_2\frac{9}{14} - \frac{5}{14}log_2\frac{5}{14} = 0.94$$

$$S_{Sunny} \leftarrow [2+, 3-] \qquad Entropy(S_{Sunny}) = -\frac{2}{5}log_2\frac{2}{5} - \frac{3}{5}log_2\frac{3}{5} = 0.971$$

$$S_{Overcast} \leftarrow [4+, 0-] \qquad Entropy(S_{Overcast}) = -\frac{4}{4}log_2\frac{4}{4} - \frac{0}{4}log_2\frac{0}{4} = 0$$

$$S_{Rain} \leftarrow [3+, 2-] \qquad Entropy(S_{Rain}) = -\frac{3}{5}log_2\frac{3}{5} - \frac{2}{5}log_2\frac{2}{5} = 0.971$$

$$Gain\ (S, Outlook) = Entropy(S) - \sum_{v \in \{Sunny, Overcast, Rain\}} \frac{|S_v|}{|S|} Entropy(S_v)$$

$$Gain(S, Outlook)$$

$$= Entropy(S) - \frac{5}{14}Entropy(S_{Sunny}) - \frac{4}{14}Entropy(S_{Overcast})$$

$$-\frac{5}{14}Entropy(S_{Rain})$$

$$Gain(S, Outlook) = 0.94 - \frac{5}{14}0.971 - \frac{4}{14}0 - \frac{5}{14}0.971 = 0.2464$$

Day	Outlook	Temp	Humidity	Wind	Play Tennis
D1	Sunny	Hot	High	Weak	No
D2	Sunny	Hot	High	Strong	No
D3	Overcast	Hot	High	Weak	Yes
D4	Rain	Mild	High	Weak	Yes
D5	Rain	Cool	Normal	Weak	Yes
D6	Rain	Cool	Normal	Strong	No
D7	Overcast	Cool	Normal	Strong	Yes
D8	Sunny	Mild	High	Weak	No
D9	Sunny	Cool	Normal	Weak	Yes
D10	Rain	Mild	Normal	Weak	Yes
D11	Sunny	Mild	Normal	Strong	Yes
D12	Overcast	Mild	High	Strong	Yes
D13	Overcast	Hot	Normal	Weak	Yes
D14	Rain	Mild	High	Strong	No

Attribute: Temp

$$Values\ (Temp) = Hot, Mild, Cool$$

$$S = [9+, 5-] \qquad Entropy(S) = -\frac{9}{14}log_2\frac{9}{14} - \frac{5}{14}log_2\frac{5}{14} = 0.94$$

$$S_{Hot} \leftarrow [2+, 2-] \qquad Entropy(S_{Hot}) = -\frac{2}{4}log_2\frac{2}{4} - \frac{2}{4}log_2\frac{2}{4} = 1.0$$

$$S_{Mild} \leftarrow [4+, 2-] \qquad Entropy(S_{Mild}) = -\frac{4}{6}log_2\frac{4}{6} - \frac{2}{6}log_2\frac{2}{6} = 0.9183$$

$$S_{Cool} \leftarrow [3+, 1-] \qquad Entropy(S_{Cool}) = -\frac{3}{4}log_2\frac{3}{4} - \frac{1}{4}log_2\frac{1}{4} = 0.8113$$

$$Gain\ (S, Temp) = Entropy(S) - \sum_{v \in \{Hot, Mild, Cool\}} \frac{|S_v|}{|S|} Entropy(S_v)$$

$$Gain(S, Temp)$$

$$= Entropy(S) - \frac{4}{14}Entropy(S_{Hot}) - \frac{6}{14}Entropy(S_{Mild})$$

$$- \frac{4}{14}Entropy(S_{Cool})$$

$$Gain(S, Temp) = 0.94 - \frac{4}{14}1.0 - \frac{6}{14}0.9183 - \frac{4}{14}0.8113 = 0.0289$$

Day	Outlook	Temp	Humidity	Wind	Play Tennis
D1	Sunny	Hot	High	Weak	No
D2	Sunny	Hot	High	Strong	No
D3	Overcast	Hot	High	Weak	Yes
D4	Rain	Mild	High	Weak	Yes
D5	Rain	Cool	Normal	Weak	Yes
D6	Rain	Cool	Normal	Strong	No
D7	Overcast	Cool	Normal	Strong	Yes
D8	Sunny	Mild	High	Weak	No
D9	Sunny	Cool	Normal	Weak	Yes
D10	Rain	Mild	Normal	Weak	Yes
D11	Sunny	Mild	Normal	Strong	Yes
D12	Overcast	Mild	High	Strong	Yes
D13	Overcast	Hot	Normal	Weak	Yes
D14	Rain	Mild	High	Strong	No

Attribute: Humidity

$$Values\ (Humidity) = High, Normal$$

$$S = [9+, 5-] \qquad Entropy(S) = -\frac{9}{14}log_2\frac{9}{14} - \frac{5}{14}log_2\frac{5}{14} = 0.94$$

$$S_{High} \leftarrow [3+, 4-] \qquad Entropy(S_{High}) = -\frac{3}{7}log_2\frac{3}{7} - \frac{4}{7}log_2\frac{4}{7} = 0.9852$$

$$S_{Normal} \leftarrow [6+, 1-] \qquad Entropy(S_{Normal}) = -\frac{6}{7}log_2\frac{6}{7} - \frac{1}{7}log_2\frac{1}{7} = 0.5916$$

$$Gain\ (S, Humidity) = Entropy(S) - \sum_{v \in \{High, Normal\}} \frac{|S_v|}{|S|} Entropy(S_v)$$

$$Gain(S, Humidity)$$

$$= Entropy(S) - \frac{7}{14}Entropy(S_{High}) - \frac{7}{14}Entropy(S_{Normal})$$

$$Gain(S, Humidity) = 0.94 - \frac{7}{14}0.9852 - \frac{7}{14}\ 0.5916 = 0.1516$$

Day	Outlook	Temp	Humidity	Wind	Play Tennis
D1	Sunny	Hot	High	Weak	No
D2	Sunny	Hot	High	Strong	No
D3	Overcast	Hot	High	Weak	Yes
D4	Rain	Mild	High	Weak	Yes
D5	Rain	Cool	Normal	Weak	Yes
D6	Rain	Cool	Normal	Strong	No
D7	Overcast	Cool	Normal	Strong	Yes
D8	Sunny	Mild	High	Weak	No
D9	Sunny	Cool	Normal	Weak	Yes
D10	Rain	Mild	Normal	Weak	Yes
D11	Sunny	Mild	Normal	Strong	Yes
D12	Overcast	Mild	High	Strong	Yes
D13	Overcast	Hot	Normal	Weak	Yes
D14	Rain	Mild	High	Strong	No

Attribute: Wind

$$Values\ (Wind) = Strong, Weak$$

$$S = [9+, 5-] \qquad Entropy(S) = -\frac{9}{14}log_2\frac{9}{14} - \frac{5}{14}log_2\frac{5}{14} = 0.94$$

$$S_{Strong} \leftarrow [3+, 3-] \qquad Entropy(S_{Strong}) = 1.0$$

$$S_{Weak} \leftarrow [6+, 2-] \qquad Entropy(S_{Weak}) = -\frac{6}{8}log_2\frac{6}{8} - \frac{2}{8}log_2\frac{2}{8} = 0.8113$$

$$Gain\ (S, Wind) = Entropy(S) - \sum_{v \in \{Strong, Weak\}} \frac{|S_v|}{|S|} Entropy(S_v)$$

$$Gain(S, Wind) = Entropy(S) - \frac{6}{14} Entropy(S_{Strong}) - \frac{8}{14} Entropy(S_{Weak})$$

$$Gain(S, Wind) = 0.94 - \frac{6}{14}\ 1.0 - \frac{8}{14}\ 0.8113 = 0.0478$$

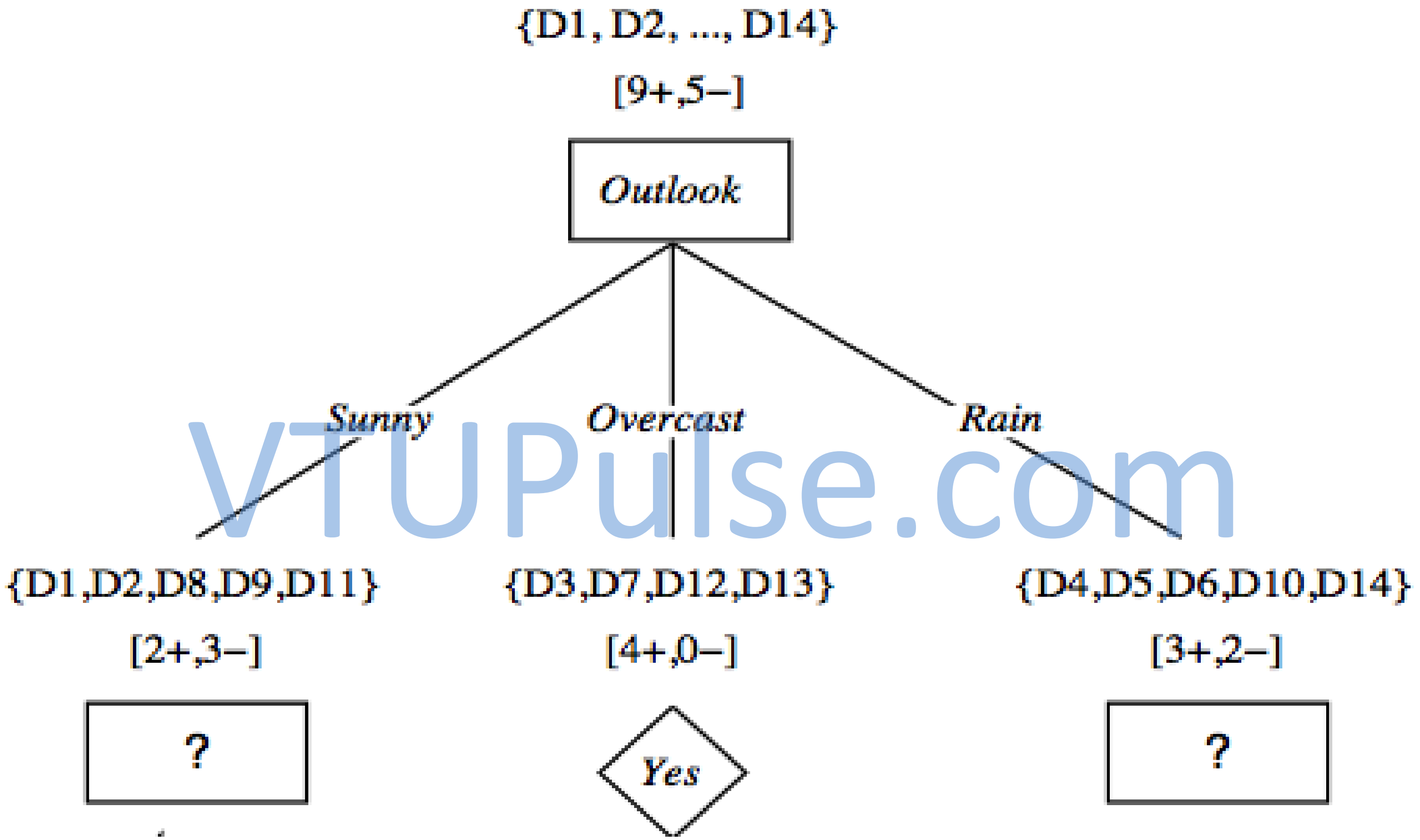
Day	Outlook	Temp	Humidity	Wind	Play Tennis
D1	Sunny	Hot	High	Weak	No
D2	Sunny	Hot	High	Strong	No
D3	Overcast	Hot	High	Weak	Yes
D4	Rain	Mild	High	Weak	Yes
D5	Rain	Cool	Normal	Weak	Yes
D6	Rain	Cool	Normal	Strong	No
D7	Overcast	Cool	Normal	Strong	Yes
D8	Sunny	Mild	High	Weak	No
D9	Sunny	Cool	Normal	Weak	Yes
D10	Rain	Mild	Normal	Weak	Yes
D11	Sunny	Mild	Normal	Strong	Yes
D12	Overcast	Mild	High	Strong	Yes
D13	Overcast	Hot	Normal	Weak	Yes
D14	Rain	Mild	High	Strong	No

$$\boldsymbol{Gain(S, Outlook) = 0.2464}$$

$$\boldsymbol{Gain(S, Temp) = 0.0289}$$

$$\boldsymbol{Gain(S, Humidity) = 0.1516}$$

$$\boldsymbol{Gain(S, Wind) = 0.0478}$$



{D1, D2, ..., D14}
[9+,5−]
Outlook
Sunny
Overcast
Rain
{D1,D2,D8,D9,D11}
[2+,3−]
{D3,D7,D12,D13}
[4+,0−]
{D4,D5,D6,D10,D14}
[3+,2−]
?
Yes
?

Day	Outlook	Temp	Humidity	Wind	Play Tennis
D1	Sunny	Hot	High	Weak	No
D2	Sunny	Hot	High	Strong	No
D3	Overcast	Hot	High	Weak	Yes
D4	Rain	Mild	High	Weak	Yes
D5	Rain	Cool	Normal	Weak	Yes
D6	Rain	Cool	Normal	Strong	No
D7	Overcast	Cool	Normal	Strong	Yes
D8	Sunny	Mild	High	Weak	No
D9	Sunny	Cool	Normal	Weak	Yes
D10	Rain	Mild	Normal	Weak	Yes
D11	Sunny	Mild	Normal	Strong	Yes
D12	Overcast	Mild	High	Strong	Yes
D13	Overcast	Hot	Normal	Weak	Yes
D14	Rain	Mild	High	Strong	No

Day	Temp	Humidity	Wind	Play Tennis
D1	Hot	High	Weak	No
D2	Hot	High	Strong	No
D8	Mild	High	Weak	No
D9	Cool	Normal	Weak	Yes
D11	Mild	Normal	Strong	Yes

Attribute: Temp

$$Values\ (Temp) = Hot, Mild, Cool$$

$S_{Sunny} = [2+, 3-]$ $\quad Entropy(S_{Sunny}) = -\frac{2}{5}log_2\frac{2}{5} - \frac{3}{5}log_2\frac{3}{5} = 0.97$

$S_{Hot} \leftarrow [0+, 2-]$ $\quad Entropy(S_{Hot}) = 0.0$

$S_{Mild} \leftarrow [1+, 1-]$ $\quad Entropy(S_{Mild}) = 1.0$

$S_{Cool} \leftarrow [1+, 0-]$ $\quad Entropy(S_{Cool}) = 0.0$

$$Gain\ (S_{Sunny}, Temp) = Entropy(S_{Sunny}) - \sum_{v \in \{Hot, Mild, Cool\}} \frac{|S_v|}{|S|} Entropy(S_v)$$

$$Gain(S_{Sunny}, Temp)$$

$$= Entropy(S_{Sunny}) - \frac{2}{5}Entropy(S_{Hot}) - \frac{2}{5}Entropy(S_{Mild}) - \frac{1}{5}Entropy(S_{Cool})$$

$$Gain(S_{sunny}, Temp) = 0.97 - \frac{2}{5}0.0 - \frac{2}{5}\ 1 - \frac{1}{5}0.0 = 0.570$$

Day	Temp	Humidity	Wind	Play Tennis
Dl	Hot	High	Weak	No
D2	Hot	High	Strong	No
D8	Mild	High	Weak	No
D9	Cool	Normal	Weak	Yes
Dl1	Mild	Normal	Strong	Yes

Attribute: Humidity

$$Values\ (Humidity) = High, Normal$$

$$S_{Sunny} = [2+, 3-] \qquad Entropy(S) = -\frac{2}{5}log_2\frac{2}{5} - \frac{3}{5}log_2\frac{3}{5} = 0.97$$

$$S_{high} \leftarrow [0+, 3-] \qquad Entropy(S_{High}) = 0.0$$

$$S_{Normal} \leftarrow [2+, 0-] \qquad Entropy(S_{Normal}) = 0.0$$

$$Gain\ (S_{Sunny}, Humidity) = Entropy(S_{Sunny}) - \sum_{v \in \{High, Normal\}} \frac{|S_v|}{|S|} Entropy(S_v)$$

$$Gain(S_{Sunny}, Humidity)$$

$$= Entropy(S_{Sunny}) - \frac{3}{5}Entropy(S_{High}) - \frac{2}{5}Entropy(S_{Normal})$$

$$Gain(S_{sunny}, Humidity) = 0.97 - \frac{3}{5}\ 0.0 - \frac{2}{5}0.0 = 0.97$$

Day	Temp	Humidity	Wind	Play Tennis
Dl	Hot	High	Weak	No
D2	Hot	High	Strong	No
D8	Mild	High	Weak	No
D9	Cool	Normal	Weak	Yes
Dl1	Mild	Normal	Strong	Yes

Attribute: Wind

$Values\ (Wind) = Strong, Weak$

$S_{Sunny} = [2+, 3-]$

$Entropy(S) = -\frac{2}{5} log_2 \frac{2}{5} - \frac{3}{5} log_2 \frac{3}{5} = 0.97$

$S_{Strong} \leftarrow [1+, 1-]$

$Entropy(S_{Strong}) = 1.0$

$S_{Weak} \leftarrow [1+, 2-]$

$Entropy(S_{Weak}) = -\frac{1}{3} log_2 \frac{1}{3} - \frac{2}{3} log_2 \frac{2}{3} =$

0.9183

$$Gain\ (S_{Sunny}, Wind) = Entropy(S_{Sunny}) - \sum_{v \in \{Strong, Weak\}} \frac{|S_v|}{|S|} Entropy(S_v)$$

$$Gain(S_{Sunny}, Wind)$$

$$= Entropy(S_{Sunny}) - \frac{2}{5} Entropy(S_{Strong}) - \frac{3}{5} Entropy(S_{Weak})$$

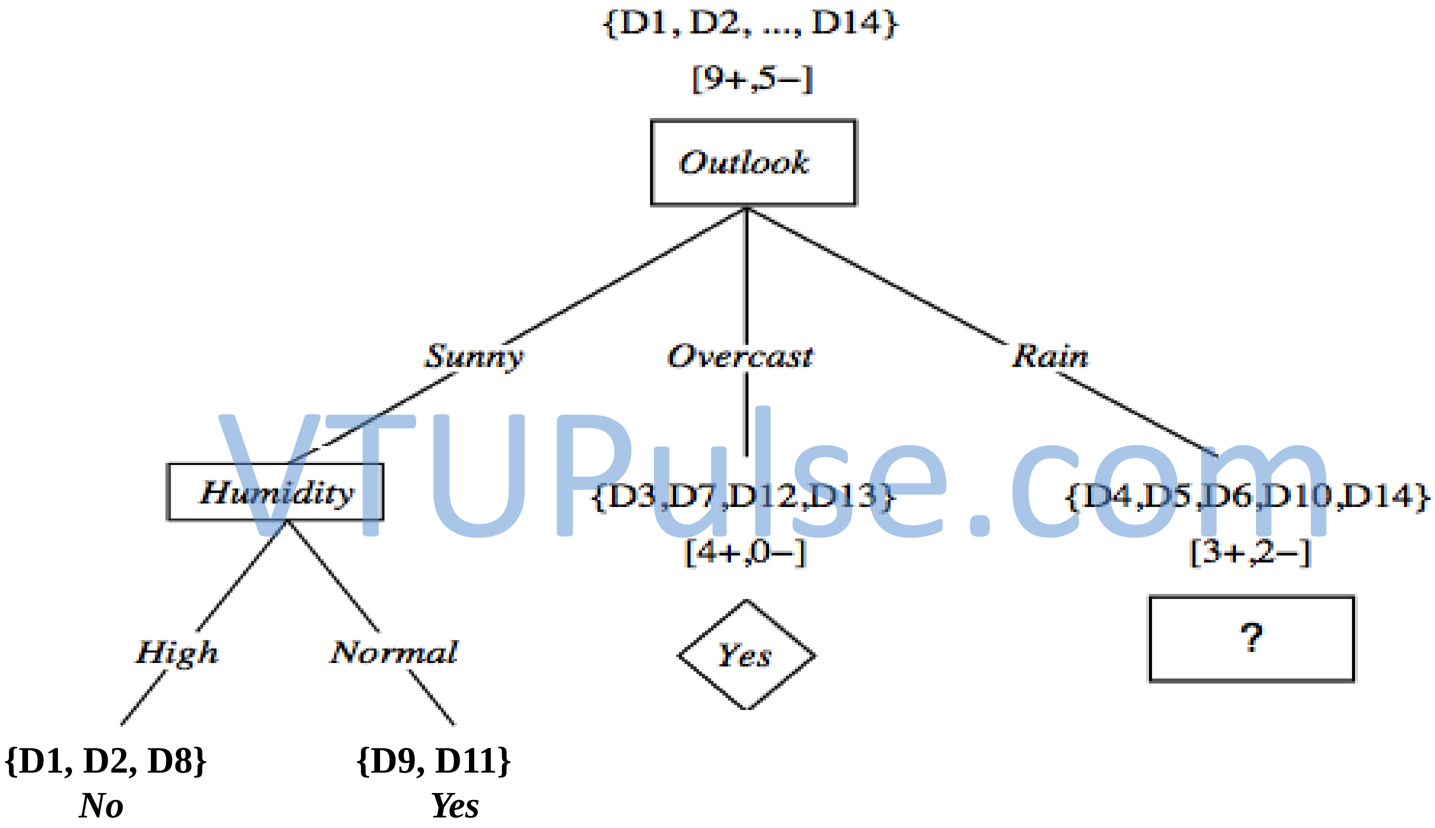
$$Gain(S_{sunny}, Wind) = 0.97 - \frac{2}{5} 1.0 - \frac{3}{5}\ 0.918 = 0.0192$$

Day	Temp	Humidity	Wind	Play Tennis
D1	Hot	High	Weak	No
D2	Hot	High	Strong	No
D8	Mild	High	Weak	No
D9	Cool	Normal	Weak	Yes
D11	Mild	Normal	Strong	Yes

$$Gain(S_{sunny}, Temp) = 0.570$$

$$Gain(S_{sunny}, Humidity) = 0.97$$

$$Gain(S_{sunny}, Wind) = 0.0192$$



{D1, D2, ..., D14}
[9+,5–]
Outlook
Sunny
Overcast
Rain
Humidity
{D3,D7,D12,D13}
[4+,0–]
{D4,D5,D6,D10,D14}
[3+,2–]
High
Normal
Yes
?
{D1, D2, D8}
No
{D9, D11}
Yes

Day	Outlook	Temp	Humidity	Wind	Play Tennis
D1	Sunny	Hot	High	Weak	No
D2	Sunny	Hot	High	Strong	No
D3	Overcast	Hot	High	Weak	Yes
D4	Rain	Mild	High	Weak	Yes
D5	Rain	Cool	Normal	Weak	Yes
D6	Rain	Cool	Normal	Strong	No
D7	Overcast	Cool	Normal	Strong	Yes
D8	Sunny	Mild	High	Weak	No
D9	Sunny	Cool	Normal	Weak	Yes
D10	Rain	Mild	Normal	Weak	Yes
D11	Sunny	Mild	Normal	Strong	Yes
D12	Overcast	Mild	High	Strong	Yes
D13	Overcast	Hot	Normal	Weak	Yes
D14	Rain	Mild	High	Strong	No

Day	Temp	Humidity	Wind	Play Tennis
D4	Mild	High	Weak	Yes
D5	Cool	Normal	Weak	Yes
D6	Cool	Normal	Strong	No
D10	Mild	Normal	Weak	Yes
D14	Mild	High	Strong	No

Attribute: Temp

$Values\ (Temp) = Hot, Mild, Cool$

$S_{Rain} = [3+, 2-]$

$Entropy(S_{Sunny}) = -\frac{3}{5} log_2 \frac{3}{5} - \frac{2}{5} log_2 \frac{2}{5} = 0.97$

$S_{Hot} \leftarrow [0+, 0-]$

$Entropy(S_{Hot}) = 0.0$

$S_{Mild} \leftarrow [2+, 1-]$

$Entropy(S_{Mild}) = -\frac{2}{3} log_2 \frac{2}{3} - \frac{1}{3} log_2 \frac{1}{3} =$

0.9183

$S_{Cool} \leftarrow [1+, 1-]$

$Entropy(S_{Cool}) = 1.0$

$$Gain\ (S_{Rain}, Temp) = Entropy(S_{Rain}) - \sum_{v \in \{Hot, Mild, Cool\}} \frac{|S_v|}{|S|} Entropy(S_v)$$

$$Gain(S_{Rain}, Temp)$$

$$= Entropy(S_{Rain}) - \frac{0}{5} Entropy(S_{Hot}) - \frac{3}{5} Entropy(S_{Mild})$$

$$- \frac{2}{5} Entropy(S_{Cool})$$

$$Gain(S_{Rain}, Temp) = 0.97 - \frac{0}{5} 0.0 - \frac{3}{5}\ 0.918 - \frac{2}{5}\ 1.0 = 0.0192$$

Day	Temp	Humidity	Wind	Play Tennis
D4	Mild	High	Weak	Yes
D5	Cool	Normal	Weak	Yes
D6	Cool	Normal	Strong	No
Dl0	Mild	Normal	Weak	Yes
Dl4	Mild	High	Strong	No

Attribute: Humidity

$$Values\ (Humidity) = High, Normal$$

$S_{Rain} = [3+, 2-]$ $\quad Entropy(S_{Sunny}) = -\frac{3}{5}log_2\frac{3}{5} - \frac{2}{5}log_2\frac{2}{5} = 0.97$

$S_{High} \leftarrow [1+, 1-]$ $\quad Entropy(S_{High}) = 1.0$

$S_{Normal} \leftarrow [2+, 1-]$ $\quad Entropy(S_{Normal}) = -\frac{2}{3}log_2\frac{2}{3} - \frac{1}{3}log_2\frac{1}{3} = 0.9183$

$$Gain\ (S_{Rain}, Humidity) = Entropy(S_{Rain}) - \sum_{v \in \{High, Normal\}} \frac{|S_v|}{|S|} Entropy(S_v)$$

$$Gain(S_{Rain}, Humidity)$$

$$= Entropy(S_{Rain}) - \frac{2}{5}Entropy(S_{High}) - \frac{3}{5}Entropy(S_{Normal})$$

$$Gain(S_{Rain}, Humidity) = 0.97 - \frac{2}{5}1.0 - \frac{3}{5}0.918 = 0.0192$$

Day	Temp	Humidity	Wind	Play Tennis
D4	Mild	High	Weak	Yes
D5	Cool	Normal	Weak	Yes
D6	Cool	Normal	Strong	No
Dl0	Mild	Normal	Weak	Yes
Dl4	Mild	High	Strong	No

Attribute: Wind

$$Values\ (wind) = Strong, Weak$$

$$S_{Rain} = [3+, 2-] \qquad Entropy(S_{Sunny}) = -\frac{3}{5}log_2\frac{3}{5} - \frac{2}{5}log_2\frac{2}{5} = 0.97$$

$$S_{Strong} \leftarrow [0+, 2-] \qquad Entropy(S_{Strong}) = 0.0$$

$$S_{Weak} \leftarrow [3+, 0-] \qquad Entropy(S_{weak}) = 0.0$$

$$Gain\ (S_{Rain}, Wind) = Entropy(S_{Rain}) - \sum_{v \in \{Strong, Weak\}} \frac{|S_v|}{|S|} Entropy(S_v)$$

$$Gain(S_{Rain}, Wind) = Entropy(S_{Rain}) - \frac{2}{5}Entropy(S_{Strong}) - \frac{3}{5}Entropy(S_{Weak})$$

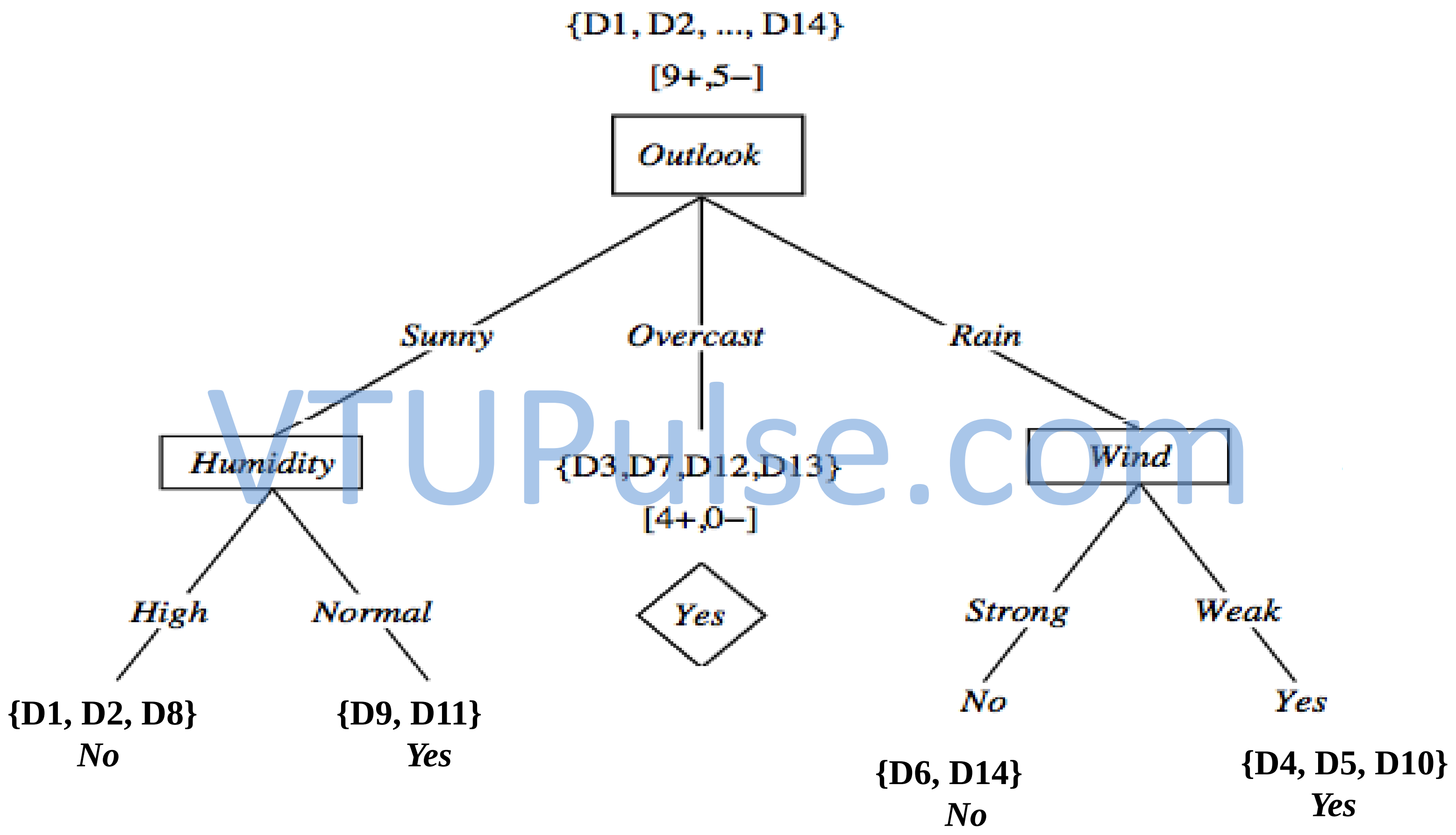
$$Gain(S_{Rain}, Wind) = 0.97 - \frac{2}{5}\ 0.0 - \frac{3}{5}0.0 = 0.97$$

Day	Temp	Humidity	Wind	Play Tennis
D4	Mild	High	Weak	Yes
D5	Cool	Normal	Weak	Yes
D6	Cool	Normal	Strong	No
Dl0	Mild	Normal	Weak	Yes
Dl4	Mild	High	Strong	No

$$\boldsymbol{Gain(S_{Rain}, Temp) = 0.0192}$$

$$\boldsymbol{Gain(S_{Rain}, Humidity) = 0.0192}$$

$$\boldsymbol{Gain(S_{Rain}, Wind) = 0.97}$$


{D1, D2, ..., D14}
[9+,5−]
Outlook
Sunny
Overcast
Rain
Humidity
{D3,D7,D12,D13}
[4+,0−]
Yes
Wind
High
Normal
Strong
Weak
{D1, D2, D8}
No
{D9, D11}
Yes
No
Yes
{D6, D14}
No
{D4, D5, D10}
Yes

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DECISION TREE EXAMPLE - 2

Instance	Classification	a1	a2
1	+	T	T
2	+	T	T
3	-	T	F
4	+	F	F
5	-	F	T
6	-	F	T

1. What is the entropy of this collection of training examples with respect to the target function classification?
2. What is the information gain of *a2* relative to these training examples?
3. Draw decision tree for the given dataset.

Decision Tree Algorithm – ID3 Solved Example

1. What is the entropy of this collection of training examples with respect to the target function classification?
2. What is the information gain of *a1* and *a2* relative to these training examples?
3. Draw decision tree for the given dataset.

Instance	Classification	a1	a2
1	+	T	T
2	+	T	T
3	-	T	F
4	+	F	F
5	-	F	T
6	-	F	T

Instance	Classification	a1	a2
1	+	T	T
2	+	T	T
3	-	T	F
4	+	F	F
5	-	F	T
6	-	F	T

Attribute: a1

$$Values\ (a1) = T, F$$

$$S = [3+,\ 3-] \qquad Entropy(S) = 1.0$$

$$S_T = [2+,\ 1-] \qquad Entropy(S_T) = -\frac{2}{3}log_2\frac{2}{3} - \frac{1}{3}log_2\frac{1}{3} = 0.9183$$

$$S_F \leftarrow [1+,\ 2-] \qquad Entropy(S_F) = -\frac{1}{3}log_2\frac{1}{3} - \frac{2}{3}log_2\frac{2}{3} = 0.9183$$

$$Gain\ (S, a1) = Entropy(S) - \sum_{v \in \{T,F\}} \frac{|S_v|}{|S|} Entropy(S_v)$$

$$Gain(S, a1) = Entropy(S) - \frac{3}{6}Entropy(S_T) - \frac{3}{6}Entropy(S_F)$$

$$Gain(S, a1) = 1.0 - \frac{3}{6} * 0.9183 - \frac{3}{6} * 0.9183 = 0.0817$$

Example - 2

Decision Tree Algorithm – ID3

Solved Example

Instance	Classification	a1	a2
1	+	T	T
2	+	T	T
3	-	T	F
4	+	F	F
5	-	F	T
6	-	F	T

Attribute: a2

$$Values\ (a2) = T, F$$

$$S = [3+,\ 3-] \qquad Entropy(S) = 1.0$$

$$S_T = [2+,\ 2-] \qquad Entropy(S_T) = 1.0$$

$$S_F \leftarrow [1+,\ 1-] \qquad Entropy(S_F) = 1.0$$

$$Gain\ (S, a2) = Entropy(S) - \sum_{v \in \{T,F\}} \frac{|S_v|}{|S|} Entropy(S_v)$$

$$Gain(S, a2) = Entropy(S) - \frac{4}{6} Entropy(S_T) - \frac{2}{6} Entropy(S_F)$$

$$Gain(S, a2) = 1.0 - \frac{4}{6} * 1.0 - \frac{2}{6} * 1.0 = 0.0$$

Example - 2

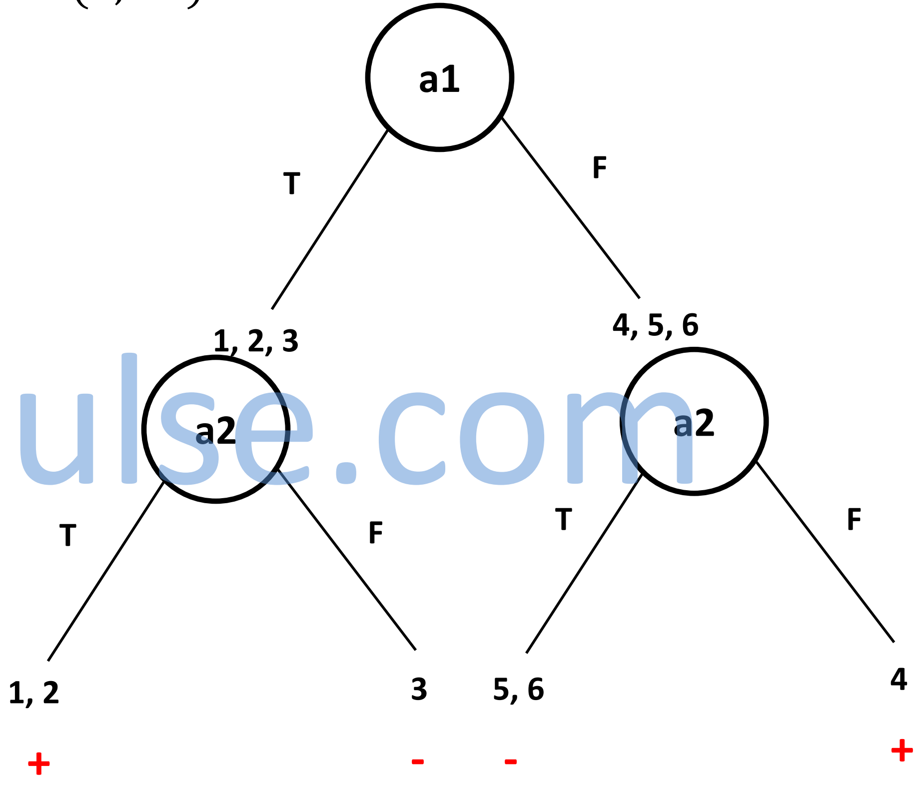
Decision Tree Algorithm – ID3

Solved Example

Instance	Classification	a1	a2
1	+	T	T
2	+	T	T
3	-	T	F
4	+	F	F
5	-	F	T
6	-	F	T

$$Gain(S, a1) = 0.0817 \quad - \quad Maximum\ Gain$$

$$Gain(S, a2) = 0.0$$



Example - 2
Decision Tree Algorithm – ID3
Solved Example

DECISION TREE EXAMPLE - 3

Instance	a1	a2	a3	Classification
1	True	Hot	High	No
2	True	Hot	High	No
3	False	Hot	High	Yes
4	False	Cool	Normal	Yes
5	False	Cool	Normal	Yes
6	True	Cool	High	No
7	True	Hot	High	No
8	True	Hot	Normal	Yes
9	False	Cool	Normal	Yes
10	False	Cool	High	Yes

1. Construct the decision tree for the following tree using ID3 Algorithm

Decision Tree Algorithm – ID3 Solved Example

Instance	a1	a2	a3	Classification
1	True	Hot	High	No
2	True	Hot	High	No
3	False	Hot	High	Yes
4	False	Cool	Normal	Yes
5	False	Cool	Normal	Yes
6	True	Cool	High	No
7	True	Hot	High	No
8	True	Hot	Normal	Yes
9	False	Cool	Normal	Yes
10	False	Cool	High	Yes

Instance	a1	a2	a3	Classification
1	True	Hot	High	No
2	True	Hot	High	No
3	False	Hot	High	Yes
4	False	Cool	Normal	Yes
5	False	Cool	Normal	Yes
6	True	Cool	High	No
7	True	Hot	High	No
8	True	Hot	Normal	Yes
9	False	Cool	Normal	Yes
10	False	Cool	High	Yes

Attribute: a1

$Values\ (a1) = True,\ False$

$S = [6+,\ 4-]$ $\qquad Entropy(S) = -\frac{6}{10}log_2\frac{6}{10} - \frac{4}{10}log_2\frac{4}{10} = 0.9709$

$S_{True} = [1+, 4-]$ $\qquad Entropy(S_{True}) = -\frac{1}{5}log_2\frac{1}{5} - \frac{4}{5}log_2\frac{4}{5} = 0.7219$

$S_{Flase} \leftarrow [5+,\ 0-]$ $\qquad Entropy(S_{False}) = 0.0$

$$Gain\ (S, a1) = Entropy(S) - \sum_{v \in \{True, False\}} \frac{|S_v|}{|S|} Entropy(S_v)$$

$$Gain(S, a1) = Entropy(S) - \frac{5}{10}Entropy(S_{True}) - \frac{5}{10}Entropy(S_{False})$$

$$Gain(S, a1) = 0.9709 - \frac{5}{10} * 0.7219 - \frac{5}{10} * 1 = 0.6099$$

Example - 3
Decision Tree Algorithm – ID3
Solved Example

Instance	a1	a2	a3	Classification
1	True	Hot	High	No
2	True	Hot	High	No
3	False	Hot	High	Yes
4	False	Cool	Normal	Yes
5	False	Cool	Normal	Yes
6	True	Cool	High	No
7	True	Hot	High	No
8	True	Hot	Normal	Yes
9	False	Cool	Normal	Yes
10	False	Cool	High	Yes

Attribute: a2

$$Values\ (a2) = Hot,\ Cool$$

$$S = [6+,\ 4-] \qquad Entropy(S) = -\frac{6}{10}log_2\frac{6}{10} - \frac{4}{10}log_2\frac{4}{10} = 0.9709$$

$$S_{Hot} = [2+, 3-] \qquad Entropy(S_{Hot}) = -\frac{2}{5}log_2\frac{2}{5} - \frac{3}{5}log_2\frac{3}{5} = 0.9709$$

$$S_{Cool} \leftarrow [4+,\ 1-] \qquad Entropy(S_{Cool}) = -\frac{4}{5}log_2\frac{4}{5} - \frac{1}{5}log_2\frac{1}{5} = 0.7219$$

$$Gain\ (S, a2) = Entropy(S) - \sum_{v\in\{Hot,Cool\}} \frac{|S_v|}{|S|} Entropy(S_v)$$

$$Gain(S, a2) = Entropy(S) - \frac{5}{10}Entropy(S_{Hot}) - \frac{5}{10}Entropy(S_{Cool})$$

$$Gain(S, a2) = 0.9709 - \frac{5}{10} * 0.9709 - \frac{5}{10} * 0.7219 = 0.1245$$

Example - 3
Decision Tree Algorithm – ID3
Solved Example

Instance	a1	a2	a3	Classification
1	True	Hot	High	No
2	True	Hot	High	No
3	False	Hot	High	Yes
4	False	Cool	Normal	Yes
5	False	Cool	Normal	Yes
6	True	Cool	High	No
7	True	Hot	High	No
8	True	Hot	Normal	Yes
9	False	Cool	Normal	Yes
10	False	Cool	High	Yes

Attribute: a3

$$Values\ (a3) = High,\ Normal$$

$$S = [6+,\ 4-] \qquad Entropy(S) = -\frac{6}{10}log_2\frac{6}{10} - \frac{4}{10}log_2\frac{4}{10} = 0.9709$$

$$S_{High} = [2+, 4-] \qquad Entropy(S_{High}) = -\frac{2}{6}log_2\frac{2}{6} - \frac{4}{6}log_2\frac{4}{6} = 0.9183$$

$$S_{Normal} \leftarrow [4+,\ 0-] \qquad Entropy(S_{Normal}) = 0.0$$

$$Gain\ (S, a3) = Entropy(S) - \sum_{v \in \{High, Normal\}} \frac{|S_v|}{|S|} Entropy(S_v)$$

$$Gain(S, a3) = Entropy(S) - \frac{6}{10} Entropy(S_{High}) - \frac{4}{10} Entropy(S_{Normal})$$

$$Gain(S, a3) = 0.9709 - \frac{6}{10} * 0.9183 - \frac{4}{10} * 0.0 = 0.4199$$

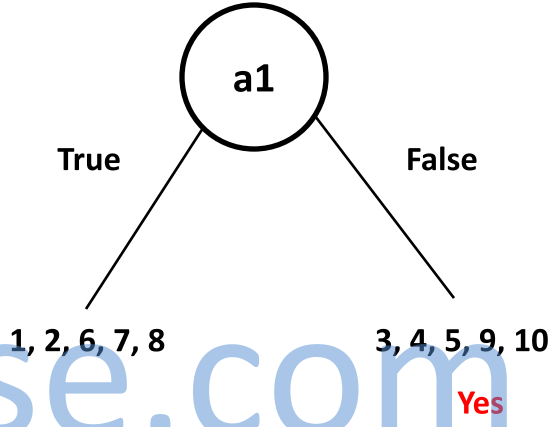
Example - 3
Decision Tree Algorithm – ID3
Solved Example

Instance	a1	a2	a3	Classification
1	True	Hot	High	No
2	True	Hot	High	No
3	False	Hot	High	Yes
4	False	Cool	Normal	Yes
5	False	Cool	Normal	Yes
6	True	Cool	High	No
7	True	Hot	High	No
8	True	Hot	Normal	Yes
9	False	Cool	Normal	Yes
10	False	Cool	High	Yes

$$Gain(S, a1) = 0.6099 \quad - \; Maximum\ Gain$$

$$Gain(S, a2) = 0.1245$$

$$Gain(S, a3) = 0.4199$$



Example - 3
Decision Tree Algorithm – ID3
Solved Example

Attribute: a2

Instance	a2	a3	Classification
1	Hot	High	No
2	Hot	High	No
6	Cool	High	No
7	Hot	High	No
8	Hot	Normal	Yes

$$Values\ (a2) = Hot,\ Cool$$

$$S_{a1} = [1+,\ 4-] \qquad Entropy(S_{a1}) = -\frac{1}{5}log_2\frac{1}{5} - \frac{4}{5}log_2\frac{4}{5} = 0.7219$$

$$S_{Hot} = [1+, 3-] \qquad Entropy(S_{Hot}) = -\frac{1}{4}log_2\frac{1}{4} - \frac{3}{4}log_2\frac{3}{4} = 0.8112$$

$$S_{Cool} \leftarrow [0+,\ 1-] \qquad Entropy(S_{Cool}) = 0.0$$

$$Gain\ (S, a2) = Entropy(S) - \sum_{v \in \{Hot, Cool\}} \frac{|S_v|}{|S|} Entropy(S_v)$$

$$Gain(S, a2) = Entropy(S) - \frac{4}{5} Entropy(S_{Hot}) - \frac{1}{5} Entropy(S_{Cool})$$

$$Gain(S, a2) = 0.9709 - \frac{4}{5} * 0.8112 - \frac{1}{5} * 0.0 = 0.3219$$

Example - 3
Decision Tree Algorithm – ID3
Solved Example

Attribute: a3

Instance	a2	a3	Classification
1	Hot	High	No
2	Hot	High	No
6	Cool	High	No
7	Hot	High	No
8	Hot	Normal	Yes

$$Values\ (a3) = High,\ Normal$$

$$S_{a1} = [1+,\ 4-] \qquad Entropy(S_{a1}) = -\frac{1}{5}log_2\frac{1}{5} - \frac{4}{5}log_2\frac{4}{5} = 0.7219$$

$$S_{High} = [0+, 4-] \qquad Entropy(S_{High}) = 0.0$$

$$S_{Normal} \leftarrow [1+,\ 0-] \qquad Entropy(S_{Normal}) = 0.0$$

$$Gain\ (S, a3) = Entropy(S) - \sum_{v \in \{High, Normal\}} \frac{|S_v|}{|S|} Entropy(S_v)$$

$$Gain(S, a3) = Entropy(S) - \frac{4}{5} Entropy(S_{High}) - \frac{1}{5} Entropy(S_{Normal})$$

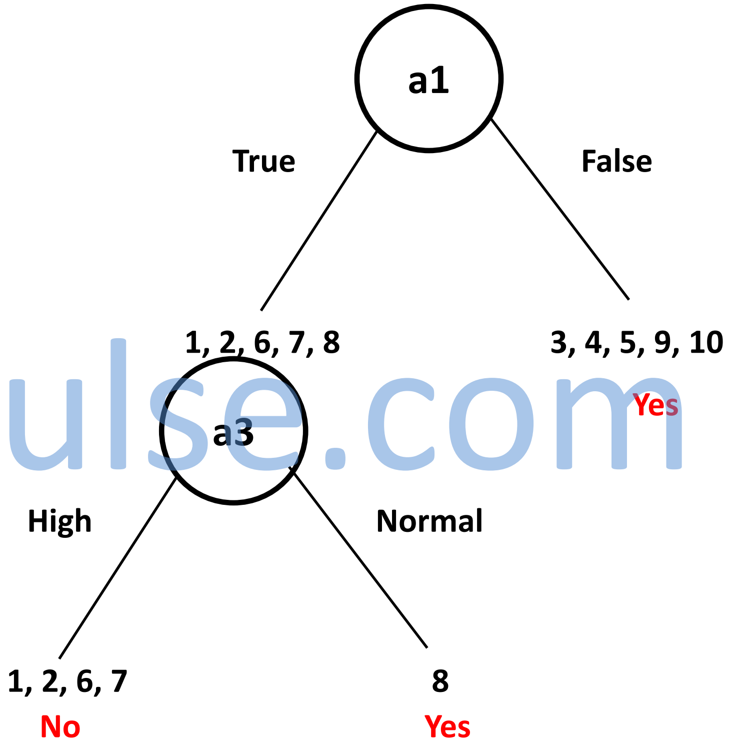
$$Gain(S, a3) = 0.9709 - \frac{4}{5} * 0.0 - \frac{1}{5} * 0.0 = 0.7219$$

Example - 3
Decision Tree Algorithm – ID3
Solved Example

$$Gain(S_{a1}, a2) = 0.3219$$

$$Gain(S_{a1}, a3) = 0.7219 - \text{ Maximum Gain}$$

Instance	a2	a3	Classification
1	Hot	High	No
2	Hot	High	No
6	Cool	High	No
7	Hot	High	No
8	Hot	Normal	Yes



Example - 3
Decision Tree Algorithm – ID3
Solved Example

When to use Decision Trees

- Problem characteristics:
  - Instances can be described by attribute value pairs
  - Target function is discrete valued
  - Disjunctive hypothesis may be required
  - Possibly noisy training data samples
    - Robust to errors in training data
    - Missing attribute values
- Different classification problems:
  - Equipment classification
  - Medical diagnosis
  - Credit risk analysis
  - Several tasks in natural language processing

Issues in decision trees learning

- Overfitting
  - Reduced error pruning
  - Rule post-pruning
- Extensions
  - Continuous valued attributes
  - Alternative measures for selecting attributes
  - Handling training examples with missing attribute values
  - Handling attributes with different costs
  - Improving computational efficiency
  - Most of these improvements in C4.5 (Quinlan, 1993)

Overfitting: definition

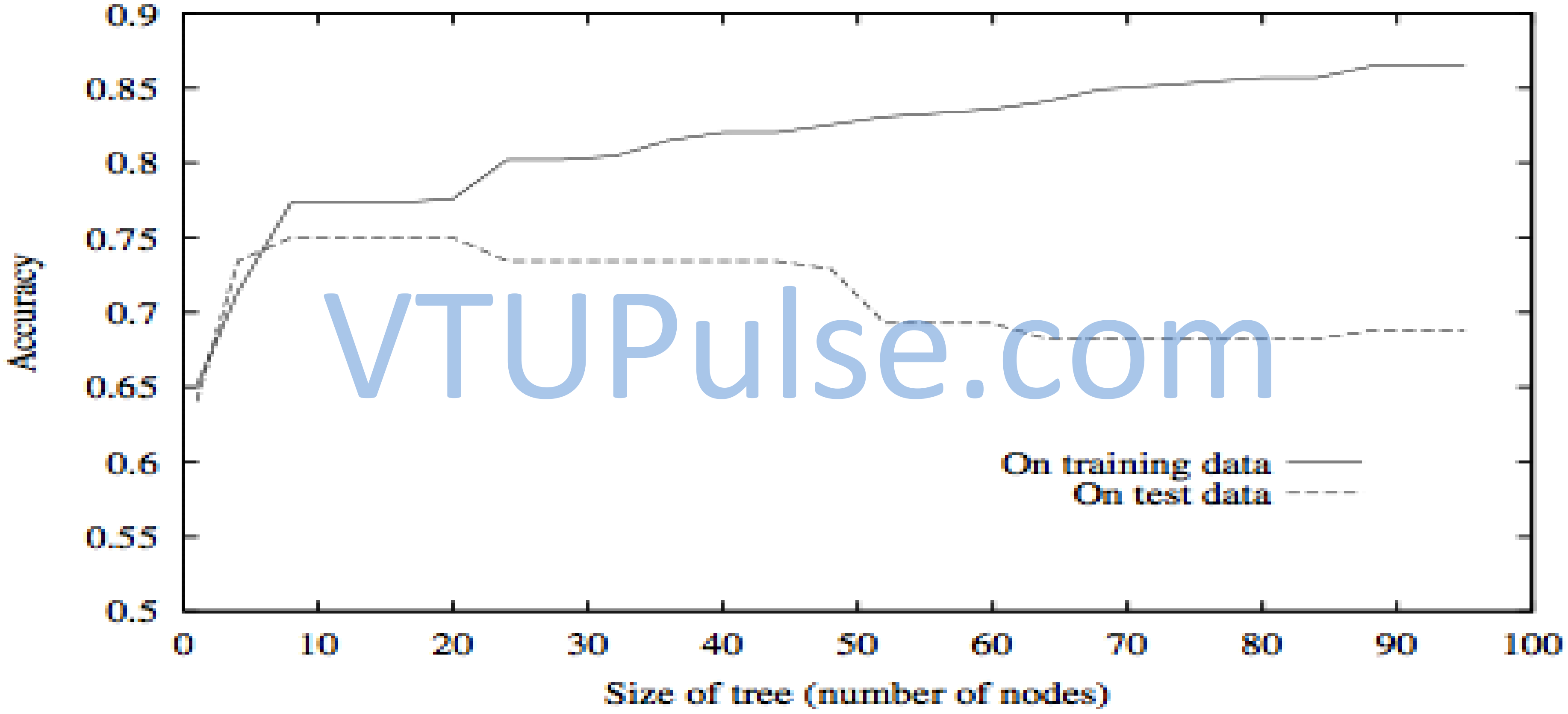
- Building trees that "adapt too much" to the training examples may lead to "overfitting".
- Consider error of hypothesis h over
  - training data: $error_D(h)$ empirical error
  - entire distribution X of data: $error_X(h)$ expected error
- Hypothesis h *overfits* training data if there is an alternative hypothesis $h' \in H$ such that

$$error_D(h) < error_D(h') \quad \text{and}$$

$$error_X(h') < error_X(h)$$

i.e. h' behaves better over unseen data

Overfitting in decision tree learning



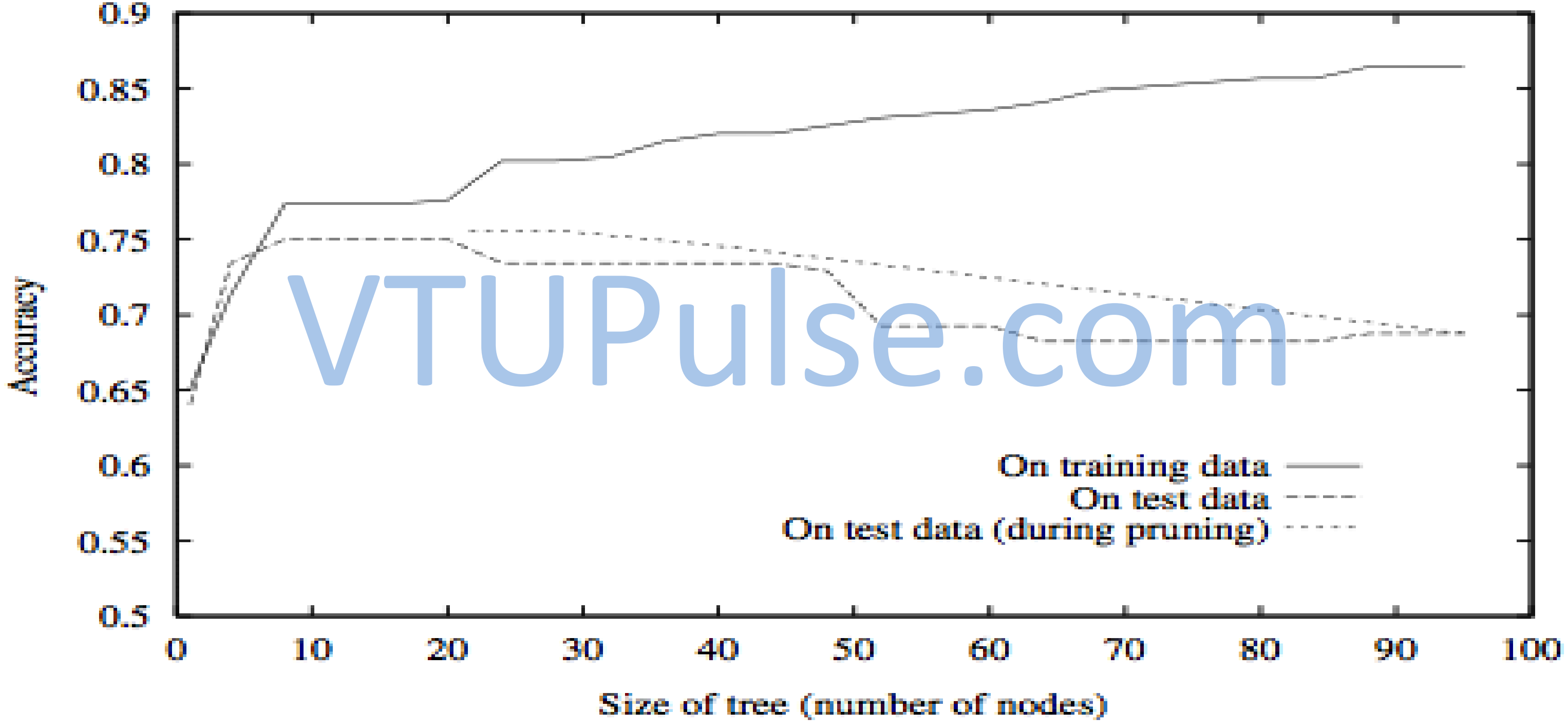
Avoid overfitting in Decision Trees

- **Two strategies:**
  1. Stop growing the tree earlier, before perfect classification
  2. Allow the tree to *overfit* the data, and then *post-prune* the tree
     - Training and validation set: split the training in two parts (training and validation) and use validation to assess the utility of *post-pruning*
       - *Reduced error pruning*

Reduced-error pruning (Quinlan 1987)

- Each node is a candidate for pruning
- *Pruning* consists in removing a subtree rooted in a node: the node becomes a leaf and is assigned the most common classification
- Nodes are removed only if the resulting tree performs no worse on the validation set.
- Nodes are pruned iteratively: at each iteration the node whose removal most increases accuracy on the validation set is pruned.
- Pruning stops when no pruning increases accuracy

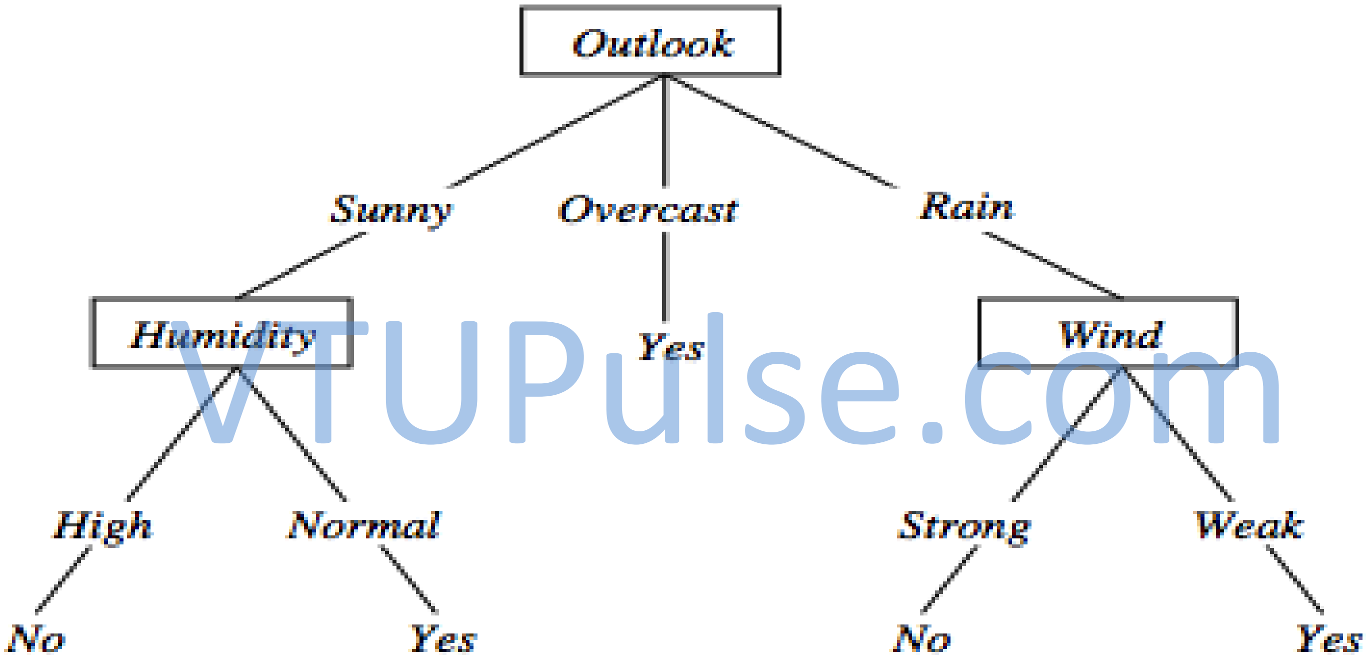
Effect of reduced error pruning



Rule post-pruning

1. Create the decision tree from the training set
2. Convert the tree into an equivalent set of rules
   - Each path corresponds to a rule
   - Each node along a path corresponds to a pre-condition
   - Each leaf classification to the post-condition
3. Prune (generalize) each rule by removing those preconditions whose removal improves accuracy over validation set
4. Sort the rules in estimated order of accuracy, and consider them in sequence when classifying new instances

Converting to rules



Rule Post-Pruning

- Convert tree to rules (one for each path from root to a leaf)
- For each antecedent in a rule, remove it if error rate on validation set does not decrease
- Sort final rule set by accuracy


```
Outlook=sunny ^ humidity=high -> No
Outlook=sunny ^ humidity=normal -> Yes
Outlook=overcast -> Yes
Outlook=rain ^ wind=strong -> No
Outlook=rain ^ wind=weak -> Yes
```

Compare first rule to:

```
Outlook=sunny->No
Humidity=high->No
```

Why converting to rules?

- Each distinct path produces a different rule: a condition removal may be based on a local (contextual) criterion. Node pruning is global and affects all the rules
- In rule form, tests are not ordered and there is no book-keeping involved when conditions (nodes) are removed
- Converting to rules improves readability for humans

Dealing with continuous-valued attributes

- So far discrete values for attributes and for outcome.
- Given a continuous-valued attribute A, dynamically create a new attribute A_c

  A_c = True *if* $A < c$, False *otherwise*

- How to determine threshold value c ?
- Example. *Temperature* in the *PlayTennis* example
  - Sort the examples according to *Temperature*

Temperature	40	48	60	72	80	90
PlayTennis	No	No	Yes	Yes	Yes	No

  - Determine candidate thresholds by averaging consecutive values where there is a change in classification: (48+60)/2=54 and (80+90)/2=85
  - Evaluate candidate thresholds (attributes) according to information gain. The best is $Temperature_{>54}$. The new attribute competes with the other ones

Tid	Refund	Marital Status	Taxable Income	Cheat
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

Handling incomplete training data

- How to cope with the problem that the value of some attribute may be missing?
  - *Example*: Blood-Test-Result in a medical diagnosis problem
- The strategy: use other examples to guess attribute
  1. Assign the value that is most common among the training examples at the node
  2. Assign a probability to each value, based on frequencies, and assign values to missing attribute, according to this probability distribution
- Missing values in new instances to be classified are treated accordingly, and the most probable classification is chosen (C4.5)

Handling attributes with different costs

- Instance attributes may have an associated cost: we would prefer decision trees that use low-cost attributes
- ID3 can be modified to take into account costs:

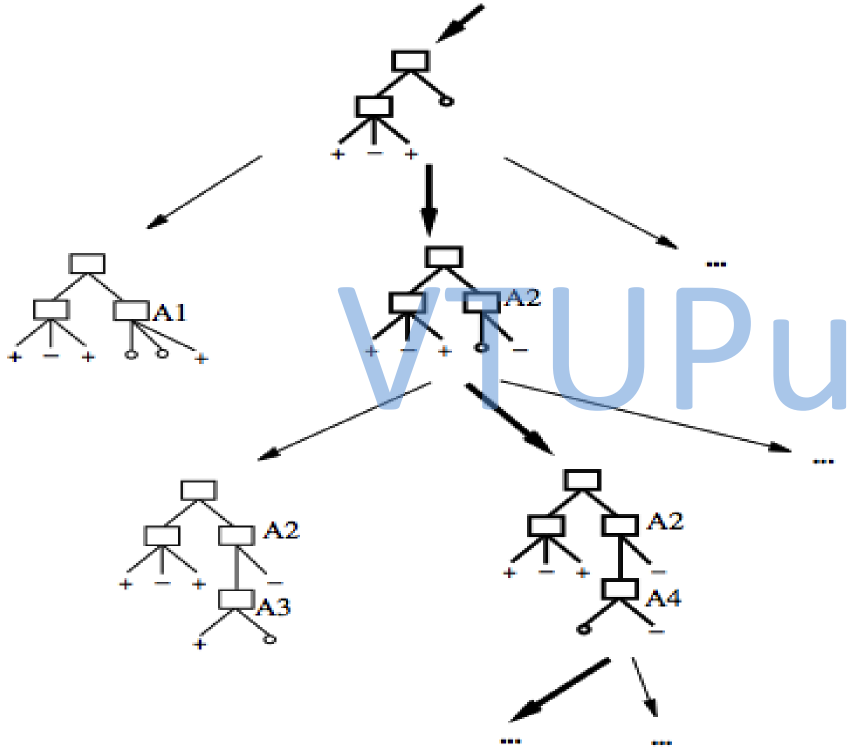
1. Tan and Schlimmer (1990)

$$\frac{Gain^2(S, A)}{Cost(S, A)}$$

2. Nunez (1988)

$$\frac{2^{Gain(S, A)} - 1}{(Cost(A) + 1)^w} \quad w \in [0,1]$$

Search space in Decision Tree learning



- The search space is made by partial decision trees
- The algorithm is *hill-climbing*
- The evaluation function is *information gain*
- The hypotheses space is complete (represents all discrete-valued functions)
- No backtracking; no guarantee of optimality
- It uses all the available examples (not incremental)

Inductive bias in decision tree learning

What is the inductive bias of DT learning?

1. *Shorter trees are preferred over longer trees*

   Not enough. This is the bias exhibited by a simple breadth first algorithm generating all DT's e selecting the shorter one

2. *Prefer trees that place high information gain attributes close to the root*

Prefer shorter hypotheses: Occam's razor

- Why prefer shorter hypotheses?
- Arguments in favor:
  - There are fewer short hypotheses than long ones
  - If a short hypothesis fits data unlikely to be a coincidence
- Arguments against:
  - Not every short hypothesis is a reasonable one.
- Occam's razor: "*The simplest explanation is usually the best one.*"
  - a principle usually (though incorrectly) attributed14th-century English logician and Franciscan friar, William of Ockham.
  - The term razor refers to the act of *shaving away* unnecessary assumptions to get to the simplest explanation.

Two kinds of biases

- Preference or search biases
  - ID3 searches through incompletely through *complete* hypotheses space; from simple to complex hypotheses, until termination condition is reached.
- Restriction or language biases
  - *It searches the hypotheses space completely*
  - *Candidate-Elimination* search is *complete*

References

- Machine Learning, Tom Mitchell, Mc Graw-Hill International Editions, 2013, India Edition.