RECONSTRUCTION OF ANALOG BANDPASS SIGNAL

Department of Electrical and Computer Engineering

ELEC 534 Application of Digital Signal Processing Techniques

PROJECT REPORT

SUBMITTED BY

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Date: April 22nd 2020

ABSTRACT

In Signal processing, Bandpass sampling is a technique where one samples a bandpass filtered signal at a sample rate below its Nyquist rate(which is twice the upper cutoff frequency),but is still able to reconstruct the signal. An analytical formula of the filter to reconstructions of an analog bandpass signal has been derived and discussed. The theoretical background, the algorithmic implementation and the MATLAB code for implementing the formula to reconstruct the analog bandpass signal will be discussed in this project.

1. Introduction

Bandpass signals are a kind of band-limited signals with a higher frequency range(as upper limit). These signals can be found in sonar and audio broadcasting,coherent radar where an audio signal is modulated by a higher frequency carrier and the modulated signal will be a bandpass signal with a narrow bandwidth. These signals contain two bands, which are symmetrically located across the zero(described ahead). Here the theory of sampling bandpass signal and its reconstruction are discussed. Also, the time domain and the frequency domain of the reconstructed signal and original signals are compared to check the result.

2. Theory

2.1 Sampling of Bandpass Signal

There are many continuous time signals with the higher frequency range $\Omega_L \leq \Omega \leq \Omega_H$ where $\Omega_L > 0$. Such signals are known as bandpass signals. The range of bandpass signals is defined as $\Delta\Omega=\Omega_H - \Omega_L$.

Here we are considering the highest frequency contained in a bandpass signal x(t) is a multiple of the bandwidth, I.e.,

$$\Omega_H = M.\boldsymbol{\Delta}\Omega \tag{1.1}$$

And choose the sampling frequency Ω_S

$$\Omega_S = 2.\boldsymbol{\Delta}\Omega = 2.\Omega_H/M \tag{1.2}$$

Then, the Fourier transform of the sampled analog signal $x_s(t)$ becomes:

$$X_s(j\Omega) = \frac{1}{T_s}.\sum_{k=-\infty}^{\infty} X(j(\Omega - 2\mathrm{k}\boldsymbol{\Delta}\Omega)) \tag{1.3}$$

From figure 1, we can see that signal x(t) (figure 1.a) with a frequency range in $\Omega_L \leq \Omega \leq \Omega_H$ is sampled at a frequency $\Omega_S = 2.\boldsymbol{\Delta}\Omega$ where $\boldsymbol{\Delta}\Omega = \Omega_H - \Omega_L$. The Fourier transform of the sampled signal $X_s(j\Omega)$ is shown in the figure (1.c). Comparing to normal cases (lowpass signals), the frequency content of the signal is in the range $0 \leq \Omega \leq \Omega_{Max}$, and the sampling frequency should be greater than 2*Ω_{Max}, because M should be greater integer than 1, however the sampling frequency set by (1.2) is less than 2$*\ \Omega_H$. In a lowpass signals, the bandwidth is chosen as $\boldsymbol{\Delta}\Omega = \Omega_{Max} - 0$, thus the $\Omega_S \geq 2.\Omega_{Max}$. So shannon's sampling theorm can be understood as $\Omega_S \geq 2.\boldsymbol{\Delta}\Omega$.

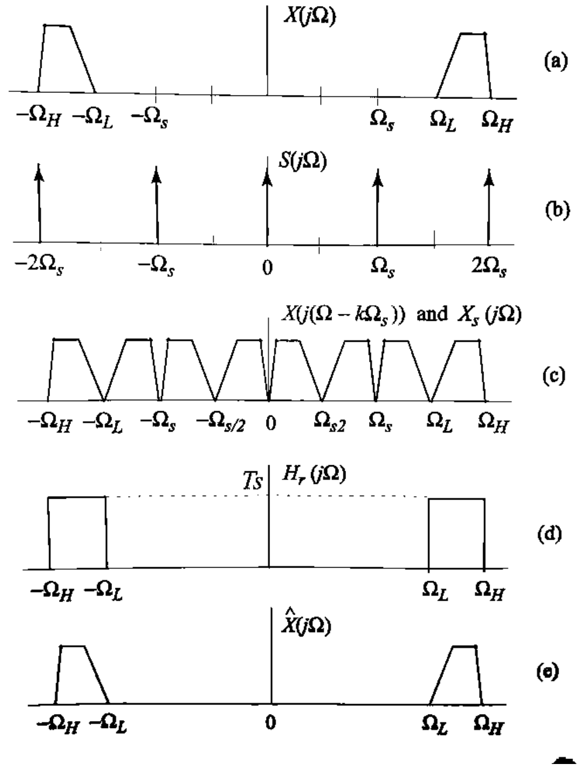

Figure 1

2.2: Reconstruction of analog bandpass signal and formulation of bandpass filter:

The original bandpass signal x(t) can be recovered passing by sampled signal $x_s(t)$ through an appropriate bandpass filter $H_r(j\Omega)$ (shown in figure 1(d)) that has cut of frequency Ω_L and Ω_H. An expression of the bandpass filter to reconstruct $x_s(t)$ is derived below.

To recover the signal x(t) back from the bandpass sampled signal $x_s(t)$, a bandpass filter is designed with a frequency response $H_r(j\Omega)$.

The frequency response of the bandpass filter is given below, this design is with respect to the Figure:1

$$H_r(j\Omega) = \begin{cases} T_s \ for & -\Omega_H \le \Omega \le -\Omega_L \\ T_s \ for & \Omega_L \le \Omega \le \Omega_H \\ 0 \ for & else \end{cases} \tag{1.4}$$

The impulse response of the above filter is given below:

$$h_r(t) = \frac{T_S}{2\pi}\left[\int_{\Omega_L}^{\Omega_H} e^{j\Omega t} d\Omega + \int_{-\Omega_H}^{-\Omega_L} e^{j\Omega t} d\Omega \qquad \right] \tag{1.5}$$

$$h_r(t) = \frac{T_S}{2\pi}\left[\frac{e^{j\Omega_H t} - e^{j\Omega_L t} + e^{-j\Omega_L t} - e^{-j\Omega_H t}}{t*j}\right] \tag{1.6}$$

$$h_r(t) = \frac{T_S}{2\pi}\left[\frac{e^{j\Omega_H t} - e^{-j\Omega_H t} - (e^{j\Omega_L t} - e^{-j\Omega_L t})}{t*j}\right]$$

$$h_r(t) = \frac{T_S}{2\pi}\left[\frac{2\mathrm{j} * \sin(\Omega_H t) - 2j * \sin(\Omega_L t)}{t*j}\right]$$

$$h_r(t) = \frac{\sin(\Omega_H t) - \sin(\Omega_L t)}{((t) * \Omega_S)/2}$$

Where sampling frequency $\Omega_S = \frac{2\pi}{T_S}$, $\Omega_H = \frac{M * \Omega_S}{2}$ and $\Omega_L = \frac{(M-1) * \Omega_S}{2}$

$$h_r(t) = \frac{\sin((\mathrm{M} * \Omega_S * t)/2) - \sin(((\mathrm{M}-1) * \Omega_S * t)/2)}{((t) * \Omega_S)/2}$$

$$\mathrm{h_r(t)} = \frac{\sin((\mathrm{M*t})\pi)/\mathrm{T_S}) - \sin(((\mathrm{M}-1)*\mathrm{t})\pi)/\mathrm{T_S})}{((\mathrm{t})*\pi)/\mathrm{T_S})} \tag{1.7}$$

The output of the bandpass filter is the convolution of the input signal x[n] with the impulse response $h_r(t)$. The bandpass signal can be recovered by an ideal bandpass filter. The Formula 1.8 shows the way to reconstruct the bandpass signal x[n]. The output of the filter is represented as $\hat{x}$. The output of the filter is given below.

$$\hat{x} = \sum_{n=-\infty}^{\infty} x[n] * \frac{\sin((\mathrm{M}*\pi(\mathrm{t-n*Ts}))/\mathrm{T}_S)) - \sin(((\mathrm{M}-1)*(\mathrm{t-n*Ts})*\pi)/\mathrm{T}_S)}{(t)*\pi/\mathrm{T}_S)} \tag{1.8}$$

The impulse response of the filter can be represented as Sinc function as follows:

$$h_r(t) = \frac{\sin((\mathrm{M} * \mathrm{t})\pi/\mathrm{T}_S)) - \sin(((\mathrm{M}-1) * \mathrm{t})\pi/\mathrm{T}_S))}{((t) * \pi)/\mathrm{T}_S)}$$

$$h_r(t) = M * Sinc\left(\frac{Mt}{T_s}\right) - (M-1) * Sinc\left(\frac{(M-1)t}{Ts}\right)$$

$$\hat{x} = \sum_{n=-\infty}^{\infty} x[n] * \left(M * Sinc\left(\frac{Mt}{T_s}\right) - (M-1) * Sinc\left(\frac{(M-1)t}{Ts}\right)\right) \tag{1.9}$$

3.1 Implementation

Following denotations are used in this project to implement , a function called ‘bandrecover’ is created. The function accepts three inputs such as ‘fl’, ‘fh’ ,’sta’ , which are lower cut off frequency, upper cut off frequency and state of the function that generates random number respectively. For the simulation, a bandpass signal is created. The Sampling frequency is made equal to two times the difference between the upper cut off frequency and lower cut off frequency, which is given as input parameters to function, and the sampling frequency chosen is less than the Nyquist frequency. Then a loop is started to create signal ‘s’ with a frequency of a range from lower cut off frequency (fl) to from upper cut off frequency (fh). The signal consists of many sin waves with various amplitudes which depend upon random state chosen. From the bandpass sampling theorem, M is calculated (equation 1.1 discussed above). To compare the final reconstructed result, a signal ‘ss’ is created with the same frequency range but sampled at a high-frequency rate. Here, the sampling frequency is chosen as 500Hz for the reference signal.

‘fl’: lower cut off frequency.
‘fh’:upper cut off frequency.
‘sta’: state of the function which generates random numbers.

Then to reconstruct the analog from the signal sampled at low frequency. A loop is started to implement the equation(1.9) given above. To compare the result, both reconstructed analog signal and high frequency sampled signal are plotted in the same figure. Finally, the FFT of both signals is plotted.

The main function is used to call the bandpass recovery function and pass various parameters.

3.2 Matlab Code:

```
%This function 'bandrecover' accepts the lower and upper cut off
%frequency of bandpass signal and state of random signal.

%**********************************************************
function bandrecover(fl,fh,sta)

fs=2*(fh-fl);
N=100;
Ts = 1/fs;
tn = Ts*(-N:1:N);
tdash=tn';

randn('state',sta)
a=0.25*randn(fh-fl,1);
s=zeros(2*N+1,1);
%This loop to construct bandpass signal with sum of sin wave
with frequency from fl to fh at sampling frequency tdash.
for i=1:(fh-fl)

     s1=a(i)*sin(2*pi*(i+fl)*tdash);
      s=s+s1;

end

ss=zeros(1000,1);
t = -1:2/999:1;

%Bandpass signal sampled at a frequency at 500Hz
for i=1:(fh-fl)
    s1=a(i)*sin(2*pi*(200+i)*t);
    ss=ss+s1;
end



%M =2Fm/8;
M=(2*(fh)/fs);
xh = 0;
```

```
%This loop computes reconstruction of bandpass signal
for i = 1:(2*N+1),
    n = i - N - 1;
    tw1 = M*(t - n*Ts)/Ts;
    tw2 = (M-1)*(t - n*Ts)/Ts;
    xh = xh + s(i)*(M*sinc(tw1(:)) -  (M-1)*sinc( tw2(:) ));
end

%Plot of both original and reconstructed on same figure
plot(t,xh,'-','linewidth',1.5)
hold on;
plot(t,ss','--','linewidth',1.5)
axis([-0.05 0.05 -2 2])
xlabel('Time in seconds')
grid

%FFT of reconstructed signal
Y = fft(xh);
L=1000;
P2 = abs(Y/L);
P1 = P2(1:L/2+1);
figure
Fs=500;
f = Fs*(0:(L/2))/L;
P1(2:end-1) = 2*P1(2:end-1);
plot(f,P1)
title('FFT of reconstructed signal')
xlabel('f (Hz)')
ylabel('Amplitude')

%FFT of original signal
Y1=fft(ss(1,:));
P2 = abs(Y1/L);
P1 = P2(1:L/2+1);
figure
Fs=500;
f = Fs*(0:(L/2))/L;
P1(2:end-1) = 2*P1(2:end-1);
plot(f,P1)
title('FFT of orginal signal')
xlabel('f (Hz)')
ylabel('Amplitude')

end
```

```
%Main function which calls the bandrecover function to pass the cut
%off frequencies.

 bandrecover(200,204,17)
 bandrecover(200,250,10)
 bandrecover(200,208,6)
```

4. Results and Evaluations

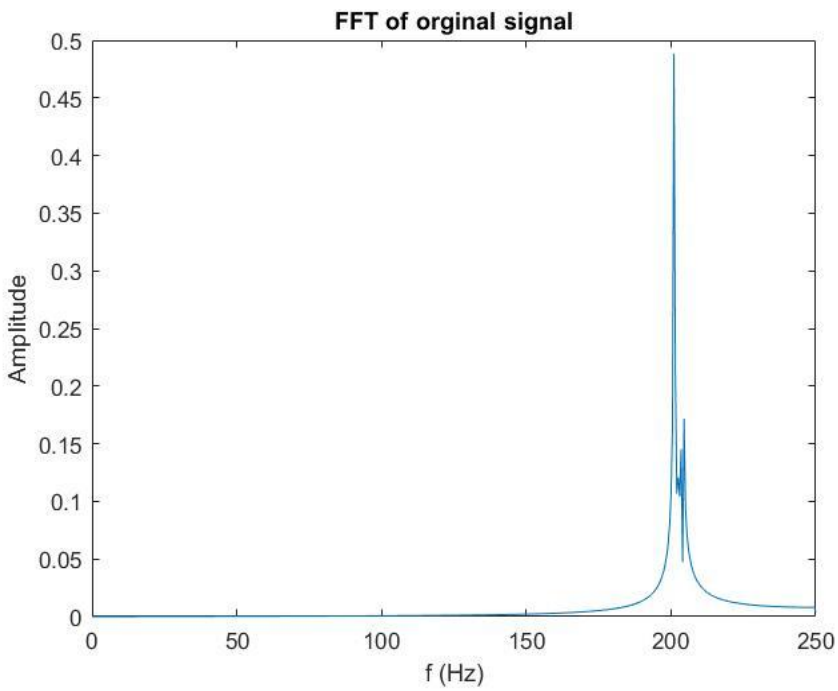

Figure 1:FFT of the original signal with a frequency range from 200Hz to 204Hz

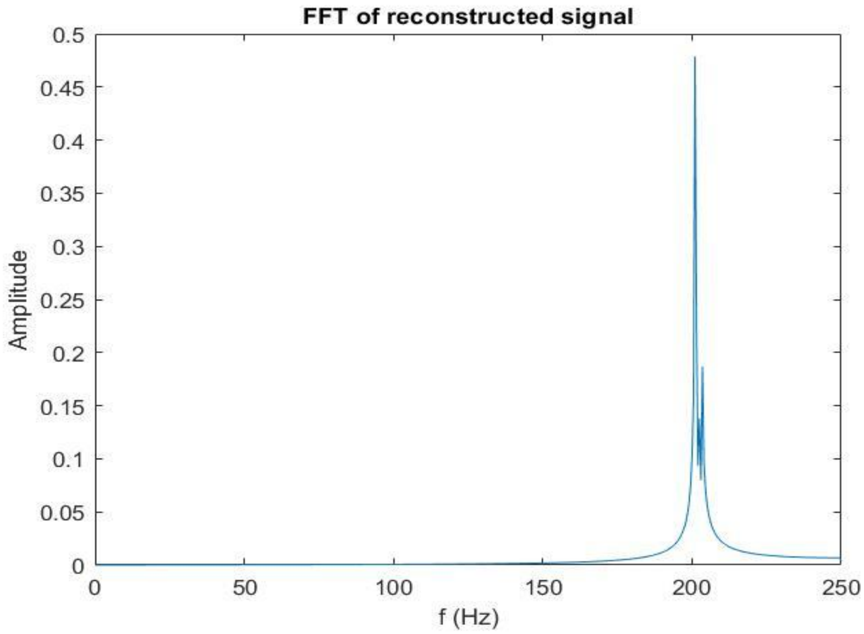

Figure 2: FFT of the reconstructed signal with a frequency range from 200Hz to 204Hz

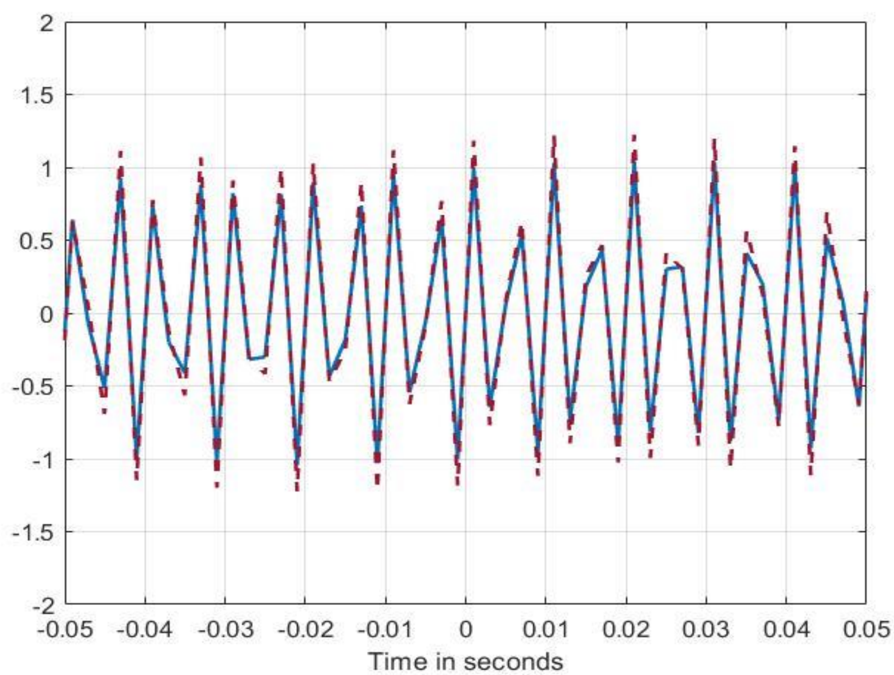

Figure 3: Bandpass signal of frequency between 200Hz and 204Hz with random state of 17:

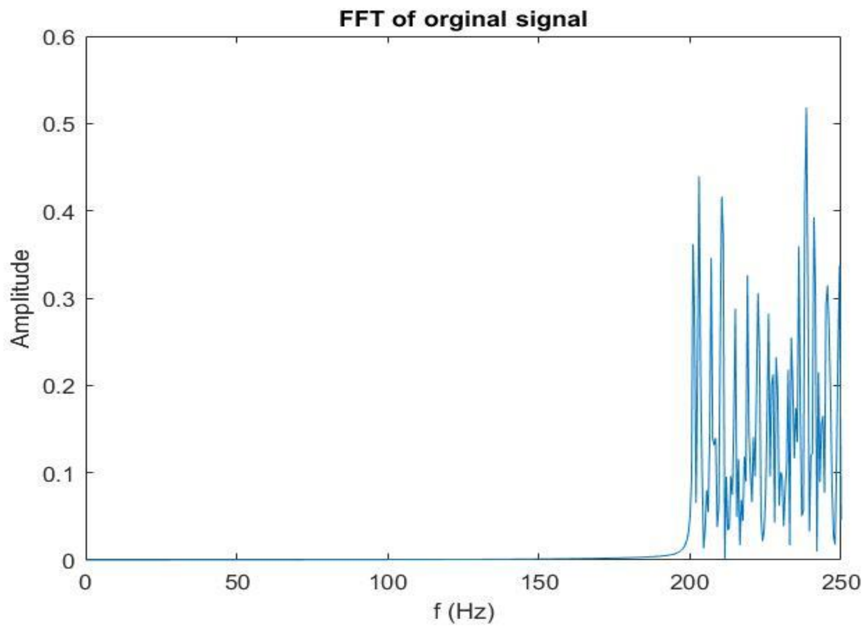

Figure 4: FFT of the original signal with a frequency range from 200Hz to 250Hz

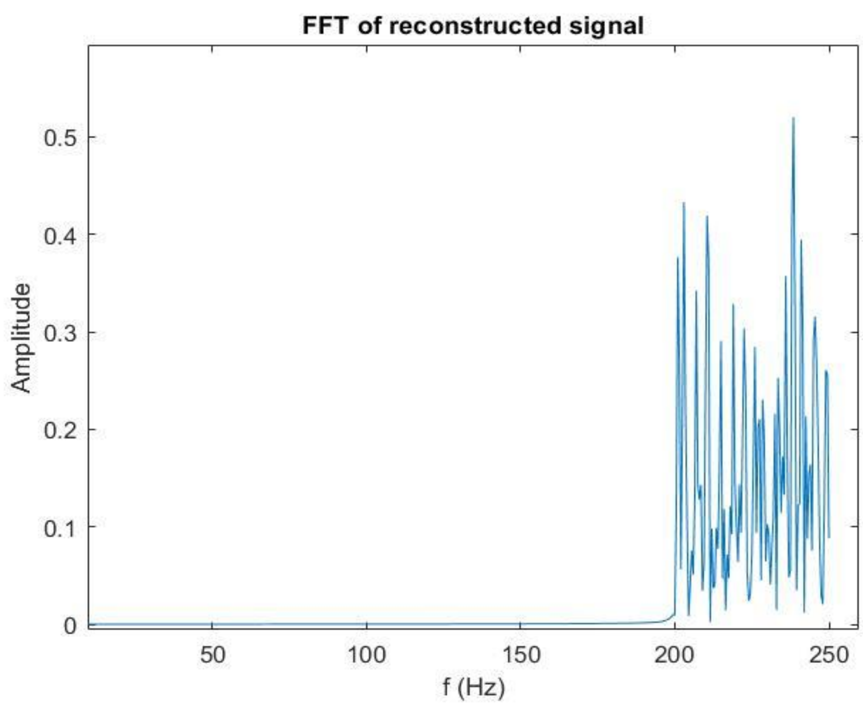

Figure 5: FFT of the reconstructed signal with a frequency range from 200Hz to 250Hz

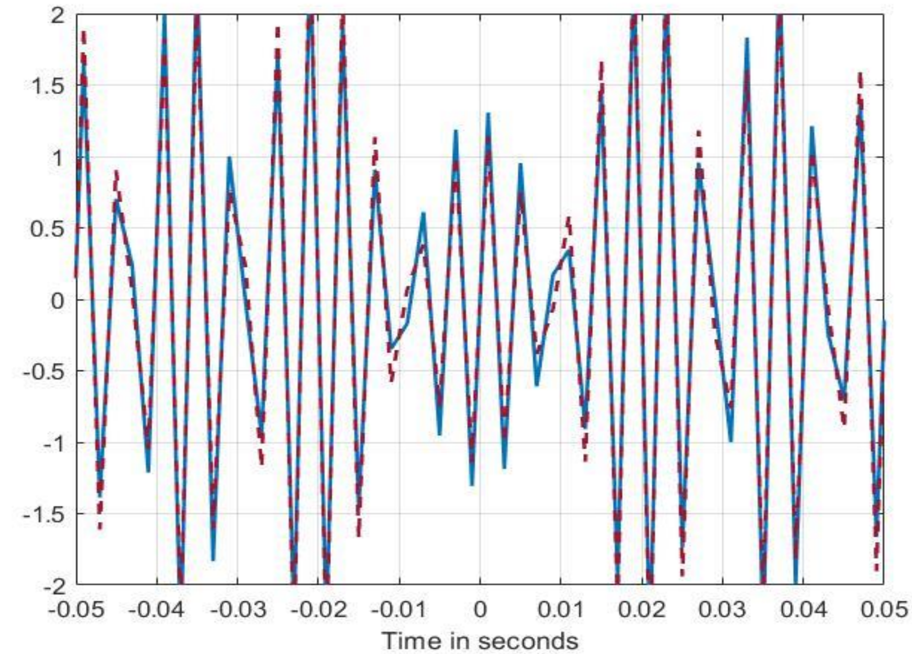

Figure 6: Bandpass signal of frequency between 200Hz and 250Hz with a random state of 10

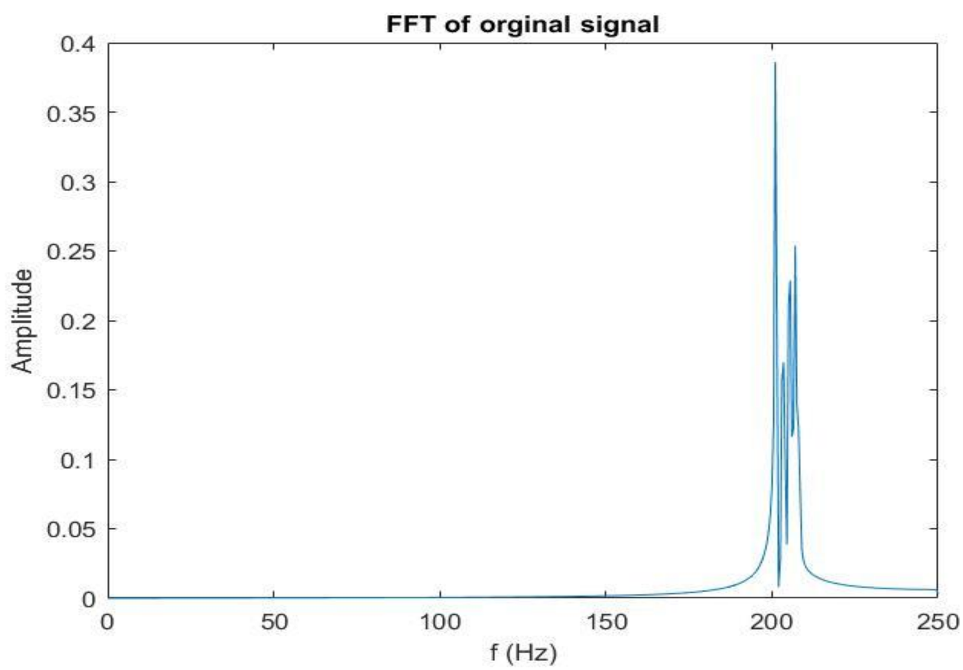

Figure 7: FFT of the original signal with a frequency range from 200Hz to 208Hz

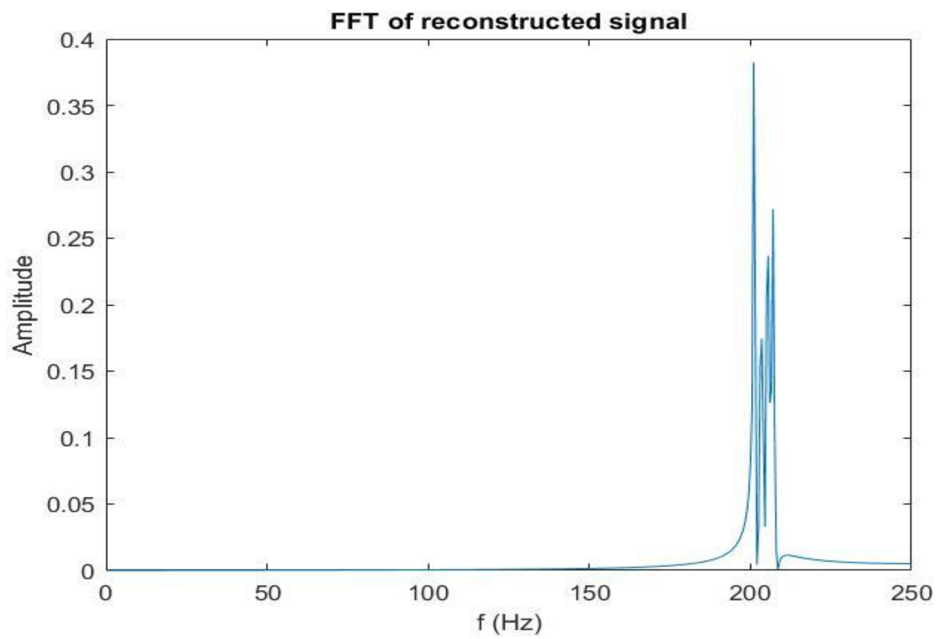

Figure 8: FFT of the reconstructed signal with a frequency range from 200Hz to 208Hz

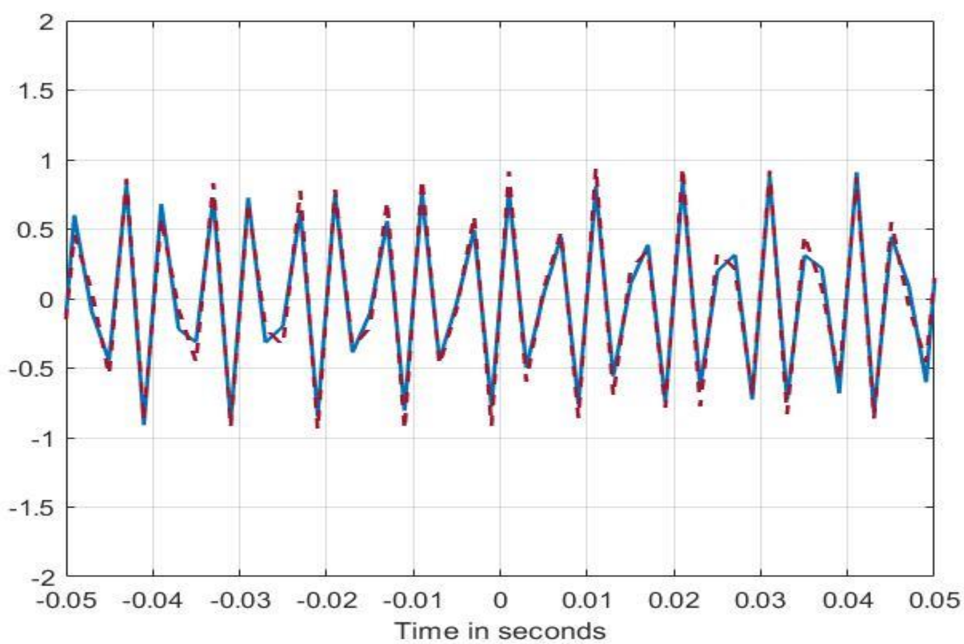

Figure 9: Bandpass signal of frequency between 200Hz and 208Hz with random state of 6

5. Discussion

In order to compare the reconstructed bandpass signal and the original signal, both signals are plotted in the same figure for different in ranges of frequencies. Here, we have considered three frequency ranges to such as 200Hz to 204Hz, 200Hz to 250Hz and 200Hz to 208Hz. Considering the amplitude response of all case(shown in Figure:3, Figure:6 and Figure:9 respectively shown above), reconstructed signal resembles the original signal with almost perfect amplitude. Reconstructed signal has the same frequency as the original signal which can be seen from the fourier transform of original and reconstructed signal.

6. Conclusion

Project has focused on the reconstruction of the analog bandpass signal. A system is successfully designed and implemented in MATLAB. The performance of the system is verified by comparing the amplitude response of the original and the reconstructed signals.

7. References

[1] W.-S. Lu, Applications of Digital Processing Techniques.
[2]Meng Xiangwei, "Sampling and reconstruction of bandpass signal," Proceedings of Third International Conference on Signal Processing (ICSP'96)*, Beijing, 1996, pp. 80-82 vol.1.*
doi: 10.1109/ICSIGP.1996.566977