1) Is $T : R^2 \to R$ defined by $T(x, y) = xy$ is a linear transformation

Soln:

Let $u, v \in R^3$

$u = (a, b)$; $v = (c, d)$

$T(u) = ab$ $T(v) = cd$

i) $T(u, +v) = T((a, b) + (c, d))$

$= T(a+c, b+d)$

$= (a+c)(b+d) = ab + ad + bc + cd$

$\neq ab + cd.$

$\neq T(u) + T(v)$

Hence T is not linear.

2) Verify that $T : R^3 \to R$ and $T(u) = \|u\|$ is a Linear Transformation or not.

Let $u, v \in R^3$

$T(u+v) = \|u+v\|$

$\leq \|u\| + \|v\|$ $[\because \|u+v\| \leq \|u\| + \|v\|]$

$= T(u) + T(v)$

$T(u+v) \neq T(u) + T(v)$

$\therefore T$ is not a linear Transformation.

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3) Is there a linear transformation $T: R^3 \to R^2$ such that $T(1,0,3) = (1,1)$ and $T(-2,0,-6) = T(2,1)$? Justify

Soln

Standard basis of R^3

$e_1 = (1,0,0) \quad e_2 = (0,1,0) \quad e_3 = (0,0,1)$

$(1,0,3) = 1e_1 + 0e_2 + 3e_3$

$T(1,0,3) = T(e_1 + 3e_3)$

$(1,1) = T(e_1) + 3T(e_3)$ — ①

$(-2,0,-6) = -2e_1 + 0e_2 - 6e_3$

$T(-2,0,-6) = T(-2e_1 - 6e_3)$

$(2,1) = -2T(e_1) - 6T(e_3)$ - ②

② $\Rightarrow$ $-2[(T(e_1) + 3T(e_3))] = (2,1)$

i.e, $T(e_1) + 3T(e_3) = (-1, -1/2)$ - ③

From ① & ③

Linear Transformation not possible.

4) Let $T: R^2 \to R^3$ be the linear transformation defined by $T(x,y) = (x+y, x-y, y)$. Then find the Range space of T.

Soln

$T: R^2 \to R^3$

$T(x,y) = (x+y, x-y, y)$

To find:

Range space of T:

$T(1,0) = (1,1,0)$

$T(0,1) = (1,-1,1)$

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$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 1 \end{bmatrix} \quad R_2 \to R_1 - R_2$$

$$\sim \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & -1 \end{bmatrix}$$

Basis of $R(T) = \{(1,1,0), (0,2,-1)\}$

Rank $T = 2$

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5 If $T: R^3 \to R^2$ defined by $T(x,y,z) = (x-y, 2z)$ is a linear transformation. Then find the nullity of T?

Soln

$$N(T) = \{(x,y,z) \mid T(x,y,z) = 0\}$$

$$T(x,y,z) = 0$$

$$\Rightarrow (x-y, 2z) = (0,0)$$

$$x - y = 0$$

$$2z = 0$$

$$z = 0, \quad x = y$$

$$N(T) = \{(x,x,0) \mid x \in R\}$$

Nullity of $T = 1$

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6) Let $T: R^2 \to R^2$ defined by $T(a_1, a_2) = (a_1 + a_2, a_1)$.

Find the dim $N(T)$?

Null space of T:-

$$N(T) = \{(a_1, a_2) \in R^2 \,/\, T(a_1, a_2) = 0\}$$

$$T(a_1, a_2) = 0$$

$$\Rightarrow (a_1 + a_2, a_1) = (0, 0)$$

$$a_1 + a_2 = 0 \quad — ①$$

$$a_1 = 0 \quad - ②$$

$$a_2 = 0.$$

$$N(T) = \{(0,0) \,/\, T(a_1, a_2) = 0\}$$

$$\dim N(T) = 0.$$

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7) Define Kernel of the transformation

Kernel (or) Null space:-

Let V and W be vector spaces and let $T: V \to W$ be linear transformation. Then the null space (or) kernel of T is defined by

ie, $N(T) = \{x \in V \,/\, T(x) = 0_W\}$

Here 0_W is the zero element of W.

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8) State Rank Nullity Theorem (or) Dimension theorem

Ans

Let $T: V \to W$ be linear transformation and V be a finite dimensional vector space Then

$$\dim[R(T)] + \dim[N(T)] = \dim(V)$$

$$\text{Rank}(T) + \text{Nullity}(T) = \dim(V)$$

9) Obtain the matrix representing the linear transformation $T: V_3(R) \to V_3(R)$ defined by $T(a,b,c) = (3a, a-b, 2a+b+c)$ with respect to the standard basis $\{e_1, e_2, e_3\}$

Soln:

The basis are

$B = \{(1,0,0), (0,1,0), (0,0,1)\}$

$B' = \{(1,0,0), (0,1,0), (0,0,1)\}$

$T(a,b,c) = (3a, a-b, 2a+b+c)$

$T(1,0,0) = (3,1,2)$

$T(0,1,0) = (0,-1,1)$

$T(0,0,1) = (0,0,1)$

The required matrix is

$$T = \begin{bmatrix} 3 & 0 & 0 \\ 1 & -1 & 0 \\ 2 & 1 & 1 \end{bmatrix}$$

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10) Let $T: P_3(R) \rightarrow P_2(R)$ be a linear transformation defined by $T(F(x)) = F'(x)$. Let B_1 and B_2 be the standard bases of $P_3(R)$ and $P_2(R)$ respectively. Then find $[T]$

Soln:

The standard basis are

$B_1 = \{1, x, x^2, x^3\}$, $B_2 = \{1, x, x^2\}$

$T(1) = 0 = 0.1 + 0.x + 0.x^2$

$T(x) = 1 = 1.1 + 0.x + 0.x^2$

$T(x^2) = 2x = 0.1 + 2.x + 0.x^2$

$T(x^3) = 3x^2 = 0.1 + 0.x + 3.x^2$

The required matrix is

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

11) Find the linear transformation $T: R^2 \rightarrow R^3$ determined by the matrix $\begin{bmatrix} 1 & 3 \\ 0 & 0 \\ 2 & -4 \end{bmatrix}$ with respect to the standard bases

soln:

$T: R^2 \rightarrow R^3$

The standard basis of R^2 is $(1,0)$ and $(0,1)$

$T(1,0) = (1,0,2)$

$T(0,1) = (3,0,-4)$

Let $(x,y) \in R^2$

$(x,y) = x(1,0) + y(0,1)$

$T(x,y) = x\,T(1,0) + y\,T(0,1)$

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$$= x(1,0,2) + y(3,0,-4)$$

$$T(x,y) = (x+3y, 0, 2x-4y)$$

The required Linear transformation is

$$T(x,y) = (x+3y, 0, 2x-4y)$$

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12) Find the linear transformation $T : V_3(R) \rightarrow V_3(R)$ determined by the matrix $\begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ -1 & 3 & 4 \end{pmatrix}$ with respect to the standard bases $\{e_1, e_2, e_3\}$

<u>Soln</u>

The standard basis of R^3 is $(1,0,0)$ $(0,1,0)$, $(0,0,1)$

$$T(1,0,0) = (1,0,-1)$$

$$T(0,1,0) = (2,1,3)$$

$$T(0,0,1) = (1,1,4)$$

Let $(x,y,z) \in R^3$

$$(x,y,z) = x(1,0,0) + y(0,1,0) + z(0,0,1)$$

$$T(x,y,z) = x\,T(1,0,0) + y\,T(0,1,0) + z\,T(0,0,1)$$

$$= x(1,0,-1) + y(2,1,3) + z(1,1,4)$$

$$= (x,0,-x) + (2y,y,3y) + (z,z,4z)$$

$$T(x,y,z) = (x+2y+z,\ 0+y+z,\ -x+3y+4z)$$

The required Linear transformation is

$$T(x,y,z) = (x+2y+z,\ 0+y+z,\ -x+3y+4z)$$

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13) Find the eigen values of $-3A$ are if $A = \begin{pmatrix} 3 & 1 & 2 \\ 0 & -1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$.

Soln

The eigen values of A are $3, -1, 1$

Because of upper Triangular matrices.

The eigen values of $-3A$ are $-9, 3, -3$

14) Test the matrix $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \in M_{2\times 2}(R)$ for diagonalizable

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

Characteristic Equation is $\lambda^2 - S_1\lambda + S_2 = 0$

$S_1 = 2, \quad S_2 = 0$

Characteristic Equation is $\lambda^2 - 2\lambda = 0$

$\lambda(\lambda - 2) = 0$

$\lambda = 0, \lambda = 2$

The eigen values are distinct.

So, A is diagonalizable.

15) Test the matrix $A = \begin{bmatrix} -2 & 3 \\ -10 & 9 \end{bmatrix} \in M_{2\times 2}(R)$ for diagonalizable

$A = \begin{bmatrix} -2 & 3 \\ -10 & 9 \end{bmatrix}$ C.E is $\lambda^2 - S_1\lambda + S_2 = 0$.

$S_1 = -2 + 9 = 7$

$S_2 = -18 + 30 = 12$

C.E is $\lambda^2 - 7\lambda + 12 = 0$.

$(\lambda - 3)(\lambda - 4) = 0$

$\lambda = 3, \lambda = 4$

The eigen values are distinct.

A is diagonalizable.

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