DATA SCIENCE NAIVE BAYES CLASSIFICATION

Suppose we have a dataset with features $x_1, ..., x_n$ and a class label C. What can we say about classification using Bayes' theorem?

$$P(\text{class } C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \text{class } C) \cdot P(\text{class } C)}{P(\{x_i\})}$$

Bayes' theorem can help us to determine the probability of an observation belonging to a class, given the data we observe.

This term is the **prior probability** of C. It represents the probability of an observation belonging to class C before the data is taken into account.

$$P(\text{class } C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \text{class } C) \cdot P(\text{class } C)}{P(\{x_i\})}$$

This term is the likelihood function. It represents the joint probability of observing features $\{x_i\}$ given that the observation belongs to class C.

$$P(\text{class } C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \text{class } C) \cdot P(\text{class } C)}{P(\{x_i\})}$$

This term is the normalization constant. It doesn't depend on C, and is generally ignored.

$$P(\text{class } C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \text{class } C) \cdot P(\text{class } C)}{P(\{x_i\})}$$

This term is the **posterior probability** of C. It represents the probability of an observation belonging to class C after the data is taken into account.

$$P(\operatorname{class} C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \operatorname{class} C) \cdot P(\operatorname{class} C)}{P(\{x_i\})}$$

The idea of Bayesian inference, then, is to **update** our beliefs about the distribution of c using the data ("evidence") at our disposal.

Q: Which one of the terms on the right side of the equation looks like it would be impossibly difficult to estimate?

A: The likelihood function.

$$P({x_i}|C) = P({x_1, x_2, ..., x_n})|C)$$

Observing this exactly would require us to have enough data for every possible combination of features to make a reasonable estimate.

Q: So what can we do about it?

A: Make a simplifying assumption. In particular, we assume that the features x_i are conditionally independent from each other:

$$P({x_i}|C) = P({x_1, x_2, ..., x_n}|C) \approx P({x_1}|C) * P({x_2}|C) * ... * P({x_n}|C)$$

This "naïve" assumption simplifies our estimation of the likelihood function.

$$P(\operatorname{class} C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \operatorname{class} C) \cdot P(\operatorname{class} C)}{P(\{x_i\})}$$

In summary, the training phase of the model involves computing the likelihood function, which is the conditional probability of each feature given each class.

The prediction phase of the model involves computing the posterior probability of each class given the observed features, and choosing the class with the highest probability.