Newton's Law of Cooling and Finding the Death Time of a person

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Abstract

The time estimate of a person since death is the most important data for many of the police and detective cases. It is known to all that a dead body started cooling after the circulation of the blood stop and our body started to lose heat energy from our body. In the early age many detective used the Mathematical Equation of Newton Laws of Cooling to find the correct time of death and by the help of this information they solve many cases. By this process we can only estimate the time of the death and our result will be not 100% accurate, but its result will be nearly to the actual death time of a body. Through this research paper we want to explain the Newton's Law of cooling. The second part has some examples and charts for finding the death time and also we have found the importance of mathematics in the Newton Law of Cooling.

Keywords: Energy, Newton's Law, Law of cooling

Introduction

Physicists Issac Newton gave a mathematical equation to find the temperature of an object when it loses heat energy. The heat energy transfer basically from the body to its surroundings. The rate of decreasing or increasing of the temperature of the body is directly proportional to the difference in the temperature between the body and the surroundings. Basically here is the formula to find the temperature of the body at a given time is given as below:

Temperature of the body at a specific time = surrounding temperature
+ (starting temperature – surrounding temperature) x exponential to the power of (-cooling constant x time)

Here is the mathematical equation of the above formula:

$$T(t) = Ts + (T_o - Ts)\, e^{(-kt)}$$

$T(t)$ = temperature of a body at a specific time (Kelvin, K)
t = time (s)
Ts = temperature of the surrounding area (Kelvin, K)
T_o = initial temperature of the body (Kelvin, K)
k = cooling constant, specific to the object ($1/s$)

Equation Methodology

First-order linear differential equation are widely used by scientists and engineers to solve the problems related to the temperature of the body.

Newton's Law of Cooling is modelled as a first-order initial-value problem.

$$\frac{dT}{dt} = k(T - T_S)$$

$$T(0) = T_0$$

Where T_0 isthe initial temperature of the body and k is is the constant of proportionality.

If T_S is constant by separating variable

$$\left(\frac{1}{T - T_S}\right) dT = k dt$$

Integrating both sides,

$$\int \left(\frac{1}{T - T_S}\right) dT = \int k dt$$

$$\ln(T - T_S) = kt + C$$

$$T - T_S = e^{kt+C}$$

$$T(t) = T_S + Ce^{kt+C}$$

Applying the initial condition,

$$T(0) = T_S + C_{e^0} \rightarrow C = T_0 - T_S$$

Hence,

$$T(t) = T_S + (T_0 - T_s)e^{kt}$$

If $k < 0, \lim_{t\to\infty} e^{kt} = 0$ Therefore, $\lim_{t\to\infty} T(t) = T_S$ and the temperature of the body approaches to that of its surroundings.

Literature Review

1.1 Newton's Law of cooling

It defined as the rate of loss of heat energy from a body is proportional to the difference in the temperature between the body and its environment. Simply put a cup of hot tea, it will cool down faster in a cold room then in a hot room. So Newton's Law states that about an instantaneous rate of change of the temperature. Hence, he applied one principle that has main function to calculate the temperature of hot iron ball, by analysis of the time taken by ball to cool down at particular temperature and comparing this value to the time taken to cool down through known value at ordinary temperature (The Encyclopaedia Britannica, 1910).Given law states that temperature of the body, which is excess from it surrounding and observed in equal interval of time then it's found to be in a mathematical common ratio. The inaccuracy of Newton's law becomes considerable at high temperatures.

1.2. Introduction to the Principle of Heat Transfer

Heat transfer is a science that studies the energy transfer between two bodies due to temperature difference.Basically in common language we can say that the rate of flow of transfer of heat energy is directly depend on temperature(amount of thermal energy) and flow of heat energy(movement of heat energy from place to place).On a microscopic scale thermal heat energy is directly proportional to kinetic energy of molecules of the body, means the greater the thermal energy of body causes greater thermal agitation of the body. This states that heat energy is transfer from a body of higher thermal energy to lower thermal energy. Many materials have properties of transferring thermal energy between two region having different temperature. This process is very important in surface emissivities, fluid viscosities, specific heat, thermal conductibility and many more. We may conclude this that this is solution many heat problem issue.Heat transfer mechanisms can be grouped into three broad categories: Conduction, Convection and Radiation.

Application

Finding the time of death in the following case:-

A man shot dead at Place Dadar, Mumbai (Date: 12 Oct 2018)

The Detective arrives at the scene at 10:23 pm. immediately; the temperature of the body is taken and is found to be 80° F. The detective finds that the room temperature has been constant for 68^oF.

From the crime scene evidence is obtained. Again the temperature of the body is taken and found to be 78.5^oC. This last temperature reading was taken exactly one hour after the first one.

Now using Newton's Law of cooling we have:

$T(t) = Te + (T0 - Te)\, e^{-kt}$,

Where $T(t)$ is the temperature of the object at time t, Te is the constant temperature of the environment, $T0$ is the initial temperature of the object, and k is a constant that depends on the material properties of the object.

Using Newton's law of cooling we have the following two equations,

$T(x) = 80 = 68 + (98.6 - 68)\, e^{-kt}$,   (1)

$T(x+1) = 78.5 = 68 + (98.6 - 68)\, e^{-k(x+1)}$,   (2)

Simplifying (1) gives,

$12 = 30.6\, e^{-kx}$,   (3)

Simplifying and using laws of exponents,

$10.5 = 30.6\, e^{-kx}\, e^{-k}$.   (4)

Solving (3) and (4) equations. These are two equations having two unknowns.

$$e^{-kr} = \frac{12}{30.6}$$

Substituting the value of e^{-kx} in (4) we get,

$$10.5 = 30.6 \, \frac{12}{30.6} e^{-k}$$

$$e^{-k} = \frac{10.5}{12}$$

$$\text{-k} = \ln\frac{10.5}{12}$$

$$k = 0.13353$$

Now that we have a value for *k*, we can use this to solve for the remaining unknown, x. Substituting $k \approx 0.134$ into (3) yields,

$$\frac{12}{30.6} \approx e^{-0.13353r}$$

$$\ln\frac{12}{30.6} \approx -0.13353r$$

$$r \quad \approx \ 7.0$$

Thus, the death time is found to be 7 hours. Therefore the individual have died 7 hours before the detective has arrived.

Conclusion

Basically, by this experiment we solve the real life problems with Mathematics and Science. We predict the death time of a body by the help of some information about body's surroundings temperature and Newton's Laws of Cooling. There are many equation and formula in physics which are incomplete without Mathematics. So, Mathematics is also a important part of Physics. And this Newton's Law of cooling is a very important law given by Sir Issac Newton.

Recommendation

This Newton Law of cooling is directly related and dependent to thermal radiation of a body. So in future if you want to get deeper knowledge about Newton's Law of Cooling then you have to study Thermal Radiation of Body.

Acknowledgement

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