

SOME PRIME LABELING OF GRAPH

Dissertation submitted to **H.H.THE RAJAH'S COLLEGE (Autonomous B⁺)**

In partial fulfillment of the requirements for the award of the degree of

MASTER OF SCIENCE IN APPLIED MATHEMATICS

Submitted by

R. SARANRAJ

Register No. 22PMT 4116

Under the Guidance of

Mrs. K. NACHAMMAL, M.Sc., M.Phil., PGDCA.,

Assistant Professor



PG AND RESEARCH DEPARTMENT OF MATHEMATICS

H.H.THE RAJAH'S COLLEGE (AUTONOMOUS – B⁺)

(AFFILIATED TO BHARATHIDASAN UNIVERSITY, TIRUCHIRAPPALLI)

PUDUKKOTTAI – 622 001

MARCH 2024

Mrs. K. NACHAMMAL, M.Sc., M.Phil., PGDCA.,

Assistant Professor,

Department of Mathematics,

H.H. The Rajah's College (Autonomous B⁺),

Pudukkottai – 622 001.

CERTIFICATE

This is to certify that the dissertation entitled “**SOME PRIME LABELING OF GRAPH**” submitted in partial fulfillment of the requirements for the award of the Degree of **MASTER OF SCIENCE** In Mathematics, H.H. THE RAJAH'S COLLEGE (Autonomous B⁺), Pudukkottai, Bharathidasan University, Tiruchirappalli, is a record of this research work done by **R. SARANRAJ (Reg. No. 22PMT 4116)** under my supervision and guidance during the academic period (2023 - 2024).

Signature of the Guide

Signature of the HOD

Signature of the Examiner

R. SARANRAJ,
Reg. No. 22PMT 4116,
M.Sc., MATHEMATICS,
H.H. The Rajah's College (Autonomous B⁺),
Pudukkottai – 622 001.

DECLARATION

I hereby declare that the dissertation entitled, “ **SOME PRIME LABELING OF GRAPH** ” submitted to H. H. The Rajah's college (Autonomous B⁺), Pudukkottai, Bharathidasan University, Tiruchirappalli in partial fulfillment of the requirements for the award of the Degree of **MASTER OF SCIENCE** in Mathematics a record of work done by me under the supervision and guidance. **Mrs. K. NACHAMMAL, M.Sc., M.Phil., PGDCA.,** Assistant Professor, Department of Mathematics, H.H. The Rajah's College (Autonomous B⁺), Pudukkottai and this dissertation has not formed the basis for the award of any Degree/ Diploma/ Fellowship or other similar titles to any candidate of any University.

Station : Pudukkottai

Signature of the Candidate

Date :

R. SARANRAJ

ACKNOWLEDGEMENT

First I wish to render my most heart full and sincere thanks to **MY MOTHER** and **FATHER** whose love, support and encourage throughout my life has made me achieve this cherished good.

I would like to thank **THE GOD ALMIGHTY** for this wonderful presence with me for the preparation of dissertation work.

I also express my sincere thanks to **Dr. B. BUVANESWARI, M.Com., M.Phil., MBA., Ph.D.**, principal H.H. The Rajah's College (Autonomous B⁺), Pudukkottai for her encouragement and advice during the course of dissertation. & for having granted necessary permission & facilities to do this dissertation.

I express my sincere thanks to **Dr. KR. BALASUBRAMANIAN, M.Sc., M.Phil., Ph.D.**, Associate Professor & Head of the Department of mathematics, H.H. The Rajah's College (Autonomous B⁺), Pudukkottai for his encouragement and advice during the course of dissertation & helped me a lot to develop this dissertation and complete successfully.

Now I wish to express my deep sense of gratitude to my respected guide **Mrs. K. NACHAMMAL, M.Sc., M.Phil., PGDCA.**, Assistant Professor, Department of mathematics, H.H. The Rajah's College (Autonomous B⁺), Pudukkottai for having suggested the topic.

Further I also like to thank all the **Faculty members of Mathematics Department and my Friends** for their support. As an important factor I wish to thank all my friends for their constant support & encouragement completion of this dissertation report.

R. SARANRAJ

TABLE OF CONTENTS

CHAPTER NO	TITLE	PAGE NO
I	INTRODUCTION	1
II	PRELIMINARIES	3
III	PRIME LABELING OF CERTAIN GRAPHS	13
IV	PRIME LABELING OF DIFFERENT GRAPHS	24
V	CONCLUSION	35
	REFERENCE	37

CHAPTER I

INTRODUCTION

Throughout this project, we consider only finite simple undirected graph. The graph G has vertex set $V = V(G)$ and edge set $E = E(G)$.

The labeling of a graph G is an assigning of integers either to the vertices or edges or both subject to certain conditions.

The notion of a prime labeling was introduced by Roger Entringer and was discussed in a paper by Tout.A. Many researchers have studied prime graph for example in Fu.H. have proved that the path P_n on n vertices is a prime graph.

In Dretsky.T have proved that the C_n on n vertices is a prime graph. In Lee.S have proved that the wheel W_n is a prime graph if and only if n is even. In Vaidya.k have proved the prime labeling for some cycle related graphs.

In this project, we study some prime labeling of graph.

CHAPTER II

CHAPTER II

PRELIMINARIES

Definition 2.1

A graph G consist of a pair $(V(G), E(G))$ where $V(G)$ is a non empty finite set whose elements are called points or vertices and $E(G)$ is another set of unordered pairs of distinct elements of $V(G)$. The elements of $E(G)$ are called edges of graph.

Example 2.2

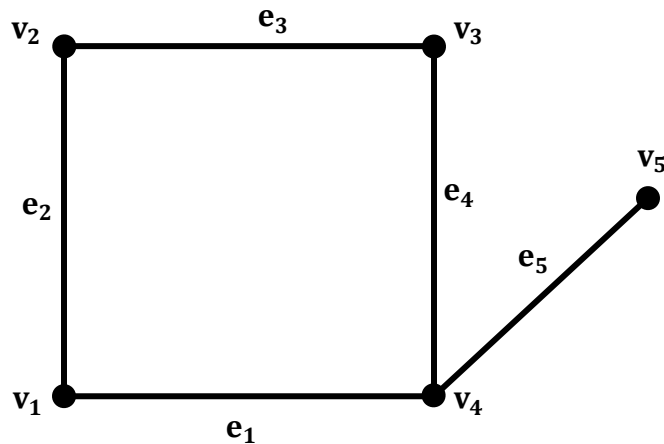


Fig 2.1: Graph (G)

Definition 2.3

A graph g is said to be a subgraph of a graph G if all the vertices and all the edges of g are in G , and each edge of g has the same end vertices in g as in G .

Example 2.4

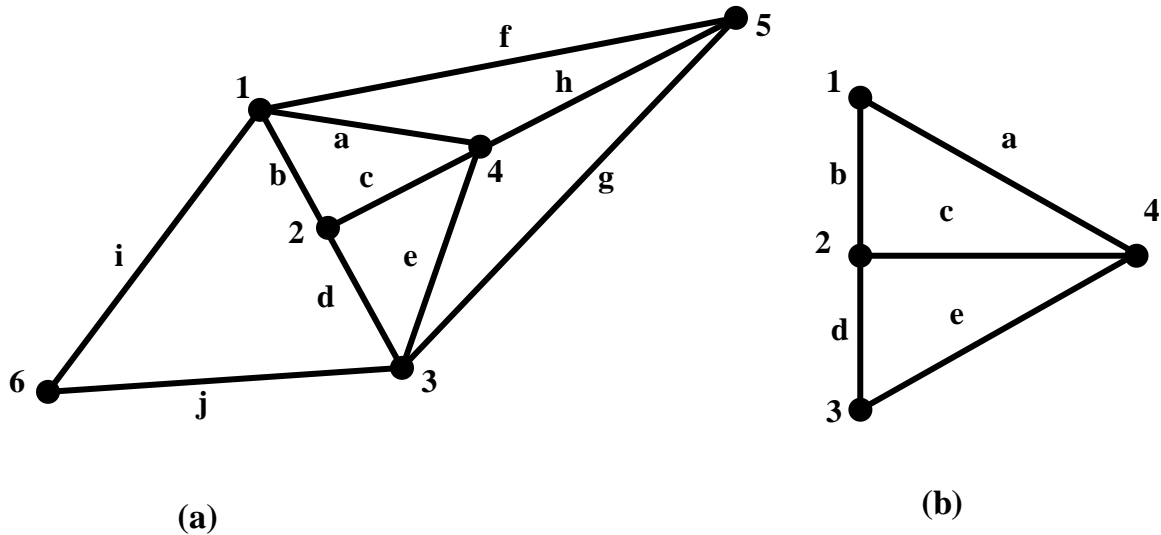


Fig 2.2: Graph (a) and one of its subgraph (b)

Definition 2.5

Let G be a graph. A walk is defined as a finite alternating sequence of vertices and edges. Beginning and ending with vertices, such that each edge is incident with the vertices preceding and following it.

For example,

Fig 2.3 **a 5 b 2 c 3 d 1 e** is a walk.

Example 2.6

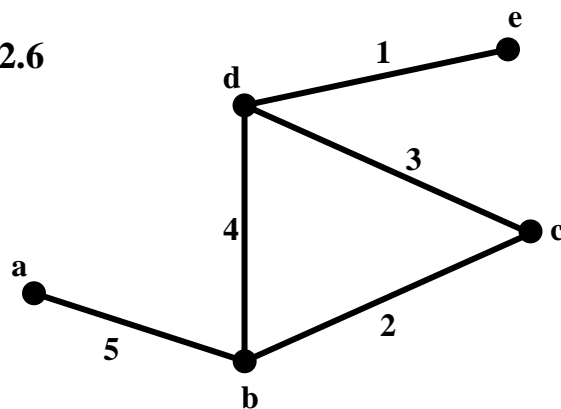


Fig 2.3: Graph (G)

Definition 2.7

A walk to begin and end at the same vertex, such a walk is called a closed walk. A walk is not closed is called an open walk.

Definition 2.8

An open walk in which no vertex appears more than once is called a path.

For example,

Fig 2.4 **a 1 b 3 e 5 d 8 c** is a path and **a 1 b 3 e 5 d 6 b 2 c** is not a path.

Definition 2.9

A closed walk in which no vertex (except the initial and the final vertex) appears more than once is called a circuit or cycle. That is, a cycle is a closed nonintersecting walk.

For example,

Fig 2.4 **a 1 b 2 c 8 d 5 e 7 a** is a cycle.

Example 2.10

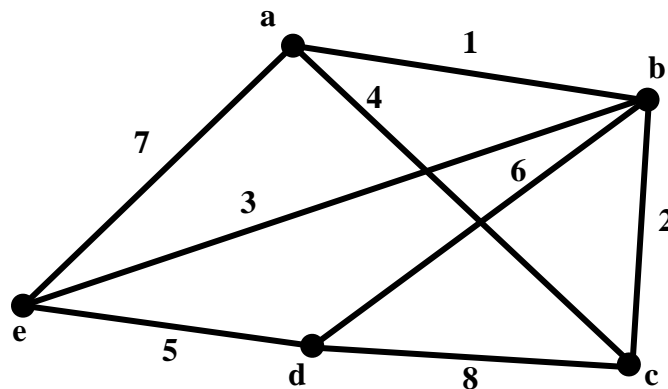


Fig 2.4: Graph (G)

Definition 2.11

The number of edges incident on a vertex v_i , with self-loops counted twice, is called the degree, $d(v_i)$ of vertex v_i .

For example,

Fig 2.4 $d(a) = d(c) = d(d) = d(e) = 3$ and $d(b) = 4$

Definition 2.12

The sum of the degree of all vertices in G is twice the number of edges in G . That is,

$$\sum_{i=1}^n d(v_i) = 2e$$

Definition 2.13

A vertex of degree one is called a pendent vertex.

For example,

Fig 2.5 $d(A) = d(C) = d(D) = d(E) = 1$. Hence A,C,D,E is a pendent vertex.

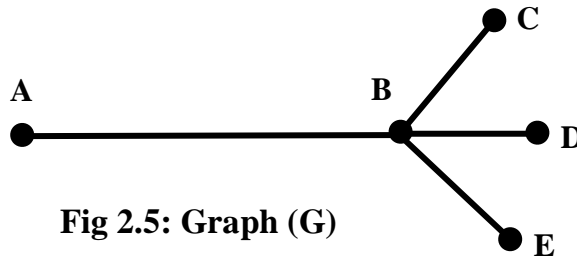
Example 2.14

Fig 2.5: Graph (G)

Definition 2.15

A graph in which all vertices are of equal degree is called a regular graph or simply a regular. If every vertex in a graph G has the same degree r , then the graph G is called a regular graph of degree r , or r -regular graph.

Example 2.16

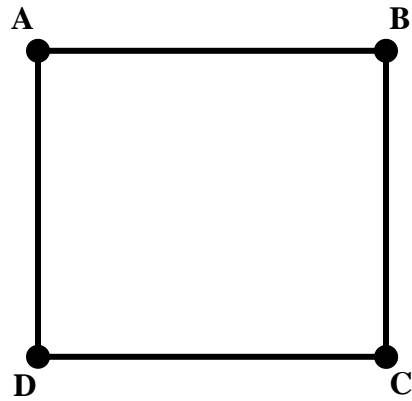


Fig 2.6: r -regular graph

Definition 2.17

A graph G is said to be connected if there is at least one path between every pair of vertices in G . Otherwise, G is disconnected.

Example 2.18

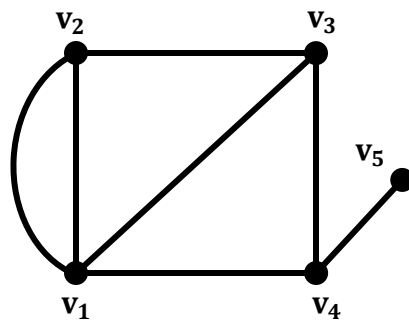


Fig 2.7: Connected graph

Definition 2.19

A graph that has no self-loops and parallel edges is called a simple graph.

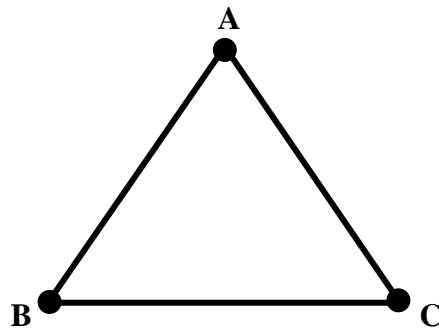
Example 2.20

Fig 2.8: Simple graph

Definition 2.21

An undirected graph is graph. That is, a set of vertices are connected together, where all the edges are bidirectional.

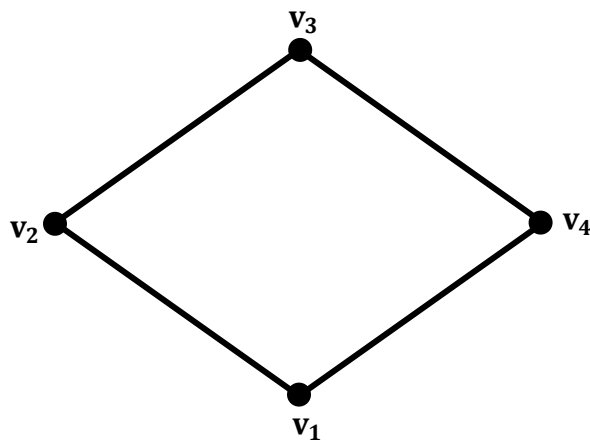
Example 2.22

Fig 2.9: Undirected graph

Definition 2.23

A bipartite graph is one whose vertex set can be partitioned into two subsets X and Y , So that each edge has one end in X and one end in Y ; Such a partition (X,Y) is called a bipartition of the graph.

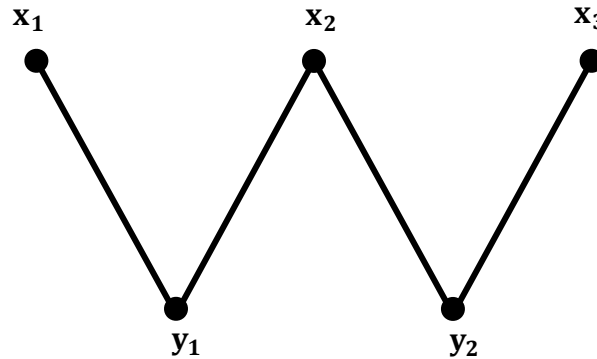
Example 2.24

Fig 2.10: Bipartite graph

Definition 2.25

A complete bipartite graph is a simple bipartite graph with bipartition (X,Y) in which each vertex of X is joined to each vertex of Y ; if $|X| = m$ and $|Y| = n$, such a graph is denoted by $K_{m,n}$.

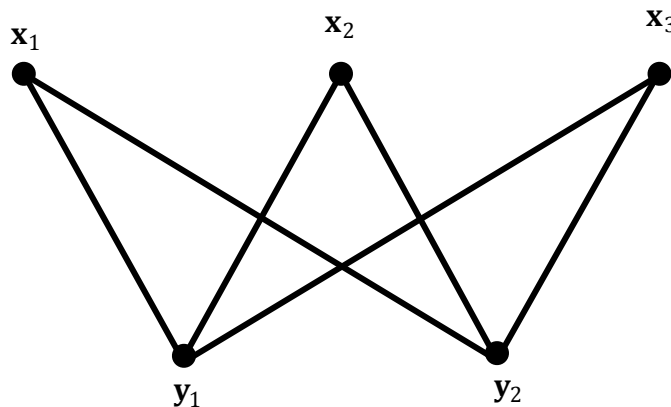
Example 2.26

Fig 2.11: Complete Bipartite graph $K_{3,2}$

Definition 2.27

A simple graph in which there exists an edge between every pair of vertices is called a complete graph.

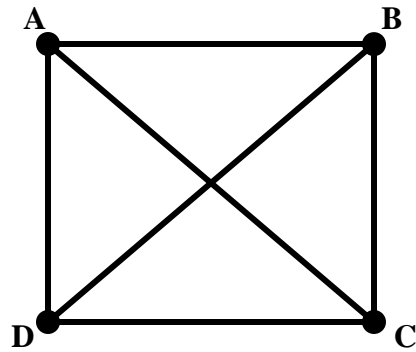
Example 2.28

Fig 2.12: Complete graph

Definition 2.29

A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions.

Definition 2.30

A graph in which each vertex is assigned unique name or label that graph is called the labeled graph.

Example 2.31

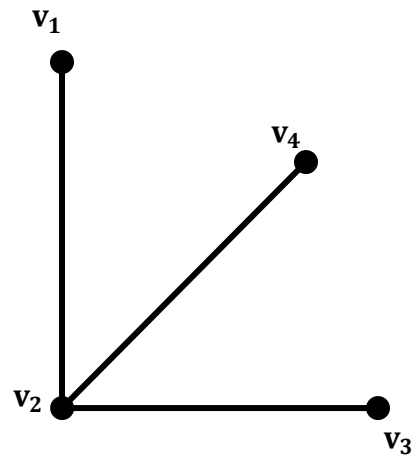


Fig 2.13: Labeled graph

CHAPTER III

CHAPTER III

PRIME LABELING OF CERTAIN GRAPHS

Definition 3.1

Let $G(V, E)$ be a graph with p -vertices and q -edges. A bijection $f: V(G) \rightarrow \{1, 2, 3, \dots, P\}$ is called a prime labeling if for each edge $e = \{uv\}$, such that $\gcd\{f(u), f(v)\} = 1$. A graph which admits prime labeling is called a prime graph.

Definition 3.2

The Friendship graph T_n is a set on n triangles having a common central vertex.

Example 3.3

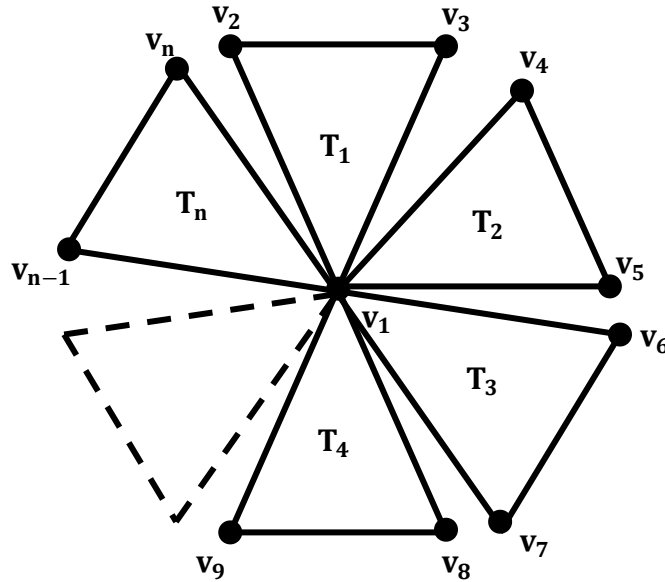


Fig 3.1: Friendship graph T_n

Theorem 3.4

The Friendship graph T_n is a prime graph.

Proof:

Let $V(T_n) = \{v_1, v_2, \dots, v_{2n+1}\}$ with v_1 as the centre vertex.

Let $E(T_n) = \{v_1v_i / 2 \leq i \leq 2n+1\} \cup \{v_{2i}v_{2i+1} / 1 \leq i \leq n\}$

Define a labeling f by

$f: V(T_n) \rightarrow \{1, 2, 3, \dots, 2n+1\}$ as follows

Let $f(v_i) = i$ for $1 \leq i \leq 2n+1$

Now,

$\gcd\{f(v_1), f(v_i)\} = 1$ for $2 \leq i \leq 2n+1$

$\gcd\{f(v_{2i}), f(v_{2i+1})\} = 1$ for $1 \leq i \leq n$

Then f admits prime labeling.

Hence T_n is a prime graph.

Example 3.5

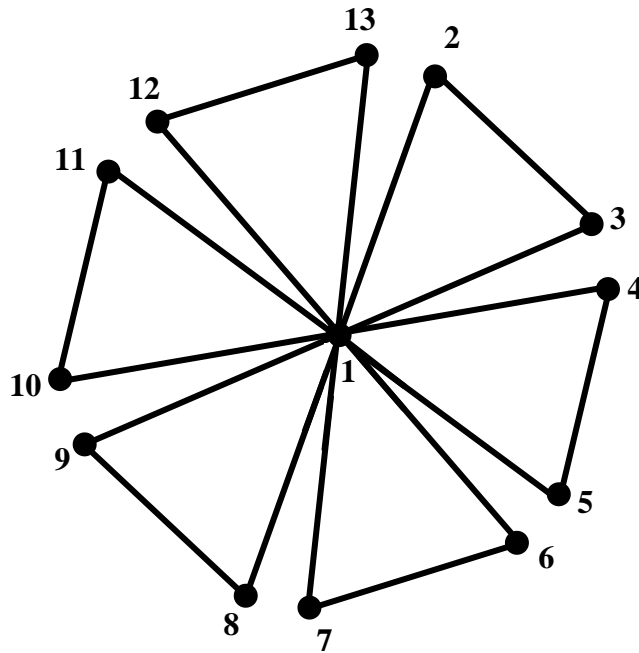
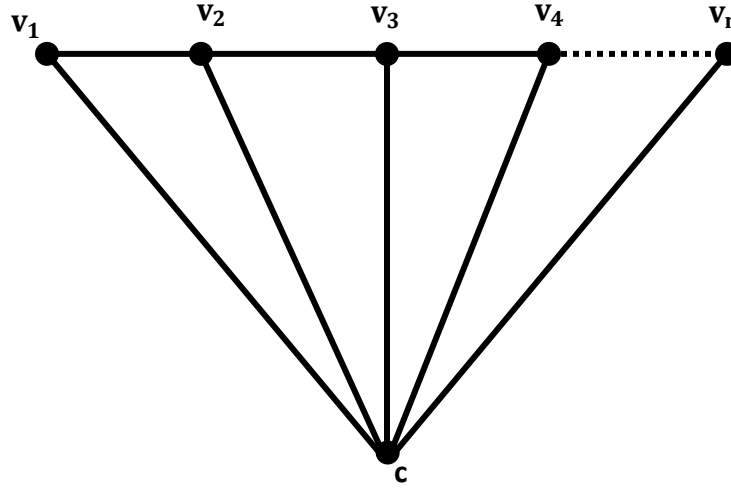


Fig 3.2: Prime labeling of friendship graph T_6

Definition 3.6

A fan graph obtained by joining all vertices of F_n , $n \geq 2$ is a path P_n to a further vertex called the center.

Thus F_n contains $n+1$ vertices say $c, v_1, v_2, v_3, \dots, v_n$ and $(2n-1)$ edges say cv_i , $1 \leq i \leq n$ and $v_i v_{i+1}$, $1 \leq i \leq n-1$.

Example 3.7**Fig 3.3: Fan graph F_n** **Theorem 3.8**

The Fan graph F_n is a prime graph.

Proof:

Let $G = F_n$ be a fan graph.

Let $V(G) = \{v_1, v_2, v_3, \dots, v_n, c\}$

Let $E(G) = \{cv_i / 1 \leq i \leq n\} \cup \{v_i v_{i+1} / 1 \leq i \leq n-1\}$

Define a labeling f by

$f: V(G) \rightarrow \{1, 2, 3, \dots, n+1\}$ as follows

Let $f(c) = 1$

$f(v_1) = 2$

$f(v_2) = 3$

$f(v_3) = 4$

$$f(v_i) = i + 1 \quad \text{for } 1 \leq i \leq n$$

Now,

$$\gcd \{f(c), f(v_i)\} = 1 \quad \text{for } 1 \leq i \leq n$$

$$\gcd \{f(v_i), f(v_{i+1})\} = 1 \quad \text{for } 1 \leq i \leq n-1$$

Then f admits prime labeling.

Hence G is a prime graph.

Example 3.9

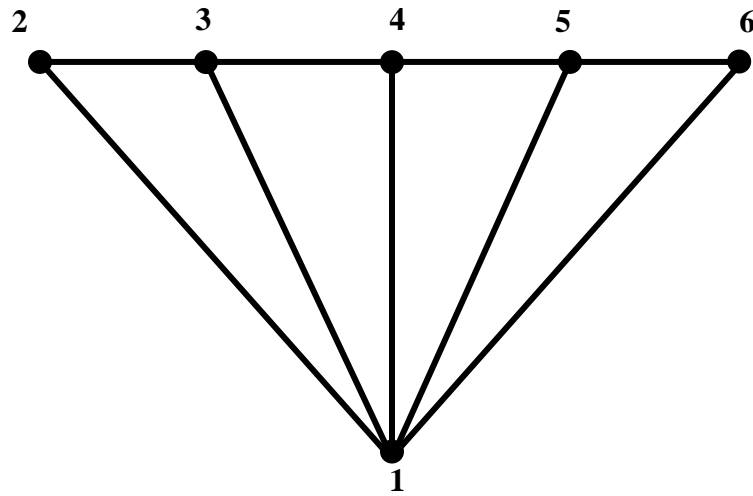


Fig 3.4: Prime labeling of fan graph F_5

Definition 3.10

The Franklin graph is a 3-regular graph with 12 vertices and 18 edges.

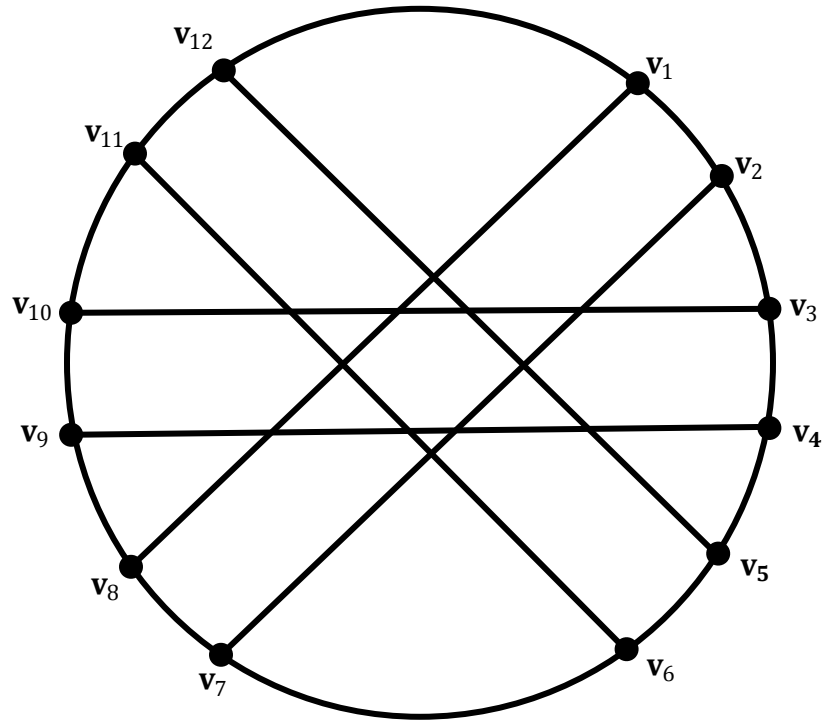
Example 3.11

Fig 3.5: Franklin graph (FG)

Theorem 3.12

The Franklin graph FG is a prime graph.

Proof:

Let FG be the Franklin graph with 12 vertices and 18 edges.

$$\text{Let } V(\text{FG}) = \{v_1, v_2, v_3, \dots, v_{12}\}$$

$$\text{Let } E(\text{FG}) = \{v_i v_{i+1} / 1 \leq i \leq 11\} \cup \{v_{12} v_1\} \cup \{v_i v_{9-i} / 1 \leq i \leq 2\} \cup \{v_i v_{13-i} / 3 \leq i \leq 4\} \cup \{v_i v_{17-i} / 5 \leq i \leq 6\}$$

$$|V(\text{FG})| = 12 \quad \text{and} \quad |E(\text{FG})| = 18$$

Define a labeling f by

$$f: V(\text{FG}) \rightarrow \{1, 2, 3, \dots, 12\} \quad \text{as follows}$$

$$\text{Let } f(v_i) = i \quad \text{for } 1 \leq i \leq 12$$

Now,

$$\gcd \{f(v_i), f(v_{i+1})\} = 1 \quad \text{for } 1 \leq i \leq 11$$

$$\gcd \{f(v_{12}), f(v_1)\} = 1$$

$$\gcd \{f(v_i), f(v_{9-i})\} = 1 \quad \text{for } 1 \leq i \leq 2$$

$$\gcd \{f(v_i), f(v_{13-i})\} = 1 \quad \text{for } 3 \leq i \leq 4$$

$$\gcd \{f(v_i), f(v_{17-i})\} = 1 \quad \text{for } 5 \leq i \leq 6$$

Then f admits prime labeling.

Hence FG is a prime graph.

Example 3.13

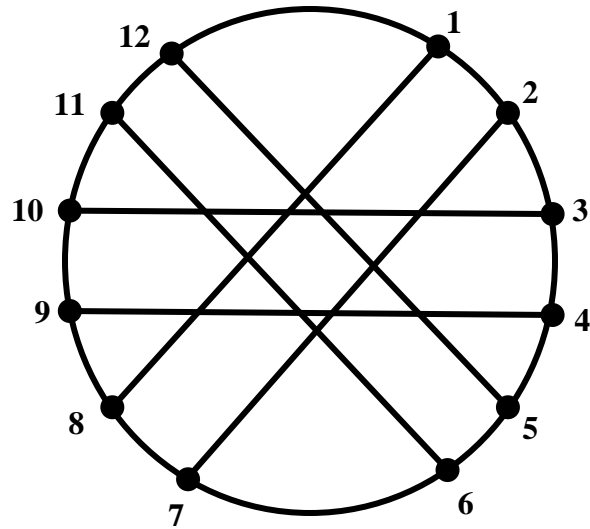


Fig 3.6: Prime labeling of Franklin graph

Definition 3.14

The Heawood graph is an undirected graph with 14 vertices and 21 edges. Heawood graph is a 3-regular graph.

Example 3.15

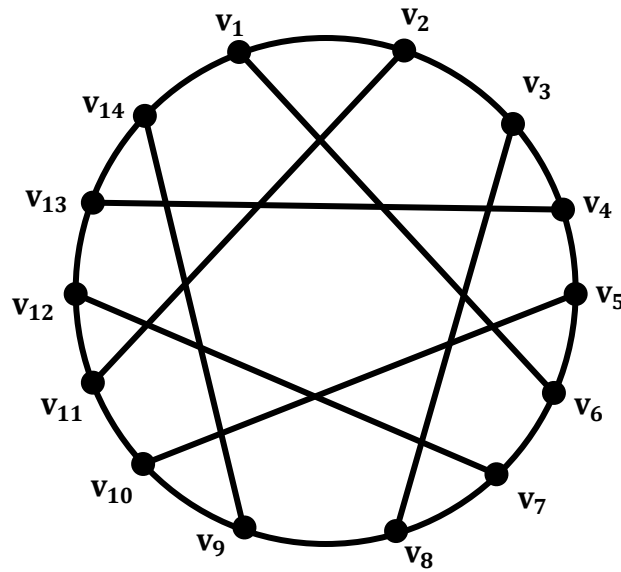


Fig 3.7: Heawood graph

Theorem 3.16

The Heawood graph is a prime graph.

Proof:

Let G be the heawood graph with 14 vertices and 21 edges.

$$\text{Let } V(G) = \{v_1, v_2, v_3, \dots, v_{14}\}$$

$$\text{Let } E(G) = \{v_i v_{i+1} / 1 \leq i \leq 13\} \cup \{v_1 v_{14}\} \cup \{v_i v_{i+5} / i = 1, 3, 5, 7, 9\} \cup \{v_2 v_{11}\} \cup \{v_4 v_{13}\}$$

$$|V(G)| = 14 \quad \text{and} \quad |E(G)| = 21$$

Define a labeling f by

$$f: V(G) \rightarrow \{1, 2, 3, \dots, 14\} \quad \text{as follows}$$

$$\text{Let } f(v_i) = i \quad \text{for } i = 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 13, 14$$

$$f(v_1) = 5$$

$$f(v_5) = 1$$

Now,

$$\gcd \{f(v_i), f(v_{i+1})\} = 1 \quad \text{for } 1 \leq i \leq 13$$

$$\gcd \{f(v_1), f(v_{14})\} = 1$$

$$\gcd \{f(v_i), f(v_{i+5})\} = 1 \quad \text{for } i = 1, 3, 5, 7, 9$$

$$\gcd \{f(v_2), f(v_{11})\} = 1$$

$$\gcd \{f(v_4), f(v_{13})\} = 1$$

Then f admits prime labeling.

Hence G is a prime graph.

Example 3.17

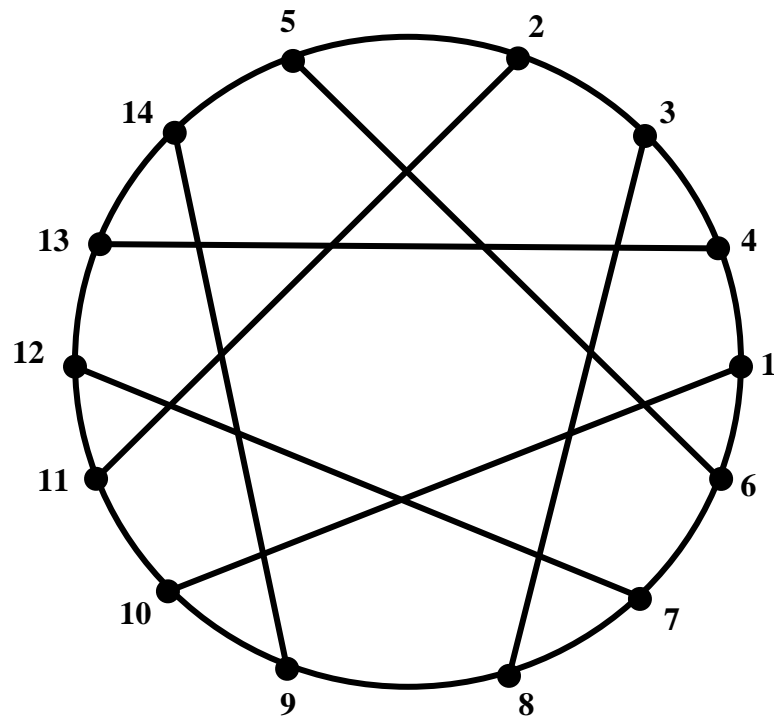


Fig 3.8: Prime labeling of Heawood graph

CHAPTER IV

CHAPTER IV

PRIME LABELING OF DIFFERENT GRAPHS

Definition 4.1

A simple graph of 'n' vertices ($n \geq 3$) and n edges forming a cycle of length 'n' is called as a cycle graph. In a cycle graph, all the vertices are of degree is 2. By adding the path, the new vertices of v_1, v_2, \dots, v_m , and the new graph G is denoted by $C_n P_m$.

Theorem 4.2

The cycle and path graph are a prime graph. Then the graph $C_n P_m$ is a prime labeling of the graph.

Proof:

Let G be the graph obtained by joining cycle C_n and a path P_m .

To prove the graph $C_n P_m$ admits to prime labeling of the graph.

Let u_1, u_2, \dots, u_n be the vertices of cycle C_n .

Let v_1, v_2, \dots, v_m be the vertices of path P_m .

Define a labeling,

Let $f(u_i) = i$ for $1 \leq i \leq n$

Assume $u_n = v_1$

Let $f(v_{i+1}) = n + i$ for $1 \leq i \leq (m-1)$

Now,

$$\gcd \{f(u_i), f(u_{i+1})\} = 1 \quad \text{for} \quad 1 \leq i \leq n$$

$$\gcd \{f(u_1), f(u_n)\} = 1$$

$$\gcd \{f(v_{i+1}), f(v_{i+2})\} = 1 \quad \text{for} \quad 1 \leq i \leq (m-1)$$

$$\gcd \{f(v_1), f(v_2)\} = 1$$

Then f admits prime labeling.

Hence $C_n P_m$ is a prime graph.

Example 4.3

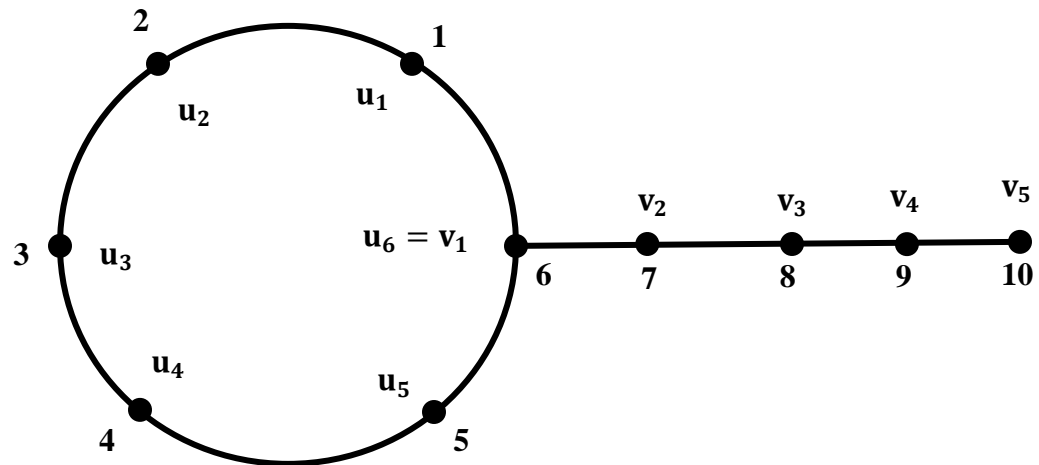


Fig 4.1: Prime labeling of graph $C_6 P_5$

Definition 4.4

The crown graph on $2n$ vertices is an undirected graph with two set of vertices $\{u_1, u_2, \dots, u_n\}$ and $\{v_1, v_2, \dots, v_n\}$ and with on edge from u_i to v_j whenever $i \neq j$. By adding the path, the new vertices of w_1, w_2, \dots, w_m , and the new graph G is denoted by crown $C_n P_m$.

Theorem 4.5

The crown and path graph are a prime graph. Then the graph $C_n P_m$ is a prime labeling of the graph.

Proof:

Let G be the graph obtained by joining crown C_n by a path P_m .

To prove the graph $C_n P_m$ admits to prime labeling of the graph.

Let u_1, u_2, \dots, u_n be the vertices of crown C_n .

Let v_1, v_2, \dots, v_n be the vertices of crown C_n .

Let w_1, w_2, \dots, w_n be the vertices of path P_m .

Define a labeling,

$$\text{Let } f(u_i) = 2i - 1 \quad \text{for} \quad 1 \leq i \leq n$$

$$f(v_i) = 2i \quad \text{for} \quad 1 \leq i \leq n$$

Assume $v_n = w_1$

$$\text{Let } f(w_{i+1}) = 2n + i \quad \text{for} \quad 1 \leq i \leq (m-1)$$

Now,

$$\gcd \{f(u_i), f(u_{i+1})\} = 1 \quad \text{for} \quad 1 \leq i \leq n$$

$$\gcd \{f(u_1), f(u_n)\} = 1$$

$$\gcd \{f(u_i), f(v_i)\} = 1 \quad \text{for } 1 \leq i \leq n$$

$$\gcd \{f(w_{i+1}), f(w_{i+2})\} = 1 \quad \text{for } 1 \leq i \leq (m-1)$$

$$\gcd \{f(w_1), f(w_2)\} = 1$$

Then f admits prime labeling.

Hence $C_n P_m$ is a prime graph.

Example 4.6

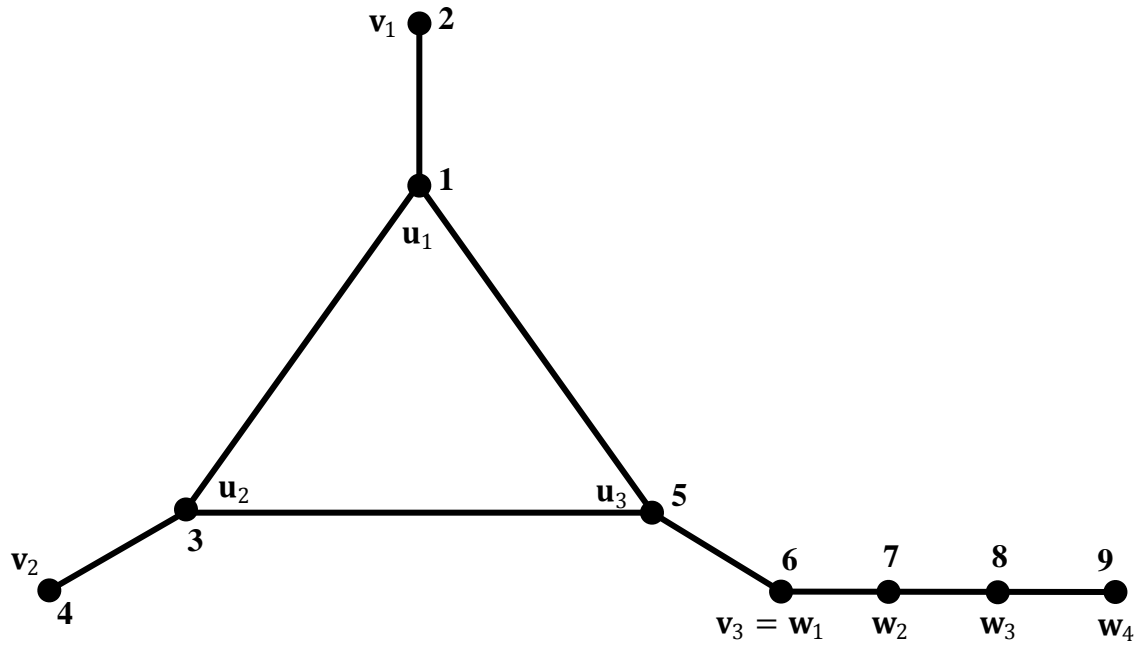


Fig 4.2: Prime labeling of graph $C_3 P_4$

Definition 4.7

The Friendship graph F_n is a graph which consists of n -triangles with a common vertex. If $V(G) = 2n + 1$ and $E(G) = 3n$ by adding the path, the new vertices of v_1, v_2, \dots, v_m , and the new graph G is denoted by $F_n P_m$.

Theorem 4.8

The friendship and path graph are a prime graph. Then the graph $F_n P_m$ is a prime labeling of the graph.

Proof:

Let G be the graph obtained by joining friendship F_n by a path P_m .

To prove the graph $F_n P_m$ admits to prime labeling of the graph.

Let u_1, u_2, \dots, u_n be the vertices of friendship F_n .

Let v_1, v_2, \dots, v_n be the vertices of path P_m .

Define a labeling,

$$\text{Let } f(u_i) = i \quad \text{for } 1 \leq i \leq (2n+1)$$

$$\text{Assume } u_1 = v_1$$

$$\text{Let } f(v_{i+1}) = (2n + 1) + i \quad \text{for } 1 \leq i \leq (m-1)$$

Now,

$$\gcd \{f(u_1), f(u_i)\} = 1 \quad \text{for } 2 \leq i \leq (2n+1)$$

$$\gcd \{f(u_{2i}), f(u_{2i+1})\} = 1 \quad \text{for } 1 \leq i \leq n$$

$$\gcd \{f(v_1), f(v_2)\} = 1$$

$$\gcd \{f(v_{i+1}), f(v_{i+2})\} = 1 \quad \text{for} \quad 1 \leq i \leq (m-1)$$

Then f admits prime labeling.

Hence $F_n P_m$ is a prime graph.

Example 4.9

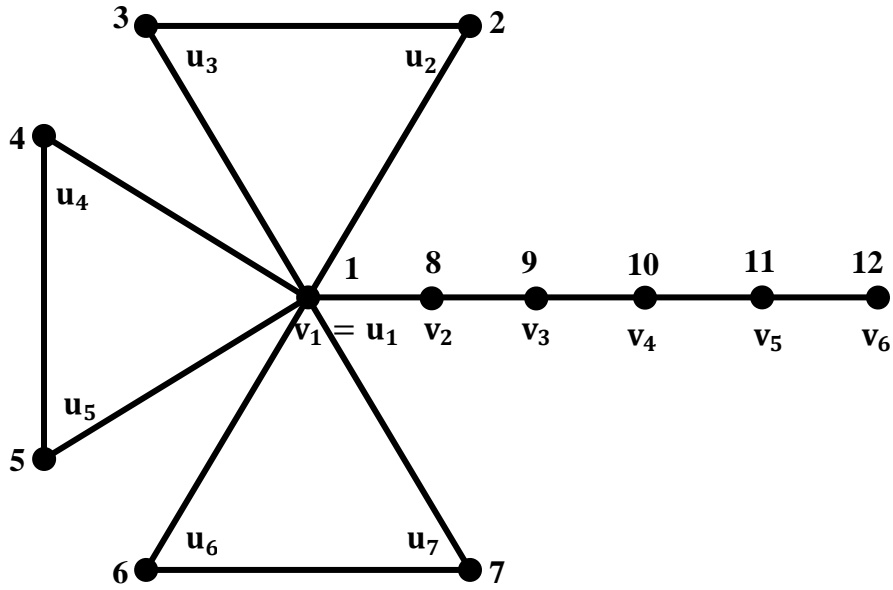


Fig 4.3: Prime labeling of graph $F_3 P_6$

Definition 4.10

The star graph S_n is special type of graph in which $n-1$ vertices have degree 1 and single vertex have $n-1$ degree. This look like $n-1$ vertex is connected to central vertex. A star graph with total n vertex is termed as S_n . By adding the path, the new vertices of v_1, v_2, \dots, v_m , and the new graph G is denoted by $S_n P_m$.

Theorem 4.11

The star and path graph are a prime graph. Then the graph $S_n P_m$ is a prime labeling of the graph.

Proof:

Let G be the graph obtained by joining star S_n by a path P_m .

To prove the graph $S_n P_m$ admits to prime labeling of the graph.

Let $u_0, u_1, u_2, \dots, u_n$ be the vertices of star S_n .

Let v_1, v_2, \dots, v_m be the vertices of path P_m .

Define a labeling,

Let $f(u_0) = 1$

$$f(u_i) = 1 + i \quad \text{for} \quad 1 \leq i \leq n$$

Assume $u_n = v_1$

$$\text{Let } f(v_{i+1}) = (n + 1) + i \quad \text{for} \quad 1 \leq i \leq (m-1)$$

Now,

$$\gcd \{f(u_0), f(u_i)\} = 1 \quad \text{for} \quad 1 \leq i \leq n$$

$$\gcd \{f(v_1), f(v_2)\} = 1$$

$$\gcd \{f(v_{i+1}), f(v_{i+2})\} = 1 \quad \text{for} \quad 1 \leq i \leq (m-1)$$

Then f admits prime labeling.

Hence $S_n P_m$ is a prime graph.

Example 4.12

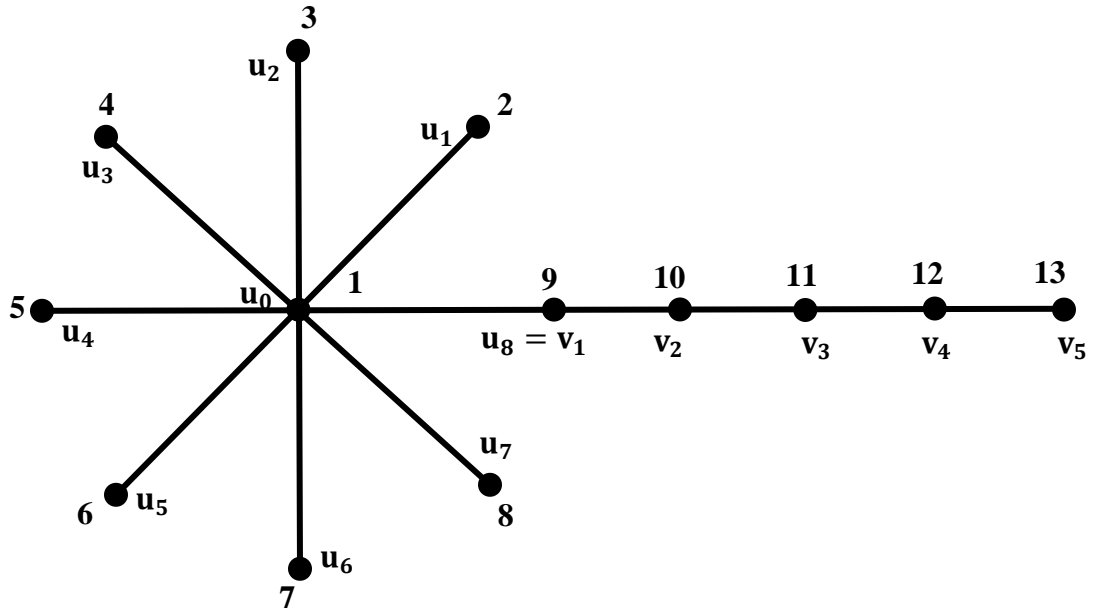


Fig 4.4: Prime labeling of graph $S_8 P_5$

Definition 4.13

The Gear graph G_n also known as a bipartite wheel graph is a wheel graph with a vertex added between each pair of adjacent vertices of the outer cycle, further vertex is called center. Gear graph G_n has $2n+1$ vertices and $3n$ edges. By adding the path, the new vertices of w_1, w_2, \dots, w_m , and the new graph G is denoted by $G_n P_m$.

Theorem 4.14

The Gear and path graph are a prime graph. Then the graph $G_n P_m$ is a prime labeling of the graph.

Proof:

Let G be the graph obtained by joining Gear G_n by a path P_m .

To prove the graph $G_n P_m$ admits to prime labeling of the graph.

Let u_1, u_2, \dots, u_n and $v_0, v_1, v_2, \dots, v_n$ be the vertices of Gear G_n .

Let w_1, w_2, \dots, w_n be the vertices of path P_m .

Define a labeling,

Let $f(v_0) = 1$

$$f(v_i) = 2i \quad \text{for} \quad 1 \leq i \leq n$$

$$f(u_i) = (2i + 1) \quad \text{for} \quad 1 \leq i \leq n$$

Assume $u_n = w_1$

$$\text{Let } f(w_{i+1}) = (2n + 1) + i \quad \text{for} \quad 1 \leq i \leq (m-1)$$

Now,

$$\gcd \{f(v_0), f(v_i)\} = 1 \quad \text{for} \quad 1 \leq i \leq n$$

$$\gcd \{f(v_i), f(u_i)\} = 1 \quad \text{for} \quad 1 \leq i \leq n$$

$$\gcd \{f(u_i), f(v_{i+1})\} = 1 \quad \text{for} \quad 1 \leq i \leq n$$

$$\gcd \{f(v_1), f(u_n)\} = 1$$

$$\gcd \{f(w_1), f(w_2)\} = 1$$

$$\gcd \{f(w_{i+1}), f(w_{i+2})\} = 1 \quad \text{for} \quad 1 \leq i \leq (m-1)$$

Then f admits prime labeling.

Hence $G_n P_m$ is a prime graph.

Example 4.15

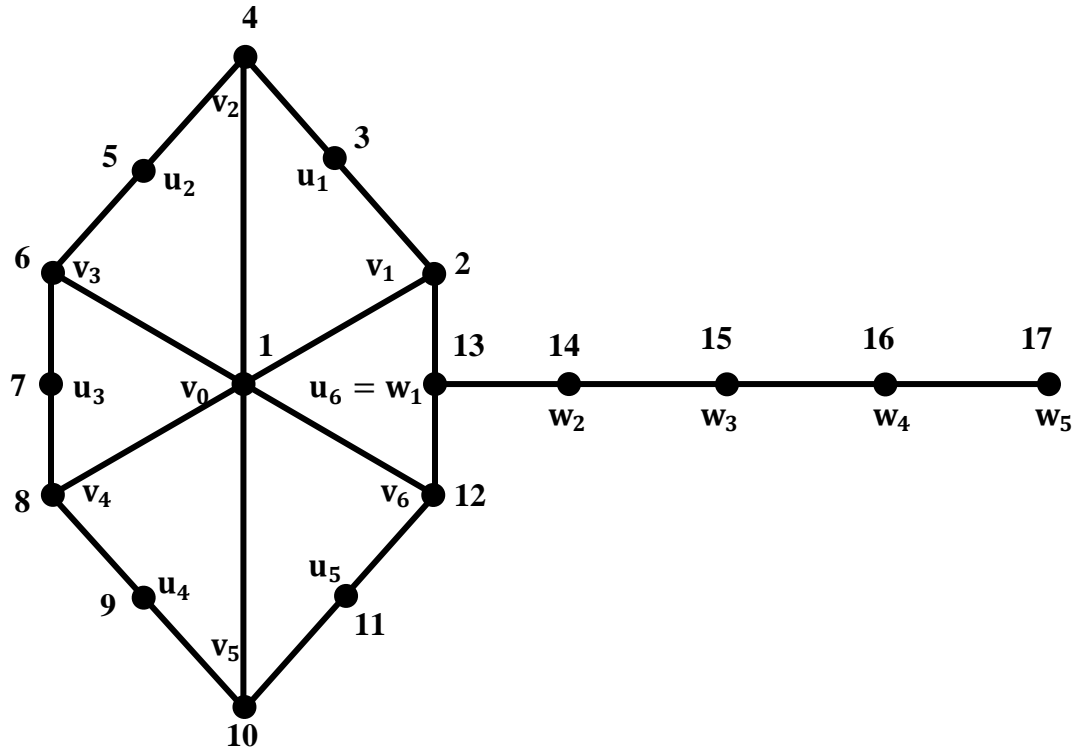


Fig 4.5: Prime labeling of graph $G_6 P_5$

CHAPTER V

CONCLUSION

Prime labeling has been studied for the five decades. A huge number of research articles published in the area of graph theory and discrete mathematics.

In the first chapter, we discuss, “Introduction to the labeling of a graph” , “Introduction to the prime labeling of a graph”.

In the second chapter, we present some basic definitions which are needed to the subsequent chapters.

In the third chapter, we study the prime labeling of certain graphs for, friendship graph T_n , fan graph F_n , franklin graph FG , heawood graph.

In the fourth chapter, we study the prime labeling of different graphs for, C_nP_m , F_nP_m , crown C_nP_m , S_nP_m and G_nP_m in necessary conditions, In future work for some connected graphs.

REFERENCE

REFERENCE

- [1] Gallian JA , et al. A Dynamic survey of Graph Labeling. Electron J Comb. 2017.
- [2] Tout A, et al. Prime labeling of Graphs, Nati Acada Sci Lett. 1982;11:365-368.
- [3] Vaidya K, et al. Prime Labeling for some cycle related graphs. J Mat Res.2010;23:98-104.
- [4] Lee SM et al. On the amalgamation of prime graphs. Bull Malays Math Sci Soc.1988;11:59-67.
- [5] Bondy JA, et al. Graph theory with applications. Elsevier North Holland. New York.(1976).
- [6] Vaithilingam K. et al. Prime Labeling For Some Crown Related Graphs. Int J Sci. 2013;2.
- [7] Dretsky et al. Vertex prime labeling of graphs in graph theory. Combinatorics and applications 1991;1:299-359.
- [8] Meena S et al. Prime labeling for some fan related graphs. Int J Eng Res. Technol 2012;1.
- [9] Parmar YM. Edge Vertex Prime Labeling for Wheel, Fan and Friendship Graph. Int J Math Stat Invent. 2017;5:23-29.
- [10] Fu HC, et al. On prime labeling discrete math. 1994;127:181-186.
- [11] Ramya N, et al. On prime labeling of some classes of graphs. Int J Comput Appl. 2012;44:975-8887.

Evaluated and conducted vivavoce on

.....

External Examiner

Internal Examiner and Guide