

Q1

Numerically integrate the following using Midpoint, Trapezoidal and Simpson techniques for $N = 8, 16$ and 24 and compare the result (in tabular format) with the actual analytical result.

$$\int_1^4 \sqrt{1 + \frac{1}{x}} dx$$

In [2]:

```

from My_Lib import*
import math

def f(x): #Defining the function
    return math.sqrt(1+1/x)

N = [8, 16, 24] #values of N
Analytic_solution = 3.620184280 #Actual Value of Integral

#Listing the erros and solutions
solution = []
Modulus_of_Error=[]

for i in N:
    sol_mid = integration_midpoint(f,4,1,i)
    solution.append(sol_mid)
    Modulus_of_Error.append(abs(Analytic_solution-sol_mid))

print("Actual analytical value of the integral", Analytic_solution )

print ("\n~~~~~MIDPOINT METHOD~~~~~")
print ("|| Value via Midpoint\t\t|| Modulus of Error\t||")
print ("~~~~~")
for i in range (0,len(N)):
    print ("||\t%.9f"%solution[i], "(N =", N[i], ")\t\t", "%.6f"%Modulus_of_Error[i], "\t|")

print ("~~~~~")

#####Trapezoidal Rule#####
#####

N = [8, 16, 24] #value of N
Analytic_solution = 3.620184280 # actual value

#Listing the erros and solutions
solution = []
Modulus_of_Error=[]

for i in N:
    sol_trap = integration_trap(f,4,1,i)
    solution.append(sol_trap)
    Modulus_of_Error.append(abs(Analytic_solution-sol_trap))

print ("\n~~~~~Trapezoidal METHOD~~~~~")
print ("|| Value via Trapezoidal\t\t|| Modulus of Error\t||")
print ("~~~~~")
for i in range (0,len(N)):
    print ("||\t%.9f"%solution[i], "(N =", N[i], ")\t\t", "%.5f"%Modulus_of_Error[i], "\t|")

print ("~~~~~")

#####

```

```
#####
#Simpson Method
N = [8, 16, 24]
Analytic_solution = 3.620184280

#Listing the erros and solutions
solution = []
Modulus_of_Error=[]

for i in N:
    sol_simp = integral_simp(f,4,1,i)
    solution.append(sol_simp)
    Modulus_of_Error.append(abs(Analytic_solution-sol_simp))

print ("\n~~~~~Simpson METHOD~~~~~")
print("|| Value via Simpson   \t\t|| Modulus of Error\t||")
print ("~~~~~")
for i in range (0,len(N)):
    print("||\t%.9f"%solution[i],"(N =",N[i]," )\t\t||", "%.6f"%Modulus_of_Error[i]," \t|
|")

print ("~~~~~")
```

Actual analytical value of the integral 3.62018428

```
~~~~~MIDPOINT METHOD~~~~~
|| Value via Midpoint          || Modulus of Error      ||
~~~~~
||      3.618313859 (N = 8 )   ||      0.001870        ||
||      3.619709762 (N = 16 )  ||      0.000475        ||
||      3.619972786 (N = 24 )  ||      0.000211        ||
~~~~~
```

```
~~~~~Trapezoidal METHOD~~~~~
|| Value via Trapezoidal       || Modulus of Error      ||
~~~~~
||      3.623956949 (N = 8 )   ||      0.00377         ||
||      3.621135404 (N = 16 )  ||      0.00095         ||
||      3.620607687 (N = 24 )  ||      0.00042         ||
~~~~~
```

```
~~~~~Simpson METHOD~~~~~
|| Value via Simpson          || Modulus of Error      ||
~~~~~
||      3.620330143 (N = 8 )   ||      0.000146        ||
||      3.620194889 (N = 16 )  ||      0.000011        ||
||      3.620186450 (N = 24 )  ||      0.000002        ||
~~~~~
```

Q2

Numerically integrate the following using Midpoint, Trapezoidal and Simpson techniques with maximum error of 0.001.

$$\int_0^1 x\sqrt{1+x} dx$$

In [4]:

```

from My_Lib import*
import math

def f(x):
    return x*math.sqrt(1+x)

#defining error expressions for three methods

def error_mid(f_X,upper_limit,lower_limit):
    N = (((upper_limit-lower_limit)**3/(24*0.001))*abs(f_X(0)))**0.5
    return N

def error_trap(f_X,upper_limit,lower_limit):
    N = (((upper_limit-lower_limit)**3/(12*0.001))*abs(f_X(0)))**0.5
    return N

def error_simp(f_X,upper_limit,lower_limit):
    N = (((upper_limit-lower_limit)**5/(180*0.001))*abs(f_X(0)))**0.25
    return N

# manually calculated expressions
# defining Second derivative of f(x)
def f_2(x):
    return (3*x+4)/(4*(x+1)**(3/2))

# defining Fourth derivative of f(x)
def f_4(x):
    return (9*x+24)/(16*(x+1)**(7/2))

N_mid = math.ceil(error_mid(f_2,1,0))
N_trap = math.ceil(error_trap(f_2,1,0))
N_simp = math.ceil(error_simp(f_4,1,0))

sol_mid = integration_midpoint(f, 1,0, N_mid)
sol_trap = integration_trap(f,1,0, N_trap)
sol_simp = integral_simp(f,1,0, N_simp)

print("Hence the value of the integral by three methods are")
print("Midpoint Method: ", sol_mid, "\nTrapezoid Method: ", sol_trap, "\nSimpson Method: ", sol_simp)
print("Original Analytic value is 0.643790283299492")

```

Hence the value of the integral by three methods are
Midpoint Method: 0.643137698010569
Trapezoid Method: 0.6444300126935937
Simpson Method: 0.6439505508593788
Original Analytic value is 0.643790283299492

Q3

Use Monte Carlo to estimate the value of π from the following integral:

$$\int_0^1 \frac{4}{1+x^2} dx = \pi$$

Use system built random number generator and sample N starting from 10 and keep increasing in multiple of 10. Go as far as possible within a reasonable time. Plot π vs. N .

In [26]:

```

import matplotlib.pyplot as plt

from My_Lib import*

Range = 10**4

def f(x):
    return 4/(1+x**2)

list_1 = [ i for i in range(10,Range, 10) ]
list_2 = []
for n in list_1:
    answer ,p = monte_carlo(f,0,1,n)
    list_2.append(answer)

print("The Following table shows the convergence of the Integral values..for N=10^4")
print ("\n~~~~~Monte CARLO~~~~~")
print ("||\tN\t\t|| Value of Integral\t\t||")
print ("~~~~~")
for i in range (0,len(list_1)):
    print("||\t%.9f"%list_1[i],"\t||\t",list_2[i],"\t||")

print ("~~~~~")

plt.title("$\pi$ vs $N$")
plt.plot(list_1,list_2,'r-.')
plt.xlabel("N")
plt.ylabel("Value of the Integral$")
plt.show()

```

The Following table shows the convergence of the Integral values..for $N=10^4$

~~~~~Monte CARLO~~~~~			
N		Value of Integral	
~~~~~			
	10.000000000		3.1263985025717886
	20.000000000		3.352380258137071
	30.000000000		2.9988357894458213
	40.000000000		2.9903854306842854
	50.000000000		3.1697606348852108
	60.000000000		3.2486719116115883
	70.000000000		3.2815721956096744
	80.000000000		3.0772262521366893
	90.000000000		3.1819774684733786
	100.000000000		3.2107157721808273
	110.000000000		3.1749241304495057
	120.000000000		3.1130268325500245
	130.000000000		3.201740689298924
	140.000000000		3.1568314674478857
	150.000000000		3.132686555427452
	160.000000000		3.110778214730597
	170.000000000		3.14893778352554
	180.000000000		3.219734177372277
	190.000000000		3.1262393544107057
	200.000000000		3.157448403365798
	210.000000000		3.1404189566520504
	220.000000000		3.150816311983262
	230.000000000		3.1897909677126353
	240.000000000		3.0985946327991023
	250.000000000		3.1265917275239508
	260.000000000		3.1086196327856848
	270.000000000		3.137943635996579
	280.000000000		3.1465569836935683
	290.000000000		3.1629317282835308
	300.000000000		3.1413970800757367
	310.000000000		3.0986918027462353
	320.000000000		3.1753192516319526
	330.000000000		3.1213805647263695
	340.000000000		3.153820259748186
	350.000000000		3.096870937974358
	360.000000000		3.0863608226645582
	370.000000000		3.1327428693340345
	380.000000000		3.1675895481840675
	390.000000000		3.133781649506093
	400.000000000		3.103498846444085
	410.000000000		3.1961448354437727
	420.000000000		3.1319177640938487
	430.000000000		3.0814712100656716
	440.000000000		3.1926774646504485
	450.000000000		3.1601816778816763
	460.000000000		3.1508252287014207
	470.000000000		3.1457861225181882
	480.000000000		3.065556149259455
	490.000000000		3.147282080447906
	500.000000000		3.166324340162784
	510.000000000		3.138176302130102
	520.000000000		3.1556222607463553
	530.000000000		3.170787267248266
	540.000000000		3.1502753527669807
	550.000000000		3.146381285311545

560.000000000	3.1279894689173293
570.000000000	3.101754243798621
580.000000000	3.180550806439261
590.000000000	3.1483739332230036
600.000000000	3.14652493592085
610.000000000	3.155244846245229
620.000000000	3.17403642274191
630.000000000	3.1632287102560404
640.000000000	3.1848680457809806
650.000000000	3.128298900979253
660.000000000	3.1512629912873624
670.000000000	3.141682931100235
680.000000000	3.1548710177210255
690.000000000	3.16575596704473
700.000000000	3.1075612675924167
710.000000000	3.0915422998165654
720.000000000	3.16825358995738
730.000000000	3.1777817190033697
740.000000000	3.110766561687031
750.000000000	3.177216601272785
760.000000000	3.1495487025921065
770.000000000	3.12988436932269
780.000000000	3.132537041277648
790.000000000	3.096643901654115
800.000000000	3.100136085785554
810.000000000	3.1474064936590005
820.000000000	3.140545307517365
830.000000000	3.123064153786547
840.000000000	3.1311611280979705
850.000000000	3.1565631690940057
860.000000000	3.1402795264857293
870.000000000	3.09250934162807
880.000000000	3.1546899782090985
890.000000000	3.1422480275616604
900.000000000	3.12949409686715
910.000000000	3.1175269496707005
920.000000000	3.14805568353907
930.000000000	3.154799096230094
940.000000000	3.1236358783464544
950.000000000	3.164727484337088
960.000000000	3.1599117729406503
970.000000000	3.155481721959013
980.000000000	3.117605524981184
990.000000000	3.131784905014192
1000.000000000	3.157073670258384
1010.000000000	3.1303107927116374
1020.000000000	3.1451543629713536
1030.000000000	3.119363675409134
1040.000000000	3.126606517706828
1050.000000000	3.14759025889441
1060.000000000	3.136933400572886
1070.000000000	3.1324706581452295
1080.000000000	3.140385022321381
1090.000000000	3.138483511951092
1100.000000000	3.146784726921453
1110.000000000	3.097721635727517
1120.000000000	3.152613037493482
1130.000000000	3.1543022793397393
1140.000000000	3.1630932685517625
1150.000000000	3.1333984952129237
1160.000000000	3.1435677891149316

1170.000000000	3.1207105018265886
1180.000000000	3.1424252124003003
1190.000000000	3.143558450003597
1200.000000000	3.153107755831585
1210.000000000	3.1442061833300663
1220.000000000	3.1235972792465008
1230.000000000	3.1502009310199024
1240.000000000	3.1504119241617836
1250.000000000	3.1622343131144706
1260.000000000	3.135647645597401
1270.000000000	3.1390915182748387
1280.000000000	3.1555199636010585
1290.000000000	3.1346066000710016
1300.000000000	3.136922961292197
1310.000000000	3.160901820377861
1320.000000000	3.1220851442515762
1330.000000000	3.179802305575618
1340.000000000	3.134428229543088
1350.000000000	3.1394934844525486
1360.000000000	3.1442650831186745
1370.000000000	3.1456090702241597
1380.000000000	3.141788548314022
1390.000000000	3.1399261528934868
1400.000000000	3.1397740236982634
1410.000000000	3.139608527648239
1420.000000000	3.1874710399868973
1430.000000000	3.133219791032915
1440.000000000	3.1206821817029122
1450.000000000	3.1301768028264356
1460.000000000	3.135881190844234
1470.000000000	3.139906974489728
1480.000000000	3.1367922842279645
1490.000000000	3.133355116798889
1500.000000000	3.153088225992854
1510.000000000	3.150557092206166
1520.000000000	3.155695506562442
1530.000000000	3.13924081480636
1540.000000000	3.128645136344771
1550.000000000	3.1497157738378916
1560.000000000	3.1576787130861215
1570.000000000	3.163747276898657
1580.000000000	3.1175213275804943
1590.000000000	3.1625794081480443
1600.000000000	3.121133858594395
1610.000000000	3.1514365924495724
1620.000000000	3.129777338922157
1630.000000000	3.152081672923504
1640.000000000	3.135465700892391
1650.000000000	3.1477554224778164
1660.000000000	3.0969550699895674
1670.000000000	3.1549495335138427
1680.000000000	3.114026716277657
1690.000000000	3.1225863196516923
1700.000000000	3.151017766130744
1710.000000000	3.174330687960184
1720.000000000	3.1241500604883776
1730.000000000	3.157673404878284
1740.000000000	3.137490522286939
1750.000000000	3.148353759270733
1760.000000000	3.133968548777835
1770.000000000	3.124246206601634

1780.000000000	3.148618469100261
1790.000000000	3.1623153327519384
1800.000000000	3.139187410636695
1810.000000000	3.1402910443888583
1820.000000000	3.1511054360901403
1830.000000000	3.1305243635561526
1840.000000000	3.144760793508552
1850.000000000	3.1668105253317997
1860.000000000	3.1533966545460674
1870.000000000	3.1424122550697424
1880.000000000	3.158567868598446
1890.000000000	3.124208732872257
1900.000000000	3.1121940704021553
1910.000000000	3.149978446145561
1920.000000000	3.146858367213617
1930.000000000	3.134803908398246
1940.000000000	3.1501321564431297
1950.000000000	3.1421364541423156
1960.000000000	3.1442133997264423
1970.000000000	3.1336940880026116
1980.000000000	3.133105866942592
1990.000000000	3.130145482057157
2000.000000000	3.131819841564967
2010.000000000	3.1416750230761656
2020.000000000	3.1365911708418723
2030.000000000	3.1387738762733215
2040.000000000	3.1248810738695316
2050.000000000	3.1200744482838334
2060.000000000	3.118230110235705
2070.000000000	3.134029201918748
2080.000000000	3.1543165589695312
2090.000000000	3.1268745705963057
2100.000000000	3.144933149515301
2110.000000000	3.1445654019962785
2120.000000000	3.122253585182161
2130.000000000	3.1441824779688976
2140.000000000	3.1429850897580236
2150.000000000	3.1171158947014357
2160.000000000	3.1272231035864206
2170.000000000	3.127098607363318
2180.000000000	3.129389862972662
2190.000000000	3.1446999234698225
2200.000000000	3.157267894262325
2210.000000000	3.1531329363616996
2220.000000000	3.162908271267494
2230.000000000	3.112224007330447
2240.000000000	3.154170213997103
2250.000000000	3.163767047128806
2260.000000000	3.156644596255627
2270.000000000	3.1568541866137125
2280.000000000	3.131829263849237
2290.000000000	3.1318403023426735
2300.000000000	3.1377028331870833
2310.000000000	3.1567803259286173
2320.000000000	3.130402477541938
2330.000000000	3.1479939098985006
2340.000000000	3.1421335443830243
2350.000000000	3.149161745287879
2360.000000000	3.1516746937938294
2370.000000000	3.1200572049127726
2380.000000000	3.1390056900403462

2390.000000000	3.143103327590663
2400.000000000	3.1233104451355103
2410.000000000	3.1566005897405964
2420.000000000	3.1295384082239437
2430.000000000	3.1316134459813427
2440.000000000	3.1450435451111733
2450.000000000	3.146635862808236
2460.000000000	3.111293889989928
2470.000000000	3.1564372204279394
2480.000000000	3.1556054317745024
2490.000000000	3.129117954604536
2500.000000000	3.142163074724802
2510.000000000	3.1576290897459725
2520.000000000	3.131793923903424
2530.000000000	3.128852761173829
2540.000000000	3.143851571432197
2550.000000000	3.1311866149948813
2560.000000000	3.1204660907307002
2570.000000000	3.124455309430758
2580.000000000	3.1356071379250636
2590.000000000	3.1581251463653857
2600.000000000	3.1518866064749615
2610.000000000	3.145561823752418
2620.000000000	3.133098092757346
2630.000000000	3.147503770914837
2640.000000000	3.150906691839077
2650.000000000	3.1327404628294038
2660.000000000	3.151414719657813
2670.000000000	3.1004772972254693
2680.000000000	3.1339818839613858
2690.000000000	3.1448761369143416
2700.000000000	3.1459887491532053
2710.000000000	3.1383525358766837
2720.000000000	3.1519675705594294
2730.000000000	3.1495435494597728
2740.000000000	3.1390287148581444
2750.000000000	3.138948266503779
2760.000000000	3.122444226494334
2770.000000000	3.147734347183339
2780.000000000	3.1401118287887586
2790.000000000	3.1305821807479113
2800.000000000	3.149569631924307
2810.000000000	3.1446083087012147
2820.000000000	3.149311641159701
2830.000000000	3.1461876943976654
2840.000000000	3.1308437295809513
2850.000000000	3.152046417239517
2860.000000000	3.136507979251022
2870.000000000	3.1386386687923635
2880.000000000	3.1329672689692614
2890.000000000	3.131277140246237
2900.000000000	3.1524378492533436
2910.000000000	3.134009911980498
2920.000000000	3.1267601576376047
2930.000000000	3.159603869299518
2940.000000000	3.161843010056328
2950.000000000	3.146632711270667
2960.000000000	3.1433618917736017
2970.000000000	3.141898034916244
2980.000000000	3.129044185821932
2990.000000000	3.129404725943917

3000.000000000	3.11813913344195
3010.000000000	3.1513190621015137
3020.000000000	3.13157933881565
3030.000000000	3.1507724084113873
3040.000000000	3.1499499490435974
3050.000000000	3.140867616155775
3060.000000000	3.1356042843414516
3070.000000000	3.135934276851215
3080.000000000	3.134197223719009
3090.000000000	3.158450234111311
3100.000000000	3.145245074659746
3110.000000000	3.159453498382093
3120.000000000	3.1416562372944017
3130.000000000	3.1415570731910325
3140.000000000	3.1172654837532137
3150.000000000	3.1376642610114125
3160.000000000	3.1105633825584422
3170.000000000	3.146174702178162
3180.000000000	3.1245919058712204
3190.000000000	3.121969699702527
3200.000000000	3.1489330919346883
3210.000000000	3.151999306349956
3220.000000000	3.1422745350902233
3230.000000000	3.143337946440998
3240.000000000	3.1504929743306658
3250.000000000	3.1606012870282796
3260.000000000	3.1286133369685247
3270.000000000	3.162263348642636
3280.000000000	3.1362122189484944
3290.000000000	3.1585670325310935
3300.000000000	3.1647335992292875
3310.000000000	3.116611001114305
3320.000000000	3.145129003299934
3330.000000000	3.1387910380662665
3340.000000000	3.137101496172164
3350.000000000	3.12819666978829
3360.000000000	3.1474461701679863
3370.000000000	3.1368463140624305
3380.000000000	3.138902595985074
3390.000000000	3.135903123319844
3400.000000000	3.159748320131207
3410.000000000	3.1287674337474862
3420.000000000	3.1452720287583826
3430.000000000	3.132755500125502
3440.000000000	3.125271139837033
3450.000000000	3.136790845091765
3460.000000000	3.145858094783407
3470.000000000	3.1496509512837463
3480.000000000	3.1459880960907367
3490.000000000	3.1374790578819547
3500.000000000	3.1218328758435847
3510.000000000	3.152112622106831
3520.000000000	3.1333174108546378
3530.000000000	3.137692321050882
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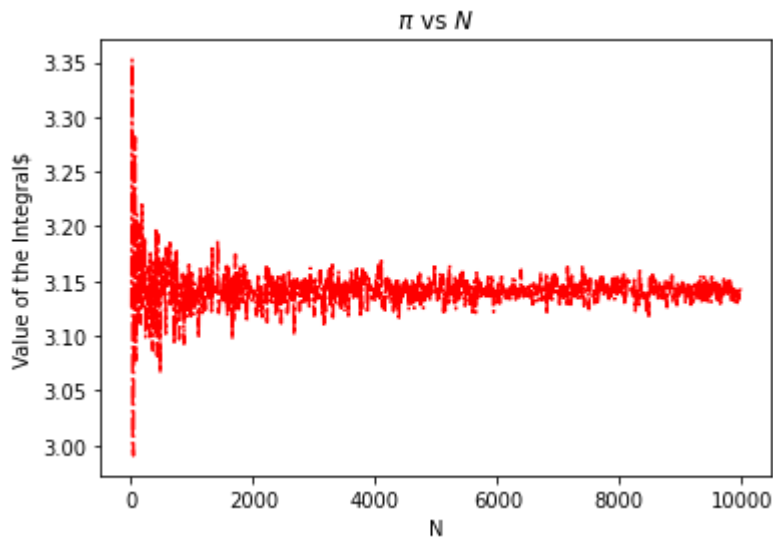
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8430.000000000	3.1373431398046554
8440.000000000	3.119930833349265
8450.000000000	3.1343327453563035
8460.000000000	3.1389967924584607
8470.000000000	3.1398847213994534
8480.000000000	3.118063771853109

8490.000000000	3.1460461472008525
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8510.000000000	3.138012980296634
8520.000000000	3.1484480479938646
8530.000000000	3.1364724499211913
8540.000000000	3.1558435209270264
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8560.000000000	3.144056667664796
8570.000000000	3.137843414785595
8580.000000000	3.1473352209720113
8590.000000000	3.1539301834907194
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8620.000000000	3.146551544785513
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8670.000000000	3.1492447382854474
8680.000000000	3.1458748210053677
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9700.000000000	3.1344432530956348

9710.000000000	3.1516322862649275
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9820.000000000	3.15241453408275
9830.000000000	3.133055295150643
9840.000000000	3.130435810614177
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9960.000000000	3.1427604995729093
9970.000000000	3.141061227894
9980.000000000	3.1402081853390706
9990.000000000	3.142924900780944

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## Q4

A beam 2 meter long has linear mass density  $\lambda(x) = x^2$ , where  $x$  is measured from one of the ends.

Let's say the beam is  $L$  meter long then in general, So the center of Mass will be at

$$X_{CM} = \frac{\int_0^L x dm}{\int_0^L dm}$$

Now here  $L = 2$  and

$$\begin{aligned} dm &= \lambda(x) dx \\ dm &= x^2 dx \end{aligned}$$

Hence, the center of mass will be at

$$X_{CM} = \frac{\int_0^2 x \cdot x^2 dx}{\int_0^2 x^2 dx}$$

In [5]:

```
from My_Lib import*
import math
def f_1(x):
    return x**3
def f_2(x):
    return x**2

#solving the problem using three methods
sol_mid_1 = integration_midpoint(f_1,2,0,500000)
sol_mid_2 = integration_midpoint(f_2,2,0,500000)

sol_trap_1 = integration_trap(f_1,2,0,500000)
sol_trap_2 =integration_trap(f_2,2,0,500000)

sol_simp_1 = integral_simp(f_1,2,0,500000)
sol_simp_2 = integral_simp(f_2,2,0,500000)
print("The center of mass will be at,", "%.5f"%(sol_mid_1/sol_mid_2),"meters from the e
nd of the beam. Evaluated by Midpoint method." )
print("The center of mass will be at,", "%.5f"%(sol_trap_1/sol_trap_2),"meters from the
end of the beam. Evaluated by Trapezoidal method." )
print("The center of mass will be at,", "%.5f"%(sol_simp_1/sol_simp_2),"meters from the
end of the beam. Evaluated by Simpson method." )
```

The center of mass will be at, 1.50000 meters from the end of the beam. Evaluated by Midpoint method.

The center of mass will be at, 1.50000 meters from the end of the beam. Evaluated by Trapezoidal method.

The center of mass will be at, 1.50000 meters from the end of the beam. Evaluated by Simpson method.