1. How I tried to get $\tilde{\gamma}(r)$?

I want that the Fourier transform of $\tilde{\gamma}(r)$ gives the proper scattering intensity. Often only $\gamma_0(r) = \tilde{\gamma}(r)/\tilde{\gamma}(0)$ is supplied in text books. For example for spherical particles one find

$$\gamma_0(r) = \begin{cases} \left(1 - \frac{3}{4}\frac{r}{R} + \frac{1}{16}\left(\frac{r}{R}\right)^3\right) & \text{for } r \leq 2R \\ 0 & \text{for } r > 2R \end{cases} \tag{1}$$

The Fourier transform of $\gamma_0(r)$ yields

$$\mathcal{F}[\gamma_0(r)] = \int_0^\infty 4\pi r^2 \frac{\sin Qr}{Qr}\gamma_0(r)\mathrm{d}r \tag{2}$$

$$= \frac{4}{3}\pi R^3 \left(3\frac{\sin QR - QR\cos QR}{(QR)^3}\right)^2 \tag{3}$$

Here the factor $\Delta\eta^2 \frac{4}{3}R^3$ is missing to get the correct scattering intensity of a single sphere. It seems that $\tilde{\gamma}(r) = \Delta\eta^2 V_p \gamma_0(r)$ and

$$V_p = \int_0^\infty 4\pi r^2 \gamma_0(r)\mathrm{d}r \tag{4}$$

Therefore I used the following definition for $\tilde{\gamma}(r)$:

$$\tilde{\gamma}(r) = \gamma_0(r)\Delta\eta^2 \int_0^\infty 4\pi r^2 \gamma_0(r)\mathrm{d}r \tag{5}$$

2. Transformation cycle at the example of randomly distribute two phase system (DAB)

Now lets have a look on the DAB example. It has the normalized correlation function $\gamma_{0,\mathrm{DAB}}(r) = \exp(-r/\xi)$. Following the above procedure I get

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$$\tilde{\gamma}_{\mathrm{DAB}}(r) = \gamma_{0,\mathrm{DAB}}(r)\Delta\eta^2 \int_0^\infty 4\pi r^2 \gamma_{0,\mathrm{DAB}}(r)\mathrm{d}r = 8\pi\xi^3\Delta\eta^2 \exp(-r/\xi)$$

and from that the scattering intensity $I_{\mathrm{DAB}}(Q)$

$$I_{\mathrm{DAB}}(Q) = \mathcal{F}[\tilde{\gamma}_{\mathrm{DAB}}(r)] = \int_0^\infty 4\pi r^2 \frac{\sin Qr}{Qr}\tilde{\gamma}_{\mathrm{DAB}}(r)\mathrm{d}r \tag{7}$$

$$= \frac{(8\pi\xi^3\Delta\eta)^2}{(1+Q^2\xi^2)^2} \tag{8}$$

From this function one also can calculate the Hankel transform $\mathcal{H}[I_{\mathrm{DAB}}(Q)]$ analytically. For this see the enclosed Mathematica file. So one can calculate $\tilde{G}(z)$ directly from the intensity which reads as

$$\tilde{G}(z) = 2\pi\mathcal{H}[I_{\mathrm{DAB}}(Q)] = 64\Delta\eta^2\pi^3\xi^3 z\mathrm{BesselK}_1(z/\xi) \tag{9}$$

However, one also can calculate $\tilde{G}(z)$ via the Abel transform of the correlation $\mathcal{A}[\tilde{\gamma}_{\mathrm{DAB}}(r)]$ which reads as

$$\tilde{G}(z) = (2\pi)^2\mathcal{A}[\tilde{\gamma}_{\mathrm{DAB}}(r)] = 64\Delta\eta^2\pi^3\xi^3 z\mathrm{BesselK}_1(z/\xi) \tag{10}$$

I had to multiply the Abel transform by $(2\pi)^2$ to get consistency in the results. It seems, we now have closed the transformation cycle for a specific example. It would be interesting to proof it for any correlation function.

3. Mathematica output

gamma0DAB[r_, xi_]:=Exp[−r/xi]

gamma0DAB[r, xi]

$e^{-\frac{r}{\mathrm{xi}}}$

Integrate[gamma0DAB[r, xi] * 4 * Pi * r^2, {r, 0, Infinity},

Assumptions → xi > 0]

$8\pi\mathrm{xi}^3$

gammaDAB[r_, xi_, eta_]:=

Integrate[gamma0DAB[r, xi] * 4 * Pi * r^2, {r, 0, Infinity},

Assumptions → xi > 0] * gamma0DAB[r, xi] * eta^2

IDAB =

FullSimplify[

Integrate[gammaDAB[r, xi, eta] * Sin[Q * r]/(Q * r) * 4 * Pi * r^2,

{r, 0, Infinity}], Assumptions → xi > 0&&Q > 0]

$\frac{64\mathrm{eta}^2\pi^2\mathrm{xi}^6}{\left(1+Q^2\mathrm{xi}^2\right)^2}$

GzHF = Integrate[2 * Pi * IDAB * BesselJ[0, Q * z] * Q, {Q, 0, Infinity}, Assumptions → z > 0&&xi > 0]

$64\mathrm{eta}^2\pi^3\mathrm{xi}^3 z\mathrm{BesselK}\left[1, \frac{z}{\mathrm{xi}}\right]$

GzA = Integrate[2 * (2 * Pi)^2 * gammaDAB[r, xi, eta]r/Sqrt[r^2 − z^2], {r, z, Infinity}, Assumptions → z > 0&&xi > 0]

$64\mathrm{eta}^2\pi^3\mathrm{xi}^3 z\mathrm{BesselK}\left[1, \frac{z}{\mathrm{xi}}\right]$