Gandaki University

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Bachelor of Information Technology

BSM 102

Exercise on Complex Number

1. Express given complex number in the form of $x + iy$

   a. $\dfrac{1+2i}{1-3i}$ b. $\dfrac{4+\sqrt{-25}}{2-\sqrt{-9}}$

2. Write the following complex number into polar form.

   a. $3-3i$ b. $2i, -2i$ c. -5 d. $\dfrac{1}{2}+\dfrac{1}{4}\pi i$

   e. $\dfrac{1+i}{1-i}$ f. $\dfrac{3\sqrt{2}+2t}{-\sqrt{2}-(2/3)i}$ g. $\dfrac{-6+5i}{3i}$ h. $\dfrac{2+3i}{5+4i}$

   PRINCIPAL ARGUMENT: Let $z = x + iy$ be any complex number then modulus of z is denoted by $|z| = \sqrt{x^2+y^2} = r$, and $\theta = \arctan(y/x)$ is called argument of z, is denoted as "arg z."
   θ is the directed angle from the positive x-axis to the number $z = x + iy$. For $z = 0$ this angle θ is undefined. (Why?) For a given $z \neq 0$ it is determined only up to integer multiples of 2π since cosine and sine are periodic with period 2π. But one often wants to specify a unique value of $\arg z$ of a given $z \neq 0$. For this reason one defines the principal value $\operatorname{Arg} z$ (with capital A!) of $\arg z$ by the double inequality

   $$-\pi < \operatorname{Arg} z \leqq \pi.$$

   Example: Principal Value of z, i.e Arg z, if $z = 1 + i$

   $z = 1 + i$ has the polar form $z = \sqrt{2}\left(\cos\frac{1}{4}\pi + i\sin\frac{1}{4}\pi\right)$.
   Hence we obtain

   $$|z| = \sqrt{2}, \quad \arg z = \frac{1}{4}\pi \pm 2n\pi (n = 0, 1, \cdots), \quad \text{and} \quad \operatorname{Arg} z = \frac{1}{4}\pi \quad \text{(the principal value).}$$

   $$\text{Similarly, } z = 3 + 3\sqrt{3}i = 6\left(\cos\frac{1}{3}\pi + i\sin\frac{1}{3}\pi\right), |z| = 6, \text{ and } \operatorname{Arg} z = \frac{1}{3}\pi.$$

3. Determine the principal value of the argument.

   a. $-1-i$ b. $-20+i, -20-i$ c. $4 \pm 3i$ d. $-\pi^2$

   e. $7 \pm 7i$ f. $(1+i)^{12}$ g. $(9+9i)^3$

4. Represent in the form $x + iy$ and graph it in the complex plane.

   a. $\cos\dfrac{1}{2}\pi + i\sin\dfrac{1}{2}\pi$

   b. $4\left(\cos\frac{1}{3}\pi \pm i\sin\frac{1}{3}\pi\right)$

   c. $12\left(\cos\frac{3}{2}\pi + i\sin\frac{3}{2}\pi\right)$

5. Find all roots in the complex plane.

   a. $\sqrt{-i}$ b. $\sqrt[8]{1}$ c. $\sqrt[4]{-1}$ d. $\sqrt[3]{3+4i}$ e. $\sqrt[5]{-1}$