Mathematics II (BSM 102)

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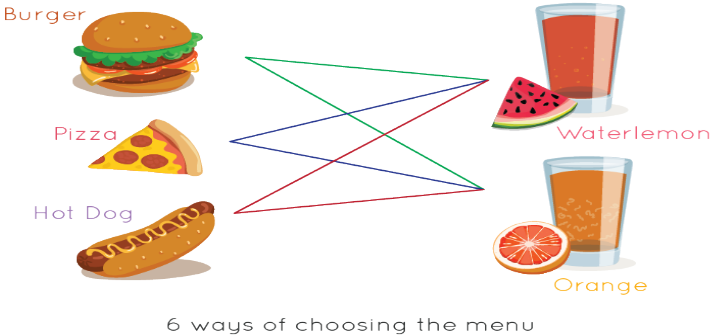
Outlines

- Introduction
- Basic Principles of Counting
- Factorial Notation
- Definition and problems of Permutation
    - permutation of objects alike,
    - permutation with restrictions,
    - circular permutation,
- Definition and problems of Combination

Why do we want to study permutation and combination?

1.Very often we will find situations where one thing can be done in different ways.
2. In order to find the best way, we need to know how many possible ways are there in total.
3. For example, you can have menu that offer combo snacks showing as image below:



Basic Principles of Counting

Fundamental Counting Principle can be used to determine the number of possible outcomes when there are two or more characteristics .

Fundamental Counting Principle states that if an event has **m** possible outcomes and another independent event has **n** possible outcomes, then there are $\mathbf{m} \times \mathbf{n}$ possible outcomes for the two events together.

Example: A student is to roll a die and flip a coin. How many possible outcomes will there be?

1H	2H	3H	4H	5H	6H
1 T	2 T	3 T	4 T	5 T	6 T

12 outcomes

Also, from counting principle: $\mathbf{6} \times \mathbf{2} = \mathbf{12}$ outcomes

Exercise: For a job interview, Rohit has to choose what to wear from the following: 4 pants, 3 shirts, 2 shoes and 5 ties. How many possible outfits does he have to choose from?

Solution: $\mathbf{4} \times \mathbf{3} \times \mathbf{2} \times \mathbf{5} = \mathbf{120}$ outfits to choose.

Permutations

Permutation is an arrangement of items in a particular order.
To find the number of Permutations of n items, we can use the Fundamental Counting Principle or formula with factorial notation.

Factorial Notation

Let n be any natural number then factorial n is denoted by $n!$ is the continued product from 1 to n.

$n! = n \cdot (n-1) \cdots 3 \cdot 2 \cdot 1$

For example, $4! = 4 \times 3 \times 2 \times 1 = 24$

$0! = 1$

Example: The number of ways to arrange the letters ABC: 3 Number of choices for first blank? 2 Number of choices for second blank? 1 Number of choices for third blank?

$3 \times 2 \times 1 = 6 \quad or \quad 3! = 3 \times 2 \times 1 = 6$ ways

ABC ACB BAC BCA CAB CBA

Permutations

Rule 1: Permutation of objects all different: The number of permutation of n different type of things taken r at a time is denoted by $^{n}p_{r}$ or $\mathrm{P(n,r)}$ and given by

$$\mathrm{P(n,r)} = \frac{\mathrm{n!}}{\mathrm{(n-r)!}} \quad \mathrm{(r \leq n)}$$

The number of permutations of n different things taken all at a time is $n!$

Example: $^{5}p_{3} = \frac{5!}{(5-3)!} = \frac{5!}{2!} = 5 \times 4 \times 3 = 60$

Rule 2: Permutation of Objects not at All Different

The number of permutation of 'n' objects taken all at a time, when 'p' objects are of one kind, 'q' objects are of second kind, 'r' objects are of third kind, 's' objects are of fourth kind and so on is given by

$$\frac{\mathrm{n!}}{\mathrm{p!q!r!s!\ldots}}$$

Permutations Continue......

Example: Find the total number of arrangements of the letters of the word 'STATISTICS' taken all at a time.

Solution:

There are 10 letters in the word "STATISTICS". The letters S, T and I repeats 3,3 and 2 times respectively.

So, $n = 10, p = 3, q = 3, r = 2$.

We know, Total no of arrangements $= \dfrac{\mathrm{n!}}{\mathrm{p!q!r!}} = \dfrac{10!}{3!3!2!} = 50400$

Note: Permutation of n objects taken r at a time is given by n^r if the repetition is allowed.

Example: How many pin code consisting 4 digits can be constructed from the integers 0 to 9?

Solution: here $n = 10, \quad r = 4$

No. of possible pin code consisting 4 digits out of 10 integers:

$n^r = 10^4 = 10,000$

Rule 3: Circular Permutation

The number of circular permutations of n objects $= \dfrac{n!}{n} = (n-1)!$

$\therefore$ Circular Permutations of n objects $= (\mathrm{n}-1)!$

Example: In how many ways can 7 boys be arranged in a (i) circle (ii) line

Solution

(i) Here, $\mathrm{n} = 7$ and $\mathrm{r} = 7$

We know,

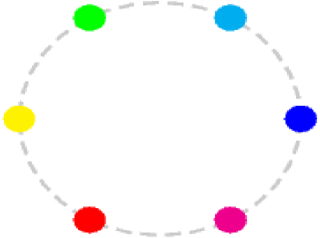
No. of permutations $= (\mathrm{n}-1)! = (7-1)! = 6! = 720$

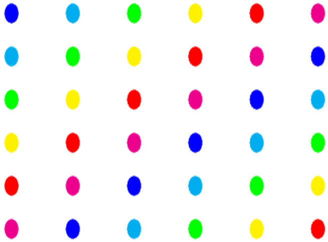
(ii) We know, No. of permutations

$$= p(n, r) = \frac{n!}{(n-r)!} = \frac{7!}{(7-7)!} = \frac{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{0!} = 5040$$

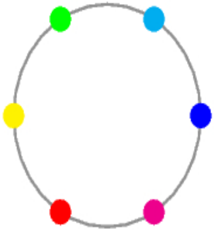
Circular Permutation(clockwise/counterclockwise)

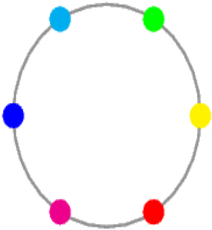
Note: The number of ways of arranging n discinct objects along a circle when clockwise and anticlockwise arrangements are considered alike is $\frac{1}{2}(n-1)!$





Rule 4: Restricted Permutation





Permutation of n things taken r at a time when m particular objects are

1. excluded $= {}^{\mathrm{n-m}}\mathrm{P}_{\mathrm{r}}$.
2. included $= r \cdot {}^{\mathrm{n-m}}\mathrm{P}_{r-\mathrm{m}}$

Combination

Combination is the selection of objects taking some or all of them at a time where ORDER DOES NOT MSATTER.
The number of combinations of n different thing taken r at a time is denoted by ${}^{n}C_r$ or $C(n,r)$ or $\begin{pmatrix} n \\ r \end{pmatrix}$ and given by

$$C(n,r) = \frac{n!}{(n-r)!r!}$$

Example: Six men and eight women are qualified to serve on a project team to revise operational procedures. How many different teams can be formed containing three men and three women?
Solution: Three men can be selected from six in $C(6,3)$ ways, and three women can be selected from eight in $C(8,3)$ ways. Therefore, three men and three women can be selected in

$$C(6,3) \cdot C(8,3) = \frac{6!}{3!3!} \cdot \frac{8!}{5!3!} = 1120 \text{ ways}$$

Example: In how many ways can a committee of 15 members formed out of 8 engineers, 5 businessmen and 4 lawyers so that all the lawyers always have a representation.

Solution:

For selection, the following scheme is used so that the minority representation from lawyers.

8 engineers	5 businessmen	4 lawyers
8	3	4
7	4	4
6	5	4

Thus, the number of different ways in which a committee of 15 members formed

$= {}^8C_8 \times {}^5C_3 \times {}^4C_4 + {}^8C_7 \times {}^5C_4 \times {}^4C_4 + {}^8C_6 \times {}^5C_5 \times {}^4C_4$

$= 10 + 40 + 28$

$= 78$

Combination Continue.....

Some Important Relations

(i) ${}^{\mathrm{n}}\mathrm{C}_{\mathrm{r}} = {}^{\mathrm{n}}\mathrm{C}_{\mathrm{n-r}}$ (Complementary combination)

(ii) ${}^{\mathrm{n}}\mathrm{C}_{\mathrm{r}} + {}^{n}\mathrm{C}_{\mathrm{r-1}} = {}^{\mathrm{n+1}}\mathrm{C}_{\mathrm{r}}$

(iii) ${}^{n}\mathrm{C}_{1} = \mathrm{n}$

(iv) ${}^{n}\mathrm{C}_{0} = 1 = {}^{n}\mathrm{c}_{\mathrm{n}}$

(v) If ${}^{n}C_{p} = {}^{n}C_{q}$, then $p = q$ or $p + q = n$

Restricted Combinations

The number of combinations of n different objects taken 'r' at a time in which

1. m particular objects are excluded $= {}^{n-m}\mathrm{C}_{\mathrm{r}}$
2. m particular objects are included $= {}^{n-m}\mathrm{C}_{\mathrm{r-m}}$
3. Number of ways of selection of r different things from n things, where k things are always selected, and m things are always rejected $= {}^{n-k-m}C_{r-k}$

Thank You