Mathematics II (BSM 102)

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Alternating Series Test

An **alternating series** is any series, $\sum a_n$, for which the series terms can be written in one of the following two forms.

$$a_n = (-1)^n b_n \qquad b_n \geq 0$$
$$a_n = (-1)^{n+1} b_n \qquad b_n \geq 0$$

Altenating Series Test: Suppose that we have a series $\sum a_n$ and either $a_n = (-1)^n b_n$ or $a_n = (-1)^{n+1} b_n$ where $b_n \geq 0$ for all n. Then if,
1. $\lim_{n\to\infty} b_n = 0$ and,
2. $\{b_n\}$ is a decreasing sequence

the series $\sum a_n$ is convergent.

Examples:
$\sum_{n=1}^{\infty}(-1)^{n-1}\frac{2}{n+1} = 1 - \frac{2}{3} + \frac{2}{4} - \frac{2}{5} + \ldots$
$\sum_{n=1}^{\infty}(-1)^{n}\frac{2}{n+1} = -1 + \frac{2}{3} - \frac{2}{4} + \frac{2}{5} - \ldots$
$\sum_{n=0}^{\infty}(-1)^{n}2^n = 1 - 2 + 4 - 8 + \ldots$
$\sum_{n=4}^{\infty}(-1)^{n-1}\frac{n}{n+2} = -\frac{4}{6} + \frac{5}{7} - \frac{6}{8} + \frac{7}{9} - \ldots$

Alternating Harmonic Series:

A series of the form

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots + \frac{(-1)^{n-1}}{n} + \cdots$$

is called an alternating harmonic series.

Example:

Determine if the following series is convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$$

Solution: First, identify the b_n for the test.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n} \quad b_n = \frac{1}{n}$$

Now, all that we need to do is run through the two conditions in the test.

$$\lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{1}{n} = 0$$

$$b_n = \frac{1}{n} > \frac{1}{n+1} = b_{n+1}$$

Both conditions are met and so by the Alternating Series Test the series must converge.

classwork 1: Determine if the following series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{7+2n}$$

classwork 2: Determine if the following series converges or diverges.

$$\sum_{n=0}^{\infty} \frac{1}{(-1)^n \left(2^n+3^n\right)}$$

classwork 3: Determine if the following series converges or diverges.

$$\sum_{n=0}^{\infty} \frac{(-1)^{n+3}}{n^3+4n+1}$$

Exercise

Determine if the following series converges or diverges.

$$a.\quad \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n+5} \qquad b.\quad \sum_{n=4}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n-3}}$$

$$c.\quad \sum_{n=1}^{\infty} (-1)^{n-1} \frac{n}{3n-2} \qquad d.\quad \sum_{n=0}^{\infty} \frac{(-1)^{n-2}}{3^n+3n}$$

$$e.\quad \sum_{n=0}^{\infty} \frac{(-1)^{n+3}}{n^3+4n^2+8} \qquad f.\quad \sum_{n=0}^{\infty} \frac{(-1)^{n+7}}{(n^2+3)}$$

Thank You