Mathematics II (BSM 102)

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Outlines:

- Introduction to function
- Function of two or more variables
- Partial first order derivatives
- Higher order partial derivatives
- Homogeneous function
- Maxima & minima of a function

Introduction

Definition: Suppose D is a set of n-tuples of real numbers $(x_1, x_2, \ldots, x_n)$. **A realvalued function** f on D is a rule that assigns a real number

$$w = f\left(x_1, x_2, \ldots, x_n\right)$$

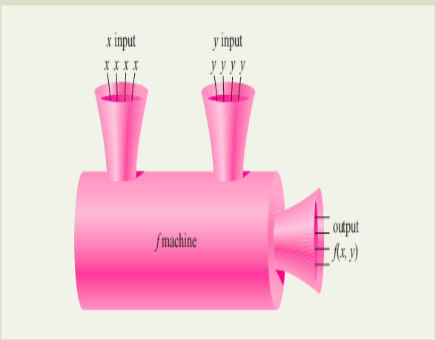
to each element in D.
The set D is the function's domain.
The set of w values taken on by f is the function's range.
The symbol w is the dependent variable of f, and f is said to be a function of the n independent variables x_1 to x_n.
We also call the x 's the function's input variables and call w the function's output variable.

Function: A function is an equation for which any x that can be plugged into the equation will yield exactly one y out of the equation. The common notation for the function is

$$y = f(x)$$

to express a functional relationship between x and y.
$y = f(x)$ is read as 'y equals f of x' or 'y is a function of x'

Function of Two Variables: A function f of the two independent variables x and y is a rule that assigns to each ordered pair (x, y) in a given set D (the domain of f) exactly one real number, denoted by $f(x, y)$.



Domain: Excluding inputs that lead to complex numbers or division by zero.If $f(x,y)=\sqrt{y-x^2}, y$ cannot be less than x^2. If $f(x,y)=1/(xy), xy$ cannot be zero. The domains of functions are otherwise assumed to be the largest sets for which the defining rules generate real numbers.

Functions of two Variables:

Function	Domain	Range
$w=\sqrt{y-x^2}$	$y\geq x^2$	$[0,\infty)$
$w=\frac{1}{xy}$	$xy\neq 0$	$(-\infty,0)\cup(0,\infty)$
$w=\sin xy$	Entire plane	$[-1,1]$

Functions of three Variables:

Function	Domain	Range
$w=\sqrt{x^2+y^2+z^2}$	Entire space	$[0,\infty)$
$w=\frac{1}{x^2+y^2+z^2}$	$(x,y,z)\neq(0,0,0)$	$(0,\infty)$
$w=xy\ln z$	Half-space $z>0$	$(-\infty,\infty)$

Partial Derivatives

Definition: The partial derivative of $\boldsymbol{f(x, y)}$ with respect to $\boldsymbol{x}$ at the point (x_0, y_0) is

$$\left.\frac{\partial f}{\partial x}\right|_{(x_0,y_0)} = \left.\frac{d}{dx} f\left(x, y_0\right)\right|_{x=x_0} = \lim_{h \to 0} \frac{f\left(x_0 + h, y_0\right) - f\left(x_0, y_0\right)}{h},$$

provided the limit exists. (Think of ∂ as a kind of d.)
Definition: The partial derivative of $\boldsymbol{f(x, y)}$ with respect to $\boldsymbol{y}$ at the point (x_0, y_0) is

$$\left.\frac{\partial f}{\partial y}\right|_{(x_0,y_0)} = \left.\frac{d}{dy} f\left(x_0, y\right)\right|_{y=y_0} = \lim_{h \to 0} \frac{f\left(x_0, y_0 + h\right) - f\left(x_0, y_0\right)}{h},$$

provided the limit exists.

Example: Find the values of $\partial f/\partial x$ and $\partial f/\partial y$ at the point $(4, -5)$ if

$$f(x, y) = x^2 + 3xy + y - 1.$$

Solution: To find $\partial f/\partial x$, we regard y as a constant and differentiate with respec to x :

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\left(x^2 + 3xy + y - 1\right) = 2x + 3 \cdot 1 \cdot y + 0 - 0 = 2x + 3y.$$

The value of $\partial f/\partial x$ at $(4, -5)$ is $2(4) + 3(-5) = -7$.
pause To find $\partial f/\partial y$, we con x as a constant and differentiate with respect to y :

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\left(x^2 + 3xy + y - 1\right) = 0 + 3 \cdot x \cdot 1 + 1 - 0 = 3x + 1.$$

The value of $\partial f/\partial y$ at $(4, -5)$ is $3(4) + 1 = 13$.

Classwork

1. 1. Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.
   - $f(x, y) = 2x^2 - 3y - 4$
   - $f(x, y) = x^2 - xy + y^2$
   - $f(x, y) = \left(x^2 - 1\right)(y + 2)$
2. Find $f_x,\ f_y,\ f_z$ of the following functions:
   - $f(x, y, z) = ax^2 + cy^2 + ez^2$
   - $f(x, y, z) = -x^4 + 2x^2 - y^4 + 2y^2 - z^4 + 2z^2$
   - $f(x, y, z) = e^{x+y-z}$
3. Suppose that at a certain factory, output is given by the Cobb-Douglas production function $Q(K, L) = 60K^{1/3}L^{2/3}$ units, where K is the capital investment measured in units of \$1,000 and L the size of the labor force measured in worker-hours.
   a. Compute the output if the capital investment is \$512,000 and 1,000 worker-hours of labor are used.
   b. Show that the output in part (a) will double if both the capital investment and the size of the labor force are doubled.

Exercise

Find $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$ and $\frac{\partial f}{\partial z}$ from the following functions

- $f(x,y) = 5xy - 7x^2 - y^2 + 3x - 6y + 2$
- $f(x,y) = (xy-1)^2$
- $f(x,y) = (2x-3y)^3$
- $f(x,y) = \sqrt{x^2+y^2}$
- $f(x,y) = \left(x^3 + (y/2)\right)^{2/3}$
- $f(x,y) = \dfrac{1}{(x+y)}$
- $f(x,y) = \dfrac{x}{(x^2+y^2)}$
- $f(x,y) = \dfrac{x+y}{xy-1}$
- $f(x,y,z) = \dfrac{2xyz}{x^2+y^2+z^2}$

Concept of Partial Derivatives

Output Q at a factory is often regarded as a function of the amount K of capital investment and the size L of the labor force. Output functions of the form

$$Q(K, L) = AK^{\alpha}L^{\beta}$$

where A, α, and β are positive constants with $\alpha + \beta = 1$, have proved to be especially useful in economic analysis and are known as Cobb-Douglas production functions.

Thank You