Mathematics II (BSM 102)

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BSM 102

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Maxima & Minima of a Function of Several Variables

Let $z = f(x, y)$ be a function for which both $\frac{\partial z}{\partial x} = 0$ and $\frac{\partial z}{\partial y} = 0$ at a point (a, b) and suppose that all second partial derivatives are continuous there.

Evaluate D at the critical point (a, b), and conclude the following:

$$D = \frac{\partial^2 z}{\partial x^2} \cdot \frac{\partial^2 z}{\partial y^2} - \left(\frac{\partial^2 z}{\partial x \partial y} \right)^2 = z_{xx} \cdot z_{yy} - (z_{xy})^2$$

1. If $D > 0$ and $\dfrac{\partial^2 z}{\partial x^2} = z_{xx} > 0$ at (a, b), and $\dfrac{\partial^2 z}{\partial y^2} = z_{yy} > 0$ at (a, b) then a relative minimum occurs at (a, b).
2. If $D > 0$ and $\dfrac{\partial^2 z}{\partial x^2} = z_{xx} < 0$ at (a, b), and $\dfrac{\partial^2 z}{\partial y^2} = z_{yy} < 0$ at (a, b) then a relative maximum occurs at (a, b)
3. If $D < 0$ at (a, b), there is neither a relative maximum nor a relative minimum at (a, b).
4. If $D = 0$ at (a, b), the test fails; investigate the function near the point.

Procedure:

To find relative maxima of $z = f(x, y)$

1. Find $\partial z/\partial x$ and $\partial z/\partial y$.
2. Find the point(s) that satisfy both $\partial z/\partial x = 0$ and $\partial z/\partial y = 0$
3. Find all second partial derivatives.
4. Evaluate D at each critical point.
5. Use the test for maxima and minima to determine whether relative maxima or minima occur.

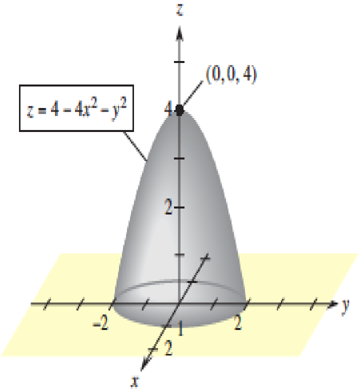
Example

Example:
Test $z = 4 - 4x^2 - y^2$ for relative maxima and minima.

Solution:
1. $\frac{\partial z}{\partial x} = -8x; \frac{\partial z}{\partial y} = -2y$
2. $\frac{\partial z}{\partial x} = 0$ if $x = 0$, $\frac{\partial z}{\partial y} = 0$ if $y = 0$.
3. $\dfrac{\partial^2 z}{\partial x^2} = -8$; $\quad \dfrac{\partial^2 z}{\partial y^2} = -2$;

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x} = 0$$

4. At critical point (0,0),
$D = (-8)(-2) - 0^2 = 16.$
5. $D > 0$, $\quad \partial^2 z/\partial x^2 < 0$, and
$\partial^2 z/\partial y^2 < 0$. A relative maximum occurs at (0,0).



Example:

Test $z = x^2 + y^2 - 2x + 1$ for relative maxima and minima.

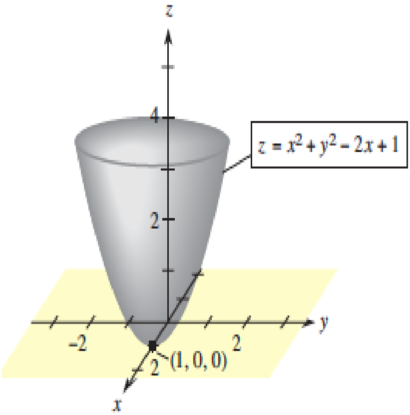
Solution:

1. $\frac{\partial z}{\partial x} = 2x - 2; \quad \frac{\partial z}{\partial y} = 2y$
2. $\frac{\partial z}{\partial x} = 0$ if $x = 1$. $\quad \frac{\partial z}{\partial y} = 0$ if $y = 0$.

Both are 0 if $x = 1$ and $y = 0$, so the critical point is $(1, 0, 0)$.

3. $\frac{\partial^2 z}{\partial x^2} = 2; \quad \frac{\partial^2 z}{\partial y^2} = 2; \quad \frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x} = 0$
4. At $(1, 0), D = 2 \cdot 2 - 0^2 = 4$.
5. $D > 0, \dfrac{\partial^2 z}{\partial x^2} > 0$, and $\dfrac{\partial^2 z}{\partial y^2} > 0$.

A relative minimum occurs at $(1, 0)$.



Classworks

Test for maxima and minima.

a. $z = 24 - x^2 + xy - y^2 + 36y$

b. $z = 46 - x^2 + 2xy - 4y^2$

c. $z = x^3 + y^2 + 6xy + 24x$

Thank You