MA 201(Autumn 2025-26)
Tutorial 3

Q1. In a parallel universe, two powerful creatures, Xorak and Yvandor, control two worlds. The values of X and Y represent the energy levels of their worlds. These energy levels are given by the following joint probability mass function (PMF):

$$p(x,y) = \begin{cases} \frac{2}{15} & \text{for } (x,y) = (1,1) \\ \frac{3}{15} & \text{for } (x,y) = (1,2) \\ \frac{4}{15} & \text{for } (x,y) = (2,1) \\ \frac{6}{15} & \text{for } (x,y) = (2,2) \\ 0 & \text{otherwise} \end{cases}$$

The creatures have discovered that understanding their world's energy will give them an edge in future battles, but the calculations are tricky.
a) Compute the expected energy level E(X) and E(Y).
b) Find the variance Var(X) and Var(Y).
c) Compute the covariance COV(X,Y) and the correlation coefficient between X and Y.
d) Calculate the joint expectation E(XY).
e) Suppose Xorak uses a Bernoulli random variable Z with probability p=0.7 to decide whether to increase his world's energy. If X is the energy level of Xorak's world, calculate the expected value and variance of X given that Z=1.

Q2. A quality control engineer tests the quality of produced computers. Suppose that 5% of computers have defects, and defects occur independently of each other.
a) Find the probability of exactly 3 defective computers in a shipment of twenty.
b) Find the probability that the engineer has to test at least 5 computers in order to find 2 defective ones.

Q3.In a mystical realm, a trickster spirit offers a hero two challenges to prove their worth. The first challenge involves flipping a trick coin where the probability of heads is p=0.7. The outcome X is either 1 (heads) or 0 (tails). In the second challenge, the hero performs a ritual consisting of 6 coin flips, where the number of heads Y follows a Binomial distribution with parameters n=6 and p=0.7. However, the number of heads influences the outcome of a new flip, which follows a conditional Bernoulli distribution.
The trickster requires the hero to prove their calculation skills:
a) Calculate the expected value and variance of a single coin flip X.
b) Compute the expected value and variance of the number of heads Y from the 6-flip ritual.
c) The trickster introduces a dependency such that

$$P(X=1|Y=k) = \frac{k+1}{7}$$

where k is the number of heads from the ritual. Calculate E(X|Y), E(XY), COV(X,Y) and the correlation coefficient between X and Y.
d) Determine the probability that the final coin flip results in heads given that the hero obtained exactly 4 heads in the ritual.

Q4. a) Suppose $X \sim \text{Bernoulli}(p)$. Find $E[X]$. (This is important! Remember it!)

b) Suppose $Y = X1 + X2 + ... + X12$, where each $Xi \sim \text{Bernoulli}(0.25)$. Find $E[Y]$.

Q5. a) Let $X \sim \text{Bernoulli}(p)$. Compute $Var(X)$.

b) Let $Y \sim Bin(n, p)$. Show $Var(Y) = n\, p(1 - p)$.

c) Suppose $X1, X2, ... , Xn$ are independent and all have the same standard deviation $\sigma = 2$. Let X be the average of $X1, ... , Xn$. What is the standard deviation of X?