Joint Probability Table

Cheated in exam

	Yes	No	
Male	0.32 ← Joint	0.22	0.54 ← Marginal
Female	0.28	0.18	0.46
	0.60	0.40	

1] Joint Probability

$$P(\text{male} = 1 \cap C = 1) = 0.32$$

2] Marginal Probability

$$P(\text{Male}) = P(C = 1, \text{Male}) + P(C = 0, \text{Male})$$

$$= 0.54$$

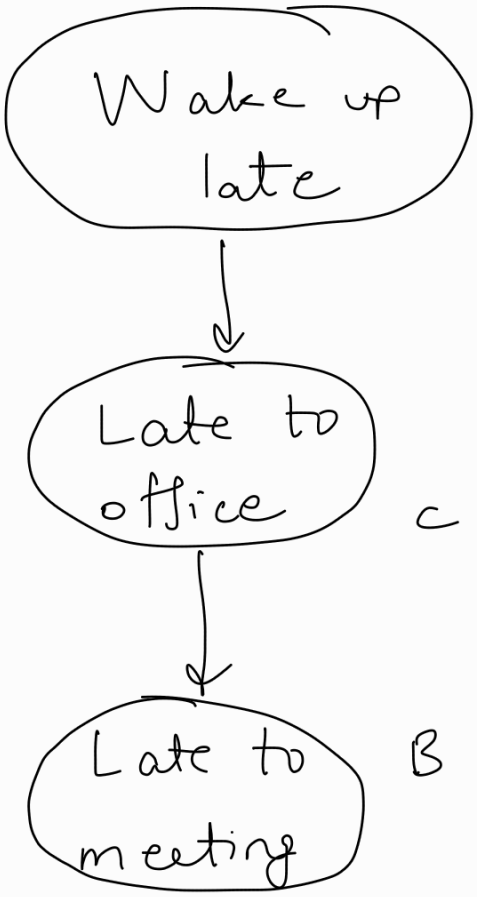
3] Conditional Probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

State = Random Variable = Node

Edge = Relationship

Bayesian nets represent joint probability compactly



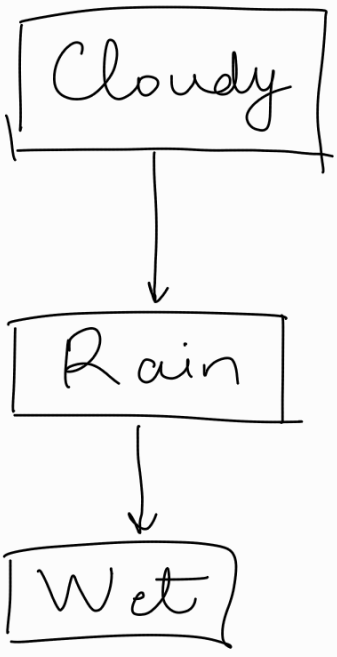
When observing C, B becomes independent of A.

$$P(A \cap B) = P(A) \cdot P(B) \cdots (A \text{ \& } B \text{ independent})$$

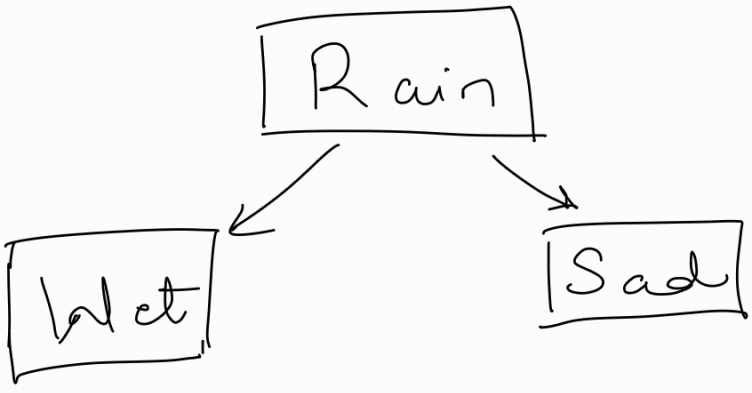
$$P(A, B \mid C) = P(A \mid C) \cdot P(B \mid C)$$

Bayesian Networks are DAGs

eg]

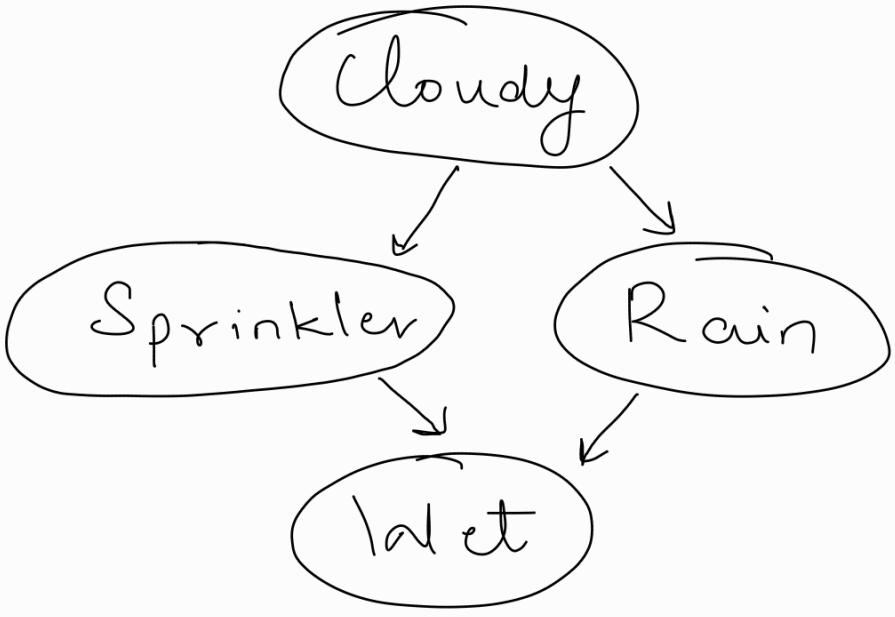


$$P(C, R, W) = P(C)\ P(R \mid C)\ P(W \mid R)$$



$$P(R, W, S) = P(R)\ P(W \mid R)\ P(S \mid R)$$

eg]



General Rule:

$$P(C, S, R, W) = P(C)\ P(S|C)\ P(R|C,S)\ P(W|C,S,R)$$

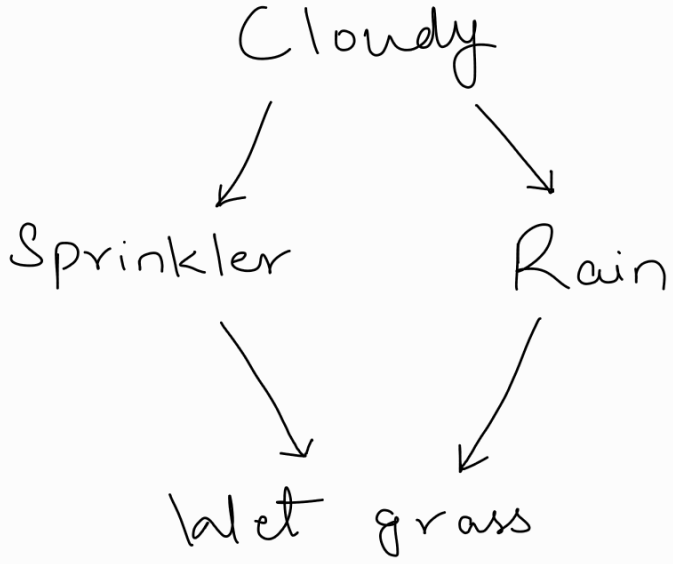
Using Conditional Independence,

$$P(C,S,R,W) = P(C)\ P(S|C)\ P(R|C)\ P(W|S,R)$$

Wet is conditionally independent of Cloudy, given Sprinkler & Rain

eg]

P(C=F)	P(C=T)
0.5	0.5



C	S=F	S=T
F	0.5	0.5
T	0.9	0.1

C	R=F	R=T
F	0.8	0.2
T	0.2	0.8

S	R	W=F	W=T
F	F	1	0
T	F	0.1	0.9
F	T	0.1	0.9
T	T	0.01	0.99

$$P(C,S,R,W) = P(C)\,P(S|C)\,P(R|C)\,P(W|S,R)$$

1] $P(C=T, S=T)$

$$P(C,S) = P(S|C) \cdot P(C)$$

$$P(C,S) = \underbrace{\sum_{W,R} P(C=1, S=1, W, R)}_{\text{Marginalization}}$$

2] $P(C=0, S=0, R=0, W=0)$

$$= P(C=0)\, P(S=0|C=0)\, P(R=0|C=0)$$

$$P(W=0|S=0, R=0)$$

$$= \frac{1}{2} \times \frac{1}{2} \times \frac{8}{10} \times 1 = \frac{2}{10} = 0.2$$

3] Inference

$$P(S=1, W=1) = 0.180 + 0.050 + 0.009 + 0.040$$

$$= 0.279$$

$$P(R=1, W=1) = 0.045 + 0.050 + 0.324 + 0.040$$

$$= 0.459$$

$$P(S=1 \mid W=1) = \frac{P(S=1, W=1)}{P(W=1)}$$

$$= \frac{\sum_{c,r} P(C=c, S=1, R=r, W=1)}{P(W=1)}$$

$$= \frac{0.279}{0.6471}$$