Consider a change of basis, or a transformation

Consider transformation matrix

$$\begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}$$

The columns of this matrix denote the new basis

$\hat{i}\ (1, 0)$ changed to $\hat{i}\ (3, 0)$

$\hat{j}\ (0, 1)$ changed to $\hat{j}\ (1, 2)$

→ During this transformation, most vectors are knocked off their span.

→ But vectors on the x-axis and the diagonal line passing through $(-1, 1)$ are only stretched by a factor of 3 and 2 respectively.

The vectors which stay on their span after a LT are the eigen vectors for that LT and the factor by which they scale is the eigenvalue.

Note: Eigen value of $-\frac{1}{2}$ denotes that the eigen vectors are flipped and shrunk to half

Why to think about Eigen Vectors?

Consider a 3D rotation. If you find the eigen vector with value 1, you have found the axis of rotation.

Basically, eigen vectors and values tell us what a LT is doing

The Mathematical Aspect

$$\underset{\substack{\uparrow \\ \text{Transformation} \\ \text{matrix}}}{A}\ \underset{\substack{\uparrow \\ \text{Eigen vector}}}{\vec{v}} = \overset{\text{Eigen value}}{\lambda}\ \vec{v}$$

Applying LT (A) on $\vec{v}$ scales $\vec{v}$ by a factor of λ.

$$Av = \lambda v$$

$$Av = \lambda I v$$

$$\therefore Av - \lambda I v = 0$$

$$\therefore (A - \lambda I) v = 0$$

We want a non-zero solution for v. The only way this is possible is if the transformation associated with A squishes space into lower dimension.

Squishification $\underset{\text{implies}}{\Longrightarrow}$ $\det(A - \lambda I) = 0$