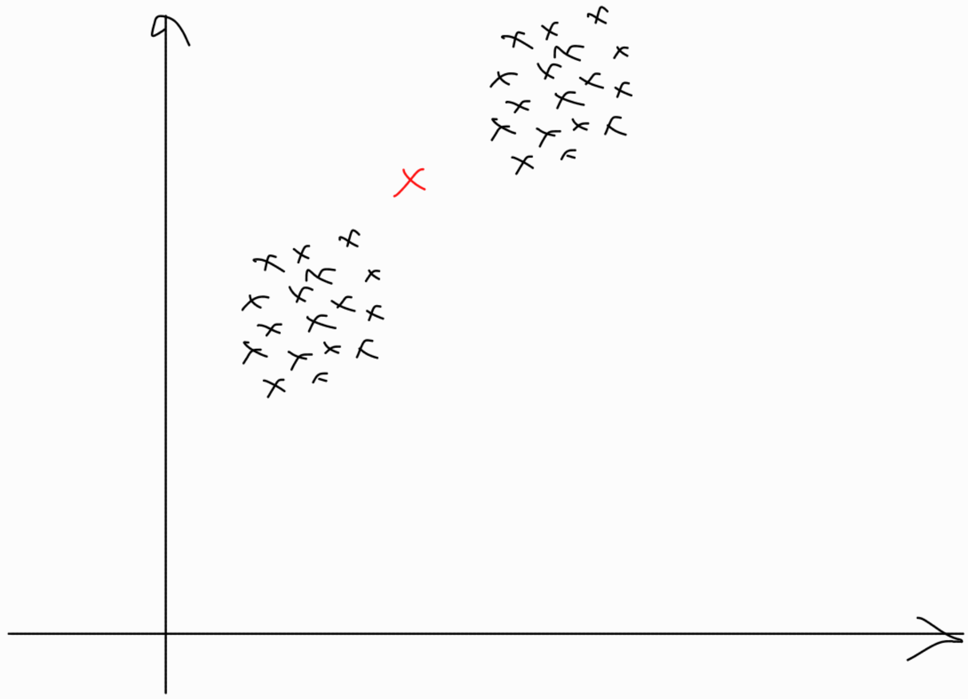
13/02/24



→ How do you figure out if the data point belongs to my data and where?

→ Multiple Gaussians here

$$p(x) = \sum_{\forall c} \underline{\pi_c} \, \mathcal{N}(x_i \; \mu_c, \sigma_c)$$

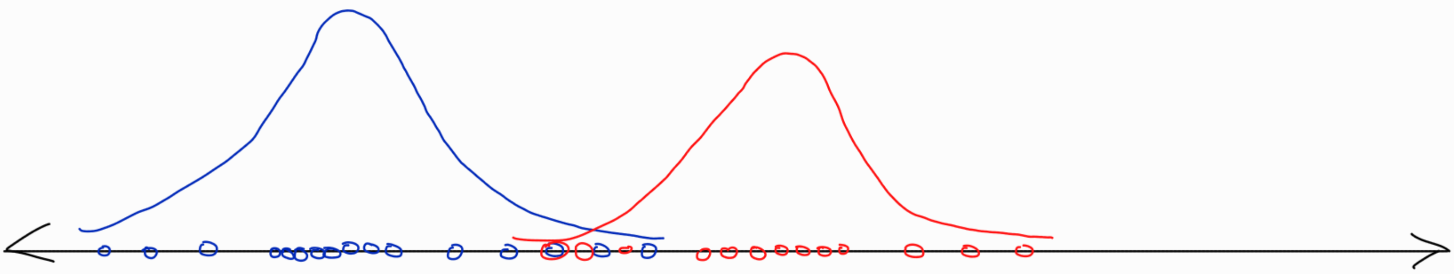
latent variable (weight)

Maximize $p(x)$ using MLE.

Scenario 1

(We know which point belongs to which cluster)

→ Consider 2 distributions A and B.



$$a_i = P(a|x_i)$$

$$b_i = P(b|x_i)$$

$$\mu_a = \frac{\sum a_i x_i}{N}$$

$$\mu_b = \frac{\sum b_i x_i}{N}$$

$$\sigma_a = \frac{\sum\left[a_i(x_i-\mu_a)^2\right]}{N-1}$$

$$\boxed{a_i = 1 - b_i}$$

$$\pi_a = \frac{\sum a_i}{\sum a_i + b_i}$$

Distribution with more points will have higher π_c

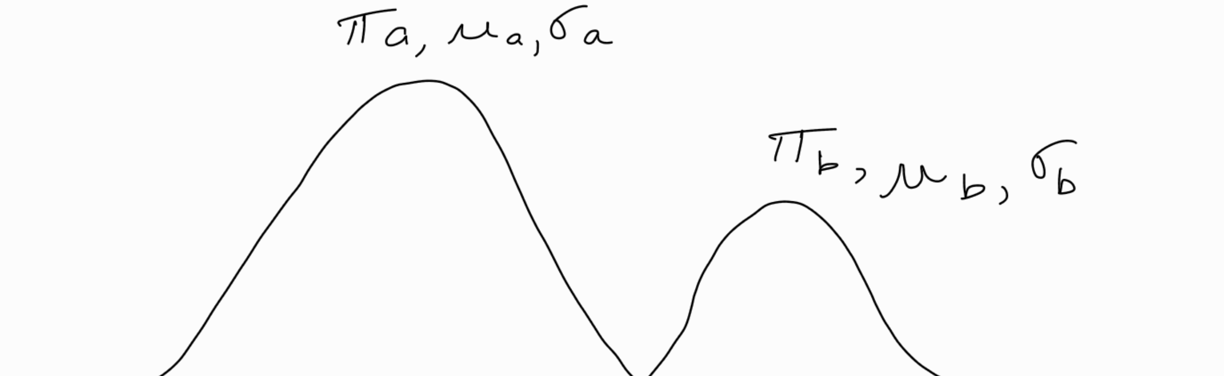
Q] Now, how do we sample from multiple Gaussian distributions?

$$P(x) = \sum \underbrace{\pi_c}_{\uparrow} N(x_i, \mu_c, \sigma_c)$$

A] Sample on π_c first. The Gaussian with higher π_c will be more likely to be picked.

Now, you sample normally from the picked distribution.

Scenario 2 (We know μ and σ for each distribution)



$$a_i = P(a|x_i) = \frac{P(x_i|a) \cdot P(a)}{p(x_i|a)\,p(a) + p(x_i|b)\,p(b)}$$

Expectation Maximization (EM)

1) Start with 2 random Gaussians

Randomly take $\mu_a, \sigma_a, \mu_b, \sigma_b, \pi_a, \pi_b$

2] Now this is scenario 2. Find association for every point (a_i and b_i)

3] Now find $\mu_a = \frac{\sum a_i x_i}{N}$, $\mu_b = \frac{\sum b_i x_i}{N}$

π, σ

$$p(x) = \sum \pi_c \, N(x_i, \mu_c, \sigma_c)$$

$$= \max \prod \left[\pi_c \, N(x, \mu, \sigma_c) \right]$$

$$= \max \sum \log(\qquad)$$

GMM for background subtraction

Cluster with lowest $\left(\frac{\pi_c}{\sigma_c}\right)$

Smaller cluster and higher variance.