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For non-linear boundaries, add another dimension. eg: 2D → 3D

Now, we might be able to fit a plane to classify. ϕ is dimension increaser

Let $\phi(x) = \phi\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 x_1 \\ x_1 x_2 \\ x_2 x_1 \\ x_2 x_2 \end{bmatrix}$

Let $K(X, Y) = \phi(X) \cdot \phi(Y) = \begin{bmatrix} x_1^2 \\ x_1 x_2 \\ x_2 x_1 \\ x_2^2 \end{bmatrix} \begin{bmatrix} y_1^2 \\ y_1 y_2 \\ y_2 y_1 \\ y_2^2 \end{bmatrix}$

$K(X, Y) = (X \cdot Y)^2$

where $X \cdot Y = x_1 y_1 + x_2 y_2$

$= x_1^2 y_1^2 + x_2^2 y_2^2 + 2 x_1 y_1 x_2 y_2$

K is kernel function

Why use Kernel Functions?

You can calculate $\phi(x)$ and $\phi(Y)$ and then multiply them.

The kernel function is a shortcut function which saves compute.

SVM Post Mapping

Training:

$$Q(\alpha) = \sum \alpha - \frac{1}{2} \sum \sum \alpha_i \alpha_j d_i d_j \, \phi(x_i) \cdot \phi(x_j)$$