The University of Georgia

Mathematics Education Program

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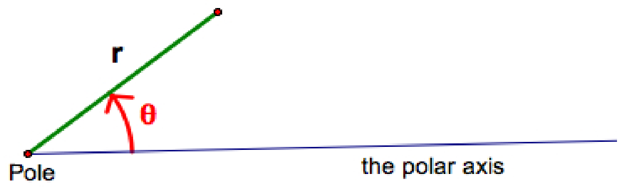
Polar Equations

by

Hee Jung Kim

There are two coordinate systems in the plane: the Cartesian coordinate system and the polar coordinate system. Any point in the plane can have the Cartesian coordinates (x, y) or the polar coordinates (r, θ) in the corresponding system. The polar coordinates (r, θ) of a point consists of the radial coordinate r and the angular coordinate θ. The r coordinate represents the directional distance from the origin called the pole, and the θ coordinate represents the counterclockwise angle from the the initial ray called the polar axis. The polar axis usually coincide with the x-axis in the Cartesian coordinate system.

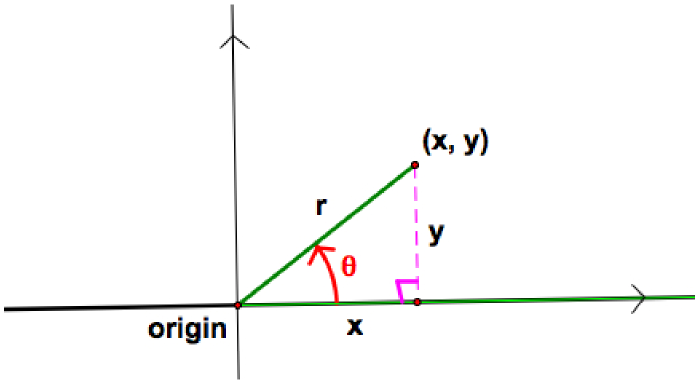


When we overlap two coordinate systems, we can easily convert the polar coordinates (r, θ) of a point to the Cartesian coordinates (x, y).

$$x = r \cos \theta$$

$$y = r \sin \theta$$

$$r = x^2 + y^2$$



I. Now, let's investigate the behavior of the graph of the polar equation r = 2**a**sin (**k**θ) + **b** varying the values of (nonzero) **a**, **b**, and **k** with $0 \le \theta \le 2\pi$.

1.1. The effect of **a**

Let's set **b** = 0 and **k** = 1, and observe the graph of r = 2**a**sinθ as **a** varies. In the animation below, **a** varies from 0 to 10.

$r = 2\sin\theta$

$r = 4\sin\theta$

$r = 10\sin\theta$

$r = 2a\sin\theta$

The graph of r = 2**a**sinθ is a circle with center (0, 1) and radius 1. It can be also algebraically shown by converting the polar equation into the equation in the Cartesian coordinate system.