1). Hydraulic grade line (HGL) = $\underbrace{\frac{P}{\gamma}}_{\text{pressure head}} + \underbrace{z}_{\text{elevation head}}$

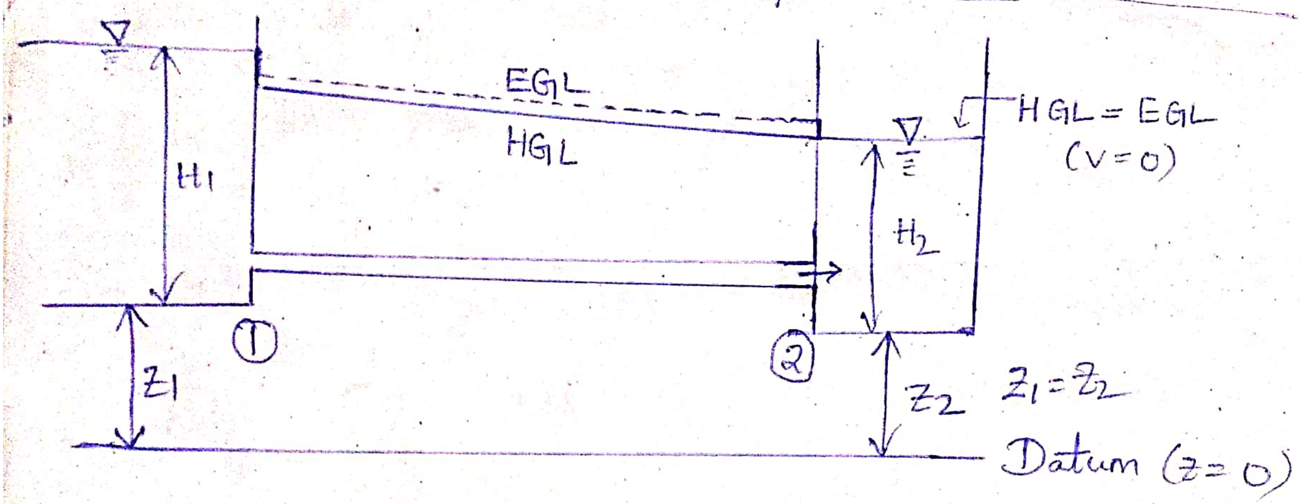
- Hydraulic grade line is measured by piezometer (standing pipe)

2). Energy line (EGL) = HGL + Velocity head

$$= \frac{P}{\gamma} + z + \frac{v^2}{2g}$$

- Energy grade line is measured by using pitot tube

I). HGL and EGL for Reservoirs at upstream and downstream



At point ①

$$EGL_1 = H_1 - K_{entrance} \cdot \frac{v^2}{2g} + z_1$$

$$HGL_1 = EGL - \frac{v^2}{2g}$$

$$= H_1 + z_1 - K_{entrance} \cdot \frac{v^2}{2g} - \frac{v^2}{2g}$$

At point ②

$$EGL_2 = H_2 + K_{exit} \cdot \frac{v^2}{2g} + z_2$$

$$HGL_2 = H_2 + z_2 + K_{exit} \frac{v^2}{2g} - \frac{v^2}{2g}$$

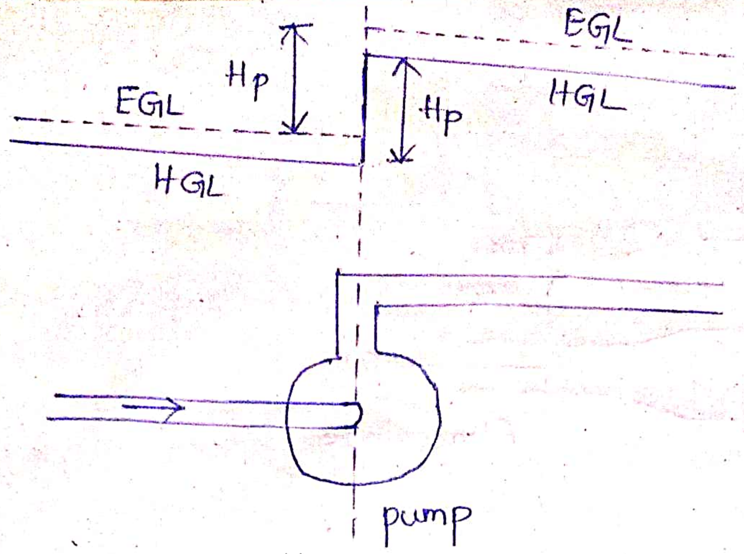
II Pump

$$EGL_2 = EGL_1 - H_{LP}$$

$$HGL_2 = EGL_2 - \frac{v^2}{2g}$$

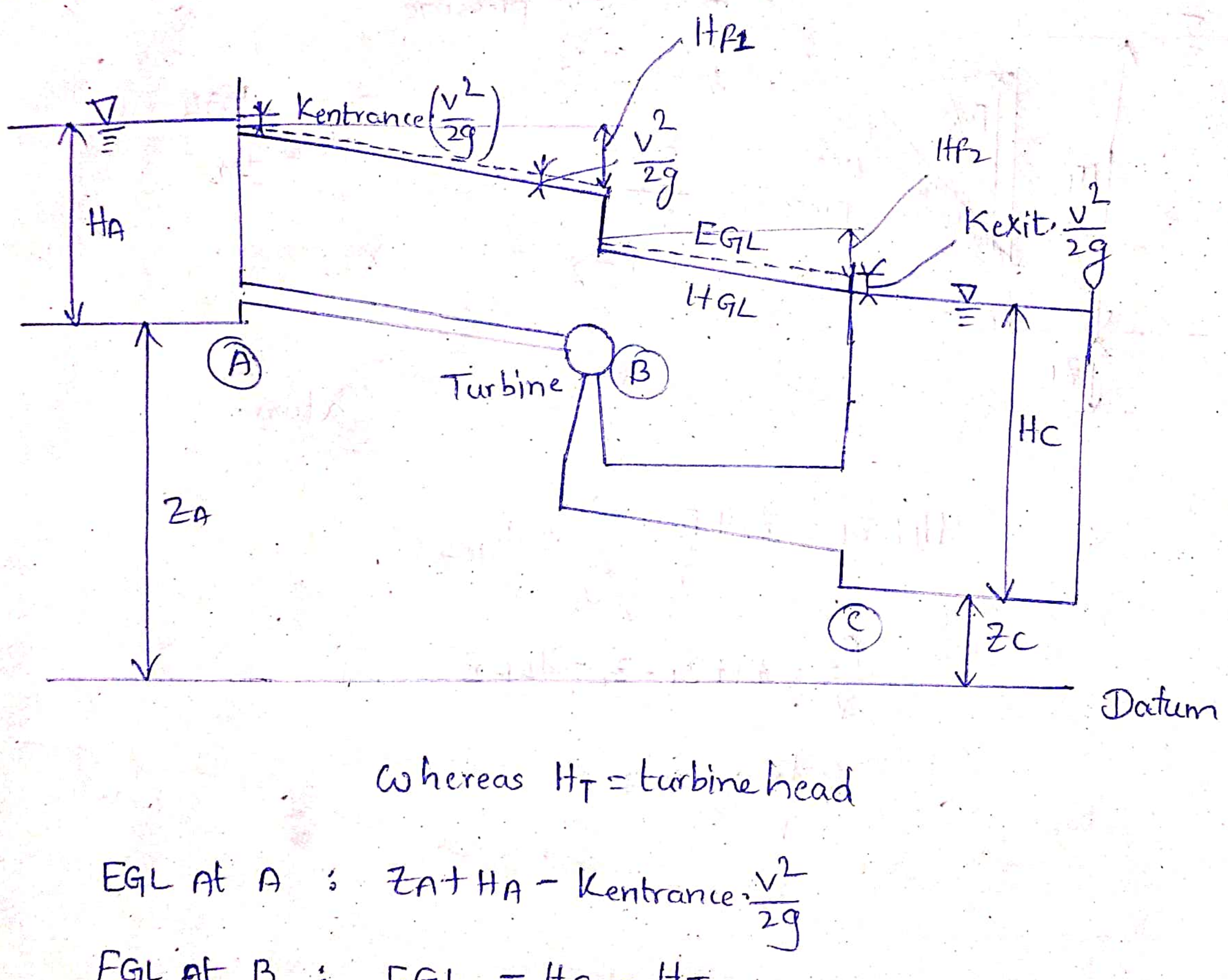
$$H_2 + z_2 = EGL_2 - K_{exit} \cdot \frac{v^2}{2g}$$

II pump



whereas H_p = pump head

III Turbine



whereas H_T = turbine head

EGL At A : $Z_A + H_A - K_{entrance} \cdot \frac{v^2}{2g}$

EGL At B : $EGL_A - H_{f1} - H_T$

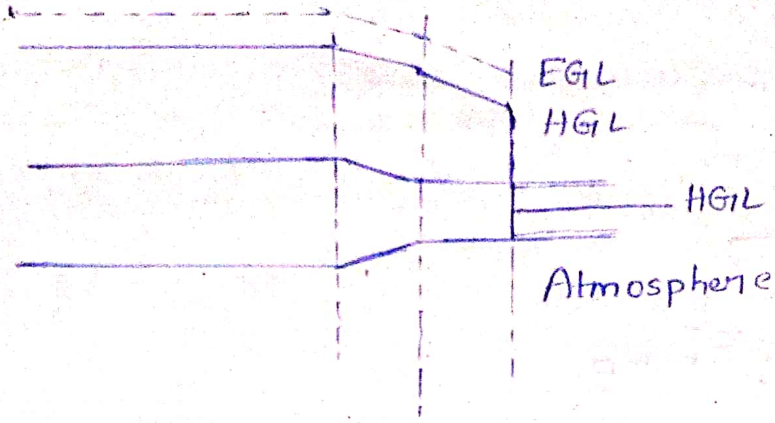
EGL At C : $EGL_B - H_{f2} - K_{exit} \cdot \frac{v^2}{2g}$

HGL At A : $EGL_A - \frac{v^2}{2g}$

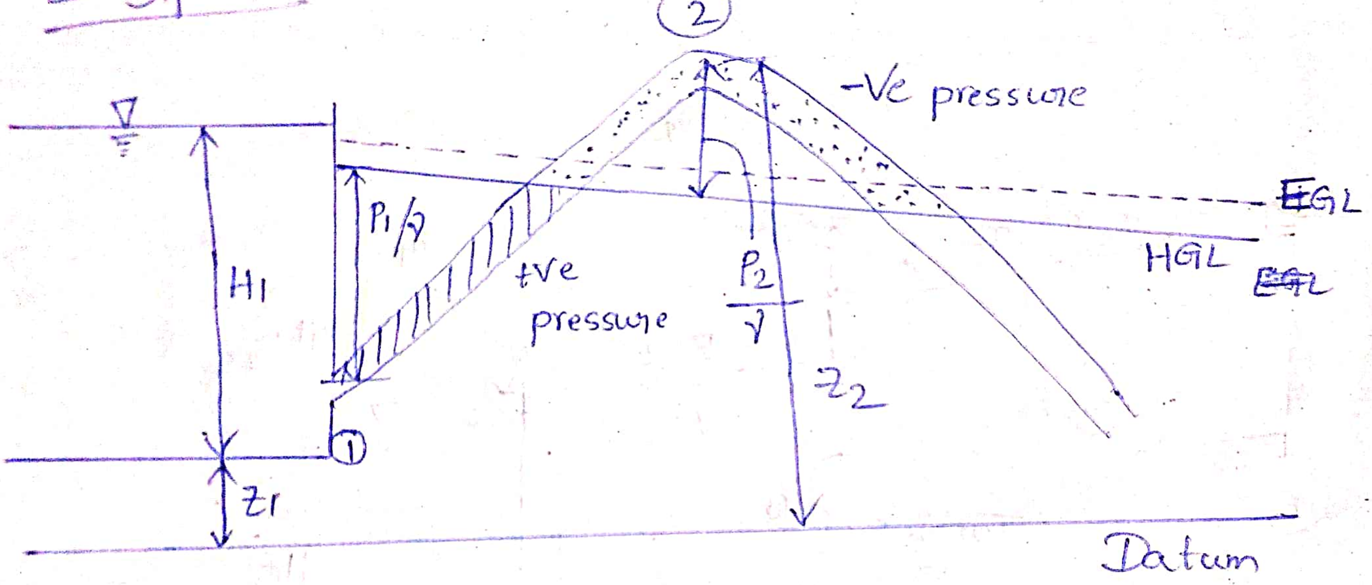
HGL At B : $EGL_B - \frac{v^2}{2g}$

HGL At C : EGL_C

IV Nozzle



V Siphon



$$H_1 + Z_1 = Z_2 + \frac{P_2}{\gamma} + \cancel{\frac{V^2}{2g}}^{\,0} + H_{L1-2}$$

$$\frac{P_2}{\gamma} = H_1 + Z_1 - Z_2 - H_{L1-2}$$

$$\frac{P_2}{\gamma} > -0.7\,m$$

$$Q_{P_{n+1}} = C_P - C_a H_{P_{n+1}}$$

$$Q_{P_{n+1}} = C_P - C_a \left[H_{res} - \frac{Q_{P_{n+1}}^2}{2gA^2}(1-K) \right]$$

Let $K_2 = \dfrac{C_a(1-K)}{2gA^2}$

$$Q_{P_{n+1}} = C_P - C_a H_{res} + K_2 Q_{P_{n+1}}^2$$

$$K_2 Q_{P_{n+1}}^2 - Q_{P_{n+1}} + (C_P - C_a H_{res}) = 0$$

$$Q_{P_{n+1}} = \frac{1 - \sqrt{1 - 4K_2(C_P - C_a H_{res})}}{2K_2}$$

$$H_{P_{n+1}} = H_{res} - \frac{Q_{P_{n+1}}^2}{2gA^2}(1-K)$$

Dead end

Dead end is at the downstream end.

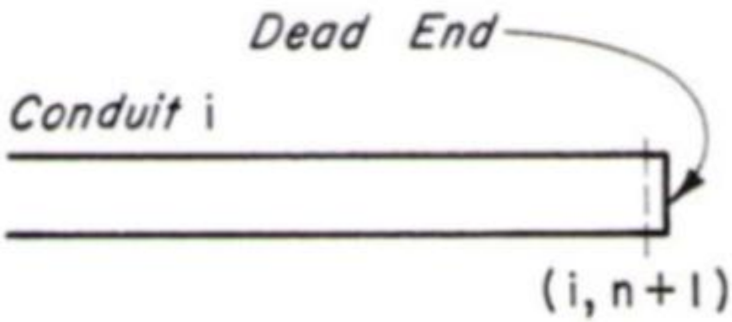


Fig 3.5 Dead end at the downstream

$$Q_{P_{n+1}} = C_P - C_a H_{P_{n+1}}$$

$$Q_{P_{n+1}} = 0 \Rightarrow H_{P_{n+1}} = \frac{C_P}{C_a}$$

Downstream valve

The condition imposed by a valve boundary which is a relationship between the head and discharge through the valve.

Steady state flow through a valve discharging into air.

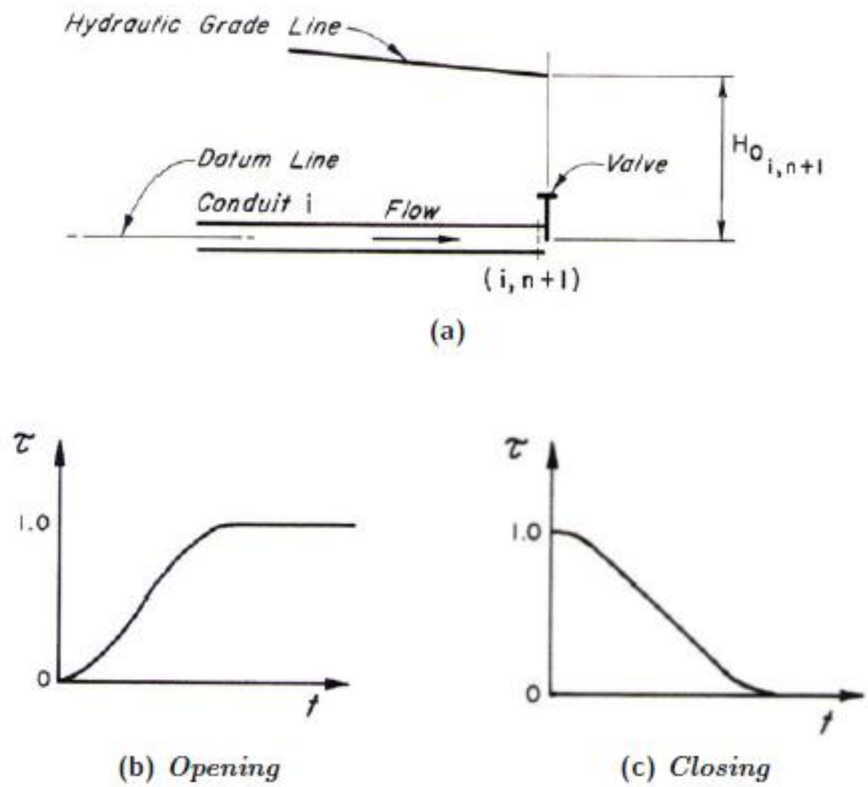


Fig 3.6 Valve at the downstream end and valve characteristics

$Q_{0_{i,n+1}} = (C_d A_V)_0 \sqrt{2gH_{0,n+1}}$

$C_d =$ Coefficient of discharge

$H_{0,n+1} =$ Head upstream of the valve

$A_V =$ area of the valve opening

At transient state

$Q_{P_{n+1}} = (C_d A_V)\sqrt{2gH_{P_{n+1}}}$

From the above two equations

$$\frac{Q_{P_{n+1}}}{Q_{0,n+1}} = \frac{(C_d A_V)_P}{(C_d A_V)_0}\sqrt{\frac{H_{P_{n+1}}}{H_{0,n+1}}}$$

Let $\tau = \frac{(C_d A_V)_P}{(C_d A_V)_0}$

$$\frac{Q_{P_{n+1}}}{Q_{0,n+1}} = \tau\sqrt{\frac{H_{P_{n+1}}}{H_{0,n+1}}}$$

$$Q_{P_{n+1}}^2 = \frac{(Q_{0,n+1}\tau)^2}{H_{0,n+1}} H_{P_{n+1}}$$

PCE at the end = $Q_{P_{n+1}} = C_P - C_a H_{P_{n+1}}$

$$Q_{P_{n+1}} = C_P - C_a Q_{P_{n+1}}^2 \cdot \frac{H_{0,n+1}}{(Q_{0,n+1}\tau)^2}$$

Let $C_V = \frac{(Q_{0,n+1}\tau)^2}{C_a H_{0,n+1}}$

$$C_V Q_{P_{n+1}} = C_P C_V - Q_{P_{n+1}}^2$$

$$Q_{P_{n+1}}^2 + C_V Q_{P_{n+1}} - C_P C_V = 0$$

$$Q_{P_{n+1}} = \frac{-C_V \pm \sqrt{C_V{}^2 + 4C_P C_V}}{2}$$

$$H_{P_{n+1}} = \frac{C_P - Q_{P_{n+1}}}{C_a}$$

τ vs time may be given either in a tabular form or by an algebraic equation.

Orifice downstream

It is similar to valve at downstream, but $\tau = 1.0$

Series junction

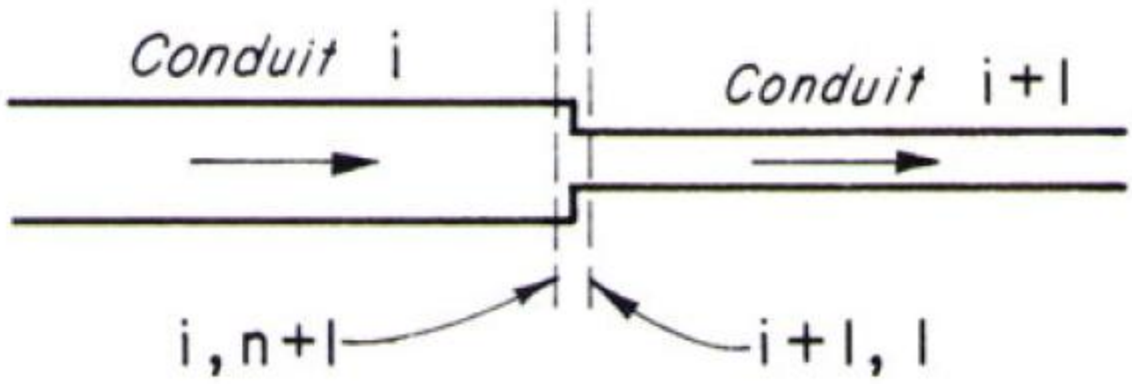


Fig 3.7 series junction

$$H_{P_{i,n+1}} = H_{P_{i+1,1}}$$
$$Q_{P_{i,n+1}} = Q_{P_{i+1,1}}$$

Solve PCE & NCE

$$Q_{P_{i,n+1}} = C_{P_i} - C_{a_i} H_{P_{i,n+1}} \qquad (1)$$
$$Q_{P_{i+1,1}} = C_{n_{i+1}} + C_{a_{i+1}} H_{P_{i+1,1}} \qquad (2)$$
$$H_{P_{i,n+1}} = \frac{C_{P_i} - C_{n_{i+1}}}{C_{a_i} + C_{a_{i+1}}} \qquad (3)$$
$$Q_{P_{i,n+1}} = Q_{P_{i+1,1}} = C_{P_i} - C_{a_i} H_{P_{i,n+1}}$$

Branching junction

$$Q_{P_{i,n+1}} = Q_{P_{i+1,1}} + Q_{P_{i+2,1}}$$
$$H_{P_{i,n+1}} = H_{P_{i+1,1}} = H_{P_{i+2,1}}$$
$$Q_{P_{i,n+1}} = C_{P_i} - C_{a_i} H_{P_{i,n+1}}$$
$$Q_{P_{i+1,1}} = C_{n_{i+1}} + C_{a_{i+1}} H_{P_{i+1,1}}$$
$$Q_{P_{i+2,1}} = C_{n_{i+2}} + C_{a_{i+2}} H_{P_{i+2,1}}$$

From above relations

$$H_{P_{i,n+1}} = \frac{C_{P_i} - C_{n_{i+1}} - C_{n_{i+2}}}{C_{a_i} + C_{a_{i+1}} + C_{a_{i+2}}}$$

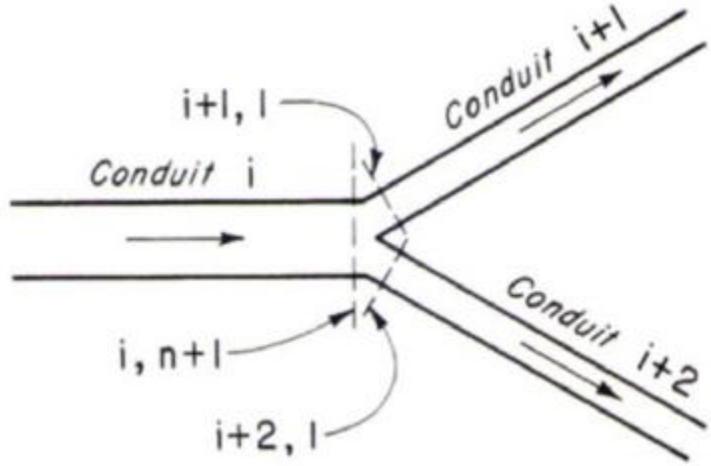


Fig 3.8 branching junction

Centrifugal pump

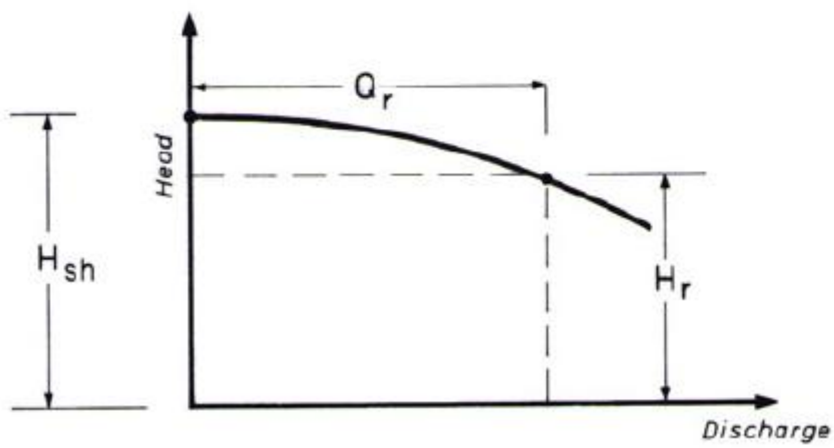


Fig 3.9 Pump characteristics (Centrifugal pump)

Centrifugal pump located at the upstream end

Pump performance curve (quadratic)

$$H_{P_{i,1}} = H_{Shut} - CQ_{P_{i,1}}^{2}$$

$$C = \frac{\left(H_{Shut} - H_{r}\right)}{Q_{r}^{2}}$$

Negative characteristics equation

$$Q_{P_{i,1}} = C_{n_i} - C_{a_i} H_{P_{i,1}}$$

$$H_{P_{i,1}} = \frac{Q_{P_{i,1}} - C_{n_i}}{C_{a_i}}$$

$$CQ_{P_{i,1}}^2 + \frac{Q_{P_{i,1}}}{C_{a_i}} - \left(\frac{C_{n_i}}{C_{a_i}} + H_{Shut} \right) = 0$$

$$Q_{P_{i,1}} = \frac{-1 + \sqrt{1 + 4C_{a_i} C \left(C_{n_i} + C_{a_i} H_{Shut} \right)}}{2C_{a_i} C}$$

Francis turbine

The typical head discharge curve for a Francis turbine

$$H_{P_{i,n+1}} = C_1 + C_2 Q_{P_{i,n+1}}^2$$

Solving this equation with PCE

$$Q_{P_{i,n+1}} = C_{P_i} - C_{a_i} H_{P_{i,n+1}}$$

$$H_{P_{i,n+1}} = \frac{C_{P_i} - Q_{P_{i,n+1}}}{C_{a_i}}$$

$$C_2 Q_{P_{i,n+1}}^2 + \frac{Q_{P_{i,n+1}}}{C_{a_i}} + C_1 - \frac{C_{P_i}}{C_{a_i}} = 0$$

$$Q_{P_{i,n+1}} = \frac{-1 + \sqrt{1 + 4C_{a_i} C_2 \left(C_{P_i} - C_{a_i} C_1 \right)}}{2C_{a_i} C_2}$$

Convergence and stability

Finite difference approximation of PDIs must satisfy the convergence and stability conditions.

<u>Discretization error</u>

$$\underbrace{U(x,t)}_{\text{Exact solution}} - \underbrace{u(x,t)}_{\text{FD solution}} = \text{ Discretization error}$$

<u>Truncation error</u>

$F_i^j(u)=0$ is FD solution at grid point $i\Delta x$ and $j\Delta t$ in which i, j represent grid location in x and t directions in $x-t$ plane.

$U(i, j)$ is the exact solution.

$F_i^j(U)=$ Local truncation error at grid point (i, j)

<u>Consistency</u>

As $(\Delta x, \Delta t)\rightarrow 0 \Rightarrow$ local truncation error tends to zero, then FD equation is said to be consistent.

<u>Convergence</u>

If the scheme is stable and consistent then it is convergent.

For example, u is FD solution, U is exact solution as $(\Delta x, \Delta t)\rightarrow 0,\ u\rightarrow U$ then scheme is said to be convergent.

<u>Stability</u>

A numerical scheme is said to be stable if the amplification of round off error for all sections remains bounded as time $'t'$ tends to infinity.

Explicit FD methods is stable only if $C_N \leq 1$

$$C_N = \frac{\text{Actual wave speed}}{\text{Numerical wave speed}}$$

$$C_N = \frac{a}{\Delta x/\Delta t} = \frac{a\Delta t}{\Delta x} \leq 1$$

Wave propagation

Transient flow in a piping system with constant head reservoir at the upstream end and a valve at the downstream end is shown in the figure given below. Figs illustrate the propagation of a wave in a pipe and the reflections of the wave from a reservoir and a closed valve.

Valve is closed at time $(t)=0$

1. $0 < t \leq \frac{L}{a}$