2. UNSTEADY FLOW GOVERNING EQUATIONS

Important definitions

Steady flow: Pressure and velocity at a point do not change with time then the flow is called steady flow.

Unsteady flow: If the flow conditions change with time, the flow is termed as unsteady.

The intermediate stage between two steady states is called as **transient state**.

Steady oscillatory flow: If the flow conditions are varying with time and if they repeat after a fixed time interval. The time interval at which conditions are repeating is called period. If T is period in seconds, the frequency is $1/T$.

Column separation: Pressure in a closed conduit drops to the vapor pressure of a liquid resulting cavities formed.

Pressure surges: Transients involving slowly varying pressure oscillations are referred as pressure surges.

What causes pressure surges?

Rapid changes in flowrate is caused by

(1) Valves open/close
(2) Pumps (startup, stop, power failure)
(3) Check valve (slamming)
(4) Presence of air

Worst cases are (1) sudden valve closure (2) power failure at a fixed discharge.

What happens when pump stops or valve closes suddenly?

upstream	downstream
Pressure head rises	(1) Pressure head drops (2) Column separation (-14.5 psi, -1.0 atm) (3) Flow reverses and water column rejoins (4) Vapor pocket collapses

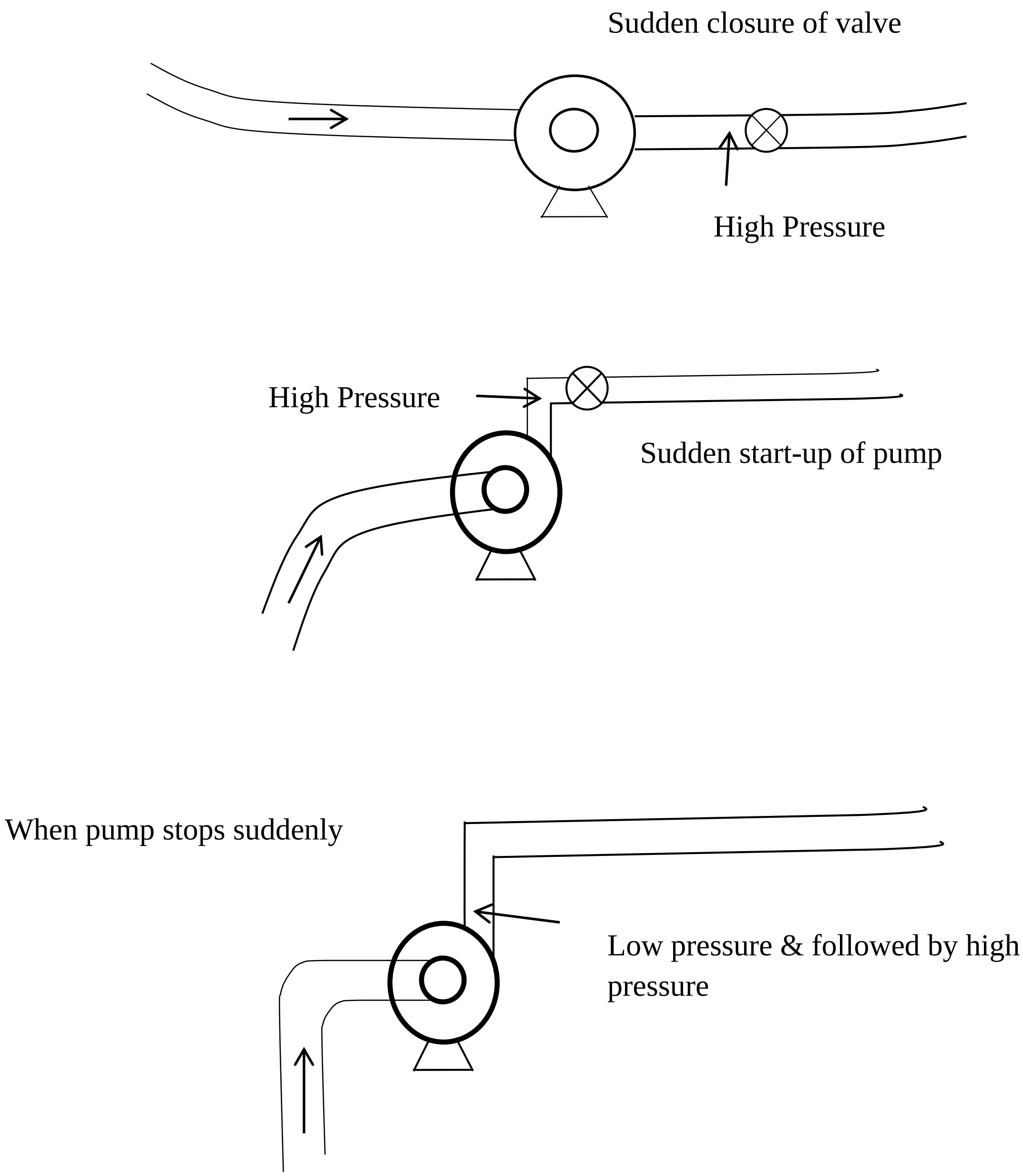


Fig. 2.1 Pump transients

Repercussions:

1. Damaged valve at the downstream of pump
2. Cavitation on pump impeller (Dynamic cavitation)
3. Transient cavitation in delivery and suction pipes

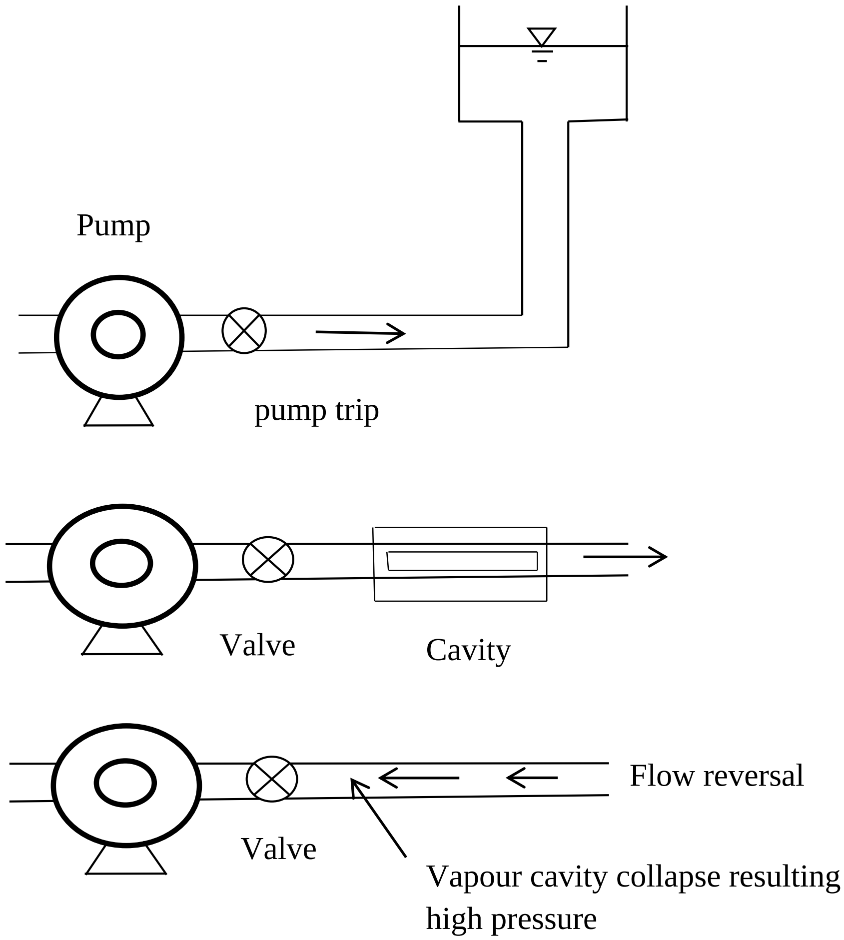


Fig. 2.2 Pump trip

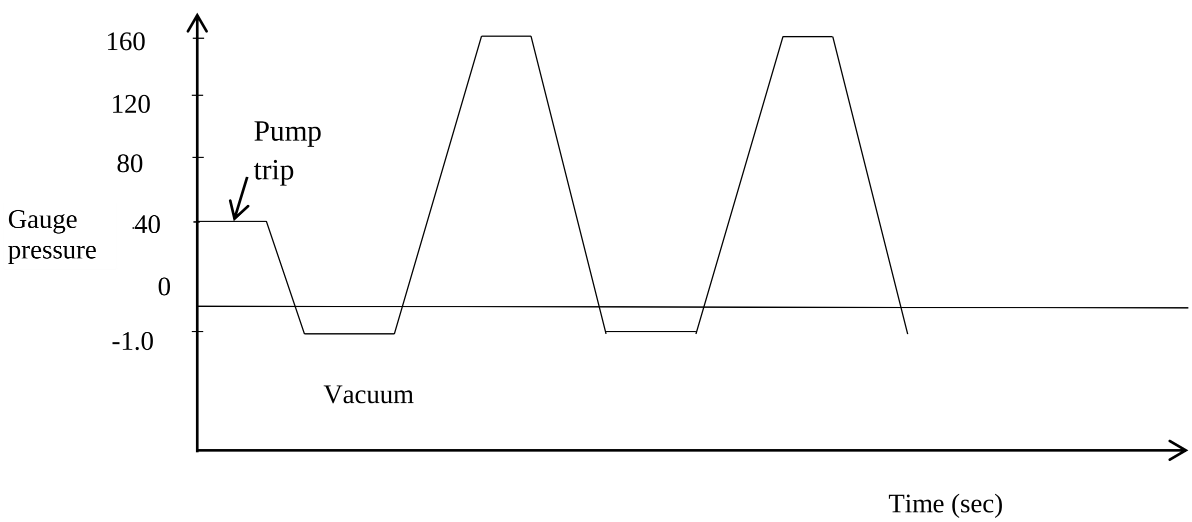


Fig. 2.3 Positive and negative pressures caused by a pump trip

Dynamic cavitation

Cavitation in pumps:

Cavitation in pumps is an undesirable phenomenon; cavitation causes a great deal of noise, vibrations, damage. When the cavitation occurs, cavities collapse into small bubbles thereby creating high temperature spots, emitting shock waves which cause noise. As the water move around the blades, low pressure zone is created. The faster blade moves, the lower the pressure around it. As the lower pressure reaches vapor pressure, fluid vaporizes and form small bubbles. Ultimately the bubbles collapse and create local shock waves in the fluid, which damage the blades.

Transient cavitation

Transient cavitation in pumps is two types, (1) suction cavitation, and (2) delivery cavitation.

Suction cavitation:

Low pressure on the suction side creates vapor cavities at the eye of the pump impeller. The vapor is carried over to the discharge side of the pump, due to the high pressure on the discharge side, vapor cavities implode on the face of the impeller. An impeller that has been operating under a suction cavitation condition can have large chunks of material removed from its face, which cause premature failure of the pump.

Discharge cavitation:

Discharge cavitation occurs when the pump discharge pressure is very high. This occurs in a pump when it is running at less than 10% of its efficiency. The high discharge pressure causes the most of fluid to circulate inside the pump instead of being allowed to flow out of the discharge. As the liquid flows around the impeller, it must pass through the small clearance between the impeller and pump housing at very high flow velocity. This flow velocity causes a vacuum to develop at the housing wall, which turns the liquid into a vapor. A pump operating under these conditions shows premature wear of the impeller blades; pump housing, failure of seals, bearings and impeller shaft.

Cavitation inside pipes:

Vapor cavities formed on the pipe wall collapse resulting high pressure at the pipe wall resulting following negative effects.

(1) Erosion of pipe material (2) fatigue of the pipeline (3) pipe wall becomes thinner (4) ultimately pipe break (5) pipe lining damaged (6) pipe collapse.

Causes of transients

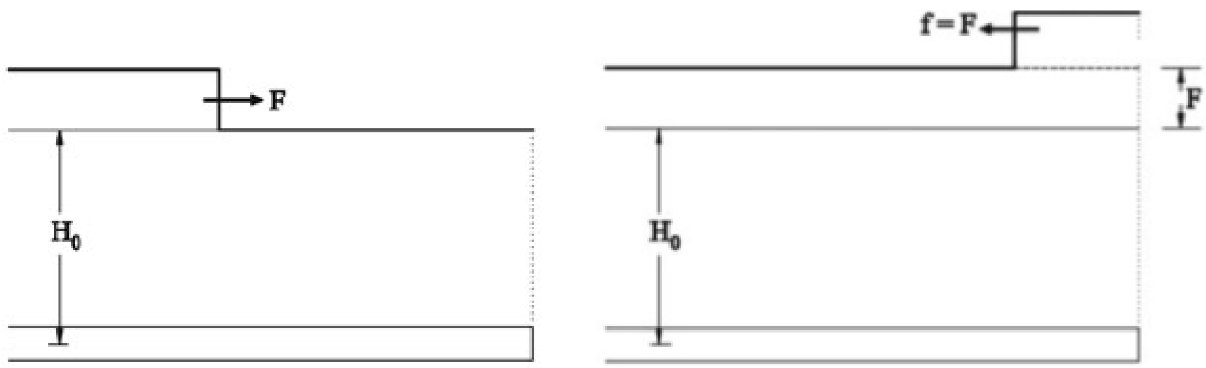
1. Origin of transients is the rapid variation of flow velocity.
2. Pressure waves created by transients propagate at a speed of 1500 m/s. Speed of sound wave propagation depends on the pipe material, pipe diameter, pipe thickness and pipe joints.

3. Pressure waves are reflected at the ends of the system. They move back and forth.
4. Especially high points in the system are more at risk because susceptibility of water column separation.

   Reflection of the pressure wave propagation at various boundaries is shown in the Fig. 2.4.

(a) Incident pressure wave (b) Reflected wave

A. Reflection at the reservoir



(a) Incident wave (b) Reflected wave

B. Reflection at the dead end

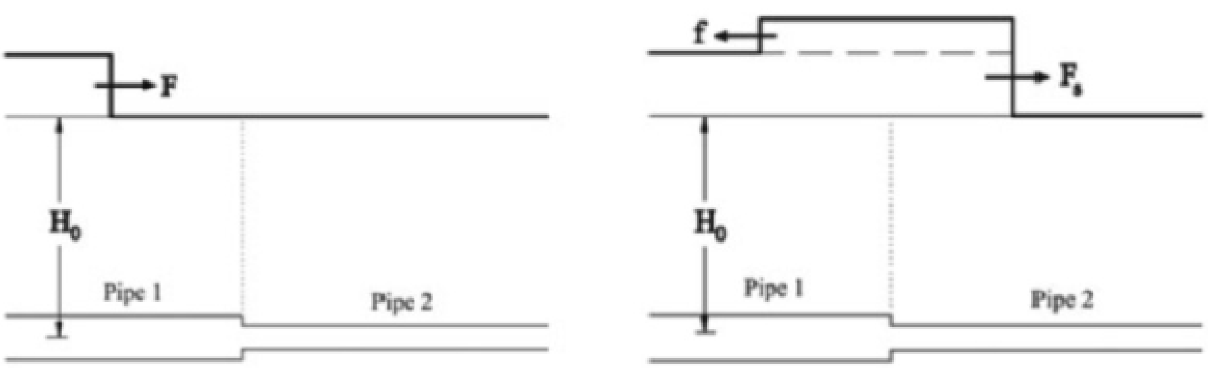


Fig. 2.4 C Reflection at the series junction

Following devices may be used reduce the water hammer

(1) Fly wheel
(2) Vacuum valves
(3) Surge relief valves
(4) Variable frequency drive (VFD)
(5) Stand pipe
(6) Bypass check valve
(7) Surge vessel

Fly wheel:

1. Flywheel increases rotating inertia of the pump thereby increases energy requirements of the pump.
2. Cannot be used for submersible pumps

Vacuum breaker:

1. Vacuum breaker is used for preventing column separation.
2. Vacuum breakers require high maintenance.
3. Reaction time designing is very important, time between transient and time to fully open the valve.
4. When it is fully open large volume of air admitted into the pipe.

Surge relief valve or surge anticipation valve:

1. It is good for relieving high pressure by releasing water into atmosphere.
2. Ineffective against low pressure (no use of surge relief valves during column separation)

Variable frequency drive (VFD):

1. The VFD is used for (i) controlling pump startup and (ii) controlling pump shutdown.

2. Variable frequency drive is ineffective during power failure.

Stand pipe:

1. Stand pipe needs to be higher than HGL so it is a tall structure and very expensive solution.
2. Suitable for only high pressures, not suitable for column separation.

Bypass Check Valve:

(1) Good for overcoming negative pressures.
(2) Needs positive pressure upstream,
(3) Not suitable for positive pressure surges.

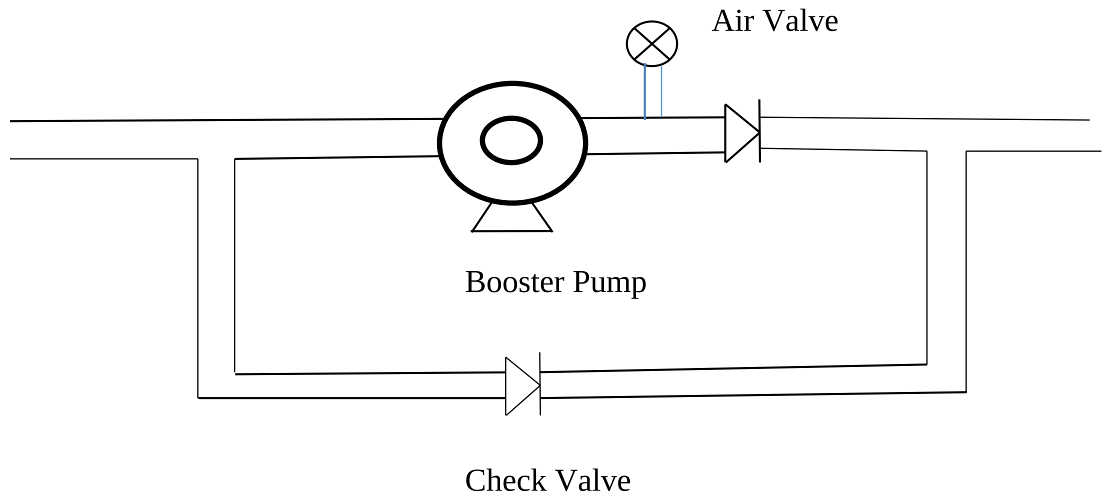


Fig. 2.5 Bypass Check Valve

Surge vessel:

1. This is the best solution for negative and positive surges.
2. Gives energy and receives energy.
3. Low maintenance.

Required information to do the surge analysis

1. Detailed profile of the pipe
2. Maximum and minimum allowed pressures
3. Maximum flowrate
4. Pipe characteristics (diameter, length, thickness and material)

Pipe characteristics

Welded steel pipe

Allowed negative pressure = -10.0 psi = -0.70 bar
Speed of pressure waves = 488.0 m/s

Concrete pipe

Allowed negative pressure = -10.0 psi = -0.70 bar
Speed of pressure waves = 549.0 m/s

Prestressed concrete

Allowed negative pressure = 0 bar (g)
Speed of pressure waves = 549.0 m/s

HDPE pipe

Allowed negative pressure = -10.0 psi = -0.70 bar = -7.0 m water head
Speed of pressure waves = 243.80 m/s

PVC pipe

Allowed negative pressure = -3.0 psi = -0.21 bar = -2.1 m water head
Speed of pressure waves = 213.00 m/s

Ductile Iron pipe

Allowed negative pressure = -6.0 psig = -0.42 bar = -4.2 m water head
Speed of pressure waves = 1372.00 m/s

Surge tank design criteria (What needs to be computed)

1. Surge tank volume
2. Design pressure
3. Outlet diameter
4. Constant pressure during steady state

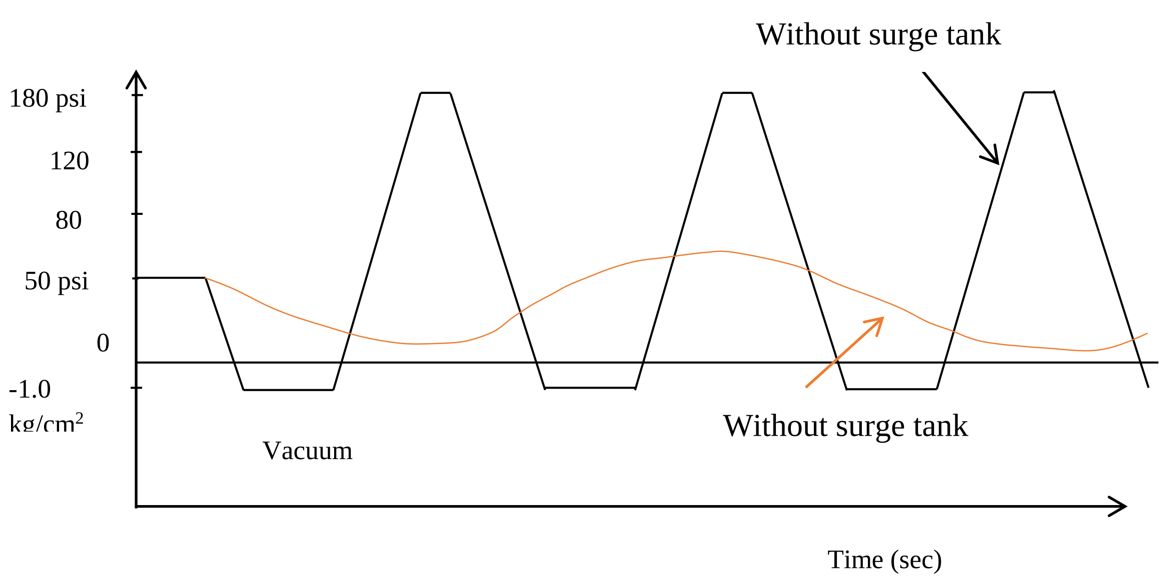


Fig. 2.6. Surge pressures resulting from a pump trip with and without surge tank

History of water hammer

Joukowsky first published basic theory of water hammer as explained below.

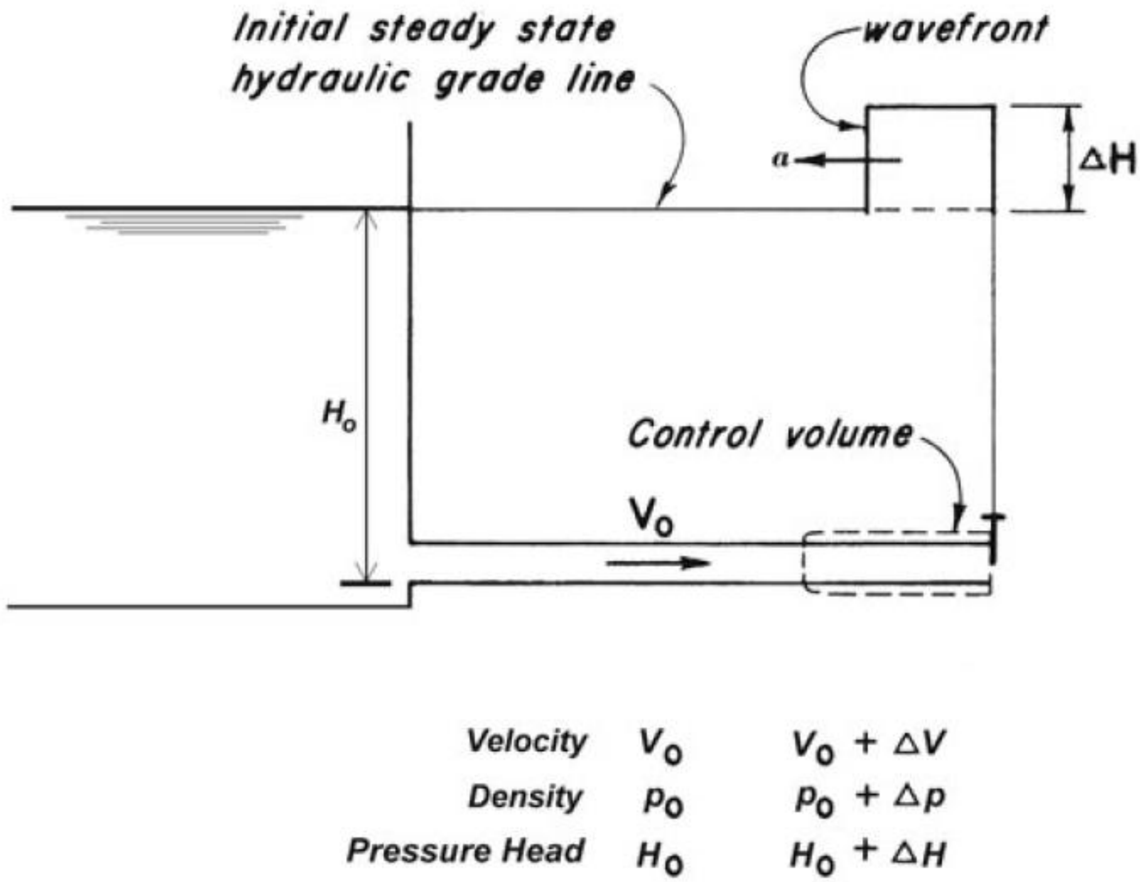


Fig. 2.7. Unsteady flow

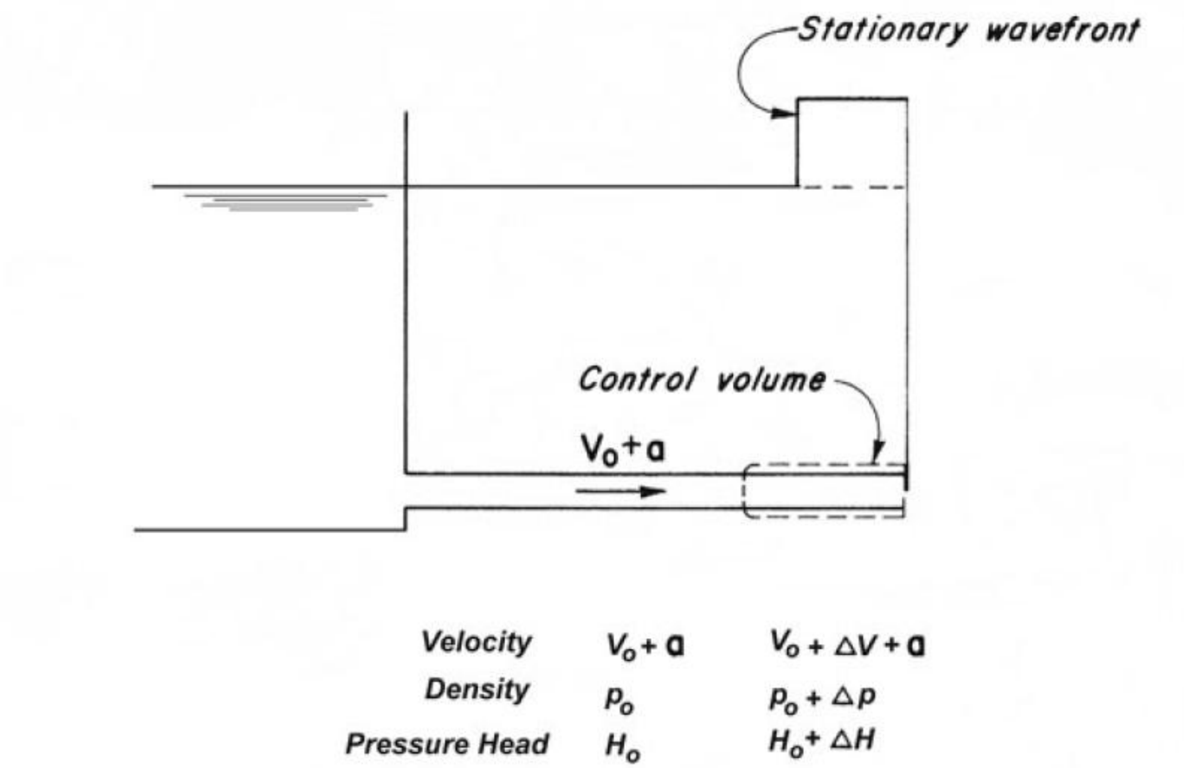


Fig. 2.8. Control volume moving upstream with a velocity

The time rate of change of momentum in the positive x-direction is

$$\rho_0(V_0 + a)A[(V_0 + \Delta V + a) - (V_0 + a)]$$

$$\rho_0(V_0 + a)A\Delta V$$

Negative friction resultant pressure force acting on the control volume

$$F = \rho_0 g H_0 A - \rho_0 g(H_0 + \Delta H)A$$

$$= -\rho_0 g \Delta H A$$

Equating forces

$$\rho_0(V_0 + a)A\Delta V = -\rho_0 g \Delta H A$$

$$\therefore \Delta H = -\frac{1}{g}(V_0 + a)\Delta V$$

The wave velocity a in metal or concrete pipes = 1000 m/s whereas typical flow velocity (V_0) = 1-10 m/s therefore V_0 is neglected.

$$\therefore \Delta H = -\frac{1}{g}a\Delta V$$

The negative sign indicates increase in pressure head as a result of the decrease in velocity on the upstream direction.

In the downstream direction,

$$\therefore \Delta H = \frac{1}{g} a \Delta V$$

This indicates pressure head on the downstream increases with increase in velocity.

Derivation of pressure wave velocity:

Applying continuity equation

$$\rho_0 (V_0 + a) A = (\rho_0 + \Delta\rho)(V_0 + \Delta V + a)$$

$$\Delta V = -\frac{\Delta\rho}{\rho_0}(V_0 + \Delta V + a)$$

But $V_0 + \Delta V \ll a$

$$\Delta V = -\frac{\Delta\rho}{\rho_0} a$$

Derivation of wave velocity

The bulk modulus of Elasticity

$$K = \frac{\Delta P}{\frac{\Delta\rho}{\rho_0}}$$

$$\frac{\Delta\rho}{\rho_0} = \frac{\Delta P}{K}$$

$$\Delta V = -\frac{\Delta P}{K} a \Rightarrow a = -\frac{K \Delta V}{\Delta P}$$

$$\Delta P = \rho_0 g \Delta H$$

$$a = -K \frac{\Delta V}{\rho_0 g \Delta H} \quad ,$$

however, $\Delta V = -\frac{g \Delta H}{a}$

$$a = +K \frac{g \Delta H}{a \rho_0 g \Delta H}$$

$$a = \sqrt{\frac{K}{\rho_0}}$$

This formula is applicable only for slightly compressible fluid in a rigid pipe.

Reynolds Transport Theorem (RTT)

$$\frac{dB_{sys}}{dt} = \frac{d}{dt}\int_{v} \beta\rho dv + \left(\beta\rho AV_s\right)_{out} - \left(\beta\rho AV_s\right)_{in}$$

$B =$ extensive property

$\beta =$ intensive property

$v =$ Total volume

Note that the velocity V_s is with respect to the control surface since it accounts for the inflow or outflow from the control volume.

For fixed control volume, $V_s =$ fluid flow velocity V. If control volume is moving with 'w' velocity then $V_s = V - w$

$$\frac{dB_{sys}}{dt} = \frac{d}{dt}\int_{v} \beta\rho dv + \left[\beta\rho A(V-w)\right]_{out} - \left[\beta\rho A(V-w)\right]_{in}$$

Derivation of Continuity Equation

Assumptions

1) Flow is slightly compressible
2) Conduit walls are elastic
3) Flow is one directional.

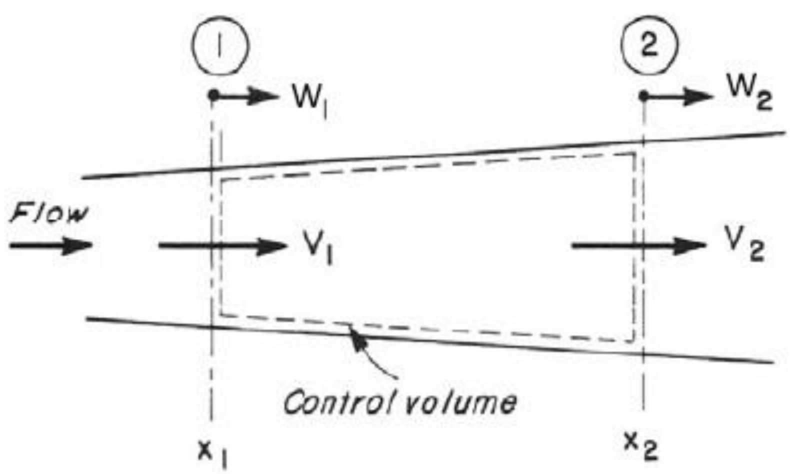


Fig. 2.9. Application of continuity equation