2. UNSTEADY FLOW GOVERNING EQUATIONS

Important definitions

Steady flow: Pressure and velocity at a point do not change with time then the flow is called steady flow.

Unsteady flow: If the flow conditions change with time, the flow is termed as unsteady.

The intermediate stage between two steady states is called as **transient state**.

Steady oscillatory flow: If the flow conditions are varying with time and if they repeat after a fixed time interval. The time interval at which conditions are repeating is called period. If T is period in seconds, the frequency is $1/T$.

Column separation: Pressure in a closed conduit drops to the vapor pressure of a liquid resulting cavities formed.

Pressure surges: Transients involving slowly varying pressure oscillations are referred as pressure surges.

What causes pressure surges?

Rapid changes in flowrate is caused by

(1) Valves open/close
(2) Pumps (startup, stop, power failure)
(3) Check valve (slamming)
(4) Presence of air

Worst cases are (1) sudden valve closure (2) power failure at a fixed discharge.

What happens when pump stops or valve closes suddenly?

upstream	downstream
Pressure head rises	(1) Pressure head drops (2) Column separation (-14.5 psi, -1.0 atm) (3) Flow reverses and water column rejoins (4) Vapor pocket collapses

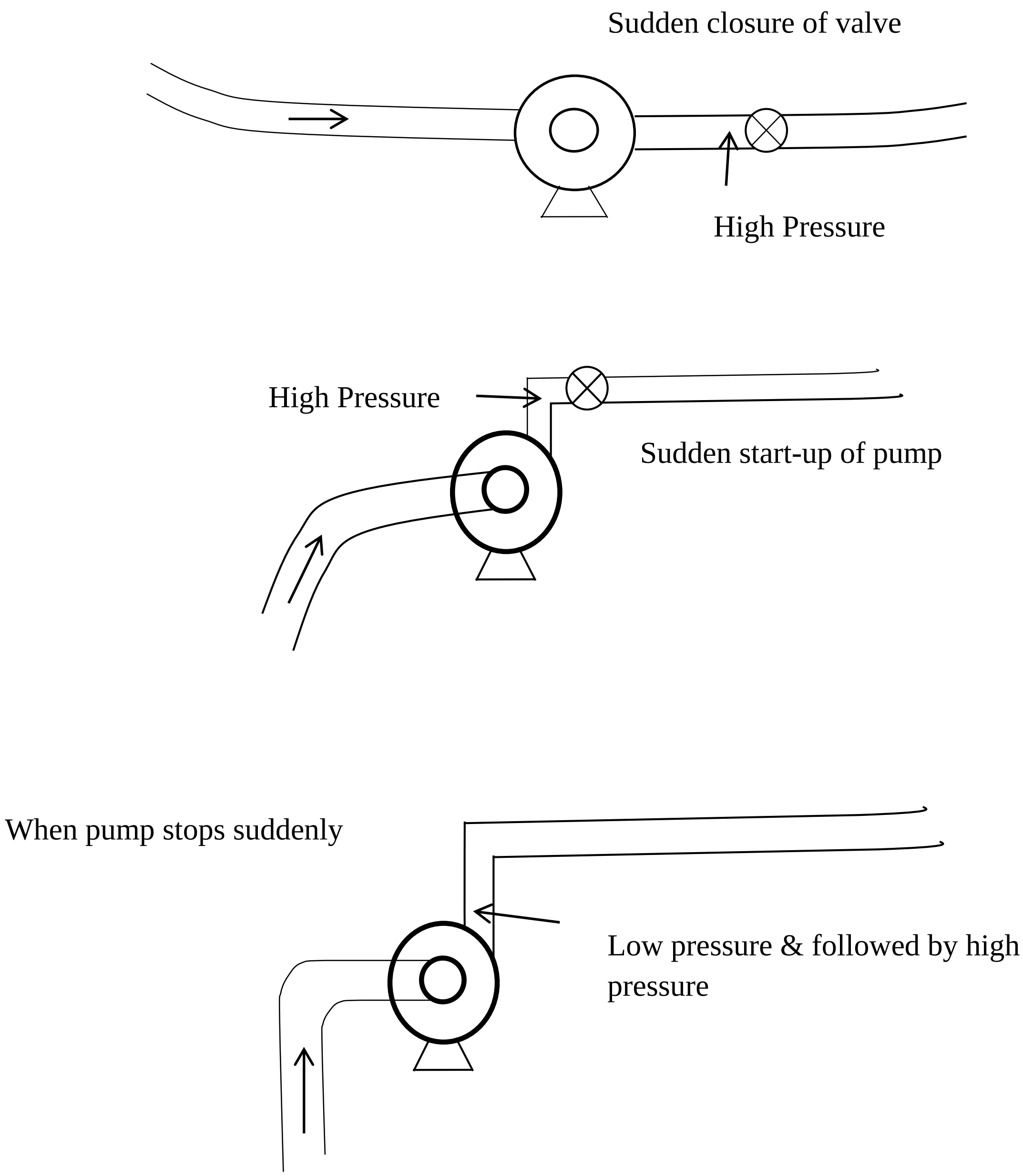


Fig. 2.1 Pump transients

Repercussions:

1. Damaged valve at the downstream of pump
2. Cavitation on pump impeller (Dynamic cavitation)
3. Transient cavitation in delivery and suction pipes

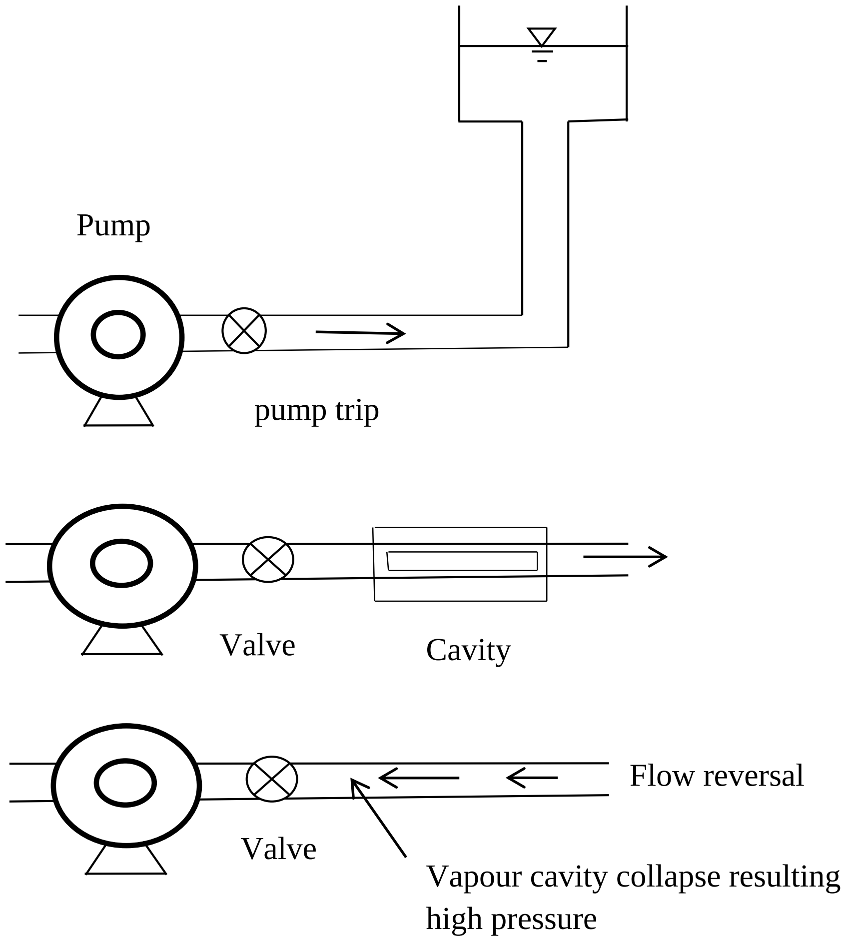


Fig. 2.2 Pump trip

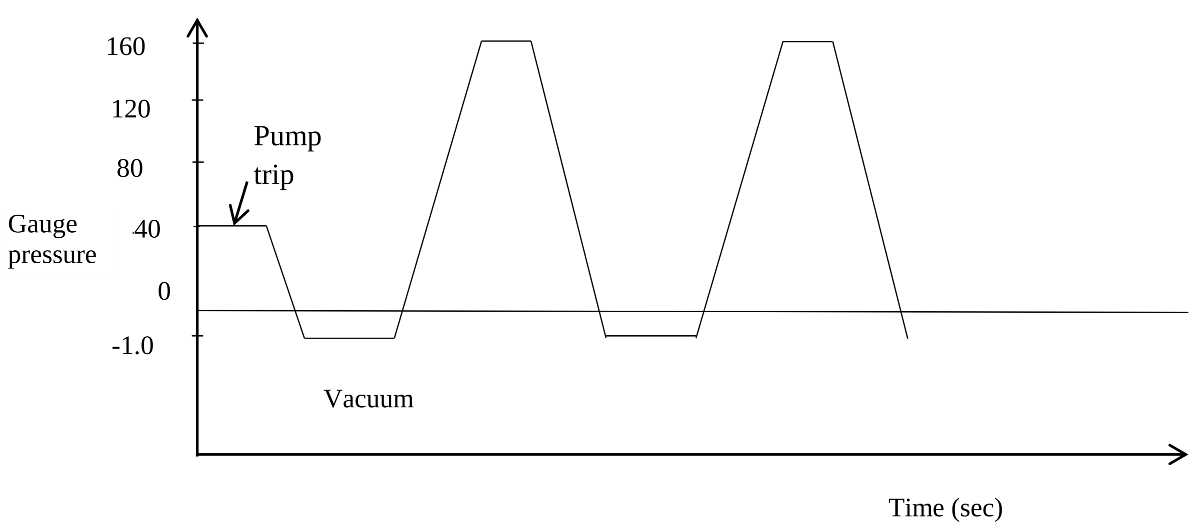


Fig. 2.3 Positive and negative pressures caused by a pump trip

Dynamic cavitation

Cavitation in pumps:

Cavitation in pumps is an undesirable phenomenon; cavitation causes a great deal of noise, vibrations, damage. When the cavitation occurs, cavities collapse into small bubbles thereby creating high temperature spots, emitting shock waves which cause noise. As the water move around the blades, low pressure zone is created. The faster blade moves, the lower the pressure around it. As the lower pressure reaches vapor pressure, fluid vaporizes and form small bubbles. Ultimately the bubbles collapse and create local shock waves in the fluid, which damage the blades.

Transient cavitation

Transient cavitation in pumps is two types, (1) suction cavitation, and (2) delivery cavitation.

Suction cavitation:

Low pressure on the suction side creates vapor cavities at the eye of the pump impeller. The vapor is carried over to the discharge side of the pump, due to the high pressure on the discharge side, vapor cavities implode on the face of the impeller. An impeller that has been operating under a suction cavitation condition can have large chunks of material removed from its face, which cause premature failure of the pump.

Discharge cavitation:

Discharge cavitation occurs when the pump discharge pressure is very high. This occurs in a pump when it is running at less than 10% of its efficiency. The high discharge pressure causes the most of fluid to circulate inside the pump instead of being allowed to flow out of the discharge. As the liquid flows around the impeller, it must pass through the small clearance between the impeller and pump housing at very high flow velocity. This flow velocity causes a vacuum to develop at the housing wall, which turns the liquid into a vapor. A pump operating under these conditions shows premature wear of the impeller blades; pump housing, failure of seals, bearings and impeller shaft.

Cavitation inside pipes:

Vapor cavities formed on the pipe wall collapse resulting high pressure at the pipe wall resulting following negative effects.

(1) Erosion of pipe material (2) fatigue of the pipeline (3) pipe wall becomes thinner (4) ultimately pipe break (5) pipe lining damaged (6) pipe collapse.

Causes of transients

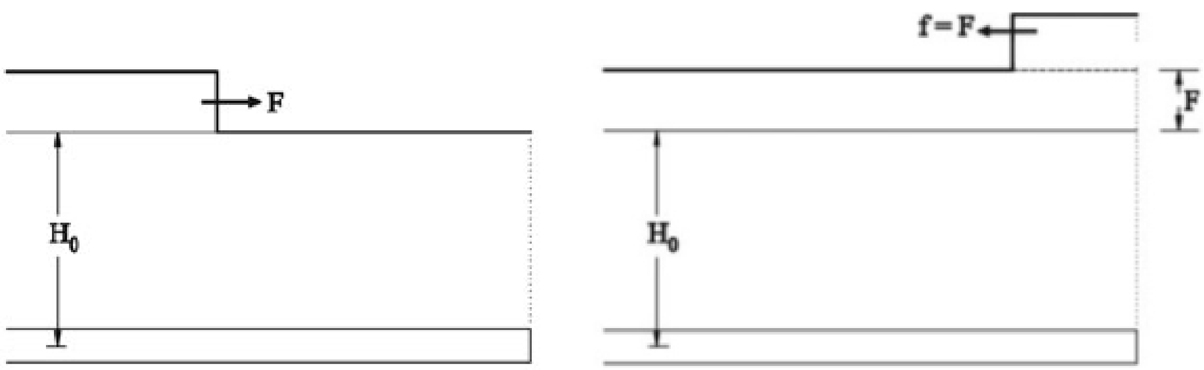
1. Origin of transients is the rapid variation of flow velocity.
2. Pressure waves created by transients propagate at a speed of 1500 m/s. Speed of sound wave propagation depends on the pipe material, pipe diameter, pipe thickness and pipe joints.

3. Pressure waves are reflected at the ends of the system. They move back and forth.
4. Especially high points in the system are more at risk because susceptibility of water column separation.

   Reflection of the pressure wave propagation at various boundaries is shown in the Fig. 2.4.

(a) Incident pressure wave (b) Reflected wave

A. Reflection at the reservoir



(a) Incident wave (b) Reflected wave

B. Reflection at the dead end

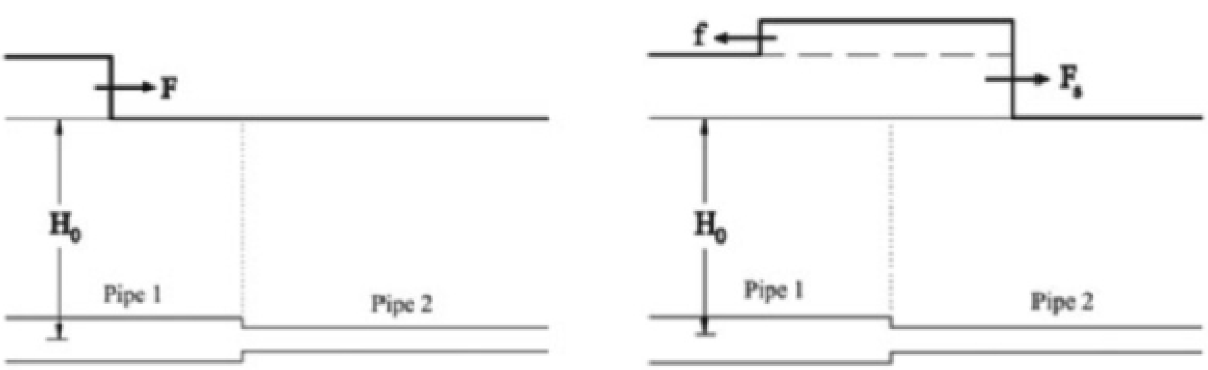


Fig. 2.4 C Reflection at the series junction

Following devices may be used reduce the water hammer

(1) Fly wheel
(2) Vacuum valves
(3) Surge relief valves
(4) Variable frequency drive (VFD)
(5) Stand pipe
(6) Bypass check valve
(7) Surge vessel

Fly wheel:

1. Flywheel increases rotating inertia of the pump thereby increases energy requirements of the pump.
2. Cannot be used for submersible pumps

Vacuum breaker:

1. Vacuum breaker is used for preventing column separation.
2. Vacuum breakers require high maintenance.
3. Reaction time designing is very important, time between transient and time to fully open the valve.
4. When it is fully open large volume of air admitted into the pipe.

Surge relief valve or surge anticipation valve:

1. It is good for relieving high pressure by releasing water into atmosphere.
2. Ineffective against low pressure (no use of surge relief valves during column separation)

Variable frequency drive (VFD):

1. The VFD is used for (i) controlling pump startup and (ii) controlling pump shutdown.

2. Variable frequency drive is ineffective during power failure.

Stand pipe:

1. Stand pipe needs to be higher than HGL so it is a tall structure and very expensive solution.
2. Suitable for only high pressures, not suitable for column separation.

Bypass Check Valve:

(1) Good for overcoming negative pressures.
(2) Needs positive pressure upstream,
(3) Not suitable for positive pressure surges.

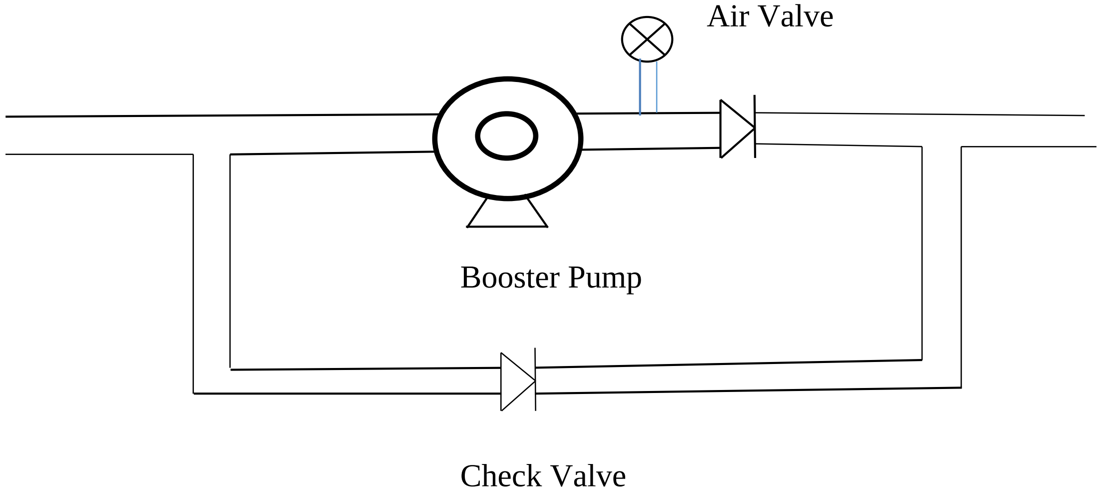


Fig. 2.5 Bypass Check Valve

Surge vessel:

1. This is the best solution for negative and positive surges.
2. Gives energy and receives energy.
3. Low maintenance.

Required information to do the surge analysis

1. Detailed profile of the pipe
2. Maximum and minimum allowed pressures
3. Maximum flowrate
4. Pipe characteristics (diameter, length, thickness and material)

Pipe characteristics

Welded steel pipe

Allowed negative pressure = -10.0 psi = -0.70 bar
Speed of pressure waves = 488.0 m/s

Concrete pipe

Allowed negative pressure = -10.0 psi = -0.70 bar
Speed of pressure waves = 549.0 m/s

Prestressed concrete

Allowed negative pressure = 0 bar (g)
Speed of pressure waves = 549.0 m/s

HDPE pipe

Allowed negative pressure = -10.0 psi = -0.70 bar = -7.0 m water head
Speed of pressure waves = 243.80 m/s

PVC pipe

Allowed negative pressure = -3.0 psi = -0.21 bar = -2.1 m water head
Speed of pressure waves = 213.00 m/s

Ductile Iron pipe

Allowed negative pressure = -6.0 psig = -0.42 bar = -4.2 m water head
Speed of pressure waves = 1372.00 m/s

Surge tank design criteria (What needs to be computed)

1. Surge tank volume
2. Design pressure
3. Outlet diameter
4. Constant pressure during steady state

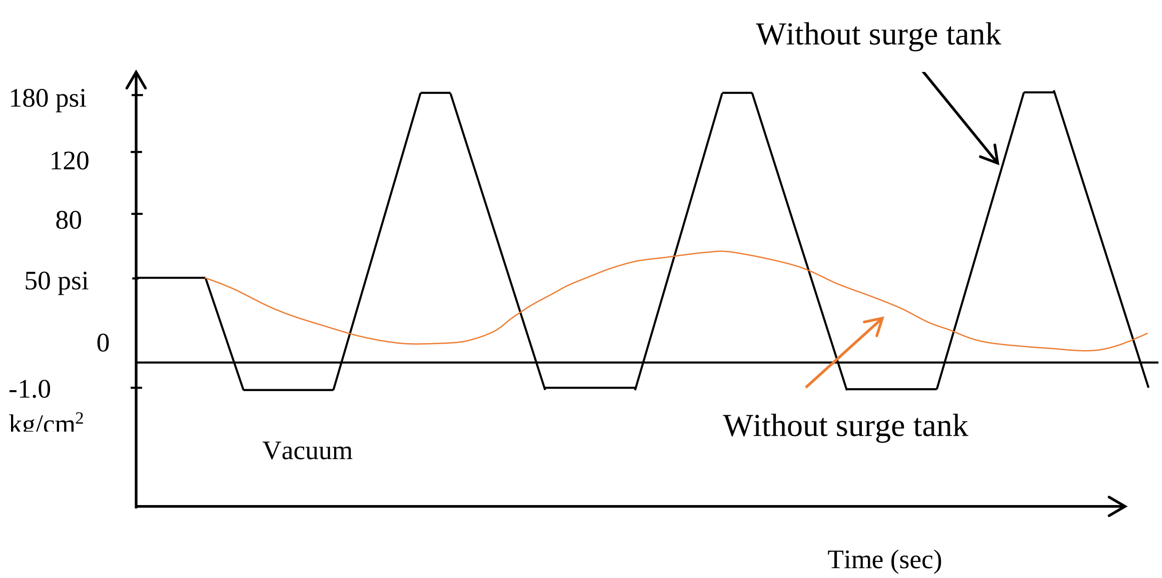


Fig. 2.6. Surge pressures resulting from a pump trip with and without surge tank

History of water hammer

Joukowsky first published basic theory of water hammer as explained below.

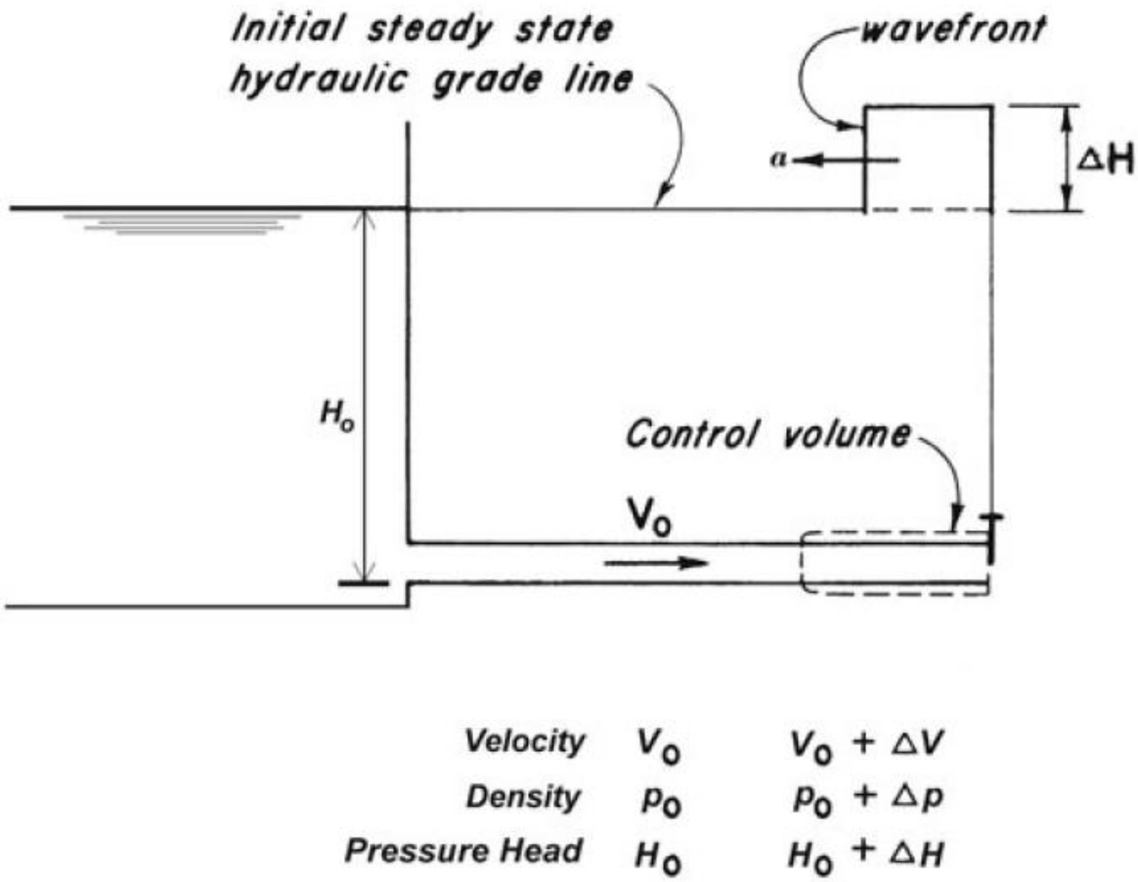


Fig. 2.7. Unsteady flow

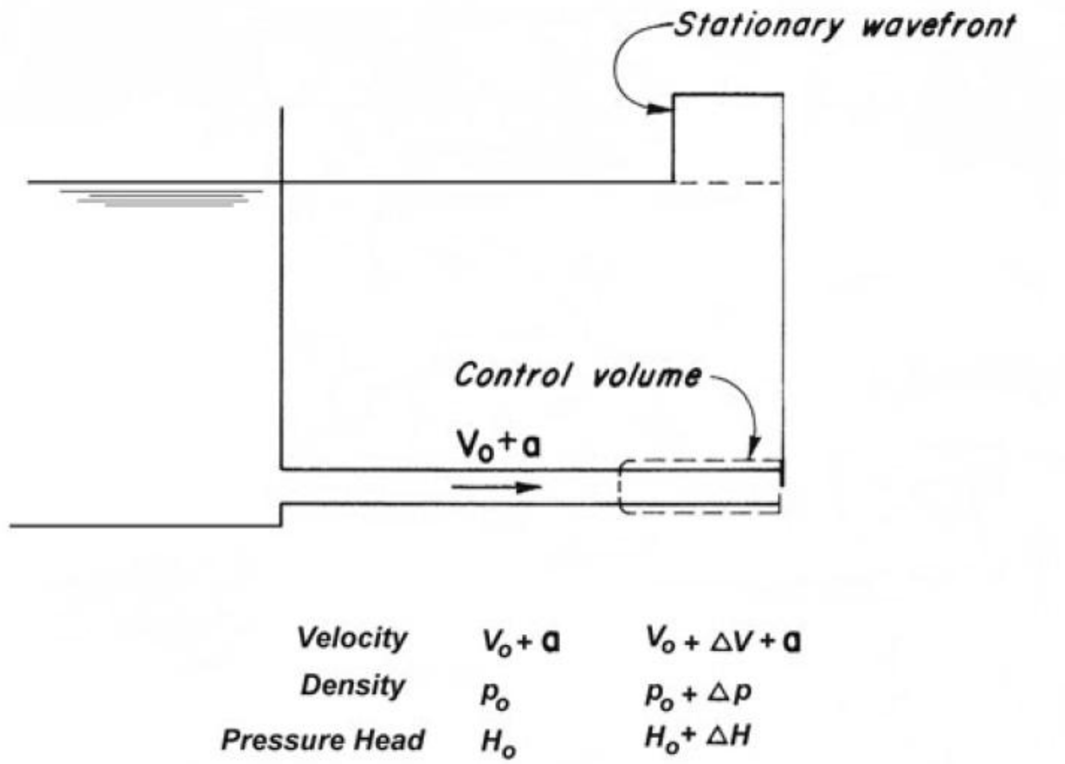


Fig. 2.8. Control volume moving upstream with a velocity

The time rate of change of momentum in the positive x-direction is

$$\rho_0(V_0 + a)A[(V_0 + \Delta V + a) - (V_0 + a)]$$

$$\rho_0(V_0 + a)A\Delta V$$

Negative friction resultant pressure force acting on the control volume

$$F = \rho_0 g H_0 A - \rho_0 g(H_0 + \Delta H)A$$

$$= -\rho_0 g \Delta H A$$

Equating forces

$$\rho_0(V_0 + a)A\Delta V = -\rho_0 g \Delta H A$$

$$\therefore \Delta H = -\frac{1}{g}(V_0 + a)\Delta V$$

The wave velocity a in metal or concrete pipes = 1000 m/s whereas typical flow velocity (V_0) = 1-10 m/s therefore V_0 is neglected.

$$\therefore \Delta H = -\frac{1}{g}a\Delta V$$

The negative sign indicates increase in pressure head as a result of the decrease in velocity on the upstream direction.

In the downstream direction,

$$\therefore \Delta H = \frac{1}{g} a \Delta V$$

This indicates pressure head on the downstream increases with increase in velocity.

Derivation of pressure wave velocity:

Applying continuity equation

$$\rho_0 (V_0 + a) A = (\rho_0 + \Delta\rho)(V_0 + \Delta V + a)$$

$$\Delta V = -\frac{\Delta\rho}{\rho_0}(V_0 + \Delta V + a)$$

But $V_0 + \Delta V \ll a$

$$\Delta V = -\frac{\Delta\rho}{\rho_0} a$$

Derivation of wave velocity

The bulk modulus of Elasticity

$$K = \frac{\Delta P}{\frac{\Delta\rho}{\rho_0}}$$

$$\frac{\Delta\rho}{\rho_0} = \frac{\Delta P}{K}$$

$$\Delta V = -\frac{\Delta P}{K} a \Rightarrow a = -\frac{K \Delta V}{\Delta P}$$

$$\Delta P = \rho_0 g \Delta H$$

$$a = -K \frac{\Delta V}{\rho_0 g \Delta H} \quad ,$$

however, $\Delta V = -\frac{g \Delta H}{a}$

$$a = +K \frac{g \Delta H}{a \rho_0 g \Delta H}$$

$$a = \sqrt{\frac{K}{\rho_0}}$$

This formula is applicable only for slightly compressible fluid in a rigid pipe.

Reynolds Transport Theorem (RTT)

$$\frac{dB_{sys}}{dt} = \frac{d}{dt}\int_{v} \beta\rho dv + \left(\beta\rho AV_s\right)_{out} - \left(\beta\rho AV_s\right)_{in}$$

$B =$ extensive property

β = intensive property

$v =$ Total volume

Note that the velocity V_s is with respect to the control surface since it accounts for the inflow or outflow from the control volume.

For fixed control volume, V_s = fluid flow velocity V. If control volume is moving with 'w' velocity then $V_s = V - w$

$$\frac{dB_{sys}}{dt} = \frac{d}{dt}\int_{v} \beta\rho dv + \left[\beta\rho A(V - w)\right]_{out} - \left[\beta\rho A(V - w)\right]_{in}$$

Derivation of Continuity Equation

Assumptions

1) Flow is slightly compressible
2) Conduit walls are elastic
3) Flow is one directional.

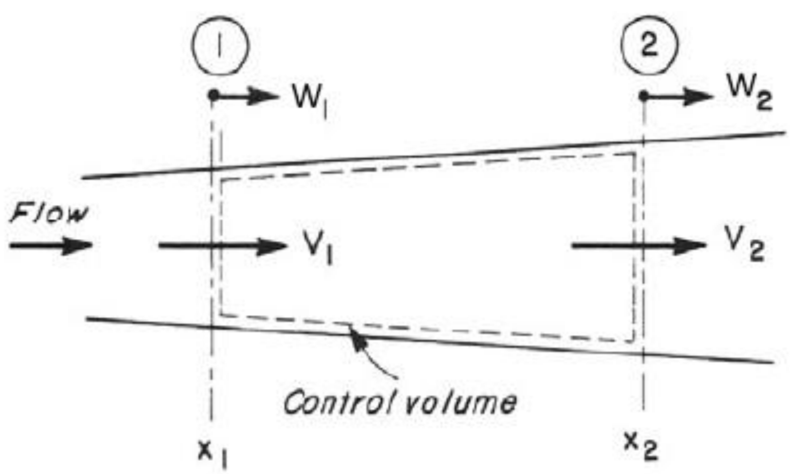


Fig. 2.9. Application of continuity equation

1-4 Basic Waterhammer Equations

In this section, we derive the basic waterhammer equations — expressions for the velocity of pressure waves in a conduit and for the change in pressure due to instantaneous change in the flow velocity.

Let us consider the flow in a frictionless pipe (Fig. 1-1) in which a slightly compressible fluid is flowing with velocity V_o, and the initial steady state pressure head upstream of the valve is H_o. Let the flow velocity V_o be changed instantaneously at time $t = 0$ to $V_o + \Delta V$. An increase in the flow velocity ΔV and an increase in pressure ΔH are considered positive and a decrease, as negative. As a result of this change in the flow velocity, the pressure head H_o changes to $H_o + \Delta H$, the fluid density ρ_o changes to $\rho_o + \Delta\rho$, and a pressure wave of magnitude ΔH travels in the upstream direction. Let us designate the velocity of the pressure wave (commonly called *wave velocity*) by a, and, to simplify the derivation, let us assume the pipe walls are rigid, i.e., the pipe area, A, does not change due to pressure changes. In the next chapter, an expression for the wave velocity is derived in which the fluid is slightly compressible and the pipe walls are slightly deformable.

The unsteady flow of Fig. 1-1a may be converted into steady flow by superimposing velocity a in the downstream direction. This is equivalent to an observer traveling in the upstream direction with velocity a to whom the moving wave front appears as stationary (Fig. 1-1b), and the inflow and outflow velocities from the control volume are $(V_o + a)$ and $(V_o + \Delta V + a)$, respectively.

Let us consider distance, x, and velocity, V, in the downstream direction as positive. Referring to Fig. 1-1b, the time rate of change of momentum in the positive x-direction

$$\begin{aligned} &= \rho_o (V_o + a) A [(V_o + \Delta V + a) - (V_o + a)] \\ &= \rho_o (V_o + a) A \Delta V \end{aligned} \tag{1-4}$$

Neglecting friction, the resultant force, F, acting on the fluid in the control volume in the positive x-direction

$$F = \rho_o g H_o A - \rho_o g (H_o + \Delta H) A = -\rho_o g \Delta H A \tag{1-5}$$

According to Newton's second law of motion, the time rate of change of momentum is equal to the resultant force. Hence, it follows from Eqs. 1-4 and 1-5 that

$$\Delta H = -\frac{1}{g} (V_o + a) \Delta V \tag{1-6}$$

The wave velocity a in metal or concrete pipes or in the rock tunnels, is approximately 1000 m/s while typical flow velocity is about 10 m/s or less.

Therefore, V_o is significantly smaller than a and may thus be neglected. Then, Eq. 1-6 becomes

$$\Delta H = -\frac{a}{g}\Delta V \tag{1-7}$$

The negative sign on the right-hand side of Eq. 1-7 indicates that the pressure head increases (i.e., ΔH is positive) for a reduction in velocity (i.e., for negative ΔV) and vice versa. Also note that Eq. 1-7 is derived for an instantaneous velocity change at the downstream end of a pipe and for the wave front moving in the upstream direction. Proceeding similarly, it can be proven that for a velocity change at the upstream end and for the wave front to move in the downstream direction,

$$\Delta H = \frac{a}{g}\Delta V \tag{1-8}$$

Note that, unlike Eq. 1-7, there is no negative sign on the right-hand side of Eq. 1-8. This means that the pressure head in this case increases for an increase in velocity and decreases with a decrease in velocity.

If the fluid density change $\Delta\rho$ is caused by the change in pressure, Δp, then referring to Fig. 1-1b,

$$\text{Rate of mass inflow} = \rho_o A (V_o + a) \tag{1-9}$$

$$\text{Rate of mass outflow} = (\rho_o + \Delta\rho) A (V_o + \Delta V + a) \tag{1-10}$$

If the fluid is slightly compressible, the increase in the mass of control volume due to the change in fluid density is small and may be neglected. Therefore, the rate of mass inflow is equal to the rate of mass outflow. Hence,

$$\rho_o A (V_o + a) = (\rho_o + \Delta\rho) A (V_o + \Delta V + a) \tag{1-11}$$

which upon simplification becomes

$$\Delta V = -\frac{\Delta\rho}{\rho_o}(V_o + \Delta V + a) \tag{1-12}$$

Since $(V_o + \Delta V) << a$, Eq. 1-12 may be written as

$$\Delta V = -\frac{\Delta\rho}{\rho_o} a \tag{1-13}$$

The bulk modulus of elasticity, K, of a fluid is defined as [Streeter, 1966]

$$K = \frac{\Delta p}{\Delta\rho/\rho_o} \tag{1-14}$$

Hence, it follows from Eqs. 1-13 and 1-14 that

$$a = -K\frac{\Delta V}{\Delta p} \tag{1-15}$$

By utilizing Eq. 1-7 and noting that $\Delta p = \rho_o g \Delta H$, we may write this equation as

$$a = \frac{K}{a\rho_o} \tag{1-16}$$

or

$$a = \sqrt{\frac{K}{\rho_o}} \tag{1-17}$$

Note that this expression for the wave velocity is for a slightly compressible fluid confined in a rigid pipe. In the next chapter, we will discuss how this expression is modified if the pipe walls are elastic.

Example

Compute the velocity of pressure waves in a 0.5-m-diameter pipe conveying oil. Determine the pressure rise if a steady flow of 0.4 m^3/s *is instantaneously stopped at the downstream end. Assume that the pipe walls are rigid, the density of the oil,* $\rho = 900\ kg/m^3$*, and the bulk modulus of elasticity of the oil,* $K = 1.5\ GPa$*.*

Solution

$$\begin{aligned}
A &= \frac{\pi}{4}(0.5)^2 = 0.196\ \text{m}^2 \\
V_o &= \frac{Q_o}{A} = \frac{0.4}{0.196} = 2.04\ \text{m/s} \\
a &= \sqrt{\frac{K}{\rho}} \\
&= \sqrt{\frac{1.5\times 10^9}{900}} = 1291\ \text{m/s}
\end{aligned}$$

Since the flow is completely stopped, $\Delta V = 0 - 2.04 = -2.04$ m/s. Therefore,

$$\begin{aligned}
\Delta H &= -\frac{a}{g}\Delta V \\
&= -\frac{1291}{9.81}(-2.04) = 268.5\ \text{m}
\end{aligned}$$

A positive sign for ΔH means the pressure rises as a result of this reduction in the flow velocity.

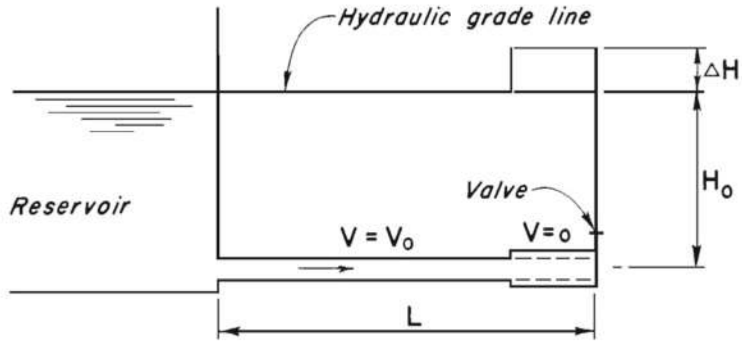
1-5 Wave Propagation

In this section, we discuss transient flow in a piping system with a constant-level reservoir at the upstream end and a valve at the downstream end (Fig. 1-2) to illustrate the propagation of a wave in a pipe and the reflections of the wave from a reservoir and from a closed valve. The pipe walls are considered elastic. Therefore, the pipe expands as the inside pressure increases and contracts as the pressure decreases.

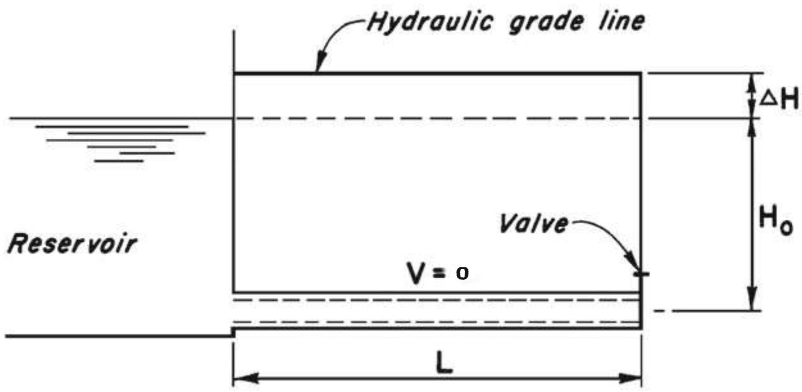
Let the flow conditions in the piping system be steady prior to instantaneous closure of the downstream valve at time $t = 0$. If the system is assumed frictionless, then the initial steady-state pressure head along the length of the pipeline is H_o. Let us consider the distance x and the velocity V as positive in the downstream direction. The upstream and downstream directions are with respect to the initial steady flow.

The sequence of events following valve closure may be divided into four parts (Fig. 1-2) as follows:

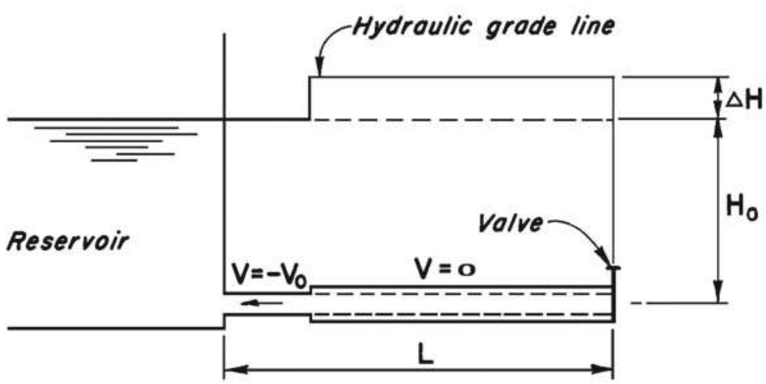
1. $0 < t \leq L/a$
   The flow velocity at the valve is reduced to zero as soon as the valve is completely closed. This increases the pressure at the valve by $\Delta H = (a/g)\,V_o$. Because of this increase in pressure, the pipe expands (the initial steady-state pipe diameter in the expanded or contracted parts of the pipe is shown by dotted lines in Fig. 1-2), the fluid is compressed which increases the fluid density, and a positive pressure wave propagates towards the reservoir. Behind this wave front, the flow velocity is zero, and the kinetic energy has been converted into elastic energy (Fig. 1-2a). If a is the velocity of the pressure wave and L is the length of the pipeline, then the wave front reaches the upstream reservoir at time $t = L/a$. At this time, along the entire length of the pipeline, the pipe is expanded, the flow velocity is zero, and the pressure head is $H_o + \Delta H$ (Fig. 1-2b).
2. $L/a < t \leq 2L/a$
   Just as the wave reaches the upstream reservoir, pressure at a section on the reservoir side is H_o while the pressure at an adjacent section in the pipe is $H_o + \Delta H$. Because of this difference in pressure, the fluid flows from the pipeline into the reservoir with velocity $-V_o$. Thus, the flow velocity at the pipe entrance is reduced from zero to $-V_o$. This causes the pressure to drop from $H_o + \Delta H$ to H_o and a negative wave travels towards the valve (Fig. 1-2c). The pressure behind this wave front (i.e., on the reservoir side) is H_o and the fluid velocity is $-V_o$. At $t = 2L/a$, the wave front reaches the closed valve, and the pressure head in the entire pipeline is H_o, and the fluid velocity is $-V_o$ (Fig. 1-2d).
3. $2L/a < t \leq 3L/a$
   Since the valve is completely closed, a negative velocity cannot be maintained at the valve. Therefore, the velocity changes instantaneously from $-V_o$ to 0, the pressure drops to $H_o - \Delta H$, and a negative wave propagates



(a) Time, $t = \epsilon$

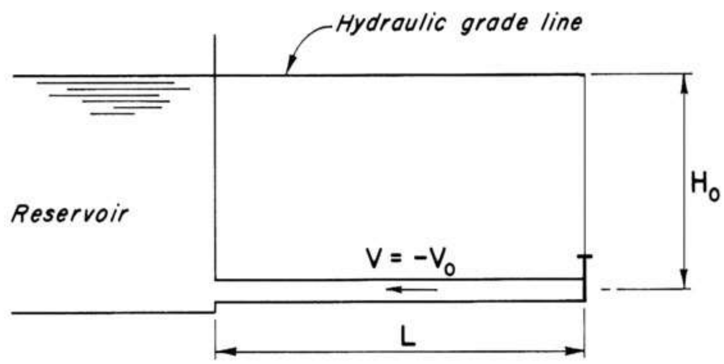


(b) Time, $t = \frac{L}{a}$

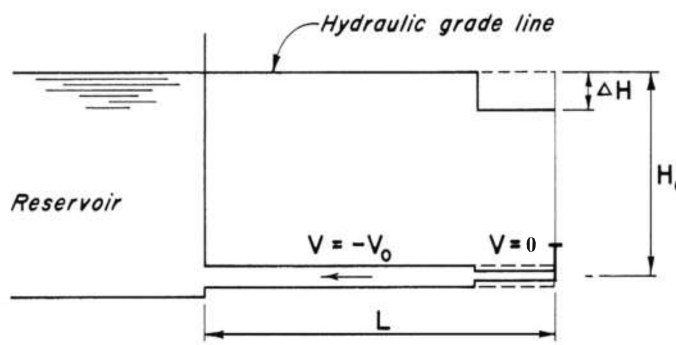


(c) Time, $t = \frac{L}{a} + \epsilon$

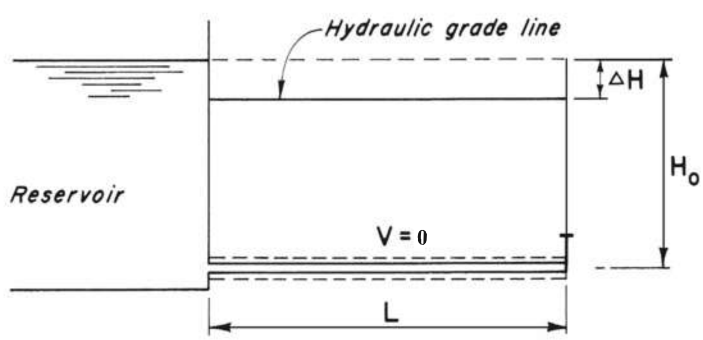
Fig. 1-2. Propagation and Reflection of pressure waves.



(d) Time, $t = \frac{2L}{a}$

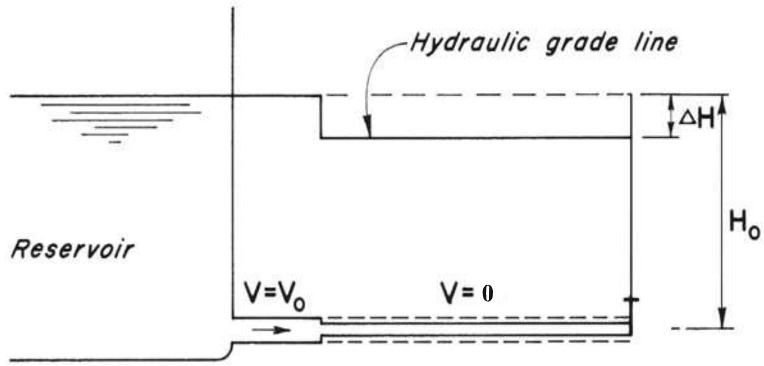


(e) Time, $t = \frac{2L}{a} + \epsilon$

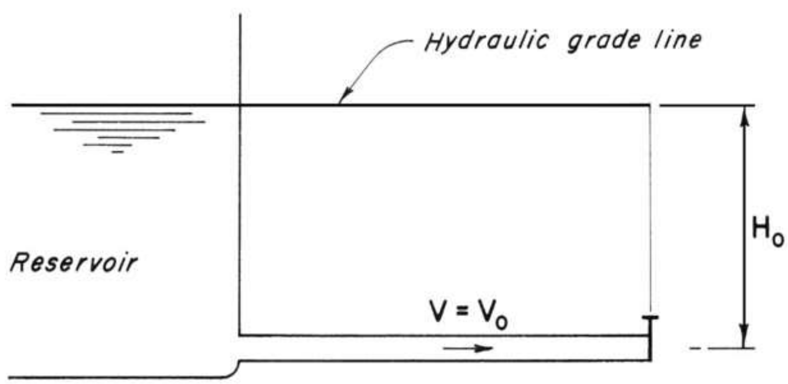


(f) Time, $t = \frac{3L}{a}$

Fig. 1-2. *(Continued)*



(g) Time, $t = \frac{3L}{a} + \epsilon$



(h) Time, $t = \frac{4L}{a}$

Fig. 1-2. *(Continued)*

towards the reservoir (Fig. 1-2e). Behind this wave front, the pressure is $H_o - \Delta H$ and the fluid velocity is zero. The wave front reaches the upstream reservoir at time $t = 3L/a$, the pressure head in the entire pipeline is $H_o - \Delta H$, and the fluid velocity is zero (Fig. 1-2f).

4. $3L/a < t \leq 4L/a$
   As soon as this negative wave reaches the reservoir, an unbalanced condi-

tion is created again at the upstream end. Now the pressure is higher on the reservoir side than at an adjacent section in the pipeline. Therefore, the

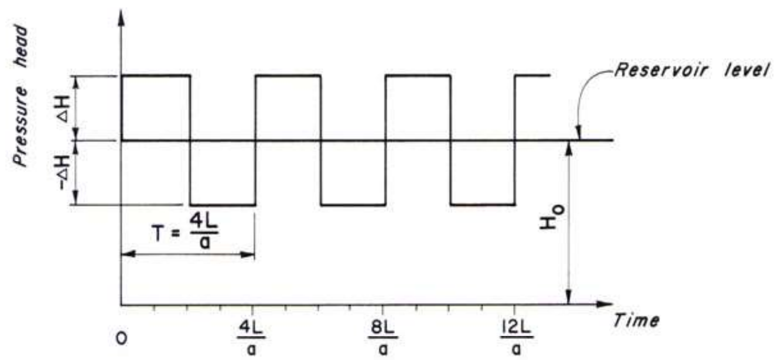


Fig. 1-3. Pressure variation at valve; friction losses neglected.

fluid now flows from the reservoir into the pipeline with velocity V_o, and the pressure head increases to H_o (Fig. 1-2g). At time $t = 4L/a$ the wave front reaches the downstream valve, the pressure head in the entire pipeline is H_o, and the flow velocity is V_o. Thus, the conditions in the pipeline at this time are the same as those during the initial steady-state conditions except that the valve is now closed (Fig. 1-2h).

Since the valve is completely closed, the preceding sequence of events starts again at $t = 4L/a$. Figure 1-2 illustrates the sequence of events along the pipeline, while Fig. 1-3 shows the variation of pressure at the valve with time. Since we assumed the system to be frictionless, this process continues with the conditions repeating at an interval of $4L/a$. The interval at which conditions are repeated is termed the *theoretical period* of the pipeline. In a real system, however, pressure waves are dissipated due to energy losses as the waves propagate back and forth in the pipeline, and the fluid becomes stationary after some time. The variation of pressure at the valve with time is shown in Fig. 1-4 if the friction losses are taken into consideration.

1-6 Wave Reflection and Transmission

In the previous section we discussed the propagation of pressure waves in a pipe and the reflection of the wave from a reservoir and from a closed valve. In this section, we introduce the concept of reflection and transmission coefficients which defines the magnitude and the sign of reflected and transmitted

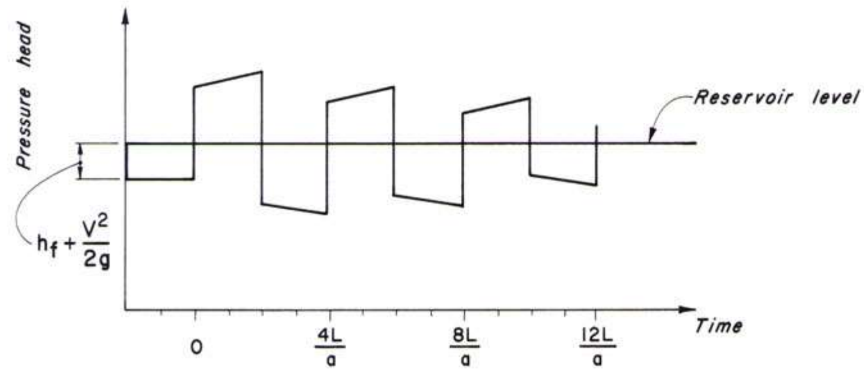


Fig. 1-4. Pressure variation at valve; friction losses considered.

waves from or at a boundary. For simplification, we assume no energy loss as a wave is reflected from or is transmitted at a boundary.

Let us designate an incident pressure wave approaching a boundary as F and the wave reflected from the boundary as f. Then, the reflection coefficient, r, is defined as $r = f/F$. The pressure in a positive pressure wave is higher behind the wave front than ahead of the wave front while the pressure in a negative wave is lower behind the wave front than ahead of the wave front.

Constant-Level Reservoir

A large lake, reservoir, tank or other storage facility may be considered to have a constant level if its water surface level remains unchanged irrespective of the changes in the flow in the pipeline connected to the facility. The reflected pressure wave, f, from a constant-level reservoir is equal in magnitude to that of the incident wave, F, but is of the opposite sign. Therefore, the reflection coefficient for a constant-level reservoir, $r = -1$ (Fig. 1-5). For example, a 10-m positive pressure wave is reflected from a reservoir as a 10-m negative pressure wave.

The reflection of a velocity wave at a reservoir has the same magnitude and the same sign as that of the incident wave.

Dead End

At a dead end or at a fully closed valve, a reflected pressure wave has the same magnitude and the same sign as that of the incident wave (Fig. 1-6), i.e., the refelction coefficient, $r = 1$. Thus, if a 10-m pressure wave is approaching a dead end with the initial pressure head of 100 m, the pressure increases to

110 m as the wave arrives at the dead end and then increases to 110+10 = 120 m after the pressure wave is reflected from the dead end.

Series Junction

A junction of two pipes having different diameters, wall thicknesses, wave velocities, and/or friction factors is called a series junction. A pressure wave F traveling in one of the series pipes, say pipe 1, is reflected back into pipe 1 and another wave, f_s is transmitted into the second pipe, say pipe 2 (Fig. 1-7). The refelection coefficient, r and transmission coefficient, s, for this junction are:

$$r = \frac{f}{F} = \frac{\dfrac{A_1}{a_1} - \dfrac{A_2}{a_2}}{\dfrac{A_1}{a_1} + \dfrac{A_2}{a_2}}$$

$$s = \frac{f_s}{F} = \frac{\dfrac{2A_1}{a_1}}{\dfrac{A_1}{a_1} + \dfrac{A_2}{a_2}} \tag{1-18}$$

in which A = pipe cross-sectional area; a = wave velocity; and subscripts 1 and 2 refer to the quantities for pipe 1 and 2, respectively. Note that f_s is a positive wave since the diameter of pipe 1 is larger than that of pipe 2. If the diameter of pipe 2 is larger than that of pipe 1, then f_s will be a negative wave.

Branching Junction

At the junction, pipe 1 branches into pipe 2 and pipe 3. The incident wave, F, in pipe 1 is reflected back as f from the junction into pipe 1 and a transmitted wave f_s is transmitted into pipe 2 and pipe 3. The reflection and transmission coefficiets for a branching junction are:

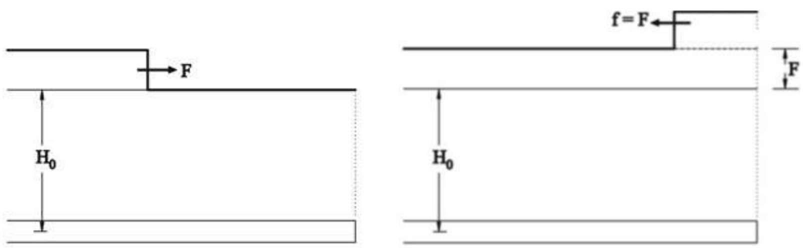
$$r = \frac{f}{F} = \frac{\dfrac{A_1}{a_1} - \dfrac{A_2}{a_2} - \dfrac{A_3}{a_3}}{\dfrac{A_1}{a_1} + \dfrac{A_2}{a_2} + \dfrac{A_3}{a_3}}$$

$$s = \frac{f_s}{F} = \frac{\dfrac{2A_1}{a_1}}{\dfrac{A_1}{a_1} + \dfrac{A_2}{a_2} + \dfrac{A_3}{a_3}} \tag{1-19}$$

Similar equations for r and s may be written for a branching junction of two incoming pipes and one outgoing pipe.

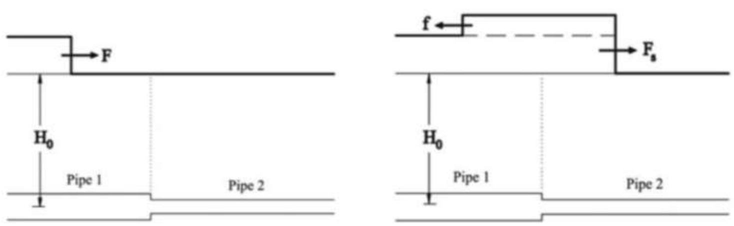
(a) Incident pressure wave **(b) Reflected wave**

Fig. 1-5. Incident and reflected waves at a reservoir.



(a) Incident wave **(b) Reflected wave**

Fig. 1-6. Incident and reflected waves at a dead end.



(a) Incident pressure wave **(b) Reflected and transmitted waves**

Fig. 1-7. Incident, reflected, and transmitted waves at a series junction.

1-7 Transient Flow Analysis

For analysis, transient flow may be classified as closed-conduit flow, open-channel or free-surface flow and combined free-surface-pressurized flow. We consider each of these flows in the following paragraphs.

The transient flow in closed-conduits may be analyzed by using the distributed-system or lumped-system approach. In the distributed-system approach, transient phenomenon is in the form of traveling waves. Examples are transients in water-supply pipes, in power plant conduits, and in gas-transmission lines. In the lumped-system approach, any change in the flow conditions is assumed to travel instantaneously throughout the system, i.e., the fluid is considered as a solid body. An example of such a system is the slow oscillations of water level in a surge tank following a load change on the turbines in a hydroelectric power plant.

Mathematically speaking, the transients in a distributed system are represented by partial differential equations, whereas the transients in a lumped system are described by ordinary differential equations. As discussed in Chapter 8, the system may be analyzed as a lumped system if $\omega L/a$ is significantly less than 1, [Chaudhry, 1970]; otherwise, the system should be analyzed as a distributed system. In the preceding expression, ω = frequency of the flow oscillations, L = length of the pipeline, and a = wave velocity.

Transients in an open channel may be classified based on the time rate at which they occur: gradual, such as flood waves in rivers, and rapid, such as surges in power canals. If the wave front in the rapidly varied flow is steep, it is referred to as a *bore*.

Free-surface flow may become pressurized due to priming of the conduit during the transient-state conditions. Such flows are called combined free-surface-pressurized flows. Examples of such flows are the flow in a sewer following a large rainstorm, and the flow in the tailrace tunnel of a hydroelectric power plant following rapid acceptance of load on turbines.

1-8 Causes of Transients

As defined previously, the intermediate-stage flow when the flow conditions change from one steady state to another, is termed *transient-state flow*. In other words, the transient conditions are initiated whenever the steady state conditions are disturbed. Such a disturbance may be caused by planned or accidental changes in the settings of the control equipment of a man-made system or by changes in the inflow or outflow of a natural system.

Common causes of transients in engineering systems are:

Opening, closing, or “chattering” of valves in a pipeline;
Starting or stopping the pumps in a pumping system;
Starting-up a hydraulic turbine, accepting or rejecting load;

Vibrations of the vanes of a runner or an impeller;
Sudden opening or closing the control gates of a canal;
Failure of a dam, and
Sudden increases in the river or sewer inflow due to a flash storm.

1-9 System Design and Operation

No generalized procedures are presently available to design a system directly that gives an acceptable transient response. Therefore, following trial-and-error approach is employed.

The system layout and parameters are first selected, and the system is analyzed for transients caused by various possible operating conditions. If the system response is unacceptable, e.g., the maximum and minimum pressures are not within the prescribed limits, then the system layout and/or parameters are changed, or various control devices are provided and the system is analyzed again. This procedure is repeated until a desired response is obtained. For a particular system, several different control devices may be suitable for transient control. If possible it may be economical in some cases to modify the operating conditions, or the acceptable response. However, the final objective is to have an *overall* economical system that yields an acceptable response.

The system is designed for the normal operating conditions expected to occur during its life. And, similarly, it is essential that the system be operated strictly according to the operating guidelines. Failure to do so may cause spectacular accidents and result in extensive property damage and many times in loss of life [Rocard, 1937; Jaeger, 1948; Bonin, 1960; Jaeger, 1963; Kerensky, 1965-1966; Pulling, 1976; Trenkle, 1979; Serkiz, 1983].

If the data for a system are not precisely known, e.g., wave speed, friction factors, reservoir levels, etc., then the system should be analyzed for the expected range of various variables. This is usually referred to as sensitivity analysis. For example, such an analysis may be done by varying the variable by $\pm 10\%$.

During the commissioning of a newly built system or after major modifications have been completed on an existing project, the system should be tested for possible operating conditions. To avoid accidents and failures, it is usually advisable to conduct the tests in a progressive manner. For example, if there are four parallel pumping-sets on a pipeline, the tests for power failure should begin with one pumping-set and progressively increase to all four.

1-10 Accidents and Incidents

In this section, a number of photographs of failures caused by transients are presented.

1. Introduction

Pipeline design and pipeline operation should address both safety and minimum cost of operation. Pipeline design mainly concerned with pipeline sizing, equipment sizing and equipment location, whereas system operation is concerned with pipeline system or facility startup and shutdown, flowrate change, emergency shutdown and equipment failure.

A pipeline state is defined as a condition of a pipeline expressed in terms of pressure and flowrate at given location and time. steady state is a condition of a pipeline system that flow variable doesn't change with time, whereas transient state is an unsteady condition that changes with time between two steady states. A surge or water hammer is a transient that occurs abruptly during flow changes from normal steady state flow. Positive surge or upsurge occurs if the pipeline pressure increases above the steady state pressure in the pipeline. A down surge occurs if the pipeline pressure decreases below the steady state pressure in the pipeline.

In steady state, pressure and flowrate are independent of time. Generally a pipeline system can be based on a steady state operation. This assumption is applicable only when the system is not subjected to sudden changes in the flowrates. Steady state conditions are not applicable to test capability of the system to test the level of safety. The steady state is not valid during transient state because flowrates and pressures are changing therefore the following problems cannot be addressed.

(1) Positive and negative surges
(2) Pump tripping
(3) Column separation

Why transient analysis is required for water pipelines?

1. Many pipeline failures occur because of improper provisions are made to manage transient related problems such as pump trip, check valve slam.
2. The following operating conditions should be taken into account in design and operational analysis
   (a) Changes in valve settings
   (b) Power failures
   (c) Pump startup and stopping
   (d) Air entrainment

Transient analysis is beneficial because

1. Steady state design for water pipeline systems tends to give larger pipe diameter, high capacity pump requirement, and incorrect booster pump location.
2. Steady state design is based on peak load whereas transient state design may be based on average load.

Transient (Dynamic) model

Transient model calculates time dependent flow and pressure behavior by solving the time dependent flow equations. Transient model generates hydraulically more realistic results than a steady state model, it predicts system behavior such as effect of changes in supply flow rates, supply pressure and delivery demands.

Transient (Dynamic) model capabilities

Analyze startup or shutdown procedure, i.e., different combinations of startup or shutdown procedures. A transient model can be used to study demand changes. Transient model is used to evaluate corrective strategies by modeling pump failures, pipe rupture and stoppage of flow. Transient model is required to study when a leak or rupture occurs. A transient model is more complex to use and execution time is longer than that of a steady state model. It requires extensive data particularly equipment and operational data.

Governing laws and equations

Dependent variables Q , P, independent variables x, t

Equations: continuity equation and momentum equation

Continuity equation:

Mass cannot be created or destroyed. For incompressible flows in a pipeline, inflow = outflow.

Momentum equation:

The rate of change of momentum equals to the sum of external forces. Newton's second law of motion is applied to the fluid element in pipelines to derive the momentum equation.

A general form of the one dimensional continuity and momentum equations are given below.

$$\frac{\partial P}{\partial t} + \rho a^2 \frac{\partial V}{\partial x} = 0 \tag{1.1}$$

$$\frac{\partial V}{\partial t} + V \frac{\partial V}{\partial x} + \frac{1}{\rho} \frac{\partial P}{\partial x} + g \frac{\partial h}{\partial x} + \frac{fV|V|}{2D} = 0 \tag{1.2}$$

Where f = friction factor, h = elevation, g = gravitational acceleration, x = displacement, t = elapsed time, V = cross sectional average velocity, P = pressure, a = wave velocity, ρ = mass density.

In the momentum equation, the first time represents a force due to acceleration, second term is a force due to convective acceleration 'or' force due to kinetic energy. These two terms together called as inertia force. The third term is a force due to pressure difference between two points in a pipe segment. The fourth term is gravitation force and the last term is frictional force, or Darcy Weisbach equation which is nothing but viscous force opposing the flow in the pipeline.

Reynolds number

Reynolds number relates density, viscosity, fluid velocity and pipe diameter. Reynolds number is dimensionless.

$$R_e = \frac{\rho V D}{\mu} = \frac{V D}{\nu}$$

Where D = inside diameter; V = fluid velocity; μ = absolute or dynamic viscosity; ν = kinematic viscosity; ρ = fluid mass density.

If R_e < 2000, flow is laminar, flow is in transition (critical) for 2000 <R_e< 4000, and flow is fully turbulent for R_e> 4100.

Friction factor (f)

Friction factor f is a function of R_e for laminar flows. Friction factor is a function of (R_e, and pipe relative roughness) for smooth turbulent flow. For highly turbulent flows, friction factor is function of only relative roughness. The moody diagram (Fig. 1.1) relates the friction factor in terms of Reynolds number and relative roughness.

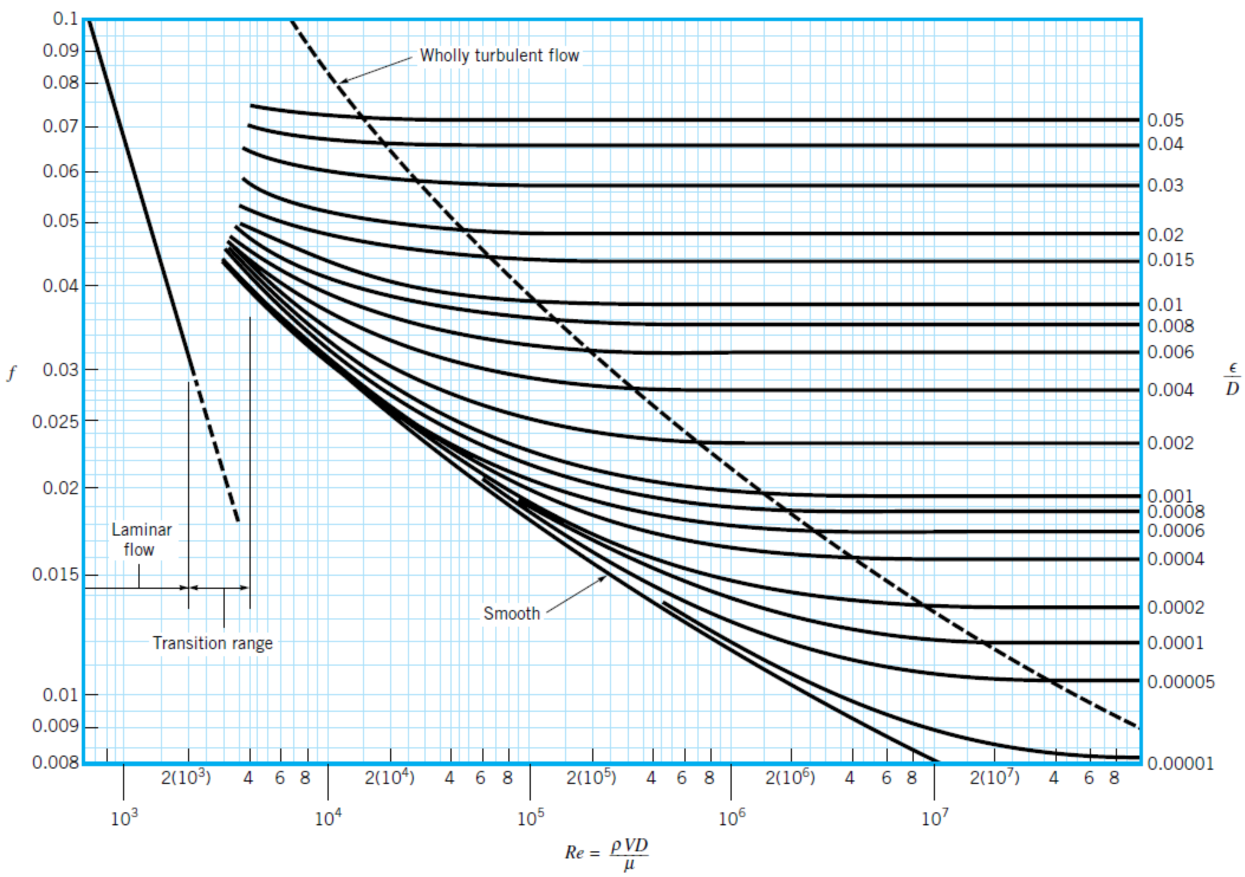


Fig 1.1: The Moody Diagram

In laminar flows friction factor decreases with increasing Reynolds number. Similarly, in smooth turbulent flows, for a given relative roughness, friction factor decreases with increasing Reynolds number. However, for highly turbulent flows (R_e> 1000000), friction factor is function of only relative roughness. Fluid rheological property viscosity is a measure of fluid resistance to shear force, expressed in absolute or kinematic viscosity.

Solution methods

The continuity and momentum equations are non-linear so they can be solved only in a numerical approach using a computer. The set of these equations are discretized dividing pipelines into finite intervals. The discretized equations are numerically solved for pressure and flowrate over a time step. Common solutions approaches are explicit solutions, implicit solutions and method of characteristics.

Assumptions

Flow in a pipeline can be represented in 1-D equations and angular momentum is negligibly small.

Explicit methods

The values at the current time are explicitly calculated from the values at the previous time step with the boundary conditions at current time. An explicit FD method converges only when distance and time steps are small. These methods are subjected to CFL (Courant Frederik Law) stability criteria.

Implicit Solutions

The values at the current time are implicitly calculated from the values at the current time and previous time and boundary conditions at the current time. The implicit FD method equations are expressed in large matrices which are solved simultaneously for pressure and flowrate at every discretized point. The large matrices are solved by a sparse matrix technique. Implicit solution techniques are flexible with time steps and inherently stable always.

Method of Characteristics

The characteristic method converts the continuity and momentum equations into four total differential equations. The four characteristic equations are solved explicitly for pressure and flowrate. The solution methodology is simple for a single fluid pipeline. Time step is constrained by Δx and acoustic speed of the fluid a.

$$\frac{\Delta x}{\Delta \mathrm{t}} \geq a$$

Boundary and initial conditions

1. Pressure and Pressure boundary condition
2. Pressure and flow boundary condition

The initial conditions can be

(1) A steady state
(2) Transient state available from a previous simulation run

Causes of transients

Transients are basically two types (1) pressure transients (2) flow transients. Pressure transients occur when a change in pressure occurs in the pipeline which adds or remove energy from the pipeline. Flow transients occur when there is a change in flowrate by change in energy.

The main causes of transients in a pipeline are

(1) Change in the valve settings (open or close)
(2) Startup or shutting down the pump
(3) Changes in pump speed or head
(4) Pipeline rupture, column separation or trapped air
(5) Vibration of impeller in a radial pump

Transient properties

1. A transient (surge or water hammer) is a pressure wave.
2. Pressure wave propagates at the sound velocity of the fluid and velocity of fluid in a pipe.
3. Initial magnitude of pressure wave is proportional to acoustic velocity and fluid velocity.
4. The magnitude attenuates as the pressure wave moves away from the source of the transient.
5. Small amount of air entrained in the liquid can alter acoustic velocity.

Water hammer

1. Water hammer occurs because the fluid mass before the stoppage is still moving forward with its fluid velocity, building up a very high pressure.
2. On the other hand when an upstream flow in a pipe is suddenly stopped, the fluid downstream will attempt to continue flowing which creates a vacuum that may cause the pipe to collapse. This problem particularly occurs if the pipe is on a downhill slope.
3. Other causes of water hammer are (i) pump failure and (2) check valve slam.

Acoustic speed

The acoustic speed in a buried pipe can be calculated by following formula. Acoustic speed is a function of void fraction in the water as shown in Fig. 1.2.

$$a = \frac{\sqrt{K/\rho}}{\sqrt{1 + (K/E)(D/e)(1 - \upsilon^2)}}$$

Where K = bulk modulus of fluid; E = Young's modulus; D = inside diameter of pipe; e=thickness and υ= poison ratio = 0.3.

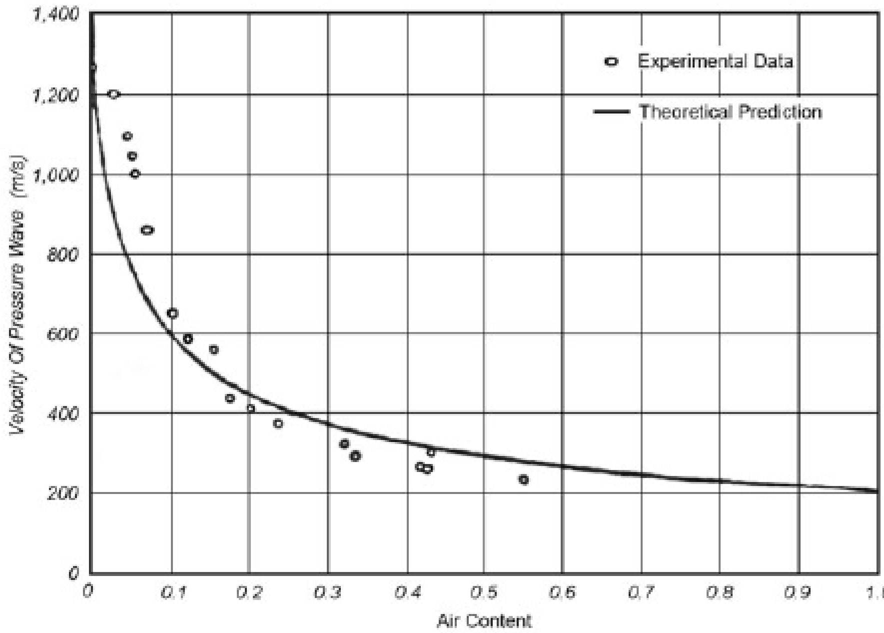


Fig 1.2: Acoustic speed as a function of void fraction.

Potential surge

The initial pressure increase following flow stoppage is referred to as the potential surge.

$\Delta P = -\rho a V$; $\Delta H = -\frac{aV}{g}$; V=change in velocity.

Where ΔP and ΔH are change in pressure; ρ=mass density of water; a = acoustic wave velocity.

Pressure wave propagates away from the transient location. The wave reflects back at a boundary point and the reflected wave has a negative head.

Critical period (t_c)

The critical period is defined as the time that an acoustic wave travels from the transient location to the end point and travels back to the transient location.

$$t_c = \frac{2L}{a}$$

L= pipe length.

Attenuation of surge

The magnitude of the potential surge decreases as it travels, this reduction is called attenuation. At high pressures, increase of fluid stored occurs, which is called as line packing. The pressure increases, the pipe wall expands and the fluid is compressed.

Colum separation

1. On the downstream side of the closed valve, the pressure drops as given by the Joukowsky equation. If the pressure drops below the vapor pressure, the liquid vaporizes and column separation occurs.
2. Column separation is a phenomenon that results from water hammer. It happens when part of the pipe is subjected to low pressure.
3. Column separation accompanies the down surge. It is more likely to occur at high points or knees (sharp changes in slope) in the pipeline.
4. Column separation can buckle the pipelines and should be prevented from happening through proper design and operation.

Column separation and rejoining of water columns is shown in Fig. 1.3

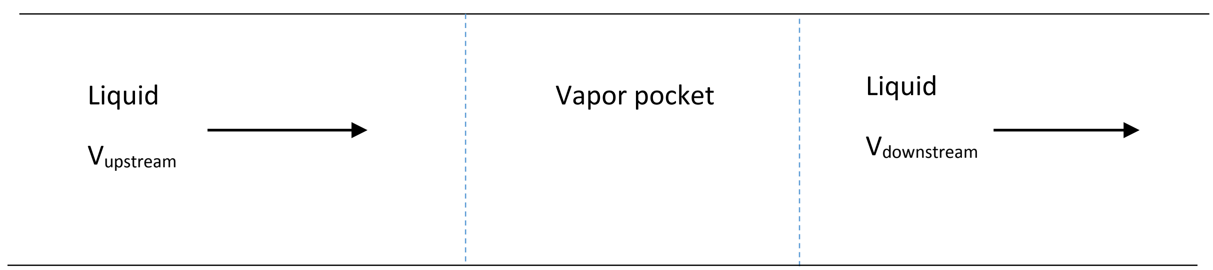


Fig 1.3: Schematic of column separation

The head increase which results from the collapse (ΔH):

$$\Delta H = \frac{a\left(V_{upstream} - V_{downstream}\right)}{g}$$

a = acoustic velocity.

Collapse of vapor column

The upstream column will be accelerated and the downstream column is decelerated if the back pressure increases, and the upstream column overtakes the downstream column. If the process occurs quickly, the vapor column collapses. When the vapor column collapses, it can be destructive, this head increase may be sufficient magnitude to rupture the pipe.

Consequences of transients

1. Unstable pressures
2. Column separation
3. Check valve slam or control valve oscillation

4. Resonance in pipe system
5. Pipeline shutdown due to pipe rupture or collapse.

Objectives of surge control

1. The main objective of the surge control is to limit the magnitude of the surge to within the allowable limits of the pipe system, including joints and equipment on the pipeline system.
2. There are two ways of managing pressure transients
   (a) Control the surge
   (b) Extra protection of the pipeline

Surge control is important for the following operations

1. Pump startup and shutdown
2. Valve operations sudden opening and sudden closure
3. Changes in supply and demands of flow

Protection of the pipeline and equipment

The main objective of pipeline and equipment protection is to preserve the integrity of the pipeline and to prevent system failure. When events occur beyond the control of operations such as power failure, valve failure and operation error.

Transient control

1. Control strategies include timing of control of pumps and valve operations.
2. Surges expected to be severe, the magnitudes of surges should be determined.
3. Pipeline should be reinforced with proper joints and anchoring system.
4. Computer simulations are necessary to select reliable and responsible control system.
5. The continuous pounding of small surges can cause leaks and eventually lead to pipe failure.
6. Knee points are avoided, thicker pipes need to be installed and surge control devices are installed.

Operation phase

1. Open and close valve gradually.
2. Fixed speed drives for automatic control valves which open and close control valves gradually
3. Variable speed drives for pumps which ramp speed up and ramp down the speed in a controlled way.

Mechanisms to control surges in the pipeline systems

(1) Valve movement (2) check valve (3) pump startup/shutdown (4) power failure

Surge control devices

1. Pressure relief valves

2. Pressurized surge tanks
3. Rupture disks
4. Control valves

Valve movement

Surge magnitude depends on the (1) type of valve (2) valve movement (3) elastic restraint properties of the pipeline system (4) valve coefficient.

It is safe to close the valve in time longer than $2L/a$

Check valve

1. Check valves can cause large transient pressure if the flow reversal through them can occur before the valve closure is complete.
2. Modern check valves closes gradually
3. Some check valves closes at the instant forward flow stops.
4. Check valve must close quickly before reverse flow is considerable or close gradually, i.e., more than critical time ($2L/a$)

Pump Startup

As the pump starts up and comes "online" a positive surge is created in the downstream line. The magnitude of the surge depends on the sudden increase in speed which occurs when the check valve is forced open and the liquid in the line begins to move. When there is no entrapped air, the pressure increase is generally not large. If there is a vapor in the delivery line, substantial transient pressure can be developed. In general upsurges are followed by down surges and vice versa. Upsurges and down surges are shown in Fig. 1.4.

Pump power failure

1. Severe transients occur upon power failure where the static lift is large, where pipeline profile rises rapidly immediately downstream of the pump.
2. If power is cutoff, the pressure just downstream of the pump drops rapidly and this pressure drop propagates downstream at the wave speed.
3. The drop in pressure can cause extensive column separation and lead to subsequent cavity collapse and shock of large magnitude.
4. Moreover a flow reversal in the system occurs and lead to significant overpressures in the vicinity of the pumps.

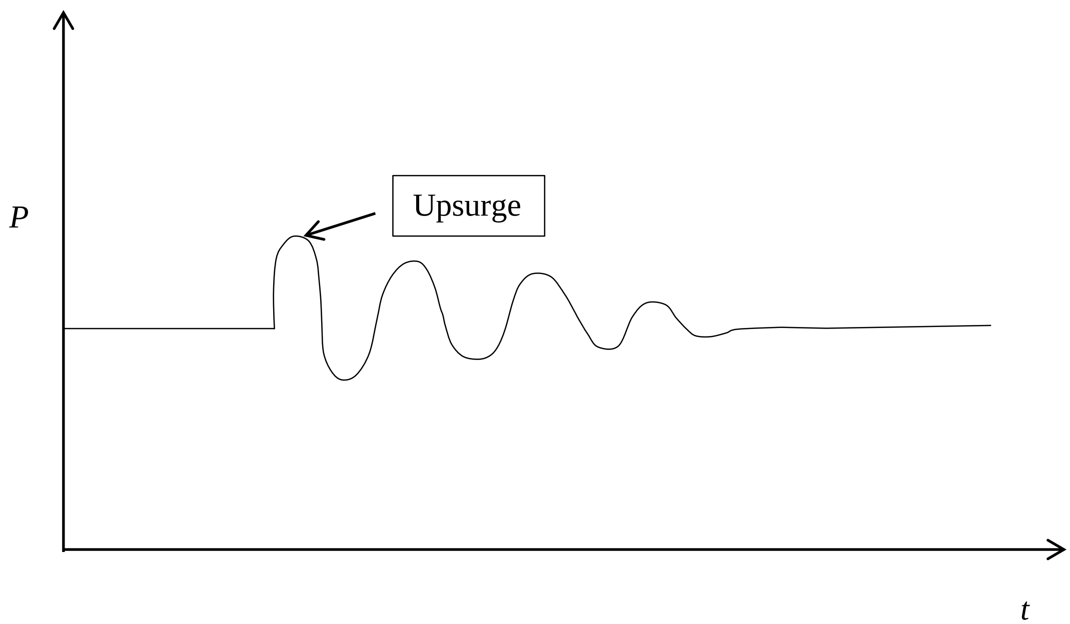
Pressure relief valve (PRV)

1. PRVs are mechanical devices that are used to protect the pipeline system from excessive pressures. They are installed along the pipeline where maintenance is easy.

2. PRV opens when the pressure exceeds a specified preset pressure. The valve closes when the pressure in the line decreases below the set value.
3. PRV can release some of the flow during pump startup to avoid over pressurizing the system.
4. PRV installed in the pump discharge line between the pump and the discharge control valve acts like a bypass valve during pump startup to prevent a pump from operating near shutoff head.
5. PRVs can be used only to control upsurges and not suitable to control down surges.
6. PRVs are very useful in short and steep pipe profile where reversal of flow quickly follows power failure or pump trip.

Limitations of PRV

1. Upsurge and downsurge (Fig 1.4) travel at acoustic speed, a PRV may not open that quickly therefore ineffective.
2. PRV is designed for a certain flowrate, undersized PRV may not be able to discharge at a high enough flowrate.



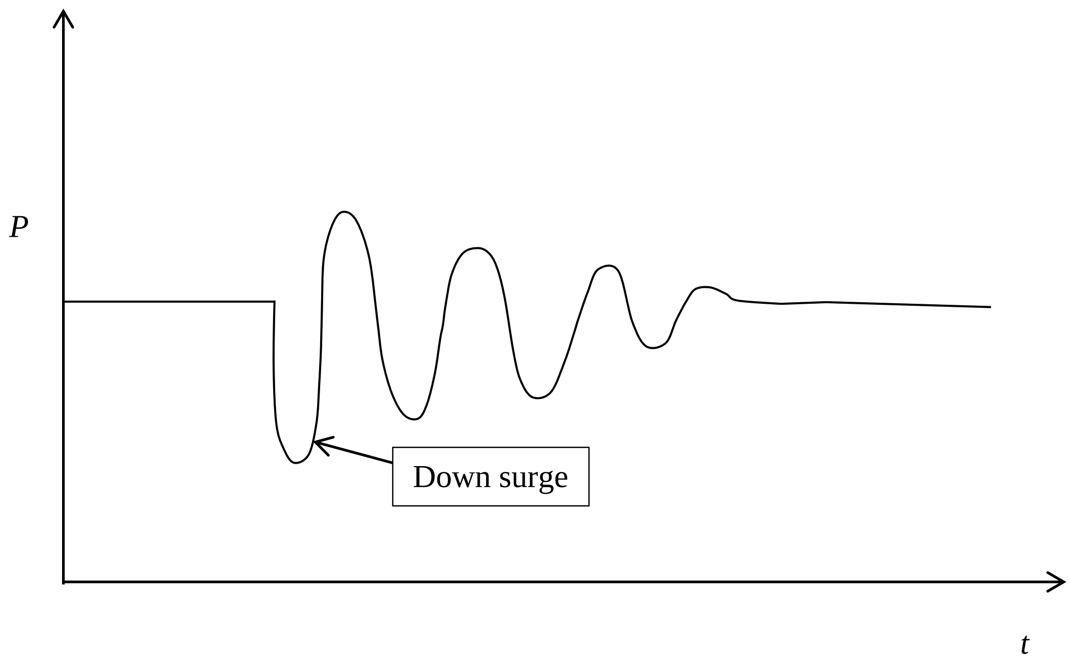


Fig 1.4: Upsurge and downsurge

Pressurized surge tank (Air vessel or accumulator or hydrophore tank)

An accumulator (air vessel) contains a gas that absorbs the pressure surges and prevents the transfer of pressure waves that arise in one section to other parts of the pipeline system. Pressurized surge tank (air vessel) are reliable and require no repairs because there are no moving parts. They are expensive, regular maintenance is required to maintain the volume of gas in the tank.

Rupture disks

Rupture disks are non-mechanical pressure surge control devices which consists of bursting membrane designed to rupture at preset conditions of pressure. Rupture disks are inexpensive and they work similar to PRVs, they need storage tanks to accept relief flow. Rupture disks must be replaced after being ruptured therefore they must be available in stock.

Transient control (pump startup)

Pump startup operations can cause a rapid increase in fluid velocity that may result in undesirable surge, but they seldom cause a problem in actual operation.

(1) If there are several pumps, start them one at a time at intervals at least two times the critical period.
(2) Open a control valve slowly after the motor starts (at least two times the critical period)
(3) Use a variable speed drive for each pump ramped upto full speed slowly enough to avoid high surges.

(4) Pumps and control valves should be operated in systematic to greatly reduce the transients.
(5) Upon start-up of pump, pump operates against a closed valve. As the valve opens, the flow into the pipeline gradually increases to the full capacity of the pump.

Transient control pump shutdown

1. Normal pump shutdown may also cause surges. They can be controlled to keep the pressures within acceptable limits.
2. Turn pumps off one at a time at intervals at least two times the critical period.
3. Close a control valve slowly (at least two times the critical period) before the stopping the motor.
4. If pumps are equipped with variable speed drives, motor is completely stopped in ramp down approach.
5. At the time of pump shutdown, the control valve must be closed slowly to give gradual deceleration to the flow, after the valve is fully closed after which power to the pump is shutoff.

Transient control after power failure

Power failure at a pumping station causes pump tripping, and results in an initial rapid down surge in the discharge header and piping close to the pump station. The power failure cannot be fully avoided but its effect can be reduced by increasing the inertia of pump and motor with a fly wheel.

Important note: Control valves cannot prevent down surge on power failure

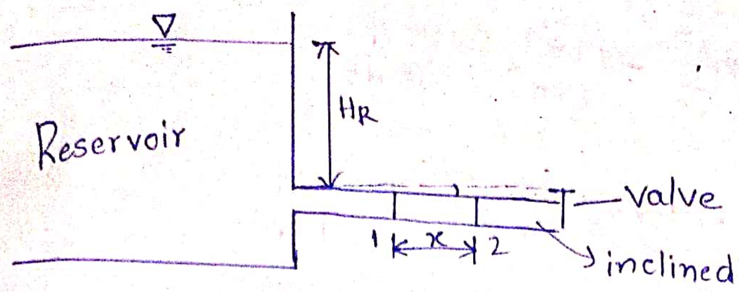
Uses of Transient Analysis

1. Determine pipe wall thickness by locating high pressure points in steady and unsteady flow situations (especially worst case scenarios).
2. Determine the location and sizing of pressure relief valve and air vessel.
3. Determine maximum and minimum working pressures (worst case scenarios).
4. To design ramping up and down of pumps (variable speed).

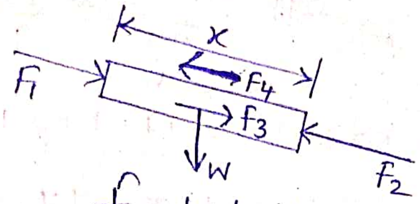
Worst case scenarios must be studied

1. Pump trip, 2. Check valve slam, 3. Control valve sudden closure

Lumped system modelling - Rigid column theory



L = Length
D = diameter



free body diagram

where $f_1 = \gamma(H_R - H_E - H_v)A = \gamma\left(H_R - H_E - \frac{v^2}{2g}\right)A$

$f_2 = \gamma H A$

F_3 = gravitational force component

$= \gamma A x S_0$

$f_4 = \gamma H_f A$

where H_f = frictional losses
H_E = Entrance losses

Applying the newton's Second Law of motion,

Applied Forces $= ma = m\frac{dv}{dt}$

$= \frac{W}{g}\frac{dv}{dt}$

$$F_1 - F_2 + F_3 - F_4 = \frac{\gamma A x}{g}\frac{dv}{dt}$$

$$\Rightarrow \gamma\left(H_R - H_E - \frac{v^2}{2g}\right)A - \gamma H A + \gamma A x S_0 - \gamma H_f A = \frac{\gamma A x}{g}\frac{dv}{dt}$$

$$\Rightarrow H_R - \frac{v^2}{2g} - \frac{K_e v^2}{2g} - H + x S_0 - H_f = \frac{x}{g}\frac{dv}{dt}$$

$$\Rightarrow H_R - \frac{v^2}{2g} - \frac{K_e v^2}{2g} - H + x S_0 - \frac{f x v^2}{2gD} = \frac{x}{g}\frac{dv}{dt}$$

~~After~~ including ~~valve head loss~~, when we write this equation for whole pipe System ie, @ $x = L$, $H = 0$ & valve head loss

$\frac{K_v V^2}{2g}$ is to be added. And the pipe is also horizontal

$$\Rightarrow H_R - \frac{V^2}{2g} - \frac{K_e V^2}{2g} - \frac{K_v V^2}{2g} - \frac{FLV^2}{2gD} = \frac{L}{g}\frac{dv}{dt}$$

$$H_R - \left(1 + K_e + K_v + \frac{FL}{D}\right)\frac{V^2}{2g} = \frac{L}{g}\frac{dv}{dt}$$

$$H_R - \frac{C_1 V^2}{2g} = \frac{L}{g}\frac{dv}{dt}$$

where $C_1 = 1 + K_e + K_v + \frac{FL}{D}$

Once the flow is Established, steady state is Achieved

$$\therefore \frac{L}{g}\frac{dv}{dt} = 0$$

$$H_R = \frac{C_1 V^2}{2g}$$

And $V_0 = \sqrt{\frac{2gH_R}{C_1}}$

To determine the discharge behaviour as a function of time during the Establishment time interval

$$\frac{L}{g}\frac{dv}{dt} = H_R - \frac{C_1 V^2}{2g}$$

$$\frac{L}{g}dv = \left(H_R - \frac{C_1 V^2}{2g}\right)dt$$

$$\frac{L}{g} \cdot \frac{1}{\left(H_R - \frac{C_1 V^2}{2g}\right)} dv = dt$$

Applying Integration,

$$\int_0^t dt = \frac{L}{g}\int_0^V \frac{dv}{H_R - \frac{C_1 V^2}{2g}}$$

$$t = 2L \int_0^V \frac{dv}{C_1 (V_0^2 - V^2)}$$

$$t = \frac{L}{V_0 C_1} \ln\left[\frac{V_0 + V}{V_0 - V}\right]$$

for ideal fluid, $C_1 = 1$

A horizontal pipe 61 cm in diameter 3048 m long leaves a reservoir 30.48 m below its Surface through a valve. f is constant = 0.018

a). If the valve is suddenly opened completely, what is the time that is required to attain 99% of the steady state velocity, neglect frictional head losses & other losses

Sol:

$$C_1 = 1.0$$

$$\therefore V_0 = \sqrt{2gH_R} = \sqrt{2 \times 9.81 \times 30.48}$$

$$= 24.45 \text{ m/sec}$$

The time to reach 99% of V_0

$$t = \frac{L}{V_0} \ln\left[\frac{V_0 + V}{V_0 - V}\right]$$

$$= \frac{3048}{24.45} \ln\left[\frac{V_0 + 0.99V_0}{V_0 - 0.99V_0}\right]$$

$$= \frac{3048}{24.45} \ln[199]$$

$$= 660 s$$

b). When the pipe friction $f = 0.018$, $K_E = K_V = 0$

$$C_1 = 1 + 0 + 0 + \frac{0.018 \times 3048}{61 \times 10^{-2}}$$

$$= 91$$

$$V_0 = \sqrt{\frac{2gH_R}{C_1}} = \sqrt{\frac{2 \times 9.81 \times 30.48}{91}}$$

$$= 2.56 \text{ m/s}$$

$$t = \frac{L}{C_1 V_0} \ln\left[\frac{V_0 + V}{V_0 - V}\right]$$

$$= \frac{3048}{2.56 \times 91} \ln[199]$$

$= 69.28$ Sec

c). when pipe Friction $f = 0.018$, $K_e = 0.5$ and $K_v = 50$

$$C_1 = 1 + K_e + K_v + \frac{fL}{D}$$

$$= 1 + 0.5 + 50 + \left(\frac{0.018 \times 3048}{61 \times 10^{-2}}\right)$$

$$= 96.5$$

$$V_0 = \sqrt{\frac{2 \times 9.81 \times 3048}{96.5}}$$

$$= 2.49 \, m/sec$$

$$t = \frac{3048}{(2.49)(96.5)} \ln[199]$$

$$= 67.2 \text{ Sec}$$

prob: 2

The reservoir head on the pipeline is 18.29m. The 30.48cm diameter Pipeline is 914.4m long with an equivalent roughness $k_s = 0.0003m$. Since the valve has been fully opened for a long time, the flow of water is steady

a). Calculate the steady-state velocity in the pipeline assuming there is no loss at the valve & $K_e = 0.5$

b). Compute the maximum pressure in the pipeline if the valve closes so that the rate of decrease in velocity is linear in time from its steady state value to zero in 20 seconds.

Sol:

Steady state i.e., $\frac{dv}{dt} = 0$

There is no loss in the valve i.e., $K_v = 0$

$$H_R = \left(1 + K_e + K_v + \frac{fL}{D}\right)\frac{v^2}{2g} \rightarrow (1)$$

Here f is unknown & V_0 is unknown

Haaland Equation,

$$\frac{1}{\sqrt{f}} = 1.8 \log_{10}\left[\left(\frac{k_s/D}{3.7}\right)^{1.11} + \frac{6.9}{Re}\right] \rightarrow (2)$$

$$Re = \frac{VD}{\nu}$$

Substitute ② in ① & Solving iteratively

After Assuming $f = 0$ in eq① initially,

$$f = 0.0201, \quad V_0 = 2.41 \, m/sec$$

b). When deceleration is constant

$$a = \frac{L}{g}\frac{dv}{dt} = \frac{914.4}{9.81} \cdot \frac{(-2.41)}{20}$$

$$= -11.23 \, m/s^2$$

$$\Rightarrow \frac{2L}{C} = \frac{2 \times 914.4}{1250} = 1.46 \, sec$$

$$\therefore 1.46 < 20 sec$$

$\Rightarrow$ Slow valve closure

$$\frac{L}{g}\frac{dv}{dt} = H_R - \left(1 + K_e + \frac{fL}{D}\right)\frac{v^2}{2g} - \frac{P_2}{\gamma}$$

$$-11.23 = 18.29 - \left(1 + 0.5 + \frac{f(914.4)}{0.3048}\right)\frac{v^2}{2g} - H_2$$

$$29.52 = \left(1.5 + \frac{f(914.4)}{0.3048}\right)\frac{v^2}{2g} - H_2$$

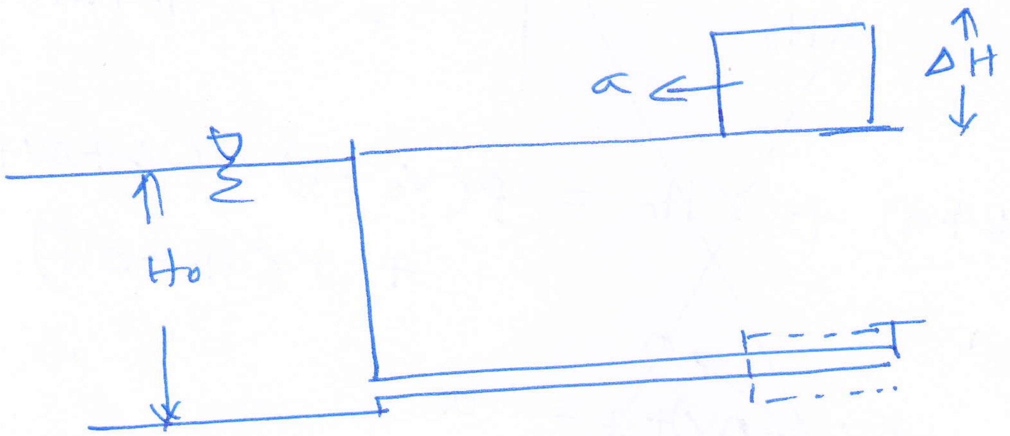
$$H_2 = 29.52 - \left(1.5 + \frac{f(914.4)}{0.3048}\right)\frac{v^2}{2g}$$

as V decreases, H_2 increases

when flow completely stops, $V = 0$

$$\therefore H_2 = 29.52 \, m$$

Pressure wave velocity and water hammer pressure rise of slightly compressible fluid in a rigid pipe



ρ $\rho + \Delta\rho$

$V_0 + a$ $V_0 + \Delta V + a$

A A

H $H_0 + \Delta H$

Apply continuity equation.

$$V_0 + \Delta V = 0 \quad \text{(valve is closed)}$$

$$\rho \left(V_0 + a \right) A = \left(\rho + \Delta\rho \right) a A$$

divide above equation $\rho a A$

$$\frac{V_0}{a} + 1 = 1 + \frac{\Delta\rho}{\rho}$$

$$\boxed{\frac{V_0}{a} = \frac{\Delta\rho}{\rho}} \rightarrow (1)$$

Apply momentum equation

$$\rho_1 Q V_{1x} + P_1 A_1 = \rho_2 Q V_{2x} + P_2 A_2$$

$Q = (V_0 + a) A$

$P_1 = \gamma H_0$

$V_{x1} = \dfrac{V_0 + a}{A}$

$Q = (V_0 + a) A$

$P_2 = \gamma (H_0 + \Delta H)$

$V_{x2} = \dfrac{V_0 + \Delta V + a}{A}$

$$\rho (V_0 + a) A (V_0 + a) + \gamma H_0 A = \rho (V_0 + a) A (V_0 + \Delta V + a) + \gamma (H_0 + \Delta H) A$$

$$\rho (V_0 + a) A \left[V_0 + a - V_0 - \Delta V - a \right] = \gamma (H_0 + \Delta H - H_0) A$$

$$\rho (V_0 + a) \left[-\Delta V \right] = \rho g \Delta H$$

$$\rho (V_0 + a)(-\Delta V) = \Delta p$$

$V_0 << a$

$$\boxed{\Delta p = -\rho a \Delta V} \rightarrow (2)$$

$$\Delta H = \frac{-a \Delta V}{g} \rightarrow (3)$$

for sudden closure of valve

$\Delta V = -V_0$

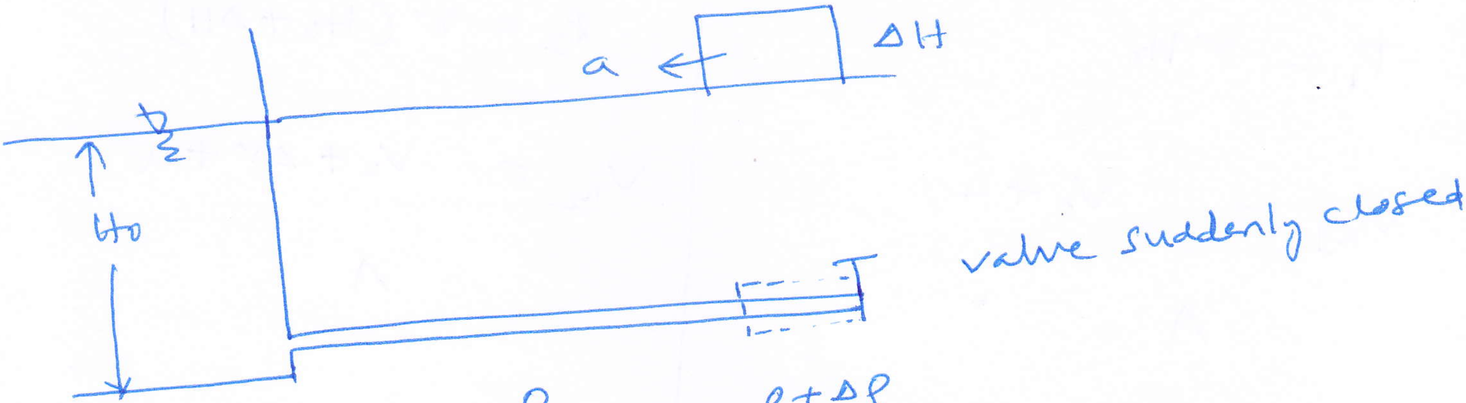
$$\boxed{\Delta H = \frac{a V_0}{g}} \rightarrow (4)$$

from the definition of bulk modulus

$$K = \frac{\Delta p}{\frac{\Delta \rho}{\rho}} \Rightarrow \Delta p = K \frac{\Delta \rho}{\rho}$$

$$\boxed{\Delta p = \rho a V_0} \rightarrow (5) \Rightarrow \rho a V_0 = K \frac{\Delta \rho}{\rho} \Rightarrow \boxed{a = \sqrt{\frac{K}{\rho}}} \rightarrow (6)$$

Pressure wave velocity of slightly compressible fluid in non-rigid pipe



ρ	$\rho + \Delta\rho$
$V_0 + a$	$(V_0 + \Delta V + a) = a$
A	$A + \Delta A$
H_0	$H_0 + \Delta H$

Apply continuity equation

$$\rho (V_0 + a) A = (\rho + \Delta\rho) a (A + \Delta A)$$

divide above equation by $\rho a A$

$$\frac{V_0}{a} + 1 = \left(1 + \frac{\Delta\rho}{\rho}\right)\left(1 + \frac{\Delta A}{A}\right)$$

$$\Rightarrow \quad 1 + \frac{V_0}{a} = 1 + \frac{\Delta\rho}{\rho} + \frac{\Delta A}{A} + \cancelto{0}{\frac{\Delta\rho}{\rho}\frac{\Delta A}{A}} \rightarrow (7)$$

from compressibility $\quad K = \dfrac{\Delta p}{(\Delta\rho/\rho)}$

$$\Rightarrow \quad \frac{\Delta\rho}{\rho} = \frac{\Delta p}{K} \rightarrow (8)$$

from Eq. (5) $\quad V_0 = \dfrac{\Delta p}{\rho a} \rightarrow (9)$

substitute (8) & (9) in (7)

$$1+\frac{V_0}{a} = 1+\frac{\Delta p}{K}+\frac{\Delta A}{A}$$

Substitute (9) in (10) $\Rightarrow$ $\boxed{1+\frac{\Delta p}{\rho a^2} = 1+\frac{\Delta p}{K}+\frac{\Delta A}{A}} \rightarrow (10)$

from hoop stress

$$\sigma = \frac{\Delta p\cdot(D/2)}{t}$$

t = thickness

σ = circumferential stress

stress = Young's modulus x strain

$$\sigma = E\,\frac{\Delta D}{D}$$

strain $= \frac{\pi\Delta D}{\pi D}$

$$E\,\frac{\Delta D}{D} = \frac{\Delta p\cdot D}{2t}$$

$$\boxed{\frac{\Delta D}{D} = \frac{D}{2Et}\cdot\Delta p} \rightarrow (11)$$

$$A = \frac{\pi D^2}{4} \Rightarrow \Delta A = \frac{2\pi D\,dD}{4}$$

$$\frac{\Delta A}{A} = \frac{2\,dD}{D} = 2\,\frac{\Delta D}{D}$$

$$\frac{\Delta A}{A} = 2\cdot\frac{D}{2Et}\,\Delta p = \frac{D}{Et}\,\Delta p$$

$$\boxed{\frac{\Delta A}{A} = \frac{D}{Et}\,\Delta p} \rightarrow (12)$$

from (10)

$$1+\frac{\Delta p}{\rho a^2} = 1+\frac{\Delta p}{K}+\frac{D\,\Delta p}{Et}$$

$$\frac{1}{\rho a^2} = \frac{1}{K} + \frac{D}{Et}$$

$$\frac{1}{\rho a^2} = \frac{1}{K'}$$ where $\frac{1}{K'} = \frac{1}{K} + \frac{D}{Et}$

$$\frac{1}{K'} = \frac{Et + DK}{KEt}$$

$$a = \sqrt{\frac{K'}{\rho}} \rightarrow (13)$$

$$K' = \frac{KEt}{Et+DK}$$

$$a = \sqrt{\frac{KEt}{\rho(Et+DK)}} \rightarrow (14)$$

pressure wave velocity of compressible fluid in an elastic pipe

$$a = \sqrt{\frac{K}{\rho_0}}$$

This formula is applicable only for slightly compressible fluid in a rigid pipe.

Reynolds Transport Theorem (RTT)

$$\frac{dB_{sys}}{dt} = \frac{d}{dt}\int_v \beta\rho dv + \left(\beta\rho A V_s\right)_{out} - \left(\beta\rho A V_s\right)_{in}$$

B = extensive property

β = intensive property

v = Total volume

Note that the velocity V_s is with respect to the control surface since it accounts for the inflow or outflow from the control volume.

For fixed control volume, V_s = fluid flow velocity V. If control volume is moving with 'w' velocity then $V_s = V - w$

$$\frac{dB_{sys}}{dt} = \frac{d}{dt}\int_v \beta\rho dv + \left[\beta\rho A\left(V - w\right)\right]_{out} - \left[\beta\rho A\left(V - w\right)\right]_{in}$$

Derivation of Continuity Equation

Assumptions

1) Flow is slightly compressible
2) Conduit walls are elastic
3) Flow is one directional.

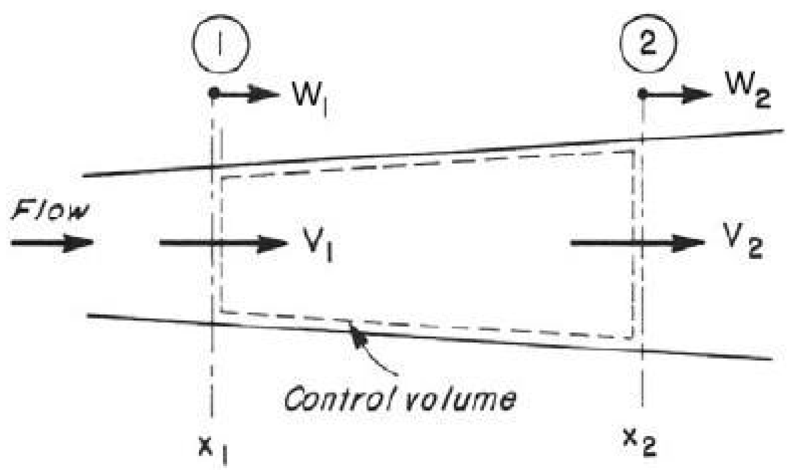


Fig. 2.9. Application of continuity equation

Control surfaces are moving with w_1 and w_2 velocities, that means control volume is either contracting or expanding. Rapid expansion and contraction are taken into account in this derivation.

Applying RTT, $\beta = 1$

$$\frac{d}{dt}\int_{x_1}^{x_2} \rho A dx + \rho_2 A_2 (V_2 - w_2) - \rho_1 A_1 (V_1 - w_1) = 0$$

Then apply Leibnitz's rule to the first term on the LHS.

We know, Leibnitz theorem

$$\frac{d}{dt}\int_{f_1(t)}^{f_2(t)} F(x,t)dx = \int_{f_1(t)}^{f_2(t)} \frac{\partial}{\partial t} F(x,t)dx + F(f_2(t),t)\frac{df_2}{dt} - F(f_1(t),t)\frac{df_1}{dt}$$

Therefore first term on LHS can be written as

$$\frac{d}{dt}\int_{x_1}^{x_2} \rho A dx = \int \frac{\partial}{\partial t}(\rho A)dx + \rho_2 A_2 \frac{dx_2}{dt} - \rho_1 A_1 \frac{dx_1}{dt}$$

Total equation can be written as

$$\int \frac{\partial}{\partial t}(\rho A)dx + \rho_2 A_2 w_2 - \rho_1 A_1 w_1 + \rho_2 A_2 (V_2 - w_2) - \rho_1 A_1 (V_1 - w_1) = 0$$

$$\frac{\partial}{\partial t}(\rho A)\Delta x + \rho_2 A_2 V_2 - \rho_1 A_1 V_1 = 0$$

$$\frac{\partial}{\partial t}(\rho A) + \frac{\partial}{\partial x}(\rho A V) = 0$$

Expansion of terms inside the parenthesis

$$A\frac{\partial \rho}{\partial t} + \rho\frac{\partial A}{\partial t} + \rho A\frac{\partial V}{\partial x} + \rho V\frac{\partial A}{\partial x} + AV\frac{\partial \rho}{\partial x} = 0$$

Divide it by (ρA)

$$\frac{1}{\rho}\frac{\partial \rho}{\partial t} + \frac{1}{A}\frac{\partial A}{\partial t} + \frac{\partial V}{\partial x} + \frac{V}{A}\frac{\partial A}{\partial x} + \frac{V}{\rho}\frac{\partial \rho}{\partial x} = 0$$

$$\frac{1}{\rho}\frac{d\rho}{dt} + \frac{1}{A}\frac{dA}{dt} + \frac{\partial V}{\partial x} = 0$$

We need to convert the above equation interms of pressure and velocity V.

The bulk modulus of Elasticity, $$K = \frac{dP}{\frac{d\rho}{\rho_0}} = -\frac{dP}{\frac{\Delta V}{V_0}}$$

The equation may be written as $\dfrac{d\rho}{dt}=\rho\dfrac{dP}{Kdt}$

For circular conduits, $A=\pi R^2$, $R=$radius.

$$\frac{dA}{dt}=2\pi R\frac{dR}{dt}$$

The above equation may be written as

$$\frac{dA}{dt}=2\pi R^2\frac{1}{R}\frac{dR}{dt}$$

$\dfrac{dR}{R}=d\varepsilon$ Where $\varepsilon=$ Strain

$$\frac{1}{A}\frac{dA}{dt}=2\cdot\frac{d\varepsilon}{dt}$$

For circular conduits, $\varepsilon=\dfrac{\sigma_h-\sigma_a\mu}{E}$

Where $\sigma_h=$ hoops stress, $\sigma_a=$ axial stress and $\mu=$ poison ratio.

Pipe has expansion joints, therefore $\sigma_a=0$.

So, $\varepsilon=\dfrac{\sigma_h}{E}$

Hoop stress is defined as, $\sigma_h=\dfrac{PD}{2e}$

By taking the time derivative on both sides, we can get

$$\frac{d\sigma_h}{dt}=\frac{P}{2e}\frac{dD}{dt}+\frac{D}{2e}\frac{dP}{dt}$$

$\sigma_h=E\varepsilon$, where E is a constant.

$$E\frac{d\varepsilon}{dt}=\frac{P}{2e}\frac{dD}{dt}+\frac{D}{2e}\frac{dP}{dt}$$

We know that, $\dfrac{1}{A}\dfrac{dA}{dt}=2\cdot\dfrac{d\varepsilon}{dt} \quad\Rightarrow \dfrac{1}{\frac{\pi D^2}{4}}\cdot\dfrac{1}{4}\dfrac{d\left(\pi D^2\right)}{dt}=2\dfrac{d\varepsilon}{dt}$

$$\frac{1}{D}\frac{dD}{dt} = \frac{d\varepsilon}{dt}$$

$$E\frac{d\varepsilon}{dt} = \frac{P}{2e}D\frac{d\varepsilon}{dt} + \frac{D}{2e}\frac{dP}{dt}$$

$$\frac{d\varepsilon}{dt} = \frac{\frac{D}{2e}\frac{dP}{dt}}{E - \frac{PD}{2e}}$$

We already know that, $\frac{1}{A}\frac{dA}{dt} = 2 \cdot \frac{d\varepsilon}{dt}$

$$\frac{1}{2A}\frac{dA}{dt} = \frac{\frac{D}{2e}\frac{dP}{dt}}{E - \frac{PD}{2e}}$$

Continuity equation

$$\frac{1}{\rho}\frac{d\rho}{dt} + \frac{1}{A}\frac{dA}{dt} + \frac{\partial V}{\partial x} = 0$$

$$\frac{d\rho}{dt} = \frac{\rho}{K}\frac{dP}{dt}$$

$$\frac{1}{\rho}\frac{\rho}{K}\frac{dP}{dt} + \left(\frac{\frac{D}{e}}{E - \frac{PD}{2e}}\right) \cdot \frac{dP}{dt} + \frac{\partial V}{\partial x} = 0$$

$$\left(\frac{1}{K} + \frac{1}{\frac{eE}{D} - \frac{P}{2}}\right) \cdot \frac{dP}{dt} + \frac{\partial V}{\partial x} = 0\,, \qquad \text{Where } \frac{P}{2} << \frac{eE}{D}$$

$$\frac{1}{K}\left(1 + \frac{DK}{eE}\right) \cdot \frac{dP}{dt} + \frac{\partial V}{\partial x} = 0$$

$$\frac{K}{\left(1 + \frac{DK}{eE}\right)} \cdot \frac{\partial V}{\partial x} + \frac{dP}{dt} = 0$$

$$\rho \frac{\frac{K}{\rho}}{\left(1+\frac{DK}{eE}\right)} \cdot \frac{\partial V}{\partial x} + \frac{\partial P}{\partial t} + V \frac{\partial P}{\partial x} = 0$$

$$\Rightarrow \rho a^2 \cdot \frac{\partial V}{\partial x} + \frac{\partial P}{\partial t} + V \frac{\partial P}{\partial x} = 0$$

Derivation of momentum equation

$$\frac{d}{dt}\int_v \beta\rho dv + \beta\rho(V-w)A\big|_{outlet} - \beta\rho(V-w)A\big|_{inlet} = 0$$

For momentum equation $\beta = V$

$$\frac{d}{dt}\int_v V\rho dv + V\rho(V-w)A\big|_{outlet} - V\rho(V-w)A\big|_{inlet} = \sum F$$

By applying Leibnitz rule to the first term

$$\sum F = \int_{x_1}^{x_2} \frac{\partial}{\partial t}(\rho AV)dx + \rho AV\big|_{out}\frac{dx_2}{dt} - \rho AV\big|_{in}\frac{dx_1}{dt} + \rho A(V-w)V\big|_2 - \rho A(V-w)V\big|_1$$

$$\sum F = \int_{x_1}^{x_2} \frac{\partial}{\partial t}(\rho AV)dx + (\rho AV)_2 w_2 - (\rho AV)_1 w_1 + \left[\rho A(V-w)V\right]_2 - \left[\rho A(V-w)V\right]_1$$

$$\frac{\sum F}{\Delta x} = \frac{\partial}{\partial t}(\rho AV) + \frac{(\rho AV^2)_2 - (\rho AV^2)_1}{\Delta x}$$

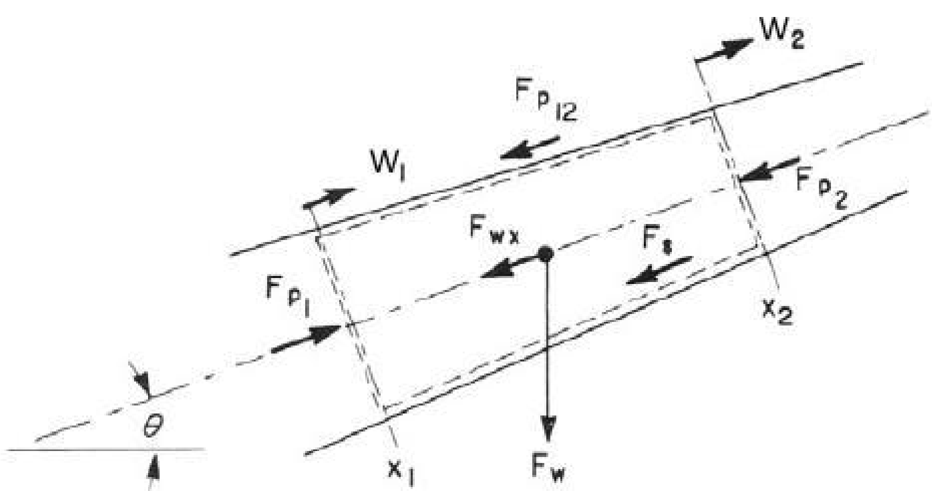


Fig. 2.10. Application of momentum equation