Parameter Estimation

Geohydraulics| CE60113

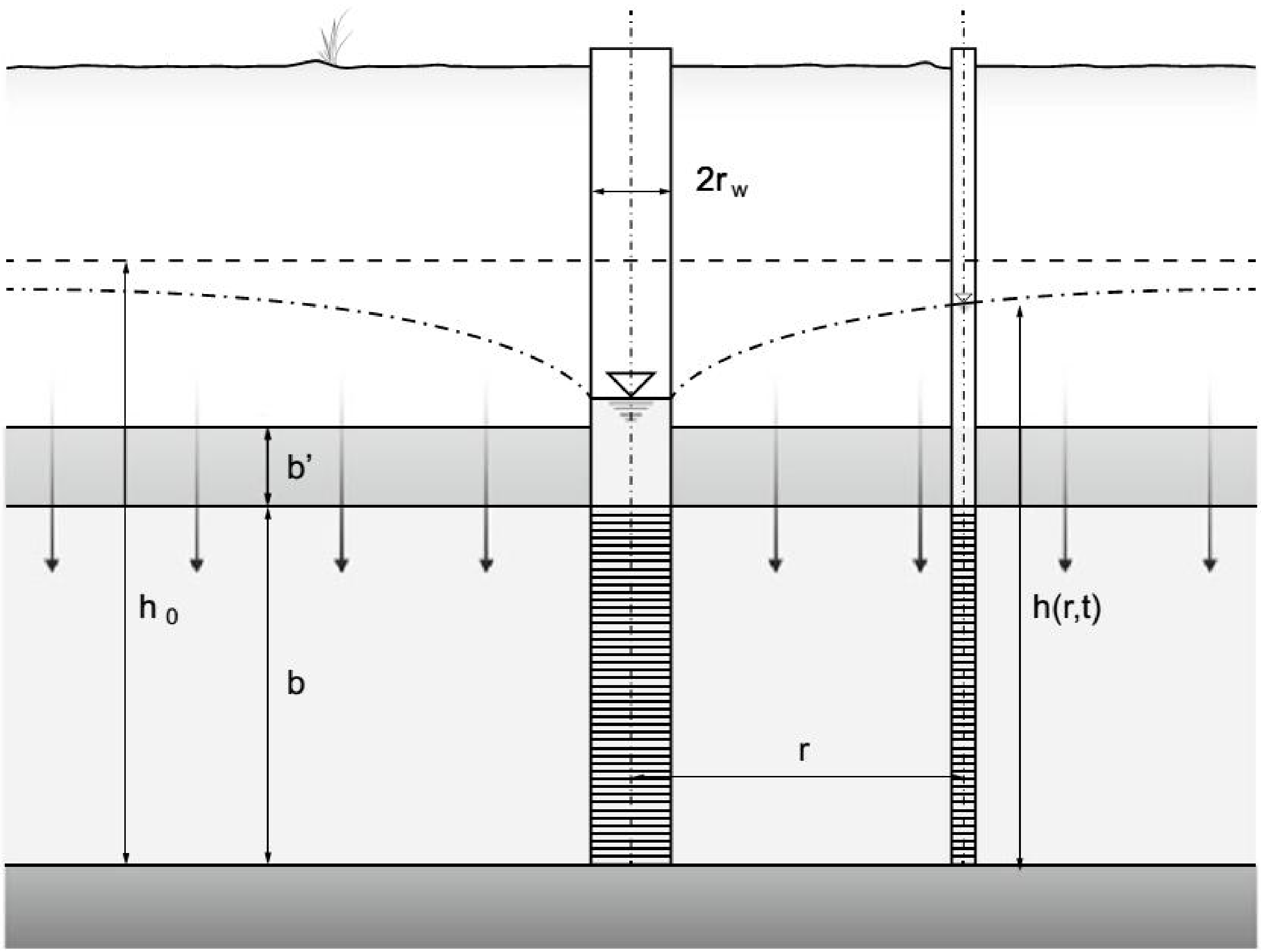
Lecture:16

Learning Objective(s)

- **To estimate aquifer parameter under unsteady semiconfined flow condition**
- **To utilize unsteady pumping solutions for different boundary conditions**

Unsteady Radial Flow in Confined Aquifer

- Semiconfined Aquifer



Unsteady Radial Flow in Confined Aquifer (Contd.)

- Under unsteady state radial flow condition in semiconfined aquifer

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r}\frac{\partial h}{\partial r} - \frac{h_0 - h}{B^2} = \frac{S}{T}\frac{\partial h}{\partial t}$$

Subject to

Initial Condition (IC)

$$h(r,0) = h_0$$

Boundary Conditions (BCs)

$$h(r \to \infty, t) = h_0$$

$$\lim_{r \to 0}\left(r\frac{\partial h}{\partial r}\right) = \frac{Q}{2\pi T}$$

Hantush-Jacob Solution

$$h_0 - h(r,t) = s(r,t) = \frac{Q}{4\pi T}\int_u^\infty \frac{1}{\omega} e^{-\left(\omega + \frac{\beta^2}{4\omega}\right)} d\omega = W(u, \beta\ )$$

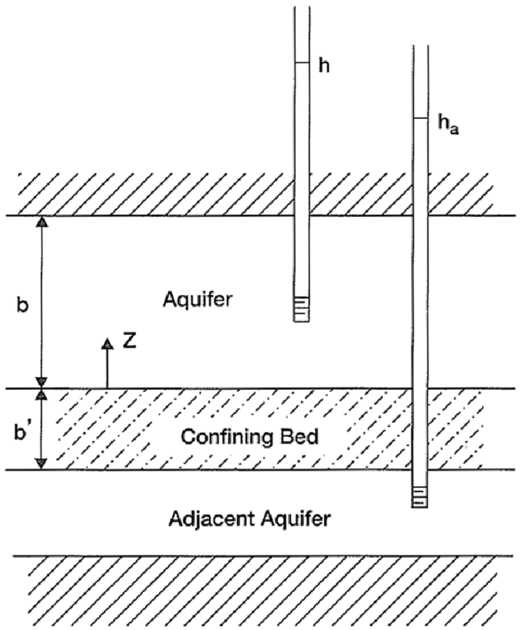
$$u = \frac{r^2 S}{4Tt}, \beta = \frac{r}{B}$$

$$B = \sqrt{\frac{T}{c}} = \sqrt{\frac{Tb'}{K'}} = \sqrt{\frac{K}{K'}bb'}$$

Unsteady Radial Flow in Confined Aquifer (Contd.)

- Under unsteady state radial flow condition in semiconfined aquifer

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r}\frac{\partial h}{\partial r} - \frac{h - h_a}{B^2} = \frac{S}{T}\frac{\partial h}{\partial t}$$



Subject to

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$$h(r,0) = h_a$$

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Hantush Solution

$$h_a - h(r,t) = s(r,t) = \frac{Q}{4\pi T}\int_u^\infty \frac{1}{\omega} e^{-\left(\omega + \frac{\beta^2}{4\omega}\right)} d\omega = W(u, \beta\ )$$

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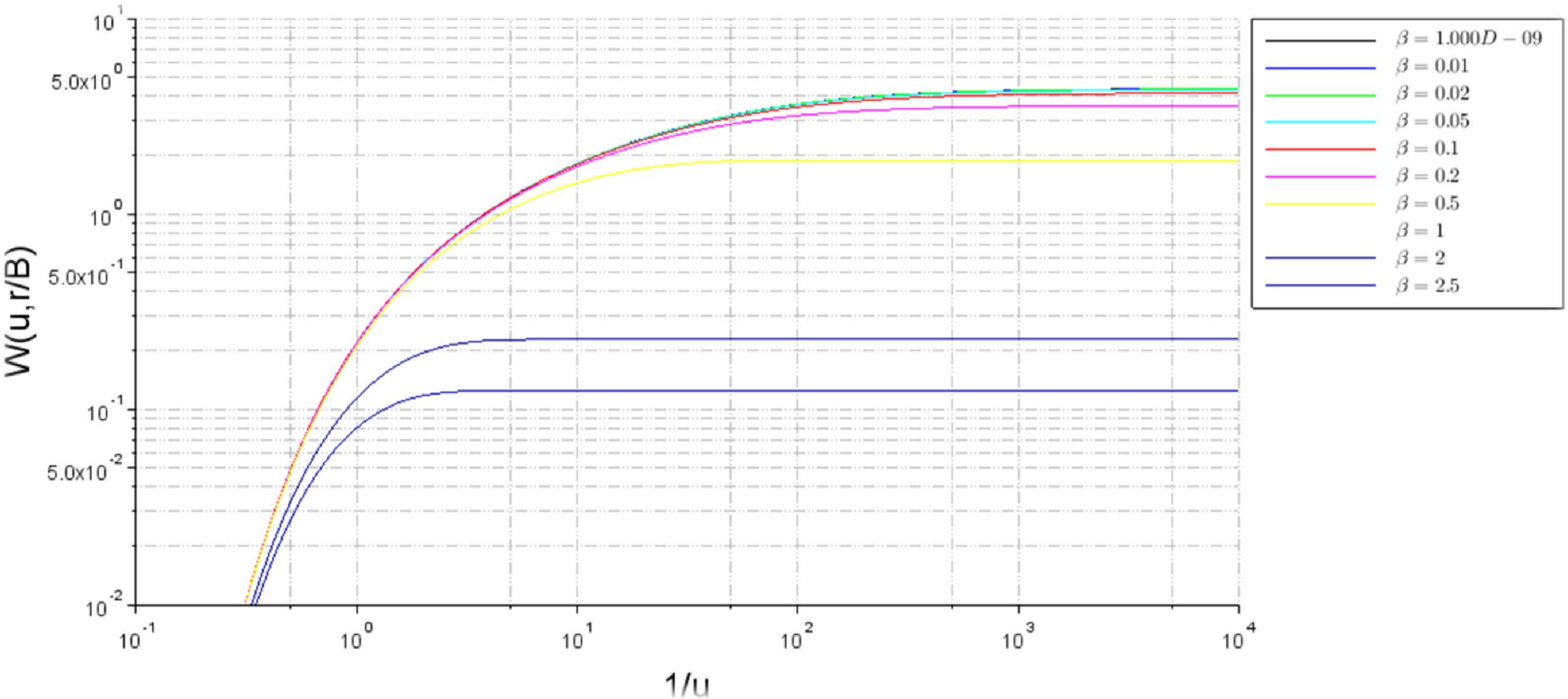
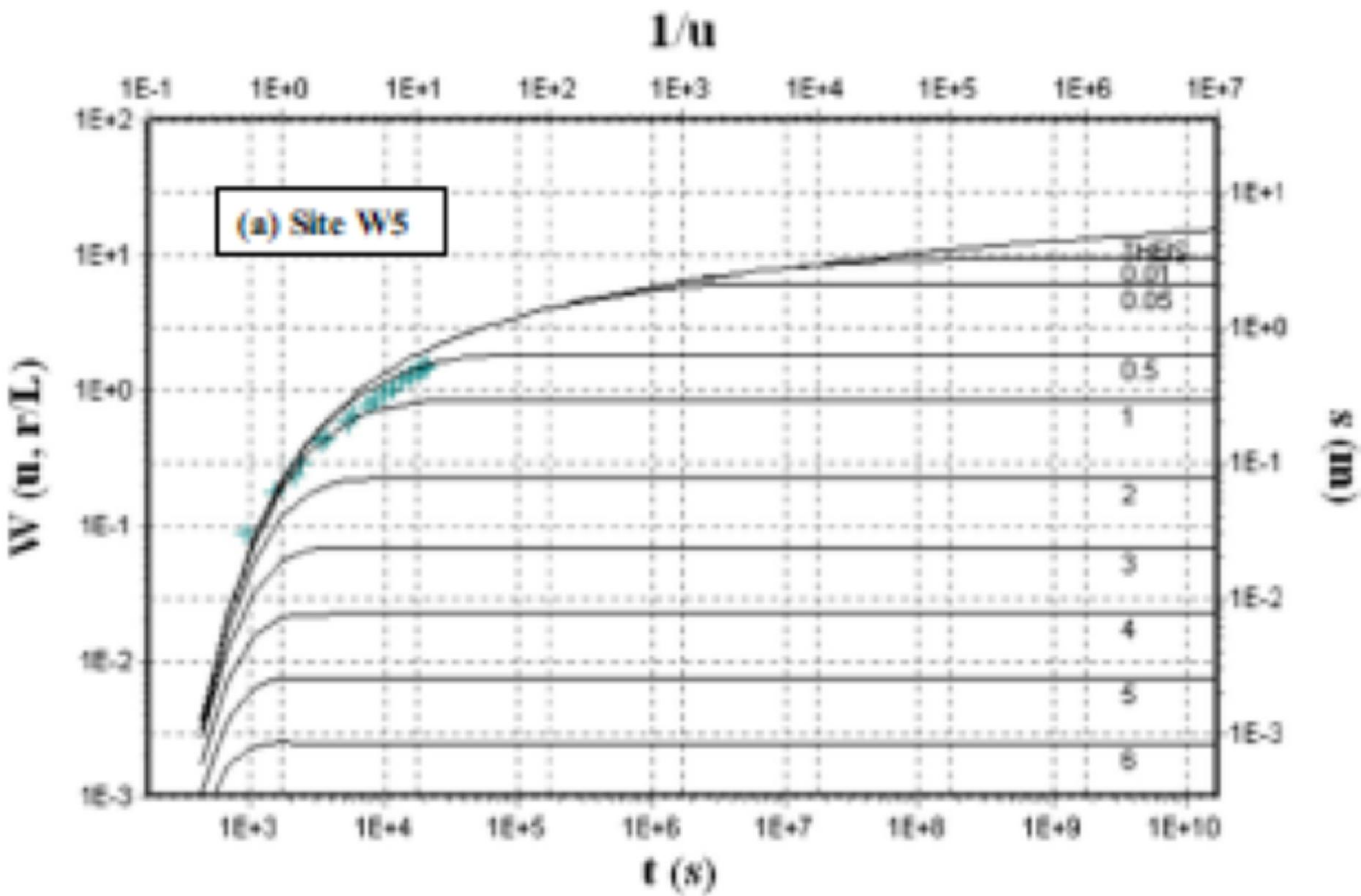
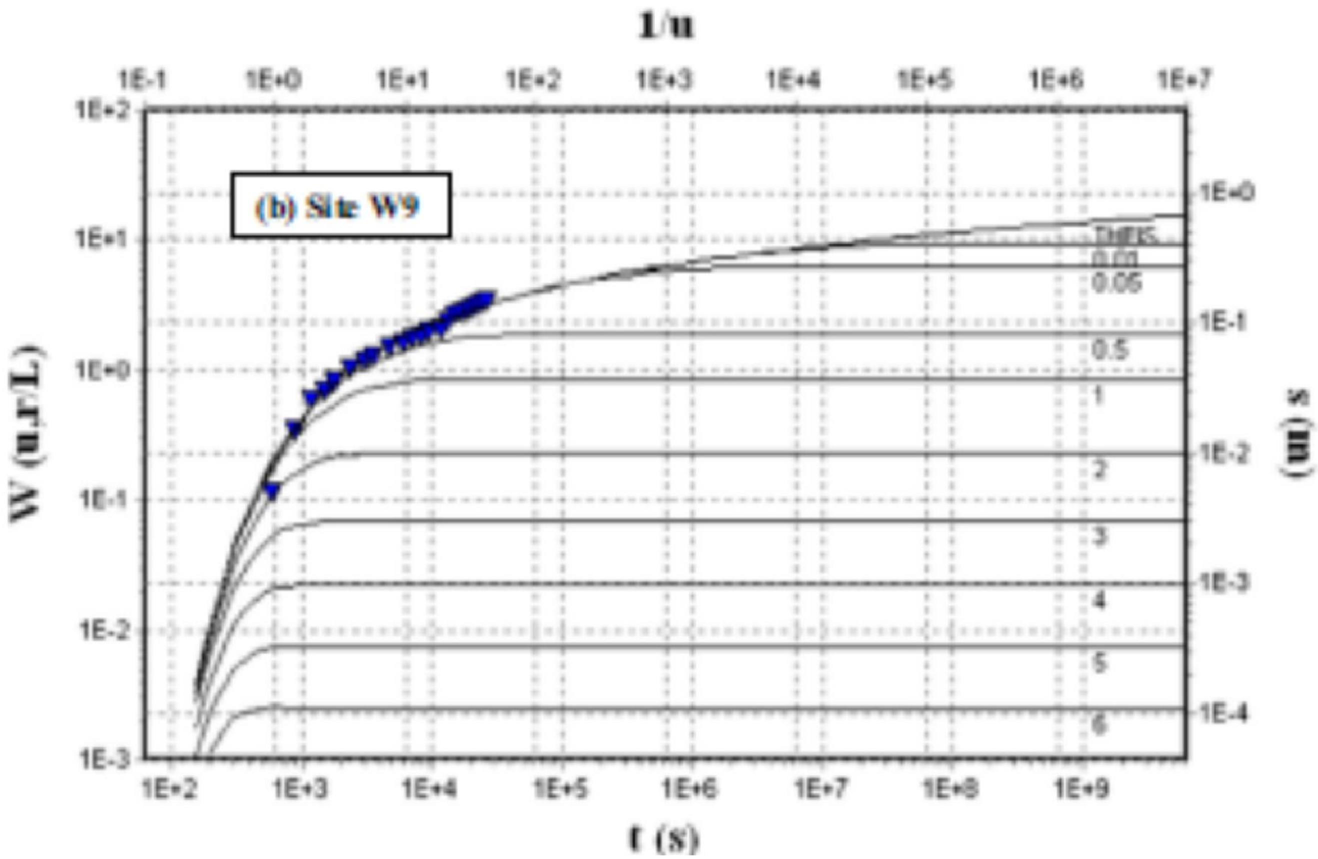
Unsteady Radial Flow in Confined Aquifer (Contd.)



Unsteady Radial Flow in Confined Aquifer (Contd.)



Unsteady Radial Flow in Confined Aquifer (Contd.)



Unsteady Radial Flow in Confined Aquifer (Contd.)

1. Match field data with Nonequilibrium type curve and identify β
2. For a pair of arbitrary points $[t^+, s^+]$ and $\left[\left(\frac{1}{u}\right)^+, W(u,\beta)^+\right]$, calculate

$$T = \frac{Q}{4\pi}\frac{W(u,\beta)^+}{s^+}$$

$$B = \frac{r}{\beta}$$

$$c = \frac{T}{B^2}$$

$$S = 4T\frac{t^+}{r^2\left(\frac{1}{u}\right)^+}$$

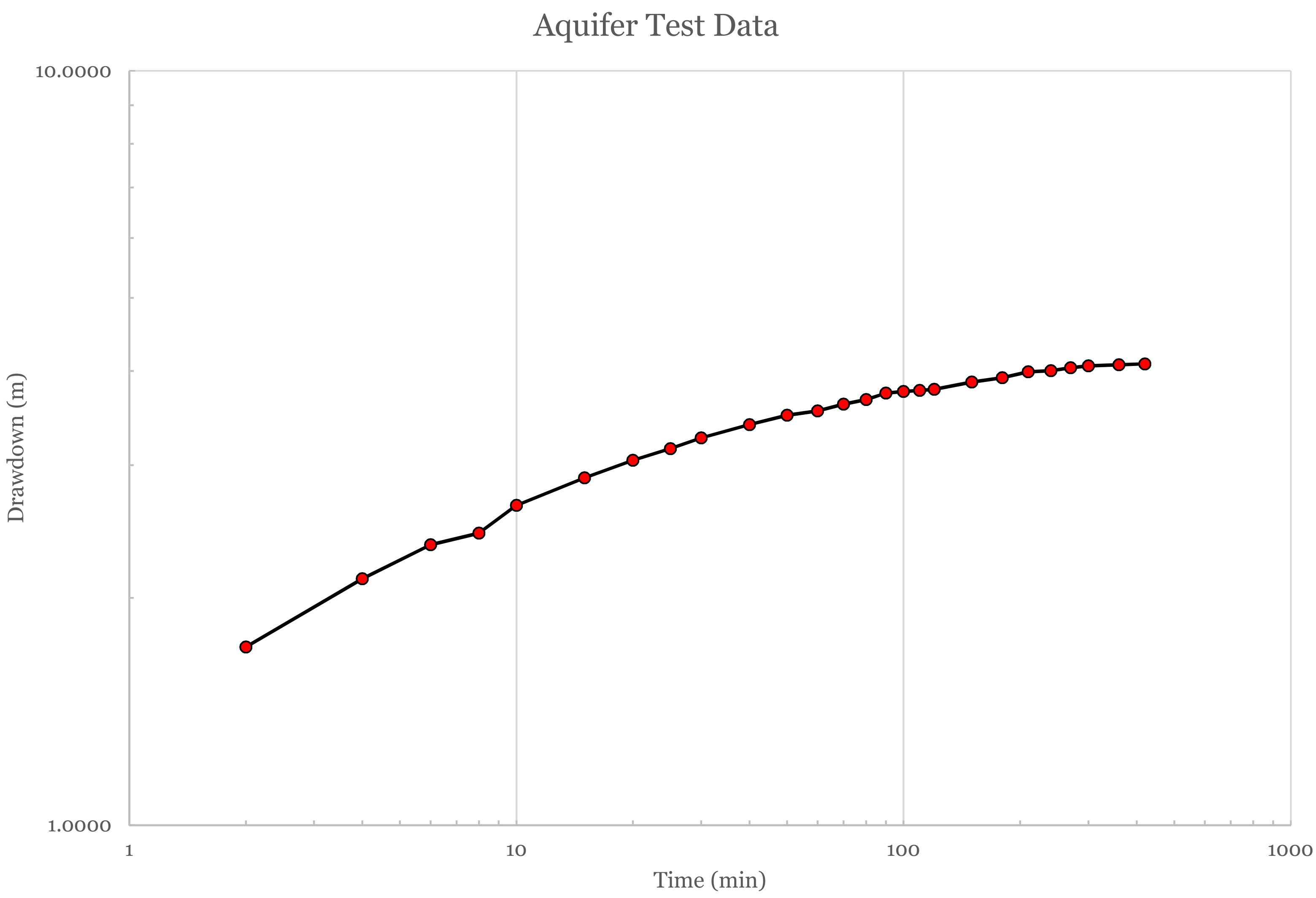
Unsteady Radial Flow in Confined Aquifer (Contd.)

- Semiconfined Aquifer

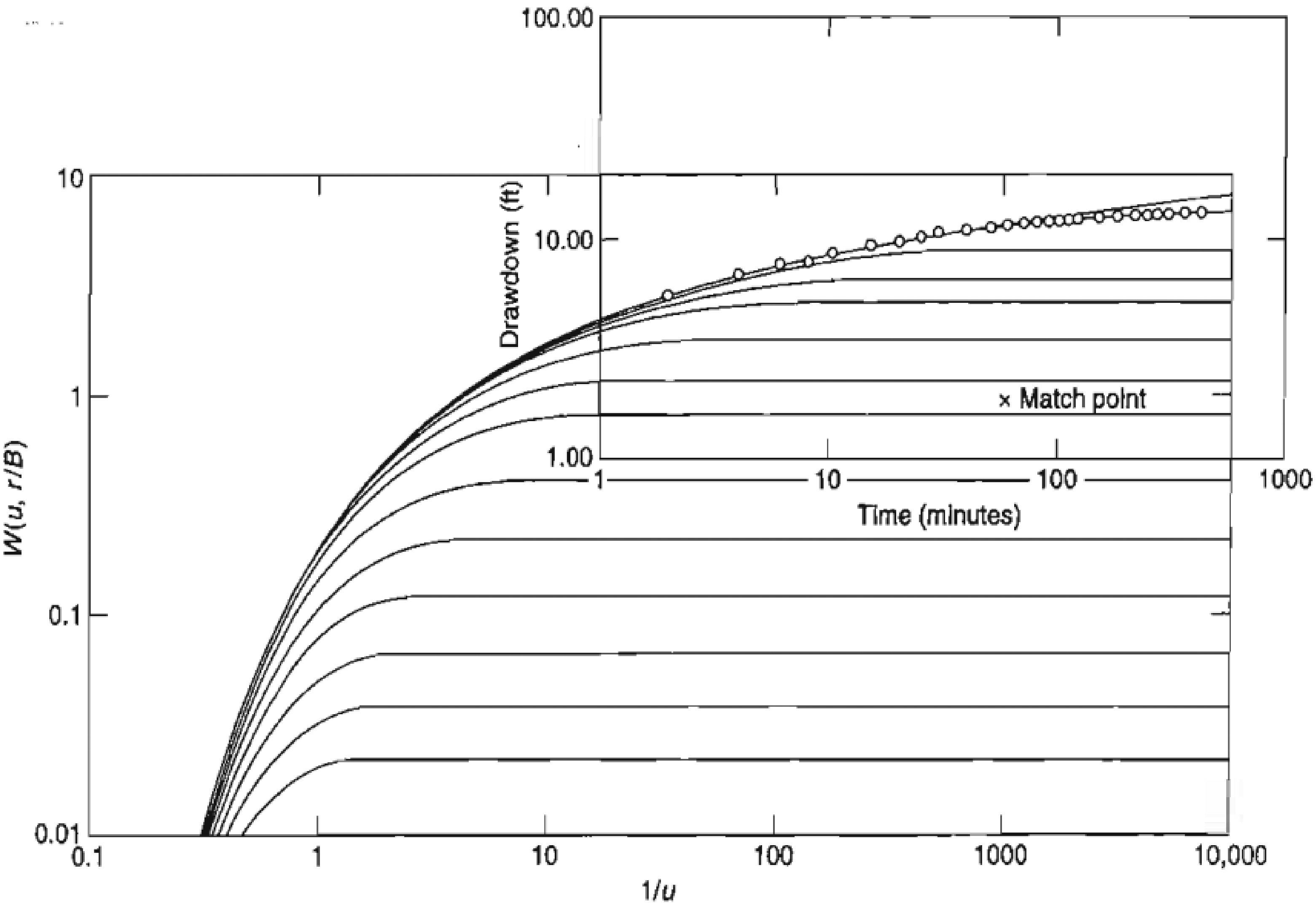
- Q= 17 m³/min
- b' = 4.25 m
- r = 12.2 m

t	s	t	s
min	m	min	m
0	0.00	80	3.664
2	1.722	90	3.737
4	2.121	100	3.758
6	2.353	110	3.770
8	2.438	120	3.783
10	2.655	150	3.868
15	2.886	180	3.917
20	3.045	210	3.990
25	3.155	240	4.002
30	3.261	270	4.039
40	3.395	300	4.063
50	3.493	360	4.075
60	3.542	420	4.087
70	3.615		

Unsteady Radial Flow in Confined Aquifer (Contd.)



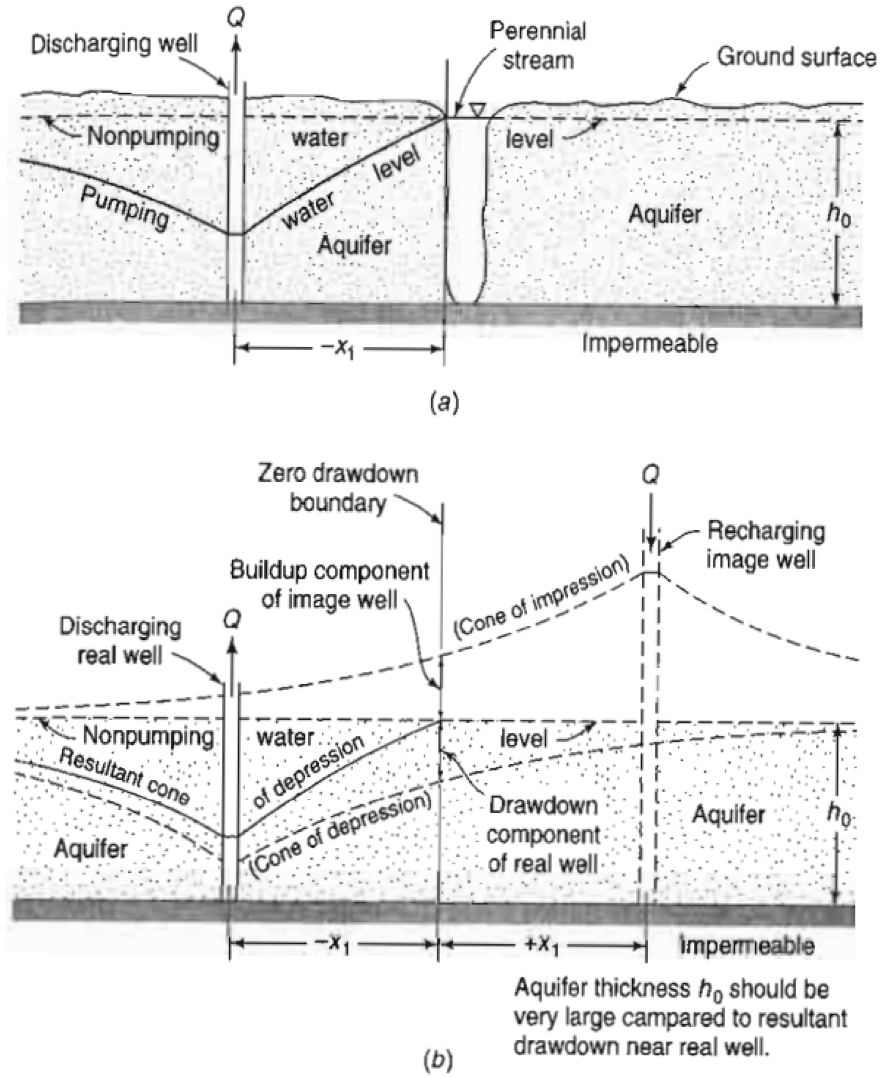
Unsteady Radial Flow in Confined Aquifer (Contd.)



Well Flow for Special Conditions

- Constant well drawdown
- Varying, cyclic, and intermittent well discharges
- Sloping aquifers
- Aquifers of variable thickness
- Two-layered aquifer/multilayered aquifers
- Anisotropic aquifers
- Aquifer conditions varying with depth
- Large-diameter wells
- Collector wells
- Wells with multiple-sectioned well screens
- Effect of boundary conditions

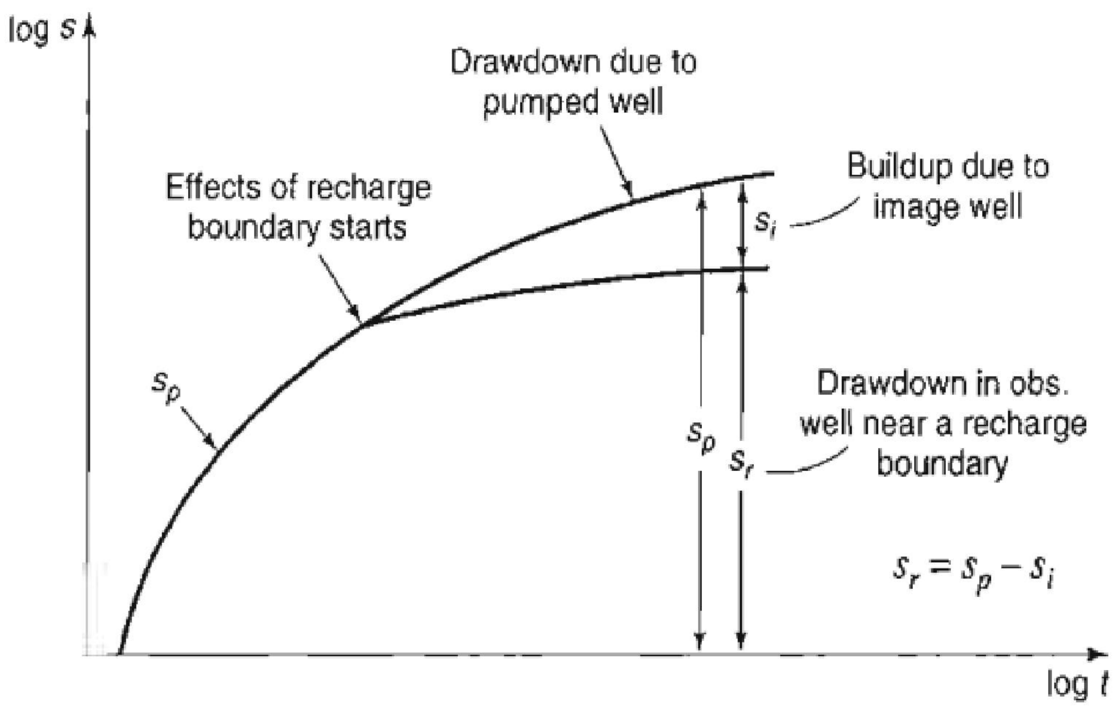
Well Flow Near a Stream



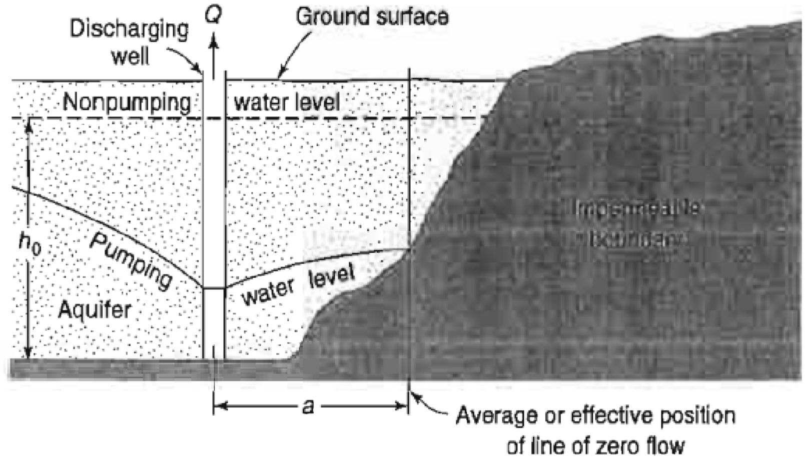
Well Flow Near a Stream (Contd.)

The drawdown equation can be written as

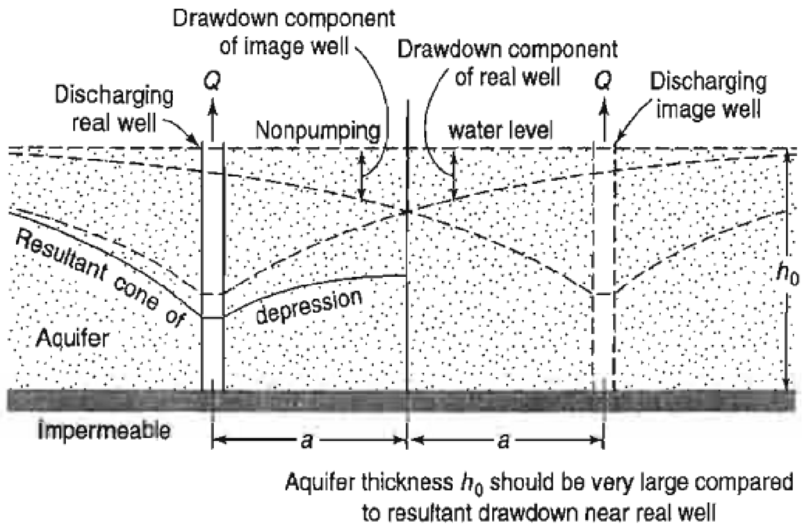
$$s_r = \frac{Q}{4\pi T}\left[W\left(u_p\right) - W\left(u_i\right)\right]$$



Well Flow Near an Impermeable Boundary



(a)

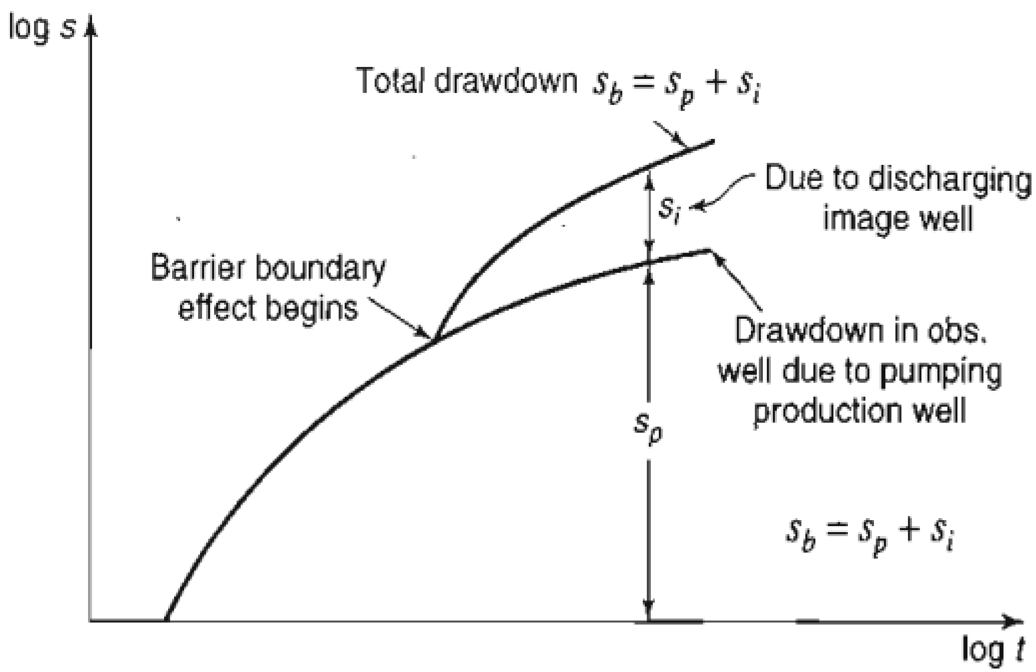


(b)

Well Flow Near an Impermeable Boundary (Contd.)

The total drawdown can be expressed as

$$s_b = \frac{Q}{4\pi T} W(u_p) + \frac{Q}{4\pi T} W(u_i)$$



Well Flow Near an Impermeable Boundary (Contd.)

The total drawdown can be expressed as

$$s_b = \frac{Q}{4\pi T} W(u_p) + \frac{Q}{4\pi T} W(u_i)$$

Now suppose that we choose drawdowns at times t_p and t_i such that $s_p = s_i$, then $W(u_p) = W(u_i)$ and $u_p = u_i$. Then

$$\frac{r_i^2 S}{T t_i} = \frac{r_p^2 S}{T t_p}$$

which reduces to

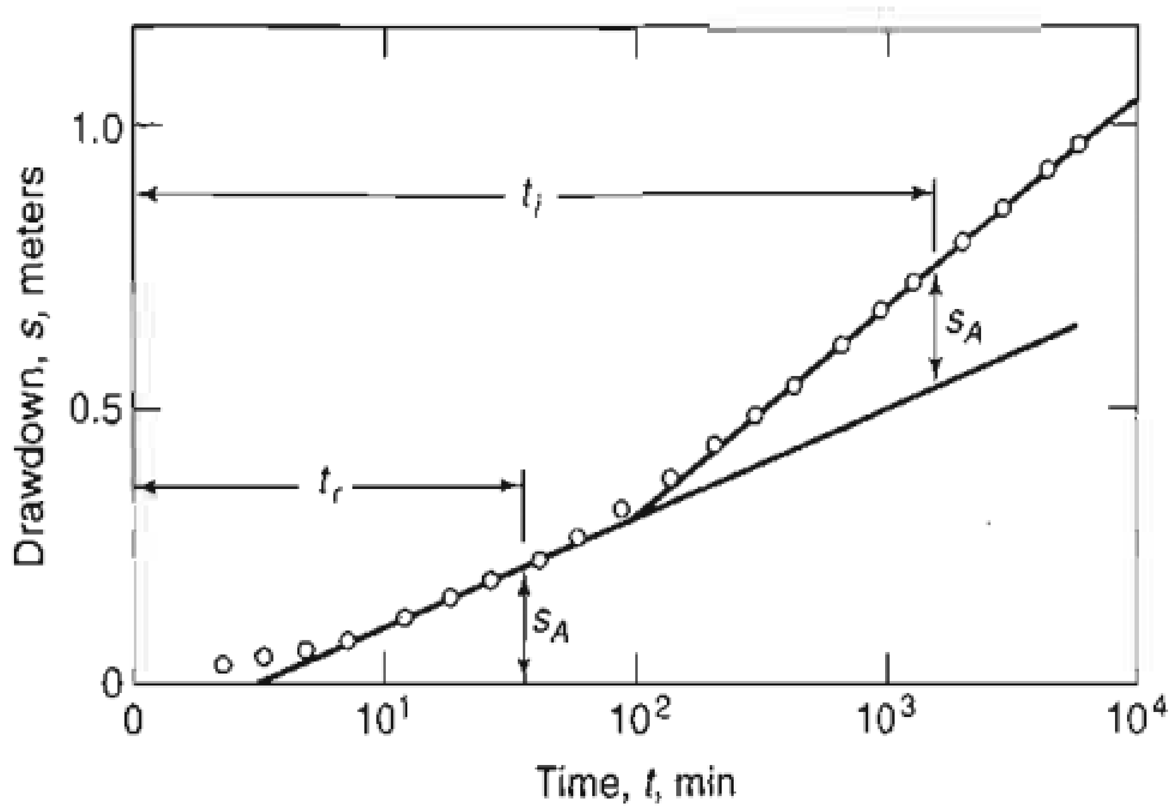
$$\frac{r_i^2}{t_i} = \frac{r_p^2}{t_p}$$

law of times

distance from an image well to an observation well

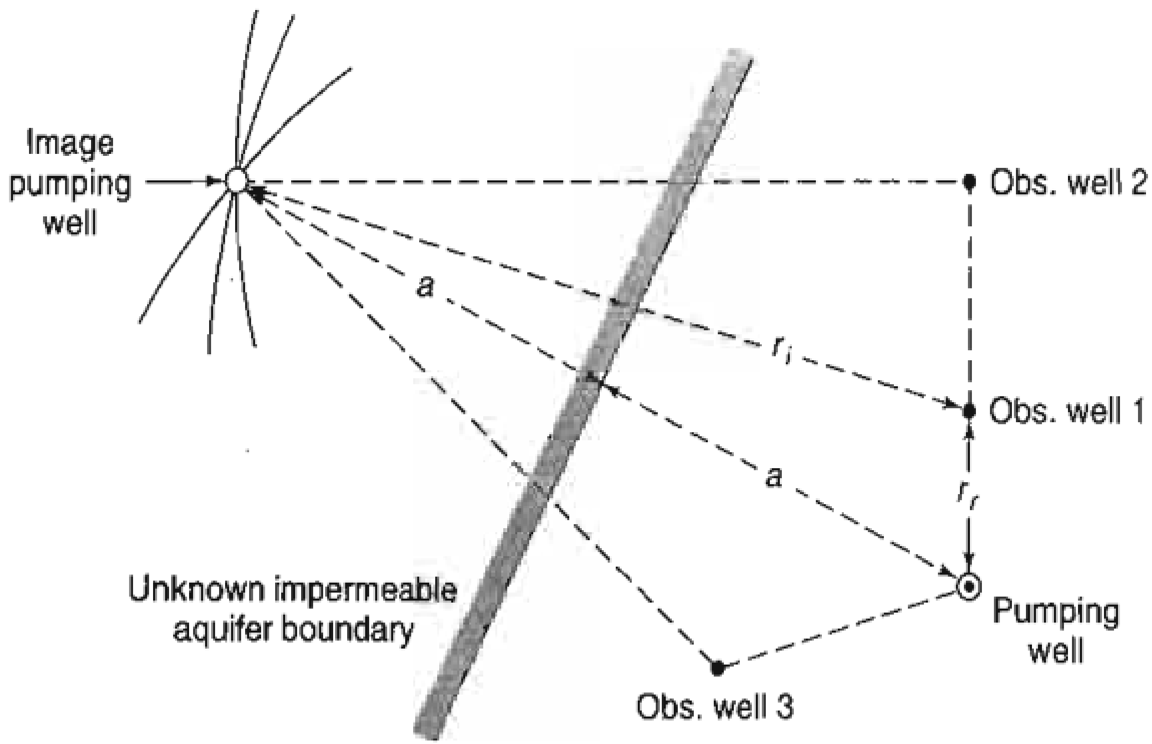
$$r_i = r_p \sqrt{\frac{t_i}{t_p}}$$

Well Flow Near an Impermeable Boundary (Contd.)

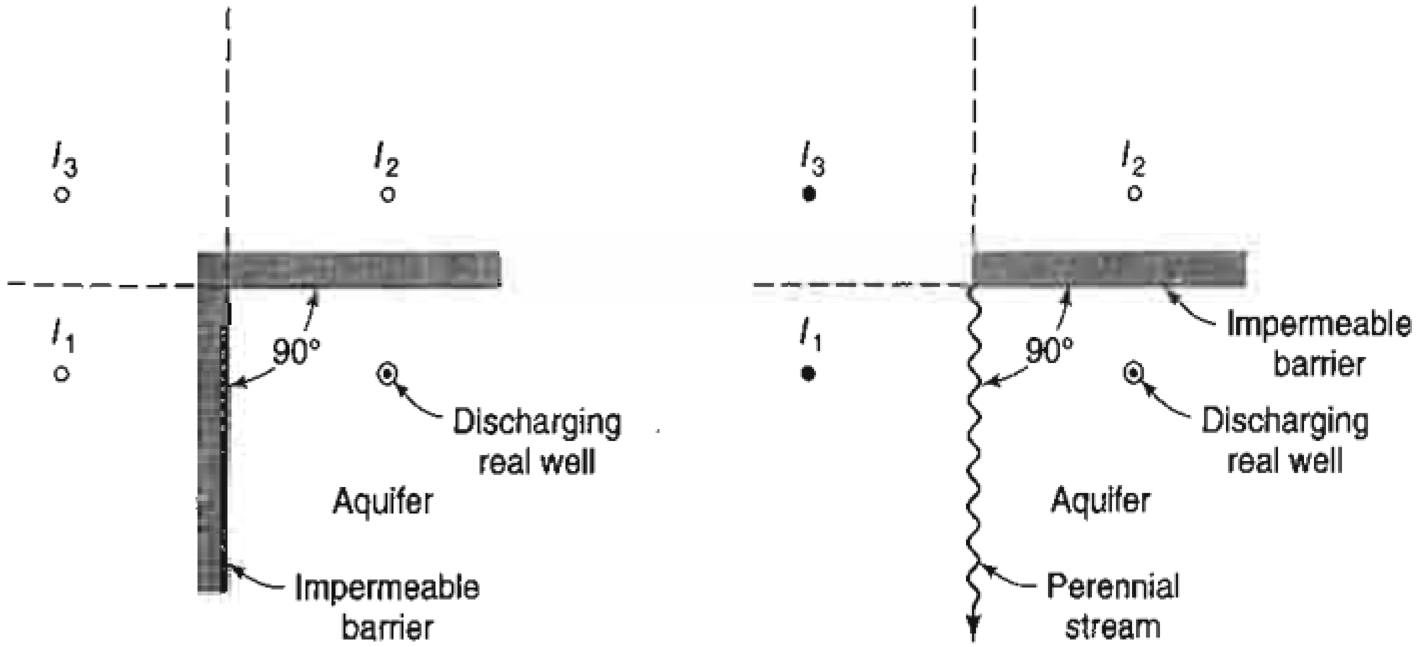


Cooper–Jacob drawdown curve showing effect of an impermeable boundary

Well Flow Near an Impermeable Boundary (Contd.)



Combined Effect of Stream and Impermeable Boundaries



Open circles are discharging image wells

filled circles are recharging image wells

Superposition with Multiple Pumping Rates

A well with effective radius of 0.3 m produces from an aquifer with $T = 300\ m^2/d$ and $S = 0.0006$. The well pumps at a rate of $550\ m^3/d$ for 30 days, and then at a rate of $800\ m^3/d$ for an additional 30 days. The well is then shut off. Calculate the residual drawdown after 80 days from the start of pumping (20 days after the well was shut off).

- **Basic Rule:** Once a well function has been "turned on," it cannot be turned off.

	Period 1	Period 2	Period 3
Time (days)	0-30	30-60	60-80
Q (m^3/d)	550	800	0
ΔQ (m^3/d)	550	250	-800
Duration t (days)	80	50	20

$$u_1 = \frac{r^2 S}{4Tt_1}$$
$$u_2 = \frac{r^2 S}{4Tt_2}$$
$$u_3 = \frac{r^2 S}{4Tt_3}$$

Superposition with Multiple Pumping Rates (Contd.)

	Period 1	Period 2	Period 3
Time (days)	0-30	30-60	60-80
Q (m^3/d)	550	800	0
ΔQ (m^3/d)	550	250	-800
Duration t (days)	80	50	20

$$s = \frac{Q_1}{4\pi T} W(u_1) + \frac{\Delta Q_{12}}{4\pi T} W(u_2) + \frac{\Delta Q_{23}}{4\pi T} W(u_3)$$

$$u_1 = \frac{r^2 S}{4Tt_1} = \frac{(0.3\ m)^2 \times 0.0006}{4 \times 300\ m^2/d \times 80d}$$

$$u_2 = \frac{r^2 S}{4Tt_2} = \frac{(0.3\ m)^2 \times 0.0006}{4 \times 300\ m^2/d \times 50d}$$

$$u_3 = \frac{r^2 S}{4Tt_3} = \frac{(0.3\ m)^2 \times 0.0006}{4 \times 300\ m^2/d \times 20d}$$

Thank you