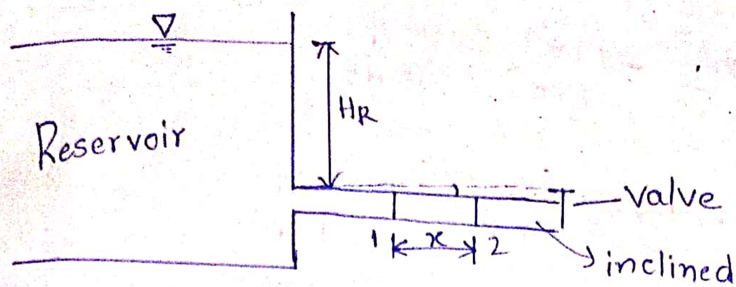
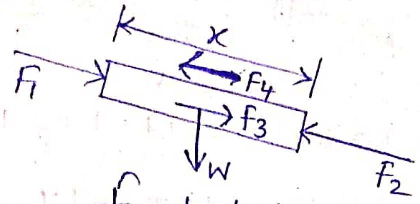
Lumped system modelling - Rigid column theory



L = Length
D = diameter



free body diagram

where $f_1 = \gamma(H_R - H_E - H_V)A = \gamma\left(H_R - H_E - \frac{V^2}{2g}\right)A$

$f_2 = \gamma H A$

F_3 = gravitational force component

$= \gamma A x S_0$

$f_4 = \gamma H_f A$

where H_f = frictional losses
H_E = Entrance losses

Applying the newton's Second Law of motion,

$$\text{Applied Forces} = ma = m\frac{dv}{dt}$$

$$= \frac{W}{g}\frac{dv}{dt}$$

$$F_1 - F_2 + F_3 - F_4 = \frac{\gamma A x}{g}\frac{dv}{dt}$$

$$\Rightarrow \gamma\left(H_R - H_E - \frac{V^2}{2g}\right)A - \gamma H A + \gamma A x S_0 - \gamma H_f A = \frac{\gamma A x}{g}\frac{dv}{dt}$$

$$\Rightarrow H_R - \frac{V^2}{2g} - K_e\frac{V^2}{2g} - H + x S_0 - H_f = \frac{x}{g}\frac{dv}{dt}$$

$$\Rightarrow H_R - \frac{V^2}{2g} - K_e\frac{V^2}{2g} - H + x S_0 - \frac{f x V^2}{2gD} = \frac{x}{g}\frac{dv}{dt}$$

~~After~~ including ~~valve head loss~~, when we write this equation for whole pipe System ie, @ $x = L$, $H = 0$ & valve head loss

$\frac{K_v V^2}{2g}$ is to be added. And the pipe is also horizontal

$$\Rightarrow H_R - \frac{V^2}{2g} - \frac{K_e V^2}{2g} - \frac{K_v V^2}{2g} - \frac{FLV^2}{2gD} = \frac{L}{g}\frac{dv}{dt}$$

$$H_R - \left(1 + K_e + K_v + \frac{FL}{D}\right)\frac{V^2}{2g} = \frac{L}{g}\frac{dv}{dt}$$

$$H_R - \frac{C_1 V^2}{2g} = \frac{L}{g}\frac{dv}{dt}$$

where $C_1 = 1 + K_e + K_v + \frac{FL}{D}$

Once the flow is Established, steady state is Achieved

$$\therefore \frac{L}{g}\frac{dv}{dt} = 0$$

$$H_R = \frac{C_1 V^2}{2g}$$

And $V_0 = \sqrt{\frac{2gH_R}{C_1}}$

To determine the discharge behaviour as a function of time during the Establishment time interval

$$\frac{L}{g}\frac{dv}{dt} = H_R - \frac{C_1 V^2}{2g}$$

$$\frac{L}{g}dv = \left(H_R - \frac{C_1 V^2}{2g}\right)dt$$

$$\frac{L}{g} \cdot \frac{1}{\left(H_R - \frac{C_1 V^2}{2g}\right)} dv = dt$$

Applying Integration,

$$\int_0^t dt = \frac{L}{g}\int_0^V \frac{dv}{H_R - \frac{C_1 V^2}{2g}}$$

$$t = 2L \int_0^V \frac{dv}{C_1 (V_0^2 - V^2)}$$

$$t = \frac{L}{V_0 C_1} \ln\left[\frac{V_0 + V}{V_0 - V}\right]$$

for ideal fluid, $C_1 = 1$

A horizontal pipe 61 cm in diameter 3048m long leaves a reservoir 30.48 m below its Surface through a valve. f is constant = 0.018

a). If the valve is suddenly opened completely, what is the time that is required to attain 99% of the steady state velocity, neglect frictional head losses & other losses

Sol:

$$C_1 = 1.0$$

$$\therefore V_0 = \sqrt{2gH_R} = \sqrt{2 \times 9.81 \times 30.48}$$

$$= 24.45 \text{ m/sec}$$

The time to reach 99% of V_0

$$t = \frac{L}{V_0} \ln\left[\frac{V_0 + V}{V_0 - V}\right]$$

$$= \frac{3048}{24.45} \ln\left[\frac{V_0 + 0.99V_0}{V_0 - 0.99V_0}\right]$$

$$= \frac{3048}{24.45} \ln[199]$$

$$= 660 s$$

b). When the pipe friction $f = 0.018$, $K_E = K_V = 0$

$$C_1 = 1 + 0 + 0 + \frac{0.018 \times 3048}{61 \times 10^{-2}}$$

$$= 91$$

$$V_0 = \sqrt{\frac{2gH_R}{C_1}} = \sqrt{\frac{2 \times 9.81 \times 30.48}{91}}$$

$$= 2.56 \text{ m/s}$$

$$t = \frac{L}{C_1 V_0} \ln\left[\frac{V_0 + V}{V_0 - V}\right]$$

$$= \frac{3048}{2.56 \times 91} \ln[199]$$

$= 69.28$ Sec

c). when pipe Friction $F = 0.018$, $K_e = 0.5$ and $K_v = 50$

$$C_1 = 1 + K_e + K_v + \frac{FL}{D}$$

$$= 1 + 0.5 + 50 + \left(\frac{0.018 \times 3048}{61 \times 10^{-2}}\right)$$

$$= 96.5$$

$$V_0 = \sqrt{\frac{2 \times 9.81 \times 3048}{96.5}}$$

$$= 2.49 \text{ m/sec}$$

$$t = \frac{3048}{(2.49)(96.5)} \ln[199]$$

$$= 67.2 \text{ Sec}$$

prob: 2

The reservoir head on the pipeline is 18.29m. The 30.48cm diameter Pipeline is 914.4m long with an equivalent roughness $K_s = 0.0003$m Since the valve has been fully opened for a long time, the flow of water is steady

a). Calculate the steady-state velocity in the pipeline assuming there is no loss at the valve & $K_e = 0.5$

b). Compute the maximum pressure in the pipeline if the valve closes So that the rate of decrease in velocity is linear in time from its steady state value to Zero in 20 Seconds.

Sol:

Steady state i.e., $\frac{dv}{dt} = 0$

There is no loss in the valve i.e., $K_v = 0$

$$H_R = \left(1 + K_e + K_v + \frac{FL}{D}\right)\frac{v^2}{2g} \rightarrow ①$$

Here F is unknown & V_0 is unknown

Haaland Equation,

$$\frac{1}{\sqrt{F}} = 1.8 \log_{10}\left[\left(\frac{K_s/D}{3.7}\right)^{1.11} + \frac{6.9}{Re}\right] \rightarrow ②$$

$$Re = \frac{VD}{\nu}$$

Substitute (2) in (1) & Solving iteratively

After Assuming $f = 0$ in eq(1) initially,

$$f = 0.0201, \quad V_0 = 2.41\, m/sec$$

b). When deceleration is constant

$$a = \frac{L}{g}\frac{dv}{dt} = \frac{914.4}{9.81} \cdot \frac{(-2.41)}{20}$$

$$= -11.23\, m/s^2$$

$$\Rightarrow \frac{2L}{C} = \frac{2 \times 914.4}{1250} = 1.46\, sec$$

$\therefore 1.46 < 20 sec$

$\Rightarrow$ Slow valve closure

$$\frac{L}{g}\frac{dv}{dt} = H_R - \left(1 + K_e + \frac{fL}{D}\right)\frac{v^2}{2g} - \frac{P_2}{\gamma}$$

$$-11.23 = 18.29 - \left(1 + 0.5 + \frac{f(914.4)}{0.3048}\right)\frac{v^2}{2g} - H_2$$

$$29.52 = \left(1.5 + \frac{f(914.4)}{0.3048}\right)\frac{v^2}{2g} - H_2$$

$$H_2 = 29.52 - \left(1.5 + \frac{f(914.4)}{0.3048}\right)\frac{v^2}{2g}$$

as V decreases, H_2 increases

when flow completely stops, $V = 0$

$$\therefore H_2 = 29.52\, m$$