

Water in Subsurface Environment

Groundwater Engineering| CE60205

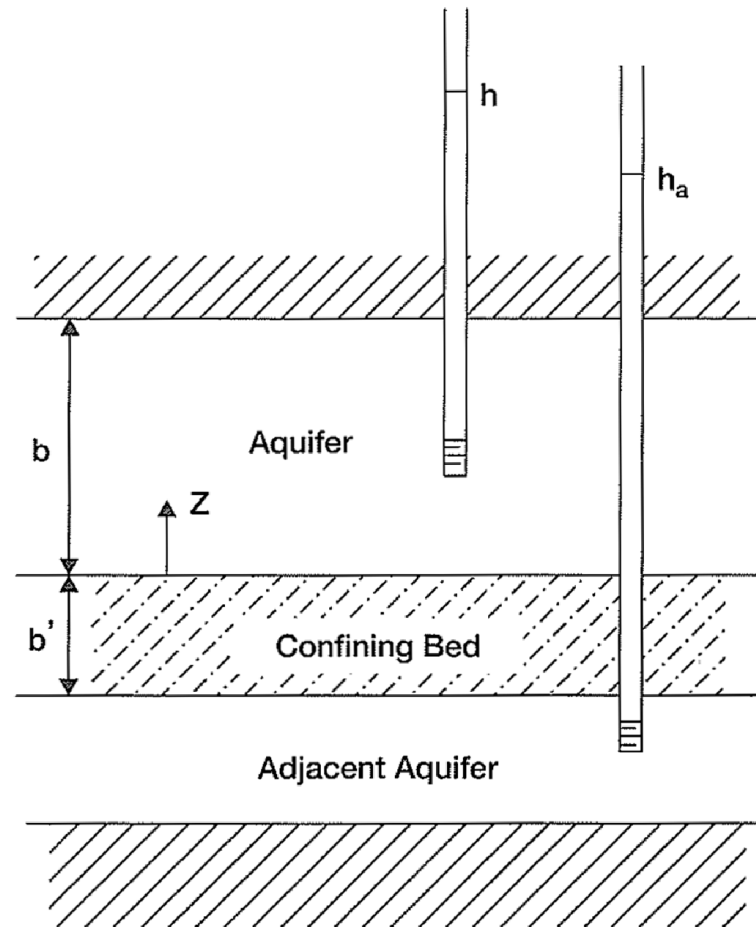
Lecture:05

Learning Objective(s)

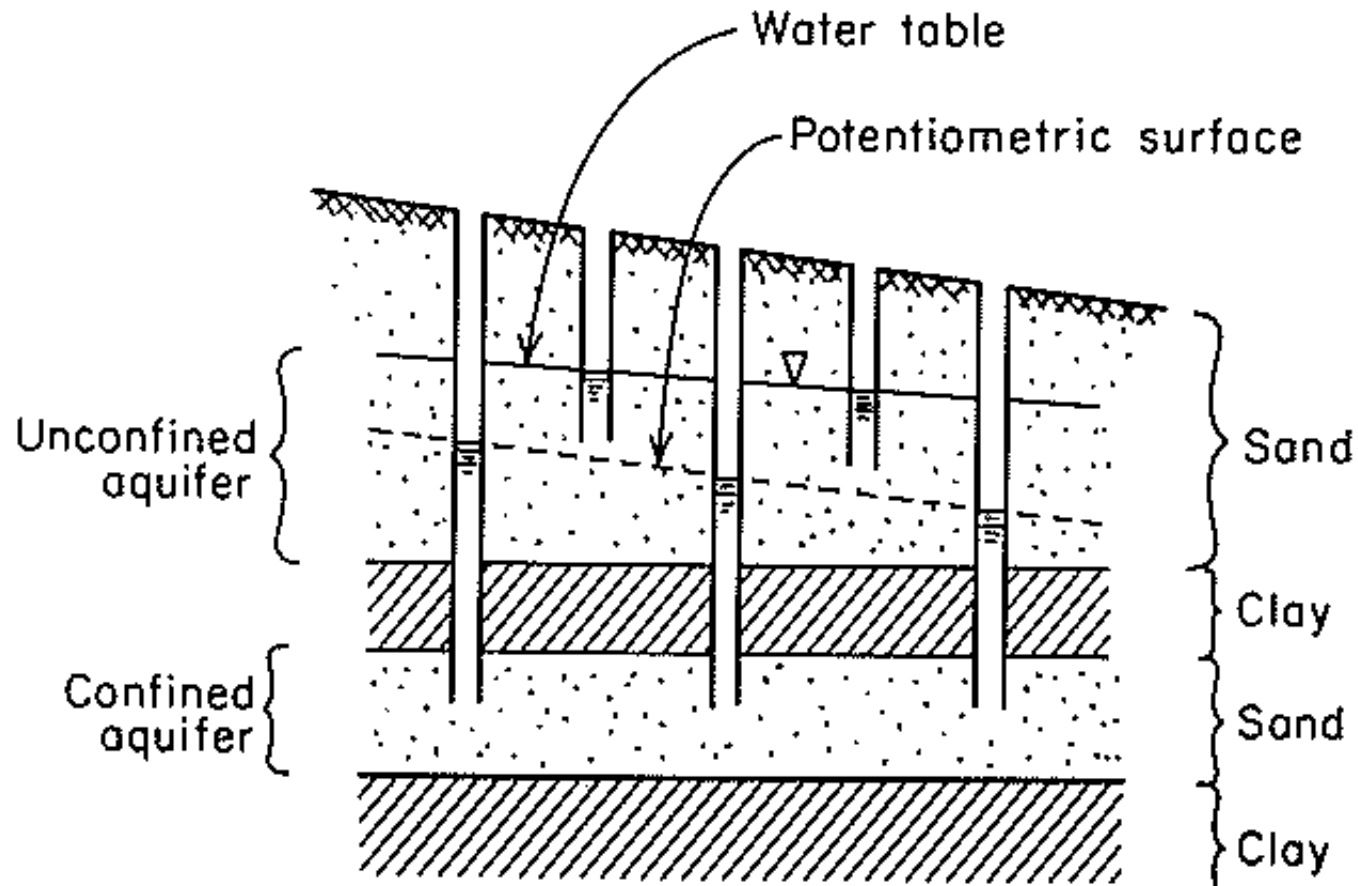
- To calculate storage coefficient, specific yield, specific retention.

Two Confined Aquifers with Different Heads

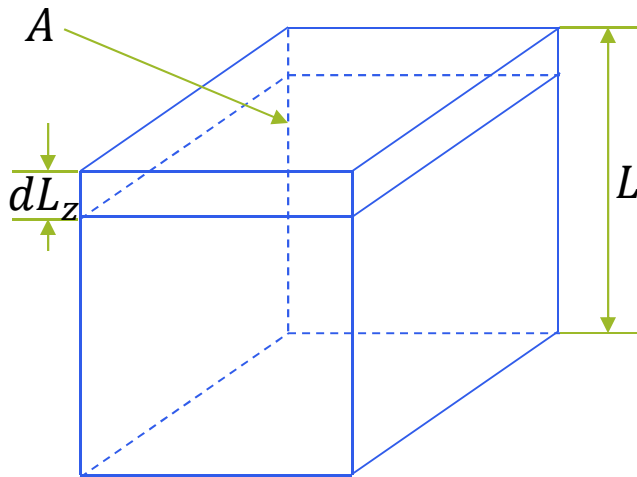
- Groundwater will tend to flow from the top aquifer to the bottom aquifer.
- We can't make any conclusion about horizontal head gradients from this picture.



Horizontal and Vertical Head Gradients



Specific storage



- Total mass of water contained within the control volume

$$V_T = AL_z$$

$$V_s = (1 - \eta)AL_z$$

$$V_v = \eta AL_z$$

$$M = \rho_w \eta AL_z$$

- If A remains unchanged

$$dM = \eta AL_z d\rho_w + \rho_w AL_z d\eta + \rho_w \eta A dL_z$$

- Under the assumption $dV_s = 0$

$$dV_s = (1 - \eta)A dL_z - AL_z d\eta = 0$$

- Replacing $L_z d\eta = (1 - \eta)dL_z$

$$dM = \eta AL_z d\rho_w + \rho_w A(1 - \eta)dL_z + \rho_w \eta A dL_z$$

or

$$dM = \eta AL_z d\rho_w + \rho_w A dL_z$$

or

$$dM = V_v d\rho_w + \rho_w dV_T = \eta V_T d\rho_w + \rho_w dV_T = \rho_w V_T (\eta \beta + \alpha) dp$$

$$\beta = -\frac{1}{V_w} \frac{dV_w}{dp} = \frac{1}{\rho_w} \frac{d\rho_w}{dp}$$

$$\alpha = \frac{1}{V_T} \frac{dV_T}{dp}$$

$$dp = \rho_w g dh$$

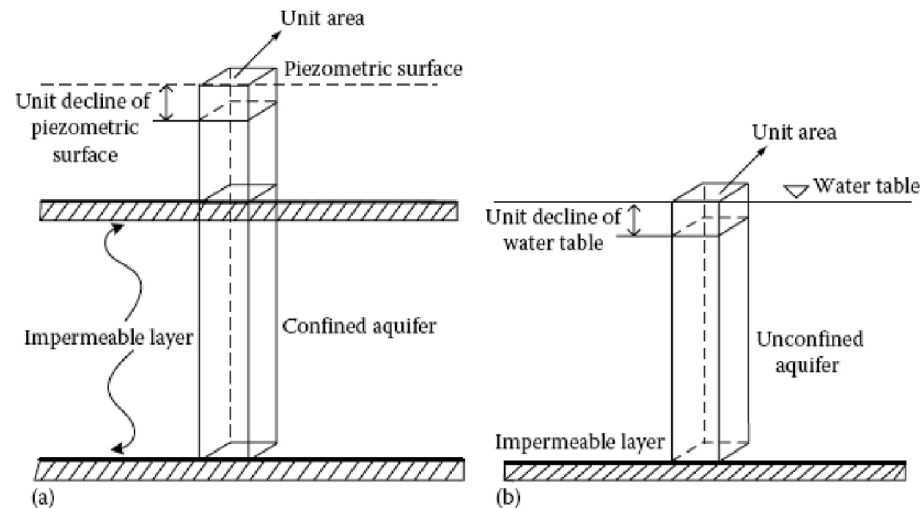
$$V_w = \frac{dM}{\rho_w} = V_T (\eta \beta + \alpha) \rho_w g dh$$

Storage Coefficient

$$S_s = \frac{1}{V_T} \frac{dV_w}{dh} = (\eta\beta + \alpha)\rho g$$

$$S = S_s b = \frac{1}{A} \frac{dV_w}{dh}$$

- Common range of the storage coefficient in confined aquifers is from 0.00005 to 0.005



- Another way to calculate the specific storage coefficient

$$S_s = \gamma_w \left[(1-n)\alpha + n\beta \right]$$

Storage Coefficient (Contd.)

• Using chain rule of differentiation $\frac{\partial (\varepsilon\rho)}{\partial t} = \varepsilon \frac{\partial \rho}{\partial t} + \rho \frac{\partial \varepsilon}{\partial t}$

$$\frac{\partial \varepsilon}{\partial t} = \frac{d\varepsilon}{dp} \frac{\partial p}{\partial t}$$

$$= C_v \frac{\partial p}{\partial t},$$

C_v is related to the classical *coefficient of consolidation*

$$d\varepsilon = d\left(\frac{V_v}{V_T}\right) = \frac{V_T dV_v - V_v dV_T}{V_T^2} \approx \frac{V_s}{V_T^2} dV_v$$

$$\frac{d\varepsilon}{dp} = \frac{V_s}{V_T^2} \frac{dV_v}{dp} = \frac{V_s}{V_T} \alpha = (1 - \varepsilon)\alpha$$

$$\alpha = \frac{1}{V_T} \frac{dV_T}{dp} \approx \frac{1}{V_T} \frac{dV_v}{dp}$$

$$\beta \equiv \frac{1}{\rho} \frac{d\rho}{dp}$$

$$\frac{\partial (\varepsilon\rho)}{\partial t} = \varepsilon \frac{\partial \rho}{\partial t} + \rho \frac{\partial \varepsilon}{\partial t}$$

$$= \varepsilon \frac{d\rho}{dp} \frac{\partial p}{\partial t} + \rho \frac{d\varepsilon}{dp} \frac{\partial p}{\partial t}$$

$$= \rho (\varepsilon\beta + C_v) \frac{\partial p}{\partial t}.$$

$$\frac{d(\varepsilon\rho)}{dp} = \varepsilon \frac{d\rho}{dp} + \rho \frac{d\varepsilon}{dp}$$

$$= \varepsilon \frac{d\rho}{dp} \frac{dp}{dt} + \rho \frac{d\varepsilon}{dp} \frac{dp}{dt}$$

$$= \rho [\varepsilon\beta + (1 - \varepsilon)\alpha] \frac{dp}{dt}$$

$$S_s = \gamma_w [(1 - n)\alpha + n\beta]$$

Specific Yield

- Specific yield is the volume of water that is released from storage in an unconfined aquifer per unit surface area of the aquifer per unit decline in the water table.

$$S_y = \frac{1}{A} \frac{dV_w}{dh}$$

- The ratio of the volume of water discharged from the sample, considering the gravity, over total volume of the sample is its specific yield.

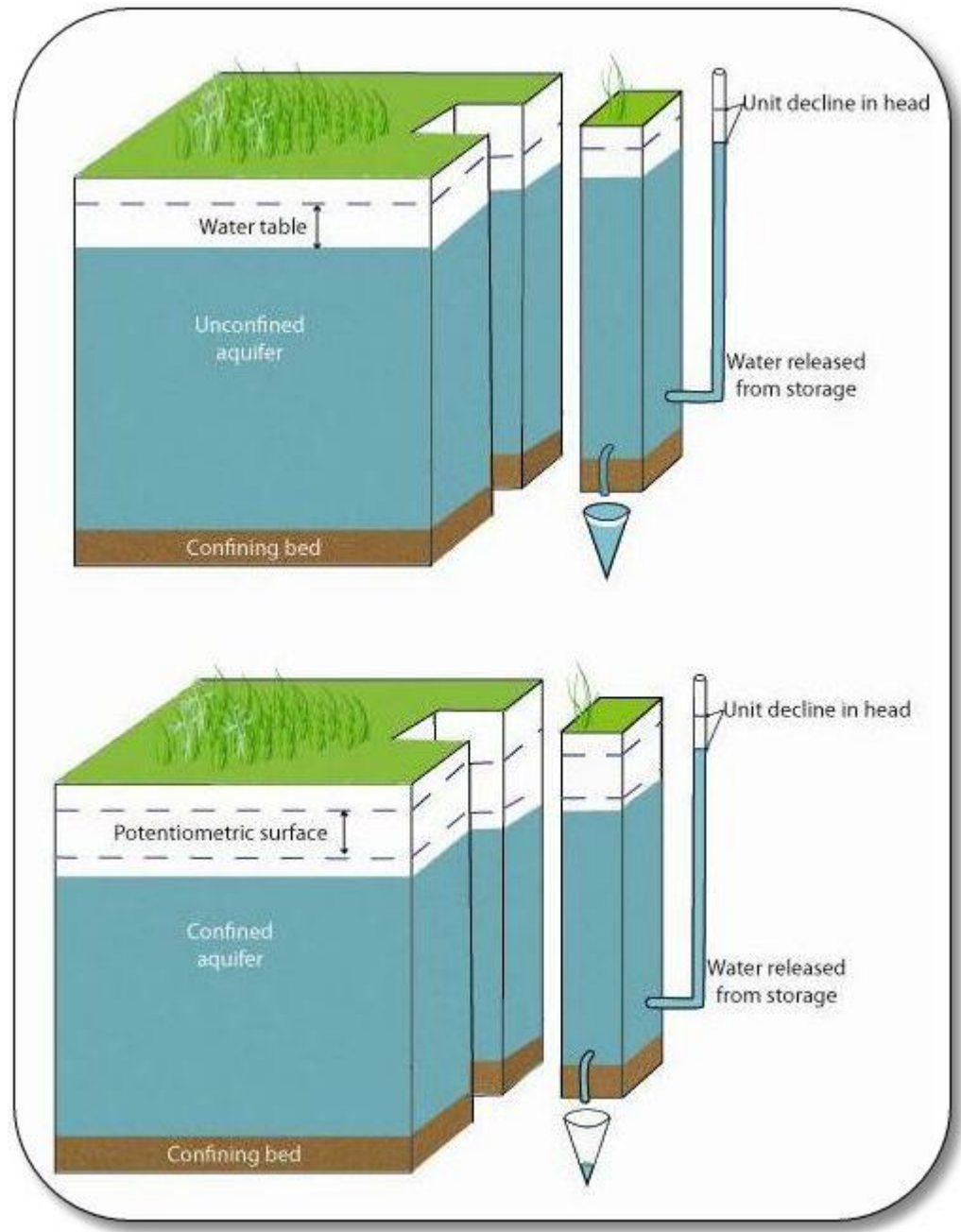
$$S_y = \frac{\text{Gravity Drainage Volume}}{\text{Bulk Volume}}$$

- For unconfined aquifers, the concept of specific yield could be expressed as the difference between storage coefficient and specific storage times the aquifer thickness.

$$S = S_y + bS_s$$

- The storage coefficient for confined aquifers is usually equal to or less than 0.005.
- The storage coefficient for unconfined aquifers is almost between 0.2 and 0.3.

Water Yield



Specific Retention

- Specific retention:

$$S_r = \eta - \eta_e$$

- Effective Porosity

$$\eta_e = \frac{\text{Volume of water able to circulate}}{\text{Total volume of rock}}$$

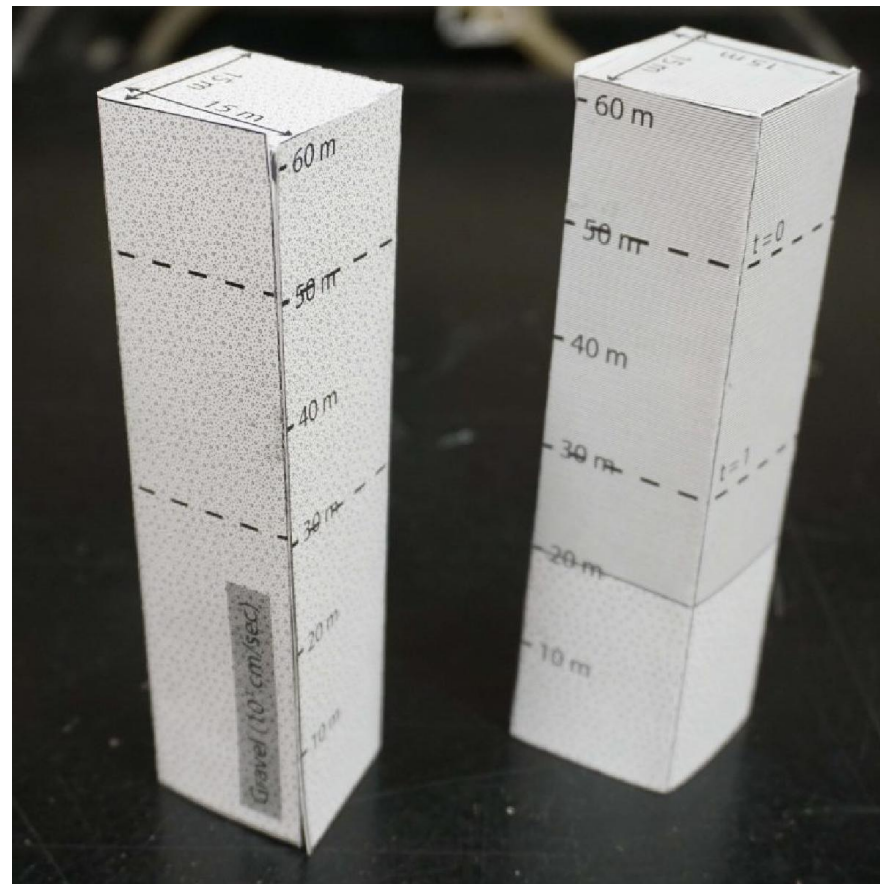
- Clay has a good retention and low yield

$$\eta = S_y + S_r$$

- Specific retention -> Field Capacity

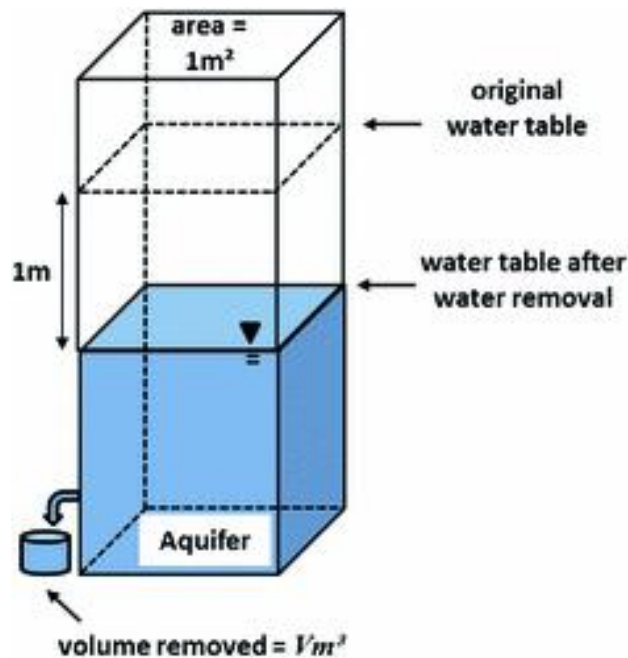
Home Lab

- Foldable Aquifer Project -<http://aquifer.geology.buffalo.edu/>
- Paper aquifer model
 - Groundwater Storage

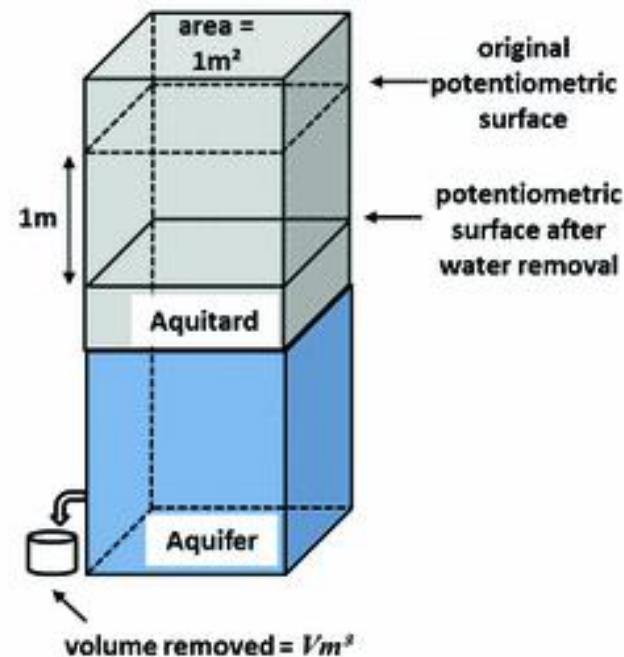


Home Lab (Contd.)

- Using the foldable aquifer models given below answer the following questions assuming that the volume of water released in each of the aquifers is 945 m^3 .
 - A. Quantify the specific yield in the unconfined aquifer.
 - B. Quantify the specific storage in the confined aquifer.
 - C. Describe the difference between the storage mechanism in the unconfined aquifer as compared to the confined aquifer.



(a) Specific yield



(b) Storativity

Problem 1

- Determine the total stress acting on the top of the aquifer underlying a clayey aquitard, which is 30 m thick, and calculate the effective stress in the aquifer. Assume the bulk density of the aquitard is 2500 kg/m³ and there is an average of 40 m pressure head in it. If pumping causes a reduction of 1.5 m in the hydraulic head in the aquifer, what will be the resulting changes in the pore water pressure and effective stress. Consider the density of water equal to 1000 kg/m³.

$$\sigma_T = \rho_b g b = 2500 \times 9.81 \times 30 = 735750 \text{ N/m}^2$$

$$p = \rho_w g h = 1000 \times 9.81 \times 40 = 392400 \text{ N/m}^2$$

$$\sigma_e = \sigma_T - p = 343350 \text{ N/m}^2$$

$$\Delta p = \rho_w g \Delta h = 1000 \times 9.81 \times (-1.5) = -14715 \text{ N/m}^2$$

$$\Delta \sigma_e = \Delta \sigma_T - \Delta p = -\Delta p = 14715 \text{ N/m}^2$$

Problem 2

- Given a porosity of 0.22 and a dry bulk density of 1.95 g/cm³, what is the particle density?

$$\rho_d = \rho_s = \frac{m_s}{V_s} = \frac{m_s}{V} \times \frac{V}{V_s} = \frac{\rho_b}{(1 - \eta)} = \frac{1.95}{(1 - 0.22)} = 2.5 \text{ g/cm}^3$$

Problem 3

- An aquifer with an area of 7 km² experiences a head drop of 0.85 m after 8 years of pumping. If the pumping rate is 5.5 m³/day, determine the specific yield of the aquifer.

$$S_y = \frac{V_w}{A\Delta h} = \frac{Q\Delta t}{A\Delta h} = \frac{5.5 \frac{m^3}{day} \times 8 \text{ year} \times 365 \frac{day}{year}}{7 \text{ km}^2 \times 10^6 \frac{m^2}{km^2} \times 0.85 \text{ m}} = 2.7 \times 10^{-3}$$

Problem 4

- A soil sample has a volume of 180 cm³. The volume of voids in the sample is estimated equal to 67 cm³. Out of the volume of voids, water can move through only 45 cm³. Determine the porosity, effective porosity, specific retention, and specific yield of the soil. What is the area of the aquifer, if pumping at a rate 6 m³/day causes 1 m head drop in the aquifer in 5 years?

$$\eta = \frac{V_v}{V} = \frac{67}{180} = 0.37$$
$$\eta_e = \frac{V_{cv}}{V} = \frac{45}{180} = 0.25$$

$$S_r = \eta - \eta_e = 0.12$$

$$S_y = \eta - S_r = \eta_e = 0.25$$

$$A = \frac{V_w}{S_y \Delta h} = \frac{Q \Delta t}{A \Delta h} = \frac{6.0 \frac{m^3}{day} \times 5 \text{ year} \times 365 \frac{day}{year}}{0.25 \times 1.00 \text{ m}} = 43800 \text{ m}^2$$

Problem 5

- The thickness of the saturated layer in a field is 8.2 m. The porosity of the soil was measured equal to 0.28. The elasticity modulus soil and water are $5.3 \times 10^7 \text{ N/m}^2$ and $2.1 \times 10^9 \text{ N/m}^2$, respectively. Determine the corresponding storage coefficient for this layer. Assume the temperature is 20°C .

$$\beta = \frac{1}{E_w} = 4.76 \times 10^{-10} \text{ m}^2/\text{N}$$

$$\alpha = \frac{1}{E_s} = 1.89 \times 10^{-8} \text{ m}^2/\text{N}$$

$$\begin{aligned} S = S_s b &= (\eta\beta + \alpha)\rho g b = (0.28 \times 4.76 \times 10^{-10} + 1.89 \times 10^{-8}) \times 1000 \times 9.81 \times 8.2 \\ &= 1.53 \times 10^{-3} \end{aligned}$$

Problem 6

- The porosity of a confined aquifer is 42%. The thickness of this aquifer is 78 m. The compressibility of water and the granular matrix are 4.76×10^{-10} and 6.3×10^{-8} , respectively. Estimate the specific storage and storativity of the aquifer. Also, calculate how much water is released if there is a total head drop of 100 m and the area is 1200 km².

$$S = S_s b = (\eta\beta + \alpha)\rho g b = (0.42 \times 4.76 \times 10^{-10} + 6.3 \times 10^{-8}) \times 1000 \times 9.81 \times 78 \\ = 4.84 \times 10^{-2}$$

$$V_w = S\Delta h A = 4.84 \times 10^{-2} \times 100 \times 1200 \text{ km}^2 \times 10^6 \text{ m}^2/\text{km}^2 = 5.81 \times 10^9 \text{ m}^3$$

Problem 7

- The total volume of a soil sample is 280 cm³. The volume of water and volume of voids were measured equal to 65 and 115 cm³, respectively. Determine the porosity, volumetric water content, and Degree of saturation of water for the current sample.

$$\eta = \frac{V_v}{V} = \frac{115}{280} = 0.41$$

$$\theta_v = \frac{V_w}{V} = \frac{65}{280} = 0.23$$

$$S_w = \frac{V_w}{V_v} = \frac{\theta_v}{\eta} = 0.56$$

Learning Strategy

Chapter 6: Deformation, Storage, and General
Flow Equations



Thank you