Parameter Estimation

Geohydraulics| CE60113

Lecture:13

Learning Objective(s)

- **To estimate aquifer parameter under steady unconfined flow condition**
- **To estimate aquifer parameter under steady leaky confined flow condition**

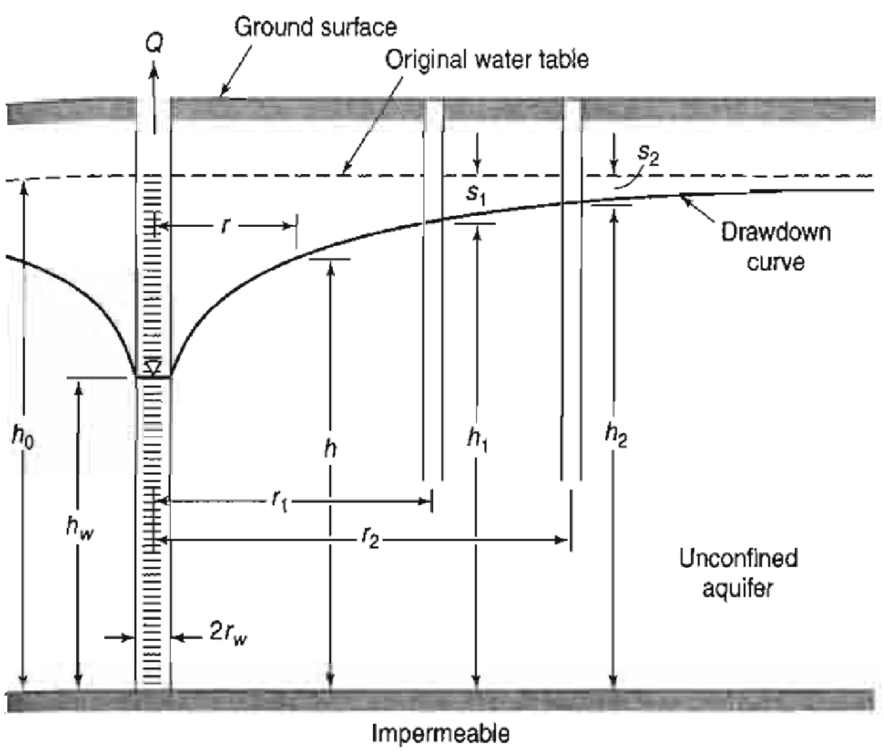
Basic Aquifer Types

- Confined Aquifer
- Unconfined Aquifer
- Semiconfined/Leaky Aquifer

Steady Radial Flow in Unconfined Aquifer

- Well discharge can be written as

$$Q = Aq_r = (2\pi r H)\left(-K_r \frac{\partial H}{\partial r}\right) = -2\pi r K_r H \frac{\partial H}{\partial r}$$



- Head difference is

$$H_0^2 - H_w^2 = \frac{Q}{\pi K_r} ln\left(\frac{r_0}{r_w}\right)$$

- Discharge can be written as

$$Q = \pi K_r \frac{H_0^2 - H_w^2}{ln\left(\frac{r_0}{r_w}\right)}$$

- Hydraulic conductivity

$$K_r = \frac{Q}{\pi(H_2^2 - H_1^2)} ln\left(\frac{r_2}{r_1}\right)$$

Steady Radial Flow in Unconfined Aquifer (Contd.)

$$\cancelto{0}{\frac{2S_y}{K}\frac{\partial H}{\partial t}} = \nabla^2 H^2 + \cancelto{0}{\frac{2N}{K}}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial H^2}{\partial r}\right) = 0$$

Solution

$$H^2(r) = Aln(r) + B$$

$$Q = -2\pi r H q_r(r) = 2\pi r K_r H \frac{\partial H}{\partial r} = \pi K_r A$$

$$H^2(r) = \frac{Q}{\pi K_r} ln(r) + B$$

Radius of influence: $H = H_0\ at\ r = r_0$

$$H^2(r) = H_0^2 - \frac{Q}{\pi K_r} ln\left(\frac{r_0}{r}\right)$$

Steady Radial Flow in Unconfined Aquifer (Contd.)

- Let us consider $s_1 = H_0 - H_1$ and $s_2 = H_0 - H_2$

$$H_1^2(r) = (H_0 - s_1)^2 = \frac{Q}{\pi K_r} ln(r_1) + B$$
$$H_2^2(r) = (H_0 - s_2)^2 = \frac{Q}{\pi K_r} ln(r_2) + B$$

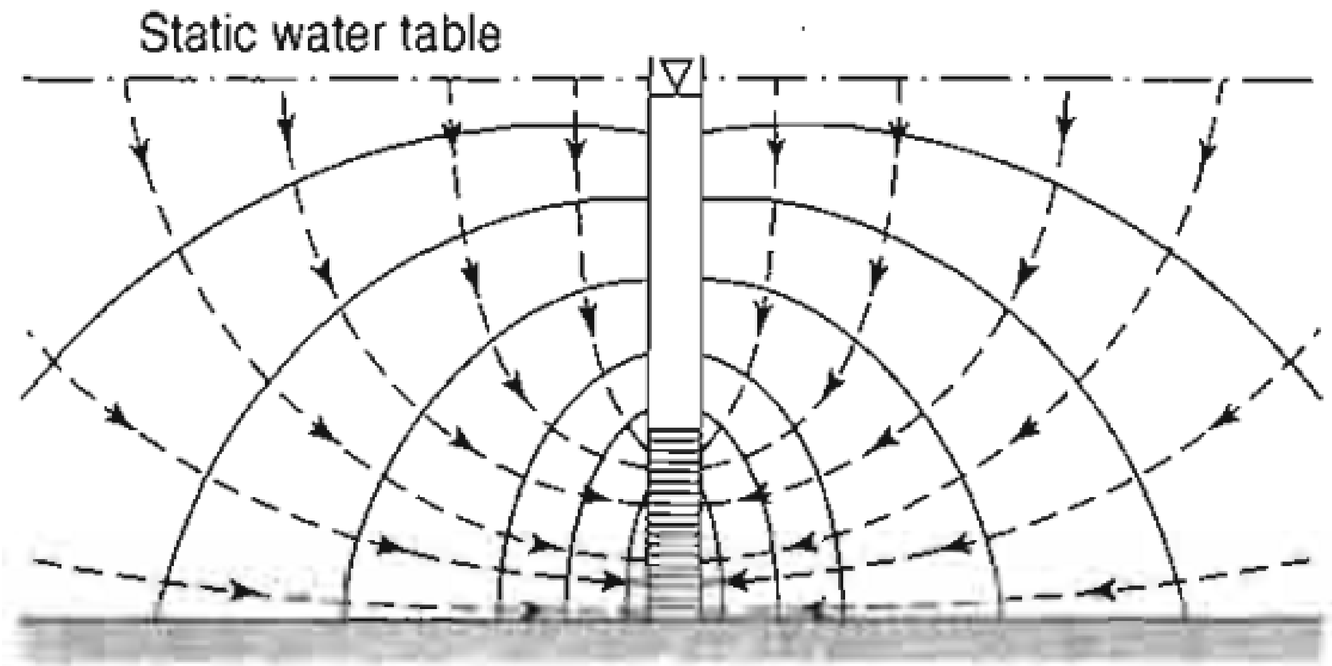
Subtracting the expressions

$$2H_0\left[\left(s_1 - \frac{s_1^2}{2H_0}\right) - \left(s_2 - \frac{s_2^2}{2H_0}\right)\right] = \frac{Q}{\pi K_r} ln\left(\frac{r_2}{r_1}\right)$$

Transmissivity for the full thickness

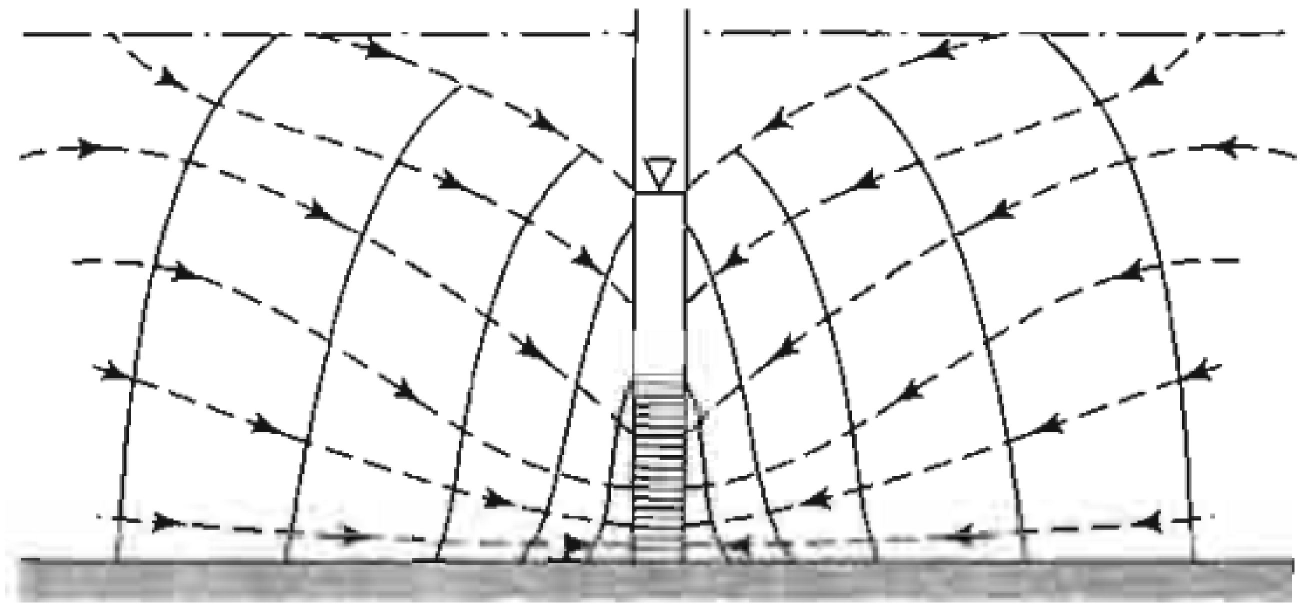
$$T = K_r H_0 = \frac{Q}{2\pi\left[\left(s_1 - \frac{s_1^2}{2H_0}\right) - \left(s_2 - \frac{s_2^2}{2H_0}\right)\right]} ln\left(\frac{r_2}{r_1}\right)$$

Steady Radial Flow in Unconfined Aquifer (Contd.)



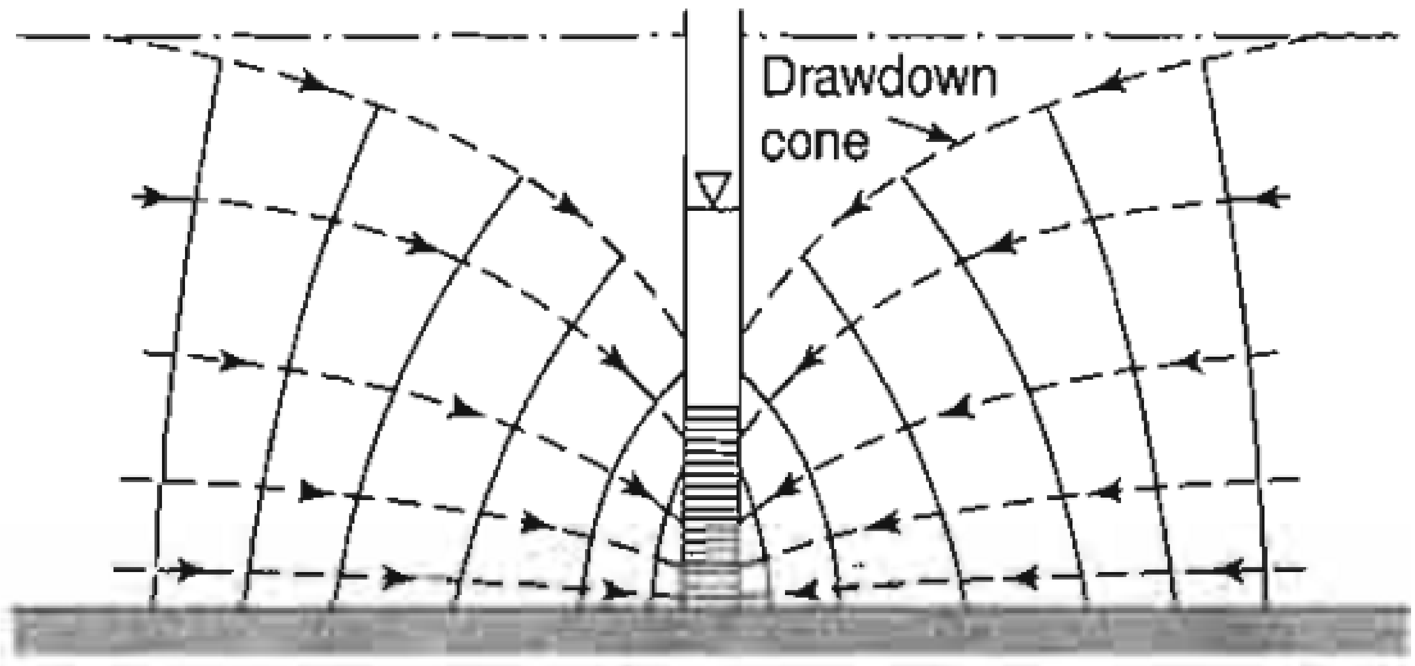
(a) Initial stage in pumping a free aquifer. Most water follows a path with a high vertical component from the water table to the screen.

Steady Radial Flow in Unconfined Aquifer (Contd.)



(*b*) Intermediate stage in pumping a free aquifer. Radial component of flow becomes more pronounced but contribution from drawdown cone in immediate vicinity of well is still important.

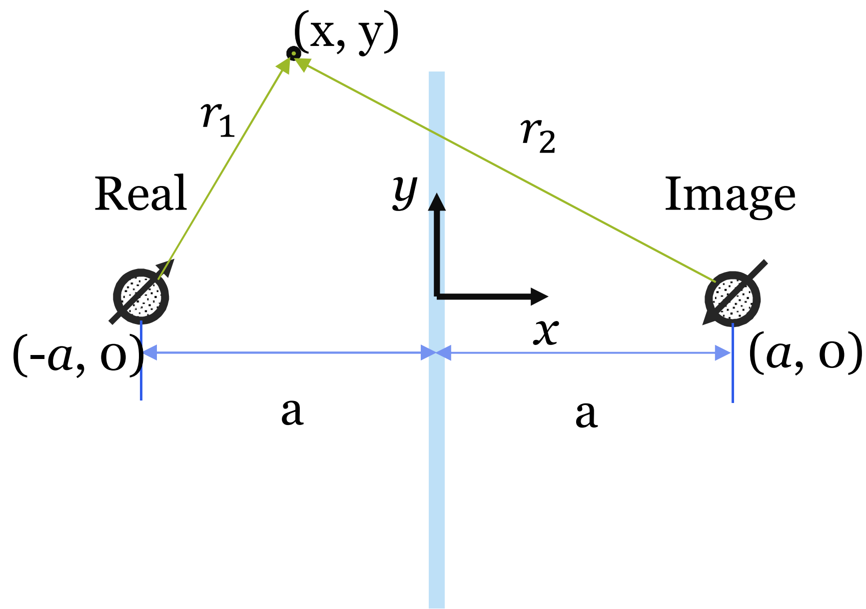
Steady Radial Flow in Unconfined Aquifer (Contd.)



(c) Approximate steady-state stage in pumping a free aquifer. Profile of cone of depression is established. Nearly all water originating near outer edge of area of influence and stable primarily radial flow pattern established.

Steady Radial Flow in Unconfined Aquifer (Contd.)

- River or surface water body (constant head)



$$H_0^2 - H^2(r) = \frac{Q}{\pi K_r} ln\left(\frac{r_0}{r_1}\right) - \frac{Q}{\pi K_r} ln\left(\frac{r_0}{r_2}\right)$$
$$= \frac{Q}{\pi K_r} ln\left(\frac{r_2}{r_1}\right)$$
$$r_1 = \sqrt{(x+a)^2 + y^2}$$
$$r_2 = \sqrt{(x-a)^2 + y^2}$$

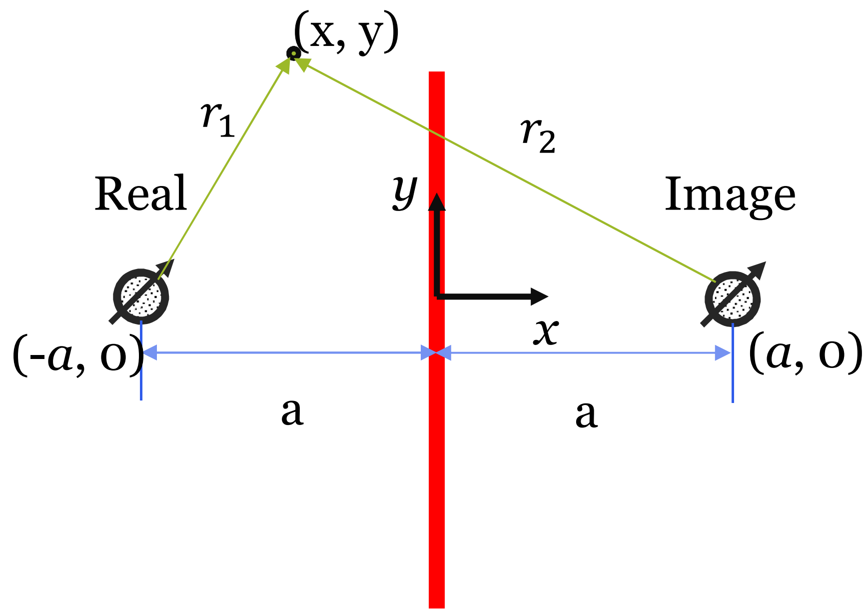
Along y-axis, $r_1 = r_2$

$$H^2 = H_0^2$$

Steady Radial Flow in Unconfined Aquifer (Contd.)

- Impermeable faults (no-flow boundaries)

$$r_1 = \sqrt{(x+a)^2 + y^2}$$
$$r_2 = \sqrt{(x-a)^2 + y^2}$$
$$H^2(r) = \frac{Q}{\pi K_r} ln(r_1) + \frac{Q}{\pi K_r} ln(r_2) + B$$
$$= \frac{Q}{\pi K_r} ln(r_1 r_2) + B$$



x-component of flow

$$U_x = Hq_x = -KH\frac{\partial H}{\partial x} = -\frac{K}{2}\frac{\partial H^2}{\partial x}$$
$$= -\frac{Q}{2\pi}\left(\frac{d}{dr_1}(\ln(r_1))\frac{\partial r_1}{\partial x} + \frac{d}{dr_2}(\ln(r_2))\frac{\partial r_2}{\partial x}\right)$$
$$= -\frac{Q}{2\pi}\left(\frac{1}{r_1}\frac{x+a}{r_1} + \frac{1}{r_2}\frac{x-a}{r_2}\right)$$
$$= -\frac{Q}{2\pi}\left(\frac{x+a}{(x+a)^2+y^2} + \frac{x-a}{(x-a)^2+y^2}\right)$$

Along x=0, the aquifer flux

$$U_x(0,y) = -\frac{Q}{2\pi}\left(\frac{0+a}{(0+a)^2+y^2} + \frac{0-a}{(0-a)^2+y^2}\right)$$

Steady Radial Flow in Leaky Confined Aquifer

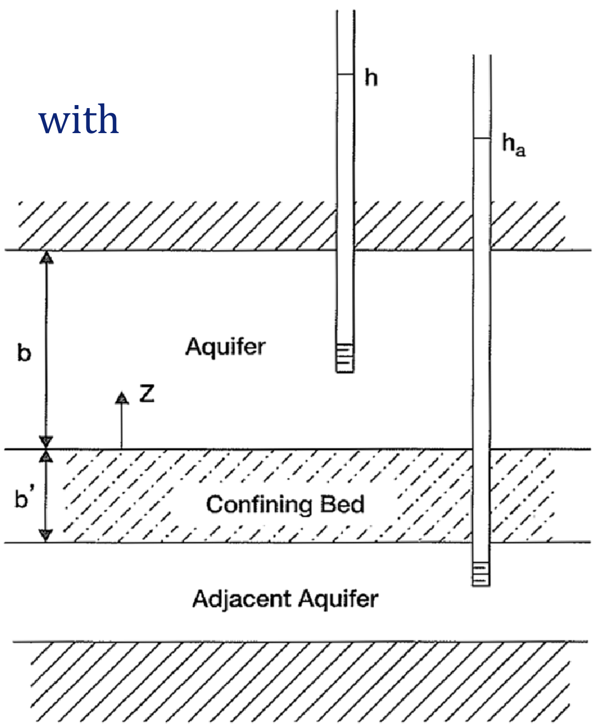
$$S\frac{\partial\bar{h}}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left(T_r\frac{\partial\bar{h}}{\partial r}\right)-\frac{K'}{b'}(\bar{h}-\bar{h}_a)$$

Under steady state radial flow condition in semiconfined aquifer

$$T\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial h}{\partial r}\right)-\frac{K'}{b'}(h-h_a)=0$$

$$\frac{d^2h}{dr^2}+\frac{1}{r}\frac{dh}{dr}-\frac{h}{B^2}=-\frac{h_a}{B^2}$$

with



$$B=\sqrt{\frac{Tb'}{K'}}$$

- Bessel's modified differential equation of order zero

$$Q=-2\pi rbq_r(r)=2\pi rK_rb\frac{\partial h}{\partial r}$$

Solution

$$s(r)=h_a-h(r)=\frac{Q}{2\pi T}K_0\left(\frac{r}{B}\right)$$

$K_0\left(\frac{r}{B}\right)$ is modified Bessel function of the second kind of order zero

Steady Radial Flow in Leaky Confined Aquifer (Contd.)

- Transmissivity

$$T = \frac{Q}{2\pi s} K_0\left(\frac{r}{B}\right) = \frac{Q}{2\pi s} K_0\left(\frac{r}{\sqrt{T\frac{b'}{K'}}}\right)$$

- Nonlinear function can be written as

$$F(T) = T - \frac{Q}{2\pi s} K_0\left(\frac{r}{\sqrt{T\frac{b'}{K'}}}\right) = 0$$

$$\frac{dF}{dT} = 1 - \frac{Q}{4\pi s} \frac{\left(r\frac{b'}{K'}\right)}{\left(T\frac{b'}{K'}\right)^{3/2}} K_1\left(\frac{r}{\sqrt{T\frac{b'}{K'}}}\right)$$

- For p^{th} iteration

$$T^{(p)} = T^{(p-1)} - \frac{F\left(T^{(p-1)}\right)}{\left(\frac{dF}{dT}\right)^{(p-1)}}$$

where $p \geq 1$

Thank you