Parameter Estimation

Geohydraulics| CE60113

Lecture:12

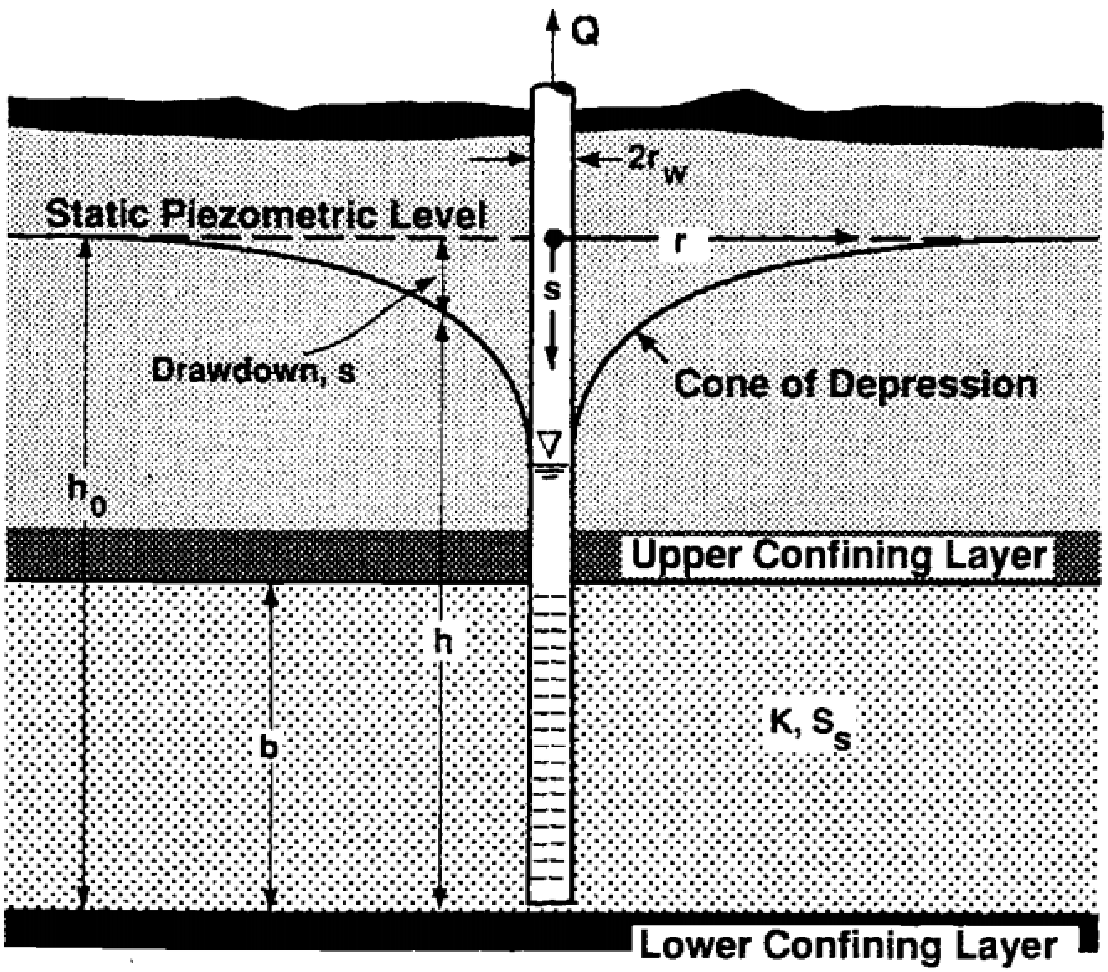
Learning Objective(s)

- **To derive the governing equation for saturated leaky confined aquifer**
- **To derive the governing equation for saturated leaky unconfined aquifer**
- **To estimate aquifer parameter under steady confined flow condition**

Basic Aquifer Types

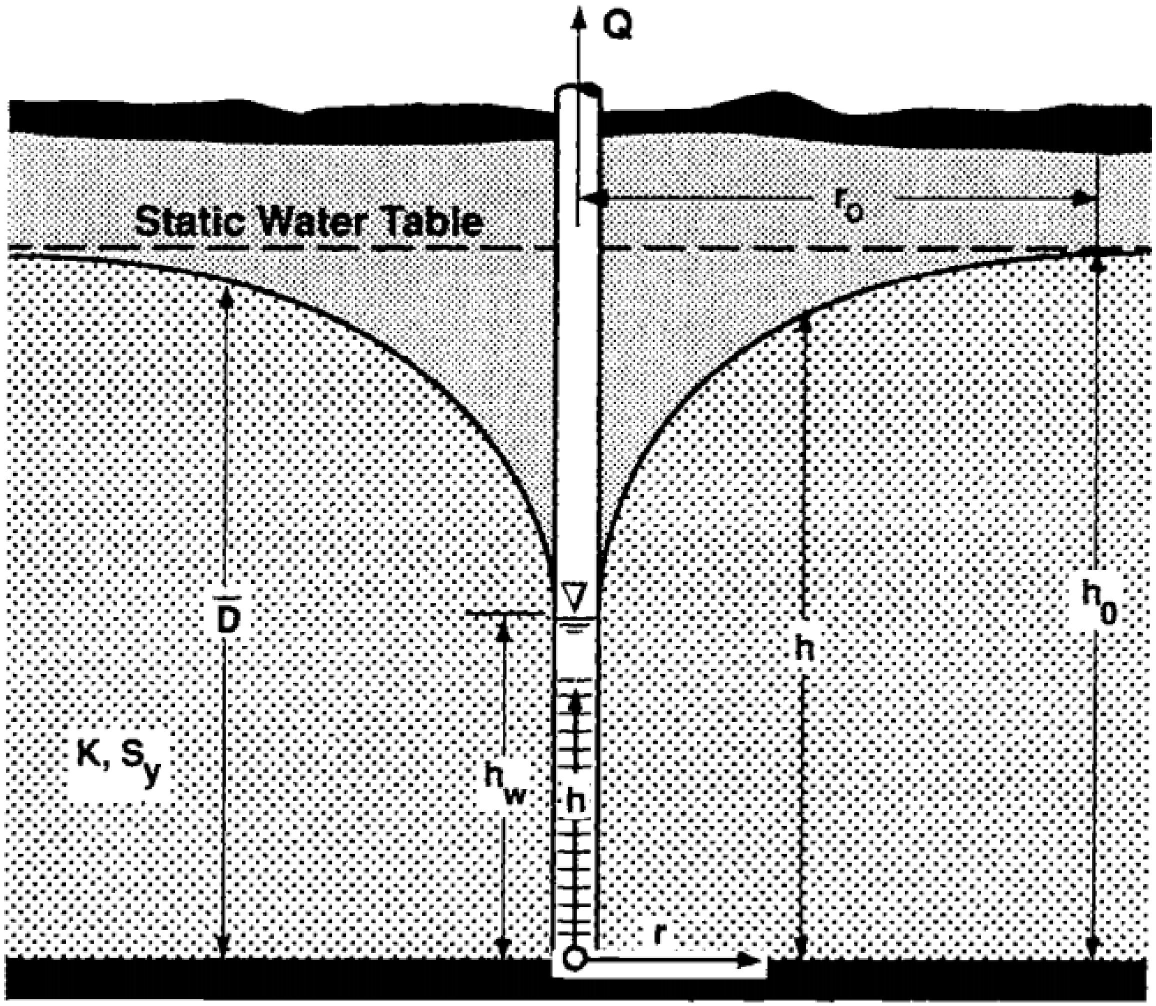
- Confined Aquifer
- Unconfined Aquifer
- Semiconfined/Leaky Aquifer

Confined Aquifer



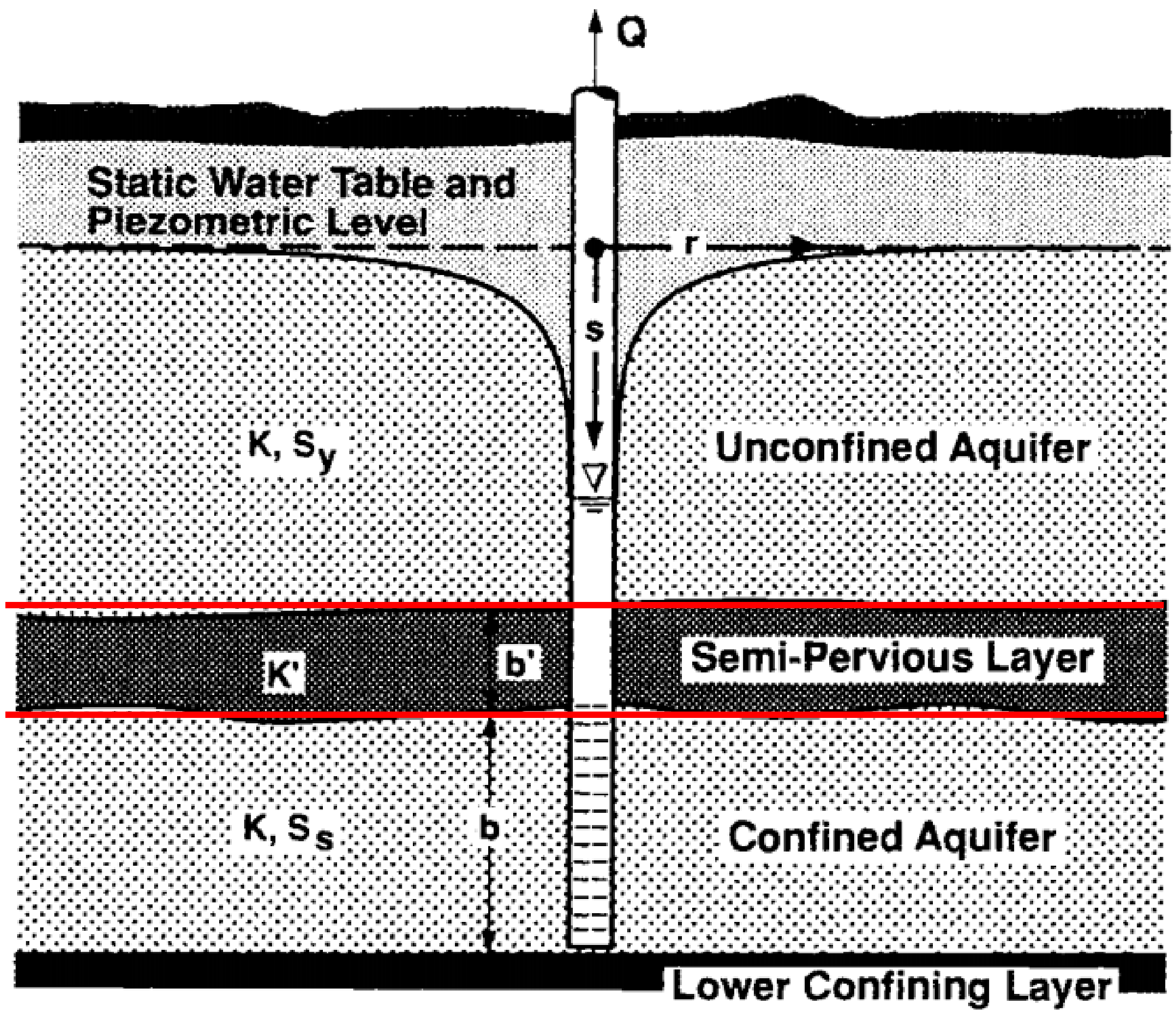
a. Confined

Unconfined Aquifer



b. Unconfined

Semiconfined Aquifer



c. Semi-Confined

Flow in a Leaky Phreatic Aquifer

- The governing equation for phreatic aquifer

$$S_y \frac{\partial H}{\partial t} = -\nabla_{xy} \cdot \mathbf{U} - q_B + N$$

Or,

$$S_y \frac{\partial H}{\partial t} = \frac{\partial}{\partial x}\left[K_x (H - \xi) \frac{\partial H}{\partial x}\right] + \frac{\partial}{\partial y}\left[K_y (H - \xi) \frac{\partial H}{\partial y}\right] - q_B + N$$

- Leaky condition: If we assume steady flow across the horizontal leaky confining layer ($\nabla \xi = 0$)

$$q_B = -\mathbf{q}|_{z=\xi} \cdot \nabla(z - \xi) = -q_z|_{z=\xi} = K' \frac{H - h_{CA}}{b'}$$

By applying leaky condition

$$S_y \frac{\partial H}{\partial t} = \frac{\partial}{\partial x}\left[K_x (H - \xi) \frac{\partial H}{\partial x}\right] + \frac{\partial}{\partial y}\left[K_y (H - \xi) \frac{\partial H}{\partial y}\right] - K' \frac{H - h_{CA}}{b'} + N$$

- $\frac{K'}{b'}$ is called the *'leakance'*

Flow in a Leaky Confined Aquifer

- 2D governing equation for confined aquifer

$$S\frac{\partial h}{\partial t} + q_T + q_B + q_{ext} = \nabla_{xy} \cdot \left[\mathbf{T} \cdot \nabla_{xy} h\right]$$

where

$$q_B = -\mathbf{q}|_{z=a} \cdot \nabla(z-a) = 0$$
$$q_T = \mathbf{q}|_{z=b} \cdot \nabla(z-b)$$

- Leaky condition: If we assume steady flow across the horizontal leaky confining layer ($\nabla b = 0$)

$$q_T = \mathbf{q}|_{z=b} \cdot \nabla(z-b) = q_z|_{z=b} = -K' \frac{H_{UA} - h}{b'}$$

By applying leaky condition

$$S\frac{\partial h}{\partial t} = \frac{\partial}{\partial x}\left[T_x \frac{\partial h}{\partial x}\right] + \frac{\partial}{\partial y}\left[T_y \frac{\partial h}{\partial y}\right] - K' \frac{h - H_{UA}}{b'} - q_{ext}$$

- $\frac{K\prime}{b'}$ is called the *'leakance'*

Parameter Estimation

- Mathematical Method
  - Analytical Method
  - Numerical Method
- Aquifer Test- controlled stress-response experiment at field level for determination of hydraulic conductivity, transmissivity, storage coefficient
  - Pumping Test
    - Aquifer is stressed using controlled extraction or injection from a pumping well
  - Slug Test
    - Aquifer is stressed using sudden change in water level in the control well
  - Constant Head Test
    - Aquifer is stressed using constant head (no change in head) in the control well

URL: http://www.aqtesolv.com/aquifer-tests/aquifer-tests.htm

Parameter Estimation (contd.)

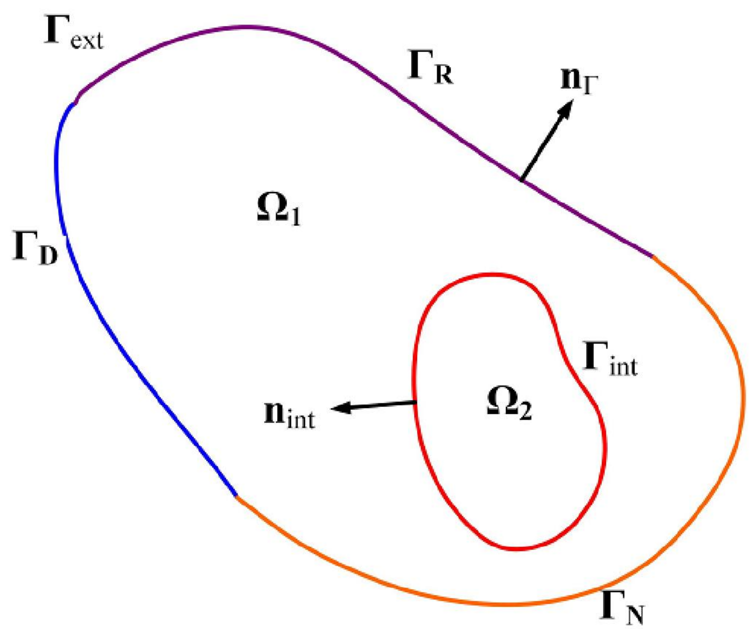
- Groundwater flow equation

$$S_s \frac{\partial h}{\partial t} = \frac{\partial}{\partial x}\left(K_x \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y \frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_z \frac{\partial h}{\partial z}\right) - W$$

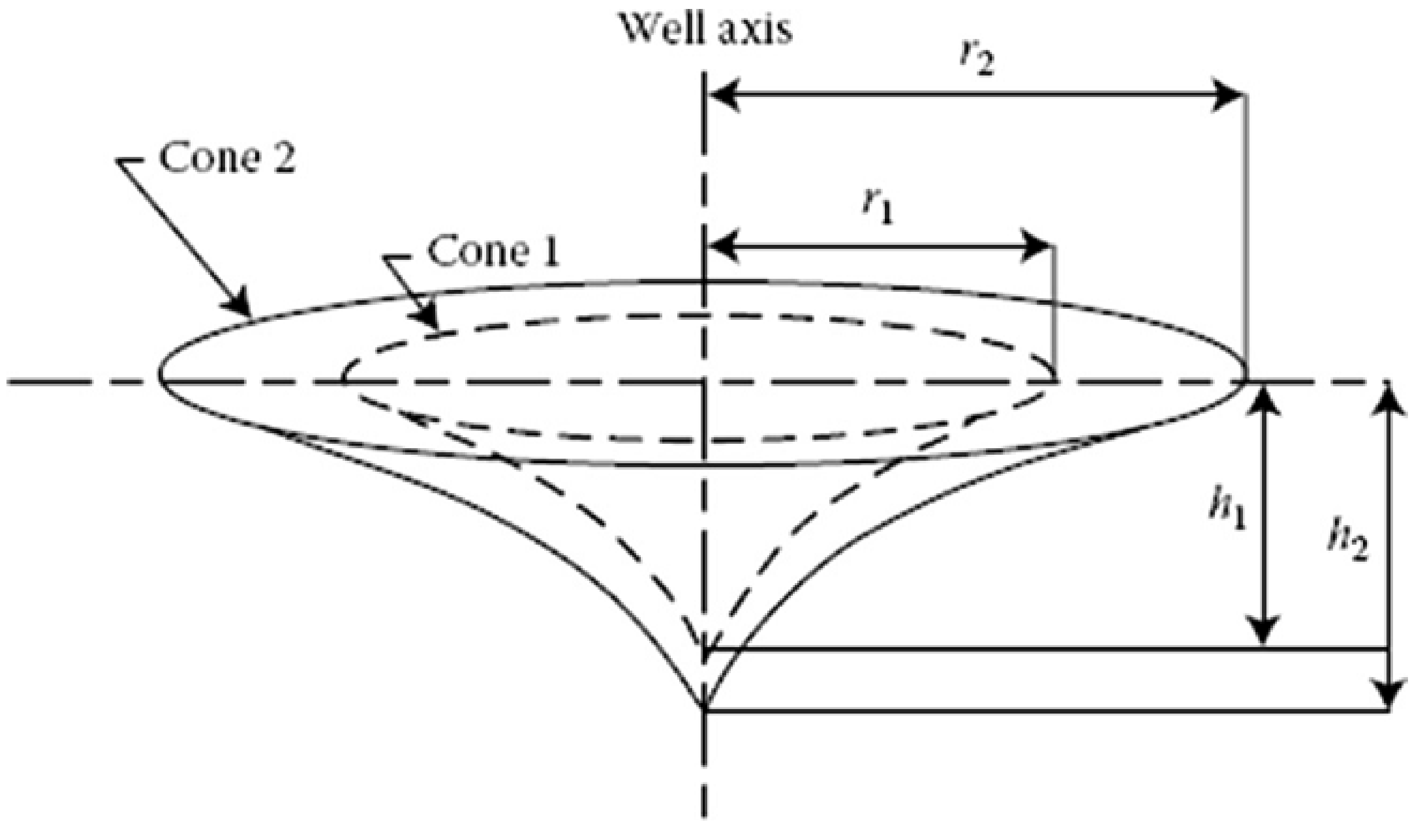
or,

$$S_s \frac{\partial h}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(K_r r \frac{\partial h}{\partial r}\right) + \frac{1}{r}\frac{\partial}{\partial \theta}\left(\frac{K_\theta}{r} \frac{\partial h}{\partial \theta}\right) + \frac{\partial}{\partial z}\left(K_z \frac{\partial h}{\partial z}\right) - W$$

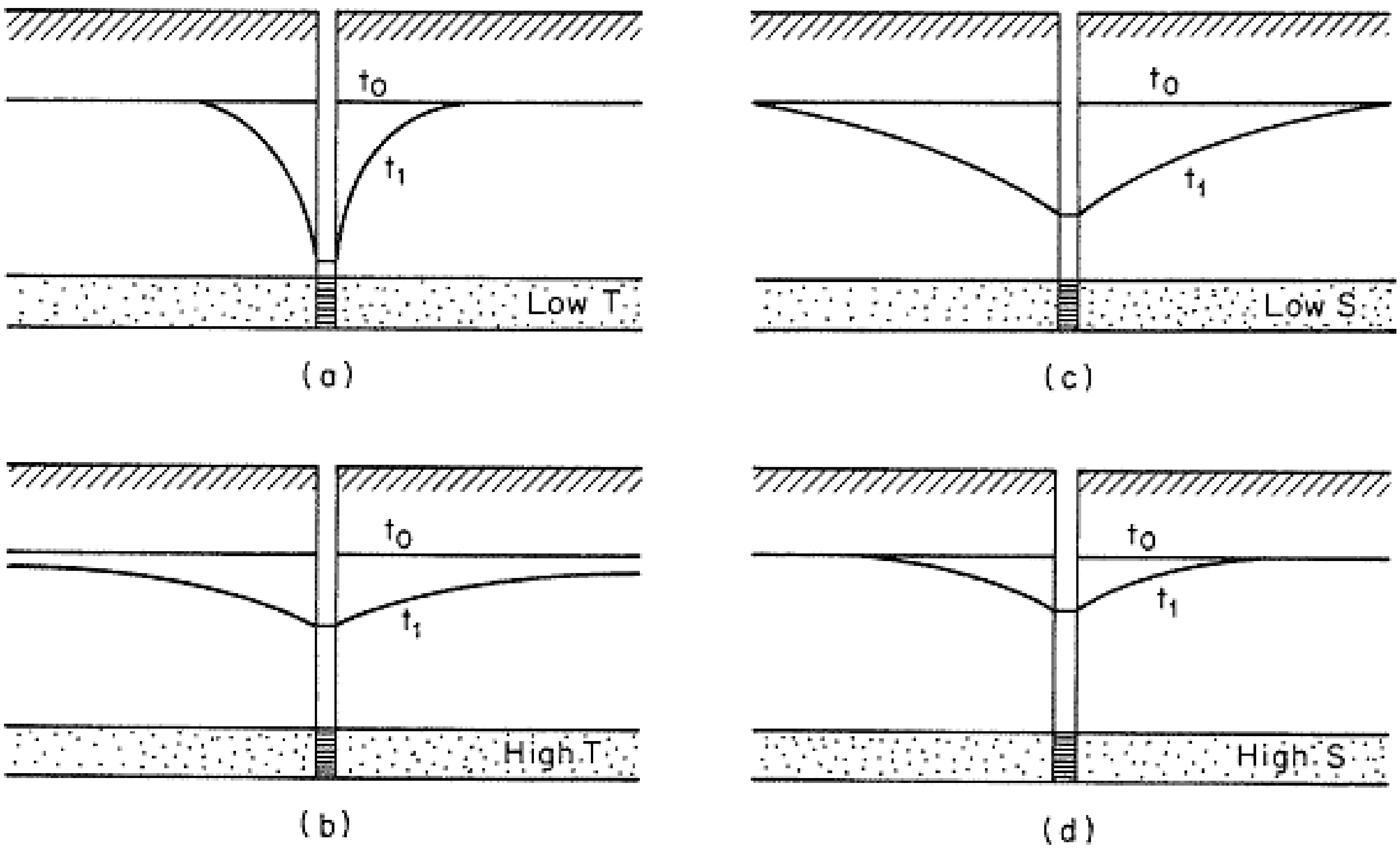
- Two parameters
  - hydraulic conductivity tensor **K**
  - specific storage S_s
- Initial Condition
- Boundary Conditions
  - Dirichlet, also known as fixed head
  - Neumann, also known as fixed flux
  - Robbins, also known as induced flux



Parameter Estimation (contd.)



Parameter Estimation (contd.)



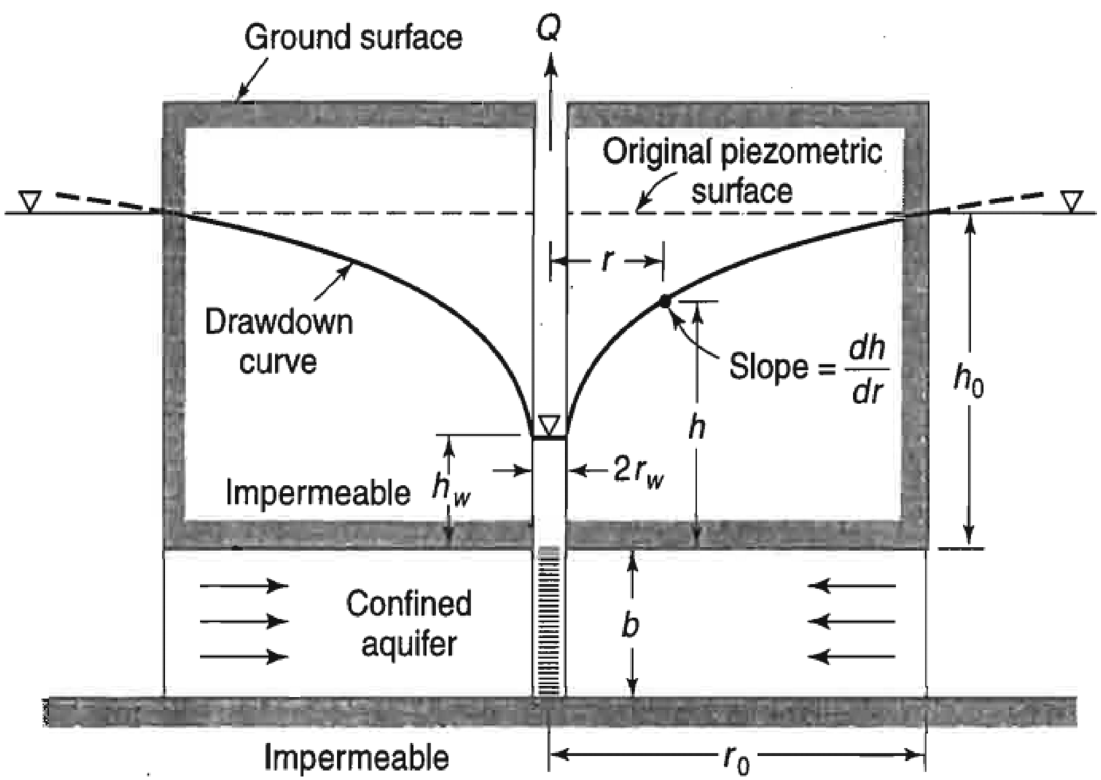
Parameter Estimation (Contd.)

- Inverse Problems
  - Backward or retrospective problem
    - The initial conditions are to be found.
  - Coefficient inverse problem
    - This is the classical parameter estimation problem where a constant multiplier in a governing equation is to be found.
  - Boundary inverse problem
    - Some missing information at the boundary of a domain is to be found.

Steady Radial Flow in Confined Aquifer

- Confined Aquifer
- Well discharge can be written as

$$Q = Aq_r = (2\pi rb)\left(-K_r \frac{\partial h}{\partial r}\right) = -2\pi rbK_r \frac{\partial h}{\partial r}$$



- Head difference is

$$h_0 - h_w = \frac{Q}{2\pi bK_r} ln\left(\frac{r_0}{r_w}\right)$$

- Discharge can be written as

$$Q = 2\pi bK_r \frac{h_0 - h_w}{ln\left(\frac{r_0}{r_w}\right)}$$

Steady Radial Flow in Confined Aquifer (Contd.)

$$\cancelto{0}{S_s \frac{\partial h}{\partial t}} = \frac{1}{r}\frac{\partial}{\partial r}\left(K_r r \frac{\partial h}{\partial r}\right) + \cancelto{0}{\frac{1}{r}\frac{\partial}{\partial \theta}\left(\frac{K_\theta}{r}\frac{\partial h}{\partial \theta}\right)} + \cancelto{0}{\frac{\partial}{\partial z}\left(K_z \frac{\partial h}{\partial z}\right)} - \cancelto{0}{W}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial h}{\partial r}\right) = 0$$

Solution

$$h(r) = Aln(r) + B$$

$$Q = -2\pi r b q_r(r) = 2\pi r b K_r \frac{\partial h}{\partial r} = 2\pi T A$$

- Negative sign is used to represent inward flow

$$h(r) = \frac{Q}{2\pi T} ln(r) + B$$

Steady Radial Flow in Confined Aquifer (Contd.)

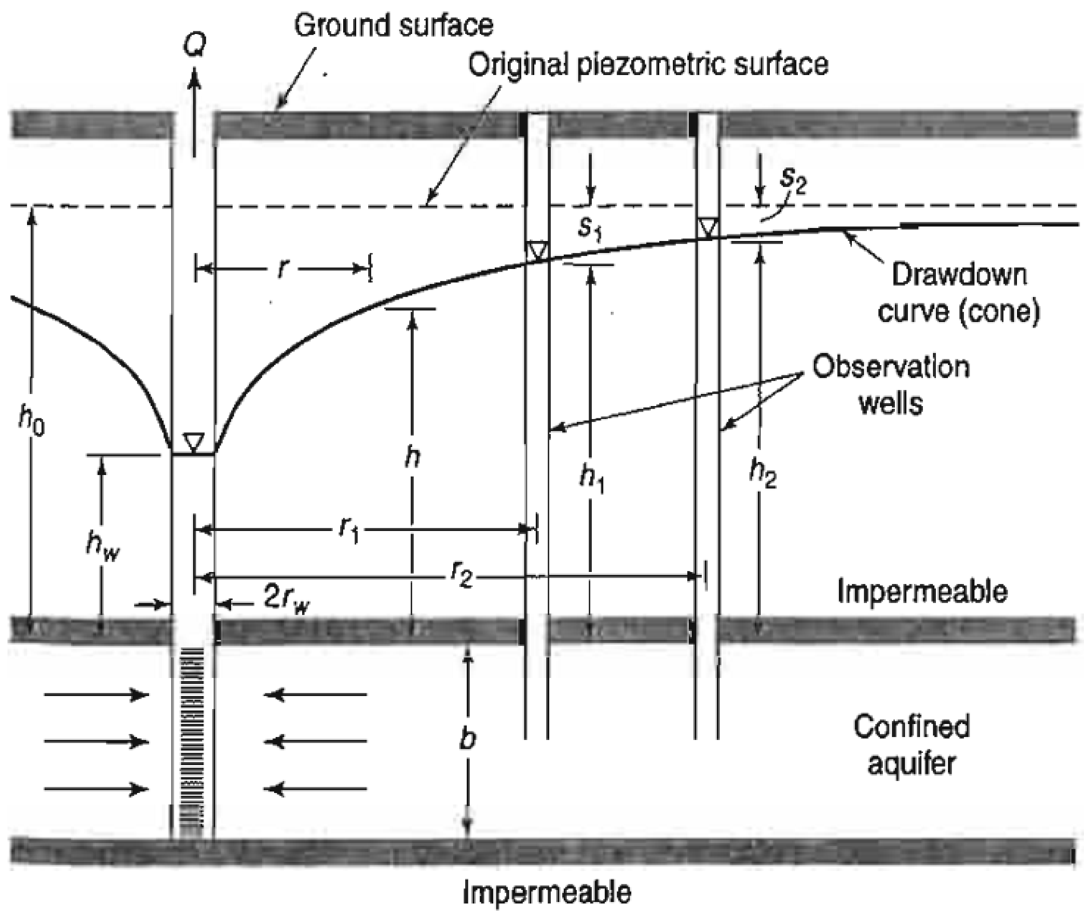
- Thiem Equation

$$Q = 2\pi b K_r \frac{h - h_w}{ln\left(\frac{r}{r_w}\right)}$$

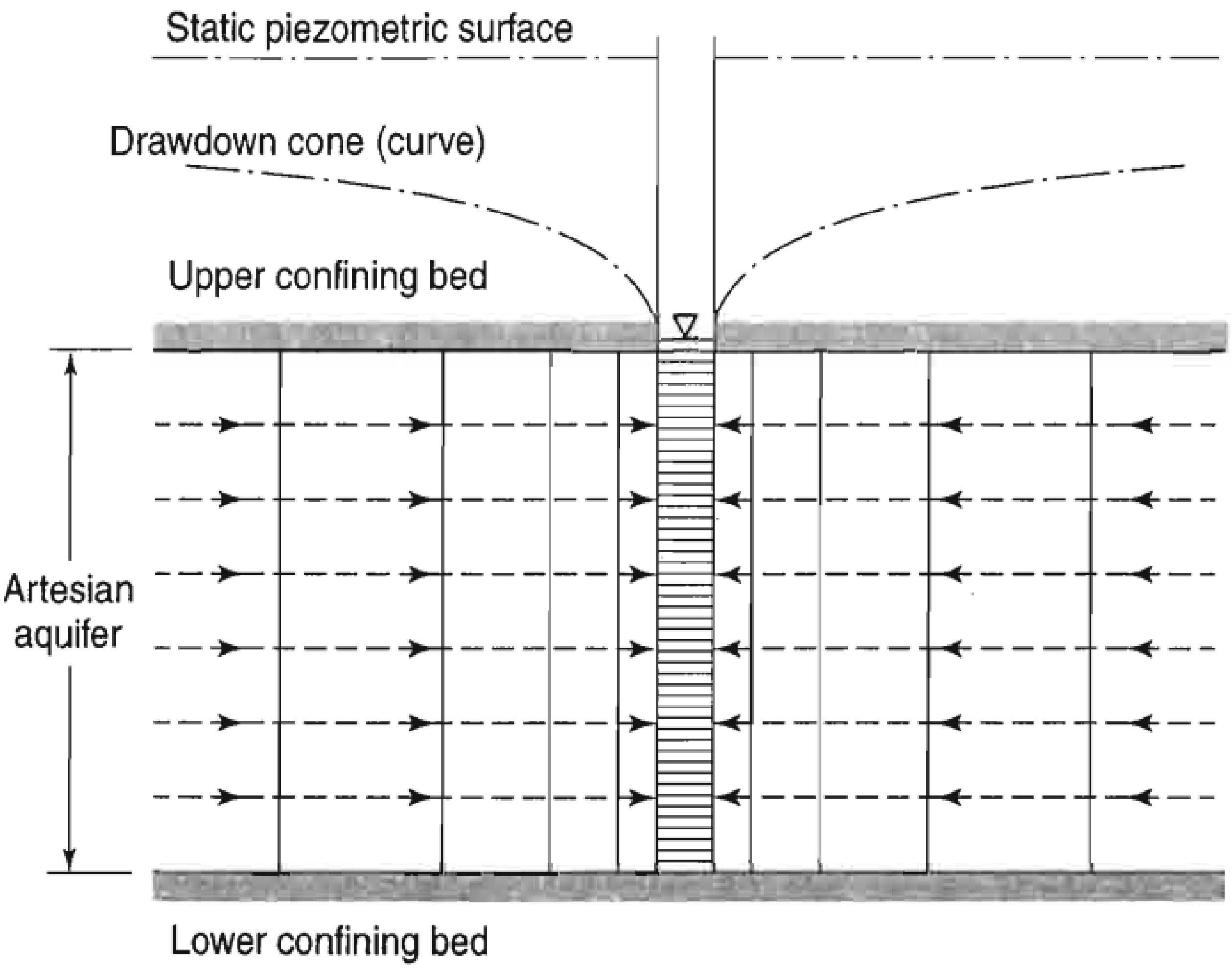
$$h_2 - h_1 = \frac{Q}{2\pi T} ln\left(\frac{r_2}{r_1}\right)$$

$$s_1 - s_2 = \frac{Q}{2\pi T} ln\left(\frac{r_2}{r_1}\right)$$

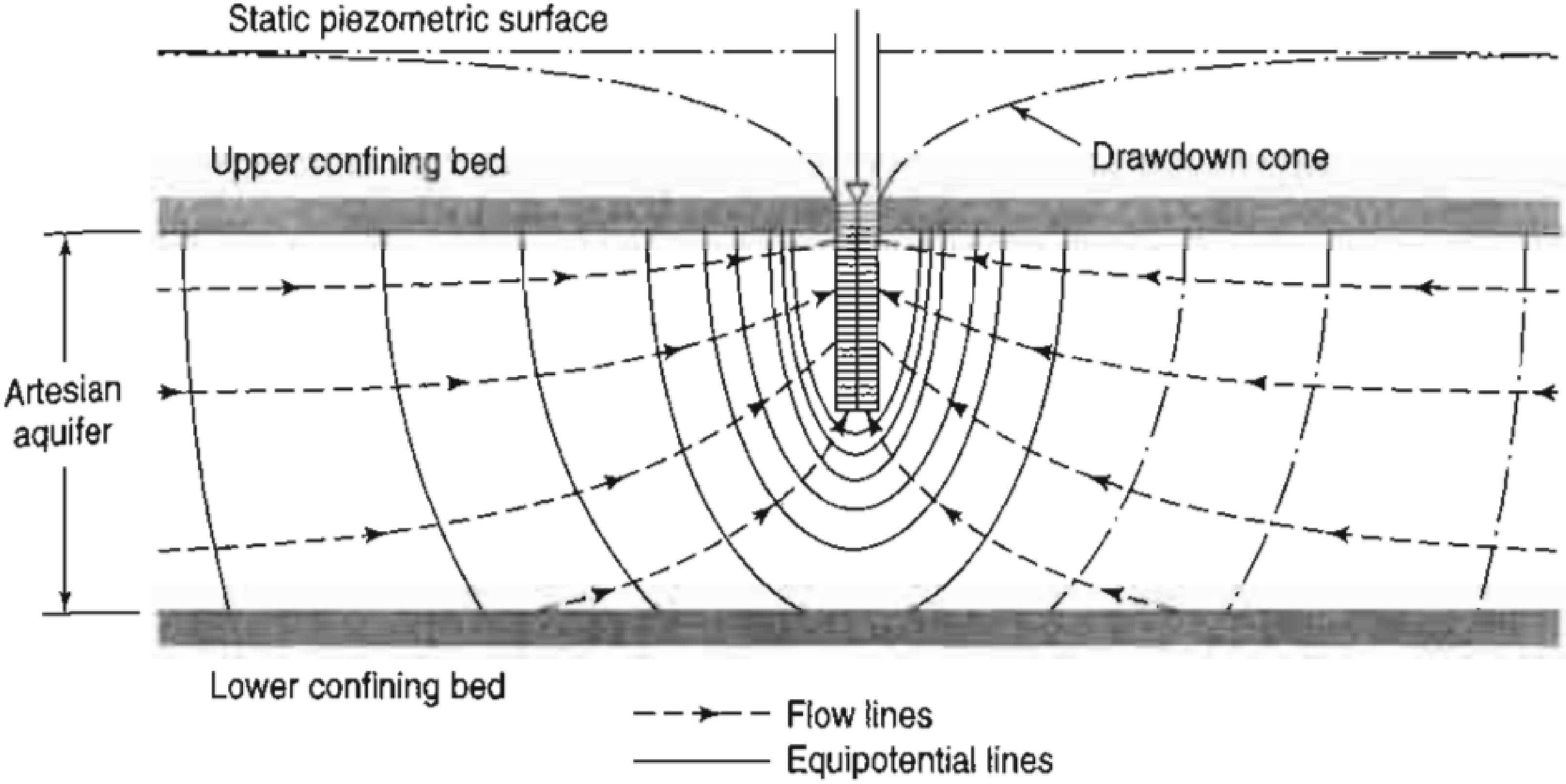
$$T = bK_r = \frac{Q}{2\pi(s_1 - s_2)} ln\left(\frac{r_2}{r_1}\right)$$



Steady Radial Flow in Confined Aquifer (Contd.)



Steady Radial Flow in Confined Aquifer (Contd.)



Thank you