Unsaturated Flow

Geohydraulics| CE60113

Lecture:17

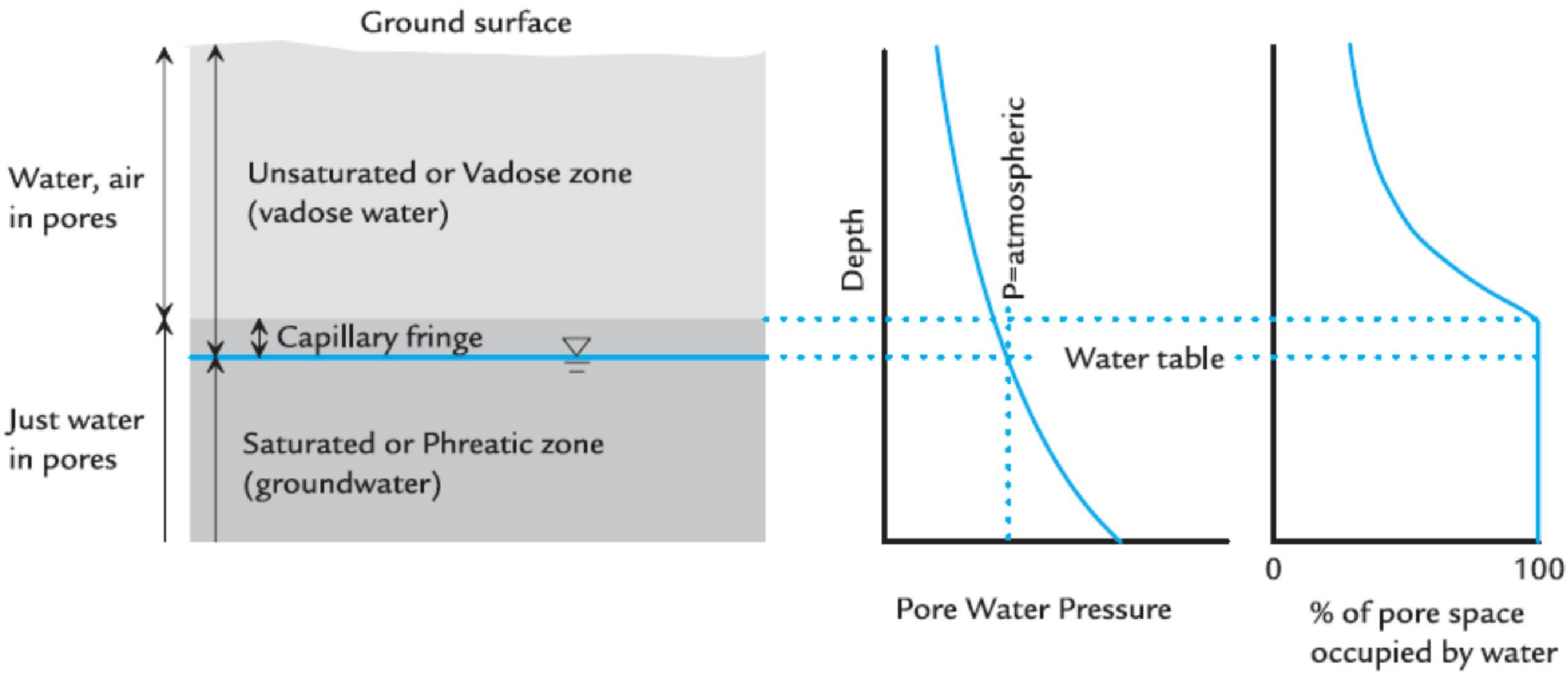
Learning Objective(s)

- **To explain unsaturated flow**
- **To estimate infiltration**

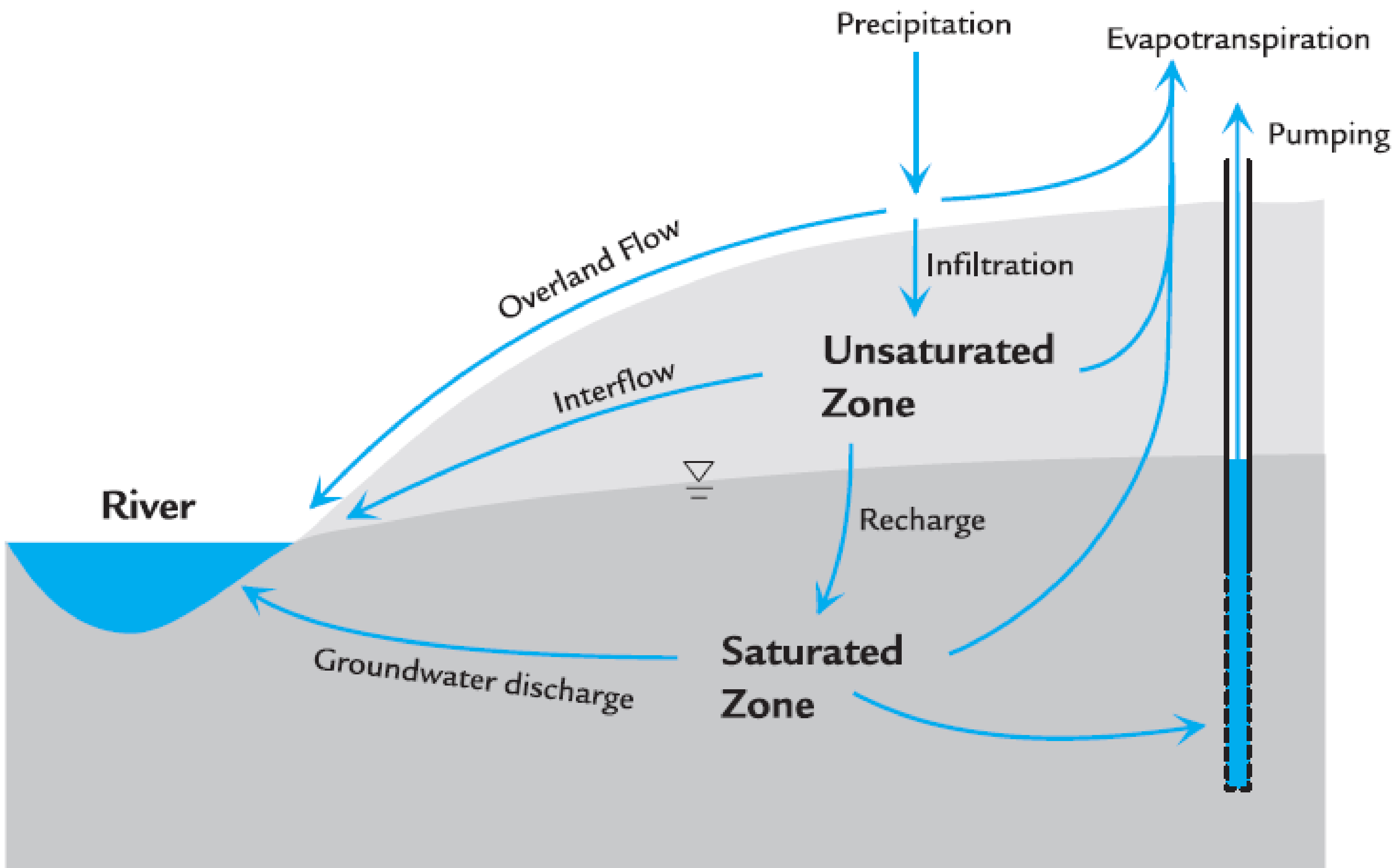
Unsaturated Flow

- Partially Saturated Flow
- Unsaturated Flow
- Vadose Zone Flow

Unsaturated Flow(Contd.)



Unsaturated Flow(Contd.)



Unsaturated Flow(Contd.)

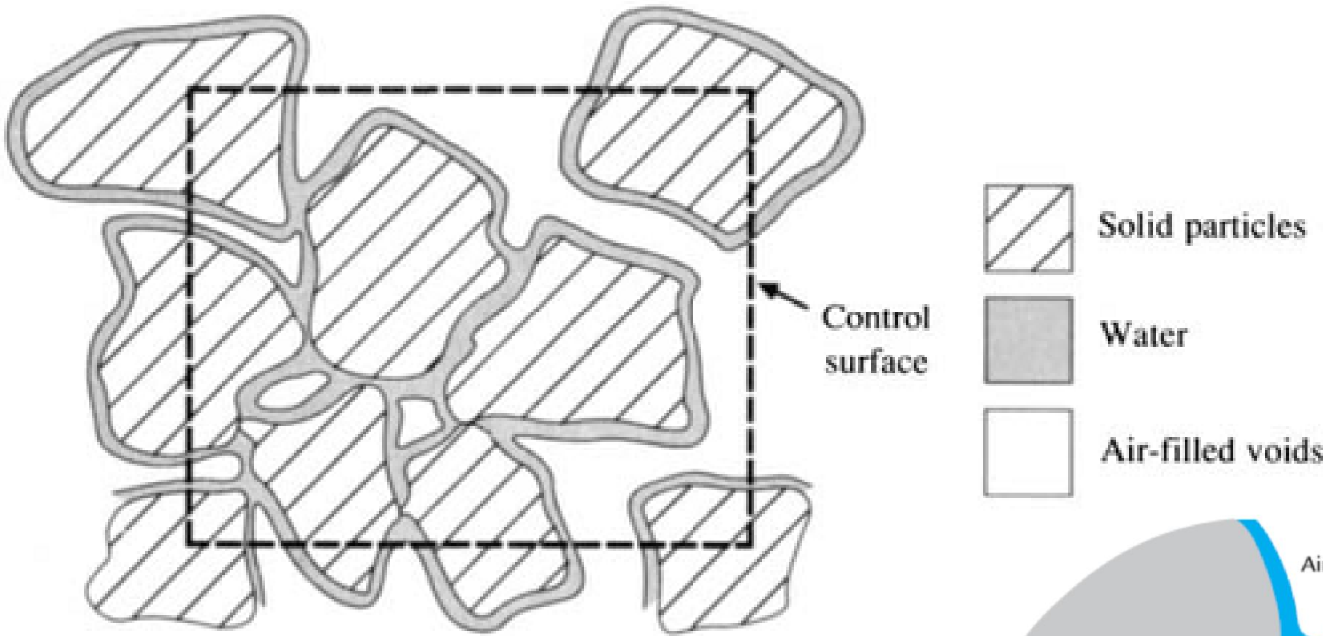
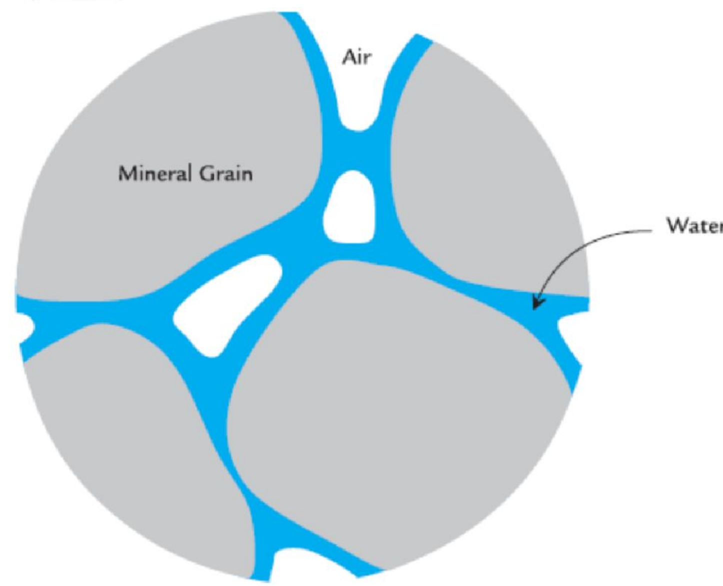
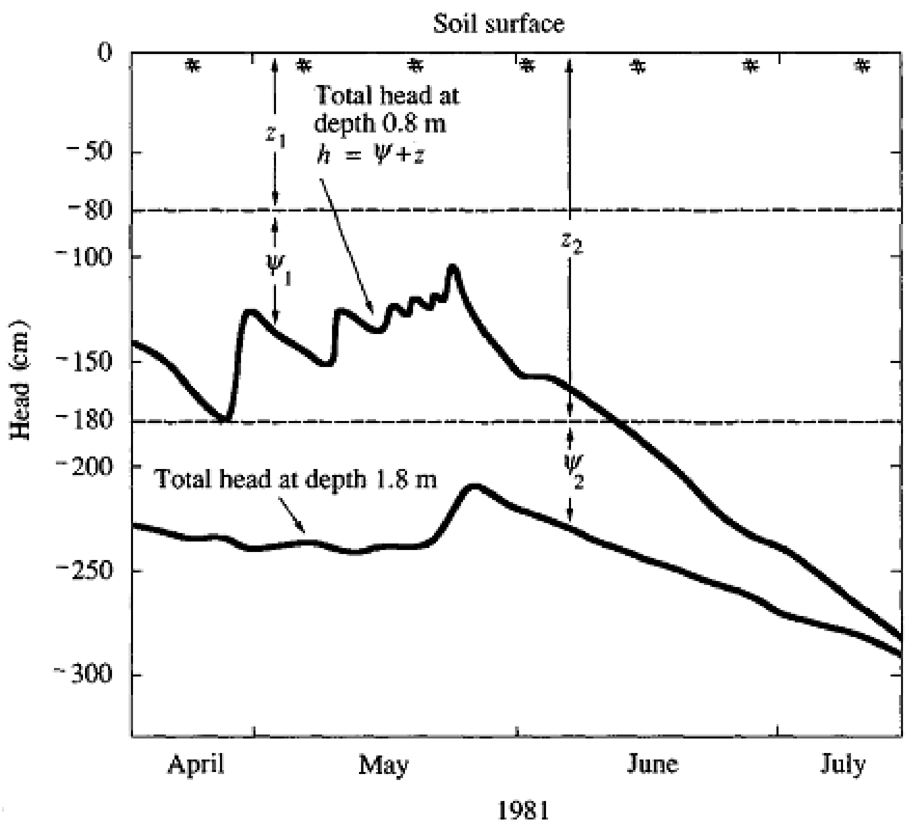


FIGURE 4.1.2
Cross section through an unsaturated porous medium.



Unsaturated Flow(Contd.)



(*a*)

FIGURE 4.1.5(*a*)
Profiles of total soil moisture head through time at Deep Dean in Sussex, England. (*Source:* Research Report 1981–84, Institute of Hydrology, Wallingford, England, Fig. 36, p. 33, 1984. Used with permission.)

Unsaturated Flow(Contd.)

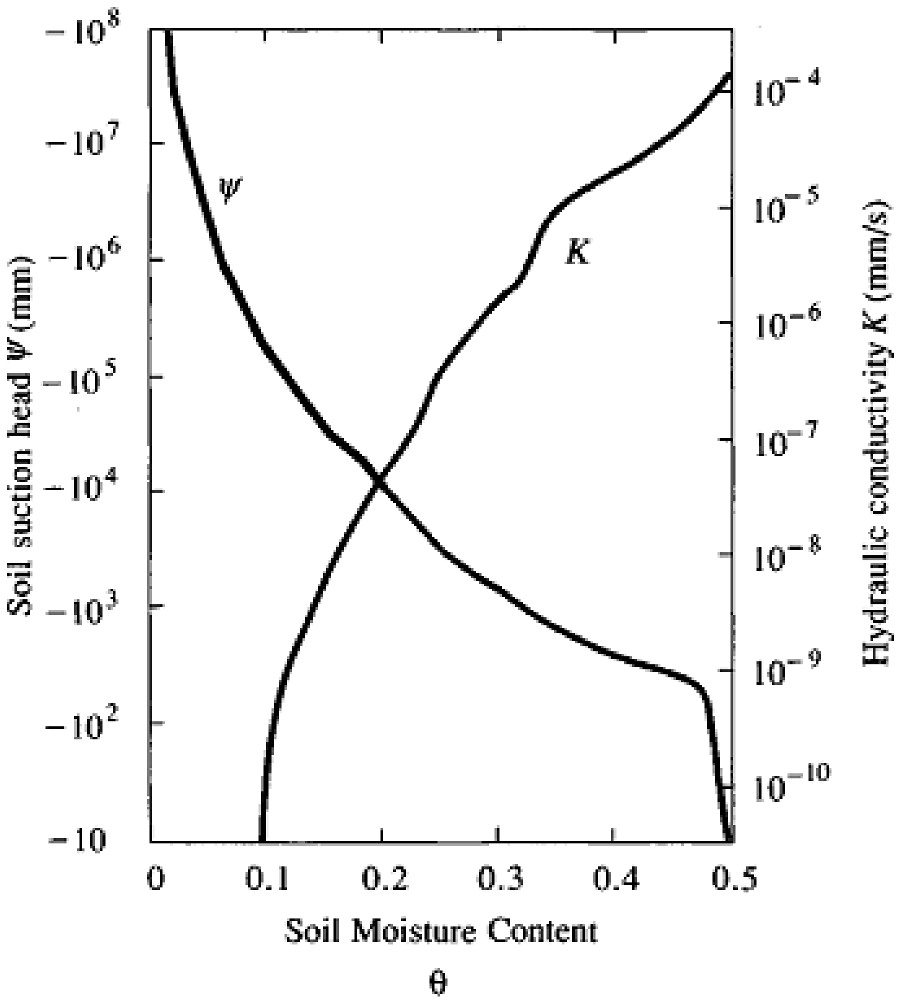


FIGURE 4.1.4
Variation of soil suction head ψ and hydraulic conductivity K with moisture content θ for Yolo light clay. (Reprinted with permission from A. J. Raudkivi, *Hydrology*, Copyright 1979, Pergamon Books Ltd.)

Unsaturated Flow(Contd.)

- Total Head

$$h = z + \psi$$

- Darcy's Law

$$\mathbf{q} = -K(\theta)\nabla h$$

- Relative Permeability

$$k_r(\theta) = \frac{K(\theta)}{K_s}$$

$$0 \leq k_r(\theta) \leq 1$$

- Expanded form of Darcy's Law

$$\begin{aligned} \mathbf{q} &= -K_s k_r(\theta)\nabla(z + \psi) \\ &= -K_s k_r(\theta)\mathbf{e_z} - K_s k_r(\theta)\nabla\psi \end{aligned}$$

Unsaturated Flow (Contd.)

- Continuity Equation

$$\frac{\partial \theta}{\partial t} + \nabla \cdot \mathbf{q} = S'$$

- Mixed Form (h $-$ θ based)

$$\frac{\partial \theta}{\partial t} - \nabla \cdot (K(\theta)\nabla h) = 0$$

or,

$$\frac{\partial \theta}{\partial t} = \nabla \cdot (K_s k_r(\theta)\mathbf{e_z} + K_s k_r(\theta)\nabla \psi\ )$$

- One equation with two unknowns

Unsaturated Flow(Contd.)

- Pressure Form (ψ based)

$$C(\psi)\frac{\partial\psi}{\partial t} = \nabla \cdot (K(\psi)\nabla\psi\ ) + \frac{\partial K(\psi)}{\partial z}$$

where

$$C(\psi) = \frac{d\theta}{d\psi}$$

- $C(\psi)$-Specific capacity or capillary (soil water) capacity
- Moisture Form (θ based)

$$\frac{\partial\theta}{\partial t} = \nabla \cdot (D(\theta)\nabla\theta) + \frac{\partial K(\theta)}{\partial z}$$

where

$$D(\theta) = K(\theta)\frac{d\psi}{d\theta}$$

- $D(\theta)$-Specific diffusivity or capillary (soil water) diffusivity

Unsaturated Flow(Contd.)

- van Genuchten–Mualem model
  - Degree of saturation- $S_e(\theta)$

$$S_e(\theta) = \frac{\theta - \theta_r}{\theta_s - \theta_r} = \begin{cases} \dfrac{1}{(1 + |\alpha\psi|^n)^m}, & \psi < 0 \\ 1, & \psi \geq 0 \end{cases}$$

  - Hydraulic Conductivity

$$K(\psi) = \begin{cases} K_s S_e^{0.5} \left[1 - \left(1 - S_e^{1/m}\right)^m\right]^2, & \psi < 0 \\ K_s, & \psi \geq 0 \end{cases}$$

  - Soil Water Capacity

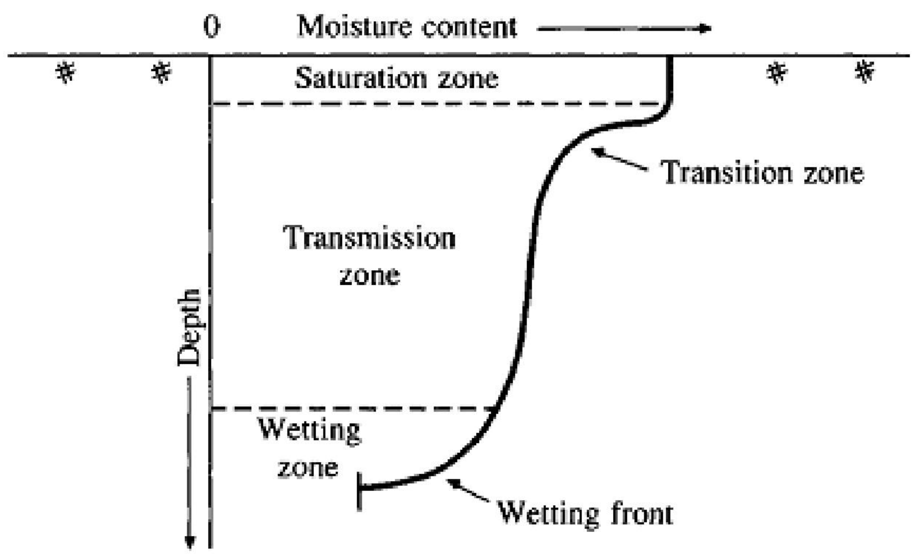
$$C(\psi) = \frac{d\theta}{d\psi} = \begin{cases} -\dfrac{\alpha m n(\theta_s - \theta_r)|\alpha\psi|^{n-1}}{(1 + |\alpha\psi|^n)^{m+1}}, & \psi < 0 \\ 0, & \psi \geq 0 \end{cases}$$

  - Soil Water Diffusivity

$$D(\theta) = K(\theta)\frac{d\psi}{d\theta}$$

Unsaturated Flow(Contd.)

- Cumulative Infiltration



$$F(t) = \int_0^t f(\tau)\, d\tau$$

$$f(t) = \frac{dF(t)}{dt}$$

FIGURE 4.2.1
Moisture zones during infiltration.

Unsaturated Flow(Contd.)

- Horton's Equation

$$f(t) = f_c + (f_0 - f_c)e^{-kt}$$

- Governing Equation

$$\frac{\partial\theta}{\partial t} = \nabla \cdot (D(\theta)\nabla\theta) + \frac{\partial K(\theta)}{\partial z}$$

- K and D are constants independent of the moisture content of the soil

$$\frac{\partial\theta}{\partial t} = D\ \frac{\partial^2\theta}{\partial z^2}$$

- Cumulative Infiltration

$$F = f_c t + \frac{f_0 - f_c}{k}\left(1 - e^{-kt}\right)$$

Unsaturated Flow(Contd.)

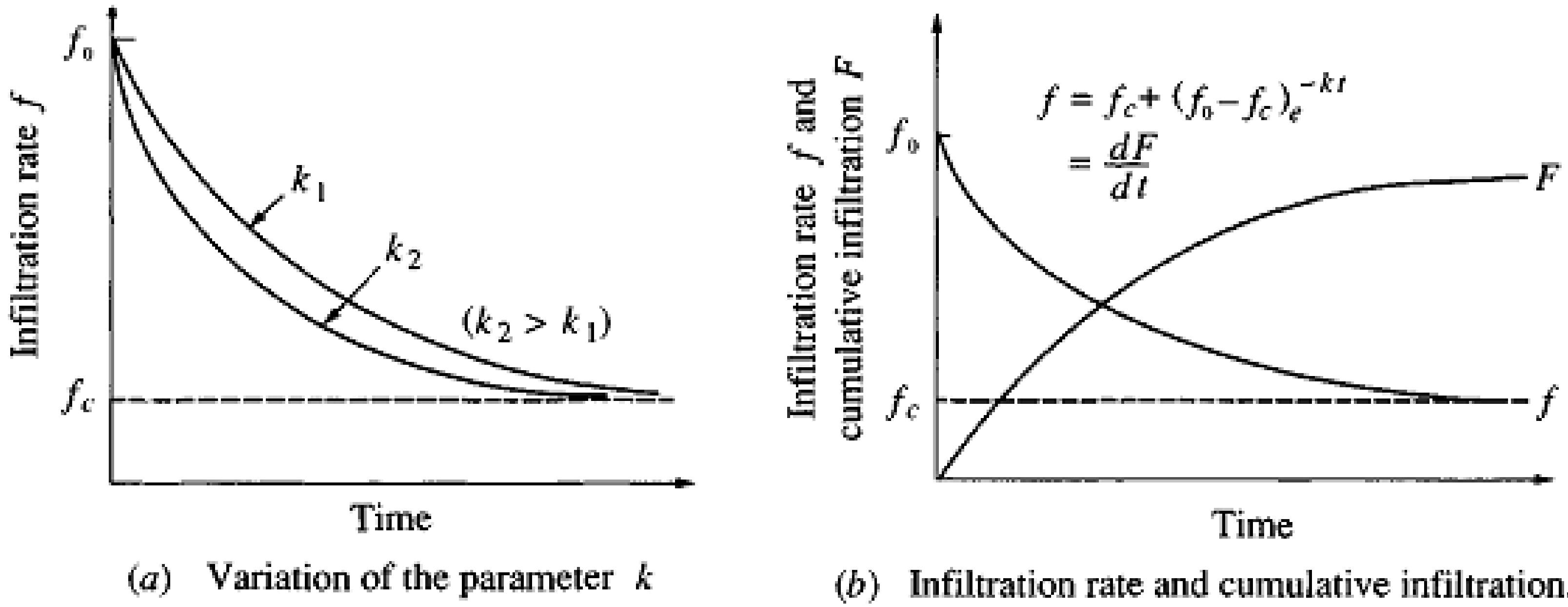


FIGURE 4.2.2
Infiltration by Horton's equation.

Unsaturated Flow(Contd.)

- Philip's Equation

$$F(t) = St^{1/2} + Kt$$

$$f(t) = \frac{1}{2}St^{-1/2} + K$$

As $t \longrightarrow \infty$, $f(t)$ tends to K.

For a horizontal column

$$F(t) = St^{1/2}$$

Unsaturated Flow(Contd.)

- Green-Ampt Model

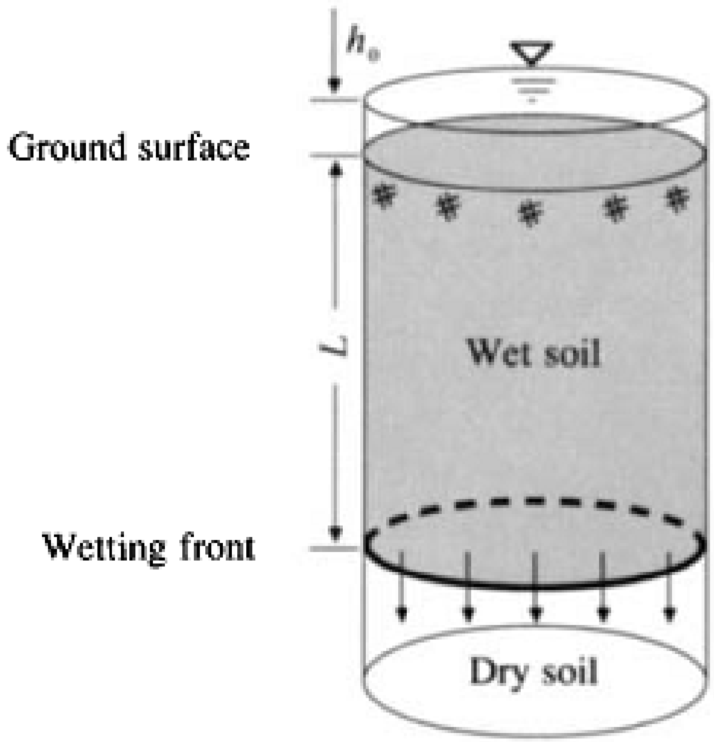


FIGURE 4.3.2
Infiltration into a column of soil of unit cross-sectional area for the Green-Ampt model.

Unsaturated Flow(Contd.)

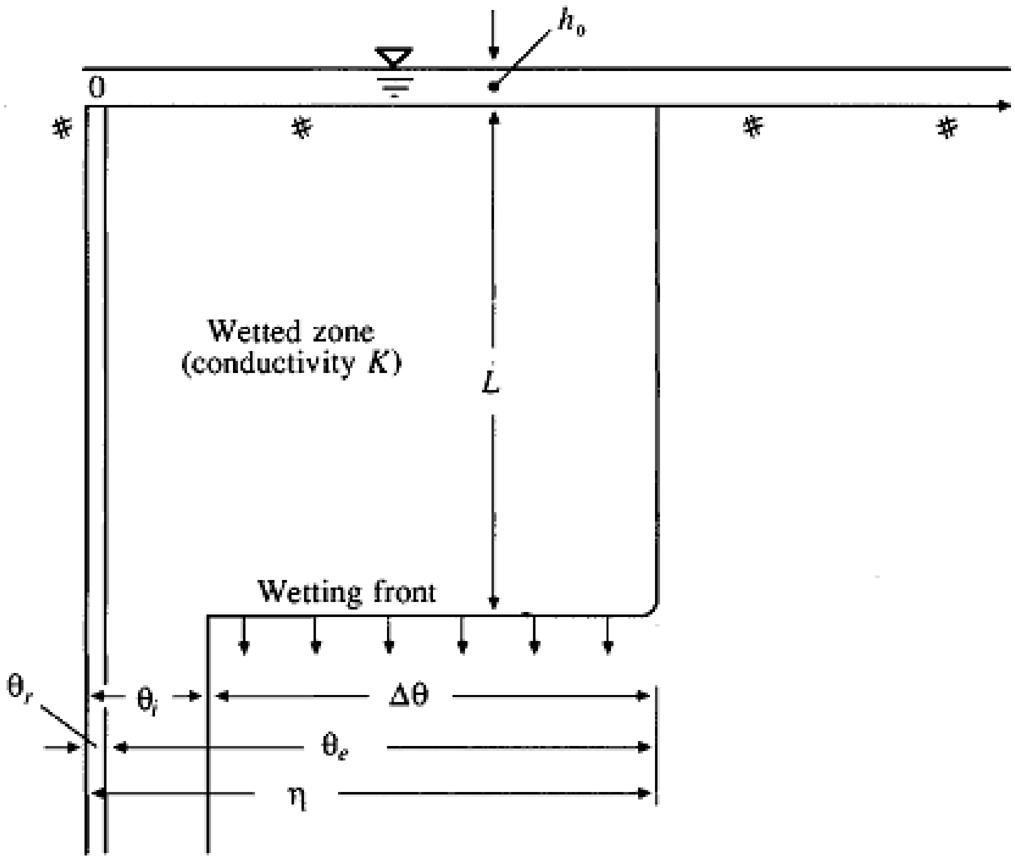


FIGURE 4.3.1
Variables in the Green-Ampt infiltration model. The vertical axis is the distance from the soil surface, the horizontal axis is the moisture content of the soil.

Unsaturated Flow(Contd.)

- Continuity

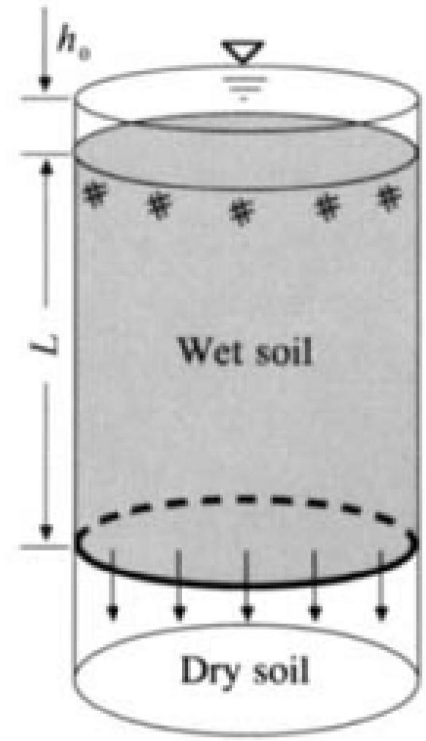
$$F(t) = L(\eta - \theta_i)$$
$$= L\Delta\theta$$

where $\Delta\theta = \eta - \theta_i$.

- Momentum

$$q = -K\frac{\partial h}{\partial z}$$

$$f = K\left[\frac{h_1 - h_2}{z_1 - z_2}\right]$$



$$f = K\left[\frac{h_0 - (-\psi - L)}{L}\right]$$
$$\approx K\left[\frac{\psi + L}{L}\right]$$

$$f = K\left[\frac{\psi\Delta\theta + F}{F}\right]$$

$$\frac{dF}{dt} = K\left[\frac{\psi\Delta\theta + F}{F}\right]$$

Unsaturated Flow(Contd.)

$$\left[\frac{F}{F + \psi\Delta\theta}\right]dF = K\,dt$$

$$\left[\left(\frac{F + \psi\,\Delta\theta}{F + \psi\,\Delta\theta}\right) - \left(\frac{\psi\,\Delta\theta}{F + \psi\,\Delta\theta}\right)\right]dF = K\,dt$$

$$\int_0^{F(t)} \left(1 - \frac{\psi\,\Delta\theta}{F + \psi\,\Delta\theta}\right) dF = \int_0^t K\,dt$$

$$F(t) - \psi\,\Delta\theta\left\{\ln\left[F(t) + \psi\,\Delta\theta\right] - \ln\,(\psi\,\Delta\theta)\right\} = Kt$$

or

$$F(t) - \psi\,\Delta\theta\;\ln\left(1 + \frac{F(t)}{\psi\,\Delta\theta}\right) = Kt$$

Unsaturated Flow(Contd.)

- Infiltration rate

$$f(t) = K\left(\frac{\psi \Delta\theta}{F(t)} + 1\right)$$

- Cumulative Infiltration

$$F(t) = Kt + \psi \Delta\theta \ln\left(1 + \frac{F(t)}{\psi \Delta\theta}\right)$$

$$s_e = \frac{\theta - \theta_r}{\eta - \theta_r}$$

$$\Delta\theta = \eta - \theta_i = \eta - (s_e \theta_e + \theta_r)$$

$$\Delta\theta = (1 - s_e)\theta_e$$

Unsaturated Flow(Contd.)

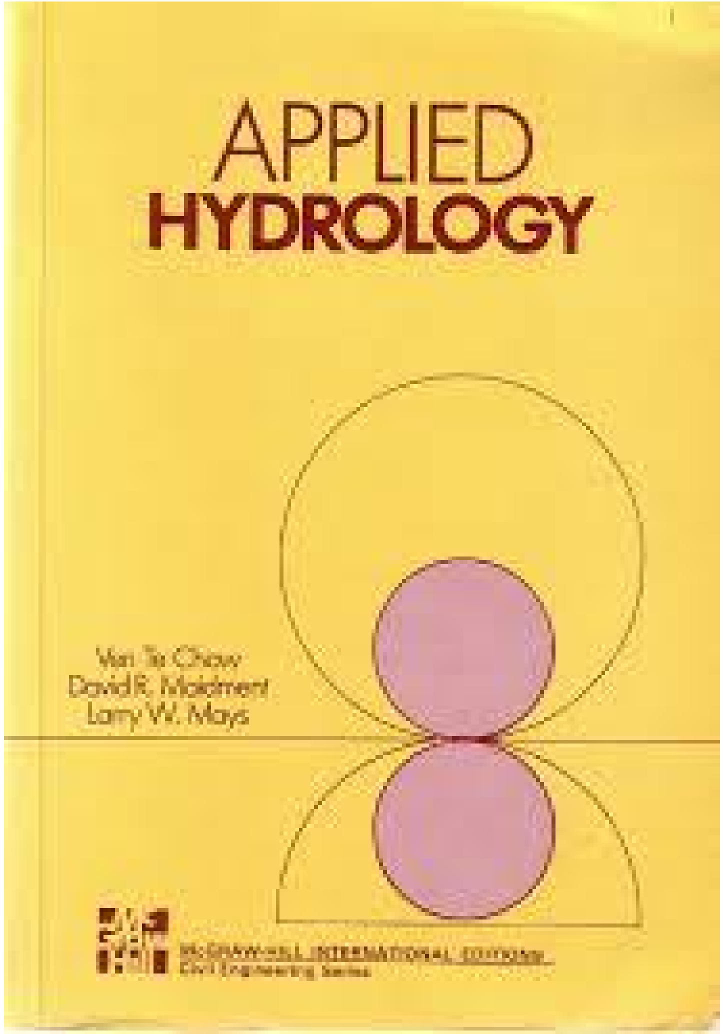
TABLE 4.3.1
Green-Ampt infiltration parameters for various soil classes

Soil class	Porosity η	Effective porosity θ_e	Wetting front soil suction head ψ (cm)	Hydraulic conductivity K (cm/h)
Sand	0.437 (0.374–0.500)	0.417 (0.354–0.480)	4.95 (0.97–25.36)	11.78
Loamy sand	0.437 (0.363–0.506)	0.401 (0.329–0.473)	6.13 (1.35–27.94)	2.99
Sandy loam	0.453 (0.351–0.555)	0.412 (0.283–0.541)	11.01 (2.67–45.47)	1.09
Loam	0.463 (0.375–0.551)	0.434 (0.334–0.534)	8.89 (1.33–59.38)	0.34
Silt loam	0.501 (0.420–0.582)	0.486 (0.394–0.578)	16.68 (2.92–95.39)	0.65
Sandy clay loam	0.398 (0.332–0.464)	0.330 (0.235–0.425)	21.85 (4.42–108.0)	0.15
Clay loam	0.464 (0.409–0.519)	0.309 (0.279–0.501)	20.88 (4.79–91.10)	0.10
Silty clay loam	0.471 (0.418–0.524)	0.432 (0.347–0.517)	27.30 (5.67–131.50)	0.10
Sandy clay	0.430 (0.370–0.490)	0.321 (0.207–0.435)	23.90 (4.08–140.2)	0.06
Silty clay	0.479 (0.425–0.533)	0.423 (0.334–0.512)	29.22 (6.13–139.4)	0.05
Clay	0.475 (0.427–0.523)	0.385 (0.269–0.501)	31.63 (6.39–156.5)	0.03

The numbers in parentheses below each parameter are one standard deviation around the parameter value given. *Source:* Rawls, Brakensiek, and Miller, 1983.

Learning Strategy

Chapter 4: Subsurface Water



Thank you