Aquifer Properties

Geohydraulics| CE60113

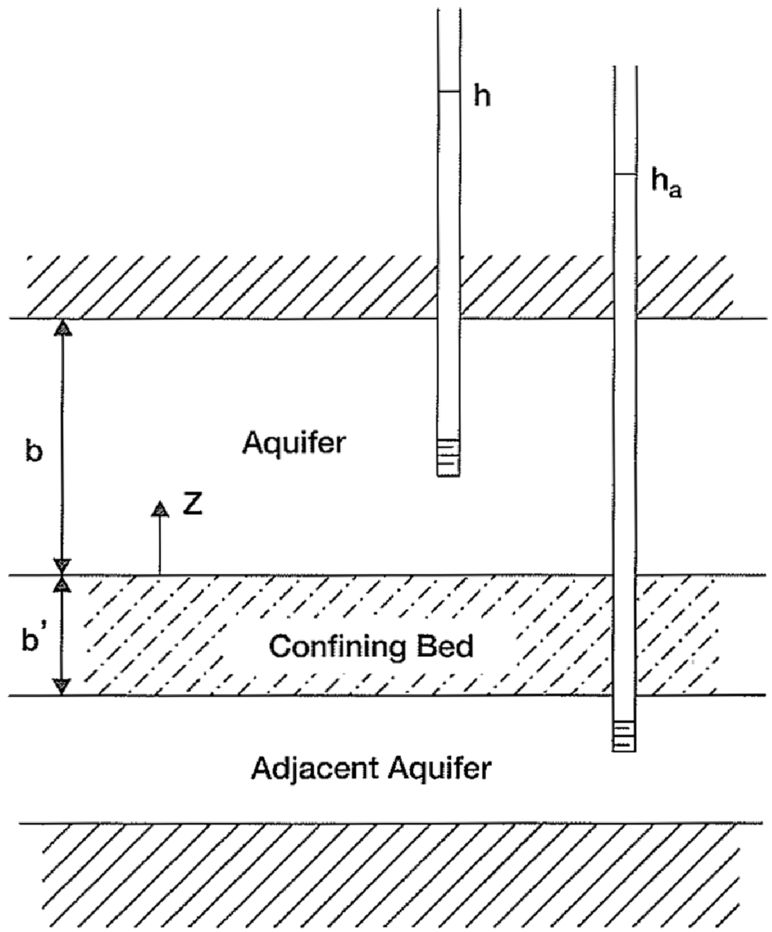
Lecture:06

Learning Objective(s)

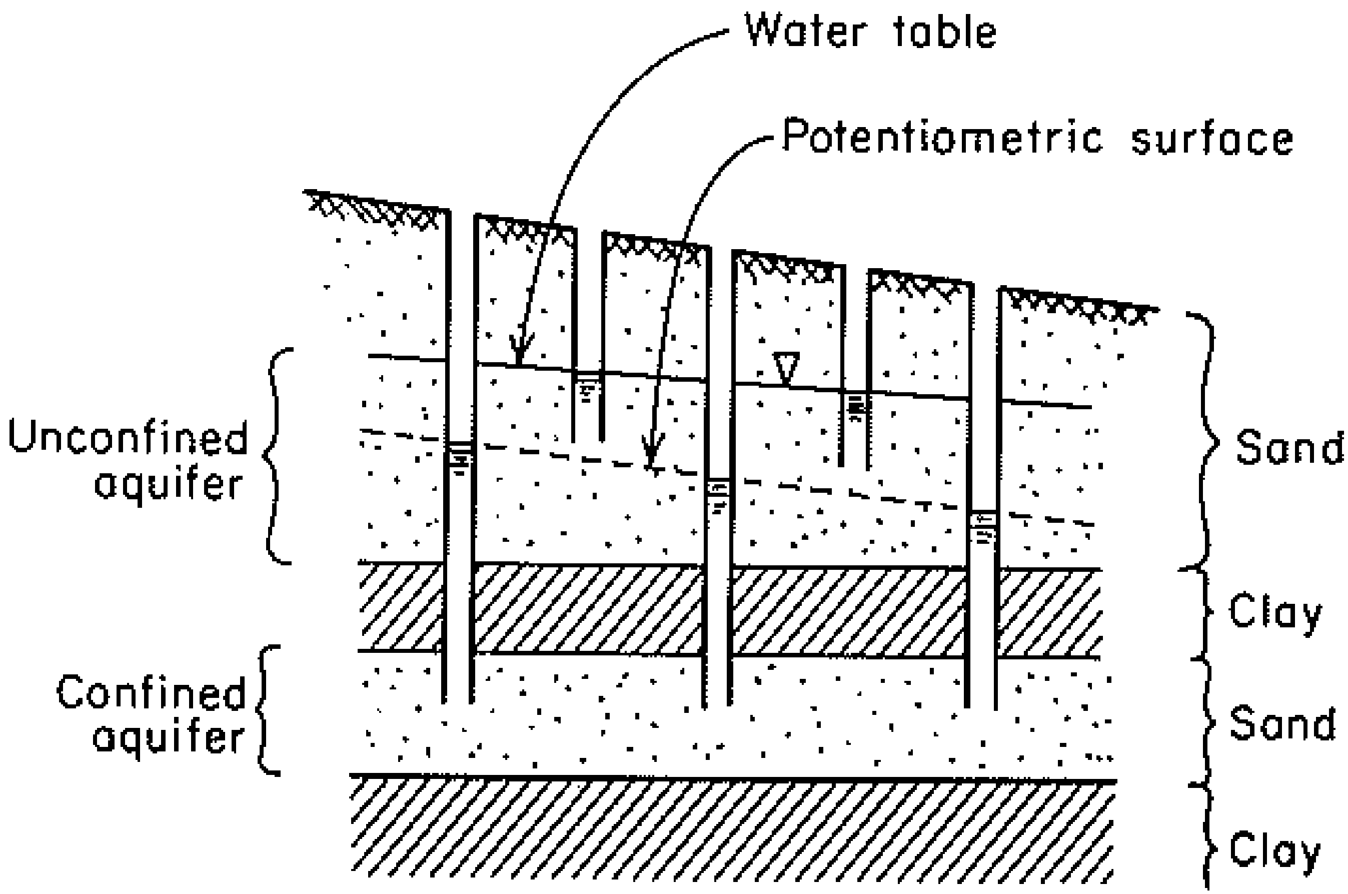
- To calculate storage coefficient, specific yield, specific retention.

Two Confined Aquifers with Different Heads

- Groundwater will tend to flow from the top aquifer to the bottom aquifer.
- We can't make any conclusion about horizontal head gradients from this picture.

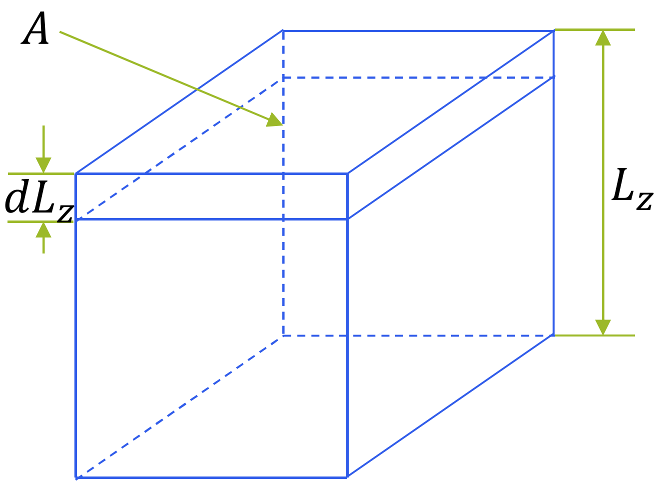


Horizontal and Vertical Head Gradients



Specific storage

$$V_T = AL_z$$
$$V_s = (1-\eta)AL_z$$
$$V_v = \eta AL_z$$



- Total mass of water contained within the control volume

$$M = \rho_w \eta A L_z$$

- If A remains unchanged

$$dM = \eta A L_z d\rho_w + \rho_w A L_z d\eta + \rho_w \eta A dL_z$$

- Under the assumption $dV_s = 0$

$$dV_s = (1-\eta)AdL_z - AL_z d\eta = 0$$

- Replacing $L_z d\eta = (1-\eta)dL_z$

$$dM = \eta A L_z d\rho_w + \rho_w A(1-\eta)dL_z + \rho_w \eta A dL_z$$

or

$$dM = \eta A L_z d\rho_w + \rho_w A dL_z$$

or

$$dM = V_v d\rho_w + \rho_w dV_T = \eta V_T d\rho_w + \rho_w dV_T = \rho_w V_T(\eta\beta + \alpha)dp$$

$$\beta = -\frac{1}{V_w}\frac{dV_w}{dp} = \frac{1}{\rho_w}\frac{d\rho_w}{dp}$$

$$\alpha = \frac{1}{V_T}\frac{dV_T}{dp}$$

$$dp = \rho g dh$$

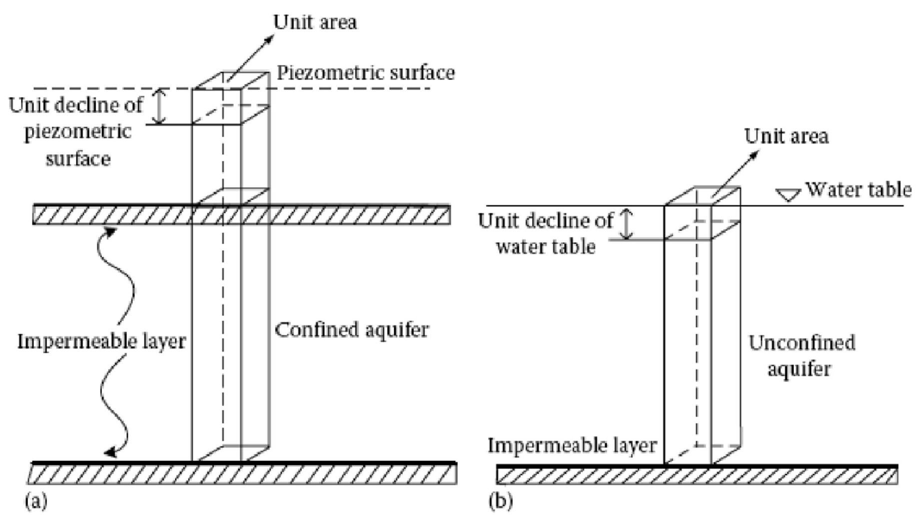
$$V_w = \frac{dM}{\rho_w} = V_T(\eta\beta + \alpha)\rho g dh$$

Storage Coefficient

$$S_s = \frac{1}{V_T}\frac{dV_w}{dh} = (\eta\beta + \alpha)\rho g$$

$$S = S_s b = \frac{1}{A}\frac{dV_w}{dh}$$

- Common range of the storage coefficient in confined aquifers is from 0.00005 to 0.005



- Another way to calculate the specific storage coefficient

$$S_s = \gamma_w\left[(1-n)\alpha + n\beta\right]$$

Storage Coefficient (Contd.)

- Using chain rule of differentiation

$$\frac{\partial(\varepsilon\rho)}{\partial t} = \varepsilon\frac{\partial\rho}{\partial t} + \rho\frac{\partial\varepsilon}{\partial t}.$$

$$\frac{\partial\varepsilon}{\partial t} = \frac{d\varepsilon}{dp}\frac{\partial p}{\partial t}$$

$$= C_v\frac{\partial p}{\partial t},$$

C_v is related to the classical *coefficient of consolidation*

$$d\varepsilon = d\left(\frac{V_v}{V_T}\right) = \frac{V_T dV_v - V_v dV_T}{V_T^2} \approx \frac{V_s}{V_T^2}dV_v$$

$$\frac{d\varepsilon}{dp} = \frac{V_s}{V_T^2}\frac{dV_v}{dp} = \frac{V_s}{V_T}\alpha = (1-\varepsilon)\alpha$$

$$\boxed{\alpha = \frac{1}{V_T}\frac{dV_T}{dp} \approx \frac{1}{V_T}\frac{dV_v}{dp} \qquad \beta \equiv \frac{1}{\rho}\frac{d\rho}{dp}.}$$

$$\frac{\partial(\varepsilon\rho)}{\partial t} = \varepsilon\frac{\partial\rho}{\partial t} + \rho\frac{\partial\varepsilon}{\partial t}$$

$$= \varepsilon\frac{d\rho}{dp}\frac{\partial p}{\partial t} + \rho\frac{d\varepsilon}{dp}\frac{\partial p}{\partial t}$$

$$= \rho(\varepsilon\beta + C_v)\frac{\partial p}{\partial t}.$$

dt

$$\frac{d(\varepsilon\rho)}{dp} = \varepsilon\frac{d\rho}{dt} + \rho\frac{d\varepsilon}{dt}$$

$$= \varepsilon\frac{d\rho}{dp}\frac{dp}{dt} + \rho\frac{d\varepsilon}{dp}\frac{dp}{dt}$$

$$= \rho[\varepsilon\beta + (1-\varepsilon)\alpha]\frac{dp}{dt}$$

$$S_s = \gamma_w\left[(1-n)\alpha + n\beta\right]$$

Specific Yield

- Specific yield is the volume of water that is released from storage in an unconfined aquifer per unit surface area of the aquifer per unit decline in the water table.

$$S_y = \frac{1}{A}\frac{dV_w}{dh}$$

- The ratio of the volume of water discharged from the sample, considering the gravity, over total volume of the sample is its specific yield.

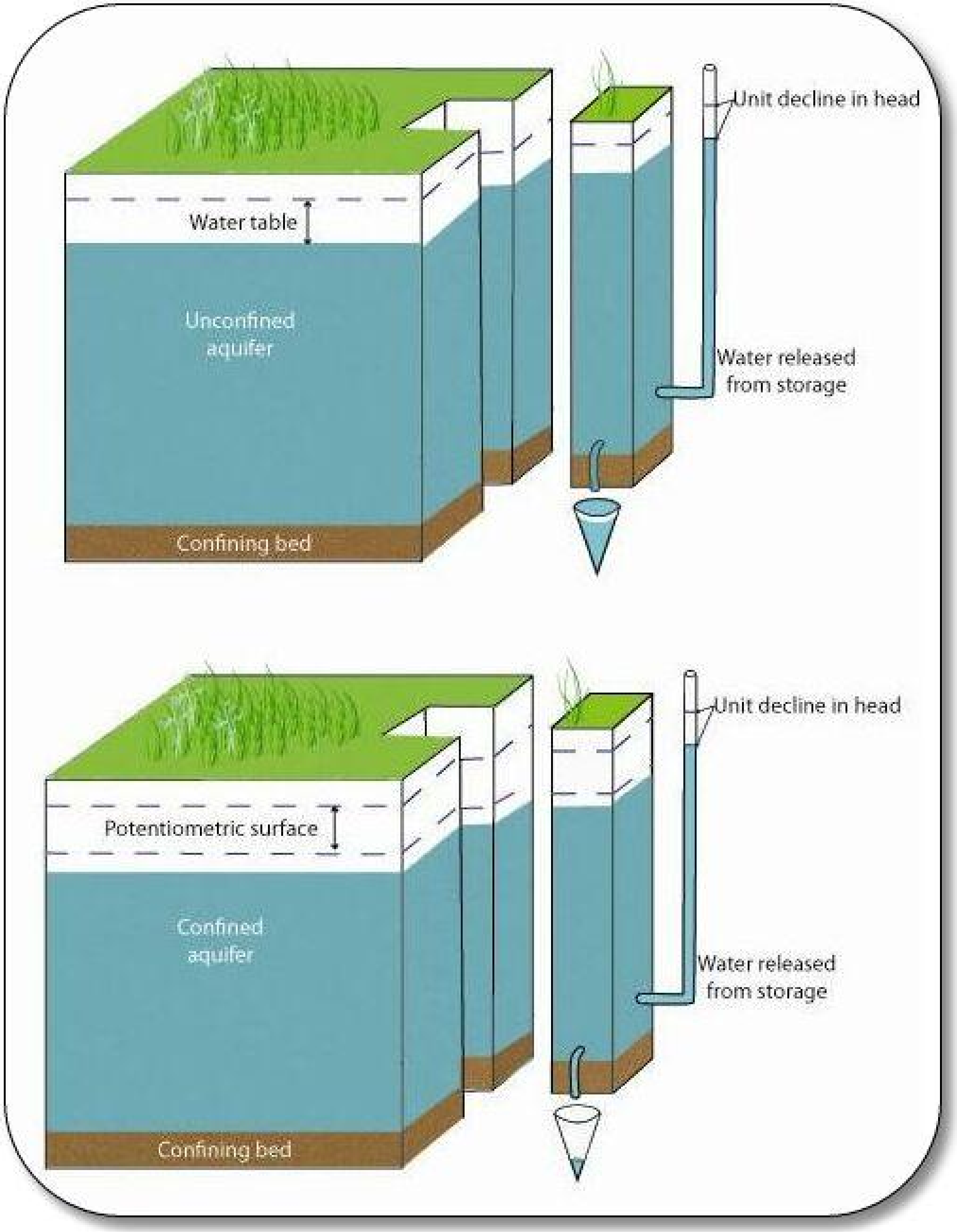
$$S_y = \frac{Gravity\ Drainage\ Volume}{Bulk\ Volume}$$

- For unconfined aquifers, the concept of specific yield could be expressed as the difference between storage coefficient and specific storage times the aquifer thickness.

$$S = S_y + bS_s$$

- The storage coefficient for confined aquifers is usually equal to or less than 0.005.
- The storage coefficient for unconfined aquifers is almost between 0.2 and 0.3.

Water Yield



Unit decline in head
Water table
Unconfined aquifer
Water released from storage
Confining bed
Unit decline in head
Potentiometric surface
Confined aquifer
Water released from storage
Confining bed

Specific Retention

- Specific retention:

$$S_r = \eta - \eta_e$$

- Effective Porosity

$$\eta_e = \frac{Volume\ of\ water\ able\ to\ circulate}{Total\ volume\ of\ rock}$$

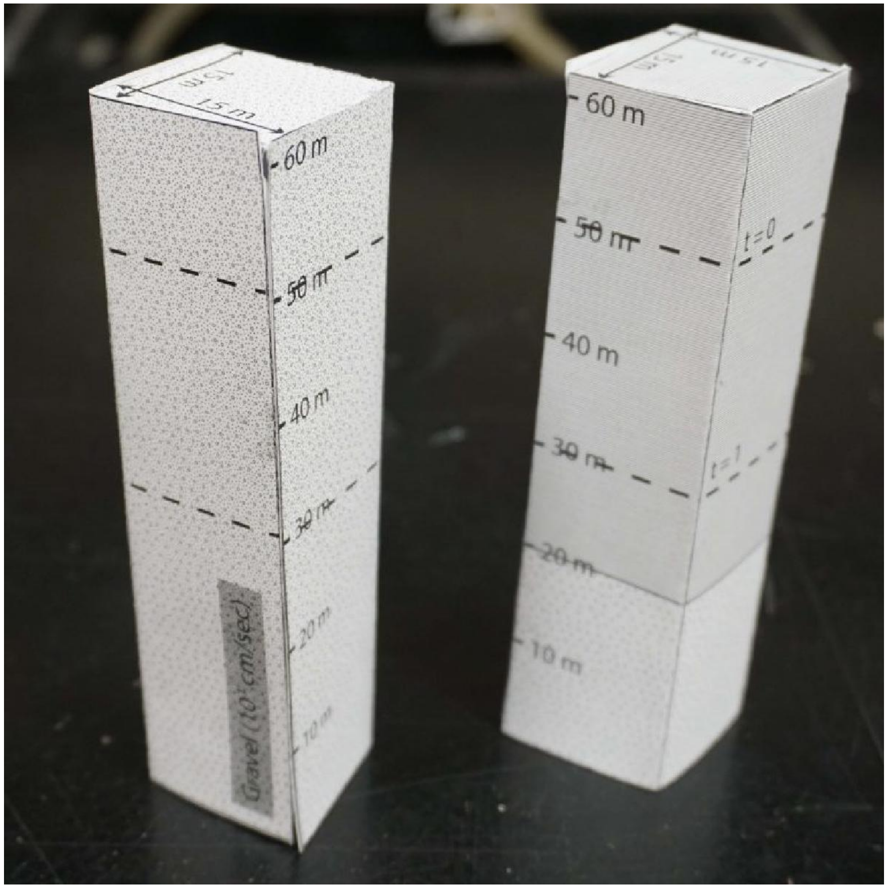
- Clay has a good retention and low yield

$$\eta = S_y + S_r$$

- Specific retention -> Field Capacity

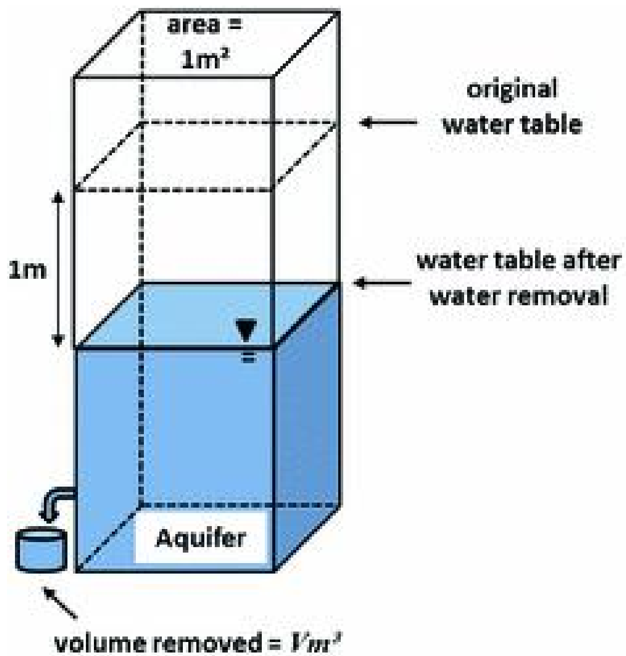
Home Lab

- Foldable Aquifer Project -http://aquifer.geology.buffalo.edu/
- Paper aquifer model
  - Groundwater Storage

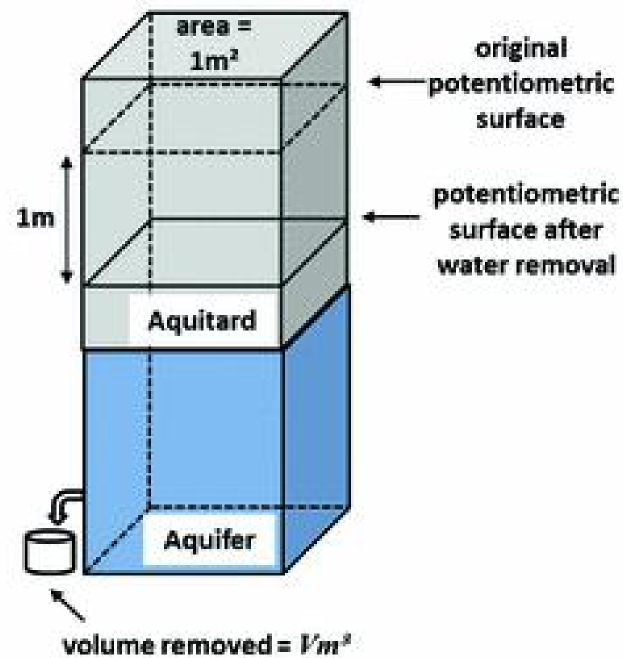


Home Lab (Contd.)

- Using the foldable aquifer models given below answer the following questions assuming that the volume of water released in each of the aquifers is 945 m^3.
  - A. Quantify the specific yield in the unconfined aquifer.
  - B. Quantify the specific storage in the confined aquifer.
  - C. Describe the difference between the storage mechanism in the unconfined aquifer as compared to the confined aquifer.



(a) Specific yield



(b) Storativity

Problem 1

- Determine the total stress acting on the top of the aquifer underlying a clayey aquitard, which is 30 m thick, and calculate the effective stress in the aquifer. Assume the bulk density of the aquitard is 2500 kg/m³ and there is an average of 40 m pressure head in it. If pumping causes a reduction of 1.5 m in the hydraulic head in the aquifer, what will be the resulting changes in the pore water pressure and effective stress. Consider the density of water equal to 1000 kg/m³.

$$\sigma_T = \rho_b g b = 2500 \times 9.81 \times 30 = 735750\ N/m^2$$

$$p = \rho_w g h = 1000 \times 9.81 \times 40 = 392400\ N/m^2$$

$$\sigma_e = \sigma_T - p = 343350\ N/m^2$$

$$\Delta p = \rho_w g \Delta h = 1000 \times 9.81 \times (-1.5) = -14715\ N/m^2$$

$$\Delta\sigma_e = \Delta\sigma_T - \Delta p = -\Delta p = 14715\ N/m^2$$

Problem 2

- Given a porosity of 0.22 and a dry bulk density of 1.95 g/cm3, what is the particle density?

$$\rho_d = \rho_s = \frac{m_s}{V_s} = \frac{m_s}{V} \times \frac{V}{V_s} = \frac{\rho_b}{(1-\eta)} = \frac{1.95}{(1-0.22)} = 2.5\ g/cm^3$$

Problem 3

- An aquifer with an area of 7 km² experiences a head drop of 0.85 m after 8 years of pumping. If the pumping rate is 5.5 m³/day, determine the specific yield of the aquifer.

$$S_y = \frac{V_w}{A\Delta h} = \frac{Q\Delta t}{A\Delta h} = \frac{5.5\,\frac{m^3}{day} \times 8\;year \times 365\,\frac{day}{year}}{7\;km^2 \times 10^6\,\frac{m^2}{km^2} \times 0.85\;m} = 2.7 \times 10^{-3}$$

Problem 4

- A soil sample has a volume of 180 cm³. The volume of voids in the sample is estimated equal to 67 cm³. Out of the volume of voids, water can move through only 45 cm³. Determine the porosity, effective porosity, specific retention, and specific yield of the soil. What is the area of the aquifer, if pumping at a rate 6 m³/day causes 1 m head drop in the aquifer in 5 years?

$$\eta = \frac{V_v}{V} = \frac{67}{180} = 0.37$$

$$\eta_e = \frac{V_{cv}}{V} = \frac{45}{180} = 0.25$$

$$S_r = \eta - \eta_e = 0.12$$

$$S_y = \eta - S_r = \eta_e = 0.25$$

$$A = \frac{V_w}{S_y \Delta h} = \frac{Q\Delta t}{A\Delta h} = \frac{6.0\,\frac{m^3}{day} \times 5\ year \times 365\ \frac{day}{year}}{0.25 \times 1.00\ m} = 43800\ m^2$$

Problem 5

- The thickness of the saturated layer in a field is 8.2 m. The porosity of the soil was measured equal to 0.28. The elasticity modulus soil and water are 5.3×10^7 N/m^2 and 2.1×10^9 N/m^2, respectively. Determine the corresponding storage coefficient for this layer. Assume the temperature is 20ºC.

$$\beta = \frac{1}{E_w} = 4.76 \times 10^{-10}\ m^2/N$$

$$\alpha = \frac{1}{E_s} = 1.89 \times 10^{-8}\ m^2/N$$

$$S = S_s b = (\eta\beta + \alpha)\rho g b = (0.28 \times 4.76 \times 10^{-10} + 1.89 \times 10^{-8}) \times 1000 \times 9.81 \times 8.2$$
$$= 1.53 \times 10^{-3}$$

Problem 6

- The porosity of a confined aquifer is 42%. The thickness of this aquifer is 78 m. The compressibility of water and the granular matrix are 4.76×10^{-10} and 6.3×10^{-8}, respectively. Estimate the specific storage and storativity of the aquifer. Also, calculate how much water is released if there is a total head drop of 100 m and the area is 1200 km^2.

$$S = S_s b = (\eta\beta + \alpha)\rho g b = (0.42 \times 4.76 \times 10^{-10} + 6.3 \times 10^{-8}) \times 1000 \times 9.81 \times 78$$
$$= 4.84 \times 10^{-2}$$

$$V_w = S\Delta hA = 4.84 \times 10^{-2} \times 100 \times 1200\ km^2 \times 10^6 m^2/km^2 = 5.81 \times 10^9 m^3$$

Problem 7

- The total volume of a soil sample is 280 cm^3. The volume of water and volume of voids were measured equal to 65 and 115 cm^3, respectively. Determine the porosity, volumetric water content, and Degree of saturation of water for the current sample.

$$\eta = \frac{V_v}{V} = \frac{115}{280} = 0.41$$

$$\theta_v = \frac{V_w}{V} = \frac{65}{280} = 0.23$$

$$S_w = \frac{V_w}{V_v} = \frac{\theta_v}{\eta} = 0.56$$

Learning Strategy

Chapter 6: Deformation, Storage, and General Flow Equations

Thank you