Principles of Flow

Geohydraulics| CE60113

Lecture:08

Learning Objective(s)

- To estimate hydraulic conductivity

Hydraulic Conductivity

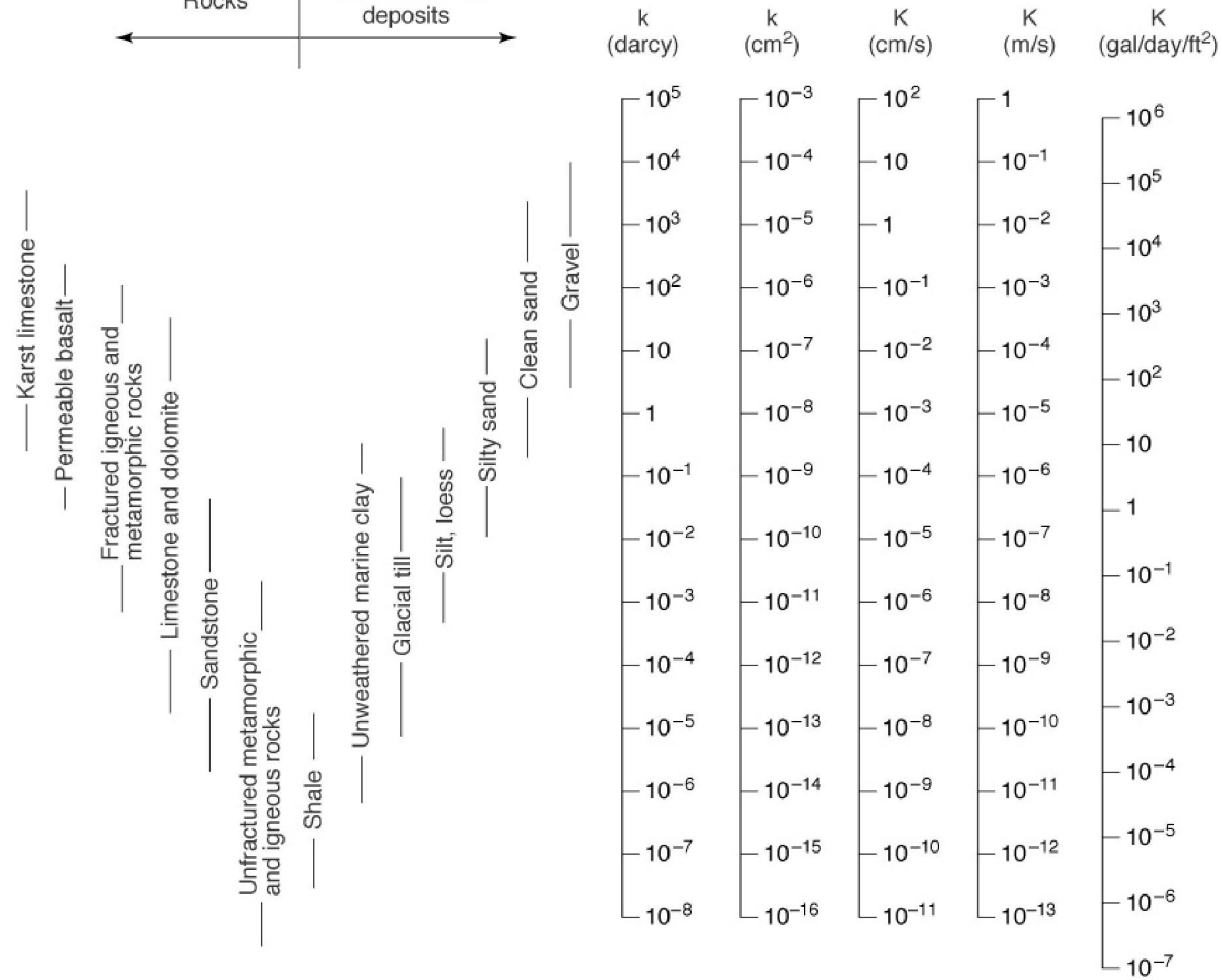
- Other factors being equal, the average velocity of groundwater migration is proportional to K
- Hydraulic conductivity is an empirical constant measured in laboratory or field experiments.
- Historically, *Permeability* ≡ *Hydraulic Conductivity*
- Now its usage is associated with *intrinsic permeability*

$$K = k\frac{\rho g}{\mu}$$

k	=	intrinsic permeability	Porous medium property
ρ	=	density	Fluid properties
μ	=	dynamic viscosity	
g	=	Gravitational acceleration constant	

Hydraulic Conductivity (Contd.)

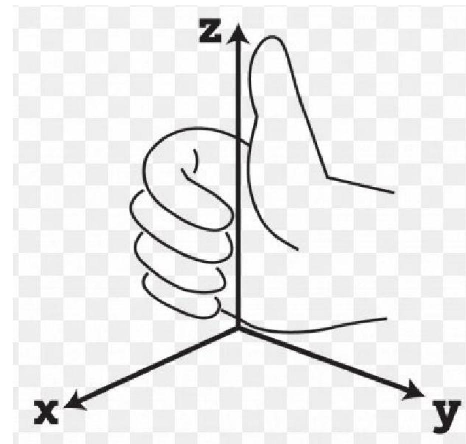
$$1 \text{ darcy} = 9.87 \times 10^{-9} \text{ cm}^2$$



Darcy’s Law in Three Dimensions

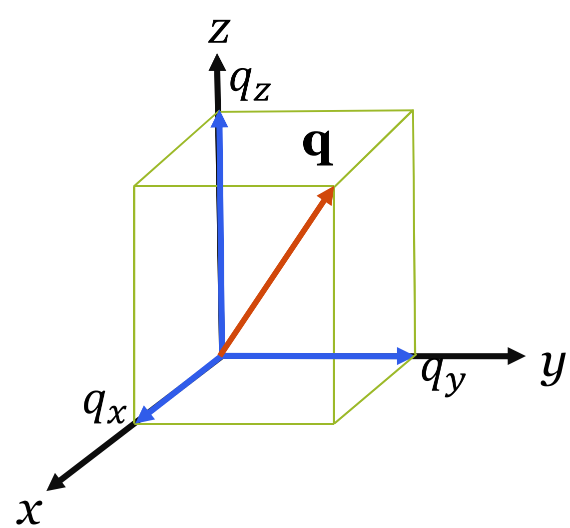
- Components of flow

$$q_x = -K_x \frac{\partial h}{\partial x}$$
$$q_y = -K_y \frac{\partial h}{\partial y}$$
$$q_z = -K_z \frac{\partial h}{\partial z}$$
$$\mathbf{q} = q_x \hat{\imath} + q_y \hat{\jmath} + q_z \hat{k}$$



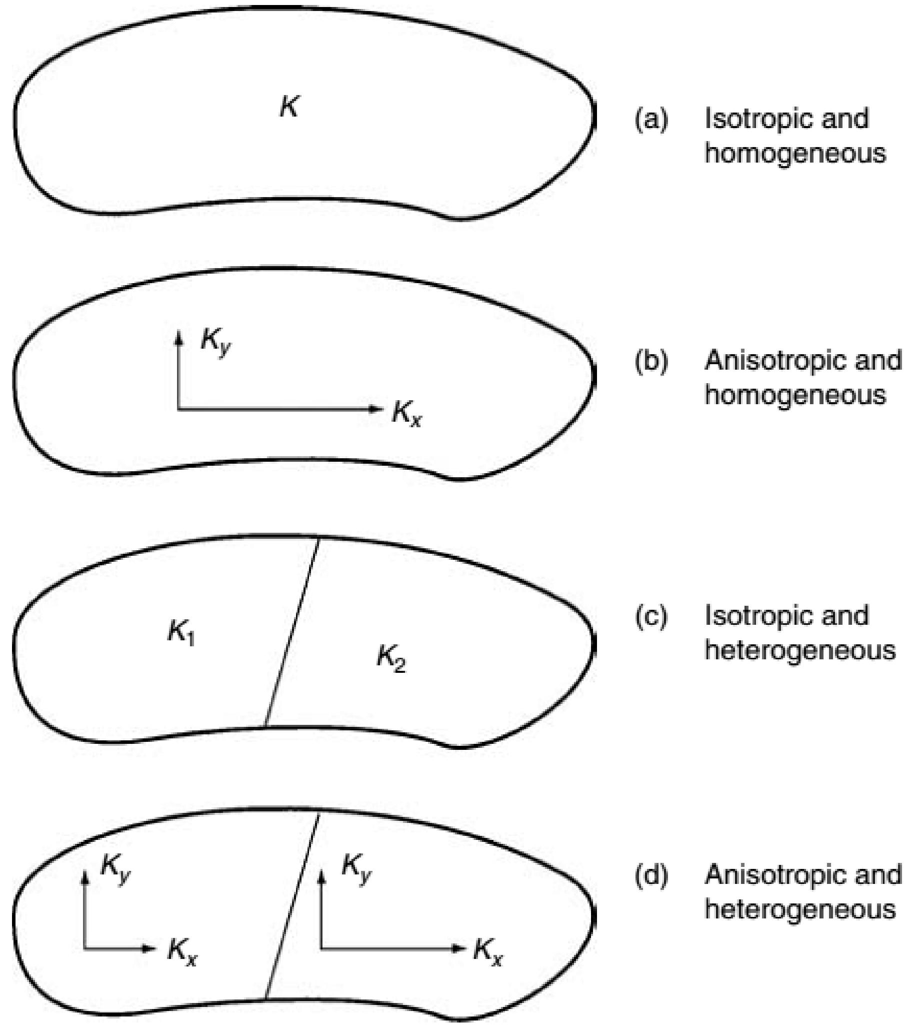
- The magnitude of specific discharge vector

$$|\mathbf{q}| = \sqrt{q_x^2 + q_y^2 + q_z^2}$$



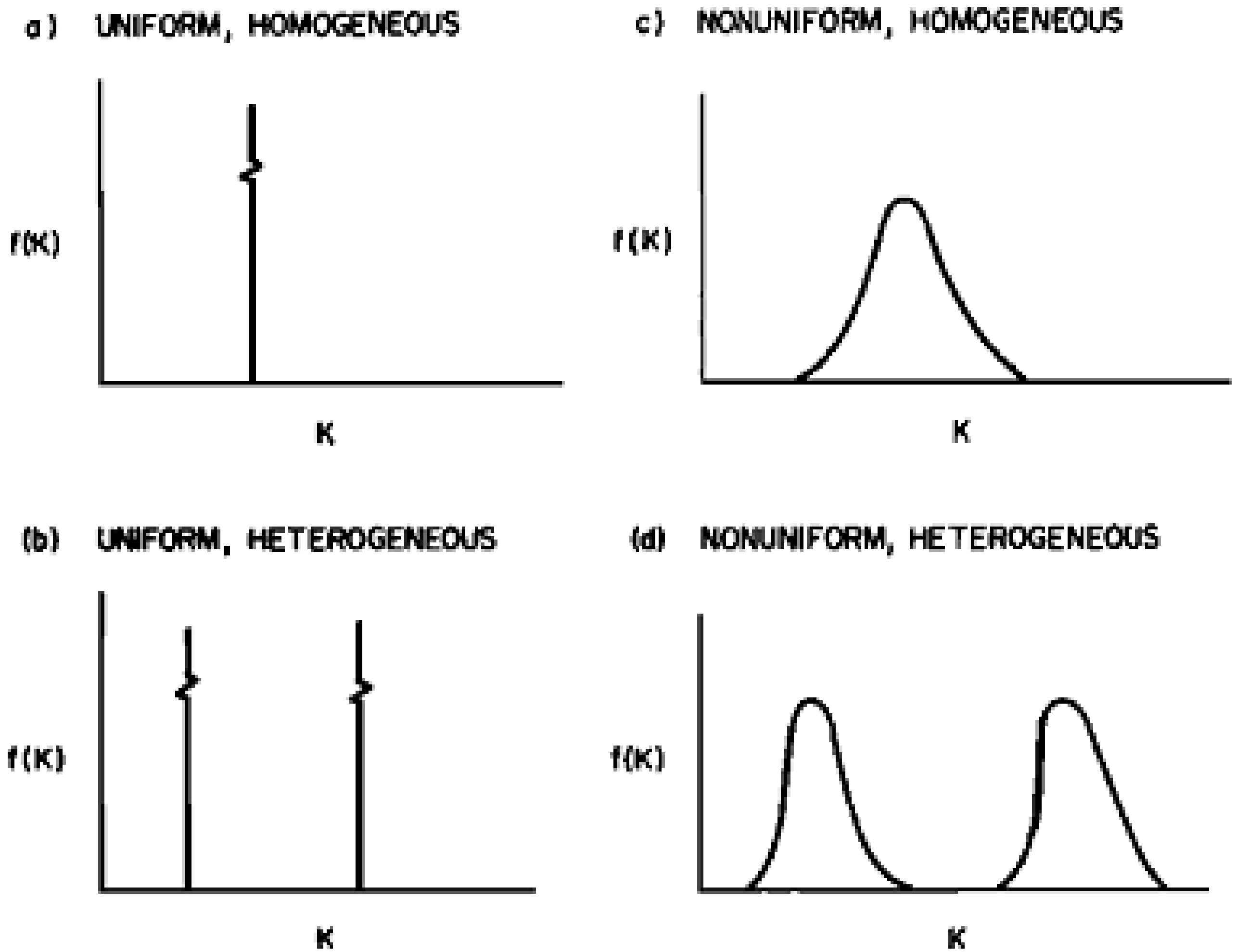
Heterogeneity and Anisotropy of Hydraulic Conductivity

- Deterministic Approach



Heterogeneity and Nonuniformity of Hydraulic Conductivity

- Stochastic Approach

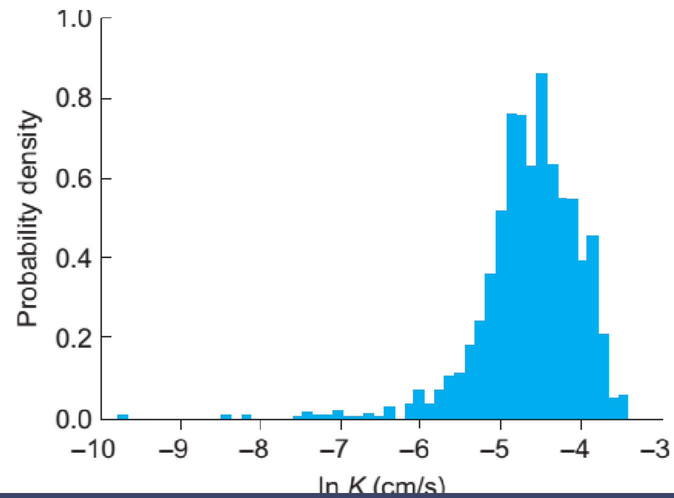


Probabilistic Analysis

- Matherton (1967) determined that the geometric mean of small-scale K measurements gives the appropriate large-scale average K under the following circumstances:
- The K histogram is log-normal.
- K has a statistically isotropic distribution in space.
- Flow is two-dimensional.
- Flow is uniform (one-dimensional on a large scale).

The geometric mean K_g of n K measurements is calculated as

$$K_g = (K_1 K_2 K_3 \cdots K_n)^{1/n}$$



Specific Discharge Vectors at an Interface

- At the boundary between two materials with differing hydraulic conductivities, the flow paths are bent in a manner similar to optical refraction.
- At the material interface, two conditions must be met:
  - The specific discharge normal to the interface is the same on both sides of the interface to preserve continuity of flow.

$$q_{n1} = q_{n2}$$

  - Pressure must be continuous in a fluid. Therefore head must also be continuous across the interface.

$$\left(\frac{\partial h}{\partial t}\right)_1 = \left(\frac{\partial h}{\partial t}\right)_2$$

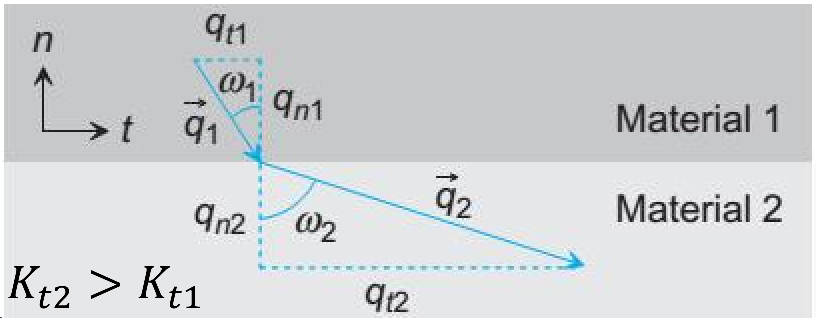
- The angles $\omega_1$and ω_2 are related to the specific discharge components

$$\tan \omega_1 = \frac{q_{t1}}{q_{n1}}, \tan \omega_2 = \frac{q_{t2}}{q_{n2}}$$

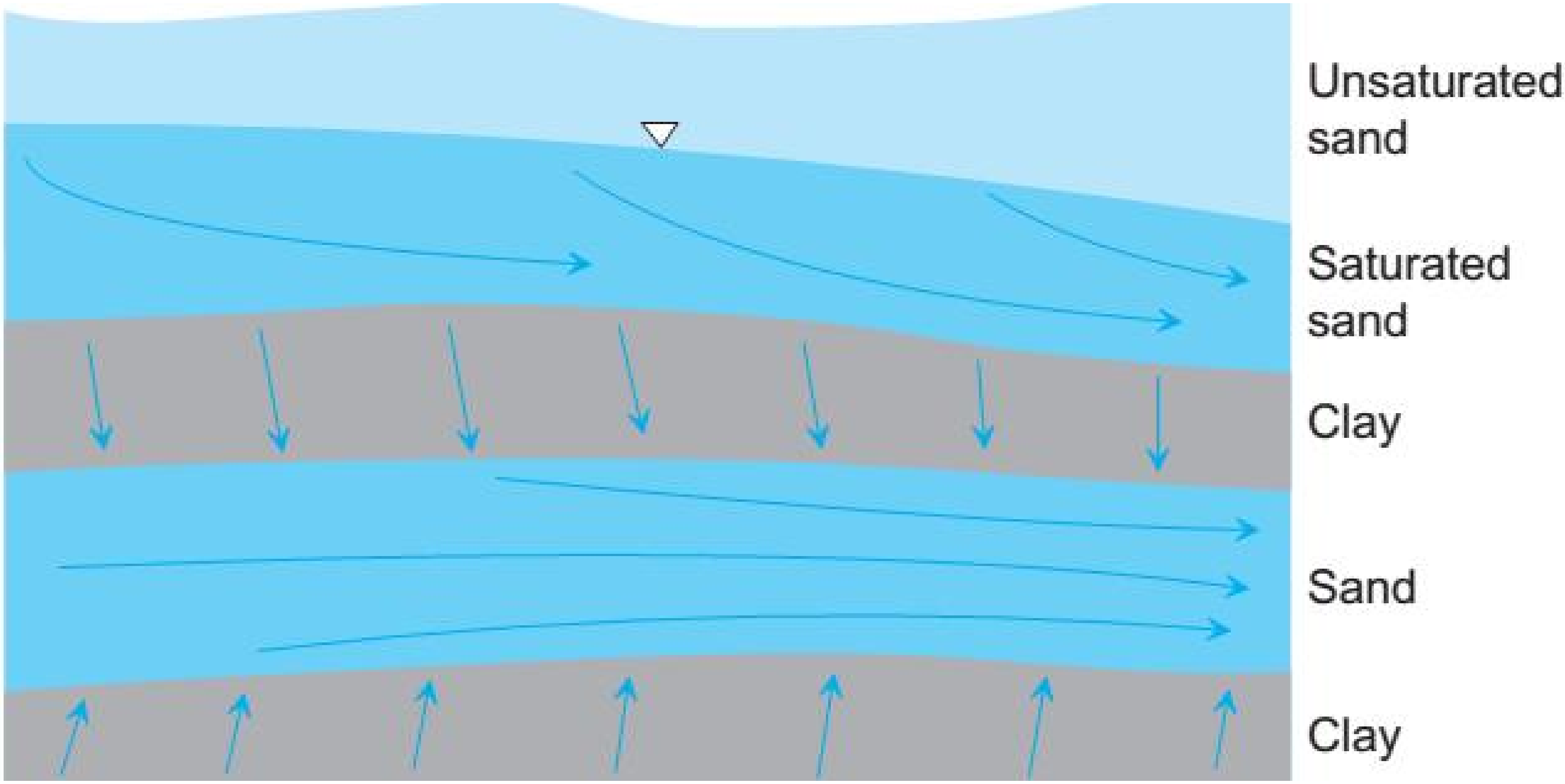
- Using Darcy's law

$$\frac{\tan \omega_1}{\tan \omega_2} = \frac{K_{t1}}{K_{t2}}$$

- When $K_{t1} \ll K_{t2}$, $\omega_1 \to 0$ and $\omega_2 \to \frac{\pi}{2}$



Specific Discharge Vectors at an Interface (Contd.)



Estimating Average Hydraulic Conductivities

- Specific Discharge

$$q_z = q_{z1} = q_{z2} = q_{z3} = \cdots = q_{zn}$$

- The specific discharge q_{zi} for the i^{th} layers is given by

$$q_{zi} = -K_{zi}\frac{\Delta h_i}{d_i}$$

- Total head loss

$$\Delta h = \Delta h_1 + \Delta h_2 + \Delta h_3 + \cdots + \Delta h_n$$

Or,

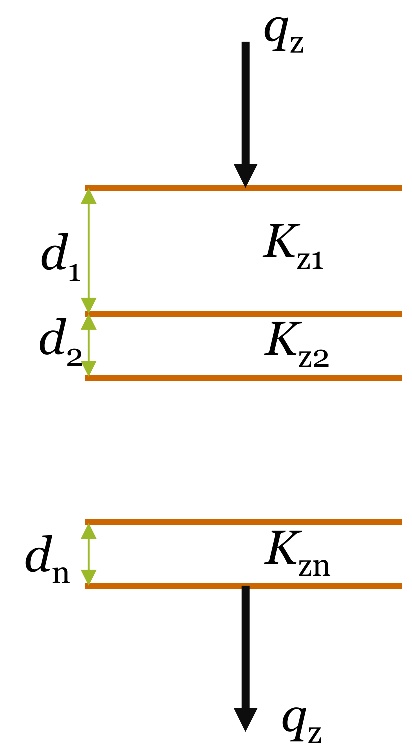
$$q_z\frac{d}{K_{ze}} = q_{z1}\frac{d_1}{K_{z1}} + q_{z2}\frac{d_2}{K_{z2}} + q_{z3}\frac{d_3}{K_{z3}} + \cdots + q_{zn}\frac{d_n}{K_{zn}}$$

Or,

$$\frac{d}{K_{ze}} = \frac{d_1}{K_{z1}} + \frac{d_2}{K_{z2}} + \frac{d_3}{K_{z3}} + \cdots + \frac{d_n}{K_{zn}}$$

Or,

$$K_{ze} = \frac{\sum_{i=1}^{n} d_i}{\sum_{i=1}^{n}\frac{d_i}{K_{zi}}}$$



Estimating Average Hydraulic Conductivities (Contd.)

- Discharge Q_{xi} for the i^{th} layers is given by

$$Q_{xi} = -K_{xi}\frac{\partial h}{\partial x}d_i$$

- Total Discharge

$$Q_x = Q_{x1} + Q_{x2} + Q_{x3} + \cdots + Q_{xn}$$

Or,

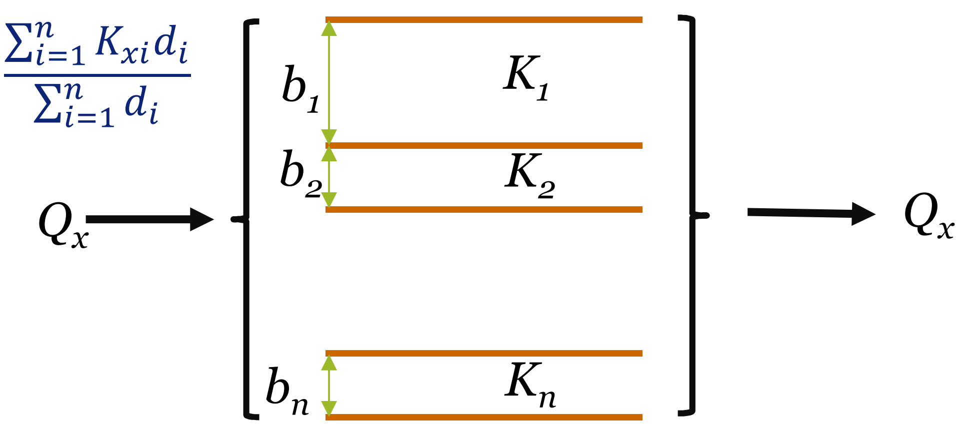
$$K_{xe}\frac{\partial h}{\partial x}d = K_{x1}\frac{\partial h}{\partial x}d_1 + K_{x2}\frac{\partial h}{\partial x}d_2 + K_{x3}\frac{\partial h}{\partial x}d_3 + \cdots + K_{xn}\frac{\partial h}{\partial x}d_n$$

Or,

$$K_{xe}d = K_{x1}d_1 + K_{x2}d_2 + K_{x3}d_3 + \cdots + K_{xn}d_n$$

Or,

$$K_{xe} = \frac{\sum_{i=1}^{n} K_{xi}d_i}{\sum_{i=1}^{n} d_i}$$



Horizontal Hydraulic Conductivity

The horizontal hydraulic conductivity in alluvium is normally greater than that in the vertical direction:

$$K_H > K_V$$

Imagine a two-layer case:

$$\frac{K_1 d_1 + K_2 d_2}{d_1 + d_2} > \frac{d_1 + d_2}{\frac{d_1}{K_1} + \frac{d_2}{K_2}}$$

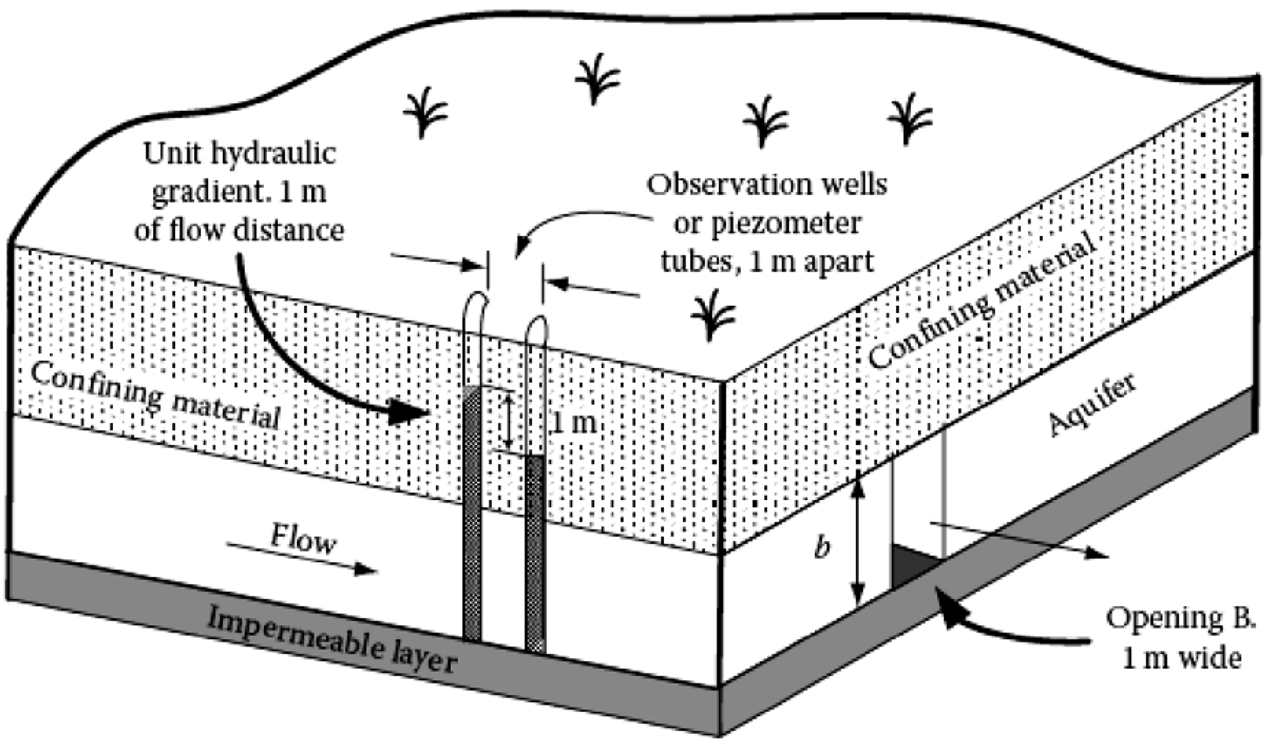
- Ratios of K_H/K_V, usually fall in the range of 2–10 for alluvium, but values up to 100 or more occur where clay layers are present.

Transmissivity

- If the hydraulic conductivity tangential to the layer K can be assumed constant over the thickness b of a layer, the transmissivity T of the layer is simply

$$T = Kb$$

- Transmissivity is the amount of water that moves horizontally through a unit width of a saturated aquifer as a result of a unit change in gradient.



Transmissivity (Contd.)

- If a layer is composed of m strata of thickness b_i and hydraulic conductivity K_i, the total transmissivity of the layer is the sum of the transmissivities of each stratum:

$$T = \sum_{i=1}^{m} T_i$$

- In an unconfined aquifer, transmissivity is not as well defined as in a confined aquifer.

Measuring Hydraulic Conductivity

- Correlations of Grain Size to Hydraulic Conductivity
- Hazen (1911) proposed the following empirical relation, based on experiments with various sand samples:

$$K = C(d_{10})^2$$

where K is hydraulic conductivity in cm/sec, C is a constant with units of $(cm\ sec)^{-1}$, and d_{10} is the grain diameter in centimeters such that grains this size or smaller represent 10% of the sample mass.

- This equation requires a fixed set of units.
- The constant C varies from about 40 to 150 for most sands.
- C is at the low end of this range for fine, widely graded sands, and C is near the high end of the range for coarse, narrowly graded sands.
- Kozeny–Carman equation

$$K = \left(\frac{\rho_w g}{\mu}\right)\left(\frac{n^3}{(1-n)^2}\right)\left(\frac{(d_{50})^2}{180}\right)$$

- Kozeny-Carman equation is dimensionally consistent

Laboratory Measurement of Hydraulic Conductivity

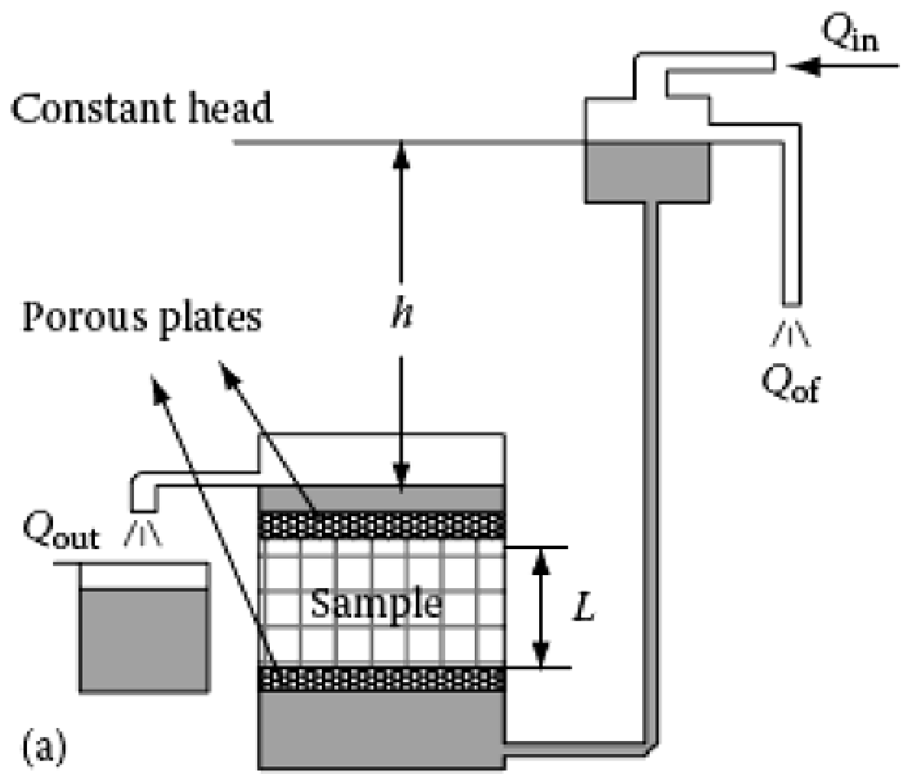
- Constant head permeameter
- Q_{of} is the rate of overflow. From Darcy's law we have

$$Q = -KA\frac{h_2 - h_1}{L}$$

$$K = \frac{QL}{Ah}$$

where:

Q is the flow rate
A is the cross-sectional area
L is the length of the sample
h is the constant head.



Laboratory Measurement of Hydraulic Conductivity (contd.)

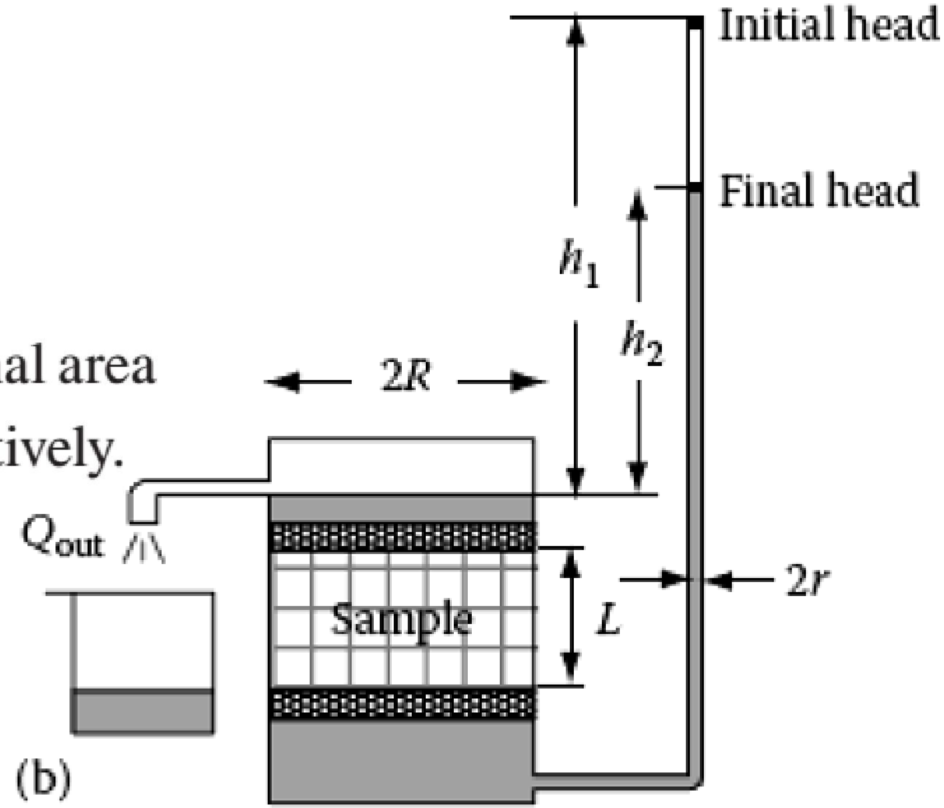
- Falling head

$$Q_{\text{in}} = -A_t \frac{\mathrm{d}h}{\mathrm{d}t}$$

$$Q_{\text{out}} = KA_c \frac{h}{l}$$

where A_t and A_c are the cross-sectional area of the tube and the container, respectively.

In steady-state condition, $Q_{\text{in}} = Q_{\text{out}}$:



Falling head permeameter

$$-A_t \frac{\mathrm{d}h}{\mathrm{d}t} = KA_c \frac{h}{l}$$

$$\int_{h_1}^{h_2} \frac{\mathrm{d}h}{h} = \frac{-K}{L} \frac{A_c}{A_t} \int_{t_1}^{t_2} \mathrm{d}t$$

$$\left[\ln h\right]_{h_1}^{h_2} = \frac{-K}{L} \frac{A_c}{A_t} \left(t_2 - t_1\right)$$

$$\ln \frac{h_2}{h_1} = \frac{-K}{L} \frac{A_c}{A_t} \left(t_2 - t_1\right)$$

$$K = \frac{A_t L}{A_c \left(t_2 - t_1\right)} \ln \frac{h_1}{h_2}$$

where:

L is the length of the sample

h_1 and h_2 are the heads at the beginning, t_1 and at time t_2 later.

Learning Strategy

Chapter 3: Principles of Flow

Thank you