Parameter Estimation

Geohydraulics| CE60113

Lecture:14

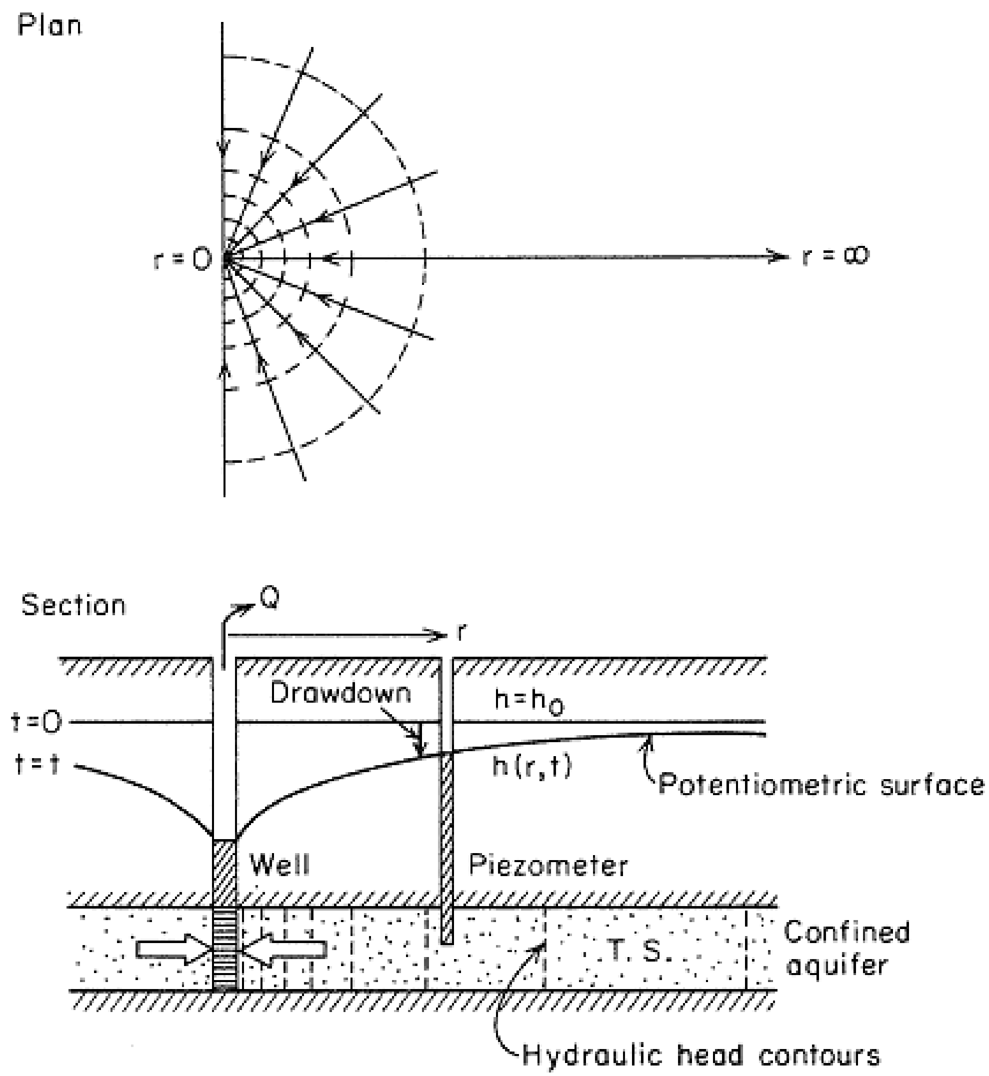
Learning Objective(s)

- **To estimate aquifer parameter under unsteady confined flow condition**

Unsteady Radial Flow

- Confined Aquifer
  - Assumptions
    - Aquifer→ Homogeneous, Isotropic, Infinite extent
    - Initial piezometric surface → Horizontal
    - Pumping at well → Constant rate
    - Well → fully penetrating
    - Flow → Horizontal
    - Well diameter → infinitesimal (storage can be neglected)
    - Aquifer storage release → Instantaneous
- Unconfined Aquifer
- Semiconfined/Leaky Aquifer

Unsteady Radial Flow in Confined Aquifer



Unsteady Radial Flow in Confined Aquifer (Contd.)

- Confined Aquifer

$$S_s \frac{\partial h}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(K_r r \frac{\partial h}{\partial r}\right) + \cancelto{0}{\frac{1}{r}\frac{\partial}{\partial \theta}\left(\frac{K_\theta}{r}\frac{\partial h}{\partial \theta}\right)} + \cancelto{0}{\frac{\partial}{\partial z}\left(K_z \frac{\partial h}{\partial z}\right)} - \cancelto{0}{W}$$

or,

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r}\frac{\partial h}{\partial r} = \frac{S}{T}\frac{\partial h}{\partial t}$$

Subject to

Initial Condition (IC)

$$h(r, 0) = h_0$$

Boundary Conditions (BCs)

$$h(r \to \infty, t) = h_0$$

$$Q = \lim_{r \to 0}[2\pi r b(-q_r)] = \lim_{r \to 0}\left[2\pi r \left(T \frac{\partial h}{\partial r}\right)\right]$$

or,

$$\lim_{r \to 0}\left(r \frac{\partial h}{\partial r}\right) = \frac{Q}{2\pi T}$$

Unsteady Radial Flow in Confined Aquifer (Contd.)

- Theis Solution

$$h_0 - h(r,t) = \frac{Q}{4\pi T}\int_u^\infty \frac{1}{\omega}e^{-\omega}d\omega$$

with

$$u = \frac{r^2 S}{4Tt}$$

In terms of drawdown

$$s(r,t) = \frac{Q}{4\pi T}W(u)$$

Theis Well Function

$$W(u) = \int_u^\infty \frac{1}{\omega}e^{-\omega}d\omega$$

Incomplete Gamma Function

$$\Gamma(a,u) = \int_u^\infty \omega^{a-1}e^{-\omega}d\omega$$

Well Function

$$W(u) = \Gamma(0,u)$$

Unsteady Radial Flow in Confined Aquifer (Contd.)

- Analysis

Well Function in terms of infinite series

$$W(u) = -\gamma - ln(u) + \sum_{k=1}^{\infty} \frac{(-1)^{k+1} u^k}{k\, k!}$$

where γ = Euler–Gamma Constant =0.5772156649

As $t \to \infty, u \to 0, W(u) \to 0$

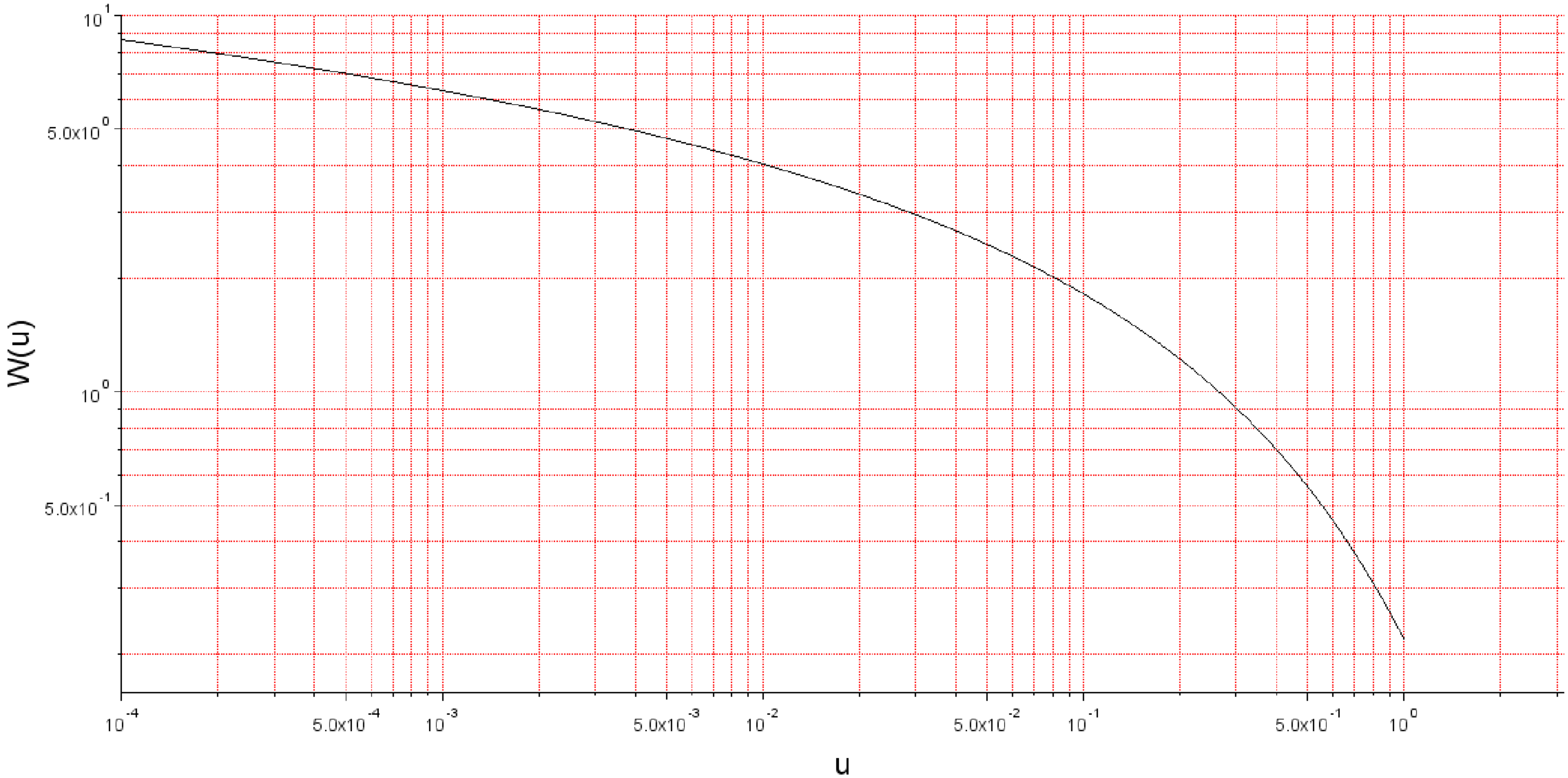
$$\frac{\partial s}{\partial t} = \frac{Q}{4\pi T}\left(-\frac{1}{u}e^{-u}\right)\frac{\partial u}{\partial t} = \frac{Q}{4\pi T}\left(\frac{1}{t}e^{-u}\right)$$

For large t or small r, $e^{-u} \to 1$

$$\frac{\partial s}{\partial t} = \frac{Q}{4\pi T}\left(\frac{1}{t}\right)$$

Unsteady Radial Flow in Confined Aquifer (Contd.)

- Well Function



Unsteady Radial Flow in Confined Aquifer (Contd.)

- Pumping Test Data: $Q = 2500\ m^3/day$

($r = 60$ m)					
t, min	s, m	r^2/t, m^2/min	t, min	s, m	r^2/t, m^2/min
0	0	∞	18	0.67	200
1	0.20	3,600	24	0.72	150
1.5	0.27	2,400	30	0.76	120
2	0.30	1,800	40	0.81	90
2.5	0.34	1,440	50	0.85	72
3	0.37	1,200	60	0.90	60
4	0.41	900	80	0.93	45
5	0.45	720	100	0.96	36
6	0.48	600	120	1.00	30
8	0.53	450	150	1.04	24
10	0.57	360	180	1.07	20
12	0.60	300	210	1.10	17
14	0.63	257	240	1.12	15

Unsteady Radial Flow in Confined Aquifer (Contd.)

- Numerical Solution

$$Min\ F(S,T) \equiv \sum_{l=1}^{N_T} \sum_{i=1}^{N_l} \left[s_i^l - \frac{Q}{4\pi T} W\left(\frac{r_i^2 S}{4T t_l}\right) \right]^2$$

First derivative:

$$F_1(S,T) = \frac{\partial F}{\partial S} = 0$$
$$F_2(S,T) = \frac{\partial F}{\partial T} = 0$$

Taylor series expansion

$$F_1(S+\Delta S, T+\Delta T) = F_1(S,T) + \frac{\partial F_1}{\partial S}\Delta S + \frac{\partial F_1}{\partial T}\Delta T + \mathcal{O}(\Delta S^2, \Delta T^2)$$
$$F_2(S+\Delta S, T+\Delta T) = F_2(S,T) + \frac{\partial F_2}{\partial S}\Delta S + \frac{\partial F_2}{\partial T}\Delta T + \mathcal{O}(\Delta S^2, \Delta T^2)$$

In compact form

$$\begin{Bmatrix} F_1(S+\Delta S, T+\Delta T) \\ F_2(S+\Delta S, T+\Delta T) \end{Bmatrix} \approx \begin{Bmatrix} F_1(S,T) \\ F_2(S,T) \end{Bmatrix} + \begin{bmatrix} \dfrac{\partial F_1}{\partial S} & \dfrac{\partial F_1}{\partial T} \\ \dfrac{\partial F_2}{\partial S} & \dfrac{\partial F_2}{\partial T} \end{bmatrix} \begin{Bmatrix} \Delta S \\ \Delta T \end{Bmatrix}$$

Unsteady Radial Flow in Confined Aquifer (Contd.)

- Jacobian

$$\mathbf{J}(S,T)=\begin{bmatrix}\dfrac{\partial F_1}{\partial S} & \dfrac{\partial F_1}{\partial T}\\ \dfrac{\partial F_2}{\partial S} & \dfrac{\partial F_2}{\partial T}\end{bmatrix}$$

- Newton-Raphson

$$\begin{Bmatrix}S^{(p)}\\ T^{(p)}\end{Bmatrix}=\begin{Bmatrix}S^{(p-1)}\\ T^{(p-1)}\end{Bmatrix}-\mathbf{J}\left(S^{(p-1)},T^{(p-1)}\right)^{-1}\begin{Bmatrix}F_1\left(S^{(p-1)},T^{(p-1)}\right)\\ F_2\left(S^{(p-1)},T^{(p-1)}\right)\end{Bmatrix}$$

where p is the iteration number (≥ 1)

Unsteady Radial Flow in Confined Aquifer (Contd.)

- Graphical Solution: Theis Method
- CASE-I

$$\frac{r^2}{t} = \frac{4T}{S} u$$

$$s(r,t) = \frac{Q}{4\pi T} W(u)$$

- CASE-II

$$t = \frac{r^2 S}{4T} \frac{1}{u}$$

$$s(r,t) = \frac{Q}{4\pi T} W(u)$$

- CASE-III

$$\frac{t}{r^2} = \frac{S}{4T} \frac{1}{u}$$

$$s(r,t) = \frac{Q}{4\pi T} W(u)$$

Unsteady Radial Flow in Confined Aquifer (Contd.)

- CASE-I

$$log\left(\frac{r^2}{t}\right) = log\left(\frac{4T}{S}\right) + log(u)$$

$$log(s) = log\left(\frac{Q}{4\pi T}\right) + log\big(W(u)\big)$$

- Relation between $log(s)$ and $log\left(\frac{r^2}{t}\right)$ has the same form as the relation between $log\big(W(u)\big)$ and $log(u)$
- Two relations differ by a constant factor
- A plot of $log(s)$ vs. $log\left(\frac{r^2}{t}\right)$ should look the same as a plot of $log\big(W(u)\big)$ vs. $log(u)$
- Valid for multiple well case

Unsteady Radial Flow in Confined Aquifer (Contd.)

- Steps for CASE-I

$$log\left(\frac{r^2}{t}\right) = log\left(\frac{4T}{S}\right) + log(u)$$

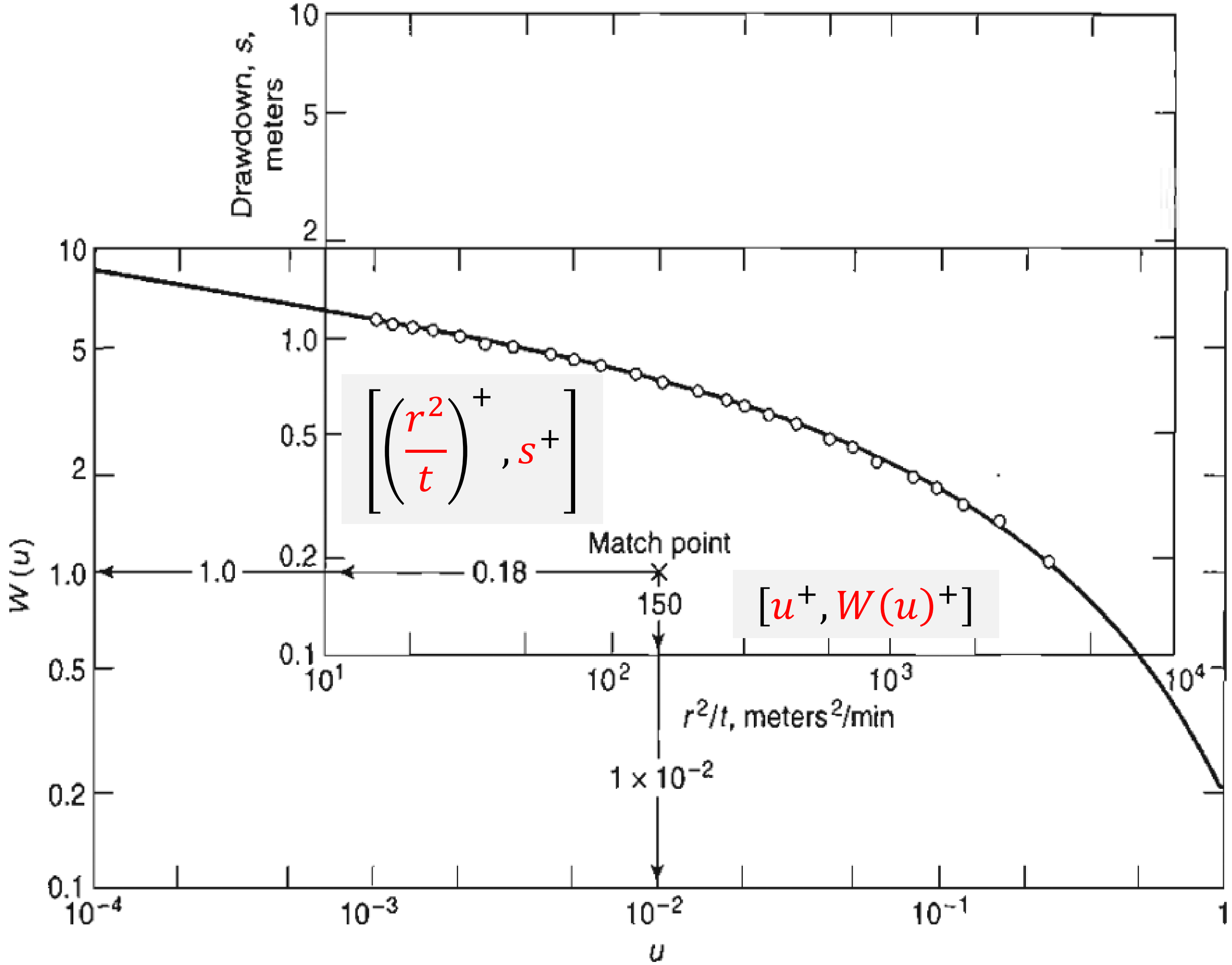
$$log(s) = log\left(\frac{Q}{4\pi T}\right) + log\big(W(u)\big)$$

- Plot the function $log\big(W(u)\big)$ vs. $log(u)$ on log-log paper. This is known as type curve.
- Plot the measured $log(s)$ and $log\left(\frac{r^2}{t}\right)$ on log-log paper of the same size and scale as the type curve. This plot is known as data curve.
- Match the data curve with type curve.
- For a pair of arbitrary points $\left[\left(\frac{r^2}{t}\right)^+, s^+\right]$ and $[u^+, W(u)^+]$, calculate

$$T = \frac{Q}{4\pi}\frac{W(u)^+}{s^+}$$

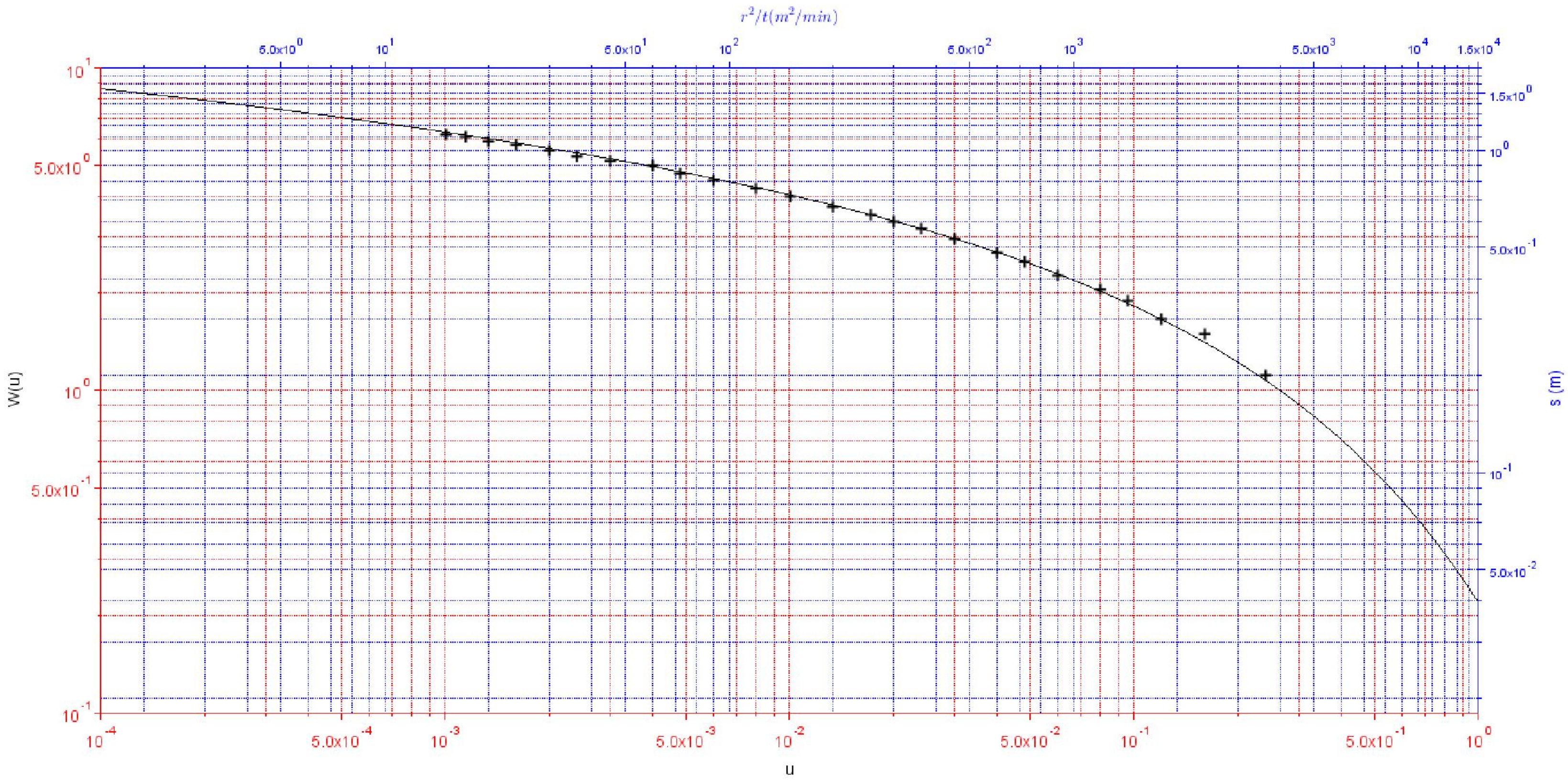
$$S = 4T\frac{u^+}{\left(\frac{r^2}{t}\right)^+}$$

Unsteady Radial Flow in Confined Aquifer (Contd.)



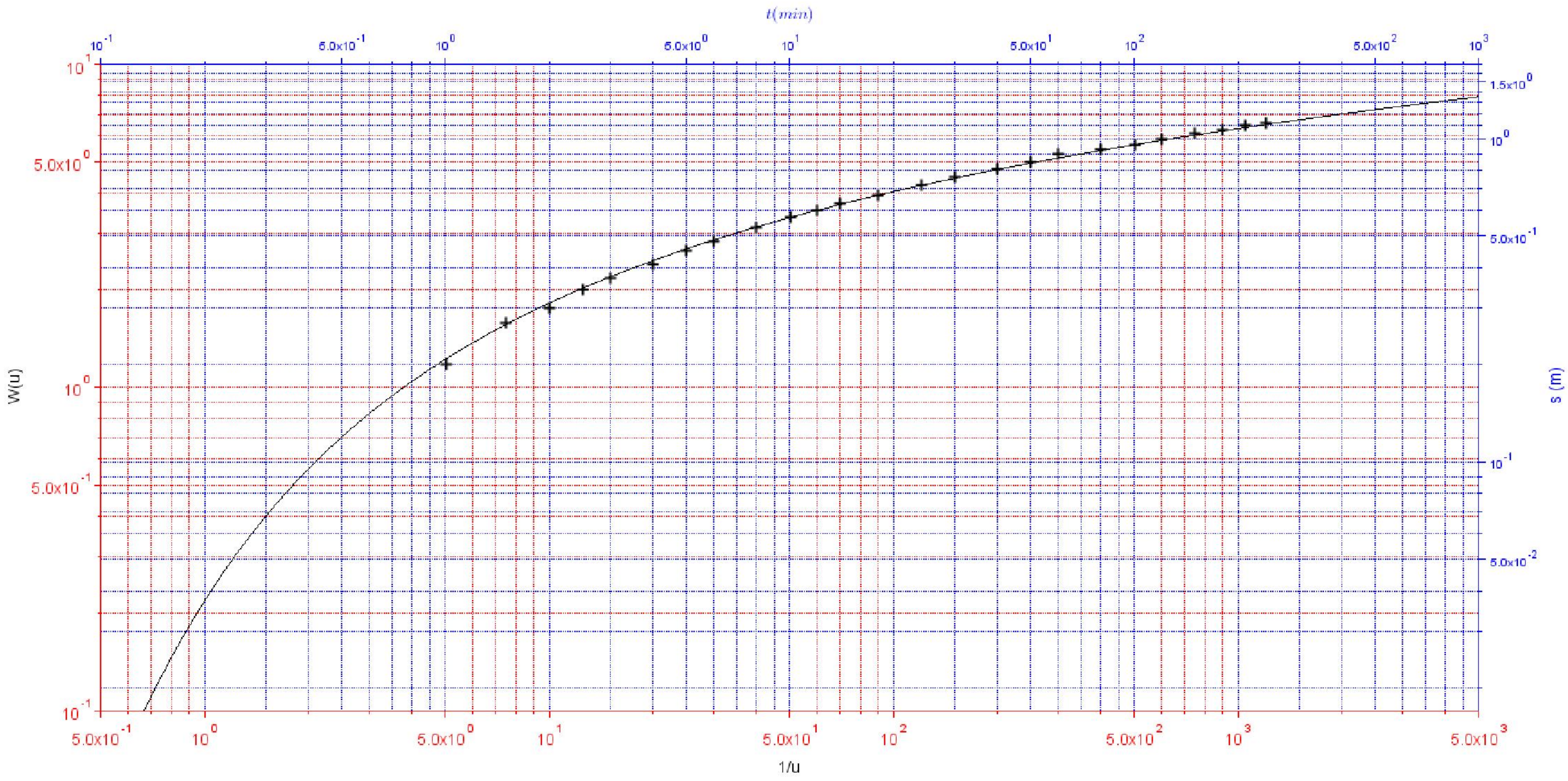
Unsteady Radial Flow in Confined Aquifer (Contd.)

- CASE-I



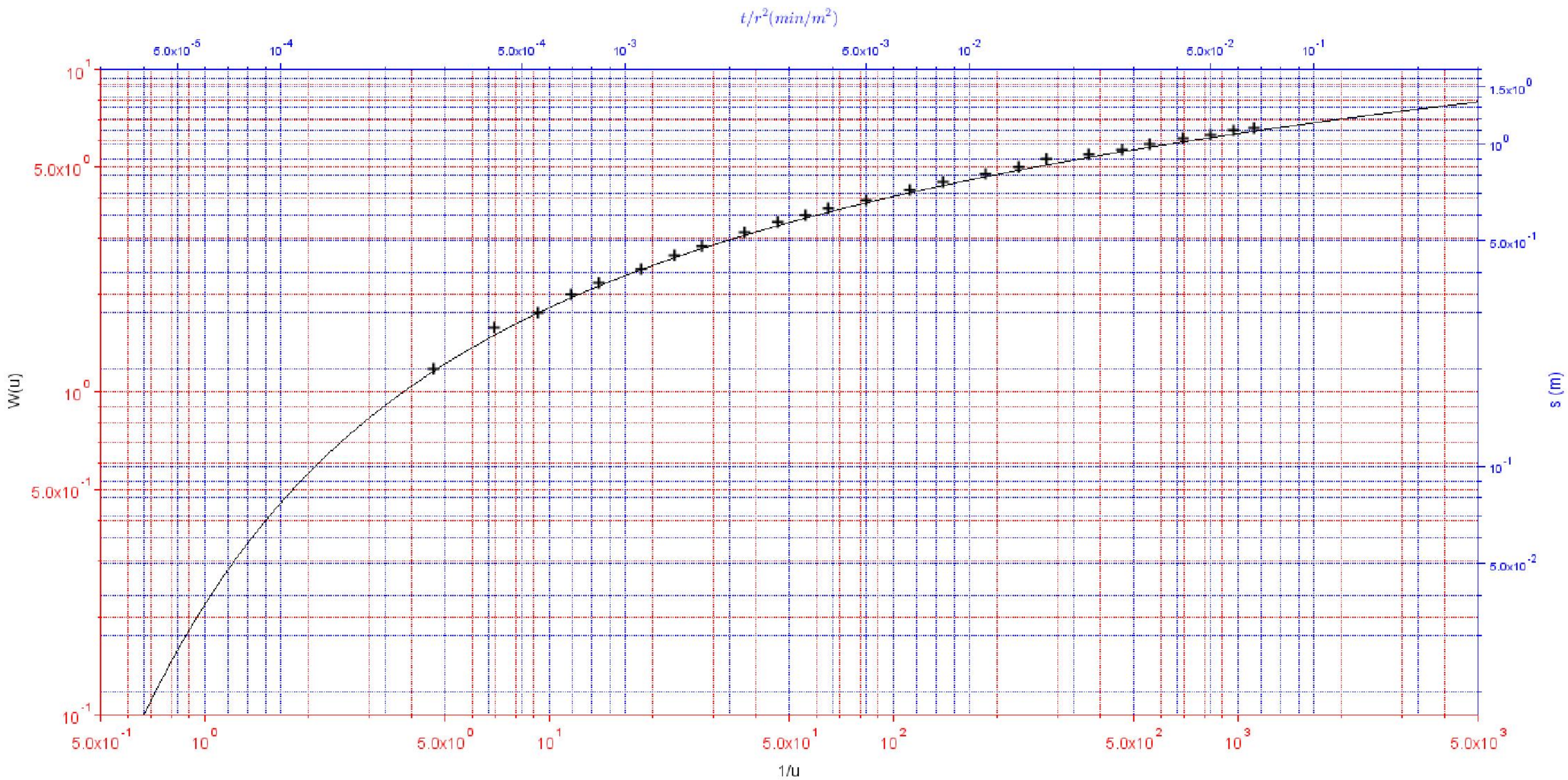
Unsteady Radial Flow in Confined Aquifer (Contd.)

- CASE-II



Unsteady Radial Flow in Confined Aquifer (Contd.)

- CASE-III



Thank you