Surface and Subsurface Water System

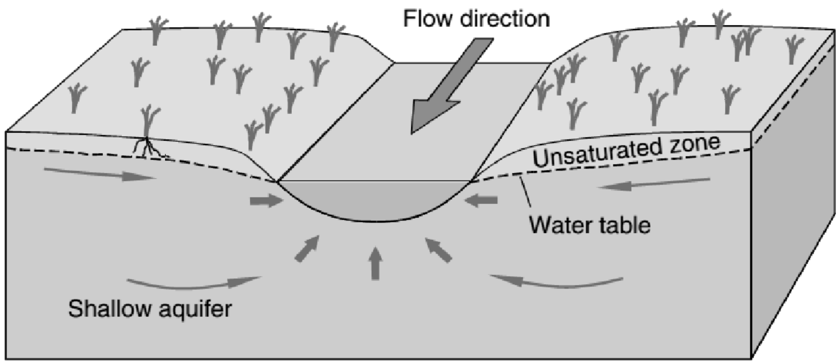
Groundwater Engineering| CE60205

Lecture: 23

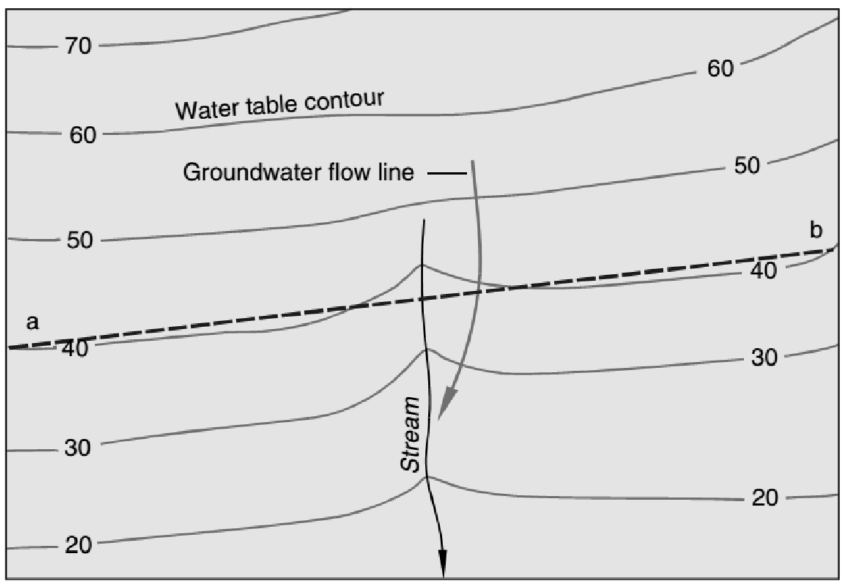
Learning Objective(s)

- **To explain surface and subsurface water system**

Gaining Stream

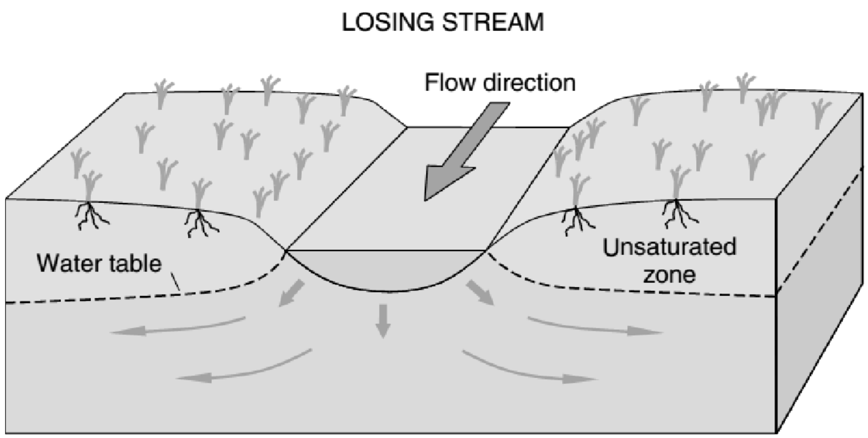


(a)

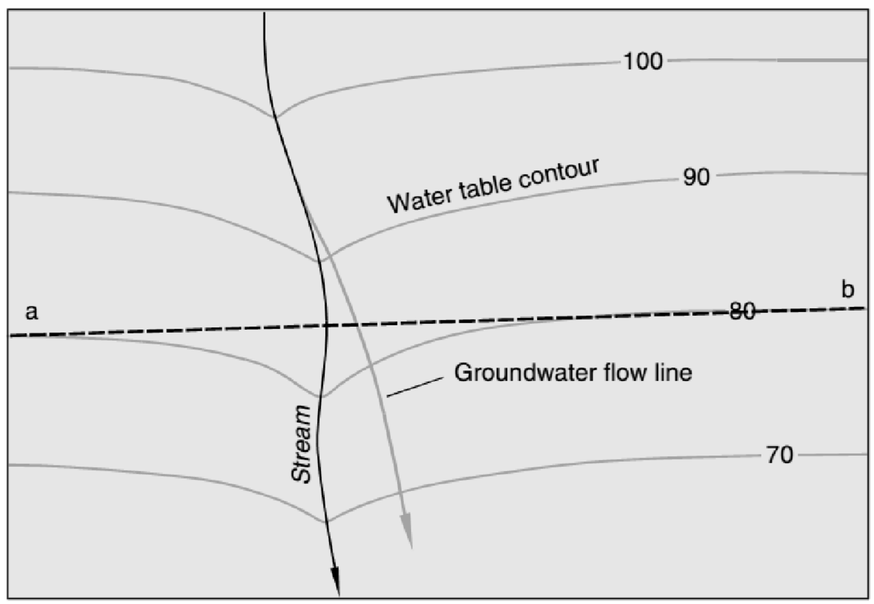


(b)

Losing Stream

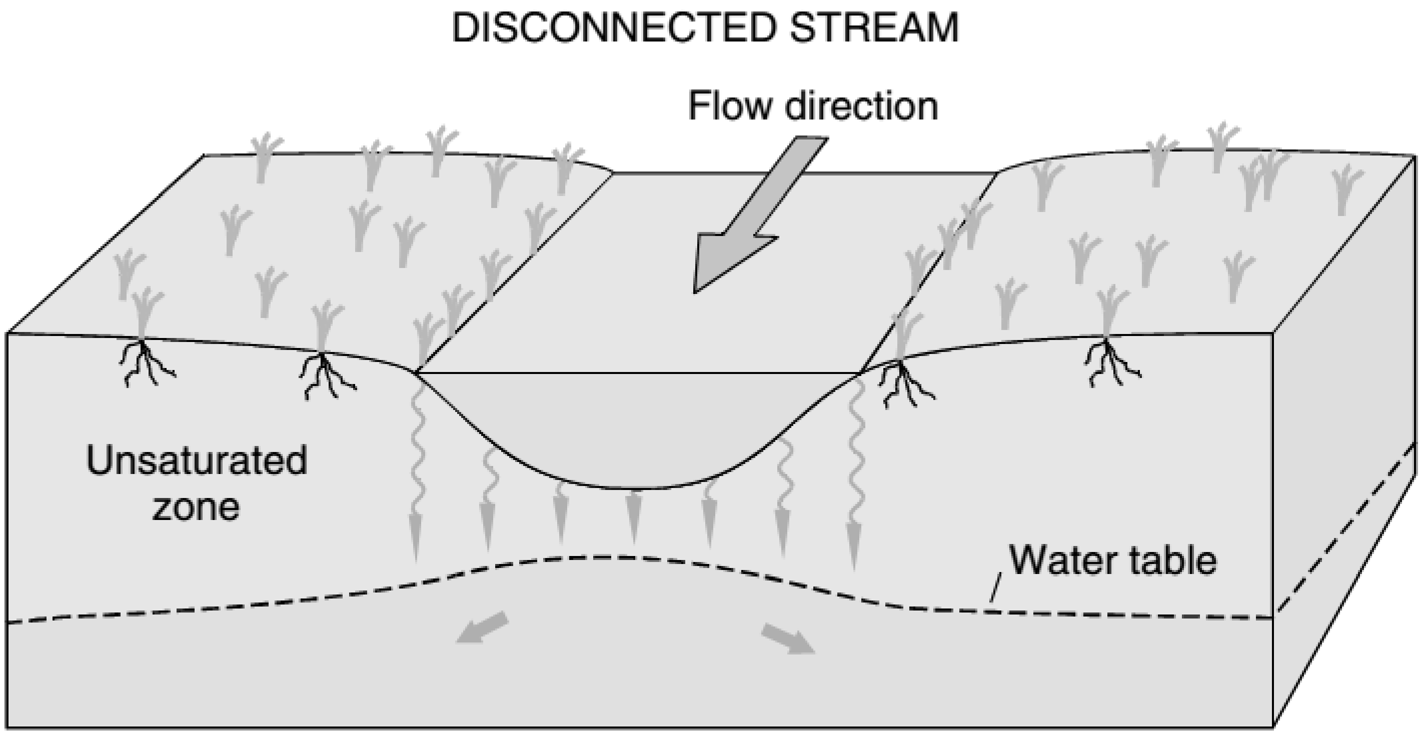


(a)

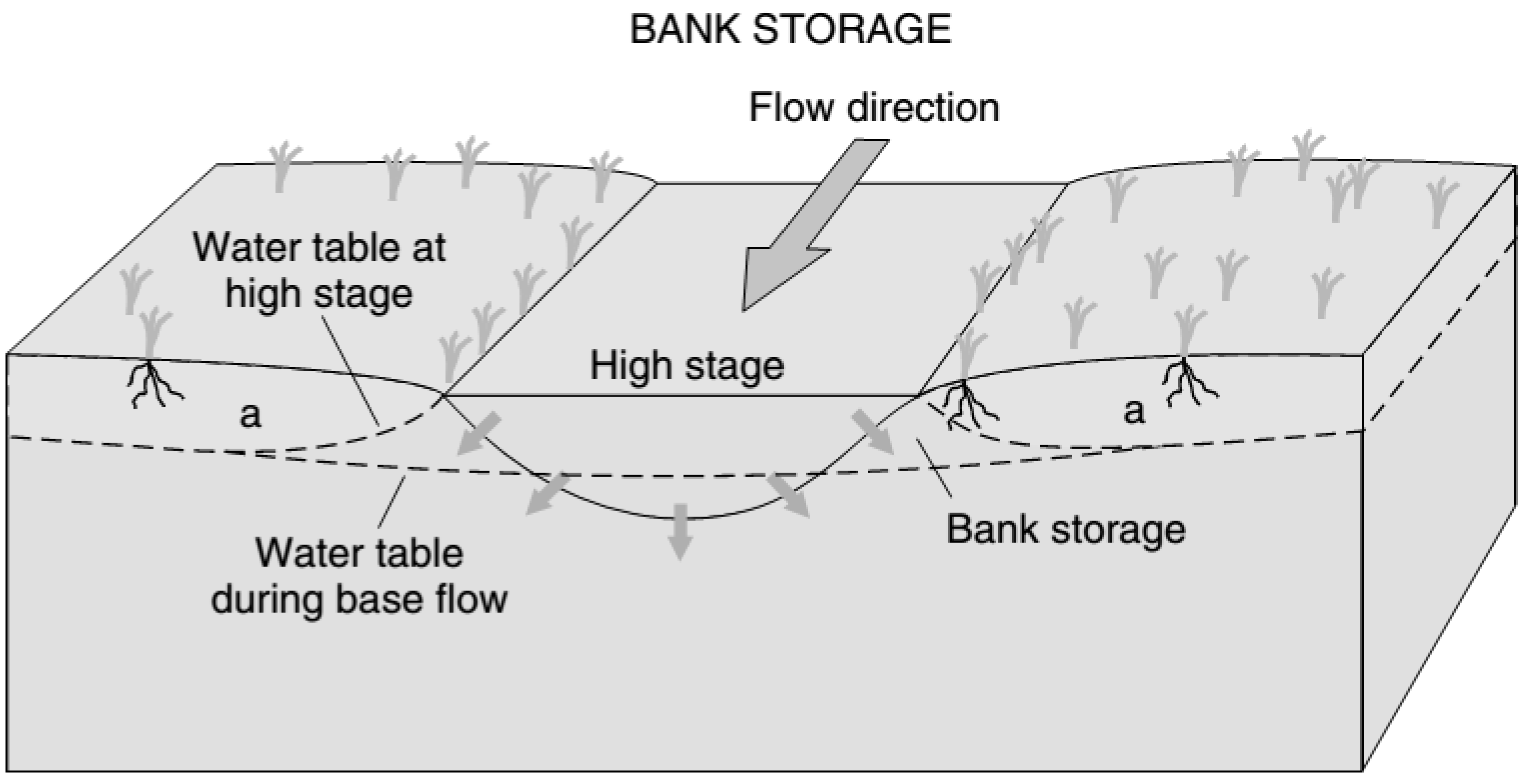


(b)

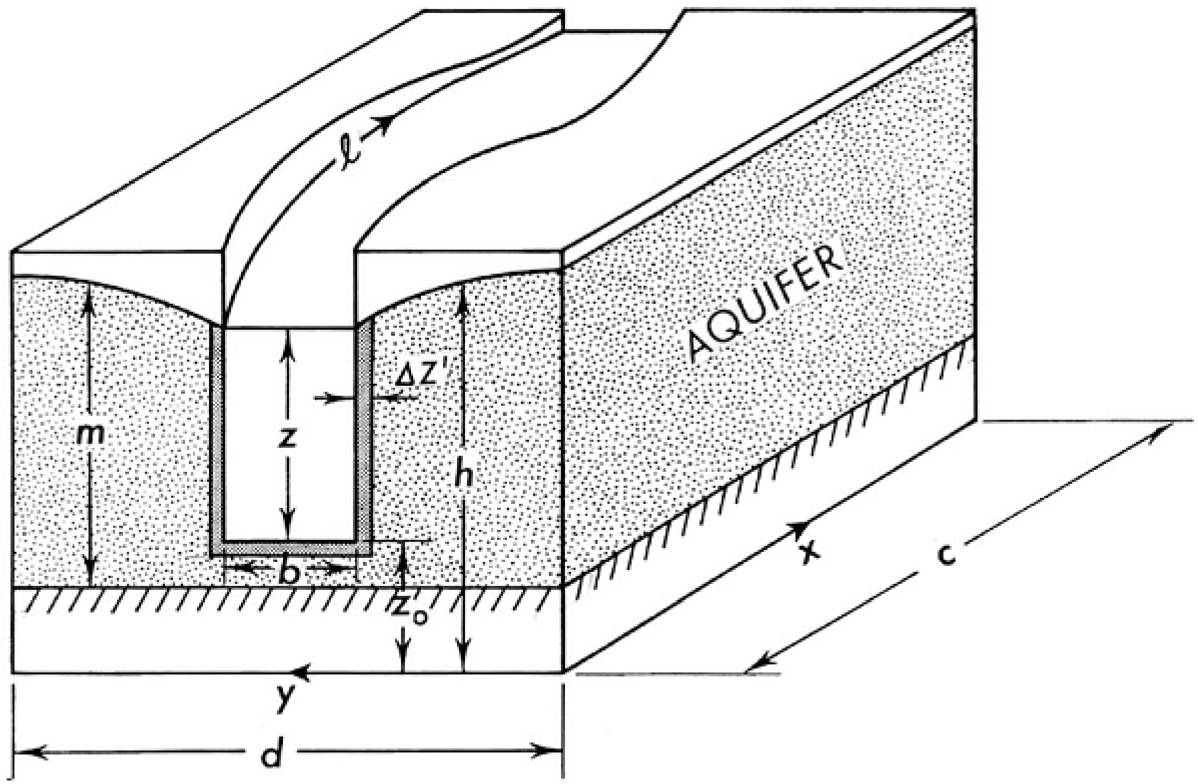
Disconnected Stream



Bank Storage



Idealized cross section of a river-aquifer system

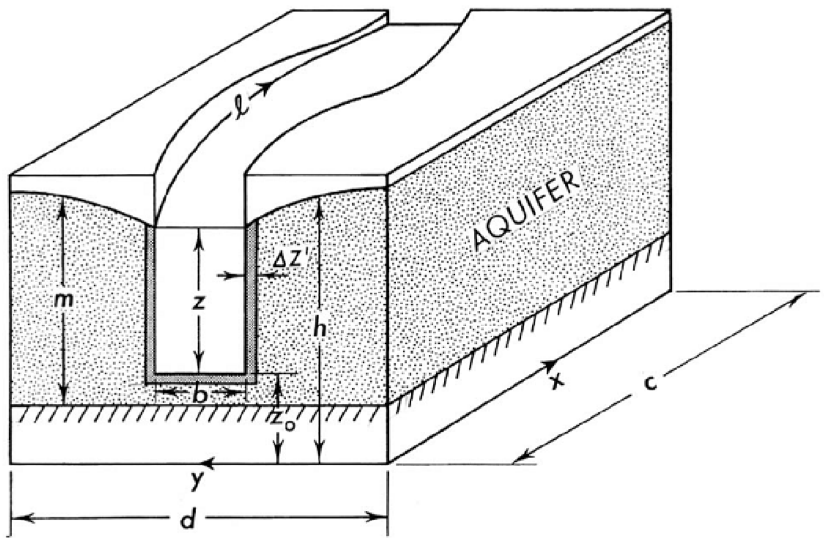


River-Aquifer System

The equation that describes the flow of water in the stream is given by

$$z\frac{\partial v}{\partial \ell} + v\frac{\partial z}{\partial \ell} + \frac{\partial z}{\partial t} = \frac{q_\ell + q_v}{b},$$

$$v\frac{\partial v}{\partial \ell} + g\frac{\partial z}{\partial \ell} + v\frac{q_\ell + q_v}{bz} + \frac{\partial v}{\partial t} = g\left(S_0 - S_f\right),$$



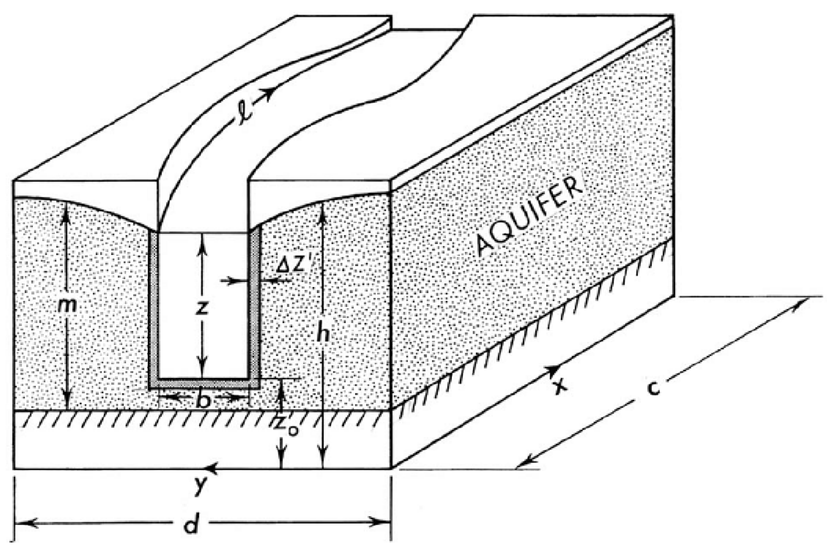
River-Aquifer System (Contd.)

The groundwater flow equation used in this analysis is given by

$$\nabla \cdot (\mathbf{T}\nabla h) = S\frac{\partial h}{\partial t} + \frac{q_v}{b + 2z}$$

below the channel, and elsewhere the governing equation is

$$\nabla \cdot \mathbf{K}m\nabla h = S_y\frac{\partial h}{\partial t},$$

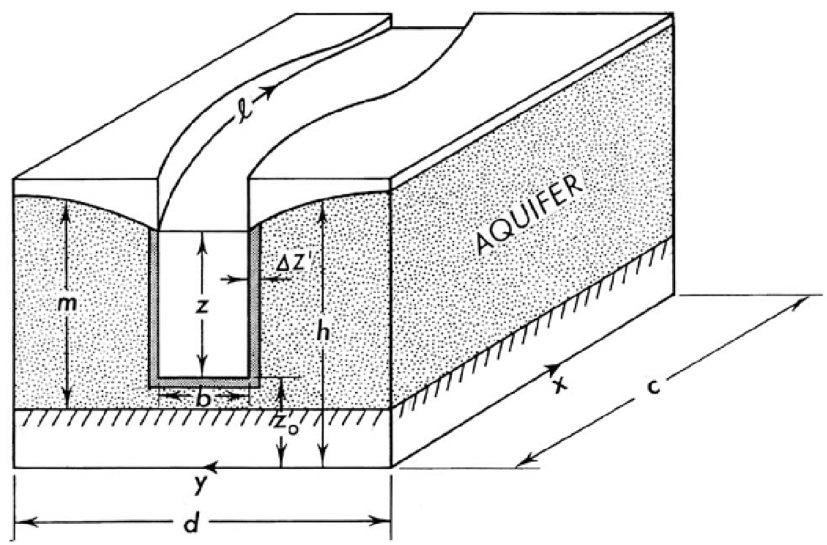


River-Aquifer System (Contd.)

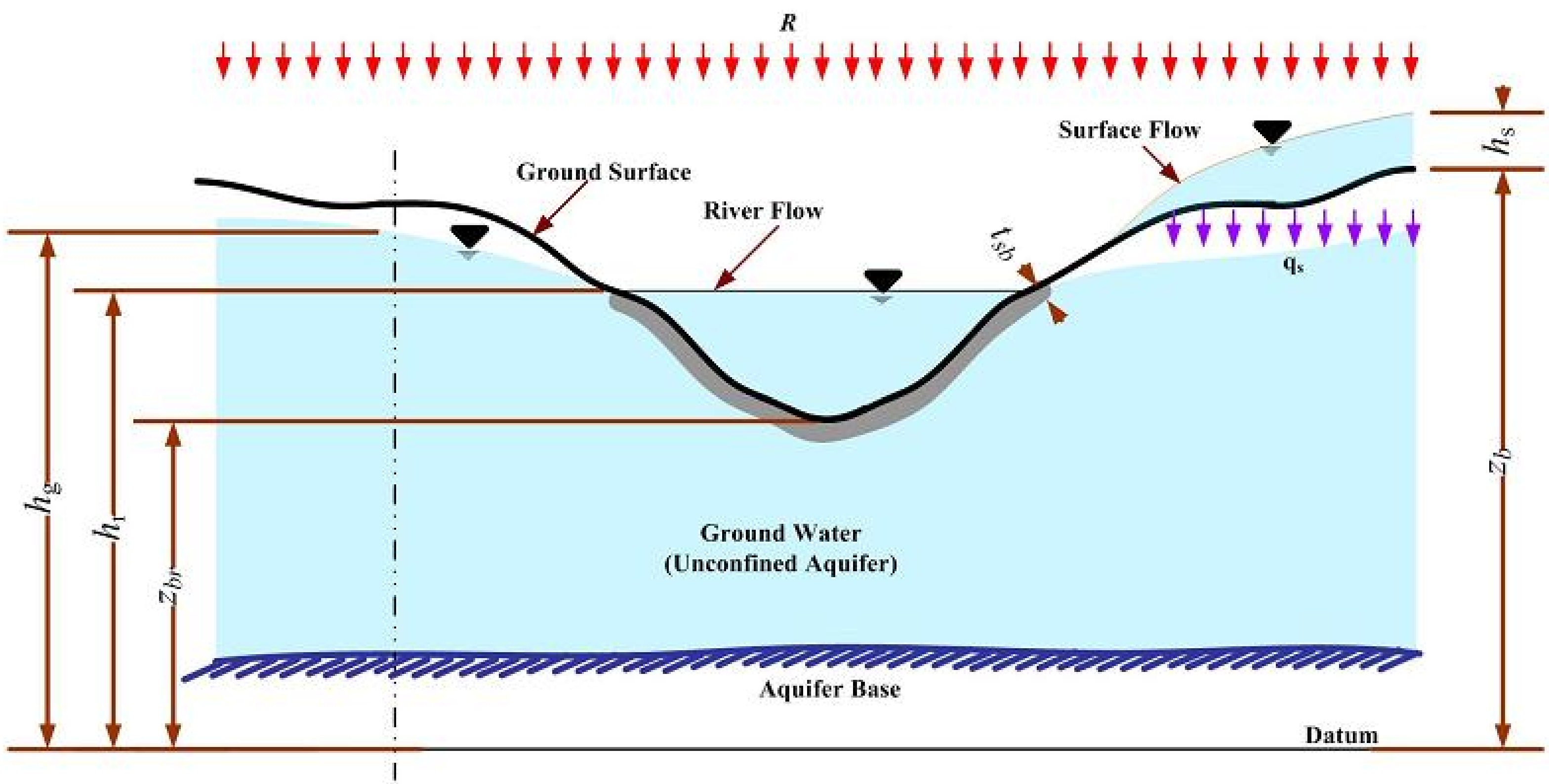
Darcy's law

$$\frac{q_v}{b+2z} = -K_p \frac{z + z_0 - h}{\Delta z'}$$

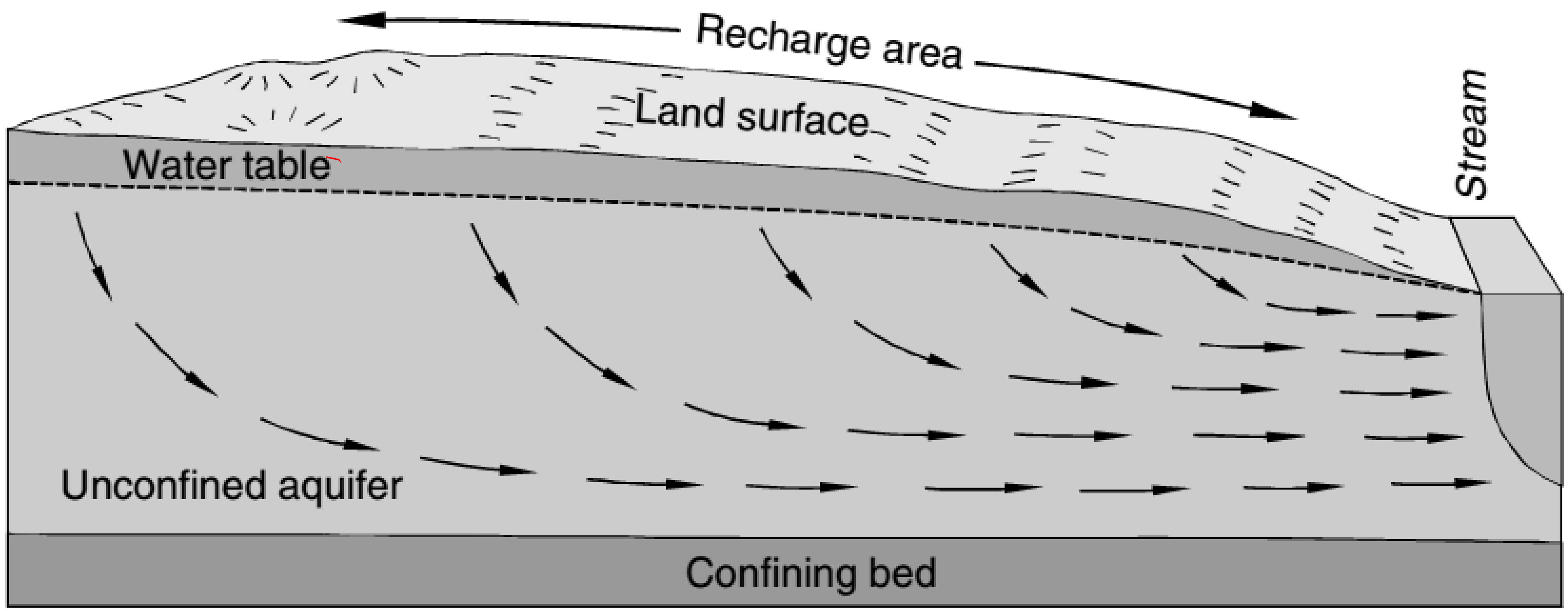
where K_p is the hydraulic conductivity of the bottom sediments of the channel, $\Delta z'$ is the thickness of the bottom sediments along the wetted perimeter of the channel, and z_0 is the elevation of the stream bottom measured from the same datum as h.



River-Surface Water-Groundwater



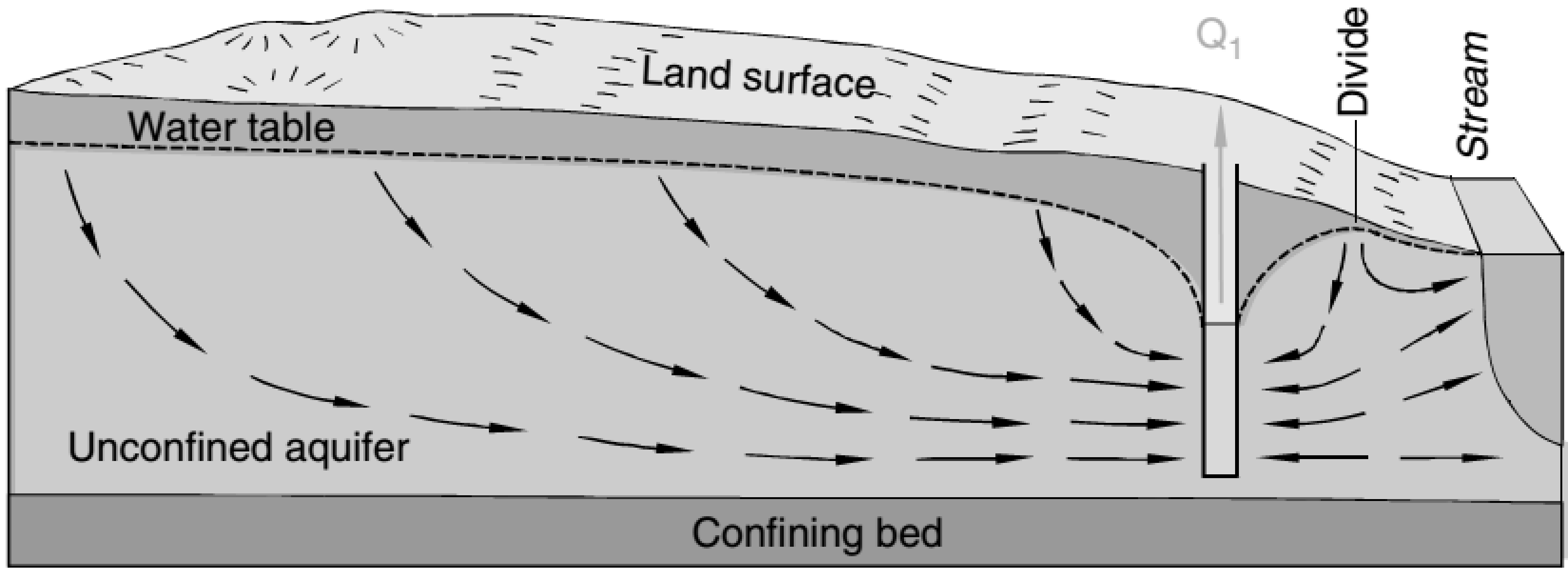
River-Aquifer System (Contd.)



(A)

discharge pattern to the stream is unaffected by a well

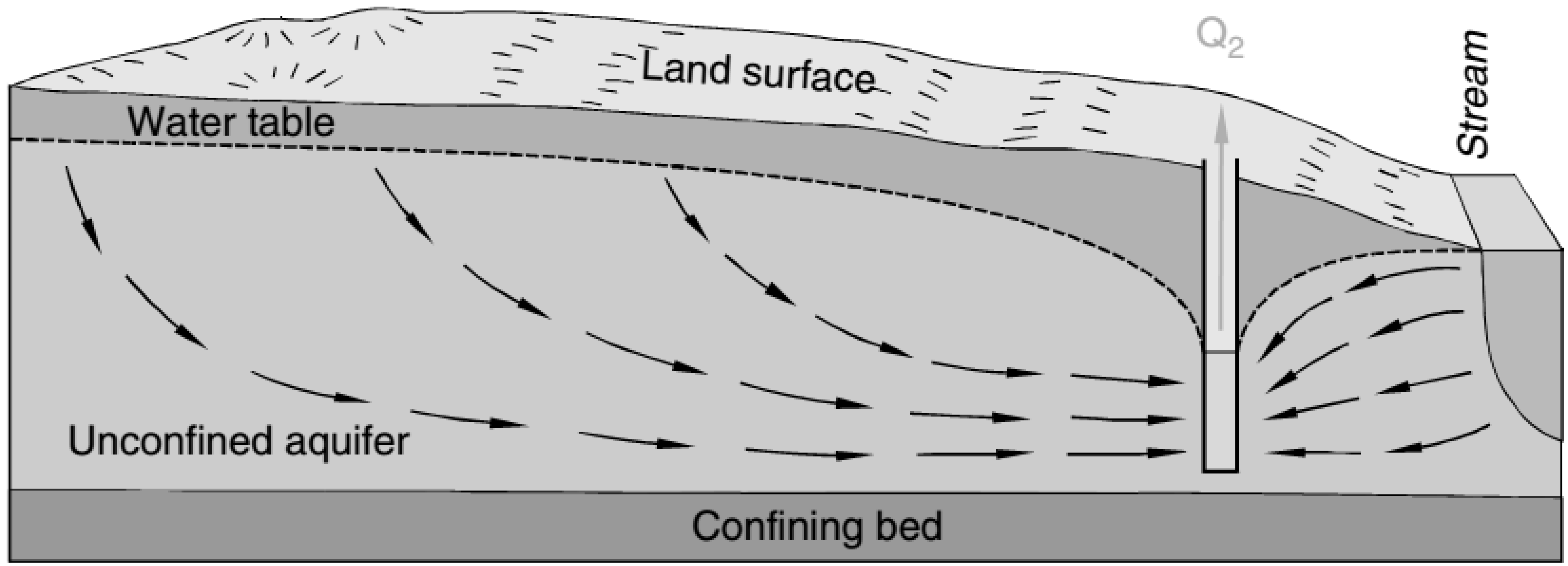
River-Aquifer System (Contd.)



(B)

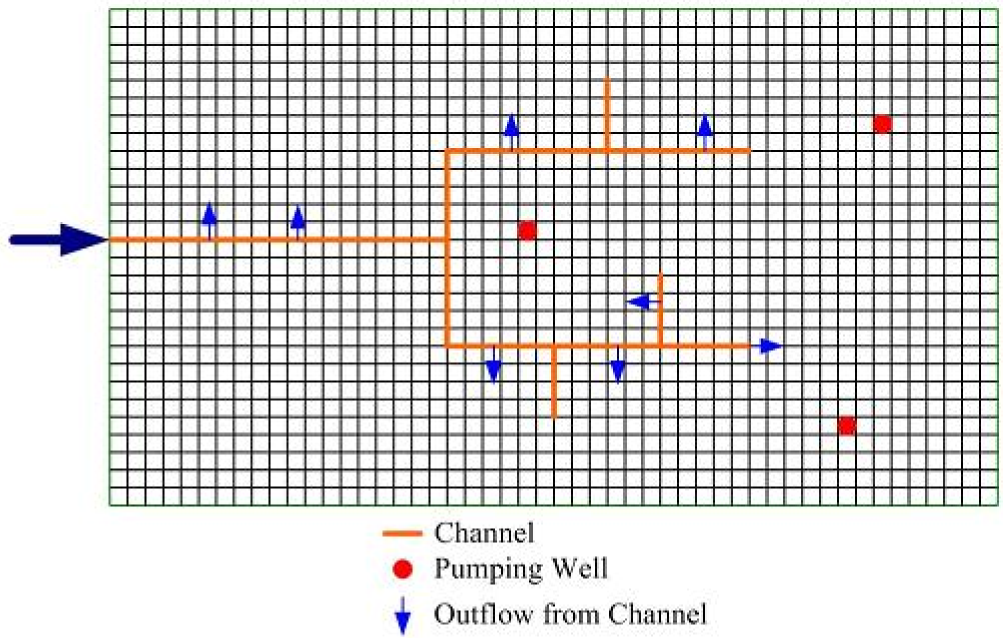
a well captures a portion of the water that would normally reach the stream,

River-Aquifer System (Contd.)



(C)

Canal-Surface Water-Groundwater



Required

- Unsteady Channel Flow: $Q_c(x,t)$, $y_c(x,t)$
- Unsteady Free-surface Flow (Shallow water): $h_s(x,y,t)$, $u_s(x,y,t)$, $v_s(x,y,t)$
- Unsteady Unconfined Aquifer Flow: $h_g(x,y,t)$

Canal-Surface Water-Groundwater (Contd.)

Governing Equation for unsteady 1D channel flow (St. Venant Equations) can be written as (Weiming, 2007),

Initial Boundary Value Problem

Continuity Equation:

$$\frac{\partial A}{\partial t} + \frac{\partial Q_c}{dx} = -q_c$$

Momentum Equation:

$$\frac{\partial}{\partial t}\left(\frac{Q_c}{A}\right) + \frac{\partial}{\partial x}\left(\frac{\alpha Q_c^2}{2A^2}\right) + g\frac{\partial H}{\partial x} + gS_f = 0$$

where

y_c= depth of flow
S_f= friction slope $\left(= \frac{n^2 Q^2}{R^{4/3} A^2}\right)$
A= cross-sectional area
q_c= lateral outflow
z= elevation of the channel bottom w.r.t. datum

H= water surface elevation $(= y_c + z)$
α= momentum correction factor
Q_c= discharge
g= acceleration due to gravity

Canal-Surface Water-Groundwater (Contd.)

Depth-integrated mass and momentum conservation equations for surface water flow can be written as,

Governing equation

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{E}}{\partial x} + \frac{\partial \mathbf{G}}{\partial y} = \mathbf{S} \tag{1}$$

$$\mathbf{U} = \begin{bmatrix} h_s \\ h_s u_s \\ h_s v_s \end{bmatrix}, \quad \mathbf{E} = \begin{bmatrix} h u_s \\ h_s u_s^2 + \frac{g h_s^2}{2} \\ h_s u_s v_s \end{bmatrix}, \quad \mathbf{G} = \begin{bmatrix} h_s v_s \\ h_s u_s v_s \\ h_s v^2 + \frac{g h_s^2}{2} \end{bmatrix}, \quad \mathbf{S} = \begin{bmatrix} R + q_c - q_s \\ g h_s (S_{0x} - S_{fx}) \\ g h_s (S_{0y} - S_{fy}) \end{bmatrix}$$

where h_s = water height, u_s, v_s = velocity at x and y directions.

Canal-Surface Water-Groundwater (Contd.)

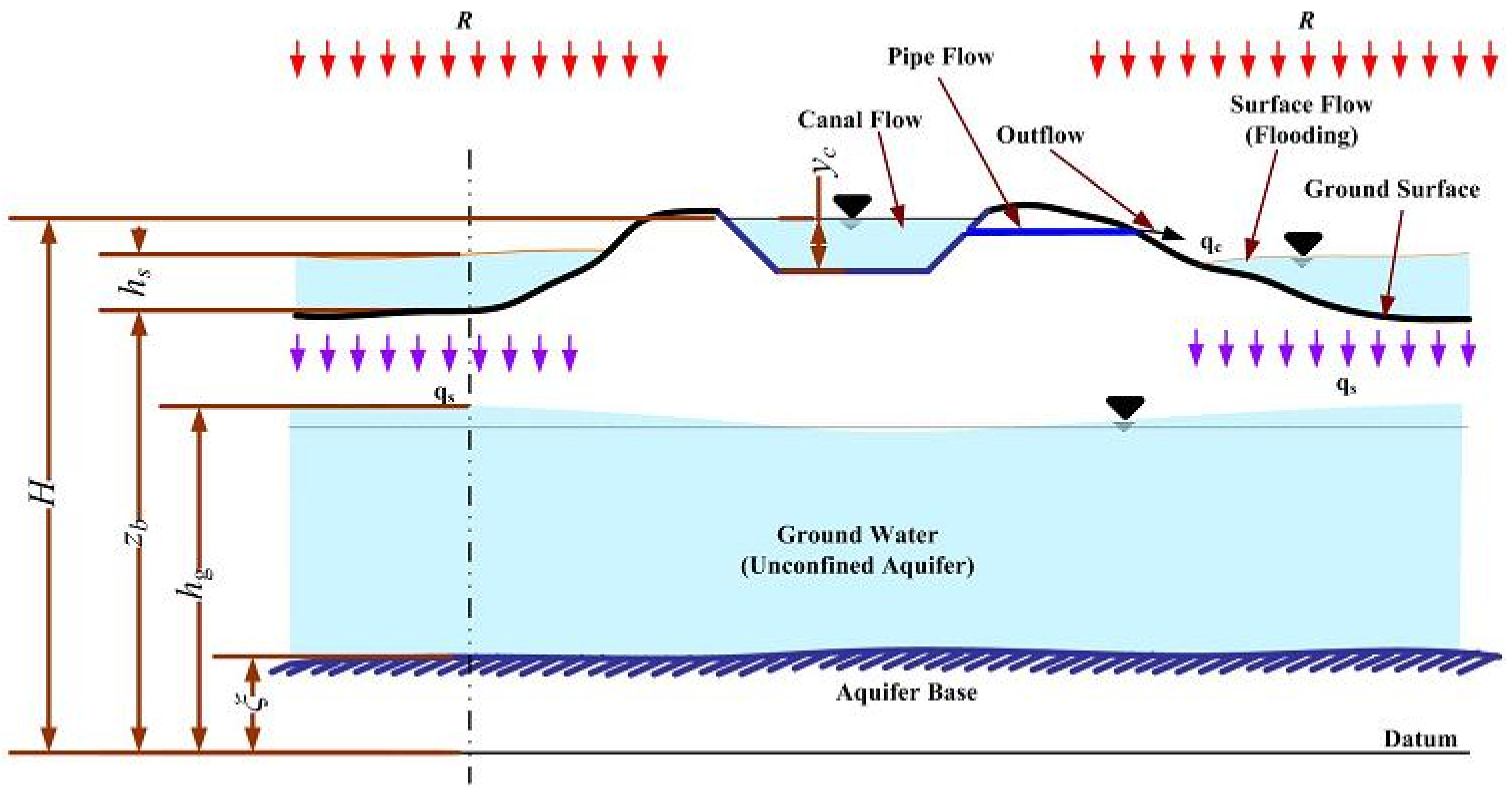
Governing equation

In two-dimension groundwater flow in unconfined aquifer can be written as,

$$S_y \frac{\partial h_g}{\partial t} = \frac{\partial}{\partial x}\left(K_x (h_g - \xi)\frac{\partial h_g}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y (h_g - \xi)\frac{\partial h_g}{\partial y}\right) - W_P + W_I + q_s$$

where $K_x, K_y =$ hydraulic conductivity at x and y directions $W_I =$ injection rate, $W_P =$ pumping rate, $\xi =$ elevation of aquifer base.

Canal-Surface Water-Groundwater (Contd.)



Thank you