# Module 02: Numerical Methods

## Unit 01: Overview

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# Learning Objective

- To identify the connection between governing equation and numerical discretizations.

# Introduction

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- Ordinary and Partial Differential Equations (ODE & PDE) are defined for continuous domain.
- Continuous domain can be divided into parts/ sub-parts for discrete representation.
- Numerical discretization defines the mathematical relation between parts or sub-parts in terms of field variables.

# Problem Definition to Solution

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| |
|---|
| Hydraulic System |

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Hydraulic System

**Mathematical Conceptualization**

Governing Equation (ODE/PDE)
Initial Condition (IC)
Boundary Condition (BC)

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Hydraulic System

**Mathematical Conceptualization**

Governing Equation (ODE/PDE)
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**Domain Discretization**

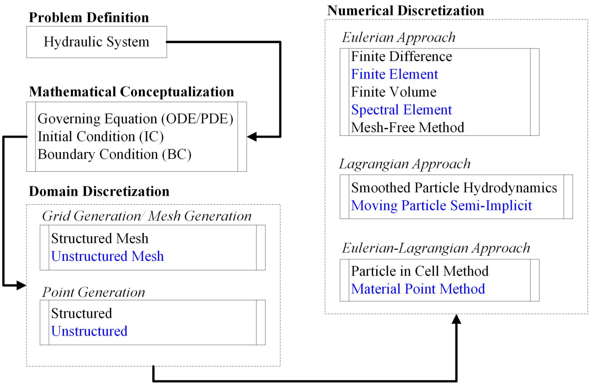
*Grid Generation/ Mesh Generation*

Structured Mesh
Unstructured Mesh

*Point Generation*

Structured
Unstructured

# Problem Definition to Solution



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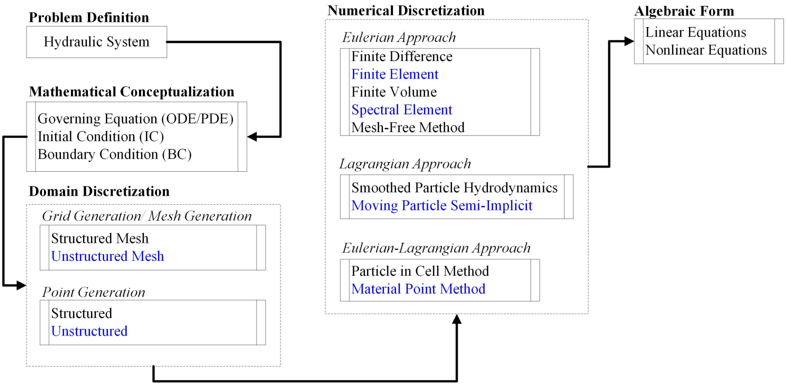

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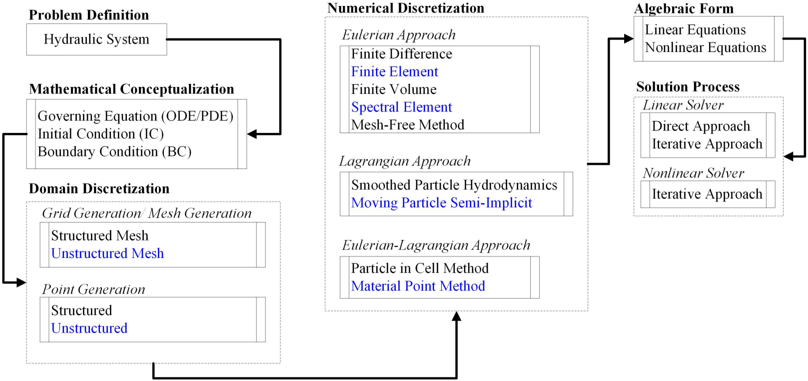

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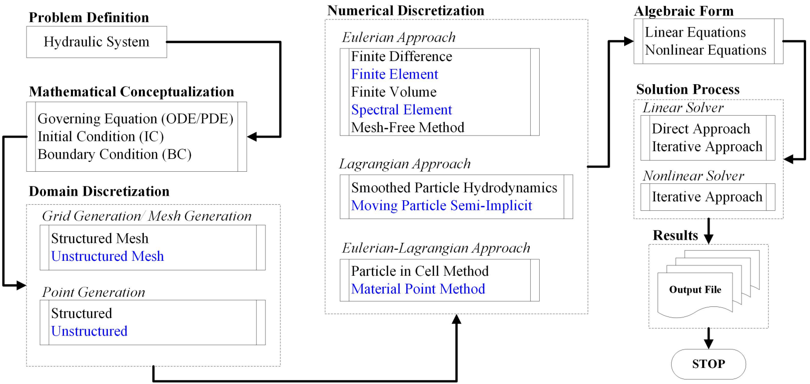


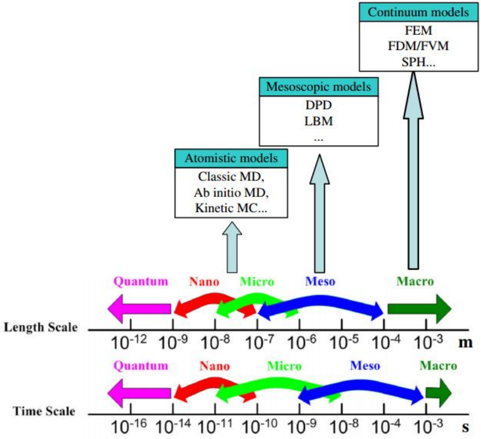
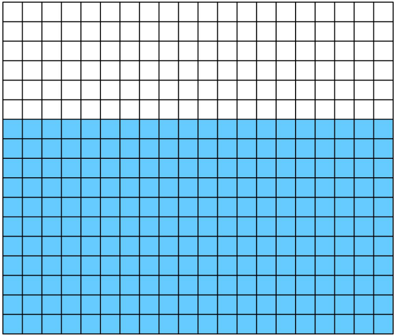


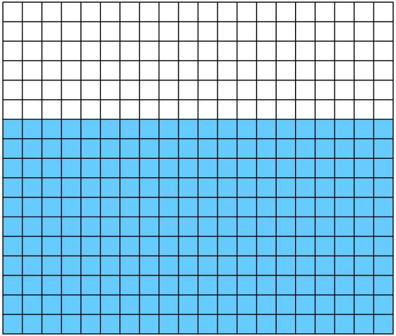

Figure: Different length, time scales and corresponding computational methods (Liu et al., 2014)

# Eulerian Description



(a) Reference Condition

# Eulerian Description



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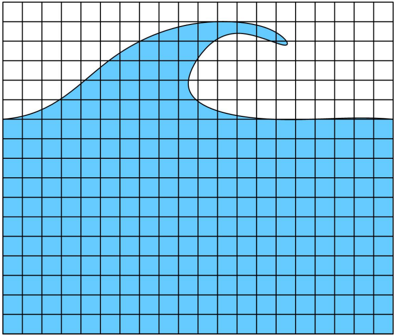
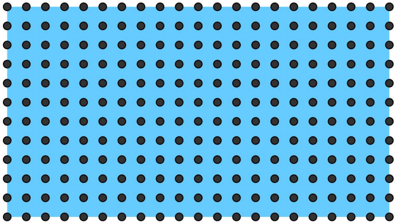


(b) Current Condition

# Lagrangian Description



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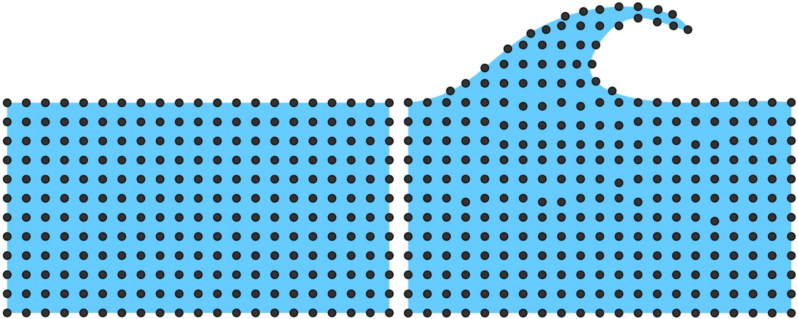
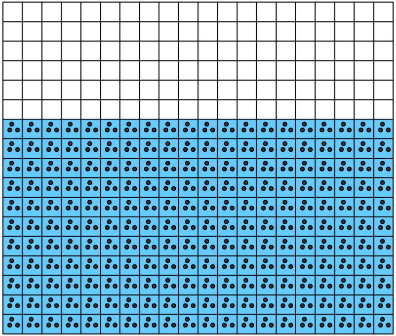


(a) Reference Condition (b) Current Condition

# Eulerian-Lagrangian Description



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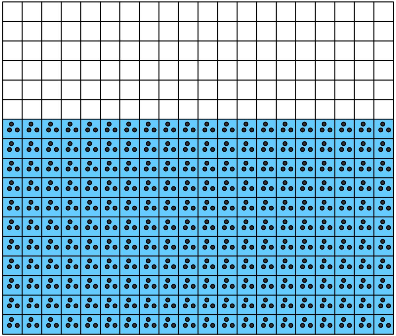

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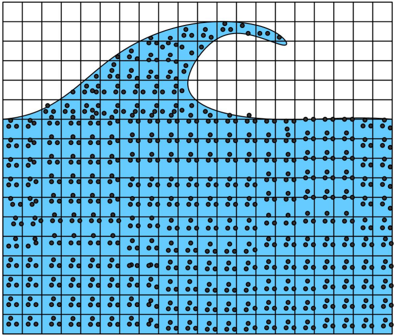
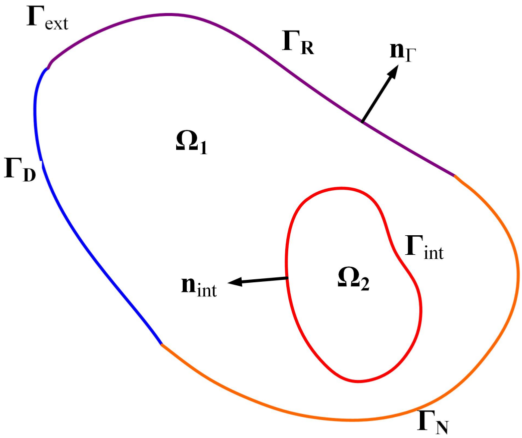


(b) Current Condition

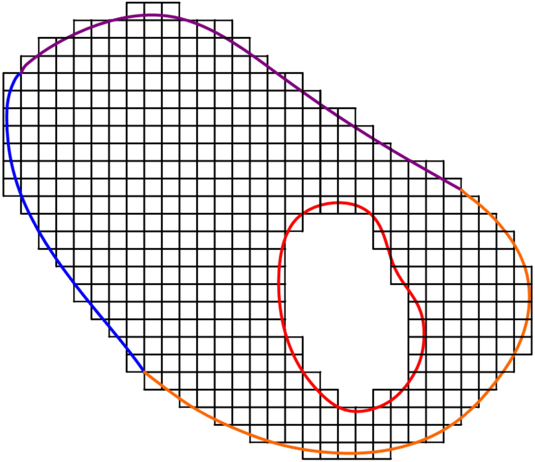
# Physical Domain
## Continuous

$$\phi(x, y, z, t) \rightarrow \phi(x_i, y_j, z_k, t_l)$$



# Discretized Domain

## Finite Difference Method



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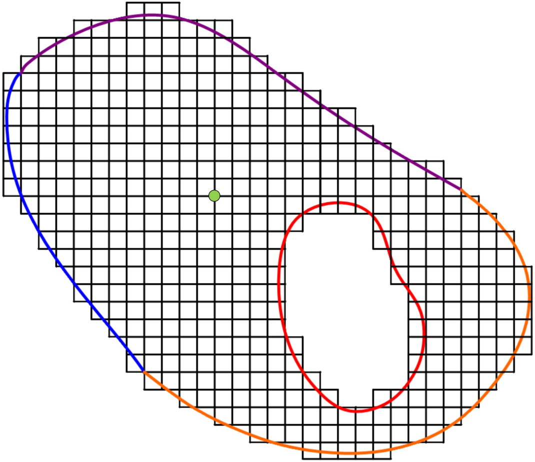
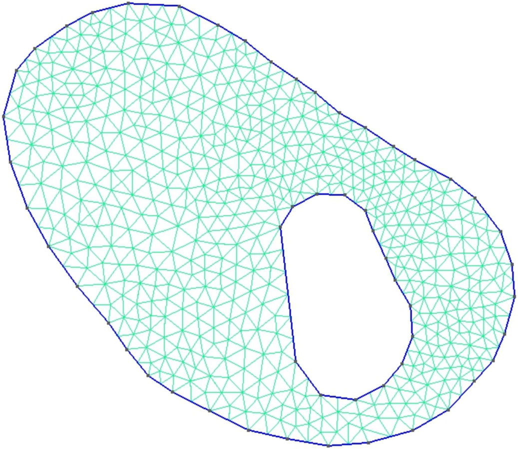


# Discretized Domain

## Finite Volume Method



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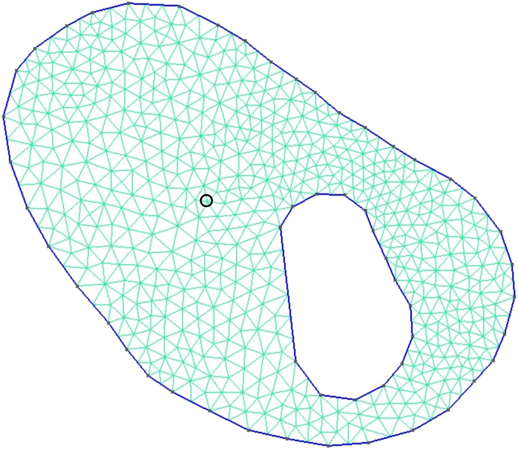
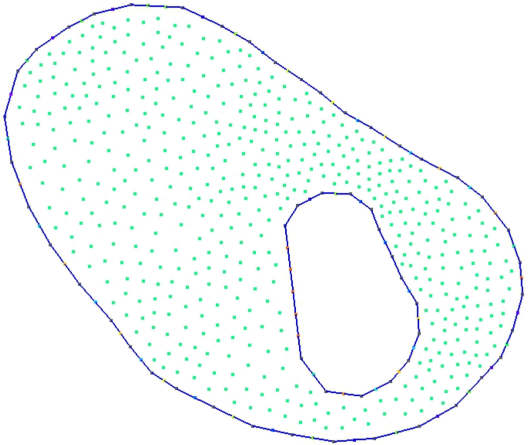


# Discretized Domain
## Mesh-free Method



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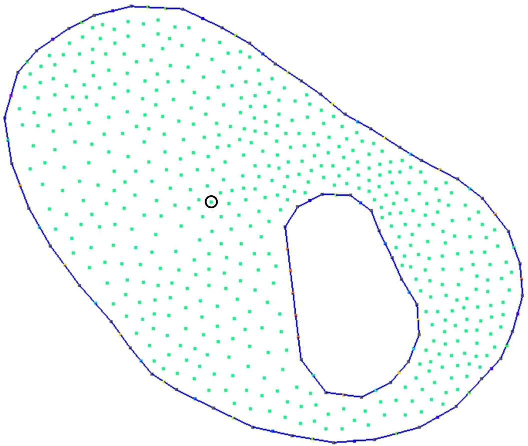

# Thank You

# References

Liu, M. B., Liu, G. R., Zhou, L. W., and Chang, J. Z. (2014). Dissipative Particle Dynamics (DPD): An Overview and Recent Developments. *Archives of Computational Methods in Engineering*, 22(4):529–556.