Module 04: Surface Water Hydraulics

Unit 05: Steady Channel Flow: Channel Network without Reverse Flow

Anirban Dhar

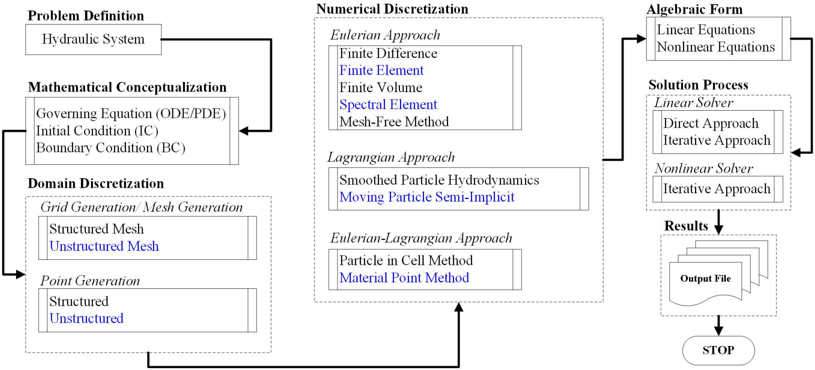
Department of Civil Engineering
Indian Institute of Technology Kharagpur, Kharagpur

National Programme for Technology Enhanced Learning (NPTEL)

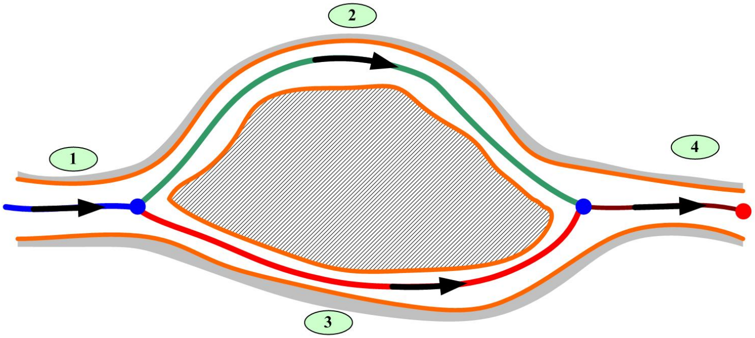
Learning Objective

- To solve steady channel flow for channel network problem without reverse flow using implicit method.

Problem Definition to Solution



Problem Statement



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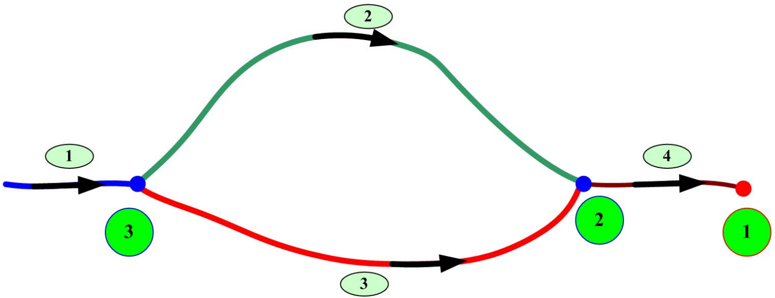
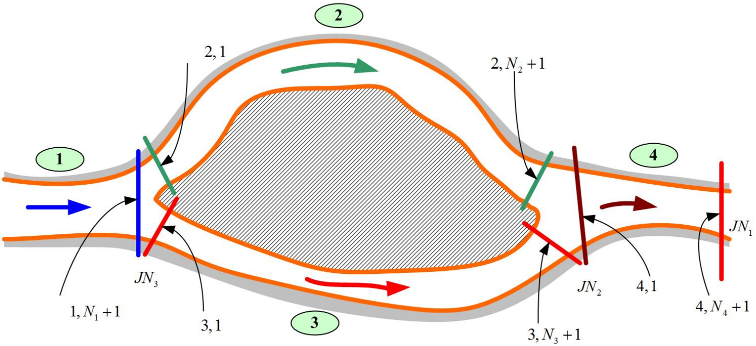
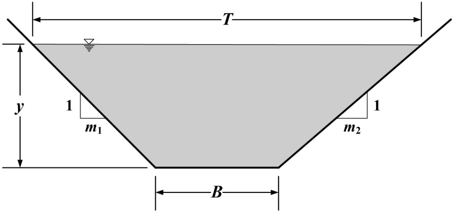


Problem Statement



Problem Statement
Trapezoidal Cross-section



$$A = By + \frac{1}{2}(m_1 + m_2)y^2$$

$$P = B + \left(\sqrt{1+m_1^2} + \sqrt{1+m_2^2}\right) y$$

$$R = \frac{A}{P}$$

$$T = B + (m_1 + m_2)y$$

where $P =$ wetted perimeter.

Problem Statement
Channel Data

Channel	length (m)	width (m)	Side Slope		reach(m)	n	S_0	Connectivity	
			m_1	m_2				JN_1	JN_2
1	100	50	2	2	25	0.012	0.0005	0	3
2	1500	30	2	2	75	0.0125	0.0004	3	2
3	500	20	2	2	25	0.013	0.0012	3	2
4	100	20	2	2	25	0.0135	0.0005	2	1

Problem Statement
Channel Data

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Junction Number	Depth (m)	Discharge (m^3/s)
1	3	-250
2	-99999	-99999
3	-99999	-99999

Problem Statement

Channel Data

Channel	length (m)	width (m)	Side Slope m_1	Side Slope m_2	reach(m)	n	S_0	Connectivity JN_1	Connectivity JN_2
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Required

Estimate the flow depth and discharge across the channels.

Problem Definition

Governing Equation for Channel Flow can be written as,

Boundary Value Problem

Continuity Equation:

$$\frac{dQ}{dx} = 0$$

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Momentum Equation:

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with

$$\mathbb{E} = y + z + \frac{\alpha Q^2}{2gA^2}$$

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with

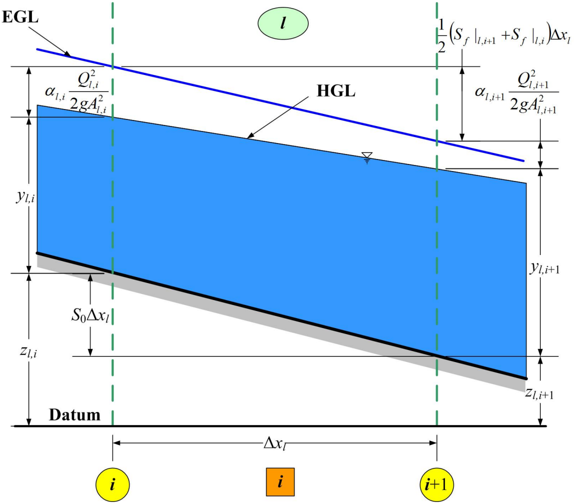
$$\mathbb{E} = y + z + \frac{\alpha Q^2}{2gA^2}$$

where

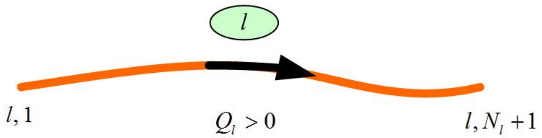
$y=$ depth of flow
$S_f=$ friction slope $\left(= \frac{n^2Q^2}{R^{4/3}A^2}\right)$
$A=$ cross-sectional area
$R=$ hydraulic radius
$z=$ elevation of the channel bottom w.r.t. datum

$x=$ coordinate direction
$\alpha=$ momentum correction factor
$Q=$ discharge
$g=$ acceleration due to gravity

Channel Flow



Channel Flow Conventions



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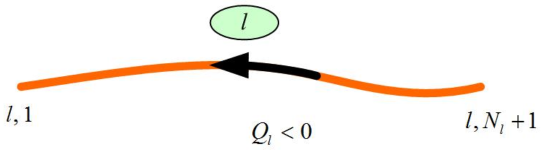
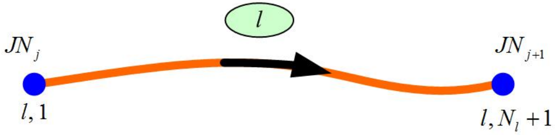
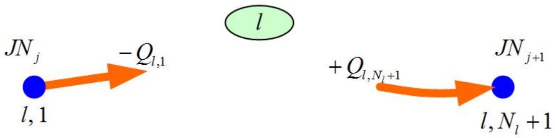


Channel Flow Conventions



Channel Flow Conventions



Algebraic Form
Continuity Equation

Discretized form of continuity equation

$$C_{l,i} = Q_{l,i+1} - Q_{l,i} = 0, \forall i \in \{1, \dots, N_l\}$$

Algebraic Form

Continuity Equation

Discretized form of continuity equation

$$C_{l,i} = Q_{l,i+1} - Q_{l,i} = 0, \forall i \in \{1, \ldots, N_l\}$$

$$\frac{\partial C_{l,i}}{\partial y_{l,i}} = 0$$

$$\frac{\partial C_{l,i}}{\partial Q_{l,i}} = -1$$

$$\frac{\partial C_{l,i}}{\partial y_{l,i+1}} = 0$$

$$\frac{\partial C_{l,i}}{\partial Q_{l,i+1}} = 1$$

Discretization

Momentum Equation

In discretized form of momentum equation for i^{th} segment of the l^{th} channel reach,

$$M_{l,i} = (y_{l,i+1} - y_{l,i}) + (z_{l,i+1} - z_{l,i}) + \frac{\alpha_l}{2g}\left(\frac{Q^2_{l,i+1}}{A^2_{l,i+1}} - \frac{Q^2_{l,i}}{A^2_{l,i}}\right)$$
$$+ \frac{n_l^2 \Delta x_l}{2}\left[\frac{Q^2_{l,i+1}}{R^{4/3}_{l,i+1}A^2_{l,i+1}} + \frac{Q^2_{l,i}}{R^{4/3}_{l,i}A^2_{l,i}}\right], \quad \forall i \in \{1, \ldots, N_l\}$$

$2N_l$ non-linear equations with $2(N_l + 1)$ unknowns (discharge + flow-depth)

Algebraic Form
Momentum Equation

$$\frac{\partial M_{l,i}}{\partial y_{l,i}} = -1 + D_1 \frac{2Q_{l,i}^2}{A_{l,i}^3} \frac{dA}{dy}\Big|_{l,i} - D_2 \left[\frac{2Q_{l,i}^2}{A_{l,i}^3 R_{l,i}^{\frac{4}{3}}} \frac{dA}{dy}\Big|_{l,i} + \frac{4Q_{l,i}^2}{3A_{l,i}^2 R_{l,i}^{\frac{7}{3}}} \frac{dR}{dy}\Big|_{l,i} \right]$$

$$\frac{\partial M_{l,i}}{\partial Q_{l,i}} = -D_1 \frac{2Q_{l,i}}{A_{l,i}^3} + D_2 \frac{2Q_{l,i}}{A_{l,i}^2 R_{l,i}^{\frac{4}{3}}}$$

$$\frac{\partial M_{l,i}}{\partial y_{l,i+1}} = 1 - D_1 \frac{2Q_{l,i+1}^2}{A_{l,i+1}^3} \frac{dA}{dy}\Big|_{l,i+1} - D_2 \left[\frac{2Q_{l,i+1}^2}{A_{l,i+1}^3 R_{l,i+1}^{\frac{4}{3}}} \frac{dA}{dy}\Big|_{l,i+1} + \frac{4Q_{l,i+1}^2}{3A_{l,i+1}^2 R_{l,i+1}^{\frac{7}{3}}} \frac{dR}{dy}\Big|_{l,i+1} \right]$$

$$\frac{\partial M_{l,i}}{\partial Q_{l,i+1}} = D_1 \frac{2Q_{l,i+1}}{A_{l,i+1}^3} + D_2 \frac{2Q_{l,i+1}}{A_{l,i+1}^2 R_{l,i+1}^{\frac{4}{3}}}$$

with

$$D_1 = \frac{\alpha_l}{2g} \quad \text{and} \quad D_2 = \frac{1}{2} n_l^2 \Delta x_l$$

Trapezoidal Section

For trapezoidal channel cross-section,

$$\frac{dA}{dy} = B + (m_1 + m_2)y$$

Trapezoidal Section

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$$\frac{dA}{dy} = B + (m_1 + m_2)y$$

$$\frac{dR}{dy} = \frac{T}{P} - \frac{R}{P}\frac{dP}{dy}$$

with

$$T = B + (m_1 + m_2)y$$

$$P = B + \left(\sqrt{1 + m_1^2} + \sqrt{1 + m_2^2}\right) y$$

$$R = \frac{A}{P}$$

$$\frac{dP}{dy} = \left(\sqrt{1 + m_1^2} + \sqrt{1 + m_2^2}\right)$$

Discretization
Boundary Condition

For downstream flow-depth condition at junction 1,

$$y_{4,N_4+1} = y_d$$

$$DBy_{4,N_4+1} = y_{4,N_4+1} - y_d = 0$$

Discretization

Boundary Condition

For downstream flow-depth condition at junction 1,

$$y_{4,N_4+1} = y_d$$
$$DBy_{4,N_4+1} = y_{4,N_4+1} - y_d = 0$$

Elements of Jacobian Matrix can be written as,

$$\frac{\partial DBy_{4,N_4+1}}{\partial y_{4,N_4}} = 0$$
$$\frac{\partial DBy_{4,N_4+1}}{\partial Q_{4,N_4}} = 0$$
$$\frac{\partial DBy_{4,N_4+1}}{\partial y_{4,N_4+1}} = 1$$
$$\frac{\partial DBy_{4,N_4+1}}{\partial Q_{4,N_4+1}} = 0$$

Discretization
Boundary Condition

For downstream discharge condition at junction 1,

$$Q_{4,N_4+1} + Q_d = 0$$
$$DBQ_{4,N_4+1} = Q_{4,N_4+1} + Q_d = 0$$

Discretization
Boundary Condition

For downstream discharge condition at junction 1,

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$$DBQ_{4,N_4+1} = Q_{4,N_4+1} + Q_d = 0$$

Elements of Jacobian Matrix can be written as,

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$$\frac{\partial DBQ_{4,N_4+1}}{\partial Q_{4,N_4+1}} = 1$$

Channel Networks
Internal Boundary condition

Junction 2

$$JC_{JN_2,1} = Q_{2,N_2+1} + Q_{3,N_3+1} - Q_{4,1} = 0$$
$$JC_{JN_2,2} = y_{4,1} - y_{2,N_2+1} + z_{4,1} - z_{2,N_2+1} = 0$$
$$JC_{JN_2,3} = y_{4,1} - y_{3,N_3+1} + z_{4,1} - z_{3,N_3+1} = 0$$

Channel Networks

Internal Boundary condition

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$$JC_{JN_2,2} = y_{4,1} - y_{2,N_2+1} + z_{4,1} - z_{2,N_2+1} = 0$$
$$JC_{JN_2,3} = y_{4,1} - y_{3,N_3+1} + z_{4,1} - z_{3,N_3+1} = 0$$

Elements of Jacobian Matrix can be written as,

$$\frac{\partial JC_{JN_2,1}}{\partial Q_{2,N_2+1}} = 1 \quad \frac{\partial JC_{JN_2,1}}{\partial Q_{3,N_3+1}} = 1 \quad \frac{\partial JC_{JN_2,1}}{\partial Q_{4,1}} = -1$$
$$\frac{\partial JC_{JN_2,2}}{\partial y_{2,N_2+1}} = -1 \quad \frac{\partial JC_{JN_2,2}}{\partial y_{4,1}} = 1$$
$$\frac{\partial JC_{JN_2,3}}{\partial y_{3,N_3+1}} = -1 \quad \frac{\partial JC_{JN_2,3}}{\partial y_{4,1}} = 1$$

Channel Networks

Internal Boundary condition

Junction 3

$$JC_{JN_3,1} = Q_{1,N_1+1} - Q_{2,1} - Q_{3,1} = 0$$
$$JC_{JN_3,2} = y_{1,N_1+1} - y_{2,1} + z_{l,N_1+1} - z_{2,1} = 0$$
$$JC_{JN_3,3} = y_{1,N_1+1} - y_{3,1} + z_{l,N_1+1} - z_{3,1} = 0$$

Channel Networks

Internal Boundary condition

Junction 3

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$$JC_{JN_3,3} = y_{1,N_1+1} - y_{3,1} + z_{l,N_1+1} - z_{3,1} = 0$$

Elements of Jacobian Matrix can be written as,

$$\frac{\partial JC_{JN_3,1}}{\partial Q_{1,N_1+1}} = 1 \quad \frac{\partial JC_{JN_3,1}}{\partial Q_{2,1}} = -1 \quad \frac{\partial JC_{JN_3,1}}{\partial Q_{3,1}} = -1$$
$$\frac{\partial JC_{JN_3,2}}{\partial y_{1,N_1+1}} = 1 \quad \frac{\partial JC_{JN_3,2}}{\partial y_{2,1}} = -1$$
$$\frac{\partial JC_{JN_3,3}}{\partial y_{l,N_1+1}} = 1 \quad \frac{\partial JC_{JN_3,3}}{\partial y_{3,1}} = -1$$

Algebraic Form

In general form, continuity and momentum equations can be written as,

$$\frac{\partial C_{l,i}}{\partial y_{l,i}}\Delta y_{l,i} + \frac{\partial C_{l,i}}{\partial Q_{l,i}}\Delta Q_{l,i} + \frac{\partial C_{l,i}}{\partial y_{l,i+1}}\Delta y_{l,i+1} + \frac{\partial C_{l,i}}{\partial Q_{l,i+1}}\Delta Q_{l,i+1} = -C_{l,i}$$

$$\frac{\partial M_{l,i}}{\partial y_{l,i}}\Delta y_{l,i} + \frac{\partial M_{l,i}}{\partial Q_{l,i}}\Delta Q_{l,i} + \frac{\partial M_{l,i}}{\partial y_{l,i+1}}\Delta y_{l,i+1} + \frac{\partial M_{l,i}}{\partial Q_{l,i+1}}\Delta Q_{l,i+1} = -M_{l,i},$$

$$\forall i \in \{1, \ldots, N_l\}$$

Algebraic Form

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$$\frac{\partial M_{l,i}}{\partial y_{l,i}}\Delta y_{l,i} + \frac{\partial M_{l,i}}{\partial Q_{l,i}}\Delta Q_{l,i} + \frac{\partial M_{l,i}}{\partial y_{l,i+1}}\Delta y_{l,i+1} + \frac{\partial M_{l,i}}{\partial Q_{l,i+1}}\Delta Q_{l,i+1} = -M_{l,i},$$

$$\forall i \in \{1, \ldots, N_l\}$$

At junction 1 (Downstream Boundary),

$$\frac{\partial DBy_{4,N_4+1}}{\partial y_{4,N_4+1}}\Delta y_{4,N_4+1} = -DBy_{4,N_4+1}$$

$$\frac{\partial DBQ_{4,N_4+1}}{\partial Q_{4,N_4+1}}\Delta Q_{4,N_4+1} = -DBQ_{4,N_4+1}$$

Algebraic Form

At junction 2 (Internal Boundary),

$$\frac{\partial JC_{JN_2,1}}{\partial Q_{2,N_2+1}}\Delta Q_{2,N_2+1} + \frac{\partial JC_{JN_2,1}}{\partial Q_{3,N_3+1}}\Delta Q_{3,N_3+1} + \frac{\partial JC_{JN_2,1}}{\partial Q_{4,1}}\Delta Q_{4,1} = -JC_{JN_2,1}$$

$$\frac{\partial JC_{JN_2,2}}{\partial y_{2,N_2+1}}\Delta y_{2,N_2+1} + \frac{\partial JC_{JN_2,2}}{\partial y_{4,1}}\Delta y_{4,1} = -JC_{JN_2,2}$$

$$\frac{\partial JC_{JN_2,3}}{\partial y_{3,N_3+1}}\Delta y_{3,N_3+1} + \frac{\partial JC_{JN_2,3}}{\partial y_{4,1}}\Delta y_{4,1} = -JC_{JN_2,3}$$

Algebraic Form

At junction 2 (Internal Boundary),

$$\frac{\partial JC_{JN_2,1}}{\partial Q_{2,N_2+1}}\Delta Q_{2,N_2+1}+\frac{\partial JC_{JN_2,1}}{\partial Q_{3,N_3+1}}\Delta Q_{3,N_3+1}+\frac{\partial JC_{JN_2,1}}{\partial Q_{4,1}}\Delta Q_{4,1}=-JC_{JN_2,1}$$
$$\frac{\partial JC_{JN_2,2}}{\partial y_{2,N_2+1}}\Delta y_{2,N_2+1}+\frac{\partial JC_{JN_2,2}}{\partial y_{4,1}}\Delta y_{4,1}=-JC_{JN_2,2}$$
$$\frac{\partial JC_{JN_2,3}}{\partial y_{3,N_3+1}}\Delta y_{3,N_3+1}+\frac{\partial JC_{JN_2,3}}{\partial y_{4,1}}\Delta y_{4,1}=-JC_{JN_2,3}$$

At junction 3,

$$\frac{\partial JC_{JN_3,1}}{\partial Q_{1,N_1+1}}\Delta Q_{1,N_1+1}+\frac{\partial JC_{JN_3,1}}{\partial Q_{2,1}}\Delta Q_{2,1}+\frac{\partial JC_{JN_3,1}}{\partial Q_{3,1}}\Delta Q_{3,1}=-JC_{JN_3,1}$$
$$\frac{\partial JC_{JN_3,2}}{\partial y_{1,N_1+1}}\Delta y_{1,N_1+1}+\frac{\partial JC_{JN_3,2}}{\partial y_{2,1}}\Delta y_{2,1}=-JC_{JN_3,2}$$
$$\frac{\partial JC_{JN_3,3}}{\partial y_{l,N_1+1}}\Delta y_{1,N_1+1}+\frac{\partial JC_{JN_3,3}}{\partial y_{3,1}}\Delta y_{3,1}=-JC_{JN_3,3}$$

Program Implementation

$$\text{chl\_inf} = \begin{bmatrix} 1 & 100 & 50 & 2 & 2 & 25 & 0.0120 & 0.0005 & 0 & 3 \\ 2 & 1500 & 30 & 2 & 2 & 75 & 0.0125 & 0.0004 & 3 & 2 \\ 3 & 500 & 20 & 2 & 2 & 25 & 0.0130 & 0.0012 & 3 & 2 \\ 4 & 100 & 40 & 2 & 2 & 25 & 0.0135 & 0.0005 & 2 & 1 \end{bmatrix}$$

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$$\text{jun\_inf} = \begin{bmatrix} 3 & -250 \\ -99999 & -99999 \\ -99999 & -99999 \end{bmatrix}$$

Program Implementation

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$$\text{jun\_inf} = \begin{bmatrix} 3 & -250 \\ -99999 & -99999 \\ -99999 & -99999 \end{bmatrix}$$

$$\text{jun\_con} = \begin{bmatrix} 1 & -4 & 0 & 0 \\ 3 & 4 & -3 & -2 \\ 3 & -1 & 2 & 3 \end{bmatrix}$$

Source Code

Channel Flow

- Channels network
  - steady_1D_channel_network_without_reverse.sci

Thank You