Module 06: Interaction of Different Types of Flow

Unit 01: Surface Water and Groundwater Interaction

Anirban Dhar

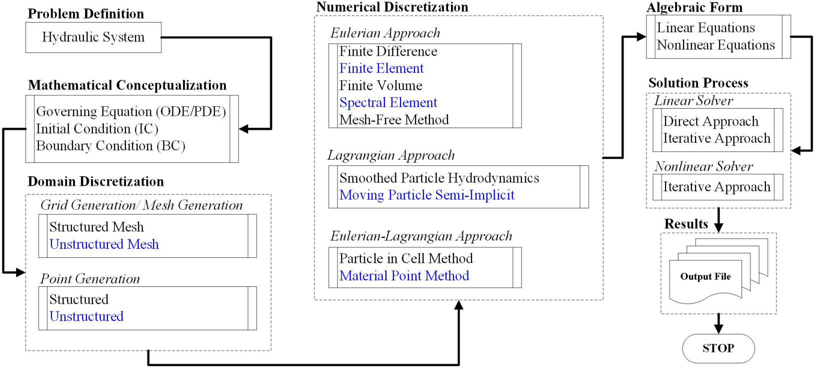
Department of Civil Engineering
Indian Institute of Technology Kharagpur, Kharagpur

National Programme for Technology Enhanced Learning (NPTEL)

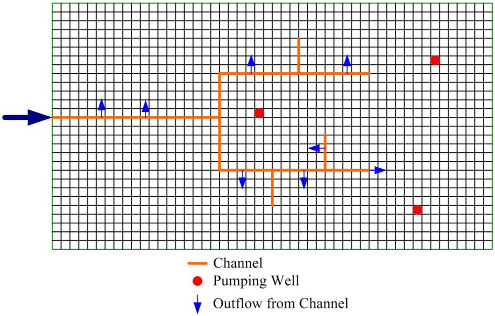
Learning Objective

- To solve unsteady interaction problem between channel flow, surface flow and groundwater flow.

Problem Definition to Solution



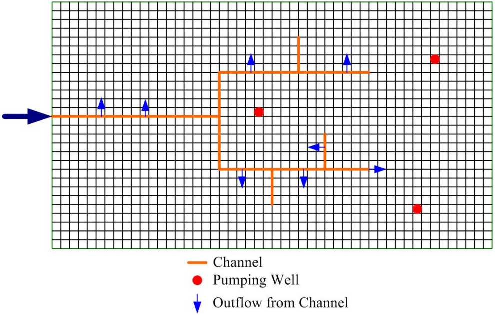
Problem Statement



Required

- Unsteady Channel Flow: $Q_c(x,t)$, $y_c(x,t)$

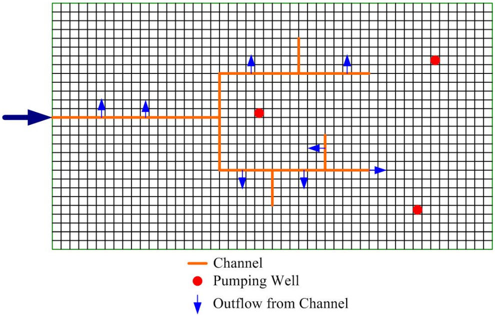
Problem Statement



Required

- Unsteady Channel Flow: $Q_c(x,t)$, $y_c(x,t)$
- Unsteady Free-surface Flow (Shallow water): $h_s(x,y,t)$, $u_s(x,y,t)$, $v_s(x,y,t)$

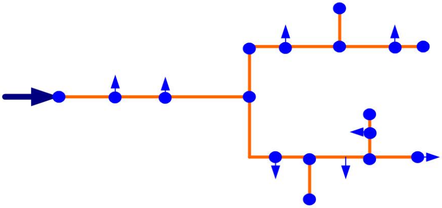
Problem Statement



Required

- Unsteady Channel Flow: $Q_c(x,t)$, $y_c(x,t)$
- Unsteady Free-surface Flow (Shallow water): $h_s(x,y,t)$, $u_s(x,y,t)$, $v_s(x,y,t)$
- Unsteady Unconfined Aquifer Flow: $h_g(x,y,t)$

Problem Definition
Channel Flow



Required

- Unsteady Channel Flow: $Q_c(x,t)$, $y_c(x,t)$

Problem Definition

Channel Flow

Governing Equation for unsteady 1D channel flow (St. Venant Equations) can be written as (Weiming, 2007),

Initial Boundary Value Problem

Continuity Equation:

$$\frac{\partial A}{\partial t} + \frac{\partial Q_c}{dx} = -q_c$$

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Momentum Equation:

$$\frac{\partial}{\partial t}\left(\frac{Q_c}{A}\right) + \frac{\partial}{\partial x}\left(\frac{\alpha Q_c^2}{2A^2}\right) + g\frac{\partial H}{\partial x} + gS_f = 0$$

Problem Definition

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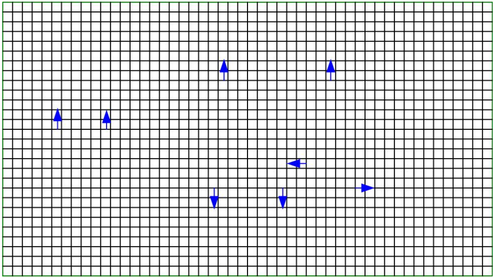
where

y_c= depth of flow
S_f= friction slope $\left(= \frac{n^2 Q^2}{R^{4/3} A^2}\right)$
A= cross-sectional area
q_c= lateral outflow
z= elevation of the channel bottom w.r.t. datum

H= water surface elevation $(= y_c + z)$
α= momentum correction factor
Q_c= discharge
g= acceleration due to gravity

Problem Definition

Unsteady Free-surface Flow



Required

- Unsteady Free-surface Flow (Shallow water): $h_s(x, y, t)$, $u_s(x, y, t)$, $v_s(x, y, t)$

Problem Definition
Unsteady Free-surface Flow

Depth-integrated mass and momentum conservation equations for surface water flow can be written as,

Governing equation

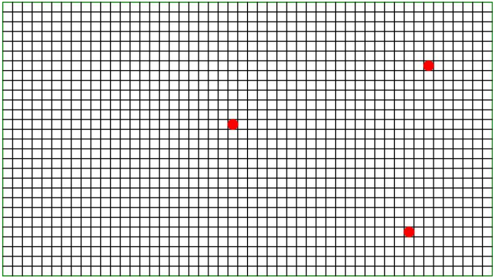
$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{E}}{\partial x} + \frac{\partial \mathbf{G}}{\partial y} = \mathbf{S} \tag{1}$$

$$\mathbf{U} = \begin{bmatrix} h_s \\ h_s u_s \\ h_s v_s \end{bmatrix}, \quad \mathbf{E} = \begin{bmatrix} h u_s \\ h_s u_s^2 + \frac{g h_s^2}{2} \\ h_s u_s v_s \end{bmatrix}, \quad \mathbf{G} = \begin{bmatrix} h_s v_s \\ h_s u_s v_s \\ h_s v^2 + \frac{g h_s^2}{2} \end{bmatrix}, \quad \mathbf{S} = \begin{bmatrix} R + q_c - q_s \\ g h_s (S_{0x} - S_{fx}) \\ g h_s (S_{0y} - S_{fy}) \end{bmatrix}$$

where h_s = water height, u_s, v_s = velocity at x and y directions.

Problem Definition

Unsteady Unconfined Aquifer Flow



Required

- Unsteady Unconfined Aquifer Flow: $h_g(x, y, t)$

Problem Definition

Unsteady Unconfined Aquifer Flow

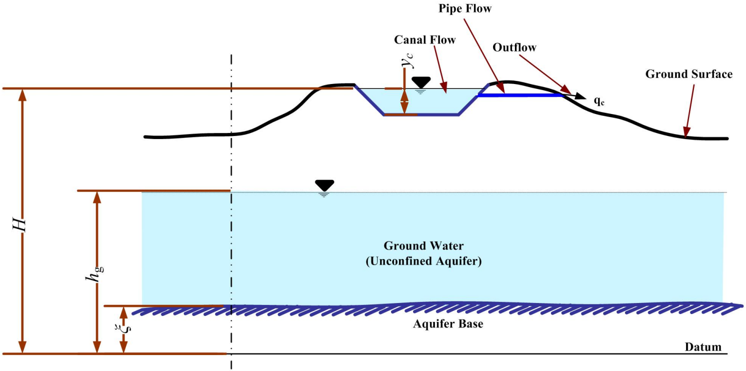
Governing equation

In two-dimension groundwater flow in unconfined aquifer can be written as,

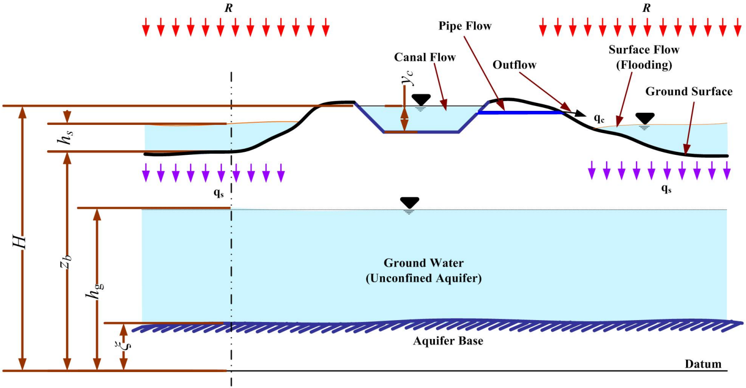
$$S_y \frac{\partial h_g}{\partial t} = \frac{\partial}{\partial x}\left(K_x(h_g - \xi)\frac{\partial h_g}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y(h_g - \xi)\frac{\partial h_g}{\partial y}\right) - W_P + W_I + q_s$$

where $K_x, K_y =$ hydraulic conductivity at x and y directions $W_I =$ injection rate, $W_P =$ pumping rate, $\xi =$ elevation of aquifer base.

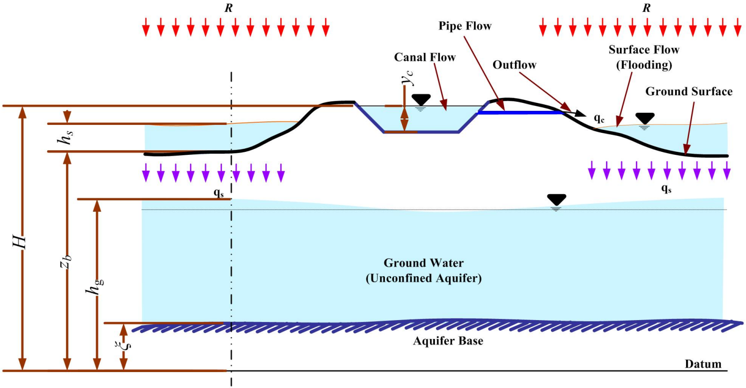
Canal-Surface Water-Groundwater Interaction



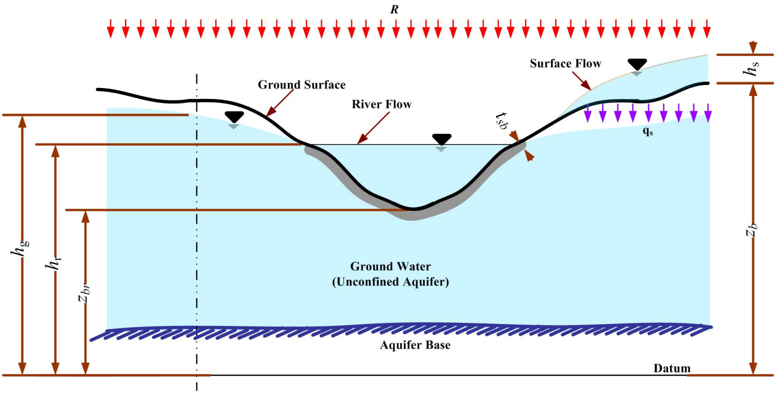
Canal-Surface Water-Groundwater Interaction



Canal-Surface Water-Groundwater Interaction



River-Surface Water-Groundwater Interaction



Algorithm Structure

Time-stepping

Data: $Q_c(x, t_n)$, $y_c(x, t_n)$, $h_s(x, y, t_n)$, $u_s(x, y, t_n)$, $v_s(x, y, t_n)$, $h_g(x, y, t_n)$

Algorithm Structure

Time-stepping

Data: $Q_c(x,t_n)$, $y_c(x,t_n)$, $h_s(x,y,t_n)$, $u_s(x,y,t_n)$, $v_s(x,y,t_n)$, $h_g(x,y,t_n)$

Result: Updated $Q_c(x,t_{n+1})$, $y_c(x,t_{n+1})$, $h_s(x,y,t_{n+1})$, $u_s(x,y,t_{n+1})$, $v_s(x,y,t_{n+1})$, $h_g(x,y,t_{n+1})$

Algorithm Structure
Time-stepping

Data: $Q_c(x, t_n)$, $y_c(x, t_n)$, $h_s(x, y, t_n)$, $u_s(x, y, t_n)$, $v_s(x, y, t_n)$, $h_g(x, y, t_n)$
Result: Updated $Q_c(x, t_{n+1})$, $y_c(x, t_{n+1})$, $h_s(x, y, t_{n+1})$, $u_s(x, y, t_{n+1})$, $v_s(x, y, t_{n+1})$, $h_g(x, y, t_{n+1})$

while *t <end time* **do**

  Solve Channel Flow with Δt_c

Algorithm Structure

Time-stepping

Data: $Q_c(x, t_n)$, $y_c(x, t_n)$, $h_s(x, y, t_n)$, $u_s(x, y, t_n)$, $v_s(x, y, t_n)$, $h_g(x, y, t_n)$
Result: Updated $Q_c(x, t_{n+1})$, $y_c(x, t_{n+1})$, $h_s(x, y, t_{n+1})$, $u_s(x, y, t_{n+1})$, $v_s(x, y, t_{n+1})$, $h_g(x, y, t_{n+1})$

while *t* < *end time* **do**
  Solve Channel Flow with Δt_c
  Calculate q_c

Algorithm Structure

Time-stepping

Data: $Q_c(x, t_n)$, $y_c(x, t_n)$, $h_s(x, y, t_n)$, $u_s(x, y, t_n)$, $v_s(x, y, t_n)$, $h_g(x, y, t_n)$
Result: Updated $Q_c(x, t_{n+1})$, $y_c(x, t_{n+1})$, $h_s(x, y, t_{n+1})$, $u_s(x, y, t_{n+1})$, $v_s(x, y, t_{n+1})$, $h_g(x, y, t_{n+1})$

while *t* <*end time* **do**
    Solve Channel Flow with Δt_c
    Calculate q_c
    Solve Surface Flow with Δt_s

Algorithm Structure
Time-stepping

Data: $Q_c(x,t_n)$, $y_c(x,t_n)$, $h_s(x,y,t_n)$, $u_s(x,y,t_n)$, $v_s(x,y,t_n)$, $h_g(x,y,t_n)$
Result: Updated $Q_c(x,t_{n+1})$, $y_c(x,t_{n+1})$, $h_s(x,y,t_{n+1})$, $u_s(x,y,t_{n+1})$, $v_s(x,y,t_{n+1})$, $h_g(x,y,t_{n+1})$

while *t* < *end time* **do**
- Solve Channel Flow with Δt_c
- Calculate q_c
- Solve Surface Flow with Δt_s
- Calculate q_s

Algorithm Structure

Time-stepping

Data: $Q_c(x, t_n)$, $y_c(x, t_n)$, $h_s(x, y, t_n)$, $u_s(x, y, t_n)$, $v_s(x, y, t_n)$, $h_g(x, y, t_n)$
Result: Updated $Q_c(x, t_{n+1})$, $y_c(x, t_{n+1})$, $h_s(x, y, t_{n+1})$, $u_s(x, y, t_{n+1})$, $v_s(x, y, t_{n+1})$, $h_g(x, y, t_{n+1})$

while *t* $<$*end time* **do**

  Solve Channel Flow with Δt_c
  Calculate q_c
  Solve Surface Flow with Δt_s
  Calculate q_s
  Solve Groundwater Flow with Δt_g

Algorithm Structure

Time-stepping

Data: $Q_c(x, t_n)$, $y_c(x, t_n)$, $h_s(x, y, t_n)$, $u_s(x, y, t_n)$, $v_s(x, y, t_n)$, $h_g(x, y, t_n)$
Result: Updated $Q_c(x, t_{n+1})$, $y_c(x, t_{n+1})$, $h_s(x, y, t_{n+1})$, $u_s(x, y, t_{n+1})$, $v_s(x, y, t_{n+1})$, $h_g(x, y, t_{n+1})$

while *t* $<$*end time* **do**
    Solve Channel Flow with Δt_c
    Calculate q_c
    Solve Surface Flow with Δt_s
    Calculate q_s
    Solve Groundwater Flow with Δt_g
    $n \leftarrow n + 1$
end

Algorithm Structure

Time-stepping

Data: $Q_c(x, t_n)$, $y_c(x, t_n)$, $h_s(x, y, t_n)$, $u_s(x, y, t_n)$, $v_s(x, y, t_n)$, $h_g(x, y, t_n)$
Result: Updated $Q_c(x, t_{n+1})$, $y_c(x, t_{n+1})$, $h_s(x, y, t_{n+1})$, $u_s(x, y, t_{n+1})$, $v_s(x, y, t_{n+1})$, $h_g(x, y, t_{n+1})$

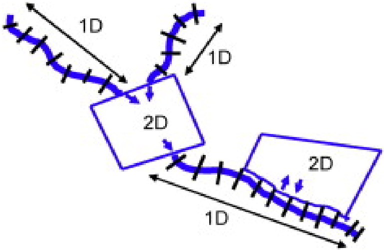
while *t* <*end time* **do**
- Solve Channel Flow with Δt_c
- Calculate q_c
- Solve Surface Flow with Δt_s
- Calculate q_s
- Solve Groundwater Flow with Δt_g
- $n \leftarrow n + 1$

end

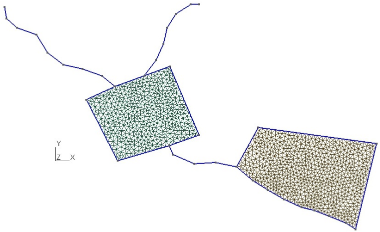
Time-Step

$\Delta t_c < \Delta t_s < \Delta t_g$

1D-2D Integrated System



(a) Integrated 1D-2D simulations with lateral and flow direction connections (Blade et al., 2012)



(b) Discretization of computational domain

Thank You

References

Blade, E., Gomez-Valentn, M., Dolz, J., Aragon-Hernandez, J., Corestein, G., and Sanchez-Juny, M. (2012). Integration of 1d and 2d finite volume schemes for computations of water flow in natural channels. *Advances in Water Resources*, 42:17 – 29.

Weiming, W. (2007). *Computational River Dynamics*. Taylor & Francis, London, UK.