Module 04: Surface Water Hydraulics

Unit 06: Steady Channel Flow: Channel Network with Reverse Flow

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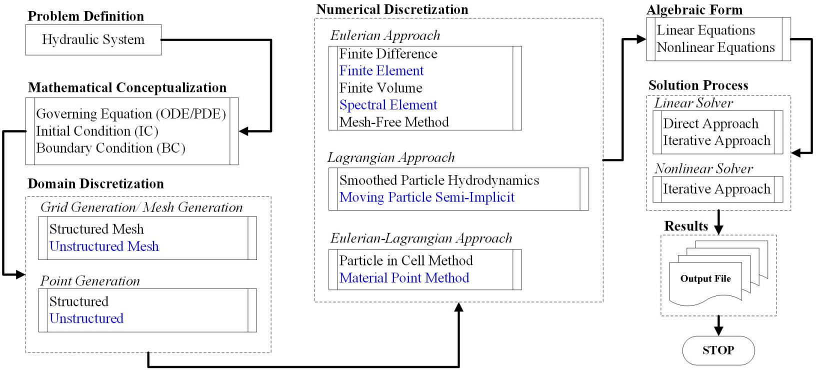
National Programme for Technology Enhanced Learning (NPTEL)

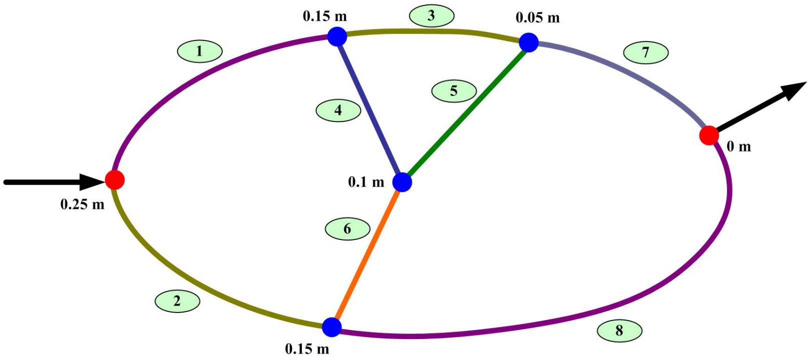
Learning Objective

- To solve steady channel flow for channel network problem with reverse flow using implicit method.

Problem Definition to Solution



Problem Statement



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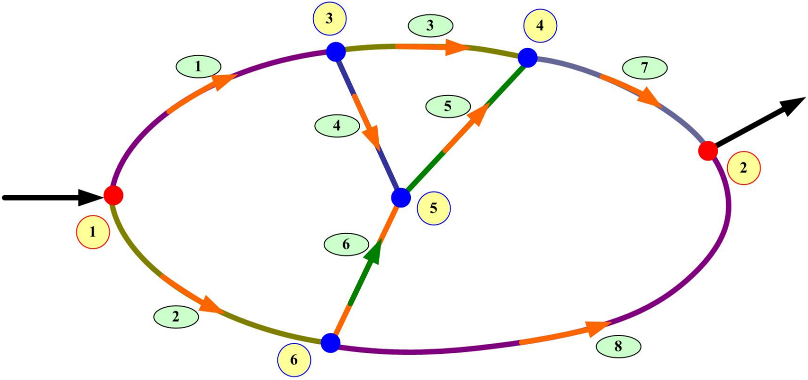

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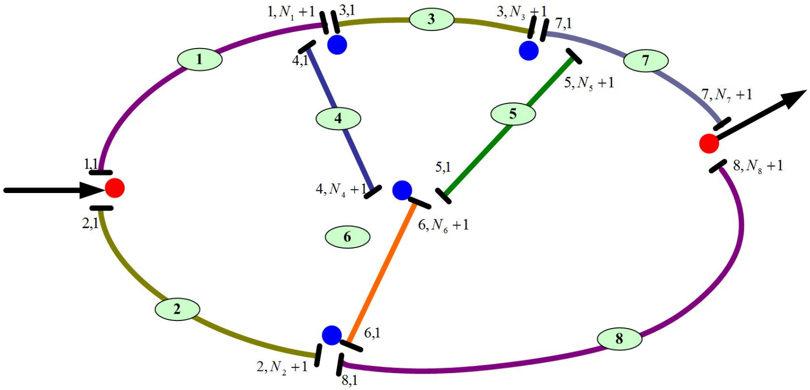
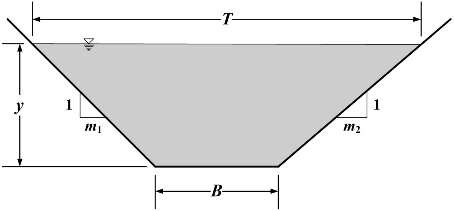


Problem Statement

Trapezoidal Cross-section



$$A = By + \frac{1}{2}(m_1 + m_2)y^2$$

$$P = B + \left(\sqrt{1 + m_1^2} + \sqrt{1 + m_2^2}\right) y$$

$$R = \frac{A}{P}$$

$$T = B + (m_1 + m_2)y$$

where $P =$ wetted perimeter.

Problem Statement

Channel Data

Channel	length (m)	width (m)	Side Slope m_1	Side Slope m_2	reach(m)	n	S_0	Connectivity JN_1	Connectivity JN_2
1	200	30	0	0	50	0.013	0.0005	1	3
2	200	40	0	0	50	0.013	0.0005	1	6
3	200	20	0	0	50	0.012	0.0005	3	4
4	100	20	0	0	25	0.014	0.0005	3	5
5	100	20	0	0	25	0.013	0.0005	5	4
6	100	25	0	0	25	0.013	0.0005	6	5
7	100	30	0	0	25	0.014	0.0005	4	2
8	300	30	0	0	75	0.014	0.0005	6	2

Problem Statement
Junction Data

Junction Number	Depth (m)	Discharge (m^3/s)	Bed Elevation (m)
1	-99999	250	0.25
2	5	-250	0
3	-99999	-99999	0.15
4	-99999	-99999	0.05
5	-99999	-99999	0.10
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Problem Statement

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Required

Estimate the flow depth and discharge across the channels.

Problem Definition

Governing Equation for Channel Flow can be written as (Chaudhry, 2008),

Boundary Value Problem

Continuity Equation:

$$\frac{dQ}{dx} = 0$$

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with

$$\mathbb{E} = y + z + \frac{\alpha Q^2}{2gA^2}$$

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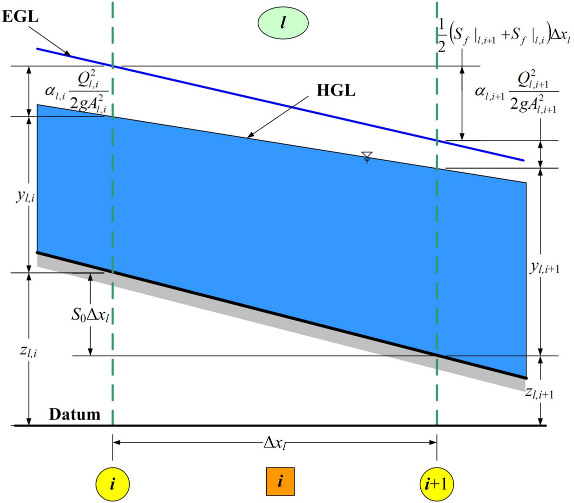
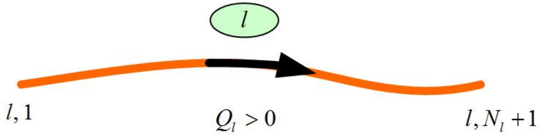
where

- $y=$ depth of flow
- $S_f=$ friction slope $\left(= \frac{n^2 Q^2}{R^{4/3} A^2}\right)$
- $A=$ cross-sectional area
- $R=$ hydraulic radius
- $z=$ elevation of the channel bottom w.r.t. datum
- $x=$ coordinate direction
- $\alpha=$ momentum correction factor
- $Q=$ discharge
- $g=$ acceleration due to gravity

Channel Flow



Channel Flow Conventions



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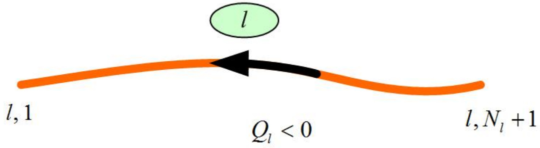

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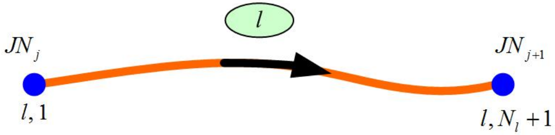

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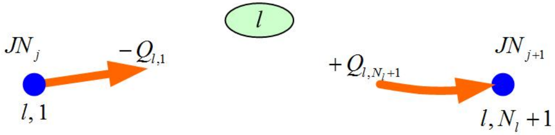

Algebraic Form

Continuity Equation

Discretized form of continuity equation

$$C_{l,i} = Q_{l,i+1} - Q_{l,i} = 0, \forall i \in \{1, \ldots, N_l\}$$

Algebraic Form

Continuity Equation

Discretized form of continuity equation

$$C_{l,i} = Q_{l,i+1} - Q_{l,i} = 0, \forall i \in \{1, \ldots, N_l\}$$

$$\frac{\partial C_{l,i}}{\partial y_{l,i}} = 0$$

$$\frac{\partial C_{l,i}}{\partial Q_{l,i}} = -1$$

$$\frac{\partial C_{l,i}}{\partial y_{l,i+1}} = 0$$

$$\frac{\partial C_{l,i}}{\partial Q_{l,i+1}} = 1$$

Discretization

Momentum Equation

In discretized form of momentum equation for i^{th} segment of the l^{th} channel reach,

$$M_{l,i} = (y_{l,i+1} - y_{l,i}) + (z_{l,i+1} - z_{l,i}) + \frac{\alpha_l}{2g}\left(\frac{Q_{l,i+1}^2}{A_{l,i+1}^2} - \frac{Q_{l,i}^2}{A_{l,i}^2}\right) + \frac{n_l^2 \Delta x_l}{2}\left[\frac{Q_{l,i+1}^2}{R_{l,i+1}^{4/3} A_{l,i+1}^2} + \frac{Q_{l,i}^2}{R_{l,i}^{4/3} A_{l,i}^2}\right], \quad \forall i \in \{1, \ldots, N_l\}$$

Considering reverse flow situation,

$$M_{l,i} = (y_{l,i+1} - y_{l,i}) + (z_{l,i+1} - z_{l,i}) + \frac{\alpha_l}{2g}\left(\frac{Q_{l,i+1}^2}{A_{l,i+1}^2} - \frac{Q_{l,i}^2}{A_{l,i}^2}\right) + \frac{n_l^2 \Delta x_l}{2}\left[\frac{Q_{l,i+1}|Q_{l,i+1}|}{R_{l,i+1}^{4/3} A_{l,i+1}^2} + \frac{Q_{l,i}|Q_{l,i}|}{R_{l,i}^{4/3} A_{l,i}^2}\right], \quad \forall i \in \{1, \ldots, N_l\}$$

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$2N_l$ non-linear equations with $2(N_l + 1)$ unknowns (discharge + flow-depth)

Algebraic Form
Momentum Equation

$$\frac{\partial M_{l,i}}{\partial y_{l,i}} = -1 + D_1 \frac{2Q_{l,i}^2}{A_{l,i}^3} \frac{dA}{dy}\Big|_{l,i} - D_2 \left[\frac{2Q_{l,i}|Q_{l,i}|}{A_{l,i}^3 R_{l,i}^{\frac{4}{3}}} \frac{dA}{dy}\Big|_{l,i} + \frac{4Q_{l,i}|Q_{l,i}|}{3A_{l,i}^2 R_{l,i}^{\frac{7}{3}}} \frac{dR}{dy}\Big|_{l,i} \right]$$

$$\frac{\partial M_{l,i}}{\partial Q_{l,i}} = -D_1 \frac{2Q_{l,i}}{A_{l,i}^3} + D_2 \frac{2|Q_{l,i}|}{A_{l,i}^2 R_{l,i}^{\frac{4}{3}}}$$

$$\frac{\partial M_{l,i}}{\partial y_{l,i+1}} = 1 - D_1 \frac{2Q_{l,i+1}^2}{A_{l,i+1}^3} \frac{dA}{dy}\Big|_{l,i+1} - D_2 \left[\frac{2Q_{l,i+1}|Q_{l,i+1}|}{A_{l,i+1}^3 R_{l,i+1}^{\frac{4}{3}}} \frac{dA}{dy}\Big|_{l,i+1} + \frac{4Q_{l,i+1}|Q_{l,i+1}|}{3A_{l,i+1}^2 R_{l,i+1}^{\frac{7}{3}}} \frac{dR}{dy}\Big|_{l,i+1} \right]$$

$$\frac{\partial M_{l,i}}{\partial Q_{l,i+1}} = D_1 \frac{2Q_{l,i+1}}{A_{l,i+1}^3} + D_2 \frac{2|Q_{l,i+1}|}{A_{l,i+1}^2 R_{l,i+1}^{\frac{4}{3}}}$$

with

$$D_1 = \frac{\alpha_l}{2g} \quad \text{and} \quad D_2 = \frac{1}{2} n_l^2 \Delta x_l$$

Trapezoidal Section

For trapezoidal channel cross-section,

$$\frac{dA}{dy} = B + (m_1 + m_2)y$$

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For trapezoidal channel cross-section,

$$\frac{dA}{dy} = B + (m_1 + m_2)y$$

$$\frac{dR}{dy} = \frac{T}{P} - \frac{R}{P}\frac{dP}{dy}$$

with

$$T = B + (m_1 + m_2)y$$
$$P = B + \left(\sqrt{1+m_1^2} + \sqrt{1+m_2^2}\right)y$$
$$R = \frac{A}{P}$$
$$\frac{dP}{dy} = \left(\sqrt{1+m_1^2} + \sqrt{1+m_2^2}\right)$$

Algebraic Form

In general form, continuity and momentum equations can be written as,

$$\frac{\partial C_{l,i}}{\partial y_{l,i}}\Delta y_{l,i} + \frac{\partial C_{l,i}}{\partial Q_{l,i}}\Delta Q_{l,i} + \frac{\partial C_{l,i}}{\partial y_{l,i+1}}\Delta y_{l,i+1} + \frac{\partial C_{l,i}}{\partial Q_{l,i+1}}\Delta Q_{l,i+1} = -C_{l,i}$$

$$\frac{\partial M_{l,i}}{\partial y_{l,i}}\Delta y_{l,i} + \frac{\partial M_{l,i}}{\partial Q_{l,i}}\Delta Q_{l,i} + \frac{\partial M_{l,i}}{\partial y_{l,i+1}}\Delta y_{l,i+1} + \frac{\partial M_{l,i}}{\partial Q_{l,i+1}}\Delta Q_{l,i+1} = -M_{l,i},$$

$$\forall i \in \{1, \ldots, N_l\}$$

Program Implementation

Configuration 1

$$\text{chl\_inf} = \begin{bmatrix} 1 & 200 & 30 & 0 & 0 & 50 & 0.0130 & 0.0005 & 1 & 3 \\ 2 & 200 & 40 & 0 & 0 & 50 & 0.0130 & 0.0005 & 1 & 6 \\ 3 & 200 & 20 & 0 & 0 & 50 & 0.0120 & 0.0005 & 3 & 4 \\ 4 & 100 & 20 & 0 & 0 & 25 & 0.0140 & 0.0005 & 3 & 5 \\ 5 & 100 & 20 & 0 & 0 & 25 & 0.0130 & 0.0005 & 5 & 4 \\ 6 & 100 & 25 & 0 & 0 & 25 & 0.0130 & 0.0005 & 6 & 5 \\ 7 & 100 & 30 & 0 & 0 & 25 & 0.0140 & 0.0005 & 4 & 2 \\ 8 & 300 & 50 & 0 & 0 & 75 & 0.0140 & 0.0005 & 6 & 2 \end{bmatrix}$$

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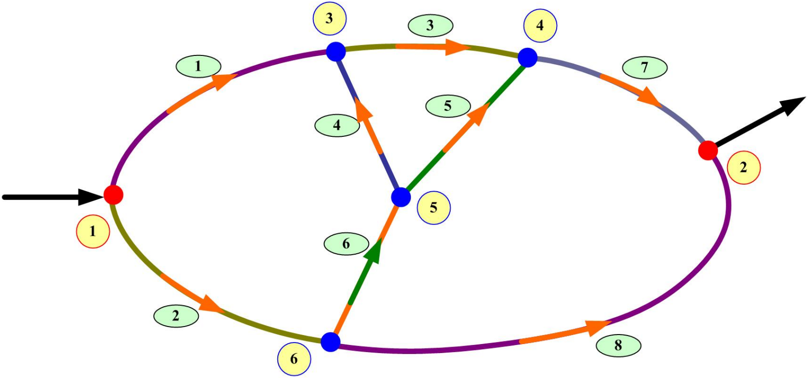
$$\text{jun\_inf} = \begin{bmatrix} -99999 & 250 & 0.25 \\ 5 & -250 & 0 \\ -99999 & -99999 & 0.15 \\ -99999 & -99999 & 0.05 \\ -99999 & -99999 & 0.10 \\ -99999 & -99999 & 0.15 \end{bmatrix} \quad \text{jun\_con} = \begin{bmatrix} 2 & 1 & 2 & 0 \\ 2 & -7 & -8 & 0 \\ 3 & -1 & 3 & 4 \\ 3 & -3 & -5 & 7 \\ 3 & -4 & -6 & 5 \\ 3 & -2 & 6 & 8 \end{bmatrix}$$

Problem Statement
Configuration 2



Problem Statement

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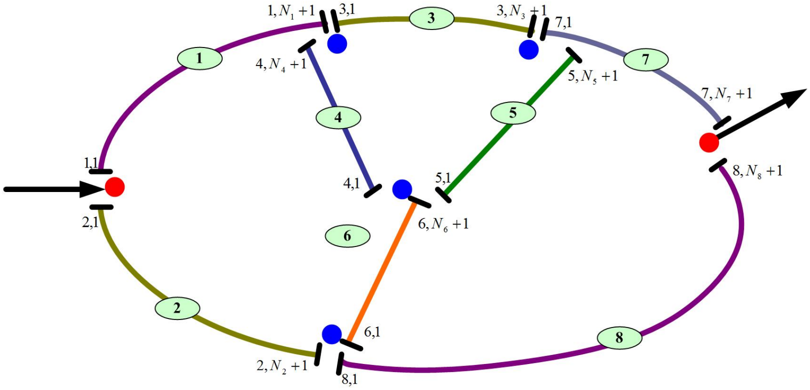

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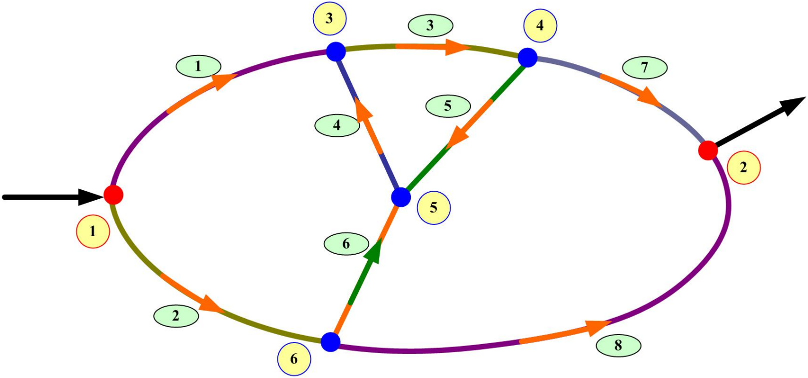
Configuration 3



Problem Statement

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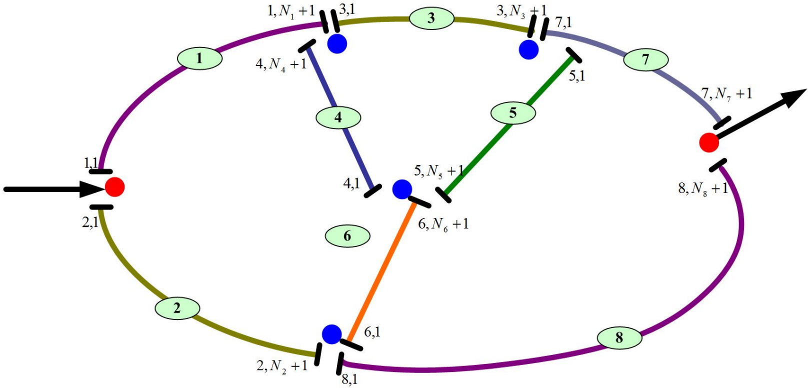

Program Implementation

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List of Source Codes

Channel Flow with Reverse Flow

- Channel Network with Configuration 1
  - steady_1D_channel_network_with_reverse_cfg1.sci

List of Source Codes

Channel Flow with Reverse Flow

- Channel Network with Configuration 1
  - steady_1D_channel_network_with_reverse_cfg1.sci
- Channel Network with Configuration 2
  - steady_1D_channel_network_with_reverse_cfg2.sci

List of Source Codes

Channel Flow with Reverse Flow

- Channel Network with Configuration 1
  - steady_1D_channel_network_with_reverse_cfg1.sci
- Channel Network with Configuration 2
  - steady_1D_channel_network_with_reverse_cfg2.sci
- Channel Network with Configuration 3
  - steady_1D_channel_network_with_reverse_cfg3.sci

Thank You

Reference

Chaudhry, M. H. (2008). *Open-Channel Flow*. Springer, India.