Module 01: Introduction to Computational Hydraulics

Unit 03: Classification of Problems based on Initial Condition (IC) and/or Boundary Condition (BC)

Anirban Dhar

Department of Civil Engineering
Indian Institute of Technology Kharagpur, Kharagpur

National Programme for Technology Enhanced Learning (NPTEL)

Learning Objectives

- To identify the initial and boundary conditions for hydraulic systems.

Learning Objectives

- To identify the initial and boundary conditions for hydraulic systems.
- To distinguish between the problems based on initial and boundary conditions.

Initial Condition

Initial Condition

Describes the initial state of the system in terms of dependent variables.

Initial Condition

Initial Condition

Describes the initial state of the system in terms of dependent variables.

- Initial water level and velocity in a channel network

Initial Condition

Initial Condition

Describes the initial state of the system in terms of dependent variables.

- Initial water level and velocity in a channel network
- Initial groundwater level in an aquifer region

Types of Boundary Conditions

Location Based

External boundary condition

defined for external/outermost locations

Types of Boundary Conditions

Location Based

External boundary condition

defined for external/outermost locations

- Upstream and downstream locations of a river

Types of Boundary Conditions
Location Based

External boundary condition

defined for external/outermost locations

- Upstream and downstream locations of a river
- River boundary for an aquifer region

Types of Boundary Conditions

Location Based

External boundary condition

defined for external/outermost locations

- Upstream and downstream locations of a river
- River boundary for an aquifer region

Internal boundary condition

defined for internal locations

Types of Boundary Conditions
Location Based

External boundary condition

defined for external/outermost locations

- Upstream and downstream locations of a river
- River boundary for an aquifer region

Internal boundary condition

defined for internal locations

- Operating conditions for hydraulic structures within channel network

Types of Boundary Conditions

Location Based

External boundary condition

defined for external/outermost locations

- Upstream and downstream locations of a river
- River boundary for an aquifer region

Internal boundary condition

defined for internal locations

- Operating conditions for hydraulic structures within channel network
- Constant water level maintained in a pond of an aquifer region

Types of Boundary Conditions

Physical Nature Based

Dirichlet/ Specified Boundary

Types of Boundary Conditions

Physical Nature Based

Dirichlet/ Specified Boundary

discharge specified at the inlet/ outlet in channel network.

Types of Boundary Conditions

Physical Nature Based

Dirichlet/ Specified Boundary

discharge specified at the inlet/ outlet in channel network.

Neumann/ Flux Boundary

Types of Boundary Conditions

Physical Nature Based

Dirichlet/ Specified Boundary

discharge specified at the inlet/ outlet in channel network.

Neumann/ Flux Boundary

no-flow boundary near impermeable region in aquifer system.

Types of Boundary Conditions

Physical Nature Based

Dirichlet/ Specified Boundary

discharge specified at the inlet/ outlet in channel network.

Neumann/ Flux Boundary

no-flow boundary near impermeable region in aquifer system.

Robin/ mixed Boundary

Types of Boundary Conditions

Physical Nature Based

Dirichlet/ Specified Boundary

discharge specified at the inlet/ outlet in channel network.

Neumann/ Flux Boundary

no-flow boundary near impermeable region in aquifer system.

Robin/ mixed Boundary

weighted combination of Dirichlet and Neumann conditions

Classification of Differential Equations

Differential Equation

- Ordinary Differential Equation

Classification of Differential Equations

Differential Equation

- Ordinary Differential Equation
  - Initial Value Problem (IVP): GE + IC

Classification of Differential Equations

Differential Equation

- Ordinary Differential Equation
  - Initial Value Problem (IVP): GE + IC
  - Boundary Value Problem (BVP): GE + BC

Classification of Differential Equations

Differential Equation

- Ordinary Differential Equation
  - Initial Value Problem (IVP): GE + IC
  - Boundary Value Problem (BVP): GE + BC
- Partial Differential Equation

Classification of Differential Equations

Differential Equation

- Ordinary Differential Equation
  - Initial Value Problem (IVP): GE + IC
  - Boundary Value Problem (BVP): GE + BC
- Partial Differential Equation
  - Boundary Value Problem (BVP): GE + BC

Classification of Differential Equations

Differential Equation

- Ordinary Differential Equation
  - Initial Value Problem (IVP): GE + IC
  - Boundary Value Problem (BVP): GE + BC
- Partial Differential Equation
  - Boundary Value Problem (BVP): GE + BC
  - Initial Boundary Value Problem (IBVP): GE + IC + BC

Gradually Varied Flow in Open Channel

Ordinary Differential Equation

Initial Value Problem

Governing Equation:

$$\frac{dy}{dx} = \frac{S_0 - S_f}{1 - Fr^2} \tag{1}$$

Gradually Varied Flow in Open Channel

Ordinary Differential Equation

Initial Value Problem

Governing Equation:

$$\frac{dy}{dx} = \frac{S_0 - S_f}{1 - Fr^2} \tag{1}$$

Initial Condition:

$$y|_{x=0} = y_0 \tag{2}$$

Steady One-Dimensional Groundwater Flow

Ordinary Differential Equation

Steady one-dimensional groundwater flow in unconfined aquifer

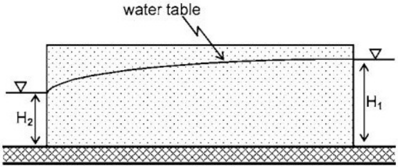


Figure: one-dimensional groundwater flow

Ordinary Differential Equation

Steady one-dimensional groundwater flow in unconfined aquifer

Boundary Value Problem

Governing Equation:

$$-\frac{d}{dx}\left(T\frac{dh(x)}{dx}\right) = f \tag{3}$$

Ordinary Differential Equation

Steady one-dimensional groundwater flow in unconfined aquifer

Boundary Value Problem

Governing Equation:

$$-\frac{d}{dx}\left(T\frac{dh(x)}{dx}\right) = f \tag{3}$$

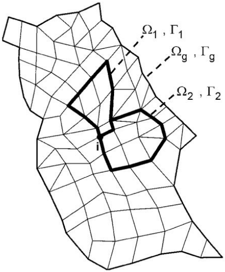
Boundary Condition:

$$h|_{x=0} = H_2 \tag{4a}$$

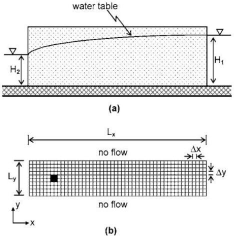
$$h|_{x=L_x} = H_1 \tag{4b}$$

Groundwater Movement in Aquifers

Variable: h(x,y,t)



(a) Descriptive schematics of discretizations of global domain and two subdomains (Dogrul and Kadir, 2006)



(b) cross section of heterogeneous aquifer between two lakes and simulation grids (Dogrul and Kadir, 2006)

Contaminant Transport

Variables: h(x,y,t), C(x,y,t)

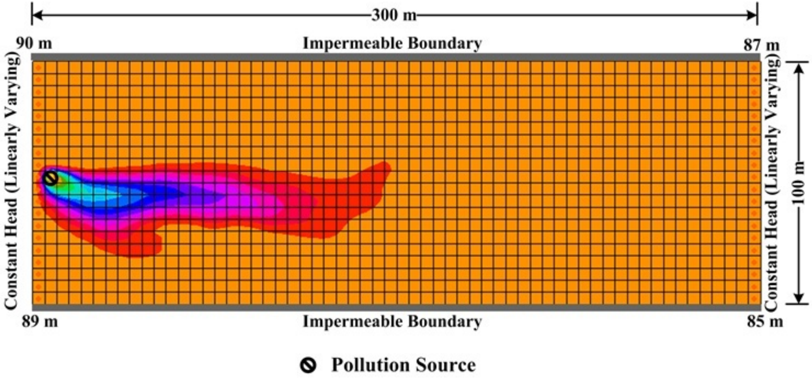


Figure: Contaminant Transport in Aquifer (Dhar and Patil, 2012)

Channel Networks

Variables: Q(x,t), h(x,t)

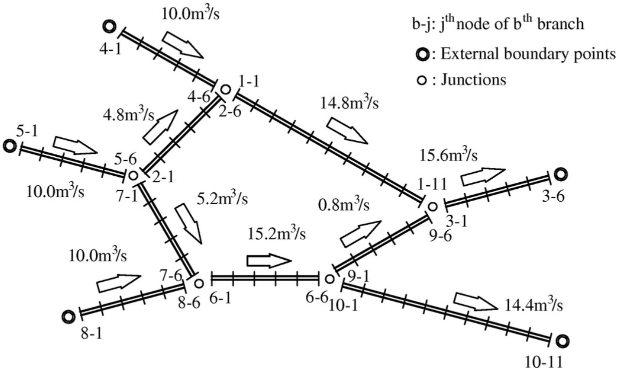


Figure: Typical channel network system (Garg and Sen, 2002)

Channel Networks

Internal Boundary condition

The junction conditions can be written as,

Mass conservation

$$\sum Q_i = \sum Q_o \tag{5}$$

where

Q_i =discharge of channel bed at inflow branch $[L^3/T]$

Q_o =discharge of channel bed at outflow branch $[L^3/T]$

Energy conservation

$$h_i + Z_i = h_o + Z_o \tag{6}$$

where

Z_i =elevation of channel bed at inflow branch $[L]$

Z_o =elevation of channel bed at outflow branch $[L]$

Surface Flooding

Variable: h(x,y,t)

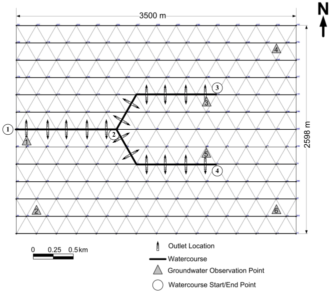
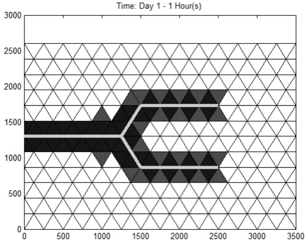
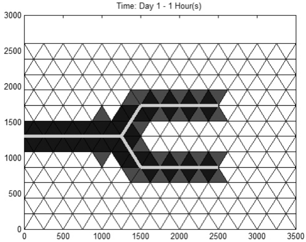


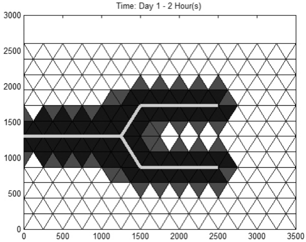
Figure: Initial Condition (Biswas, 2016)

Surface Flooding

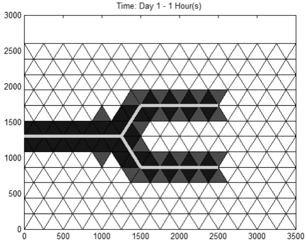


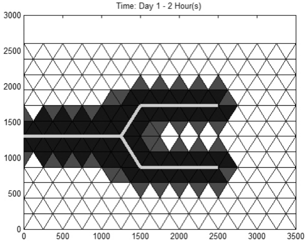
Surface Flooding

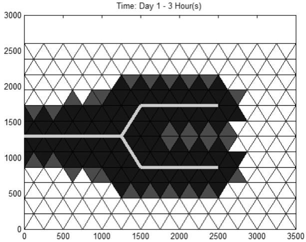




Surface Flooding







Surface Flooding

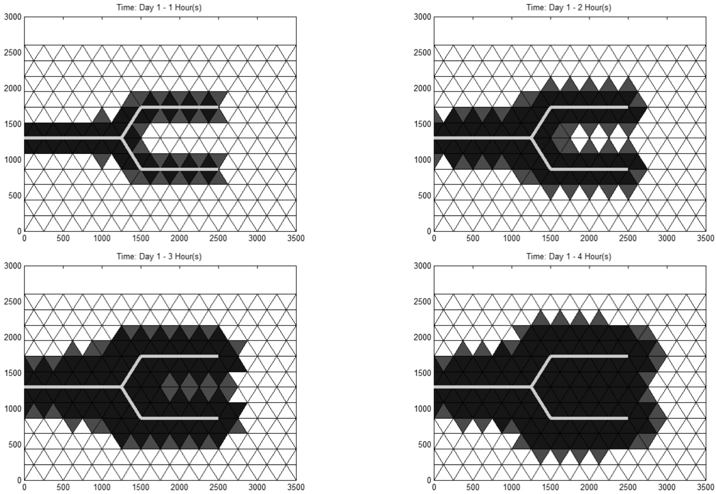


Figure: Surface flooding (Biswas, 2016)

Open Channel Flow

Hydraulic jump

Variables: u(x,z,t), w(x,z,t)

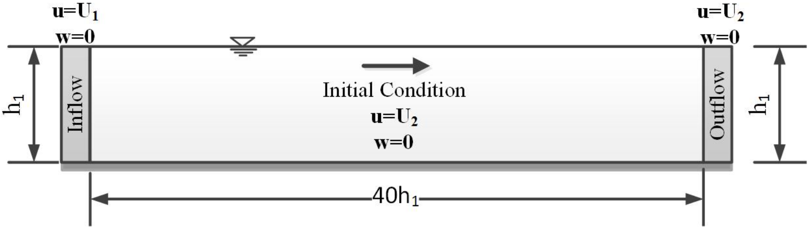
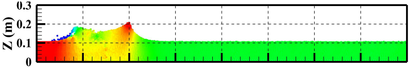
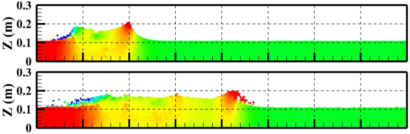


Figure: Initial condition of hydraulic jump (Pahar and Dhar, 2017)

Open Channel Flow
Hydraulic jump

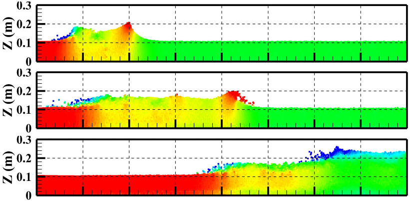


Open Channel Flow
Hydraulic jump

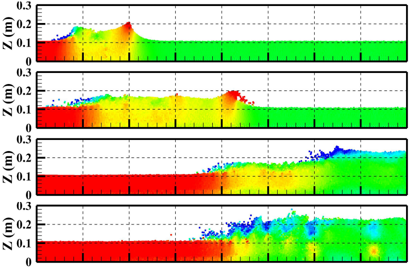


Open Channel Flow

Hydraulic jump

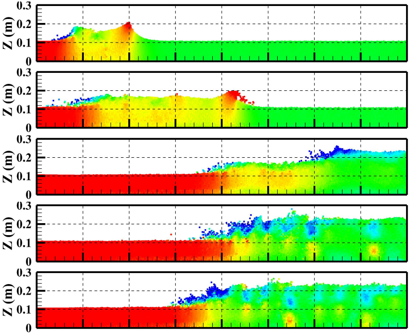


Open Channel Flow
Hydraulic jump



Open Channel Flow

Hydraulic jump



Open Channel Flow

Hydraulic jump

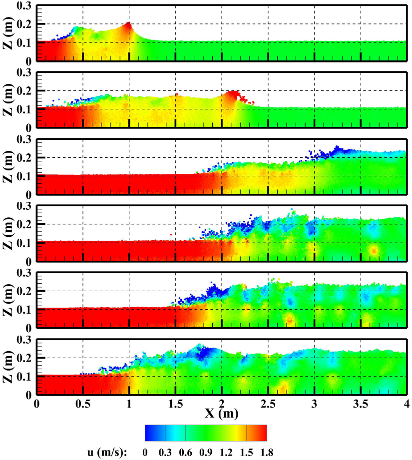


Figure: Velocity evolutions of hydraulic jump (Pahar and Dhar, 2017)

Pressurized Conduits

Variables: H(x,t), Q(x,t)

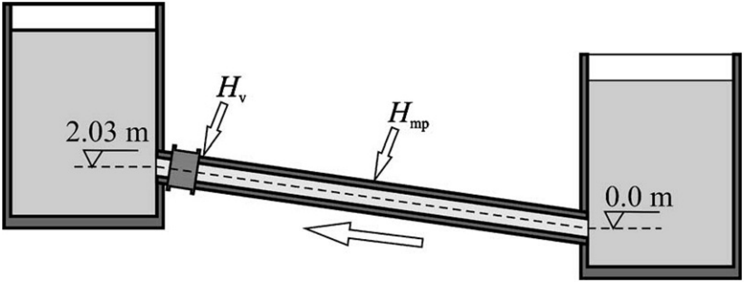


Figure: Connection between two reservoirs (Skific et al., 2010)

Pressurized Conduits

Variables: p(x,t), q(x,t)

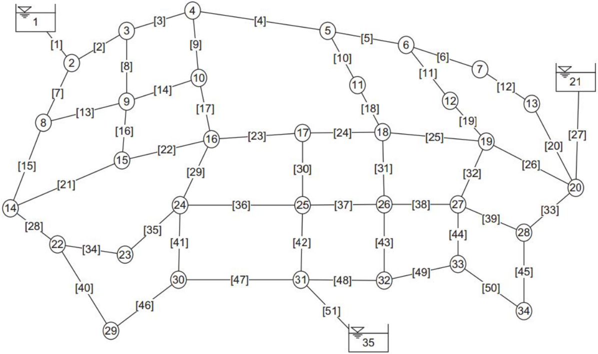


Figure: Pipe Networks (Zecchin et al., 2009)

Surface water-groundwater interaction

Variables: $\zeta^s(x,t)$, $p^s(x,t)$, $\zeta^g(x,t)$, $p^g(x,t)$,

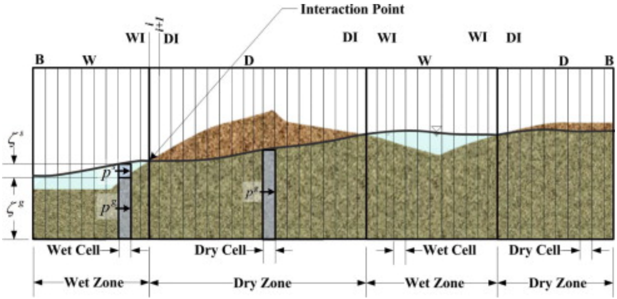
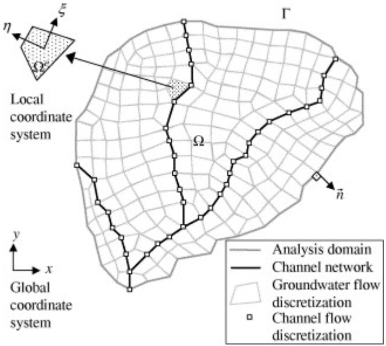


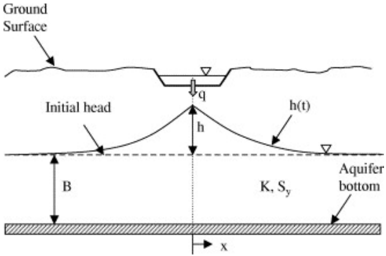
Figure: Conceptual representation of dry cell-wet cell theory (Pahar and Dhar, 2014)

Surface water-groundwater interaction

Variables: $h_s(x,t)$, $Q(x,t)$, $h_g(x,y,t)$



(a) Coupled modeling domain (Gunduz and Aral, 2005)



(b) Stream aquifer interaction (Gunduz and Aral, 2005)

Surface water-groundwater interaction

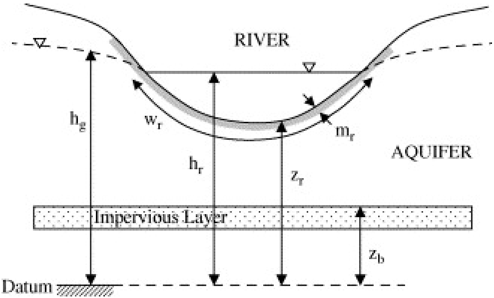


Figure: Channel flow/groundwater flow interaction (Gunduz and Aral, 2005)

Surface water-groundwater interaction

Boundary Condition

Specified Head Boundary

$$h_g(x, y, t) = H_D \tag{7}$$

Surface water-groundwater interaction

Boundary Condition

Specified Head Boundary

$$h_g(x, y, t) = H_D \tag{7}$$

Flux Boundary

$$-\mathbf{n}.((h_g - z_b))\mathbf{K}.\nabla h_g) = q_N(x, y, t) \tag{8}$$

Surface water-groundwater interaction

Boundary Condition

Specified Head Boundary

$$h_g(x, y, t) = H_D \tag{7}$$

Flux Boundary

$$-\mathbf{n}.((h_g - z_b))\mathbf{K}.\nabla h_g) = q_N(x, y, t) \tag{8}$$

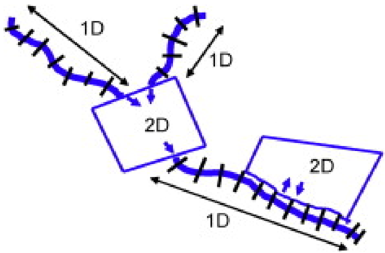
Mixed Boundary

$$-\mathbf{n}.((h_g - z_b))\mathbf{K}.\nabla h_g) = q_C(x, y, t) \tag{9}$$

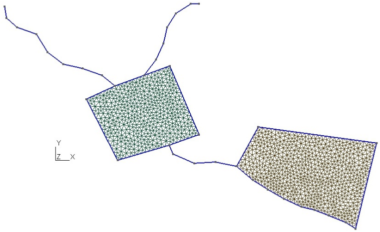
$$q_C(x, y, t) = \begin{cases} -K_r w_r \dfrac{h_r - h_g}{m_r}, & h_g > (z_r - m_r) \\ -K_r w_r \dfrac{h_r - (z_r - m_r)}{m_r}, & h_g \leq (z_r - m_r) \end{cases}$$

1D-2D integrated system

Variables: $h_c(x,t)$, $Q_c(x,t)$, $h_f(x,y,t)$, $u_f(x,y,t)$, $v_f(x,y,t)$



(a) Integrated 1D-2D simulations with lateral and flow direction connections (Blade et al., 2012)



(b) Discretization of computational domain

Thank You

References

Biswas, P. (2016). *Physically Based Numerical Simulation Model for Conjunctive water Use in Basin Irrigated Canal Command Areas*. PhD thesis, Indian Institute of technology Kharagpur.

Blade, E., Gomez-Valentn, M., Dolz, J., Aragon-Hernandez, J., Corestein, G., and Sanchez-Juny, M. (2012). Integration of 1d and 2d finite volume schemes for computations of water flow in natural channels. *Advances in Water Resources*, 42:17 – 29.

Dhar, A. and Patil, R. S. (2012). Multiobjective design of groundwater monitoring network under epistemic uncertainty. *Water Resources Management*, 26(7):1809–1825.

Dogrul, E. C. and Kadir, T. N. (2006). Flow computation and mass balance in galerkin finite-element groundwater models. *Journal of Hydraulic Engineering*, 132(11):1206–1214.

Garg, N. K. and Sen, D. J. (2002). Efficient algorithm for gradually varied flows in channel networks. *Journal of Irrigation and Drainage Engineering*, 128(6):351–357.

Gunduz, O. and Aral, M. M. (2005). River networks and groundwater flow: a simultaneous solution of a coupled system. *Journal of Hydrology*, 301(14):216 – 234.

Pahar, G. and Dhar, A. (2014). A dry zone-wet zone based modeling of surface water and groundwater interaction for generalized ground profile. *Journal of Hydrology*, 519, Part B:2215 – 2223.

Pahar, G. and Dhar, A. (2017). Robust boundary treatment for open-channel flows in divergence-free incompressible SPH. *Journal of Hydrology*, 546:464 – 475.

Skific, J., Macesic, S., and Crnjaric-Zic, N. (2010). Nonconservative formulation of unsteady pipe flow model. *Journal of Hydraulic Engineering*, 136(8):483–492.

Zecchin, A. C., Simpson, A. R., Lambert, M. F., White, L. B., and Vtkovsky, J. P. (2009). Transient modeling of arbitrary pipe networks by a laplace-domain admittance matrix. *Journal of Engineering Mechanics*, 135(6):538–547.