Principles of Flow

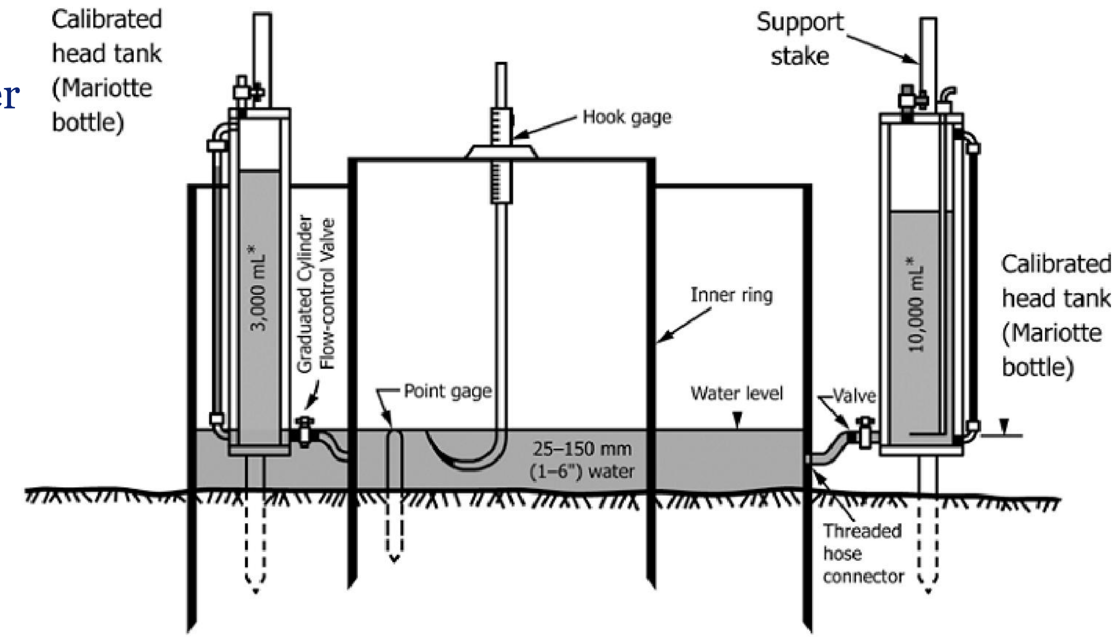
Groundwater Engineering| CE60205 Lecture:07

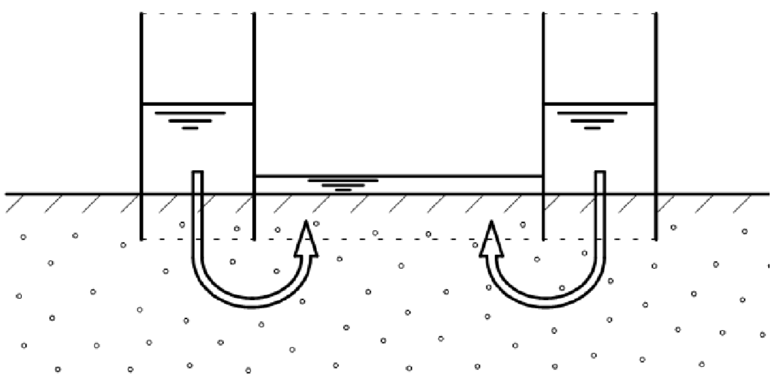
Learning Objective(s)

- To estimate field level saturated hydraulic conductivity
- To determine directional hydraulic conductivity

Field Measurement of Hydraulic Conductivity

- Double-Ring Infiltrometer
- Same water height in Outer and Inner cylinders
- Outer cylinder → function of reducing the three-dimensional (3D) radial flow from the inner cylinder
- Inner cylinder → one-dimensional (1D) infiltration process





Decreasing infiltration caused by different water levels in the inner and the outer rings.

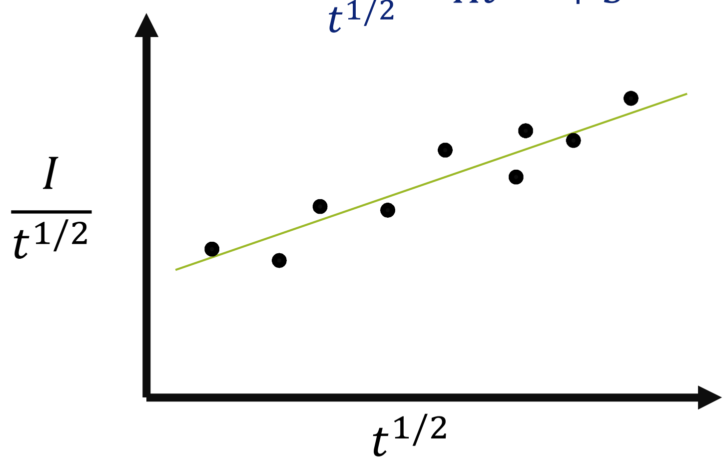
Field Measurement of Hydraulic Conductivity (Contd.)

- Cumulative infiltration

$$I = St^{1/2} + At$$

- $S(L\,T^{-1/2}) \rightarrow$ Soil sorptivity
- $A(L\,T^{-1}) \rightarrow$ a constant
- Plot I vs. t
- $K = A$ as $t \rightarrow \infty$
- For shorter times, $A = 0.5K$ or, $A = 2K/3$
- Plotting issue

$$\frac{I}{t^{1/2}} = At^{1/2} + S$$



$$y = mx + c$$

Darcy's Law in Three Dimensions

- In general form Darcy's law can be written as

$$q_x = -K_{xx}\frac{\partial h}{\partial x} - K_{xy}\frac{\partial h}{\partial y} - K_{xz}\frac{\partial h}{\partial z}$$
$$q_y = -K_{yx}\frac{\partial h}{\partial x} - K_{yy}\frac{\partial h}{\partial y} - K_{yz}\frac{\partial h}{\partial z}$$
$$q_z = -K_{zx}\frac{\partial h}{\partial x} - K_{zx}\frac{\partial h}{\partial y} - K_{zz}\frac{\partial h}{\partial z}$$

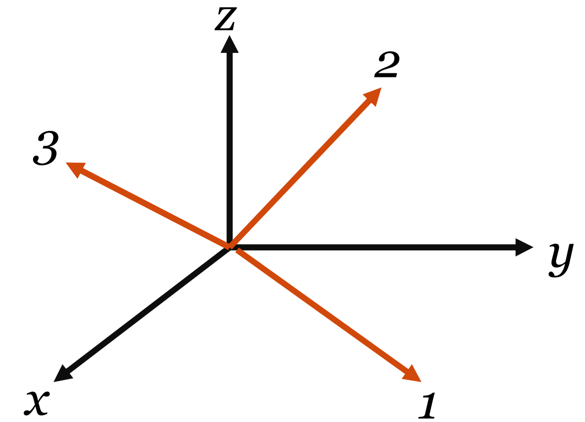
- In terms of piezometric head

$$\mathbf{q} = -\mathbf{K} \cdot \nabla h$$

Darcy's law (Contd.)

$$K=\begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix}$$

$$|K-\lambda I|=\begin{bmatrix} K_{11}-\lambda & K_{12} & K_{13} \\ K_{21} & K_{22}-\lambda & K_{23} \\ K_{31} & K_{32} & K_{33}-\lambda \end{bmatrix}$$



- Characteristic equation

$$\lambda^3 - I_1\lambda^2 - I_2\lambda - I_3 = 0$$

where

$$I_1 = tr(K),\ tr(\mathbf{K}) = \sum_{i=1}^{3} K_{ii}$$

$$I_2 = \frac{1}{2}\left[tr(\mathbf{K}^2) - [tr(\mathbf{K})]^2\right]$$

$$I_3 = \det(\mathbf{K})$$

$$q_1 = -K^{(1)}\frac{dh}{dx_1}$$

$$q_2 = -K^{(2)}\frac{dh}{dx_2}$$

$$q_3 = -K^{(3)}\frac{dh}{dx_3}$$

$$K' = \begin{bmatrix} K^{(1)} & 0 & 0 \\ 0 & K^{(2)} & 0 \\ 0 & 0 & K^{(3)} \end{bmatrix}$$

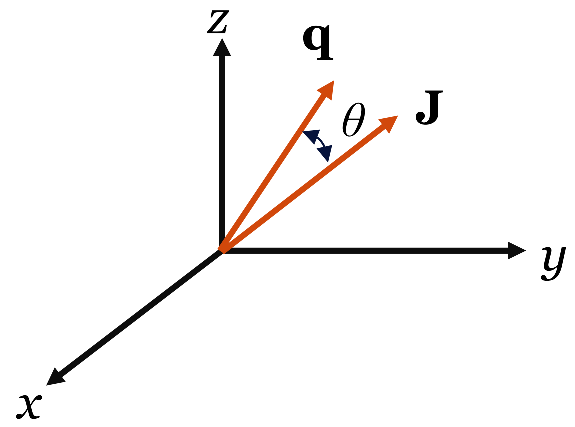
Darcy's law (Contd.)

- Isotropic condition

$$K' = \begin{bmatrix} K & 0 & 0 \\ 0 & K & 0 \\ 0 & 0 & K \end{bmatrix} = K \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{q} \cdot \mathbf{J} = q\ J\ cos\theta$$

$$cos\theta = \frac{\mathbf{q} \cdot \mathbf{J}}{q\ J} = \frac{K_x J_x^2 + K_y J_y^2 + K_z J_z^2}{q\ J}$$



Directional Hydraulic Conductivity

- In the direction of flow

$$K_q = \frac{q}{J\ cos\theta} = \frac{q^2}{\mathbf{q} \cdot \mathbf{J}} = \frac{\mathbf{q} \cdot \mathbf{q}}{\mathbf{q} \cdot \mathbf{J}} = \frac{K_x^2 J_x^2 + K_y^2 J_y^2 + K_z^2 J_z^2}{K_x J_x^2 + K_y J_y^2 + K_z J_z^2}$$

$$q_x = K_x\ J_x = q\ cos\beta_1$$

$$q_y = K_y\ J_y = q\ cos\beta_2$$

$$q_z = K_z\ J_z = q\ cos\beta_3$$

$$\frac{1}{K_q} = \frac{cos^2\beta_1}{K_x} + \frac{cos^2\beta_2}{K_y} + \frac{cos^2\beta_3}{K_z}$$

$$x = r\ cos\beta_1$$

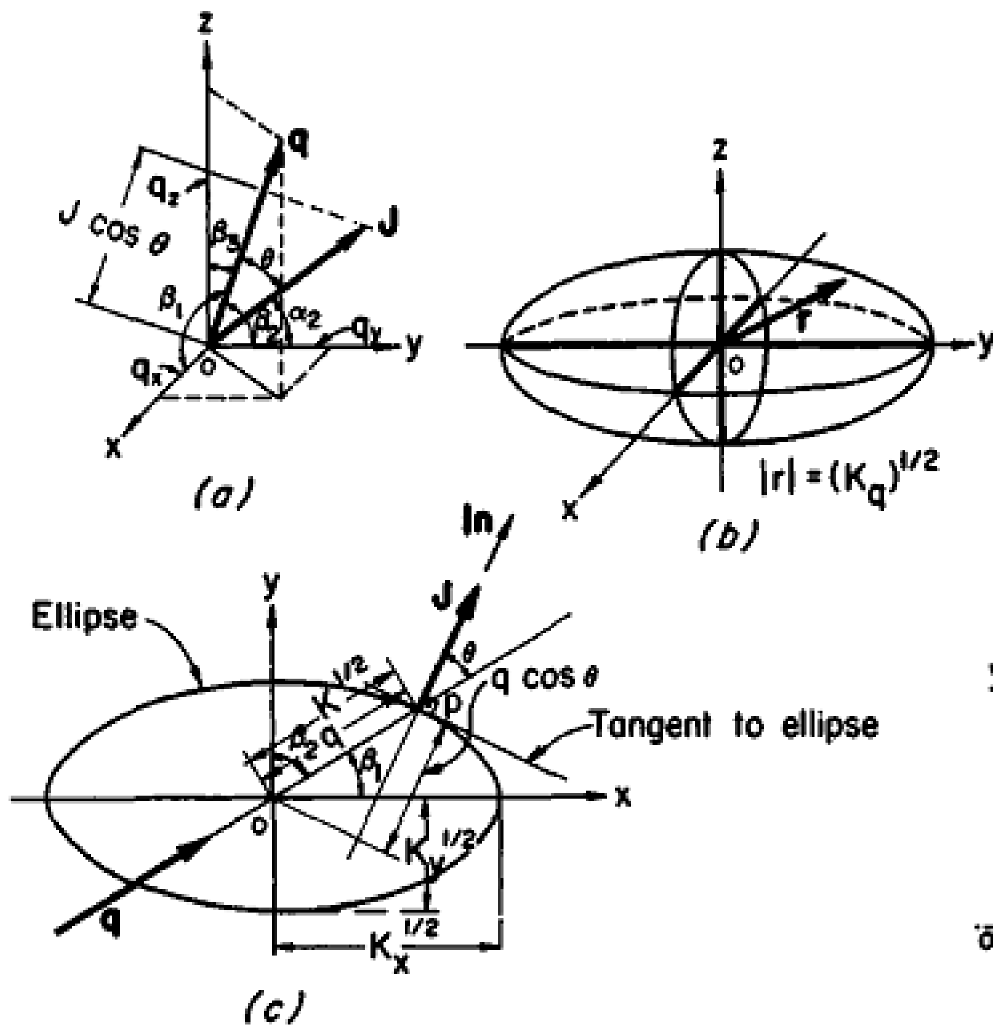
$$y = r\ cos\beta_2$$

$$z = r\ cos\beta_3$$

$$\frac{r^2}{K_q} = \frac{x^2}{K_x} + \frac{y^2}{K_y} + \frac{z^2}{K_z}$$

$$r = (K_q)^{1/2}$$

Directional Hydraulic Conductivity



Directional Hydraulic Conductivity

- In the direction of gradient

$$K_J = \frac{q\ cos\theta}{J} = \frac{\mathbf{q} \cdot \mathbf{J}}{J^2} = \frac{1}{J^2}(K_x J_x^2 + K_y J_y^2 + K_z J_z^2)$$

$$J_x = J\ cos\alpha_1$$

$$J_y = J\ cos\alpha_2$$

$$J_z = J\ cos\alpha_3$$

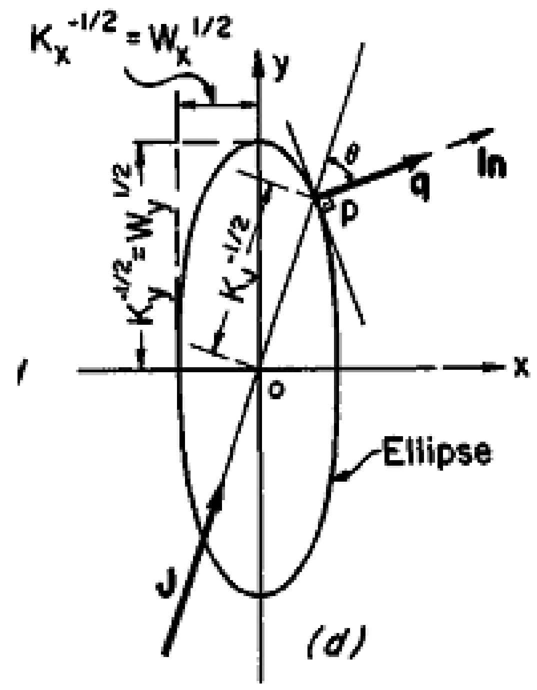
$$x = r\ cos\alpha_1,\ y = r\ cos\alpha_2,\ \ z = r\ cos\alpha_3$$

$$\frac{r^2}{1/K_J} = \frac{x^2}{1/K_x} + \frac{y^2}{1/K_y} + \frac{z^2}{1/K_z}$$

$$r = (K_J)^{-1/2}$$

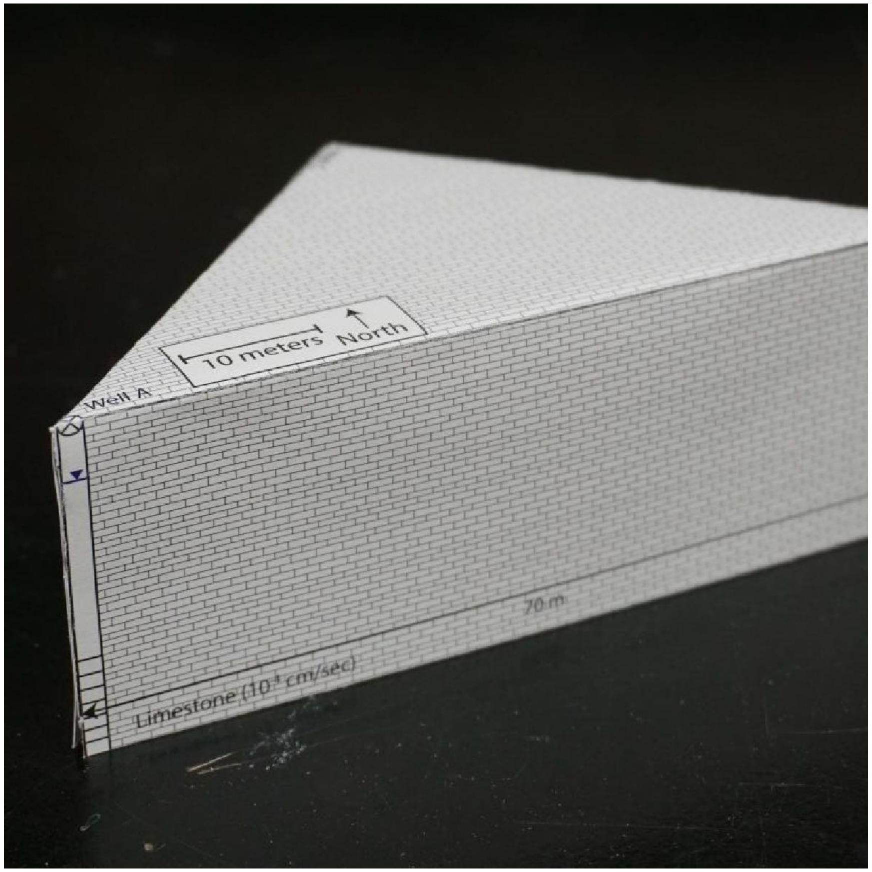
$$\frac{x^2}{W_x} + \frac{y^2}{W_y} + \frac{z^2}{W_z} = 1$$

$$W_x = \frac{1}{K_x}, W_y = \frac{1}{K_y}, W_z = \frac{1}{K_z}$$



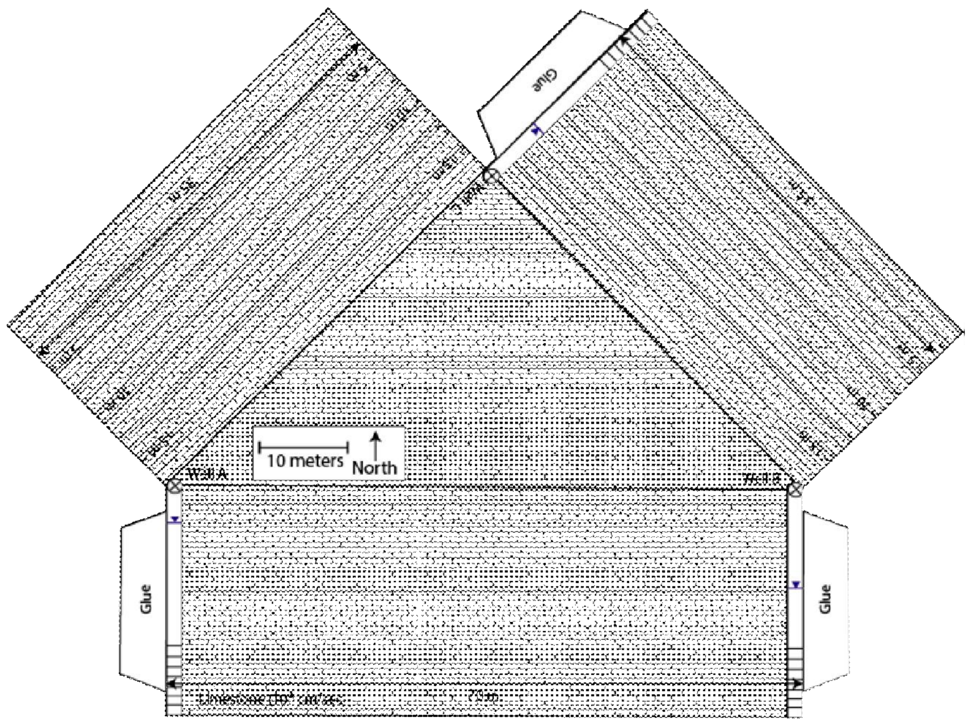
Home Lab

- Foldable Aquifer Project -http://aquifer.geology.buffalo.edu/
- Paper aquifer model
  - Three Point Problem



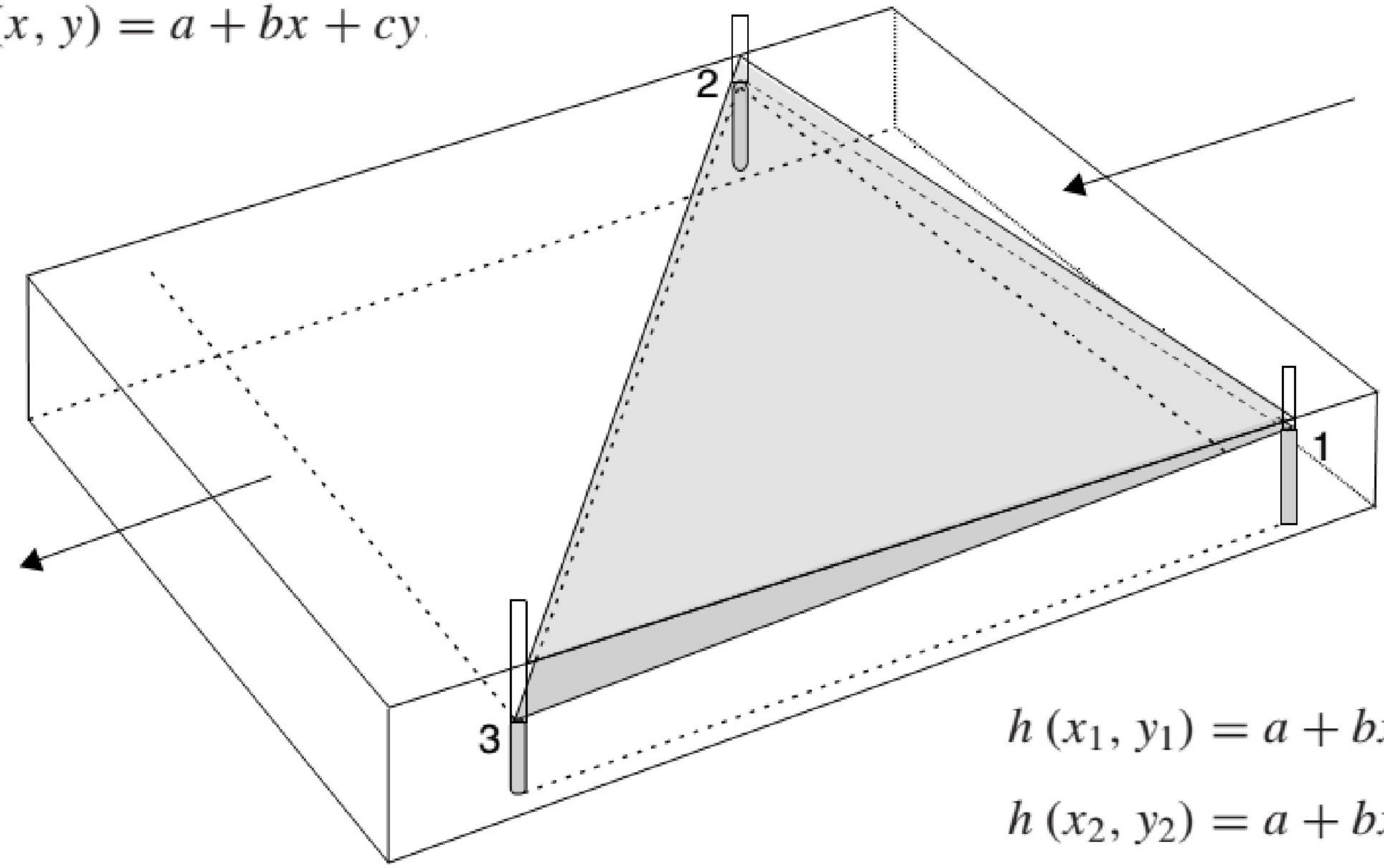
Home Lab (Contd.)

- The following problem contains of an unconfined limestone aquifer with three monitoring wells. The monitoring wells are positioned at each of the corners of the triangular aquifer. Using the water levels shown in wells complete the following tasks.
- A. Determine the direction of groundwater flow based on the water levels in wells A-C.
- B. Calculate the groundwater gradient across the unconfined aquifer.
- C. Quantify the specific discharge (q) in the unconfined aquifer.



Groundwater Flow Velocity Calculation in 2D

$$h(x, y) = a + bx + cy$$



$$h(x_1, y_1) = a + bx_1 + cy_1$$

$$h(x_2, y_2) = a + bx_2 + cy_2$$

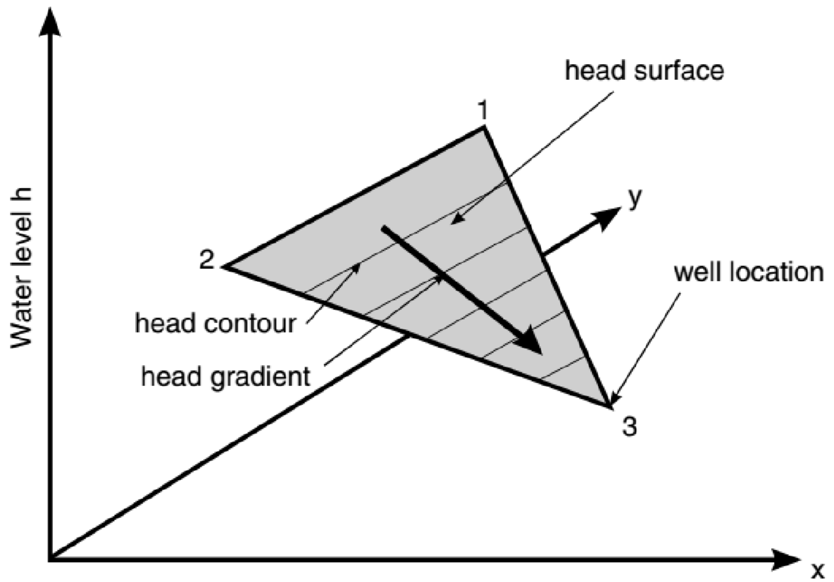
$$h(x_3, y_3) = a + bx_3 + cy_3$$

Groundwater Flow Velocity Calculation in 2D

- This provides three equations in the three unknowns, a, b , and c.

$$b = \frac{(h_1 - h_2)(y_2 - y_3) - (h_2 - h_3)(y_1 - y_2)}{(x_1 - x_2)(y_2 - y_3) - (x_2 - x_3)(y_1 - y_2)}$$

$$c = \frac{(h_1 - h_2)(x_2 - x_3) - (h_2 - h_3)(x_1 - x_2)}{(y_1 - y_2)(x_2 - x_3) - (y_2 - y_3)(x_1 - x_2)}$$



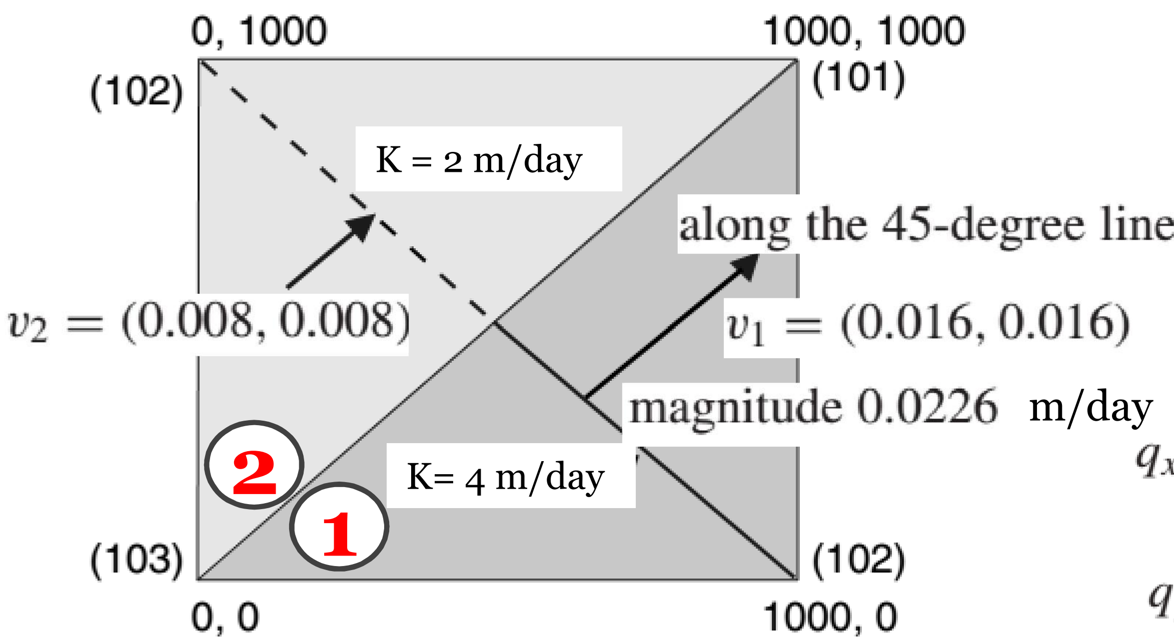
The groundwater gradient is given by

$$\nabla h = \frac{\partial h}{\partial x}\mathbf{i} + \frac{\partial h}{\partial y}\mathbf{j},$$

$$\frac{\partial h}{\partial x} = b \qquad \frac{\partial h}{\partial y} = c.$$

Problem

- The numbers appearing at the four corners of the square are the coordinates. The [0,1000] location is, for example, 1000 m north of the [0,0] location. The bracketed quantities represent the water levels observed in the wells located at the corners of the square. Calculate the groundwater velocity.



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$$\frac{dh}{dx} = b = -1/1000$$

$$\frac{dh}{dy} = c = -1/1000.$$

$$q_x = -K_{xx} \times \frac{dh}{dx} = 0.004,$$

$$q_y = -K_{yy} \times \frac{dh}{dx} = 0.004.$$

porosity 0.25

Learning Strategy

Chapter 3: Principles of Flow

Thank you