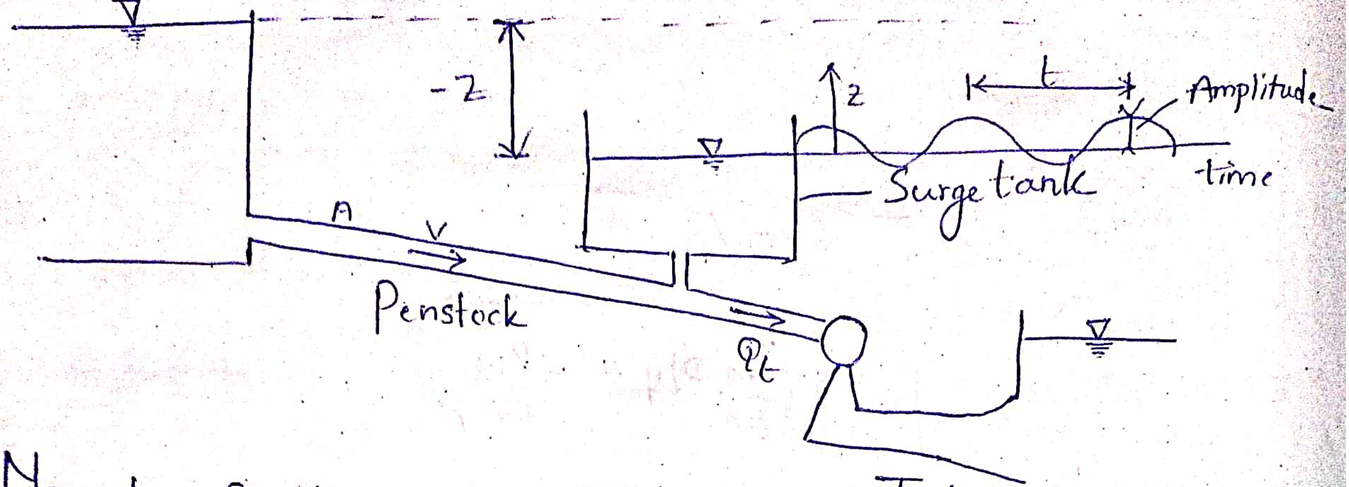
Surge tank - Lumped system approach



Momentum Equation (Neglecting minor losses and gravitational forces)

$$\frac{L}{g}\frac{dv}{dt} = H_R - \frac{FLv|v|}{2gD}$$

$$\Rightarrow \frac{L}{g}\frac{dv}{dt} = -Z - \frac{FLv|v|}{2gD} \longrightarrow (1)$$

Continuity Equation

Input - Output = storage (at Surge tank)

$$A_s\left(\frac{dz}{dt}\right) = VA - Q_t \longrightarrow (2)$$

for steady state, eq (1) & (2) becomes

$$-Z = \frac{FLv|v|}{2gD} \longrightarrow (3)$$

$$V_0A = Q_t \longrightarrow (4)$$

By neglecting pipe friction Equations (1) & (2) yield Simple harmonic motion about the level of water in the reservoir. It is a periodic motion or oscillatory motion where the restoring force is directly propotional and acts in the direction opposite to that of displacement.

$$\frac{L}{g}\frac{dv}{dt} = -Z \longrightarrow (5)$$

$$A_s\frac{dz}{dt} = VA - Q_t \longrightarrow (6)$$

Differentiating eq ⑥ again with respect to time 't'

$$A_s \frac{d^2 z}{dt^2} = \frac{dv}{dt} \cdot A - \cancel{\frac{dQ_t}{dt}}^{\,0}$$ (In Lumped system, change of discharge is Sudden)

$$\frac{L}{g} A_s \frac{d^2 z}{dt^2} = -zA$$

$$\frac{d^2 z}{dt^2} = -\left(\frac{A}{A_s} \cdot \frac{g}{L}\right) z$$

$$\frac{d^2 z}{dt^2} = -\omega^2 \cdot z$$

$$\omega^2 = \frac{g}{L} \cdot \frac{A}{A_s}$$

where $\omega = \sqrt{\frac{g}{L} \cdot \frac{A}{A_s}}$

$$\frac{d^2 z}{dt^2} + \omega^2 z = 0$$

$$(D^2 + \omega^2) z = 0$$

This is in the form of $aD^2 + bD + c = 0$, $c = \omega^2$

$$b^2 - 4ac < 0$$

$$z = e^{\alpha x}\left(c_1 \cos\beta t + c_2 \sin\beta t\right)$$

$$\alpha = \frac{-b}{2a} = 0$$

$$\beta = \frac{\sqrt{4ac - b^2}}{2a} = \frac{\sqrt{4\omega^2}}{2(1)} = \omega$$

$$\therefore \alpha = 0, \ \beta = \omega$$

$$z = c_1 \cos\omega t + c_2 \sin\omega t \longrightarrow ⑦$$

Boundary Conditions

1). @ $t = 0$, $z = 0$

2). @ $t = 0$, $\frac{dz}{dt} = \frac{Q - Q_t}{A_s}$

$\therefore C_1 = 0$

$$\frac{Q - Q_t}{A_s} = \omega C_1 (-\sin \cancel{\omega t}) + \omega C_2 \cos \omega t$$

$$C_2 = \frac{Q - Q_t}{A_s \omega}$$

$$\boxed{Z = \frac{Q - Q_t}{A_s \omega} \sin \omega t}$$

$$Z_{max} = \frac{Q - Q_t}{A_s \omega}.$$

where $\omega = \sqrt{\frac{g}{L} \cdot \frac{A}{A_s}}$ & $T = \frac{1}{\omega}$

Prob: A pipe of length 500m and dia 1.5m is used to deliver water from a reservoir to turbine at $Q = 2m^3/sec$. The turbine is protected by cylindrical surge tank with diameter 5m.

Find i). Maximum Surge

ii). Time period of pressure wave

frictional losses are neglected

Case a). Entire flow to the turbine is stopped

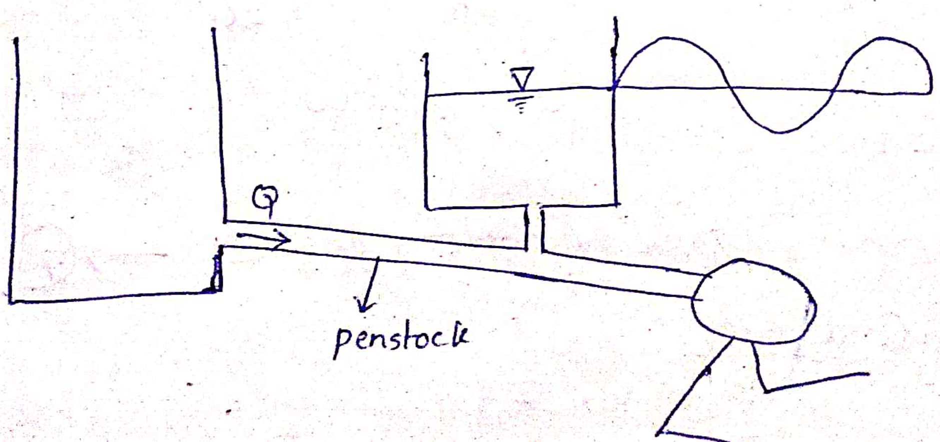
b). Only 50% of the flow to turbine is stopped

$$A_p = \frac{\pi}{4} \times 1.5^2 = 1.77 m^2$$

$$A_s = \frac{\pi}{4} \times 5^2 = 19.63 m^2$$

$$\omega = \sqrt{\frac{g}{L} \cdot \frac{A_p}{A_s}} = \sqrt{\frac{9.81 \times 1.77}{500 \times 19.63}} = 0.042 m/sec$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{0.042} = 150 \text{ sec}$$



a). Entire flow to the turbine is stopped

$$Z_{max} = \frac{Q_0 - Q_t}{A_s \omega} = \frac{2-0}{19.63 \times 0.042} = 2.42m$$

$$Z_{max} = \frac{Q_0 - Q_t}{A_s \omega} = \frac{2-1}{19.63 \times 0.042} = 1.21m$$

Mathematical modelling of Surge tank

Continuity Equation:

$$A_s \frac{dz}{dt} = Q_{tunnel} - Q_{turbine}$$

$$= Q_p - Q_t$$

$$\Rightarrow \frac{dz}{dt} = \frac{Q_p - Q_t}{A_s} = \lambda$$

Momentum Equation:

$$\frac{L}{g}\frac{dv}{dt} = -z - \frac{fL Q_p |Q_p|}{2gDA_p^2}$$

$$\frac{L}{g}\frac{dv}{dt} = -z - C_f Q_p |Q_p|$$

where $C_f = \frac{fL}{2gDA_p^2}$

$$\Rightarrow k = \frac{dv}{dt} = \frac{g}{L}\left(-z - C_f Q_p |Q_p|\right)$$

Using Runge kutta method,

$$k_1 = \frac{g}{L}\left(-z^t - C_f Q_p^t |Q_p^t|\right)$$

$$\lambda_1 = \frac{1}{A_s}\left(Q_p^t - Q_t^t\right)$$

$$Q_{1p} = Q_p^t + 0.5\Delta t . k_1 A_p$$

$$Z_1 = z^t + 0.5 \lambda_1 \Delta t$$

$$K_2 = \frac{g}{L}\left(-Z_1 - C_F Q_{1p}|Q_{1p}|\right)$$

$$\lambda_2 = \frac{1}{A_s}\left(Q_{1p} - Q_t^t\right)$$

$$Q_{2p} = Q_p^t + 0.5\Delta t\, K_2 A_p$$

$$Z_2 = z^t + 0.5\lambda_2 \Delta t$$

$$K_3 = \frac{g}{L}\left(-Z_2 - C_F Q_{2p}|Q_{2p}|\right)$$

$$\lambda_3 = \frac{1}{A_s}\left(Q_{2p} - Q_t^t\right)$$

$$Q_{3p} = Q_p^t + 0.5\Delta t\, K_3 A_p$$

$$Z_3 = z^t + 0.5\lambda_3 \Delta t$$

$$K_4 = \frac{g}{L}\left(-Z_3 - C_F Q_{3p}|Q_{3p}|\right)$$

$$\lambda_4 = \frac{1}{A_s}\left(Q_{3p} - Q_t^t\right)$$

$$Q_{4p} = Q_p^t + \Delta z\, K_4 A_p$$

$$Z_4 = z^t + \Delta t\, \lambda_4$$

$$Q_p^{t+\Delta t} = \frac{1}{6}\left(Q_{1p} + 2Q_{2p} + 2Q_{3p} + Q_{4p}\right)$$

$$z^{t+\Delta t} = \frac{1}{6}\left(Z_1 + 2Z_2 + 2Z_3 + Z_4\right)$$