Water in Subsurface Environment

Groundwater Engineering| CE60205 Lecture:02

Learning Objective(s)

To estimate the physical properties of water, air, and porous media

Properties of Water

- Density
  - The mass density of fresh water ρ_w varies within a narrow range
  - Weight density $\rho_w g$ equals the mass density times the gravitational acceleration at the earth's surface, g
- Water density does vary slightly with temperature, pressure, and chemistry if the concentration of solute molecules is high enough.
- The density of pure water at atmospheric pressure varies between 0.998 and 1.000 g/cm^3 in the range of temperatures typical for groundwater (0°C to 20°C)
- As the temperature of a liquid rises, it usually becomes less dense as molecules move with greater velocity and molecular attraction forces are overcome to a greater extent.
- Water is an unusual liquid because the maximum density does not occur at the freezing temperature, but instead slightly above freezing at 4°C.

Property	Symbol	Dimensions[†]	Value
Mass density	ρ_w	$[M/L^3]$	1.00 g/cm^3 1000 kg/m^3 1.94 slugs/ft^3
Weight density	$\rho_w g$	$[F/L^3]$	9810 N/m^3 62.4 lb/ft^3
Compressibility	β	$[L^2/F]$	4.5×10^{-10} m^2/N
Dynamic viscosity	μ	$[FT/L^2]$	1.4×10^{-3} N·sec/m^2

[†] L = length, M = mass, T = time, F = ML/T^2 = force.

Properties of Water (Contd.)

- Compressibility
  - Water is often considered incompressible, but it does have a finite, low compressibility.
  - As water pressure P rises an amount dP at a constant temperature, the density of water increases $d\rho_w$ from its original density ρ_w, and a given volume of water V_w will decrease in volume by dV_w in accordance with

$$\beta dP = \frac{d\rho_w}{\rho_w} = -\frac{dV_w}{V_w}$$

  - where β is the isothermal compressibility of water.

Property	Symbol	Dimensions†	Value
Mass density	ρ_w	$[M/L^3]$	$1.00\ g/cm^3$ $1000\ kg/m^3$ $1.94\ slugs/ft^3$
Weight density	$\rho_w g$	$[F/L^3]$	$9810\ N/m^3$ $62.4\ lb/ft^3$
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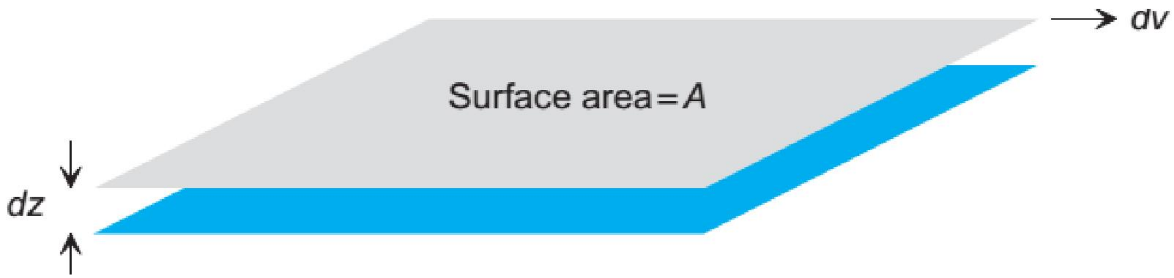
†L = length, M = mass, T = time, F = ML/T^2 = force.

Properties of Water (Contd.)

- Viscosity
  - Viscosity is friction within a fluid that results from the strength of molecule-to-molecule attractions.
  - The resisting force *F*

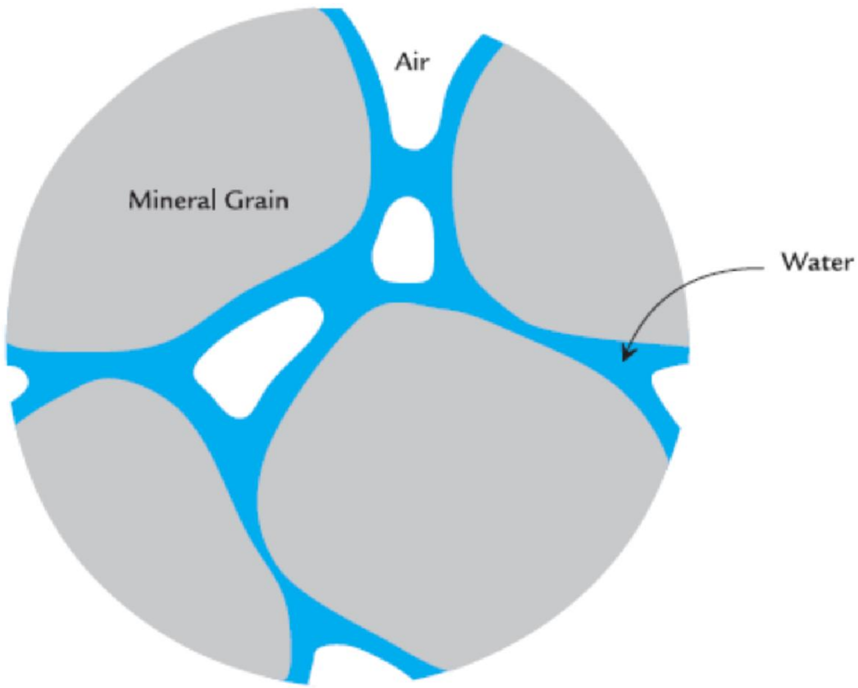
$$F = A\mu\frac{dv}{dz}$$



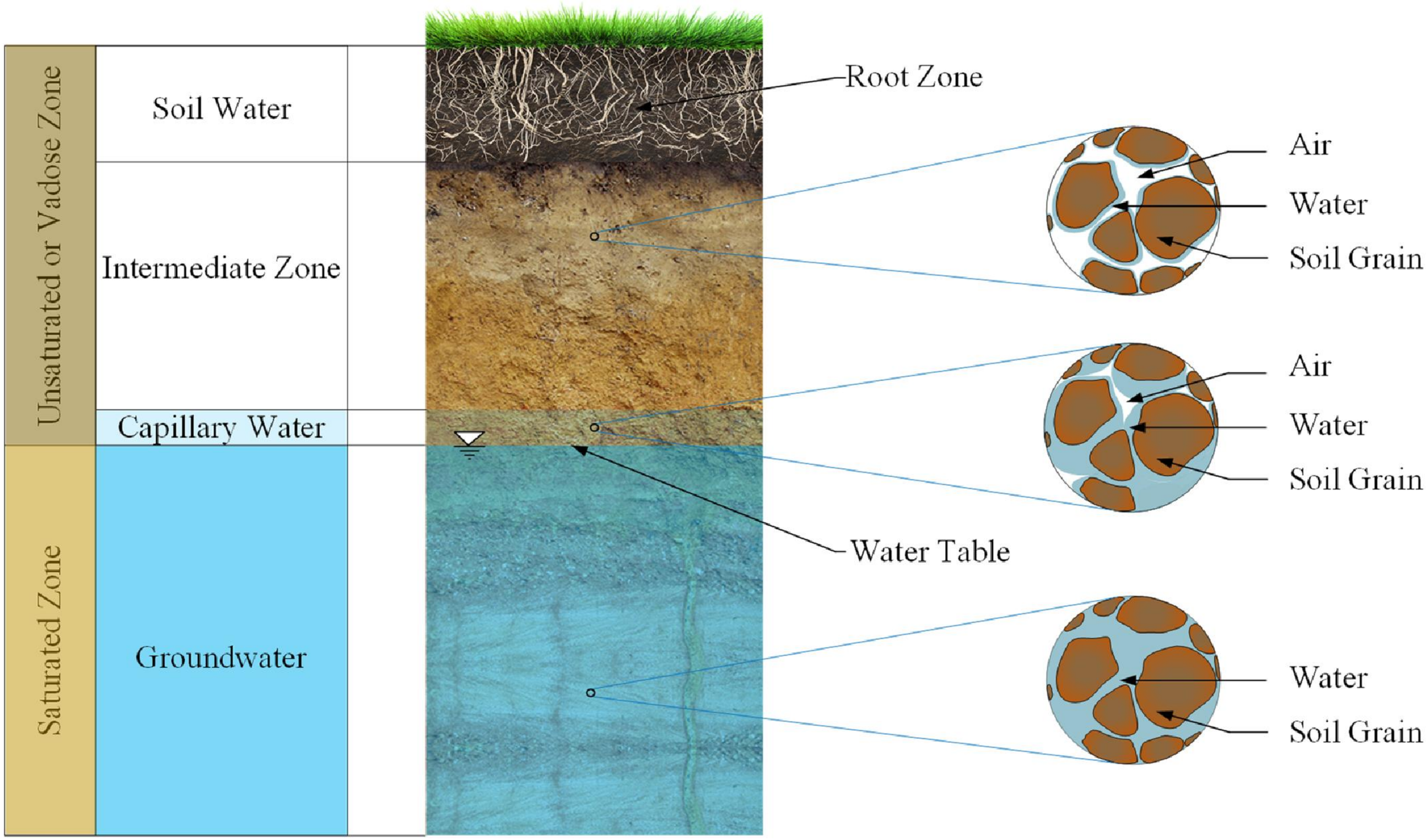
  - This resistance to internal shear causes water to resist flow through geologic materials.
  - In order to flow through pores or fractures, a packet of water must change shape and shear as it flows.
  - pore size is analogous to dz
  - water encounters greater viscous resistance flowing through materials with smaller pores
  - The viscosity of a liquid generally decreases with increasing temperature
  - The dynamic viscosity of water ranges from $\mu = 1.79\times10^{-3}$ N sec/m^2 at 0°C to $\mu = 1.01\times10^{-3}$ N sec/m^2 at 20°C.
  - Kinematic viscosity ν $\nu = \dfrac{\mu}{\rho}$

Properties of Water (Contd.)

- Surface Tension and Capillarity
  - polar water molecules are attracted to each other -> a mass of water has internal cohesion that tends to hold it together
  - water drops tend to form spheres as they fall through the air
  - In pore spaces containing both air and water, the water will generally wet the mineral surfaces, leaving the central parts of the pores for the air
  - A layer of water molecules on the order of 0.1 to 0.5 μm (10^{-6} m) thick is so strongly attracted to mineral surfaces that it is essentially immobile
  - the forces of attraction are not strong enough to prevent movement of water molecules and water outside that distance is free to move
  - Surface attraction forces are stronger for clay minerals -> due to the charged nature of clay mineral surfaces.



Properties of Water (Contd.)



Properties of Water (Contd.)

- The attraction of water to mineral surfaces causes water to pull and spread itself across the surfaces -> the pressure within the water is less than the air pressure within the pores
- As amount of water present decreases -> the pull of the mineral surface attraction forces increases -> the pressure within the water decreases -> the air–water interface develops a more contorted shape conforming to the mineral grains
- This attraction of water to mineral surfaces in partly saturated materials is called capillarity
- Capillarity allows water to wet pore spaces above the water table
- Capillary forces tend to be greater in finer-grained granular materials, due to a greater amount of mineral surface area.

Properties of Water (Contd.)

Table 2.4.1 Capillary Rise in Samples of Unconsolidated Materials (after Lohman[34])

Material	Grain size (mm)	Capillary rise (cm)
Fine gravel	5–2	2.5
Very coarse sand	2–1	6.5
Coarse sand	1–0.5	13.5
Medium sand	0.5–0.2	24.6
Fine sand	0.2–0.1	42.8
Silt	0.1–0.05	105.5
Silt	0.05–0.02	200[a]

Note: Capillary rise measured after 72 days; all samples have virtually the same porosity of 41 percent.

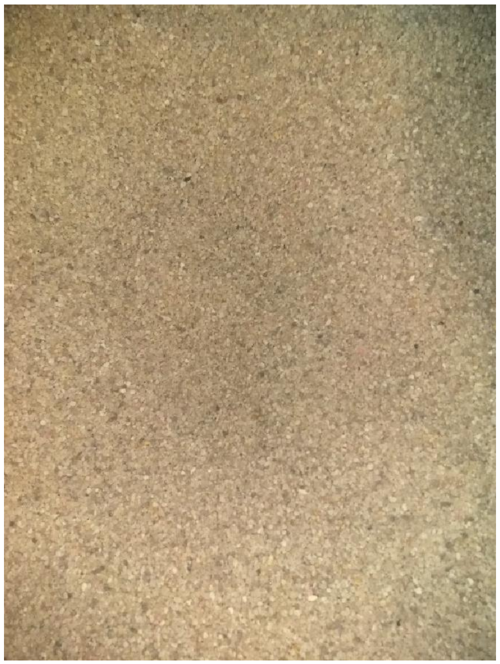
[a]Still rising after 72 days.

Properties of Air

- Air occupies some amount of the subsurface pore spaces
- The pressure in the atmosphere at the earth's surface varies with the weather and with elevation
- At sea level, atmospheric pressure averages about 1.013×10^5 N/m^2
- Groundwater pressures are often measured as gage pressure -> the amount of pressure in excess of atmospheric pressure
- The density of the atmosphere also varies with weather and altitude.
- At the earth's surface, the atmospheric density averages about 1.2 kg/m^3

Properties of Porous Media

- Soil Texture



Grade-I

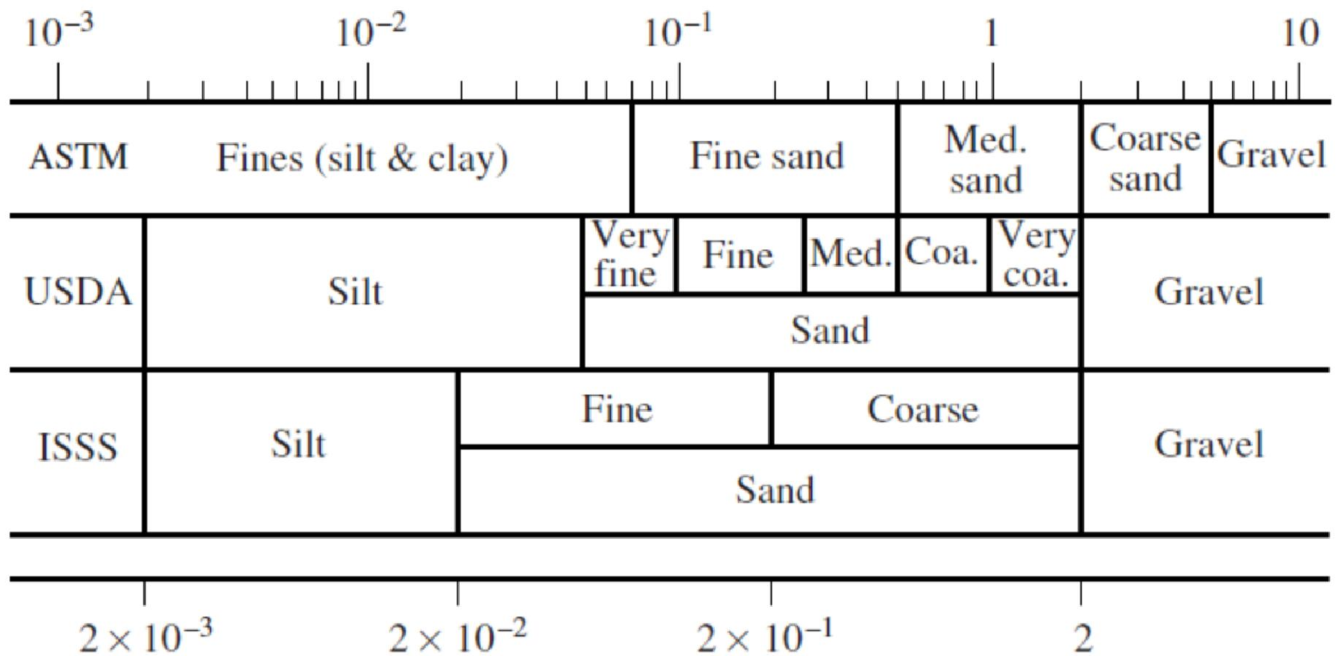
Grade-II

Grade-III

Indian Standard Sand

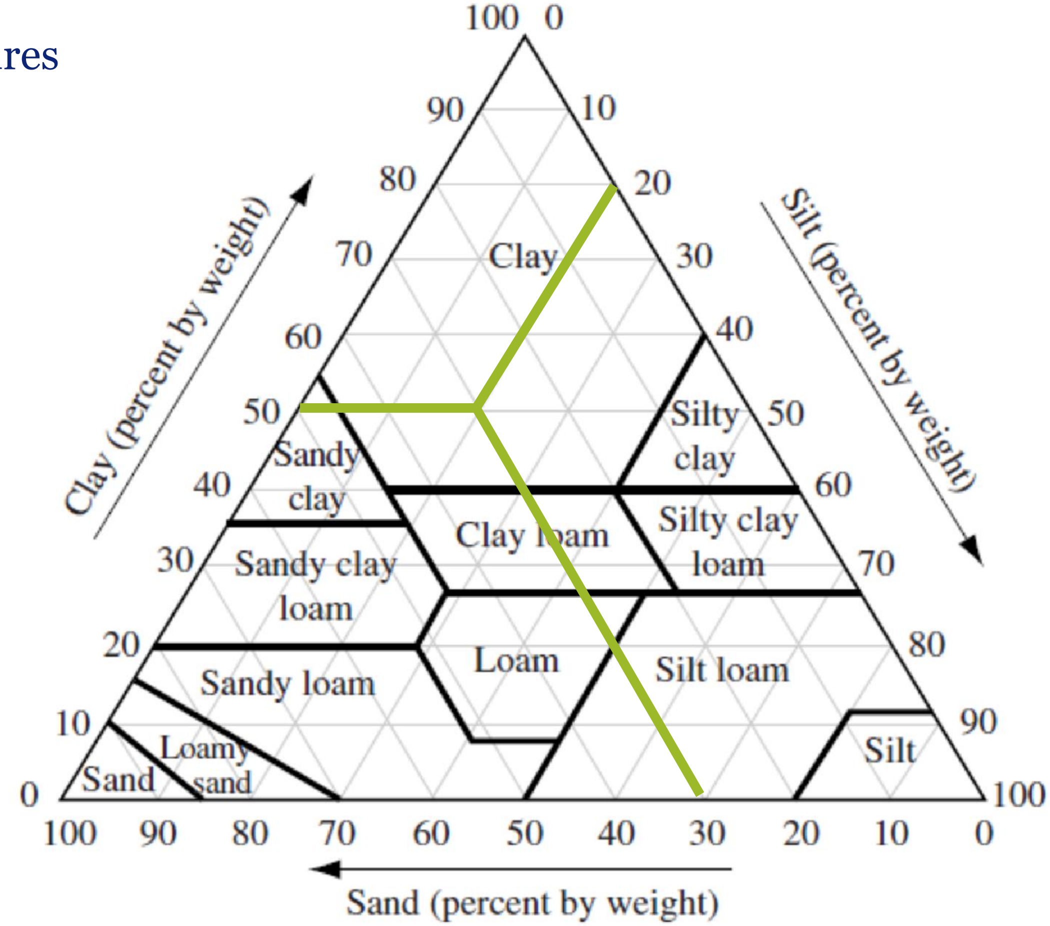
Properties of Porous Media (Contd.)

- Soil classification



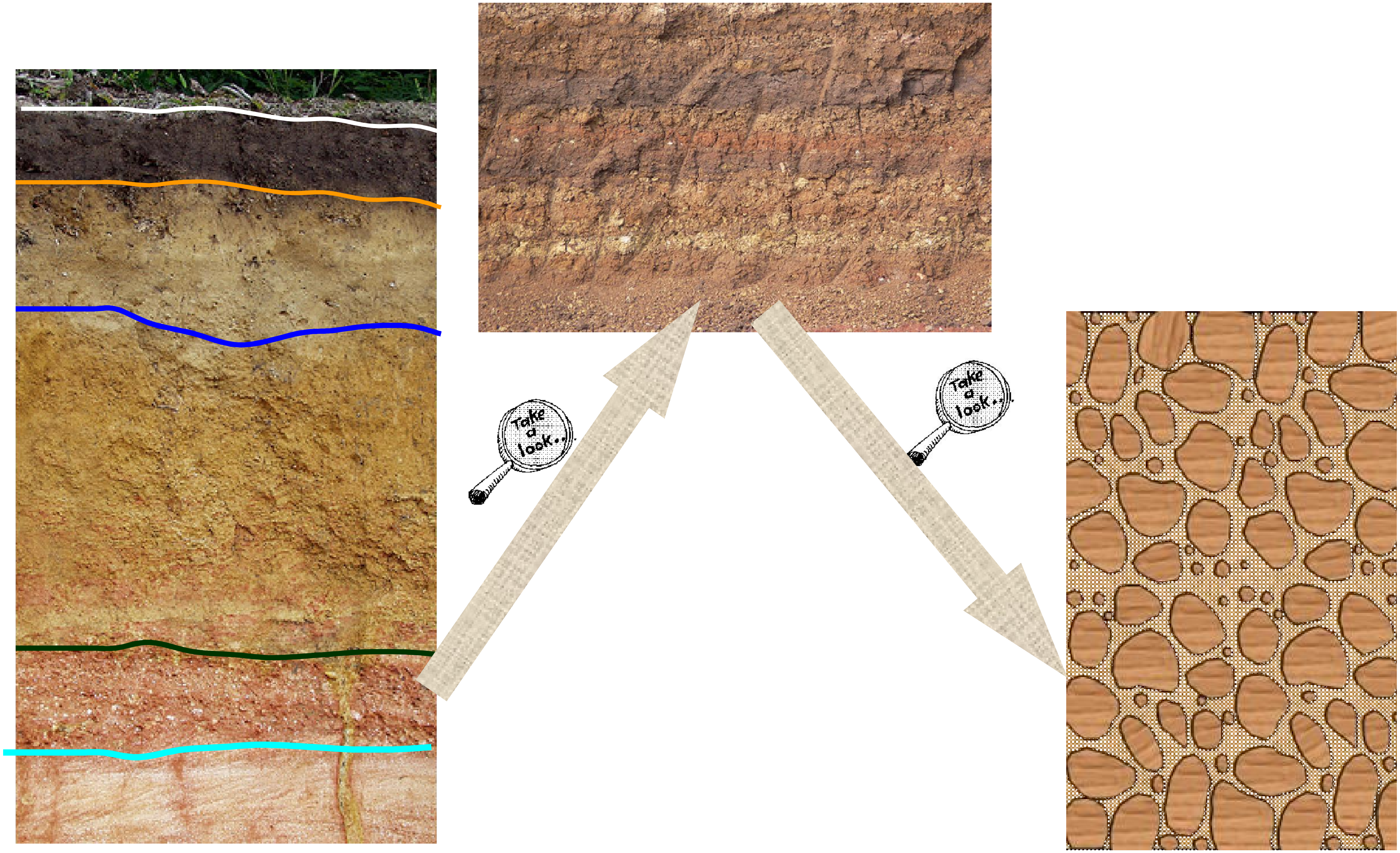
Properties of Porous Media (Contd.)

- Triangle of Soil Textures
  - **Clay:** 50%
  - **Sand:** 30%
  - **Silt:** 20%

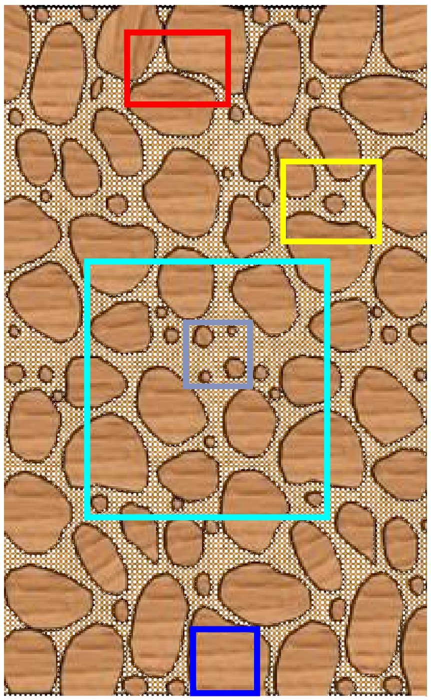


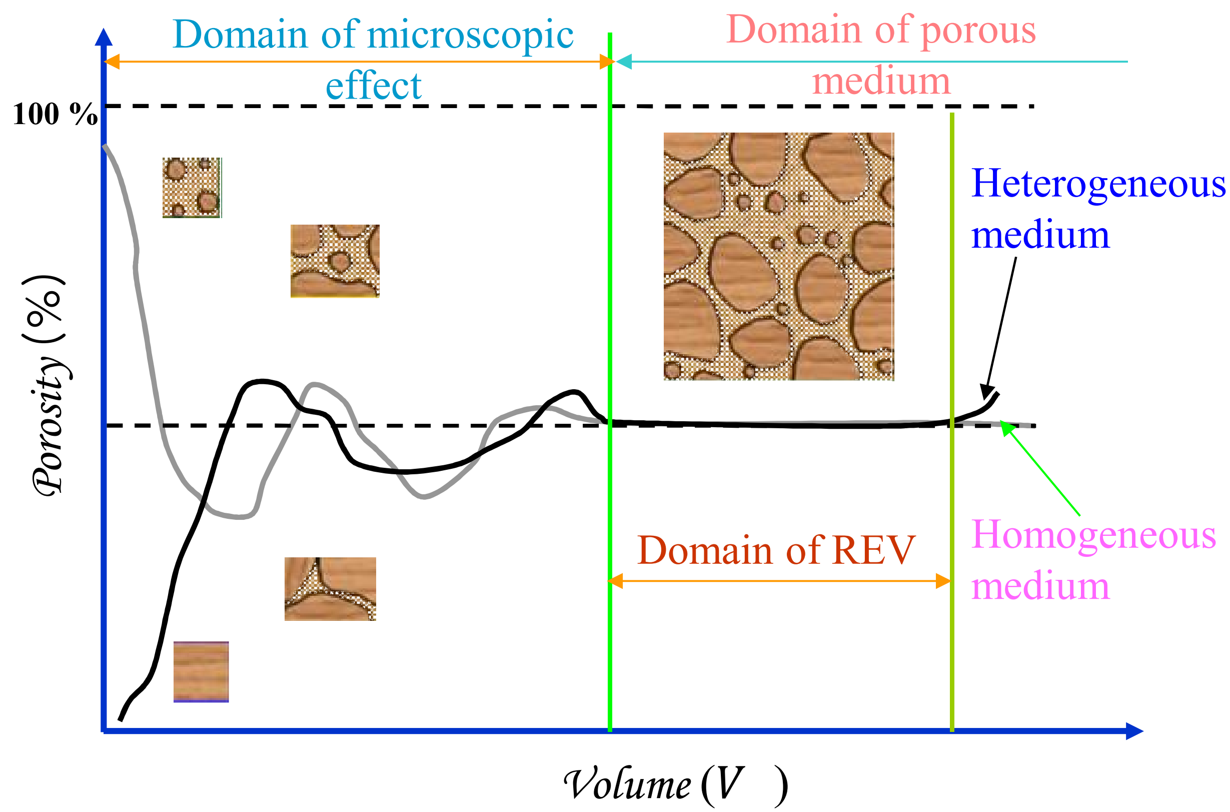
USDA soil textural triangle.

Properties of Porous Media (Contd.)



Properties of Porous Media (Contd.)

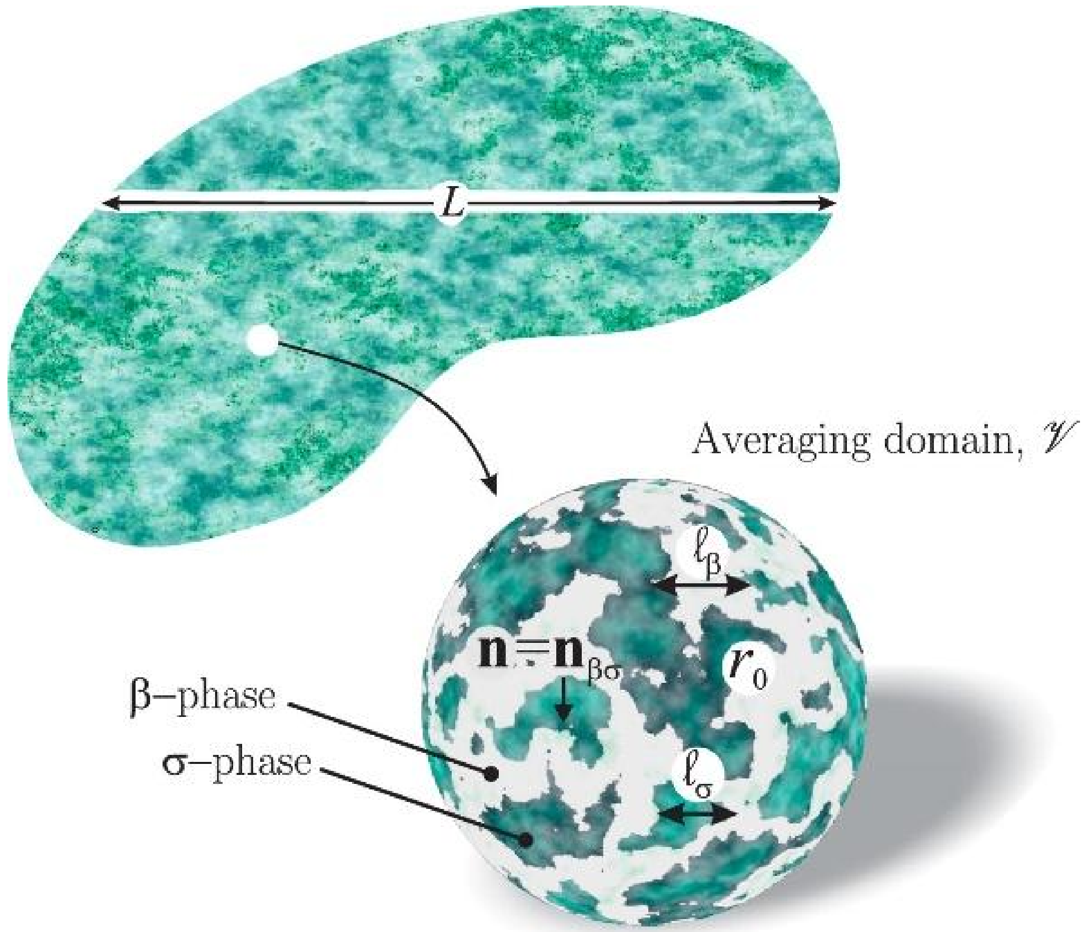




A sub-volume of a porous medium that has the "same" geometric configuration as the medium at a macroscopic scale.

Properties of Porous Media (Contd.)

- $l \ll D \ll L$



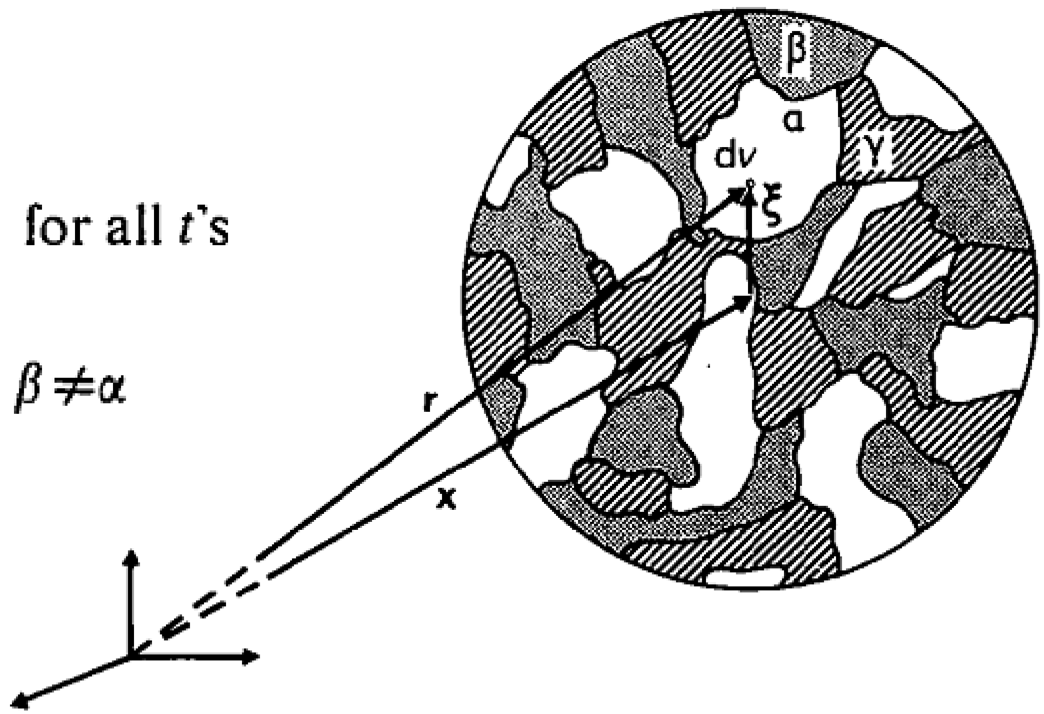
Properties of Porous Media (Contd.)

$$\mathbf{r} = \mathbf{x} + \xi$$

$$\gamma_\alpha = \gamma_\alpha(\mathbf{r}, t) = \begin{cases} 1 & \text{if } \mathbf{r} \in V_\alpha \\ 0 & \text{if } \mathbf{r} \in V_\beta \end{cases} \quad \text{for all } t\text{'s}$$

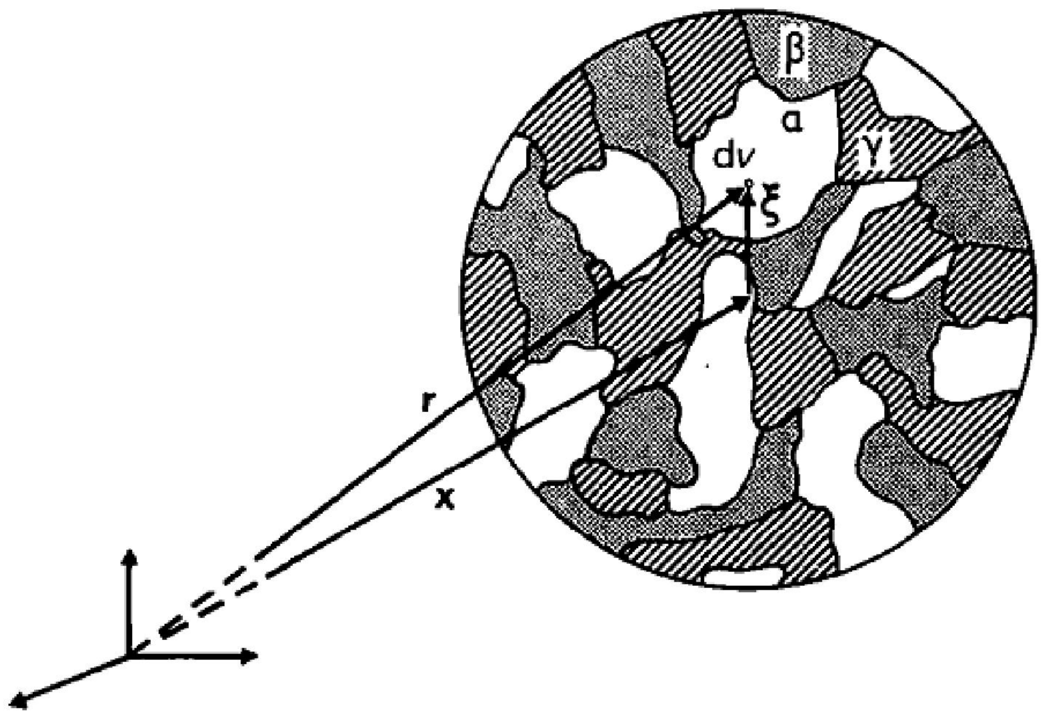
$$\alpha, \beta = 1, 2, \ldots, N \qquad \beta \neq \alpha$$

$$\mathrm{d}V_\alpha(\mathbf{x}, t) = \int_{\mathrm{d}V} \gamma_\alpha(\mathbf{x} + \xi, t)\,\mathrm{d}v$$



Properties of Porous Media (Contd.)

$$\int_{dV_s} f \, dv = \int_{dV} f \gamma_\alpha \, dv$$



$$dA_\alpha(\mathbf{x}, t) = \int_{dA} \gamma_\alpha(\mathbf{x}, \xi, t) \, da$$

- Volume fraction

$$\varepsilon_\alpha(\mathbf{x}, t) = \frac{dV_\alpha}{dV} = \frac{1}{dV} \int_{dV} \gamma_\alpha(\mathbf{r}, t) \, dv$$

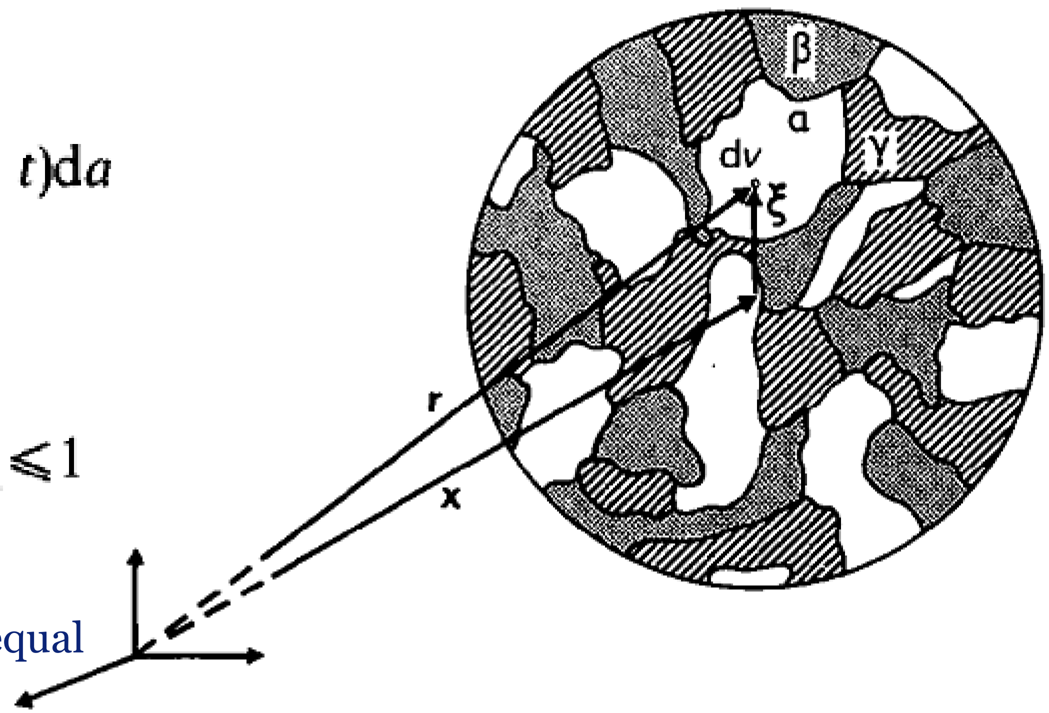
$$\sum_\alpha \varepsilon_\alpha = 1 \quad \text{and} \quad 0 \leqslant \varepsilon_\alpha \leqslant 1$$

Properties of Porous Media (Contd.)

- Areal fraction

$$\bar{\varepsilon}_{\alpha}(\mathbf{x},\ t)=\frac{\mathrm{d}A_{\alpha}}{\mathrm{d}A}=\frac{1}{\mathrm{d}A}\int_{\mathrm{d}A}\gamma_{\alpha}(\mathbf{r},\ t)\mathrm{d}a$$

$$\Sigma\bar{\varepsilon}_{\alpha}=1 \qquad \text{and} \qquad 0\leqslant\bar{\varepsilon}_{\alpha}\leqslant 1$$



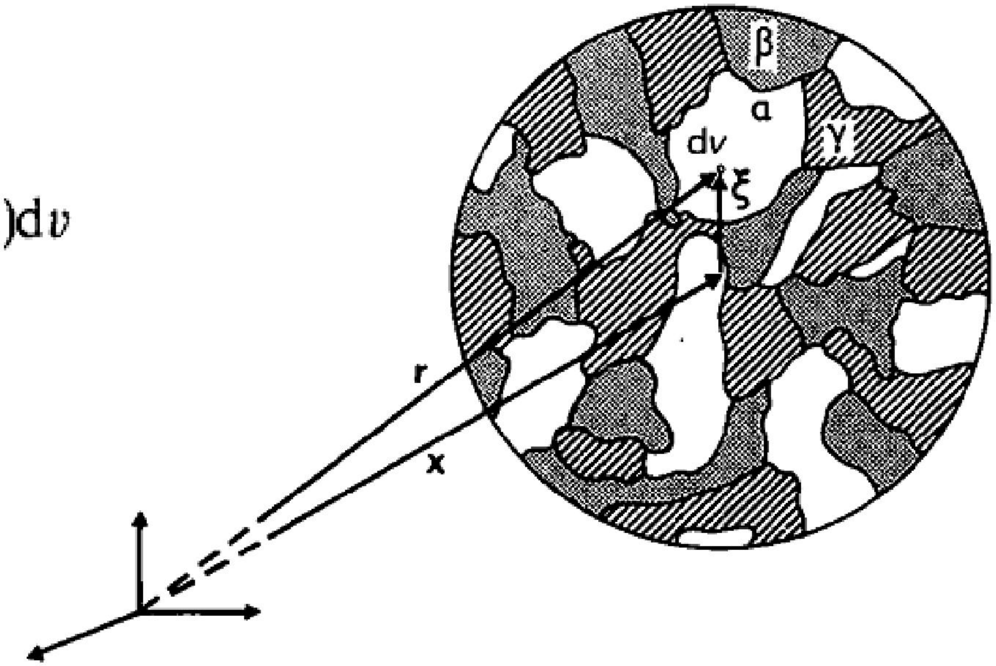
- In general ε_{α} and $\bar{\varepsilon}_{\alpha}$ need not be equal

Properties of Porous Media (Contd.)

- Volume average operator $\langle \quad \rangle_\alpha$

$$\langle f \rangle_\alpha(\mathbf{x}, t) = \frac{1}{\mathrm{d}V} \int_{\mathrm{d}V} f(\mathbf{r}, t)\gamma_\alpha(\mathbf{r}, t)\mathrm{d}v$$

$$\langle \rho \rangle_\alpha = \frac{1}{\mathrm{d}V} \int_{\mathrm{d}V} \rho\gamma_\alpha \mathrm{d}v$$



- Intrinsic volume average operator $\langle \quad \rangle$

$$\langle f \rangle_\alpha^\alpha(\mathbf{x}, t) = \frac{1}{\mathrm{d}V_\alpha(\mathbf{x}, t)} \int_{\mathrm{d}V} f(\mathbf{r}, t)\gamma_\alpha(\mathbf{r}, t)\mathrm{d}v$$

$$\langle f \rangle_\alpha = \varepsilon_\alpha \langle f \rangle_\alpha^\alpha$$

$$\langle \rho \rangle_\alpha^\alpha = \frac{1}{\mathrm{d}V_\alpha} \int_{\mathrm{d}V} \rho\gamma_\alpha \mathrm{d}v = \frac{1}{\varepsilon_\alpha} \langle \rho \rangle_\alpha$$

Properties of Porous Media (Contd.)

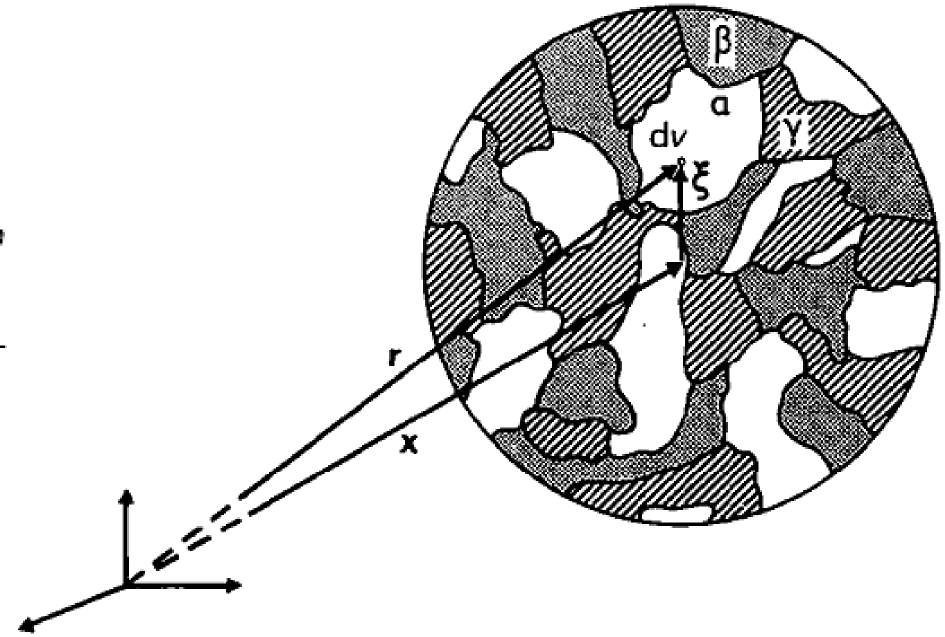
- Only when the mass density of the α-phase, is microscopically constant will the intrinsic volume average mass density function be equal to the microscopic mass density, or

$$\langle \rho \rangle_{\alpha}^{\alpha} = \rho_{\alpha}$$

Properties of Porous Media (Contd.)

- Mass average operator $\overline{}^{\alpha}$

$$\bar{f}^{\alpha}(\mathbf{x}, t) = \frac{\int_{dV} \rho(\mathbf{r}, t) f(\mathbf{r}, t) \gamma_{\alpha}(\mathbf{r}, t) \mathrm{d}v}{\int_{dV} \rho(\mathbf{r}, t) \gamma_{\alpha}(\mathbf{r}, t) \mathrm{d}v}$$



- Simplified form

$$\bar{f}^{\alpha}(\mathbf{x}, t) = \frac{1}{\langle \rho \rangle_{\alpha}(\mathbf{x}, t) \mathrm{d}V} \int_{dV} \rho(\mathbf{r}, t) f(\mathbf{r}, t) \gamma_{\alpha}(\mathbf{r}, t) \mathrm{d}v$$

Properties of Porous Media (Contd.)

- The porosity of a rock or soil is simply the fraction of the material volume that is pore space.
- In quantitative terms the porosity η is defined as

$$n = \frac{V_v}{V_t}$$

where V_v is the volume of voids in a total volume of material V

- The porosity is a dimensionless parameter in the range $0 < \eta < 1$.
- Geotechnical engineers often use a related dimensionless parameter called the void ratio e, which is defined as

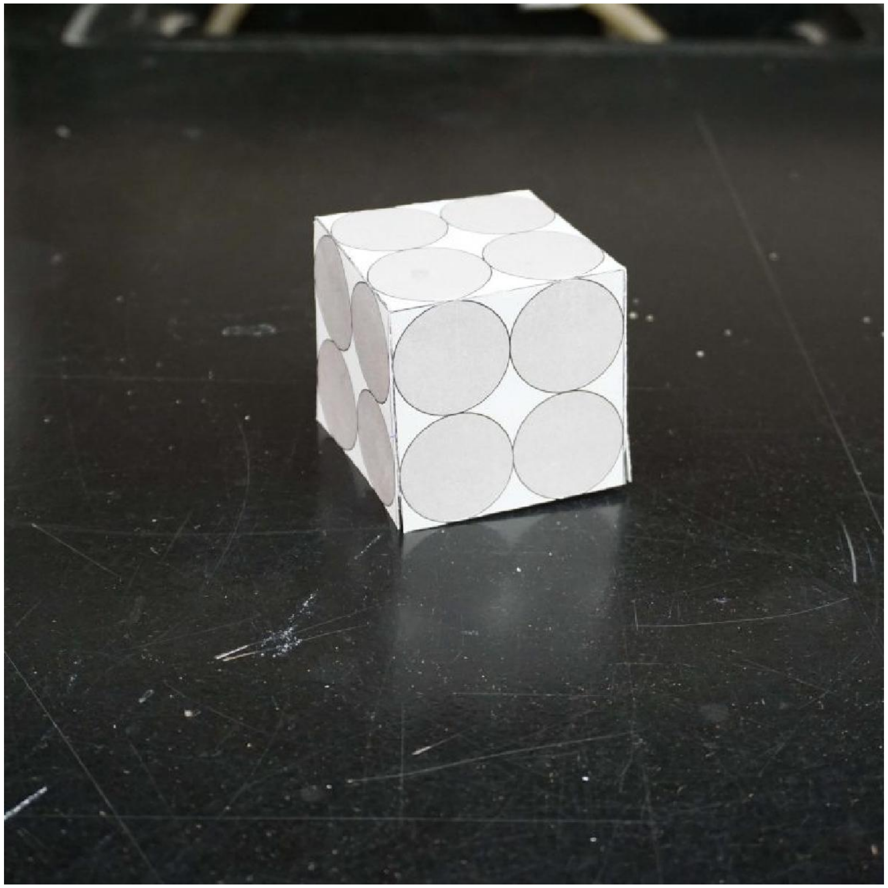
$$e = \frac{V_v}{V_s}$$

where V_s is the volume of mineral solids in a given volume of material.

$$n = \frac{e}{1+e}, \quad e = \frac{n}{1-n}$$

Properties of Porous Media (Contd.)

- Home Lab
- Foldable Aquifer Project -http://aquifer.geology.buffalo.edu/
- Paper aquifer model
  - Porosity and Grain Packing



Learning Strategy

Chapter 2: Physical Properties

Section 2.1, 2.2, 2.3, 2.4

Thank you