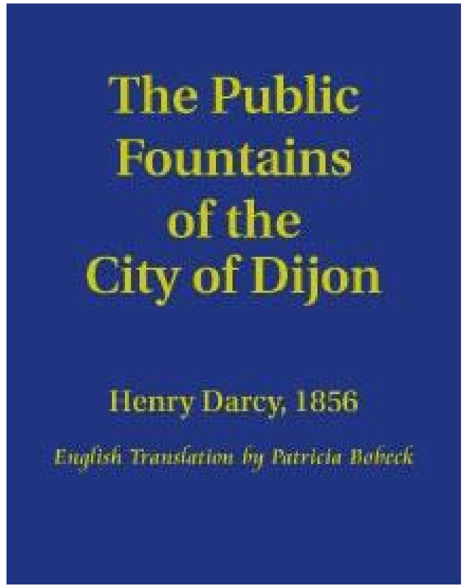
Principles of Flow

Groundwater Engineering| CE60205 Lecture:06

Learning Objective(s)

- **To apply Darcy's law**
- **To estimate hydraulic conductivity**

Darcy’s Law


The Public Fountains of the City of Dijon
Henry Darcy, 1856
English Translation by Patricia Bobeck

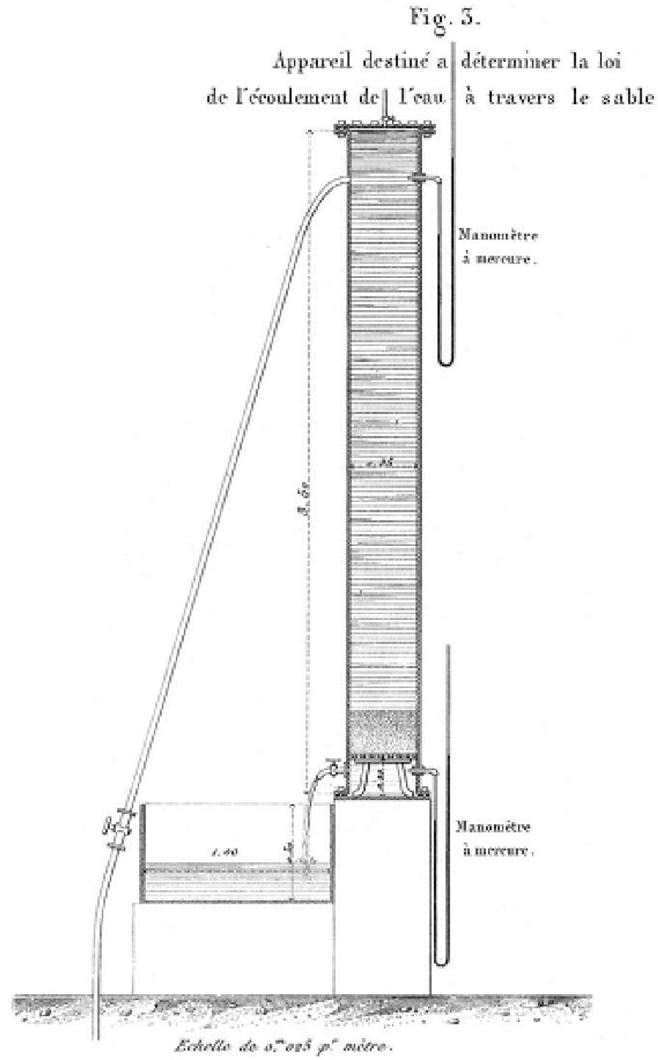
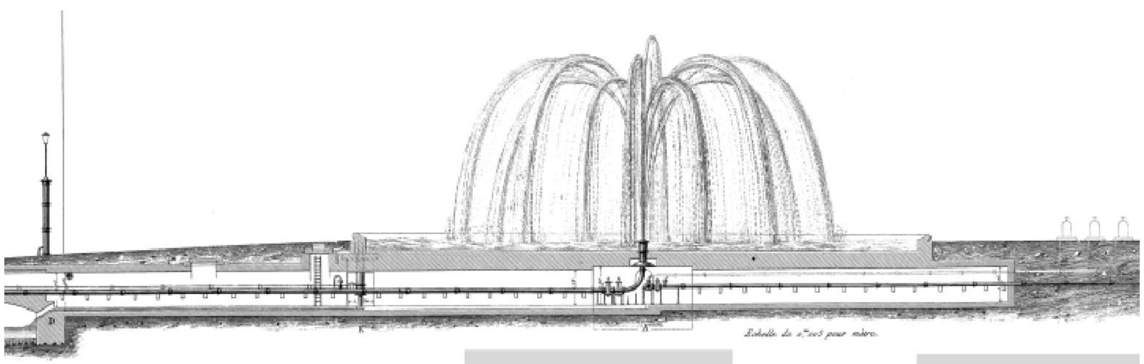
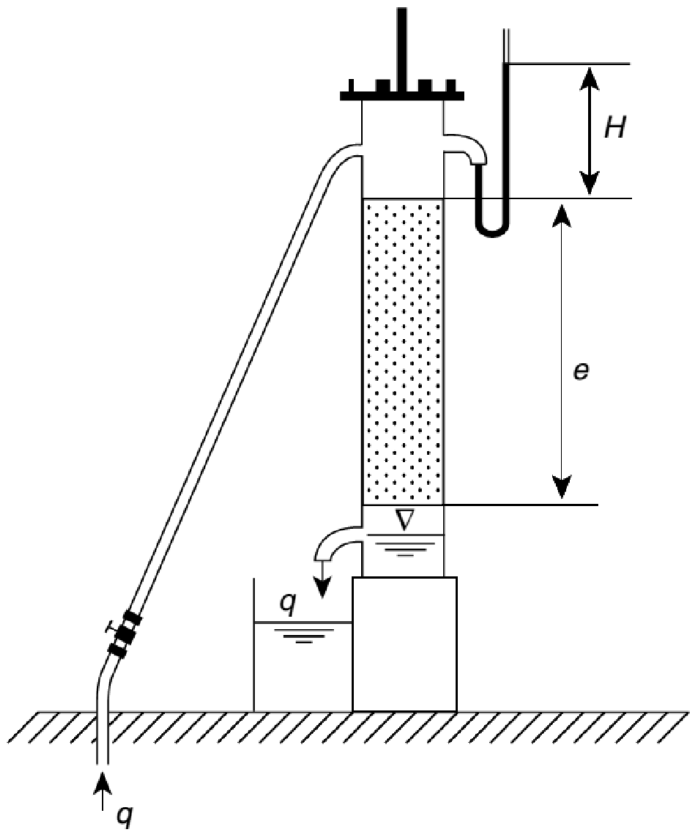


Fig. 3.
Appareil destiné à déterminer la loi de l'écoulement de l'eau à travers le sable.
Manomètre à mercure.
Manomètre à mercure.



Darcy's Law (Contd.)



Darcy's Experiment

- For percolation through a cylinder with outflow under atmospheric pressure

$$q = Ks\frac{H + e}{e}$$

  - q=percolation flux $[L^3T^{-1}]$
  - K=permeability factor$[LT^{-1}]$
  - s=surface area of the sand filled cylinder $[L^2]$
  - e=length of the sand column [L]
  - H=hydraulic pressure at the upper boundary of the sand column [L]
  - $(H+e)$=hydraulic head
  - $(H+e)/e$=hydraulic gradient

- Non-steady flow equation for the condition of a falling pressure head

$$q_t = q_0 e^{(-Kt/e)}$$

  - t =time [T]
  - q_0=percolation flux $[L^3T^{-1}]$ at t = 0

Darcy's Law (Contd.)

- Darcy found through repeated experiments with a specific sand that Q was proportional to the head difference Δh between the two manometers and inversely proportional to ($\propto$) the distance between manometers Δs:

$$Q \propto \Delta h \text{ and } Q \propto \frac{1}{\Delta s}$$

- Q is also proportional to the cross-sectional area of the column A

$$Q \propto A$$

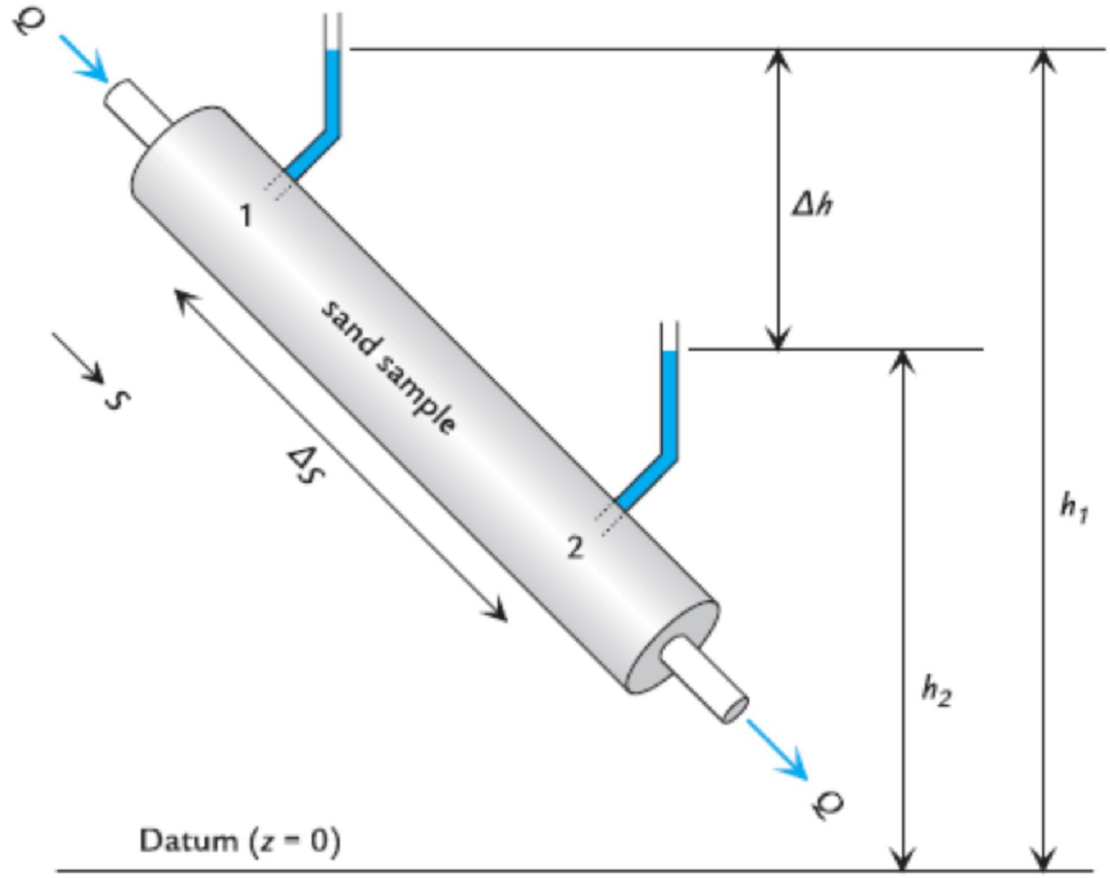
- Combining these observations **Darcy's law** for one-dimensional flow can be written as:

$$Q = -K_s \frac{\Delta h}{\Delta s} A$$

- The constant of proportionality K_s is the **hydraulic conductivity** in the s direction, a property of the geologic medium.
- Hydraulic conductivity is a measure of the ease with which a medium transmits water; higher K_s materials transmit water more easily than low K_s materials.

Darcy's Law (Contd.)

- The minus sign on the right side of Darcy's equation is necessary because head decreases in the direction of flow.
  - Q is positive and $\frac{\Delta h}{\Delta s}$ is negative
  - Q is negative and $\frac{\Delta h}{\Delta s}$ is positive



Darcy's Law (Contd.)

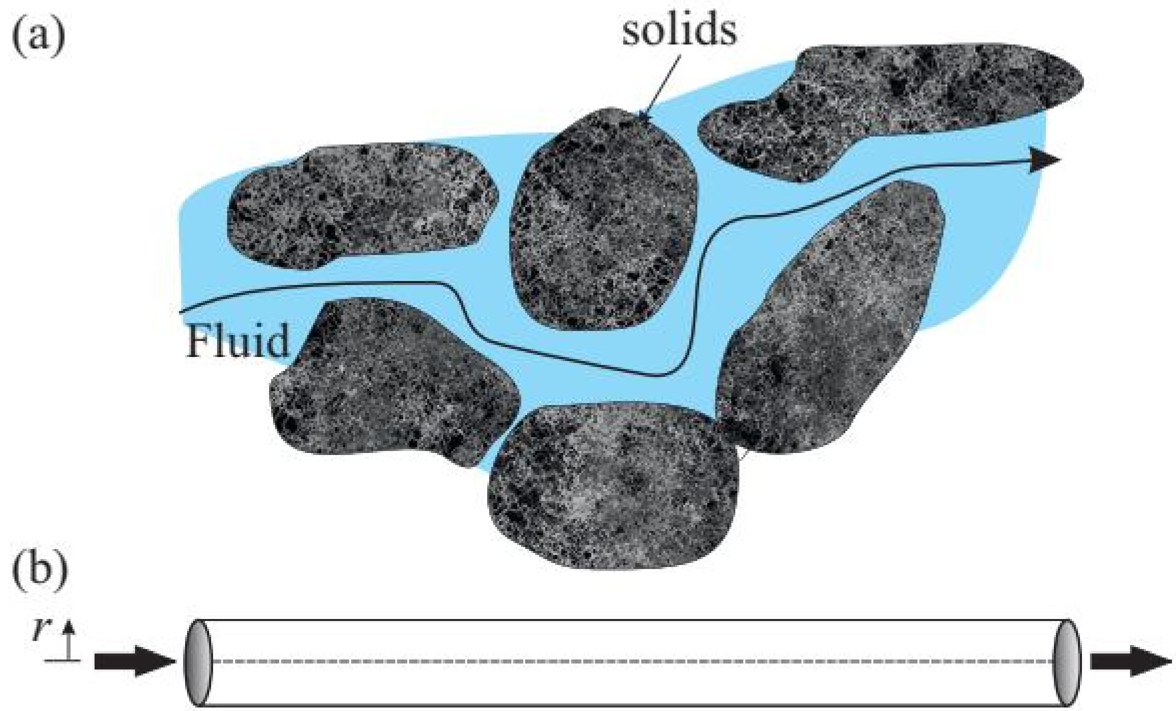
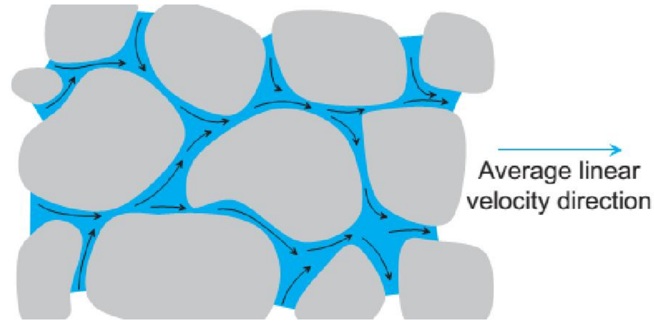


Figure Illustration of an actual pathway for a fluid particle in a) a real porous medium and b) an idealized capillary tube with a constant radius.

Darcy's Law (Contd.)

- Specific discharge or Darcy velocity
- Darcy velocity

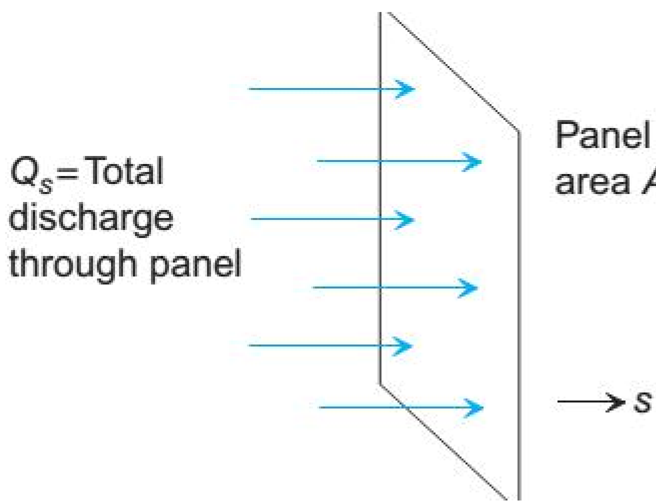


$$q_s = \frac{Q_s}{A} = -K_s \frac{dh}{ds}$$

Q_s = flow

A = total cross-sectional area through which flow occurs

- Specific discharge has units of velocity
- The specific discharge is a macroscopic concept
- It can be easily measured



Darcy's Law (Contd.)

- Darcy velocity is a fictitious velocity
  - Flow occurs across the entire cross-section of the soil sample
  - Linear flow paths assumed in Darcy's law
  - Flow actually takes place only through interconnected pore channels
- The average linear velocity of water motion is directly proportional to the specific discharge and inversely proportional to the effective porosity

$$\bar{v}_s = \frac{q_s}{\eta_e} = \frac{Q_s}{A\eta_e}$$

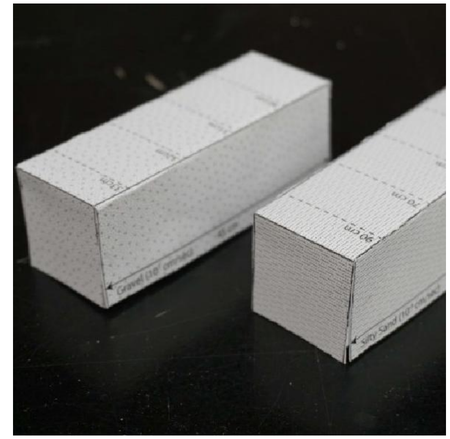
- Discharge can be written as

$$Q_s = Aq_s = A_v\bar{v}_s$$

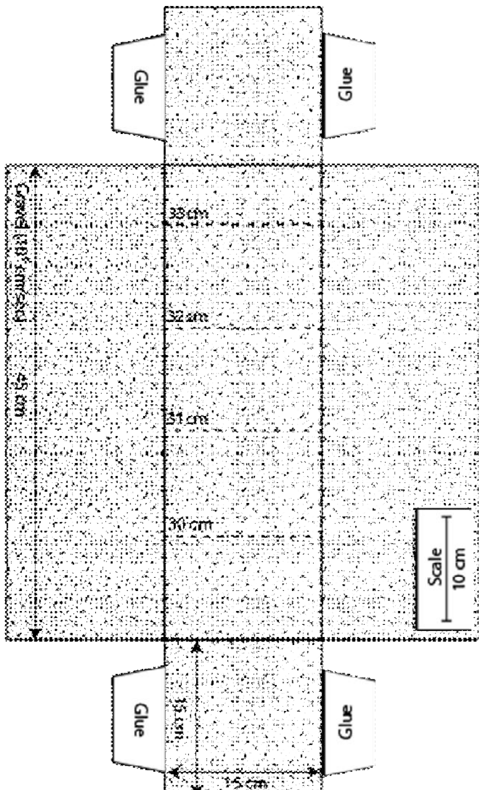
Darcy's Law (Contd.)

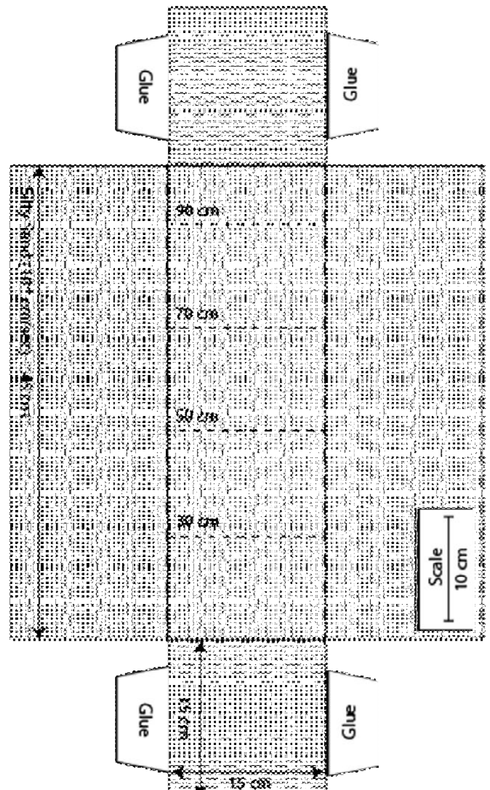
- Darcy's law holds for
  - Saturated flow and unsaturated flow
  - Steady-state and transient flow
  - Flow in aquifers and aquitards
  - Flow in homogeneous and heterogeneous systems
  - Flow in isotropic or anisotropic media
  - Flow in rocks and granular media

Home Lab



- Foldable Aquifer Project -http://aquifer.geology.buffalo.edu/
- Paper aquifer model
  - Darcy Columns
  - Objectives: To explore the relationship between discharge (Q), specific discharge (q), and average linear velocity (v) in homogenous aquifers.



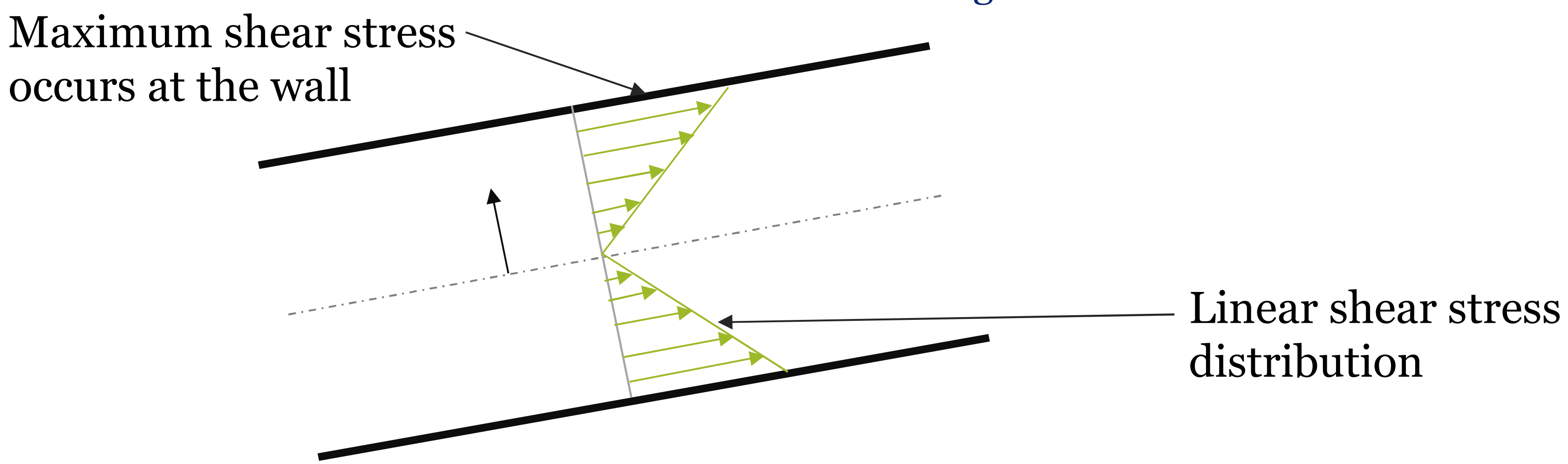


Home Lab (Contd.)

- The problem is based on water flow through two Darcy columns. The first column is filled with a gravel that has a hydraulic conductivity of 10^1 cm/sec, and the second is filled with silty sand with a hydraulic conductivity of 10^{-3} cm/sec. The equipotential lines are shown on the top of the models represented as dashed lines with their associated hydraulic heads. Note that each column has a different distribution of equipotential lines. The total length of the columns are 45 cm and a scale bare is shown lower right corner of the back panel of the model. Using these models please answer the following questions.
- A. Based on the two Darcy's columns provided quantify the difference in groundwater discharge [cm^3/sec] between the Gravel column and the Silty Sand column.
- B. Assuming the gravel column has an effective porosity of 0.30 and the silty sand column has an effective porosity of 0.25. Determine which column has a higher average linear velocity and by how much.
- C. Explain which of the following column would make a better aquifer.

Hagen-Poiseuille Flow

- Laminar flow in a round tube is called Poiseuille flow or Hagen-Poiseuille flow.



$$\tau = \mu \frac{dV}{dy}$$

- y is the distance from the pipe wall. Let us consider $y = r_0 - r$

$$\tau = \mu \frac{dV}{dy} = \mu \frac{dV}{dr}\frac{dr}{dy} = -\mu \frac{dV}{dr}$$

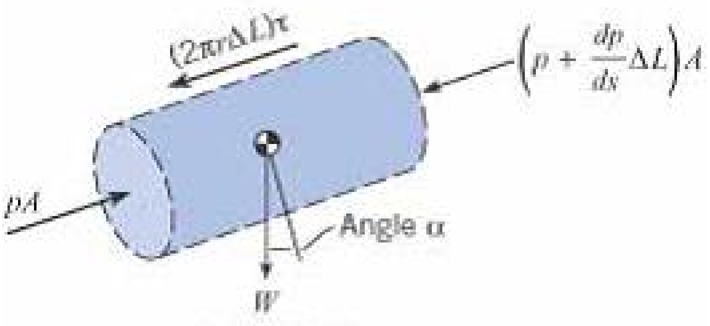
Hagen-Poiseuille Flow (Contd.)

- Considering force balance for the control volume

$$pA - \left(p + \frac{dp}{ds}\Delta L\right)A - W sin\alpha - (2\pi r \Delta L)\tau = 0$$

where $W = \gamma A \Delta L$ and $sin\alpha = \Delta z / \Delta L$

$$\tau = \frac{r}{2}\left[-\frac{d}{ds}(p + \gamma z)\right]$$



Force diagram

Shear stress distribution

$$\left(\frac{2\mu}{r}\right)\frac{dV}{dr} = \frac{d}{ds}(p + \gamma z)$$

This can be true iff

$$Constant = \frac{d}{ds}(p + \gamma z) = \frac{\Delta(p + \gamma z)}{\Delta L} = \frac{\gamma \Delta h}{\Delta L}$$

Combining

$$\frac{dV}{dr} = \left(\frac{r}{2\mu}\right)\left(\frac{\gamma \Delta h}{\Delta L}\right)$$

$$V = \left(\frac{r^2}{4\mu}\right)\left(\frac{\gamma \Delta h}{\Delta L}\right) + C$$

Hagen-Poiseuille Flow (Contd.)

No-slip condition: velocity of the fluid at wall is zero

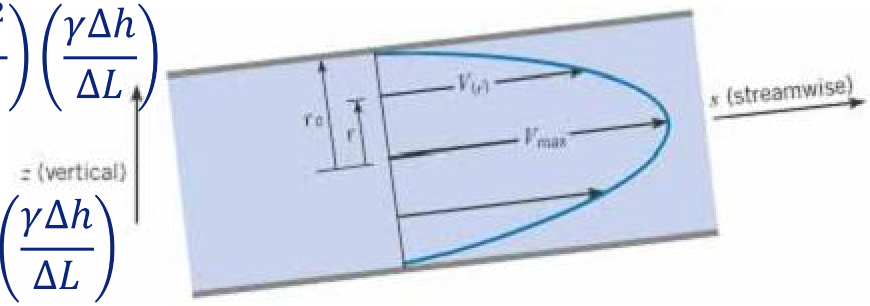
$$V(r = r_0) = 0$$

Thus constant C

$$C = -\left(\frac{r_0^2}{4\mu}\right)\left(\frac{\gamma\Delta h}{\Delta L}\right)$$

Velocity can be expressed as

$$V(r) = -\left(\frac{r_0^2 - r^2}{4\mu}\right)\left(\frac{\gamma\Delta h}{\Delta L}\right)$$



Maximum velocity

$$V_{max} = -\left(\frac{r_0^2}{4\mu}\right)\left(\frac{\gamma\Delta h}{\Delta L}\right)$$

Combining maximum velocity

$$V(r) = V_{max}\left(1 - \frac{r^2}{r_0^2}\right)$$

Hagen-Poiseuille Flow (Contd.)

- Average velocity

$$V_{avg} = \frac{1}{A}\iint VdA$$

$$V_{avg} = \frac{1}{A}\iint VdA = \frac{1}{\pi r_0^2}\int_0^{r_0} -\left(\frac{r_0^2 - r^2}{4\mu}\right)\left(\frac{\gamma\Delta h}{\Delta L}\right)(2\pi r dr) = -\left(\frac{r_0^2}{8\mu}\right)\left(\frac{\gamma\Delta h}{\Delta L}\right)$$

$$V_{avg} = -\left(\frac{r_0^2}{8\mu}\right)\left(\frac{\gamma\Delta h}{\Delta L}\right) = \frac{1}{2}V_{max}$$

Hagen-Poiseuille Flow (Contd.)

$$Q = \frac{-\pi R^4}{8\mu}\frac{\Delta p}{\Delta x} = \frac{-\pi R^4}{8\mu}\frac{dp}{dx}$$

The average velocity, $\bar{v}$, over the cross-sectional area, A, is then equal to

$$\bar{v} = \frac{Q}{A} = \frac{-R^2}{8\mu}\frac{dp}{dx}$$

Now, let us summarize what we have learned from Poiseuille's law for flow through a single flow tube embedded in a porous medium:

- The discharge Q varies as the fourth power of the radius of the tube ($Q \propto R^4$).
- The average velocity is proportional to the pressure gradient, dp/dx, for the horizontal flow scenario.

Hagen-Poiseuille Flow (Contd.)

For nonhorizontal flow,

$$\bar{v} = -\frac{\rho g R^2}{8\mu}\frac{dH}{dx}$$

where H is the total head (or hydraulic head), and it is the sum of pressure head, h, and elevation head, z, that is,

$$H = \frac{p}{\rho g} + z = h + z$$

Darcy’s law & General Fluid Flow

- Specific discharge q
- *Hagen-Poiseuille Flow*

$$V_{avg} = -\left(\frac{r_0^2}{8\mu}\right)\left(\frac{\gamma \Delta h}{\Delta L}\right)$$

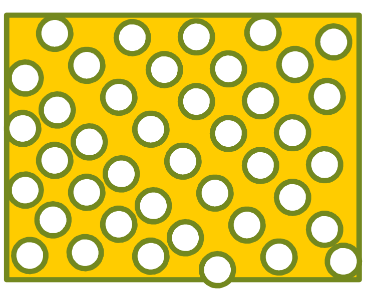
- Specific discharge q

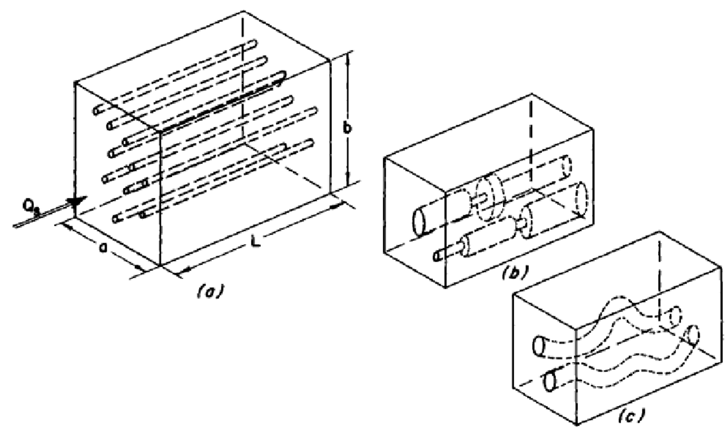
$$V_{avg} = \frac{q}{n}$$

$$q = nV_{avg} = -n\left(\frac{r_0^2}{8\mu}\right)\left(\frac{\gamma \Delta h}{\Delta L}\right) = -\frac{\rho g}{\mu}\frac{n\, r_0^2}{8}\frac{\Delta h}{\Delta L} = -K\frac{\Delta h}{\Delta L}$$

$$K = \frac{\rho g}{\mu}\frac{n\, r_0^2}{8} = \frac{\rho g}{\mu} k$$

$$k = \frac{n\, r_0^2}{8}$$





Poiseuille's Law for Flow Through a Bundle of Capillary Tubes

We therefore consider a simplified porous medium consisting of only two types of tubes: N_1 number of parallel tubes having radius a_1 and N_2 number of parallel tubes having radius a_2. The cross-sectional area of the porous medium is A (Figure 2.4). The total cross-sectional area of all the pores, A_p is

$$A_p = \pi(N_1 a_1^2 + N_2 a_2^2)$$

Then the porosity, n, of the entire medium is defined as

$$n = \frac{A_p}{A} = \frac{\pi(N_1 a_1^2 + N_2 a_2^2)}{A} \quad \text{and} \quad A = \frac{\pi(N_1 a_1^2 + N_2 a_2^2)}{n}$$

Poiseuille's Law for Flow Through a Bundle of Capillary Tubes (Contd.)

Specifically, porosity is the volume fraction of voids in a porous medium. The total discharge Q through the medium is $Q = N_1 Q_1 + N_2 Q_2$. Applying Poiseuille's law to Q_1 and Q_2, we obtain

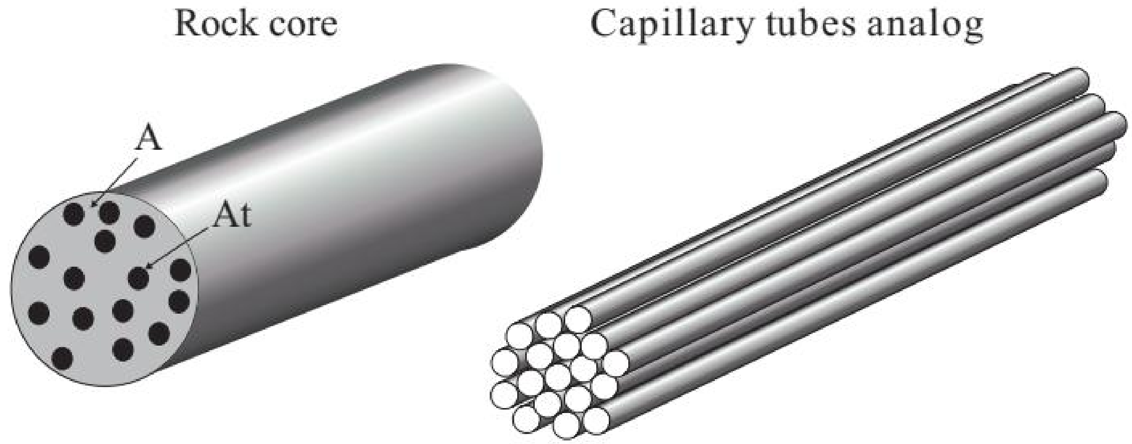


Figure Illustrations of the conceptualization of idealized porous media. The cross-sectional area of each tube is denoted by A_t.

Poiseuille's Law for Flow Through a Bundle of Capillary Tubes (Contd.)

$$Q = -\left[N_1\left(\rho g \frac{\pi a_1^4}{8\mu}\right) + N_2\left(\rho g \frac{\pi a_2^4}{8\mu}\right)\right]\frac{dH}{dx}$$

$$= -\left(\frac{\rho g \pi}{8\mu}\right)[N_1 a_1^4 + N_2 a_2^4]\frac{dH}{dx}$$

The discharge, q, per unit cross-sectional area of the medium then takes the form

$$q = \frac{Q}{A} = -\frac{\rho g}{8\mu}\left(\frac{\pi(N_1 a_1^2 + N_2 a_2^2)}{A}\right)\frac{[N_1 a_1^4 + N_2 a_2^4]}{[N_1 a_1^2 + N_2 a_2^2]}\frac{dH}{dx}$$

where q is the ***specific discharge***, the average discharge per unit cross-sectional area of the porous medium, and it has a dimension of [L/T]. If we let

Poiseuille's Law for Flow Through a Bundle of Capillary Tubes (Contd.)

$$\overline{R}^2 = \frac{[N_1\, a_1^4 + N_2 a_2^4]}{[N_1\, a_1^2 + N_2 a_2^2]}$$

we then have the formula for flow through the porous medium

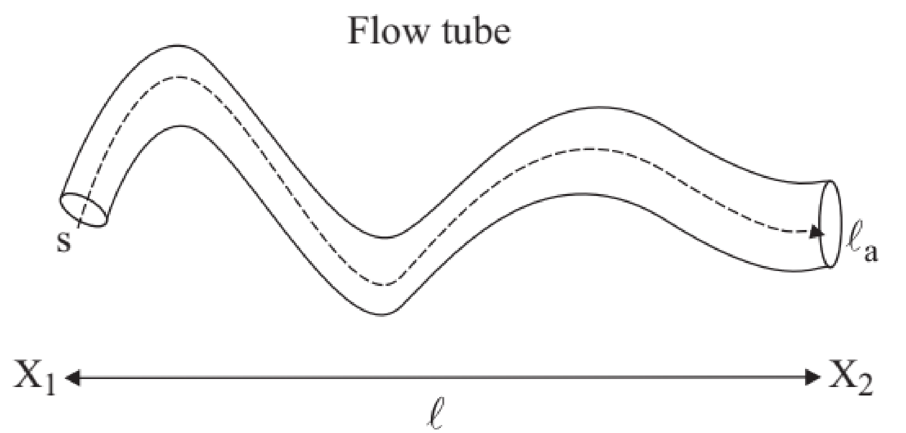
$$q = \frac{Q}{A} = -\frac{\rho g n \overline{R}^2}{8\mu}\frac{dH}{dx}$$

where $\overline{R}$ denotes an average pore radius for the porous medium.

- Specific discharge is "linearly" proportional to the total head gradient, dH/dx, and the constant of proportionality, $\rho g(n\,\overline{R}^2)/8\mu$, is called "conductivity."
- The conductivity depends on both fluid specific weight ($\gamma = \rho g$) and viscosity (μ) and the geometry of the fluid path, $(n\,\overline{R}^2)/8$.
- The driving force consists of two components:
  a. The component due to gravity, $\rho g dz/dx$, which exists only in the vertical direction.
  b. The component due to pressure head gradient, dp/dx.

Tortuosity

- While the idealized medium (a bundle of straight capillary tubes) contains many of the same characteristics of a porous medium, it lacks the tortuous ("crooked") flow paths of a porous medium. Therefore, we subsequently need to introduce a correction factor (tortuosity) to Poiseuille's law to make it a more realistic representation of the tortuous flow through an actual porous medium



$$T = \left(\frac{\ell_a}{\ell}\right)^2 > 1$$

Figure An illustration of the tortuosity concept. ℓ = straight line distance between X_1 and X_2; ℓ_a = actual distance between X_1 and X_2 along the flow path (streamline) described by a curvilinear coordinate system; X = Cartesian coordinates; S = curved streamline.

Tortuosity (Contd.)

Consider now the flow along the streamline, which can be described by

$$\mathbf{q_s} = -\rho g \frac{nR^2}{8\mu} \frac{\mathbf{dH}}{\mathbf{ds}}$$

where the specific discharge and the head gradient are vectors (bold fonts denote vectors). This equation is not useful since (a) we always use the Cartesian coordinate system for our observations and (b) we do not and cannot measure the specific discharge and the total head gradient along a streamline. For practical purposes, equation (2.3.8) should be expressed in terms of the Cartesian coordinate system. This can be accomplished by projecting the vectors in equation (2.3.8) onto the Cartesian planes. Projection of the hydraulic head gradient onto the Cartesian x-axis leads to

$$\frac{dH}{ds} = \frac{dH}{dx}\frac{dx}{ds} = \frac{dH}{dx}\left(\frac{\ell}{\ell_a}\right)$$

where ℓ/ℓ_a is the "average" cosine of the angle of q_s with respect to the direction x. The average here implies that it is a mean value of the angle over the entire streamline.

Tortuosity (Contd.)

streamline. Similarly, the specific discharge component along the x-axis can be related to q_s.

$$q_x = q_s \frac{dx}{ds} = q_s \left(\frac{\ell}{\ell_a} \right) \tag{2.3.10}$$

Combining equations (2.3.8) and (2.3.9), we obtain

$$q_x \left(\frac{\ell_a}{\ell} \right) = -\rho g \frac{nR^2}{8\mu} \left(\frac{\ell}{\ell_a} \right) \frac{dH}{dx} \tag{2.3.11}$$

and

$$q_x = -\rho g \frac{nR^2}{8\mu T} \frac{dH}{dx}, \tag{2.3.12}$$

where $T = (\ell_a/\ell)^2$ is the tortuosity, which is always greater than or equal to 1.

Tortuosity (Contd.)

- For a bundle of tortuous capillary tubes, after the application of a spatial averaging procedure takes the following form:

$$q_x = -\rho g \frac{n\overline{R}^2}{8\mu\overline{\overline{T}}} \frac{dH}{dx}$$

where $\overline{T}$ is the average tortuosity.

- The constant of proportionality now can formally be called the hydraulic conductivity of a porous medium, that is,

$$K = \rho g \frac{n\overline{R}^2}{8\mu\overline{\overline{T}}}$$

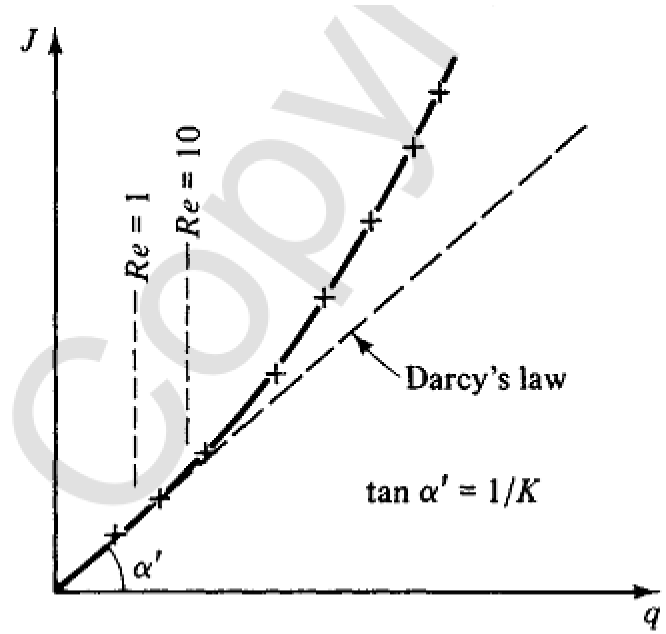
Darcy's law

$$q = -K \frac{dH}{dx}$$

Validity of Darcy's Law

- Validity of Darcy's Law
  - ignored kinetic energy (low velocity)
  - assumed laminar flow
- Reynolds Number for the flow

$$N_R = \frac{\rho q d_{10}}{\mu}$$



  - q = Specific discharge
  - d_{10} = effective grain size diameter
- Darcy's Law is valid for $N_R < 1$

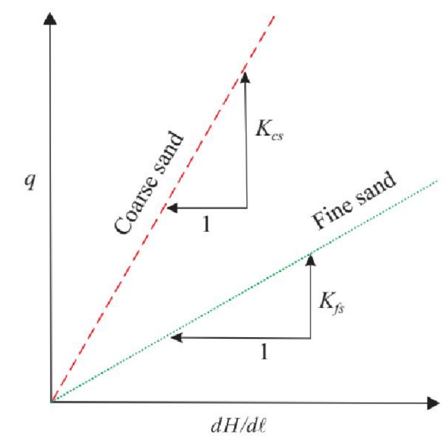


Figure 2.14. Schematic illustration of the relationship between the Darcian flux, q, and hydraulic gradient, dH/dl, for a coarse sand and a fine sand.

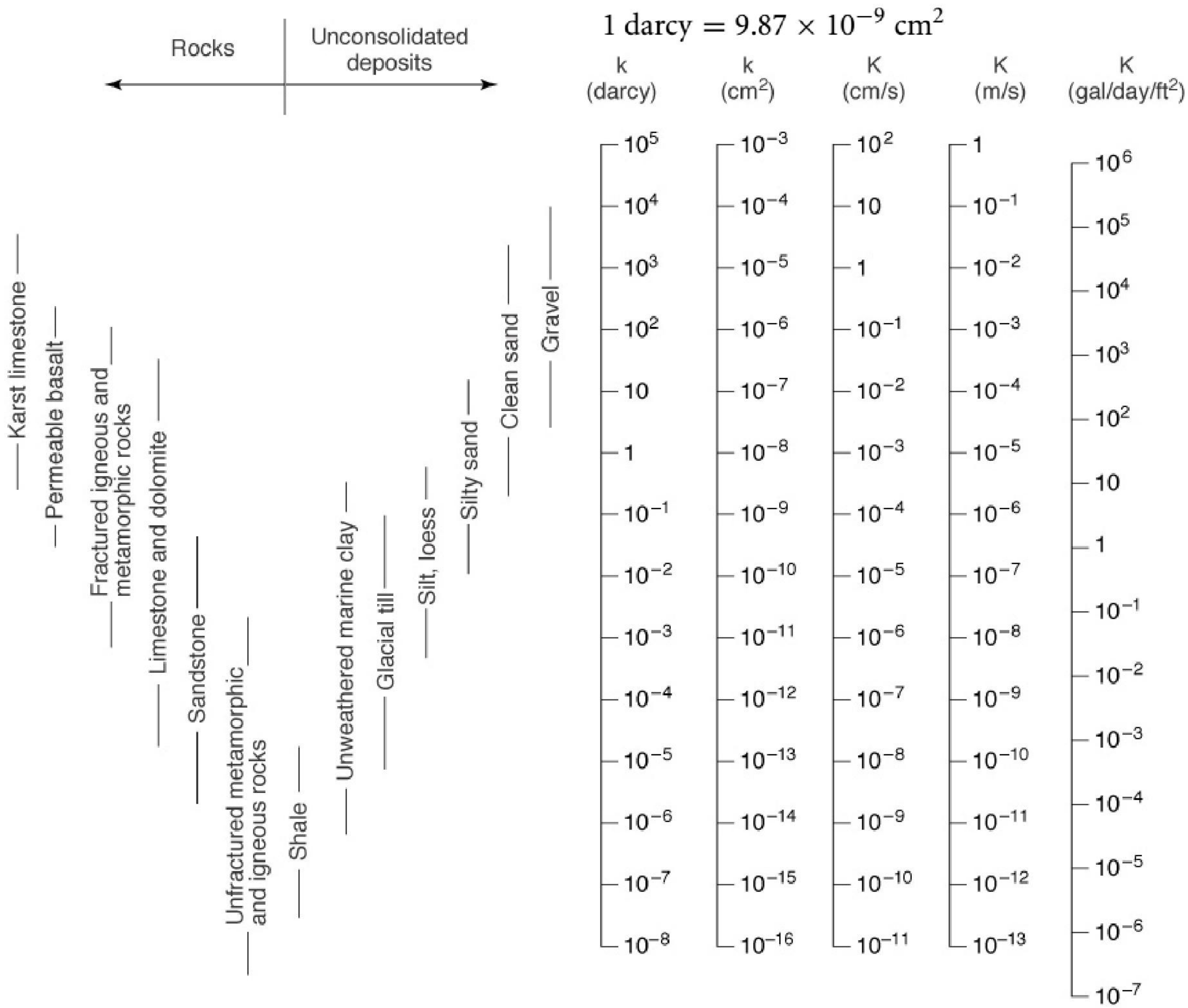
Hydraulic Conductivity

- Other factors being equal, the average velocity of groundwater migration is proportional to K
- Hydraulic conductivity is an empirical constant measured in laboratory or field experiments.
- Historically, *Permeability ≡ Hydraulic Conductivity*
- Now its usage is associated with *intrinsic permeability*

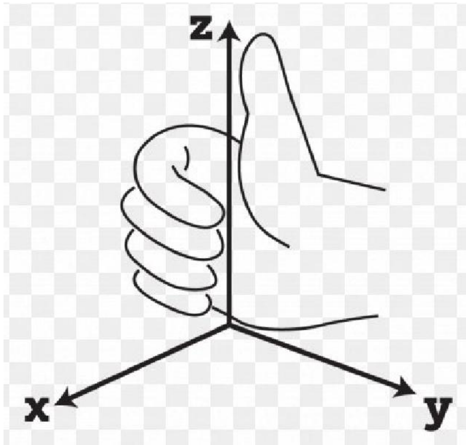
$$K = k\frac{\rho g}{\mu}$$

k	=	intrinsic permeability	Porous medium property
ρ	=	density	Fluid properties
μ	=	dynamic viscosity	
g	=	Gravitational acceleration constant	

Hydraulic Conductivity (Contd.)



Darcy's Law in Three Dimensions

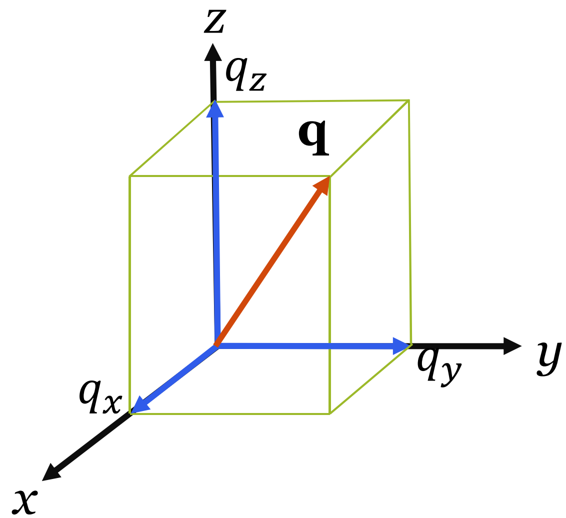


- Components of flow

$$q_x = -K_x \frac{\partial h}{\partial x}$$
$$q_y = -K_y \frac{\partial h}{\partial y}$$
$$q_z = -K_z \frac{\partial h}{\partial z}$$
$$\mathbf{q} = q_x\hat{\imath} + q_y\hat{\jmath} + q_z\hat{k}$$

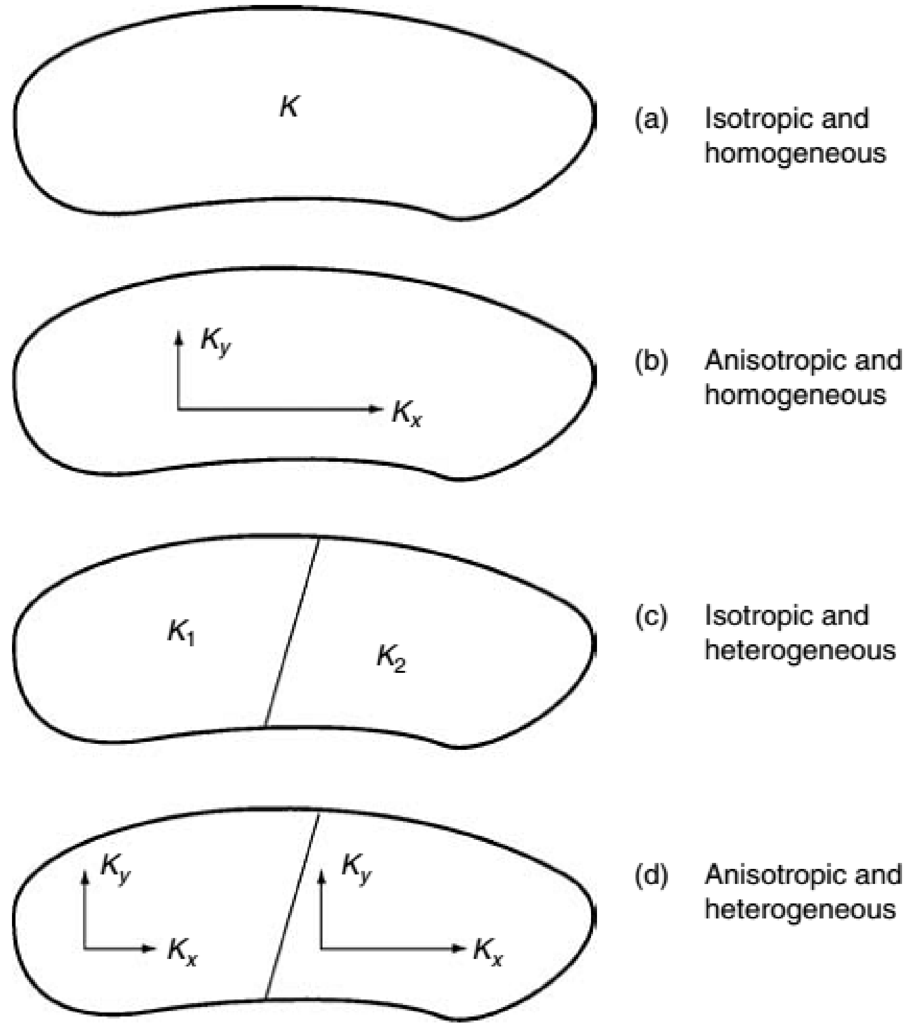
- The magnitude of specific discharge vector

$$|\mathbf{q}| = \sqrt{q_x^2 + q_y^2 + q_z^2}$$



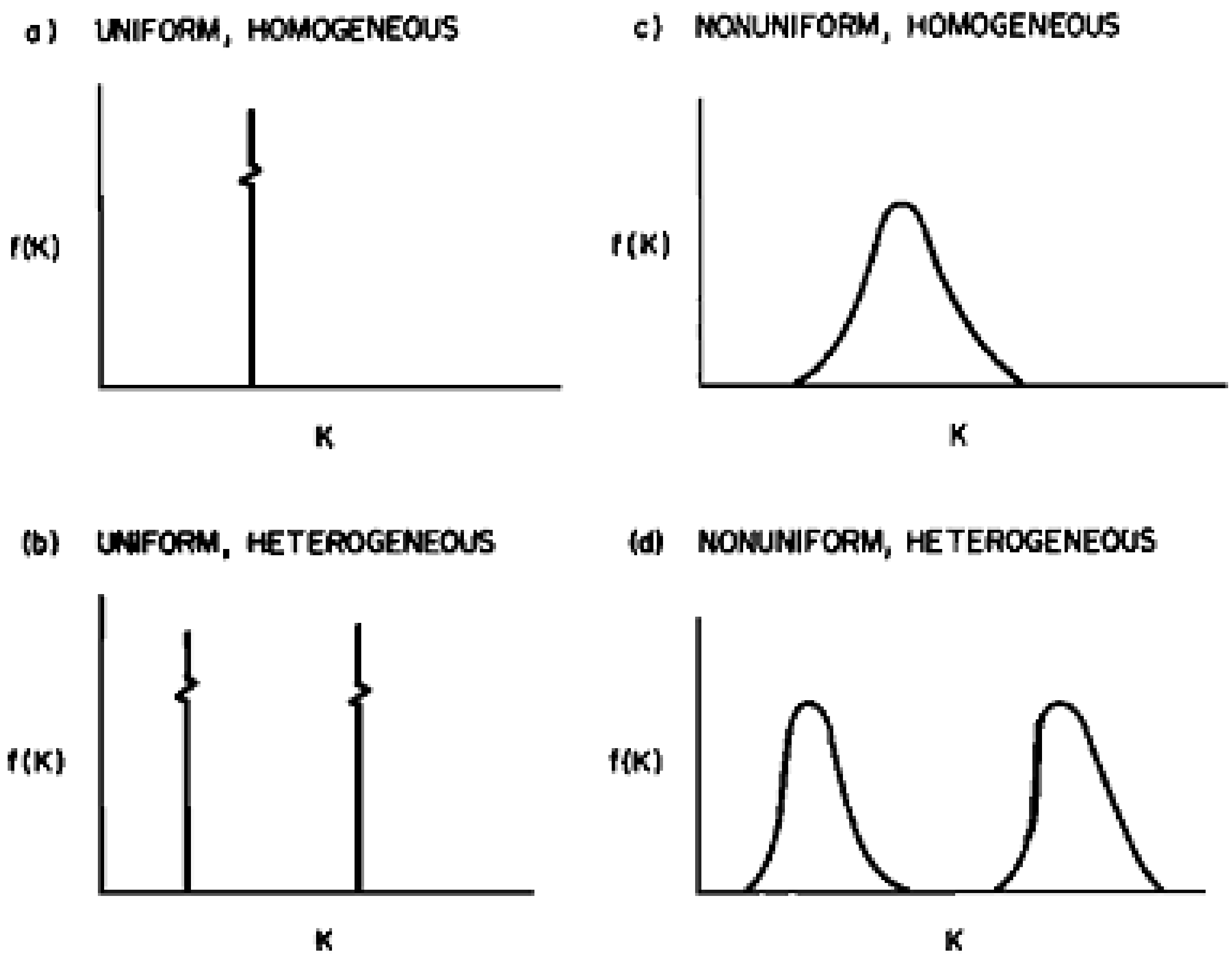
Heterogeneity and Anisotropy of Hydraulic Conductivity

- Deterministic Approach



Heterogeneity and Nonuniformity of Hydraulic Conductivity

- Stochastic Approach

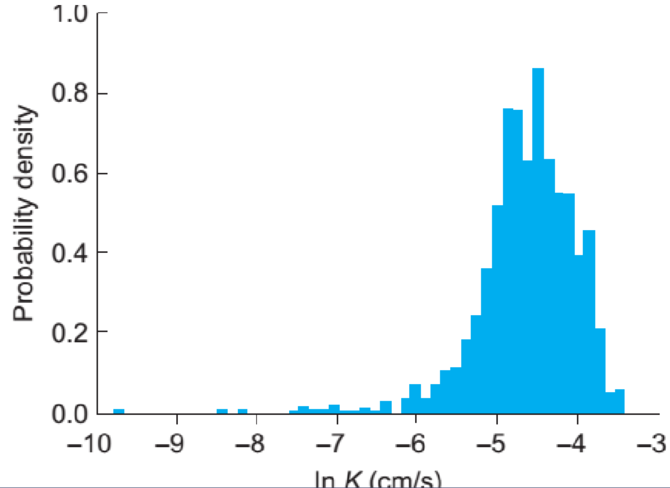


Probabilistic Analysis

- Matherton (1967) determined that the geometric mean of small-scale K measurements gives the appropriate large-scale average K under the following circumstances:
- The K histogram is log-normal.
- K has a statistically isotropic distribution in space.
- Flow is two-dimensional.
- Flow is uniform (one-dimensional on a large scale).

The geometric mean K_g of n K measurements is calculated as

$$K_g = (K_1 K_2 K_3 \cdots K_n)^{1/n}$$



Specific Discharge Vectors at an Interface

- At the boundary between two materials with differing hydraulic conductivities, the flow paths are bent in a manner similar to optical refraction.
- At the material interface, two conditions must be met:
  - The specific discharge normal to the interface is the same on both sides of the interface to preserve continuity of flow.

$$q_{n1} = q_{n2}$$

  - Pressure must be continuous in a fluid. Therefore head must also be continuous across the interface.

$$\left(\frac{\partial h}{\partial t}\right)_1 = \left(\frac{\partial h}{\partial t}\right)_2$$

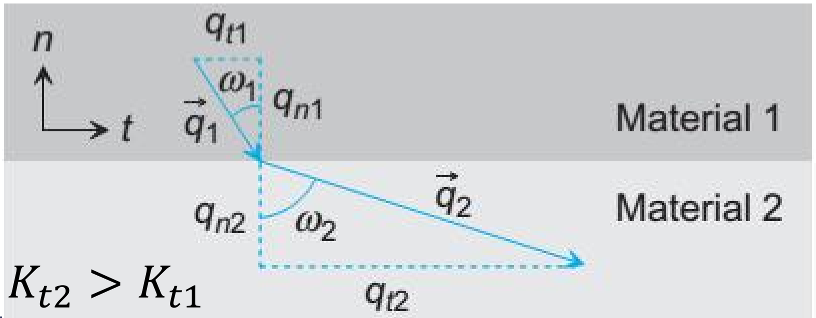
- The angles ω_1 and ω_2 are related to the specific discharge components

$$\tan \omega_1 = \frac{q_{t1}}{q_{n1}}, \tan \omega_2 = \frac{q_{t2}}{q_{n2}}$$

- Using Darcy's law

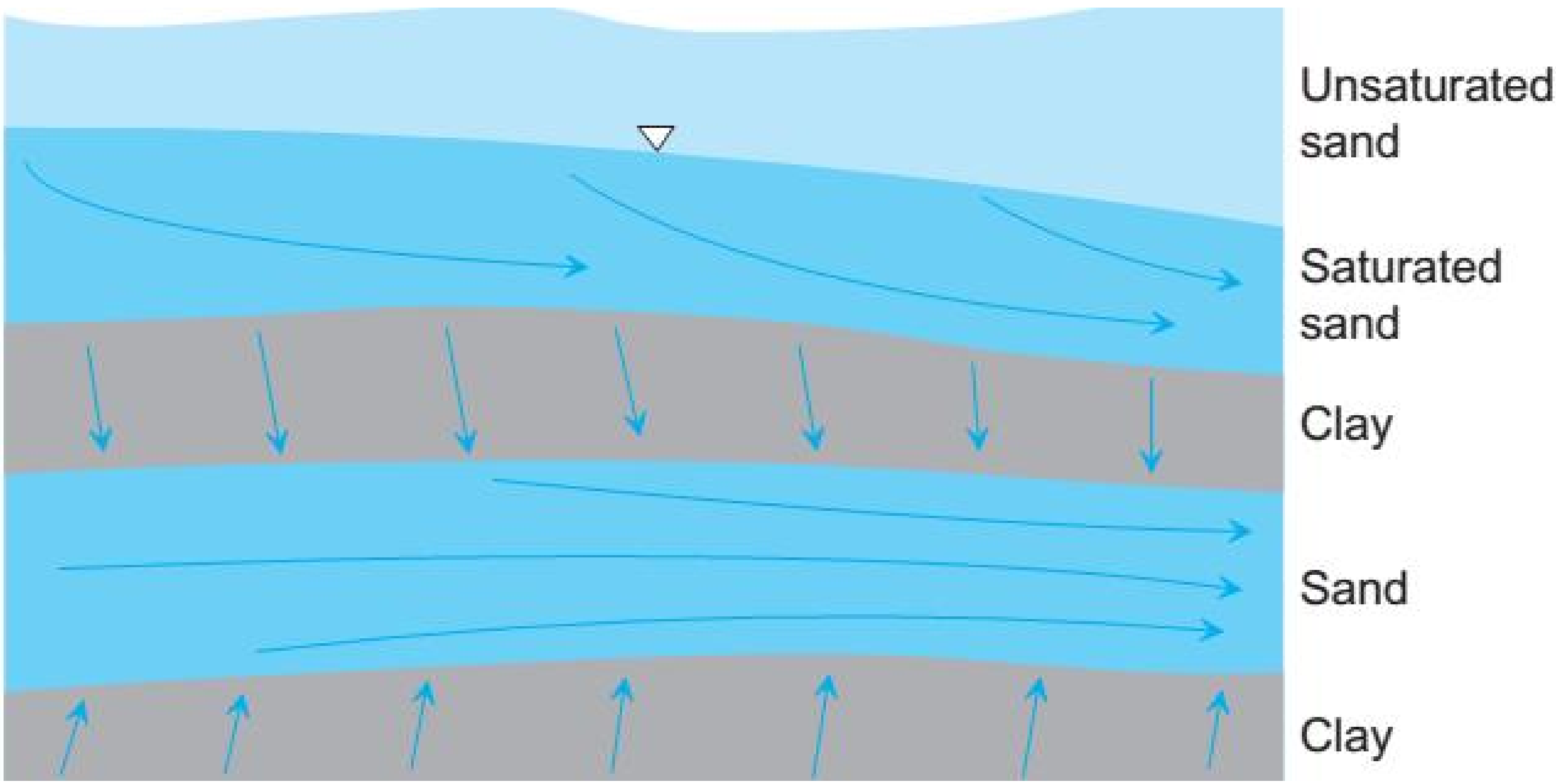
$$\frac{\tan \omega_1}{\tan \omega_2} = \frac{K_{t1}}{K_{t2}}$$

- When $K_{t1} \ll K_{t2}$, $\omega_1 \to 0$ and $\omega_2 \to \frac{\pi}{2}$



$K_{t2} > K_{t1}$

Specific Discharge Vectors at an Interface (Contd.)



Estimating Average Hydraulic Conductivities

- Specific Discharge

$$q_z = q_{z1} = q_{z2} = q_{z3} = \cdots = q_{zn}$$

- The specific discharge q_{zi} for the i^{th} layers is given by

$$q_{zi} = -K_{zi}\frac{\Delta h_i}{d_i}$$

- Total head loss

$$\Delta h = \Delta h_1 + \Delta h_2 + \Delta h_3 + \cdots + \Delta h_n$$

Or,

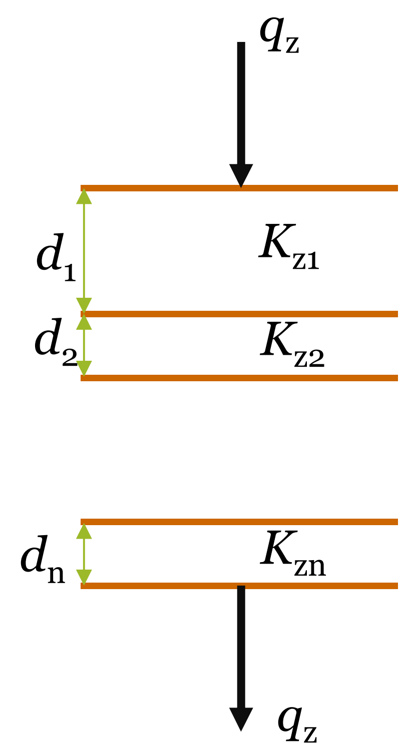
$$q_z\frac{d}{K_{ze}} = q_{z1}\frac{d_1}{K_{z1}} + q_{z2}\frac{d_2}{K_{z2}} + q_{z3}\frac{d_3}{K_{z3}} + \cdots + q_{zn}\frac{d_n}{K_{zn}}$$

Or,

$$\frac{d}{K_{ze}} = \frac{d_1}{K_{z1}} + \frac{d_2}{K_{z2}} + \frac{d_3}{K_{z3}} + \cdots + \frac{d_n}{K_{zn}}$$

Or,

$$K_{ze} = \frac{\sum_{i=1}^{n} d_i}{\sum_{i=1}^{n}\frac{d_i}{K_{zi}}}$$



Estimating Average Hydraulic Conductivities (Contd.)

- Discharge Q_{xi} for the i^{th} layers is given by

$$Q_{xi} = -K_{xi} \frac{\partial h}{\partial x} d_i$$

- Total Discharge

$$Q_x = Q_{x1} + Q_{x2} + Q_{x3} + \cdots + Q_{xn}$$

Or,

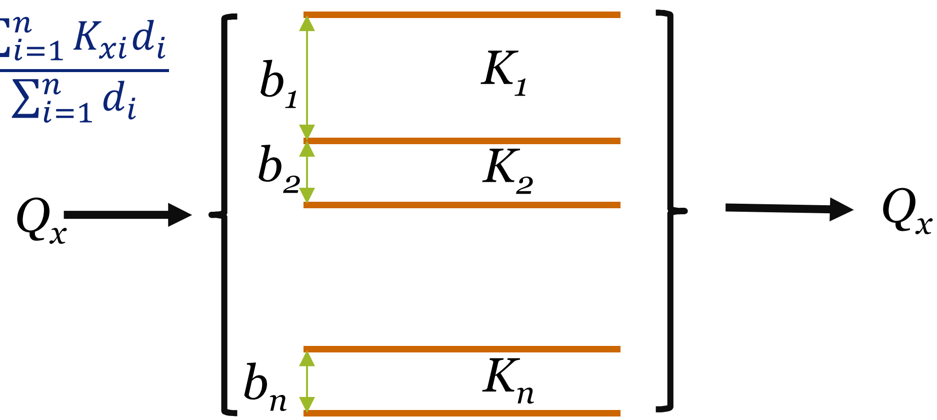
$$K_{xe} \frac{\partial h}{\partial x} d = K_{x1} \frac{\partial h}{\partial x} d_1 + K_{x2} \frac{\partial h}{\partial x} d_2 + K_{x3} \frac{\partial h}{\partial x} d_3 + \cdots + K_{xn} \frac{\partial h}{\partial x} d_n$$

Or,

$$K_{xe} d = K_{x1} d_1 + K_{x2} d_2 + K_{x3} d_3 + \cdots + K_{xn} d_n$$

Or,

$$K_{xe} = \frac{\sum_{i=1}^{n} K_{xi} d_i}{\sum_{i=1}^{n} d_i}$$



Horizontal Hydraulic Conductivity

The horizontal hydraulic conductivity in alluvium is normally greater than that in the vertical direction:

$$K_H > K_V$$

Imagine a two-layer case:

$$\frac{K_1 d_1 + K_2 d_2}{d_1 + d_2} > \frac{d_1 + d_2}{\frac{d_1}{K_1} + \frac{d_2}{K_2}}$$

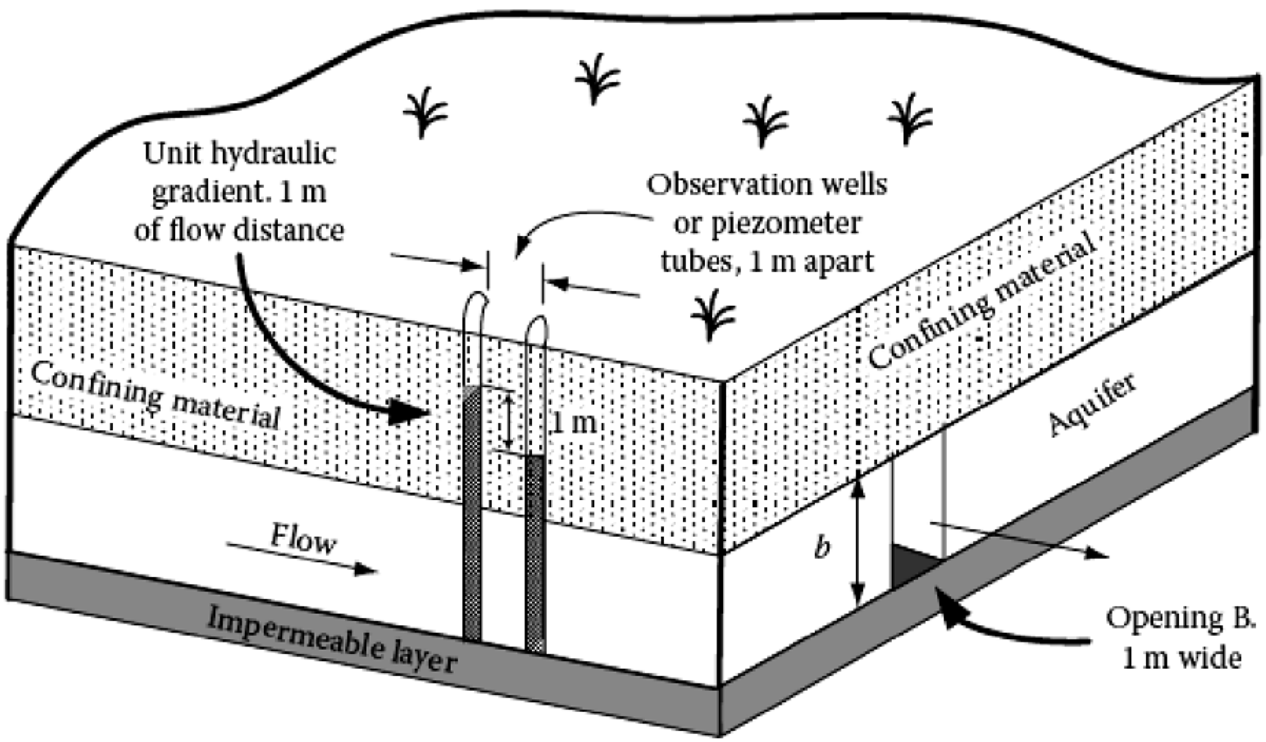
- Ratios of K_H/K_V, usually fall in the range of 2–10 for alluvium, but values up to 100 or more occur where clay layers are present.

Transmissivity

- If the hydraulic conductivity tangential to the layer K can be assumed constant over the thickness b of a layer, the transmissivity T of the layer is simply

$$T = Kb$$

- Transmissivity is the amount of water that moves horizontally through a unit width of a saturated aquifer as a result of a unit change in gradient.



Transmissivity (Contd.)

- If a layer is composed of m strata of thickness b_i and hydraulic conductivity K_i, the total transmissivity of the layer is the sum of the transmissivities of each stratum:

$$T = \sum_{i=1}^{m} T_i$$

- In an unconfined aquifer, transmissivity is not as well defined as in a confined aquifer.

Measuring Hydraulic Conductivity

- Correlations of Grain Size to Hydraulic Conductivity
- Hazen (1911) proposed the following empirical relation, based on experiments with various sand samples:

$$K = C(d_{10})^2$$

where K is hydraulic conductivity in cm/sec, C is a constant with units of $(cm\ sec)^{-1}$, and d_{10} is the grain diameter in centimeters such that grains this size or smaller represent 10% of the sample mass.

- This equation requires a fixed set of units.
- The constant C varies from about 40 to 150 for most sands.
- C is at the low end of this range for fine, widely graded sands, and C is near the high end of the range for coarse, narrowly graded sands.
- Kozeny–Carman equation

$$K = \left(\frac{\rho_w g}{\mu}\right)\left(\frac{n^3}{(1-n)^2}\right)\left(\frac{(d_{50})^2}{180}\right)$$

- Kozeny-Carman equation is dimensionally consistent

Laboratory Measurement of Hydraulic Conductivity

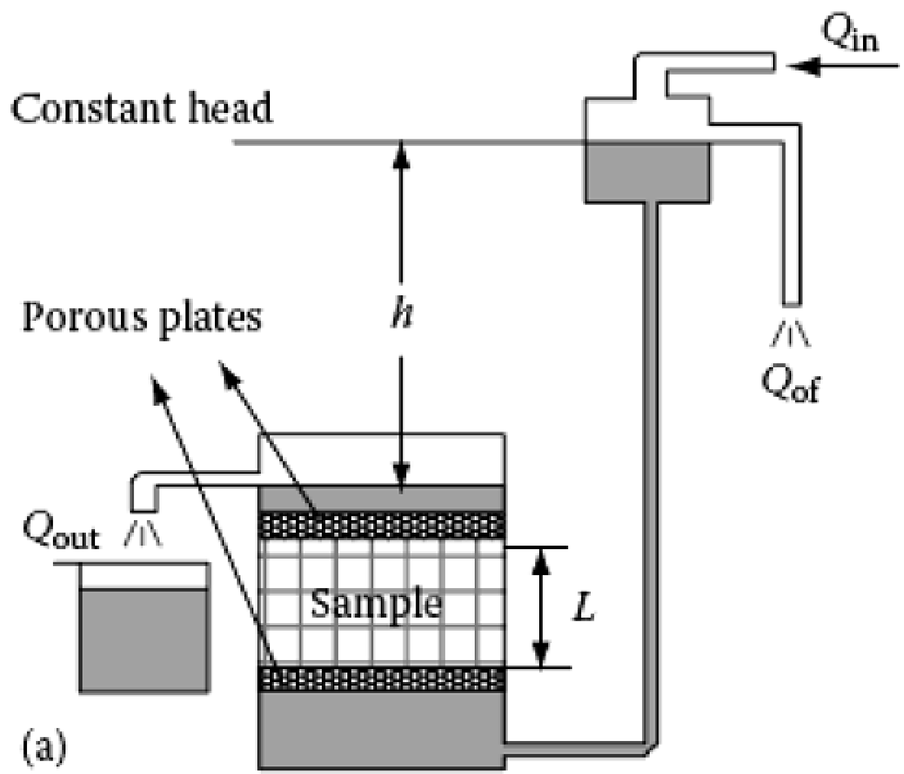
- Constant head permeameter
- Q_{of} is the rate of overflow. From Darcy's law we have

$$Q = -KA\frac{h_2 - h_1}{L}$$

$$K = \frac{QL}{Ah}$$

where:

Q is the flow rate
A is the cross-sectional area
L is the length of the sample
h is the constant head.



Laboratory Measurement of Hydraulic Conductivity (contd.)

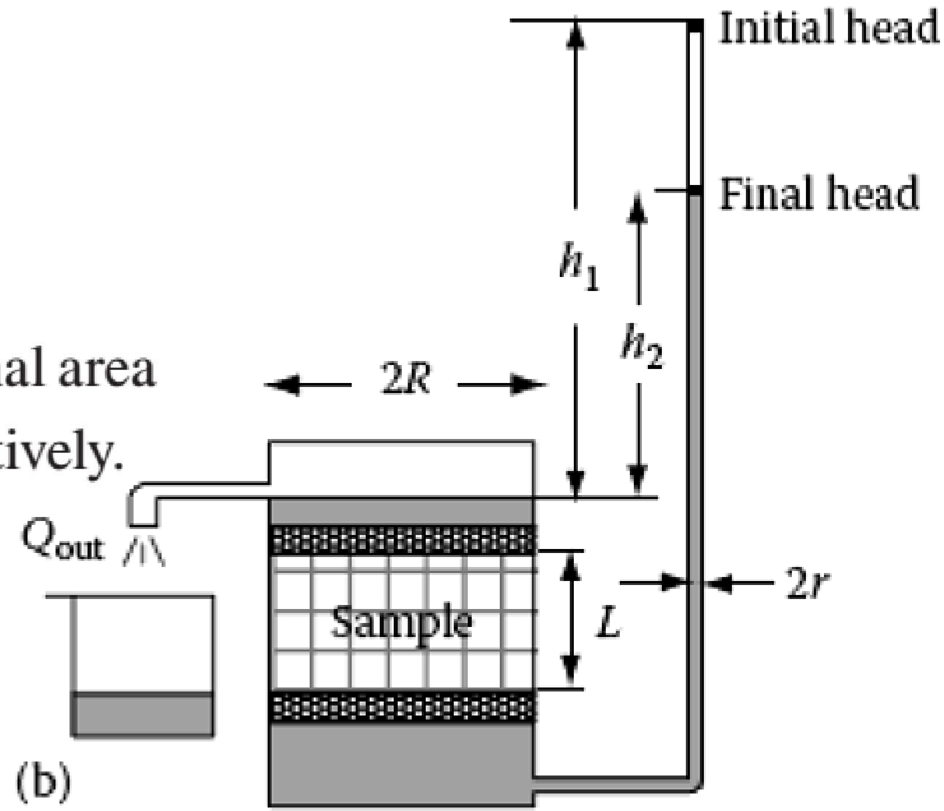
- Falling head

$$Q_{\text{in}} = -A_t \frac{\mathrm{d}h}{\mathrm{d}t}$$

$$Q_{\text{out}} = KA_c \frac{h}{l}$$

where A_t and A_c are the cross-sectional area of the tube and the container, respectively.

In steady-state condition, $Q_{\text{in}} = Q_{\text{out}}$:



Falling head permeameter

$$-A_t \frac{\mathrm{d}h}{\mathrm{d}t} = KA_c \frac{h}{l}$$

$$\int_{h_1}^{h_2} \frac{\mathrm{d}h}{h} = \frac{-K}{L}\frac{A_c}{A_t}\int_{t_1}^{t_2} \mathrm{d}t$$

$$\left[\ln h\right]_{h_1}^{h_2} = \frac{-K}{L}\frac{A_c}{A_t}\left(t_2 - t_1\right)$$

$$\ln\frac{h_2}{h_1} = \frac{-K}{L}\frac{A_c}{A_t}\left(t_2 - t_1\right)$$

$$K = \frac{A_t L}{A_c\left(t_2 - t_1\right)}\ln\frac{h_1}{h_2}$$

where:

L is the length of the sample

h_1 and h_2 are the heads at the beginning, t_1 and at time t_2 later.

Learning Strategy

Chapter 3: Principles of Flow

Thank you