Seawater Intrusion

Groundwater Engineering| CE60205 Lecture:21

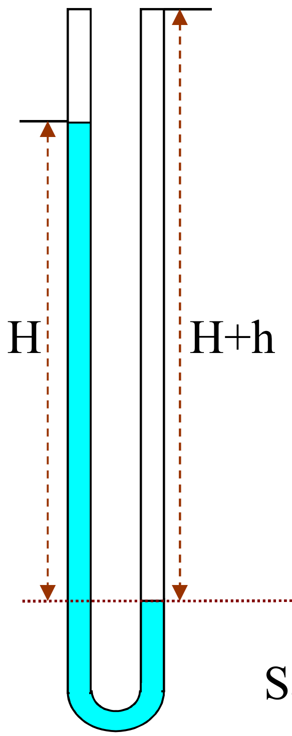
Learning Objective(s)

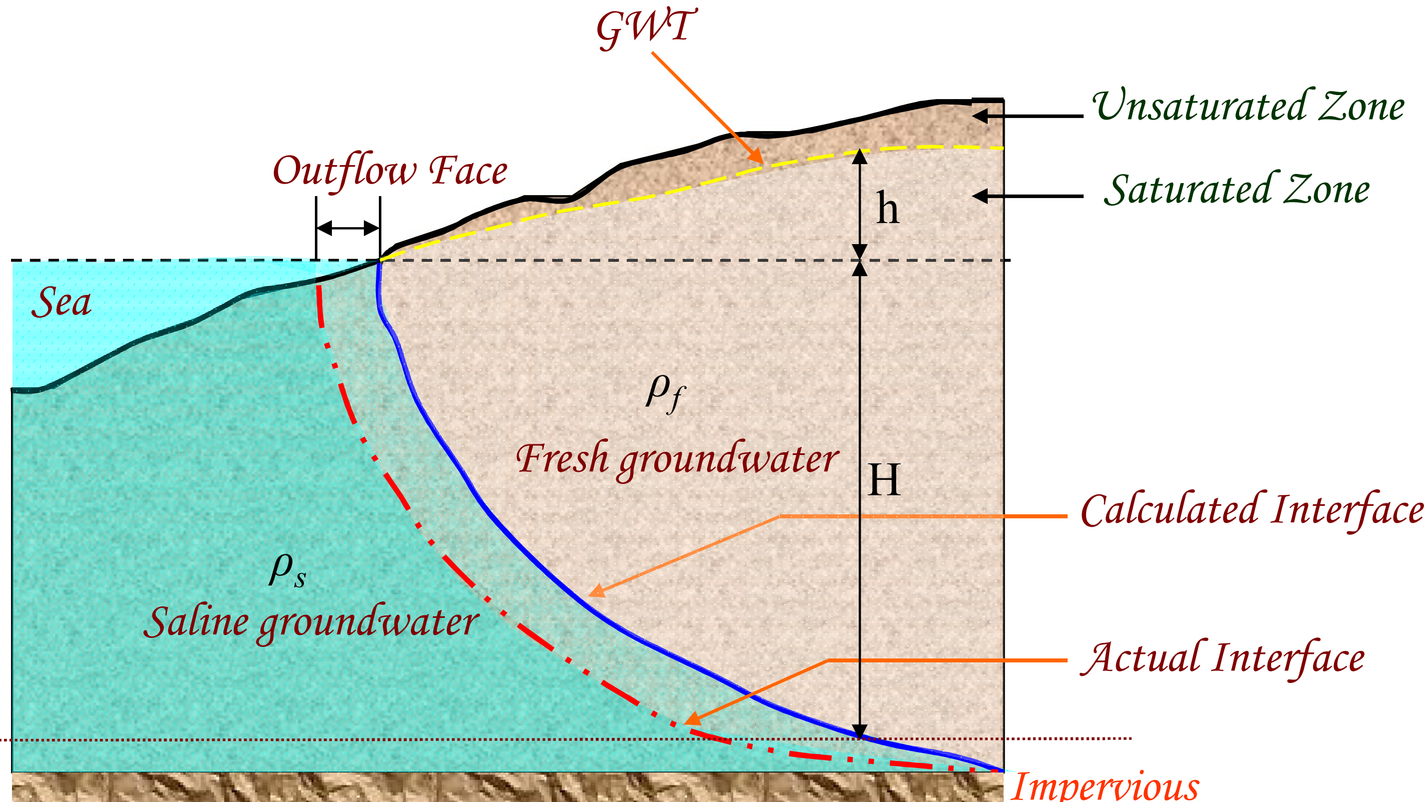
- To explain saltwater intrusion in the context of flow and transport models.

Descriptive modeling of saltwater intrusion

- Simple approximation based solution.
- Sharp interface simulation.
- Density dependent flow simulation

Bodon Ghyben-Herzberg Principle





Saline groundwater pressure = fresh groundwater pressure

$$\rho_s g \, H = \rho_f g \, (h + H)$$

$$\boxed{h = \frac{\rho_s - \rho_f}{\rho_f} H}$$

$\rho_s = 1025 \ kg/m^3$
$\rho_f = 1000 \ kg/m^3$

Sharp Interface Approximation (Contd.)

- Darcy's law for a homogeneous isotropic aquifer

$$q_x = -k\frac{\partial \phi}{\partial x} \qquad q_y = -k\frac{\partial \phi}{\partial y}$$

- Discharge vector

$$Q_x = hq_x = -kh\frac{\partial \phi}{\partial x} \qquad Q_y = hq_y = -kh\frac{\partial \phi}{\partial y}$$

- Thickness of the aquifer may be represented as a linear function

$$h = \alpha\phi + \beta$$

Case where $\alpha \neq 0$

$$Q_x = -\frac{\partial}{\partial x}\left[\frac{1}{2\alpha}k(\alpha\phi + \beta)^2\right] \qquad Q_y = -\frac{\partial}{\partial y}\left[\frac{1}{2\alpha}k(\alpha\phi + \beta)^2\right]$$

Case where $\alpha = 0$

$$Q_x = -\frac{\partial}{\partial x}[\beta k\phi] \qquad Q_y = -\frac{\partial}{\partial y}[\beta k\phi]$$

Sharp Interface Approximation (Contd.)

By defining the potential Φ as

Case where $\alpha \neq 0$

$$\Phi = \tfrac{1}{2}k\alpha(\phi + \beta/\alpha)^2 + C$$

where C stands for a constant

Case where $\alpha = 0$

$$\Phi = k\beta\phi + C$$

- Discharge Vector

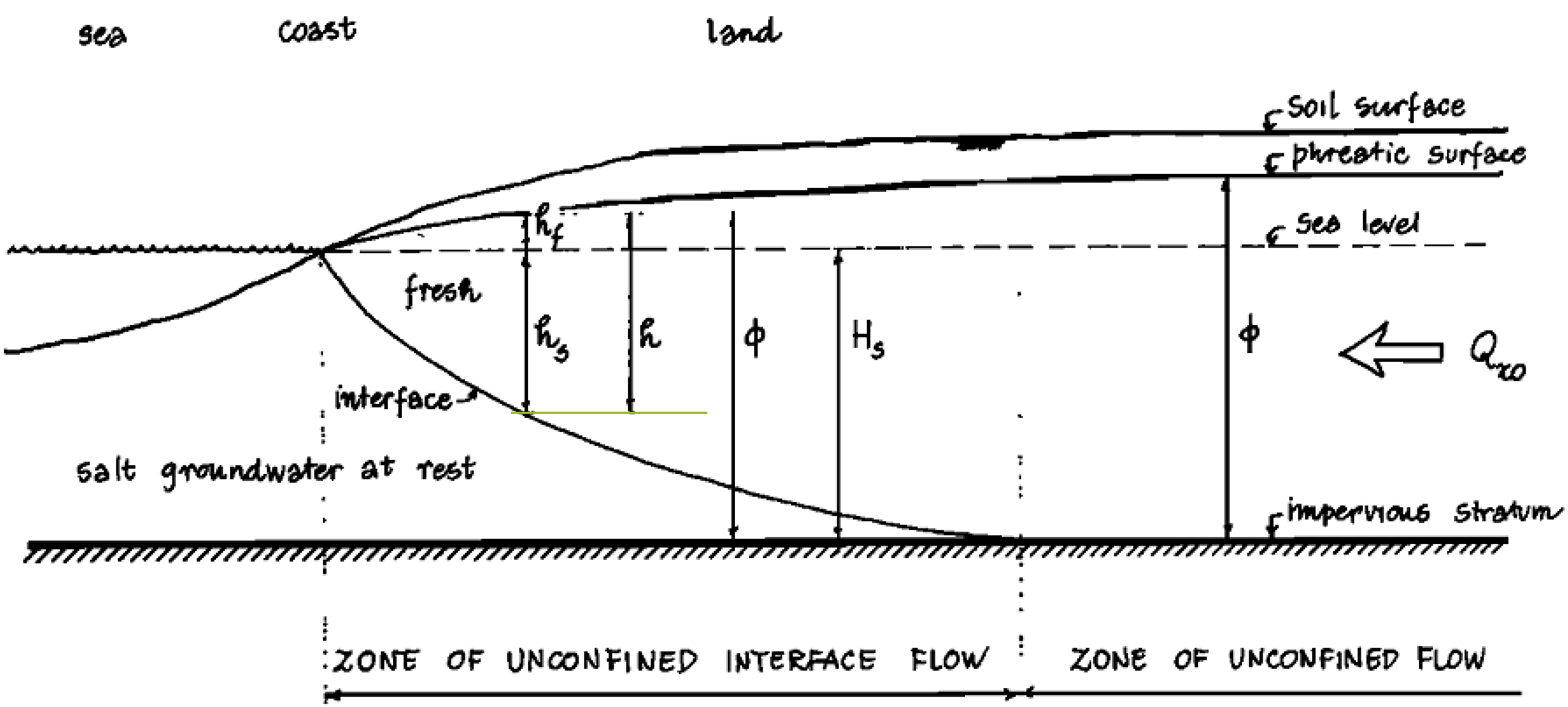
$$Q_x = -\partial\Phi/\partial x \qquad Q_y = -\partial\Phi/\partial y$$

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} = I$$

where I represents some constant influx into the aquifer from either above or below.

$$\frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2} = -I$$

Sharp Interface Approximation



Interface flow in a shallow coastal aquifer.

Sharp Interface Approximation (Contd.)

- The total height of the flow region

$$h = h_f + h_s$$

- Ghyben-Herzberg formula

$$h_s = h_f[\rho_f/(\rho_s - \rho_f)]$$

- Head

$$\phi = h_f + H_s$$

- Total height

$$h = h_f[\rho_f/(\rho_s - \rho_f)] = h_s(\rho_s/\rho_f)$$

- Potential Φ

$$\Phi = \frac{1}{2}K[\rho_s/(\rho_s - \rho_f)](\phi - H_s)^2 + C_{ui}$$

- Dupuit-Forchheimer assumption

$$\phi = h \qquad \alpha = 1 \qquad \beta = 0$$

$$\Phi = \tfrac{1}{2}k\phi^2 + C_u$$

Sharp Interface Approximation (Contd.)

zone 1

$$\Phi = \tfrac{1}{2}k\frac{l_s}{l_s - l_f}[\phi - H_s]^2 + C_{ul}$$

zone 2

$$\Phi = \tfrac{1}{2}k\phi^2 + C_u$$

- The location of the tip of the tongue is defined by the condition

$$h_s = H_s$$

$$\phi = h_f + H_s = \frac{l_s - l_f}{l_f}h_s + H_s$$

$$= \frac{l_s - l_f}{l_f}H_s + H_s = \frac{l_s}{l_f}H_s$$

Sharp Interface Approximation (Contd.)

- Along the tip of the tongue

$$\Phi = \frac{1}{2}\frac{l_s}{l_s - l_f}k\left[\frac{l_s}{l_f} - 1\right]^2 H_s^2 + C_{ut}$$

$$= \frac{1}{2}\frac{l_s}{l_f}\frac{l_s - l_f}{l_f}kH_s^2 + C_{ut}$$

and

$$\Phi = \tfrac{1}{2}k\frac{l_s^2}{l_f^2}H_s^2 + C_u$$

$$C_{ut} - C_u = \tfrac{1}{2}kH_s^2\frac{l_s}{l_f}$$

Choosing $C_{ut} = 0$,

$$C_{ut} = 0 \qquad C_u = -\tfrac{1}{2}kH_s^2\frac{l_s}{l_f}$$

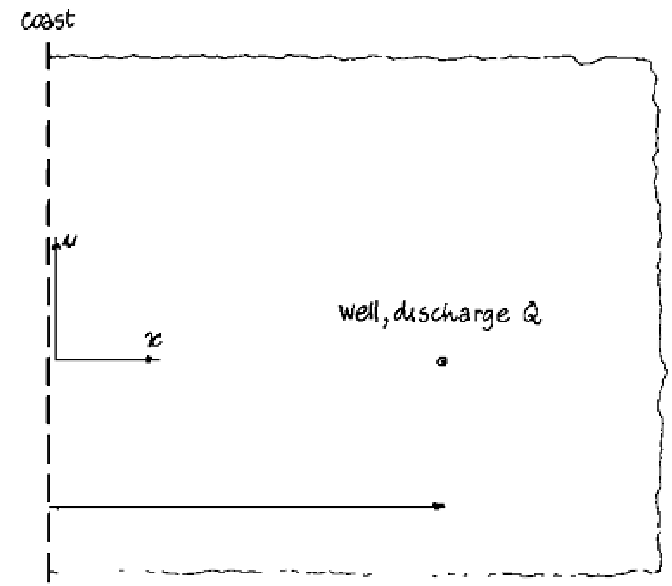
Sharp Interface Approximation (Contd.)

Zone 1

$$\Phi = \tfrac{1}{2}k\frac{l_s}{l_s - l_f}(\phi - H_s)^2$$

Zone 2

$$\Phi = \tfrac{1}{2}k\phi^2 - \tfrac{1}{2}kH_s^2\frac{l_s}{l_f}$$



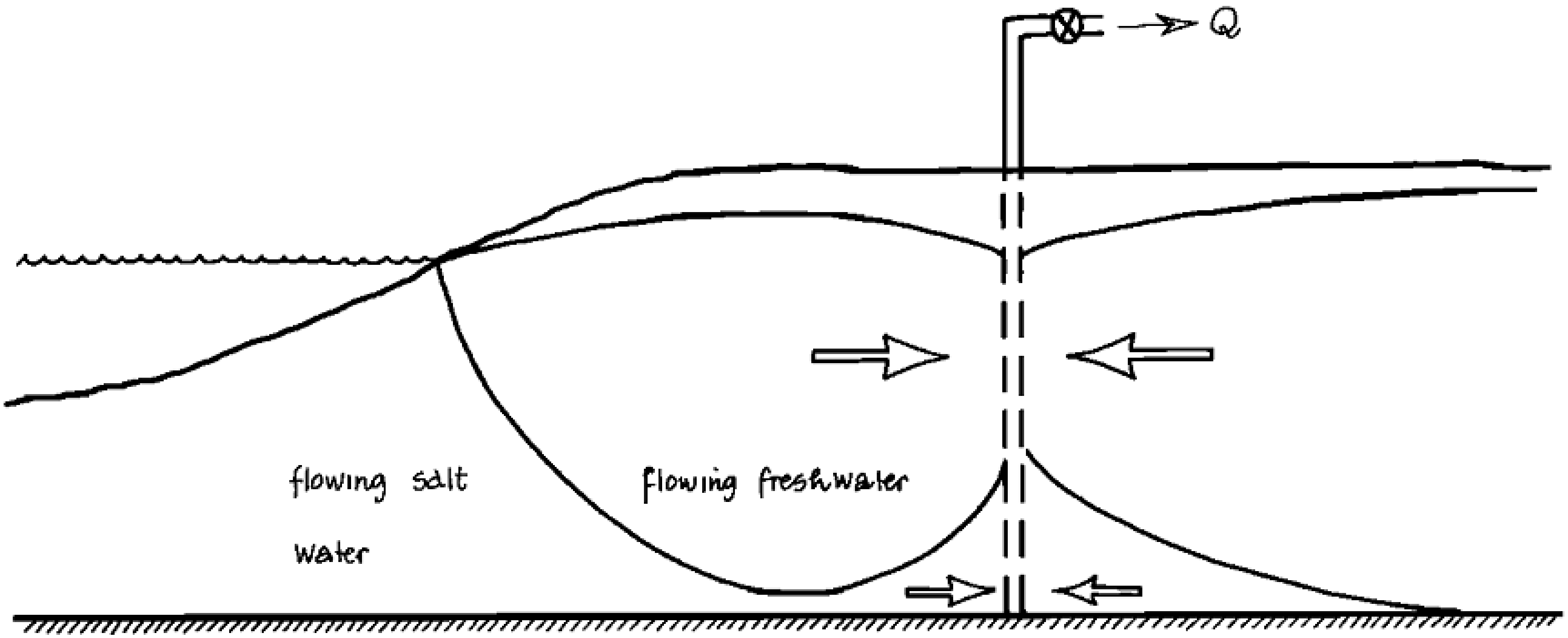
$$\Phi = Q_{x0}x$$

$$\Phi = 0 \qquad x = 0 \qquad -\infty < y < +\infty$$

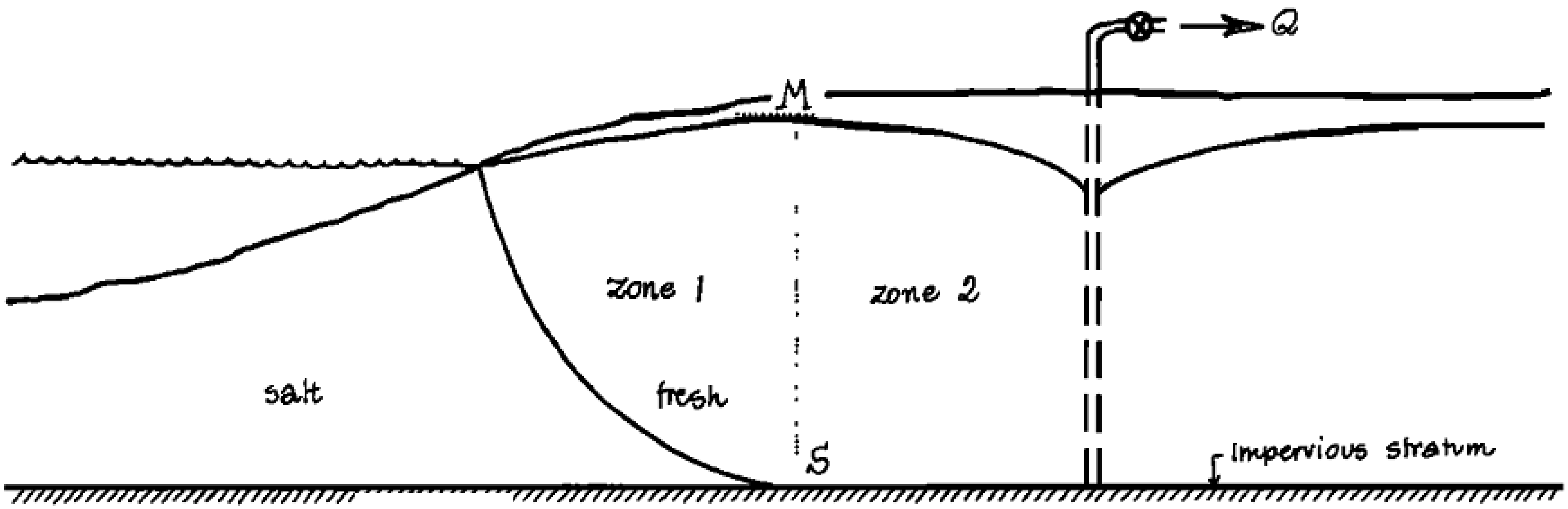
$$\Phi = \tfrac{1}{2}kH_s^2\frac{l_s}{l_f}\,\frac{l_s - l_f}{l_f}$$

$$\Phi = Q_{x0}x + \frac{Q}{2\pi}\ln\left[\frac{(x - x_w)^2 + y^2}{(x + x_w)^2 + y^2}\right]^{1/2}$$

Sharp Interface Approximation (Contd.)



Sharp Interface Approximation (Contd.)



Instability of the interface at point S.

Sharp Interface Approximation (Contd.)

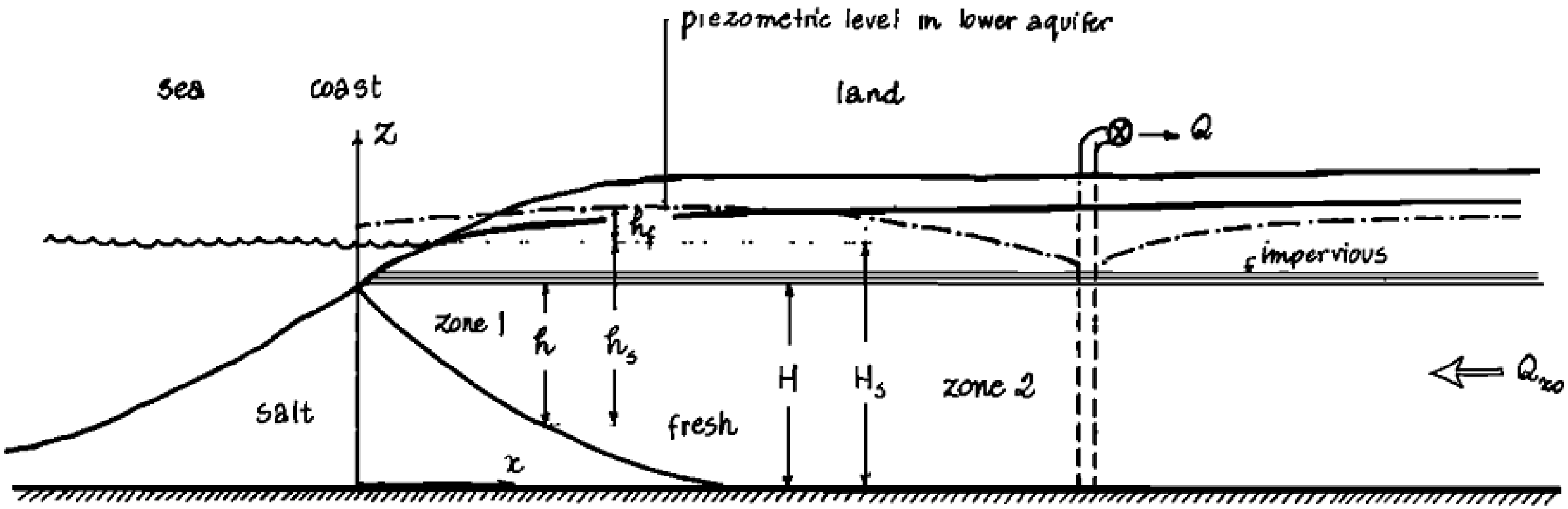


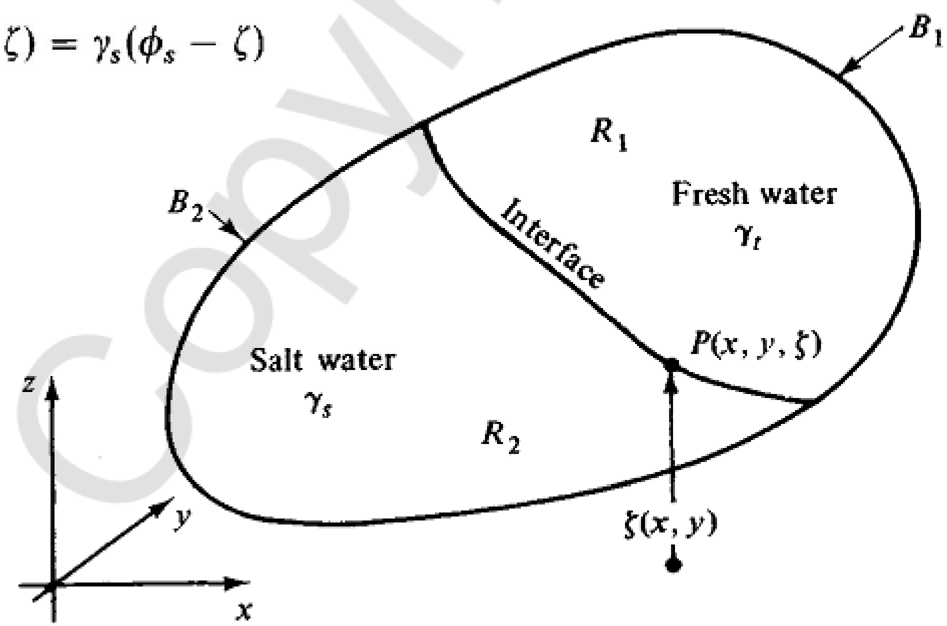
Fig. 10. Mixed confined–confined interface flow.

Sharp Interface Approximation (Contd.)

Determine ϕ_f in R_1 and ϕ_s in R_2 such that

$$\nabla\cdot(\boldsymbol{K}_f\cdot\nabla\phi_f) = S_0\frac{\partial\phi_f}{\partial t} \quad \text{in } R_1; \qquad \nabla\cdot(\boldsymbol{K}_s\cdot\nabla\phi_s) = S_0\frac{\partial\phi_s}{\partial t} \quad \text{in } R_2$$

$$\gamma_f(\phi_f-\zeta)=\gamma_s(\phi_s-\zeta)$$



$$F(x, y, z, t) = 0$$

$$F \equiv z - \zeta(x, y, t) = 0$$

Figure 9-3 An abrupt interface between regions occupied by fresh water and by salt water.

Sharp Interface Approximation (Contd.)

$$\zeta(x, y, t) = \phi_s \frac{\gamma_s}{\gamma_s - \gamma_f} - \phi_f \frac{\gamma_f}{\gamma_s - \gamma_f}$$
$$= \phi_s(1 + \delta) - \phi_f \delta; \qquad \delta = \gamma_f/(\gamma_s - \gamma_f)$$

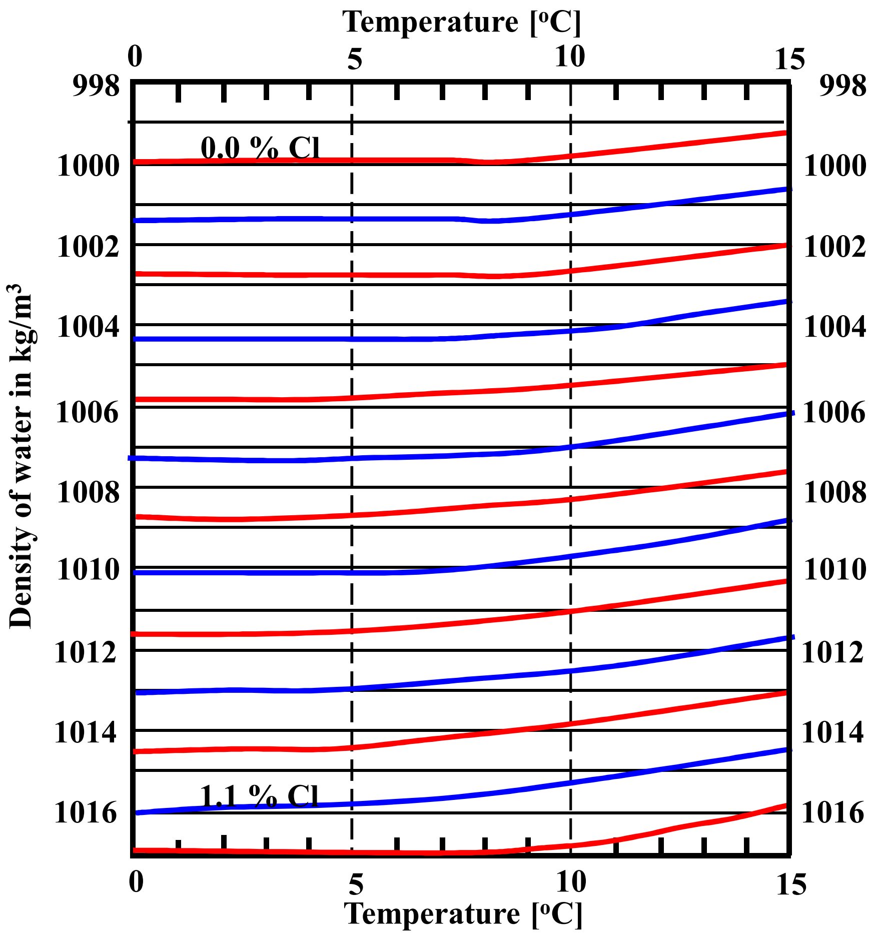
$$F \equiv z - \phi_s(1 + \delta) + \phi_f \delta = 0$$

$$dF/dt \equiv \partial F/\partial t + \mathbf{V}_f \cdot \nabla F = 0; \qquad \partial F/\partial t + \mathbf{V}_s \cdot \nabla F = 0$$

$$n\mathbf{V}_f = -\boldsymbol{K}_f \cdot \nabla \phi_f; \; n\mathbf{V}_s = -\boldsymbol{K}_s \cdot \nabla \phi_s$$

$$n\delta \frac{\partial \phi_f}{\partial t} - n(1 + \delta) \frac{\partial \phi_s}{\partial t} - \boldsymbol{K}_f \cdot [\nabla z - (1 + \delta) \nabla \phi_s + \delta \nabla \phi_f] \cdot \nabla \phi_f = 0$$
$$n\delta \frac{\partial \phi_f}{\partial t} - n(1 + \delta) \frac{\partial \phi_s}{\partial t} - \boldsymbol{K}_s \cdot [\nabla z - (1 + \delta) \nabla \phi_s + \delta \nabla \phi_f] \cdot \nabla \phi_s = 0$$

Density of water as a function of the chlorinity and temperature



Equation of state

Density

$$\rho = f\left(\rho_0, T, c\right)$$

Reference density *Temperature* *Concentration*

$$\mu = g\left(\rho_0, T, c\right)$$

Dynamic viscosity

Diffused Interface Approach (FEMWATER)

- Flow equation:

$$\frac{\rho}{\rho_0} F \frac{\partial \hbar}{\partial t} + \frac{\theta}{\rho_0} \frac{\partial \rho}{\partial C} \frac{\partial C}{\partial t} = \nabla \cdot \left[\mathbf{K} \cdot \left(\nabla \hbar + \frac{\rho}{\rho_0} \nabla z \right) \right] + \frac{\rho^*}{\rho_0} q$$

- Transport equation:

$$\theta \frac{\partial C}{\partial t} + \rho_b \frac{\partial \bar{C}}{\partial t} + \mathbf{V} \cdot \nabla C - \nabla \cdot (\theta \mathbf{D} \, \nabla \mathrm{C})$$
$$= \left(\alpha' \frac{\partial \hbar}{\partial t} + \lambda \right) (\theta C + \rho_b \bar{C}) - (\theta K_w C + \rho_b K_s \bar{C}) + q C_{in} - \frac{\rho^*}{\rho_0} q + \left(F \frac{\partial \hbar}{\partial t} + \frac{\rho}{\rho_0} \mathbf{V} \cdot \nabla \left(\frac{\rho}{\rho_0} \right) - \frac{\partial \theta}{\partial t} \right) C$$

Thank you