Parameter Estimation

Geohydraulics| CE60113

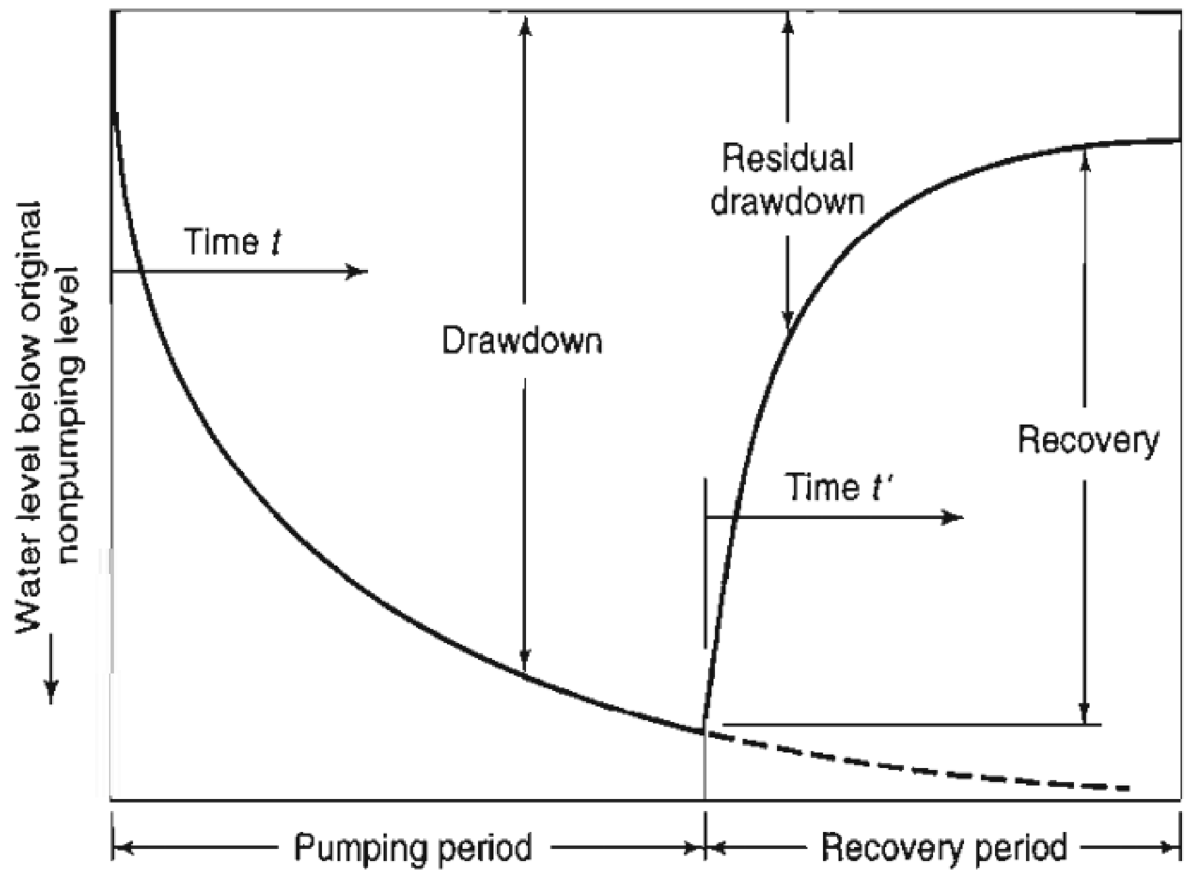
Lecture:15

Learning Objective(s)

- To estimate aquifer parameter under unsteady confined flow condition with recovery test
- To estimate aquifer parameter under unsteady unconfined flow condition

Unsteady Radial Flow in Confined Aquifer

- Recovery Test -> pumping well is shut off



Unsteady Radial Flow in Confined Aquifer (Contd.)

- Superposition with Transient Flow

$$\nabla^2 h = 0$$

$$\nabla^2 h = \frac{S}{T}\frac{\partial h}{\partial t}$$

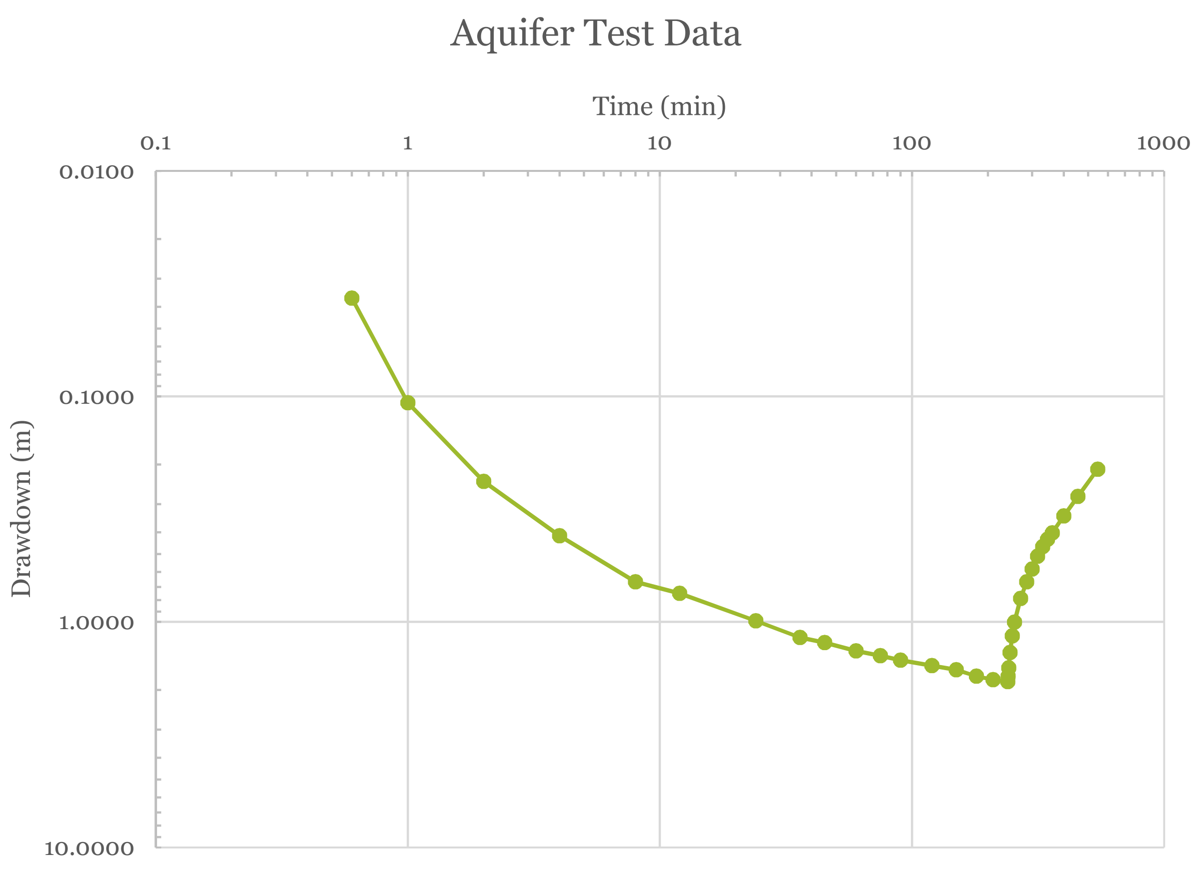
- Equations are linear in h
- If h_1 and h_2 are solutions, then so is $h = h_1 + h_2$
- The principle of superposition is not affected by transient condition.

Unsteady Radial Flow in Confined Aquifer (Contd.)

- Aquifer Test Data
- $Q = 504\ m^3/day$
- $r = 18.3\ m$

t (min)	s (m)	t (min)	s (m)
0.6	0.0366	241	1.7404
1	0.1067	242	1.6002
2	0.2377	245	1.3655
4	0.4145	250	1.1521
8	0.6645	255	1.0028
12	0.7468	270	0.7864
24	0.9906	285	0.6645
36	1.1735	300	0.5822
45	1.2344	315	0.5121
60	1.3442	330	0.4633
75	1.4143	345	0.4298
90	1.4783	360	0.4023
120	1.5606	400	0.3383
150	1.6307	455	0.2774
180	1.7404	545	0.2103
210	1.8014		
240	1.8379		

Unsteady Radial Flow in Confined Aquifer (Contd.)



Unsteady Radial Flow in Confined Aquifer (Contd.)

- Jacob Approximation

$$W(u) \approx -\gamma - ln(u) = ln\left(\frac{e^{-\gamma}}{u}\right) = ln\left(\frac{0.561}{u}\right) = ln\left(\frac{2.25Tt}{r^2S}\right)$$

Jacob's Equation

$$s(r,t) = \frac{Q}{4\pi T} ln\left(\frac{2.25Tt}{r^2S}\right) = \frac{2.303Q}{4\pi T} log\left(\frac{2.25Tt}{r^2S}\right)$$

Criteria for validity $u < 0.01$

$$s(r,t) = \frac{Q}{4\pi T}[W(u_1) - W(u_2)]$$

$$u_1 = \frac{r^2S}{4Tt}$$

$$u_2 = \frac{r^2S}{4T(t-\Delta t)}$$

$$s(r,t) = \frac{Q}{4\pi T} ln\left(\frac{u_2}{u_1}\right) = \frac{2.303Q}{4\pi T} log\left(\frac{t}{t-\Delta t}\right)$$

- Horner Plot → T

Unsteady Radial Flow in Confined Aquifer (Contd.)

- Recovery Test

t (min)	s (m)	t/(t-Δt)
241	1.7404	241.0000
242	1.6002	121.0000
245	1.3655	49.0000
250	1.1521	25.0000
255	1.0028	17.0000
270	0.7864	9.0000
285	0.6645	6.3333
300	0.5822	5.0000
315	0.5121	4.2000
330	0.4633	3.6667
345	0.4298	3.2857
360	0.4023	3.0000
400	0.3383	2.5000
455	0.2774	2.1163
545	0.2103	1.7869

Unsteady Radial Flow in Confined Aquifer (Contd.)

Unsteady Radial Flow in Confined Aquifer (Contd.)

$$s(r,t) = \frac{Q}{4\pi T} ln\left(\frac{u_2}{u_1}\right) = \frac{2.303Q}{4\pi T} log\left(\frac{t}{t-\Delta t}\right)$$

From the Horner plot

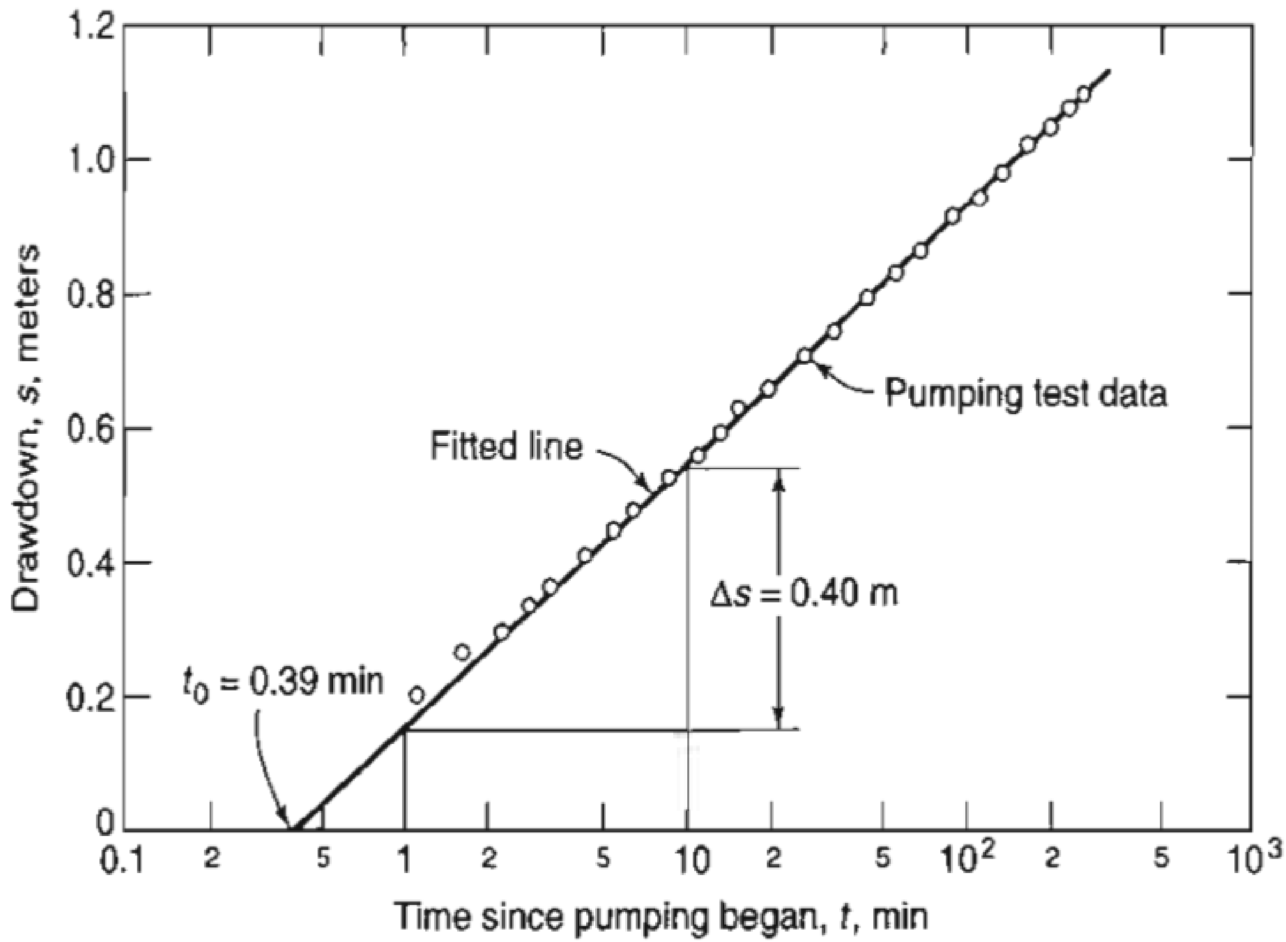
$$\frac{t}{t-\Delta t} = 10$$

$$Q = 504\ m^3/day$$

$$T = \frac{2.303Q}{4\pi s} = \frac{2.303 \times 504}{4\pi \times 0.84} = 109.96\ m^2/d$$

Unsteady Radial Flow in Confined Aquifer (Contd.)

Jacob's Method



Unsteady Radial Flow in Confined Aquifer (Contd.)

Zero drawdown

$$0 = \frac{Q}{4\pi T} ln\left(\frac{2.25Tt_0}{r^2 S}\right)$$

or,

$$\frac{2.25Tt_0}{r^2 S} = 1$$

Semi-logarithmic plot

$$\Delta s = \frac{2.303Q}{4\pi T} log\left(\frac{t_{10}}{t_1}\right) = \frac{2.303Q}{4\pi T}$$

or,

$$T = \frac{2.303Q}{4\pi \Delta s}$$

Radius of influence: $r = r_0$

Thiem Equation

$$s(r,t) = \frac{Q}{4\pi T} ln\left(\frac{r_0^2}{r^2}\right) = \frac{Q}{2\pi T} ln\left(\frac{r_0}{r}\right)$$

Unsteady Radial Flow in Confined Aquifer (Contd.)

- Radius of Influence

$$r_0 = 1.5\sqrt{\frac{Tt}{S}}$$

It violates Jacob Approximation.

Jacob Approximation is valid for $u < 0.01$

$$0.01 = \frac{r_J^2 S}{4Tt}$$

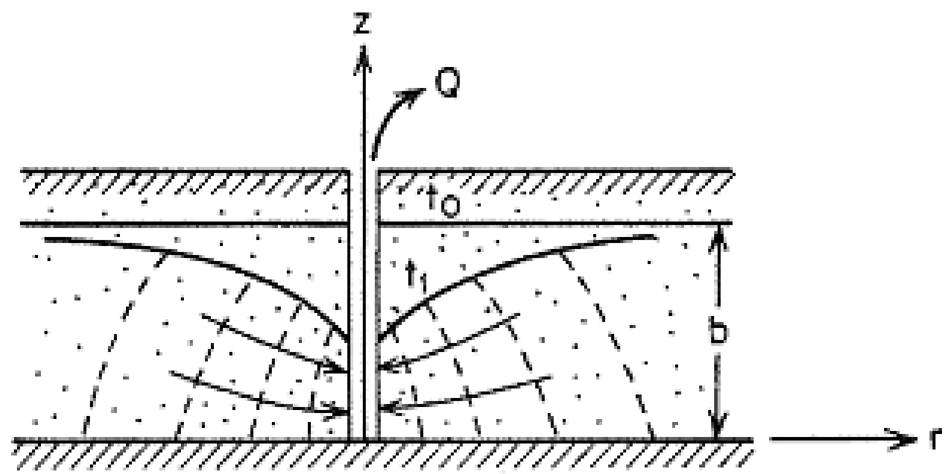
$$r_J = 0.2\sqrt{\frac{Tt}{S}}$$

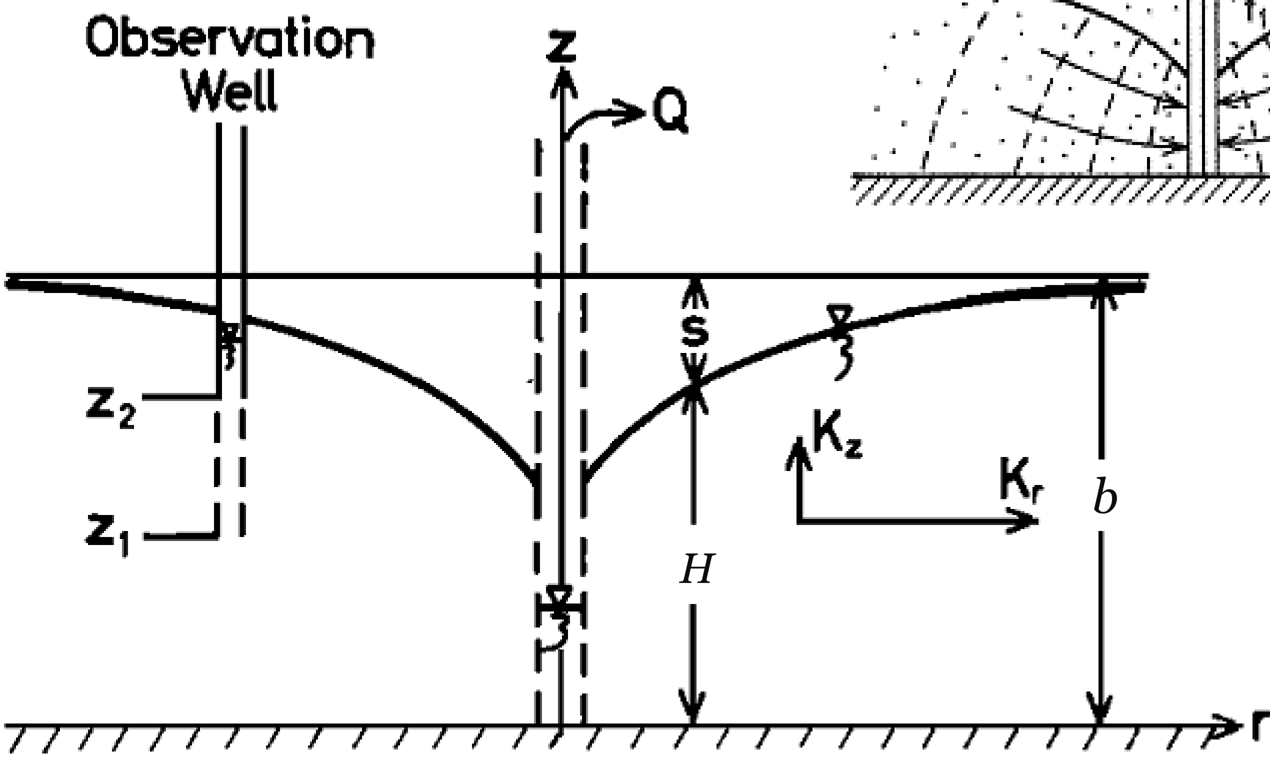
Jacob Approximation can be suitably applied for

$$r < r_J$$

Unsteady Radial Flow in Confined Aquifer

- Unconfined Aquifer





Unsteady Radial Flow in Confined Aquifer (Contd.)

$$S_s \frac{\partial h}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(K_r r \frac{\partial h}{\partial r}\right) + \cancelto{0}{\frac{1}{r}\frac{\partial}{\partial \theta}\left(\frac{K_\theta}{r}\frac{\partial h}{\partial \theta}\right)} + \frac{\partial}{\partial z}\left(K_z \frac{\partial h}{\partial z}\right) - \cancelto{0}{W}$$

or,

$$K_r \frac{\partial^2 h}{\partial r^2} + \frac{K_r}{r}\frac{\partial h}{\partial r} + K_z \frac{\partial^2 h}{\partial z^2} = S_s \frac{\partial h}{\partial t}$$

or,

$$\frac{\partial^2 s}{\partial r^2} + \frac{1}{r}\frac{\partial s}{\partial r} + \frac{K_z}{K_r}\frac{\partial^2 s}{\partial z^2} = \frac{S_s}{K_r}\frac{\partial s}{\partial t}$$

Subject to

Initial Condition (IC)

$$s(r, z, 0) = 0$$

Boundary Conditions (BCs)

$$s(r \to \infty, z, t) = 0$$

$$\frac{\partial s}{\partial z}(r, 0, t) = 0$$

$$\lim_{r \to 0} \int_0^b r \frac{\partial s}{\partial r} dz = \frac{Q}{2\pi K_r}$$

Unsteady Radial Flow in Confined Aquifer (Contd.)

- Two boundary conditions must be satisfied simultaneously on FS
  - continuity of energy which, assuming zero gauge pressure on FS

$$h(r,z,t) = H(r,t)$$

  - continuity of mass which, assuming that water and medium are incompressible

$$(\mathbf{q} - \mathbf{N}) \cdot \hat{\boldsymbol{n}} = S_y \mathbf{v_s} \cdot \hat{\boldsymbol{n}} = n_e \mathbf{v_s} \cdot \hat{\boldsymbol{n}} = n_D \mathbf{v_s} \cdot \hat{\boldsymbol{n}}$$

$$\frac{DF}{Dt} = \frac{D}{Dt}[z - H(r,t)\,]$$

$$\frac{DF}{Dt} = \frac{\partial}{\partial t}[z - H(r,t)\,] + \mathbf{v_s} \cdot \nabla[z - H(r,t)\,] = 0$$

If $\mathbf{N} = \mathbf{0}$, $\mathbf{q} = q_z \hat{\boldsymbol{e}}_z$

$$q_z = S_y \frac{\partial H}{\partial t} = -K_z \frac{\partial h}{\partial z}$$

or,

$$K_z \frac{\partial s}{\partial z} = -S_y \frac{\partial s}{\partial t} \; at \; z = H$$

Assumption $S_y \gg S$

Unsteady Radial Flow in Confined Aquifer (Contd.)

- Drawdown

$$s(r,t) = \frac{Q}{4\pi T} W(u_A, u_B, \beta)$$

with

$$\frac{1}{t_s} = u_A = \frac{r^2 S}{Tt}$$

$$\frac{1}{t_y} = u_B = \frac{r^2 S_y}{Tt}$$

$$\beta = \frac{K_z r^2}{K_r b^2}$$

Unsteady Radial Flow in Confined Aquifer (Contd.)

- Solution has two components
  - Transition from elastic storage release to drainage associated with vertical flow near water table

$$W(u_A, u_B, \beta) = W(u_A, \beta)$$

  - Transition from drainage associated with vertical flow to drainage associated with primarily horizontal flow when water table decline has slowed down sufficiently

$$W(u_A, u_B, \beta) = W(u_B, \beta)$$

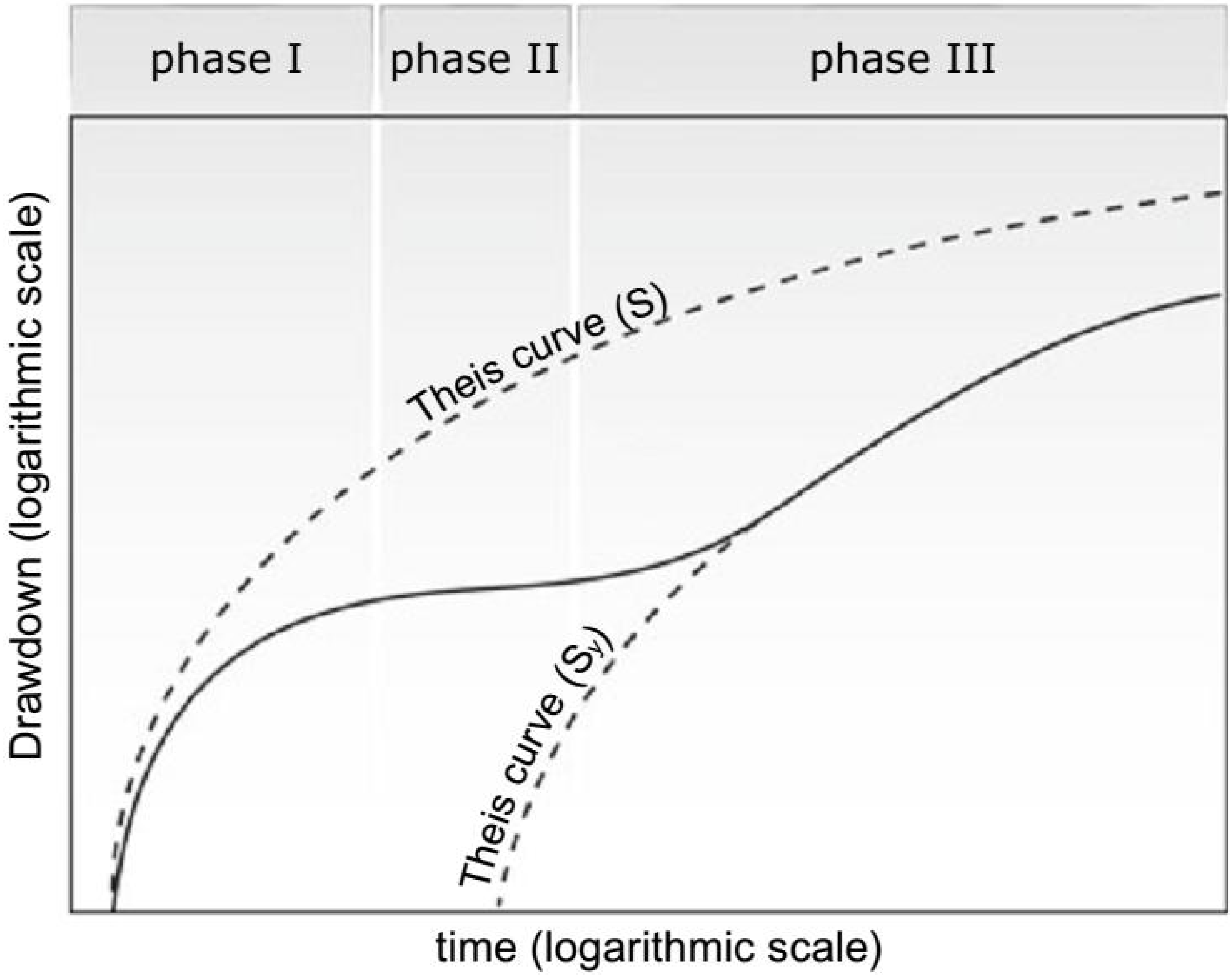
Mechanism:

(a) initially, the water supply derives from the elastic storage, S, of the aquifer, analogously to confined aquifers;

(b) a phase dominated by delayed gravity drainage follows, during which the time drawdown curve flattens out somewhat;

(c) once the effect of gravity drainage ends, the drawdown starts to increase again depending on the value of the specific yield, Sy.

Unsteady Radial Flow in Confined Aquifer (Contd.)



Unsteady Radial Flow in Confined Aquifer (Contd.)

$$s(r,z,t) = \frac{Q}{4\pi T} \cdot \int_0^\infty 4y J_0(y\beta^{\frac{1}{2}}) \cdot \left[\omega_0(y) + \sum_{n=1}^{\infty} \omega_n(y) \right] dy$$

where J_0 is the zero order Bessel function of the first kind,

$$\omega_0(y) = \frac{\left\{1 - \exp\left[-t_s \beta \left(y^2 - \gamma_0^2\right)\right]\right\} \cdot \cosh(\gamma_0 z_D)}{\left\{y^2 + (1+\sigma)\gamma_0^2 - \left[\frac{\left(y^2 - \gamma_0^2\right)^2}{\sigma}\right]\right\} \cdot \cosh(\gamma_0)},$$

$$\omega_n(y) = \frac{\left\{1 - \exp\left[-t_s \beta \left(y^2 + \gamma_n^2\right)\right]\right\} \cdot \cosh(\gamma_n z_D)}{\left\{y^2 + (1+\sigma)\gamma_n^2 - \left[\frac{\left(y^2 + \gamma_n^2\right)^2}{\sigma}\right]\right\} \cdot \cosh(\gamma_n)},$$

and the terms γ_0 and γ_n the solutions of the equations:

$$\sigma\gamma_0 \cdot \sinh(\gamma_0) - \left(y^2 - \gamma_0^2\right) \cdot \cosh(\gamma_0) = 0 \text{ with } \gamma_0^2 < y^2$$

Unsteady Radial Flow in Confined Aquifer (Contd.)

$$\sigma \gamma_n \cdot \sin(\gamma_n) + \left(y^2 + \gamma_n^2\right) \cdot \cos(\gamma_n) = 0,$$

where

$$(2n - 1) \cdot \frac{\pi}{2} < \gamma_n < n\pi, \qquad n \geq 1.$$

$$s(r,\ z,\ t) = \frac{Q}{4\pi T} \cdot s_D(\sigma,\ \beta,\ t_s,\ z_D)$$

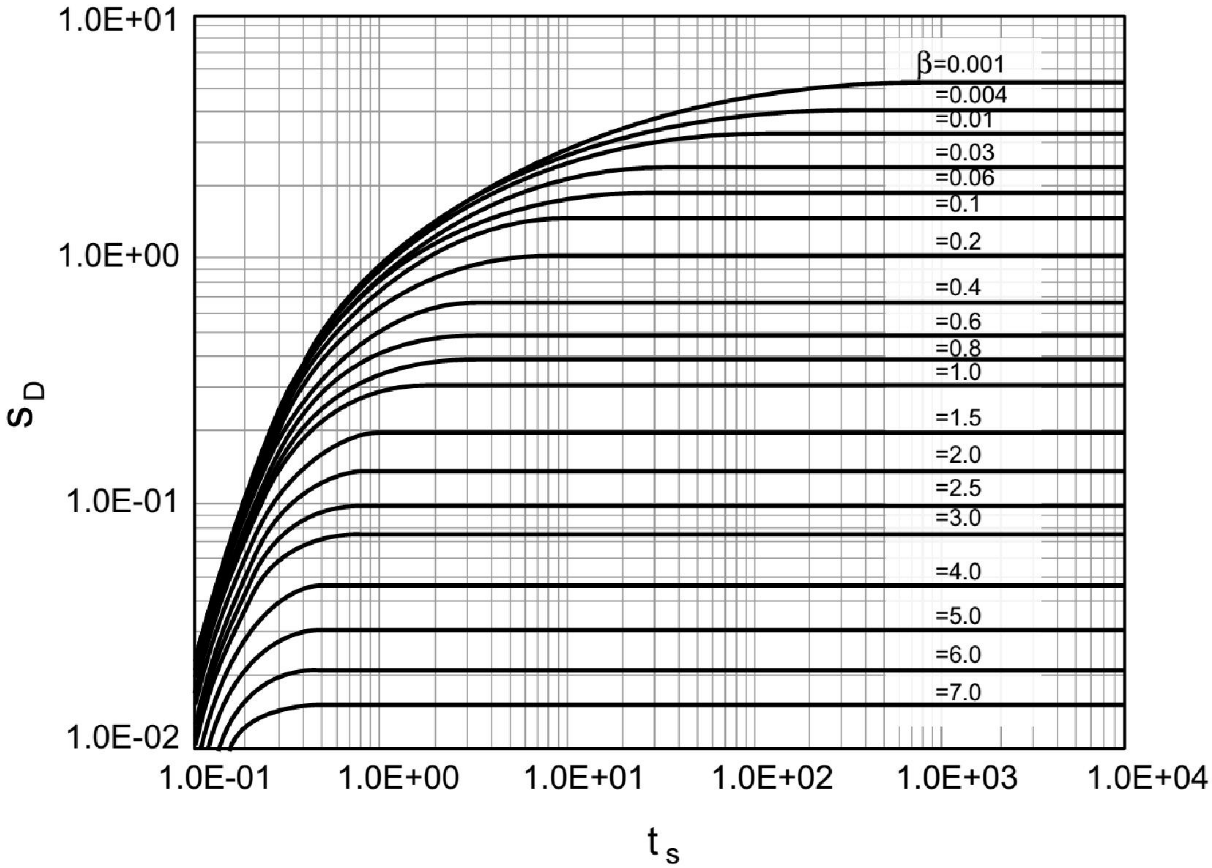
$$\sigma = \frac{S}{S_y} = \frac{t_y}{t_s} \qquad \beta = \frac{K_z}{K_r}\left(\frac{r}{b}\right)^2$$

$$t_D = t_s = \frac{T}{Sr^2} \cdot t \quad \text{for short times,}$$

$$t_D = t_y = \frac{T}{S_y r^2} \cdot t \quad \text{for long times.}$$

Unsteady Radial Flow in Confined Aquifer (Contd.)

- Type-A Curve $s_D = s_D\,(t_s,\ \beta)$ which is valid for short periods of time

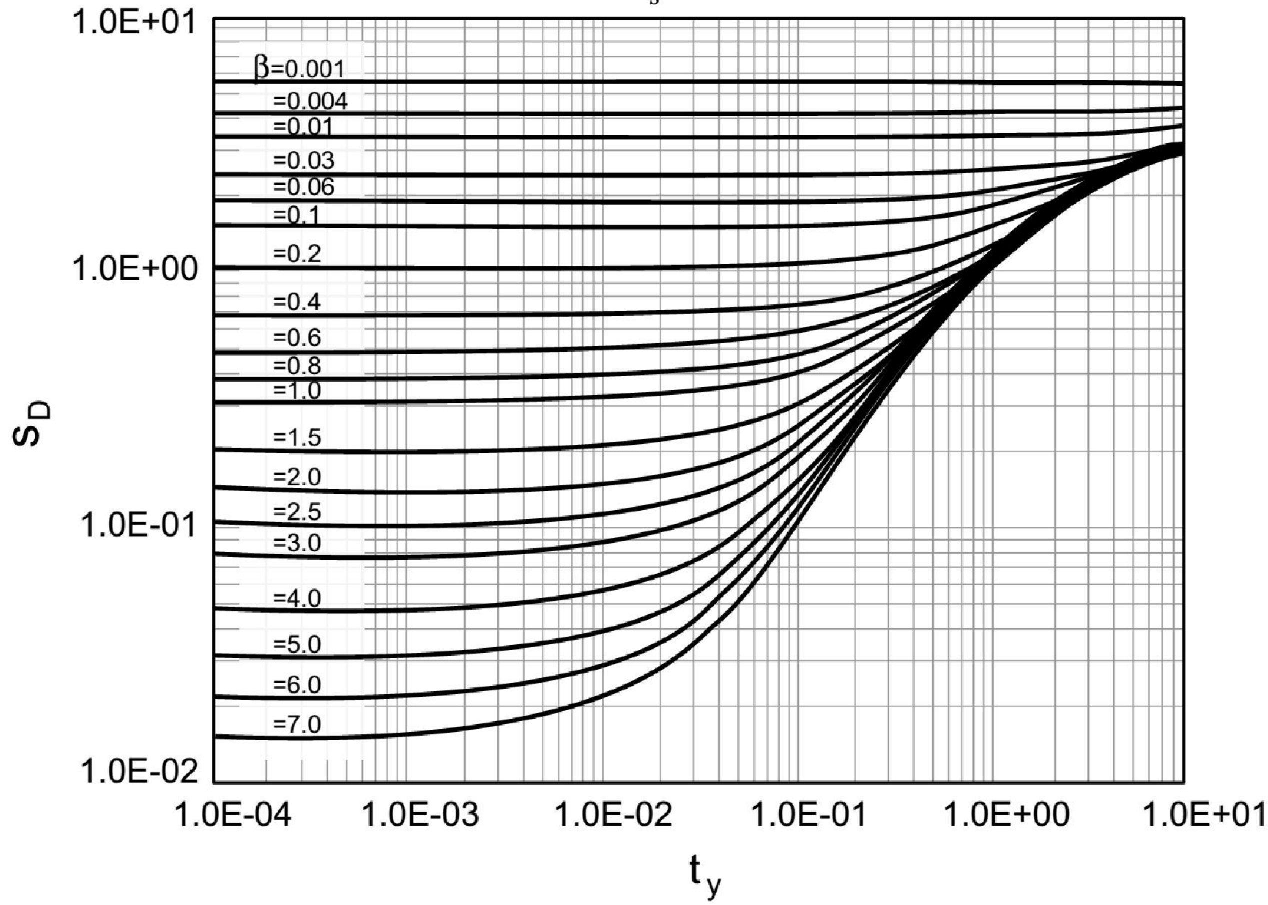


Neuman's dimensionless function $s_D\,(t_s,\ \beta)$ valid for short times

Unsteady Radial Flow in Confined Aquifer (Contd.)

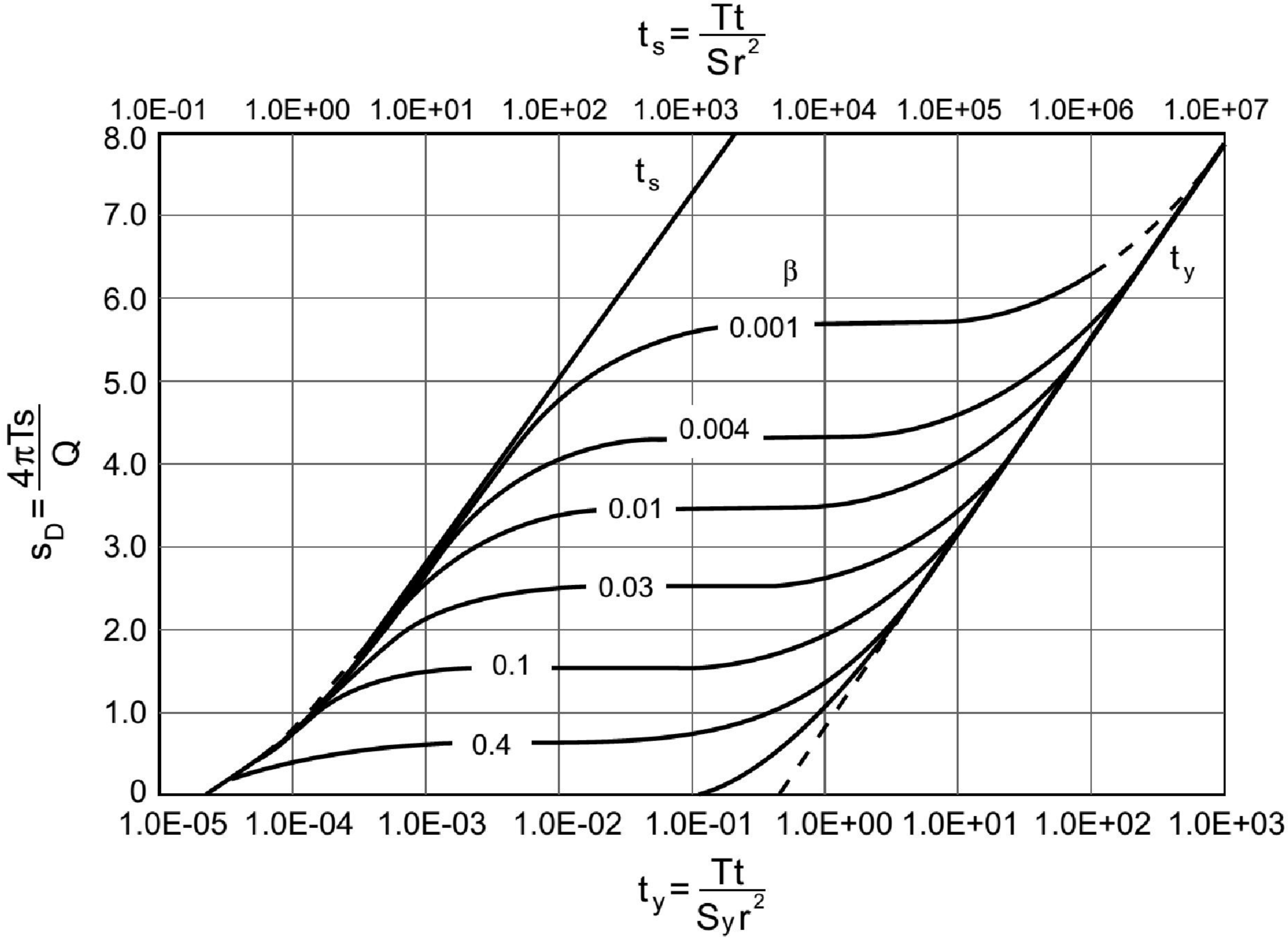
- Type-B Curve

$s_D = s_D(t_y, \ \beta)$ valid for longer periods of time and obtained by imposing $t_y = 10^{-9} t_s$.



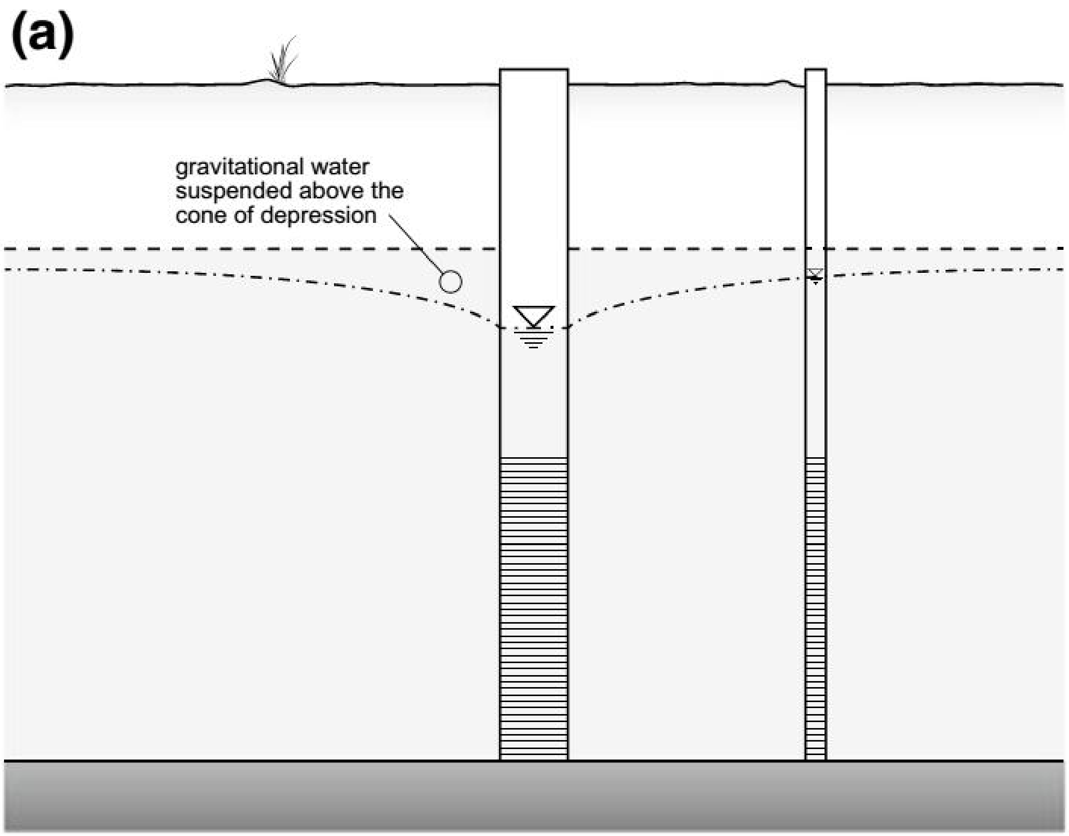
Neuman's dimensionless function $s_D(t_y, \ \beta)$ valid for extended periods of time

Unsteady Radial Flow in Confined Aquifer (Contd.)

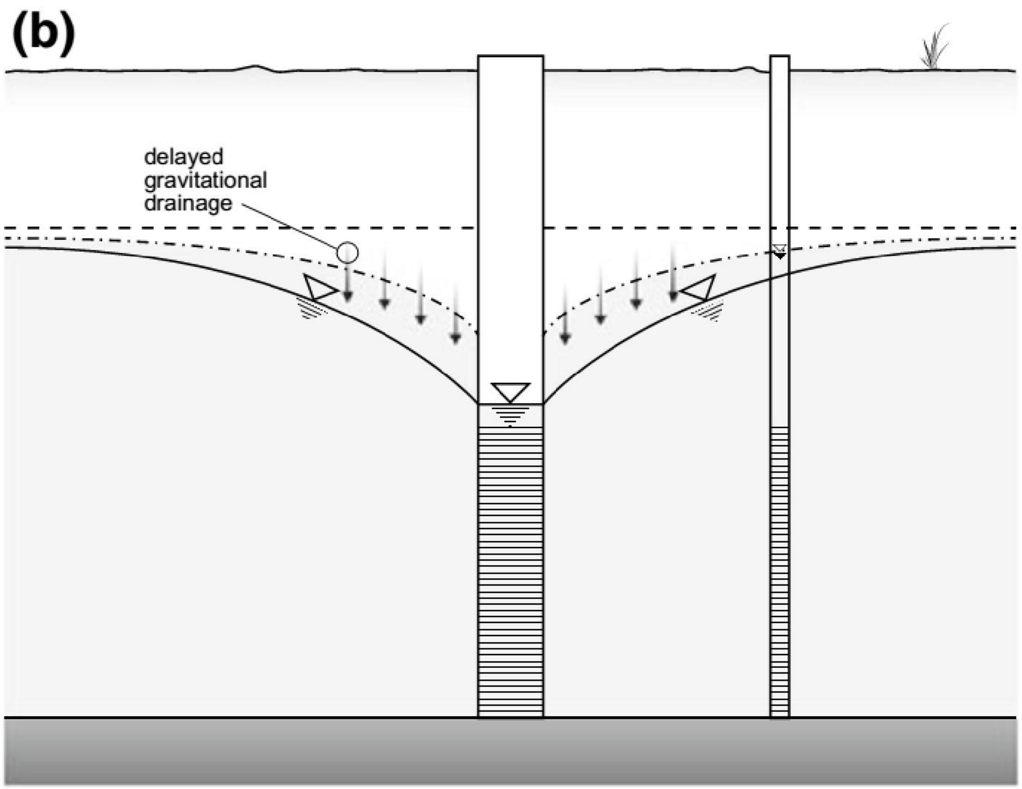


Neuman's dimensionless curves $s_D\left(t_s\ t_y,\ \beta\right)$ represented for $\sigma = 10^{-4}$

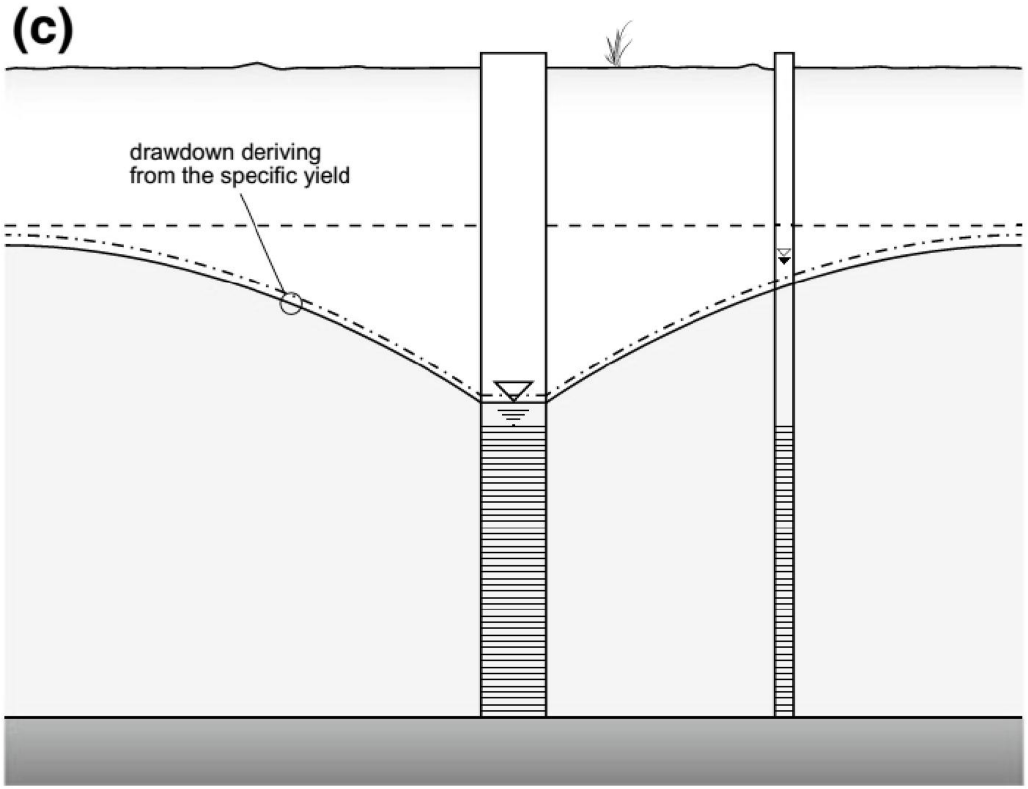
Unsteady Radial Flow in Confined Aquifer (Contd.)



Unsteady Radial Flow in Confined Aquifer (Contd.)



Unsteady Radial Flow in Confined Aquifer (Contd.)



Unsteady Radial Flow in Confined Aquifer (Contd.)

1. Match field data with Type-B curve and identify β
2. For a arbitrary point $\left(t_y^*, s_D^*\right)$ or (t^*, s^*), calculate

$$T = \frac{Q}{4\pi}\frac{s_D^*}{s^*}$$

$$S_y = \frac{Tt^*}{r^2 t_y^*}$$

3. Match field data with Type-A curve corresponding to β identified in Step 1
4. For a arbitrary point (t_s^+, s_D^+) or (t^+, s^+), calculate

$$S = \frac{Tt^+}{r^2 t_s^+}$$

5. Calculate horizontal hydraulic conductivity

$$K_r = \frac{T}{b}$$

6. Calculate vertical hydraulic conductivity

$$K_z = \beta K_r \left(\frac{b}{r}\right)^2$$

7. Calculate σ

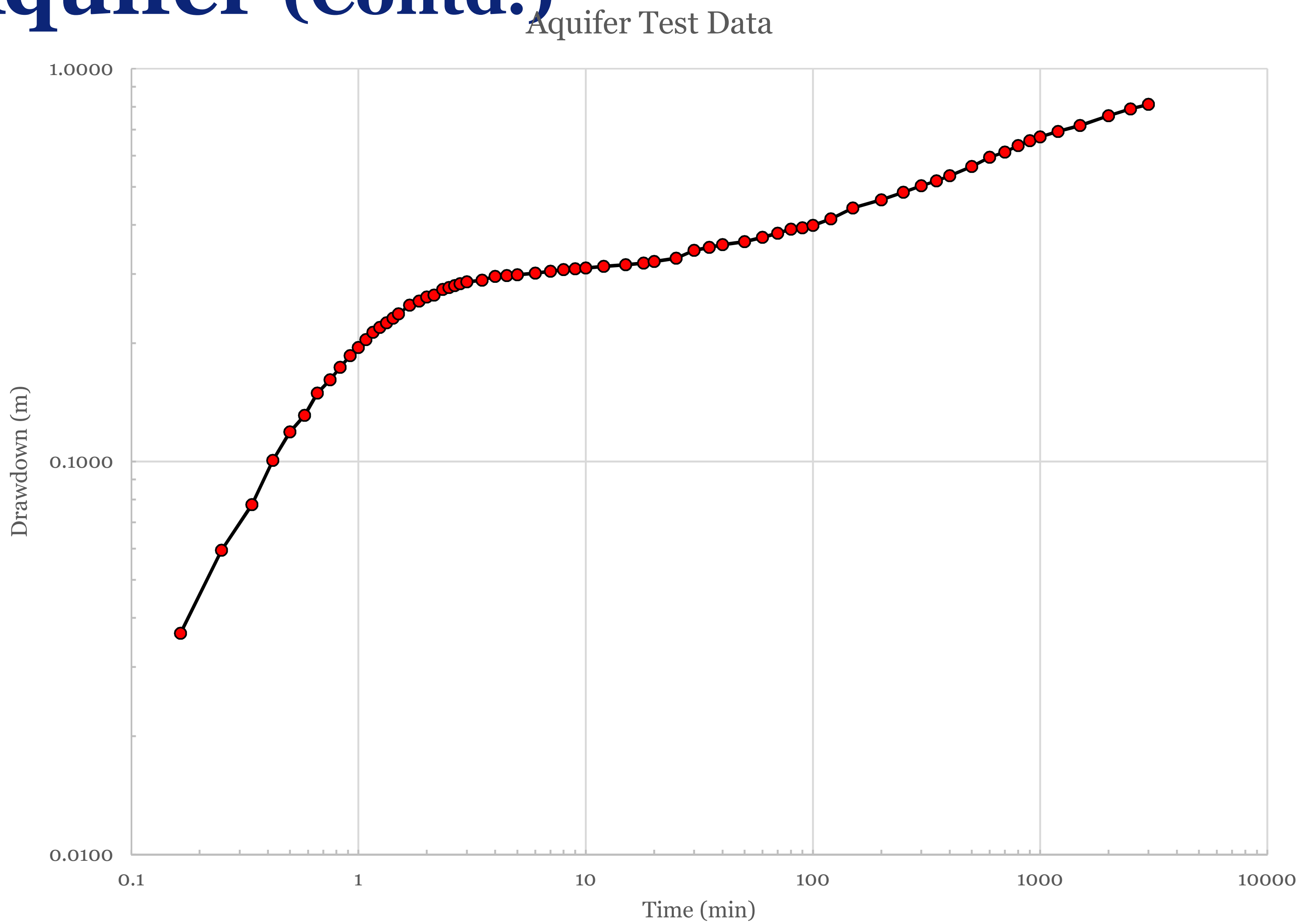
$$\sigma = \frac{S}{S_y}$$

Unsteady Radial Flow in Confined Aquifer (Contd.)

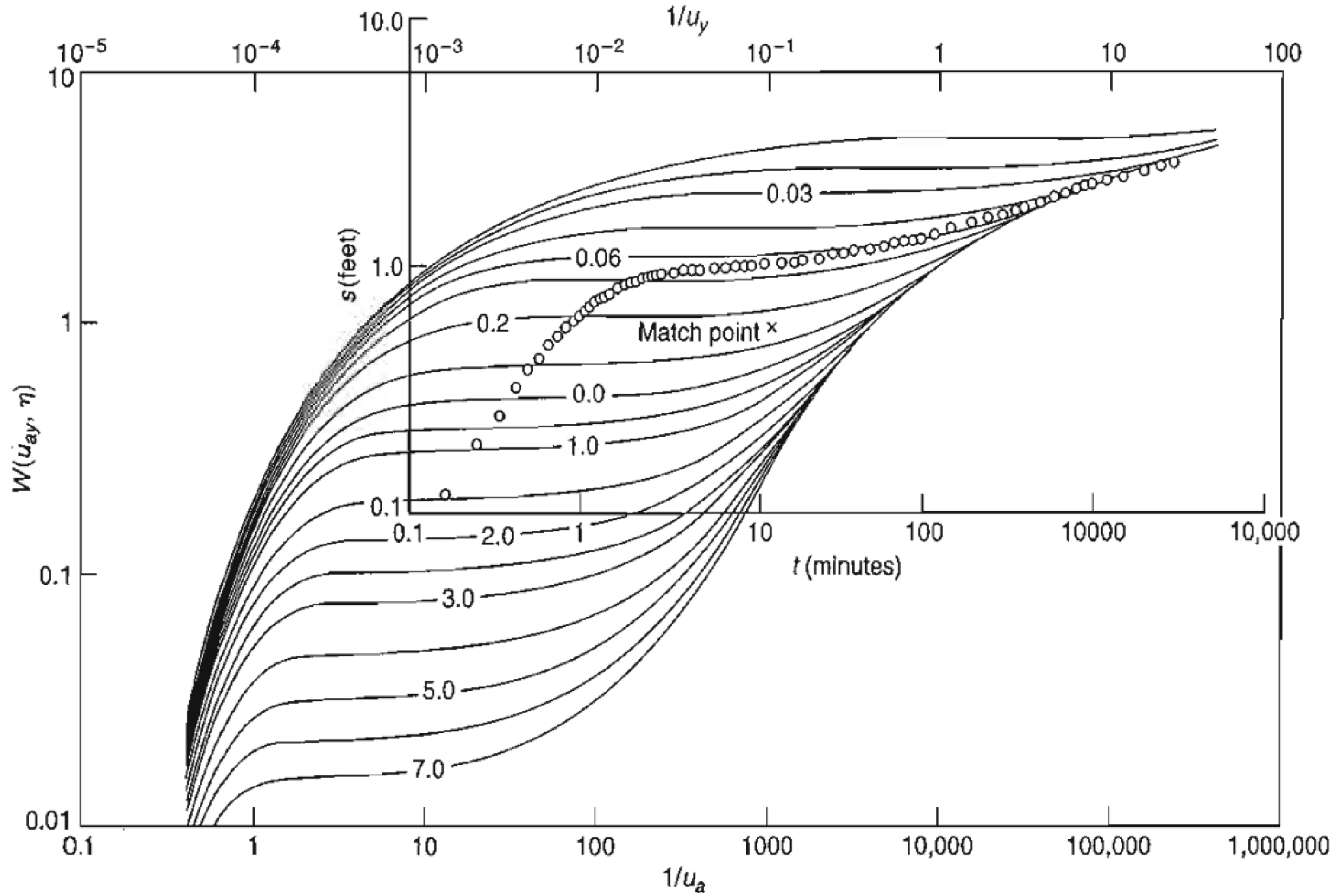
- Q= 4.1 m^3/min

t	s	t	s	t	s
min	m	min	m	min	m
0.165	0.037	2.65	0.280	80	0.390
0.25	0.059	2.8	0.283	90	0.393
0.34	0.078	3	0.287	100	0.399
0.42	0.101	3.5	0.290	120	0.415
0.5	0.119	4	0.296	150	0.442
0.58	0.131	4.5	0.297	200	0.463
0.66	0.149	5	0.299	250	0.485
0.75	0.162	6	0.302	300	0.503
0.83	0.174	7	0.305	350	0.518
0.92	0.186	8	0.308	400	0.533
1	0.195	9	0.309	500	0.564
1.08	0.204	10	0.311	600	0.594
1.16	0.213	12	0.314	700	0.613
1.24	0.219	15	0.317	800	0.637
1.33	0.226	18	0.320	900	0.655
1.42	0.232	20	0.323	1000	0.671
1.5	0.238	25	0.329	1200	0.692
1.68	0.250	30	0.344	1500	0.716
1.85	0.256	35	0.351	2000	0.759
2	0.262	40	0.357	2500	0.789
2.15	0.265	50	0.363	3000	0.811
2.35	0.274	60	0.372		
2.5	0.277	70	0.381		

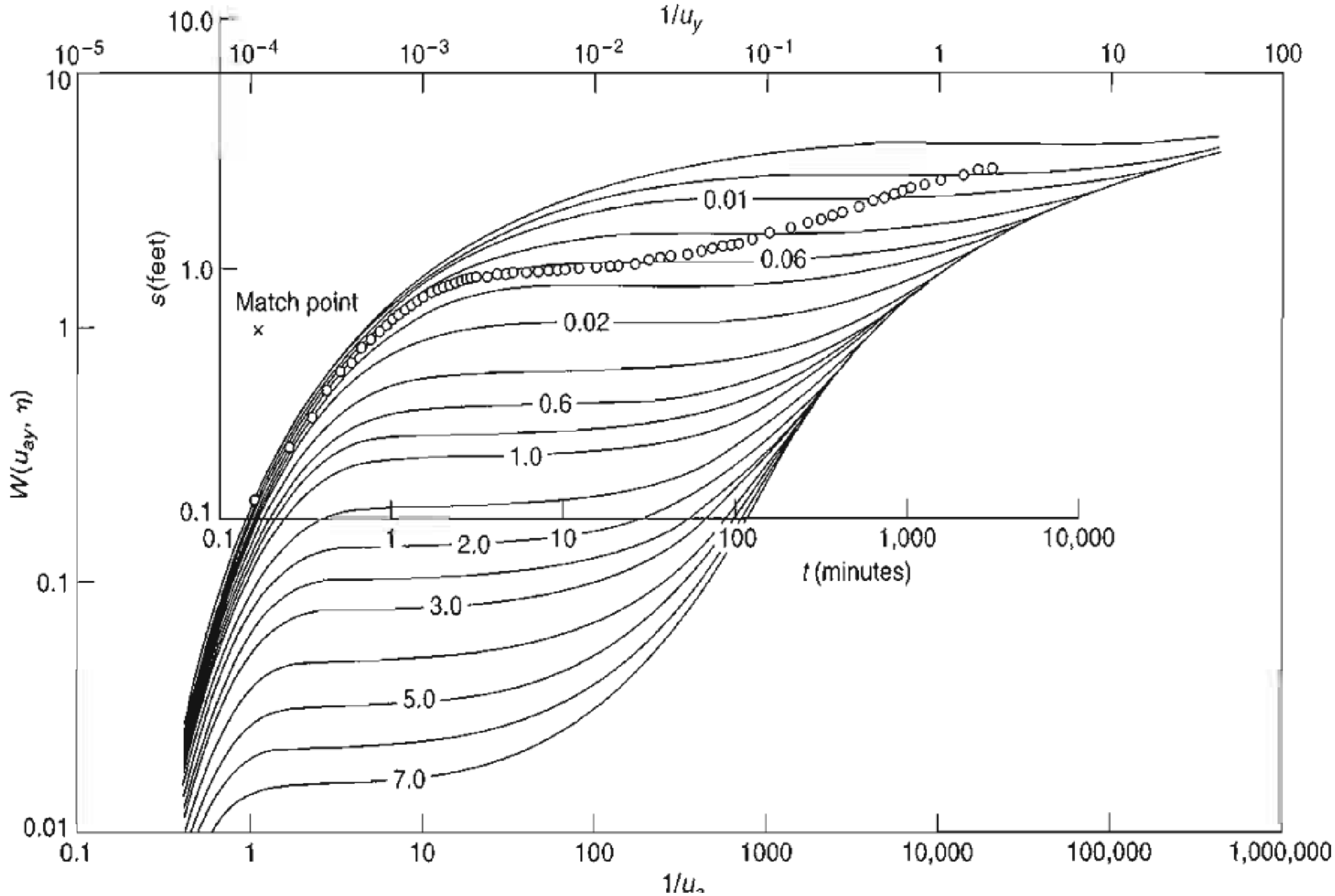
Unsteady Radial Flow in Confined Aquifer (Contd.)



Unsteady Radial Flow in Confined Aquifer (Contd.)



Unsteady Radial Flow in Confined Aquifer (Contd.)



General Comments

Recommended drawdown measurement frequency in the observation wells in confined and unconfined aquifers

Time from test start (min)	Measurement interval (confined aquifers) (min)	Measurement interval (unconfined aquifers) (min)
0–2	0.5	0.25
2–5	1	0.5
5–15	5	1
15–60	5	5
60–120	15	10
120–240	30	30
240–360	60	30
360–720	60	60
720–2880	60	180
>2880	60	480

Thank you