(*) Rate of decrease in velocity is linear with time

$$H_R - C_1\frac{v^2}{2g} = \frac{x}{g}\cdot\frac{dv}{dt}$$

$$v_0 = \sqrt{\frac{2gH_R}{C_1}}$$

$$C_1 = \left(1 + K_e + K_v + \frac{fL}{D}\right)$$

$$t = \frac{1}{C_1 v_0}\ln\left(\frac{v_0+v}{v_0-v}\right)$$

$$a = \frac{L}{g}\cdot\frac{dv}{dt}$$

$$= \frac{1829}{9.81}\times\frac{(-2.41)}{20}$$

$$= -11.23\ m/s^2$$

$$\frac{2L}{c} = \frac{2\times914.4}{1250} = 1.46 < 20\ sw$$ (slow valve closure)

$$\frac{L}{g}\frac{dv}{dt} = H_R - (1+K_e+K_v)\frac{v^2}{2g}$$

2nd Week:-

Pipeline design ⇒ Pipeline sizing,
Equipment sizing
" location

System operation ⇒ Pipeline system → startup and shutdown
flowrate change
emergency shutdown
equipment failures

Transient state:-

Unsteady condition changes with time between two steady state.

Positive surge/upsurge ⇒ Pipeline pressure increase from steady state pressure.

☑ Steady state ⇒ Pressure and flowrates are time independent.

Steady state cannot address:-

(i) Positive and negative surges

(ii) Pump tripping

(iii) Column seperation

Motors or drivers turned of purposely (Pump shutdown) or unplanned (loss of power)
It leads to waterhammers. Pump speed decay after trip.

Water column seperation:-

Breaking of liquid column in fully filled pipelines.

When pressure in a pipeline drops to the vapour pressure at specific locations such as closed ends, high points, or kness (change in pipe slope).

Two liquid column seperated by vapour cavity

↓

Collision of two liquid column or liquid column with close end cause large and instanteneous change in pressure.

↓

Severe load for hydraulic machinery.

Check valve slam:- (জোরে সজোরে বন্ধ হওয়া)

Check valve ⇒ Automatically open to forward flow and closed to reverse flow.

Occurs when reverse flow passes through the check valve before it is fully closed.

☑ When reverse flow stopped instanteneously, by the closed check valve → Causes loud shock-wave and water hammer in the pipe.

Check valve should be closed immediately or over several second or minutes (depending upon length of pipe).

Why transient analysis required:-

(i) Manage transient related problems ⇒ Pump trip, check valve slam.

(ii) changes in valve settings
Power failures
Pump startup and stopping.
Air entrainment.

Steady-state design of water pipeline systems
- → Larger pipe diameter.
- → high capacity pump requirement.
- → Incorrect booster pump location.

Steady state based on peak load
transient state → Average load.

☑ Transient model predicts →
- change Supply flow rates
- Supply pressure
- Delivery demand.

Models pump failure, pipe rupture, stoppage of flow (corrective strategies)

Trainsent model → More complex

Governing laws

depending variables ⇒ Q, P

Independent variables ⇒ x, t

a = wave velocity

Continuity equation:- $\frac{\partial P}{\partial t} + \rho a^2 \frac{\partial V}{\partial x} = 0$

Momentum " :- $\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x} + \frac{1}{\rho}\frac{\partial P}{\partial x} + g\frac{\partial h}{\partial x} + \frac{fV|V|}{2D} = 0$

- $\frac{\partial V}{\partial t}$ → Force due to acceleration
- $V\frac{\partial V}{\partial x}$ → Force due to convective acceleration / KE
- $\frac{1}{\rho}\frac{\partial P}{\partial x}$ → Force due to pressure difference
- $g\frac{\partial h}{\partial x}$ → Gravitational force
- $\frac{fV|V|}{2D}$ → Frictional force

+ = Inertia force

Non linear ⇒ when (i) Dependent variables and its derivatives are not of degree 1

(ii) coefficient of a term depends upon dependent variable.

Eg. $\frac{d^3y}{dx^3} + \left(\frac{dy}{dx}\right)^4 + 6y = 3$

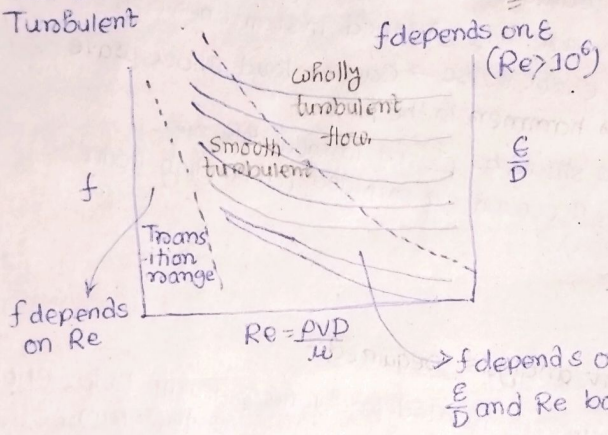
$y\frac{dy}{dx} + 2xy = 2$

$Re = \frac{\rho V D}{\mu}$

Laminar

Transition $2000 < Re < 400$

Turbulent



Moody's Diagram

Solution methods:-

Explicit method: Explicit FD converges when distance and time steps are small.

Methods subjected to CFL (Courant Friedrick law) stability criteria.

Implicit method: Values calculated from previous time step and BC's at current time.

→ values at current and previous time and BC's at previous current time

→ Flexible with time step and a inherently stable.

Method of characteristics:-

Continuity and momentum equations $\xrightarrow{\text{Converted to}}$ 4 DE's

↓

Solved explicitly for pressure and flowrates

↓

Constrained time step:-

$\frac{\Delta x}{\Delta t} \geq a$

Courran number $= \frac{a\Delta t}{\Delta x}$

BC and IC:- (i) pressure and pressure BC

(ii) Pressure and flow Bes

Ie:- (i) A steady state
(ii) Transient state available from a previous simulation run.

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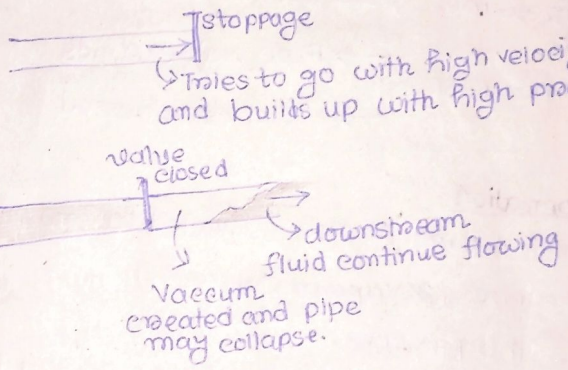
Causes of transients:- Rapid change in pipe velocity.

(1) Change of valve setting
(2) Startup or setting down of pump.
(3) Change in pump speed or head
(4) Pipeline rupture, column seperation, trapped air.
(5) Impeller vibration in radial pump.

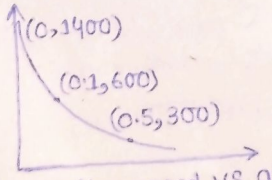
Properties:
- Pressure wave propagates → Sound velocity of the fluid and velocity of fluid in a pipe.
- Magnitude attenuates ⇒ Pressure wave moves away from the source of the transient.
- Entrained air ⇒ alters acoustic velocity.

Water hammer:-



$$\text{Acoustic speed} = \frac{\sqrt{K/\rho}}{\sqrt{1 + \frac{KD}{Ee}(1-\nu^2)}}$$

ν = Poison's ratio = 0.3
K = Bulk modulus of fluid
D = Inside diameter of the pipe.



Acoustic speed vs air content.

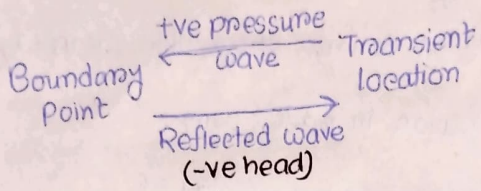
Potential surge:-

Initial pressure increase following flow stoppage is referred to as the potential surge.

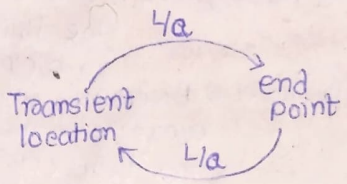
$\Delta P = -\rho a V$, V = change in velocity

$$\Delta H = \frac{\Delta P}{\rho g} = -\frac{\rho a V}{\rho g} = -\frac{aV}{g}$$

$$\Delta H = -\frac{aV}{g}$$



☑ Critical period $:= \frac{2L}{a}$



Attenuation of Surge:-

Wave travels → Magnitude decreases

At high pressure, fluid stores occurs → called 'line packing'

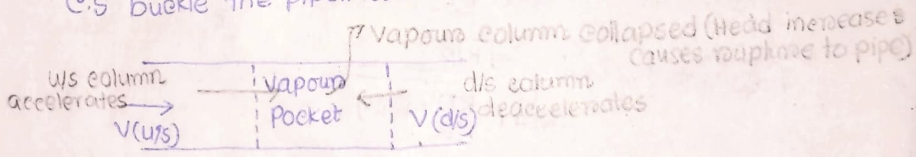
Pressure increases → Pipe wall expands
→ Fluid compressed.

Column seperation

Occurs when part of pipe subjected to low pressure

Liquid vapourizes ($P < P_{vapour}$ pressure)

C.S buckle the pipelines.



Head increase results from collapse

$$\Delta H = \frac{a(V_{u/s} - V_{d/s})}{g}$$

☑ consequences of transition:

Unstable pressure
Column seperation
(Check valve slam on
control valve oscillation)
Resonance in pipe system
Pipeline shutdown due to pipe rupture

☑ Surge control
Extra pipeline protection.

Transient control

[✓] Magnitude of severe surge → to be determined

Pipeline ⇒ Reinforcing, proper joints, anchoring.

Computer simulation ⇒ Reliable and responsible control system

Continuous pounding of small surges ⇒ Leaks in pipe.

Knee points avoided, thicker pipes, Surge control device.

Operational phase:-

✓ Gradual open and close.

✓ Fixed speed drives for automatic control valves

✓ Variable speed drives for speed up and ramp down the pumps in controlled way.

Wide range of input speed, ~~input~~, but a constant speed output

Surge control device:

(1) Pressure relief valve

(2) Pressurized surge tank

(3) Rupture disk

(4) Control valves.

Check valve:-

(?) (i) Large transient pressure → if flow reversal occurs before complete valve closure.

(ii) Modern C.V closes gradually $> \frac{2L}{a}$. Some stops at the instant when forward flow stops.

Pump startup:-

Positive surge in downstream line. (created)

Magnitude of surge depends on sudden increase in speed. If there is not entrapped air, pressure increase will not be too large.

Pump power failure:-

(i) Severe transient upon power failure — when Static lift is large. Pipeline profile rises rapidly immediately d/s.

(ii) downstream — Power off — Pressure drop — Column seperation — Cavity collapse — Shock of large magnitude — Significant overpressure near the pump.

Pressure relief valve: (PRV):

Protect pipeline system from excessive pressure.

Valve closes when pressure decreases than the set values.

Controls upsurge not, downsurge

Useful for short and steep pump profile — flow reversal — Power failure. and pump trip.

Limitations:-

Up/down surge → Travels at aquostic speed - PRV may not open that quickly therefore ineffective

PRV → undersized - Not carry-di able to discharge at high flowrate.

☑ Pressurized surge tank
(Air vessel/ Accumulator/ Hydrophore tank)

Contains gas that absorb pressure surge

During pump shutdown A.V provides water to the pipeline to minimize -ve pressure.

Rupture disk:-

Brusting membrane, high pressure relief
Needs storage tank to accept relief flow.

Transient control pump startup:

Several pumps - Start time intervals $> \frac{2L}{a}$ (Atleast)

Slow open of control valve $(t > \frac{2L}{a})$

* Variable speed drive for pump - get full speed slowly - Avoids high surge. $(\because \Delta H = -\frac{a\Delta V}{g})$
* Control valve → close to open slowly → To get the full capacity of pump.

Just deceleration for pump shutdown. other conditions are same.

Transient control after power failure:

Power failure at pumping station causes pump tripping.
Effect reduced by increasing inertia of pump and motor using fly wheel.

Week-2:-

Unsteady flow governing Equations

Steady Oscillatory flow:-
Flow condition changes with time but repeats after fixed time interval.

Column seperation:-
Pressure < Vapour pressure
cavities formed.

Pressure Surges:-

By Rapid change in flowrates:- → Most severe condition

(i) Valve open/close (sudden)
(ii) Pump (Startup/ stop/ Power failure)
(iii) check valve (Slamming)
(iv) Presence of air.

Pump stops / valve closes suddenly:-

U/S → Pressure head rise

downstream

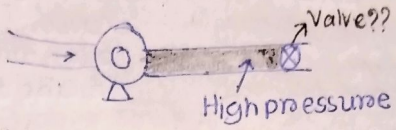
Pressure drop

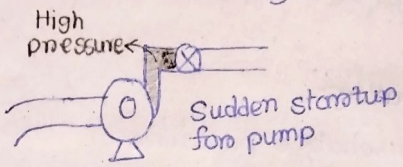
Column seperation.

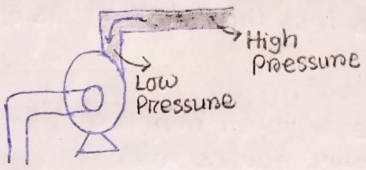
Flow reverses and water column rejoins.

Vapour pocket collapses.

Sudden closure of valve:







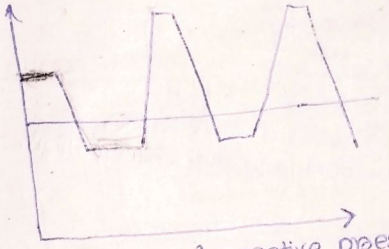
Pump trip ⇒ -ve pressure at d/s.

↓

cavity forms

↓

Flow reversal, vapour cavity collapse resulting high pressure.



Positive and negative pressure caused by a pump trip.

Dynamic cavitation:-

Cavitation in pumps:-

Low pressure zone → Cavity forms → Collapes and creates small bubbles

↓

creates high temperature spots, emits shock wave creating noise.

(when high speed of impeller, Low pressure zone is created around it (water vapourizes and bubble collapses. creates local shock wave, damages the blades)

Transient Cavitation in pumps:-

(i) Two types:-

(i) Suction cavitation:-

Low pressure at the suction side → creates vapour cavities → Duto to high pressure, vapour cavities implode on the face of the impeller. → Large chunks of material removed from the face of impeller causes premature failure of the pump.

↳ vapour carried to the discharge side.

(ii) Discharge cavitation:-

When very high discharge pressure running at less less than 10% of its efficiency

High discharge
↓
small clearance between impeller and pump housing causes very high velocity ⇒ Low pressure zone/vacuum created
↓
Premature wear of impeller blades.

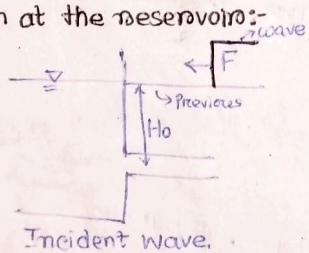
Cavitation inside pipes:-

Erosion
Fatigue
Thinner pipe walls
Pipe break
Pipe collapse.

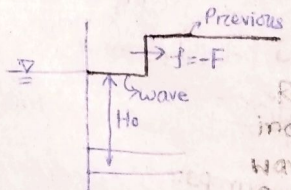
Transient

High points in the system is at more risk, because susceptibility of water column seperation.

Reflection at the reservoir:-



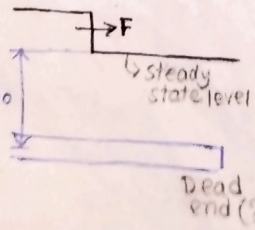
Incident wave.

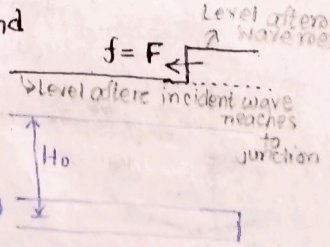


Reflected wave.

Reservoir কোনো incoming pressure wave কে value বসিয়ে দিয়ে Reflect করে।

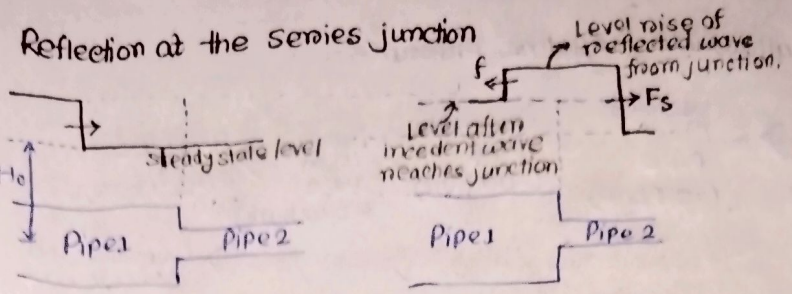
Reflection at the dead end





কিন্তু কোনো Dead end এসেই value কে সমপরিমাণে বাড়িয়ে দেয়।

Reflection at the series junction



Device used to reduce water hammer:-

Fly wheel:-

Increase rotating inertia of the pump ⇒ Increases energy requirement

! (So, pump may close or open slowly. Negative pressure generates slowly).

Vaccum breaker:

Prevents column seperation. (Negative pressure generation when fully open, large volume of air admitted into the pipe.

Surge relief valve:-

High pressure released to atmosphere.

Ineffective against low pressure.

Variable frequency drive (VFD):

Controls pump startup and shutdown

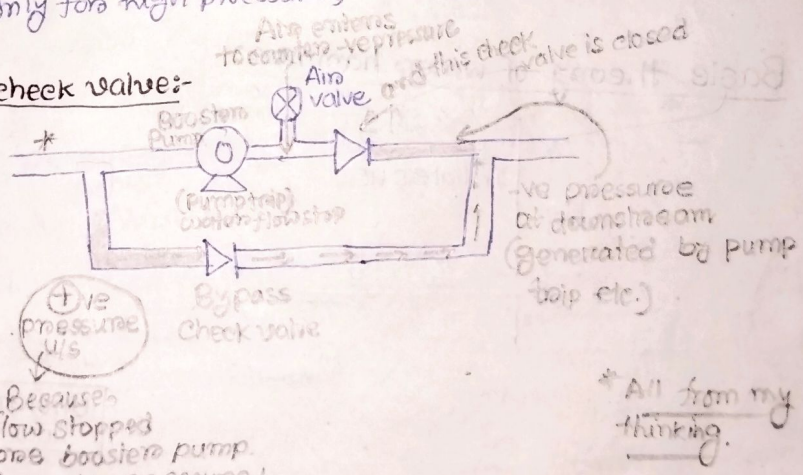
Ineffective during power failure.

Standpipe:-

Need to be higher than HGL, Tall structure.

Only for high pressure, not columnseperation.

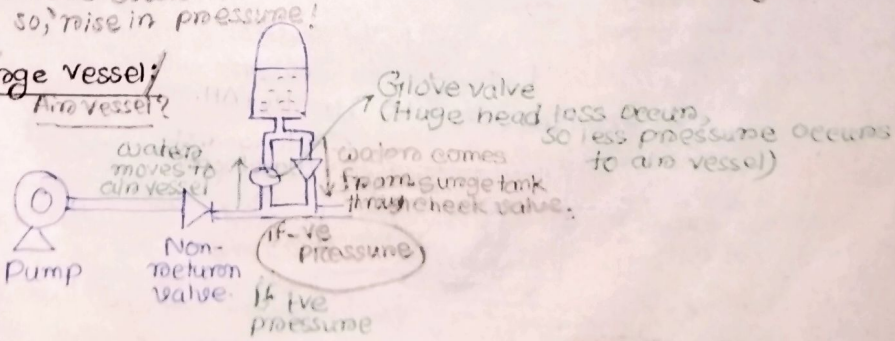
Bypass check valve:-



* All from my thinking.

Surge vessel:

Air vessel?



Sequence of allowable negative pressure:-

HDPE (High density Polyethelene Pipe) (-7 m water head) (-0.7 bars) = Welded steel pipe (-0.7 bars) = Concrete pipe (-0.7 bars) > Ductile iron pipe (-0.42 bars) > PVC Pipe (-0.21 bars) > Prestressed concrete bars (0 bars)

☑ Speed of pressurewave:-

Max: Ductile iron pipe = 1372 m/s

PVC pipe = 213 m/s.

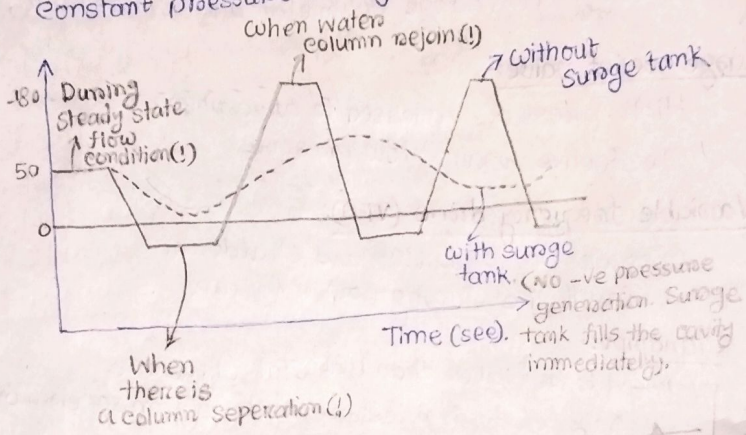
Surge tank design criteria:-

Surge tank volume

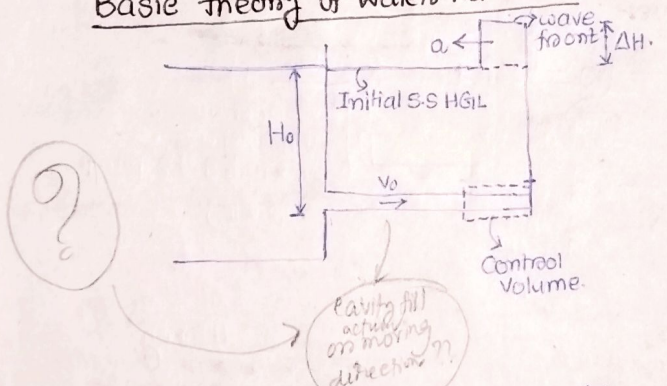
Outlet diameter

Design pressure

constant pressure during steady state.

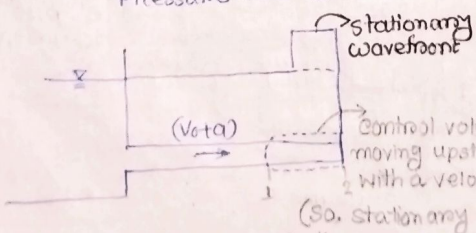


Basic theory of water hammer:



Unsteady flow

Velocity	V_o	$V_o+\Delta V$
Density	ρ_o	$\rho_o+\Delta\rho$
Pressure head	H_o	$H_o+\Delta H$.



(So, stationary observer on the control volume sees the water is moving with velocity V_o+a)

Velocity	V_o+a	$V_o+\Delta V+a$
Density	ρ_o	$\rho_o+\Delta\rho$
Pressure head	H_o	$H_o+\Delta H$.

ΔH for Water hammer: (Uses momentum eqⁿ, $\frac{dM}{dt} = F$)

The time rate of change of momentum in +ve x direction ($\rho Q V$)

$= F_2 - F_1 = \frac{dM_2}{dt} - \frac{dM_1}{dt}$

$\frac{\rho Q V}{\text{mass} \times \text{velocity}}$

Considering incompressible fluid, $\Delta\rho = 0$

$M_1 = \rho_0 A(a+v_0)(a+v_0)$

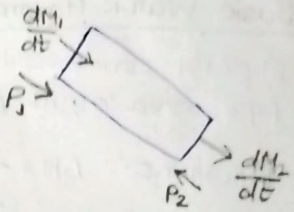
$= \rho_0 A(v_0+a)\{(v_0+\Delta v+a) - (v_0+a)\}$

$= \rho_0 A \Delta v (v_0+a)$

$\frac{dM_2}{dt} = \rho_2 Q_2 V_2$
$= \rho_2 A_2 V_2 \cdot V_2$
$= \rho_2 (A_1 V_1) V_2$ → Continuity equation
$= (\rho_0 + \Delta\rho) A (v_0+a)(v_0+\Delta v+a)$

$\frac{dM_1}{dt} = \rho_1 Q_1 v_1$
$= \rho_1 A_1 v_1 v_1$
$= \rho_0 A (v_0+a)(v_0+a)$

Negative friction resultant pressure force acting on the control volume in x direction



$= P_1 - P_2$ (P_1 in +ve x).

$= \rho_1 g H_1 A_1 - \rho_2 g H_2 A_2$

$= \rho_0 g H_0 A - \rho_0 g (H_0 + \Delta H) A$

$= -\rho_0 g \Delta H A$

[ρ_0, A constant. So, incompressible fluid, rigid pipe)

∴ Rate of change of momentum (in x dir) = Applied force (in x dir)

$\Rightarrow \rho_0 A \Delta v (v_0+a) = -\rho_0 g \Delta H A$

$\Rightarrow \Delta H = -\frac{\Delta v (v_0+a)}{g}$ Now, $v_0 \ll a$

$\therefore \Delta H = -\frac{a \Delta v}{g}$

Here, Δv is negative. So, increase of pressure head at section ② is due to decrease of velocity

Derivation of pressure wave velocity:- (Uses continuity equation):-

$\rho_0 (v_0+a) A = (\rho_0 + \Delta\rho)(v_0 + \Delta v + a) A$

$\Rightarrow \rho_0 (v_0+a) A = \rho_0 (v_0 + \Delta v + a) A + \Delta\rho (v_0 + \Delta v + a) A$

$\Rightarrow \rho_0 \Delta v A + \Delta\rho (v_0 + \Delta v + a) A = 0$ [Here, $v_0 + \Delta v \ll a$]

$\Rightarrow \rho_0 \Delta v A + \Delta\rho a A = 0$

$\Rightarrow \Delta v = -\frac{\Delta\rho}{\rho_0} a$ .....(i)

$K = \frac{\Delta P}{-\left(\frac{\Delta V}{V}\right)}$ (Because, if pressure increases, volume decrease, but K itself a +ve quantity)

Now, Bulk modulus of elasticity,

$K = \frac{\Delta P}{-\left(\frac{\Delta V}{V}\right)}$

$\Rightarrow K = \frac{\Delta P}{+\frac{\Delta\rho}{\rho_0}}$

$\Rightarrow +\frac{\Delta\rho}{\rho_0} = \frac{\Delta P}{K}$ ..... (ii)

$PV = \text{constant}$

$\Rightarrow P\Delta V + V\Delta P = 0$

$\Rightarrow \frac{P}{\Delta P} = -\frac{V}{\Delta V}$. (P, V is the initial density and volume, here ρ_0, V_0)

V = Volume
v = Velocity.

Putting the value in eqⁿ (i),

$\Delta v = -\frac{\Delta P}{K} a$

$\Rightarrow a = -\frac{K \Delta v}{\Delta P} = -\frac{K\left(-\frac{g\Delta H}{a}\right)}{\rho g \Delta H} \Rightarrow a^2 = \frac{K}{\rho} \Rightarrow a = \sqrt{\frac{K}{\rho}}$ ✓

Reynold's transport theorem:-

$$\frac{dB_{sys}}{dt} = \frac{d}{dt}\int_{v} \beta \rho dv + (\beta \rho A V_s)_{out} - (\beta \rho A V_s)_{in}$$

B = Extensive property

β = Intensive property

v = Total volume

For fixed control volume, $V_s = V$

If CV is moving with 'ω', $V_s = V - \omega$.

Basic Water Hammer equation:

Slightly compressible fluid.

Pipe area rigid. $\Delta A = 0$

Equations:- $\Delta H = -\frac{a \Delta V}{g}$

and $a = \sqrt{\frac{K}{\rho_0}}$

Eg:-

$d = 0.5m$

$Q = 0.4 m^3/s$

Rigid pipe walls.

$\rho = 900 kg/m^3$

$K = 1.5 \times 10^9$ GPa.

$A = \frac{\pi}{4} \times 0.5^2 = 0.1963 m^2$

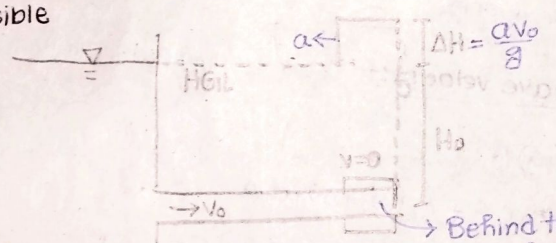
$V = \frac{Q}{A} = \frac{0.4}{0.1963} = 2.0377 m/s$

Velocity of pressure wave $= \sqrt{\frac{K}{\rho}} = \sqrt{\frac{1.5 \times 10^9}{900}} = 1291 m/s$

Pressure rise as a reduction of flow velocity

$$\Delta H = -\frac{a \Delta V}{g} = -\frac{1291 \times \cancel{0.1963} (-2.0377)}{9.81} = 268.16 m.$$

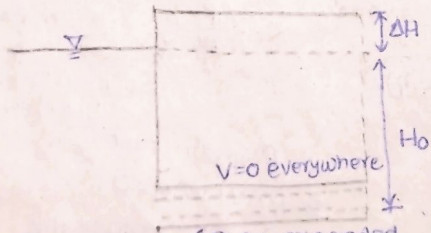
Compressible fluid, Elastic pipes:-



Time, $t = \varepsilon$.

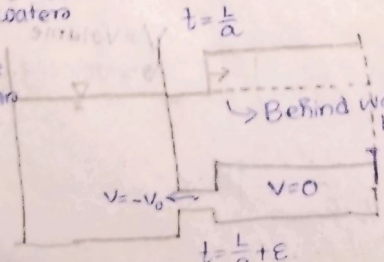
→ Behind the wave front, Wave velocity becomes zero (KE converted to elastic energy)

☑ Wave front reaches reservoir at $t = \frac{L}{a}$

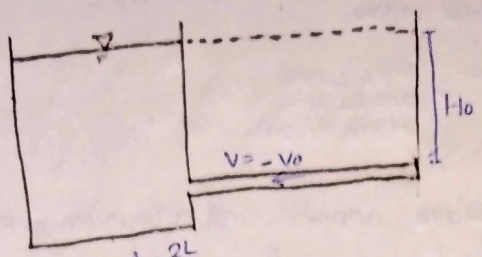


(Pipe expanded due to increased pressure)

Due to increase of HGL, water moves from pipe to reservoir ($-V_0$)

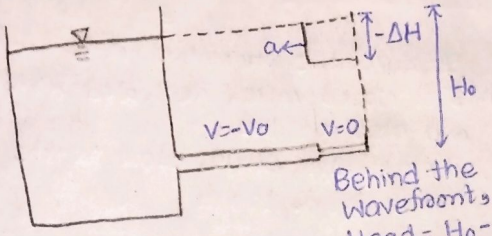


Negative wave travels towards the valve



Time, $t = \frac{2L}{a}$

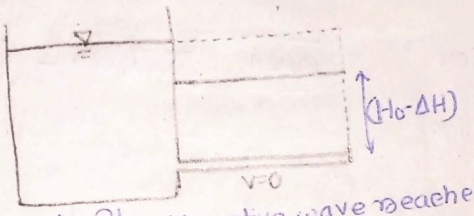
At $t = \frac{2L}{a}$, velocity at valve $= -V_0$. But valve is closed. So negative velocity becomes zero. ($\because$ velocity increase, Pressure will drop. Let think, cavity is formed near the valve). So, -ve pressure wave will travel towards reservoir.



Behind the wavefront, Head $= H_0 - \Delta H$ and $V = 0$

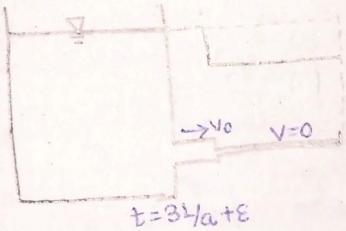
Pressure decreases, so contracted pip

$t = \frac{2L}{a} + \varepsilon$.

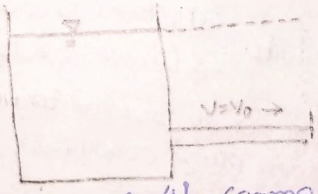


$t = \frac{3L}{a}$, Negative wave reaches to the reservoir.

Due to higher head at reservoir, water flows at +ve direction



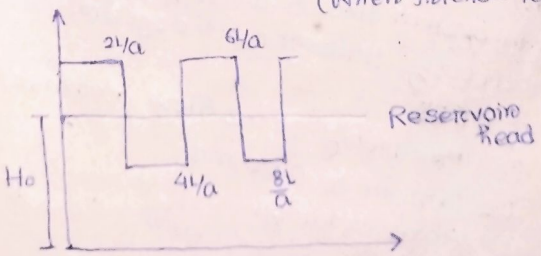
$t = 3L/a + \varepsilon$



$t = \frac{4L}{a}$ (same condition as $t=0$)

Pressure variation at the valve:-

(When friction loss is neglected)



☑ For a positive pressure wave

Pressure behind wave front > Pressure ahead the wave front.

☑ Incident pressure wave approaching a boundary = F

Reflected from the boundary = f.

Reflection coefficient, $n = \frac{f}{F}$.

For large lake, reservoirs, tank, called constant head reservoir, $n = -1$ → attacks, Reflected as

$H_0 = 100$, $\Delta H = 10$ ∴ $H = 110$

$H = 110 - 10 = 100$ m.

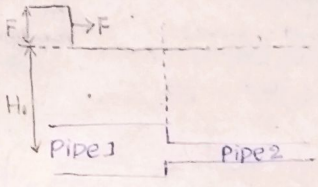
For Dead end (fully closed valve), $n = 1$

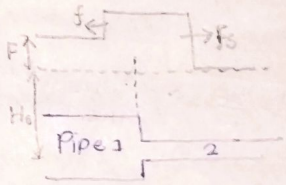
If a positive pressure wave hits a closed valve, reflected wave will also positive.

Eg. When $H_0 = 100$, $\Delta H = 10$, 110m attacks reflected as 120 m

☑ Series junction:-

Two pipes having and/or (different diameters, wall thickness, friction factor, wave velocity) called as series junction.





Reflection coff for pipe 1 = $n = \frac{f}{F} = \frac{\frac{A_1}{a_1} - \frac{A_2}{a_2}}{\frac{A_1}{a_1} + \frac{A_2}{a_2}}$

Transmission coefficient for pipe 2,

$$s = \frac{f_s}{F} = \frac{\frac{2A_1}{a_1}}{\frac{A_1}{a_1} + \frac{A_2}{a_2}}$$

Transient Flow Analysis:

Transient flow in closed conduits → Distributed system approach
→ Lumped system "

☑ For DSA: Transient phenomenon in form of travelling wave. (Water supply pipes, power plant conduits)

☑ Represented by PDE's

* $\frac{\omega L}{a}$ should be large.

☑ For Lumped S.A:-

☑ Any change in system flow condition assumed to travel instantaneously throughout the system.

☑ Described by ODE's

☑ $\omega L/a \ll 1$ ω = Frequency of the flow oscillation

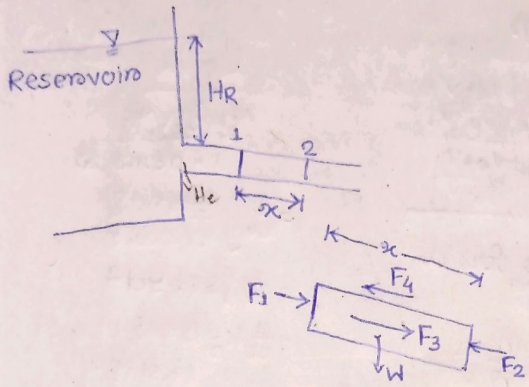
L = Length of pipe line

a = wave velocity.

☑ Free surface flow may become pressurized due to priming of the conduit during transient state condition. --> combined free-surface - pressurized flow.

Eg. Flow in a sewer following a large rainstorm.

Lumped System modelling (Rigid Column theory):



$F_1 = \rho g H A = \gamma (H_R - H_E - H_V) A$
$(\rho \times A)$

$F_2 = \gamma H A$ (H = Head remained at the outlet, for whole pipe, later we assume atmospheric, H = 0)

H_E = Entrance loss $= \frac{K_e V^2}{2g}$

H_V = Static Head loss due to velocity gain $= \frac{V^2}{2g}$

F_3 = Gravitational force component $= \gamma A x S_0$ (For horizontal, $S_0 = 0$)

$F_4 = \gamma H_f A = \gamma A \cdot \frac{f x V^2}{2gD}$

Newton's law, $F = m \frac{dv}{dt}$

$\Rightarrow F_1 + F_3 - F_2 - F_4 = \frac{\gamma A x}{g} \cdot \frac{dv}{dt}$ $\left(m = \rho V = \rho A x = \frac{\gamma A x}{g}\right)$

$\Rightarrow \gamma (H_R - H_E - H_V) A + \gamma A x S_0 - \gamma H A - \gamma H_f A = \frac{\gamma A x}{g} \frac{dv}{dt}$

$\Rightarrow H_R - H_E - H_V + x S_0 - H - H_f = \frac{x}{g} \cdot \frac{dv}{dt}$ (All terms are expressed in terms of γA to eliminate)

$$\Rightarrow \boxed{H_R - K_e \frac{V^2}{2g} - \frac{V^2}{2g} - H + x S_0 - \frac{f x V^2}{2gD} = \frac{x}{g} \cdot \frac{dv}{dt}} \quad \ldots (i)$$

H = 0 for whole system, $x = L$, $S_0 = 0$ and valve head loss is to be added.

$$\therefore H_R - K_e \frac{V^2}{2g} - \frac{V^2}{2g} - 0 + 0 - \frac{f L V^2}{2gD} + K_v \cdot \frac{V^2}{2g} = \frac{L}{g} \cdot \frac{dv}{dt}$$

[K_v = valve loss coefficient]

$$\Rightarrow \boxed{H_R - \left(1 + K_e + K_v + \frac{fL}{D}\right) \frac{V^2}{2g} = \frac{L}{g} \cdot \frac{dv}{dt}} \quad \ldots (ii)$$

Now, $1 + K_e + K_v + \frac{fL}{D} = C_1$ (Let)

$$\therefore H_R - C_1 \cdot \frac{V^2}{2g} = \frac{L}{g} \cdot \frac{dv}{dt}$$

Once the flow is established, steady state has been achieved

$\therefore \frac{dv}{dt} = 0$ and $v = v_0$

$$\therefore H_R - C_1 \frac{V_0^2}{2g} = 0$$

$$\Rightarrow \boxed{V_0 = \sqrt{\frac{2gH_R}{C_1}}} \quad \ldots (iii).$$

Now,

$$H_R - \frac{C_1 V^2}{2g} = \frac{L}{g}\frac{dv}{dt}$$

$$H_R - \frac{C_1 V_0^2}{2g} = 0$$

$$\Rightarrow \frac{C_1 V_0^2}{2g} - \frac{C_1 V^2}{2g} = \frac{L}{g}\cdot\frac{dv}{dt}$$

$$\Rightarrow \frac{C_1}{2g}(V_0^2 - V^2) = \frac{L}{g}\frac{dv}{dt}$$

$$\Rightarrow \frac{C_1}{2L}\int_0^t dt = \int \frac{dv}{V_0^2 - V^2}$$

$$\Rightarrow \frac{C_1 t}{2L} = \frac{1}{2V_0}\log_e\left|\frac{V_0+V}{V_0-V}\right|$$

$$\Rightarrow \boxed{t = \frac{1}{C_1 V_0}\log_e\left|\frac{V_0+V}{V_0-V}\right|}$$

Establishment time for a particular velocity.

$$\int \frac{dx}{a^2-x^2} \qquad x = a\sin\theta \Rightarrow dx = a\cos\theta\, d\theta$$

$$\int \frac{a\cos\theta\, d\theta}{a^2 - a^2\sin^2\theta\, d\theta}$$

$$= \frac{1}{a}\int \frac{\cos\theta\, d\theta}{\cos^2\theta}$$

$$= \frac{1}{a}\int \sec\theta\, d\theta$$

$$= \frac{1}{a}\log|\sec\theta + \tan\theta|$$

$$= \frac{1}{a}\left|\frac{1+\sin\theta}{\cos\theta}\right|$$

$$\left|\frac{1+\frac{x}{a}}{\sqrt{1-\frac{x^2}{a^2}}}\right|$$

$$\frac{1}{a}\log\left|\frac{\sqrt{1+\frac{x}{a}}}{\sqrt{1-\frac{x}{a}}}\right|$$

$$= \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right|$$

Now, $C_1 = 1 + K_V + K_E + \frac{fL}{D}$

K_V = valve loss coff

K_E = Entrance loss.

Ideal fluid, $K_V = K_E = \frac{fL}{D} = 0$

$\therefore C_1 = 1$

Problem:-1

(1) Horizontal
$D = 0.61$ m.
$L = 3048$ m
$H_R = 30.48$ m
$f = 0.018$ (case-2)
$V = 0.99 V_0$
Time required, $t = ?$

Case-1 $f = 0$.

(a) $t = \frac{L}{C_1 V_0}\log_e\left|\frac{V_0+V}{V_0-V}\right|$

$$H_R - \frac{C_1 V_0^2}{2g} = 0$$

$$\Rightarrow C_1 = 1 + K_e + K_v + \frac{fL}{D} = 1+0+0+0 = 1$$

$$V_0 = \sqrt{2gH_R} = \sqrt{2\times 9.81\times 30.48}$$
$$= 24.45 \text{ m/s}$$

$$\therefore t = \frac{3048}{1\times 24.45}\log_e\left|\frac{1.99}{0.01}\right|$$
$$= 660 \text{ s.}$$

(b) When, $f = 0.018$,

$$C_1 = 1 + 0 + 0 + \frac{0.018\times 3048}{0.61} \simeq 91.$$

$$V_0 = \sqrt{\frac{2gH_R}{C_1}} = \sqrt{\frac{2\times 9.81\times 30.48}{91}} = 2.56 \text{ m/s.}$$

$$\therefore t = \frac{L}{V_0 C_1}\ln\left|\frac{V_0+V}{V_0-V}\right| = \frac{3048}{2.56\times 91}\ln(199) = 69.26 \text{ sec}$$

(c) When, $f = 0.018$, $K_e = 0.5$ and $K_v = 5$,

$$C_1 = 1 + 0.5 + 5 + \frac{0.018\times 3048}{0.61} = 96.5$$

$$V_0 = \sqrt{\frac{2gH_R}{C_1}} = \sqrt{\frac{19.62\times 30.48}{96.5}} = 2.49 \text{ m/s}$$

$$\therefore t = \frac{L}{C_1 V_0}\ln\left|\frac{V_0+V}{V_0-V}\right| = \frac{3048}{96.5\times 2.49}\ln|199| = 67.2 \text{ s.}$$

Problem 2:

Reservoir head = $H_R = 18.29\,m$. $L = 914.4\,m$

$d = 30.48\,cm = 0.3048\,m$

$k_s = 0.0003\,m$, flow of water is steady

(a) Steady state velocity - ?

Assuming, no loss at the valve:

$$H_R - C_1\frac{v_0^2}{2g} = 0$$

Now, $C_1 = 1 + k_e + k_v + \frac{fL}{D}$

$$= 1 + 0.5 + 0 + \frac{fL}{D} = 1.5 + \frac{fL}{D}$$

$$\Rightarrow H_R - \left(1.5 + \frac{fL}{D}\right)\frac{v_0^2}{2g} = 0$$

$$\Rightarrow 18.29 - \left(1.5 + \frac{f \times 914.4}{30.48}\right)\frac{v_0^2}{19.6} = 0 \Rightarrow \cancel{16.79} \quad \ldots\ldots (i)$$

Two unknowns, f and v_0.

Haaland equation,

$$\frac{1}{\sqrt{f}} = 1.8\log_{10}\left[\left(\frac{k_s/D}{3.7}\right)^{1.11} + \frac{6.9}{Re}\right] \quad \ldots\ldots (ii)$$

$$Re = \frac{VD}{\nu}$$

$$\frac{1}{\sqrt{f}} = 1.8\log_{10}\left[\left(\frac{k_s/D}{3.7}\right)^{1.11} + \frac{6.9}{Re}\right]$$

$$= 1.8\log_{10}\left\{\left(\frac{k_s}{3.7D}\right)^{1.11} + \frac{6.9}{Re}\right\}$$

1st iteration, take $f = 0$.

$$18.29 - 1.5 \times \frac{v_0^2}{19.6} = 0$$

$$\Rightarrow v_0 = 15.46\,m/s.$$

$$\therefore Re = \frac{15.46 \times 0.3048}{10^{-6}} = 4712208$$

$$\therefore \frac{1}{\sqrt{f}} = -7.132 \Rightarrow f = 0.0196$$

Now, From (i)

$$v_0^2 = \frac{18.29 \times 19.6}{15 + \frac{f \times 914.4}{30.48}} = \frac{358.5}{15 + 30f}$$

$$\Rightarrow v_0 = 4.7957\,m/s.$$

$$Re = 1461729.$$

$$\therefore \frac{1}{\sqrt{f}} = 1.8\log_{10}\left(1.0756 \times 10^{-4} + \frac{6.9}{Re}\right)$$

$$\Rightarrow f = 0.01978$$

After solving, $f = 0.0201$, $v_0 = 2.4\,m/s$

(b) compute the maximum pressure in the pipeline if the valve closes so that rate of decrease in velocity is linear in time from its steady state value to zero in 20 seconds.

Compute

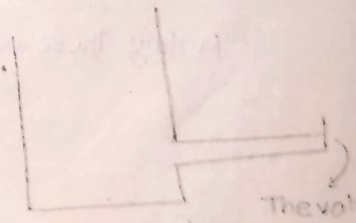
Deacceleration is constant,

$$a = \frac{L}{g}\cdot\frac{dv}{dt} = \frac{914.4}{9.81} \times \frac{(-2.41)}{20}$$

$$= -11.23\,m/s^2$$

$$\frac{2L}{c} = \frac{2 \times 914.4}{1250} = 1.46\,sec < 20\,s$$

$\therefore$ Slow valve closure.



The valve is slowly closing. So there would be some pressure head remaining at this point.

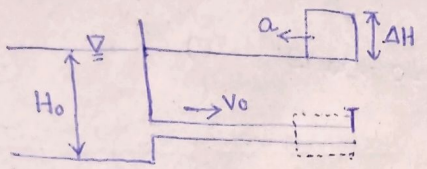
$$\frac{L}{g}\frac{dv}{dt} = H_R - \left(1 + K_e + K_v + \frac{fL}{D}\right)\frac{V^2}{2g} - H_2$$ (H_2 = Pressure head remaining at the valve outlet).

$$\Rightarrow -11.23 = 18.29 - \left(1 + 0.5 + 0 + \frac{0.0201 \times 914.4}{0.3048}\right)\frac{V^2}{2g} - H_2$$

When, valve closes, $V = 0$

$\therefore H_2 = 18.29 + 11.23 = 29.52$ m w

Pressure wave velocity and water hammer pressure rise of slightly compressible fluid in a rigid pipe.



ρ	$\rho + \Delta\rho$
$V_0 + a$	$V_0 + \Delta V + a$
A	A
H_0	$H_0 + \Delta H$

Applying continuity equation,

$$\rho(V_0 + a)A = (\rho + \Delta\rho)(V_0 + \Delta V + a) \cdot A$$

$$\Rightarrow \cancel{\rho(V_0 + a)} = \cancel{\rho(V_0 + \Delta V + a)} + \Delta\rho(V_0 + \Delta V + a)$$

$$= \rho\Delta V + \cancel{\Delta\rho V_0} + \Delta\rho a$$

$$\Rightarrow \rho(V_0 + a) = (\rho + \Delta\rho)a$$

$$\Rightarrow \rho V_0 + \rho a = \rho a + a\Delta\rho$$

$$\Rightarrow \boxed{\frac{V_0}{a} = \frac{\Delta\rho}{\rho}}$$

This equation is same as the previous one.

$$\frac{\Delta V}{a} = -\frac{\Delta\rho}{\rho_0}$$ where, $V_0 + \Delta V = 0$

$\Rightarrow V_0 = -\Delta V$ (substituted)

Velocity at section 2, $V_2 = V_0 + \Delta V = 0$ (As fluid stops completely).

Applying momentum equation,

$$\rho_1 Q V_{1x} + P_1 A_1 = \rho_2 Q V_{2x} + P_2 V_2$$

Now, $Q = (V_0 + a)A$

$P_1 = \gamma H_0$

$V_{1x} = (V_0 + a)$

$Q = (V_0 + a)A$

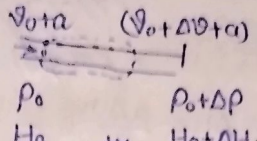
$P_2 = \gamma(H_0 + \Delta H)$

$V_{2x} = V_0 + \Delta V + a$.

Putting these values, we have

Pressure rise for ~~Slightly~~ ~~incom~~ Rigid pipe, incompressible fluid:-

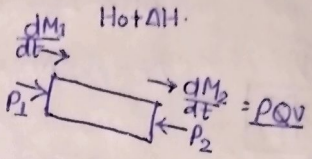
$\Delta H \rightarrow$ Momentum $\rightarrow$



$$P_1 - P_2 = \frac{dM_2}{dt} - \frac{dM_1}{dt}$$

$\Rightarrow \rho_1 g H_1 A_1 - \rho_2 g H_2 A_2$

$= \rho_2 Q_2 v_2 - \rho_1 Q_1 v_1$



$P_1 =$

Now, $\rho_1 = \rho_2 = \rho_0$

$A_1 = A_2 = A.$

$H_1 = H_0$

$H_2 = H_0 + \Delta H$

$Q_1 = Q_2 = A_1 v_1 = A(v_0 + a)$

$$\therefore \rho_0 g H_0 A - \rho_0 g (H_0 + \Delta H) A = \rho_0 A (v_0 + a) \{(v_0 + \Delta v + a) - (v_0 + a)\}$$

$$\Rightarrow -\rho_0 g \Delta H A = \rho_0 A (v_0 + a) \Delta v$$

$$\Rightarrow \Delta H = -\frac{(v_0 + a) \cdot \Delta v}{g} = -\frac{a \Delta v}{g}.$$

$$\boxed{\therefore \Delta H = -\frac{a \Delta v}{g}}$$

For sudden closure of valve $\Delta v = 0 - v_0 = -v_0$

$\therefore \Delta H = \frac{a v_0}{g} \Rightarrow \Delta P = \rho g \Delta H = \rho g \times \frac{a v_0}{g} = \rho a v_0.$

When fluid is slightly compressible,

Mass conservation eq^n.

$\rho_1 A_1 V_1 = A_2 V_2 \rho_2$

$\Rightarrow \rho_1 = \rho_0$

$\rho_2 = \rho_0 + \Delta \rho$

$A_1 = A_2 = A$

$v_1 = (v_0 + a)$

$v_2 = (v_0 + \Delta v + a)$

$$\rho_0 A (v_0 + a) = A (v_0 + \Delta v + a)(\rho_0 + \Delta \rho)$$

$$\Rightarrow \rho_0 (v_0 + a) = \rho_0 (v_0 + a) + \rho_0 \Delta v + \Delta \rho (v_0 + \Delta v + a)$$

$$\Rightarrow \rho_0 \Delta v = -\Delta \rho (v_0 + \Delta v + a)$$

$a \gg v_0 + \Delta v$

$$\Rightarrow \rho_0 \Delta v = -\Delta \rho a$$

$$\Rightarrow a = -\frac{\rho_0}{\Delta \rho} \cdot \Delta v$$

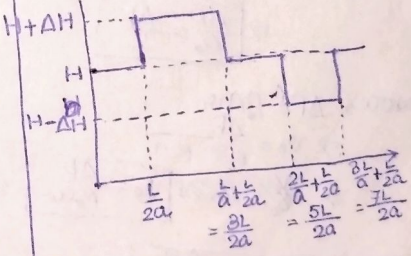
Now, Bulk modulus of elasticity, $K = -\frac{\Delta p}{\left(\frac{\Delta v}{v}\right)}$

Now, $\frac{\Delta v}{v} = -\frac{\Delta \rho}{\rho_0}$

$$K = \frac{\Delta p}{\left(\frac{\Delta \rho}{\rho_0}\right)} \Rightarrow$$

$$\Rightarrow \frac{\Delta \rho}{\rho_0} = \frac{\Delta p}{K}$$

Pressure variation at the midway between valve and reservoir.



$$\boxed{\varphi = \frac{\partial \phi}{\partial t} \quad \varphi(t, \phi)}$$

$\therefore a = -\frac{\Delta p}{K}\cdot\Delta v$

$= \frac{a\rho\Delta v}{K}$

$\Delta H = -\frac{a\Delta v}{g}$

$\Rightarrow \Delta p = \rho g \Delta H$

$= \rho g\left(-\frac{a\Delta v}{g}\right)$

$= a\rho\Delta v$

$\Rightarrow a = -\frac{K}{\Delta p}\Delta v$

$= -\frac{K\cdot\Delta v}{(-a\rho_0\Delta v)}$

$\Delta H = -\frac{a\Delta v}{g}$

$\Delta P = \rho_0 g\Delta H = \rho_0 g\left(-\frac{a\Delta v}{g}\right)$

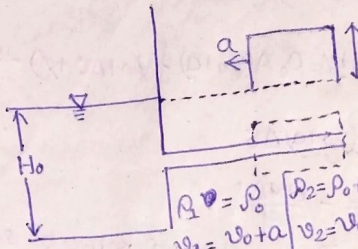
$= (-a\rho_0\Delta v)$

$\Rightarrow a^2 = \frac{K}{\rho_0} \Rightarrow \boxed{a = \sqrt{\frac{K}{\rho_0}}}$

$\Downarrow$

Pressure wave velocity for rigid pipe and slightly compressible fluid.

Slightly compressible fluid for ~~non~~-rigid pipe:-
non.



Apply continuity equation,

$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$

$\Rightarrow \rho_0 A_0(v_0+a) = (\rho_0+\Delta\rho)(A_0+\Delta A)(v_0+a+\Delta v)$

$\rho_1 = \rho_0$	$\rho_2 = \rho_0 + \Delta\rho$
$v_1 = v_0 + a$	$v_2 = v_0 + a + \Delta v \approx a$
$A_1 = A_0$	$A_2 = A_0 + \Delta A$
$H_1 = H_0$	$H_2 = H_0 + \Delta H$

$\Rightarrow \rho_0 A_0(v_0+a) = (\rho_0+\Delta\rho)(A_0+\Delta A)a$

$\Rightarrow \frac{\rho_0 A_0(v_0+a)}{\rho_0 A_0 a} = \frac{(\rho_0+\Delta\rho)}{\rho_0}\cdot\frac{A_0+\Delta A}{A_0}\cdot\frac{a}{a}$

$\Rightarrow \left(1+\frac{v_0}{a}\right) = \left(1+\frac{\Delta\rho}{\rho_0}\right)\left(1+\frac{\Delta A}{A_0}\right)$

$\Rightarrow \boxed{1+\frac{v_0}{a} = 1+\frac{\Delta A}{A_0}+\frac{\Delta\rho}{\rho_0}+\cancel{\frac{\Delta\rho\cdot\Delta A}{\rho_0 A_0}}}$ $\to 0$

Now, compressibility, $K = \frac{\Delta p}{\frac{\Delta\rho}{\rho_0}}$

$\Rightarrow \boxed{\frac{\Delta\rho}{\rho_0} = \frac{\Delta p}{K}}$

ρ_0 কে ρ এবং A_0 কে A ও,

Now, $\Delta P = \rho_0 A v_0$

$\Rightarrow v_0 = \frac{\Delta p}{\rho_0 a}$

$\Rightarrow \boxed{v_0 = \frac{\Delta p}{\rho_0 a}}$

$\therefore \boxed{1+\frac{\Delta p}{\rho_0 a^2} = 1+\frac{\Delta A}{A_0}+\frac{\Delta p}{K}}$

Now, hoop stress, $\sigma = \frac{\Delta p D}{2t}$.

Stress = Young's modulus × Strain

$\Rightarrow \frac{\Delta p D}{2t} = E \times \frac{\Delta D}{D}$

$\Rightarrow \boxed{\frac{\Delta D}{D} = \frac{D}{2tE}\Delta P}$

Reynold's Transport Theorem:-

$$\frac{dB_{sys}}{dt} = \frac{d}{dt}\int \beta\rho d\forall + (\beta\rho A V_s)_{out} - (\beta\rho A V_s)_{in}$$

$B \Rightarrow$ Extensive property.

β = Intensive property

$\forall$ = Total volume.

V = Fluid flow velocity.

$V_s = V$, for fixed cv.

$V_s = V - W$, when cv is moving with velocity W.

$$\therefore \frac{dB_{sys}}{dt} = \frac{d}{dt}\int \beta\rho d\forall + [\beta\rho A(V-W)]_{out} - [\beta A\rho(V-W)]_{in}.$$

From chatgpt

RTT → Relates the rate of change in quantity in a cv to the rate of change in quantity in a moving reference frame. It is used to describe the transport of fluid properties such as mass, momentum and energy through a fixed cv.

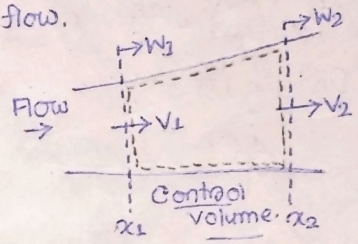
$$\frac{dQ}{dt} = -\iiint (\rho V)\, dV + \iint (\rho V)\hat{n}\, ds$$

Net outflow of Q from the cv.

Rate of change of Q within the cv due to motion of the cv.

Derivation of continuity equation:-

Slightly compressible flow.
Elastic Conduit.
1-D flow.



Control surfaces
cv moving W_1, W_2 velocity.
Either contracting or expanding.

For $\beta = 1$,

$$\frac{d}{dt}\int \rho \underbrace{d\forall}_{volume} + [\rho A(V-W)]_{out} - [\rho A(V-W)]_{in} = 0$$

$\forall$ = volume
V = Velocity.

$$\Rightarrow \frac{d}{dt}\int_{x_1}^{x_2} \rho A dx + \rho_2 A_2 (V_2 - W_2) - \rho_1 A_1 (V_1 - W_1) = 0 \quad \ldots\ldots (1)$$

Leibnitz's rule:-

$$\frac{d}{dt}\int_{f_1(t)}^{f_2(t)} F(x,t)\, dx = \int_{f_1(t)}^{f_2(t)} \frac{\partial}{\partial t} F(x,t)\, dx + F(f_2(t),t)\frac{df_2}{dt} - F(f_1(t),t)\frac{df_1}{dt}$$

$$\frac{d}{dt}\int_{x_1}^{x_2} \rho A dx = \int_{x_1}^{x_2} \frac{\partial}{\partial t}(\rho A)\, dx + \rho_2 A_2 \frac{dx_2}{dt} - \rho_1 A_1 \frac{dx_1}{dt} \quad \ldots (2)$$

Here, $\frac{dx_1}{dt} = W_1$ and $\frac{dx_2}{dt} = W_2$

$F(x,t) = \rho A$
$f_1(t) = x_1$
$f_2(t) = x_2$

Eq^n (1) & {substituting values from (2)},

$$\int \frac{\partial}{\partial t}(\rho A)\, dx + \rho_2 A_2 W_2 - \rho_1 A_1 W_1 + \rho_2 A_2 (V_2 - W_2) - \rho_1 A_1 (V_1 - W_1) = 0$$

$$\Rightarrow \int \frac{\partial}{\partial t}(\rho A)\, dx + \rho_2 A_2 V_2 - \rho_1 A_1 V_1 = 0$$

$$\Rightarrow \frac{\partial}{\partial t}(\rho A)\Delta x + \Delta(\rho A V) = 0$$

$$\Rightarrow \frac{\partial}{\partial t}(\rho A) + \frac{\Delta(\rho A V)}{\Delta x} = 0$$

$$\Rightarrow \frac{\partial}{\partial t}(\rho A) + \frac{\partial}{\partial x}(\rho A V) = 0$$

$$\Rightarrow A\frac{\partial\rho}{\partial t}+\rho\frac{\partial A}{\partial t}+\rho A\frac{\partial V}{\partial x}+AV\frac{\partial\rho}{\partial x}+\rho V\frac{\partial A}{\partial x}=0$$

$\Rightarrow$ Dividing by ρA,

$$\frac{1}{\rho}\frac{\partial\rho}{\partial t}+\frac{1}{A}\frac{\partial A}{\partial t}+\frac{\partial V}{\partial x}+\frac{V}{\rho}\frac{\partial\rho}{\partial x}+\frac{V}{A}\frac{\partial A}{\partial x}=0$$

$$\Rightarrow \frac{1}{\rho}\left(\frac{\partial\rho}{\partial t}+V\frac{\partial\rho}{\partial x}\right)+\frac{1}{A}\left(\frac{\partial A}{\partial t}+V\frac{\partial A}{\partial x}\right)+\frac{\partial V}{\partial x}=0$$

$$\Rightarrow \frac{1}{\rho}\left(\frac{\partial\rho}{\partial t}+\frac{\partial x}{\partial t}\cdot\frac{\partial\rho}{\partial x}\right)+\frac{1}{A}\left(\frac{\partial A}{\partial t}+\frac{\partial x}{\partial t}\cdot\frac{\partial A}{\partial x}\right)+\frac{\partial V}{\partial x}=0$$

$$\Rightarrow \boxed{\frac{1}{\rho}\frac{d\rho}{dt}+\frac{1}{A}\frac{dA}{dt}+\frac{\partial V}{\partial x}=0} \quad \cdots\cdots (3)$$

Bulk modulus,

$$K=\frac{dP}{\left(-\frac{\Delta V}{V_0}\right)}=\frac{dP}{\left(\frac{\Delta\rho}{\rho_0}\right)}$$

$$\Rightarrow \frac{K\Delta\rho}{\rho_0}=dP \quad \left(\text{Take, } \rho_0=\rho\right)$$

$$\Rightarrow K\,d\rho=\rho\,dP$$

$$\Rightarrow K\frac{d\rho}{dt}=\rho\frac{dP}{dt}$$

$$\Rightarrow \frac{d\rho}{dt}=\frac{\rho\,dP}{K\,dt} \quad \cdots\cdots (4)$$

$\rho V = \text{constant}$

$\Rightarrow \Delta(\rho V)=0$

$\Rightarrow V\Delta\rho+\rho\Delta V=0$

$\Rightarrow \frac{\Delta\rho}{\rho}+\frac{\Delta V}{V}=0$

$\Rightarrow \frac{\Delta\rho}{\rho}=-\frac{\Delta V}{V}$

Circular conduit, $A=\pi R^2$

$$\Rightarrow \frac{dA}{dt}=2\pi R\cdot\frac{dR}{dt}$$

$$\Rightarrow \frac{1}{A}\frac{dA}{dt}=\frac{1}{\pi R^2}\cdot 2\pi R\frac{dR}{dt}$$

$$\Rightarrow \frac{1}{A}\frac{dA}{dt}=2\times\frac{1}{R}\frac{dR}{dt} \quad \left(\frac{dR}{R}=d\varepsilon,\ \varepsilon=\text{strain}\right)$$

$$\Rightarrow \frac{1}{A}\frac{dA}{dt}=2\frac{d\varepsilon}{dt}=\frac{2}{D}\cdot\frac{dD}{dt} \quad \cdots\cdots (5)$$

$$\frac{1}{A}\frac{dA}{dt}=\frac{1}{\pi D^2}\times\frac{\pi}{4}\times 2D\times\frac{dD}{dt}=\frac{2}{D}\frac{dD}{dt}$$

Now, $\varepsilon=\frac{\sigma_h-\sigma_a\mu}{E}$ [Pipe has expansion joints. So, axial stress, $\sigma_a=0$]

$$\Rightarrow \varepsilon=\frac{\sigma_h}{E}$$

$$\Rightarrow \sigma_h=\varepsilon E$$

$$\Rightarrow \frac{d\sigma_h}{dt}=\frac{d}{dt}(E\varepsilon)$$

$$\Rightarrow \frac{d}{dt}\left(\frac{PD}{2e}\right)=E\frac{d\varepsilon}{dt}$$

$$\Rightarrow \frac{P}{2e}\frac{dD}{dt}+\frac{D}{2e}\frac{dP}{dt}=E\frac{d\varepsilon}{dt}$$

$$\Rightarrow \frac{P}{2e}\frac{dD}{dt}+\frac{D}{2e} \quad \text{From eq}^n(5),$$

#All the parts need to be found in terms of $\frac{dP}{dt}$ in eq^n(5).

$$\frac{P}{2e}\frac{D}{2A}\frac{dA}{dt}+\frac{D}{2e}\frac{dP}{dt}=E\times\frac{1}{2A}\frac{dA}{dt}$$

$$\Rightarrow \frac{1}{A}\frac{dA}{dt}\left(\frac{E}{2}-\frac{PD}{4e}\right)=\frac{D}{2e}\frac{dP}{dt}$$

$$\Rightarrow \frac{1}{A}\frac{dA}{dt}=\frac{\frac{D}{e}\frac{dP}{dt}}{\left(E-\frac{PD}{2e}\right)} \quad \cdots\cdots (6)$$

$$\Rightarrow \frac{P}{2e}D\cdot\frac{d\varepsilon}{dt}+\frac{D}{2e}\cdot\frac{dP}{dt}=E\frac{d\varepsilon}{dt}$$

$$\Rightarrow \frac{d\varepsilon}{dt}\left(E-\frac{PD}{2e}\right)=\frac{D}{2e}\frac{dP}{dt}$$

$$\Rightarrow \frac{d\varepsilon}{dt}=\left(\frac{\frac{D}{2e}\frac{dP}{dt}}{E-\frac{PD}{2e}}\right)$$

$$\Rightarrow \frac{1}{2A}\frac{dA}{dt}=\left(\frac{\frac{D}{2e}\frac{dP}{dt}}{E-\frac{PD}{2e}}\right)$$

$$\Rightarrow \frac{1}{A}\frac{dA}{dt}=\frac{\frac{D}{e}\frac{dP}{dt}}{\left(E-\frac{PD}{2e}\right)} \quad \cdots\cdots (6)$$

Not necessary this part.

Put values from (4) and (6) to (3),

$$\frac{1}{K}\frac{dP}{dt} + \frac{\frac{D}{e}\frac{dP}{dt}}{\left(E - \frac{PD}{2e}\right)} + \frac{\partial V}{\partial x} = 0$$

P = Pressure

$$\Rightarrow \left(\frac{1}{K} + \frac{\frac{D}{e}}{E - \frac{PD}{2e}}\right)\frac{dP}{dt} + \frac{\partial V}{\partial x} = 0$$

$$\Rightarrow \left(\frac{1}{K} + \frac{1}{\frac{eE}{D} - \frac{P}{2}}\right)\frac{dP}{dt} + \frac{\partial V}{\partial x} = 0 \quad \left[\frac{P}{2} \ll \frac{eE}{D}\right]$$

$$\Rightarrow \left(\frac{1}{K} + \frac{1}{\frac{eE}{D}}\right)\frac{dP}{dt} + \frac{\partial V}{\partial x} = 0$$

$$\Rightarrow \left(\frac{1}{K} + \frac{D}{eE}\right)\frac{dP}{dt} + \frac{\partial V}{\partial x} = 0$$

$$\Rightarrow \left(1 + \frac{KD}{eE}\right)\frac{dP}{dt} + K\frac{\partial V}{\partial x} = 0$$

$$\Rightarrow \frac{dP}{dt} + \frac{K}{1 + \frac{KD}{eE}}\frac{\partial V}{\partial x} = 0$$

$$\Rightarrow \frac{dP}{dt} + \rho\left(\frac{K/\rho}{1 + \frac{KD}{eE}}\right)\frac{\partial V}{\partial x} = 0$$

$$\Rightarrow \frac{dP}{dt} + \rho a^2 \frac{\partial V}{\partial x} = 0$$

$$\Rightarrow \boxed{\rho a^2 \frac{\partial V}{\partial x} + \frac{\partial P}{\partial t} + V\frac{\partial P}{\partial x} = 0} \quad \ldots (7)$$

$P(x,t)$

$$\Rightarrow \frac{dP}{dt} = \frac{\partial P}{\partial t} + \frac{\partial P}{\partial x}\frac{\partial x}{\partial t}$$

$$= \frac{\partial P}{\partial t} + V\frac{\partial P}{\partial x}$$

c

Derivation of Momentum equation:-

RTT, $\frac{d}{dt}\int_{\forall} \beta\rho d\forall + (\beta\rho A V_s)_{out} - (\beta\rho A V_s)_{in} = \frac{dB_{sys}}{dt}$ $\quad \begin{cases} \forall = \text{volume} \\ V = \text{velocity}. \end{cases}$

$\Rightarrow$ For momentum eq^n, $\beta = V$ (velocity)

$V_s = V - \omega$

$$\Sigma F = \frac{d}{dt}\int_{\forall} V\rho d\forall + V\rho A(V-\omega)\Big|_{outlet} - V\rho A(V-\omega)\Big|_{inlet} \ldots (8)$$

Applying Libenitz rule to the 1st term.

$$\Sigma F = \frac{d}{dt}\int_{x_1}^{x_2} V\rho A dx + V\rho A(V-\omega)\Big|_{out} - V\rho A(V-\omega)\Big|_{in}$$

$$\Sigma F = \int_{x_1}^{x_2}\frac{\partial}{\partial t}(\rho A V)dx + (\rho A V)_2\frac{dx_2}{dt} - (\rho A V)_1\frac{dx_1}{dt} + [\rho A V(V-\omega)]_2 - [\rho A V(V-\omega)]_1$$

$$\Sigma F = \frac{\partial}{\partial t}(\rho A V)\Delta x + (\rho A V\omega)_2 - (\rho A V\omega)_1 + [\rho A V(V-\omega)]_2 - [\rho A V(V-\omega)]_1$$

$$\frac{\Sigma F}{\Delta x} = \frac{\partial}{\partial t}(\rho A V) + \frac{(\rho A V^2)_2 - (\rho A V^2)_1}{\Delta x} \quad \ldots (9)$$

Intensive properties:- Independent of mass or volume. T, P, ρ, viscosity (μ).

Extensive properties: Mass, volume, energy, entropy.

External forces:-

Body force ← (2) Gravity force

(1) Pressure force, (3) Shear stress → Surface forces

Press forces at two sections 1 and 2, P_1A_1, P_2A_2.

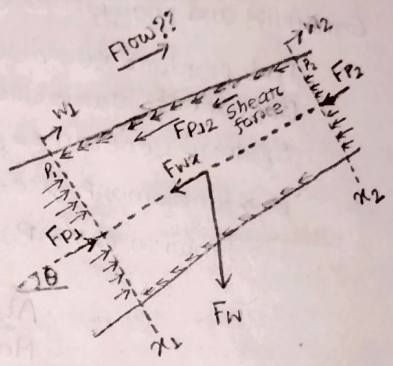
Pressure force at converging sides,

?? $F_{P12} = \frac{1}{2}(P_1+P_2)(A_1-A_2)$ ⇒ I think ⇒ Extra pressure force from side wall. Avg press $= \frac{1}{2}(P_1+P_2)$, area (A_1-A_2)

Gravitational force $= \gamma A(x_2-x_1)\sin\theta$

$\theta \rightarrow$ Angle, horizontal and pipe centreline.

Shear force $= \tau_0 \pi D(x_2-x_1)$



$$\sum F = P_1A_1 - P_2A_2 - \frac{1}{2}(P_1+P_2)(A_1-A_2) - \rho g A(x_2-x_1)\sin\theta - \tau_0\pi D(x_2-x_1)$$

$$\Rightarrow \sum F = P_1A_1 - P_2A_2 - \frac{1}{2}P_1A_1 + \frac{1}{2}P_1A_2 - \frac{1}{2}P_2A_1 + \frac{1}{2}P_2A_2 - \rho g A\Delta x\sin\theta - \tau_0\pi D\Delta x$$

$$= \frac{1}{2}(P_1A_1 + P_1A_2 - P_2A_1 - P_2A_2) - \rho g A\sin\theta\,\Delta x - \tau_0\pi D\Delta x$$

$$= \frac{1}{2}(P_1-P_2)(A_1+A_2) - \rho g A\sin\theta\,\Delta x - \tau_0\pi D\Delta x$$

$$\frac{\sum F}{\Delta x} = \frac{(P_1-P_2)(A_1+A_2)}{2\Delta x} - \rho g A\sin\theta - \tau_0\pi D \quad \ldots\ldots (10)$$

Putting value from (10) to (9),

$$\frac{(P_1-P_2)(A_1+A_2)}{2\Delta x} - \rho g A\sin\theta - \tau_0\pi D = \frac{\partial}{\partial t}(\rho A V) + \frac{(\rho A V^2)_2 - (\rho A V^2)_1}{\Delta x}$$

$$\Rightarrow -\left(\frac{A_1+A_2}{2}\right)\left(\frac{P_2-P_1}{\Delta x}\right) - \rho g A\sin\theta - \tau_0\pi D = \frac{\partial}{\partial t}(\rho A V) + \frac{\partial}{\partial x}(\rho A V^2)$$

$$\Rightarrow -A\frac{\partial P}{\partial x} - \rho g A\sin\theta - \tau_0\pi D = \frac{\partial}{\partial t}(\rho A V) + \frac{\partial}{\partial x}(\rho A V^2)$$

$$\Rightarrow \frac{\partial}{\partial t}(\rho A V) + \frac{\partial}{\partial x}(\rho A V^2) + A\frac{\partial P}{\partial x} + \rho g A\sin\theta + \tau_0\pi D = 0$$

$\tau_0 = \frac{1}{8}\rho f V|V|$

$$\frac{\tau_0\pi D}{\rho A} = \frac{\frac{1}{8}\rho f V|V| \times \pi D}{\rho \times \frac{\pi}{4}D^2} = \frac{f V|V|}{2D}$$

? $$\Rightarrow V\frac{\partial}{\partial t}(\rho A) + \rho A\frac{\partial V}{\partial t} + V\frac{\partial}{\partial x}(\rho A V) + \rho A V\frac{\partial V}{\partial x} + A\frac{\partial P}{\partial x} + \rho g A\sin\theta + \frac{1}{8}\rho f V|V|\pi D = 0$$

$$\Rightarrow V\left[\frac{\partial}{\partial t}(\rho A) + \frac{\partial}{\partial x}(\rho A V)\right] + \rho A\left(\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x}\right) + A\frac{\partial P}{\partial x} + \rho g A\sin\theta + \frac{1}{8}\rho f V|V|\cdot\pi D = 0$$

Dividing by ρA,

? $$\Rightarrow \underbrace{\frac{V}{\rho A}\left[\frac{\partial}{\partial t}(\rho A) + \frac{\partial}{\partial x}(\rho A V)\right]}_{=0?} + \left(\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x}\right) + \frac{1}{\rho}\frac{\partial P}{\partial x} + g\sin\theta + \frac{f V|V|}{2D} = 0$$

$$\Rightarrow \boxed{\frac{dV}{dt} + \frac{1}{\rho}\frac{\partial P}{\partial x} + g\sin\theta + \frac{f V|V|}{2D} = 0} \quad \ldots\ldots (11)$$

PDE ⇒ Parabolic, Hyperbolic or elliptic.

Wave eqn, $u_{tt} - u_{xx} = 0$

$$\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = 0 \qquad A=1, B=0, C=-1 \therefore B^2-4AC = +ve, \text{Hyperbolic}$$

Heat equation, $u_t = K u_{xx}$

$$\Rightarrow \frac{\partial u}{\partial t} - K\frac{\partial^2 u}{\partial x^2} = 0 \qquad A=-K,\ B=0,\ C=0 \quad \therefore B^2-4AC = 0, \text{Parabolic}$$

$$Au_{xx} + Bu_{xy} + Cu_{yy} + Du_x + Eu_y + F = 0$$

Continuity and momentum equation,

Independent variables = x, t

Dependent variables = P, V

System constants = a, ρ, f, D

$a \rightarrow$ function of $P \rightarrow a = \sqrt{K/\rho}$, where, $K = \frac{\Delta P}{(\Delta\rho/\rho)} \rightarrow$ But neglected

$f \rightarrow$ function of $v \rightarrow$ But neglected.

At low Reynold's number, (<2300),
Hagen-Poiseuille equation:-

$$f = \frac{64}{Re}$$

When Re > 4000, Colebrook equation,

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{\varepsilon}{3.7D} + \frac{2.51}{Re\sqrt{f}}\right)$$

Where, $Re = \frac{VD}{\nu}$

∴ Flow velocity and friction factor is related to each other by Reynold's number.

Classification of GE:

CE:- $\frac{\partial P}{\partial t} + V\frac{\partial P}{\partial x} + \rho a^2 \frac{\partial V}{\partial x} = 0$ ..... (From (7)),

ME:- $\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x} + \frac{1}{\rho}\frac{\partial P}{\partial x} + g\sin\theta + \frac{fV|V|}{2D} = 0$ .... (From (11)),

In matrix form,

$$\begin{Bmatrix} \frac{\partial P}{\partial t} \\ \frac{\partial V}{\partial t} \end{Bmatrix} + \begin{bmatrix} V & \rho a^2 \\ \frac{1}{\rho} & V \end{bmatrix} \begin{Bmatrix} \frac{\partial P}{\partial x} \\ \frac{\partial V}{\partial x} \end{Bmatrix} = \begin{Bmatrix} 0 \\ -g\sin\theta - \frac{fV|V|}{2D} \end{Bmatrix}$$

$$\Rightarrow \frac{\partial}{\partial t}\begin{Bmatrix} P \\ V \end{Bmatrix} + \begin{bmatrix} V & \rho a^2 \\ 1/\rho & V \end{bmatrix} \frac{\partial}{\partial x}\begin{Bmatrix} P \\ V \end{Bmatrix} = \begin{Bmatrix} 0 \\ -g\sin\theta - \frac{fV|V|}{2D} \end{Bmatrix}$$

Taking, $U = \begin{Bmatrix} P \\ V \end{Bmatrix}$

$$B = \begin{bmatrix} V & \rho a^2 \\ 1/\rho & V \end{bmatrix}$$

and $E = \begin{Bmatrix} 0 \\ -g\sin\theta - \frac{fV|V|}{2D} \end{Bmatrix}$

$$\Rightarrow \boxed{\frac{\partial U}{\partial t} + B\frac{\partial U}{\partial x} = E} \quad \ldots\ldots (12)$$

Eigen value (λ)
Eigen vectors (v)

$$|A-\lambda I|=0$$

$$\Rightarrow \begin{vmatrix} V-\lambda & \rho a^2 \\ 1/\rho & V-\lambda \end{vmatrix}=0$$

$$\Rightarrow (V-\lambda)^2-a^2=0$$

$$\Rightarrow V-\lambda=\pm a$$

$$\Rightarrow \lambda=V\pm a$$

HPR sir's notes:-

$\lambda=0 \Rightarrow$ Elliptical system

Real and distinct eigenvalues $\Rightarrow$ Hyperbolic

Real and equal eigenvalues $\Rightarrow$ Parabolic.

(Two notes are contradictory) ??

If system of linear equations has non zero eigenvalues, $\rightarrow$ unique solution

One or more zero eigenvalues $\rightarrow$ No sol^n or infinitely many solutions

Eg. $\begin{cases} x+2y=3 \\ 3x+4y=6 \end{cases}$

$\lambda_1=4.8,\ \lambda_2=0.2$

Solution $(x,y)=(0,3/2)$

$$\begin{cases} x+y+z=1 \\ 2x+2y+2z=2 \\ 3x+3y+3z=3 \\ \lambda_1=6,\ \lambda_2=\lambda_3=0 \\ (x,y,z)=(1-y-z,\ y,\ z) \end{cases}$$

Infinitely many solutions.

From A-D sir's slide:-

Parabolic: One or more zero eigenvalues.

Elliptic: Non zero, same sign.

$\rightarrow$ Hyperbolic:- Non-zero, opposite sign.

$\lambda=V\pm a \quad (a\gg V)$

$\therefore \lambda_1=V+a \quad \lambda_2=V-a$

$\quad =+ve \qquad =-ve$

So, our PDE's are hyperbolic.

GIEs are types of wave eq^{n}s $\Rightarrow$ Describes wave properties.

IC's corresponding to steady state:-

$$\frac{\partial P}{\partial t}=0,\ \frac{\partial V}{\partial t}=0$$

For IC,
$$\begin{cases} \text{CE: } \cancel{\frac{\partial P}{\partial t}}^{0}+V\frac{\partial P}{\partial x}+\rho a^2\frac{\partial V}{\partial x}=0 \\ \text{ME:- } \cancel{\frac{\partial V}{\partial t}}^{0}+V\frac{\partial V}{\partial x}+\frac{1}{\rho}\frac{\partial P}{\partial x}+g\sin\theta+\frac{fV|V|}{2D}=0. \end{cases}$$

For most problems,
convective acceleration terms are small, negligible.

$$\boxed{V\frac{\partial P}{\partial x},\ V\frac{\partial V}{\partial x} \rightarrow \text{negligible} \leftarrow g\sin\theta}$$

$\therefore$ Simplified eq^{n}s:-

CE:- $\frac{\partial P}{\partial t}+\rho a^2\frac{\partial V}{\partial x}=0$

ME:- $\frac{\partial V}{\partial t}+\frac{1}{\rho}\frac{\partial P}{\partial x}+\frac{fV|V|}{2D}=0$

Putting, $P=\rho gH$ and $V=\frac{Q}{A}$.

CE $\Rightarrow \rho g\frac{\partial H}{\partial t}+\rho a^2\frac{\partial}{\partial x}\left(\frac{Q}{A}\right)=0$

$\Rightarrow \rho g\frac{\partial H}{\partial t}+\frac{\rho a^2}{A}\cdot\frac{\partial Q}{\partial x}=0$

$\Rightarrow \frac{\partial H}{\partial t}+\frac{a^2}{gA}\cdot\frac{\partial Q}{\partial x}=0 \quad \ldots (13)$

ME $\Rightarrow \frac{1}{A}\frac{\partial Q}{\partial t}+\frac{\rho g}{\rho}\frac{\partial H}{\partial x}+\frac{fQ|Q|}{2A^2D}=0$

$\Rightarrow \frac{\partial Q}{\partial t}+gA\frac{\partial H}{\partial x}+\frac{fQ|Q|}{2AD}=0 \quad \ldots (14)$

Wave velocity:-

Function of bulk modulus of fluid, ρ_{fluid}, elastic property (D, e, E), external conditions (type of joints, anchoring conditions).

$$a=\sqrt{\frac{K}{\rho\left(1+\frac{K}{E}\psi\right)}}$$

$\psi \rightarrow$ External conditions.

Air present ⇒ Wave speed ↓ ⇒ Water hammer pressure ↓.

Air-water mixture is uniformly distributed throughout the pipeline.

(A) Pipe rigidly anchored ⇒ $\sigma_1 = \mu\sigma_2$, $\varepsilon_1 = 0$.

(B) Pipe with expansion joints ⇒ $\sigma_1 = 0$.

(C) Pipe anchored at one end and free to stretch longitudinally. $\sigma_1 = \frac{\sigma_2}{2}$.

$$a = \sqrt{\frac{K}{\rho\left(1 + \frac{KD}{Ee}\psi\right)}}$$

ψ = Non-dimensional parameter depends on restraint type.

(I) Thin walled elastic $\left(\frac{e}{D} < 0.06\right)$

(1) Pipe anchored only at u/s:-

$$\psi = \left(\frac{5}{4} - \mu\right) \text{ or } (1 - 0.5\mu)$$

(2) Full pipe restraint from axial movement:-

$$\psi = 1 - \mu^2$$

(3) Long. expansion joints along pipeline:-

$$\psi = 1.0$$

(4) Rigid conduit, ... (15)'s

$$\psi = 0$$

(II) Thick walled elastic:-

(1) Conduits anchored against longitudinal movement throughout its length:-

$$\psi = \frac{2e}{D}(1+\mu)\left[\frac{R_o^2 + R_i^2}{R_o^2 - R_i^2} - \frac{2\mu R_i^2}{R_o^2 - R_i^2}\right]$$

where, $e = R_o - R_i$

(2) Anchored against long. movement at u/s end:-

$$\psi = \frac{2e}{D}\left[\frac{R_o^2 + 1.5R_i^2}{R_o^2 - R_i^2} + \frac{\nu(R_o^2 - 3R_i^2)}{R_o^2 - R_i^2}\right]$$

(3) Conduits with frequent ex (expansion joints)?

$$\psi = \frac{2e}{D}\left(\frac{R_o^2 + R_i^2}{R_o^2 - R_i^2} + \mu\right) \quad \text{..... (16)'s}$$

(III) Rock tunnel:-

(1) Unlined tunnel:-

$$\psi = \frac{e}{D}$$

$$a = \sqrt{\frac{K}{\rho\left(1 + \frac{KD}{Ee}\cdot\frac{e}{D}\right)}} = \sqrt{\frac{K}{\rho\left(1 + \frac{K}{E}\right)}}$$

Don't worry about E. It got cancelled out

and $E = G_1$, G_1 = Modulus of rigidity of rock.

(2) Steel lined tunnel:-

$$\psi = \frac{e}{D}\left(\frac{DE}{GD + Ee}\right)$$

e = Thickness of the steel liner.

E = Modulus of elasticity of steel.

(IV) Reinforced Concrete pipe:-

RCC pipe is replaced by an equivalent steel pipe having equivalent thickness.

$$e_e = E_r e_c + \frac{A_s}{l_s}$$

e_c = Concrete pipe thickness

$E_r = \frac{E_c}{E_s}$

$A_s, l_s \rightarrow$ C/s area and spacings of steel bars. ..... (17)3

Eg:- Wave velocity at steel penstock=?

$E_{steel} = 207 \times 10^9$ Pa.

$G_{concrete} = 20.7 \times 10^9$ Pa.

$K_{water} = 2.19 \times 10^9$ Pa.

$\rho_{water} = 999$ kg/m³.

Pipe no	length (m)	Diameter (m)	Wall thickness (mm)	Type of anchoring
1	244	6.771	19	Expansion coupling at one end.
2	36.5	5.55	22	Encased in concrete.

Pipe 1:-

$$\frac{e}{D} = \frac{19 \times 10^{-3}}{6.771} = 2.806 \times 10^{-3} < 0.006.$$

∴ Thin-walled pipe.

Expansion coupling at one end ≡ Pipe anchored only at u/s.

$\therefore \psi = 1 - 0.5\mu = 1 - 0.5 \times 0.3 = 0.85$ → This is taken.

or, $\psi = \frac{5}{4} - \mu$ → but, this is not taken. ?

$= \frac{5}{4} - 0.3 = 0.95$

$$a = \sqrt{\frac{K}{\rho\left(1 + \frac{KD}{Ee}\right)}} = \sqrt{\frac{2.19 \times 10^9}{999\left(1 + \frac{2.19 \times 10^9 \times 6.771}{207 \times 10^9 \times 0.019} \times 0.85\right)}}$$

$= 722.055$ m/s

Pipe 2:-

For steel line channel,

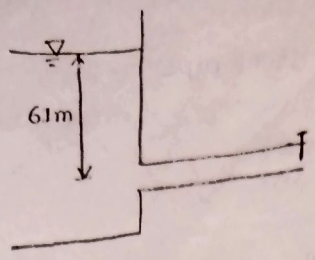
$$\psi = \frac{e}{D}\left(\frac{DE}{GD + Ee}\right)$$

$$= \frac{0.022}{5.55} \times \frac{5.55 \times 207 \times 10^9}{20.7 \times 10^9 \times 5.55 + 207 \times 10^9 \times 0.022}$$

$= 0.0381$

$$a = \sqrt{\frac{2.19 \times 10^9}{999\left(1 + \frac{2.19 \times 10^9 \times 5.55}{207 \times 10^9 \times 0.022} \times 0.0381\right)}}$$

$= 1400.45$ m/s.



$E = 207 \times 10^9$ Pa
$K = 2.19 \times 10^9$ Pa
$\rho = 1000$ Kg/m^3
Dia $= 0.61$ m
$e = 0.00635$ m.

$\frac{e}{D} = \frac{0.00635}{0.61} = 0.0104 < 0.06$
(Thin walled pipe)

Wave velocity for different cases:-

(a) Anchored only at u/s end:-

$\psi = 1 - 0.5\mu = 1 - 0.5 \times 0.3 = 0.85$

$$a = \sqrt{\frac{2.19 \times 10^9}{1000\left(1 + \frac{2.19 \times 10^9 \times 0.61}{0.00635 \times 207 \times 10^9} \times 0.85\right)}}$$

$= 1083.96$ m/s

(b) Pipe restrained from axial movement:-

$\psi = 1 - \mu^2 = 1 - 0.3^2 = 0.91$

$$a = \sqrt{\frac{2.19 \times 10^9}{1000\left(1 + \frac{2.19 \times 10^9 \times 0.61}{0.00635 \times 207 \times 10^9} \times 0.91\right)}}$$

$= 1066.65$ m/s.

(c) Long. expansion joints along the pipelines:-

$\psi = 1.0$

$a = 1042.17$ m/s

(d) Rigid pipes, $\psi = 0$

$\therefore a = \sqrt{\frac{K}{\rho}} = \sqrt{\frac{2.19 \times 10^9}{1000}} = 1479.86$ m/s.

Steady state velocity is 2 m/s and valve closed suddenly. determine pressure rises.

$\Delta H = -\frac{a\Delta V}{g} \Rightarrow \Delta P = \rho g \Delta H = -\rho g \times \frac{a\Delta V}{g} = -\rho a \Delta V$

Case(a):- $\Delta P = -1083.96 \times (0-2) \times 1000 = 2.168 \times 10^6$ Pa

Case(b): $\Delta P = 2.133 \times 10^6$

Case (c): $\Delta P = 2.08 \times 10^6$

Determine change in pipe wall stress due to increase of pressure:-

Case(a):- Anchored at u/s end:-

$\Delta\sigma_2 = \frac{\Delta P D}{2e}$

σ_2 = Hoop or Circumferential stress.
σ_1 = Longitudinal stress

? $\Delta\sigma_1 = \frac{1}{2}\Delta\sigma_2$

$= \frac{2.168 \times 10^6 \times 0.61}{4 \times 0.00635} = 52.06 \times 10^6$ Pa.

Case(b):- Pipe restrained from both end / Anchored at both end / restrained from axial movement.

? $\Delta\sigma_1 = \mu\Delta\sigma_2$

$\Delta\sigma_2 = \frac{\Delta P D}{2e} = \frac{\overset{2.13 \times 10^6}{1067} \times 0.61}{2 \times 0.00635} = 102.3 \times 10^6$ Pa.

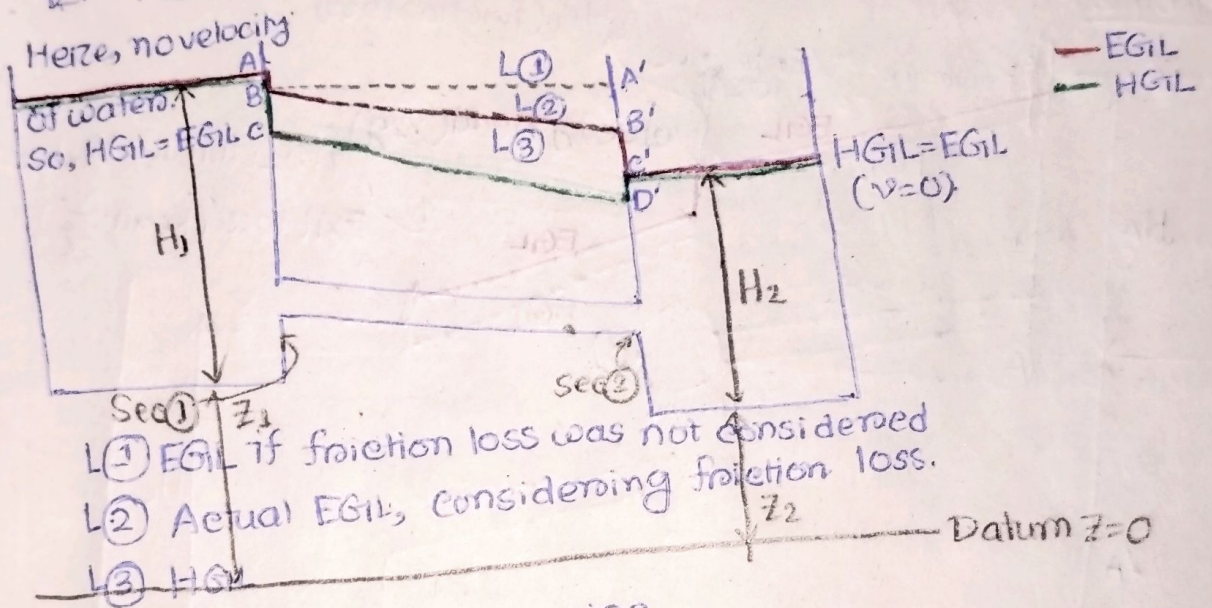
9th Week:

$$HGIL = \underbrace{\frac{P}{\gamma}}_{\text{Pressure head}} + \underbrace{z}_{\text{Elevation head}}$$

☑ HGIL measured by piezometer ⇒ standing pipe.

$$EGIL = HGIL + \text{velocity head}$$
$$= \left(\frac{P}{\gamma} + z\right) + \frac{v^2}{2g}.$$

☑ Measured using pitot tube.



L① EGIL if friction loss was not considered

L② Actual EGIL, considering friction loss.

L③ HGIL

AB = Entrance loss in pipe

$BC = \frac{v^2}{2g}$

A'B' = Loss due to friction

B'C' = Loss due to exit from pipe.

B'D' = velocity head $= \frac{v^2}{2g}$

At section 1,

$$EGIL_1 = \underbrace{H_1 + Z_1}_{\text{EGIL at reservoir 1}} - \underbrace{K_{entrance} \cdot \frac{v^2}{2g}}_{\text{Entrance loss}} \quad \text{.....(36)}$$

$$HGIL_1 = EGIL_1 - \frac{v^2}{2g}$$ [Loss of HGIL due to water velocity changes from 0 (at reservoir) to v (at pipe)] .....(37)

At section 2,

$$\cancel{EGIL_2 = H_2 + Z_2 - K_{exit}\frac{v^2}{2g}}$$

$$\underbrace{EGIL_2}_{\text{Energy at pipe 2's}} - \underbrace{K_{exit}\frac{v^2}{2g}}_{\text{Exit loss}} = \underbrace{H_2 + Z_2}_{\text{EGIL at reservoir 2}}$$

$$\Rightarrow EGIL_2 = H_2 + Z_2 + K_{exit}\frac{v^2}{2g} \quad \text{.....(38)}$$

$$HGIL_2 = \underbrace{H_2 + Z_2 + K_{exit}\cdot\frac{v^2}{2g}}_{EGIL_2} - \underbrace{\frac{v^2}{2g}}_{\text{velocity head}} \quad \text{...(39)}$$

$\{(36)-(38)\}$,

$$-EGIL_1 + EGL_2 = \left(H_1 + Z_1 - K_{entrance}\frac{V^2}{2g}\right) + \left(H_2 + Z_2 + K_{exit} \times \frac{V^2}{2g}\right)$$

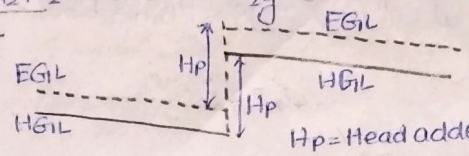
$$\Rightarrow EGIL_2 = EGIL_1 - \underbrace{\left\{\underbrace{\left(H_1+Z_1 - K_{entrance} \times \frac{V^2}{2g}\right)}_{\text{Total energy at u/s pipe end}} - \underbrace{\left(H_2+Z_2+K_{exit} \times \frac{V^2}{2g}\right)}_{\text{Total energy at d/s pipe end.}}\right\}}_{\text{This difference means head loss due to friction.}}$$

$$\therefore EGIL_2 = EGIL_1 - H_{Lf} \quad \text{------} (40)$$

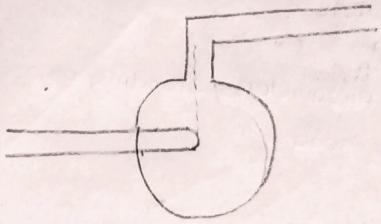
$$HGIL_2 = EGIL_2 - \frac{V^2}{2g} \quad (\text{From } (39))$$

$$H_2 + Z_2 = EGIL_2 - K_{exit}\frac{V^2}{2g} \quad (\text{From } (38)).$$

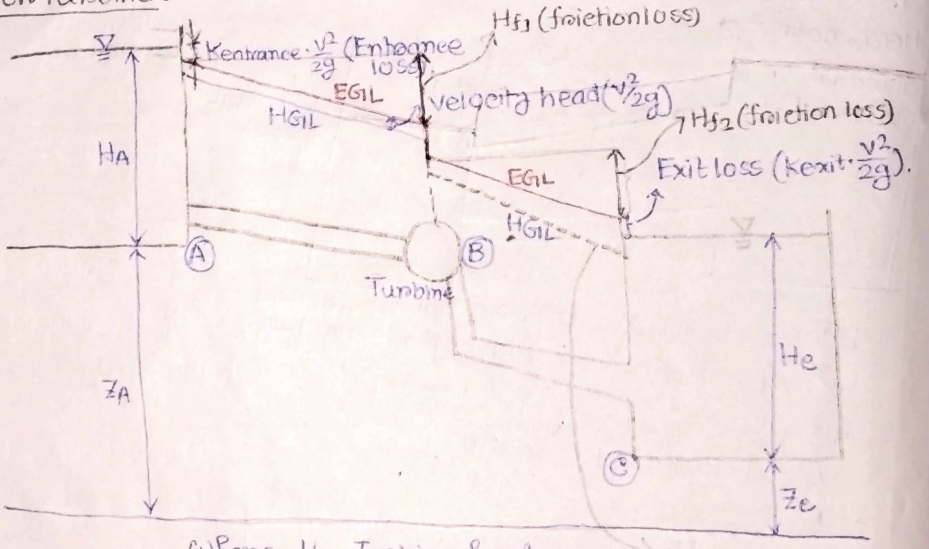
For pump:-



H_p = Head added by the pump.



For turbine:-



where, H_T = Turbine head.

I think, this HGIL line may touch below/above the water level of reservoir?.

$$EGIL_A = Z_A + H_A - K_{entrance}\frac{V^2}{2g}.$$

$$EGIL_B = EGIL_A - H_{f_1} - H_T$$

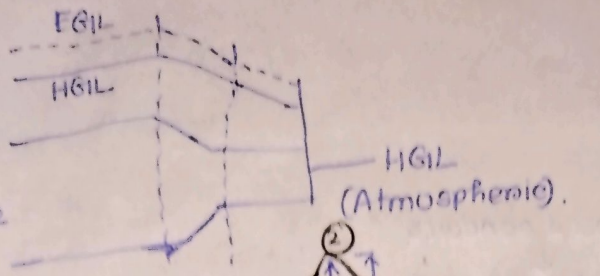
$$EGIL_C = EGIL_B - H_{f_2} - K_{exit} \cdot \frac{V^2}{2g}.$$

$$HGIL_A = EGIL_A - \frac{V^2}{2g}$$

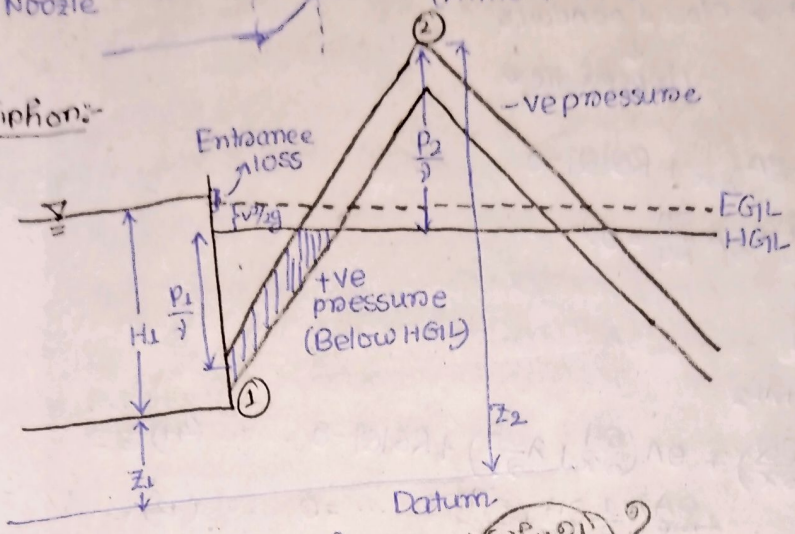
$$HGIL_B = EGIL_B - \frac{V^2}{2g}$$

$$HGIL_C = EGIL_C$$

Nozzle:-



Siphon:-



$$H_1 + Z_1 = Z_2 + \frac{P_2}{\gamma} + \cancel{\frac{V^2}{2g}}^{\,0} + \underbrace{H_{L_{1-2}}}_{\text{Head loss between node 1 and node 2.}}$$

why?!

$$\Rightarrow \frac{P_2}{\gamma} = H_1 + Z_1 - Z_2 - H_{L_{1-2}}.$$

$$\frac{P_2}{\gamma} > -0.7m$$

See (R1-P.43) for these eqns.

$$L_1 = \frac{\partial Q}{\partial t} + gA\frac{\partial H}{\partial x} + RQ|Q| = 0 \quad \cdots (3\text{-}1)$$

$$L_2 = a^2\frac{\partial Q}{\partial x} + gA\frac{\partial H}{\partial t} = 0 \quad \cdots (3\text{-}2) \quad R = \frac{f}{2DA}$$

These 2 equations valid everywhere in x-y plane

$L = L_1 + \lambda L_2$ λ = Unknown multiplier.

$$\left(\frac{\partial Q}{\partial t} + \lambda a^2\frac{\partial Q}{\partial x}\right) + gA\left(\frac{\partial H}{\partial x} + \lambda\frac{\partial H}{\partial t}\right) + RQ|Q| = 0$$

This term should not have any coefficient

$$\Rightarrow \left(\frac{\partial Q}{\partial t} + \lambda a^2\frac{\partial Q}{\partial x}\right) + \lambda gA\left(\frac{1}{\lambda}\frac{\partial H}{\partial x} + \frac{\partial H}{\partial t}\right) + RQ|Q| = 0 \quad \cdots (3\text{-}3)$$

Our target is to omit $\frac{\partial Q}{\partial x}$ ⇒ i.e. x related partial derivative terms

~~H(H,t)~~, ~~Q(Q,t)~~

$H = H(x,t)$, $Q = Q(x,t)$.

$$\frac{dQ}{dt} = \frac{\partial Q}{\partial t} + \frac{\partial Q}{\partial x}\cdot\frac{dx}{dt} \quad \cdots\cdots (3\text{-}4)$$

$$\frac{dH}{dt} = \frac{\partial H}{\partial t} + \frac{\partial H}{\partial x}\cdot\frac{dx}{dt} \quad \cdots\cdots (3\text{-}5)$$

$$\frac{dx}{dt} = \lambda a^2 \qquad \frac{\partial H}{\partial x}\cdot\frac{\partial x}{\partial t} = \frac{1}{\lambda} \quad (3\text{-}6)$$

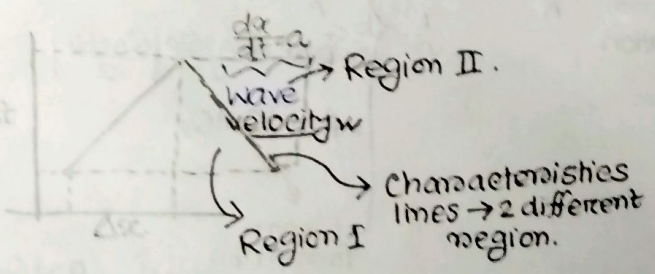
$$\lambda a^2 = \frac{1}{\lambda} \Rightarrow \lambda^2 = \frac{1}{a^2} \Rightarrow \lambda = \pm\frac{1}{a} \quad \cdots\cdots (3\text{-}7)$$

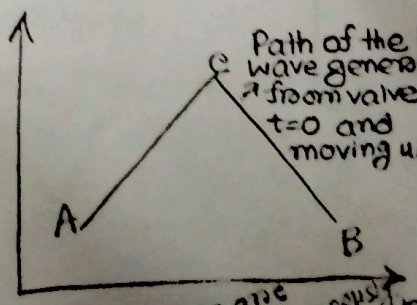
From (3-6) and (3-7), $\frac{dx}{dt} = a \quad \cdots (3\text{-}9)$ $\Rightarrow \frac{dQ}{dt} + \frac{gA}{a}\frac{dH}{dt} + RQ|Q| = 0 \quad \cdots\cdots (3\text{-}8)$

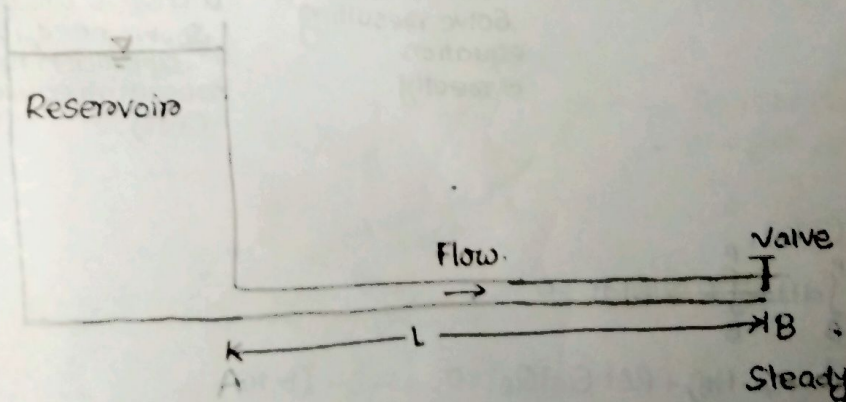
and $\frac{dx}{dt} = -a \quad \cdots (3\text{-}11)$ $\Rightarrow \frac{dQ}{dt} - \frac{gA}{a}\frac{dH}{dt} + RQ|Q| = 0 \quad \cdots\cdots (3\text{-}10)$

These two are valid only along straight lines:- $\frac{dx}{dt} = a$ and $\frac{dx}{dt} = -a$.

3-8 and 3-10 are called compatibility equations:-







(Excitations are imposed simultaneously so, C lines are generated from both left and right boundary A & B simultaneously)!

Steady at t=0. Instantly closed. wave generated and moves toward u/s.

Compute Transient pressure and discharge:-

(Head = H, discharge = Q at $t = t_0$ known at A and B)

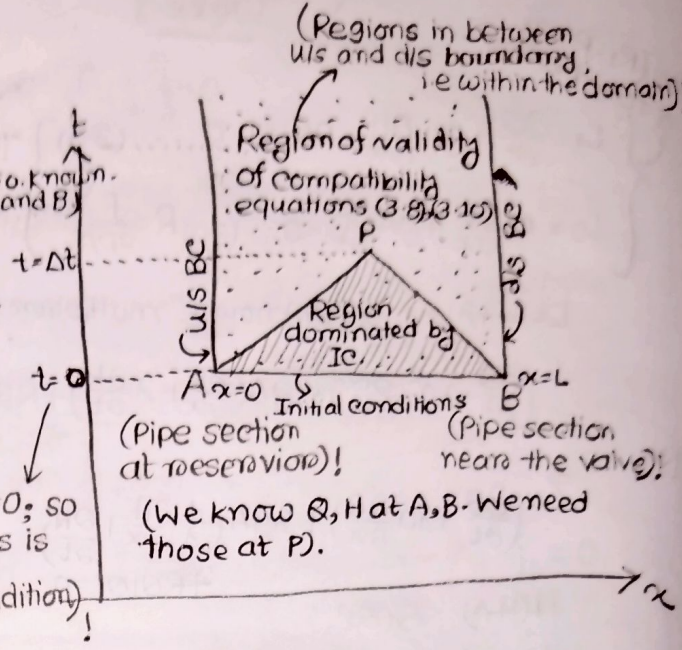
(We need H & Q at $t = t_0 + \Delta t$ at point P.)

Multiply (3.8) by dt and integrate.

Positive characteristic: (line AP).

$$\int_A^P dQ + \frac{gA}{a}\int_A^P dH + \int_A^P RQ|Q|\,dt = 0 \quad \text{.....(3-12)}$$

(Eqn is valid only along C-line. So, limit is taken from A to P)



(Pipe section at reservoir)!

(Pipe section near the valve)!

(t = 0; so this is initial condition)

(We know Q, H at A, B. We need those at P).

$$\Rightarrow (Q_P - Q_A) + \frac{gA}{a}(H_P - H_A) + RQ_A|Q_A|(t_P - t_A) = 0$$

We don't know variation of Q. wrt Δt. So, approximate discharge = Q_A

$$\Rightarrow Q_P - Q_A + \frac{gA}{a}(H_P - H_A) + R\Delta t\, Q_A|Q_A| = 0 \quad \text{.....(3-14)}$$

This 1st order approximation may give unstable result if friction term $R = \frac{f}{2DA}$ is very large.

To avoid, we need Short Δt / Higher order approximation/iterative procedure.

Second order apporixation of $\int_A^P RQ|Q|\,dt$ (3rd term of equation) $= R\int_A^P Q|Q|\,dt = \frac{1}{2}R\Delta t\left(Q_A|Q_A| + Q_P|Q_P|\right)$ (3-15a)

$$= R\Delta t\,\frac{Q_A + Q_P}{2}\left|\frac{Q_A + Q_P}{2}\right| \quad \text{.....(3.15.b)}$$

$$= R\Delta t\,|Q_A|Q_P \quad \text{.....(3.15.c)}$$

Linear form. can solve resulting equation directly.

non-linear form. Q_A and Q_P is unknown. So, we need iterative approach to solve resulting equation (3-12).

These are for reference purpose. Let us keep 1st order approximation i.e (3.14).

Similarly, for eqn (3.10),

Negative characteristic:- (line BP)

$$\int_B^P dQ - \frac{gA}{a}\int_B^P dH + \int_B^P RQ|Q|\,dt = 0$$

$$\Rightarrow Q_P - Q_B - \frac{gA}{a}(H_P - H_B) + R\Delta t\, Q_B|Q_B| = 0 \quad \text{.....(3-16)}$$

From (3.14) and (3.16).

$$Q_P = -\underbrace{\frac{gA}{a}}_{C_a} H_P + \underbrace{Q_A + \frac{gA}{a} H_A - R\Delta t\, Q_A|Q_A|}_{C_p}$$

$$\Rightarrow Q_P = -C_a H_P + C_p \quad \text{.....} \quad (3.17) \Rightarrow \text{PCE}$$

C_a = constant = $\frac{gA}{a}$ (depends on conduit property)

C_p, C_n constant for each time step only.

From (3.16).

$$Q_P = \underbrace{\frac{gA}{a}}_{C_a} H_P + \underbrace{Q_B - \frac{gA}{a} H_B - R\Delta t\, Q_B|Q_B|}_{C_n}$$

$$\Rightarrow Q_P = C_a H_P + C_n \quad \text{.....} \quad (3.18) \Rightarrow \text{NCE}$$

Solve 3.17 and 3.18

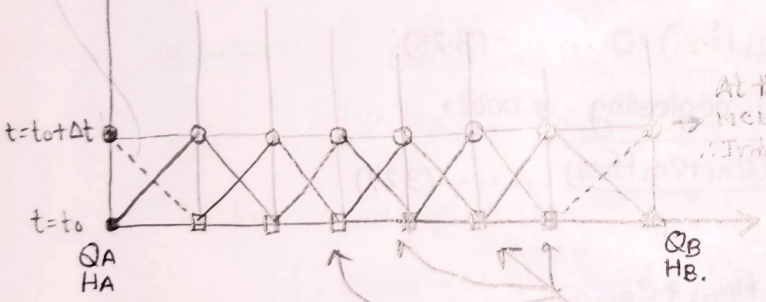
$$Q_P = \frac{1}{2}(C_p + C_n) \quad \text{.....} \quad (3.22)$$

$$H_P = \frac{Q_P - C_n}{C_a} = \frac{0.5(C_p + C_n) - C_n}{C_a} = \frac{0.5(C_p - C_n)}{C_a} \quad \text{...} \quad (3.22.1)$$

PCE (3.17) is used for d/s boundary
NCE (3.18) is used for u/s boundary.

$Q_{P_{i,j}}$ = Flow of jth junction of ith conduit
C_{a_i} = constant for i^{th} conduit.
$C_p, C_n \rightarrow$ Though may vary from one section to another, one subscript may be used to indicate the conduit number.

Why vary? $\Rightarrow$ Because, R is present $R = \frac{f}{2DA}$ $\rightarrow$ Friction factor may vary if different parts of pipe is



At this time level, interior nodes have PCL, and NCL both. But, u/s and d/s node have only one. ∴ Interior node evaluated. u/s, d/s not.

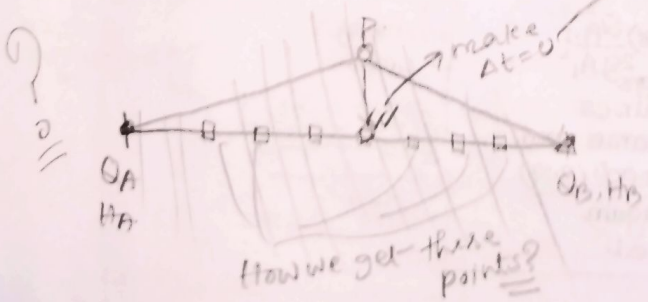
At t=0 to, (Steady state condition), we know (Q_A, H_A) and (Q_B, H_B)
So, to find these two variables for each interior nodes, we use (3.22) and (3.22.1). i.e. $Q_P = 0.5(C_p + C_n)$

$$\Rightarrow Q_P = 0.5\left\{\left(Q_A + \frac{gA}{a} H_A - R\Delta t\, Q_A|Q_A|\right) + \left(Q_B + \frac{gA}{a} H_B - R\Delta t\, Q_B|Q_B|\right)\right\}$$

(u/s and d/s ends are designated using initial flow direction, though flow may reverse during transient state)

(As it is initial time step, put Δt = 0 here. and get Q_P for all these points)?! Really!

Will give Q_P a constant value then! Not possible for all node points.



Boundary Conditions:

BC- Constant level u/s reservoir:-

entrance loss $= \frac{K Q_{P_{i,1}}^2}{2gA_i^2}$ → i^{th} pipe, 1st junction ..... (3.23)

Assumption valid when reservoir volume is large.

$$H_{P_{i,1}} = \underbrace{H_{res}}_{\text{Reservoir height above datum}} - \underbrace{\frac{Q_{P_{i,1}}^2}{2gA_i^2}}_{\text{Velocity head}} - \underbrace{\frac{K Q_{P_{i,1}}^2}{2gA_i^2}}_{\text{entrance loss.}}$$

$$H_{P_{i,1}} = H_{res} - (1+K)\frac{Q_{P_{i,1}}^2}{2gA_i^2} \quad \text{..... (3.24)}$$

To develop the boundary condition, we solve this equation simultaneously with negative characteristic equation (3.18). ⇒ ~~why -ve characteristic equation?~~

(At u/s, we don't have any +ve characteristics line. So, NCE and head specified BC (eqn 3.24) these 2 conditions are useful) → I.T.

From (3.18), $Q_P = C_a H_P + C_n$

$$\Rightarrow H_{P_{i,1}} = \frac{Q_{P_{i,1}} - C_{n_i}}{C_{a_i}}$$

$$H_{res} - (1+K)\frac{Q_{P_{i,1}}^2}{2gA_i^2} = \frac{Q_{P_{i,1}} - C_{n_i}}{C_{a_i}}$$

$$\Rightarrow C_{a_i} \cdot H_{res} - \frac{C_{a_i}(1+K)}{2gA_i^2} Q_{P_{i,1}}^2 = Q_{P_{i,1}} - C_{n_i}$$

where, $\frac{C_{a_i}(1+K)}{2gA_i^2} = K_1$ and K = entrance loss coefficient.

$$\Rightarrow C_{a_i} H_{res} - K_1 Q_{P_{i,1}}^2 = Q_{P_{i,1}} - C_{n_i}$$

$$\Rightarrow K_1 Q_{P_{i,1}}^2 + Q_{P_{i,1}} - (C_{n_i} + C_{a_i} \cdot H_{res}) = 0 \quad \text{..... (3.25)}$$

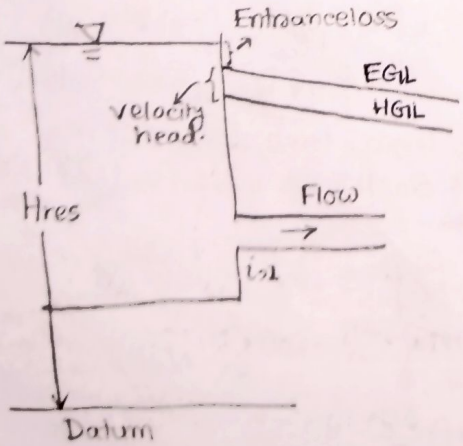
Solving quadratic equation (3.25), neglecting -ve root.

$$Q_{P_{i,1}} = \frac{-1 + \sqrt{1 + 4K_1(C_{n_i} + C_{a_i} H_{res})}}{2K_1} \quad \text{..... (3.27)}$$

From eqn (3.18), $Q_{P_{i,1}} = C_{a_i} H_{P_{i,1}} + C_{n_i}$

$$\Rightarrow H_{P_{i,1}} = \frac{Q_{P_{i,1}} - C_{n_i}}{C_{a_i}}$$

[Putting $Q_{P_{i,1}}$ value from (3.27), we get $H_{P_{i,1}}$ value].

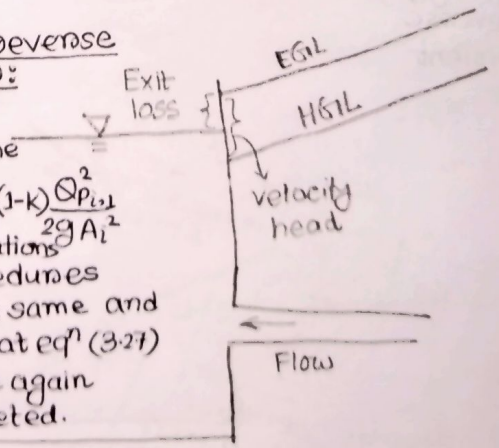


For reverse flow:

Eqn (3.24) will become

$$H_{P_{i,1}} = H_{res} - (1-K)\frac{Q_{P_{i,1}}^2}{2gA_i^2}$$

Other equations and procedures should be same and -ve root at eqn (3.27) should be again neglected.



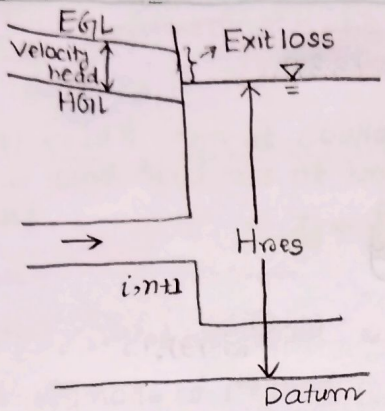
Special case:- When velocity head and entrance losses are negligible,

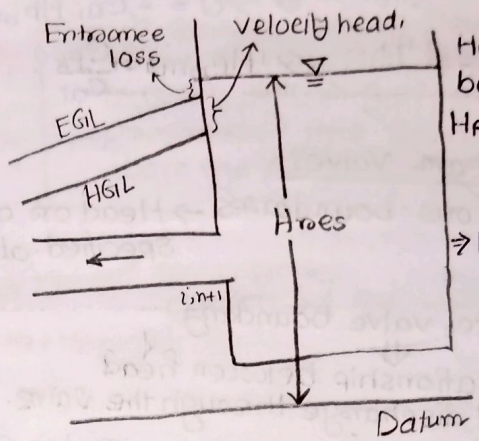
from eq^n (3.24), $H_{P_{i,n+1}} = H_{res}$ ----- (3.28)

∴ From -ve characteristics equation,

$$Q_{P_{i,n+1}} = C_{ai} \cdot H_{res} + C_{ni} \quad \text{----- (3.29)}$$

3e- Constant level dis reservoir:-





(Reverse flow condition).

Here, equation (3.31) becomes,

$H_{P_{i,n+1}} = H_{res}$ - Entrance loss - velocity head.

$\Rightarrow H_{P_{i,n+1}} = H_{res} - (1+K)\dfrac{Q^2_{P_{i,n+1}}}{2gA_i^2}$

For initial or steady-state flow direction, we have,

$$\text{Exit loss}, = \frac{KQ^2_{P_{i,n+1}}}{2gA_i^2} \quad \text{----------- (3.30)}$$

$$H_{P_{i,n+1}} = H_{res} + \frac{KQ^2_{P_{i,n+1}}}{2gA_i^2} - \frac{Q^2_{P_{i,n+1}}}{2gA_i^2}$$

$$H_{P_{i,n+1}} = H_{res} - (1-K)\frac{Q^2_{P_{i,n+1}}}{2gA_i^2} \quad \text{-------- (3.31).}$$

PCE:- (3.17), $Q_{P_{i,n+1}} = -C_{ai} H_{P_{i,n+1}} + C_{Pi}$

$$\Rightarrow H_{P_{i,n+1}} = \frac{C_{Pi} - Q_{P_{i,n+1}}}{C_{ai}}$$

Put this value to equation (3.31),

$$H_{res} - (1-K)\frac{Q^2_{P_{i,n+1}}}{2gA_i^2} = \frac{C_{Pi} - Q_{P_{i,n+1}}}{C_{ai}}$$

$$\Rightarrow C_{ai} \cdot H_{res} - \frac{C_{ai}(1-K)}{2gA_i^2} Q^2_{P_{i,n+1}} = C_{Pi} - Q_{P_{i,n+1}}.$$

$$\Rightarrow K_2 Q^2_{P_{i,n+1}} - Q_{P_{i,n+1}} + C_{Pi} - C_{ai} \cdot H_{res} = 0 \quad \text{-----(3.32)}$$

where, $K_2 = \dfrac{C_{ai}(1-K)}{2gA_i^2}$

Solving quadratic equation of (3.32),

Why positive (+) root is neglected??

$$\rightarrow Q_{P_{i,n+1}} = \frac{1 - \sqrt{1 - 4K_2(C_{Pi} - C_{ai} \cdot H_{res})}}{2K_2} \quad \text{----- (3.34).}$$

But why? লেখানোর $ax^2+bx+c=0$ সমীকরণে যদি $b=+ve$ হয়, তাহলে $\dfrac{-b+\sqrt{b^2-4ac}}{2a}$ root নেওয়া হবে। $b=-ve$ হলে, $\dfrac{-b-\sqrt{b^2-4ac}}{2a}$ root টি নেওয়া হয়।

I.T ⇒ observe, other BCs

Special case:- When velocity head and entrance losses are negligible, $H_{P_{i,n+1}} = H_{res}$

From PCE, $Q_{P_{i,n+1}} = C_{Pi} - C_{ai} \cdot H_{res}$.

BC- Dead end at d/s

Dead end located at the d/s of the ith conduit, then,

$$Q_{P_{i,n+1}} = 0$$

From PCE, (3.17), $Q_{P_{i,n+1}} = -C_{ai} H_{P_{i,n+1}} + C_{P_i}$

$$\Rightarrow \quad 0 = -C_{ai} H_{P_{i,n+1}} + C_{P_i}$$

$$\Rightarrow H_{P_{i,n+1}} = \frac{C_{P_i}}{C_{ai}} \quad \text{.....} (3.37).$$

BC- Downstream Valve:-

Previous boundaries → Head or discharge specified at boundary

Here, valve boundary
⇓
Relationship between head and discharge through the valve.

Steady-state flow through a valve discharging into air,

$$Q_{o_{i,n+1}} = (C_d A_v)_o \sqrt{2g H_{o_{i,n+1}}} \quad \text{.....} (3.38)$$

↓ Area of valve opening.

Transient state,

$$Q_{P_{i,n+1}} = (C_d A_v) \sqrt{2g H_{P_{i,n+1}}} \quad \text{.....} (3.39)$$

$\{(3.39) \div (3.38)\}^2$

$$\frac{Q^2_{P_{i,n+1}}}{Q^2_{o_{i,n+1}}} = \underbrace{\left\{\frac{C_d A_v}{(C_d A_v)_o}\right\}^2}_{\tau} \frac{H_{P_{i,n+1}}}{H_{o_{i,n+1}}}$$

where,
$$\tau = \left\{\frac{C_d A_v}{(C_d A_v)_o}\right\}^2$$

$$\Rightarrow Q^2_{P_{i,n+1}} = (Q_{o_{i,n+1}} \tau)^2 \cdot \frac{H_{P_{i,n+1}}}{H_{o_{i,n+1}}} \quad \text{.....} (3.40)$$

From +ve characteristics equation (PCE),

$$H_{P_{i,n+1}} = \frac{C_{P_i} - Q_{P_{i,n+1}}}{C_{ai}} \quad \text{.....} (3.40.1)$$

From (3.40), $H_{P_{i,n+1}} = \left(\frac{Q_{P_{i,n+1}}}{Q_{o_{i,n+1}} \cdot \tau}\right)^2 H_{o_{i,n+1}} = \frac{C_{P_i} - Q_{P_{i,n+1}}}{C_{ai}}$

$$\Rightarrow Q^2_{P_{i,n+1}} = \frac{(Q_{o_{i,n+1}} \cdot \tau)^2}{H_{o_{i,n+1}} \cdot C_{ai}} (C_{P_i} - Q_{P_{i,n+1}})$$

$$\Rightarrow Q^2_{P_{i,n+1}} = C_v (C_{P_i} - Q_{P_{i,n+1}})$$

where,
$$C_v = \frac{(Q_{o_{i,n+1}} \cdot \tau)^2}{H_{o_{i,n+1}} \cdot C_{ai}}$$

$$\Rightarrow Q^2_{P_{i,n+1}} + C_v Q_{P_{i,n+1}} - C_v \cdot C_{P_i} = 0 \quad \text{...} (3.41)$$

Neglecting root with negative sign with radical term,

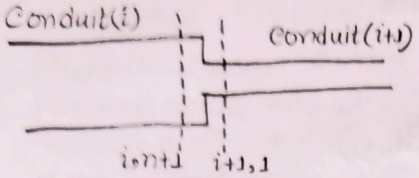
$$Q_{P_{i,n+1}} = \frac{-C_v + \sqrt{C_v^2 + 4 C_v \cdot C_{P_i}}}{2} \quad \text{....} (3.42)$$

Put this value to PCE (3.40.1) to get $H_{P_{i,n+1}}$.

For orifice located at d/s end, $\tau = \left\{\frac{C_d A_v}{(C_d A_v)_o}\right\}^2 = 1$, because orifice opening remains constant

Series Junction:-

Two conduits having different diameters, friction factor, wall thickness.



Let, Velocity head is same at section $(i,n+1)$ and $(i+1,1)$ and head loss at junction neglected

$$\underline{\underline{I.T}} \Rightarrow \frac{P_1}{\rho g} + \cancel{\frac{V_1^2}{2g}} + \cancel{Z_1}$$

$$= \frac{P_2}{\rho g} + \cancel{\frac{V_2^2}{2g}} + \cancel{Z_2} + \cancel{H_L}^{\,0} \quad (Z_1 = Z_2 = \text{same elevation for both pipes.})$$

$$\Rightarrow \frac{P_1}{\rho g} = \frac{P_2}{\rho g} \Rightarrow H_1 = H_2$$

$$H_{P_{i,n+1}} = H_{P_{i+1,1}} \quad \cdots\cdots (3.43)$$

PCE for d/s node of ith pipe $(i,n+1)$,

$$Q_{P_{i,n+1}} = -C_{a_i} \cdot H_{P_{i,n+1}} + C_{P_i} \quad \cdots\cdots (3.44)$$

NCE for u/s node of $(i+1)^{th}$ pipe,

$$Q_{P_{i+1,1}} = C_{a_{i+1}} \cdot H_{P_{i+1,1}} + C_{n_{i+1}} \quad \cdots\cdots (3.45)$$

Continuity equation at the junction,

$$Q_{P_{i,n+1}} = Q_{P_{i+1,1}}$$

$$\Rightarrow -C_{a_i} \cdot H_{P_{i,n+1}} + C_{P_i} = C_{a_{i+1}} \cdot H_{P_{i+1,1}} + C_{n_{i+1}} \quad \text{(From 3.44 and 3.45)}$$

$$\Rightarrow H_{P_{i,n+1}} (C_{a_{i+1}} + C_{a_i}) = C_{P_i} - C_{n_{i+1}} \quad \text{(From 3.43).}$$

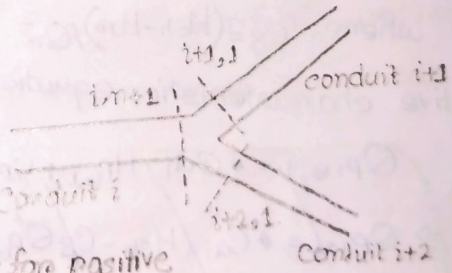
$$\Rightarrow H_{P_{i,n+1}} = \frac{C_{P_i} - C_{n_{i+1}}}{C_{a_{i+1}} + C_{a_i}} \quad \cdots\cdots (3.47)$$

Values of $H_{P_{i+1,1}}$, $Q_{P_{i,n+1}}$, $Q_{P_{i+1,1}}$ are determined by putting value of $H_{P_{i,n+1}}$ (from 3.47) to (3.43), (3.44), (3.45).

Branching Junction:-

Continuity equation,

$$Q_{P_{i,n+1}} = Q_{P_{i+1,1}} + Q_{P_{i+2,1}} \quad \cdots\cdots (3.48)$$



PCE for node $(i,n+1)$ and NCE for nodes $(i+1,1)$ and $(i+2,1)$, we have,

$$Q_{P_{i,n+1}} = -C_{a_i} H_{P_{i,n+1}} + C_{P_i} \quad \cdots\cdots (3.49)$$

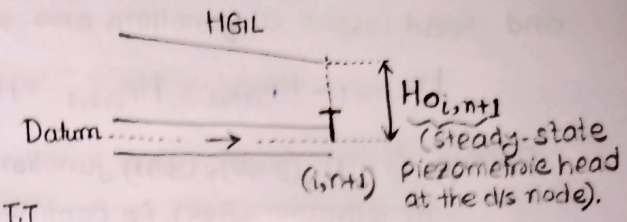
This eqn is for positive characteristics equation.

These two P's are from point P.

$$Q_{P_{i+1,1}} = C_{a_{i+1}} \cdot H_{P_{i+1,1}} + C_{n_{i+1}} \quad \cdots\cdots (3.50)$$

$$Q_{P_{i+2,1}} = C_{a_{i+2}} \cdot H_{P_{i+2,1}} + C_{n_{i+2}} \quad \cdots\cdots (3.51)$$

Valve at d/s end



I.T

At surface water flow on reservoir, HGL is the top water surface. But, pipe flow is pressurized flow. So, HGL would be above the pipe.

Let, velocity head is same for these 3 junctions and head losses at junctions are neglected then,

$$H_{P_{i,n+1}} = H_{P_{i+1,1}} = H_{P_{i+2,1}} = H \text{ (Let)} \quad \cdots\cdots (3.52)$$

Putting (3.49), (3.50), (3.51) junction discharge values in equation (3.48). i.e continuity equation.

$$-C_{ai} \cdot H + C_{Pi} = (C_{ai+1} \cdot H + C_{n,i+1}) + (C_{ai+2} \cdot H + C_{n,i+2})$$

$$\Rightarrow C_{Pi} - C_{ni+1} - C_{ni+2} = H(C_{ai} + C_{ai+1} + C_{ai+2})$$

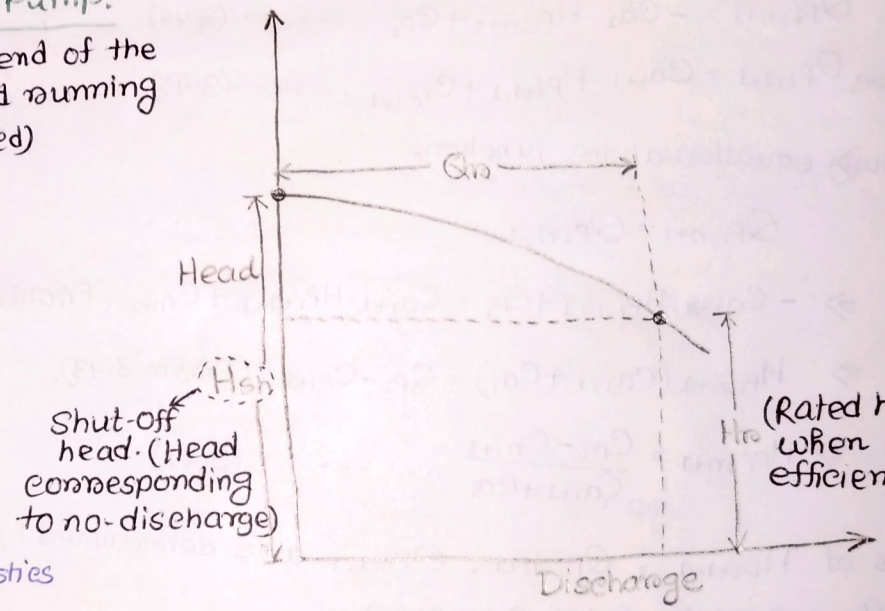
$$\Rightarrow H = \frac{C_{Pi} - C_{ni+2} - C_{ni+2}}{C_{ai} + C_{ai+1} + C_{ai+2}} \quad \cdots\cdots\cdots (3.53)$$

$\Downarrow$ Head value for all junctions.

To get all discharge values put H in equation (3.49), (3.50), (3.51).

UBC- Centrifugal Pump:-

(Located at u/s end of the ith conduit and running at constant speed)



Head-discharge curve for Centrifugal pump. (Pump-performance curve)

Pump characteristics equation,

u/s boundary, (i,1).

$$H_{P_{i,1}} = H_{sh} - C_8 Q^2_{P_{i,1}} \quad \cdots\cdots (3.54)$$

where, $C_8 = (H_{sh} - H_{ro})/Q_{ro}^2$

Negative characteristics equation for u/s end,

$$Q_{P_{i,1}} = C_{ai} \cdot H_{P_{i,1}} + C_{ni}$$

$$\Rightarrow Q_{P_{i,1}} = C_{ai}(H_{sh} - C_8 Q^2_{P_{i,1}}) + C_{ni}$$

$$\Rightarrow C_{ai} \cdot C_8 Q^2_{P_{i,1}} - Q_{P_{i,1}} + C_{ni} + C_{ai} H_{sh}$$

$$Q_{P_{i,1}} = C_{ai} H_{P_{i,1}} + C_{ni}$$

$$\Rightarrow Q_{P_{i,1}} = C_{ai}(H_{sh} - C_8 Q^2_{P_{i,1}}) + C_{ni}$$

$$\Rightarrow C_{ai} \cdot C_8 \cdot Q^2_{P_{i,1}} + Q_{P_{i,1}} - (C_{ai} \cdot H_{sh} + C_{ni}) = 0$$

$$\therefore Q_{P_{i,1}} = \frac{-1+\sqrt{1+4C_{a_i}C_8(C_{a_i}\cdot H_{sn}+C_{m_i})}}{2C_{a_i}\cdot C_8} \quad \ldots\ldots (3.55).$$

and $H_{P_{i,1}}$ can be determined from equation (3.54).

Francis Turbine:-

Head-discharge curve of francis turbine located at d/s end of ith conduit running at constant speed and constant gate opening:-

$$H_{P_{i,n+1}} = C_9 + C_{10} Q^2_{P_{i,n+1}} \quad \ldots (3.56)$$

Now, +ve characteristics line equation for d/s.

$$Q_{P_{i,n+1}} = -C_{a_i} H_{P_{i,n+1}} + C_{P_i}$$

$$\Rightarrow Q_{P_{i,n+1}} = -C_{a_i}(C_9 + C_{10} Q^2_{P_{i,n+1}}) + C_{P_i}$$

$$\Rightarrow C_{a_i} C_{10} Q^2_{P_{i,n+1}} + Q_{P_{i,n+1}} + (C_9 C_{a_i} - C_{P_i}) = 0$$

Solving, $$Q_{P_{i,n+1}} = \frac{-1+\sqrt{1-4C_{a_i}C_{10}(C_9 C_{a_i} - C_{P_i})}}{2C_{a_i}C_{10}} \quad \ldots (3.57)$$

$H_{P_{i,n+1}}$ obtained by putting this value to (3.56).

Main equation is:-

$$Q_P = -C_a H_P + C_P$$

Then add subscript for particular pipe number and junction number

$$Q_{P_{i,n+1}} = -C_{a_i} H_{P_{i,n+1}} + C_{P_i}$$

For accurate numerical solution of PDE's FD approximations should satisfy convergence and stability criteria.

Discretization error = $U(x,t) - u(x,t)$

$U(x,t)$ → Exact soln, $u(x,t)$ → FD solution

Local truncation error:- (C.G)

When we approximate the solution of a differential equation using numerical method, error that arises in each step of of a numerical method.

This error may be reduced by using more accurate numerical methods or by taking smaller steps in the numerical method.

The local truncation error is typically defined as difference between the exact solution and numerical solution at the next step, the exact solution as the initial condition.

Example:-

Suppose, we approximate derivative of a function $f(x)$ at point x_0 using FD formula:-

$$f'(x_0) = \frac{f(x_0+h) - f(x_0)}{h}.$$

Using Taylor's series:-

$$f(x_0+h) = f(x_0) + h f'(x_0) + \frac{h^2}{2!} f''(x_0) + \frac{h^3}{3!} f'''(x_0) + \cdots\cdots$$

Substituting this into forward difference formula,

$$\underbrace{f'(x_0)}_{\text{Forward difference derivative at } x_0} = \frac{f(x_0+h) - f(x_0)}{h}$$

$$= \frac{\left\{\cancel{f(x_0)} + h f'(x_0) + \frac{h^2}{2!} f''(x_0) + \frac{h^3}{3!} f'''(x) + \cdots\cdots\right\} - \cancel{f(x_0)}}{h}$$

$$= \underbrace{f'(x_0)}_{\text{This term is the exact derivative at } x_0.} + \underbrace{\frac{h}{2!} f''(x) + \frac{h^2}{3!} f^3(x) + \cdots\cdots}_{\text{This is the error came due to forward difference approximation.}}$$

= Local truncation error = $O(h)$

Truncation error ↓ → Take smaller steps, Use more accurate method

↓

But, computational cost and round off error may increase. So, we need an balance between accuracy and efficiency.

Round off error = Numerical solution of FDE from a real computer with finite accuracy - Exact solution of FDE

Discretization or Truncation error = Exact analytical solution of PDE - Exact solution of FDE

Truncation error (notes):-

$F_i^j(u) = 0$ ⇒ Finite difference solution

$U(i,j)$ is exact solution

$F_i^j(U)$ = Local truncation error

Consistency:-

If $(\Delta x, \Delta t) \to 0$ ⇒ Local truncation error tends to zero, then FD equation is called consistent.

Stability:-

Amplification of round-off errors for all sections remains bounded with progress of time.

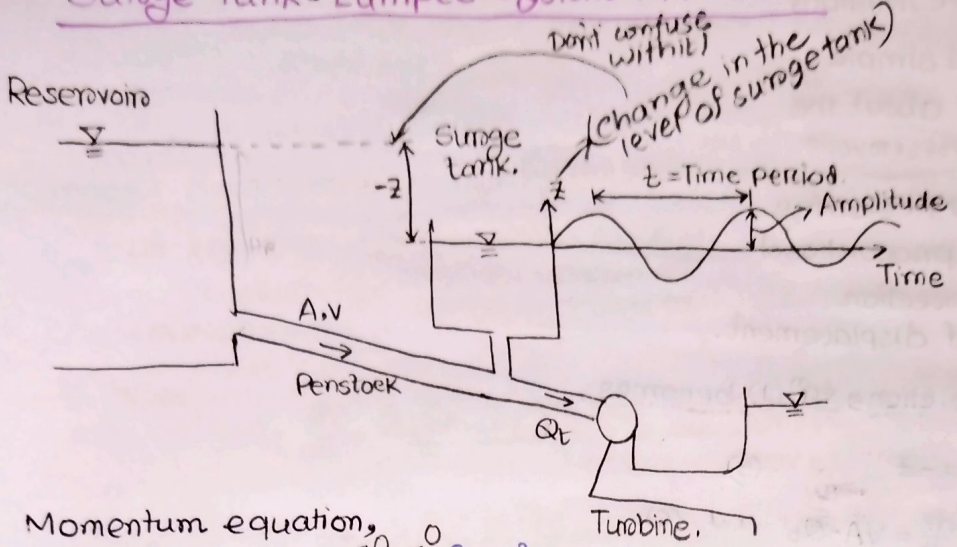
Explicit FD methods are stable, only if,

Courant number, $C_N \leq 1$

$$\Rightarrow \frac{\text{Actual wave speed}}{\text{Numerical wave speed}} \leq 1$$

$$\Rightarrow \frac{a}{\Delta x / \Delta t} \leq 1.$$

Surge Tank - Lumped System Approach



Momentum equation,

$$H_R - \left(1 + \cancel{K_e}^{0} + \cancel{K_v}^{0} + \frac{fL}{D}\right)\frac{V^2}{2g} = \frac{L}{g}\cdot\frac{dV}{dt}$$

neglect minor losses, velocity head and gravitational terms, we have, (Gravitational term was $= x S_0$, ↳ Neglected previously)

$$\Rightarrow H_R - \frac{fLV^2}{2gD} = \frac{L}{g}\cdot\frac{dV}{dt}$$

$$\Rightarrow \frac{L}{g}\cdot\frac{dV}{dt} = H_R - \frac{fLV|V|}{2gD}$$

Friction loss accounting the direction of the flow.

$$\Rightarrow \frac{L}{g}\cdot\frac{dV}{dt} = -z - \frac{fLV|V|}{2gD} \quad \text{------(1)}$$

why this is -z ??

Continuity equation,

Input - Output = storage at surge tank.

$$\Rightarrow VA - Q_t = A_s\frac{dz}{dt} \quad \text{------(2)}$$

V: Velocity at u/s of the surge tank

A: c/s area of the penstock

Q_t: Flow to the turbine

A_s: c/s area of the surge tank.

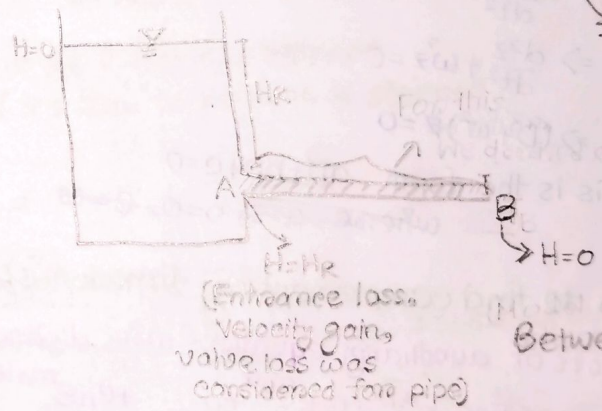
For steady state, equation (1) and (2) becomes,

$$-z = \frac{fLV|V|}{2gD} \quad \text{------(3)}$$

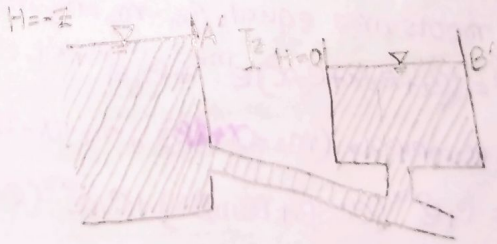
$$V_0A = Q_t \quad \text{------(4)}$$

I think, ?!

In previous problem,



Between A & B



Between A' & B'

By neglecting pipe friction,
Eq^n (1) and (2) yeild simple harmonic motion about the water level of the reservoir. → or surge tank?!

Periodic motion → Restoring force is directly proportional and acts in the direction opposite to that of displacement.

∴ Neglecting friction, eq^n (1) becomes,

$$\frac{L}{g}\frac{dv}{dt} = -z \quad \ldots\ldots (5)$$

Re-write eq^n (2), $A_s\frac{dz}{dt} = VA - Q_t \quad \ldots\ldots (6)$.

Differentiating equation (6) wrt t,

$$A_s\frac{d^2z}{dt^2} = A\cdot\frac{dv}{dt} - \cancel{\frac{dQ_t}{dt}}^{\,0}$$

(In lumped system, change of discharge is sudden)
↳ why? Turbine is closed suddenly??!

$$\Rightarrow A_s\frac{d^2z}{dt^2} = A\left(-\frac{gz}{L}\right)$$

↳ Putting value from eq^n (5)

$$\Rightarrow \frac{L}{g}A_s\frac{d^2z}{dt^2} = -zA.$$

$$\Rightarrow \frac{d^2z}{dt^2} = -\left(\frac{A}{A_s}\cdot\frac{g}{L}\right)z$$

$$\Rightarrow \frac{d^2z}{dt^2} = -\omega^2 z \quad \text{where, } \omega^2 = \frac{g}{L}\cdot\frac{A}{A_s} \Rightarrow \omega = \sqrt{\frac{gA}{LA_s}}$$

$$\Rightarrow \frac{d^2z}{dt^2} + \omega^2 z = 0$$

$$\Rightarrow (D^2+\omega^2)z = 0$$

This is the form $aD^2+bD+c=0$
~~where, a=z, b=0, c=ω~~

Methods to find complementary function (CF):-

If roots of auxiliary equations are distinct,

$$CF = c_1e^{m_1x} + c_2e^{m_2x} + \ldots\ldots + c_ne^{m_nx}$$

When some roots are equal, $(m_1=m_2=m_3=m, m_4, \ldots)$

$$CF = (c_1+c_2x+c_3x^2)e^{mx} + c_4e^{m_4x} + \ldots\ c_ne^{m_nx}$$

Roots are complex, $(m_1=\alpha+i\beta, m_2=\alpha-i\beta)$

$$CF = c_1e^{\alpha x}(\cos\beta + i\sin\beta x) + c_2e^{\alpha x}(\cos\beta x - i\sin\beta x)$$

Here, auxiliary equation, $D^2+\omega^2=0$ $\quad a=1, b=0, c=\omega^2$

$\therefore b^2-4ac = 0-4\times1\times\omega^2 = -4\omega^2<0$

$\therefore$ Roots are $= \dfrac{0\pm\sqrt{0-4\omega^2}}{2\times1} = 0\pm i\omega.$

$\therefore z = c_1\cos\omega t + c_2 \overset{\sin}{\cancel{\cos}}\,\omega t$ ..... (7).

Apply Boundary Conditions to equation (7),

at $t=0, z=0$ (Steady-state condition, No oscillation of water in surge tank) $\Rightarrow$ I.T.

$\therefore 0 = c_1\cos 0 + c_2 \sin 0$

$\Rightarrow c_1 = 0$

Also, at $t=0$, $\dfrac{dz}{dt} = \dfrac{Q-Q_t}{A_s}$ (Similar to equation 2, inflow - outflow = storage).

From eq^n (7), $\dfrac{dz}{dt} = -c_1\omega\sin\omega t + c_2\omega\cos\omega t$

$\Rightarrow \dfrac{Q-Q_t}{A_s} = c_2\omega\cos 0$

$\Rightarrow c_2 = \dfrac{Q-Q_t}{A_s\cdot\omega}$

Putting c_1, c_2 values from equation (7), $\boxed{z = \dfrac{Q-Q_t}{A_s\cdot\omega}\sin\omega t}$

$\therefore$ Amplitude $= z_{max} = \dfrac{Q-Q_t}{A_s\cdot\omega}$

☑ Pipe length = 500m, dia = 1.5m
Reservoir to turbine = $2\,m^3/s$.
Surge tank dia = 5m

Find $\begin{cases}\text{Max surge}\\ \text{Time period of pressure wave.}\end{cases}$

For $\begin{cases}\text{Case I: Entire flow to the turbine is stopped}\\ \text{Case-II: Only 50\% of the flow to turbine is stopped.}\end{cases}$

$A_p = \frac{\pi}{4}\times1.5^2 = 1.77\,m^2$

$A_s = \frac{\pi}{4}\times5^2 = \cancel{0.042}\ 19.63\,m^2$

Same for case I and case II.

$\Rightarrow \omega = \sqrt{\dfrac{g}{L}\cdot\dfrac{A_p}{A_s}} = \sqrt{\dfrac{9.81}{500}\times\dfrac{1.77}{19.63}} = 0.042\ m/s.$

Time period of pressure wave $= \dfrac{2\pi}{\omega} = \dfrac{2\pi}{0.042} = 150$ sec. I.T. $\Rightarrow$ Steady state discharge.

When entire flow to turbine is stopped, max surge $= \dfrac{Q_0-Q_t}{A_s\cdot\omega} = \dfrac{2-0}{19.63\times0.042} = 2.42\,m.$

When 50% of " " " " , max surge $= \dfrac{2-1}{19.63\times0.042} = 1.21\,m.$

I.T. $\Rightarrow$ Amplitude of surge wave.

Mathematical Modelling of surge tank:-

Continuity equation:- $A_s\dfrac{dz}{dt} = Q_{tunnel} - Q_{turbine}$ (From eq^n (2) and $Q_{tunnel} = AV = Q_p$).

$= Q_p - Q_t$

Penstock.

$\Rightarrow \boxed{\dfrac{dz}{dt} = \dfrac{Q_p-Q_t}{A_s} = \lambda}$ ..... (7)

Momentum equation:-

From equation (1),

$$\frac{L}{g}\cdot\frac{dv}{dt} = -z - \frac{fL|V|V|}{2gD}$$

$$\Rightarrow \frac{L}{g}\cdot\frac{dv}{dt} = -z - \frac{fL\,Q_p|Q_p|}{2gD\,A_p^2}$$

$$\Rightarrow \frac{L}{g}\cdot\frac{dv}{dt} = -z - C_F\,Q_p|Q_p|$$

where, $C_F = \frac{fL}{2gDA_p^2}$

$$\Rightarrow \boxed{K = \frac{dv}{dt} = \frac{g}{L}\left(-z - C_F\,Q_p|Q_p|\right)} \quad \ldots (8)$$

$$\boxed{\lambda = \frac{dz}{dt}} \quad K = \frac{dv}{dt} \Rightarrow A_p\frac{dv}{dt} = KA_p \Rightarrow \boxed{K' = \frac{dQ_p}{dt}}$$

$$z_1 = \Delta t\ \lambda(t_n, z_n)$$

$$= \Delta t\ \lambda_1$$

$$z_2 = \Delta t\ \lambda(t_n + 0.5\Delta t,\ z_n + 0.5 z_1)$$

Something different (0.5) from Computational hydraulics ?!

$$\boxed{\lambda = \frac{dz}{dt} = \frac{Q_p - Q_t}{A_s}}$$

$$\lambda_1 = \left(\frac{dz}{dt}\right)_{t=t} = \frac{1}{A_s}\left(Q_p^t - Q_t^t\right)$$

$$z_1 = \Delta t\,\lambda_1$$

$$\lambda_2 = \left(\frac{\partial z}{\partial t}\right)$$

Effect of air entrainment on wave speed:-

Free air or any gas → wave speed decreases
⇓
Resulting pressure substantially changed $\left(\Delta H = -\frac{a\Delta V}{g}\right)$ or, $\Delta P = -\frac{\rho a \Delta V}{g}$.

Wave speed in uniform air-water mixture,

$$a = \sqrt{\frac{K_{mix}/\rho_{mix}}{1 + \frac{K_{mix} D}{eE}\psi}}$$

ψ = Non-dimensional parameter depends on restraint type and pipe property.

$$K_{mix} = \frac{K_{liq}}{1+\alpha\left(\frac{K_{liq}}{K_{air}} - 1\right)}$$ → If $\frac{K_{liq}}{K_{air}} \gg 1$, then $K_{mix} = \frac{K_{liq}}{1+\alpha \cdot \frac{K_{liq}}{K_{air}}}$

$\rho_{mix} = (1-\alpha)\rho_{liq}$, α = void fraction.

$$\rho_{mix} = \frac{\rho_{liq}(1-\alpha) + \rho_{air} \times \alpha}{(1-\alpha)+\alpha}$$ ⇒ Total mass / ⇒ Total volume

$$= \rho_{liq}(1-\alpha) + \underbrace{\cancel{\rho_{air}}^{\,0} \times \alpha}_{\text{This is negligible.}}$$

For this case, wave velocity becomes,

$$a = \sqrt{\frac{K_{liq}}{\rho_{mix}\left(1+\alpha \cdot \frac{K_{liq}}{K_{air}}\right)\left(1 + \frac{K_{mix} \cdot D}{eE} \cdot \psi\right)}}$$

$$= \sqrt{\frac{K_{liq}/\rho_{mix}}{1 + \frac{K_{mix} \cdot D}{eE}\psi + \alpha \frac{K_{liq}}{K_{air}}}}$$

In notes, $\frac{K_{liq}}{E} \cdot \frac{D}{e}$ was omitted why?

$$\frac{K_{mix} D}{eE}\psi + \alpha \cdot \frac{K_{liq}}{K_{air}} \cdot \frac{K_{mix} D \cdot \psi}{eE} \times ?$$
$$= \frac{D\psi}{eE}\left(K_{mix} + \alpha \frac{K_{liq} \cdot K_{mix}}{K_{air}}\right) \stackrel{?}{=}$$
$$= \frac{D\psi}{eE} \cdot K_{mix} \times \frac{K_{air} + \alpha K_{liq}}{K_{air}}$$

From this equation, it is clear that wave speed in the pipeline depends on α and K_{air}. Both of these quantities depend on pipeline pressure.
void fraction

From $\Delta P = -\frac{\rho a \Delta V}{g}$, pressure depends on wave speed (a). But, when we calculate a, it also depends on pressure. So, relation of them is quite complicated.

Kair for different process:-

Isothermal:- Energy may flow in and out, but $\Delta T = 0$. $K_{air} = P$.

Isentropic:- Entropy remains constant since no energy is dissipated. Both adiabatic and reversible process. $K_{air} = 1.4P$.

Adiabatic process:-

No heat is transferred from system to surrounding and vice-versa. Irreversible process.

$K_{air} = 1.4P$

Polytropic process:

$PV^n = constant$

$K_{air} = 1.2P$.

This process models behaviors like compression and expansion of gases.

n is called polytropic constant.

$n = \frac{C_p}{C_v}$ (আমরা যাকে γ বলে জানি)

For isothermal process, $n=1$ $PV = n_{RT}$

For isobaric process, $n=0$

For adiabatic process of ideal gas

$\frac{C_p}{C_v}$ ratio for,

Monoatomic gases $= 1.67$

Diatomic " $= 1.4$

Tri-atomic " $= 1.28$.

Numerical: compute wave velocity.

Provided data:- $D = 6.78m$,

$e = 19\,mm = 0.019\,m$.

$L = 244m$

$E = 207\,GPa$

$\rho_{liq} = 1000\,kg/m^3$

$K = 2.19\,GPa \Rightarrow K_{liq} = 2.19 \times 10^9\,Pa$

longitudinal expansion joint, $\psi = 1.0$

air fraction, $\alpha = 0.1\% = 0.001$.

$H = 50m$

Assume isothermal process.

$$a = \frac{\sqrt{K_{liq}/\rho_{mix}}}{\sqrt{1 + \frac{K_{liq}}{E} \cdot \frac{D}{e}\psi + \alpha \cdot \frac{K_{liq}}{K_{air}}}}$$

$\rho_{mix} = 1000(1-\alpha) = 1000(1-0.001) = 999\,kg/m^3$

$K_{air} = P =$ Atmospheric pressure + Water height in reservoir.

For isothermal process, $= $ (Absolute pressure)

$= 10m + 50m$

(Actually, 10.36m)

$= 60m$. of water

$= 60 \times 1000 \times 10$

$= 6 \times 10^5\,Pa$ (ρ, g)

$$\therefore a = \sqrt{\frac{2.19 \times 10^9 / 999}{1 + \frac{2.19 \times 10^9}{207 \times 10^9} \cdot \frac{6.78}{0.019} \cdot 1 + 0.001 \times \frac{2.19 \times 10^9}{6 \times 10^5}}}$$

$= 510\,m/s$.

Transient Analysis:

Surge:- ☑ Mass oscillation constant along pipeline, but changes with time.

☑ For longer period or slow transients. Numerically modelled with large time step.

Hydraulic transients → changes in flow, pressure and velocity due to valve closure, pump shutdown etc. These transients may cause pressure surge, water hammer, cavitation, pipe vibration.

High head turbine runner ⇒ Even number runner Blade + even " wicket gate } Result: Some blades cracked.

American school of practice:-
Involves extensive use of surge protection device.
Relying more on cure than prevention.

European school of practice:-
Avoid unacceptable surge at the first place.

Rigid column theory is used for finding preliminary dimensions for pressure vessels and surge tanks.
Selection of appropriate check valve

Why transient analysis:-
Flow Behaviour → Safe operating environment
Max, min pressure - probability of occurance - Extreme event.
↳ (Avoid sub atmospheric pressure, protective measures for transient pressures).
Time scale → establish steady state flow time after pump startup, after alteration of valve setting.

Pay attention for booster pump behavior, air valve operations, fast acting valves.

Test pressure = Working pressure + Transient pressure.

☑ Failure modes ⇒ (1) Collapse
(2) Brittle failure

long, fast brittle cracks run along the pipelines in wavy manner.
Brittle fracture depends on hoop stress. (<0.5%) w

Minimum pressure:-
☑ Thin walled pipes
↳ Small deformation, modulus - buckling
☑ Flexible pipes - ovalization (Fatigue damage) - reduction of pipe stiffness.
☑ Low pressure - gas cavity - Collapse of cavity causes excessive pressure.
Min pressure = +2 m water
For deeply buried pipe pressure > 2 m.

Pipe friction: (1) steady state friction:
When Pumping main ⇒ Roughness coefficient ⇒ upper limit of the range of the likely variables. - develop necessary head.
↳ If we can obtain necessary head against high friction factor, we can easily get that under other values.

When unsteady flow ⇒ Severe pressure transient ⇒ Roughness factor - Lower end

If allowable transient pressure is generated under low friction factor, then for high friction factor it will be always less in value.

☑ Study surge pressure using alternate friction coff. Trial and error method.

$$h_L = \frac{fLV^2}{2Dg}$$

$$f = f\left(\frac{K}{D}, Re\right)$$

$Re < 2100$, for laminar flow, $f = \frac{64}{Re}$.

Friction factor for transition and fully developed turbulent flow,

Colebrook-white equation ⇒ Implicit formulation

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{2.51}{Re\sqrt{f}} + \frac{K}{3.71D}\right)$$

Haaland equation ⇒ (Explicit form to approximate the colebrook-white equation)

$$\frac{1}{\sqrt{f}} = -1.8\log_{10}\left\{\left(\frac{(K/D)}{3.7}\right)^{1.11} + \frac{6.9}{Re}\right\}$$

⊛ For determining K value, corrosion and sliming must be taken care of.

Corrosion:-

Due to corrosion, Roughness height (K) increases over the years.

Present value of $K = K_0 + \Delta K \cdot t$

K_0 → new pipe roughness height; ΔK → $\frac{ft}{year}$; t → year.

ΔK is obtained from equation, $2\log_{10}(12\Delta K) = 3.8 - pH$

Sliming:-

Occurs in sewage and waste-water mains.

Roughness formula of sliming pipe is a function of velocity.

$$K = \alpha V^{(-2.34)}$$

$\alpha = 0.054 - 3.660$ (Mean value = 0.446)

(2) Unsteady friction:

Steady state friction factor ⇒ good result for for computing 1st peak of unsteady pressure.

But not reliable for multiple operation

(i) Pump start following power failure,

(ii) load acceptance by turbine following load rejection.

Three types unsted friction modelling:-

(a) Quasi 2D model:- (computationally intensive) → used for simple piping system

CE and ME are solved in cylindrical co-ordinate.

Shear stress
- for laminar flow → Newton's law
- for turbulent flow →
  - ~~Sim~~ in viscous sub layer → Newton's law
  - In turbulent region → Prandtl mixing length hypothesis

(b) Convolutional integral method:

Suitable for 1D models → use past histories of local accelerations and waiting functions.

Method requires large computational memory.

Gives exact unsteady friction for ⇒ laminar unsteady flow.

This method recently used for smooth and rough turbulent flows.

(c) Instantaneous acceleration based method:-

$$\frac{\partial H}{\partial x} + \frac{1}{g}\frac{\partial V}{\partial t} + J_s + J_u = 0$$

J_s → steady friction term

J_u → unsteady friction term } → when velocity and pressure of pipe flow changes with time.

Here, Damping is attributable using local $\left(\frac{\partial V}{\partial t}\right)$ and convective $\left(a\frac{\partial V}{\partial x}\right)$ acceleration.

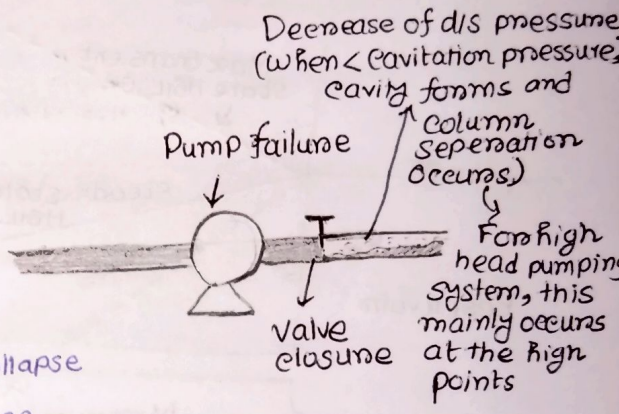
$$J_s = \frac{f V|V|}{2gD}$$

$$\begin{cases} J_u = \frac{k}{g}\left[\frac{\partial V}{\partial t} + \text{sign}(V)\, a \left|\frac{\partial V}{\partial x}\right|\right] & \text{.... Single coefficient method} \\ J_u = \frac{1}{g}\left[k_{ut}\frac{\partial V}{\partial t} + k_{ux}\,\text{sign}(V)\, a \left|\frac{\partial V}{\partial x}\right|\right] & \text{.... Double coefficient method.} \end{cases}$$

a → wave velocity

Column Seperation:

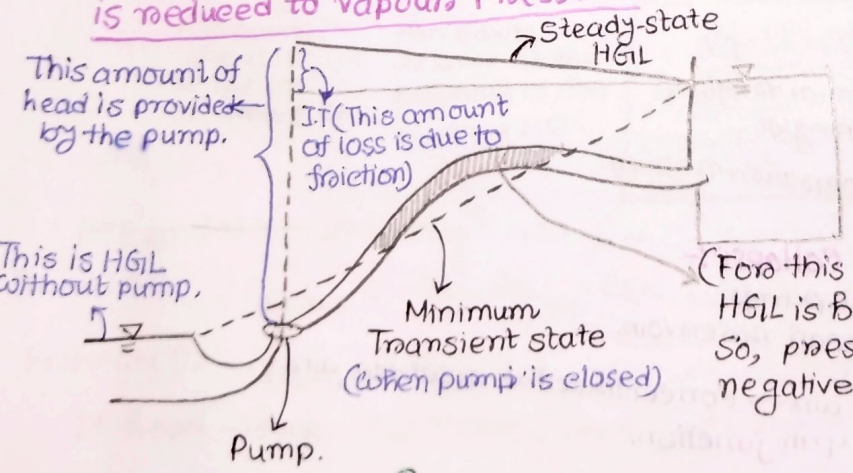
Cause:- pressure < Vapour pressure
⇓
Cavitation
⇓
Liquid-column seperation.
⇓
Backward flow and cavity collapse
⇓
Generation of huge pressure.



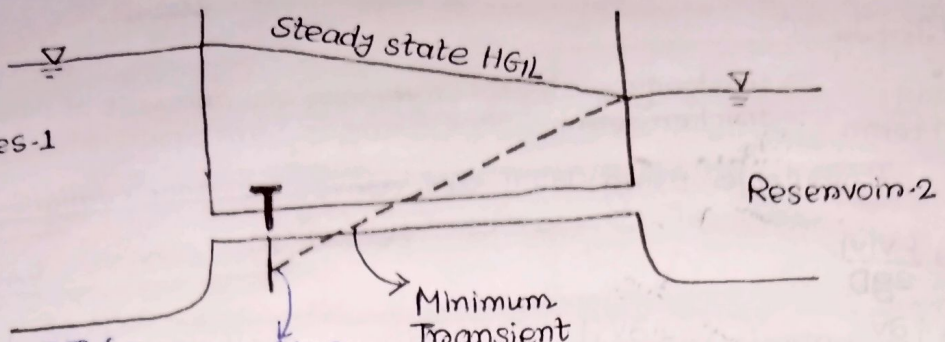
Energy dissipation:

In cavitating flow, dissipation of pressure wave in gas-liquid mixture is more than pure liquid. This additional dissipation is due to heat transfer to the liquid when bubbles expand and contract.

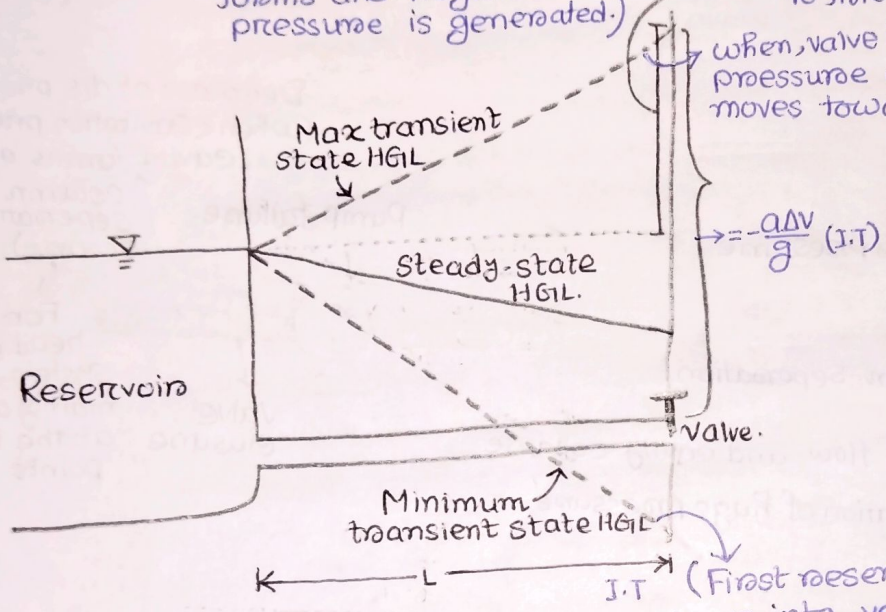
Positions of HGL when transient pressure is reduced to Vapour pressure:



Power failure in a pumping system.



I.T (Due to sudden stoppage of the valve, here cavity forms and negative pressure is generated.)



I.T (This amount of head lost is due to friction along the pipe)

when, valve is suddenly closed, +ve pressure wave is generated and moves towards the reservoir

$= -\frac{a\Delta V}{g}$ (I.T)

I.T (First reservoir turns +ve pressure wave into -ve pressure wave and reflects back towards the valve (∵ constant head reservoir has reflection coefficient $r=-1$). When, that -ve pressure wave reaches to the valve, again -ve head is generated (as, for dead end, $r=1$) and -ve wave propagates towards the reservoir.)

I.T (For max transient state HGL, water is pushed towards '←' this direction. So, frictional head loss is from right to left).

(For min. transient state HGL, driving force of water in '→' direction. So, h_f is from left to right)

☑ System with cavitating flow.

Three regions:- water hammer region.
Cavitation region.
Column - seperation region.

Mathematical model of cavity collapse:-

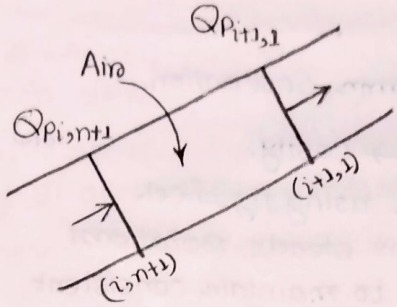
u/s boundary → centrifugal pump
d/s " → constant-head reservoir.

Total volume of entrained air → concentrated at discrete air cavity → at 'i'th junction.

$\forall_i$ = Volume of air cavity $= \alpha A_i L_i$ ..... (9.20)

α → void fraction

A_i, L_i → Area and length of ith pipe.



Expansion and contraction of air pocket follows polytropic equation. For a perfect gas.

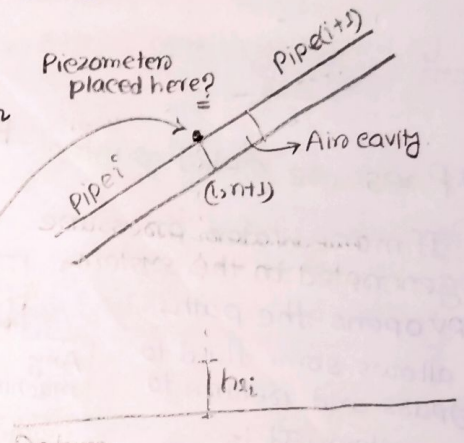
I.T (Measured absolute piezometric head).

$$(H_{P_{i,n+1}} - h_{\ell_i})\, \forall_{P_i}^{m} = C \quad \text{..... (9.21)}$$

$\forall_i$ ⇒ Volume of air pocket at the end of time step.

I.T. (correct Absolute piezometric head). $H_{P_{i,n+1}}$ ⇒ Piezometric head above the datum at section $(i,n+1)$ at the end of the time-step.

I.T (This is the error by the gauge) h_{ℓ_i} ⇒ Pressure head between datum and lowest possible absolute-head pressure head level that a gauge at ~~this level~~ ~~can measure~~. the top of the pipe at junction i can measure.



$m = 1$ (for isothermal process) (Slow)

$= 1.4$ (" adiabatic process) (Fast)

Average value $m = 1.2$ may be used.

Continuity equation

volume of air cavity and discharges at the next time-steps are unknown.

$$\forall_{P_i} = \forall_i + 0.5\Delta t\left[(Q_{P_{i+1,1}} - Q_{i+1,1}) - (Q_{P_{i,n+1}} - Q_{i,n+1})\right] \quad \text{..... (9.22).}$$

$\forall_{P_i}$ → Air cavity volume at the end of time step

$\forall_i$ → Air cavity volume at the begining of the time step.

$$\underbrace{\forall_{P_i} - \forall_i}_{\text{Volume increase of air cavity}} = 0.5\Delta t\left\{\underbrace{(Q_{P_{i+1,1}} - Q_{P_{i,n+1}})}_{\text{Air cavity volume increase rate at next time step}} - \underbrace{(Q_{i+1,1} - Q_{i,n+1})}_{\text{Air cavity volume Increase rate at current time step.}}\right\}$$

→ I.T.

PCE and NCE for $(i,n+1)^{th}$ and $(i+1,1)^{th}$ sections are,

From (3.17), $Q_{P_{i,n+1}} = -C_{a_i} H_{P_{i,n+1}} + C_{P_i}$ .... (9.23)

From (3.18), $Q_{P_{i+1,1}} = C_{a_{i+1}} H_{P_{i+1,1}} + C_{n_i}$ .... (9.24)

If head losses at junctions are neglected,

$$H_{P_{i,n+1}} = H_{P_{i+1,1}} \quad \text{..... (9.25)}$$

So, we have 5 unknowns at previous time step.

Those are:- V_{P_i}, $Q_{P_{i,n+1}}$, $Q_{P_{i+1,1}}$, $H_{P_{i,n+1}}$, $H_{P_{i+1,1}}$

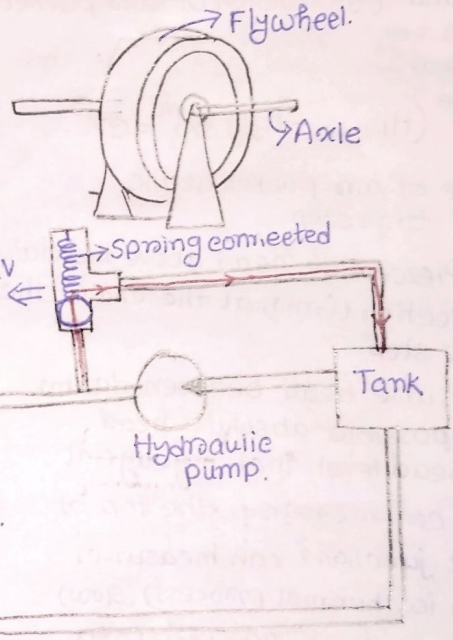
So, these are 5 unknowns.

We have 5 equation, (9.21) to (9.25)

Design Considerations:-

- Prevent cavitation and column Seperation.

1. Air chamber, Surge tank → costly.
2. Increasing motor Interitia using fly wheel. (Flywheel in pump maintains steady rotational speed of the impeller, helps to maintain consistent flow rate of the liquid. Flywheels store rotational energy).



(3) Pressure relief valve:-

If more water pressure is generated in the system, PRV opens the path. It allows some fluid to bypass and return to the system. It is generally provided at the outlet of the pump.

(4) Air Inlet Valve:-

Air inlet valve prevents sub-atmospheric pressure to pipeline. But, air need to be removed then.

- Transient Control:-

Based on frequency of occurance operating controls are classified as:-

(i) Normal operating condition ⇒ starting and shutting down of multiple pumps.

Check valve slam:- Sudden and forceful closing of a check valve due to backflow or pressure reversal in a pipeline or system. It may cause damage to valve and pipeline, create noise and vibration.

FOS=3 recommended based on ultimate bursting strength and collapse.

(ii) Emergency:-

(a) Surge suppressors/surge tanks/Relief valves is inoperative.

(b) closure of check valve delayed in power failure

(c) Air inlet valve → Inoperative.

FOS = 2.

(iii) Exceptional:

Loss of all air in air chamber.

Very sudden abnormal opening and closing of a valve or a gate.

Pump shaft failure.

Probability of occurance is very less. So, FOS = 1.25

☑ Max. transient state pressure - steady state pressure ≤ 10% rated head of the pump.

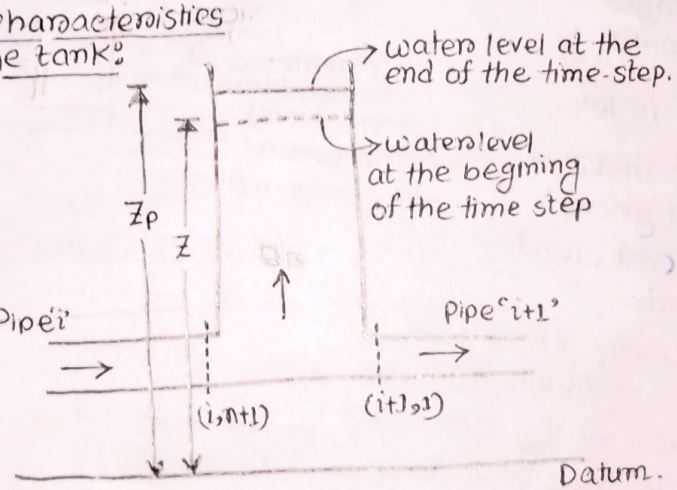
So, a number of alternatives with or without control devices should be considered.

☑ Surge tank:-

Rapid transient in pipe system having a surge tank → analysed by method of characteristics

Slow transient (eg. oscillation of water level in a surge tank following a load change on a turbine → analysed as a lumped system.

Method of characteristics for a surge tank:



Our variables:- $H_{P_{i,n+1}}$; $Q_{P_{i,n+1}}$; $H_{P_{i+1,1}}$; $Q_{P_{i+1,1}}$; Z_P.

We have following equations:-

PCE :- $Q_{P_{i,n+1}} = -C_{a_i} H_{P_{i,n+1}} + C_{P_i}$ ..... (i)

NCE :- $Q_{P_{i,n+1}} = C_{a_i} H_{P_{i,n+1}} + C_{n_i}$ .... (ii)

CE :- $Q_{P_{i,n+1}} = Q_{P_{i+1,1}} + Q_{PS}$ → Discharge to the surge tank. .... (iii)

Energy equation: (Head loss at the junction is being neglected):-

$$H_{P_{i,n+1}} = H_{P_{i+1,1}} = Z_P \quad \text{.... (iv)}$$

Surge tank water level:-

$$(Z_P - Z) = \frac{\overbrace{\frac{1}{2}(Q_{PS} + Q_S)}^{\text{Average discharge at time} = \Delta t} \times \Delta t}{A_S}$$

$(Z_P - Z)$ → water level change between 2 time steps.

A_S → Area of surge tank

$$Z_p = Z + \frac{\Delta t}{2A_s}(Q_{ps} + Q_s) \quad \ldots (V)$$

We have 5 eqns to solve 5 variables.

Solving we get, $$\boxed{H_{p_{i,n+1}} = \frac{C_p - C_n + Q_s + \left(\frac{2A_s Z}{\Delta t}\right)}{C_{a_i} + C_{a_{i+1}} + \left(\frac{2A_s}{\Delta t}\right)}}$$ ⇒ Try to solve it later!

Air chamber:

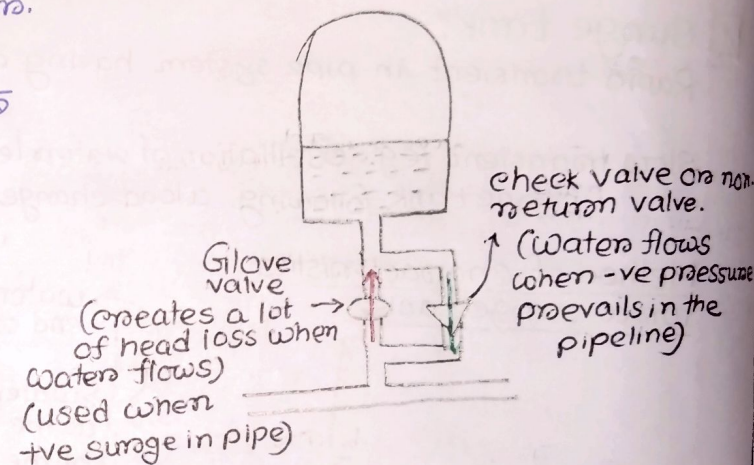
{ Compressed air ⇒ At top
  Water ⇒ Lower part.

Differential orifice ⇒ Provided between the chamber and pipeline.
↳ creates more head loss for inflow to chamber, than outflow

To prevent column seperation, outflow should be quick (Energy loss efficient).

Inflow may be restricted to reduce the size of the chamber.

$$\frac{(\text{Head loss})_{inflow}}{(\text{Head loss})_{outflow}} = 2.5$$



Air chamber advantages compared to surge tank:-

(i) Volume of chamber is less.

(ii) Air chamber can be installed with axis parallel to ground. Reduce foundation cost, wind and earthquake loads.

I.T. (Air chamber reduces +ve pressure surge energy by workdone with compressing air. But, surge tank do that by increasing water level. i.e pressure energy is converted to potential energy)!?

(iii) It can be located near to pump. due to its reduced weight and smaller volume. Transient controlled easily.

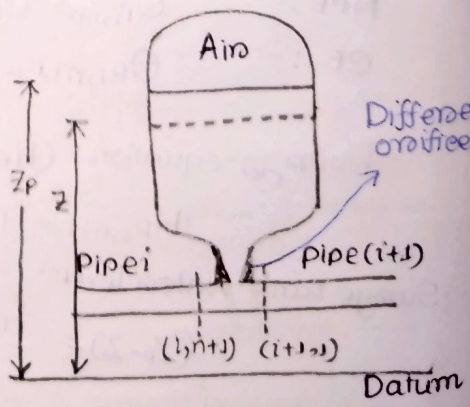
Method of characteristics for transient analysis of air vessel:-

PCE:- $Q_{p_{i,n+1}} = -C_{a_i} H_{p_{i,n+1}} + C_{p_i}$ .... (10.8)

NCE:- $Q_{p_{i+1,1}} = +C_{a_{i+1}} H_{p_{i+1,1}} + C_{n_{i+1}}$ .... (10.9)

CE:- $Q_{p_{i,n+1}} = Q_{p_{i+1,1}} + Q_{p_{air}}$ ..... (iii) 10.11)

↓ Flow into the air vessel at the end of time step.



Energy equation:-

$$H_{P_{i,n+1}} = H_{P_{i+1,1}} \quad \ldots (10.10)$$

Q_0 = Flow into air vessel at the start of the time step.

Q_{P_0} = " " " at the end of time st

Air enclosed at the top of the chamber follows the polytropic relation of the perfect gas.

$$H^*_{P_{air}} \forall_{P_{air}}^m = C. \quad \ldots (10.12)$$

$H^*_{P_{air}}$: Absolute pressure head at the end of the time-step

$\forall_{P_{air}}$: Volume of air enclosed at the end of the time-step.

$m = 1.0$ for isothermal process.
$m = 1.4$ for adiabatic process.
Take, average, $m = 1.2$

#Because, transients are rapid at the begining and slow at the end.

Value of the constant C determined by, $C = \forall_{0_{air}}^m \cdot H_{0_{air}}^m$ initial steady-state condition.

Head loss for the flow through orifice,

$$h_{P_{orf}} = C_{orf} Q_{P_{orf}} |Q_{P_{orf}}| \quad \ldots (10.13).$$

Equations can be written for enclosed air volume,

At the end of Δt,

$$H^*_{P_{air}} = H_{P_{i,n+1}} + H_b - Z_P - h_{orf} \quad \ldots (10.14)$$

$H^*_{P_{air}}$: Absolute pressure head at air chamber

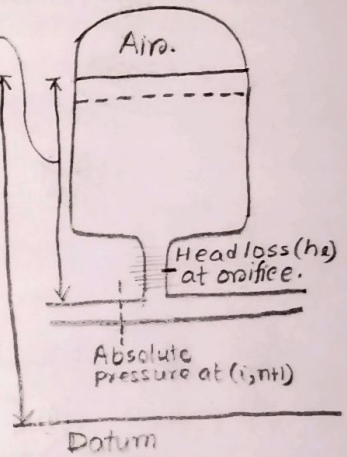
$H_{P_{i,n+1}}$: Pressure head at section $(i,n+1)$

H_b: Barometric + head at section $(i,n+1)$

$H_{P_{i,n+1}} + H_b$: Absolute pressure head at section $(i,n+1)$

Z_P

h_{orf}: head loss at orifice



? I think, this height should be subtracted. Why Z_P?!

#Decrease of air volume = Increase of water volume.

$$\forall_{P_{air}} - \forall_{air} = -\{A_c(Z_P - Z)\}$$

$$\forall_{P_{air}} = \forall_{air} - A_c(Z_P - Z) \quad \ldots (10.15)$$

A_c: C/S area of air chamber.

$(Z_P - Z)$: Increase of water surface elevation.

$$\underbrace{(Z_P - Z)A_c}_{\text{Increase of water volume at air chamber}} = \underbrace{0.5(Q_{orf} + Q_{P_{orf}})}_{\text{Average discharge through the orifice at time-step } \Delta t} \cdot \Delta t$$

$$\Rightarrow Z_P = Z + 0.5(Q_{orf} + Q_{P_{orf}}) \cdot \frac{\Delta t}{A_c} \quad \ldots (10.16).$$

#Now we have 9 variables, and we have 9 equations to solve. (10.8 to 10.16).

$$\left\{ \begin{array}{lll} Q_{P_{i,n+1}} & H_{i,n+1} & h_{P_{orf}} \quad \forall_{P_{air}} \quad Z_P \\ Q_{P_{i+1,1}} & H_{P_{i+1,1}} & \\ Q_{P_{orf}} & H^*_{P_{air}} & \end{array} \right\}$$

#→ But, all of them are not linear algebric equations. So, N-R method should be used (iterative). Not, normal matrix inversion.

☑ Let, try to simplify this system of equations:-

from (10.10), Put in (10.8) and (10.9)

$$Q_{P_{i,n+1}} = -C_{ai} H_{P_{i,n+1}} + C_{Pi}$$

$$Q_{P_{i+1,1}} = +C_{ai+1} H_{P_{i,n+1}} + C_{ni}$$

Put these values to equation (10.11),

$$-C_{ai} H_{P_i,n+1} + C_{P_i} = C_{a_{i+1}} H_{P_i,n+1} + C_{ni} + Q_{Pout}$$

$$\Rightarrow Q_{Pout} = (C_{P_i} - C_{ni}) - (C_{ai} + C_{a_{i+1}}) H_{P_i,n+1} \quad \text{------} \quad (10.17)$$

So, we have eliminated the following unknowns,

$Q_{P_i,n+1}$

$Q_{P_{i+1},1}$

$H_{P_{i+1},1}$

Now, we have 6 equations (10.12 to 10.17) and 6 variables

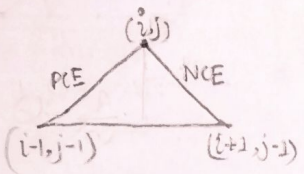
$$C_a = \frac{gA}{a}$$

$$C_P = Q_A + \frac{gA}{a} H_A - R\Delta t\, Q_A |Q_A|$$

$$C_n = Q_B - \frac{gA}{a} H_B - R\Delta t\, Q_B |Q_B|$$

$$R = \frac{f}{2DA}$$

$$h = H_{res} - \frac{fLQ^2}{2dgA^2}$$



$$Tau(2) = Tau(1) + \{Tau(3) - Tau(1)\} \times \frac{1}{4}$$

$$Tau(3) = Tau(1) + \{Tau(3) - Tau(1)\} \times \frac{3}{4}$$

$$Q_{P_i,n+1} = 0.5\left(-C_v + \sqrt{C_v^2 + 4C_{P_i}C_v}\right) \ldots (1)$$

where, $C_v = \dfrac{(\tau \cdot Q_{oi,n+1})^2}{C_a H_{oi,n+1}}$

+ve characteristics equation,

$$Q_{P_i,n+1} = C_{P_i} - C_{ai} \cdot H_{P_i,n+1}$$

↓

Now these obtained (at $t = \Delta t$) $H_{P_i,n+1}$ and $Q_{P_i,n+1}$ are stored as conditions at the begining of the next time step.

These procedure repeated untill the desired duration

↓

These conditions are printed at every second time steps. (0.5, 0.75, 1.0, 1.5, 2.0, ...)

By specifying IPRINT=2.

*

Valves:-

Transient controls by valve:-

(i) Reduces max pressure by opening

(ii) Flow controls.

(iii) Admit air to pipelines prevent pressure dropping to the liquid vapour pressure.

(i) Safety valve:-

Spring or Weight loaded valve.

Opens when pressure > Specified value.
closed " " < " " } Either fully open or fully closed.

(ii) PRessure regulating valve

(iii) Air-inlet valve

(iv) check valve.

Transit time
$= \frac{550}{1100} + \frac{450}{900} = 1s$

$\Delta t = \frac{1}{16}$ to $\frac{1}{24}$ of transit time

$\boxed{\frac{a\Delta t}{\Delta x} \le 1}$ → Courant stability (i) Criterion

$\boxed{\Delta t = \frac{L_i}{a_i n_i}}$ $i = 1$ to N, N = no of pipes. ... (2) ... (3.69)

↳ condition to be satisfied to avoid interpolation.

n_i must be integer and = no. of reaches at which i^{th} conduit is divided.

(Wave velocity may need to be adjusted To make $C_N = 1$ and eqn (2) is satisfied).

Actually,
$\Delta t = \frac{(L_i/n_i)}{a_i}$
$= \frac{\Delta x_i}{a_i}$

Then characteristics lines should pass through grid points.

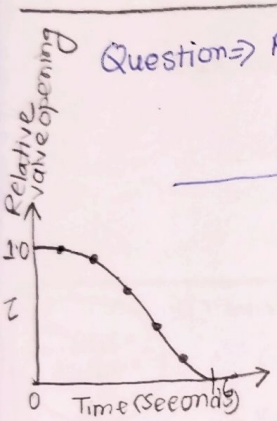
Question ⇒ Page-104- ⇒ $\frac{a_i \Delta t}{\Delta x_i} = 1$

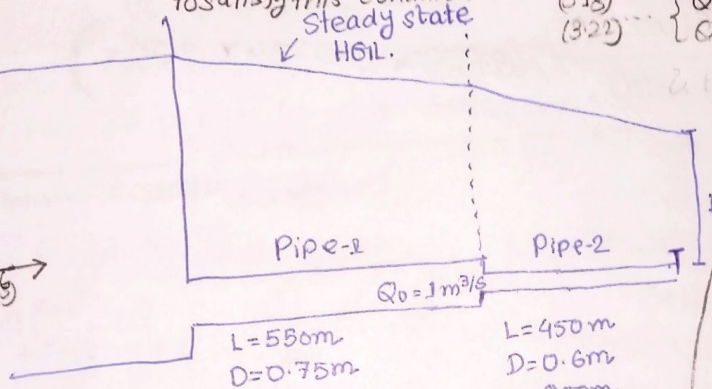
So (3.69) is mentioned to satisfy this condition.

Steady state discharge and pressure head

Transient state ↓

(3.18) $Q_P = 0.5(C_P + C_N)$
(3.22) $Q_P = C_N + C_a H_P$ } At time level Δt, Q, H for internal nodes.





Pipe-1	Pipe-2
L = 550m	L = 450 m
D = 0.75m	D = 0.6m
a = 1100 m	a = 900m
f = 0.01	f = 0.012

Valve closure time (6s) > wave transit time (1s)

$\Delta t = \frac{450}{a_2 \times 2} \Rightarrow 0.25 = \frac{450}{a_2 \times 2}$
$\Rightarrow a_2 = 900$

$0.25 = \frac{550}{a_2 \times 2} \Rightarrow a_2 = \underline{1100}$ m

Read and print input data
↓
Compute Δt and pipe constants
↓
Determine steady state condition in pipe system
↓
$T = 0$, $K = 0$.
↓
Print flow and pressure head in the system.
↓
$K = K + 1$
$T = T + \Delta t$
↓
$T = T_{last}$
↓
Compute H_P and Q_P at interior sections
↓
Compute H_P and Q_P at interior junctions
↓
Compute H_P and Q_P at u/s and d/s boundary.
↓
Store Q_P and H_P
↓
Print ?!

Boundary Conditions:-

(i) u/s reservoir: $H_{P_{i,1}} = H_{res}$ ... (3.28)
$Q_{P_{i,1}} = C_{a_i} \cdot H_{res} + C_{n_i}$ ... (3.29)

(ii) Junctions for Pipe (1) and pipe (2):-

$H_{P_{i,n+1}} = \frac{C_{P_i} - C_{n_{i+1}}}{C_{a_i} + C_{a_{i+1}}}$ ... (3.47)

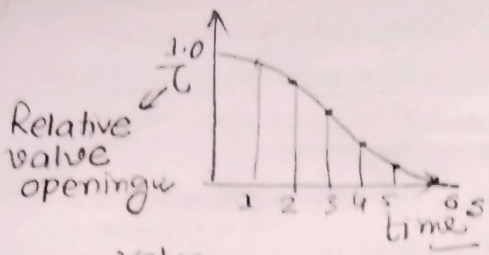
$H_{P_{i+1,1}} = H_{P_{i,n+1}}$ ... (3.43)

$Q_{P_{i,n+1}} = C_{P_i} - C_{a_i} H_{P_{i,n+1}}$ ... (3.44)

$Q_{P_{i+1,1}} = Q_{P_{i,n+1}}$ ... (3.46)

(iii) Valve BC:

Seven points in τ (percentage opening) - t curve has been stored [For 0, 1, 2, 3, 4, 5 and 6 s]. For intermediate times, values are interpolated.



Valve closure curve

~~He~~ Each pipe has 100 nodes

Define matrices:-

$C_N = 1$

$\Delta x \geq a\Delta t$

$\Rightarrow \Delta x \geq$

Transit time $= \frac{550}{1100} + \frac{450}{900} = 1s$

$\Delta t = \frac{1}{16}$ to $\frac{1}{24}$ s

$$C_p(2,3) = \cancel{Q(1,2)}\left(\frac{9.81 \times 0.4418}{1100} \times 66.6955 - 0.0151 \times 0.25 \times 1\right)$$

$$\left(-\frac{9.81 \times 6.2827}{900} \times 62.869 - 0.0354 \times 0.25 \times 1\right)$$

Indian standard:-

(a) (upsurge protection) → 1.25x Maximum operating pressure → Example:- Pump delivery pressure = 45m. (1.25)x45 = 56.25m head for (upsurge control).

(minimum of these two?!) → FOS based on ultimate bursting or collapsing strength of pipeline.

(b) Collapsing pressure of a pipe

$$P_c = \frac{2E}{1-\mu^2}\left(\frac{e}{D}\right)^3 \qquad e = \text{Pipe thickness}$$

For example MS pipe of 6 mm thick, 600 mm dia:-

$E = 207\,GPa$, $\mu = 0.303$ for MS pipe.

$$P_c = \frac{2\times 207\times 10^9}{1-(0.303)^2}\times\left(\frac{6\times 10^{-3}}{0.6}\right)^3 = 4.5\times 10^5\,Pa = 4.5\text{ bar.}$$

MAC-CORMACK SCHEME:- (comprises of two steps, Predictor, corrector)

Features:- (1) Explicit scheme

(2) Conditionally stable

(3) Second-order accurate (Both in space and time)

(4) Can be used for linear and non-linear equations.

Class note:-

1D wave equation, $\frac{\partial u}{\partial t} + a\frac{\partial u}{\partial x} = 0$

Predictor step:- (FTFS)

$$\frac{u_i^{n+1*} - u_i^n}{\Delta t} + a\,\frac{u_{i+1}^n - u_i^n}{\Delta x} = 0$$

$$\Rightarrow u_i^{n+1*} = u_i^n - \frac{a\Delta t}{\Delta x}\left(u_{i+1}^n - u_i^n\right)$$

Provisional value at next time step / Predictor value.

? ** Above mentioned step is unstable because both are Forward difference and forward step. Therefore, it has to be corrected.

$$\frac{u_i^{**} - u_i^n}{\Delta t} + a\,\frac{u_i^* - u_{i-1}^*}{\Delta x} = 0$$

$$\Rightarrow u_i^{**} = u_i^n - \frac{a\Delta t}{\Delta x}\left(u_i^* - u_{i-1}^*\right)$$

$$\boxed{u_i^{n+1} = \frac{1}{2}\left(u_i^* + u_i^{**}\right)}$$

(Meaning of * → Predicted value and ** ?)

Mac-Cormack Scheme → Hanif Chowdhury

Momentum and Continuity equation:- $\frac{\partial Q}{\partial t} + gA\frac{\partial H}{\partial x} + RQ|Q| = 0$ ..... (3-1)

Momentum equation:- $a^2\frac{\partial Q}{\partial x} + gA\frac{\partial H}{\partial t} = 0$ ..... (3-2)

↳ H and Q at $(t_0+\Delta t)$ or $(n+1)^{th}$ time level need to be determined.

Second order accurate both at space and time and inherently dissipative

Inherently dissipative Scheme:-

IDS includes dissipation in its numerical scheme, artificially introduces certain amount of viscosity and diffusion into the simulation.

In an IDS, dissipation is determined by the numerical scheme itself and does not depend on the physical properties of the fluids being simulated.

Dissipation helps to stabilize the simulation and prevent oscillations and numerical unstabilities.

IDS is used in CFD simulations of turbulent flows, where chaotic and complex behaviour of flow causes numerical unstability.

But, as it introduces some artificial viscosity or diffusion, it affects the accuracy of simulation, particularly in the region where flow is highly variable or contains sharp gradient.

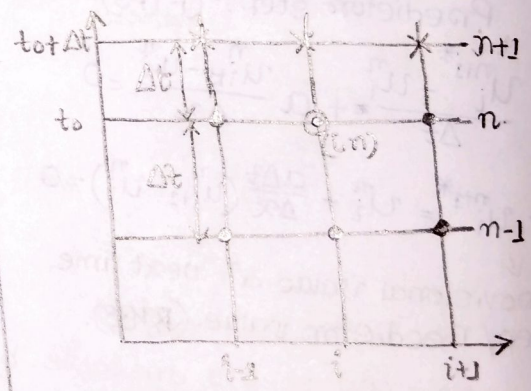
1st alternative:

FD approximation in Predictor part.
BD " in Corrector part.

2nd alternative:-

BD → Predictor step
FD → Corrector step.

Mac-Cormack suggests ~~one~~ scheme 1st alternative at one time step, 2nd alternative at next time-step again 1st alternative for next time-step



⊙ Known conditions
× Unknown "

For 1st alternative,

(i) Predictor:-

H_i^*/H_i

1st Alternative

Predictor step: $\frac{\partial Q}{\partial t} + gA\frac{\partial H}{\partial x} + RQ|Q| = 0$

$$\Rightarrow \frac{Q_i^* - Q_i^n}{\Delta t} + gA\frac{H_{i+1}^n - H_i^n}{\Delta x} + RQ_i^n|Q_i^n| = 0$$

$$\Rightarrow Q_i^* = Q_i^n - gA\frac{\Delta t}{\Delta x}(H_{i+1}^n - H_i^n) - R\Delta t\, Q_i^n|Q_i^n| \quad \text{..... (3.99.1)}$$

In Chowdury's books this Δt is missing. ?

$$a^2\frac{\partial Q}{\partial x} + gA\frac{\partial H}{\partial t} = 0$$

$$\Rightarrow a^2\frac{Q_{i+1}^n - Q_i^n}{\Delta x} + gA\frac{H_i^* - H_i^n}{\Delta t} = 0$$

$$\Rightarrow \frac{gA}{\Delta t}H_i^* = \frac{gA}{\Delta t}H_i^n - \frac{a^2}{\Delta x}(Q_{i+1}^n - Q_i^n)$$

(Here, $i = 1, 2, \ldots m$)

$$\Rightarrow H_i^* = H_i^n - \frac{a^2\Delta t}{gA\Delta x}(Q_{i+1}^n - Q_i^n) \quad \text{------- (3.99.2)}$$

Corrector step:- For this step,

$$H_i^{n+1} = \frac{1}{2}(H_i^n + H_i^{**})$$

$$Q_i^{n+1} = \frac{1}{2}(Q_i^n + Q_i^{**})$$

, (Q_i^{**} means RHS of equation (3.99.1) only in the place of time level 'n', it will be time level '*' and opposite FDM method is used).

$$\therefore Q_i^{n+1} = \frac{1}{2}\left\{Q_i^n + Q_i^* - gA\frac{\Delta t}{\Delta x}(H_{i+1}^* - H_{i-1}^*) - R\Delta t\, Q_i^*|Q_i^*|\right\} \quad \text{.... (3.100.1)}$$

and $$H_i^{n+1} = \frac{1}{2}\left\{H_i^n + H_i^* - \frac{a^2\Delta t}{gA\Delta x}(Q_{i+1}^* - Q_{i-1}^*)\right\} \quad \text{------- (3.100.2)}$$

(Here, $i = 2, 3, \ldots m+1$)
↳ otherwise, backward difference would not be valid. $Q_{i-1} = Q_{2-1} = Q_1$

2nd Alternative:

Equation format is same.
But, ~~forward~~ backward difference for predictor step
and ~~backward~~ forward difference " corrector ".

Predictor step:-

$$Q_i^* = Q_i^n - gA\frac{\Delta t}{\Delta x}(H_{i+1}^n - H_{i-1}^n) - R\Delta t\, Q_i^n|Q_i^n| \quad \text{..... (3.101.1)}$$

$$H_i^* = H_i^n - \frac{a^2\Delta t}{gA\Delta x}(Q_i^n - Q_{i-1}^n) \quad \text{..... (3.101.2)}$$

Corrector step:-

$$Q_i^{n+1} = \frac{1}{2}(Q_i^n + Q_i^{**})$$

$$= \frac{1}{2}\left\{Q_i^n + Q_i^* - gA\frac{\Delta t}{\Delta x}(H_{i+1}^* - H_i^*) - R\Delta t\, Q_i^*|Q_i^*|\right\} \quad \text{.... (3.102.1)}$$

$$H_i^{n+1} = \frac{1}{2}(H_i^n + H_i^{**})$$

$$= \frac{1}{2}\left\{H_i^n + H_i^* - \frac{a^2\Delta t}{gA\Delta x}(Q_{i+1}^* - Q_i^*)\right\} \quad \text{..... (3.102.2)}$$

Numerically solving St. Venant eq^ns using Finite differences

Shallow water equations to describe rapidly varied flow and shocks.
These equations work for flows with hydrostatic pressure distribution.

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = 0 \quad \ldots\ldots (1)$$

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}(QV + gA\bar{y}) = gA(S_0 - S_f). \quad \ldots\ldots (2)$$

CE:-

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = 0$$

$$\Rightarrow \frac{\partial}{\partial x}(VA) + \frac{\partial A}{\partial t} = 0$$

$$\Rightarrow A\frac{\partial V}{\partial x} + V\frac{\partial A}{\partial x} + \frac{\partial A}{\partial t} = 0$$

$$\Rightarrow Ty\frac{\partial V}{\partial x} + V\frac{\partial}{\partial x}(Ty) + \frac{\partial}{\partial t}(Ty) = 0$$

$$\Rightarrow y\frac{\partial V}{\partial x} + V\cdot\frac{\partial y}{\partial x} + \frac{\partial y}{\partial t} = 0$$

Hydraulic mean depth ($\bar{y}$) $= \frac{\text{Area}(A)}{\text{Top width}(T)}$

$\Rightarrow A = Ty.$

'D' in notes ⇒ 'D' is hydraulic depth. see {P-38, Eq=(8)}.

Momentum equation:-

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\{QV + gA\bar{y}\} = gA(S_0 - S_f)$$

$$\Rightarrow \frac{\partial}{\partial t}(AV) + \frac{\partial}{\partial x}\left(AV^2 + gA\frac{y}{2}\right) = gA(S_0 - S_f)$$ [For rectangular channel $\bar{y} = \frac{y}{2}$].

$$\Rightarrow A\frac{\partial V}{\partial t} + A\frac{\partial}{\partial x}\left(V^2 + g\frac{y}{2}\right) = gA(S_0 - S_f)$$

$$\Rightarrow \frac{\partial V}{\partial t} + g\frac{\partial}{\partial x}\left(\frac{V^2}{g} + \frac{y}{2}\right) = g(S_0 - S_f).$$

মিলছে না?! In both cases, SP. energy term is not coming. ?

$$\frac{\partial}{\partial t}(AV) + \frac{\partial}{\partial x}\{AV^2 + gA\bar{y}\} = gA(S_0 - S_f)$$

$$\Rightarrow \frac{\partial V}{\partial t} + \frac{\partial}{\partial x}(V^2) + \frac{\partial}{\partial x}(gy) = g(S_0 - S_f)$$

$$\Rightarrow \frac{\partial V}{\partial t} + 2V\frac{\partial V}{\partial x} + g\frac{\partial y}{\partial x} = g(S_0 - S_f)$$

Equations (1) and (2) are written as, interms of velocity and depth.

$$\frac{\partial y}{\partial t} + D\cdot\frac{\partial V}{\partial x} + V\cdot\frac{\partial y}{\partial x} = 0 \quad \ldots\ldots (3)$$

$$\frac{\partial V}{\partial t} + g\cdot\frac{\partial}{\partial x}\left(\frac{V^2}{2g} + y\right) = g(S_0 - S_f) \quad \ldots\ldots (4).$$

$$\Rightarrow \frac{\partial V}{\partial t} + V\cdot\frac{\partial V}{\partial x} + g\cdot\frac{\partial y}{\partial x} = g(S_0 - S_f).$$

In vector form,

$$\begin{Bmatrix} \frac{\partial y}{\partial t} \\ \frac{\partial v}{\partial t} \end{Bmatrix} + \begin{bmatrix} v \cdot \frac{\partial y}{\partial x} & D \cdot \frac{\partial v}{\partial x} \\ g \cdot \frac{\partial y}{\partial x} & v \cdot \frac{\partial v}{\partial x} \end{bmatrix} = \begin{bmatrix} 0 \\ g(S_0 - S_f) \end{bmatrix}$$

$$\frac{\partial}{\partial t}\begin{Bmatrix} y \\ v \end{Bmatrix} + \begin{Bmatrix} v & D \\ g & v \end{Bmatrix} \cdot \begin{Bmatrix} \frac{\partial y}{\partial x} \\ \frac{\partial v}{\partial x} \end{Bmatrix} = \begin{Bmatrix} 0 \\ g(S_0 - S_f) \end{Bmatrix}$$

$$\Rightarrow \frac{\partial}{\partial t}\begin{pmatrix} y \\ v \end{pmatrix} + \begin{bmatrix} v & D \\ g & v \end{bmatrix} \frac{\partial}{\partial x}\begin{Bmatrix} y \\ v \end{Bmatrix} = B.$$

$$\Rightarrow \frac{\partial G}{\partial t} + A \frac{\partial G}{\partial x} = B \qquad \text{where, } A = \begin{bmatrix} v & D \\ g & v \end{bmatrix}, \; G = \begin{Bmatrix} y \\ v \end{Bmatrix}, \; B = \begin{Bmatrix} 0 \\ g(S_0 - S_f) \end{Bmatrix}.$$

Eigen values of vector A, $\begin{vmatrix} v-\lambda & D \\ g & v-\lambda \end{vmatrix} = 0$

$$\Rightarrow (V-\lambda)^2 - gD = 0$$

$$\Rightarrow V - \lambda = \pm\sqrt{gD}.$$

$\lambda_1 = V + \sqrt{gD}$ and $\lambda_2 = V - \sqrt{gD}$.

λ_1, λ_2 are real and distinct values.

Therefore, the equations are set of hyperbolic PDE's. (Hyperbolic PDE's are kind of wave equations, only marching procedures are suitable for numerical integration of these equations.) Governing equations are solved by using method of charact

[* From (R_2-Page 43), if coefficient matrix of space derivative term has real and distinct eigenvalues
↳ Hyperbolic eq^{n}s
Zero eigenvalues → Elliptical system.
Real and equal λ's → Parabolic system.]

Rewrite equation (3) and (4), in form of $L_1 + \lambda L_2 = 0$, we have.

$$\left(\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x} + g\frac{\partial y}{\partial x}\right) + \lambda\left(\frac{\partial y}{\partial t} + Dh\frac{\partial V}{\partial x} + V\frac{\partial y}{\partial x}\right) = g(S_0 - S_f)$$

Seperating V and y related terms,

$$\left(\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x} + \lambda Dh\frac{\partial V}{\partial x}\right) + \left(g\frac{\partial y}{\partial x} + \lambda\frac{\partial y}{\partial t} + \lambda V\frac{\partial y}{\partial x}\right) = g(S_0 - S_f).$$

$$\Rightarrow \underbrace{\left\{\frac{\partial V}{\partial t} + (V + \lambda D_h)\frac{\partial V}{\partial x}\right\}}_{\text{1st part}} + \lambda \underbrace{\left\{\frac{\partial y}{\partial t} + \left(V + \frac{g}{\lambda}\right)\frac{\partial y}{\partial x}\right\}}_{\text{2nd part.}} = g(S_0 - S_f) \quad \text{.....(5)}$$

We know, $\frac{dV}{dt} = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} \cdot \frac{dx}{dt}$ ..... (6.1)

and $\frac{dy}{dt} = \frac{\partial y}{\partial t} + \frac{\partial y}{\partial x} \cdot \frac{dx}{dt}$. ..... (6.2)

Comparing 1st part of equation (5), with (6.1),

$$\frac{dx}{dt} = (V + \lambda D_h) \quad \ldots\ldots (7.1)$$

and for 2nd part and (6.1), we have,

$$\frac{dx}{dt} = \left(V + \frac{g}{\lambda}\right) \quad \ldots\ldots\ldots (7.2)$$

Eq^n (5) can be written as,

$$\boxed{\frac{dV}{dt} + \lambda \frac{dy}{dt} = g(S_0 - S_f)} \quad \ldots\ldots (7.3)$$

Comparing (7.1) and (7.2), (Because, both are $\frac{dx}{dt}$ values for same equation),

$$\cancel{V} + \lambda D_h = \cancel{V} + \frac{g}{\lambda}$$

$$\Rightarrow \lambda^2 = \frac{g}{D_h}$$

$$\Rightarrow \lambda = \pm\sqrt{\frac{g}{D_h}} = \pm\sqrt{\frac{gT}{A}} \quad \ldots\ldots (8)$$

$$\Rightarrow D_h = \frac{g}{\lambda^2} \quad \ldots\ldots (8.1)$$

Hydraulic depth.

$$\left[D_h = \text{Hydraulic depth} = \frac{A}{T}\right]$$

Celerity of gravity wave, $c = \sqrt{gD_h}$

$$= \sqrt{g \cdot \frac{A}{T}}$$

$$\Rightarrow D_h = \frac{c^2}{g} \quad \ldots (8.2)$$

Comparing these two,

$$\frac{g}{\lambda^2} = \frac{c^2}{g} \Rightarrow \lambda^2 = \frac{g^2}{c^2}$$

$$\Rightarrow \lambda_1 = \frac{g}{c}, \quad \lambda_2 = -\frac{g}{c} \quad \ldots\ldots (9)$$

For two different values of λ (i.e. λ_1 and λ_2) equation (7.3) can be written as:-

$$\frac{dV}{dt} + \frac{g}{c} \cdot \frac{dy}{dt} = g(S_0 - S_f) \quad \ldots\ldots (10.1)$$

and it's $\frac{dx}{dt}$ value (from 7.1 or 7.2, Put λ values in any of these equations).

$$\frac{dx}{dt} = V + g \cdot \frac{c}{g} = V + c. \quad \ldots\ldots (10.2)$$

→ Compatibility equations.

and

$$\frac{dV}{dt} - \frac{g}{c} \cdot \frac{dy}{dt} = g(S_0 - S_f) \quad \ldots\ldots (11.1)$$

and $\frac{dx}{dt} = V + \lambda D_h = V - \frac{g}{c} \cdot D_h$

$$= V - \frac{g}{c} \cdot \frac{c^2}{g}$$

$$= V - c \quad \ldots\ldots (11.2)$$

Positive and negative

→ Remember, equation (11.1) is valid if equation (11.2) is satisfied.

In equation (3) and (4), from journey from eq^n (5), to eq^n (10.1) and (11.1), we removed space variable x from governing equation and converted PDE to ODE.

Some point regarding Method of characteristics:-

It solves 1st order linear or quasi-linear PDE's in the form,

$$a(x,y)\underbrace{u_x}_{} + b(x,y)\underbrace{u_y}_{} = c(x,y,u).$$

These terms are may be λ.

where, $u(x,y)$ are unknown functions and a, b, c are given functions.

The method of characteristics can be used when the PDE can be reduced to system of ODE's by introducing set of curves, known as Characteristics curves.

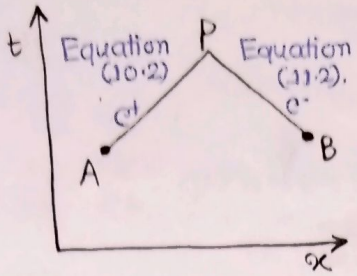
These curves are defined by the differential equations.

$$\frac{dx}{dt} = a(x,y)$$

$$\frac{dy}{dt} = b(x,y)$$

$$\frac{du}{dt} = c(x,y,u)$$

The solution of PDE's can be obtained by solving the system of ODE's and using the characteristics curve to reconstruct the solution.



Original governing equations (3) and (4)

i.e, $\frac{\partial y}{\partial t} + D.\frac{\partial v}{\partial x} + v.\frac{\partial y}{\partial x} = 0$

and $\frac{\partial v}{\partial t} + g.\frac{\partial}{\partial x}\left(\frac{v^2}{2g} + y\right) = g(S_o - S_f)$

are valid for any value of x and t or valid throughout (x,t) plane.

But, (10.1) and (11.1) are valid only along characteristics line (10.2) and (11.2) respectively.

Multiply (10.1) and (11.1) by dt and integrating along the characteristics line AP and BP,

From (10.1), $\int_A^P dv + \int_A^P \frac{g}{c} dy = \int_A^P g(S_o - S_f) dt$

$\Rightarrow (v_P - v_A) + \left(\frac{g}{c}\right)_A (y_P - y_A) = g(S_o - S_f)_A (t_P - t_A)$

$\Rightarrow \underline{\underline{v_P}} = v_A - \left(\frac{g}{c}\right)_A \underline{\underline{(y_P}} - y_A) + g(S_o - S_f)_A (t_P - t_A)$

(v_P and y_P are the unknowns here)

$\Rightarrow v_P = -\underbrace{\left(\frac{g}{c}\right)_A}_{C_A} y_P + \underbrace{v_A + \left(\frac{g}{c}\right)_A y_A + g(S_o - S_f)_A (t_P - t_A)}_{C_P}$

$\Rightarrow \boxed{v_P = -C_A y_P + C_P}$ ..... (12.1)

where, $C_A = -\left(\frac{g}{c}\right)_A$ ..... (12.1)

and $C_P = v_A + C_A y_A + g(S_o - S_f)_A (t_P - t_A)$ ..... (12.2)

For line BP, (From 11.1),

$\int_B^P dv - \int_B^P \frac{g}{c} dy = \int_B^P g(S_o - S_f) dt.$

$\Rightarrow (v_P - v_B) - \left(\frac{g}{c}\right)_B (y_P - y_B) = g(S_o - S_f)_B (t_P - t_B)$

$\Rightarrow v_P = \left(\frac{g}{c}\right)_B y_P + v_B - \left(\frac{g}{c}\right)_B y_B + g(S_o - S_f)_B (t_P - t_B)$

$\Rightarrow \boxed{v_P = C_B y_P + C_n}$ ..... (13.2)

$v_P, y_P \Rightarrow$ values of secondary variables at next time level.

$C_B \Rightarrow$ constant at point B (current time level) $= \left(\frac{g}{c}\right)_B$ ..... (13.1)

$C_n \Rightarrow$ constant regarding negative characteristics line.

$= v_B - C_B y_B + g(S_o - S_f)_B (t_P - t_B)$ ..... (13.2)

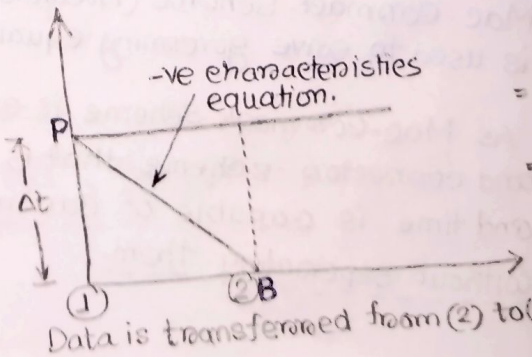
12.1 and 12.2 are final algebraic equations which are valid along positive and negative characteristics line respectively.

Upstream Boundary Conditions

(12.2) for NCE, $v_P = C_B y_P + C_n$

From (13.1)

$C_n = v(2) - C_B y(2) + g(S_o - S_{f_2}).\Delta t$



Data is transferred from (2) to (P).

$\frac{g}{c} = \frac{g}{\sqrt{gy}} = \sqrt{\frac{g}{y}} = \sqrt{\frac{gT}{A}}$

$$S_{f_2} = \frac{n^2 Q^2}{K^{8/3} A^2} = \frac{n^2 Q(2) \cdot |Q(2)|}{[R_H(2)]^{4/3} [A(2)]^2}$$

$$C_B = \sqrt{\frac{gT(2)}{A(2)}}$$

$y_{new}(1) = y_0 \longrightarrow$ How (?) Boundary condition?

$v_p = C_B y_p + C_n$

after getting v_p, we get discharge as:- $Q_p = v_p A_p$ (where A_p means A_{new})

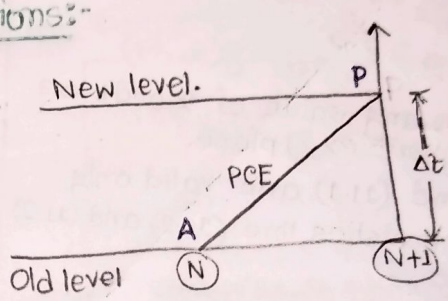
Downstream Boundary conditions:-

$v_p = -C_A y_p + C_p$

$Q_{new}(N+1) = 0$ ..... (15.1)

$C_A = \sqrt{\frac{gT(N)}{A(N)}}$

$S_{f(A)}$ or $S_{f(N)} = \frac{n Q(N) \cdot |Q(N)|}{[A(N)]^2 \cdot [R_H(N)]^{4/3}}$



Data is transferring from N^{th} node of (t_0) time level to $(N+1)^{th}$ node of $(t_0 + \Delta t)$ time level

$C_p = v_A + C_A y_A + g(S_0 - S_f)_A \cdot (t_p - t_A)$

$= \frac{Q(N)}{A(N)} + C_A \cdot y(N) + g(S_0 - S_{f_N}) \cdot \Delta t$

Note that, node numbers are represented as 1, 2, N, N+1 etc.
But, the coefficients are always C_A, C_B, C_p, C_n.

We know that, for positive characteristics line,

$v_p = -C_A y_p + C_p$, But here $v_p = 0$

(# Because, from (15.1), $Q_{new}(N+1) \equiv Q_p(N+1) = 0$, I.e, it is a boundary condition)!

$\Rightarrow 0 = -C_A y_p + C_p$

$\Rightarrow y_p = \frac{C_p}{C_A}$

$A_{new}(N+1) = y_p \cdot T_p$

$= y_{new}(N+1) \cdot T_{new}(N+1)$

y_p can also be represented as, $y_{new}(N+1)$.
↳ y for node (N+1) of new time level.
y for point 'P' of PCE.
Both are same points

(?) Interior nodes:- (Why we don't use PCE and NCE here?)!

Mac-Cormack Scheme (Predictor and Corrector method) is used to solve governing equations at interior cells.

As Mac-Cormack scheme is explicit, two step prediction and corrector scheme that is 2nd order accurate in space and time is capable of capturing the se' shocks without seperating them.

(?)

Governing equations:-

CE: $\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = 0$ ..... (16.1)

ME: and $\frac{\partial Q}{\partial t} + \frac{\partial E}{\partial x} = gA(S_o - S_f)$. where, $E = QV + gA\bar{y}$. ..... (16.2)

Predictor step:-

Continuity equation,

$Q_i^* + \cancel{Q}$ unknown.

$$\frac{Q_{i+1}^n - Q_i^n}{\Delta x} + \frac{A_i^* - A_i^n}{\Delta t} = 0$$

$$A_i^* = A_i^n - \frac{\Delta t}{\Delta x}(Q_{i+1}^n - Q_i^n) \quad \text{... (16.4)}$$

From momentum equation,

$$\frac{\partial Q}{\partial t} + \frac{\partial E}{\partial x} = gA(S_o - S_f)$$

$$\Rightarrow \frac{Q_i^* - Q_i^n}{\Delta t} + \frac{E_{i+1}^n - E_i^n}{\Delta x} = gA_i^n(S_o - S_{f_i}^n)$$

unknown.

$$\Rightarrow Q_i^* = Q_i^n - \frac{\Delta t}{\Delta x}(E_{i+1}^n - E_i^n) + gA_i^n(S_o - S_{f_i}^n) \quad \text{.... (16.5)}$$

Friction slope varies with node to node and also for different time steps. (In includes variables like Q, A, which varies)

But, bed slope is taken as constant throughout the channel. !

Now, $V_i^* = \frac{Q_i^*}{A_i^*}$

$\bar{y}_i^* = \frac{A_i^*}{T_i^*}$ → How to get top with for predicted time step?

$$S_{f_i}^* = \frac{n^2 Q_i^{*2}}{A_i^{*2} \cdot R_i^{4/3}} \quad \left[R = \frac{A}{P} = \frac{A_i^*}{T_i^* + 2y_i^*}\right]$$

General steps of Predictor and Corrector step (Recap)

Predictor step (FD:- $\frac{\partial f}{\partial t} = \frac{f_i^* - f_i^n}{\Delta t}$

$\frac{\partial f}{\partial x} = \frac{f_{i+1}^n - f_i^n}{\Delta x}$

Corrector step (BD):-

$$\boxed{\frac{\partial f}{\partial t} = \frac{f_i^{**} - f_i^n}{\Delta t}}$$

$$\boxed{\frac{\partial f}{\partial x} = \frac{f_i^* - f_{i-1}^*}{\Delta x}} \quad \text{.... (16.3)}$$

Now, value of discharge and area at node no. i at next time step,

$$Q_i^{n+1} = \frac{1}{2}(Q_i^* + Q_i^{**}) \longrightarrow ?$$

(#But, at page 35, formula was, $Q_i^{n+1} = \frac{1}{2}(Q_i^n + Q_i^{**})$)

Corrector step:-

Continuity equation:- (& Formula mentioned at 16.3),

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = 0$$

$$\Rightarrow \frac{Q_i^* - Q_{i-1}^*}{\Delta x} + \frac{A_i^{**} - A_i^n}{\Delta t} = 0$$

[Here, in corrector step, * is known, ** unknown!]

$$\Rightarrow A_i^{**} = A_i^n - \frac{\Delta t}{\Delta x}(Q_i^* - Q_{i-1}^*)$$

Momentum equation:- $\frac{\partial Q}{\partial t} + \frac{\partial E}{\partial x} = gA(S_o - S_f)$

$$\Rightarrow \frac{Q_i^{**} - Q_i^n}{\Delta t} + \frac{E_i^* - E_{i-1}^*}{\Delta x} = gA_i^*(S_o - S_{f_i}^*)$$

$$\Rightarrow Q_i^{**} = Q_i^n - \frac{\Delta t}{\Delta x}(E_i^* - E_{i-1}^*) + g A_i^* (S_0 - S_{f_i}^*)\Delta t$$

Do this for all nodes from 2 to N.

Now, finally variable values at $(n+1)^{th}$ time level,

$$Q_i^{n+1} = \frac{1}{2}(Q_i^* + Q_i^{**})$$

$$A_i^{n+1} = \frac{1}{2}(A_i^* + A_i^{**})$$

$$V_i^{n+1} = \frac{Q_i^{n+1}}{A_i^{n+1}}$$

$$y_i^{n+1} = \frac{A_i^{n+1}}{T_i^{n+1}}$$

Stability criteria:-

Mac-Cormack scheme and method of characteristics is stable if CFL is less than 1. $\left(\frac{a\Delta t}{\Delta x} \leq 1\right)$ $\Rightarrow$ This condition is to be satisfied at each grid point and each computational interval.

Find, $dt = \frac{dx}{V_{max} + C_{max}}$

In computations, minimum value of dt is chosen among all the grid points.

But, if we take a particular Δt value from courant stability condition and do the calculations for each node and every time-step then only we can know V_{max} and C_{max}. So, how is it possible to calculate dt using V_{max} and C_{max}

?