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On the use of analytical solutions to design pumping tests in leaky aquifers connected to a stream

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SUMMARY

Christensen et al. (2009) studied the optimal design of pumping tests whose goal is to estimate hydrogeologic parameters affecting stream depletion caused by pumping in a leaky aquifer. Their analysis relies on the analytical solutions of Zlotnik and Tartakovsky (2008), which are based on the assumptions of negligible drawdown in the source bed of a leaky aquifer and of horizontal flow in an aquifer of infinite extent. We conduct a series of two- and three-dimensional simulations to identify the validity and range of applicability of these assumptions, and to quantify their impact on estimation of aquifer drawdown, stream depletion, their sensitivities to the hydrogeologic parameters, and on selection of optimal locations to observe drawdown during a pumping test. Quantitative criteria for the use of these analytical solutions are formulated in terms of the source-bed transmissivity and the distance from the well to prescribed-head boundaries other than the stream.

Introduction

Sensitivity analysis and optimal design (e.g., Christensen, 2000; Christensen et al., 2009) of pumping tests under various hydrogeological conditions often rely on analytical solutions. For example, Christensen (2000) used the analytical solution of Hunt (1999) to conduct a sensitivity analysis of a pumping test conducted in a non-leaky aquifer near a stream; and Christensen et al. (2009) relied on the analytical solution of Zlotnik and Tartakovsky (2008) to carry out a sensitivity analysis and global optimization for the optimal design of a pumping test in a leaky aquifer, which is separated from a deep source bed by an aquitard. Analytical solutions typically require a certain level of abstraction, i.e. a number of physical assumptions, to represent complex hydrogeological conditions in a mathematical form that is amenable to analytical treatment. In particular, the analytical solution of Zlotnik and Tartakovsky (2008) assumes that flow is horizontal, flow domain is infinite, and drawdown in the deep source bed is negligible.

The validity and limitation of the latter assumption for pumping in leaky-aquifer systems are well understood (e.g. Hantush, 1967; Neuman and Witherspoon, 1969a,b). Analyzing pumping in leaky aquifers connected to a stream, Hunt (2008) argued that this assumption is unrealistic on two accounts. First, it implies that the source bed supplies an infinite amount of water for recharge of the pumped aquifer. Second, it suggests that groundwater pumping is compensated by a combination of stream depletion, leakage from the source bed through the aquitard, and change of storage; with the contribution of the second source increasing with the distance between the well and the stream. However, for the infinite domain studied by Zlotnik and Tartakovsky (2008) there can only be two sources to compensate pumping, a stream and storage. In the (very) long term the stream depletion rate will therefore become equal to the pumping rate and not just a fraction of it as modeled by the depletion solution given by Zlotnik and Tartakovsky (2008). (This was also demonstrated in the recent discussion paper of Scott, 2009, which first became available during our study.) While agreeing that the assumption of negligible drawdown is never rigorously met in actual field situations, Butler et al. (2008) point out that various pumping tests in leaky-aquifer systems have shown this assumption to be reasonable for pumping periods of limited durations.

The preceding discussion leads one to conclude that the analytical solution of Zlotnik and Tartakovsky (2008) might be more appropriate for pumping-test analyses than for simulation of long-term drawdown and stream depletion. The main goal of this

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analysis is to quantify this qualitative distinction. Specifically, we investigate the impact of the key assumptions underlying the analytical solution of Zlotnik and Tartakovsky (2008) on predictions of stream depletion and drawdown, as well as on their sensitivities with respect to relevant hydrogeologic parameters. The latter are of particular importance, since they are used in uncertainty analysis and optimization (e.g., Christensen et al., 2009).

The paper is organized as follows. The first section presents the methodologies used for computing drawdown, rate of stream depletion, their sensitivities, and the prediction variance from hydrogeologic parameter values estimated by pumping-test analysis. We also briefly summarize the pumping-test design optimization methodology described by Christensen et al. (2009). After this follows a presentation of results that illustrate the implications of the simplifying assumptions. Finally we discuss some of the implications of our results and draw practical conclusions.

Methodology

We study stream depletion and aquifer drawdown induced by groundwater extraction from a homogeneous leaky aquifer, in which a pumping well is adjacent to a stream that is hydraulically connected to an aquifer (Fig. 1). The aquifer is separated from a deeper source bed by an aquitard. Such hydrogeological conditions are quite common, as discussed by Zlotnik (2004). The following three subsections present two alternative approaches to compute depletion rate, drawdown, and their sensitivities under conditions of either negligible ("Solutions for negligible source-bed drawdown") or non-negligible ("Two-dimensional simulations for non-negligible source-bed drawdown" and "Three-dimensional simulations for non-negligible source-bed drawdown") drawdown in the source bed. "Optimization of pumping-test design" summarizes the optimization methodology of Christensen et al. (2009) for pumping-test design. In the following, we use the terms "stream depletion" and "stream depletion rate" interchangeably to improve readability.

Solutions for negligible source-bed drawdown

Zlotnik and Tartakovsky (2008) derived analytical solutions to compute rate of stream depletion and drawdown in the aquifer by assuming that flow in the aquifer is horizontal, drawdown in the source bed is negligible, and the flow domain is infinite. Analytical expressions for the dimensionless stream depletion rate, q_d, and the dimensionless drawdown in the aquifer, ϕ_d, are given respectively by Eqs. (22) and (17) of Zlotnik and Tartakovsky (2008),

$$q_d = \frac{a_1}{2}E\left(-\frac{1}{B_d}\right) - \frac{a_2}{2}E\left(\frac{1}{B_d}\right) + a_3 e^{\lambda_d^2 t_d/4 - t_d/B_d^2} E\left(\frac{\lambda_d}{2}\right), \quad (1a)$$

where

$$E(\xi) = e^{\xi} erfc\left(\frac{1}{2\sqrt{t_d}} + \sqrt{t_d}\xi\right). \quad (1b)$$

and

$$\phi_d(x_d, y_d, t_d) = \frac{1}{4\pi}\left[W(u,z) - \int_0^\infty e^{-\theta} W(u_\lambda, z_\lambda) d\theta\right], \quad (2a)$$

where

$$W(u,z) = \int_u^\infty \frac{1}{y} \exp\left(-y - \frac{z^2}{4y}\right) dy. \quad (2b)$$

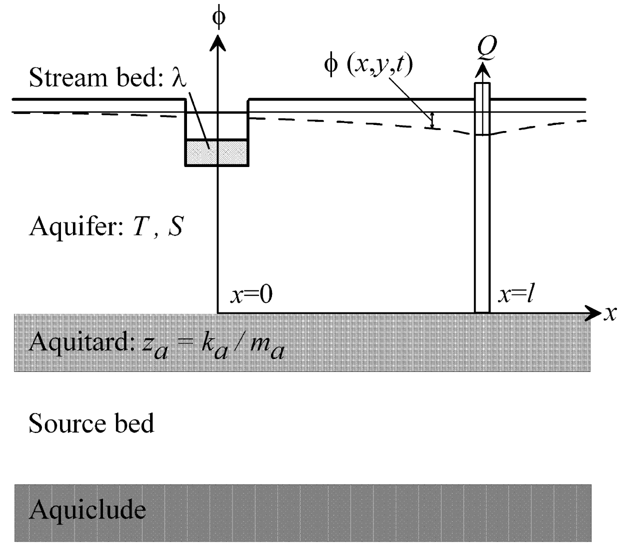

Fig. 1. A schematic representation of the stream–aquifer–aquitard–source bed system and the major hydrological parameters for the solutions of Zlotnik and Tartakovsky, 2008. Explanation of the symbols follows Eq. (3).

These expressions contain dimensionless parameters

$$\begin{aligned} &q_d = \tfrac{q}{Q}, \quad \phi_d = \tfrac{\phi T}{Q}, \quad t_d = \tfrac{Tt}{Sl^2}, \quad x_d = \tfrac{x}{l}, \quad y_d = \tfrac{y}{l}, \quad B_d^2 = \tfrac{T}{z_a l^2}, \\ &\lambda_d = \tfrac{\lambda l}{T}, \quad a_1 = \tfrac{B_d}{2/\lambda_d + B_d}, \quad a_2 = \tfrac{B_d}{2/\lambda_d - B_d}, \quad a_3 = a_1 a_2, \\ &u = \tfrac{r_d^2}{4t_d}, \quad z = \tfrac{r_d}{B_d}, \quad u_\lambda = \tfrac{r_\lambda^2}{4t_d}, \quad z_\lambda = \tfrac{r_\lambda}{B_d}, \\ &r_d^2 = (x_d - 1)^2 + y_d^2, \quad r_\lambda^2 = \left(1 + |x_d| + \tfrac{2\theta}{\lambda_d}\right)^2 + y_d^2 \end{aligned} \quad (3)$$

where Q is the pumping rate, q is the stream depletion rate, ϕ is the drawdown, T is the aquifer transmissivity, S is the aquifer storativity, l is the distance between the well and the stream, x and y are the Cartesian coordinates, t is the time elapsed since pumping commenced, z_a is the leakage coefficient of the aquitard, and λ is the stream-bed conductance. The leakage coefficient can be computed as $z_a = k_a/m_a$, where k_a and m_a are the hydraulic conductivity and the thickness of the aquitard, respectively. The stream-bed conductance can be represented as $\lambda \approx k_s w_s / m_s$ where k_s, w_s, and m_s are the hydraulic conductivity, the width, and the thickness of the stream bed, respectively.

The sensitivity of stream depletion with respect to the transmissivity T quantifies the change in stream depletion in response to slight changes in T. It is defined by the derivative $\partial q/\partial T$ and can be computed analytically by differentiating (1) with respect to T. Similarly one can derive analytical expressions for the sensitivities of the depletion rate with respect to the other hydrogeologic parameters, $\partial q/\partial S$, $\partial q/\partial \lambda$, and $\partial q/\partial z_a$. Drawdown sensitivities with respect to the hydrologic parameters, $\partial \phi/\partial T$, $\partial \phi/\partial S$, $\partial \phi/\partial \lambda$, $\partial \phi/\partial z_a$ can be obtained by differentiating (2). These analytical expressions, which are presented in Christensen et al. (2009), are used below to compute the depletion and drawdown sensitivities when drawdown is negligible.

The limit of $B_d \to \infty$ corresponds to an impermeable aquitard, in which cases (1) and (2) reduce to the non-leaky aquifer solutions for depletion and drawdown given by Hunt (1999). Analytical expressions for sensitivities for the Hunt solutions can be found in Christensen (2000).

Two-dimensional simulations for non-negligible source-bed drawdown

In the absence of analytical solutions for flow regimes with non-negligible source-bed drawdown (Fig. 1), we rely on numerical simulations to compute stream depletion and aquifer drawdown in both "infinite" and finite flow domains. These simulations were carried out with MODFLOW-2000 (Harbaugh et al., 2000) for the following values of the hydrogeologic parameters. The aquifer (upper layer) has transmissivity $T = 1000\,m^2/d$ and storativity $S = 0.1$. The distance between the pumping well and the stream is $l = 1000$ m, and the pumping rate is $Q = 1000\,m^3/d$. The stream-bed conductance is $\lambda = 1$ m/d or 0.1 m/d, which corresponds to $\lambda_d = 1$ or 0.1, respectively. The aquitard (middle layer) has storativity 10^{-4} and leakage coefficient $z_a = 10^{-5}\,d^{-1}$ or $10^{-7}\,d^{-1}$, which corresponds to $B_d = 10$ or 100, respectively. The deep source bed (bottom layer) has storativity 10^{-4} and transmissivity which is a factor $T_d = 1$, 10, 100, or 1000 times greater than the aquifer transmissivity T. The stream stage and the initial hydraulic head in all layers are set to 0 m. (The simulation results were identical to those obtained by using a two layer MODFLOW-2000 model with a confining bed.)

Constant head in all three layers was imposed along the boundaries of a square flow domain. Most of the results reported below correspond to a square flow domain with the distance from the well to the boundaries equal to $50{,}000 \times l$ in both the x and y directions. This precludes the simulated drawdown from reaching the constant-head boundaries. The domain of this size is referred to as "infinite" in the following. Another set of simulations was conducted on a smaller square domain with the distance from the well to the boundaries equal to $50 \times l$. This domain is referred to as "finite", since it allows drawdown to reach the boundaries and cause inflow. Simulations in both domains employed a fine spatial discretization around the well and along the stream (the order of 1 m), which is gradually made coarser (by using a factor of 1.5) when moving towards the boundaries. The initial time step is 54.4 s and the subsequent 119 time steps increase by a factor of 1.2, adding up to a 10^7 day simulation period. Finally, the sensitivities were computed numerically by solving the sensitivity equations with MODFLOW-2000 (Hill et al., 2000).

For both infinite and bounded domains, the numerical simulations with constant head in layer 3, i.e. without drawdown in the source bed, yielded results of drawdown, stream depletion rate, and their sensitivities that differ by less than a few percents from their analytical counterparts given by (1) and (2) and the sensitivities derived from them. The maximum differences between the numerical and analytical solutions are 1% for depletion, 5% for drawdown, and up to 6% for their sensitivities.

Allowing hydraulic head in the bottom layer to vary, we reran these two-dimensional (2D) numerical models to simulate the development of drawdown in all three model-layers, rate of stream depletion and the corresponding sensitivities with respect to T, S, λ, z_a, and T_d.

Although the numerical model is set up in terms of dimensional variables, the results are presented in dimensionless form. This is done to indicate general applicability of the results for given dimensionless input values, and it is the general results that are of interest here. Dimensional results and conversion from dimensionless to dimensional results can be found in Christensen et al. (2009).

Three-dimensional simulations for non-negligible source-bed drawdown

To assess the impact of the assumption of horizontal flow, we simulated three-dimensional (3D) flow with a numerical model in which the aquifer, aquitard, and source bed are each subdivided into five computational layers. The horizontal resolution for the infinite domain is the same as that described in "Two-dimensional simulations for non-negligible source-bed drawdown". The upper five model-layers, each 2 m thick, represent the pumped aquifer with horizontal hydraulic conductivity $K_h = 100$ m/d, vertical hydraulic conductivity $K_v = 1$ m/d, specific yield $S_y = 0.1$, and specific storage $S_s = 10^{-5}\,m^{-1}$. This corresponds to transmissivity $T = 1000\,m^2/d$ and storativity $S = 0.1$ used in the two-dimensional simulations described in "Three-dimensional simulations for non-negligible source-bed drawdown". The pumping well extracts $200\,m^3/d$ from each of the five model-layers at the distance $l = 1000$ m from the stream. The stream is connected to model layer 1 and has bed conductance $\lambda = 1$ m/d, i.e. $\lambda_d = 1$.

The aquitard is represented by the five middle layers, each of which is 2 m thick and has the horizontal and vertical hydraulic conductivities $K_h = K_v = 10^{-4}$ m/d. This corresponds to the aquitard leakage coefficient $z_a = 10^{-5}\,d^{-1}$. The specific storage is either $S_s = 10^{-5}\,m^{-1}$ or $10^{-4}\,m^{-1}$, which results in storativity 10^{-4} or 10^{-3}, respectively.

The deep source bed is represented with the five bottom layers, each of which is 20 m thick and has the horizontal hydraulic conductivity $K_h = 10$ m/d or 10^4 m/d, which yields $T_d = 1$ or 1000, respectively. In both cases, the vertical to horizontal hydraulic conductivity anisotropy is 0.01 and the specific storage is $S_s = 10^{-6}\,m^{-1}$, which corresponds to source-bed storativity $S = 10^{-4}$.

Optimization of pumping-test design

Christensen et al. (2009) describe an optimization procedure that can be used to identify optimum locations to observe drawdown for pumping tests, whose purpose is either to make a prediction dependent on the hydrogeologic parameters (here stream depletion rate), or to estimate a particular hydrogeologic parameter, or to estimate a linear combination of parameters. We use this procedure to find optimal locations for observing drawdown, when the purpose of the pumping test is to predict rate of stream depletion caused by pumping.

Consider a pumping-test analysis that aims to estimate hydrogeologic parameters, primarily T, S, λ, and z_a, by fitting Eq. (2) to a set of n observations of drawdown made at varying times and locations. Let us assume that measurement errors in the n observations are uncorrelated and have zero mean and variance σ^2. Then the 4×4 covariance matrix of the estimated (fitted) parameter values can be approximated by (Seber and Wild, 1989)

$$\mathbf{C} = \mathbf{C}(T, S, \lambda, z_a) = \sigma^2(\mathbf{X}^T\mathbf{X})^{-1}. \tag{4}$$

The ith row of the $n \times 4$ sensitivity matrix $\mathbf{X} = [\partial\phi_i/\partial T, \partial\phi_i/\partial S, \partial\phi_i/\partial\lambda, \partial\phi_i/\partial z_a]_{i=1,n}$ contains the sensitivities of the computed drawdown corresponding to the time and location of the ith observation. The superscripts T and -1 in (4) indicate matrix transpose and matrix inverse, respectively. The sensitivities are computed either analytically or numerically following the procedures described in "Solutions for negligible source-bed drawdown" and "Two-dimensional simulations for non-negligible source-bed drawdown", respectively.

Stream depletion is predicted by using either Eq. (1) or numerical modeling. The corresponding predictive uncertainty is quantified by the standard deviation of the prediction,

$$\sigma_{qd} = \sigma\sqrt{\mathbf{Z}^T\mathbf{C}\mathbf{Z}}, \tag{5}$$

where $\mathbf{Z}^T = [\partial q_d/\partial T,\ \partial q_d/\partial S,\ \partial q_d/\partial\lambda,\ \partial q_d/\partial z_a]$ is the (transposed) vector of sensitivities of depletion with respect to the hydrogeologic parameters.

A scaled standard deviation of the dimensionless depletion is defined as

$$\sigma_{qds} = \sigma_{qd}Q/\sigma T. \tag{6}$$

If parameters T_d and K_v are to be estimated in conjunction with T, S, λ, and z_a, columns of $\partial\phi_i/\partial T_d$ and $\partial\phi_i/\partial K_v$ must be included in the matrix **X** in (4), and the vector **Z** in (5) must be expanded to include elements $\partial q_d/\partial T_d$ and $\partial q_d/\partial K_v$.

The optimal locations to observe drawdown are defined as locations that minimize (5), i.e., make the uncertainty of the stream-depletion prediction as small as possible. The approach to identify these locations consists of the following steps:

1. Define the time at which the stream depletion rate q_d is to be predicted. This is the target prediction.
2. Define the number of observation wells, duration of observations, and frequency with which drawdown is to be observed.
3. Find locations of the observation wells, which minimize the standard deviation of the target prediction σ_{qd}.

The results presented below correspond to predictions of stream depletion at short, intermediate, and long times after pumping has been initiated. Unless indicated otherwise, we use two observation wells and an observation period starting at $t_d = 10^{-3}$ and ending at $t_d = 10^2$ (these values were found to be nearly optimal by Christensen et al., 2009). In all cases, the observation frequency is 10 observations per decade evenly spaced when time is $\log_{10}$-transformed. To find the global minimum of σ_{qd}, we used the CMAES_P code (Doherty, 2008), which is an implementation of the CMA-ES global optimization scheme by Hansen and Ostermeier (2001). (Note that minimization of σ_{qd} is nontrivial since it is a nonlinear function of the well locations.) We applied the search strategy suggested by Christensen et al. (2009), in which a search for optimum well locations is conducted only along the line perpendicular to the stream passing through the pumping well. This proved to be an efficient and accurate strategy (Christensen et al., 2009).

Results and discussion

The analytic solutions for drawdown and depletion developed by Zlotnik and Tartakovsky (2008) are based on the assumptions that flow in the aquifer is 2D and that drawdown is insignificant in the deep source bed. To test the validity of these assumptions, and to analyze their impact on the predicted stream depletion rate, drawdown, and sensitivities, as well as on identification of the optimal locations to observe drawdown during a pumping test, we conduct a series of numerical experiments. The analytical solutions and their numerical counterparts are compared for several hydrogeologic settings that were chosen from those presented by Christensen et al. (2009).

Stream depletion rate and its sensitivities

Fig. 2 compares the dimensionless stream depletion rate and its sensitivities computed analytically (the solid curves) with their numerical counterparts (the dashed curves). Two analytical solutions are presented: the Hunt (1999) solution, which both disregards leakage in the aquifer and ignores the source bed; and the Zlotnik and Tartakovsky (2008) solution, which allows for leakage in the aquifer but assumes that drawdown in the source bed is negligible. Consequently, the former solution is independent of both z_a and T_d, while the latter solution is independent of T_d only. The numerical solutions are for the infinite domain in which drawdown is allowed to develop in all three hydrogeologic units. The transmissivity of the source bed varies from being equal to the transmissivity of the pumped aquifer ($T_d = 1$) to being 1000 times greater ($T_d = 1000$). In all simulations, we set $B_d = 10$ and $\lambda_d = 1$.

One can see that the depletion and its sensitivities computed with the Hunt and Zlotnik–Tartakovsky solutions are similar at early times, diverge after some time of pumping, and reach different large-time asymptotes (except for the sensitivity with respect to storativity S). The differences in the two sets of curves stem from different physical processes captured by the corresponding models. In the Hunt solution pumping at steady state is compensated only by leakage from the stream, while in the Zlotnik–Tartakovsky solution the deep source bed represents another potential source.

For early times, the analytical solutions are in good agreement with their numerical counterparts. The agreement deteriorates at intermediate times, with the Zlotnik–Tartakovsky solutions providing a reasonable approximation for longer periods of time. At late time, the numerical simulations yield the asymptotic values predicted by the Hunt solution. This occurs after drawdown has developed so that all pumping is compensated by leakage from the stream; at this stage, the source bed acts as another layer through which water can flow from the stream towards the well.

Let $[t_1, t_2]$ denote the time interval between time t_1, at which the analytical and numerical solutions start to diverge, and time t_2, at which the numerical solutions reach the asymptotes predicted by the Hunt solution. Fig. 2 reveals that the length of this interval depends on the transmissivity of the source bed. If the source-bed transmissivity is equal to the aquifer transmissivity, then the numerical solutions are generally very similar to the Hunt curves for the entire time range. On the other hand, if the source bed has a transmissivity that is three orders of magnitude larger than that of the aquifer, then the numerical solutions follow the Zlotnik–Tartakovsky curves closely until dimensionless time $t_d \approx 100$ while it takes further four-to-six orders of magnitude before they converge towards the Hunt curves.

We have obtained results that are qualitatively similar to those presented in Fig. 2 for other values of B_d and λ_d. As the value of stream-bed conductance λ_d decreases, the disagreement between the Hunt and Zlotnik–Tartakovsky solutions increases for intermediate and large times. The numerical solutions for various values of T_d remain close to the Zlotnik–Tartakovsky curves at early times, and converge towards the Hunt curves at late times.

For smaller B_d (i.e. for less permeable aquitards), the disagreement between the Hunt and Zlotnik–Tartakovsky curves is less pronounced, and both are closer to the numerical solutions. The value of t_1 is larger, because drawdown in less permeable aquitards takes longer to develop.

The numerical simulations reported in Fig. 2 are based on the assumptions that flow in the aquifer and the source bed is horizontal and flow through the aquitard is vertical (see "Two-dimensional simulations for non-negligible source-bed drawdown"). These assumptions of quasi-2D flow were verified with the 3D simulations described in "Three-dimensional simulations for non-negligible source-bed drawdown". We found the difference between the solutions obtained with the 2D and 3D simulations to be negligible. The largest discrepancies are between the predictions of sensitivities with respect to λ (less than 3%) and T (less than 6%).

The 3D flow model was also used to compute the sensitivity of depletion to parameters that are not included in the 2D model. We found that the dimensionless sensitivity of depletion with respect to the vertical hydraulic conductivity of the aquifer, K_v, is an order of magnitude smaller than all other sensitivities shown in Fig. 2. Likewise, the sensitivities with respect to the vertical hydraulic conductivity of the source bed and to the specific storage of the aquifer, aquitard, and source bed are three orders of magnitude smaller. This suggests that stream depletion is only mildly dependent on K_v and, for practical purposes, is independent of both the specific storage of the three hydrostratigraphic units and the vertical conductivity of the source bed.

Fig. 3 shows the dimensionless stream depletion rate and its sensitivities computed in a hydrogeological setting that is identical to that used in Fig. 2, except that the flow domain is finite. In

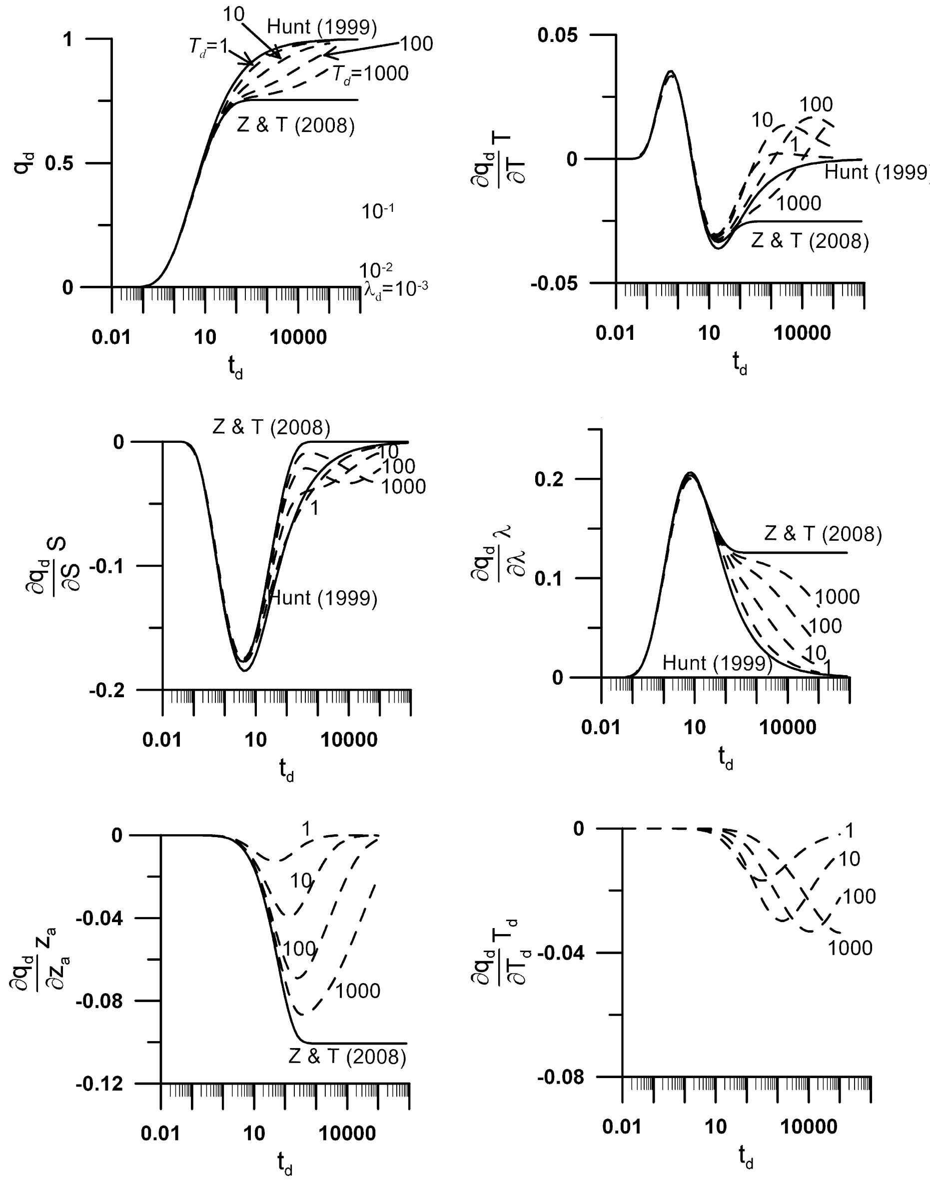

Fig. 2. The dashed curves show dimensionless stream depletion rate and dimensionless sensitivities of stream depletion rate when $B_d = 10$, $\lambda_d = 1$, drawdown can develop in the source bed, and the flow domain is "infinite". Curves are shown for four different values of dimensionless source-bed transmissivity, $T_d = 1$, 10, 100, and 1000. The solid line curves show results computed on basis of the analytical models of Hunt (1999) and Zlotnik and Tartakovsky (2008), respectively.

this case, drawdown at steady state extends to the boundaries, which therefore act as sources of groundwater flow. Fig. 3 demonstrates that for $T_d = 1000$ and 100 the depletion and sensitivity curves predicted with the Zlotnik–Tartakovsky solution and numerical simulations are nearly identical, for $T_d = 10$ the numerical solutions fall between the Zlotnik–Tartakovsky and Hunt curves, and for $T_d = 1$ the numerical solutions are close to those predicted with the Hunt solution. Thus, in situations where the deep source-bed drawdown is not negligible, the closer the constant-head boundaries are to the pumping well, the more accurate the predictions of depletion (and its sensitivities) based on the Zlotnik–Tartakovsky solution become. For a given distance between the boundaries and the well, the Zlotnik–Tartakovsky curves provide a good approximation of stream depletion rate as long as T_d, which represents the ratio of the transmissivities of the source bed and the aquifer, exceeds a certain threshold value (which in Fig. 3 is about 100). The smaller the distance, the smaller is this threshold value.

Aquifer drawdown and its sensitivities

Fig. 4 shows dimensionless drawdown and sensitivities in the aquifer at a location beneath the stream, $(x_d, y_d) = (0, 0)$. The hydrogeologic setting is the same as that used in Fig. 2: the flow domain is infinite, $B_d = 10$, and $\lambda_d = 1$. The analytical and numerical predictions of dimensionless drawdown and its sensitivities with respect to transmissivity T and storativity S are in good agreement. Differences between the numerical and analytical curves of the sensitivity with respect to the stream-bed conductance λ and the aquitard leakage coefficient z_a are more pronounced at later times, say, at $t_d \geqslant 10$. For a relatively large source-bed transmissivity (e.g., for $T_d \geqslant 10$), the numerically computed sensitivity curves, which account for the source-bed drawdown, are very similar to the sensitivity curves derived from the Zlotnik and Tartakovsky (2008) solution, which assumes drawdown in the source bed to be negligible. For source beds with small T_d, the numerical curves are more similar to the Hunt curves. It is worthwhile pointing out that these

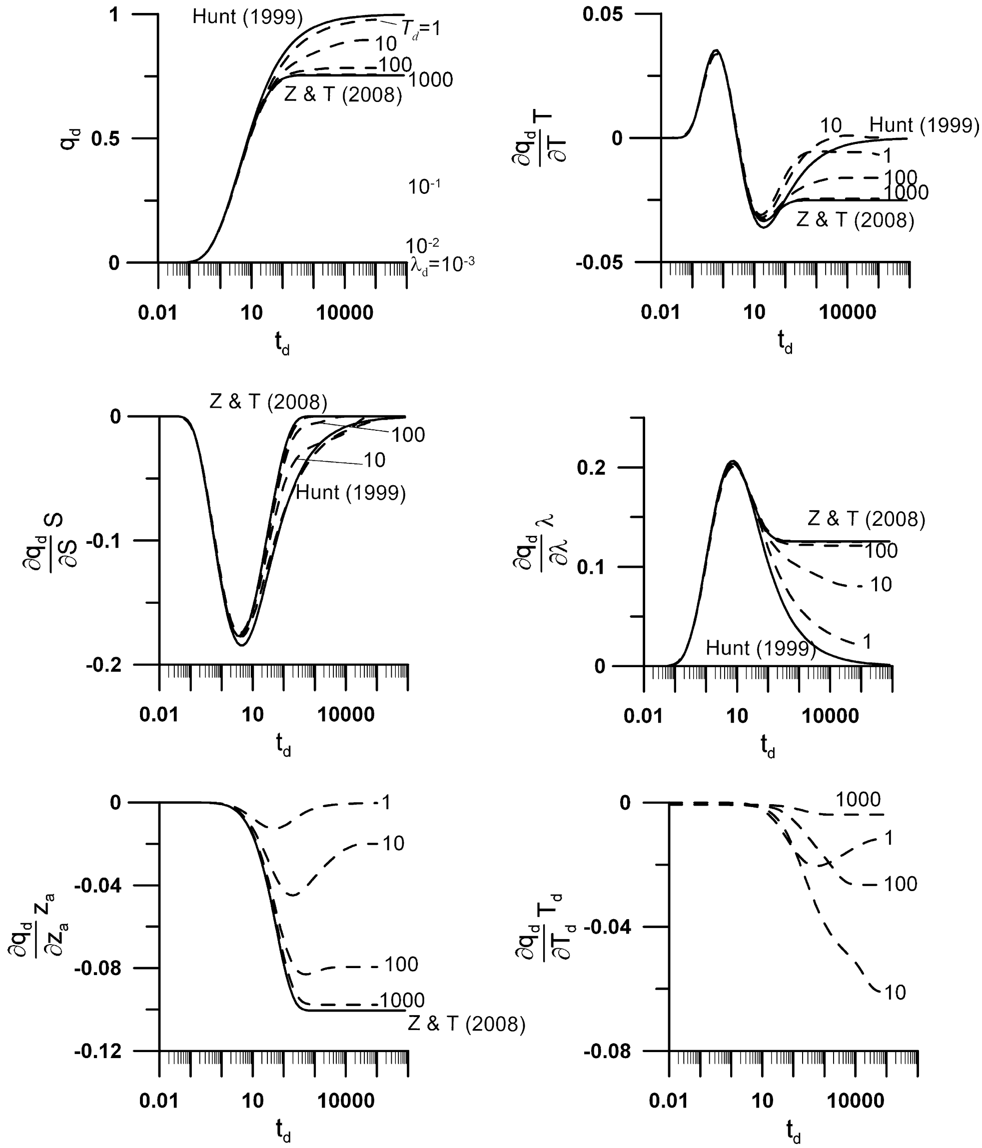

Fig. 3. The dashed curves show dimensionless stream depletion rate and dimensionless sensitivities of stream depletion rate when B_d = 10, $\lambda_d = 1$, drawdown can develop in the source bed, and the flow domain is finite. Curves are shown for four different values of dimensionless source-bed transmissivity, T_d = 1, 10, 100, and 1000. The solid line curves show results computed on basis of the analytical models of Hunt (1999) and Zlotnik and Tartakovsky (2008), respectively.

findings are, to a large extent, in agreement with the results of Neuman and Witherspoon (1969a), who studied pumping in a two-aquifer system without a stream. Their results suggest that drawdown in the pumped aquifer can be neglected if T_d > 100.

Fig. 4 also demonstrates that drawdown sensitivity with respect to source-bed transmissivity T_d is two orders of magnitude smaller than that with respect to aquifer transmissivity T. The T_d-sensitivity curves are similar in shape to the z_a-sensitivity curves, but are shifted to the right by about a decade. This implies that the pumping test duration that is required to estimate T_d from drawdown observations has to be an order of magnitude larger than that required to estimate z_a. Consequently, it is difficult (if not impossible) to estimate the source-bed transmissivity from observations of the aquifer drawdown.

While not shown in Fig. 4, the aquifer drawdown and its sensitivities for other values of B_d and λ_d exhibit similar behavior. As λ_d decreases, the discrepancy between the Zlotnik–Tartakovsky curves, the Hunt curves, and the numerical solutions increases. The discrepancy between the three solutions decreases as B_d increases.

The effect of the finite size of the flow domain is to shift slightly the drawdown and its sensitivities computed numerically towards the Zlotnik–Tartakovsky curves. For the hydrogeologic parameters used in Fig. 4, the shift is barely visible. This suggests that drawdown observations alone make it difficult (or impossible) to identify head-dependent sources of groundwater flow other than the nearby stream.

We conducted a series of 3D simulations to investigate the validity of the assumptions of quasi-2D flow underlying the analytical and numerical solutions shown in Fig. 4. In the 3D simulations, observation wells were placed in the top layer of the numerical model (see "Three-dimensional simulations for non-negligible source-bed drawdown"), reflecting typical field conditions. The discrepancy between the 2D and 3D solutions did not exceed 2% throughout much of the flow domain. Close to the pumping well (dimensionless distance smaller than 0.05 for the hydrogeologic parameter values used to compute the curves in Fig. 4), the 2D and 3D simulations lead to slightly more pronounced differences in the early-time part of the drawdown and sensitivity curves. The difference between the drawdown (and its sensitivities)

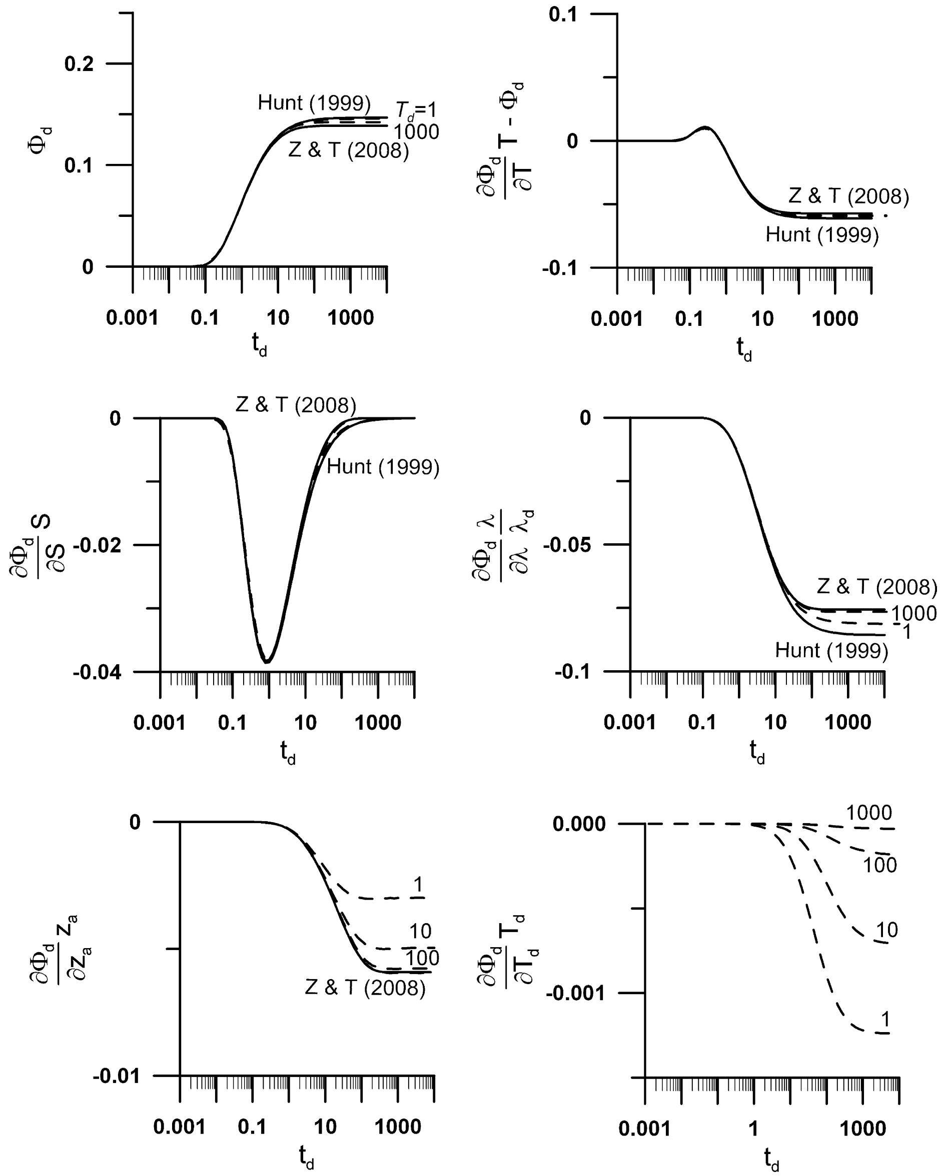

Fig. 4. The dashed curves show dimensionless drawdown and dimensionless sensitivities of drawdown at $(x_d, y_d) = (0, 0)$ when $B_d = 10$, $\lambda_d = 1$, drawdown can develop in the source bed, and the flow domain is "infinite". Curves are shown for four different values of dimensionless source-bed transmissivity, $T_d = 1$, 10, 100, and 1000. The solid line curves show results computed on basis of the analytical models of Hunt (1999) and Zlotnik and Tartakovsky (2008), respectively.

computed with the 2D and 3D simulations is smaller, if "observations" are made at the bottom of the pumped aquifer (in the fifth model-layer). It is worthwhile pointing out that the value of 0.05 for the dimensionless distance from the pumping well might be different for other values of hydrogeologic parameters.

The 3D simulations were used to compute drawdown sensitivity with respect to vertical hydraulic conductivity K_v, which cannot be computed with 2D simulations. The sensitivity of drawdown to K_v beneath the stream, $(x_d, y_d) = (0, 0)$, has the opposite sign and the same order of magnitude as the sensitivity to z_a. We therefore conclude that drawdown in the upper part of the aquifer near the stream, where flow has a vertical component, is sensitive to K_v.

Finally, we used the 3D simulations to quantify the effects of different values of aquitard storativity on predictions of aquifer drawdown. These simulations revealed that for large source-bed transmissivity ($T_d = 1000$), drawdown and its sensitivities are unaffected by an order-of-magnitude change in aquitard storativity. For small source-bed transmissivity ($T_d = 1$), an order-of-magnitude change in aquitard storativity causes late-time aquifer drawdown to increase slightly at the stream, as do its sensitivities to T, S_y, K_v, and z_a; while its sensitivity to λ slightly decreases. At the same time, early-time (e.g., for $t_d < 0.1$ for a dimensionless distance of 0.05) aquifer drawdown near the pumping well, and its sensitivity to T, increase somewhat, while its sensitivity to S_y decreases. Elsewhere in the flow domain, drawdown and its sensitivities (e.g. the ones shown in Fig. 4) are insensitive to an order-of-magnitude change of aquitard storativity.

Optimal locations to observe drawdown

Table 1 shows the optimal locations to observe drawdown, which were identified with the numerical simulations for the infinite flow domain. In the optimization procedure used to obtain these results we assumed that only T, S, λ, and z_a have to be estimated from the pumping-test analysis; that the number of observation wells during the test is two; and that observations of drawdown stop at dimensionless time $t_{d\_stop} = 100$. We also set $B_d = 10$, λ_d to 1.0 or 0.1, and T_d to 1, 10, 100, or 1000. In each case,

Table 1
CMAES_P optimization of well locations when prediction variance, σ_{qds}, is minimized for stream flow prediction, $q_d(t_{d\_qd})$. Optimization is based on 2D flow simulations for an "infinite" domain (length and width equal 100,000 × l) and σ_{qds} dependency on T, S, λ, and z_a sensitivities. However, for $T_d \to \infty$ the sensitivities were computed using analytical solutions derived from Zlotnik and Tartakovsky (2008) solutions.

B_d	λ_d	t_{d_qd}	T_d	Two observation wells		
				x_{d_w1}	x_{d_w2}	σ_{qds}
10	1	10	∞	0.00	3.30	1.83
			1000	0.00	3.36	1.82
			100	0.00	3.34	1.81
			10	0.00	3.31	1.78
			1	0.00	3.28	1.75
10	1	10^3	∞	0.10	4.86	4.73
			1000	0.10	4.67	3.99
			100	0.09	4.69	2.86
			10	0.04	4.69	1.28
			1	0.00	3.46	0.40
10	1	10^5	∞	0.10	4.86	4.73
			1000	0.05	4.69	1.16
			100	0.02	4.05	0.29
			10	0.07	2.64	.084
			1	0.00	2.49	.034
10	0.1	10	∞	0.00	2.02	2.23
			1000	0.00	2.03	2.20
			100	−0.01	2.04	2.20
			10	0.00	2.07	2.21
			1	0.00	2.22	2.17
10	0.1	10^3	∞	0.00	2.37	7.06
			1000	−0.01	2.38	7.00
			100	0.00	2.34	6.94
			10	−0.01	2.28	6.66
			1	0.00	2.25	5.68
10	0.1	10^5	∞	0.00	2.37	7.06
			1000	0.00	2.24	6.48
			100	0.00	2.10	4.70
			10	−0.01	2.07	2.24
			1	0.00	2.16	1.04

Table 2
CMAES_P optimization of well locations when prediction variance, σ_{qds}, is minimized for stream flow prediction, $q_d(t_{d\_qd})$. Optimization is based on 2D flow simulations for a finite domain (length and width equal 100 × l) and σ_{qds} dependency on T, S, λ, and z_a sensitivities. However, for $T_d \to \infty$ the sensitivities were computed using analytical solutions derived from Zlotnik and Tartakovsky (2008) solutions.

B_d	λ_d	t_{d_qd}	T_d	Two observation wells		
				x_{d_w1}	x_{d_w2}	σ_{qds}
10	1	10	∞	0.00	3.30	1.83
			1000	0.00	3.36	1.84
			100	0.00	3.33	1.83
			10	0.00	3.27	1.80
			1	0.00	3.20	1.76
10	1	10^3	∞	0.10	4.86	4.73
			1000	0.08	5.12	4.74
			100	0.07	5.09	4.11
			10	0.09	4.35	2.18
			1	0.00	3.61	0.51
10	1	10^5	∞	0.10	4.86	4.73
			1000	0.08	5.09	4.74
			100	0.07	5.07	4.00
			10	0.05	4.49	1.29
			1	0.00	2.51	0.13

the optimization was carried out for three target predictions of stream depletion rate, i.e., for depletion at dimensionless times t_{d_qd} = 10, 10^3, and 10^5. Table 1 also contains the optimal locations predicted from the Zlotnik–Tartakovsky solutions with infinitely large source-bed transmissivity, $T_d \to \infty$.

Table 1 reveals that for predictions of early-time stream depletion (t_{d_qd} = 10) the optimal locations to observe drawdown do not depend on the source-bed transmissivity: one observation well should be located near the stream ($x_{d\_w1} \approx 0.0$), and the other should be located behind the pumping well (x_{d_w2} > 1.0) with the distance from the well increasing with λ_d. The optimal locations are insensitive to the source-bed transmissivity, because the early-time stream depletion is not affected by leakage from the source bed. This is confirmed by Fig. 2, which shows that at t_d = 10 the values of depletion q_d predicted with the Hunt (1999) solutions, the Zlotnik and Tartakovsky (2008) solutions, and the numerical simulations are nearly identical; the sensitivity of depletion with respect to T, S, and λ is significant, while the sensitivity with respect to z_a is small.

For predictions of intermediate-time stream depletion ($t_{d\_qd} = 10^3$), the optimal locations to observe drawdown depend weakly on the source-bed transmissivity: the minimized value of σ_{qd} decreases slowly with the source-bed transmissivity T_d, and the optimized location of the second well (x_{d_w2}) is somewhat closer to the pumping well for T_d = 1 and λ_d = 1.0 or 0.1, as well as for T_d = 10 and $\lambda_d = 0.1$. However, for practical purposes, the optimized locations can be viewed as independent from a value of the source-bed transmissivity.

For predictions of late-time stream depletion ($t_{d\_qd} = 10^5$), the optimal locations depend quite significantly on the source-bed transmissivity T_d, especially when the stream-bed conductance is large, $\lambda_d = 1$. For T_d = 1 or 10, the optimized distance between the stream and the second observation well (x_{d_w2}) is almost half of that for T_d = 1000 and $T_d \to \infty$. This is because the sensitivities of $q_d(t_d = 10^5)$, especially with respect to λ and z_a, depend significantly on T_d (Fig. 2).

Table 2 presents the optimal locations to observe drawdown computed with the numerical simulations for the finite flow domain. Comparison of the corresponding results presented in Tables 1 and 2 reveals that for $t_{d\_qd} = 10^5$ the optimal locations identified from the Zlotnik and Tartakovsky (2008) solution are closer to those computed with the numerical simulations for the finite domain (Table 2) than to those for the infinite domain (Table 1). The dependence of the optimal observation locations on the source-bed transmissivity diminishes as the distance between the pumping well and the constant-head boundaries becomes smaller.

The optimization results discussed above are based on the assumption that only T, S, λ, and z_a have to be estimated by drawdown analysis. This is problematic since we found predictions of stream depletion to be sensitive to T_d and K_v as well, and the sensitivity of aquifer drawdown to T_d to be small. The latter finding suggests that estimations of T_d from drawdown observations are highly uncertain, as are predictions of stream depletion based on such estimates. A way to reduce this predictive uncertainty is to observe drawdown in the source bed during the pumping test.

This strategy yields the optimal drawdown observation locations presented in Table 3. The optimization procedure used to obtain these results relied on three observation wells and assumed that T_d has to be estimated by drawdown analysis together with T, S, λ, and z_a. Two of the observation wells have to be located in the aquifer (to estimate T, S, λ, and z_a), and one has to be located in the source bed (to estimate T_d). Table 3 shows locations for the three wells that have been obtained alternatively with either a simultaneous optimization or a two-step sequential optimization procedure. The sequential optimization treats the locations of the two wells in the aquifer (Table 1) as optimal, and optimizes only the location of the deep well, x_{d_wdeep}.

For early-time predictions of stream depletion (t_{d_qd} = 10), the optimal locations to observe aquifer drawdown (Table 3) are identical to those reported in Table 1. This is because early-time stream depletion is insensitive to the source-bed transmissivity T_d. The prediction uncertainty σ_{qd} is therefore nearly unaffected by the estimation of T_d and the location of the deep observation well.

Table 3
CMAES_P optimization of well locations when prediction variance, σ_{qds}, is minimized for stream flow prediction, $q_d(t_{d\_qd})$. Optimization is based on 2D flow simulations for an "infinite" domain (length and width equal 100,000 × l) and σ_{qds} dependency on T, S, λ, z_a, and T_d sensitivities. Three well locations are optimized: two wells are located in the aquifer (x_{d_w1} and x_{d_w2}) and one well is located in the source bed (x_{d_wdeep}). The locations are either optimized simultaneously or sequentially. For sequential optimization the location of the two wells in the aquifer are identical to those listed in Table 1.

B_d	λ_d	t_{d_qd}	T_d	Simultaneous optimization				Sequential optimization	
				x_{d_w1}	x_{d_w2}	x_{d_wdeep}	σ_{qds}	x_{d_wdeep}	σ_{qds}
10	1	10	1000	0.00	3.36	1.33	1.86	1.33	1.86
			100	0.00	3.33	0.28	1.82	1.46	1.82
			10	0.00	3.31	8.00	1.80	9.22	1.80
			1	0.00	3.28	1.10	1.77	15.6	1.77
10	1	10^3	1000	0.00	7.52	1.34	33.4	1.33	33.5
			100	0.00	6.43	1.34	11.0	1.34	11.0
			10	0.10	4.69	1.39	3.33	1.47	3.62
			1	0.08	4.69	3.46	1.17	9.33	2.50
10	1	10^5	1000	1.03	23.1	1.35	118.	1.34	120.
			100	0.00	8.16	1.31	11.2	1.34	11.4
			10	0.01	6.44	1.39	0.74	1.49	1.07
			1	0.08	4.69	4.02	.135	9.36	.345

Table 4
CMAES_P optimization of well locations when prediction variance, σ_{qds}, is minimized for stream flow prediction, $q_d(t_{d\_qd})$. Optimization is based on 3D flow simulations for an "infinite" domain (length and width equal 100,000 × l) and σ_{qds} dependency on T, S, λ, z_a, and K_v sensitivities. The wells are all located in the aquifer.

B_d	λ_d	t_{d_qd}	T_d	Two observation wells, 3D flow, Z_1 unknown			Three observation wells, 3D flow, Z_1 unknown			
				x_{d_w1}	x_{d_w2}	σ_{qds}	x_{d_w1}	x_{d_w2}	x_{d_w3}	σ_{qds}
10	1	10	1000	−0.22	3.58	2.04	0.00	0.92	4.68	1.45
			1	−0.22	3.45	1.95	0.00	0.93	3.45	1.46
10	1	10^3	1000	0.19	3.84	4.26	0.00	1.00	4.53	3.68
			1	−0.22	3.96	0.48	0.00	1.04	4.35	0.37
10	1	10^5	1000	−0.22	6.08	1.32	0.00	1.04	4.69	1.10
			1	−0.22	2.69	.040	0.00	0.93	3.45	.026

For predictions of intermediate- and late-time stream depletion ($t_{d\_qd} = 10^3$ and $t_{d\_qd} = 10^5$), the optimization results in Table 3 show that the location of the observation well in the deep source bed can be optimized sequentially. Indeed, the prediction uncertainty is hardly reduced by using simultaneous instead of sequential estimation. The optimal location of the deep well is behind the pumping well. The values of σ_{qd} in Table 3 are one-to-two orders of magnitude larger than the corresponding values in Table 1. Thus even if the source-bed drawdown were observed, estimates of T_d will be highly uncertain, and the uncertainty in T_d will dominate prediction uncertainty. The transmissivity of the deep source bed should therefore be estimated by means other than a pumping test conducted in the upper aquifer. The optimization results in Table 1 show that for $B_d = 10$ and $\lambda_d = 1$ the error of a T_d estimate must be less than an order of magnitude in order not to bias the optimization results, whereas for $B_d = 10$ and $\lambda_d = 0.1$ one order of magnitude error in T_d does not cause significant optimization bias.

Table 4 provides the optimal observation locations based on the assumption that K_v, the vertical hydraulic conductivity of the aquifer, has to be estimated by drawdown analysis together with T, S, λ, and z_a. For two observation wells, it is optimal to locate one well at some distance across from the stream, and the other behind the pumping well. Thus if K_v is to be estimated from the pumping test, the optimal location of the first well shifts from the vicinity of the stream (Table 1) to some distance away (Table 4). This shift is an optimal compromise between observing drawdown at locations most sensitive to λ and observing drawdown at locations most sensitive to K_v. As a consequence, the prediction uncertainty σ_{qd} increases by up to 20%.

If three observation wells are planned to be used to estimate T, S, λ, z_a, and K_v it is optimal to locate the first at the stream, the second close to the pumping well, and the third behind the pumping well (Table 4). Observing drawdown close to the pumping well provides information about K_v and other parameters, so the prediction uncertainty is reduced by up to 30% relative to that reported in Table 1.

The results in Table 4 are found to be insensitive to an order-of-magnitude (from 10^{-4} to 10^{-3}) change of aquitard storativity.

Summary and conclusions

Accurate and reliable predictions of stream depletion caused by groundwater extraction require an accurate parameterization of a mathematical model that reflects the actual hydrogeologic conditions within a studied basin. Since parameter estimates must represent the scale of the cone of depression caused by pumping, they are usually obtained from pumping tests.

Christensen et al. (2009) investigated an optimal design of pumping tests in a leaky aquifer near a stream, with a goal to estimate hydrogeologic parameters required for predictions of stream depletion caused by pumping. Their analysis is based on the analytical solutions developed by Zlotnik and Tartakovsky (2008) under assumptions of negligible drawdown in an underlying source bed of a leaky aquifer, and horizontal flow in an infinite aquifer.

We conducted a series of two- (2D) and three-dimensional (3D) numerical simulations to investigate the validity and applicability range of these assumptions, as well as their implications for the reliability of stream-depletion predictions. The (quasi)-2D simulations relied on the assumptions that flow in the aquifer and the source bed is horizontal and flow through the aquitard is vertical. The fully 3D simulations remove these assumptions. We focused on stream depletion rate, aquifer drawdown, their sensitivities to hydrogeologic parameters, and on optimization of locations to observe drawdown during a pumping test. Our analysis leads to the following major conclusions:

1. The 2D and 3D simulations yield essentially analogous predictions of stream depletion, aquifer drawdown, and their sensitivities to major hydrogeologic parameters. This validates the assumptions that flow in the aquifer and the source bed is horizontal and flow through the aquitard is vertical.
2. If the source-bed transmissivity T_d is much smaller than the aquifer transmissivity T (e.g., $T_d \leqslant 0.1T$), stream depletion, aquifer drawdown, and their sensitivities are best described by the Hunt (1999) solutions for a non-leaky aquifer. The Zlotnik and Tartakovsky (2008) solutions should be used when T_d is at least of the same order of magnitude as T.
3. In infinite flow domains, i.e., when external prescribed-head boundaries do not affect groundwater pumping, the Zlotnik–Tartakovsky solutions provide accurate predictions of stream depletion and its sensitivities at small dimensionless times, $0 \leqslant t_d < t_1$. The agreement between the Zlotnik–Tartakovsky solutions and their numerical counterparts starts to deteriorate after time t_1, which increases with the ratio T_d/T. At late times, $t_d > t_2$, the Hunt solutions coincide with their numerical counterparts; the value of t_2 increases with T_d.
4. In bounded (finite) flow domains, the Zlotnik–Tartakovsky solutions provide accurate predictions of stream depletion and its sensitivities at all times, provided T_d is sufficiently large (two orders of magnitude larger than T in our example). If this condition is not met, the numerical predictions of stream depletion and its sensitivities fall between those derived from the Zlotnik–Tartakovsky and Hunt solutions.
5. Stream depletion is sensitive not only to the stream-bed conductance, the aquifer transmissivity and storativity, and the aquitard leakage coefficient. It is also sensitive to the source-bed transmissivity and, to a lesser degree, to the vertical hydraulic conductivity of the aquifer. The latter two parameters are not accounted for in the analytical solutions of Zlotnik and Tartakovsky (2008) or of Hunt (1999). For practical purposes, stream depletion can be viewed as independent of both the specific storage of the aquifer, aquitard, and source bed and the vertical hydraulic conductivity of the source bed.
6. Aquifer drawdown and its sensitivities obtained from the analytical solutions of Zlotnik and Tartakovsky (2008) and Hunt (1999) are generally in close agreement with those obtained with numerical simulations, being less affected by T_d and the flow domain size (finite or infinite). The latter finding indicates that from observations of drawdown alone it might be impossible to identify head-dependent groundwater sources other than the nearby stream.
7. The low sensitivity of drawdown to the source-bed transmissivity T_d indicates that it is not feasible to estimate T_d solely from drawdown data. Such estimates are likely to be highly uncertain, even if drawdown is observed in the source bed. This conclusion is more far-reaching than that made by Neuman and Witherspoon (1969b) for a system of leaky aquifers not connected to a stream: "relying entirely on drawdown data from the pumped aquifer is not sufficient to characterize a leaky system".
8. Neglecting drawdown within the source bed *might* affect computed sensitivities and, hence, identification of optimal locations to observe drawdown during a pumping test with the optimization procedure of Christensen et al. (2009). This procedure minimizes the target prediction variance, which depends on drawdown sensitivities and prediction sensitivities.
9. If the optimization target is to predict early-time stream depletion, the assumption of negligible drawdown does not affect the selection of observation locations, because stream depletion at this stage is not significantly influenced by the source bed leakage. If the target is to predict intermediate-time stream depletion, the implications of this assumption are of no practical importance. If the target is to predict late-time stream depletion, this assumption is important when the aquifer has a strong hydraulic connection to both the stream and the deep source bed, and T_d is sufficiently small (not exceeding $100T$). However, the significance of this assumption is considerably weakened by the presence within the cone of depression of head-dependent sources of groundwater recharge other than the stream.
10. In principle, two and/or three-dimensional numerical simulations can replace their analytical counterparts in the optimization procedure of Christensen et al. (2009), obviating the need for simplifying assumptions of the negligible source-bed drawdown and horizontal flow. However, this would require one to obtain independent estimates of the source-bed transmissivity T_d and the vertical hydraulic conductivity of the pumped aquifer K_v. Our analysis showed that estimates of T_d are usable as long as their estimation errors do not exceed an order of magnitude, and that including K_v into the optimization procedure does not lead to significant changes in the optimal design of an observational network.

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