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Determination of Hydrogeological Parameters Using Sinusoidal Pressure Tests: A Theoretical Appraisal

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A method for determining the hydrogeological parameters of formations of interest based on a sinusoidal pressure fluctuation in an excitation borehole is proposed. Equations are derived which describe the dependence of pressures and phase lags outside the excitation borehole on distance, signal frequency, specific storage, hydraulic conductivity, and flow rates. These cover two distinct configurations: that of a point source deep within a water-saturated elastic formation and that of a line source totally penetrating a confined aquifer. The ranges of application of these two arrangements are appraised using existing data on hydraulic diffusivities to derive likely excitation and response pressures for various flow rates and signal frequencies. The two configurations require the accurate measurement of phase shifts in the case of the point source and of amplitudes in the line source case, and it is concluded that the ranges of applications are useful but limited.

INTRODUCTION

The need to carry out groundwater investigations in widely varying rocks and geological environments has arisen from hydrogeological work in fields such as pollution movement, geothermal energy, and radioactive waste. At the same time economics have encouraged petroleum engineers to consider increasingly low-permeability reservoirs and to map permeability variations in ever greater detail in order to optimize secondary and tertiary recovery techniques. It is therefore probably an appropriate time to examine an alternative to the step change, constant discharge aquifer test.

The objective of this study was to evaluate the feasibility of employing some type of periodic pressure or flow inducement in a water-saturated porous medium to derive the relevant hydrogeological parameters. In other words, having excited the formation with a wave of a particular amplitude and frequency, would there be a measurable signal at a useful distance, and would this be suitably dependent on a limited number of hydraulic parameters? Also, would any phase shifts of the wave be measurable and uniquely characteristic of the medium through which it was traveling? Since the principal object was to use the method for measuring the hydrogeologic parameters of potential host rocks for radioactive waste disposal, the range of hydraulic conductivities and storativities was selected with this in mind. Two basic cases were considered: that of a point source within an elastic porous medium of effectively infinite extent and that of a line source totally penetrating a confined formation. The elastic properties of both the water and the formation determine the response to the sinusoidal excitation.

The sinusoidal pressure test, where a sinusoidal variation of pressure is induced in a source well and the signal is measured in an observation well, is in some ways more limited than the step change method. These limitations are basically fourfold:

1. The distance of penetration of measurable pressure fluctuations is less than the step change method.
2. The property measured is hydraulic diffusivity: a combined function of hydraulic conductivity and specific storage rather than the two separately.
3. The equipment necessary for the test is not easily available.
4. There are no readily available methods of analysis (i.e., type curves).

However, there are certain unique advantages, such as the potential ability to measure hydraulic parameters in three dimensions and the fact that there is no net discharge. The diagnostic measurements are phase shift and amplitude, so the start time for the test is irrelevant, which enables movements of the measuring position (i.e., up or down within observation boreholes) without having to stop the excitation. In addition, overall equilibration times should be shorter, since there is no net discharge.

To some extent these questions have already been answered in the petroleum literature, where 'pulse testing' was devised and reported by *Johnson et al.* [1966]. Pulse testing involves alternating periods of flow and shut-in for producing wells combined with observations of the pressure disturbance in surrounding wells. This effectively creates a square wave, but since production is not reinjected, there is a net discharge, and the periodic response is superimposed on an overall drawdown trend. Evaluations have been made of the effect of altering the ratio of production to shut-in time [*Kamal and Brigham,* 1976] and the idea of using the method between two intervals in the same well [*Hirasaki,* 1974; *Falade and Brigham,* 1974]. The use of pulse tests for three-dimensional reservoir descriptions has also been reported covering the orientation of hydraulic fractures [*Pierce et al.,* 1975], natural permeability variations [*McKinley et al.,* 1968; *Kamal,* 1979], and the prediction of water flood performance [*Pierce,* 1977]. The groundwater equivalent of this literature was given by *Williams et al.* [1972], but the reported results were of limited applicability.

The groundwater literature concerning periodically fluctuating water levels or pressures in porous media is limited. It deals mostly with planar and linear sources involved with tidal fluctuations and was reviewed by *van der Kamp* [1973].

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The oscillatory response of open observation wells to variations of hydraulic pressure due to certain causes has been reported by several authors: earth tides by *Bredehoeft* [1967], seismic waves by *Cooper et al.* [1965], and underdamped slug tests by *van der Kamp* [1976]. Work reported by *Dagan* [1967] contained some interesting concepts, such as a point source with variable and constant discharge rates, but the analysis only applied to unconfined conditions, that is nonelastic storage with a free water table and vertical flow of water.

ANALYSIS

Two cases are evaluated in this study: one is that of a periodic point source deep in the interior of a saturated elastic porous medium, and the other is that of a periodic line source which totally penetrates a confined elastic porous medium of effectively infinite areal extent. The assumptions used in formulating the mathematical model are as follows:

1. Flow is described by Darcy's law.
2. The medium is saturated with water.
3. The porous medium is homogeneous and isotropic.
4. The compressibility of both the porous medium and the water are taken into account.
5. Isothermal conditions prevail.
6. The fluid density is constant (neglecting small variations resulting from fluid compressibility).
7. Inertial effects of flow into the borehole or cavity and flow within the formation are neglected.

1-2
2 notes:

3-4
2 notes:

The Case of a Point Source of Periodic Flow/Pressure

something that is anticipated, visualized, or imagined as a future possibility.

It is assumed that the periodic point source is deep within an elastic porous medium and that flow is therefore three dimensional. Obviously, a finite cavity is envisaged in practice, but a point source is convenient for the analysis.

The vectorial form of the approximate partial differential equation governing the unsteady state flow of groundwater in a homogeneous and isotropic medium is given by *Bear* [1972] as

$$\nabla^2 h = \frac{S_s}{K}\frac{\partial h}{\partial t} \tag{1}$$

where h is potential in terms of hydraulic head, K is hydraulic conductivity, and S_s is specific storage:

$$S_s = \rho g\{[1-\theta]\alpha + \theta\beta\} \tag{2}$$

(This is the definition of elastic storage related to fixed coordinates.)

With the periodic point source at $r = 0$ and assuming as $r \to \infty$, $h \to 0$,

$$\lim_{r_0 \to 0} 4\pi r_0^2 K \left.\frac{dh}{dr}\right|_{r=r_0} = -Q_0 e^{i\omega t} \tag{3}$$

It can be shown that the steady periodic solution to (1) is

$$h(r, t) = \frac{Q_0}{4\pi K r}\exp\left[-\left(\frac{1+i}{2^{1/2}}\right)\left(\frac{\omega S_s r^2}{K}\right)^{1/2}\right]e^{+i\omega t} \tag{4a}$$

$$h(r, t) = G(r, \omega)e^{i\omega t} \tag{4b}$$

where $G(r, \omega)$ is the complex frequency response function for this system, r is the distance from the source, and ω is the angular frequency. (Note that Q_0 positive is water being withdrawn from the porous medium).

Equation (4*a*) relates the head response h at a distance r from the source of the periodic excitation to the source function. This complex function can be written in polar form with an amplitude and phase shift, for example,

Amplitude

$$|G| = \frac{Q_0}{4\pi K r}\exp\left[-r\left(\frac{\omega S_s}{2K}\right)^{1/2}\right] \tag{5a}$$

Phase shift

$$\phi = -r\left(\frac{\omega S_s}{2K}\right)^{1/2} \tag{5b}$$

Although (5*a*) relates a head to a flow function, it is more useful if (5*a*) is expressed as a ratio of head amplitudes at two discrete radii, so (5*a*) is changed to (5*c*) by considering the amplitude at r compared to r_0:

$$\frac{|G|}{|G_0|} = \frac{r_0}{r}e^{(r_0 - r)\left(\frac{\omega S_s}{2K}\right)^{1/2}} \tag{5c}$$

This is a more convenient form than (5*a*) and enables the attenuation of the signal to be examined (see discussion below).

The case of a Line Source of Periodic Flow/Pressure

It is assumed that the line source of periodic flow completely penetrates an elastic porous medium which is confined both above and below. The mathematical model assumes infinite lateral extent and a line source, but in reality a finite diameter borehole is envisaged in a medium which extends a long way laterally in relation to the position of observation measurments.

If (1) is written for two-dimensional flow in a medium of thickness l, then

$$\nabla^2 \bar{h} = \frac{S}{Kl}\frac{\partial \bar{h}}{\partial t} \tag{6}$$

where S is the storage coefficient

$$S = \rho g l\{\alpha + \theta\beta\} = lS^* \tag{7}$$

$\bar{h}$ is the average hydraulic head, and S^* is the two-dimensional confined specific storage. (Note that the minor difference between the three-dimensional specific storage (S_s in (2)) and the two-dimensional specific storage (S^* in (7)) is a result of averaging the hydraulic potential throughout the whole formation thickness and allowing the impermeable boundary to move slightly as the formation deforms elastically.)

With the periodic line source at $r = 0$, and taking $h \to 0$ as $r \to \infty$,

$$\lim_{r_0 \to 0} 2\pi r_0 K \left.\frac{d\bar{h}}{dr}\right|_{r=r_0} = -Q_0 e^{i\omega t} \tag{8}$$

It can be shown that the steady periodic solution to this equation is

$$h(r, t) = \frac{Q_0}{2\pi K l}K_0\left[\left(\frac{\omega r^2 S^*}{K}\right)^{1/2} e^{i\pi/4}\right]e^{i\omega t} \tag{9a}$$

$$h(r, t) = G^*(r, \omega)e^{i\omega t} \tag{9b}$$

where G^* is the complex frequency response function for this

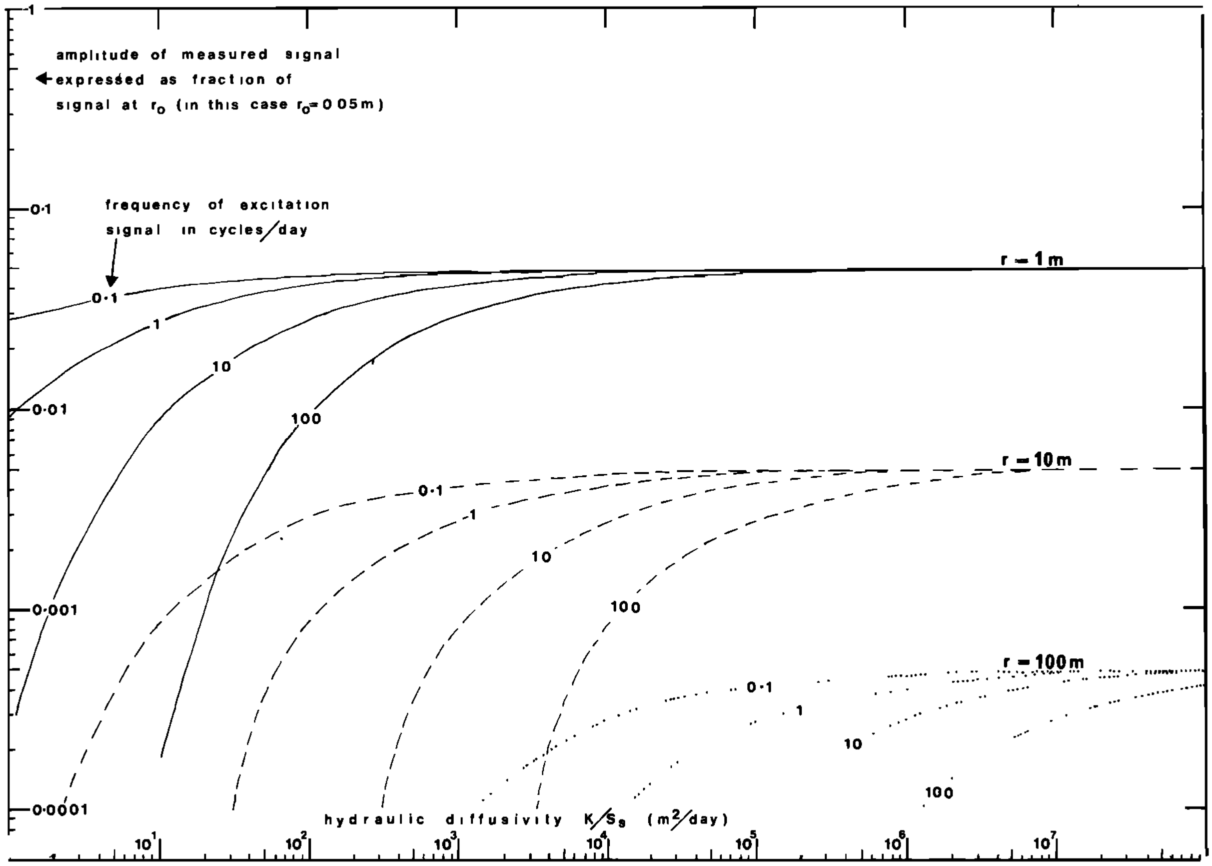

Fig. 1. Dependence of signal amplitude attenuation on signal frequency, distance from the source, and hydraulic diffusivity: the point source case.

system and K_0 is the modified Bessel function of the second kind. Since the argument of the Bessel function has a phase angle of $\pi/4$, the following definition can be used to express the results in terms of modified Kelvin functions of the second kind [*Abramowitz and Stegun,* 1964]:

$$K_0(xe^{i\pi/4}) = \text{Ker}\,(x) + i\,\text{Kei}\,(x) \tag{10}$$

Thus after transformation to the polar form, the amplitude and phase shift are

Amplitude

$$|G^*| = \frac{Q_0}{2\pi Kl} N_0\left[r\left(\frac{\omega S^*}{K}\right)^{1/2}\right] \tag{11a}$$

Phase shift

$$\phi^* = \phi_0\left[r\left(\frac{\omega S^*}{K}\right)^{1/2}\right] \tag{11b}$$

As for the point source case a ratio of pressures is more useful than a pressure related to a flow function, so

Amplitude ratio

$$\frac{|G^*|}{|G_0^*|} = \frac{N_0\left[r\left(\dfrac{\omega S^*}{K}\right)^{1/2}\right]}{N_0\left[r_0\left(\dfrac{\omega S^*}{K}\right)^{1/2}\right]} \tag{11c}$$

The significance of these functions is examined below.

RESULTS

The primary questions to be answered by this study are how far from the source the pressure signal can be expected to be detected and what are the factors affecting the rate of attenuation. The method envisaged only requires the measurement of pressure, and this means that hydraulic diffusivity cannot be separated into its component parts. It is likely that separate tests, using a more standard procedure, would be a better method of defining either of the two components. The device employed in (5*c*) and (11*c*) of comparing the pressures at two radii from the source avoids the need to define flow amplitudes at any point. In order to depict the attenuation of the signal amplitude a reference radius of 0.05 m (a commonplace borehole radius) was chosen for Figures 1 and 2. In these figures the amplitude at 0.05 m is taken as a reference value, and the amplitude at three particular distances (i.e., 1, 10, and 100 m) is plotted as a percentage of this reference amplitude. It is clear in Figure 1, the point source case, that attenuation is pronounced even with the higher diffusivities and that for low frequencies and high diffusivities an asymptotic value of r_0/r is approached, as the exponential function tends toward 1. It is also clear that the frequency of the signal has an increasing importance as greater distances are involved. For the line source case (Figure 2) all the amplitudes are greater, and there are much greater differences in observable amplitudes caused by varying the signal frequency. Within the depicted range of hydraulic diffusivities the amplitudes do not tend toward an asymptotic value, unlike the point source case.

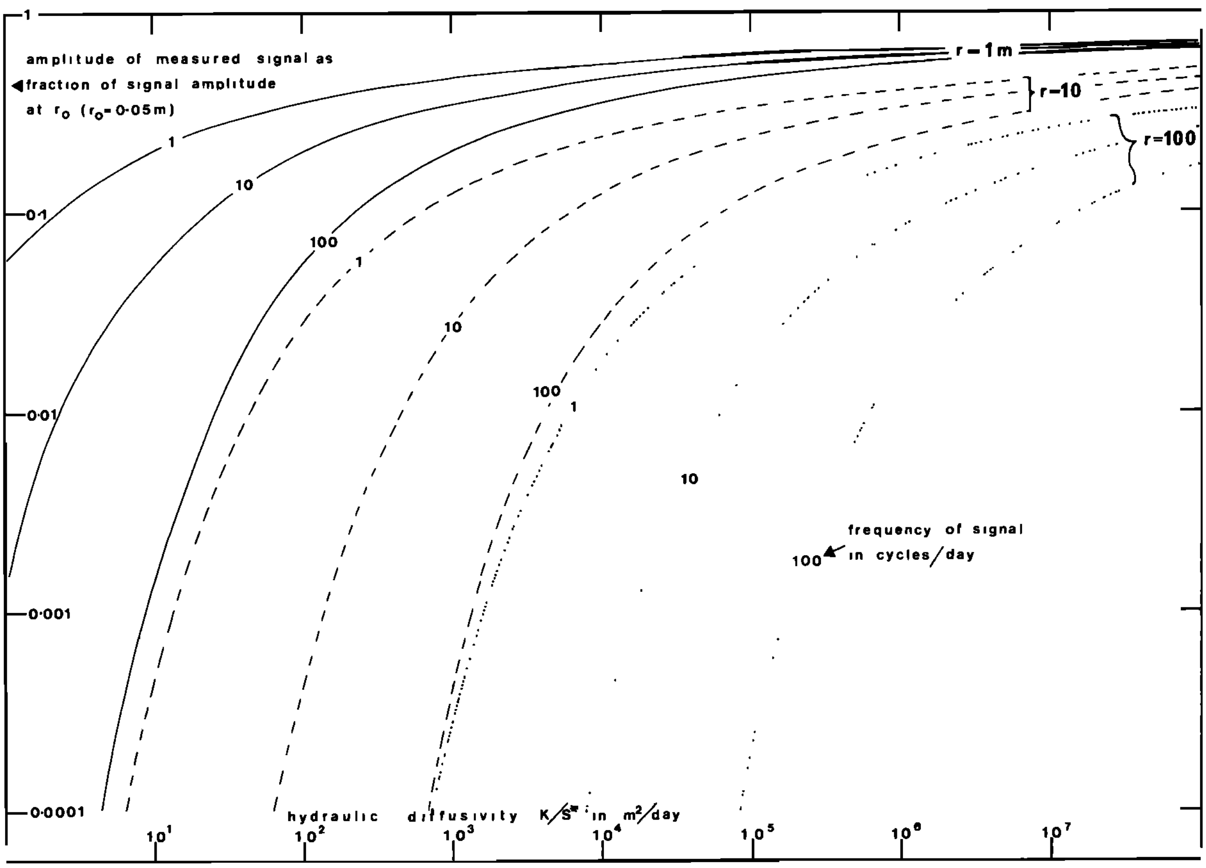

Fig. 2. Dependence of signal amplitude attenuation on signal frequency, distance from the source, and hydraulic diffusivity: the line source case.

The phase shifts are shown in Figures 3*a* and 3*b*, which, like Figures 1 and 2, have common axes. The phase shift of 1 cycle (2π rad) is marked, as it effectively represents the upper limit for a unique experimental result, since if the source and observed signals are displaced by more than 1 cycle, this is not readily identifiable. It is notable that within the depicted range of diffusivities the line source rather than the point source tends toward an asymptotic value around 0.1 rad. Indeed, the phase shifts for the line source case are much less variable than those for the point source case, with only small differences caused by frequency changes.

The results in Figures 1–3 are obtained from equations which apply once a steady state has been reached which would follow some undefined period of transience. It can be shown that the time taken to reach steady state amplitude and phase shift is approximately $r^2S_s/4K$. If this approximation is evaluated for the two cases, then it can be seen that long transient periods (e.g., in excess of 5 days) only apply to configurations of diffusivity and distance where the signal amplitude would be greatly attenuated, so attenuated in fact that for all practical purposes the signal would be below detection limits.

In summary therefore, the mathematical results suggest that although both amplitude and phase shift should be measured during tests using periodic signals, a different emphasis is required depending on the test geometry. For the point source configuration the phase shift is more likely to be diagnostic than the amplitude attenuation, while for the line source configuration the reverse is the case. This conclusion becomes more important when more than one excitation frequency is applied. It is fortunate that with both analyses, as phase shifts exceed one cycle, so amplitudes diminish rapidly, probably below measurement limits.

Applications

The point source and line source cases were chosen with different applications in mind. In the case of the point source the rock to be examined would have low hydraulic conductivity, would have no preferentially higher horizontal or vertical hydraulic conductivity due to sedimentary bedding, and the distance from the excitation source to an upper boundary (either ground surface or a water table) would be in excess of 200 m. Therefore a test is envisaged in which a source well with a very limited length of open borehole gives out a sinusoidal pressure signal which is detected in nearby boreholes by pressure transducers with packers. The packers and transducers could be moved up and down the observation borehole while the test was in progress, giving measurements of hydraulic diffusivity at various orientations within the plane of the two boreholes. In crystalline rocks the relationship between hydraulic parameters and such linear and planar features as jointing, fabric, and shear zones could be investigated.

The use of the periodic line source is envisaged as a useful method in places such as industrial plants where contaminated groundwater might be encountered in 'aquifer-type' formations. The sinusoidal pressure test could solve the problem of determining hydrogeological parameters without producing vast quantities of contaminated discharge. In these

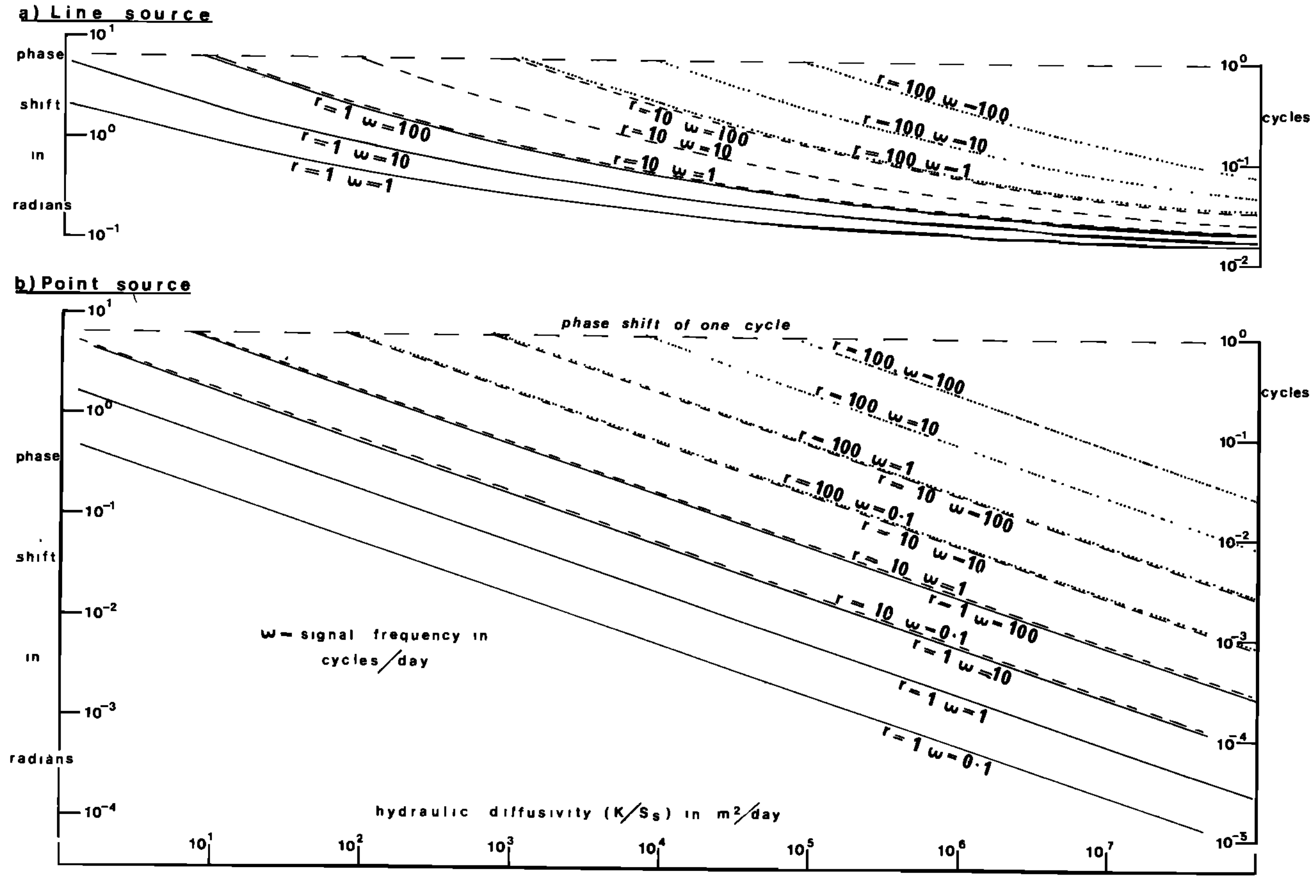

Fig. 3. Dependence of observed signal phase shift on signal frequency, distance from the source, and hydraulic diffusivity: (*a*) line source case and (*b*) point source case.

cases, where it is unlikely that high hydraulic heads would be possible, a line source technique is probably more appropriate in order to get measurable fluctuations over large distances. The contaminated water would be pumped out onto the surface to be held in a large tank only to be recharged to produce the other half of the sinusoidal signal.

The viability of these two schemes depends on a number of factors, such as the range of hydraulic diffusivities likely to be encountered and the resultant implications for volumes and rates of discharge and detection limits of pressure transducers. Hydraulic diffusivities are seldom quoted in the literature, but a few representative examples are given in Table 1. This shows a relatively small range of specific storages (3×10^{-4} to 2×10^{-7} m^{-1}) compared to the range of hydraulic conductivities (5.6×10^1 to 4.5×10^{-7} m/d) which combine to produce the range of hydraulic diffusivities (8×10^6 to $3 \times$

TABLE 1. Values of Hydraulic Diffusivity for Various Rock Types

Rock Type	Porosity, %	Specific Storage, m^{-1}	Hydraulic Conductivity, m/d	Hydraulic Diffusivity, m^2/d	Reference or Source
			Aquifers		
Unconsolidated sand	(30)	3×10^{-5}	56	1.9×10^6	*van der Kamp* [1973]
Unconsolidated sand	(30)	1.1×10^{-5}	32	2.8×10^6	*van der Kamp* [1973]
Semiconsolidated sand and clay	30	6.6×10^{-6}	20	3×10^6	*Marine* [1975]
Sandstone (Permian)	18	2.2×10^{-6}	1	4.5×10^5	*Carr and van der Kamp* [1969]
Limestone (Lincolnshire)	(20)	1.3×10^{-5}	200	1.5×10^7	*Rushton* [1975]
			'Nonaquifers'		
Clay and peat	(40)	3×10^{-4}	(0.01) (1×10^{-6})	(3×10^1) (3×10^{-3})	*van der Kamp* [1973]
Mudstone (Triassic)	3	4×10^{-7}	4.5×10^{-7}	1.1	*Marine* [1975]
Granite	0.5	2.1×10^{-7}	4×10^{-4}	2×10^3	*Black* [1979]
Metamorphic schists	0.13	2.1×10^{-7}	1.2×10^{-5}*	5.7×10^1	*Marine* [1975]
Gneisses and quartzites			3.3×10^{-2}†	1.6×10^5	

Values in parentheses are estimated.
*Value derived from virtually impermeable zone measured by packer test.
†Value derived from hydraulically transmissive zone measured by pumping test.

TABLE 2. Peak Flow Rates and Half-Cycle Flow Volumes for Likely Combinations of Parameters

Hydraulic Diffusivity, m^2/d	Hydraulic Conductivity, m/d	Peak Flow Rate, l/s		Total Volume per Half Cycle, l	
		1 cpd	100 cpd	1 cpd	100 cpd
			Aquifers		
10^8	10^2	63.8	79.6	1.7×10^6	2.2×10^4
	1	0.64	0.8	1.7×10^4	2.2×10^2
10^6	10^2	79.6	106	2.2×10^6	2.9×10^4
	1	0.80	1.1	2.2×10^4	2.9×10^2
	10^{-2}	8.0×10^{-3}	1.1×10^{-2}	2.2×10^2	2.9
			'Nonaquifers'		
10^4	1	1.06	1.6	2.9×10^4	4.4×10^2
	10^{-2}	1.1×10^{-2}	1.6×10^{-2}	2.9×10^2	4.4
	10^{-4}	1.1×10^{-4}	1.6×10^{-4}	2.9	4.4×10^{-2}
10^2	10^{-2}	1.6×10^{-2}	3.1×10^{-2}	4.4×10^2	8.5
	10^{-4}	1.6×10^{-4}	3.1×10^{-4}	4.4	8.5×10^{-2}
	10^{-6}	1.6×10^{-6}	3.1×10^{-6}	4.4×10^{-2}	8.5×10^{-4}
1	10^{-4}	3.1×10^{-4}	4.6×10^{-3}	8.5	1.3
	10^{-6}	3.1×10^{-6}	4.6×10^{-5}	8.5×10^{-2}	1.3×10^{-2}
	10^{-8}	3.1×10^{-8}	4.6×10^{-7}	8.5×10^{-4}	1.3×10^{-4}

All values are based on a 1-m length of 0.1-m-diameter borehole producing a pressure at the borehole wall of 100 m of water head.

10^{-3} m^2/d). If the value for clay and peat is excluded, the range is reduced by 3 orders of magnitude to approximately the range used in Figures 1–3. For the point source test the 'nonaquifers' envisaged would probably fall in the range 1–10^4 m^2/d. The question of the discharge required to give measurable pressures in observation boreholes can be examined by using the formula from the line source case. If a 1-m length of 0.1-m diameter is pressurized to 100 m of positive or negative (by abstraction) water pressure, the maximum flow rates and total half-cycle flows for likely combinations of hydraulic diffusivity, hydraulic conductivity, and selected signal frequencies are given in Table 2.

For the nonaquifer range the peak flow rates are well within the capability of available pumps, and even for the 1-cpd frequency the total volume needing storage is not unreasonable. The only question is that of pressure measurement. A borehole pressure transducer is available in the United Kingdom in 1979 having a combined nonlinearity and hysteresis of 0.06%, though resolution is quoted as infinite. Certainly, using differential rather than absolute pressure measuring transducers and measuring phase shift rather than amplitude, hydraulic diffusivities of the order encountered in Cornish granite [*Black*, 1979] could be measured over distances of about 100 m.

Although the present analysis is based on a homogeneous isotropic porous medium, there seems no bar to assuming that hydraulic diffusivity values apply only between source and measurement point and that the existence of differing values in nearby vectors would not detract from the validity of individual values. Indeed, the real value of the method lies in this possibility of producing a group of diffusivity vectors in the plane of the excitation and observation boreholes. This could be accomplished if the position of the source is kept static and the packer system containing the measuring pressure transducer is moved within neighbouring observation boreholes. A series of varying responses are likely to be produced which, when compensated for distance, represent variously oriented diffusivities.

Where a fracture directly connects the source and measurement points, flow will be concentrated within this thin layer, and responses will be indicative of the line source case within a very thin confined aquifer. In other words, in a fractured medium the whole range of responses is possible. However, bearing in mind these possibilities, once hydraulic diffusivity has been measured, groups of hydraulic conductivity vectors can be computed if specific storage is known or can be estimated.

The use of a line source in aquifer-type materials is a somewhat different application, but Table 2 can be used again, and the same figures will apply for a 10-m length of 0.1-m-diameter borehole pressurized to ±10 m of water head. It would appear from Table 2 that some of the higher hydraulic conductivities involve pumping rates and stored volumes which are beyond the range of practical equipment. However, the Permian sandstone of *Carr and van der Kamp* [1969] is within the bounds of possibility, requiring a pump delivering about 1 l/s and a half-cycle stored volume for 1 cpd of about 20 m^3. Such a signal would be easily detected at 100 m, though in all probability a higher frequency such as 100 cps could be detected at that range.

Although it is implicit throughout these considerations that hydraulic diffusivity is a worthwhile parameter to measure in its own right, there remains the problem of separating it into its components. It is therefore suggested that slug tests and injection tests can define near-borehole hydraulic conductivities and an estimate of specific storage. Additionally, in low-porosity crystalline rocks, geophysically derived elastic properties can provide an approximation of specific storage. Also neglected in these considerations are complications due to inertial effects, such as resonant oscillations in the boreholes, since it is envisaged that none would have open water levels and only the longer excitation frequencies would be useful.

CONCLUSIONS

The method of measuring hydraulic diffusivity using sinusoidal pressure fluctuations in an excitation borehole is possible over useful distances given the right equipment in a limited range of rock types. The method would not be useful for high storage/low hydraulic conductivity media such as peat or clay but certainly has uses in fissured crystalline rocks.

For the three-dimensional examination of crystalline rock the signal frequency should be no more than 1 cpd, and measurement should concentrate on the phase shift. Signals will be measurable up to about 100 m from the source, which could be a 1-m length of open borehole. A large source amplitude of about 100 m of water head is envisaged so the source should be at least double this distance below the water surface so that only elastic storage is operative.

For the two-dimensional examination of aquifer-type rocks in pollution studies or similar applications the line source has a much more penetrating signal. In this case, lower excitation amplitudes would be brought about by practical limitations of pumping equipment and storage facilities, though because of the more penetrating signal, reliable measurements could be undertaken. Unlike the point source case, measurement would concentrate on the amplitude of the signal, and the excitation frequencies could be higher (of the order of 100 cpd).

In summary, it must be said that the method of sinusoidal pressure testing is not seen as a substitute for classical aquifer testing but as a useful tool to be used in certain specific applications. The method would undoubtedly be costly in both time and equipment, and the problem of producing the sinusoidal pressure fluctuation is not underestimated. However, the opportunity of mapping hydrogeological parameters in three dimensions awaits the investigator willing to invest his time and money.

NOTATION

- ρ density of water, ML^{-3}.
- g acceleration due to gravity, LT^{-2}.
- β compressibility of water, L^2F^{-1}.
- α compressibility of rock, L^2F^{-1}.
- ϕ porosity.
- h hydraulic potential, L.
- S_s specific storage (applying to the three-dimensional case), L^{-1}.
- S storage coefficient.
- S^* two-dimensional specific storage under confined conditions, L^{-1}.
- K hydraulic conductivity, LT^{-1}.
- r vectorial distance from the point or line source, L.
- Q_0 amplitude of the periodic volumetric flow rate, L^3T^{-1}.
- ω frequency of the periodic functions, T^{-1}.
- r_0 radius of a sphere or cylinder enveloping the source, L.
- l thickness of the porous material, L.
- $|G|$ amplitude of the point source frequency response, L.
- G frequency response function for the point source.
- $|G^*|$ amplitude of the line source frequency response.
- N_0 amplitude of the Kelvin function.
- ϕ_0 phase angle of the Kelvin function.
- K_0 modified Bessel function of the second kind.
- ϕ phase of the point source frequency response.
- ϕ^* phase angle of the line source frequency response.
- Ker real part of the Kelvin function of the second kind.
- Kei imaginary part of the Kelvin function of the second kind.

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Determination of hydrogeological parameters using sinusoidal pressure tests: A theoretical appraisal

Black, J. H.; Kipp, K. L.

01 Tanmoy Das Page 2

5/2/2019 12:43

a line source (a well screened over the entire thickness of the aquifer).

02 Tanmoy Das Page 2

5/2/2019 12:43

03 Tanmoy Das Page 2

5/2/2019 12:43

a point source (a well screened over a very small section relative to the entire thickness of an aquifer) and a line source (a well screened over the entire thickness of the aquifer).

04 Tanmoy Das Page 2

5/2/2019 12:43