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Strategies for avoiding errors and ambiguities in the analysis of oscillatory pumping tests

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ABSTRACT

Oscillatory pumping tests have recently seen a resurgence in interest as a strategy for aquifer characterization. In a cross-well pumping test, measured responses to oscillatory pumping tests consist of the amplitude and phase delay of pressure changes at an observation well. This information can be used to obtain estimates of effective aquifer parameters (conductivity and storage coefficients), by fitting field data with an analytical model through parameter estimation. Alternately, multiple pumping tests can be fit simultaneously through tomographic analyses. However, in both cases, analysis of obtained test results may be ambiguous if "phase wrapping" occurs, i.e. if signals are delayed by more than one period. In this work, we demonstrate scenarios under which phase wrapping can make analysis of oscillatory testing difficult, and present guidelines for avoiding ambiguity in oscillatory testing results.

1. Introduction

Oscillatory pumping tests, in which water is alternately injected into and extracted from an aquifer sinusoidally, have seen increasing interest recently as a useful strategy for characterizing both sedimentary aquifers (Rasmussen et al., 2003; Zhou et al., 2016) and fractured bedrock environments (Guiltinan and Becker, 2015). Relative to standard constant-rate pumping tests, oscillatory tests have several advantages that make them practical for field experiments including:

- The ability to perform testing with no net injection/extraction of water, thus minimizing disturbance of solute plumes and changes to effective stress in the aquifer.
- The ability to perform testing at several different frequencies for any given pair of wells, thus obtaining additional characterization information.
- Simplified data signal extraction in the presence of significant background signals or noises.

That said, oscillatory pumping tests are far less common in practice than traditional pumping test (e.g., Rasmussen et al., 2003; Renner and Messar, 2006; Becker et al., 2016), and the practicing hydrogeologist is likely much less fluent in the analysis of obtained oscillatory testing data, and the possible pitfalls that may be encountered.

After performing oscillatory pumping in one well and measuring pressure response in an observation well, the "steady periodic" response can be summarized in terms of the amplitude and phase delay. Analytical solutions, as presented in several prior works (Black and Kipp, 1981; Rasmussen et al., 2003), may then be used to estimate effective aquifer parameters for a given pumping well/observation well combination. Alternatively, instead of performing standard one-at-a-time processing of pumping test data, the responses at several observation wells (and, possibly, from multiple pumping tests) can be analyzed simultaneously using numerical, frequency-domain groundwater flow models. This process, dubbed Oscillatory Hydraulic Tomography or Oscillatory Hydraulic Imaging (Cardiff et al., 2013; Zhou et al., 2016) can take advantage of the differing sensitivities of tests performed across a range of pumping locations, observation locations, and frequencies in order to produce maps of heterogeneous aquifer properties. Blind application of either of these approaches, however, can lead to nonsensical results if "phase wrapping" occurs – i.e. if the measured signal at an observation location is delayed by more than one full period – which can occur under some realistic aquifer scenarios.

The purpose of this work is to examine the issues associated with phase wrapping in oscillatory hydraulic tests and their associated analysis – an issue which has not been discussed in detail

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in the past – and thus to encourage the effective use of oscillatory pumping test data. Because phase wrapping has not been clearly accounted for in analytical formulations to-date, we first present slightly modified analytical solutions for oscillatory flow in 1-D, 2-D, and 3-D aquifers, and briefly compare these to solutions developed in earlier works. These solutions also address some minor errors and ambiguities that are present in prior analytical solutions (as discussed in the Supplementary Material). We then discuss how, under certain scenarios, phase wrapping can present issues for parameter estimation, when using either analytical or numerical modeling techniques. Finally, we discuss strategies that can be used to help reduce the likelihood of phase wrapping negatively impacting data analysis.

2. Problem statement

For purposes of simplicity, we consider the case of a fully confined, isotropic aquifer subject to oscillatory pumping by a well with a small radius. It should be noted that other more complex cases, such as that presented by a leaky confining layer, have been developed in prior derivations by Rasmussen et al. (2003). The standard equations governing changes in head (from an initial steady-state) as a function of radial distance in 1-D, 2-D, or 3-D are:

3-D (Spherical flow)	2-D (Cylindrical flow)	1-D (Linear flow)	
$S_s \frac{\partial h}{\partial t} = K \frac{1}{r^2} \frac{\partial}{\partial r}\left(r^2 \frac{\partial h}{\partial r}\right)$	$S_s \frac{\partial h}{\partial t} = K \frac{1}{r} \frac{\partial}{\partial r}\left(r \frac{\partial h}{\partial r}\right)$	$S_s \frac{\partial h}{\partial t} = K \frac{\partial^2 h}{\partial r^2}$	(1)
$\lim_{r_0 \to 0}\left(-4\pi r_0^2 K \frac{\partial h}{\partial r}\right) = Q(t)$	$\lim_{r_0 \to 0}\left(-2\pi r_0 K l \frac{\partial h}{\partial r}\right) = Q(t)$	$\lim_{r_0 \to 0}\left(-2Klw \frac{\partial h}{\partial r}\right) = Q(t)$	(2)
$h(r, 0) = 0$			(3)
$h(\infty, t) = 0$			(4)

where $h[L]$ represents changes in hydraulic head from an initially steady-state head distribution before pumping begins, with positive h indicating increased head (i.e., increased aquifer pressure); $r[L]$ represents the radius, i.e. distance from the center of the oscillatory pumping well; $t[T]$ represents time; $S_s[L^{-1}]$ represents specific storage; and $K[LT^{-1}]$ represents hydraulic conductivity. The boundary condition at the pumping well (2) is dependent on the dimension of the problem, i.e. whether the well effectively serves as a point source (generating 3-D flow), line source (2-D flow), or plane source (1-D flow) in the given geometry. For the line and plane source cases, where the well is fully-penetrating one or more dimension of the aquifer, $l[L]$ is the aquifer thickness and $w[L]$ is aquifer width. The standard assumptions of zero initial head change and no head change at infinitely large distances are implemented in (3) and (4), respectively.

Likewise, for simplicity, we assume that a conservative oscillatory pumping scheme (i.e., no net injection or extraction) is implemented. The net flowrate into the aquifer is then given by:

$$Q(t) = Q_{\max} \cos(\omega t) \tag{5}$$

where $Q(t)[L^3T^{-1}]$ is a time-varying volume flowrate, with positive values representing flow into the aquifer (injection); $Q_{\max}[L^3T^{-1}]$ is the maximum flowrate achieved during a pumping cycle; ω $[T^{-1}]$ is the angular frequency of pumping, equal to $\frac{2\pi}{P}$ where $P[T]$ is the period of the pumping signal; and $t[T]$ is time.

After an initial period where head change amplitudes and phases within the aquifer are changing, the aquifer will eventually obtain "steady periodic" conditions throughout, i.e. the transient head changes at each point within the aquifer will have a consistent amplitude and phase. In a homogeneous aquifer, Rasmussen et al. (2003) developed an integral expression describing the initial period of non-steady-periodic response, which can be evaluated numerically. Practically, Cardiff et al. (2013) showed through numerical experiments that these conditions are often obtained within 5–10 periods of oscillation for reasonable aquifer conditions. Similarly, Zhou et al. (2016) showed that in laboratory sandbox conditions, similarly quick attainment of steady-periodic conditions was observed.

In analyzing traditional, constant-rate aquifer pumping tests, one may be quite loose with specifying directions or sign of quantities. If, for example, a pumping test is performed, yet an analytical solution gives positive values for head change, then it is simple to catch the error and understand that the solution must be giving drawdown values rather than head change values. We should note at this point that sign conventions are quite important – and errors more difficult to catch – in oscillatory testing. For example, a sign error in specifying the well flowrate Q would result in a solution that may look perfectly correct to the untrained eye, but is actually 180° out of phase with the correct solution; such an error would propagate into incorrect estimates of aquifer parameters. Thus we re-iterate that for our solution Q positive corresponds to injection into an aquifer and h represents head change. Obviously, a completely correct solution is also obtained if Q positive is alternatively interpreted as pumping (extraction) from an aquifer while h is alternatively interpreted as drawdown.

3. Solutions for 1D/2D/3D geometries

Through Laplace or Fourier transform techniques, solutions for the head field at steady-periodic conditions can be readily obtained. Fundamentally, all solutions have the following form:

$$h(r,t) = \mathrm{Re}[\mathbf{\Phi}(r)\exp(i\omega t)] \tag{6}$$

where $\Phi[L]$, the wave phasor, is a complex field variable and i is the imaginary unit, $\sqrt{-1}$. By use of Euler's formula, the solution can be written equivalently as:

$$\begin{aligned} &h(r,t) = \mathbf{\Phi}_R(r)\cos(\omega t) - \mathbf{\Phi}_I(r)\sin(\omega t) \\ &\text{where} \\ &\mathbf{\Phi}(r) = \mathbf{\Phi}_R(r) + i\mathbf{\Phi}_I(r) \end{aligned} \tag{7}$$

The amplitude, $|h|$, of the head waves at any point is given by the modulus of the phasor:

$$|h|(r) = |\mathbf{\Phi}(r)| = \sqrt{\mathbf{\Phi}_R^2(r) + \mathbf{\Phi}_I^2(r)} \tag{8}$$

If the phase of the oscillatory pumping flowrate is considered as the baseline – i.e., a cosine signal represents zero phase delay – then the phase delay of the head signal at a point in the aquifer is given as the negative argument of the phasor:

$$\begin{aligned} \theta(r) &= \arctan_2(-\mathbf{\Phi}_I(r), \mathbf{\Phi}_R(r)) \\ &= \arg(\bar{\mathbf{\Phi}}(r)) \\ &= -\arg(\mathbf{\Phi}(r)) \end{aligned} \tag{9}$$

where θ is the wave phase delay in radians (always positive); $\arctan_2$ represents the four-quadrant arctangent returning angles between 0 and 2π, as implemented in many programming languages as atan2(y,x); $\bar{\mathbf{\Phi}}$ represents the complex conjugate of the phasor $\bar{\mathbf{\Phi}} = \mathbf{\Phi}_R - i\mathbf{\Phi}_I$; and arg() is the complex argument function.

To simplify notation, we first define the auxiliary variable u as

$$u = \sqrt{\frac{\omega S_s r^2}{2K}} = \sqrt{\frac{\omega r^2}{2D}} \tag{10}$$

where $D[L^2T^{-1}]$ is hydraulic diffusivity. The solutions for the head change, for the cases of 3-D, 2-D, and 1-D flow are, respectively:

$$h_{3D} = \mathrm{Re}\left[\underbrace{\frac{Q_{\max}}{4\pi Kr}\exp[-(u+iu)]}_{\Phi_{3D}(r)}\exp(i\omega t)\right] \tag{11}$$

$$h_{2D} = \mathrm{Re}\left[\underbrace{\frac{Q_{\max}}{2\pi Kl}K_0(u+iu)}_{\Phi_{2D}(r)}\exp(i\omega t)\right] \tag{12}$$

$$h_{1D} = \mathrm{Re}\left[\underbrace{\frac{Q_{\max}}{2lw(KS_s\omega)^{1/2}}\exp\left(\frac{-i\pi}{4}\right)\exp[-(u+iu)]}_{\Phi_{2D}(r)}\exp(i\omega t)\right] \tag{13}$$

where the wave phasor in each case has been highlighted with under-brackets, and K_o represents the zeroth-order modified Bessel function of the second kind. Based on these solutions for head, amplitudes and phase delays for all three solutions can be obtained by taking the modulus and argument, respectively, of the given phasor solutions, as shown in Table 1.

The solutions given above, for 2-D and 3-D flow dimensions, are similar to those presented in earlier works (Black and Kipp, 1981; Rasmussen et al., 2003), but contain some minor corrections and simplifications. Supplementary material addressing these minor differences is provided online, for the interested reader. Briefly, however, the solutions above utilize consistent sign conventions and definitions for phase delay, which help to ensure correct treatment of monitoring data.

4. Analysis issues

An important issue highlighted by our revised formulas for signal phase – (15), (17), and (19) – is that several different values of u may produce the same measured phase due to phase wrapping, i.e. when the phase delay exceeds 2π radians. For certain realistic frequencies ω and well distances r, then, the phase response at a well *can be consistent with several different values for the aquifer diffusivity*, D. While this issue was briefly mentioned by Black and Kipp (1981), to the best of our knowledge it has not been dealt with formally, and it has not been shown that phase wrapping can occur for realistic experimental designs (as we show in the following sections).

4.1. An abstract 3-D example

As an instructive, but artificial example in the case of 3-D flow, consider a well that happens to be located at exactly the position of:

$$r = \sqrt{\frac{2D_{true}(2\pi+\varepsilon)^2}{\omega}}$$

Table 1
Amplitude and phase of head oscillations expected in homogeneous, confined aquifers subject to pumping. Signal phase delay θ is defined by convention as a value between [0, 2π], representing phase delay from a perfect cosine. For example, a sine wave will have a phase delay of $\pi/2$.

Flow dimension	Amplitude		Phase	
3-D	$\lvert h\rvert = \frac{Q_{\max}}{4\pi Kr}\exp[-u]$	(14)	$\theta = rem(u, 2\pi)$	(15)
2-D	$\lvert h\rvert = \frac{Q_{\max}}{2\pi Kl}\lvert K_0(u+iu)\rvert$	(16)	$\theta = -\arg(K_0(u+iu))$	(17)
1-D	$\lvert h\rvert = \frac{Q_{\max}}{2lw(KS_s\omega)^{1/2}}\exp[-(u)]$	(18)	$\theta = rem(\frac{\pi}{4}+u, 2\pi)$	(19)

rem is the remainder function, giving the remainder after the first argument is divided by the second.

where D_{true} is the true aquifer diffusivity and ε is a small constant. Substituting into Eq. (10), this gives exactly $u = 2\pi + \varepsilon$, which due to the phase wrapping inherent in Eq. (15), would give an apparent phase delay of ε. Blind application of the Black and Kipp (1981) equation relating phase delay to diffusivity would yield an estimate of diffusivity that approaches infinity as $\varepsilon \to 0^+$. In essence, since phase wrapping makes phase delays of ε indistinguishable from phase delays of $n2\pi + \varepsilon$ under steady-periodic (where n is an integer), the diffusivity parameter in theory cannot be uniquely determined without other information.

4.2. Concrete 2-D examples

4.2.1. Processing with an analytical (homogeneous) model

The discussion above shows that, at least theoretically, using amplitude and phase data from an observation well to estimate aquifer parameters can produce incorrect results due to the phenomena of phase wrapping. Here we present concrete examples of this effect, using the 2-D solution. To demonstrate the issue, we utilize the same analysis approach applied by Rasmussen et al. (2003), and later by Guiltinan and Becker (2015). As a review, the approach consists of the following steps which are used to estimate aquifer transmissivity $T = Kl$, storativity $S = S_s l$, and diffusivity $D = T/S$:

1. Given field data, determine the cosinusoidal and sinusoidal coefficients that best fit the data using ordinary least squares fitting or other approaches. Then calculate the observed phase delay θ_0 and the observed normalized amplitude as $|h_o|_n = |h_o|/Q_{\max}$ (where $|h_o|$ is observed head change amplitudes).
2. Approximate the inverse function for obtaining an estimate of diffusivity, D_{est}, from θ_0 as:

$$D_{est} = \omega r^2 \Bigg/ \left[\exp\left(\sum_{j=0}^{5} c_{(j+1)}(\ln\theta_0)^j\right)\right]$$

where the vector of coefficients for the 5th-order logarithmic polynomial is:

$$c_j = \begin{bmatrix} -0.12665 \\ 2.8642 \\ -0.47779 \\ 0.16586 \\ -0.076402 \\ 0.03089 \end{bmatrix}$$

This approximation was developed by Rasmussen et al. (2003), and represents a close approximation to the inverse function of (17) on the section of the curve where apparent phase delays match true phase delays (i.e., wrapping has not occurred).

3. Once the diffusivity estimate is obtained, estimate transmissivity T by using the observed normalized head change amplitudes $|h_o|_n$, and inverting the amplitude Eq. (16), as:

$$T_{est} = \frac{\left|K_0\left(r\sqrt{\frac{i\omega}{D}}\right)\right|}{2\pi|h_o|_n}$$

4. Finally, calculate the estimated storativity S_{est} as $S_{est} = T_{est}/D_{est}$

Fundamentally, the approach above is very useful, though a possible problem with the approach above is that Step 2 assumes a unique relationship between phase delay and diffusivity, i.e., that

Eq. (17) is a one-to-one relationship that can be inverted. Using a few carefully chosen example cases, we will show that this assumption may be violated in some realistic aquifer scenarios.

We have generated example data (amplitudes and phases) for a few different aquifer scenarios analytically using the solutions presented above. Implementing steps 1–4 above in MATLAB, we then attempted to estimate aquifer parameters using the approach used by Rasmussen et al. (2003) and Guiltinan and Becker (2015). The results are contained in Table 2. When phase wrapping does not occur due to high aquifer diffusivity (as in Case 3 or Case 4), it is apparent that the approach is working correctly and performs very well, since accurate parameter estimates are obtained within ~1% of the true values. However, when phase wrapping is present (as in Cases 1 and 2), we note that the estimated diffusivities, transmissivities, and storativities can be incorrect by orders of magnitude.

To demonstrate that this is not simply a theoretical concern, we have generated synthetic data using the true parameters obtained in one of the aquifer tests performed by Rasmussen et al. (2003). Specifically, the testing results given by the last row of Table 6 and Table 7 in Rasmussen et al. (2003) were synthetically generated, corresponding to pumping in well SWP 300A, while observing in well SWP 303A. Case 5 (Table 2) demonstrates that when the true aquifer parameters are used, they are recovered after application of the algorithm described above. However, Case 6 shows that comparable data would be measured (and thus, the same parameter estimates obtained) under a drastically different set of "true" aquifer parameters, demonstrating again the non-unique relationship between observed phase/amplitude and estimated aquifer parameters.

4.2.2. Processing via OHT inversion

The issues of data processing introduced by phase wrapping are not limited to the case where data is processed via an analytical model. While analytical processing techniques presented to-date generally use the amplitude and phase of the obtained head signal to produce aquifer parameter estimates, the Oscillatory Hydraulic Tomography (OHT)/Oscillatory Hydraulic Imaging (OHI) approach advocated by Cardiff et al. (2013), which uses phasor coefficients instead of amplitude and phase as measurements, faces similar issues. To review this technique, the basic steps are as follows:

1. For each observation of head change, utilize a steady-periodic portion of the time series and process the data to extract the Fourier Coefficients or "phasor" associated with the input pumping frequency. Data from each observation location is thus summarized as $\Phi_o = A_o + B_o i$, where A_o and B_o represent coefficients that produce the best fit between the model $A_o \cos(\omega t) - B_o \sin(\omega t)$ and the observed field data.
2. Numerically simulate all observations using a heterogeneous numerical model based on a discretization of the equation:

$$i\omega S_s(\mathbf{x})\mathbf{\Phi} = \nabla \cdot (K(\mathbf{x})\nabla\mathbf{\Phi}) + Q_{\max}(\mathbf{x})$$

which is the governing equation for the steady-periodic wave phasor in the case where the stimulation occurs at a single frequency with $Q(\mathbf{x}, t) = Q_{\max}(\mathbf{x})\cos(\omega t)$.

3. Through inverse modeling, numerically alter the heterogeneous values of $S_s(\mathbf{x})$ and $K(\mathbf{x})$ in order to improve phasor data fit, while also meeting given regularization goals (e.g., smoothing or geostatistical reasonableness of the $K(\mathbf{x})$ and $S_s(\mathbf{x})$ fields).

More comprehensive details on the practice of OHT can be found in Cardiff et al. (2013). For a 2D model, as in the example presented below, the parameters $K(\mathbf{x})$ and $S_s(\mathbf{x})$ are replaced by the depth-integrated quantities transmissivity $T(\mathbf{x})[L^2/T]$ and $S(\mathbf{x})[-]$, where $Q_{\max}(\mathbf{x})$ represents the associated depth-integrated pumping signal.

The issue of phase wrapping is not obviated by the use of the wave phasor Φ_o, as described above, to represent observed signals. To demonstrate this, we have generated synthetic data from a 2-D numerical model with a homogeneous T and S field matching the parameters of Case 6 in Table 2 (T = 2.583e−6[m^2/s] and S_s = 2.213e−5[−]). A single oscillating pumping well is located at the origin, with observation wells located at (0, 139.8), (150, 0), (0, −200), and (−250, 0), with all distances specified in meters. As demonstrated in Table 2, a distance of 139.8 m or greater produces phase wrapping for this set of true aquifer parameters, thus all observation wells in this model are subject to phase wrapping.

In beginning the inversion, an initial guess must be supplied. We have naively assumed that we can use the results from the time series measured at the shortest distance (139.8 m) in order to produce a good initial guess, without taking into account the issue of phase wrapping. In effect, a homogeneous parameter field of T = 3e−3[m^2/s] and S = 8e−4[−] was used as an initial parameter field guess. Inverse modeling was performed using a MATLAB phasor-based numerical model (available from the lead author upon request) and a geostatistical inversion, as described in Cardiff et al. (2013). Briefly, the geostatistical inversion algorithm is a standard, gradient-based inversion approach in which the objective function to be minimized is a combination of the sum of squared data misfits and a regularization penalty function based on geostatistics. In order to ensure non-negativity, the parameters altered during optimization are ln(T) and ln(S) values. After obtaining convergence through linearized iteration, the estimated heterogeneous parameter fields (shown in Fig. 1), which have a geometric mean of T = 2.8e−3[m^2/s] and S = 2.3e−3[−], obtain a

Table 2
Example analysis cases with parameter estimates obtained by applying available analytic solutions. Estimates that are of the wrong order of magnitude are highlighted.

	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
Inputs						
r [m]	31	10	100	10	139.8	139.8
D [m^2/s]	0.37	0.01	1000	10	3.5648	0.1167
T [m^2/s]	1e−4	1e−6	1e−2	1e−3	3.08e−3	2.583e−6
S [−]	3.7e−5	1e−4	1e−5	1e−4	8.64e−4	2.213e−5
P [s]	30	500	3600	20	9000	9000
$Q_{\max}$ [m^3/s]	1e−3	5e−5	1e−4	5e−5	1.079e−3	1.079e−3
Observations						
Amplitude $\lvert h_o \rvert$ [m]	0.0015	0.0011	0.0036	0.0020	0.0120	0.0120
Phase delay θ_o [rad]	0.2021	2.0287	0.3456	1.6098	1.7424	1.7474
Estimates						
D_{est}	3.3e5	0.2283	999.4	10.01	3.576	3.551
T_{est}	**4.136e−1**	**1.1e−3**	9.999e−3	1e−3	3.088e−3	**3.065e−3**
S_{est}	**1.221e−6**	**4.9e−3**	1.001e−5	9.9e−5	8.638e−4	**8.633e−4**

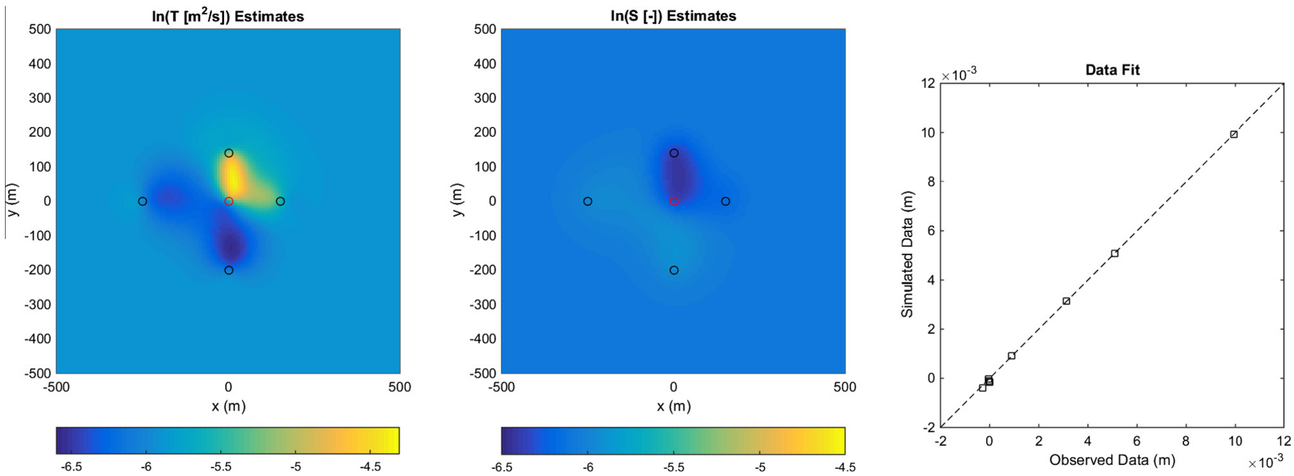

Fig. 1. Parameter estimates for T (left) and S (center) obtained when using an initial homogeneous guess of $T = 2.8\text{e}{-3}[\text{m}^2/\text{s}]$, $S = 2.3\text{e}{-3}[-]$. Central red location represents pumping well, with black open circles representing observation wells. Data fit is represented (right) as the observed phasor coefficients compared to simulated phasor coefficients. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

near-exact fit to all monitoring data. In order to obtain good fits to monitoring wells at a variety of distances, the inversion algorithm has introduced heterogeneity within the parameter field. However, since the inversion algorithm is gradient-based, and thus searches locally within the neighborhood of the current parameter guess during each iteration, the solution has been affected by the order-of-magnitude inaccurate initial guess and thus obtains only a locally optimal solution. Near-perfect data fit has been obtained, but at the expense of introducing heterogeneity within the parameter field.

In contrast, if a more correct initial guess is used, with $T = 1\text{e}{-6}\ [\text{m}^2/\text{s}]$ and $S = 1\text{e}{-5}[-]$, the inversion algorithm is able to correctly converge to the global minimum which obtains perfect data fit and requires no heterogeneity (i.e., perfect smoothness). This converged solution, consisting of a homogeneous parameter field with $T = 2.583\text{e}{-6}[\text{m}^2/\text{s}]$ and $S = 2.213\text{e}{-5}[-]$, is shown in Fig. 2.

5. Guidelines and lessons learned

While oscillatory pumping tests are a valuable strategy for aquifer characterization, they must be performed and analyzed with care in order to ensure that meaningful results are obtained. One important issue is that computing a well's phase response requires careful attention to make sure that the phase is defined in a consistent manner, and all observed quantities (flow rates, head changes, etc.) must be carefully inspected for the correct sign. A second issue is that, as demonstrated above, there are peculiar aquifer scenarios where developed analysis strategies may produce inconsistent results. Especially if a single oscillatory pumping test/observation well pair is used to characterize an aquifer, and no other information is utilized, there is the opportunity for automated parameter estimation algorithms to make large errors in aquifer parameter estimates. As more oscillatory tests and observations become available, it is likely that these errors will become apparent and consistent results can be generated. However, as demonstrated in our tomography example, inversion algorithms may also be impacted by phase wrapping issues if a poor initial guess for aquifer parameter estimates is produced.

Fortunately, there are a few common-sense "fixes" to the problems associated with phase wrapping. If early time transient data before attainment of steady-periodic conditions is visualized, it may be visually apparent that phase wrapping has not an approach that was employed by Guiltinan (pers. comm.). Another approach is, if possible, to observe head changes in the pumping well during

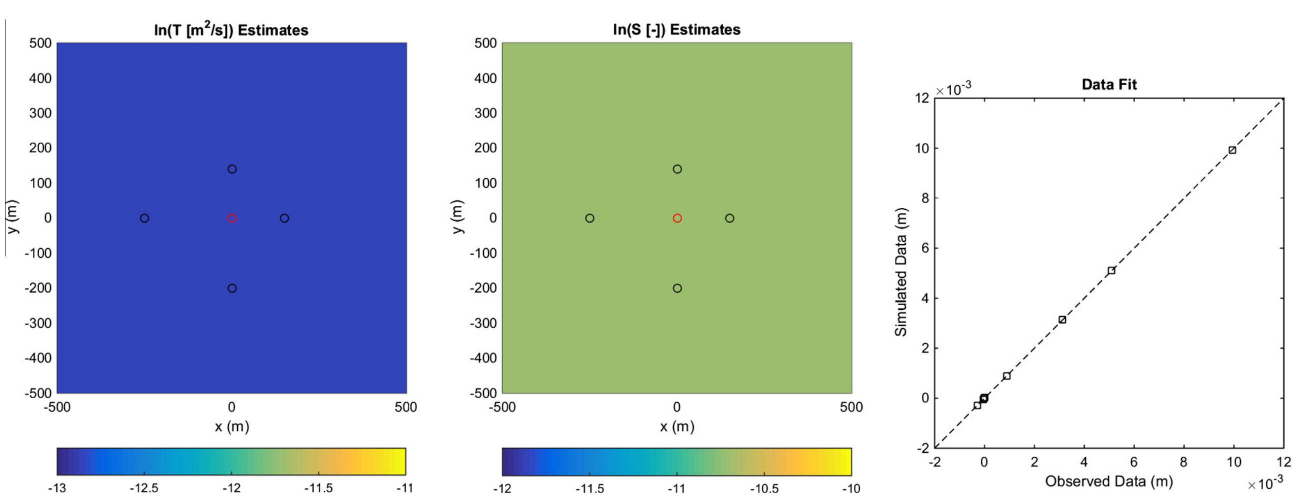

Fig. 2. Parameter estimates for T (left) and S (center) obtained when using an initial homogeneous guess of $T = 1\text{e}{-6}[\text{m}^2/\text{s}]$, $S = 1\text{e}{-5}[-]$. Central red location represents pumping well, with black open circles representing observation wells. Data fit is represented (right) as the observed phasor coefficients compared to simulated phasor coefficients. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Table 3
Guidelines for maximum radius to observation well that should be analyzed, to avoid phase wrapping issues.

	Phase formula	First phase wrap	Limiting radius
3-D	$\theta = rem(u, 2\pi)$	$u = 2\pi$	$r_{\max} = \sqrt{\frac{2D_{\min}P(2\pi)^2}{\pi}}$
2-D	$\theta = -\arg(K_0(u + iu))$	$u \approx 5.8$	$r_{\max} = \sqrt{\frac{D_{\min}P(5.8)^2}{\pi}}$
1-D	$\theta = rem(\frac{\pi}{4} + u, 2\pi)$	$u = \frac{7\pi}{4}$	$r_{\max} = \sqrt{\frac{D_{\min}P(\frac{7\pi}{4})^2}{\pi}}$

oscillatory pumping tests. Measured at or near the pumping well, head change magnitude should be able to clearly differentiate between high-diffusivity and low-diffusivity aquifers that otherwise produce the same response at an observation well. This would eliminate confusion between solutions with different diffusivities (e.g., Case 5 vs. Case 6 in Table 2).

Finally, using the phase formulas (15)–(19), one can determine the value of u that will produce the first phase wrap, i.e. a phase of exactly 2π. Then, if the period P of testing and a conservative lower bound on diffusivity $D_{\min}$ (i.e., as small as is reasonable) is provided for the given aquifer, a maximum radius can be calculated for which signals can be analyzed without fear of phase wrapping. Alternately, the period of testing P may be increased in order to avoid phase wrapping given known inter-well distances. The guidelines based on this logic are found in Table 3. If there is an indication that tomographically-analyzed testing data could experience phase wrapping, then testing multiple initial guesses in inverse modeling may be called for in order to ensure a global optimum solution is obtained. In designing oscillatory tests, we suggest that these guidelines be considered in addition to previously-developed guidelines for test sizing presented by Cardiff and Barrash (2015).

6. Conclusions

Oscillatory pumping tests have several advantages for aquifer characterization, relative to traditional constant rate pumping tests. Oscillatory pumping tests can be performed without net extraction of water from an aquifer, and they produce signals that are highly identifiable in the presence of background noise (i.e., head changes from other aquifer forcings). They can also be performed at multiple frequencies in order to assess impacts of heterogeneity or check for "frequency dependence" in aquifer parameters. While hardware for performing oscillatory pumping tests is more complex than the hardware needed for simple constant rate pumping tests, computer-controlled pumping systems using oscillating slugs (Guiltinan and Becker, 2015) or pneumatic systems (the Pneusine – http://hydroresolutions.com/hr_press/wp-content/uploads/2013/02/PneuSine-Test-HydroResolutions-LLC.pdf) are beginning to be "packaged" as tools for aquifer characterization.

That said, oscillatory pumping tests are not yet commonly applied, and practitioners using these tests must be aware of the care necessary to interpret collected data. In particular, sign conventions must be carefully observed in order to correctly estimate "phase delay" at observation wells. Minor errors and inconsistencies identified in prior solutions, likewise, can cause additional confusion. Finally, in some scenarios, the issue of phase wrapping – where a true phase delay of $n2\pi + \varepsilon$ is observed as an apparent phase delay of ε – has the potential to produce unrealistic aquifer parameter estimates in some cases.

In this work, we have sought to remove any ambiguities that could prevent the successful analysis of oscillatory pumping tests. We have also provided a simple analytical formula that can be used to ensure that phase wrapping issues will be avoided. When combined with earlier guidelines presented in Cardiff and Barrash (2015), we believe these simple tools provide a template for the successful performance and analysis of oscillatory pumping tests.

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Appendix A. Supplementary material

Supplementary data associated with this article can be found, in the online version, at http://dx.doi.org/10.1016/j.jhydrol.2016.06.045.

References

Becker, M.W., Cole, M., Ciervo, C., Coleman, T., Mondanos, M., 2016. Measuring hydraulic connection in fractured bedrock with periodic hydraulic tests and distributed acoustic sensing. In: 41st Workshop on Geothermal Reservoir Engineering. Stanford University, Stanford, CA, USA, SGP-TR-209.
Black, J.H., Kipp Jr., K.L., 1981. Determination of hydrogeological parameters using sinusoidal pressure tests: a theoretical appraisal. Water Resour. Res. 17 (3), 686–692.
Cardiff, M., Bakhos, T., Kitanidis, P.K., Barrash, W., 2013. Aquifer heterogeneity characterization with oscillatory pumping: sensitivity analysis and imaging potential. Water Resour. Res. 49 (9), 5395–5410.
Cardiff, M., Barrash, W., 2015. Analytical and semi-analytical tools for the design of oscillatory pumping tests. Ground Water 53 (6), 896–907.
Guiltinan, E., Becker, M.W., 2015. Measuring well hydraulic connectivity in fractured bedrock using periodic slug tests. J. Hydrol. 521, 100–107.
Rasmussen, T.C., Haborak, K.G., Young, M.H., 2003. Estimating aquifer hydraulic properties using sinusoidal pumping at the Savannah River site, South Carolina, USA. Hydrogeol. J. 11, 466–482.
Renner, J., Messar, M., 2006. Periodic pumping tests. Geophys. J. Int. 167, 479–493.
Zhou, Y., Lim, D., Cupola, F., Cardiff, M., 2016. Aquifer imaging with pressure waves – evaluation of low-impact characterization through a sandbox study. Water Resour. Res. 52 (3), 2141–2156. http://dx.doi.org/10.1002/2015WR017751.