On Methods of Determining Specific Yield

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ABSTRACT

A comparative discussion of several methods for the determination of specific yield is presented. The discussion relies on a pumping test conducted recently by Nwankwor and coworkers from the University of Waterloo in an unconfined aquifer at Borden, Ontario, Canada. We show that a water-balance method promulgated by these workers and used earlier by others, including Wenzel in 1942 and 1946, is invalid because it overlooks a major component of the water budget. This method suggests erroneously that specific yield increases with time during a pumping test. The method can therefore lead to arbitrarily large specific yield values which may lack physical significance. By relying on this water-balance method and on laboratory drainage experiments, Nwankwor and coworkers conclude incorrectly that type-curve methods, such as those proposed by Prickett in 1965 and Neuman in 1975, give unreasonably low specific yield values. We show instead that these specific yield values are consistent with water-balance considerations when all the components of the water budget are properly taken into account. We further point out that whereas the larger specific yields usually obtained from laboratory drainage experiments may be well suited for the evaluation of ground-water reserves that can potentially be recovered from an unconfined aquifer over long periods of time, they are not directly relevant to the problem of relating ground-water level variations to pumpage which is characterized by a shorter time scale. This is especially true when specific yield is taken to be the difference between water content at saturation and residual water content at high suctions as done by Nwankwor and coworkers. The rate at which ground-water levels drop or fluctuate in response to pumpage is controlled by the smaller specific yield that one obtains from time-drawdown analyses.

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INTRODUCTION

Specific yield is the volume of water released from a unit area of unconfined aquifer when the water table drops by one unit of height. Methods to determine specific yield include the analysis of time-drawdown data from pumping tests by techniques such as those of Prickett (1965) and Neuman (1975), water-balance calculations based on measured fluctuations of the water table in response to given rates of recharge and discharge, laboratory drainage experiments on samples of aquifer material, and parameter estimation by means of numerical models. A recent study by Nwankwor *et al.* (1984) compares the time-drawdown methods of Prickett and Neuman with a water-balance method applied to the cone of depression around a pumping well and an approach in which specific yield is computed as the difference between the saturated and residual water contents of a soil sample in the laboratory. The study notes that specific yields determined by these two time-drawdown methods are generally much smaller than those obtained from water-balance calculations and laboratory experiments. Water-balance calculations on cones of depression suggest that the specific yield increases with time during a pumping test. Nwankwor *et al.* consider the water-balance and laboratory methods to provide more direct, and therefore more reliable, means of determining specific yield than do the time-drawdown methods. They therefore conclude that the latter methods "provide values of specific yield that are not suitable for use in the context of long-term aquifer yield analysis" and recommend that one rely instead on "the volume-balance method at late time or the laboratory method."

The perceived poor performance of the Prickett (1965) and Neuman (1975) time-drawdown methods is attributed by Nwankwor *et al.* (1984) to weaknesses in the theories of Boulton (1954, 1963) and Neuman (1972, 1974) which, respectively, underlie these methods. The theory of Boulton is said to be deficient because it does not consider vertical flow in the saturated zone while supposedly accounting for unsaturated drainage above the water table. The theory of Neuman is considered weak because it neglects such unsaturated drainage. Nwankwor *et al.* suggest that "a more appropriate model would be one that includes both of these effects," as it might resolve the discrepancy between the results of time-drawdown analyses and specific yield obtained from water-balance and laboratory experiments.

The implications of not considering vertical flow under the water table on Boulton's (1954, 1963) theory have been discussed in detail by Neuman (1979). One important implication is that the theory cannot deal with vertical anisotropy and data from partially penetrating wells. However, when applied to data from fully penetrating wells according to the method of Prickett (1965), it yields practically the same values of transmissivity, storativity, and specific yield as does the method of Neuman (1975). Neuman (1979) has also shown that, contrary to popular belief and despite the fact that Boulton viewed delayed drainage as a process taking place above the water table, Boulton's mathematical model does not in any way account for unsaturated flow. Instead, his model is fully consistent with that of Streltsova (1972a, b; 1973) which, like Neuman's, completely disregards drainage above the water table. While in reality such drainage does take place, Kroszynski and Dagan (1975) have demonstrated very clearly that its effect on drawdowns in the saturated zone is negligible during most pumping tests of practical interest.

Since unsaturated flow has no significant effect on the interpretation of pumping test data, what may be causing the discrepancy between specific yield values obtained from time-drawdown analyses [by methods such as Neuman's (1975) which does account for vertical flow] and those based on water-balance calculations or laboratory experiments? In this brief note, we rely on the pumping test and laboratory data published by Nwankwor *et al.* (1984) to help answer this question. We show that the specific yield values they calculated from time-drawdown data by the methods of Prickett (1965) and Neuman are consistent with the available field data and with considerations of water balance. On the other hand, the specific yield values they obtained in the laboratory and by performing a water budget calculation on the observed cone of depression are consistent neither with the field results nor with proper water-balance considerations. This leads to conclusions that are quite different from those reached by Nwankwor *et al.* regarding the utility of various methods for the evaluation of specific yield.

THE BORDEN PUMPING TEST REVISITED

Nwankwor *et al.* (1984) described the details of two well-instrumented pumping tests conducted in a shallow unconfined sand aquifer at the Canadian Forces Base in Borden, Ontario. The sand is 9 m thick and is underlain by a thick deposit of clayey silt. The water table was about 2.3 m below ground surface before and after the tests. The pumping well had a casing with an inner diameter of 15 cm and was screened in the lower four meters of the aquifer. Water levels were measured in a series of piezometers completed at various distances and elevations. The piezometers were made from 3.5 cm I.D. PVC pipes perforated over the bottom 35 cm and screened with nylon cloth. They were distributed in such a way as to facilitate monitoring the shape of the water table at all times.

One of the pumping tests lasted about four hours; the other lasted 65 hours. We limit our discussion to the longer test which gave enough data to evaluate the specific yield by time-drawdown methods. During this test the discharge was maintained at 60 l/min, resulting in a maximum drawdown between 50 and 60 cm. The drawdown recorded in the various piezometers was qualitatively consistent with that predicted by the theory of Neuman (1972, 1974). This was clear to Nwankwor *et al.* (1984) who stated that "the hydraulic head data from below the water table supports the conceptual basis of aquifer flow inherent in the Neuman model." When the late time-drawdown data were analyzed with the aid of Neuman's (1975) type curves, they gave a specific yield of 0.07. A similar analysis based on Prickett's (1965) type curves gave a specific yield of 0.08. In what follows, we adopt the average value of 0.075 as representative of the time-drawdown method.

The water-balance method was applied by Nwankwor *et al.* (1984) to the cone of depression as delineated by the observed water-table elevations. The specific yield, S_y, at any given time was calculated according to the formula

$$S_y = V_w/V_c \qquad (1)$$

where V_w is the cumulative volume of discharge from the pumping well, and V_c is the volume of the observed cone of water-table depression. The resultant S_y values increased with time from 0.02 at 15 min. through 0.05 at 40 min., 0.12 at 600 min., 0.20 at 1560 min., 0.23 at 2690 min., to 0.25 at 3870 min. corresponding to the latest time on record. Nwankwor *et al.* concluded that S_y tends asymptotically to the value of 0.30 obtained from a sample of the aquifer material in the laboratory by subtracting the residual water content at high suctions from the water content at saturation.

Unfortunately, the water-balance calculations of Nwankwor *et al.* (1984) are invalid because the observed cone of depression accounts for only a small percentage of the water released from storage in the aquifer. Of some minor significance is the possibility that, during the first few minutes of the test, much of the discharge came from a release of artesian storage due to the compaction of aquifer material and expansion of water in the deeper portions of the aquifer near the pumping well. This is suggested by the time-drawdown curves in Figure 3 of Nwankwor *et al.* corresponding to the deeper piezometers which appear to follow an early (artesian) Theis curve during these first few minutes. More relevant to our discussion is the aquifer response at later times when the artesian contribution became relatively unimportant. At these later times, most of the water released from storage originated outside the radius of the observed cone of water-table depression. This is evident from a close examination of the water-table profiles in Figure 5 of Nwankwor *et al.* and, as we shall see, is strongly supported by theory.

Figure 5 of Nwankwor *et al.* (1984) suggests that, at 600 min., the cone of depression considered by the authors extended to a radial distance, r, of approximately 35 m. If one enlarges their figure and draws a tangent to the drawdown curve at r = 30 m, one obtains a radial hydraulic gradient of approximately 3.56×10^{-3}. We take a more conservative approach by measuring a drawdown of 1.78×10^{-2} m at r = 30 m, a zero drawdown at r = 42.2 m, and assigning the corresponding radial hydraulic gradient of 1.46×10^{-3} to r = 35 m. Though this calculation is highly inaccurate and disregards the nonlinear shape of the drawdown cone, it is nevertheless enough to demonstrate our point. Nwankwor *et al.* computed a transmissivity for the aquifer from the late time-drawdown data (transmissivities obtained by them from earlier data do not represent vertically averaged drawdowns) of approximately 1.5×10^{-3} m^2/sec. Since the drawdown was very small in comparison to the saturated thickness of about 7 m, we safely ignore it in calculating that lateral flow rate across a cylinder of radius r = 35 m, centered about the pumping well, was approximately 28.9 l/min. This constitutes over 48 percent of the instantaneous discharge rate from the well, implying that less than 52 percent was contributed to this rate by water-table decline within the visible cone of depression (i.e., within $r \leq 35$ m).

At 3870 min., the observed cone of depression extended to about 65 m. From Figure 5 of Nwankwor *et al.* (1984), we read a drawdown of 4.45×10^{-2} m at r = 46 m and zero at r = 74 m. These numbers yield a rough estimate for the hydraulic gradient of 1.59×10^{-3} which we assign to the edge of the visible cone of depression at r = 65 m. Thus, the lateral flow rate across a cylinder of radius 65 m centered about the pumping well was approximately 58.3 l/min. This constitutes over 97 percent of the instantaneous discharge rate from the well, implying that less than 3 percent of this rate was now contributed by water-table decline within the observed cone of depression. We see that as the measurable cone expands, its contribution to the release of water from storage relative to the release outside the area which this cone occupies diminishes with time. Even though drawdowns outside the observable cone of depression are too small to measure with any degree of accuracy, they are spread over large enough an area to explain the supply of water from outside the radius of this cone by gravity alone (i.e., without considering compressibility as a storage mechanism). This will be demonstrated theoretically later in the text.

Based on the relatively slow change in the radial gradient at the edge of the observed cone as it expands with time, we introduce (for illustration purposes only) a very rough approximation by assuming that the lateral flow rate, Q(t), from outside the cone area varies linearly with time. Let f(t) = 1 − Q(t)/Q, where Q is the pumping rate. Then f(t) represents the fraction of the instantaneous discharge rate from the pumping well that is released from storage by the falling observable cone of depression at time t. Since this ratio was found to be roughly 0.52 at time t = 600 min. and 0.03 at t = 3870 min., our assumption of linearity implies that $f(t) = 6.099 \times 10^{-1} - 1.498 \times 10^{-4}\, t$, where t is in minutes. Points at other times such as 40 min., 1560 min., and 2690 min. that can be obtained on the basis of Figure 5 in Nwankwor *et al.* (1984) fall not far from this straight line. The cumulative fraction, F(t), of the instantaneous pumping rate that is attributable to the observed cone of depression is the integral of f(t) divided by t. It is given by $F(t) = 6.099 \times 10^{-1} - 7.49 \times 10^{-5}\, t$. Clearly, the proper way to write equation (1) is

$$S_y^* = F\, V_w / V_c = F\, S_y \qquad (2)$$

where S_y^* is a specific yield value that accounts for contribution from storage inside and outside the observed cone of depression, and S_y is the value obtained from (1). If we keep S_y fixed at 0.075 as determined from the time-drawdown analyses, we obtain the F and S_y values in Table 1. The reader can see that, despite the approximate nature of the calculations, our S_y values are not too different from those computed by Nwankwor *et al.* (1984) which are also listed in the table for comparison. These values, however, are an artifact of having disregarded the large quantities of water released from storage outside the observed cone of depression. The specific yield that actually controls the observed drawdowns (including the observed shape of the water table) is S_y^*.

Our observation that large amounts of water are contributed from storage areas lying outside the measurable cone of depression should come as no surprise; it is entirely consistent with theory. The time-drawdown data in Figure 3 of Nwankwor *et al.* (1984) suggest that, at t = 600 min. and beyond, the vertically averaged drawdowns at radial distances of 15 m and less follow the late Theis curve. These drawdowns are given by the equation

$$s(r,t) = \frac{Q}{4\pi T} \int_u^\infty \frac{e^{-y}}{y}\, dy \qquad (3)$$

where s is drawdown, T is transmissivity, and $u = r^2 S_y / 4Tt$. Note that S_y in (3) represents release from storage due to gravity drainage alone and does not account for compressibility as a storage mechanism. By using the Leibnitz rule of differentiation, we find that

$$\frac{Q(r,t)}{Q} = e^{-u} \qquad (4)$$

where Q(r, t) is the radial flow rate at time t across a cylinder of radius r centered about the pumping well.

Table 1. F and S_y Values for S_y^* = 0.075

t, min.	F	S_y	S_y [1]
40	0.607	0.124	0.05
600	0.565	0.133	0.12
1560	0.493	0.152	0.20
2690	0.408	0.184	0.23
3870	0.320	0.234	0.25

[1] From Nwankwor *et al.* (1984).

Since the drawdowns during the Borden test are controlled by a delayed yield process, we have no assurance that the Theis equation and (4) which derives from it are valid beyond r = 15 m. We will, however, assume for illustration purposes that (4) is valid for all r values. With this crude approximation and an S_y of 0.075 we find from (3) and (4) that, at t = 600 min., s(r, t) and Q(r, t)/Q vary with r according to Table 2. The calculated vertically averaged drawdowns at r = 10 m and r = 20 m correspond quite closely to those shown in Figure 5 of Nwankwor *et al.* (1984) even though the latter represent conditions at the water table. This confirms that flow at t = 600 min. is essentially horizontal within the observed cone of depression and the late Theis curve applies at $r \leqslant 20$ m, though possibly not beyond this distance. Table 2 tells us that the cone of depression beyond the observed radius of 35 m contributes significant amounts of water from storage due to gravity drainage even though the drawdowns there are imperceptible. The theoretical rate of flow across this radius is roughly 65 percent, more than the 48 percent estimated earlier from the shape of the water table. The discrepancy between these numbers may reflect the approximate nature of both our empirical and theoretical analyses.

Table 3 lists the values of s(r, t) and Q(r, t)/Q for various radial distances at t = 3870 min. At this time the average drawdowns correspond

Table 2. Theoretical s(r, t) and Q(r, t)/Q at t = 600 min.

r, m	s(r, t), m	Q(r, t)/Q
10	1.49×10^{-1}	0.966
20	8.12×10^{-2}	0.870
30	4.67×10^{-2}	0.732
40	2.60×10^{-2}	0.574
50	1.46×10^{-2}	0.420
60	7.97×10^{-3}	0.287
70	3.96×10^{-3}	0.182

Table 3. Theoretical s(r, t) and Q(r, t)/Q at t = 3870 min.

r, m	s(r, t), m	Q(r, t)/Q
10	2.47×10^{-1}	0.995
20	1.75×10^{-1}	0.979
30	1.33×10^{-1}	0.953
40	1.04×10^{-1}	0.918
50	8.39×10^{-2}	0.874
60	6.69×10^{-2}	0.824
70	5.31×10^{-2}	0.768

quite closely to those shown for the water table in Figure 5 of Nwankwor *et al.* (1984) at $r \leqslant 30$ m, but there is no assurance that the analysis is accurate beyond this distance. The table shows that the area outside the radius of r = 65 m, i.e., outside the observed cone of depression, contributes well over 77 percent to the pumping rate at this time. While this theoretical rate is less than the 97 percent estimated from the experimental shape of the water table (again due to the approximate nature of both the empirical and the theoretical analyses), it is nevertheless enough to invalidate the water balance concept underlying the method promulgated by Nwankwor *et al.*

CONCLUSIONS

The following conclusions can be drawn from this discussion:

1. The 65-hour Borden pumping test described by Nwankwor *et al.* (1984) lends excellent experimental support to Neuman's (1972, 1974, 1975, 1979) theory of delayed gravity response.

2. Methods of time-drawdown analysis based on this theory and the delayed yield theory of Boulton (1954, 1963; see also Prickett, 1965) lead to specific yield values that are consistent with water-balance calculations based on the shape of the water table observed during the above test.

3. Water-balance methods in which specific yield is computed from pumping test data by assuming that all the discharge derives from drainage within the observed cone of depression may lead to errors of several hundred percent. Theory shows and the Borden test confirms that significant amounts of water are released from storage by gravity drainage outside the observed cone of depression even though the drawdowns there may be imperceptible. Neglecting this source of water results in exaggerated values of specific yield. It further leads to the erroneous impression that the specific yield grows with time as the pumping test progresses.

4. Specific yields determined from drainage experiments on samples of aquifer material in the laboratory are often much larger than values obtained from pumping tests in the field. This is especially true when the specific yield is taken to be the difference between the water content at saturation and the residual content at high suctions. According to Nwankwor *et al.* (1984), the time required for equilibrium to be reached when the sandy Borden aquifer material was subjected to stepwise increments of suction ranged from tens of minutes at high values of water content to one to two days at low values of water content. In the pumping test which lasted less than three days, the most rapid fall of the water table occurred during the first ten hours. This shows that water-table response to pumpage is a much faster phenomenon than drainage in the unsaturated zone above it. Since the two phenomena are characterized by very different time scales, the variation of ground-water levels in response to pumpage is relatively insensitive to residual drainage in the unsaturated zone. A similar conclusion can be reached on the basis of a theoretical study conducted more than a decade ago by Kroszynski and Dagan (1975). It thus becomes clear that whereas specific yields obtained in the laboratory may be useful for the evaluation of ground-water reserves that may be potentially recoverable over long time periods, they are generally not relevant to the problem of relating ground-water level fluctuations to pumpage. This latter problem, which arises in the majority of ground-water studies, requires specific yield values of the kind obtained from the analysis of time-drawdown data.

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