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Research paper

On the practical application of the Cooper and Jacob distance-drawdown method to analyse aquifer-pumping test data

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ABSTRACT

Studies have illustrated how Infinite Acting Radial Flow (IARF) can be used as an objective criterion than the u condition to evaluate the valid of applying the Cooper and Jacob time-drawdown (1946) method. The Cooper and Jacob (1946) time-drawdown and distance-drawdown methods work on the same principle which incorporates the u condition. A similar study to assess the use of IARF as a criterion to determine the applicability of the Cooper and Jacob distance-drawdown (1946) method to analyse aquifer-pumping tests would be beneficial to groundwater practitioners. This study uses numerical groundwater flow modelling to assess and evaluate the application of the Cooper and Jacob (1946) distance-drawdown method to analyse multi-well aquifer pumping tests in ideal homogenous porous and isotropic confined aquifers under unsteady-state flow. Three multi-well aquifer-pumping tests are simulated in ideal one-layer homogeneous and isotropic confined aquifer models using Processing MODFLOW and the data is analysed by the Cooper and Jacob (1946) distance-drawdown method. The results shows that the Cooper and Jacob (1946) distance-drawdown method can only estimate correct transmissivity and storativity parameters when it is applied to observation data after IRAF. The u values calculated for the analysis vary over a wider range and do not satisfy all the literature prescribed u values. The study demonstrates and recommends how IRAF should be used as an objective criterion to determine the validity of applying the Cooper and Jacob (1946) distance-drawdown method.

1. Introduction

Recent studies on the application of the Cooper and Jacob time-drawdown method (1946) by Gomo (2019) have demonstrated that there is a need to continue improving the understanding of the application of some of the conventional analytical methods used to analyse and interpret aquifer pumping tests data. While the Cooper and Jacob time-drawdown method (1946) had been in use for decades as a conventional method to analyse data in typical homogenous porous and isotropic confined under unsteady flow, Gomo (2019) have illustrated that only analysing the pumping-well data with the method will yield true formation transmissivity. Gomo (2019) further showed that the use of the Cooper and Jacob time-drawdown (1946) method to determine transmissivity and storage parameters from observation-well data cannot give the true formation parameters and groundwater practitioners need to understand the degree of variation between the true and incorrect estimates. These findings reiterated the need to continue asking research questions, even on what sometimes seems obvious or appears to be widely understood as part of the quest to improve the understanding of aquifer pumping test data analysis and provide scientific practical guidance to field hydrogeologists/geohydrologists.

In most groundwater science textbooks, aquifer pumping tests guides and manuals, the Cooper and Jacob time-drawdown (1946) method is often presented together with the Cooper and Jacob distance-drawdown (1946) method. The Cooper and Jacob distance-drawdown (1946) method is used to determine transmissivity (Equation (1), Table 1) and storativity (Equation (2), Table 1) in typical homogenous porous and isotropic confined aquifers under unsteady flow.

$$T = \frac{2.3Q}{4\pi\Delta s} \tag{Eq1}$$

$$S = \frac{2.25Tt}{r_0^2} \tag{Eq2}$$

$$u = \frac{r^2 S}{4Tt} \tag{Eq3}$$

For determining transmissivity and storativity from observation well data, the Cooper and Jacob time-drawdown and distance-drawdown

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Table 1
Summary of the Cooper and Jacob method (1946).

Parameter	Description	Units
T	Transmissivity of the aquifer	[m^2/day]
Q	Constant discharge of the pumping well	[m^3/day]
Δs	The gradient of a straight line fit on the semi-log plot of drawdown against observation distance between a 1 log cycle	[m]
S	Storativity of the aquifer	[ ]
r_0	The distance defined by the intercept of the straight-line fit of the data and zero-drawdown axis	[m]
r	Distance of the observation well from the pumping well	[m]
T	The time at which the set of drawdown data is taken	[days]

Source: Kruseman and de Ridder (1991).

(1946) methods are said to be applicable based on the u condition (Equation (3)). Conventional groundwater science textbooks give different threshold values for this u condition; Freeze and Cherrry (1979) – u $\leq$ 0.02; Kruseman and de Ridder (1991), and Schwartz and Zhang (2003) – u $\leq$ 0.01; Fetter (2001) and Sterrett (2007) – u $\leq$ 0.05 and Fitts (2012) – u $<$ 1. Gomo (2019) has demonstrated that the transmissivity and storage parameters estimated from observation well data using the Cooper and Jacob time-drawdown (1946) method, will always vary from the true formation parameters irrespective of the u value and that one need to understand the degree of deviation of estimates from the true parameters if the method is to be meaningfully used. The calculation of u (Equation (3)) incorporates transmissivity and storativity whose correctness needs to be evaluated on the same u value. If these two parameters are wrong, then their potential effects on the u value and general evaluation process are unknown.

In contrast, the Infinite Acting Radial Flow (IARF) condition (Spane, 1993; Renard et al., 2008) has been illustrated by (Gomo, 2019) to be a practically objective evaluation criterion for the application of the Cooper and Jacob time-drawdown (1946) method as compared to u. This study is extension of the previous work of Gomo (2019) aimed at improving the practical understanding of issues surrounding the practical application of Cooper and Jacob (1946) methods to estimate aquifer parameters from multi-well aquifer pumping tests. The Cooper and Jacob (1946) time-drawdown which Gomo (2019) has assessed uses time and drawdown data from pumping and observation wells while the Cooper and Jacob (1946) distance-drawdown approach uses time and observation well distance data. The two methods work on the basis of the same principles that incorporate the u value (Equation (3)) as the determining criteria for their validity, but in a different way. Recently, Gomo (2019) illustrated how IARF can be used as an objective criterion to determine the applicability of the Cooper and Jacob time-drawdown (1946). There is also research need to assess and illustrate the effects of using IARF as a criterion to determine the applicability of the Cooper and Jacob (1946) distance-drawdown method to analyse aquifer pumping tests data. The practical effects of the observation distance and therefore u on estimation of parameters using the Cooper and Jacob (1946) distance-drawdown method also need to be understood. Knowledge from this study provides additional practical guidance to groundwater practitioners on the use of the method.

The study uses numerical groundwater flow modelling to assess and evaluate the application of the Cooper and Jacob (1946) distance-drawdown method to analyse multi-well aquifer pumping tests in ideal homogenous porous and isotropic confined aquifers under unsteady-state flow. Many studies have successfully used numerical modelling as a tool for evaluating the application of analytical methods to interpret aquifer pumping test data (Meier et al., 1998; Sanchez-Vila et al., 1999; Osiensky et al., 2006; Halford et al., 2006; Calvache et al., 2016; Gomo, 2019).

2. Methods and materials

2.1. Model development

Three multi-well aquifer-pumping tests were simulated in one-layer homogeneous and isotropic confined aquifer models using Processing MODFLOW (Chiang and Kinzelbach, 2001). Groundwater flow numerical modelling is one of ways available to create an ideal homogenous and isotropic aquifer, which can be used for testing the application of analytical models. With the exception to the condition of aquifer infinite areal extent, other assumptions and conditions underlying the application of analytical methods used to evaluate pumping tests in confined aquifers for unsteady-state flow can be satisfied in a groundwater flow numerical model (Chiang and Kinzelbach, 2001; Gomo, 2019). Table 2 summaries the configurations and parameters of the three models.

The pumping and observation wells fully penetrate the aquifers to ensure that groundwater flow is horizontal towards the pumping well. Observation wells were placed at random within the aquifer and a pumping well is placed in the centre cell of the model. The cell size of 100 m was for the general model while the grids around the pumping wells were refined to 0.15 m cell size pumping well. This is done to improve simulation of drawdown around the pumping well. Fixed head boundary conditions are prescribed on all the model boundaries to ensure there was equal amount of inexhaustible water from each boundary. In order to approximate, the assumption of aquifer infinite areal extent, the fixed boundaries was placed as far as was possible from the wells of interest. In models 1 and 2, the fixed head boundaries are placed at distances of at 2500 m from the centre of the pumping well and 5000 m in model 3. A plan layout showing the model design is shown in Fig. 1.

Theis drawdown of 0.0033 m, 0.0031 m and 0.00031 m were predicted at the radial distance where the boundaries are placed in each of the three models for the expected duration pumping simulations. These Theis (1935) predicted drawdowns at the boundaries were regarded to be small and negligible to provide an approximation of an infinite areal extend aquifer, with a boundary at which the Theis drawdown becomes zero. Furthermore, the drawdown data did not show any influence of the fixed boundary, indicating that the fixed head hydraulic boundary did not influence the pumping tests.

2.2. Data interpretation

This study is an extension of the recent work of Gomo (2019), hence similar steps are used for data interpretation. However, the studies are different in that they investigate different methods. Gomo (2019) investigated the Cooper and Jacob (1946) time-drawdown method, while the current study is focused on the Cooper and Jacob (1946) distance-drawdown method.

The simulated aquifer-pumping test observation data was first

Table 2
Summary of model configuration and parameters.

Parameter	Model 1	Model 2	Model 3
Model dimensions (length × width) [m x m]	5000x5000	5000x5000	10000x10000
Aquifer thickness (b) [m]	15	18	20
Initial hydraulic head (h) [m]	26	30	35
Storativity (S) []	0.0003	0.0018	0.005
Transmissivity (T) [m^2/day]	14	100	520
Constant well discharge rate (Q) [m^3/day]	43.20	864.00	2591.99
Theis predicted drawdown at the fixed head boundary (s) [m]	0.0033	0.0031	0.00031
Distance from pumping well to fixed head boundary (R) [m]	2500	2500	5000
Duration of pumping (t) [days]	10.83	7.05	10.83
Number of observation wells []	36	20	29

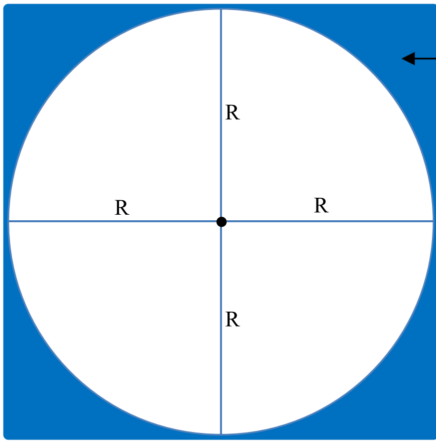

Fig. 1. Plan layout of the model illustrating the fixed head boundary design, where R is distance from pumping well to fixed head boundary.

interpreted using the Theis (1935) radial flow model (Equation (4), Table 3) for observation wells. This was done in order to assess the ability of the simulated models to satisfy the assumptions and conditions that underlie the use of analytical methods to evaluate pumping tests in porous, homogeneous and isotropic confined aquifers under unsteady-state flow conditions (Gomo, 2019). The models were deemed to satisfactorily approximate the Theis (1935) underlying assumptions when the estimated transmissivity and storativity were equal to the model prescribed parameters. For the Theis (1935) analysis, data from a closer well (within 50 m from the pumping well) and the furthest from the pumping well are presented in this paper for illustration purposes.

$$s = \frac{Q}{4\pi T} W(u) = \frac{Q}{4\pi T}\left(-0.5772 - In\ u + u - \frac{u^2}{2.2!} + \frac{u^3}{3.3!} - \ldots \right) \quad \text{(Eq4)}$$

Theis (1935) analysis was followed by interpreting the simulated drawdown from observation well using the Cooper and Jacob (1946) distance-drawdown method. The main goal was to assess the use of IRAF as a criterion to determine the applicability of the Cooper and Jacob (1946) distance-drawdown method as compared to the u condition. For this assessment, sets of data from two arbitrarily selected time steps was analysed with the Cooper and Jacob (1946) distance-drawdown method for each model. The analysis of data is done before and after IRAF is attaining. The IRAF was identified when the log drawdown derivative becomes constant as indicated by a slope of zero (horizontal line) on a log-log plot of derivative drawdown against time (See Fig. 2 for illustration). From the analysis, the use IRAF as a criterion to determine the applicability of the Cooper and Jacob (1946) distance-drawdown method was assessed.

The derivative of the drawdown was calculated using the algorithm developed by Bourdet et al. (1989) (Equation (5)) with the help of Excel Microsoft. The algorithm utilises a simple three-point formula to compute drawdown derivatives from drawdown data by numerical differentiation.

Table 3
Summary of Theis (1935) method.

Parameter	Parameter description	Units
s	Drawdown measured in a piezometer at a distance r	[m]
u	Dimensionless parameter (Equation (3))	[]
Q	Constant discharge of the pumping well	[m^3/day]
T	Transmissivity of the aquifer	[m^2/day]
S	Storativity of the aquifer	[]
t	Time since pumping started	[days]
$W(u)$	Well function $= -0.5772 - In\ u + u - \frac{u^2}{2.2!} + \frac{u^3}{3.3!} - \ldots$	[]

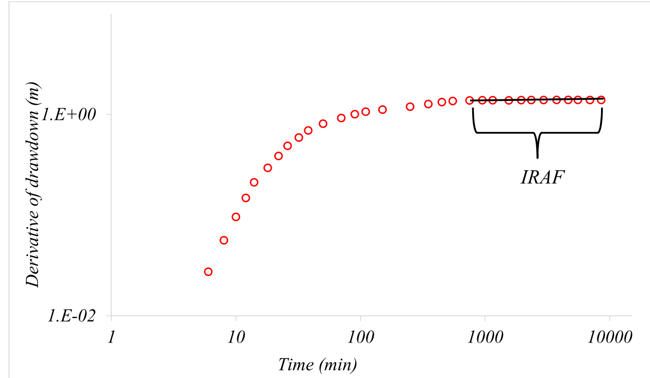

Fig. 2. Log-log graph of derivative of drawdown against time of pumping showing an example of IRAF.

$$\left(\frac{\partial}{\partial \ln T}\right)_i = \frac{\left(\delta_{s_{i-1}}/\delta \ln T_{i-1}\right)\delta \ln T_{i+1} + \left(\delta_{s_{i+1}}/\delta \ln T_{i+1}\right)\delta \ln T_{i-1}}{\delta \ln T_{i-1} + \delta \ln T_{i+1}} \quad \text{(Eq5)}$$

where: s is the drawdown (m) and T is an appropriate time function (e.g. elapsed time) (days).

3. Results

3.1. Theis analysis

3.1.1. Model 1

Graphs showing the log-log plot of drawdown (s) against time from Model 1 and the best fit of the Theis model for observation wells located at 25.35 m and 648.49 m from the pumping well are presented in Figs. 3 and 4 respectively. Storativity values of 2.5×10^{-4} and 2.8×10^{-4} are estimated for from these two observation wells respectively. Storativity estimates from all 30 observation wells ranges from 0.00025 to 0.00031 $\times 10^{-4}$ which corresponds to the prescribed storativity of 0.0003. A transmissivity value of 14 m^2/day is estimated from all the observation wells and is equal the one prescribed for the model input. Model 1 was judged to satisfy the underlying assumptions for the application of Theis (1935) and therefore the Cooper and Jacob (1946) methods to analyse aquifer pumping test data in homogenous and isotropic confined aquifers under unsteady-state flow.

3.1.2. Model 2

Figs. 5 and 6 shows the log-log plot of drawdown (s) against time from Model 2 and the best fit of the Theis model on data from

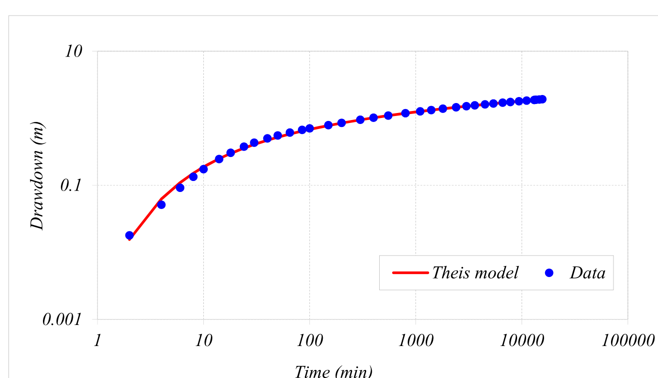

Fig. 3. Scatter graph showing log-log plot of drawdown (s) against time from Model 1 and the best fit of the Thesis model on data from an observation well located at 25.35 m from the pumping well.

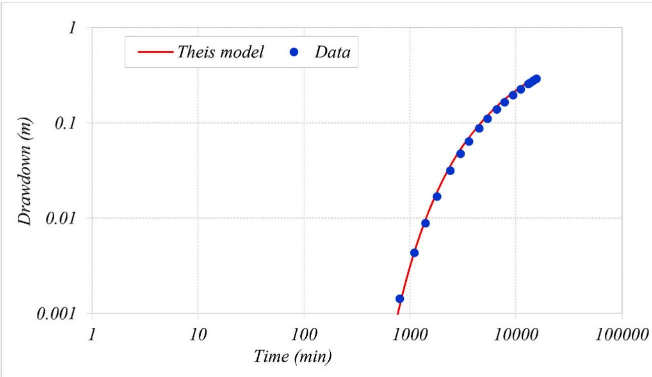

Fig. 4. Scatter graph showing log-log plot of drawdown (s) against time from Model 1 and the best fit of the Theis model on data from an observation well located at 648.49 m from the pumping well.

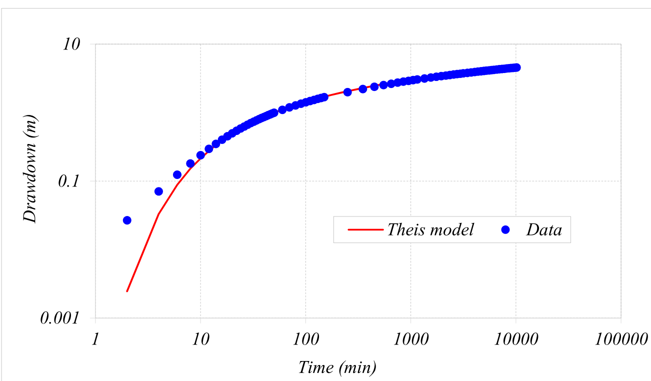

Fig. 5. Scatter graph showing log-log plot of drawdown (s) against time from Model 2 and the best fit of the Theis model on data from an observation well located at 40.42 m from the pumping well.

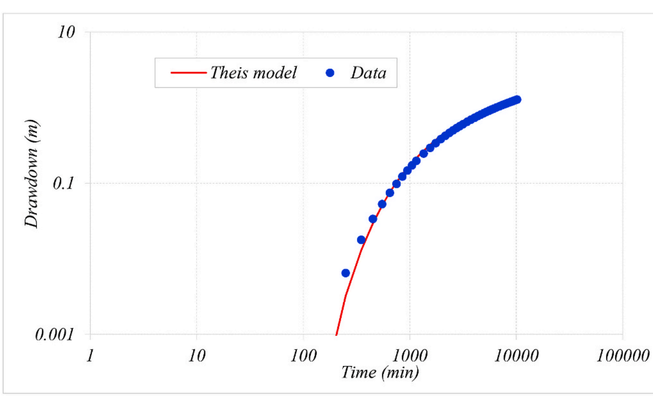

Fig. 6. Scatter graph showing log-log plot of drawdown (s) against time from Model 2 and the best fit of the Theis model on data from an observation well located at 420.83 m from the pumping well.

observation wells located at 40.42 m and 420.83 m from the pumping well. The Theis model matches to the data. Storativity and transmissivity value of 0.0015 and 100 m^2/day are respectively estimated for these two and the rest of the observation wells respectively. The estimates corresponds to the model prescribed input thus Model 2 was therefore judged to satisfy the underlying assumptions for the application of Theis (1935) and therefore the Cooper and Jacob (1946) methods to analyse aquifer pumping test data in homogenous and isotropic confined aquifers under unsteady-state flow.

3.1.3. Model 3

Graphs showing the log-log plot of drawdown (s) against time from Model 3 and the best fit of the Theis model for observation wells located at 38.45 m and 466.41 m from the pumping well are presented in Figs. 7 and 8 respectively. Storativity values of 0.005 and 0.0052 are estimated for from these two observation wells respectively and these correspond to model prescribed storativity of 0.005. A transmissivity value of 520 m^2/day is estimated from all the observation wells and is equal to the model prescribed model input.

Model 3, just like 1 and 2 was judged to satisfy the underlying assumptions for the application of Theis (1935) radial flow model and therefore the Cooper and Jacob (1946) methods to analyse aquifer pumping test data in homogenous and isotropic confined aquifers under unsteady-state flow.

3.2. Cooper and Jacob distance-drawdown method

3.2.1. Use of IRAF data

3.2.1.1. Model 1. Fig. 9 shows a semi-log scatter graph of drawdown (s) against observation wells distance from the pumping well simulated in Model 1 at 300 min time-step. The Cooper and Jacob (1946) distance-drawdown straight-line model is only matched to the data from observation wells experiencing IRAF. The observation wells with IRAF after 300 min of pumping are identified using the derivative of drawdown as explained under Section 2.2. Transmissivity and storativity estimates of 13.68 m^2/day and 0.00031 are obtained. The parameter estimates are equally in line with prescribed model parameters of 14 m^2/day and 0.0003.

The u values calculated for all observation wells ranges from 0.0018 to 0.11. From the different u values given in literature; (Freeze and Cherry (1979) – $u \leq 0.02$; Kruseman and de Ridder (1991), and Schwartz and Zhang (2003) – $u \leq 0.01$; Fetter (2001) and Sterrett (2007) – $u \leq 0.05$, this would imply that some of the wells would have been removed from the analysis. However according to the Fitts (2012) criteria of $u < 1$, all well the wells will be appropriate yet is clear in Fig. 9 that not all can be used in this case. If we assumed that criteria $u \leq 0.05$ is used, then four of the wells were not going to be included in the analysis. If these are the only wells available for observations, one can easily conclude that Cooper and Jacob distance-method is not applicable on them, which would be wrong. This potential pitfall can be avoided by using IRAF as the validating criteria.

After 13600 min, all the observation wells were experiencing IRAF and the entire data plot on a straight-line (Fig. 10). The data is analysed using the Cooper and Jacob (1946) distance-drawdown method to give transmissivity and storativity estimates of 13.89 m^2/day and 0.00032 and these are equal to the prescribed model parameters. Calculated u

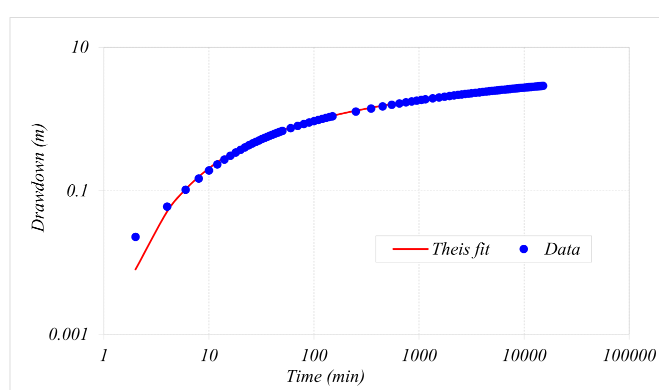

Fig. 7. Scatter graph showing log-log plot of drawdown (s) against time from Model 3 and the best fit of the Theis model on data from an observation well located at 38.45 m from the pumping well.

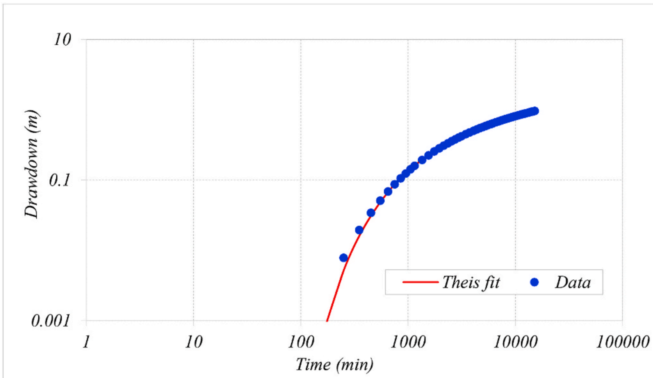

Fig. 8. Scatter graph showing log-log plot of drawdown (*s*) against time from Model 3 and the best fit of the Theis model on data from an observation well located at 466.41 m from the pumping well.

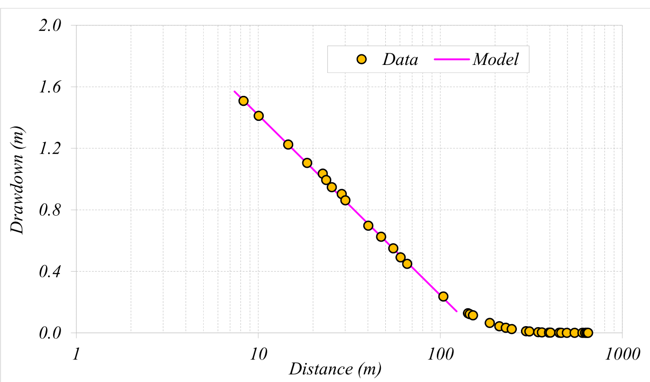

Fig. 9. Scatter graph showing a semi-log plot of drawdown (*s*) against observation distance from the pumping well simulated in Model 1 at 300 min time-step and the best fit line of the Cooper and Jacob distance-drawdown model on observation wells experiencing IRAF.

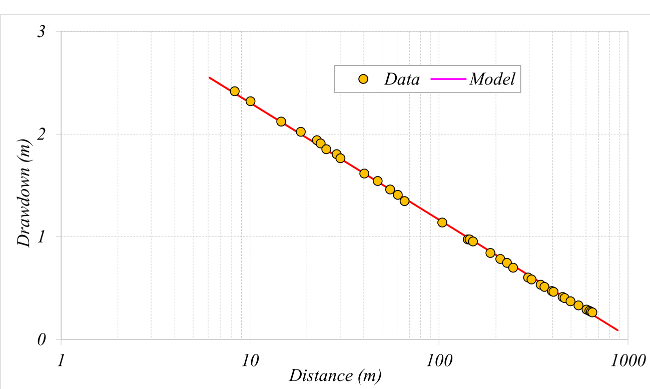

Fig. 10. Scatter graph showing a semi-log plot of drawdown (*s*) against observation distance from the pumping well simulated in Model 1 at 13600 min time-step and the best fit of the Cooper and Jacob distance-drawdown model on the data from observation well experiencing IRAF.

values for the data sets range from 0.00004 to 0.26 and these does not satisfy all the u values given in literature yet estimates the correct aquifer parameters.

3.2.1.2. Model 2. From model two, data from those observation wells experiencing IRAF after 850 min was used for analysis with the Cooper and Jacob (1946) distance-drawdown method and gave transmissivity and storativity estimates of 100.63 m^2/day and 0.0018 respectively (Fig. 11). After 4050 min, all the wells were now experiencing IRAF and as result all, the data points plot on a straight line (Fig. 12). Transmissivity and storativity of 100.43 m^2/day and 0.0019 are estimated using the Cooper and Jacob (1946) distance-drawdown method. Same as in model 1, the parameters estimated after IRAF correspond to model prescribed parameters.

3.2.1.3. Model 3. Fig. 13 shows a semi-log plot scatter graph of drawdown (*s*) against observation distance from the pumping well simulated in Model 3 taken at 1050 min time-step and the best-fit line of the Cooper and Jacob distance-drawdown model on the data from observation wells experiencing IRAF. Transmissivity and storativity values of 522.44 m^2/day and 0.0057 respectively are estimated. For these boreholes, the value of u ranges from 0.0056 to 0.33. If the u values are the basis for the analysis, the data from some of these boreholes would be invalid, yet with IRAF, they all estimated transmissivity and storativity correctly Fig. 14.

At the end of the simulated pumping test after 15150 min, the observation wells were all experiencing IRAF and the entire data plot on a straight-line. When the data is analysed using the Cooper and Jacob (1946) distance-drawdown method, transmissivity and storativity estimates of 521.78 m^2/day and 0.0061 and these correspond to the prescribed model parameters. The calculated u values for the data set range from 0.00041 to 0.060 and these does not satisfy all the u values given in literature. Although, the calculated u values do not satisfy all the literature prescribed u values, the data gave correct parameters on the basis of the IRAF criteria.

3.2.2. Use of drawdown data prior to IRAF

It is also important as part of the study to assess and illustrate the effects of using the Cooper and Jacob (1946) distance-drawdown method to analyse observation well data prior to IRAF. This is done by using wells whose data falls on a straight line but before IRAF. Since the trend is similar in all the models, only some of the data from model 1 and 3 is used for illustration purposes.

3.2.2.1. Model 1. Observation drawdown data from 6 wells at time step 300-min model is plotted against observation distance on a semi-log pot (Fig. 15). The data points on a semi-log plot of drawdown against time (Fig. 15) falls on a straight line. If this data sets is used to estimate parameters using the Cooper and Jacob (1946) distance-drawdown method based on a straight and late time criteria, incorrect estimate transmissivity (33.98 m^2/day) is obtained while storativity remains in the same order of magnitude (0.00024).

The transmissivity estimate of 33.98 m^2/day is more than double of

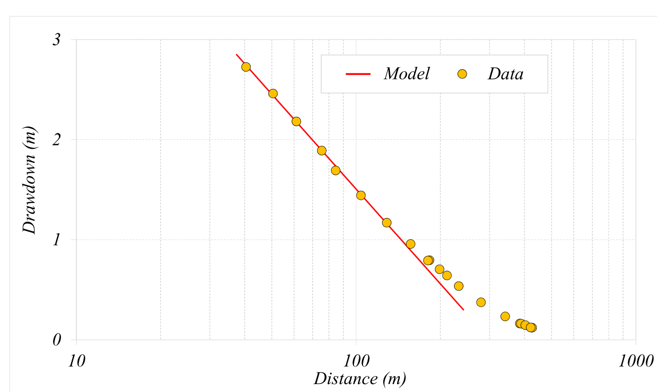

Fig. 11. Scatter graph showing a semi-log plot of drawdown (*s*) against observation distance from the pumping well simulated in Model 2 at 850 min time-step and the best fit of the Cooper and Jacob distance-drawdown model on the data from observation wells experiencing IRAF.

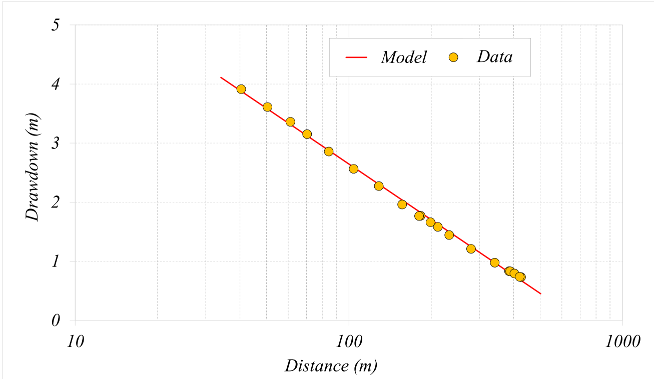

Fig. 12. Scatter graph showing a semi-log plot of drawdown (*s*) against observation distance from the pumping well simulated in Model 2 at 4050 min time-step and the best fit of the Cooper and Jacob distance-drawdown model on the data from observation wells experiencing IRAF.

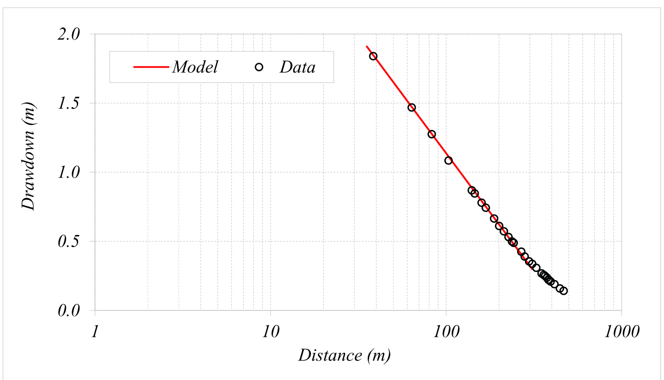

Fig. 13. Scatter graph showing a semi-log plot of drawdown (*s*) against observation distance from the pumping well simulated in Model 3 at 1050 min time-step and the best fit line of the Cooper and Jacob distance-drawdown model on the data from observation wells experiencing IRAF.

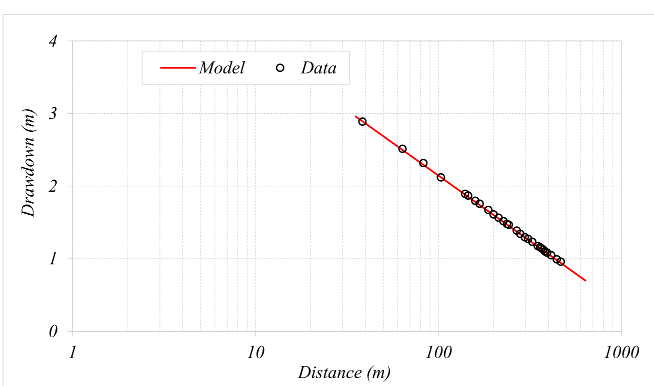

Fig. 14. Scatter graph showing a semi-log plot of drawdown (*s*) against observation distance from the pumping well simulated in Model 3 at 15150 min time-step and the best-fit line of the Cooper and Jacob distance-drawdown model on the data from observation all wells experiencing IRAF.

the one prescribed model 1 of 14 m^2/day while storativity estimate is less sensitive. The u values calculated for each observation well in Fig. 15 ranges from 0.17 to 0.44. The results these would be considered valid based on the Fitts (2012) criteria of u < 1.

The second example to illustrate the effects of using the Cooper and Jacob (1946) distance-drawdown method to analyse observation data before the IRAF is presented. Observation drawdown data from 9 wells at time step 3000-min in Model 1 is plotted against observation distance on a semi-log pot (Fig. 16) where the data falls on a straight line. Transmissivity and storativity of 21.43 m^2/day and 0.0003 are estimated. Transmissivity is larger than model prescribed parameters while the storativity is within the same order of magnitude of 10^{-4}. The u values were calculated for each observation well in Fig. 16, these range from 0.14 to 0.40 and would be only acceptable according to the criteria suggested by Fitts (2012) of u < 1 despite giving incorrect parameter estimates.

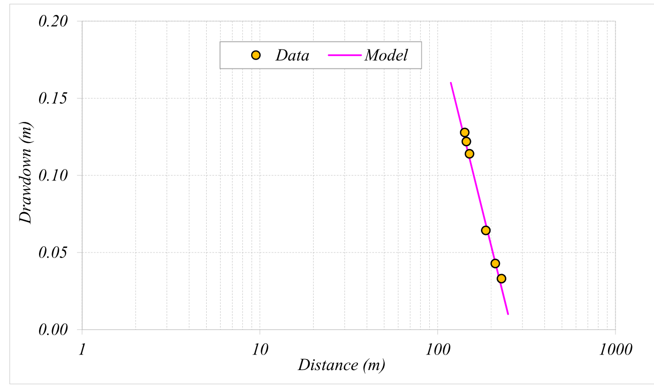

Fig. 15. Scatter graph a showing semi-log plot of drawdown (*s*) against observation distance from the pumping well at 300 min time-step simulated in Model 1 and the best-fit line of the Cooper and Jacob distance-drawdown model on six observation wells without IRAF.

3.2.2.2. Model 3. A third example to illustrate the effects of using the Cooper and Jacob method (1946) distance-drawdown method to analyse observation data before the IRAF is based on Model 3 simulations. Scatter graph showing a semi-log plot of drawdown (*s*) against observation distance from the pumping well at 100 min time-step simulated in Model 3 is presented in Fig. 17. The Cooper and Jacob (1946) distance-drawdown model is fit on the data from the first four observation wells that falls on a straight line. Although the data falls on a straight line, this is before IRAF, as a result incorrect transmissivity of 616 m^2/day is estimated while storativity of 0.0054 is very close to the model input. Prescribed transmissivity and storativity for Model 3 are 520 m^2/day and 0.005. The u values calculated for this analysis ranges from 0.05 to 0.34, and this will only satisfy some of the u values given in literature to validate the application of the Cooper and Jacob (1946) distance-drawdown model thereby making decisions difficult.

The three examples above can be used to emphasise that the

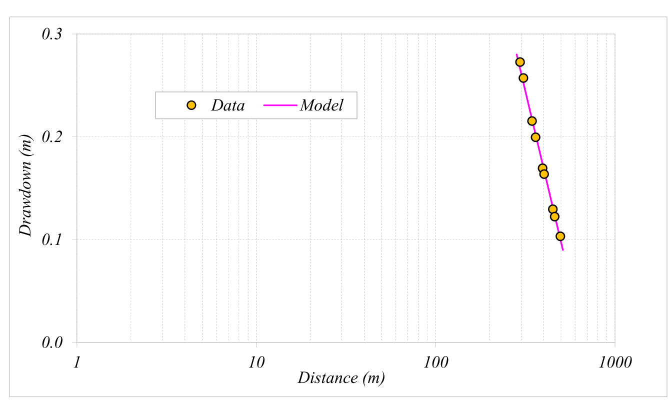

Fig. 16. Scatter graph a showing semi-log plot of drawdown (*s*) against observation distance from the pumping well at 3000 min time-step simulated in Model 1 and the best fit of the Cooper and Jacob distance-drawdown model on six observation wells without IRAF.

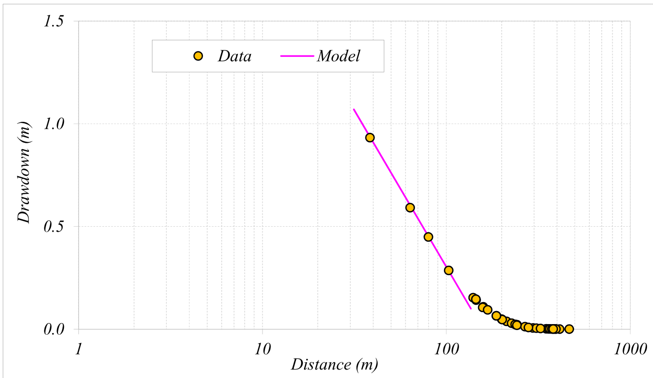

Fig. 17. Scatter graph a showing semi-log plot of drawdown (*s*) against observation distance from the pumping well at 100 min time-step simulated in Model 3 and the best-fit line of the Cooper and Jacob (1946) distance-drawdown model first six observation wells before IRAF.

occurrence of data points on a straight line does not always indicate IRAF and neither does it justify the application of the Cooper and Jacob (1946) methods.

4. Discussion

The results illustrates that when the Cooper and Jacob (1946) distance-drawdown method is used to analyse observation data from an aquifer pumping test after IRAF it gives correct estimates of transmissivity and storativity. The u values calculated for these analyses vary over a wider range and do cannot satisfy all the literature prescribed u values. The findings therefore suggest that IRAF is more objective criteria to determine the validity of also applying the Cooper and Jacob (1946) distance-drawdown method as compared to u which is clearly subjective and dependent on which literature source is being used. While the results presented above might seem so obvious, it is the opinion of the author that the absence of this information in literature reflects a world-wide limited understanding on the application of the Cooper and Jacob (1946) distance-drawdown method.

Besides the conventional groundwater science textbooks earlier highlighted, there are international examples of software technology applications in which the key information highlighted in this paper is not explained or mentioned. This helps to further illustrate that the phenomenon being discussed in this paper is not well understood by some groundwater practitioners. A few examples can be given. AquiferTest Pro 9.0 Help (Waterloo Hydrogeologic, 2018) is a very popular software technology for graphical analysis and reporting of aquifer pumping test data, but it does not prescribe the condition of IRAF as a requirement when analysing data in observation wells using the Cooper and Jacob (1946) method. For application of the Cooper-Jacob (1946) distance-drawdown method using AquiferTest Pro 9.0 Help, Waterloo Hydrogeologic (2018) indicates the data requirements as; drawdown vs time data at three or more observation wells, distance from the pumping well to the observation wells and constant pumping rate. However, this study illustrates that with the use of IRAF criteria; even the distance-drawdown data from two observation wells can be successfully used to estimate the aquifer transmissivity and storativity parameters. AQTESOLV by HydroSOLVE, Inc. (1998-2019) indicates the requirements for Cooper-Jacob (1946) distance-drawdown analysis as; a single drawdown measurement per each observation well recorded at the same time that is plotted on a distance-drawdown graph. These two popular software technology for graphical analysis and reporting of pumping does not include the IRAF flow condition, which this study has demonstrated to be an objective criteria. The failure to include the IRAF requirement is not by coincidence but rather a true reflection of the current international state of the art on the application of the method. It is therefore possible that groundwater practitioners could be incorrectly applying the method. Just like many groundwater science textbooks, these software technologies for graphical analysis and reporting of pumping tests also stipulates of u condition (values of u are small i.e., when r is small and t is large) as determining criterion. It is true that the smaller the values of u, the more likely applicable are the Cooper and Jacob (1946) methods. However, how small is small enough is relative. It is for this reason why values of u given in literature vary over a wider range (0.01–1). While the actual effect of these different u values has not been investigated, it is a fact that they will have some effects.

It must be emphasised again that practically, the u criterion is not meaningful (Gomo, 2019) because it is determined based on the estimated T and S parameters, but if these parameters are wrong then u will also be wrong (Gomo, 2019). This implies that, prescription of u criteria for the validation of the Cooper and Jacob (1946) methods is practically limited. The most likely reason why practitioners worldwide have continued to use this criterion is that up to this day, no studies have questioned it or attempted to demonstrate its limitations while suggesting a criterion that are more objective. The other aspect is that aquifer pumping tests guidelines and conventional textbooks seem to emphasise that for the application of this method is that the “data points will fall on straight line”. However, this study has illustrated that the data points can fall on a straight line but without attaining IRAF conditions, the use of such data will give incorrect parameters. At the same time, the falling of data points on the straight-line does not always imply the IRAF. The IRAF is evaluated by the analysing the derivatives of drawdown (Spane, 1993; Renard et al., 2008; Gomo, 2019).

While the application of both the Cooper and Jacob (1946) time-drawdown and distance-drawdown methods requires the attaining IRAF, some differences have to be highlighted. For the analysis of pumping well data, once the IRAF is attained, the Cooper and Jacob (1946) time-drawdown uniquely give the correct transmissivity (Halford et al., 2006; Gomo, 2019). However, it is not the case with observation well data. The estimated transmissivity increases exponentially with observation distance while storativity decrease but within the order of magnitude of the correct parameter irrespective of IRAF conditions (Gomo, 2019). For the Cooper and Jacob (1946) distance-drawdown, this study demonstrates that once the IRAF conditions occurs, applying the method gives correct transmissivity and storativity irrespective of observation distance from the pumping well.

The author also wishes to reiterate that the Cooper and Jacob (1946) methods are and will remain some of the popular conventional methods for interpreting aquifer-pumping test but it is the understanding of how they must be practically applied which needs improvement. In other words, the author is not claiming that there is something wrong with the Cooper and Jacob (1946) methods besides the known inherent limitation from its original development. It must be emphasised that this study and the previous one of Gomo (2019) are not attempting to down-grade the Cooper and Jacob (1946) methods but rather trying to upgrade them by improving the practical understanding of their application. This study has therefore provided additional knowledge by illustrating how the IRAF can be applied as criteria to determine the validity of the Cooper and Jacob (1946) distance-drawdown method’s application to analysing aquifer-pumping test data. The study further also illustrated the practical problems associated with the use of u condition to validate the application of the Cooper and Jacob (1946) distance-drawdown method. It is therefore the hope of the author that the findings of this paper will further improve the knowledge of the application of the Cooper and Jacob (1946) methods to optimise their use by groundwater practitioners in more objective and consistent ways.

5. Conclusions

The results illustrates that the Cooper and Jacob (1946) distance-drawdown method can only estimate correct transmissivity and storativity when it is applied to observation data after IRAF conditions have been attained. The u values calculated for these analyses vary over a wider range and in most cases does not satisfy the literature prescribed u criteria. These findings therefore reiterate that Infinite Radial Acting Flow (IRAF) is a more objective criterion to determine the validity of applying the Cooper and Jacob (1946) distance-drawdown method as compared u, which is clearly subjective and dependent on which literature source is being used.

When the groundwater flow field of the observation boreholes has attained the IRAF, the data points will all fall on a straight-line on a semi-log of drawdown against distance. However, when the data falls on a straight-line prior to IRAF the use Cooper and Jacob (1946) distance-drawdown method incorrectly estimates of transmissivity and storativity. The transmissivity is more sensitive to this effect than storativity, which remain within the same order of magnitude. Contrary to what has been presented in some of the literature, the falling of the data points on a straight line without attainment of IRAF does not therefore justify the application of Cooper and Jacob (1946) methods. It is recommended that the validity of applying the Cooper and Jacob (1946) distance-drawdown method to estimate transmissivity and storativity from multi-well aquifer pumping tests should be evaluated on the basis of IRAF conditions being attained in observation wells during a pumping test using the following steps:

- Determine the time when all the observation wells to be used for analysis have attained IRAF conditions,
- The IRAF is attained when the log drawdown derivative becomes constant as indicated by a slope of zero (horizontal line) on a log-log plot of derivative drawdown against time,
- To determine the time when all the observation wells to be used for analyses have attained IRAF conditions, use the furthest located observation well from the pumping borehole,
- For any time selected during the IRAF conditions take the drawdown from each observation well of interest and the corresponding observation distance to estimate the aquifer transmissivity and storativity using the Cooper and Jacob (1946) distance-drawdown method.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.gsd.2020.100478.

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