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Technical Note

# Step-drawdown test for identifying aquifer and well loss parameters in a partially penetrating well with irregular (non-linear increasing) pumping rates

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ABSTRACT

A step-drawdown test with an increasing pumping rate at each step in a fully penetrating well is a typical procedure for estimating aquifer parameters and well losses. However, partially penetrating wells in closed aquifers have also been adopted mainly due to economic constraints with a stepwise increase in the pumping rate. In this study, a new empirical method is proposed based on the stepwise drawdown defined by irregular pumping rates in a partially penetrating well. The method was validated by fitting drawdown data from classical works. The characteristics of the drawdowns and well losses were discussed for varying pumping scenarios. The results indicate that the new empirical method interprets previous works more accurately for a fully penetrating well and also effectively estimates the aquifer and well loss parameters. A new coefficient, the pumping rate-varying index $\alpha$, was introduced to indicate the pumping rate difference ($\Delta Q$) between the two steps. As such, a negative pumping rate difference would decrease the well-loss and result in a negative $\alpha$. In addition, the effect of the ratio of the well's screen length to aquifer thickness demonstrates that a longer screen would cause a larger well loss. Finally, the proposed empirical method was applied to a fieldwork conducted in Xiangyang city, central China, to investigate the aquifer and well loss parameters using the particle swarm optimization (PSO) method.

## 1. Introduction

The step-drawdown test is commonly designed to estimate aquifer parameters and well losses in a single well pumping test (Jacob, 1946; Clark, 1977; Kawecki 1995). A step-drawdown is generally defined as a constant-rate pumping test starting from an initial constant pumping rate while observing the water level that successively reaches a quasi-steady state. Afterward, the water level is allowed to decrease further due to a relatively high constant pumping rate for which the water level is monitored until a new quasi-steady state is achieved. By standards, the process is recommended to be repeated at least three times, which means that a step-drawdown test commonly contains a minimum of four steps (Mathias and Todman, 2010).

Jacob (1946) suggested that the aquifer loss is proportional to the pumping rate while the well-loss is assumed to be proportional to the square of the pumping rate. Subsequently, Rorabaugh (1953) proposed an extended equation where the well-loss is a power function of the pumping rate with order $P$. Detailed information about the classical models is presented in the **Appendix**.

Rorabaugh's (1953) model was later on widely recognized by researchers who made efforts to determine the range of $P$. For instance, Rorabaugh (1953) proposed that the value of $P$ varies between 2.43 and 2.82 in a high discharge and may be equal to unity in cases of low discharge. Lennox (1966) reported that the value of $P$ is as high as 3.5. Sheahan (1971) presented a set of type curves by simplifying Rorabaugh's (1953) solution and suggested $P$ ranges from 1.7 to 4.0. It is notable that most previous research mainly focused on the well-loss function and well loss power, but they neglected the aquifer properties and the form of step-drawdown test.

In the last few decades, several methods have been developed to

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estimate the parameters of aquifer loss and well loss by analyzing the step-drawdown tests based on the empirical equation. Since Jacob (1946) introduced the definition of well-loss, graphical methods were commonly used to estimate the well-loss coefficient (Rorabaugh, 1953; Sheahan, 1971; Birsoy and Summers, 1980). Thereafter, numerical methods and optimization methods were introduced into well loss estimation (Labadie and Helweg, 1975; Sheahan, 1975; Gupta, 1989; Avci, 1992). Singh (2002) presented a new optimization method to estimate well loss by analyzing the variable rate pumping instead of a step-drawdown test. Mathias and Todman (2010) also used the Forchheimer equation to analyze the step-drawdown tests and provided an approximate method to estimate well loss by considering $P = 2$. Karami and Younger (2002) presented a new transformation method to record the homogenization drawdown data in a confined heterogeneous aquifer during a step-drawdown test.

However, a careful review of relevant literature highlights that most of these methods are based on the following two assumptions: (1) the well is a fully penetrating well in a confined aquifer; (2) the pumping rate rises for each step. For assumption (1), researchers did have methods of analyzing well losses in a partially penetrating well but these came with limitations. For example, Wang and Zhan (2015, 2017) discussed the effect of intra-wellbore head loss in a partially penetrating well with different boundaries in a vertical well and a horizontal well by the integral method. This was done without applying a step-drawdown pumping test, which made the method hard to apply in fieldwork. Lyu et al. (2021) and Wu et al. (2021) calculated the groundwater head distribution by analyzing the effects of a close barrier which would significantly affect the head distribution during the pumping test. Besides, there are limited works that consider the well structure when exploring the well losses. Houben (2015) explained the mechanism of head loss in the well screen by establishing equations associated with the well screen diameter and flow velocity inside the well screen. Meanwhile, the influence of the well structure still has not been well elucidated, particularly for a partially penetrating well. Despite the different methods, the common thread is that the linear aquifer loss component is applied for Theis's (1935) equation which is well-known and established for a fully penetrating well. The differences between each method are commonly in the form of the well-loss coefficient or the exponent of the well discharge *P*. As a result, there is still a need for a more convenient method for well loss estimation for a partially penetrating well in fieldwork.

As for the second assumption (2), Clark (1977) made a step-drawdown test in a fully penetrating well with detailed data for six steps that offered experimental data to validate the other methods. After that, Gupta (1989) developed an interactive computer code to estimate aquifer characters based on the procedure of Clark (1977). Avci (1992) proposed a method considering the time-dependency of the aquifer loss coefficient by analyzing two data sets. Kawecki (1995) proposed a new method to calculate the total well loss by considering the discharge differential ($\Delta Q$) in two steps. As mentioned in assumption (2), the $\Delta Q$ must be a positive value, otherwise, the method cannot be applied.

Although the previous methods can solve most step-drawdown problems and get highly accurate parameters of the aquifer, there are still some real-world problems. For instance, the existing classical models cannot estimate the well-loss parameters in a partially penetrating well or if the pumping rate cannot be increased in each step.

In this study, we propose a new method based on previous empirical equations for estimating aquifer and well loss parameters by a step-drawdown test in a partially penetrating well with irregular (non-linear increased) pumping rate, which is an extension of the classical models with an increasing pumping rate step in a fully penetrating well. The proposed method can be simplified to Rorabaugh's (1953) method and it is validated by testing the results from Clack (1977). Finally, the proposed method is applied to the data from actual fieldwork in Xiangyang city, central China. The overall finding shows that the proposed method can provide new insights for the step-drawdown tests and extend the application of step-drawdown tests in the field.

## 2. Materials and methods

As shown in.Fig. 1, a partially penetrating well in a confined aquifer is considered. Several assumptions are adopted in this study: (1) the aquifer is homogeneous, radially isotropic, extends infinitely and is closed with a finite constant thickness; (2) the pumping rate changes instantaneously and keeps constant in each step; (3) the well partially penetrates the aquifer with an infinitesimal radius; (4) the system is hydrostatic before the pumping starts. Two main factors must be considered when solving the problem of parameter estimation by step-drawdown tests in a partially penetrating well with irregularly changing pumping rates. The first is the definition of the aquifer loss function *A* (Appendix, Eq.(A1)) since it is not a fully penetrating well in the main aquifer. This means that the problem does not fulfill the conditions of the Theis (1935) solution. The other issue is the components of the well-loss function *B* (Appendix, Eq.(A1)), which should reveal the irregular change in pumping rate and the relationship of well screen with the aquifer.

With these considerations, the governing equation for drawdowns in a fully penetrating well is similar to the previous work (Chang and Chen, 2003) except for the inner boundary condition, which turns the constant pumping rate *Q* to an irregular changing pumping rate with time $Q(t)$ (Wen et al. 2014). Due to the complexity of solving a three-dimensional flow problem around a partially penetrating well (compared to a radial flow in a fully penetrating well), most previous works turned the mathematical problems of a partially penetrating well into semi-analytical solutions by Fourier transform and Laplace transform or numerical solutions by finite element method and finite difference method (Chiu et al., 2010; Mishra et al., 2012; Wen et al., 2013). However, in this study, aquifer loss and well loss are considered simultaneously in a partially penetrating well, making it hard to derive a mathematical solution. Hence, an empirical equation like Jacob (1946) method is commonly used for aquifer and well loss estimation. The main innovation of this study is the determination of the aquifer loss function and well loss function in a partially penetrating well with irregular pumping rates. As a result, the aquifer loss function should conform to the fundamental seepage law and the classical models. Hantush (1961) proposed systematically the drawdown around a partially penetrating well in different aquifer situations which can be applied in this study for the aquifer head loss part (Eq.(A3)). 47According to Hantush (1961), the

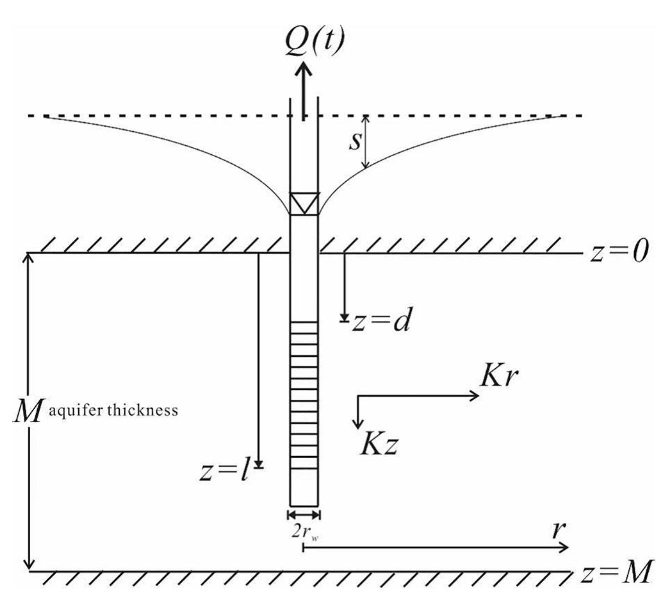


**Fig. 1.** Schematic diagram of the flow system.

aquifer loss function $A$ based on the settings of Fig. 1 could be expressed as:

$$A = \frac{1}{4\pi T}\left\{\begin{array}{c} W(u_r) + \dfrac{2M}{\pi(l-d)}\sum_{n=1}^{\infty}\dfrac{1}{n}\left[\sin\left(\dfrac{n\pi l}{M}\right) - \sin\left(\dfrac{n\pi d}{M}\right)\right] \\ \times\cos\left(\dfrac{n\pi z}{M}\right)W\left(u_r, \dfrac{n\pi r}{M}\sqrt{\dfrac{K_z}{K_r}}\right) \end{array}\right\} \quad (1)$$

Where $n$ is the finite cosine Fourier transform variable (Mishra et al., 2012), $W(u_r)$ and $W\left(u_r, \frac{n\pi r}{M}\sqrt{\frac{K_z}{K_r}}\right)$ are the well function and partially penetrating well function, respectively. There are two advantages to adopting Eq.(1) as aquifer loss function $A$. On one hand, $A$ is derived from a partially penetrating well, which can cover the fully penetrating well by adjusting the well length $l$ and $d$. On the other hand, since it is a step-drawdown test, a real space domain expression $A$ was easier to calculate by the superposition method in the postprocessing compared with a previous semi-analytical solution in Laplace domain (Avci, 1992; Avci et al., 2010; Jacob, 1946; Singh, 2002).

As for the well-loss, the commonly used processing method was to make the drawdown divided by a parametric constant such as pumping rate (Clark, 1977; Gupta, 1989) or time (Singh 2002), which aims to turn the problem into a problem with a lower order. These methods are then implemented by graphical techniques such as the linear graphic method (Eden and Hazel, 1973), which complicates the estimation process due to the difficulties in the characterization of a feasible linear relationship between pumping rate and time. Another way is to define a new expression for the well loss by introducing a coefficient with a certain physical significance (Mathias and Todman, 2010). For instance, Mathias and Todman (2010) proposed an approximate solution by introducing a variable involving Forchheimer parameter which indicated the flow seepage situation in a well. Similarly, this study considers a new form of temporal pumping rate, that is, a negative $\Delta Q$ value may exist during step-drawdown test which is different from the test with a continuously increasing pumping rate in each step. Nevertheless, a negative $\Delta Q$ would lead to a negative well loss value which conflicts with its physical meaning. As such, it is critical to propose an appropriate expression of $\Delta Q$ and quantify the influences of both $\Delta Q$ and $Q$ on the well-loss parameter. Thus, a new coefficient referred to as the pumping rate-varying index $\alpha$ is introduced to adjust the well-loss differences caused by the pumping rate between two steps. In order to keep consistent with the form of the pumping rate in the well-loss function, the power function with the exponent $\alpha$ is also used in the expression of $\Delta Q$. The $\alpha$ value is dependent on the $\Delta Q$. Typically, a positive $\Delta Q$ would increase the well-loss while a negative $\Delta Q$ would decrease the well-loss. This implies that the $\alpha$ would also be a negative value in some cases. Besides, the well-loss part should also contain the partially penetrating well factor. Hence, we also considered the influences of the well screen length on well-loss parameter. Overall, the expression of well-loss in each step should comprise the pumping rate differences, well screen length factor, and the pumping rate, which can be expressed as:

$$s_w = [(Q_i - Q_{i-1})\mathrm{sign}(Q_i - Q_{i-1})]^{\alpha} C\left(\frac{l-d}{M}\right)^{p} Q_i^{p} \quad (2)$$

where $s_w$ is the total well loss, $Q_i$ is the pumping rate during the $i$th step of the pumping test; $[(Q_i - Q_{i-1})\mathrm{sign}(Q_i - Q_{i-1})]^{\alpha}$ is the irregular pumping rate variation; $\mathrm{sign}(Q_i - Q_{i-1})$ is the sign function that keeps the well-loss as a positive value, which is particularly designed to solve a negative well loss problem. $C$ is the common well loss coefficient; $\left(\frac{l-d}{M}\right)^{p}$ indicating the influence of well screen length on well loss. Hence, this solution is different from previous ones as it includes a term for variable pumping rate loss and a term for partially penetrated screen loss, which have been neglected in most previous studies.

Therefore, the $i$th step drawdown that considers the well loss can be summed up as:

$$s = \frac{1}{4\pi T}\left\{\begin{array}{c} W(u_r) + \dfrac{2M}{\pi(l-d)}\sum_{n=1}^{\infty}\dfrac{1}{n}\left[\sin\left(\dfrac{n\pi l}{M}\right) - \sin\left(\dfrac{n\pi d}{M}\right)\right] \\ \times\cos\left(\dfrac{n\pi z}{M}\right)W\left(u_r, \dfrac{n\pi r}{M}\sqrt{\dfrac{K_z}{K_r}}\right) \end{array}\right\}Q_i(t) + [(Q_i - Q_{i-1})\mathrm{sign}(Q_i - Q_{i-1})]^{\alpha} C\left(\frac{l-d}{M}\right)^{p} Q_i(t)^{p} \quad (3)$$

Above all, Eq. (3) is a new empirical approach summarized and innovated from previous studies of step-drawdown pumping test models in a fully penetrating well. It extends the classical step-drawdown pumping test model, which accounts for a continuously rising pumping rate in a fully penetrating well, by considering an irregularly changed pumping rate in a partially penetrating well.

Finally, the Particle Swarm Optimization (PSO) algorithm (Chen et al. 2020; Wang et al. 2021) is employed to validate the model and estimate the aquifer and well loss parameters by comparing the objective function given as:

$$R = \min_{h}\sqrt{\frac{1}{N}\sum_{i=1}^{n}(s_i(h) - s_c)^2} \quad (4)$$

Where $R$ is the objective function which means a lower $R$ indicates a higher accuracy. $N$ is the number of the experimental data, $s_i$ is the observed drawdown, $s_c$ is the calculated drawdown. The advantages of the PSO method (Chen et al. 2020) are: (1) PSO is flexible for a multiple parameters estimation and allows for quick modification of the number of parameters by only changing the initial setting which would not expand the calculations; (2) PSO method is rapidly convergent because it evolves by comparing the local extremum value with the global extremum value which would reduce the iteration quantity. The basic hyperparameters setting of the PSO in this study are $N = 40, 60, 80$ (Number of particle swarm), $C_1 = C_2 = 2$ (learning factors), $w = 0.4$ (the inertia factor), $M = 50$ (the iterations), respectively.

## 3. Results and discussion

### 3.1. Model validation

Firstly, the field data from Clark (1977) was used to validate the model. However, as mentioned earlier, the proposed solution is designed for a partially penetrating well whereas the previous methods were applied to a case where the well was fully penetrating. For the case of a fully penetrating well, the aquifer part which employed the Theis (1935) model implies that the partially penetrating well term $(l\text{-}d)/M$ would be set as 1. The Eq.(3) can be transformed as:

$$s = \frac{1}{4\pi T}\{W(u_r)\}Q_i + [(Q_i - Q_{i-1})\mathrm{sign}(Q_i - Q_{i-1})]^{\alpha} C Q_i^{p} \quad (5)$$

The discharge rates from Clark (1977) were set as: $Q_1 = 1306$ m$^3$/day, $Q_2 = 1693$ m$^3$/day, $Q_3 = 2423$ m$^3$/day, $Q_4 = 3261$ m$^3$/day, $Q_5 = 4094$ m$^3$/day, $Q_6 = 5019$ m$^3$/day, respectively. The pumping rate differences between the two steps were $\Delta Q = 1306, 387, 730, 838, 833$ and 925 m$^3$/d for this case, respectively. Table 1 shows the results of aquifer

**Table 1**
The estimated results from the new solution compared with Jacob's method based on data from Clark (1977).

| Parameters | Eden and Hazel (1973) | Singh (2002) | This study |
|---|---|---|---|
| $T$(m$^2$/d) | 362 | 252 | 295 |
| $r_w^2S$(m$^2$) | $5.70 \times 10^{-4}$ | $1.39 \times 10^{-3}$ | $1.08 \times 10^{-4}$ |
| $P$ | 2.00 | 2.19 | 2.98 |
| $C$ | $6.40 \times 10^{-8}$ | 0.31 | $8.36 \times 10^{-2}$ |
| $R$ | 0.60 | 0.51 | 0.24 |

and well loss parameters estimated by different models.

For the proposed method in this study, the pumping rate-varying index $\alpha$ for each step was estimated as: $\alpha_1 = 1.68$, $\alpha_2 = 9.83$, $\alpha_3 = 1.59$, $\alpha_4 = 1.50$, $\alpha_5 = 0.88$, $\alpha_6 = 0.97$, respectively. Table 1 shows that the aquifer parameters $T$ and $S$ are in the same order of magnitude. However, for the well loss coefficient, the results vary because each model considered diverse influencing factors. Eden and Hazel (1973) used the linear graphic method by analyzing the relationship between the ratio of drawdown and flow rate with discharge rate. Singh (2002) calculated the well loss considering a variable pumping rate at each time step. This study takes the pumping rate differentials into account. Fig. 2 illustrates the drawdowns of each solution fitted with the experimental data of Clark (1977). It can be concluded that these three models all provided superb fitting curves. However, it is noteworthy that the results estimated by this study fitted the data better according to the objective function $R$ in Table 1, which highlights that the estimation accuracy of the aquifer and well loss parameters would be increased when considering the pumping rate differences. Fig. 2 also proves that this solution can be used for aquifer and well loss estimation in a fully penetrating well. Table 1 and Fig. 2 illustrate that this study can be compatible with the previous work and improve the accuracy of aquifer and well loss parameters. Furthermore, the proposed empirical approach could also be applied in common step-drawdown cases.

### 3.2. Analysis of each influence factor of the new method

The following subsections aim to discuss the effects of pumping rate-varying index $\alpha$, pumping rate difference ($\Delta Q$) and well screen length $(l\text{-}d)/M$ on drawdowns and well losses. Previous research assumed that the well-loss was mainly related to the pumping rate instead of the pumping rate difference ($\Delta Q$). What is different in this study is that we even account for the case of pumping rate with a non-linear temporal variation, and the partially penetrating well is also considered. According to Eq. (2), when analyzing the effect of one parameter, the other parameters are given constant.

#### 3.2.1. Pumping rate-varying index ($\alpha$)

When analyzing the effects of $\alpha$, the other parameters are given as: $K_x = 5$ m/d, $K_z = 0.5$ m/d, $S = 10^{-4}$, $P = 2$, $C = 10^{-6}$, $Q_n = 1000$, 1200, 1400,1600, 1800 $m^3$/d, respectively. Additionally, the aquifer thickness is set as 20 m and the well screen is assumed to be located in the middle of the aquifer with $(l\text{-}d)/M = 0.25$. The effects of the pumping rate-varying index $\alpha$ on the drawdown are shown in Fig. 3. It is notable that whatever the sign of $\alpha$ is, the well-loss would objectively exist.

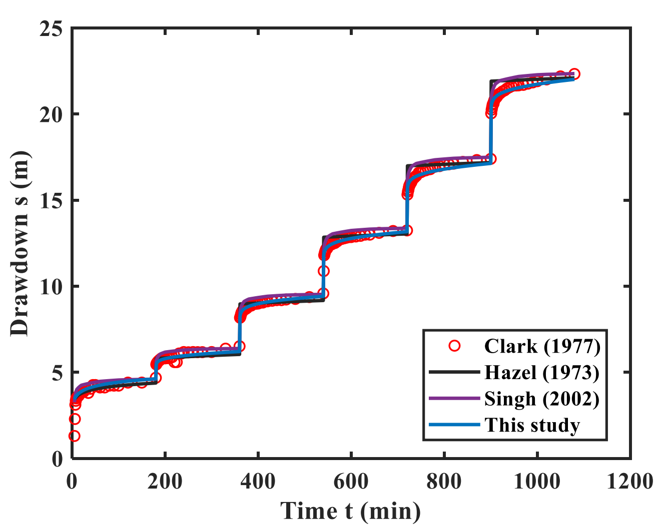


**Fig. 2. Model validation for a special case of fully penetrating well ($(l\text{-}d)/M = 1$) by using the data from Clark (1977).**

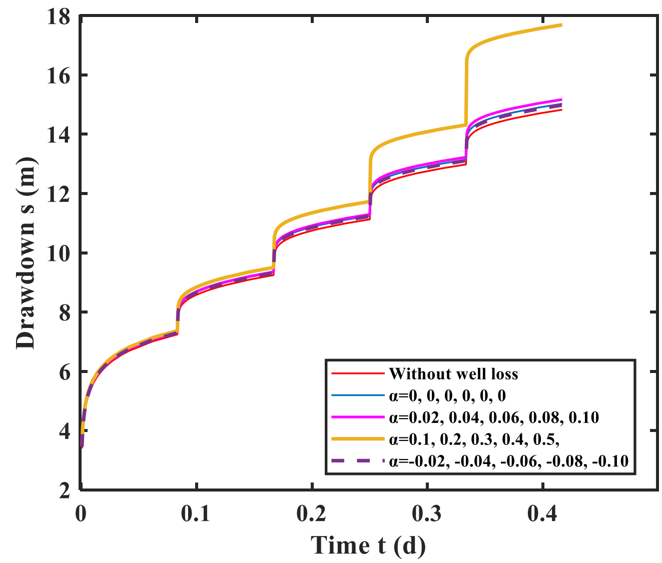


**Fig. 3.** Effect of pumping rate-varying index $\alpha$ with $Q = 1000$, 1200, 1400, 1600, 1800 $m^3$/d.

Consequently, a larger $\alpha$ causes a larger well loss with the same $\Delta Q$. In addition, the value of $\alpha$ was calculated from the pumping rate in each step, which enables the newly proposed model to characterize an irregularly changed step-drawdown test. With the consideration of a pumping rate-varying index $\alpha$, this new model can be used to solve the problem in An et al. (2020) that the well loss parameter could not be estimated with an exceptionally higher $\Delta Q$ or a negative $\Delta Q$.

In fact, it would be more precise to give a feasible range of $\alpha$ when it is introduced to improve the calculation accuracy of well loss caused by different $\Delta Q$, but there is no previous work considering or deriving the relationship between $\alpha$ and $\Delta Q$. Meanwhile, the current research indicates that the aquifer parameters are determined by the properties of the aquifer formation, while the well loss parameters are dependent on the specific operational parameters of the pumping test. Therefore, it is difficult to derive a constitutive relationship between aquifer and well loss parameters. Besides, it can be speculated that $\alpha$ in each step would not vary tremendously because the pumping rate has very limited variation in the field step-drawdown test due to pumping equipment.

#### 3.2.2. Pumping rate difference ($\Delta Q$)

Fig. 4 shows the well loss differences for increasing and decreasing pumping rate cases where $\alpha = 0.01$. The other parameters are the same to those used in Fig. 3 except for the pumping rates. As shown in Fig. 4 (a), the well loss can be represented by the deviation between the curves that consider and those that do not consider well loss, denoted in the legends of Fig. 4(a). It is notable that the well loss increases with the increase of $\Delta Q$, especially in the higher pumping rate stage since the Eq. (3) also indicates a direct relationship between the well loss and $\Delta Q$ with a positive $\alpha$. However, when $\Delta Q$ is negative as Fig. 4(b) indicates, drawdowns decrease mainly due to the decreasing pumping rate, while the well losses also decrease even for larger values of $\Delta Q$. In Fig. 4(c), the well losses clearly varied by plus or minus of $\Delta Q$, which is different from the continuously increasing pumping rate models. Hence, it can be concluded that the well loss is mainly influenced by the pumping rate. In addition, an introduction of the $\Delta Q$ will improve the estimation accuracy of the step-drawdown test with irregular pumping rates.

#### 3.2.3. Well screen length (l-d)

Fig. 5 illustrates the relationship between well losses and the length of the well screen. The parameters here are the same as those used in Fig. 4 except for $(l\text{-}d)/M = 0.25$, 0.5, 0.5, 1. It is important to note that regardless of the sign of $\Delta Q$, the well loss is inversely proportional to the

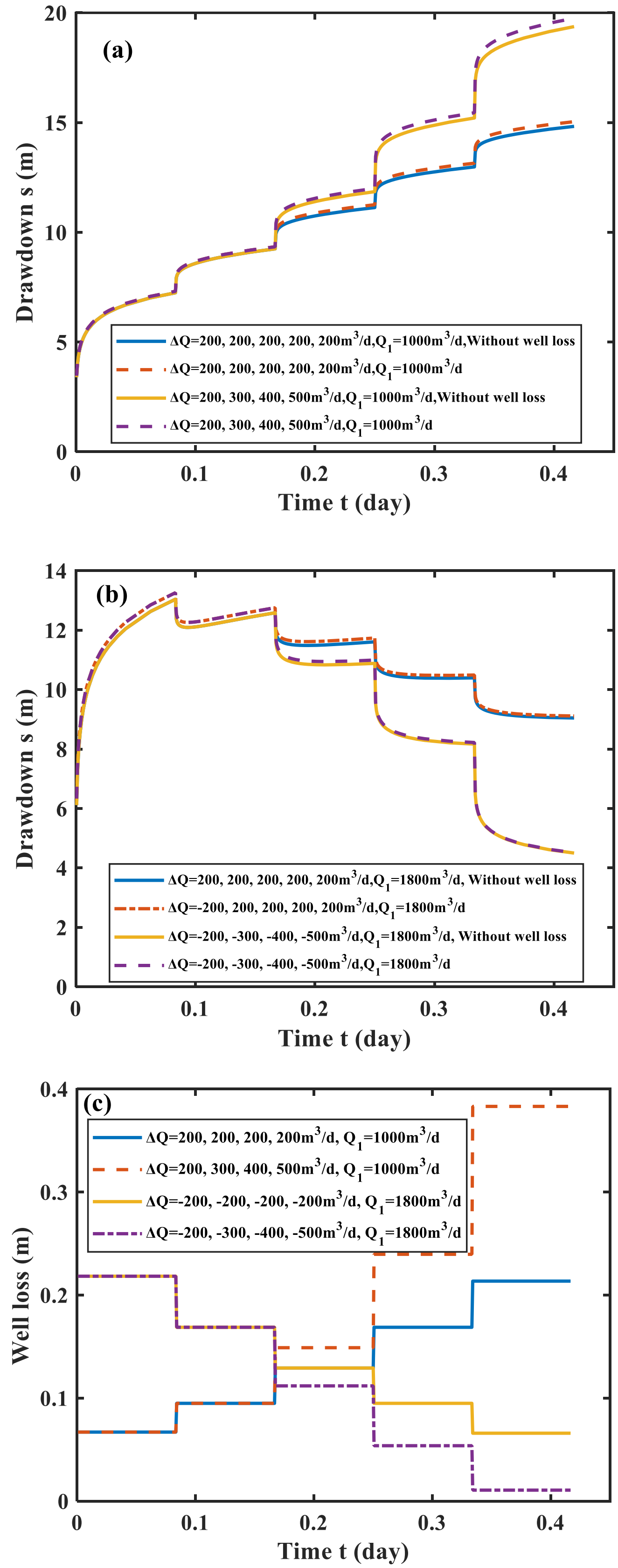


**Fig. 4.** Effect of pumping rate in different cases. (a) increasing pumping rates with different $\Delta Q$. (b) decreasing pumping rates with different $\Delta Q$. (c) Well loss values in increasing and decreasing pumping rate cases.

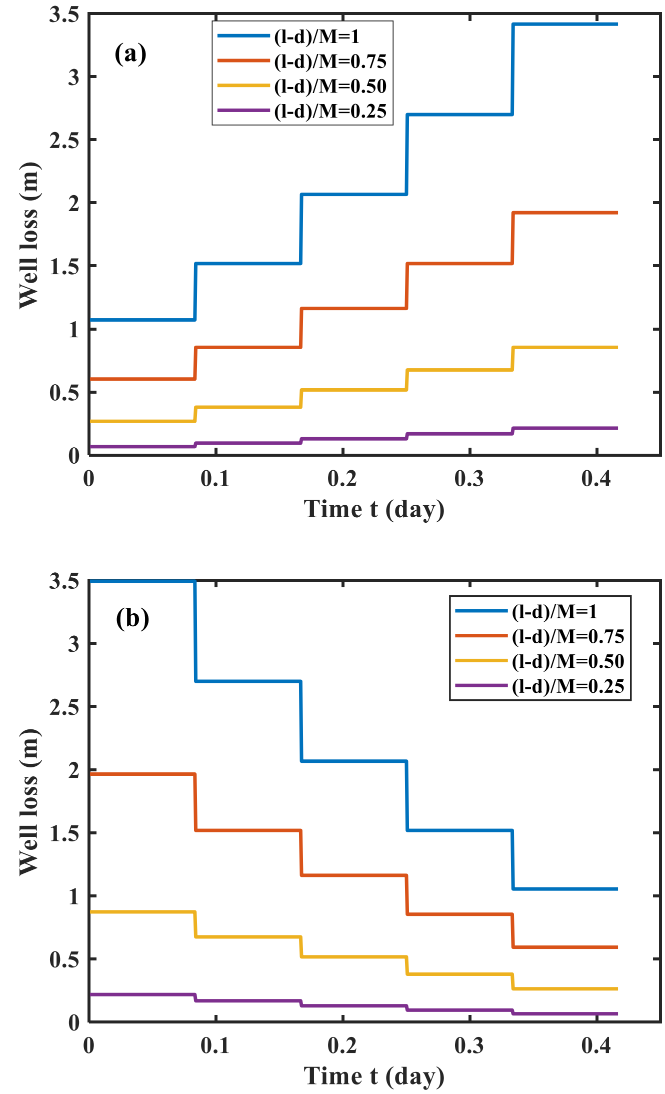


**Fig. 5.** Well loss with different partially penetrating terms. (a) $\Delta Q$ = 200, 200, 200, 200 m$^3$/d ($Q_1$ = 1000 m$^3$/d). (b) $\Delta Q$ = -200, −200, −200, −200 m$^3$/d ($Q_1$ = 1800 m$^3$/d).

ratio of the well screen and the aquifer thickness. In previous works, the well loss in the well screen was generally related to the flow length and flow velocity for the same material (Chen et al. 2003, Houben, 2015). A longer flow path and a faster velocity increase the well loss, but it is hard to evaluate which of these factors has a greater influence. Hence, in this study, we treat the ratio of well screen length and aquifer thickness as a whole to discuss the influence of the partially penetrating term on the well loss. Overall, the well loss only increased or decreased evidently for large pumping rate cases ($(l\text{-}d)/M$ = 0.75, 1) while the well loss varied inconspicuously for low pumping rate cases ($(l\text{-}d)/M$ = 0.25, 0.5). This suggests that for a water supply well in a high-yield aquifer area, a shorter well screen could be designed to satisfy the demand. If the water supply well is located in a poor-yield aquifer, a longer well screen is recommended to increase the water extraction capacity.

### 3.3. Application of this study to data from a field experiment

As mentioned previously, there is limited research on aquifer parameter estimation for partially penetrating wells based on a step-drawdown test. Generally, it is challenging to perform an increasing pumping rate test with at least four steps in the field, especially in the case of large pumping rates. Hence, we conducted an irregular pumping rates step-drawdown test with limited equipment in Xiangyang city, located in Central China. The proposed model was used to interpret the experimental data obtained from the field. The data was obtained from the pumping well, numbered XY001 located nearby the Hanjiang river. The geological formation is fluvial with lacustrine sedimentary conditions. According to Fig. 6, the confined aquifer is composed of coarse sand and gravel with upper and lower clay aquiclude which indicates that the aquifer has a large water yield property. Hence, it can be speculated that the water supply would satisfy a step-drawdown test with different pumping rates even in a partially penetrating well. The basic information about the test is as follows: the testing aquifer begins from a depth of 13 m to 87 m; the filter is between 60 and 72 m, making it a partially penetrating well; the well diameter is 0.3 m and the groundwater level was constant before the test. The step-drawdown test lasted for 10 h with 5 equal time interval steps, with the pumping rates reported as $Q_1 = 1803\ m^3/d$, $Q_2 = 1461\ m^3/d$, $Q_3 = 899\ m^3/d$, $Q_4 = 1380\ m^3/d$, $Q_5 = 1750\ m^3/d$, respectively.

Since it is a multi-parameters problem, we introduced the particle swarm optimization (PSO) algorithm to estimate the aquifer loss and well loss parameters. The objective function was set according to Eq.(4). The PSO method was designed in MATLAB version R2014a program, and it is freely available upon request from the authors. The population size $N$ was set as 40, 60 and 80 to avoid a local solution and to ensure accuracy. The results of each population size are shown in Table 2.

Fig. 7 illustrates the best-fitting curve of the proposed method. It is notable that the new empirical equation results in a good matching degree for irregularly changed pumping rates. According to the calculated parameters, the proportions of aquifer loss and well loss can be estimated as 85.06 % and 14.94 % (85.5 % and 14.5 % for $N = 40$; 84.3 % and 15.7 % for $N = 60$), respectively. The results indicate that the new form step-drawdown test could also estimate the aquifer loss and well loss parameters accurately. It is an extension of the classic step-drawdown test, which is generally limited to four increasing pumping rates and the new form test would reduce the equipment requirements for field pumping tests. This method would give new insights into the step-drawdown pumping test design in the later fieldwork.

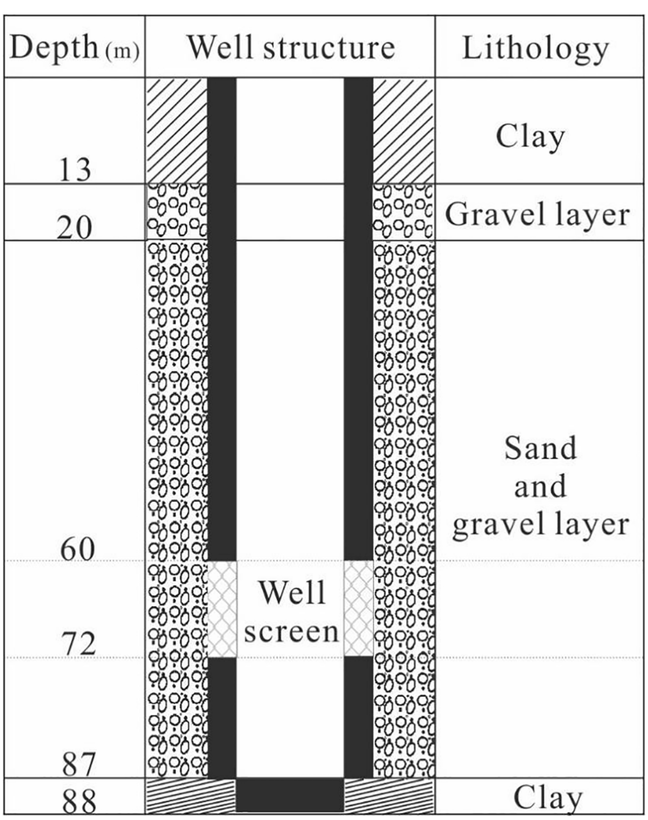


**Fig. 6.** The diagram of the experiment well XY001 in Xiangyang city.

**Table 2**
The best parameters of this study estimated by PSO method.

| Parameters | $N = 40$ | $N = 60$ | $N = 80$ |
|---|---|---|---|
| $R$ | 0.13 | 0.14 | 0.12 |
| $K_x$ (m/d) | 41.97 | 45.27 | 40.09 |
| $K_z$ (m/d) | 2.98 | 1.71 | 2.78 |
| $S$ | 0.0094 | 0.0003 | 0.009 |
| $P$ | 2.00 | 2.49 | 3.12 |
| $C$ | 0.076 | 0.071 | 0.0012 |
| $\alpha_1$ | −1.18 | −1.54 | −1.47 |
| $\alpha_2$ | −2.47 | −2.24 | −2.17 |
| $\alpha_3$ | −2.42 | −2.69 | −4.80 |
| $\alpha_4$ | −2.33 | −3.53 | −2.28 |
| $\alpha_5$ | −1.50 | −1.96 | −1.88 |

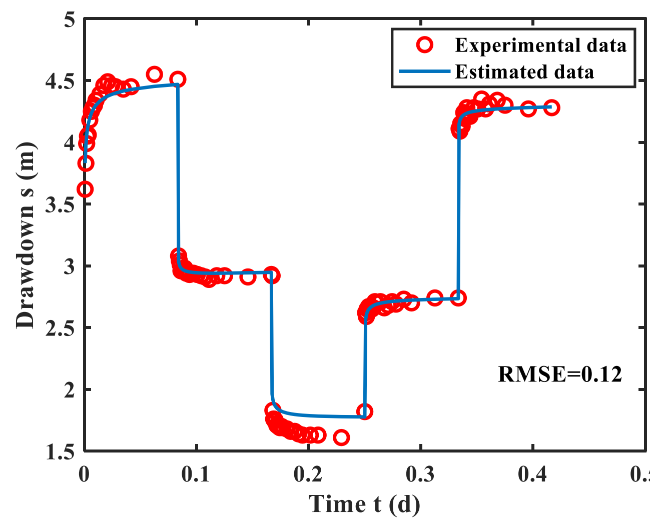


**Fig. 7.** Comparison of observed and estimated drawdowns of well in Xiangyang city.

## 4. Limitations

Theoretically, a clear quantitative relationship between $\alpha$ and $\Delta Q$ should have been provided in this study. However, the analyses in the above sections have shown that $\alpha$ changes synergistically with the value of $\Delta Q$. Similarly, the same absolute value of $\Delta Q$ does not indicate the same $\alpha$e because $\pm\ \Delta Q$ would lead to a different pumping rate $Q$. As such, the range of $\alpha$ also remains a subject for further research. It is noteworthy that the evaluation with respect to the $\alpha$ is similar to the approach by Mathias et al. (2008) (Eq.3) and Wen et al. (2017) (Eq.4), where the parameters related to the pumping rate (such as the $\alpha$ in this study) are objective but their optimal ranges were not presented. Nevertheless, the $\alpha$ outcomes from the field pumping test in this study are dominant at certain ranges between −10 and 10 based on the iterative and recursive technique of the PSO employed in the step-drawdown test with irregular pumping rates. Overall, we posit that the range of $\alpha$ would vary for other contexts.

Accordingly, further research based on actual field tests should be conducted to evaluate the quantitative relationship between $\alpha$ and $\Delta Q$ as well as the optimal range of $\alpha$. In addition, although the PSO method shows good convergence on the estimated aquifer parameters, it may have some uncertainty for the results. A Monte Carlo simulation (Metropolis and Ulam, 1949; Zhang, et al., 2017) is recommended to analyze the uncertainty of the estimation results, which will be done in the future and reported elsewhere.

## 5. Conclusions

This study proposed a new step-drawdown test model based on an irregularly variable pumping rate with a continuously rising pumping rate, which extends the concept of the traditional step-drawdown test. The new model defines the well-loss estimation for a partially penetrating well by considering an additional term, the well screen length which significantly increases the applicability of the model in fieldwork. As such, the new model is more general compared to previous works. The model was validated by applying it to interpret pumping test data obtained from the field and comparing the interpretive results with two other models (Eden and Hazel, 1973; Singh, 2002). The results show that the new model performs and produces smaller modeled data. Finally, the method was applied to estimate the aquifer loss and well loss parameters in an actual field test. The major conclusions from this study are presented as follows:

(1) The proposed method extends the concept of the increasing pumping test to that of a step-drawdown test with an irregular variable rate. Also, consideration for $\Delta Q$ by introducing a pumping-rate varying index ($\alpha$) increases the accuracy of the aquifer parameter estimation compared to methods from previous studies (Eden and Hazel, 1973; Singh, 2002).

(2) Both positive and negative $\Delta Q$ values influence the estimation of well-loss. For example, a positive $\Delta Q$ causes more distinct changes than negative $\Delta Q$ case, particularly where the pumping rate is high. Meanwhile, it is noteworthy that an increasing pumping rate creates a positive $\alpha$ while the $\alpha$ value would tend to be negative when there is a negative $\Delta Q$ during the pumping period.

(3) The well-loss value is proportional to the length of the well screen in a partially penetrating well, indicating that a larger ratio of the well screen length to aquifer thickness results in a larger well loss.

(4) Through the fitting between the new solution and the observed experimental data in Xiangyang city, the best parameters obtained are as follows: $K_x = 40.09$ m/d, $K_z = 2.78$ m/d, $S = 9.0 \times 10^{-3}$, $P = 3.12$, $C = 1.2 \times 10^{-3}$, $\alpha$ = -1.47, -2.17, -4.80, -2.18, -1.88. The drawdown is composed of 85 % of aquifer loss and 15 % well loss.

The findings of this work provide insight into the form of step-drawdown test with non-linear increasing pumping rates, which would reduce the requirements of pumping power. In addition, the study elucidates the significance of the well structure, suggesting that a partially penetrating well could also result in a high well efficiency compared to a fully penetrating well. This also implies that the well screen length can be optimized to be half or a third of the aquifer thickness to save cost, especially for a centralized water supply well construction.

However, it is important to highlight that there is room for more improvement and research on the new model. For example, the optimal range of the $\alpha$ is not elucidated in this work and the applicability of this method in a poor-yielding aquifer is untested, which may be the subject of future research.

### CRediT authorship contribution statement

**Chen Chen:** Conceptualization, Methodology, Writing – original draft. **Quanyu Tao:** Software, Validation. **Zhang Wen:** Supervision, Writing – review & editing. **Anders Wörman:** Supervision. **Hamza Jakada:** .

### Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

### Data availability

Data will be made available on request.

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## Appendix

The empirical equation proposed by Jacob (1946) is expressed as:

$$s_w = AQ_i + BQ_i^2 \tag{A1}$$

Where $s_w$ is the drawdown in the pumping well [L], $A$ [$L^{-2}$ T] and $B$ [$L^{-5}T^2$] are represented the formation loss and well loss coefficients, respectively, $Q_i$ is the pumping rate of $i$th step [$L^3T^{-1}$]. Jacob (1946) suggested that the first term on the right side of the equation is due to the aquifer head loss and the second term is due to the well loss. After that, Rorabaugh (1953) extend Jacob (1946) equation to:

$$s_w = AQ_i + BQ_i^P \tag{A2}$$

Where $P$ is the well loss power, an empirical exponent, and accordingly, $B$ now has the dimension of [$L^{1-3P}T^P$].

The general solution of Hantush (1967) is given as:

$$s(r,z,t) = \frac{Q}{4\pi KM}\left[W(u) + f\left(u, \frac{r}{M}, \frac{l}{M}, \frac{d}{M}, \frac{z}{M}\right)\right] \tag{A3}$$

Where $s(r, z, t)$ is the drawdown [L], $Q$ is the pumping rate [$L^3$/T]; $r$ and $z$ are the radial coordinate and vertical coordinate, respectively. $K$ is the radial hydraulic conductivity [L/T], $M$ is the thickness of the aquifer [L], $l$ is the distance from the top of the aquifer to the bottom of the well screen [L], $d$ is the distance from the top of the aquifer to the top of the well screen [L] (Fig. 1). $f$ is the added resistance coefficient decided by the features of the partially penetrating well which means $f$ would be varied in different well geometries.

The expression of each function in Eq.(1) can be given by:

$$W(u_r) = \int_{u_r}^{\infty} \frac{\exp(-y)}{y} dy \tag{A4}$$

$$W\left(u_r, \frac{n\pi r}{M}\sqrt{\frac{K_z}{K_r}}\right) = \int_{u_r}^{\infty} \frac{1}{y} \exp\left( - y - \frac{K_z}{K_r} \frac{(n\pi r/M)^2}{4y}\right) dy \tag{A5}$$

$$u_r = \frac{r^2 M S_s}{4Tt} \tag{A6}$$

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