

Pulse Testing:

Method: Rapidly changes the water level in the well and monitors the pressure response (pulse) in the aquifer over time.
Analysis: Measures only the amplitude and time lag of the pressure pulse, focusing on transmissivity and wellbore storage.

Difference between pumping test and interference test:

Essentially, a pumping test focuses on the drawdown in the pumped well and nearby observation wells, while an interference test focuses on the pressure response in a specifically placed observation well due to pumping in another well.

Interference Testing:

Method: Continuously pumps water from one well (pumping well) and monitors the pressure changes in both the pumping well and an observation well located at a distance.

Analysis: Utilizes the complete pressure response curve over time to estimate various aquifer properties like transmissivity, storativity, and boundaries.

Pulse-Testing: A New Method for Describing Reservoir Flow Properties Between Wells

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HOUSTON, TEX.**ABSTRACT**

A new method of reservoir evaluation called pulse-testing has been developed for describing formation properties between wells. Pulse-testing utilizes a sensitive differential-pressure gauge at a responding well to measure and record the response generated by a series of flow rate changes (pulses) at an adjacent or pulsing well. Since the pulse-test instruments have a sensitivity of about 0.001 psi, pulses of several hours or less in duration will generate a measurable response in most reservoirs. For this reason, many well pairs can be tested in a short period of time with little interference in field operations.

Comparison of pulse-test results to conventional testing methods shows that the pulses obey unsteady-state, compressible-flow theory and thus provide a measure of both transmissibility (kh/μ) and storage (ϕch). In addition, the method can be used qualitatively to describe communication across faults and between zones, and direction and magnitude of fracture trends.

INTRODUCTION

To obtain maximum usefulness of new reservoir analysis methods as well as to aid the field engineer in understanding local reservoir anomalies, improved methods are needed to describe heterogeneities in specific reservoirs. Optimum well spacing, equipment specifications, well completion design and especially the economics of pressure maintenance and secondary recovery programs depend on the extent and location of the heterogeneities that affect flow behavior in the field.

One method for obtaining a measure of areal reservoir heterogeneity is an interference test. In the simplest interference test, constant-rate production or injection is initiated at one well and the effect of this flow is measured as pressure vs time at another well. Thus, during interference tests the flow characteristics of the formation are determined in situ.

Despite their obvious usefulness in reservoir delineation,

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interference tests have not enjoyed frequent usage. Reasons for this have been the weeks or months required in many fields to obtain a pressure response measurable with conventional gauges, and the interruption of routine field operation during the field-wide shut-in normally required during these long times. Also, only average properties of the relatively large volume of the reservoir disturbed during these long times can normally be obtained—variations in properties between individual wells cannot be delineated. To eliminate these drawbacks of conventional tests, a special reservoir description technique called pulse testing¹ has been developed.

Pulse-testing has several advantages that make it more convenient and practical for field use than interference tests. Valid tests are more certain because use of a series of flow disturbances gives rise to a diagnostic pressure response that can more readily be distinguished from unknown trends in reservoir pressure and other "noise." The time required is only a fraction of that for an interference test because a special, high-sensitivity differential pressure gauge can detect a much smaller pressure change. Variations in properties between individual wells can be delineated because of the relatively small volume of the reservoir disturbed during these short times. Finally, wells other than the pulsing and responding wells do not have to be shut in or carefully regulated.

¹References given at end of paper.

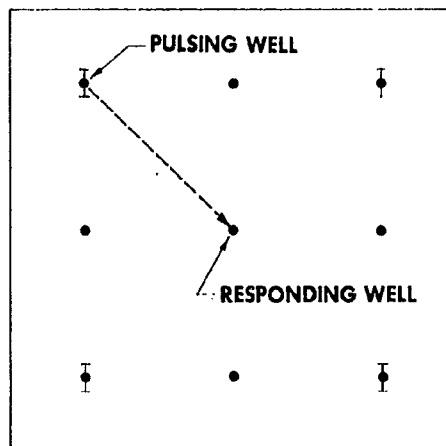


FIG. 1—PULSING AND RESPONDING WELL PAIR.

1. Time Lag (t_L): It measures the delay between a flow pulse (a rapid change in flow) and the peak response in a system's pressure.

Tangent Lines: Straight lines drawn to approximate the initial and final slopes of the response curve.

PULSE-TESTING PROCEDURE

The basic element in pulse-testing is the *well pair* consisting of a pulsing well and an adjacent responding well (Fig. 1). At the pulsing well, a series of flow disturbances is generated by alternate intervals of flow and shut-in.* This well can be a flowing or pumping producer or an injection well. Sometimes, auxiliary pumping equipment is required to generate a sufficient pulse. Measured characteristics of this flow disturbance are the pulse rate, pulse interval and between-pulse interval (Fig. 2) for alternate intervals of flow and shut-in.

Consider the results of a sequence of flow-rate changes at a pulsing well. A corresponding series of pressure transients propagates through the reservoir and arrives at an adjacent communicating well with greatly diminished amplitude after a certain elapsed time. If this responding well has some wellhead pressure, the slight pressure changes in the reservoir are transmitted to the wellhead. Here a sensitive differential pressure gauge records the pressure responses which contain intelligence introduced into the reservoir with the series of flow disturbances. Correspondence of input and output intelligence (Fig. 2) insures certainty in the detection of small pressure changes in the presence of unknown trends in reservoir pressure and other noise. From the record of pressure response, the response amplitude and the time lag (Fig. 2) are determined.

An important feature of pulse testing is the use of an extremely sensitive instrument for measuring differential pressures. One such instrument* has been tested under a variety of conditions in fields in Oklahoma, Kansas, Louisiana, Texas, Illinois, Canada, France and Peru. Because of its high sensitivity (about 0.001 psi) pressure response can be detected much more quickly than with conventional gauges.

PULSE-TEST DATA ANALYSIS

The pressure transients that are detected by the pulse-test apparatus installed at the responding well travel through the reservoir as a true diffusion response using only the fluid as a transmitting medium. For this reason, established equations for unsteady-state flow through porous media can be applied to pulse-testing.

*This paper is limited to the sequence caused by alternate flow and shut-in. In practice, any sequence of rate changes can be used.

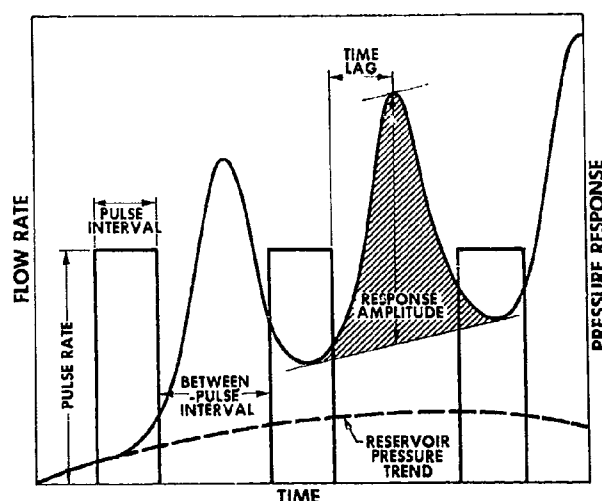


FIG. 2—PULSE-TEST TERMINOLOGY.

One unsteady-state flow model — the line source for an infinite, homogeneous, single-phase, slightly compressible reservoir — has been used widely to describe and interpret build-up, fall-off and interference tests. The pressure response of this model as given by the exponential integral (Ei) solution can also be used to describe pulse-test pressure response.** For a series of injection pulses (Fig. 2) the total pressure response is given by**

$$p = p_o + \frac{70.6B}{T} \sum_{j=1}^n (q_j - q_{j-1}) Ei \left(\frac{-56,900 Sr^2}{T(t - \tau_j)} \right) \quad (1)$$

where n = number of production rate changes prior to time, t

p_o = initial pressure, psi

q = production rate, STB/D ($q_o = 0$)

τ_j = time of initiation of each rate change, min.

Pulse-test data could be analyzed by several techniques involving Eq. 1; for instance, the data could be curve-fitted by the least-squares method. Another method called the tangent method (Fig. 2) lends itself to routine analysis in addition to acting as a simple linear filter to remove the linear components of reservoir pressure trends. This technique is based upon the fact that the two rock-fluid parameters in Eq. 1 (transmissibility T and storage S) can be calculated if the rate and timing of the flow disturbances are known and if at least two independent characteristics of the pressure response curve are measured. The time lag and response amplitude have been chosen as these two independent characteristics.

The time lag t_L for multiple pulses such as those in Fig. 2 is defined as $t_L = t(\Delta p) - \tau_k$ with $t(\Delta p)$ being the time at which the pressure increment between the response curve and the tangent line is a maximum and τ_k being the time at which the corresponding flow pulse ends. An equivalent definition results from the fact that $t(\Delta p)$ is the time where the time derivative of the pressure-response curve is equal to the slope of the lower tangent line (Fig. 2). The dimensionless time lag ($t_{DL} = t_L / \Delta t$) of the first pulse of a multiple-pulse series may be related to reservoir properties by Eq. 2:

$$t_{DL} \approx 20,000 \frac{Sr^2}{T\Delta t} \quad (2)$$

with t_L and Δt being the time lag and the pulse length, respectively, in minutes. This approximation is useful for estimating times required for pulse-testing in a particular reservoir. Its development and range of validity are given in the Appendix.

The response amplitude of a multiple pulse Δp_m is defined as the maximum pressure increment between the tangent and the pressure-response curve (Fig. 2). The response amplitude, like the pressure response given by Eq. 1, is implicitly dependent upon well spacing, transmissibility, storage, pulse interval and between-pulse interval; it is also directly proportional to pulse rate. Therefore, difficult testing conditions can be made more favorable by increasing pulse rate, for example, by alternate production and injection. Equations for time lag and pulse-response amplitude for the unit impulse, single pulse and multiple pulse are developed in the Appendix.

Pulse-tests give average values for transmissibility and storage for the portion of the reservoir sampled during

**Throughout the paper the symbols T for transmissibility and S for storage have been used even though these symbols normally designate the quantities temperature and saturation, respectively. To avoid confusion Roman-type faces have been used rather than italics.

the trade-off between the improved sensitivity and data detail obtained with high-sensitivity pulse tests and the potentially limited sampling area due to their shorter testing times.

the test. Use of the high-sensitivity pulse-test instrument (as opposed to conventional pressure gauges with sensitivities of 0.1 psi or higher) allows detection of low-level pressure responses and use of short testing times. These short times, in turn, limit the portion of the reservoir sampled to a relatively small area around the tested well pair. In a field developed on a regular pattern, for example, the area sampled is approximately equal to the drainage areas of both pulsing and responding wells.

Figs. 3 through 5 have been prepared to show how storage, transmissibility, well spacing and pulse interval affect time lag and response amplitude when the pulse interval is equal to the between-pulse interval. Fig. 3 shows that the time lag decreases and that the response amplitude passes through a maximum as transmissibility increases. Reservoirs with low transmissibility are difficult to test because the response amplitude becomes small and the time lag (and total testing time) becomes long. Fig. 4 shows that either increase of storage or well spacing will cause a corresponding increase in time lag and decrease of response amplitude. This means that in reservoirs on wide well spacing, or where depletion has greatly increased the compressibility of the reservoir by the release of solution gas, pulse-tests are more difficult and require longer testing times. Increasing pulse interval (Fig. 5) helps in restoring the response amplitude, but does not significantly shorten time lag. The figures were derived for the reservoir conditions shown but the general statements can be applied to other reservoir conditions as well. Using the *Ei* equation, the effect of other variables or specific reservoir conditions can be examined.

EXPERIMENTAL TESTS

Extensive experimental testing in a single-phase (brine), single-zone, sandstone test reservoir near Chandler, Okla., was used to evolve the method and to confirm the reliability of analysis by the *Ei* equation. In these tests, where the reservoir features of transmissibility, storage and distribution of heterogeneity were known from core analysis⁵ and interference tests,^{5,6} one well (H-5, Fig. 6) was pulsed and the response was measured at both Well A-30 and Well A-29. Thirteen separate tests with pulse intervals ranging from 10 to 60 minutes showed a probable error of ± 9 per cent in values of transmissibility and storage. For comparison, a 15-hour interference test was made in which the drawdown occurring at Wells A-29 and A-30 due to production at Well H-5 was measured. In Table 1 the range of transmissibility and storage obtained from conventional tests is in agreement with the range from pulse-tests.

In theory, if the response amplitude and time lag of a response curve are used to obtain values of *T* and *S* for the reservoir system, the *Ei* equation can also be used to generate the remainder of the response curve. Fig. 7 indicates for one case how well the solution of the *Ei* equation reproduces the pressure-time response observed in a pulse-test between Wells H-5 and A-30. Shown as open circles are the actual response data resulting from a pulse

TABLE 1—COMPARISON OF INTERFERENCE AND PULSE-TEST DATA FROM CHANDLER

Measurement	Transmissibility (md-ft/cp)	Storage (ft/psi)
Conventional tests		
Core analysis (25 wells)	3,318 ⁵	—
Interference tests (at wells P-5 and D-5)	3,400 ^{5,6}	7.5 × 10 ⁻⁵
Interference test (at well H-5)	4,325	6.2 × 10 ⁻⁵
Pulse tests		
H-5 → A-30	2,860 ± 210*	(5.41 ± 0.39) × 10 ⁻⁵
H-5 → A-29	4,100 ± 270	(9.01 ± 0.82) × 10 ⁻⁵

* ± values on this table are the standard deviation.

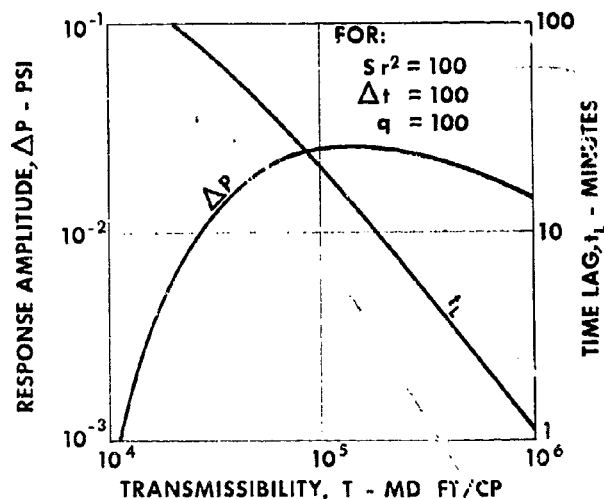


FIG. 3—EFFECT OF TRANSMISSIBILITY ON RESPONSE AMPLITUDE AND TIME LAG.

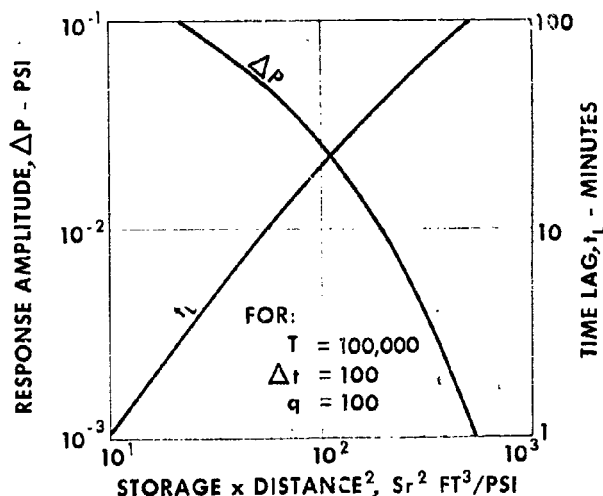


FIG. 4—EFFECT OF STORAGE-DISTANCE² ON RESPONSE AMPLITUDE AND TIME LAG.

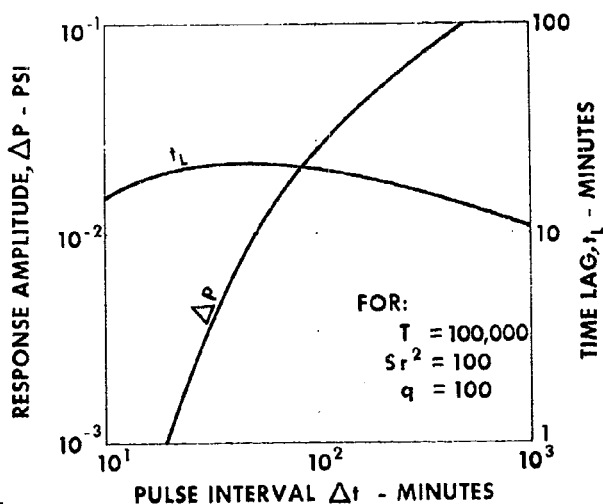


FIG. 5—EFFECT OF PULSE INTERVAL ON RESPONSE AMPLITUDE AND TIME LAG.

cycle composed of a 15-minute production pulse followed by a 30-minute shut-in interval. Transmissibility and storage values obtained from the experimental data by the tangent method were used in the Ei -equation to yield the calculated response curve shown as a solid line.

APPLICATION

Pulse-testing can provide a description of heterogeneities in many reservoirs. However, it may not be applicable to those reservoirs with properties that require long testing times. Pulse-tests requiring several days can be invalidated by changes in reservoir pressure trend, instrument drift and random pressure noise. Experience gained by applying pulse-testing to more than 25 oil and gas reservoirs has shown that when the time lag t_L from Eq. 2 exceeds 1,000 minutes, the pulse response will be difficult to detect. One full pulse-test cycle requires a testing time of about $3 t_L$. Often, t_L is on the order of 20 minutes so that a full pulse-test cycle can be completed in one hour.

In simple reservoirs, the assumptions* of the analytical model are most nearly met, and a pulse-test survey will provide a map of the areal distribution of field heterogeneities. If these heterogeneities are larger than about one well spacing, this areal map can be quantitative; i.e., not only are heterogeneities located but values of transmissibility and storage can be assigned to them. Since most highly fractured reservoirs act as simple reservoirs, the trend and magnitude of their anisotropy can also be described by pulse-testing.

In complex systems, such as multizoned or highly-faulted reservoirs where assumptions of the analytical model may be in doubt, pulse-testing can be used qualitatively to determine communication between wells or between zones and locate fault barriers or avenues of flow, fault trends and gross areal heterogeneities.

Some conditions, such as low permeability, high compressibility and wide well spacing, place a reservoir (or part of a reservoir) outside of present testing capability. Many reservoirs that currently fall into this unfavorable category are fields with solution gas drive that have suffi-

*Ideally, the interpretation model is that of a single-phase, single-zone, homogeneous, isotropic, constant-thickness porous medium; however, in a practical sense these assumptions do not preclude the extension of the Ei equation to multiphase, heterogeneous, anisotropic reservoirs.

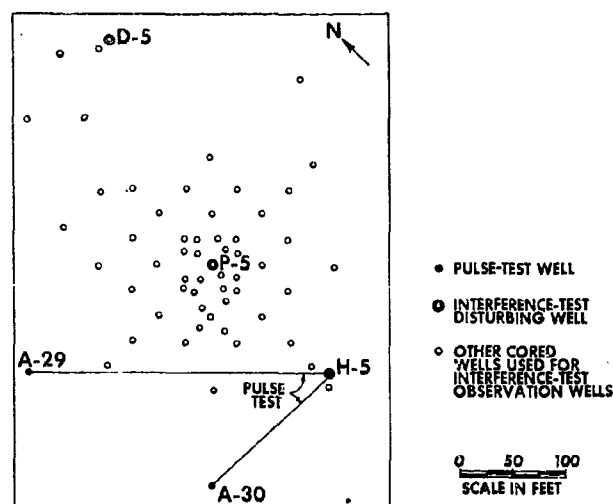


FIG. 6—WELL LOCATION MAP OF CHANDLER, OKLA., TEST SITE.

cient free gas to raise compressibility to a high level. On the other hand, these could have been tested quite easily during the period of primary development when the field was undersaturated. However, a lack of response to pulse-tests may still give useful information since a certain limiting value of transmissibility or storage is necessary to attenuate the response below the detection level.

CONCLUSIONS

Present pulse-test techniques are capable of providing an areal description of reservoir heterogeneity in many oil and gas reservoirs. An important feature of pulse-test method is use of a special, highly sensitive differential pressure gauge to measure response.

1. Pulse-testing is a reservoir description technique that can be performed in shorter times and with greater ease and certainty than conventional interference tests.

2. Pulse-testing can be used to provide qualitative information about between-well characteristics such as the presence of heterogeneities between wells, fracture orientation and between-zone communication.

3. In reservoirs that are not complex, pulse-testing can be used to provide quantitative values for transmissibility and storage for the formation between a tested well pair.

4. Field experience in over 25 reservoirs has shown that a reservoir can be pulse-tested successfully if the value of time lag from Eq. 2 is less than 1,000 minutes.

NOMENCLATURE

- B = formation volume factor
- c = compressibility, psi^{-1}
- h = effective reservoir thickness, ft
- k = permeability, md
- p = reservoir pressure, psi
- Δp = response amplitude, psi
- q = pulse rate, B/D
- r = distance between wells, ft
- S = storage ($=\phi ch$), ft/psi
- T = transmissibility ($=kh/\mu$), md ft/cp
- Δt = pulse interval, min
- t = time, min
- t_L = time lag, min
- η = hydraulic diffusivity ($=0.00633 k/1,440 \phi c\mu$), sq ft/min
- μ = viscosity, cp
- ϕ = porosity, fraction
- τ = time of initiation of a flow rate change, min

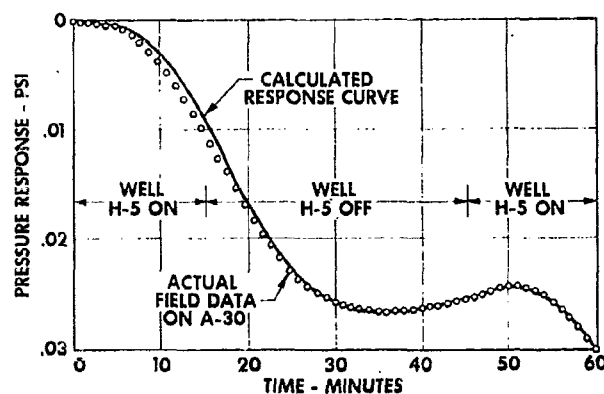


FIG. 7—COMPARISON OF A MEASURED PULSE RESPONSE WITH A PULSE RESPONSE CALCULATED FROM RESPONSE AMPLITUDE AND TIME LAG OBTAINED FROM FIELD DATA.

ACKNOWLEDGMENTS

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REFERENCES

1. Greenkorn, R. A. and Johnson, C. R.: "Method for Defining Reservoir Heterogeneities", U. S. Patent No. 3,285,064 (Nov. 15, 1966).
2. Johnson, C. R. and Raynor, R.: "System for Measuring Low Level Pressure Differential", U. S. Patent No. 3,247,712 (April 26, 1966).
3. Craft, B. C. and Hawkins, M. F.: *Applied Petroleum Reservoir Engineering*, Prentice-Hall Publishing Co., Englewood Cliffs, N. J. (1959) 331.
4. Collins, R. E.: *Flow of Fluids Through Porous Materials*, Reinhold Publishing Corp., New York, N. Y. (1961) 113.
5. Greenkorn, R. A. and Johnson, C. R.: "Variation of a Natural Reservoir Element: An Objective Analysis of Core Measurements", Paper No. 1577G presented at SPE Annual Fall Meeting held in Denver, Colo. (Oct. 2-5, 1960).
6. Johnson, C. R. and Greenkorn, R. A.: "Comparison of Core Analysis and Drawdown Test Results from a Water-Bearing Upper Pennsylvania Sandstone of Central Oklahoma", *Bull., International Society of Scientific Hydrology* (Sept., 1962).
7. Carslaw, H. S. and Jaeger, J. C.: *Conduction of Heat in Solids*, Oxford U. Press, London, England (1959) 258.
8. LiYun-Shan: "A Method for Determining the Coefficient of Piezo Conductivity of a Stratum by the Point of Maximum Change of Formation Pressure in a Responding Well". *Sb. Podzemnaya Gidrodinamika. Tr. MINKh i GP, Gostoptekhlizdat* (1961) vyp. 33.

APPENDIX

The implications of time lag and pulse-response amplitude for multiple pulses are more readily apparent if one begins with the mathematical concept of a unit impulse.⁷ The unit impulse corresponds to a unit volume of fluid produced instantaneously at $t = 0$. From the unit impulse case, the time lag and response-amplitude concepts will be extended to single and multiple pulses.

UNIT IMPULSE

The pressure response of a unit impulse from a line source is given by Eq. A-1. This equation

$$p = p_o - \frac{70.6}{Tt} \text{Exp} \left(\frac{-56,900Sr^2}{Tt} \right) \quad (A-1)$$

is equivalent to the first time derivative of Eq. 1 with a unit rate $n = 1$ and $\tau_i = 0$. The pressure-time response given by this function is analogous to the response shown in Fig. 8. However, for the unit impulse the pulse interval Δt approaches zero.

The time lag (characteristic diffusion time) of the unit impulse is found by equating the first time derivative of Eq. A-1 to zero to find the time of the maximum response amplitude at a radius r . This time lag t_L (minutes) is given by:

$$t_L = \frac{56,900Sr^2}{T} = \frac{r^2}{4\eta} \quad (A-2)$$

where the hydraulic diffusivity η is $\eta = \frac{0.00633}{1,440} \frac{k}{\phi c \mu}$.

The response amplitude of the unit impulse Δp_u can be determined by substituting Eq. A-2 into Eq. A-1

$$\Delta p_u = - \frac{0.000456}{Sr^2} \quad (A-3)$$

In view of Eqs. A-3 and A-2, it becomes obvious that measurement of response amplitude Δp_u , time lag t_L , and well spacing r allows one to solve explicitly for two reservoir rock-fluid parameters (storage S and transmissibility T).

SINGLE PULSE

For the case of a single pulse (Fig. 8), the reservoir is disturbed for a time Δt while pressure vs time is recorded at a distance r from the pulsing well. The pressure response for the single pulse is given by superposition of the solutions for each rate change; i.e., for $t \leq \Delta t$, the value of n in Eq. 1 is equal to one and for $t > \Delta t$, n is equal to two.

In a manner completely analogous to that used for the unit impulse, the time of maximum pressure response t can be found by equating the first time derivative of Eq. 1 to zero.⁸ The resulting equation for the dimensionless time lag t_{DL} is given by

$$(t_{DL} + 1) t_{DL} \ln \left(1 + \frac{1}{t_{DL}} \right) = \frac{56,900Sr^2}{T\Delta t} = \frac{r^2}{4\eta\Delta t} \quad (A-4)$$

where $t_{DL} = (t - \Delta t)/\Delta t = t_L/\Delta t$.

The pulse-response amplitude for the single pulse is obtained by evaluating Eq. 1 at $t_{DL} + 1$

$$\Delta p_s = \frac{70.6qB}{T} \left[Ei \left(\frac{-r^2}{4\eta\Delta t(t_{DL} + 1)} \right) - Ei \left(\frac{-r^2}{4\eta\Delta t t_{DL}} \right) \right],$$

where Δp_s is the response amplitude.

Reservoir transmissibility and storage can be obtained from experimental data as follows. The observed dimensionless time lag ($t_{DL} = t_L/\Delta t$), pulse interval Δt and well

⁸Note that as t_{DL} increases in Eq. A-4, the log term approaches $1/t_{DL}$. Therefore, the time lag approaches a limit when the time lag becomes long compared to the pulse length. This limit is: t_{DL} is approximately equal to $(r^2/4\eta\Delta t) - 1$. This limit, in turn, reduces to the time lag of a unit impulse (Eq. A-2) when pulse interval Δt approaches zero.

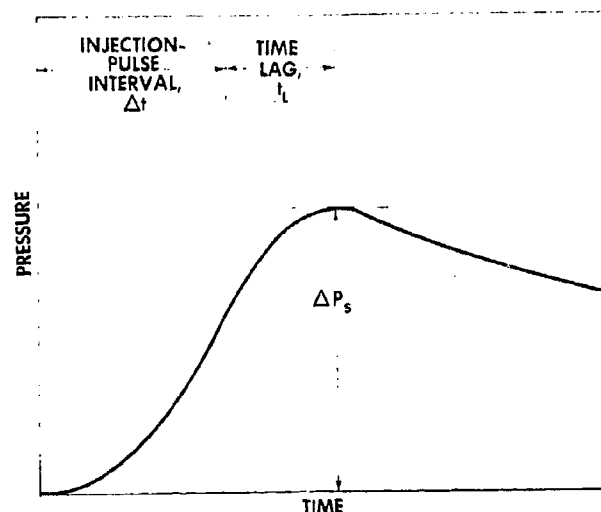


FIG. 8—PRESSURE RESPONSE FOR A SINGLE PULSE.

spacing r are used to solve Eq. A-4 for hydraulic diffusivity η . This value of η is substituted into Eq. A-5 along with pulse rate q and response amplitude Δp , to obtain transmissibility T . T and η can then be used to obtain storage S .

In practice, data generated by a single pulse will be superimposed upon pressure trends normally present because of routine reservoir production operations. In this case, it is often extremely difficult to obtain accurate estimates of the time lag and response amplitude. However, by generating multiple pulses these pressure trends can be compensated for by the tangent method of analysis discussed below.

MULTIPLE PULSE

A multiple pulse consists of a series of pulses introduced into the reservoir for pulse intervals Δt with between-pulse intervals Δt_b . The flow and pressure response configuration is shown in Fig. 2. The pressure-time relationship is given by Eq. 1 with the appropriate number of flow rate changes n being applied at each time.

As previously stated, several techniques of analysis could be used to analyze multiple-pulse data. The tangent method was chosen because it lends itself to routine analysis by the use of interpretation charts. In addition, it is independent of linear pressure trends which may result from pressure build-up of the responding well or instrument drift.

TANGENT METHOD DEVELOPMENT

The time lags for multiple pulses are obtained from Eq. 1 and its time derivatives. The conditions to be satisfied are that the derivatives at the tangent points and the peak must equal the slope of the tangent (Fig. 2) as expressed by

$$\left(\frac{\partial p}{\partial t}\right)_1 = \left(\frac{\partial p}{\partial t}\right)_m = \left(\frac{\partial p}{\partial t}\right)_2 = \frac{p_2 - p_1}{t_2 - t_1}, \quad \dots \quad (\text{A-6})$$

where subscript 1 corresponds to the first tangent point on

the base tangent line, subscript 2 to the second tangent point and m corresponds to the point where the incremental pressure response between the tangent line and the response curve is a maximum. Eq. A-6 represents three independent equations which are implicit in time lag t_L , first tangent time t_1 and second tangent time t_2 .

Eq. 2 is an approximation of the solution of Eq. A-6 when $0.2 < t_{DL} < 1$:

$$t_{DL} \approx \frac{20,000Sr^2}{T\Delta t} = \frac{r^2}{11.36\eta\Delta t} \quad \dots \quad (\text{2})$$

Eq. 2 applies to the first pulse of a multiple-pulse series when the ratio of between-pulse interval to pulse interval is one. Tangent times t_1 and t_2 also yield estimates of diffusivity η .

The response amplitude for the first pulse of the series may be found in the following manner. First, evaluate Eq. 1 at time $\Delta t + t_L$ and let this value of pressure be $70.6qB\bar{p}_m/T$; then let the pressure-response level represented by the tangent line at this time be $70.6qB\bar{p}_{rm}/T$. The pulse-response amplitude relative to the base tangent can now be defined as

$$\Delta p_m = \frac{70.6qB}{T} (\bar{p}_m - \bar{p}_{rm}) \quad \dots \quad (\text{A-7})$$

The dimensionless pressure function $(\bar{p}_m - \bar{p}_{rm})$ can be correlated with a dimensionless pulse interval $(\Delta t_p = 4\eta\Delta t/r^2)$ and between-pulse interval with the aid of Eq. 1 and the times derived from the solution of the implicit Eq. A-6.

Now, having response amplitude and time lag, the hydraulic diffusivity η can be evaluated with Eq. 2 (or a correlation based upon the implicit Eq. A-6). Transmissibility can then be found from response amplitude and hydraulic diffusivity by using correlations based upon Eq. A-7. Storage S is then readily obtained from T and η . ★★★