Journal of Hydrology 375 (2009) 554–565

Contents lists available at ScienceDirect

Journal of Hydrology

journal homepage: www.elsevier.com/locate/jhydrol

Optimal design of pumping tests in leaky aquifers for stream depletion analysis

Steen Christensen [a,*], Vitaly A. Zlotnik [b], Daniel M. Tartakovsky [c]

[a] Department of Earth Sciences, University of Aarhus, Ny Munkegade Building 1520, DK-8000 Aarhus C, Denmark
[b] Department of Geosciences, University of Nebraska, Lincoln, NE 68588, USA
[c] Department of Mechanical and Aerospace Engineering, University of California, San Diego, 9500 Gilman Dr., La Jolla, CA 92093, USA

ARTICLE INFO

Article history:
Received 13 November 2008
Received in revised form 30 June 2009
Accepted 7 July 2009

This manuscript was handled by P. Baveye, Editor-in-Chief

Keywords:
Pumping test
Stream depletion
Leaky aquifer
Optimal design
Sensitivity analysis
Global optimization

SUMMARY

We analyze the optimal design of a pumping test for estimating hydrogeologic parameters that are subsequently used to predict stream depletion caused by groundwater pumping in a leaky aquifer. A global optimization method is used to identify the test's optimal duration and the number and locations of observation wells. The objective is to minimize predictive uncertainty (variance) of the estimated stream depletion, which depends on the sensitivities of depletion and drawdown to relevant hydrogeologic parameters. The sensitivities are computed analytically from the solutions of Zlotnik and Tartakovsky [Zlotnik, V.A., Tartakovsky, D.M., 2008. Stream depletion by groundwater pumping in leaky aquifers. ASCE Journal of Hydrologic Engineering 13, 43–50] and the results are presented in a dimensionless form, facilitating their use for planning of pumping test at a variety of sites with similar hydrogeological settings. We show that stream depletion is generally very sensitive to aquitard's leakage coefficient and stream-bed's conductance. The optimal number of observation wells is two, their optimal locations are one close to the stream and the other close to the pumping well. We also provide guidelines on the test's optimal duration and demonstrate that under certain conditions estimation of aquitard's leakage coefficient and stream-bed's conductance requires unrealistic test duration and/or signal-to-noise ratio.

Introduction

Accurate and reliable predictions of stream depletion (aka stream flow depletion) caused by groundwater extraction are becoming increasingly important due to droughts, proliferation of irrigation wells, and consequently disruption of stream flow regimes. This is the case in the alluvial plains of USA (Sophocleous, 1997; Kollet and Zlotnik, 2003, 2005, 2007), the outwash plains of western Denmark (Nyholm et al., 2002, 2003), and in sand and gravel environments in wetlands (Hunt et al., 2001; Lough and Hunt, 2006). Reliable predictions of stream depletion require mathematical models that reflect actual hydrogeologic conditions and utilize accurate parameter estimates.

Classical analytical models of stream depletion (Theis, 1941; Hantush, 1965; Jenkins, 1968) are limited to streams that fully penetrate an aquifer. Such streams are rare, especially on the alluvial plains and outwash plains mentioned above. This led to the development of analytical models that consider the effects of shallow aquifer penetration by streams (e.g., Hunt, 1999; Zlotnik and Huang, 1999; Butler et al., 2001). Additional phenomena that have been analyzed analytically include pumping from a well in a semi-confined aquifer (Hunt, 2003) and the finite widths of streams and aquifers (Hunt, 2008). All of these models predict that the steady-state rate of stream depletion is equal to the groundwater withdrawal rate.

The stream depletion rate might correspond to only a fraction of the pumping rate, whereas the remaining fraction is supplied from a deeper aquifer through an aquitard. Hantush (1955, 1964), Zlotnik (2004), and Butler et al. (2007) demonstrated that this occurs when a well pumps from leaky aquifers adjacent to a fully penetrating stream. Zlotnik and Tartakovsky (2008) reached a similar conclusion by deriving an analytical solution for stream depletion in a leaky two-aquifer system in which a lower aquifer (source bed) has negligible drawdown (Fig. 1). This solution, which includes the solutions of Theis (1941), Hantush (1955, 1964), and Hunt (1999) as special cases, was used to demonstrate that both hydraulic stream/aquifer connection and hydraulic aquifer/source-bed connection determine the fraction of the pumping rate that is supplied by stream depletion. The solution also coincides with the Hunt (2003) solution when the storage coefficient of the source bed becomes infinitely large, i.e. when pumping in an aquifer does not cause drawdown in the deeper source bed.

Predictions of stream depletion require accurate estimates of the stream-bed conductance and the hydraulic conductivities and storage coefficients of hydrostratigraphic layers, all of which

* Corresponding author.
E-mail address: sc@geo.au.dk (S. Christensen).

0022-1694/$ - see front matter © 2009 Elsevier B.V. All rights reserved.
doi:10.1016/j.jhydrol.2009.07.006

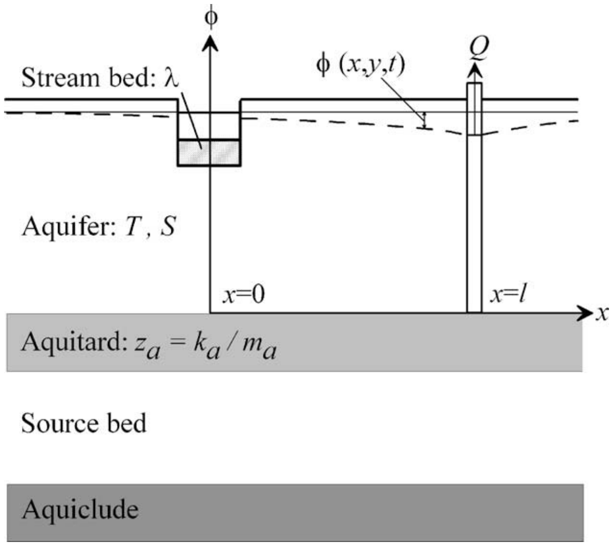

Fig. 1. A schematic representation of the stream–aquifer–aquitard–source bed system and the major hydrological parameters for the solutions of Zlotnik and Tartakovsky (2008). Explanation of the symbols follows Eq. (1).

influence the drawdown dynamics. Since these estimates must represent a spatial scale corresponding to the cone of depression caused by pumping, they are typically obtained from pumping tests. A drawdown analysis is often done by using analytical solutions (e.g., Theis, 1941; Hantush, 1965; Hunt, 1999, 2003; Hunt et al., 2001; Kollet and Zlotnik, 2007; Zlotnik and Tartakovsky, 2008), although hydrogeological conditions sometimes make it necessary to use numerical models (e.g., Nyholm et al., 2002; Kollet and Zlotnik, 2005). In all cases, the importance of each parameter must be evaluated, especially considering resources needed for parameter acquisition.

The sensitivity analysis of Christensen (2000) revealed that actual hydrogeological conditions significantly influence a pumping test's design and analysis. The analysis relied on the Hunt (1999) analytical solutions for drawdown and depletion caused by a well adjacent to a shallow stream extracting groundwater from a non-leaky aquifer. The uncertainty of the stream-bed conductance, which quantifies the hydraulic connectivity between the aquifer and the stream, was shown to be a significant, and often the major, source of uncertainty of stream depletion. The analysis of Christensen (2000) also demonstrated that different parts of the time-drawdown curve are sensitive to the aquifer transmissivity T and storativity S, as well as to the stream-bed conductance λ. This implies that these three parameters can be estimated using drawdown data from just one observation well. Lough and Hunt (2006) used the Hunt (2003) solution to arrive at the same conclusion.

The duration of a pumping test that is required for such estimations should be short enough to avoid or minimize transient disturbances from varying weather or other sources and sinks, and to reduce costs. This led Christensen (2000) to recommend that a pumping well be located relatively close to a stream, accurate drawdown measurements be made both near the pumping well and near the stream, and these measurements be used simultaneously to estimate T, S, and λ. Unfortunately, the necessary duration of a pumping test can be months or years if S is large and either T or λ is small, especially if parameter estimates obtained from a drawdown analysis are to be used for reliable predictions of stream depletion (Christensen, 2000). While such tests are currently uncommon, future groundwater use will lead to interpretation and re-interpretation of the aquifer and stream depletion rate parameters after prolonged exploitation of pumping wells.

We analyze the three-layered leaky aquifer system considered by Zlotnik and Tartakovsky (2008), in which groundwater is extracted from a top aquifer with a pumping well adjacent to a stream that is hydraulically connected to the top aquifer. An aquitard separates the top aquifer from a deeper source bed (Fig. 1), and it is assumed that drawdown does not develop in the source bed. We use the method of Christensen (2000) and the analytical solutions of Zlotnik and Tartakovsky (2008) to study the sensitivities of stream depletion and aquifer drawdown to the hydrogeologic parameters of the streambed, the aquifer, and the aquitard. We expand the analysis of Christensen (2000) to determine the optimal locations and duration of drawdown observations during a pumping test, which goals are to estimate hydrogeologic parameters and to make accurate predictions of stream depletion.

Methodology

We rely on the Zlotnik and Tartakovsky (2008) solutions to derive analytically the sensitivities of depletion and drawdown with respect to relevant hydrogeologic parameters. The sensitivities are employed to compute both the covariance matrix of hydrogeologic parameters estimated from drawdown data and the standard deviation of depletion using the parameter values estimated by drawdown analysis. Minimization of the standard deviation for the depletion prediction is used to determine the optimal locations for measuring drawdown during a pumping test for varying durations of the test.

In the following we use dimensionless parameters:

$$q_d = \frac{q}{Q}, \quad \phi_d = \frac{\phi T}{Q}, \quad t_d = \frac{Tt}{Sl^2}, \quad x_d = \frac{x}{l}, \quad y_d = \frac{y}{l}, \quad B_d^2 = \frac{T}{z_a l^2},$$
$$\lambda_d = \frac{\lambda l}{T}, \quad a_1 = \frac{B_d}{2/\lambda_d + B_d}, \quad a_2 = \frac{B_d}{2/\lambda_d - B_d}, \quad a_3 = a_1 a_2 \tag{1}$$

where Q is the pumping rate, q is the stream depletion rate, ϕ is the drawdown, T is the aquifer transmissivity, S is the aquifer storativity, l is the distance between the well and the stream, x and y are the Cartesian coordinates, t is the time since pumping started, z_a is the leakage coefficient of the aquitard, and λ is the stream-bed conductance. The leakage coefficient can be computed as $z_a = k_a/m_a$ where k_a and m_a are the hydraulic conductivity and the thickness of the aquitard, respectively. The stream-bed conductance can be represented as $\lambda \approx k_s w_s/m_s$ where k_s, w_s, and m_s are the stream-bed's hydraulic conductivity, width, and thickness, respectively. It is worthwhile noting that Hunt (1999) refers to λ as a constant of proportionality between the seepage flow rate and the hydraulic head difference between the aquifer and the stream.

Depletion and depletion sensitivities

Dimensionless stream depletion rate q_d can be computed as (Zlotnik and Tartakovsky, 2008, Eq. (22))

$$q_d = \frac{a_1}{2} E\left(-\frac{1}{B_d}\right) - \frac{a_2}{2} E\left(\frac{1}{B_d}\right) + a_3 e^{\lambda_d^2 t_d/4 - t_d/B_d^2} E\left(\frac{\lambda_d}{2}\right) \tag{2a}$$

where

$$E(\xi) = e^{\xi} erfc\left(\frac{1}{2\sqrt{t_d}} + \sqrt{t_d}\xi\right). \tag{2b}$$

Differentiation of (2) gives dimensionless sensitivities of the depletion rate to the hydrogeologic parameters

$$\frac{\partial q_d}{\partial T}T = \frac{a_1}{2B_d}\left(\frac{1}{2}-\frac{a_1}{\lambda_d}\right)E\left(-\frac{1}{B_d}\right)+\frac{a_1 e^{-1/4t_d - t_d/B_d^2}}{4T\sqrt{\pi t_d}}+\frac{a_2}{2B_d}\left(\frac{1}{2}+\frac{a_2}{\lambda_d}\right)E\left(\frac{1}{B_d}\right)$$
$$-\frac{a_2 e^{-1/4t_d - t_d/B_d^2}}{4T\sqrt{\pi t_d}} - a_3\left(\frac{4a_3}{\lambda_d^2 B_d^2}+\frac{\lambda_d}{2}+\frac{\lambda_d^2 t_d}{4}\right)e^{\lambda_d^2 t_d/4 - t_d/B_d^2}E\left(\frac{\lambda_d}{2}\right)$$
$$+\frac{a_3 e^{-1/4t_d - t_d/B_d^2}}{2\sqrt{\pi}}\left(\frac{1}{\sqrt{t_d}}+\lambda_d\sqrt{t_d}\right) \tag{3}$$

$$\frac{\partial q_d}{\partial S}S = \left(\frac{a_2 - a_1 - 2a_3}{2\sqrt{t_d}} - \frac{a_1 + a_2}{B_d}\sqrt{t_d} + \lambda_d a_3\sqrt{t_d}\right)\frac{e^{-1/4t_d - t_d/B_d^2}}{2\sqrt{\pi}}$$
$$-a_3 t_d\left(\frac{\lambda_d^2}{4}-\frac{1}{B_d^2}\right)e^{\lambda_d^2 t_d/4 - t_d/B_d^2}E\left(\frac{\lambda_d}{2}\right) \tag{4}$$

$$\frac{\partial q_d}{\partial \lambda}\lambda = \frac{a_1^2}{B_d\lambda_d}E\left(-\frac{1}{B_d}\right)-\frac{a_2^2}{B_d\lambda_d}E\left(\frac{1}{B_d}\right)$$
$$+a_3\left(\frac{8}{4-\lambda_d^2 B_d^2}+\frac{\lambda_d+\lambda_d^2 t_d}{2}\right)e^{\lambda_d^2 t_d/4 - t_d/B_d^2}E\left(\frac{\lambda_d}{2}\right)$$
$$-\frac{a_3\lambda_d\sqrt{t_d}}{\sqrt{\pi}}e^{-1/4t_d - t_d/B_d^2} \tag{5}$$

and

$$\frac{\partial q_d}{\partial z_a}z_a = -\frac{a_1}{4B_d}\left(1+\frac{2a_1}{\lambda_d}\right)E\left(-\frac{1}{B_d}\right)+\frac{a_1+a_2}{2}\sqrt{t_d}\frac{e^{-1/4t_d - t_d/B_d^2}}{\sqrt{\pi}B_d}$$
$$-\frac{a_2}{4B_d}\left(1-\frac{2a_2}{\lambda_d}\right)E\left(\frac{1}{B_d}\right)$$
$$-a_3\left(\frac{4}{4-\lambda_d^2 B_d^2}+\frac{t_d}{B_d^2}\right)e^{\lambda_d^2 t_d/4 - t_d/B_d^2}E\left(\frac{\lambda_d}{2}\right) \tag{6}$$

Drawdown and drawdown sensitivities

Dimensionless drawdown in the aquifer caused by the pumping is given by (Zlotnik and Tartakovsky, 2008, Eq. (17))

$$\phi_d(x,y,t) = \frac{1}{4\pi}\left[W(u,z) - \int_0^\infty e^{-\theta}W(u_\lambda, z_\lambda)d\theta\right] \tag{7}$$

where

$$W(u,z) = \int_u^\infty \frac{1}{y}\exp\left(-y-\frac{z^2}{4y}\right)dy \tag{8}$$

and

$$u = \frac{r_d^2}{4t_d},\quad z = \frac{r_d}{B_d},\quad u_\lambda = \frac{r_\lambda^2}{4t_d},\quad z_\lambda = \frac{r_\lambda}{B_d},$$
$$r_d^2 = (x_d - 1)^2 + y_d^2,\quad r_\lambda^2 = \left(1+|x_d|+\frac{2\theta}{\lambda_d}\right)^2 + y_d^2 \tag{9}$$

Differentiation of (7) gives dimensionless sensitivities of the drawdown to the hydrogeologic parameters

$$\frac{\partial\phi}{\partial T}\frac{T^2}{Q} = \frac{\partial\phi_d}{\partial T}T - \phi_d$$
$$= -\phi_d + \frac{u}{4\pi}W_0(u,z) + \frac{z^2}{16\pi}W_2(u,z) + \frac{1}{4\pi}$$
$$\times\int_0^\infty e^{-\theta}\left[\frac{\theta^2}{\lambda_d^2} - \frac{(1+|x_d|)^2 + y_d^2}{4}\right]\left[\frac{1}{t_d}W_0(u_\lambda, z_\lambda) + \frac{z_\lambda^2}{r_\lambda^2}W_2(u_\lambda, z_\lambda)\right]d\theta \tag{10}$$

$$\frac{\partial\phi}{\partial S}\frac{T}{Q}S = \frac{\partial\phi_d}{\partial S}S = -\frac{u}{4\pi}W_0(u,z) + \frac{1}{4\pi}\int_0^\infty e^{-\theta}u_\lambda W_0(u_\lambda, z_\lambda)d\theta \tag{11}$$

$$\frac{\partial\phi}{\partial\lambda}\frac{T^2}{Ql} = \frac{\partial\phi_d}{\partial\lambda}\frac{T}{l}$$
$$= -\frac{1}{4\pi\lambda_d}\int_0^\infty e^{-\theta}\frac{\theta}{\lambda_d}\left[\frac{2\theta}{\lambda_d}+1+|x_d|\right]$$
$$\times\left[\frac{1}{t_d}W_0(u_\lambda, z_\lambda) + \frac{1}{B_d^2}W_2(u_\lambda, z_\lambda)\right]d\theta \tag{12}$$

$$\frac{\partial\phi}{\partial z_a}\frac{T}{Q}z_a = \frac{\partial\phi_d}{\partial z_a}z_a$$
$$= -\frac{z^2}{16\pi}W_2(u,z) + \frac{1}{16\pi}\int_0^\infty e^{-\theta}z_\lambda^2 W_2(u_\lambda, z_\lambda)d\theta \tag{13}$$

where

$$W_0(u,z) = \frac{1}{u}\exp\left(-u-\frac{z^2}{4u}\right) \tag{14}$$

and

$$W_2(u,z) = \int_0^{u^{-1}}\exp\left(-\frac{1}{y}-\frac{z^2}{4}y\right)dy \tag{15}$$

Covariance of parameter estimates obtained by drawdown analysis

Following Christensen (2000), we assume that the hydrogeologic parameters T, S, λ, and z_a are estimated by pumping test analysis, i.e. by fitting (7) to a set of n observations of drawdown made at varying times and locations. We also assume that measurement errors in the n observations are uncorrelated and have zero mean and variance σ^2. Then a 4×4 covariance matrix of the estimated (fitted) parameter values can be approximated (Seber and Wild, 1989) by

$$\mathbf{C} = \mathbf{C}(T,S,\lambda,z_a) = \sigma^2(\boldsymbol{X}^T\boldsymbol{X})^{-1} \tag{16}$$

where $\boldsymbol{X} = [\partial\phi_i/\partial T,\ \partial\phi_i/\partial S,\ \partial\phi_i/\partial\lambda,\ \partial\phi_i/\partial z_a]_{i=1,n}$ is the $n \times 4$ sensitivity matrix in which the ith row contains the sensitivities of the computed drawdown corresponding to the time and location of the ith observation. In (16), the superscripts T and -1 indicate matrix transpose and matrix inverse, respectively. The sensitivities are computed using Eqs. (10)–(13).

Uncertainty of depletion prediction

Depletion is predicted by using (2). The corresponding predictive uncertainty is quantified by the standard deviation of the prediction, which depends on the uncertainty of the estimated hydrogeologic parameters T, S, λ, and z_a. Since we assume that these parameters are estimated by drawdown analysis, the parameter uncertainty can be quantified by the covariance $\mathbf{C}$ in (16), which depends on when and where drawdown was measured and used to estimate T, S, λ, and z_a. The following sub-sections provide details about computation of the standard deviation of a depletion prediction and an analysis of the dependency of this standard deviation on the locations and duration of drawdown observations.

Standard deviation of depletion prediction

The standard deviation of a predicted dimensionless depletion q_d is computed (e.g. Seber and Wild, 1989, p. 192–193) as

$$\sigma_{qd} = \sigma\sqrt{\boldsymbol{Z}^T\mathbf{C}\boldsymbol{Z}} \tag{17}$$

where $\boldsymbol{Z}^T = [\partial q_d/\partial T,\ \partial q_d/\partial S,\ \partial q_d/\partial\lambda,\ \partial q_d/\partial z_a]$ is the (transposed) vector of sensitivities of depletion with respect to the hydrogeologic parameters (c.f. Eqs. (3)–(6)). Eq. (17) implies that the predicted variable q_d is free of measurement errors (we predict stream depletion, not a measurement of depletion). If needed, one can easily modify (17) to include measurement errors in the predicted variable (e.g. Seber and Wild, 1989, p. 193).

A scaled standard deviation of dimensionless depletion is defined as

$$\sigma_{qds} = \sigma_{qd} Q / \sigma T \tag{18}$$

where σ is the standard deviation of the drawdown measurement error, defined in "Covariance of parameter estimates obtained by drawdown analysis".

Optimizing drawdown observations

We use (17) to analyze the dependence of the uncertainty of depletion predictions on the location and duration of drawdown observations that are used to estimate T, S, λ, and z_a. Our approach consists of the following steps.

1. Define the time over which the stream depletion q_d is to be predicted. This is the target prediction.
2. Define the number of observation wells, and define the time period and the frequency for which drawdown is observed in these wells.
3. Optimize the location of the observation wells by minimizing the standard deviation of the target prediction σ_{qd} in (17).

Our goal is to predict stream depletion at different dimensionless times t_{d_qd} after pumping has been initiated. The optimal placement of observation wells is defined by the minimum standard deviation of stream depletion at time t_{d_qd}. We considered up to three observation wells, and used observation periods starting at $t_d = 10^{-3}$and ending at either $t_d = 1$, $t_d = 10$, $t_d = 10^2$, or $t_d = 10^3$. In all cases we assumed an observation frequency of 10 observations per decade evenly spaced when time is $\log_{10}$-transformed.

The minimization of σ_{qd} is nontrivial, since it is a nonlinear function of the well locations. To find its global minimum, we used the CMAES_P code (Doherty, 2008), which is an implementation of the iterative evolutionary stochastic search (CMA-ES) global optimization algorithm (Hansen and Ostermeier, 2001). The algorithm uses the parameter covariance matrix to generate parameter realizations, and then adapts the covariance matrix as the optimization process progresses (see Doherty, 2008, for a brief introduction and Hansen and Ostermeier, 2001 for a thorough description). We used the default values of the optimization parameters in the CMAES_P code (Doherty, 2008), which led to a good performance.

We deployed the CMAES_P code to implement two search strategies. The first strategy is to search for optimum well locations along the line perpendicular to the stream passing through the pumping well, i.e. along the line $y_d = 0$. This strategy is based on a practical consideration: no matter what the x coordinate of a candidate location is, it is advantageous to choose a location close to the pumping well (i.e. at $y_d = 0$) where the drawdown will be larger, than at a more distant location ($y_d \neq 0$) where drawdown will be smaller. In other words, for a given x coordinate the location with the largest drawdown is advantageous, since this will produce the most accurate parameter estimates.

The second strategy is to use CMAES_P for a search over all locations in the horizontal domain. This strategy makes no assumptions about possible locations of the minimum, but is clearly more expensive computationally than the first strategy. With a sole exception, the well locations found to yield the smallest value of σ_{qd} always fell on the line $y_d = 0$, thus confirming the validity of the first search strategy.

In both search strategies, we rejected locations whose dimensionless distance to the pumping well was less than 0.001. This is because under certain conditions the optimization procedure might predict observations at the pumping well to be optimal. In practice, one would usually avoid using such observations, since they may be significantly affected by well skin, well-bore storage, etc. Skin effect and well-bore storage can also affect drawdown measured in piezometers and observation wells located close to the pumping well (Moench, 1997). If these effects are expected to be significant, one can change the exclusion radius from the 0.001 used in our simulations to a larger value.

When a search for observation points is conducted over a wide range of candidate locations, CMAES_P might converge to a local minimum, i.e. yield suboptimal observation locations. To facilitate convergence to the global minimum, we implemented an additional refined search, which in some cases produced locations that were slightly more optimal than those found with the basic searches.

This optimization procedure described above is "global" in the sense that it aims to find the optimal locations of all observation wells simultaneously. It can be simplified by choosing optimal observation locations sequentially, one by one. First, find an optimal observation location by minimize σ_{qd} for a single well. Second, find an optimal location of the second well, while keeping the location of the first observation well fixed. This procedure can be repeated to place as many observation wells as necessary. This sequential optimization is similar to those used by Hill et al. (2001) and Tiedeman et al. (2004), among others. Our simulations revealed that the observation locations identified with the sequential optimization differ from, and are less optimal than, those obtained by the global optimization. In the following, we only present and discuss the results obtained with the global optimization.

The global optimization can also be used to identify optimal conditions for pumping tests whose goal is either to make another prediction, to estimate a parameter, or to estimate a linear combination of several parameters. In this case, it is enough to redefine the sensitivity vector $\boldsymbol{Z}$ in (17) as follows. To make an additional prediction, $\boldsymbol{Z}$ must contain sensitivities of that prediction to the four parameters; to estimate a particular parameter, the element of $\boldsymbol{Z}$ pertaining to that parameter must be set to 1.0 and all other elements be set to 0.0; to estimate a linear combination of the parameters each element of $\boldsymbol{Z}$ must equal the linear combination coefficient pertaining to the corresponding parameter.

Results

In the following we investigate the dependence of depletion and drawdown on transmissivity T, storativity S, stream-bed conductance λ, and confining bed leakage coefficient z_a. We also present optimal observation locations, which provide estimates of these hydrogeological parameters that lead to most accurate predictions of stream depletion.

Variation and sensitivity of depletion

Fig. 2 shows the magnitude and temporal variation of dimensionless stream depletion q_d and its sensitivity with respect to T, S, λ and z_a for $B_d = 10$. The depletion curves show that for values of the stream-bed conductance λ_d varying over several orders of magnitude, the steady-state (maximum) level of depletion is reached after a pumping duration corresponding to dimensionless time t_d between 100 and 1000. The sensitivity curves show that depletion is most sensitive to transmissivity T and storativity S when the depletion changes fast (the steep part of the depletion curve) while the maximum depletion is insensitive to S. (Storativity influences how fast drawdown and depletion develop, rather than how large the long-term or steady-state drawdown and

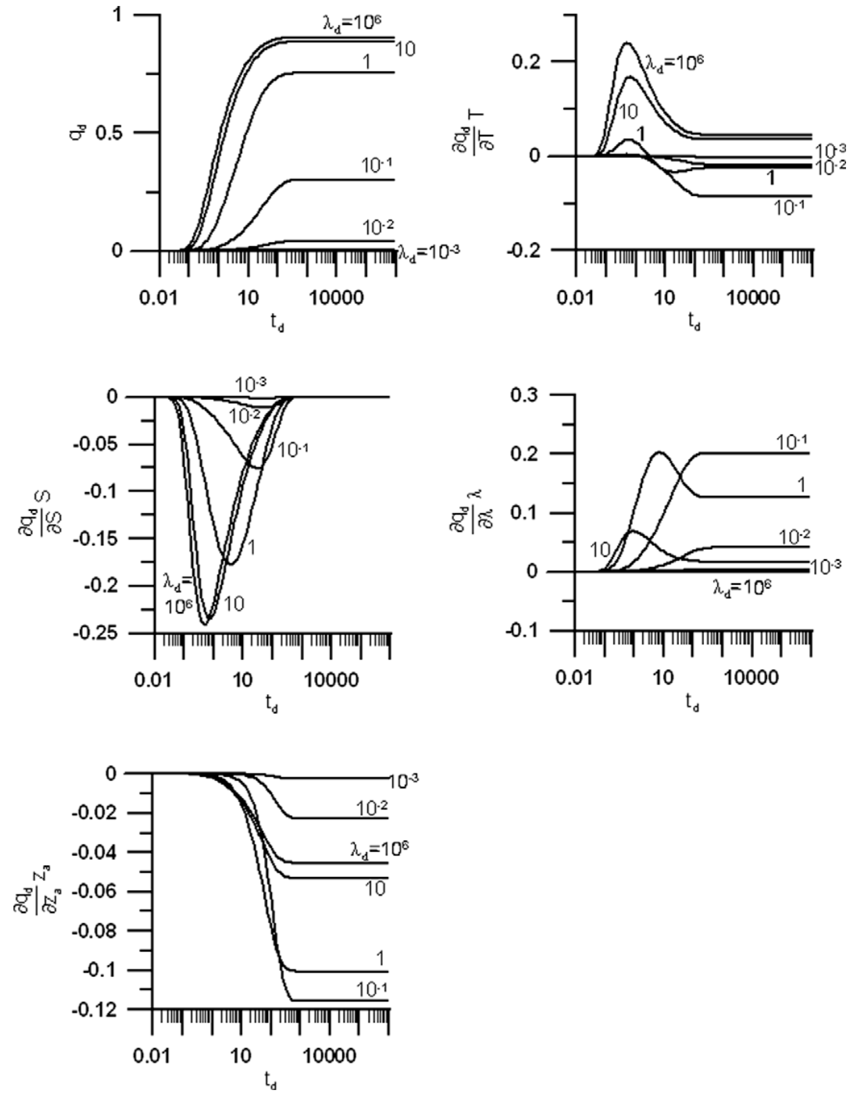

Fig. 2. Curves of dimensionless stream depletion and of dimensionless sensitivities of stream depletion when $B_d = 10$.

depletion will be.) Maximum depletion is sensitive to T, λ and z_a, except when λ_d is small, i.e. when the hydraulic connection between the stream and the aquifer is poor. In this case, stream depletion is small for a wide range of T and z_a.

Fig. 3 exhibits the dimensionless stream depletion and sensitivity curves for $B_d = 100$, which corresponds to the aquitard's leakage coefficient being one hundred times smaller than that used in Fig. 2 (other hydrogeologic parameters being equal). Since larger B_d implies that the source bed supplies less water to the aquifer, the stream depletion in Fig. 3 is larger than that in Fig. 2. Larger B_d also means that the maximum depletion occurs later because drawdown has to develop for a longer time and over a larger area before pumping is fully compensated. The compensation is to a larger degree caused by stream depletion and to a lesser degree by leakage from the source bed.

The depletion and depletion sensitivity curves for other values of B_d lead to the following observations. For $B_d \geqslant 1000$, leakage from the source bed is negligible and the model of Zlotnik and Tartakovsky (2008) coincides with the model of Hunt (1999).

For $B_d \leqslant 0.316$, stream depletion is negligible even if λ_d is large ($\lambda_d = 10^6$ yields q_d as small as 0.04), and practically all pumping is compensated by leakage from the source bed. This represents an aquifer that is hydraulically well connected to the source bed. Pumping in such an aquifer would lead to a highly localized drawdown cone, which does not reach the stream and does not cause drawdown in the source bed. The latter cannot occur in practice, since a strong hydraulic connection between the aquifer and the underlying source bed means that the aquitard is either absent or its leakage coefficient is very high. The upper aquifer and the deeper source bed would respond to pumping as a single aquifer system, in which drawdown develops in the entire depth and flow is horizontal beyond a certain distance from the well and the stream. This represents an aquifer system that is hydraulically connected to the stream via the upper aquifer, a hydrogeologic setting that is described better by the Hunt (1999) model than by the model of Zlotnik and Tartakovsky (2008).

For $0.316 < B_d < 1000$, the magnitude of λ_d determines the importance of both stream depletion and leakage through the source bed. Drawdown in the source bed might or might not develop, depending on the relative magnitudes of the transmissivities of the source bed and the aquifer and on the distance to head-dependent sources of groundwater flow other than the stream. We analyze the validity and implications of the Zlotnik and Tartakovsky (2008) assumption of the lack of drawdown in the source bed in the companion paper (Christensen et al., in preparation).

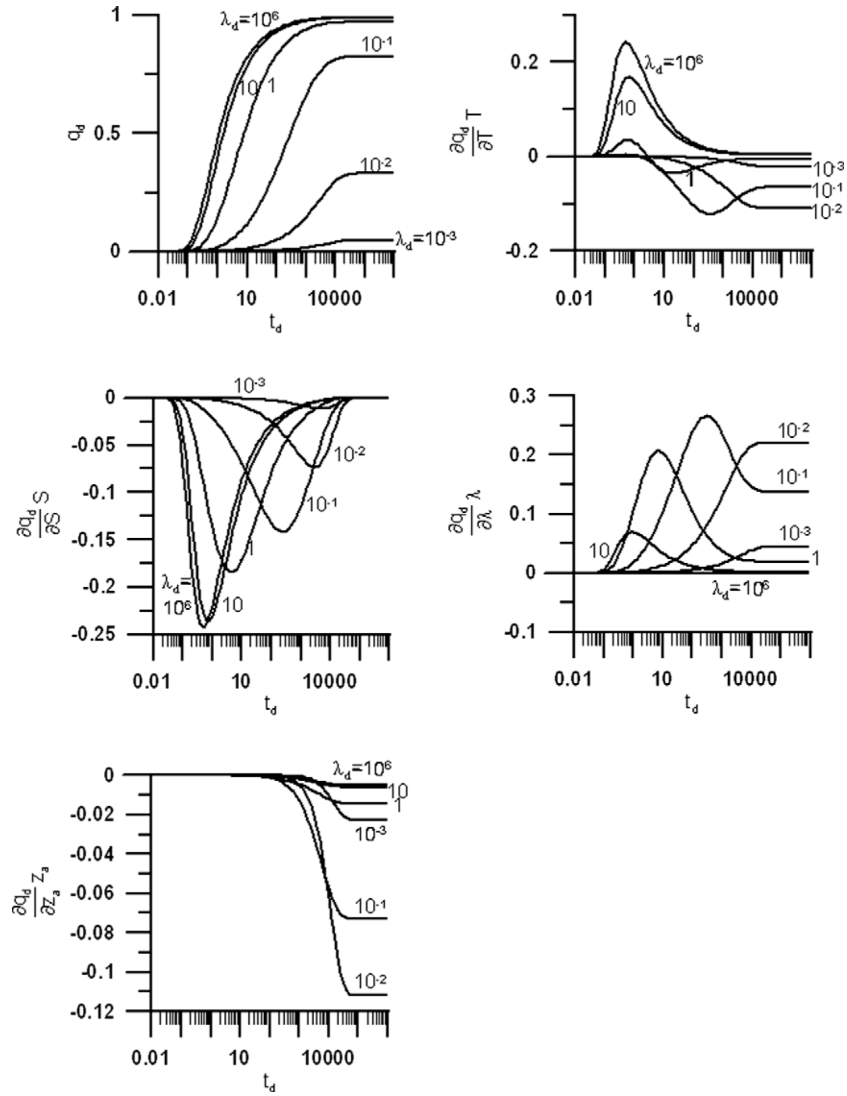

Fig. 3. Curves of dimensionless stream depletion and of dimensionless sensitivities of stream depletion when $B_d = 100$.

The dimensionless sensitivities in Figs. 2 and 3 vary over the same orders of magnitude. To obtain the sensitivity with respect to a parameter at a given time, the corresponding dimensionless sensitivity in Figs. 2 or 3 has to be divided by the value of the parameter. Since the aquitard leakage coefficient z_a is usually orders of magnitude smaller than T and S, the computed stream depletion is very sensitive to the value of z_a. Likewise, the computed stream depletion can be very sensitive to the value of the stream-bed conductance λ. This finding suggests that accurate predictions of the stream depletion caused by pumping from a leaky aquifer requires accurate estimates of both z_a and λ.

Variation and sensitivity of drawdown

Fig. 4 illustrates the magnitude and temporal variation of dimensionless drawdown ϕ_d and its sensitivities near the pumping well, $(x_d, y_d) = (0.95, 0.00)$, for $B_d = 10$. Drawdown at this location is discernable at early times t_d, and the shape of the drawdown curve is sensitive to the stream-bed conductance λ_d and the aquitard leakage coefficient z_a. After some time, drawdown gradually stabilizes when stream depletion and leakage from the deeper aquifer compensate pumping. The time and level of the stabilization depend on λ_d and z_a.

Fig. 4 also reveals that drawdown is sensitive to S from early dimensionless times ($t_d < 0.001$) to the time when drawdown stabilizes, and is sensitive to T for all times $t_d \geqslant 0.001$. This indicates that drawdown measurements over a relatively short period of time in the vicinity of the pumping well are sufficient to estimate both T and S. For λ_d and z_a the drawdown sensitivities are significant only at large times, $t_d \geqslant 1$. This suggests that the estimation of λ_d and z_a require long-term pumping tests. Finally, since the sensitivity curves with respect to λ_d and z_a are almost indistinguishable, it is difficult (or impossible) to obtain independent estimates of λ_d and z_a from drawdown observations made only near the pumping well.

Fig. 5 presents the dimensionless drawdown and sensitivity curves beneath the stream, at $(x_d, y_d) = (0, 0)$, for $B_d = 10$. Drawdown and sensitivities to T and S are smaller at this location than those close to the well (Fig. 4), and the curves deviate from zero at later times. The sensitivity curves with respect to λ_d are shifted toward smaller times as compared to their counterparts close to the well, and the sensitivities are larger beneath the stream. The sensitivity curves with respect to z_a beneath the stream are very similar to the corresponding curves near the well. Further analysis has shown that the dimensionless sensitivity curves with respect to z_a are relatively location-independent as long as the distance

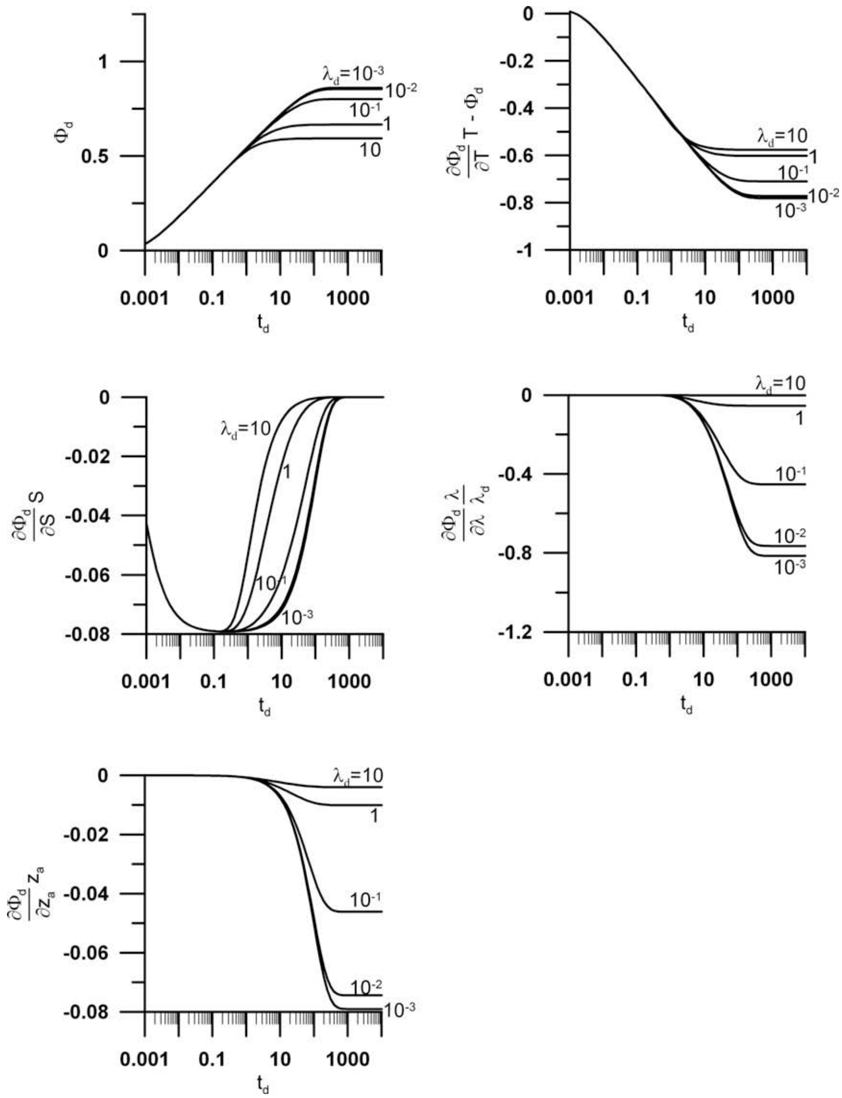

Fig. 4. Curves of dimensionless drawdown and of dimensionless sensitivities of drawdown at $(x_d, y_d) = (0.95, 0.00)$ for $B_d = 10$.

from the pumping well does not exceed 5l, where l is the distance between the pumping well and the stream.

Fig. 6 exhibits the dimensionless drawdown and sensitivity curves beneath the stream for $B_d = 100$, i.e. for the aquitard leakage coefficient z_a that is two orders of magnitude smaller than its counterpart in Fig. 5. Comparison of Figs. 5 and 6 shows that both dimensionless drawdown and absolute values of dimensionless sensitivities increase significantly with B_d. Larger values of B_d signify less leakage from the deeper aquifer, leading to increased drawdown in the pumped aquifer. The sensitivity curves with respect to z_a are shifted by between one and two orders of magnitude to the right as compared to Fig. 5, which indicates that longer pumping is required for drawdown development to be affected by leakage from the deeper aquifer. This implies that the observation time that is necessary to estimate z_a from a drawdown analysis increases significantly with B_d.

The results indicate that in order to obtain independent estimates of hydrogeologic parameters, T, S, λ and z_a, by drawdown analysis, observations should be made in the vicinity of the stream as well as at a location more distant to the stream in the direction of (or behind) the pumping well. The observations should be made for a relatively long period (to allow estimation of λ and z_a).

Duration of the pumping test

We employ a simplified Christensen (2000) approach to select the pumping test duration that minimizes the uncertainty of stream depletion predictions by estimating the minimum time required for accurate inference of hydrogeologic parameters from drawdown data. The approach uses the analytical sensitivities of depletion and drawdown derived in "Variation and sensitivity of depletion" and "Variation and sensitivity of drawdown" as input.

For $B_d = 10$, Fig. 2 indicates that stream depletion occurs if $\lambda_d > 10^{-2}$ and that stream flow predictions are sensitive to all four hydrogeologic parameters; except for maximum depletion, which is insensitive to storativity S, and for depletion at early dimensionless times t_d, which is relatively insensitive to the leakage coefficient of the aquitard z_a. Figs. 4 and 5 show that drawdown is only sensitive to z_a for $t_d > 10$. This indicates that drawdown observations during a pumping test must continue until dimensionless time $10 < t_{d\_stop} < 100$ in order to obtain reliable estimates of the hydrogeological parameters used to predict stream depletion. Otherwise, observations do not contain sufficient information to infer z_a.

A similar analysis for $B_d = 1$ revealed that significant stream depletion occurs only if $\lambda_d \geqslant 1$, in which case the duration of the pumping test should be $t_{d\_stop} \geqslant 1$.

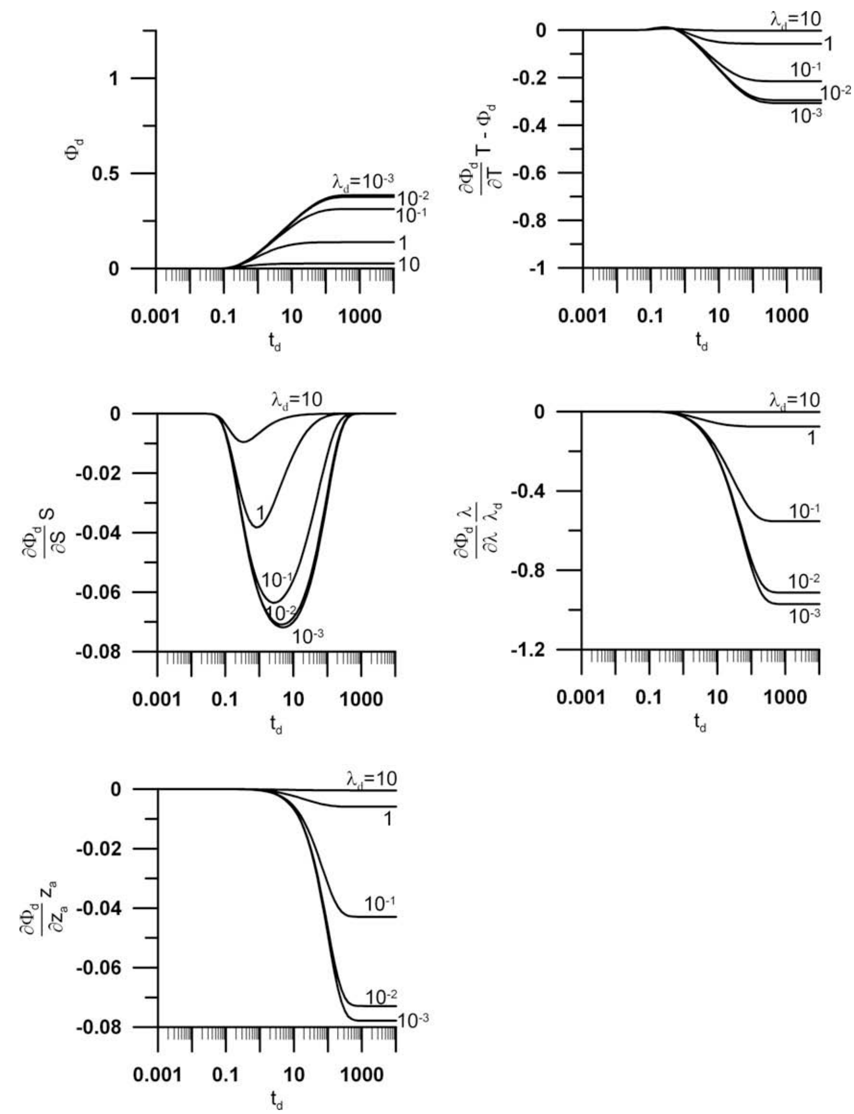

Fig. 5. Curves of dimensionless drawdown and of dimensionless sensitivities of drawdown at $(x_d, y_d) = (0.00, 0.00)$ for $B_d = 10$.

For $B_d = 100$, stream depletion is significant, while leakage through the aquitard is significant only for smaller values of stream-bed conductance, $\lambda_d < 1$ (Fig. 3). If $\lambda_d < 1$, then estimation of the leakage coefficient z_a requires the pumping test to be continued until at least $t_{d\_stop} \approx 1000$; otherwise estimates of z_a are expected to be highly uncertain. For some hydrogeologic conditions, this might require an unrealistically long pumping test. For example, if $T = 10^{-2}$ m^2/s, $S = 0.2$, $l = 100$ m, and $z_a = 10^{-10}$ s^{-1}, then the dimensionless time $t_d = Tt/Sl^2 = 1000$ corresponds to time $t = 2 \times 10^8$ s = 6.3 years.

For $B_d \geqslant 1000$, or for $B_d = 100$ and $\lambda_d \geqslant 1$, leakage from the source bed is negligible and the duration of the pumping test is equal to that obtained for non-leaky aquifers ($t_{d\_stop} \geqslant 10$). This is in agreement with the results of Christensen (2000, Fig. 10), which were obtained using the depletion and drawdown models of Hunt (1999).

Optimum locations to observe drawdown

Table 1 presents optimized locations of either two or three observation wells for a number of combinations of B_d, λ_d, dimensionless test duration t_{d_stop}, and dimensionless time of the target predicted stream depletion t_{d_qd}. The results in Table 1 are obtained for $t_{d\_qd} = 10^5$, which corresponds to a time when steady state has been reached; $t_{d\_qd} = 10$, which is a time when the stream depletion is about 50% of the pumping rate; and an intermediate time $t_{d\_qd} = 10^3$. In all cases, the steady-state level of stream depletion, as computed by the solution of Zlotnik and Tartakovsky (2008), varies from 0.12 (for $B_d = 1$ and $\lambda_d = 1$) to 0.97 (for $B_d = 100$ and $\lambda_d = 1$).

As discussed earlier, optimal observation locations lie along the line perpendicular to the stream passing through the pumping well. Comparison of the values of σ_{qds} (the scaled version of σ_{qd}) in Table 1 reveals that the addition of the third observation well leads to a relatively minor (less than a factor of two) reduction of uncertainty in depletion prediction. In contrast, the reliance on a single observation well results in predictive uncertainty that is an order of magnitude higher than predictive uncertainty arising from the use of two observation wells. This is in agreement with the above analysis of the drawdown sensitivity curves in Figs. 4 and 5, which is to be expected since σ_{qd} is a function of drawdown and depletion sensitivities.

When two observation wells are used, one of them should be placed near the stream ($x_{d\_w1} \approx 0$) if the test duration is sufficiently long to provide a fairly small value of σ_{qds}. However, in some cases, mainly for shorter test durations, it should be located between the pumping well and the stream at a dimensionless distance of 0.2–

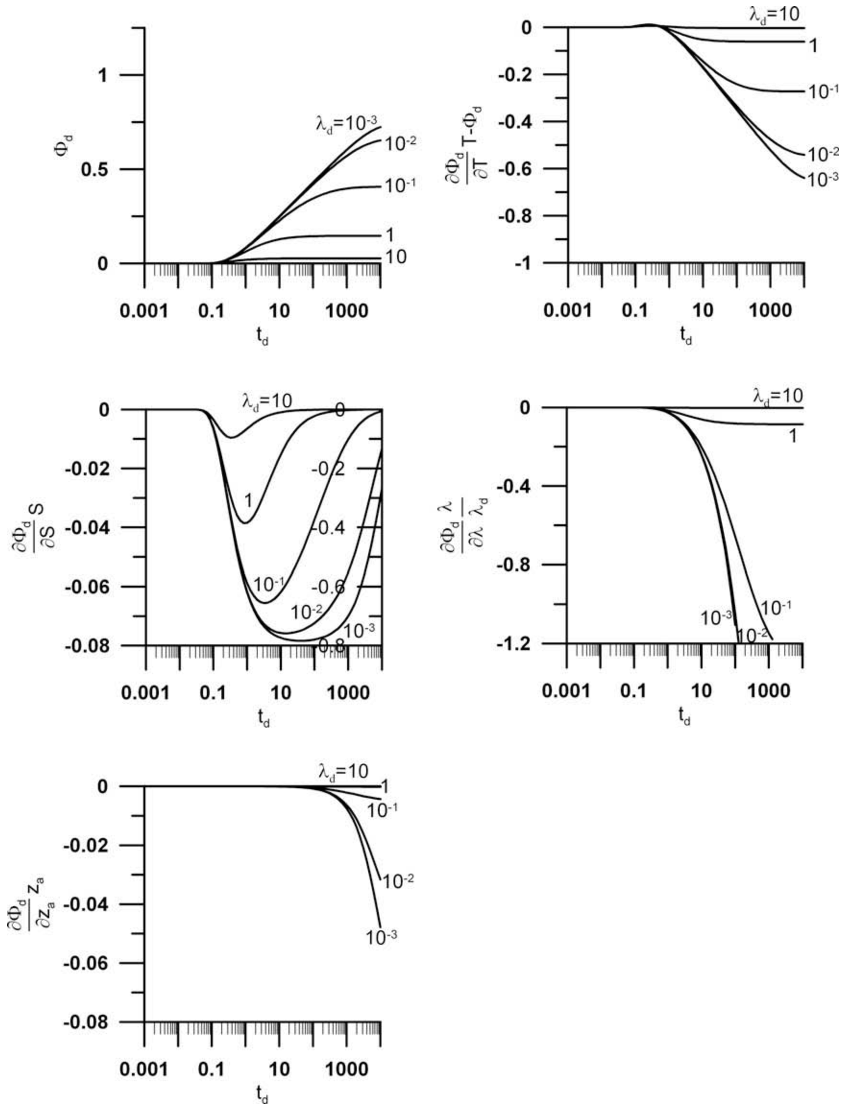

Fig. 6. Curves of dimensionless drawdown and of dimensionless sensitivities of drawdown at $(x_d, y_d) = (0.00, 0.00)$ for $B_d = 100$.

0.5 from the stream. The optimal location for the second observation well is behind the pumping well ($x_{d\_w2} > 1$, i.e. further away from the stream than the pumping well). When the target prediction is the steady-state depletion ($t_{d\_qd} = 10^5$), x_{d_w2} increases with the duration of the pumping test, i.e. the longer the test, the more distant is the optimal location. To a lesser degree, x_{d_w2} increases with B_d and λ_d. If the target prediction is a short-term depletion corresponding to the steep part of the depletion curve ($t_{d\_qd} = 10$ in Table 1), then the optimal location is practically insensitive to values of B_d. This is because short-term predictions are not affected by leakage through the aquitard, i.e. by values of z_a and, thus, B_d (see Figs. 2 and 3). The optimal observation for estimation of the three remaining parameters $T, S,$ and λ, are not significantly influenced by the test duration.

Table 1 also shows that when three observation wells are used, it is almost always optimal to locate one of them near the stream ($x_{d\_w1} \approx 0$). In many cases, the optimal location of the second observation well is near the pumping well ($x_{d\_w2} \approx 1$) and the third well should be behind the pumping well ($x_{d\_w3} > 1$). The optimal distance from the pumping well to the third observation well increases with the pumping test duration. In a few cases reported in Table 1, the optimal locations of the second and third observation wells nearly coincide with each other ($x_{d\_w2} \approx x_{d\_w3}$). In these cases, the optimized values x_{d_w2} and x_{d_w3} are similar to the value of x_{d_w2} for the corresponding two-well observation campaign, and the prediction variance σ_{qd} increases by 5–60% if one sets $x_{d\_w1} = 0.0$ and $x_{d\_w1} = 1.0$ and optimizes x_{d_w3} only.

Analysis of predictive uncertainty

Table 1 shows that the predictive uncertainty, as quantified by σ_{qds}, decreases with t_{d_stop}, the duration of the drawdown observation period. The rate of decrease becomes small beyond a certain value of t_{d_stop}, which can serve to determine a maximum duration of the pumping test. The maximum duration thus defined depends on the hydrogeologic parameters and the time of the target stream flow prediction t_{d_qd} (see Table 1) .

For example, uncertainty in predictions of steady-state depletion ($t_{d\_qd} = 10^5$ for $B_d = 10$ and $\lambda_d = 1$) declines rapidly from $\sigma_{qds} = 164.3$ at $t_{d\_stop} = 1$ to $\sigma_{qds} = 22.48$ at $t_{d\_stop} = 10$ to $\sigma_{qds} = 4.73$ at $t_{d\_stop} = 100$, but much slower after that ($\sigma_{qds} = 2.48$ at $t_{d\_stop} = 1000$). Therefore, one can terminate the test at $t_{d\_stop} = 100$, which is in agreement with the test duration estimated above by analyzing the sensitivity curves.

All the values in Table 1 are for dimensionless variables. Suppose that for a given field site one can expect the hydrogeological

Table 1
CMAES_P optimization of well locations when uncertainty, σ_{qd}, is minimized for stream prediction, $q_d = q_d(t_{d\backslash qd})$. The underlying sensitivities were computed using analytical solutions derived from Zlotnik and Tartakovsky (2008) solutions.

B_d	λ_d	t_{d_stop}	t_{d_qd}	q_d	Two observation wells			Three observation wells			
					x_{d_w1}	x_{d_w2}	σ_{qds}	x_{d_w1}	x_{d_w2}	x_{d_w3}	σ_{qds}
1	1	1	10^5	0.12	0.00	1.04	10.56	0.00	1.00	1.18	6.25
		10			0.00	2.42	3.68	0.00	1.00	1.29	2.56
1	10	10	10^5	0.31	0.00	1.04	4.85	0.00	1.00	1.29	1.81
		100			0.26	2.21	2.82	0.00	1.00	1.34	1.38
10	0.1	10	10^5	0.30	0.00	2.74	28.11	0.00	2.81	2.82	22.30
		100			0.00	2.37	7.06	0.00	1.65	4.94	5.70
		1000			0.00	2.30	4.28	0.00	1.00	7.10	3.05
10	1	1	10^5	0.75	0.34	1.00	164.3	0.00	1.00	1.13	76.51
		10			0.36	2.59	22.48	0.00	1.00	1.98	14.77
		100			0.10	4.86	4.73	0.00	5.14	5.17	3.54
		1000			0.00	6.44	2.48	0.00	6.57	6.57	1.84
10	10	10	10^5	0.89	0.55	2.47	13.51	0.55	2.54	2.55	10.18
		100			0.42	4.49	3.59	0.42	4.68	4.74	2.62
100	0.1	10	10^5	0.83	0.26	2.61	1290	0.00	1.00	2.34	872.8
		100			0.00	3.40	146.0	0.00	3.08	4.38	113.9
		1000			0.95	19.83	29.97	0.00	1.00	14.81	19.66
100	1	1	10^5	0.97	0.42	1.00	2132	0.00	1.00	1.12	1055
		10			0.41	2.55	295.3	0.00	1.00	2.21	193.3
		100			0.20	5.29	49.47	0.22	5.49	5.49	36.60
		1000			0.07	13.39	12.98	0.08	13.84	13.97	9.31
10	1	10	10	0.53	0.06	3.02	4.84	0.04	3.19	3.20	3.81
		100			0.00	3.30	1.83	0.00	0.95	4.50	1.53
		1000			0.01	3.24	1.43	0.00	1.03	4.92	1.03
100	1	10	10	0.54	0.07	3.04	4.87	0.04	3.22	3.22	3.84
		100			0.00	3.31	1.78	0.00	0.95	5.44	1.38
		1000			0.05	2.79	1.34	0.00	0.98	15.54	0.73
10	0.1	100	10	0.11	0.00	2.02	2.23	0.00	1.03	4.69	1.76
			10^2	0.27	0.00	2.23	5.67	0.00	1.43	4.43	4.59
			10^3	0.30	0.00	2.37	7.06	0.00	1.62	4.69	5.71
			10^5	0.30	0.00	2.37	7.06	0.00	1.65	4.94	5.70
10	1	100	10	0.53	0.00	3.30	1.83	0.00	0.95	4.50	1.53
			10^2	0.74	0.06	4.95	3.76	0.04	5.26	5.31	2.82
			10^3	0.75	0.10	4.86	4.73	0.09	5.04	5.10	3.54
			10^5	0.75	0.10	4.86	4.73	0.00	5.14	5.17	3.54

parameters to be on the order of $T \approx 10^{-2}$ m^2/s, $S \approx 0.2$, $\lambda \approx 10^{-4}$ m/s, and $z_a \approx 10^{-8}$ s^{-1}. Furthermore, let us suppose that the pumping well is located l = 100 m from the stream. Then $B_d = \sqrt{T/z_a l^2} \approx 10$ and $\lambda_d = \lambda l/T \approx 1$, and $t_{d\_stop} = Tt/Sl^2 = 100$ corresponds to time $t = t_{d\_stop} Sl^2/T \approx 2 \times 10^7$ s = 231 days. This represents a very long, and often an unrealistically long, pumping test. Basing the prediction on data from a pumping test that has lasted for only one tenth of the time, 23 days, is more realistic, but leads to an almost five-fold increase in the predictive uncertainty.

Table 2 contains dimensional values of z_a, λ, t_{stop}, t_{qd}, and σ_{qd} computed from the B_d, $\lambda_d, t_{d\_stop}, t_{d\_qd}$, and σ_{qds} values in Table 1 by setting Q = 10 m^3/h, σ = 0.01 m, $T = 10^{-2}$ m^2/s, S = 0.2, and l = 100 m. In most cases, the uncertainty σ_{qd} in long-term predictions of depletion ($t_{qd} > 2 \times 10^5$ days) represents a significant fraction (over 25%) of the predicted depletion q_d. Scenarios with uncertainty $\sigma_{qd} < 0.25$ have significant test duration ($t_{stop} > 231$ days), moderate or high aquitard leakage coefficient ($z_a \geqslant 10^{-8}$ s^{-1}), and moderate or high stream-bed conductance ($\lambda \geqslant 10^{-4}$ m/s). For identical test durations, the uncertainty in short-term predictions of stream flow is smaller than the uncertainty in their long-term counterparts because the former predictions depend less on z_a. This is to be expected, since an accurate estimation of z_a requires relatively long pumping tests.

The uncertainties in Table 2 may seem disappointingly high even for long pumping tests. One obvious way to reduce the uncertainty would be to increase the signal-to-noise ratio during the pumping test; that is to increase drawdown caused by pumping (by increasing the pumping rate) and/or to reduce the drawdown measurement error (by making more accurate measurements). For example, if Q/σ were to increase by an order of magnitude (in Table 2, from 10^3 to 10^4 m^2/h) then the uncertainties σ_{qd} are reduced to one tenth of the values shown in Table 2. The resulting predictive uncertainty will be acceptable (1–10%) for all short-term and many long-term predictions listed in Table 2, and a test duration of 23 days is sufficient in many of these cases.

Summary and conclusions

We considered a three-layered leaky aquifer system, in which groundwater is extracted from a top aquifer with a pumping well adjacent to a stream that is hydraulically connected to the top aquifer, and a leaky aquitard separates the top aquifer from a deeper source bed. The Zlotnik and Tartakovsky (2008) model adopted to describe this phenomenon is characterized by the four hydrogeological parameters: the aquifer transmissivity T and storativity S, the stream-bed conductance λ, and the aquitard leakage coefficient z_a. We analyzed an optimal design of a pumping test, whose aim is to estimate these hydrogeologic parameters that are subsequently used to predict stream depletion. The optimal design consists of identifying the number and locations of observation wells and the test duration in a way that minimizes the uncertainty in predictions of stream depletion. The analysis employed an expansion of the optimization procedure of Christensen (2000), with parameter sensitivities computed analytically from the Zlotnik and Tartakovsky (2008) solutions for drawdown and stream depletion.

Our analysis leads to the following major conclusions.

1. Global optimization over all observation locations simultaneously yields locations that are more optimal than those obtained with sequential optimization strategies (e.g. Hill et al., 2001; Tiedeman et al., 2004) that search for optimal locations one well at a time.

Table 2
Dimensionless results in Table 1 recomputed to correspond with a situation where: pumping rate $Q = 10\ m^3/h$; standard deviation of drawdown measurement error $\sigma = 0.01$ m; transmissivity $T = 10^{-2}\ m^2/s$; storativity $S = 0.2$; and distance from pumping well to stream $l = 100$ m. The values in the table are for: leakage coefficient of the aquitard z_a; stream-bed conductance λ; duration of pumping test t_{stop}; predicted stream depletion $q_d = q_d(t_{qd})$ at time t_{qd}; and uncertainty/standard deviation σ_{qd} of q_d prediction based on parameter values determined by pumping test analysis using drawdown observations from two observation wells.

z_a (s^{-1})	λ (m/s)	t_{stop} (d)	t_{qd} (d)	q_d (−)	σ_{qd} (−)	σ_{qd}/q_d (−)
10^{-6}	10^{-4}	2.315	2,314,821	0.12	0.38	3.17
		23.15			0.13	1.10
10^{-6}	10^{-3}	23.15	231,482	0.31	0.17	0.56
		231.5			0.10	0.33
10^{-8}	10^{-5}	23.15	231,482	0.30	1.01	3.37
		231.5			0.25	0.85
		2315			0.15	0.51
10^{-8}	10^{-4}	2.315	231,482	0.75	5.91	7.89
		23.15			0.81	1.08
		231.5			0.17	0.23
		2315			0.09	0.12
10^{-8}	10^{-3}	23.15	231,482	0.89	0.49	0.55
		231.5			0.13	0.15
10^{-10}	10^{-5}	23.15	231,482	0.83	46.44	55.95
		231.5			5.26	6.33
		2315			1.08	1.30
10^{-10}	10^{-4}	2.315	231,482	0.97	76.75	79.13
		23.15			10.64	10.96
		231.5			1.78	1.84
		2315			0.47	0.48
10^{-8}	10^{-4}	23.15	23.15	0.53	0.17	0.33
		231.5			0.07	0.12
		2315			0.05	0.10
10^{-10}	10^{-4}	23.15	23.15	0.54	0.18	0.32
		231.5			0.06	0.12
		2315			0.05	0.09
10^{-8}	10^{-5}	231.5	23.15	0.11	0.08	0.73
			231.5	0.27	0.20	0.76
			2315.	0.30	0.25	0.85
			231,482	0.30	0.25	0.85
10^{-8}	10^{-4}	231.5	23.15	0.53	0.07	0.12
			231.5	0.74	0.14	0.18
			2315.	0.75	0.17	0.23
			231,482	0.75	0.17	0.23

2. The reliance on a single observation well results in predictive uncertainty that is an order of magnitude higher than predictive uncertainty arising from the use of two observation wells. The addition of the third observation well leads to a relatively minor (less than a factor of two) reduction of uncertainty in depletion prediction.
3. Regardless of the number of observation wells, their optimal placement is on the line perpendicular to the stream passing through the pumping well.
4. If two observation wells are envisioned, it is often optimal to place one of them at the stream and the other behind the pumping well. For short test durations, it is optimal to shift the location of the first observation well from the stream in the direction of the pumping well.
5. If an observation network consists of three wells, it is almost always optimal (or nearly optimal) to locate one of them near the stream, another near the pumping well, and the third one behind the pumping well. The optimal distance between the observation wells increases with test duration.
6. The predictive uncertainty (variance of the predicted stream depletion) decreases with the duration of the drawdown observation period. The rate of decrease becomes small beyond a certain time, which can serve to determine a maximum duration of the pumping test. The maximum test duration thus defined depends on the hydrogeologic parameters and the time of the target stream flow prediction.
7. Variance of the predicted stream depletion is inversely proportional to the square of the signal-to-noise ratio of the pumping test. Increasing the pumping rate and/or decreasing the drawdown measurement error can be used to reduce the predictive uncertainty.
8. Drawdown measurements over a relatively short period of time in the vicinity of the pumping well are sufficient to estimate both the aquifer transmissivity T and storativity S.
9. To obtain accurate estimates of the stream-bed conductance λ and the aquitard leakage coefficient z_a, drawdown observations must be made in the vicinity of the stream as well as at a location more distant from the stream in the direction of (or behind) the pumping well, and over a relatively long period of time.
10. Under certain hydrogeological conditions (e.g. relatively small stream-bed conductance and/or aquitard leakage coefficient, or relatively large distance between the pumping well and the stream), the pumping test duration has to be unrealistically long, and the signal-to-noise ratio has to be unrealistically high, to reduce the prediction variance of stream depletion to acceptable levels. In such cases T and S can typically be estimated with the pumping test analysis, while λ and/or z_a have to be estimated by other methods.

Our analysis and conclusions are based on the assumptions that the flow domain is infinite, and drawdown in the deep source bed is negligible. In the companion paper (Christensen et al., in preparation), we investigate the implications of these assumptions on stream depletion, drawdown, their sensitivities, and on pumping test design optimization. Heterogeneity of the aquifer, the aquitard, and the stream-bed is another factor that can significantly affect the reported results.

References

Butler Jr., J.J., Zhan, X., Zlotnik, V.A., 2007. Pumping-induced drawdown and stream depletion in a leaky aquifer system. Ground Water 45 (2), 178–186.
Butler Jr., J.J., Zlotnik, V.A., Tsou, M.S., 2001. Drawdown and stream depletion produced by pumping in the vicinity of a partially penetrating stream. Ground Water 39 (5), 651–659.
Christensen, S., 2000. On the estimation of stream flow depletion parameters by drawdown analysis. Ground Water 38 (5), 726–734.
Christensen, S., Zlotnik, V.A., Tartakovsky, D.M., in preparation. Numerical analysis of implications of designing a pumping test in a leaky aquifer connected to a stream using analytical solutions, Journal of Hydrology.
Doherty, J., 2008. Addendum to the PEST 11 Manual. Watermark Numerical Computing, Brisbane, Queensland, Australia. 2008..
Hansen, N., Ostermeier, A., 2001. Completely derandomized self-adaptation in evolution strategies. Evolutionary Computation 9 (2), 159–195.
Hantush, M.S., 1955. Discussion of river depletion resulting from pumping a well near a river by Robert E. Glover and Glenn G. Balmer. Transaction of the American Geophysical Union 36 (2), 345–346.
Hantush, M.S., 1964. Hydraulics of wells. In: Chow, V. (Ed.), Advances in Hydroscience, vol. 1. Academic Press, New York, pp. 281–432.
Hantush, M.S., 1965. Wells near streams with semipervious beds. Journal of Geophysical Research 70 (12), 2829–2838.
Hill, M.C., Ely, D.M., Tiedeman, C.R., O'Brien, G.M., D'Agenese, F.A., Faunt, C.C., 2001. Preliminary evaluation of the importance of existing hydraulic-head observation locations to advective-transport predictions. Death Valley regional flow system, California and Nevada, US Geological Survey Water Resources Investigations Report 00-4282, 62 p.
Hunt, B., 1999. Unsteady stream depletion from ground water pumping. Ground Water 37 (1), 98–102.
Hunt, B., 2003. Unsteady stream depletion when pumping from semiconfined aquifer. Journal of Hydrologic Engineering 8 (1), 12–19. doi:10.1061/(ASCE)1084-0699.
Hunt, B., 2008. Stream depletion for streams and aquifers of finite widths. Journal of Hydrologic Engineering 13 (2), 80–89. doi:10.1061/(ASCE)1084-0699.
Hunt, B., Weir, J., Clausen, B., 2001. A stream depletion field experiment. Ground Water 39 (2), 283–289.
Jenkins, C.T., 1968. Techniques for computing rate and volume of stream depletion by wells. Ground Water 6 (2), 37–46.
Kollet, S.J., Zlotnik, V.A., 2003. Stream depletion predictions using pumping test data from a heterogeneous stream–aquifer system (a case study from the Great Plains, USA). Journal of Hydrology 281, 96–114.

Kollet, S., Zlotnik, V.A., 2005. Influence of aquifer heterogeneity and return flow on pumping test interpretation. Journal of Hydrology 300, 267–285.
Kollet, S.J., Zlotnik, V.A., 2007. Evaluation of the streambed leakage concept in analytical models using data from three pumping tests. Hydrogeology Journal 15, 1–12. doi:10.1007/s10040-006-0156-7.
Lough, H.K., Hunt, B., 2006. Pumping test evaluation of stream depletion parameters. Ground Water 44 (4), 540–546. doi:10.1111/j.1745-6584.2006.00212.x.
Moench, A.F., 1997. Flow to a well of finite diameter in a homogeneous, anisotropic water table aquifer. Water Resources Research 33 (6), 1397–1407.
Nyholm, T., Christensen, S., Rasmussen, K.R., 2002. Flow depletion in a small stream caused by ground water abstraction from wells. Ground Water 40 (4), 425–437.
Nyholm, T., Rasmussen, K.R., Christensen, S., 2003. Estimation of stream flow depletion and uncertainty from discharge measurements in a small alluvial stream. Journal of Hydrology 274 (1–4), 129–144.
Seber, G.A.F., Wild, C.J., 1989. Nonlinear Regression. John Wiley, New York. 768 pp.
Sophocleous, M., 1997. Managing water resources systems: why "safe yield" is not sustainable. Ground Water 35 (4), 561.
Theis, C.V., 1941. The effect of a well on the flow of a nearby stream. Transactions – American Geophysical Union 22 (Part 3), 734–738.
Tiedeman, C.R., Ely, D.M., Hill, M.C., O'Brien, G.M., 2004. A method of evaluating the importance of system state observations to model predictions, with application to the Death Valley regional groundwater flow system. Water Resources Research 40, W12411. doi:10.1029/2004WR003313.
Zlotnik, V.A., 2004. A concept of maximum stream depletion rate for leaky aquifers in alluvial valleys. Water Resources Research 40, W06507. doi:10.1029/2003WR002932.
Zlotnik, V.A., Huang, H., 1999. Effect of shallow penetration and streambed sediments on aquifer response to stream stage fluctuations (analytical model). Ground Water 37 (4), 599–605.
Zlotnik, V.A., Tartakovsky, D.M., 2008. Stream depletion by groundwater pumping in leaky aquifers. ASCE Journal of Hydrologic Engineering 13, 43–50.