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# Response of Well-Aquifer Systems to Earth Tides

A semidiurnal wave is a type of tidal wave that exhibits two high tides and two low tides during a 24-hour period.

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The water level in a well open to an artesian aquifer responds to pressure-head fluctuations caused by the dilatation of the aquifer. Based on hydrologic considerations, it is shown that (1) most well-aquifer systems respond to disturbances with periods of less than several days as if the well were drilled into a medium of infinite or partially bounded extent, and (2) the representation of an aquifer as a finite cavity is unrealistic for most well-aquifer systems. The amplitude of tidal water level fluctuations in well-aquifer systems depends on the dilatation and the specific storage of the aquifer. Analysis of the dilatation caused by the earth tide is based on the assumptions that (1) the latitudinal and longitudinal strains caused by the earth tide are determined by the elastic properties of the earth as a whole and are largely independent of the elastic properties of a near surface aquifer, and (2) the vertical strain of a near surface aquifer depends on Poisson's ratio (or the Lamé constants) for the aquifer and the latitudinal and longitudinal strains. The tidal dilatation can be computed from equilibrium tide theory provided that Poisson's ratio is known. The amplitude of the tidal dilatation produced by the large semidiurnal wave, $M_2$, is approximately $1 \times 10^{-8}$. It is not unusual to have earth-tide fluctuations in wells corresponding to $M_2$ with an amplitude of 1 to 2 cm. The fact that tidal water-level fluctuations depend upon the specific storage of the aquifer explains the variation in amplitude which troubled others. The specific storage and the porosity of the aquifer can be computed from an analyses of earth-tide fluctuations if Poisson's ratio for the aquifer is known. Computations of specific storage and porosity are presented for three artesian wells for which tidal harmonic analysis of hydrograph data was published by Melchior. The results of these calculations appear to be better than one might reasonably expect.

## NOTATION

$a$, radius of the earth $(L)$.

$B$, barometric efficiency (dimensionless).

$b$, saturated thickness of water table aquifer $(L)$.

$d$, aquifer thickness $(L)$.

$E_s$, modules of compression of soil skelton confined in situ $(M/LT^2)$.

$E_w$, bulk modulus of elasticity of water $(M/LT^2)$.

$G$, gravitational constant $(L^3/MT^2)$.

$g$, gravity field strength $(L/T^2)$.

$H$, height of water column in well casing $(L)$.

$h$, pressure head $(L)$.

$\bar{h}$, $\bar{k}$, $\bar{l}$, Love number at earth's surface (dimensionless).

$H(r)$, $L(r)$, $K(r)$, $F(r)$, Love numbers (dimensionless).

$n$, porosity (dimensionless).

$P$, fluid pressure $(M/LT^2)$.

$Q$, quantity of water $(L^3)$.

$r$, radial distance in spherical coordinates $(L)$.

$r_w$, radius of well $(L)$.

$S_s$, specific storage $(1/L)$.

$t$, time $(T)$.

$u_r$, $u_\theta$, $u_\varphi$, radial, latitudinal, and longitudinal displacements $(L)$.

$V$, volume of effective cavity $(L^3)$.

$W_2$, lunar/solar disturbing potential $(L^2/T^2)$.

$\Delta$, total dilatation (dimensionless).

$\Delta_h$, dilatation produced by change in fluid pressure (dimensionless).

$\Delta_t$, tidal dilatation (dimensionless).

$\epsilon_{xx}$, $\epsilon_{yy}$, $\epsilon_{zz}$, strains in $x$, $y$, and $z$ directions (dimensionless).

$\epsilon_{rr}$, $\epsilon_{\theta\theta}$, $\epsilon_{\varphi\varphi}$, strains in radial, latitudinal, and longitudinal directions (dimensionless).

$\theta$, colatitude (radians).

$\varphi$, longitude (radians).

$\lambda$ and $\mu$, Lamé constants $(M/LT^2)$.

$\nu$, Poisson's ratio (dimensionless).

$\rho$, density of water $(M/L^3)$.

$\sigma_{rr}$, $\sigma_{\theta\theta}$, $\sigma_{\varphi\varphi}$, stresses in radial, latitudinal, and longitudinal directions $(M/LT^2)$.

## INTRODUCTION

*General.* It is a well established fact that the water levels in many wells are affected by earth tides. This effect is of interest to geophysicists because it may provide a means of studying earth tides. In many instances tidal water-level fluctuations in wells are sufficiently large to be easily measured and recorded. To the hydrologist the tidal response of wells provides another means of studying aquifer mechanics.

Advances in the theory of aquifer mechanics since earlier investigations permit treatment of more realistic aquifer models. The purpose of this paper is to reexamine the fluctuation of water levels in wells, caused by earth tides, in terms of what we now consider a realistic hydrologic model.

The study of earth tides in wells has been the subject of a number of investigations. *Klonne* [1880] reported on water-level fluctuations (which were of a tidal nature) in a flooded coal mine near Duchov, Czechoslovakia. *Grablovitz* [1880], considering Klonne's observation, attributed the fluctuations to the dilatation produced by the tide.

*Young* [1913] reported on a well near Cradock, South Africa, in which fluctuations of a tidal nature in excess of 6 cm were observed. Young, following the results of *Michelson and Gale* [1919], recognized that the fluctuations observed at Cradock were due to earth tides. *Theis* [1939] pointed out that Young had penciled the following note in the copy of his paper sent to the U. S. Geological Survey: 'Since the publication of the Michelson-Gale results, I have had little doubt that the Cradock tides result from earth tides · · ·' April 1927.

*Robinson* [1939] published hydrographs of several wells in New Mexico and Iowa which show earth tides. Barometric effects were removed in these investigations, producing a residual water level that was clearly related to earth tides.

*Theis* [1939] working with Robinson's data and other data from Carlsbad, New Mexico, made an analysis of the mechanism causing the earth-tide fluctuation. Theis recognized that the water-level fluctuations could only be caused by the dilatation that accompanies the tidal bulge.

*Lambert* [1940] devoted one chapter of his *Report on Earth Tides* to the fluctuations in wells. He discussed the previous work by Klonne, Grablovitz, Young, Robinson, and Theis.

*Pekeris* [1940] in an appendix to Lambert's report gave a theoretical analysis of the problem. Pekeris based his analysis on previous solutions of the earth-tide problem by *Hoskins* [1920] and *Stoneley* [1926]. Unfortunately Stoneley's solution does not lead to realistic values of the Love numbers; therefore, Pekeris' values for the necessary constants are in error. Pekeris did, however, show the relationship between the dilatation and the lunar-solar disturbing potential. He pointed out that the magnitude of lunar semidiurnal tidal dilatation is approximately $1.5 \times 10^{-8}$. In relating his results to wells, he assumed that an aquifer can be represented by a finite open cavity that contracts and expands during the passage of disturbances causing dilatation. According to this theory, the magnitude of the fluctuations depends on the volume of the effective cavity.

*George and Romberg* [1951] investigated the tidal response of a well at Fort Stockton, Texas. They made simultaneous water-level measurements and gravity observations for a 48-hour period. Their comparison indicates that the water-level fluctuations are in part the result of earth tides.

*Richardson* [1956] reported on the tidal fluctuations in a well at Oak Ridge, Tennessee. This well penetrates a water table aquifer and has a semidiurnal tidal fluctuation with an amplitude of approximately 2 cm.

*Melchior's investigations.* Recently Melchior has made a number of investigations of tidal fluctuations in wells [*Melchior*, 1956, 1960; *Melchior et al.*, 1964]. His work has been extensive, but, unfortunately, the papers are published in French and German and may not have received the attention they deserve in this country.

*Melchior* [1956] discussed the earth tides in a deep well (2000 meters) at Turnhout, Belgium, and in a hot spring at Kiabukwa, Belgian Congo. The tidal fluctuations at Turnhout have a total amplitude of 6 cm; the amplitude of tidal fluctuation at Kiabukwa is 17 cm. In this paper Melchior reviewed the earlier work, indicated that tidal fluctuations in wells and mines are due to the tidal dilatation, and made some theoretical calculations on the magnitude of the dilatation. He performed a harmonic analysis of the fluctuation in the hot spring at Kiabukwa and compared the amplitude of the major tidal component waves with the amplitude of $M_2$, the large semidiurnal lunar wave. The amplitude ratios for the larger waves agree reasonably well with values predicted from the equilibrium tide theory.

In 1960, Melchior published the results of further harmonic analyses of tidal fluctuations reported by previous investigators. He analyzed Robinson's and Theis' data from Carlsbad, New Mexico; Robinson's data from Iowa City, Iowa; Richardson's data from Oak Ridge, Tennessee; Klonne's data from Duchov, Czechoslovakia; and his own data from Turnhout, Belgium, and Kiabukwa, Belgian Congo. Comparison of the relative amplitudes of the major waves showed a reasonable agreement with the ratios predicted from equilibrium theory. Melchior's studies indicate that the well is responding to dilatation produced by the earth tide.

Melchior's ideas on the response of a well to earth tides are best expressed in his 1960 paper. His analysis is based on the representation of the aquifer as a finite cavity, an assumption made previously by the geophysicists *Blanchard and Byerly* [1935] in considering the fluctuations in water wells caused by the dilatation accompanying some seismic waves. *Melchior* [1960], drawing on the analysis of Blanchard and Byerly, developed the following relationship:

$$dH = \frac{dV}{\pi r_w^2 + (\rho g V/E_w)} \tag{1}$$

where $dH$ is the displacement of the water caused by a change $dV$ in the volume, $V$, of the finite cavity. Melchior pointed out that as the volume, $V$, becomes increasingly large, the term $\pi r_w^2$ has less and less importance, and (1) reduces to

$$dH \cong \Delta E_w/\rho g \tag{2}$$

where $\Delta = dV/V$ is the dilatation.

Using this relationship, Melchior indicated that for the semidiurnal lunar component of the tide, $M_2$, which produces a dilatation with an amplitude of approximately $2 \times 10^{-8}$, one would expect the amplitude of the corresponding water-level fluctuation to be approximately 0.4 cm. Melchior pointed out, however, that the observed amplitude of the $M_2$ fluctuation in the wells for which he had data is closer to 2 cm. This discrepancy between the observed and expected amplitudes is fully discussed below.

In another study *Melchior et al.* [1964] made simultaneous gravity and water-level measurements in a well near Basecles, Belgium, from June to November 1962. These data were studied by harmonic analysis. The ratio of the observed amplitude of the principal waves was compared with the amplitude of the change in the gravitational acceleration computed from equilibrium theory. This ratio remained essentially constant, again indicating that the observed tidal fluctuations are produced by the earth tide. In this investigation a shift in phase of up to 25° was found between the theoretical dilatation produced by the individual waves and the harmonic components reduced from the data. This phase shift is difficult to explain.

*Melchior et al.* [1964] compared the amplitude of the $M_2$ wave in a number of wells by adjusting all the observations to a common latitude, the equator. When this adjustment is made, the amplitude apparently increases with the well depth.

## Hydrologic Considerations

It is convenient in hydrology to think in terms of two ideal aquifer systems: (1) the confined or artesian system and (2) the unconfined or water table system.

*Confined aquifers.* The problem of groundwater flow in confined aquifers is the classic problem in hydrology. The ideal system, shown in Figure 1, consists of an extensive porous and permeable water saturated medium, an aquifer, that is overlain and underlain by impermeable material. In the usual case the distance to the aquifer outcrop is sufficiently large, and the system is considered infinite.

The fluid pressure in the aquifer is usually such that water will rise above the top of the aquifer in a well drilled into the aquifer. The height of a fluid column, $H$, is just sufficient to balance the pressure in the aquifer and is given by $H = p/\rho g$, where $p$ is the fluid pressure at the base of column.

It is commonly assumed [see *Jacob*, 1940, p. 583] that, when the aquifer is compressed, the change in volume of the solid material due to deformation of the individual particles is small in comparison with the change in volume of the water. This assumption is apparently valid for granular aquifers, but, depending on the pore geometry and Poisson's ratio, it may not be valid for such aquifers as limestone and basalt. To the extent that the above assumption holds

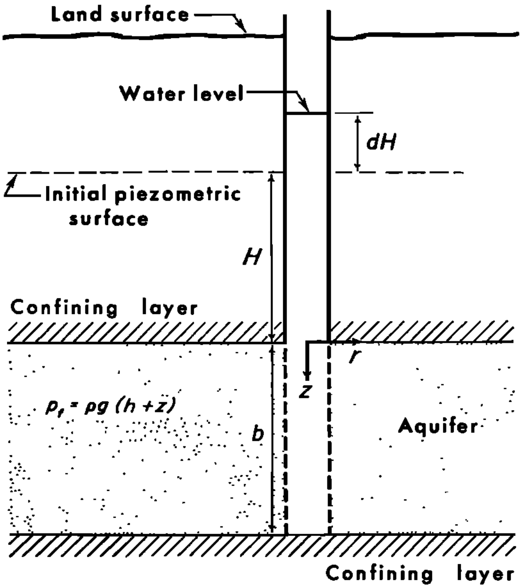


Fig. 1. Idealized representation of an observation well drilled into a confined aquifer.

for a given aquifer, nearly all the volume change of a dilated aquifer goes into a volume change of the water in its pores. Hence, the dilatation of the water is nearly equal to that of the aquifer divided by the porosity. The change in pressure head in the aquifer, $-dh$, due to a given dilatation, $\Delta$, is therefore

$$-dh = -dp/\rho g = \frac{(\Delta E_w/n)}{\rho g}$$

where $h$ is defined as positive upward (Figure 1).

The dilatation $\Delta$ is defined in the conventional manner as the sum of the normal strains

$$\Delta = \epsilon_{xx} + \epsilon_{yy} + \epsilon_{zz}$$

It is often assumed for problems of well hydraulics that the well has an infinitesimal diameter. Clearly when this assumption is made, the change of water level in the well, $dH$, is equal to the change in pressure head in the aquifer, $dh$. When the well is of finite diameter, however, the effect of the inertia of the mass of water in the well and the capacity of the well to store water must be considered. In recent investigations of well hydraulics by *Cooper et al.* [1965] and *Bredehoeft et al.* [1966] these effects have been considered and evaluated.

The theory necessary to describe the response of a finite-diameter well to a sinusoidal fluctuation of artesian pressure is presented in the paper by *Cooper et al.* [1965]. Although the analysis was developed for seismic disturbances, it applies equally well to earth tides. If the friction losses in the well are neglected, the equation of motion of the column of water in the cased portion of the well is closely approximated by [*Cooper et al.*, 1965, p. 3920]

$$\frac{H_e}{g}\frac{d^2x}{dt^2} + x = -s + \frac{p_0}{\rho g}\sin(\omega t - \eta)$$

where $H_e = H + (3/8)d$ = effective column height, $d$ = thickness of screened interval, $x$ = displacement of the water level (positive upward), $s$ = drawdown in the aquifer immediately outside the well screen (positive downward), $p_0 \sin(\omega t - \eta)$ = periodic fluctuation in pressure in the aquifer, $\omega = 2\pi/\tau$ = angular frequency of seismic wave, $\tau$ = period of seismic wave, $\eta$ = phase angle, and $t$ = time. After no more than one oscillation the drawdown $s$ is closely described by

$$s_x = \frac{r_w^2}{2T}\left(\frac{dx}{dt}\operatorname{Ker}\alpha_w - \omega x \operatorname{Kei}\alpha_w\right)$$

where $T$ = transmissibility of aquifer, $S$ = storage coefficient of aquifer, and $\alpha_w = r_w\sqrt{\omega S/T}$. The functions Ker $\alpha$ and Kei $\alpha$, sometimes called Kelvin functions, are the real and imaginary parts of $K_0(\alpha i^{1/2})$, which is the modified Bessel function of the second kind of order zero.

The dynamics of the well-aquifer system are such that the water-level fluctuations in the well can magnify or diminish the pressure-head fluctuations in the aquifer. The amplification factor $A$ is defined as the ratio of the amplitude $x_0$ of the oscillation of the water level in the well to the amplitude, $h_0 = p_0/\rho g$, of the pressure-head fluctuation in the aquifer. Accordingly,

$$A = \frac{x_0}{h_0} = \frac{\rho g x_0}{p_0} = \text{amplification factor}$$

From the equation of motion this amplification factor is found [*Cooper et al.*, 1965, p. 3921] to be

$$A = 1\bigg/\left[\left(1 - \frac{\pi r_w^2}{T\tau}\operatorname{Kei}\alpha_w - \frac{4\pi^2 H_e}{\tau^2 g}\right)^2 + \left(\frac{\pi r_w^2}{T\tau}\operatorname{Ker}\alpha_w\right)^2\right]^{1/2}$$

It is clear from the work by Cooper et al. that for artesian aquifers where $S$ is usually about 0.0001 the amplification factor for an artesian well with $T/r_w^2 \approx 1.0$ sec$^{-1}$ is equal to 1.0 for waves with periods in excess of several minutes. However, the question arises for earth tides as to how small the factor $T/r_w^2$ can become before the amplification factor becomes smaller than 1. Figure 2 is a graph of the amplification factor $A$ versus the wave period $\tau$ for artesian wells with small $T/r_w^2$ ratios computed from the amplification factor given above. The results of these computations indicate, for example, that for a well of radius $r_w = 30.5$ cm (1 ft) the transmissibility of the aquifer would have to be less than 0.929 cm$^2$/sec (0.001 ft$^2$/sec or approximately 600 gpd/ft) in order for the amplification factor for a 12-hour wave to be less than 0.9.

The computations shown in Figure 2 are for a well with $H_e = 30.5$ meters. For waves with period in excess of 1000 sec, however, the height of the fluid column has little or no effect on the amplification curves. For $\tau > 1000$ sec the amplification curve for a well with an $H_e$ of 3.05 meters is essentially the same as the amplification curve for a well with an $H_e$ of 3050 meters.

For the very long period waves, such as earth tides, the inertial effects are negligible, and the motion in the well is in phase with the pressure-head change in the aquifer.

It follows that for wells in aquifers with transmissibilities in excess of about 1 cm$^2$/sec (approximately 0.001 ft$^2$/sec) the change in pressure head due to the earth tide is equal to the change in water level in the well; therefore,

$$-dH = -dh = \frac{\Delta E_w/n}{\rho g} \tag{3}$$

*Unconfined aquifers.* An idealized unconfined aquifer system is shown in Figure 3. In this case a dilatation of the aquifer produces a change in height of the free water surface, $db$. For a well of infinitesimal diameter penetrating a permeable isotropic aquifer of limited vertical extent, the change in height of the water surface is related to the dilatation in the following manner:

$$-db = (\Delta/n)b \tag{4}$$

Equation 4 implies that, as the saturated

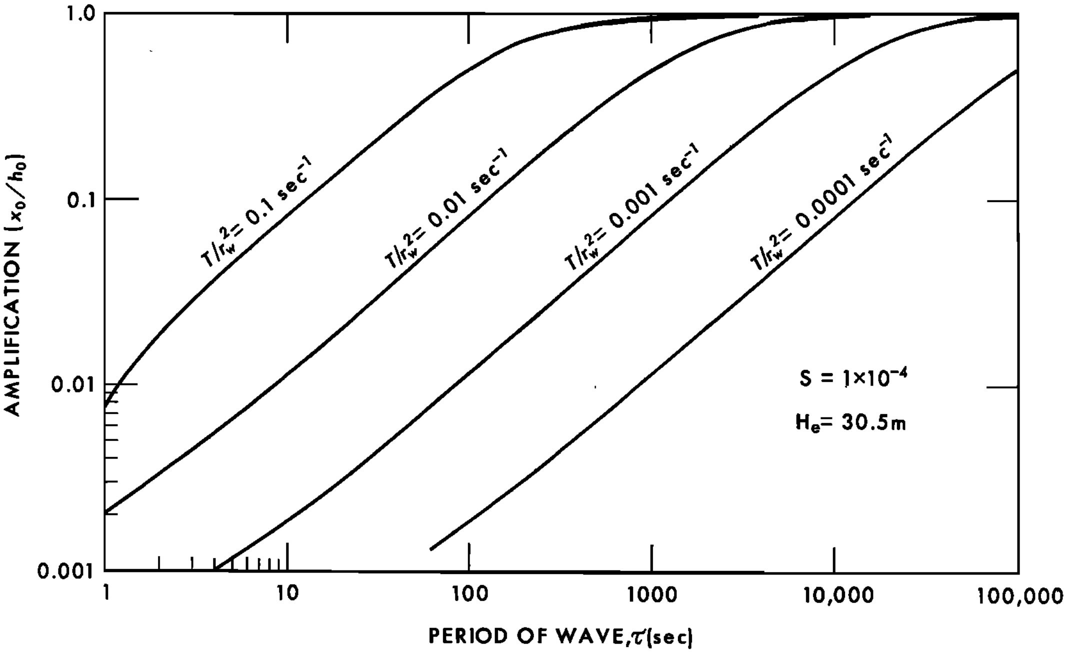


**Fig. 2.** Amplification of pressure-head fluctuation.

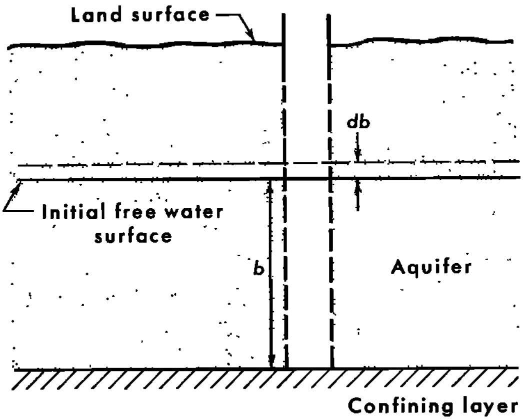


Fig. 3. Idealized representation of an observation well drilled into an unconfined aquifer.

thickness $b$ increases, the change in height of the free water surface $db$ produced by a given dilatation also increases. In a well-aquifer system, which behaves approximately as (4) implies, the porosity must be small or the thickness must be large in order for a tidal fluctuation to be observed. Consider an aquifer in which the saturated thickness is 100 meters, $b = 1 \times 10^4$ cm. Since the earth tide dilatation is approximately $1 \times 10^{-8}$, (4) becomes

$$-db = (1 \times 10^{-4})/n$$

This relation indicates that the porosity must be of the order of $10^{-4}$ to have tidal fluctuations with an amplitude of 1 cm. The tidal fluctuations reported by *Richardson* [1956] for a water table well at Oak Ridge, Tennessee, suggest that the aquifer there must have a small porosity.

As $b$ becomes large or the permeability becomes small, the above relationship ceases to apply, and the problem must be treated as a nonsteady phenomenon. The problem is further complicated by the fact that many aquifers are stratified, which commonly reduces the vertical permeability in an undetermined manner.

Both the confined and the unconfined aquifer models discussed above only approximate actual field conditions. All earth materials have some permeability; therefore, the aquifer confinement is never perfect. However, many geologic units, such as clay, shale, evaporite deposits, and some carbonate rocks, have small permeabilities and often provide nearly perfect confinement for an aquifer. Many aquifer systems behave, at least for periods of days or years, as the idealized artesian system pictured in Figure 1. The remainder of the paper deals almost exclusively with artesian systems.

## Tidal Deformation

The problem of the tidal deformation of the earth is simplified by the introduction of the Love numbers $H(r)$, and $L(r)$, which relate the actual displacements to the displacements predicted from equilibrium tide theory. $K(r)$ a third Love number relates the tide producing potential to the change in potential of the earth produced by the deformation. Although the Love numbers are functions of $r$ at the earth's surface, they are considered constants and designated by $\bar{h}$, $\bar{k}$, and $\bar{l}$. The following relationships describe the tidal displacements of the earth's surface

$$u_r = \bar{h}\, W_2/g \tag{5}$$

$$u_\theta = (\bar{l}/g)(\partial W_2/\partial \theta) \tag{6}$$

$$u_\varphi = (\bar{l}/g) \sin \theta(\partial W_2/\partial \varphi) \tag{7}$$

where $r$, $\theta$, and $\varphi$ are the spherical coordinates of a point referred to the center of the earth and the polar axis, $\theta$ is the colatitude, and $\varphi$ is east longitude.

In theory the Love numbers may be determined if the density distribution and the distribution of elastic properties within the earth are known. *Takeuchi* [1950] using two earth models proposed by Bullen evaluated the Love numbers by numerical integration of the basic differential equations of deformation. In practice, however, knowledge of the Love numbers comes from observations of the various tidal disturbances.

*Relationship between stress and strain.* For an isotropic medium the normal stresses are related to the normal strains in the following manner [see *Sokolnikoff*, 1956, p. 180]:

$$\begin{aligned} \sigma_{rr} &= \lambda\Delta + 2\mu\epsilon_{rr} \\ \sigma_{\theta\theta} &= \lambda\Delta + 2\mu\epsilon_{\theta\theta} \\ \sigma_{\varphi\varphi} &= \lambda\Delta + 2\mu\epsilon_{\varphi\varphi} \end{aligned} \tag{8}$$

At a free surface the radial stress is equal to zero, and

$$\sigma_{rr} = 0 = \lambda\Delta + 2\mu\epsilon_{rr}$$

Therefore, at the free surface the radial strain is equal to

$$-\epsilon_{rr} = \Delta(\lambda/2\mu) \tag{9}$$

In spherical coordinates the dilatation is

$$\Delta = \epsilon_{rr} + \epsilon_{\theta\theta} + \epsilon_{\varphi\varphi} \tag{10}$$

In the vicinity of the free surface the radial strain is given as

$$-\epsilon_{rr} = \frac{\lambda}{2\mu}(\epsilon_{rr} + \epsilon_{\theta\theta} + \epsilon_{\varphi\varphi})$$

which reduces to

$$-\epsilon_{rr} = \frac{\lambda}{2\mu + \lambda}(\epsilon_{\theta\theta} + \epsilon_{\varphi\varphi}) \tag{11}$$

For convenience the Lamé constants can be expressed in terms of Poisson's ratio $\nu$ [see *Jaeger,* 1956, p. 57]

$$\lambda/\mu = 2\nu/(1 - 2\nu)$$

The radial strain (equation 11) is rewritten in terms of Poisson's ratio as

$$\epsilon_{rr} = \frac{-\nu}{1 - \nu}(\epsilon_{\theta\theta} + \epsilon_{\varphi\varphi}) \tag{12}$$

The dilatation near the free surface is expressed in terms of the latitudinal and longitudinal strains as

$$\Delta = \frac{-\nu}{1 - \nu}(\epsilon_{\theta\theta} + \epsilon_{\varphi\varphi}) + \epsilon_{\theta\theta} + \epsilon_{\varphi\varphi}$$

which simplified is

$$\Delta = \left(\frac{1 - 2\nu}{1 - \nu}\right)(\epsilon_{\theta\theta} + \epsilon_{\varphi\varphi}) \tag{13}$$

*Dilatation in a 'dry' formation.* Dilatation in an aquifer will depend not only on the tidal strain but also on the effect of the change in internal fluid pressure produced by the tidal dilatation. For analysis it is convenient (1) to consider the dilatation that would occur if the fluid were not present, and (2) then to consider the dilatation produced by the change in pressure head.

Neglecting for the moment the saturating fluid, consider an aquifer near the earth's surface subjected to earth tides. The aquifer will be subjected to tidal strains in the latitudinal and longitudinal directions that are almost entirely determined by the elastic properties of the earth as a whole. These strains in the plane of the aquifer are assumed independent of the elastic properties of the aquifer and are in spherical coordinates [*Love,* 1944, p. 56].

$$\epsilon_{\theta\theta} = \frac{1}{r}\left(\frac{\partial u_\theta}{\partial \theta} + u_r\right) \tag{14}$$

$$\epsilon_{\varphi\varphi} = \frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial u_\varphi}{\partial \varphi} + u_\theta\frac{\cos\theta}{\sin\theta} + u_r\right) \tag{15}$$

The displacements $u_r$, $u_\theta$, and $u_\varphi$ are given by (5), (6), and (7) in terms of the Love numbers $\bar{h}$ and $\bar{l}$.

Although the aquifer is displaced radially an amount $u_r$, we assume that the radial strain within the aquifer, $\epsilon_{rr}$, is independent of this displacement and depends only on the strains in the plane of the aquifer ($u_\theta$ and $u_\varphi$) and Poisson's ratio for the aquifer itself. The tidal dilatation based on these assumptions is given by (13) and can be expressed in terms of the displacements $u_\theta$ and $u_\varphi$ (equations 14 and 15) as

$$\Delta_t = \left(\frac{1 - 2\nu}{1 - \nu}\right)\left\{\frac{1}{r}\left[2u_r + \frac{\partial u_\theta}{\partial\theta} + u_\theta\frac{\cos\theta}{\sin\theta} + \frac{1}{\sin\theta}\frac{\partial u_\varphi}{\partial\varphi}\right]\right\} \tag{16}$$

*Love* [1911, p. 52] indicated that the tide generating potential $W_2$ may be approximated with sufficient accuracy as a spherical solid harmonic of second degree. It is convenient to express $W_2$ as

$$W_2 = r^2 S_2$$

where $S_2$ is a spherical surface harmonic. The displacements may then be expressed in terms of $W_2$ and $S_2$ as [*Melchior,* 1956, p. 24]

$$u_r = \bar{h}\frac{W_2}{g} \tag{17}$$

$$u_\theta = \bar{l}\frac{r^2}{g}\frac{\partial S_2}{\partial\theta} \tag{18}$$

$$u_\varphi = \bar{l}\frac{r^2}{g\sin\theta}\frac{\partial S_2}{\partial\varphi} \tag{19}$$

where $\theta$ is the colatitude

The dilatation (15) is then given as

$$\Delta_t = \left(\frac{1-2\nu}{1-\nu}\right)\left\{\frac{1}{r}\left[2\bar{h}\,\frac{W_2}{g} + \frac{\partial}{\partial\theta}\left(\frac{\bar{l}r^2}{g}\,\frac{\partial S_2}{\partial\theta}\right)\right.\right.$$

$$\left.\left. + \frac{\bar{l}r^2}{g}\,\frac{\cos\theta}{\sin\theta}\,\frac{\partial S_2}{\partial\theta} + \frac{1}{\sin\theta}\,\frac{\partial}{\partial\varphi}\left(\frac{\bar{l}r_2}{g}\,\frac{1}{\sin\theta}\,\frac{\partial S_2}{\partial\varphi}\right)\right]\right\}$$

which simplified becomes

$$\Delta_t = \left(\frac{1-2\nu}{1-\nu}\right)$$

$$\cdot\frac{1}{r}\left\{2\bar{h}\,\frac{W_2}{g} + \frac{\bar{l}r^2}{g}\left[\frac{1}{\sin\theta}\,\frac{\theta}{\partial\theta}\left(\sin\theta\,\frac{\partial S_2}{\partial\theta}\right)\right.\right.$$

$$\left.\left. + \frac{1}{\sin\theta}\,\frac{\partial}{\partial\varphi}\left(\frac{1}{\sin\theta}\,\frac{\partial S_2}{\partial\varphi}\right)\right]\right\}$$

*Melchior* [1956, p. 24] pointed out that, since the function is a spherical harmonic of second degree,

$$\left[\frac{1}{\sin\theta}\,\frac{\partial}{\partial\theta}\left(\sin\theta\,\frac{\partial S_2}{\partial\theta}\right) + \frac{1}{\sin\theta}\,\frac{\partial}{\partial\varphi}\left(\frac{1}{\sin\theta}\,\frac{\partial S_2}{\partial\varphi}\right)\right] = -6S_2$$

The dilatation is, therefore,

$$\Delta_t = \left(\frac{1-2\nu}{1-\nu}\right)\frac{1}{r}\left\{2\bar{h}\,\frac{W_2}{g} + \frac{\bar{l}r^2}{g}\,[-6S_2]\right\}$$

and, since $S_2 = W_2/r^2$, the dilatation at the earth's surface is

$$\Delta_t = \left(\frac{1-2\nu}{1-\nu}\right)\left\{(2\bar{h} - 6\bar{l})\,\frac{W_2}{ag}\right\} \qquad (20)$$

where $a$ is the earth's radius.

*Love* [1911, p. 53] showed that the dilatation can be related to the disturbing potential by introducing a fourth Love number, $F(r)$, where

$$\Delta_t = F(r)\,\frac{W_2}{g}$$

*Takeuchi* [1950] evaluated $F(r)$ by numerical integration of the differential equations that relate the displacement to the density, $\rho(r)$, and the Lamé constants, $\mu(r)$ and $\lambda(r)$, all of which are functions of $r$. *Melchior* [1956] outlined Takeuchi's method and summarized his results relating to the dilatation. Takeuchi's numerical calculations indicate that near the earth's surface the dilatation is given by

$$\Delta_t = 0.49(W_2/ag)$$

This value obtained by Takeuchi

$$F(r)\Big|_{r=a} = 0.49$$

depends on the earth model assumed, i.e. the assumed distribution of $\rho(r)$, $\lambda(r)$, and $\mu(r)$. Near the surface Takeuchi used values for $\lambda \approx 0.6 \times 10^{12}$ dynes/cm² and $\mu \approx 0.6 \times 10^{12}$ dynes/cm², which means that Poisson's ratio is $\nu \approx \frac{1}{4}$. Using this Poisson's ratio and the values of the Love numbers $\bar{h}$ and $\bar{l}$ considered by *Takeuchi* [1950, p. 688] as most reliable ($\nu = \frac{1}{4}$, $\bar{h} = 0.60$, and $\bar{l} = 0.07$), equation 20 gives for the dilatation

$$\Delta_t \approx 0.5(W_2/ag)$$

which agrees rather well with Takeuchi's calculations.

*Total dilatation.* The total dilatation in a saturated confined aquifer is the sum of the tidal dilatation $\Delta_t$ plus the dilatation produced by the change in fluid pressure $\Delta_h$

$$\Delta = \Delta_t + \Delta_h \qquad (21)$$

The dilatation produced by the change in fluid pressure is

$$\Delta_h = -dp/E_s \qquad (22)$$

where $E_s$ is defined as the modulus of compression of the soil skeleton as confined in situ [*Jacob*, 1940]. The modulus of compression $E_s$ is defined by

$$E_s = \sigma_{rr}/\Delta_h$$

where the horizontal strains $\epsilon_{\theta\theta}$ and $\epsilon_{\varphi\varphi}$ are equal to zero. It can be shown that $E_s$ is given in terms of the Lamé constants as

$$E_s = \lambda + 2\mu$$

and relates $E_s$ to the more commonly used elastic constants such as Young's modulus and Poisson's ratio.

The total dilatation $\Delta$ is related to the change in pressure by (3); and, using (3), (21), and (22), we obtain an expression for the dilatation

$$\Delta_t = n\,\frac{dp}{E_w} + \frac{dp}{E_s}$$

which, when we introduce the pressure head $p = \rho g h$, reduces to

$$\Delta_t = -\rho g\left(\frac{1}{E_s} + \frac{n}{E_w}\right) dh \qquad (23)$$

*Jacob* [1940] and *Cooper* [1966] have shown that the specific storage $S_s$, used in groundwater hydrology, is

$$S_s = \rho g\left(\frac{1}{E_s} + \frac{n}{E_w}\right) \qquad (24)$$

The specific storage $S_s$ is defined as the quantity of water that is released or taken into storage from a unit aquifer volume per unit change in head, or

$$S_s = \frac{1}{V}\frac{dQ}{dh}$$

where $dQ/V$ is the volume of water released or taken into storage per unit volume.

The change in head produced by the tidal dilatation $\Delta_t$ is from (23)

$$-dh = \Delta_t/S_s \qquad (25)$$

*Tidal potential.* Classically, the potential is separated into various purely harmonic functions. *Melchior* [1964] points out that, although a great number of waves are obtained in the harmonic development, only five of the main waves are of real importance geophysically. These five are: $M_2$, a lunar wave with a period of 12h 25m 14s; $S_2$, a solar wave with a period of 12 h 00m; $N_2$, a lunar wave with a period of 12h 39m 30s (due to the eccentricity of the lunar orbit); $K$, a lunisolar wave with a period of one sidereal day 23h 56m 4s; and $O$, a lunar wave with a period 25h 49m 10s. Together these waves account for approximately 95% of the tidal potential. The potential is readily computed from the equations for the height of the equilibrium tide [*Munk and MacDonald*, 1960. p. 68; *Doodson and Warburg*, 1941].

## Discussion

By combining (20) and (25) we obtain an expression for the displacement of the water level in terms of the tidal disturbing potential and the specific storage of the aquifer

$$-dh = \frac{1}{S_s}\left(\frac{1-2\nu}{1-\nu}\right)\left[\left(\frac{2\bar{h}-6\bar{l}}{ag}\right) dW_2\right] \qquad (26)$$

With the exception of the specific storage and Poisson's ratio the parameters in (26) are known. Using this relationship, the specific storage can be determined from the tidal response of a well if Poisson's ratio for the aquifer is known.

Jacob [1940] introduced the idea of the 'barometric efficiency' of an artesian well as an index of the elasticity of an aquifer system. The fluctuation of barometric pressure causes rises and declines of the water level in an open artesian well. A constant of proportionality, the barometric efficiency $B$, relates the two effects

$$dp_b = -B\, dp$$

where $p_b$ is the atmospheric pressure. The barometric efficiency, of course, varies from well to well and ranges between 0 and 1. *Jacob* [1940] showed that the barometric efficiency is related to the specific storage as

$$B = \rho g n/E_w S_s \qquad (27)$$

The result as given by (26) indicates that, in theory, the unknown quantity $(1/S_s)$ $([1 - 2\nu)/(1 - \nu)]$ can be computed from observations of earth-tide fluctuations in artesian wells. If Poisson's ratio for the aquifer is known the specific storage $S_s$ may be determined. Using (27), it is possible to compute the aquifer porosity $n$ if the barometric efficiency is known. This porosity would represent an average value for a large aquifer volume in the vicinity of the well, a quantity which interests hydrologists and which is difficult, if not impossible, to determine by other means.

The expression for the fluctuation in terms of the dilation (3) differs from Melchior's expression (2) by the factor $(1/n)$. This arises from the fact that we treat the aquifer as an infinite porous medium rather than as a finite cavity. Using this theory, Melchior's unexplained observation that the amplitude of many of the observed $M_2$ waves is greater than the 0.4 cm predicted by (2) can be explained. The amplitude of the tidal fluctuations in artesian wells would vary depending on the aquifer porosity.

The fact that the amplitude of the $M_2$ fluctuations, corrected for latitude [*Melchior*, 1964], increases with the depth of the well probably depends on the porosity of the aquifer. In general, the porosity of geologic deposits decreases with depth, which would tend to increase the amplitude of the tidal fluctuation. The permeability of the confining layers also generally decreases with depth, so that deeper aquifers more closely approximate ideal artesian conditions. This too would tend to produce larger earth tides in deeper wells. These are, of course, generalities; the geology of each site must be examined independently.

*Response of a finite cavity.* In some instances the finite cavity model is a better approximation than the infinite aquifer model. For example, the fluctuations of water level in a shaft ending in an inundated mine can probably be analyzed better by representing the mine as a finite buried cavity with impermeable walls.

An idealized finite cavity is pictured in Figure 4. The mass of water in the system is

$$M = \rho(V + \pi r_w^2 h)$$

where $M$ is the total mass of water in the cavity and well. A change in the total volume of the cavity $dV$ produces a corresponding change of the height of water in the well, $dH$, and, since the mass remains constant,

$$dM = 0 = d\rho(V + \pi r_w^2 H) + \rho(dV + \pi r_w^2\, dH) \qquad (28)$$

The change in density is given as

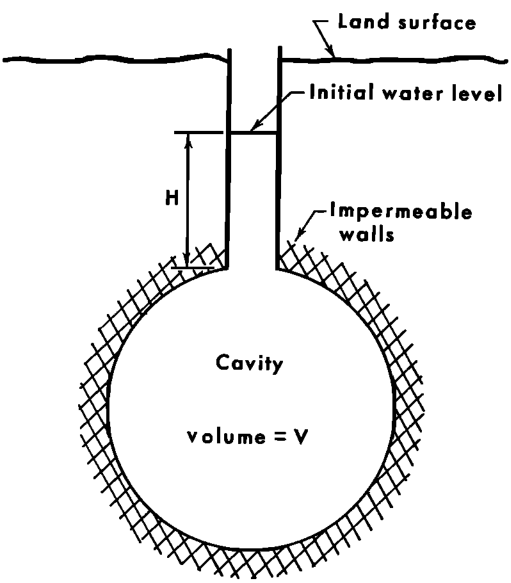


Fig. 4. Idealized representation of an observation well ending a finite cavity.

$$d\rho = dp(\rho/E_w)$$

where $dp = \rho g\, dH$ is the change in pressure. Substituting in (28) and simplifying we obtain

$$-dH = dV \Big/ \frac{\rho g V}{E_w} + \pi r^2 + \frac{\rho g \pi r_w^2 H}{E_w}$$

The last term in the denominator, $\rho g \pi r_w^2 H/E_w$, which is the change in the mass of water in the well due to the change in pressure, $\rho g\, dH$, is small and may be neglected; therefore,

$$-dH = \frac{dV}{\pi r_w^2 + (\rho g V/E_w)} \qquad (29)$$

This is the same as Melchior's results (1) except that we have defined $H$ as positive upward.

Equation 29 can be expressed in terms of the dilation as

$$-dH = \frac{\Delta}{(\pi r_w^2/V) + (\rho g/E_w)} \qquad (30)$$

This result can be used to compute the volume of an unknown cavity provided that the dilation can be computed with sufficient accuracy.

It follows from (30) that the density of wells can become a factor in the tidal response. The term in the denominator $\pi r_w^2/V$ must be less than approximately $1 \times 10^{-8}$ in order for the tidal response not to be diminished by too small a volume of water available to the well. To illustrate this consider an artesian aquifer 10 meters in thickness with a porosity of 0.10 in which observation wells 30 cm in diameter are drilled on a square grid. If all the wells respond by an equal change in water level $-dH$ to an earth tide dilatation $\Delta = 1 \times 10^{-8}$, the volume of water in the aquifer contributing to each well is

$$V = nbL^2 = 0.10(10^3 \text{ cm})L^2$$

where $L$ is the distance between wells. In order for $-dH$ to be approximately 0.5 cm the volume must be $V > 1 \times 10^{11}$ cm$^3$. For this hypothetical aquifer the well spacing must be $L > 300$ meters.

*Analysis of observed data.* There are two possible approaches for analyzing observed hydrographs for the effects of tidal fluctuations. One can either (1) compare the fluctuation in the well—adjusted for trends, barometric effects, and the effects of pumping—with the

TABLE 1. Comparison of Data for the $M_2$, $S_2$, $N_1$, $K_1$, and $O_1$ Waves for Artesian Wells at Turnhout, Iowa City, and Carlsbad

| Tidal Component | Amplitude of Associated Water Level from Harmonic Analyses, cm* | Theoretical Amplitude of Tidal Potential, $W_2 \times (1/g)$, cm | Theoretical Amplitude of Tidal Dilatation $\Delta_t \times 10^{-8}$ ($\nu$ = 0.25) | Specific Storage $S_s$, $\times 10^{-8}$ cm$^{-1}$ | Barometric Efficiency $B$ | Porosity $n$ ($\nu$ = 0.25) |
|---|---|---|---|---|---|---|
| | | *Turnhout, Belgium, latitude + 51°19′*† | | | | |
| $M_2$ | 1.48 | 10 2 | 0.83 | 0.6 | 0.77 | 0.10 |
| $S_2$ | 0.68 | 4.7 | 0.38 | 0.6 | | 0.10 |
| $N_2$ | 0.24 | 1.9 | 0.16 | 0.7 | | 0.11 |
| $K_1$ | 1.41 | 14 8 | 1.2 | 0.9 | | 0.15 |
| $O_1$ | 1.20 | 10 5 | 0 86 | 0.7 | | 0.12 |
| | | *Iowa City, Iowa, latitude + 41°39′*‡ | | | | |
| $M_2$ | 1.15 | 14.5 | 1.2 | 1 | 0.75 | 0.18 |
| $S_2$ | 0.50 | 6.8 | 0.55 | 1 | | 0.19 |
| $N_2$ | 0.33 | 2.7 | 0 22 | 0.7 | | 0 12 |
| $K_1$ | 1.41 | 15.1 | 1.2 | 0.9 | | 0.15 |
| $O_1$ | 0.59 | 10.7 | 0.87 | 1 | | 0.25 |
| | | *Carlsbad, New Mexico, latitude + 32°18′*§ | | | | |
| $M_2$ | 0.45 | 18.6 | 1.5 | 0.3 | 0.65 | 0.42 |
| $S_2$ | 0.25 | 8.6 | 0.70 | 0.3 | | 0.36 |
| $N_2$ | 0.09 | 3.5 | 0 29 | 0.3 | | 0.40 |
| $K_1$ | 0.16 | 13.7 | 1.1 | 0.7 | | 0.78 |
| $O_1$ | 0.15 | 9.7 | 0.79 | 0.5 | | 0.65 |

* *Melchior* [1960].
† Analysis of 62 days, 1956.
‡ Analysis for 28 days, 1939.
§ Analysis of 60 days, 1938.

fluctuations one would expect from tidal theory, or (2) compare the amplitude of the various tidal components obtained by harmonic analysis of the hydrograph with the theoretical amplitude of particular waves. Since the components of the tide potential are harmonic functions whose periods are well established, it is convenient to make a harmonic analysis of the observations. The analysis of the well fluctuations is complicated by barometric effects which also have diurnal and semidiurnal components. It is necessary to adjust the hydrograph data for barometric effects before making a harmonic analysis.

It follows from (26) that the ratio of changes in water level to the changes in disturbing potential is constant, or

$$-\frac{dh}{dW_2} = \frac{1}{S_s}\left[\frac{1-2\nu}{1-\nu}\right]\left[\frac{2\bar{h}-6\bar{l}}{ag}\right] \tag{31}$$

This implies that either (1) the ratio of the observed amplitude for any particular component to the theoretical amplitude of the tide potential for that component, or (2) the ratio of the total change in water level to the theoretical total change in tide potential, should equal the same constant.

*Melchior* [1964] analyzed his data from the Basecles well by comparing the amplitude of the various tide components obtained by harmonic analysis of the hydrograph data with the theoretical amplitude of the change in gravity associated with each of the waves. He found that the ratios for the $M_2$, $N_2$ $K + P_1$, $O$, and $Q$ components were approximately constant and, in this case, equal to 0.15 ± .22. One would expect some variation because some of the observed amplitudes were small.

*Melchior* [1960] reports the results of harmonic analyses of hydrograph data from artesian wells at Turnhout, Belgium; Iowa City, Iowa; and Carlsbad, New Mexico. The theoretical amplitude of the various components are computed for these wells and compared with the results of Melchior's harmonic analyses in Table 1. The theoretical dilatation, the specific storage, and the porosity are computed, assuming a Poisson's ratio of $\nu = 0.25$. Only data from these three wells, which the geologic information indicates are clearly artesian, are considered in Table 1.

The computations for Turnhout and Iowa City give good results. The various component waves give reasonably constant values for specific storage and porosity. The specific storage and porosity values for these wells are reasonable.

The results for the Carlsbad well are not as good. The specific storage is reasonably constant for the semidiurnal waves; however, the diurnal components yield values almost twice as large as the semidiurnal values. The values for the computed porosity seem unreasonably large even for the semidiurnal computations. The results suggest that the tidal fluctuations should have been larger than observed; certainly the diurnal components are smaller than one would expect. Either this well does not fit our assumptions for an ideal artesian system, or something is operating to reduce the magnitude of the tidal fluctuations.

## Conclusions

Although a number of factors in well-aquifer systems tend to reduce the magnitude of earth-tide fluctuations, earth tides should be present in most wells which penetrate a well-confined aquifer. Indeed, those wells that respond to earthquakes usually respond to earth tides.

Because the dilatation produced by the earth tide $\Delta_t$ is approximately $1 \times 10^{-8}$ and earth tides in wells often have magnitudes of 1 to 2 cm, we have a direct measure of sensitivity of the well-aquifer system. This means that with a sensitive measuring device, such as a pressure transducer capable of measuring 1 mm of water, it would be possible to observe dilatations of the earth of the order of $1 \times 10^{-9}$. This would indicate that the artesian well is as sensitive to strains of the crust (i.e., dilatation, the sum of the normal strains) as the strain seismometer. One can conceive of instances in which a properly instrumented well might be sensitive to even smaller dilatations of approximately $1 \times 10^{-10}$ to $1 \times 10^{-11}$. The potential for making observations of such small strains exists in the artesian well.

Analyses of the water-level fluctuations caused by the earth tide can be used to compute the specific storage and the porosity of the aquifer, parameters that greatly interest groundwater hydrologists.

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