Check for updates

Water Resources Research

RESEARCH ARTICLE
10.1029/2021WR030717

Well-Type Steady Flow in Strongly Heterogeneous Porous Media: An Experimental Study

G. F. A. Brunetti[1], C. Fallico[1], S. De Bartolo[2], and G. Severino[3]

[1]Department of Civil Engineering, University of Calabria, Rende, Italy, [2]Department of Engineering for Innovation, University of Salento, Lecce, Italy, [3]Department of Agricultural Sciences, University of Naples Federico II, Naples, Italy

Key Points:

- Steady well-flow through a strongly heterogeneous formation is investigated by means of an experimental study
- The formation was artificially packed to reproduce a given, statistical distribution of the hydraulic conductivity
- The equivalent conductivity is proved to be a proper parameter to simulate the average flow

Correspondence to:

G. Severino,
gerardo.severino@unina.it

Citation:

Brunetti, G. F. A., Fallico, C., De Bartolo, S., & Severino, G. (2022). Well-type steady flow in strongly heterogeneous porous media: An experimental study. *Water Resources Research*, *58*, e2021WR030717. https://doi.org/10.1029/2021WR030717

Received 29 JUN 2021
Accepted 2 APR 2022

Author Contributions:

Conceptualization: G. Severino
Data curation: G. F. A. Brunetti, C. Fallico, G. Severino
Formal analysis: G. Severino
Funding acquisition: C. Fallico
Investigation: G. F. A. Brunetti, C. Fallico, S. De Bartolo, G. Severino
Methodology: C. Fallico, G. Severino
Resources: G. F. A. Brunetti, C. Fallico
Software: G. F. A. Brunetti
Supervision: C. Fallico, S. De Bartolo, G. Severino
Validation: C. Fallico, S. De Bartolo
Writing – original draft: G. Severino

© 2022. The Authors.
This is an open access article under the terms of the Creative Commons Attribution-NonCommercial-NoDerivs License, which permits use and distribution in any medium, provided the original work is properly cited, the use is non-commercial and no modifications or adaptations are made.

Abstract Steady well-type flow was monitored in an aquifer that was artificially packed in order to reproduce a given, highly heterogeneous, statistical distribution of the log-conductivity Y. In particular, we focus on pumping tests carried out at 10 volumetric flow rates. The experimental arrangement was composed by a pumping well and several surrounding observation piezometers. The unique feature of this experimental study is that the high heterogeneity structure of Y is known fairly well. Thus, the study lends itself as a valuable tool to corroborate theoretical findings about flows driven by sources through porous formations, where the variance σ_Y^2 (in the present study equal to 3.79) of Y is large. Besides discussing experimental findings, we tackle the crucial issue of upscaling the hydraulic conductivity in a well-flow configuration. In particular, we deal with the *equivalent conductivity* (EC) as that pertaining to a homogeneous (fictitious) medium which conveys the same volumetric flow rate of the real one, under the same boundary conditions. Hence, the EC can be identified straightforwardly by means of head measurements. Even if we show that the EC is a proper parameter to reproduce measurements, it is experimentally shown (in line with the theoretical results) to be position-dependent, and therefore, it cannot be regarded (unlike groundwater-type flow) as a formation's property. This implies that the EC applies only to the configuration at stake. Then, we show that the EC, combined with a recent model of effective conductivity in well-flows through highly heterogeneous porous formation, leads to a reasonably reliable estimate of σ_Y^2, some limitations and approximations, notwithstanding. It is hoped that the present experimental study will be useful for other researchers who are engaged with similar research-topics.

1. Introduction

Many hydrological applications, such as pumping tests or the set up of in situ remediation protocols, depend to a great extent upon the spatial variability (generically referred as *heterogeneity*) of the aquifer's hydraulic properties (Rubin, 2003). In particular, it is a common tenet to regard the heterogeneity of the hydraulic conductivity K as the controlling parameter of flow and transport (Dagan, 1989). The variability of K implies that the flow variables are affected by a spatial uncertainty, and therefore developing efficient and reliable methodologies to quantify the K-heterogeneity becomes of paramount importance. This is particularly relevant for strongly heterogeneous formations, where the difference with the homogeneous formation is not evident, sometimes counterintuitive (see, e.g., Janković et al., 2006, and references therein).

Among the plethora of methods used to identify the hydraulic conductivity (a comprehensive critical review can be found in Cardiff et al., 2013), pumping tests are (thanks to their minimal equipment's requirement, and ease of implementation), by far, the most prominent tools. Owing to several (mainly economic and logistic) limitations, most of the efforts have focused on the identification of average properties (Copty et al., 2008, 2011; Desbarats, 1994; Zech et al., 2012). However, this is not satisfactory at all, especially to achieve meaningful predictions on solute transport (Dagan, 1989; Rubin, 2003). For this reason, recently it was suggested to carry out measurements over dense borehole spacings in order to achieve accurate three dimensional estimates for mildly heterogeneous aquifers (Cardiff et al., 2013). In line with this trend, the natural question here is whether pumping tests enable one to identify efficiently the heterogeneity structure of the conductivity even in strongly heterogeneous formations. In fact, with the exception of a single pumping test carried out as part of the well known MADE-experiment (Bohling et al., 2012), other experimental investigations under similar conditions limit to deal with "mildly heterogeneous formations" (see, e.g., Fernández-Garcia et al., 2004; Cardiff et al., 2013). Thus, from the experimental point of view, very little has been done when the degree of the formation's heterogeneity is high, and the present study constitutes an attempt to (partially) fill the gap.

10.1029/2021WR030717

19447973, 2022, 5, Downloaded from https://agupubs.onlinelibrary.wiley.com/doi/10.1029/2021WR030717 by Indian Institute Of Technology, Kharagpur Central Library, Wiley Online Library on [17/01/2024]. See the Terms and Conditions (https://onlinelibrary.wiley.com/terms-and-conditions) on Wiley Online Library for rules of use; OA articles are governed by the applicable Creative Commons License

Identification of the effective conductivity, K_{eff}, in convergent flows through a heterogeneous formation has become a topic of intensive analytical studies (e.g., S. P. Neuman & Orr, 1993; S. P. Neuman et al., 2004; Sánchez-Vila, 1997; Zech & Attinger, 2016). The drawback related to K_{eff} is that it is not a medium's property (non-locality). Instead, it depends upon the geometrical configuration of the set-up. In addition, K_{eff} depends upon quantities (like the head gradient) that for radial-type flows are very difficult (and expensive) to measure in the field. To alleviate this logistic burden, an alternative approach, relying upon the EC concept, has been developed (Desbarats, 1992; Indelman et al., 1996; Sánchez-Vila et al., 1999; Schneider & Attinger, 2008). Its main advantage is that the EC can be computed by means of the head measurements, solely. However, like the effective conductivity, even the equivalent one is a non-local property (a detailed review is given by Sanchez-Vila et al., 2006).

Another viable option is by means of *Monte Carlo simulations* (MCs). However, this approach has two serious drawbacks. First, to generate a K-distribution reproducing the degree of heterogeneity of a strongly heterogeneous medium, very dense numerical grids are required. This would generate a very huge system of algebraic equations to solve, most of the times far beyond the standard computational power. Second, to account accurately for the statistics of the flow variables, many realizations are needed. Thus, these two requirements are too demanding for the majority of the real world situations. Furthermore, the scarcity of field data and the lack of precision in the measurement-devices render the identification of the statistics of K quite uncertain. As a consequence, a poorly accurate, that is, large uncertainty (especially when the formation possesses a high degree of heterogeneity), input data set can produce only an equally poor output data set (no matter how efficient per *sé* is the MC-method). This issue plays dawn significantly the ability of the MCs to come up with accurate results, de facto deteriorating the advantage of computationally intensive numerical simulations.

Despite theoretical progresses, understanding how to relate field measurements to the conductivity field can not be considered definitively over (see, e.g., Leven & Dietrich, 2006). This is mainly due to the fact that, especially for strongly heterogeneous formations, detailed experimental studies are still lacking. Besides dealing with a conductivity that varies considerably over the observation scale (Schad & Teutsch, 1994), it should be also considered that an excess in the pumping may eventually lead to compaction of the porous materials, and concurrently to a modification of the K-values (Chao et al., 2000). For these reasons, developing characterization-strategies, that are cost-effective for repeated and/or continuous implementation, will provide valuable information for reducing overall costs (Ptak & Teutsch, 1994). This has stimulated development of pressure-based methods, that is, methods in which changes in the hydraulic head provide information content for mapping heterogeneity (see, e.g., Fallico et al., 2018; Fernandez-Garcia et al., 2002; Severino, De Bartolo, et al., 2019).

In the present study, we have carried out several pumping tests into a largely heterogeneous porous medium, which was artificially packed. Even if the experimental set up does not resemble exactly a real setting, it nevertheless exhibits statistical properties close to those observed (see, e.g., Istok et al., 1994; Rehfeldt et al., 1992). Like similar studies dealing with mildly heterogeneous porous media created in a laboratory (Fernández-Garcia et al., 2004), our study has the unique advantage that the hydraulic properties and in particular the log-conductivity $Y \equiv \ln K$ are well defined. In this way, one overcomes the lack of spatial information that renders the analysis very complex, sometimes hiding conclusions (Boggs et al., 1992; Wu et al., 2005; Zech et al., 2015). In addition, in our experiments the heterogeneity structure of Y is well known at point scale, therefore minimizing the uncertainty in the parameters. In this way, one can test the capability of the predicting models to adequately reproduce real data. On the other hand, results of the identification could be matched against known values of the Y-field in order to define the proper strategy that makes use of a steady-state solution.

The paper is organized as follows. The procedure to pack the medium according to a known statistical distribution is described; then, we discuss flow experiments (carried out at several flow rates), consisting of recording pressure heads at several piezometers radially distributed around a pumping well. Pressure-head data are employed to demonstrate (in agreement with the theoretical results) that the EC is the proper parameter to mimic radial flows taking place in strongly heterogeneous formations. Then, we illustrate how the present study is also useful for the so-called inverse (i.e., identification) problem. We end up by highlighting some concluding remarks and recommendations for future investigations.

19447973, 2022, 5, Downloaded from https://agupubs.onlinelibrary.wiley.com/doi/10.1029/2021WR030717 by Indian Institute Of Technology, Kharagpur Central Library, Wiley Online Library on [17/01/2024]. See the Terms and Conditions (https://onlinelibrary.wiley.com/terms-and-conditions) on Wiley Online Library for rules of use; OA articles are governed by the applicable Creative Commons License

Table 1
K-Values (in m/s) Attached to Each Inclusion (I, II, … , XII), Along With the Fluctuation $Y' = \ln(K/K_G)$ of the Log-Conductivity Y, Supplemented With Their Uncertainties Within Each Class

Inclusion	I	II	III	IV	V	VI
$K \cdot 10^{-6}$	375.0 ± 6.0	366.0 ± 4.5	68.6 ± 0.1	$(344.0 \pm 7.0) \cdot 10$	74.9 ± 1.0	86.1 ± 0.7
Y' (−)	1.65 ± 0.02	1.62 ± 0.01	-0.05 ± 0.01	3.86 ± 0.02	0.0369 ± 0.01	0.177 ± 0.01
Inclusion	VII	VIII	IX	X	XI	XII
$K \cdot 10^{-6}$	13.2 ± 0.6	63.2 ± 1.9	20.9 ± 2.8	241.0 ± 21.5	8.6 ± 0.7	2.7 ± 0.2
Y' (−)	-1.70 ± 0.04	-0.133 ± 0.03	-1.24 ± 0.13	1.21 ± 0.02	-2.13 ± 0.08	-3.30 ± 0.06

2. Experimental Set-Up and Flow Experiments

Experiments were carried out at the laboratory of *Hydraulics* (University of Calabria, Italy) in a steel box (2×2 m-base, and 1 m-height). Inside the box, and all around the perimeter, vertical metallic supports were installed (5 cm away from the walls) to anchor a metal net over which a layer of geotextile was placed. The porous medium was packed by using 12 inclusions (10 cm × 10 cm × 5 cm), which were obtained by combining different quantities (i.e., silt, sand, fine gravel and coarse gravel) of soil material. The conductivity of each inclusion was determined by a permeameter operating at constant head. The K-value of each inclusion was obtained as mean over three replicates over cylindrical samples (diameter 6.4 cm and height 15.3 cm). The outcome is summarized in Table 1.

The log-conductivity $Y \equiv \ln K$, of geometric mean $K_G \equiv \exp\langle Y \rangle = 7.22 \times 10^{-5}$ m/s and variance $\sigma_Y^2 = 3.79$, was such that the fluctuation $Y' = \ln(K/K_G)$ is Gaussian (Figure 1). The hypothesis $Y' \sim \mathcal{N}(0, 3.79)$ was also quantitatively confirmed by the Kolmogorov–Smirnov test which lead to $D = 0.516$ (to be compared with the value $D^\star = 0.895$ at the 5%-level of confidence). From Table 1, it is seen that the variability within the K-class is characterized by a coefficient of variation $\mathtt{CV}_K^c$ much smaller than its counterpart at the formation scale, that is, $\mathtt{CV}_K^f$. Specifically, it is seen that, in "the most limiting" case (i.e., largest $\mathtt{CV}_K^c$), it yields $\mathtt{CV}_K^c/\mathtt{CV}_K^f = 0.134/6.54 = 0.02$. For this reason, one can disregard the variability within the class as compared with that at aquifer-scale, and concurrently in the subsequent developments we shall consider the local conductivity value as an exact one. The porosity n was $(46.4 \pm 0.2) \times 10^{-2}$. Likewise, since the coefficient of variation $\mathsf{CV}_n = 4.31 \cdot 10^{-5}$ results much smaller than that of K, the porosity can be regarded (in line with a common tenet, see e.g., Dagan, 1989; Rubin, 2003) as a given (equal to its mean value) parameter.

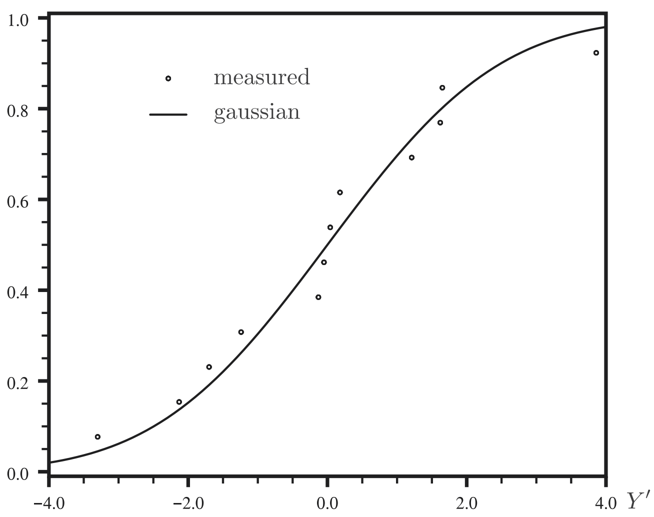

Figure 1. Cumulative Distribution Function (CDF) of measured (symbols) and Gaussian (continuous line) fluctuation Y'.

The porous formation was assembled by subsequent superpositions of 7 strata, each one composed by 19×19 inclusions. Location of each inclusion was determined according to a generator (NumPy) of random numbers uniformly distributed (due to the lack of correlation between each inclusion) in the set of integers {I, *II*, …, *XII*} (each one corresponding to a conductivity-value in Table 1). In order to place correctly inclusions (i.e., without leaving any gap in the between, and/or other disturbances), a metal grid (with a surface 1.0 m × 0.9 m × 0.05 m equal to the quarter of that of the whole caisson's surface) with the mesh sizes equal to that of a single inclusion (first picture in Figure 2) was used. The grid was removed at the completion of the inclusions' placement (second picture in Figure 2), and placed nearby repeating the same filling till to form a layer (third picture in Figure 2). Then, to favor compaction (therefore eliminating any possible gap between two adjacent inclusions as well as air bubbles), multiple wetting cycles were supplied. This procedure was iterated for seven times to build up the whole aquifer (fourth picture in Figure 2). At this stage, it is crucial to remind that inclusions' placement occurred with the piezometers already installed (first picture in Figure 2), and therefore, there was no disturbance due to the application of piezometers. At the completion, the formation (1.90 m × 1.90 m × 0.35 m) resulted is composed of $7 \times 19 \times 19$ blocks of conductivity lying in the set of Table 1 (Figure 3). To check that the statistical properties of Y after pack-

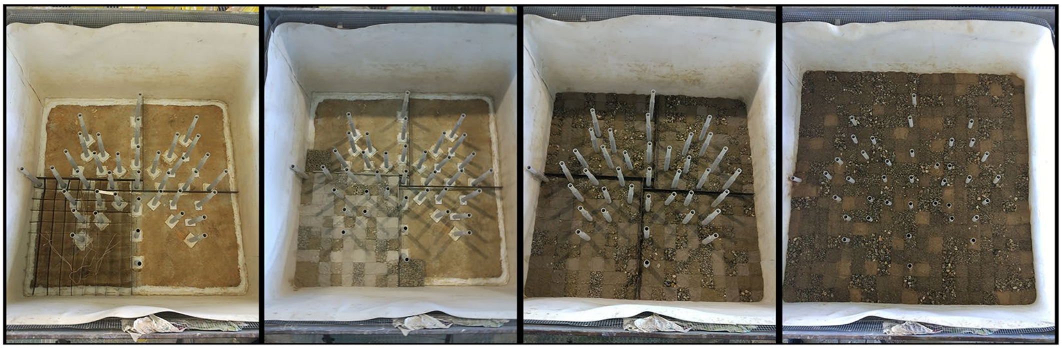

Figure 2. Illustration (from the left to the right) of the sequence leading to the layers of inclusions making up the formation.

ing were consistent with those in Table 1, several flow experiments were conducted, and measurements were compared with the expected ones. The purpose of this comparison is to confirm that there were no significant discrepancies (such as settling and disturbances during the packing) between the designed and the actual medium.

The medium was packed such to resemble the heterogeneity's structure considered within the self-consistent approximation: the formation is a multiphase material made up of 12 homogeneous blocks of given log-permeability Y arranged at random in the space. Thus, two points at a relative distance x larger than $\mathcal{L}$ (being this latter either the horizontal, I, or the vertical, I_v, integral scale) will exhibit completely different values of Y. To the contrary, two points at a relative distance x lesser than $\mathcal{L}$ will have the same Y-value. As a consequence, the autocorrelation function ρ is written as:

$$\rho(x) = \begin{cases} 1 & x < \mathcal{L} \\ 0 & x > \mathcal{L}, \end{cases} \tag{1}$$

with the two integral scales coincident with the characteristic sizes of the inclusion (i.e., $I \simeq 10$ cm and $I_v \simeq 5$ cm, respectively). The autocorrelation (1) is typical of the so-called "structureless formations" [examples can be found in Severino et al. (2009) and references therein], and therefore, it is relevant for the applications.

A battery of piezometers (inner radius equal to 1.4 cm) surrounding a pumping well of the same diameter at the center of the tank was placed according to a radial configuration (Figure 4). Each piezometer was screened till to

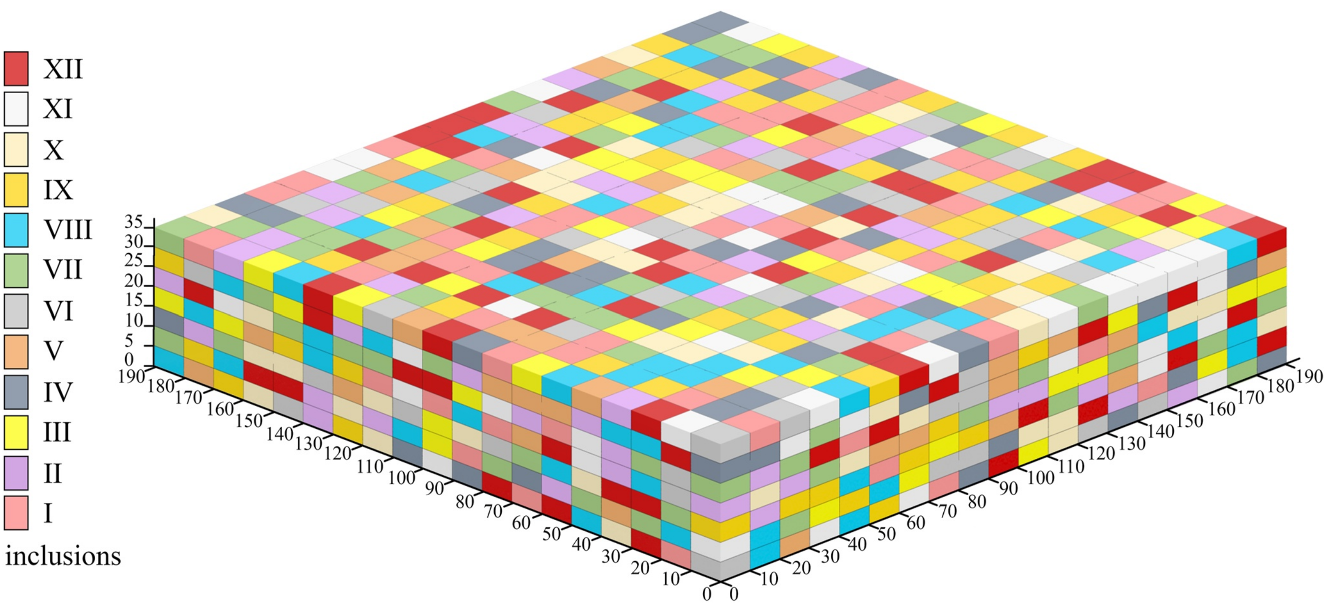

Figure 3. Three dimensional view of the distribution of the inclusions listed in Table 1.

19447973, 2022, 5, Downloaded from https://agupubs.onlinelibrary.wiley.com/doi/10.1029/2021WR030717 by Indian Institute Of Technology, Kharagpur Central Library, Wiley Online Library on [17/01/2024]. See the Terms and Conditions (https://onlinelibrary.wiley.com/terms-and-conditions) on Wiley Online Library for rules of use; OA articles are governed by the applicable Creative Commons License

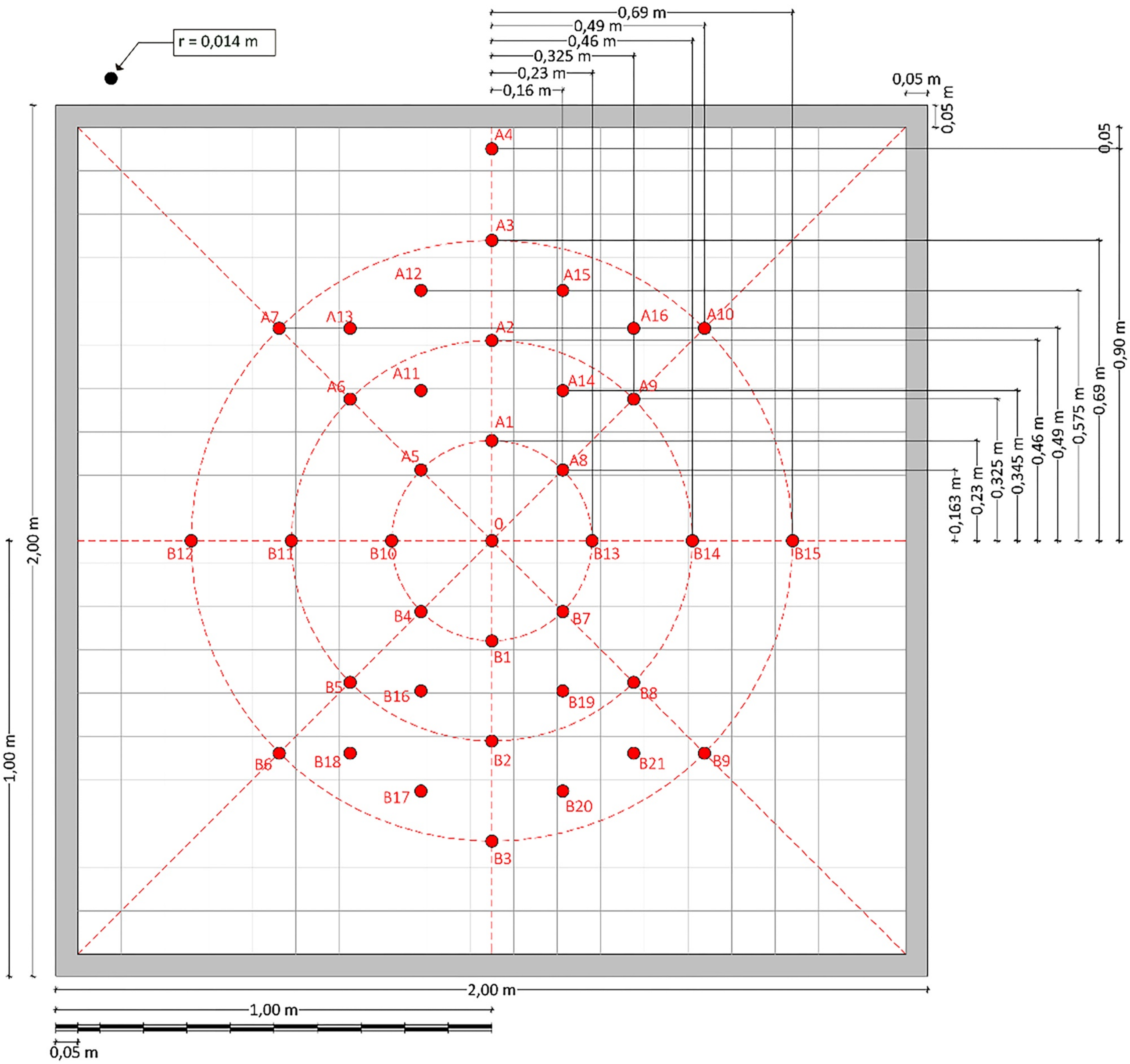

Figure 4. Arrangement (planar view) of the piezometers around the (central) pumping well.

35 cm. To avoid intrusions of particles within the piezometers, these latter were also coated by a geotextile. The external boundary condition was that of given head $\left(\bar{H}_w = 35\,\text{cm}\right)$ along the perimeter of the hosting box. This $\bar{H}_w$-value was fixed by a constant level of water between two (PVC) vertical walls (forming a cavity) around the perimeter of the caisson (Figure 5). To prevent lowering of the water level in the cavity due to the pumping (*a*) as shown in Figure 5, a new flow rate, slightly larger than the pumping one, was injected from the bottom of the box (*b*) in Figure 5. Finally, to avoid any excess in the water rising, the cavity was in hydraulic connection with two tanks (*c*) in Figure 5 whose level (kept constant thanks to a weir) was at 35 cm. The internal (i.e., at the pumping well) boundary condition is of given head along the well's axis, supplemented by the mass conservation, that is,

$$\frac{r_w}{B}\int_0^{2\pi}\int_0^{B} \mathrm{d}\theta\,\mathrm{d}z\, q_r(r_w, z, \theta) = Q_w = \text{const}, \tag{2}$$

being Q_w is the specific (per unit depth) flow rate, whereas $q_r(r_w, z, \theta)$ is the radial flux at the well's envelope $r = r_w$. Pressure transducers, placed at the bottom of each piezometer, were used to monitor pressure-head, p/γ.

19447973, 2022, 5, Downloaded from https://agupubs.onlinelibrary.wiley.com/doi/10.1029/2021WR030717 by Indian Institute Of Technology, Kharagpur Central Library, Wiley Online Library on [17/01/2024]. See the Terms and Conditions (https://onlinelibrary.wiley.com/terms-and-conditions) on Wiley Online Library for rules of use; OA articles are governed by the applicable Creative Commons License

19447973, 2022, 5, Downloaded from https://agupubs.onlinelibrary.wiley.com/doi/10.1029/2021WR030717 by Indian Institute Of Technology, Kharagpur Central Library, Wiley Online Library on [17/01/2024]. See the Terms and Conditions (https://onlinelibrary.wiley.com/terms-and-conditions) on Wiley Online Library for rules of use; OA articles are governed by the applicable Creative Commons License

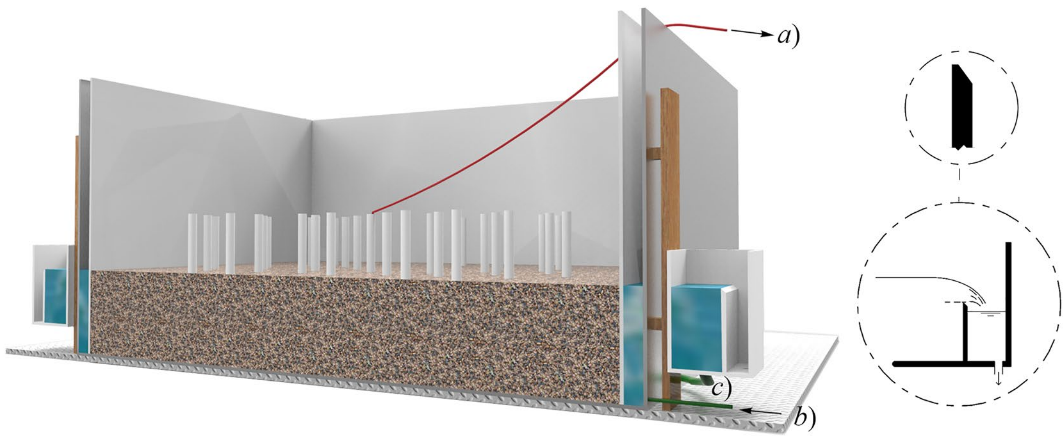

Figure 5. A three-dimensional view of the experimental set-up, together with the device used to fix the boundary condition of given head around the perimeter of the caisson.

Pressure ports, equal to the aquifer's thickness $B = 35$ cm (thus with no screening), were connected to a precision-transducer. Finally, prior to any pumping the apparatus was tested (from both the hydraulic and electronic point of view) by checking that the hydraulic head at the piezometers was the static one, that is, $p/\gamma = 35$ cm. In the flow experiments, water was pumped at the central well (Figure 4) by 10 constant flow rates, that is, $Q = 20$; 25; 30; 35; 40; 47.5; 50; 55; 60; 70 (liter/hour). Recording of pressure-heads initiated after reaching the steady state regime. The latter was achieved when water levels (and concurrently pressure head) at the piezometers and at the pumping well did not change in the time.

To show the transition in the output from pumping at the smallest flow rate to pumping at the highest one, in Figure 6 we have depicted contour levels pertaining to a single realization (corresponding to $Q = 20, 35, 50, 70$ ℓ/h, respectively) of the drawdown s_w. Besides the tremendous impact of the medium's heterogeneity upon the spatial distribution of the cone of depression, the striking feature is that in the close vicinity of the pumping well, the isolines are circular. This is explained by recalling that within a portion of domain surrounding the well (whose size is roughly a few horizontal integral scales), the medium behaves like a uniform one of constant (equal to the harmonic mean K_H) conductivity (Severino, Leveque, & Toraldo, 2019). Since for the formation at stake the horizontal integral scale was $I \simeq 10$ cm, it is not surprising that the isolines are circular inside an annulus of radius $\simeq$20 cm. Instead, away from the well the distortion mechanism due to the heterogeneity is clearly observed.

Before proceeding further, it is worth noting that, in the flow experiments at stake, the drawdown s_w at the pumping well is always relatively small as compared with the characteristic hight $\ell_c = 35$ cm of the aquifer, that is, $s_w/\ell_c = 0.025 \div 0.10$ (where the smallest and highest values correspond to the smallest/highest flow-rate, respectively). As a consequence, in the subsequent derivations assuming the flow domain as unbounded results a reasonable approximation.

3. Applications and Discussion

We wish to exploit results analyzed so far to address one of the central problems in the effective medium theory (Choy, 2015): defining an effective, constitutive (Darcy) model $\langle \mathbf{q} \rangle = -\mathbf{K}_{\text{eff}} \nabla \langle H \rangle$ relating the mean flux to the mean head gradient. In a different manner, it is assumed that the mean flow can be tackled as a homogeneous (fictitious) one, by dealing with an effective conductivity $\mathbf{K}_{\text{eff}}$. It is well known that the presence of a well (and more generally of spatially distributed sources) prevents obtaining an effective Darcy's law in a classical sense (Indelman, 1996; Sánchez-Vila et al., 1999). In fact, for a well-type flow, the head gradient does not vary slowly (which is the condition for obtaining a local effective law, see Dagan et al., 2013) especially in the zone surrounding the well. This is particularly relevant in strongly heterogeneous media, where the impact of the flow non-uniformity is enhanced by the huge variability of the conductivity (see, e.g., Desbarats, 1994; Durlofsky, 2000; Firmani et al., 2006). As a consequence, a central problem is whether, in this case, it is still possible dealing with an *upscaled conductivity*, relating the mean flow variables.

10.1029/2021WR030717

19447973, 2022, 5, Downloaded from https://agupubs.onlinelibrary.wiley.com/doi/10.1029/2021WR030717 by Indian Institute Of Technology, Kharagpur Central Library, Wiley Online Library on [17/01/2024]. See the Terms and Conditions (https://onlinelibrary.wiley.com/terms-and-conditions) on Wiley Online Library for rules of use; OA articles are governed by the applicable Creative Commons License

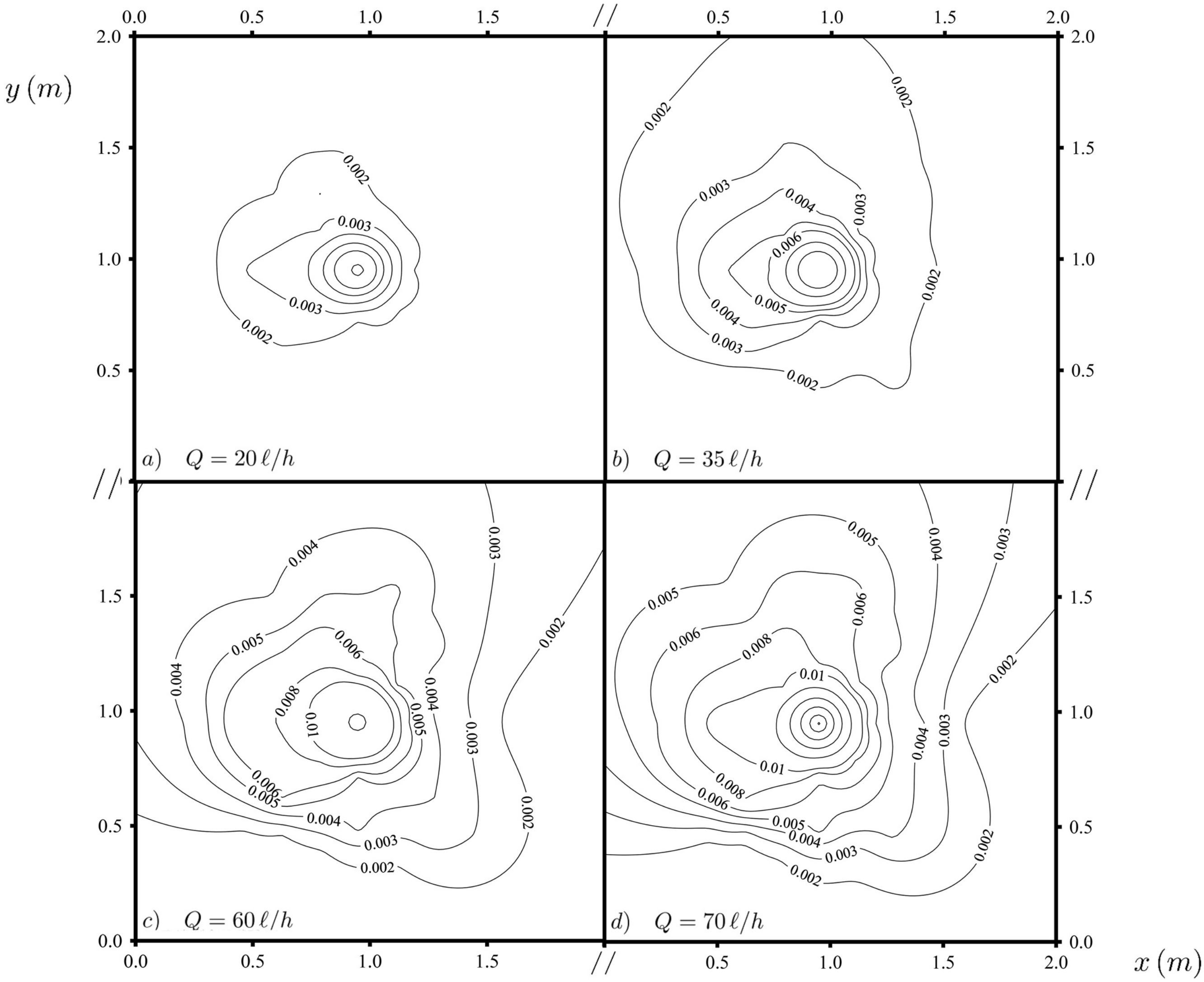

Figure 6. Iso-values (in m) of the single realization of the drawdown s_w (m), as detected in the flow experiment for: a) $Q = 20\,\ell/h$; b) $Q = 35\,\ell/h$; c) $Q = 60\,\ell/h$; and d) $Q = 70\,\ell/h$.

Identification from real data of the upscaled conductivity is not straightforward, and the set up of a fairly general methodology is still matter of debate (a wide review can be found in Dagan and Lessoff (2007), Dagan et al. (2009, 2013), Pechstein et al. (2016) and Demir et al. (2017)). In the present study, we follow an avenue similar to that of Indelman et al. (1996). Thus, we consider an equivalent conductivity, EC, as that of a fictitious (homogeneous) formation that conveys the same specific (per unit length) flow rate Q_w as the real (heterogeneous) one, that is,

$$K_{\text{eq}} = \frac{Q_w \ln(2r/L)}{2\pi\left[\langle H_w(r)\rangle - \bar{H}_w\right]}, \tag{3}$$

under the same boundary condition: $\langle H_w(L/2)\rangle = \bar{H}_w$. In Equation 3, $0 < r \leq L/2$ is the radial distance from the well's axis, whereas $L = 2$ m is the side of the caisson's base (see Figure 4). Note that the replacement of the head's spatial average with the ensemble one is allowed by virtue of ergodicity. This point shall be discussed to a deeper extent later on. Finally, since the ratio between the well's radius r_w and its length L_w is equal to 0.04, the well can be regarded as a line of singularity (Dagan, 1978), and this authorizes adoption of 3 (in agreement with Indelman et al. (1996)).

19447973, 2022, 5, Downloaded from https://agupubs.onlinelibrary.wiley.com/doi/10.1029/2021WR030717 by Indian Institute Of Technology, Kharagpur Central Library, Wiley Online Library on [17/01/2024]. See the Terms and Conditions (https://onlinelibrary.wiley.com/terms-and-conditions) on Wiley Online Library for rules of use; OA articles are governed by the applicable Creative Commons License

Table 2
Values (Blue Face) of the Spatial Average of the Head (m) at Several Radial Distances r, and for Each Volumetric Flow Rate Q (ℓ/h)

N_p	r (m)	$Q = 20$	$Q = 25$	$Q = 30$	$Q = 35$	$Q = 40$
8	0.23	0.297 ±0.001	0.297 ± 0.001	0.296 ± 0.001	0.296 ± 0.001	0.296 ± 0.001
4	0.38	0.298 ± 0.001	0.297 ± 0.001	0.297 ± 0.001	0.297 ± 0.001	0.296 ± 0.001
8	0.46	0.298 ± 0.001	0.298 ± 0.001	0.297 ± 0.001	0.297 ± 0.001	0.297 ± 0.001
8	0.60	0.299 ± 0.001	0.298 ± 0.001	0.298 ± 0.001	0.298 ± 0.001	0.298 ± 0.001
8	0.69	0.299 ± 0.001	0.299 ± 0.001	0.298 ± 0.001	0.298 ± 0.001	0.298 ± 0.001
N_p	r (m)	$Q = 47.5$	$Q = 50$	$Q = 55$	$Q = 60$	$Q = 70$
8	0.23	0.295 ± 0.001	0.294 ± 0.001	0.293 ± 0.001	0.293 ± 0.001	0.291 ± 0.001
4	0.38	0.296 ± 0.001	0.295 ± 0.001	0.295 ± 0.001	0.295 ± 0.001	0.293 ± 0.001
8	0.46	0.296 ± 0.001	0.296 ± 0.001	0.295 ± 0.001	0.295 ± 0.001	0.293 ± 0.001
8	0.60	0.297 ± 0.001	0.297 ± 0.001	0.297 ± 0.001	0.297 ± 0.001	0.296 ± 0.001
8	0.69	0.297 ± 0.001	0.297 ± 0.001	0.297 ± 0.001	0.297 ± 0.001	0.296 ± 0.001

Note. The Integer N_p is the number of local measurements $H_i^{(\mathrm{exp})}$ appearing into (4).

The utility of the definition (3) is that it requires only head-values that are easily measurable in the field. To the contrary, the drawback of 3 is that the EC is not a medium's property, but instead it also depends upon the distance, as well as the flow configuration (S. Neuman et al., 1996). In other words, the EC depends on the problem at stake, and therefore, it can not be used for other configurations (such as battery of wells, partially penetrating wells, injecting/pumping well-systems).

In order to apply the definition (3) to the flow experiments, in Table 2, we have listed the ensemble head $\langle H_w \rangle$ as computed from the N_p-measurements of the head $H_i^{(\mathrm{exp})}$ at the piezometers located at the same radial distance, that is,

$$\langle H_w \rangle \simeq \frac{1}{N_p} \sum_{i=1}^{N_p} H_i^{(\mathrm{exp})}. \tag{4}$$

The resulting EC has been depicted in Figure 7 as function of r. The striking evidence is that the EC defies completely the well-known bounds: $K_H \leq K_{\mathrm{eq}} \leq \langle K \rangle$ ($K_H \equiv \langle K^{-1} \rangle^{-1}$ is the harmonic mean) that are valid for mean uniform flows (Dagan, 1989; Matheron, 1967), being the smallest EC-value bigger than $\langle K \rangle = 3.97 \times 10^{-4}$ m/s. Moreover, the K_{eq}-values also defy the limits [Equation (19) in Indelman et al. (1996)] that apply to a well-type flow taking place in weakly heterogeneous (i.e., $\sigma_Y^2 \ll 1$) porous media (see, also Dagan & Lessoff, 2007). This behavior is clearly due to the coupling between the strong heterogeneity with the flow non-uniformity. A deep theoretical analysis is required in order to explain this out coming.

Our results corroborate theoretical findings: the EC varies with distance between the piezometer and the pumping well (Pechstein & Copty, 2021). As a consequence, if the EC, as determined by piezometers, is applied in simulations as representative of the effective conductivity, K_{eff}, the latter may result significantly different from the correct value. The discrepancy between conductivities determined by pumping tests and those to be used in simulations to fit measured heads has been observed (see, e.g., Dagan & Lessoff, 2007; Dagan et al., 2009; Severino, 2011).

3.1. Implementation

To illustrate how the identification-methodology of K_{eq} leads to predictions which are in agreement with the experimental measurements, in Figure 8 we have depicted the non-dimensional quantity:

$$\alpha(r) \equiv \frac{2\pi r}{\bar{H}_w - \langle H_w(r) \rangle} = \frac{4\pi^2 \left(r/\ell_{\mathrm{eq}} \right)}{\ln \left[L/(2r) \right]} \qquad \left(\ell_{\mathrm{eq}} \equiv \frac{Q_w}{K_{\mathrm{eq}}} \right) \tag{5}$$

10.1029/2021WR030717

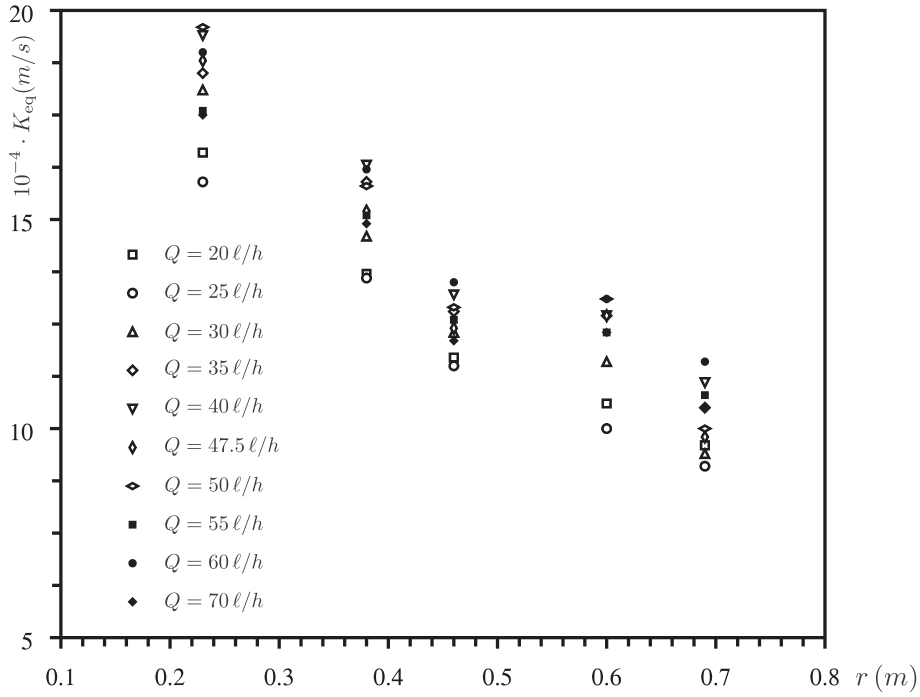

Figure 7. Values of the equivalent conductivity (3) at the different distances r, and for the entire set of the pumping volumetric flow rates Q.

as function of the radial distance r. The α-function has been obtained from 3 after dividing each side by the mean velocity $V(r) = Q_w/(2\pi r)$ (in analogy to Firmani et al. (2006)). Since the latter is valid for an unbounded domain, the quantity (5) applies here approximately for $r \ll L/2$. The utility of using 5 stems from the fact that in this way the 10 plots $\langle H_w \rangle \equiv \langle H_w(r) \rangle$ (each one corresponding to a given volumetric flow rate) collapse into a single one,

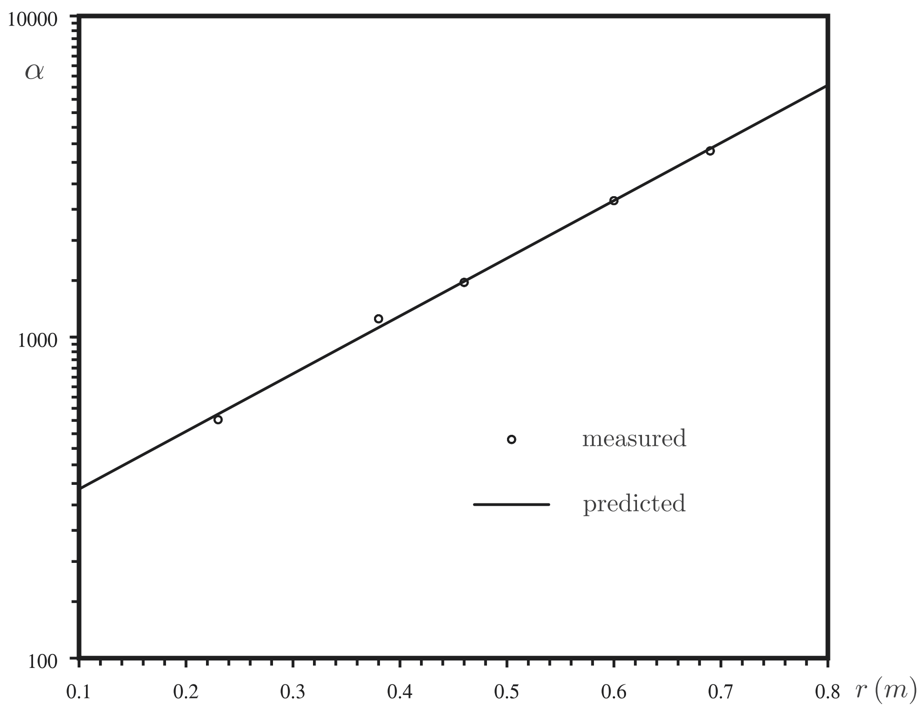

Figure 8. Values of the α-parameter at several radial distances r predicted (continuous line) and measured (symbols). The length ℓ_{eq} is equal to 2 cm.

19447973, 2022, 5, Downloaded from https://agupubs.onlinelibrary.wiley.com/doi/10.1029/2021WR030717 by Indian Institute Of Technology, Kharagpur Central Library, Wiley Online Library on [17/01/2024]. See the Terms and Conditions (https://onlinelibrary.wiley.com/terms-and-conditions) on Wiley Online Library for rules of use; OA articles are governed by the applicable Creative Commons License

Table 3
Estimates of the Mean and Standard Deviation (sdev) of the Variance σ_Y^2 Determined by the Calibration Procedure and by Measurements

Methodology	Mean	sdev	$CI_{0.68}$	$CI_{0.95}$
Calibration	2.92	0.21	[2.71; 3.13]	[2.51; 3.33]
Direct measurements	3.78	0.41	[3.37; 4.19]	[2.98; 4.58]

Note. The corresponding confidence interval CI (at 68% and 95% level of confidence) is also indicated.

therefore providing a compact insight about the capability of the K_{eq}-concept to reproduce the experimental data-set.

"Identifying the non-local K_{eff} with the aid of field measurements or relating it to K_{eq} is not a simple matter" (Indelman et al., 1996). Toward this aim, we propose in the sequel a methodology that makes use of a self-consistent approximation of the effective conductivity in a well-type flow (Severino, 2019). Thus, the physical model underlying the self-consistent expression of the effective conductivity regards the porous formation as a collection of many homogeneous, non-overlapping inclusions each of conductivity K set at random in space (a somewhat similar methodology has been employed by Attinger (2003)). Hence, the effective conductivity K_{eff} is computed by adapting a procedure, which was originally developed for mean uniform flows (see Dagan, 1989, Section 3.4.3), and the final result reads as follows:

$$\int_0^\infty \int_0^r \mathrm{d}K\, \mathrm{d}\mathcal{R}\, f(K,\mathcal{R}) \frac{K_{eff}-K}{K_{eff}+K}\mathcal{R} = 0, \tag{6}$$

being $\mathcal{R}$ the characteristic size of the single inclusion. Since the model regards the medium as isotropic, the effective conductivity has a scalar nature. The non-linear Equation 6 allows computing K_{eff}, once the joint probability distribution function $f \equiv f(K,\mathcal{R})$ is selected. The most important feature which is detected from 6 is the dependence of K_{eff} upon the radial distance r that, like the EC, prevents *de facto* considering K_{eff} as a local medium's property. Equation 6 allows investigating flow in the near and far field. Thus, close to the well (small r) it writes as: $\int_0^\infty \mathrm{d}K\, f(K,r)\,(K_{eff}-K)/(K_{eff}+K) \simeq 0$, which clearly shows the non-locality of the EC. To the contrary, away from the well (large r), one has:

$$\int_0^\infty \int_0^\infty \mathrm{d}K\, \mathrm{d}\mathcal{R}\, f(K,\mathcal{R}) \frac{K_{eff}-K}{K_{eff}+K}\mathcal{R} = 0. \tag{7}$$

In this case, the K_{eff} is not anymore a function of the position, and therefore, one can claim that in the far field it is a medium's property. A similar result was found to be valid also for transport (Di Dato et al., 2017).

With this prerequisite, the identification of the effective conductivity comes from the requirement that $K_{eq} \simeq K_{eff}$, which easily leads to the estimate of the variance σ_Y^2. To elucidate "in practice" the use of such a methodology, we consider the above described flow experiments. Preliminarily we note that, in this case, inclusions have the same characteristic size $\mathcal{R}_0$, and therefore, their distribution may be approximated by a δ-function centered at $\mathcal{R}_0$. Then, by regarding Y as a normally distributed random variable, the joint distribution function f is written as follows:

$$f(Y,\mathcal{R}) \simeq \frac{1}{\sqrt{2\pi}\,\sigma_Y}\exp\left[-\frac{(Y-\langle Y\rangle)^2}{2\sigma_Y^2}\right]\delta(\mathcal{R}-\mathcal{R}_0). \tag{8}$$

Insertion of (8) into (6) and switching to the fluctuation $Y' = Y - \langle Y\rangle$ lead to:

$$H(r-\mathcal{R}_0)\int_{-\infty}^{+\infty} \mathrm{d}Y' \exp\left[-\frac{1}{2}\left(\frac{Y'}{\sigma_Y}\right)^2\right]\frac{\kappa - \exp Y'}{\kappa + \exp Y'} = 0, \tag{9}$$

being $\kappa = K_{eff}/K_G$, whereas $H \equiv H(x)$ is the Heaviside step-function. The solution of 9 is found in a fairly good agreement (the error is less than 10^{-4}) with $\sigma_Y^2 = H(r-\mathcal{R}_0)\ln\kappa$. Then, by estimating κ as the ratio between the previously determined EC (see Figure 7) and $K_G = 7.22 \cdot 10^{-5}$ m/s, one can easily identify σ_Y^2. Results from such a calibration procedure are summarized in Table 3, where we have also included the statistics of σ_Y^2 as obtained from measurements. Unlike the confidence intervals at 95%, the two at 68% do not overlap. The explanation for such a discrepancy is multi-fold:

1. The self-consistent model (6) is valid "in a strict sense" for a two-dimensional formation with circular inclusions, whereas the packed aquifer has a three dimensional structure. Nevertheless, this does not represent a serious limitation, since the radial flow pattern holds the Dupuit's assumption, due to the fact that streamlines are practically horizontal (owing to the absence of screening). The feasibility of a 2D-modeling has been addressed recently in a couple of studies (Dagan et al., 2009; Dagan & Lessoff, 2007), as follow up of the

19447973, 2022, 5, Downloaded from https://agupubs.onlinelibrary.wiley.com/doi/10.1029/2021WR030717 by Indian Institute Of Technology, Kharagpur Central Library, Wiley Online Library on [17/01/2024]. See the Terms and Conditions (https://onlinelibrary.wiley.com/terms-and-conditions) on Wiley Online Library for rules of use; OA articles are governed by the applicable Creative Commons License

19447973, 2022, 5, Downloaded from https://agupubs.onlinelibrary.wiley.com/doi/10.1029/2021WR030717 by Indian Institute Of Technology, Kharagpur Central Library, Wiley Online Library on [17/01/2024]. See the Terms and Conditions (https://onlinelibrary.wiley.com/terms-and-conditions) on Wiley Online Library for rules of use; OA articles are governed by the applicable Creative Commons License

earlier analysis from Meier et al. (1998). Moreover, the main effect of the space dimensionality is mostly influential within a tiny region surrounding the well whose characteristic length is of $\mathcal{O}(I)$ (Indelman, 1996). Since, in the present study $I = 10$ cm, and the nearest battery of sampling piezometers is at $r = 23$ cm (Figure 4), the issue of the space dimensionality is an unwarranted complication (see, also Riva et al., 2006; Severino et al., 2011).

Overall, the $2D$ (approximated) approach should be adopted cautiously (better avoided) in those circumstances in which the presence of screening, or any other disturbance, clearly defines the Dupuit's assumption. In these cases, the $3D$ modeling remains the only viable option;

2. While the model (6) relies upon the assumption of circular inclusions, the real aquifer is made up of inclusions of parallelepiped-shape. Even if the shape of inclusions does not constitute *per sé* a crucial flaw (Jankovic et al., 2003), the impact of the formation's anisotropy (neglected in our approach) could be a possible argument to explain the difference in the confidence intervals at 68%;
3. Given the geometrical configuration of the packed aquifer, we have taken for granted the requirement of ergodicity, that is *the ratio between the aquifer's thickness and the vertical integral scale is much larger than one.* In the present study, this ratio is 35/5 that is not large enough to regard the flow as fully ergodic [an extended treatment on this requirement can be found in Sanchez-Vila and Tartakovsky (2007) and Di Dato et al. (2017)]. This leaves a certain degree of uncertainty upon the mean values of the hydraulic heads;
4. We have dealt with a zero correlated joint probability distribution (8). This makes the algebraic Equation 6, providing the effective conductivity, quite similar to that known for mean uniform flows (Dagan, 1989; Severino & Santini, 2005). Nevertheless, away from the well the flow behaves like a mean uniform one (Indelman, 2001; Severino, 2011), and therefore, we conclude that the neglect of correlation in $f \equiv f(K, R)$ becomes a reasonable working hypothesis in the far field, that is, $r/I \gg 1$. For the experiment at stake, the minimum sampling distance r is such that $r/I = 23/10$, and therefore, the above condition of "far field" is not fulfilled completely (at least for the nearest battery of piezometers);
5. The number N_p of measurements (see Table 2) at each radial distance adopted to compute the spatial average in 4 is certainly not extremely large (owing to the many logistic as well as economic limitations). This, together with the experimental measurement errors, represents another way to address the discrepancy in Table 3;
6. The plan view of the real setting is square, whereas the flow model relies upon a domain with a polar symmetry. Although, this issue is certainly of minor relevance for the applications (see, e.g., Fernández-Garcia et al., 2004), it nevertheless has to be considered as a warning against to straightforward generalizations and/or adaptions to domains of different shape (Severino et al., 2011).

To summarize, even if the experimental conditions do not match perfectly the assumptions underlying the effective conductivity model (6), they nevertheless lead to predictions that are in relatively good agreement with real data. Hence, we believe that our approach lends itself as a robust tool to identify the variance σ_Y^2, the approximations and experimental limitations, notwithstanding.

4. Conclusions and Recommendations for Future Studies

We have presented a baseline experimental study on well-type steady flows in a strongly heterogeneous aquifer. The latter was artificially packed in order to reproduce a given, statistical distribution of the log-conductivity Y, with a high degree of heterogeneity $\left(\sigma_Y^2 = 3.79\right)$. The density of sampling locations (piezometers), and the scrupulous control of the experimental conditions, permitted to minimize the uncertainties, which normally prevent achieving definite and conclusive insights. The experiments were designed to investigate the combined influence of the flow configuration and the large spatial variability of the conductivity. Hence, the head measurements collected during pumping tests at different volumetric flow rates provide a unique data-set to grasp what can be expected in similar, field scale situations.

Our analyses have focused upon the problem of upscaling the conductivity K, a topic that, unlike the case of weakly heterogeneous formations (i.e., $\sigma_Y^2 \ll 1$), has received a scarce number of experimental studies. In particular, a salient question is whether one can still implement the concept of equivalent conductivity (similarly to Severino & Coppola, 2012, in the case of unsaturated flows). It is shown that the latter depends on the mean flow pattern (in agreement with theoretical studies), and therefore, the EC can not be regarded as a formation's

19447973, 2022, 5, Downloaded from https://agupubs.onlinelibrary.wiley.com/doi/10.1029/2021WR030717 by Indian Institute Of Technology, Kharagpur Central Library, Wiley Online Library on [17/01/2024]. See the Terms and Conditions (https://onlinelibrary.wiley.com/terms-and-conditions) on Wiley Online Library for rules of use; OA articles are governed by the applicable Creative Commons License

property. In spite of these limitations, the EC lends itself as a robust parameter to simulate steady well-flows in strongly heterogeneous formations. Then, we have taken advantage from the data set to validate a new methodology enabling one to identify the variance σ_Y^2 of the log-hydraulic conductivity. Such a procedure leads to predictions close to the benchmark of the real data.

Now, we wish to illustrate, from the point of view of the practitioners, how results of the present studies can be used in the real-world applications. In particular, we ask whether radial flow occurring in a strongly heterogeneous formation can be solved ”only once”, by dealing with a medium with the same behavior of the real setting, similarly to the stand point adopted for flow under natural gradient conditions (Jankovic et al., 2003). Owing to the non-locality of the conductivity, the critical issue concerns the K-values to be assigned to each point of the flow domain. This is accomplished straightforwardly by means of Figure 8, which allows one to compute the proper nodal $K_{\mathrm{eq}} = \alpha(r)V(r)$, once the radial distance r is selected.

While the above example shows straightforwardly how findings of our study can be readily applied to solve practical problems, there are numerous aspects which require further investigations. Among these, we believe that the priority should be given to the impact of the aquifer's boundaries, and to the attainment of the ergodicity requirement (along the lines traced in Di Dato et al., 2017). Another central topic is the extension of the self-consistent solution to $3D$. Unlike the case of mean uniform flows (Dagan, 1979), this is a rather difficult problem, and one can not make use of the potential theory, like in two-dimensional formations (Severino, 2019). A few of these challenges are part of ongoing research projects.

To conclude, we hope that the data-set exploited in the present study, and the preliminary check of theoretical results, will be useful for other researchers, and stimulate new theoretical/experimental developments on the topic.

Conflict of Interest

The authors declare no conflicts of interest relevant to this study.

Data Availability Statement

Although the exploited information is contained in the figures and tables provided in the manuscript, the used data set is also available at https://doi.org/10.5281/zenodo.5773764.

Acknowledgments

We are indebted to the Editor in Chief (Xavier Sanchez-Vila) and to the Associate Editor (Daniel Fernández-Garcia) for providing numerous and fruitful suggestions. We thank Aldo Fiori and an anonymous Reviewer for their comments, which have tremendously improved the early version of the manuscript. Open Access Funding provided by Università degli Studi di Napoli Federico II within the CRUI-CARE Agreement.

References

Attinger, S. (2003). Generalized coarse graining procedures for flow in porous media. *Computational Geosciences*, *7*(4), 253–273. https://doi.org/10.1023/B:COMG.0000005243.73381.e3

Boggs, J. M., Young, S. C., Beard, L. M., Gelhar, L. W., Rehfeldt, K. R., & Adams, E. E. (1992). Field study of dispersion in a heterogeneous aquifer: 1. Overview and site description. *Water Resources Research*, *28*(12), 3281–3291. https://doi.org/10.1029/92WR01756

Bohling, G. C., Liu, G., Knobbe, S. J., Reboulet, E. C., Hyndman, D. W., Dietrich, P., & Butler, J. J., Jr. (2012). Geostatistical analysis of centimeter-scale hydraulic conductivity variations at the MADE site. *Water Resources Research*, *48*(2). https://doi.org/10.1029/2011WR010791

Cardiff, M., Bakhos, T., Kitanidis, P., & Barrash, W. (2013). Aquifer heterogeneity characterization with oscillatory pumping: Sensitivity analysis and imaging potential. *Water Resources Research*, *49*(9), 5395–5410. https://doi.org/10.1002/wrcr.20356,2013

Chao, H.-C., Rajaram, H., & Illangasekare, T. (2000). Intermediate-scale experiments and numerical simulations of transport under radial flow in a two-dimensional heterogeneous porous medium. *Water Resources Research*, *36*(10), 2869–2884. https://doi.org/10.1029/2000WR900096

Choy, T. C. (2015). *Effective medium theory: Principles and applications*. Oxford: Oxford University Press.

Copty, N. K., Trinchero, P., & Sanchez-Vila, X. (2011). Inferring spatial distribution of the radially integrated transmissivity from pumping tests in heterogeneous confined aquifers. *Water Resources Research*, *47*(5). https://doi.org/10.1029/2010WR009877

Copty, N. K., Trinchero, P., Sanchez-Vila, X., Sarioglu, M. S., & Findikakis, A. N. (2008). Influence of heterogeneity on the interpretation of pumping test data in leaky aquifers. *Water Resources Research*, *44*(11). https://doi.org/10.1029/2008WR007120

Dagan, G. (1978). A note on packer, slug, and recovery tests in unconfined aquifers. *Water Resources Research*, *14*(5), 929–934. https://doi.org/10.1029/WR014i005p00929

Dagan, G. (1979). Models of groundwater flow in statistically homogeneous porous formations. *Water Resources Research*, *15*(1), 47–63. https://doi.org/10.1029/WR015i001p00047

Dagan, G. (1989). *Flow and transport in porous formation*. New York: Springer-Verlag.

Dagan, G., Fiori, A., & Jankovic, I. (2013). Upscaling of flow in heterogeneous porous formations: Critical examination and issues of principle. *Advances in Water Resources*, *51*, 67–85. https://doi.org/10.1016/j.advwatres.2011.12.017

Dagan, G., & Lessoff, S. C. (2007). Transmissivity upscaling in numerical aquifer models of steady well flow: Unconditional statistics. *Water Resources Research*, *43*(5), 1–12. https://doi.org/10.1029/2006WR005235

Dagan, G., Lessoff, S. C., & Fiori, A. (2009). Is transmissivity a meaningful property of natural formations? Conceptual issues and model development. *Water Resources Research*, *45*(3), 1–12. https://doi.org/10.1029/2008WR007410

Demir, M. T., Copty, N. K., Trinchero, P., & Sanchez-Vila, X. (2017). Bayesian estimation of the transmissivity spatial structure from pumping test data. *Advances in Water Resources*, *104*, 174–182. https://doi.org/10.1016/j.advwatres.2017.03.021

Desbarats, A. (1992). Spatial averaging of transmissivity in heterogeneous fields with flow toward a well. *Water Resources Research*, *28*(3), 757–767. https://doi.org/10.1029/91WR03099

Desbarats, A. (1994). Spatial averaging of hydraulic conductivity under radial flow conditions. *Mathematical Geology*, *26*(1), 1–21. https://doi.org/10.1007/BF02065873

Di Dato, M., Fiori, A., de Barros, F. P., & Bellin, A. (2017). Radial solute transport in highly heterogeneous aquifers: Modeling and experimental comparison. *Water Resources Research*, *53*(7), 5725–5741. https://doi.org/10.1002/2016WR020039

Durlofsky, L. J. (2000). An approximate model for well productivity in heterogeneous porous media. *Mathematical Geology*, *32*(4), 421–438. https://doi.org/10.1023/A:1007521831889

Fallico, C., Ianchello, M., De Bartolo, S., & Severino, G. (2018). Spatial dependence of the hydraulic conductivity in a well-type configuration at the mesoscale. *Hydrological Processes*. https://doi.org/10.1002/hyp.11422

Fernández-Garcia, D., Illangasekare, T. H., & Rajaram, H. (2004). Conservative and sorptive forced-gradient and uniform flow tracer tests in a three-dimensional laboratory test aquifer. *Water Resources Research*, *40*(10), 1–17. W10103. https://doi.org/10.1029/2004WR003112

Fernandez-Garcia, D., Sánchez-Vila, X., & Illangasekare, T. H. (2002). Convergent-flow tracer tests in heterogeneous media: Combined experimental–numerical analysis for determination of equivalent transport parameters. *Journal of Contaminant Hydrology*, *57*(1–2), 129–145. https://doi.org/10.1016/S0169-7722(01)00214-5

Firmani, G., Fiori, A., & Bellin, A. (2006). Three-dimensional numerical analysis of steady state pumping tests in heterogeneous confined aquifers. *Water Resources Research*, *42*(3), W03422. https://doi.org/10.1029/2005WR004382

Indelman, P. (1996). Averaging of unsteady flows in heterogeneous media of stationary conductivity. *Journal of Fluid Mechanics*, *310*, 39–60. https://doi.org/10.1017/S0022112096001723

Indelman, P. (2001). Steady-state source flow in heterogeneous porous media. *Transport in Porous Media*, *45*(1), 105–127. https://doi.org/10.1023/A:1011814418142

Indelman, P., Fiori, A., & Dagan, G. (1996). Steady flow toward wells in heterogeneous formations: Mean head and equivalent conductivity. *Water Resources Research*, *32*(7), 1975–1983. https://doi.org/10.1029/96WR00990

Istok, J., Blout, D., Barker, L., Johnejack, K., & Hammermeister, D. (1994). Spatial variability in alluvium properties at a low-level nuclear waste site. *Soil Science Society of America Journal*, *58*(4), 1040–1051. https://doi.org/10.2136/sssaj1994.03615995005800040007x

Jankovic, I., Fiori, A., & Dagan, G. (2003). Effective conductivity of an isotropic heterogeneous medium of lognormal conductivity distribution. *Multiscale Modeling and Simulation*, *1*(1), 40–56. https://doi.org/10.1137/S1540345902409633

Janković, I., Fiori, A., & Dagan, G. (2006). Modeling flow and transport in highly heterogeneous three-dimensional aquifers: Ergodicity, Gaussianity, and anomalous behavior-1. conceptual issues and numerical simulations. *Water Resources Research*, *42*(6), 1–9. W06D12. https://doi.org/10.1029/2005WR004734

Leven, C., & Dietrich, P. (2006). What information can we get from pumping tests?-comparing pumping test configurations using sensitivity coefficients. *Journal of Hydrology*, *319*(1–4), 199–215. https://doi.org/10.1016/j.jhydrol.2005.06.030

Matheron, G. (1967). *Eléments pour une théorie des milieux poreux*. Masson.

Meier, P. M., Carrera, J., & Sánchez-Vila, X. (1998). An evaluation of jacob's method for the interpretation of pumping tests in heterogeneous formations. *Water Resources Research*, *34*(5), 1011–1025. https://doi.org/10.1029/98WR00008

Neuman, S. P., Guadagnini, A., & Riva, M. (2004). Type-curve estimation of statistical heterogeneity. *Water Resources Research*, *40*(4). https://doi.org/10.1029/2003WR002405

Neuman, S. P., & Orr, S. (1993). Prediction of steady state flow in nonuniform geologic media by conditional moments: Exact nonlocal formalism, effective conductivities, and weak approximation. *Water Resources Research*, *29*(2), 341–364. https://doi.org/10.1029/2003WR002405

Neuman, S., Tartakovsky, D., Wallstrom, T., & Winter, C. (1996). Correction to prediction of steady state flow in nonuniform geologic media by conditional moments: Exact nonlocal formalism, effective conductivities, and weak approximation? *Water Resources Research*, *32*(5), 1479–1480. https://doi.org/10.1029/96WR00489

Pechstein, A., Attinger, S., Krieg, R., & Copty, N. K. (2016). Estimating transmissivity from single-well pumping tests in heterogeneous aquifers. *Water Resources Research*, *52*(1), 495–510. https://doi.org/10.1002/2015WR017845

Pechstein, A., & Copty, N. K. (2021). Interpretation of pumping tests in heterogeneous aquifers with constant head boundary. *Groundwater*, *59*(4), 517–523. https://doi.org/10.1111/gwat.13072

Ptak, T., & Teutsch, G. (1994). Forced and natural gradient tracer tests in a highly heterogeneous porous aquifer: Instrumentation and measurements. *Journal of Hydrology*, *159*(1–4), 79–104. https://doi.org/10.1016/0022-1694(94)90250-X

Rehfeldt, K. R., Boggs, J. M., & Gelhar, L. W. (1992). Field study of dispersion in a heterogeneous aquifer: 3. Geostatistical analysis of hydraulic conductivity. *Water Resources Research*, *28*(12), 3309–3324. https://doi.org/10.1029/92WR01758

Riva, M., Sánchez-Vila, X., Guadagnini, A., De Simoni, M., & Willmann, M. (2006). Travel time and trajectory moments of conservative solutes in two-dimensional convergent flows. *Journal of Contaminant Hydrology*, *82*(1–2), 23–43. https://doi.org/10.1016/j.jconhyd.2005.06.014

Rubin, Y. (2003). *Applied stochastic hydrogeology*. Oxford: Oxford University Press.

Sánchez-Vila, X. (1997). Radially convergent flow in heterogeneous porous media. *Water Resources Research*, *33*(7), 1633–1641. https://doi.org/10.1029/97WR01001

Sánchez-Vila, X., Axness, C. L., & Carrera, J. (1999). Upscaling transmissivity under radially convergent flow in heterogeneous media. *Water Resources Research*, *35*(3), 613–621. https://doi.org/10.1029/1998WR900056

Sanchez-Vila, X., Guadagnini, A., & Carrera, J. (2006). Representative hydraulic conductivities in saturated groundwater flow. *Reviews of Geophysics*, *44*(3). https://doi.org/10.1029/2005RG000169

Sánchez-Vila, X., Meier, P. M., & Carrera, J. (1999). Pumping tests in heterogeneous aquifers: An analytical study of what can be obtained from their interpretation using Jacob's method. *Water Resources Research*, *35*(4), 943–952. https://doi.org/10.1029/1999WR900007

Sanchez-Vila, X., & Tartakovsky, D. M. (2007). W0341 Ergodicity of pumping tests. *Water Resources Research*, *43*(3), 1–5. https://doi.org/10.1029/2006WR005241

Schad, H., & Teutsch, G. (1994). Effects of the investigation scale on pumping test results in heterogeneous porous aquifers. *Journal of Hydrology*, *159*(1–4), 61–77. https://doi.org/10.1016/0022-1694(94)90249-6

Schneider, C. L., & Attinger, S. (2008). Beyond Thiem: A new method for interpreting large scale pumping tests in heterogeneous aquifers. *Water Resources Research*, *44*(4). https://doi.org/10.1029/2007WR005898

19447973, 2022, 5, Downloaded from https://agupubs.onlinelibrary.wiley.com/doi/10.1029/2021WR030717 by Indian Institute Of Technology, Kharagpur Central Library, Wiley Online Library on [17/01/2024]. See the Terms and Conditions (https://onlinelibrary.wiley.com/terms-and-conditions) on Wiley Online Library for rules of use; OA articles are governed by the applicable Creative Commons License

10.1029/2021WR030717

Severino, G. (2011). Stochastic analysis of well-type flows in randomly heterogeneous porous formations. *Water Resources Research*, *47*(3), 1–12. https://doi.org/10.1029/2010WR009840

Severino, G. (2019). Effective conductivity in steady well-type flows through porous formations. *Stochastic Environmental Research and Risk Assessment*, *33*(3), 827–835. https://doi.org/10.1007/s00477-018-1639-5

Severino, G., & Coppola, A. (2012). A note on the apparent conductivity of stratified porous media in unsaturated steady flow above a water table. *Transport in Porous Media*, *91*(2), 733–740. https://doi.org/10.1007/s11242-011-9870-2

Severino, G., De Bartolo, S., Brunetti, G., Sommella, A., & Fallico, C. (2019). Experimental evidence of the stochastic behavior of the conductivity in radial flow configurations. *Stochastic Environmental Research and Risk Assessment*, 1–7. https://doi.org/10.1007/s00477-019-01704-z

Severino, G., Leveque, S., & Toraldo, G. (2019). Uncertainty quantification of unsteady source flows in heterogeneous porous media. *Journal of Fluid Mechanics*, *10*(7), 5–26. https://doi.org/10.1017/jfm.2019.203

Severino, G., & Santini, A. (2005). On the effective hydraulic conductivity in mean vertical unsaturated steady flows. *Advances in Water Resources*, *28*, 964–974. https://doi.org/10.1016/j.advwatres.2005.03.003

Severino, G., Santini, A., & Monetti, V. M. (2009). Modelling water flow and solute transport in heterogeneous unsaturated porous media. In *Advances in modeling agricultural systems* (pp. 361–383). Boston, MA: Springer. https://doi.org/10.1007/978-0-387-75181-8_17

Severino, G., Santini, A., & Sommella, A. (2011). Macrodispersion by diverging radial flows in randomly heterogeneous porous media. *Journal of Contaminant Hydrology*, *123*(1), 40–49. https://doi.org/10.1016/j.jconhyd.2010.12.005

Wu, C.-M., Yeh, T.-C. J., Zhu, J., Lee, T. H., Hsu, N.-S., Chen, C.-H., & Sancho, A. F. (2005). Traditional analysis of aquifer tests: Comparing apples to oranges? *Water Resources Research*, *41*(9). https://doi.org/10.1029/2004WR003717

Zech, A., Arnold, S., Schneider, C., & Attinger, S. (2015). Estimating parameters of aquifer heterogeneity using pumping tests–implications for field applications. *Advances in Water Resources*, *83*, 137–147.

Zech, A., & Attinger, S. (2016). Analytical drawdown solution for steady-state pumping tests in two-dimensional isotropic heterogeneous aquifers. *Hydrology and Earth System Sciences*, *20*(5), 1655–1667. https://doi.org/10.5194/hess-20-1655-2016

Zech, A., Schneider, C. L., & Attinger, S. (2012). The extended thiem's solution: Including the impact of heterogeneity. *Water Resources Research*, *48*(10). https://doi.org/10.1002/2015WR018509

19447973, 2022, 5, Downloaded from https://agupubs.onlinelibrary.wiley.com/doi/10.1029/2021WR030717 by Indian Institute Of Technology, Kharagpur Central Library, Wiley Online Library on [17/01/2024]. See the Terms and Conditions (https://onlinelibrary.wiley.com/terms-and-conditions) on Wiley Online Library for rules of use; OA articles are governed by the applicable Creative Commons License