

Aufgabe 02a

$$(\lambda f. \underbrace{x.fx}_{GV=\{x\}})(\lambda zy. \underbrace{xy}_{FV=\{x\}})x$$

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$$\begin{aligned} & (\lambda f \underbrace{x.fx}_{GV=\{x\}}) (\lambda zy.\underbrace{xy}_{FV=\{x\}})x \\ \Rightarrow_{\alpha} & (\lambda f \underbrace{x'.fx'}_{GV=\{x'\}}) (\lambda zy.\underbrace{xy}_{FV=\{x\}})x \end{aligned}$$

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$$\begin{aligned} & (\lambda f \underbrace{x.fx}_{GV=\{x\}}) (\lambda zy.\underbrace{xy}_{FV=\{x\}})x \\ \Rightarrow_{\alpha} & (\lambda f \underbrace{x'.fx'}_{GV=\{x'\}}) (\lambda zy.\underbrace{xy}_{FV=\{x\}})x \\ \Rightarrow_{\beta} & (\lambda x'.\underbrace{(\lambda zy.xy)x'}_{GV=\{y,z\}}) \underbrace{x}_{FV=\{x\}} \end{aligned}$$

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$$\begin{aligned}& (\lambda f \underbrace{x.fx}_{GV=\{x\}}) (\lambda zy.\underbrace{xy}_{FV=\{x\}})x \\& \Rightarrow_{\alpha} (\lambda f \underbrace{x'.fx'}_{GV=\{x'\}}) (\lambda zy.\underbrace{xy}_{FV=\{x\}})x \\& \Rightarrow_{\beta} (\lambda x'.\underbrace{(\lambda zy.xy)x'}_{GV=\{y,z\}}) \underbrace{x}_{FV=\{x\}} \\& \Rightarrow_{\beta} (\lambda z \underbrace{y.xy}_{GV=\{y\}}) \underbrace{x}_{FV=\{x\}}\end{aligned}$$

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$$\begin{aligned} & (\lambda f \underbrace{x.fx}_{GV=\{x\}}) (\lambda zy.\underbrace{xy}_{FV=\{x\}})x \\ \Rightarrow_{\alpha} & (\lambda f \underbrace{x'.fx'}_{GV=\{x'\}}) (\lambda zy.\underbrace{xy}_{FV=\{x\}})x \\ \Rightarrow_{\beta} & (\lambda x'.\underbrace{(\lambda zy.xy)x'}_{GV=\{y,z\}}) \underbrace{x}_{FV=\{x\}} \\ \Rightarrow_{\beta} & (\lambda z \underbrace{y.xy}_{GV=\{y\}}) \underbrace{x}_{FV=\{x\}} \\ \Rightarrow_{\beta} & (\lambda y.xy) \end{aligned}$$

Aufgabe 2b

$g :: \mathbf{Int} \rightarrow \mathbf{Int} \rightarrow \mathbf{Int}$

$g\ a\ 0 = a$

$g\ a\ b$

| $b == 1 = g\ (a + 1)\ (b - 1)$

| **otherwise** = $g\ (a + 2)\ (b - 2)$

$\langle G \rangle = ((\lambda gxy. \langle ite \rangle (\langle iszero \rangle y)$

$(\langle ite \rangle (\langle iszero \rangle (\langle pred \rangle y)))$

$(g(\langle succ \rangle x)(\langle pred \rangle y))$

$(g(\langle succ \rangle (\langle succ \rangle x))(\langle pred \rangle (\langle pred \rangle y)))$

$))$