Summer Work Number Systems Homework

Read the following article on the number systems. Then answer all the problems that follow in the Self-Review & Exercises. Then complete the worksheets that follow. Will discuss & go over the first week of class.



Number Systems (on CD)

Objectives

- To understand basic number systems concepts such as base, positional value, and symbol value.
- To understand how to work with numbers represented in the binary, octal, and hexadecimal number systems
- To be able to abbreviate binary numbers as octal numbers or hexadecimal numbers.
- To be able to convert octal numbers and hexadecimal numbers to binary numbers.
- To be able to covert back and forth between decimal numbers and their binary, octal, and hexadecimal equivalents.
- To understand binary arithmetic, and how negative binary numbers are represented using two's complement notation.

Here are only numbers ratified. William Shakespeare

Nature has some sort of arithmetic-geometrical coordinate system, because nature has all kinds of models. What we experience of nature is in models, and all of nature's models are so beautiful.

It struck me that nature's system must be a real beauty, because in chemistry we find that the associations are always in beautiful whole numbers—there are no fractions.

Richard Buckminster Fuller



Outline

- E.1 Introduction
- E.2 Abbreviating Binary Numbers as Octal Numbers and Hexadecimal Numbers
- E.3 Converting Octal Numbers and Hexadecimal Numbers to Binary Numbers
- E.4 Converting from Binary, Octal, or Hexadecimal to Decimal
- E.5 Converting from Decimal to Binary, Octal, or Hexadecimal
- E.6 Negative Binary Numbers: Two's Complement Notation

Summary • Terminology • Self-Review Exercises • Answers to Self-Review Exercises • Exercises

E.1 Introduction

In this appendix, we introduce the key number systems that Java programmers use, especially when they are working on software projects that require close interaction with "machine-level" hardware. Projects like this include operating systems, computer networking software, compilers, database systems, and applications requiring high performance.

When we write an integer such as 227 or -63 in a Java program, the number is assumed to be in the decimal (base 10) number system. The digits in the decimal number system are 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. The lowest digit is 0 and the highest digit is 9—one less than the base of 10. Internally, computers use the binary (base 2) number system. The binary number system has only two digits, namely 0 and 1. Its lowest digit is 0 and its highest digit is 1—one less than the base of 2.

As we will see, binary numbers tend to be much longer than their decimal equivalents. Programmers who work in assembly languages and in high-level languages like Java that enable programmers to reach down to the "machine level," find it cumbersome to work with binary numbers. So two other number systems the octal number system (base 8) and the hexadecimal number system (base 16)—are popular primarily because they make it convenient to abbreviate binary numbers.

In the octal number system, the digits range from 0 to 7. Because both the binary number system and the octal number system have fewer digits than the decimal number system, their digits are the same as the corresponding digits in decimal.

The hexadecimal number system poses a problem because it requires sixteen digits—a lowest digit of 0 and a highest digit with a value equivalent to decimal 15 (one less than the base of 16). By convention, we use the letters A through F to represent the hexadecimal digits corresponding to decimal values 10 through 15. Thus in hexadecimal we can have numbers like 876 consisting solely of decimal-like digits, numbers like 8A55F consisting of digits and letters, and numbers like FFE consisting solely of letters. Occasionally, a hexadecimal number spells a common word such as FACE or FEED—this can appear strange to programmers accustomed to working with numbers.

Each of these number systems uses positional notation—each position in which a digit is written has a different positional value. For example, in the decimal number 937 (the 9, the 3, and the 7 are referred to as symbol values), we say that the 7 is written in the ones position, the 3 is written in the tens position, and the 9 is written in the hundreds position.

Notice that each of these positions is a power of the base (base 10), and that these powers begin at 0 and increase by 1 as we move left in the number (Fig. E.3).

Binary digit	Octal digit	Decimal digit	Hexadecimal digit	
0	0	0	0	
1	1	1	1	
	2	2	2	
	3	3	3	
	4	4	4	
	5	5	5	
	6	6	6	
	7	7	7	
		8	8	
		9	9	
			A (decimal value of 10)	
			B (decimal value of 11)	
			c (decimal value of 12)	
			D (decimal value of 13)	
			E (decimal value of 14)	
			F (decimal value of 15)	

Flg. E.1 Digits of the binary, octal, decimal and hexadecimal number systems.

Attribute Binary		Octal	Decimal	Hexadecimal				
	CONTRACTOR VICTOR CONTRACTOR							
Base	2	8	10	16				
Lowest digit	0	0	0	0				
Highest digit	1	7	9	F				

Fig. E.2 Comparing the binary, octal, declmal and hexadecimal number systems.

NAME OF STREET OF STREET STREET, STREE	CONTRACTOR OF THE STATE OF THE	
e decimal numbe	r system	
DOMESTIC CONTRACTOR OF THE PARTY.		
9	3	7
Hundreds	Tens	Ones
100	10	1
10 ²	10 ¹	100
	9 Hundreds 100	9 3 Hundreds Tens 100 10

Fig. E.3 Positional values in the decimal number system.

For longer decimal numbers, the next positions to the left would be the thousands position (10 to the 3rd power), the ten-thousands position (10 to the 4th power), the hundred-thousands position (10 to the 5th power), the millions position (10 to the 6th power), the ten-millions position (10 to the 7th power), and so on.

In the binary number 101, we say that the rightmost 1 is written in the ones position, the 0 is written in the twos position, and the leftmost 1 is written in the fours position. Notice that each of these positions is a power of the base (base 2), and that these powers begin at 0 and increase by 1 as we move left in the number (Fig E.4).

For longer binary numbers, the next positions to the left would be the eights position (2 to the 3rd power), the sixteens position (2 to the 4th power), the thirty-twos position (2 to the 5th power), the sixty-fours position (2 to the 6th power), and so on.

In the octal number 425, we say that the 5 is written in the ones position, the 2 is written in the eights position, and the 4 is written in the sixty-fours position. Notice that each of these positions is a power of the base (base 8), and that these powers begin at 0 and increase by 1 as we move left in the number (Fig. E.5).

For longer octal numbers, the next positions to the left would be the five-hundred-and-twelves position (8 to the 3rd power), the four-thousand-and-ninety-sixes position (8 to the 4th power), the thirty-two-thousand-seven-hundred-and-sixty eights position (8 to the 5th power), and so on.

In the hexadecimal number 3DA, we say that the A is written in the ones position, the D is written in the sixteens position, and the 3 is written in the two-hundred-and-fifty-sixes position. Notice that each of these positions is a power of the base (base 16), and that these powers begin at 0 and increase by 1 as we move left in the number (Fig. E.6).

		· · · · · · · · · · · · · · · · · · ·	A MILLON CONTROL TO COMMUNICATE MATERIAL SERVICE DE CONTROL DE CON				
Positional values in the binary number system							
Binary digit	1	0	1				
Position name	Fours	Twos	Ones				
Positional value	4	2	1				
Positional value as a power of the base (2)	2 ²	2 ¹	2°				

Fig. E.4 Positional values in the binary number system.

Positional values in the octal number system							
		COMMUNICATION CONTRACTOR AND AND AND ASSESSMENT	WAR A PROCESSION OF THE PARTY O				
Decimal digit	4	2	5				
Position name	Sixty-fours	Eights	Ones				
Positional value	64	8	1				
Positional value as a power of the base (8)	82	81	80				

Fig. E.5 Positional values in the octal number system.

	e hexadecimai numb		
Decimal digit	3	D	A
Position name	Two-hundred-and- fifty-sixes	Sixteens	Ones
Positional value	256	16	1
Positional value as a power of the base (16)	16 ²	16 ¹	16 ⁰

Fig. E.6 Positional values in the hexadecimal number system.

For longer hexadecimal numbers, the next positions to the left would be the four-thousand-and-ninety-sixes position (16 to the 3rd power), the sixty-five-thousand-five-hundred-and-thirty-six position (16 to the 4th power), and so on.

E.2 Abbreviating Binary Numbers as Octal Numbers and Hexadecimal Numbers

The main use for octal and hexadecimal numbers in computing is for abbreviating lengthy binary representations. Figure E.7 highlights the fact that lengthy binary numbers can be expressed concisely in number systems with higher bases than the binary number system.

Decimal number	Binary{ representation	Octal representation	Hexadecimal representation
0	0	0	0
1	1	1	1
2	10	2	2
3	11	3	3
4	100	4	4
5	101	5	5
6	110	6	6
7	111	7	7
8	1000	10	8
9	1001	11	9
10	1010	12	Α
11	1011	13	В
12	1100	14	С
13	1101	15	D
14	1110	16	E
15	1111	17	F
16	10000	20	10

Fig. E.7 Decimal, binary, octal, and hexadecimal equivalents.

A particularly important relationship that both the octal number system and the hexadecimal number system have to the binary system is that the bases of octal and hexadecimal (8 and 16 respectively) are powers of the base of the binary number system (base 2). Consider the following 12-digit binary number and its octal and hexadecimal equivalents. See if you can determine how this relationship makes it convenient to abbreviate binary numbers in octal or hexadecimal. The answer follows the numbers.

Binary Number Octal equivalent Hexadecimal equivalent 100011010001 4321 8D1

To see how the binary number converts easily to octal, simply break the 12-digit binary number into groups of three consecutive bits each, and write those groups over the corresponding digits of the octal number as follows

Notice that the octal digit you have written under each group of thee bits corresponds precisely to the octal equivalent of that 3-digit binary number as shown in Fig. E.7.

The same kind of relationship may be observed in converting numbers from binary to hexadecimal. In particular, break the 12-digit binary number into groups of four consecutive bits each and write those groups over the corresponding digits of the hexadecimal number as follows

Notice that the hexadecimal digit you wrote under each group of four bits corresponds precisely to the hexadecimal equivalent of that 4-digit binary number as shown in Fig. E.7.

E.3 Converting Octal Numbers and Hexadecimal Numbers to Binary Numbers

In the previous section, we saw how to convert binary numbers to their octal and hexadecimal equivalents by forming groups of binary digits and simply rewriting these groups as their equivalent octal digit values or hexadecimal digit values. This process may be used in reverse to produce the binary equivalent of a given octal or hexadecimal number.

For example, the octal number 653 is converted to binary simply by writing the 6 as its 3-digit binary equivalent 110, the 5 as its 3-digit binary equivalent 101, and the 3 as its 3-digit binary equivalent 011 to form the 9-digit binary number 110101011.

The hexadecimal number FAD5 is converted to binary simply by writing the F as its 4-digit binary equivalent 1111, the A as its 4-digit binary equivalent 1010, the D as its 4-digit binary equivalent 1101, and the 5 as its 4-digit binary equivalent 0101 to form the 16-digit 11111010110101.

E.4 Converting from Binary, Octal, or Hexadecimal to Decimal

Because we are accustomed to working in decimal, it is often convenient to convert a binary, octal, or hexadecimal number to decimal to get a sense of what the number is "really" worth. Our diagrams in Section E.1 express the positional values in decimal. To convert a number to decimal from another base, multiply the decimal equivalent of each digit by its

positional value, and sum these products. For example, the binary number 110101 is converted to decirnal 53 as shown in Fig. E.8.

To convert octal 7614 to decimal 3980, we use the same technique, this time using appropriate octal positional values as shown in Fig. E.9.

To convert hexadecimal AD3B to decimal 44347, we use the same technique, this time using appropriate hexadecimal positional values as shown in Fig. E.10.

E.5 Converting from Decimal to Binary, Octal, or Hexadecimal

The conversions of the previous section follow naturally from the positional notation conventions. Converting from decimal to binary, octal, or hexadecimal also follows these conventions.

Suppose we wish to convert decimal 57 to binary. We begin by writing the positional values of the columns right to left until we reach a column whose positional value is greater than the decimal number. We do not need that column, so we discard it. Thus, we first write:

Converting a bina	ry number to	decimal				
Positional values:	32	16	8	4	2	1
Symbol values:	1	1	0	1	0	1
Products:	1*32=32	1*16=16	0*8=0	1*4=4	0*2=0	1*1=1
Sum:	= 32 + 1	16 + 0 +	4 + 0 +	1 = 53		

Fig. E.8 Converting a binary number to decimal.

Converting an oct	al number to decir	nal		
Positional values:	512	64	8	1
Symbol values:	7	6	1	4
Products	7*512=3584	6*64=384	1*8=8	4*1=4
Sum:	= 3584 + 384	+ 8 + 4 = 3	98,0	

Fig. E.9 Converting an octal number to decimal.

Products	A*4096=40960	D*256=3328	3*16=48	B*1=11
Symbol values:	A	D	3	В
Positional values:	4096	256	16	1
Converting a hexa		decimal		

Fig. E.10 Converting a hexadecimal number to decimal.

Converting a hexadecimal number to decimal

Sum: = 40960 + 3328 + 48 + 11 = 44347

Fig. E.10 Converting a hexadecimal number to decimal.

Positional values: 64 32 16 8 4 2 1

Then we discard the column with positional value 64 leaving:

Positional values: 32 16 8 4 2 1

Next we work from the leftmost column to the right. We divide 32 into 57 and observe that there is one 32 in 57 with a remainder of 25, so we write 1 in the 32 column. We divide 16 into 25 and observe that there is one 16 in 25 with a remainder of 9 and write 1 in the 16 column. We divide 8 into 9 and observe that there is one 8 in 9 with a remainder of 1. The next two columns each produce quotients of zero when their positional values are divided into 1 so we write 0s in the 4 and 2 columns. Finally, 1 into 1 is 1 so we write 1 in the 1 column. This yields:

Positional values: 32 16 8 4 2 1 Symbol values: 1 1 1 0 0 1

and thus decimal 57 is equivalent to binary 111001.

To convert decimal 103 to octal, we begin by writing the positional values of the columns until we reach a column whose positional value is greater than the decimal number. We do not need that column, so we discard it. Thus, we first write:

Positional values: 512 64 8 1

Then we discard the column with positional value 512, yielding:

Positional values: 64 8 1

Next we work from the leftmost column to the right. We divide 64 into 103 and observe that there is one 64 in 103 with a remainder of 39, so we write 1 in the 64 column. We divide 8 into 39 and observe that there are four 8s in 39 with a remainder of 7 and write 4 in the 8 column. Finally, we divide 1 into 7 and observe that there are seven 1s in 7 with no remainder so we write 7 in the 1 column. This yields:

Positional values: 64 8 1 Symbol values: 1 4 7

and thus decimal 103 is equivalent to octal 147.

To convert decimal 375 to hexadecimal, we begin by writing the positional values of the columns until we reach a column whose positional value is greater than the decimal number. We do not need that column, so we discard it. Thus, we first write

Positional values: 4096 256 16 1

Then we discard the column with positional value 4096, yielding:

Positional values: 256 16 1

Next we work from the leftmost column to the right. We divide 256 into 375 and observe that there is one 256 in 375 with a remainder of 119, so we write 1 in the 256 column. We divide 16 into 119 and observe that there are seven 16s in 119 with a remainder of 7 and write 7 in the 16 column. Finally, we divide 1 into 7 and observe that there are seven 1s in 7 with no remainder so we write 7 in the 1 column. This yields:

Positional values: 256 16 1 Symbol values: 1 7 7

and thus decimal 375 is equivalent to hexadecimal 177.

E.6 Negative Binary Numbers: Two's Complement Notation

The discussion in this appendix has been focussed on positive numbers. In this section, we explain how computers represent negative numbers using two's complement notation. First we explain how the two's complement of a binary number is formed, and then we show why it represents the negative value of the given binary number.

Consider a machine with 32-bit integers. Suppose

```
int value = 13;
```

The 32-bit representation of value is

```
00000000 00000000 00000000 00001101
```

To form the negative of **value** we first form its *one's complement* by applying Java's bitwise complement operator (~):

```
onesComplementOfValue = ~value;
```

Internally, **~value** is now **value** with each of its bits reversed—ones become zeros and zeros become ones as follows:

```
value:
00000000 00000000 00000000 00001101
~value (i.e., value's ones complement):
11111111 11111111 11111111 11110010
```

To form the two's complement of **value** we simply add one to **value**'s one's complement. Thus

```
Two's complement of value: 11111111 11111111 11110011
```

Now if this is in fact equal to -13, we should be able to add it to binary 13 and obtain a result of 0. Let us try this:

The carry bit coming out of the leftmost column is discarded and we indeed get zero as a result. If we add the one's complement of a number to the number, the result would be all 1s. The key to getting a result of all zeros is that the twos complement is 1 more than the one's complement. The addition of 1 causes each column to add to 0 with a carry of 1. The carry keeps moving leftward until it is discarded from the leftmost bit, and hence the resulting number is all zeros.

Computers actually perform a subtraction such as

```
x = a - value;
```

by adding the two's complement of value to a as follows:

```
x = a + (\sim value + 1);
```

Suppose **a** is 27 and **value** is 13 as before. If the two's complement of **value** is actually the negative of **value**, then adding the two's complement of value to a should produce the result 14. Let us try this:

which is indeed equal to 14.

SUMMARY

- When we write an integer such as 19 or 227 or -63 in a Java program, the number is automatically assumed to be in the decimal (base 10) number system. The digits in the decimal number system are 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. The lowest digit is 0 and the highest digit is 9—one less than the base of 10.
- Internally, computers use the binary (base 2) number system. The binary number system has only
 two digits, namely 0 and 1. Its lowest digit is 0 and its highest digit is 1—one less than the base of
 2.
- The octal number system (base 8) and the hexadecimal number system (base 16) are popular primarily because they make it convenient to abbreviate binary numbers.
- The digits of the octal number system range from 0 to 7.
- The hexadecimal number system poses a problem because it requires sixteen digits—a lowest digit of 0 and a highest digit with a value equivalent to decimal 15 (one less than the base of 16). By convention, we use the letters A through F to represent the hexadecimal digits corresponding to decimal values 10 through 15.
- Each number system uses positional notation—each position in which a digit is written has a different positional value.
- A particularly important relationship that both the octal number system and the hexadecimal number system have to the binary system is that the bases of octal and hexadecimal (8 and 16 respectively) are powers of the base of the binary number system (base 2).
- To convert an octal number to a binary number, simply replace each octal digit with its three-digit binary equivalent.

- To convert a hexadecimal number to a binary number, simply replace each hexadecimal digit with its four-digit binary equivalent.
- Because we are accustomed to working in decimal, it is convenient to convert a binary, octal or hexadecimal number to decimal to get a sense of the number's "real" worth.
- To convert a number to decimal from another base, multiply the decimal equivalent of each digit
 by its positional value, and sum these products.
- · Computers represent negative numbers using two's complement notation.
- To form the negative of a value in binary, first form its one's complement by applying Java's bitwise complement operator (-). This reverses the bits of the value. To form the two's complement of a value, simply add one to the value's one's complement.

TERMINOLOGY

base digit base 2 number system hexadecimal number system base 8 number system negative value base 10 number system octal number system base 16 number system one's complement notation binary number system positional notation bitwise complement operator (~) positional value conversions symbol value decimal number system two's complement notation

SELF-REVIEW EXERCISES

E.1	The	bases	of the	decimal,	binary,	octal,	and	hexadecimal	number	systems	are	,
	,			and	*********	respec	ctive	ly.				

- **E.2** In general, the decimal, octal, and hexadecimal representations of a given binary number contain (more/fewer) digits than the binary number contains.
- **E.3** (True/False) A popular reason for using the decimal number system is that it forms a convenient notation for abbreviating binary numbers simply by substituting one decimal digit per group of four binary bits.
- **E.4** The (octal / hexadecimal / decimal) representation of a large binary value is the most concise (of the given alternatives).
- **E.5** (True/False) The highest digit in any base is one more than the base.
- **E.6** (True/False) The lowest digit in any base is one less than the base.
- **E.7** The positional value of the rightmost digit of any number in either binary, octal, decimal, or hexadecimal is always _____.
- **E.8** The positional value of the digit to the left of the rightmost digit of any number in binary, octal, decimal, or hexadecimal is always equal to ______.
- **E.9** Fill in the missing values in this chart of positional values for the rightmost four positions in each of the indicated number systems:

decimal	1000	100	10	1
hexadecimal	• • •	256	• • •	
binary				
octal	512		R	

E.10 Convert binary 110101011000 to octal and to hexadecimal.

- E.11 Convert hexadecimal FACE to binary.
- E.12 Convert octal 7316 to binary.
- **E.13** Convert hexadecimal 4FEC to octal. (Hint: First convert 4FEC to binary then convert that binary number to octal.)
- E.14 Convert binary 1101110 to decimal.
- E.15 Convert octal 317 to decimal.
- E.16 Convert hexadecimal EFD4 to decimal.
- E.17 Convert decimal 177 to binary, to octal, and to hexadecimal.
- **E.18** Show the binary representation of decimal 417. Then show the one's complement of 417, and the two's complement of 417.
- **E.19** What is the result when the one's complement of a number is added to itself?

EXERCISES

- **E.20** Some people argue that many of our calculations would be easier in the base 12 number system because 12 is divisible by so many more numbers than 10 (for base 10). What is the lowest digit in base 12? What might the highest symbol for the digit in base 12 be? What are the positional values of the rightmost four positions of any number in the base 12 number system?
- **E.21** How is the highest symbol value in the number systems we discussed related to the positional value of the first digit to the left of the rightmost digit of any number in these number systems?
- **E.22** Complete the following chart of positional values for the rightmost four positions in each of the indicated number systems:

```
decimal1000100 10 1 base 6 ..... 6... base 13 ... 169... ... base 3 27......
```

- E.23 Convert binary 100101111010 to octal and to hexadecimal.
- E.24 Convert hexadecimal 3A7D to binary.
- E.25 Convert hexadecimal 765F to octal. (Hint: First convert 765F to binary, then convert that binary number to octal.)
- E.26 Convert binary 1011110 to decimal.
- E.27 Convert octal 426 to decimal.
- E.28 Convert hexadecimal FFFF to decimal.
- E.29 Convert decimal 299 to binary, to octal, and to hexadecimal.
- **E.30** Show the binary representation of decimal 779. Then show the one's complement of 779, and the two's complement of 779.
- **E.31** What is the result when the two's complement of a number is added to itself?
- **E.32** Show the two's complement of integer value -1 on a machine with 32-bit integers.





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Binary to Decimal Conversion Worksheet

Name:		
10101001	=	
00110010	=	
00111000	=	
01100010	=	
11101110	=	
11100001	=	
00101101	=	
00011000	=	
11010110	=	
01110010	=	





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Decimal to Binary Conversion Worksheet

Name:		
55	=	
147	=	
227	=	
235	=	
217	=	
252	=	
122	=	
105	=	
215	=	
41	=	





Save this file and use it offline, by simply clicking on the colored area. Save Paper & Trees, if you wish you can also print this document for later use.

Hexadecimal to Decimal Conversion Worksheet

Name:

11E	=	197	=
1B0	=	150	=
198	=	191	Ξ
146	=	1D2	=
73	=	1A2	=
86	=	18D	=
71	=	83	=
47	=	1DA	=

Number Systems Homework

1.	Convert binary 110101011000 to octal and to hexadecimal.
2.	Convert hexadecimal FACE to binary.
3.	Convert 7316 ₁₀ to binary.
4.	Convert hexadecimal 4FEC to octal.
5.	Convert binary 1101110 to decimal.

6.	Convert octal 317 to decimal.
7.	Convert hexadecimal EFD4 to decimal.
8.	Convert decimal 177 to binary, octal and hexadecimal.
9.	Show the binary representation of decimal 417. Then show the one's complement of 417, and the two's complement of 417.
10.	What is the result when one's complement of a number is added to itself?

Extra Number Base Problems

1-19 000

	-		1	12	1 .
Convert	from	hexad	ecimal	to	binary.

5A_{sixteen}
 479_{sixteen}

2. B6_{sixteen}

3. EF_{sixteen}

4. DB_{sixteen}

6. 807_{sixteen}

7. $39A5_{\text{sixteen}}$

8. 526A_{sixteen}

Convert from binary to hexadecimal.

9. 110111_{two}

12. 10111011100_{two}

10. 110110_{two}

11. 101101110_{two}

13. 10111011111011000_{two}

14. 110110111011111000_{two}

In one very common method of converting symbols into binary digits for a computer (a method called ASCII), the letters A through Z are assigned the numbers 65 through 90, so A has binary code 1000001 and Z has code 1011010. Lower case letters a through z have codes 97 through 122.

Write the binary code for each letter.

15. D

16. K

17. R

18. T

Break the following into groups of seven digits and write as letters.

19. 1001000100010110011001010000

20. 10000111001000101010110000111001011