SUPPORT VECTOR MACHINES IN R

RBF Kernels: Generating a complex dataset

A bit about RBF Kernels

- Highly flexible kernel.
  - Can fit complex decision boundaries.
- Commonly used in practice.

Generate a complex dataset

- 600 points (x1,x2)
- x1 and x2 distributed differently

```
n <- 600
set.seed(42)

df <- data.frame(x1 = rnorm(n, mean = -0.5, sd = 1),
                 x2 = runif(n, min = -1, max = 1))
```

Generate boundary

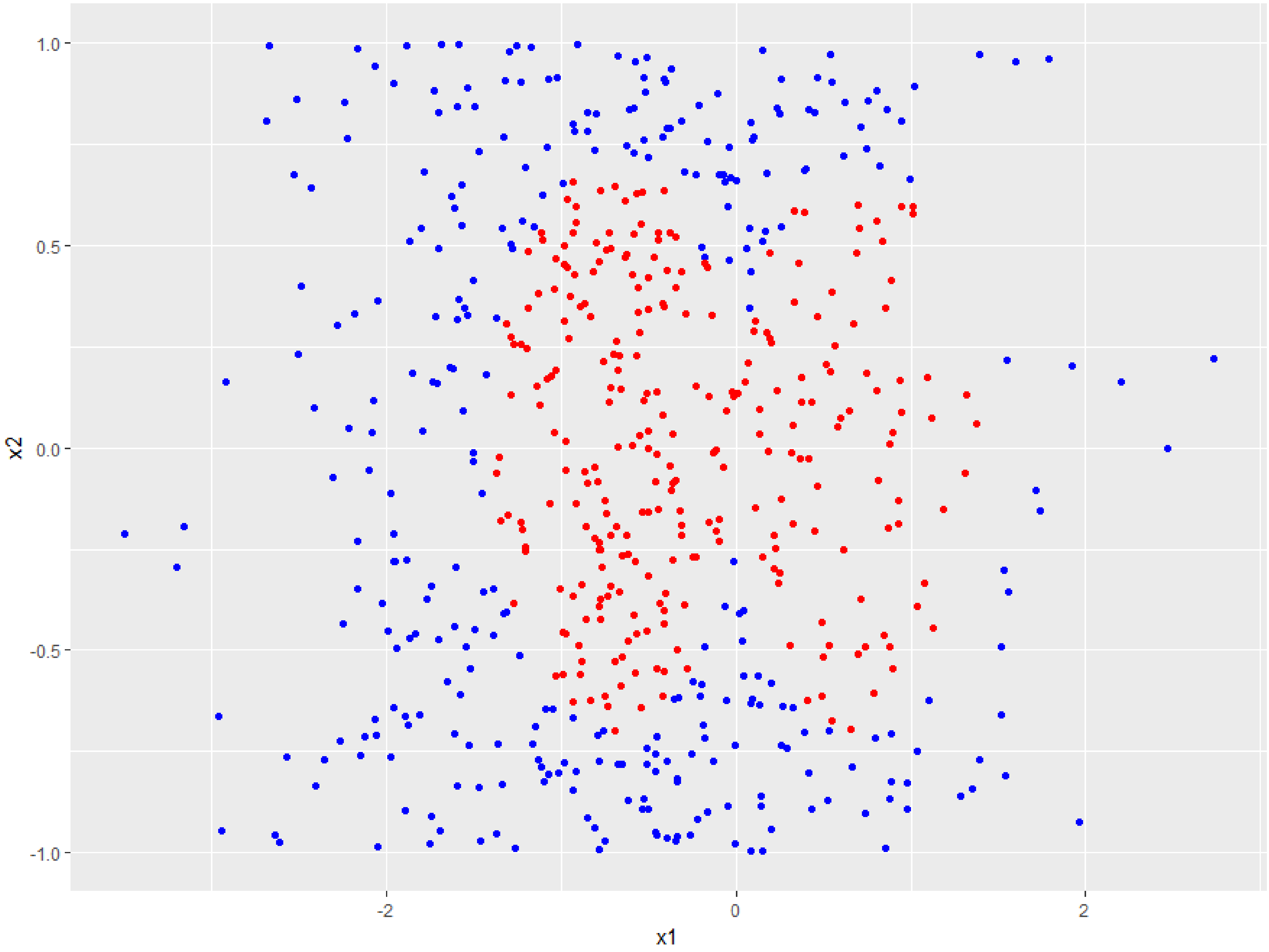
- Boundary consists of two equi-radial circles with a single point in common.

```
#set radius and centers
radius <- 0.7
radius_squared <- radius^2
center_1 <- c(-0.7,0)
center_2 <- c(0.7,0)

#classify points
df$y <-
    factor(ifelse(
        (df$x1-center_1[1])^2 + (df$x2-center_1[2])^2 < radius_squared|
        (df$x1-center_2[1])^2 + (df$x2-center_2[2])^2 < radius_squared,
        -1,1), levels = c(-1,1))
```

Visualizing the dataset

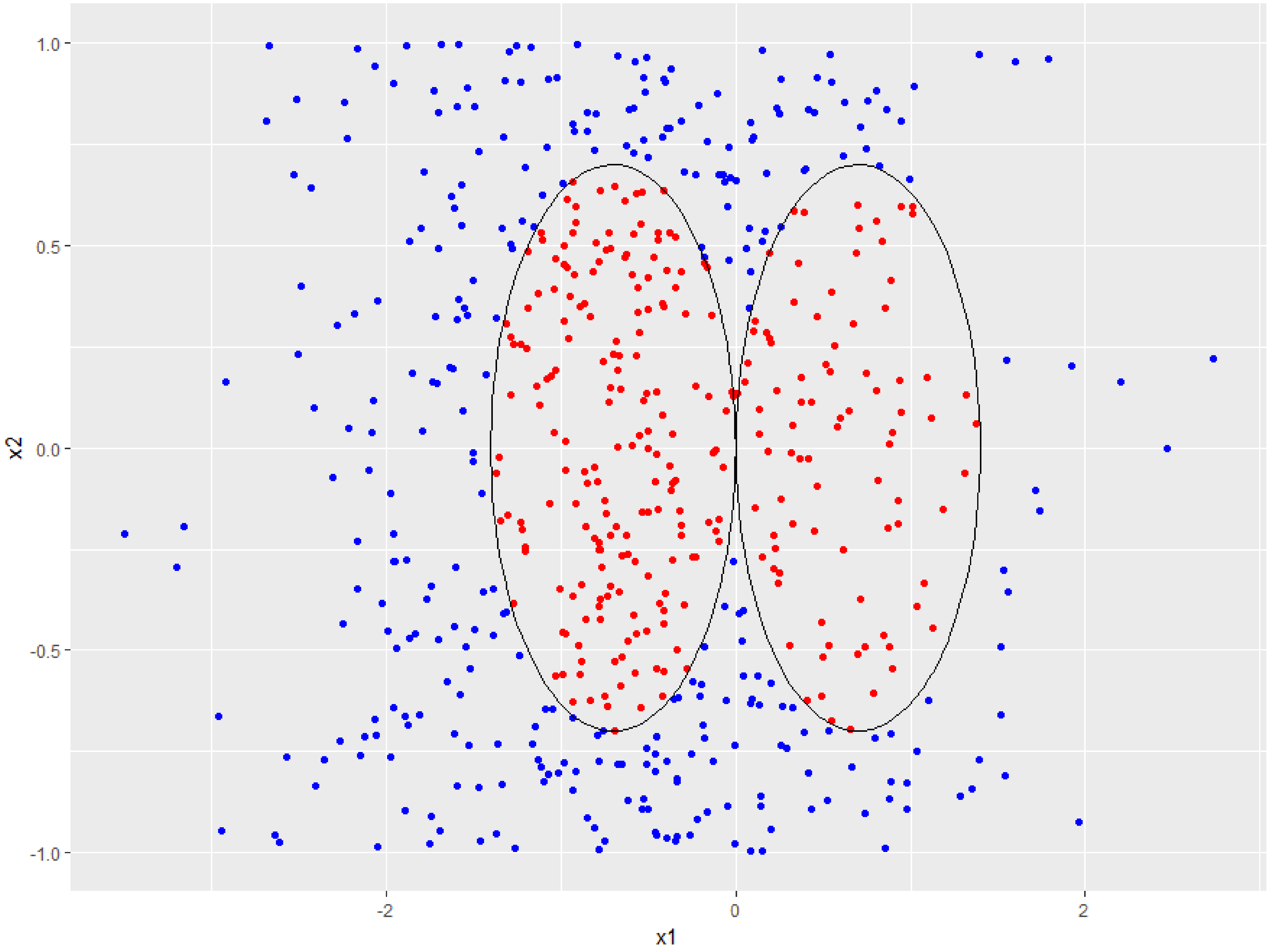
- Visualize the dataset using ggplot; distinguish classes by color

```
library(ggplot2)

p <- ggplot(data = df, aes(x = x1, y = x2, color = y)) +
     geom_point() +
     guides(color = FALSE) +
     scale_color_manual(values = c("red","blue"))

p
```


1.0
0.5
0.0
-0.5
-1.0
x2
-2
0
2
x1

Code to visualize the boundary

```
#function to generate points on a circle
circle <- function(x1_center, x2_center, r, npoint = 100){
   theta <- seq(0,2*pi, length.out = npoint)
   x1_circ <- x1_center + r * cos(theta)
   x2_circ <- x2_center + r * sin(theta)
   return(data.frame(x1c = x1_circ, x2c = x2_circ))
}

# generate boundary and plot it
boundary_1 <- circle(x1_center = center_1[1],
                     x2_center = center_1[2],
                     r = radius)
p <- p +
     geom_path(data = boundary_1,
               aes(x = x1c, y = x2c),
               inherit.aes = FALSE)
boundary_2 <- circle(x1_center = center_2[1],
                     x2_center = center_2[2],
                     r = radius)
p <- p +
     geom_path(data = boundary_2,
               aes(x = x1c, y = x2c),
               inherit.aes = FALSE)
p
```


1.0
0.5
0.0
-0.5
-1.0
x2
-2
0
2
x1

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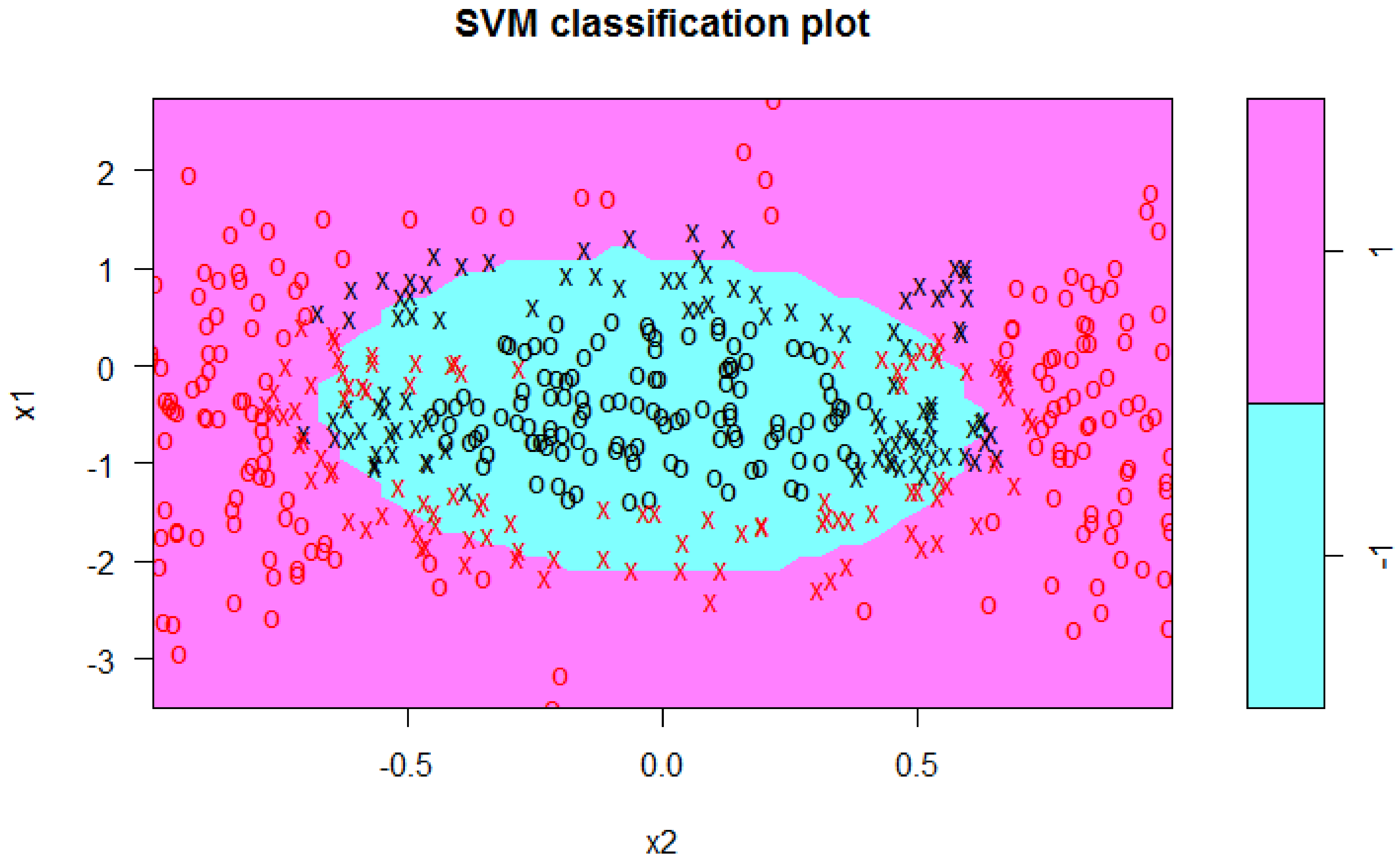
Motivating the RBF kernel

Quadratic kernel (default parameters)

- Partition data into test/train (not shown)
- Use degree 2 polynomial kernel (default params)

```
svm_model<-
  svm(y ~ .,
      data = trainset,
      type = "C-classification",
      kernel = "polynomial",
      degree = 2)
svm_model
....
Number of Support Vectors:  204
#predictions
....
pred_test <- predict(svm_model, testset)
mean(pred_test==testset$y)
[1] 0.8666667

#plot
plot(svm_model, trainset)
```

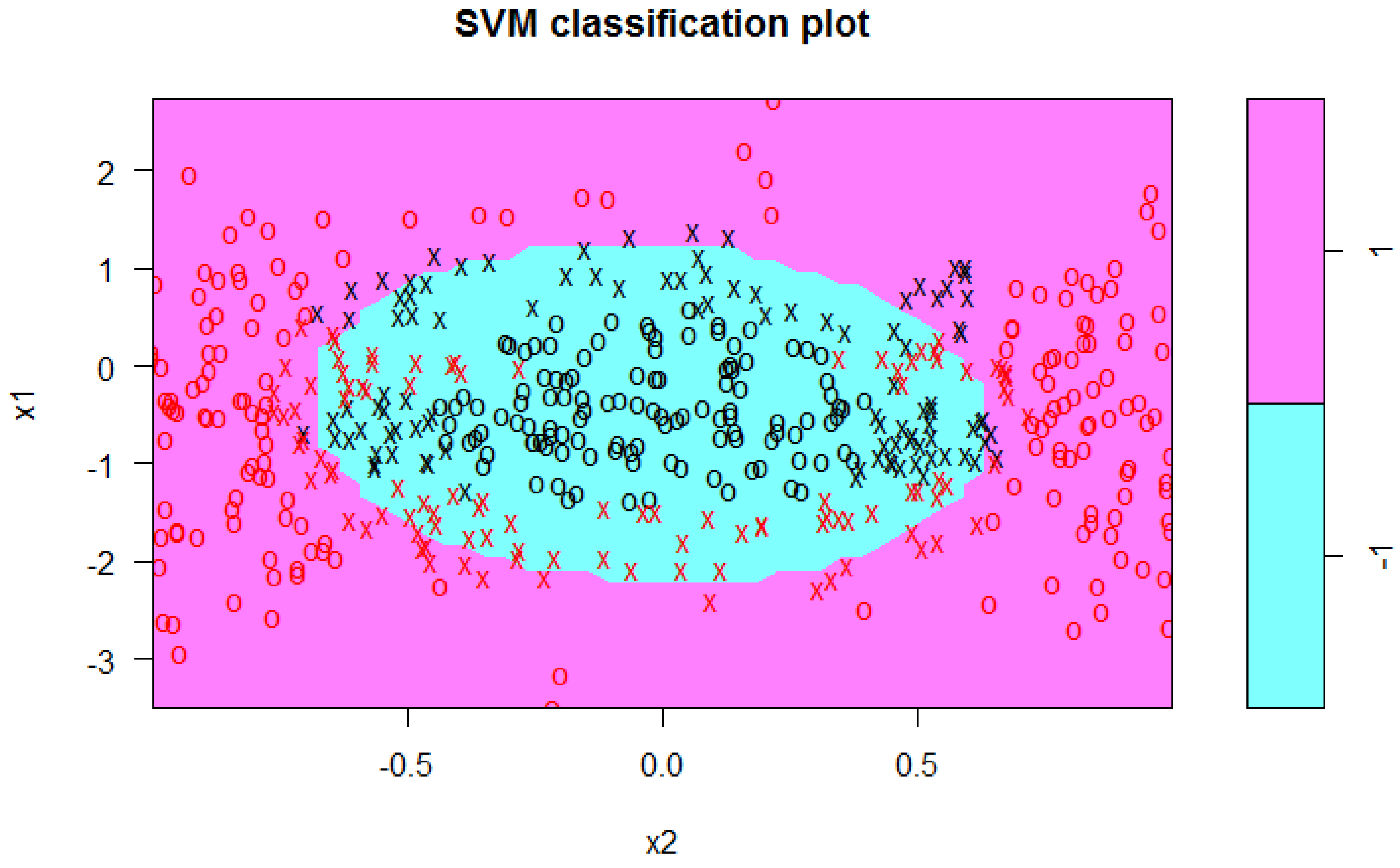

SVM classification plot
2
1
0
-1
-2
-3
x1
-0.5
0.0
0.5
x2
1
-1

Try higher degree polynomial

- Rule out odd degrees -3,5,9 etc.
- Try degree 4

```
svm_model<- svm(y ~ .,
                data = trainset,
                type = "C-classification",
                kernel = "polynomial",
                degree = 4)
svm_model
..............
Number of Support Vectors: 203
...
pred_test <- predict(svm_model, testset)
mean(pred_test==testset$y)
[1] 0.8583333

#plot
plot(svm_model, trainset
```


SVM classification plot
x1
x2
2
1
0
-1
-2
-3
-0.5
0.0
0.5
1
-1

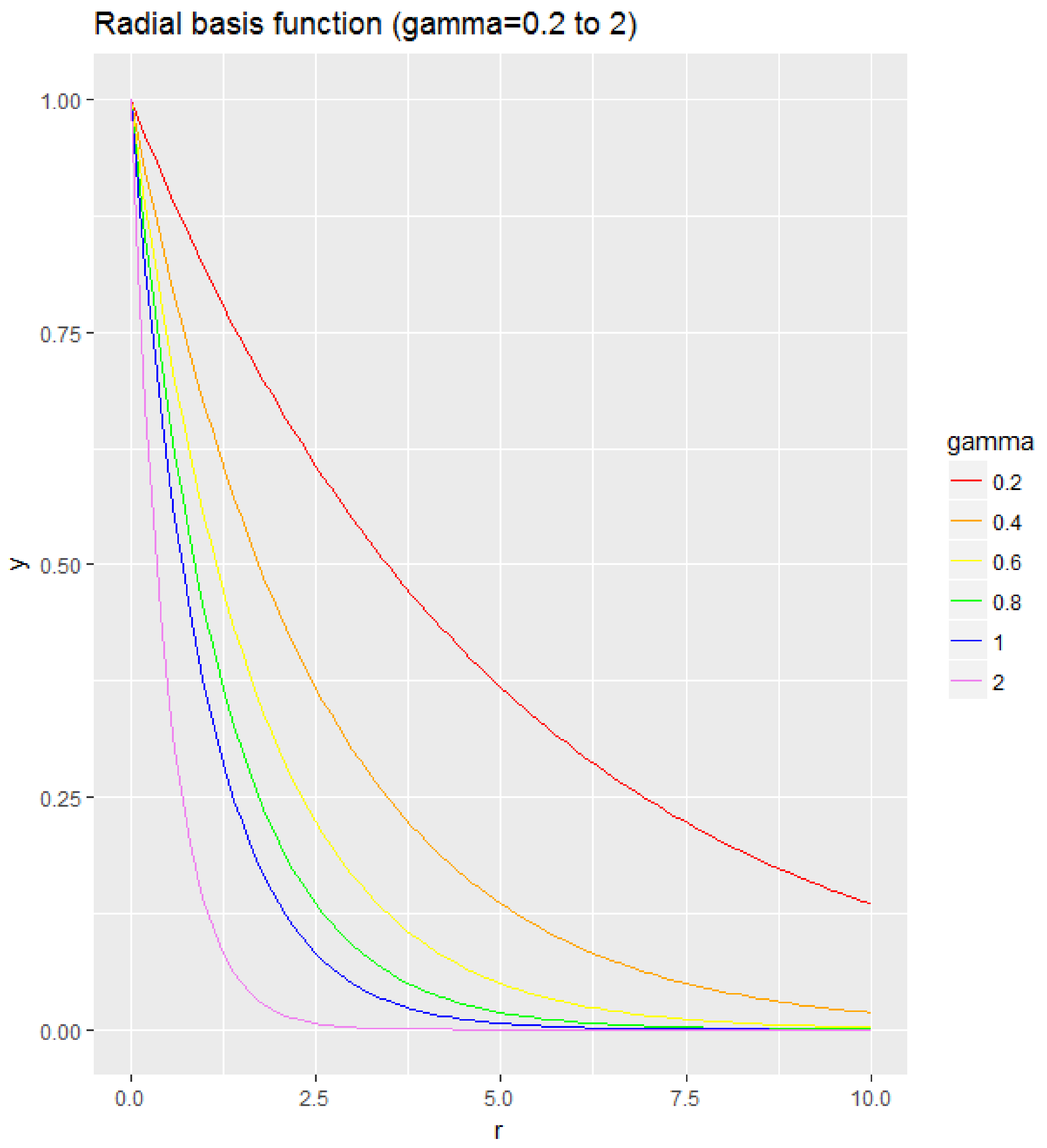
Another approach

- **Heuristic**: points close to each other have the same classification:
  - Akin to K-Nearest Neighbors algorithm.
- For a given point in the dataset, say **X1**=(a,b):
  - The kernel should have a maximum at (a,b)
  - Should decay as one moves away from (a,b)
  - The rate of decay should be the same in all directions
  - The rate of decay should be tunable
- A simple function with this property is **`exp(-gamma*r)`**, where `r` is the distance between X1 and any other point X

How does the RBF kernel vary with gamma (code)

```
#rbf function
rbf <- function(r, gamma) exp(-gamma*r)

ggplot(data.frame(r = c(-0, 10)), aes(r))+
  stat_function(fun = rbf, args = list(gamma = 0.2), aes(color = "0.2")) +
  stat_function(fun = rbf, args = list(gamma = 0.4), aes(color = "0.4")) +
  stat_function(fun = rbf, args = list(gamma = 0.6), aes(color = "0.6")) +
  stat_function(fun = rbf, args = list(gamma = 0.8), aes(color = "0.8")) +
  stat_function(fun = rbf, args = list(gamma = 1), aes(color = "1")) +
  stat_function(fun = rbf, args = list(gamma = 2), aes(color = "2"))+
  scale_color_manual("gamma",
                     values = c("red","orange","yellow",
                                "green","blue","violet")) +
  ggtitle("Radial basis function (gamma=0.2 to 2)")
```


Radial basis function (gamma=0.2 to 2)
y
1.00
0.75
0.50
0.25
0.00
0.0
2.5
5.0
7.5
10.0
r
gamma
0.2
0.4
0.6
0.8
1
2

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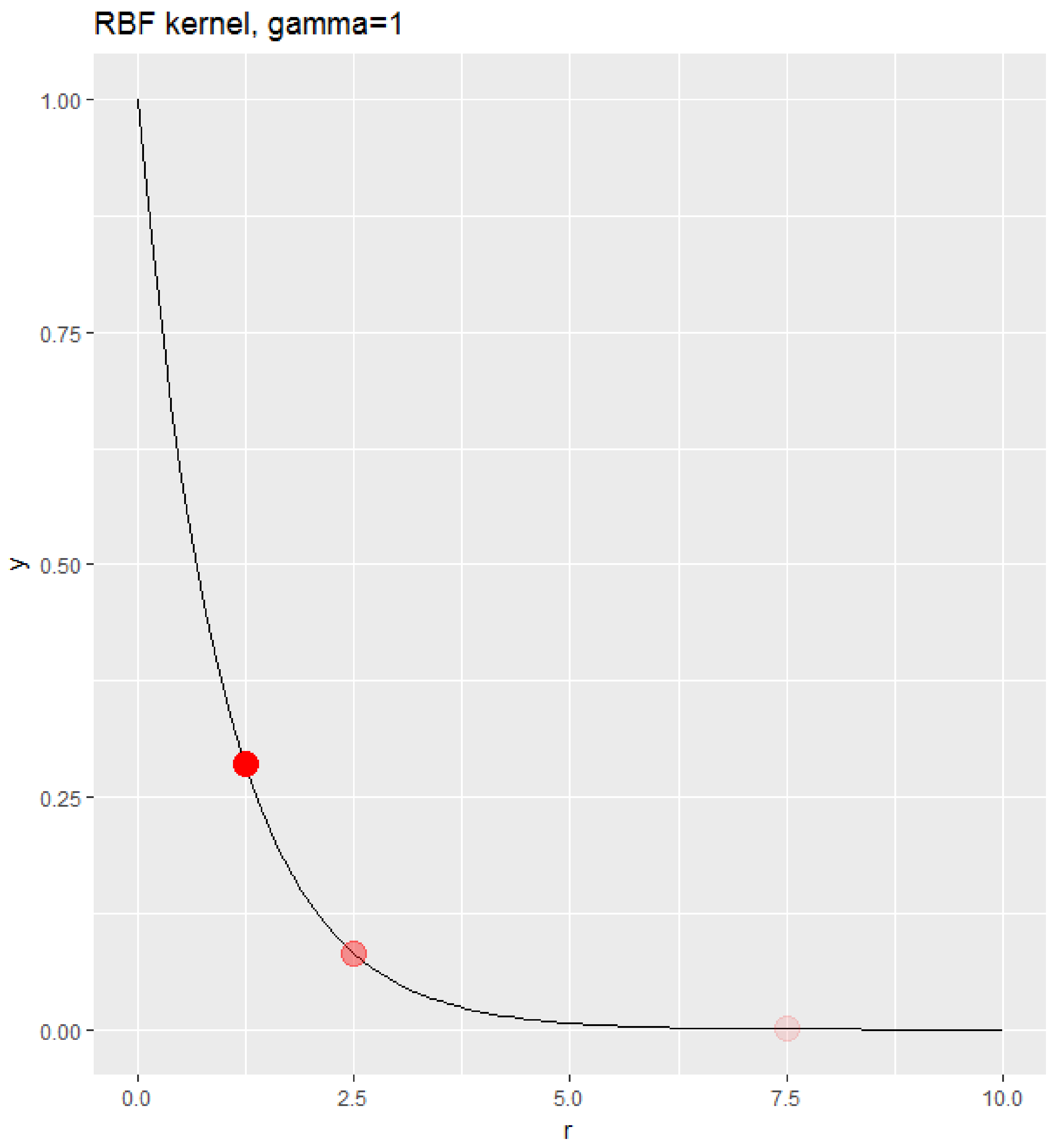
The RBF Kernel

RBF Kernel in a nutshell

- Decreasing function of distance between two points in dataset.
- Simulates k-NN algorithm.


RBF kernel, gamma=1
1.00
0.75
0.50
0.25
0.00
y
0.0
2.5
5.0
7.5
10.0
r

Building an SVM using the RBF kernel

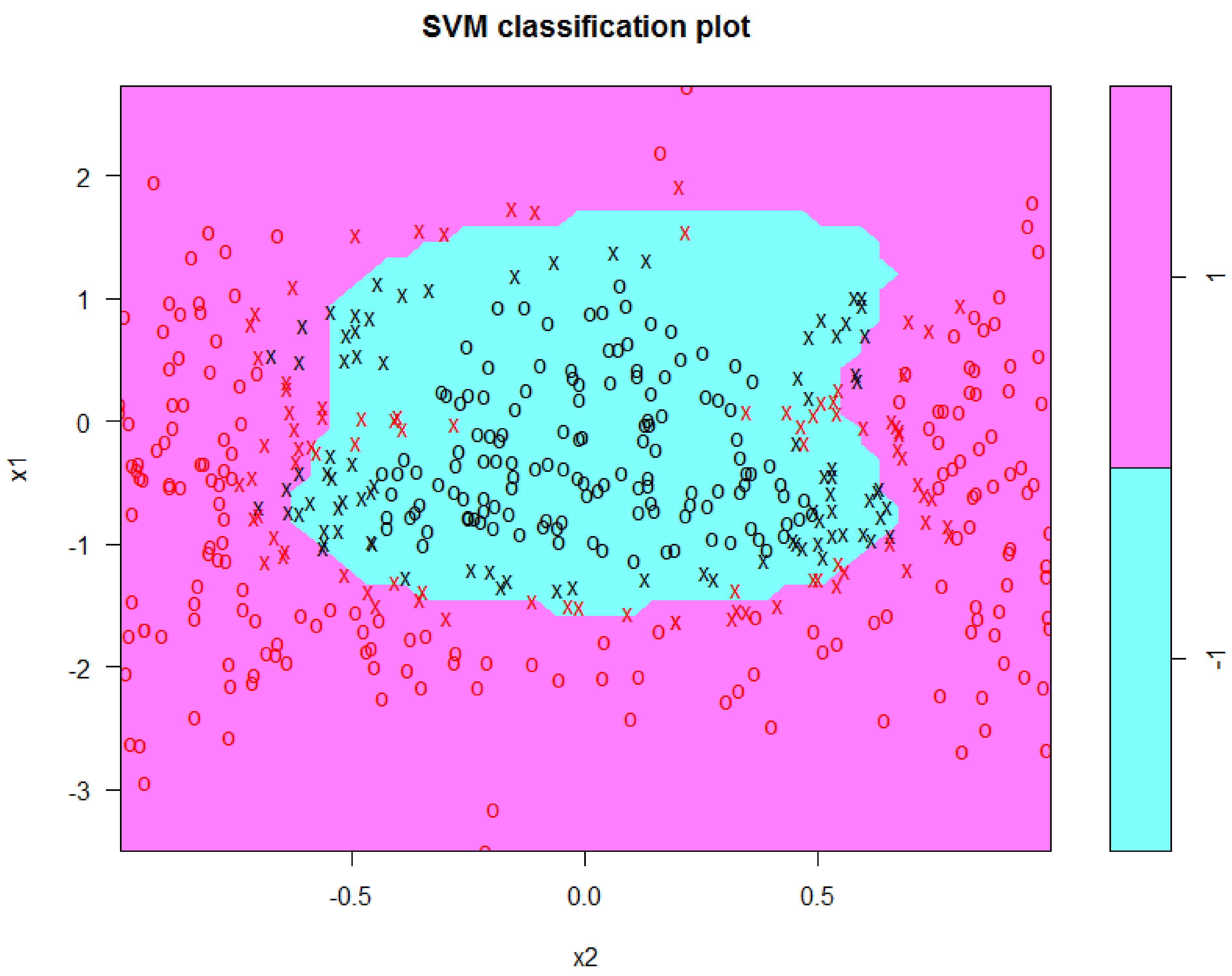
- Build RBF kernel SVM for complex dataset

```
svm_model<- svm(y ~ .,
                data = trainset,
                type = "C-classification",
                kernel = "radial")
```

- Calculate training/test accuracy and plot against training dataset.

```
pred_train <- predict(svm_model, trainset)
mean(pred_train==trainset$y)
[1] 0.93125
pred_test <- predict(svm_model, testset)
mean(pred_test==testset$y)
[1] 0.9416667

#plot decision boundary
plot(svm_model, trainset)
```


SVM classification plot
x1
x2
2
1
0
-1
-2
-3
-0.5
0.0
0.5
1
-1

Refining the decision boundary

- Tune `gamma` and `cost` using `tune.svm()`

```
#tune parameters
tune_out <- tune.svm(x = trainset[,-3],
                     y = trainset[,3],
                     gamma = 5*10^(-2:2),
                     cost = c(0.01,0.1,1,10,100),
                     type = "C-classification",
                     kernel = "radial")
```

- Print best parameters

```
#print best values of cost and gamma
tune_out$best.parameters$cost
[1] 1
tune_out$best.parameters$gamma
[1] 5
```

The tuned model

- Build tuned model using `best.parameters`

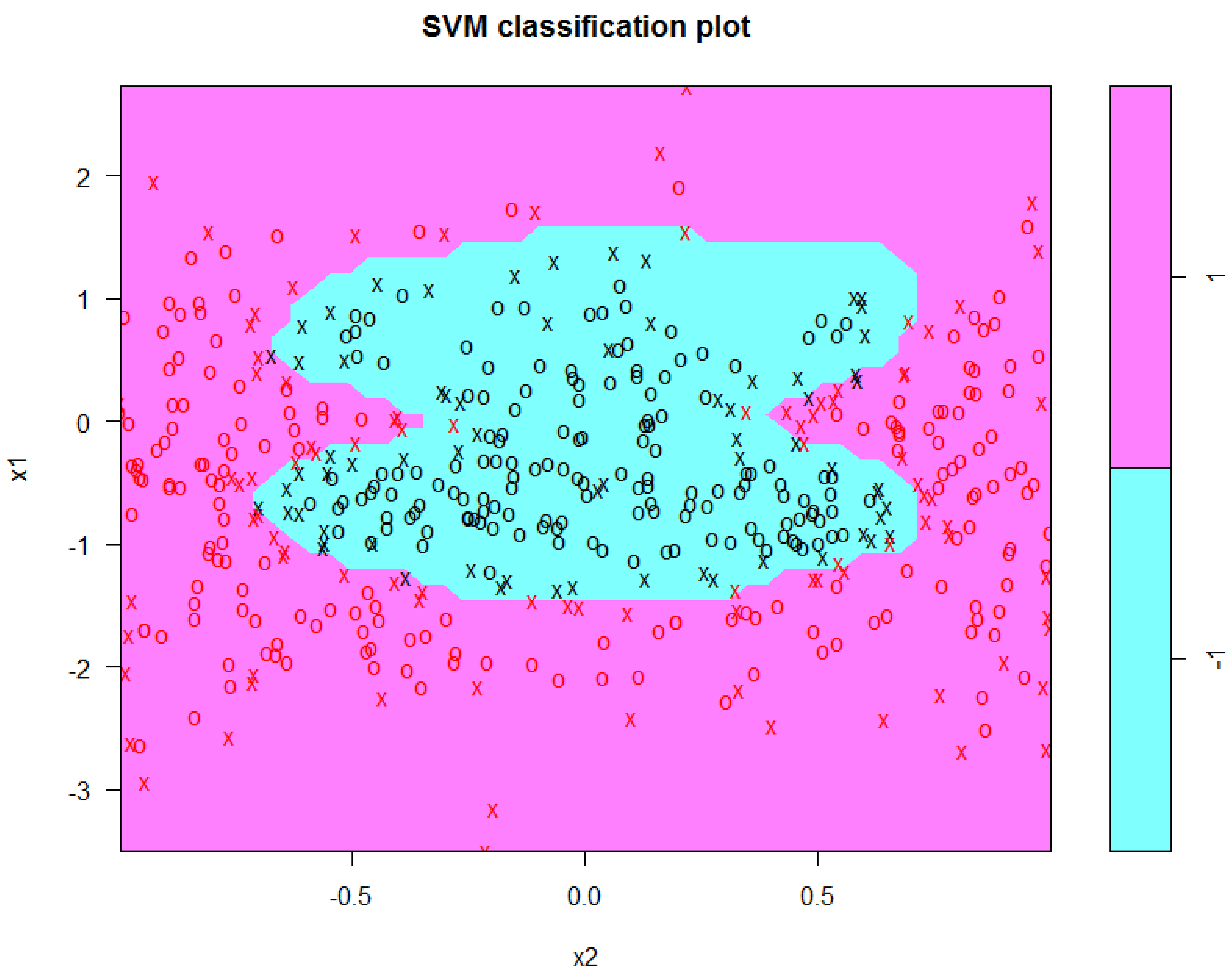
```
svm_model <- svm(y~ .,
                 data=trainset,
                 type="C-classification",
                 kernel="radial",
                 cost=tune_out$best.parameters$cost,
                 gamma=tune_out$best.parameters$gamma)
```

- Calculate test accuracy

```
mean(pred_test==testset$y)
[1] 0.95
```

- plot decision boundary

```
plot(svm_model, trainset)
```


SVM classification plot
x1
x2
2
1
0
-1
-2
-3
-0.5
0.0
0.5
1
-1

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