Vishwa Mohan Singh
1032170273

Open Elective 2
Soft Computing

Lab Assignment 1

Title: Implementation of Fuzzy Sets Operations and Creation of Fuzzy Relation by Cartesian Product

Aim: Implement Union, Intersection, Compliment & Difference operation on fuzzy sets. Also create fuzzy relation by Cartesian product of any two fuzzy sets and max-min composition.

Objective:
1) To implement Union, Intersection, Compliment & Difference operations on fuzzy sets
2) To create fuzzy relation by cartesian product of any 2 fuzzy sets & perform min max composition

Theory:

Different Fuzzy set operations:

1) Compliment: In this operation, we find the converse of the data/fuzzy value.

Expression for complement:

$$\boxed{\mu_{\bar{A}}(u) = 1 - \mu_A(u)}$$

Fuzzy set Union: This is similar to the crisp set operation. Then, in order to unionize, we need to take the maximum of the 2 or more fuzzy value

Expression:

$$\boxed{\mu_{A \cup B}(x) = \max\{\mu_A(u), \mu_B(u)\}}$$

Fuzzy set intersection: In intersection, we select the minimum of fuzzy values.

Expression

$$\boxed{\mu_{A \cap B}(u) = \min\{\mu_A(u), \mu_B(u)\}}$$

Bounded Difference: It is defined as the or between 0 & the difference in of the fuzzy values

Expression :

$$\boxed{\mu_{A \ominus B} = 0 \vee (\mu_A - \mu_B)}$$

Cartisian Product :

If $A_1, A_2, \ldots, A_n$ are fuzzy sets $U_1, U_2 \ldots U_2$ respectively. The cartisian product of $A_1, A_2 \ldots A_n$ is a fuzzy sets in the space $U_2 \times V_2 \ldots U_n$ with the membership function as:

$$\boxed{\mu_{A_1 \times A_2 \ldots \times A_n}(x_1, x_2, \ldots, x_n) = \min[\mu_{A_1}(x_1), \mu_{A_2}(x_2) \ldots \mu_{A_n}(x_n)]}$$

Max - Min composition :

Consider 2 fuzzy relation ; $R(x \times Y)$ and $S(Y \times Z)$, then a relation $T(X \times Z)$ can be expressed as (max-min composition)

$$T = R \circ S$$
$$\mu_T(x,z) = \text{max-min}[\mu_R(x,y), \mu_S(y,z)]$$
$$= \vee[\mu_R(x,y) \wedge \mu_S(y,z)]$$

This composition can be interpreted as indicating the strength of the existance of relation between the elements of X & Z.

Programming Platform: Python
OS: Linux

Conclusion: Thus, we successfully implemented union, intersection, compliment and difference operations on fuzzy set.

FAQ

Question 1 Difference between set & fuzzy set

	Crisp Set	Fuzzy Set
1)	Classes of object with sharp boundaries	Classes of object with unsharp boundaries
2)	Membership value of elements are either 0 or 1	Membership value of elements are between 0 to 1
3)	Used in digital system design	Used in fuzzy controller
4)	Example $X = \{1, 2, 3, 4\}$ $A = \{1, 3, 4\}$	Example $X = \{1, 2, 3, 4\}$ $A = \{(1,1), (2,0), (3,1), (4,0)\}$

Question 2 Define cartesian product?

=> **Cartisian product:**

If $A_1, A_2, \ldots, A_n$ are fuzzy sets defined over $U_1, U_2, \ldots, U_n$ respectively.

The product is

$$\mu_{A_1 \times A_2 \times \ldots \times A_n}(x_1, x_2, x_3 \ldots x_n)$$

$$= \min[\mu_{A_1}(x_1), \mu_{A_2}(x_2) \ldots \mu_{A_n}(x_n)]$$

Question 3 What is ~~max~~ max-min composition on two relation?

=> Max-min composition:

Consider 2 fuzzy relation ; $R(X \times Y)$ and $S(Y \times Z)$

Then a relation $T(X \times Z)$ can be expressed as:

$$T = R \circ S$$

$$\mu_T(x,z) = \text{max-min}[\mu_R(x,y), \mu_S(y,z)]$$

$$= \vee[\mu_R(x,y) \wedge \mu_S(y,z)]$$

Example :

$$R = \begin{array}{cc} y_1 & y_2 \\ \end{array}$$
$$R = \begin{bmatrix} 0.6 & 0.3 \\ 0.2 & 0.4 \end{bmatrix}$$

and

$$\begin{array}{ccc} z_1 & z_2 & z_3 \end{array}$$
$$S = \begin{bmatrix} 1 & 0.5 & 0.3 \\ 0.8 & 0.4 & 0.7 \end{bmatrix}$$

$$T = R \circ S = \begin{array}{c|ccc} & z_1 & z_2 & z_3 \\ \hline x_1 & 0.6 & 0.5 & 0.3 \\ x_2 & 0.8 & 0.4 & 0.7 \end{array}$$

$$\uparrow$$

$$M_T(x_2, z_1) = \max[\min(0.2, 1), \min(0.4, 0.8)]$$